Dynamic Price Quotation in a Responsive Supply Chain for One-of-a-Kind Production

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Abstract

This paper studies the setting in which a one-of-a-kind production (OKP) firm o ers two types of orders (due-date guaranteed and due-date unguaranteed) at di erent prices to the sequentially arriving customers, who are also OKP production firms. The prices for two types of orders are quoted to each customer on its arrival. We study two problems in this setting. First, we model a dynamic pricing strategy (DPS) and compare our DPS with a constant pricing strategy (CPS). Through a numerical test, we show that both the firm and its customers are better o when our DPS is employed, so that the DPS improves overall performance of the supply chain. Through an industry case, a custom window production firm, we show how to apply the proposed DPS when products are complex. We also develop a method to adaptively estimate the firm's available capacity, the number of future arrivals and the distributions of the customers' willingness to pay and impatience factor. The simulation result shows that, when multiple distribution parameters are unknown, the proposed parameter estimating method results in estimates close to the true values.

Keywords: Dynamic pricing, make-to-order, parametric learning, capacity management, responsive supply chain, one-of-a-kind production

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1. Introduction

Constrained by global competition in prices and costs, the large firms have limited incentives to provide high-variety customization (Cavusoglu et al. 2007), which becomes the core competence of local small or medium sized enterprises (SMEs). One-of-a-kind production (OKP) is a strategy that allows local firms to provide highly customized products at an effective production rate. OKP features a "once" successful approach for product development and production according to specific customer requirements, and accordingly no prototype or specimen is made (Tu et al. 2000). The firm usually does not keep inventory of some inputs (e.g., parts) and only makes orders on demand. As a consequence, this requires that the entire supply chain provides fast and reliable delivery.

We study the OKP supply chain, in which the customers of an OKP firm are also OKP firms. An OKP supply chain is characterized by SMEs, batches of orders that are small or just one item, make-to-order production, downstream firms that cannot guarantee future orders to upstream firms, and detailed requirements that change from order to order. Because of these features, it is difficult for firms within the same OKP supply chain to form a reliable cooperative relationship through contract or integration based on revenue and information sharing (Giannoccaro and Pontrandolfo 2004, Disney et al. 2008, Kelle et al. 2009). In this work, we examine supply chain coordination without revenue and information sharing.

Because it is usually the case that customers in a supply chain are heterogenous in leadtime requirements, different types of customers should be treated with different priority to realize coordination. However, as an OKP firm usually receives discrete orders which arrive sequentially, then the problem is to allocate capacity to customers from different priority classes. That is, whether to allocate capacity to the current customer or save it for future arrivals that might be from higher priority classes and hence generate higher profits (Keskinocak and Tayur 2004). In the literature, a well-accepted method to solve this problem is price differentiation between priority classes. The research in pricing under sequentially arriving customers starts from the pricing problem in a priority queue. That is, the customers pay different prices for different positions within the queue, but the firm does not guarantee the due dates e.g., (Kleinrock 1967, Adiri and Yechiali 1974, Mendelson and Whang 1990, etc). A detailed review of the research on priority pricing can be found in Bitran and Caldentey (2003).

We focus on the case where some customers have strict requirements on leadtime guarantee while others do not. In this case the firm offers two types of orders: due-date-guaranteed (G-order) and due-date-unguaranteed (U-order), and only the G-orders are promised due-date delivery. We compare two pricing strategies: a dynamic pricing strategy (DPS) and a constant pricing strategy (CPS). G-orders and U-orders are priced differently, but in DPS the firm dynamically changes the price for each type of order to maximize its profit. That is, the price quotes to a customer are also determined by its arrival time. We analyze how the supply chain benefits when each firm prices each order accounting for the order's arrival time together with the firm's production capacity and schedule. To match the DPS for larger-scale problems, we also introduce a periodic pricing strategy (PPS), which is a compromise between the features of DPS and CPS. Our results suggest a promising approach to integrate (or coordinate) a two-echelon supply chain with no dominant echelon.

A number of articles on dynamic pricing can be found in the literature (Yano and Gilbert

2005), but most research focus on pricing in make-to-stock (MTS) production, e.g., Transchel and Minner (2009), Ray et al. (2005), etc. In contrast, we use the Bellman equation (Bellman 1957) to compute the price quote for OKP make-to-order firms. The earliest work we have found on pricing using Bellman equations is Kinacaid and Darling (1963), and more recent studies used similar methods to model supply chain dynamics, e.g., Stadje (1990), Gallego and Ryzin (1994) and Zhao and Lian (2011),etc. Using numerical analysis, we compare price quotes under the two different pricing strategies. The results show that in large-scale problems the CPS prices can be a good approximation for DPS prices. We explain the application of the proposed DPS through an industrial case from a custom window producing firm. In the case study, we propose a method to evaluate the firm's available capacity and future order arrivals, which we require to compute prices. We chose a custom window manufacturer for case study because it is a typical OKP firm whose products have hierarchical structures and are processed through multiple production lines. The DPS studied in this work can also be applied in other OKP firms where leadtime is a factor of quality but the leadtime is constrained by the firm's capacity, e.g. molding companies, high-tech component companies, or service providers such as transportation companies.

We also develop a set of practical parameter estimation methods for our proposed pricing strategy. In the literature, most authors assume that the distribution of the customers' "impatience factors", which can be defined as how much it costs a customer for each time unit that the lead time of its order is increased, is exogenous and known by the firm. Based on this distribution of impatience factors, optimal prices are computed following the discipline of "third degree price discrimination"(Perloff 2009). However, in practice, this impatience factor is usually hard to measure or obtain. In the supply chain management literature the distribution of random variables are usually obtained from a learning process. For example, Chen and Plambeck (2008) develop a learning method to obtain the probability of a customer choosing a substitute, and Tomlin (2009) uses a Bayesian learning process to dynamically update the supplier's yield distribution. In our model, we develop a maximum-likelihood-estimation (MLE) based learning method to estimate the distribution of the customer's willingness to pay (WTP) as well as the distribution of the impatience factor. There are also several articles that focus on estimating the distribution of the customer's WTP. Bishop and Heberlein (1979), Hanemann (1984) and Cameron (1988) proposed methods to estimate the mean of the customer's WTP when the distribution type is known. Kriström (1990) studied the case where the distribution type is not known and proposed a non-parametric estimator of the distribution of the customer's WTP, which requires larger sample sizes. Due to sample size limitations in OKP supply chains, we extend the previous literature and study the case where the distribution type is known, but the distribution parameters, such as the mean and variance of a normal distribution, must be estimated. This is realistic when the firm has some rough information on its customers' WTP distribution. As for the customer's impatience factor, we have not found any work studying the estimation of its distribution.

Our article is organized as follows. In Section 2, we describe the problem and define the notation and assumptions. In Section 3 a dynamic pricing method is presented with a polynomial algorithm to find the optimal solution. In Section 4, a CPS is presented. In Section 5, we compare the firm and its customers' welfare under the two different pricing strategies. In Section 6, we propose a method to estimate parameters required to compute the dynamic prices. In Section 6.1, we present an industrial case to show how to evaluate the firm's available capacity and future customer arrivals, while in Section 6.2, a learning process is designed to estimate the distribution of the a customer's WTP and impatience factor. In Section 6.2, we discuss the management insight of this research by referring a practical case study. Section 8 contains final remarks and a summary of future work.

2. Notation and Assumptions

We study an OKP firm which accepts two types of orders, G-orders and U-orders. These orders are for the next production period as shown in Figure 1. Our problem is to decide the optimal price quote for each type of order. Every newly received G-order is guaranteed delivery by the end of the next period. To guarantee the promised due date of the G-orders, the firm dispatches a higher priority to the G-orders in production. The quantity of unallocated capacity is used to guarantee the delivery of G-orders. We use the term *available capacity* to represent the quantity of unallocated capacity. The firm does not accept G-orders when it has no available capacity. In addition, the firm stops accepting G-orders at the beginning of the next period, which we name as *deadline*. After the deadline, the production schedule in that period is frozen. The firm does not allow new orders to be inserted into a frozen schedule because at the beginning of each production period, the firm needs to reallocate the available resources, e.g., the number of workers at each machine, internal and external logistics, etc. Inserting an order usually incurs extra cost. The superiority of freezing production schedules has been proven by Sridharan et al. (1987).

[Figure 1 about here.]

Notation. We denote the two different price quotes the firm offers for G-order and U-order by $p^G, p^U : p^G, p^U \in \mathbb{R}^+$, respectively. The available capacity, which represents the number of capacity units (i.e., man-day, man-hour, etc), is denoted by $m : m \in \mathbb{Z}^+$. The number of future arrivals before the deadline is denoted by $n : n \in \mathbb{Z}^+$, which is adaptively estimated, noting that each arrival does not necessarily result in an accepted order. (m, n) represents the case in which the firm has m available capacity and expects n future arrivals.

We use $r : r \in \mathbb{R}$ to represent a customer's WTP for a G-order. A customer's WTP is determined by two factors, i.e., its valuation on the firm's product and the substitutes from the firm's competitors. Supposing that a customer values the firm's product at $V : V \in \mathbb{R}$ and the profit it can obtain by choosing the best substitute is $S : S \in \mathbb{R}^+$, then we define a customer's WTP as $r = V - S$. Because V and S are both random variables, then r is a random variable. We use $v : v \in \mathbb{R}^+$ to represent the customer's impatience factor. We define the impatience factor as the cost incurred to the customer when there is no due-date guarantee. Without loss of generality, we use $f(r, v)$ to denote the joint probability density function (JPDF). We also use the notation $f(r, v; \vec{\theta})$ to represent the JPDF of r and v when the form of the distribution functions depends on the vector $\vec{\theta}$.

