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PETROLEUM LEASING CONTRACTS AND UNCERTAINTY

by

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ABSTRACT

This thesis deals with the financial aspects of petroleum leasing contracts in Alberta, in conjunction with the uncertainty of exploration. In particular we solve for the Pareto Optimal allocation of risk between a firm and the provincial government, on a lease of given size, for a specific payment schedule. The major factors influencing the allocation of risk include: the attitudes of the firm and the government towards risk, the characteristics of the lease in question, and the possibility of expectation asymmetries.

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CHAPTER ONE

INTRODUCTION

This study is concerned with one issue in the allocation of publicly owned mineral rights for the private exploration and production of oil and gas. Mineral leasing policy arises out of the fact that some governments, which own mineral rights, do not involve themselves with the exploration, development or production of their resources. In Alberta, the mineral rights to almost 86% of potential oil and gas bearing land are owned by the provincial government. The remaining 14% is owned primarily by the private sector with a small percentage in the hands of the federal government[1].

The rights to potential oil and gas reserves are transferred from the government to the oil industry by way of a complex system of leases. Different type leases or contracts may be applicable to different stages of development from preliminary exploration to production. The lease is defined over a tract of land of given size. The location and pattern are closely regulated. In addition, the lease includes a list of obligations to the company which are both financial (the lease specifies the type of payment, such as a bonus bid) and non-financial (such as drilling requirements). Our main concern is with the financial aspects of a lease.

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[1]For a more complete discussion of the allocation of mineral rights in Alberta see Crommelin, Pearse and Scott, Management of Oil and Gas Resources in Alberta: An Economic Evaluation of Public Policy, University of British Columbia, Dept. of Economics, June 1976, p. 16-23.

Most exploration rights are linked directly to production rights in the sense that exploration rights may be converted in whole or in part to production rights. Leases are usually offered for sale in response to a request by a firm. The lease is then auctioned to the industry by way of sealed bids. In general, the firm making the highest bid obtains the lease, although the government retains the right to reject all bids. In addition to the bonus bid payment, most Alberta leases require rental payments as well as the obligation for a gross royalty payment if production ensues. Initially, a lease had fixed terms under established royalties. Now, the government reserves the right to alter the specifications of the lease at any time. Profits from oil and gas production are subject to the Canadian corporate profit tax, divided between the federal and provincial governments.

In general, there are two types of payments made on a lease: the fixed payment (annual rent or bonus bid) and the profit or royalty tax. The royalty scheme in Alberta uses a gross royalty: a proportion of the gross value of the recovered resource. A gross royalty is to be distinguished from a net royalty: a proportion of the net value of the recovered resource. The gross royalty is essentially a revenue tax and the net royalty a profit tax. The economic implications of a gross royalty and a net royalty will be discussed in later chapters.

Any government leasing policy that regulates the allocation of oil and gas rights may be evaluated with respect to several criteria. The major 'over-riding' objective may be the maximization of government utility derived from oil and gas revenue. In addition to revenue the components that may be considered when evaluating a leasing policy include: the timing of resource development, self-sufficiency in energy, environmental impact, distribution of income and so on. All these concerns have, no doubt, influenced mineral leasing policy in Alberta.

Perhaps the most important element is the government utility derived from revenue earned through exploitation of its resources. Although we do not overlook the other issues, our attention will be focused on the maximization of government utility derived from revenue. Directly related to this issue is the timing problem. Clearly, the government wishes to maximize utility over the life of the resource, and so is concerned with the rate of development and depletion of the petroleum resource base. This in turn implies some rate of investment in exploratory activity (for example, the rate at which the industry accumulates new reserves) and some rate of production. However, we will refrain from considerations of this aspect of the problem. Rather, we will assume that the timing implicatons on any tract of land have already been considered by the government prior to actual leasing of that tract. Our focus is upon the financial terms which should be attached to a particular lease.

If the government wishes to maximize its utility of oil and gas revenue there are two areas of concern with respect to economic efficiency. First, the government should try and preserve the total value of the lease by choosing a payment schedule that is as neutral as possible with respect to the firm's production decisions. Second, it must minimize the adverse effects the taxation scheme, when combined with the uncertainty of exploration, may have on the firm's production decisions.

Both these topics will be discussed in this study. Through a careful selection of payment schemes it can be ensured that the firm will not be affected on its marginal decisions. Having satisfied the first problem of efficiency, our focus will be on the influence of uncertainty in the determination of the lease contract.

The existence of uncertainty in exploration is one of the most notable characteristics of the oil industry. As put by Peterson[2]:

> Instead of a world of perfect information we have a world dominated by risk and uncertainty, and mineral exploitation is one of the most risky businesses, with costly exploration programs and low success ratios.

The existence of risk has a notable influence on the firm's production decisions. Leland[3] notes three specific influences risk will have on a risk averse firm:

(a) they will tend to explore less as exploration is a risky investment;

(b) they may produce resources too quickly if there is price uncertainty;

(c) any fixed payments made to the government, such as a bonus bid,

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[2]Peterson, "The Government Role in Mineral Exploration", in <u>Mineral Leasing as an Instrument of Public Policy</u>, edited by Crommelin and Thompson, University of British Columbia Press, 1977, p. 34.

[3]Leland, "Comment" on "Cash Bonus Bidding for Mineral Resources" by Mead, in <u>ibid</u>., p. 56-57.

will tend to be lower.

One response by the government to uncertainty in the crude petroleum industry has been to utilize two payments: conditional and unconditional payments.

> Most lease contracts contain unconditional and conditional payment clauses. Unconditional payments such as lease bonuses do not depend on subsequent events, such as the discovery of resources on the tract. If there are no further conditional payments, the firm which wins the bid bears the entire uncertainty regarding the amount of resources, selling price and cost of production. Thus, the outright sale (or lease with only unconditional payments) transfers none of the risk from the buyer to the seller. Conditional payments are dependent on conditions which are unknown at the time the lease is sold. Royalties, for example, depend on future production (if any) and on the market price at the time of production. Properly chosen conditional payments reduce undertainty to the firm, since they involve large amounts only in conditions favorable to the firm, and small amounts otherwise.[4]

In general, risk sharing by the government, in the form of conditional payments in conjunction with unconditional payments will:

> reduce social welfare loss resulting from risk aversion. Firms will make more economically efficient exploration, development, and production decisions. And the government will enjoy greater expected revenues.[5]

That risk sharing is beneficial is clear: the relevant questions

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[4]Leland, <u>op cit</u>., p. 57.

[5]Leland, <u>op cit</u>., p. 58.

become how much risk should the government bear and how the financial terms of the lease are affected by a change in the characteristics of the lease. In order to examine these issues we will develop a model of leasing under uncertainty. In it we will solve for the optimal leasing contract under a variety of conditions. From the general theory developed, it will be seen that the characteristics of a given tract of land will be a major determinant of the lease arrangement. The relevant topics of discussion include: attitudes of the firm and the government toward risk, uncertainty on the tract of land in question and symmetric and asymmetric expectations of the companies and the government.

In order to facilitate the reader's understanding of the analysis presented in this study the second chapter will be devoted to a short summary of the relevant aspects of the state preference theory used in subsequent chapters.

In Chapter Three we will develop the model and discuss its major features. In Chapter Four we derive the optimal characteristics of one of the payment schedules developed in the third chapter. This chapter examines the optimum under conditions of symmetric expectations between the firm and the government. In Chapter Five we relax this assumption and consider the optimum under conditions of asymmetric expectations. Chapter Six incorporates the conclusions of this study.

CHAPTER TWO

STATE PREFERENCE THEORY

The model used in this thesis is based on the State Preference Theory. In order to assist the reader with the model developed in subsequent chapters we will provide some of the underlying theory by developing a simple uncertainty model in this chapter. We will focus our attention on those elements in the State Preference Theory that will aid in the understanding of later analysis. The model we will use in this chapter is based on a model developed by Greene in his book Consumer Theory.[1]

A. The Basic Model

Greene's model has the following specifications. Let there be one consumer with an initial stock of wealth $\overline{W}$. The consumer wishes to maximize his utility by allocating his wealth between a safe asset holding, S, and a risky asset holding, R. The safe asset pays a guaranteed rate of return r. The risky asset pays the safe rate of return r, plus either a capital gain, g, or a capital loss, l. Accordingly, we may specify two states of nature: state A if a capital gain is realized and state B if a capital loss is realized.

The consumer has subjective expectations about the occurance of

[1]John Greene, Consumer Theory, MacMillian Press Ltd., 1976, p. 249-274. The reader may also be interested in Kenneth Arrow (1974), Jack Hirshleifer (1964-66) and Tobin (1958).

either state of nature. State A occurs with probability π_A and state B occurs with probability π_B, where $\pi_A + \pi_B = 1$. From the above conditions we may write:

(2-1) $$R + S = \overline{W} \qquad \text{or} \qquad R = \overline{W} - S \ .$$

(2-2) If state A occurs the consumer receives:

$$W_a = \overline{W}(1+r) + Rg \quad .$$

If state B occurs the consumer receives:

$$W_b = \overline{W}(1+r) - Rl \quad .$$

By definition wealth in state A is greater than or equal to wealth in state B.

Associated with the two states of nature is the expected value of future wealth, $E(W)$, where:

(2-3) $$\begin{aligned} E(W) &= W_a\pi_A + W_b\pi_B \\ &= [\overline{W}(1+r) + Rg]\pi_A + [\overline{W}(1+r) - Rl]\pi_B \\ &= \overline{W}(1+r) + R(g\pi_A - l\pi_B) \quad . \end{aligned}$$

A bet may be defined as being fair, unfair, or favorable. In terms of our model we may specify a bet as being fair, unfair, or favorable as follows:

if $E(W) - \overline{W}(1+r) = 0$ and $R(g\pi_A - l\pi_B) = 0$ then the bet is fair;
if $E(W) - \overline{W}(1+r) < 0$ and $R(g\pi_A - l\pi_B) < 0$ then the bet is unfair;
if $E(W) - \overline{W}(1+r) > 0$ and $R(g\pi_A - l\pi_B) > 0$ then the bet is favorable.

This implies that if $1/g < \pi_A/\pi_B$ then the bet is favorable. Using this definition of a bet, Arrow[2] defines a risk averter as:

> one who starting from a position of certainty is unwilling to take a bet which is actually fair to him (a fortiori, he is unwilling to take a bet which is actually unfair to him).

If the consumer is risk averse then unless $1/g < \pi_A/\pi_B$ (the bet is favorable) the consumer will invest no wealth in the risky asset R.

We may define the expected utility hypothesis with respect to a risk averter with a utility of wealth function, U(W), having the following properties:

(a) $W_a > W_b$ if and only if $U(W_a) > U(W_b)$

(b) if $\overline{W}(1+r) = E(W)$ (the bet is fair) then

$$U[\overline{W}(1+r)] > E[U(W)] = U_a(W_a)\pi_A + U_b(W_b)\pi_B \quad .$$

These two conditions imply that a risk averter's utility function is strictly concave. U(W) is assumed to be at least twice differentiable:

(2-4) $U'(W) > 0$ is the marginal utility of wealth

$U''(W) < 0$ is the rate of change of the marginal utility of wealth

where the primes denote differentiation. Wealth is assumed to be desirable so that the marginal utility of wealth $U'(W)$, is positive.

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[2]Kenneth Arrow, Essays in the Theory of Risk Bearing, North Holland Publishing Co., 1974, p. 94.

For a risk averter $U'(W)$ is strictly decreasing as wealth increases hence, $U''(W) < 0$. If the consumer is risk neutral then $U(W)$ will be linear ($U''(W) = 0$) and if he prefers risk $U(W)$ will be strictly convex ($U''(W) > 0$).

Much of our later analysis will depend on how risk averse an individual is; we will introduce two measures of risk aversion[3]:

(a) $-\frac{U''(W)}{U'(W)}$ the absolute degree of risk aversion

(b) $-\frac{U''(W)}{U'(W)} W$ the relative degree of risk aversion .

These are local measures of risk aversion, defined at a specific point on the utility function. Clearly, a constant level of absolute risk aversion implies an increasing level of relative risk aversion as wealth rises. Further, a constant or decreasing level of relative risk aversion implies a decreasing level of absolute risk aversion. The absolute degree of risk aversion may be interpreted as a measure of an individual's demand for a favorable bet. The relative degree of risk aversion, a somewhat 'finer' measure, measures the bet not in absolute terms but in terms proportional to $\overline{W}$.

These two measures of risk aversion will be used extensively in future chapters. In the next section we demonstrate the importance of the absolute and the relative degrees of risk aversion. These points and other issues will be clearly demonstrated as we work through

[3]Kenneth Arrow, op cit., p. 94; and see J.W. Pratt, "Risk Aversion in the Small and in the Large", in Econometrica, 32 (January-April, 1964).

the model.

The relationships described by equations (2-1) to (2-3) are illustrated in Figure (2-1). The axes show the levels of wealth in the two alternative states of nature, A and B. Along the certainty line all wealth is held in the safe asset (R = 0). Expected wealth is constant along this 45° line and is equal to $\overline{W}(1+r)$. Alternatively, if all wealth is invested in the risky asset, R, his possible gains are shown along the line $R = \overline{W}$. The possible divisions of wealth between holdings of the risky asset and the safe asset will yield future levels of wealth in state A or B as shown by the line CD (ignoring the possibilities of buying long or selling short). This line is analogous to the budget constraint of consumer theory. The slope of this 'budget constraint' is $-1/g$.

The problem facing the consumer is to choose a portfolio so as to maximize expected utility. Expected utility may be expressed as:

(2-5) $$E(U) = U_A(W_A)\pi_A + U_B(W_B)\pi_B \quad .$$

To find the consumer's indifference curves differentiate the above equation with respect to W_A and W_B while holding expected utility constant.

(2-6) $$dE(U) = U_A'\pi_A dW_A + U_B'\pi_B dW_B = 0$$

$$\text{or} \qquad \frac{dW_B}{dW_A} = -\frac{U_A'\pi_A}{U_B'\pi_B}$$

where U_i' is the marginal utility of wealth in state i.

From equation (2-6) we see that as W_A increases W_B must decrease in order for the consumer to maintain a constant level of utility. Further differentiation of (2-6) yields:

$$\frac{d^2W_B}{dW_A^2} = -\frac{\pi_A}{\pi_B}\left[\frac{U''_A U'^2_B \pi_B + U''_B U'^2_A \pi_A}{U'^3_B \pi_B}\right] \quad . \tag{2-7}$$

Equation (2-7) is positive if the consumer is risk averse ($U'' < 0$), zero if risk neutral ($U'' = 0$) and negative if the consumer prefers risk ($U'' > 0$).

From equation (2-6) we see that $U'_A = U'_B$ on the certainty line, where $W_A = W_B$. The slope of the consumer's indifference curve on the certainty line will therefore equal $-\pi_A/\pi_B$. In figure (2-1), that is indicated by the slope of the consumer's indifference curve, I_1, on the certainty line. We see from the slope of CD that the bet is favorable ($1/g < \pi_A/\pi_B$). Hence, the consumer would maximize utility at point G, where I_2 is tangent to the budget line.

To obtain the optimal point G we solve the following problem for the given level of wealth $\overline{W}$, and the given subjective probabilities:

$$\text{Max } E(U) = U_A[\overline{W}(1+r) + Rg]\pi_A + U_B[\overline{W}(1+r) - R1]\pi_B \quad . \tag{2-8}$$

As R is the only variable we may write the first order conditions as:

$$\frac{dE(U)}{dR} = U'_A g\pi_A - U'_B 1\pi_B = 0 \tag{2-9}$$

or

$$\frac{U'_A \pi_A}{U'_B \pi_B} = \frac{1}{g} \quad .$$

Equation (2-9) states that at the optimum the individual's indifference curve must be tangent to the budget line, as is indicated by point G in figure (2-1).

The second-order condition for a maximum requires

$$\frac{d^2E(U)}{dR^2} = U_A''g^2\pi_A^g + U_B''1^2\pi_B < 0 \tag{2-10}$$

which is satisfied if the consumer is risk averse ($U'' < 0$).

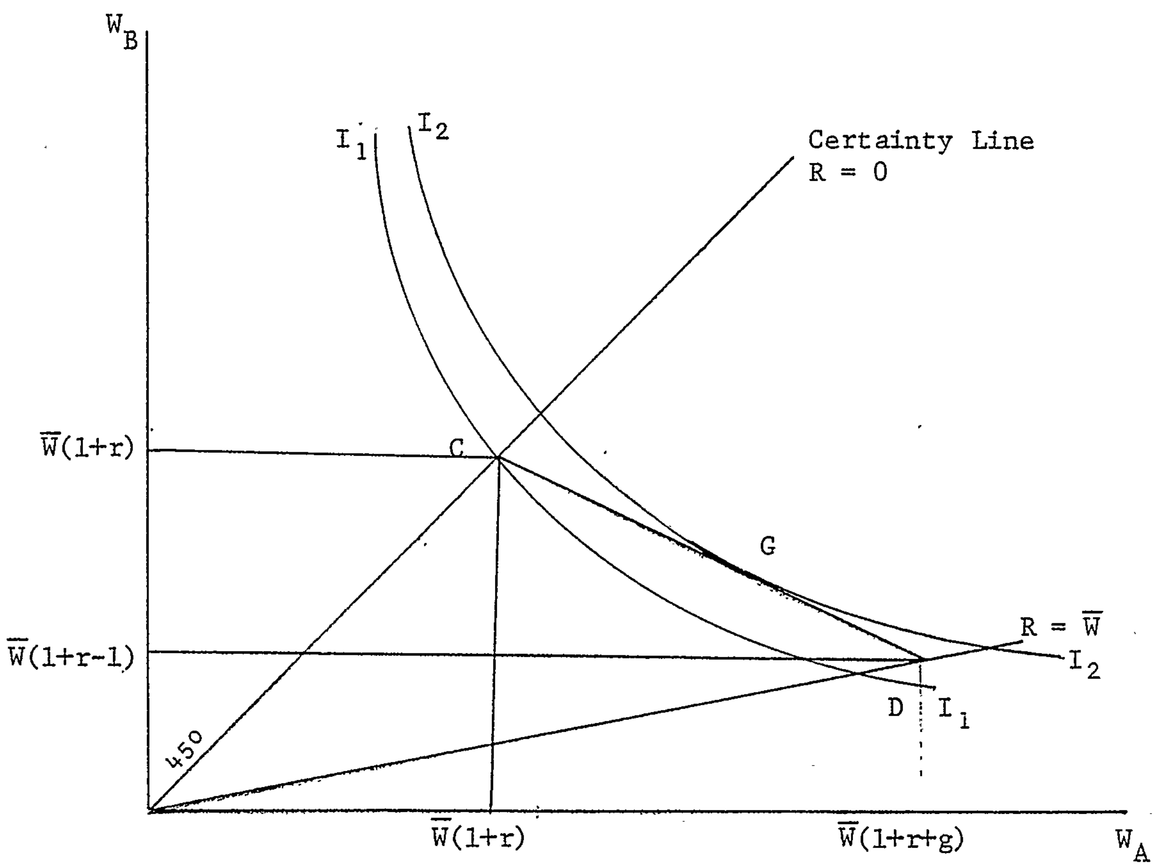

FIGURE (2-1)

