THE UNIVERSITY OF CALGARY

Modelling Assistance for Project Procurement and Disposal Decisions

by

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A DISSERTATION

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF

DOCTOR OF PHILOSOPHY

FACULTY OF MANAGEMENT

CALGARY, ALBERTA

November, 1998

0-612-38514-0

ABSTRACT

Managers must make important decisions when attempting to effectively coordinate large-scale projects. Uncertainty arising due to quality problems in materials, vendor shipment delays and labour disruptions contribute to the complexity involved in project management.

In this dissertation, we develop mathematical models to guide the acquisition and disposal of items in a project context. Specifically, with respect to a particular facility, we divide time into two distinct phases: a "construction phase" (during which the facility is erected) and an "ongoing phase" (during which the facility is in operation). We shall consider an important, expensive item (e.g. pipe for a pipeline project, or valves and electronic control devices in a compressor station) that exhibits uncertainty with regard to total requirements during the construction phase. Materials managers are allowed to place a single procurement order for this item at the beginning of the construction phase. We consider such costs as procurement costs, holding charges and stockout penalties.

Surplus units on-hand after the construction phase completion may be salvaged (disposed for revenue), or retained for usage (as a spare part or for routine replacement) in the ongoing operations of the constructed facility.

The decision variables, then, include the appropriate quantity to procure at the beginning of the construction phase, and the proper amount to dispose, in the event of on-hand surplus, after construction completion. The procurement and disposal quantities are to be selected so as to satisfy project and ongoing requirements at lowest possible cost.

We use a spreadsheet model to determine the best procurement and disposal quantities.

We consider the effects of non-constant salvage values. "Marginally decreasing" salvage values consist of those situations in which larger disposal quantities generate smaller "per unit" salvage revenues. We also examine the scenario in which, over a limited range, "per unit" salvage values may rise as additional units are disposed ("increasing" salvage values). This higher unit price may still be beneficial to the buyer for it saves the negotiation and logistics hassles involved in procuring smaller quantities from several companies, as well as providing transportation economies.

We numerically explore the advantage of considering both construction phase and ongoing phase issues when making one's original construction phase procurement decision. We determine the least-cost procurement quantities that would result if one considered solely construction phase issues (the "myopic" approach), or construction phase issues plus the disposal of all surplus stock (the "all-disposal" strategy). We determine the percentage cost penalties of following these "non-integrated" approaches, and illustrate those cases that lead to higher penalties.

We further consider the impact of a future project occurring at a random time after completion of the initial project's construction phase. We show how this scenario affects the initial project's procurement quantity. Two specific cases are illustrated: no inter-project usage, and deterministic, level inter-project usage.

We also analyze two separate extensions to our mathematical model; namely, the incorporation of deterministic, time-varying ongoing phase usage, and multiple procurement opportunities in the construction phase.

ACKNOWLEDGEMENTS

I will be forever grateful for the exceptional supervision provided by Professor Edward Silver. His wisdom, derived from academic and practitioner perspectives, offered me continual sources of strength throughout my doctoral studies. Besides an outstanding teacher, he proved to be an wonderful friend. I shall always retain fond memories of our association, and am honoured to be considered his colleague.

The additional members of my Examination Committee deserve special recognition for contributing their keen insights into this research: Dr. T. Grossman, Dr. F. Hartman, Dr. R. Russell and Dr. P. Kelle (Louisiana State University).

Without implicating them, I wish to express gratitude for the practical assistance provided by the following employees of NOVA Gas Transmission Ltd., Calgary: Mr. D. Brocklebank, Mr. B. Brown, Mr. J. Ewanek, Mr. M. Kearns and Mr. R. Mayer.

During the course of this research, several other individuals provided valuable advice, on such topics as stochastic dynamic programming software packages, experimental design approaches, and convexity proofs. I am grateful for the following: Dr. D. Costa (Nestle Inc.), Dr. G. Curry (Texas A&M University), Dr. B. Deuermeyer (Texas A&M University), Dr. P. Ehlers (University of Calgary), Dr. R. Feldman (Texas A&M University), Dr. T. Fung (University of Calgary), Dr. Y. Gerchak (University of Waterloo), Dr. G. Jergeas (University of Calgary), Dr. K. Klassen (California State University, Northridge), Ms. K. May (The Ottawa Citizen), Dr. D. Montgomery (Arizona State University), Dr. T. Morton (Carnegie Mellon University), Dr. E. Petersen (Queen's

University), Dr. E. Porteus (Stanford University), Dr. T. Rohleder (University of Calgary), Dr. G. Sick (University of Calgary), Dr. M. Sobel (SUNY at Stony Brook), Dr. A. Taylor (Queen's University), Dr. R. Teunter (University of Groningen, The Netherlands) and Dr. J. Wei (University of Toronto).

The research leading to this dissertation was partially supported by the Natural Sciences and Engineering Research Council of Canada under Grant No. A1485, and by the Carma Chair at the University of Calgary.

DEDICATION

I dedicate this dissertation to my precious wife, Leanne. Her faithful support and cherished devotion throughout my years of study offered a powerful foundation of strength. Thank you, Leanne, for believing in me.

I also wish to mention my beloved father-in-law, Mr. Frank Schlosser (1937-1997). Successful project management would be difficult without people like him.

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1. INTRODUCTION

1.1 Motivation

> "Except in the midst of a battlefield, nowhere must men coordinate the movement of other men and all materials in the midst of such chaos and with such limited certainty of present facts and future occurrences as in a huge construction project".
>
> (Blake Construction Co. vs. C.J. Coakley Co. (1981); emphasis added)

Clearly, large-scale projects are subject to a tremendous amount of uncertainty. Delays in vendor shipments, quality problems, mid-stream engineering design changes (even project cancellation), environmental conditions (e.g. unpredictable project "windows" in remote, harsh climates) and labour disruptions, among others, contribute to the extreme difficulty in effectively managing projects.

In this dissertation, we shall develop mathematical models to guide the acquisition and disposal of items in a project context. Specifically, with regard to a particular facility, we divide time into two distinct phases: a "construction phase" (during which the facility is erected) and an "ongoing phase" (during which the facility is in operation). We shall consider an important, expensive item (e.g. pipe for a pipeline project, or valves and electronic control devices in a compressor station) that exhibits uncertainty with respect to total requirements during the construction phase. Silver (1989) indicated that these uncertainties may arise due to subsequent engineering design changes, the inherent uncertainty in specific types of projects (e.g. subsurface work), errors in the material takeoff, failures on installation (e.g. electrical equipment), or damage/loss of items.

Surplus units on-hand after the construction phase completion may be salvaged (disposed for revenue), or retained for usage (as a spare part or for routine replacement) in the ongoing operations of the constructed facility. As far as we have been able to determine, no previous attempt has been made to jointly analyze the effects of acquisition and disposal in a project context.

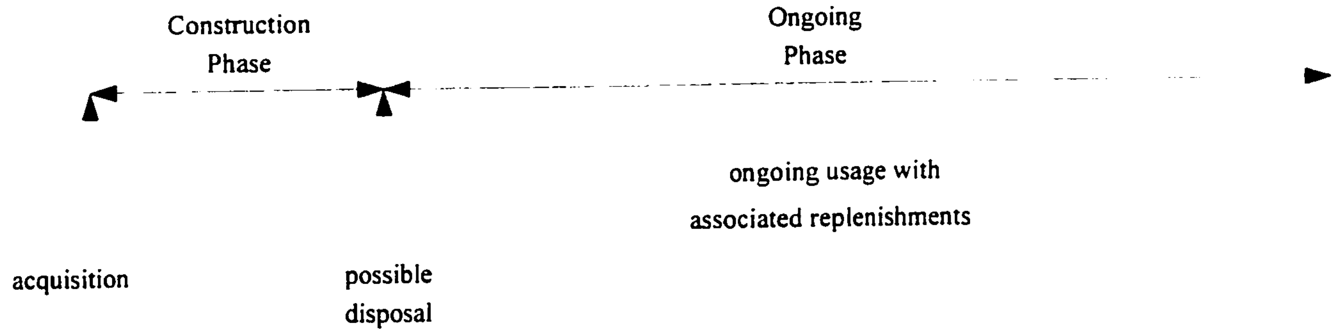
The vital decision variables, then, include the appropriate quantity to procure at the beginning of the construction phase, and the proper amount to dispose, in the event of on-hand surplus, after construction completion. The procurement and disposal quantities are to be selected so as to satisfy project and ongoing requirements at lowest possible cost. Figure 1 illustrates a timeline of the respective acquisition and disposal decisions during both phases.

Silver (1986, 1987b) conducted a survey of procurement and logistics managers involved in large-scale oil and gas projects in the Province of Alberta. Critical decision areas faced by these professionals involved coping with uncertainty surrounding total project requirements, and the disposal of project surplus. Options for disposal included, but were not limited to, return to supplier (using a buyback clause in the purchase contract), sell (usually at a discount), or trade-in on a future purchase.

Subsequent discussions with several materials management personnel at a major gas transmission company have further illustrated the pivotal nature of these respective decisions. These are by no means trivial issues. Effective materials management within large-scale projects is critical to project success. Hence, there would appear to be a practitioner-oriented motivation for a systematic method to tackle the acquisition and disposal problem. The relationship between construction phase requirements and

Figure 1

Timeline of Decision Environment



ongoing usage is an important managerial issue examined in this dissertation.

The use of the "real world" for validating various parameters in order to create an approach that is realistic, practical and able to lead to improved decision-making ought not to be treated lightly. Besides an approach applicable to large-scale project situations, we shall describe (in Chapter 9) how our approach can be applied in other decision-making scenarios.

A principal element of this research is an analysis of the relative importance of the various parameters involved in this decision-making environment. Obviously, the "optimal solution" has benefit from an analytical standpoint. However, the ability to determine the effect on the optimal solution of changes in parameter values (ie. sensitivity analysis) is critical from a managerial perspective. Wagner (1980) suggested that effective inventory management research ought to provide an indication of the quantitative tradeoffs accompanying different strategic assumptions. Several experiments will illustrate how the optimal solution varies under different conditions, and the concomitant cost penalties of deviating from a best solution.

Some of the different decision-making scenarios illustrated in this dissertation will now be described. We will begin by examining a single opportunity to procure at the beginning of the construction phase. A further scenario could allow for the existence of multiple procurement opportunities. To the extent that the firm is able to acquire items at various times during construction, project procurement may become more "just-in-time" (JIT) in nature.

The quantitative effect of product standardization represents an important issue for procurement managers. This will be treated in the dissertation by changing the salvage revenue received for surplus units disposal. We propose that the more (less) standard an item, the higher (lower) the salvage value it can earn when disposed. Silver (1987a) reported that, among company owners and procurement contractors, substantial differences of opinion exist with respect to item standardization. The former group, due to concomitant operating advantages (standard spare parts, maintenance procedures, etc.), tends to favor increased standardization. Contractors, on the other hand, dislike the concept because of its restrictive nature. Further, they suggest that some of the benefits of standardization may be eroded in the event of subsequent product changes made by the supplier. Mitchell (1962) suggested that excessive standardization could discourage the technical progress which follows the experience gained from variation in design.

Large firms dealing in the construction industry often include a department devoted to surplus disposals (under the name of "Investment Recovery Analysis", or other such title). Several trade journals are published, listing companies willing to purchase or sell various surplus items. Market conditions undoubtedly play a role in the disposal of surplus items. As a result of differing market circumstances, one may not always obtain a constant salvage value for disposals. Salvage values could be "marginally decreasing", in the sense that additional disposals yield lower per unit salvage values. Salvage values could also be "increasing" as additional units are disposed. A firm desiring to purchase a given number of units on the surplus trade market may pay a higher per unit price to a supplier that could deliver the appropriate number of units required by the firm. Purchasing from this supplier saves the negotiation hassles of buying smaller amounts from many different organizations. (We will allow salvage values to be increasing up to a point, after which the marginal values will begin to decrease).

Various scenarios in the ongoing phase will be examined. A critical managerial issue involves establishing the appropriate ongoing replenishment strategy. Increased costs (presumably when a facility operates in a more remote location) may imply different ordering strategies, such as a firm ordering larger quantities of required items in a single replenishment.

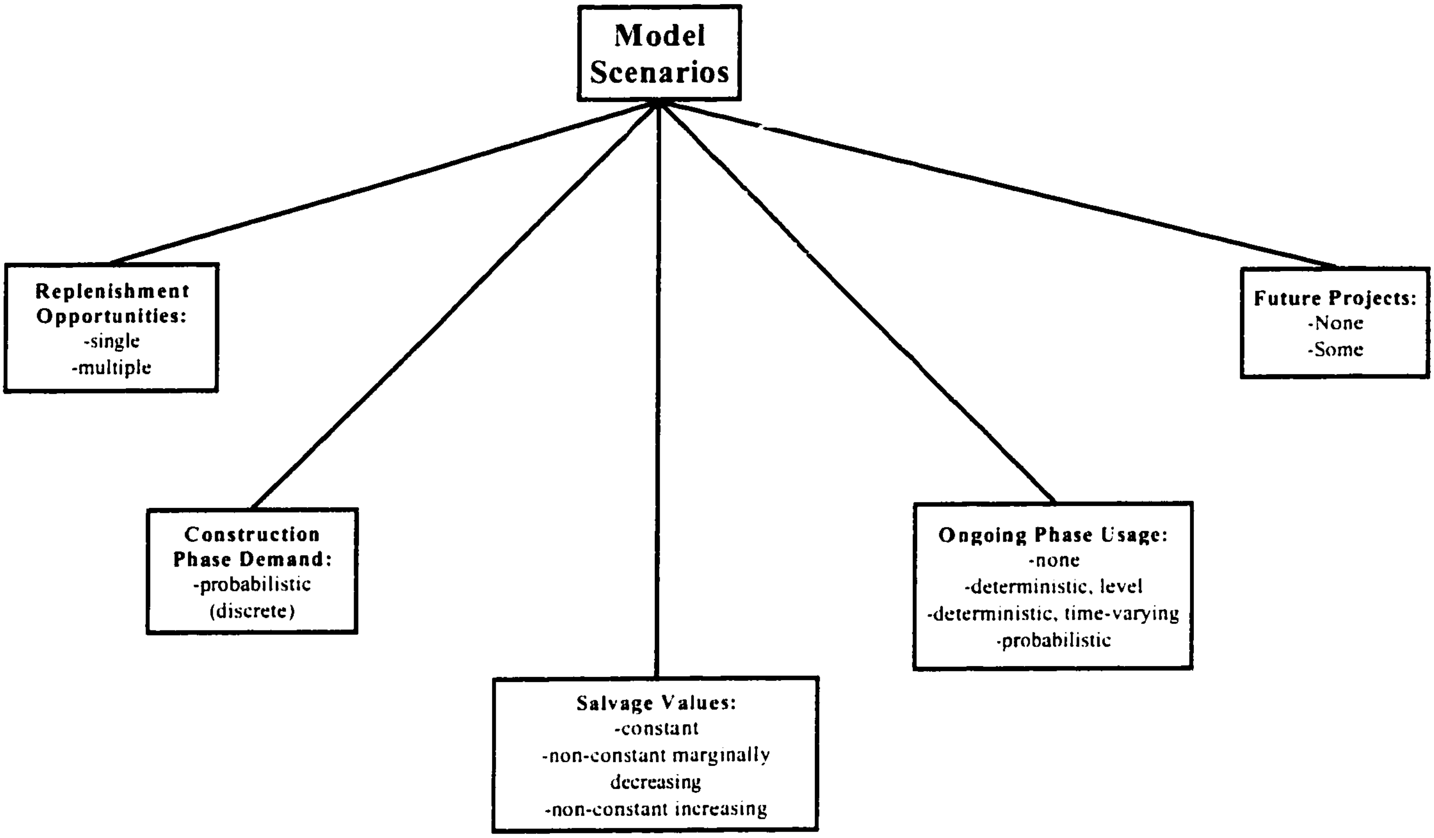
We will also explore distinct usage patterns for these spare parts in the ongoing phase. Besides deterministic level usage, we shall analyze probabilistic (Poisson) and deterministic time-varying usage. The latter case incorporates a relatively high usage of the item during the startup phase of the facility (due to the so-called "infant mortality" effect). This usage tapers over time (as the facility matures), then begins to increase as items start to wear out. In a comprehensive review of inventory management research, Silver (1981) contended that the procurement of service (spare) parts is an important research problem.

Our model will be expanded to include subsequent projects, occurring at random points in the future. The procurement and disposal of material in the original project when faced with the possibility of future projects represents an additional managerial issue examined in this research.

Figure 2 provides an illustration of the various model scenarios to be analyzed in this dissertation.

Figure 2

Taxonomy of Model Scenarios



1.2 Assumptions

The following assumptions will be used in our development of the various acquisition and disposal models. Additional assumptions will be described when required.

- We will only examine a single important item during the construction and ongoing phases.
- Without loss of generality, initial inventory, prior to the acquisition at the beginning of the construction phase, is zero.
- Any surplus is either disposed or kept on-site (ie. we do not consider multiple locations).
- Any surplus remaining after a project would be used up during the subsequent project. That is, the surplus after any project would never be sufficient to cover the complete requirements during the next project. This implies that each project requires at least some level of procurement.

1.3 Overview

In Chapter 2, we provide a review of the literature pertinent to this problem area. We reference key articles in project management, materials management, and inventory modelling.

Chapter 3 then describes the development of models to analyze decisions in the ongoing phase. We begin by treating deterministic level usage, then showing its equivalence to probabilistic (Poisson) usage. We illustrate how to calculate the optimal replenishment quantity and show that it can be approximated by the economic order quantity (EOQ). In Chapter 4, we develop models to derive optimal disposal quantities at the conclusion of the construction phase. We launch our discussion by examining constant salvage values, then show how these decisions are affected by non-constant salvage values.

Chapter 5 combines the disposal decision rules with a model to examine the acquisition of items at the beginning of the construction phase. We explore both an exact and an approximate approach for determining the expected total project costs. Chapter 6 then provides numerical results for the models developed in Chapters 3 through 5. We investigate the effect of various parameters by running our models with high, middle, and low values for each parameter.

Chapter 7 analyzes the case of subsequent projects occurring at random points in the future. We begin by exploring the situation in which there exists no "inter-project" usage, then examine the circumstance of deterministic level "inter-project" usage. Chapter 8 provides a description of two extensions to our acquisition and disposal models: deterministic time-varying usage in the ongoing phase, and multiple procurement opportunities in the construction phase.

We summarize our findings in Chapter 9 by indicating pertinent conclusions and suggesting some relevant directions for further study.

In the Appendices, we provide a glossary of notation as well as various analytical proofs supporting the text of the dissertation. We also illustrate the results of several numerical experiments designed to evaluate the heuristic used for approximating the level of on-hand inventory in the case of future projects and deterministic level ongoing usage.

2. LITERATURE REVIEW

This dissertation combines three dimensions of operations management (OM) research: project management, materials management, and inventory modelling. Each will be discussed in turn, with special attention given to the problem of excess stock disposal. Figure 3 provides an overview for this chapter. We shall begin our discussion at the top of the framework, and proceed in a clockwise fashion.

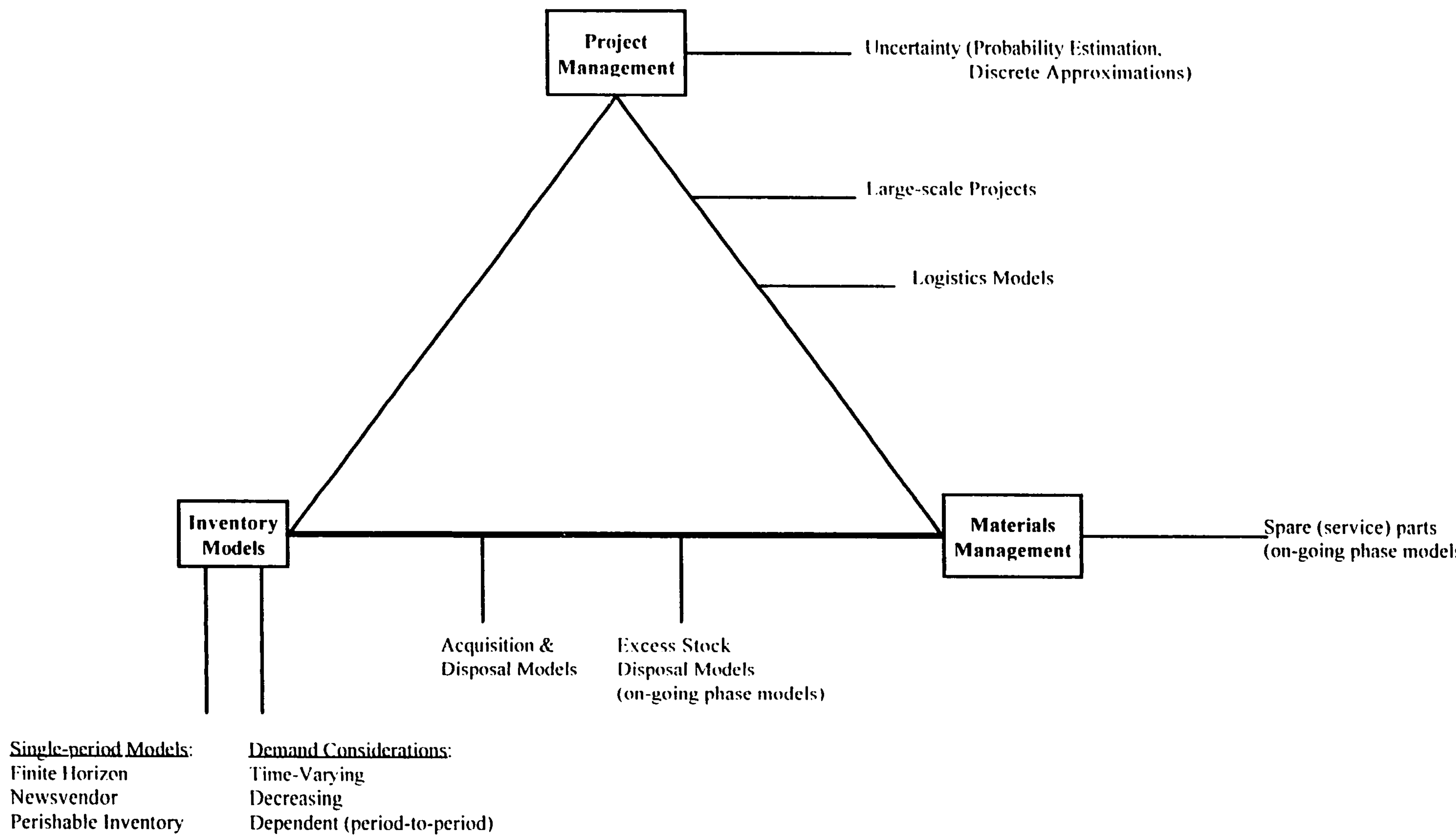
2.1 Project Management

The management of projects has received considerable attention in the literature. Several textbooks, including Harrison (1992), Meredith and Mantel (1985), Lewis (1995), and Morris (1994), have described analytical approaches in project management as well as organizational structures required to effectively administer projects. Projects are often judged on the cost or time taken to complete the project, the project's technical performance (ie. how well does it do what it is supposed to do), or the degree to which stakeholders are satisfied.

Fox (1984) described a framework for evaluating the management of large, complex projects. He based his framework on eight mega-projects (costing more than $1 billion each), including the TransAlaska pipeline. This 800 mile-long project was initially budgeted at $900 million, but ballooned to $8.5 billion due to engineering and regulatory changes. Fox illustrated the appropriate criteria one must apply when

Figure 3

Dissertation Research Framework



evaluating large-scale project management as well as common errors to avoid. He suggested that one ought to judge a project manager's decisions on the basis of the information possessed by the manager when the original decision was made. Morris (1986) described some of the lessons learned from the successful management of projects. In particular, planning and design work ought to consider the future engineering and construction portions of the project as well as a careful regard for any geophysical uncertainties.

The treatment of uncertainty within a project management setting will now be illustrated. AbouRizk and Halpin (1992) surveyed the durations of 71 different activities and tasks in North American construction projects. They found that the beta distribution would appear to be a suitable candidate for modelling construction task durations. Shtub (1986) represented project uncertainty via stochastic activity durations and lead times. He developed a heuristic procedure which could calculate the probability of finishing the project late and the concomitant net present cost of the project. He then generated an efficient frontier of net present costs versus the risk of not completing the project on time. This allowed managers to examine the trade-offs involved in their projects. Recognizing a nonlinear distribution of work completed versus the duration of the activity, Gilyutin (1993) suggested that project management software needs to be refined. According to the author, software tools are ineffective if they only allow linear completion plans (ie. work would be completed at a linear pace).

Shachter and Kenley (1989) reported on the use of influence diagrams, a graphical representation for a decision problem under uncertainty. In an attempt to develop a model for construction projects with uncertain activity durations and costs, Padilla and Carr (1991) used influence diagrams. Resource assignments were dynamically adjusted while the project was under construction.

Skitmore, Stradling and Tuohy (1989) developed a "psychological" model to assist human decision-makers in coping with the uncertainty they face. Crucial to dealing with risk was effective communication within a project environment and the equal distribution of responsibility amongst participants.

Silver and Jain (1994) examined uncertainty within a project context. Specifically, they allowed uncertainty with respect to period-by-period resource requirements and available period-by-period supplier capacities. Procurement quantities were selected at the beginning of individual periods, taking into account the current inventory level and having observed the actual requirements and capacities in previous periods. Costa and Silver (1996) developed an exact procedure for this problem, based on a branch and bound approach with a dynamic programming algorithm. Since the exact formulation could be only used on problems of relatively small size, various heuristic procedures were also examined.

By reducing potential solution spaces, discrete approximations of continuous random variables can be used to tackle uncertainty within a project management framework. Miller III and Rice (1983) suggested that, in order to gauge the accuracy of a discrete approximation, it must preserve as many moments of the original distribution as possible. Claiming that typical discrete approximations often underestimate all moments, they proposed a numerical integration procedure to determine more accurate approximations. Zaino, Jr. and D'Errico (1989) emphasized using Taguchi's work on tolerance analysis to provide better approximations.

Keefer and Bodily (1983) highlighted the importance of searching for an approximation which provides reasonable results for estimating expected utilities over a variety of utility functions. Suggesting that one can encounter problems if decision variables are made arbitrarily discrete, Stonebraker and Kirkwood (1994) proposed using mathematical programming solution methods to handle continuous decision variables. They used lower and upper bound functions to represent the potential range of values for these variables.

Poland (1993) discussed "certainty equivalents". These represented the amount of certain payoff one would be indifferent in receiving versus an uncertain payment from a decision analysis problem. He recommended using these in assessing the "goodness" of a discrete approximation. Keefer (1994) further alluded to the use of certainty equivalents. He suggested that, instead of solely concentrating on the estimation of moments of the underlying distribution, one ought to closely match the certainty equivalents.

Estimating the uncertain elements of project management problems often requires carefully eliciting various values from those most intimately familiar with the system under study. Merkhofer (1987) proposed some formalized procedures for obtaining, among other items, a discrete representation of continuous probability distributions. Keeney and von Winterfeldt (1991) suggested a seven-step process for eliciting probabilities. They adopted their method for a large-scale study involving nuclear safety.

Logistics is an integral component of large-scale projects. Hax (1976) surveyed the design of large-scale logistics systems. Keys for effective systems included, but were not limited to, delivering the requisite quantities of goods where and when required, and with appropriate levels of quality. Assuming stochastic usage, Luxhoj and Rizzo (1988) analyzed spare part inventory levels within large-scale logistics models.

Fabrycky and Banks (1966) developed a hierarchy of logistics systems, based upon the nature of the item and source (either single and multiple). Sobel (1988) analyzed dependent period-by-period demands in a logistics model.

Previous researchers have illustrated the significance of effective materials management within a project management context. Some of these articles will be discussed in the following section.

2.2 Materials Management

Materials management, according to Bennett (1985), involves the coordination of several functions: purchasing, inventory control, warehousing, distribution and the disposal of surplus materials. He showed that item standardization can lead to a reduction in total inventory investment. Kathawala and Nauo (1989) suggested that integrated materials management ought to be viewed from a holistic perspective. They remarked that the efficient management of the disposal function, once regarded as an incidental task, has gained substantial importance due to a better recognition of the key benefits it can generate in an organization.

Diekmann (1981) emphasized the critical aspect of accurately determining the quantity of both materials and labour required in a project. In an effort to more effectively integrate the project management and materials management functions in the pulp and paper industry, Mendel (1986) developed an efficient decision support system.

Smith-Daniels and Aquilano (1984) claimed that managers need to simultaneously consider project schedules and material requirements. They developed a heuristic scheduling procedure, applicable to multi-project environments. A heuristic scheduling procedure was described by Shtub (1988). He addressed the problem of scheduling a project in which expensive, long lead time inventory items are ordered from outside vendors. Shtub (1991) analyzed a heuristic procedure for the scheduling of programs within a number of identical manufacturing or construction projects (e.g. ships, houses). He discussed the Line of Balance (LOB) technique and assumed unconstrained resource availability.

Recognizing the "lumpy" nature of materials requirements within a project, Smith-Daniels and Smith-Daniels (1986) examined the performance of several lot-sizing heuristics. Assuming unconstrained resource availability, the same authors developed in a later article (1987) a mixed-integer programming formulation to find the optimal project scheduling and materials lot-sizing solution.

A vital materials management issue involves the provision of spare or service parts. Numerous articles have appeared in the literature, attempting to determine the appropriate level of spare parts inventory. Since the item we propose to analyze in our acquisition and disposal models could be used as a spare part in the ongoing operations of the firm, the literature on spare parts is worthy of consideration.

Melese, Barache, Comes, Elina and Hestaux (1960) reported on a decision rule for the inventory control of spare parts in the French steel industry. They considered two types of costs (inventory carrying charges and stockout costs) and assumed a constant lead time. Johnson (1962) allowed for stochastic lead times in his analysis of spare parts control procedures at Canadian army workshops. The optimal solution showed a 34% reduction in operating costs combined with a 24% increase in spare part availability.

Taking into account the possible repair of spare units, Howard (1984) analyzed spare parts inventories of machinery for the National Coal Board (Britain). Jung (1993) also considered spare units repair, but used a non-stationary failure rate. This implied reliability amelioration as products were improved. He used the commercial airline industry as his case study.

Foote (1995) described a forecasting method, based on maximum likelihood estimation, for the procurement of spare parts in a Philadelphia-based Aviation Supply Office. Considering inventory carrying, ordering and shortage costs, Bartakke (1981) developed a technique based on mathematical programming and simulation to determine appropriate spare parts inventory levels for Sperry-Univac. A multi-echelon provisioning system was utilized within this organization. An additional study of a multi-echelon system was provided by Shtub and Simon (1994). Applying the problem in a military context, the authors developed an algorithm that would maximize fill rates at those maintenance facilities deemed to possess the highest priority.

A large Belgian chemical plant, involving about 34,000 different types of inventoried spare parts, was the subject of an analysis by Vereecke and Verstraeten (1994). Demand occurrences were assumed to be Poisson distributed. Jensen (1992) conducted a case study to examine spare parts inventory systems for the automobile workshops within the Catena Group, an organization which manages the European operations of Volvo. Another study on spare parts systems within the automobile industry was conducted by Moore, Jr. (1971). He developed a forecasting technique and dynamic inventory model to schedule the production runs of service parts.

Due to the inherent incompleteness, inconsistency and imprecise nature of many of the parameters involved in determining optimal spare parts levels, Petrovic and Petrovic (1992) suggested using heuristics. They developed an expert system to provide the appropriate assortment and quantities of spare parts in electronic systems.

Phelps (1962) developed a dynamic programming method to determine the optimal number of spare parts to procure, repair or dispose. His state variables included the number of "serviceables" (repaired units not yet used) and the number of "repairables" (units no longer working, but able to be repaired) available at the beginning of each period. A simulation model, developed by Matta (1985), determined the appropriate inventory policy for repairable spare parts. He also determined the optimal number of repair stations to include within a manufacturing facility.

Thomas and Osaki (1978) analyzed a maintenance problem, where there existed a lead time on orders and only sufficient room to store <u>one</u> spare unit in stock. They considered shortage, holding and ordering costs. Their work was expanded by Kabir and Al-Olayan (1996). The latter authors explored the situation in which an item is replaced upon reaching a specific age, or on failure (whichever comes first). Optimal values were determined by minimizing the expected total cost per unit time.

Geurts and Moonen (1992) analyzed insurance-type spares. These expensive spares are essential to the operation of the equipment they serve, but have a rather considerable probability of never being needed during the lifetime of the equipment. The costs of a stockout (requiring a spare, but having none available) are enormous. Providing spares for equipment which is used on a scheduled, periodic basis (e.g. the Space Shuttle) was the subject of an article by Bridgman and Mount-Campbell (1993). Since the machinery in question is only used periodically, spares need not be available at all times. They are only required when the next scheduled usage would begin.

Faced with the situation of small and decreasing usage (ie. "slow-movers"), Brown (1967) determined the "all-time" supply that one ought to order. Fortuin (1980) calculated the final order for a service part in a discontinued, manufactured product. In this "resource-saving age", Yamashina (1989) considered it a "social responsibility" for a firm to supply its customers with service parts, even for products manufactured and sold many years previously.

Hollier (1980) concentrated on the distribution component of the spare parts problem. He examined the situation in which a manufacturer needs to provide an acceptable level of after-sales service at least cost.

Gross and Ray (1964) discussed two techniques for controlling spare parts inventories. Bulk control (items are expensed on a periodic basis) was compared to item control (each unit is accounted as it is used). The unit cost and demand of an item helped to determine the appropriate costing technique.

Obsolescence considerations are important within materials management problems. Incorporating stochastic demands and obsolescence rates, Pierskalla (1969) suggested a discrete-time model to deal with this problem. After obsolescence occurs, there is no longer any demand for the item. Masters (1991) likened obsolescence to anticipated demand failing to materialize. He allowed a salvage value for obsolete units, hinting that even though items may be obsolete to one organization, they may still be quite useful for another. Song and Zipkin (1996) also equated obsolescence with deteriorating demand. They determined appropriate inventory management policies for such problems. Given that availability of items was deemed essential, the authors showed that simply "cutting" inventories in the face of obsolescence may not be the answer.

2.3 Excess Stock Disposal

A critical inventory management decision arises when an organization finds itself with an excess of stock on-hand. Specifically, the problem is to determine the appropriate amount of stock to dispose. Disposal creates benefits in at least two ways; namely, the salvage revenue obtained from surplus unit disposal, and the savings in inventory carrying charges due to the reduction in on-hand stock. However, due to ongoing operational usage of the item, the organization may be required to eventually repurchase (or remanufacture) units. Eliminating "too much" of this stock may, thus, force the company to make premature repurchasing arrangements. As a result, the cost tradeoff exists between salvage revenue and reduced inventory carrying charges versus future repurchasing costs.

The potential causes of excess stock are legion. An abrupt decrease in demand or changing business conditions may lead to an excess stock situation. Similarly, price increases, forecasting errors, customer cancellations, the introduction of a new (competing) product, production overruns, overpurchasing (to protect against stockouts), or even simple goofs (e.g. errors in the transmission of an order request) may be the basis for the excess occurrence. Poor quality in final product assembly could lead to an over-supply of a sub-component. Ultimately, inadequate materials planning and execution systems are at the root of the excess stock problem.

Tersine and Toelle (1984) suggested that excess inventory is a "dead weight". Among other adverse effects, it uses valuable storage space, inflates assets, diminishes working capital, and causes a reduction in return on investment (ROI). Toelle and Tersine (1989) claimed that inventory is in fact a liability if it costs more than it earns.

Gottlieb (1994) submitted that two-thirds of the U.S. national defense stockpile is wholly or in part excess. This surplus stock represented an investment of a few billion dollars. He further alluded to the political difficulties and economic disruptions that can be created should a country be perceived as "dumping" excessive amounts of key materials. Bolwijn and Kumpe (1986) cited Martin Kuilman (a Philips Vice-President), who maintained that the company has a substantial investment tied up in unnecessary inventories of subassemblies, finished products and raw materials. May (1996) reported on a Canadian Government Treasury Board study which indicated that at least $1.25 billion could be annually saved by eliminating over-supply situations. Police hats, scientific equipment and National Defense inventories represented some of the excess items. We even recall the story of a local firm which contracted with a large construction company to purchase all of its excess pipeline during a given year. The local firm received such a quantity of surplus pipeline that it quickly filled its own warehouse facilities. It was eventually required to rent storage space in order to house the excess stock!

Silver, Pyke and Peterson (1998) reported that, given the current increases in the rate of technological change (which imply a shortening of the typical product life cycle), the general area of excess stock disposal is likely to continue to increase in importance.

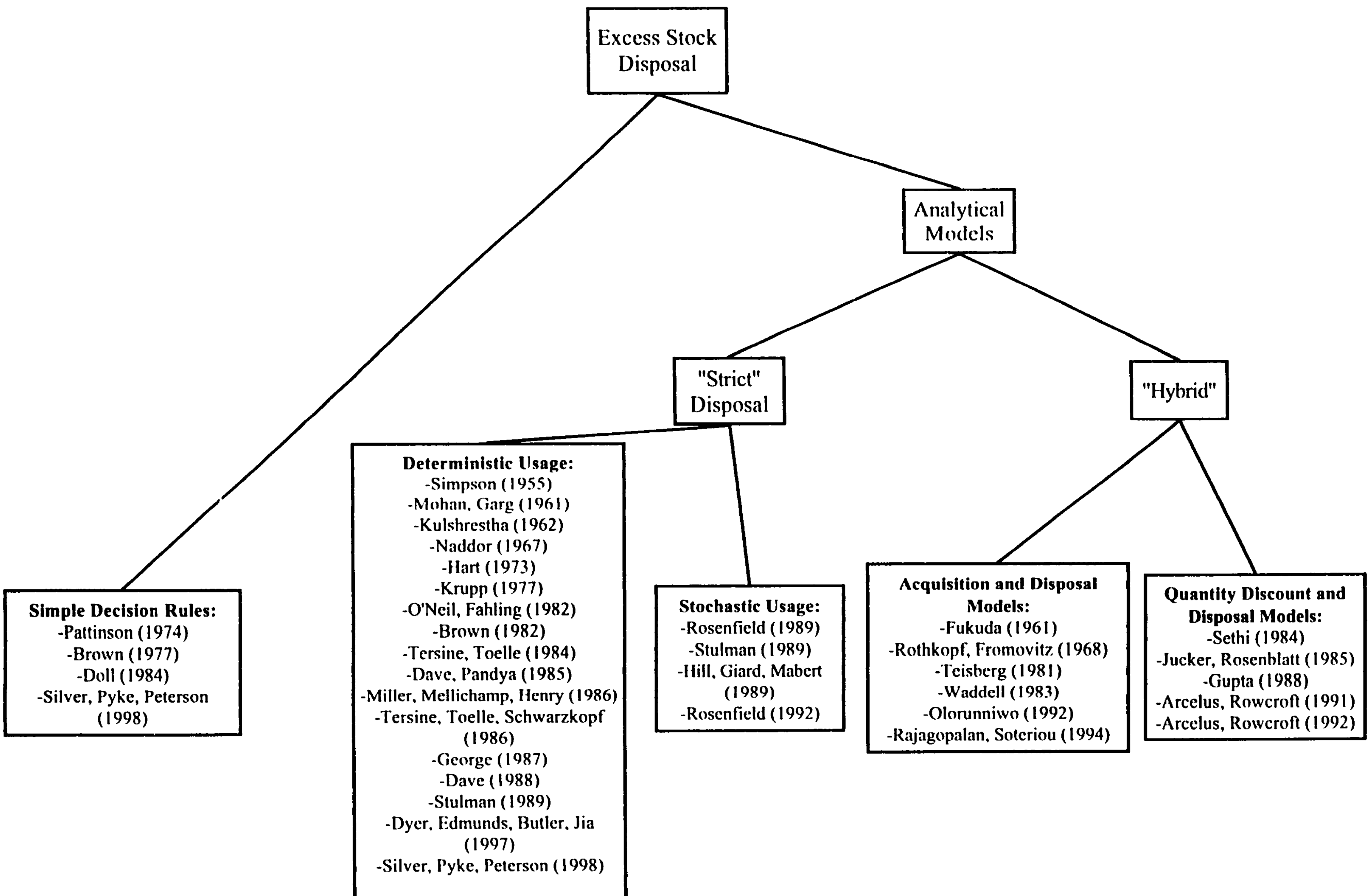
We begin by describing simple decision rules for the disposal of excess stock. We then relate analytical models which consider the disposal decision in isolation ("strict" disposal), with either deterministic or stochastic usage. We conclude our discussion with a treatment of "hybrid" models, ie. those analytical models in which the disposal choice is combined with an acquisition decision. We examine general acquisition and disposal models, as well as models that link quantity discounts and disposals. Figure 4 illustrates the framework adopted for this discussion.

2.3.1 Simple Decision Rules

These decision rules can be viewed as simple (mainly subjective) "rules of thumb". They pay little (or no) attention to such details as inventory carrying charges and future reordering costs. Their value is not so much in "to-the-penny" exactitude, but rather in their ability to offer managers a conceptually simple, easy to implement technique of determining the amount of excess stock an organization ought to dispose.

Figure 4

Excess Stock Disposal Framework



Pattinson (1974) prescribed a cross-functional approach to this problem, involving representatives from such departments as marketing, operations, finance and engineering. He suggested that one closely monitor all inventory in excess of 12 months' supply. Any stock exceeding that time supply would be considered surplus to current requirements.

Brown (1977) offered a description of the excess stock issue for a general managerial audience. He advocated using managerial intuition in setting two limits, the "number-of-weeks supply" and the "dollar-value-of supply". Any stock that surpassed either of these two limits would be regarded as excess inventory.

A managerial examination of the nature of excess stocks was provided by Doll (1984). He proposed the Inventory Evaluation and Review Technique (INVERT), a process for reviewing the present state of an organization's inventory position and providing guidelines for improvement plans. While he suggested that an economic analysis be performed to determine the most beneficial disposal strategy (retain, sell, segregate, write-off), he failed to indicate any analytical details of this procedure.

A more quantitative, yet still relatively simple, excess stock rule was given by Silver, Pyke and Peterson (1998). They suggested that one calculate, on an item-by-item basis, the expected time at which the inventory level would be depleted. This quantity was known as the item's "coverage". By listing each item in descending order of coverage and also maintaining a record of the item's unit value, managers could obtain a quick indication of the "cost" of excess stock. For instance, they could easily determine the percentage of total inventory value tied up in stock with coverage of at least, say, 30 months. This would provide decision-makers with evidence of the significance of the excess stock problem. Disposal of a portion of the inventories of items with at least a certain coverage would "free up" a specific amount of inventory investment.

2.3.2 "Strict" Disposal - Deterministic Usage

Although simple decision rules can provide a quick basis for decisions regarding the disposal of excess stock, analytical models can consider a variety of specific inventory details. The outcome of these modelling efforts is the quantity (and in some cases, the "timing") of excess stock disposal.

Several models have been developed to examine the disposal of excess stock, given that the organization is currently in a surplus inventory situation. We refer to these as "strict" disposal models. Researchers have attempted to determine either an economic retention quantity (or economic retention time period). Any stock that is found exceeding either the best retention quantity or time supply ought to be disposed. Marginal salvage values for stock disposal have been assumed to be constant in all cases. These models have analyzed the problem from different perspectives (e.g. usage distributions, cost components used, and the manner in which inflation and the time value of money are addressed).

Assuming known and constant future item usage, Simpson (1955) was an early contributor to the excess stock problem. Basing his analysis on inventories held at Naval supply stores, he calculated an economic retention time period. His break-even examination featured a tradeoff between storage and obsolescence costs versus the expenses of repurchasing the material in the future (if and when required). The author used a constant time until obsolescence, and ignored inflation (ie. the future unit acquisition cost was assumed to be equivalent to the current price).

Mohan and Garg (1961) expanded some features of Simpson's model. Besides considering inflation, they allowed the time until obsolescence to follow a general probability distribution. In fact, they used the exponential distribution and constructed an appropriate economic retention period. Kulshrestha (1962) expanded Simpson's model by incorporating an exponential probability distribution during initial deterioration and obsolescence, up until the time at which one could model the time until obsolescence as a normal distribution.

Naddor (1967) developed an excess stock disposal model for the cases of both finite and infinite horizons. However, he did not include any present value considerations in his analysis. Dave and Pandya (1985) expanded Naddor's model by allowing the stock to exhibit a constant rate of deterioration. They examined a classical lot-size inventory system, in which the EOQ was used for future, ongoing replenishments. Assuming no shortages and zero leadtime, they developed expressions for the best amount of surplus stock to retain. Under no conditions would an organization dispose a quantity of such a size as to leave themselves with less than the EOQ on hand. Dave (1988) elaborated on the previous work, by developing models in which shortages were permitted to be completely backlogged.

Hart (1973) recognized that demand rates may be variable during the planning horizon. However, he assumed that this horizon could be divided into a given number of subperiods (which would not necessarily be of the same length), and that a separate forecast of demand could be generated for each of these respective time periods. The demand rate, then, was presumed to be constant within each of these subperiods. In this way, item usage was deterministic, yet time-varying. Hart heuristically determined a future procurement schedule for the item, and noted that the heuristic performed quite well when compared to the optimal schedules produced by a dynamic programming algorithm (this latter technique required considerably longer computing times, a major consideration when thousands of inventory items would be examined). Specific costs considered included inventory holding charges, fixed and variable procurement costs, and scrap value of disposed units. After discounting all future costs to the present, he was able to find the optimal retention quantity by using a sequential search procedure.

An additional effort to recognize deterministic, time-varying demand was produced by Miller, Mellichamp and Henry (1986). Basing their research on a financially troubled General Motors carburetor assembly plant in Tuscaloosa, Alabama, they attempted to find minimum cost time supplies for surplus items. Due to the multiproduct nature of the facility, the same product could go into several "kits". As a result, there existed an additional manner in which excess stock could be disposed - surplus units could be "remade" into a different product for which there was a "good" demand. Their present value model considered inventory carrying charges as well as future procurement costs. Since future replenishments of the item would most likely be produced on a smaller lot-size production run, the unit acquisition cost was assumed to be higher in the future. Other costs considered included salvage revenue and the tax savings associated with inventory write-offs. Using the technique of differencing, the authors determined the <u>integer</u> value of time supply that yielded the smallest total discounted cost. Adoption of the analytical method generated savings of approximately $1 million at the GM plant.

Krupp (1977) illustrated the manner in which obsolescence can lead to excess inventory. He defined "fiscal obsolescence" as the gradual depletion in a product's value, resulting from the effect of accrued carrying charges over an extended period of time. An item becomes "fiscally obsolete" at the specific point in time at which the cumulative carrying charges exceeded the net unrecoverable value of the item (standard cost less resale or salvage value). Any stock which exceeded this economic time supply would be disposed. However, he neglected to include such factors as time value considerations, or the effect of future reordering and repurchasing costs.