The adaptive control process is described as in Figure 2. As shown in Figure 2, the dynamic pricing module computes the prices p^G and p^U, and then the firm quotes the prices to arriving customers. The sample collecting module gathers customers' choices and their arrival rate, and the production monitoring module monitors the workload of the firm's each production line in real time. The sample collecting module and the production monitoring module periodically pass the information to the parameter estimating module. Based on the current firm's production status, customer arrival rate and customer choices, the parameter learning module adjusts the estimation of m, n, $\vec{\theta}_r$ and $\vec{\theta}_v$, and then passes the new estimators back to the dynamic pricing module.

[Figure 2 about here.]

Assumptions. We make the following assumptions to form our model:

Assumption 1. *The firm makes a take-it-or-leave-it offer.*

We assume that the firm makes price quote to each customer. If the customer accepts the price quote, then it places the order; otherwise the customer leaves and does not come back. A similar assumption can be found in previous research, e.g., Gallego and Ryzin (1994), where they assumed that customers do not act strategically by adjusting their buying behavior in response to the firm's pricing strategy. This is also common in practice. As an OKP firm usually keeps little or no parts inventory and only makes orders on demand, but when the firm orders parts, it requires fast delivery. Therefore, if the customer does not accept the current offer, then a substitution has to be found immediately and hence long term strategic behavior does not happen.

Assumption 2. *The processing time of an order is constant.*

Here we assume that the workload of a single order is constant and equal to unity, which we take as a capacity unit. This assumption is consistent with the characteristic of OKP that the batch size of an order is small, or even just a single unit. For the case where the orders are heterogeneous in processing time, we can approximate the optimal prices through our model, and the method is shown in Section 6.1. We abstract from the issues of differing set-up costs between orders.

Assumption 3. *The variable cost of production is zero.*

As mentioned in earlier, we treat the labor cost as a fixed cost, that is, an added order does not incur additional labor cost. We do not consider material cost as the pricing strategy is our focus recognizing that the problem can be easily generalized by subtracting a constant unit production cost from the unit price. A similar assumption can be found in the literature on production planning and scheduling when pricing is considered, for example, Chen and Hall (2010) and Deng and Yano (2006). There is little loss of generality as with a constant processing time, we can take the variable cost of each order to be fixed, and reinterpret prices as net of costs.

3. Dynamic pricing strategy (DPS)

Our setting can be regarded as an extension of the literature in dynamic pricing, which studies cases in which a firm has a stock of goods to dispose of within a specified time, potential customers arrive sequentially and stochastically, and the probability distribution of prices they are willing to pay is known. The firm sets prices so as to maximize expected cash receipts during the sale, recognizing that unsold items are worthless to the firm (Bitran and Caldentey 2003). Our setting is similar because if the available capacity is not fully allocated after the beginning of a production period, then it is worthless. We use the Bellman equation to compute the optimal price quotes when the firm offers two types of orders.

Suppose that a customer arrives and receives price quotations p^G for G-order and p^U for U-order. If the customer chooses a G-order, then its net gain, denoted by ξ^G, is $\xi^G = r - p^G$. Otherwise, if the customer chooses a U-order, then its net gain, denoted by ξ^U, is $\xi^U = r - p^U - v$. The concept of "net gain" associates with the price-setting firm. If a customer has positive net gain by purchasing from the target firm, it means the net profit to be obtained from the target firm is higher than the one to be obtained from the best offer of other firms (outside options). We define the term "absolute gain" as the difference between the customer's valuation of the product and the product's price. Then the net gain equals to the difference of the absolute gains the customer makes by choosing the price-setting firm's product rather than its best outside option. The net gain measures the difference between the net profits that can be obtained from a supply chain with the target firm and a supply chain without the target firm. The customer chooses the option that creates the higher net gain and only purchases when its net gain is non-negative. Otherwise it leaves without purchasing and its net gain (from the firm) is zero. First, we compute the probabilities of the customer choosing each type of order.

The customer chooses the G-order only if $\xi^G \geq 0$ and $\xi^G \geq \xi^U$. Thus, given p^G, p^U and the distribution of r and v, the probability of the customer choosing the G-order, denoted by $\mathbb{P}_G(p^G, p^U)$, can be obtained as

$$\begin{aligned}
\mathbb{P}_G(p^G, p^U) &= \text{Prob}(\xi^G \geq 0 \text{ and } \xi^G \geq \xi^U) \\
&= \text{Prob}(r - p^G \geq 0 \text{ and } r - p^G \geq r - v - p^U) \\
&= \int_{p^G}^{+\infty} \int_{p^G - p^U}^{+\infty} f(r, v) dv dr \qquad (1)
\end{aligned}$$

The customer chooses the U-order under two circumstances, i.e., $\xi^G \geq 0$ and $\xi^G < \xi^U$, or $\xi^G < 0$ and $\xi^U \geq 0$. Thus, given p^G, p^U and the distribution of r and v, the probability of the customer choosing the U-order, denoted by $\mathbb{P}_U(p^G, p^U)$, can be obtained as

$$\begin{aligned}
\mathbb{P}_U(p^G, p^U) &= \text{Prob}(\xi^G \geq 0 \text{ and } \xi^G < \xi^U) + \text{Prob}(\xi^G < 0 \text{ and } \xi^U \geq 0) \\
&= \text{Prob}(r - p^G \geq 0 \text{ and } r - p^G < r - v - p^U) \\
&\quad + \text{Prob}(r - p^G < 0 \text{ and } r - v - p^U \geq 0) \\
&= \int_{p^G}^{+\infty} \int_0^{p^G - p^U} f(r, v) dv dr + \int_{p^U}^{p^G} \int_0^{r - p^U} f(r, v) dv dr \qquad (2)
\end{aligned}$$

The Bellman equation used to compute the optimal price quotes is based on the probabilities obtained from (1) and (2).

Suppose that when a customer arrives, the firm faces an (m, n) case. For clarity of expression, we let n include the current customer. We examine the optimal price quote under two different situations: when capacity is greater or equal to the number of future arrivals, and when capacity is less, i.e., $m \geq n$ and $m < n$.

When $m \geq n$, the firm estimates that the number of future arrivals will not exceed the current available capacity. Then the (m, n) case can be treated as a straightforward uncapacitated problem. Because there is no capacity constraint, a single arrival has no impact on the price quotation to the other arrivals. Thus, all the arrivals are quoted the same price. The firm optimizes the price quotation by maximizing the expected profit contributed by each customer. Thus the optimal price quotation for problem (m, n), denoted by $\{p^G_{mn}, p^U_{mn}\}$, can be obtained by

$$\{p^G_{mn}, p^U_{mn}\} = \arg\max_{\{p^G, p^U\}} \Big[p^G \mathbb{P}_G(p^G, p^U) + p^U \mathbb{P}_U(p^G, p^U)\Big], \tag{3}$$

which can be solved through the first-order conditions. Because there are n future arrivals, the maximum expected total profit, denoted by π^m_n, is

$$\pi^m_n = n\big[p^G_{mn}\mathbb{P}_G(p^G_{mn}, p^U_{mn}) + p^U_{mn}\mathbb{P}_U(p^G_{mn}, p^U_{mn})\big]. \tag{4}$$

When $m < n$, the (m, n) case is a constrained problem. Because the price quoted to a later customer depends on the behavior of earlier customers, we use a Bellman equation to solve the optimal price quote for the current customer. The impact of the current customer is summarized as follows:

- If the current customer chooses the G-order, a unit of available capacity is allocated. The firm earns p^G and then faces an $(m - 1, n - 1)$ problem;
- If the current customer chooses the U-order, it does not affect available capacity preserved for G-orders. Thus, the firm earns p^U and then faces an $(m, n - 1)$ problem;
- If the current customer does not choose either type of order, then the firm earns no profit and faces an $(m, n - 1)$ problem.

Considering the three possibilities, the optimal price quote for the current customer, $\{p^G_{mn}, p^U_{mn}\}$, can be obtained by solving the following Bellman equation:

$$\begin{aligned}\pi^m_n &= \max_{\{p^G, p^U\}} \Big[\mathbb{P}_G(p^G, p^U)[p^G + \pi^{m-1}_{n-1}] + \mathbb{P}_U(p^G, p^U)[p^U + \pi^m_{n-1}] \\ &\qquad + [1 - \mathbb{P}_G(p^G, p^U) - \mathbb{P}_U(p^G, p^U)]\pi^m_{n-1}\Big],\end{aligned} \tag{5}$$

where π^m_n can be solved recursively.

When the firm has no available capacity, the customers can only purchase U-orders. Thus π_n^0 for any n can be obtained as

$$\begin{aligned}\pi_n^0 &= \max_{p^U} np^U \text{Prob}(\xi^U \geq 0) \\ &= \max_{p^U} np^U \text{Prob}(r - v - p^U \geq 0) \\ &= \max_{p^U} np^U \int_{p^U}^{+\infty} \int_0^{r-p^U} f(r,v)dvdr.\end{aligned}$$

The example in Figure 3 illustrates the process of solving for expected profit when available capacity is 3 and the number of future arrivals is 5, π_5^3, through recursion. To solve each π_j^i where $i \leq m$ and $j \leq n$, π_{j-1}^i and π_{j-1}^{i-1} are first solved sequentially. Note that the values of π_2^1 and π_3^2 are known when accessed the second time. Thus, a "tabu list" can be employed to substantially reduce the computational time. We preset an $m \times n$ array as a tabu list to store every solved value of π_j^i. At the beginning of each recursion for computing π_j^i, we check the tabu list first and see if it is already solved. If π_j^i is solved already, then the current recursion stops and directly returns the solution stored in the tabu list. In Figure 3, each node represents a recursion, and there are 13 recursions in the entire computation process.

