THE UNIVERSITY OF CALGARY

The Term Structure of Canadian Interest Rates

by

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ABSTRACT

This study uses weekly Canadian short and long-term interest rate data from 1980 to 1998 to test the pure expectations hypothesis of the term structure of interest rates. The Augmented Dickey-Fuller (1981) unit root test, the Engle and Granger (1987) cointegration test and the Granger (1969) causality test are applied. An autoregressive model is estimated in each case, revealing a significant relationship between the short rate and long rate by way of the interest rate spread. Evidence is found that short-term interest rates and long-term interest rates are cointegrated and that there exists bi-directional causality between them. This supports the pure expectations hypothesis of the term structure of interest rates.

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I. INTRODUCTION

Numerous studies have attempted to explain the relationship between short and long-term interest rates. The pure expectations hypothesis suggests that the long rate is an average of expected future short-term interest rates. The goal of this thesis is to test the pure expectations hypothesis of the term structure of interest rates.

The question of whether the term structure of interest rates can be explained by the pure expectations hypothesis, poses important implications for any economic agent with exposure to interest rate risk. Policy makers, for instance, have an interest in term structure theories, in particular, through the understanding of monetary transmission effects. Managers benefit from term structure theories when calculating discount rates for project analysis. Moreover, for the purpose of eliminating interest rate risk, financial institutions use term structure theories when trading derivative securities. As such, a valid theory of the term structure of interest rates presents useful knowledge to these economic agents.

The primary application of the term structure of interest rates for policy makers is the monetary transmission mechanism. The conventional analysis in macroeconomics is that an increase in the money supply reduces the interest rate. More specifically though, the Bank of Canada implements monetary policy by directly effecting short-term interest rates. In expansionary monetary policy, for instance, the supply of loanable funds increases and the short-term interest rate falls. As a result of the low short-term interest rate, borrowing and spending increase which in turn increases output. The long-term interest rate, on the other hand, is assumed to increase to accommodate higher expected inflation in the long run. Thus, the conventional analysis suggests monetary expansion is likely to reduce short-term interest rates but increase long-term interest rates. This is the foundation of the orthodox view where short maturity interest rates cause long maturity interest rates. By seeking an accurate term structure theory, policy makers can better understand the monetary transmission mechanism.

Another reason for investigating theories of the term structure of interest rates pertains to project analysis. Since the aim of a corporation's management is to maximise shareholder wealth, project analysis is paramount. Such criteria for evaluating projects are: net present value, internal rate of return, weighted average cost of capital, beta and the real options value approach. Central to all of these are expectations of future interest rates which are embedded in the term structure of interest rates. The choice of the wrong project may result, unless information in the term structure of interest rates is taken into consideration. Corporations can benefit from an accurate theory of the term structure of interest rates for conducting project analysis.

Financial institutions, on the other hand, trade derivative securities with the direct purpose of eliminating interest rate risk. Taking a futures contract position opposite to a position in the spot market is called hedging. This is to reduce exposure to risk by protecting oneself from unexpected price changes of the underlying security. If the security is a bond, the unexpected price change is a result of interest rate volatility. By buying a futures contract today, an investor can lock in a future interest rate today with certainty. Expectations of interest rates in the future are important to financial institutions whose goal is eliminating interest rate risk.

Chapter II of this thesis provides an examination of theoretical concepts, the purpose of which is to develop the terminology and notation for which various term structure theories can be discussed. The chapter begins by reviewing generally how interest rates are determined. Since this study uses the yield to maturity on bonds of the highest quality as the observable interest rate, Section 2.1 outlines fixed income securities. The chapter then proceeds to Section 2.2, which assigns notation to a bond's yield to maturity. This enables a definition of the term structure of interest rates, the yield curve, and the yield spread, in Section 2.3. Holding period return and forward rates are defined in Section 2.4 and Section 2.5, respectively. The last two concepts are necessary for a detailed examination of the pure expectations hypothesis in Section 3.1. This section asserts the important implications of the pure expectations hypothesis for testing in Chapter IV and V. Section 3.2 and Section 3.3 discuss other term structure theories which are variations of the pure expectations hypothesis. As a whole, Chapter II provides an understanding of the theory underlying the pure expectations hypothesis.

Chapter III undertakes a history of the pure expectations hypothesis. Included in Chapter III is a summary of one hundred years of the pure expectations hypothesis colourful past, a mention of a variety of its applications in research, as well as a survey of literature that tests it. Section III.2 will provide the brief history and Section III.3 reviews selected papers chosen for their similarity to the analysis done here.

The first test of the pure expectations hypothesis is performed in Chapter IV. Because of the nature of macroeconomic time series data, conventional econometric testing is deficient. The testing methodology in this study requires incorporation of recent advancements in econometrics. Chapter IV addresses these concerns arising from the unique properties of times series data as well as outlines in detail the methodology of applying these powerful econometric tests. As such, before cointegration testing, the stationarity of the individual series is considered. This is done using the autocorrelation and partial autocorrelation functions and the augmented Dickey-Fuller (1981) unit root test. Once these results are obtained, the chapter continues with cointegration testing. This is the first direct test of the theoretical relationship proposed by the pure expectations hypothesis in Chapter II. As has been said, the concept of cointegration and its methodology and procedure are outlined in detail.

Chapter V attempts to resolve two additional issues concerning the validity of the pure expectations hypothesis. Because of the conditions established in Chapter IV the question of forecasting and causality are addressed. Section V.2 theoretically introduces these two issues, and discusses the econometric reasoning. As a result, the theory from Chapter II is expanded to incorporate the yield spread. Section V.3 comprehensively details the econometric procedure used to test for Granger causality. The results of these tests are shown in Section V.4.

Various outcomes of this thesis are defined in the last chapter. Such outcomes are organised into a summary of results, criticisms regarding this and other studies and directions for further research.

As a whole, the question of whether the term structure of interest rates can be explained by the pure expectations hypothesis is tested. It will be shown that if this theory prevails, the long-term interest rate is a weighted average of expected future short-term interest rates. Consequently, if this relation holds then forward rates implied in the term structure will be unbiased estimates of expected future short-term rates. This hypothesis is of obvious interest to any economic agent with exposure to interest rate risk.

II. THEORY

II.1. Introduction

Defining the theoretical concepts of the term structure of interest rates is critical to an accurate analysis of the pure expectations hypothesis. Throughout this chapter, the basic tools for examining the term structure of interest rates will be presented. Since Irving Fisher first postulated the unbiased expectations hypothesis in 1896, a wide variety of term structure theories have been proposed. Underlying all these theories are several common foundational concepts. An understanding of the methods of interpreting interest rate behaviour is essential in forming any conclusions.

The chapter begins with a discussion of how interest rates are determined and concludes with an account of various term structure theories. Included in Section II.2 is a development of the notation for defining such concepts as fixed income securities, yield to maturity, yield curve, holding period return and forward rates. Section II.3 discusses four theories of the term structure of interest rates. In particular, the pure expectations hypothesis and expectations hypothesis are presented in full and brief discussions of the market segmentation theory and preferred habitats theory follow. Lastly, Section II.5 will offer some concluding remarks. As a whole, the claims of the pure expectations hypothesis are developed in detail; these assertions will be tested in later chapters.

II.2. Theoretical Concepts

Generally speaking, the interest rate is the primary mechanism whereby supply and demand of savings in an economy are brought into balance. The "interest rate" is often quoted and used by economists as an indicator of macroeconomic activity. This "interest rate" is actually determined in the market for fixed income securities of the highest quality.

II.2.1. Fixed Income Securities

This study looks at bonds with no call provisions or default risk, so that their payments are fully specified in advance. No default risk means that there is no uncertainty about the nominal payments promised by the bond. For example, Government of Canada bonds and Treasury bills are assumed to be default free. Bonds such as these are known as fixed income securities. There are two types of fixed income securities: zero-coupon bonds and coupon bonds. Zero coupon bonds, also called discount bonds, make a single payment in the future, called the face value of the bond. The maturity date is the date in the future this final payment is due. The length of time to the maturity date is known as the term to maturity of the bond. Coupon bonds, on the other hand, make equal payments of a fraction of the face value at equally spaced dates up to the maturity date. At maturity, the bond's face value is also paid. Coupon bonds, are often thought of as packages of discount bonds, one discount bond corresponding to each coupon payment and one discount bond corresponding to the final coupon payment and repayment of the principle. This study uses the yield to maturity on zero-coupon bonds of the highest quality as the observable interest rate.

II.2.2. Yield to Maturity

The yield to maturity on a bond is the rate that equates the present value of future cash flows plus the redemption price with the current market price. The notation and development in this subsection and later subsections follows closely that of Campbell, Lo, and MacKinlay (1997). The price of a bond in terms of the yield to maturity is

$$P_{nt} = \frac{1}{(1+Y_{nt})^{n}}, \qquad \ldots(2.1)$$

where P_{nt} is the price at time t of a zero coupon bond that makes a single payment of $1 at time $t+n$, and Y_{nt} is the bond's yield to maturity in units of percent per annum. In Y_{nt}, the first subscript refers to the term to maturity of the bond and the second subscript is the date at which the yield is set. The interest rate thus, is the rate at which the single payment of a dollar at time $t+n$, is discounted to the present. Equivalently, the yield can be found from the price,

$$(1+Y_{nt}) = P_{nt}^{-\left(\frac{1}{n}\right)}. \qquad \ldots(2.2)$$

To transform equation (2.2) into a linear relation for empirical use, the logarithmic operator is applied to get,

$$y_{nt} = -\left(\frac{1}{n}\right)p_{nt}, \qquad \ldots(2.3)$$

where $y_{nt} = \log(1+Y_{nt})$ and $p_{nt} = \log P_{nt}$. This is the log yield to maturity at time t of an n-period bond. Taking the logarithm of the yield imposes the assumption of continuously compounded returns which makes empirical testing possible. When the yields to maturity of zero coupon bonds at differing terms to maturity are viewed as a whole, this is called the term structure of interest rates.

II.2.3. Term Structure and Yield Curve

As Burton Malkiel (1966) observes, "The term structure of interest rates is perhaps the most intriguing structural relationship among market interest rates." The relationship between interest rates on bonds with the same default risk but different terms to maturity is known as the term structure of interest rates. That is, bonds with identical risk, liquidity, tax characteristics etc., may have different interest rates because the time remaining to maturity is different. All factors other than maturity must be held constant if the relationship studied is to be meaningful. A term structure may be approximated graphically by plotting yield and maturity for like bonds at a moment in time. A plot of the yields on bonds with differing terms to maturity but same risk, liquidity, and tax characteristics etc., is called a yield curve. Figure 2.1 shows some various yield curve shapes. The yield curve describes the term structure of interest rates for similar bonds.

Curve (A) in Figure 2.1 depicts an upward sloping yield curve. The yield on long-term bonds is greater than the yield on short-term bonds. Curve (B) shows that the yields on short-term and long-term bonds are the same. Curve (C) shows that the yields on short-term bonds are greater than long-term bonds. Yield curves can also have more complicated shapes in which they first slope down then up, or vice versa. The difference between the yield on an n-period bond and a one-period bond is the yield spread. The yield spread in log terms is:

$$s_{nt} = y_{nt} - y_{1t} \qquad \ldots(2.4)$$

which is also a measure of the shape of the yield curve. Therefore, when the yield curve is upward sloping, for example, the yield spread is positive. The next two subsections define two final concepts necessary before term structure theories can be examined.

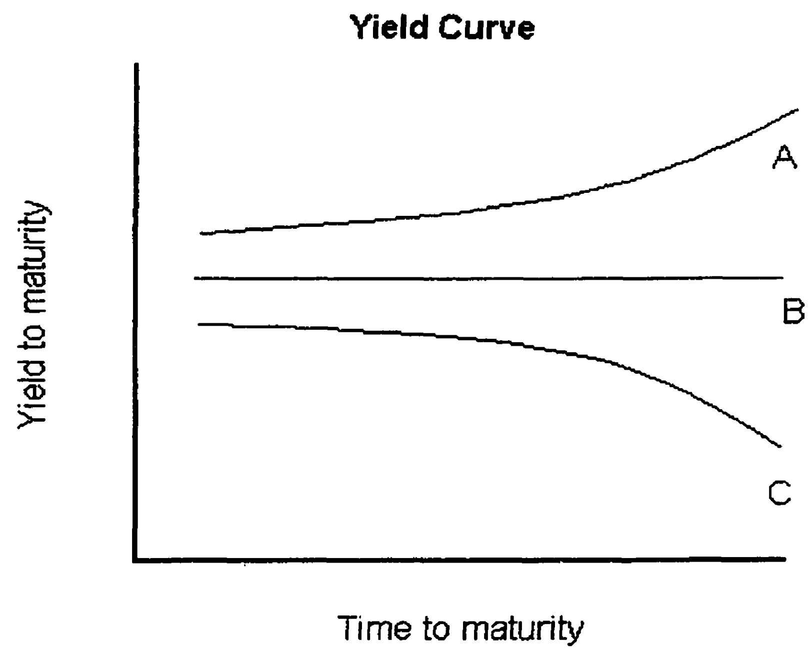

Figure 2.1: Yield Curve Shapes

II.2.4. Holding period return

The yield realised by an economic agent over some period of which she has funds to invest is called the holding period return. For example, an economic agent can buy a 20-year bond and hold it for one year then sell it at the end of the first year, realising a one-period holding period return on a 20-year bond. In general, the one-period holding period return on an n-period bond purchased at time t and sold at time t+1 is $R_{n,t+1}$. As can be seen from equation (2.1) the holding period return for this strategy will be

$$\left(1+R_{n,t+1}\right)=\frac{P_{n-1,t+1}}{P_{nt}}=\frac{\left(1+Y_{nt}\right)^{n}}{\left(1+Y_{n-1,t+1}\right)^{n-1}}. \qquad \ldots(2.5)$$

This relation shows that the holding period return is high if the yield is high, when the bond is purchased at time t, and if the yield is low when the bond is sold at time t+1. The logarithmic operator is applied to equation (2.5) to get the log holding period return

$$\begin{aligned} r_{n,t+1} &= \log\left(\frac{P_{n-1,t+1}}{P_{nt}}\right) \\ &= p_{n-1,t+1}-p_{nt} \\ &= ny_{nt}-(n-1)y_{n-1,t+1} \\ &= y_{nt}-(n-1)\left(y_{n-1,t+1}-y_{nt}\right) \end{aligned} \qquad \ldots(2.6)$$

Analogous to equation (2.5) this shows the relation between the log holding period return and time t yield as well as the time t+1 yield. The log holding period return is high when the time t yield is high or the change in the yield over the period is negative.