B. Comparative Static Analysis

1. Change in Wealth

To illustrate the importance of the absolute and relative degrees of risk aversion let us undertake an exercise in comparative statics. Consider the movement in equilibrium if both $\overline{W}$ and R are allowed to vary. The first-order condition requires that at equilibrium $dE(U)/dR = 0$. Further a change in $\overline{W}$ will effect a change in R unless the absolute degree of risk aversion is constant. Take the total derivative of the first order condition in equation (2-9) with respect to R and $\overline{W}$. This yields:

$$\text{(2-11)} \qquad [U''_A g^2 \pi_A + U''_B l^2 \pi_B]dR + [U''_A g(1+r)\pi_A - U''_B l(1+r)\pi_B]d\overline{W} = 0$$

$$\text{or} \qquad \frac{dR}{d\overline{W}} = \frac{-U''_A g(1+r)\pi_A + U''_B l(1+r)\pi_B}{U''_A g^2 \pi_A + U''_B l^2 \pi_B} \; .$$

The denominator is the second order condition for a maximum which we know to be negative. So, the sign of the numerator determines the sign of $dR/d\overline{W}$. Specifically we have:

$$\frac{dR}{d\overline{W}} \gtreqless 0 \qquad \text{as} \qquad U''_A g(1+r)\pi_A \gtreqless U''_B l(1+r)\pi_B$$

$$U''_A \gtreqless \frac{U''_B l(1+r)\pi_B}{g(1+r)\pi_A}$$

$$\frac{U''_A}{U'_A} \gtreqless \frac{U''_B}{U'_B}$$

$$\text{(2-12)} \qquad \frac{dR}{d\overline{W}} \gtreqless 0 \qquad \text{as} \qquad -\frac{U''_A}{U'_A} \gtreqless -\frac{U''_B}{U'_B} \; .$$

The second inequality in equation (2-12) compares the absolute degrees of risk aversion, in the respective states of nature. We may define the relationship between the degrees of absolute risk aversion as follows:

$$\text{if} \quad -\frac{U''_A}{U'_A} < -\frac{U''_B}{U'_B} \qquad \text{decreasing absolute risk aversion}$$

$$-\frac{U''_A}{U'_A} = -\frac{U''_B}{U'_B} \qquad \text{constant absolute risk aversion}$$

$$-\frac{U''_A}{U'_A} > -\frac{U''_B}{U'_B} \qquad \text{increasing absolute risk aversion.}$$

Thus, if our consumer has decreasing absolute risk aversion then an increase in his initial stock of wealth will cause him to increase his absolute holdings of bonds. It is generally assumed that an individual will have decreasing absolute risk aversion[4]. In order to see what proportion this increase in R will take relative to the increase in wealth we must examine the relative degree of risk aversion. Multiply equation (2-11) by $\overline{W}/R$ to obtain:

$$\text{(2-13)} \qquad \frac{dR}{d\overline{W}}\,\frac{\overline{W}}{R} = \frac{-\,U''_A\overline{W}(1+r)g\pi_A + U''_B\overline{W}(1+r)1\pi_B}{U''_A R g^2\pi_A + U''_B R 1^2\pi_B} \quad .$$

We have already determined that equation (2-12) is positive if the consumer has decreasing absolute risk aversion. From equation (2-13) we may determine that:

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[4]Kenneth Arrow, <u>op cit</u>., p. 96.

$$\frac{dR}{d\overline{W}}\,\frac{\overline{W}}{R} \gtreqless 1 \text{ as } U_A^{g''}Rg^2\pi_A + U_B''Rl^2\pi_B \gtreqless -\,U_A''\overline{W}(1+r)g\pi_A + U_B''\overline{W}(1+r)l\pi_B$$

$$U_A''[\overline{W}(1+r) + Rg]g\pi_A \gtreqless U_B''[\overline{W}(1+r) - Rl]l\pi_B$$

$$U_A''W_A \gtreqless U_B''W_B\,\frac{l\pi_B}{g\pi_A}$$

$$\frac{U_A''}{U_A'}\,W_A \gtreqless \frac{U_B''}{U_B'}\,W_B \quad .$$

Hence the following condition holds:

$$\text{(2-14)} \qquad \frac{dR}{d\overline{W}}\,\frac{\overline{W}}{R} \gtreqless 1 \text{ as } -\frac{U_A''}{U_A'}\,W_A \lesseqgtr -\frac{U_B''}{U_B'}\,W_B \quad .$$

The second inequality in equation (2-14) compares the relative degrees of risk aversion in alternative states of nature. As with the absolute degree of risk aversion, a consumer has increasing constant or decreasing relative risk aversion as the relative degree of risk aversion in state A is greater than, equal to, or less than the relative degree of risk aversion in state B. If our consumer has decreasing relative risk aversion then an increase in $\overline{W}$ will increase the optimal holdings of the risky asset R, and the increase in R will be proportionally greater than the increase in $\overline{W}$.

2. Change in Degree of Risk Aversion

Another useful piece of information is how a change in the degree of risk aversion will affect the portfolio choice. First, it is useful to know that the relative degree of risk aversion is equivalent to the reciprocal of the elasticity of substitution of wealth in state A for wealth in state B[5]. In order to examine the effects of a change in the degree of risk aversion we may simplify the problem. Assume that our consumer has a constant degree of relative risk aversion (a constant elasticity of substitution and a decreasing absolute degree of risk aversion)[6]. This allows us to draw indifference curves that are homothetic to the origin. As noted above, the proportion of the risky asset in his portfolio will be constant at all levels of wealth, for budget lines of a given slope.

The effects of a change in the relative degree of risk aversion as shown in figure (2-2). Let π_A/π_B be such that the indifference curves I_1 and I_2 both intersect the certainty line with the slope indicated. Initially the relative degree of risk aversion is assumed to be such that an indifference curve like I_1 will be tangent to the budget line CD at G_1. Now allow the relative degree of risk aversion to decrease (the elasticity of substitution to increase) while holding all other variables constant; in other words allow the consumer to become less risk averse.

The initial indifference curve I_1 will flatten out to say I_2, as the relative degree of risk aversion decreases. At the limit, as the relative degree of risk aversion approaches zero, the indifference

[5]John Greene, ibid., p. 319-32.

[6]Arrow suggests that the appropriate value for a constant relative degree of risk aversion is one, ibid., p. 98.

curve will be linear and have a constant slope equal to $- \pi_A/\pi_B$ (see equation (2-6)): the consumer has become risk neutral).

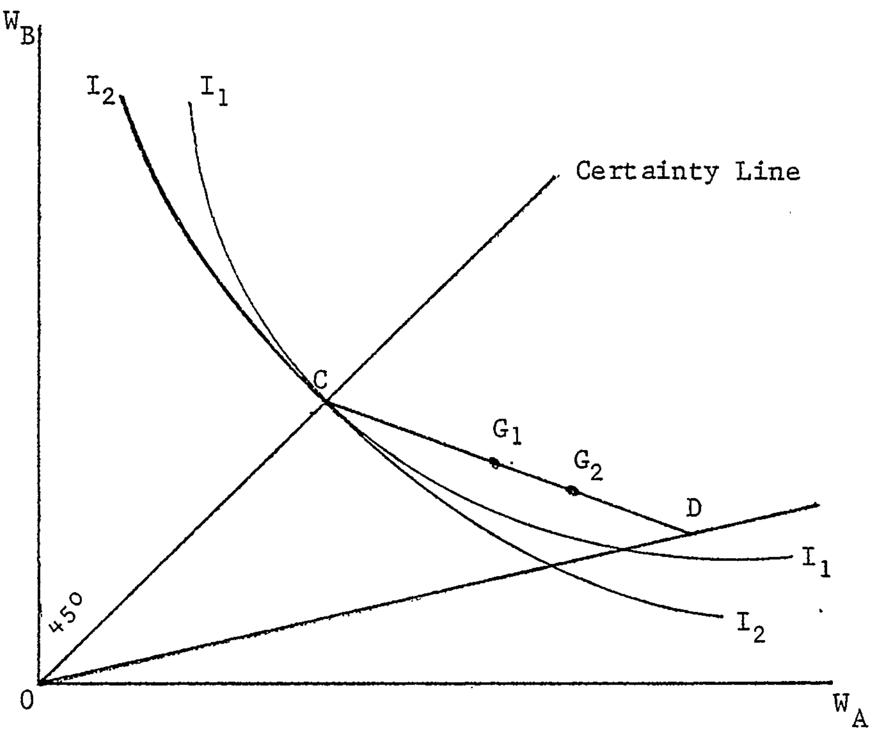

FIGURE (2-2)

In order for the I_2 curve to be tangent to the budget line the ratio W_A/W_B must increase, from that at a point as G_1 to that at a point as G_2. As the elasticity of substitution increases the marginal rate of substitution, dW_A/dW_B on an indifference curve, decreases at all points off the certainty line. A greater increase in the ratio W_A/W_B is necessary to reduce the marginal rate of substitution of an I_2 curve to the slope of the budget line, $- 1/g$, than is necessary for an I_1 curve. Hence, G_2 is further away from the certainty line than G_1.

In general, a decrease in the relative degree of risk aversion (an increase in the elasticity of substitution) will cause an increase in the optimal holdings of the risky asset. This movement is always away from the certainty line.

3. Change in Expectations

Another comparative static result is of interest. Examine the effects of a change in π_A (and π_B, since $\pi_B = 1 - \pi_A$) on the optimal holdings of R[7]. Differentiation of (2-9) with respect to π_A and R yields:

$$[U_A'' g^2 \pi_A + U_B'' 1^2 (1-\pi_A)] dR + [U_A' g + U_B' 1] d\pi_A = 0 \tag{2-15}$$

$$\text{or} \qquad \frac{dR}{d\pi_A} = \frac{-(U_A' g + U_B' 1)}{U_A'' g^2 \pi_A + U_B'' 1^2 (1-\pi_A)} > 0$$

if the individual is risk averse.

A more optimistic level of subjective expectations about future wealth will increase the holdings of the risky asset at the optimum. Note that at the limit as π_A approaches one the uncertainty is eliminated and all assets will be held in R (which is, of course, no longer a risky asset, but yields a positive return equal to r+g).

The results of equation (2-14) are depicted in figure (2-3). In figure (2-3) the slope of an I_1 curve on the certainty line is $-\pi_A/\pi_B$. The slope of an I_2 curve on the certainty line is $-\pi_A'/\pi_B'$, where $\pi_A' > \pi_A$.

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[7]John Greene, ibid., p. 266.

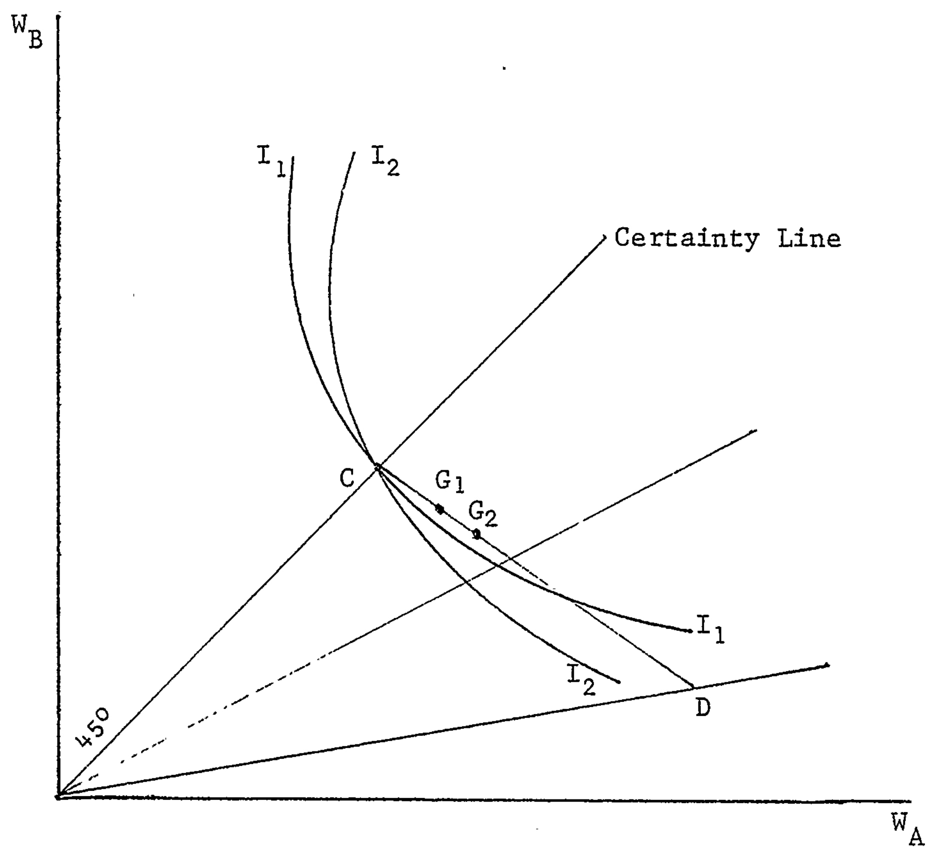

FIGURE (2-3)

From figure (2-3) and equation (2-6) we see that the marginal rate of substitution is less for an I_2 curve than for an I_1 curve everywhere below the certainty line. The result is the same as for an increase in the degree of relative risk aversion: in order for both I_1 and I_2 to be tangent to CD the ratio of W_A/W_B must be smaller at G_1 than at G_2.

In general, we may conclude, that as our consumer becomes more 'optimistic' about the future (E(W) increases) then he will increase his holdings of the risky asset. Conversely if he becomes 'pessimistic' about the future (E(W) decreases) he will decrease his holdings of the risky asset.

C. The Certainty Equivalent and the Risk Premium

Two additional concepts that will be useful in later chapters are the cash or certainty equivalent and the risk premium[8]. Essentially, the certainty equivalent, CE, is an amount such that an individual will be indifferent between receiving this amount with certainty and undertaking a particular gamble $E(\tilde{W})$ (i.e. select CE such that $U(CE) = E[U(\tilde{W})]$). For example, in our two-state model, with a certainty equivalent level of $\overline{W}(1+r)$, the consumer would be indifferent between receiving $\overline{W}(1+r)$ with certainty and taking any gamble along the indifference curve that intersects the certainty line at $\overline{W}(1+r)$. For any particular gamble the value of the certainty equivalent will depend on the amount of risk undertaken (as commonly measured, for example, by the variance of W_A and W_B around the mean $E(\tilde{W})$: the greater the variance the greater the risk) and on the degree of risk aversion.

The risk premium, P, is defined as the difference between the expected value of any given risk (for example, $E(\tilde{W}) = \tilde{W}_A\pi_A + \tilde{W}_B\pi_B$) and the certainty equivalent.

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[8]For a more indepth discussion of these two concepts see Stone, Risk, Return and Equilibrium, The Colonial Press Inc., 1970, p. 12-21 and J.W. Pratt, ibid.

(2-16) $$P = E(\tilde{W}) - CE \quad .$$

From equation (2-16) and the definition of certainty equivalent an individual will be indifferent from receiving $CE = E(\tilde{W}) - P$ with certainty and taking the risk $E(\tilde{W})$. Hence we may write:

(2-17) $$U[E(\tilde{W}) - P] = E[U(\tilde{W})] \quad .$$

Equation (2-17) states that the expected utility for any given risk is equal to the utility of the certainty equivalent for that risk. If the consumer is risk averse it must be the case that

(2-18) $$U([E(\tilde{W})] > E[U(\tilde{W})] \quad .$$

From equations (2-17) and (2-18) we see that for a risk averter the risk premium must be positive ($P > 0$). If the consumer is risk neutral ($U'' = 0$) then the risk premium is zero. Hence, starting from a position of certainty, the risk premium is the amount a risk averter is willing to pay to avoid taking any particular risk.

These concepts are illustrated in figure (2-4). In this graph wealth is measured along the horizontal axis and the total utility of wealth along the vertical axis. $U(W)$ is strictly concave as is required for a risk averter. Assume that for a particular gamble W_a and W_b have the values indicated in figure (2-4). The points G and H indicate the expected utility and wealth levels in state B and state A respectively. The chord between G and H, the straight line GH, indicates the expected wealth and the expected utility of wealth for different subjective probabilities associated with the occurance of either state A or state B. Let the values of π_A and π_B be such that the expected value of the gamble is determined by the point F. The point F yields us the expected value of the gamble $E(\tilde{W})$ as well as the expected utility of the gamble $E[U(\tilde{W})]$. Notice that the utility of the gamble, $U[E(\tilde{W})]$ is greater than the expected utility of the gamble, $E[U(\tilde{W})]$. The point C indicates an amount of wealth, CE, such that the consumer is indifferent between receiving CE with certainty and taking the gamble $E(\tilde{W})$: $U(CE) = E[U(\tilde{W})]$. The difference between $E(\tilde{W})$ and CE is, of course, the risk premium, P.

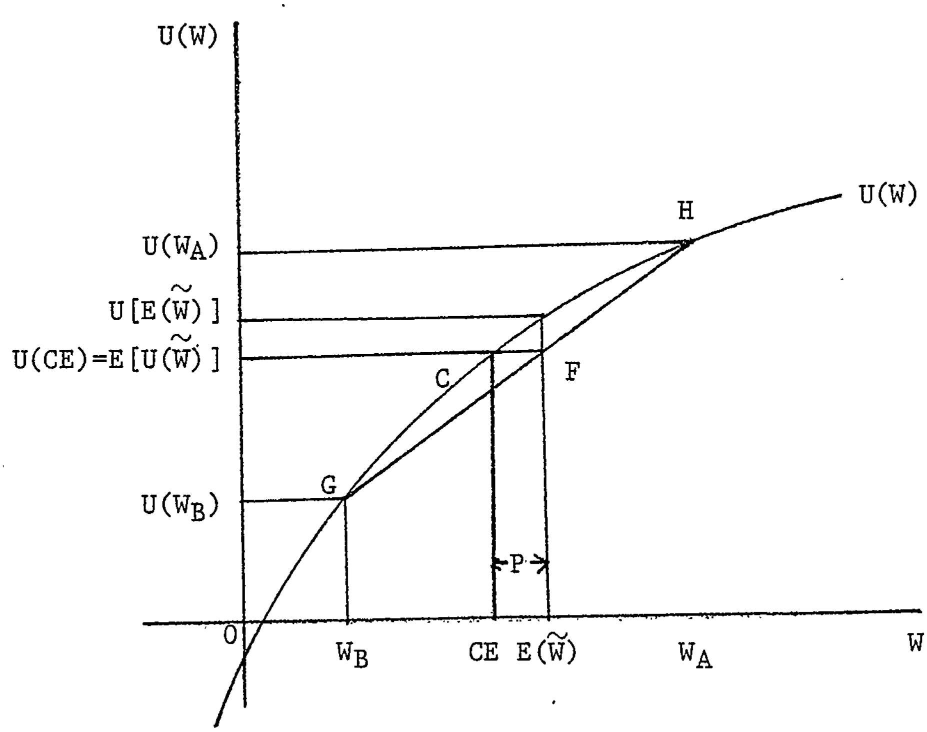

FIGURE (2-4)

We may translate these concepts into our earlier consumer uncertainty model. In figure (2-5) the indifference curve I_1 intersect the certainty line at point C, where R = 0 and future wealth is known with certainty and equal to $\overline{W}(1+r)$. The line CD is the budget constraint. If we allow W_a and W_b to vary but hold E(W) constant and equal to $\overline{W}(1+r)$ we obtain the line CH. This is, of course, an indifference curve for a risk neutral consumer and has a slope equal to $- \pi_A/\pi_B$ and describes points of constant expected wealth. From figure (2-5) we see that the consumer is indifferent between taking any risk along the indifference curve I_1 and receiving $\overline{W}(1+r)$ with certainty. Hence, the certainty equivalent $\overline{W}(1+r)$, is identified with respect to the indifference curve I_1.