[Figure 3 about here.]

Let $T(m,n)$ be the number of recursions required in computing $\pi_n^m|_{n>m}$ when the tabu list is incorporated. Then by analyzing the branched figure as in Figure 3, it is not difficult to obtain:

$$T(m,n) = 2m(n-m) + 1 \in O(mn). \tag{6}$$

From (6), we can conclude that when the tabu list is employed, $\pi_n^m|_{n>m}$ can be computed within polynomial time, which means that solving p_{mn}^G and p_{mn}^U is not an NP-hard problem.

4. Constant pricing strategy (CPS)

In this section we present a CPS in which the firm does not adjust the price quoted to the customers, and then we compare the prices obtained by the CPS and our DPS.

Suppose with a CPS, the firm quotes prices p^G and p^U to every arriving customer. The probability of a customer choosing the G-order is $\mathbb{P}_G(p^G, p^U)$, and so the number of customers that choose the G-order follows a binomial distribution with n trials, each of which yields success with probability $\mathbb{P}_G(p^G, p^U)$. As the number of G-orders is constrained by m, then the expected number of G-orders, denoted by $N_G(p^G, p^U)$, is obtained as

$$N_G(p^G, p^U) = m\big[1 - B\big(m; n, \mathbb{P}_G(p^G, p^U)\big)\big] + \sum_{i=1}^{m} ib\big(i; n, \mathbb{P}_G(p^G, p^U)\big), \tag{7}$$

where $b(k; n, Pr)$ and $B(k; n, Pr)$ are respectively the pmf (probability mass function) and the cdf of k when k is a random integer following a binomial distribution with n trials and success probability Pr.

Similar to the G-orders, the number of customers that choose the U-order follows a binomial distribution with n trials, each of which yields success with probability $\mathbb{P}_U(p^G, p^U)$. Note that besides the customers that initially prefer U-orders, some customers choose U-orders because the firm has no available capacity. Thus, the expected number of U-orders, denoted by $N_U(p^G, p^U)$, is obtained as

$$N_U(p^G, p^U) \quad = \quad n\mathbb{P}_U(p^G, p^U) + \sum_{i=m+1}^{n} [i-m] b\big(i; n, \mathbb{P}_G(p^G, p^U)\big) \kappa(p^G, p^U), \tag{8}$$

where $\kappa(p^G, p^U) = \big[\int_{p^G}^{+\infty} \int_{p^G-p^U}^{r-p^U} f(r,v) dv dr\big] / \mathbb{P}_G(p^G, p^U)$. κ computes the probability that a customer's net gain from choosing a U-order is positive, given that it initially prefers a G-order.

Based on (7) and (8), we can solve the optimal p^G and p^U by

$$\max_{p^G, p^U} p^G N_G(p^G, p^U) + p^U N_U(p^G, p^U). \tag{9}$$

We present a numerical analysis to compare the prices obtained from the DPS and CPS. In the numerical analysis, we set r and v to be distributed following three different distributions as shown in Figure 4. In the case where r and v are independent, as in Figure 4(a), the pdf of each random variable is not affected by the value of the other random variable. In Figure 4(b) and Figure 4(c), v and r are correlated. We use the distribution in Figure 4(b) as an example where r and v are positively correlated because the expected value of v is increasing in the value of r, and use the distribution in Figure 4(c) as an example where r and v are negatively correlated because the expected value of v is decreasing in the value of r. In the independent case, we set r and v to be both distributed from $U(0, \alpha_1)$, while in the positively correlated and negatively correlated cases, we set the r and v to be jointly distributed from the uniform distribution within the area as shown in Figure 4(b) and Figure 4(c). We set $\alpha_1 = 10$, $\alpha_2 = 0.8$ and $\alpha_3 = 0.8$ for the numerical test. In the three cases, we set $m/n = 2$. We use numerical methods to solve p^G and p^U from (9). In order to avoid the complexity incurred from computing the pdf and cdf of a binomial distribution, we approximate the binomial distribution with normal distribution when n is large ($n \geq 30$).

[Figure 4 about here.]

The prices for G-order and U-order under different problem scales in the three cases are displayed in Figure 8. We observe that when the problem scale is small (m and n are small), the difference between two pricing strategies is substantial. The results also show that with increased problem scale, the stochastic problem approaches to the deterministic problem. That is, for both DPS and CPS, the price quotes converges to the optimal solution of deterministic problem.

$$\max_{p^G, p^U} p^G \mathbb{P}_G(p^G, p^U) + p^U \mathbb{P}_U(p^G, p^U), \text{ Subject to: } n\mathbb{P}_G(p^G, p^U) \leq m.$$

[Figure 5 about here.]

In the numerical analysis, we can also observe that when the problem scale is larger than a certain point, the difference between the prices obtained in DPS and CPS is very small. Under the constraint of constant pricing, computing the CPS prices can be much more efficient. Thus, based on the results of numerical analysis, we can conclude that CPS prices can be a good approximation for DPS prices when the problem scale is large.

Although in large scale problems the CPS price is a good approximation for DPS prices, the welfare of the firm and of some customers might be significantly different because in CPS the firm does not change price as time passes. Thus, the optimal price quote at any time may deviate from the original setting. In practice, a more common pricing strategy, a periodic pricing strategy (PPS), can be found where the firm changes the price setting periodically when the problem scale is large and it is not feasible to dynamically compute a price quote for every customer arrival. In our version of PPS, we assume that the firm adjusts price quote each time the new customer arrivals reach a fixed number, which we name as the "pricing interval". In the next section, we also investigate performance of the supply chain under different settings of pricing interval in PPS.

5. Welfare analysis

In this section, we analyze the firm's profit and the customers' welfare when different pricing strategies are employed.

5.1. The firm's profit in DPS and CPS

The firm's expected profit can be obtained through (4), (5) and (9). In principle, the DPS should always yield weakly greater profits for the firm because the dynamic price could be set to a constant. The gap between the firm's profits obtained from our DPS and CPS in the three distribution cases are shown in Figure 8. In Figure 8, the curves show the percentage by which the expected profit obtained from our DPS is higher than the one obtained from the CPS. We observe that when the expected value of v increases, the superiority of DPS is more substantial.

[Figure 6 about here.]

5.2. The customer's net welfare obtained from the price-setting firm in DPS and CPS

We define the customers' net welfare obtained from the price-setting firm as the difference of welfare obtained from a supply chain with the price-setting firm and the one from a supply chain without the price-setting firm. The customers' net welfare in case (m, n) is denoted by W_n^m. In an extreme case, if all no customers buy from the target firm, the customers' net welfare (from the price-setting firm) is zero.

First we compute any arriving customer's expected net gain which can be divided into three parts.

1. If $r \geq p^G$ and $v \geq p^G - p^U$, then the customer chooses the G-order. The expected net gain, denoted by $\xi_1(p^G, p^U)$, can be computed as

$$\xi_1(p^G, p^U) = \int_{p^G}^{+\infty} \int_{p^G - p^U}^{+\infty} [r - p^G] f(r, v) dr.$$

2. If $r \geq p^G$ and $v < p^G - p^U$, then the customer chooses the U-order. The expected net gain, denoted by $\xi_2(p^G, p^U)$, can be computed as

$$\xi_2(p^G, p^U) = \int_{p^G}^{+\infty} \int_0^{p^G - p^U} [r - v - p^U] f(r, v) dv dr.$$

3. If $p^U \leq r < p^G$ and $r - v \geq p^U$, then the customer chooses the U-order. The expected net gain, denoted by $\xi_3(p^G, p^U)$, can be computed as

$$\xi_3(p^G, p^U) = \int_{p^U}^{p^G} \int_0^{r - p^U} [r - v - p^U] f(r, v) dv dr.$$

When the customer's r and v are not within the ranges specified above, it does not purchase and its net gain is zero. Based on the expected net gain of each arriving customer, the customers' net welfare can be obtained as

$$\begin{aligned} W_n^m \quad = \quad & \xi_1(p_{mn}^G, p_{mn}^U) + \xi_2(p_{mn}^G, p_{mn}^U) + \xi_3(p_{mn}^G, p_{mn}^U) \\ & + \mathbb{P}_G(p_{mn}^G, p_{mn}^U) W_{n-1}^{m-1} + [1 - \mathbb{P}_G(p_{mn}^G, p_{mn}^U)] W_{n-1}^m. \end{aligned} \tag{10}$$

In (10), the first line computes the welfare of the current customer when price quotes are p_{mn}^G and p_{mn}^U, respectively, while the second line computes the total welfare of later arriving customers.

When $m \geq n$, all customers are quoted the constant price regardless of the pricing strategy. Thus, the customers' net welfare can be obtained as

$$W_n^m = n[\xi_1(p^G, p^U) + \xi_2(p^G, p^U) + \xi_3(p^G, p^U)], \tag{11}$$

where p^G and p^U are computed by (3).

When there is no available capacity, $m = 0$, the customers can only choose U-orders, and then the customers' net welfare is

$$W_n^0 = n \int_{p^U}^{+\infty} \int_0^{r - p^U} [r - v - p^U] f(r, v) dv dr. \tag{12}$$

In order to obtain the customers' net welfare in DPS and CPS, we substitute p_{mn}^G and p_{mn}^U in (10), (11) or (12) with the prices computed under DPS and CPS for case (m, n). Note that in CPS, p_{mn}^G and p_{mn}^U are constant during the recursion.