II.2.5. Forward Rates

The yield on an actual bond of a particular maturity at time t is known as its spot rate of interest. Spot rates of interest on bonds of various maturities can be used to infer forward rates of interest. Although forward rates of interest do not actually exist, they are a useful concept in understanding how expectations of future spot rates are formed.

To make a specified return with certainty some time in the future, an economic agent can lock in an interest rate on a one-period investment in the future called the forward rate. To do this she proceeds as follows. She sells one n-period bond at time t and buys one (n+1)-period bond at time t. Until time $t+n$, the obligation on the n-period bond is offset by the return on the $(t+n+1)$-period bond. Hence, the return on the one-period investment from time $t+n$ to time $t+n+1$, is defined to be the forward rate. Following from equation (2.5), if this one period investment pays $1 at time $t+n+1$, then the forward rate is

$$(1+F_{nt}) \equiv \frac{(1+Y_{n+1,t})^{n+1}}{(1+Y_{nt})^{n}}. \qquad \ldots(2.7)$$

This is analogous to the one period holding period return but at some time $t+n$ in the future. In the notation F_{nt}, the subscripts have a different meaning than the subscripts on the yield to maturity. The first subscript refers to the number of periods ahead that the one-period investment is to be made, and the second subscript refers to the date at which the forward rate is set. Once again, by applying the logarithmic operator, the n-period ahead log forward rate can be found from equation (2.7) to be

$$\begin{aligned} f_{nt} &\equiv (n+1)y_{n+1,t} - ny_{nt} \\ &\equiv y_{n+1,t} + n(y_{n+1,t} - y_{nt}) \\ &\equiv y_{nt} + (n+1)(y_{n+1,t} - y_{nt}) \end{aligned} \qquad \ldots(2.8)$$

This relation shows that the forward rate will be greater than the n-period bond yield and the $(n+1)$-period bond yield whenever the $(n+1)$-period yield is higher than the n-period yield. If the $(n+1)$-period bond yield is greater than the n-period bond yield then there exists a positive yield spread and consequently an upward sloping yield curve. The forward rate is an important concept for forming expectations about future spot rates of interest. This is discussed in the next section.

II.3. Theories of the Term Structure

Investigation of term structure theories begins with the pure expectations hypothesis. Using this theory as a strict form, additional theories are considered by relaxing assumptions. The pure expectations theory is a special case of all theories where bonds of different maturities are substitutes and no risk premium is required by investors to hold bonds of different maturities. In what follows we concentrate on pure discount bonds of the highest quality. The pure expectations theory and expectations theory will be examined closely and the additional theories will be briefly mentioned in the final subsection.

II.3.1. The Pure Expectations Hypothesis

The pure expectations theory implies that the bond markets are highly efficient.[1] Efficient financial markets exist when security prices reflect all available information. In an efficient market, market participants are assumed to be risk-neutral and willing and able to rapidly exploit profit opportunities. In exploiting profit opportunities, market participants cause security prices to be valued according to all available information. As a result, market prices of securities adjust quickly to new information. The actions of these market participants seeking profit, results in the term structure being determined completely by expectations regarding future interest rates. In short, efficient markets imply an absence of market imperfections that impede the rapid dissemination of information and the rapid reaction to this information by the market participants.

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[1] The terms, "pure expectations hypothesis" and "pure expectations theory" are used interchangeably.

In the context of the pure expectations theory, all relevant information is incorporated into market participants expectations concerning the future course of interest rates. Should forward rates differ from expected future spot rates, market participants would exploit the opportunity until it was eliminated. As a result forward rates implied in the term structure would be unbiased estimates of expected future spot rates.

Moreover, the prominent assumption behind the pure expectations theory is that buyers of bonds do not prefer bonds of one maturity over another, so they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity. This means that if bonds with different maturities are perfect substitutes, the expected return on these bonds for a given holding period will be equal. Examination will begin by defining the two different forms of the pure expectations hypothesis.

The pure expectations theory has two forms. Underlying both these definitions are two important implications of an efficient market. Firstly, forward rates of interest embodied in the term structure are unbiased estimates of expected future spot rates of interest.[2] This is because bonds of different maturities are assumed to be substitutes. Hence, the expected returns on these bonds for a given holding period in the future must be equal. Secondly, for a given holding period, the expected holding period return at the time of the initial investment will be the same for all possible maturity strategies. This is because the interest rate on a long-term bond will equal an average of short-term interest

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[2] This is how the pure expectations hypothesis derived its original name, "unbiased expectations hypothesis," first coined by Irving Fisher in 1896.

rates that people expect to occur over the life of the long-term bond.[3] The two forms of the theory go by the names "one-period pure expectations hypothesis" and "n-period pure expectations hypothesis," respectively.

Under the pure expectations theory the return on a one-period bond should equal the expected return from holding an n-period bond for the same period. Equating these two returns shows the first form of the pure expectations theory. Since the return from holding a one-period bond is $(1+Y_{1t})$, and the return from holding an n-period bond for one-period is given by equation (2.5), the relation becomes:

$$(1+Y_{1t}) = E_t\left[1+R_{n,t+1}\right] = \frac{(1+Y_{nt})^n}{E_t\left[(1+Y_{n-1,t+1})^{(n-1)}\right]}. \qquad \ldots(2.9)$$

The expectations operator applies to the yield on the (n-1)-period bond because expectations are formed at time t but the rate isn't known with certainty until time t+1. Whereas the yield on the one-period and n-period bonds are observable spot rates. This form is also known as the "one-period pure expectations hypothesis."

The second form of the pure expectations hypothesis is called the "n-period pure expectations hypothesis." This form takes the reasoning that the return on an n-period bond should equal the expected return from investing in n successive one-period bonds for n periods. Equating the two strategies can be shown:

$$(1+Y_{nt})^n = E_t\left[(1+Y_{1t})(1+Y_{1,t+1})\ldots(1+Y_{1,t+n-1})\right]. \qquad \ldots(2.10)$$

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[3] In the case of a coupon bond, the interest rate on a long term bond will equal a weighted average of short-term interest rates that people expect to occur over the life of the long-term bond with greater weights on near term maturities. Short rates in the near future should carry more weight in determining long bond yields than do expected short-term interest rates further in the future. This is because a greater part of the value of a coupon bond is derived from coupon payments made in the near future.

Sometimes the strategy on the right hand side of the equation above is referred to the expected return from "rolling over" one-period bonds for n-periods.

The equivalence of the forward rate and the expected future spot rate can be shown by first substituting for (n-1) one-period short rates on the right hand side of equation (2.10) with an (n-1)-period bond,

$$(1+Y_{nt})^n = (1+Y_{n-1,t})^{n-1} E_t\left[(1+Y_{1,t+n-1})\right], \qquad \ldots(2.11)$$

and by re-arranging, the expected (n-1)-period-ahead short rate can be expressed as equivalent to the forward rate

$$\frac{(1+Y_{nt})^n}{(1+Y_{n-1,t})^{n-1}} = E_t\left[1+Y_{1,t+n-1}\right] \equiv (1+F_{n-1,t}). \qquad \ldots(2.12)$$

This is the essence of the pure expectations hypothesis. Equation (2.12) says that forward rates implied in the term structure are unbiased estimates of the expected future spot rates of interest.

It has been argued that the two forms of the pure expectations hypothesis are not exactly equivalent. One problem is time inconsistency. Another problem is that there exits a mathematical contradiction between the two forms.[4] The differences between these forms of the pure expectations hypothesis are not crucial to examining its validity in the term structure of interest rates.

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[4] Shiller, Campbell, and Schoenholtz (1983) state, "The technical problem that has confronted model builders is that, if mathematical expectations are taken to represent market expectations, so long as there is uncertainty about future interest rates these models contradict."

Most researchers use neither form, but a log form of the pure expectations hypothesis. This model has been argued to be equivalent to either of the conflicting equations.[5] Firstly, the implication of equation (2.9) is restated in log form:

$$y_{1t} = E_t\left[r_{n,t+1}\right]. \qquad \ldots(2.13)$$

This says that the one-period log yield should equal the expected log holding return on a longer n-period bond held for the same period. This form, of course, implies that the expected difference between the log one-period short rate at time t and the log one-period holding period return on an n-period bond, for the same period, equals zero.

Equation (2.13) can be expanded out for longer maturities. To do this, recall that the strategy of rolling over n one-period bonds for n periods should equal the return on an n-period bond held until maturity. Therefore, the log yields of either strategy are equivalent as well. This can be shown by applying the logarithmic operator to equation (2.10):

$$y_{nt} = (1/n)\sum_{i=0}^{n-1} E_t\left(y_{1,t+i}\right). \qquad \ldots(2.14)$$

Lastly, as in equation (2.12), forward rates of interest implied in the term structure at time t would be unbiased estimates of expected future short rates:

$$f_{n-1,t} \equiv E_t\left[y_{1,t+n-1}\right]. \qquad \ldots(2.15)$$

Therefore, the (n-1)-period ahead one-period log forward rate equals the expected one-period log spot rate (n-1) periods ahead. In this study the log form of the pure expectations hypothesis is used.

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[5] Shiller (1981), Shiller et al. (1983), and Campbell (1986) all argue that the equivalence is exact after linearization.

An important inference of the pure expectations hypothesis in equation (2.14) is that interest rates on bonds of different maturities will move together over time. Because long-term interest rates are related to the average of expected future short-term interest rates, a rise in short-term interest rates will raise long-term interest rates, causing short and long-term interest rates to move together over time.

The pure expectations hypothesis also provides an explanation of why interest rates on bonds of different maturities vary. It explains how the yield curve changes at different times and why short-term interest rates are expected to have different values at future dates. When the yield curve is upward sloping, the pure expectations hypothesis suggests that short-term rates are expected to rise in the future. In this situation, in which the long-term rate is currently higher than current short rate, the average of future short-term rates is expected to be higher than the current short-term rate, which can occur only if short-term interest rates are expected to rise. When the yield curve slopes downward, the average of future short-term interest rates is expected to be below the current short-term interest rate, implying that short-term interest rates are expected to fall, on average, in the future.

Furthermore, the pure expectations hypothesis proposes that bonds of different maturities are substitutes and no premium or excess return is required to induce investors to hold bonds of different maturities. The expectations hypothesis, on the other hand, suggests the existence of a term premium. This theory is discussed next.

II.3.2. Expectations Hypothesis

If complete certainty existed in the market, forward rates would be exact forecasts of expected future spot rates. Efficient markets would cause all bond maturities to be consistent with interest rate expectations. Investors would receive the same return, for a given holding period, regardless of the maturity of the bond held. The forward rate would contain no compensation for risk.

The expectations hypothesis allows the existence of expected excess returns or term premiums.[6] The argument to incorporate term premiums is that future interest rates become more uncertain the further into the future one tries to predict. The longer the maturity of the bond, the greater is the risk of interest rate fluctuations.[7] Therefore, the expectations hypothesis suggests that a term premium is needed to persuade investors to hold long-term bonds. The expectations hypothesis is exactly the same as the pure expectations hypothesis with the exception that it allows the existence of expected excess returns or term premiums.

Similarly, the expectations hypothesis can be related to the pure expectations hypothesis in that it can be formulated in one-period returns, n-period returns, or log returns. The log form can be shown just as equation (2.14) was shown for the pure

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[6] The expectations hypothesis is also known as the liquidity premium theory. The liquidity premium theory was first developed by Lutz (1940) and additional work was done by Hicks (1946) and Kessel (1965).

[7] Hicks argues that a liquidity premium exists because a given change in the interest rates will have a greater effect on the price of long-term bonds than short-term bonds. Hence there is a greater risk to the value of the principal invested with long-term bonds.

expectations hypothesis. The log yield of an n-period bond will equal an average of n log one-period yields plus a term premium,

$$y_{nt} = (1/n)\sum_{i=0}^{n-1} E_t\left(y_{1,t+i}\right) + \Lambda_{nt}, \qquad \ldots(2.16)$$

where Λ_{nt} is the term premium on an n-period bond at time t. The forward rate can also be found similar to equation (2.15),

$$f_{n-1,t} = E_t\left[y_{1,t+n-1}\right] + \Lambda_{n-1,t} \qquad \ldots(2.17)$$

Where $\Lambda_{n-1,t}$ is a premium on the log forward rate to account for investor's risk considerations. Two yield curves are displayed in Figure 2.2, one based on expectations alone and one based on expectations plus the term premiums.

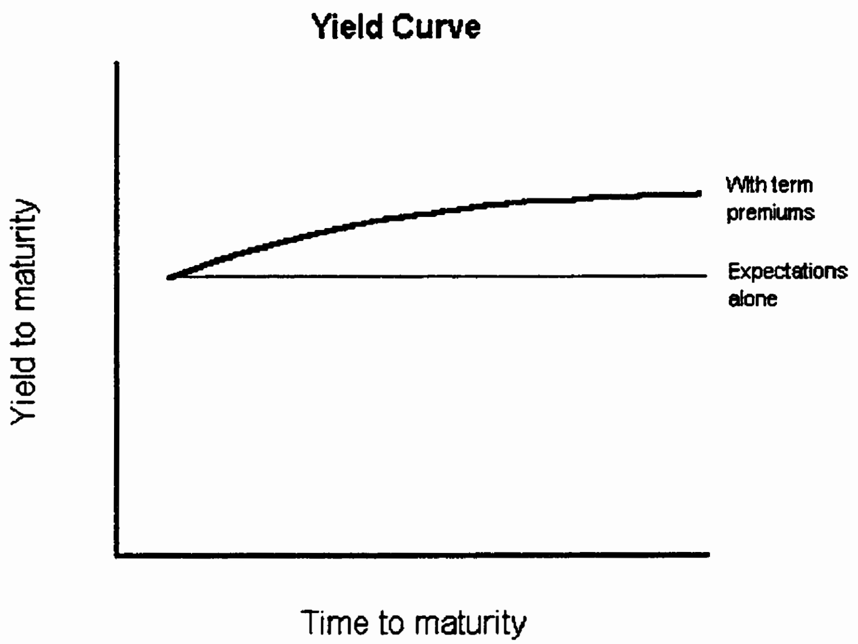

Figure 2.2: Expectations or Liquidity Premium Yield Curve

The expectations theory provides an explanation why yield curves have been historically upward sloping. In times of uncertainty or interest volatility investors would require a premium to hold bonds of longer maturities. An important assumption about the term premium in the expectations theory is that it must be constant or time invariant. Other assumptions about the term premium would lead to other theories of the term structure.

