

Conditional Variability

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1.0 Introduction

Conditional statements are presumed to be understood by both speaker and listener. If the conditional statement is true, all parties concerned presume that the statement is true for some reason. When a speaker asserts, "If kangaroos had no tails they would topple over," then both speaker and listener know exactly what the statement means and whether or not it is true even if there are in fact no kangaroos in the world without tails. Asserting that conditional statements are understood and known to be true or false is one thing. Stating exactly what it is that is understood or known is quite another.

In this paper it will be shown that when a conditional statement is understood or known to be true, a number of implicitly specified variables are given more or less concrete values. Each of the variables will be defined and examples will be employed to demonstrate their use in conditional evaluation. From time to time this analysis in terms of variables will be contrasted with a 'possible worlds' analysis of conditionals. The purpose of this paper is not to argue against the possible worlds analysis but rather to provide an alternative to that analysis.

2.0 Background

In logic conditional statements are symbolized ($A \rightarrow B$) and are of the general form "If A is the case then B is the case." They are false if and only if the antecedent, A, is true and the consequent, B, is false.

There are two types of conditional statements: the material conditional, symbolized as above or sometimes with a 'hook' symbol; and the strict conditional, which asserts the necessity of the corresponding material conditional, symbolized variously with the 'fish-hook' or entailment symbol.

Conditional statements were intended to correspond with similar statements in natural language; the idea was that sentences like "If it rains I'll get wet" could be represented in formal notation and given truth values

by deduction from other formalized sentences. This ambition was never realized. A large class of conditional statements, called variously 'subjunctive conditionals' or 'counterfactuals' resisted analysis into the strict or material conditional form. By 'counterfactuals' I mean the following forms of conditional statements: statements with false antecedents such as "If Oswald had not shot Kennedy then he would be alive today", causal statements and statements which predict into the future such as "If it rains the river will rise", and subjunctive statements such as "If he had ambition he would go far."

The failure to analyze counterfactuals in terms of material or strict conditionals has two related causes. First, many counterfactuals, although true, are not necessarily true. There are some instances in which the antecedent may be true, the consequent false, and the statement as a whole true. Second, many laws of inference such as 'strengthening the antecedent' which are valid for material and strict conditionals are not valid for counterfactuals.

A recent development in philosophy has been the analysis of counterfactuals not as material or strict conditionals but rather as a distinct conditional connective with its own rules of inference: the variably strict conditional (see Lewis 1973a and Stalnaker 1968). This analysis comes with a price: the truth of a variably strict conditional is determined on the new analysis not by the state of affairs in the world but rather by the state of affairs in a possible world. The possible world selected is one in which the conditional is no longer counterfactual - what was false has become true, what was in the future has now occurred - and is selected on the basis of relevant similarity with the actual world if the corresponding conditional is true in the possible world.

The possible worlds analysis of conditional statements has severe problems. How can we select a possible world on the basis of similarity if at least part of that similarity might depend on the truth of the very counterfactuals we are trying to analyze? I am not concerned to press that argument here. Rather, I wish to focus on an alternative. The suggestion is this: counterfactuals are variable because of implicit variables in the counterfactual conditional relation. These variables, if stated explicitly, may be employed in part to provide the framework of an analysis of counterfactuals which occurs in this world and not in some possible world.

3.0 Component Strength

Conditional statements may vary in strength according to the truth values of the components A (the antecedent) and C (the consequent). I am not concerned in this paper with how the truth of A and C is established. I merely wish to indicate that, if determinable, it is determinable in a variety of manners.

Let us first consider the antecedent A. The antecedent A may be true or false. In the latter case, the conditional is called a counterfactual, or more precisely, contrary-to-fact conditional. In cases where A is true, although the term 'counterfactual' still misleadingly applies, perhaps the term 'factual conditional' is more appropriate.

There are also some cases in which the antecedent may be undetermined or undeterminable; consider, for example:

- (1) If it rains tomorrow the crops will grow.