Figure 8 shows the percentage by which the customers' net welfare obtained by DPS is higher than the one obtained by CPS. We observe that in both the independent and correlated cases, the customers' net welfare obtained by DPS is higher. We can also observe that when the mean of v increases, the customer benefit more from DPS. This is because the CPS increases the chance that the price is underestimated or overestimated when the future supply-demand ratio (m/n) changes. If the future m/n decreases, as mentioned in Section 4, p^G should increase while p^U should decrease, and hence in CPS G-orders will be underpriced and U-orders will be overpriced. When this happens, customers that arrive early and purchase G-orders, are better off. However, despite the customers that are better off, the net gain of the customers that purchase U-orders are reduced, and also because the G-orders are underpriced, it increases the risk that the firm does not have enough available capacity for the late arriving customers that would purchase G-orders. On the other hand, if the future m/n decreases, G-orders will be overpriced and U-orders will be underpriced when CPS is employed. In this case, customers that purchase U-orders will be better off, but the net gain of the customers that purchase G-orders will be reduced.

[Figure 7 about here.]

Consider an extreme case in which there are 1 unit available capacity and 2 arriving customers whose r and v are both distributed from $U(0, 10)$ as shown in Table 1. We compute the price quotes for each customer when DPS and CPS are employed. We can observe that although there is a reduction for the welfare of the customer who arrives earlier, the increase of the welfare of the customer who arrives later leads to the increase of the total customer welfare. As the firm's expected profit also increases in DPS, we conclude that the welfare for the entire supply chain is increased.

[Table 1 about here.]

Because in DPS, both the firm's and the customer's net welfare are increased, the net welfare of the global supply chain is increased, and that is the value of DPS over CPS. In the following section, we propose a learning method to estimate the required parameters. These parameters are usually hard to be measured or observed in practice.

5.3. The customers' absolute welfare obtained from the price-setting firm in DPS and CPS

We define the customers' absolute welfare as the welfare obtained by the customers supposing all the customers choose their outside options after rejecting the price-setting firm. Thus, when computing the customers' absolute welfare, a customer who rejects the price-setting firm may obtain positive profit.

We use a numerical test to show the customers' absolute welfare in both DPS and CPS. In the numerical test, we suppose that by choosing their best outside option, a customer's absolute gain, S, is distributed from $U(0, 5)$. We set a customer's valuation of the price-setting firm's product, R, to be distributed from $U(5, 10)$. Supposing that a customer's impatience factor, v, is distributed from $U(0, 10)$, then based on the definition of a customer's WTP, $r = R - S$, in Section 2, we have $f(r, v)$ as in Figure 8. If a customer chooses G-order, then its absolute gain is $R - p^G$; if it chooses U-order, then his absolute gain is $R - p^U - v$; if it rejects the price-setting firm, then its absolute gain is S. It is straightforward that without the price-setting firm, the absolute welfare of n customers would be $nE(S)$, where $E(S)$ is the expected value of S.

[Figure 8 about here.]

We keep $n/m = 2$ and compare the customers' absolute welfare for problems at different scales. Due to the complexity of computing the customers' absolute welfare, we evaluate the customers' absolute welfare based on simulation. That is, we simulate the case where a number of customers arrives sequentially. In DPS, each customer are quoted prices based on current m and n, while in CPS, each customer are quoted the same prices computed for the first customer. By summation, we can obtain the customers' total absolute gain. We repeat the simulation experiment for 100 times, and then the mean of the total absolute gain in each repetition approximately equals to the customers' absolute welfare.

The gap between customers' absolute welfare using DPS and CPS under different problem scales is shown in Figure 9. The dashed line in Figure 9 is the power trendline generated by MS Excel. It can be observed that in DPS the customers' absolute welfare is higher the one in CPS, which is consistent with the result of comparison of customers' net welfare between in DPS and in CPS.

[Figure 9 about here.]

5.4. The relationship between pricing interval and customer welfare

In large scale problems a compromise pricing strategy, PPS, is often employed. We examine the relationship between the price setting interval and the customer's welfare is studied through a simulation experiment.

In the simulation experiment, we set a customer's WTP and impatience factor to be both identically and independently distributed from $U(0, 10)$. We set $m = 250$ and $n = 500$ to simulate a large scale problem. We show in Figure 10 the percentage by which the customers' net welfare obtained by PPS is higher than the one obtained by CPS under different settings of the pricing interval, denoted by l. From Figure 10, we can see that the PPS increases the customer's welfare, and the superiority of l decreases when l is increased. We observe in Figure 10 that when l is less than 100, the change of the superiority of PPS is not very significant. This observation indicates that in practice, the firm can increase the pricing interval within certain range without significantly affecting the customer welfare.

[Figure 10 about here.]

6. Parametric estimation for dynamic pricing

As stated in the description of the control system in Section 2, the parameter learning module needs to adaptively change the estimates of m, n and other parameters related to the distribution of r and v.

6.1. Estimate available capacity (m) and future arrivals (n)

In this work, we consider a case where the overall production system within the firm is composed by many subsystems, and the overall demand has various requirements for capacity of each subsystem. We use an industrial case from a custom window manufacturer, which has been referred to in a previous study (Hong et al. 2010), as an example to demonstrate the process of identifying the estimators of m and n.

[Figure 11 about here.]

As introduced by Hong et al., the design of a custom window can be described by an AND-OR tree as shown in Figure 11. For the purpose of clarity, We use $N\#$ to name a node in the tree. When a node of the tree, say node N, is chosen, the customer then need to choose its child node(s). Let the set of N's direct child nodes be $DC(N)$, and then the conditional probability of choosing a child node $e : e \in DC(N)$ is denoted by $PROB_e$. If $DC(N)$ is dominated by an AND relationship, then $PROB_e = 1\ \forall e \in DC(N)$. If $DC(N)$ is dominated by an OR relationship, then $\sum_{e \in DC(N)} PROB_e = 1$

We suppose that any node in the AND-OR tree, N, must be processed by a production line, which is denoted by L_N. Note that the production line here can be a virtual production line. For example, $N1321$ and $N1322$ are just nodes representing feature options, no real production line is assigned to these nodes. The window production is accomplished through five real production lines, of which three produce frames (i.e., wood, metal and vinyl), one cuts glass, and one is for final assembling. In Figure 11, each node which requires a real production line is marked with '*'.

Here we define two types of available capacity relating to a node (N), i.e., the available capacity of the production line, denoted by CL_N, and the overall available capacity of node, denoted by C_N.

Suppose that the maximum workload that can be processed on L_N in a production period is $MAXW_N$, and the current total workload of the jobs for G-orders to be processed on L_N is $CURW_N$. Then if N is a node with '*', CL_N can be obtained as

$$CL_N = \frac{MAXW_N - CURW_N}{MAXW_N}$$

If N is a node with no '*', since no real production line is required, then $CL_N = 1$.

C_N is mutually determined by CL_N and the available capacity of N's direct child nodes. C_N can be obtained as

$$C_N = \min\{CL_N, \sum_{e \in DC(N)} PROB_e C_e\} \text{ if } DC(N) \text{ is dominated by OR relationship,} \tag{13}$$

$$\text{or } C_N = \min\{CL_N, \min_{e \in DC(N)} C_e\} \text{ if } DC(N) \text{ is dominated by AND relationship.}$$

In (13), we use $PROB_e$ as the weight of C_e when computing C_N because when the customers are more prone to choose child node e, then e is more critical when computing the overall capacity.

Because each customer may choose different customization for their products, the maximum number of windows the firm can produce varies from period to period. Thus, we denote the average of the maximum number of windows produced in each period by $AMAX$. Then the available capacity, m, can be obtained as

$$m = C_{N1} \times AMAX$$

The future arrivals n is the future arrivals before the deadline. In most theoretical work, the assumption is made that the customers arrive according to a Poisson process, and so n can be estimated as λt, where λ is the arrival rate and t is the time left to the deadline. However, in many cases, the arrival rate is not just a function of time. For example, when the firm's target market is limited, then the number of future arrivals can also be related to the past arrivals. A variety of methods to predict the expected number of arrivals can be found in Armstrong and Green (2005).

Even though we assumed the processing time of an order is constant (Assumption 2), in practice a customer's order may vary in its capacity requirement (or processing time). In such cases, we can use our method to compute the price for an order with average capacity requirement. Supposing that after accepting the order, the available capacity becomes m', then the prices for G-order and U-order can be approximately obtained at $p^G_{mn}(m-m')$ and $p^U_{mn}(m-m')$, respectively.

6.2. Learning the distribution of r and v

Because it is hard to observe the customer's WTP and impatience factor in practice, in this section we focus on developing a learning process to estimate the distributions of r and v.

The form of a distribution might be determined by many features, such as the distribution type (i.e., exponential, normal, uniform, etc.). In practice, the firm might have more information about some particular features but less information about others. We study the case in which the firm knows the distribution type of the customer's WTP and impatience factor, but is uncertain about values of the distribution parameters. Without loss of generality, we suppose that the firm knows that r and v are independent, and the distribution types of r and v are normal, but the means (μ_r and μ_v) and the variances (σ_r^2 and σ_v^2) are unknown. Thus, the goal of the learning process is to estimate μ_r, μ_v, σ_r and σ_v so that the distribution functions required for computing the optimal price can be formed.