II.3.3. Other Theories of the Term Structure

The most discussed theories of the term structure of interest rates are the pure expectations theory and expectations theory. Apart from these, often mentioned are the market segmentation and preferred habitats theories. Though less popular they are equally insightful. Brief discussions of both these theories follow.

The segmented markets theory suggests that the behaviour of borrowers and lenders determine the shape of the yield curve.[8] Lenders have preferred maturity ranges in which they operate due to the nature of their business, legal restrictions or other reasons. On the other hand, borrowers relate the maturity of their debt to their needs for funds. This means that borrowers and lenders have maturity preferences regardless of yields on other maturities. Market segmentation theory implies that the rate of interest for a particular maturity is determined exclusively by demand and supply for that maturity. To illustrate, if there were four distinct maturity ranges their would be four sets of supply and demand curves. Linking together equilibrium interest rates across maturity ranges would determine the yield curve, as shown in Figure 2.3.

The main assumption in the segmented markets theory is that bonds of different maturities are not substitutes at all, so the expected return from holding a bond of one maturity has no effect on the demand for a bond of another maturity. This theory is at the opposite extreme to the pure expectations hypothesis where bonds of different maturities are perfect substitutes.

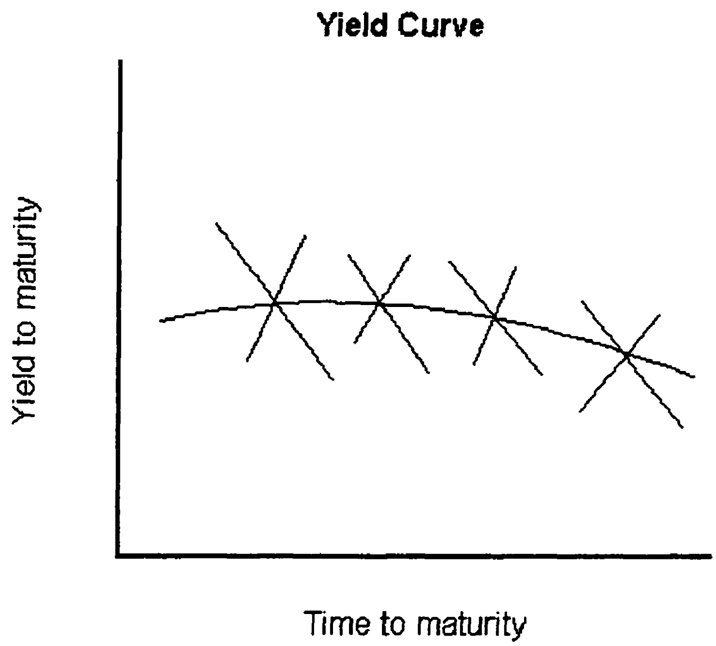

Figure 2.3: Segmented Markets Yield Curve

A variation of the market segmentation theory is the preferred maturity habitats theory. In fact this theory is a combination of the expectations theory and the segmented markets view. It asserts that the interest rate on a long-term bond will equal an average of short-term interest rates expected to occur over the life of the long bond plus a term

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[8] First work on the Segmented Markets theory is attributed to Culbertson (1957).

premium. The term premium, unlike the expectations theory, will also be a function of supply and demand conditions for that bond. That is, lenders and borrowers have maturity preferences and won't invest in other maturities unless sufficient excess returns can be obtained. Therefore, the premiums in the preferred habitats theory are a function of term structure uncertainty as well as investor maturity preferences. In other words, the term premiums in the expectations hypothesis are a function of the term to maturity only while the term premiums in the preferred maturity habitats theory are a function of the term to maturity as well as the maturity preferences of lenders and borrowers.

II.4. Conclusion

This chapter has outlined the pure expectations hypothesis in its entirety. For this purpose, theoretical concepts such as yield to maturity, yield spread, holding period return, and the forward rate were developed. Additional theories, which were shown to be modifications of the pure expectations hypothesis, were also briefly discussed.

Recent empirical research typically concentrates on the log form of the pure expectations hypothesis. The pure expectations hypothesis differs from the expectations hypothesis only in the existence of expected excess returns. The pure expectations theory says that expected excess returns on long-term over short-term bonds are zero. This is in contrast to the expectations hypothesis that allows the existence of expected excess returns. Although not unanimously accepted, the pure expectations hypothesis is definitely the most discussed theory in the term structure of interest rates literature. Other theories have attempted to explain some of the pure expectations hypothesis' shortcomings. The expectations theory, for instance, better explains the historical fact that yield curves typically slope upward. Some truth has been found that each theory explains facts that the other cannot. The following chapter will focus on how the recent empirical research has been applied to testing the pure expectations hypothesis of the term structure of interest rates.

III. LITERATURE REVIEW

III.1. Introduction

This chapter provides a survey of the literature that performs tests of the pure expectations theory of the term structure of interest rates. Early research started before the turn of the century; however, this research was very crude. Either there wasn't any data available or sufficient statistical tools to conduct an empirical test. Recent studies have benefited from the accumulation of data over time, as well as, the ability to incorporate advancements in econometrics. In addition, an understanding of monetary policy effects has made examination easier. Most research of the term structure of interest rates uses the pure expectations hypothesis or the expectations hypothesis. These theories have been used to study and forecast inflation, growth, fiscal and monetary policy effects, term premiums, or simply the structural relationship as done in this study.

The layout of the rest of the chapter is as follows. Section III.2 provides an overview of the history of research pertaining to the pure expectations hypothesis of the term structure of interest rates. Included are discussions of the terminology used to distinguish the different theories, a brief chronological development of the theories, a review of the pure expectations hypothesis' applications, and the outcome of various studies. Section III.3 reviews three particular papers chosen for their similarity to the analysis done here. Lastly, Section III.4 offers some concluding remarks.

III.2. Brief History

There is a long history of research into the pure expectations hypothesis of the term structure of interest rates. The history begins in 1896 with Irving Fisher's unbiased expectations hypothesis. In the literature, what was once called the unbiased expectations hypothesis is now referred to as the pure expectations hypothesis. For first half of this century the unbiased expectations hypothesis was called the expectations hypothesis and the theory that allowed the existence of term premiums was called the liquidity premium theory. In the second half of the century the expectations hypothesis referred to both the pure expectations hypothesis and the expectations hypothesis and researchers would discuss term premiums separately. Even more recently the introduction of the efficient markets (EM) theory of the term structure of interest rates, adding to the obscurity, has been used to refer to the expectations hypothesis plus rational expectations. Now the expectations hypothesis of the term structure of interest rates is defined by interest rate expectations plus the existence of excess returns or term premiums and the pure expectations hypothesis of the term structure of interest rates is simply defined by interest rate expectations. This terminology is consistent with Campbell et al. (1997).

Since Irving Fisher first proposed the unbiased expectations hypothesis, there has been extensive research into the question of whether the term structure of interest rates can be explained by the pure expectations hypothesis. The first researcher to produce empirical evidence that related long-term interest rates to expectations of future short-term rates was Macaulay (1938). He found that time money rates anticipate the seasonal rise in call money rates and concluded that this constituted "...evidence of definite and relatively successful forecasting."[9] Hickman (1942) reasoned, if the expectations hypothesis is valid then the expected yield curves will be correlated with observed yield curves. Hickman, like Macaulay, sought evidence of successful forecasting; unlike Macaulay he failed to find it. Culbertson (1957) found it difficult to believe that speculators would operate in government securities markets and predict as badly as his results suggested. He rejected the pure expectations hypothesis. Meiselman (1962) showed that expectations, whether or not they are correct, nevertheless affect the term structure of interest rates. He provided information relevant to evaluating the segmented markets theory of the term structure but the market was not segmented enough to invalidate the expectations theory of the term structure. Since Macaulay (1938) much of the research found no evidence to support the pure expectations theory or the expectations theory, including Modigliani and Sutch (1966), Sargent (1979), Shiller (1979), Shiller, Campbell and Schoenholtz (1983), Fama (1984), Campbell and Shiller (1984), Campbell and Shiller (1986), and Mankiw (1986). Recently, however, MacDonald and Speight (1988), Froot (1989), McFadyen et al. (1991), Hall, Anderson and Granger (1992), Lutkepohl and Reimers (1992) and Wallace and Warner (1993) have all found evidence supporting the expectations theory.

The term structure of interest rates has been examined for diverse objectives. One such objective, is to forecast inflation. Fama (1975) has argued the best estimate of expected inflation is provided by the term structure of interest rates. Other studies have used the term structure of interest rates to examine the interest rate spread. Papers such as these are: Campbell and Shiller (1984), Fama (1984), and Fama and Bliss (1987). In

---

[9] Frederick R. Macaulay, *Movements of Interest Rates*, page 63.

addition, Harvey (1997) has recently been able to forecast economic growth by incorporating expectations of future interest rates from the term structure of interest rates. Apart form those mentioned in the introduction: project analysis, trading derivatives, and monetary policy there are many other uses of the term structure of interest rates.

Studies that use the term structure of interest rates to study monetary and fiscal policy are numerous and the variety of their results are as well. Jones and Roley (1983), for instance, found higher levels of foreign holdings in Treasury securities associated *with lower term premiums*. They also found that the term premium is about 35% of the yield spread. Shiller, Campbell and Schoenholtz (1983), on the other hand concentrated on the apparent overreaction of long-term rates to money surprises; however their paper is inconclusive. Mankiw and Summers (1984) found no support for the expectations hypothesis, but suggest a need to develop a theory to explain the liquidity premiums. Plosser (1987) and Benninga and Possen (1988) use the term structure to investigate deficits and interest rates. More specifically, Plosser (1982) looks at "Ricardian Equivalence" where individuals view deficits as simply postponed tax liabilities and therefore deficits do not alter wealth or desired consumption paths. Similar to Shiller, Campbell and Schoenholtz (1983), Hsu and Kugler (1997) derive a policy response function and find the spread to have predictive power for the short rate. They argue their result is attributed to the adoption of the spread as an indicator of monetary policy. The Bank of Canada and the Federal Reserve-MIT group in the United States have also constructed elaborate general equilibrium econometric models to study the level and the structure of interest rates. The next section looks at recent research into the term structure of interest rates, as it applies to the structural relationship only.

III.3. Selected Papers

There are three papers reviewed here, chosen for their similarity to the approach taken in this study. They share the aim of testing the pure expectations hypothesis and offer insights regarding monetary policy and the forecasting ability of the spread. The review begins with Hall, Anderson, and Granger (1992) and is followed by McFadyen, Pickerill, and Devaney (1991), and MacDonald and Speight (1988).

Hall, Anderson, and Granger (1992)

This study uses the nominal yields to maturity data from the Fama Twelve Month Treasury Bill Term Structure File of the Centre for Research in Security Prices at the University of Chicago. The sample has 228 observations from 1970 to 1988 and covers three monetary regimes. The first regime being the period 1970 - 1979 where the Federal Reserve targeted interest rates, the second period is 1979 -1982 where the Federal Reserve controlled the growth of reserves, and the third period after 1982 where the Federal Reserve resumed targeting the interest rates. They try to explain the cointegrating vector with the spreads between yields of different maturities. First, they define the Fisher-Hicks formula and postulate a general relationship between yields of different maturities as

$$
\begin{aligned}
&R(k,t) = \frac{1}{k}\left[\sum_{j=1}^{k} E_t\left[R(1,t+j\text{-}1)\right]\right] + L(k,t) \\
&where \;\; L(k,t) = \frac{1}{k}\left[\sum_{j=1}^{k} \Lambda(j,t)\right]
\end{aligned}
\qquad \ldots(3.1)
$$

This shows that the long rate is a weighted average of expected one-period short rates plus a term premium. Although this equation is not a general equilibrium model, it indicates that the yields of bonds with similar maturities will move together. The unit root tests were conducted in Hall, Anderson and Granger (1990), and found the interest rate series to be I(1). To test the hypothesis that the yields move together over time, they employ the Johansen (1991) cointegration test. They find that the spreads between yields of different maturities define the cointegrating vector in this system and conclude that the error correction model seems to provide more accurate forecasting than the naïve no-change forecasts.

They conclude, that during the periods when the Federal Reserve has targeted interest rates their tests support the predictions of the expectations theory. During the period where the Federal Reserve controlled the growth of reserves and expanded the band on interest rate fluctuations, the cointegrating relationship didn't hold. They argue that this is because the liquidity premiums became non-stationary from increased uncertainty caused by volatility in monetary growth, interest rates, and economic activity. In addition, their error correction model is unstable over the Federal Reserves policy regime changes but stable when using post 1982 data and shown to be useful for forecasting changes in yields. In conclusion, they suggest much can be learned about the term structure if the common factor underlying the time series behaviour can be related to economic variables such as monetary growth or inflation. Nevertheless, they state, "The estimated model is statistically significant and is shown to be potentially useful for forecasting yields of Treasury bills."

McFadyen, Pickerill, and Devaney (1991)

McFadyen, Pickerill, and Devaney (1991) use a bi-variate autoregressive system. Although they argue that they are testing the efficient markets (EM) theory, this can be understood as the expectations hypothesis plus rational expectations. They use monthly average yields on U.S. Treasury issues for the 90-day T-bill, the 5-year note, 10-year note and 20-year bond from 1953 to 1984. This autoregressive system incorporates the rate "spread" (difference in the 90-day T-bill rate and the long rate) and the change in the 90-day T-bill rate from the cointegrating equations.