The antecedent "it rains tomorrow" is neither true nor false, for tomorrow has not yet occurred. An antecedent which is a tautology will always be true; an antecedent which is a contradiction will always be false. Depending on the semantics chosen, there may be a wide range in between.

Like the antecedent, the consequent may be of varying truth value. In many cases (and most especially in many of the examples we choose to discuss) the consequent is known to be true or false. Even if the consequent is false the conditional itself may be true. This is most clearly demonstrated by the material conditional: if A is false and C is false then $A \rightarrow C$ is true.

For our purposes the most interesting cases are those in which the truth value of the consequent is not known, undetermined, or in some other way not certainly true and not certainly false. The recognition that the truth values of the components of conditionals may vary serves almost immediately to prevent some philosophical errors. For example, Eisenberg (1969) argues that all counterfactuals must be explicable by the 'conjunction analysis' as follows:

- (2) $[(x) (Mx \rightarrow Px)] \& [Mz \rightarrow Pz] \& [-Mz] \& [-Pz]$

The relevant portions of this analysis for this discussion are the negations $[-Mz]$ and $[-Pz]$. As Williamson (1970) points out, the antecedent and the consequent need not be false for the statement to qualify as a counterfactual.

Suppose the following example:

(3) If Wayne had been here it would have been a good party.

The counterfactual may be true, but someone may respond,

(4) Wayne was here and it was a good party, but you were in the kitchen all night, didn't see him, and missed all the fun.

We may thus allow that the truth values of the components may vary. The components may be absolutely true or false or, depending on the semantics, anywhere in between.

Let us examine a little more precisely the ways in which the truth values of the components may vary. First, the components of the counterfactual may vary because the truth of various propositions is variable in the world. For example, something may be 'possibly' true. It might rain tonight, for example. That does not mean that it is true, but it is also misleading to say that it is false. That "it will rain" is possibly true is a fact about the world; the truth value of "it will rain" is therefore variable.

Sometimes propositions which are 'possibly' true might have their truth values fixed more precisely in terms of 'probability'. For example, "This atom of uranium will decompose" is a statement which has a certain precise probability of being true. Such a probability value is not arbitrary; the rate of uranium decomposition just is a probability function.

Second, components of a counterfactual may also vary because of how much (or how little) we know about the world. The most common occurrence of this is in the statement of statistical hypotheses such as "The NDP is supported by forty per cent of Canadians." Variable truth values in such instances are explicitly stated: "samples of this size are accurate to five per cent nineteen times out of twenty."

In other cases the variable quality of our knowledge of some statement cannot be so precisely measured. Statements like "I'm reasonably certain" or "I have little doubt" express this. The nature of our determination affects this variable; if we see that the car is red we are quite certain that the car is in fact red; if we are told by a friend that the car is red then we are less certain.

4.0 Saliency

Counterfactual truth may vary as determined by relevant or 'salient' factors. These factors are best described using an example. Consider the following pair of counterfactuals (from Quine 1960:222):

(5) If Caesar had been in command (in Korea) he would have used the atom bomb.

(6) If Caesar had been in command he would have used catapults.

By 'context' we mean the situation in which one or another of these counterfactuals would have been asserted: a political science class, perhaps, or a history seminar. 'Saliency' is determined by context. It refers to those qualities of Caesar which are the most important to the discussion taking place. Which of (5) or (6) is true will depend on what quality of Caesar's is most salient. In this case, if Caesar's primitive knowledge of technology is most salient, then (6) will be true. If Caesar's ruthlessness is most salient, then (5) would be true.

On the possible worlds analysis, statements about Caesar's use of the bomb or catapults are analyzed as above in terms of saliency and context. The possible world selected for reference will be the one which is most similar to the actual world with respect to these salient qualities. On the analysis presented in this paper, saliency is employed directly in the determination of truth values for counterfactuals. Saliency is presented in terms of a closely related notion, vagueness.

To show how this works, let me consider an example.

(7) If it reaches -40 tonight, Calgary will be the coldest city on the Prairies.

It might reasonably be argued that Calgary is not on the Prairies; rather it is in the foothills, and so could not be the coldest city on the Prairies no matter what. Whether or not this counterfactual is true depends on how 'Prairies' is defined. Since it is not a precise geographical region its boundaries are vague. On some accounts, Calgary is on the Prairies, on others it is not. In fact 'Prairies' refers to not just one geographical area but many, each differently defined. Some such definitions are not complete definitions; the eastern border of the Prairies is not defined at all but the western border is defined as 'east of Calgary'.

On this analysis we mean by 'salience' the specification of exactly which of the varying specifications of some vague term will be employed. What we know of Caesar is vague at best. We know that Caesar lived in ancient Rome and that he was a brilliant though somewhat ruthless tactician. To assess the pair of counterfactuals above we must define Caesar more precisely: "Caesar the ancient Roman" or "Caesar the ruthless". If choosing between either of the two options we may have to consider the truth values of each proposed definition of Caesar. These may vary just as truth values for the different components of a counterfactual vary.

5.0 Connective Strength

The strongest form of the conditional connective is necessary implication. That is, if A is true and known to be true and the connective is expressed $A \rightarrow C$ then C must be true and known to be true. Both the material conditional and the strict conditional are conditional connectives of this form. If the conditional is true, then if the antecedent is true, the consequent must be true. Showing one instance in which the antecedent is true and the consequent false shows that the conditional is false.

As discussed in section 2 above, both Lewis and Stalnaker propose a third type of conditional, the variably strict conditional. This conditional is employed to symbolize what we mean when we use counterfactuals. It should be clear that the strength of the variably strict conditional does not lie somewhere between the strength of the material and strict conditionals, for the strength of the latter two is identical. The variably strict conditional is a form of conditional which has a weaker connective strength than either the material or strict conditional. This difference may be characterized as follows. If A is the antecedent and C is the consequent and $A \rightarrow B$ is the variably strict conditional, then if A is true C might not be. The variably strict conditional is not necessarily truth or falsity preserving. We may illustrate this using the previously mentioned rule of strengthening the antecedent.

Suppose some conditional statement ($A \rightarrow B$) is true. According to the rule of strengthening the antecedent, if some C is conjoined with the antecedent A then the resultant conditional $((A \& C) \rightarrow B)$ remains true. This law is valid for material and strict conditionals but not valid for variably strict conditionals. Conjoining some C to the antecedent can change the truth value of the corresponding conditional. The variably strict conditional may be more or less strong depending on how much or how little needs to be added to the antecedent to cause a change in truth value.

It would be a mistake, I suggest, to suppose that there is only one type of variably strict conditional. They might be quite strong or they may have no strength at all. The failure to recognize this latter possibility lies at the heart of many criticisms of Lewis and Stalnaker. Consider, for example, the following argument proposed by Bennett (1974). According to Bennett, Lewis's analysis fails in the case of the 'accidental' even-if conditional.

Consider the following conditional:

(8) If London is a large city then Jupiter has twelve moons.

If is the case that Jupiter would have twelve moons whether or not London were a large city. If London actually is a large city then, on the possible world account, we should check and see whether Jupiter actually has twelve moons; if it does the conditional is true. If London is not a large city, then according to the possible worlds theory we should consult the nearest possible world in which London is large and count the moons of Jupiter; if there are twelve then the counterfactual is true. On the possible worlds story there would in fact be twelve moons since the size of London does not affect the number of moons possessed by Jupiter.

Bennet argues as follows. While it is consistent to maintain that, in the nearest possible world, Jupiter has twelve moons, it is also consistent to maintain that, in the nearest possible world, Jupiter has thirteen moons. The truth of the accidental conditional is thus, to Bennett, undetermined. Bennett employs this argument to support the alternative 'regularity' theory of counterfactuals. But the regularity theory demands that, if there is no regular relation between the antecedent and the consequent, the counterfactual is false. But why should we say that? It is not determined that Jupiter has twelve or thirteen moons given that London is or is not a large city and so the conditional is neither true nor false.

