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The Construction of Aggregation Theoretic Money Measures Using Canadian Data

By

Terence Molik

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ABSTRACT

The aim of this thesis is to review the recent literature on aggregation theory and apply the suggestions of this literature to the construction of the monetary aggregates M1, M2 and M3. Specifically, superlative indexes such as the Divisia and Fisher are used for this purpose. The need to use alternative techniques to construct M1, M2 and M3 is prompted by the evidence that the simple sum monetary aggregates published by the Bank of Canada make highly unrealistic assumptions about the nature of the money market. The issue of whether the Divisia and Fisher aggregates constitute an improvement on the simple sum aggregates is briefly examined through a cursory set of empirical tests, including tests for a causality relationship between money and GDP or prices. The results provide little evidence that one aggregation methodology produces aggregates that are superior in terms of forecasting macroeconomic fluctuations.

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Chapter 1: A Brief Introduction

There is already a substantial amount of literature dealing with the issue of how to measure the aggregate quantity of money in the economy. Much has been made in this literature of the potential inadequacies of the existing official monetary aggregates constructed by a method (so-called simple sum aggregation) that does not take advantage of the recommendations of existing aggregation theory. (The aggregation theory of most interest here is the aggregation of the monetary assets held by a single representative consumer.)

Typically, in constructing the four popular aggregates of M1, M2, M3 and M2+, the Bank of Canada merely adds the component assets and deposit accounts contained in the definition of the aggregate to form the aggregate. The weakness of this simple method of measuring the money supply is that it involves no consideration of the relative prices of these component monetary assets. In so doing the implicit assumption is being made that the relative prices of these monetary assets are assumed to be constant and equal over time, and further that the component monetary assets are perfect (and dollar-for-dollar) substitutes for each other. The empirical evidence indicates otherwise however. It is obviously the case that most economic agents hold a portfolio of monetary assets (that have significantly different opportunity costs), rather than a single asset with the lowest cost. Given this, it is highly unlikely that economic agents regard component monetary assets as perfect substitutes for each other.

Since most monetary assets are not regarded as perfect substitutes, it is necessary to consider what determines the elasticity of substitution between assets. Generally, the degree of "moneyness" of the asset is believed to be a factor in deciding the degree of substitution between monetary assets. Ultimately, an improved means of constructing monetary aggregates would assign weights to each component asset in accordance to the differing degrees to which the asset contains "money" properties. In the context of statistical index theory, indexes used to construct monetary aggregates in this fashion are referred to as "weighted" indexes. However, these indexes must weight monetary assets nonlinearly in the aggregation process otherwise assets must still be perfect (but not dollar-for-dollar) substitutes. The Divisia and Fisher Ideal indexes both weight component monetary asset data nonlinearly and both indexes will be discussed in the thesis.

The use of Divisia and Fisher Ideal indexes (also known as superlative indexes) has its foundation in the microeconomic theory of a monetary economy. The monetary economy is thus represented in a microeconomic framework through the consumer's optimization problem in which the vector of monetary assets is included in the representative consumer's utility function along with all other goods and services available in the economy. The price of monetary assets (required in the budget constraint of the model) is given by the user cost of these assets. (Later, it will be shown that these user costs will become the weights used in the Divisia index). The user cost may easily be calculated by using the expected nominal holding period yield on the given asset, the expected holding period yield on an alternative "benchmark" asset and the true cost-of-living (price) index.[1]

[1] See Barnett, Fisher and Serletis (1992, p.2093)

The two-stage theory of optimization is used to more effectively focus on the demand for monetary services. This theory is used as follows: In the first stage of optimization, the consumer allocates their expenditure among broad categories (such as consumption goods, leisure and monetary services) while in the second stage, expenditure is allocated within each category, or in this special case, among the monetary assets individually. In the first stage, the allocation decision is guided by the price indices of the broad categories. For the second stage, the relative user costs of the individual monetary assets determine the allocation of consumption among the assets.[2]

For the above general framework to hold, two principal assumptions must be satisfied. First, the use of a representative agent can only be allowed if Gorman's condition is satisfied. Gorman's condition requires that all consumers have linear and parallel Engel curves.[3] This assumption (if satisfied) basically ensures that economic agents in the economy will not be so diverse and heterogeneous that the use of a representative consumer in the consumption allocation model for the economy is invalid. The second important assumption is that of weak separability. That is to say, it must be the case that the utility function in the optimization model described above is weakly separable in the services of monetary assets.[4] For a simple utility function where c represents the vector of the services of consumption goods, L is leisure time and m is the vector of monetary assets, weak separability allows the utility function to be given as

$$u = U[c, L, f(m)]$$

[2] See Barnett, Fisher and Serletis (1992, p.2094)

[3] See Anderson, Jones and Nesmith (1997, p.33)

[4] See Barnett, Fisher and Serletis (1992, p.2094)

in which f defines the monetary subutility function. Technically, the property of weak separability means that the marginal rate of substitution between any two monetary assets is independent of the levels of c and L consumed.[5] The weak separability condition is absolutely necessary for the two-stage optimization approach to be possible.

As long as the monetary subutility function, $f(m)$ is first-degree homogeneous, it is a monetary quantity aggregator function.[6] The second stage of the optimization problem is completed by determining the optimum quantities of each monetary asset; this is done by maximizing the subutility function subject to the expenditure constraint implied by the first-stage. Once these optimum quantities are inputted to the subutility function, an aggregation theoretic money quantity aggregate is determined. Thus the aggregator function itself provides a way to construct more accurate aggregates for M1, M2, M3 and M2+.

The problem here is that the parameters of the quantity aggregator function must be estimated empirically unless statistical index number theory is applied. Estimating the aggregator function directly requires that specific assumptions be made about the functional forms of the subutility functions. Statistical index numbers obviate the need for the estimation of the specific function. Statistical index numbers contain no unknown parameters and are said to be exact if the index number tracks the aggregator function, evaluated at the optimum, without error.[7]

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[5] See Barnett, Fisher and Serletis (1992, p.2094)

[6] See Anderson, Jones and Nesmith (1997, p.38)

[7] See Anderson, Jones and Nesmith (1997, p.90)

The Divisia and Fisher Ideal indexes are examples of statistical index numbers. The continuous-time versions of these indexes are exact for the given quantity aggregator function. The discrete-time versions of the Divisia and Fisher indexes used here are not exact. There exist mathematical functions however that can provide second-order approximations to the unknown aggregator functions. These mathematical functions are known as flexible functional forms.[8] There are also statistical index numbers that are exact for some of these flexible functional forms and these are referred to as superlative index numbers. (The discrete-time Divisia index is exact for the translog flexible functional form, while the Fisher index is exact for the homogeneous quadratic functional form.)[9]

Thus the superlative index numbers (such as the Divisia and Fisher Ideal indexes) are able to provide second-order approximations to the unknown aggregator function. It is the aim of this thesis to use superlative indexes (primarily the Divisia, but also the Fisher and currency equivalent) to provide an aggregation theoretic means of measuring M1, M2, and M3[10], for the purpose of providing an economic comparison with those aggregates.

Chapter 2 will briefly describe the basic definitions and component data of the monetary aggregates in Canada. Chapter 3 will serve as a thorough framework describing the microeconomic, aggregation theoretic and statistical index number foundations behind the aggregation approach used in this project. Chapter 4 will briefly describe a theoretical extension of the Divisia index to allow for the existence of risk in money

[8] See Anderson, Jones and Nesmith (1997, p.41)

[9] See Anderson, Jones and Nesmith (1997, p.42)

markets and for the risk aversion of the consumer. Chapter 5 will describe in detail the data series and methodology used in the construction of monetary aggregates for M1, M2 and M3, and will briefly compare the aggregates as generated by the use of different index numbers. Finally, Chapter 6 will examine the money measures obtained for unit roots, and perform cointegration and causality tests on the money measures and their relationship with aggregate price and income variables. Chapter 7 will serve as a brief conclusion.

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[10] M2+ aggregates are not calculated in this thesis due to data availability problems.

Chapter 2: A Discussion of the Definition of Money

I. Introduction

While the definition of what money consists of is not something that has to be discussed in most contexts, for the purpose of the monetary aggregation project performed in this thesis, a brief review of the nature of money may be useful. In the exchange and acquisition of resources, no development has contributed quite so much to efficiency and social welfare as the introduction of money. Money, in its most generic sense, is defined by whatever asset performs at least one of the following three functions: money can serve as a standard unit of account, a medium of exchange and a store of value.

The alternative to the money economic system is the use of barter. Barter is the direct exchange of goods and services for other goods and services, and the barter system preceded the introduction of money back to ancient times. Even though it has been displaced by the introduction of money for a variety of reasons, it still can be seen in even highly industrialized countries on a small scale. Any informal direct exchange of one good or service for another between individuals in the economy would qualify as an example of the use of barter.

Finally, the issue of how the money supply is defined will be discussed. For the Bank of Canada, there are several official measures of the money supply which are published. Primarily, these consist of definitions (ranging from narrow to broad) M1 through M3 and M2+. M1 consists of currency in circulation plus current and personal chequing accounts. M2 adds personal savings deposits and notice deposits to M1, while

M3 adds non-personal fixed term deposits and foreign currency deposits to M2. M2+ includes the deposit accounts of so-called alternative financial institutions in addition to M2. In addition, there are several methods that can be used to measure these definitions of the money supply, and these also will briefly be mentioned.

II. Monetary Standards

The first type of money used, in the move away from barter was commodity money. Its first function was to serve as a medium of account.

> *"In the early pastoral societies everything was valued in terms of cattle, which were a symbol of wealth and prestige. It was only after their use as a standard unit of account that they also came to serve as a medium of exchange and a store of a value."*[11]

Obviously, however this kind of commodity money was inefficient and arbitrary in a number of respects. The simplest reason for this is that there is no standard cow, bull or ox, and furthermore small transactions are difficult to arrange because cattle are not divisible. And, as a store of value, cattle are only as useful as their life expectancy.

The next development in the use of money was the move from the use of commodities such as cattle to the use of metals. Metals such as gold, silver and copper came to be minted and used in the form of coins or bars, and this became a long-lasting standard medium of exchange mostly because metals possess all of the essential characteristics required of any medium of exchange. Metals are relatively homogeneous, easily recognizable, highly durable, easily portable, and are divisible into any size

[11] See Binhammer (1993, p.5)

(facilitating the use of small transactions).[12] With the development of banks, the transportation of large deposits of metal from one bank to another created costs (in terms of the risk of robbery and the weight of transportation) which were avoidable by using paper receipts to represent a certain weight of a given precious metal. From such receipts the use of paper currency was born. From this, more sophisticated monetary standards were developed.

In cases where the circulating medium of exchange is paper notes, such notes themselves have no intrinsic value, but are backed by, and convertible into, a given precious commodity such as gold or are backed by the authority of the government. From this, development of the gold standard was achieved in which the market for gold dictates the value of money.[13] The use of a relatively strict gold standard was maintained by the most of the western world from the time of the Napoleonic Wars until 1914.

> *"Each major nation specified the gold content of its currency, and anyone holding notes backed by that currency could redeem them on demand receiving the face value in gold. In effect, this regime fixed the price of gold, and, at least theoretically, held exchange rate fluctuations within narrow limits."*[14]

The stability of the exchange rate and of the price level were the principal contributions of the gold standard. However, this stability was bought at the price of leaving no room for discretionary monetary policy to be practiced by governments and central banks. The

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[12] See Binhammer (1993, p.6)

[13] See Siklos (1997, p.24)

[14] See Siklos (1997, p.24)

lack of room for discretionary policymaking and the inflexibility of the strict rules of the gold standard, can have serious consequences by exacerbating the difficulties imposed on an economy by recessions. Furthermore, since no country has enough gold to back all of its currency, the degree of backing by the government is an issue that creates the possibility of non-cooperation among countries participating in the gold standard.

The alternative to the commodity backing of paper currency seen in the gold standard is the use of a fiat-money standard. With fiat money, the circulating medium is typically paper notes and coins with virtually no non-monetary function. The value of these coins and notes are determined by whatever the government says they are worth. Under such a paper money standard, "*the value of the circulating currency is guaranteed, not by some precious commodity but by the taxing and borrowing powers of the government.*"[15] Today, the fiat money standard is easily the dominant monetary standard of the entire industrialized world.

With the adoption of fiat money, the national central banks became increasingly important. The first duty of any central bank is the management and control of the country's money supply. Central banks are typically a part of the public sector and have a legally defined relationship with the government of the day. In Canada, the United States, and many European countries, the relative independence of the central bank from the fiscal policy authority and other political authorities is seen as the ideal legally defined relationship to the government. The reason for this view is the example provided by

[15] See Siklos (1997, p.23)

crises such as wars, in which a government may push its central bank to issue ever-larger quantities of money resulting in hyperinflation. [16]

III. Measuring the Money Supply

If the central bank is to effectively manage and control the money supply, it must first attempt to measure the size of the money supply. The issue is not an easy one, if only because disagreement exists as to which financial assets should be incorporated into the money supply. The simplest standard definition of the money supply (or the money stock) used by the Bank of Canada, is the amount of money that is easily available, at a given point in time, for use in the payment of other assets including other forms of money.

Any asset that satisfies all three functions of money can be accepted without question, as part of the money supply. Finding such an asset that ideally satisfies all these functions simultaneously is quite difficult however. Hence, it has become the custom to measure the money supply according to either narrow or broad definitions.

> *"Narrow definitions include only those assets that the public holds as generally acceptable and immediately available media of exchange. These may be said to be the public's transaction balances, which can be transferred at little or no cost. Broad definitions of money, on the other hand, include assets that are held as stores of value, but may with some inconvenience and at some costs be sued as media of exchange."*[17]

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[16] See Siklos (1997, p.27)

[17] See Binhammer (1993, p.9)

The money supply as measured by the Bank of Canada, is fully defined by four categories, M1, M2, M3 and M2+. These categories are regularly supplied by the Bank of Canada in its Weekly Financial Statistics and monthly in the Bank of Canada Review. M1 is officially the narrowest definition of the money supply (or aggregate) provided by the Bank of Canada, while M2+ is the broadest monetary aggregate.[18]

Before giving the specific breakdown and the definitions of M1, M2, M3 and M2+ in terms of what assets and deposits are included in these measures of money, it is useful to review the types of banks and deposits found in Canada.

IV. Types of Banks and Deposits

In Canadian law, there used to be a strict distinction made between chartered banks (large institutions such as the Bank of Montreal, the Royal Bank, and the Bank of Nova Scotia) and other financial institutions such as trust companies, credit unions and Quebec's caisses populaires. These distinctions are no longer as strict as they used to be, but they still do make a difference in how the Bank of Canada measures its monetary aggregates.

Distinctions are also made between deposits made by individuals and firms at the banks. A depositor is essentially loaning his or her money to the deposit-taking institution, which is responsible for repaying it in future with interest. Of course, deposits are assets to the lenders and liabilities to the deposit-takers. These types of deposits have vastly proliferated since the 1970's but today the Bank of Canada basically classifies all deposits as being one of three types, chequing (or demand) deposits, savings (or notice) deposits, and term deposits. Their definitions are described further below.

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[18] See Binhammer (1993, p.9)

1. Chequing/Demand Deposits typically allow depositors to write cheques against them but in turn pay little or no interest. As the word demand in the title indicates, the bank or financial institution must repay the deposit to the holder on demand, without requiring that prior notice is given for deposit withdrawal.
2. Savings/Notice Deposits pay more interest than demand accounts and need not be chequable. Unlike demand deposits, prior notice may be required (at the discretion of the bank) before deposit withdrawals can be made by the depositor.
3. Term Deposits are made for a fixed term. They pay typically fairly high interest, which may be forfeited upon early withdrawal. To further penalize early withdrawal of fixed term deposits prior notice (often not enforced) is required. These accounts are rarely, if ever, chequable.[19]

These deposits can be made by everyone, but statistics typically differentiate between "personal" deposits, made in the name of an individual or small group of individuals and "nonpersonal" deposits (corporate or chartered bank deposits).

As mentioned previously however, the nature of these types of deposits have evolved considerably over time. More specifically changes have occurred with the passage of federal legislation in 1992 that phased out the reserve requirements of banks over two years ending in June 1994. Since 1994, the characteristics of demand deposits and notice deposits have become very similar. The reason is that most chartered banks typically no longer offer personal deposits accounts that are non-chequable. They did not eliminate the old non-chequable accounts, but because there is very little growth in these types of accounts, they are no longer offered. Second, although most popular savings

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[19] See Siklos (1997, p.29)

deposits have a notice requirement in the contractual agreement with the depositor, most banks never enforce this clause. So with regard to the two most important characteristics of notice and demand accounts (whether they are chequable or not, and whether or not they require prior notice of withdrawal) these two types of accounts seem to be practically identical at the present time.

V. Cheques and Cheque Clearing

The writing and cashing of cheques is a complicated process has some bearing on the definitions of the money supply, and so it is useful to briefly describe the process by which cheques are written and cleared here. A cheque is a written order for a bank to transfer a specific amount of funds from the writer's account to another account.

This process would not be overly complicated if only one bank existed in the national economy. Whether a person takes their cheque payment in cash or whether they simply allow the funds to be deposited in their account makes little difference for the bank. It is just a matter of accurately recording the transactions in the form of bookkeeping entries. However none of the industrialized economies has only one bank. It will be more than commonplace to find a situation in which a cheque is written from an account at one bank, (say the Royal Bank) with the amount involved to be deposited in an account at the CIBC with the branches of these two banks being separated geographically. To add another complication, consider that in the very same day, the recipient at the CIBC could write a cheque against the new money deposited in his account for somebody who uses yet another bank. It is easy to see that keeping track of all the myriad transactions accomplished through the use of cheques that occur in a single day becomes very complicated.

To deal with this problem, a clearinghouse would be used to sort out net interbank payments and keep everyone's account straight. In Canada, all banks and trust companies belong to the Canadian Payments Association, which uses a computerized automated cheque-clearing system. Computer technology has vastly improved the speed and accuracy of cheque clearing. Even today however, the process still takes time, and some cheques remain "in transit" for periods greater than a day. As an example of this, consider that it is quite possible for a bank account receiving funds from another via a cheque may allow those funds to be withdrawn before those funds have actually been withdrawn from the first bank. Hence, when the Bank of Canada is measuring the amount of the money contained in financial institutions, it would be easy to double count various deposits. To remove double counting from the official monetary aggregates, the Bank of Canada uses a so-called "private sector float" to adjust the demand deposit (and now the notice deposit) figures.[20]

VI. The Definitions of the Canadian Money Supply

The definitions of the money supply as reported by the Bank of Canada have always been changing over time, and so here only the current definitions will be given. Typically the quantities of various monetary asset are reported in the form of both seasonally adjusted and unadjusted figures. For the aggregation project performed in this thesis primarily seasonally adjusted figures were used but the differences between the two methods of reporting will be briefly discussed later in this chapter. Also, the structure

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[20] See Siklos (1997, p.30)

of Canadian monetary aggregates is that one aggregate measure is nested within the next broadest measure. For example, the assets that comprise M1 are also included in M2 along with a set of additional assets, and all the assets included in M2 are also included in M3 along with another set of asset categories. Hence, it is more convenient to describe these nested aggregates starting with the narrowest definition of the money supply, M1, and describing the assets contained within this measure as shown below.

1. Currency in circulation. The very narrowest definition of the money supply is the amount of currency physically in circulation (that is to say currency outside of the banking system). These paper notes and coins are fiat money acceptable as a medium of exchange because it has been declared as legitimate by the government of Canada. The actual amount of currency in circulation is typically estimated by subtracting bank holdings of notes and coins from the amounts reported outstanding by the Bank of Canada and the Royal Canadian Mint.
2. Current accounts + Personal chequing accounts + Currency = M1. Current accounts and personal chequing accounts basically comprise the demand deposits held by the chartered banks. These chequable accounts (net of the private sector float) are the other major component of M1. This narrow measure of money was long popular in economic studies of the impact of monetary policy because it is the definition of the money supply that contains only those monetary assets that most closely fulfill the medium of exchange function.
3. M1 + Personal savings deposits + Non-personal notice deposits = M2. Personal savings deposits (which also are personal notice deposits) have gradually become

like demand deposits over time in the sense that they are mostly chequable and hence can serve as a medium of exchange. Yet despite the similarity with demand deposits nowadays, they are still included in M2. The so-called non-personal notice deposits are the notice deposits of firms. Another similarity between the notice deposits which are included in M1 and these notice deposits, as mentioned before, is that, in practice, banks practically never require a notice of withdrawal for someone to obtain the finds held in their notice account.

Most of the assets contained in M2 have sufficient liquidity to serve as a medium of exchange, and the M2 categories are designed to better take account of financial innovations than M1. This is exemplified by the fact that notice deposits have grown particularly rapidly in recent years, much larger than the components of M1. The basic innovation of notice deposits that made them so attractive to depositors was the combination of the chequing features of demand deposits with the high interest rates that were the traditional purview of savings deposits. The period of primary growth was the 1980's and was of such a magnitude that by 1989, 85% of M2 was composed of notice deposits.[21]

4. M2 + Chartered bank non-personal fixed term deposits + the Canadian dollar value of chartered bank deposits denominated in foreign currencies (mostly US dollars) owned by Canadian residents = M3. This definition of the money supply is considerably broader than are provided by either M1 or M2. In fact, the assets that compose M3 net of M2 tend to function less as a medium of exchange and more as a store of value which is why the definition is considered to be a broader one.

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[21] See Siklos (1997, p.31)

5. M2 + Deposits at trust and mortgage loan companies + Deposits at credit unions and caisses populaires + Life insurance company's annuities + Personal deposits at government savings institutions + Money market mutual funds = M2+. As financial institutions other than the chartered banks assumed greater importance with time, the Bank of Canada started reporting a new monetary aggregate M2+ which was defined by adding the deposits held by a variety of alternative financial institutions to M2. Since M2+ is indeed about 60% larger than M2, this suggests that ignoring the deposits of the alternative financial institutions would ignore a great deal of relevant economic activity to the economy as a whole. This is the reason why the Bank of Canada prefers to monitor movement in M2+ when making its decisions as to the course of monetary policy. In 1992, the Bank of Canada sought to broaden the definition of M2+ to include money market mutual funds (funds invested in short term instruments), deposits at the Province of Ontario Savings Office, personal deposits at Alberta Treasure branches, and life insurance company annuities.[22] Finally, it is worth mentioning that although M2 had a greater rate of growth than M1 in the 1980's, M2+ had by far the greatest growth of all the measures of the money supply.

[22] See Binhammer (1993, p.10)

TABLE 1. THE DEFINITIONS OF THE CANADIAN MONEY SUPPLY

Money Accounts	CANSIM Series Seasonally Unadjusted	Seasonally Adjusted
Currency Outside Banks	B2001	B1604
Personal Chequing Accounts	B486	B1643
Current Accounts	B487	B1644
Adjustments to M1	B2050	B2050
Gross M1	**B2054**	**B1642**
Of which: Chartered Bank Net Demand Deposits	B478	B1601
Net M1	**B2033**	**B1627**
Chartered Bank Non-Personal Notice Deposits	B472/73	None
Chartered Bank Personal Savings Deposits	B451	None
Adjustments to M2	B2051	B2051
M2	**B2031**	**B1630**
Chartered Bank Non-Personal Term Deposits plus Foreign Currency Deposits	B475/82	None
Adjustments to M3	B2052	B2052
M3	**B2030**	**B1628**
(With M2 and not including M3)		
Deposits of Trust and Mortgage Loan Company	B2038	B1639
Deposits of Credit Unions and Caisses Populaires	B2042	B1640
Life Insurance Company Annuities	B2046	None
Personal Deposits at Government Savings Institutions	B2047	None
Money Market Mutual Funds	B2048	None
Adjustments to M2+	B2053	B2053
M2+	**B2037**	**B1633**

As a final note on the various definitions of the money supply shown above (and in Table 1), all of the measures exclude interbank and other inter-institution deposits as well as federal government deposits.[23] Interbank and inter-institution deposits are frequently the temporary repository site for cheques "in transit", as described above, and hence are

[23] See Binhammer (1993, p.11)

excluded to avoid double courting certain deposits in the money supply. Federal government deposits are typically excluded on the grounds that they are not public and thus do not satisfy the medium of exchange function. It is not the case however that the deposits at other levels of government are excluded. This serves as a reminder that the choice of deposits to be included in money supply measures is somewhat arbitrary. [24]

VII. Possible Refinements to the Money Measures

The extent to which one could broaden the definitions of the money supply, as it stands today, are practically unlimited. For example, it has been suggested by some that highly liquid assets such as Canada Savings Bonds (which are cashable anytime at little cost and are sometimes used as an alternative to bank deposits) be included in a broader money measure.

