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TEACHERS' AWARENESS OF VARIATION

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Here, we report on a study of teachers' evolving awareness of how they work with patterns of variation to structure and teach mathematics lessons. We identify a number of critical features regarding teachers' awareness of variation.

Keywords: Mathematical Knowledge for Teaching; Enactivism; Variation Theory of Learning; STEM; Design Experiments

The Math Minds Initiative is a five-year initiative exploring how critical use of high quality curriculum resources might help mathematics teachers support student achievement and confidence. As part of this work, we have identified three key principles as significant: (a) mastery learning, which emphasizes success-for-all, formative assessment, and enrichment (Guskey, 2010); (b) structured variation (Marton, 2015; Watson & Mason, 2006); and (c) intrinsic motivation (Blackwell, Trzesniewski, & Dweck, 2007; Pink, 2011). We treat these features as design principles that allow flexibility and responsiveness to evolving relationships between teachers, students, and mathematics. While the resources used in the initiative embody important features of variation, teachers must select from, highlight, and extend what is offered. Here, we propose five aspects critical to teachers' awareness of variation.

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THEORETICAL FRAMEWORK: THE VARIATION THEORY OF LEARNING

Our approach is based on Marton's (2015) Variation Theory of Learning (VTL). The core principle of the VTL is that *new understanding requires experiencing difference against a background of sameness, rather than experiencing sameness against a background of difference*. Note that *experience* of variation is of key significance here; just because certain elements are available to be experienced does not mean that they will be. This is consistent an enactivist perspective of knowing (*cf*. Simmt & Kieren, 2015), in that both enactivism and the VTL emphasize the significance of an observer and of perceptually guided action in bringing forth a world of significance. Marton has used the VTL to develop practical implications for directing attention in ways that make it more likely for learners to attend to particular aspects of a chosen object of learning. This offers a way to build sufficient commonality for learners' worlds of significance to interact and continue to co-evolve with each other and with the object of learning. The VTL has similarities with the theory of concept attainment elaborated by Bruner, Goodnow, and Austin, (1967), but it further elaborates particular patterns of variation for drawing learners' attention to aspects needed to perceive a particular situation in a particular way. These patterns include *separation* of key aspects (through contrast), *generalization* (through induction), and *fusion* (through combination). For example, to discern "green," we must *separate* it by *contrast* with other colours (which is easier when colour is the only thing that varies), *generalize* "green" to other objects that share that colour, then *fuse* it with other aspects that must be discerned *simultaneously* to perceive something in a certain way (e.g. "green" and "plant").

Clear focus on an object of learning is central to the VTL. Each such object has *critical* aspects that learners must discern and that teachers must present in a way that makes them distinguishable from the background and eventually simultaneously discernable. Marton (2015) and Runesson (2005) emphasized that variation offers an important lens for analyzing what is *possible* for students to learn, which may be different from what is *actually* learned.

METHODOLOGY

In this study we wanted to better understand how teachers' use of variation evolved over time. Our data were based on weekly classroom observations (typically by a single researcher) of six teachers' mathematics classes, ranging from Grade 1 to 6. As we observed, we attended particularly to (a) task sequences, (b) the ways teachers drew attention to variation within those tasks, and (c) student engagement with and responses to those tasks. Observations were structured such that each time the teacher presented a task for which student feedback was invited, it was recorded, along with visible student responses. Sometimes individual students would respond to a prompt, while at other times all students would respond in a way that the teacher could receive feedback via their responses (for example, answers written on mini-whiteboards, fingers in the air to indicate a particular number). In some cases, the teacher and a researcher met afterwards to discuss the lessons, with a particular focus on task sequence and student engagement. Task sequences were analysed in terms of the patterns of variation available for discernment in the sequences, for student responses to those patterns of variation, and for how student feedback was used to inform further evolution of the sequences. In the following section, we describe five features of teachers' awareness of variation that were identified through this analysis.

FINDINGS: CRITICAL FEATURES FOR AWARENESS OF VARIATION

To present our results, we elaborate on contrasting task sequences that draw attention to particular features of teachers' use of variation. In some cases we contrast a particular teacher's sequences at different points in time (with different objects of learning); in others we contrast the lesson-as-taught with a potential alternative. Through these contrasts, we wish to draw attention to five features that we deem critical to teachers' awareness of variation: (a) a focus on a clear object of learning; (b) attention to mathematical structure; (c) prompts that highlight change against a background of sameness; (d) efforts to draw attention to variation; and (e) a distinction between micro-*steps* and micro-*discernments*.

Clear Object of Learning

Figure 1 shows a task sequence used by Teacher 1 and a modified version of that sequence.

A	B
5+7=12 5+8=13 5+9=14 4+9=13 3+9=12 2+9=11 3+10=13 4+11=15 5+12=17 50+120 500+1200 5000+12000	5+7=12 5+8=13 5+9=14 4+9=13 3+9=12 2+9=11 3+10=13 4+11=15 5+12=17 105+12 = 117 205+12 = 217 305+12 = 317 405+112 = 517 505+112 = 617

Figure 1: What is the object of learning? (Teacher 1: Apr. 13, 2015)

In Sequence A in Figure 1, the intent was to *separate* the impact of each addend on the sum, to then *generalize* this effect, and finally to *fuse* the impact of varying both addends. In the last three items, however, the object of learning shifted to consider the effect of shifting place values. Sequence B offers an alternative that retains a focus on the original object of learning.

Attention to Mathematical Structure

Figure 2 shows two sequences used by the same teacher on different days. In Sequence A, spotting a pattern (Hewitt, 1992) took precedence over insight into mathematical structure, whereas in Sequence B, attention was drawn to the impact on the sum of altering both addends.

A	B
3 + 7 = 10 4 + 7 = 11 5 + 7 = 12 5 + 8 = 13 5 + 10 = 15 T: Predict the next answer without knowing the question.	0 + 7 = 7 1 + 6 = 7 2 + 5 = 7 S1: One side is adding 1 number and the other side is losing one number [many students started to extend the sequence at this point; S2 extended to 7+0] T: What happens to the sum? S3: The beginning is going up, the middle is going down, and the total staying the same! S2: There's nowhere left to count. [after 7 + 0] S3: You could switch to 8 – 1, then 9 – 2. T: Is there a limit? S1: There's no limit. S3: You could go up to 100! T: Here's a bonus: 53 + 15 = 68. What is 54 + 14? [several students extended to 68 + 0]

Figure 2: Pattern spotting vs. mathematical structure (Teacher 1: May 1, May 6, 2015)

In Sequence A, the teacher's focus on the patterns in the sums may be seen in her prompt to "predict the next answer without knowing the question." The discussion following Sequence B, on the other hand, explored the implications of one child's observation that when one addend goes up by 1 and other goes down by 1, the sum stays the same. After noting the structure of the initial sequence, several students spontaneously extended it beyond what the teacher had asked and one noted a way it might be continued past zero. Later, the teacher made a choice to offer a similar problem with larger numbers. Had the emphasis in Sequence B remained on "predict the next answer," it would have been a trivial exercise for the students to merely note the repeating "sum of 7" pattern.

Change against a background of sameness (one thing changing)

Figure 3 shows two sequences used by the same teacher at different times.

A	B
6 x 6 = 36 6 x 7 = 42 (another 6) 6 x 8 = 48 (another 6; most students successful) 8 x 8 = 64 (2 more 8s; several students stumped)	7 x 2 = 14 7 x 3 = 21 (another 7) 7 x 4 = 28 (another 7) 7 x 6 = 42 (two more 7s) 7 x 12 = 84 (twice as many 7s) 7 x 24 = 168 (twice as many 7s) 7 x 48 = 336 (twice as many 7s) 7 x 47 = 329 (one less 7) 7 x 23 = 161 (one less 7 than 7 x 24) 7 x 22 = 154 (one less 7) (high success and engagement)

Figure 3: Changing 2 things vs. changing 1 thing (Teacher 2: Mar. 30, May 1, 2015)

In Sequence A of Figure 3, the fourth item introduced two elements of variation that were not first varied on their own (i.e. not separated): There was a switch from increasing by 6s to increasing by 8s, and there was a switch from jumping by 1 multiple to jumping by 2 multiples. Had the sequence moved from 6 x 8 to 7 x 8 (highlighting a shift from another *6* to another *8*) and *then* to 8 x 8 (highlighting a shift from *one* more 8 to *two* more 8s), this may have been more effective. In Sequence B, the teacher offered a pattern of variation that separated the effects of adding or subtracting 7s and doubling the number of 7s. Once students can confidently work with such changes, they might work with both simultaneously (an example of *fusion*), perhaps by doubling *and* adding 7; e.g. 7 x 22 → 7 x 43. Or they might try varying either factor: 7 x 22 → 7 x 23 (one more 7) → 7 x 25 (two more 7s) → 8 x 25 (one more 25) → 12 x 25 (four more 25s) → 24 x 25 (twice as many 25s), etc.