The risk premium, P, is the difference between the expected wealth as measured along I_1 and the certainty equivalent, $\overline{W}(1+r)$.

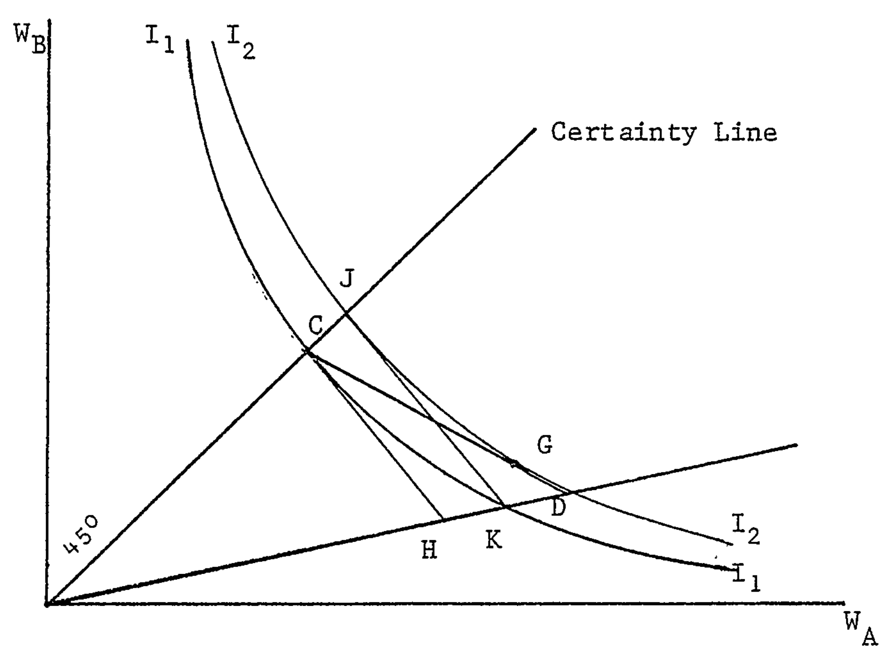

FIGURE (2-5)

Consider the optimal allocation of risk in figure (2-5) as described by the tangency between I_2 and the budget constraint CD. This point is G. The consumer is indifferent between taking any risk described along I_2 and receiving the amount of wealth described at J with certainty (J defines the value of the certainty equivalent for all points on I_2). The line JK describes a constant level of expected wealth equivalent to receiving the amount J with certainty. The line JK is parallel to the line CH. The risk premium, P, as defined at the optimal point G, is equal to the expected value of wealth as evaluated at G less the certainty equivalent as evaluated at J. In one sense, the risk premium at the optimum is a measure of the risk averseness of the individual. It is essentially the 'difference' between the indifference curve for a risk averter and the indifference curve for a risk neutral consumer. This difference may be visualized in figure (2-5) as the "distance" between the constant expected wealth line JK[9] (a constant utility line for a risk neutral consumer) and the indifference curve for a risk averse individual I_2.

It is also important to note that as an individual becomes less risk averse the certainty equivalent increases and the risk premium decreases for any given risk. Furthermore, as the amount of risk undertaken decreases (W_A decreases and W_B increases or π_A rises) then the certainty equivalent increases and the risk premium decreases.

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[9]The constant wealth line JK has a slope of $-\pi_A/\pi_B$ and along JK wealth is constant: $dE(W) = 0$.

Having completed this section we have all the basic tools necessary for developing our undertainty model. Let us proceed, therefore, to Chapter Three where we describe the basic model.

CHAPTER THREE

THE MODEL

In this chapter we will outline a particular uncertainty model based on the State Preference Theory developed in the last chapter. This model is a variation of a model developed by Markusen[1]. The purpose of the model is to yield the characteristics, in the oil industry, of an optimal leasing contract between the government and the firm. A variety of different conditions will be examined. With the help of comparative static analysis we will also be able to consider the type of contract that may be optimal under different stages of development of any particular lease. This analysis will be conducted in the next chapter. For this chapter we will be satisfied with outlining the model and illuminating the characteristics of profit-sharing contracts in the context of oil leases.

Much of the analysis will be done by comparing a general profit-sharing contract with two extreme forms of a profit-sharing contract. In addition we will examine the characteristics of a bonus bid lease arrangement. These four major contract types may be generally defined as follows:

(a) Fixed-Rent Contract: this contract denotes a lease wherein the government receives a fixed-rent per unit of land leased. The firm

[1] James Markusen, Personal and Job Characteristics as Determinants of Employee-Firm Contract Structure, 1977 and 1978, an unpublished manuscript.

receives all the net income from the sale of the oil less the rent paid to the government. This implies that the government assumes no risk and the firm assumes all risk.

(b) Fixed-Profit Contract: this contract arrangement is opposite to a fixed-rent contract. The government 'hires' the firm at a fixed rate per unit of land on the lease in question. The government's revenue will vary depending on the amount of oil produced. The firm's profits are, of course, fixed.

(c) Bonus Bid Contract: in order for the firm to acquire a lease from the government it submits a bid to the government. The bid is a 'front-end' payment, made at the time the lease is acquired. It is assumed that the amount of the bid will be such that the firm will maintain a constant level of utility, given the associated risk. In a very competitive bidding environment the firm would presumably make maximum bids: those which keep its utility level equal to what it would be in the absence of the project[2]. Hence, the firm assumes all the risk and the government no risk. Thus, the bonus bid contract will generally be analogous to a fixed-rent contract at least as far as risk sharing is concerned.

(d) Profit-Sharing Contract: under this contract type the revenue the government receives and the profit the firm receives are both a

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[2]Crommelin, Pearse and Scott, Management of Oil and Gas Resources in Alberta: An Economic Evaluation of Public Policy, Resource Paper 1, June 1976, University of B.C., Department of Economics, p. 22-23; Leland, "Optimal Risk Sharing and the Leasing of Natural Resources, with Application to Oil and Gas Leasing on the OCS," The Quarterly Journal of Economics, August 1978, p. 415-416.

function of the net return from the lease in question. In our model a profit-sharing contract will generally have two components: a fixed payment per unit of land leased and a profit or net royalty tax.

In Alberta, land leased solely for exploration purposes utilizes a fixed-rent contract arrangement but with an option to convert partially to a lease with production rights. However, most land is leased for exploration and production purposes. Usually, there is a royalty tax and a profit tax, as well as a front-end payment[3]. This model will not deal with the optimal size of land that should be leased by the government, however, the acreage of any given lease used for exploration will, in general, tend to be greater than the acreage on a given lease used for production: that is, unproductive acreage will not be held.

The fixed-profit contract and the fixed-rent contract are two extreme forms of the more general profit-sharing contract. Most of the analysis on profit-sharing contracts will focus on a net royalty tax with a fixed-payment. Two other types of profit-sharing contracts will also be considered.

The use of a net royalty tax or profit tax requires some explanation as the government is in the habit of using a gross royalty tax (a gross royalty does not take into account production costs when assessing taxes). The major problem with most royalty schemes is that they use a gross royalty. A gross royalty promotes production inefficiences by the firm: in particular they effect the firm's decisions

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[3]Crommelin, Pearse and Scott, op cit., p. 16-24.

at the margin, which induces early abandonment of a well. Further they will lead to underinvestment in exploration and development activity.

Leland[4] demonstrates that a Pareto optimal production decision will be made by the firm only if the firm is not affected, at the margin, by the leasing contract. Clearly a gross royalty tax will not fulfill this requirement. However, in practice the government does have an influence on the firm's production decisions. In particular the North American governments have exercized price control, established maximum efficient rates of production, introduced market demand prorationing, offered exploration incentives and so on. These tools allow the government some degree of control over the firm's production decisions. Also, many gross royalty schemes in use involve a sliding scale royalty designed to reduce the marginal impact. Accordingly, it would seem that deviations from productive efficiency, due to specific tax equations, have been regarded by governments in North America as being relatively minor.

Hence, the analysis implicitly assumes that any loss in profits, due to changes in the firm's production decisions, is negligible. When we solve for Pareto optimal conditions we will be referring to the optimal allocation of risk between the firm and the government as well as the optimal production decisions by the firm.

In order to get to the issues at hand we will develop an uncertainty model based on the following specifications.

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[4]Leland, op cit., p. 426-431. For a theoretical approach to this problem see Ross, "The Economic Theory of Agency: the Principal's Problem", The American Economic Review, May 1973, p. 134-139.

(a) There exists one government that owns all the mineral rights in question.

(b) There exists one firm that desires access to these particular mineral rights, or a number of firms with identical utility of wealth functions, defined over the possible income from the property concerned.

(c) There is only one variable which affects the level of production: the size of the lease in question L. We may specify the production function as:

$$Q = \alpha F(L) \quad , \tag{3-1}$$

where F has the usual neoclassical properties ($F'(L) > 0$, $F''(L) < 0$) and α allows us to distinguish between alternative states of nature. A production function dependent only on land (and the state of nature) implicitly assumes that there are perfectly fixed proportions between land and capital and labor inputs. Also, the land input is the constraining input on capital and labor. The assumption might be interpreted as implying that each lease requires an exploratory well, and that such a well has fixed capital and labor inputs.

We further assume that the lease in question is on a tract of land of specified size so that $L = \overline{L}$. This also implies given quantities of capital and labor. We will not explicitly consider the optimal quantity of its total land the government should lease in each period; rather, given that the government has decided to lease a particular plot of land, we ask what the optimal lease conditions should be.

Although the lease in question is of a fixed area, the size of the underlying oil deposit is uncertain. Both the firm and the government associate the same size of deposit with the same state of nature: they have identical expectations about the quantity of oil discovered in any given state of nature. For simplicity, we assume there are only two possible states of nature: state A, the 'good' state and state B, the 'bad' state. In other words, the size of the deposit found if state A occurs is larger than the size of the deposit found if state B occurs. We may specify the following quantities for each state of nature:

$$\text{State A} \quad Q_A = \alpha_A F(\overline{L}) \quad \text{occurs with frequency } (\pi_A^g, \pi_A^f)$$

$$\text{State B} \quad Q_B = \alpha_B F(\overline{L}) \quad \text{occurs with frequency } (\pi_B^g, \pi_B^f) \quad .$$

The π's are the subjective probabilities of either state A or B occuring, where the superscripts f and g denote the probabilities as preceived by the firm and the government respectively.

(d) We assume that the net price p_i (net of production costs but not lease payments in state i) is determined by the state of nature. The well head price of oil is assumed to be fixed and known with certainty. The difference in p_A and p_B will reflect production cost differences between the two states of nature. Consistent with our assumption that state A is the preferred state, net income in state A is greater than net income in state B: $p_A\alpha_A F(\overline{L}) > p_B\alpha_B F(\overline{L})$. In the event of a dry well in state B it must be the case that $p_B\alpha_B F(\overline{L}) < 0$. We will, however, assume that both $p_A\alpha_A F(\overline{L})$ and $p_B\alpha_B F(\overline{L})$ are positive. For simplicity of notation let $E(V) = E[p\alpha F(\overline{L})]$. We may write the expected net income from the lease in question from the government's and the firm's point of view as:

$$E(V^g) = p_A\alpha_A F(\bar{L})\pi_A^g + p_B\alpha_B F(\bar{L})\pi_B^g$$
$$= V_A\pi_A^g + V_B\pi_B^g$$

(3-2) $$E(V^f) = p_A\alpha_A F(\bar{L})\pi_A^f + p_B\alpha_B F(\bar{L})\pi_B^g$$
$$= V_A\pi_A^f + V_B\pi_B^f \quad .$$

Note that $E(V^g) = E(V^f)$ if the firm and the government have symmatric expectations ($\pi_i^g = \pi_i^f$). Also, the government and the firm perceive the same net income from the lease in each alternative state: V_i is the same for both the firm and the government.

The net expected value of the lease may be different for the government and the firm if the subjective probabilities attached to each state of nature by the firm and the government differ (recall that both the firm and the government preceive the same V for each alternative state of nature). The case of asymmetric expectations ($\pi_i^g \neq \pi_i^f$) will not be dealt with until Chapter Five. Rather, the analysis in this chapter and in Chapter Four will be restricted to the case of symmetric expectations ($\pi_i^g = \pi_i^f$).

(e) The firm maximizes a utility of profit function written as:

(3-3) $$W = W(\phi); \qquad W' > 0$$

where W is the total utility of the firm and ϕ denotes profits. In the event of uncertainty the firm acts to maximize the expected utility value, $E[W(\phi)] = \Sigma W_i\pi_i$ for states of nature i.

(f) Similarly, the government maximizes the total utility of revenue.

(3-4) $$U = U(R); \qquad U' > 0$$

where U is the total government utility and R is revenue. Under conditions of uncertainty the government maximizes total expected utility, $E[U(R)] = \Sigma U_i \pi_i$ for states of nature i.

If the firm and the government are risk averse then $W'' < 0$ and $U'' < 0$.

(g) By definition the income from the lease in the alternative states of nature must be captured by either the firm or the government. That is:

$$R_A + \phi_A = V_A$$

and

$$R_B + \phi_B = V_B$$

where R_i and ϕ_i denote government revenue and firm profits and V_i total 'economic rent' in state i.

With the above information we may give specific functional form to our contract types.

Fixed-Rent Contract

(3-5) $$E(R_s) = R_s = r_s\overline{L}$$

$$E(\phi_s) = (V_a - r_s\overline{L})\pi^f_A + (V_b - r_s\overline{L})\pi^f_B \quad .$$

The unit rental rate r_s determines the fixed level of revenue for the government R_s. The expected value of firm profits is denoted by $E(\phi_s)$.

Fixed-Profit Contract

(3-6)
$$E(R_f) = (V_A - r_f\overline{L})\pi_A^g + (V_B - r_f\overline{L})\pi_B^g$$
$$E(\phi_f) = \phi_f = r_f\overline{L}$$

where the subscript f denotes a fixed-profit contract.

Bonus Bid Contract

Assume that the firm maintains some fixed level of utility, $\overline{W}$. Then, the bonus bid made by the firm will represent the amount the firm is willing to pay for the gamble E(V) rather than not have it. If the firm is risk neutral, then the bid, $r_n\overline{L}$, made by the firm will equal the certainty equivalent necessary to maintain $\overline{W}$ (the risk premium is zero for any given risk). So we may write:

(3-7)
$$E(R_n) = r_n\overline{L}$$
$$E(\phi_n) = \overline{\phi}_n = (V_A - r_n\overline{L})\pi_A^f + (V_B - r_n\overline{L})\pi_B^f$$

where $\overline{\phi}_n$ is the fixed level of profits that a risk neutral firm will maintain under a bonus bid scheme. Notice that the firm will submit a bid $r_n\overline{L}$, such that all its profits are bid away up to the point where profits equal $\overline{\phi}_n$.

If, however, the firm is risk averse, then in order to maintain a constant level of utility, $E[W(\phi)] = \overline{W}$ the bid made by the firm will equal its certainty equivalent (the amount of profit the firm can expect to receive with certainty, necessary to maintain a level of utility equal to $\overline{W}$) less the risk premium for any associated risk. Hence, the bid made by a risk neutral firm will be greater than the bid made by a risk averse firm: the difference between the two bids being the risk premium. The concept of a bonus bid contract will be discussed in more detail as we proceed with our analysis.

Profit-Sharing Contract

$$(3\text{-}8) \qquad E(R_p) = [\beta(V_A - \gamma r_p\overline{L})]\pi_A^g + [\beta(V_B - \gamma r_p\overline{L})]\pi_B^g + r_p\overline{L}$$

$$E(\phi_p) = [(1 - \beta)V_A]\pi_A^f + [(1 - \beta)V_B]\pi_B^f - (1 - \gamma\beta)r_p\overline{L}$$

where the subscript, p, denotes a profit-sharing contract. The government tax rate is denoted by β, where $0 \leq \beta \leq 1$; the tax rate is constrained by being no less than zero per cent and no more than 100 per cent. The inclusion of γ allows us to alter the payment schedule by making the fixed payment, $r_p\overline{L}$, deductable from, non-deductable from, or added to income before assessing taxes. Specifically, we will examine the above three cases by allowing γ to take on the values one, zero, and negative one.

Notice that the fixed-rent contract and the fixed-profit contract are two extreme cases of the more general profit-sharing.

The fixed-rent contract is obtained from equation (3-8) by setting $\beta = 0$ and $r_p = r_s > 0$. The fixed-profit contract is obtained by setting $\beta = 1$, $r_p = r_f < 0$, and $\gamma = 0$. If $r_p = 0$ then we have a 'pure' profit-sharing contract. Otherwise there is a rental payment made to the government by the firm ($r_p > 0$), or a rental payment made to the firm by the government ($r_p < 0$).

Risk in this model is defined in terms of income variability. Holding the expected value of the lease constant, expected government revenue constant and expected firm profit constant, if, for example, R_A increases and R_B decreases then income variability increases for the government (and decreases for the firm since ϕ_A must decrease and ϕ_B must increase). Hence, the amount of risk undertaken by the government has increased (and the amount undertaken by the firm has decreased). As we move from a fixed-rent contract to a fixed-profit contract (β increases and r_p decreases) the variability of government revenue increases and the variability of firm profit decreases. At the one extreme, when the government bears no risk and the firm bears all risk, we are restricted to a fixed-rent contract (or a bonus bid contract). At the other extreme, a fixed-profit contract, the firm bears no risk and the government bears all risk. Between these two extremes lies the set of profit-sharing contracts, where risk is shared by the firm and the government.

Let us now examine in detail three profit-sharing contracts, as defined by assigning γ the value one, negative one and zero.

(a) Assign γ the value one and rewrite equation (3-8) to read:

$$(3\text{-}9) \qquad E(R_p) = \beta(V_A - r_p\overline{L})\pi_A^g + \beta(V_B - r_p\overline{L})\pi_B^g + r_p\overline{L}$$

$$E(\phi_p) = (1 - \beta)(V_A - r_p\overline{L})\pi_A^f + (1 - \beta)(V_B - r_p\overline{L})\pi_B^f \quad .$$

In this instance we see that the fixed payment $r_p\overline{L}$ is deductable from income before assessing taxes. If, for example, r_p is positive then the government receives the fixed-rental payment plus a percentage of net income as determined by the tax rate β.