We let $f^r(r;\mu_r,\sigma_r)$ and $F^r(r;\mu_r,\sigma_r)$ respectively be the pdf and cdf of r, and let $f^v(v;\mu_v,\sigma_v)$ and $F^v(v;\mu_v,\sigma_v)$ respectively be the pdf and cdf of v. Because extra parameters (μ_r, σ_r, μ_v and σ_v) are needed as inputs in the cdf and pdf, we modify the argument in the notation for the probabilities that a customer purchases a G-order or a U-order in (1) and (2). Supposing that the firm has recorded K customers' purchases and customer $k : k \in \{1,\dots,K\}$ is quoted prices p^G_k and p^U_k for each type of orders, then the probabilities that a customer purchases a G-order or a U-order can be represented as $\mathbb{P}^{\vec{\theta}}_G(p^G,p^U)$ and $\mathbb{P}^{\vec{\theta}}_U(p^G,p^U)$ respectively where $\vec{\theta} = (\mu_r,\sigma_r,\mu_v,\sigma_v)$.

We develop a maximum-likelihood-estimation (MLE) based parametric estimation method to find the estimates of μ_r, μ_v, σ_r and σ_v. The likelihood, denoted by $\mathcal{L}$, is constructed as

$$\mathcal{L} = \prod_{k=1}^{K}\left[\left[1-\mathbb{P}^{\vec{\theta}}_G(p^G,p^U)-\mathbb{P}^{\vec{\theta}}_U(p^G,p^U)\right]^{y^0_k}\left[\mathbb{P}^{\vec{\theta}}_G(p^G,p^U)\right]^{y^G_k}\left[\mathbb{P}^{\vec{\theta}}_U(p^G,p^U)\right]^{y^U_k}\right], \qquad (14)$$

where

$$y^0_k = \begin{cases} 1 & \text{if customer } k \text{ leaves without purchase;} \\ 0 & \text{otherwise.} \end{cases}$$

$$y^G_k = \begin{cases} 1 & \text{if customer } k \text{ chooses G-order;} \\ 0 & \text{otherwise.} \end{cases}$$

$$y^U_k = \begin{cases} 1 & \text{if customer } k \text{ chooses U-order;} \\ 0 & \text{otherwise.} \end{cases}$$

To simplify the form of the likelihood, we take natural logs both sides of (14). After simple algebra, we can obtain

$$\ln \mathcal{L} = \sum_{k=1}^{K} \left[y_k^0 \ln \left[1 - \mathbb{P}_G^{\vec{\theta}}(p_k^G, p_k^U) - \mathbb{P}_U^{\vec{\theta}}(p_k^G, p_k^U)\right] + y_k^G \ln \mathbb{P}_G^{\vec{\theta}}(p^G, p^U) + y_k^U \ln \mathbb{P}_G^{\vec{\theta}}(p_k^G, p_k^U) \right]. \quad (15)$$

Because of the complexity of solving the first-order condition of (15), we use trust-region method (Conn et al. 2000) to search the optimal setting of parameters.

We present a neural network (NN) based regression method as a benchmark for the MLE based parameter estimation method. In NN, we train the network to regress the probability of a customer choosing each type of orders with respect to the price quote for each type of orders. That is, p_k^G and p_k^U are the inputs, and $\mathbb{P}_G^{\vec{\omega}}(p^G, p^U)$ and $\mathbb{P}_U^{\vec{\omega}}(p^G, p^U)$ are the outputs, where $\vec{\omega}$ is the vector of weights which will be determined through Levenberg-Marquardt backpropagation training. In the training process, y^G and y^U are the target values for $\mathbb{P}_G^{\vec{\omega}}$ and $\mathbb{P}_U^{\vec{\omega}}$, respectively. We set 1 hidden layer which includes 20 neurons for the NN.

In the simulation, r and v are each normally distributed such that $r \sim N(10, 2^2)$ and $v \sim N(3, 2^2)$. The prices quoted to each customer are randomly generated such that $p^G \sim U(0, 15)$ and $p^U \sim U(0, p^G)$. Given a combination of r, v, p^G and p^U, we obtain each customer's choice indicators, y^0, y^G and y^U.

We test the two regression methods under different volumes of historical sale records. Then we compare the outputs of MLE based method and NN based method with the true result computed by (1) and (2) in Section 3. In the comparison, we randomly choose p^G and p^U from $U(0, 15)$ and $U(0, p^G)$, respectively, and then show the coefficient of variation of the root mean square error, $CVRMSE$, computed as

$$CVRMSE = \frac{\sqrt{E\left[(y - \hat{y})^2\right]}}{\bar{\hat{y}}},$$

where y is the output of the regressed probability functions, $\hat{y}$ is the output of the true probability functions ((1) and (2)), and $\bar{\hat{y}}$ is the mean of $\hat{y}$. The comparison of MLE based regression and NN based regression is shown in Table 2. In Table 2, each value of $CVRMSE$ is the average of 20 rounds of simulation. We observe that the MLE based regression method can achieve an output much closer to the true value computed by (1) and (2), which supports the superiority of the MLE based regression. However, this MLE based regression method is only applicable when the distribution types of r and v are known. In the case where r and v are distributed with unknown distribution, the NN based regression is the only option because r and v are not required to form $\mathbb{P}_G^{\vec{\omega}}(p^G, p^U)$ and $\mathbb{P}_U^{\vec{\omega}}(p^G, p^U)$.

[Table 2 about here.]

Figure 12 graphically shows the outputs of the true probability functions and the regressed probability functions using different regression methods when the sample size is 500. We expect that the outputs of NN based regression approximates better when the number of neurons and sample sizes are increased, whereas it is usually not realistic to obtain a large enough sample. However, because the NN based regression is an substitute of the MLE based regression when the firm cannot estimate the distribution type of r and v, then it is a good practice for the firm to simultaneously run the two regressions so that the firm can capture the failure of estimating the distribution type of r and v.

[Figure 12 about here.]

7. A practical case study of a local window and door company

As a practical case study we investigate a local door and window company and analyze its sales data to study the potential to increase the company's expected profit and the customers' welfare. Hereafter, we use the term "principal company" to refer the local door and window being studied. As an OKP company, the principal company provides customized doors and windows to the local customers that include local builders and renovators. Customer orders can be divided into two categories: New-Construction (NC) and Sales&Installation (SI). In the NC category, the customers order the customized doors and windows for building new homes or other kinds of new estate properties, while in the SI category, the customers order replacements of broken, inefficient, or out-of-style windows or doors. In the company's business, the NC orders and SI orders take up the proportions of 55% and %45 (by dollar sales), respectively. The two categories of orders have different features.

In the NC category, the customers usually order the products early before the due dates direct from the company. For example, some builders usually order the doors or windows with specified requirements several months before the due dates and request on-time delivery services. Because of the long-term cooperation with the builders, the principal company schedule the NC orders with priority.

In the SI category the windows and doors are usually ordered for immediate use. Thus, the customers are sensitive to the leadtime. In the principal company the production is planned in weeks. At any time, the production schedule for the current business week is determined and frozen. Thus, new orders can only be scheduled after the current business week, which means that at earliest, the principal company can only guarantee the delivery before the end of the next week. In the SI category, the principal company treats the orders which require delivery before the end of the next week as G-orders, and treats the other orders as U-orders. For the U-orders, the company only promises a leadtime of 2-3 weeks.

The principal company produces products of multiple product families, and the products in different product families are processed in different production lines. Thus the capacity of the production line determines the production capacity for a product family. The principal company prices each product family separately. The customers make U-orders by default, and for each product family, there is a fixed price for every detail settings of product features. If a customer requires a G-order, then an extra amount is charged on top of the U-order prices.

The number of G-orders is constrained by the company's available capacity. The available capacity is computed by subtracting the workload of pre-scheduled jobs, which include NC orders and the U-orders which have been placed for 2 weeks, from the capacity of the production line. Here, we only show the company's sales data of one product family, vinyl-framed windows. The shop floor layout of the designated production line, V10, is displayed as in Figure 13.

[Figure 13 about here.]

Figure 14 shows the 4-week sales data of V10 orders in April 2009. Note that the sales data is scaled in this article because of the non-disclosure agreement. At the beginning of the week, the production manager communicates with the sales department the available capacity to guarantee the delivery of G-orders received in that week. The sales department stops accepting G-orders when the available capacity is completely allocated. The numbers of G-orders and U-orders as well as the pre-scheduled orders are displayed in the form of accumulated bar-chart. The dash line in Figure 14 shows the V10 production line's capacity. The principle company employs a simple differential pricing strategy for G-orders and U-orders, where the customers are charged $25 more than the U-order price for a G-order.

[Figure 14 about here.]

From Figure 14, we can see that among the SI orders received within the four weeks, the proportion of G-orders takes up 35%, 36%, 30% and 26%, respectively. It is obvious that in the first week, the available capacity is not fully utilized while in the third and the forth weeks, the proportions of G-orders are bounded by the company's available capacity. From Section 5, we know that in this case the company and the customers' welfare are not maximized due to improper pricing. Thus, a potential exists where both the company and the customers' welfare can be increased by employing DPS or PPS.

In order to estimate the distributions of the customers' WTPs and impatience factors, the company can first dynamically change the extra charge for G-orders by adding or subtracting a constant amount. The company can also add a periodically varying promotion discount to U-orders. When the company has enough sales data under varying prices, the customers' WTPs and impatience factors can be estimated using MLE, and then the more efficient extra charge for G-orders and promotion discount for U-orders can be computed through the method introduced in this work.