One interesting and slightly radical alteration to the definition of money would be to include the unused portion of credit card balances and other lines of credit, such as consumer or residential credit. These monetary services are unlike no others mentioned or included by the Bank of Canada in their measures of money in that they are assets to the banking system, not liabilities.[25] If one looks at methods of constructing monetary aggregates (such as M1, M2 etc.) other than the so-called "simple sum" approach described above, the radical nature of the introduction of lines of credit becomes clear. In this project, the Divisia index will be used to aggregate across the various components of M1, M2 and M3 and for this method of aggregation, a collection of user costs (which

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[24] See Binhammer (1993, 11)

[25] See Siklos (1997, p. 32)

correspond to the individual monetary components of M1, M2 and M3) must be calculated. At the present time, there is no formula available for the calculation of these user costs when dealing with liabilities to the representative consumer.[26] This might be a suggested area of future research concerning measures of the money supply.

The Divisia index mentioned above is only one means of constructing alternate monetary measures to those provided by the Bank of Canada. The fundamental reason why alternative measures are sought at all is because there is a definite drawback to all of the simple sum money supply aggregates provided currently by the Bank of Canada: Each monetary aggregate such as M1, M2, etc. contains what people perceive as qualitatively different assets.

> *"For example, most people view currency and daily-interest accounts as distinct kinds of assets, yet the M2 measure adds them together. Partly as a response to such concerns, analysts have developed divisia indexes, which weigh the components of the various money measures according to the degree to which they provide liquidity."*[27]

Each monetary asset included in the official money aggregates is perceived to have a different degree of "moneyness" to a consumer, since each asset functions as money to a greater or lesser degree. Hence the simple sum index used by the Bank of Canada in aggregating across these assets (the method of simply adding up the component assets to form the desired aggregate) is obviously inadequate. Weighted indexes such as the Divisia construct the weights for the component monetary assets

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[26] Lines of credit really are liabilities to the representative consumer, and as such can be considered to be "negative assets". The calculation of prices and user costs for negative assets is rarely performed in the context of the basic microeconomic utility maximization problem.

through the user cost and the corresponding "price" of each asset. These weighted indexes are given a stronger microeconomic foundation since they are exact for flexible functional forms, which "approximate" the demand functions for the various monetary assets produced from the utility maximization of a representative consumer (representing the economy). The derivation of the Divisia index (and other weighted indexes) will be discussed in greater detail in subsequent chapters.

As discussed above, the main area of contention in this discussion is the issue of what degree of liquidity or moneyness is required for an asset's inclusion in the measures of money. Some economists have attempted to resolve these issues by accepting currency and demand deposits as money and then employing various statistical techniques to calculate the degree of substitution between these and other potential money assets.[28] These economists then define the degree of moneyness of other potential assets according to their degree of substitution with currency and demand deposits. The problem with this method is the same as for those who use liquidity to define what assets should be included in money measures in the first place: What degree of substitutability is required to give an asset money properties.[29]

Milton Friedman has used an entirely different approach to defining and measuring the money supply. He has proposed that the money supply "*should include financial assets whose aggregate amount the monetary authorities can control and which best describe the course of economic activity or important variables that directly*

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[27] See Siklos (1997, p.33)

[28] See Feige, Edgar L. and Douglas K. Pearce, "The Substitutability of Money and Near-Monies: A Survey of the Time Series," Journal of Economic Literature, June 1977, pp. 439-69

[29] See Binhammer (1993, p.12)

influence it." Central banks have, in fact, used this kind of logic to decide which monetary aggregate should be most closely watched in setting money supply growth targets. The preference of the Bank of Canada used to be to use the narrowest measure of money M1, but now with the tremendous growth of broader money measures which are seen to better represent economic activity. M2 and M2+ are the preferred money measures for economic analysis.

This chapter has discussed two slightly different issues simultaneously; the issue of what monetary assets to include in the money measures, and the separate issue of how to measure the aggregates constructed from those money assets. This thesis project mostly focuses on the latter issue. The final issue of what money assets to include in the money measures was bypassed because the same money assets included in M1, M2 and M3 by the Bank of Canada were used here in the construction of our aggregates measures. This was done for the purpose of comparing the Divisia monetary aggregates constructed in this project to the simple sum aggregates given by the Bank of Canada.

VIII. Seasonal Adjustment

The figures published by the Bank of Canada and used in this project are frequently *seasonally adjusted* because the public's use of cash and chequing accounts is not evenly distributed over the course of a year. (For example, consumption activity is almost always at a peak during the Christmas holiday season.) Seasonal adjustments are made to smooth out some of the "normal" variations that occur in the course of a year long period.

Unfortunately, more data is available in the seasonally unadjusted form than otherwise, making the consistent use of seasonally adjusted data in projects such as these somewhat difficult. The reason for the greater availability of unadjusted data is that the Bank of Canada collects information for the broader aggregates, and only publishes seasonally adjusted data for the narrow aggregates. Further the methods used by the Bank of Canada to effect seasonal adjustment are kept confidential within the institution.

The disadvantage of using seasonally adjusted data is that it removes some information which can be of use to some analysts, mostly for those trying to make short term forecasts that depend on the amount of cash or the volume of cheques written in the economy.[30] The discrepancies between seasonally adjusted and unadjusted data grow less and less noticeable as one moves from narrow to broad definitions of the money supply.

IX. Conclusion

One of the most fundamental developments in economic history was the move from the barter system to the money system. Money greatly facilitates and simplifies exchange transactions over what would be possible in a barter economy. This is principally accomplished in two ways. First, money greatly reduces the number of prices in the economy that consumers must monitor. Second, money eliminates the double coincidence of wants between the parties of a transaction by separating buyers from sellers. Money typically serves as a medium of account, as a medium of exchange and as a store of value. Several empirical definitions of money are provided by the Bank of Canada. The narrowest, M1, consists of currency and demand deposits. One of the broadest, M3, includes savings deposits, fixed term and foreign currency deposits. A relatively new broad definition, M2+, is designed to account for the deposits of

[30] See Binhammer (1993, p.34)

alternative financial institutions such as trust companies and credit unions. Finally, the construction of aggregates such as M1 through M3 may be accomplished using many different statistical index number formulas such as the simple sum index number or so-called weighted indexes such as the Divisia index (indexes which weight the components of the aggregate according to their degree of "moneyness").

Chapter 3: A Theoretical Framework

I. Introduction

In order to construct measures of Canadian monetary aggregates consistent with aggregation theory, it is necessary to review the accepted precepts of recent research in such areas. One precept advanced by monetary aggregation theory today is that the use of simple summation in constructing monetary aggregates is generally inconsistent with microeconomic theory. Typically in applying the simple sum aggregation technique over a finite set of monetary assets, the relative prices of these monetary assets are assumed to be constant and equal over time. (Indeed with the simple sum method of aggregation, relative prices are normalized to unity). For this to be an accurate method of aggregation, microeconomic theory indicates that the component monetary assets must be not only perfect substitutes, but dollar-for-dollar substitutes. The empirical evidence points to the fact that relative prices are neither constant nor equal across time, and thus, in fact monetary sub-aggregates are not dollar-for dollar substitutes for each other.[31]

Microeconomic theory generally regards each monetary asset as having different degrees of "moneyness". Thus at very least, it would be an improvement in the construction of monetary aggregates to assign weights to each monetary asset, that reflect the differing degrees to which assets contain "money" properties. The weights being between zero and unity. This assigning of different weights to each monetary asset differs from the procedure followed in the simple sum method, where each asset is given a

[31] See Barnett, Fisher and Serletis (1992)

weight of either zero or unity. Even if and when the weights can be determined, there remains the question of whether a linear function for the construction of an aggregate is really the best approach that can be taken. Again the latest work in monetary aggregation theory suggests that as long as the summation process involves a linear weighting of the component assets, the assumption of perfect substitutability among the component assets must hold. However dollar-for-dollar substitutability need no longer hold for a linear weighting of assets.

Despite these difficulties, attempts have been made to adjust the approach without having to abandon the simple sum method of aggregation entirely, Hicksian aggregation in fact attempts to reconcile the simple sum technique with more realistic assumptions regarding the demand for monetary assets. Hicksian aggregation itself makes rather restrictive assumptions. For example, for Hicksian aggregation to be possible, the relative prices (user costs) of various component monetary assets must remain constant over the sample period.[32] It must also be the case that the constant relative user cost between any two assets be equal to 1.0. As before, this condition can only be true if the component monetary assets are perfect substitutes. This is highly unlikely given that most financial assets provide different services and have different own rates of return and thus the user costs that depend upon these yields. Additionally, these rates of return (and hence user costs) change over time.

Microeconomic aggregation theory advocates superior techniques to the simple sum method. This theory has two branches, one advocating the derivation of money demand functions from a series of equations "*modeling the wealthholder's allocation of*

[32] See Barnett, Fisher and Serletis (1992)

funds among money and nonmoney assets"[33] the other branch advocating the use of nonparametric statistical index number approximations to the parametric money demand functions mentioned above. Ultimately the use of statistical index numbers is the preferred approach used in this project, and in particular, a discrete-time version of the Divisia index formula will be used in the construction of the monetary aggregates.

To justify the use of statistical index numbers, a brief review of recent monetary aggregation literature will require that both approaches be discussed in the following section.

II. The Microeconomic Theory of Monetary Aggregation

In aggregation theory, there are two distinct types of problems, aggregation across heterogeneous agents and the aggregation of the various goods purchased by a single consumer. This paper focuses on the aggregation of the monetary assets held by a single representative consumer. In other words, the discussion of microeconomic aggregation theory presented in this section proceeds from the assumption that consumers and firms are not so heterogeneous that Gorman's condition is not satisfied. Gorman's condition requires that all consumers have linear and parallel Engel curves.[34] In such a case, the aggregate expenditure variable is merely the sum or mean of expenditures of all individual consumers and redistribution effects may be ignored. As a result, aggregate consumer demand may be modeled as if all decisions are made by a single consumer, which is similar in practice to assuming that the economy is populated by identical

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[33] See Barnett, Fisher and Serletis (1992, p.2092)

[34] See Anderson, Jones and Nesmith (1997, p.33)

individuals.[35] The weaknesses of assuming the existence of a representative consumer will be discussed at the end of the chapter.

In addition to assuming the existence of a representative agent, three other assumptions are sufficient conditions for the aggregation of a group of economic decision variables. These are:

> *"(1) the existence of a theoretical aggregator function defined over the group variables - that is, the existence of a subfunction defined over the group of variables that can be factored out of the economic agent's decision problem; (2) the efficient allocation of resources over the group of variables and, (3) the absence of quantity rationing within the group of variables."*[36]

As stated above, the following discussion will focus on the aggregation of monetary assets held by a price-taking representative consumer. The consumer maximizes an intertemporal utility function (in which current-period monetary assets are weakly separable from all other decision variables) subject to a set of multi-period budget constraints.

The inclusion of monetary assets in the consumer's utility function is a somewhat controversial issue in microeconomics. There is however, a long history of the approach going back at least to the work of Walras (trans. 1954) and Sidrauski (1967). According to Arrow and Hahn (1971), if money has positive value in general equilibrium, largely from its role in facilitating transactions, there exists a derived utility function containing

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[35] See Anderson, Jones and Nesmith (1997, p.34)

[36] See Anderson, Jones and Nesmith (1997, p.34)

money. Any system that produces an incentive for holding money in equilibrium does indeed allow for the inclusion of money in the utility function. For such a system any model that does not include money in the utility function is functionally equivalent to one that does. "*Hence, no generality is lost, or gained, by including money in the utility function.*"[37]

For the infinite horizon utility maximization problem, Clower (1967) advocated that the role of money in carrying out transactions be captured by the explicit introduction of a "transactions technology". This is the so-called cash-in-advance constraint. The other approach is to include money directly in the utility function, as done by Sidrauski (1967). Feenstra (1986) finds that the two approaches are equivalent, a result consistent with the work of Arrow and Hahn (1971). [38]

From this basic model, it can be shown that the weak separability assumption implies the existence of a theoretical aggregator function that can be defined over current-period monetary assets. Quantity rationing is simply ruled out while the maximization of utility means that the allocation of resources over the weakly separable group will be efficient.[39]

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[37] See Anderson, Jones and Nesmith (1997, p.34)

[38] Turnovsky (1995, p.265)

[39] As a final note before expounding upon the consumption model, it is worth mentioning that very similar models can be derived illustrating the microeconomic foundations of monetary aggregation from the vantage point of profit-maximizing firms and financial intermediaries. See Barnett, Jones and Nesmith (1997).

Assume that in each period, the representative consumer maximizes intertemporal utility over a finite planning horizon of T periods. The consumer's intertemporal utility function in any period, t, is

$$U(m_t, m_{t+1}, ..., m_{t+T}; c_t, c_{t+1}, ..., c_{t+T}; l_t, ..., l_{t+T}; A_{t+T}) \qquad (1)$$

where for the set of periods contained in $\{t, t+1, ..., t+T\}$,

$m_t = (m_{1t}, ..., m_{nt})$ is a vector of real stocks of n monetary assets

$c_t = (c_{1t}, ..., c_{ht})$ is a vector of quantities of h non-monetary consumption goods and services

l_t is the desired amount of leisure time

A_{t+T} is the real stock of a benchmark financial asset, held in the final period of the planning horizon, at date $t+T$.

The utility function shown above is maximized subject to a set of $T+1$ period budget constraints, shown below,

$$p_t' c_t + \pi_t' m_t + w_t l_t = y_t \qquad (2)$$

where y is full income (income accounting for expenditures on time as well as on goods and services), c is a vector of the prices of consumption goods and services, π is a vector of the user costs of the monetary assets, and w is the shadow price of leisure, all for periods t. The formula for the user cost of the monetary services provided by asset i is given by,

$$\pi_i = p_s^* \left(\frac{R - r_{is}}{1+R} \right) \qquad (3)$$

Here, the user cost measures the opportunity cost in terms of the interest foregone by holding monetary asset i for the given period. r_{it} is the expected nominal holding period yield on the ith asset at time t, while R is the expected holding-period yield on the benchmark (alternative) asset, and p_t^* is the true cost-of-living index at time t. The formula for the user cost of monetary assets given above can be derived from the intertemporal optimization problem itself.[40]

A more appropriate way to structure the budget constraint from an intertemporal perspective would be the following:

$$\sum_{i=1}^{h} p_{it} c_{it} = w_t L_t + \sum_{i=1}^{n} [(1+r_{i,t-1}) p_{t-1}^* m_{i,t-1} - p_t^* m_{i,t}] + [(1+R_{t-1}) p_{t-1}^* A_{t-1} - p_t^* A_t] \quad (4)$$

where A_t is the real quantity of the benchmark asset that appears in the utility function in the final period, and L_t is the number of hours of labor supplied, (related to leisure time by the formula $l_t = H - L_t$ where H is the total number of hours in a period). A_t appears in each budget constraint because the benchmark asset is used to transfer wealth from period to period. However, the benchmark asset is only included in the utility function in the final period because only then does it contribute to utility as a monetary asset itself.

To simplify notation, let the set of current-period monetary assets be represented by the vector $m_t = (m_{1t}, \ldots, m_{nt})$ and let the vector $x_t = (m_{t+1}, \ldots, m_{t+T}; c_t, \ldots, c_{t+T}; l_t, \ldots, l_{t+T}; A_{t+T})$ include all remaining decision variables which contribute to utility. The utility function can then properly be written as $U(m_t, x_t)$. Finally, the vectors, m_t^* and x_t^* will be the solution to the consumer's optimization

[40] See Barnett, Fisher and Serletis (1992, p.2094)

problem, with m_t^* representing the consumer's optimal holdings of current-period monetary assets, while x_t^* represents the optimal holdings of all other decision variables.

The first order conditions resulting from the constrained optimization problem allow us to state that the marginal rate of substitution between current-period monetary assets m_{it} and m_{jt} evaluated at the optimum takes the form

$$\frac{\dfrac{\partial U(m_t,x_t)}{\partial m_{it}}}{\dfrac{\partial U(m_t,x_t)}{\partial m_{jt}}} = \frac{\left(p_t^* \dfrac{R_t - r_{it}}{1+R_t}\right)}{\left(p_t^* \dfrac{R_t - r_{jt}}{1+R_t}\right)} \qquad (5)$$

The marginal rate of substitution between the current-period non-monetary consumption good c_{kt}, and the current-period monetary asset m_{it} evaluated at the optimum is

$$\frac{\dfrac{\partial U(m_t,x_t)}{\partial m_{it}}}{\dfrac{\partial U(m_t,x_t)}{\partial c_{kt}}} = \frac{\left(p_t^* \dfrac{R_t - r_{it}}{1+R_t}\right)}{p_{kt}} \qquad (6)$$

As a general rule in simple microeconomics, marginal rates of substitution between two goods rate are equal to the goods' relative prices. The result above is consistent with the earlier statement that the opportunity cost (or user cost) of the current-period monetary asset is indeed

$$\frac{p_t^*(R_t - r_{it})}{(1+R_t)}. \qquad (7)$$

The intuition behind the derivation of user costs for monetary assets is that such assets are treated in this procedure as durable goods. Hence, monetary assets simultaneously provide services and depreciate (to some partial degree) during each period of time. The user cost is derived following the Diewert (1974, 1980) procedure in which the purchase price of the durable good, the depreciation rate of the good, and a discount factor are required in the calculation.[41] Assuming an agent buys one unit of a durable good and later sells the good one period later, the cost to the agent of "renting" that good for the one period (or the cost of the services the agent obtains through the use of the good for the period), is determined by the difference between the purchase price of the good and the present value of the amount received by the agent when the non-depreciated part of the one unit of the durable good is sold. As such, an explicit rental market for the good does not exist, thus the user cost determined by the formula is considered to be an implicit rental price or user cost, implicit as if one were renting the good to one for the period.[42]

Barnett derived the general formula for the user cost (or rental price) of monetary assets through the following procedure: The $T+1$ period budget constraints of the general intertemporal consumption model are combined, solving each equation for A_s (for each period s) and recursively substituting the equations backwards in time beginning with A_{t+T}. The general form of the discounted nominal user cost, π_{is} of each monetary asset (for all s contained in $\{t, t+1, ..., t+T\}$ is therefore

$$\pi_{is} = \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}}\right] \qquad (8)$$

where r_{is} is the nominal holding period yield and the discount factor ρ_s is defined by

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[41] See Anderson, Jones and Nesmith (1997,p.36)

[42] See Anderson, Jones and Nesmith (1997, p.36)

$$\rho_s = \left\{1, \prod_{u=t}^{s-1}(1+R_u) when, t+1 \le s \le t+T\right\} \qquad (9)$$

The general form for π_{it} given above can be modified somewhat for the special case of the current-period nominal user cost π_{it} of monetary asset m_{it}, as

$$\pi_{it} = \frac{p_t^*(R_t - r_{it})}{1+R_t} \qquad (10)$$

The above is the same formula as the one given previously as the formula for the price of current period monetary assets.

To proceed further in the development of the demand for monetary assets, the theory of two-stage optimization should be discussed. To begin the two-stage optimization process, it is necessary to assume that the intertemporal utility function is weakly separable in the group of current monetary assets. In the first stage of the two-stage optimization process, the consumer allocates his expenditures optimally among the broad categories that contribute to utility (nonmonetary consumption goods and services, leisure and monetary services) for each period of the process. The second stage allocates expenditures optimally within each category. In the first stage of the problem, the expenditure allocation decision of the consumer is influenced by the relative prices among the general categories of goods in the utility function. For the second stage, the decision of how to allocate expenditure among the component monetary assets is guided by their relative prices.[43] From an intertemporal standpoint, the first stage of the two-stage optimization process will aim to allocate expenditure between two broad categories, current-period monetary assets and the rest of the goods relevant to the consumer,

[43] See Barnett, Fisher and Serletis (1992, p.4094)

including monetary assets consumed in future periods. For this to be possible the vector of current-period monetary assets must be weakly separable from the rest of the consumption goods in the utility function as shown below.

$$U[f(m_t), m_{t+1}, \ldots, m_{t+T}; c_t, c_{t+1}, \ldots, c_{t+T}; l_t, \ldots, l_{t+T}; A_{t+T}] \qquad (11)$$

Which may be more conveniently be written as $u(f(m_t), x_t)$ where x_t was defined previously.[44]

Only the vector of current-period monetary assets $m_t = (m_{1t}, \ldots, m_{nt})$ is included in the category subutility function $f(m_t)$. The weak separability assumption in this case does not also imply symmetry, which is to say that no other group of assets within the utility function is necessarily weakly separable. Without the weak separability assumption, the optimization process for the representative consumer cannot be broken into two stages. Due to the ability to divide the expenditure allocation problem into two stages, as weak separability allows, the marginal rate of substitution among current-period monetary assets is independent of the quantities of other goods chosen outside of the weakly separable group. This independence of the marginal rate of substitution as a result is illustrated mathematically as,

$$\frac{(\partial U / \partial m_{it}) / (\partial U / \partial m_{jt})}{\partial \phi} = 0, i \neq j \qquad (12)$$

where ϕ is any component of the super-vector x_t.[45]

[44] See Anderson, Jones and Nesmith (1997)

[45] See Barnett, Fisher and Serletis (1992, p.2094)

Much of the monetary demand literature uses the weakly separable assumption as an untested hypothesis due to the lack of effective methods for conducting an empirical test of whether the utility function is appropriately separable in monetary services. Whether the weakly separability of the utility function actually holds or not is an empirical question and the difficulties of performing such tests must count as a factor against using this microeconomic approach in constructing monetary aggregates.

The optimal demand for the set of current-period monetary assets may thus be found through the maximization of the category subutility function subject to the income constraint derived from the first-stage of the larger consumer optimization problem. This is shown below.

Max $\quad f(m_t) \qquad (13)$

Subject to $\quad \sum_{i=1}^{n} m_{it}\pi_{it} = y_t$

Where $\quad y_t = \sum_{i=1}^{n} m_{it}^{*}\pi_{it}$

{ y_t is the amount of expenditure on the broad category of current-period monetary assets derived from the 1st stage.}

This establishes how the two-stage optimizing procedure is used to determine the optimal quantities of the individual current-period monetary assets. To review, the consumer chooses the optimal total expenditure y_t and also the optimal quantities of the other monetary assets, goods, services, and leisure that comprise the utility function in the first stage. For the second stage, the consumer determines his(her) optimal demand for the individual current-period monetary assets subject to the optimal total expenditure on current-period monetary assets chosen in the first stage.[46]

The category subutility function $f(.)$ may be termed a monetary quantity aggregator function dependent on the assumption that the function is first-degree homogeneous. A monetary quantity aggregate is obtained by inputting the optimal quantities of all the individual current-period monetary assets into the monetary quantity aggregator function (category subutility function), $M_t = f(m_t^*)$. This money quantity aggregate may be treated as if it were the optimum quantity of a single-period elementary good, the elementary good being current-period monetary services.[47] In Barnett's terminology, the 1st stage optimizing process involves a choice between two broad groups; monetary services (the money aggregate) and all other decision variables.

The monetary quantity aggregate can be found as described above using the quantity aggregator function. If quantity and price aggregates are dual with respect to each other, then the price aggregate may easily be determined once the quantity aggregate is known. Duality (which follows from a property known as factor reversal) implies that the quantity aggregate multiplied by the price aggregate equals the total expenditure on all individual assets within the aggregate. The price aggregate (or user cost aggregate) in this case is Π_t and may be defined from the vector of nominal user costs of the set of current-period monetary assets in the form of a unit expenditure function as shown below.

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[46] See Anderson, Jones and Nesmith (1997,p.36)

[47] See Anderson, Jones and Nesmith (1997,p.38)

$$\Pi_t = E(\pi,1) \qquad (15)$$

$$= \min\{\sum_{i=1}^{n} m_i \pi_{it} : u(m) = 1\}$$

where $\pi_t = \{\pi_{1t},...,\pi_{nt}\}$ is the vector of user costs to correspond to the vector of current-period monetary assets. The duality of the quantity and price aggregates implies that the budget constraints in both the 1st and 2nd stages of the optimizing problem can be rewritten in terms of the aggregates M_t and Π_t. Barnett showed that the T+1 multi-period budget constraints of the 1st stage could be combined into one budget constraint. Current-period monetary assets enter this single budget constraint as

$$\sum_{i=1}^{n} m_{it}\pi_{it} \qquad (16)$$

The budget constraint evaluated at the optimum can be rewritten in terms of the aggregates M_t and Π_t as shown below.

$$\Pi_t M_t = \sum_{i=1}^{n} \pi_{it} m_{it}^{*} = y_t \qquad (17)$$

Following duality and the conditions for the factor reversal property, the dual user cost is implicitly defined by

$$\Pi_t = \left[\sum_{i=1}^{n} m_{it}^{*}\pi_{it}\right] / M_t \qquad (18)$$

To review, the two-stage budget/optimizing process allows expenditure to be allocated between the optimal quantities of monetary services M_t and all other consumption goods outside the weakly separable group of current-period monetary assets, subject to prices and a budget constraint where the price of monetary services is given by the dual opportunity cost Π_t. The first-stage decision determines the optimal total expenditures on current-period monetary assets, $y_t = M_t \pi_t$. In the second-stage decision, this optimal expenditure is allocated among the individual current-period monetary assets. This allocation is irrelevant to the allocation of expenditures among the other goods and services contained in the utility function. Therefore, the aggregates M_t and Π_t contain "*all the information about the consumer's portfolio of current-period monetary assets that is relevant to other aspects of the representative consumer's problem.*"[48] This result is ultimately dependent on the assumption that current-period monetary assets are weakly separable from all other decision variables, that the function f(.) is first-degree homogeneous so that it can serve as a quantity aggregator function, and finally on the assumption that the quantity and price aggregates are dual with respect to each other.