As mentioned previously we will assume that the firm and the government have symmetric expectations about alternative states of nature ($\pi_i^f = \pi_i^g$). For the moment also assume that both the government and the firm are risk neutral (W" and U" = 0). Looking at the government first, differentiate equation (3-9) with respect to r_p and β, while holding expected revenue constant. This yields:

$$\text{(3-10)} \qquad dE(R_p) = E(V - r_p\bar{L})\,d\beta + (1 - \beta)\bar{L}dr_p = 0$$

$$\Rightarrow \left(\frac{dr_p}{d\beta}\right)^g = - \frac{E(V - r_p\bar{L})}{(1 - \beta)\bar{L}} < 0$$

assuming the numerator is positive and β is not equal to one.

From the above expression we see that the marginal rate of substitution of r_p for β is negative. In other words, for expected government revenue to remain constant, the unit rental rate, r_p, must decrease as the tax rate, β, increases. At the limit, as β approaches one, $(dr_p/d\beta)^g$ approaches infinity. This implies that the government will never impose a one hundred per cent tax rate under this contract type.

Upon further differentiation of equation (3-10) we obtain

$$\text{(3-11)} \qquad \left(\frac{d^2 r_p}{d\beta^2}\right)^g = - \frac{E(V - r_p\bar{L})}{(1 - \beta)^2\bar{L}} < 0 \quad .$$

The relationship between r_p and β as described in equations (3-10) and (3-11) is, in fact, an indifference curve for a risk neutral government. Pictured in figure (3-1), the curve r_sX defines the locus of points along which the risk neutral government has constant utility. Assigning r_p an initial value equal to r_s and setting $\beta = 0$, we see that along r_sX the government is indifferent between various profit-sharing contracts with different rental rates and $0 < \beta < 1$ and a fixed-rent contract with rental payment equal to $r_s\overline{L}$. The pure profit-sharing contract is located at the point T.

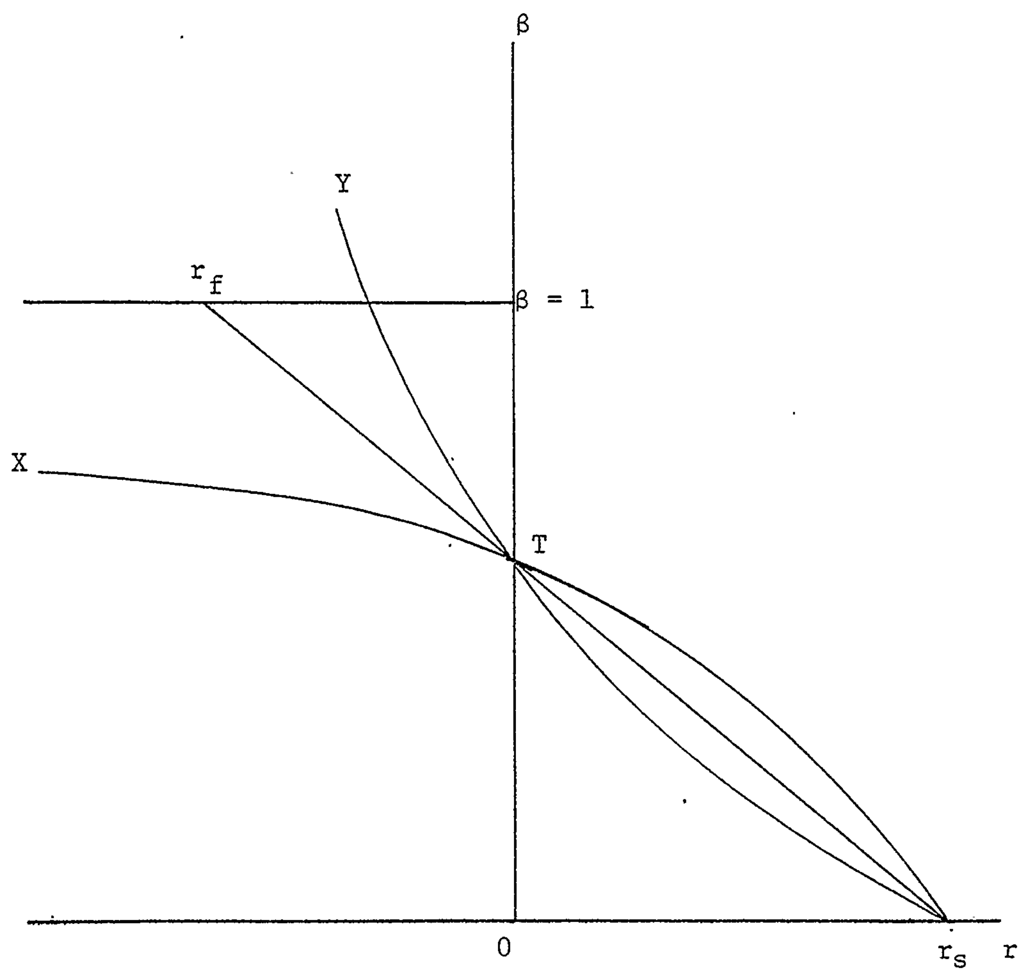

FIGURE (3-1)

Performing the same calculations for the firm we obtain, from equation (3-9):

$$\left(\frac{dr_p}{d\beta}\right)^f = -\frac{E(V - r_p\overline{L})}{(1 - \beta)\overline{L}} < 0 \tag{3-12}$$

and

$$\left(\frac{d^2r_p}{d\beta^2}\right)^f = -\frac{E(V - r_p\overline{L})}{(1 - \beta)^2\,\overline{L}} < 0 \quad . \tag{3-13}$$

If we compare equation (3-10) with (3-12) and (3-11) with (3-13) we see that for a risk neutral firm and a risk neutral government:

$$\left(\frac{dr_p}{d\beta}\right)^g = \left(\frac{dr_p}{d\beta}\right)^f \qquad \text{and} \qquad \left(\frac{d^2r_p}{d\beta^2}\right)^g = \left(\frac{d^2r_p}{d\beta^2}\right)^f \quad .$$

Thus, given an initial fixed-rent contract with $r_p = r_s$ and $\beta = 0$ the curve r_xX describes the locus of points along which a risk neutral firm and a risk neutral government have constant utility and constant income.

(b) If we wish to add the fixed payment onto the taxable income we set $\gamma < 0$. In this case we will let $\gamma = -1$. In this instance the fixed payment is taxable in addition to all pre-rent profits earned by the firm. In effect, rents are taxed twice. We may rewrite equation (3-9) to read:

$$E(R_p) = E[\beta(V + r_p\overline{L})] + r_p\overline{L} \tag{3-14}$$

$$E(\phi_p) = E[(1 - \beta)(V)] - (1 + \beta)r_p\overline{L} \quad .$$

From equation (3-14) if, for example, r_p is greater than zero then the fixed payment made to the government is added to corporate profits before taxes are levied. As done in case (a) differentiate equation (3-14) with respect to r_p and β while holding expected government revenue constant. This yields:

$$\left(\frac{dr_p}{d\beta}\right)^g = - \frac{E(V + r_p\overline{L})}{(1 + \beta)\overline{L}} < 0 \tag{3-15}$$

and upon further differentiation

$$\left(\frac{d^2 r_p}{d\beta}\right)^g = \frac{E(V + r_p\overline{L})}{(1 + \beta)^2\overline{L}} > 0 \quad . \tag{3-16}$$

The government indifference curve resulting from equations (3-15) and (3-16) is described in figure (3-1) by the curve r_sY (given an initial fixed-rent contract where $r_p = r_s$ and $\beta = 0$). Like the previous case the MRS of r_p for β is negative. However, the rate of change of the government's MRS is positive. This is due, of course, to the fact that the fixed payment $r_p\overline{L}$ is added to income before assessing taxes.

Deriving the firm's indifference curve from equation (3-14) yields:

$$\left(\frac{dr_p}{d\beta}\right)^f = - \frac{E(V + r_p\overline{L})}{(1 + \beta)\overline{L}} < 0 \tag{3-17}$$

and

(3-18) $$\left(\frac{d^2 r_p}{d\beta^2}\right)^f = \frac{E(V + r_p\overline{L})}{(1+\beta)^2\overline{L}} > 0 \quad .$$

Hence, in case (b), as in case (a), if both the firm and the government are risk neutral and have symmetric expectations then their indifference curves will be the same, given initial values for $r_p = r_s$ and $\beta = 0$.

(c) In the final case the rental payment is neither deductable from or added to income before levying taxes. We set γ equal to zero. This case is the simplest of the three cases. For a risk neutral firm and government we shall see the MRS will, in fact, be constant for all values of r_p and β. From equation (3-9) this case is described by:

(3-19) $$E(R_p) = E(\beta V) + r_p\overline{L}$$

$$E(\phi_p) = E[(1-\beta)V] - r_p\overline{L} \quad .$$

If r_p is positive the government receives a percentage of corporate income plus a fixed payment. The fixed payment is not deductable from firm profit when calculating taxes. This case approximates the situation for petroleum production in Canada since the November 1974, federal budget.

We may describe the indifference curves for a risk neutral firm and government with symmetrical expectations as:

(3-20) $$\left(\frac{dr_p}{d\beta}\right)^g = -\frac{E(V)}{\overline{L}} = \left(\frac{dr_p}{d\beta}\right)^g < 0$$

and

$$\left(\frac{d^2 r_p}{d\beta^2}\right)^g = 0 = \left(\frac{d^2 r_p}{d\beta^2}\right)^f \quad . \tag{3-21}$$

This linear relationship between r_p and β is depicted in figure (3-1) as the straight line $r_s r_f$. Along the line $r_s r_f$ expected revenue and expected profits are constant. It describes the profit-sharing contract, fixed-rent contract and fixed-profit contract between which the firm and government are indifferent.

Furthermore, if the government is utilizing a bonus bid scheme with a royalty tax then for any value of β along $r_s r_f$ the corresponding value of r_p, in figure (3-1) will represent the bonus bid the firm will make for that value of β.

On particularly important result that is derived from this analysis is summarized in the following proposition:

<u>Proposition 1</u>

> If the firm and the government are risk neutral and have symmetric expectations about alternative states of nature then for any given type of profit-sharing contract, points of constant government revenue and constant company profit will be identified by the same curve (where $E(V) = E(R) + E(\phi)$).

If we move along any curve in figure (3-1) from the point where $r_p = r_s$ and $\beta = 0$ we see that as r_p decreases β increases until at the point T, $r_p = 0$. Any profit-sharing contract where $r_p = 0$ and $0 < \beta < 1$ is termed a pure profit-sharing contract. The set of all pure profit-sharing contracts divide risk evenly between the firm and the government. However, this does not imply an equal division of expected income or, of course, equal utility levels. This is determined by the tax rate β. In order for the tax rate to reach higher levels than indicated by T, the government must, in a sense, 'hire' the firm.

Consider an increase in β for any pure profit-sharing contract ($r_p = 0$). This will cause R_A and R_B to increase and ϕ_A and ϕ_B to decrease. This implies that government utility has increased and that firm utility has decreased, but that the allocation of risk has not changed. In a sense, the fixed-payment is equivalent to the safe asset used in the previous model (in Chapter Two). The fixed payment is made or paid with certainty. Of course, the tax rate used in this model is not directly analogous to the risky asset used in Chapter Two. In the previous model an increase in the holdings of the risky asset implied a decrease in the holdings of the safe asset as well as an increase in W_A and a decrease in W_B. However, in this model, there is no such trade-off between the 'safe' asset and the 'risky' asset.

Note that the points along $\beta = 0$ are identical to the certainty line for the government, and points along $\beta = 1$ are identical to the certainty line for the firm. When $r_p = 0$ neither the government nor the firm are holding any 'safe' assets. Hence, as stated, any pure profit-sharing contract defines points of equal risk sharing, but not necessarily points of equal expected income or equal utility levels.

In fact, as will be proved in the next chapter, if the firm and the government are equally risk averse then the optimal contract will be a pure profit-sharing contract, regardless of the utility levels that the government and the firm are able to maintain.

As r_p decreases and β increases, along any constant revenue (profit) line, income variability is increased for the government and decreased for the firm. The amount of risk borne by either party is, of course, positively correlated to income variability. Thus, if r_p is positive more risk is carried by the firm, and if r_p is negative more risk is carried by the government.

For the rest of this analysis we will focus on case (c), with the fixed payment non-deductable[5]. Now let us derive the indifference curves for a risk averse firm and government. Rewriting equation (3-19) we have:

$$\text{(3-22)} \qquad E(U) = E[U(\beta V + r_p\bar{L})]; \qquad U'' < 0$$

$$E(W) = E[W((1 - \beta)V - r_p\bar{L})]; \qquad W'' < 0 \quad .$$

First solving for the government's indifference curve, differentiate equation (3-22) with respect to r_p and β while holding expected government utility constant.

$$\text{(3-23)} \qquad dE(U) = E(U'V)d\beta + E(U')Ldr_p = 0$$

$$\left(\frac{dr_p}{d\beta}\right)^g = -\frac{E(U'V)}{E(U')L} < 0 \quad .$$

[5]see Leland, <u>ibid</u>., p. 418-420 for certain restrictions on the utility functions that are given in equation (3-22).

From the above equation we see that, as in the risk neutral case, a decrease in r_p must be met by an increase in β in order for government utility to remain constant. Further differentiation of equation (3-23) yields:

$$\text{(3-24)} \qquad \left(\frac{d^2 r_p}{d\beta^2}\right)^g = - \frac{\pi_A^g \pi_B^g (V_A - V_B)^2 (U_A'' U_B'^2 \pi_B^g + U_B'' U_A'^2 \pi_A^g)}{E(U')^3 \bar{L}} > 0 \quad .$$

The relationship between r_p and β for a risk averse government is drawn in figure (3-2) as the curve $r_s r_f'$. It shows the locus of points along which expected government utility under a profit-sharing contract is equal to government utility under a particular fixed-rent contract ($r_p = r_s$, $\beta = 0$) or is equal to expected government utility under a particular fixed-profit contract ($r_p = r_f'$, $\beta = 1$).

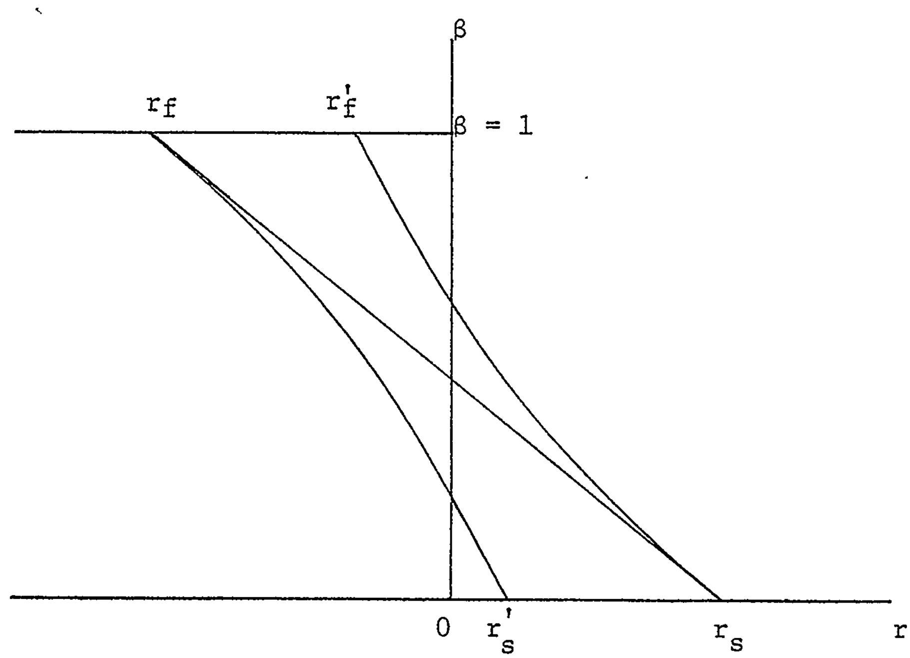

FIGURE (3-2)

If we compare the MRS for a risk neutral government with the MRS for a risk averse government we see that:

$$\left(\frac{dr_p}{d\beta}\right)^g_{\text{RISK AVERSE}} \leq \left(\frac{dr_p}{d\beta}\right)^g_{\text{RISK NEUTRAL}}$$

since U is strictly concave by definition of a risk averter. The strict inequality holds for all values of β, where β is greater than zero. The indifference curves for a risk neutral government and a risk averse government will be tangent on the certainty line ($\beta = 0$). Hence, for all points where β is greater than zero the slope of the curve $r_s r_f'$, in figure (3-2) will be greater than the slope of the line $r_s r_f$ for any given tax rate β. Notice that in order for a risk averse government to remain indifferent between receiving $r_s\overline{L}$ with certainty and taking any given risk ($r_p > r_s$ and $0 < \beta \leq 1$) the expected revenue must be higher; revenue will increase at all points as r_p decreases and β increases along the curve $r_s r_f'$. The horizontal distance between $r_s r_f$ and $r_s r_f'$ is a measure of the amount a risk averse government is willing to pay (receive) for any $\beta > 0$ in order to avoid (assume) the associated risk. In fact, this value is the government's risk premium associated with any given risk.

Now let us apply the above methodology to derive the indifference curve for a risk averse firm. First solve for the tax conditions which yield a constant utility for the firm. Equation (3-19) yields the following marginal rate of substitution for the firm:

$$(3\text{-}25) \qquad \left(\frac{dr_p}{d\beta}\right)^f = -\frac{E(W'V)}{E(W')\overline{L}} < 0$$

and upon further differentiation

$$(3\text{-}26) \qquad \left(\frac{d^2 r_p}{d\,\beta^2}\right)^f = \frac{\pi_A^f \pi_B^f (V_A - V_B)^2 (W_A'' W_B'^2 \pi_B^f + W_B'' W_A'^2 \pi_A^f)}{E(W')^3 \overline{L}} < 0 \quad .$$

We immediately know from equation (3-25) that for any given value of β the MRS for a risk averse firm is greater than the MRS for a risk neutral firm. This is due to the fact that W is strictly concave. However, on the firm's certainty line ($\beta = 1$) these two indifference curves will be tangent.

The relationship between r_p and β as described by equations (3-25) and (3-26) is represented in figure (3-2) as the curve $r_s' r_f$. Along the curve $r_s' r_f$ the firm maintains a level of utility equal to receiving $r_f \overline{L}$ with certainty. In order for a risk averse firm to maintain $\overline{W} = r_f \overline{L}$ as it bears more and more risk (r_p increases and β decreases) the firm's expected profits must increase. The difference between the certainty equivalent, $r_f \overline{L}$, and the expected profits for any given tax rate along the curve $r_s' r_f$ is equal to the firm's risk premium for that given risk. That is the horizontal distance between $r_s r_f$ and $r_s' r_f$.

Further, if r_p is positive, then the locus of points along $r_s' r_f$ describe the bonus bid the firm would make for any given tax rate, assuming there is no rental. If there is a rental payment, the bonus bid would be the difference between this amount and the rental. For any given royalty rate the firm will bid just enough to maintain a level of utility equal to receiving $r_f\bar{L}$ with certainty.

Having completed our discussion about the specifications and characteristics of our model, we are now ready to solve for the optimal contract. This will be accomplished in the next chapter.

