2015-06

One Step Back, Three Forward: Success Through Mediated Challenge

Metz, Martina

University of Calgary

Metz, M., Sabbaghan, S., Preciado, P. & Davis, B. "One Step Back, Three Forward: Success Through Mediated Challenge" (2015). In Preciado Babb, P., Takeuchi, M., & Lock, J. (Eds.). Proceedings of the IDEAS: Designing Responsive Pedagogy Conference, pp. 178-186. Calgary, Canada: Werklund School of Education, University of Calgary.

http://hdl.handle.net/1880/50872

ONE STEP BACK, THREE FORWARD: SUCCESS THROUGH MEDIATED CHALLENGE

Martina Metz, Soroush Sabbaghan, Paulino Preciado, Brent Davis
Werklund School of Education, University of Calgary

How can you keep all students engaged in deepening their mathematical understanding without overwhelming the weakest students or boring the strongest? Teachers in the Math Minds project design lessons around structured sequences that seek to engage all students with questions on which they can succeed, and to then proceed through increasingly sophisticated variations. Teachers attend closely to student responses so that they can adjust difficulty in a manner that allows success and challenge for all. In this paper, we describe key principles that have emerged from the Math Minds initiative. We draw particular attention to variation theory (Marton, 2015) and consider how it plays out in interaction with the other principles.

Keywords: Mathematics education; Mastery learning; Variation theory

WHAT IS MATH MINDS?

The ideas we report here are based on our work as researchers for the Math Minds project, a five-year partnership between the Werklund School of Esducation, JUMP Math, the Calgary Catholic School District, the Calgary Public Library, and the Boys' and Girls' Club of Calgary and sponsored by Canadian Oil Sands Limited. The project aims to enhance early numeracy, and our research is framed within the broad goal of understanding what teachers need to know to effectively teach elementary mathematics. Our research team is particularly interested in how

2015. In Preciado Babb, Takeuchi, and Lock (Eds.). *Proceedings of the IDEAS: Designing Responsive Pedagogy*, pp. 178-186. Werklund School of the Education, University of Calgary.

access to a particular resource (in this case, JUMP Math, 2015) and related professional development pertaining to effective use of that resource might support (a) the development of teachers' knowledge for teaching mathematics and (b) student achievement. We begin with a brief summary of core principles emerging as significant to our work, with a particular focus on how variation theory (Marton, 2015; Runesson, 2005; Watson & Mason, 2006) both supports and is informed by the other principles.

CORE PRINCIPLES

After one year of teachers' supported use of JUMP materials with all students in a K-6 school with a history of low mathematical performance, we observed a number of changes in how students engaged with mathematics. Based on weekly observations and student interviews, we noticed that students who previously struggled become more willing to take part, and many students became excited to keep pushing their understanding to new levels. At the same time, students' scores for mathematics on the Canadian Test of Basic Skills (Nelson, 2014) showed a significant increase in national percentile rankings from 2013 ($M = 30.28$, $SD = 25.06$) to 2014 ($M=36.37$, $SD=23.88$); $t(66)=-2.97$, $p = 0.004$.

Here, we elaborate the core principles that we believe are significant to the successes we have observed. While each is connected to a significant body of educational research in its own right, it appears that together they can inform mathematics education in ways that go beyond the sum of their parts. Our work is grounded in the assumption that all students can succeed at challenging mathematics. Blackwell, Trzesniewski, and Dweck (2007) emphasized the importance of a *growth mindset*: They showed that teaching students that mathematical ability is learnable (not innate) supports achievement in mathematics. Following JUMP Math, we take this one step further by structuring instruction in ways that *demonstrate* to students that they *are* successful—importantly, without extensive remediation. JUMP's methods (JUMP Math 2015; Mighton, 2007) adhere to key principles of *mastery learning* (Guskey, 2010), which emphasizes the importance of pre-assessment to determine an appropriate entry level, group instruction, regular formative assessment, instruction that continually responds to assessment, and opportunities for extension. In our work, this has included a clear emphasis on starting at a point that every student can be included and engaged, stepping up in small increments while ensuring nobody gets left behind, and offering frequent extensions so that all remain sufficiently challenged and engaged. If students struggle, the teacher creates a smaller step rather than attempting to remediate (*cf*. Preciado-Babb, McInnis, Metz, Sabbaghan, & Davis, 2015; Sabbaghan, Metz, Preciado-Babb, & Davis, 2015). *Formative assessment* (Wiliam, 2011), then, is key: Student responses continually influence next steps in instruction—i.e., many times during a single lesson. All of this is consistent with the literature on *intrinsic motivation*, which emphasizes the importance of autonomy, mastery, and purpose (Pink, 2011). Data we have gathered from interviews with students from Grades 1 through 6 has shown that students at all levels like math best when they can succeed with little assistance from the teacher. Key aspects of purpose include appropriate challenge and connecting to others, both of which become evident in a classroom where all students confidently engage together in a lesson that climbs steadily to higher levels. By building a strong base of common understanding, students more easily form a *learning collective* in which they are further able to engage together in challenging work that draws on their diverse interests and abilities (Davis & Simmt, 2003) and provides a context in which *emergent mathematical understanding* (Davis & Renert, 2015) can flourish. By starting at a level that includes everybody and moving steadily forward together (with some extending further than others), it is possible for all students to move further than they might otherwise have done. *Variation theory* provides a way of attending to mathematical structure that supports each of these principles.