III. Statistical Index Number Theory

The problem of constructing monetary aggregates within the microeconomic framework of a utility maximization problem of a representative agent depends clearly upon being able to estimate the parameters of $f(.)$. Solving the system of demand equations for individual current-period monetary assets and using the available specific monetary data (user costs and so forth), the parameters of $f(.)$ could be estimated using econometric techniques. This dependency upon parametric estimation has discouraged policymakers and government agencies from using such an approach (as described in the previous section) in the construction of monetary aggregates. The reason is that for such

[48] See Anderson, Jones and Nesmith (1997, p.40)

parametric techniques to be employed, one must make very specific assumptions about the functional forms of the utility or cost functions of the model.

Statistical index numbers are specification and estimation free functions giving the relation between price and quantity aggregates across two or more time periods. "*Unlike aggregator functions, statistical index numbers contain no unknown parameters. A statistical index number is said to be exact for an aggregator function if the index number tracks the aggregator function, evaluated at the optimum, without error.*" [49]

It is a general result of the monetary aggregation literature that the continuous-time version of the Divisia quantity index is an exact approximation for the monetary aggregate, M_t. Here the continuous-time version of the Divisia quantity index for monetary assets is denoted by M_t^D and is defined for the differential equation shown below.

$$\frac{d\log(M_t^D)}{dt} = \sum_{i=1}^{n} w_{it} \frac{d\log(m_{it}^{\bullet})}{dt} \qquad (19)$$

where for $i = 1,\ldots,n$,

$$w_{it} = \frac{m_{it}^{\bullet}\pi_{it}}{\sum_{j=1}^{n} m_{jt}^{\bullet}\pi_{jt}} \qquad (20)$$

w_{it} here represents the expenditure shares of the set of individual monetary assets. The Divisia continuous-time user cost index is defined below.

$$\frac{d\log(\Pi_t^D)}{dt} = \sum_{i=1}^{n} w_{it} \frac{d\log(\pi_{it})}{dt} \qquad (21)$$

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[49] See Anderson, Jones and Nesmith (1997, p.40)

The Divisia continuous-time quantity and price indexes also satisfy the desirable (and necessary) property of factor reversal. This means that the total expenditure on all assets included in the index is the product of the Divisia quantity and price indexes as shown below.

$$M_t^D \Pi_t^D = \sum_{i=1}^{n} m_{it}^{*} \pi_{it} = y_t \qquad (22)$$

The growth path of the monetary quantity aggregate itself may be found using only the 2nd stage of the two-stage optimizing process, even though the functional form of the category subutility function is unknown. The quantity aggregate $M_t = f(m_t^{*})$ emerges from the second-stage consumer optimization process as follows.

$$\text{Max } f(m) \text{ subject to } \sum_{i=1}^{n} m_i \pi_{it} = y_t \qquad (23)$$

The variables of this optimization problem include only current-period monetary assets and their corresponding user costs. The Lagrangian first-order conditions are

$$\frac{\partial u(m)}{\partial m_i} = \lambda \pi_{it} \text{ for i=1,...,n} \qquad (24)$$

λ above is obviously the Kuhn-Tucker multiplier of the Lagrangian function.[50] From Euler's equation, we can define the monetary quantity aggregate algebraically as follows.

$$M_t = f(m_t^{*}) = \sum_{j=1}^{n} \left[\left(\frac{\partial u(m)}{\partial m_j} \right) (m_{jt}^{*}) \right] \qquad (25)$$

$$= \sum_{j=1}^{n} \lambda \pi_{jt} m_{jt}^{*}$$

[50] See Anderson, Jones and Nesmith (1997, p.41)

$$= \lambda \sum_{j=1}^{n} \pi_{jt} m_{jt}^{*}$$

The aim of going through this algebraic procedure is to show that the continuous-time Divisia quantity index may be directly defined from the representative consumer's second-stage maximization problem. To continue, the next step is to take the derivative of the above algebraic definition of the monetary quantity aggregate with respect to time, as shown below.

$$\frac{dM_t}{dt} = \frac{df(m_t^{*})}{dt} \qquad (26)$$

$$= \sum_{i=1}^{n} \left[\left(\frac{\partial u(m)}{\partial m_i} \right) \left(\frac{dm_{it}^{*}}{dt} \right) \right]$$

$$= \sum_{i=1}^{n} \lambda \pi_{it} \frac{d \log(m_{it}^{*})}{dt}$$

$$= \lambda \sum_{i=1}^{n} \pi_{it} m_{it}^{*} \frac{d \log(m_{it}^{*})}{dt}$$

Dividing the above expression by the previous one given in (25), the following expression is obtained.

$$\frac{d \log(M_t)}{dt} = \sum_{i=1}^{n} w_{it} \frac{d \log(m_{it}^{*})}{dt} \qquad (27)$$

$$= \frac{d \log(M_t^{D})}{dt}$$

The above equation is exactly equal to the definition of the growth rate of the continuous-time Divisia quantity index, M_t^D. Thus it has been shown that the Divisia continuous-time quantity index is indeed an exact approximate for the money quantity aggregate defined in the representative consumer's optimization problem.

IV. The Discrete-Time Case

In the case of discrete time, the above rules do not apply for there is no statistical index number which functions as an exact approximation for any given aggregator function. This does not mean that the use of statistical index numbers in the construction of aggregates is unnecessary. The reason is that there exist mathematical functions that can provide second-order approximations for unknown aggregator functions. These mathematical functions may be classified as flexible functional forms. In turn, statistical index numbers are available which exactly approximate these mathematical functions. Statistical index numbers of this type (that are exact for certain specific flexible functional forms) have been termed superlative by Diewert (1976). For example, the discrete-time version of the Divisia index (which will be derived below) is an exact approximation for the linearly homogeneous flexible translog functional form and is therefore a member of the superlative class of statistical index numbers.

There are two basic types of superlative index numbers; chained and fixed base indexes. A chained statistical index number is the description applied to an index number formula in which the prices and quantities used are taken from adjacent periods, (for example, monetary quantity data such as m_{it} and $m_{i,t+1}$). Fixed base statistical index numbers use price and quantity data from the current period and a fixed base period such as monetary quantity data m_{it} and fixed base, m_{io}.[51] In general, the use of chained superlative statistical index numbers is preferred over the fixed base indexes. The reason for this is that the remainder term of the second-order approximation for the chained indexes is typically smaller than for the fixed base indexes because changes in prices and

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[51] See Anderson, Jones and Nesmith (1997,p.41)

quantities are typically smaller between adjacent periods than between the current period and a fixed base. Thus, chained superlative statistical index numbers will typically provide a better approximation for the unknown monetary aggregator function than does the fixed base statistical index number. Hence, the discrete-time Divisia index formula shown here will be in chained form.

Two important superlative statistical indexes are the Fisher ideal index and the Tornqvist-Theil discrete-time approximation to the Divisia index (otherwise known as the discrete-time Divisia index). The Fisher Ideal index generates results exactly identical to those of the quadratic flexible functional form. For the purposes of constructing a monetary quantity aggregate, M_t^F, the Fisher ideal quantity index takes the form of the formula given below.

$$M_t^F = M_{t-1}^F \sqrt{\frac{\sum_{i=1}^{n} m_{it}^{\bullet} \pi_{it} * \sum_{i=1}^{n} m_{it}^{\bullet} \pi_{i,t-1}}{\sum_{i=1}^{n} m_{i,t-1}^{\bullet} \pi_{it} * \sum_{i=1}^{n} m_{i,t-1}^{\bullet} \pi_{i,t-1}}} \qquad (28)$$

It is commonly observed that the growth rate of the Fisher ideal index is the geometric mean of the growth rates of the Paasche and Lasperyres quantity indexes which themselves are not superlative, since they are only first-order approximations of the underlying aggregator function.

The discrete-time version of the Divisia index as mentioned previously is exact for the translog flexible functional form. In the context of monetary aggregation, the quantity index M_t^{DD} will take the following form.

$$M_t^{DD} = M_{t-1}^{DD} \prod_{i=1}^{n} \left(\frac{\dot{m}_{it}}{\dot{m}_{i,t-1}} \right)^{1/2(w_{it}+w_{i,t-1})} \qquad (29)$$

where w_{it} and $w_{i,t-1}$ are share equations for the individual monetary assets as described before. The user cost index dual to the discrete-time Divisia index is given by

$$\Pi_t^{Dual} = \Pi_{t-1}^{Dual} \left[\frac{\sum_{i=1}^{n} \pi_{it} \dot{m}_{it}}{\sum_{i=1}^{n} \pi_{i,t-1} \dot{m}_{i,t-1}} \right] \Bigg/ \left[\frac{M_t^{DD}}{M_{t-1}^{DD}} \right], \qquad (30)$$

which is based on a weak form of factor reversal.[52]

V. Monetary Service Flows and Monetary Wealth

So far, the discrete-time Divisia index as used in the monetary aggregation theory advanced by Barnett, is only capable of measuring the flow of monetary services provided by the individual monetary assets contained in the representative consumer's utility function. However, this measurement is not the only perspective of interest to an economist. For example, it would also be quite useful to derive an expression for the stock of the consumer's monetary wealth.

As for all economic goods, changes in the price of monetary services will affect both the supply and demand of monetary and nonmonetary goods and services. This raises the possibility that substitution and income effects must be considered with respect to the consumption of monetary services.

A simple way to construct a measure of monetary wealth for the representative consumer is to find the discounted present value of the expected expenditure on monetary

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[52] See Anderson, Jones and Nesmith (1997, p.42)

services for a given length of time. In fact, the discounted present value of expenditure on monetary services for the consumer's intertemporal optimization decision is included in the simple budget constraint, constructed (using Barnett's methodology (1978, 1987)) from the multi-period budget constraint. The formula for the discounted present value of expenditure on monetary services is given below.

$$V_t = \sum_{s=t}^{T}\sum_{i=1}^{n}\left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}}\right]m_{is} \qquad (31)$$

$$= \sum_{s=t}^{T}\sum_{i=1}^{n}\pi_{is}m_{is}$$

for the periods $\{t, t+1, ..., t+T\}$

The discount factor, shown above in the formula for V_t is defined as follows.

$$\rho_s = \left\{1, \prod_{u=1}^{s-1}(1+R_u) for, t+1 \leq s \leq t+T \right. \qquad (32)$$

The discounted nominal user costs π_{is} have been defined previously.

Expanding the length of the time horizon in the intertemporal decision problem from T to infinity, and evaluating V_t at the optimum, the formula given above becomes

$$V_t = \sum_{s=t}^{\infty}\sum_{i=1}^{n}\pi_{is}m_{is}^* = \sum_{s=t}^{\infty}y_s \qquad (33)$$

where y_s is the optimal total expenditure on current-period monetary assets for period s. When V_t is expressed as an infinite time horizon sum of future expenditures on monetary services, its direct measurement becomes impossible.[53]

If the admittedly unrealistic assumption of static expectations with regard to the future prices and own rates of various assets is satisfied, the computation of V_t becomes possible. Specifically, the assumption of static expectations entails that the consumer does not believe that future prices and own rates will depart from their current values. Thus for s contained in {t,t+1, ...,t+T}, $E_t[r_{is}] = r_{it}$ and $E_t[R_s] = R_t$ where E is the expectations operator, based upon the information set available at time t. If this assumption holds (if future prices and rates remain unchanged from today) the consequence will be that the consumer's optimal holdings of individual assets will also remain unchanged with the passage of time. Therefore, $E_t(m^{*}_{is}) = m^{*}_{it}$ where t is for the current period and s covers all future time periods.

Given the static expectations assumption, Barnett (1991) has shown that a formula known as the Rotemberg currency-equivalent index, CE_t, will calculate the value of the stock of monetary wealth. The formula for the CE_t is shown below,

$$CE_t = p_t \sum_{i=1}^{n} \frac{(R_r - r_{it})}{R_t} m^{*}_{it} \qquad (34)$$

As it becomes possible to obtain a measure of the stock of monetary wealth, it also becomes possible to examine how consumer behaviour is affected by changes in this stock.

To review, with the Divisia discrete-time index, the measurement of the flow of monetary services in the intertemporal context is possible. On the other hand, a measure of the stock of monetary wealth through the use of the Rotemberg currency equivalent index only becomes possible if the assumption of static expectations is made. In one special case, these two measurements will coincide and the currency equivalent index is able to measure the flow of monetary services. If the category subutility function $f(.)$, is

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[53] See Anderson, Jones and Nesmith (1997, p.43)

quasi-linear in a monetary asset whose own rate is always zero, in addition to the assumptions that underlie the Divisia index (weak separability, etc.) then the CE index will measure the flow of monetary services.[54] Given that these assumptions are more restrictive (and thus harder to satisfy), the CE index is statistically inferior compared to the discrete-time Divisia index as a measure of the flow of monetary services.

There exists another index that can under the correct circumstances provide a measure both of the flow of monetary services and the stock of monetary wealth. The simple sum index as described at some length previously, is only capable of functioning as a means of monetary aggregation (and thus as a measure of the flow of monetary services following Barnett's logic) if the component monetary assets included in the index are perfect substitutes. This means, in microeconomic theory, that the indifference curves generated by the comparison between two or more of these monetary assets would be linear and parallel. If these assets have different user costs, the relevant budget constraint, (the slope of which is determined by the ratio of these user costs) would differ from the indifference curve and hence, a corner solution would be obtained at the optimum. The result of differing user costs for monetary assets that are perfect substitutes would be that the consumer would hold only one monetary asset in equilibrium, the one with the lower user cost. As this is obviously not empirically observed the evidence does not support the use of the simple sum index as a measure of the flow of monetary services.

The CE index (itself a measure of the stock of monetary wealth of the representative agent) is contained within the simple sum index formula, under the static

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[54] See Anderson, Jones and Nesmith (1997, p.44)

expectations assumption. Hence, indirectly, the simple sum index itself becomes a stock measure. The derivation of the CE_t index from within the simple sum (SS_t) index is shown below.

$$SS_t = \sum_{i=1}^{n} p_t^{*} m_{it}^{*}$$

$$= p_t \sum_{i=1}^{n} \frac{(R_t - r_{it}) m_{it}^{*}}{R_t} + p_t \sum_{i=1}^{n} \frac{r_{it} m_{it}^{*}}{1+R_t} \left[1 + \frac{1}{(1+R_t)} + \frac{1}{(1+R_t)^2} + \ldots \right] \qquad (35)$$

The first term in the decomposition above is the CE index while the second term is the discounted present value of all current and future interest received on monetary assets again under the assumption of static expectations. Thus the simple sum index can have a role as a stock variable (one aspect being the measure of the stock of monetary wealth) but the simple sum index cannot however serve as an accurate measure of the flow of monetary services.

VI. Limitations and Extensions

The microeconomic and statistical index theory methodology used so far in the construction of monetary aggregates has been dependent upon some relatively strong microeconomic assumptions. To review, these were:

> *(1) the existence of representative agent, (2) blockwise weak separability of current-period monetary assets, (3) homotheticity of the category subutility function and (4) perfect certainty* [55]

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[55] See Anderson, Jones and Nesmith (1997, p.44)

All of these assumptions, if satisfied, will lead to the result that the discrete-time Divisia index will serve as a second-order approximation to the unknown aggregator function, M_t.

> *"If the conditions that are necessary for the existence of a representative agent are not satisfied, or if the weak separability of current-period monetary assets from an agent's other decision variables is violated during some sample periods, then the growth rates of the optimal quantities of monetary assets, $\dot{m}_{1t},\ldots,\dot{m}_{nt}$, user costs, $\dot{\pi}_{1t},\ldots,\dot{\pi}_{nt}$ are expenditure shares, $\dot{w}_{1t},\ldots,\dot{w}_{nt}$ may contain information that is not contained in the discrete-time Tornqvist-Theil index."*[56]

Tests are available which allow for a measurement of the degree to which the discrete-time Divisia index does contain all the available information. Thus indirectly these tests can provide some indication as to whether the four assumptions given above in fact hold. One test of this variety which will in fact be covered here is the *dispersion dependency test*, suggested by Barnett and Serletis (1991), which in particular examines the Divisia index second moments (variances) of the growth rates of quantities, user costs and expenditure shares.

To review, the log change or growth rate of the discrete-time Divisia quantity index is

$$\Delta\log(M_t^{DD}) = \sum_{i=1}^{n} \overline{w_{it}}\Delta\log(\dot{m}_{it}) \qquad (36)$$

where for i = 1,…,n

[56] See Anderson, Jones and Nesmith (1997, p.45)

$$\overline{w_{it}} = \frac{1}{2}(w_{it} + w_{i,t-1}) \qquad (37)$$

are the average expenditure shares of the monetary assets across adjacent periods. The growth rate of the quantity index here is basically expressed as the average expenditure share-weighted mean of the growth rates of the individual monetary components. The average individual expenditure shares here introduce of valid measure of probability, which thus allows the discrete-time Divisia quantity index to be thought of as the moment (mean) of a probability distribution.

The log change or growth rate of the Divisia user cost index, Π_t^{DD}, is defined below.

$$\Delta \log(\Pi_t^{DD}) = \sum_{i=1}^{n} \overline{w_{it}} \Delta \log(\pi_{it}) \qquad (38)$$

Once again, from an intuitive standpoint, the growth rate for the whole index may be thought of as the average expenditure share-weighted mean for the user costs of the component monetary assets.

The growth rate of the discrete-time Divisia expenditure share index (not shown previously) S_t^{DD} is defined by the formula below.

$$\Delta \log(S_t^{DD}) = \sum_{i=1}^{n} \overline{w_{it}} \Delta \log(w_{it}) \qquad (39)$$

This too is expressed as an average share weighted mean of (in this case) the component expenditure share growth rates of the monetary assets.[57]

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[57] See Anderson, Jones and Nesmith (1997, p.45)

Theil (1967) illustrated that the three growth rates of the Divisia discrete-time indexes given above are related to each other by the expression

$$\sum_{i=1}^{n} \overline{w_{it}} \Delta \log(w_{it}) + \Delta \log(y_t) = \sum_{i=1}^{n} \overline{w_{it}} \Delta \log(m_{it}^{*}) + \sum_{i=1}^{n} \overline{w_{it}} \Delta \log(\pi_{it}). \qquad (40)$$

Now to conduct the dispersion-dependency test, the second moments or variances of the growth rates of the discrete-time Divisia indexes given above will have to be determined. The formula for the Divisia quantity growth rate variance is shown to be

$$K_t = \sum_{i=1}^{n} \overline{w_{it}} [\Delta \log(m_{it}^{*}) - \Delta \log(M_t^{DD})]^2 \qquad (41)$$

which basically is the variance of the growth rates of the component monetary assets. The Divisia user-cost variance formula given below.

$$J_t = \sum_{i=1}^{n} \overline{w_{it}} [\Delta \log(\pi_{it}) - \Delta \log(\Pi_t^{DD})]^2 \qquad (42)$$

is defined as the variance of the growth rates of the user costs for the component assets. The formula for the Divisia expenditure-share growth-rate variance is

$$\Psi_t = \sum_{i=1}^{n} \overline{w_{it}} [\Delta \log(w_{it}) - \Delta \log(S_t^{DD})]^2 . \qquad (43)$$

The variance is for the growth rates of the expenditure shares of the component monetary assets. A final formula useful for the dispersion-dependency test is the covariance of the growth rates of the component quantities and user costs which is given as

$$\Gamma_t = \sum_{i=1}^{n} \{ \overline{w_{it}} [\Delta \log(\pi_{it}) - \Delta \log(\Pi_t^{DD})] * [\Delta \log(m_{it}^{*}) - \Delta \log(M_t^{DD})] \} . \qquad (44)$$

Theil (1967) gave a second identity relating the four second moments in the formula,

$$\psi = K_t + J_t + 2\Gamma_t . \qquad (45)$$

Dispersion-dependency tests conducted by Barnett and Serletis and presented in Barnett and Serletis (1990) and Barnett, Jones and Nesmith (1996), suggest that the Divisia second moments given above do in fact possess economic information not captured in the growth rates of the discrete-time Divisia quantity, user cost and expenditure share indexes.

This result has the significant implication that some of the assumptions given at the beginning of the section, (requiring the existence of a representative agent, weak separability of current-period monetary assets, perfect certainty and the homotheticity of the category subutility function) may not hold. The dispersion-dependency test above is particularly designed to test for the existence of a representative consumer and blockwise separability. The results of the tests suggest that "*for at least some time periods, movements in the observed quantities and prices of monetary assets are not consistent with the movements that would be implied by the actions of a representative agent with a weakly separable utility function.*"[58]

Since this is evidence against the soundness of the assumptions behind the aggregation process, the dispersion-dependency test results must be counted as slight evidence that maybe the discrete-time Divisia index is not an entirely satisfactory method of monetary aggregation. However rather than discarding the discrete-time Divisia index, (due to the current lack of an accepted superior alternative), Barnett and Serletis (1990) propose including the Divisia second moments in macroeconomic models to provide some degree of correction for the aggregation error observed.[59]

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[58] See Anderson, Jones and Nesmith (1997, p.46)

[59] See Anderson, Jones and Nesmith (1997, p.46)

VII. Extensions

A. Homothetic Preferences

In this section, the case when the assumption of homothetic preferences is not satisfied will be considered. Unfortunately, it is rather unlikely that a category subutility function (utility function) will satisfy linear homogeneity (and thus homotheticity) since linear homotheticity is a rather strong assumption.[60] In the case, that the category subutility function for current-period monetary assets, $f(.)$, is not homothetic, the Divisia index will not track the utility function in continuous time. However, other indexes can correctly track the category subutility function even if the assumption of homotheticity is violated. So long as the representative agent's utility function is weakly separable in current-period monetary assets, the Konus and Malmquist indexes, for instance, correctly approximate the quantity and user cost aggregator functions even in the absence of homotheticity.

Given a nonhomothetic category subutility function, $f(.)$, the monetary quantity aggregate becomes defined by the *distance function*, $d(m_t^*, \tilde{u})$ which is defined implicitly as follows

$$u\left(\frac{m}{d(m,\tilde{u})}\right) = \tilde{u}\,. \qquad (46)$$

This distance function will always be linearly homogeneous in current period monetary assets (m) whether or not the utility function (category subutility function) itself is linearly homogeneous.[61]

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[60] See Barnett, Fisher and Serletis (1992, p.2110)

[61] See Barnett, Fisher and Serletis (1992, p.2112)

The dual use cost aggregate is the expenditure function evaluated at the reference utility level, $\tilde{u}$, and its formula is given as

$$e(\pi_t,\tilde{u}) = \min_{m_t}\{\sum_{i=1}^{n}\pi_{it}m_{it} : u(m_t)=\tilde{u}\} \quad . \qquad (47)$$

The quantity and price aggregates can be made exact by normalizing the indexes to equal one in a base period. The normalized quantity index is known as the Malmquist quantity index while the normalized user cost index is known as the Konus user-cost index.

The Malmquist quantity index is defined by the following:

$$M(m_t^{\bullet},m_o^{\bullet},\tilde{u}) = \frac{d(m_t^{\bullet},\tilde{u})}{d(m_o^{\bullet},\tilde{u})} \qquad (48)$$

The Konus user-cost index is defined in the formula below.

$$K(\pi_t,\pi_0,\tilde{u}) = \frac{e(\pi_t,\tilde{u})}{e(\pi_0,\tilde{u})} \qquad (49)$$

Both indexes as can clearly be seen above will equal unity in the base period, t=0.