The accidental conditional is an extreme case. There is no strength to the connection. The conditional is therefore possibly true and possibly false, nothing more. At the other extreme are the material and strict conditionals. The conditional is necessarily true or necessarily false. It is reasonable to suggest that a range of possibilities lies in between. I will suggest just a few of them. Natural or physical laws may be one example. The laws of nature, as Hume demonstrated, are not necessary laws. Many such laws, such as Newton's laws, once considered true, are now generally considered false. We consider the possibility of failure to be a factor when evaluating currently accepted laws. It is not a logical contradiction to entertain their falsity. The

variable strength of such laws is sometimes expressed in an explicitly conditional form: if true, a law. Though weaker than the strict or material conditional, the conditional which expresses a law of nature is nonetheless stronger than an accidental generalization.

A further variation of strength may be the case of non-lawlike non-accidental conditionals. The "dimes in the pocket case" is one such case. Suppose I put my hand in my pocket on Canada Day, 1987, and retrieve a handful of dimes. It is true that, in Canada, all dimes are made of nickel. I could then say:

(9) If I had put my hand in my pocket on Canada Day, 1987, all the coins I would have found would have been made of nickel.

This clearly is not necessarily true. It does not even appear to have the strength of a law of nature. But neither is the conditional an accidental conditional; there is some sort of connection between placing my hand in my pocket and touching nickel.

Although it seems clear that different strengths of a conditional connective are possible, it is not clear how to quantify that variable. What we want is a syntax which will first allow for such a range of values and second determine a syntactic relation between the varying strengthened conditionals within that range. In the next section I shall outline a syntactic structure which permits this determination.

6.0 The Domain of the Conditional

What makes a necessary statement necessary? On the Leibnizian thesis a statement is necessary if it is true in all possible worlds. In conditional terms, a conditional is necessarily true (is of greatest strength) if it is a universal statement. We have seen that not all conditionals are necessarily true; there are varying shades of strength. Therefore universality, a condition suggested by a number of analyses and the first conjunct of Eisenberg's, above, will be sufficient to describe only some small number of counterfactuals.

A law of nature is not a necessary statement. On some possible worlds a law of nature might be different from the laws of nature in the actual world. But laws of nature are expressed in the general form

(10) For all x , if Fx then Gx .

Both necessary truths and laws of nature employ the universal quantifier. Mere use of the universal quantifier will not be sufficient to distinguish between the two. A finer distinction is required. Let me suggest the following.

Consider the size of the domain of the conditional: that is, within what world, worlds, or parts of worlds a conditional is intended to be true. A necessary conditional is intended to be true in all possible worlds. A lawlike statement is intended to be true all over this, the actual, world. We might say that universality expresses the success rate of a conditional within its intended domain. The strength of the conditional may therefore be evaluated according to these two variables: the size of its domain, and its success rate within that domain. A reduction of the domain or a reduction of the success rate may weaken the conditional connective. Exactly how this is to be spelled out is probably a fascinating task and I hope one day to finish it.

7.0 Propositions

During the course of this paper I have not clearly distinguished between counterfactual propositions and counterfactual statements. Let me accomplish this now. The proposition

(11) Brakeless trains are dangerous.

does not refer only to one train but rather to a large number of trains. It is expressed counterfactually as follows:

(12) If any train has no brakes then it is dangerous.

The proposition expressed by (12) is intended to correspond with specific 'instances', in this case, specific trains, as follows:

(13) If train 1 has no brakes it would be dangerous.

(14) If train 2 has no brakes it would be dangerous.

(15) If train n has no brakes it would be dangerous.

The idea is that if each of the instances is true then the proposition as a whole is true. But propositional truth is not an all-or-nothing venture; some instances may be false while the proposition is true. Suppose, for example,

(16) Train 4489 has no brakes and is not dangerous.