8. Conclusion

In this research we modeled a DPS when the firm's capacity is limited and a due-date guarantee may be required or favored by some customers. We formulated a dynamic Bellman model to compute the optimal price quotes. We analyzed the computational complexity of the proposed dynamic method, proving that the complexity of the dynamic method to compute the optimal price quotes is polynomial.

Usually it is believed that if the firm dynamically changes prices to maximize its profit, the market demand is exploited and so the customers' welfare is lowered. However, by comparing DPS with the constant pricing strategy (CPS), we show that when the firm dynamically change the prices for each type of orders, both the firm and the customers are better off. This finding strongly supports the superiority of DPS. In the literature, we have not found any research studying the how the DPS increases benefits to the entire supply chain.

For practical applications, we also proposed methods to estimate the parameters, which are complementary to the proposed pricing strategy. We introduced the methods to evaluate the firm's current available capacity, future arrivals, and the distribution of the customer's WTP and impatience factor. In the estimation of the distribution of WTP and impatience factor, the proposed estimation method can be easily extended so that the distribution with more unknown parameters can be estimated. We also presented a case study where DPS and CPS could be employed.

As future work, the proposed method can be studied within a supply chain which includes one supplier that supplies parts for multiple manufacturers. In this setting, the supplier is OKP manufacturer and faces a pricing problem with respect to the delivery date, while the downstream manufacturers choose the best option and then face a job sequencing problem.

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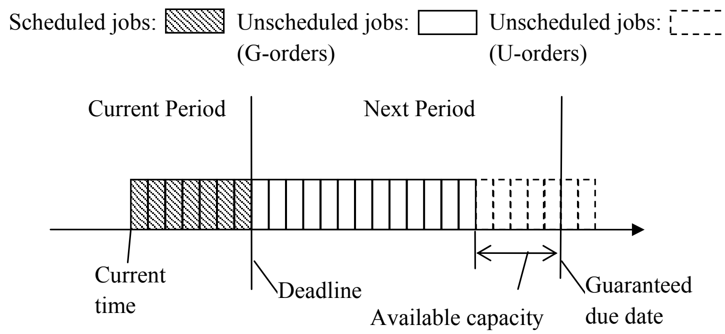


Figure 1: Problem description

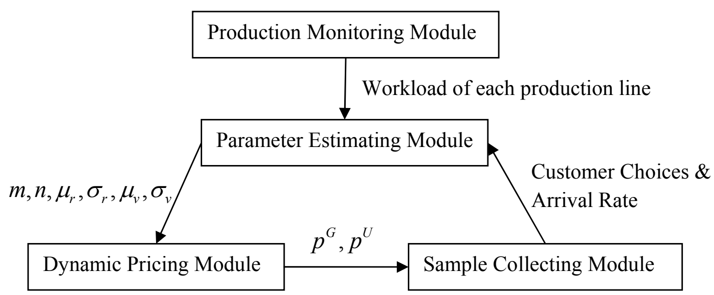


Figure 2: The adaptive control process

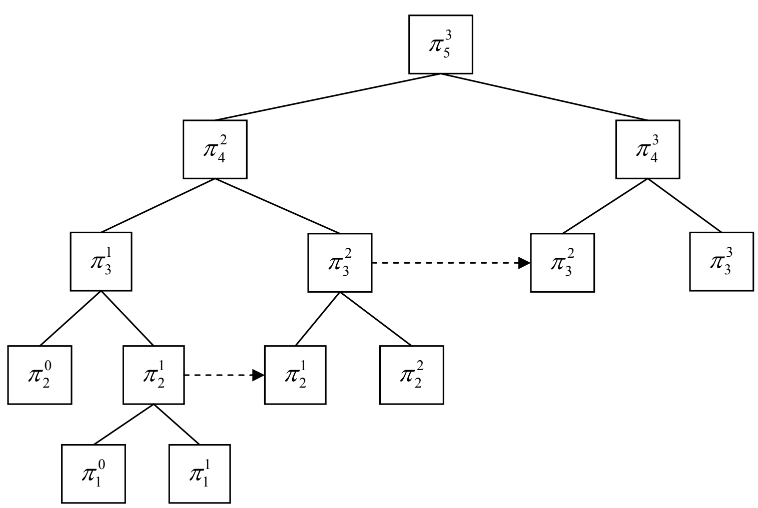


Figure 3: Solving π_5^3.

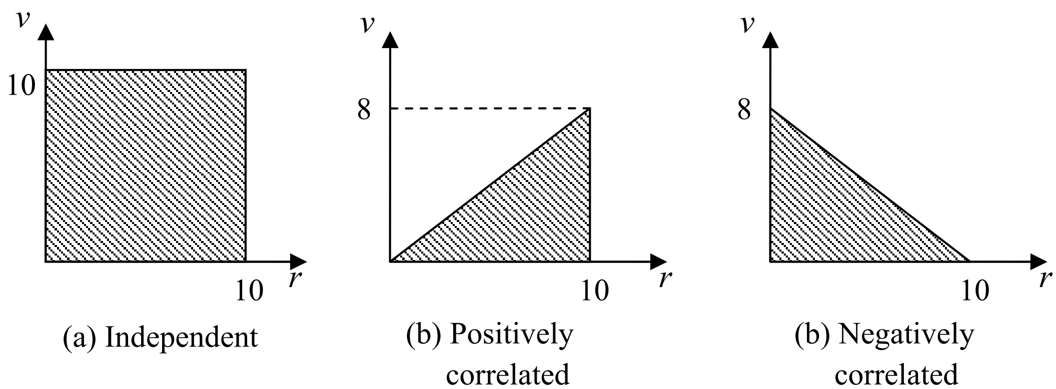
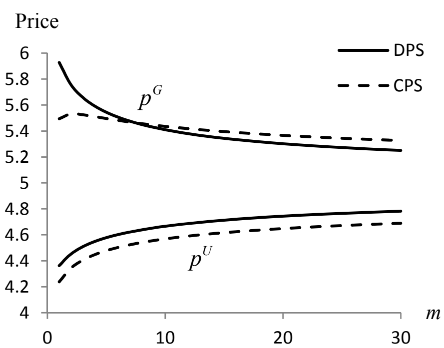
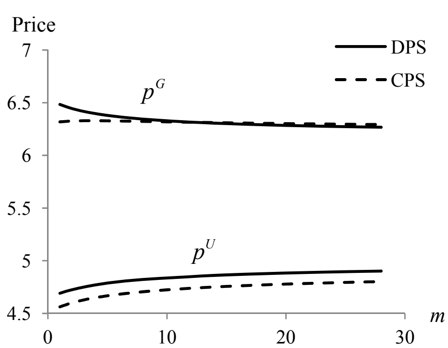


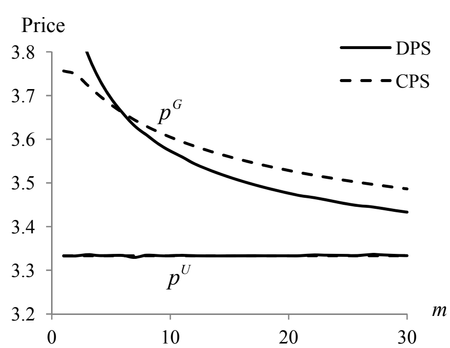
Figure 4: Distributions of r and v



(a) Independent

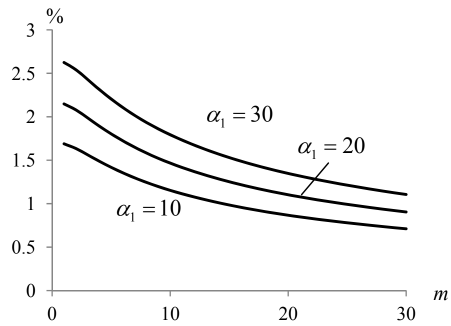


(b) Positively correlated

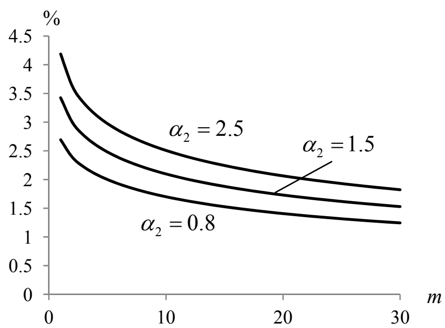


(c) Negatively correlated

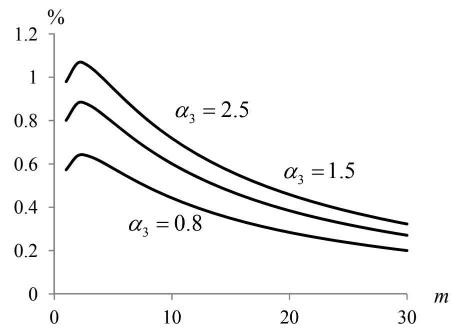
Figure 5: Price quotes obtained from DPS and CPS in different cases