CHAPTER FOUR

CHARACTERISTICS OF AN OPTIMAL CONTRACT

In this chapter we solve for the Pareto Optimal allocation of risk between the firm and the government. This analysis will be performed in the first section. In the second section we analyze the conditions under which the fixed payment is received by either the firm or the government, or is zero. Comparative static analysis is done in the third section. We are particularly interested in the changes in the optimal contract resulting from a change in the expected value of the lease. Throughout this chapter we assume that the firm and the government have symmetric expectations about alternative states of nature ($\pi_i^g = \pi_i^f$). The case of asymmetric expectations ($\pi_i^g \neq \pi_i^f$) will be dealt with in the fifth chapter.

A. Optimal Characteristics

We wish to maximize government utility while constraining the utility level of the firm. Let us first consider the case where the government is risk averse but the firm is risk neutral. For this section and the remainder of this thesis we will use the linear profit-sharing equation: the net royalty contract with a fixed payment.

The maximization problem may be formulated as follows:

(4-1) $\quad$ Max $E(U)$ subject to $E(\phi) = \overline{\phi}$; $U'' < 0$; $W'' = 0$; $\pi_i^g = \pi_i^f$.

Notice that we are constraining the firm's profits to $\overline{\phi}$, which is an amount that the firm receives with certainty. Hence, the contract that identifies, $\overline{\phi}$, is a fixed-profit contract.

Form the Lagrange equation, T, from equation (4-1):

$$T = U_A[\beta V_A + r_p\overline{L}]\pi_A^g + U_B[\beta V_B + r_p\overline{L}]\pi_B^g \tag{4-2}$$

$$+ \lambda[(1-\beta)V_A\pi_A^f + (1-\beta)V_B\pi_B^f - r_p\overline{L} - \overline{\phi}] \quad .$$

Solving for the first order conditions for a maximum yields:

$$\frac{\partial T}{\partial \beta} = U_A'V_A\pi_A^g + U_B'V_B\pi_B^g - \lambda[V_A\pi_A^f + V_B\pi_B^f] = 0$$

$$\frac{\partial T}{\partial r_p} = U_A'\overline{L}\pi_A^g + U_B'\overline{L}\pi_B^g - \lambda\overline{L} = 0 \tag{4-3}$$

$$\frac{\partial T}{\partial \lambda} = \phi_A\pi_A^f + \phi_B\pi_B^f - \overline{\phi} = 0 \quad .$$

Dividing the first two equations of (4-3) we obtain:

$$\frac{U_A'}{U_B'} = 1 \quad . \tag{4-4}$$

Equation (4-4) states that the government's marginal utility in state A must equal the marginal utility in state B, at the constrained optimum. Or in other words, the government's marginal rate of substitution of revenue in state A for revenue in state B must equal one at the optimum. This condition will be satisfied only if $R_A = R_B$. Hence, the optimal contract will be either a fixed-rent contract or a bonus bid contract. At this optimal contract the firm bears all risk and the government bears no risk. These results are summarized in the following proposition:

<u>Proposition 2</u>

> Assume (a) the firm is risk neutral and the government is risk averse and (b) they have symmetric expectations about alternative states of nature. With these conditions then at the optimum any profit-sharing contract is Pareto inferior to some fixed-rent contract or bonus bid contract.

The above proposition is well known in risk theory. If one agent is risk neutral and the other agent is risk averse then the risk neutral agent should bear all risk. The results in Proposition 1 will be reversed if the firm is risk averse and the government is risk neutral.

The more interesting case is where both the government and the firm are risk averse. Accordingly we may reformulate the problem in equation (4-1).

(4-5) $\quad$ Max $E(U)$ subject to $E(W) = \overline{W}$; U'' and $W'' < 0$; $\pi_i^g = \pi_i^f$.

Form the Lagrange function:

$$J = U_A[\beta V_A + r_p\overline{L}]\pi_A^g + U_B[\beta V_B + r_p\overline{L}]\pi_B^g + \lambda[W_A\left((1-\beta)V_A - r_p\overline{L}\right)\pi_A^f + W_B\left((1-\beta)V_B - r_p\overline{L}\right) - \overline{W}] \quad (4\text{-}6)$$

Solving equation (4-6) for the first-order conditions we obtain:

$$\frac{\partial J}{\partial \beta} = E[U'V] - \lambda E[W'V] = 0$$

(4-7) $$\frac{\partial J}{\partial r_p} = E[U']L - \lambda E[W']L = 0$$

$$\frac{\partial J}{\partial \lambda} = W_A(\phi_A)\pi_A^f + W_B(\phi_B)\pi_B^f - \overline{W} = 0 \quad .$$

Dividing and expanding the terms in the first two equations of (4-7) we obtain:

(4-8) $$\frac{U_A'}{U_B'} = \frac{W_A'}{W_B'} \quad .$$

Equation (4-8) states that at the optimal contract the marginal rates of substitution between income in the two states of nature will be equal. This implies that the indifference curves for the firm and the government will be tangent at the optimal contract. Furthermore, since the MRS for the firm and the government equal one only on their respective certainty lines it must be the case that $R_A > R_B$ and $\phi_A > \phi_B$ at the optimal contract.

The second order condition for a constrained optimum is that the relevant boardered Hessian determinant be positive:

(4-9) $$\begin{vmatrix} E[U''V^2] + \lambda E[W''V^2] & E[U'']\overline{L} + \lambda E[W'']\overline{L} & -E[W'V] \\ E[U''V] + \lambda E[W''V] & E[U'']\overline{L} + \lambda E[W'']\overline{L} & -E[W'] \\ -E[W'V] & -E[W']\overline{L} & 0 \end{vmatrix} > 0 \quad .$$

Expanding equation (4-9) we obtain:

$$(4\text{-}10) \qquad - \pi_A\pi_B(V_A-V_B)^2\bar{L}[U_A''W_B'^2\pi_B + U_B''W_A'\pi_A$$

$$+ W_A''U_B'W_B'\pi_B + W_B''U_A'W_A'\pi_A] > 0$$

or

$$- \pi_A\pi_B(V_A-V_B)^2\bar{L}U_A'W_B'\left[\frac{U_A''}{U_A'}W_B'\pi_B + \frac{U_B''}{U_B'}W_A'\pi_A\right.$$

$$\left.+ \frac{W_A''}{W_A'}W_B'\pi_B + \frac{W_B''}{W_B'}W_A'\pi_A\right] > 0$$

where (4-10) is strictly positive if the firm and the government are risk averse (U" and W" are negative).

The above results are summarized in the following proposition:

Proposition 3

> If (a) the firm and the government are risk averse and (b) they have symmetric expectations about alternative states of nature then any fixed-rent contract or fixed-profit contract is Pareto inferior to some profit-sharing contract.

Like Proposition 2, Proposition 3 has a basis in risk theory. It is well known that if both agents are risk averse then the optimal contract will involve risk sharing by both agents.

The graphic solution to the optimal contract implied by Proposition 3 is indicated at point C in figure (4-1).

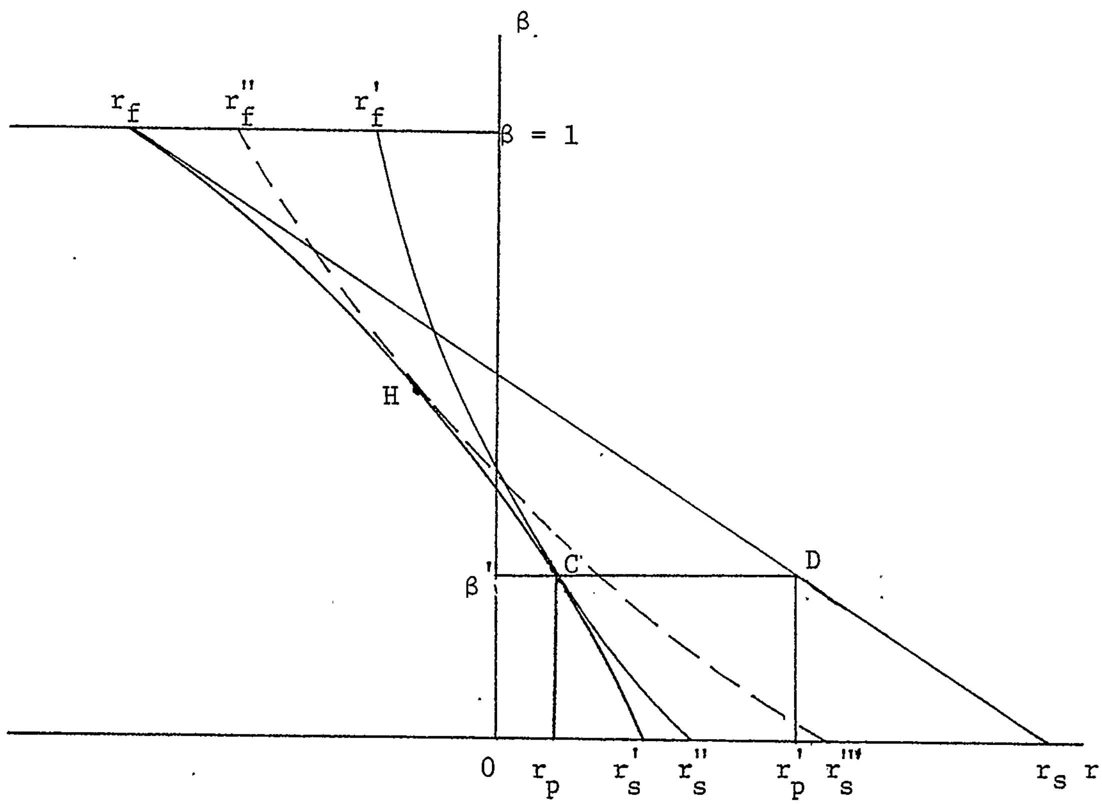

FIGURE (4-1)

There are several interesting features about the particular representation depicted in figure (4-1). First we notice that at the optimal contract, as depicted by the tangency between the firm's $(r_s' r_f)$ and government's $(r_s'' r_f')$ indifference curves, point C, the fixed-payment, r_p, is positive. In other words the government receives a fixed sum from the firm. This fixed sum could be either a rental payment or a bonus bid. We see that the firm is constrained to an expected utility level equivalent to $r_f\overline{L}$ with certainty: the government is constrained to points along the firm's indifference curve $r_s'r_f$. This allows the government to maximize its utility at the point C. Note that the point C gives the government a level of expected utility equivalent to receiving the amount $r_s'\overline{L}$ with certainty. Furthermore, the expected revenue the government receives when the firm is risk averse is less than the certain revenue the government receives when the firm is risk neutral (total expected revenue at point C is less than total revenue at point $r_p = r_s$ and $\beta = 0$).

We see that at the optimal rate of taxation, β', the fixed payment is equal to $r_p\overline{L}$. If the government was using a bonus bid payment then the bonus bid made by the firm for this level of taxes is equal to the firm's certainty equivalent $r_f\overline{L}$, less the firm's risk premium, P^f, at this level of risk ($P^f = (r_p' - r_p)\overline{L}$).

B. Determining the Recipient of the Fixed Payment

One issue we have not yet dealt with is whether the sign of r_p is greater than, less than or equal to zero. A casual inspection of figure (4-1) shows that as the government becomes less risk averse relative to the firm (and holding the firm's utility fixed at $r_s'r_f$) then the optimal contract will move up along the firm's utility curve $r_s'r_f$. In other words, if the government becomes less risk averse, at the old optimum C, $U_A'/U_B' < W_A'/W_B'$. In order for the firm's MRS of r_p for β and the government's MRS of r_p for β to be equal, the ratio r_p/β must decrease from point C so that the firm's MRS will decrease and the government's MRS will increase. Such a new optimal point is H in figure (4-1), where the firm's indifference curve is tangent to the government's new indifference curve $r_s'''r_f''$. Furthermore we see that as the government becomes less risk averse its expected revenue, at the optimal contract, will increase (expected revenue at H is greater than expected revenue at C).

Before proving this more formally we may advance the following proposition:

Proposition 4

> If (a) the firm and the government are risk averse, (b) they have symmetric expectations about alternative states of nature and (c) they have constant relative risk aversion then the fixed payment, $r_p\overline{L}$, will be zero, paid to the government or paid to the firm as the firm and the government are equally risk averse, the goverment is more risk averse, or the government is less risk averse.

This proposition has been demonstrated by Stiglitz[1]. We will present a somewhat different proof. This proof requires an alternative approach to the one we have used so far in this paper. We may identify an expected government utility function and an expected firm utility function:

[1]Stiglitz, Joseph; "Incentives and Risk Sharing in Sharecropping", The Quarterly Journal of Economics, 1974, p. 231-232.

(4-11)
$$E[U] = U_A(R_A)\pi_A^g + U_B(R_B)\pi_B^g$$

$$E[W] = W_A(\phi_A)\pi_A^f + W_B(\phi_B)\pi_B^f$$

where E(R) and E(ϕ) are described by equation (3-19). Also recall that, by definition:

(4-12)
$$\phi_A = V_A - R_A \qquad \text{and} \qquad \phi_B = V_B - R_B \ .$$

Solving for the government's indifference curves from equation (4-11) yields:

(4-13)
$$\frac{dR_B}{dR_A} = -\frac{U_A'\pi_A^g}{U_B'\pi_B^g} < 0$$

and upon further differentiation:

(4-14)
$$\frac{d^2R_B}{dR_A^2} = -\frac{\pi_A^g}{\pi_B^g}\left[\frac{U_A''U_B'^2\pi_B^g + U_B''U_A'^2\pi_A^g}{U_B'^3\pi_B^g}\right] > 0$$

where equation (4-14) is positive if the government is risk averse.

The corresponding indifference curves for the firm are described by:

(4-15)
$$\frac{d\phi_B}{d\phi_A} = -\frac{W_A'\pi_A^f}{W_B'\pi_B^f} < 0$$

and

(4-16)
$$\frac{d^2\phi_B}{d\phi_A^2} = -\frac{\pi_A^f}{\pi_B^f}\left[\frac{W_A''W_B'^2\pi_B^f + W_B''W_A'^2\pi_A^f}{W_B'^3\pi_B^f}\right] > 0$$

if the firm is risk averse.

Solving for the optimal contract as we have done earlier, we may formulate the following optimization problem:

(4-17) Max $E(U)$ subject to $E(W) = \overline{W}$; U'' and $W'' < 0$; $\pi_i^g = \pi_i^f$.

From equation (4-17) we may write the following Lagrange equation:

$$S = U_A(R_A)\pi_A^g + U_B(R_B)\pi_B^g + \lambda[W_A(V_A - R_A)\pi_A^f + W_B(V_B - R_B)\pi_B^f - \overline{W}] \quad (4\text{-}18)$$

From equation (4-18) the following first order conditions for a maximum are obtained:

$$\begin{aligned} \frac{\partial S}{\partial R_A} &= U_A'\pi_A^g - \lambda W_A'\pi_A^f = 0 \\ \frac{\partial S}{\partial R_B} &= U_B'\pi_B^g - \lambda W_B'\pi_B^f = 0 \\ \frac{\partial S}{\partial \lambda} &= W_A\pi_A^f + W_B\pi_B^f - \overline{W} = 0 \, . \end{aligned} \quad (4\text{-}19)$$

As earlier results dictate, dividing the first two equations of (4-19) yields:

$$\frac{U_A'}{U_B'} = \frac{W_A'}{W_B'} \quad . \quad (4\text{-}20)$$

The second order conditions require that the determinant of the relevant boardered Hessian be positive for a maximum.

$$\begin{vmatrix} U_A''\pi_A^g + \lambda W_A''\pi_A^f & 0 & -W_A'\pi_A^f \\ 0 & U_B''\pi_B^g + W_B''\pi_B^f & -W_B'\pi_B^f \\ -W_A'\pi_A^f & -W_B'\pi_B^f & 0 \end{vmatrix} > 0 \quad . \quad (4\text{-}21)$$

Expanding the determinant in (4-21) we obtain:

$$(4\text{-}22) \qquad - U_A' W_B' \left[\frac{U_A''}{U_A'} W_B' \pi_B + \frac{U_B''}{U_B'} W_A' \pi_A + \frac{W_A''}{W_A'} W_B' \pi_B + \frac{W_B''}{W_B'} W_A' \pi_A \right] > 0$$

where (4-22) is strictly positive if the firm and the government are risk averse.

The results of this analysis are graphed, in figure (4-2), utilizing an Edgeworth box diagram. In figure (4-2) 0^g and 0^f are the origins for the government and the firm respectively. The dimensions of the box are V_a, measured along the horizontal axis, and V_b measured along the vertical axis. Any point in the box implies a division of income and risk. The government's certainty line, 0^gD, is drawn at a 45^o angle from the origin 0^g. It represents the set of all fixed-rent or bonus bid contracts ($r_p = r_s$ or $r_n > 0$, $\beta = 0$). Movement along 0^gD away from the government's origin 0^g will increase government utility.

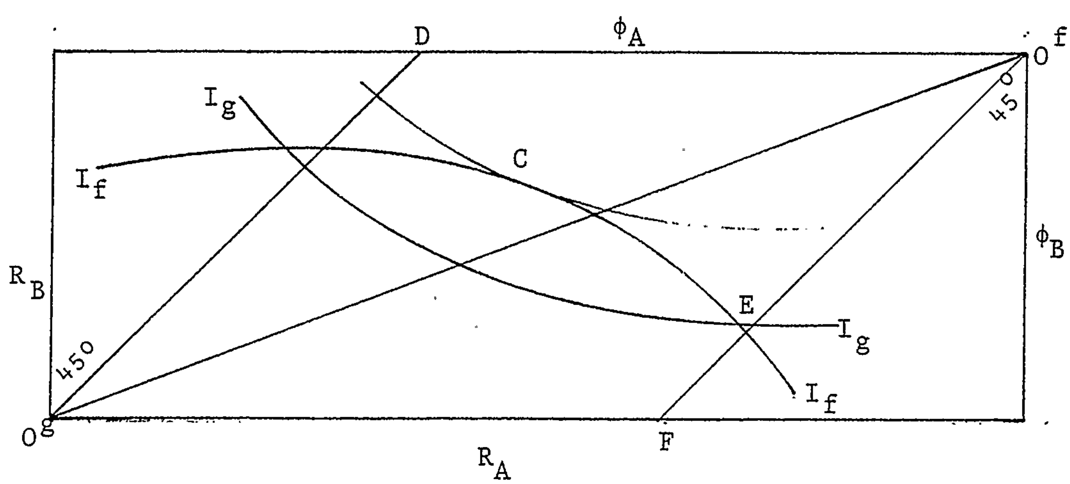

FIGURE (4-2)

The corresponding certainty line for the firm is the line 0^fF. It defines the set of fixed-profit contracts ($r_p = r_f < 0$, $\beta = 1$). Movement along 0^fF away from the firm's origin 0^f will increase firm utility. In this graph we have constrained the firm's expected utility to be equal to receiving $r_f\bar{L}$ with certainty, where r_f is evaluated at the point E. The set of points that yield this constraint are defined by the firm's indifference curve I_f.

Any point between the government's certainty line and the firm's certainty line describes some profit-sharing contract ($0 < \beta < 1$). The diagonal 0^g0^f defines the set of pure profit-sharing contracts for any given tax rate β. Any point to the right of the pure profit-sharing line has the fixed payment made to the firm ($r_p < 0$). Conversely, any point to the left of the pure profit-sharing line has the fixed payment made to the government ($r_p > 0$).