Sequence A in Figure 4 shows a sequence used by Teacher 3.

A	B	C	D	E	F
2 + 3 = __	2 + 1 = __	4 + 2 = __	4 + 2 = __	3 + 4 = __	4 + __ = 7
6 = 4 + __	2 + 2 = __	__ = 4 + 2	__ + 2 = 6	7 = __ + 4	4 + __ = 6
7 = __ + 2	2 + 3 = __		4 + __ = 6	__ + 3 = 7	4 + __ = 5
4 + 4 = __	3 + 3 = __	__ = 3 + 5			3 + __ = 5
6 + 2 = __	4 + 3 = __	5 + 3 = ___	3 + 5 = __	__ = 2 + 3	2 + __ = 5
3 + 1 = __	5 + 4 = __		__ + 5 = 8	2 + __ = 5	
	6 + 5 = __		3 + __ = 8	5 = __ + 3	__ + 1 = 3
	5 + 5 = __				__ + 1 = 4
	4 + 5 = __				__ + 1 = 5
					__ + 2 = 5
					__ + 3 = 5

Figure 4: Separating critical features (Teacher 3: Nov. 18, 2015)

Here, we see evidence that the teacher was aware of critical features pertaining to students' understanding of equivalency in addition. This was evident both in her choice of examples that highlighted various patterns and in her use of verbal cues to draw attention to the features she wanted students to notice. Nonetheless, these features were not separated through contrast in the chosen task sequence. In the lesson leading up to Sequence A, the teacher reviewed the meaning of the equal sign and the plus sign, then worked with the entire class on a sequence intended to draw attention to the significance of the placement of the missing addend. When presented with Sequence A, students were asked to model each statement with counters, draw a picture of their model, and write a number sentence to represent the picture. While explaining the instructions, the teacher juxtaposed the various representations to highlight the connection between them and worked through one example of each kind.

For this task, representations changed while a particular addition statement was held constant. While most students quickly discerned the distinction between representations, the shifting position of the missing addend and equal sign remained difficult for a significant number of students. Some students struggled with even the first question in Sequence A (2 + 3 = ____); for them, Sequence B may have helped draw attention to the structure of the relationship between sum and addends prior to engaging with varying placement of the addends (Sequence C), the equal sign (Sequence D), and both (Sequence E). Sequence F offers possibilities for fusing ideas in B and D.

Attention to Variation

Teachers in our study further drew attention to variation by commenting on or asking children to consider what was changing and what was staying the same, and by visually juxtaposing particular features in a manner that allowed this sort of comparison—approaches similar to those described by Kullberg, Runesson, and Mårtensson (2014). Initially, Teacher 1 (Figure 1) would erase each item in her sequences prior to putting up another item. As she reflected on one lesson after class, she recognized that students would likely be better able to detect variation if they could see relevant examples one above the other. Similarly, positioning variations in such a manner that the aspects that were similar / different were visually aligned proved helpful in some cases. Teacher 3 was particularly attentive to highlighting significant changes with visual (e.g. colour) and / or verbal (e.g. tone) cues. Direct requests to attend to what changed and what stayed the same were also effective in some cases.

Distinguishing Micro-*steps* from Micro-*discernments*

The resource being used in the project places a high emphasis on breaking ideas into small steps, assessing at every step, having the class move together to greater challenges (with further extensions for those who need them), and stepping back if it becomes clear that too big a leap has been made. Invoking the language of variation has helped us to draw attention to the significance of micro-*discernments* rather than merely micro-*steps*. In other words, separating material into manageable pieces is not just about using smaller pieces or easier numbers; it is about creating the conditions necessary for discernment of critical features. While the resource often models this, it is not made explicit: Micro-discernments are themselves a critical feature that initially was not adequately separated from micro-steps. Although it is important to start where all students can engage, to assess continuously, and to adapt task sequences in response to student responses, it is also important that such adaptations offer meaningful mathematical variation, which is not defined by difficulty. Often, it is failure to distinguish critical mathematical features that impede learning, and simply using (for example) smaller numbers does not solve this problem.

SUMMARY

We have observed that combining the practices of mastery learning with careful attention to mathematical variation allows students to make important distinctions in mathematical understanding. Teachers' awareness of this variation is therefore of central importance to effective teaching, and we believe that the five critical features presented in this paper can inform teachers' daily practices. While the examples given here involved very elementary number sequences, our continuing work with teachers will also support the extension and combination of concepts to allow for consideration of increasingly complex problems.

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