The testing finds cointegration and bi-directional causality. In particular, the 90-day T-bill, 5-year note, 10-year note and the 20-year bond were found to be integrated of order one and the short rate (90-day T-bill) to be cointegrated with the 5-year note, 10-year note, and 20-year bond at the 5% level of significance. In constructing the bi-variate autoregressive model they specify an 8 month lag for the 10 and 20-year bonds and a 14 month lag on the 5-year note. Using Granger causality the "spreads" at each of the long rates was found to Granger cause changes in the short rate. Their results support the expectations theory but, more specifically, provide evidence that forecasts of future changes in the short rate can be improved by including the spread.

The data consists of the monthly average of each rate which, over 31 years, totals 312 data points. The monthly averages artificially smoothes the data and biases the results in favour of the Efficient Markets theory, thus reducing the robustness of testing. In addition, since the Efficient Markets theory assumes a perfect capital market then all arbitrage opportunities would be eliminated instantaneously, suggesting correction at every announcement. A stronger test would be to use the higher frequency.

MacDonald and Speight (1988)

This study by MacDonald and Speight is the first to find convincing results in support of the efficient markets theory, which is the expectations theory plus rational expectations. They use UK data reported in the Bank of England Quarterly Bulletin from 1963 to 1987 and construct a bi-variate system consisting of the spread and the UK Treasury Bill rate.

They begin with the Meiselman (1962) formula:

$$R_t^n = \frac{1}{n}\left[r_t + E_t r_{t+1} \ldots + E_t r_{t+n-1} \right] \qquad \ldots(3.2)$$

This shows that an agent may invest in an n-period bond, R_t^n, and hold it until maturity or invest in or "roll over" a succession of one period bonds, r_t, r_{t+1}, ... r_{t+n-1}, and earn the same rate of return. In addition, investors are assumed to be rational, risk neutral, and there are no transactions costs. Since their long bond is a consol, short rates in the near future should carry more weight in determining long yields than do expected short term interest rates in the more distant future. Therefore, R_t^n relates to the present value of future short term rates discounted geometrically by $\overline{R}$, the mean long bond rate,

$$R_t^n = \frac{1-\alpha}{1-\alpha^n} \sum_{k=0}^{n-1} \alpha^k E_t(r_{t+k}) \qquad \ldots(3.3)$$
$$where \quad \alpha = 1/1 + \overline{R}$$

Given the standard assumption, $n \rightarrow \infty$ for the long bond,

$$R_t^n = (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_t(r_{t+k}) \qquad \ldots(3.4)$$

By constructing a bi-variate autoregressive model based on this equation they are able to test all the restrictions of the efficient markets theory. By defining the spread as S_t

$= R_t - r_t$ and subtracting r_t from both sides of the equation above and re-arranging, they show that the spread is the optimal forecast of the weighted average of future changes in r. They suggest that these individual equations of the autoregressive system have the analogous interpretation to error correction equations. As expected, they find that the interest rate series are indeed all I(1) and cointegrated. Therefore, the spreads can be taken from the cointegrating regression (as the error correction term) and used along with Δr_t to construct the bi-variate autoregressive system.

Testing the bi-variate autoregressive system, they find evidence in support of the efficient markets theory of the term structure. After specifying an optimal lag length of 6 on both variables, they conduct causality tests. The results of which reveal bi-directional Granger Causality, indicating that the long and short-term interest rates are determined by the efficient markets model of the term structure of interest rates.

Even though many studies of the term structure have been done since MacDonald and Speight, this paper remains the pivotal study. Until this paper, no other authors found evidence in support of the expectations theory of the term structure. Furthermore, they state that "...support for the efficient markets view of the term structure of interest rates legitimises the use of a single interest rate - 'the' interest rate - in a macroeconomic model such as the IS-LM model."

III.4. Conclusion

The aim of this chapter was to provide a brief review of the literature investigating the pure expectations hypothesis of the term structure of interest rates. The chapter showed that the term structure of interest rates has been examined for a number of reasons. Some of these include the over-reaction or under-reaction of long-term rates to fiscal or monetary policy, forecasting inflation, or identifying term or liquidity premiums. From this chapter, one can tell that the history of research into the pure expectations hypothesis is quite extensive.

As a result of researching the literature testing the pure expectations hypothesis or other theories of the term structure of interest rates, a few criticism arise. Firstly, there is a need for a universal terminology regarding the particular names given to the variety of theories. The terminology of Campbell et al. (1997) seems most appropriate. The pure expectations hypothesis should refer to the term structure of interest rates determined entirely by interest rate expectations where term premiums are zero and the expectations hypothesis should refer to the term structure of interest rates determined by interest rate expectations with constant term premiums. If term premiums are time varying then an additional name should be used. In at least one sample period each of these theories has held, yet to interpret which one from the authors work is another matter. Second, it is important that the expectations hypothesis requires time invariant term premiums. The uncertainty in the term structure should be only a function of the term to maturity and not related to the business cycle or other macroeconomic volatility. Third, collectively there appears to be a need for statistical tools to identify and decompose the term premiums, if they do exist. Only a few papers have claimed to have done this; however, the presumable gains from so doing have clearly outweighed the attempts. One final observation or criticism is, with the exception of Harvey (1997), Canadian data has never been used in any of the literature, including Canadian authors.

To conclude, the pure expectations hypothesis has been and will remain important in the analysis of the term structure of interest rates. This chapter has briefly revealed studies investigating the term structure with diverse objectives. There is much that can be learned from an understanding of the structural relationship of bonds differing only in their term to maturity. The next chapter will move forward and complete the first test of the pure expectations hypothesis.

IV. COINTEGRATION BASED TESTS OF THE TERM STRUCTURE OF INTEREST RATES

IV.1. Introduction

The motivation behind this chapter is to perform the cointegration based testing of the term structure of interest rates. As shown in chapter two, cointegration between the short rate and long rate is necessary if the pure expectations hypothesis holds.

In chapter two the implications of the pure expectations hypothesis were presented. A model of interest rates determined by interest rate expectations was constructed. Equation (2.14) showed that the yield to maturity on a long-term bond is the average of expected future short rates. This model implies that short and long rates of interest would move together over time. Such a phenomenon can be tested by utilising a cointegrating equation. Conclusions based on this will provide evidence of the pure expectations theory in the term structure of interest rates as well as establish conditions to examine causality in the next chapter.

Conventional econometric modelling is invalid here because of the nature of macroeconomic time series data. It is well known that most macroeconomic time series data are non-stationary. Before cointegration can be established the underlying behaviour of the individual time series is considered. This chapter completes the necessary conditions for any model specification to be sound.

A detailed outline of the methodology in developing the cointegration-based tests is provided. Firstly, the data characteristics such as source, frequency, and summary statistics are discussed in Section IV.2. Unlike other studies, which use one variable for the short rate, two proxies are used for the short rate. Secondly, Section IV.3 investigates the stationarity of the individual series. The visual technique incorporating the autocorrelation function is used as well as the Augmented Dickey Fuller unit root tests. Lastly, Section IV.4 discusses in detail the cointegration methodology and then proceeds with testing for cointegration. Lastly, Section IV.5 summarises the results of these tests.

IV.2. Data

The data used in this study are the weekly rates for one-month and three-month Canadian Treasury bills and twenty-year Government of Canada bonds, as listed in the Bank of Canada Review. The data were retrieved via the Cansim database. The Cansim series numbers for the one-month T-bill rate, three-month T-bill rate and 20-year bond rate are B113883, B113884 and B113896 respectively. The observations are realised on Wednesdays for the period from January 9th 1980 to June 3rd 1998. Short-term rates are measured by the weekly yields to maturity of the one month Canadian Treasury Bill and the three-month Canadian Treasury Bill. Long-term interest rates are measured by the weekly yields to maturity of the Government of Canada twenty-year bond. The following results where obtained using the Econometrics Views (1997) package. All series are in units of percent per annum. From this point on the logarithm of each series is used. That is, for all graphs, statistics and testing the log of the raw data is used. Furthermore, cointegration testing uses the logarithm of the percentage per annum yield. For all testing the full sample period is used. There are a total of 961 observations. This is a large sample size compared to ones used in the literature. No smoothing or filtering is performed; therefore, if evidence in support of the pure expectations hypothesis is found the results will be more robust.

Table 4.1 provides some summary statistics of each series. Skewness[10] values give some display of each series symmetry. A symmetrical distribution has a skewness value of zero, a right skewed distribution has a positive value, and a left skewed distribution has a negative value. Kurtosis[11] values provide some comparison of the series distribution to the normal distribution. The normal distribution is characterised by a typical bell-shape and holds a kurtosis value of three. Kurtosis values of higher than three are characteristic of distributions with fat tails and kurtosis values of less than three indicate light tails.

TABLE 4.1: SUMMARY STATISTICS

	30-day T-bill	90-day T-bill	20-year bond
Mean	2.084569	2.111563	2.298394
Maximum	3.070376	3.054001	2.898119
Minimum	0.862890	1.004302	1.717395
Std. Deviation	0.482820	0.458408	0.241729
Skewness	-0.511587	-0.493716	0.061108
Kurtosis	2.703290	2.666707	2.769170

Approximate normal distributions and near symmetry for each series can be inferred from Table 4.1. The skewness values are close to zero indicating near symmetry. The kurtosis values indicate each series is very close to a normal distribution but with

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[10] The skewness of a random variable x is also known as the normalised third moment. Skewness is defined by $S[x] \equiv E\left[\frac{(x-\mu)^3}{\sigma^3}\right]$ where μ is the mean, or otherwise known as the first moment, and σ is the variance of the variable or second moment.

slightly lighter tails. Kurtosis values less than three indicate excess kurtosis. Campbell et al. (1997) note that it is common when assuming continuously compounded returns to find excess kurtosis in historical returns.

Plots of each series and their first differences follow. In Figure 4.1 to 4.3 each series is presented. In Figures 4.4 to 4.6 the levels of the logarithm of the individual series are plotted along with their first differences of log levels.

A visual interpretation of Figures 4.1 to 4.4 can suggest how testing can proceed. All series trend down over this period and the long rate in general is greater than the short rate. The short rate shows a lot of volatility and substantial peaks and troughs as compared to the long rate. This may suggest seasonal fluctuations. There are quite definite structural breaks throughout the sample period. The post 1980's recession effects can be seen as well as the uncertainty during the early 1990's recession. Structural breaks are usually attributed to a change in monetary regimes. Since this sample period is of one regime, no allowance is made for structural breaks. In addition, since the 20-year bond rate is on the whole greater than the 30-day T-bill rate this indicates a predominate positive yield spread although both series trend down over this period. Recall that a positive yield spread at a particular point in time implies an upward-sloping yield curve.

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[11] The kurtosis of a random variable x is also known as the normalised fourth moment. Kurtosis is defined: $K[x] \equiv E\left[\frac{(x-\mu)^4}{\sigma^4}\right]$.