The optimal values of β and r_p are determined by the tangency point of the firm's and the government's state preference indifference curves. Such a point is C in figure (4-2). Notice that since the firm is constrained the optimal contract must lie along I_f between 0^gD and 0^fF.

In order to demonstrate the content of Proposition 4 assume that the firm and the government have constant degrees of relative risk aversion (constant elasticities of substitution). This allows us to draw the government's and the firm's indifference curves homothetically to their respective origins. Hence, along any ray from the origin the MRS will be constant.

We may identify three cases as r_p is equal to, greater than or less than zero at the optimal contract.

(a) Assume that the value of r_p at the optimal contract is equal to zero. This implies that the optimal contract will be a pure profit-sharing contract. Hence, the firm's MRS and the government's MRS will be equal on the pure profit-sharing line. This point is depicted by the tangency of I_g and I_f at point C in figure (4-3).

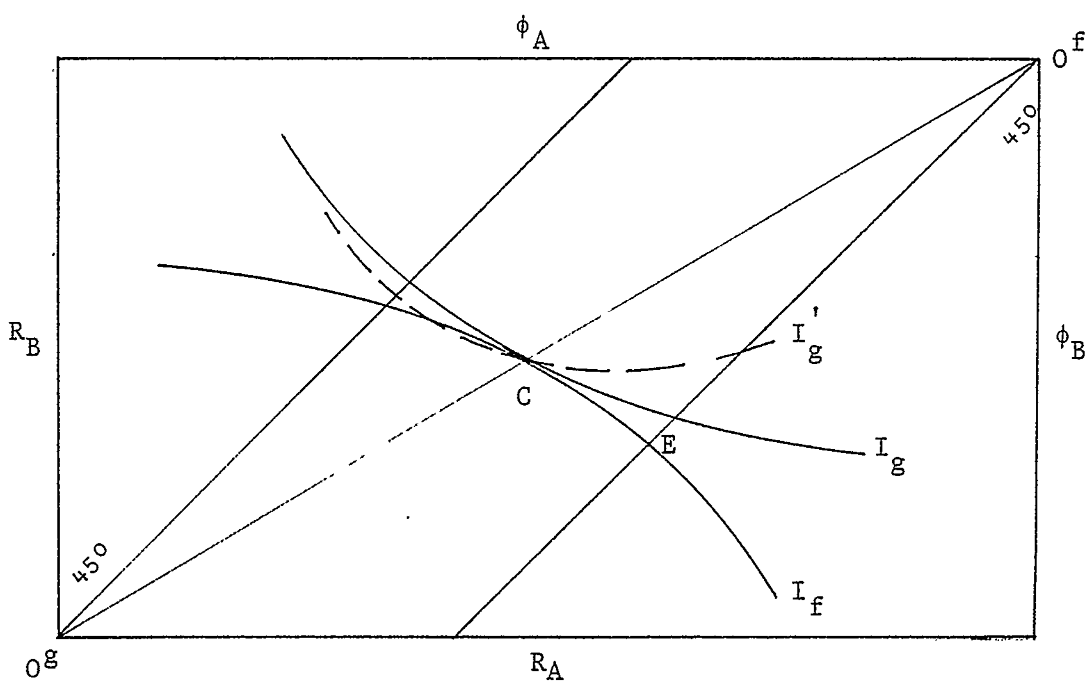

FIGURE (4-3)

Since I_g and I_f are homothetic and since the pure profit-sharing line 0^g0^f is a common ray to the firm's and the government's origins, it must be the case that the firm and the government have equal and constant degrees of relative risk aversion (equal and constant elasticities of substitution). Thus, if the firm and the government are equally risk averse then the optimal allocation of risk will always yield a pure profit-sharing contract. The firm and the government will share risk equally. This proves the first case of proposition 4.

To prove the second part of Proposition 4 allow the government to become more risk averse (decrease the elasticity of substitution), but hold the firm's indifference curve fixed. From the definition of the degree of relative risk aversion we may write that the government is relatively more risk averse than the firm if:

$$- \frac{U''}{U'} R > - \frac{W''}{W'} \phi \ . \qquad (4\text{-}23)$$

We may draw the government's new indifference curve through C as I_g', in figure (4-3). We see that at C the MRS for the government is less than the MRS for the firm. In order for an I_g' type curve to be tangent to I_f the government must decrease its share of risk from the point C. Thus the ratio r_p/β must decrease in order for the firm's MRS to decrease and the government's MRS to increase. In effect, the optimal contract will move to a point, along I_f, closer to the government's certainty line. Notice that this also implies an increase in R_B and a decrease in R_A and a corresponding decrease in ϕ_B and increase in ϕ_A. Furthermore, we see that the firm's risk premium at the new optimum will be greater than was previously experienced. As the risk premium is a measure of the amount of risk undertaken (as well as the degree of risk aversion) we see that as the government becomes more risk averse it assumes less risk and the firm assumes more risk.

These results will, of course, be reversed for a decrease in the degree of relative risk aversion for the government. The new optimum in this case will have the government assuming more risk and the firm assuming less risk. This analysis confirms the statements contained in Proposition 4.

In general, if an agent experiences an increase (decrease) in the relative degree of risk aversion then that agent will assume less (more) risk at the optimal contract, all other things being equal.

These results have some interesting implications for the oil industry in Alberta. It is evident that Alberta does not in any direct sense 'hire' firms for exploration and production activity, although one might suggest that government exploration incentives to the industry are, in fact, an indirect payment. However, in general, we may assume that front-end payments are being received by the government. From the above analysis this suggests that Alberta is relatively more risk averse than the oil industry as a whole[2].

Examination of the sources of government revenue from oil in Alberta reveal the increasing dominance of gross royalty payments over rental payments and bonus bid payments[3]. This tendency reflects the

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[2]For a discussion of government's attitude toward risk see Peterson, "The Government Role in Mineral Exploration", in Mineral Leasing as an Instrument of Public Policy, edited by Crommelin and Scott, p. 149-158.

[3]For the relevant figures see Alberta Department of Mines and Minerals, Annual Reports, Edmonton.

fact that Alberta has become more dominated by production rather than exploration. In addition, rising oil prices have increased the rent base available for taxation. Furthermore, the province has been steadily raising the gross royalty rate. The increases in the royalty rate suggest that Alberta has become less risk averse. (This assumes, of course, that the dispersion of expected outcomes has remained relatively constant. Although this is not the case, one must also consider the fact that Alberta has begun to get more directly involved with the development of her oil resources, through the establishment of the Alberta Energy Company and her equity participation in Syncrude. Hence, in all likelihood, Alberta has become somewhat less risk averse over time.) As we proceed with our analysis we will be able to gain further insights into the above discussion.

C. Comparative Static Analysis

In this section we wish to examine the effects on the optimal allocation of risk due to a change in the expected value of the lease, E(V). A change in E(V) can be produced by allowing the subjective probabilities (of the firm and the government) to change. A change in either the net price, p (reflecting changes in production costs or the well head price), or the quantities of oil, α_A or α_B, will also change E(V). In particular we will examine a change in π_A and π_B ($\pi_B = 1 - \pi_A$) and in V_a ($= \rho_A \alpha_A F(\overline{L})$). The change in the allocation of risk will be determined by the movements in β, r_p, R_A and R_B due to a change in E(V).

1. A Change in π_A and π_B

Recall that we are still operating under the assumption that both the firm and the government are risk averse and that they have symmetric expectations about future alternative states of nature. Let us first find the directional changes in R_A and R_B due to a change in π_A and $\pi_B = (1 - \pi_A)$. To accomplish this take the total differentials of equations (4-19) to obtain the following system:

$$\begin{bmatrix} U''_A\pi^g_A + \lambda W''_A\pi^f_A & 0 & - W'_A\pi^f_A \\ 0 & U''_B\pi^g_B + \lambda W''_B\pi^f_B & - W'_B\pi^f_B \\ - W'_A\pi^f_A & - W'_B\pi^f_B & 0 \end{bmatrix} \begin{bmatrix} dR_A \\ dR_B \\ d\lambda \end{bmatrix} =$$

(4-24)

$$\begin{bmatrix} \lambda W''_A\pi^f_A dV_A \\ 0 \\ - (W_A - W_B)d\pi_A - W'_A\pi^f_A dV_A \end{bmatrix} .$$

Noting that the first order conditions require $U'_A/U'_B = W'_A/W'_B$ and that $\lambda > 0$, hold $dV_A = 0$ and solve via Crammer's rule for $dR_A/d\pi_A$ and $dR_B/d\pi_A$.

(4-25) $$\frac{dR_A}{d\pi_A} = \frac{- (W_A - W_B)\ U'_A W'_B \left[\frac{U''_B}{U'_B} + \frac{W''_B}{W'_B}\right]}{G} > 0$$

and

$$\text{(4-26)} \qquad \frac{dR_B}{d\pi_A} = \frac{-(W_A - W_B)\, U_A' W_B' \left[\frac{U_A''}{U_A'} + \frac{W_A''}{W_A'}\right]}{G} > 0$$

where G is the determinant of equation (4-21) and must be positive.

From equation (4-25) and (4-26) we see that a decrease in the uncertainty for any given lease (π_A increases and π_B decreases) will increase R_A and R_B at the optimum. This implies that ϕ_A ($\phi_A = V_A - R_A$) will increase and ϕ_B ($\phi_B = V_B - R_B$) will decrease, since the company is constrained to be at the same utility level as before ($E(W) = \overline{W}$). If both R_A and R_B increase then the government has moved to a higher level of utility. The firm, of course, is constrained to a constant level of utility ($E(W) = \overline{W}$).

An increase in R_A and R_B does not, a prori, indicate a particular movement for the tax rate, β, or the fixed payment, r_p. In fact, as we shall see, the movement of r_p and β will be very conditional indeed. In order to find the effects on r_p and β due to a change in π_A and π_B take the total differentials of equation (4-7) and form the following system:

$$\begin{bmatrix} E(U''V^2) + \lambda E(W''V^2) & E(U''V)L + \lambda E(W''V)L & -E(W'V) \\ E(U''V) + \lambda E(W''V) & E(U'')L + \lambda E(W'')L & -E(W') \\ -E(W'V) & -E(W')L & 0 \end{bmatrix} \begin{bmatrix} d\beta \\ dr_p \\ d\lambda \end{bmatrix} =$$

$$\begin{bmatrix} -[U_A''\beta\pi_A^g - \lambda W_A''(r\beta)\pi_A^f]V_A dV_A \\ -[U_A''\beta\pi_A^g - \lambda W_A''(1-\beta)\pi_A^f]dV_A \\ -(W_A - W_B)d\pi_A - W_A'\pi_A^f dV_A \end{bmatrix} .$$

Make appropriate substitutions for $U'_A/U'_B = W'_A/W'_B$ and hold $dV_A = 0$ and solve (4-27) for $d\beta/d\pi_A$ and $dr_p/d\pi_A$ using Crammer's rule.

$$\text{(4-28)} \qquad \frac{d\beta}{d\pi_A} = \frac{(V_A - V_B)\; U'_A W'_B \pi_A \pi_B \left[\frac{U''_A}{U'_A} - \frac{U''_B}{U'_B} + \frac{W''_A}{W'_A} - \frac{W''_B}{W'_B}\right]}{H}$$

and

$$\frac{dr_p}{d\pi_A} = \frac{-\,(V_A - V_B)\; U'_A W'_B\; \pi_A \pi_B \left[\frac{U''_A}{U'_A} V_A - \frac{U''_B}{U'_B} V_B + \frac{W''_A}{W'_A} V_A - \frac{W''_B}{W'_B} V_B\right]}{H}$$

where H is the determinant of equation (4-9) and must be positive for a maximum.

First let us determine the sign of $d\beta/d\pi_A$, as written in equation (4-28). Notice that the terms in the large bracket are the degrees of absolute risk aversion. From the definition of absolute risk aversion we may write:

$$\text{(4-30)} \qquad \frac{U''_A}{U'_A} - \frac{U''_B}{U'_B} \gtreqless 0 \qquad \text{and} \qquad \frac{W''_A}{W'_A} - \frac{W''_B}{W'_B} \gtreqless 0$$

as the government and the firm have decreasing, constant or increasing absolute risk aversion (recall that U''/U' and W''/W' are negative). The normal assumption in risk theory is that individuals have decreasing absolute risk aversion[4]. This assumption is compatable with the

[4]see Arrow, Essays in the Theory of Risk Bearing, North Holland Publishing Company, 1974, p. 96.

case of constant relative risk aversion assumed elsewhere in this study. We may write:

$$\text{(4-31)} \qquad \frac{U''_A}{U'_A} - \frac{U''_B}{U'_B} > 0 \qquad \text{and} \qquad \frac{W''_A}{W'_A} - \frac{W''_B}{W'_B} > 0 \quad .$$

Hence, from equation (4-31) the sign of $d\beta/d\pi_A$ is positive if both the firm and the government have decreasing absolute risk aversion[5].

This result has strong implications for the oil industry. As the uncertainty of discovering oil diminishes (π_A increases and all else remains constant) the government should impose a higher tax rate.

When a tract of land is first being explored the probability of a successful well is relatively low. However, as the land becomes more delineated (and in this case assuming expectations about the size of the deposit remain unchanged, $dV = 0$) then the probability of a successful well may increase. In other words, as the information about a tract of land increases, the probability of a dry hole diminishes. This implies a low royalty on land that is initially being explored relative to land that has been more fully explored, and still appears profitable. This helps explain why Alberta has consistantly increased royalty rates as Alberta has become a mature producing region.

Furthermore, due to the general uniformity of royalty taxes on leases in Alberta (the royalties on old and new oil are still uniformly

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[5]The results here concur with those found by Leland, _ibid_., p. 421.

applied), it is evident that the government relies on bonus bidding to capture the additional rent on more prolific leases relative to less prolific leases. The above analysis suggests less reliance on bonus bids and more reliance on royalties to capture these rent differentials. These results will equally apply to a change in the quantity of oil as is demonstrated in the second half of this section.

Now let us examine a change in the fixed payment, r_p, with respect to a change in π_A as described by equation (4-29). Looking at equation (4-29) we see that the sign of $dr_p/d\beta$ will take on the opposite sign of the large term in brackets. Hence, we may write:

$$\text{(4-32)} \qquad \frac{dr_p}{d\pi_A} \gtreqless 0 \quad \text{as} \quad \left[\frac{U''_A}{U'_A} V_A - \frac{U''_B}{U'_B} V_B + \frac{W''_A}{W'_A} V_A - \frac{W''_B}{W'_B} V_B\right] \lesseqgtr 0 \ .$$

In order to simplify the above expression assume that both the firm and government have constant relative risk aversion. From the definition of the degree of relative risk aversion we may write:

$$\text{(4-33)} \qquad \frac{U''_A}{U'_A} R_A = \frac{U''_B}{U'_B} R_B \Rightarrow \frac{U''_B}{U'_B} = \frac{U''_A}{U'_A} \frac{R_A}{R_B}$$

$$\text{(4-34)} \qquad \frac{W''_A}{W'_A} \phi_A = \frac{W''_B}{W'_B} \phi_B \Rightarrow \frac{W''_B}{W'_B} = \frac{W''_A}{W'_A} \frac{\phi_A}{\phi_B} \ .$$

Substituting for U''_B/U'_B and W''_B/W'_B into equation (4-32) yields:

$$\text{(4-35)} \qquad \frac{U''_A}{U'_A} [V_A R_B - V_B R_A] \frac{1}{R_B} + \frac{W''_A}{W'_A} [V_A \phi_B - V_B \phi_A] \frac{1}{\phi_B} \lesseqgtr 0$$

$$\frac{U''_A}{U'_A}\frac{1}{R_B}(V_A-V_B)r_p\bar{L} + \frac{W''_A}{W'_A}\frac{1}{\phi_B}(V_A-V_B)r_p\bar{L} \lesseqgtr 0$$

$$\left[\frac{U''_A}{U'_A}\frac{1}{R_B} - \frac{W''_A}{W'_A}\frac{1}{\phi_B}\right](V_A-V_B)r_p\bar{L} \lesseqgtr 0 \quad .$$

Looking at equation (4-35) it is evident that the sign of $dr_p/d\pi_A$ will be determined by the term in brackets and the sign of r_p. Multiply the equation by $R_A\phi_A$ and rearrange terms to obtain the following condition:

$$\text{(4-36)} \qquad \frac{dr_p}{d\pi_A} \gtreqless 0 \qquad \text{as} \qquad \left[\frac{U''_A}{U'_A}R_A - \frac{W''_A}{W'_A}\phi_A\frac{R_AR_B}{\phi_A\phi_B}\right](V_A-V_B)r_p\bar{L} \lesseqgtr 0 \quad .$$

We may consider three separate cases that will assist us in determining the sign of equation (4-36) and thus the sign of $dr_p/d\pi_A$.

(a) Assume that the government is relatively more risk averse than the firm so that the optimum contract will have r_p greater than zero. From the definition of the relative risk aversion it must be the case that:

$$\text{(4-37)} \qquad \frac{U''_A}{U'_A}R_A - \frac{W''_A}{W'_A}\phi_A < 0 \quad .$$

Looking back to equation (4-36) we see that the values of R_A, R_B, ϕ_A, and ϕ_B are important. If, for example $R_AR_B \le \phi_A\phi_B$ then equation (4-36) is clearly negative and hence $dr_p/d\pi_A > 0$. However if $R_AR_B > \phi_A\phi_B$ then $dr_p/d\pi_A$ may be positive, negative or zero. Thus the sign of $dr_p/d\pi_A$ is dependent on the income sharing as well as the degrees of risk aversion. The difficult case is where the government has a higher revenue share and is only slightly more risk averse than the firm.

Notice that if the government becomes less risk averse then the value of equation (4-37) increases. Hence, from equation (4-36) the absolute change in r_p due to a change in π_A decreases. This brings us to the second case.

(b) Assume that the government and the firm are equally risk averse. Thus at the optimal contract $r_p = 0$ and

$$\frac{U''_A}{U'_A} R_A - \frac{W''_A}{W'_A} \phi_A = 0 \quad . \tag{4-38}$$

Hence $dr_p/d\pi_A = 0$.

(c) Assume the government is less risk averse than the firm. At the optimal contract $r_p < 0$ and

$$\frac{U''_A}{U'_A} R_A - \frac{W''_A}{W'_A} \phi_A > 0 \quad . \tag{4-39}$$

If, for example $R_A R_B \geq \phi_A \phi_B$, then from equation (4-36) $dr_p/d\pi_A > 0$.

Considering the above three cases we see that we can only be definite about the optimal movement in r_p with respect to a change in π_A if the firm and the government are equally risk averse: in which case $dr_p/d\pi_A = 0$. More information about who receives what proportion of lease value would be useful. In addition, we would like to know how much more risk averse the government is than the firm (or vice versa).

