Manuscript Number: ENEECO-D-08-00311

Title: A 'Simple' Hybrid Model for Power Derivatives

Article Type: Full Length Article

Section/Category:

Keywords: Electricity Pricing; Power Derivatives; Seasonality

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Manuscript Region of Origin:

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Suggested Reviewers: Matt Davison
University of Western Ontario

Matt is well versed in the field of Electricity pricing, and some of his work is cited in our paper.

Opposed Reviewers:

A 'Simple' Hybrid Model for Power Derivatives

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Abstract

This paper presents a method for valuing power derivatives using a supply-demand approach. Our method extends work in the field by incorporating randomness into the base load portion of the supply stack function and equating it with a noisy demand process. We obtain closed form solutions for European option prices considering two different supply models: a mean-reverting model and a Markov chain model. The results are extensions of the classic Black-Scholes equation. The model provides a relatively simple approach to describe the complicated price behaviour observed in electricity spot markets and also allows for computationally efficient derivatives pricing.

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1. introduction

Electricity is essential for the normal course of life for almost every human in the industrialized world. The distribution of power to households and businesses is a complicated process involving significant engineering design and planning.

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Complexities from both engineering and financial perspectives are largely caused by the inability to store electricity economically. This lack of storage causes electricity markets to have the most complicated spot price behavior of all the energy markets, ((6), (15)). In order to construct a reasonable model for the electricity spot price it is necessary to account for as many of the price peculiarities as possible.

The standard approach to handling the behaviour of electricity markets has normally consisted of increasing the complexity of standard financial models. That is, a common approach is to use variations of mean-reverting jump diffusion, (MRJD), models to try to capture the price dynamics within the electricity markets. (See (6) and (11) for an overview). However, increasing the complexity of these models has some negative effects: i) they begin to lose mathematical tractability, ii) they become increasingly more computationally demanding, and iii) they are difficult to calibrate. A relatively new approach is to incorporate fundamental drivers, such as temperature and supply constraints into the dynamics of electricity spot prices. Indeed, a number of these "Hybrid Models" have been proposed in the past. (See: Anderson (2), Barlow (3), Davison et. al. (8), and Eydeland and Wolyniec (10)). Their main drawback, as noted by Weron (16), for instance, is the amount of time these models take to calibrate. Therefore, they suffer from some of the same disadvantages as complex MRJD models. If time for pricing and calibration is of minimal concern then these models work well; however, when time is of great importance, such as on a trading floor, then these disadvantages are critical.

The goal of this paper is to create a model that captures the dynamics of the electricity markets and can also remain mathematically trackable. We shall be using a model similar to that of Barlow (3). This sets the stage for reduced form supply-demand type modeling, and as it has the added benefit of being relatively simple in mathematical terms, the parameters can be easily estimated. Our model includes some extensions which makes the Barlow model more realistic. For instance, we use Fourier analysis to find the deterministic behavior in the demand and supply portions of the model. On the supply side of the model we use a two step method to determine the supply stack. 1) We modify a method outlined by Elliott et. al. in (9) to model the baseload portion of the supply stack, and 2) we consider a case when the baseload follows a mean-reverting process. We also develop a price dependent function which represents the "peak-load" portion of supply. Our dynamics solve many of the issues found in standard pricing models and is fully scalable up to a high level of complexity, such as that used by Anderson, (2), or Eydeland and Wolyniec, (10). More importantly our model is easy to implement when compared to some of the complex MRJD models and it yields closed form pricing equations for derivatives.

The paper is organized as follows: Section two focuses on the development of the price model, section three investigates the implications for option pricing, section four discusses the estimation of the model. Section five looks at the empirical results of the model, and section six concludes the paper and provides some suggestions for future work.

2. Model Construction

By observing Fig. 1, it is obvious that the modeling of electricity spot behaviour is very different from modeling stock prices. Likewise, from Table 1 we can see that the first four moments of baseload peak supply and demand are much

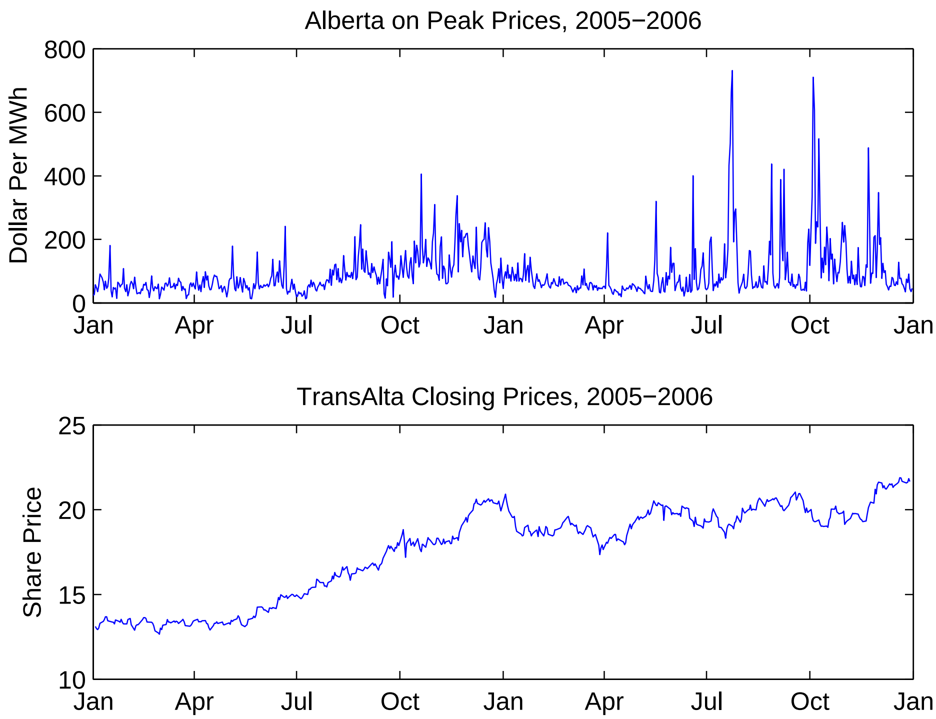

Figure 1: Daily prices of the Alberta electricity spot price market and the closing prices of an electricity generator, TransAlta

more 'normal' than the first four moments of the electricity spot price. Consequently, we base our model on the premise that supply equals demand, from which an equilibrium price can be determined. This type of pricing is consistent with standard economic arguments and allows an intuitive exploration of price construction. In doing so we are also able to avoid trying to construct a reduced form model for the extreme spot price behaviour observed in Fig. 1. Our first step is to model the demand portion. We then construct the supply curve and from the combination of these, obtain a price process.

	Price	Supply	Demand
Mean	92.1956	7636.1646	8027.4981
Stdev	80.8988	339.9402	417.2096
Skewness	3.6229	-0.3112	-0.0125
Kurtosis	21.7703	2.6280	2.404

Table 1: Market parameters for price, supply, and demand over 2005-2006, where baseload supply is defined as all supply bid into the stack at $\$50$ per MWh or less.

2.1. The Demand Side

Power demand is highly sensitive to human behaviour which depends on such variables as the time of day and heating and cooling seasons. Though it is common to use the term ‘seasonal’ in other markets such as natural gas, electricity markets may be better thought of as multi-cyclical, rather than seasonal. The price of power is effected not only by the seasons but also by the working and sleeping habits of consumers over the course of months, weeks, days, and hours. This type of behaviour is fairly predictable or deterministic and should be modeled as such. Therefore, we suppose that system demand $D(t)$ has dynamics:

$$D(t) = f(t) + \hat{D}(t) \; . \tag{1}$$

Here $f(t)$ is the deterministic component that can be estimated via signal processing methods or other techniques, and $\hat{D}(t)$ is demand minus the deterministic portion. In this paper we develop a model that assumes $\hat{D}(t)$ follows a mean-reverting process described by the solution of the following stochastic differential equations (SDE):

$$d\hat{D}(t) = \kappa(\mu - \hat{D}(t))dt + \sigma dB_d(t) \; . \qquad (2)$$

Here $B_d(t)$ is standard Brownian motion, σ is the "volatility" of demand, κ is the speed of mean-reversion and μ is the long term mean.