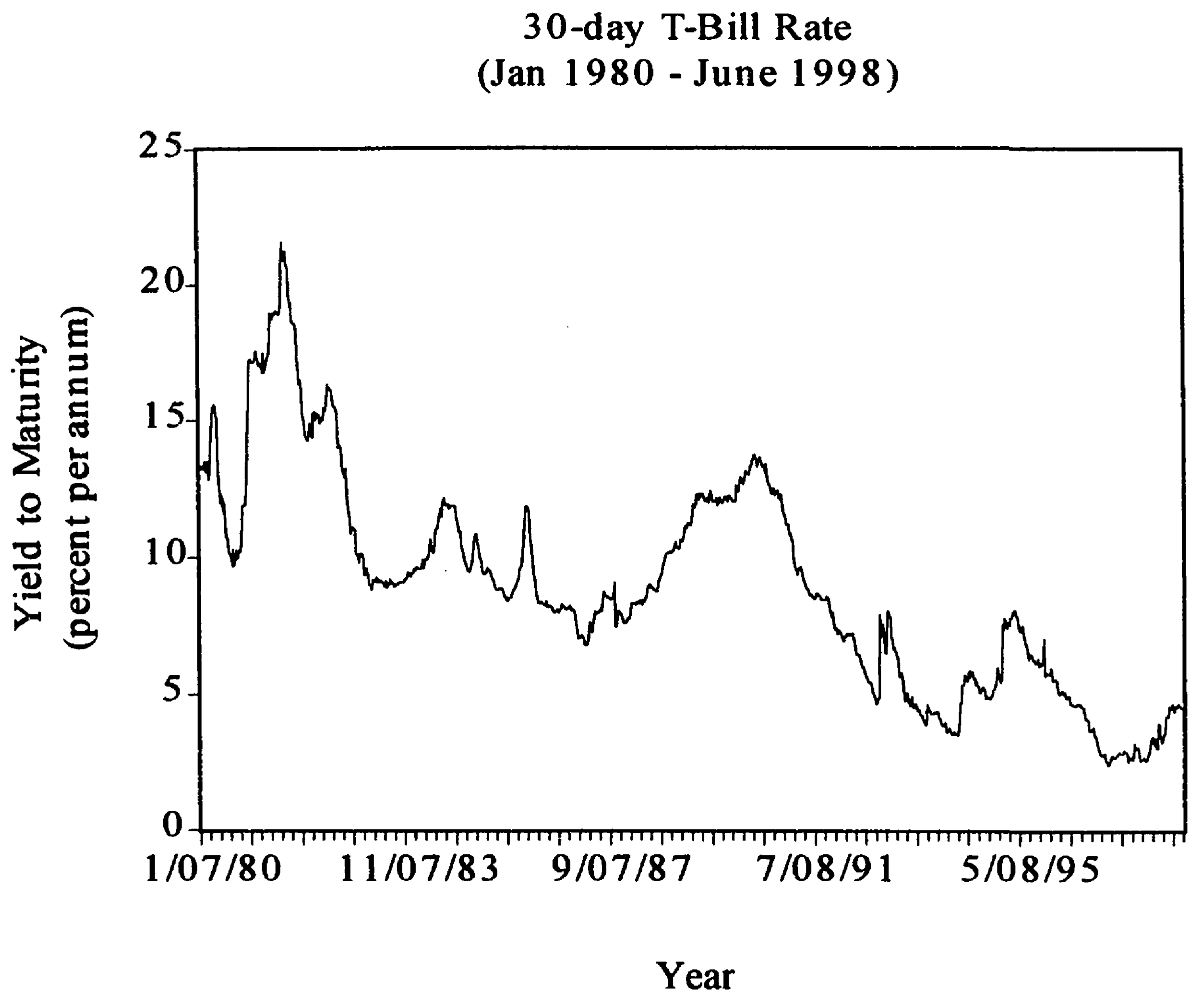
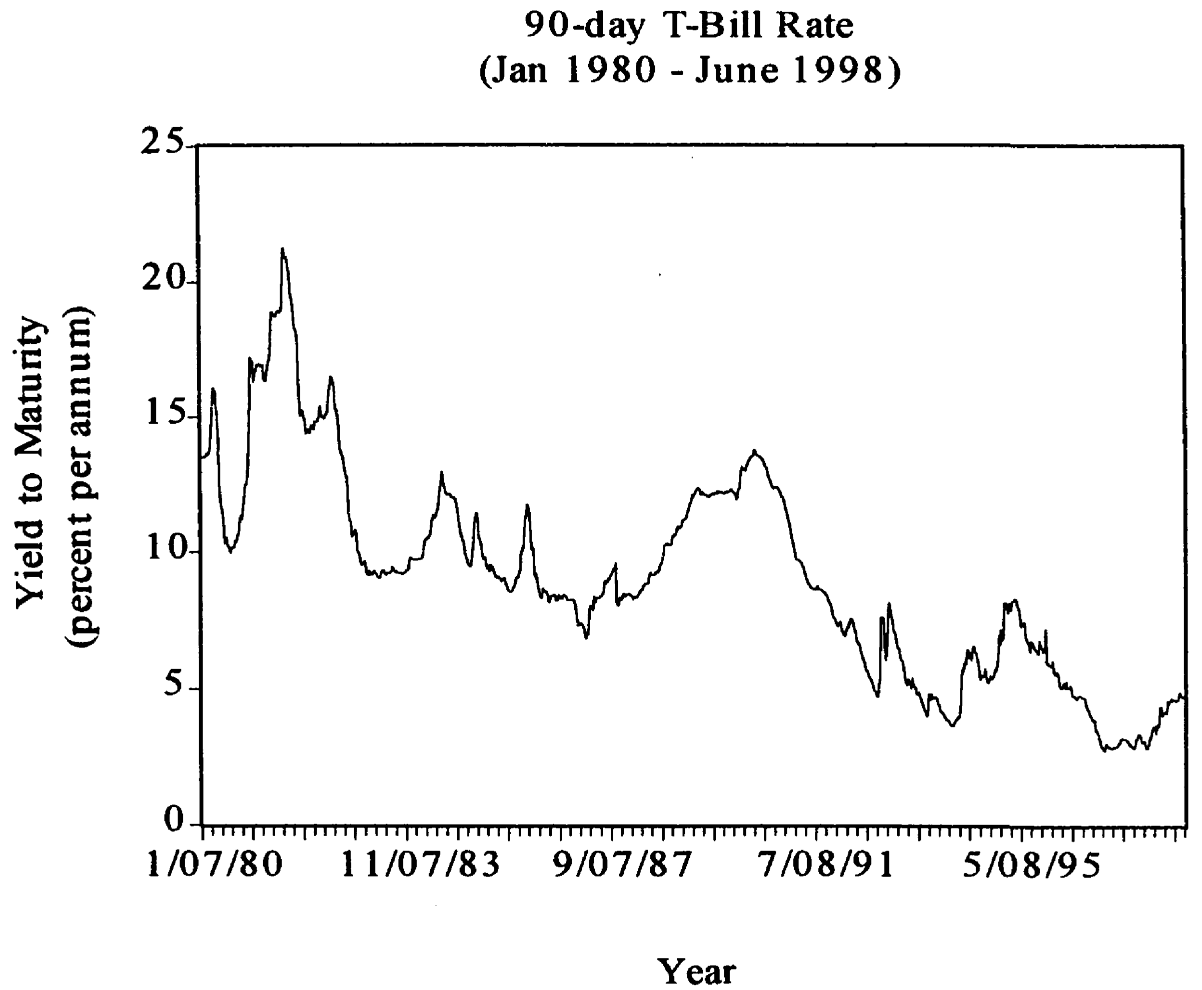

30-day T-Bill Rate
(Jan 1980 - June 1998)
Yield to Maturity
(percent per annum)
25
20
15
10
5
0
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.1



90-day T-Bill Rate
(Jan 1980 - June 1998)
Yield to Maturity
(percent per annum)
25
20
15
10
5
0
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.2

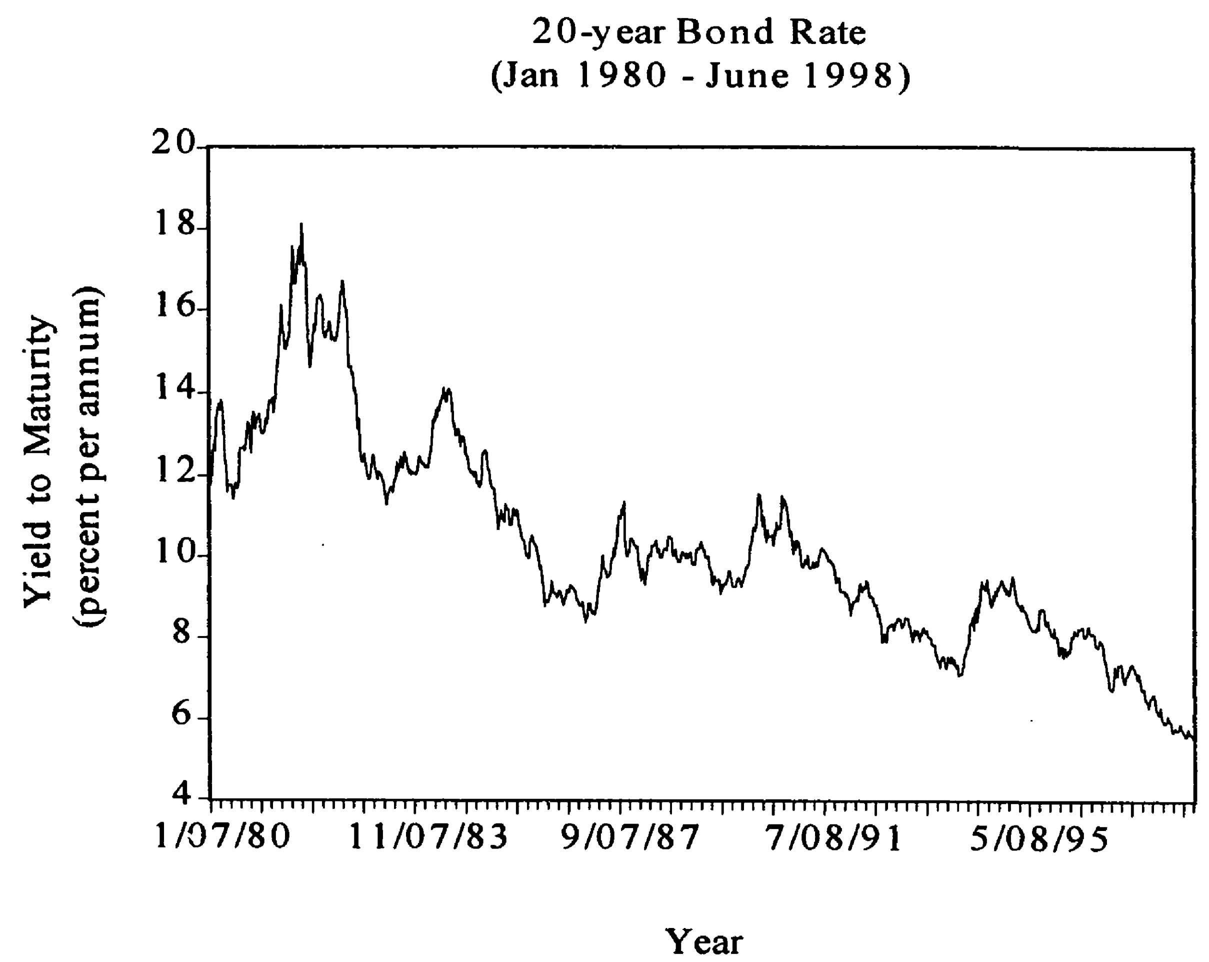

20-year Bond Rate
(Jan 1980 - June 1998)
Yield to Maturity
(percent per annum)
20
18
16
14
12
10
8
6
4
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.3

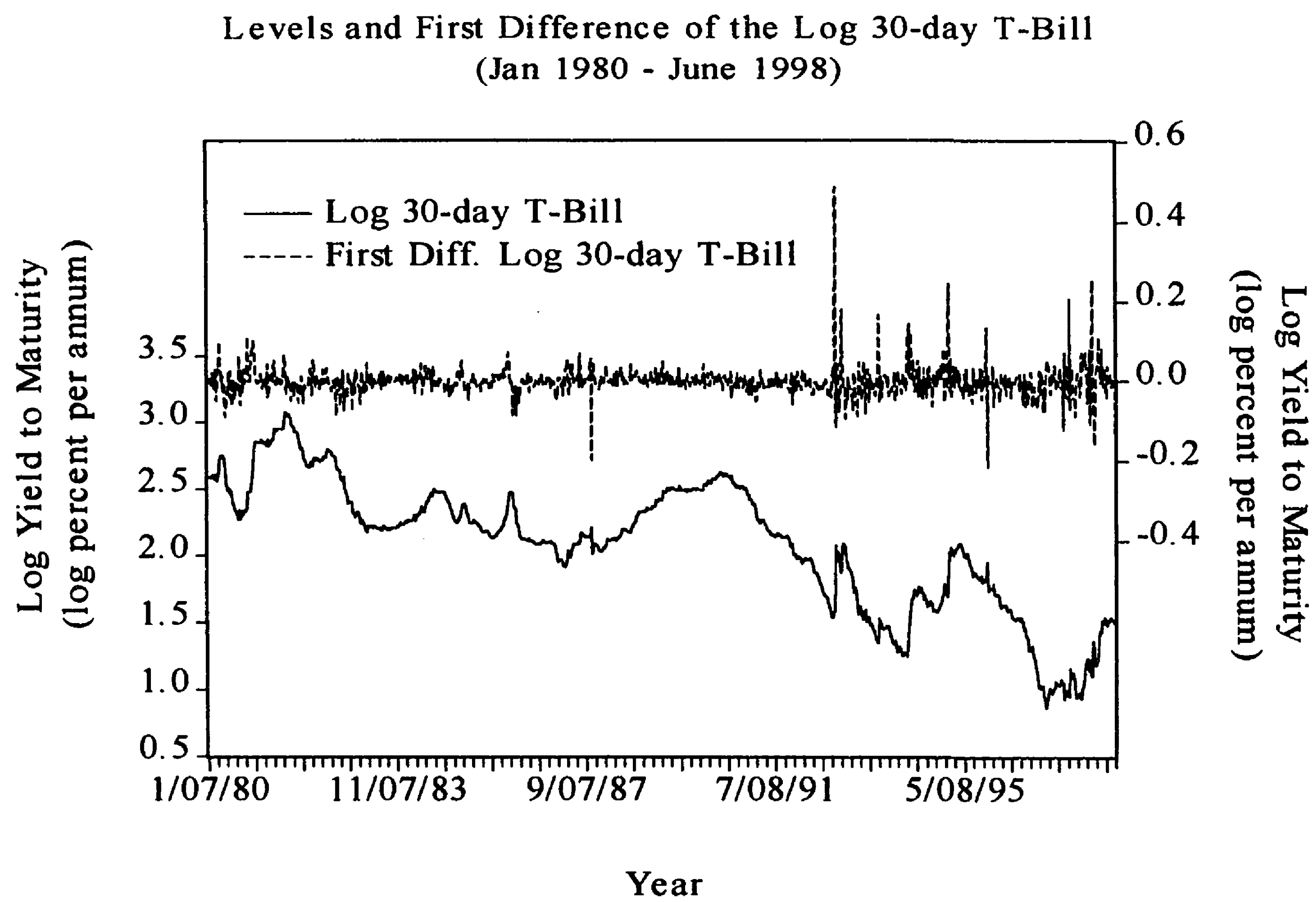

Levels and First Difference of the Log 30-day T-Bill
(Jan 1980 - June 1998)
Log 30-day T-Bill
First Diff. Log 30-day T-Bill
Log Yield to Maturity
(log percent per annum)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.6
0.4
0.2
0.0
-0.2
-0.4
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.4

