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Quantification, Opacity and Modality

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ABSTRACT

QUANTIFICATION, OPACITY AND MODALITY

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This paper is about whether or not quantification is permissible in contexts which are referentially opaque. Quine argues that such quantification is impermissible and, a large part of this paper is devoted to understanding that claim and the arguments for its support. The alleged arguments, however, fail to establish Quine's claim. But Quine also argues that quantification into a modal context leads to nonsense. This raises interesting issues regarding the intelligibility of quantified modal logic, and issues which have to do with essentialism; these concerns are addressed in the second half of this paper.

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For my mother and father.

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Introduction

In natural language, we normally expect that two expressions with the same referent are intersubstitutable in a given sentence *salva veritate*. It is reasonable to suppose that if the sentence 'Somebody owns John Merrick's bones' is true, then the sentence 'Somebody owns the elephant man's bones' must be true as well; given, of course, that John Merrick *is* the elephant man. However, there are linguistic contexts which limit our freedom to substitute terms which are coreferential; such contexts are dubbed by Quine as being *referentially opaque*. A referentially opaque context is such that if there is an occurrence of a singular term in this context, then a substitution of that occurrence of the singular term with a coreferential one yields a sentence which differs in truth-value from the original sentence.

Why bother worrying about referentially opaque contexts? What does the phenomenon teach us? Well, some philosophers[1] suggest that a term which occurs in a referentially opaque context behaves differently than when it occurs in a non-referentially opaque context. These philosophers have articulated theories which purport to explain the role of an occurrence of a singular term in an opaque context. Quine, roughly speaking, thinks that whatever the semantic contribution of the occurrence of a singular term in an

[1] Quine and Frege, for example.

opaque context is, it is not merely the specification of the object denoted by the singular term in question. According to Quine, a singular term which occurs in an opaque context does not merely refer to an object, while it may and normally does in non-opaque contexts. Frege, however, thinks that the reference of an occurrence of a singular term in an opaque context (or *oblique*, in Frege's terminology) refers to what he calls its *customary sense*. But in any case, the behavior of occurrences of singular terms has been widely discussed in the last century. These discussions have been fruitful in, but not limited to, semantics in the philosophy of language, metaphysics and linguistics.

The relevance of discussing singular terms in referentially opaque contexts becomes very apparent in discussions which involve quantification. Existential Generalization is a rule of 1st-order logic which allows us to infer the sentence 'Something is mortal' from the sentence 'Thales is mortal'. More generally, EG tells us that if α *is P*, then *something is P*. But the existence of referentially opaque contexts seems to suggest that EG is an invalid rule of inference, and Quine has argued that, consequently, quantification into opaque contexts leads simply to nonsense. Quantified modal logic, for instance, has been the object of much criticism by Quine. Quine says, for example, that: If to a variable which occurs inside the scope of a modal operator, we prefix a quantifier with the intention that it govern that variable from outside the referentially opaque context, then what we end up with is nonsense.[2] Surely, then, if we are inclined to maintain that quantified modal discourse *is* intelligible, then we have got to address Quine's charge of the apparent nonsense of quantified modal logic. And one way of addressing this charge is to consider first and foremost the phenomenon of referential opacity itself.

In *Chapter One*, Section 1, I characterize the phenomenon of referential opacity and discuss its connection with the related notion of a *purely referential occurrence of a singular term.* In Section 2, I consider two main arguments in defense of Quine's claim, and argue that both fail to establish it. In *Chapter Two*, Section 1, I examine Smullyan's response to the claim that the modal case violates the principle of identity. I conclude with Smullyan that the modalities are not a counterexample to the principle of identity. Smullyan's response invokes theorem *14.18 of Russell and Whitehead's *Principia Mathematica.* Consequently, I consider and reject an argument which takes the apparent failure of EG in modal contexts as evidence for the claim that quantification into modal contexts is not permissible. Here, I show that a plausible version of EG is derivable from theorem *14.18 of *Principia*; and if theorem *14.18 is not threatened by the modal case, and if the plausible version of EG is derivable from *14.18, then the derived principle of EG is

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[2] Quine, W. From A Logical Point Of View. (Cambridge: Harvard University Press, 1996) 148. I use the phrase 'Quine's claim', or 'Quine's claim that quantification into opaque contexts is illegitimate', or '...simply leads to nonsense' interchangeably and, is meant to reflect what he said on page 148 of "Reference and Modality." (*See footnote 9 on page 27 of this paper.*)

not threatened by the modal case as well. In Section 2, I articulate some of the key features of a formal semantics for quantified modal logic. In Section 3, I give a general discussion of Aristotelian essentialism, the view that among the traits of an object, some are essential to it and some are not. While epistemic concerns about essentialism present challenges to quantified modal discourse, I argue that these concerns are independent of the *intelligibility* of quantified modal discourse, and conclude that discourse which involves essential properties (or quantified modal discourse) is, contrary to what Quine thinks, intelligible.

Chapter One

Section 1

In "Reference and Modality," Quine writes: "[*G*]*iven a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true.*"[3] Assuming that what Quine means by the word 'statement' is what we would normally mean by the word 'sentence', it seems that the idea here is that singular terms which denote the same object are intersubstitutable in any true sentence *salva veritate.* Let us suppose that the name 'Bugs Bunny' and the description 'the long-eared galoot' both denote Bugs Bunny. Then, the sentence 'Bugs Bunny always outsmarts his foes' is true only if the sentence 'The long-eared galoot always outsmarts his foes' is true. The way in which the principle is articulated above leaves room for a substitution of coreferential terms in *any* true sentence. In other words, the principle does not specify what kind of sentence is open to substitution, and what kind of sentence is not open to substitution- the principle simply tells us that *all* true sentences are open to substitution of coreferential terms. One might propose then to understand Quine as enunciating the following principle, variously known as *Leibniz's Law*:

(1) For any singular terms α, β, $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, only if, for any sentences S and S', if S' is orthographically the same as S except that S' contains one

[3] *Ibid.*, 139.

> (or more) occurrences of β where S contains an occurrence of α, S expresses a true proposition only if S' expresses a true proposition.

However as Quine has sometimes stressed, it is easy to find cases contrary to (1). For instance, whereas sentences

(2) Giorgione was so-called because of his size

and

(3) Giorgione = Barbarelli

are both true, a substitution of the occurrence of 'Giorgione' in (2) for 'Barbarelli' leads to the false sentence

(4) Barbarelli was so-called because of his size.

Similarly, though

(5) 'Cicero' begins with the letter 'c'

and

(6) Cicero = Tully

are both true,

(7) 'Tully' begins with the letter 'c'

is not true; hence (1) is false.

Another example in which there is substitution failure is illustrated by the modal case. For instance, while sentences

(8) The number of planets = nine

and

(9) Necessarily, nine is greater than seven

are both true, a substitution of 'nine' with 'the number of planets' yields the false sentence:

(10) Necessarily, the number of planets is greater than seven.[4]

Similarly, (8) and

(11) Possibly, the number of planets is even

are both true, but

(12) Possibly, nine is even

is patently false.

It may be tempting to think that the falsity of (1) entails the falsity of another principle, namely:

(13) Every instance of the schema:
(B) $\forall x \forall y(x = y \rightarrow (Fx \rightarrow Fy))$, is true.

But notice that whereas (1) involves talk of expressions and substitution, instances of **(B)** need not involve expressions and substitution. That (1) is false just shows that not all sentences are open to a substitution of coreferential terms. To put things differently, (1) is linguistic whereas instances of **(B)** are not- (1) is about expressions and sentences of natural language and **(B)** is about an object and its properties.

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[4] Expressions like 'necessarily' or 'possibly', etc. which occur in sentences are, unless otherwise stated, interpreted as operators on the sentences.

It seems to me that (13) expresses a self-evident truth and should be accepted. However, sentences (2), (3) and (4) appear problematic for the following instance of (**B**):

(**B′**) $(\forall x)(\forall y)(x = y \rightarrow$ every property of x is a property of $y)$

(**B′**) is false, only if there is an x and there is a y, such that x and y are identical, but there is a property which applies to x yet does not apply to y. However, we are considering *one* object, so how can it be that some property applies to that object and does not apply to the very same object at the same time? Richard Cartwright offers an argument which can be seen as justification for (**B′**). He says, "[i]f we let P be the property which a thing x has just in case the proposition that x is so called because of its size is true, then since the proposition that Giorgione was so called because of his size is true, Giorgione has P, and since the proposition that Barbarelli was so-called because of his size is false, Barbarelli lacks P...But the contention that there is such a property as P, possessed by Giorgione though not by Barbarelli, can be seen to be incoherent."[5] Cartwright's argument is as follows. Suppose that:

(i) Giorgione has P
(ii) Giorgione is called 'Barbarelli'.

If (i) and (ii) are true, then their conjunction must be true:

(iii) Giorgione has P and Giorgione is called 'Barbarelli'.

[5] Richard Cartwright. "Identity and Substitutivity." Identity and Individuation 1971. Rpt. in Philosophical Essays. Ed. Richard Cartwright. (Cambridge: MIT Press, 1987) 138.

One application of EG gives us

(iv) $(\exists x)$ x has P & x is called 'Barbarelli'.

What does (iv) mean? Well, recall that P stood for *the property which a thing x has just in case the proposition that x is so-called because of its size is true.* The sentence we are considering, then, is equivalent to the following sentence:

(v) $(\exists x)$ x is called 'Barbarelli' and the proposition that x is so-called because of his size is true.

But (v) is surely false and, as Cartwright suggests, no one called 'Barbarelli' is so-called because of his size. Thus, the supposition that Giorgione has P is false; there is no property that applies to Giorgione but that does not apply to Barbarelli.