Solving (2) for $\hat{D}$ yields:

$$\hat{D}(t) = e^{-\kappa t}[\hat{D}(0) + \mu(e^{\kappa t} - 1)] + \int_0^t \sigma e^{\kappa(s-t)} dB_d(s) \; . \qquad (3)$$

Substituting (3) back into (1) gives a model for the total demand in the system.

$$D(t) = f(t) + e^{-\kappa t}[\hat{D}(0) + \mu(e^{\kappa t} - 1)] + \int_0^t \sigma e^{\kappa(s-t)} dB_d(s) \; . \qquad (4)$$

Thus, total demand is normally distributed with mean

$$E[D(t)|\mathcal{F}_0] = f(t) + e^{-\kappa t}[\hat{D}(0) + \mu(e^{\kappa t} - 1)] = \mu_D(t) \; .$$

and variance $V[D(t)|\mathcal{F}_0] = (1 - e^{-2\kappa t})\dfrac{\sigma^2}{2\kappa} = \sigma_D(t)$.

Equation (4) describes the demand at any time t. As demand is almost inelastic, (demand does not change with price), we use the value $D(t)$ as the market demand.

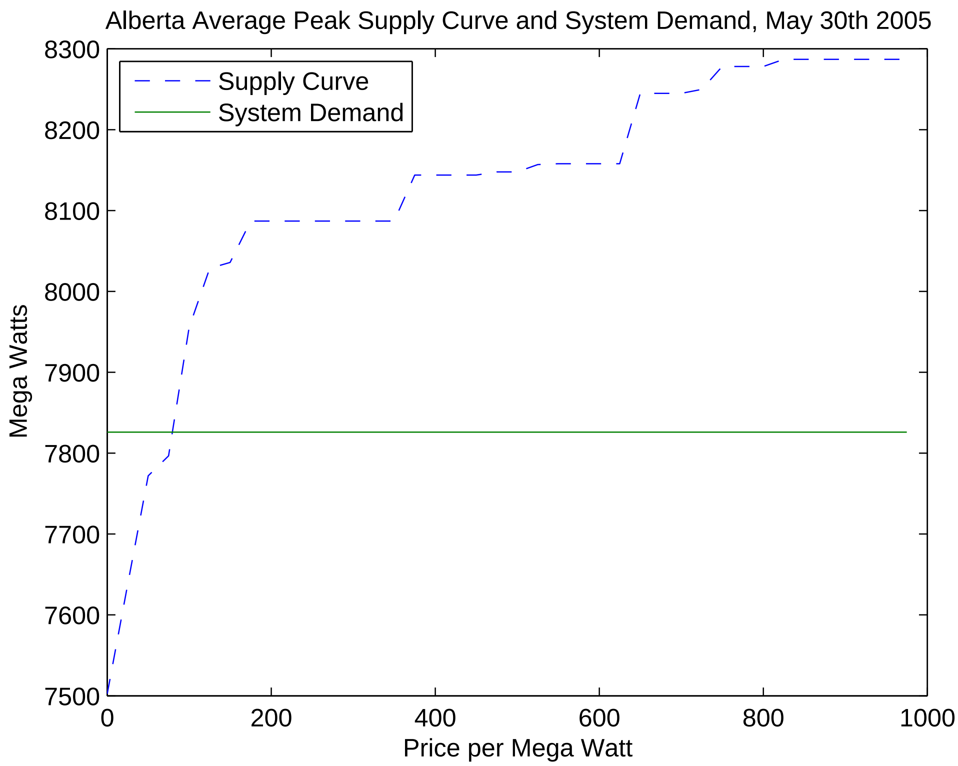

Figure 2: Alberta average supply curve and system demand on May 30 2005

2.2. The Supply Side

We suppose that the supply side[3] is composed of two distinct components, the "baseload" and the "mid and peaking load". These will be modeled separately. Each of these components varies in time and quantity which allows for a dynamical simulation and derivative pricing environment. There are two choices of randomness we wish to explore for the supply side. The first, (see (9) for a similar approach), is a model which uses a Markov chain to describe baseload noise dynamics. The second is a model which uses mean-reversion for the baseload. In each case, we assume that base supply also has a cyclical component. It follows that suppliers must increase supply to meet demand and, since demand has a strong cyclical component, then so should supply.

Thus, we model supply as,

$$S(t, P(t)) = S_b(t, P(t)) + S_k(t, P(t)) \,. \tag{5}$$

Here $S_b(\cdot)$ is the baseload portion of system supply, and $S_k(\cdot)$ is the mid to peaking, (high cost), portion of the system supply curve. $P(t)$ is the price of power at time t.

It can be costly to stop baseload power stations so they are usually run at a certain capacity for long periods of time, except for maintenance outages and other planned stoppages. Therefore, we assume that the baseload portion of supply does not vary with price and is a function only of time $S_b(t)$. The high cost portion of

[3]Throughout the paper, our analysis is done with price as the independent variable so that in price/quantity space, price is along the horizonal axis, and quantity is represented by the vertical axis, as in Fig. 2.

the supply has more flexibility and is able to adjust to price at a much faster rate. Thus, it is natural to have the high cost portion $S_k(P(t))$ depend on price.

We would like our supply curve to have the following economically sensible requirements.

1. Supply increases with Price, $\frac{\partial S}{\partial P} \geq 0$
2. The capable marginal increases in supply are decreasing in price $\frac{\partial^2 S}{\partial P^2} \leq 0$

For simplicity we assume a basic model for $S_k(\cdot)$ that satisfies the above concavity requirement, and take $S_k(P(t)) = b \log(cP(t) + \xi)$.
Then

$$S(t, P(t)) = aS_b(t) + b \log(cP(t) + \xi) \ . \tag{6}$$

Here, a, b, c and ξ are real positive constants. The supply function has the additional benefit that when prices $P(t)$ are at the market minimum[4], the only supply available is from the baseload portion. For instance, if the market minimum price is zero then the portion of the supply stack that is bid in at the minimum price is $aS_b(t) + b \log(\xi)$. The next two sections will be concerned with developing the models for the baseload portion of the supply function. From Fig. 2 the logarithmic function used here does seem a reasonable choice for the concave portion of the supply curve and thus we will use the same high cost portion of the function throughout the rest of the paper.

[4]Because of the non storable nature of electricity, producers sometimes have to pay to dispose of any excess power, inducing a negative price.

2.2.1. *The Markov Chain Model*

We supposes that the generators, (suppliers), who provide baseload do so in a fairly predictable manner except in some circumstances where there might be a plant failure. Therefore, we model the baseload portion of the supply stack using a method similar to that outlined in (9). However, we deviate from their method which models the number of baseload suppliers using a homogeneous Markov chain. Instead we use a Markov chain to describe the noise in the baseload supply. Suppose $G = \{G_t, t \geq 0\}$ is a Markov chain whose state space is the set of unit vectors,

$$\{e_1, e_2, \ldots, e_n\},\ e_i = (0, \ldots, 1, \ldots, 0)' \in \mathbb{R}^N \ .$$

Suppose that A, the transition rate matrix of G is independent of time. Then,

$$G_t = G_0 + \int_0^t AG_s ds + M_t \ . \tag{7}$$

Where M_t is a martingale.

Using (7) we can establish a simple way of describing the noise in baseload supply. From that we can establish the baseload curve at any time t.

We suppose the baseload has dynamics:

$$S_b^{mc}(t, P(t)) = B(t) + \langle \alpha, G_t \rangle \ . \tag{8}$$

Here, $B(t)$ is the deterministic component of supply, $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_{N-1}) \in \mathbb{R}^N$ are supply weights that affect the price when the chain is in a given state.