Measuring alternative disposal strategies in terms of their effects on relevant cash flows, O'Neil and Fahling (1982) presented a decision model for excess inventories. Their cash flow liquidation model, possibly appropriate for a retailing or distribution enterprise, evaluated the present values of inventory carrying charges and cash from disposal (net of tax). However, they did not incorporate future reordering and repurchasing costs. They allowed disposal of stock at "discrete" points in time (the end of each month). A rather cumbersome procedure was developed to determine the best disposal strategy. The authors evaluated the total discounted cash flow of retaining all inventory, then the total discounted cash flow of liquidating one month of inventory (and retaining the remainder), and so on until all possible liquidation quantities up to and including the quantity on hand had been evaluated. The optimal disposal amount, then, was the one leading to the maximum discounted cash flow. Tersine, Toelle and

Schwarzkopf (1986) expanded O'Neil and Fahling's liquidation model. They adopted continuous compounding of future cash flows (the earlier authors had suggested that all cash flows occurred at the end of discrete time periods). In addition, the later researchers were able to develop an analytical, closed-form result for the optimal number of months of stock to retain.

Brown (1982) determined economic retention quantities by comparing current salvage revenue with the future costs of repurchasing the item. He permitted the consideration of the time value of money, but disregarded holding costs and the fixed cost of placing a future replenishment order.

Tersine and Toelle (1984) generated relationships for the economic time supply of an item, under the existence or non-existence of present value and inflation considerations. Backorders were not allowed. Their "net benefit" for the disposal of excess stock may be conceptualized as:

Net Benefit = Salvage Revenue + Holding Cost Savings - Repurchase Costs - Reorder Costs

The economic time supply produced by the present value model was shorter than the one given by the model in which present value was not considered. Future reorder and repurchase charges can be heavily discounted when considering present values. This would tend to reduce the appeal of retaining more units of excess stock. As an outgrowth of their models, Tersine and Toelle computed the minimum economic salvage value, the lowest price for which a unit of excess stock would be disposed. This has considerable managerial appeal, since it provides some indication of the sensitivity of solutions to changes in model parameters.

Silver, Pyke and Peterson (1998) described a method to calculate the amount of excess stock which ought to be disposed. Neglecting any present value considerations, they considered such parameters as the current inventory level of the item, replenishment lot size (EOQ), annual item usage, unit acquisition cost, inventory carrying charge, and salvage value. The authors note that, when salvage value is equivalent to unit acquisition cost, the best disposal strategy is to dispose down to the EOQ level, ie. in this case, it is optimal to put the inventory into the same situation as immediately subsequent to the receipt of a replenishment.

George (1987) determined the minimum disposal price one must receive to obtain immediate disposal of an <u>entire stock</u> of surplus units. Restricting attention to a slow-moving, non-replenishable item, he assessed different pricing strategies by comparing the net return on invested funds. He found that, in most practical situations, a special price of at least 80% of the normal price would be required for entire surplus disposal.

Stulman (1989), ignoring the possibility of obsolescence or spoilage, developed an expression for the optimal retention quantity. His net benefit of excess stock disposal involved surplus revenue (received immediately) as well as the present values of associated carrying charges and ongoing replenishment costs.

Dyer, Edmunds, Butler and Jia (1997) applied utility theory to the disposal of fifty metric tons of surplus weapons-grade plutonium, an important decision faced by the U.S. Department of Energy. The authors considered such issues as cost, nonproliferation and an assortment of environmental, health and safety factors. Disposal alternatives such as immobilization, “borehole” (direct disposal) and the use of plutonium to create nuclear reactor fuel were examined.

2.3.3 "Strict" Disposal - Stochastic Usage

Rosenfield (1989) was the initial researcher to study the problem of excess stock disposal, given stochastic usage. The number of units demanded per "demand episode" was assumed to follow a Poisson distribution. While he assumed no stockouts and no additions to inventory, the author did consider such factors as the immediate salvage value of surplus unit disposal, holding costs resulting from carried items, and the ultimate sales value of surplus stock. He applied his methodology to an actual distributor of durable goods faced with excessive amounts of slow-moving items. The model showed that substantial savings could be earned by the judicious disposal of surplus stock. Rosenfield also examined the effect of inventory perishability on the excess stock disposal decision. He obtained closed-form results for the cases of complete (all-units) perishability at random or known times, and for individual item perishability at random times.

In a later paper, Rosenfield (1992) showed the optimality of a myopic policy when disposing excess stock. Assuming that the same disposal opportunity is available at any subsequent time and given Poisson demand, the optimum threshold remains the same. Notwithstanding the opportunities to change one's mind in the future, one still disposes the same number of items. Stulman (1989) also found an expression for the optimal retention quantity given probabilistic (Poisson) usage.

Hill, Giard and Mabert (1989) analyzed service parts inventory retention levels in a Fortune 100 company. They developed an integrated, menu-driven, database decision support system (DSS) which permitted the forecasting of future demand and the determination of optimal retention stocks. The authors permitted disposal to occur immediately, or at the "product termination date". The latter time was established by marketing as the period after which no parts required for the specific product were to be kept in stock. Various components such as both tax savings and after-tax revenue of surplus unit disposal (either immediately, or at the termination date) and after-tax back-ordering and carrying costs were included in their model. The Fortune 100 enterprise examined by the researchers had an original investment of $50 million in service parts inventories. Over a two-year period, the organization was able, with help of the analytical model, to dispose $13 million worth of service parts. This resulted in a tax savings of approximately $6 million.

2.3.4 Acquisition and Disposal Models

Now our attention shall turn to a consideration of those analytical approaches combining the acquisition and disposal decisions. We note that the disposal decision now consists of the quantity of excess items to dispose, as well as the timing of disposals.

Fukuda (1961) was perhaps the first to jointly consider acquisition and disposal decisions. He examined ordering and disposal policies in a multiechelon, multiperiod inventory environment. Considering such details as ordering costs, disposal values, shortage penalties and holding costs, he was able to determine optimal policies for the planning horizon. Essentially, the decision made at the beginning of each time period was always one of the following: an order of a certain amount is placed, a given quantity is disposed, or no ordering or disposal choice is made (the "do nothing" alternative).

Rothkopf and Fromovitz (1968) discussed the rental of container units, and the decision as to when to return the container to the supplier. They considered a commodity, purchased in bulk using containers that must be rented. Rental fees for the container stop when the container is returned. However, returning the container (to terminate the rental charges) requires discarding unused contents. Under what circumstances, therefore, ought the container to be returned? The authors analyzed constant and exponentially distributed demand sizes, as well as the discounting of future costs. They further considered the decision as to the size of the container to rent.

Teisberg (1981) illustrated a model to guide the ongoing acquisitions and disposals (releases) of the U.S. strategic petroleum reserve. He developed a multi-period, stochastic dynamic programming tool to analyze this situation, incorporating potential "states" of the oil market in a given time period. His methodology is from a rather "economics" viewpoint as he considered "consumer surplus" and the supply and demand functions for oil in both a domestic and world context. For each entering stockpile size and each possible oil market state, and using the present value of all relevant costs, he was able to determine the optimal stockpile acquisition or release rates for a specific time period.

Models treating the maintenance and replacement of equipment have been examined in the literature. These models, although examining an "acquisition and disposal" scenario, consider a varied set of costs when making appropriate decisions. Maintenance charges, operating costs, lease costs, license fees and road use expenditures are a few of the many parameters considered. Furthermore, these approaches often examine the timing of equipment disposal, rather than how many units of excess stock ought to be disposed. Models often use the shortest path method to determine when to procure or dispose an expensive piece of equipment (Eppen, Gould and Schmidt (1993) contain a description of this methodology for the equipment replacement problem).

Waddell (1983) used this methodology to analyze the replacement of highway trucks within the Phillips Petroleum Company. Annuals savings of $90,000 were reported due to the implementation of his approach. Olorunniwo (1992) examined several issues surrounding the maintenance of equipment (e.g. when should maintenance be performed, and of what type (corrective, preventive, overhaul) ought it to be). Adopting an integer programming methodology, Rajagopalan and Soteriou (1994) treated equipment procurement as a "capacity addition". Due to breakdowns caused by physical deterioration and technological obsolescence, the effective capacity of a piece of equipment may diminish over time.

2.3.5 Quantity Discount and Disposal Models

An additional class of models in which acquisition and disposal decisions are combined involves those approaches featuring quantity discounts and disposals. An important type of quantity discount treated in the literature concerns "all-unit" structures (see Johnson and Montgomery (1974) or Silver, Pyke and Peterson (1998) for a treatment of their effects on inventory management and control). An all-units discount, as opposed to an incremental one, offers the reduced cost on all procured units. Without loss of generality, one can assume a situation in which two unit prices are possible (c_0 at the lower quantity, and c_1 at the larger quantity, where $c_1 < c_0$). In order to take advantage of the discount, a certain number of units, Q_d, must be purchased.

Sethi (1984) proposed certain situations in which it may be better to purchase the larger number of units (Q_d) at the lower unit price, then dispose of a given quantity (even if there exists a cost in making such disposals). Jucker and Rosenblatt (1985) extended the work of Sethi. They evaluated the disposal of excess stock in a quantity discount context, for a single-period situation. They determined a range ($Q_r \leq Q < Q_d$) just before the breakpoint, Q_d, such that it would be better for the purchaser to order the larger quantity. Jucker and Rosenblatt also discuss the implication of probabilistic demand in this quantity discount - disposal decision. For stochastic usage, the purchaser will always wait until the end of the period to dispose excess units. This occurs due to the relative uncertainty surrounding total demand and since, in the single-period newsvendor formulation, the period is assumed to be so short in duration that holding costs may be ignored. In the deterministic case, the purchaser is indifferent as to the time of disposal.

In an effort to determine optimal procurement quantities when faced with all-units price discounts, Gupta (1988) developed upper bounds on the total annual relevant costs. He determined a relation which a specific price level must satisfy in order to yield the optimal procurement quantity. Should the price violate that relation, then it could be ignored from further consideration. This greatly reduced the computational effort required to find the best solution.

Arcelus and Rowcroft (1991) examined the integration of purchasing and stock-control policies in the presence of secondary markets, a rather important practical problem which has received little attention in the academic literature. They considered both quantity and freight-rate discounts with the possibility of disposals. Their research allowed a price-dependent (downward-sloping) demand function. They assumed that there exists only one price break. Firms have the option of taking advantage of a larger quantity purchase, at lower unit costs. Since a constant "markup" is applied to purchase cost (to produce retail price), a lower unit cost will generate higher demand for the product. Arcelus and Rowcroft considered purchasing, ordering and holding costs. They derived the net profit resulting from either taking advantage of the quantity discount, or ordering the smaller quantity. Sensitivity analysis was performed to observe the effects of the various model parameters on resulting profit. A later article (1992) by the same authors discussed multiple price breaks. For this problem scenario, a computationally efficient, simple-to-use, two-stage algorithm was developed. First, one derived the solution for the generalized all-units discount structure, when disposals are not allowed. Then, the disposal decision was incorporated into the model. The profit or return on investment (ROI) of following each strategy was determined.

2.4 Inventory Models

The literature on inventory modelling is expansive. Porteus (1990) offered an excellent review of various stochastic topics and methodologies discussed in this vital

OM area. (A precise treatment on several stochastic issues is given by Ross (1993)).

Heyvaert and Hurt (1956), analyzing the inventory management of slow-moving items, used a Poisson distribution to model item usage. They attempted to maximize the "customer's satisfaction", the ratio of the number of items delivered to the number requested.

Scarf (1960) considered the dynamic inventory problem. Holding and shortage costs were charged at the end of every period, and one attempted to minimize the expectation of the discounted value of all costs. Azoury (1985) commented that most dynamic inventory models with stochastic usage make the assumption that the underlying demand distribution is known with certainty. Suggesting that some decision-makers may have considerable uncertainty regarding the nature of the distribution of demand, he proposed a Bayesian formulation to this problem.

Hadley and Whitin (1961) proposed an inventory system that is only reviewed at discrete, equally spaced intervals of time. They considered review, procurement, storage and stockout costs while assuming that the mean rate of demand does not change with time. A later article by the same authors (1962) allowed demand to change with time, although demands in each period were assumed to be independent.

Kaplan (1970) analyzed a single-product problem wherein lead time is a discrete random variable with a known distribution. Veinott (1965) described a multi-product problem with a fixed lead time. A general demand process (no stationarity, no independence assumptions) was used in his analysis. He proposed sufficient conditions that the optimal policy must satisfy. Naddor (1975) developed optimal solutions and heuristic decision rules for single and multi-item inventory systems. He allowed uncertainty with respect to product demand and lead time.

Zipkin (1986) evaluated the cost performance of inventory systems under the conditions of random lead time and demands. He allowed all stockouts to be backordered. Vinson (1972) claimed that substantial losses can result if we ignore lead time unreliability. This factor can be more important in determining inventory cost behavior than either mean lead time, or the variability of demand during lead time.

A procurement strategy we shall analyze in our modelling efforts consists in solely considering only one phase in the project, namely construction phase requirements. This shall help us determine the efficacy of combining construction phase and ongoing phase decisions. There has been considerable research in so-called "single-period" models.

Ward, Chapman and Klein (1991) claimed that one of the difficulties with theoretical models is that often they cannot be used in a given practical context. The authors, using an approximate, discrete specification of demand, illustrated a practical approach for obtaining solutions to the newsvendor problem. Lau (1980) considered two alternative optimization objectives for this problem: maximizing expected utility, or maximizing the probability of achieving a budgeted profit.

In specifying the underlying demand distribution for the newsvendor problem, Shore (1986) did not require any assumptions as to the form of the distribution. His approximate solutions, however, required that one specify the first three or four moments of the distribution. Recognizing that demand may not be solely an exogenous variable, Moon and Choi (1995) considered the situation in which demand depends upon the inventory position. As inventory falls, some potential customers may choose to go elsewhere (in the hopes of greater selection). Like Shore, the authors did not make any distributional assumptions; they only required that the first two moments of a distribution are known.

"Lump-sum" penalty costs represent an important issue within single-period inventory problems. Bellman, Glicksberg and Gross (1955) were perhaps the first to consider these types of penalty costs. Examining a single commodity, finite horizon, periodic-review inventory problem, Aneja and Noori (1987) considered both a fixed and variable component in stockout costs. For all nonincreasing demand density functions, the optimal approach is shown to follow an "order-up-to" policy.

The perishability inventory area is related to the newsvendor problem. In this former problem, products become worthless after a specific (perhaps known) time. Nahmias (1982) offered an exceptional review of perishable inventory theory. He considers both fixed and random lifetimes, as well as deterministic and stochastic demand for either single or multiple product models. Nahmias (1978) assumed zero lead time and backlogging of all unsatisfied demand. He evaluated the effect of the fixed ordering cost on the nature of the optimal ordering policy.

Nose, Ishii and Nishida (1984) examined perishable inventory management under two different selling prices and two different lead times. Allowing the commodity to possess a fixed-life perishability, they developed optimal ordering policies for all cases. Nadakumar and Morton (1993) also examined the fixed-life perishability problem. They derived efficient "near-myopic" bounds on the order quantities, and then use these bounds to suggest some good heuristics.

Barbosa and Friedman (1981) describe the finite-time horizon inventory lot-size model. They determine the optimal replenishment schedule when examining a single commodity, assuming no backlogs and no lead time. A model to derive optimal order quantities for a finite horizon, multi-item inventory system was prescribed by Mehrez and Ben-Arieh (1991). They allowed demand to be normally distributed and adopted a goal programming, mixed-integer technique to solve the problem.

Chand (1982) considered the situation arising in a production shop in which the product would be discontinued or replaced after a certain (known) number of periods. Further, he extended his algorithm to deal with non-constant demand rates. Goyal, Morin and Nebebe (1992) suggested a replenishment policy for an inventory item that displays a linearly increasing demand rate over a finite time horizon.

Morton and Pentico (1995) tested four heuristics against a stochastic dynamic programming optimal solution for a finite horizon, nonstationary stochastic inventory problem. Almost 1,000 test problems were examined. The best of the heuristics averaged only 0.02% above the optimal solution.

The nature of the underlying product demand has received attention in the literature. Using a continuous-time discrete-state dynamic program, Song and Zipkin (1993) considered inventory control within a fluctuating demand environment. Their model allowed two state variables (inventory position, state of the world). Smith (1977) analyzed the problem of specifying the timing and sizing of all production runs required to satisfy all future demand. He explored the situation in which the demand rate is exponentially decreasing. Brosseau (1982) also investigated the optimal number and timing of future replenishments, but uses a demand rate that is linearly decreasing.

Silver (1978) examined the timing and sizing of replenishments for an item displaying time-varying demand. This demand was assumed to be probabilistic, with an average value that varies considerably over time. Due to the complexities of developing an explicit optimization model, a heuristic was offered.

Examining dependent period-by-period demands, Johnson and Thompson (1975) showed the optimality of myopic inventory policies. They assumed proportional holding and shortage costs, no fixed ordering costs, and zero lead time. Heyman and Sobel (1982) developed a two-vector Markov decision process to examine the dependent demand inventory problem. The vectors represented the on-hand inventory at the beginning of the period and the previous period's demand. Miller (1986) considered linear holding, shortage and ordering costs in his dependent demand model. Average demand was described by an exponential smoothing expression.

3. ONGOING PHASE ANALYSIS

Figure 5 illustrates the respective decisions encountered throughout the construction and operation of the facility. We shall restrict our attention, for the time being, to those aspects directly related to ongoing phase replenishments.

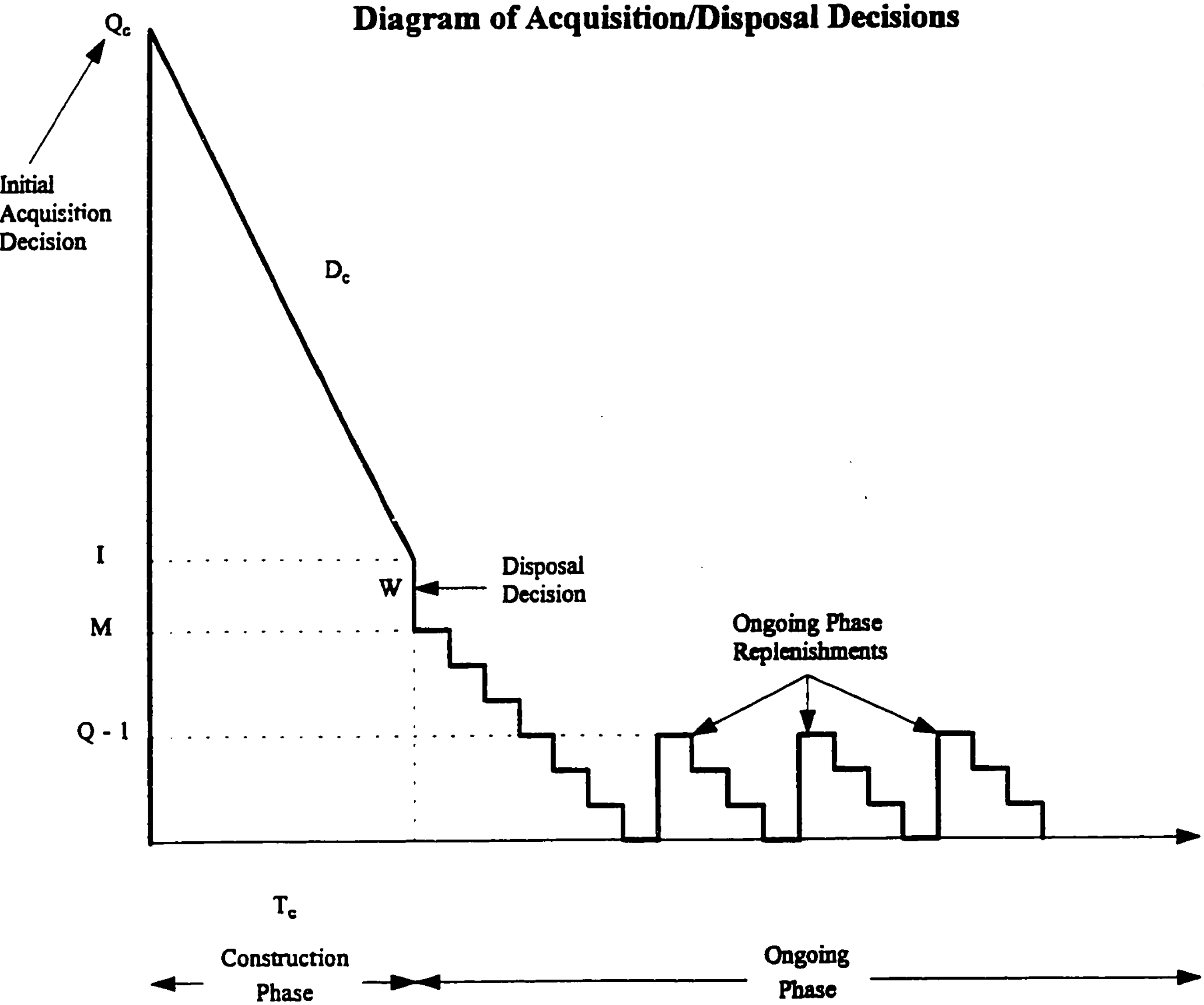
Our analysis will feature deterministic, level usage of this expensive, important item. (We illustrate in Appendix B the equivalence of our cost expressions under both deterministic, level usage and probabilistic (Poisson) usage. Moreover, the case of deterministic, time-varying usage will be treated as a model extension in Chapter 8).

A vital issue involves the determination of the optimal (ie. least-cost) replenishment quantity in the ongoing phase. We represent a particular replenishment amount by the variable Q, with the optimal quantity being denoted by Q_o. In addition, the following parameters are used in evaluating the costs of replenishments in the ongoing phase (for ease of reference, a glossary of notation is provided in Appendix A):

A:	Fixed cost of acquisition
α:	Continuous discount rate
D_o:	Annual usage rate
h:	"Out-of-pocket" inventory carrying charges, expressed in \$ per unit of inventory per unit time
v_o:	Unit acquisition cost

Note that we use a constant carrying charge (h) per unit of inventory per unit time. Alternatively, we could have used the acquisition cost of the item (v_o) multiplied by an appropriate annual carrying cost percentage (often referred to as "r") to derive the holding costs. However, (as we shall see later), unit acquisition costs in the ongoing phase are

Figure 5
Diagram of Acquisition/Disposal Decisions



higher than those in the construction phase. Had we used the carrying cost percentage approach, this would have produced different holding costs for the same item in either phase! To be more realistic, we decided to use the parameter *h* for our carrying charges.

The continuous discount rate, α, includes the firm's cost of capital (the so-called opportunity cost). A further purpose of the discount rate, in our study, is to "bring all costs back" to a specific time, so that the present value of relevant costs may be determined. The holding cost parameter *h* includes out-of-pocket expenses such as those costs associated with running a warehouse, insurance, taxes, and the costs of any special storage requirements.

Operational usage of this item occurs in discrete amounts, with one unit being used every $1/D_o$ time units. Consequently, the usage pattern portrayed in Figure 5 involves an infinite series of "staircases". Vertical segments of this "step-wise" pattern occur when an item is used and, thus, on-hand inventory drops by one unit. Since usage is constant and known under this deterministic, level pattern, we propose that the receipt of new replenishments (of size Q_o) can be timed so as to arrive precisely when a unit is used. As a result, the top of each step-wise cycle comprises Q_o-1 units (Q_o units are received just as one unit is used). Modelling replenishments and ongoing usage in such a way also allows a time interval of $1/D_o$ time units (prior to the receipt of an order) in which on-hand inventory is zero.

We now illustrate the procedures involved in developing an analytical expression for the present value of this infinite step-wise pattern of ongoing phase replenishments. Our approach is similar to that found in Hadley (1964). We shall let $Z(Q)$ denote the present value of an infinite series of replenishments of size Q.

Replenishments of size Q are received every Q/D_o time units. Consequently, we incur an $A+Qv_o$ charge each time a replenishment is received. Carrying charges are also incurred throughout the step-wise pattern. In an effort to accurately model ongoing phase costs, we will continuously discount these carrying charges. Hadley and Whitin (1963) and Gurnani (1983) offer a discussion of the continuous discounting of these costs.

We recognize that one unit is carried, in a particular step-wise cycle, from time 0 to time $1/D_o$. The continuously discounted carrying charges of this unit, then, are:

$$h \int_0^{\frac{1}{D_o}} e^{-\alpha t} dt$$

This is evaluated as:

$$h \left[\frac{1}{\alpha} - \frac{e^{-\frac{\alpha}{D_o}}}{\alpha} \right]$$

Another unit is carried from time 0 to time $2/D_o$. In a similar fashion to the analysis depicted above, its carrying charges are:

$$h \int_0^{\frac{2}{D_o}} e^{-\alpha t} dt$$

which is evaluated as:

$$h\left[\frac{1}{\alpha} - \frac{e^{\frac{-2\alpha}{D_o}}}{\alpha}\right]$$

Finally, one unit is carried from time 0 to time $(Q\text{-}1)/D_o$. The evaluation of the continuously discounted carrying charges for this unit yields:

$$h\left[\frac{1}{\alpha} - \frac{e^{\frac{-(Q-1)\alpha}{D_o}}}{\alpha}\right]$$

Letting j be an index variable, we can represent the total (out-of-pocket) inventory carrying charges in one cycle as:

$$h\left(\sum_{j=1}^{Q-1} \frac{1}{\alpha} - \frac{e^{\frac{-j\alpha}{D_o}}}{\alpha}\right) \tag{3.1}$$

Ignoring, for the moment, the parameter h and pulling out of the summation any terms which are independent of j, we obtain the following:

$$\frac{Q-1}{\alpha} - \frac{1}{\alpha}\sum_{j=1}^{Q-1} e^{\frac{-j\alpha}{D_o}} \tag{3.2}$$

We may write (3.2) as:

$$\frac{Q-1}{\alpha} - \frac{e^{\frac{-\alpha}{D_o}}}{\alpha}\left(1 + e^{-\frac{\alpha}{D_o}} + e^{\frac{-2\alpha}{D_o}} + \ldots + e^{\frac{-(Q-2)\alpha}{D_o}}\right) \tag{3.3}$$

Recognizing that the term in parentheses in (3.3) is equivalent to:

$$\frac{1 - r^{k-1}}{1 - r}$$

where

$$r = e^{-\frac{\alpha}{D_o}}$$

the total carrying charges per cycle become:

$$h\left[\frac{Q - 1}{\alpha} - \frac{e^{-\frac{\alpha}{D_o}}}{\alpha}\left(\frac{1 - e^{\frac{-(Q-1)\alpha}{D_o}}}{1 - e^{-\frac{\alpha}{D_o}}}\right)\right] \tag{3.4}$$

The total replenishment cost incurred in one step-wise cycle then can be written as:

$$A + Qv_o + h\left[\frac{Q - 1}{\alpha} - \frac{e^{-\frac{\alpha}{D_o}}}{\alpha}\left(\frac{1 - e^{\frac{-(Q-1)\alpha}{D_o}}}{1 - e^{-\frac{\alpha}{D_o}}}\right)\right] \tag{3.5}$$

Since each replenishment cycle is separated by Q/D_o time units, we can use the following expression to obtain the present value of this infinite stream of costs (see Silver, Pyke and Peterson (1998) and Trippi and Lewin (1977) for an analysis of the present value of costs in inventory systems). To provide notational simplicity, we now define α' as α/D_o and h' as h/α.

$$Z(Q) = \frac{A + Qv_o + h'\left[Q - 1 - e^{-\alpha'}\left(\frac{1 - e^{-(Q-1)\alpha'}}{1 - e^{-\alpha'}}\right)\right]}{1 - e^{-Q\alpha'}} \tag{3.6}$$

Although (3.6) provides an accurate depiction of the present value of a series of inventory cycles associated with replenishments of size Q, we can make the expression somewhat less cumbersome. Algebraic manipulation yields:

$$Z(Q) = \frac{A + Q(v_o+h')}{1 - e^{-Q\alpha'}} - h'\left[\frac{\dfrac{1-e^{-\alpha'} + e^{-\alpha'}\left(1-e^{-(Q-1)\alpha'}\right)}{1-e^{-\alpha'}}}{1 - e^{-Q\alpha'}}\right]$$

which can be written as:

$$Z(Q) = \frac{A + Q(v_o+h')}{1 - e^{-Q\alpha'}} - \frac{h'}{1 - e^{-\alpha'}} \tag{3.7}$$

In Appendix B, we illustrate that the same functional form of $Z(Q)$ is obtained under Poisson usage. Hence, adopting either a deterministic, level usage pattern (with annual rate D_o units) or a Poisson one (with an annual usage rate of λ units) will lead to the same results for the present value of all future costs. We are only required, thus, to perform complete numerical analyses with one of the usage patterns. We shall use deterministic, level usage.

In order to adopt the optimal replenishment strategy in the ongoing phase, we must select the integer Q that minimizes $Z(Q)$. Since Q is evaluated at discrete points, we must use the method of "differencing" to find the best Q (we shall denote this optimal replenishment quantity by Q_o). In other words, we need to find the smallest integer Q such that $\Delta Z(Q) = Z(Q+1) - Z(Q) > 0$. This corresponds to the first place where the $Z(Q)$ function begins to "turn up". Pursuant to finding Q_o through differencing is the notion that the function in question be convex. We must be certain that once the function begins to increase, it will continue to increase. Convexity also implies that the function reaches one (and only one) minimum point. Appendix C offers a proof that $Z(Q)$ is indeed convex.

From (3.7), we have:

$$\Delta Z(Q) = \frac{A + (Q+1)(v_o+h')}{1 - e^{-(Q+1)\alpha'}} - \frac{A + Q(v_o+h')}{1 - e^{-Q\alpha'}} > 0 \qquad (3.8)$$

Using basic algebra to write (3.8) under a common denominator, and then multiplying both sides of the inequality by that common denominator yields:

$$\left[1 - e^{-Q\alpha'}\right]\left[A + (Q+1)(v_o+h')\right] - \left[1 - e^{-(Q+1)\alpha'}\right]\left[A + Q(v_o+h')\right] > 0$$

Expanding terms and simplifying the result gives:

$$v_o + h' - Ae^{-Q\alpha'} - Q(v_o+h')e^{-Q\alpha'} - v_o e^{-Q\alpha'} - h'e^{-Q\alpha'} + Ae^{-(Q+1)\alpha'} + Q(v_o+h')e^{-(Q+1)\alpha'} > 0$$

Dividing through by v_o and then multiplying all terms by $e^{Q\alpha'}$ gives:

$$e^{Q\alpha'} + \frac{h'}{v_o}e^{Q\alpha'} - \frac{A}{v_o} - Q - \frac{Qh'}{v_o} - 1 - \frac{h'}{v_o} + \frac{A}{v_o}e^{-\alpha'} + Qe^{-\alpha'} + \frac{Qh'}{v_o}e^{-\alpha'} > 0 \qquad (3.9)$$

This may be simplified as:

$$e^{Q\alpha'}\left(1+\frac{h'}{v_o}\right) > \frac{A}{v_o}+Q\left(1+\frac{h'}{v_o}\right)+1+\frac{h'}{v_o}-\frac{A}{v_o}e^{-\alpha'}-Q\left(1+\frac{h'}{v_o}\right)e^{-\alpha'} \qquad (3.10)$$

Equation (3.10) can be written as:

$$e^{Q\alpha'}\left(1+\frac{h'}{v_o}\right) > \left[1-e^{-\alpha'}\right]\left[\frac{A}{v_o}+Q\left(1+\frac{h'}{v_o}\right)\right]+\left(1+\frac{h'}{v_o}\right) \qquad (3.11)$$

which becomes:

$$e^{Q\alpha'} > 1+\frac{\left(1-e^{-\alpha'}\right)\left(\frac{A}{v_o}+Q\left(1+\frac{h'}{v_o}\right)\right)}{1+\frac{h'}{v_o}} \qquad (3.12)$$

Thus, equation (3.12) is the nonlinear equation that the best Q, Q_o, must satisfy. One simply finds the <u>first</u> (ie. smallest) integer Q such that the left-hand side of (3.12) exceeds the right-hand side. The fact that Q appears on both sides of the above inequality does not present us with serious problems. We have shown (see Appendix D) that the Economic Order Quantity (EOQ) is often a good place at which to initiate the process of finding Q_o. The EOQ is given as follows:

$$EOQ = \sqrt{\frac{2AD_o}{h+\alpha v_o}}$$

Consequently, we are not required to search individual Q's from Q=1, 2, and so on until we find the smallest Q such that (3.12) is satisfied. This limits our search procedure by allowing us to quickly detect the optimal replenishment quantity in the ongoing phase.

Numerical Example:

Consider the following parameter values:

A = \$250
D_o = 20 units per year
h = \$13 per unit of inventory per year
$\alpha = 0.10$
v_o = \$190

Using equation (3.12) gives us the following values for the inequality:

Q = 16 has left-hand side = 1.08328 and right-hand side = 1.08369.

Q = 17 has left-hand side = 1.08871 and right-hand side = 1.08868.

Thus, Q_o = 17 units and $Z(Q_o)$ = \$43761.42.

The value given by the EOQ expression is 17.68, which is very close to Q_o.

4. DISPOSAL DECISIONS

We shall now address those inventory decisions encountered at the conclusion of the project's construction phase. As we illustrated in Figure 5 (see page 44), a critical decision involves determining the optimal number of units to dispose (or retain for ongoing phase usage), given a specific quantity of on-hand surplus upon completion of the construction phase. We shall examine the case of constant salvage values in the initial section of this chapter. Later sections will be devoted to various types of non-constant salvage value functions.

We shall use the following notation in our analysis:

g: per unit salvage value for surplus disposals
I: on-hand surplus (if any) upon completion of project construction phase
M: number of units retained
W: number of units disposed

To evaluate the present value of concluding the construction phase with I units and disposing W of them (to leave I - W, or M units on-hand), we must consider the revenue generated from disposals as well as the costs of carrying retained units in the "transition" period (prior to ongoing phase replenishments). When these retained units have all been used, the replenishment pattern (with Q_o units bought in each cycle) is inaugurated. Consequently, we must also include all costs associated with an infinite stream of ongoing phase replenishments (of size Q_o). This approach is similar to that prescribed in Stulman (1989), although the author halts his transition period when on-hand inventory reaches the top of its normal operating range (ie. Q_o). In our analysis, the transition period concludes when the M units are all used (ie. when on-hand inventory reaches zero).

When the number of units disposed increases, higher revenues result (assuming a strictly positive g). Carrying charges in the "transition" period (prior to ongoing phase replenishments) are reduced since less units are then carried. However, the future replenishment pattern is inaugurated earlier. Since its costs ($Z(Q_o)$) are incurred sooner, the present value of these costs (when brought back to the conclusion of the construction phase) increases.

4.1 Constant Salvage Values

In this case, disposing W units earns a revenue of gW. Since we wish to evaluate the present value of decisions as of the conclusion of the construction phase and because disposals occur immediately upon project completion, gW represents the present value of disposal revenue.

Carrying costs in the transition period are continuously discounted. Using the analysis discussed in Chapter 3, we hold one unit for $1/D_o$ time units, another unit for $2/D_o$ time units, and so on until the Mth unit, which is held for M/D_o time units. The present value of these carrying charges, then, are:

$$h'\left[M - e^{-\alpha'}\left(\frac{1 - e^{-M\alpha'}}{1 - e^{-\alpha'}}\right)\right] \tag{4.1}$$

As described in Chapter 3, we can set the replenishments in the ongoing phase to arrive exactly when required since item usage is known and constant. Hence, the inauguration of the future replenishment pattern is "offset" by $(M+1)/D_o$ time units (we have a time interval of $1/D_o$ time units in which on-hand inventory is zero, prior to the receipt of the initial replenishment in the ongoing phase).

The present value of the future replenishment pattern, when brought back to the conclusion of the construction phase, is:

$$e^{-(M+1)\alpha'}(Z(Q_o)) \tag{4.2}$$

Combining these various revenue and cost components yields the present value of concluding the construction phase with I units and disposing W of them:

$$PV(W|I) = -gW + h'\left[M - e^{-\alpha}\left(\frac{1 - e^{-M\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M+1)\alpha'}(Z(Q_o)) \tag{4.3}$$

Since $W = I - M$, we may write (4.3) as:

$$PV(M) = -gI + gM + h'\left[M - e^{-\alpha}\left(\frac{1 - e^{-M\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M+1)\alpha'}(Z(Q_o)) \tag{4.4}$$

This is the present value of retaining M units upon completion of the construction phase, given an initial surplus of I units.

Since retention quantities can only assume integer values, we will use the method of differencing to find the optimal retention amount (ie. the value of M, denoted by M^*, which minimizes $PV(M)$). In order to determine an analytical expression for the value of M^*, we will need to show (as we did in finding the best ongoing phase replenishment quantity in Chapter 3) that the function under question is convex. To do this, we will need to show that $\Delta^2 PV(M) = PV(M+2) + PV(M) - 2PV(M+1)$ is strictly positive.

Expansion of (4.4) gives:

$$PV(M) = -gl + gM + h'M - \frac{h'e^{-\alpha'}}{1 - e^{-\alpha'}} + \frac{h'e^{-(M+1)\alpha'}}{1 - e^{-\alpha'}} + e^{-(M+1)\alpha'}(Z(Q_o)) \tag{4.5}$$

Note that the first and fourth terms of this expression are independent of M. As well, the second and third terms are linear with respect to M. Consequently, these terms disappear when determining $\Delta^2 PV(M)$.

Evaluating $\Delta^2 PV(M)$ gives:

$$\left[\frac{h'}{1 - e^{-\alpha'}} + Z(Q_o)\right]\left[e^{-(M+3)\alpha'} + e^{-(M+1)\alpha'} - 2e^{-(M+2)\alpha'}\right]$$

which can be expressed as:

$$\left[\frac{h'}{1 - e^{-\alpha'}} + Z(Q_o)\right]\left[\frac{e^{-(M+3)\alpha'} + e^{-(M+1)\alpha'}}{2} - e^{-(M+2)\alpha'}\right] \tag{4.6}$$

Obviously, the terms within the first set of squared brackets are strictly positive. In addition, the expression within the second set of squared brackets is strictly positive. (We know that e^{-x} is convex for $x > 0$, so the arithmetic mean of two points (j,l) along the e^{-x} function, less the value of a point in between (k), will be positive). Thus, $PV(M)$ is convex, implying that the function reaches a minimum point.

In developing an analytical expression for M^*, we need to find the first (ie. smallest) integer M such that $\Delta PV(M) = PV(M+1) - PV(M) > 0$.

Evaluating the first difference of (4.5), and setting it strictly positive, yields:

$$g + h' - e^{-(M+1)\alpha'}\left(\frac{h'}{1 - e^{-\alpha'}}\left(1 - e^{-\alpha'}\right)\right) - e^{-(M+1)\alpha'}\left(Z(Q_o)\left(1 - e^{-\alpha'}\right)\right) > 0 \qquad (4.7)$$

which may be expressed as:

$$e^{-(M+1)\alpha'}\left[h' + Z(Q_o)\left(1 - e^{-\alpha'}\right)\right] < g + h' \qquad (4.8)$$

Solving for M in (4.8) gives the following inequality:

$$-(M+1)\alpha' < \ln\left[\frac{g + h'}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}\right]$$

which becomes:

$$M > \frac{1}{\alpha'}\ln\left[\frac{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}{g + h'}\right] - 1 \qquad (4.9)$$

This expression is used to find the optimal retention quantity. Since M only occurs on one side of the inequality, the procedure to find the best value of M is quite straight-forward. One simply calculates, from the parameter values given, the right-hand side of (4.9). The smallest integer greater than or equal to that right-hand side becomes M^*.

Determining appropriate disposal (or retention) quantities depends on the amount of on-hand inventory, I, at the conclusion of the construction phase and the value of M^*.

The optimal policy is to retain any surplus inventory up to and including the quantity M^*. One would never want to retain more than M^* units for ongoing phase usage. Although the present value of the future inventory cycles would be lessened by higher amounts of M, these cost savings would be eroded by additional carrying charges and foregone salvage revenue. Should I be less than or equal to M^*, no stock is disposed (all surplus units are retained). For any values of inventory greater than M^*, one disposes the excess above M^* (and, thus, retains M^* units). Thus, we may express the optimal disposal quantity decision rules, for the case of constant salvage values, as:

$$\text{If } I \leq M^*,\ W^* = 0$$

$$\text{If } I > M^*,\ W^* = I - M^*$$

Figure 6 expresses these decision rules in graphical format. Disposal of surplus units begins when inventory values exceed the optimal retention quantity.

Numerical example:

Let us expand on the one provided in Chapter 3. Recall that we had:

$Z(Q_o) = \$43761.42$
$\alpha = 0.10$
$D_o = 20$ units per year
$h = \$13$ per unit of inventory per year

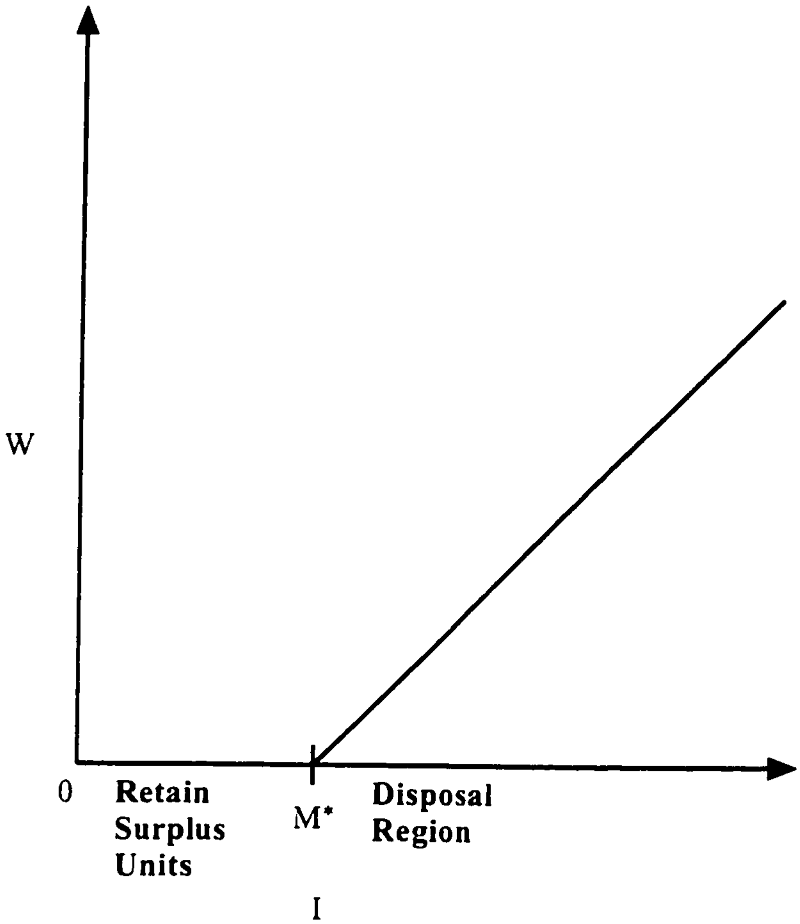
Let us introduce a value of \$35 per unit for g.

Evaluating (4.9) shows that $M^* = 149$. Consequently, we would only begin to dispose surplus units when inventory values exceeded 149. If the on-hand inventory after the construction phase was, say, 160 units, then our optimal materials management policy would be to dispose 11 units.

Figure 6

Optimal Disposal Decisions

Constant Salvage Values



We present, in Appendix E, a proof that the expression within the logarithmic argument of (4.9) is always > 1. Thus, the logarithmic value will always be defined, and the resulting optimal retention quantities will be positive. Appendix F illustrates that, when salvage values are equivalent to ongoing phase unit acquisition costs, the optimal retention quantity becomes Q_o-1. This implies that when one can salvage a unit for as much as one pays for it, the best policy is to put the inventory into the same situation as immediately subsequent to the receipt of an order.

4.2 Marginally Decreasing Salvage Values

As far as we have been able to determine, previous excess stock disposal research has always assumed constant salvage values. We now explore the situation in which a firm finds itself unable to earn constant marginal salvage values as total disposals increase. In this scenario, a certain salvage value (g_1) is earned for a specific number of units disposed, followed by a lower salvage value (g_2) for units disposed beyond that amount (up to a certain limit). An even lower salvage value (g_3) is obtained for any disposals beyond the most allowed under g_2, etc.

We can define the following model parameters:

n: number of different unit salvage values
g_i: marginal salvage value earned for *ith* "group" of disposals ($i = 1,2,...,n$)
N_i: maximum number of units that can be disposed for \$$g_i$ per unit ($i = 1,2,...,n$)
M_i^*: the best remaining stock level (after possible disposal) when there is a marginal salvage value g_i ($i = 1,2,...,n$)

The value M_i^* is calculated by replacing g with g_i in (4.9). We thus have that M_i^*

is the smallest integer M such that:

$$M > \frac{1}{\alpha'} \ln\left[\frac{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}{g_i + h'}\right] - 1 \tag{4.10}$$

Since $g_1 > g_2 > ... > g_n$, it follows that $M_1^* \leq M_2^* \leq ... \leq M_n^*$. Strict inequalities do not necessarily apply due to the integer requirement on M_i^*.

For disposals at the largest salvage value (g_1), the disposal decision is determined relatively easily. For any I (on-hand inventory prior to any disposal decision) $\leq M_1^*$, $W^* = 0$ (in other words, there is no disposal). Note that this follows a similar approach to the one used in the constant salvage value case. Recall that in that situation, no units were disposed if I was $\leq M^*$.

For the marginally decreasing salvage value scenario, we know that the most we can dispose <u>and still earn g_1 per unit</u> is N_1 units. Consequently, when I exceeds M_1^*, we will dispose at g_1 per unit up until the point at which $I = M_1^* + N_1$. Thus, for I such that $M_1^* < I \leq M_1^* + N_1$, $W^* = I - M_1^*$ (we dispose the excess above M_1^*).

While disposing at g_1 per unit, retained inventory remains at M_1^*. When we reach the maximum possible disposals at g_1, we know that $I = M_1^*$ (retained stock) + N_1 (disposed stock). From the definition of M_i^*, we cannot inaugurate disposals at g_2 until the <u>retained inventory</u> exceeds M_2^*. Disposing N_1 units implies that retained inventory equals $I - N_1$. Thus, until $I - N_1 > M_2^*$, we will only dispose the N_1 units (at g_1 per unit). Once we initially reach the maximum possible disposals at g_1 (when $I = M_1^* + N_1$), we will stay at a disposal quantity of N_1 units (until $I = M_2^* + N_1$). As a result, total disposals remain at N_1 units for a "distance" $M_2^* - M_1^*$.

We will dispose excess stock at g_2 per unit for I such that $M_2^* + N_1 < I \leq M_2^* + N_1 + N_2$. Figure 7 displays a pictorial representation of the optimal disposal policies, when facing diminishing marginal salvage values. We obtain an alternating pattern of plateaus (no additional disposals are taking place) and ramps (additional disposals are occurring at a certain g_i). The plateau between the use of g_i and g_{i+1} is of width $M_{i+1}^* - M_i^*$ (corresponding to the "distance" notion described in the previous paragraph). Ramps at a certain g_i continue for N_i units. We note that there will be no plateau between two ramps if and only if $M_{i+1}^* = M_i^*$.