Train 4489 also has no engine and has not moved since 1959. Even though (16) is an exception to the general rule that does not mean that the proposition is false. It is true in most cases.

A proposition is a statement that corresponds to more than one instance. Since not all instances need be true for the proposition to be true the strength of a proposition may vary. Propositional variability may be quantified according to the domain of the proposition and the success rate (proportion of true instances) within that domain.

8.0 Causal Counterfactuals

A great number of the counterfactual propositions we assert every day are causal propositions. By that I mean the assertion that some A causes some B to occur. Causal propositions, like other propositions, correspond to a set of instances. If we assert that A causes B then we assert that A1 causes B2, and so on.

There remains a problem to be resolved. Suppose you heat some water. The water boils; that is, little bubbles form and steam rises. The cause of the water boiling is the heat; the symptoms are the steam and bubbles. We could say, quite accurately, that the heat caused the bubbles and the steam. But now it is equally possible to say that, if there are bubbles, then there will be steam; that is, that the bubbles cause the steam to rise. The relation between the heat and the steam is quite different from the relation between the bubbles and the steam; the first is a causal relation, the second an apparently accidental relation.

At the same time, however, the strengths of the two conditionals will be the same. That is, the domain in both cases will be the same (the system described above). The universality will be the same. The truth values of each instance of this proposition will be the same. Yet typically we assert that the causal relation is stronger than the accidental relation. The distinction between the causal conditional and the accidental conditional is contained in the idea of 'causal dependency'. The idea is that the steam and the bubbles depend on the heat, and not each other, in order to occur. A

relation of dependency is an asymmetric relation. That is, if A depends on B then B does not depend on A. Accordingly we test for dependency by testing pairs of counterfactuals: $(A \rightarrow B)$ and $(B \rightarrow A)$. But both $(A \rightarrow B)$ and $(B \rightarrow A)$ will be true in exactly the same instances even in relations of dependency.

We have to consider the contraries of both: $(\neg A \rightarrow \neg B)$ and $(\neg B \rightarrow \neg A)$ (Lewis 1973b). If a relation of dependency exists then in some instances where the effect B is not present the cause A will be present and yet in very few instances where the cause A is not present will the effect B be present. The causal proposition is therefore a complex proposition which depends on the truth values of four corresponding counterfactual propositions. More formally if

(17) $A \rightarrow B$

is a causal proposition then the four corresponding propositions will be

(18) $A \rightarrow B$

(20) $\neg A \rightarrow \neg B$

(19) $B \rightarrow A$

(21) $\neg B \rightarrow \neg A$

each of which will be given a truth value which corresponds to the number of instances in which it is true.

Lewis (1973b) expresses this theory within the context of a possible worlds analysis of counterfactuals but it is not necessary to refer to a possible world to establish the variably strict truth of each of the propositions in question. We therefore retain the strength of Lewis's proposal while omitting the weakness.

9.0 Summary

In this paper it has been shown that a number of variables are implicitly given concrete values when a conditional statement or proposition is asserted. First, the antecedent and the consequent of the conditional may have varying truth values depending on how certain they are in the world and how well they are known. Second, features of the world which are relevant to the evaluation of the conditional which are more or less vaguely defined will be defined precisely. Third, the strength of the conditional connective will vary depending on its intended domain and its intended success rate within that domain. Fourth, conditional propositions which correspond to sets of instances will vary with respect to the number of instances over which the conditional is intended to be true. Fifth, some

conditional propositions will correspond to sets of several other conditional propositions and will be evaluated with respect to the truth value of each of the other conditional propositions.

Given a clear specification of each of these variables it is possible to state exactly what is understood when a conditional statement is understood. In addition, such a clear specification of the variables will specify exactly what must be true for the conditional to be true. It should be understood that conditional truth is not an all-or-nothing venture and that some conditionals will be partly true or even have no truth value at all, depending on the variables. None of these variables requires reference to some possible world for specification. Therefore the analysis proposed in this paper provides a viable alternative to the possible worlds analysis.

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