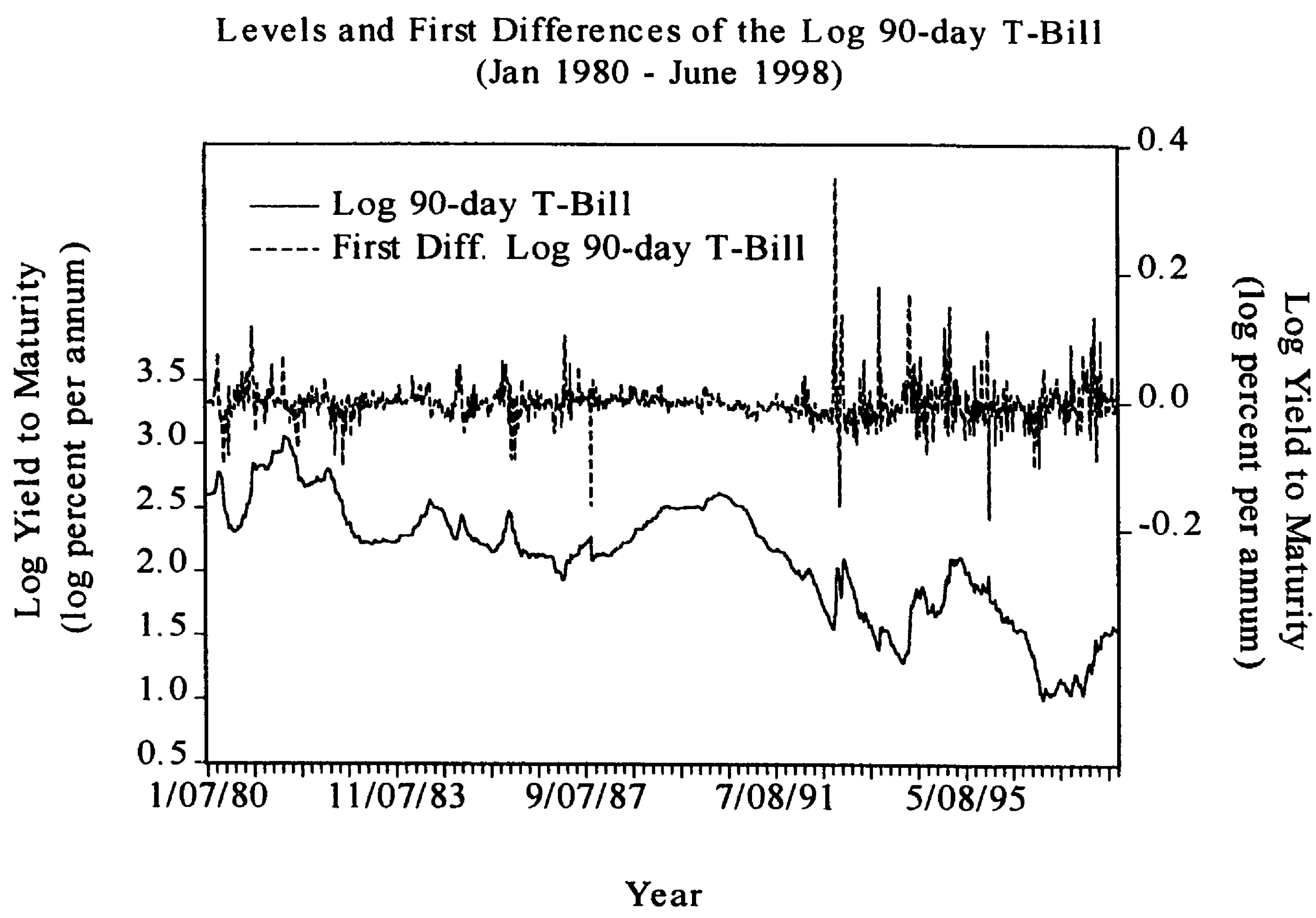

Levels and First Differences of the Log 90-day T-Bill
(Jan 1980 - June 1998)
Log 90-day T-Bill
First Diff. Log 90-day T-Bill
Log Yield to Maturity
(log percent per annum)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.4
0.2
0.0
-0.2
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.5

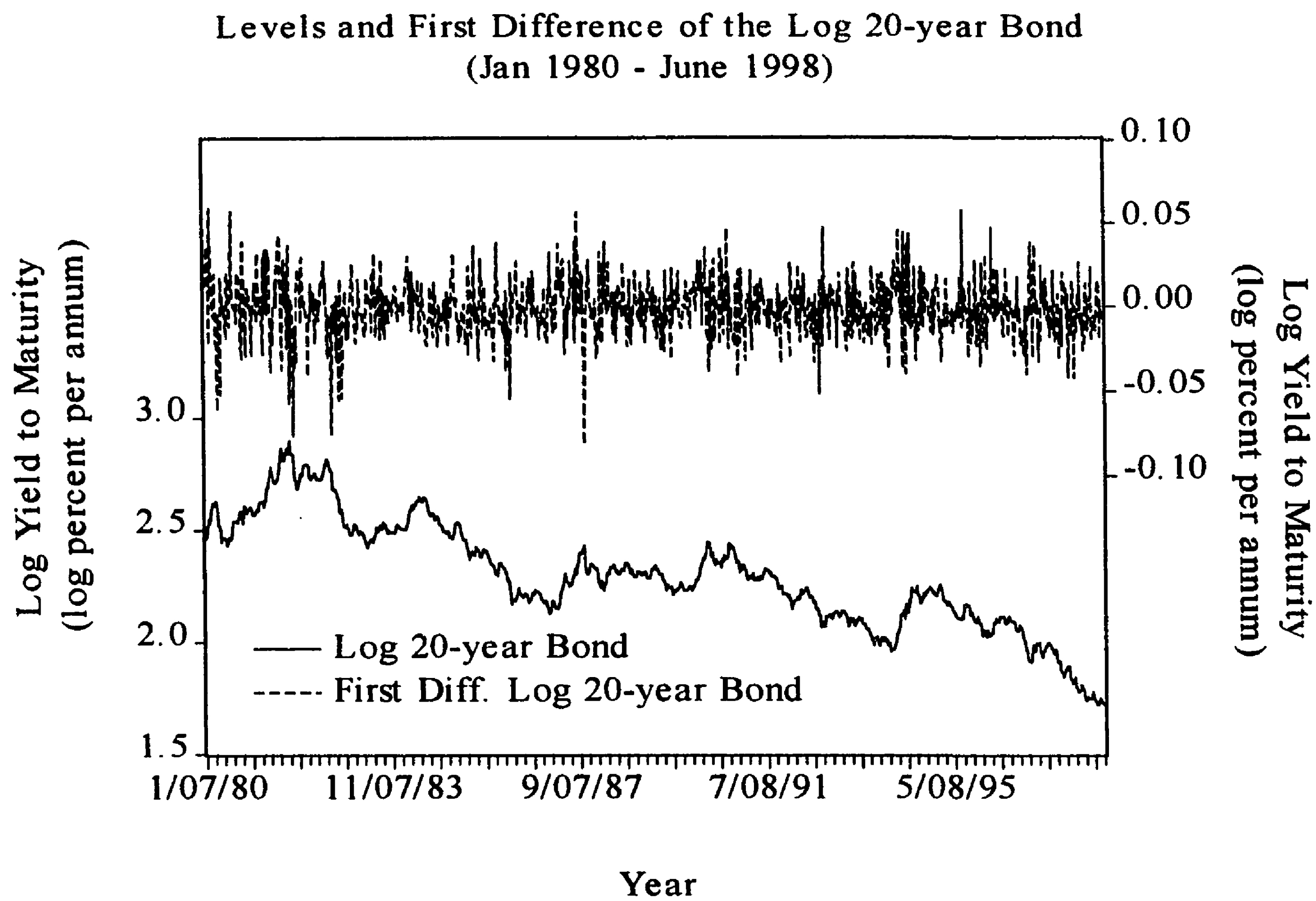

Levels and First Difference of the Log 20-year Bond
(Jan 1980 - June 1998)
Log Yield to Maturity
(log percent per annum)
3.0
2.5
2.0
1.5
0.10
0.05
0.00
-0.05
-0.10
Log 20-year Bond
First Diff. Log 20-year Bond
1/07/80
11/07/83
9/07/87
7/08/91
5/08/95
Year

FIGURE 4.6

The graphs of each series already indicate the presence of non-stationarity. The graphs of the first difference of the data predominately indicate a value fluctuating about zero. This presents evidence of stationarity in first differences.

IV.3. Order of Integration

Time series data are typically non-stationary and it wouldn't be surprising to find it here.[12] Non-stationarity presents problems when regressing one time series variable on another. Often a seemingly good fit and high R^2 is obtained although no meaningful relationship exists. Regression models involving time series are often used for forecasting. Unless non-stationarity is taken into account, regression models will be mis-specified and forecasting invalid.

The assumption that errors corresponding to different observations are uncorrelated (independent) breaks down in time series data. When errors from different time periods are correlated ordinary least squares regression estimators are no longer efficient.[13]

Interest rates are commonly assumed to be non-stochastic and constant in a multi-period setting. This is convenient but misleading. Interest rates are not constant and are stochastic. A stochastic process is strictly stationary if the joint and conditional

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[12] Many studies have found that most macroeconomic time series are non-stationary. It would be impossible to list them all but the most recognised of these studies is Nelson and Plosser (1982). Indeed, most macroeconomic time series are found to be first difference stationary.

[13] One estimator is more efficient than another if it has a smaller variance. An efficient unbiased estimator has the smallest variance of all unbiased estimators.

probability distributions of the process are the same at all points in time. Because this condition is difficult to establish in practice, a relaxed definition of weak stationarity is used, in which the series' mean and variance are constant over time and the value of the covariance between two time periods depends only on the distance or lag between the two time periods and not on the actual time at which the covariance is computed. To be stationary Y_t must have mean, variance and covariance (autocovariance) invariant with respect to time. In other words, after a random shock to the series, the series will tend to return to its mean, and the variance about the mean will be constant and independent of time. The mean, variance and covariance of a series Y_t are:

$$E_t(Y_t) = \mu$$
$$\text{var}(Y_t) = \sigma^2$$
$$\text{cov}(y_t, y_{t+k}) = E[(Y_t - \mu)(Y_{t+k} - \mu)]$$

where μ is the mean, σ^2 is the variance, and $cov(y_t, y_{t+k})$ is the covariance between observation y_t and the observation at lag k, y_{t+k}. Specifically, the simplest example of a stochastic time series is a random walk process without drift:

$$y_t = y_{t-1} + \varepsilon_t$$

The error terms are assumed to be white noise with zero mean and constant variance. The variance of y_t increases with time (var $y_t = t\sigma^2$). Since the variance isn't finite, the series y_t is non-stationary. The series can be made stationary by taking the first difference:

$$y_t - y_{t-1} = \varepsilon_t$$

Where $\Delta y_t = y_t - y_{t-1}$ has a constant mean,

$$E_t(\Delta y_t) = E_t(y_t - y_{t-1}) = \mu_\varepsilon$$

and finite variance,

$$\mathrm{var}(\Delta y_t) = \mathrm{var}(\varepsilon_t) = \sigma_\varepsilon^2$$

and therefore difference stationary, or I(1).[14] Two methods are used here to investigate the stationarity of the 30-day T-bill rate, 90-day T-bill rate, and the 20-year bond rate. Firstly, the autocorrelation and partial autocorrelation functions are examined and secondly, the Augmented Dickey-Fuller unit root test is applied.

IV.3.1. Autocorrelation and Partial Autocorrelation Function

A visual test for stationarity involves examination of a plot of the autocorrelation function[15] and partial autocorrelation function[16] of the series. The plots of the autocorrelation and partial autocorrelation functions for each series in levels and first differences are shown in Tables 4.2 to 4.4. The autocorrelation (AC) and partial autocorrelation (PAC) values are listed for lags up to twelve.

Since a non-stationary series in levels will have a long memory, we would expect the series to take a very long time to return to its mean after a random shock. This shows up as correlation between adjacent data points in the series. Thus the autocorrelation function will decline very slowly, as the number of lags becomes very large. On the other hand if the series is stationary then we would expect the plot of the autocorrelation

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[14] Engle and Granger (1987) developed the notation I(d), to indicate how many times a series must be differenced to render it stationary. This is discussed in depth later.

[15] The autocorrelation function shows how much correlation there is between two data points in a series y_t. The autocorrelation at lag k is: $\rho_k = \dfrac{Cov(y_t, y_{t+k})}{\sigma_{y_t}\sigma_{y_{t+k}}}$.

[16] For a discussion of the derivation of the partial autocorrelation function the reader is referred to Pindyck and Rubinfeld (1998).

function in levels to decline rapidly. In other words, a stationary series will return rapidly to its mean after a random shock.

The autocorrelation and partial autocorrelation functions of each series are quite indicative of the persistence of a unit root in each series. The autocorrelation statistic is reported in the column AC and the partial autocorrelation statistic is reported in the column PAC. The AC values die out slowly with each successive lag. The PAC of each series also shows a strong value at one lag and then convergence to zero thereafter. This clearly represents a unit root in each series as discussed in Pindyck and Rubinfeld (1998). The Augmented Dickey Fuller test for stationarity is used as well but since it is common to find interest rate series to be first difference stationary, it is incorporated into the next section's discussion on the order of integration.

TABLE 4.2
ACF AND PACF IN LEVELS AND FIRST DIFFERENCE OF LOG 30-DAY T-BILL

Log 30-day T-bill in Levels

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat[17]	Prob
.|********	.|********	1	0.996	0.996	955.52	0.000
.|********	.|	2	0.991	-0.003	1903.6	0.000
.|********	.|	3	0.986	-0.038	2843.6	0.000
.|********	.|	4	0.981	-0.051	3774.7	0.000
.|********	.|	5	0.976	-0.004	4697.0	0.000
.|*******	.|	6	0.971	-0.027	5610.0	0.000
.|*******	.|	7	0.965	-0.004	6513.7	0.000
.|*******	*|	8	0.959	-0.098	7406.6	0.000
.|*******	.|	9	0.952	-0.041	8288.1	0.000
.|*******	.|	10	0.945	-0.007	9158.0	0.000
.|*******	.|	11	0.939	-0.018	10016.	0.000
.|*******	.|	12	0.932	0.029	10863.	0.000

30-day T-bill in First Differences

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|	.|	1	-0.001	-0.001	0.0016	0.968
.|	.|	2	0.058	0.058	3.2574	0.196
.|*	.|*	3	0.071	0.072	8.1417	0.043
.|	.|	4	0.005	0.002	8.1668	0.086
.|	.|	5	0.037	0.029	9.5119	0.090
.|	.|	6	0.004	-0.001	9.5287	0.146
.|*	.|*	7	0.136	0.133	27.568	0.000
.|	.|	8	0.047	0.045	29.730	0.000
.|	.|	9	0.015	0.001	29.946	0.000
.|	.|	10	0.042	0.018	31.672	0.000
.|	.|	11	-0.044	-0.052	33.580	0.000
.|	.|	12	-0.022	-0.036	34.045	0.001

[17] The Box Pierce Q-statistic tests the joint hypothesis that all the autocorrelation coefficients are zero, $\rho_1 \dots \rho_k$. The Q-statistic is distributed as a chi-square with k degrees of freedom (where k is equal to the number of lags) and the null is rejected if the Q-statistic is greater than the critical value.