The greatest shortcoming of these two statistical indexes is their reliance upon a reference utility level, $\tilde{u}$. However, given an appropriate method for selecting the reference utility level, Diewert (1976, pp.123-24) has shown that the discrete-time Divisia index itself does indeed provide a superlative approximation to the distance function. Therefore, the discrete-time Divisia index (unlike the continuous time Divisia index) provides a second-order approximation to the correct aggregator function regardless of whether the category subutility function is homothetic. This reliance on a reference utility level can be shown to vanish in the Malmquist and Konus indexes if the homotheticity assumption is satisfied, which clearly illustrates just how useful a property homotheticity is. If homotheticity applies (if the category subutility function is first-degree homogeneous) the distance function can be expressed as the product of the category subutility function and the proportionality factor, the reference utility level.

The Malmquist index may thus be rewritten as follows:

$$M(m_t^{\bullet}, m_0^{\bullet}, \tilde{u}) = \frac{d(m_t^{\bullet}, \tilde{u})}{d(m_0^{\bullet}, \tilde{u})} = \frac{u(m_t^{\bullet}) * \tilde{u}}{u(m_0^{\bullet}) * \tilde{u}} \qquad (50)$$

$$= \frac{u(m_t^{\bullet})}{u(m_0^{\bullet})}$$

The expenditure function may also be rewritten (if the assumption of a first-degree homogeneous utility function is satisfied) as

$$e(\pi, \tilde{u}) = e(\pi, 1) * \tilde{u} \qquad (51)$$

and thus the Konus index may be expressed as

$$K(\pi_t, \pi_0, \tilde{u}) = \frac{e(\pi, 1)}{e(\pi_0, 1)} \qquad (52)$$

Both indexes are now clearly independent of a reference utility level. To review, in the case that the assumption of a homothetic category subutility function is not satisfied, the Divisia index, will no longer form a satisfactory approximation to the aggregator function in continuous time. However, indexes do exist which will be exact for the quantity and user cost indexes and these are known as the Malmquist quantity index and the Konus user cost index respectively. Their only drawback is the reliance on a reference utility level which makes their empirical application considerably more troublesome.

B. Perfect Certainty

The fourth assumption required for the basic approach to monetary aggregation outlined in this literature review is clearly an unrealistic one, and therefore economists have been quite interested in constructing aggregates based upon assets with risky returns, the more common situation in reality.

Fortunately, introducing risk to the given model constructing monetary aggregates in the case of risk-neutral consumers is easy. The user costs used in the discrete-time Divisia index (in the case of risk-neutral consumers) must be redefined as the expected value of nominal current-period perfect certainty user costs. The altered user cost formula is given below.

$$\pi_{it} = E_t\{p_t^* \frac{R_t - r_{it}}{1+R_t}\} \qquad (53)$$

Constructing monetary aggregates while allowing for the possibility of risk averse consumers is considerably more difficult however. In such a case, the modification to the user costs (shown above) does not hold because the component assets user costs become dependent on the representative consumer's marginal utility, which empirically speaking is quite difficult to determine. Generally speaking, for the case of risk aversion, the representative consumer's two-stage intertemporal decision problem as discussed in Section II becomes a stochastic optimal control problem for which dynamic programming methods are required.[62] Several papers (among them Barnett and Liu (1994) and Barnett, Liu and Jensen (1997)) use a modified Divisia quantity index in which the user costs are adjusted to account for risk.

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[62] See Barnett, Fisher and Serletis (1992, p.2112)

> *"The size of the adjustment depends on the agent's degree of risk aversion and on the covariance between the asset's rate of return and the agent's consumption stream.* "[63]

It is noteworthy that in the above mentioned papers, little difference is found empirically between the original Divisia index and the version modified to account for risk. This provides evidence that the need to undertake an adjustment for risk will not be a significant issue in the construction of monetary assets using the Divisia index as in this project.

C. Weak Separability

The key indispensable assumption, as clearly shown by the above sections is the weak separability assumption. It "*is the fundamental existence condition without which aggregates and sectors do not exist.*"[64]The requirement of this particular monetary aggregation approach is quite modest; only one group of assets or goods must satisfy weak separability. Considering this and the fact that nearly all current microeconomic intertemporal decision models are strongly separable, the kind of intertemporal weak separability of monetary assets required by this method of monetary aggregation is not an unrealistic requirement.

D. Final Extension

It is worth mentioning that aggregation over economic agents has not been discussed at all in this chapter. Although the theory for aggregation over goods is

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[63] See Anderson, Jones and Nesmith (1997, p.48)

[64] See Barnett, Fisher and Serletis (1992, p.2113)

independent from the theory of aggregation across economic agents, the theory for aggregation over goods for one economic agent remains relevant for any means of aggregation across economic agents. (Unlike the case of aggregating across goods, a unique solution does not exist for aggregation across agents. Any number of solutions exist for the problem of aggregation over agents conditional upon the model's "*dependence on distribution effects*".)[65] Assuming all agents possess linear and parallel Engel curves, the solution can be obtained by solving the standard microeconomic utility maximization problem by means of a representative consumer. If agents do not have linear and parallel Engel curves (in other words, they are sufficiently heterogeneous), Pareto's general stratification approach must be used, *"which integrates utility functions over the distribution functions of all variables that can produce distribution effects,"*[66] distribution functions for income or wealth or the demographic characteristics of the population.

A compromise approach has been advanced by John Muellbauer that allows for the use of a representative consumer, although the second and first moments of the distribution functions remain necessary. There is also Barnett's (1981, pp.58-68) stochastic convergence method, which does not assume the existence of a representative consumer but which at least requires that fewer assumptions are satisfied than John Muellbauer's method.

The use of a representative consumer is more or less inappropriate depending on the importance of the distribution effects. If the distribution effects are considered to be

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[65] See Barnett, Fisher and Serletis (1992, p.2113)

significant, then one must use the Pareto stratification method. Barnett and Serletis (1990) have actually tested for the statistical significance of such distribution effects (using Divisia second moments). Their results suggest that they actually had little significance in improving the chances that Gorman's condition of parallel and linear Engel curves across agents can be satisfied. Thus, it is possible for the purposes of aggregation across agents that a representative consumer can be used in a microeconomic utility maximization model structure.[67]

VIII. Conclusion

This above chapter has reviewed the most recent developments in monetary aggregation theory, focussing mostly on recent attempts to develop approaches to monetary aggregation that have an improved foundation in microeconomic theory. The microeconomic foundation involved is the maximization of the utility of a representative consumer over a vector of consumption goods and services, monetary assets and leisure. It is generally required that current-period quantities of monetary assets be weakly separable from other assets, goods and services and leisure.

Once the weak separability condition is satisfied, the utility maximization process may occur in two stages The first stage allows the consumer to optimally allocate expenditure between current-period monetary assets as one block and all other assets and goods. For the second stage, the utility function used is defined over current-period monetary assets (and is typically termed the category subutility function). The consumer

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[66] See Barnett, Fisher and Serletis (1992, p.2113)

[67] See Barnett, Fisher and Serletis (1992, p.2114)

chooses the optimal quantities of the individual current-period monetary assets subject to the optimal total expenditure on current-period monetary assets.

The category subutility function is in fact the correct aggregator function for the purposes of monetary aggregation. Typically, the form of the aggregator function is unknown and so parametric estimation becomes necessary. However, statistical index numbers do exist which exactly track certain flexible functional forms which in turn are second or third-order approximations for the unknown aggregator function. Both the quantity index and the user cost index are required for monetary aggregation.

Finally, the four microeconomic assumptions used in this approach to monetary aggregation were discussed and reviewed. (These four assumptions were: (1) the existence of a representative consumer; (2) the blockwise weak separability of current-period monetary assets; (3) the homotheticity of the category subutility function and (4) perfect certainty.)

Chapter 4: Extension to Adjust the Divisia Index in the Case of Risk Aversion

I. Introduction

The case of perfect certainty (in which the Divisia index exactly tracks any aggregator function through the flexible functional form), was thoroughly described in chapter 3. Risk aversion however, has not been discussed in any detail in this paper, yet it is worth showing that there indeed does exist a form of the Divisia index that will exactly track the aggregator function in the case of risk aversion. Further, this extended or generalized Divisia index, given additional assumptions, has a relation to the capital asset pricing model (CAPM).[68] It is the aim of this chapter to show the process by which Barnett's extended Divisia index may be derived and also its connection to the capital asset pricing model.

II.(1) Risk Aversion

In the case of risk aversion, the first-order conditions of the representative consumer's utility maximization problem are Euler equations. This has the resulting implication that the ability of the usual Divisia index to track the aggregator function will be compromised. "*The degree to which the tracking ability degrades is a function of the degree of risk aversion and the amount of risk.*"[69] Nevertheless, the possibility exists that the Euler equations could be estimated by the generalized method of moments and by solving for the estimated exact rational expectations monetary aggregator function. Any

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[68] See Barnett, Liu and Jensen (1997, p.486)

[69] See Barnett, Liu and Jensen (1997, p.486)

attempt at the estimation of the aggregator function however creates a situation where results are dependent on the particular parametric specification of the aggregator function and also, on the choice of regression estimator in determining the parameters of the aggregator function. This is precisely the situation that the introduction of statistical index number theory into the aggregation methodology is designed to avoid. The whole point in using the Divisia index in monetary aggregation is to produce a parameter-free method of aggregation that depends only on the relevant data. Therefore, another method is given here which is used to derive the extended Divisia index (for the case of risk aversion). Indeed it will also be shown that the extended Divisia index will reduce to the usual discrete-time Divisia index in the case of perfect certainty.

II.(2) The CAPM Connection to Index Number Theory

The connection between the capital asset pricing model (CAPM) and the extended Divisia index discussed here is simple; According to the treatment of this given in Barnett, Liu and Jensen [1997], both the Divisia index and the CAPM are special nested cases of a broader theoretical framework. While the capital asset pricing model deals with the trade-off between the expected return and risk of consumption assets, the Divisia index is structured in terms of a trade-off between investment return and liquidity. The broader theoretical framework involves a three-dimensional trade-off between average return, risk and liquidity.[70]

This structure fits particularly well into existing monetary aggregation theory, because

[70] See Barnett, Liu and Jensen (1997, p.487)

> "*money-market assets are characterized by substantially differing degrees of each of the three characteristics: mean rates of return, risk and liquidity especially when the collection of money-market assets includes those subject to prepayment penalties, such as series EE bonds and nonnegotiable certificates of deposit, and those subject to regulated low rates of return, such as currency.*"[71]

The original set of monetary aggregates produced by central banks had fewer components and those components yielded no interest. Since the rate of return was exactly known for each component in this situation, the perfect certainty assumption was easily satisfied. Given zero rates of return for each component monetary asset, it was further the case that the *user costs* of each asset were known and identical. Thus, it was perfectly reasonable to use the simple summation index number to produce monetary aggregates since with zero user costs, all assets could be assigned equal weights of unity because all component assets were perfect (and dollar-for-dollar) substitutes for each other.

However, with nonzero and differing rates of return for all component monetary assets, monetary assets can by no means be viewed as perfect substitutes and therefore the assumptions required of the simple summation aggregation method are no longer satisfied. An additional wrinkle in the fabric of traditional monetary aggregation theory is provided by the fact that the rate of return on a component monetary asset is not a contribution to monetary services. Thus the capitalized value of the interest yield, "*although embedded in the value of the stock of such assets, is not part of the economic*

[71] See Barnett, Liu and Jensen (1997, p.487)

monetary stock. The capitalized value of the monetary service flow net of that interest yield is the economy's monetary stock."[72]

The assumption of perfect certainty in turn is undermined by the fact that there is almost always, some degree of uncertainty about what the rate of return on component assets will emerge as, the rate of return of course being required in the calculation of the user costs (foregone interest of holding any interest-yielding monetary asset). With the introduction of risk and uncertainty into the model, it becomes clear that any index number used for aggregation must be capable of tracking a nonlinear aggregator function.[73]

Much of the recent literature on monetary aggregation maintains that the usual discrete-time Divisia index numbers produced from perfect-certainty first-order conditions may be sufficient to measure the monetary service flows of the current collection of component monetary assets, since these assets mostly have rates of return with low variance and low correlation with the consumption of other goods and services. However, as individuals receive a greater portion of their monetary service flows from common stock and bond funds, (which are characterized by substantial risk), the usual Divisia index, seems less and less likely to be a sufficient index number for the production of accurate, economically valid monetary aggregates. In short, monetary aggregation stands to benefit a great deal by the development of statistical index numbers that can account for the existence of risk and the extent of risk aversion among consumers.

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[72] See Barnett, Liu and Jensen (1997, p.487)

[73] See Barnett, Liu and Jensen (1997, p.487)

III. Consumer Demand for Monetary Assets

To develop the extended Divisia index, it will be necessary to briefly review the representative consumer's utility optimization problem and the demand for monetary assets. Here, the representative consumer's stochastic decision problem is formulated over consumer goods and monetary assets, leaving leisure out of the utility function here for simplicity and convenience.

The problem will be set for an infinite planning horizon broken into discrete time intervals, $t, t+1, \ldots, s, \ldots, \infty$ where t is the current-time period, and may also be used to represent any of the possible time periods in the range. The variables are defined as follows:

> "x_s = *n-dimensional vector of real consumption of goods and services during period* s , p_s = *n-dimensional vector of goods and services prices and of the durable-goods rental prices during period* s, a_s =*k-dimensional vector of real balances of monetary assets during period* s, ρ_s =*k-dimensional of vector of nominal holding-period yields on monetary assets.* A_s =*holdings of the benchmark asset during period* s. R_s =*the one-period holding yield on the benchmark asset during period* s. I_s =*the sum of all other sources of income during period* s, *and* $p_s^* = p_s^*(p_s)$ *the true cost of living index.*"[74]

The consumer's consumption possibility set $S(s)$ for period s (where y is defined to be the consumer's survival set is given below.

[74] See Barnett, Liu and Jensen (1997, p.489)

$$S(s) = \{(a_s, x_s, A_s) \in y : \sum_{i=1}^{n} p_{is} x_{is}$$

$$= \sum_{i=1}^{k} [(1 + p_{i,s-1}) p^{*}_{s-1} a_{i,s-1} - p^{*}_{s} a_{is}] + (1 + R_{s-1}) p^{*}_{s-1} A_{s-1} - p^{*}_{s} A_s + I_s\}$$

Due to the fact that the benchmark asset A_s, provides no services other than its yield R_s, the benchmark asset is not included in the consumer's contemporaneous utility function. The consumer's subjective rate of time preference ξ is assumed to be constant and the single-period utility function $u(a_t, x_t)$ is assumed to be increasing (in both monetary and consumption goods) and quasi-concave (as required by the Generalized Axiom of Revealed Preference).

The consumer's decision problem (set up as an optimal control problem) is to choose the deterministic point (a_t, x_t, A_t) and the stochastic process (a_s, x_s, A_s), where $s = t+1, \ldots, \infty$ to maximize

$$U(a_t, x_t) + E_t[\sum_{s=t+1}^{\infty} \left(\frac{1}{1+\xi}\right)^{s-t} U(a_s, x_s)] \qquad (1)$$

subject to $(a_s, x_s, A_s) \in S(s)$ for s, t and also subject to the transversality condition

$$\lim_{s \to \infty} E_t \left(\frac{1}{1+\xi}\right)^{s-t} A_s = 0. \qquad (2)$$

"*The transversality condition rules out perpetual borrowing at the benchmark rate.*"[75]

In the definition of the monetary aggregate, the model being considered here involves a partition of the monetary asset vector as into two distinct subvectors as shown by $a_s = (m_s, h_s)$. (Here the dimension of the subvector m_s is defined to be k_1 and the dimension of h_s is k_2. This means that the monetary asset interest rate vector ρ_s must in turn be partitioned such that $\rho_s = (r_s, i_s)$, r_s corresponding to the set of monetary assets contained in m_s, i_s corresponding to the set of assets contained in h_s. As in the traditional model given in chapter 3, u is here assumed to be blockwise weakly separable in some subset of monetary assets, m_s. Unlike the model shown last chapter however, the utility function is also weakly separable in the nonmonetary goods and services vector x_s and strongly separable in the monetary subset vector h_s. This breakdown of the utility function takes the Gorman form as shown below.

$$u(m_s, h_s, x_s) = F[M(m_s), X(x_s)] + H(h_s) \qquad (2)$$

The aggregate consumption of nonmonetary consumption goods and services is defined by $c_s = X(x_s)$, while the exact monetary aggregate is given by

$$M_s = M(m_s) \qquad (3)$$

Let the implied utility function $V(m_s, c_s)$ be defined by

$$V(m_s, c_s) = F[M(m_s), c_s].$$

M_s here is the monetary aggregator function and the category subutility function as defined in chapter 3. In the case of perfect certainty, if the monetary aggregator function is linearly homogeneous and the weak separability condition is satisfied, two-stage budgeting may be employed in the consumer's maximization problem. Further, the two-stage budgeting procedure makes it clear that the consumer behaves as if the exact monetary aggregate were itself an elementary good. However, given weak separability

[75] See Barnett, Liu and Jensen (1997, p.490)

and a linearly homogeneous aggregator function, two-stage budgeting theory is not applicable in any case in which risk is introduced. Nonetheless, even with risk, $M(m_s)$ can still be treated as a quantity aggregate.

Two important Euler equations (from the consumer's maximization problem in the case of risk), are given below. The first Euler equation shown is the general form of the equations for each of the monetary assets, m_{is}.

$$E_t[\frac{\partial V}{\partial m_{is}} - \rho \frac{p_t^*(R_t - r_{it})}{p_{s+1}^*} * \frac{\partial V}{\partial c_{s+1}}] = 0 \qquad (4a)$$

for $i = 1,...k$ and for all s with $\rho = 1/(1+\zeta)$ as before where p_s^* is the exact price aggregate that corresponds to the consumer good quantity aggregate, c_s. The Euler equation given below is for the nonmonetary goods aggregate c_s,

$$E_t\left[\frac{\partial V}{\partial c_s} - \rho \frac{p_s^*(1+R_s)}{p_{s+1}^*} * \frac{\partial V}{\partial c_{s+1}}\right] = 0 \qquad (4b)$$

The derivations of the two Euler equations given above are arrived at using Bellman's method of dynamic programming.

To briefly review the results of the perfect-certainty case, the formula for the real user cost of the individual monetary assets for the current period t is

$$\pi_{it} = \frac{R_t - r_{it}}{1+R_t} \qquad (5)$$

The exact value of the quantity monetary aggregate can be exactly determined using the Divisia index, shown in continuous time notation.

$$d\log M_t = \sum_{i=1}^{k_1} s_{it} d\log m_{it} \qquad (6)$$

The expenditure shares, s_{it}, given above are defined as

$$s_{it} = \pi_{it} m_{it} \Big/ \sum_{j=1}^{k_1} \pi_{jt} m_{jt} .$$

The Divisia index is capable of tracking the monetary aggregator function, M, even if its functional form is unknown, up to a third-order remainder term in the changes. "*That remainder term usually is less than the roundoff error in the component data and typically is negligible for data that are available with at least annual frequency.*"[76] However, as mentioned before, the Divisia index in the form above is incapable of exactly tracking the unknown aggregator function.

IV. The Extended Divisia Index

To begin the process of deriving the new extended Divisia index, it is necessary to use the Euler equations given in (4a) and (4b) to find a new form of real user cost that is appropriate for an accounting of risk. The new definition for the contemporaneous risk-adjusted real user cost price is expressed as the marginal rate of substitution between a given monetary asset and consumer goods, as shown below.

Definition 1. $$\Pi_{it} = \frac{\partial V}{\partial m_{it}} \Big/ \frac{\partial V}{\partial c_t}$$

No expectations operator is needed in the above definition because the marginal utilities are known with certainty in the current period t. Where interest rates are used however, the presence of uncertainty prevents exact interest rates from being known

[76] See Barnett, Liu and Jensen (1997, p.491)

contemporaneously and hence these expressions (requiring the use of interest rates) become stochastic. In the notation used here, nominal rates of return, r_{it} and R_t are expressed as real total rates of return $1+r_{it}^*$ and $1+R_t^*$ by conversion through the following formulas.

$$1+r_{it}^* = \frac{p_t^*(1+r_{it})}{p_{t+1}^*},\; 1+R_t^* = \frac{p_t^*(1+R_t)}{p_{t+1}^*} \qquad (7)$$

r_{it}^* and R_t^* are known by themselves as the real rates of excess return.

This alteration in notation allows the Euler equations given in (4a) and (4b) to be simplified to the following two equations.

$$\frac{\partial V}{\partial m_{it}} - \rho E_t[(R_t^* - r_{it}^*)\frac{\partial V}{\partial c_{t+1}}] = 0 \qquad (8)$$

and

$$\frac{\partial V}{\partial c_t} - \rho E_t[(1+R_t^*)\frac{\partial V}{\partial c_{t+1}}] = 0 \qquad (9)$$

Giving a more specific form to the definition of the contemporaneous risk-adjusted real user cost (from Definition 1), it can be shown that the risk adjusted user cost of the services of monetary asset i under risk is given by

$$\Pi_{it} = \pi_{it} + \psi_{it} \qquad (10)$$

where $\pi_{it} = \dfrac{E_t R_t - E_t r_{it}}{1+E_t R_t}$ $\qquad (11)$

and $\psi_{it} = \rho(1-\pi_{it}) * \dfrac{\mathrm{cov}\left(R_t^*, \dfrac{\partial V}{\partial c_{t+1}}\right)}{\dfrac{\partial V}{\partial c_t}} - \rho \dfrac{\mathrm{cov}\left(r_{it}^*, \dfrac{\partial V}{\partial c_{t+1}}\right)}{\dfrac{\partial V}{\partial c_t}}$ $\qquad (12)$

To show that (10) holds, recall that it is possible to rewrite equation (8) as

$$\frac{\partial V}{\partial m_{it}} = \rho E_t\left[\left(R_t^{*} - r_{it}^{*}\right)\frac{\partial V}{\partial c_{t+1}}\right] \tag{13}$$

Assuming that the marginal utility and interest rates used in equation (13) above are uncorrelated, the rules of expectations operators can be used to rewrite the expectation of the product as the product of the expectations. However, under risky conditions, it is unlikely that the marginal utility of nonmonetary consumption and interest rates will be uncorrelated. Equation (13) must therefore be expanded into the following inelegant expression shown below.

$$\frac{\partial V}{\partial m_{it}} = \rho E_t\left[\frac{\partial V}{\partial c_{t+1}}\right]\left(E_t R_t^{*} - E_t r_{it}^{*}\right) + \rho \operatorname{cov}\left(R_t^{*}, \frac{\partial V}{\partial c_{t+1}}\right) - \rho \operatorname{cov}\left(r_{it}^{*}, \frac{\partial V}{\partial c_{t+1}}\right) \tag{14}$$

With risk neutrality in the above expression, the covariances would become zero. However, since we are aiming to account for the consumer's degree of risk aversion, the covariances must remain in the expression. The Euler equation given in (9) can be used to get the following expression (again assuming the absence of risk neutrality).

$$\frac{\partial V}{\partial c_t} = \rho E_t\left[\frac{\partial V}{\partial c_{t+1}}\right] + \rho E_t\left[R_t^{*}\right]E_t\left[\frac{\partial V}{\partial c_{t+1}}\right] + \rho \operatorname{cov}\left(R_t^{*}, \frac{\partial V}{\partial c_{t+1}}\right) \tag{15}$$

With some more manipulation of equations (14) and (15), $\rho E_t\left[\partial V / \partial c_{t+1}\right]$ can be eliminated, and $\frac{\partial V}{\partial m_{it}}$ is related to $\frac{\partial V}{\partial c_t}$ as follows.

$$\frac{\partial V}{\partial m_{it}} = \left(\pi_{it} + \psi_{it}\right)\frac{\partial V}{\partial c_t} \tag{16}$$

where

$$\pi_{it} = \frac{E_t R_t^{*} - E_t r_{it}^{*}}{1 + E_t R_t^{*}} \tag{17}$$

and

$$\psi_{it} = \rho(1-\pi_{it})\frac{\text{cov}\left(R_t^{\bullet}, \frac{\partial V}{\partial c_{t+1}}\right)}{\frac{\partial V}{\partial c_t}} - \rho\frac{\text{cov}\left(r_{it}^{\bullet}, \frac{\partial V}{\partial c_{t+1}}\right)}{\frac{\partial V}{\partial c_t}} \qquad (18)$$

Converting the real rate of return in equations (18) and (17), (using the conversion formula implied by equation (7)), equation (18) becomes identical to equation (12) and equation (17) becomes equivalent to equation (11). Also it is clear that equation (16) becomes $\dfrac{\partial V}{\partial m_{it}} \Big/ \dfrac{\partial V}{\partial c_t} = \pi_{it} + \psi_{it} = \Pi_{it}$

which in turn means that equation (10) logically follows from Definition 1.