(a) Independent

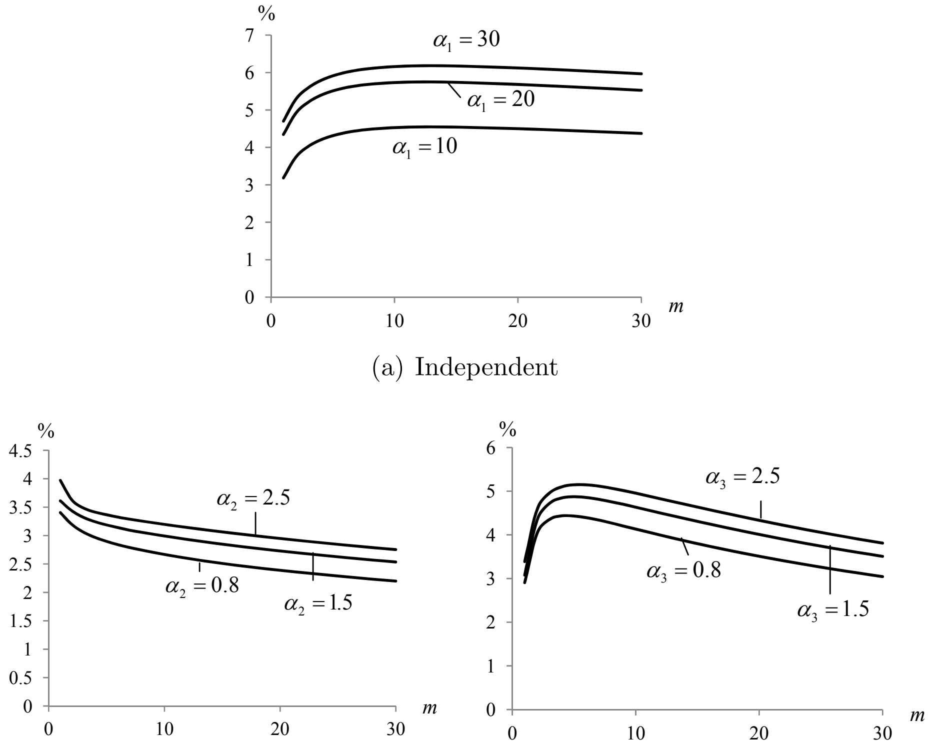


(b) Positively correlated



(c) Negatively correlated

Figure 6: The gap of the firm's expected profit under different pricing strategies in different cases



(a) Independent

(b) Positively correlated

(c) Negatively correlated

Figure 7: The gap of customers' net welfare under different pricing strategies in different cases

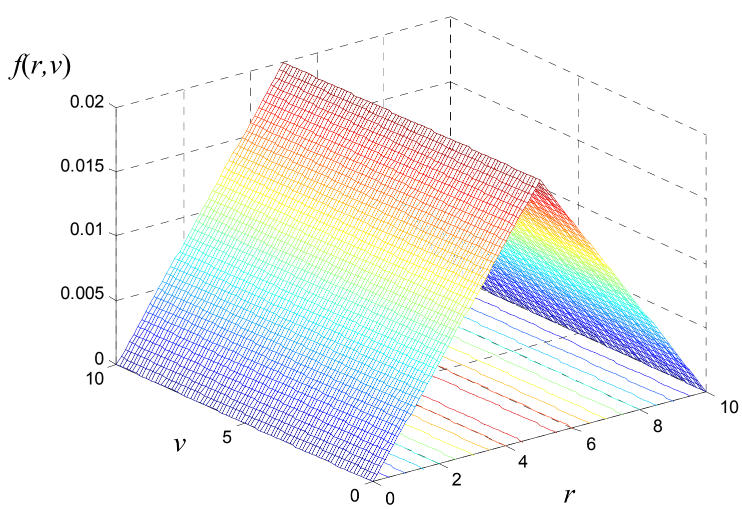


Figure 8: $f(r,v)$ when $r = R - S$, $s \sim U(0,5)$, $R \sim U(5,10)$ and $v \sim U(0,10)$

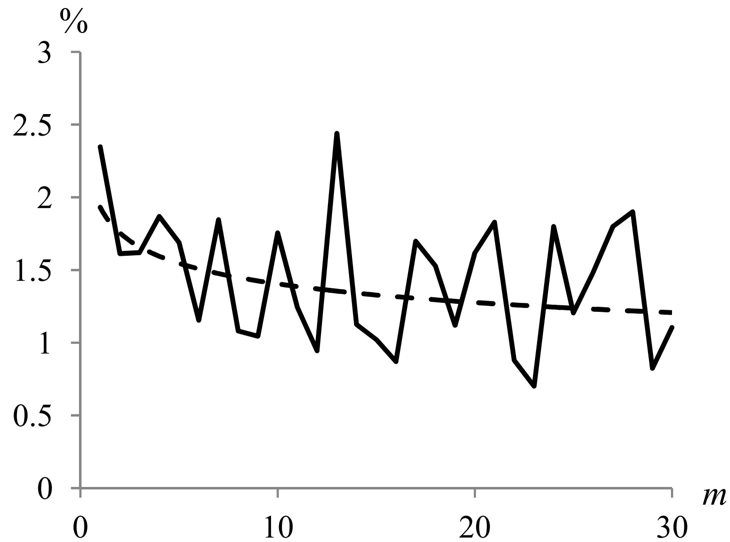


Figure 9: The gap of customers' absolute welfare under in DPS and CPS

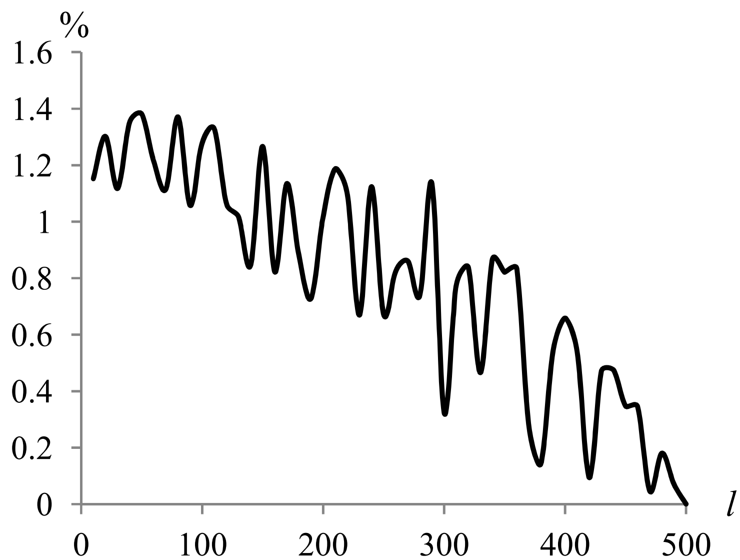


Figure 10: The gap of customers' net welfare under DPS and CPS for different setting of l