JUMP Math (2015) is a carefully sequenced mathematics program that demonstrates aspects of effective mathematical variation. The resources were developed and are frequently refined in response to student engagement and understanding (Mighton, 2007). In our analysis of and work with these materials, we have explicitly invoked variation theory, while maintaining a clear emphasis on student engagement and understanding as key litmus tests of effectively sequenced instruction. Variation theory has proven particularly helpful in our attempts to support teachers in extending what is offered in the resources, both in terms of creating easier steps and creating extensions. This is important to students both psychologically in terms of creating high levels of success and challenge and mathematically in terms of portraying mathematics as connected and extendable.

In a nutshell, variation theory includes (a) clear focus on a particular "object of learning"; (b) identification of features that can vary within that object of learning ("dimensions of possible variation"); (c) the development of sequences of exercises that systematically vary these features, first one at a time (while holding others constant), then simultaneously; and (d) attention to contrasts and generalizations that can become apparent in this space of variation.

Consider how the items in Table 1 vary from one to the next (we encourage the reader to work through them). In addition to number facts, what might be learned here? In other words, what is the object of learning?

Addition Sequence	Subtraction Sequence
7+5=	7-5=
7+6=	7-6=
7+7=	7-7=
8+7=	8-7=

9+7=	9-7=
10+7=	10-7=
11+8=	11-8=
12+9=	12-9=
13+10=	13-10=
14+9=	14-9=
15+8=	15-8=
16+7=	16-7=

Table 1: Addition and subtraction sequences.

In each sequence, first one number, then the other, and then both increase by one, and then one goes up while the other goes down (and further note that the effect of this variation is not the same for the addition sequence as for the subtraction sequence). The point of such variation is to draw attention to relationships between changing addends (or minuends/subtrahends) and sums (or differences). Since stumbling at one step could create significant difficulty at the next, assessment of all students at each step is essential. It may be important to include more repetition within particular sections or to insert smaller steps. For example, it may be necessary to explore *decreasing* addends or subtrahends/minuends before attempting "one-up-one-down." The list can be extended as needed with more challenging items, for example by increasing or decreasing the numbers by amounts larger than one. Much larger numbers could be used to help draw attention to relationships rather than to isolated calculations. For example, given 73+55=128, what is 74+55? Consider a second example. Recently, Metz was working with a student to count money. After ensuring that he could skip-count by 5s, 10s, and 25s, she was tempted to ask him to find all the ways to make a particular amount of money with nickels, dimes and quarters. This could prompt him to think more flexibly about coin combinations, to work systematically, and to consider how he knew when he was finished. Instead, she asked him to "find a way to make 30 cents that uses 3 coins." Although this variation has a single correct answer, trying something and then adjusting also provides a context where flexibly grouping coins becomes likely, and without adding the additional difficulty of considering "all combinations." Extending the question to "find a way to make 30 cents that uses *4* coins" is a small enough variation that it might prompt adjustment to the first problem to find a solution to the second; e.g., trade one dime for 2 nickels to get an extra coin. Note that this would be less likely if the next question were not closely connected to the first. Students might then be asked to find solutions with 5 coins or 6 coins…then perhaps 2, which would involve re-combining nickels into a quarter. They might try a similar progression for 35 cents. Having thus had some experience with *systematically* manipulating the coins, they might *then* be asked to find *all* the ways to make 40 cents. Anything from 50 to 74 cents adds the possibility of a second quarter and allows many more possibilities. In this way, the size of step can easily be tailored to the level of challenge that a particular child finds engaging, but everybody gets to experience success at the base level, everybody has the opportunity to master the object of learning for the lesson, and everybody can get a "bonus." Maybe somebody will find that finding all the ways to make $0.55 is an interesting challenge. However, starting there with the idea that everybody can at least find an entry point can sap motivation and overwhelm.

Now consider the previous examples in light of the questions in Table 2. In doing so, we hope the significance of the *set* becomes apparent. Here, variation theory is informed by other principles deemed significant to Math Minds. For example, Questions 4 and 5 attend to intrinsic motivation and mastery learning, and Questions 6 and 7 attend specifically to emergent knowing.