Therefore, we have a total supply curve given as:

$$S(t, P(t)) = aS_b(t, P(t)) + S_k(t, P(t)) = a(B(t) + \langle \alpha, G_t \rangle) + b \log(cP(t) + \xi) \ . \tag{9}$$

2.2.2. *The Mean Reverting Model*

In this setting, instead of using a Markov chain to model the randomness of the baseload, we now suppose that it follows a mean reverting process.

Under this setting the baseload $S_b^{mr}(t) = B(t) + S_b^1(t)$ where, $B(t)$ is the deterministic portion of baseload and S_b^1 is the solution of the mean reverting stochastic differential equation:

$$dS_b^1(t) = \kappa_s(\mu_s - S_b^1(t))dt + \sigma_s dB_s(t). \tag{10}$$

Then,

$$\begin{aligned} S_b^{mr}(t) &= B(t) + S_b^1(t) \\ &= B(t) + e^{-\kappa_s t}[{S_b}^1(0) + \mu_s(e^{\kappa_s t} - 1)] + \int_0^t \sigma_s e^{\kappa_s(\nu - t)} dB_s(\nu) \ . \end{aligned} \tag{11}$$

So, in the mean-reverting model, we have base supply normally distributed, with mean

$$E[S_b^{mr}(t)|\mathcal{F}_0] = B(t) + e^{-\kappa_s t}[{S_b}^1(0) + \mu_s(e^{\kappa_s t} - 1)] = \mu_{mr}(t) \ .$$

and variance $V[S_b^{mr}(t)|\mathcal{F}_0] = (1 - e^{-2\kappa_s t})\dfrac{\sigma_s^2}{2\kappa_s} = \sigma_{mr}(t)$.

This gives the equation for the supply curve as:

$$\begin{aligned} S(t, P(t)) = B(t) &+ e^{-\kappa_s t}[{S_b}^1(0) + \mu_s(e^{\kappa_s t} - 1)] \\ &+ \int_0^t \sigma_s e^{\kappa_s(\nu - t)} dB_s(\nu) + b\log(cP(t) + \xi) \ . \end{aligned} \tag{12}$$

2.3. *Equilibrium Price*

We now use the above supply and demand equations to obtain a market price for power $P(t)$. In equilibrium,

$$D(t) = S(t, P(t)) = aS_b(t, P(t)) + S_k(t, P(t)) \; . \tag{13}$$

The market clearing condition (13) can be used to solve for the price $P(t)$ at any time t. One noticeable issue with this model is that it does not necessarily put caps and floors on the price. In power markets both price caps and floors do exist depending on various markets. For instance, in the Alberta market, there is an upper bound at $\$999.99$ and a lower bound of $\$0$ per Megawatt hour. For other markets, there may be no price caps or floors, or they may be more extreme values then those of Alberta.

In this paper we will be looking at average peak power prices which are often well within the market caps and floors. Thus, we avoid creating a piecewise type of function for which it would be extremely difficult to obtain closed form solutions.

For simplicity we assume a basic model of $S_k(t, P)$ that meets the concavity conditions, and use the market clearing equation,

$$D(t) = aS_b(t) + b\log(cP(t) + \xi) \tag{14}$$

to obtain a solution for the price:

$$P(t) = \frac{1}{c}(\exp(-\frac{aS_b(t) - D(t)}{b}) - \xi) \; . \tag{15}$$

Exploring this model we see that price decreases as each of the parameters a, b, c and ξ increases. Likewise, as one would expect, as S_b increases, price falls, and as D increases we obtain a price increase. Therefore, large price movements occur when the distance between baseload supply and market demand increases.

Thus, we have derived a model which appears to address most of the required attributes that are observed in the power spot markets.

3. Derivatives Pricing

Derivatives are an important part of the energy markets. For instance, it is nearly impossible to buy and hold electric power so derivative contracts are used extensively. Standard risk neutral pricing arguments, (e.g. cost of carry), are not appropriate in the case of power markets, given the inability to store the underlying. Consequently, we price all of the derivatives under the physical or real world measure.

3.1. The Price of a European Call Option

We price claims using the stochastic discount factor, (SDF), approach as in (7). The price of a claim $g(\cdot)$ at time t is the discounted expected payoff under the physical measure at some time $T > t \geq 0$ in the future,

$$V(t) = E[\frac{m(T)}{m(t)} g(T) | \mathcal{F}_t] \, .$$

Here $\mathcal{F}_t$ is the information known at time t, $m(\cdot)$ is the SDF, and $g(\cdot)$ is the claim on the underlying.

For simplicity we suppose that the SDF is a riskless bond with constant interest rate r, minus the market price of risk variable[5], γ. If g is a European call option with strike price K, then the price of the call at time t, is:

[5]More complicated versions of the discount factor can be used, and given the large literature on term structure one can easily extend this model.

$$V(t) = e^{-(r-\gamma)(T-t)} E[(P-K)^+|\mathcal{F}_t] = e^{-(r-\gamma)(T-t)} \int_{-\infty}^{\infty} (P-K)^+ dF(P) \, .$$

Here $F(P)$ is the cumulative density function of P.

This leads the following results for the price of a call, using either model:

Proposition 1. *With a total system demand following the mean reverting process (4) and supply given by the Markov chain model (12), the price of a European call option for the price of power P with strike price K, expiry time T and a discount factor that is a deterministic riskless bond with interest rate $r-\gamma$, where γ is the market price of risk,*

$$V_t = e^{-(r-\gamma)(T-t)} \langle C_t, e^{A(T-t)} G_t \rangle. \tag{16}$$

Here

$$C_t = (C_t^1, \ldots, C_t^N)'$$

where

$$C_t^i = \frac{1}{c}(e^{-\lambda_i + \mu_z + \sigma_z^2/2} \Phi(d1) - (\xi + cK)\Phi(d2)),$$

and

$$\begin{aligned}
d1 &= \frac{\mu_z + \sigma_z^2 - \lambda_i - \log(cK + \xi)}{\sigma_z} \\
d2 &= \frac{\mu_z - \lambda_i - \log(cK + \xi)}{\sigma_z} = d1 - \sigma_z \\
\mu_z &= \frac{\mu_D(T-t)}{b} \\
\sigma_z^2 &= \frac{\sigma_D(T-t)^2}{b^2} \\
\lambda_i &= \frac{a(B(T-t) + \alpha_i)}{b} \, .
\end{aligned}$$

α_i is the weight parameter in state i.

Proof.

$$V_t = e^{-(r-\gamma)(T-t)} \int_{-\infty}^{\infty} (\frac{1}{c}(\exp(-\frac{aS_b^{mc}(T-t) - D(T-t)}{b}) - \xi) - K)^+ dF(P)\ . \tag{17}$$

For each of the i states of the Markov chain G we have,

$$P_t^i = \frac{1}{c}(\exp(-\frac{a(B(t)+\alpha_i) - D(t)}{b}) - \xi)$$

where

$$\bar{P}_t = (P_t^1, \ldots, P_t^N)'$$
$$\text{and}\ \ P_t = \langle \bar{P}_t, G_t \rangle\ .$$

We must find:

$$\begin{aligned} E[e^{-(r-\gamma)(T-t)}(P_T - K)^+ | \mathcal{F}_t] &= e^{-(r-\gamma)(T-t)} E[(P_T - K)^+ | \mathcal{F}_t] \\ &= e^{-(r-\gamma)(T-t)} E[E[(\langle \bar{P}_t, G_t \rangle - K)^+ | \mathcal{F}_t^B \vee \mathcal{F}_T^G] | \mathcal{F}_t]\ , \end{aligned}$$

where $\mathcal{F}_t^B = \sigma\{\mathcal{B}_u : u \leq t\}$ and $\mathcal{F}_t^G = \sigma\{G_u : u \leq t\}$. Note that,

$$E[G_t | G_s] = e^{A(t-s)} G_s,$$

and consider,

$$E[(\langle \bar{P}_t, G_T \rangle - K)^+ | \mathcal{F}_t^B \vee \mathcal{F}_T^G]\ .$$

Suppose that $G_T = e_i$.