TABLE 4.3
ACF AND PACF IN LEVELS AND FIRST DIFFERENCE OF LOG 90-DAY T-BILL

Log 90-day T-bill in Levels

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|********	.|********	1	0.996	0.996	956.66	0.000
.|********	*|	2	0.992	-0.087	1905.8	0.000
.|********	*|	3	0.987	-0.095	2845.9	0.000
.|********	.|	4	0.981	-0.017	3776.6	0.000
.|********	.|	5	0.976	0.010	4698.1	0.000
.|*******	.|	6	0.970	-0.026	5609.8	0.000
.|*******	*|	7	0.964	-0.078	6510.9	0.000
.|*******	.|	8	0.957	-0.045	7400.5	0.000
.|*******	.|	9	0.950	-0.048	8277.7	0.000
.|*******	.|	10	0.943	0.007	9142.6	0.000
.|*******	.|	11	0.935	-0.031	9994.7	0.000
.|*******	.|	12	0.928	-0.004	10834.	0.000

Log 90-day T-bill in First Differences

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|*	.|*	1	0.144	0.144	19.906	0.000
.|*	.|*	2	0.149	0.131	41.186	0.000
.|	.|	3	0.048	0.011	43.417	0.000
.|	.|	4	0.007	-0.021	43.464	0.000
.|	.|	5	0.046	0.041	45.473	0.000
.|*	.|*	6	0.094	0.089	54.097	0.000
.|*	.|	7	0.081	0.051	60.520	0.000
.|*	.|	8	0.094	0.054	69.090	0.000
.|	.|	9	0.002	-0.038	69.095	0.000
.|	.|	10	0.043	0.027	70.852	0.000
.|	.|	11	0.001	-0.009	70.853	0.000
.|	.|	12	-0.019	-0.036	71.205	0.000

TABLE 4.4
ACF AND PACF IN LEVELS AND FIRST DIFFERENCE OF LOG 20-YEAR BOND

Log 20-year bond in Levels

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|********	.|********	1	0.994	0.994	952.77	0.000
.|********	.|	2	0.988	-0.021	1895.0	0.000
.|********	.|	3	0.982	-0.037	2825.8	0.000
.|*******	.|	4	0.975	-0.044	3744.5	0.000
.|*******	.|	5	0.968	-0.007	4651.0	0.000
.|*******	.|	6	0.961	0.001	5545.4	0.000
.|*******	.|	7	0.953	-0.026	6427.2	0.000
.|*******	.|	8	0.946	-0.014	7296.3	0.000
.|*******	.|	9	0.939	0.002	8152.7	0.000
.|*******	.|	10	0.932	0.031	8997.4	0.000
.|*******	.|	11	0.925	0.016	9830.7	0.000
.|*******	.|	12	0.918	-0.003	10653.	0.000

Log 20-year bond in First Differences

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|	.|	1	0.046	0.046	2.0590	0.151
.|*	.|*	2	0.074	0.072	7.3257	0.026
.|*	.|*	3	0.085	0.079	14.358	0.002
.|	.|	4	-0.009	-0.022	14.443	.006
.|	.|	5	0.045	0.035	16.404	0.006
.|	.|	6	-0.013	-0.021	16.560	0.011
.|	.|	7	-0.008	-0.010	16.622	0.020
.|	.|	8	-0.025	-0.030	17.250	0.028
.|	.|	9	-0.043	-0.036	19.007	0.025
.|	.|	10	-0.033	-0.027	20.057	0.029
.|	.|	11	-0.011	0.003	20.166	0.043
.|	.|	12	0.012	0.023	20.308	0.061

IV.3.2. Unit Root Tests

The most common test for order of integration is the Augmented Dickey-Fuller unit root test, [Dickey and Fuller (1981)]. Both the short rate and long rate series are tested for a unit root using the Augmented Dickey-Fuller (ADF) test. The testing is conducted over the full sample period. Three equations are run using OLS

no constant $$\Delta^k Y_t = \gamma\Delta^{k-1}Y_{t-1} + \sum_{j=1}^{l} \beta_j \Delta^k Y_{t-j} + \varepsilon_t \qquad \ldots(4.1)$$

no trend $$\Delta^k Y_t = \alpha + \gamma\Delta^{k-1}Y_{t-1} + \sum_{j=1}^{l^*} \beta_j \Delta^k Y_{t-j} + \varepsilon_t \qquad \ldots(4.2)$$

with trend $$\Delta^k Y_t = \alpha + \beta t + \gamma\Delta^{k-1}Y_{t-1} + \sum_{j=1}^{l^{**}} \beta_j \Delta^k Y_{t-j} + \varepsilon_t \qquad \ldots(4.3)$$

Where $\Delta Y_t = Y_t - Y_{t-1}$ when k=1 and $\Delta^2 Y_t = \Delta Y_t - \Delta Y_{t-1}$ when k=2 (indicating the second difference of the series). The lag length, l, allows for autocorrelation and is chosen such that the error terms are white noise. Said and Dickey (1984) suggest setting l equal to the number of observations raised to the power of one third. Here the Akaike[18] information criterion and Schwartz[19] model selection criterion are used to select the optimal lag length at which to evaluate the test.

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[18] The Akaike Information Criterion selects the optimal lag length by minimising AIC $= \ln \hat{\sigma}^2 + \exp\left(\frac{2k}{N}\right)$. See Akaike (1969) for more details.

[19] The Schwartz Criterion selects the optimal lag length where $\log \mathrm{S.C} = \ln \hat{\sigma}^2 + \frac{k \ln N}{N}$ is minimised. See Schwartz (1978) for more details.

The procedure begins with k=1 and tests the null hypothesis, H_o:I(1),[20] vs. the alternative hypothesis, H_a: I(0), on equation (4.1). If the null hypothesis, γ=0, [or I(1)] is rejected the series is stationary, I(0), and the test is terminated. However, failure to reject the null is inconclusive and H_o:I(1) vs. H_a: I(0) is tested on equation (4.2). Rejecting the null in this case results in the series being stationary with a drift, I(0), and the test is terminated. However, failure to reject, leads to the test H_o:I(1) vs. H_a: I(0) on equation (4.3). If the null is rejected then the series is trend stationary, I(0).

Failure to reject the null of stationarity in levels means that the series is at least I(1). Testing in the same fashion as above is now conducted on the first difference of the series (k=2) where the null hypothesis is, H_0: I(2), vs. the alternative hypothesis, H_a: I(1). If the null is rejected for all three equations, testing is terminated, and the series is said to be integrated of order one. If the null cannot be rejected then we repeat the process for k=3 and continue in that fashion. In general, if a time series has to be differenced d times to render it stationary then the series is integrated of order d or I(d).

Under the null, the t-statistic is conventionally called the τ (tau) statistic and doesn't follow a student's t distribution even in large samples. The τ (tau) statistic critical values have been tabulated by MacKinnon (1991) where the null is rejected if $|\tau| > |\tau_c|$, where τ_c is the MacKinnon critical value.

The results from the unit root testing are presented in Tables 4.5 to 4.8. Only the ADF with trend in levels is shown since, if the null hypothesis of possible stationarity in first differences cannot be rejected then the null hypothesis of the ADF test with no trend

[20] I(d) means integrated of order d. If d=0 then the series is stationary in levels and requires no differencing to render it stationary. If d=1 than the series is stationary only

or no constant could not be rejected either. These results are shown in Table 4.5. Of course the null hypothesis of trend stationary in first differences cannot be rejected. The testing proceeds to the null hypothesis of possible second difference stationarity and the results are reported in Table 4.6 to 4.8. The Schwartz and AIC criterion are listed but not discussed here. They will become more important in the next section's discussion of cointegration.

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after it has been differenced once. See Engle and Granger (1987) for more details.

TABLE 4.5
AUGMENTED DICKEY - FULLER (ADF) UNIT ROOT TESTS IN "LOGGED LEVELS" WITH TREND

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{j=1}^{J**} \beta_j \Delta Y_{t-j} + \varepsilon_t$$

Ho: at least I(1)
Ha: I(0) stationary

Series	Lags	Coefficient	ADF Stat	AIC	Schwartz
30-day T-Bill	0	-0.009605	-2.168482	-6.545090	-6.529880
	1	-0.009625	-2.165429	-6.541981	-6.521685
	2	-0.010234	-2.298902	-6.542887	-6.517497
	3	-0.010981	-2.464396	-6.545785	-6.515291
	4	-0.011099	-2.480403	-6.542793	-6.507187
90-day T-Bill	0	-0.007480	-1.913568	-6.895605	-6.880396
	1	-0.008550	-2.204867	-6.914619	-6.894323
	2	-0.009685	-2.511842	-6.930201	-6.904811
	3	-0.009849	-2.543545	-6.927370	-6.896876
	4	-0.009719	-2.499086	-6.924423	-6.888817
20-year Bond	0	-0.012992	-2.502925	-8.108121	-8.092911
	1	-0.013476	-2.583971	-8.107930	-8.087634
	2	-0.014314	-2.736100	-8.111408	-8.086017
	3	-0.015017	-2.866929	-8.119501	-8.089007
	4	-0.014520	-2.753778	-8.117640	-8.082034

NOTES: ADF critical values: at the 1% significance level -3.9726, at the 5% significance level -3.4168, at the 10% significance level -3.1304. Cannot reject the null at the 1% significance level at all lags.

TABLE 4.6
AUGMENTED DICKEY - FULLER (ADF) UNIT ROOT TESTS IN "FIRST DIFFERENCES OF LOGGED LEVELS" NO CONSTANT

$$\Delta^2 Y_t = \gamma \Delta Y_{t-1} + \sum_{j=1}^{l} \beta_j \Delta^2 Y_{t-j} + \varepsilon_t$$

H_0: at least I(2), second difference stationary
H_a: I(1) first difference stationary

Series	Lags	Coefficient	ADF Stat	AIC	Schwartz
30-day T-Bill	0	-1.000388	-30.96391	-6.542421	-6.537347
	1	-0.941385	-20.61363	-6.542810	-6.532654
	2	-0.873323	-15.93575	-6.544973	-6.529726
	3	-0.870763	-14.10553	-6.541906	-6.521560
	4	-0.844623	-12.43678	-6.539974	-6.514520
90-day T-Bill	0	-0.855164	-26.74669	-6.914870	-6.909796
	1	-0.742800	-17.71346	-6.929152	-6.918996
	2	-0.734027	-15.17333	-6.926158	-6.910911
	3	-0.748617	-13.87679	-6.923412	-6.903066
	4	-0.717481	-12.12589	-6.922028	-6.896574
20-year Bond	0	-0.951823	-29.50025	-8.103729	-8.098655
	1	-0.882311	-19.83436	-8.106853	-8.096697
	2	-0.813864	-15.46509	-8.114633	-8.099386
	3	-0.833625	-14.16124	-8.113309	-8.092962
	4	-0.796744	-12.30741	-8.113970	-8.088516

NOTES: ADF critical values: at the 1% significance level -2.5679, at the 5% significance level -1.9397, at the 10% significance level -1.6158. All reject the null of second difference stationary at the 1% significance level.

TABLE 4.7

AUGMENTED DICKEY - FULLER (ADF) UNIT ROOT TESTS IN "FIRST DIFFERENCES OF LOGGED LEVELS"

NO TREND

$$\Delta^2 Y_t = \alpha + \gamma \Delta Y_{t-1} + \sum_{j=1}^{l^*} \beta_j \Delta^2 Y_{t-j} + \varepsilon_t$$

H_0: at least I(2), second difference stationary

H_a: I(1) first difference stationary

Series	Lags	Coefficient	ADF Stat	AIC	Schwartz
30-day T-Bill	0	-1.001287	-30.97569	-6.541239	-6.531091
	1	-0.943065	-20.62942	-6.541516	-6.526282
	2	-0.875588	-15.95406	-6.543588	-6.523259
	3	-0.873595	-14.12613	-6.540502	-6.515069
	4	-0.848013	-12.46036	-6.538560	-6.508015
90-day T-Bill	0	-0.856190	-26.76127	-6.913697	-6.903549
	1	-0.744369	-17.72934	-6.927766	-6.912532
	2	-0.736096	-15.19231	-6.924760	-6.904430
	3	-0.751225	-13.89900	-6.922037	-6.896604
	4	-0.720506	-12.14988	-6.920607	-6.890062
20-year Bond	0	-0.953753	-29.54354	-8.103540	-8.093392
	1	-0.885844	-19.88262	-8.106494	-8.091259
	2	-0.818789	-15.52114	-8.114221	-8.093892
	3	-0.840227	-14.22880	-8.113081	-8.087648
	4	-0.804247	-12.37235	-8.113482	-8.082937

NOTES: ADF critical values: at the 1% significance level -3.4399, at the 5% significance level -2.8650, at the 10% significance level -2.5686. All reject the null of second difference stationary at the 1% significance level.

TABLE 4.8

AUGMENTED DICKEY - FULLER (ADF) UNIT ROOT TESTS IN "FIRST DIFFERENCES OF LOGGED LEVELS" WITH TREND

$$\Delta^2 Y_t = \alpha + \beta t + \gamma \Delta Y_{t-1} + \sum_{j=1}^{l**} \beta_j \Delta^2 Y_{t-j} + \varepsilon_t$$

H_0: at least I(2), second difference stationary

H_a: I(1) first difference stationary

Series	Lags	Coefficient	ADF Stat	AIC	Schwartz
30-day T-Bill	0	-1.001303	-30.95997	-6.539168	-6.523947
	1	-0.943095	-20.61915	-6.539445	-6.519132
	2	-0.875622	-15.94610	-6.541509	-6.516097
	3	-0.873642	-14.11925	-6.538423	-6.507904
	4	-0.848050	-12.45409	-6.536473	-6.500837
90-day T-Bill	0	-0.856204	-26.74774	-6.911627	-6.896405
	1	-0.744392	-17.72046	-6.925690	-6.905378
	2	-0.736124	-15.18474	-6.922680	-6.897269
	3	-0.751260	-13.89214	-6.919956	-6.889436
	4	-0.720539	-12.14384	-6.918520	-6.882885
20-year Bond	0	-0.955450	-29.57743	-8.103048	-8.087826
	1	-0.888675	-19.91506	-8.105671	-8.085358
	2	-0.821984	-15.54517	-8.112985	-8.087574
	3	-0.843934	-14.25204	-8.111773	-8.081253
	4	-0.808751	-12.40407	-8.112249	-8.076613

NOTES: ADF critical values: at the 1% significance level -3.9726, at the 5% significance level -3.4168, at the 10% significance level -3.1304. All reject the null of second difference stationary at the 1% significance level.