Continuing our reasoning described above for determining optimal disposal quantities for g_1 and g_2, we can develop the following decision rules (we note that $M_0^* = 0$ and $N_0 = 0$):

$$PLATEAU : For\ M_i^* + \sum_{j \leq i} N_j < I \leq M_{i+1}^* + \sum_{j \leq i} N_j,\ \ W^* = \sum_{j \leq i} N_j$$

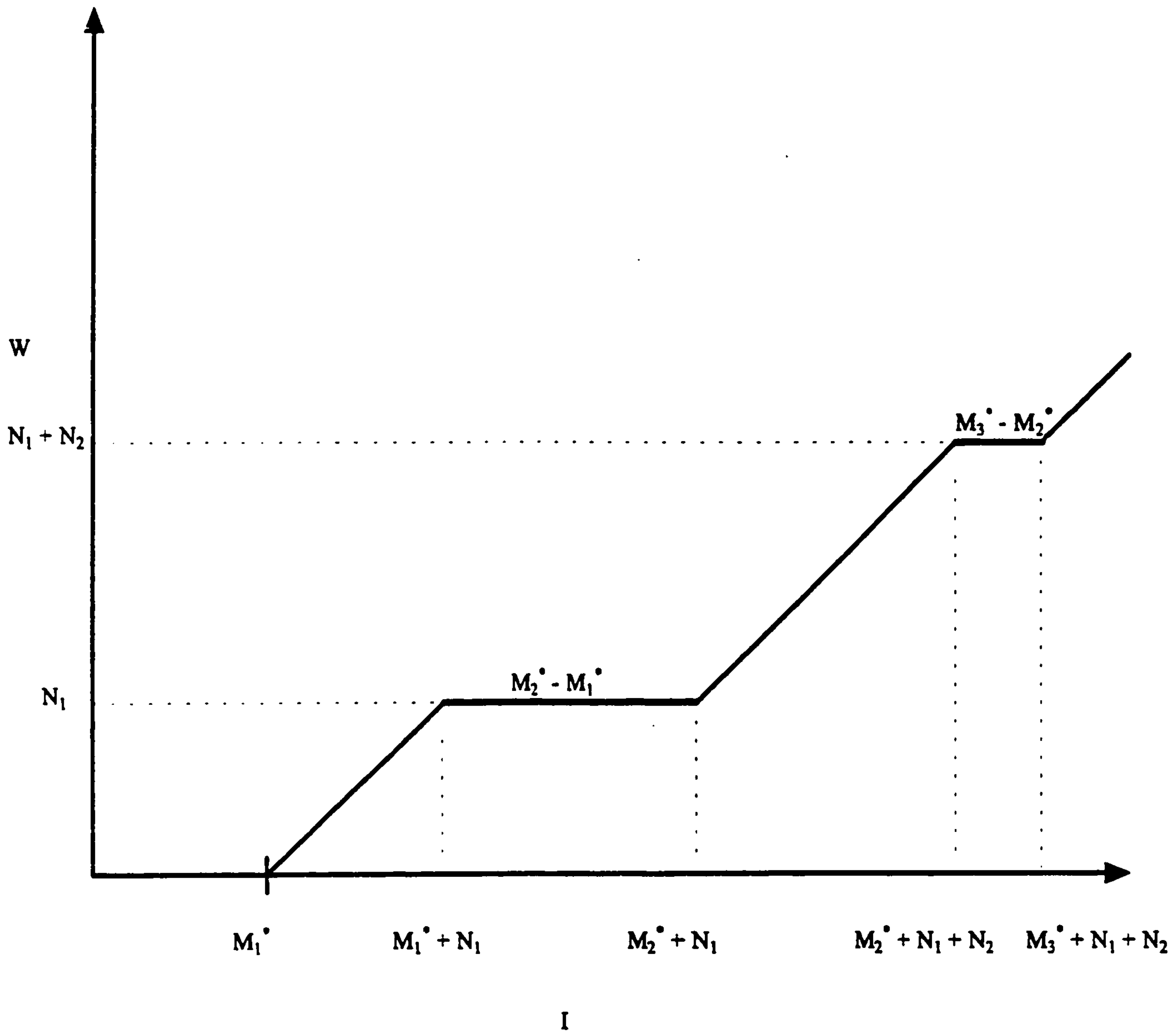
$$RAMP : For\ M_i^* + \sum_{j < i} N_j < I \leq M_i^* + \sum_{j < i} N_j + N_i,\ \ W^* = I - M_i^*$$

These decision rules allow a relatively straight-forward procedure in finding the optimal disposal quantities. We begin by calculating, for a given marginally decreasing salvage value function, each of the M_i^*'s. Then, for any on-hand inventory amount, we can quickly determine whether that specific I value corresponds to a plateau or a ramp. We can then ascertain the optimal disposal quantity, given that I value. We can use the cost expressions outlined in Section 4.1 to obtain the present value of having a specific

Figure 7

Optimal Disposal Decisions

Marginally Decreasing Salvage Values



amount of on-hand inventory after the completion of the project's construction phase, and proceeding optimally regarding any possible disposals.

Numerical Example:

Consider the following marginally decreasing salvage value function (ongoing phase costs remain as outlined in prior numerical examples):

g_1: \$50 per unit	N_1: 40 units
g_2: \$35 per unit	N_2: 30 units
g_3: \$20 per unit	N_3: 40 units
g_4: \$1 per unit	N_4: infinite (we receive \$1 per unit for any disposals made beyond 110 units)

Using equation (4.10) yields the following values for M_i^*:

M_1^*: 131
M_2^*: 149
M_3^*: 168
M_4^*: 195

For $I \leq 131$, we make no disposals. Disposals at g_1 per unit (ramp) occur until $I = M_1^* + N_1$ (131 + 40 = 171). At that point, there is a plateau of width $M_2^* - M_1^* = 149 - 131 = 18$ units. Thus, disposals at g_2 per unit are not begun until I exceeds 171 + 18 = 189. A ramp at g_2 continues until $I = 219$. Then, there is a plateau of 19 units (M_3^* - M_2^*); that is, until $I = 238$. A ramp at g_3 occurs from $I = 238$ to $I = 278$, after which there is a plateau of 27 units (until $I = 305$). From that point onward, a ramp at g_4 continues indefinitely (additional units are disposed for \$1 each).

4.3 Increasing Salvage Values

We shall now provide an analytical treatment of optimal disposal decisions given increasing salvage values. In this scenario, we suggest that per unit salvage values may rise as additional units are disposed. A firm desiring to purchase a given number of units on a surplus trade market may pay a higher unit price to a supplier that has the capability of delivering the quantity of units required by the firm. This higher unit price is beneficial to the buyer since purchasing from this supplier saves it the negotiation and logistics hassles involved in procuring smaller quantities from several companies, as well as providing transportation economies.

Our analytical models will allow salvage values to be increasing up to a point, after which the marginal values will begin to decrease. This features recognizes that, in practice, salvage values will not display this increasing behaviour forever. (In a somewhat related perspective, Das (1984) derived economic order quantities in the presence of price "premiums", whereby unit acquisition prices increased as additional units were purchased).

Increasing salvage values imply that $g_i < g_{i+1} < \ldots < g_m$ (where m represents the number of increasing salvage values). We will use the following notation in our analysis:

L_i: minimum quantity of units which must be disposed to earn g_i per unit disposed

U_i: maximum quantity of units which can be disposed to earn g_i per unit disposed

These L_i and U_i values represent ranges over which a certain g_i is valid. Thus, if $L_i = 11$, $U_i = 50$ and $g_i = \$30$, this would imply that, should total disposals be anywhere from 11 to 50 (inclusive), the firm would earn $30 on every unit disposed. Note that this represents an "all-units" revenue function. This, then, is a vital difference between the increasing and marginally decreasing salvage value functions. In the latter case, we used an "incremental" revenue function. The firm obtained the particular salvage value only on those units for which the range was valid.

The ranges for increasing salvage values obey the following restrictions:

$L_{i+1} = U_i + 1$
$L_i < U_i$
$L_1 = 1$

The analytical treatment in this section is perhaps the most complex of any in the dissertation. Our findings, however, can be summarized by noting that since the "all-units" revenue function is discontinuous, the graph of optimal disposal quantities may show some discontinuities. Figure 8 illustrates a possible graph for this salvage value case. Various notation for this graph will be described later.

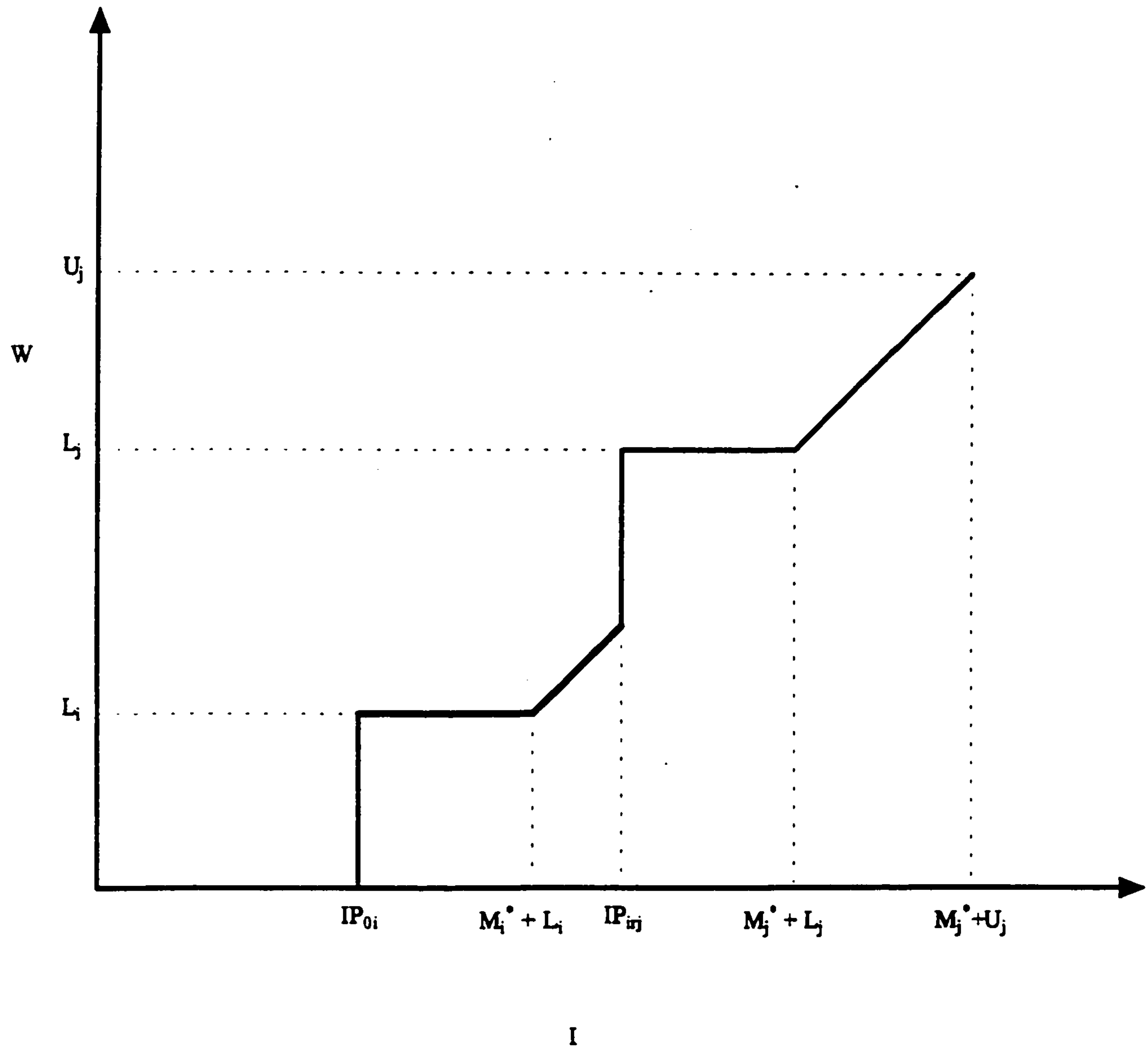
We still have the "ramps" and "plateaus" (additional units are either disposed or retained as inventory is increased) as described in earlier sections of this chapter. However, in an effort to earn the larger revenue (on all units disposed) associated with a higher per unit salvage value, it may become attractive (at certain inventory values) to "jump up" to a higher plateau. Jumping up to the higher plateau implies, obviously, that one must dispose the number of units required to earn the higher salvage value.

We shall now derive an expression for the inventory level at which it first becomes attractive to jump from a plateau at L_k (total disposals equal L_k) to a plateau at L_i

Figure 8

Optimal Disposal Decisions

Increasing Salvage Values



(W is L_i). We earn g_k per unit disposed when at the L_k plateau, and g_i per unit disposed when we jump to the L_i plateau ($g_k < g_i$). We will derive the smallest inventory level, I, such that the present value of the costs associated with disposing L_i units at g_i (and retaining I - L_i units) is less than the present value of the costs associated with disposing L_k units at g_k (and, thus, retaining I - L_k units). This smallest inventory level shall be noted by IP_{ki} (the indifference point in jumping from an L_k plateau to an L_i plateau).

Using our notation from earlier sections of this chapter, we wish to find the smallest I such that:

$$-g_iL_i + h'\left[(I - L_i) - e^{-\alpha'}\left(\frac{1 - e^{-(I-L_i)\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(I-L_i+1)\alpha'}Z(Q_o) <$$

$$-g_kL_k + h'\left[(I - L_k) - e^{-\alpha'}\left(\frac{1 - e^{-(I-L_k)\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(I-L_k+1)\alpha'}Z(Q_o) \qquad (4.11)$$

Expansion of the terms in (4.11) and some algebraic simplification yields:

$$-L_i(g_i+h') + e^{-(I-L_i+1)\alpha'}\left[\frac{h'}{1 - e^{-\alpha'}} + Z(Q_o)\right] <$$

$$-L_k(g_k+h') + e^{-(I-L_k+1)\alpha'}\left[\frac{h'}{1 - e^{-\alpha'}} + Z(Q_o)\right] \qquad (4.12)$$

Letting:

$$\frac{h'}{1 - e^{-\alpha'}} + Z(Q_o) = c$$

we may write (4.12) as:

$$e^{-(I+1)\alpha'} e^{L_i \alpha'}(c) - e^{-(I+1)\alpha'} e^{L_k \alpha'}(c) < L_i(g_i + h') - L_k(g_k + h') \tag{4.13}$$

Equation (4.13) can further be simplified as:

$$e^{-(I+1)\alpha'} < \frac{L_i(g_i + h') - L_k(g_k + h')}{\left(e^{L_i \alpha'} - e^{L_k \alpha'}\right)c} \tag{4.14}$$

Solving for I yields IP_{ki}:

$$I > \frac{1}{\alpha'} \ln\left[\frac{\left(e^{L_i \alpha'} - e^{L_k \alpha'}\right)c}{L_i(g_i + h') - L_k(g_k + h')}\right] - 1 \tag{4.15}$$

Thus, the smallest integer greater than or equal to the right-hand side of (4.15) becomes the indifference point in jumping from a plateau at L_k to a plateau at L_i. This inventory level is the first place at which it becomes attractive to jump to the higher plateau.

An important special case of (4.15) involves the situation in which we jump from making no disposals to a certain L_i plateau. With $W = 0$, we have that $L_k = 0$ and $g_k = 0$. This changes the above expression in the following manner:

$$I > \frac{1}{\alpha'} \ln\left[\frac{\left(e^{L_i \alpha'} - 1\right)c}{L_i(g_i + h')}\right] - 1 \tag{4.16}$$

This inventory indifference point shall be noted by IP_{0i} (the inventory level at which it first becomes attractive to move from making no disposals to disposing a certain L_i).

We have thus far discussed situations in which optimal disposal decisions involve jumping from plateau to plateau. There remains one additional inventory indifference point to examine. As shown in Figure 8, there may be situations in which disposals involve a "ramp" at a certain g_k, followed by moving to a higher plateau. Recall from our earlier analysis in this chapter that disposals on a ramp occur when the retained inventory equals M_k^*. As total inventory increases, additional units are disposed (total disposals become $I - M_k^*$, while retained stock remains at M_k^* units).

Thus, we need to determine the smallest inventory level at which it becomes attractive to jump from a ramp with salvage value g_k (we dispose $I - M_k^*$ and retain M_k^*) to a plateau earning g_i per unit disposed (L_i units are disposed while $I - L_i$ are retained). Pursuing our cost expressions used earlier, we may write this as:

$$-g_i L_i + h'\left[(I - L_i) - e^{-\alpha'}\left(\frac{1 - e^{-(I-L_i)\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(I-L_i+1)\alpha'}Z(Q_o) <$$

$$-g_k(I-M_k^*) + h'\left[M_k^* - e^{-\alpha'}\left(\frac{1 - e^{-M_k^*\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M_k^*+1)\alpha'}Z(Q_o) \qquad (4.17)$$

Similar algebraic manipulation to that used earlier gives:

$$e^{-(I+1)\alpha'} < \frac{L_i(g_i+h') - (I-M_k^*)(g_k+h') + e^{-(M_k^*+1)\alpha'}(c)}{e^{L_i\alpha'}(c)} \qquad (4.18)$$

which becomes:

$$I > \frac{1}{\alpha'}\ln\left[\frac{e^{L_i\alpha'}(c)}{L_i(g_i+h') - (I-M_k^*)(g_k+h') + e^{-(M_k^*+1)\alpha'}(c)}\right] - 1 \qquad (4.19)$$

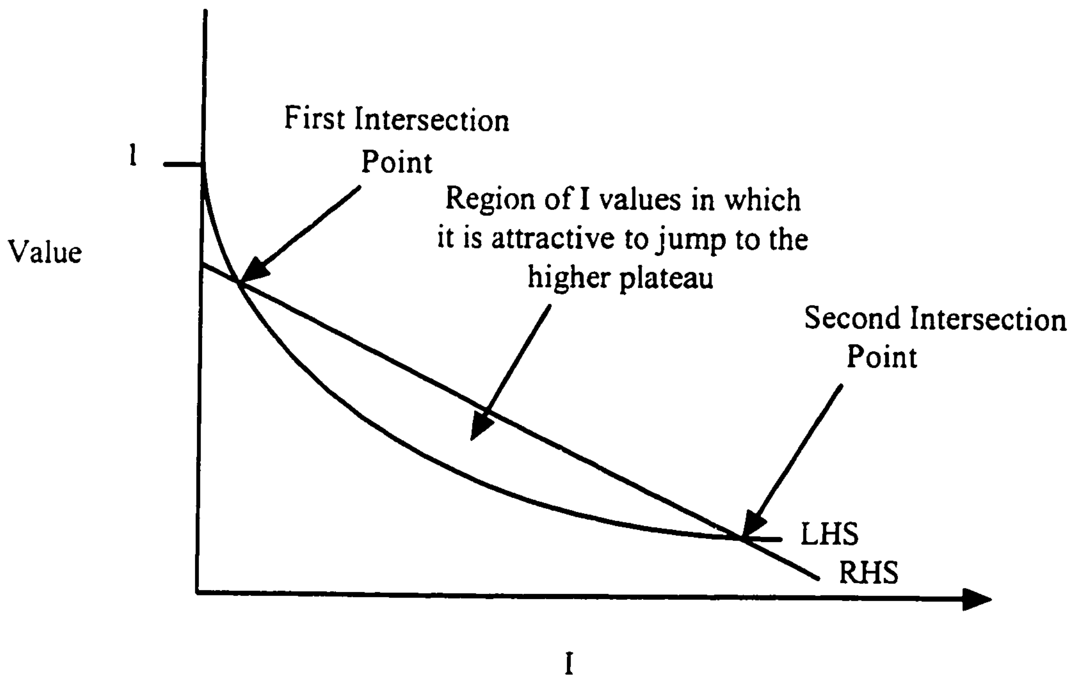
The smallest integer greater than or equal to the right-hand side of (4.19) represents the inventory point at which it first becomes less costly to jump from a ramp involving salvage value g_k (total disposals equal I - M_k^*) to a plateau using (a higher) salvage value g_i (total disposals become L_i). We shall denote this indifference point by IP_{kri} (where the "r" signifies "ramp").

A potentially troublesome feature of (4.19) is that the term I appears on both sides of the inequality. Thus, the effort to find IP_{kri} is not as straight-forward as that used in finding either IP_{0i} or IP_{ki} (recall that in our earlier indifference points, I appeared on only one side of the inequality). As a result, we are required to evaluate values of I until we find the first I such that the inequality in (4.19) is satisfied. We know, however, that in order for us to be on the ramp with salvage value g_k, the retained stock must equal M_k^* units. Since we are required to make at least L_k disposals in order to earn g_k per unit disposed, total stock must be greater than M_k^* + L_k (retained + disposed). Since we cannot dispose more than U_k units at the g_k salvage value, total inventory cannot exceed M_k^* + U_k (retained + disposed). Consequently, total inventory (I) is bounded between M_k^* + L_k (on the low side) and M_k^* + U_k (on the high side).

There is another feature of IP_{kri} worth noting. As illustrated in Figure 9, we note that the left-hand side (LHS) of the inequality in (4.18) is nonlinear with respect to I (it is decreasing exponentially). The right-hand side (RHS) is linear with respect to I (as inventory increases, the right-hand side decreases linearly). In addition, when I = 0, the

Figure 9

Behavior of Expression (4.18)



left-hand side is quite close to 1 ($\alpha' << 1$). Analyzing the right-hand side of this expression, we note a relatively large value of the denominator (due to the positive exponential term). This implies that it is possible, when $I = 0$, for the right-hand side to be much lower than 1. Since it is decreasing linearly from an intercept much lower than 1, it will intersect the left-hand side of (4.18) in two places. When the left-hand side of (4.18) is smaller than its right-hand side (for a given I), this implies that it is attractive to jump from the ramp with lower salvage value g_k to a plateau involving L_i disposals (but at higher unit value, g_i). However, when the left-hand side becomes greater than the right-hand side of (4.18), the reverse holds true. It then becomes attractive to "jump back" to the ramp. This suggests a rather counter-intuitive behaviour (ie. moving back to a lower salvage value). But, we have been able to show in a detailed proof (see Appendix G) that the "second" indifference point would occur when I exceeds $M_k^* + U_k$! Such an inventory value is beyond the relevant range of consideration for the g_k salvage value. As a result, when applying our analytical models, we would never observe a situation in which we would "jump back" to the lower salvage value. If it is attractive to jump to the higher plateau (and earn g_i per unit disposed) somewhere along the ramp, it will still be attractive to do so at the end of the ramp (when $I = M_k^* + U_k$).

We shall now present an efficient algorithm for determining optimal disposal quantities under the presence of increasing salvage values. As inventories on-hand at the conclusion of the construction phase increase, we recognize that additional units can either be retained or disposed. Thus, as we did with marginally decreasing salvage values, our procedure shall establish W^* as one "builds up" I.

Obviously for $I = 0$, no units are available to be disposed. As a result, $W^* = 0$. Our initial step in this algorithm, consequently, is to determine at what inventory level we will first find it attractive to make disposals. If we are currently disposing no units, then we can either jump to a higher plateau (at some inventory level IP_{0i}), or we can begin a ramp at the lowest increasing salvage value. We note that this ramp at g_1 would begin when I exceeds M_1^* (ie. when $I \geq M_1^* + 1$). Thus, we initially find:

$$\min\left(IP_{0i},\ M_1^* + 1\right)\ \forall i > 1 \tag{4.20}$$

For example, if there were four increasing salvage values, then we would initially determine IP_{02}, IP_{03} and IP_{04}. These values correspond to the inventory levels at which it becomes attractive to jump up to a higher plateau (where we would dispose either L_2, L_3 or L_4 units). The value $M_1^* + 1$ is the on-hand inventory level at which we would begin a ramp with salvage value g_1 (we would dispose, in this case, $I - M_1^*$ units).

The minimum value as found in (4.20) represents the on-hand inventory level at which we start making disposals. For any inventory values less than the minimum in (4.20), we would fail to dispose any of the surplus stock (ie. $W^* = 0$).

Suppose the minimum as given in (4.20) corresponds to a higher plateau (one of the IP_{0i} values). Further, let IP_{0k} represent this minimum value (in other words, at inventory level IP_{0k}, we change from disposing 0 units to disposing L_k at g_k per unit). We now are required to determine at what inventory level it would first become attractive to jump to a higher plateau, or enter a ramp with salvage value g_k. Thus, we need to find:

$$\min\left(IP_{ki},\ M_k^* + L_k + 1\right)\ \forall i > k \tag{4.21}$$

The indifference points IP_{ki} signify jumping from a plateau at $W^{*} = L_k$ to one at which we dispose L_i units. We would enter a ramp with salvage value g_k when the on-hand inventory, I, less any required disposals (L_k) exceeded M_k^{*}. Thus, we would start a ramp at g_k when $I \geq M_k^{*} + L_k + 1$ (we dispose I - M_k^{*} units). The minimum value as found in (4.21) gives the inventory level at which we would begin making <u>more</u> than L_k total disposals.

Suppose that the minimum as established in (4.20) was $M_l^{*} + 1$. Thus, we would enter a ramp with salvage value g_l. We know, from our previous analysis in this section, that <u>somewhere</u> along this ramp (ie. on or before reaching $I = M_l^{*} + U_l$), we will jump to a higher plateau. Consequently, we need to find:

$$\min \left(IP_{1ri}\right) \forall i > 1 \tag{4.22}$$

The minimum value as given by the expression in (4.22) represents the on-hand inventory level at which we leave the ramp with salvage value g_l, jumping to a higher plateau (with larger per unit salvage values).

As an extension to (4.22), suppose the $M_k^{*} + L_k + 1$ value represented the minimum in (4.21). Then, we would enter a ramp with salvage value g_k. The next step in our procedure would be to determine:

$$\min \left(IP_{kri}\right) \forall i > k \tag{4.23}$$

Using expressions (4.20) through (4.23), one can efficiently determine the optimal disposal quantity for any given level of on-hand inventory. We begin by establishing the inventory level at which we would start making disposals. When we jump to a higher plateau, we determine if the next jump will be to an even higher plateau, or if we will start a ramp at the current salvage value. If we ever find ourselves on a ramp, all we need to do is determine at what inventory level it becomes attractive to jump to a higher plateau (with larger per unit salvage values).

Two critical issues regarding this procedure ought to be discussed. One is that should we jump from, say, zero disposals to making L_i disposals, we will never find it attractive to make disposals at any (increasing) salvage values k, where $k < i$. For example, if we initially jumped to an L_3 plateau from zero disposals, we would never find it attractive to make disposals for g_1 or g_2 per unit. This tends to reduce the computational effort of our algorithm. Once a salvage value has been passed over, it can be removed from consideration. We only need to consider higher salvage values than the one currently under analysis.

A second point is that when we begin making disposals for g_m per unit, our approach becomes somewhat more simplified. Since this is the highest increasing salvage value, we are not required to use our earlier expressions [(4.20) through (4.23)]. Simply put, there are no higher salvage values to consider. We will initiate a ramp at g_m when $I \geq M_m^* + L_m + 1$. This ramp will conclude when $I = M_m^* + U_m$ (retained stock plus disposed units).

As explained at the outset of this section, we propose that the increasing salvage value behaviour will not be observed indefinitely. Eventually, we will allow marginally decreasing salvage values. Thus, once we conclude a ramp using the largest increasing salvage value, we can adopt our earlier approach for marginally decreasing salvage values. That is, for $I > M_m^* + U_m$, there will be a plateau with total length corresponding to the difference in M_i^* values between the largest increasing (g_m) and largest marginally decreasing salvage values (call it g_{m+1}). We then initiate a ramp using the highest marginally decreasing salvage value. This ramp extends until we have disposed the maximum possible number of units at g_{m+1}, after which the usual interchanging pattern of plateaus and ramps continues.

Numerical Example:

Consider the following salvage value function. The first four salvage values represent the increasing ones (ie. m = 4), while the latter four correspond to the marginally decreasing salvage values. Values for various ongoing phase parameters are identical to those used in earlier numerical examples (remember that a glossary of notation is included in Appendix A).

Table 1
Numerical Example for Increasing Salvage Values

i	g_i	L_i	U_i	N_i	M_i^*
1	3	1	20		192
2	40	21	50		143
3	45	51	100		137
4	65	101	140		115
5	35	141	200	60	149
6	25	201	240	40	161
7	20	241	260	20	168
8	1	261			195

We initiate our procedure by determining IP_{0i} as well as M_i^*. We have the following values:

IP_{02}: 154 (*** minimum value ***)
IP_{03}: 164
IP_{04}: 169
M_1^*: 192

Thus, for $I < 154$, we fail to dispose any units. When I reaches 154, we begin making disposals for g_2 per unit by jumping to the L_2 plateau (21 total disposals).

Our next step is to determine when we leave the L_2 plateau. We would begin a ramp at g_2 when $I = M_2^* + L_2 + 1$. Furthermore, the IP_{2i} values illustrate the specific inventory levels at which we jump from a plateau at L_2 to a plateau at L_i. We have the following results:

$M_2^* + L_2 + 1 = 165$ (*** minimum value ***)
IP_{23}: 170
IP_{24}: 172

Thus, we begin a ramp with salvage value g_2 when $I = 165$. Our next step involves determining the inventory level at which it becomes attractive to jump from this ramp (to a higher plateau). Using (4.19), we observe the following values for jumping from the ramp at g_2 to a plateau involving L_3 total disposals:

For $I = 169$: LHS of (4.19) = 169, RHS of (4.19) = 169.106
For $I = 170$: LHS of (4.19) = 170, RHS of (4.19) = 169.992

Thus, when on-hand inventory at the conclusion of the construction phase equals 170, it is attractive to jump from a ramp at g_2 to the L_3 plateau.

For the L_4 plateau, we observe the following values for (4.19) when $I = 170$:

$I = 170$. LHS of (4.19) = 170, RHS of (4.19) = 170.459

The inequality involving a jump to the L_4 plateau is <u>not</u> satisfied when $I = 170$. As a result, the first movement from the ramp at g_2 occurs when we jump to the L_3 plateau at $I = 170$.

From the L_3 plateau, we will either enter a ramp with salvage value g_3, or proceed to a plateau involving total disposals of L_4 units. The inventory level at which it becomes attractive to enter the ramp at g_3 is 189 ($M_3^* + L_3 + 1$), while determining the value for IP_{34} yields the point at which we would move to the higher plateau. It turns out that IP_{34} equals 173, thus indicating that we will stay along the L_3 plateau for only a narrow interval of inventory values (from $I = 170$ to $I = 173$).

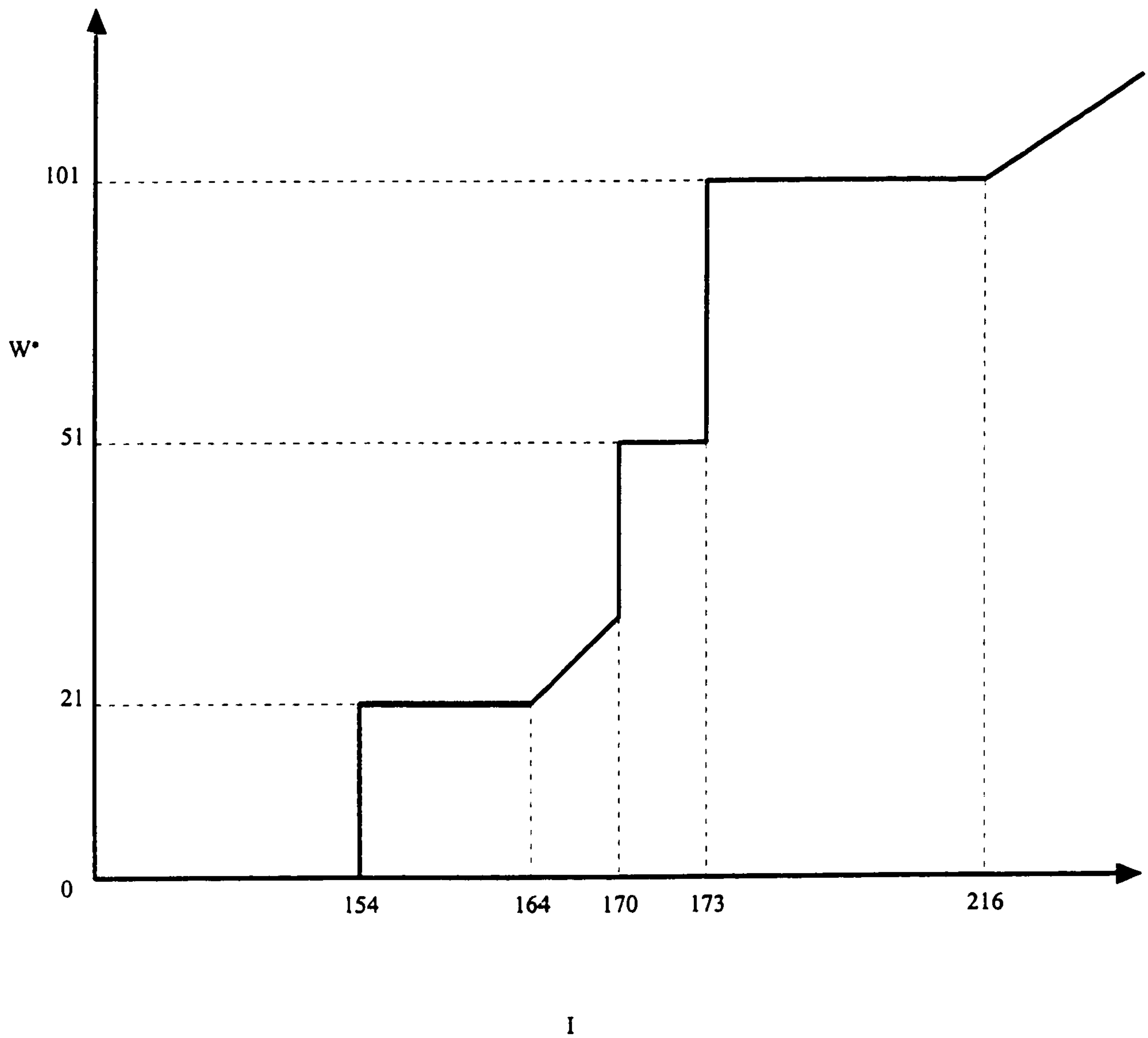
We are now making disposals for the highest increasing salvage value. Our further analysis of optimal disposal decisions becomes quite straight-forward. A ramp with salvage value g_4 begins when $I = M_4^* + L_4 + 1$ ($115 + 101 + 1 = 217$). Thus, for $I \geq 217$, we dispose $I - M_4^* = I - 115$ units. This ramp concludes when $I = M_4^* + U_4 = 115 + 140 = 255$. Then, we initiate disposals on the marginally decreasing side. There is a plateau of length $149 - 115 = 34$ units. Thus, for I such that $255 \leq I \leq 255 + 34 = 289$, $W^* = 140$. Then, a ramp using a salvage value of \$35 per unit begins. The remainder of this behaviour is similar to that described in our earlier numerical example for marginally decreasing salvage values. Figure 10 illustrates the behavior of W^* versus I for the increasing salvage values of this numerical example.

We have now completed a critical chapter of this dissertation. The implication of these findings is that, for any salvage value function (whether it involve constant or non-constant values) <u>and</u> for any level of on-hand inventory, we can quickly determine the

Figure 10

Numerical Example - Graph of W* vs. I

in the Increasing Salvage Value Range



optimal disposal decision. We can also calculate the present value of the relevant costs of this best decision. This capability will be crucial to our model development in Chapter 5.

5. CONSTRUCTION PHASE DECISIONS

Having completed our analysis of disposal decisions as well as replenishment strategies in the ongoing phase of the project, we will now examine construction phase procurement decisions. Faced with uncertainty as to the total requirements of an item during the construction phase, what quantity ought a materials manager to procure at the outset of the project? This procurement decision shall reflect acquisition, holding and shortage costs during the construction phase as well as the costs (as discussed in Chapter 4) of concluding construction with a specific amount of on-hand inventory, and proceeding in an optimal fashion from thereon.

The following notation will be used in this chapter:

B_1:	Fixed cost per stockout occasion
B_2:	Penalty (expressed as a fraction of the unit value) per unit short
D_c:	Total requirements in the construction phase
$EPV^*(I)$:	Expected present value of all future costs associated with concluding the construction phase with I units of inventory on-hand, and proceeding in an optimal fashion from thereon (with respect to disposal and ongoing phase replenishment decisions)
Q_c:	Construction phase procurement quantity
T_c:	Duration of construction phase (in years)
v_c:	Unit acquisition cost in the construction phase

To reflect uncertainty with respect to total construction phase requirements, we shall allow D_c to follow a discrete probability distribution (where $P_D(D_c)$ represents the probability of observing a specific value of total requirements). Using a discrete distribution, as opposed to a continuous one, reduces the effort involved in optimizing this analytical model and also makes it easier for the practitioner to subjectively specify

difference between the smooth and discrete patterns diminishes as an inventory cycle increases in height. Total requirements in the construction portion of the project tend to dominate annual ongoing usage, so allowing requirements in the construction phase to display the linear pattern will have only a minor, secondary effect. We were more concerned with modelling the precise nature of item usage during the ongoing phase (when annual usage rates were relatively lower); hence our use of the step-wise pattern during that portion of our analysis.

We initialized our description of construction phase costs by using an "approximate" approach. This approach allowed a more straightforward illustration of cost function convexity. Later, we developed an "exact" approach. Sections of this chapter will be devoted to each approach.

5.1 Approximate Approach

We now wish to determine the expected total costs for the entire project (construction and ongoing phases) as a function of the quantity procured before construction is initiated. We shall use the following notation:

$ETC(Q_c)$: Expected total discounted costs, as a function of Q_c

There are two important differences between how we determine $ETC(Q_c)$ in either approach. The first difference consists in how we derive construction phase holding costs. The approximate approach uses the average requirements in the construction phase, $\overline{D}_c$, in its computation of holding costs. This average value is computed as follows (noting that n represents the number of different possible requirements in the construction phase):

$$\overline{D_c} = \sum_{i=1}^{n} \left(D_i * P_D(D_i)\right)$$

Under the exact approach, we explicitly recognize each separate requirements value, D_c, when computing holding costs.

The other major difference concerns the calculation of the present value of stockout penalties. Under the approximate approach, we discount the stockout costs from the end of the construction phase. With the exact approach, these stockout charges are discounted from the specific moment at which they might occur.

As an aside, we note that these differences suggest that the construction phase procurement quantity provided by the approximate approach is a lower bound on the optimal quantity given by the exact approach. The approximate approach underestimates the cost of a stockout by discounting this event from a point in time later than its occurrence. Moreover, the approximate approach overpenalizes carrying charges. For very large construction phase requirements, we recognize that smaller carrying charges would actually be incurred than those given in the approximate approach (which uses average construction phase requirements).

We shall now describe the various components of $ETC(Q_c)$ under the approximate approach. The acquisition costs are simply:

$$Q_c v_c \tag{5.1}$$

Since there is only a single procurement opportunity during the construction phase, the fixed cost of procurement (the "A" term) has been ignored.

The holding charges can be evaluated as:

$$h \int_0^{T_c} \left(Q_c - \frac{\overline{D_c}}{T_c} t \right) e^{-\alpha t} \, dt \tag{5.2}$$

A common table of integrals may be used to evaluate (5.2). It becomes:

$$h \left(\frac{Q_c}{\alpha} \left(1 - e^{-\alpha T_c}\right) + \frac{\overline{D_c}}{T_c} \left[e^{-\alpha T_c} \left(\frac{T_c}{\alpha} + \frac{1}{\alpha^2} \right) - \frac{1}{\alpha^2} \right] \right) \tag{5.3}$$

The fixed costs per stockout occasion, B_1, must be weighted by the probability of a stockout, then discounted from the end of the construction phase. This gives:

$$B_1 \, e^{-\alpha T_c} \, P_D(D_c > Q_c) \tag{5.4}$$

The per unit stockout costs consist of an expediting premium ($B_2 v_c$) in addition to the normal unit acquisition costs (in order to satisfy construction phase usage, we must bring in the number of units by which we are short). The present value of these costs is:

$$(1 + B_2) v_c \, e^{-\alpha T_c} \sum_{D_c > Q_c} (D_c - Q_c) \, P_D(D_c) \tag{5.5}$$

The term:

$$\sum_{D_c > Q_c} (D_c - Q_c) \, P_D(D_c)$$

represents the expected number of units short.

We must properly represent the present value of concluding the construction phase with a given level of on-hand stock, and proceeding from thereon in an optimal fashion with respect to disposal and ongoing phase replenishment decisions. Obviously, if the total requirements in the construction phase exceeded the procurement quantity (ie. a stockout occurred), then we would expedite the number of units by which we were short, thus beginning the ongoing phase with 0 units on-hand. No units are available for disposal; hence, we would simply bring in Q_o units (this replenishment would be timed so as to arrive when the next usage occurs). The expected present value of this is:

$$e^{-\alpha T_c} EPV^*(0) \; P_D(D_c \geq Q_c) \tag{5.6}$$

Suppose, however, that the construction phase concluded with I units on-hand, where $I > 0$. This could only happen when the procurement quantity exceeded total construction phase requirements (ie. $I = Q_c - D_c > 0$). The expected present value of these costs becomes:

$$e^{-\alpha T_c} \sum_{D_c < Q_c} EPV^*(Q_c - D_c) \, P_D(D_c) \tag{5.7}$$

Combining expressions (5.1) and (5.3 - 5.7) yields an expression for the entire project costs as a function of the initial procurement quantity.

In order to provide a search method for determining the optimal procurement quantity (as will be described later), we must show that $ETC(Q_c)$ is convex <u>within a range between two adjacent requirements values</u>. If it is, then we will know that the function will reach one (and only one) local minimum in each range. We observe that those costs specifically related to the construction phase (ie. expressions (5.1) and (5.3 - 5.5)) are, for $D_i \leq Q_c < D_{i+1}$, linear with respect to Q_c. Hence, these costs are convex between adjacent requirements values.

It is vital that we accurately understand the behaviour of $EPV^*(I)$. Recall that units on-hand at the conclusion of the construction phase may be either disposed or retained. Assuming that $g > 0$, disposal of surplus units implies that $EPV^*(I+1) < EPV^*(I)$. Salvage revenue lowers the expected present value of costs. The retention of surplus units, then, can only be attractive if it generates a greater reduction in costs than that provided by stock disposal. Thus, if retention of surplus stock is optimal, we must have that $EPV^*(I+1) < EPV^*(I)$. Consequently (provided that $g > 0$), $EPV^*(I)$ is a <u>non-increasing</u> function in I. For constant salvage values, it displays a nonlinear pattern during the "retention region" (ie. when it is attractive to retain stock) and a linear shape during the "disposal region" (ie. when we dispose surplus units). Specifically, we showed in Chapter 4 that $PV(M)$, the present value of retaining M units to satisfy ongoing usage, was convex in M. Since the disposal region exhibits a linear shape, it is thus convex. Hence, for Q_c within an interval such that $D_i \leq Q_c < D_{i+1}$ and for constant salvage values, we know that $ETC(Q_c)$ is convex.

We shall now address the convexity of $ETC(Q_c)$ for the case of marginally decreasing salvage values. Note that those costs specifically related to the construction phase are identical under either constant or non-constant salvage value functions; hence, these costs are convex regardless of the specific form of salvage values. The key issue involves determining if $EPV^*(I)$ remains convex when marginally decreasing salvage values are introduced.

As was shown in Figure 7 (see page 63), optimal policies in the presence of marginally decreasing salvage values exhibit an interchanging pattern of plateaus and ramps (retentions and disposals). Recall that the distance of a specific plateau corresponds to the difference between adjacent M_i^* and M_{i+1}^* values, while the length of a ramp is determined by the maximum number of disposals that can be made for g_i per unit. The function $EPV^*(I)$ displays a nonlinear but convex shape during any such region, while it has a linear pattern when it is attractive to dispose stock (although the slopes of these respective linear segments become less negative as I increases, due to the marginally decreasing nature of the salvage values). From our earlier explanation for constant salvage values, we showed that the present value of retaining any quantity of units to satisfy ongoing usage was convex with respect to the retention amount. Since linear functions (which occur when disposals become attractive) are also convex, we see that $EPV^*(I)$ remains convex. As a result, we can say $ETC(Q_J)$ is convex, within a range between two adjacent requirements values, for the case of marginally decreasing salvage values.

With increasing salvage values, our analysis must be modified somewhat. Now, we have discontinuous places (denoted by the values IP_{0i}, IP_{ki}, or IP_{kri}) such that we jump from a plateau or ramp to a plateau with a higher per unit salvage value. The function $EPV^*(I)$, as before, is convex within specific ramps or plateaus; however, when analyzing the convexity of $ETC(Q_J)$, we must give special attention to these points at which discontinuities occur in the optimal disposal graph.

A discontinuity occurs when on-hand inventory equals one of the IP values as given above, say, IP_{0i}. Thus, a discontinuity would be generated when $I = Q_c - D_c = IP_{0i}$, or $Q_c = D_c + IP_{0i}$. This implies that when a construction phase procurement quantity equals a specific requirements value plus a given inventory indifference point, a discontinuity will occur in the $EPV^*(I)$ function (and, by association, in the $ETC(Q_c)$ function). Let us denote by Q_{ip} any procurement quantity that yields a discontinuity in $ETC(Q_c)$, due to the presence of increasing salvage values.

When evaluating the convexity of $ETC(Q_c)$, we can no longer restrict our attention simply to ranges between adjacent requirements values. There will be discontinuous jumps in the $ETC(Q_c)$ function, whenever $Q_c = Q_{ip}$. This means that the $ETC(Q_c)$ function is now convex between any D_c or Q_{ip} value and the next higher value, whether it be a D_c or Q_{ip} point. With the inclusion of increasing salvage values, the number of ranges within which the total cost function is convex increases.

5.2 Exact Approach

We shall now provide an examination of the exact approach for determining the expected total project costs. The acquisition costs are identical to those given under the approximate approach, namely:

$$Q_c v_c \tag{5.8}$$

We explicitly consider each total requirements value when evaluating construction phase holding costs. We have, for any procurement quantity Q_c, the following expression for carrying charges:

$$h \sum_{D_c \le Q_c} P_D(D_c) \int_0^{T_c} \left(Q_c - \frac{D_c}{T_c} t \right) e^{-\alpha t}\, dt \tag{5.9}$$

This is evaluated as:

$$h \sum_{D_c \le Q_c} P_D(D_c) \left(\frac{Q_c}{\alpha} \left(1 - e^{-\alpha T_c} \right) + \frac{D_c}{T_c} \left[e^{-\alpha T_c} \left(\frac{T_c}{\alpha} + \frac{1}{\alpha^2} \right) - \frac{1}{\alpha^2} \right] \right) \tag{5.10}$$

Under the exact approach, we also recognize any carrying charges incurred <u>prior</u> to the occurrence of a stockout. Since a stockout, if it occurs, would happen at time $Q_c T_c / D_c$, this is given as:

$$h \sum_{D_c > Q_c} P_D(D_c) \int_0^{\frac{Q_c T_c}{D_c}} \left(Q_c - \frac{D_c}{T_c} t \right) e^{-\alpha t}\, dt \tag{5.11}$$

which becomes:

$$h \sum_{D_c > Q_c} P_D(D_c) \left(\frac{Q_c}{\alpha} \left(1 - e^{-\frac{\alpha Q_c T_c}{D_c}} \right) + \frac{D_c}{T_c} \left[e^{-\frac{\alpha Q_c T_c}{D_c}} \left(\frac{Q_c T_c}{\alpha D_c} + \frac{1}{\alpha^2} \right) - \frac{1}{\alpha^2} \right] \right) \tag{5.12}$$

Expression (5.12) may be simplified to yield:

$$h \sum_{D_c > Q_c} P_D(D_c) \left[\frac{Q_c}{\alpha} + \frac{D_c}{T_c} \left(e^{-\frac{\alpha Q_c T_c}{D_c}} \left(\frac{1}{\alpha^2} \right) - \frac{1}{\alpha^2} \right) \right] \tag{5.13}$$

Although we explicitly recognize holding costs prior to a stockout, we shall ignore any carrying charges incurred subsequent to the receipt of expedited stock. In all likelihood, the relatively large stockout penalties would dominate these holding costs. We note, however, that these costs could be included in our total cost function if deemed necessary.

Since the exact approach discounts stockouts from the specific moment at which they occur, our expressions for the stockout penalties must be modified somewhat. The B_1 stockout penalty is given as:

$$B_1 \sum_{D_c > Q_c} P_D(D_c)\, e^{-\frac{\alpha Q_c T_c}{D_c}} \tag{5.14}$$

while the B_2 stockout penalty is:

$$(1+B_2) v_c \sum_{D_c > Q_c} (D_c - Q_c)\, P_D(D_c)\, e^{-\frac{\alpha Q_c T_c}{D_c}} \tag{5.15}$$

The expected present value of concluding the construction phase with I units, and proceeding in an optimal fashion regarding any future inventory decisions, is the same under either the approximate or exact approach. Thus, we have (for $D_c \geq Q_c$):

$$e^{-\alpha T_c} EPV^*(0)\ P_D(D_c \geq Q_c) \tag{5.16}$$

and for $D_c < Q_c$:

$$e^{-\alpha T_c} \sum_{D_c < Q_c} EPV^*(Q_c - D_c)\, P_D(D_c) \tag{5.17}$$

Combining expressions (5.8), (5.10) and (5.13 - 5.17) yields our exact approach for determining the expected total project costs, as a function of the initial procurement quantity.

Earlier, we showed that the approximate approach yielded a cost function which was convex between adjacent D_c values (or, in the case of increasing salvage values, a convex cost function between any D_c or Q_{ip} value and the next higher D_c or Q_{ip} point). We shall now show that the same behaviour is observed under the exact approach. Recall that those expressions which comprise $ETC(Q_c)$ include (5.8), (5.10) and (5.13) - (5.17) inclusive.

Expressions (5.8), (5.16) and (5.17) are identical to those given under the approximate method. In addition, expression (5.10) is linear with respect to Q_c, so the convexity of $ETC(Q_c)$ between adjacent D_c or Q_{ip} values will not be affected by this new expression.

The convexity of expressions (5.13) - (5.15) can be checked by determining the second difference of each expression. If a second difference is strictly positive, then we know that the specific expression is convex. To illustrate this, consider expression (5.13). Evaluating its second difference for each D_c yields (after all linear terms are eliminated):

$$h \; P_D(D_c) \left[\frac{D_c}{T_c \alpha^2} \right] \left[e^{\frac{-\alpha(Q_c+2)T_c}{D_c}} + e^{\frac{-\alpha(Q_c)T_c}{D_c}} - 2e^{\frac{-\alpha(Q_c+1)T_c}{D_c}} \right]$$

which can be expressed as:

$$h \; P_D(D_c) \left[\frac{2\,D_c}{T_c \alpha^2} \right] \left[\frac{e^{\frac{-\alpha(Q_c+2)T_c}{D_c}} + e^{\frac{-\alpha(Q_c)T_c}{D_c}}}{2} - e^{\frac{-\alpha(Q_c+1)T_c}{D_c}} \right] \qquad (5.18)$$

Obviously, the term within the first set of squared brackets is strictly positive. As we showed in Chapter 4, e^{-x} is convex for x > 0. Consequently, the arithmetic mean of two points (j,l) along the e^{-x} function, less the value of a point in between (k) will be positive. Thus, expression (5.13) is indeed convex. Similar reasoning can be applied to functions (5.14) and (5.15) to illustrate their convexity.

Since $ETC(Q_c)$ is convex between adjacent D_c or Q_{ip} values, we know that the function will reach one (and only one) local minimum within each range. Thus, we can now prescribe a simple search procedure to find the optimal construction phase procurement quantity (using the exact approach).

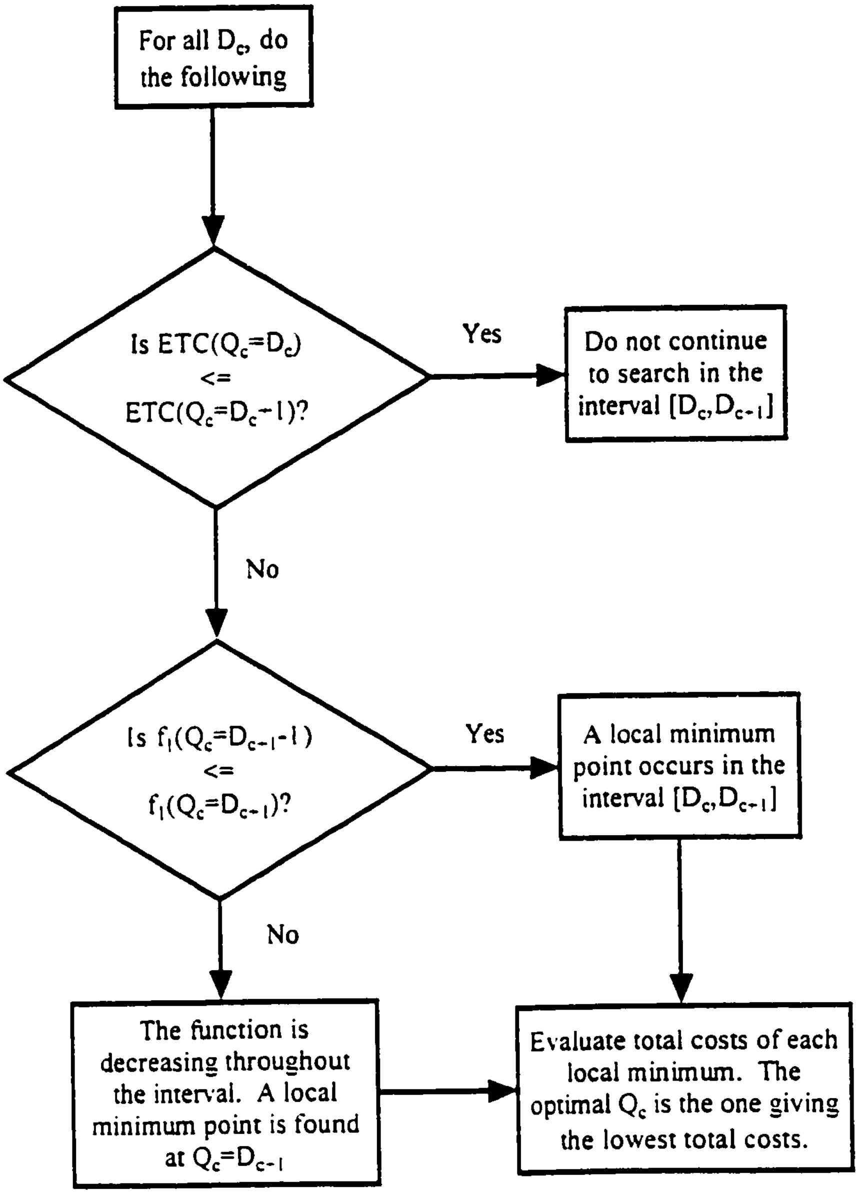
We shall begin with the case of constant or marginally decreasing salvage values. Figure 11 illustrates the various steps to follow in finding the best Q_c. We proceed through each of the requirements values, beginning with the lowest D_c. We evaluate the expected total costs of procuring an amount equal to a D_c value ($Q_c = D_c$), then the expected total costs of procuring one unit above a D_c point ($Q_c = D_c + 1$). If the expected total costs increase from the lower to higher quantity, then we know (from the convexity property) that these costs will continue increasing throughout the entire range between adjacent D_c values (ie. between D_c and D_{c+1}). Consequently, we do not need to evaluate any additional procurement quantities within this range.

If, on the other hand, the costs fail to increase from D_c to $D_c + 1$, then we need to evaluate costs at the right end of this range (ie. from D_{c+1}-1 to D_{c+1}). Recognizing that proceeding from one unit below a requirements value to a specific D_c will yield a "drop" in the expected total cost function (due to a reduction in B_l stockout penalties), we evaluate the total costs between D_{c+1}-1 and D_{c+1} without the B_l component (we shall

Figure 11
Search Procedure for Finding Q_c^*
Constant or Marginally Decreasing Salvage Values

Define:

$ETC(Q_c)$ = Expected Total Costs (all components)
$f_1(Q_c)$ = ETC without the B_1 component



denote this cost expression by $f_l(Q_c)$. If the total costs, without the B_l component, were not increasing between these two points, then the expected total cost function has been decreasing throughout the entire $[D_c, D_{c+1}]$ range. A local minimum is then found at $Q_c = D_{c+1}$.

Should the total costs without the B_l component increase between these two points (D_{c+1}-1 and D_{c+1}), then a local minimum is found somewhere within the interval $[D_c, D_{c+1}]$. A "Fibonacci" search could be used to efficiently find this interior minimum. Due to the convexity of the expected total cost function within adjacent requirements values, once the function begins to "turn up", we know that a local minimum has been found.

The optimal procurement quantity, Q_c^*, is simply the local minimum with the lowest expected total costs.

Numerical Example:

Consider the following parameter values for the construction phase (ongoing phase parameter values remain as previously given, with a constant salvage value for disposals of $35 per unit):

B_l = \$3000	
B_2 = 0.5	
D_1 = 200	$P_D(D_1)$ = 0.10
D_2 = 300	$P_D(D_2)$ = 0.20
D_3 = 400	$P_D(D_3)$ = 0.40
D_4 = 500	$P_D(D_4)$ = 0.20
D_5 = 600	$P_D(D_5)$ = 0.10
T_c = 1 year	
v_c = \$100	

Using our search procedure, we evaluate the following points:

$ETC(200) = \$91,337.10$
$ETC(201) = \$91,288.85$
$f_l(299) = \$85,148.82$
$f_l(300) = \$85,122.48$

Thus, the cost function is decreasing throughout the [200,300] interval.

$ETC(300) = \$87,086.20$
$ETC(301) = \$87,047.55$
$f_l(399) = \$83,251.95$
$f_l(400) = \$83,248.51$

The cost function is decreasing throughout the [300,400] interval.

$ETC(400) = \$84,083.03$
$ETC(401) = \$84,055.59$
$f_l(499) = \$83,870.92$
$f_l(500) = \$83,903.01$

There is an interior minimum within the [400,500] range. It is found at $Q_c = 436$ units, with $ETC(436) = \$83,595.21$

$ETC(500) = \$84,179.02$
$ETC(501) = \$84,199.42$

The function is increasing throughout the [500,600] interval.

$ETC(600) = \$88,550.73$
$ETC(601) = \$88,606.83$

The function is increasing for any points beyond the largest requirements value, 600. On a range-by-range basis, we have local minima at the right ends of the [200,300] and [300,400] ranges, as well as the left ends of the [500,600] and [600+] intervals. In addition, we have an interior minimum at $Q_c = 436$. The local minimum point giving the lowest expected total costs is $Q_c = 436$. Thus, this quantity becomes our optimal procurement amount.

The search procedure is relatively similar for the case of increasing salvage values. As depicted in Figure 12, wc sort all the D_c and Q_{ip} values in ascending order and evaluate cost differences at the left end of each interval. (These intervals are defined between any D_c or Q_{ip} value, and the next higher point).

If the total costs are decreasing at the beginning of an interval, then we perform a cost evaluation at the right end of the range. Noting that proceeding from one unit below a Q_{ip} point to a specific Q_{ip} value will yield a "drop" in the expected total cost function (due to the inclusion of higher per unit salvage values), we evaluate the total costs between Q_{ip+1}-1 and Q_{ip+1} assuming that all disposals earn the marginal salvage value obtained prior to the indifference point (we shall denote this cost expression by $f_2(Q_c)$). If the total costs, without the higher per unit salvage values, were not increasing between these two points, then the expected total cost function has been decreasing throughout the entire $[Q_{ip}, Q_{ip+1}]$ range.

Numerical Example:

Consider the same construction and ongoing phase parameter values as illustrated in the previous example in this chapter. The following increasing salvage value function is used:

Figure 12
Search Procedure for Finding Q_c^*
Increasing Salvage Values

<u>Define:</u>

$ETC(Q_c)$ = Expected Total Costs (all components)
$f_1(Q_c)$ = ETC (Q_c) without the B_1 component
$f_2(Q_c)$ = ETC (Q_c), assuming that disposals earn the
marginal salvage value obtained <u>before</u> the IP is reached

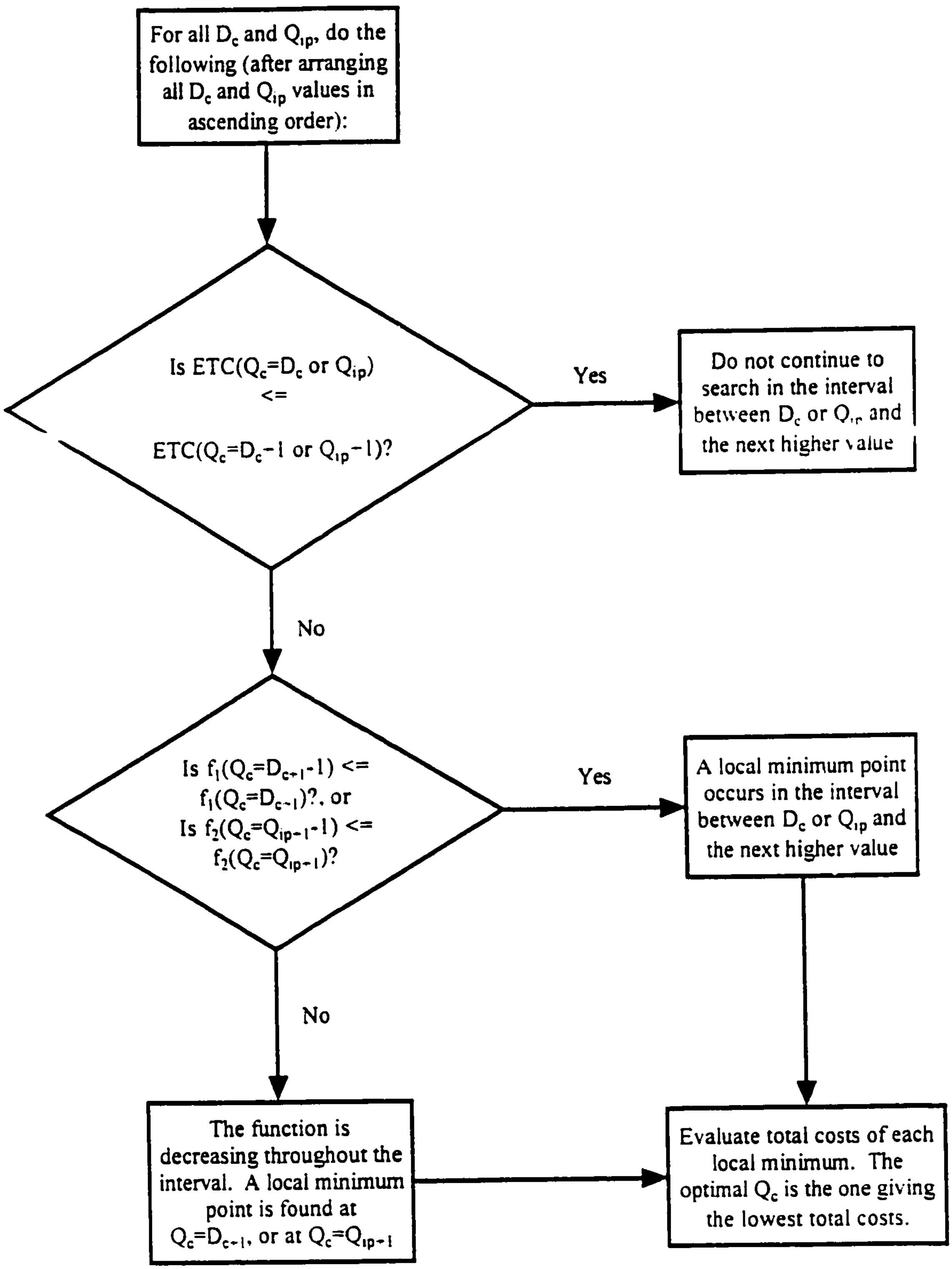


Table 2
Numerical Example for Search Procedure Involving Increasing Salvage Values

i	g_i	L_i	U_i	N_i	M_i^*
1	5	1	10		189
2	20	11	30		168
3	25	31	60		161
4	60	61	100		121
5	40	101	130	30	143
6	20	131	170	40	168
7	10	171	200	30	182
8	1	201			195

We have chosen this specific increasing salvage value function in order to make our numerical example relatively concise. Specifically, this given function has a single inventory indifference point at $I = 152$. For inventory levels less than this quantity, no excess stock is disposed. When $I = 152$, we begin making disposals for g_4 per unit. We jump up to the L_4 plateau (61 total disposals), remaining at that disposal level until I exceeds $M_4^* + L_4$. A ramp with salvage value g_4 then begins, proceeding until total disposals equal U_4. Subsequently, disposals are initiated on the marginally decreasing side.

Recall that $Q_{ip} = D_c + IP$. Since our lone indifference point (IP) is at $I = 152$, we will have to consider the following Q_{ip} values (352, 452, 552, 652 and 752) in addition to the usual five requirements values (200, 300, 400, 500 and 600). Using our search procedure, we evaluate the following points:

$ETC(200) = \$91,337.10$
$ETC(201) = \$91,288.85$
$f_1(299) = \$85,148.82$
$f_1(300) = \$85,122.48$

Thus, the cost function is decreasing throughout the [200,300] interval.

$ETC(300) = \$87,086.20$
$ETC(301) = \$87,047.55$
$f_2(351) = \$85,675.63$
$f_2(352) = \$85,658.60$

The cost function is decreasing throughout the [300,352] interval.

$ETC(352) = \$85,658.39$
$ETC(353) = \$85,636.51$
$f_1(399) = \$83,105.96$
$f_1(400) = \$83,100.26$

The cost function is decreasing throughout the [352,400] interval.

$ETC(400) = \$83,934.78$
$ETC(401) = \$83,905.08$
$f_2(451) = \$83,443.56$
$f_2(452) = \$83,453.62$

There is an interior minimum within the [400,452] range. It is found at $Q_c = 437$ units, with $ETC(437) = \$83,374.69$.

$ETC(452) = \$83,453.20$
$ETC(453) = \$83,453.44$

The function is increasing throughout the [452,500] interval.

$ETC(500) = \$83,658.65$
$ETC(501) = \$83,675.88$

The function is increasing throughout the [500,552] interval.

$ETC(552) = \$85,448.83$
$ETC(553) = \$85,521.79$

The function is increasing throughout the [552,600] interval.

$ETC(600) = \$87,481.29$
$ETC(601) = \$87,534.14$

The function is increasing throughout the [600,652] interval.

$ETC(652) = \$90,812.44$
$ETC(653) = \$90,879.76$

The function is increasing throughout the [652,752] interval.

$ETC(752) = \$99,202.45$
$ETC(753) = \$99,295.56$

The function is increasing for any points beyond the largest Q_{ip} value, 752.

On a range-by-range basis, we have local minima at the right ends of the [200,300], [300,352] and [352,400] ranges. Furthermore, there are local minima at the left ends of the [452,500], [500, 552], [552,600], [600,652], [652,752] and [752+] intervals. We also have an interior minimum point at $Q_c = 437$. The local minimum point providing the lowest expected total costs is $Q_c = 437$. Consequently, this quantity becomes our optimal procurement amount.

Although our search procedures will find the optimal quantity, we can also develop some bounds on Q_c^* to further narrow the range of possible procurement quantities to evaluate. For a specific set of parameter values and for constant salvage values, we can obtain an upper bound on Q_c^*. This bound is developed by noting that we would continue to procure in excess of the largest construction phase requirements value as long as the marginal benefits of so doing exceeded the concomitant marginal costs.

Since stockout charges are nil when Q_c exceeds the largest D_c, the present value of the marginal benefits consist of the reduction in $EPV^*(I)$ associated with larger procurement amounts. These become:

$$e^{-\alpha T_c}\left(\Delta\, EPV^*(I)\right) \tag{5.19}$$

The present value of the marginal costs comprise acquisition and carrying charges. These are:

$$v_c + h'\left(1 - e^{-\alpha T_c}\right) \tag{5.20}$$

Thus, we would continue to procure until the absolute value in (5.19) fell below (5.20). This would give the largest inventory amount that we ever need consider. Adding this I to the largest D_c provides us with the largest possible Q_c. Such an upper bound on the procurement quantity could prove helpful in situations for which it becomes attractive to exceed the largest D_c (e.g. very high stockout penalties, high likelihood of largest D_c occurring, and so forth).

An additional procedure to narrow the search is to use solutions obtained from prior constant salvage value cases as bounds on Q_c^* for the case of non-constant salvage value functions. Suppose various cases have been run in which different constant salvage values were used. The solution from a situation in which the constant value represented the largest marginally decreasing one provides an upper bound on Q_c^* in the case of the specific marginally decreasing salvage value function. When one can earn the largest salvage value over all disposals (instead of over a limited range of disposals), higher procurement quantities become attractive. Of course, a lower bound on Q_c^* is obtained when the constant salvage value represents the lowest marginally decreasing one.

For the case of increasing salvage values, we can use the lowest salvage value to provide a lower bound on Q_c^*. An upper bound on the optimal procurement quantity is somewhat more complex. We could use the reasoning presented with respect to (5.19) and (5.20) to find the largest inventory amount that we ever need consider. As before, this largest inventory amount would then be added to the largest D_c to provide the largest possible Q_c. However, we must recognize that, for the case of increasing salvage values, the $EPV^*(I)$ function is not convex. In particular, we could find that, once the absolute value in (5.19) fell below (5.20), there may be a larger I value for which the absolute value in (5.19) exceeded (5.20). Thus, a decision-maker may need to consider a relatively large number of I values before being confident that an upper bound on Q_c^* has been obtained.

6. MODEL RESULTS

The mathematical model developed in Chapters 3 through 5 of this dissertation provides a mechanism to determine the expected total costs of procuring a specific quantity of a given item in a project context. Moreover, our model also illustrates the calculation of costs arising due to disposal and ongoing usage decisions. We have also shown the manner in which least-cost decisions may be determined.

At various places during the previous three chapters, we have used numerical examples to demonstrate our model. However, the true value of any optimization exercise lies not simply in generating the optimal solution for a particular set of parameter values, but rather in being able to analyze a range of parameter values and their impact on managerial decision-making. Extensive sensitivity analysis avoids the problem of making broad generalizations based on a single numerical example.

Table 3 lists levels for the various parameters that will be used in our model analysis (remember that a glossary of notation is included in Appendix A). The specific ranges of values have been elicited from materials management personnel involved in large-scale construction projects. Note that the three respective levels for each parameter's value are equally spaced. Observe further that we use only a single representative value for v_c, the unit acquisition cost in the construction phase. This serves as our "point of reference" for other (dollar-valued) model parameters (ie. one could normalize everything in terms of v_c).

Table 3
Parameter Values

Parameter	Low Value	Middle Value	High Value
v_c		\$100	
B_1	\$2,000	\$3,000	\$4,000
B_2	0.5	1.0	1.5
h	\$10	\$13	\$16
a	0.08	0.10	0.12
T_c	0.5	1.0	1.5
A	\$100	\$250	\$400
D_o	10	20	30
v_o	\$160	\$190	\$220

Although we restrict each scenario to minimum and maximum construction phase requirements of 200 and 600 units, respectively, the probabilities of various requirements do change. Table 4 shows the different discrete probability distributions of construction phase requirements. The "base case" provides a distribution peaked in the middle, while the increasing (decreasing) case offers an upward-sloping (downward-sloping) probability distribution. The level scenario suggests that each requirements value is equally likely, while the bi-modal case introduces a situation in which high and low values are most likely (the probability distribution is peaked at both ends).

Table 4
Probability Distributions of Construction Phase Requirements

Requirements	200	300	400	500	600
Base Case	0.1	0.2	0.4	0.2	0.1
Increasing	0.06	0.13	0.2	0.27	0.34
Decreasing	0.34	0.27	0.2	0.13	0.06
Level	0.2	0.2	0.2	0.2	0.2
Bi-modal	0.35	0.12	0.06	0.12	0.35

Recall that procurement, disposal and ongoing operational usage decisions are combined in our model. Is it worthwhile for materials managers to jointly consider these issues when making procurement decisions? Or, can managers simply examine construction phase factors when determining appropriate procurement quantities? In order to increase the appeal of our approach in managing real-world project inventories, we must provide a mechanism for illustrating the benefit of combining project procurement, disposal and ongoing usage decisions. To accomplish this, we will test three different inventory management strategies:

- Integrated Strategy: determines optimal procurement quantities by considering construction phase costs as well as subsequent disposal and ongoing phase replenishment decisions
- All-disposal Strategy: determines best procurement decisions by considering construction phase costs plus the disposal of <u>all</u> surplus units on-hand after the construction phase (no retention of excess stock)
- Myopic Strategy: determines best procurement decisions by considering only construction phase costs

We require the following notation:

Q_c^*: optimal procurement quantity produced by the integrated inventory management strategy (using the exact costing approach in the construction phase)
Q_a^*: optimal procurement quantity produced by following the all-disposal inventory management strategy
Q_m^*: optimal procurement quantity produced by following the myopic inventory management strategy

The integrated strategy is the approach adopted by our mathematical model. Both of the "non-integrated" strategies fail to consider ongoing phase replenishment decisions. The difference between these two strategies is that the all-disposal scheme disposes any surplus stock while the myopic approach, as the name implies, is the most short-sighted of the alternatives. It simply looks at construction phase costs when determining the best procurement quantities.

With the exception of one case (which shall be explained later), we shall test the effects of the different strategies by varying one model parameter at a time and observing the resulting best construction phase procurement quantity under each inventory management strategy. We choose to vary a single parameter during each model run in order to assess the change in an optimal solution directly attributable to that specific parameter. For instance, we begin by determining the best procurement quantities for the various strategies when all parameters are at their "middle" values and the "base case" setting is used for the probability distribution of construction phase requirements (note that in all model analyses, this shall be referred to as the "intermediate" treatment combination). Then, we change the value of one parameter (say, B_2) to its low value and observe the resulting Q_c^*, Q_a^* and Q_m^* values keeping all other model parameters at their middle or base case settings. We then change the value of B_2 to its high value and observe the optimal procurement quantities. This process is repeated for all settings of model parameters.

These "one-way" tests allow us to observe how various decisions change for different levels of model parameters. While it may be tempting to simply compare the different optimal procurement quantities produced by any of the respective strategies, we ought to determine the penalty cost associated with following each non-integrated inventory management strategy.

To find the penalty costs, we take either Q_a^* or Q_m^* and find the expected total discounted costs of this procurement quantity under the integrated approach. In other words, we find the total costs of procuring such a quantity (using the exact costing approach in the construction phase), and then proceeding in the best possible fashion in the future with respect to disposal and ongoing phase replenishment decisions. The percentage difference between this total cost and the expected total discounted costs of Q_c^* is referred to as the percentage cost penalty. Obviously, if an integrated and non-integrated strategy produce identical optimal procurement quantities for a specific setting of model parameters, then the percentage cost penalty of following a non-integrated strategy (for that setting of model parameters) would be nil. Moreover, we point out that our percentage cost penalties are somewhat "conservative" in nature in that, when determining the total costs of following a non-integrated strategy, we assume that the best possible disposal and ongoing replenishment decisions will be made.

This chapter devotes individual sections to each of the three types of salvage value functions: constant, marginally decreasing and increasing. We shall begin by examining the model results associated with constant salvage values.

6.1 Constant Salvage Values

Table 5 shows the range of constant salvage values considered in our model analysis.

Table 5
Range of Constant Salvage Values

Parameter	Low Value	Middle Value	High Value
g	$20	$35	$50

Table 6 (see pages 111-112) provides the optimal procurement quantities for each treatment combination. The percentage cost penalties of following each non-integrated strategy, as well as the integrated strategy (using approximate costing in the construction phase), are given in parentheses below each best procurement quantity. Recall that using these one-way tests allows us to pinpoint the effect on the percentage cost penalty directly attributable to that single setting of the specific parameter.

Table 6
Results - Constant Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	500	500	400 (1.54)	400 (1.54)
B_1 low	459	455 (0.005)	400 (1.35)	400 (1.35)
B_1 high	500	500	400 (1.76)	400 (1.76)
B_2 low	436	433 (0.004)	400 (0.58)	300 (4.18)
B_2 high	500	500	400 (3.17)	400 (3.17)
h low	500	500	400 (2.31)	400 (2.31)
h high	450	445 (0.06)	400 (0.99)	400 (0.99)
α low	500	500	400 (1.94)	400 (1.94)
α high	451	446 (0.01)	400 (1.15)	400 (1.15)
T_c low	500	500	400 (2.87)	400 (2.87)
T_c high	441	435 (0.02)	400 (0.72)	400 (0.72)
D_c incr.	600	600	500 (0.99)	500 (0.99)
D_c decr.	400	400	300 (2.21)	300 (2.21)
D_c level	500	500	400 (1.10)	400 (1.10)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
D_c bi-modal	500	422 (0.32)	400 (0.54)	300 (3.27)
A low	500	448 (0.05)	400 (1.13)	400 (1.13)
A high	500	500	400 (1.83)	400 (1.83)
D_o low	433	430 (0.01)	400 (0.88)	400 (0.88)
D_o high	500	500	400 (2.53)	400 (2.53)
v_o low	441	436 (0.01)	400 (0.65)	400 (0.65)
v_o high	500	500	400 (2.67)	400 (2.67)
g low	455	451 (0.01)	400 (1.26)	400 (1.26)
g high	500	500	400 (1.90)	400 (1.90)

We note that, for all treatment combinations, the optimal procurement quantities produced by the integrated approach are never smaller than the best quantities given by the non-integrated inventory management strategies. This would seem to suggest that, when ongoing phase replenishment issues are factored into the decision, it becomes attractive to procure extra units in the construction phase. Procuring these units during project construction saves one from having to purchase these units (at much costlier unit prices) during subsequent operations. Moreover, we note the close similarity in best procurement quantities given by either integrated approach (using exact or approximate construction phase costing). As we described in Chapter 5, the approximate approach provides a lower bound on the optimal procurement quantities used under the exact method.

A few comments will now be made on the effects of various parameters. Note that when the B_2 parameter is at its low setting (indicating a relatively small stockout penalty), Q_m^* falls to 300 (when B_2 was at its middle setting, Q_m^* was 400). Observe further that Q_c^* falls from 500 to 436 when B_2 has a low value. The percentage cost penalty of procuring this smaller quantity is relatively large (we note that cost penalties tend to rise as the percentage difference between integrated and non-integrated best procurement quantities increases).

When either the B_1 or B_2 parameters are at their high settings, Q_a^* and Q_m^* do not change from their "intermediate" level of 400. Moreover, Q_c^* does not change from its intermediate treatment combination quantity of 500. However, when stockouts are more costly (as results when either B_1 or B_2 take on higher values) and ongoing operations are considered in the original procurement choice, it becomes more attractive to procure extra units. This leads to larger percentage cost penalties associated with following the non-integrated approaches.

Lower values of the holding cost parameter, h, increase the advantage of procuring additional units during project construction since these units are less costly to hold. Since both Q_a^* and Q_m^* do not change with either setting of the h parameter, the cost penalties are larger for smaller holding costs.

Lower values of either the continuous discount rate, α, or project duration, T_c, produce smaller discount factors. As a result, the costs associated with ongoing phase decisions are discounted less. Thus, from a present value perspective, the ongoing phase becomes more "costly". Failure to consider ongoing phase decisions (as both non-integrated strategies suggest) leads to larger percentage cost penalties.

Ongoing operations also become costlier as the parameters A, D_o and v_o take on their high values. Consequently, adopting a non-integrated strategy produces larger cost penalties. We note that the "intermediate" Q_a^* or Q_m^* value of 400 units is used for Q_a^* or Q_m^* in these cases, since the above parameters are not considered in either non-integrated strategy.

If larger values of construction phase requirements are more likely (as is reflected in the increasing requirements distribution), even the non-integrated inventory management strategies will want to over-procure. The advantage of procuring extra units to provide for ongoing usage would be partly eroded by the desire to over-procure to safeguard against construction phase stockouts. As a result, the cost penalty of following a non-integrated approach drops. On the other hand, if construction phase requirements follow a decreasing probability distribution, then non-integrated strategies would feel less of a need to over-procure. This leads to larger cost penalties associated with not following the integrated approach. Note the large percentage cost penalty involved with the myopic strategy and a bi-modal requirements distribution. This distribution, with its relatively high likelihood of either large or small requirements, provides significant benefit for procuring larger quantities during the project. Surplus units (a substantial quantity of them would result if the smallest requirements value occurred) could be either disposed or used to satisfy ongoing usage. Neither of these possibilities are included in the myopic strategy.

Finally, we note that larger values of the salvage value parameter, g, combined with consideration of ongoing usage, leads one to procure additional units during project construction. This results in a larger percentage cost penalty associated with following a non-integrated strategy.

For several scenarios, the all-disposal and myopic policies provide the same optimal procurement quantity. This would appear to indicate the utility of retaining excess stock. Given the parameter values considered in our analysis, there is not a great deal of additional benefit in disposing surplus items. The true benefit comes from the ability to retain stock to satisfy ongoing operational usage. In other words, the option to simply dispose all surplus stock after the construction phase does not change procurement decisions that much when compared to solely considering construction phase costs.

These comments indicate the general effects of various parameters. However, they do not assess the statistical importance of each parameter. Is the increase in percentage cost penalty attributable to a specific parameter truly "significant"? Which of the model parameters are important in contributing to the benefit of adopting an integrated inventory management strategy?

A factorial design is a useful approach for determining the statistical significance of various factors (Montgomery (1991)). This method involves the replication of a model for all possible combinations of the levels of the parameters. Recall that our project procurement and disposal model involves nine factors with three levels each (B_1, B_2, h,a, T_c, A, D_o, v_o and g), plus an additional model parameter (construction phase requirements distributions) at five levels. Consequently, the number of replications required to fully analyze all possible combinations of model parameters is enormous. The issue of computational time, coupled with the complexity involved with interpreting results when several parameters are simultaneously varied, forces us to adopt a fractional factorial design. In this experimental approach, only a limited number of the replications are tested.

Our experimental approach shall use both the one-way results previously obtained plus results for "two-way" tests. These latter tests shall involve each possible pair of parameters, keeping all other model parameters at their middle or base case settings. For example, we shall take two parameters (say, B_1 and h) and determine model results for all possible combinations of these factors (B_1 high and h high, B_1 low and h high, B_1 high and h low, and so on). Table 7 provides an example of the pair-wise results which are generated when varying B_1 and h. Appendix H lists results for the complete set of pair-wise comparisons.

Table 7
Example of Pair-wise Comparisons - Constant Salvage Values

B_l* vs. *h

		All-Disposal			Myopic		
B_l	High	400 (2.53)	400 (1.76)	400 (1.07)	400 (2.53)	400 (1.76)	400 (1.07)
	Middle	400 (2.31)	400 (1.54)	400 (0.99)	400 (2.31)	400 (1.54)	400 (0.99)
	Low	400 (2.09)	400 (1.35)	400 (0.99)	400 (2.09)	400 (1.35)	400 (0.99)
		Low	Middle	High	Low	Middle	High
				h			

Note that we have two separate penalty cost comparisons (one for the all-disposal strategy, and another for the myopic strategy). We have chosen to not analyze the percentage cost penalties resulting from the approximate integrated strategy. Recall that our initial motivation for the approximate method, as outlined in Chapter 5, was that it allowed a more straightforward illustration of cost function convexity. In addition, the best procurement quantities obtained with this approach serve as lower bounds on the Q_c^* provided with the exact method. The percentage cost penalties of using the approximate method are extremely small.

Observe that the middle cell in each 3*3 "square" is either Q_a^* or Q_m^*, as well as the percentage cost penalty, produced by the intermediate treatment combination (all parameter values are at their middle or base case settings). The results given at either end of the middle row in the all-disposal or myopic tables were previously obtained as part of the one-way tests (in the middle row, one of the parameters is held at its middle value, while the other is varied). Similarly, the one-way tests provided the results for the top and bottom cells in each of the middle columns.

One of the principal advantages of the 3*3 tables is that they provide a visual indication of the direction or "pull" of the percentage cost penalty. Note, for example, the cost penalties of following either non-integrated strategy when B_l and h are varied. As B_l takes on higher values or h takes on lower values, the percentage cost penalty increases.

The tables varying the construction phase requirements distributions and another model parameter are not 3*3 tables, as shown in the latter portions of Appendix H. For each of the non-integrated inventory management strategies, we obtain a 5*3 table. The values in the middle column of the table, as well as those in the middle row, were obtained as part of the one-way analysis.

Our one-way and two-way tests provide a sample size of 239 “observations” for each non-integrated strategy. This specific number of observations is obtained in the following manner:

- One observation is provided by the "intermediate" treatment combination.
- 22 observations are given by one-way tests (four construction phase requirements distribution values, plus nine other factors (B_l, B_2, h,a, T_c, A, D_o, v_o and g) at high and low levels each).
- 144 observations are obtained when we perform two-ways tests on any pair of model parameters (excluding the requirements distributions). This is derived by taking pairs of these nine factors (we have ${}_9C_2$ = 36 such pairs), then multiplying this result by four (the number of additional

observations for each pair of parameters).

- 72 observations are provided when we vary the requirements distribution and any of the nine other model parameter (B_1, B_2, h,α, T_c, A, D_o, v_o and g). We have 36 different comparisons (four distributions * nine other parameters). For each of these comparisons, we have two observations (resulting when any of the nine other parameters are at either their high or low settings).

The average percentage cost penalty of following the myopic strategy, considering all 239 observations, was 2.006. The average percentage penalty of following the all-disposal strategy was 1.542.

In our model scenarios, the dependent variable is represented by the percentage cost penalty derived as a result of the one-way and two-way tests. The different settings of the model parameters constitute levels for the independent variables. The statistical package *SAS/STAT®* (using the General Linear Models procedure) can indicate the significance of the various model parameters. These are also called main effects. This statistical package uses the analysis of variance method to determine the portion of variability in cost penalties ("sums of squares") attributed to a specific model parameter. An F test is used to compute factor significance. Table 8 indicates our statistical results (*** indicates significance at the 1% level, while * indicates significance at the 5% level).

Table 8
Statistical Significance Results - Constant Salvage Values

Parameter	Inventory Management Strategy			
	All-Disposal		Myopic	
	Sum of Squares	F value	Sum of Squares	F value
B_1	0.9498	2.57	0.3226	0.49
B_2	42.1248	113.83 ***	32.6998	49.98 ***
h	12.1865	32.93 ***	11.1212	17.00 ***
α	3.793	10.25 ***	2.1823	3.34 *
T_c	42.7671	115.56 ***	36.9611	56.50 ***
A	2.8042	7.58 ***	3.0805	4.71 ***
D_o	27.408	74.06 ***	30.1504	46.09 ***
v_o	32.9824	89.12 ***	36.2171	55.36 ***
g	1.6754	4.53 *	4.1002	6.27 ***
D_c Incr.	2.7315	14.76 ***	4.81	14.71 ***
D_c Decr.	0.4764	2.57	4.826	14.75 ***
D_c Level	3.2228	17.42 ***	3.1988	9.78 ***
D_c Bi-Modal	15.4128	83.30 ***	18.3586	56.13 ***

For the particular ranges of parameter values used in our model, the following conclusions can be provided. Several of the parameters are highly significant (ie. at the 1% level). It would appear that many of our parameters account for large portions of the variability in the dependent variable. The exceptions are as follows: in the myopic strategy, the discount rate (α) is only significant at the 5% level, while in the all-disposal strategy, the salvage value (g) is also significant at this level. The B_1 parameter is not significant under either strategy, while the decreasing requirements distribution is insignificant under the all-disposal strategy. Again, we wish to stress that concluding that a certain variable is not significant does not mean it is not important. Rather, it simply suggests that, given the range of values considered for that parameter, it did not contribute to an extensive portion of variability in percentage cost penalties.

Using *SAS/STAT®*, we can generate a normal probability plot to determine if the residuals from the myopic or all-disposal cases follow a normal distribution. These plots are illustrated in Figures 13 and 14. The "*" symbols correspond to the actual residual values in our data sets, while the "+" symbols are used to illustrate a straight (45°) line.

If the underlying residual distribution is normal, then the normal probability plots ought to resemble a straight line. The residuals from the all-disposal case (Figure 13) appear to bend down slightly on the left side and turn up slightly on the right side. According to Montgomery (1991), this would indicate that the left tail is somewhat thinner than would be expected in a normal distribution while the right tail is somewhat thicker. The negative residuals are not quite as large (in absolute value) as we would generally see in a normal distribution, while the positive residuals are larger than would usually be observed. The residuals from the myopic case (Figure 14) appear to bend down slightly on the left. Although each plot does display some irregularity, the distribution is reasonably close to normal. Moreover, the analysis of variance approach (used to obtain the F test statistics in Table 8) is only slightly affected by variations from the normality assumption of the residuals. Thus, since the analysis of variance approach is robust, we are confident in the statistical results obtained in our cases.

Although the results provided in Table 8 indicate the significance of main effects

Figure 13
Normal Probability Plot
All-Disposal Residuals

Variable=RESID

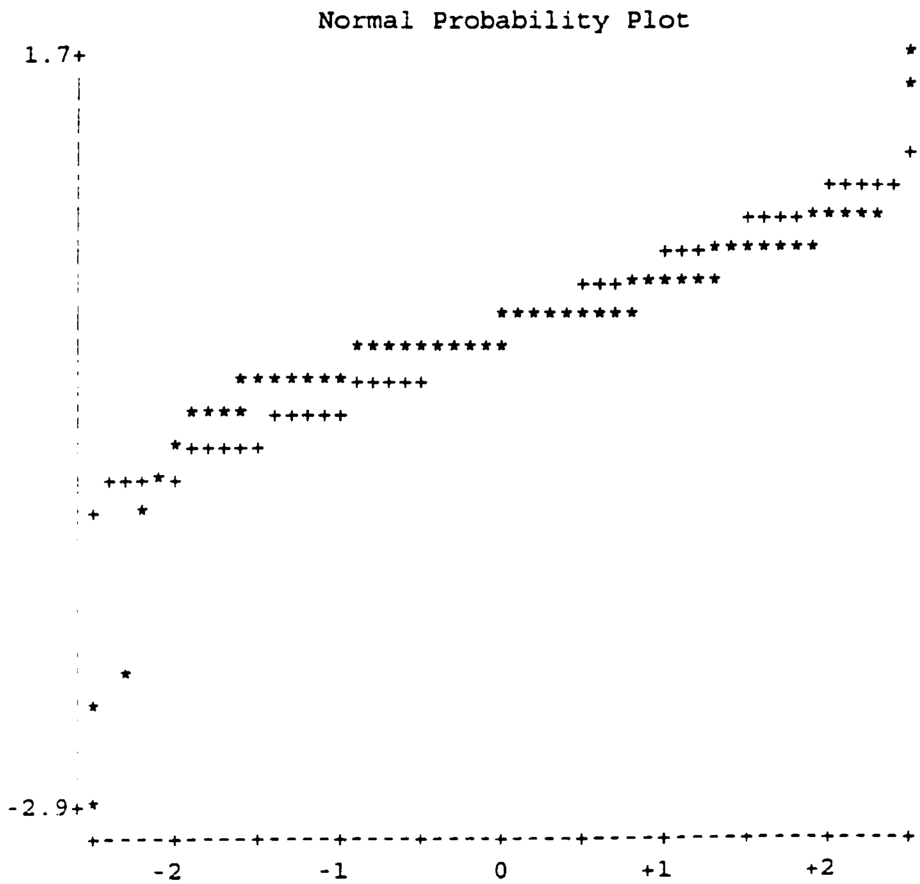


Figure 14
Normal Probability Plot
Myopic Residuals

```
Variable=RESID

                               Normal Probability Plot
             1.5+                                                  *
                |                                                +++
                |                                            +**** *
                |                                        ++***
                |                                     +++**
                |                                 ++++***
                |                             +++******
                |                         ********
                |               ***********
                |          ******  ++++
                |        **    ++++
                |      **   +++
                |       ++++
                |   ++++
                |+++
                |
                |    *
                |
                |
                |
                |
                |   **
                |* *
            -3.1+*
                 +----+----+----+----+----+----+----+----+----+----+
                     -2        -1         0        +1        +2
```

in our model, we were unable to use our analysis of variance approach to determine the significance of any two-way interaction effects (ie. the contribution to percentage cost penalty due to one parameter multiplied by another). Since we determined percentage cost penalties arising from only one-way and two-way tests, we did not have a large enough sample size to test all of the two-way interaction effects. Had we attempted to statistically test the interaction effects, we would not have had enough degrees of freedom to determine appropriate F tests.

We note, however, two ways of overcoming this problem (both with some drawbacks). Firstly, we could have used our analysis of variance approach to statistically test "some" of the interaction effects. However, this would have required *a priori* knowledge of the supposedly important interactions. If we knew ahead of time the significant interactions, then why would we require statistical tests? Further, we were rather reluctant to simply test some of the interactions, for this still could have missed important two-way interaction effects. Our second approach would have involved determining percentage cost penalties arising from three-way tests. This would have offered a large enough sample size to test two-way interactions. However, as indicated earlier, the issue of computational time, coupled with the complexity involved with interpreting results when several parameters are simultaneously varied, precluded us from using this approach.

Nonetheless, there is a valid approach that one can use to obtain the significance of two-way interaction effects. As long as we have three values for each parameter, we note that a quadratic function (using the Response Surface Regression procedure available on *SAS/STAT*®) could be fit to this data. Our model analysis has used three values (high, middle and low) for each of the following parameters: B_1, B_2, $h_{:}a$, T_c, A, D_o, v_o and g. The only model parameters that cannot be used are the respective D_c distributions. These parameters are actually "qualitative" independent variables, for we used a 1 to indicate the presence of a particular requirements distribution, 0 otherwise. As a result, choosing to fit a quadratic function to this data involves eliminating any data points obtained when the D_c distribution was varied from the "base case" scenario. This reduces our sample size from 239 observations to 163.

Our response surface regression results are provided in Table 9.

Table 9
Response Surface Regression Results

Inventory Management Strategy			
All-Disposal		Myopic	
1%	5%	1%	5%
B_2*B_2	B_2*g	B_1*B_2	
B_2*h	$h*a$	B_2*B_2	
B_2*T_c	$h*T_c$	B_2*h	
B_2*D_o	$h*g$	B_2*a	
B_2*v_o	T_c*g	B_2*T_c	
$h*h$	D_o*D_o		
$h*D_o$	$a*v_o$		
$h*v_o$	v_o*v_o		
T_c	$g*g$		
T_c*T_c			
T_c*D_o			
T_c*v_o			
D_o*v_o			

Table 9 illustrates that there are several significant two-way interactions and quadratic effects (ie. a parameter multiplied by itself). For the all-disposal case, it would appear that a handful of the parameters (B_2, h, T_c, D_o and v_o) have extensive interaction as well as quadratic effects. An examination of our percentage cost penalty results in Table 6 shows that various settings of these five particular parameters contributed to relatively large percentage cost penalties (the other four parameters led to smaller percentage cost penalties). Consequently, these five parameters can be deemed rather important in determining the penalty of adopting an all-disposal versus integrated inventory management strategy. Interactions between these parameters (e.g. high value of B_2 and a low value of T_c) would result in even larger penalties (as shown in Appendix H). We note that the only parameters to not appear significant at either level were B_1 and A.

For the myopic case, we note the substantial interaction involving the B_2 parameter. This is rather understandable since, as shown in Appendix H, varying other parameters with B_2 played a significant role in changing the value of Q_m^* and the concomitant cost penalties. (We note that there were no effects significant at just the 5% level).

We shall limit our two-way tests to the case of constant salvage values. We are confident that the set of significant model parameters observed in this section would not change substantially under new scenarios. As shall be shown later, incorporating non-constant salvage value functions does not drastically alter the percentage cost penalties from those observed with constant salvage values.

6.2 Marginally Decreasing Salvage Values

We now consider the optimal procurement quantities in the construction phase, as well as percentage cost penalties, obtained when salvage values are marginally decreasing. Table 10 illustrates the various marginally decreasing salvage functions included in our analysis (for comparison, recall that under the case of constant salvage values, we used g values of 20, 35 and 50).

Table 10
Marginally Decreasing Salvage Value Cases

Case 1		Case 2		Case 3	
g_i	N_i	g_i	N_i	g_i	N_i
50	40	55	50	50	30
35	30	50	40	20	70
20	40	25	40	10	40
1	Infinite	1	Infinite	1	Infinite

The infinite value in the N_i column for the lowest salvage value indicates that we receive \$1 per unit for any disposals made beyond $N_1 + N_2 + N_3$ total disposals.

Model results are provided in Table 11 (see pages 128-130). All treatment combinations, except the final two, use "Case 1" for the salvage value function.

Table 11
Results - Marginally Decreasing Salvage Values

Treatment Combination	**Integrated (exact)**	**Integrated (approx.)**	**All-disposal**	**Myopic**
Intermediate	500	500	400 (1.53)	400 (1.53)
B_1 low	462	458 (0.004)	400 (1.37)	400 (1.37)
B_1 high	500	500	400 (1.75)	400 (1.75)
B_2 low	435	432 (0.004)	400 (0.57)	300 (4.23)
B_2 high	500	500	400 (3.17)	400 (3.17)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
h low	500	500	400 (2.36)	400 (2.36)
h high	452	448 (0.01)	400 (1.01)	400 (1.01)
a low	500	500	400 (1.97)	400 (1.97)
a high	453	448 (0.01)	400 (1.17)	400 (1.17)
T_c low	500	500	400 (2.87)	400 (2.87)
T_c high	441	434 (0.02)	400 (0.71)	400 (0.71)
D_c incr.	600	600	500 (0.75)	500 (0.75)
D_c decr.	400	400	300 (2.46)	300 (2.46)
D_c level	500	500	400 (0.97)	400 (0.97)
D_c bi-modal	500	420 (0.05)	327 (2.08)	300 (3.19)
A low	500	451 (0.01)	400 (1.11)	400 (1.11)
A high	500	500	400 (1.84)	400 (1.84)
D_o low	430	427 (0.01)	400 (0.79)	400 (0.79)
D_o high	500	500	400 (2.59)	400 (2.59)
v_o low	444	439	400 (0.67)	400 (0.67)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
v_o high	500	500	400 (2.71)	400 (2.71)
g_i Case 2	500	500	400 (1.78)	400 (1.78)
g_i Case 3	500	500	400 (1.41)	400 (1.41)

Generally, these one-way results are quite similar to those we observed under the constant salvage value scenario. This would appear to suggest that, given the parameter ranges considered in our analysis, adopting a marginally decreasing salvage value function provides little difference in procurement decisions and cost penalties.

There is one interesting difference, however. Recall that Q_a^* ,faced with a bi-modal requirements distribution and constant salvage values, was 400 (see Table 6). Now, the similar scenario with marginally decreasing salvage values yields an optimal procurement decision of 327 units. With the bi-modal requirement distribution, there is a relatively high likelihood of observing very low construction phase requirements. For moderate procurement quantities, this would result in a substantial quantity of surplus stock. When larger disposal quantities earn less revenue per unit (as is the case with marginally decreasing salvage values), it becomes less attractive to over-procure during project construction when one is assuming that all remaining items will be disposed. This reduces the optimal procurement quantity.

6.3 Increasing Salvage Values

Finally, we consider model results for the case of increasing salvage values. Table 12 illustrates the various increasing salvage functions analyzed (again, recall that for the case of constant salvage values, the settings of g were 20, 35 and 50).

Table 12
Increasing Salvage Value Cases

Case 1				Case 2				Case 3			
g_i	L_i	U_i	N_i	g_i	L_i	U_i	N_i	g_i	L_i	U_i	N_i
5	1	10		3	1	20		25	1	100	
20	11	30		40	21	50		30	101	110	
25	31	60		45	51	100		40	111	140	
60	61	100		65	101	140		60	141	170	
40	101	130	30	35	141	200	60	40	171	220	50
20	131	170	40	25	201	240	40	30	221	250	30
10	171	200	30	20	241	260	20	15	251	290	40
1	201		∞	1	261		∞	1	291		∞

The model results are given in Table 13 (see pages 132-133). All treatment combinations, except the final two, use "Case 1" for the salvage value function.

Table 13
Results - Increasing Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	500	500	400 (1.99)	400 (1.99)
B_1 low	500	500	400 (1.77)	400 (1.77)
B_1 high	500	500	400 (2.21)	400 (2.21)
B_2 low	437	434 (0.01)	400 (0.67)	300 (4.45)
B_2 high	500	500	400 (3.63)	400 (3.63)
h low	500	500	400 (2.74)	400 (2.74)
h high	500	459 (0.07)	400 (1.30)	400 (1.30)
α low	500	500	400 (2.34)	400 (2.34)
α high	500	500	400 (1.55)	400 (1.55)
T_c low	500	500	400 (3.36)	400 (3.36)
T_c high	442	437 (0.01)	400 (0.81)	400 (0.81)
D_c incr.	600	600	500 (1.30)	500 (1.30)
D_c decr.	403	400 (0.004)	300 (2.88)	300 (2.88)
D_c level	500	500	400 (1.63)	400 (1.63)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
D_c bi-modal	500	500	400 (1.03)	300 (4.38)
A low	500	500	400 (1.60)	400 (1.60)
A high	500	500	400 (2.27)	400 (2.27)
D_o low	437	434 (0.01)	400 (1.24)	400 (1.24)
D_o high	500	500	400 (2.75)	400 (2.75)
v_o low	455	451 (0.01)	400 (0.87)	400 (0.87)
v_o high	502	501 (0.0001)	400 (3.08)	400 (3.08)
g_i Case 2	500	500	401 (2.16)	400 (2.21)
g_i Case 3	503	502 (0.001)	401 (1.89)	400 (1.95)

We observe that percentage cost penalties are somewhat higher than those previously provided for constant, or marginally decreasing salvage values. Increasing salvage values may not have a substantial effect on procurement quantities, but they do increase the penalty of following a non-integrated inventory management strategy. If surplus units can be either disposed for increasing (attractive) salvage values or be used to satisfy ongoing usage, the impact of solely considering the construction phase, or restricting one's post-project decision to disposal only, becomes more severe.

There are, however, some optimal procurement quantities which are somewhat larger under increasing salvage values. Recall that results for constant, marginally decreasing and increasing salvage values are provided in Tables 6, 11 and 13, respectively (also, remember that a list of tables is provided on pages x and xi of the dissertation). Note that when B_l is at its low value, the Q_c^* was 459 (constant salvage values), 462 (marginally decreasing) and 500 (increasing). For the high setting of α, the Q_c^* was 450 (constant salvage values), 453 (marginally decreasing) and 500 (increasing). In these cases, non-constant salvage values tend to increase Q_c^* since relatively large disposal quantities generate larger total salvage revenues than those that could be earned with constant salvage values. For example, with increasing salvage values, one has to dispose at least 61 units (but no more than 100) to earn a salvage value of $60 per unit. This attractive disposal opportunity provides the incentive to procure extra units. Note that, with the low value of B_l or the high setting of α, the desire to over-procure to protect against construction phase stockouts or to provide for ongoing operational usage, is somewhat eroded. It is the (initially) higher disposal revenue that makes it worthwhile to over-procure.

Moreover, note Q_a^* for Cases 2 and 3 (respective increasing salvage value functions). These decisions are both 401. This quantity does not correspond to a specific requirements value (as most of the non-integrated strategies do), but rather a "breakpoint" in the salvage value function. In both of these functions, there is an L_i (on the "increasing" side) equal to 101. If one wants to obtain the salvage values associated with L_i equal to 101, then one would need to procure exactly 101 units above a requirements value. This leads to the procurement decision of 401 units.

7. FUTURE PROJECTS ANALYSIS

Thus far, we have examined the procurement of an item to satisfy requirements during a project's construction phase. Surplus units on-hand at the conclusion of construction may be disposed, or used to satisfy ongoing operational usage. As it stands, the only source of requirements for this item in the future consists of this ongoing usage.

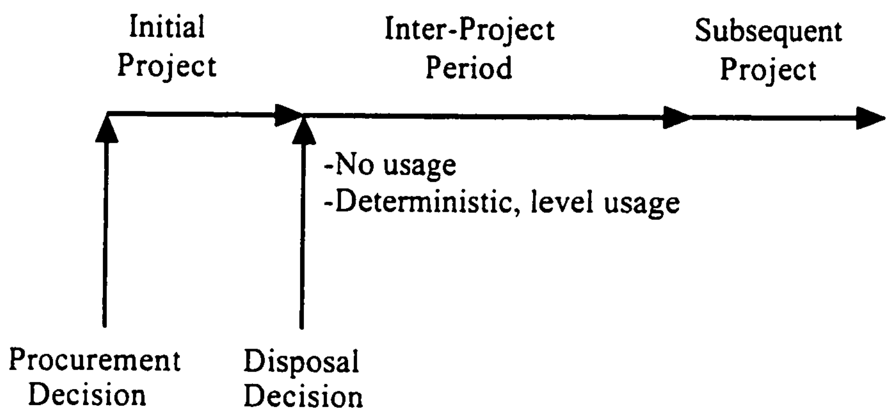
However, companies would hope to be around long enough for more than just one project! Suppose one considers the impact of a subsequent large-scale project, occurring at some random time in the future. What effect does the presence of this next project have on the procurement decision in the initial project? How are disposal decisions at the conclusion of the first project affected? Presumably, if unit acquisition costs in the subsequent project rise substantially from their levels in the first project, then one may find it advantageous to procure more (and/or dispose less) in the initial project, assuming that the same item is used in the subsequent project. Figure 15 illustrates a timeline for this decision-making situation.

We shall model two scenarios of inter-project usage: no usage, or deterministic-level, (ongoing) usage. We note that suggesting there is no usage during the inter-project period resembles the situation involved in the procurement of pipe for large pipeline projects. In this context, pipe is not retained for ongoing "spare parts" usage. Its only source of usage is during large-scale projects.

We shall begin by examining the (less complex) case of no inter-project usage.

Figure 15

Timeline of Decision Environment - Future Projects



Then, we shall examine the case of deterministic, level ongoing usage. Within each section, we shall treat the three respective salvage value functions (constant, marginally decreasing and increasing).

7.1 No Inter-Project Usage

After initial project completion, there is a random time until the subsequent project. This random time follows a discrete probability distribution. We introduce the following notation:

- t_i: the end-point of time interval i associated with a subsequent project ($i = 1,2,\ldots,n$)
- i: a particular time interval
- p_i: probability of a subsequent project beginning within $[t_{i-1}, t_i]$

Common probability rules maintain that each p_i is non-negative. However, we note that the sum of the respective probabilities over all time intervals does not necessarily have to equal one (a sum less than one would indicate, with a certain likelihood, the case of no future projects). We assume that the subsequent project is equally likely to occur any time within $[t_{i-1}, t_i]$ and that $t_o = 0$.

As an example, suppose we had three potential time intervals (in years) for the subsequent project with t_1, t_2 and t_3 values of 1.0, 2.5 and 3.5. Suppose further that the respective probabilities were 0.6, 0.25 and 0.15. This probability and time interval combination suggests that there is a 60% chance of a subsequent project beginning at any time within 1.0 years after initial project completion, a 25% likelihood that a subsequent project will begin between 1.0 and 2.5 years after the initial project and a 15% chance that a subsequent project will commence between 2.5 and 3.5 years in the future.

Surplus stock may be retained after completion of the initial project in the hope of satisfying part of the requirements of the subsequent project. Maintaining available on-hand stock, then, means that those specific units will not need to be procured during the future project. This represents a cost savings. However, one must pay holding charges to carry these units in inventory. As we have done in earlier chapters of this dissertation, we shall use continuous discounting to determine these holding costs. Let us introduce some further notation:

v_s: Unit acquisition cost in a subsequent project

Due to varying market conditions, this future unit procurement cost may be greater than, the same as or less than the unit cost in the initial project. (However, it will most certainly be lower than v_o, the applicable unit price should one be required to replenish for any ongoing usage).

The present value of the inter-project costs for each surplus unit retained may be modelled as:

$$\int_0^{t_{max}} f_i(t_o)\left[R(t_o)\right]dt_o \tag{7.1}$$

where t_{max} represents the largest t_i value under consideration and:

$$R(t_o) = \int_0^{t_o} h\ e^{-\alpha z} dz - v_s e^{-\alpha t_o} \tag{7.2}$$

and, for $t_{i-1} \leq t_o \leq t_i$,

$$f_t(t_o) = \frac{p_i}{t_i - t_{i-1}}$$

Evaluation of (7.2) gives (recall that $h' = h/\alpha$):

$$h'(1 - e^{-\alpha t_o}) - v_s e^{-\alpha t_o} \tag{7.3}$$

Since we are dealing with a discrete probability distribution, we may rewrite (7.1) as:

$$\sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} R(t_o) \frac{p_i}{t_i - t_{i-1}} \, dt_o$$

which is:

$$\sum_{i=1}^{n} \frac{p_i}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} R(t_o) \, dt_o \tag{7.4}$$

Substituting (7.3) into (7.4) gives:

$$\sum_{i=1}^{n} \frac{p_i}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \left[h'(1 - e^{-\alpha t_o}) - v_s e^{-\alpha t_o} \right] dt_o \tag{7.5}$$

Evaluation of the integral in (7.5) gives:

$$h' t_o \Big|_{t_{i-1}}^{t_i} - \int_{t_{i-1}}^{t_i} \left(h' e^{-\alpha t_o} + v_s e^{-\alpha t_o} \right) dt_o$$

which can be expressed as:

$$h'(t_i - t_{i-1}) - \left(\frac{h' + v_s}{\alpha} \right) \left[e^{-\alpha t_{i-1}} - e^{-\alpha t_i} \right] \tag{7.6}$$

Thus, the expected present value of the inter-project costs per surplus unit retained ($EIPC$) becomes:

$$EIPC = \sum_{i=1}^{n} \frac{p_i}{t_i - t_{i-1}} \left(h'(t_i - t_{i-1}) - \left(\frac{h' + v_s}{\alpha} \right) \left[e^{-\alpha t_{i-1}} - e^{-\alpha t_i} \right] \right) \tag{7.7}$$

Note that the unit inter-project costs are linear with respect to the quantity of retained stock. In essence, they are independent of the on-hand surplus after initial project completion (assuming that all units retained are needed in the subsequent project, a rather reasonable assumption).

Numerical Example:

Consider the following parameter values:

$p_1 = 0.05$ $\quad t_1 = 1.0$ years
$p_2 = 0.05$ $\quad t_2 = 3.0$ years
$p_3 = 0.90$ $\quad t_3 = 4.5$ years
$h = \$13$ per unit of inventory per year
$v_s = \$100$
$\alpha = 0.10$

Using equation (7.7), we determine that $EIPC = -32.777$.

We note that, for realistic settings of our parameters, $EIPC$ will have a negative value (ie. ignoring any immediate salvage value, it is attractive to retain surplus items). Observe, however, that if the unit price in the subsequent project is quite low, then the $EIPC$ value could be positive.

7.1.1 Constant Salvage Values

Disposing surplus stock upon completion of the initial project generates immediate revenues. Since we have constant marginal inter-project costs and disposal revenues, the determination of optimal disposal quantities upon completion of the initial project is quite straight-forward. We simply compare the associated costs and revenues in the following fashion:

- If $EIPC + g \leq 0$ (this implies that $EIPC \leq -g$), then $W^* = 0$. No surplus stock is disposed (equivalently, all units are retained) since the benefit (cost reduction) of retention is higher than the benefit of disposal.
- If $EIPC + g > 0$ (this implies that $EIPC > -g$) then $W^* = I$. All surplus stock is disposed (equivalently, no units are retained) since the benefit (cost reduction) of retention is less than the benefit of disposal.

Figure 16 illustrates this decision rule. The numerical example provided earlier gave a result of -32.777 for EIPC. Consequently, we would need to obtain a unit salvage value of more than \$32.777 in order to make disposal attractive. If our salvage values were below \$32.777, then retention of surplus stock would prove the best choice.

The disposal choice after the initial project is, essentially, an "all-or-nothing" decision. We either dispose everything on-hand, or retain all of it to satisfy future project requirements. One could suggest that, if one were indifferent between surplus stock retention or disposal (ie. $EIPC = -g$), then it could be attractive to dispose a portion of the on-hand surplus. However, in this extreme case, observe that total costs would be equivalent under any disposal strategy. The key finding is that an organization could never be better off by disposing a portion of the on-hand surplus.

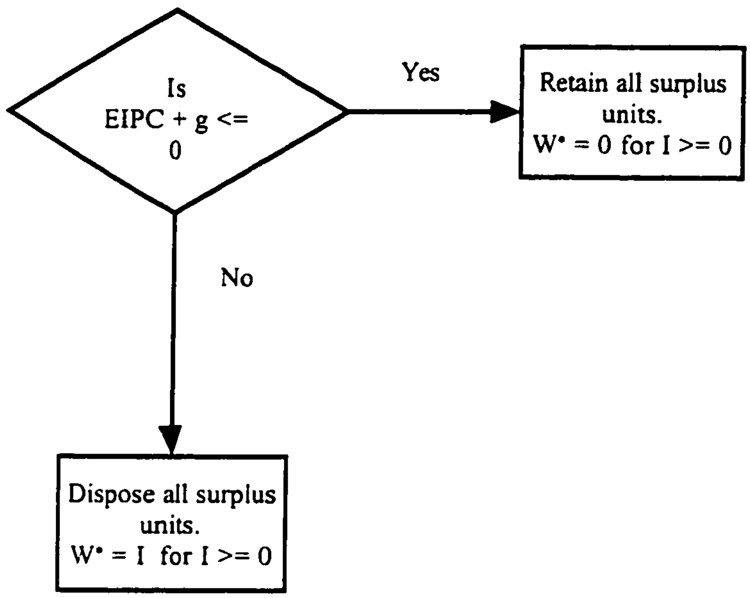
Since disposal decisions are independent of on-hand stock, the $EPV^*(I)$ values are

Figure 16

Optimal Disposal Decision Rules

Future projects, No inter-project usage

Constant salvage values



either:

- $EIPC * I$, if $W^* = 0$ (all surplus stock is retained)
- $-g * I$, if $W^* = I$ (all surplus stock is disposed)

As we did in Chapter 6, we shall calculate the percentage cost penalties of following non-integrated inventory management strategies. We shall use the same values for construction phase parameters as our previous analysis (see Tables 3 and 4). However, since we do not have any ongoing usage, the parameters A, D_o and v_o will not be used in this section.

Table 14 lists the respective inter-project period probability distributions used in our analysis. Table 15 gives the range of values for the parameter v_s, the unit acquisition cost in a subsequent project.

Table 14
Inter-Project Period Probability Distributions

Case 1		Case 2		Case 3	
p_i	t_i	p_i	t_i	p_i	t_i
0.6	1	0.2	0.5	0.05	1
0.3	2	0.6	3	0.05	3
0.1	3.5	0.2	4	0.9	4.5

Table 15
Range of Unit Acquisition Costs in a Subsequent Project

Parameter	Low Value	Middle Value	High Value
v_s	$80	$100 (equal to v_c)	$120

Table 16 (see pages 144-145) provides the procurement quantities, selected by the various approaches, in the initial project's construction phase, as well as any applicable percentage cost penalties. All treatment combinations, unless otherwise indicated, use "Case 1" for the inter-project period probability distribution.

Table 16
Results - Future Projects, No Inter-project Usage, Constant Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	400	400	400	400
B_1 low	400	400	400	400
B_1 high	500	400 (0.23)	400 (0.23)	400 (0.23)
B_2 low	400	400	400	300 (4.59)
B_2 high	500	500	400 (2.72)	400 (2.72)
h low	500	500	400 (0.80)	400 (0.80)
h high	400	400	400	400
a low	500	500	400 (0.42)	400 (0.42)
a high	400	400	400	400
T_c low	500	500	400 (2.12)	400 (2.12)
T_c high	400	400	400	400
D_c incr.	600	600	500 (1.20)	500 (1.20)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
D_c decr.	400	400	300 (2.75)	300 (2.75)
D_c level	500	500	400 (2.33)	400 (2.33)
D_c bi-modal	600	600	400 (5.23)	300 (10.74)
g low	400	400	400	400
g high	400	400	400	400
t_i Case 2	400	400	400	400
t_i Case 3	400	400	400	400
v_s low	400	400	400	400
v_s high	500	500	400 (2.31)	400 (2.31)

One of the main conclusions of this analysis is that the desire to over-procure in the initial project's construction phase has been somewhat dampened. For several of the scenarios, adopting a non-integrated inventory management strategy yields identical optimal procurement decisions to those given under an integrated approach. The percentage cost penalties for these scenarios are zero. When the only source of item usage comes from large-scale project requirements, simply considering the needs of one's initial project may represent an attractive policy.

However, there are instances in which considering future projects does lead to differences in the procurement decisions produced by the integrated and non-integrated strategies. We note that the following scenarios: B_1 high, B_2 high, h low, α low, T_c low and v_s high each lead to Q_c^*'s of 500 units. Intuitively, this makes sense for each of these scenarios suggests a greater attractiveness in over-procuring during the initial project.

Observe also that the integrated optimal procurement quantity under a bi-modal requirements distribution is rather large (600 units). When we have future projects and no ongoing usage, there is a considerable constant marginal benefit to retaining surplus stock (recall that under the cases presented in Chapter 6, the marginal benefit of retaining extra units decreased as one retained more and more units). Given that the bi-modal distribution provides a good chance of observing low construction phase requirements, it is not surprising that the optimal procurement quantity is rather large. Procuring a large quantity in the construction phase would yield a fair amount of excess stock, should the lowest requirements value be observed. Since the bi-modal distribution could also produce relatively high requirements, over-procurement is further warranted due to its ability to hedge against costly construction phase stockouts. Note that following either non-integrated approach leads to fairly large cost penalties.

7.1.2 Marginally Decreasing Salvage Values

Recall that with constant salvage values, the disposal choice after the initial project was an "all-or-nothing" decision. With marginally decreasing salvage values, the "all-or-nothing" disposal decision is now made at each of the respective salvage values. Consequently, we ought to investigate each particular salvage value to determine if disposal is warranted.

However, as Figure 17 shows, we can take advantage of a simpler process. We

Figure 17
Optimal Disposal Decision Rules
Future projects, No inter-project usage
Marginally decreasing salvage values

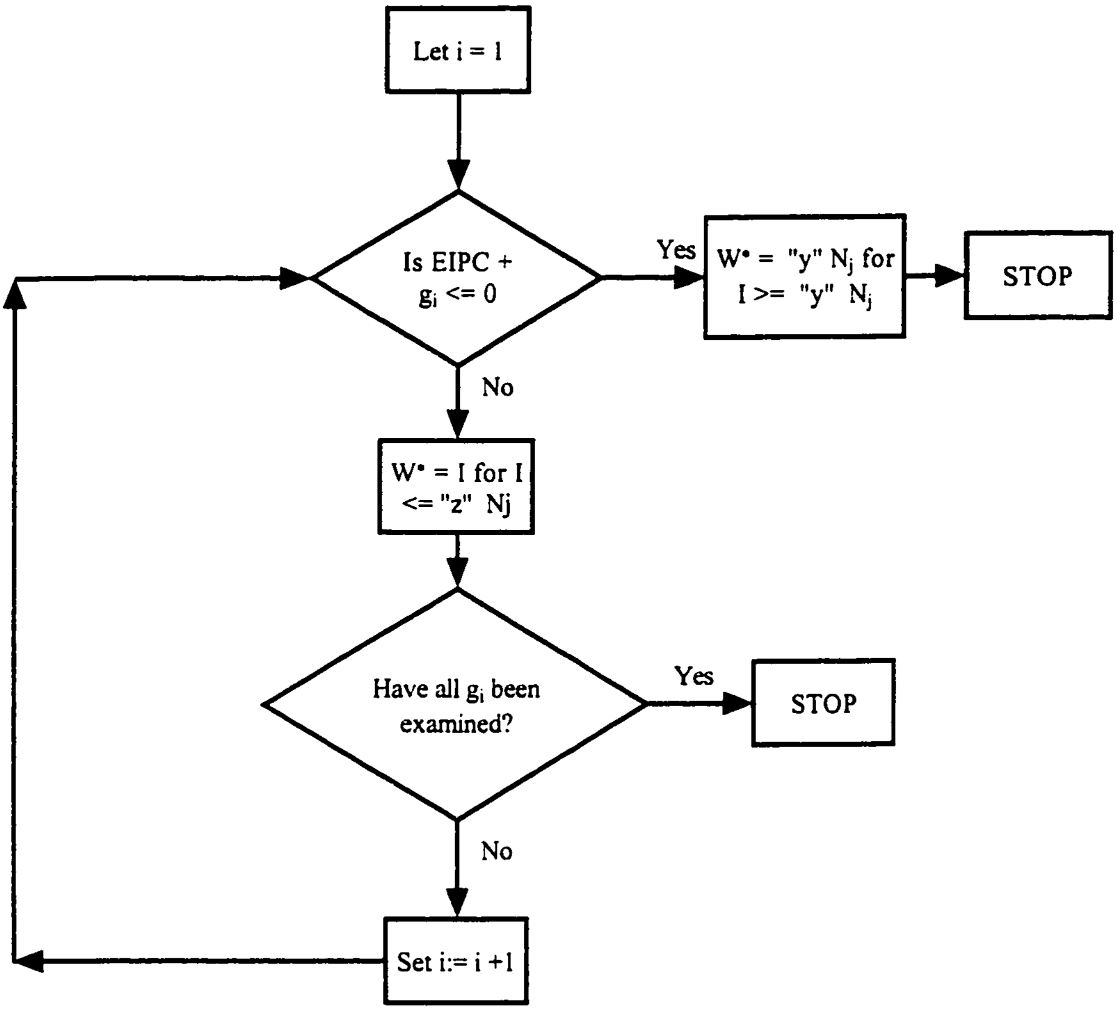
Note the symbols ("y" and "z") used to represent summation terms

Define:
$N_0 = 0$

$$\sum_{j<1} = 0$$

$$y = \sum_{j<i}$$

$$z = \sum_{j \le i}$$



know that if disposal is not warranted for a certain salvage value, it will never be attractive to dispose for lower salvage values. As a result, we begin our determination of optimal disposal decisions by examining the first (largest) salvage value. We compute the quantity $EIPC + g_i$. If this value is ≤ 0, then we would never find it attractive to dispose surplus units, whatever the salvage value. Thus, we would have $W^* = 0$ for all quantities of surplus stock. The key conclusion is that whenever we obtain a value of $EIPC + g_i \leq 0$, our procedure can stop. Should it be beneficial to dispose surplus stock at a particular unit salvage value, then we will continue to dispose until at least the point where the surplus equals the maximum disposal quantity at that salvage value. In the extreme case wherein it is attractive to dispose stock at the lowest salvage value (g_i = \$1 per unit), then $W^* = I$ for all quantities of surplus stock.

The values of $EPV^*(I)$ are determined in a straight-forward manner. For any quantity of surplus stock, we obtain the total salvage revenue (if any) and add this to $EIPC\ (I - W^*)$.

The results for the case of marginally decreasing salvage values are given in Table 17 (see pages 149-150). All treatment combinations, unless otherwise indicated, use "Case 1" for the salvage value (see Table 10) and inter-project probability distributions (see Table 14).

Table 17
Results - Future Projects, No Inter-Project Usage, Marginally Decreasing Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	400	400	400	400
B_1 low	400	400	400	400
B_1 high	500	400 (0.23)	400 (0.23)	400 (0.23)
B_2 low	400	400	400	300 (4.59)
B_2 high	500	500	400 (2.72)	400 (2.72)
h low	500	500	400 (0.80)	400 (0.80)
h high	400	400	400	400
α low	500	500	400 (0.42)	400 (0.42)
α high	400	400	400	400
T_c low	500	500	400 (2.12)	400 (2.12)
T_c high	400	400	400	400
D_c incr.	600	600	500 (1.20)	500 (1.20)
D_c decr.	400	400	300 (2.75)	300 (2.75)
D_c level	500	500	400 (2.33)	400 (2.33)
D_c bi-modal	600	600	327 (9.25)	300 (10.74)
g_i Case 2	400	400	400	400

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
g_i Case 3	400	400	400	400
t_i Case 2	400	400	400	400
t_i Case 3	400	400	400	400
v_s low	400	400	400	400
v_s high	500	500	400 (2.31)	400 (2.31)

Generally, these one-way results are quite similar to those we observed under the constant salvage value scenario. This would appear to suggest that, given the parameter ranges considered in our analysis, adopting a marginally decreasing salvage value function provides little difference in procurement decisions and cost penalties.

There is one interesting difference, however. Recall that Q_a^*, faced with a bi-modal requirements distribution and constant salvage values, was 400 (see Table 16). Now, the similar scenario with marginally decreasing salvage values yields an optimal procurement decision of 327 units. With the bi-modal requirement distribution, there is a relatively high likelihood of observing very low construction phase requirements. For moderate procurement quantities, this would result in a substantial quantity of surplus stock. When larger disposal quantities earn less revenue per unit (as is the case with marginally decreasing salvage values), it becomes less attractive to over-procure during project construction. This reduces the optimal procurement quantity.

7.1.3 Increasing Salvage Values

The process of finding optimal disposal quantities for the case of increasing salvage values is somewhat similar to the procedure adopted in the previous section. Figure 18 provides an illustration of the appropriate decision rules.

As we established in section 7.1.2, we know that if disposal is not warranted for a specific salvage value, it will never be attractive to dispose for lower salvage values. Consequently, we begin our determination of optimal disposal decisions by examining the largest increasing salvage value. We compute the quantity $EIPC + g_i$. If this value is ≤ 0, then we would never find it attractive to dispose surplus units, whatever the salvage value. Thus, we would have $W^* = 0$ for all quantities of surplus stock.

If it is beneficial to dispose surplus stock for a certain increasing salvage value, we will continue to dispose I units as long as $L_i \leq I \leq U_i$. As an example, suppose we found it worthwhile to dispose stock for the largest increasing salvage value, but for none of the others. In that case, we would have $W^* = 0$ for all levels of surplus stock until $I = L_m$ (where L_m represents the L_i value for the largest increasing salvage value). Then, we would have $W^* = I$ (dispose all stock) for $L_m \leq I \leq U_m$ (where U_m represents the U_i value for the largest increasing salvage value)

If we do find it attractive to dispose stock for at least one of the increasing salvage values, then we will need to examine the largest salvage value on the decreasing side. Suppose that, in the example under consideration, none of the decreasing salvage values were found to be sufficiently large to permit disposal. As a result, our W^* values would be $W^* = U_m$ for all $I \geq U_m$.

Figure 18
Optimal Disposal Decision Rules
Future Projects, No inter-project usage
Increasing Salvage Values

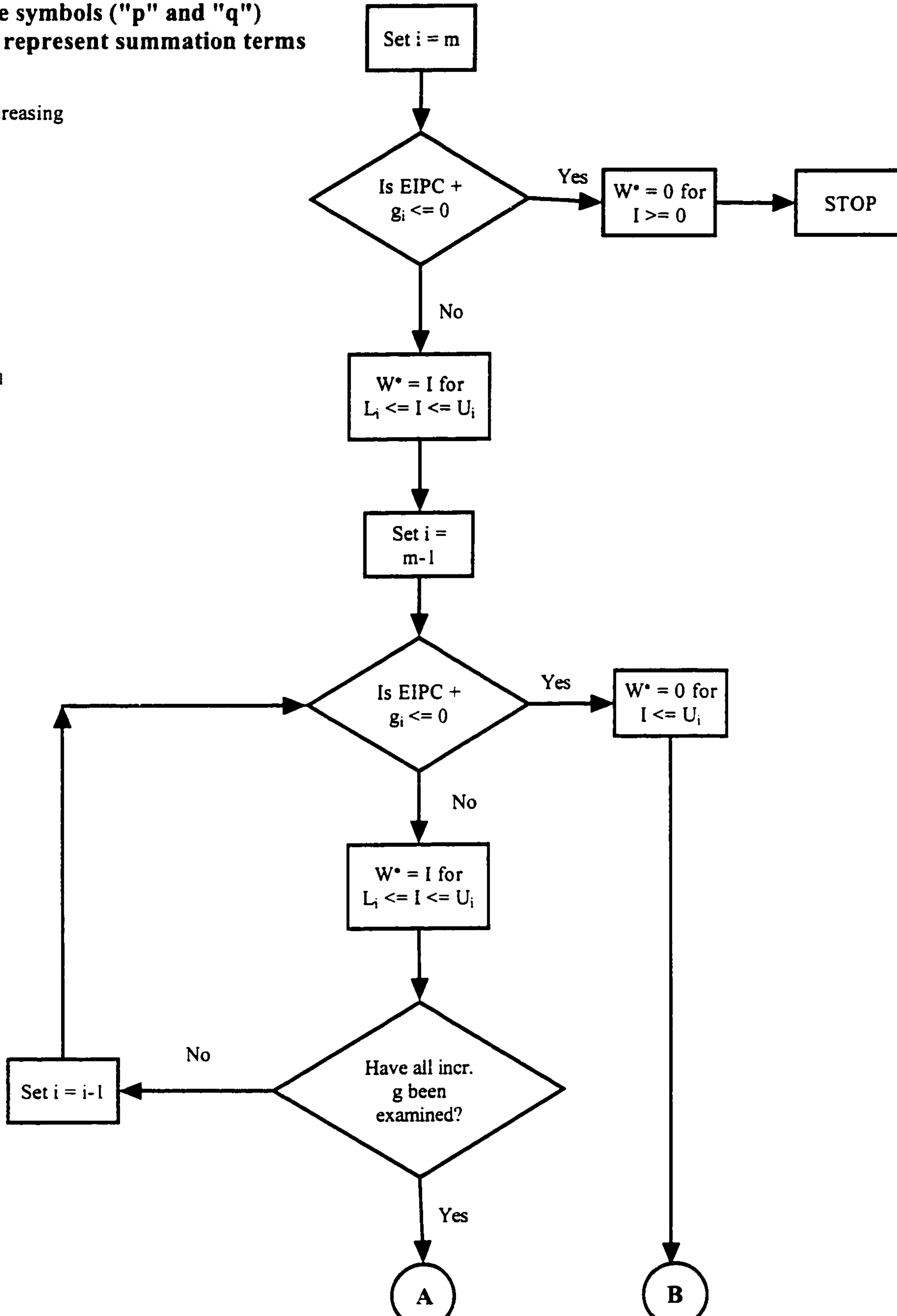
Note the symbols ("p" and "q") used to represent summation terms

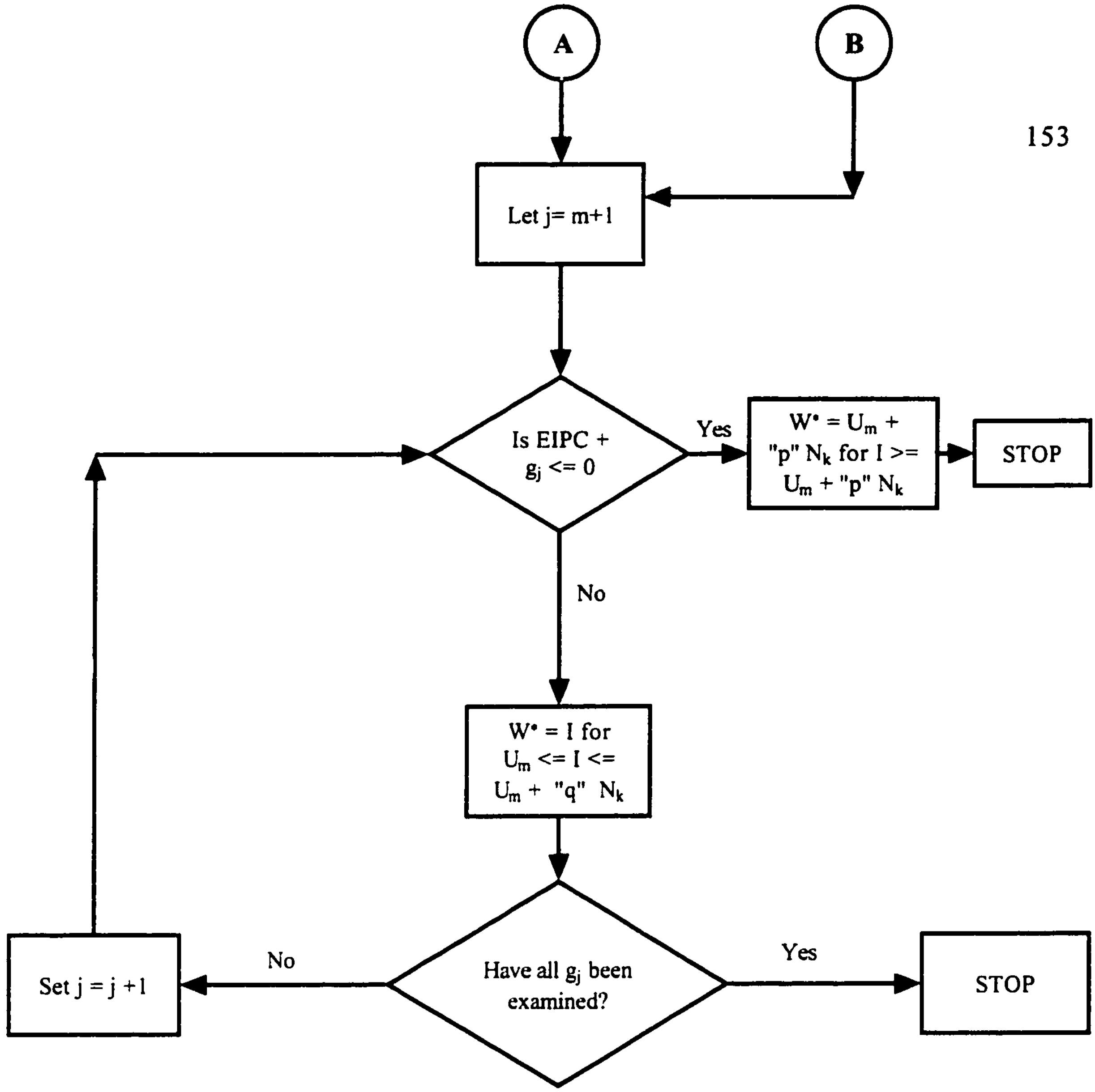
Define:
m = number of increasing salvage values

$$\sum_{k=m+1}^{m} = 0$$

$$p = \sum_{k=m+1}^{j-1}$$

$$q = \sum_{k=m+1}^{j}$$




A
B
Let j= m+1
Is EIPC + g_j <= 0
Yes
W* = U_m + "p" N_k for I >= U_m + "p" N_k
STOP
No
W* = I for U_m <= I <= U_m + "q" N_k
Have all g_j been examined?
No
Set j = j +1
Yes
STOP

As we found in the previous section, for cases in which it is attractive to dispose stock at the lowest salvage value, then $W^* = I$ for all quantities of surplus stock.

The values of $EPV^*(I)$ are determined in a straight-forward manner. For any quantity of surplus stock, we obtain the total salvage revenue (if any) and add this to $EIPC\,(I - W^*)$.

The results for the case of increasing salvage values are given in Table 18 (see pages 154-155). All treatment combinations, unless otherwise indicated, use "Case 1" for the salvage value and inter-project probability distributions. The increasing salvage value functions are included in Table 12, while Table 14 provides the inter-project probability distributions.

Table 18
Results - Future Projects, No Inter-project Usage
Increasing Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	400	400	400	400
B_1 low	400	400	400	400
B_1 high	500	400 (0.23)	400 (0.23)	400 (0.23)
B_2 low	400	400	400	300 (4.59)
B_2 high	500	500	400 (2.72)	400 (2.72)
h low	500	500	400 (0.80)	400 (0.80)
h high	400	400	400	400

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
α low	500	500	400 (0.42)	400 (0.42)
α high	400	400	400	400
T_c low	500	500	400 (2.12)	400 (2.12)
T_c high	400	400	400	400
D_c incr.	600	600	500 (1.20)	500 (1.20)
D_c decr.	400	400	300 (2.75)	300 (2.75)
D_c level	500	500	400 (2.33)	400 (2.33)
D_c bi-modal	600	600	400 (5.23)	300 (10.74)
g_i Case 2	400	400	401 (0.01)	400
g_i Case 3	400	400	401 (0.01)	400
t_i Case 2	400	400	400	400
t_i Case 3	400	400	400	400
v_s low	400	400	400	400
v_s high	500	500	400 (2.31)	400 (2.31)

Again, we observe similar results to those obtained under the constant salvage value scenario. However, note the behaviour of the various inventory management strategies for Cases 2 and 3 of the salvage value function. In these cases, we obtain a higher procurement quantity for the all-disposal case than we do for the integrated strategy. This marks the first time that a non-integrated approach has produced larger procurement quantities than those given under an integrated method. It would appear that when one is limited to the disposal of all surplus units, it can become attractive to "jump up" to a higher $D_c + L_i$ value. Note that both cases 2 and 3 have L_i values of 101 on the "increasing" side. When we consider future projects and no inter-project usage, we have often found it best (given the range of parameter values in our model) to retain all surplus units upon completion of the original construction phase. Consequently, there is no desire to "jump up" to a higher L_i when adopting an integrated strategy.

7.2 Deterministic, Level Ongoing Usage

We shall now extend our analysis to consider the case of deterministic, level ongoing usage of an item during the inter-project period. In this case, materials managers may retain surplus stock after completion of the initial project to satisfy two sources of future usage of an item; namely, operational ongoing usage as well as requirements in a subsequent project.

Recall that in the case of no inter-project usage, all of the stock retained (after any disposal decision) was available to meet requirements in a subsequent project. This resulted in a future cost savings since those specific units did not need to be procured during the future project.

When we introduce ongoing operational usage during the inter-project period, the quantity of stock on-hand when the next project occurs can still be transferred to the subsequent project. As before, this provides a cost savings. However, the specific level of on-hand stock will likely not be equal to the amount available at the start of the inter-project period. Operational usage runs down on-hand stock, while replenishments of size Q_o (made every Q_o/D_o time units) build it back up. We require the following notation:

I_s: (Approximate) on-hand inventory when a subsequent project occurs

As an aside, we note that procurement for ongoing operations can be affected by the subsequent project. When the next project occurs, materials managers are provided a special opportunity to procure items at v_s ($v_s < v_o$) to satisfy ongoing usage from the earlier project. Others who have examined special procurement opportunities include Whitin (1953), Burton and Morgan (1982), Lev and Soyster (1971) and Hall (1992). We wish to note an assumption that we make regarding this scenario. Faced with future projects, we realize that (in practice) materials managers may adjust their ongoing replenishment quantities, particularly if a subsequent project is imminent and a replenishment is required. Since regular replenishments cost v_o per unit, a materials manager may opt to bring in very little at the regular price, realizing that in a very short time, one will have the opportunity to replenish at reduced unit prices. However, we will ignore these adjustments of Q_o which may occur as a future project draws near. These are rather complicated effects. Besides, assuming that Q_o is established without consideration of future projects ought to have very little impact on the overall Q_c^* and W^* decisions.

In order to accurately calculate overall costs, we need to determine the quantity of on-hand stock when a subsequent project occurs. Obviously, I_s is affected by, among other factors, the number of units retained after the disposal decision (M), the annual usage rate in the ongoing phase (D_o), the optimal ongoing replenishment quantity (Q_o)

and the time until a subsequent project. However, determining exact on-hand stock levels for each and every potential moment at which a future project could occur, by using simulation, could prove extremely time-consuming. Due to this complexity, we shall develop a heuristic approach to approximate I_s (note that a general treatment of heuristic procedures is provided in Barr, Golden, Kelly, Resende and Stewart, Jr. (1995) as well as Silver, Vidal and de Werra (1980)). Further, we note that solely for purposes of determining the subsequent amount of time over which to discount (in order to calculate present values of various costs), we assume that the subsequent project occurs half-way within $[t_{i-1}, t_i]$.

Our heuristic approach is illustrated in Figure 19. Recall that in Chapter 4, we determined the present value of beginning the ongoing phase of a constructed facility with M units of on-hand inventory. Recall further that we had a "transition" period during which these M units were depleted. When these units were eventually used up by ongoing operations, we began making replenishments of size Q_o. Since ongoing usage was assumed to be constant and known, we showed (in Chapter 3) that materials managers could time their replenishments so as to arrive precisely when a unit was used in ongoing usage. Consequently, this meant that each replenishment cycle was Q_o-1 units high. These ongoing replenishments continued indefinitely.