We then first evaluate: $E[(P_T^i - K)^+ | \mathcal{F}_t^B \vee \{G_T = e_i\}]$

$$\text{Now: } P_T^i = \frac{e^{-\lambda_i} e^Z - \xi}{c}$$
$$\text{where } \lambda_i = \frac{a(B(T) + \alpha_i)}{b}$$
$$\text{and } Z = \frac{D(t)}{b} \sim N(\frac{\mu_D}{b}, \frac{\sigma_D^2}{b^2}) = N(\mu_z, \sigma_z^2).$$

Consequently,

$$P_T^i > K \text{ when}$$

$$\frac{e^{-\lambda_i} e^Z - \xi}{c} > K$$
$$\text{So, } \; Z > \log[e^{\lambda_i}(cK + \xi)] = y$$

$$C_t^i = e^{-(r-\gamma)(T-t)} E[C_T^i | \mathcal{F}_t^B \vee \{G_T = e_i\}]$$

Let $m = e^{-(r-\gamma)(T-t)}$, then,

$$\begin{aligned}
C_t^i &= mE[(P_T^i - K)^+ | \mathcal{F}_t^B \vee \{G_T = e_i\}] \\
&= m \int_y^\infty (\frac{e^{-\lambda_i} e^Z - \xi}{c} - K) \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2}) dZ \\
&= m \int_y^\infty \frac{1}{c\sqrt{2\pi}\sigma_D(t)} e^{-\lambda_i} e^Z \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2}) dZ - m(\xi/c + K)(1 - \Phi(\frac{y-\mu_z}{\sigma_z})) \\
&= m \frac{e^{-\lambda_i}}{c} \int_y^\infty \frac{1}{\sqrt{2\pi}\sigma_z} e^Z \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2}) dZ - m(\xi/c + K)(1 - \Phi(\frac{y-\mu_z}{\sigma_z})).
\end{aligned}$$

Completing the square,

$$
\begin{aligned}
\frac{C_t^i}{m} &= \frac{e^{-\lambda_i+\mu_z+\sigma_z^2/2}}{c}\int_y^\infty \frac{1}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{(Z-\mu_z-\sigma_z^2)^2}{2\sigma_z^2}\right)dZ - (\xi/c+K)\left(1-\Phi\left(\frac{y-\mu_z}{\sigma_z}\right)\right) \\
&= \frac{e^{-\lambda_i+\mu_z+\sigma_z^2/2}}{c}\left(1-\Phi\left(\frac{y-(\mu_z+\sigma_z^2)}{\sigma_z}\right)\right) - (\xi/c+K)\left(1-\Phi\left(\frac{y-\mu_z}{\sigma_z}\right)\right).
\end{aligned}
$$

Now since, $1-\Phi(x) = \Phi(-x)$

$$
\begin{aligned}
\frac{C_t^i}{m} &= \frac{e^{-\lambda_i+\mu_z+\sigma_z^2/2}}{c}\left(\Phi\left(\frac{\mu_z+\sigma_z^2-\lambda_i-\log(cK+\xi)}{\sigma_z}\right)\right) - (\xi/c+K)\left(\Phi\left(\frac{\mu_z-\lambda_i-\log(cK+\xi)}{\sigma_z}\right)\right) \\
&= \frac{e^{-\lambda_i+\mu_z+\sigma_z^2/2}}{c}\left(\Phi\left(\frac{\mu_z+\sigma_z^2-\lambda_i-\log(cK+\xi)}{\sigma_z}\right) - (\xi/c+K)\Phi\left(\frac{\mu_z-\lambda_i-\log(cK+\xi)}{\sigma_z}\right)\right) .
\end{aligned}
$$

Therefore,

$$
(\langle P_T, G_T\rangle - K)^+ = \langle \bar{C}_T, G_T\rangle
$$

where $\bar{C}_T = (C_T^1, \ldots C_T^N)'$.

Consequently,

$$
\begin{aligned}
E[(P_T-K)^+|\mathcal{F}_t] &= E[(\langle \bar{P}_T, G_T\rangle - K)^+|\mathcal{F}_t] \\
&= E[\langle \bar{C}_T, G_T\rangle|\mathcal{F}_t] \quad (**) \\
&= E[E[\langle \bar{C}_T, G_T\rangle|\mathcal{F}_t^B \vee \mathcal{F}_T^G]|\mathcal{F}_t]
\end{aligned}
$$

and

$$
\begin{aligned}
E[\langle \bar{C}_T, G_T\rangle|\mathcal{F}_t^B \vee \mathcal{F}_T^G] &= \langle E[\bar{C}_T|\mathcal{F}_t^B \vee \mathcal{F}_T^G], G_T\rangle \\
&= \langle C_t, G_T\rangle
\end{aligned}
$$

where $C_t = (C_t^1, C_t^2, \dots, C_t^N)'$

Then $(**)$ gives

$$E[\langle \bar{C}_T, G_T \rangle | \mathcal{F}_t] = \langle C_t, e^{A(T-t)} G_t \rangle.$$

$\square$

$\square$

Proposition 2. *With a total system demand following the mean-reverting process (4) and supply given by the mean reverting model (12), the price of a European call option for the price of power P with strike price K, expiry time T and a discount factor that is a deterministic riskless bond with interest rate $r - \gamma$, where γ is the market price of risk,*

$$V_t = \frac{e^{-(r-\gamma)(T-t)}}{c} [e^{\sigma_z^2/2 - \mu_z} \Phi(d_1) - (\xi + Kc)\Phi(d_2)] \ . \tag{18}$$

Where,

$$\begin{aligned} d_1 &= \frac{\sigma_z^2 - \mu_z - \log(\xi + Kc)}{\sigma_z} \\ d_2 &= -\frac{\mu_z + \log(\xi + Kc)}{\sigma_z} = d_1 - \sigma_z \\ \mu_z &= \frac{a\mu_{sm}(T-t) - \mu_D(T-t)}{b} \\ \sigma_z^2 &= (\frac{\sigma_D(T-t)}{b})^2 + (\frac{a\sigma_{sm}(T-t)}{b})^2 \end{aligned}$$

Proof. The price of the claim, is the expected discounted payoff.

$$V_t = e^{-(r-\gamma)(T-t)} \int_{-\infty}^{\infty} (\frac{1}{c}(\exp(-\frac{aS_b^{mr}(T-t) - D(T-t)}{b}) - \xi) - K)^+ dF(P) \ . \tag{19}$$

In our model we have both S_b^{mr} and D are independently normally distributed. Thus we have the difference between two normal random variables.

Let ,

$$
\begin{aligned}
Z &= \frac{aS_b^{mr} - D}{b} \\
&\sim N(\frac{a}{b}\mu_{sm} - \frac{1}{b}\mu_D, (a/b)^2\sigma_{sm}^2 + (1/b)^2\sigma_D^2) \\
&= N(\mu_z, \sigma_z^2).
\end{aligned}
$$

Then (19) becomes,

$$
\begin{aligned}
V_t &= \frac{e^{-(r-\gamma)(T-t)}}{c} \int_{-\infty}^{\infty} (\exp(-Z) - (cK + \xi))^+ \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ \\
&= \Psi \int_{-\infty}^{\infty} (\exp(-Z) - (cK + \xi))^+ \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ.
\end{aligned}
$$

Then the integrand is non-zero when,

$$
Z < -\log(\xi + Kc) = y.
$$

Thus,

$$
\begin{aligned}
V_t &= \Psi \int_{-\infty}^{y} (\exp(-Z) - (cK + \xi)) \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ \\
&= \Psi[\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-Z) \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ \\
&- \int_{-\infty}^{y} (cK + \xi) \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ].
\end{aligned}
$$

Write $w = \frac{Z-\mu_z}{\sigma_z}$. Then,

$$
\begin{aligned}
V_t &= \Psi[\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-Z) \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ - \int_{-\infty}^{\frac{y-\mu_z}{\sigma_z}} (cK + \xi) \frac{1}{\sqrt{2\pi}} \exp(-\frac{w^2}{2})dw] \\
&= \Psi[\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-Z) \exp(-\frac{(Z-\mu_z)^2}{2\sigma_z^2})dZ - (cK + \xi)\Phi(\frac{y-\mu_z}{\sigma_z})].
\end{aligned}
$$

Here $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\frac{x^2}{2})dx$.