It is easy to show that the risk-adjusted user cost formula becomes equal to equation (11), (something very close to the original user cost formula in the absence of any risk), assuming risk neutrality on the part of the consumer. If the consumer is not to any degree risk averse but is in fact risk neutral, the covariances in equation (18) become equal to zero, and hence the risk-adjusted user cost Π_{it} reduces to π_{it} as suggested above. [77] The only difference between risk-adjusted risk neutral user costs and user costs as defined in the perfect certainty case is that in the presence of risk, known risk interest rates are replaced by expected interest rates.

"*However, even under risk aversion the utility function is strictly concave in consumption, so that marginal utility is inversely related to consumption.*"[78] In the case of

[77] See Barnett, Liu and Jensen (1997, p.494)

[78] See Barnett, Liu and Jensen (1997, p.494)

a slightly risky investment, it is possible that the interest rate on such an asset might actually reduce the risk of the investment. If the rate of return is positively correlated with marginal utility, the total risk facing a consumer is reduced by an investment, assuming an inverse relationship between consumption and marginal utility.

In the case of a monetary asset with low risk and low volatility, the asset would contribute little to the riskiness of the household's consumption stream. On the other hand, a very risky monetary asset such as common stock will probably add to the mean risk of the household's consumption stream. In such a situation, the rate of return on the asset is positively correlated with consumption, and in turn is negatively correlated with the marginal utility of consumption, which is one of the central results of the Consumption Capital Asset Pricing (CCAPM). [79]

In the risk-adjusted user cost formula, equation (12) can be considered the adjustment for risk aversion. Assume that the expression is normalized with respect to $\dfrac{\partial V}{\partial c_t}$ so that the denominator of equation (12) can be ignored.

$$\psi_{it} = \rho(1-\pi_{it})\frac{\operatorname{cov}\left(R_t^{*}, \dfrac{\partial V}{\partial c_{t+1}}\right)}{\dfrac{\partial V}{\partial c_t}} - \rho\frac{\operatorname{cov}\left(r_{it}^{*}, \dfrac{\partial V}{\partial c_{t+1}}\right)}{\dfrac{\partial V}{\partial c_t}} \qquad (12)$$

Supposing that the own rate of return on monetary asset i, r_{it}^{*}, is positively correlated with the marginal utility of consumption the monetary asset should decrease the amount of risk facing the consumer. This should increase the covariance in the second term of equation (12) shown above. Therefore holding this monetary asset should reduce the risk-

[79] See Barnett, Liu and Jensen (1997, p.494)

adjusted user cost price that the consumer would have to pay to hold that asset as the positive covariance increases.[80] The opposite result holds if the covariance between the interest rate and the marginal utility of consumption is negative. An asset in that case will be increasing the risk facing the consumer and hence the risk-adjusted user costs of the asset will increase.

In the case that there is a positive covariance between the benchmark rate and the marginal utility of consumption, the benchmark asset decreases consumption risk. With the decrease in consumption risk, the opportunity cost of foregoing the benchmark asset return by holding the monetary asset is increased. Thus, a positive covariance of the type described above will increase the risk-adjusted user cost, which is indeed the effect of the first term of equation (12).

> *"Conversely, if that covariance is negative, so that holding the benchmark asset increases the consumer's risk, then foregoing the benchmark asset in favor of monetary asset i decreases risk, and hence results in a subtraction from the risk-adjusted user cost,* Π_{it} *of holding asset* i*."*[81]

V. Generalized Divisia Index Under Risk Aversion

We have shown above how the formula for the user costs of various individual monetary assets must be adapted to account both for the presence of risk and the presence of risk aversion on the part of the consumer. In this section, the necessary modifications to the usual Divisia index to account for risk aversion will also be shown. To do this, the

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[80] See Barnett, Liu and Jensen (1997, p.495)

[81] See Barnett, Liu and Jensen (1997, p.495)

share equations used in the definition of the Divisia index must be appropriately modified.

Previously, the share equations s_{it} (used in the Divisia index formula $d\log M_t = \sum_{i=1}^{k_1} s_{it} d\log m_{it}$) were defined as $s_{it} = \pi_{it} m_{it} \Big/ \sum_{j=1}^{k_1} \pi_{jt} m_{jt}$. Now, to account for risk and risk aversion, the unadjusted user costs π_{it} must be replaced by the risk-adjusted user costs, Π_{it} as given in Definition 1. This will produce the adjusted share formula,

$$S_{it} = \Pi_{it} m_{it} \Big/ \sum_{j=1}^{k_1} \Pi_{jt} m_{jt}.$$

If the weak separability assumption is satisfied (as reflected in $V(m_t, c_t) = F[M(m_t), c_t]$) and if it is still true that the monetary aggregator function M is linearly homogeneous, then the generalized Divisia index formula we have been seeking is shown below.

$$d\log M_t = \sum_{i=1}^{k_1} S_{it} d\log m_{it} \qquad (19)$$

In the next steps, this result will now be proven in a more rigorous fashion. Under the weak separability assumption, (given $V(m_t, c_t) = F[M(m_t), c_t]$), it must be the case that

$$\frac{\partial V}{\partial m_{it}} = \frac{\partial F}{\partial M_t} * \frac{\partial M_t}{\partial m_{it}} \qquad (20)$$

Using equation (16) and (20) given above, the following expression holds.

$$\frac{\partial M}{\partial m_{it}} = (\pi_{it} + \psi_{it}) \left(\frac{\partial V}{\partial c_t} \Big/ \frac{\partial F}{\partial M_t} \right) \qquad (21)$$

The total differential of $M_t = M(m_t)$ is

$$dM_t = \sum_{i=1}^{k_1} \frac{\partial M}{\partial m_{it}} dm_{it} \qquad (22)$$

It is clear that (21) can now be substituted into (22) to produce

$$dM_t = \left(\frac{\partial V}{\partial c_t} \bigg/ \frac{\partial F}{\partial M_t} \right) \sum_{i=1}^{k_1} (\pi_{it} + \psi_{it}) dm_{it} \quad (23)$$

If the function M is indeed linearly homogeneous, then by Euler's theorem for homogeneous functions, equation (22) may be rewritten as

$$M_t = \sum_{i=1}^{k_1} \frac{\partial M}{\partial m_{it}} m_{it} \qquad (24)$$

This now means that equation (23) may be modified into

$$M_t = \left(\frac{\partial V}{\partial c_t} \bigg/ \frac{\partial F}{\partial M_t} \right) \sum_{i=1}^{k_1} (\pi_{it} + \psi_{it}) m_{it} \quad (25)$$

The modified or generalized Divisia index may easily be obtained by dividing (23) by (25).

A thoroughly satisfactory result is obtained through this process as the exact tracking of the Divisia index in its monetary application is not compromised so long as the adjusted user costs, $\pi_{it} + \psi_{it}$ are used in the share equations of the index. The index shown above is applicable in both the cases of perfect certainty and risk aversion; that is why it is referred to as the generalized Divisia index.) To verify that this is indeed true, remember that it has already been shown that the adjusted user costs reduce to the usual user costs in the instance of perfect certainty and hence the share equations are defined as in the usual case (and the generalized Divisia index (19) consequently reduces to the usual Divisia index, (6)).

For the case of risk neutrality, $\psi_{it} = 0$ and so the user cost used in the Divisia index will be defined as shown in equation (17). Thus it is clear that the case of perfect certainty and risk neutrality are strictly nested special cases of the generalized Divisia index. This situation is further described by Barnett, Liu and Jensen (1997) as a three-dimensional decision in terms of asset characteristics.

> *"The monetary assets having nonzero own rates of return produce investment returns, contribute to risk, and provide liquidity services. Our objective is to track the nested utility function $M(m_t)$, which measures only liquidity and is the true economic monetary aggregate. To do so, we must remove the other two motives: investment yield and risk aversion.* "[82]

The ordinary Divisia monetary aggregate is capable of removing the investment motive and can track the liquidity service flow above only in the absence of risk. The generalized Divisia index on the other hand removes both the investment motive and the aversion-to-risk motive to extract the liquidity service flow, with the consumer involved facing a three-way trade-off among mean investment return, risk aversion, and liquidity service consumption.[83]

VI. Conclusion

In this chapter, the issue of risk and adjusting for risk within the Divisia index was extensively dealt with. Such an adjustment is of obvious value in the context of monetary aggregation projects such as the one presented in this paper. So far, little research has been conducted in the area of constructing statistical index numbers that can account and

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[82] See Barnett, Liu and Jensen (1997, p.496)

compensate for the degree of risk aversion of the consumer. No monetary aggregates have yet been constructed which aimed to use such index numbers. The aggregates constructed in this project will not compensate for risk aversion although the theoretical approach suggested in this chapter suggests a useful methodology by which it can be done.

Basically the pioneering work of Barnett, Liu and Jensen (1997) suggests that the normal Divisia index in the perfect certainty case is a nested case that can be derived from a generalized Divisia index described in their paper and shown here. The maximization problem of a representative consumer is set up as an optimal control problem, in which the standard assumption of the weak separability of current period monetary assets is satisfied. Using Bellman's method of dynamic programming, two Euler equations are derived. Using these two Euler equations, a user cost formula is developed which accounts both for the presence of risk and the degree of risk aversion. If this generalized user cost formula is used in the share equations of the Divisia index, the statistical index number is capable of fully accounting for risk.

[83] See Barnett, Liu and Jensen (1997, p.497)

Chapter 5: Concepts, Data and Methods in the Construction of the New Monetary Aggregates

I. Introduction

The purpose of this chapter is to provide a practical, detailed discussion of the methodology used in the construction of new monetary aggregates (using Canadian data) and of the results of such efforts. These new monetary aggregates are created in accordance with the latest aggregation and statistical index number theory (discussed previously in Chapter 2).

It will be important to review the distinction between real and nominal monetary indexes. In particular, it must be shown that the nominal monetary quantity aggregate generates not a nominal dual user cost index but rather a real dual user cost index. Another issue of importance is the difference between nominal monetary asset and real asset stock data. While the aggregation theory as developed in chapter 3 used real asset stock data in the aggregation approach, the actual monetary asset stock data are collected in nominal terms. [84] Ultimately, as shown in this chapter, it is just as easy to use nominal data as real data in the aggregation of monetary assets.

With regard to the data itself, the own rate of return data used in the construction of the indexes is of particular interest. This discussion fits directly into the means by which the calculation of the user costs of the monetary assets included in the sample is

[84] See Anderson, Jones and Nesmith (1997, p.53)

achieved. Also relevant is the concept and definition of a benchmark asset and the means by which the benchmark rate of return is constructed.

Finally there are several methodological difficulties encountered in the construction of the various monetary indexes which must be carefully considered. A first and obvious difficulty encountered is the introduction of new monetary assets, and the resulting changes in the definition of what does and does not constitute a monetary asset. Lastly, the use of seasonally adjusted data must be discussed and justified.

II. The Construction of the New Monetary Aggregates

The theory behind the construction of the Divisia, Fisher and CE monetary aggregates has already been fully discussed in the theoretical framework of this project. For a brief review of some basic concepts, the monetary quantity aggregate is a measure of the flow of monetary services received by the holders of monetary assets.[85] Nonparametric estimation of the correct unknown aggregator function (produced from the standard representative consumer's utility maximization framework), is possible using superlative statistical index numbers. Specifically, these statistical index numbers can provide second-order approximations to monetary aggregates in discrete time. There are many superlative index numbers, yet the discrete-time Divisia index (or the Tornqvist-Theil index number) is the only one capable of retaining its second-order tracking capabilities when some common aggregation theoretic assumptions are violated. [86]

The new monetary quantity aggregates, M_t^{real} are constructed by the following formula.

[85] See Anderson, Jones and Nesmith (1997, p.55)

[86] See Caves, Christensen and Diewert (1982) or Anderson, Jones and Nesmith (1997)

$$M_t^{real} = M_{t-1}^{real} \prod_{i=1}^{n} \left(\frac{m_{it}^{real}}{m_{i,t-1}^{real}} \right)^{\overline{w_{it}}}$$

where $w_{it} = \left(\pi_{it}^{nom} m_{it}^{real} / y_t\right)$ and $w_{i,t-1} = \left(\pi_{i,t-1}^{nom} m_{i,t-1}^{real} / y_{t-1}\right)$ are the expenditure shares of monetary asset i in periods t and $t-1$, while the average expenditure share used in the formula is $\overline{w_{it}} = \frac{1}{2}\left(w_{it} + w_{i,t-1}\right)$.

The price of a unit of monetary services can be measured with an index that is dual to M_t^{real}. If the product of the price index and the quantity index is equal to the total expenditure on the component assets included in the indexes, then the price index may be said to be dual to the quantity index. This property is known as factor reversal.

> *"Dual to MSI_t^{real}, is the nominal dual user cost index, Π_t^{nom}, which is defined using Fisher's (1992) weak factor criterion by the formula*
>
> $$\Pi_t^{nom} = \Pi_{t-1}^{nom} \left(\frac{y_t / y_{t-1}}{MSI_t^{real} / MSI_{t-1}^{real}} \right)$$ *[where the monetary services index, MSI, here is equivalent to the Divisia monetary aggregate used above, M]"* [87]

In this case, the real monetary quantity index, M_t^{real} and its nominal dual user cost index Π_t^{nom} are constructed as chained superlative indexes. Indexes are said to be chained if the prices and quantities used in the index number formula are the prices and quantities of adjacent periods.[88]

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[87] See Anderson, Jones and Nesmith (1997, p.56)

[88] See Anderson, Jones and Nesmith (1997, p.41)

The simple sum, currency equivalent, and Fisher Ideal indexes were also calculated from the component monetary asset and interest rate data available, mostly for the purposes of comparison to the discrete-time Divisia index. To review, the simple sum index is generated by the following formula

$$SS_t = \sum_{i=1}^{n} p_t^* m_{it}^*$$

where n is the number of monetary assets. The simple sum index measures the flow of monetary services only if the representative consumer's indifference curves are parallel lines for monetary assets. In such a case, the agent regards all monetary assets as perfect substitutes. If monetary assets have different user costs however, the economic agent would arrive at a corner solution - a situation in which only one monetary asset is held by the consumer, an outcome that is not substantiated empirically.

As such, in practical use by the Bank of Canada in the construction of M1, M2, M3 and M2+, each monetary asset is assigned an equal unitary weight, thus $p_t^* = 1$ for each m_{it}^* in the formula above. Therefore the simple sum aggregate is created through the simple addition of each of its components.

Simple sum indexes were created for M1, M2 and M3 here, for the purpose of providing a comparison with the Divisia aggregates generated (and not so much for comparing our simple sum aggregates to those of the Bank of Canada). As shown in the theoretical review of chapter 3, the simple sum index can be decomposed into two-terms; one measures the discounted present value of all current and future interest received on monetary assets, and the other is the currency equivalent (CE) index, the formula for which is shown below.

$$CE_t = p_t^* \sum_{i=1}^{n} \frac{R_t - r_{it}}{R_t} m_{it}^*$$

The above formula assumes the use of real asset stock data. Since nominal monetary asset stock data was instead used, the formula used for the calculation of the CE index numbers becomes simply

$$CE_t = \sum_{i=1}^{n} \left(\frac{R_t - r_{it}}{R_t} \right) m_{it}^{nom}.$$

The intuitive basis for the CE index is that it provides a measure of the stock of monetary wealth. The CE index is only capable of measuring the flow of monetary services in the case, where, in addition to the assumptions required to satisfy the Divisia index, the category subutility function (or aggregator function)[89] u(.) is quasi-linear in a monetary asset whose own rate is always zero.[90] Since the CE index requires additional assumptions to provide the same measurement as the Divisia index, it is viewed as statistically inferior to the Divisia index, but is provided here for the purposes of contrast and comparison nonetheless.

Finally, a full set of Fisher Ideal index numbers for M1, M2 and M3 were derived. (The full formula for the Fisher Ideal index is given in the section of this chapter devoted to the introduction of new monetary assets). The Divisia index is superlative in a stronger sense than the Fisher Ideal index and is therefore regarded as the superior index number. However, both indexes are in theory supposed to closely approximate each other and thus calculating the Fisher Ideal indexes for M1, M2, M3 is a useful means of verifying that

[89] This is described in Chapter 3.

[90] See Anderson, Jones and Nesmith (1997, p.44)

the Divisia index numbers provided were indeed accurate. This was the principal reason that the Fisher index numbers were constructed.

While the M1, M2 and M3 aggregates were calculated using all four index numbers (simple sum, currency equivalent, Fisher Ideal, and Divisia), M2+ was not derived. The reason was that most of the desirable asset categories were only available at quarterly frequencies. Further, it was considerably difficult to obtain suitable and accurate interest rate data for the various M2+ asset categories, in particular the credit union and money market mutual fund assets.

III. Extension of Theory to Allow for the Use of Nominal Monetary Data

This section will briefly cover the distinction between real and nominal stocks of monetary assets. The reason this must be discussed is that the Bank of Canada collects its monetary asset stock data in nominal terms while the aggregation theory as described and developed here has dealt with the conditions required for the aggregation of real stocks of monetary assets. The aggregation of nominal rather than real stocks of monetary assets should require some elaboration of the existing aggregation theory.

Of fundamental importance is the fact that the total expenditure on monetary assets can be defined in two ways. First, total expenditure may be expressed as the sum of the products of the real asset stocks and their nominal user costs, or alternatively, as the sum of the products of the nominal monetary asset stocks and their corresponding real user costs. The fact that both means of defining total expenditure are ultimately equivalent is easily shown below.

$$y_t = \sum_{i=1}^{n} \pi_{it}^{nom} m_{it}^{real}$$

$$= \sum_{i=1}^{n} p_t^* \pi_{it}^{real} \left(\frac{m_{it}^{nom}}{p_t^*} \right)$$

$$= \sum_{i=1}^{n} \pi_{it}^{real} m_{it}^{nom}$$

The implication of this is that the total expenditure on monetary asset stocks can be calculated without needing the price index p_t^* and further that real monetary assets stocks are not required. Instead nominal monetary asset stocks (such as those provided by the Bank of Canada) are enough to obtain total expenditure, so long as real user costs can be accurately observed. It is now clear that the expenditure shares can be expressed in one of two ways; as either expenditure on real monetary asset stocks based on nominal user costs (as shown below)

$$w_{it} = \left(\pi_{it}^{nom} m_{it}^{real}\right) / y_t$$

$$= \left(R_t - r_{it}\right) m_{it}^{real} \Big/ \sum_{j=1}^{n} \left(R_t - r_{jt}\right) m_{jt}^{real}$$

or as total expenditure on nominal monetary asset stocks given real user costs, as illustrated below.

$$w_{it} = \left(\pi_{it}^{real} m_{it}^{nom}\right) / y_t$$

$$= \left(R_t - r_{it}\right) m_{it}^{nom} \Big/ \sum_{j=1}^{n} \left(R_t - r_{jt}\right) m_{jt}^{nom}$$ [91]

It is clear given all this, that a monetary asset quantity index may be constructed using only the available nominal asset stock data, and this index in turn can later be

[91] See Anderson, Jones and Nesmith (1997, p.56)

deflated using the price index. In practice, the initial discrete-time Divisia monetary quantity index is defined by

$$M_t^{nom} = M_{t-1}^{nom} \prod_{i=1}^{n} \left(\frac{m_{it}^{nom}}{m_{i,t-1}^{nom}} \right)^{\overline{w_{it}}}$$

where $\overline{w_{it}} = \frac{1}{2}\left(w_{it} + w_{i,t-1}\right)$. The individual expenditure shares are the same regardless of whether nominal or real asset stock data are used, and thus the only modification made here to the discrete-time Divisia index as shown previously, is the use of nominal asset stocks in place of real monetary asset stocks in the index formula. Of course, the nominal dual user cost can no longer be used in the total expenditure formula. Instead the real dual user cost index, Π_t^{real} (as defined below) becomes especially useful

$$\Pi_t^{real} = \Pi_{t-1}^{real} \left(\frac{y_t / y_{t-1}}{M_t^{nom} / M_{t-1}^{nom}} \right)$$

(As a final reminder, the real dual user cost index is of course dual only to the nominal monetary asset quantity index, M_t^{nom}).

To show how the nominal quantity index may be deflated, the log change operator (defined as $\Delta \ln(z_t) = \ln(z_t) - \ln(z_{it-1})$) is necessary. The following identity shows how the nominal monetary quantity index is related to the real monetary quantity index.

$$\Delta \ln\left(M_t^{nom} / p_t^{*}\right) = \Delta \ln\left(M_t^{real}\right)$$

Basically, the log change of the real monetary quantity index is only equivalent to the log change of the nominal monetary quantity index deflated by the price index. In other words, just deflating the nominal quantity index by the price index is not sufficient to produce the real monetary quantity index.

Similarly, the real and nominal dual user cost indexes are related according to a similar identity.

$$\Delta \ln\left(\Pi_t^{nom} / p_t^{*}\right) = \Delta \ln\left(\Pi_t^{real}\right)$$

Using the expressions shown above, it is easily possible to obtain the real monetary quantity index by deflating the first-obtained nominal monetary quantity index. The actual deflating however, can be left to the user since the appropriate price index (deflator) used depends on the context and models in which these new monetary aggregates are being used.

> *"In consumer demand models, the appropriate price index is the measure of the true cost of living. In firm factor demand models, the appropriate price index is an index of factor input prices."*[92]

There are a wide array of published indexes which could be used as a deflator, among them the Canadian Consumer Price index, producer price indexes and the GDP deflator. Or for some purposes, the monetary quantity indexes could be deflated using a measure of the real wage rate.

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[92] See Anderson, Jones and Nesmith (1997, pp.58-59)

IV. The Implicit Rate of Return on Demand Deposits

While it is traditionally assumed that the own rate of return on demand deposits (current accounts and personal chequing accounts) is zero, it is possible to construct a user cost for demand deposits. This is done by attempting to calculate the *implicit rate of return* on demand deposits. While typically explicit interest rates were not provided for demand deposits, there is still a possibility that financial institutions have paid implicit interest in the form of free or reduced-cost bank services, (or possibility of easier access to credit). Using an approach pioneered by Klein (1974) and later Startz (1979), it is possible to find a simple formula which allows the implicit rate of return on demand deposits to be approximated knowing only the rate of return on an alternative asset, and the maximum required reserve ratio for demand deposits. The formula is given below.

$$r_D = (1-c)r_A$$

where r_D is the implicit interest rate on demand deposits, r_A is the interest rate on an alternative asset, and where c is an estimate of the maximum required reserve ratio on demand deposits.

For the calculation of the implicit interest rate on demand deposits (in this case personal chequing accounts and current accounts included in M1), the interest rate on an alternative asset was given by the rate on 3-5 year Government of Canada marketable bonds (CANSIM series B14010). The maximum required reserve ratio was constructed from both the primary reserve ratio on demand deposits and the secondary reserve ratio. The primary reserve ratio on demand deposits was dictated directly by the Bank of Canada from 1974 until June 1992, when the requirement was eliminated. In contrast to the primary reserve requirements, the secondary reserve ratio was maintained on total statutory deposits rather than on specific types of deposits such as demand, notice and foreign currency deposits. To calculate the secondary reserve ratio on demand deposits, it was necessary to multiply the secondary reserve ratio by the proportion of total statutory deposits consisting of demand deposits.

Although the required reserve ratios were ended in 1992, the dollar amount of requirements was reduced from the average level of the 12 months preceding June 1992 by 3 percent every 6 months until June 1994, when the remaining requirement was entirely eliminated. The required ratios for the phase out period were calculated by finding the dollar amount of required reserves for each month of the period from June 1992 to June 1994, and dividing the dollar amount of requirements by the size of demand deposits.

V. Yield Curve Adjustment

Interest rates on monetary assets that have different maturities may have different term or liquidity premiums, and these term premiums must be removed from each own rate to make them comparable. This process is described as the *yield curve adjustment* of the own rates on monetary assets, and it is accomplished by using the yield curve for Treasury securities. The Treasury yield curve provides a relatively pure estimate of the term premium because these securities have no default risk.

The process is accomplished as follows. Let r_n be the own rate for a monetary asset with a maturity of n months, let r_n^T be the own rate on Treasury securities that mature in n months, and let r_3^T be the secondary-market rate on a Treasury bill with a 3-month maturity. The interest rate r_n is yield curve adjusted by subtracting the estimated liquidity premium $\left(r_n^T - r_3^T\right)$ from the interest rate such that the yield curve adjusted own rate, r_n^{YCA}, is equal to $r_n - \left(r_n^T - r_3^T\right)$. This process is only achievable if all interest rates involved have already been converted to the same basis.