Recall that as the government becomes less risk averse, then at the optimum the fixed payment, r_p, received by the government, decreases and the tax rate, β, increases, if the government is initially more risk averse than the firm. If the government is less risk averse than the firm the fixed payment is received by the firm. In this case, as the government becomes less risk averse then, at the optimum, the government's fixed payment to the firm will increase and the tax rate will increase. At the limit, when the government becomes risk neutral the optimal contract will have a 100 per cent tax rate, and the firm will receive fixed profits equal to its certainty equivalent. The less risk averse the government is the higher will be E(R) and the lower will be E(ϕ). Hence, the more risk averse the government is relative to the firm the more likely $R_A R_B \leq \phi_A \phi_B$. Conversely, the less risk averse the government is relative to the firm the more likely $R_A R_B \geq \phi_A \phi_B$.

Given the tendency for the above relationship to hold we suggest that the optimal fixed payment will be likely to increase with respect to an increase in π_A. If the government is more risk averse than the firm the fixed payment it receives will likely increase. If the firm is more risk averse than the government the fixed payment it receives will likely increase. Furthermore the more risk averse the government is relative to the firm the greater will be the expected increase in the fixed payment and the smaller the increase in the tax rate. If, for example, the firm is risk neutral, the tax rate remains constant and equal to zero. The entire increase in government revenue due to the increase in π_A is received as a fixed payment.

The above results are graphed in figure (4-4). Let π_A increase and π_B decrease such that the firm's constant profit (government's constant revenue) line shifts from $r_s r_f$ to $r_s' r_f$. The corresponding shift in the firm's utility curve will be from $r_f A$ to $r_f B$. Suppose that the government is more risk averse than the firm such that the initial optimal contract is at C. We see that ($\beta < 1$) all points on the firm's new indifference curve $r_f B$, are to the right of all points on the old indifference curve, $r_f A$, for any given tax rate. Hence, for any given tax rate, a decrease in the uncertainty will increase the fixed payment the firm is willing to pay the government (or decrease the amount the firm is demanding from the government).

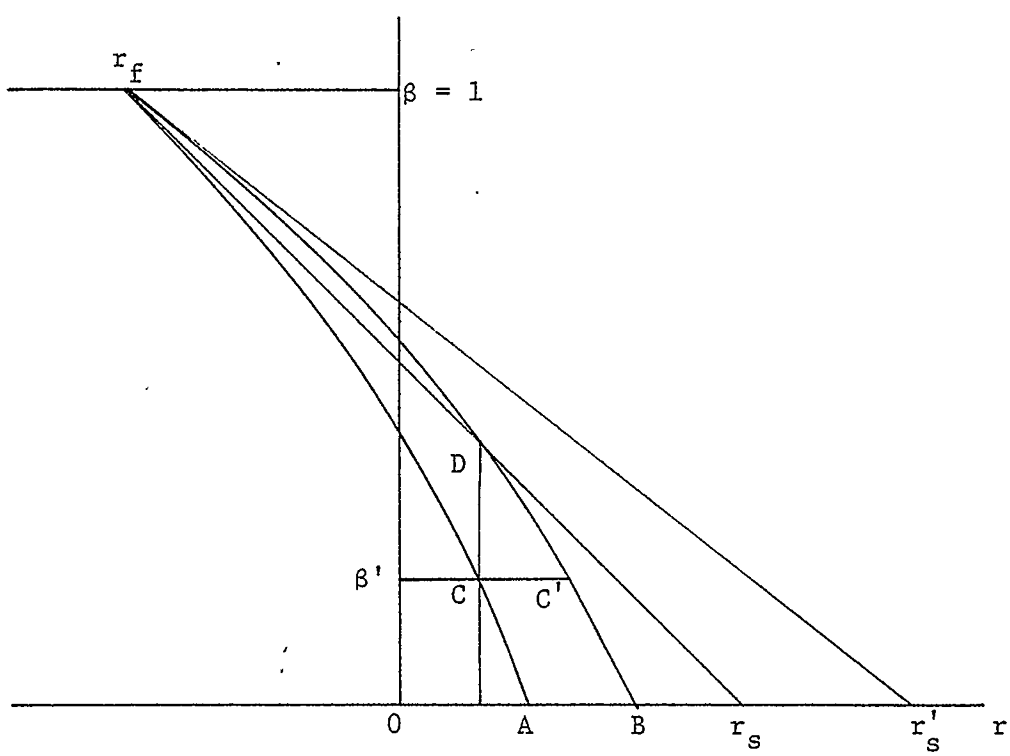

FIGURE (4-4)

Furthermore, assuming β will increase (the firm and the government have decreasing absolute risk aversion) the new optimal contract must have a value for β greater than β' in figure (4-4) and lie on the firm's new indifference curve, $r_f B$, above C'. We would expect the fixed-payment to increase which would further restrict the new contract to lie on $r_f B$ below D and above C'. As a final point, if the firm is risk neutral ($W'' = 0$) then $dr_p/d\pi_A > 0$ and $d\beta/d\pi_A = 0$. The optimal contract will move from r_s ($r_p = r_s$ and $\beta = 0$) to r_s' ($r_p = r_s'$ and $\beta = 0$).

In general, we see that the changes in r_p and β will be determined by the risk averseness of the government relative to the risk averseness of the firm, and the division of income between the firm and the government in each alternative state of nature. If both the firm and the government are risk averse then the tax rate will increase with an increase in π_A. The fixed payment will remain constant and equal to zero if the firm and the government are equally risk averse. We expect the fixed payment to increase to the government: if the government is relatively more risk averse than the firm. Conversely, we expect the fixed payment received by the firm to increase if the firm is more risk averse relative to the government.

2. A Change in V_A

Although a change in V_A will produce results similar to a change in the subjective probabilities: it is not totally analogous to a change in π_A. Either an increase in V_A or an increase in π_A will increase E(V) and thus reduce the existing uncertainty. An increase in π_A reduces the uncertainty but does not change the available rent in either alternative state of nutre. An increase in V_A, however, increases the available rent, should state A occur, although the actual probability of state A occuring has not changed. As we shall see, this fact will make it more difficult to determine the change in the optimal contract resulting from a change in V_A.

First let us look at the movement in R_a and R_b due to a change in V_A. Solve equation (4-24) for dR_A/dV_A and dR_B/dV_A via Crammer's rule and hold $d\pi_A = 0$. Recalling that optimality conditions require $U_A'/U_B' = W_A'/W_B'$ and making appropriate substitutions for λ we obtain:

$$\text{(4-40)} \qquad \frac{dR_A}{dV_A} = \frac{-\,W_A'W_B'\left[\frac{U_B''}{U_B'}\,U_A'\pi_A^g + \frac{W_A''}{W_A'}\,U_B'\pi_B^g + \frac{W_B''}{W_B'}\,U_A'\pi_A^g\right]}{G} > 0$$

and

$$\text{(4-41)} \qquad \frac{dR_B}{dV_A} = \frac{-\,U_A''W_A'W_B'\pi_B^f}{G} > 0 \quad .$$

From equation (4-40) and (4-41) we see that an increase in V_A will increase both R_A and R_B. Hence, government utility will increase. Inspection of the above two equations also yields $dR_A/dV_A > dR_B/dV_A$. These results concur with the results obtained for an increase in π_A.

Now let us examine the movement in r_p and β at the optimum due to a change in V_A. Solve equation (4-27) for $d\beta/dV_A$ and dr_p/dV_A while holding π_A constant. This yields:

$$(4\text{-}42)\quad \frac{d\beta}{dV_A} = \frac{\bar{L}\pi_A\pi_B(V_A-V_B)U_A'W_B'\left[\left(\frac{U_A''}{U_A'}-\frac{U_B''}{U_B'}+\frac{W_A''}{W_A'}-\frac{W_B''}{W_B'}\right)W_A'\pi_A+\left(\frac{U_B''}{U_A'}\beta-\frac{W_A''}{W_A'}(1-\beta)\right)E(W')\right]}{H}$$

$$(4\text{-}43)\quad \frac{dr_p}{dV_A} = \frac{-\pi_A\pi_B(V_A-V_B)U_A'W_B'\left[\left(\frac{U_A''}{U_A'}V_A-\frac{U_B''}{U_B'}V_B+\frac{W_A''}{W_A'}V_A-\frac{W_B''}{W_B'}V_B\right)W_A'\pi_A+\left(\frac{U_A''}{U_A'}\beta-\frac{W_A''}{W_A'}(1-\beta)\right)E(W'V)\right]}{H}$$

Let us determine the sign of $d\beta/dV_A$. The first term in the large bracket in equation (4-42) will be positive if both the firm and the government and the firm have decreasing absolute risk aversion. The sign of the second term in the large bracket poses more of a problem. Assume that both the firm and the government have constant relative risk aversion. Hence we may write:

$$(4\text{-}44)\qquad \frac{U_A''}{U_A'}\,[\beta V_A + r_p\bar{L}] \gtreqless \frac{W_A''}{W_A'}\,[(1-\beta)V_A - r_p\bar{L}]$$

as the government is relatively less risk averse, equally risk averse or more risk averse than the firm.

Rearranging terms in equation (4-44) yields:

$$(4\text{-}45)\qquad \left[\frac{U_A''}{U_A'}\,\beta - \frac{W_A''}{W_A'}\,(1-\beta)\right] \gtreqless - \left[\frac{U_A''}{U_A'} + \frac{W_A''}{W_A'}\right]\frac{r_p\bar{L}}{V_A} \quad .$$

The term in brackets on the RHS of equation (4-45) is negative (U_A''/U_A' and W_A''/W_A' are negative) so that the RHS of the above equation assumes the sign of r_p. We can identify three cases as $r_p \gtreqless 0$ at the optimum.

(a) Let the government be more risk averse than the firm. At the optimum $r_p > 0$. Hence, the LHS of equation (4-45) must be less than the RHS, by virtue of equation (4-44), if the government is more risk averse than the firm. Hence, in this case:

$$\left[\frac{U''_A}{U'_A}\beta - \frac{W''_A}{W'_A}(1-\beta)\right] \gtreqless 0 \quad . \tag{4-46}$$

(b) Let the government and the firm be equally risk averse. At the optimum risk must be allocated evenly between the firm and the government so that the optimal contract will be a pure profit-sharing contract ($r_p = 0$). By virtue of equation (4-45):

$$\left[\frac{U''_A}{U'_A}\beta - \frac{W''_A}{W'_A}(1-\beta)\right] = 0 \quad . \tag{4-47}$$

Substitution of equation (4-47) into equation (4-42) will yield $d\beta/dV_A > 0$.

(c) Finally assume the optimal contract yields a negative fixed payment ($r_p < 0$). Hence, the government must be less risk averse than the firm. The RHS of equation (4-45) must be negative. By virtue of equation (4-44) the LHS of equation (4-45) must be greater than the RHS. As in case (a) we have:

$$\left[\frac{U''_A}{U'_A}\beta - \frac{W''_A}{W'_A}(1-\beta)\right] \gtreqless 0 \quad . \tag{4-48}$$

From the above three cases $d\beta/dV_A$ is assuredly positive only if the firm and the government are equally risk averse. However, meaning can be attached to the other two cases. First, note that even if the expression in equations (4-46) and (4-48) were negative, it would have to outweigh the first expression in brackets in equation (4-42), which is clearly positive.

Initially assume the government is more risk averse than the firm. At the optimum r_p will be positive. If we allow the government to become less risk averse then the RHS of equation (4-45) is positive and will decrease, at the optimum, until it equals zero, when the firm and the government are equally risk averse. If the government becomes still less risk averse the RHS of equation (4-45) will become negative ($r_p < 0$). The government is now less risk averse than the firm. A similiar change occurs to the LHS of equation (4-45). As the government becomes less and less risk averse relative to the firm the LHS of equation (4-45) decreases to zero then becomes negative. As previous analysis shows, information regarding utility levels and the relative degrees of risk aversion would be useful. However, we would expect that equation (4-46) is strictly positive and equation (4-48) is strictly negative.

Referring back to equation (4-42) we may infer from the above discussion that, if the government is more risk averse than the firm, then $d\beta/dV_A > 0$. However, as the government becomes less risk averse, the increase in the tax rate due to an increase in V_A (the second term in brackets of equation (4-42)) is decreasing as well as the first term).

If the government is sufficiently less risk averse than the firm then the two terms in brackets of equation (4-42) may cancel each other out such that $d\beta/dV_A = 0$. At the limit, as the government becomes risk neutral, it will indeed by the case that $d\beta/dV_A = 0$.

We will now sort out the conditions that determine the sign of dr_p/dV_A as is written in equation (4-43). The two terms in brackets in equation (4-43) have been previously analyzed. The firm term in brackets may be rewritten as in equation (4-35) (assuming the firm and the government have constant relative risk aversion). This new expression will have a tendency to conform to the following restrictions:

(4-49) $$\left[\frac{U_A''}{U_A'}\frac{1}{R_B} - \frac{W_A''}{W_A'}\frac{1}{\phi_B}\right](V_A-V_B)r_p\bar{L} < 0$$ if the government is more risk averse than the firm

$= 0$ if the government and the firm are equally risk averse

> 0 if the government is less risk averse than the firm.

Analysis of the second term in brackets will yield a tendency to conform to the following restrictions.

(4-50) $$\left[\frac{U_A''}{U_A'}\beta - \frac{W_A''}{W_A'}(1-\beta)\right]$$ > 0 if the government is more risk averse than the firm

$= 0$ if the government and the firm are equally risk averse

< 0 if the government is less risk averse than the firm.

Given the tendencies expressed in equations (4-49) and (4-50) we may determine the sign of dr_p/dV_A as follows: if the government and the firm are equally risk averse then clearly $dr_p/dV_A = 0$. Regardless of the changes on the expected value of the lease the optimal contract will be a pure profit-sharing contract.

If the government is less risk averse than the firm then there will be a pronounced tendency for the government to increase the fixed payment it makes to the firm. Conversely, if the government is more risk averse than the firm the two bracketed terms in equation (4-43) will likely have the opposite signs. We would expect the first term to outweigh the second term thus increasing the fixed payment the government receives from the firm.

Intuitively these results are appealing. If the government is more risk averse than the firm then, at the optimal contract, the firm will bear the most risk. The fixed payment is made to the government. If the fixed payment actually decreases, due to an increase in V , then the government is increasing its share of the risk. This is not a result we would normally expect. Rather, we would expect the fixed payment to increase such that the government maintains a constant share of the risk. Leland[4] demonstrates that given the revenue and profit functions we are analyzing this will be the case.

These results are summarized in the following proposition:

Proposition 5

If the firm and the government have (a) constant

[4]Leland, Ibid., 1978, p. 419-420.

relative risk aversion and (b) symmetric expectations about future alternative state of nature then an increase in the expected value of the lease will generally cause, at the optimal contract:

(i) an increase in the tax rate β,

(ii) an increase in the fixed payment the government receives from the firm, if the firm is less risk averse than the government,

(iii) an increase in the fixed payment the firm receives if the firm is more risk averse than the government,

(iv) or no change in the fixed payment if the firm and the government are equally risk averse.

This section completes this chapter. In the next chapter we consider how asymmetric expectations will affect the optimal contract.

CHAPTER FIVE

OPTIMALITY UNDER CONDITIONS OF ASYMMETRIC EXPECTATIONS

One assumption we have maintained throughout this study is that the firm and the government have identical subjective probabilities about alternative states of nature ($\pi_i^g = \pi_i^f$). However, there is no reason to suppose that this is actually the case. It is quite possible that there will exist a divergence between the subjective probabilities of the firm and of the government. These differences will arise due to differences in information about the lease in question and from differences in estimation techniques. It is generally agreed that the firm is in possession of better information than the government. Whether this will lead to higher expectation or lower expectations by the firm relative to the government will, of course, depend on the characteristics of the lease in question.

In this chapter we relax the assumption of symmetric expectations and allow for the firm and the government to have different subjective probabilities ($\pi_i^g \neq \pi_i^f$). They are still assumed to face identical states of nature: $V_A > V_B$. With asymmetric expectations, the firm and the government will assign different values for the expected income from the lease in question: $E(V^g) \neq E(V^f)$. If the government is 'pessimistic' relative to the firm then $\pi_A^g < \pi_A^f$ and thus $E(V^g) < E(V^f)$. On the other hand, if the government is 'optimistic' relative to the firm then $\pi_A^g > \pi_A^f$ and $E(V^g) > E(V^f)$.

To solve for the optimal contract under conditions of asymmetric expectation we formulate the same maximization problem as used last chapter. We initially assume the government is risk averse but the firm is risk neutral.

(5-1) Max: $E(U)$ subject to $E(\phi) = \overline{\phi}$, $U'' < 0$, $W'' = 0$, $\pi_i^g \neq \pi_i^f$.

The resulting first order conditions from the above maximization problem are identical to those stated in equation (4-3). However, the following condition is obtained from the first two equations of (4-3):

(5-2) $$\frac{U_A'}{U_B'} = \frac{\pi_B^g}{\pi_A^g}\frac{\pi_A^f}{\pi_B^f} .$$

As before, optimality conditions require tangency between the government's indifference curve and the firm's indifference curve (in this case the firm's indifference curve is identified by a linear constant profit line since the firm is risk neutral). From the above condition we see that if $\pi_i^g = \pi_i^f$ we have the same optimal contract as described by equation (4-4): a fixed-rent or a bonus bid contract.

If the government is more pessimistic than the firm ($\pi_A^g < \pi_A^f$) it must be the case that $U_A'/U_B' > 1$. In order for this condition to be satisfied $R_A < R_B$ in order for $U_A' > U_B'$ at the optimum. This result can be obtained only with a negative tax rate ($\beta < 0$). Since we restrict the tax rate to be greater than or equal to zero we must have a corner solution. Such a point is depicted in Figure (5-1) at $r_p = r_s'$ and $\beta = 0$.

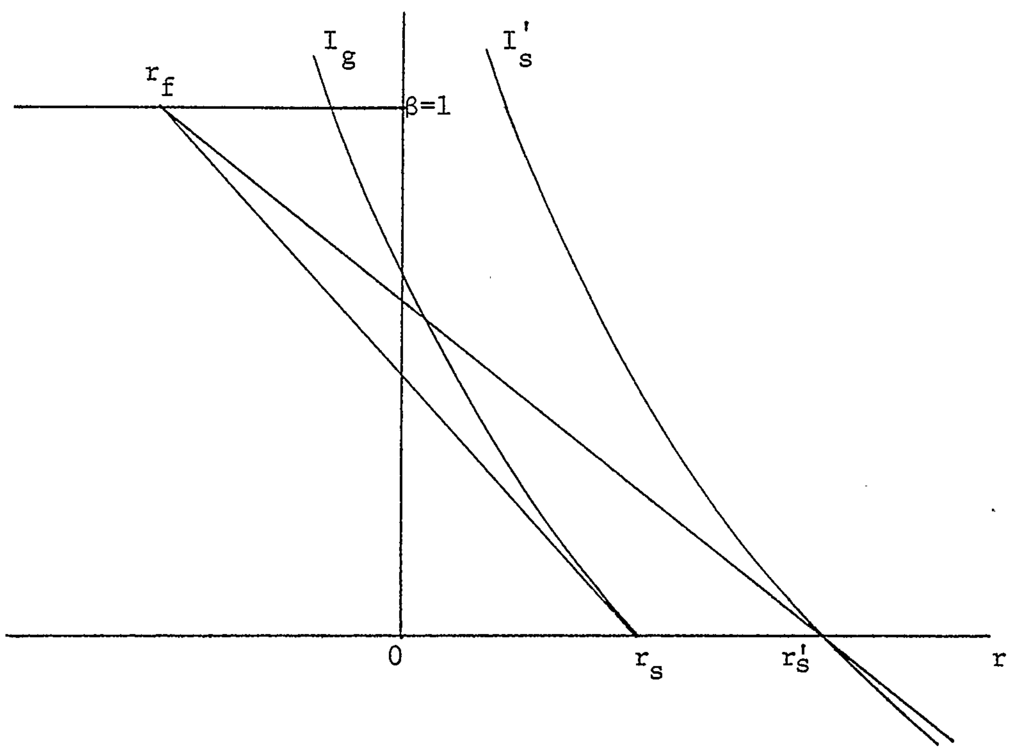

FIGURE (5-1)

In figure (5-1) π^g_A and π^g_B are such that $r_s r_f$ denotes a constant revenue line (constant utility for a risk neutral government). The firm, having higher expectations than the government ($\pi^g_A < \pi^f_A$) has a constant profit line lying above the government's constant revenue line. Such a line is represented by $r'_s r_f$. We assume the firm is constrained by its certainty equivalent such that $\overline{\phi} = r_f\overline{L}$. The indifferences curves of the government, I_g and I'_g, are drawn so as to be tangent to the constant revenue line on the government's certainty line ($\beta = 0$). Assuming the tax rate may not be negative the optimal contract will be at the point $r_p = r_s'$ and $\beta = 0$.