Completing the square in the two exponentials,

$$V_t = \Psi[\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-(\mu_z-\sigma_z^2))^2 + 2\mu_z\sigma_z^2 - \sigma_z^4}{2\sigma_z^2})dZ - (cK+\xi)\Phi(\frac{y-\mu_z}{\sigma_z})]$$
$$= \Psi e^{\sigma_z^2/2-\mu_z} \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_z} \exp(-\frac{(Z-(\mu_z-\sigma_z^2))^2}{2\sigma_z^2})dZ - \Psi(cK+\xi)\Phi(\frac{y-\mu_z}{\sigma_z}).$$

Writing $\psi = \frac{Z-(\mu_z-\sigma_z^2)}{\sigma_z}$, we have:

$$V_t = \Psi e^{\sigma_z^2/2-\mu_z} \int_{-\infty}^{\frac{y-(\mu_z-\sigma_z^2)}{\sigma_z}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{\psi^2}{2})dZ - \Psi(cK+\xi)\Phi(\frac{y-\mu_z}{\sigma_z})$$
$$= \Psi e^{\sigma_z^2/2-\mu_z} \Phi(\frac{y-(\mu_z-\sigma_z^2)}{\sigma_z}) - \Psi(cK+\xi)\Phi(\frac{y-\mu_z}{\sigma_z}).$$

Consequently, the price of a call option under the mean-reverting model is,

$$V_t = \frac{e^{-(r-\gamma)(T-t)}}{c}[e^{\sigma_z^2/2-\mu_z}\Phi(d_1) - (\xi+Kc)\Phi(d_2)]\ .$$

where,

$$d_1 = \frac{\sigma_z^2 - \mu_z - \log(\xi+Kc)}{\sigma_z}$$
$$d_2 = -\frac{\mu_z + \log(\xi+Kc)}{\sigma_z} = d_1 - \sigma_z,$$

under the conditions that $\xi + Kc > 0$ and $c > 0$.

□ □

3.2. Finding the Market Price of Risk

Suppose we have a Forward contract with a delivery price F at some time T priced as follows.

$$F(t) = E[\frac{m(T)}{m(t)}P(T)|\mathcal{F}_t] = e^{-(r-\gamma)(T-t)}E[P(T)|\mathcal{F}_t].$$
$$\text{Then } \gamma = \frac{\log(F(t)) - \log(E[(P(T))|\mathcal{F}_t]) + r(T-t)}{T-t}. \tag{20}$$

So

$$\begin{aligned}
E[P(T)|\mathcal{F}_t] &= \frac{1}{c}E[\exp(-\frac{aS_b(T-t)}{b})\exp(\frac{D(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c} \\
&= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})E[\exp(-\frac{aS_b(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c} \\
&= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})E[\exp(-\frac{aS_b(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c}.
\end{aligned}$$

This provides a formula for both the Markov chain and Mean-reverting supply models.

For the Markov chain model

$$\begin{aligned}
E[P(T)|\mathcal{F}_t] &= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})E[\exp(-\frac{aS_b(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c} \\
&= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})E[\exp(-\frac{aS_b^{mc}(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c} \\
&= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})\langle\exp(-\frac{a\alpha}{b}), \exp(A(T-t))G_t\rangle - \frac{1}{c}.
\end{aligned}$$

For the mean reverting model

$$\begin{aligned}
E[P(T)|\mathcal{F}_t] &= \frac{1}{c}\exp(\frac{\mu_D(T-t)}{b} + \frac{\sigma_D(T-t)^2}{2b^2})E[\exp(-\frac{aS_b(T-t)}{b})|\mathcal{F}_t] - \frac{1}{c} \\
&= \frac{1}{c}\exp(\frac{\mu_D(T-t) - a\mu_{sm}(T-t)}{b} + \frac{\sigma_D(T-t)^2 + a^2\sigma_{sm}(T-t)^2}{2b^2}) - \frac{1}{c}\ .
\end{aligned}$$

Using these results and (20), in each case a market price of risk can be determined according to the specific model selected. This allows the the parameters of the model to be estimated with real world data, and then use derivatives, (in this case, the forward curve), to obtain the market price of risk. Once the market price of risk is obtained, the price of various other derivative contracts can be obtained using standard asset pricing arguments.

4. Estimation

In this section we describe the estimation of our model. There are a number of steps that should be taken to obtain a logical calibration. The first is to estimate the parameters for demand and baseload supply. We wish to extract the cyclical components within both the demand and supply and we do this by fitting two sinusoids to the data. We then use spectral analysis to determine how many more sinusoids are required to extract other cyclical components. After this is done, we fit our mean reverting and Markov chain processes to the decycled data. This gives us the parameters for both demand and baseload supply. Finally we use non-linear regression to find the remaining values.

4.1. Cycle Extraction

Much of the cyclical behavior of electricity prices is due to the fact that electricity can not be economically stored, (other than using hydroelectric dams). This prevents inventories from being accumulated to smooth rapid price movements. Consequently, power prices are highly sensitive to electricity demand. For instance, in much of North America there are two extreme seasons, winter and summer, where electricity demand is much different and prices react accordingly.

In the winter, days are shorter requiring increased use of electrical lighting. In addition, homes and businesses often require heating during the winter months.

Though much of the heating in North America comes from either natural gas or heating oil, electric heaters do exist and are used. Coupled with the fact that there are increasing numbers of natural gas fired generators coming online, this also drives up the cost of production during winter, given that natural gas prices are also historically higher in the winter months. In the summer the increased demand comes from the use of air conditioning units. Though many consider these two cycles the most dominant, there are also several others that are significant, including weekly, daily, and intra-daily cycles in demand.

During the week there are low and high times for electricity demand. Weekdays require more electricity than the weekends or holidays. These are considered weekly cycles. The working hours in the day cause electricity demand to rise, while the late evening and early morning hours see reduced demand. These are considered intra-day cycles.

There are several methods authors have used model the cyclical components within electricity prices. Cartea and Figueroa (5) use a Fourier curve fit to account for the seasonal components and Burger et al. (4) use a load forecast to determine the seasonal component. A sinusoidal method is used by Pilipović (15). She considers seasonal effects and suggests modeling them by using two sinusoidal functions with frequencies that correspond to both annual and semi-annual cycles within the spot prices. Her approach is both intuitive and sensible, as it accounts for two of the most prominent cycles within electricity prices, the extreme seasons within a year. Unfortunately, there are limitations with this approach, as the seasons may not be purely simple sinusoidal and may instead require a greater number of frequencies to account for a more complex seasonal curve. In addition, as discussed above, there are other cycles that may need to be accounted for.

The method we propose is a combination of the method used by (15) and the inclusion Fourier analysis to identify the number of sinusoids that will need to be used to extract the cyclical component for higher frequencies such as weekly or daily cycles. We construct an extension of a usual method for separating deterministic signals from noise using the discrete Fourier transform (DFT). The (DFT), or the fast Fourier transform (FFT), which is an optimized version of the DFT (12), is used extensively throughout the sciences. Engineers, for example use it to identify the frequency at which a signal is being transmitted and then design a filter to isolate the frequency and suppress the unwanted noise that has contaminated the signal. This method of cycle detection provides several benefits: 1) It allows for visual interpretation of the dominant cycles seen in electricity prices. 2) The number of cycles can be properly identified, which is helpful given the different characteristics in different regions. 3) Complicated curve fitting techniques are not required to model the cyclical component. 4) The Fourier transform is very popular, is used in a variety of disciplines, and is easy to access in a wide range of computational packages. Lastly, 5) the cyclical and stochastic components can be modeled separately which is useful in many circumstances. Alvarado and Rajaraman (1) proposed the use of a similar method for electricity price volatility. However, they use the DFT without considering stationarity. This is of concern when using Fourier methods, (see (13)). We circumvent this by simply fitting a curve, as in (15), to capture the seasonal, (annual and semi-annual), portion of the prices. We then difference the residuals and use the DFT to determine the remaining cyclical components[6].