The implicit rates on demand deposits, the own rates on chequable and non-chequable personal savings deposits, chequable and non-chequable non-personal notice deposits and foreign currency deposits were not yield curve adjusted because these monetary assets do not have given maturities. Of those monetary assets which have fixed maturities, (personal fixed term deposits and non-personal term deposits), only personal fixed term deposits need be yield curve adjusted. The suggested own rate used for non-personal term deposits, the prime rate, is very closely tied to the interest rate on 3-month T-Bills. The size of the term premium in this case is so small as to be insignificant. However for personal fixed term deposits, the 5-year term deposit rate is adjusted because the term premium in this case is not insignificant.

VI. A Set of Methodological Difficulties

There are several relevant methodological problems that must be faced and dealt with in the process of obtaining new monetary aggregates with a sounder basis in statistical index number theory. These are:

> *"(1) the introduction of new monetary assets, (2) changes in the definition of the underlying monetary asset stock data, (3) the calculation of monetary service indexes and related indexes at different frequencies and (4) seasonal adjustment of the indexes."*[93]

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[93] See Anderson, Jones and Nesmith (1997, p.77)

Issue (2) was not dealt with because, for the project, the accounts used to construct the monetary aggregates were consistently defined across the sample period. Also issue (3) was not a concern because the data were given in monthly terms and we wanted to construct monthly aggregates, and therefore there was no need to convert these aggregates into quarterly frequencies. Issues (1) and (4) will be discussed in the following two sections.

VI.(1) The Problem of the Introduction of New Monetary Assets

Over the past thirty years (the approximate time frame of this study), there have been a great many financial innovations in the means of payment and the forms in which wealth may be stored. As a result of these innovations, many new monetary assets have come into existence, and the indexes used in the aggregation process must be modified to include them.[94]

It is a well-known fact that the discrete-time Divisia index (suggested as the best overall method of aggregation in Chapter 3) is not well defined when new monetary assets enter the index. It is suggested by Anderson, Jones and Nesmith (1997) and by Farr and Johnson (1985) that the Fisher Ideal index is more suitable for use under these conditions. The reason for this is that it remains well-defined over all periods in which new assets are introduced. (However, it remains true, as suggested by Coves and Christensen and Diewert (1982), that the discrete-time Divisia index is superior to the Fisher Ideal index in all periods for which data is available, because the Divisia index is superlative in a broader sense.) In any case, the real Fisher Ideal user cost index is given by

[94] See Anderson, Jones and Nesmith (1997, p.77)

$$P_t^F = P_{t-1}^F \sqrt{\left(\frac{\sum_{j=1}^{n} \pi_{jt}^{real} m_{jt}^{nom}}{\sum_{j=1}^{n} \pi_{j,t-1}^{real} m_{jt}^{nom}} * \frac{\sum_{j=1}^{n} \pi_{jt}^{real} m_{j,t-1}^{nom}}{\sum_{j=1}^{n} \pi_{j,t-1}^{real} m_{j,t-1}^{nom}} \right)}$$

and the corresponding quantity index may be constructed using Fisher's factor reversal formula.

To use the Fisher Ideal index in periods when new monetary assets are introduced, an estimate can be obtained for the new asset's user cost during the period prior to its introduction.[95] One method that could be used in this situation is to define the user cost in this period as the reservation user cost, that is to say, the user cost that would be sufficient to ensure that the quantity demanded of the new monetary asset would be zero in the period prior, if the asset had in fact existed. The problem with this method is that it requires that the actual aggregator function be empirically estimated. This undermines the advantage provided by statistical index numbers, which are used in the first place to avoid the need for parametric estimation.

Anderson, Jones and Nesmith (1997) suggest an alternative method (to obtain the reservation user cost), introduced by Diewert (1980) and used in Diewert and Smith (1997). They describe this method as follows:

> *"In the period when a new monetary asset is introduced, we calculated a Fisher Ideal user cost index over all monetary assets except the new one, which we call P_t^{**}. If monetary asset i is introduced in period t, P_t^{**} will be defined by*

[95] See Anderson, Jones and Nesmith (1997, p.78)

$$P_t^{**} = P_{t-1}^{**}\sqrt{\left(\frac{\sum_{j\neq i}\pi_{jt}^{real}m_{jt}^{nom}}{\sum_{j\neq i}\pi_{j,t-1}^{real}m_{jt}^{nom}} * \frac{\sum_{j\neq i}\pi_{jt}^{real}m_{j,t-1}^{nom}}{\sum_{j\neq i}\pi_{j,t-1}^{real}m_{j,t-1}^{nom}}\right)}$$ [96]

Diewert (1980) suggests that this procedure will generate lower bias than any of the other available alternatives, and in fact the procedure (and P_t^{**}) will be exactly correct if the new asset i in period t divided by the reservation user cost is equal to

$$\sum_{j\neq i}\pi_{jt}^{real}m_{jt}^{nom} \Big/ \sum_{j\neq i}\pi_{j,t-1}^{real}m_{jt}^{nom}.$$

The best suggestion of all this previous literature is that during periods in which new monetary assets are introduced, the real user cost indexes are obtained by the Fisher Ideal index, (as opposed to the real dual price index of the discrete-time Divisia monetary quantity index). During these periods, the dual monetary services index is calculated according to Fisher's weak factor reversal formula.

When the Fisher Ideal index for M3 was calculated here, quantity data on foreign currency deposits was unavailable from 1974 until October 1981, while the interest rate proxy, B54415, was available for all periods. The problem emerges in that the rate for the period before foreign currency deposits are introduced is not estimated to ensure the demand on the new asset is zero. Therefore we used the second method suggested above, that of ignoring the new monetary asset for the period of its introduction in the calculation of the Fisher index. The Divisia index is then normalized to this value of the Fisher Index for the period when the new monetary asset is introduced. The Divisia index is calculated as normal from this period on. In our results, for the whole sample period,

[96] See Anderson, Jones and Nesmith (1997, p.78)

the Fisher index corresponds very closely with the Divisia index for the aggregates M1, M2 and M3, which suggests that the methodology used to calculate both the Fisher and the Divisia indexes is at least approximately correct.

VI.(2) Seasonal Adjustment

With regard to the issue of seasonality, it must be said that index number theoretic methods for dealing with seasonality do exist in treatments given by Diewert (1980, 1983, 1996). Some of these methods are similar in some respects to the methodology used to transform indexes from one frequency to another. While the Bank of Canada does provide seasonally adjusted series for many monetary assets (mostly for the M1 assets) many assets needed for the construction of M2 and M3 were only available in seasonally unadjusted terms. Therefore, taking the *unadjusted* monetary asset series for M1, M2 and M3, the data were seasonally adjusted using the SAMA command of TSP 4.3 (Time Series Processor version 4.3) *before* the construction of the aggregates. TSP performs the seasonal adjustment of time series data by using a moving average method.

VII. The Monetary Assets

To begin constructing the Divisia, Fisher, currency equivalent and simple sum indexes for M1, M2, M3, it is necessary to describe exactly which monetary asset categories will be used to construct each aggregate.

For M1, essentially three asset categories are used to define the aggregate. These are: currency outside banks (CANSIM series number B2001), personal chequing accounts (CANSIM series number B486) and current accounts (CANSIM series number B487). Included in M2 less M1 are personal savings deposits at banks and non-personal savings or notice deposits. Personal savings deposits are broken down into chequable deposits (CANSIM series number B452), non-chequable deposits (CANSIM series number B453), and fixed term deposits (CANSIM series B454). The non-personal notice deposits are broken down into chequable (CANSIM series B472) and non-chequable deposits (CANSIM series B473). Finally, in M3 less M2, we have two additional deposit categories, non-personal term deposits and foreign currency deposits (CANSIM series numbers B475 and B482 respectively).

For the purposes of this project all the data series described above (with one exception) begin in January 1974 and end in the month of August 1998. The exception described above are the foreign currency deposits, for which the CANSIM series begins in the month of November 1981.

All of the data series described above are shown in Table 2, given below.

TABLE 2.

Money Stock Data	**CANSIM Series Number**
M1	
Currency Outside Banks	B2001
Personal Chequing Accounts	B486
Current Accounts	B487
M2 Less M1	
Personal Savings Deposits	
Chequable	B452
Non-Chequable	B453
Fixed Term	B454
Non-Personal Notice Deposits	
Chequable	B472
Non-Chequable	B473
M3 Less M2	
Non-Personal Term Deposits	B475
Foreign Currency Deposits	B482

VIII. The Interest Rates

Concerning the corresponding interest rates on M1, the interest rates on the currency outside banks account are by assumption, zero. However the interest rates on personal chequing accounts and current accounts (demand deposits) are implicit and must be calculated according to the methodology mentioned previously in this chapter. To do this, the required reserve ratios (ended as of June 1994), and the own rate on an alternative asset (in our case the rate on 3-5 year marketable Government of Canada bonds, CANSIM series B14010) are used.

For M2 less M1, the interest rates on personal savings deposits with the exception of fixed term deposits are a composite of several interest rate series available from Statistics Canada and the Bank of Canada. The interest rate on chequable personal savings deposits (as used in the calculation of the user costs) is series B14035 from the beginning of 1974 to September of 1982. From October of 1982 onwards to 1998, the interest rate on daily interest chequing accounts of $5000 and over (DICA 5K+) is used.

For non-chequable personal savings deposits (B453), the interest rate used for the calculation of user costs is a complex compilation of series B14019, the rate on daily interest savings deposits for accounts in excess of 25,000 (DISA 25) and the daily interest savings rate on deposits in excess of 75,000 (DISA 75). From the beginning of 1974 to December of 1986, series B14019 alone is used as the rate for non-chequable personal savings deposits. For the year of 1987 up to and including January 1988, the account DISA 25 is employed as the interest rate. From February of 1988 until August of 1998, the interest rate used is an average of DISA 25 & 75. For personal term deposits (B454), the 5-year term deposit rate (B14045) is selected as the only interest rate needed. For the other portion of M2 less M1, non-personal notice (savings) deposits, the interest rate on 90-day Canadian bonds (B14043) is selected as the proxy for the natural rate on both the chequable (B472) and non-chequable (B473) types of these non-personal deposits.

Concerning M3 less M2, non-personal term deposits (B479) are given the prime rate (B14020) as the proxy for the natural rate on this asset category. Finally, for foreign currency deposits (B482), the three-month Eurodollar deposit rate in London (B54415), which is closely linked to the wholesale deposit rate in Canada, is used as the corresponding interest rate. All of the own rates that correspond to the given monetary stock data are shown below in Table 3.

TABLE 3.

Money Stock Data	Interest Rate Data	CANSIM Series Number
M1		
Currency Outside Banks	Zero	None
Personal Chequing Accounts	Implicit Interest Rate	See Bank of Canada Review an B14010
Current Accounts	Implicit Interest Rate	See Bank of Canada Review an B14010
M2 Less M1		
Personal Savings Deposits		
Chequable	Selected Cda. Bond Yields, Daily Interest Chequing Accounts	B14035 or DICA 0-2K (1974:1 t 1982:9) , DICA 5K+ (1982:10-1998:8)
Non-Chequable	Selected Cda. Bond Yields, Daily Interest Savings Accounts in excess of 25K and 75K	B14019 (1974:1 to 1986:12), DI 25 (1987:1 to 1988:1), Average DISA 25 & DISA 75 (1988:2 to 1998:8)
Fixed Term	5 Year Term Deposit Rate	B14045 from 1974:1 to 1998:8
Non-Personal Notice Deposits		
Chequable	90-Day Canadian Bonds	B14043 from 1974:1 to 1998:8
Non-Chequable	90-Day Canadian Bonds	B14043 from 1974:1 to 1998:8
M3 Less M2		
Non-Personal Term Deposits	Prime Rate	B14020 from 1974:1 to 1998:8
Foreign Currency Deposits	Euro-US Deposit Rate	B54415 from 1974:1 to 1998:8

Concerning the benchmark rate used for the calculation of the user costs, it must be mentioned that the benchmark series used here is not the rate on one asset. The reason is that there seemed to be no obvious relevant asset that generated an interest rate higher than all of the interest rates described above for every period in the sample. Instead, the benchmark rate is a construct of the following interest rates: the rate on Government of Canada marketable bonds with a term to maturity of over 10 years (B14013), the rate on 5 year guaranteed investment certificates (B14080), the rate on other long term bonds (all corporates, Scotia-McLeod, series B14048) and the rate on other mid-term bonds (all corporates, Scotia-McLeod, series B14049), and finally all the other rates series as chosen and constructed for all the monetary assets for M1 through M3 mentioned previously.

The procedure used to construct the benchmark rate was simple: In each period, a program was created to select the highest available interest rate from the list given above, and add a small increment of 0.01. (This last step is done to avoid the possibility of zero user costs in the event that the benchmark rate and the rate on the given asset would be identical).

IX. A Comparison

A set of graphs are provided in the appendix showing how each of the indexes for M1, M2 and M3 compare to each other. For M1, comparing the simple sum, currency equivalent and Divisia indexes, we see that these indexes closely correspond in the early periods of the sample. In later periods the Divisia index reached the highest values, while the currency equivalent and simple sum indexes follow the growth path of the Divisia index closely from underneath, though the currency equivalent seems to follow more closely than the simple sum aggregates. This is shown in Figure 1.

As for the comparison between the Divisia, CE and simple sum index numbers for M2 and M3 (shown in Figures 3 and 5 respectively), one does see considerable divergence between the behavior of these three index aggregates over time (for both M2 and M3). Divisia M2 and M3 are more stable than any of their counterparts while CE M2 and M3 behave most erratically. The simple sum measures are more stable than the CE aggregates but also rise more sharply over the sample period than the Divisia aggregates.

Figures 2, 4 and 6 show the comparison between the Divisia and Fisher indexes for M1, M2 and M3 respectively. It is a well-known property in the literature on statistical index number theory that these two superlative indexes often correspond very closely in their practical application. As these graphs show, the same is true for this project in that Divisia and Fisher M1, M2 and M3 are almost exactly identical. Indeed, the differences between the Divisia and Fisher indexes for M1 through M3 were within the round-off error as Barnett (1980) showed. Each index provides a check for the other in the sense that a large discrepancy between the Divisia and Fisher aggregates for some set of data might indicate the presence of a mistake in the calculation of one or both of the indexes. Again, the Divisia index is regarded as the superior compared to the Fisher Ideal index because it is superlative in a stronger sense for all periods in which data is available.[97] However, the Fisher Ideal index provides the advantage of remaining well-defined even for periods in which new assets are introduced unlike the Divisia index and that is another reason why the Fisher Ideal index was constructed.[98]

X. Conclusion

The purpose of this project was to construct a measure of monetary aggregates with a stronger microeconomic foundation than the simple sum aggregates currently

[97] See Caves, Christensen and Diewert (1982)

[98] See Farr and Johnson (1985)

published by the Bank of Canada. Following the pioneering work of Barnett and Diewert and closely following the study conducted by Anderson, Jones and Nesmith (1997), the discrete-time Divisia index provides a means of creating more "economically sound" aggregates. The monetary asset data were found in the Bank of Canada Review. The corresponding own rates to the stock data were selected carefully based partly on the helpful suggestions of the Bank of Canada. The benchmark rate was constructed from a number of interest series, but in simple terms was merely the highest interest rate available for each period in the sample.

From this data, the Divisia index, the Fisher Ideal index, the currency equivalent index and finally, the simple sum index were all used to construct measures of the aggregates M1, M2 and M3. As the theory predicts, the Fisher and Divisia aggregates were nearly identical while there was considerable divergence between the Divisia aggregates on the one hand and the simple sum and currency equivalent indexes on the other.

FIGURE 1

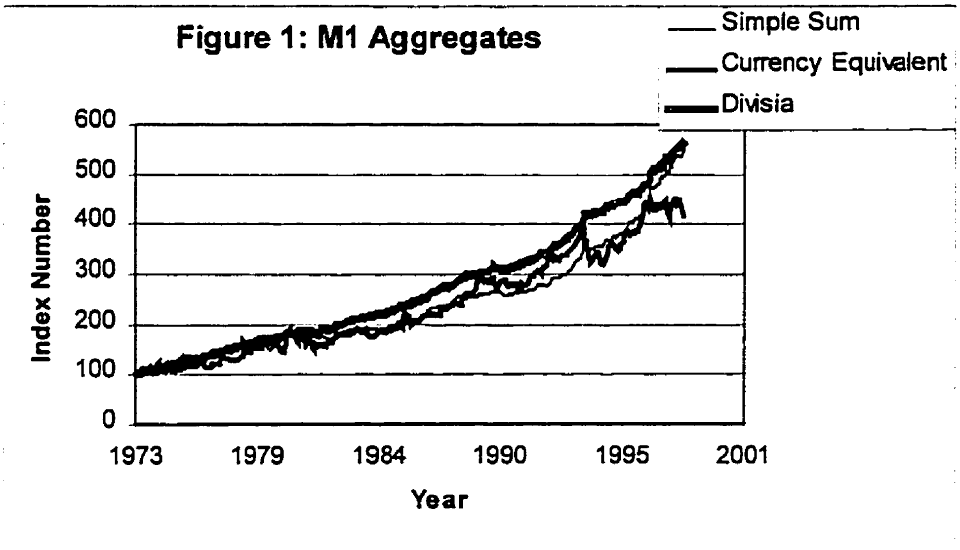


FIGURE 2

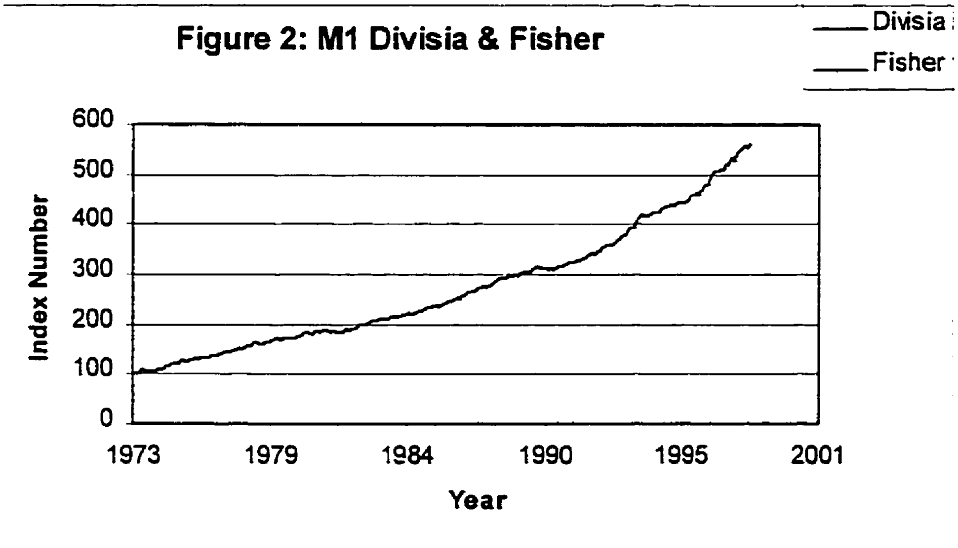


FIGURE 3

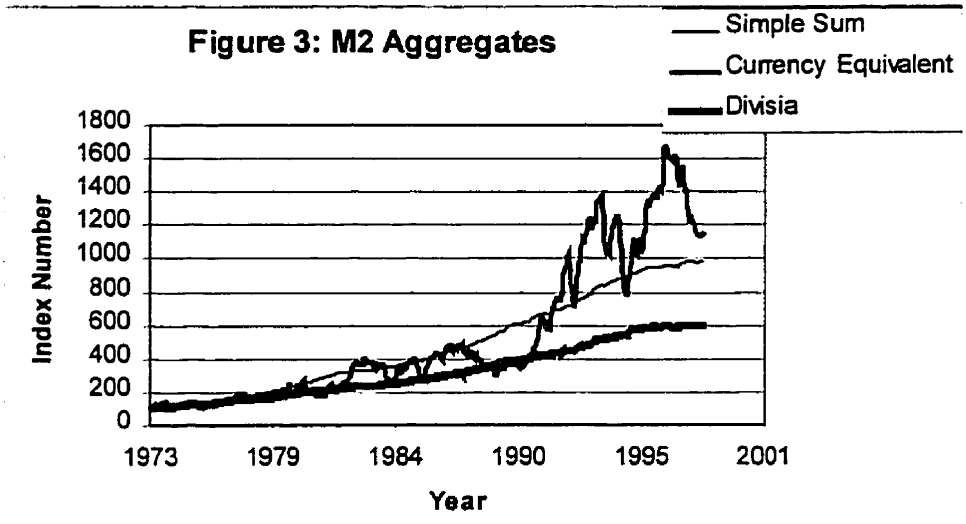


FIGURE 4

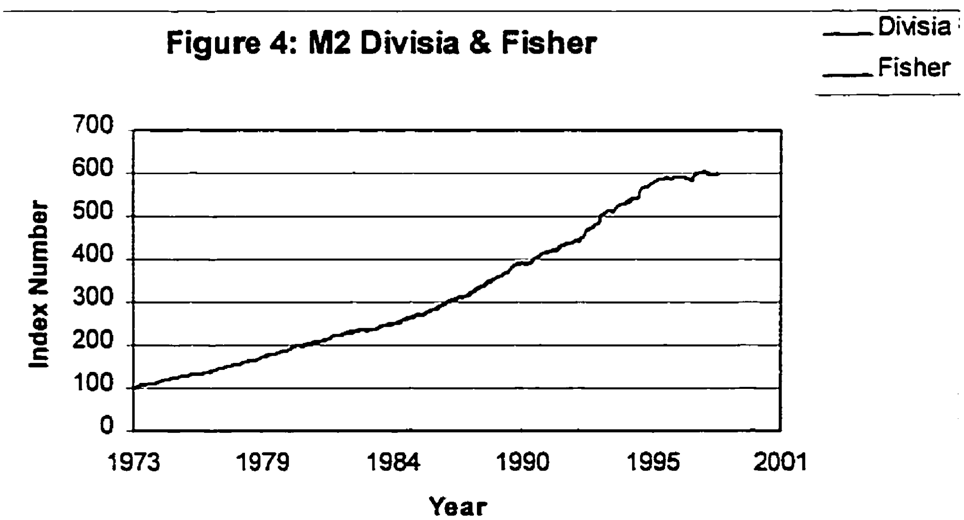


FIGURE 5

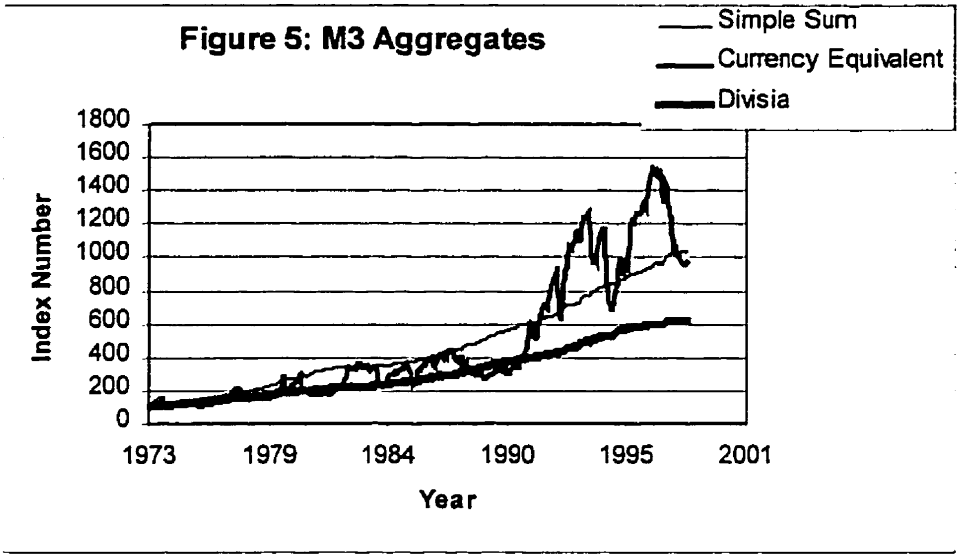
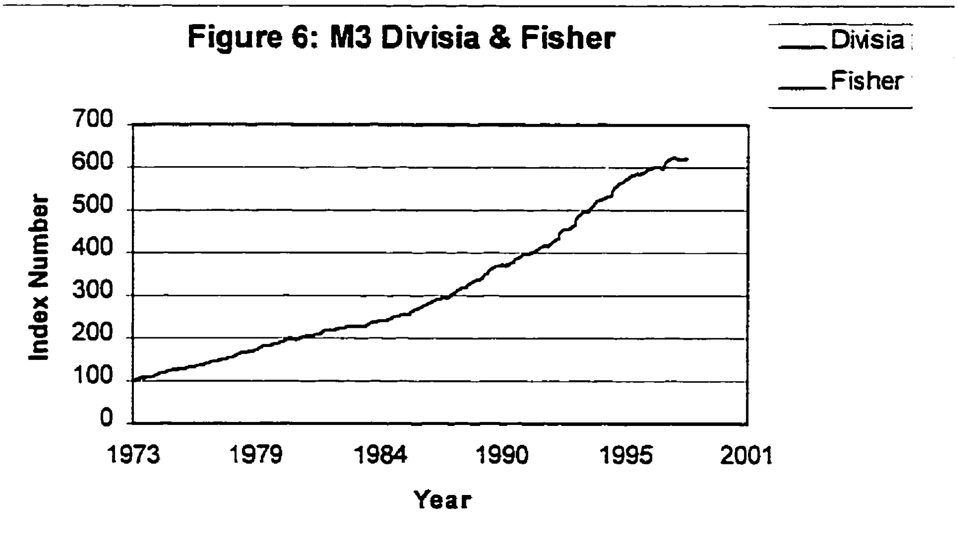


FIGURE 6



Chapter 6: Some Empirical Tests

I. Introduction

The purpose of this final chapter is to investigate the empirical relationship between money on the one hand and income and prices on the other. As with earlier papers investigating these relationships (specifically Serletis (1988) and Serletis and King (1993)), the focus of interest is on the aggregation-theoretic monetary aggregates (or monetary services indexes) such as the Divisia and Fisher indexes calculated here. Since the Bank of Canada only publishes simple sum aggregates for M1, M2 and M3, most empirical investigations into the relationships between money, prices and income define money according to the non-aggregation theoretic simple sum indexes. Hence, the alternative indexes provided here afford a richer perspective on the comparison between money, prices and income.