If the fixed payment is set by the government (as opposed to a bonus bid made by the firm) then the government must be aware of the difference between π_A^g and π_A^f in order to maximize its utility. If the government underestimates the firm's 'optimism' it will set the rental rate too low and allow the firm to increase its utility above $\overline{\phi}(E(\phi) > \overline{\phi})$. With a bonus bid scheme, and assuming the firm makes a bid equal to its certainty equivalent then the government will always reach its highest possible level of revenue and utility. (Of course, this rests on the assumption that the firm will maximize its bid consistant with its 'target' utility level $\overline{\phi}$.) For the purposes of this study we will not enter the realm of strategic bidding by the firm and always assume the firm makes its maximum bid.

Another case we may encounter is when the government has 'optimistic' expectations relative to the firm $(\pi_A^g > \pi_A^f)$. If this is the situation then from equation (5-2) it must be the case that $U_A'/U_B' < 1$. In order for $U_A' < U_B'$ then $R_A > R_B$. This implies that although the government is risk averse and the firm is risk neutral the optimal contract will have risk being shared by both parties.

A solution to this case is depicted by point C in figure (5-2). At this point the government's indifference curve I_g' is tangent to the firm's indifference curve $r_s' r_f$. The firm's pessimistic outlook leads its constant profit (utility) line to be below the government's constant revenue line $r_s r_f$.

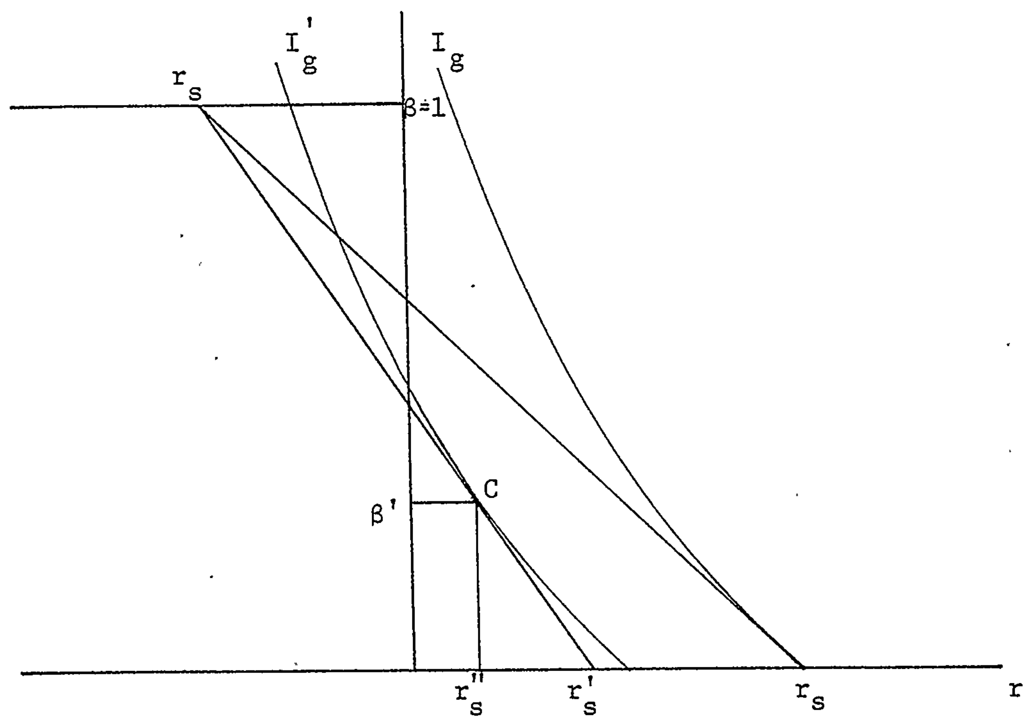

FIGURE (5-2)

The problem, discussed above, associated with a fixed-rent scheme versus a bonus bid scheme is not as pronounced in this case. If the government sets a rental payment equal to r_s and $\beta = 0$, in figure (5-2), then clearly the firm will not accept the lease. The government may lower the fixed payment to r_s'' and impose a royalty tax equal to β'. Or it may chose to wait until the firm's expectations correspond to its own. If a bonus bid scheme is used they will immediately move to point C with a bonus bid equal to $r_s''\overline{L}$ and a tax rate of β'. Although the government still has the option of rejecting the bid. These results are summarized in the following proposition.

Proposition 6

> If (a) the government is risk averse and (b) the firm is risk neutral then some fixed-rent contract will be Pareto superior to any profit-sharing contract if the government is more pessimistic than the firm. If the government is more optimistic than the firm then any fixed-rent contract will be Pareto inferior to some profit-sharing contract.

Now consider the case where both the firm and the government are risk averse. Reformulate the maximization problem of equation (5-1) to read:

$$\text{(5-3)} \qquad \text{Max } E(U) \text{ subject to } E(W) = \overline{W},\ U'' \text{ and } W'' < 0,\ \pi^g \neq \pi_i^f \ .$$

The first order conditions for (5-3) are identical to those stated in Chapter Four in equation (4-7). From the first two equations in (4-7) we obtain the following optimality condition:

$$\text{(5-4)} \qquad \frac{U_A'}{U_B'} = \frac{W_A' \, \pi_A^f \, \pi_B^g}{W_B' \, \pi_B^f \, \pi_A^g} \ .$$

From the above condition we see that the more pessimistic the government is relative to the firm ($\pi_A^g < \pi_A^f$) the smaller the amount of risk the government is willing to bear. In other words, as the government becomes more and more pessimistic the optimal contract will tend towards a fixed-rent or a bonus bid contract. Conversely, the more optimistic the government becomes the optimal contract will tend towards a fixed-profit contract.

These results are easily demonstrated by the comparative static exercise done below. In this case we will examine the movements in R_A, R_B, r_p and β resulting from a change in the governments expectations: π_A^g and π_B^g.

First consider the movement in R_A and R_B. From equation (4-19) we obtain the following system.

$$(5\text{-}5)\qquad \begin{bmatrix} U_A''\pi_A^g + W_A''\pi_A^f & 0 & -W_A'\pi_A^f \\ 0 & U_B''\pi_B^g + W_B''\pi_B^f & -W_B'\pi_B^f \\ -W_A'\pi_A^f & -W_B'\pi_B^f & 0 \end{bmatrix} \begin{bmatrix} dR_A \\ dR_B \\ d\lambda \end{bmatrix}$$

$$= \begin{bmatrix} -U_A'd\pi_A^g \\ U_B'd\pi_A^g \\ 0 \end{bmatrix} .$$

Solving for $dR_A/d\pi_A^g$ and $dR_B/d\pi_A^g$ via Crammer's rule we obtain:

$$(5\text{-}6)\qquad \frac{dR_A}{d\pi_A^g} = \frac{W_B'\pi_B^f[U_A'W_A'\pi_B^f + U_B'W_A'\pi_A^f]}{G} > 0$$

and

$$\frac{dR_B}{d\pi_A^B} = - \frac{W_A'\pi_A^f[U_A'W_B'\pi_B^f + U_B'W_A'\pi_A^f]}{G} < 0 \quad . \tag{5-7}$$

From equations (5-6) and (5-7) we see that as the government's expectations increase (π_A^g increases) R_A will increase and R_B will decrease. Hence, the government is increasing its share of risk at the optimum. From this fact we immediately know that the tax rate will increase and the fixed payment will decrease. Conversely, if the government's expectations fall (π_A^g decreases) the government will decrease its share of risk (R_A decreases and R_B increases). Hence the tax rate will decrease and the fixed payment will increase.

To check on these predicted movements in r_p and β solve the following system obtained from equations (4-7):

$$\begin{bmatrix} E(U''V^2) + \lambda E(W''V^2) & E(U''V)\overline{L} + \lambda E(W''V)\overline{L} & -E(W'V) \\ E(U''V) + \lambda E(W''V) & E(U'')\overline{L} + \lambda E(W'')\overline{L} & -E(W') \\ -E(W'V) & -E(W')\overline{L} & 0 \end{bmatrix} \begin{bmatrix} d\beta \\ dr_p \\ d\lambda \end{bmatrix}$$

$$= \begin{bmatrix} -(U_A'V_A - U_B'V_B)\,d\pi_A^g \\ -(U_A' - U_B')\,d\pi_A^g \\ 0 \end{bmatrix} \quad .$$

Using Crammer's rule to solve for $dr_p/d\pi_A^g$ and $d\beta/d\pi_A^g$ results in:

$$\text{(5-9)} \qquad \frac{d\beta}{d\pi_A^g} = \frac{E(W')\overline{L}(V_A - V_B)[U_A'W_B'\pi_B^f + U_B'W_A'\pi_A^f]}{H} > 0$$

and

$$\text{(5-10)} \qquad \frac{dr_p}{d\pi_A^g} = - \frac{E(W')(V_A - V_B)[U_A'W_B'\pi_B^f + U_B'W_A'\pi_A^f]}{H} < 0 \quad .$$

As surmised, if the government's expectations increase the tax rate β increases and the fixed payment r_p decreases. If the government's expectations fall then the tax rate β decreases and the fixed payment r_p decreases. The results of this analysis are summarized below:

<u>Proposition 7</u>

> A fixed-rent contract or a bonus bid contract becomes relatively more attractive as the government becomes more pessimistic relative to the firm and a fixed-profit contract becomes relatively more attractive as the government becomes optimistic relative to the firm.

At this point we conclude the formal analysis of this study. However, we will discuss some of the implications of the above results as well as those results obtained in Chapter 4, in the conclusion.

CHAPTER SIX

CONCLUSION

The preceeding chapters have discussed the optimal characteristics of a petroleum leasing contract under a variety of conditions. We have demonstrated how the attitudes toward risk exhibited by the firm and the government, the subjective probabilities assigned to each state of nature, the net value of the lease in the possible states of nature, and the possibility of expectation asymmetries will all influence the optimal contract. In this chapter we will briefly reiterate the major results generated by the model. Next we will discuss some of the shortcomings of the model. Finally, we will draw on the model to make some general comments about the structure of risk bearing in Alberta.

Summary of the Model

The main results of the model were generated in Chapters Four and Five. These results are briefly summarized below. The term profit-sharing contract is used to describe an agreement which includes an element of profit sharing through a net royalty or profit tax; it may also include a fixed payment clause, like a rental payment or bonus bid.

(a) Given some profit-sharing contract and a fixed-rent or bonus bid contract between which the firm is indifferent then the government will maximize its utility with either a fixed-rent contract or a bonus bid contract if:

(i) the government is risk averse

(ii) the firm is risk neutral

and (iii) the firm and the government have symmetric expectations or the government is pessimistic relative to the firm

or (iv) the firm is also risk averse but the government is significantly more pessimistic relative to the firm.

(b) Given various profit-sharing contracts, a fixed-profit contract, and a fixed-rent or bonus bid contract among which the firm is indifferent then some profit-sharing contract will maximize government utility if:

(i) the firm and the government are risk averse

and (ii) they have symmetric expectations

or (iii) the firm is risk neutral and the government is optimistic relative to the firm.

(c) Given that the optimal contract is a profit-sharing contract and the firm and the government have symmetric expectations then the recipient of the fixed payment element in the contract is determined as follows:

(i) the government receives the fixed payment if the government is relatively more risk averse than the firm

(ii) the firm receives the fixed payment if the government is relatively less risk averse than the firm

(iii) the fixed payment will be zero if the firm and the government are equally risk averse.

(d) Given the optimal contract is a profit-sharing contract then an increase in the expected value of the lease will normally increase the tax rate. If the government is more risk averse than the firm the fixed payment made by the firm to the government will generally increase. If the government is relatively less risk averse than the firm the fixed payment received by the firm will generally increase. If the firm and the government are equally risk averse the fixed payment will be constant and equal to zero.

It is useful to note that Leland[1] demonstrates that a mean preserving decrease in the uncertainty (for example, π_A increases and V_A decreases such that $E(V)$ remains constant) will have the same effect on the optimal contract as the above conclusion.

Shortcomings of the Model

It is apparent that a number of issues have been dealt with at arm's length throughout this study. Some are more critical than others in the search for an optimal leasing policy. These simplifications were necessary to reduce an already complex problem to manageable proportions.

First, it was necessary to restrict the analysis to only four possible contract types, dealing with financial terms of the contracts. In reality, the allocation of mineral rights in Alberta is governed by a complex system of lease arrangements. The terms of these contracts

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[1]Leland, op. cit., 1978, p. 421-422.

cover a wide range of areas, from specifying the payment schedule to stipulating drilling requirements. However, the selection of contracts we have analyzed does, to a large extent, contain the major financial components of the current leasing system in Alberta.

Second, our model utilizes a net royalty tax or profit tax rather than a gross royalty, such as used by the provincial government. This allows us to solve for the Pareto optimal production decisions by the firm as well as the Pareto optimal allocation of risk. As previously discussed a gross royalty misallocates resources by affecting the firm's behavior at the margin. The major concern here is that a gross royalty will cause early abandonment of wells. It will also cause underinvestment in exploration and development. Exploration incentives offered by the government and drilling requirements are no doubt an attempt by the government to offset some of the underinvestment problems caused by a gross royalty. There is also the use, in Alberta, of a sliding scale royalty with a lower rate on wells with small production. Hence, there is some justification in using a net royalty tax rather than a gross royalty tax and so avoid the government's mistake.

Third, we do not take into account the optimal timing of resource development. Rather, we have assumed a given quantity of land and have not attempted to characterize the optimal acreage of a lease. The optimal rate at which the government brings land onto the market has not been discussed by our model. One might suppose, given the above solutions to both the Pareto optimal production decisions by the firm and the Pareto optimal allocation of risk, that the resource will be produced at the optimal rate without further government interference. Indeed, resource theory tells us that with a number of restrictive assumptions (for example, that the private and social discount rates are equal), then we will have a Pareto optimum with respect to the timing of resource development regardless of when the government selects to offer leases. Of course, these models assume the quantity of the resource is fixed and known with certainty. While work is done by the government trying to estimate the total resource base, significant uncertainty inevitable remains. In this case the government may very well wish to hold some land rather than leasing, until exploratory activity on leased tracts reduces uncertainty.

The government, through leasing policy, is able to influence both the rate of exploration and the rate of production. Through the use of exploration incentives offered to the firm and by stipulating drilling requirements the government may influence the level of exploratory activity, as well as the demand for additional leases. Given the impact of these policies on the industry we may infer that the government is attempting to establish at least a minimum size for its resource base. Once it has a rough estimate of the resource it is in a better position with which to evaluate the optimal rate of production. Through prorationing schemes and export controls the government can influence the rate of utilization. Hence, our assumption that the government has already considered the timing implications prior to releasing acreage for development would appear to be valid.

Implications for Alberta's Leasing Policy

The changing nature of Alberta's oil industry, from an unexplored 'virgin' territory to a mature producing region, has had a pronounced effect on the structure of risk bearing both within the industry and between the industry and the government. In the initial stages of development, the industry was dominated by a small number of large firms. Royalty payments were small in relation to bonus bids and rentals. The industry was clearly bearing the greater share of risk. Given the size of the companies, their ability to 'pool' risk (they explore over a number of geological areas and thereby reduce the 'riskiness' of exploration in just one area), their diversification of holdings, and their ability to spread risk among a large number of share holders suggests that they were in a better position to bear risk than the government[2]. It is also possible that the industry is more optimistic than the government. In fact, it appears that industry estimates of present and future reserves correspond closely to those of the government[3]. Of course, there may be large differences in expectations on individual tracts of land.

As Alberta became a developed region much of the initial 'riskiness' decreased. This allowed the entry, into the industry, by a number of small firms. Today, there are many small firms in operation as well as large firms. One common method of spreading risk among both small

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[2]Joseph Stiglitz, "The Efficiency of Market Prices in Long-Run Allocations in the Oil Industry", in Studies In Energy Tax Policy, edited by Brannon, Ballinger Publishing Co., 1975, p. 68-69.

[3]National Energy Board, Canadian Oil Supply and Requirements, 1977, p. 9-25.

and large firms is through joint ventures.

The risk share of the government has changed as a result of the changing environment in the oil industry. In addition to the general decrease in 'riskiness' resulting from the development of the region, the wealth of the government has greatly increased due to increased revenues from the industry. These factors should allow for the government to increase its share of risk with the industry (assuming decreasing absolute risk aversion on the part of the government). Indeed, recent provincial behavior supports this elementary premise of risk theory.

First, through the introduction of exploration incentives to the industry the province is bearing some of the risk inherent in the exploration process. These incentives increase exploratory activity and thereby increase known reserves. Hence, through this policy the government is increasing its long-run expected revenue by assuming some of the risk of exploration. In effect, the government is making an indirect fixed payment to the industry. Second, we see that gross royalty rates have been progressively increasing over the years. Our model suggests that with the increase in royalty rates there will be a decrease in the bonus bids offered by the firm. Finally, the government has directly involved itself in the industry by the establishment of the Alberta Energy Company and through its equity particiaption in Syncrude.

As a final observation, notice that Alberta maintains a nearly uniform level of gross royalties across the province at any one time. Through bonus bidding the government attempts to capture the economic rent not gained by the royalty tax. Furthermore, the bonus bid is used to capture the difference in economic rent between two different leases. As suggested earlier, the more prolific leases would ideally involve more risk sharing by the government. At the very least, the government should have higher royalty rates and lower bonus bids on the more productive leases. This suggests that the government impliment a scheme whereby the firm submit a bid on the front-end payment as well as on the gross royalty rate. This would have the desirable effect of increasing expected government revenue on the better prospects. For reasons of economic efficiency, the scheme would involve the use of a net royalty rather than a gross royalty.

The next major step in work of this nature is to make the model dynamic. Such a model would provide us insights into the stability conditions of an optimal leasing policy. In particular, we could capture the effects on the optimum due to resource depletion, production decline, and the erosion of government and industry income.

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