4.1.1. The Seasonal Component

Recall that demand is,

$$D(t) = f(t) + \hat{D}(t),$$

where $f(t)$ is the cyclical component. We assume that

$$f(t) = f_S(t) + f_W(t) + f_D(t).$$

Here, $f_S(t)$ is considered the seasonal component, $f_W(t)$ the weekly component, and $f_D(t)$ is the daily component.

We let,

$$f_S(t) = a\sin(2\pi t/365 - t_a) + b\sin(4\pi t/365 - t_b) + ct + d \tag{21}$$

with $a\sin(2\pi t/365 - t_a)$ representing the yearly variations, and $b\sin(4\pi t/365 - t_b)$[7] representing the semi-yearly variations seen in electricity prices. Here:

- a and b are the amplitude parameters
- t_a is the annual centering parameter
- t_b is the semi-annual centering parameter
- c and d are standard intercept and slope parameters

For the supply seasonal estimation, we also use (21).

[6] We cannot use this method to extract the seasonal component as differencing the data acts like a high pass filter and renders the low frequency components (seasonal components) undetectable.

[7] These equations would change to $a\sin(2\pi t/(365 \cdot 24) - t_a)$ if one is to use hourly data.

4.1.2. Higher Frequency Components

Our next step is to consider $f_W(t)$ and $f_D(t)$ components of the data,again exploiting the power of the Fourier transform.

Write

$$h(t) = \frac{d(f(t) - f_S(t))}{dt}, \tag{22}$$

which represents the cyclical function containing the weekly and daily cycles. We take the Fourier transform of h and look at its power spectral density for an indication of how many cyclical components we have in our data. For a more sophisticated method of cycle extraction, which handles noise floor estimation and spectral smoothing see the methods outlined in (1) and (14)[8].

Consequently, we can obtain the number of harmonics required for both Weekly and Daily cycles:

$$f_w = \sum_{n=1}^{N} w_n \sin(2\pi \frac{n}{365 \cdot 7} + t_{w_n}) \tag{23}$$

$$f_D = \sum_{n=1}^{N} d_n \sin(2\pi \frac{n}{365 \cdot 24} + t_{d_n}). \tag{24}$$

4.2. Finding a, b, c and ξ

Our method to find the parameters is simply to use of non-linear regression to minimize the pricing errors associated with fitting the model. That is, we use the price equation (15),

$$P(t) = \frac{1}{c}(\exp(-\frac{aS_b(t) - D(t)}{b}) - \xi) \ .$$

[8]However, both (1) and (14) ignore issues with non-stationarity.

and execute a non-linear regression to obtain the results for the parameters. Using this approach requires the use of iterative numerical optimization routines. However, many popular computational packages have the ability to allow for easy implimentation[9].

4.3. Estimating the Noise

4.3.1. The Markov Chain Model

The Markov Chain model requires us to estimate various states of the noise and then find the transition matrix and the associated weight parameters. Formally, we define certain levels representing different states. We might say that there is an normal state of the noise, and we would count the number of observations that the noise was in that state. Likewise, we would do the same for other states. Similarly, we observe the number of transitions between states. Consequently, we can obtain an estimate for our transition matrix.

The entries in the transition matrix are

$$\pi_{j,i} = P(G_{k+1} = e_j | G_k = e_i), \tag{25}$$

giving an estimate for the transition matrix Π, where

$$\Pi = \begin{pmatrix} \pi_{1,1} & \pi_{1,2} & \dots & \pi_{1,n} \\ \pi_{2,1} & \pi_{2,2} & \dots & \pi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{n,1} & \pi_{n,2} & \dots & \pi_{n,n} \end{pmatrix} = e^{A\delta t} \; . \tag{26}$$

[9]In this paper we use the 'nlinfit' command within the computational package MATLAB to conduct the non-linear regression.

Here δt is the time step used when estimating the parameters.

For each state i we estimate the α_i by taking the average of the data points within the state. That is for all observations in state i we take the arithmetic mean of those observations to determine α_i:

$$\alpha_i = \frac{1}{n}\sum_{t_i=1}^{n} X_{t_i} \ .$$

Here, $\{X_{t_i}\}$ is the set of data observations within state i given the entire sample space X_t.

4.3.2. The Mean Reverting Model

The mean reverting model is a continuous time analog of the standard AR(1) model and can be estimated using least squares or maximum likelihood methods. We use maximum likelihood estimation to estimate the parameters associated with the SDE:

$$dX_t = \kappa(\mu - X_t)dt + \sigma dB_t$$

having the solution,

$$X_t = e^{-\kappa t}[X_0 + \mu(e^{-\kappa t} - 1)] + \int_0^t \sigma e^{\kappa(t-s)} dB_s \ .$$

Discretizing we obtain an AR(1) process given by:

$$X_{t+1} = a + bX_t + \hat{\sigma}\epsilon_t$$

$$a = \mu(1 - e^{-\kappa\delta t})$$
$$b = e^{-\kappa\delta t}$$
$$\hat{\sigma}^2 = \sigma^2 \frac{(1 - e^{-2\kappa\delta t})}{2\kappa} \ .$$

This has the transition density,

$$f(X_{t+1}|X_t; a, b, \hat{\sigma}) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp(\frac{(X_{t+1} - X_t b - a)^2}{2\hat{\sigma}^2}) \ .$$

Maximizing over the logarithm of $f(\cdot)$ yields the results we desire.

5. Empirical Results

The goal of this paper is to establish a model which captures the complicated dynamics present in electricity spot markets. We have chosen as test data the Alberta power market, which is a rather small market by global standards. The choice of the Alberta market for study is two fold: firstly, there is a wealth of publicly available data[10], and secondly, Alberta is the first market in North America which has established an emissions market. This will allow researchers to investigate the behaviour of electricity markets in response to emission prices.

There are, however, some disadvantages to studying the Alberta market. The price volatility is almost unprecedented when compared to other markets. This has to do with the lack of diversified generation units within the market, and an essentially Base to Peak, (lack of mid supply available), bid stack. Additionally,

[10]Historical data can be downloaded from the Alberta Electricity System Operator (ASEO) website: http://www.aeso.ca/ and facts about the market can be found at http://www.energy.gov.ab.ca/OurBusiness/electricity.asp .

there is a lack of any available derivatives data, given the small size of the market, so the market does not have a liquid options market. However, it does have a liquid OTC Forward market.

5.1. The Market Data

For our sample estimates we look at 2 years' of daily peak (8:00-23:00) data. The data set begins January 1 2005 and ends in December 2006. This provides 730 data points for estimation. We use daily on-peak averages for computational convenience and the fact that many derivative contracts are written on blocks of on-peak hour hours. However, the method can be extended to include hour by hour analysis.

Our method depends on trying to accurately approximate the dynamics of both supply and demand within the market, Fig. 3 represents the onpeak demand and baseload supply[11]. It is clear that there is a seasonal component to the base supply. Additionally, an interesting plot can be found in Fig. 4 which represents the supply surface. It is a visual representation of the price-quantity relationship of supply over time. As mentioned earlier, we avoid the common way of plotting price versus quantity and instead plot quantity versus price. We can then see how that relationship 'moves' over time. From this supply surface plot we can observe that, as we have assumed in our model, the structure, or shape, of the concave portion of the supply curve seems to be stable over time. Thus our assumption of time invariance in the non-baseload portion of the supply curve seems to be valid.

[11]We define baseload supply as all capacity bid into the system at $50 per MWh or less.

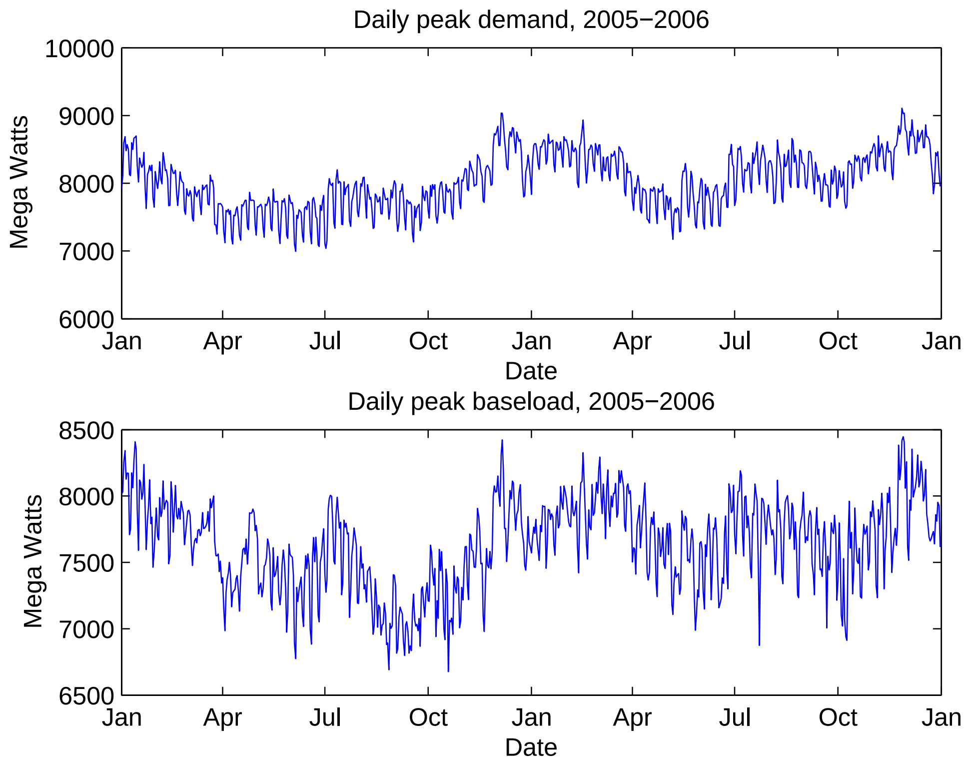

Figure 3: On-peak demand and baseload for the Alberta market

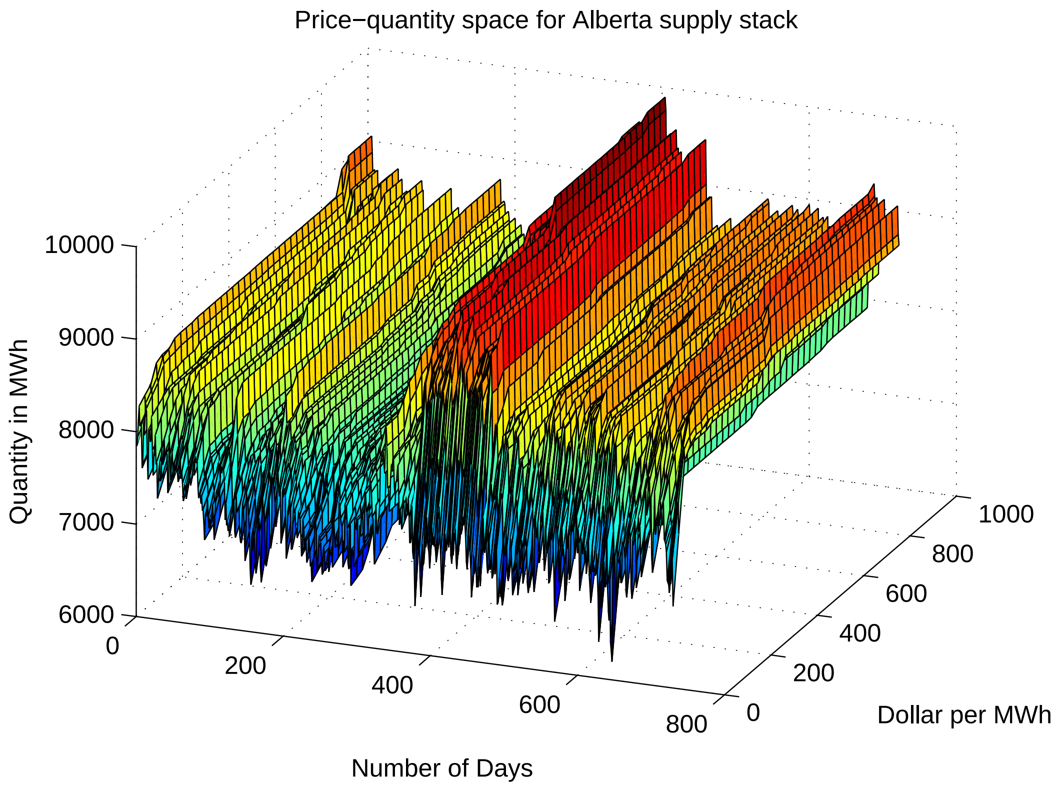

Figure 4: On-peak Alberta supply stack with $25 per MWh partitions for the years 2005-2006

5.2. Estimation Results

5.2.1. Deterministic Component

As stated above, we are only concerned with daily price behaviour and, thus, will not need to account for intra daily cycles in the supply and demand data. Taking the FFT of (22) we can observe in Fig. 5 that there are three weekly harmonics that need to be captured for demand and two for the supply. This implies that the seasonal components for both supply and demand have the following functional form:

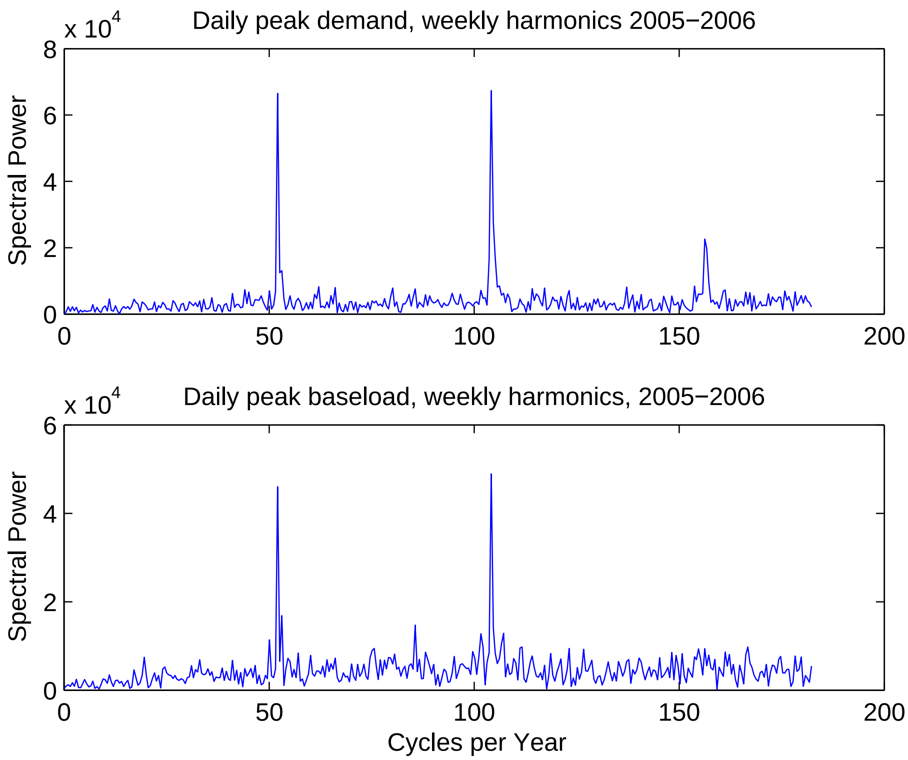

Figure 5: Frequency domain of system demand and baseload supply

$$f(t) = f_S(t) + \sum_{n=1}^{3} w_n \sin(2\pi \frac{n}{365 \cdot 7} + t_{w_n}) \tag{27}$$

$$B(t) = f_S(t) + \sum_{n=1}^{2} w_n \sin(2\pi \frac{n}{365 \cdot 7} + t_{w_n}). \tag{28}$$