There are basically three sets of tests that are conducted here on the money aggregates, prices and income. First, a series of univariate unit root tests are performed on the data, namely the Dickey-Fuller test, the Weighted-Symmetric test and the Phillips-Perron test. Second, if the possibility exists that most if not all the data are nonstationary (in this case, possess unit roots), tests for cointegration between money and prices and/or money and income become important. Finally, a set of Granger-causality tests between money, prices and income will be conducted.

II. The Unit Root Tests

Unit root tests are basically tests to see if a given variable follows a random walk. If a time series variable follows a random walk, it cannot be a stationary stochastic process (is nonstationary). The property of stationarity allows one to model the given process or variable with one simple algebraic equation with fixed coefficients that can be estimated from past data.[99] Hence it is quite important for the purposes of regression and equation estimation to know whether a given variable is or is not a random walk

There are two standard means by which nonstationary series are converted into stationary series; detrending (by regressing the series on time or a function of time) and differencing the series (using the differences between observation of the series to form the data set). In the case of a random walk series, detrending the variables will not correct for the nonstationary aspect of the series. Only first-differencing a random walk will make the series stationary.[100]

Here, three unit root tests for a random walk are performed, the Dickey-Fuller test the Weighted-Symmetric and the Phillips-Perron test. We will start with the Dickey-Fuller unit root test. Suppose there exists a time-series variable y_t, which has been growing over time. We can attempt to model this growth as shown below.

$$y_t = \alpha + \beta t + p y_{t-1} + \varepsilon_t \quad (1)$$

There are two possibilities as to why the growth in y_t occurs. The first possibility is that the series could have a positive trend, $\beta \geq 0$. In this case, detrending the series would remove the nonstationarities. Second, y_t could be growing because it follows a random

[99] See Pindyck and Rubinfeld (1997, p.493)

[100] See Pindyck and Rubinfeld (1997, p.507)

walk with a positive drift ($\alpha \geq 0, \beta = 0$ and $p = 1$). In this event, detrending would not yield a stationary series. Instead, first-differencing y_t would be the preferred method to make the series stationary.[101] If we suspect y_t of being a nonstationary series, it would not be possible to estimate equation (1), since the nonstationarities violate the required assumptions of OLS. In this case, if the true value of p is 1, the OLS estimator is biased towards zero.

The Dickey-Fuller test is basically a simple F-test of the random walk hypothesis ($\beta = 0$, $p = 1$). First, the equation given in (1) must be modified as shown below.

$$y_t - y_{t-1} = \alpha + \beta t + (p-1)y_{t-1} \qquad (2)$$

Using OLS, one estimates the above formula as the unrestricted regression and then estimates the restricted regression

$$y_t - y_{t-1} = \alpha \qquad (3)$$

From these regressions, one calculates the test statistic in the same manner as the F test statistic used in models where the joint significance of a group of variables is being tested. The critical values to which the test statistic is compared in the Dickey-Fuller test are not derived from the standard F distribution but rather according to a distribution derived by Dickey and Fuller. One important fact to note is that the Dickey-Fuller critical values are considerably higher than the corresponding F distributed critical values. This makes it considerably more difficult than it would otherwise be to reject the null hypothesis (of a unit root).

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[101] See Pindyck and Rubinfeld (1997, p.508)

One disadvantage with the simple unit root test performed on equation (1) is that it makes the implicit assumption that there is no serial correlation of any kind in the error term ε_t. The augmented Dickey-Fuller test allows for serial correlation in ε_t while testing for a unit root in the series itself.[102] The test is performed by expanding the equation given in (1) to include lagged changes in y_t in the on the right-hand side of the equation. This is shown as below

$$y_t = \alpha + \beta t + p y_{t-1} + \sum_{j=1}^{p} \lambda_j \Delta y_{t-j} + \varepsilon_t \qquad (4)$$

where $\Delta y_t = y_t - y_{t-1}$ is the first-difference of series y_t. The number of lags (p) to include in equation (4) is up to the discretion of the economist performing the test. (Usually the decision is made through an iteration of tests to see which equation best represents the expected results of a given model). The basic unit root test is performed using the same basic procedure as for the Dickey-Fuller unit root test. One runs an OLS regression on the unrestricted equation.

$$y_t - y_{t-1} = \alpha + \beta t + (p-1) y_{t-1} + \sum_{j=1}^{p} \lambda_j \Delta y_{t-j} \qquad (5)$$

and then on the restricted equation

$$y_t - y_{t-1} = \alpha + \sum_{j=1}^{p} \lambda_j \Delta y_{t-j} \qquad (6)$$

using the F ratio to test the null hypothesis of ($\beta = 0, p = 1$). One must use the distributions given by Dickey and Fuller to generate the critical values with which to compare the F-ratio.

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[102] See Pindyck and Rubinfeld (1998, p.509)

In general, one should remember that the Dickey-Fuller test only allows one to reject or fail to reject the hypothesis that a variable is not a random walk. Accepting the null hypothesis of a unit root (and thus a random walk) provides only weak evidence that the time series involved is in fact a random walk.

The Phillips-Perron test involves the ordinary least squares regression of

$$y_t = \alpha + \beta(t - T/2) + py_{t-1} + \varepsilon_t \qquad (7)$$

where T denotes the sample size. The test for unit roots involves a test of the null hypothesis, $\beta = 0$, $\alpha = 1$. The Phillips-Perron test, unlike the simple Dickey-Fuller test shown above is robust to a "*wide variety of serial correlation and time-dependent heteroskedasticity and accommodates models with a drift and a time trend so that it may be used to discriminate between unit root stationarity and stationarity about a deterministic trend.*" [103]

When examining the results of the unit root tests performed (some described above such as the Dickey-Fuller, the Phillips-Perron, and including another, the Weighted-Symmetric) the p-values are examined to determine whether the null hypothesis of a unit root can be rejected or accepted. For the purpose of our test, we compare the p-value generated by the given unit root test to see whether it is greater or smaller than the 5 % level. That is to say, we examine whether the p-value (conceptually defined here as the probability of committing a Type 1 error, the error of incorrectly rejecting the null hypothesis of a unit root) is less than 5% (0.05). If it is, then we feel safe in rejecting the hypothesis that the time series variable in question possesses a unit

[103] See King and Serletis (1993, p.95)

root. If the p-value is greater than 0.05, then we cannot reject the possibility that the given variable possesses a unit root with any certainty.

The results of the unit root tests for the log levels of the relevant monetary aggregates, the price level series CPI, real GDP (the IPI index) and nominal GDP (the product of the CPI and IPI indexes) are presented in Table 1. It is clear from these that the unit root hypothesis cannot be rejected for any of these time series variables with the sole exception of currency equivalent M1.

The first differences of the log levels of all variables are also tested for unit roots (shown in Table 5). The results of the tests performed under this circumstance show that first-differencing removes the unit root and thus generates a stationary series for all the variables except possibly simple sum M3, the CPI index, nominal GDP and real GDP. For these four variables however, the results across the three unit root tests differ with the Phillips-Perron test suggesting that these series are stationary while the Weighted-Symmetric and Dickey-Fuller tests suggest otherwise.

III. Tests For Cointegration

Once the tests for unit roots have been completed, it is useful to move on to perform cointegration tests. As stated before, first-differencing a random walk series (a series possessing unit roots) can convert the series back into a stationary form. Thus, in regressing one random walk series on another, differencing the series in question before using them in the regression is a good method to ensure that meaningful results can be obtained.

However, the cost of using first-differencing as a corrective method to allow a proper regression analysis is that one may lose information as to the true long-run relationship between two variables.[104] Therefore, it would be desirable indeed to be able to run a regression properly on the original random walk series in question without having to first-difference beforehand.

The circumstance in which this would be possible occurs when two variables are random walks yet a linear combination of these variables will be stationary.[105] In such a case, these two variables are said to be *cointegrated* which respect to each other. For example, assuming variables, x_t and y_t are random walks and cointegrated, we might have a variable $z_t = x_t - \lambda y_t$ which itself is stationary or integrated of order zero (I(0)). (λ here is termed the cointegrating parameter). Intuitively, the explanation behind the cointegration in this example is that the "trends" in x_t and y_t cancel each other out.[106] In general, if two series are integrated of the same order, (for example - if series y is I(1) and series x is I(1)), these two series can be cointegrated. Nonetheless, despite this general rule, it is necessary to formally test for the possibility of cointegration between random walk series before linear regressions between the series can be assured of generating meaningful results.

Given the above example for series x and y, this is done by running an OLS regression of x_t on y_t, (in order that the cointegrating parameter λ may be estimated). The residuals of this regression can be used to test whether x_t and y_t are indeed

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[104] See Pindyck and Rubinfeld (1997, p.513)

[105] See Pindyck and Rubinfeld (1997, p.513)

[106] See Gujarati (1995, p.756)

cointegrated.[107] This is the basis for the test for cointegration developed by Engle and Granger (1987). More formally, the Engle-Granger test is performed as follows.

Assume first of all, using the Dickey-Fuller test for unit root tests described earlier, that x_t and y_t are found to be random walks and further, that the first-differenced series Δx_t and Δy_t are stationary. It is then a simple matter to test for cointegration between x_t and y_t. First an OLS regression is run on x_t and y_t as shown below.

$$x_t = \alpha + \beta y_t + \varepsilon_t$$

One then tests for unit roots in the residuals, ε_t, (ie. for nonstationary residuals). *"If x_t and y_t are not cointegrated, any linear combination of them will be nonstationary and hence the residuals ε_t will be nonstationary."* [108] Therefore, one runs the following unrestricted regression on ε_t

$$\varepsilon_t - \varepsilon_{t-1} = \mu + \theta t + (p-1)\varepsilon_{t-1}$$

and then the restricted regression

$$\varepsilon_t - \varepsilon_{t-1} = \mu \, .$$

As before with the Dickey-Fuller test, one calculates the F ratio to test whether the null hypothesis of ($\beta = 0$, $p = 1$) holds. Unlike before, however, one cannot use the Dickey-Fuller and (augmented Dickey-Fuller) critical significance values for the cointegration test on the residuals. Engle and Granger calculated critical values appropriate for the unit root test on the residuals accounting for the dependence of the estimated residuals on the

[107] See Pindyck and Rubinfeld (1998, p.514)

[108] See Pindyck and Rubinfeld (1998, p.514)

estimated cointegrating parameter β given above.[109] The Dickey-Fuller (DF) and augmented Dickey-Fuller tests on the residuals in this context are known as the Engle-Granger (AEG) test. Here, as before the results of the Engle-Granger tests are provided with the p-values which indicate that are one can reject the null hypothesis of a unit root in the residuals when less than 0.05.

The results of the Engle-Granger cointegration tests are presented in Table 6. Six columns are given: the first shows the results of a regression of the CPI index on the given money series, the second regression is of the money series on the CPI index, the third involves the regression of the IPI index (real GDP) on the money series and so on. Given the rather sizeable p-values obtained (all being higher than 0.05), it is clear that the hypothesis that these series are jointly nonstationary (possess unit roots in the residuals) cannot be rejected. Hence it seems that none of the variables are cointegrated. The only possible exception to this is the regression between prices (and income) on the one hand and currency equivalent M1 on the other which might be cointegrated if regressed on each other in that order.

IV. Tests for Causality

The third basic issue of relevance to this empirical investigation is the question as to whether money variables provide useful information about recent or current economic conditions which is obviously a matter of great interest to the monetary policy-making authorities.

> *"Of particular concern is whether the observation of a variable like the money stock are potentially useful for anticipating future movements in*

[109] See Gujarati (1995, p.727)

macroeconomic goal variables [such as income and price levels or growth rates], in which case they could be used to indicate that instrument settings should perhaps be reconsidered to prevent undesirable future outcomes in terms of goals." [110]

The above idea is known as the information variable approach and one of its implications is that the monetary authority need not base its policy-making on the movements of one monetary aggregate alone (unless one aggregate contains all the information needed to obtain the monetary authorities objectives, an idea which is belied by the empirical evidence). Therefore, from the monetary authority's perspective, it is advantageous to maximize the use of all available information in anticipating future movements in key macroeconomic variables.[111]

The key question to evaluate the relevance of the information variable approach is whether the observed monetary aggregates (Divisia, Simple sum, CE and Fisher) can help predict future movements in income and prices, (the two key macroeconomic variables examined here through the IPI and CPI indexes respectively). To answer this question, we will use a Granger-Sims causality test to determine the order of causality between money, income and prices.

The idea behind the Granger-Sims causality test is simple: If the money stock, M_t, "causes" or influences income (IPI) or prices (CPI), then changes in M_t should precede changes in (IPI_t) or prices (CPI_t). First, M_t should help predict IPI_t (or CPI_t)

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[110] See Serletis and King (1993, p.103)

[111] See Serletis and King (1993, p.103)

and second, IPI_t (or CPI_t) should not help to predict M_t. If the second condition were not true, a situation would exist whereby changes in one or more variables other than M_t, IPI_t and CPI_t would be causing the observed changes in both M_t and IPI_t (or CPI_t).

The Granger test is designed to test whether both of these conditions hold. First one would test the null hypothesis that M_t does not cause IPI_t (or CPI_t). To do this, regress IPI_t (CPI_t) against lagged values of IPI_t (CPI_t) and lagged values of M_t (the unrestricted regression) and then regress IPI_t (CPI_t) against only lagged values of IPI_t (CPI_t) which serves as the restricted regression.

A simple F-test can indicate whether M_t contributes significantly to explaining IPI_t (CPI_t), through the unrestricted regression. If this is the case then, the null hypothesis that M_t does not cause IPI_t (CPI_t) is rejected and the data is thus consistent with M_t causing IPI_t (CPI_t). The null hypothesis that IPI_t (CPI_t) does not cause M_t is tested using the same steps given above.

The first null hypothesis, (that M_t does not cause IPI_t (CPI_t) is tested by running the two regressions shown below.

Unrestricted regression: $IPI_t(CPI_t) = \sum_{i=1}^{m} \alpha_i IPI_t(CPI_t) + \sum_{i=1}^{m} \beta_i M_{t-i} + \varepsilon_t$

Restricted regression: $IPI_t(CPI_t) = \sum_{i=1}^{m} \alpha_i IPI_{t-i}(CPI_{t-i}) + \varepsilon_t$

Using the sum of squared residuals from each regression to calculate an F-ratio, test whether the group of coefficients $\beta_1, \beta_2, ..., \beta_m$ are significantly different from zero. If they are, one can reject the null hypothesis given above.[112]

To test the second assumption of the causality test that $IPI_t\,(CPI_t)$ does not cause M_t run the same regressions as above, but this time switching the place of M_t and $IPI_t\,(CPI_t)$ and test whether the lagged values of $IPI_t\,(CPI_t)$ are a significant influence on M_t. To make the final conclusion that M_t causes $IPI_t\,(CPI_t)$, the hypothesis that M_t does not cause $IPI_t\,(CPI_t)$ must be rejected and the hypothesis that $IPI_t\,(CPI_t)$ does not cause M_t must be accepted.

As a final note on the theoretical structure of the Granger-Sims causality test, it is often the case that the direction of causality may depend on the number of lagged terms used in the test.[113] Therefore, it is generally a good idea to run the test for a number of values of m and make sure the results are not sensitive to the number of lags chosen.

The comprehensive set of results of the Granger causality tests between money and prices, and money and income are provided in Table 7 through 9. All the variables were given in terms of first-differenced log levels, which is equivalent to using the growth versions of the price, income and money variables. First differences were presumed sufficient to eliminate the unit roots in the CPI, IPI and nominal GDP series based on the results of the Phillips-Perron test shown earlier. The postulated order of causality is shown in the column heading. The Schwartz Criterion was used to determine

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[112] See Pindyck and Rubinfeld (1998, p. 243)

[113] See Gujarati (1995, p.622)

which combination of lag lengths in the band of 2 to 4 was optimal for use in the Granger-causality tests for both variables used in the unrestricted version of the regression.

Once again if the p-values shown are greater than 0.05 then the null hypothesis that money does not cause or affect prices/income (or vice versa) cannot be rejected. Since all of the p-values are significantly greater than the 5% level for all of the tests, the hypothesis that price growth (and income (GDP) growth) does not Granger-cause money growth can be accepted in each case. This is one requirement concluding that a causality relationship between money and prices (and income, both nominal and real) exists. The second requirement for the causality relationship is the rejection of the null hypothesis that money growth does not Granger-cause price or income growth. This null hypothesis cannot be rejected for any of the series. Indeed for all of the simple sum, currency equivalent and Divisia measures of money we accept the hypothesis that money growth does not Granger cause nominal or real income growth (or prices) at the 5% level.

V. A Set of Comparisons

It is useful at this point to compare the results of our empirical tests on the aggregation theoretic monetary aggregates obtained in our study to the results of similar previous studies performed in the past. First, Serletis and King (1993) examine the empirical relationships between both summation and Divisia monetary aggregates and income or prices in Canada in a very similar manner as done here. Using quarterly data for the sample period 1968 to 1989, the results of their tests concerning the stationarity of the time series data indicate that "*the unit root hypothesis cannot be rejected for any of* *the series.*"[114] Further, they find no evidence of cointegration between money and prices or income.

Regarding the causality tests, they accept the hypothesis that money does not Granger-cause prices at the 1% level for all of the aggregates except simple sum M2 and M2+. Tail area comparisons indicate that the growth rate of simple sum M2+ is the best leading indicator of inflation.

The hypothesis that money growth does not Granger-cause nominal income growth would be accepted at the 1% level with all the summation aggregates except M1 and M2+. Regarding the Divisia aggregates, the above hypothesis would be rejected for all levels of aggregation, meaning that they do find evidence that various money measures cause nominal income growth. Although, they find no evidence of a causality relationship between the broader aggregates and real income, the tail-area comparisons indicate that the narrow measures may Granger-cause real income. In contrast, they find support for the hypothesis that real income causes the broader sum aggregates such as M3. This provides some support for the view that money measures respond to changes in economic conditions, otherwise known as the endogenous money hypothesis. In the results shown in Tables 4 though 6, no support is found for the endogenous money hypothesis.

The results of the unit root and cointegration tests given in Tables 4 through 9 are consistent with those of the Serletis and King (1993) study to a limited extent. However, no evidence is obtained in this study for the view that simple sum M2 and M2+ Granger-cause prices or that simple sum M1, M2+ and the Divisia measures Granger-cause

[114] See Serletis and King (1993, p.96)

nominal income. Nor do we find evidence here that the narrow measures simple sum M1 and Divisia M1 are significantly correlated to real income. Finally, our causality results differ in that it does not appear that real income significantly affects the broader aggregates of either index.

Another important study by Longworth and Atta-Mensah (1995) performs similar empirical tests on the relationships between money, prices and income. Specifically, they compare the empirical performance of the simple sum aggregates and weighted monetary aggregates such as the Divisia and Fisher in terms of their information content and their ability to forecast variables such as prices, real output and nominal spending for the period 1971 (quarter 1) to 1989 (quarter 3).

As shown in the previous chapter, the Divisia and Fisher indexes are, under most circumstances nearly identical. That is why the Fisher results for the unit root, cointegration and causality tests were not included in the tables in this chapter. However for the Longworth and Atta-Mensah (1995) study, they chose to construct the Fisher Ideal aggregates in place of the Divisia for their comparisons. Their basic conclusion is that the simple sum monetary aggregates appear to be empirically superior to the Fisher Ideal aggregates in terms of forecasting inflation, nominal spending and real output.

Regarding unit root tests, Longworth and Atta-Mensah (1995) find that all of the monetary aggregates (simple sum and Fisher Ideal), inflation and real income are integrated of order one. Using the methodology of Johansen and Juselius, they look for cointegrating vectors between the simple sum monetary aggregates, interest rates, real income and inflation measures. Indeed, they find evidence for the existence of "*cointegrating relationships among the monetary aggregates (in real terms), real income*

R90 [the interest rate on the 90-day T-Bill] and inflation."[115] (In contrast, they find that Fisher Ideal aggregates broader than Fisher M1 are hardly ever cointegrated with price CPI or real income measures.) This is a result that differs radically from our study and the work of Serletis and King (1993) in which practically none of the variables are cointegrated.

Longworth and Atta-Mensah (1995) also perform a series of short run causality tests, but instead of using the Granger methodology, they use a vector error-correction model (VECM) and "*subject it to exclusion tests on the lags of each of the first-differenced variables and on the lagged cointegrating terms, (using the cointegrating vectors obtained earlier by the Johansen and Juselius methodology).*"[116] In summary the results of these tests relevant for the purposes of comparison to our results are as follows. 1) Fisher M1 is the only aggregate that has a significant impact on real income (GDP) in the short run. 2) Simple sum M2, M2+ and M3 are the only aggregates that significantly influence CPI in the short run.

In general the results of their forecasting tests indicate that simple sum aggregates M1, M2+ and real M1 provide the best explanation for the CPI. Fisher Ideal M1 forecast for nominal spending in the short run, while simple sum M2+ provides the best forecast for the same variable at longer horizons, while unsurprisingly simple sum real M1 provides the best forecasts for real GDP.

Several older Canadian studies also find that simple sum monetary aggregates are better correlated with price and income measures than superlative or so-called "weighted"

[115] See Longworth and Atta-Mensah (1995, p.35)

[116] See Longworth and Atta-Mensah (1995, p. 43)

index aggregates such as the Fisher Ideal and Divisia. Among these are the work of Cockerline and Murray (1981), Hostland, Poloz and Storer (1988) and Chrystal and McDonald (1994). In contrast, however, Serletis (1988) finds that Divisia monetary aggregates possess a more statistically significant causal relationship with real GNP than their simple sum counterparts. He also finds that the causality relationship for money to prices is significant for Divisia M1, M2 and L as well as simple sum M1, along with evidence of a causal relationship for prices to money. Also, recall that Serletis & King (1993) find that Divisia aggregates at all levels of aggregation serve as a better leading indicator of nominal income than their simple sum counterparts. It is not clear yet whether superlative or simple sum monetary aggregates can most effectively forecast future movements in prices and income.

The results obtained here find no evidence of a causality relationship in any direction between monetary aggregates (at all levels of aggregation and for all of the simple sum, currency equivalent and Divisia measures) and prices (or income, both real and nominal) in any direction. In this sense our results differ significantly from all of the studies discussed above. The results here do not allow any further insight into determining which money measures are superior to the others.

The absence of a causality relationship between monetary aggregates and prices or income is substantiated by several studies conducted in the U.S. most notably those of Friedman (1988, 1989) and Friedman and Kuttner (1992). They find that including data from the 1980's significantly reduces or eliminates (depending on the size of the sample period) evidence of causality between monetary aggregates (at all levels) and prices or income.

While the tests here do not seem to give any indication that superlative index aggregates are superior measures of the money supply to simple sum measures, there is another bias that factors into the calculations made by central banks regarding which aggregation methodology should be employed. Traditionally, central banks regard the simple sum index as the only practical means of constructing monetary aggregates due to the perceived high costs of obtaining interest rate data and the sophistication of the calculations required to construct Divisia money measures. However, these sort of cost/benefit calculations do not disfavor the Divisia approach as much as central banks might believe.

The computer programs required to use the Divisia index for aggregation are of trivial simplicity, and the data required for the interest rates corresponding to specific quantity series are already obtained and collected by the Bank of Canada. Therefore, it is clear that the principle area of debate in the future should focus primarily on the economic merits of alternative aggregation approaches rather than on the perceived bureaucratic difficulties of implementing a different approach.

VI. Conclusion

This chapter examines the empirical relationships between money, prices and income in the Canadian economy. While cointegration and causality tests have been performed many times before on these variables, we take advantage of the opportunity provided by the construction of a new set of aggregation theoretic (and hopefully better) money measures to re-examine some of the key assumptions of monetary policy (namely that money supply fluctuations have an observable effect upon inflation and economic growth).

First, a set of unit root tests were performed on all the variables, which not surprisingly indicate that all of the time series data save one were nonstationary. The first differenced log levels for all of the money measures corrected for the presence of unit roots. The effectiveness of first-differencing the price and income variables was more questionable given that the Phillips-Perron test strongly indicated the possibility that the nonstationarities were removed by first-differencing while the Weighted-Symmetric and Dickey-Fuller tests indicated otherwise.

The mere presence of a unit root in the money, price and income measures does not mean that a consistent linear regression of money on either price or income measures will not be possible. If it is possible, then money and prices (or income) are cointegrated. The cointegration tests provide no indication that a linear combination of the nonstationary money measures with prices (or income) will yield a stationary series. Hence it will not be possible to use an OLS regression to provide a consistent estimator of the parametric relationship between money and prices (and income).

Most significantly however, the Granger causality tests give one no reason to think that money measures (whether they are aggregation theoretic or not) possess information which may be used to forecast future movement in price or income variables. Indeed to the extent the relationship between money growth and the resulting movements in prices and income was examined, some (admittedly weak) evidence in favour of the neutrality and superneutrality hypotheses is found.

The dependability of the results of the causality tests is called into question somewhat by the conflicting indicators as to whether the first-differenced log levels of the price (CPI) and income (nominal and real GDP) series are truly stationary. A useful extension of the work done here would be to conduct additional unit root tests on the price and income variables to more conclusively indicate whether the unit roots contained in the price and income variables can be fully eliminated by first-differencing.

Concerning the comparison between the test results obtained and those of previous studies, most Canadian studies find some evidence of causal relationships between money variables and prices or income, while that is not the case for the thesis test results. Since no evidence is found for the view that money has a causality relationship with respect to prices or income in this study across all the money measures tested, no further insight is provided into the debate regarding the relative merits of aggregation theoretic money measures as opposed to simple summation, or other, measures. However, it is argued here that the costs of constructing Divisia money measures are not prohibitive as is often believed by many central banks, and this should not be used as a factor in deciding which aggregation methodology to employ.

TABLE 4. SUMMARY OF UNIT ROOT TESTS

Log Levels	p-value(lags)		
Series	**Weighted-Symmetric**	**Dickey-Fuller**	**Phillips-Perron**
Simple Sum M1	0.98681(4)	0.9856(3)	0.97328(3)
Simple Sum M2	1(4)	0.99528(5)	0.99478(5)
Simple Sum M3	0.99996(8)	0.33191(13)	0.75691(13)
Currency Equivalent M1	0.00048341(2)	0.000921(2)	0.00025146(2)
Currency Equivalent M2	0.036803(10)	0.068654(10)	0.088468(10)
Currency Equivalent M3	0.031649(10)	0.0704(10)	0.0744226(10)
Divisia M1	0.90482(13)	0.048356(13)	0.23344(13)
Divisia M2	0.99999(2)	0.98496(2)	0.97263(2)
Divisia M3	0.99801(2)	0.55078(2)	0.6201(2)
CPI	1(13)	0.97106(13)	0.99321(13)
Nominal GDP	1(13)	0.93238(13)	0.97570(13)
Real GDP	0.99999(5)	0.85395(13)	0.93262(13)

TABLE 5. SUMMARY OF UNIT ROOT TESTS

First Differences of Log Levels	p-values(lags)		
Series	**Weighted-Symmetric**	**Dickey-Fuller**	**Phillips-Perron**
Simple Sum M1	1.59185E-10(2)	6.19001E-15(2)	3.38714E-34(2)
Simple Sum M2	1.69853E-07(3)	1.40787E-07(4)	6.03893E-34(4)
Simple Sum M3	0.21592(13)	0.2712(13)	1.3197E-37(13)
Currency Equivalent M1	4.72844E-08(6)	1.49815E-10(6)	2.4515E-25(6)
Currency Equivalent M2	0.00022728(9)	0.00033952(9)	8.31701E-23(9)
Currency Equivalent M3	0.0001911(9)	0.00026775(9)	4.86053E-24(9)
Divisia M1	1.37167E-10(2)	0.011871(13)	5.70138E-38(13)
Divisia M2	1.76274E-10(2)	5.89425E-15(2)	7.79862E-32(2)
Divisia M3	4.37375E-10(2)	2.33315E-14(2)	2.14778E-31(2)
CPI	0.064172(13)	0.18614(13)	0(13)
Nominal GDP	0.079846(13)	0.13698(13)	3.25257E-38(13)
Real GDP	0.063055(13)	0.077186(13)	2.03927E-37(13)

TABLE 6. COINTEGRATION TEST RESULTS

Log Levels	p-value(lags)					
Series	**CPI to Money**	**Money to CPI**	**IPI to Money**	**Money to IPI**	**Nom.GDP to Money**	**Money to Nom.GDP**
Simple Sum M1	0.81105(2)	0.68535(5)	0.98117(2)	0.94714(8)	0.94313(2)	0.81181(5)
Simple Sum M2	0.95413(2)	0.93162(2)	0.99714(2)	0.98273(2)	0.99059(2)	0.97088(2)
Simple Sum M3	0.7419(8)	0.97534(8)	0.84868(2)	0.96625(2)	0.75297(8)	0.99044(2)
Currency Equivalent M1	0.0032963(2)	0.99499(2)	0.0032425(2)	0.96902(5)	0.0032620(2)	0.97529(2)
Currency Equivalent M2	0.17587(8)	0.99342(8)	0.1923(8)	0.98212(8)	0.18517(8)	0.99184(8)
Currency Equivalent M3	0.12318(8)	0.98949(8)	0.14604(8)	0.97536(8)	0.13526(8)	0.98672(8)
Divisia M1	0.4984(3)	0.99341(8)	0.67327(3)	0.99279(7)	0.61334(3)	0.99438(8)
Divisia M2	0.96026(2)	0.95633(2)	0.99123(2)	0.96647(2)	0.98175(2)	0.96165(2)
Divisia M3	0.7649(2)	0.98091(8)	0.81773(2)	0.95912(5)	0.79265(2)	0.97834(8)

TABLE 7. TAIL AREAS (P-VALUES) OF TESTS OF GRANGER-CAUSALITY BETWEEN MONEY AND PRICES

	Money to Prices		**Prices to Money**	
Series	**Lag Lengths**	**P-values**	**Lag Lengths**	**P-values**
Simple Sum M1	(4,2)	0.7265	(2,4)	0.75776
Simple Sum M2	(4,2)	0.34307	(4,2)	0.15277
Simple Sum M3	(4,2)	0.3844	(2,2)	0.17782
Currency Equivalent M1	(4,2)	0.33099	(4,2)	0.99496
Currency Equivalent M2	(4,2)	0.27833	(2,2)	0.96066
Currency Equivalent M3	(4,2)	0.37421	(2,2)	0.88968
Divisia M1	(4,2)	0.74347	(2,4)	0.90093
Divisia M2	(4,2)	0.73667	(2,2)	0.18375
Divisia M3	(4,2)	0.85733	(2,2)	0.32915

TABLE 8. TAIL AREAS (P-VALUES) OF TESTS OF GRANGER-CAUSALITY BETWEEN MONEY AND REAL GDP (INCOME)

	Money to IPI		IPI to Money	
Series	**Lag Lengths**	**P-values**	**Lag Lengths**	**P-values**
Simple Sum M1	(3,2)	0.98479	(2,4)	0.3554
Simple Sum M2	(3,3)	0.1602	(2,2)	0.10656
Simple Sum M3	(3,2)	0.86207	(2,2)	0.17507
Currency Equivalent M1	(3,3)	0.09072	(4,2)	0.33028
Currency Equivalent M2	(3,3)	0.13925	(2,3)	0.28259
Currency Equivalent M3	(3,3)	0.14761	(2,3)	0.20615
Divisia M1	(3,3)	0.14647	(2,4)	0.3412
Divisia M2	(3,3)	0.31566	(2,2)	0.20371
Divisia M3	(3,2)	0.9999	(2,2)	0.29319

TABLE 9. TAIL AREAS (P-VALUES) OF TESTS OF GRANGER-CAUSALITY BETWEEN MONEY AND NOMINAL GDP (INCOME)

	Money to Nominal GDP		Nominal GDP to Money	
Series	**Lag Lengths**	**P-values**	**Lag Lengths**	**P-values**
Simple Sum M1	(4,2)	0.79761	(2,4)	0.64998
Simple Sum M2	(3,3)	0.16187	(2,2)	0.07724
Simple Sum M3	(3,2)	0.94813	(2,2)	0.13193
Currency Equivalent M1	(4,2)	0.14298	(4,2)	0.44854
Currency Equivalent M2	(3,2)	0.38251	(2,2)	0.99516
Currency Equivalent M3	(3,2)	0.43495	(2,2)	0.99097
Divisia M1	(4,3)	0.19437	(4,2)	0.83422
Divisia M2	(3,2)	1	(2,2)	0.15309
Divisia M3	(3,2)	1	(2,2)	0.24349

Chapter 7: Conclusion

This thesis study has examined the microeconomic theory of monetary aggregation. The theory is based upon the foundation provided by the aggregator functions of representative agents. To obtain the aggregator functions, it must be the case that current-period quantities of monetary assets be weakly separable from all the other assets, goods, services and leisure (in the representative consumer's utility function). The aggregator functions are actually subutility functions contained in the larger utility functions in the first stage of the two-stage optimization process.

Statistical index numbers are used to provide a second-order approximation of the aggregator functions so that parametric estimations of these functions are not required. (Parametric estimations generally require that restrictive assumptions be made regarding the functional form of the given aggregator function. Thus using a non-parametric approximation of the aggregator functions is seen as a good thing.) The Divisia index is the main such index of interest here, although the Fisher and currency equivalent indexes are also described.

The Divisia, Fisher and currency equivalent indexes are used to construct a set of aggregates, using the same data series and definitions for M1, M2 and M3 as those of the official (simple sum) aggregates provided by the Bank of Canada. The Divisia aggregates have a better foundation in microeconomic and aggregation theory than the official aggregates and the work done here allows us to compare the official aggregates to those that were constructed for this study. Aggregation theory points to the superiority of the superlative indexes (such as the Divisia and Fisher Ideal) over the simple sum index (named as such because this method literally means that the component assets are added together to form the aggregate) because of some implicit assumptions of the simple sum index which are not empirically supported in the way consumers regard various monetary assets. In not taking account of the relative prices of various monetary assets in the aggregation procedure, the simple sum index assumes by implication that all monetary assets are perfect (and dollar-for-dollar) substitutes for each other. The fact that most economic agents hold a portfolio of monetary assets that have significantly different opportunity costs rather than a single asset with the lowest cost is strong evidence against the idea that owners regard these assets as perfect substitutes.[117]

The microeconomic theory of aggregation advocates that the component assets be weighted (in a nonlinear fashion) in the aggregation process, according to their degree of "moneyness". (Linear weighted aggregation still implies perfect but not dollar-for-dollar substitutability among the component assets.) The weights used in the aggregator functions described earlier (and in the Divisia and Fisher Ideal indexes) are the user costs of the assets, and these do account for the opportunity cost of holding the various component monetary assets relative to a benchmark asset.

Once the Divisia, Fisher and currency equivalent aggregates for M1, M2 and M3 were constructed, it was then possible to compare and contrast these with the simple sum aggregates.[118] The aggregation literature generally predicts that the numbers generated by the Divisia and Fisher Ideal indexes will closely correspond, and so constructing both was done here mostly as a means of verifying that the calculations regarding the Divisia

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[117] See Anderson, Jones and Nesmith (1997, p.31)

[118] M2+ aggregates were not calculated here due to the difficulty in finding satisfactorily detailed data with regard to both asset categories and interest rate proxies.

aggregates were done correctly. (The Divisia aggregates were chosen as the primary aggregation theoretic money measures over the Fisher because the Divisia index is superlative in a stronger sense. The advantage of the Fisher Ideal index over the Divisia is that it remains well defined in periods when new assets are introduced whereas this is not the case for the Divisia index. However, using the Fisher Ideal index to construct the aggregates in period for which new monetary assets are introduced and using the Divisia index to construct the aggregates for all other time periods compensates for this weakness of the Divisia index. In this sense, the Divisia aggregates discussed here have values in certain time periods which were determined using the Fisher Ideal index.) In comparing the various index numbers to each other, the Divisia and Fisher indexes correspond almost exactly as predicted, while the Divisia was more stable over time than either the currency equivalent or simple sum aggregates. The currency equivalent index is the least stable of the three.

Once all the aggregates were obtained, unit root tests were performed on the logged levels of all these variables, the price level (the CPI index), real income or GDP (the IPI index) and nominal income or GDP (the product of the CPI and IPI indexes). The null hypothesis of a unit root could not be rejected for any of the time series variables. Next, tests were performed to see whether any of the money measures were cointegrated to the price or income series. Only currency equivalent M1 was found to be cointegrated with the price and income series out of all the monetary aggregates tested.

Finally, sets of causality tests were performed on the money measures with respect to the price and income series. Once the unit roots were removed from all variables by first-differencing them, at the 5% level no evidence was found for a causality relationship between any of the money measures and prices or income. This contradicts other causality studies that have been performed in the past on the relationships between these variables have typically found some evidence of causality between money and prices or income in the economy. The results obtained here give no evidence that money measures possess information which may be used to forecast future movements in price or income variables.

The first-differenced variables allow one to look at the relationship between money growth measures and price and income growth. Since no indication was given of a significant relationship between money growth and price or income growth, some support is found for the neutrality and superneutrality of money hypotheses.

Some previous studies examining the merits of aggregation theoretic money measures such as the Divisia and Fisher as compared to the simple sum, have supported the view that simple sum aggregates generally forecast price and income movements better than the Divisia and Fisher aggregates.[119] Others, such as Barnett, Fisher and Serletis (1992) suggest that the simple sum index should be abandoned both as a source of research data and as an intermediate target or indicator for monetary policy.[120] This is because they view the grounding of aggregation methodology in microeconomic theory and the nonlinear-weighting of monetary assets as being fundamentally important and so the superlative class of aggregates is regarded as superior from this point of view. Since the causality tests give no indication that the simple sum aggregates perform better than

[119] See Longworth and Atta-Mensah (1995)

[120] See Barnett, Fisher and Serletis (1992, p.2115)

the Divisia aggregates in terms of forecasting movements in price and income, no new evidence is found to give reason to prefer one set of aggregates over another.

Some previous literature has also provided evidence in favor of the endogenous money hypothesis, the position that changes in economic conditions such as prices or income Granger-cause changes in money measures.[121] No evidence is found for this order of causality between prices or income and the money measures in the tests conducted in chapter 6. Therefore no support is found for the endogenous money hypothesis.

The empirical tests performed in this thesis were only cursory at best, and represent only the beginning of an investigation of the merits of constructing aggregation theoretic money measures. The database of Divisia, Fisher and currency equivalent aggregates form an important resource for further investigation using the kinds of tests described above. Models of money demand may be created from the money aggregates constructed here. Investigation into the elasticities of substitution between various Divisia aggregates and the investigation into the aggregation bias featured in the construction of the simple sum and Divisia aggregates represent other important avenues for research, using the aggregates constructed here as a starting point.

On the most general level, the fundamental issue concerning policymakers regarding the various methods of aggregation discussed in this thesis, are the benefits and costs of the Divisia method as opposed to the simple sum. The Bank of Canada and most central banks have refrained from constructing Divisia and Fisher aggregates because of the added costs associated with doing so. It is true that the simple sum aggregates will always be easier to construct. The fundamental belief of the central banks to date has

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[121] See Serletis and King (1993, p.96)

been that the added costs of constructing the Divisia aggregates are prohibitive. At this time, no recommendation can be submitted to the central banks one way or the other as to whether this is correct. However, the primary purpose of the database of money measures constructed here is that further empirical studies may be conducted so that a more satisfactory answer can be obtained as to whether simple sum aggregates or Divisia aggregates indeed are the more reliable and accurate money measures. It would also be desirable for the difference in quality to be sufficient to allow a recommendation to be made. If the results do indeed favour the Divisia approach then the issue of cost is less significant than the central banks might believe. It is trivial to calculate the Divisia aggregates in the computer age, once the quantity data and interest rates series are selected and found. Nor is the issue of data availability a severe obstacle, since the Bank of Canada already provides a comprehensive set of accounts for virtually every monetary aggregate to at least the quarterly frequency.

Therefore, here we stipulate that the cost issue is no longer a sufficient argument against using the Divisia index. More emphasis should instead be placed on the relative accuracy of one aggregation approach over another. There is enough evidence that the simple sum and Divisia indexes perform differently in terms of forecasting income and prices, if not from this study but rather in past studies, and this warrants further investigation.

Bibliography

Anderson, Richard G., Barry E. Jones and Travis D. Nesmith. "Introduction to the St. Louis Monetary Services Index Project," *Federal Reserve Bank of St. Louis Review*, January/February 1997, pp. 25-29.

Anderson, Richard G., Barry E. Jones and Travis D. Nesmith. "Monetary Aggregation Theory and Statistical Index Numbers," *Federal Reserve Bank of St. Louis Review*, January/February 1997, pp. 31-51.

Anderson , Richard G. Barry E. Jones and Travis D. Nesmith. "Building New Monetary Services Indexes: Concepts, Data and Methods," *Federal Reserve Bank of St. Louis Review*, January/February 1997, pp. 53-82.

Arrow, K.J. and F.Hahn. *General Competitive Analysis*. Holden-Day Inc., 1971, 452 pp.

Barnett, William A. "Developments in Monetary Aggregation Theory," *Journal of Policy Modeling*, Vol. 12(2) 1990, pp. 205-257.

Barnett, William A. *Consumer Demand and Labor Supply: Goods, Monetary Assets and Time*. Amsterdam: North-Holland, 1981.

Barnett, William A. Barry E. Jones, and Travis D. Nesmith. "Divisia Second Moments: An Application of Stochastic Index Number Theory," *International Review of Comparative Public Policy*, vol. 8 1996, pp. 115-38.

Barnett, William A., Douglas Fisher and Apostolos Serletis. "Consumer Theory and the Demand for Money," *Journal of Economic Literature*, Vol. XXX December 1992, pp.2086-2119.

Barnett, William A. and Yi Liu. "Beyond the Risk Neutral Utility Function," *paper presented at the University of Mississippi Conference on Divisia Monetary Aggregation*. October 1994. conference volume forthcoming.

Barnett, William A. Yi Liu and Mark Jensen. "CAPM Risk Adjustment For Exact Aggregation Over Financial Assets," *Macroeconomic Dynamics*, Vol. 1 1997, pp. 485-512.

Barnett, William A. and Apostolos Serletis. "A Dispersion-Dependency Diagnostic Test for Aggregation Error: With Applications to Monetary Economics and Income Distribution," *Journal of Econometrics*, January/February 1990, pp.5-34.

Binhammer, H.H. *Money, Banking and the Canadian Financial System*. 6^{th} ed. Nelson Canada, 1993, 580 pp.

Caves, Douglas W., Laurits R. Christensen, and W. Erwin Diewert. "The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity," *Econometrica*, November 1982, pp. 1393-414.

Chrystal, K.A. and R. McDonald. "Empirical Evidence on the Recent Behaviour and Usefulness of Simple Sum and Weighted Measures of the Money Stock," *Federal Reserve Bank of St. Louis Review*, 1976, pp. 73-109.

Clower, R.W. "A Reconsideration of the Macroeconomic Foundations of Monetary Theory," *Western Economic Journal*, Vol. 6, 1967, pp.1-8.

Cockerline, J.P. and J. Murray. "A Comparison of Alternative Methods of Monetary Aggregation: Some Preliminary Evidence," *Bank of Canada Technical Report 28.*

Diewert, W. Erwin. "Seasonal Commodities, High Inflation and Index Number Theory," *Working Paper DP96-06*,.University of British Columbia, January 1996.

Diewert, W. Erwin. "The Treatment of Seasonality in a Cost-Of-Living-Index," *Price Level Measurement*, W.E. Diewert and C. Montmarquette, eds., Ottawa: Statistics Canada, 1983, pp.1019-45.

Diewert, W. Erwin. "Aggregation Problems in the Measurement of Capital," *The Measurement of Capital*, Dan Usher, ed., University of Chicago Press, 1980, pp.433-539.

Diewert, W. Erwin. "Exact and Superlative Index Numbers," *Journal of Econometrics*, Vol. 4(2) May 1976, pp. 115-45.

Diewert, W. Erwin. "Intertemporal Consumer Theory and the Demand for Durables," *Econometrica*, May 1974, pp. 497-516.

Farr, Helen T., and Deborah Johnson. "Revisions in the Monetary Services (Divisia) Indexes of the Monetary Aggregates," *Staff Study 147. Board of Governors of the Federal Reserve System*, December 1985.

Feenstra, R. "Functional Equivalence between Liquidity Costs and the Utility of Money." *Journal of Monetary Economics*, Vol. 17 1986, pp.271-291.

Feige, Edgar L. and Douglas K. Pearce. "The Substitutability of Money and Near-Monies: A Survey of the Time-Series Evidence," *Journal of Economic Literature*, June 1977, pp. 439-465.

Fisher, Irving. *The Making of Index Numbers: A Study of Their Varieties, Tests, and Reliability*, Houghton Mifflin Company, 1922.

Friedman, Benjamin. "Lessons on Monetary Policy from the 1980's," *National Bureau of Economic Research*, Working Paper #2551, April 1988.

Friedman, Benjamin M. and Kenneth N. Kuttner. "Money, Income, Prices and Interest Rates," *American Economic Review*, Vol. 82, No. 3, June 1992. pp. 472-492.

Friedman, Benjamin M. and Kenneth N. Kuttner. "Money, Income and Prices after the 1980's" *National Bureau of Economic Research*, Working Paper #2852, February 1989, 22 pp.

Gujarati, Damodar N. *Basic Econometrics*. McGraw-Hill, Inc. 1995, 838 pp.

Hill, Robert J., and Kevin J. Fox. "Splicing Index Numbers," working paper, School of Economics, The University of New South Wales, 1995; *Journal of Business and Economic Statistics*, forthcoming.

Hostland, D., S. Poloz and P. Storer. "An Analysis of the Information Content of Alternative Monetary Aggregates," *Bank of Canada Technical Report 48*.

Longworth, David and Joseph Atta-Mensah. "The Canadian Experience with Weighted Monetary Aggregates," *Bank of Canada Working Paper 95-10*, 1995, 76 pp.

Klein, Benjamin. "Competitive Interest Payments on Bank Deposits and the Long-Run Demand for Money," *American Economic Review*, December 1974, pp. 931-49.

Pindyck, Robert S. and Daniel L. Rubinfeld. *Econometric Models and Economic Forecasts. 4th ed.* Irwin McGraw-Hill, 1998, 634 pp.

Serletis, Apostolos. "The Empirical Relationship Between Money, Prices and Income Revisited," *Business and Economic Statistics*, Vol. 6, No. 3 July 1988, pp.351-357.

Serletis, Apostolos and Martin King. "The Role of Money in Canada." *Journal of Macroeconomic Dynamics*, Vol. 15 No. 1, Winter 1993 pp. 91-107.

Sidrauski, Miguel. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review*, Vol. 57, 1967, pp. 534-544.

Siklos, Pierre L. *Money, Banking and Financial Institutions: Canada in the Global Environment.* 2nd ed. McGraw-Hill, Ryerson Limited, 1997, 678 pp.

Startz, Richard. "Implicit Interest on Demand Deposits," *Journal of Monetary Economics*, October 1979, pp. 515-34.

Theil, Henri. *Economics and Information Theory*. Amsterdam: North-Holland, 1967.

Thoma, Mark A. and Jo Anna Gray. "Financial Market Variables Do Not Predict Real Activity," *Economic Inquiry*, Vol. XXXVI October 1998, pp. 522-539.

Turnovsky, Stephen J. *Methods of Macroconomic Dynamics*. MIT Press, Cambridge Mass. 1995, 535 pp.

Walras, Leon. *Elements of Pure Economics*. William Jaff, trans. London: Allen and Unwin, 1954, 620 pp.