1. What is the object of learning?
2. What dimensions can vary? What should stay the same?
3. Once explored individually, are there dimensions that can co-vary?
4. What prior knowledge is assumed for the simplest variation?
5. Are variations sufficiently broken down that students can approach each step independently?
6. What more could be learned from the set than from a single example or

from a poorly structured set?
7. How could the set be extended?

Table 2: Variation in Math Minds.

Variation comes in many forms. At times, the same problem might be solved with various strategies. For this to prompt deeper understanding than what was already available to any one particular student, each strategy can first be explored with its own variations before juxtaposing *different* strategies. In this way, attention is freed to consider relationships between various strategies, rather than merely an accumulation of alternative (and sometimes inefficient) ways to do something. Variation may also occur in increasingly complex mathematical contexts as students achieve mastery of component pieces; here students' attention may shift to the parsing and sequencing of familiar pieces. Again, starting small and building up can help ensure that everyone is included without placing a ceiling on how far a sequence might be extended.

There are times when it is important for students to take part in identifying and varying potential dimensions of variation, particularly when the variables and their impact are readily discernible by students. A very simple example might be found in a vintage children's toy, the Etch-a-Sketch. Essentially, it is a screen with a magnetic "cursor" that traces a line and can be moved up/down by turning one knob clockwise or counter-clockwise or right/left by turning the other knob. When challenged to draw a simple spiral (for example), students can experiment with the dimensions of variation available in the knobs and judge their progress by the image that appears on the screen. It is also possible to make discerning, naming, and exploring potential dimensions of variation the object of learning (*cf*. Metz & Simmt, 2015). In each case, it is important to consider what might vary and how the proposed tasks will draw attention to this variation.

SUMMARY

Taken together, the Math Minds principles offer a way to attend to both deep mathematical structure and the development of mathematical fluency. When teacher and student attention remains focused on key ideas and their continuous extension (and combination), all may participate in mathematical exploration in ways that avoid some of the difficulties sometimes experienced as teachers attempt to transition away from transmission-based models of math instruction (see Swan, Peadman, Doorman, & Mooldjik, 2013). By attending closely to the mathematical variation available to be experienced in a particular context and by structuring that variation in a responsive manner that allows all to succeed and all to be challenged, learners have the opportunity to expand their mathematical understanding in a manner that supports both deeply interconnected and emergent understanding as well as mathematical fluency.

REFERENCES

Blackwell, L., Trzesniewski, K. & Dweck, C. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development 78*(1), 246-263.

Davis, B. & Renert, M. (2015). *The math teachers know: Profound understanding of emergent mathematics*. New York, NY: Routledge.

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education 34(2),* 137-167.

Guskey, T. (2010). Lessons of mastery learning. *Educational Leadership 68*(2), 52-57.

JUMP Math (2015). JUMP Math. Retrieved from https://jumpmath.org/jump/en

Marton, F. (2015). *Necessary conditions of learning*. New York, NY: Routledge.

Metz, M. & Simmt, E. (2015). Researching mathematical experience from the perspective of an empathic second-person observer. *ZDM Mathematics Education 47*(2), 197-209.

Mighton, J. (2007). *The end of ignorance: Multiplying our human potential*. Toronto, Canada: Alfred A. Knopf.

Nelson (2014). *Assessment*. Retrieved from http://www.assess.nelson.com/default.html

Pink, D. (2011). *Drive: The surprising truth about what motivates us*. New York, NY: Riverhead.

Preciado-Babb, McInnis, Metz, Sabbaghan & Davis (2015). Epiphanies in mathematics teaching: The personal learning of an elementary teacher in the Math Minds initiative. In A. P. Preciado-Babb, M. Takeuchi & J. Lock (Eds.) *Proceedings of the IDEAS: Rising to Challenge Conference*. Calgary, Alberta: Werklund School of the Education, University of Calgary.

Runesson, U. (2005). Beyond discourse and interaction. Variation: A critical aspect for teaching and learning mathematics. *Cambridge Journal of Education 35*(1), 69-87

Sabbaghan, Metz, Preciado-Babb & Davis (2015). Dynamic responsive pedagogy: Implications of micro-level scaffolding. In A. P. Preciado-Babb, M. Takeuchi & J. Lock (Eds.) *Proceedings of the IDEAS: Designing Responsive Pedagogy*. Calgary, Alberta: Werklund School of the Education, University of Calgary.

Swan, M., Pead, D., Doorman, M, & Mooldjik, A. (2013). Designing and using professional development resources for inquiry-based larning. *ZDM Mathematics Education 45*, 945-957.

Watson, A. & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning 8*(2), 91-111.

Wiliam, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree.