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AN EFFICIENT ALGORITHM FOR OPTIMAL PIPELINE DESIGN AND OPERATION

BY

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ABSTRACT

Design and operation of pipelines for long-distance transportation of oil and gas are complex engineering tasks. Even small improvements in the design and operating conditions of a pipeline system can lead to substantial savings in capital and operating costs. This makes the optimization of design and operation of pipelines an important task.

The pipeline optimization problem involves a large number of variables which may be continuous (e.g. suction pressure at a booster station and station location) or discrete (e.g. discharge head at a station and pipe diameter). The objective function can either be an expression for operating cost or energy consumption, which is inherently nonlinear primarily due to pipeline friction losses. The numerous constraints, such as lower and upper bounds for pressure limitations, add to the complexity of the problem. It is almost impossible to find the solution for this multivariable interconnected optimization problem through a trial-and-error approach or through a complete enumeration of feasible solutions. In contrast, a new algorithm for optimal design and operation of a pipeline network is developed. The approach is based on dynamic programming combined with integer programming. A 'fine-tuning' procedure is developed in order to increase the accuracy of results in an efficient way.

The model is demonstrated to be efficient for a realistic case in which only 1.50×10^6 or less candidates out of as many as 5.37×10^{39} feasible solutions have to be searched to locate the global optimum of that particular design problem. The difference in the number of iterations becomes even more dramatic with the introduction of fine-tuning. Only 3.29×10^7 iterations are necessary in order to reach the optimum with an accuracy which could be obtained by complete enumeration requiring 2.22×10^{73} iterations. The results are checked for their validity with a sensitivity analysis.

Once the system configuration is established, the operating conditions are controlled leading to the optimum with respect to either the lowest energy consumption or the least operating cost scenario. Efforts were directed to make the algorithm computationally efficient, requiring relatively short computer time. In addition to providing steady-state solutions, the approach can also be used for generating a set of optimal operating conditions in the transient (dynamic) mode of the pipeline by specifying appropriate changes at each time step.

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NOMENCLATURE

c_c : capital cost ($/W y)

c_{cp} : piping capital cost ($/in m y)

c_f : fixed cost ($/y)

c_o : operating cost ($/W y)

c_{tot} : total annual cost ($/y)

c_x : cost index

d : diameter (in)

dl : step size on length discretization (m)

dp : step size on pressure discretization (Pa)

dsl : step size on station location discretization (m)

f_i : feed stream

g : gravitational acceleration (m^2/s)

jl : a counter

jp : a counter

l : length (m)

m : number of pipe segments

m' : mass flowrate (kg/s)

mz : number of pipe segments joining at a node

n : number of booster stations

p : pressure (Pa)

P : power (W)

q : volumetric flowrate (m^3/s)

s : specific gravity

s_i : side stream

s_u : dimensionless sensitivity

sl : station location (m)

x = 0, 1 (pump off or on)

x_s : an index

v : velocity (m/s)

W : work (J/kg)

y : a counter

z : elevation (m)

Greek Symbols:

Δ : difference

η : efficiency

μ : viscosity (mPa s)

ρ : density (kg/m^3)

Subscripts:

base : base case

d : discharge

f	: friction
i	: index of a station
il	: an index
j	: index of a pipe segment
jl	: an index
k	: index of pump combination
max	: maximum value
min	: minimum value
p	: pumping
poss	: possible
s	: suction
t	: throttling
y	: index of a pump
z	: an index

CHAPTER 1

INTRODUCTION

Transportation of large amounts of fluids over long distances is a challenging problem. In this study, the problem of pipeline transportation of crude oil is investigated, and necessary modifications for the case of natural gas are introduced. When the location of the fluid is far away from the delivery point, it is not realistic to increase the pressure at the source location so that the fluid can be sent through the whole pipeline because it may not resist the high pressure. Instead, numerous booster stations with several pumps placed at each station have to be built along the pipeline which are connected through pipe segments of different sizes.

Existence of many parameters and restrictions, and the high cost of equipment make the design and operation of pipeline transmission systems complex engineering tasks. The annual cost for a 1500 km oil pipeline can easily exceed 20 million $ (Grelli and Gilmour, 1986). Considering such a high amount, even small improvements in the system - decreasing the cost by a few percent - can lead to substantial savings. Hence, optimization can play an important role by either minimizing the cost or decreasing the energy consumption of the system.

Optimization, which means to minimize or maximize an objective function subject to constraints, can be as simple as calculating a few feasible results and determining the optimum by choosing the lowest or highest value in the case of a minimization or maximization problem, respectively. Depending on the complexity of the problem - which can easily increase with nonlinearities in the equations and with the number and nature of constraints - and how close one wants to come to the global optimum, advanced mathematical skills may be necessary.

In the approach developed in the present investigation, the multivariable interconnected optimization problem is decomposed into a sequence of sub-problems that can be solved serially by use of the dynamic programming technique. Starting from the last stage which is characterized by delivery specifications, the sub-problems at each stage are solved resulting in an optimum over the stages considered. This procedure is continued until the first stage (source) is reached at which the solution represents the global optimum for the whole system. Integer programming is utilized to decrease the computation time needed for selecting the 'best' pump combination at a station. A 'fine tuning' procedure is introduced in order to increase the accuracy of the solution efficiently. The algorithm is capable of handling the following design parameters for fixed source and delivery conditions: number and location of booster stations, number and capacity of pumps (or compressors) at each station, suction and discharge pressure at each station, and the diameter of the pipeline segment.

In the following Chapter, optimization applied in the industry and the work available from literature are reviewed. Later a general pipeline system is introduced for which the optimization problem is defined and the developed solution procedure is described leading to minimum annual cost of the pipeline transmission system. The solution method to design an oil pipeline is modified to handle optimization of the operating conditions of the pipeline. Necessary modifications required for application of the solution procedure to pipeline transportation of natural gas are explained. A computer program based on the solution strategy is described. Results from several computer runs are compared to show the features of the program and to prove the success in reaching the global optimum of the system leading to immediate savings. The reliability of the results are checked with a sensitivity analysis and, finally, conclusions and recommendations are listed.

CHAPTER 2

AVAILABLE WORK ON PIPELINE OPTIMIZATION

An optimization technique is acceptable to the industry, only if it is 'safe', i.e. it is proven to bring financial profits over a relatively short period of time, whereas academic work is done to find new and improved methods which may or may not lead to immediate profit. Hence, academic work is usually not accepted and appreciated as being applicable for a real case. Since the research in the area of pipeline optimization is no exception, optimization procedures for pipeline systems which are applied in the industry and the theoretical work developed for pipeline optimization are considered separately.

2.1. OPTIMIZATION IN INDUSTRY

In general, pipeline optimization in the industry is based on the method of trial and error. Cost estimations are made for several different system configurations and the most economic solution is chosen to be the 'optimum case' (Tsal et al., 1986). It is obvious that the likelihood of being close to the global optimum is very small. For example, to reach the global optimum of a pipeline with 15 stations and three pumps at each station is almost impossible since the number of feasible solutions out of which the optimal solution has to be chosen would lie in the range of 10^{54}. At the same time, the trial-and-error approach is a very time consuming and laborious procedure. Considering that fuel cost is skyrocketing and that time is very valuable, more effective optimization procedures should be applied.

2.2. LITERATURE REVIEW

Although trial and error is the most common way of pipeline optimization, more effective methods have been developed. One of the first attempts was made by Wong and Larson (1968). They optimized an unbranched natural gas transmission pipeline by using dynamic programming. Since the system configuration was fixed, the only variable to be handled was the suction pressure at the pumping stations. Hence, their solution was only applicable to operating conditions of an already installed pipeline.

Kally (1969) used dynamic programming to design a simple unbranched pipeline with fixed pipe segment lengths connecting single pumps at different elevations. Although only two pumps were considered in sample calculations, 15 minutes run time was necessary to decide whether to place a pump at the two predetermined sites and which pipe diameter and pipe class to choose.

Cheeseman (1971) developed a program which he called 'an experience routine', since several rules of thumb were used which were adopted by experienced engineers and cost estimators. The optimization method was capable of finding local optima for different 'starting points', 'guiding' the engineer in order to come 'near' the optimal solution.

Edgar et al. (1978) applied a combination of general reduced gradient method and branch-and-bound technique to optimize a pipeline system. Assuming all variables to be continuous, they were able to use a nonlinear optimization technique giving 'odd' results, like pipe diameters which are unavailable commercially. Hence, the optimal solution was not likely to be a practical one.

Gopal (1980) combined dynamic programming and integer programming to optimize a pipeline system with fixed configuration and having only one independent variable. His major contribution has been in considering the limitations of the pump capacity by introducing discrete functions to the problem.

Deb (1981) considered the optimum energy cost design of a pipeline by concentrating on the annual cost functions. Since the pipe diameter was considered as the only variable, optimization was achieved by equating the derivative of cost with respect to pipe diameter to zero.

A survey of applications of optimization in oil and gas pipeline engineering was presented by Huang and Seireg (1985). They offered a good reference for optimal design, operation, expansion and control of pipeline systems, giving brief information about the research work without criticism, leading to the conclusion that active research was being done to improve the efficiency of optimization. They remarked correctly that 'significant benefit *could be* obtained from the use of optimization techniques in the design of pipeline systems', implying that industry does not pay much attention to newly developed methods of optimization.

Grelli and Gilmour (1986) utilized dynamic programming to optimize the operating conditions of a straight pipeline transmission system. Since this project was done with and for a pipeline company, necessary data were available to make a case study. Their calculations showed that by optimizing the operating conditions the annual expenses can be decreased by up to 10%, leading to an annual savings of approximately 3 million $.

Hathoot (1986) presented a procedure to be followed in designing a pipeline of optimum diameter. His solution method has been restricted by allowing only a pipeline with 'equally spaced, similar pumping units'.

Tsal et al. (1986) proposed a method based on a 'modification of Bellman's Dynamic Programming', defined as 'dynamic programming with variable stages'.

This definition may be an underestimation of Bellman's Principle of Optimality since it is a general idea, rather than a fixed method, applicable to a wide range of well defined multistage problems. The success of dynamic programming is related to the efficient way of using the basic rules as long as 'the curse of dimensionality' can be controlled (In the proposed method discussed in this study, dynamic programming is used efficiently for several variables, in combination with additional methods, by keeping the dimensionality problem at a minimum). Tsal et al. (1986) described their solution method in detail but instead of solving the optimization problem they left their work as 'a base for development of a computer program'.

Jha (1987) described steps to follow in order to design a pipeline, pointing out the potential problems.

Kurak (1989) documented the actual savings achieved at the Texas Eastern Products Pipeline Co. by using an optimization program which can determine pump rates required to meet the pipeline product demand requirements and to estimate the arrival times for product deliveries. Once the complete modelling of the pipeline is fixed, the program is capable of selecting which pumps should run for various flowrates in order to minimize the operating cost.

Examples, such as those shown by Grelli and Gilmour (1986), make it clear that optimization of pipeline systems can be very efficient. Most of the work available in literature concentrates on optimizing the operating conditions of a pipeline system which is simpler than the design problem, since usually only a very limited number of variables is considered. In this work, a general program is developed which can handle the optimization of the design and of operating conditions of pipeline systems including straight and tree networks.

CHAPTER 3

DEFINITION OF THE OPTIMIZATION PROBLEM

3.1 DESCRIPTION OF A GENERAL PIPELINE NETWORK

A pipeline network consists of one or more source(s) from which the fluid has to be sent to one or more delivery location(s). Booster stations are required to pump the fluid through the pipeline. Pipe segments connecting the stations may be of different length and diameter. There may be several pumps of different sizes - or compressors, in case of gas transportation - at a station. The pipeline may go through a hilly terrain where the elevation will affect the system hydraulics. The pipeline may be straight or a tree network. Throughout the pipeline, there may be side streams and intermediate feed streams changing the flowrate in the main line as is shown in Figure 3.1. The fuel sent through the pipeline may be natural gas or oil. In case of oil, different fluids can flow through the pipe. A pipeline can easily be 1500 km long whence there may be variation in fuel cost at different locations. The fuel cost may also vary with time and the season of the year.

Although pipeline design requires a larger investment, both designing and operating a pipeline involve high costs. Since optimization of both procedures can lead to substantial savings, the following two problems are considered for the

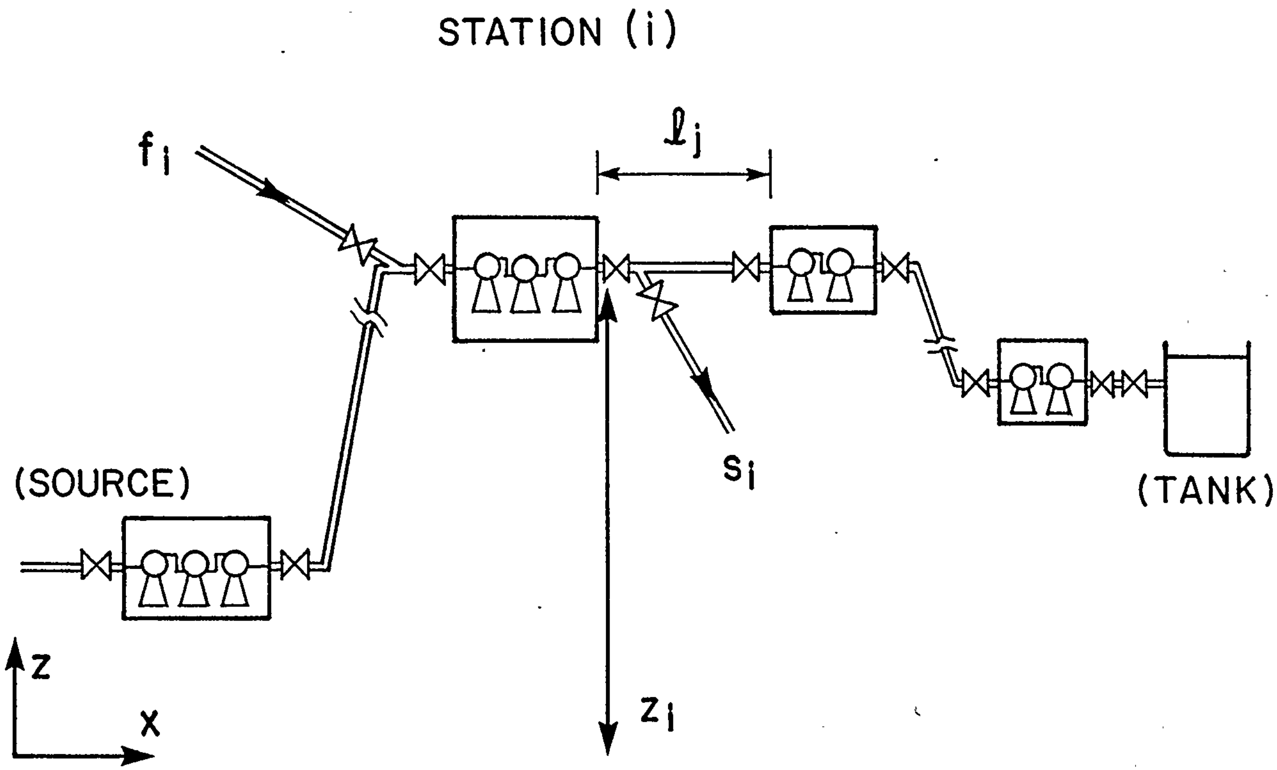


Figure 3.1 : Pipeline Configuration

pipeline: Optimal Design and Optimal Operating Conditions. Although their definitions are different, the procedure to find the most economic way of designing and operating a pipeline are very similar. First the method of optimal design of a pipeline has to be developed since this is the more complicated problem. Once this is accomplished, some modifications are required to redefine the problem for optimal operating conditions so that the new problem can be solved using the same optimization technique.

For any kind of pipeline network, the following requirements remain the same for the design problem. The number of booster stations necessary to pump the fluid to the delivery location(s) have to be determined along with their locations. At each station, the minimum number of pumps and their capacities have to be fixed. Also, the diameters of the pipe segments have to be obtained. Suction and discharge pressures at the stations have to be determined to know the operating pressure ranges. Design is made with respect to the worst possible scenario, e.g. calculations are made considering the peak load of the most viscous fluid. Once the pipeline is designed, the operating conditions have to be determined giving the suction and discharge pressures at the booster stations and the respective pump combinations. Preferably, this should be done in such a way that any change in the system can be taken into account in a convenient way and the solution procedure should lead to the result in a short time to enable optimization of the operating conditions as often as necessary. Both design and operating conditions have to be determined seeking the most economic, i.e. optimum, solution.

3.2. MATHEMATICAL FORMULATION OF THE DESIGN PROBLEM

Optimal design of the pipeline network, like any optimization problem, can be formulated so as to find the extremum of an objective function subject to constraints.

3.2.1. The Objective Function

The design problem can be defined as minimizing the annual cost (Edgar and Himmelblau, 1988):

$$c_{tot} = \sum_{i} \left[\left(c_{x,i}\, c_{o,i} + c_{c,i} \right) P_i + c_{f,i} \right] + \sum_{j} \left[c_{cp,j}\, l_j\, d_j \right] \tag{3.1}$$

The first summation covers the booster stations, where i represents the counter on the stations. The cost to operate the pumps, $c_{o,i}$, is corrected by the cost index, $c_{x,i}$, which takes into account the variation of cost with time and location. The sum of this expression and the capital cost of the station, $c_{c,i}$, is a function of power production of the pumps and is multiplied by P_i which is the total power produced at a station. The cost term for a station also includes a fixed cost $c_{f,i}$ for keeping the station intact. This term is added to the cost function for each station of an existing pipeline, whether the station is used or by-passed, since there has to be a maintenance cost of a station even if there is no pumping. This procedure is different in the case of designing a pipeline: If results show that no pumping is necessary at a station, this station does not need to be built, and the maintenance cost of that particular station is excluded from the cost function. The second summation in Equation 3.1 is made over the pipe segments connecting the booster stations. Capital cost, $c_{cp,j}$ of each pipe segment j is multiplied by the length and diameter of that segment.

The power production P_i at a station can be determined by multiplying the work done at a station by the mass flow rate.

$$P_i = W_i \, m_i' \qquad (3.2)$$

An energy balance is written between two stations to find work produced at the former station. For incompressible fluid flow, the modified Bernoulli equation is written as Equation 3.3. In case of natural gas, the energy equation has to be integrated including density as a variable.

$$\eta W_i = \frac{p_{s,i+1} - p_{s,i}}{\rho} + g\,(z_{i+1} - z_i) + \frac{1}{2}\,(v_{i+1}^2 - v_i^2) + \frac{p_{f,i}}{\rho} + \frac{p_{t,i}}{\rho} \tag{3.3}$$

Equation 3.3 represents the Bernoulli equation between Stations (i) and (i+1) which are shown in Figure 3.2. Work produced at a station can be related to the pressure and elevation difference between the stations. Velocity difference does not occur in further calculations since it would only be caused by variation in pipe diameter of a pipe segment which would make pigging impossible. The Bernoulli equation is modified by adding the $p_{t,i}$ term which represents the pressure loss due to throttling. Suction pressure, $p_{s,i}$, represents the inlet pressure at Station (i). The summation of pressure produced by pumping, given by $p_{p,i}$, and $p_{s,i}$ results in the discharge pressure at Station (i), $p_{d,i}$. If the pressure level at the exit of Station (i) has to be less than the discharge pressure, throttling is necessary. Pressure decrease due to throttling is found from the difference between produced and required pressure at the exit of a station. When the fluid flows from Station (i) to Station (i+1), the pressure decreases by an amount $p_{f,i}$ representing the frictional losses in the pipe.

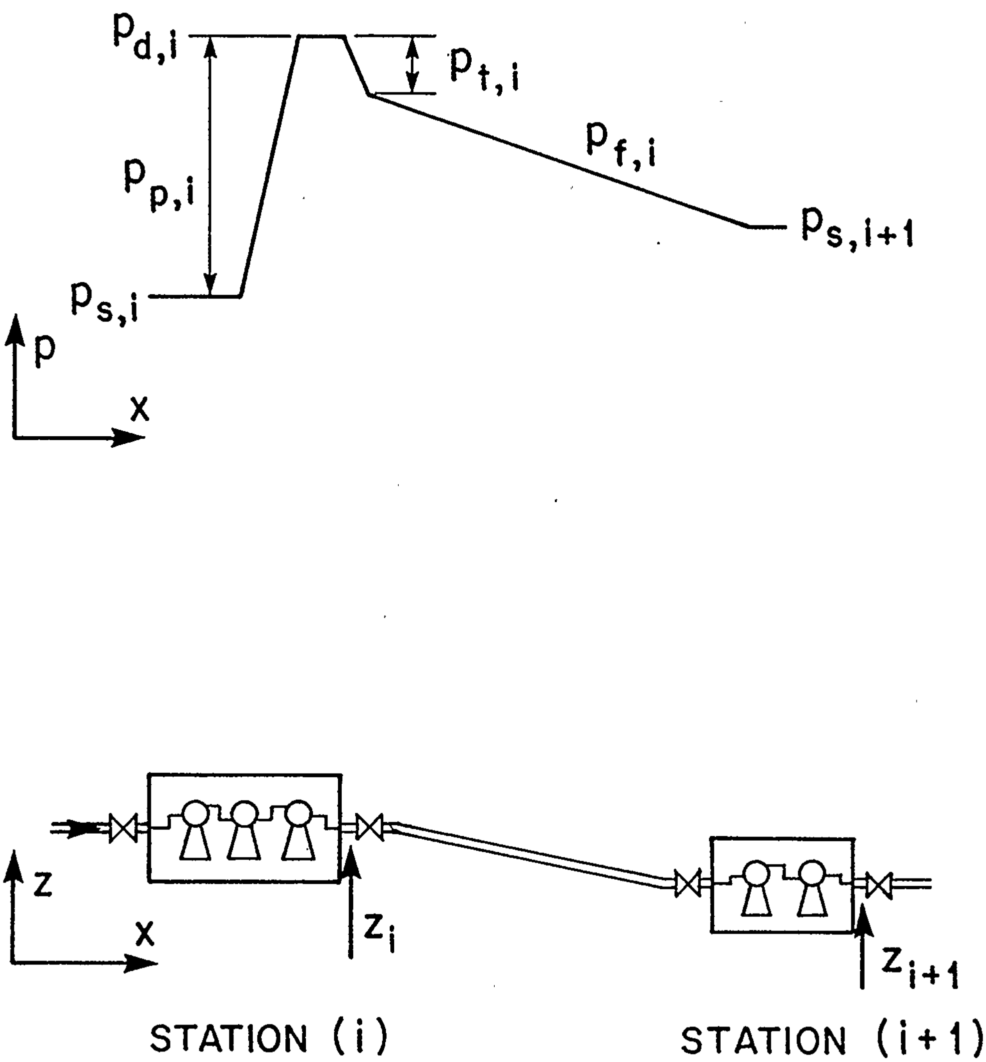


Figure 3.2 : Pressure Variation at the Booster Station (i) and in the Adjacent Pipe Segment

3.2.2. The Constraints

A pipeline system has numerous constraints. The suction and discharge pressures at a station have to be bound by certain limits determined by the net positive suction head (NPSH) and the maximum allowable pressure in the equipment.

$$p_{s_{min},i} \leq p_{s,i} \leq p_{s_{max},i} \quad , i = 1,n \tag{3.4}$$

$$p_{d_{min},i} \leq p_{d,i} \leq p_{d_{max},i} \quad , i = 1,n \tag{3.5}$$

Conditions at the source and delivery location(s) are fixed which leads to the fixed suction pressure at stations at the source and the end nodes of the network. Discharge pressure at a station is the sum of suction pressure and pressure produced by pumping, $p_{p,i}$.

$$p_{d,i} = p_{s,i} + p_{p,i} \tag{3.6}$$

The output of the pumps is given by the following equation:

$$p_{p,i} = \left[\frac{\sum x_y P_y}{q} \right]_i \tag{3.7}$$

where x_y takes the values 1 or 0 depending on whether pump y is on or off. Outputs of pumps which are actually operating (x=1) are added up. This sum is divided by the volumetric flowrate, q, to give the pressure production at a station. With $p_{p,i}$, a discrete function is introduced since it is a limitation of pumps that any arbitrary pressure cannot be produced.

When selecting equipment, commercially available sizes and materials must be considered to keep the cost at a minimum. Hence, the pipe diameter can only be one out of a discrete number of choices. It is also limited by a maximum and minimum size.

$$d_{i_{min}} \leq d_i \leq d_{i_{max}} \tag{3.8}$$

$$d_i = y_i * 12'' , \qquad y_i = 1,2,3... \tag{3.9}$$

Equation 3.9 represents the case when d can be 12", 24", 36" or any multiple of 12 lying in the boundaries.

Another important constraint is that the flow equation has to hold, which gives a correlation for the pressure decrease in the pipeline due to frictional losses. In this study, the Miller's correlation (Explorer Pipeline Company, 1989), Equation 3.10, is used for oil flow. The terms q_i, $p_{f,i}$ and l_j are modified with the addition of (') because they are given in British Units instead of SI units (only for this equation):

$$q_i' - \frac{4.06}{24}\left[\frac{d^5\, p_{f,i}'}{s\, l_j'}\right]^{1/2}\left[\log\left(\frac{d^3\, s\, p_{f,i}'}{\mu^2\, l_j'}\right) + 4.35\right] = 0 \qquad (3.10)$$

where q' is in barrels/hour, $p_{f,i}'$ is in psia and l_j' is in miles.

An obvious constraint is that the total flow has to be equal to the sum of flow in the pipe segments and side streams.

$$q_{tot} = \sum_{l} \left(q_l + q_{s,l} \right) \qquad (3.11)$$

The total length of the pipeline system is given by the following equation:

$$l_{tot} = \sum_{j} l_j \qquad (3.12)$$

where j is the counter on the pipe segments l. This pipe segment is limited by a minimum and a maximum value:

$$l_{j_{min}} \le l_j \le l_{j_{max}} \tag{3.13}$$

The lower boundary results from the fact that if two stations are located very close to each other they may as well be combined into one to save installation costs. If two adjacent stations are located far away from each other, it may only be possible to pump the fluid over that large distance by increasing the pressure to a value above allowable pressure limits whence an upper limit for distance between stations is introduced.

The maximum number of stations which may be installed in the pipeline system is predetermined. The final number of designed stations is bound by this upper limit. When calculations show that no work is required at a possible station, this station is simply skipped and the total number of booster stations is decreased by one.

$$n_{system} \le n \tag{3.14}$$

At nodes of the network where pipe segments join each other or the main stream, the pressure at the connection point has to be the same for the joining end of each segment. This constraint does not cause any complication in case of a straight pipeline where a maximum of two pipe segments can be connected. For a tree network, however, it is very important to keep this constraint in mind. Optimizing pipe segments separately and then connecting these at a node can lead to different pressure values at one location, i.e. at the node, which makes the whole calculation unrealistic. Hence, no combination of results can be declared as optimal solution unless the following restriction holds:

$$p_{il} = p_{z,il}\,, \quad z = 1, mz_{il} \tag{3.15}$$

where z specifies the pipe segments which are connected at node il. The term $p_{z,il}$ can also represent the pressure of side streams and feeds at location il.

The optimization problem is defined for incompressible fluids. Some of the equations will change if natural gas has to be transported. The Bernoulli equation can no longer be used since the energy balance cannot be integrated assuming constant fluid density. Instead, a differential energy balance is written, taking variable density into account. Miller's correlation has also to be replaced since it is specific to oil flow. The Panhandle correlation can be used for natural gas flow (Younger, 1989).

The optimization problem which is defined by the objective function and constraints results in the following variables at each station: suction (p_s) and discharge pressures (p_d), pressure loss due to throttling (p_t) and friction (p_f) . In the case of design, length of each pipe segment and elevation of a station are also variables due to variable station location. In addition, the number of pumps to be installed at a station, their capacities and the pipe diameter have to be included as variables. For an already existing pipeline, the number of pumps, pipe diameter and station locations are fixed. In order to optimize the operating conditions of such a system the flowrate in each pipe segment, fuel cost variations and fluid specifications like viscosity and specific gravity have to be taken into account.

3.2.3. Analysis of the Problem

The optimization problem is given by a nonlinear objective function with numerous constraints. Several continuous and discrete variables exist. Although suction pressure at a station is a continuous variable, discharge pressure becomes a discrete variable since there are only a fixed number of possible pressure levels which the pumps can produce. Pipe diameter is a typical example for variables of discrete type because of restriction on their availability.

The numerous variables make the system multivariable. Decisions on the variables have to be made at each station leading to a multistage decision process. Another important point is that the conditions at stations are interrelated. For example, the suction pressure at Station (i) can be varied to obtain optimal conditions at that specific station. But this variation may also lead to changes at the next station, Station (i+1), disturbing the optimal conditions at that station. Hence, simply adding up minima obtained at separate stations is not likely to lead to the overall optimum of the system. The large number of feasible solutions to the system make it extremely difficult to find the optimum through complete enumeration of these solutions. Hence, an appropriate solution technique is necessary.

CHAPTER 4

SOLUTION PROCEDURE

4.1. METHOD OF SOLUTION: DYNAMIC PROGRAMMING

The dynamic programming technique was developed by Bellman in the 1950's (Bellman, 1957). It is a powerful method to find the global optimum of a multistage decision process with many constraints and continuous as well as discrete variables.

Dynamic programming is a mathematical technique which represents a multistage decision process by dividing it into subproblems which are usually easier to solve (Rao, 1984). Although these subproblems are interrelated, they are solved in such a way that the optimum of the system can simply be obtained by summing the solutions to the subproblems. The beauty of dynamic programming lies in its capability of decomposing a complex multistage problem into simpler subproblems of which the solutions sum to the global optimum. It is of lesser significance as to how the particular sub-optimization is carried out. It can be done simply by enumeration or it may require advanced optimization techniques.

When dynamic programming is applied to a problem, it is important to build the technique well into the solution procedure. An intelligent use of this technique is usually mentioned as 'modification' of dynamic programming although it does not improve the technique itself but only widens its range of application. The dynamic programming technique is known to have the 'curse of dimensionality' problem and is usually used for systems with only one variable. The solution method developed in this work is a good example showing that - with the support of additional methods - dynamic programming can be utilized for solving a problem with several variables.

Dynamic programming is especially powerful for systems with discrete variables. Nonlinear programming techniques can only deal with continuous variables, usually leading to solutions which cannot be reached in reality because of the existence of discrete variables. In such cases, the result has to be rounded up or down to the next feasible solution but this decreases the accuracy of the result. Another important aspect of dynamic programming is that the solution gives the global optimum whereas nonlinear programming techniques can only find local optima which may not be close to the global optimum.

Analysis of the problem in Section 3.2.3 showed that a pipeline network can be represented as a multistage decision process which can be decomposed into sub-stages. The problem has many feasible solutions leading to numerous local optima. For an objective function which is not 'well behaved', these optima may be far from the value of the global optimum. For the pipeline which requires big investment this may mean a significant difference. Hence, it is of great importance to reach the global optimum rather than a local minima. On the other hand, one has to deal with discrete variables in a pipeline system. Considering the specific needs of the pipeline network problem, dynamic programming is found to be a suitable solution technique. To make the method of solution more effective, dynamic programming is combined with integer programming and 'fine tuning'.

4.2. SOLUTION STRATEGY

There is no standard mathematical formulation of the solution strategy; hence, it has to be developed for each separate problem. It is very important to define the problem and to decide what will be called 'stage', 'state', 'level' and 'decision'. Stage is a unit in the system at which decisions have to be made. Level is defined as a value which a variable can take. Each combination of the levels of separate variables forms a state. A decision is made by choosing one of the feasible states declaring values to the variables. In the case of a pipeline, a station is a stage where decisions have to be made concerning the combination of the suction pressure, station location and pipe diameter.

Having clarified the main keywords, the optimal strategy can be summarized as follows: Relatively 'best' decisions are accumulated for each stage of the system in such a way that the consideration of the vast number of inefficient alternatives is avoided. The combination of all decisions leading to the lowest value of the objective function is declared to be the optimal solution of the system.

Although a standard mathematical formulation is not available for this procedure there are steps which have to be followed:

- discretize and relate variables,
- determine decision criteria and store 'best' decisions, and
- reach the global optimum via Bellman's Principle of Optimality.

4.2.1. Discretized Variables

4.2.1.1. Discretization of Continuous Variables

Several variables in the system, like pipe diameter and output of pumps, are discrete. In order to use dynamic programming, the continuous variables like suction pressure have also to be discretized. This is achieved by increasing in steps the variable from its lowest possible value to its maximum value. Each of these steps leads to one of the discrete values, feasible 'levels', which the variable can take. This procedure can be formulated through the following equations:

The pressure at station i, p_i, is bound by lower and upper limits,

$$p_{\min,i} \leq p_i \leq p_{\max,i} \quad , \quad i = 1,n \tag{3.4}$$

and can have any of the following values as a feasible solution:

$$p_i = p_{\min,i} + (jp_i - 1)\ dp, \quad jp_i = 1,\ jp_{\max,i} \tag{4.1}$$

where jp is the integer counter increasing the value of p_i from its minimum to the maximum value.

The same procedure can be repeated for the pipe segment l_j :

$$l_{\min,j} \leq l_j \leq l_{\max,j} \quad , \quad j = 1,m \tag{3.13}$$

$$l_j = l_{\min,j} + (jl_j - 1)\ dl, \quad jl_j = 1,\ jl_{\max,j} \tag{4.2}$$

where jl increases from one to its maximum value in order to vary l_j in the boundaries given by Equation 3.13.

Discrete pipe segment length leads to discretization of station locations. In reality, the range in which the stations have to be located may already be discontinuous. The topography of the region and the distance from a town can eliminate immediately vast parts of the range in which the optimal station locations are sought. Instead of discretizing the length of the whole pipeline, a practical approach is applied: First, a station location is determined considering outside effects. Then, a search is made in the area around this location in order to determine the most convenient and economic station location thereby saving time by disregarding inconvenient areas. This method can also cover the whole area in which the pipeline is going to be built simply by increasing the search areas around the stations in such a way that the endpoint of a region is at the start point of another region. The region which is considered for a station can be discretized by adding multiples of a step size *dsl* to the initially suggested location *sl*:

$$sl_{poss,l} = sl_l + x_{s,l}\, dsl_l\,, \quad x_{s,l} = \ldots -1,0,1 \ldots \tag{4.3}$$

where sl_{poss} represents all possible locations considered during the search for the optimal solution.

The values which the discrete variables can take are schematized in Figure 4.1.

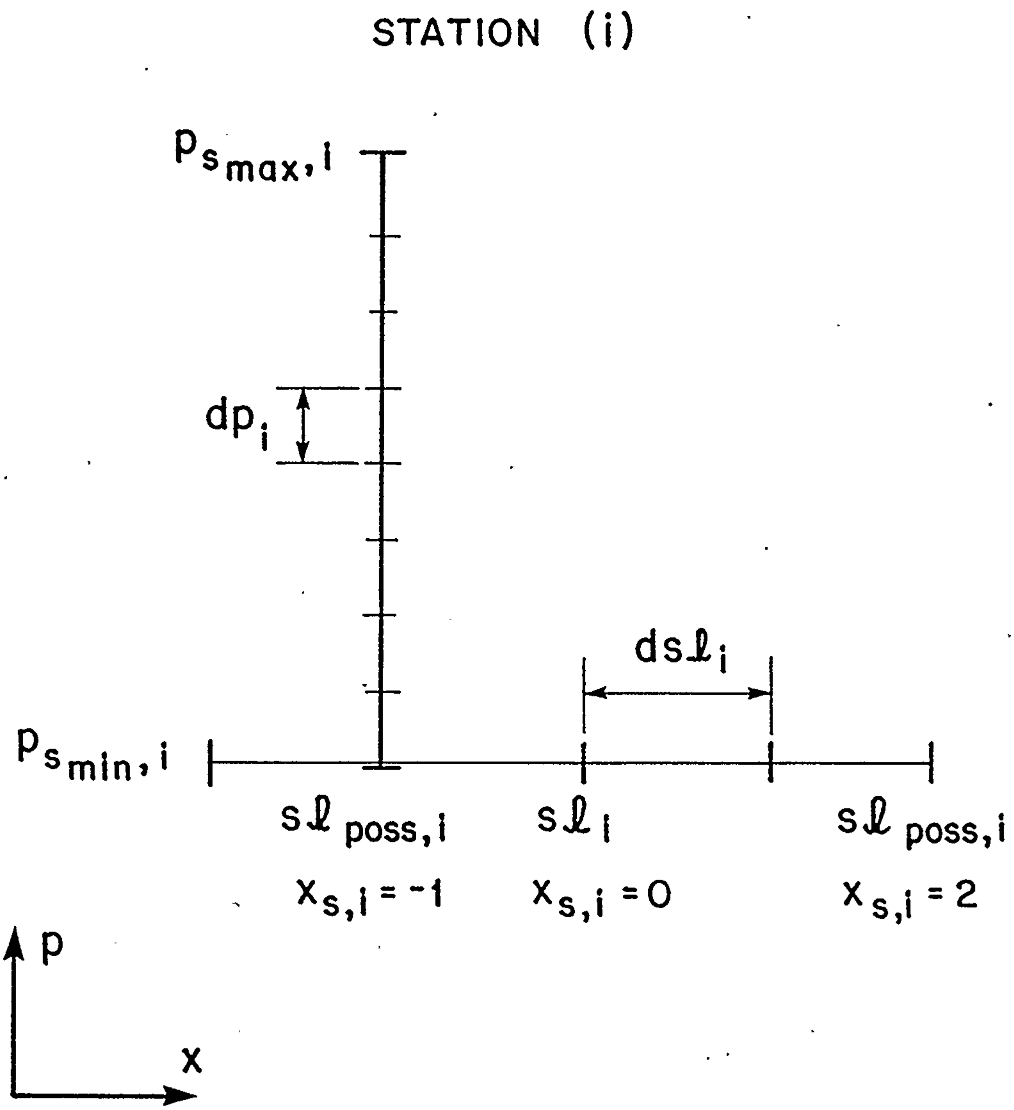


Figure 4.1 : Discretized Station Location and Pressure

4.2.1.2. Relationship Between Variables

Once all variables are expressed in a discrete manner, they should be examined for dependence on each other. This is a very important step since the successful use of dynamic programming is very sensitive to the dimension of the system because of the 'curse of dimensionality' problem. Pipe diameter is an independent variable and so is the location of a station. When the topography of a region is known, the elevation of a station can be written as a function of station location -or in a similar manner length of a pipe segment l. The capacities of the pumps at a station are independent variables and are handled in a way that they do not affect the system dimension directly: Pumps with different capacities are selected to be potential pumps for installation at a station. The required pumps for design are selected out of commercially available ones by considering their size and cost, eliminating the expensive and unnecessarily high capacity ones.

Variables like flow rate, fuel cost and fluid specifications, i.e. viscosity and density, are considered as external effects because they do not relate to the optimization procedure directly. They cannot be changed to make a case optimum, they simply change because of conditions outside the pipeline system, e.g. cost of fuel cannot be controlled or the flowrate has to be changed depending on delivery requirements. These kinds of variables are handled by keeping them in a convenient input file and updating their values after each modification on the system.

An important group of variables is formed by the pressure terms. Pressure change due to friction loss is related to the independent variables pipe length and diameter through the flow equation (Equation 3.10). Discharge pressure at a station results from the combination of suction pressure and pressure produced by pumping. The difference between the actual and required pressure at the exit of a station gives the necessary throttling. Writing the Bernoulli equation with constant fluid velocity (Equation 4.4) and rearranging gives the throttling pressure as a function of the remaining pressures (Equation 4.5).

$$\frac{p_{p,i}}{\rho} = \eta W_i = \frac{p_{s,i+1} - p_{s,i}}{\rho} + g\,(z_{i+1} - z_i) + \frac{p_{f,i}}{\rho} + \frac{p_{t,i}}{\rho} \tag{4.4}$$

$$p_{t,i} = p_{s,i} + p_{p,i} - p_{f,i} - p_{s,i+1} - \rho\, g\,(z_{i+1} - z_i) \tag{4.5}$$

Having decreased the dimension of the system as much as possible by searching for relationships among variables, the next step is to determine a decision criteria upon which elimination can be made between feasible decisions.

4.2.2. 'Best' Decision

4.2.2.1. Decision Criteria

Discrete or discretized variables are a prerequisite for dynamic programming. The disadvantage of discretizing continuous variables is that not all feasible values of a variable can be considered while searching for the optimum. This problem can be minimized by choosing small step size in the solution procedure and correcting the results by 'fine-tuning' which decreases the inaccuracy caused by larger step size in an efficient way. The details of 'fine-tuning' will be considered later.

Figure 4.2 shows how pump combinations can increase the suction pressure at a station to different levels of discharge pressure. To make it easier to understand how the decision criteria is determined, it is assumed that the location of two adjacent stations and their suction pressures in addition to all other variables except for pressure produced by pumping, $p_{p,i}$, are known. Friction loss between these two stations can be calculated from the flow equation. Since Equation 3.10 is nonlinear, it is solved by the Newton-Raphson method. Adding the frictional loss to the suction pressure at Station (i+1) gives the required pressure level to which the suction pressure at Station (i) has to be increased.

Now, the question is which pump combination to choose at a station with

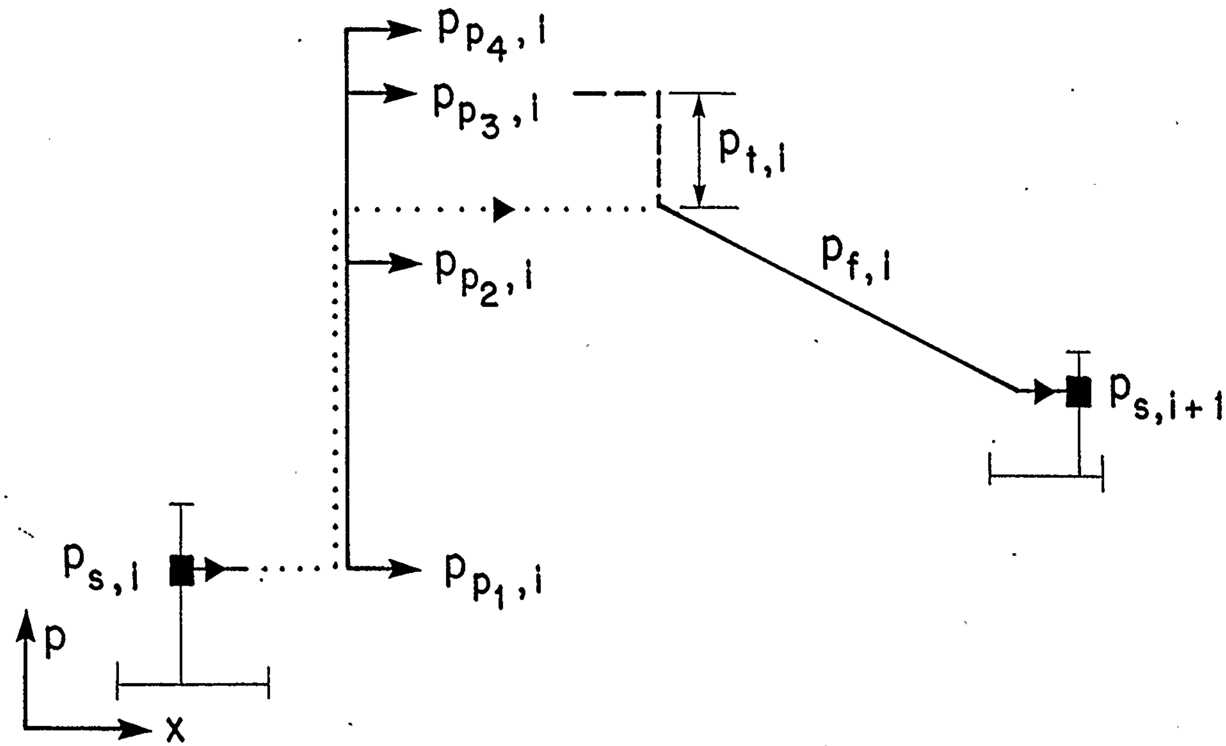


Figure 4.2 : Pump Combinations

the known suction pressure ($p_{s,i}$) and a fixed pressure level at the exit of this station. Ideally, the discharge pressure, being the sum of suction pressure and the pressure produced by pumping, should simply give the required pressure level at the exit of a station. In reality, however, this is not possible because the pumps cannot produce any arbitrary pressure. For a station with three pumps, the pressure produced by pumping can be formulated as:

$$p_{p_k,i} = (x_1 \; P_1 + x_2 \; P_2 + x_3 \; P_3) \; / \; q \quad , \quad x_y = 0{,}1 \tag{4.6}$$

where k is the index of the pump combination and x_y can take the value 1 or 0 depending whether pump y is on or off, respectively. The pumps are numbered in the order of increasing capacity.

For the case of maximum three pumps of different capacities at each station, eight pump combinations are possible. These combinations are listed in the order of increasing output, e.g. pump combination 1 represents no pumping, combination 2 stands for operation of the pump with smallest capacity and combination 8 would mean that all three pumps are working. In Figure 4.2, $p_{p1,i}$ represents the result of no pumping. The second pump combination, $p_{p2,i}$, is obtained by turning on pump number 1. This choice increases the pressure almost to the required level. The pressure $p_{p3,i}$ represents the pump combination with the second pump turned on.

This pump combination produces the first output which exceeds the requirements. Although the output of $p_{p2,i}$ is closer to the required result, only the third combination is feasible since the required pressure level can be reached after throttling. A fourth combination could be to turn on both pumps. This is also a feasible solution because the required result can be achieved by increasing the amount of throttling. Feasible solutions can be accumulated in this manner. The aim of optimization is to minimize the cost and can be achieved by choosing the pump combination with the least power requirment causing a sufficiently high pressure increase. This pump combination being equal to or exceeding the required exit level is declared as the optimal decision for the given conditions.

4.2.2.2. Elimination of Infeasible Combinations

The procedure mentioned in Section 4.2.2.1. has to be repeated many times since the required pressure level at the exit of a station is not known. There will be numerous possible levels for each - assumed to be known - suction pressure. Considering all possible outputs would obviously be very time consuming. Implicit enumeration comes into play at this point, making complete enumeration of all possibilities unnecessary.

Pump combinations are defined in the order of increasing output and possible exit levels are listed in the order of increasing pressure. For finding the least pump combination sufficient to pump the fluid from the suction pressure to the lowest (first) exit level, the combinations are tried in the order of increasing output. The first combination producing the required pressure or excess pressure is declared to be the optimal case for the given conditions. When the procedure is repeated for a higher pressure level at the exit of the station, complete enumeration of possible combinations is not necessary. The pump combination leading to the optimal solution of the previous conditions is taken as the first possible combination instead of pump combination 1, because for the next exit level, which is higher than the first, the combinations which were eliminated for the previous level will again be infeasible. Hence, the pump combinations are tried in the order of increasing capacity starting from the solution to the previous exit pressure level. This procedure helps eliminates a vast number of possible but infeasible combinations.

Another useful tool to decrease the number of calculations involves giving the number '0' to a pump combination if its output is not high enough to pump the fluid to the required pressure level. The pressure levels at the exit of a station are listed in the order of increasing value whence if a pump combination is not enough to reach an exit pressure level, it is also not sufficient to reach any remaining (higher) pressure level. Hence, as soon as a pump combination is called '0', no further calculations are done for the given conditions with higher exit pressure values. With this procedure, far less calculations have to be made compared to complete enumeration of all possibilities.

4.2.2.3. Choosing the 'Best' Decision for any State at a Station

To highlight the concept of how to define the decision criteria, a simplified system with all variables fixed but pump combination was considered in the previous section. In reality, neither the station location nor the suction pressure is fixed at the stations. There are many 'states' at a station which are characterized by any of the possible station locations, suction pressures and pipe diameter of the adjacent pipe segment for that station. Any of the grid points shown in Figure 4.3 can represent a state. All variables, other than the station location and the suction pressure, are assumed to be fixed to make a two dimensional representation possible. Without this assumption, a state would be defined in n-dimensional space where n is the number of variables.

In the previous section, it was shown how the decision criteria is used to find the minimum cost of pumping from a station with fixed conditions to the adjacent station. The cost of pumping the fluid from Station (i) to the end of the pipeline would be the sum of the cost to pump to Station (i+1) and the cost of pumping from Station (i+1) to the delivery location.

$$C_{i,n} = C_{i,i+1} + C_{i+1,n} \tag{4.7}$$

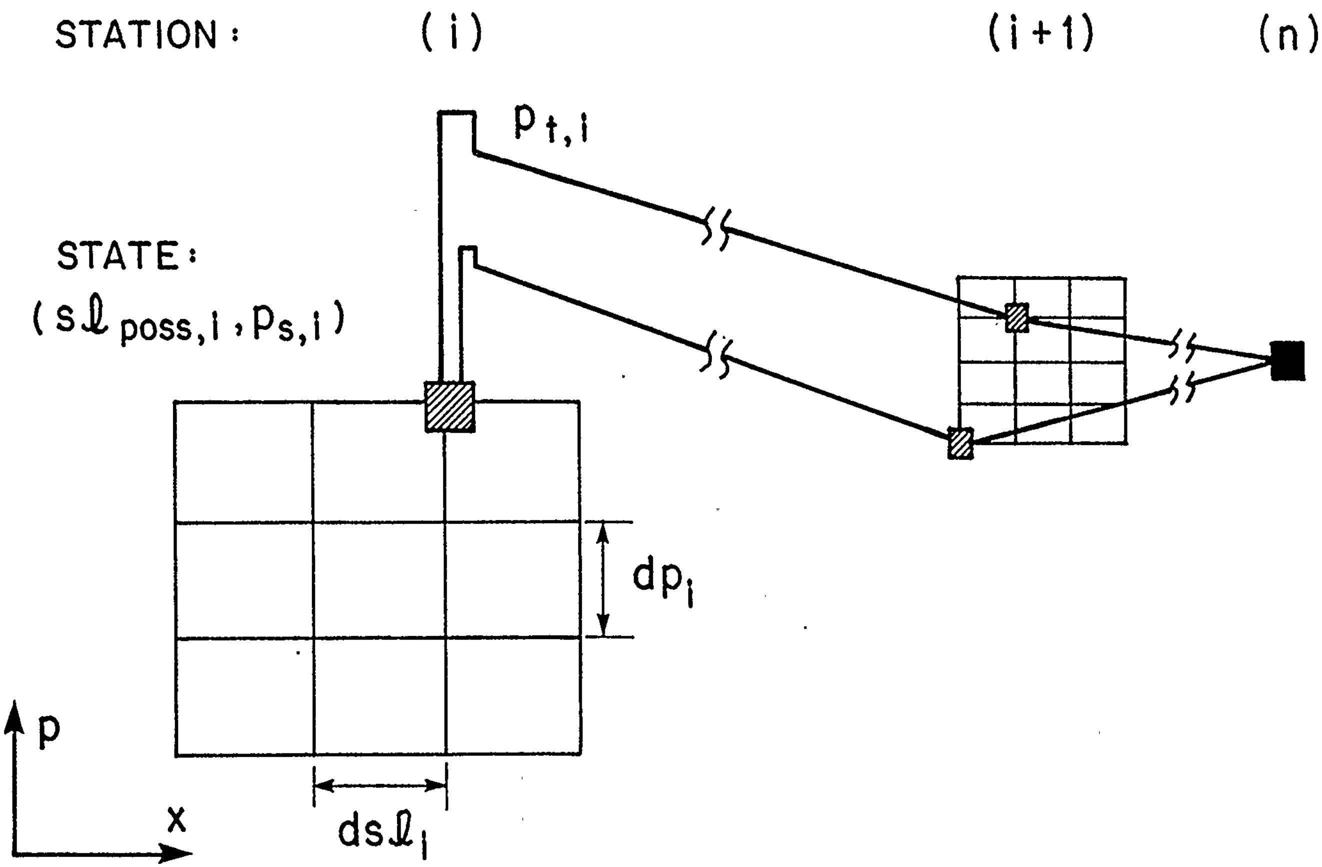


Figure 4.3 : Possible States at a Station

To make the understanding easy, it is assumed that operating cost from Station (i+1) to Station (n) can be stored for each possible state at Station (i+1). The optimal route from a state at Station (i) to any state at Station (i+1) can be obtained by utilizing the procedure explained in Section 4.2.2.1. The cost resulting from this route is added to the stored value at Station (i+1) of the respective state. The same procedure is repeated for all feasible states at Station (i+1) which can be reached from the state at Station (i). The most economic of these routes is declared as the optimal route to reach the delivery point (location n) from a given state at Station (i). As already explained, there are numerous states also at Station (i), all of which lead to different solutions. Since conditions at stations are interrelated, this procedure cannot simply be repeated for two adjacent stations separately in order to find the overall optimum. To find the global optimum of the problem, Bellman's Principle of Optimality has to be considered.

4.2.3. Global Optimum

4.2.3.1. Bellman's Principle of Optimality

For a decision to be part of a global optimum, Bellman's Principle of Optimality has to hold which is stated as follows (Bellman, 1957):

An optimal policy has the property that whatever the initial state and initial *decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*

In Bellman's definition, 'state' is used in its conventional meaning and not in the specific meaning defined in this study. Figure 4.4 shows how this principle works. Decision making occurs in the direction of information flow which is usually opposite to the material flow (Edgar et al., 1978). Stage n is the only stage which does not have any influence on a 'following' stage. The only 'remaining decisions' exist prior to this stage. Hence, if it is assumed that the input to this stage is known, no remaining decision is left and the decision at stage n can be made regardless of the rest of the system. This procedure has to be repeated for each possible input to stage n. The next stage at which a decision has to be made is (n-1). Any decision at this point will affect the n^{th} stage. To by-pass this complication, the stages (n-1) and (n) are grouped together. The new group is now called stage n'. Similar to stage n, this stage has 'remaining decisions' only towards stage 1. Once the input to stage n', or (n-1), is assumed to be known, no 'remaining' decision needs to be taken into account. Hence, it can be optimized on its own not interfering with any condition. One has to keep in mind that with this method any 'following' stage is eliminated but the input to a stage cannot be fixed. As it has been mentioned for stage n, stage n' has also to be optimized with

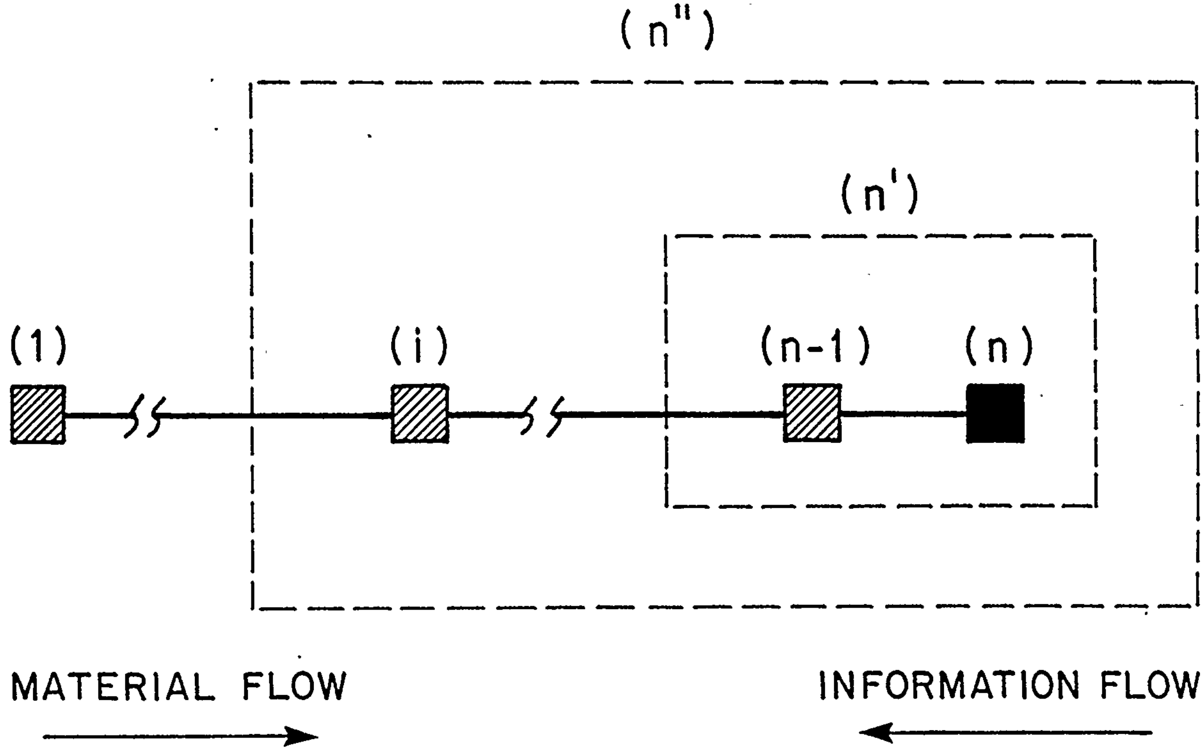


Figure 4.4 : Principle of Optimality

respect to all possible inputs to it. This procedure is continued until the whole system is grouped to a single stage. This stage also has to be optimized with respect to all possible input values. Since the conditions at stage 1 are fixed there will be only one input available. Hence, only one optimal result exists which is the global optimum of the system.

4.2.3.2. Global Optimum of the System

Bellman's Principle of Optimality has to be applied to the pipeline system. Decision making starts at location n which corresponds to stage n in Figure 4.4. No optimization is required at this location since this represents the delivery point where all conditions are fixed. Next step is to group together the last two stations. This group forms stage n' with all possible inputs being synonymous with all possible states at a station.

Figure 4.5 shows schematically the possible states in the form of grids. For the sake of simplicity, only station location and suction pressure have been shown as variables so that a state can be defined in a two dimensional space. Stage n' is optimized with respect to all possible inputs, i.e. all possible states at Station (n-1). Optimization of stage n' with respect to a state is done by computing the minimum cost to pump the fluid from that state at Station (n-1) to Station (n). This cost is added to the cost stored for pumping from Station (n) to the terminal location and

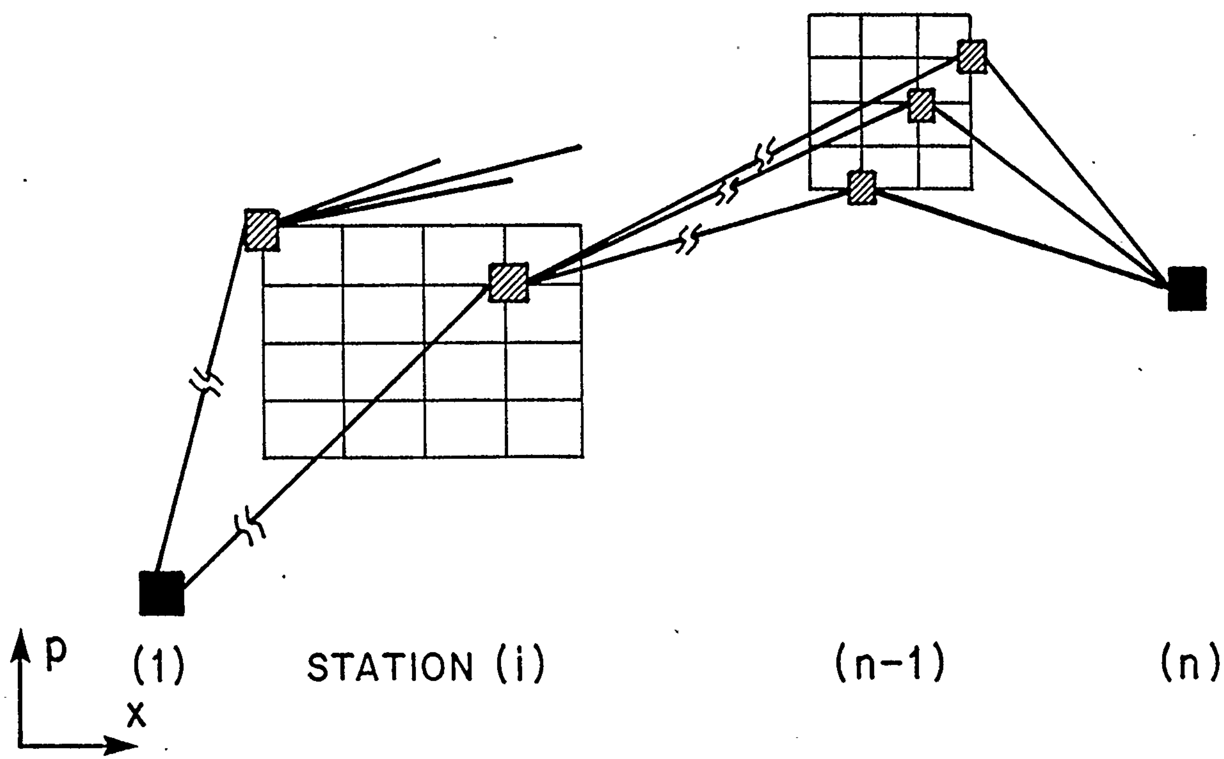


Figure 4.5 : Optimal Solution to the System

the sum is declared to be the optimum with respect to the given state. Although for Station (n) this last statement does not have any significance - since Station (n) stands for the termination of the pipeline - for all other stations it is a very important step. This procedure is repeated for all possible states at station (n-1). After the group is optimized, its size is increased by one station and it is optimized for all possible states of the most recently added station. When the whole system is gathered under one group, there is only one possible state since the conditions at the first station, i.e. the source, are fixed. Hence, the optimal solution with respect to this single input gives the global optimum of the system.

When searching for the optimal solution of the system, which is chosen to be minimum annual cost, all states (combination of levels of different variables) that could be part of the final result are stored. Once the optimal annual cost is obtained, the states being part of the global optimum are listed for all stages. The optimum number of booster stations and their locations leading to minimum cost are determined in this manner. Number and capacity of the pumps at each station are also obtained along with the optimal pipe diameter.

4.3. APPLICATION OF THE SOLUTION PROCEDURE TO TREE NETWORKS

The solution procedure is described for the case of an unbranched pipeline. In case of gathering of two or more lines, the same procedure has to be repeated separately for each of these lines. It has to be kept in mind that at the gathering node the resulting pressure may not be predetermined but it has to be the same for all gathering pipe segments.

When a pipeline is optimized between a known delivery location and a gathering node, no 'one' fixed input exists, whence an optimal solution cannot be declared immediately. Each pipe segment has to be optimized with respect to any possible input, e.g. any possible pressure at the gathering node. Once all segments are optimized, for each of the possible input or for each state, there will be as many solutions stored as there are pipe segments. Usually, these solutions are based on minimum cost. The sum of the optimal costs for all branches ending in the same state are added together. The actual cost at each of these states is found by summing up the costs stored for each pipeline diverging from this node to delivery locations, or other nodes. Finally, the state with the lowest cost is declared to be the optimal solution at that stage.

4.4. FINE-TUNING OF THE RESULTS

During the solution procedure all variables, whether already discrete or continuous of nature, are formulated in a discrete form. Hence, the results can only take discrete values. Annual cost could be a continuous function but because of discrete variables, the resulting cost cannot take any possible value and as a consequence the result may not be accurate. The accuracy of the solution is directly influenced by the step size used in discretizing the continuous variables.

The accuracy of the result can be enhanced by decreasing the step size in discretization, but this would increase the number of possible levels for each variable. The system can be represented more realistically with this modification, leading to an improvement in the accuracy of the results. At the same time, the number of feasible solutions increases, which can increase the number of calculations required to find the optimum significantly.

The accuracy of the results could be improved by increasing the original number of levels, x, of a variable by inserting x number of levels between adjacent levels. This would almost square the number of levels. But because of overlapping of some of the new created levels with the old ones, following formula applies rather than x^2:

$$(x-1)\,x - (x-2) = x^2 - 2x + 2 \tag{4.8}$$

The drastic increase in number of levels causes also the number of calculations to increase significantly. Instead, the same improvement in accuracy is achieved in a much faster way by introducing the method of 'fine tuning': First the approximate solution is found utilizing the original step size. The actual solution is expected to lie within the range of a step size bigger or smaller than the calculated value. The same is true for the actual values of the variables forming the optimal solution. The problem is solved again to 'fine-tune' the solution. For the second run of calculations the lower and upper limits of all variables are shrunk around the approximate solution, concentrating the feasible levels of variables, as well as the solution, in only two step sizes of the previous run. Inserting x number of levels between adjacent levels leads to almost doubling the original number. Following formula applies rather than $2\,x$, because of overlapping:

$$(3-1)\,x - (3-2) = 2x - 1 \tag{4.9}$$

Since the fine-tuning is only made in the neighbourhood of the approximate result, far less calculations are required for achieving the same improvement in the accuracy of the solution.

Care has to be taken, however, with highly nonlinear problems which are not 'well behaved'. If the initial step size is too large, the discretized system cannot represent the actual system accurately enough, because of which the declared optimum may only be a local minimum close to the global optimum.

The dynamic programming technique is known to have the dimensionality problem which becomes stronger with increasing number of variables. The increase in number of calculations can be partly compensated with the fine tuning procedure. This procedure can be repeated until the optimal result remains the same; but usually one or two runs should be sufficient for good accuracy.

4.5. OPTIMIZATION OF OPERATING CONDITIONS

Once the pipeline is designed, the location and number of the booster stations, capacity and number of the pumps as well as the pipe diameter are fixed. Hence, the number of variables decreases and fewer calculations have to be made to find the optimal operating condition of the system. The main concern becomes to decide which pumps to turn on and to determine the suction and discharge pressures at the booster stations.

4.5.1. Dynamic Systems

With the decreased number of calculations the solution algorithm becomes computationally efficient, requiring relatively short computation time. In addition to providing steady-state solutions, the approach presented here can also be utilized to generate a set of optimal operating conditions in the transient (dynamic) mode of the pipeline network by feeding appropriate changes to the program at each time step .

4.5.2. Different Fluids in Series

Different oils can be present in the pipeline at the same time. If these fluids have similar densities, plug flow can be assumed. Since the calculations for the pipeline system can be repeated very frequently, the transient system is assumed to be at steady state for short time intervals, which leads to fairly accurate results, especially in the case of long pipelines. The solution procedure is very similar to the case with a single type of oil flowing. The only difference remains in the friction loss calculations. In each pipe segment, it is known from the input files which portion of the pipe is occupied by which fluid. For each of these portions, the friction loss is calculated separately. Friction losses caused by all fluids present in the pipe are added to give the friction loss in the whole pipe segment.

CHAPTER 5

RESULTS OF THE DEVELOPED PROGRAM

5.1. PROGRAM DESCRIPTION

A computer program was developed based on the solution procedure stated in previous chapters. The algorithm is listed in Appendix A. Separate input files are formed for different kind of inputs, e.g. fluid specifications and capacities of pumps are stored separately so that variations in either of them can be fed to the program conveniently. The range of data as weel as a sample input file are given in Appendix B. The program developed for designing the pipeline network is used to optimize the operating conditions after some modification. The program written to optimize the design is discussed first, following which the modifications necessary to optimize the operating conditions of the system are described.

The solution procedure is repeated for each branch of the network as explained in Section 4.3. The conditions at the exit of a branch are known whereas the entrance conditions are determined after consideration of all gathering branches. Hence, calculations have to start at the end of a pipe segment and continue until the entrance is reached. First, all feasible pipe diameters for such a segment are determined. The calculations to find the optimal operating cost of the segment are repeated inside the loop on all possible diameters. The diameter leading to the most economic conditions is chosen as the optimal diameter for that specific branch. The diameter can also vary along a branch for which the loop on diameter has to be placed in the loop on specifications of a station. The following description of the program is for the case where only one diameter can be chosen for each branch of the pipeline.

Calculations for all 'potential stations' are placed in a loop inside the loop on diameter. The term 'potential station' is used because some of the considered stations may not be needed simply because no boost on pressure is required in the regions in which these stations could be built. The region in which a station can be built is predetermined. The sum of regions may cover the whole area where the pipeline is planned to exist, or only convenient areas may be considered. Each region is divided into increments with each of the grid points representing a possible location of the booster station. The range of allowed pressure at a station is predetermined by upper limitations and the net positive suction head of the pumps. This range of pressure is represented by approximately 20 'pressure levels'.

The main specification of a 'state' at any station is defined by the combination of one station location and one pressure level. The state can also include values for diameter, index for fuel cost etc. Cost of fluid transportation is optimized for each feasible state at a station for the flow from that station to the end of the pipeline. Since the solution procedure is followed in the direction of information flow, which is opposite to the material flow, the states at the following station are already defined. The specifications of the states at the present station and the next station are known including the information on station locations. Hence, for each pair of states at the present and following stations, the length of the pipe connecting these two stations can be determined easily. This value is substituted into the flow equation, which could be any expression relating the friction loss to system conditions and is chosen to be Miller's correlation. Since this expression is nonlinear the friction loss is calculated via the Newton-Raphson method. The convergence problem is taken care of by introducing lower and upper limits on the step size.

The 'best', i.e. most economic, conditions to send the fluid through the pipeline is determined for each possible state at a station. For doing so, first the possible 'routes' to reach a state at the next station from the state at the present station are determined as explained in Section 4.2.2.1. The number of 'routes' represents the number of combinations of pumps which may be placed at a station. The route itself depends on the capacity of the individual pumps. The number of calculations are minimized as explained in Section 4.2.2.2 on 'Elimination of Infeasible Combinations'. In this manner, for each state to be reached at Station (i+1), a route is chosen originating from any state at Station (i). Cost of a route is added to the cost stored at the exit of this route, leading to the cost of pumping the fluid from the state at station (i) to the exit of the pipe segment.

Once all possible costs are calculated, the conditions leading to the lowest cost are declared as the optimal conditions for the specific state. Optimal conditions are stored for all states at a station. This procedure is repeated until the first station of a pipe segment is reached. If there is no gathering of branches, only one state exists at this station whence the optimal conditions of this single state represent the optimal solution of the whole system. If, however, the first station of a pipe segment is at a node where several branches combine into one stream, the procedure described in Section 4.3 is followed.

The conditions leading to the least cost form the optimal solution and are sent to the output file. This file lists specifications of the system including the overall cost. The input files consist of the worst conditions which are expected during the lifetime of the pipeline. If, for these conditions, some of the pumps or even stations are declared to be unnecessary, they do not need to be installed, or built, at all. The output file of the design problem can be used as the input file to the program for finding the optimal operating conditions for various changes in the designed system.

Complete enumeration of all feasible solutions for a pipeline system with 5 possible pipe diameters, 5 possible locations for each station, 10 possible suction pressures and 8 pump combinations at any station of 15 possible stations (end of pipeline excluded) would require $5x(5x10x8)^{15} = 5.37x10^{39}$ calculations. With dynamic programming this number decreases to $5x(5^2x10^2x8)x15 = 1.50x10^6$. The reason for squaring the number of levels for suction pressure and station location is that these parameters are unknown at Station (i) as well as Station (i+1). Since the described solution procedure involves a combination of dynamic programming and implicit enumeration even fewer iterations than $1.50x10^6$ are required. Applying Equation 4.8 to the variables, the accuracy of the results obtained via complete enumeration could lead to $5x(17x82x50)^{15} = 2.22x10^{73}$ calculations. The same accuracy can be achieved with 'fine-tuning'(Section 4.4). Once the number of levels for the variables are increased by the formula given in Equation 4.9 the fine-tuned solution can be obtained in less than $5x(9^2x19^2x15)x15 = 3.29x10^7$ iterations. The drastic decrease in number of evaluations leads to approximately 1 minute of CPU time on the Honeywell Multics System.

The location of the stations, diameter and length of pipe segments are fixed for an existing pipeline. The developed program for the case of design is modified by feeding these specifications instead of giving only a range and repeating the calculations for any possible value of these parameters. Obviously, the number of calculations decreases substantially by doing so. The program which already runs fast, speeds up even more allowing the results to be produced very frequently (less than 20 seconds are required for the case under investigation). Hence, for a dynamic system the optimal operating conditions can be updated as often as necessary.

5.2. DISCUSSION OF RESULTS

The following procedure is adapted to design the pipeline system: First, an allowable range for each parameter is determined based on information obtained from the industry, in combination with data gathered from books and papers (references). Then, the pipeline is designed by letting the optimization program choose the 'best' value for all parameters out of the predetermined ranges.

Once the system is designed, the operating conditions of the pipeline network are optimized with respect to changing conditions based on this 'case'. For a large pipeline with many parameters, it is very difficult to collect all necessary data from a single source to make a case study. Several pipeline companies were approached to gather the information of existing pipelines. The companies supplied different sets of data but a complete set of information about an existing pipeline was not available. Hence, a direct comparison of the operating conditions determined from the optimizing program with the operating conditions of an existing pipeline was not possible.

The effectiveness of the program is demonstrated by comparing the different annual costs resulting from different runs of the program obtained for variation in the number of variables and their values. Seven runs for different optimization cases of design and operating conditions are discussed to indicate the extent of savings which can be achieved with the developed program. Data for these discussed cases are given in Table 5.3. This discussion could easily be continued for many other cases since system modifications can be fed to the program in a convenient manner and little computational time is necessary to produce the output. At this point, however, the aim is to give a general impression of how effective the features of the developed program are.

A sample output is shown in Table 5.1. It is produced for optimizing operating conditions with known flowrate, diameter and fluid specifications. At each station the suction and discharge pressures are listed. The pumping units A, B and C represent the pumps at each station where 1 or 0 indicate the pump being on or off, respectively. The cost of pumping is listed in the last row, where '0' represents that that specific station is by-passed. The optimized annual cost is listed in the last row.

Table 5.2 lists the pressure variation at a station and in the adjacent pipe segment. The pressures are consistent with the schematized pressure profile shown in Figure 3.2. The output can also be represented in graphical form. Figure 5.1 shows the result of optimal design for the fluid specifications and maximum flowrate of the

Table 5.1 : Results of the Program

flowrate (m^3/h)	diameter (in)	viscosity (mPa s)	specific gravity
1589.004	24	0.664	0.735

station number	station location (km)	suction pressure (MPa)	pumping units A B C	discharge pressure (MPa)	pumping cost (M$*)
1	0	0.1	1 0 0	2.8	0.671
2	100	0.4	1 1 0	7.2	1.342
3	200	5.1	0 0 0	5.1	0
4	300	3.4	0 0 0	3.4	0
5	400	1.1	1 0 1	9.9	1.963
6	500	8.0	0 0 0	8.0	0
7	650	5.1	0 1 0	9.2	1.006
8	700	8.0	0 0 0	8.0	0
9	870	5.1	0 0 0	5.1	0
10	920	3.4	0 0 0	3.4	0
11	930	3.1	1 0 0	5.8	0.537
12	1010	4.4	0 0 0	4.4	0
13	1070	2.6	0 0 0	2.6	0
14	1110	1.3	0 0 0	1.3	0
15	1120	1.1	0 0 0	1.1	0
16	1140	0.1	0 0 0	0.1	0
annual cost = 22.086 M$					

(*) : 1 M$ = 1×10^6 $

Table 5.2: Pressure Variation at a Station and in the adjacent Pipe Segment

pressure specification	pressure (MPa)
suction	5.100
pumping	4.054
discharge	9.154
throttling	0.143
friction	1.011

Table 5.3 : Data for Figures 5.1- 5.7

Case number	flowrate (m^3/h)	pipe diameter (in)	specific gravity	viscosity (mPa s)
1	1920	24	0.815	2.199
2	1920	24	0.815	2.199
3	1920	36	0.815	2.199
4	1920	24	0.815	2.199
5	1590	24	0.815	2.199
6	1920	24	0.815	2.199
7	1920	24	0.815	2.199

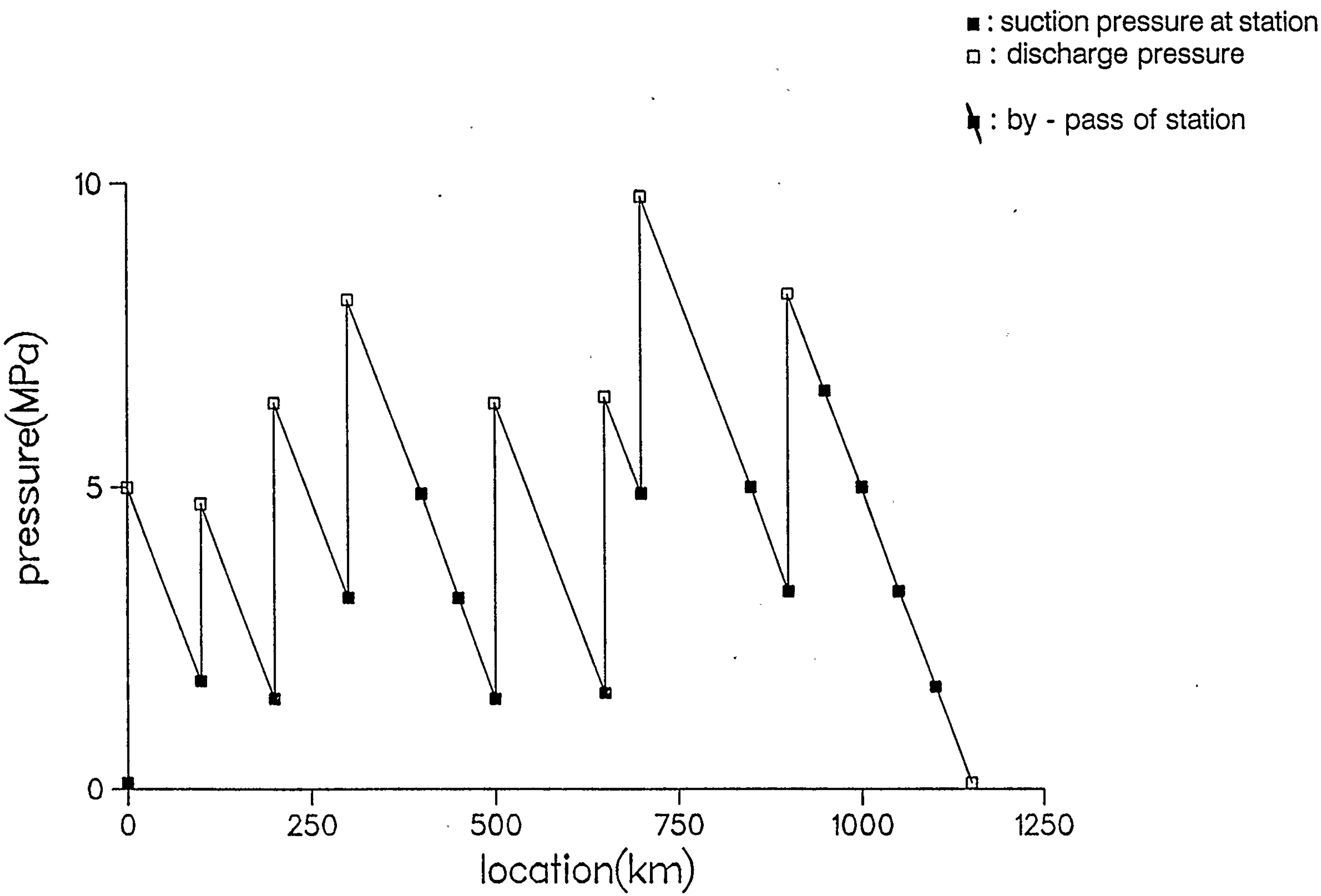


Figure 5.1 : Pressure Profile Case 1, annual cost = 29.569 million $

first case. The change in pressure with location in the pipeline is shown for the optimized case. Out of the 15 possible stations only 8 are required for the specified maximum load. Filled squares represent the booster stations and are located at the levels of the suction pressures. A crossing line through such a square means that that particular station is not needed for the given design conditions, since it indicates that no boost in pressure is required. Empty squares indicate the level to which the pressure is increased at the station as a result of pumping.

The pipeline design is optimized for variable diameter, suction, discharge and throttling pressures, number and capacity of pumps. The suction pressure at the first station and the outlet pressure at the terminal point are fixed at 1 atm. Annual fuel cost is predicted to be 0.350 \$/W. The annual operating cost of this case is calculated to be 29.569 million \$.

Figure 5.2 shows the result obtained for the optimization of the same system but with the addition of variable station location. When the location of any station is not fixed but is allowed to vary by ±6 km around each station, the necessary number of stations to pump the fluid through the 1,150 km long pipeline decreases from 8 (Figure 5.1) to 5. With this improvement the annual cost is decreased to 29.194 million \$.

Besides flexibility in station location, it is also very important to let the diameter

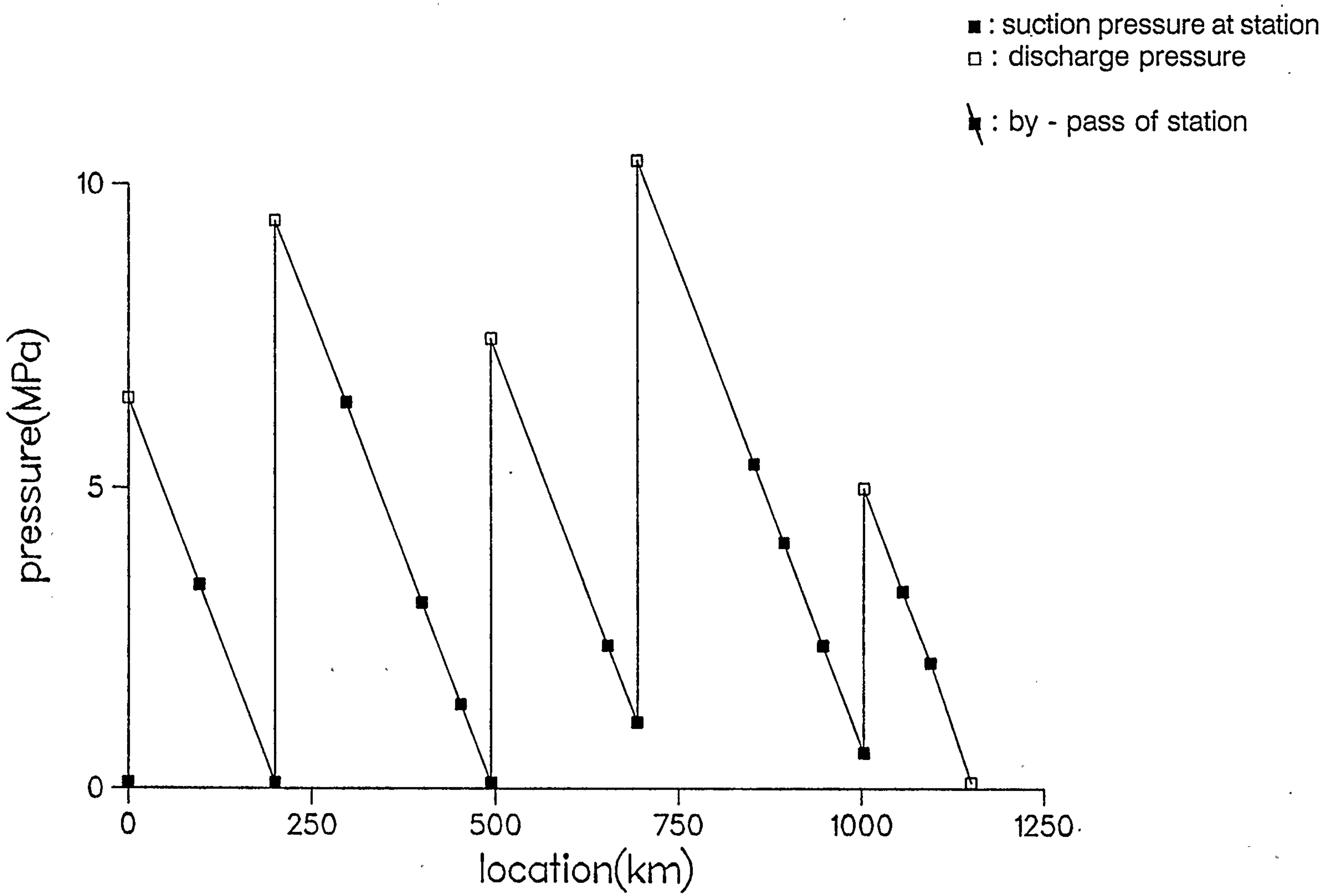


Figure 5.2 : Pressure Profile Case 2, annual cost = 29.194 million $

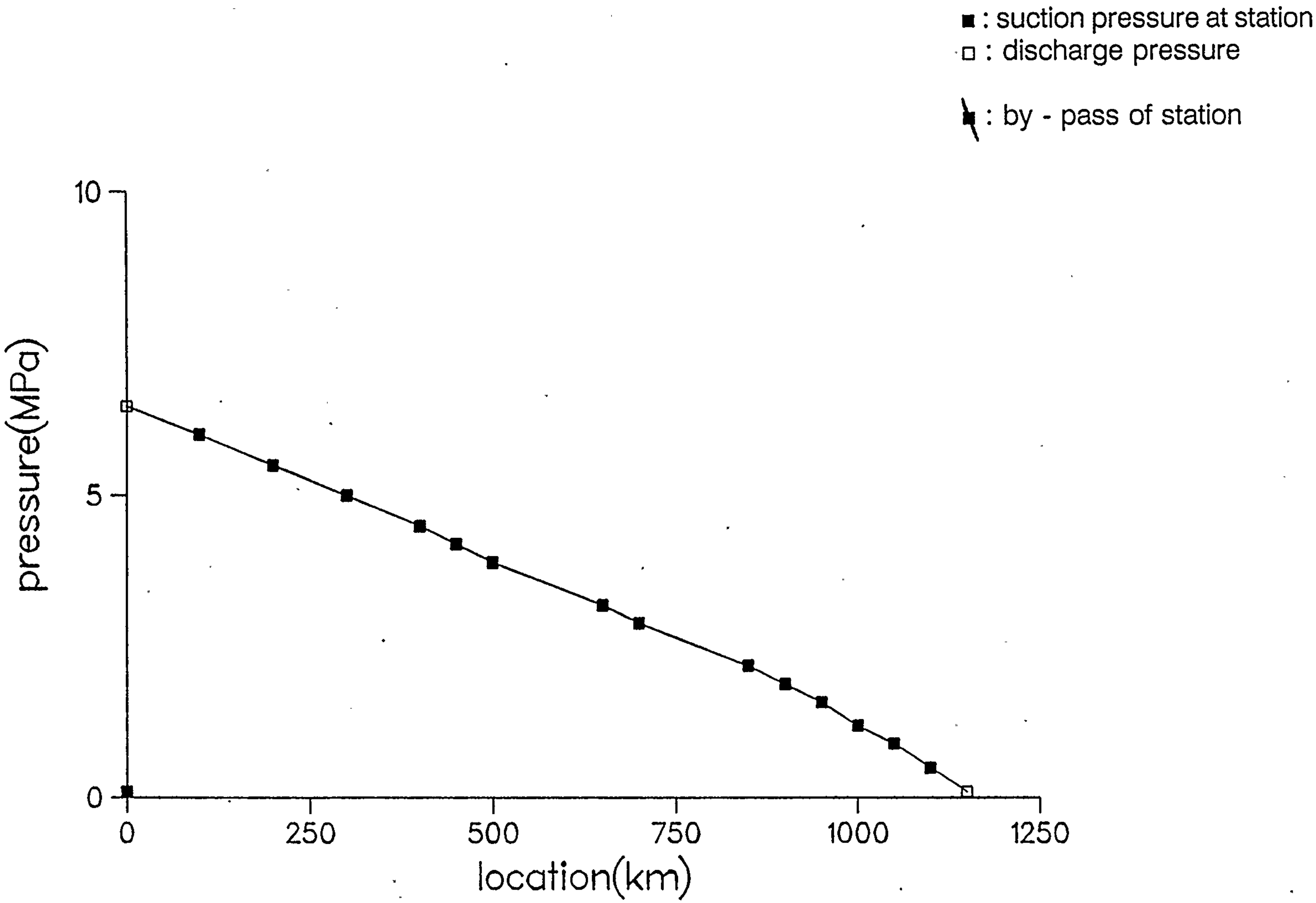


Figure 5.3 : Pressure Profile Case 3, annual cost = 27.712 million $

vary. The significance of variable diameter is shown in Figure 5.3. The optimal design calculations of the case in Figure 5.1 was repeated for variable pipe diameter. It was found that 36" pipe is more economical than 24" pipe although the capital cost of the pipe increases significantly with larger diameter. At the same time, the larger pipe diameter causes less frictional losses which decreases the pumping cost. To pump the same amount of fluid through a 36" pipe requires only a single station compared to 8 stations which are needed in case of 24" pipe. Decrease in pipeline cost and increase in pumping cost balance each other so that even with only one booster station the annual cost of the pipeline system decreases only by 1.86 million \$. Although it seems that increasing the pipe diameter simply decreases the annual cost, this conclusion is likely not to hold for a slightly different case. Since the effect of variation in pipe diameter cannot be predicted *a priori*,it is of interest to keep the pipeline diameter as a variable.

The results from a set of runs are presented in Figures 5.4 to 5.7 to compare cases of optimized operating conditions. Figure 5.4 shows the suction and discharge pressures at all stations for the case of 15 existing booster stations connected through a 24" pipe. The operating conditions are optimized for a fluid with 2.2 mPa s viscosity and 0.82 specific gravity flowing at the rate of 1920 m^3/h. Three pumps are placed at each station. The main concern in the optimization of operating conditions is to determine which pumps to turn on to send the fluid through the pipeline. In searching for the most economic case, it is made sure that

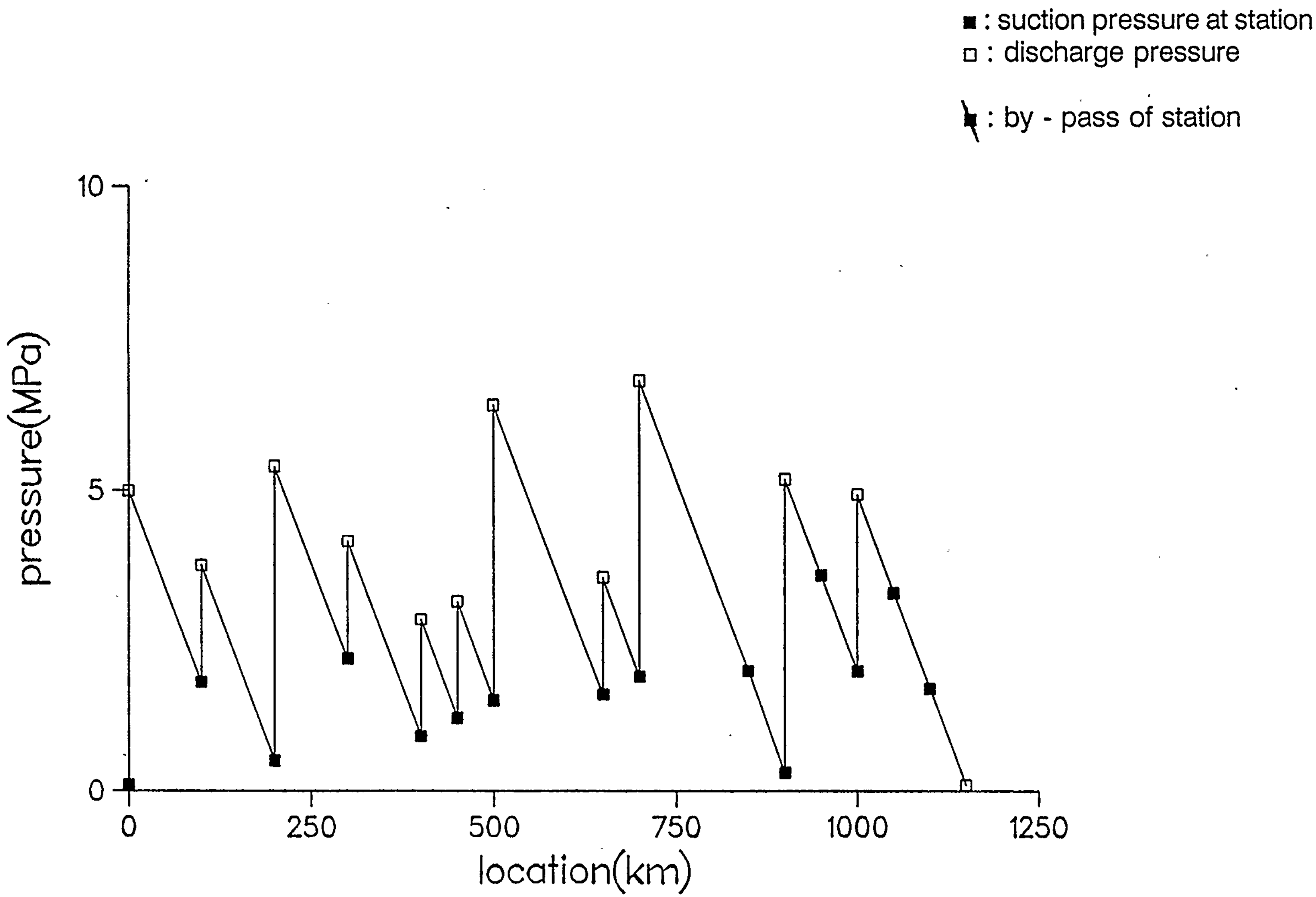


Figure 5.4 : Pressure Profile Case 4, annual cost = 29.461 million $

all restrictions, like pressure at the terminal location, are taken into account. A filled square with a crossing line through it represents stations which are by-passed. Although no pumping cost is necessary at these stations, a fix cost is required for maintaining the station. The annual cost of the described system is found to be 29.461 million $.

An important feature of the program is to be sensitive to changes in flowrate. In reality, the amount of pumping at a station is usually not varied once the pumps are installed. Even when the flowrate declines, pumps are not turned off producing unnecessarily high pressure. It is simply neglected that energy can be saved by shutting off some pumps which may not be required at lower flowrates than the design case. Figure 5.5 represents the resultant pressure profile for optimized operating conditions for the same case used for Figure 5.4, but 1590 m^3/h flowrate instead of 1920 m^3/h. The operating conditions are modified by turning off pumps which are not required to pump the fluid through the pipeline. This results in 4.93 million $ savings which is 16.7 % of the entire operating cost.

A 1500 km pipeline network can sometimes spread over different countries or provinces/ states. It is very likely that the fuel cost would be different at different locations. When the fuel cost is increased only in a specific geographical region the annual operating cost would increase but the system configuration and pressure profile would remain the same if the optimization procedure of operating

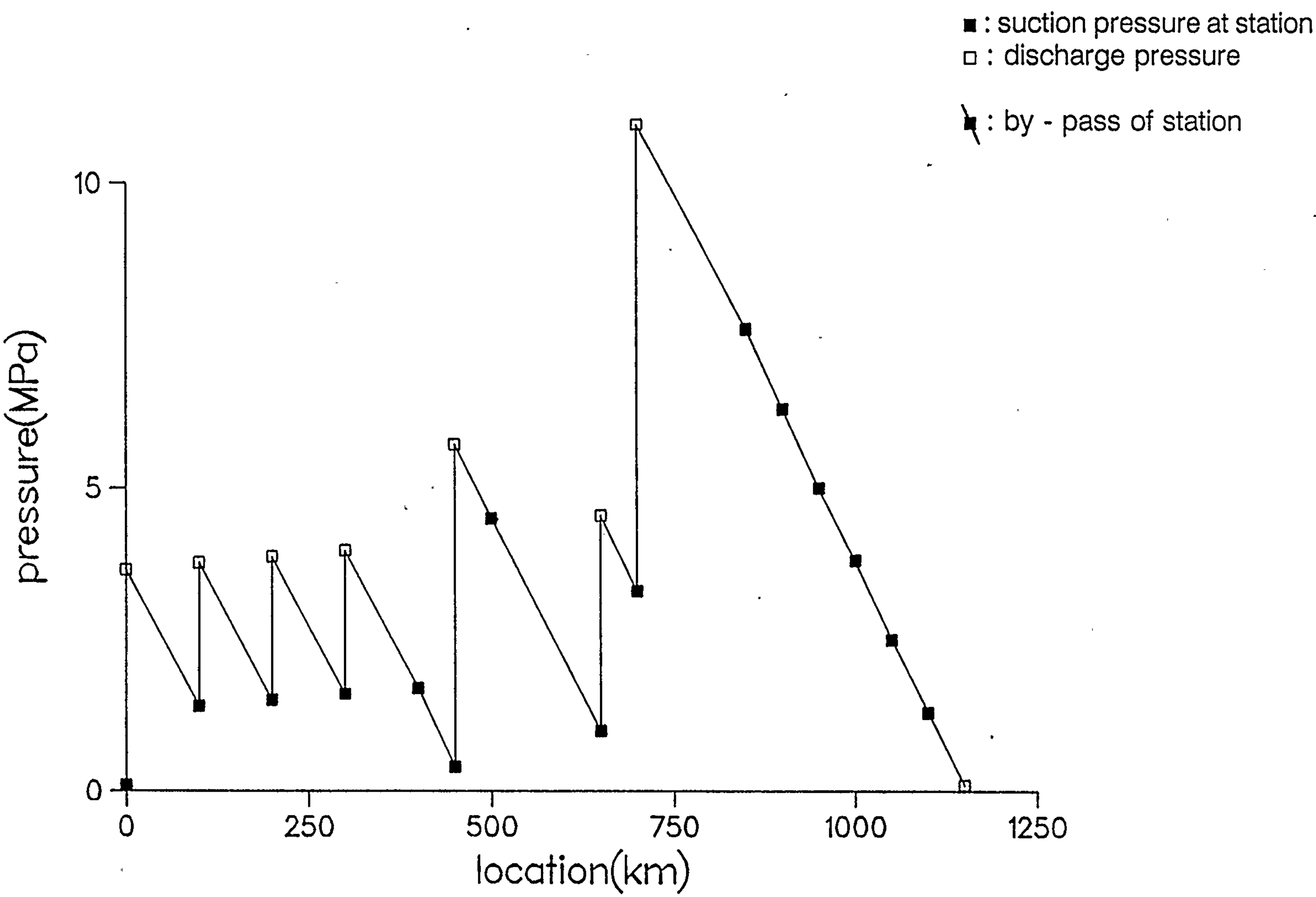


Figure 5.5 : Pressure Profile Case 5, annual cost = 24.428 million $

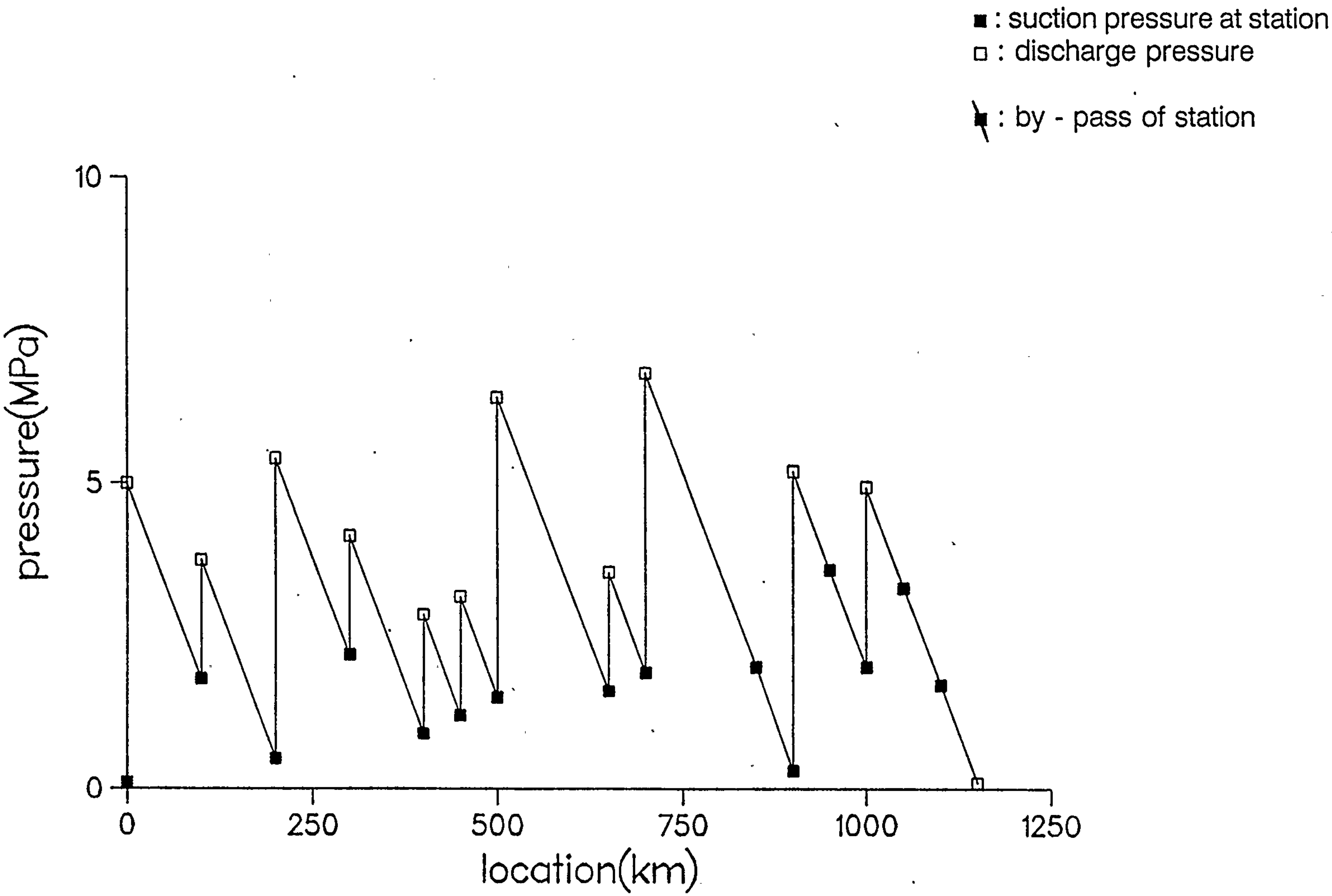


Figure 5.6 : Pressure Profile Case 6, annual cost = 33.656 million $

conditions is not sensitive to variation in fuel cost. This situation is represented in Figure 5.6 with 50% increase in fuel cost at Stations 1 to 8. The pressure profile is the same as in Figure 5.4 and the only change is a 4.2 million $ increase in annual cost. The developed program is capable of modifying the operating conditions with respect to variation in fuel cost. For the mentioned increase in fuel cost, the results suggest that the operation of pumps at three stations should be modified. Figure 5.7 shows the redefined strategy. Pumping capacity is increased at Stations 5 and 6 by turning on additional pumps. This appears to be unexpected, since these stations lie in the 'expensive' region. At the same time 2 pumps are shut off at Station 7 and 1 pump is turned off at Station 8, which are also in the region of higher energy cost. The overall decrease in pumping at the first eight stations is compensated by turning on pumps at stations where no variation in fuel cost is felt. This modification leads to 0.25 million $ savings in annual cost.

Pipeline networks are dynamic systems whence many changes occur in time, e.g. some pumps may be temporarily defective or a lighter fluid than specified in the design case may be transported. The illustrated examples show that substantial savings can be achieved by adjusting the operating conditions whenever a change occurs in or outside the system.

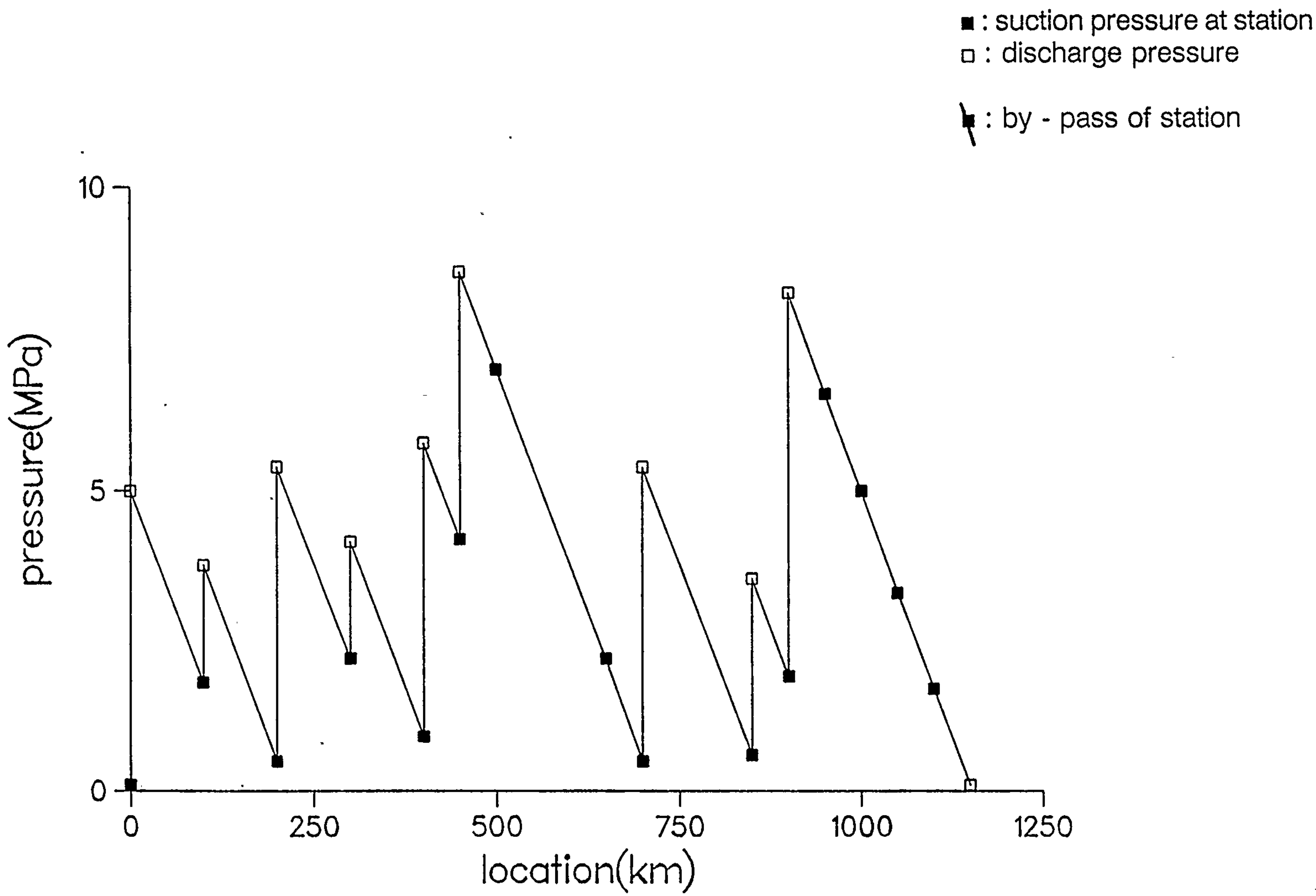


Figure 5.7 : Pressure Profile Case 7, annual cost = 33.404 million $

CHAPTER 6

SENSITIVITY ANALYSIS

The obtained optimal results can only be considered reliable after a sensitivity analysis with respect to the variables and boundary conditions. Because of the complex nature of the tackled problem, an analytical method cannot be applied for the sensitivity calculations. Instead, the sensitivity analysis has to be performed numerically. A base case is formed by running the program with known conditions and obtaining the optimal solution. The calculations are repeated with one of the variables or boundary conditions varied, e.g. increased or decreased by 10 percent. The relative change in the optimal solution with variation in the input is used as an indication of the sensitivity of results to the various variables and boundary conditions.

The nondimensionalised sensitivity of the annual cost, c, to a variable or a boundary condition, u, can be formulated as:

$$s_u = \frac{(\Delta c \;/\; c_{base})}{(\Delta u \;/\; u_{base})} \tag{6.1}$$

where Δu shows the change in the variable u. Δc represents the respective variation in annual cost. Thus, the dimensionless sensitivity coefficient, s_u, represents the sensitivity of the annual cost to the variable u.

The number of variables affects the solution directly, because with an increased number of variables more solutions can be formed whence the possibility of finding a lower minimum than the previous one increases. This was shown in Section 5.2, by comparing the results of systems with fixed and variable station locations. The effect of varying a parameter was also explained in the same section. For example, the program adjusts the operating conditions according to changes in the flowrate thereby eliminating unnecessary pump work. When the flowrate was changed from 1920 m^3/h to 1590 m^3/h, the optimal cost decreased from 29.461 million $ to 24.428 million $. When the 5 million $ saving is substituted into Equation 6.1, the result is: $s_u = 0.99$.

In optimization problems, the boundary conditions of the system may have a significant effect on the optimal solution. For example, although absolute boundaries exist for pressure limitations formed by NPSH and strength of the equipment, it is of interest to keep the pressure range small since it directly affects the number of calculations in the program. On the other hand, this range cannot be kept arbitrarily small since the objective function giving the annual cost of the pipeline network is not 'well behaved'. If the pressure in the system is bound by only a small range, the resulting optimal solution may not be very favourable. It is simply because, inside the absolute limits but outside the present limitations, lower minima may exist which are not considered as solutions. In order not to limit the efficiency of the program, appropriate boundary conditions have to be chosen. The success of a particular choice can be tested by applying sensitivity analysis to the results with respect to variation in boundary conditions.

Boundaries on pressure, station location and pipe diameter are constraints for the described pipeline system. In Section 5.2, it was shown that increasing the range of pipe diameter can have a significant effect on the results. The increase in range can only be achieved by adding another commercially available diameter out of which the optimal diameter can be chosen. A sensitivity analysis on pipe diameter may not be useful since the boundaries on diameter cannot be increased arbitrarily by a percentage. Hence, importance has been given to variation in the limits on pressure and station location.

Changes in annual cost with variations in pressure limitations are summarized in Tables 6.1-6.6. The input for all base cases are kept the same. The lower pressure limits for the case under consideration are in a narrow range of 0.1 - 0.4 MPa and the upper limits vary between 7 to 9 MPa. For the purpose of measuring the sensitivity, the lower limits are varied by ± 10 %. Larger variations are not applied because the lower pressure boundaries are usually around 1 atm. The upper limit at each station may be more flexible whence the upper boundaries are varied by ± 10 % to ± 50 %. The program is run after each change in the input and the resulting optimized annual cost is listed in the second column of the tables. Variation in the new output with respect to the result of the base case is indicated in column 3 (Table 6.1) in the form of percent variation which is simply calculated from:

$$\% variation = \left(\frac{c_{new} - c_{base}}{c_{base}} \right) * 100 \tag{6.3}$$

Once the optimal annual cost for maintaining and operating the pipeline is calculated, the program is run again to search for a lower minimum around the first 'approximate' solution in order to fine-tune the result. Results of fine-tuning are also listed along with the variation in cost with change in pressure limitations. As expected, the annual costs after fine-tuning are lower than the 'approximate' optimized annual costs. The fine-tuned results are used in calculations in order to obtain the sensitivity of cost to the pressure boundaries. The dimensionless sensitivity is found from Equation 6.1 and is calculated by dividing the percent variation in fine-tuned cost (Table 6.1, column 5) by the percent variation in lower or upper pressure limits (Table 6.1, column 1). The dimensionless sensitivity coefficient, s_u, in Table 6.1 varies between 0.0 and 0.057, which is quite

Table 6.1 : Effect of Variation in Lower Pressure Limitations on Annual Cost with dp = 1.0 MPa

variation in lower pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: s_u (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	30.254	0.0	29.415	0.0	0.0
-10	30.254	0.0	29.415	0.0	0.0
+10	30.086	-0.56	29.247	-0.57	-0.057

insignificant relative to the sensitivity of cost to volumetric flowrate (s_u = 0.99). Hence, it can be concluded that the optimized annual cost is insensitive to changes at the lower pressure limits. For this particular case, the pressures which lead to the optimal solution do not lie at the lower boundaries, supporting the conclusion of insensitivity of cost to the lower pressure limitations.

From Table 6.1, it can be read that a +10 % increase of the minimum pressures at all stations results in a lower annual cost by 0.57 %. Although this value may be considered negligible, an increase in lower limits cannot cause any decrease in the cost because the range out of which an optimal solution has to be chosen is shrunk. The slight decrease in cost is due to the discretized pressure. When the lower pressure limit is increased, discretization starts from a different pressure level than in the base case. The new discretized pressure levels may lead to a slight improvement in the solution.

Table 6.2 shows the effect of varying the upper pressure limit at all booster stations. The same base case, as listed in Table 6.1, is used for these calculations. The number of feasible solutions is increased (decreased) by expanding (shrinking) the allowable pressure range of the system; in this case by increasing (decreasing) the upper limit. Surprisingly, widening the pressure range does not seem to improve the results. Similar to the results in Table 6.1, the fluctuation in cost can be explained in terms of the discretization of pressure.

Table 6.2 : Effect of Variation in Upper Pressure Limitations on Annual Cost with dp = 1.0 MPa

variation in upper pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: s_u (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	30.254	0.0	29.415	0.0	0.0
-10	30.244	-0.03	29.573	0.54	0.054
+10	30.422	0.56	29.415	0.0	0.0
-20	30.244	-0.03	29.405	-0.03	0.0015
+20	30.422	0.56	29.415	0.0	0.0
-50	30.610	1.2	29.603	0.64	0.0128
+50	30.412	0.52	29.573	0.54	0.0108

The maximum allowable pressure level represents the upper pressure limit which is usually not one step size apart from the adjacent pressure level. When the range is expanded, the pressure level at the previous maximum is usually skipped. The results listed in Table 6.2 are produced with a pressure step size, dp, of 1 MPa which, after discretization over a range of 0.1 - 8 MPa, leads to nine pressure levels. A 10 % increase in the upper limit leads to 10 levels and the ninth level is shifted from 8 to 8.1 MPa. Although a change of 0.1 MPa does not seem to be very important, the optimal configuration may change significantly, especially because large step size leads to a discretization which cannot represent the actual range properly, and 'loss' of the previous maximum pressure level can affect the optimal solution unfavourably. An additional disadvantage of large step size is that the resulting optimum may be far off the global minimum. This is because the range in which the optimum is searched would not be well represented.

In order to improve the accuracy of results, the pressure range at each station should be divided into a larger number of levels, i.e. the step size should be decreased. The results in Tables 6.3 and 6.4 were obtained with dp = 0.5 MPa, which implies doubling the number of possible pressure levels at each station. The base case in Tables 6.3 and 6.4 is the same as in Table 6.1. With 'finer' discretization, the optimal annual cost decreases from 30.254 million \$ to 29.741 million \$. The accuracy of results for the modified cases in Tables 6.3 and 6.4 is also improved but there are still some undesirable results, such as a lower cost

Table 6.3 : Effect of Variation in Lower Pressure Limitations on Annual Cost with dp = 0.5 MPa

variation in lower pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: s_u (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	29.741	0.0	29.573	0.0	0.0
-10	29.751	0.03	29.583	0.03	-0.003
+10	29.563	-0.60	29.227	-1.17	-0.117

Table 6.4 : Effect of Variation in Upper Pressure Limitations on Annual Cost with dp = 0.5 MPa

variation in upper pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: s_u (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	29.741	0.0	29.573	0.0	0.0
-10	29.741	0.0	29.573	0.0	0.0
+10	29.731	-0.03	29.395	-0.60	-0.06
-20	29.741	0.0	29.573	0.0	0.0
+20	29.731	-0.03	29.731	0.53	0.0265
-50	29.593	-0.50	29.425	-0.50	0.01
+50	29.731	-0.03	29.731	0.53	0.0106

with shrunk pressure range. Hence, the step size dp could still be decreased some more.

Results shown in Table 6.5 and 6.6 were obtained with a 0.2 MPa step size. When the maximum pressure is decreased by 20% the new maximum pressure lies at a level which did not exist in the base case. Results show that one of these maximum pressure levels is part of the configuration leading to a optimum which is slightly lower than the optimum result for the base case. The discrepancy in results due to discretization is much less in the case of 0.2 MPa step size than in the case of 1.0 MPa. The slight variation of 0.03% in some results can be ignored since discretization of continuous functions would always introduce inaccuracy. Such inaccuracies can only be minimized by using infinitesimal step size which obviously would increase the number of calculations tremendously.

The results in Tables 6.5 and 6.6 are accurate enough that fine-tuning on them does not lead to a further improvement. The sensitivity coefficient varies between 0 - 0.015 allowing the conclusion that the annual cost is not sensitive to the pressure boundaries. Hence, the results are reliable and one does not to be concerned whether a lower minimum is missed just because of the pressure limitations. This conclusion is also indicated by the results listed in Tables 6.1, 6.2 and 6.3, 6.4 which are not accurate but may be useful to get a general idea about the system and its optimum without excessive computation time. A comparison of

Table 6.5 : Effect of Variation in Lower Pressure Limitations on Annual Cost with dp = 0.2 MPa

variation in lower pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: s_u (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	29.247	0.0	29.247	0.0	0.0
-10	29.227	-0.07	29.227	-0.07	-0.007
+10	29.217	-0.10	29.217	-0.10	-0.01

Table 6.6 : Effect of Variation in Upper Pressure Limitations on Annual Cost with dp = 0.2 MPa

variation in upper pressure limits (%)	optimized annual cost		optimized annual cost after fine tuning		sensitivity coeff.: $\dot{s}_u$ (Equation 6.1)
	(M$/y)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
0 (base case)	29.247	0.0	29.247	0.0	0.0
-10	29.247	0.0	29.247	0.0	0.0
+10	29.247	0.0	29.247	0.0	0.0
-20	29.237	-0.03	29.237	-0.03	-0.0015
+20	29.237	-0.03	29.237	-0.03	-0.015
-50	29.267	0.07	29.267	0.07	0.0014
+50	29.247	0.0	29.247	0.0	0.0

results listed in Tables 6.1-6.6 shows that accuracy of the solution is affected by the choice of step size.

Besides the boundary on pressure, the limitations on station locations should also be examined carefully in order to check the reliability of the results. Unlike the pressure range, the range in which the 'best' location for a station is searched is treated not to be continuous since inconvenient locations on a hill or far from a town can be excluded immediately. Results for 1, 3 and 5 possible location(s) at which a station can be placed are listed in Table 6.7. Allowing only one location for a station represents the case of fixed station locations. A comparison of the first and second rows shows that the annual cost of the studied case can be decreased from 30.576 million $ to 29.717 million $ by making station locations variable. This is equivalent to 2.81 % savings in annual cost. Further savings can be achieved by increasing the number of locations at which a station can be built. With 5 possible station locations per station over a range of 12 km, the annual cost can be decreased by an additional 1.71 %. The critical decision appears to be to keep the station location as a variable. Additional improvement can be achieved by increasing the range in which the optimal location has to be searched for.

Results in Table 6.8 show how a variation in the location range affects the overall cost of designing the pipeline system. The number of possible locations for each station are kept at 5 and the range is varied by changing the step size between

Table 6.7: Effect of Limitations on Station Location on Annual Cost, step size fixed at 3 km

# of possible locations for each station	range for each station location (km)	optimized annual cost (M$/y)	relative improvement in cost (%)
1	0	30.576	0.0
3	6	29.717	2.81
5	12	29.193	1.76

Table 6.8: Effect of Limitations on Station Locations on Annual Cost, number of possible locations for each station fixed at 5

range for each station location		optimized annual cost		sensitivity coeff.: s_u (Equation 6.1)
(km)	variation from the base case (%)	(M$/y)	variation from the base case (%)	
12.0 (base case)	0.0	29.194	0.0	0.0
10.8	-10	29.529	1.15	0.115
13.2	+10	29.194	0.0	0.0
9.6	-20	29.549	1.22	0.061
14.4	+20	29.194	0.0	0.0
6.0	-50	29.707	1.76	0.035
18.0	+50	29.194	0.0	0.0

the possible locations. Increasing the 3 km step size by 10, 20 or even 50 % does not yield any improvement. The strongest effect of shrinking the range is felt by decreasing the step size from 3 to 2.7 km. The sensitivity s_u in this case is 0.115, indicating that the cost is not very sensitive to the location boundaries.

Sensitivity analysis on the boundary conditions is necessary in order to determine whether the limits have to be relaxed or not. Results can be considered reliable, only if they do not vary with modifications in boundary conditions. In this Chapter, it has been shown that the annual cost does not vary much with changing the boundaries given for the 'case study', which is called 'base case' in the tables. Hence, the results obtained for the studied case are found to be reliable. The sensitivity analysis should be repeated for each different system since the sensitivity is obtained numerically and can vary with different input.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

An efficient algorithm is developed for optimal pipeline design. Several design parameters, such as pipe diameter, number of booster stations and their locations, number and capacity of pumps at each station, pressure profile of the system including suction and discharge pressures at the stations can be optimized in less than one minute of computer time.

Since the optimization procedure is built upon a main module with individual functions connected to it, the validity of the results can be assured by using sophisticated cost functions and choosing appropriate flow equations. The case study is based on a fairly generalized cost function since the aim of this study was to develop an optimization technique rather than doing accurate economic calculations. The cost function can easily be replaced with a more realistic expression to improve the applicability of the results. At the same time, separate cost functions can be provided to allow for variations in an interprovincial pipeline.

Compared to complete enumeration of possible solutions, computation time is decreased drastically, due to combined use of dynamic programming, integer programming and 'fine tuning'. The optimal solution to a pipeline system which would be achieved by complete enumeration with 2.22×10^{73} iterations can be obtained with less than 3.29×10^{7} iterations in the optimization program.

The algorithm is flexible such that it can also be used for optimizing the operating conditions of an existing pipeline. Although building a pipeline is obviously more costly than operating it, it can be significant to adjust the operating conditions according to changes inside and/or outside the system.

Substantial savings are achieved in a very short computation time. A variation in each of the basic design parameters and operating conditions can lead to 10-20% savings of the annual cost.

The described model is applicable to dynamic systems. Optimization of a pipeline can be achieved in less than 20 seconds on the Honeywell Multics System. This feature becomes significant when changes occur in the system. If, for example, some of the pumps become defective, cost of power changes, or flowrate is varied, new optimal operating conditions can be obtained almost immediately.

7.2. RECOMMENDATIONS

The applicability of the algorithm can be increased by making the cost function of the pipeline more realistic. Improvements could be made by taking the present worth of expenses into account and considering the depreciation of equipment.

Another improvement could be achieved by increasing the number of variables. Adjusting the wall thickness of the pipeline according to pressure variations could lead to additional savings but the 'curse of dimensionality' has to be kept in mind in doing so. Each additional variable increases the number of calculations required to reach the optimal annual cost. Hence, before deciding which parameters to add as variables, a sensitivity analysis should be made to ascertain their relative importance.

Necessary modifications for gas flow have been explained. Applying these changes to the algorithm should not require much effort but would increase the range of its applicability significantly.

The optimality of the solution is based on minimum cost. In reality, there may be cases where additional constraints can cause a solution with higher cost to be more favourable than the original optimum solution. Repeating the solution procedure for each change in and/or outside the system (including an increase in the number of constraints) can be time consuming for a system with many variables. Instead, suboptimal solutions could be stored along with the global optimum so that the operating engineer could understand the behaviour of the system better and could decide to operate at suboptimal conditions, if necessary.

REFERENCES

Bellman, R.E., "Dynamic Programming", Princeton University Press, Princeton, NJ, 1957.

Cheeseman, A.P., "How to Optimize Gas Pipeline Design by Computer", Oil and Gas Journal, Vol. 69, No. 46, pp 65-68, 1971.

Deb, A.K., "Optimum Energy Cost Design of a Pipeline", Journal of Pipelines, Vol. 1, No. 3, pp 191-196, 1981.

Edgar T.F. and D.M. Himmelblau, "Optimization of Chemical Processes", McGraw-Hill, 1988, Chapters 9 and 12.

Edgar, T.F., D.M. Himmelblau, and T.C. Bickel, "Optimal Design of Gas Transmission Network", Society of Petroleum Engineers Journal, pp 96-104, April 1978.

Explorer Pipeline Company Handout, Calgary, 1989.

Flanigan, O., "Constrained Derivatives in Natural Gas Pipeline System Optimization", Journal of Petroleum Technology, pp 549-556, May 1972.

Gas Processor Suppliers Association, "Engineering Data Book", 1980, Chapter 10.

Gilmour, B.J., "The Development of An Optimization Program", PSIG Annual Meeting, Albuquerque, NM, Oct. 24-25, 1985.

Gopal, N.V., "Optimizing Pipeline Operations", Journal of Petroleum Technology, Vol. 32, No. 11, pp 2063-2067, 1980.

Grelli, G.J., "Implementing An Optimization Program For A Natural Gas Transmission Pipeline", PSIG Annual Meeting, Albuquerque, NM, Oct. 24-25, 1985.

Grelli, G.J. and J. Gilmour, "Western U.S. Gas Pipeline Optimization Program Reduces Fuel Consumption, Trims Operating Cost", Oil and Gas Journal, Vol. 84, No. 28, pp 100-104, 1986.

Hathoot, H.M., "Minimum Cost Design of Pipelines", Journal of Transportation Engineering, Vol. 112, No. 2, pp 465-480, 1986.

Huang, Z. and A. Serreg, "Optimization in Oil and Gas Pipeline Engineering", Journal of Energy Resources Technology Transactions ASME, Vol. 107, No. 2, pp 264-267, 1985.

Jha, I.C., "Pipeline Design For Natural Gas Transportation", Chemical Age of India, Vol. 38, No. 10, pp 539-541, 1987.

Kally, E., "Pipeline Planning by Dynamic Programming", Journal of American Water Works Association, Vol. 61, No. 3, pp 114-118, 1969.

Kurak, S.D., "Power Optimization Yields Savings For Products Pipeline", Oil and Gas Journal, Vol. 87, No. 26, pp 65-66, 1989.

Mah, R.S.H. and M. Shacham, "Pipeline Network Design and Synthesis", Advances in Chemical Engineering, Vol. 10, pp 125-205, 1978.

Mantri, V.B., L.B. Preston and C.S. Pringle, "Computer Optimizes Natural Gas Pipeline Operation", Pipe Line Industry, Vol. 65, No. 1, pp 37-38, 41-42, 1986.

Martch, H.B. and N.J. McCall, "Optimization of the Design and Operation of Natural Gas Pipeline Systems", 47th Annual SPE of AIME Fall Meeting Preprint SPE-4006, 1972.

McCabe, W.L. and J.C. Smith, "Unit Operations of Chemical Engineering", 3rd ed., McGraw-Hill, New York, NY, 1976, Section 2.

Nicholson, T.A.J., "Optimization In Industry, Vol. 1 - Optimization Techniques", Aldine-Atherton Inc., New York, 1971.

Peters, M.S. and K.D. Timmerhaus, "Plant Design and Economics for Chemical Engineers", 4th ed., McGraw-Hill, New York, 1991, Chapter 14.

Rao, S.S., "Optimization: Theory and Applications", 2nd ed., Halsted Press, New York, 1984, Chapters 9 and 10.

Tsal, R.J. and M.S. Adler, "Evaluation of Numerical Methods For Ductwork and Pipeline Optimization", ASHRAE Transactions, USA, Vol. 93, Part 1, pp 17-34, 1987.

Tsal, R.J., E. Gordon, K.O. Simpson and R.R. Olson, "Optimal Gas Pipeline Design via Dynamic Programming With Variable Stages", PSIG Annual Meeting, New Orleans, Louisiana, Oct. 30-31, 1986.

Wong,P.J. and R.E. Larson, "Optimization of Natural Gas Systems via Dynamic Programming", Industrial and Engineering Chemistry, AC-12, No.5, pp 475-481, 1968.

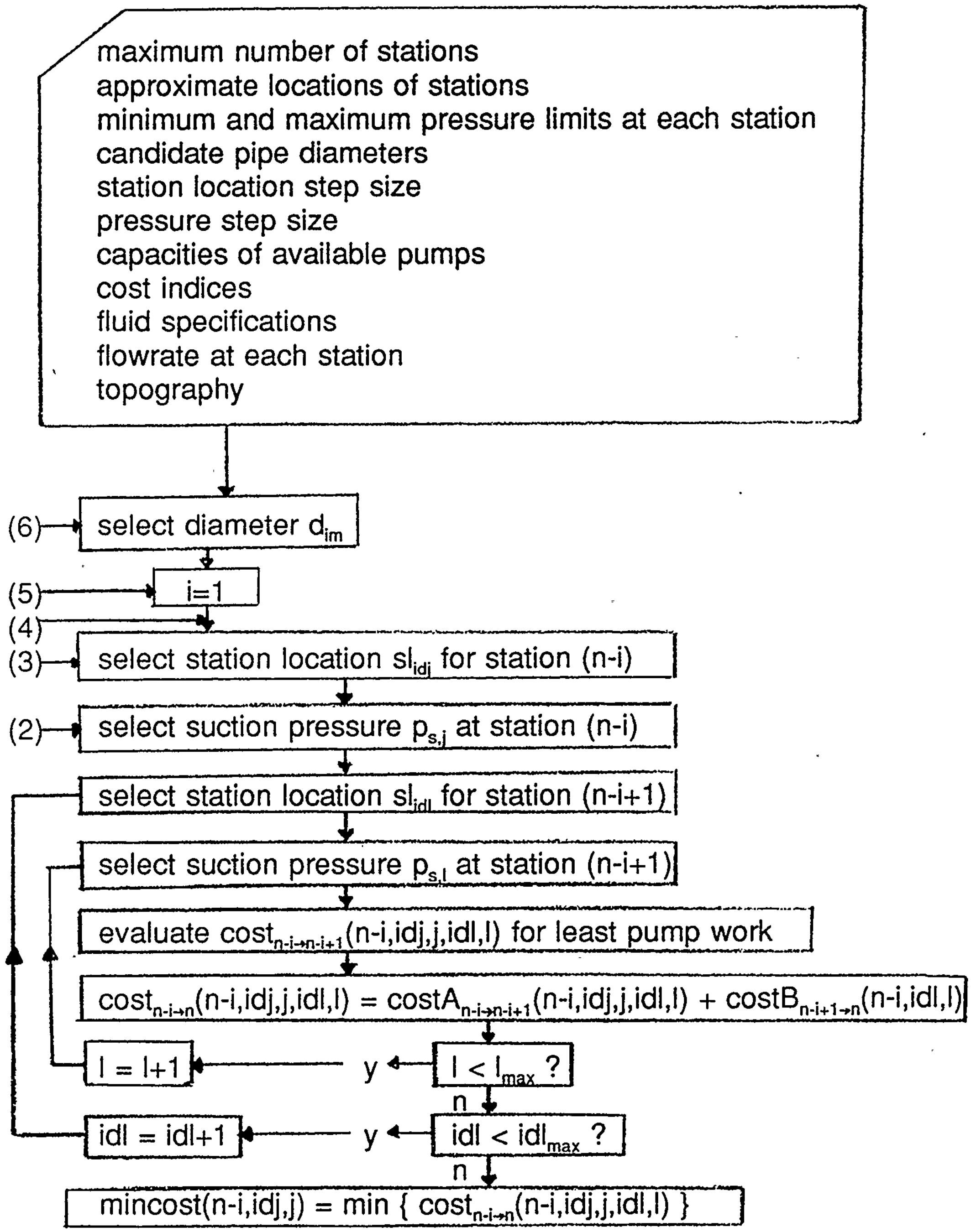
Younger, A.H., "Natural Gas Processing Principles and Technology, ENCH

607/609 Lecture Notes", University of Calgary, 1989, Chapter 6.

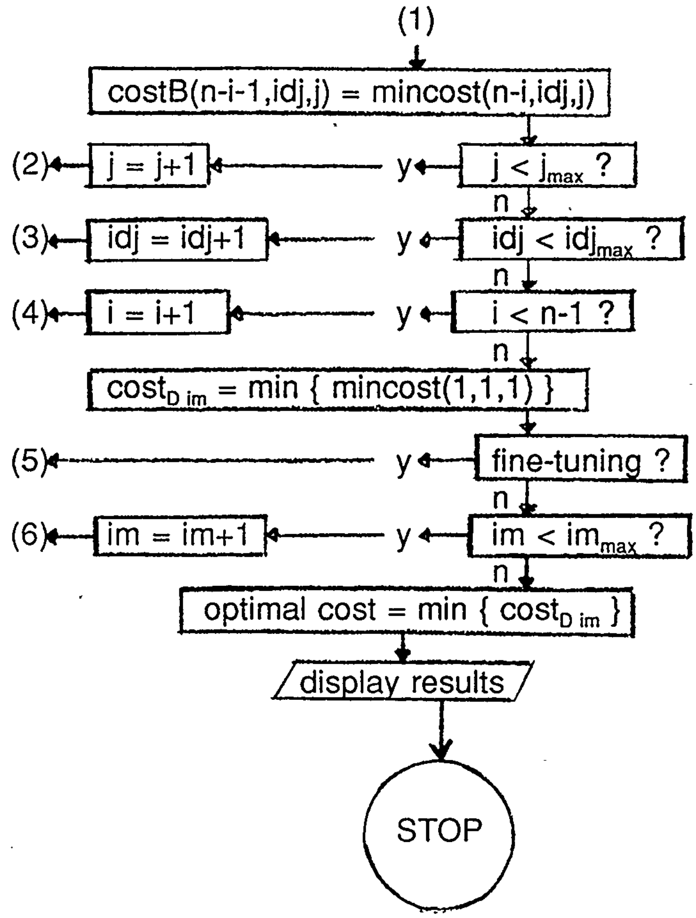
Zimmer, H., "Dynamic Programming. A New Way To Calculate Optimum Design", Pipeline Industry, Vol. 43, No. 1, pp 44-48, 1975.

APPENDIX A:

Algorithm



(1)


(1)
costB(n-i-1,idj,j) = mincost(n-i,idj,j)
(2)
j = j+1
y
j < j_{max} ?
n
(3)
idj = idj+1
y
idj < idj_{max} ?
n
(4)
i = i+1
y
i < n-1 ?
n
$cost_{D\ im}$ = min { mincost(1,1,1) }
(5)
y
fine-tuning ?
n
(6)
im = im+1
y
im < im_{max} ?
n
optimal cost = min { $cost_{D\ im}$ }
display results
STOP

APPENDIX B:

DATA

Although calculations have been repeated only in the ranges listed below, the algorithm can be used for broader ranges.

candidate pipe diameters:	12" - 36"
pressure step size:	0.2 - 1.0 MPa
flexibility around a station:	0 - 18 km
elevation variation:	0 - 1700 m
flowrate:	1000 - 2000 m^3/h
viscosity:	0.6 - 3.5 mPa s
specific gravity:	0.7 - 0.85
capacity of available pumps:	2000 - 4500 hp
cost indices:	0.5 - 1.8
minimum pressure:	0.1 - 0.6 MPa
maximum pressure:	0.1 - 9 MPa
maximum number of stations:	20

Sample Input File

Each set of data (given in a line or a column) is stored in a separate file.

three possible pipe diameters: 12", 24", 36"
pressure step size: 0.2 MPa
maximum three pumps at a station with 2000, 3000 and 4500 hp capacities.
maximum number of stations: 15(+ terminal location).
flat topography.
oil transportation with 0.815 specific gravity and 2.2 mPa s viscosity.
5 possible locations in a 12 km range for each station.

station number	approximate station location (km)	flowrate (m^3/h)	minimum pressure (MPa)	maximum pressure (MPa)	cost index
1	0	1920	0.1	0.1	1.4
2	100	1920	0.4	8.0	1.2
3	200	1920	0.1	7.0	1.0
4	300	1920	0.4	8.0	0.9
5	400	1920	0.1	7.0	0.8
6	450	1920	0.4	8.0	1.1
7	500	1920	0.1	7.0	1.3
8	650	1920	0.4	8.0	1.4
9	700	1920	0.1	7.0	1.0
10	850	1920	0.4	8.0	1.6
11	900	1920	0.1	7.0	0.9
12	950	1920	0.4	8.0	1.3
13	1000	1920	0.6	7.0	1.1
14	1050	1920	0.3	8.0	0.8
15	1100	1920	0.1	9.0	1.0
16	1150	0	0.1	0.1	1.0