The model parameters for $f(t)$ and $B(t)$ can be found in Table 2:

Variable	Supply	Demand
a	262.5	328.9
b	158.8	195.7
c	0.5737	0.8824
d	7427	7706
w_1	-149.3	216.8
w_2	93.62	136.9
w_3		47.78
t_a	-187.7	-10.97
t_b	-5.324	-5.324
t_{w_1}	6.544	9.612
t_{w_2}	2.011	58.51
t_{w_3}		0.0578

Table 2: Estimated parameters for deterministic component for demand and baseload supply

5.2.2. Supply Shaping Parameters: a, b, c, and ξ

To establish the shaping parameters used in the supply function, we ran a non-linear regression on the equilibrium pricing equation to obtain the estimates for

a, b, c and ξ[12]. These can be found in Table 3. The error within the estimates of c and ξ is very large. However, this is not that unexpected given the large jumps in price. Indeed there are almost discontinuous jumps in the supply stack, from baseload to the peaking portion of the curve.

Variable	Estimated Value
a	1.115 ± 0.0853
b	685.89 ± 298.7774
c	0.0049 ± 0.0050
ξ	0.0832 ± 0.2130

Table 3: Estimated Parameters for supply model

[12]The null hypothesis that the price data and the difference between supply and demand contained a unit root was rejected after conducting an Augmented Dickey Fuller test.

5.2.3. The Noise

In the Markov model we use a Markov chain to represent the noise in base load supply. We arbitrarily chose three states: a 'high' state, a 'normal' state, and a 'low' state. To find the transition matrix, we historically define a high state to be, when the de-cycled noise is 1.5 standard deviations above the historical mean. Similarly the low state is when the noise is 1.5 standard deviations below the historical mean. The normal state is the state in between the high and low states. The weight parameters are calculated by grouping those observations which are in each of the states, and then averaging them. That is, for each historical observation that was considered in the high state, we would take the arithmetic mean of the collection of these observations to determine the weight parameter α for the high state. Similarly the weight parameters are determined for the other states. Here, state 1 is the normal state, state 2 is the high state and state 3 is the low state. The transition matrix is estimated to be

$$\Pi = \begin{pmatrix} 0.9164 & 0.4783 & 0.6327 \\ 0.0331 & 0.5217 & 0 \\ 0.0505 & 0 & 0.3673 \end{pmatrix}$$

and

$$\alpha = (4.8954, 429.2737, -457.0059)'.$$

For the mean-reverting noise model we have the results of applying the Maximum likelihood estimation methods to the de-cycled noise as provided in Table 4:

Using our estimation results we run a Monte Carlo simulation for the three

Variable	Estimated Value Supply	Estimated Value Demand
κ	0.2723	0.1371
μ	-2.0789	1.4883
σ	188.8025	133.1162

Table 4: Estimated Parameters for Mean-Reverting supply model and Mean-Reverting portion of Demand

years 2005, 2006, and 2007. We simulated 1000 sample paths and then calculated the first four central moments of the averaged sample paths. The results are provided in Table 5. The models perform reasonably well when compared to standard models used in Finance. However, we cannot capture the extreme Skewness and Kurtosis seen in the Alberta market. This is possibly due to the smooth log function we have used. The Alberta market has almost no mid pricing plants so there is a rapid increase in price from base to peak, with only a small deviation in quantity. Thus, an extremely complicated supply function would perhaps be needed. We assume that in larger markets our model will be applicable and, as the Alberta market matures and adds a greater mixture of supply generation, we believe that this model should become a better proxy. Similarly, different estimation techniques or a larger data sample may provide better results then the ones obtained here.

Our next step is to price some Call options using our estimated parameters.

5.3. Derivatives Pricing

Our calibration period was the time period of January 1, 2005 to December 31, 2006. Using our estimates we now combine them with the results from Propositions 1 and 2 to price some European Call options. Since we would like to use

	2005^1	2006^1	2007^1	2005^2	2006^2	2007^2	2005^3	2006^3	2007^3
Mean	85.39	99.00	84.26	97.57	117.88	111.62	95.49	108.57	112.24
Std.	57.04	98.80	83.84	63.24	78.68	71.91	72.91	78.30	85.17
Skewness	1.77	3.51	4.80	1.90	2.52	2.05	2.30	2.64	2.12
Kurtosis	7.24	17.91	29.67	8.019	13.57	10.76	11.93	15.64	11.47

Table 5: Simulation results for years 2005 (in sample), 2006 (in sample), and 2007. [1] indicates market data, [2] indicates results from the Markov chain model, and [3] are the results using the Mean-reverting model.

only public data for this paper, we choose a range for the market price of risk in our pricing scenarios[13].

We wish to price European Call options where we assume that the strike price is \$92. (This is the nearest whole number to the mean on-peak power price for the 2005 to 2006 sample). We use various values for the market price of risk and the expiry time. We allow the market price of risk to take on values from 0% up to 20% continuously compounded yearly, with 5% increments. We also look at expiry times from 90 days up to one year away. The risk free rate is assumed to be 5%.

Using our current parameters, the mean reverting model commands a higher premium than does the Markov chain model. However, this can clearly change

[13]As shown in Section 3, one can easily obtain the market price of risk from current forward curve quotes that are available to virtually every trader in the power markets. Additionally, estimations of time varying market price of risk parameters can also be calculated with modest enhancements.

	T-t=90	T-t=120	T-t=210	T-t=270	T-t=365
γ=0.00	3.0792	16.6157	14.5388	73.5757	71.0398
$\gamma = 0.05$	3.1174	17.0305	14.9631	76.3480	74.6821
$\gamma = 0.10$	3.1561	17.4556	15.3998	79.2247	78.5111
$\gamma = 0.15$	3.1953	17.8914	15.8492	82.2097	82.5365
$\gamma = 0.20$	3.2349	18.3380	16.3118	85.3073	86.7682

Table 6: Call option prices for Markov chain supply model

	T-t=90	T-t=120	T-t=210	T-t=270	T-t=365
γ=0.00	10.2942	27.5875	25.2781	83.5000	80.8677
$\gamma = 0.05$	10.4219	28.2762	26.0158	86.6462	85.0139
$\gamma = 0.10$	10.5512	28.9821	26.7751	89.9109	89.3726
$\gamma = 0.15$	10.6821	29.7056	27.5565	93.2986	93.9548
$\gamma = 0.20$	10.8146	30.4472	28.3608	96.8140	98.7720

Table 7: Call option prices for mean reverting supply model

depending on the assumptions used when estimating the states and the weight parameters for the Markov chain. There is noticeable seasonality in the call prices as well as an obvious increase in call prices near the end of the year, (270 days and 365 days), for both models. This indicates that the model is indeed pricing the expected increase of the spot price during these times of the year.

6. Conclusion

In this paper we have presented a hybrid model that uses a supply demand approach for price electricity derivatives. We were able to capture most of the dynamics exhibited in the power markets and find closed form solutions for European call options. Our goal was to provide a model that captured the price dynamics of power while remaining simple enough to yield closed form solutions for derivatives. We believe that models such as the one we have proposed are of benefit to those operating in the power trading market, and represent a class of model that can be used rather than the standard Black-Scholes model or its variants. Admittedly the model is more complex than the standard financial models. However, we believe that the model is still simple enough to be used on a day-to-day basis by practitioners.

Our results are promising and we tested them on an the extremely volatile Alberta market. There are many areas of future research: exploring the implications of imposing hard price floors and caps in the model would be of interest. Additionally more complex supply functions may be appropriate and might yield better statistical results.

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