In a slightly different way, we might try and illustrate that there is no one property that applies to Giorgione but not to Barbarelli. Suppose that the property being attributed to Giorgione in (2) is the property of *being called 'Giorgione' because of his size*. This is a very natural supposition given that (2) is true and, given that the word 'so-called' in (2) derives its meaning from the antecedent occurrence of 'Giorgione'. Also, suppose that the property being attributed to Barbarelli (that is, Giorgione) in (4) is the property of *being called 'Barbarelli' because of his size*. Again, this is a natural supposition to make given that (4) is false and, given that 'so-called' derives its meaning from the antecedent occurrence of 'Barbarelli' in (4). But obviously, the property of being called 'Giorgione' is different than the property of being called

'Barbarelli'. Indeed, it is very difficult to imagine that there is something which applies to Giorgione and which does not apply to Barbarelli at a particular time, but it is very easy to imagine how that is impossible. A rejection of (**B'**) means taking *one* (but arbitrarily chosen) object, determining some arbitrary property which applies to *that* object and, arguing that that same property does not apply to that object (*at that time*), which, of course, is a contradiction. Identity is a relation which holds, if it holds at all, only between an object and itself. So, it appears to be the case that the falsity of (1) has little or no bearing on any instance of (**B**). (13) is true, but (1) is false because singular terms can sometimes fail to *purely* refer.

Quine thinks that failures of substitution are indicative that the affected singular terms are contributing something other, or in addition to the specification of their objects to the determination of the truth-value of their respective sentences. Thus, if the occurrence of 'Giorgione' in (2) simply referred to Giorgione, then (4) would likewise be true. But (4), as we have seen, is false and as such, the occurrence of 'Giorgione' in (2) seems to be doing something other than merely specifying the object denoted by the name 'Giorgione' in the determination of the truth-value of (2). Suppose, for instance, that we are given the following sentence: 'Giorgione is a painter'. In this case, the object assigned to the name 'Giorgione' falls in the extension of the predicate *is a painter*. Given (3), we would expect that if 'Giorgione is a painter' is true, then 'Barbarelli is a painter' must be true as well; and it is.

However, the situation is different with respect to sentence (2). Here, the truth of (2) and (3) does not guarantee the truth of (4). What has gone wrong? One answer, as I have remarked above, seems to be that the occurrence of 'Giorgione' in (2) is contributing something other than the specification of its denotation to the determination of the truth-value of (2). The occurrence of 'Giorgione' in (2), therefore, is not *purely referential.*

If the occurrence of 'Giorgione' in (2) does not merely refer to the man Giorgione, then what or what else does it refer to? Frege would argue that the occurrence of 'Giorgione' in (2) refers to a *sense* of 'Giorgione'. The sense of 'Giorgione', of course, is not to be confused with the *nominatum* of 'Giorgione' which for Frege is just the referent of the name 'Giorgione'. But what does Frege mean by a name's sense? Here, Frege offers us only an analogy. Suppose that the *nominatum* of the expression 'the moon' is the object denoted by that expression. The sense of the expression 'the moon', then, is likened to be something like a telescopic or even perhaps a television image of the moon. Senses, according to Frege, are not subjective things; in the analogy we are discussing, a sense of 'the moon', though like a telescopic image, is not like *our* retinal image of the moon. Nor are they entirely objective in the way that the moon is objective. Senses, as far as the analogy goes, lie in between our retinal image and the actual object denoted by the expression 'the moon'.

Frege gives us at least some idea of what a singular term in an opaque or oblique context refers to, while Quine does not seem to. What Quine offers instead is a condition for what is to count as a purely referential occurrence of a singular term. What Quine would argue is that an occurrence of a singular term in a sentence is purely referential only if it can be substituted with a coreferential term without altering the truth-value of the original sentence.

Generally speaking, we may take failure of substitutivity as implying that there are (at least) two coreferential singular terms α and β, such that $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition and, for any sentences S and S', if S and S' are orthographically the same except that where there is one (or more) occurrences of α in S, there is one (or more) occurrences of β in S', then it is not the case that S is true if and only if S' is true. Quine's remark that: "[f]ailure of substitutivity reveals merely that the occurrence to be supplanted is not *purely referential*..., that is, that the statement depends not only on the object but on the form of the name," is indicative of the following condition for a purely referential occurrence of a singular term.[6]

(14) For any sentence S, any singular term α, and any z, if z is an occurrence of α in S, then z is purely referential only if, for any sentence S', and any singular term β, if S' is the result of substituting β for z, and $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition, then S expresses a true

[6] Quine, W. From A Logical Point Of View. (Cambridge: Harvard University Press, 1996) 140.

> proposition, if and only if, S' expresses a true proposition.

According to (14), the occurrence of 'Giorgione' in sentence (2) is purely referential, only if, for any other sentence S' and any other singular term β, if S' is the result of substituting β for the occurrence of 'Giorgione' in (2), then (2) is true if and only if the sentence S' is true. Now, S' is just sentence (4), but since it is not the case that (2) is true if and only if (4) is true, the occurrence of 'Giorgione' in (2) must be not purely referential. Similarly, consider the occurrence of 'Barbarelli' in (4). Since it is not the case that (4) expresses a true proposition if and only if (2) expresses a true proposition, the occurrence of 'Barbarelli' in (4) is not purely referential. Hence, the occurrence of 'Giorgione' in (2) and the occurrence of 'Barbarelli' in (4) are *both* not purely referential. To use a different example, take sentences (9) and (10). Since it is not the case that (9) expresses a true proposition if and only if (10) expresses a true proposition, both the occurrence of '9' in (9) and the occurrence of 'the number of planets' in (10) are not purely referential. It is worth stressing that (14) is a strong principle; if (14) is true, then, for any singular terms α and β, any sentences S and S', and any z, if z is an occurrence of α in S and, S' is the result of replacing z with β, but it is not the case that S expresses a true proposition if and only if S' expresses a true proposition, then both z and the corresponding occurrence of β are not purely referential.

Are occurrences of singular terms in referentially opaque contexts *ever* purely referential? Perhaps not, but (14) does not have this consequence. However, what is a consequence of (14) is the following: If the context of a particular sentence is referentially opaque, then for any singular terms α and β, and any sentences S and S', if S and S' are orthographically the same except that where there is an occurrence of α in S there is an occurrence of β in S', but S and S' *differ* in truth-value, then both α and β are not purely referential.

In *Word and Object*, there is a discussion which involves the position of an occurrence of a singular term. Quine says, for instance: "Here we have a criterion for what may be called *purely referential position*: the position must be subject to the *substitutivity of identity*."[7] To avoid any obvious confusion between the use of the phrase 'purely referential position' and the phrase 'a purely referential occurrence', let us use the phrase 'transparent position' in place of Quine's use of 'purely referential position'. Consider, for instance, the sentence 'Bugs Bunny always outsmarts his foes'. Now, the position of the occurrence of 'Bugs Bunny' in the sentence we are considering is presumably transparent. For, it is the case that the position of the occurrence of 'Bugs Bunny' is subject to substitution with 'the long-eared galoot' *salva veritate*. But contrast the position of the occurrence of 'Bugs Bunny' in the sentence 'Bugs Bunny always outsmarts his foes' with the position of the

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[7] Quine, W. Word and Object. (Cambridge: The M.I.T. Press, 1960) 142.

occurrence of 'Cicero' in (5). The position of the latter is presumably not transparent, since a substitution of the occurrence of 'Cicero' for 'Tully' yields a sentence which *differs* from (5) in truth-value.

I shall understand the phrase 'the context of a sentence' as being the result of deleting an occurrence of a singular term in that sentence. Hence, '___ was so-called because of his size' is a context obtained from (2) by deleting the occurrence of 'Giorgione' (or by deleting the occurrence of 'Barbarelli' in (4)). The context 'Necessarily, ___ is greater than seven' is a context obtained from (9) by deleting the occurrence of '9' (or by deleting the occurrence of 'the number of planets' in (10)). But note that the position of an occurrence of a singular term just *is* the context of a sentence; for, the position of the occurrence of 'Giorgione' in (2) is '___ was so-called because of his size', where the '___' indicates where the term 'Giorgione' would occur.

A reason for suggesting that Quine would endorse (14) in the first place, however, stems (partially) from Quine's remark that *failure of substitutivity reveals merely that the occurrence to be supplanted is not purely referential.* But his further remark: *That is, the statement depends not only on the object but on the form of the name* suggests something other or in addition to (14). The thought seems to be that if an occurrence of a singular term α in a sentence S is purely referential, then the only contribution that α makes to the determination of the truth-value of S is the specification of the object denoted by the term α.

Consider the sentence 'Smith loathes Alma' which is true just in case the object assigned to the expression 'Smith' satisfies the sentence '*x* loathes Alma'. The truth-value of the sentence '*x* loathes Alma' is dependent only on the value of the variable '*x*'- which in this case is the object assigned to the expression 'Smith'- and whether the value assigned to '*x*' satisfies '*x* loathes Alma'. If the value of the variable '*x*' loathes Alma and it is Smith, then the sentence 'Smith loathes Alma' is true, but if the value of the variable '*x*' does not loathe Alma and it is Smith, then the sentence 'Smith loathes Alma' is not true. In either case, the suggestion is that the occurrence of 'Smith' is purely referential, only if, the only contribution that 'Smith' makes to the determination of the truth-value of 'Smith loathes Alma' is the specification of the object denoted by the expression 'Smith'.

In the light of Quine's further remark that *the statement depends not only on the object but on the form of the name*, we might consider a different principle than (14), and the following is substantially weaker.

> (16) For any sentence S, any singular term α, and any z, if z is an occurrence of α in S then, z is purely referential, only if, for any sentence S', and any new variable x, if S' is the result of substituting x for z, then S expresses a true proposition, if and only if S' is true of the object assigned to x.

According to (16), the occurrence of 'Giorgione' in (2) is purely referential, only if:

> (i) 'Giorgione was so-called because of his size' is true, if and only if, the sentence 'x was so-called because of his size' is true of the object assigned to the variable x and x = Giorgione.

But now consider (4), which is just like (2) except that where there is an occurrence of 'Giorgione' in (2), there is an occurrence of 'Barbarelli' in (4). Despite the fact that Giorgione = Barbarelli, we have seen that (2) and (4) differ in truth-value. From (16), the occurrence of 'Barbarelli' in (4) is purely referential, only if:

> (ii) 'Barbarelli was so-called because of his size' is true, if and only if, the sentence 'x was so-called because of his size is true' of the object assigned to the variable x and x = Barbarelli.

From (i) and (ii), it follows that the sentence 'Giorgione was so-called because of his size' is true, if and only if, the sentence 'Barbarelli was so-called because of his size' is true. But it is not the case that 'Giorgione was so-called because of his size' is true, if and only if, 'Barbarelli was so-called because of his size' is true. Thus, it must be the case that either (i) or (ii) is false. But if (i) is false, then the occurrence of 'Giorgione' in 'Giorgione was so-called because of his size' is not purely referential; and similarly, if (ii) is false, then the occurrence of 'Barbarelli' in 'Barbarelli was so-called because of his size' is not purely referential. Either (i) or (ii) is false, so either the occurrence of 'Giorgione' is not purely referential, or the occurrence of 'Barbarelli' is not purely referential. To be sure, it may turn out that both (i) and (ii) are false. However, that both (i) and (ii) are false does not follow from the fact that (2) and (4) differ in truth-value; in other words, the difference in truth-value of (2) and (4) does not imply that both the occurrence of 'Giorgione' and the occurrence of 'Barbarelli' are not purely referential.

To use a different example, consider sentences (9) and (10). If (16) is true, then, the occurrence of '9' in (9) is purely referential only if, for any sentence *S*, and any variable *x*, if *S* is the result of substituting *x* for the occurrence of '9' in (9), then (9) expresses a true proposition, if and only if *S* is true of the object assigned to *x*. Similarly, if (16) is true, then, the occurrence of 'the number of planets' in (10) is purely referential only if, for any sentence *S*, and any variable *x*, if *S* is the result of substituting *x* for the occurrence of 'the number of planets' in (10), then (10) expresses a true proposition, if and only if *S* is true of the object assigned to *x*. But (9) and (10) differ in truth-value. Therefore, either the occurrence of '9' in (9) or the occurrence of 'the number of planets' in (10) is not purely referential.

Section 2

In the last section, several principles in defense of the claim that quantification into opaque contexts is illegitimate were considered. Though no real arguments for the claim was considered there, I will discuss how and if those principles support the claim in this section. In what follows, I will be looking at one class of arguments which involves the principles articulated in (14) – (16).

Recall that a consequence of (14) is that, for any singular terms α, β, any sentences S and S', if $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition and, S contains an occurrence of α and, S' is the result of substituting some occurrence of α in S with β, but it is not the case that S is true if and only if S' is true, then, that occurrence of α in S and, the corresponding occurrence of β in S' are both not purely referential. It is worth stressing again that (14) is a strong principle. If we consider sentence (2) and, if (14) is true, then the occurrence of 'Giorgione' in (2) is not purely referential, simply because it is not the case that (2) is true if and only if (4) is true. Similarly, if we consider sentence (4) and, if (14) is true, then the occurrence of 'Barbarelli' in (4) is not purely referential, again because it is not the case that (4) is true if and only if (2) is true. Hence, neither the occurrence of 'Giorgione' in (2) nor the occurrence of 'Barbarelli' in (4) are purely referential.

Another but similar consequence of (14) might be described as follows. Let us suppose the following: $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition; a sentence S contains an occurrence of α; a sentence S' is the result of substituting some occurrence of α in S with β; and, S and S' differ in truth-value. Then, the context obtained by deleting that occurrence of α in S is referentially opaque; since S and S' differ in truth-value. But the phrase 'the context obtained by deleting some occurrence of α in S is referentially opaque' is, as I have stipulated earlier, equivalent to the phrase 'the position of an occurrence of α in S is referentially opaque'; and hence, if the position of an occurrence of α in S is referentially opaque and, if (14) is true, then that occurrence of α in S is not purely referential. Similarly, given that S' is orthographically the same as S except that where α occurs in S, β occurs in S', then, the position of an occurrence of β in S' is referentially opaque only if that occurrence of β in S' is not purely referential. Notice again that if (14) is true, then neither occurrences of α in S nor β in S' are purely referential.

One may ask what, if anything, a non-purely referential occurrence of a singular term has to do with quantification. An answer given by Quine is that quantification into opaque contexts leads simply to nonsense. But why is it nonsense? In some of the examples from the last section, quantification into opaque contexts appears to lead to nonsense because, in each case, a variable which occurs inside the opaque context fails to be bound by the initially placed quantifier; and it seems to be a common view in 1st-order logic that variables are devices of pure reference. So, let us suppose that it is true that *variables of quantification are devices of pure reference.* Hence, for any variable x and any singular term α, if $\ulcorner x = \alpha \urcorner$ expresses a true proposition, then, if the occurrence of α in some sentence S is not purely referential and, a sentence S' is the result of substituting x for α in S, then the occurrence of x in S' is not purely referential as well. Conversely, for any variable x, any sentence S, if x

occurs uniquely in S, then, the occurrence of x is purely referential only if x simply refers to whatever value was assigned to x.

Let us take the referentially opaque context which is obtained by deleting the occurrence of the term 'Cicero' from the sentence ' 'Cicero' begins with the letter 'c'', i.e., sentence (5). Now, quantification into the context ' '___' begins with the letter 'c'' means, among other things, putting a variable in place of the '___' and prefixing a quantifier to the sentence with the aim that the prefixed quantifier govern whatever variable was put in place of the '___'. Hence, quantification of (5) yields the following sentence:

(17) $(\exists x)$ 'x' begins with the letter 'c'.

What does (17) say? It says that *there is something such that 'x' begins with the letter 'c'*. An idea which underlies Quine's misgivings about quantifying into a referentially opaque context, such as ' '___' begins with the letter 'c'', is illustrated by considering the italicized sentence above. Quine thinks that the quantified phrase 'there is something such that' cannot bind the 'x' which follows it. To look at it another way, the second occurrence of the 'x' in (17) is not even a variable of quantification. Single quotes, in a sense, have the effect of drawing attention to the word or phrase that occurs inside of them. So, in (17), the fact that the expression 'x' occurs inside single quotes means that what we are talking about is the letter 'x'. Moreover, that the letter 'x' occurs in the subject position of the italicized sentence means that it is there for the rest of the sentence to say something about it. If this is the case, then the italicized sentence which is supposed to capture what is expressed in (17), is false- the letter '*x*' does not begin with the letter 'c'.

To use another example, take sentence (2). EG on the singular term 'Giorgione' in (2) gives us:

(18) $(\exists x)$ *x* was so-called because of his size.

What does (18) say? There is something such that *it* was so-called because of his size. But the word 'so-called' is supposed to be anaphoric on some antecedent expression, but there is none. Consider *the object*, that object is so-called because of his size. Surely we would want to say that no proposition has been expressed. But if we are inclined to say that a proposition has been expressed, then we shall have to regard the occurrence of the '*x*' in (18) as a proper name. For I suppose it might be true that something, or someone, was called '*x*' because of his size. But treating the variable which is supposed to be bound by some quantifier as a name is, needless to say, a position hardly worth considering. The fact that (2) and (18) differ in truth-value appears to suggest that something has gone wrong and, hence, we are confronted with Quine's charge that quantification into opaque contexts simply leads to nonsense.

Another but different way of illustrating the motivation for thinking that quantification into opaque contexts leads to nonsense is as follows. Let us suppose that (14) is true and that, for any singular terms α, β, any sentences *S* and *S'*, *S* and *S'* are orthographically the same except that where *S* contains one (or more) occurrences of α, S' contains one (or more) occurrences of β and, $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition, but S and S' differ in truth-value. Given our suppositions, some occurrences of α and β in S and S', respectively, are not purely referential. Consider now any variable x and, any sentence S'' which is the result of substituting 'x' with either the occurrence of α in S or β in S'. It must follow that either S and S'' or S' and S'' differ in truth-value.

Suppose that S and S'' differ in truth-value, and recall from (14) that an occurrence α in S is purely referential only if S is true if and only if S'' is true (of the object assigned to the variable x in S'', which in this case happens to be the object denoted by 'α'). Since it is not the case that S is true if and only if S'' is true (of the object assigned to the variable x in S''), that occurrence of α in S is not purely referential. Similarly, an occurrence of the variable x in S'' is purely referential only if S is true if and only if S'' is true (of the object assigned to the variable x in S''). But S and S'' differ in truth-value, hence, that occurrence of the variable x in S'' is not purely referential.

If Quine did, in fact, intend (14) to be used in an argument against quantifying into opaque contexts, then the case would be closed- the alleged argument against quantifying into opaque contexts that was considered above is valid, but it is questionable whether or not it is sound. Certainly, *if* (14) is true, then that would be more reason to think that the argument is indeed sound. But the truth of (14) is precisely what is at issue and, Quine, as a matter of fact, is of no help- he does not offer any argument. The argument which was considered above for the claim that quantification into opaque contexts is illegitimate rests on (14) and, (14) was motivated by some scarce remarks by Quine; there is really no strong evidence which would suggest that Quine did, in fact, intend (14) to be used. Yet what *is* confusing in Quine's writings, particularly "Reference and Modality," is, in addition to his claim that *failure of substitutivity reveals merely that the occurrence to be supplanted is not purely referential*, he adds that *the statement depends not only on the object but on the form of the name*, which prompted (16). The confusion, it seems to me, is due to the fact that Quine's claim that:

> If to a referentially opaque context of a variable we apply a quantifier, with the intention that it govern that variable from outside the referentially opaque context, then what we commonly end up with is unintended sense or nonsense...In a word, we cannot in general properly *quantify into* referentially opaque contexts,[8]

can only be established *if* (14) is true. Since there is no argument for (14), I propose we reject Quine's claim.

We are left, then, with (16). Would Quine's claim be established if (16) is used? For simplicity, suppose: $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition and α denotes some object *O*; a sentence *S* has one occurrence of α; a

[8] Quine, W. From A Logical Point Of View. (Cambridge: Harvard University Press, 1996) 148.

sentence S' is orthographically the same as S except that where S contains the occurrence of α, S' contains an occurrence of β; and, S and S' differ in truth-value. Two further suppositions: x is a variable whose value is O; and, a sentence S'' is the result of replacing the occurrence of α in S with the variable x.

Consider sentence (2) 'Giorgione was so-called because of his size', and sentence (4) 'Barbarelli was so-called because of his size' which is orthographically the same as (2) except that where 'Giorgione' occurs in (2), 'Barbarelli' occurs in (4). According to (16), the occurrence of 'Giorgione' in (2) is purely referential, only if:

> (i) 'Giorgione was so-called because of his size' is true, if and only if, the sentence 'x was so-called because of his size' is true of the object assigned to the variable x and x = Giorgione.

The occurrence of 'Barbarelli' in (4) is purely referential, only if:

> (ii) 'Barbarelli was so-called because of his size' is true, if and only if, the sentence 'x was so-called because of his size is true' of the object assigned to the variable x and x = Barbarelli.

But (i) and (ii) imply that the sentence 'Giorgione was so-called because of his size' is true, if and only if, the sentence 'Barbarelli was so-called because of his size' is true. We have supposed, however, that (2) and (4) differ in truth-value. Therefore, if (2) and (4) differ in truth-value, then either (i) or

(ii) is false. But if (i) is false, then the occurrence of 'Giorgione' in (2) is not purely referential; and if (ii) is false, then the occurrence of 'Barbarelli' in (4) is not purely referential. Hence, if (16) is true, then *either* the occurrence of 'Giorgione' in (2) *or* the occurrence of 'Barbarelli' in (4) is not purely referential. It may turn out that both of the relevant coreferential singular terms are not purely referential, but (16) does not establish that.

Consider a sentence *S''* which is orthographically the same as (2) (or (4) for that matter) except that where 'Giorgione' occurs in (2) (or where 'Barbarelli' occurs in (4)), a variable '*x*' occurs in *S''*. It is easily seen that either (2) and *S''* differ in truth-value or (4) and *S''* differ in truth-value. If (2) and *S''* differ in truth-value and, if (16) is true, then either the occurrence of 'Giorgione' in (2) or the occurrence of the variable '*x*' in *S''* is not purely referential. Similarly, if (4) and *S''* differ in truth-value and, if (16) is true, then either the occurrence of 'Barbarelli' in (4) or the occurrence of the variable '*x*' in *S''* is not purely referential. But it does not follow that quantification into the context '___ was so-called because of his size' is illegitimate. Such quantification would only be illegitimate given that *variables are devices of pure reference*, and *the occurrence of '*x*' in S'' is not purely referential.* But as we have just seen, that the occurrence of the variable '*x*' in *S''* is not purely referential is not a consequence of (16). Consider a game which involves two players and, the rules of the game are such that, in the end, there will definitely be a winner. So, either A wins or B wins. But from this, it does not follow that A will win. Quine's claim relies on it being the case that both of the relevant coreferential singular terms are not purely referential. But (16) does not have this consequence and, hence, Quine's claim cannot be established by an appeal to (16).

There is a similar argument to be found in David Kaplan's paper entitled "Opacity."[9] In his paper, Kaplan considers an argument in defense of the claim that quantification into opaque contexts leads to nonsense and, charges that the argument is fallacious, that *it rests on a logical blunder.* The argument is as follows.

> Step 1: A purely designative occurrence of a singular term in a formula is one in which the singular term is used solely to designate the object. [This is a definition]
>
> Step 2: If an occurrence of a singular term in a formula is purely designative, then the truth value of the formula depends only on *what* the occurrence designates not on *how* it designates. [From 1.]
>
> Step 3: Variables are devices of pure reference; a bindable occurrence of a variable must be purely designative. [By standard semantics.]
>
> Notation: Let ϕ be a formula with a single free occurrence of 'x', and let $\phi\alpha$, $\phi\beta$, $\phi\gamma$ be the results of proper substitution of the singular terms α, β, γ for 'x'.
>
> Step 4: If α and β designate the same thing, but $\phi\alpha$ and $\phi\beta$ differ in truth value, then the indicated occurrences of α in $\phi\alpha$ and of β in $\phi\beta$ are not purely designative. [From 2.]
>
> Now assume 5.1: α and β are co-designative singular terms, but $\phi\alpha$ and $\phi\beta$ differ in truth value.

[9] David Kaplan. "Opacity." In The Philosophy of W. V. O. Quine. Eds. Lewis Hahn and Paul Schilpp. (La Salle: Open Court Publishing Company, 1986) 229 – 289.

and 5.2: γ is a *variable* whose value is the object co-designated by α and β.

Step 6: Either $\phi\alpha$ and $\phi\gamma$ differ in truth value or $\phi\beta$ and $\phi\gamma$ differ in truth value. [From 5.1, since $\phi\alpha$ and $\phi\beta$ differ]

Step 7: The indicated occurrences of γ in $\phi\gamma$ is not purely designative. [From 5.2, 6, and 4.]

Step 8: It is semantically incoherent to claim that the indicated occurrence of γ in $\phi\gamma$ is bindable. [From 7 and 3.]

Kaplan goes on to say: "All but one of these steps seem to me to be innocuous...That one is step 4 which, of course, does *not* follow from step 2. All that follows from 2 is that at least one of the two occurrences is not purely designative. When 4 is corrected in this way, 7 no longer follows."[10]

It might be suggested that "the error of step 4" is the error in thinking that both the occurrence of α in $\phi\alpha$ and the occurrence of β in $\phi\beta$ or, equivalently, that both the occurrence of α in *S* and the occurrence of β in *S'* are not purely referential, follows from (16).[11] But as Kazmi points out, "it is not clear from Quine's writings that he is guilty of this error,"[12] but my own thought is that Quine never gave us an argument in the first place and, that what we are essentially doing is speculating- Kaplan conceded this- and there seems to be evidence which indicates that Quine would, to some degree, endorse both (14) and (16). But as I remarked earlier, it seems to me that Quine's claim can only be established if (14) is true. Could it be that

[10] *Ibid.*, 235.
[11] This point was made by Ali Kazmi in his paper "Quantification And Opacity." Linguistics and Philosophy 10. (Burnaby: Reidel Publishing Company, 1987) 92.
[12] *Ibid.*

Quine intended (14) to be used in an argument in defense of his claim, but felt it unnecessary to give an argument for (14)? Perhaps; for if Quine intended (16) to be used, it is more than likely that he would have recognized that (16) does not entail that both occurrences are not purely referential.

In the first section of the following chapter, I discuss Smullyan's paper, "Modality and Description," and conclude with Smullyan that if an occurrence of a definite description in a modal sentence is treated in accordance with Russell's theory of definite descriptions, then such modal sentences are not counterexamples to (13). In addition, I consider theorem *14.18 of *Principia Mathematica* and argue that the modal case is not a counterexample to it. Moreover, I shall argue that if the modal case is not problematic for theorem *14.18, then it is not problematic for any sentence which is derived from theorem *14.18. Specifically, I show that a (restricted) version of EG is derivable from theorem *14.18 and that the modal case does not present any challenges to this restricted version of EG. I also consider Quine's alleged conjecture that quantification into a modal context is permissible only if EG on a term which occurs in that context is truth-preserving. I shall argue that if that conjecture assumes the restricted version of EG, then we may accept the alleged conjecture, otherwise not.

In the second section of the following chapter, I highlight some of the key features of a formal semantics for quantified modal logic offered by Kripke in his paper entitled "Semantical Considerations On Modal Logic."

And in the third section, I discuss the problem of essentialism and distinguish between the intelligibility of quantified modal discourse, and the supposed truths of quantified modal sentences. I argue that issues concerning essentialism and the supposed truths of quantified modal logic are independent of its intelligibility.

Chapter Two

Section 1

In the following discussion of Smullyan's paper "Modality and Description," it will be of some benefit to consider again (13) which says that every instance of the schema: $(\forall x)(\forall y)(x = y \rightarrow (Fx \rightarrow Fy))$ is true. The modal case presents an apparent challenge to (13) and, as Smullyan suggests, "[t]here are logicians who maintain that modal logic violates Leibnitz's principle that if *x* and *y* are identical, then *y* has every property of *x*."[13] I should first point out, however, that (13) is not what I have called Leibnitz's law. I have used the expression 'Leibnitz's law' in connection with (1) which says that for any singular terms α, β, $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, only if, for any sentences *S* and *S'*, if *S'* is orthographically the same as *S* except that *S'* contains one (or more) occurrences of β where *S* contains an occurrence of α, *S* expresses a true proposition only if *S'* expresses a true proposition. Smullyan's usage of the expression 'Leibnitz's principle' or 'Leibnitz's law' presumably refers to a particular instance of (13). And though nothing of great consequence arises from these two distinct uses of the

[13] Arthur Smullyan. "Modality And Description." Rpt. in Reference And Modality. Ed. Leonard Linsky. (London: Oxford University Press, 1971) 35.

expression 'Leibnitz's principle', note that (1) has already been shown to be false in *Chapter One*.

There are (at least) three sentences which are of particular interest in a discussion of Smullyan's "Modality and Description." These three sentences are sentences (8), (9) and (10). However, it is not merely these three sentences in and of themselves that are of interest here, rather it is that sentence (10) is supposed to be inferred from sentences (8) and (9). Thus, we may view the three sentences as forming the following argument:

(8) The number of planets = nine.

(9) Necessarily, nine is greater than seven.

(10) Necessarily, the number of planets is greater than seven.

Premises (8) and (9) are true, but the conclusion (10) is not true, and the argument is invalid. "[Y]et the conclusion appears to be derived by means of the logical precept that if x is y then any property of x is a property of y. Such is the paradox of modal logic."[14] How then is the paradox to be resolved? To answer this question, we need to look at Russell's theory of definite descriptions.

One of Russell's contributions to the philosophy of language is the observation that (10) and sentences like it are ambiguous. Consider, for instance, the following sentence: 'Every man danced with some woman'. Under one interpretation, this sentence means that every man danced with

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[14] *Ibid.*

some woman (or other); but yet under another interpretation, it means that there is some *one* woman with whom every man danced. These two interpretations are different and both have different truth-conditions. The sentence 'Every man danced with some woman' is, therefore, ambiguous. To disambiguate this sentence, there are a number of different conventions one may use. For instance, one may wish to express the former interpretation by the following sentence: *For all x there is a y such that x danced with y.* Similarly, one may wish to express the latter interpretation by the following sentence: *There is an x such that for all y, y danced with x.*

Structural ambiguity is, however, not limited to sentences which contain two (or more) quantifiers. There are sentential operators such as the negation and the modal operator which generate ambiguities in a sentence which also contains a definite description (or more generally, a quantified noun phrase). Now, sentence (10) involves both a definite description and a modal operator, whereas the sentence 'Every man danced with some woman' involves the quantifiers 'every' and 'some'. But before we consider (10), let us consider another sentence similar in form to (10). Suppose that we are asked to determine the truth-value of the sentence 'It is not the case that the present king of France is bald'. One thing to notice is that in this sentence, there is an occurrence of the negation operator and an occurrence of a definite description. As such, disambiguation is needed before any questions of truth-value can be adequately answered. If the scope of the definite description in the sentence we are considering is confined to the complement sentence, i.e., if the scope of that description is narrow, then the sentence 'It is not the case that the present king of France is bald' is true. However, if the scope of that definite description extends to the entire formula, then the sentence we are considering is not true.

As one would expect, a sentence which has the operator 'necessarily' and a definite description is ambiguous according as the description is understood to have wide or narrow scope. Let us consider the form of such sentences and indicate that a description has wide scope in a sentence if the sentence has the following form:

(19) The *F* is necessarily *G*.

And let us indicate that a description has narrow scope in a sentence if the sentence has the following form:

(20) Necessarily, the *F* is *G*.

As we have already seen, the truth-value of a sentence containing both a sentential operator and a definite description may vary depending on whether that description has wide or narrow scope. Smullyan illustrates the distinction of the scope of a definite description as follows.

> I will ask the reader to believe that James is now thinking of the number 3. If, now, some one were to remark, 'there is one and only one integer which James is now thinking of and that integer is necessarily odd', then he would be stating a contingent truth. For that there is just one integer which James now thinks of, is only an empirical fact. This statement could just as well be expressed in the form, [19], 'The integer,

> which James is now thinking of, satisfies the condition that it is necessarily odd.' In contrast, the statement, 'It is necessary that James's integer is odd', which is of the form, [20] is an impossible statement and not a contingent one.[15]

According to Smullyan, modal paradoxes are immediately resolved if we keep in mind the distinction of scope of a definite description. Consider the following argument which is again illustrative of the notion of scope distinction. Suppose that we are given the truth of

(21) $E!(\iota x)(Fx)$

and

(22) $(\forall x)(N(x = x))$,

where 'N' means or is short for the expression 'necessarily'; (21) says that, as a matter of fact, there exists exactly one F, and (22) tells us that every object is such that necessarily, it is self-identical. But given that there exists exactly one F, and given that everything is necessarily self-identical, does sentence (23) follow?

(23) $N((\iota x)(Fx) = (\iota x)(Fx))$.

Note that (23) is a strong claim; it says that *necessarily*, there exists exactly one F (which is self-identical). But recall that (21) says that *as a matter of fact*, there exists just one F, and it would seem that (21) and (23) are in conflict. Is (23) derivable from (21) and (22)? Smullyan does not think so and argues that

[15] *Ibid.*, 36.

"[t]he absurdity of this derivation will soon be made apparent to the reader."[16]

In order to see what Smullyan means by 'the absurdity of such a derivation', we need to look at Russell and Whitehead's *Principia Mathematica.* In particular, there is a theorem in *Principia* written as follows:

*14.18: $E!(\iota x)(\varphi x) \rightarrow ((\forall x)(\psi x) \rightarrow \psi(\iota x)(\varphi x))$.

If we were to substitute 'F' for 'φ', 'Necessarily($x = x$)' for 'ψx' in theorem *14.18, then could we not deduce the sentence 'Necessarily[$(\iota x)(Fx) = (\iota x)(Fx)$]'? By performing the specified substitutions on *14.18, we obtain:

(24) $E!(\iota x)(Fx) \rightarrow ((\forall x)$Necessarily$(x = x) \rightarrow$ Necessarily$[(\iota x)(Fx) = (\iota x)(Fx)])$.

Since we are supposing the truth of '$E!(\iota x)(Fx)$' and '$(\forall x)$[Necessarily($x = x$)]', we should be able to infer the sentence 'Necessarily[$(\iota x)(Fx) = (\iota x)(Fx)$]' by two applications of *modus ponens*; such an inference might look like the following:

(25) $E!(\iota x)(Fx) \rightarrow ((\forall x)$Necessarily$(x = x)$ Necessarily$[(\iota x)(Fx) = (\iota x)(Fx)])$

(26) $E!(\iota x)(Fx)$

(27) $((\forall x)$Necessarily$(x = x) \rightarrow$ Necessarily$[(\iota x)(Fx) = (\iota x)(Fx)])$ [*from* (25), (26)]

(28) $((\forall x)$Necessarily$(x = x)$

[16] *Ibid.*

(29) Necessarily[$(\iota x)(Fx) = (\iota x)(Fx)$]) [*from* (27), (28)]

Thus, one might be inclined to argue that given the truth of (21) and (22), (23) is in fact deducible as evidenced by the supposed proof (25) – (29). However, as Smullyan points out, one who is inclined to make such an inference is "committing the subtle fallacy of misreading the scope of the description, '$(\iota x)(Fx)$'" in (29).[17] In *Principia Mathematica* it is assumed that the scope of a description is the smallest formula containing that description, unless it is to the contrary indicated. In the case of sentence (29), the scope of the descriptive phrase '$(\iota x)(Fx)$' extends to the sentence 'Necessarily[$(\iota x)(Fx) = (\iota x)(Fx)$]' and should be understood or written thus: $[(\iota x)(Fx)]\{\text{Necessarily}[(\iota x)(Fx) = (\iota x)(Fx)]\}$. Smullyan argues that "[i]t is only be neglecting this consideration that one is led to deduce [(29)] from [(25) – (28)]…."[18]

The parallel between the argument just considered and the argument with (8) and (9) as premises, and (10) as the conclusion is now immediately obvious. Let us perform the following substitutions on theorem *14.18: '*P*' for 'φ', '$(x = 9 \rightarrow \text{N}(x > 7))$' for '$\psi x$'; where '*P*' means or is short for the expression 'is a number of planets'. Such a substitution yields:

(30) $\text{E!}(\iota x)(Px) \rightarrow [(\forall x)(x = 9 \rightarrow \text{N}(x > 7)) \rightarrow \text{N}((\iota x)(Px) > 7)]$.

[17] *Ibid.*, 37.
[18] *Ibid.*

Suppose the truth of '$E!(\iota x)(Px)$', i.e., *the number of planets exists*, and the truth of '$(\forall x)(x = 9 \rightarrow N(x > 7))$', i.e., *anything identical with 9 is necessarily greater than seven*. Given these suppositions and sentence (30), can we deduce the sentence '$N((\iota x)(Px) > 7)$' which is equivalent to sentence (10)? Not if the scope of the second occurrence of the descriptive phrase '$(\iota x)(Px)$' in (30) extends to the entire consequent of (30), i.e., '$N((\iota x)(Px) > 7)$'. On the contrary, what we are able to legitimately infer is the sentence

(31) $[(\iota x)(Px)]N((\iota x)(Px) > 7)$

which is true, but (31) is not equivalent to (10); nor is it derivable from (10).

One problem with the argument (8), (9) and (10) is that it is an apparent counter-example to Leibnitz's law. Let us suppose that the following is an instance of (**B**):

(32) $(\forall x)(\forall y)(x = y \rightarrow$ (necessarily x is greater than seven $\rightarrow$ necessarily y is greater than seven)).

Now, if one is inclined to think that that argument is a counterexample to Leibnitz's law, then one may be inclined to think that the following sentence:

(33) (Nine = the number of planets $\rightarrow$ (necessarily, nine is greater than seven $\rightarrow$ necessarily, the number of planets is greater than seven))

falsifies (32). A sentence **B** falsifies a sentence **A** just in case **B** is deducible from **A** and, **B** is false. Thus, (33) falsifies (32) just in case (33) is derivable from (32) and, (33) is false. If, however, (33) does not falsify (32), then either

(33) is not deducible from (32), or (33) is not false. What I shall argue is that though (33) is false, it is not deducible from (32), therefore (33) does not falsify (32). The argument exposes a scope ambiguity of the second occurrence of the descriptive phrase 'the number of planets' in (33), and is due to Smullyan.

Disambiguation of (33) is needed prior to answering any questions of derivability. For instance, consider the following question: Can we deduce 'A' from 'A & B → C'? Well, any answer is not immediately obvious because 'A & B → C' is not a well-formed sentence. If by 'A & B → C' one means 'A & (B → C)', then the answer to the question is "yes." But if by 'A & B → C' one really means '(A & B) → C', then the answer is "no." Thus, we need to note that (33) is ambiguous between:

(34) $(9 = (\iota x)(Px) \rightarrow (\text{necessarily } 9 > 7 \rightarrow [(\iota x)(Nx)](\text{necessarily } (\iota x)(Nx) > 7)))$

and

(35) $(9 = (\iota x)(Px) \rightarrow (\text{necessarily } 9 > 7 \rightarrow \text{necessarily } [(\iota x)(Nx) > 7]))$.[19]

The scope of the second occurrence of the definite description '$(\iota x)(Nx)$' in (34) is 'Necessarily($(\iota x)(Nx) > 7$)', whereas the scope of that description in

[19] There are other ways in which (33) is ambiguous. For my purposes, I have noted one relevant ambiguity here, *viz.*, (34) and (35).

(35) is limited to the complement of the sentence 'necessarily($(\iota x)(Px) > 7$)', i.e., '$(\iota x)(Px) > 7$'.

Having disambiguated (33), we are now in a position to address concerns regarding whether or not (34) or (35) falsifies (32). We can immediately dispense with (34), since (34) is a true sentence and is derivable from (32) and theorem *14.18. However, does (35) falsify (32)? Certainly (35) is false, but recall that (35) falsifies (32) if and only if (35) is false *and* (35) is derivable from (32). Can (35) be derived from (32)? Again, in *Principia Mathematica* it is assumed that the scope of a definite description in a sentence is the smallest formula which contains it. But in sentence (35), the scope of the last occurrence of the definite description is '$(\iota x)(Px) > 7$'. Given *14.18, the scope of the last occurrence of the definite description in (35) should be '$[(\iota x)(Px)]$(necessarily$(\iota x)(Px) > 7$)' and not '$(\iota x)(Px) > 7$'. For the Russellian, this means that (34) is the proper interpretation of (33); hence, if the scope of the last occurrence of the description '$(\iota x)(Px)$' in (35) is 'Necessarily$(\iota x)(Nx) > 7$' then (35), though false, is not derivable from (32) and theorem *14.18. Therefore, (35) does not falsify (32).

As Smullyan notes, "[o]ne of the possible sources of the confusion which we are trying to eliminate is to be found in *Principia Mathematica* itself where the authors inadvertently assert, on p. 186, vol. I, '[i]t should be observed that the proposition in which $(\iota x)(\varphi x)$ has the larger scope always implies the corresponding one in which it has the smaller scope,...'" Smullyan adds, "[i]t is evident that this pronouncement holds good only when truth functional contexts are in question. In non-truth-functional contexts, the contention fails to hold...This is an important difference between intensional and extensional contexts."[20] For instance, if we are considering the true sentence 'the present Queen of England is not very young', then it follows that the sentence 'it is not the case that the present Queen of England is very young' is true as well. And this is simply due to the fact that the negation operator is truth-functional. However, the modal operator is not truth-functional; '$[(\iota x)(Px)]\{\text{necessarily}((\iota x)(Px) > 7)\}$' does not imply 'Necessarily$((\iota x)(Px) > 7)$'.

As we have just seen, Leibnitz's law is not threatened by the modal case. In addition, theorem *14.18 of *Principia Mathematica* which is sometimes called Universal Instantiation is not threatened by the modal case either. But as one might expect, UI is closely connected to what we have been calling EG. And with the negation operator, the one is definable in terms of the other. For example, a sentence of the form '$\sim(\forall x)\varphi$' is equivalent to a sentence of the form '$(\exists x)\sim\varphi$'. If it is the case that a plausible principle of EG can be derived from theorem *14.18 of *Principia*, then the derived

[20] *Ibid.*, 38.

principle of EG should be immune to the kind of modal case discussed above.

It is Quine's belief that the modal case presents some difficulty in that EG is thought to break down when applied to those cases. However, if EG is derivable from UI (or theorem *14.18 of *Principia*), we expect, contrary to Quine's belief, that EG will remain a valid rule of inference when applied to the modal case. In what follows, I will show that a plausible version of EG *is* derivable from UI, and that the modal case does not present any serious challenges to that version.[21]

Consider again theorem *14.18: $E!(\iota x)(\varphi x) \rightarrow ((\forall x)(\psi x) \rightarrow \psi(\iota x)(\varphi x))$, and consider the following sentence:

(36) $G[(\iota x)(Fx)] \rightarrow (\exists x)(Gx)$

which roughly says that if *the F is G, then something is G.* Does theorem *14.18 entail sentence (36)? Note first that *14.18 is a schema; the occurrences of the symbol 'φ' and the symbol 'ψ' are merely schematic for other expressions, e.g., predicate letters. So, let us replace those symbols in *14.18 with 'F' and '$\sim G$' respectively. (I will use square brackets to indicate the scope of the descriptive phrase '$(\iota x)(Fx)$'). Performing the specified substitution in *14.18 yields the following sentence:

(37) $[E!(\iota x)(Fx)] \rightarrow ((\forall x)(\sim Gx) \rightarrow [\sim G(\iota x)(Fx)])$.

[21] The derivation resulted from the help of Ali Kazmi.

Now, does (37) entail (36)? (37) is equivalent to the following sentence:

(38) $[E!(\iota x)(Fx)] \rightarrow (\sim[\sim G(\iota x)(Fx)] \rightarrow \sim(\forall x)(\sim Gx))$.

But (38) is equivalent to:

(39) $[E!(\iota x)(Fx)] \rightarrow (\sim[\sim G(\iota x)(Fx)] \rightarrow (\exists x)(Gx))$

which entails:

(40) $([E!(\iota x)(Fx)] \;\&\; (\sim[\sim G(\iota x)(Fx)])) \rightarrow (\exists x)(Gx)$.

Given Russell's theory of definite descriptions, the following sentence is valid:

(41) $[G(\iota x)(Fx)] \rightarrow ([E!(\iota x)(Fx)] \;\&\; \sim[\sim G(\iota x)(Fx)])$.

Notice that (40) is of the form 'P → Q', (41) is of the form 'R → P'. What we can legitimately infer, given these two forms, is a sentence of the form 'R → Q'. Thus, (40) and (41) entails (36) which says that if *the F is G, then something is G*. Let us, however, abstract away from any particular instances of EG. Sentence (42) is the result of replacing '*F*' and '*G*' with the schematic symbols 'φ' and 'ψ' respectively.

(42) $[\psi(\iota x)(\varphi x)] \rightarrow (\exists x)(\psi x)$.

A principle of EG, then, could be stated in a sentence thus:

(43) For any sentence *S* which contains an occurrence z of a definite description, if the scope of z extends to *S*, then *S* is true only if '$(\exists x)(Sx)$' (where z is replaced with x) is true.

Notice that there is a scope condition placed on occurrences of definite descriptions. If the condition is satisfied, then the legitimacy of EG in modal contexts is guaranteed; in other words, (43) is true.

Consider, then, (43). Let sentence S be sentence (10), and let z be the occurrence of 'the number of planets' in (10). If (43) is true, then if the scope of z extends to all of (10), then (10) is true only if the result of replacing z with a variable, say, x and prefixing an existential quantifier is true. That is, if (43) is true, then if the scope restriction is observed, then (10) is true only if '$(\exists x)$ necessarily$(x > 7)$' is true. Is the sentence '$(\exists x)$ necessarily$(x > 7)$' true? Is it the case that there is an object such that that object is necessarily greater than seven? It seems to me that it is; indeed there are many numbers which are necessarily greater than seven, and nine happens to be one of them. As expected, the modal case is not a counterexample to (43). But we would not have expected otherwise, since (43) is derivable from *14.18, and the modal case is not a counterexample to *14.18.

Let us now consider a principle of EG which places no restrictions on the scope of an occurrence of a definite description.

> (44) For any sentence S with an occurrence z of a singular term α, if S is true, then the result of replacing z with a variable x, and prefixing an existential quantifier likewise true.

To see that (44) is problematic, consider the following sentence:

(45) Nine = the number of planets & ~necessarily, the number of planets is greater than seven

which is true. Yet one application of EG on the occurrences of 'the number of planets' in that sentence yields:

(46) $(\exists x)$(nine = x & ~necessarily, x is greater than seven)

which is not true. (46) says that there is an object x, such that x is identical with nine, but it is not the case that x is necessarily greater than seven. The problem with (44) is that from the truth of (45), one application of EG on the occurrences of 'the number of planets' in that sentence yields the false sentence (46), and this is due to a failure of observing the scope of the second occurrence of 'the number of planets' in that sentence. *If*, however, one does observe the scope of the occurrence of that description and, specifies that it extends to the entire sentence, then (45) would be false and EG would not apply.

The modal case is definitely problematic for (44). In fact, (44) is more than problematic, it is simply false as the pair of sentences (45) and (46) indicates. Moreover, the falsity of (44) can be illustrated by not only the modal context, but other contexts as well. Take, for instance, sentences (2) and (18). That (44) is false is illustrated by the fact that sentence (2) is true, but a replacement of the occurrence of 'Giorgione' in (2) with a variable x, and prefixing an existential quantifier yields sentence (18) which does not express any proposition and, therefore, does not express a true proposition.

It is difficult to know exactly what Quine's worries about the failure of EG in a given context are. And it is very difficult to know what the failure of EG in a given context goes to show. In "Reference And Modality," it seems that Quine is sometimes of the opinion that a failure of EG in a given context is evidence for the claim that quantification into that context is not permissible. If this is the case, then a principle which captures this idea is the following one.

> (47) For any context C: ___α___, where α is a non-vacuous singular term, if quantification into C is permissible, then EG on α in C is truth-preserving.

But which principle of EG is alleged in (47)? Is it (43) or (44)? But notice that there was strong evidence to reject (44). (44) is false, not a principle of logic, and any principle which appeals to a false principle ought not be taken very seriously. However, if the principle of EG appealed to in (47) is the one articulated in (43), then the result of its application to modal contexts which involve definite descriptions are clear.

Quine alleges that quantification into a modal context is not permissible. If (47) is used as a premise in an argument which purports to show that quantification into a modal context is not permissible, then it needs to be shown that EG on an occurrence of a definite description in a modal sentence is not truth-preserving. But we have just seen that, if (43) is true, then EG is always going to be truth-preserving when applied to a modal context. So long as the scope condition in (43) is satisfied, it is very difficult to imagine a (relevant) example which would render EG to be an invalid rule of inference. EG on an occurrence of a definite description in a true modal sentence would be invalid, only if its application yielded a sentence which is not true. But this is impossible. Consider the sentence 'Necessarily, the present queen of England is identical with Elizabeth II' which is ambiguous. But the interpretation that I wish to discuss takes the scope of the occurrence of 'the present queen of England' in that sentence to extend to the entire sentence. If it is true that *there is a unique present queen of England such that necessarily, she is identical with Elizabeth II*, then how could it be false that there is something necessarily identical with Elizabeth II? Indeed, consider *any* sentence which satisfies the conditions in (43); so long as the scope condition is met, EG is always going to be truth-preserving. Or, to put it a little differently, if *14.18 is valid, and if (43) is derivable from *14.18, then (43) is valid as well.

As I said earlier, it is difficult to know exactly what Quine's worries about the application of EG in modal contexts are. Perhaps he is to be understood as not so much worrying about the truth preservation (or lack of truth preservation) of EG in modal contexts, but rather that EG on an occurrence of some term in a modal sentence yields a sentence which is apparently meaningless or incoherent. If we follow this line of thought, it can readily be seen why Smullyan's response to Quine is unsatisfactory. Smullyan argued that if an occurrence of a definite description in a sentence is treated in accordance with Russell's theory of definite descriptions, then worries about modal cases violating Leibnitz's law are easily dispelled. However, if Quine's concerns about quantification into propositional attitude contexts are analogous to his concerns about quantification into modal contexts, then a Quinean might counter Smullyan by suggesting that (31) and sentences like (31) all involve quantifying into a modal context from the outside, and this is a dubious business.[22] Smullyan's response, though it addresses an apparent problem for Leibnitz's law, an apparent problem for UI and hence EG, does not address Quine's worries about the intelligibility of sentences such as

(48) $(\exists x)$ necessarily$(x > 7)$

as Quine's remarks below would seem to indicate.

> What is this number which, according to [(48))] is necessarily greater than seven? According to [(9)], from which [(48)] was inferred, it was 9, that is, the number of planets; but to suppose this would conflict with the fact that [(10)] is false.[23]

Let me illustrate Quine's worry in the following way. For Frege, *sense* is distinct from *nominatum*, for he says: "A proper name (word, sign, sign-

[22] I have not discussed propositional attitude contexts in this paper, and though Quine's paper entitled "Quantifiers and Propositional Attitudes." Rpt. in Reference and Modality. Ed. Leonard Linsky. (London: Oxford University Press, 1971) 101 – 111, is worthy of discussion, I think that the problem which Quine sees as affecting the propositional attitude context and the modal context are, at base, one and the same problem. Of course, in "Quantifiers and Propositional Attitudes," Quine offers a reconstrual of quantified propositional attitude discourse which is supposed to be free from the alleged problem of quantifying in.

[23] Quine, W. From A Logical Point Of View. (Cambridge: Harvard University Press, 1996) 148.

compound, expression) expresses its sense, and designates or signifies its nominatum."[24] Let us *just* say that, in a referentially opaque context, a singular term does not refer to its nominatum, but rather to what is its ordinary or customary sense. For the Fregean, however, the following is problematic. The occurrence of 'Chrissy' in the sentence 'Chrissy is smart, but Jack thinks that she is not' refers to the object denoted by the name 'Chrissy' (that is, to the nominatum of 'Chrissy'), but the occurrence of 'she', which is supposed to be anaphoric on 'Chrissy', does not refer to the nominatum of 'Chrissy', but refers rather to the sense of 'Chrissy'. (Recall that an occurrence of a singular term in an opaque contexts designates its ordinary sense). The problem is that it is *Chrissy* of whom the assertion is made that Jack thinks that *she* is not smart and, under Frege's theory, we cannot capture the idea of anaphora. In the modal case, Quine would say that the occurrence of 'it' in 'Nine is greater than seven and, necessarily it is greater than seven' does not refer to what the occurrence of '9' refers to; for, the claim is that 'Necessarily, x is greater than seven' is not a trait of any number, therefore, is not a trait of the number nine. Anaphora seems destroyed- the 'it' in 'Nine is greater than seven and, necessarily, it is greater than seven' does not refer to an object; and therefore, does not refer to the object that 'nine' refers to. (In my opinion, any claim which has this consequence ought to be, *prima facie*, rejected).

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[24] Gottlob Frege. "On Sense and Nominatum." Rpt. in The Philosophy of Language. Ed. Martinich, A. P. (Oxford: Oxford University Press, 1996) 189.

Consider sentence (48) again. Open sentences are said to be true of objects and, if the open sentence 'Necessarily, x is greater than seven' is true of some object, then Quine's concern is which object, if any, that sentence is true of. Many numbers one might say, one of which just happens to be the number nine. But how can one legitimately say that when nine *is* the number of planets, and the sentence 'Necessarily, nine is greater than seven' expresses a true proposition, while the sentence 'Necessarily, the number of planets is greater than seven' does not express a true proposition? This is why Quine thinks that the sentence 'Necessarily, x is greater than seven' is not even an intelligible sentence. Hence, if there is no object of which 'Necessarily, x is greater than seven' is true, then given that (47) is true, quantification into 'necessarily, ___ is greater than seven' is not permissible.

For Quine, an alternative to rejecting quantified modal discourse altogether involves limiting the role of the modal operators to sentential predication.[25] This means that though the sentence 'Necessarily, x is greater than seven' is meaningless, the sentence ' 'Nine is greater than seven' is necessarily true' is not meaningless. Quine thinks that the sentence 'Nine is greater than seven' is necessarily true' *is* true, but the sentence 'The number of planets is greater than seven' is necessarily true' is *not* true, and this suggests that the truth-value of a sentence of the form 'α is F' is necessarily true'

[25] See Quine's "Three Grades of Modal Involvement." (Proceedings of the XI*th* International Congress of Philosophy, Brussels 14, 1953) 158 – 176. Cf. pages 148 – 149 of "Reference And Modality."

depends on how we choose to specify the object denoted by 'α'. The point, however, is not that a definite description cannot be substituted for 'nine'; for if we substitute the occurrence of 'nine' in that sentence with the description 'the product of three and three', the sentence ' 'The product of three and three is greater than seven' is necessarily true' is true. Rather, the point is simply that a singular term 'α' which occupies the subject position of a given sentence, to which the predicate 'necessarily true' or 'is necessary' attaches, may render that sentence true or false depending on 'α' itself. But perhaps an even more important point to stress is that Quine rejects all open sentences of the form 'Necessarily, *x* is *F*'.[26] In short, Quine thinks that to be necessarily thus-and-so makes no sense when applied to objects, but that it makes tolerable sense when applied to (closed) sentences. I am not wholly unsympathetic to this view, but in everyday discourse, we do sometimes talk about objects being necessarily thus-and-so, and this suggests that we find such discourse intelligible. Thus, let us reject the view that modal operators can only be attached to (closed) sentences, and consider the intelligibility of open sentences which attribute necessity.

Consider then the truth-conditions for sentence (48) which is true just in case *there is at least one object such that necessarily, that object is greater than seven.* Now, which object is this? Nine, that is, the number of planets? Surely, to

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[26] But it should be noted that Quine does not think such sentences are even open sentences. His charge is that such sentences are meaningless.

suppose this would *not* conflict with the fact that (10) is false; for (48) says that there *is* an object *x*, such that necessarily *x* is greater than seven. The object denoted by the expression 'nine' and the object denoted by the description 'the number of planets' is one and the same object, and it is true that it is such that, necessarily, it is greater than seven. Perhaps another way of thinking about (48) is the following. If the variable *x* ranges over objects, then clearly (48) is about an object. Which object? Well, whichever object satisfies the open sentence 'Necessarily, *x* is greater than seven'. The open sentence 'Necessarily, *x* is greater than seven' is true of many numbers, one of which is the number nine, that is, the number of planets. Of course, when things are put in this way, it is clear that a definite answer is available to Quine's question of 'which number'. Supposing that the object denoted by the expression 'nine' (that is, the object denoted by the definite description 'the number of planets') satisfies the open sentence 'Necessarily, *x* is greater than seven' does *not* conflict with the fact that (10) is false. (10) just says that it is necessarily the case that the number of planets is greater than seven, which of course is false. (We saw earlier that the scope of an occurrence of a definite description in a sentence may affect, and does affect in this case, the truth-value of that sentence).

What, then, are we to say about (47) given the above discussion of the modal case? Is EG on the singular term(s) in the modal example truth-preserving? Or better yet: Does the sentence which results from one application of EG on the expression 'nine' in (9) (or 'the number of planets' in (10)) express a true proposition? If (47) involves the principle of EG which is articulated in (43), then I think it should be granted that the resulting sentence does express a true proposition. In contrast, however, sentences such as (17) and (18) clearly do not express true propositions; therefore, (47) is true, but it is not conceptually true.

In the present section of this paper, I have tried to show that quantified modal logic does not violate theorem *14.18 (or UI), nor does it violate the (restricted) principle of EG. Indeed, if quantified modal logic did violate these principles, then that would give us some (good) reason to be suspicious of quantified modal discourse. But recall that Quine charges that quantified modal discourse is unintelligible. As such, if it turns out that it *is* intelligible, then that charge of Quine is largely without merit. In what follows, I will discuss Kripke's paper entitled "Semantical Considerations On Modal Logic." Kripke's paper articulates some of the formal aspects of the semantics of quantified modal logic, and I will highlight some of the key points which are helpful in understanding not only modal sentence, but quantified modal sentences as well.

Section 2

Imagine that there are 100 books, each of which contains sentences of English, and jointly, the 100 books give an exhaustive list of the sentences of

English. Now, consider some arbitrary sentence S, $\Box S$ is to be interpreted as follows: $\Box S$ is true if and only if S appears in all 100 books and, false otherwise. How do we know that $\Box S$ is true? Well, we look to see if S is in all of the books. Thus, if we take sentence S to be the sentence 'Necessarily, nine is greater than seven', then S is true if and only if the sentence 'Nine is greater than seven' has at least one entry in each of the 100 books. So, why is the sentence 'Necessarily, the number of planets is greater than seven' false? Simple: The sentence 'The number of planets is greater than seven' does not have at least one entry in each of the 100 books. The real motivation behind the story with the books is that Kripke's system offers a similar but more sophisticated interpretation of sentences like $\Box S$ in addition to providing the semantics for quantified modal sentences.

Under Kripke's system, an interpretation of a modal sentence requires what is called a "model structure." A model structure is a set containing the ordered triple G, K and R, defined as follows. K is the set of all (possible) worlds of which G is a member; R is a reflexive relation on K- if H_1 and H_2 are two worlds, then "H_1RH_2 means intuitively that H_2 is 'possible relative to' H_1; i.e., that every proposition true in H_2 is [possibly true] in H_1."[27] In addition to model structures, an analysis of $\Box S$ requires a model which assigns a truth-value to each and every sentence (or proposition expressed by

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[27] Saul Kripke. "Semantical Consideration on Modal Logic." Rpt. in Reference And Modality. Ed. Leonard Linsky. (London: Oxford University Press, 1971) 64.

those sentences) of every world in the set K. A model M assigns a value of T to a sentence S in a world H just in case S is true in H, but if S is not true in H, then M assigns a value of F to S. $\Box S$ is defined as follows: $\Box S$ receives a value of T in some world $H \in K$ just in case S is true in every world H′ and, HRH', and receives a value of F otherwise.

Now, the semantics for quantified modal sentences is just an extension of the semantics of modal (closed) sentences. One assumption which both requires is that for any world $W \in K$, W has a specified domain. But for the semantics of the former, it is also required that for any n-place predicate P^n in W, the extension of P^n in W is given by a list (or set) of ordered n-tuples (whose members are members of $W \in K$). Given these assumptions, the truth-conditions for quantified modal sentences are readily accessible.

If we take our sentence S to be sentence (10), then we shall have to disambiguate (10) before considering the truth-conditions. If the scope of the occurrence of 'the number of planets' in (10) is restricted to the sentence 'The number of planets is greater than seven', then (10) is true in a world W, just in case, for any world W', the sentence 'The number of planets is greater than seven' is true in W', and WRW'. But notice that if the scope of the occurrence of the description in (10) is restricted to the complement sentence in (10), then the truth-conditions which were given above cannot be satisfied;

the reason simply being that it is false that for any world W', the sentence 'The number of planets is greater than seven' is true in W'. Take, for example, the expression 'the strongest man in the world'. (Competition for the title of 'the strongest man in the world' is an annual event). In any case, the definite description 'the strongest man in the world' denotes different people from year to year. That is not to say, however, that one individual cannot be the strongest... for more than one year. But it is not difficult to imagine how the proposition that the strongest man in the world is always the same individual is not true. Bruno was last year, but died shortly after the competition. Analogously, the description 'the number of planets' denotes different individuals in different possible worlds. Had the universe turned out differently, the number of planets might have been, say, six in number.

There are, however, a class of definite descriptions which function like names. These so-called *rigid* definite descriptions denote, if they denote at all, the same individual in all possible worlds in which those individuals exist. For example, the description 'the object which is identical with α' is going to denote α in all worlds in which α exists. Or to take another example, the description 'the even prime' denotes the number two in every world in which the number two exists. Not unlike names, these descriptions are such that their referents are fixed. But whatever the case may be with this class of definite descriptions, 'the number of planets' is not a member of this class, and this is why the sentence 'Necessarily, the number of planets is greater than seven' (in which the scope of the occurrence of the description is restricted to the complement sentence) is false.

If we take the scope of the occurrence of the definite description in (10) to extend to all of (10), then (10) is going to be true in a world *W*, just in case, for any *W'*, 'Nine is greater than seven' is true in *W'*, and *W*R*W'*. The condition that for any world *W'*, 'Nine is greater than seven' is true in *W'*, is stated thus because it is not true that for any world *W'*, 'The number of planets is greater than seven' is true in *W'*. It is not true because given that in *W'*, the description 'the number of planets' may denote, if it denotes, a number which is less than nine. If we take the scope of the occurrence of the description in (10) to extend to all of (10), then the proposition which results is equivalent to the proposition expressed by the sentence 'Nine is such that necessarily it is greater than seven'. 'The number of planets' is a description which denotes an object, i.e., the object which is also denoted by the expression 'nine'. And we need to make it clear that, if it exists, it is *that* object which is necessarily greater than seven.

"What is this number which, according to (48) is necessarily greater than seven?" There are many numbers which are greater than seven, and nine is one of them. But as sometimes is the case in the literature, it is questioned how an object is to be identified across worlds. How do we know that in some possible world, the object we are considering is in fact nine? And it is sometimes suggested that one way of identifying an object across possible worlds is by its necessary traits. For instance, any object of which the sentence 'Necessarily, *x* is greater than seven' is not true is not the number nine. One might think that it is necessary for nine's existence that it is greater than seven; since, given any number which is not greater than seven, that number surely cannot be nine. However, this is problematic because it does not follow that any number which satisfies the open sentence is the number nine. But "not all [necessary] properties will serve…; a property may be too individuating to be of help...Every object *a* has the necessary property of being identical with itself, and no other object is identical with *a*. It is fatuous, however, to suppose that one can identify *a* across possible worlds by finding the object that is identical with *a* in each possible world; for our problem is how to do that."[28] I think that it is indeed fatuous, but because it is the wrong approach. The problem of identifying objects across possible worlds, I want to suggest, is a pseudo-problem.

What is meant by the utterance 'Nine is necessarily greater than seven'? Well, what is intended is that the object that we are indicating *now* (which happens to be the number nine), is such that it is greater than seven in all possible worlds relative to our own. We have, as it were, taken the object

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[28] Leonard Linsky. "Reference, Essentialism, and Modality." Rpt. in Reference And Modality. Ed. Leonard Linsky. (London: Oxford University Press, 1971) 99.

across possible worlds. Kripke, for example, says: "[W]e begin with the objects which we *have*, and can identify, in the actual world. We can then ask whether certain things might have been true of the objects."[29] Take, for example, the claim that Gore might have won the last U.S election. Not only is this claim true, but it is in fact very easy to imagine Gore as the winner of the last U.S election. Here, it seems that anyone reasonable would not have a problem identifying Gore- one simply considers *the person* named 'Gore' and, imagines a situation in which that person is the winner of the last U.S election.

I think that Kripke's work on possible world semantics verifies, to some degree, our intuitions about objects and necessary properties. And indeed it is an easy assessment of Kripke's work to make- that it verifies some of our intuitions- given the benefit of hindsight. However, I believe that given the intuitive nature of *Naming and Necessity* and, to a lesser degree, "Semantical Considerations On Modal Logic," it would be very surprising if, after Quine's literary attack on the modalities, no one provided a (coherent) rebuttal. To be sure, just how coherent and satisfactory such rebuttals are is an entirely different question. But many authors have tried to refute Quine, and this is testament to the fact that we find quantified modal discourse valuable and are not willing to give it up (not yet anyway).

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[29] Saul Kripke. Naming and Necessity. (Cambridge: Harvard University Press, 1998) 53.

Ought we give up quantified modal discourse if metaphysical questions about objects cannot be adequately answered? What does it mean to say that some object is necessarily thus-and-so? Given the benefit of Kripke's work on possible world semantics, we are able to provide some answers, but are the answers adequate? As far as the modalities go, has Quine been refuted once and for all? In what follows, I will consider what I believe to be some very pressing issues which seem to be closely connected with quantified modal discourse- the problem of essentialism. Is it an incoherent doctrine? Take, for instance, the number nine. It is obviously true, but not very interesting that in this world, the number nine is greater than seven. However, are we assured of its greater than relation to the number seven in *all* possible worlds? These are some of the questions I will be raising, and though my aim is not to answer such questions, I hope that understanding the problem of essentialism will shed some light on the direction to take.

Section 3

Essentialism is the view that among the traits or characteristics that an object has, some are essential to it and some are not. Now, consider an object *O* and suppose that it is necessary that *O* is *F*. How are we to understand the sentence 'necessarily, *O* is *F*? From the discussion in the last section, such sentences are understood thus: 'Necessarily, *O* is *F*' is true just in case for any world *W*, '*O* is *F*' is true at *W*, and *W* is possible relative to our own world. Consider then the actual world, the sentence '*O* is *F*' must be true in the actual world if it is true in all worlds. In the actual world, we know that '*O* is *F*' is true. This simply means, among other things, that the object *O* is in the extension of *F* in this world. But consider some world other than the actual one. How do we know that *O* is in the extension of *F* in *that* world? Kripke's system allows us only to say that the sentence 'Necessarily, α is *F*' is true if and only if the sentence 'α is *F*' is true in all possible worlds (relative to our own). But the question that we are considering now is not about the *truth-condition* of the sentence 'Necessarily, α is *F*'. Rather, it is about the *truth-value* of the sentence on the right-hand side of the biconditional, i.e., ' 'α is *F*' is true in all possible worlds (relative to our own)'.

A view that one might espouse is the following. It is necessary to *O*'s existence that it is *F*, just like it is necessary to humans that we are mortal. If in some possible world, there exists something which is not mortal, then, whatever it is, it is not human. So, one might say, in all possible worlds in which humans exist, they are mortal. This sounds very plausible, and it is the kind of intuition that Kripke holds in *Naming and Necessity*. I think the example he uses is that of birth. Would the person named 'Elizabeth' be the very same person if she had different parents? No Kripke thinks, and he asks rhetorically "how can this person be the very same person given different birth parents?"[30] Consider a table which is made of wood. Could anything be this very table if it were made of, say, clay? I would be inclined to answer "no, it may look like this table, but if it is not made of the same material, then it is not this table." This is an intuitive response, and it seems a reasonable one.

Consider again the object *O*. In the actual world, we know that *O* is in the extension of *F*, but in some other world, how do we know that *O* is in the extension of *F at that world*? Well, borrowing from Kripke, one might say that *O* would not be *O* if it were *not* in the extension of *F at that world*; just like a table would not be this very table if it were made of clay. So how do we know that, in some other world, *O* is *F* in that world? One might say that it is necessary to *O*'s existence that it be in the extension of *F*. But notice that in enunciating this view, one has come full circle and begged the question- 'how do we know that in some possible world *O* is *F*?' because it is necessary that *O* is *F*. And this is a way of stating the problem of essentialism.

Rather than addressing this problem directly, let me make the following distinction. Consider the sentence 'Aristotle had two slaves, but was happy with only one of them'. What does this sentence mean? One might say it means that there existed a person named 'Aristotle', that he was in possession of two slaves and, moreover, that he was pleased with only one

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[30] Saul Kripke. Naming And Necessity. (Cambridge: Harvard University Press, 1998) 113. "How could a person originating from different parents, from a totally different sperm and egg, be *this very woman*?"

of his slaves. The sentence we are considering seems perfectly intelligible. In addition to explaining what it means, one could give the truth-conditions for it. For example, the sentence 'Aristotle had two slaves, but was happy with only one of them' is true just in case Aristotle had two slaves and was happy with only one of them. But is the sentence true? *I* do not know, but it would not be difficult to find out.

Suppose now that someone utters the following: On the planet surface of Saturn, there are two obsidian-like rocks exactly two and a quarter centimeters apart. Does the utterance express a true proposition? Not that we would necessarily want to, but could we find out? Perhaps, but let us suppose that we cannot. Does that entail that the sentence 'on the planet surface…' is not an intelligible sentence? It seems to me that it does not. There are many sentences of which it is difficult, if not impossible, to determine the truth-values of. For example, consider the sentence 'God never cries on Tuesdays'. True or false? and who reasonable among us would know? Yet an understanding of that sentence does not even require a fully competent speaker of English.

It seems to me that issues concerning the intelligibility of quantified modal sentences are quite distinct from issues concerning the truth-values of those sentences. I am inclined to think that the latter kind of issue is to be settled independently of any issues regarding intelligibility. To be sure, however, a quantified modal sentence must be intelligible in order for us to assign a truth-value to it. But whether or not it is the *correct* truth-value, or whether or not it is even possible to determine if it is the correct truth-value says something about our epistemology and not our logic. Our logic gives us a representation of the form of a quantified modal sentence and does *not* purport to give the correct truth-value. For all we know, it may turn out not to be true that nine is greater than seven in *all* possible worlds. But this is quite different and independent of the issue concerning the intelligibility of quantified modal logic.

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