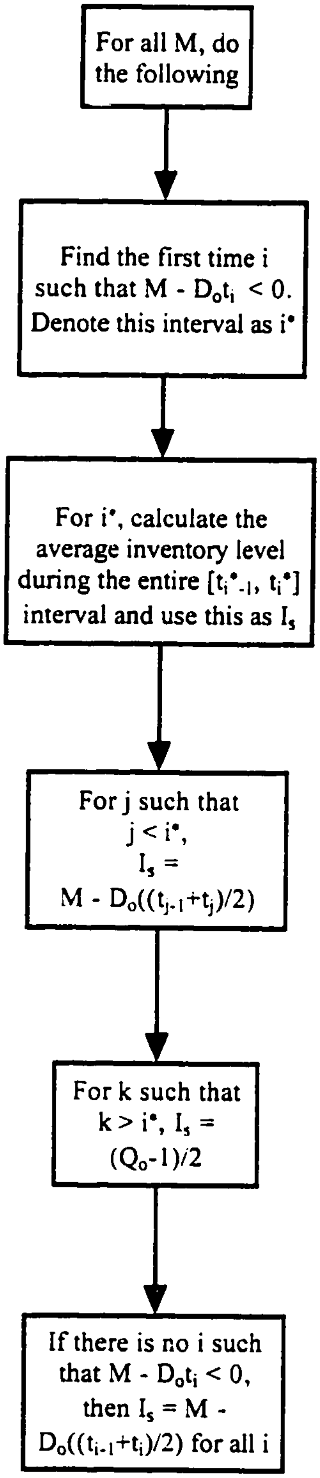
The heuristic approach we suggest for estimating I_s when a subsequent project occurs takes the above issues into consideration. In particular, we wish to carefully model inventory levels during the specific time interval in which we first enter ongoing phase replenishment cycles. It is during this time interval that the "transition" period

Figure 19

Heuristic for Approximating I_s

Define:

$t_0 = 0$



stock is depleted, and one begins replenishments of size Q_o. During this specific time interval, we shall explicitly calculate the average on-hand inventory level. However, if during all parts of a given time interval, one is in the transitional period of ongoing operations (still using on-hand stock retained after any disposal decision), or completely within the ongoing phase replenishment cycles, then we shall adopt a simpler approach for estimating I_s within that time interval.

The specific time interval i in which we first enter the ongoing phase replenishment cycles corresponds to the first t_i such that:

$$M - D_o t_i < 0 \tag{7.8}$$

If (7.8) is satisfied, then the M units of on-hand stock would have been used up by the end of this time interval, and one would have initiated the ongoing replenishment pattern. We shall call i^* the first time interval that satisfies (7.8).

We shall now describe our approach for calculating the average on-hand inventory level during this specific time interval. We recognize that on-hand stock during this time interval consists of three "types". First, we have "transitional" stock. This is stock that is used up prior to the initiation of ongoing replenishment cycles. Second, we have stock that is part of "complete" replenishment cycles. This would result if (7.8) were satisfied early enough within a time interval to allow stock from an ongoing replenishment cycle(s) to be completely used up within that time interval. Finally, we have inventory that is derived from an "incomplete" cycle. This results when a replenishment is made, but not all of the units in this cycle are depleted before the end of the time interval. For computational ease in this heuristic, we ignore the step-wise ongoing usage pattern provided in Chapter 3. We will assume that we have "linear" (ie. continuous) usage, thus creating a series of traditional "sawtooth" patterns. Further, there are Q_o units on-hand at the top of each of these replenishment cycles. For purposes of this heuristic, these are relatively minor modifications.

For the "transitional" inventory, we recognize that the total number of units on-hand at the beginning of time interval i are (with $t_o = 0$):

$$M - D_o t_{i-1}$$

Since, by definition of i^*, the "transitional" stock is completely used up within this time interval, the average number of units on-hand during the transition period is:

$$\frac{M - D_o t_{i-1}}{2} \tag{7.9}$$

These units will last for the following number of time units:

$$\frac{M - D_o t_{i-1}}{D_o}$$

or, for the following portion of time interval i:

$$\frac{M - D_o t_{i-1}}{D_o (t_i - t_{i-1})} \tag{7.10}$$

Combining (7.9) and (7.10) gives the average number of units on-hand during the time interval, resulting from "transitional" usage:

$$\left(\frac{M - D_o t_{i-1}}{2}\right)\left(\frac{M - D_o t_{i-1}}{D_o(t_i - t_{i-1})}\right) \tag{7.11}$$

We shall now calculate the average on-hand inventory level during the time interval resulting from complete cycles. Each cycle has an average inventory of:

$$\frac{Q_o}{2} \tag{7.12}$$

with each complete cycle lasting for the following number of time units:

$$\frac{Q_o}{D_o} \tag{7.13}$$

We need to determine the number of complete cycles that will occur within the specific time interval. The duration of the time interval is $t_i - t_{i-1}$ time units. The total time "used up" by the transitional stock was, as given earlier:

$$\frac{M - D_o t_{i-1}}{D_o}$$

Hence, the total time "available" for complete cycles is:

$$t_i - t_{i-1} - \left(\frac{M - D_o t_{i-1}}{D_o}\right) \tag{7.14}$$

Algebraic manipulation of (7.14) gives:

$$t_i - \frac{M}{D_o}$$

This expression makes intuitive sense for the value M/D_o gives the point in time at which the M units of retained stock would be depleted by ongoing usage. Since the end-point of this specific interval is t_i, then we know that there must be $t_i - M/D_o$ time units available for complete cycles.

Multiplying (7.14) by D_o provides the total usage during the time available for complete cycles. This is given as:

$$D_o t_i - M$$

Since each complete cycle comprises Q_o units, the number of complete cycles is:

$$\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor \tag{7.15}$$

where $\lfloor x \rfloor$ signifies the largest integer smaller than or equal to x.

Combining (7.13) and (7.15) gives the length of time covered by these complete cycles:

$$\frac{\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor Q_o}{D_o} \tag{7.16}$$

or, the following portion of time interval i:

$$\frac{\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor Q_o}{D_o (t_i - t_{i-1})} \tag{7.17}$$

Combining (7.12) and (7.17) gives the average number of units on-hand during the time interval, resulting from "complete" cycle usage:

$$\left(\frac{Q_o}{2}\right)\left(\frac{\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor Q_o}{D_o (t_i - t_{i-1})}\right) \tag{7.18}$$

When examining on-hand inventory due to an "incomplete" cycle, we must recognize that the number of units used in an incomplete cycle is:

$$MOD\left(\frac{D_o t_i - M}{Q_o}\right) \tag{7.19}$$

Since Q_o is the number of units at the top of an incomplete cycle, the following gives the number of units on-hand at the end of an incomplete cycle:

$$Q_o - MOD\left(\frac{D_o t_i - M}{Q_o}\right) \tag{7.20}$$

The average number of units on-hand during this incomplete cycle is:

$$\frac{Q_o + Q_o - MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{2}$$

which can be simplified as:

$$Q_o - \frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{2} \tag{7.21}$$

An incomplete cycle will last for the following number of time units:

$$\frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{D_o}$$

or, the following portion of time interval i:

$$\frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{D_o(t_i - t_{i-1})} \tag{7.22}$$

Combining (7.21) and (7.22) gives the average number of units on-hand during the time interval, resulting from "incomplete" cycle usage:

$$\left(Q_o - \frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{2}\right)\left(\frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{D_o(t_i - t_{i-1})}\right) \tag{7.23}$$

To calculate the average on-hand stock during the entire time interval i, we combine (7.11), (7.18) and (7.23). This results in the following expression:

$$\left(\frac{M - D_o t_{i-1}}{2}\right)\left(\frac{M - D_o t_{i-1}}{D_o(t_i - t_{i-1})}\right) + \left(\frac{Q_o}{2}\right)\left(\frac{\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor Q_o}{D_o(t_i - t_{i-1})}\right) +$$

$$\left(Q_o - \frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{2}\right)\left(\frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{D_o(t_i - t_{i-1})}\right) \tag{7.24}$$

We note that one can simplify (7.24) by taking $1/(t_i - t_{i-1})$ outside the expression. This gives:

$$\left(\frac{1}{t_i - t_{i-1}}\right)\left[\left(\frac{M - D_o t_{i-1}}{2}\right)\left(\frac{M - D_o t_{i-1}}{D_o}\right) + \left(\frac{Q_o}{2}\right)\left(\frac{\left\lfloor \frac{D_o t_i - M}{Q_o} \right\rfloor Q_o}{D_o}\right) + \right.$$

$$\left.\left(Q_o - \frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{2}\right)\left(\frac{MOD\left(\frac{D_o t_i - M}{Q_o}\right)}{D_o}\right)\right] \tag{7.25}$$

The above expression, then, calculates the average on-hand inventory (I_s) during the first time interval, i, such that the transitional stock was completely depleted and we entered the ongoing phase replenishment pattern. We note that continuous values of (7.25) are rounded to the closest integer.

The procedure we have described explicitly calculates on-hand stock levels during an important time interval. However, we still need to determine appropriate on-hand inventory levels during the other time intervals.

As shown in Figure 19, for any time interval j prior to i^*, we use the following expression to determine the average on-hand stock during this interval:

$$M - \frac{D_o(t_{j-1} + t_j)}{2} \tag{7.26}$$

Since we have yet to completely use up all of the "transitional" stock, determining the stock on-hand half-way through this time period ought to be a good estimator of I_s during this time interval.

For any time interval k subsequent to i^*, we use the following simple expression to estimate the average on-hand stock during this interval:

$$\frac{Q_o - 1}{2} \tag{7.27}$$

This expression recognizes that we would be completely within ongoing phase replenishment cycles during <u>all</u> portions of time interval k. The average on-hand stock during a single ongoing replenishment cycle ought to closely approximate the average on-hand stock during the entire time interval. We note that if (7.27) provides a continuous value, we round the value to the closest integer.

In our heuristic procedure, one unifying thread throughout all the time intervals concerns the timing of the subsequent project. We are assuming that the project occurs when the inventory is at its average level during a time interval. This average level is computed in an exact fashion for any time intervals $j \leq i^*$ while it is computed approximately for time intervals $k > i^*$. However, note that this is not equivalent to suggesting that the project occurs at the midpoint of an interval. As explained earlier, the midpoint assumption is only used when determining the duration of time over which we discount in order to calculate present values.

Numerical Example:

As a numerical example, consider the following parameter values:

$t_o = 0$
$t_1 = 1.0$
$t_2 = 2.0$
$t_3 = 3.5$
$Q_o = 17$ units
$D_o = 20$ units per year

Suppose we wish to determine the on-hand inventory levels for all three time intervals for $M = 21$.

We recognize that $M - D_o t_i$ will first be negative during the $[t_1, t_2]$ time interval, ie. $i^* = 2$. This represents the first time interval in which one begins ongoing phase replenishments.

During the $[t_0, t_1]$ time interval, we are still fully within the "transitional" period. As a result, using (7.26) provides an average on-hand stock level during this interval of 11 units.

We begin the $[t_1, t_2]$ time interval with one unit of inventory in "transitional" stock (since $M - D_o t_1 = 1$) lasting for 1/20 time units. We also have one complete replenishment cycle lasting for 17/20 time units and 15 units remaining at the end of an incomplete cycle (the incomplete cycle lasts for 2/20 time units). Using (7.25), we determine a value of 8.85 for average on-hand stock during this time interval, which is rounded up to 9.

The remaining time interval ($[t_2, t_3]$) consists completely of ongoing replenishment cycles. We estimate the average on-hand inventory during this time interval as 8 units (ie. $(Q_o-1)/2$).

After estimating the I_s levels for each of the possible time intervals until a subsequent project, our next step is to determine the costs of beginning the ongoing phase with M units of stock. This is denoted by $PV(M)$. Recall that in Chapter 4, we determined such an expression. However, that expression did not recognize the possibility of subsequent projects at random points in time. For ease of reference, we include the expression here (it appeared as (4.4) in Chapter 4):

$$PV(M) = -gI + gM + h'\left[M - e^{-\alpha'}\left(\frac{1 - e^{-M\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M-1)\alpha'}(Z(Q_o)) \quad (7.28)$$

Recall that the quantity of disposed stock, W, was equal to $I - M$. Throughout our derivation of expected costs in this section, we shall refer to (7.28) as the "old $PV(M)$".

In attempting to determine $PV(M)$ when subsequent projects are introduced, we must recognize that some costs occur prior to the subsequent project, while some costs occur after the subsequent project. The former costs shall be referred to as "pre-project" costs, while the latter shall be denoted by "post-project" costs.

During the time period between the commencement of the initial project's ongoing phase and the subsequent project, the M units of on-hand stock will be used and ongoing replenishments (if any) will be inaugurated. As illustrated in our heuristic approach, the level of on-hand inventory when the subsequent project occurs is represented by I_s. Hence, the present value of all costs during this "pre-project" period is simply the difference between the "old *PV(M)*" and the present value of having I_s units on-hand at the time of the project, using these I_s units up and beginning ongoing replenishment cycles. Figure 20 provides a sketch of these "pre-project costs". Note that in calculating the pre-projects costs, we are taking the costs out to a certain time Y (representing the time at which the subsequent project occurs) as the difference between two infinite streams of costs. One stream starts at time 0, while the other starts at time Y. Note further that Figure 20 represents the case of a time interval i fully subsequent to the initiation of ongoing phase replenishment cycles. That is, the level of on-hand stock is represented by the average inventory during a replenishment cycle, and the subsequent project is assumed to occur when the inventory is at this average level.

We can use part of (7.28) to determine $PV(I_s)$. Since the ongoing phase replenishment cycles would be delayed by $(I_s+1)/D_o$ time units, we have the following:

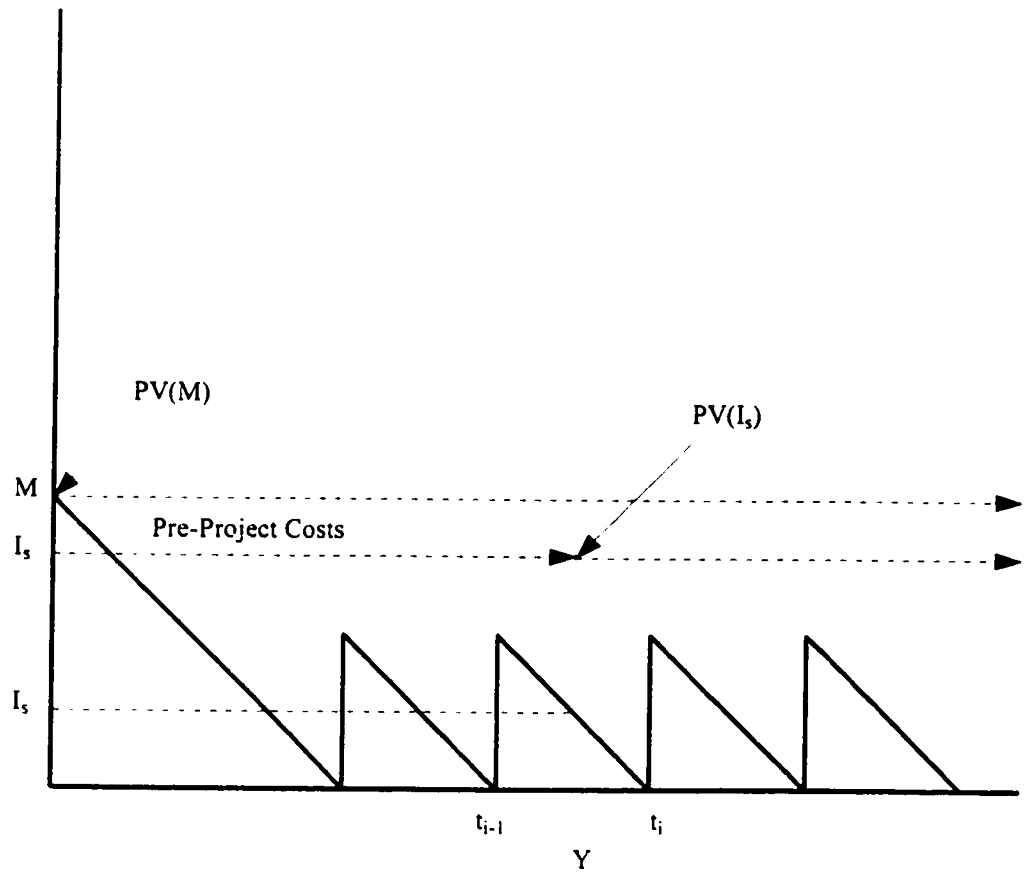
$$PV(I_s) = h'\left[I_s - e^{-\alpha'}\left(\frac{1 - e^{-I_s\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(I_s+1)\alpha'}(Z(Q_o)) \tag{7.29}$$

Since we assume (for present values purposes) that the subsequent project occurs exactly half-way within the time interval (t_{i-1} to t_i) under consideration, the present value of the "pre-project" costs may be written as:

$$Old\ PV(M) - e^{\frac{-\alpha(t_{i-1}+t_i)}{2}} PV(I_s) \tag{7.30}$$

Figure 20

Sketch of Pre-Project Costs



Now we shall explore the specific events which occur during the "post-project" period. We propose that, as we discussed at the beginning of this section, any on-hand stock when the subsequent project occurs be transferred to the subsequent project. This results in a cost savings of $I_s v_s$, since these units do not need to be procured during the subsequent project.

Moreover, as we also discussed earlier, the costs of meeting ongoing usage from the original project are also affected by the subsequent project. When the next project occurs, materials managers are provided a special opportunity to procure items at v_s ($v_s < v_o$) to satisfy ongoing usage from the first project. A vital question, therefore, is to determine the best quantity one ought to procure at this reduced price. We shall use the following notation:

S: the quantity of stock ordered at v_s to satisfy ongoing operational usage from the original project

The present value of ordering S units is comprised of the following components. Firstly, we have the costs of ordering these units. These are:

$$A + S v_s \tag{7.31}$$

We also have the costs of carrying the S units. Using the results developed in Chapter 4 (and alluded to in (7.30)), we may write these carrying charges as:

$$h' \left[S - e^{-\alpha'} \left(\frac{1 - e^{-S\alpha'}}{1 - e^{-\alpha'}} \right) \right] \tag{7.32}$$

Finally, we have the costs arising from the infinite pattern of stepwise cycles in the ongoing phase. By procuring S units, we are delaying the infinite pattern by $(S+1)/D_o$ time units. This gives the following expression:

$$e^{-(S+1)\alpha'}(Z(Q_o)) \tag{7.33}$$

Combining (7.31), (7.32) and (7.33) provides an expression for the present value, at the moment of purchase, of all costs associated with procuring S units at v_s. This expression is:

$$PV(S) = A + Sv_s + h'\left[S - e^{-\alpha'}\left(\frac{1 - e^{-S\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(S+1)\alpha'}(Z(Q_o)) \tag{7.34}$$

As we have done in earlier portions of this dissertation, we will use the method of differencing to find the optimal procurement amount (ie. the value of S, denoted by S^*, which minimizes $PV(S)$). Before doing this, however, we must show that the $PV(S)$ expression is convex. Note the close similarity between this expression and the one for $PV(M)$. This latter expression first appeared as (4.4) in Chapter 4 and was subsequently repeated as (7.28) in this Chapter. The only differences between $PV(S)$ and $PV(M)$ is that S is used in place of M in the holding cost expression (the third term) and in the offset of the present value of the ongoing replenishment cycles (the final term in the expressions). As well, $PV(S)$ includes a (linear) cost of procuring the S units, while $PV(M)$ provides salvage revenue for the disposal of surplus stock. In Chapter 4, we showed that $PV(M)$ was indeed convex. Since $PV(S)$ is structurally identical to $PV(M)$, we know that $PV(S)$ is convex.

In developing an analytical expression for S^*, we need to find the first (ie. smallest) integer S such that $\Delta PV(S) = PV(S+1) - PV(S) > 0$.

Evaluating the first difference of (7.34), and setting it strictly positive, yields:

$$v_s + h' - e^{-(S+1)\alpha'}\left(\frac{h'}{1 - e^{-\alpha'}}\left(1 - e^{-\alpha'}\right)\right) - e^{-(S+1)\alpha'}\left(Z(Q_o)\left(1 - e^{-\alpha'}\right)\right) > 0 \qquad (7.35)$$

which may be expressed as:

$$e^{-(S+1)\alpha'}\left[h' + Z(Q_o)\left(1 - e^{-\alpha'}\right)\right] < v_s + h' \qquad (7.36)$$

Solving for S in (7.36) gives the following inequality:

$$-(S+1)\alpha' < \ln\left[\frac{v_s + h'}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}\right]$$

which becomes:

$$S > \frac{1}{\alpha'}\ln\left[\frac{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}{v_s + h'}\right] - 1 \qquad (7.37)$$

This expression is used to find the optimal procurement quantity. In a fashion analogous to that used in finding M^*, the procedure to find the best value of S is quite straight-forward. One simply calculates, from the parameter values given, the right-hand side of (7.37). The smallest integer greater than or equal to that right-hand side becomes S^*.

To express all costs in a present value perspective, we discount any post-project costs back to the beginning of the ongoing phase. Since we assume that the subsequent project occurs half-way through a time interval, we have the following expression for the present value of the post-project costs:

$$e^{\frac{-\alpha(t_{i-1}+t_i)}{2}}\left(PV(S^*) - I_s v_s\right) \tag{7.38}$$

The present value of beginning the ongoing phase with M units of stock on-hand can be ascertained by combining (7.30) and (7.38). In addition, we must include the probability, p_i, of a project occurring within a given interval. This yields the following expression (we shall call it the "new *PV(M)*" to distinguish it from the "old *PV(M)*" computed without the consideration of future projects).

$$New\,PV(M) = Old\,PV(M) + \sum_{i=1}^{n} p_i\left[e^{\frac{-\alpha(t_{i-1}+t_i)}{2}}\left(PV(S^*) - PV(I_s) - I_s v_s\right)\right] \tag{7.39}$$

We have now developed an important expression which will be quite useful in obtaining the best disposal quantities (and, later, the optimal procurement quantities) for the case of future projects and ongoing usage. However, simply developing a heuristic procedure is not sufficient. One must show that this heuristic approach does a reasonably good job of estimating the on-hand inventory levels (and associated costs) when a subsequent project occurs. If it does a rather poor job, then one would not be confident in the pre-project and post-project costs obtained.

We shall use a simulation experiment to judge the appropriateness of our heuristic procedure. Recall that two critical assumptions of our heuristic approach concerned the timing at which a subsequent project would occur (we assumed it would occur exactly halfway within a time interval) and the determination of on-hand inventory levels when the project occurs, ie. the I_s values. If we wanted to use simulation to determine the exact timing of the next project and on-hand stock quantities, we would require the following notation:

τ_i: Exact time until a subsequent project that occurs in the ith interval

We note that τ_i will satisfy the following expression:

$$\tau_i = t_{i-1} + RAND(t_i - t_{i-1}) \tag{7.40}$$

where RAND is a random number drawn from a uniform distribution between 0 and 1. Moreover, we again note that $t_o = 0$.

The determination of the exact quantity of on-hand inventory is accomplished by noting that, once the ongoing phase has begun, there are $(M+1)/D_o$ time units until the ongoing replenishments (at unit price v_o) commence. Consequently, if:

$$(M + 1) > D_o\tau_i \tag{7.41}$$

then one would still be in the "transitional" period (namely, prior to the inauguration of the ongoing replenishment pattern). Thus, if (7.41) is satisfied, then the following expression represents the on-hand inventory:

$$(M + 1) - D_o\tau_i \tag{7.42}$$

Any continuous values of (7.41) are rounded to the closest integer.

However, if (7.41) is not satisfied, then one would be somewhere within the ongoing phase replenishment cycles. We recognize that the quantity:

$$D_o \tau_i - (M + 1) \tag{7.43}$$

represents the distance we have "progressed" into the infinite stepwise pattern. Recall that each replenishment cycle (following an "exact" approach) has $Q_o - 1$ units at its top. The expression:

$$MOD\left(\frac{D_o \tau_i - (M + 1)}{Q_o - 1} \right) \tag{7.44}$$

represents the number of units used in the last replenishment cycle. As a result, we can determine the on-hand inventory at the specific moment τ_i by using the following expression:

$$Q_o - 1 - MOD\left(\frac{D_o \tau_i - (M + 1)}{Q_o - 1} \right) \tag{7.45}$$

We used *Microsoft Excel* to conduct our simulation analysis. Appendix I contains results from our experiment. For a specific set of inter-project period probability distributions (in other words, a set of p_i's and t_i's), we began by computing the costs associated with following our heuristic approach. We explored $PV(M)$ values for individual M's, choosing specific values of M that we thought were most important to the effective working of our heuristic. We wanted to be sure that our heuristic accurately estimated costs arising when, for instance, specific values of M coincided with the beginning of ongoing replenishments, or close to the time at which a subsequent project occurred. We also computed the sum of all $PV(M)$ values for every value of M from 0 to 600.

We ran a series of experiments for five different inter-project period probability distributions. Our experiments in no way were meant to represent an exhaustive collection of tests. Rather, we analyzed a range of possible inter-project period distributions. Some featured a high likelihood of a subsequent project in the immediate future, while another featured a relatively strong probability of a future project quite a bit later. By testing a range of distributions, we can be confident that our heuristic approach performs well under a variety of conditions.

We performed 100 replications of our simulation model for each $PV(M)$ calculation. This specific number of replications worked well in providing accurate estimates of the exact expected costs, while also being computationally feasible. For each set of simulation replications, we determined the average $PV(M)$ value as well as the standard error, $s_{\bar{x}}$.

In order to test the appropriateness of the heuristic, we need to determine the difference between $PV(M)$ values given by the heuristic and exact approaches. A "t-value" can be computed by taking this difference and dividing it by the standard error found in the simulation replications. We note that any t-value in excess of 2.0 (in absolute value) would suggest a statistically significant difference between the heuristic and exact costs. The t-value of 2.0 corresponds to a significance level of about 5%. That is, there would be a relatively small likelihood that such a difference was due to chance occurrences.

As illustrated in Appendix I, the heuristic performs well. By assuming that the subsequent project occurs exactly halfway within a time interval, and using our heuristic to estimate I_s, there does not appear to be a significant difference in the *PV(M)* costs produced by the two approaches. None of the t-values exceeds 2.0 (in absolute value). Moreover, there does not appear to be a pattern of positive or negative t-values in our results. The heuristic does not consistently over- or under-estimate the costs produced by the exact approach. As an aside, we note that if one encountered a series of conditions under which the heuristic failed to closely approximate the exact costs, then one could "carve up" the time intervals into finer portions. This would result in relatively narrow time intervals, so the assumption that a subsequent project occurs half-way within an interval would become less prone to error. The downside to such an approach is the extra computational effort generated by the additional time intervals.

Now that we have an approach to determine the present value of beginning the ongoing phase with M units of stock while recognizing the likelihood of subsequent projects, we require an approach to determine the best amount of stock to dispose, given a specific inventory level at the conclusion of the construction phase. Recall that in Chapter 4, we developed decision rules to choose the optimal disposal quantity, W^*. We were able to do this since *PV(M)* was convex with respect to the retention amount. However, when we incorporate future projects into our analysis, we have no guarantee that the *PV(M)* expression provided in (7.39) is convex with respect to M. Indeed, we have seen in our use of this model that $PV(M)$ is not convex! As such, we are unable to adopt the decision rule methodology used in Chapter 4. In order to determine the best disposal quantities (and the very important $EPV^*(I)$ values), we must resort to a "brute force" approach. That is, we must calculate, for every I, the associated costs of disposing W units (for every W value less than or equal to I) and retaining $M = I - W$ units to satisfy ongoing operational usage or subsequent requirements in a future project. We must evaluate the following expression:

$$EPV^*(I) = \min_{W \leq I} \left(-gW + New\ PV(I - W)\right) \tag{7.48}$$

The expression, as given above, is for the case of constant salvage values. For non-constant salvage values, we would replace "$-gW$" by the total salvage revenue earned in making a total of W disposals.

7.2.1 Constant Salvage Values

The results for the case of constant salvage values are provided in Table 19 (see pages 181-182). The reader is asked to recall that Table 3 provides a list of parameter values and Table 5 gives the range of constant salvage values under consideration. The inter-project period probability distributions and ranges for the acquisition costs in a subsequent project are shown in Tables 14 and 15, respectively. All treatment combinations, unless otherwise indicated, use Case 1 for the inter-project period probability distribution.

Table 19
Results - Future Projects, Ongoing Usage, Constant Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	503	503	400 (1.44)	400 (1.44)
B_1 low	503	440 (0.06)	400 (1.22)	400 (1.22)
B_1 high	503	503	400 (1.66)	400 (1.66)
B_2 low	423	423	400 (0.62)	300 (3.98)
B_2 high	503	503	400 (3.13)	400 (3.13)
h low	502	502	400 (2.02)	400 (2.02)
h high	440	436 (0.002)	400 (0.95)	400 (0.95)
α low	501	501	400 (1.62)	400 (1.62)
α high	500	437 (0.06)	400 (1.13)	400 (1.13)
T_c low	516	516	400 (2.87)	400 (2.87)
T_c high	423	423	400 (0.69)	400 (0.69)
D_c incr.	603	603	500 (1.73)	500 (1.73)
D_c decr.	416	409 (0.01)	300 (2.71)	300 (2.71)
D_c level	516	516	400 (2.21)	400 (2.21)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
D_c bi-modal	603	603	400 (3.83)	300 (7.54)
A low	500	500	400 (1.18)	400 (1.18)
A high	500	500	400 (1.59)	400 (1.59)
D_o low	500	420 (0.11)	400 (1.01)	400 (1.01)
D_o high	500	500	400 (1.66)	400 (1.66)
v_o low	500	500	400 (1.11)	400 (1.11)
v_o high	503	503	400 (1.77)	400 (1.77)
g low	503	503	400 (1.44)	400 (1.44)
g high	503	503	400 (1.44)	400 (1.44)
t_i Case 2	444	444	400 (1.18)	400 (1.18)
t_i Case 3	457	452 (0.004)	400 (1.07)	400 (1.07)
v_s low	423	423	400 (0.86)	400 (0.86)
v_s high	516	516	400 (2.69)	400 (2.69)

One observation illustrated by this table is that the respective Q_c^* values given for the case of future projects, no ongoing usage and constant salvage values (see Table 16) provide lower bounds on the optimal procurement quantities when ongoing usage is combined with future projects. When one incorporates usage in the inter-project period (with a higher unit value), then one will procure at least as much as one did in the case of no ongoing usage. We also note that, generally, the percentage cost penalties of the non-integrated inventory management strategies are larger under the cases of ongoing usage than they are when there is no ongoing usage between projects.

However, we wish to point out the myopic percentage cost penalties observed for the case of the low setting of B_2. In particular, Table 19 shows that Q_m^* was 300 while Q_c^* was 423. The percentage cost penalty associated with this case was 3.98%. Table 16 (constant salvage values and no ongoing usage) illustrated Q_m^* and Q_c^* values of 300 and 400, respectively. Even though these procurement quantities are closer than those in Table 19, the percentage cost penalty of following the myopic strategy is larger (4.59%). This behaviour is observed since the absolute total cost figures of procuring specific quantities are substantially less when there is no ongoing usage between projects (since one does not have to make relatively costly replenishments at unit price v_o). Thus, despite the fact that the two procurement quantities in Table 16 are closer than those observed in Table 19, the smaller overall cost values in Table 16 provide larger percentage cost penalties.

We also observe that several of the Q_c^* values occurred a few units above a specific construction phase requirements value. This occurs due to the non-convexity of the $EPV^*(I)$ function. It was often worthwhile to procure a few units above a requirements value in order to capture the large cost savings associated with the decreasing (and non-convex) $EPV^*(I)$ function.

Finally, we note that for many of the treatment combinations, disposal of surplus stock was not attractive. For the ranges of parameter values considered in our analysis, the two sources of future usage of the item would appear to warrant the retention of surplus inventory after completion of the original project's construction phase.

7.2.2 Marginally Decreasing Salvage Values

The results for the case of marginally decreasing salvage values are provided in Table 20 (see pages 184-186). One can find the various marginally decreasing salvage value functions in Table 10. The other tables pertinent to this section were listed at the beginning of Section 7.2.1. As before, all treatment combinations (unless otherwise specified), use Case 1 for the marginally decreasing salvage value functions and inter-project period probability distributions.

Table 20
Results - Future Projects, Ongoing Usage, Marginally Decreasing Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	503	503	400 (1.44)	400 (1.44)
B_1 low	503	440 (0.06)	400 (1.22)	400 (1.22)
B_1 high	503	503	400 (1.66)	400 (1.66)
B_2 low	423	423	400 (0.62)	300 (3.98)
B_2 high	503	503	400 (3.13)	400 (3.13)

Treatment Combination	**Integrated (exact)**	**Integrated (approx.)**	**All-disposal**	**Myopic**
h low	502	502	400 (2.02)	400 (2.02)
h high	440	436 (0.002)	400 (0.95)	400 (0.95)
α low	501	501	400 (1.62)	400 (1.62)
α high	500	437 (0.06)	400 (1.13)	400 (1.13)
T_c low	516	516	400 (2.87)	400 (2.87)
T_c high	423	423	400 (0.69)	400 (0.69)
D_c incr.	603	603	500 (1.73)	500 (1.73)
D_c decr.	416	409 (0.01)	300 (2.71)	300 (2.71)
D_c level	516	516	400 (2.21)	400 (2.21)
D_c bi-modal	603	603	327 (6.30)	300 (7.54)
A low	500	500	400 (1.18)	400 (1.18)
A high	500	500	400 (1.59)	400 (1.59)
D_o low	500	420 (0.11)	400 (1.01)	400 (1.01)
D_o high	500	500	400 (1.66)	400 (1.66)
v_o low	500	500	400 (1.11)	400 (1.11)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
v_o high	503	503	400 (1.77)	400 (1.77)
g_i Case 2	503	503	400 (1.44)	400 (1.44)
g_i Case 3	503	503	400 (1.44)	400 (1.44)
t_i Case 2	444	444	400 (1.18)	400 (1.18)
t_i Case 3	455	450 (0.01)	400 (1.16)	400 (1.16)
v_s low	423	423	400 (0.86)	400 (0.86)
v_s high	516	516	400 (2.69)	400 (2.69)

We note a similarity between the results in Tables 19 and 20. This should not be too surprising. If the disposal of surplus stock was unattractive for the case of constant salvage values, then it will be difficult to justify it when salvage values begin to fall as the number of disposals increase.

7.2.3 Increasing Salvage Values

The results for the case of increasing salvage values are provided in Table 21 (see pages 187-188). Table 12 lists the various increasing salvage value functions used. The other tables relevant to this section were listed at the beginning of Section 7.2.1. All treatment combinations (unless otherwise specified) use Case 1 for the increasing salvage value functions and inter-project period probability distributions.

Table 21
Results - Future Projects, Ongoing Usage, Increasing Salvage Values

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
Intermediate	503	503	400 (1.44)	400 (1.44)
B_l low	503	440 (0.06)	400 (1.22)	400 (1.22)
B_l high	503	503	400 (1.66)	400 (1.66)
B_2 low	423	423	400 (0.62)	300 (3.98)
B_2 high	503	503	400 (3.13)	400 (3.13)
h low	502	502	400 (2.02)	400 (2.02)
h high	440	436 (0.002)	400 (0.95)	400 (0.95)
α low	501	501	400 (1.62)	400 (1.62)
α high	500	437 (0.06)	400 (1.13)	400 (1.13)
T_c low	516	516	400 (2.87)	400 (2.87)
T_c high	423	423	400 (0.69)	400 (0.69)
D_c incr.	603	603	500 (1.73)	500 (1.73)
D_c decr.	416	409 (0.01)	300 (2.71)	300 (2.71)
D_c level	516	516	400 (2.21)	400 (2.21)

Treatment Combination	Integrated (exact)	Integrated (approx.)	All-disposal	Myopic
D_c bi-modal	603	603	400 (3.83)	300 (7.54)
A low	500	500	400 (1.18)	400 (1.18)
A high	500	500	400 (1.59)	400 (1.59)
D_o low	500	420 (0.11)	400 (1.01)	400 (1.01)
D_o high	500	500	400 (1.66)	400 (1.66)
v_o low	500	500	400 (1.11)	400 (1.11)
v_o high	503	503	400 (1.77)	400 (1.77)
g_i Case 2	503	503	401 (1.36)	400 (1.44)
g_i Case 3	503	503	401 (1.36)	400 (1.44)
t_i Case 2	444	444	400 (1.18)	400 (1.18)
t_i Case 3	467	459 (0.01)	400 (1.47)	400 (1.47)
v_s low	423	423	400 (0.86)	400 (0.86)
v_s high	516	516	400 (2.69)	400 (2.69)

Again, we observe that there exists substantial benefit to retaining surplus stock after the original project's construction phase. Even when we incorporate increasing salvage values (for the range of parameter values considered in this case), retention is usually the best option. We note, however, that when there exists a very high likelihood of a subsequent project "far down the road" (as provided in t_f Case 3), surplus stock disposal becomes rather attractive. The desire to retain surplus units to eventually fulfill some of the requirements in a subsequent project is eroded due to the carrying charges required to hold this stock. In addition, we note that the attractiveness of surplus disposal is enhanced as the unit acquisition prices in a subsequent project fall. When "deflationary" conditions exist, one would like to have relatively little on-hand when the subsequent project arises. The cost savings obtained by transferring units to another project are reduced when unit prices fall.

This concludes the treatment of future projects. We have examined this important materials management issue through two separate cases; namely, no inter-project usage and deterministic, level ongoing usage. We have calculated the costs of retention and disposal decisions, and used this information to find the best procurement quantities in the original project's construction phase.

8. MODEL EXTENSIONS

Thus far, this dissertation has developed a mathematical model to examine the procurement and disposal of an important, expensive item within the context of a large-scale project. Non-constant salvage values and the consideration of ongoing usage and possible future projects have also been treated.

In this Chapter, we seek to extend our mathematical model to analyze two additional cases; namely, deterministic, time-varying usage in the ongoing phase and multiple construction phase procurement opportunities. We shall consider these cases in the context of constant salvage values and no future projects (in other words, the situation illustrated in Section 6.1 of this dissertation). Our goal in this Chapter shall not be to exhaustively analyze different treatment combinations, but rather to illustrate how one can modify our model to take into account these two important extensions. We shall provide some brief numerical analyses.

8.1 Deterministic, Time-Varying Ongoing Usage

Recall that our previous work in this dissertation has considered deterministic, level usage in the ongoing phase. We used the parameter D_o to represent the annual usage rate. Moreover, we were able to generate expressions (see (3.12) in Chapter 3) that provided the optimal ongoing phase replenishment quantity (Q_o) for a given set of parameter values. Recall that we also showed in Chapter 3 (and Appendix D) that the

Economic Order Quantity (*EOQ*) was often a good place at which to initiate the search process for finding Q_o. We were also able to compute the present value of all future ongoing phase costs associated with any replenishment quantity (see (3.7) in Chapter 3).

However, the assumption of deterministic, level ongoing phase usage will certainly not always be valid. Based on discussions with materials management personnel, we have found that other usage patterns may be appropriate. In particular, ongoing phase usage could follow a time-varying pattern. We submit that this case calls for a relatively high usage of an item during the startup phase of the facility (ie. immediately after the end of the construction phase). This could be due to the so-called "infant mortality" effect. Usage then tapers over time as the facility matures, then begins to increase as items wear out.

From a practical perspective, it is probably wise to assume that the time-varying usage pattern eventually becomes "level" (ie. at a constant annual usage rate). This implies that we need to sub-divide the ongoing phase into two particular phases: a "time-varying" period in which per period usage varies, and a "level" period in which per period usage is constant. A vital question, obviously, involves the specific duration of the time-varying nature of ongoing phase usage. The longer the duration of time-varying usage, the greater the computational effort involved in determining appropriate replenishment schedules. Moreover, from a present value perspective, the particular pattern of usage for periods further into the future will have less bearing on the calculation of relevant costs (for instance, the present value of beginning the ongoing phase with M units on-hand). It will have even less bearing on the calculation of the present value of procuring a specific quantity in the construction phase.

Based on our discussions with materials management personnel, we submit that an appropriate duration for the time-varying usage phase ought to be 3 years (36 months). Any usage subsequent to that time is assumed to be constant and level. We require the following notation:

d_j: number of units used in month j

Notice that we selected a month as the basic period during the time-varying usage phase. We feel this decision is appropriate since there would be enough time periods during the 3-year phase to capture the essence of time-varying usage, yet not too many time periods to make the computation of our replenishment schedules intractable. A narrow time period (days or weeks) may introduce unwanted complexities (including data needs), while a broad time period (quarters or half-years) would not adequately reflect the time-varying nature of the pattern and the opportunities for replenishment.

Recognizing that the time-varying usage pattern lasts for 36 months, we realize that either of the following two ongoing phase cases could occur:

$$Case\ A : M < \sum_{j=1}^{36} d_j$$

This case implies that at least one replenishment will be required during the time-varying portion of the ongoing phase. The quantity of on-hand stock at the beginning of the ongoing phase (M units) is not large enough to cover all of the time-varying requirements.

$$Case\ B : M \geq \sum_{j=1}^{36} d_j$$

This case implies that no replenishments are needed during the time-varying phase. We have sufficient inventory to satisfy all time-varying requirements.

We shall begin by examining the various costs incurred under case A. The determination of appropriate replenishment schedules during the time-varying usage period is not a trivial task. We cannot simply conclude, as we did in Chapter 3, that each replenishment quantity will consist of Q_o units. Since monthly usage varies, the appropriate replenishment quantities during the time-varying period are also likely to differ.

As is common practice in the literature, we will assume that replenishments can only be made at discrete points in time (namely, at the beginning of each month). We note that if one allowed continuous opportunities for replenishment, then the computational effort involved in finding the respective replenishment quantities would become immense. Furthermore, this assumption ought to be rather reasonable from a practitioner perspective. Vendors of items may limit replenishment opportunities to a set of pre-determined times. The use of discrete replenishment opportunities (and the fact that ongoing phase stockouts are not permitted) implies that an ongoing phase replenishment must cover an integer number of time periods (ie. months).

Silver, Pyke and Peterson (1998) succinctly explore the development of replenishment schedules for the case of time-varying usage. In order to provide the reader with some appreciation of the work previously done in this area, a few of the approaches will be briefly described. The Wagner-Whitin method uses dynamic programming to provide the optimal replenishment quantities for time-varying usage. However, this approach makes at least one critical assumption; namely, that there is an "ending point" where the inventory level must be at zero or some other specified value. We note that this could be worked into our mathematical model by assuming that the inventory level must be zero at the conclusion of the time-varying period. Suppose that one did the Wagner-Whitin calculations for the duration of the 36-month time-varying period and assuming that one would enter the level usage period with 0 units on-hand. We could then use our previous expressions (see (4.1) and (4.2) in Chapter 4) for calculating the present value of having 0 units on-hand and making the optimal ongoing phase replenishment decisions thereafter.

However, there would appear to be a potential difficulty with the Wagner-Whitin approach. Suppose we entered the ongoing phase with such a quantity of on-hand stock that we had 1 unit on-hand at the beginning of the final month of the time-varying portion (in other words, the 36th month). Suppose further that the usage in month 36 was 2 units. Since the Wagner-Whitin approach forces us to have 0 units on-hand at the end of the time-varying phase, we would be required to make a very small replenishment (1 unit) in the 36th month. Granted, this is an extreme case, but it does not seem attractive from an overall cost perspective to replenish such a small quantity. This issue of avoiding imposing a constraint of 0 units on-hand at the end of the time-varying phase will be handled in the heuristic solution procedure.

Silver, Pyke and Peterson (1998) also discuss the "*EOQ* approach". This method reduces the computational effort in finding replenishment schedules by ignoring the time-varying nature of the usage pattern. It determines an *EOQ* based on average demand rates, then uses this calculated *EOQ* value to guide the determination of respective replenishment quantities. Each replenishment quantity is selected so as to cover an amount at least as great as the *EOQ* value. However, this heuristic approach performs rather poorly under significantly varying usage. Consequently, we have chosen not to use it in our work.

The Silver-Meal heuristic (1973) provides an efficient approach for determining replenishment quantities in the case of time-varying usage. It minimizes total relevant costs per unit time (ordering and carrying costs), a feature possessed by the basic EOQ when the demand rate is level. However, the heuristic ignores the discounting of relevant costs. Nonetheless, Grubbstrom (1997) presents a formulation of the Silver-Meal heuristic to calculate the net present value of associated costs. Our approach, as given in this Chapter, borrows to a large extent from Grubbstrom's work.

To simplify the analysis, we shall assume that usage occurs at the conclusion of a time period (month). Without loss of generality, the material presented in the development of this approach assumes there is no on-hand inventory at the beginning of the planning period (ie. just before the replenishment in question). We shall calculate carrying charges based solely on ending monthly inventories (in other words, on the stock carried forward to the next period). Although this represents a departure from our previous use of the continuous discounting of carrying charges, it is consistent with the usual approach suggested in the Silver-Meal heuristic. Moreover, as shall be shown later, the differences (in terms of Q_c^* decisions and concomitant percentage cost penalties) are relatively minor.

Earlier, we used the notation d_j to represent monthly usages. In order to provide a succinct way of illustrating monthly ending inventories, we will use the following notation:

D_k: <u>cumulative</u> requirements up to and including month k

Mathematically, the term D_k can be expressed as:

$$D_k = \sum_{j \le k} d_j$$

We further note that D_k represents the quantity of a replenishment of sufficient size to cover all requirements up to and including month k.

Monthly ending inventories may be written as, for $j < k$:

$$D_k - D_j$$

We note that the monthly ending inventory in month k would be zero. As an example, consider the following monthly usage pattern: 5 (d_1), 3 (d_2) and 1 (d_3). Then, $D_3 = 5 + 3 + 1 = 9$ units. We also note that $D_1 = 5$ and $D_2 = 5 + 3 = 8$. If one placed an order in month 1 (of sufficient size to cover 3 months' requirements), then the respective monthly ending inventories would be $D_3 - D_1 = 4$ units (at the end of the first month), $D_3 - D_2 = 1$ unit (month 2), and 0 units (month 3).

The net present value (*NPV*) of an order covering until the end of month k (ie. a replenishment of D_k units) is:

$$NPV = A + \frac{h}{12}\sum_{j=1}^{k-1}(D_k - D_j)e^{-\alpha t_j/12} \tag{8.1}$$

We note that the term in the summation for $j=k$ is zero. This suggests that if one placed an order that was large enough to cover only one month of requirements, then the relevant inventory carrying charges would be zero (since they are based solely on ending monthly stock levels). Further, observe that we have divided the parameter h (carrying costs) by 12. This is done to represent our carrying charges on a monthly basis. Note that we also divided the t_j value (the end of the month under consideration) by 12 so as to determine appropriate present values (since the discount rate α is on an annual basis). In other words, if we were considering inventory carrying charges at the conclusion of month 6, then we would need to discount the on-hand stock costs by half a year to determine their present value.

After determining the *NPV's* associated with replenishments of various sizes, Grubbstrom then determines the heuristic replenishment schedule. He uses the following notation:

A(k): the annuity stream (the level of a constant annual cash flow generating a particular *NPV*)

His *A(k)* values are computed in the following manner:

$$A(k) = \frac{\alpha\,(NPV)}{1 - e^{-\alpha t_k/12}} \tag{8.2}$$

Once the particular *A(k)* values are computed (for $k = 1, 2, \ldots$), we determine (in a fashion analogous to the Silver-Meal heuristic) the first time that $A(k+1) > A(k)$. This represents the initial point at which the $A(k)$ function "turns up". The appropriate replenishment, consequently, is made to cover k time periods. This provides a reasonable criterion to use in obtaining "good" replenishment schedules for we end up with a "local minimum" of the annuity stream associated with the current replenishment. As pointed out in Silver, Pyke and Peterson (1998), one may obtain a lower total cost value by extending the analysis for a few more values of k, but the possibility of substantially reducing total costs is rather minimal.

Numerical Example:

Consider the following parameter values:

d_1 = 7 units
d_2 = 2 units
d_3 = 10 units
d_4 = 4 units
d_5 = 20 units
A = \$250 per replenishment
h = \$13 per unit of inventory per year
a = 0.10

Suppose we wanted to determine the associated costs of an order large enough to cover the first four months of requirements. In this case, D_4 = 23 units (assuming no units on-hand at the beginning of the first month). The particular monthly ending inventories are 16 (month 1), 14 (month 2), 4 (month 3) and 0 (month 4). Using (8.1), we obtain an NPV of \$286.33. The $A(4)$ value, as given by (8.2), is \$873.39. Table 22 provides a comparison of the $A(k)$ values for all values of k from 1 to 5. Note that $A(4)$ is the minimum $A(k)$ value, thus suggesting that one should cover the first four months of requirements with a replenishment in month one.

Table 22
Comparison of *A(k)* Values - Deterministic, Time-Varying Usage

k	Replenishment Quantity	NPV	A(k)
1	7	250.00	3012.52
2	9	252.15	1525.54
3	19	273.55	1107.92
4	23	286.33	873.39
5	43	371.22	909.61

The present value of the costs of a single inventory cycle (ordering, carrying and purchase costs) would be given as:

$$NPV + D_k v_o \tag{8.3}$$

Since we are assuming that replenishments arrive at the beginning of a time period, there is no need to discount the purchase costs (within a single cycle). However, we do note that the costs in (8.3) would need to eventually be discounted back to the beginning of the ongoing phase.

The approach outlined thus far can determine the size (and associated costs) of each replenishment during the duration of the time-varying usage pattern. However, it is not necessarily the case that the proposed replenishment schedule during the time-varying phase will <u>exactly</u> cover the 36 months worth of requirements, thus leaving 0 units on-hand at the beginning of the level portion of the usage pattern. In particular, it may be advantageous for the final replenishment during the time-varying period to cover requirements somewhat into the constant demand portion. Thus, it is important that we appropriately treat the last replenishment during this 36-month period.

An obvious way to do this would be to define $d_j = D_o/12$ for each month in the level phase. However, recall that our "base case" setting of D_o was 20 units. Dividing this quantity by 12 does not result in an integer value! Nonetheless, a rather pragmatic way around this would be to assume a monthly pattern of 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, etc. in the level phase. Observe that this 12-month usage pattern results in an annual total of 20 units, our "base case" usage rate. It is important to realize that this approach would only be used to compute the size (not the relevant costs) of the last replenishment during the time-varying usage phase. It would be necessary to compute the monthly carrying charges required to hold this stock until the conclusion of the time-varying period. Then, we would be left with a certain amount of stock on-hand (denoted by I) at the beginning of the level phase. We could use our earlier expressions (see (4.1) and (4.2) in Chapter 4) to determine the present value (36 months into the future) of having I units on-hand and proceeding in an optimal fashion regarding future (deterministic, level) replenishments.

We need to make one final point regarding the computation of relevant costs for case A (ie. M is less than cumulative requirements during the time-varying period). The initial replenishment during the time-varying phase will be made in the first time period j (denoted by j^*) such that $M - D_j < 0$. For any months prior to j^*, we must compute the relevant inventory carrying charges. Whenever we make a replenishment during the time-varying period, it is important to recognize that the decision to replenish, say, D_k units represents an "order-up-to" decision. In other words, if we enter time period j^* with some units of on-hand stock, this reduces the quantity that needs to be purchased (at unit price v_o). However, since carrying charges are based solely on monthly ending inventory levels, the determination of carrying charges is unaffected by stock on-hand at the beginning of time period j^* (or, for that matter, any other time period in which a replenishment is made).

We shall now examine (the relatively straight-forward) case B. Recall that this case does not involve replenishments during the time-varying portion of the ongoing phase. We simply have carrying charges for the on-hand stock during these 36 months. The total carrying charges are given by:

$$\sum_{j=1}^{36} \frac{h}{12}(M - D_j)e^{-\alpha j/12} \tag{8.4}$$

At the conclusion of the time-varying phase, we will have $I = M - D_{36}$ units of on-hand stock. As we noted earlier when analyzing the final replenishment during the time-varying phase, we can determine the costs (36 months into the future) of having a specific quantity of on-hand stock and proceeding in an optimal fashion with respect to future replenishment decisions.

Once we have determined the present value (at the beginning of the ongoing phase) of having M units on-hand (for all values of M), our next step is to determine the best disposal quantities as a function of the inventory level I prior to any disposal. This would allow us to generate the all-important $EPV^*(I)$ values. Recall that when we had deterministic, level ongoing usage, we were able to generate a closed-form expression (see (4.9) in Chapter 4) for determining the optimal retention quantity. However, we doubt that it is possible to do this when time-varying usage is introduced. We have seen (through numerical examples) that the $PV(M)$ expression is <u>not</u> convex with respect to M. Consequently, we will need to adopt a similar approach to what we did for the case of future projects and ongoing usage (see Section 7.2). We must compute, for every I, the associated costs of disposing W units (for every W value less than or equal to I) and retaining $M = I - W$ units to satisfy ongoing phase requirements. The following expression is used:

$$EPV^*(I) = \min_{W \leq I} \left(-gW + PV(I - W)\right) \tag{8.5}$$

We shall perform some limited numerical analysis in this Section. Table 23 provides our time-varying usage pattern.

Table 23
Monthly Time-Varying Usage - "Base Case" Example

Month	1	2	3	4	5	6	7	8	9	10	11	12
d_i	5	5	5	4	4	3	3	2	2	2	1	1
Month	13	14	15	16	17	18	19	20	21	22	23	24
d_i	1	1	1	1	1	0	0	0	0	0	1	1
Month	25	26	27	28	29	30	31	32	33	34	35	36
d_i	1	1	1	1	1	1	1	1	2	2	2	2

We wish to point out that the usage pattern portrayed in Table 23 is initially relatively high, then drops as the facility matures (eventually reaching a point of no usage for several months). Then the usage climbs, eventually levelling off at a value very close to the constant monthly usage amount.

In order to observe if time-varying usage generates substantially different optimal construction phase procurement decisions and associated penalty costs, we need to compare "apples to apples". Specifically, the total usage during the time-varying period must be equivalent to the total level usage during a time period of similar duration. For example, our "base case" usage rate was 20 units per year. This translates into 60 units during a 3-year (36-month) period. We note that the total usage during the 36 months as illustrated in Table 23 is 60 units.

Except where indicated, we shall use the "base case" settings for the various parameter values. We exclude the integrated (approximate) strategy in this Section and in Section 8.2. Throughout our previous work in Chapters 6 and 7, the approximate strategy provided very similar results to those given by the exact integrated approach.

To determine the effect of the particular pattern of time-varying usage on construction phase decisions and associated penalty costs, we shall analyze an additional usage pattern. This pattern is provided in Table 24.

Table 24
Monthly Time-Varying Usage - "Extreme" Example

Month	1	2	3	4	5	6	7	8	9	10	11	12
d_j	18	8	3	2	1	0	0	0	0	0	0	0
Month	13	14	15	16	17	18	19	20	21	22	23	24
d_j	0	0	0	0	1	1	1	1	1	1	1	1
Month	25	26	27	28	29	30	31	32	33	34	35	36
d_j	1	1	1	1	2	2	2	2	2	2	2	2

Note that this time-varying usage pattern also consists of 60 total units during the 36-month period. However, the actual time-varying pattern is quite a bit more pronounced than the one provided in Table 23. We have extremely high requirements in the first month. This drops off quite quickly, eventually reaching zero units. Usage remains at zero for a rather long time period, after which it (slowly) builds up to the constant level monthly usage amount.

Table 25 provides the results of a brief numerical analysis for the case of deterministic, time-varying ongoing phase usage. The particular treatment combinations chosen involve the ones that tended to be the more "interesting". Specifically, they provided some of the larger percentage cost penalties in our earlier work (see, for example, Table 6 in Chapter 6).

Table 25
Results - Deterministic, Time-Varying Usage

Treatment Combination	Integrated (Exact)	All-disposal	Myopic
Intermediate	500	400 (1.77)	400 (1.77)
B_2 low	443	400 (0.82)	300 (4.54)
T_c low	500	400 (3.12)	400 (3.12)
v_o high	500	400 (2.94)	400 (2.94)
Extreme usage pattern	500	400 (1.63)	400 (1.63)

The introduction of time-varying ongoing phase usage appears to generate (somewhat) higher percentage cost penalties than those observed in Table 6, for similar treatment combinations. Recall that under time-varying usage, a greater share of the ongoing phase requirements tends to occur "up-front" (near the beginning of the ongoing phase). If one selected as one's construction phase procurement decision the smaller procurement quantities suggested by the non-integrated approaches, this would lead to costly ongoing phase replenishments <u>earlier</u> in the ongoing phase. Consequently, the percentage cost penalties of these non-integrated strategies increase as compared to those observed under deterministic, level usage.

The Q_c^* decisions produced by the two types of time-varying usage pattern are identical, and the percentage cost penalties are relatively close. This would indicate that the determination of optimal procurement quantities is rather insensitive to the particular time-varying usage pattern modelled in the ongoing phase. Despite the effect of very high usage in the first month, followed by a quick drop to several months of no usage, we obtain a similar decision as the one provided by a usage pattern that is more "gradual".

We wish to make one final note regarding the case of time-varying usage. Since we did not continuously cost our inventory during each time period (instead basing it on monthly ending amounts), the respective heuristic replenishment quantities during the ongoing phase tended to be a bit larger than those observed under deterministic, level usage. Essentially, the reduction of the holding costs served to somewhat increase the size of the particular replenishment quantities.

8.2 More than One Construction Phase Procurement Opportunity

Our objective in this part of the dissertation is to introduce another procurement opportunity in the construction phase and to illustrate the manner in which the relevant costs may be calculated. Our earlier work has permitted only a single procurement opportunity at the beginning of the construction phase. As a result, decision-makers made their procurement decision, then a particular requirements value for the entire construction phase was observed. Based on the specific combination of procurement quantity and requirements value, a particular set of costs was obtained.

However, in many real-world instances, materials managers may be able to procure items at different stages during the construction of a large-scale project. Presumably, this would allow them to operate in more of a "just-in-time" environment, wherein required items could be brought in on an "as-needed" basis.

We will assume that there exists a second procurement opportunity for an important, expensive item exactly half-way through the construction phase. We note that we are simply using the case of two opportunities to illustrate how to deal with more than one procurement decision. Due to the computational complexities involved, we are not trying to establish the optimal timing of the additional procurement opportunity. Further, since the fixed costs of procurement (A) are likely dwarfed by the acquisition costs ($Q_c v_c$), we shall ignore the fixed cost parameter.

With the incorporation of an additional procurement opportunity, our mathematical model will need to include a probability distribution of requirements during either half of the construction phase. Recall that in our earlier work, we had a single set of requirements values during the construction phase (ie. the D_c values were 200, 300, 400, 500 and 600 units). To reduce complexity, we shall assume that the requirements in either half of the construction phase are independent. This implies that the specific requirements value observed in the initial half of the construction phase has no influence on the requirements value in the latter half. We note that, for a variety of reasons, real-world project requirements could be dependent between the two halves of the construction phase. Vendor quality problems or weather effects could generate inter-half dependencies. Moreover, it is possible that requirements could be negatively correlated across the two halves of the construction phase. Greater labour efficiency, learning effects or switching to a better supplier could produce this situation. Although it would provide greater model realism, it would introduce considerable additional complexity.

We shall now describe the procedure for finding the optimal procurement quantities in either half of the construction phase. We require the following notation:

I_c:	On-hand stock at the conclusion of the first half of the construction phase
$EPV^*(I_c)$:	Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units of inventory on-hand, and proceeding in an optimal fashion from thereon (with respect to any second half procurement, disposal and ongoing phase replenishment decisions)

The variable I_c is equal to max $\{Q_c - D_c, 0\}$. Note that the Q_c and D_c terms in this expression refer to the initial half of the construction phase. Our computational effort in determining best procurement decisions increases as compared to the effort involved in the single opportunity case, since for the second half decision, we must take account of all possible ("incoming") values of I_c. That is, the best procurement decision in the second half of the construction phase is a function of I_c.

However, we wish to make a few points regarding this issue. The case of $I_c = 0$ is essentially the same as the situation we have discussed throughout this dissertation. Recall that one of our assumptions described in Chapter 1 was that there was no on-hand stock at the beginning of the construction phase. As compared with our earlier model (see Chapter 5), the particular case of $I_c = 0$ simply involves a splitting in two of the parameter T_c, the construction phase duration. Moreover, we recognize that there is a limit to "all possible values" of I_c. The largest possible I_c value is equivalent to the highest possible Q_c that one could use for the first opportunity minus the smallest D_c in the first half of the construction phase. Although it is impossible to know *a priori* the "highest possible Q_c" one would encounter, we have found that using the largest D_c in its place has produced acceptable results. Using this approach, we have (through our limited numerical analyses) kept track of every single I_c value required. Once we have all the $EPV^*(I_c)$ values, then it is a rather simple procedure to find the best Q_c for the first opportunity. This best procurement decision is equivalent to finding Q_c^* in the single opportunity case (see Chapter 5), given that we know the $EPV^*(I)$ values (ie. the expected present value of all future costs given the value of the on-hand stock at the end of the construction phase).

The process of computing the best procurement decisions for the non-integrated inventory management strategies is rather similar. Let us consider the "all-disposal" approach. We begin by finding, for every value of I_c, the best procurement quantity in the second half of the construction phase, assuming that all surplus stock after construction phase completion is disposed. The following notation is useful:

$EPV_a^*(I_c)$: Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units on-hand and making the best procurement decisions in the second half of the construction phase, followed by the disposal of all surplus units

We recognize that this process creates a decision rule for determining the best all-disposal procurement decision in the latter half of the construction phase. In other words, we would know (for every possible value of I_c), the associated best procurement quantity.

Turning our attention to the first half of the construction phase, we realize that one would encounter acquisition, carrying, stockout and the $EPV_a^*(I_c)$ costs. (However, we wish to point out that no disposal is allowed at the half-way point of the construction phase. Surplus stock can only be disposed after construction completion). Using our cost expressions as developed in Chapter 5, we can determine the best all-disposal procurement quantity in the first half of the construction phase.

For the myopic strategy, the only difference would be that one must determine, for every value of I_c, the best procurement quantity in the second half of the construction phase, assuming that one considers construction phase costs only. The following notation is required:

$EPV_m^*(I_c)$: Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units on-hand and making the best procurement decisions in the second half of the construction phase, considering only construction phase costs

There exists a rather helpful feature regarding the construction phase decisions for the case of two procurement opportunities. In particular, the determination of best procurement decisions in the latter half of the construction phase (whether they are optimal, all-disposal or myopic) follows an "order-up-to" decision. We note that this is a consequence of ignoring the fixed replenishment cost parameter, A. This suggests that once we know the best procurement decision, say, for $I_c = 0$, then it is rather straight-forward to determine the best procurement quantities for any level of I_c. We simply subtract the particular I_c value from the procurement quantity found when I_c was 0! The following notation is now introduced:

S_c^*: The order-up-to level in the second half of the construction phase for the case of $I_c = 0$

Consequently, the best procurement quantity is $S_c^* - I_c$ (for $I_c \leq S_c^*$). The reason for this behaviour is quite clear. Suppose we determine S_c^*. Note that this implies that I_c

= 0. Now, suppose that one begins the second half of the construction phase with $I_c = 1$. If one procures $S_c^* - 1$ units, then the <u>only</u> cost that changes (when I_c changes from 0 to 1) is the acquisition costs ($Q_c v_c$) since we are implicitly assuming that any procurement quantities arrive at the beginning of the time period under consideration (ie. the beginning of the latter half of the construction phase). The carrying costs, stockout penalties and $EPV^*(I_c)$ values do not change since we are always dealing with the <u>same</u> on-hand stock level after the second half procurement decision. The relevant costs have simply been "scaled down" by v_c. Thus, if it was optimal to procure S_c^* units when I_c was 0, then it will be optimal to procure $S_c^* - 1$ units when I_c is 1.

Further, the computation of the expected total costs (*ETC*) of the best procurement decisions, for $I_c \leq S_c^*$, is rather straight-forward. We observe the number of units which are not required to be purchased (this is equivalent to I_c) and subtract their acquisition cost ($I_c v_c$) from $ETC(S_c^*)$. Mathematically, we can express it as:

$$ETC(Q_c^* = S_c^* - I_c) = ETC(S_c^*) - I_c v_c \tag{8.6}$$

However, what about I_c values in excess of S_c^*? At first glance, one would suppose that the best procurement decision for this case is to procure 0 units. It would seem obvious that if the on-hand stock levels before any procurement decision are rather large, then it would be attractive to procure nothing. To a large extent, this is the case. However, for rather large I_c values (particularly those close to a construction phase requirements value, D_c), it may be attractive to procure a sufficient number of units to place one at an order-up-to-level equal to the D_c value. In these cases, it would appear that the ability to reduce one's stockout penalties (by procuring an amount equivalent to a D_c value) compensates for the increased acquisition and holding costs.

Numerical Example

Consider the following parameter values:

$D_1 = 100$ with $P_D(D_1) = 0.15$
$D_2 = 200$ with $P_D(D_2) = 0.35$
$D_3 = 300$ with $P_D(D_3) = 0.35$
$D_4 = 400$ with $P_D(D_4) = 0.15$
(each half of the construction phase has the same probability distribution)
$B_1 = \$3000$
$B_2 = 1.0$
$\alpha = 0.10$
$T_c = 1$ year (recall that the construction phase is divided into two equal halves; hence, each half is 6 months long)
$h = \$13$ per unit of inventory per year
$v_c = \$100$ per unit
$v_o = \$190$ per unit
$D_o = 20$ units per year
$A = \$250$ (recall that this parameter is only applicable for ongoing usage decisions)
$g = \$35$ per unit

Using our mathematical model, we find that S_c^* is 332 units. That is, for any $I_c \leq 332$, our best procurement decision in the second half of the construction phase is $(332 - I_c)$ units. We have also observed a "breakpoint" at $I_c = 387$ units. For any I_c such that $387 \leq I_c \leq 400$, the best procurement decision in the construction phase's latter half is $(400 - I_c)$ units. For any I_c such that $332 < I_c < 387$, or for any $I_c > 400$, the best procurement decision is zero units. In these latter cases, the on-hand stock levels are simply too large (or too far away from a D_c value) to merit a procurement.

We shall perform some limited numerical analysis in this Section. As we claimed in Section 8.1, if we want to observe if a particular model extension generates substantially different results from those obtained in our earlier model, we must compare "apples to apples". Specifically, if we wish to observe the effects of two procurement opportunities in the construction phase, we need to generate a probability distribution for the single opportunity case that is equivalent to the convolution of the two "half-construction phase" independent distributions used in the two opportunities case.

We suggest values of 100, 200, 300 and 400 units as the possible construction phase requirements values during either half of the construction phase. We shall analyze the following probability distributions: a "base case" one in which the probabilities are peaked in the middle and an increasing (decreasing) distribution in which the probabilities increase (decrease) as D_c increases. We stress that each half of the construction phase has an identical probability distribution (ie. we will not examine the case where different distributions are observed in either half).

Table 26 provides a list of the equivalent probability distributions for the cases of two procurement opportunities and one opportunity, respectively. Note that the single procurement opportunity (derived from the convolution of two independent distributions) has a 7-point discrete probability distribution. The possible requirements values range from 200 to 800 units. The computation of best procurement decisions (and percentage cost penalties for the non-integrated inventory management strategies) can be done by using the mathematical model illustrated in Chapter 5. The only difference between the current case and the earlier one (in Chapter 5) is that the one in this Section involves a 7-

point distribution, while the earlier one featured a 5-point distribution.

Table 26
Probability Distributions for Examining the case of Two Procurement Opportunities

	2 Opportunities		1 Opportunity	
	D_c	$P_D(D_c)$	D_c	$P_D(D_c)$
"Base" Case	100	0.15	200	0.0225
	200	0.35	300	0.1050
	300	0.35	400	0.2275
	400	0.15	500	0.2900
			600	0.2275
			700	0.1050
			800	0.0225
Increasing	100	0.10	200	0.01
	200	0.20	300	0.04
	300	0.30	400	0.10
	400	0.40	500	0.20
			600	0.25
			700	0.24
			800	0.16
Decreasing	100	0.40	200	0.16
	200	0.30	300	0.24
	300	0.20	400	0.25
	400	0.10	500	0.20
			600	0.10
			700	0.04
			800	0.01

We shall begin by providing the numerical results for the case of a single procurement opportunity as shown in Table 27. With the exception of the specific parameter tested in a particular treatment combination, we shall use the "base case" settings for the various parameter values. As we explained in Section 8.1, the particular treatment combinations chosen involve the ones that tended to generate the larger percentage cost penalties in our earlier work.

Table 27
Results - Single Procurement Opportunity (7-point Distribution)

Treatment Combination	Integrated (Exact)	All-disposal	Myopic
Intermediate	600	500 (1.20)	500 (1.20)
B_2 low	526	400 (2.66)	400 (2.66)
T_c low	600	500 (2.36)	500 (2.36)
D_c increasing	700	600 (1.34)	600 (1.34)
D_c decreasing	500	400 (0.82)	400 (0.82)
v_o high	600	500 (1.99)	500 (1.99)

We note that these results (particularly the relative values of the respective best procurement quantities for any specific case) are generally in line with the earlier results obtained for the 5-point distribution (see Table 6).

Table 28 provides the results for the case of two procurement opportunities.

Table 28
Results - Two Procurement Opportunities

Treatment Combination	Integrated (Exact)		All-disposal		Myopic	
	Q_c^*	S_c^*	Q_a^*	S_c^*	Q_m^*	S_c^*
Intermediate	400	332	400 (0.44)	300	400 (0.44)	300
B_2 low	400	323	300 (3.61)	200	300 (3.61)	200
T_c low	400	338	400 (0.65)	300	400 (0.65)	300
D_c increasing	400	410	400 (0.04)	400	400 (3.37)	300
D_c decreasing	400	300	300 (2.08)	200	300 (2.08)	200
v_o high	400	349	400 (1.05)	300	400 (1.05)	300

The reader will probably notice that our format of Table 28 is rather different from previous tables that have provided model results. Within each inventory management strategy, the left-most column provides the best procurement decision in the initial half of the construction phase. The other column shows the order-up-to level in the second half of the construction phase. For ease of readability, we have not shown the "breakpoint" values. Recall from our earlier discussion that these values comprise rather large I_c amounts (close to a D_c value) such that one chooses to order up to the D_c value, thereby reducing one's stockout penalties.

The "benefit" of incorporating the second procurement opportunity in our model can be determined by comparing the costs of the optimal solution for the case of one procurement opportunity and a 7-point distribution versus the costs of the optimal solution for the case of two procurement opportunities. Table 29 illustrates the percentage cost savings realized by incorporating the additional procurement opportunity for the six treatment combinations analyzed.

Table 29
Comparison of Costs of Optimal Solutions
One vs. Two Procurement Opportunities

Treatment Combination	Optimal Costs		Percentage Cost Savings
	With One Opportunity	With Two Opportunities	
Intermediate	\$96,599.35	\$92,143.30	4.61%
B_2 low	\$95,212.72	\$92,143.30	3.22%
T_c low	\$96,065.53	\$92,553.20	3.66%
D_c increasing	\$107,896.42	\$96,742.26	10.34%
D_c decreasing	\$87,544.33	\$86,897.53	0.74%
v_Q high	\$100,500.73	\$95,965.43	4.51%

The increasing construction phase requirements distribution, in particular, leads to a substantial advantage associated with the additional procurement opportunity. In this scenario, failure to adequately procure in the construction phase would lead to costly stockouts. The incorporation of another procurement chance allows a decision-maker the opportunity to bring in additional stock half-way through the construction phase. Consequently, stockout penalties decrease; in addition, the decision-maker is not forced to carry a huge quantity of stock throughout the entire construction phase (as would be required with only a single procurement opportunity).

A few brief comments are in order regarding the procedure we used to calculate the various percentage cost penalties. Consider, for example, the all-disposal inventory management strategy. We noted the all-disposal best procurement decision in the first half of the construction phase and the associated costs (ordering, carrying and stockout penalties). Using this quantity, and the possible requirements values in the initial half of the construction phase, we could then determine resulting inventory levels (I_c) at the half-way point of the construction phase. We then made use of the decision rule previously determined for the all-disposal strategy. This decision rule provided the best procurement decision for any level of I_c. As we did in the initial half of the construction phase, we costed out this procurement decision in terms of ordering, carrying and stockout charges. Moreover, we used the possible requirements values in the second half of the construction phase to generate various on-hand inventory levels at the conclusion of project construction. We then could use our $EPV^*(I)$ values to determine the expected present value of concluding the construction phase with a certain quantity of on-hand stock, and proceeding in an optimal fashion with respect to any disposal or ongoing phase replenishment decisions. We could then combine all our various expected costs to arrive at a total expected cost value. This total would be compared with the (optimal) total expected cost value generated by the integrated approach to determine percentage cost penalties.

The comparison of the single and two procurement opportunities case is somewhat unclear. Based on the parameter values we considered, it would appear that there are instances in which an additional procurement opportunity lessens the percentage cost penalty of adopting a non-integrated inventory management approach. "Carving" the construction phase into smaller "decision windows" provides a materials manager with a certain element of recourse. The "gap" between best first half construction phase decisions for any of the strategies narrows (in fact, for many of the treatment combinations, there is no difference between the respective best first half procurement decisions).

However, for the cases of B_2 at its low setting or a decreasing requirements distribution, there is a noticeable cost penalty involved with either non-integrated strategy. The first half procurement choices and second half order-up-to quantities are markedly different between the integrated and non-integrated approaches. It ought to be noted that both of these cases reduce the attractiveness of over-procuring in the construction phase (since per unit stockout penalties fall, or there is a rather large likelihood of observing a relatively small requirements value).

The increasing requirements distribution displays a rather interesting behaviour. The all-disposal order-up-to decision in the second half of the construction phase is rather close to the integrated decision. However, the myopic decision is far less. With the increasing distribution, the all-disposal strategy would provide rather large salvage revenues for larger order-up-to decisions. This enhances the appeal of increased procurement in the second half of the project. Under the myopic approach, there is no such appeal for extra procurement. Increased purchasing and holding costs outweigh the benefit of reduced stockout penalties as more units are procured. This generates a larger percentage cost penalty for the myopic inventory strategy.

We need to make one final point regarding the case of two procurement opportunities. Generally, we noted that one ordered more units during the first half of the construction phase than were ordered during the latter half. The procurements were not evenly split between the halves. Presumably, over-procuring during the first half of the construction phase protected one against initial half stockouts while also allowing one the opportunity to use any surplus units during the latter half of the construction phase. Consequently, one would not be required to procure as many units during the construction phase's second half.

This concludes our treatment of two important extensions to our model. We have illustrated how deterministic, time-varying usage and an additional construction phase procurement opportunity may be separately incorporated into our analysis. Although we have not exhaustively analyzed all treatment combinations, we have still been able to draw some helpful conclusions through the testing we have done.

9. CONCLUSIONS AND DIRECTIONS FOR FURTHER STUDY

We have reached the conclusion of this dissertation. This Chapter shall offer a few comments and provide some directions for further study.

It is probably a reasonable idea, at this stage, to reflect back upon the beginning of this dissertation. Recall that we opened our initial Chapter with a citation from a legal case involving two parties in a construction dispute. Apparently, the judge had some rather strong feelings regarding the logistics of large-scale projects. Based on our experience in this dissertation, we can strongly echo the judge's sentiments!

Materials management decisions within the context of large-scale projects are by no means trivial issues. We have developed in this dissertation a mathematical model to examine a set of critical decisions that logistics personnel must make when dealing with project inventories. When an important, expensive item is subject to quantity uncertainties during a project, managers have many factors to consider. Carrying units in stock could drain funds, but the organization could face a (severe) shortage cost if required items are not available when needed.

An additional factor that materials managers must tackle involves the disposal of surplus stock. Units may be disposed for revenue by, among other possibilities, returning them to the vendor, or selling them on a secondary market. Surplus stock may also be used to satisfy ongoing operational usage.

The likelihood of future projects could also impact current procurement and inventory decision-making. When the specific item is also required during a subsequent project (a rather logical assumption), then managers may opt to retain surplus units to satisfy some of the requirements during the next large-scale project.

One of the important outcomes of this research is that it allows project managers to obtain a "wider" perspective on critical materials management issues within large-scale projects. Our mathematical model developed in this dissertation has examined factors that relate to a project's construction phase, as well as those that involve ongoing phase and future project issues. We have illustrated the relationship between construction phase requirements and ongoing operational usage. We have examined the coordination of requirements for future projects. Moreover, we have incorporated the consideration of non-constant salvage values, a feature hitherto undeveloped in previous academic research. The increasing salvage values case, in particular, presents a set of complex mathematical decisions. In addition, through two separate model extensions, we showed how additional realistic scenarios could be incorporated into our analytical framework.

Another significant result of this dissertation research is that we have been able to generate decision rules to determine, for a wide range of salvage value functions, the appropriate quantity of surplus stock to dispose upon completion of project construction. Surplus stock disposal is a key problem encountered in project management. Our decision rules considered revenue received for surplus disposal, carrying charges associated with retained stock, and future repurchase costs of the specific item. Moreover, these decision rules allowed us to observe the effects (in terms of optimal disposal decisions) as per unit salvage values changed. Recall that, as we discussed in

Chapter 1, one of the important real-world issues involved in project logistics concerns the degree of product standardization. Our model uses higher per unit salvage values to reflect a more standardized item.

A further important benefit of our research is the treatment of an additional procurement opportunity within the project's construction phase. We were able to calculate the percentage cost savings associated with the incorporation of this second procurement chance, and to highlight those cases that provided larger cost savings.

We would suppose that our ultimate goal has been to develop a model that materials management personnel can use. To a certain extent, we believe this can happen. We have described how various decisions are affected based on the different parameter values used in our study. We have calculated cost penalties that arise as one makes non-integrated inventory management decisions. Further, we have shown how these penalties change as parameter values vary. We have highlighted those cases that lend themselves to larger penalties. In some cases (particularly for future projects and no ongoing usage (Section 7.1)), we have shown that non-integrated approaches perform rather well. Presumably, there are certain situations in which managers can solely concentrate on current project requirements. This would suggest that organizations, for some of the cases cited in this dissertation, have a certain degree of flexibility when making construction phase procurement decisions. They can choose to consider or ignore the future, being fully assured that the impact of considering the future is negligible.

Even though some of our percentage cost penalties were rather small, recall that we only performed one and two-way tests (varying one or two parameters at a time, depending upon the specific case). If we were to vary three, four or more parameters simultaneously, then the cost penalties of not considering surplus stock disposal and/or ongoing phase issues would certainly increase.

We wish to make a brief comment regarding the issue of spreadsheet modelling. It would appear that academic researchers are increasingly adopting the spreadsheet both as a means of model development and pedagogical support. We can certainly attest to the utility of the spreadsheet for solving (even complex) analytical problems. Every case considered in this dissertation has been analyzed with a spreadsheet model. Although one could have used a computer language (such as *Fortran*) to develop our model, the choice of a spreadsheet was motivated by a few key factors. First, we had considerable previous experience using spreadsheet models. This removed the "learning curve" that would have been present had we used a (previously unfamiliar) approach. Further, the spreadsheet model allowed us to carve the cost calculations into small pieces so we could fully appreciate what was happening at each stage. This made model-debugging fairly straightforward when we were faced with the model providing us some rather strange outcomes. Some have commented that the applicability of spreadsheet modelling is limited due to the rather small size of problems that can be tackled. However, we analyzed a case in which the construction phase requirements and ongoing phase usage rates were multiplied by a factor of 10 (ie. the D_c values ranged from 2000 to 6000 and the base case D_o value was 200). (We wish to stress here that this particular case has not been discussed earlier in the dissertation). Our spreadsheet model was still able to handle a problem of such a dimension. The computer time spent in calculating total expected costs was a little bit higher, but still not unnecessarily long. For example, our 486 DX2 66 computer determined the optimal solution for the "usual" set of parameter values in about 5 seconds. It took roughly 15 seconds to obtain the least-cost solution for the "factor of 10" case.

As with any piece of academic research, there are some limitations to our work. Our particular model has been developed for an environment that is dynamic and ever-changing. The specific set of decisions illustrated in our approach may occur differently in other projects, or other decision areas (e.g risk assignment for surplus material ownership) may become relevant. We must not be oblivious to the fact that our approach may need to adapt in order to be applicable for continued project management decision-making.

A further limitation of our work involves the issue of external validation. Recall that we interviewed materials management personnel employed with <u>one</u> company involved in large-scale projects. To what extent is our model and its associated findings applicable to other companies and the logistics issues they face? Of course, a method to determine the external validity of our research would be to test our model with data from other companies and their projects. Frankly, we see external validation as a continued, ongoing process.

There are some future areas of study that we wish to examine within the contexts of our mathematical model. Obviously, it would be appropriate to consider those combinations of scenarios that have not been addressed in this dissertation. Among others, we have not addressed the melding of future projects and two procurement opportunities, or non-constant salvage values and two procurement opportunities.

When we examined the issue of future projects, one of our main assumptions was that the time until the next project was modelled as a random variable following a discrete probability distribution. We note that one could incorporate continuous probability distributions (such as the normal or gamma) to represent inter-project time. However, our selection of a discrete distribution was practitioner-motivated. Recall that representative parameter values in our study were obtained via interviews with materials management and logistics personnel. We felt it was more reasonable to elicit discrete data from these practitioners (ie. a certain likelihood of a subsequent project occurring within a specific time interval), rather than querying them for continuous inter-project distributions. In essence, a greater understanding (and appreciation) of model development may lead to improved prospects of model implementation.

A decision that we did not treat involved the behaviour of Q_o in the case of future projects and deterministic, level ongoing usage. Recall in our earlier work that we assumed that decision-makers would not adjust Q_o as the next project became imminent. In reality, materials managers may choose to drop this ongoing phase replenishment quantity as the subsequent project draws near. One may choose to bring in "just enough" stock to cover one's requirements until the subsequent project. In that way, one would have very little (or nothing) on-hand when the next project began. One could then replenish a larger quantity of stock at v_s ($v_s < v_o$) to satisfy ongoing operational usage. Dynamic programming could potentially be used to analytically model this situation.

We treated the future projects as sequential "entities"; that is, they occurred only one at a time. An obvious extension would be to examine the case of multiple concurrent projects. This could also lead into logistics issues surrounding the use of multi-echelon storage facilities. If several large-scale projects are underway at the same time, it may not be attractive for each project to store all units of items on-site. If the projects are rather close in distance (or if transportation costs are not prohibitive), then one may be able to make use of a few large storage facilities to serve several projects at once. This could reduce storage duplication and expense.

There are several issues one could consider with respect to construction phase decisions. One of our basic assumptions was that this model analyzed a single item. We could extend our analysis to cover the multi-item situation, wherein several items could be procured at the same time. We have also analyzed the case of two procurement opportunities and assumed that the second procurement choice occurred exactly half-way through the construction phase. A logical extension would be to examine the issue of n opportunities. Furthermore, we said nothing about the optimal timing of any of the additional procurement decisions. If one could evaluate the expected costs of procuring specific quantities at various points throughout the construction phase, then presumably one could determine the best set of times at which to procure during a large-scale project. This would assist materials managers in project scheduling. However, we recognize the complexities associated with such an approach, particularly if one allowed dependent requirements distributions during each $1/n$ portion of the construction phase. (As an aside, we note that if one analyzes the special case of $n = 2$ procurement opportunities, the "best" procurement choice is likely to occur in the latter half of the construction phase).

We could also permit quantity discounts for construction phase procurement. The approach in this dissertation allowed a single (deterministic) unit acquisition cost, v_c. Using a quantity discount schedule for an item may produce a situation in which the best construction phase procurement decision is found at a "quantity break-point", rather than at a requirements value (as was often the case in our work).

We further note that other items (besides pipe, valves and motors) could be analyzed in our model. Of particular relevance could be the modelling of brick or glass procurement. Specifically, additional procurement opportunities for this item may not be advantageous due to product quality differences in respective shipments. In this case, materials managers may opt to procure rather large quantities of these items at the outset of a project. Doing so serves to increase the likelihood that <u>all</u> units of a particular shipment will have the same product characteristics.

In terms of ongoing phase operations, we have not allowed item obsolescence. This factor would be typical of an item prone to frequent product changes. If obsolescence were considered in our mathematical model, then this would likely lead to smaller retention decisions at the conclusion of the construction phase.

There are additional environments, besides large-scale project decisions, for which our approach may be applicable. Consider the case of a movie video rental outlet. In a fashion analogous to "construction phase procurement", managers at these establishments must make key decisions as to the appropriate number of video tapes of a particular movie to order. Customer rentals of a specific movie title would most likely be subject to uncertainty. Failure to procure sufficient copies of a movie to satisfy customer demand would lead to lost sales and, perhaps, the loss of consumer goodwill. At the conclusion of a movie's "run" in a rental outlet, a manager must make a critical decision as to the number of copies to release for sale to the general public (as "previously-viewed" copies) and the quantity to keep on-hand to satisfy ongoing, occasional usage. (The reader will most likely notice the similarity between these decisions and the "disposal" and "ongoing usage" decisions described earlier in the dissertation). An important difference between this situation and the project management scenario is that the video outlet manager must decide, in addition to the <u>sizing</u> of various procurement and disposal decisions, the <u>timing</u> of disposal of movie titles (ie. the particular moment in time at which to sell off copies of the movie). Recall that, in our project management study, we assumed that the end of the construction phase occurred at a well-defined point in time. We further note that, besides video rentals, our procurement and disposal approach could be applied to the case of a retailer of fashion/style goods.

These future research suggestions, as well as our findings in this dissertation, suggest that materials management issues in large-scale projects are worthy of study. One can link academic research and practitioner issues into a mathematical model that can be used to improve decision-making.

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Appendix A. Glossary of Notation

Notation	Description
A	Fixed cost of a replenishment
$A(k)$	The annuity stream (the level of a constant annual cash flow generating a particular NPV)
B_1	Fixed cost per stockout occasion
B_2	Penalty (expressed as a fraction of the unit value) per unit short
D_c	Total requirements in the construction phase (has a discrete probability distribution $P_D(D_c)$)
d_j	Number of units used in month j
D_k	Cumulative requirements up to and including month k
D_o	Annual usage rate in the ongoing phase
$EIPC$	Expected present value of the inter-project costs per surplus unit retained
$EPV^*(I)$	Expected present value of all future costs associated with concluding the construction phase with I units of inventory on-hand, and proceeding in an optimal fashion from thereon (with respect to disposal and ongoing phase replenishment decisions)
$EPV^*(I_c)$	Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units of inventory on-hand, and proceeding in an optimal fashion from thereon (with respect to any second half procurement, disposal and ongoing phase replenishment decisions)
$EPV_a^*(I_c)$	Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units on-hand and making the best procurement decisions in the second half of the construction phase, followed by the disposal of all surplus units
$EPV_m^*(I_c)$	Expected present value of all future costs associated with concluding the first half of the construction phase with I_c units on-hand and making the best procurement decisions in the second half of the construction phase, considering only construction phase costs
$ETC(Q_c)$	Expected total discounted costs as a function of construction phase procurement quantity

Notation	Description
g	Per unit salvage value for surplus disposals (constant salvage value case)
g_i	Per unit salvage value for surplus disposals (non-constant salvage value cases)
h	Out-of-pocket inventory carrying charges ($ per unit of inventory per unit time)
h'	h/a
i	A particular time interval
i^*	The first time interval in which we begin ongoing phase replenishments
I	On-hand surplus after completion of project construction phase
I_c	On-hand stock at the conclusion of the first half of the construction phase
I_s	(Approximate) on-hand inventory when a subsequent project occurs
IP_{0i}	Inventory level at which it first becomes attractive to go from making no disposals to disposing a certain L_i (increasing salvage value case)
IP_{ki}	Inventory level at which it first becomes attractive to jump from disposing at an L_k plateau to disposing at an L_i plateau (increasing salvage value case)
IP_{kri}	Inventory level at which it first becomes attractive to jump from disposing along a ramp with salvage value g_k to disposing at an L_i plateau (increasing salvage value case)
L_i	Minimum number of units which must be disposed to earn g_i per unit disposed (increasing salvage value case)
m	Number of increasing salvage values
M	Number of units retained after disposal decision
N_i	Maximum number of units that can be disposed for g_i per unit disposed (applicable for the marginally decreasing salvage value case)
$P_D(D_c)$	Probability distribution of total requirements in the construction phase
p_i	Probability of a subsequent project beginning within $[t_{i-1}, t_i]$
Q	Replenishment quantity in the ongoing phase
Q_a^*	Optimal procurement quantity produced by following the all-disposal inventory management strategy

Notation	Description
Q_c	Construction phase procurement quantity
Q_c^*	Optimal procurement quantity produced by the integrated inventory management strategy (using the exact costing approach in the construction phase)
Q_{ip}	A procurement quantity that yields a discontinuity in $ETC(Q_c)$, due to the presence of increasing salvage values
Q_m^*	Optimal procurement quantity produced by following the myopic inventory management strategy
Q_o	Optimal replenishment quantity in the ongoing phase
S	The quantity of stock ordered at v_s to satisfy ongoing operational usage from the original project
S_c^*	The order-up-to level in the second half of the construction phase for the case of $I_c = 0$
T_c	Duration of construction phase (in years)
t_i	The end-point of time interval i associated with a subsequent project ($i = 1,2,\ldots,n$)
U_i	Maximum number of units which can be disposed to earn g_i per unit disposed (increasing salvage value case)
v_c	Unit acquisition cost in the construction phase (original project)
v_o	Unit acquisition cost in the ongoing phase
v_s	Unit acquisition cost in a subsequent project
W	Number of units disposed
$Z(Q)$	Present value of an infinite series of ongoing phase replenishments of size Q ($Z(Q_o)$ represents the present value of future replenishments associated with the optimal replenishment quantity)
α	Continuous discount rate (ie. a cost of x incurred at time t has a present value of $xe^{-\alpha t}$)
α'	α/D_o
τ_i	Exact time until a subsequent project that occurs in the ith interval

Appendix B. Equivalence of the Present Value Expressions Under Deterministic Level and Probabilistic (Poisson) Usage in the Ongoing Phase

(In a parallel development, Teunter and Haneveld (1995) generated similar results for the present value of an infinite series of inventory cycles).

When Poisson usage (with annual rate λ) is introduced in place of deterministic, level usage, the calculation of discounted carrying costs within an inventory cycle changes somewhat. We now hold one unit for time t_1, one unit for time $t_1 + t_2$, and so on until the last unit which is held for time $t_1 + t_2 + \ldots + t_{Q-1}$. Note that this assumes that we start an inventory cycle with Q-1 units (ie. we bring in Q units just as a unit is used in the ongoing phase).

With Poisson usage, the times between usage of consecutive units is exponential and the sum of independent, identical exponential variables has an Erlang distribution. Therefore, in general, we are paying continuous out-of-pocket expenses on one unit for an Erlang time and these charges must be continuously discounted to time 0. Thus, the carrying charges under Poisson usage may be represented as:

$$\sum_{k=1}^{Q-1} \int_0^{\infty} \left[\int_0^{t} he^{-\alpha x} dx \right] \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} dt \qquad \text{(B.1)}$$

The imbedded integral represents the continuous out-of-pocket expenses. Evaluating that integral gives:

$$h\left[\frac{1}{\alpha} - \frac{e^{-\alpha t}}{\alpha}\right]$$

This represents the present value of carrying one unit for a time t.

We may express (B.1) as:

$$\sum_{k=1}^{Q-1} \int_{t=0}^{\infty} \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} PV(carrying\ costs\ of\ 1\ unit\ |t)\, dt \tag{B.2}$$

which is:

$$\sum_{k=1}^{Q-1} h \int_{t=0}^{\infty} \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} \left(\frac{1}{\alpha} - \frac{e^{-\alpha t}}{\alpha}\right) dt$$

This then becomes:

$$\sum_{k=1}^{Q-1} \left[\frac{h}{\alpha} - \frac{h}{\alpha} \int_{0}^{\infty} \frac{\lambda^k t^{k-1} e^{-(\lambda+\alpha)t}}{(k-1)!} dt\right] \tag{B.3}$$

Equation (B.3) may be represented as:

$$\sum_{k=1}^{Q-1} \left[\frac{h}{\alpha} - \frac{h}{\alpha}\left(\frac{\lambda}{\lambda+\alpha}\right)^k \int_{0}^{\infty} \frac{(\lambda+\alpha)^k t^{k-1} e^{-(\lambda+\alpha)t}}{(k-1)!} dt\right] \tag{B.4}$$

The integral in (B.4) equals 1, since the integrand represents the general form of an Erlang distribution with parameter $(\lambda + \alpha)$.

As a result, the total discounted (to time 0) carrying charges, in one inventory cycle, can be given as:

$$\sum_{k=1}^{Q-1} \frac{h}{\alpha}\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^k\right]$$

which become:

$$h\left[\frac{(Q-1)}{\alpha} - \frac{1}{\alpha}\sum_{k=1}^{Q-1}\left(\frac{\lambda}{\lambda+\alpha}\right)^k\right] \tag{B.5}$$

Letting:

$$e^{-\alpha'} = \frac{\lambda}{\lambda+\alpha} \tag{B.6}$$

and:

$$\alpha' = \frac{\alpha}{D_o}$$

we can express (B.5) as:

$$h\left[\frac{Q-1}{\alpha} - \frac{1}{\alpha}\sum_{k=1}^{Q-1} e^{-k\alpha'}\right] \tag{B.7}$$

The summation term in (B.7) can be written as:

$$e^{-\alpha'}\left(1 + e^{-\alpha'} + e^{-2\alpha'} + \ldots + e^{-(Q-2)\alpha'}\right)$$

The term in parentheses can be expressed as:

$$\frac{1 - e^{-(Q-1)\alpha'}}{1 - e^{-\alpha'}}$$

which gives the following for the present value of continuously discounted inventory carrying charges within one cycle:

$$h\left[\frac{Q-1}{\alpha} - \frac{e^{-\alpha'}}{\alpha}\left(\frac{1 - e^{-(Q-1)\alpha'}}{1 - e^{-\alpha'}}\right)\right]$$

The total costs incurred in one inventory cycle, under Poisson usage, then are:

$$A + Qv_o + h\left[\frac{Q-1}{\alpha} - \frac{e^{-\alpha'}}{\alpha}\left(\frac{1 - e^{-(Q-1)\alpha'}}{1 - e^{-\alpha'}}\right)\right]$$

The present value of all future inventory cycles, *Z(Q)*, is then:

$$\frac{A + Qv_o + h\left[\frac{Q-1}{\alpha} - \frac{e^{-\alpha'}}{\alpha}\left(\frac{1 - e^{-(Q-1)\alpha'}}{1 - e^{-\alpha'}}\right)\right]}{1 - e^{-Q\alpha'}}$$

Similar algebraic manipulation as described in Chapter 3 will give the following for Z(Q):

$$\frac{A + Q(v_o + h')}{1 - e^{-Q\alpha'}} - \frac{h'}{1 - e^{-\alpha'}} \qquad \text{(B.8)}$$

Equation (B.8) is identical to (3.7), thus showing the equivalence, through the transformation of (B.6), between the present value expressions when using either deterministic level or Poisson usage.

Appendix C. Proof that Z(Q) is Convex

(Sincere appreciation is extended to Dr. Peter Ehlers, University of Calgary, for the considerable assistance provided in the development of this proof).

A simplified form of equation (3.7), recognizing only those terms which depend upon Q, can be given as follows:

$$Z(Q) = \frac{a + bx}{1 - e^{-cx}} \tag{C.1}$$

where a, b and c are (strictly positive) constants.

We are interested in the convexity of $Z(Q)$, where Q is restricted to integer values. Strictly speaking, a function defined on integers cannot be convex, since convexity requires continuity. However, it will suffice for our present purposes to show that the $Z(Q)$ function consists of points on an associated function which is convex.

The second derivative of $Z(Q)$ is:

$$\frac{ce^{-cx}}{(1-e^{-cx})^3} * 2b(1+e^{-cx}) * \left[\frac{ac}{2b} + \frac{cx}{2} - \frac{1-e^{-cx}}{1+e^{-cx}}\right]$$

which, from the definition of a hyberbolic tangent, is:

$$\frac{ce^{-cx}}{(1-e^{-cx})^3} * 2b(1+e^{-cx}) * \left[\frac{ac}{2b} + \frac{cx}{2} - \tanh\left(\frac{cx}{2}\right)\right]$$

Since $z - \tanh(z) \geq 0$ for $z \geq 0$, we can conclude that the second derivative of Z(Q) is convex.

As a result, once the function begins to increase, it will continue increasing.

Appendix D. The EOQ as an Approximation of Q_o

Recall that the optimal replenishment quantity in the ongoing phase is the smallest integer Q that satisfies the following nonlinear equation:

$$e^{Q\alpha'} > 1 + \frac{\left(1 - e^{-\alpha'}\right)\left(\frac{A}{v_o} + Q\left(1 + \frac{h'}{v_o}\right)\right)}{1 + \frac{h'}{v_o}} \tag{D.1}$$

Since $\alpha' << 1$, let us use a first-order approximation of $(1 - e^{\alpha'})$. Thus, $(1 - e^{\alpha'}) \approx \alpha'$. We can rewrite (D.1) as:

$$e^{Q\alpha}\left(1 + \frac{h'}{v_o}\right) > 1 + \frac{h'}{v_o} + \alpha'\left(\frac{A}{v_o} + Q\left(1 + \frac{h'}{v_o}\right)\right) \tag{D.2}$$

Using a second-order approximation in place of $e^{Q\alpha'}$ gives:

$$\left(1 + Q\alpha' + \frac{Q^2\alpha'^2}{2}\right)\left(1 + \frac{h'}{v_o}\right) > 1 + \frac{h'}{v_o} + \alpha'\left(\frac{A}{v_o} + Q\left(1 + \frac{h'}{v_o}\right)\right) \tag{D.3}$$

Multiplication of terms in (D.3) and simplifying the result yields:

$$\frac{Q^2\alpha'^2}{2} + \frac{Q^2\alpha'^2 h'}{2v_o} > \frac{\alpha' A}{v_o}$$

Recognizing that $\alpha' = \alpha/D_o$ and that $h' = h/\alpha$, we have:

$$\frac{Q^2\alpha^2}{2D_o^2} + \frac{Q^2\alpha h}{2v_o D_o^2} > \frac{\alpha A}{v_o D_o} \tag{D.4}$$

Multiplying all terms in (D.4) by D_o/α gives:

$$\frac{Q^2}{2D_o}\left(\alpha+\frac{h}{v_o}\right) > \frac{A}{v_o}$$

which can be expressed as:

$$Q^2 > \frac{2AD_o}{h+\alpha v_o} \tag{D.5}$$

Taking the square root of (D.5) leaves us with the expression for the EOQ:

$$Q > \sqrt{\frac{2AD_o}{h+\alpha v_o}}$$

Consequently, the value provided by the EOQ expression offers a good place at which to initiate the search for the optimal replenishment quantity in the ongoing phase.

Appendix E. Proof Regarding Logarithmic Argument in Equation (4.9)

We may write (4.9) as:

$$M > \frac{1}{\alpha'}\ln(f) - 1 \tag{E.1}$$

where:

$$f = \frac{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}{g + h'}$$

We need to show that $f > 1$. Otherwise, we could obtain an invalid result for M^*. If $f > 1$, this implies that:

$$Z(Q_o)\left(1 - e^{-\alpha'}\right) > g \tag{E.2}$$

Obviously, the largest feasible value for g is v_o (the ongoing phase unit acquisition cost). Hence, if (E.2) is valid for $g = v_o$, then it will be true for any permissible salvage value.

Substituting $g = v_o$ into (E.2) and using (3.7) yields:

$$\left[\frac{A + Q_o(v_o + h')}{1 - e^{-Q_o\alpha'}} - \frac{h'}{1 - e^{-\alpha'}}\right]\left(1 - e^{-\alpha'}\right) > v_o \tag{E.3}$$

which gives:

$$\left[A + Q_o(v_o+h')\right]\left(\frac{1 - e^{-\alpha'}}{1 - e^{-Q_o\alpha'}}\right) > v_o + h'$$

This can be expressed as:

$$A + Q_o(v_o+h') > \left(\frac{1 - e^{-Q_o\alpha'}}{1 - e^{-\alpha'}}\right)(v_o + h') \tag{E.4}$$

Thus, in order for the expression within the logarithmic argument in (4.9) to be > 1, all we need to show is that the left-hand side of (E.4) will always exceed the right-hand side. This boils down to showing that:

$$Q_o \geq \left(\frac{1 - e^{-Q_o\alpha'}}{1 - e^{-\alpha'}}\right) \tag{E.5}$$

Note that (E.5) can be written as:

$$Q_o \geq \frac{1 - r^{Q_o}}{1 - r} \tag{E.6}$$

where $r = e^{-\alpha'}$.

For $Q_o = 2$, the right-hand side of (E.6) yields (noting that $r < 1$):

$$\frac{1 - r^2}{1 - r} = \frac{(1 + r)(1 - r)}{1 - r} = 1 + r < Q_o = 2$$

For $Q_o = 3$, we have the following:

$$\frac{1 - r^3}{1 - r} = \frac{(1 + r + r^2)(1 - r)}{1 - r} = 1 + r + r^2 < Q_o = 3$$

Thus, in general, the right-hand side of (E.6) is:

$$\sum_{n=0}^{Q_o - 1} r^n \tag{E.7}$$

which is always less than Q_o since $r < 1$. As a result, the expression within the logarithmic argument of (4.9) will always be > 1. The logarithmic value will be well defined and the economic retention quantities will be positive.

Appendix F. Proof Regarding Optimal Retention Quantities When $g = v_o$

Recall that the expression for the optimal retention quantity involves selecting the smallest integer M such that:

$$M > \frac{1}{\alpha'} \ln\left[\frac{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'}{g + h'}\right] - 1 \tag{F.1}$$

where:

$$Z(Q_o) = \frac{A + Q_o(v_o + h')}{1 - e^{-Q_o\alpha'}} - \frac{h'}{1 - e^{-\alpha'}} \tag{F.2}$$

We wish to determine what one can conclude when the salvage value, g, is equal to the ongoing phase unit acquisition cost, v_o.

Since we were able to show that the associated continuous function for $Z(Q)$ was convex (see Appendix C), let us begin by evaluating the first derivative of $Z(Q)$ with respect to Q.

We have:

$$\frac{d}{dQ}Z(Q) = \frac{\left(1 - e^{-Q\alpha'}\right)\left(v_o + h'\right) - \left(A + Q(v_o + h')\right)\left(\alpha' e^{-Q\alpha'}\right)}{\left(1 - e^{-Q\alpha'}\right)^2} \tag{F.3}$$

At the minimum point, Q_o, the derivative must be equal to 0. This implies that the numerator of the derivative must equal 0.

As a result, we have:

$$\left(1 - e^{-Q_o\alpha'}\right)\left(v_o + h'\right) = \left(A + Q_o(v_o+h')\right)\left(\alpha' e^{-Q_o\alpha'}\right) \tag{F.4}$$

Simplifying (F.4) leads to:

$$\frac{A + Q_o(v_o+h')}{1 - e^{-Q_o\alpha'}} = \frac{(v_o+h')\, e^{Q_o\alpha'}}{\alpha'} \tag{F.5}$$

Observe that the left-hand side of (F.5) is equivalent to a portion of (F.2). Thus, one can substitute the right-hand side of (F.5) into (F.2). This yields the following:

$$Z(Q_o) = \frac{(v_o+h')\, e^{Q_o\alpha'}}{\alpha'} - \frac{h'}{1 - e^{-\alpha'}} \tag{F.6}$$

Replacing g with v_o and using (F.6), we can express (F.1) as:

$$M > \frac{1}{\alpha'} \ln\left(\frac{\left[\frac{(v_o+h')\, e^{Q_o\alpha'}}{\alpha'} - \frac{h'}{1 - e^{-\alpha'}}\right]\left(1 - e^{-\alpha'}\right) + h'}{v_o + h'}\right) - 1 \tag{F.7}$$

Algebraic manipulation of (F.7) gives us the following:

$$M > \frac{1}{\alpha'} \ln\left[\frac{e^{Q_o\alpha'}}{\alpha'}\left(1 - e^{-\alpha'}\right)\right] - 1 \tag{F.8}$$

Since $\alpha' \ll 1$, let us use a first-order approximation on $(1 - e^{-\alpha'})$. This means that $(1 - e^{-\alpha'})$ can be approximated as α'. We can rewrite (F.8) as:

$$M > \frac{1}{\alpha'} \ln\left[e^{Q_o \alpha'}\right] - 1$$

which becomes:

$$M > Q_o - 1$$

As a result, when salvage values equal the ongoing phase unit acquisition costs, the optimal retention quantity is the smallest integer greater than Q_o - 1 (or equal to Q_o - 1, should this quantity already be an integer value). In other words, we dispose down to the top of the optimal, ongoing inventory cycle.

Appendix G. Behaviour of Cost Function for Increasing Salvage Values

Suppose $\exists$ some I, $L_k + M_k^* < I < M_k^* + U_k$, such that it is attractive to jump from a ramp with salvage value g_k to a plateau earning g_i per unit disposed ($g_k < g_i$). Will it still be attractive to jump to this higher salvage value plateau when $I = M_k^* + U_k$? (ie. at the end of the ramp). If so, then our cost function will never display the counter-intuitive behaviour of moving back to the lower salvage value ramp.

Let us analyze the general situation in which one jumps somewhere along the g_k salvage value ramp to a plateau at a certain L_i, where $L_i = U_k + 1 + \gamma$ $(\gamma > 0)$.

At the L_i plateau, we dispose $L_i = U_k + 1 + \gamma$ units. We retain $I - L_i =$ $I - U_k - 1 - \gamma$ units.

Using our usual cost expressions from section 4.3, we have that PV (Plateau) is:

$$-(U_k+1+\gamma)g_i + h'\left[(I-U_k-1-\gamma) - e^{-\alpha}\left(\frac{1-e^{-(I-U_k-1-\gamma)\alpha}}{1-e^{-\alpha}}\right)\right] + e^{-(I-U_k-\gamma)\alpha'} Z(Q_o) \qquad \text{(G.1)}$$

Expanding (G.1) gives:

$$-U_k g_i - g_i - \gamma g_i + h'I - h'U_k - h' - h'\gamma - \frac{h'e^{-\alpha'}}{1-e^{-\alpha'}} + e^{-(I-U_k-\gamma)\alpha'}(c) \qquad \text{(G.2)}$$

where c is as defined in section 4.3.

Along the ramp with salvage value g_k, we dispose $I - M_k^*$ units and retain M_k^* units. Thus, PV (Ramp) is:

$$-(I-M_k^*)g_k + h'\left[M_k^* - e^{-\alpha'}\left(\frac{1 - e^{-M_k^*\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M_k^*+1)\alpha'}Z(Q_o) \qquad \text{(G.3)}$$

which becomes:

$$-(I-M_k^*)g_k + h'M_k^* - \frac{h'e^{-\alpha'}}{1 - e^{-\alpha'}} + e^{-(M_k^*+1)\alpha'}(c) \qquad \text{(G.4)}$$

Since PV (Plateau) < PV (Ramp), this implies that (G.2) - (G.4) < 0. Thus:

$$-U_k g_i - g_i - \gamma g_i + h'I - h'U_k - h' - h'\gamma + e^{-(I-U_k-\gamma)\alpha'}(c)$$

$$+ (I-M_k^*)g_k - h'M_k^* - e^{-(M_k^*+1)\alpha'}(c) < 0 \qquad \text{(G.5)}$$

Let us now describe the behaviour of the cost functions at $I = M_k^* + U_k$.

At the L_i plateau, we dispose $L_i = U_k + 1 + \gamma$ units. We retain $I - L_i =$ $M_k^* + U_k - U_k - 1 - \gamma = M_k^* - 1 - \gamma$ units.

Thus, PV (Plateau) is:

$$-(U_k+1+\gamma)g_i + h'\left[(M_k^*-1-\gamma) - e^{-\alpha'}\left(\frac{1 - e^{-(M_k^*-1-\gamma)\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M_k^*-\gamma)\alpha'}Z(Q_o) \qquad \text{(G.6)}$$

which is:

$$-U_k g_i - g_i - \gamma g_i + h'M_k^* - h' - h'\gamma - \frac{h'e^{-\alpha'}}{1 - e^{-\alpha'}} + e^{-(M_k^*-\gamma)\alpha'}(c) \qquad \text{(G.7)}$$

Along the ramp with salvage value g_k, we dispose $I - M_k^* = M_k^* + U_k - M_k^* = U_k$

units. A total of M_k^* units are retained. This gives PV (Ramp) as:

$$-U_k g_k + h'\left[M_k^* - e^{-\alpha'}\left(\frac{1 - e^{-M_k^*\alpha'}}{1 - e^{-\alpha'}}\right)\right] + e^{-(M_k^*-1)\alpha'}Z(Q_o) \qquad \text{(G.8)}$$

which is:

$$-U_k g_k + h'M_k^* - \frac{h'e^{-\alpha'}}{1 - e^{-\alpha'}} + e^{-(M_k^*-1)\alpha'}(c) \qquad \text{(G.9)}$$

The difference between PV (Plateau) and PV (Ramp) at $I = M_k^* + U_k$ is simply:

$$-U_k g_i - g_i - \gamma g_i - h' - h'\gamma + e^{-(M_k^*-\gamma)\alpha'}(c) + U_k g_k - e^{-(M_k^*-1)\alpha'}(c) \qquad \text{(G.10)}$$

Let us define the following terms:

$$A = -U_k g_i - g_i - \gamma g_i - h' - h'\gamma - e^{-(M_k^*-1)\alpha'}(c)$$

$$B(I) = h'I - h'U_k + e^{-(I-U_k-\gamma)\alpha'}(c) + (I-M_k^*)g_k - h'M_k^*$$

$$C(M_k^*+U_k) = e^{-(M_k^*-\gamma)\alpha'}(c) + U_k g_k$$

We know that, from (G.5):

$$A + B(I) < 0$$

The difference between PV (Plateau) and PV (Ramp) at $I = M_k^* + U_k$ can be written as:

$$A + C(M_k^*+U_k)$$

Therefore, to show that PV (Plateau) - PV (Ramp) < 0 at $I = M_k^* + U_k$, all we need to show is that:

$$F = B(I) - C(M_k^* + U_k) > 0 \tag{G.11}$$

Defining:

$$I = M_k^* + L_k + \delta$$

we can write F as:

$$F = h'(M_k^* + L_k + \delta) - h'U_k + e^{-(M_k^* + L_k + \delta - U_k - \gamma)\alpha'}(c)$$

$$+ (L_k + \delta)g_k - h'M_k^* - e^{-(M_k^* - \gamma)\alpha'}(c) - U_k g_k \tag{G.12}$$

Equation (G.12) may be simplified as:

$$F = -(h' + g_k)(U_k - L_k - \delta) + \left[e^{-(M_k^* + L_k + \delta - U_k - \gamma)\alpha'} - e^{-(M_k^* - \gamma)\alpha'}\right](c) \tag{G.13}$$

Letting:

$$R = U_k - L_k - \delta$$

we may express (G.13) as:

$$F = -(h' + g_k)R + \left[e^{-(M_k^* - R - \gamma)\alpha'} - e^{-(M_k^* - \gamma)\alpha'}\right](c)$$

which becomes:

$$F = -(h' + g_k)R + e^{-(M_k^* - \gamma)\alpha'}(c)\left[e^{R\alpha'} - 1\right] \tag{G.14}$$

In essence, to show that $F > 0$, we need to prove that:

$$e^{-(M_k^* - \gamma)\alpha'}(c)\left[e^{R\alpha'} - 1\right] > (h' + g_k)R \tag{G.15}$$

We could accomplish this by showing that the inequality in (G.15) holds for $\exp(-M_i^*)$. Obviously, if it holds for $\exp(-M_i^*)$, it will continue to hold for $\exp(-(M_i^* - \gamma))$, since the latter term is larger than the former.

From the derivation for M_i^* in Chapter 4, we know that the optimal retention quantity is the smallest integer M such that:

$$e^{-(M_k - 1)\alpha'} < \frac{g_k + h'}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'} \tag{G.16}$$

At $M_k^* - 1$, the inequality in (G.16) is reversed. Thus, we have that:

$$e^{-M_k\alpha'} > \frac{g_k + h'}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'} \tag{G.17}$$

Recognizing that $\exp(-(M_i^* - \gamma)) > \exp(-M_i^*)$, we can substitute (G.17) into (G.14) to give:

$$F > -(h' + g_k)R + \frac{(g_k + h')c\left[e^{R\alpha'} - 1\right]}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'} \tag{G.18}$$

which is:

$$\frac{F}{h' + g_k} > -R + \frac{c\left[e^{R\alpha'} - 1\right]}{Z(Q_o)\left(1 - e^{-\alpha'}\right) + h'} \tag{G.19}$$

Using the definition of c, we may express (G.19) as:

$$\frac{F}{h'+g_k} > -R + \frac{\left[\frac{h'}{1-e^{-\alpha'}} + Z(Q_o)\right]\left[e^{R\alpha'} - 1\right]}{Z(Q_o)\left(1-e^{-\alpha'}\right) + h'}$$

which gives:

$$\frac{F}{h'+g_k} > -R + \frac{\frac{1}{1-e^{-\alpha'}}\left[h' + Z(Q_o)\left(1-e^{-\alpha'}\right)\right]\left[e^{R\alpha'} - 1\right]}{Z(Q_o)\left(1-e^{-\alpha'}\right) + h'}$$

which then becomes:

$$\frac{F}{h'+g_k} > -R + \frac{e^{R\alpha'}-1}{\left(1-e^{-\alpha'}\right)} \tag{G.20}$$

Now, if we can show that:

$$\frac{e^{R\alpha'}-1}{1 - e^{-\alpha'}} > R \tag{G.21}$$

we are done. If (G.21) is true, then $F > 0$. Should F be > 0, than $B(I) > C(M_k^{*} + U_k)$.

Let us define:

$$G(R) = \frac{e^{R\alpha'}-1}{1 - e^{-\alpha'}} - R$$

If we can show that $\Delta G(R) = G(R+1) - G(R) > 0$, then the inequality in (G.21) will hold, and F will be strictly positive.

$$\Delta G(R) = \frac{e^{(R+1)\alpha'} - 1}{1 - e^{-\alpha'}} - \frac{e^{R\alpha'} - 1}{1 - e^{-\alpha'}} - 1$$

which becomes:

$$\frac{e^{(R+1)\alpha'} - e^{R\alpha'}}{1 - e^{-\alpha'}} - 1$$

This may be expressed as:

$$\frac{e^{R\alpha'}\left[e^{\alpha'} - 1\right]}{1 - e^{-\alpha'}} - 1 = \frac{e^{R\alpha'}\left[e^{\alpha'}\left(1 - e^{-\alpha'}\right)\right]}{1 - e^{-\alpha'}} - 1$$

which is:

$$e^{R\alpha'} e^{\alpha'} - 1 \qquad \text{(G.22)}$$

This term in (G.22) must be positive, since $\alpha' > 0$. Thus, for any $R > 0$, F will be strictly positive. As a result, if it is attractive to jump up to the higher plateau somewhere along a ramp, it will <u>still</u> be attractive to make the same decision when $I = M_k^* + U_k$ (at the end of the ramp).

Appendix H. Pair-wise Comparisons - Constant Salvage Values

B_1 vs. B_2

B_1	All-Disposal			Myopic		
High	400 (0.58)	400 (1.76)	400 (3.39)	400 (0.58)	400 (1.76)	400 (3.39)
Middle	400 (0.58)	400 (1.54)	400 (3.17)	300 (4.18)	400 (1.54)	400 (3.17)
Low	400 (0.58)	400 (1.35)	400 (2.96)	300 (3.74)	400 (1.35)	400 (2.96)
	Low	Middle	High	Low	Middle	High
	B_2					

B_1 vs. h

B_1	All-Disposal			Myopic		
High	400 (2.53)	400 (1.76)	400 (1.07)	400 (2.53)	400 (1.76)	400 (1.07)
Middle	400 (2.31)	400 (1.54)	400 (0.99)	400 (2.31)	400 (1.54)	400 (0.99)
Low	400 (2.09)	400 (1.35)	400 (0.99)	400 (2.09)	400 (1.35)	400 (0.99)
	Low	Middle	High	Low	Middle	High
	h					

B_1 vs. a

B_1	All-Disposal			Myopic		
High	400 (2.14)	400 (1.76)	400 (1.29)	400 (2.14)	400 (1.76)	400 (1.29)
Middle	400 (1.94)	400 (1.54)	400 (1.15)	400 (1.94)	400 (1.54)	400 (1.15)
Low	400 (1.74)	400 (1.35)	400 (1.15)	400 (1.74)	400 (1.35)	400 (1.15)
	Low	Middle	High	Low	Middle	High
	a					

B_I vs. T_c

B_I	All-Disposal				Myopic		
	High	400 (3.10)	400 (1.76)	400 (0.72)	400 (3.10)	400 (1.76)	400 (0.72)
	Middle	400 (2.87)	400 (1.54)	400 (0.72)	400 (2.87)	400 (1.54)	400 (0.72)
	Low	400 (2.65)	400 (1.35)	400 (0.72)	400 (2.65)	400 (1.35)	400 (0.72)
		Low	Middle	High	Low	Middle	High
	T_c						

B_I vs. A

B_I	All-Disposal				Myopic		
	High	400 (1.36)	400 (1.76)	400 (2.05)	400 (1.36)	400 (1.76)	400 (2.05)
	Middle	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
	Low	400 (1.09)	400 (1.35)	400 (1.62)	400 (1.09)	400 (1.35)	400 (1.62)
		Low	Middle	High	Low	Middle	High
	A						

B_I vs. D_o

B_I	All-Disposal				Myopic		
	High	400 (0.88)	400 (1.76)	400 (2.71)	400 (0.88)	400 (1.76)	400 (2.71)
	Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
	Low	400 (0.88)	400 (1.35)	400 (2.35)	400 (0.88)	400 (1.35)	400 (2.35)
		Low	Middle	High	Low	Middle	High
	D_o						

B_1 vs. v_a

B_1		All-Disposal			Myopic		
	High	400 (0.65)	400 (1.76)	400 (2.88)	400 (0.65)	400 (1.76)	400 (2.88)
	Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
	Low	400 (0.65)	400 (1.35)	400 (2.46)	400 (0.65)	400 (1.35)	400 (2.46)
		Low	Middle	High	Low	Middle	High
		v_a					

B_1 vs. g

B_1		All-Disposal			Myopic		
	High	400 (1.46)	400 (1.76)	400 (3.12)	400 (1.46)	400 (1.76)	400 (3.12)
	Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
	Low	400 (1.26)	400 (1.35)	400 (1.68)	400 (1.26)	400 (1.35)	400 (1.68)
		Low	Middle	High	Low	Middle	High
		g					

B_2 vs. h

B_2		All-Disposal			Myopic		
	High	400 (3.98)	400 (3.17)	400 (2.46)	400 (3.98)	400 (3.17)	400 (2.46)
	Middle	400 (2.31)	400 (1.54)	400 (0.99)	400 (2.31)	400 (1.54)	400 (0.99)
	Low	400 (0.88)	400 (0.58)	400 (0.37)	400 (0.88)	300 (4.18)	300 (3.46)
		Low	Middle	High	Low	Middle	High
		h					

B_2 vs. α

B_2	All-Disposal			Myopic		
	Low	Middle	High	Low	Middle	High
High	400 (3.43)	400 (3.17)	400 (2.81)	400 (3.43)	400 (3.17)	400 (2.81)
Middle	400 (1.94)	400 (1.54)	400 (1.15)	400 (1.94)	400 (1.54)	400 (1.15)
Low	400 (0.73)	400 (0.58)	400 (0.45)	400 (0.73)	300 (4.18)	300 (3.95)
	α					

B_2 vs. T_c

B_2	All-Disposal			Myopic		
	Low	Middle	High	Low	Middle	High
High	400 (4.57)	400 (3.17)	400 (1.85)	400 (4.57)	400 (3.17)	300 (1.85)
Middle	400 (2.87)	400 (1.54)	400 (0.72)	400 (2.87)	400 (1.54)	400 (0.72)
Low	400 (1.16)	400 (0.58)	400 (0.22)	400 (1.16)	300 (4.18)	300 (2.68)
	T_c					

B_2 vs. A

B_2	All-Disposal			Myopic		
	Low	Middle	High	Low	Middle	High
High	400 (2.80)	400 (3.17)	400 (3.45)	400 (2.80)	400 (3.17)	400 (3.45)
Middle	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
Low	400 (0.40)	400 (0.58)	400 (0.73)	300 (3.85)	300 (4.18)	300 (4.43)
	A					

B_2 vs. D_a

B_2	All-Disposal			Myopic		
High	400 (1.70)	400 (3.17)	400 (3.91)	400 (1.70)	400 (3.17)	400 (3.91)
Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
Low	400 (0.33)	400 (0.58)	400 (1.15)	300 (3.53)	300 (4.18)	300 (4.76)
	Low	Middle	High	Low	Middle	High
	D_a					

B_2 vs. v_a

B_2	All-Disposal			Myopic		
High	400 (2.08)	400 (3.17)	400 (4.23)	400 (2.08)	400 (3.17)	400 (4.23)
Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
Low	400 (0.33)	400 (0.58)	400 (1.22)	300 (3.31)	300 (4.18)	300 (5.24)
	Low	Middle	High	Low	Middle	High
	v_a					

B_2 vs. g

B_2	All-Disposal			Myopic		
High	400 (2.87)	400 (3.17)	400 (3.54)	400 (2.87)	400 (3.17)	400 (3.54)
Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
Low	400 (0.53)	400 (0.58)	400 (0.65)	300 (4.05)	300 (4.18)	300 (4.35)
	Low	Middle	High	Low	Middle	High
	g					

h* vs. *a

	All-Disposal				Myopic		
h	High	400 (1.26)	400 (0.99)	400 (0.84)	400 (1.26)	400 (0.99)	400 (0.84)
	Middle	400 (1.94)	400 (1.54)	400 (1.15)	400 (1.94)	400 (1.54)	400 (1.15)
	Low	400 (2.72)	400 (2.31)	400 (1.82)	400 (2.72)	400 (2.31)	400 (1.82)
		Low	Middle	High	Low	Middle	High
		a					

***h* vs. T_c**

	All-Disposal				Myopic		
h	High	400 (2.31)	400 (0.99)	400 (0.44)	400 (2.31)	400 (0.99)	400 (0.44)
	Middle	400 (2.87)	400 (1.54)	400 (0.72)	400 (2.87)	400 (1.54)	400 (0.72)
	Low	400 (3.52)	400 (2.31)	400 (1.15)	400 (3.52)	400 (2.31)	400 (1.15)
		Low	Middle	High	Low	Middle	High
		T_c					

h* vs. *A

	All-Disposal				Myopic		
h	High	400 (0.77)	400 (0.99)	400 (1.17)	400 (0.77)	400 (0.99)	400 (1.17)
	Middle	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
	Low	400 (1.91)	400 (2.31)	400 (2.61)	400 (1.91)	400 (2.31)	400 (2.61)
		Low	Middle	High	Low	Middle	High
		A					

***h* vs. D_a**

h		All-Disposal			Myopic		
	High	400 (0.70)	400 (0.99)	400 (1.89)	400 (0.70)	400 (0.99)	400 (1.89)
	Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
	Low	400 (1.11)	400 (2.31)	400 (3.27)	400 (1.11)	400 (2.31)	400 (3.27)
		Low	Middle	High	Low	Middle	High
				D_a			

***h* vs. v_a**

h		All-Disposal			Myopic		
	High	400 (0.43)	400 (0.99)	400 (1.94)	400 (0.43)	400 (0.99)	400 (1.94)
	Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
	Low	400 (1.07)	400 (2.31)	400 (3.51)	400 (1.07)	400 (2.31)	400 (3.51)
		Low	Middle	High	Low	Middle	High
				v_a			

***h* vs. g**

h		All-Disposal			Myopic		
	High	400 (0.90)	400 (0.99)	400 (1.92)	400 (0.90)	400 (0.99)	400 (1.92)
	Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
	Low	400 (2.08)	400 (2.31)	400 (2.62)	400 (2.08)	400 (2.31)	400 (2.62)
		Low	Middle	High	Low	Middle	High
				g			

a vs. T_c

a		All-Disposal			Myopic		
	High	400 (2.60)	400 (1.15)	400 (0.54)	400 (2.60)	400 (1.15)	400 (0.54)
	Middle	400 (2.87)	400 (1.54)	400 (0.72)	400 (2.87)	400 (1.54)	400 (0.72)
	Low	400 (3.05)	400 (1.94)	400 (0.93)	400 (3.05)	400 (1.94)	400 (0.93)
		Low	Middle	High	Low	Middle	High
	T_c						

a vs. A

a		All-Disposal			Myopic		
	High	400 (0.90)	400 (1.15)	400 (1.37)	400 (0.90)	400 (1.15)	400 (1.37)
	Middle	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
	Low	400 (1.57)	400 (1.94)	400 (2.21)	400 (1.57)	400 (1.94)	400 (2.21)
		Low	Middle	High	Low	Middle	High
	A						

a vs. D_a

a		All-Disposal			Myopic		
	High	400 (0.77)	400 (1.15)	400 (2.21)	400 (0.77)	400 (1.15)	400 (2.21)
	Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
	Low	400 (0.99)	400 (1.94)	400 (2.75)	400 (0.99)	400 (1.94)	400 (2.75)
		Low	Middle	High	Low	Middle	High
	D_a						

a vs. v_a

		All-Disposal			Myopic		
a	High	400 (0.53)	400 (1.15)	400 (2.20)	400 (0.53)	400 (1.15)	400 (2.20)
	Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
	Low	400 (0.80)	400 (1.94)	400 (3.04)	400 (0.80)	400 (1.94)	400 (3.04)
		Low	Middle	High	Low	Middle	High
	v_a						

a vs. g

		All-Disposal			Myopic		
a	High	400 (1.05)	400 (1.15)	400 (1.48)	400 (1.05)	400 (1.15)	400 (1.48)
	Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
	Low	400 (1.71)	400 (1.94)	400 (2.23)	400 (1.71)	400 (1.94)	400 (2.23)
		Low	Middle	High	Low	Middle	High
	g						

T_c vs. A

		All-Disposal			Myopic		
T_c	High	400 (0.53)	400 (0.72)	400 (0.80)	400 (0.53)	400 (0.72)	400 (0.80)
	Middle	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
	Low	400 (2.47)	400 (2.87)	400 (3.17)	400 (2.47)	400 (2.87)	400 (3.17)
		Low	Middle	High	Low	Middle	High
	A						

T_c vs. D_o

	All-Disposal			Myopic		
T_c High	400 (0.23)	400 (0.72)	400 (1.42)	400 (0.23)	400 (0.72)	400 (1.42)
T_c Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
T_c Low	400 (1.51)	400 (2.87)	400 (3.76)	400 (1.51)	400 (2.87)	400 (3.76)
D_o	Low	Middle	High	Low	Middle	High

T_c vs. v_o

	All-Disposal			Myopic		
T_c High	400 (0.24)	400 (0.72)	400 (1.40)	400 (0.24)	400 (0.72)	400 (1.40)
T_c Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
T_c Low	400 (1.70)	400 (2.87)	400 (4.05)	400 (1.70)	400 (2.87)	400 (4.05)
v_o	Low	Middle	High	Low	Middle	High

T_c vs. g

	All-Disposal			Myopic		
T_c High	400 (0.66)	400 (0.72)	400 (0.80)	400 (0.66)	400 (0.72)	400 (0.80)
T_c Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
T_c Low	400 (2.55)	400 (2.87)	400 (3.26)	400 (2.55)	400 (2.87)	400 (3.26)
g	Low	Middle	High	Low	Middle	High

A vs. D_o

A		All-Disposal			Myopic		
	High	400 (1.05)	400 (1.83)	400 (2.77)	400 (1.05)	400 (1.83)	400 (2.77)
	Middle	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
	Low	400 (0.66)	400 (1.13)	400 (2.20)	400 (0.66)	400 (1.13)	400 (2.20)
		Low	Middle	High	Low	Middle	High
		D_o					

A vs. v_o

A		All-Disposal			Myopic		
	High	400 (0.80)	400 (1.83)	400 (2.96)	400 (0.80)	400 (1.83)	400 (2.96)
	Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
	Low	400 (0.46)	400 (1.13)	400 (2.26)	400 (0.46)	400 (1.13)	400 (2.26)
		Low	Middle	High	Low	Middle	High
		v_o					

A vs. g

A		All-Disposal			Myopic		
	High	400 (1.55)	400 (1.83)	400 (2.18)	400 (1.55)	400 (1.83)	400 (2.18)
	Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
	Low	400 (1.00)	400 (1.13)	400 (1.52)	400 (1.00)	400 (1.13)	400 (1.52)
		Low	Middle	High	Low	Middle	High
		g					

D_o vs. v_o

D_o	All-Disposal			Myopic		
High	400 (1.33)	400 (2.53)	400 (3.80)	400 (1.33)	400 (2.53)	400 (3.80)
Middle	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
Low	400 (0.47)	400 (0.88)	400 (1.36)	400 (0.47)	400 (0.88)	400 (1.36)
	Low	Middle	High	Low	Middle	High
	v_o					

D_o vs. g

D_o	All-Disposal			Myopic		
High	400 (2.44)	400 (2.53)	400 (2.66)	400 (2.44)	400 (2.53)	400 (2.66)
Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
Low	400 (0.69)	400 (0.88)	400 (1.10)	400 (0.69)	400 (0.88)	400 (1.10)
	Low	Middle	High	Low	Middle	High
	g					

v_o vs. g

v_o	All-Disposal			Myopic		
High	400 (2.43)	400 (2.67)	400 (2.96)	400 (2.43)	400 (2.67)	400 (2.96)
Middle	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
Low	400 (0.57)	400 (0.65)	400 (0.81)	400 (0.57)	400 (0.65)	400 (0.81)
	Low	Middle	High	Low	Middle	High
	g					

D_c vs. B_1

D_c	All-Disposal			Myopic		
Incr.	500 (0.66)	500 (0.99)	500 (1.33)	500 (0.66)	500 (0.99)	500 (1.33)
Decr.	300 (1.98)	300 (2.21)	300 (2.45)	300 (1.98)	300 (2.21)	300 (2.45)
Base Case	400 (1.35)	400 (1.54)	400 (1.76)	400 (1.35)	400 (1.54)	400 (1.76)
Level	400 (0.89)	400 (1.10)	400 (1.31)	400 (0.89)	400 (1.10)	400 (1.31)
Bi-Modal	400 (0.41)	400 (0.54)	400 (0.66)	300 (3.09)	300 (3.27)	300 (3.46)
	Low	Middle	High	Low	Middle	High
	B_1					

D_c vs. B_2

D_c	All-Disposal			Myopic		
Incr.	400 (1.79)	500 (0.99)	500 (2.67)	400 (1.79)	500 (0.99)	500 (2.67)
Decr.	300 (0.51)	300 (2.21)	400 (0.01)	200 (5.46)	300 (2.21)	300 (4.52)
Base Case	400 (0.58)	400 (1.54)	400 (3.17)	300 (4.18)	400 (1.54)	400 (3.17)
Level	300 (1.89)	400 (1.10)	500 (0.002)	300 (1.89)	400 (1.10)	400 (3.22)
Bi-Modal	300 (0.37)	400 (0.54)	500 (1.21)	200 (4.66)	300 (3.27)	500 (1.21)
	Low	Middle	High	Low	Middle	High
	B_2					

D_c vs. h

D_c	All-Disposal			Myopic		
	h					
	Low	Middle	High	Low	Middle	High
Incr.	500 (1.62)	500 (0.99)	500 (0.44)	500 (1.62)	500 (0.99)	500 (0.44)
Decr.	300 (3.09)	300 (2.21)	300 (1.48)	300 (3.09)	300 (2.21)	300 (1.48)
Base Case	400 (2.31)	400 (1.54)	400 (0.99)	400 (2.31)	400 (1.54)	400 (0.99)
Level	400 (1.72)	400 (1.10)	400 (0.56)	400 (1.72)	400 (1.10)	400 (0.56)
Bi-Modal	500 (0.00)	400 (0.54)	400 (0.15)	300 (4.37)	300 (3.27)	300 (2.35)

D_c vs. a

D_c	All-Disposal			Myopic		
	a					
	Low	Middle	High	Low	Middle	High
Incr.	500 (1.35)	500 (0.99)	500 (0.59)	500 (1.35)	500 (0.99)	500 (0.59)
Decr.	300 (2.55)	300 (2.21)	300 (1.81)	300 (2.55)	300 (2.21)	300 (1.81)
Base Case	400 (1.94)	400 (1.54)	400 (1.15)	400 (1.94)	400 (1.54)	400 (1.15)
Level	400 (1.40)	400 (1.10)	400 (0.76)	400 (1.40)	400 (1.10)	400 (0.76)
Bi-Modal	400 (0.72)	400 (0.54)	400 (0.32)	300 (3.56)	300 (3.27)	300 (2.92)

D_c vs. T_c

D_c	All-Disposal			Myopic		
Incr.	500 (2.24)	500 (0.99)	500 (0.19)	500 (2.24)	500 (0.99)	500 (0.19)
Decr.	300 (3.53)	300 (2.21)	300 (0.96)	300 (3.53)	300 (2.21)	300 (0.96)
Base Case	400 (2.87)	400 (1.54)	400 (0.72)	400 (2.87)	400 (1.54)	400 (0.72)
Level	400 (2.30)	400 (1.10)	400 (0.80)	400 (2.30)	400 (1.10)	400 (0.80)
Bi-Modal	500 (0.63)	400 (0.54)	400 (0.02)	400 (2.21)	300 (3.27)	300 (1.92)
	Low	Middle	High	Low	Middle	High
	T_c					

D_c vs. A

D_c	All-Disposal			Myopic		
Incr.	500 (0.73)	500 (0.99)	500 (1.19)	500 (0.73)	500 (0.99)	500 (1.19)
Decr.	300 (1.88)	300 (2.21)	300 (2.46)	300 (1.88)	300 (2.21)	300 (2.46)
Base Case	400 (1.13)	400 (1.54)	400 (1.83)	400 (1.13)	400 (1.54)	400 (1.83)
Level	400 (0.88)	400 (1.10)	400 (1.26)	400 (0.88)	400 (1.10)	400 (1.26)
Bi-Modal	400 (0.46)	400 (0.54)	400 (0.59)	300 (3.04)	300 (3.27)	300 (3.45)
	Low	Middle	High	Low	Middle	High
	A					

D_c vs. D_o

D_c	All-Disposal			Myopic		
Incr.	500 (0.32)	500 (0.99)	500 (1.81)	500 (0.32)	500 (0.99)	500 (1.81)
Decr.	300 (0.98)	300 (2.21)	300 (3.39)	300 (0.98)	300 (2.21)	300 (3.39)
Base Case	400 (0.88)	400 (1.54)	400 (2.53)	400 (0.88)	400 (1.54)	400 (2.53)
Level	400 (0.43)	400 (1.10)	400 (1.84)	400 (0.43)	400 (1.10)	400 (1.84)
Bi-Modal	400 (0.18)	400 (0.54)	400 (0.91)	300 (2.43)	300 (3.27)	300 (4.40)
	Low	Middle	High	Low	Middle	High
	D_o					

D_c vs. v_o

D_c	All-Disposal			Myopic		
Incr.	500 (0.22)	500 (0.99)	500 (1.75)	500 (0.22)	500 (0.99)	500 (1.75)
Decr.	300 (1.25)	300 (2.21)	300 (3.21)	300 (1.25)	300 (2.21)	300 (3.21)
Base Case	400 (0.65)	400 (1.54)	400 (2.67)	400 (0.65)	400 (1.54)	400 (2.67)
Level	400 (0.47)	400 (1.10)	400 (1.73)	400 (0.47)	400 (1.10)	400 (1.73)
Bi-Modal	400 (0.32)	400 (0.54)	400 (0.76)	300 (2.63)	300 (3.27)	300 (3.98)
	Low	Middle	High	Low	Middle	High
	v_o					

D_c vs. g

D_c		All-Disposal			Myopic		
	Incr.	500 (0.59)	500 (0.99)	500 (1.45)	500 (0.59)	500 (0.99)	500 (1.45)
	Decr.	300 (1.96)	300 (2.21)	400 (0.00)	300 (1.96)	300 (2.21)	300 (2.58)
	Base Case	400 (1.26)	400 (1.54)	400 (1.90)	400 (1.26)	400 (1.54)	400 (1.90)
	Level	400 (0.66)	400 (1.10)	400 (1.61)	400 (0.66)	400 (1.10)	400 (1.61)
	Bi-Modal	400 (0.10)	400 (0.54)	500 (0.21)	300 (2.61)	300 (3.27)	300 (4.47)
		Low	Middle	High	Low	Middle	High
		g					

Appendix I. Evaluating the Heuristic for Approximating I_s

(Critical t-value is ± 2.0)

$P_1 = 0.6$ $t_1 = 1.0$
$P_2 = 0.3$ $t_2 = 2.0$
$P_3 = 0.1$ $t_3 = 3.5$
(with $t_0 = 0$)

M	PV (M) (Heuristic approach)	PV (M) (Exact approach)		
		$\bar{x}$	$s_{\bar{x}}$	t-value
Sum	$8,291,617.06	$8,228,612.46	$77,524.5026	0.813
0	40605.92	40568.62	32.6342	1.143
10	38459.63	38431.01	30.9937	0.923
30	35381.66	35354.97	19.8388	1.345
45	33792.6	33808.61	28.9563	-0.553
60	32497.1	32497.74	21.5711	-0.03

$P_1 = 0.2$ $t_1 = 0.5$
$P_2 = 0.6$ $t_2 = 3.0$
$P_3 = 0.2$ $t_3 = 4.0$
(with $t_0 = 0$)

M	PV (M) (Heuristic approach)	PV (M) (Exact approach)		
		$\bar{x}$	$s_{\bar{x}}$	t-value
Sum	$10,014,695.61	$10,040,822.47	$116,730.2648	-0.224
0	40787.89	40833.7	32.3971	-1.414
10	38610.67	38597.78	24.9565	0.517
30	35332.98	35310.7	22.1007	1.008
45	33395.89	33424.7	37.4571	-0.769
60	31919.63	31872.51	49.4638	0.953

$P_1 = 0.05$ $t_1 = 1.0$
$P_2 = 0.05$ $t_2 = 3.0$
$P_3 = 0.90$ $t_3 = 4.5$
(with $t_0 = 0$)

M	PV (M) (Heuristic approach)	PV (M) (Exact approach)		
		$\bar{x}$	$s_{\bar{x}}$	t-value
Sum	$13,295,732.30	$13,259,940.94	$72,536.4895	0.493
0	41166.76	41156.81	35.3882	0.281
10	39073.45	39081.75	26.6632	-0.311
30	35432.91	35465.06	33.7971	-0.951
45	33126.34	33136.74	32.4956	-0.32
60	31201.32	31200.08	27.0227	0.046

$P_1 = 0.3$ $t_1 = 0.5$
$P_2 = 0.5$ $t_2 = 2.0$
$P_3 = 0.2$ $t_3 = 4.5$
(with $t_0 = 0$)

M	PV (M) (Heuristic approach)	PV (M) (Exact approach)		
		$\bar{x}$	$s_{\bar{x}}$	t-value
Sum	$8,976,555.52	$8,930,469.29	$83,454.6713	0.552
0	40696.57	40697.56	30.6129	-0.032
10	38442.64	38455.71	22.2936	-0.587
30	35382.87	35387.14	22.3701	-0.191
45	33677.03	33662.96	28.5942	0.492
60	32308.16	32283.93	26.4688	0.915

$P_1 = 0.4$ $t_1 = 1.0$
$P_2 = 0.3$ $t_2 = 2.5$
$P_3 = 0.2$ $t_3 = 3.0$
(with $t_0 = 0$)

(10% chance of no future project)

M	**PV (M) (Heuristic approach)**	**PV (M) (Exact approach)**		
		$\bar{x}$	$s_{\bar{x}}$	t-value
Sum	$10,376,743.53	$10,333,360.77	$69,179.3993	0.627
0	40974.28	40972.5	20.3318	0.087
10	38847.25	38830.59	21.0916	0.79
30	35547.8	35544.26	18.9226	0.187
45	33717.97	33723.36	28.2581	-0.191
60	32161.35	32144.52	26.2904	0.64