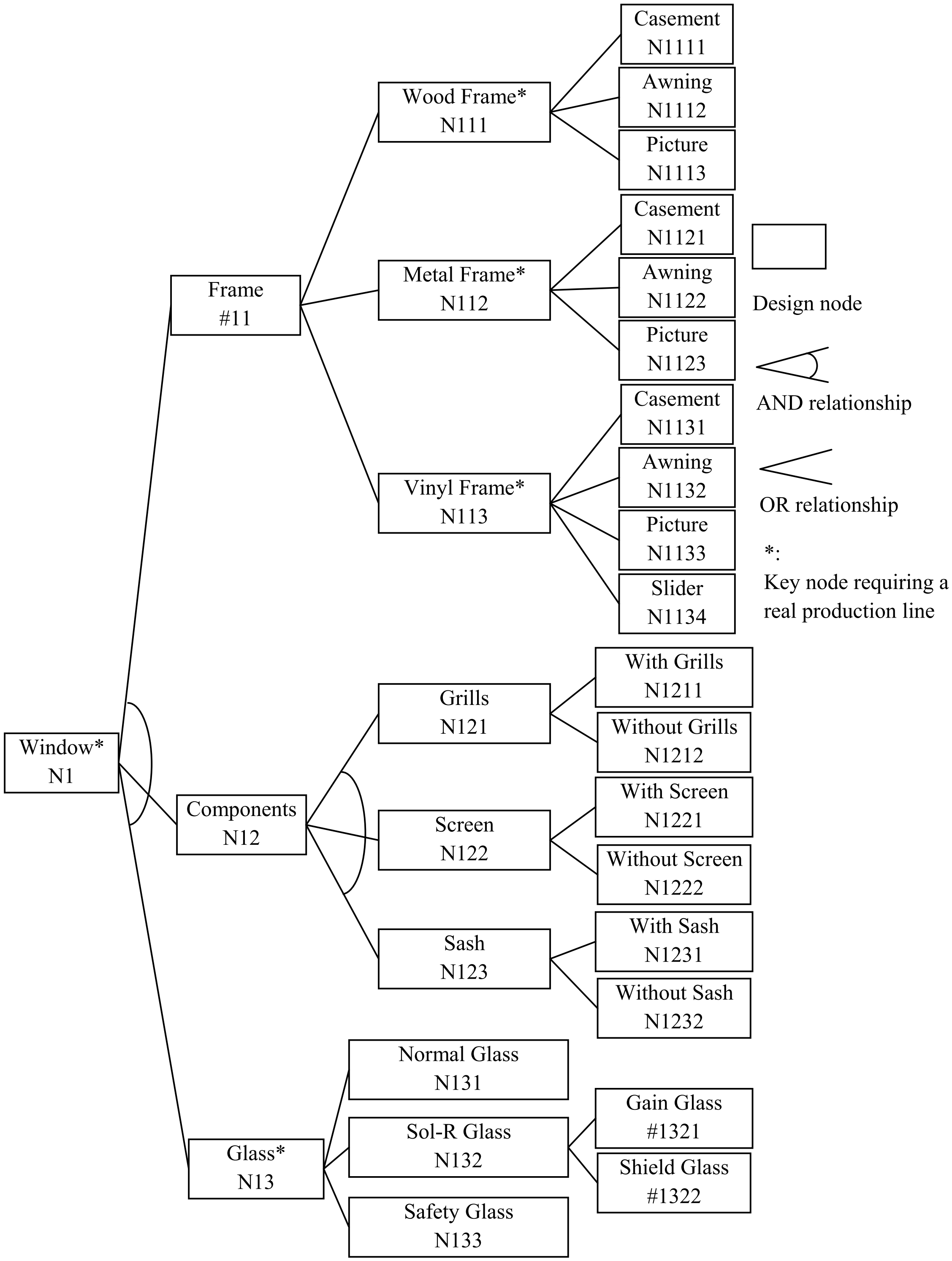


Figure 11: An AND-OR tree for modeling configuration variations of windows

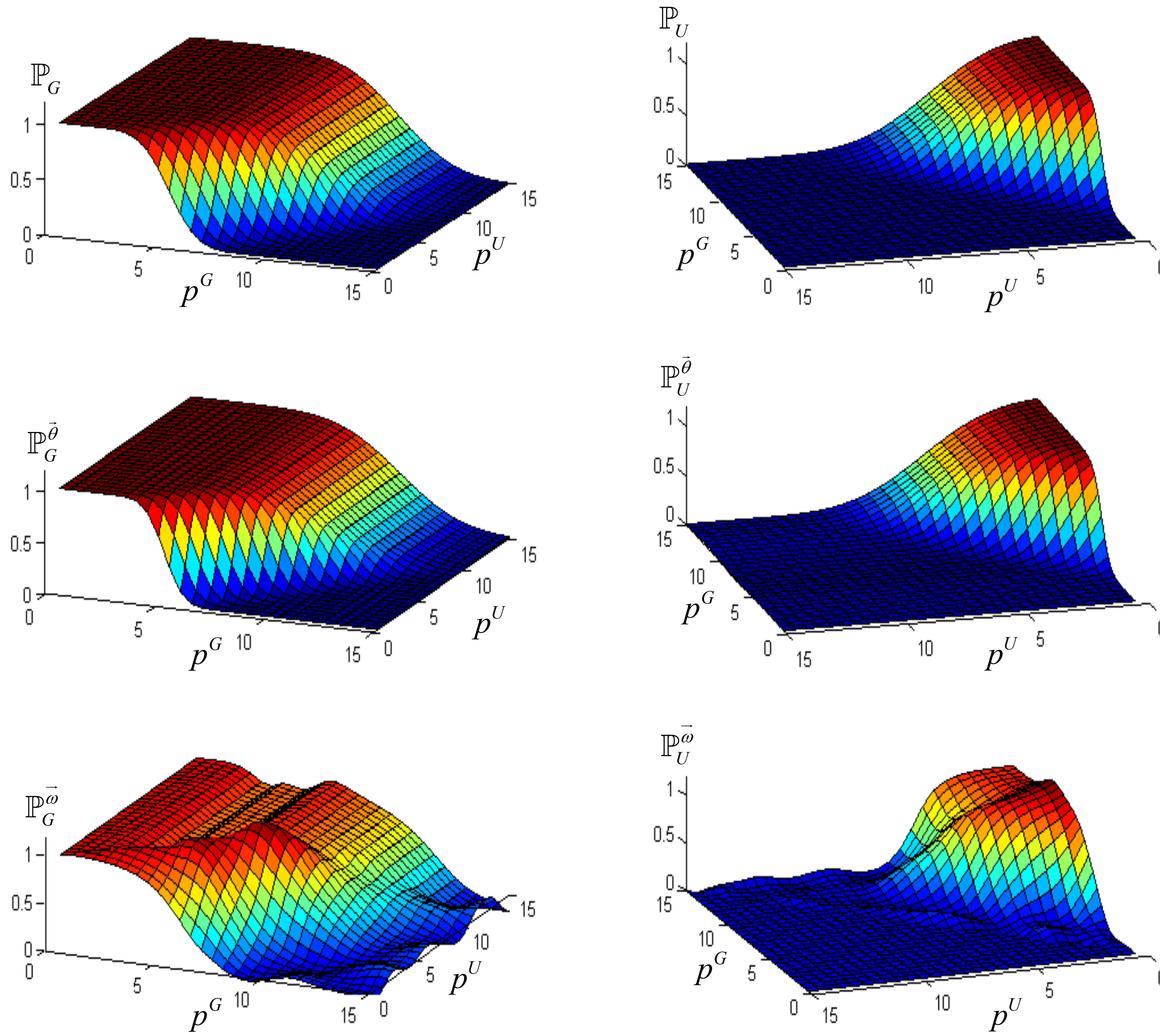


Figure 12: The outputs of true probability functions and regressed probability functions

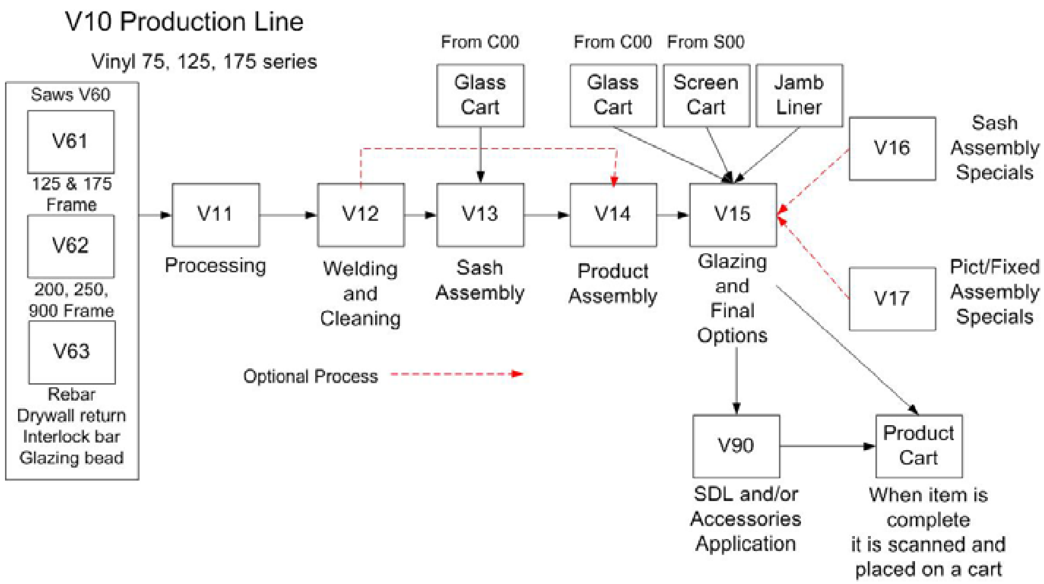


Figure 13: Shop floor layout for a production line and work centers

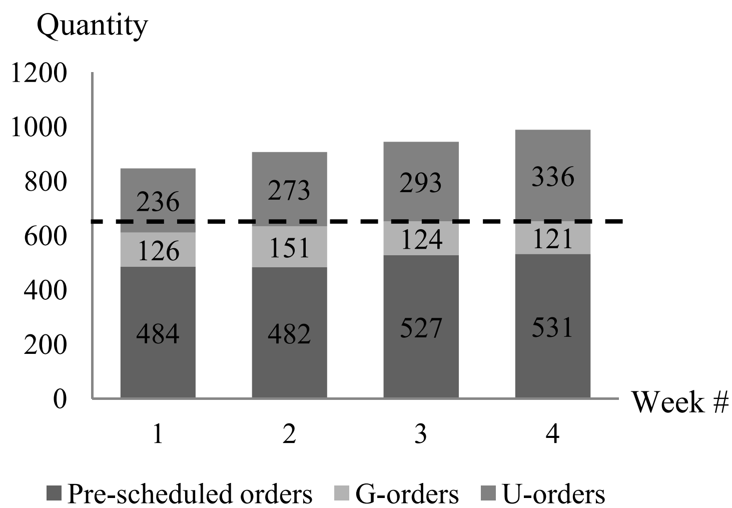


Figure 14: Shop floor layout for a production line and work centers

Table 1: An example of $(1,2)$ case

Price quote to the 1st customer:	
DPS: $\{5.93,4.36\}$	CPS: $\{5.49,4.24\}$
The welfare of the 1st customer:	
DPS: 0.886	CPS: 1.054
The probability of the 1st customer choosing G-order:	
DPS: 34.36%	CPS: 39.39%
Price quote to the 2nd customer if the 1st customer chooses G-order:	
DPS: $\{\infty,3.33\}$	CPS: $\{\infty,4.24\}$
The welfare of the 2nd customer if the 1st customer chooses G-order:	
DPS: 0.494	CPS: 0.319
Price quote to the 2nd customer if the 1st customer does NOT choose G-order:	
DPS: $\{5.00,5.00\}$	CPS: $\{5.49,4.24\}$
The welfare of the 2nd customer if the 1st customer does NOT choose G-order:	
DPS: 1.25	CPS: 1.054
The expected welfare of the 2nd customer:	
DPS: 0.990	CPS: 0.764
Total welfare of the two customers:	
DPS: **1.876**	CPS: **1.818**
The expected profit of the firm:	
DPS: **4.26**	CPS: **4.19**

Table 2: $RMSE$ of MLE based regression and NN based regression.

	Regressed function	Sample size				
		50	100	200	500	1000
MLE	$\mathbb{P}_G^{\vec{\theta}}$	15.33%	10.10%	7.29%	4.67%	3.18%
	$\mathbb{P}_U^{\vec{\theta}}$	16.08%	8.79%	6.36%	4.30%	3.37%
NN	$\mathbb{P}_G^{\vec{\omega}}$	27.50%	21.77%	16.12%	12.26%	9.00%
	$\mathbb{P}_U^{\vec{\omega}}$	27.05%	18.47%	15.10%	11.19%	9.69%