Table 4.5 unequivocally indicates non-stationarity in levels for each series regardless of lag length. The results from the second stage unarguably indicate rejection of the null of second difference stationary. This provides evidence that each series is integrated of order one. These results are significant at the 1% significance level and are quite robust. This is not surprising though since it has been quite extensively shown in the literature that interest rate series have a single unit root.

IV.4. Cointegration Tests

The tests for stationarity, performed in the previous section, found that each series is first difference stationary. The next step is to test the implication of the pure expectations hypothesis that the short and long rate move together over time as imposed by equation (2.14). This is done by employing the cointegrating regression.

Usually a linear combination of two series, which are both I(1), results in a series that is I(1). However in certain cases there exists a linear combination of the two I(1) series that yields a stationary series, I(0). In the case of cointegrating variables, the stochastic trend components of two or more variables exactly offset each other to give a stationary linear combination. In the long run, if two or more series move closely together, so that even though the series may be trended, the difference between them is stationary. In this case, the series are said to be cointegrated.[21] The Engle and Granger (1987) cointegration approach is used to test for this. The Engle and Granger approach uses the ADF test for stationarity to test for a unit root on the error terms in the cointegrating regressions. Following from equation (2.14), support for the pure expectations hypothesis is found if there exists cointegration between the long and short rate. Therefore, a rise in the short rates will also raise long rates, or vice versa, causing short and long rates to move together over time. This is because expectations are formed based on the implied forward rates in the term structure. This can be tested using the cointegration test mentioned above and discussed in detail here.

This study uses two different series as a proxy for the short rate and one proxy for the long rate. The reasons for this is the apparently tremendous volatility in the 30-day T-bill rate. McFadyen et al. (1991) and MacDonald et al. (1988) use the U.S. 90-day T-bill rate and U.K. 90-day T-bill rate respectively but use the 5, 10, 20-year bond rate as proxies for the long rate. It would seem clear that cointegration results on near term maturities would give more favourable results. However, if cointegration tests used maturities that are far apart, the tests would be more robust in evaluating whether the pure expectations theory of the term structure holds or not.

[21] See Engle and Granger (1987) for more details.

Using the full sample period, the results form the previous section show both the short and long rate to be I(1). Since the short and long rates are integrated of the same order, there exists a possibility of a cointegrating relationship between them. The first step of the Engle and Granger cointegration test is to run the following four regressions using OLS:

$$L_t = \alpha + \beta S_t + z_{t,1} \qquad \ldots(4.4)$$

$$S_t = \alpha' + \beta' L_t + z_{t,2} \qquad \ldots(4.5)$$

$$L_t = \gamma + \delta S_t + \phi t + z_{t,3} \qquad \ldots(4.6)$$

$$S_t = \gamma' + \delta' L_t + \phi' t + z_{t,4} \qquad \ldots(4.7)$$

where t is a linear time trend. Both the forward and reverse cointegrating regressions are tested although they are asymptotically equivalent. If the short and long rate are both I(1) then the residual series $z_{t,k}$ (k=1,…,4) will be stationary. Hence to test for cointegration, the ADF test is applied to test the order of integration of $z_{t,k}$. Using OLS, the following regression is run:

$$\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t \qquad \ldots(4.8)$$

If the null hypothesis, γ=0 [or I(1)] is rejected then the series is stationary and the test is terminated. Failure to reject the null provides evidence that the series is non-stationary, γ=0, and possibly I(1).The τ statistic critical values are given by the MacKinnon tables where the null is rejected if $|\tau| > |\tau_c|$. If $z_{t,k}$ is stationary then the short and long rate are cointegrated.

The results form the unit root tests on the residuals from each cointegrating equation are presented in Tables 4.9 to 4.12. The null hypothesis is that the residuals are possibly first difference stationary and the alternative is the residuals are stationary in levels. The tests were conducted for lags up to four. The AIC and Schwartz criterion are reported where an asterix indicates the optimal lag length selected by minimising the respective criteria. Tables 4.9 and 4.10 use the 30-day T-bill rate as the proxy for the short rate and Tables 4.11 and 4.12 use the 90-day T-bill as the proxy for the short rate.

TABLE 4.9
ENGLE AND GRANGER COINTEGRATION TESTS
BETWEEN THE LOG 20-YEAR BOND AND THE LOG 30-DAY T-BILL NO TREND

H_0: I(1)
H_a: I(0) cointegrated

Cointegrating equation: $L_t = \alpha + \beta S_t + z_{t,1}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.012966	-2.197677	-7.658598	-7.653528*
1	-0.013216	-2.224010	-7.655872	-7.645724
2	-0.014144	-2.365882	-7.655757	-7.640523
3	-0.015721	-2.619541	-7.660775*	-7.640446
4	-0.016005	-2.646054	-7.659363	-7.633930

Cointegrating equation: $S_t = \alpha' + \beta' L_t + z_{t,2}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.015961	-2.675273	-6.248104	-6.243034*
1	-0.016039	-2.671345	-6.245120	-6.234972
2	-0.016774	-2.779745	-6.244656	-6.229422
3	-0.018069	-2.984818	-6.248649*	-6.228319
4	-0.018299	-3.002707	-6.246970	-6.221538

Note: Critical values for all four tests; 1% (-2.5680), 5% (-1.9397), 10% (-1.6158). All reject the null at the 5% significance level. Optimal lag length indicated by *.

TABLE 4.10
ENGLE AND GRANGER COINTEGRATION TESTS
BETWEEN THE LOG 20-YEAR BOND AND THE LOG 30-DAY T-BILL WITH TREND

H_0: I(1)
H_a: I(0) cointegrated

Cointegrating equation: $L_t = \gamma + \delta S_t + \phi t + z_{t,3}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.017549	-2.799525	-8.066853	-8.061783*
1	-0.018112	-2.867896	-8.065383	-8.055235
2	-0.019116	-3.008036	-8.066810	-8.051576
3	-0.020385	-3.195792	-8.074161*	-8.053832
4	-0.020118	-3.125832	-8.072525	-8.047092

Cointegrating equation: $S_t = \gamma' + \delta' L_t + \phi t + z_{t,4}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{p} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.014112	-2.498439	-6.394896*	-6.389826*
1	-0.014122	-2.486293	-6.391834	-6.381686
2	-0.014837	-2.600873	-6.391252	-6.376017
3	-0.016078	-2.810633	-6.394568	-6.374238
4	-0.016375	-2.845496	-6.392723	-6.367290

Note: Critical values for all four tests; 1% (-2.5680), 5% (-1.9397), 10% (-1.6158). All reject the null at the 5% significance level. Optimal lag length indicated by *.

TABLE 4.11
ENGLE AND GRANGER COINTEGRATION TESTS
BETWEEN THE LOG 20-YEAR BOND AND THE LOG 90-DAY T-BILL NO TREND

H_0: I(1)
H_a: I(0) cointegrated
Cointegrating equation: $L_t = \alpha + \beta S_t + z_{t,1}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.008202	-1.646322•	-8.035902	-8.030833*
1	-0.009404	-1.882166•	-8.039671	-8.029523
2	-0.010358	-2.064012	-8.041158	-8.025924
3	-0.010784	-2.138887	-8.042707*	-8.022377
4	-0.010063	-1.983825	-8.042349	-8.016916

Cointegrating equation: $S_t = \alpha' + \beta' L_t + z_{t,2}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.010549	-2.140749	-6.773403	-6.768333
1	-0.011765	-2.387893	-6.780811	-6.770663*
2	-0.012705	-2.572000	-6.783796*	-6.768561
3	-0.012782	-2.575741	-6.783591	-6.763262
4	-0.012127	-2.431141	-6.782647	-6.757214

Note: Critical values for all four tests; 1% (-2.5680), 5% (-1.9397), 10% (-1.6158). All reject the null at the 5% significance level. Optimal lag length indicated by *.

TABLE 4.12
ENGLE AND GRANGER COINTEGRATION TESTS
BETWEEN THE LOG 20-YEAR BOND AND THE LOG 90-DAY T-BILL WITH TREND

H_0: I(1)
H_a: I(0) cointegrated

Cointegrating equation: $L_t = \gamma + \delta S_t + \phi t + z_{t,3}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.014748	-2.552335	-8.263912	-8.258842*
1	-0.015409	-2.650245	-8.263877	-8.253730
2	-0.016161	-2.763231	-8.264774	-8.249540
3	-0.016743	-2.851927	-8.270515*	-8.250185
4	-0.015814	-2.673485	-8.269671	-8.244238

Cointegrating equation: $S_t = \gamma' + \delta' L_t + \phi t + z_{t,4}$

Unit root test: $\Delta z_{t,k} = \gamma z_{t-1,k} + \sum_{j=1}^{l} \beta_j \Delta z_{t-j,k} + \varepsilon_t$

Lags	Coefficient	ADF Stat	AIC	Schwartz
0	-0.009191	-1.974572	-6.916709	-6.911640
1	-0.010512	-2.263646	-6.927378	-6.917230*
2	-0.011556	-2.485693	-6.932042*	-6.916807
3	-0.011616	-2.487069	-6.930923	-6.910594
4	-0.011115	-2.368062	-6.929557	-6.904124

Note: Critical values for all four tests; 1% (-2.5680), 5% (-1.9397), 10% (-1.6158). All reject the null at the 5% significance level. Optimal lag length indicated by *.

Tables 4.9 - 4.12 present results in support of cointegration. Specifically, cointegration is supported everywhere at the 10% level of significance. At the optimal lag length indicated by the AIC criterion, cointegration is supported at the 5% level of significance. Cointegration between the 30-day T-bill rate and the 20-year bond rate is supported at the 5% level of significance with trend and without trend according to both criterion. Cointegration is supported between the 90-day T-bill rate and the 20-year bond rate at the optimal lag length indicated by the AIC criterion at the 5% level of significance. Cointegration between the 90-day T-bill rate and the 20-year bond rate at the optimal lag length indicated by the Schwartz criterion is supported for the short rate as the dependent variable in the cointegrating equation (4.5) and (4.7).

IV.5. Summary

The motivation behind this chapter was to perform the cointegration based tests of the term structure of interest rates. The pure expectations theory of the term structure of interest rates was tested by examining a cointegrating relationship. This chapter outlined the methodology in detail behind testing for unit roots and testing for a cointegrating relationship. Tests on each of the three interest rate series were performed determining that each series is non-stationary but stationary in first differences. After finding each series to be integrated of order one, the existence of a cointegrating relationship was examined. The results from the cointegration tests suggested that the short rate and long rate are cointegrated. From the theory developed on cointegration this showed that the short and long rates of interest move together over time. This constitutes evidence in support of the pure expectations theory of the term structure.

Whenever a cointegrating relationship exists there must be a causal relationship as well. With first difference stationarity established in each series and the existence of cointegration, further evidence of the pure expectations hypothesis can be examined by performing causality tests. This is the topic of the next chapter.

V. CAUSALITY AND THE SPREAD

V.1. Introduction

The motivation behind this chapter is to address two additional issues concerning the validity of the pure expectations hypothesis of the term structure of interest rates. Since cointegration was established in the previous chapter, an analysis of causality and forecasting can be conducted. It will be shown that cointegration is necessary for examining these two issues. Investigation of causality and forecasting will offer more insight into the relationship between the short rate and long rate.

The conventional (orthodox) understanding of the relationship between long maturity bonds and short maturity bonds is that changes in short rates precede changes in long rates. The pure expectations hypothesis, implies this as well as that changes in long rates cause changes in short rates. This will be tested using the Granger (1969) test for causality. If Granger causality can be established this implies that forecasts of future interest rate changes can be improved by adding interest rate expectations by way of the interest rate spread. Thus, the result of establishing Granger causality is twofold. It means forecasts of future changes in the short rate can be improved by including the spread and a rejection of the orthodox understanding of the relationship between short rates and long rates.

Section V.2 discusses the basic concepts behind the possible existence of a causal relationship. It develops the theory to incorporate the yield spread and explains the reasons for doing this. In addition, this section discusses the implications of the yield spread for causality and forecasting. An autoregressive model is constructed to test for causality in Section V.3. The results from testing are reported in Section V.4. Lastly, Section V.5 brings the chapter together. The implications of the pure expectations hypothesis, the relationship between the long and short maturity interest rates and the connection between causality and forecasting is reviewed.

V.2. Background Theory

Government bond market participants have generally held the conventional wisdom that changes in long-maturity interest rates follow the changes in the short-maturity interest rates. This view has been supported by Ayers and Barry (1979), Mankiw (1984), and Mankiw and Summers (1984). It is widely believed that the monetary authority most directly controls interest rates near the short end of the maturity spectrum and aggregate demand depends primarily on long-term interest rates.[22] This point of view has its basis in the assumption that monetary policy is typically executed near the short end of the maturity spectrum. However, the short to long interpretation is by no means a consensus.

Laurence Weiss (1984) feels that the importance of the pure expectations hypothesis for understanding the role of monetary policy is overstated. He says that this point of view is consistent with Keynesian analysis and knows of no empirical support for the proposition. Yet, Mankiw and Summers (1984) argue that understanding the term structure of interest rates is critical to the evaluation of the effects of alternative macroeconomic policies.

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[22] This can be seen in the simple IS-LM model of Clarida and Friedman where short rates enter the LM curve and long rates enter the IS curve.

Nevertheless, any monetary transmission will inevitably rely on the behaviour of the term structure of interest rates. Since Froot (1989), research has focused on resolving the relationship between short maturity interest rates and long maturity interest rates by employing the Granger (1987) test for causality.

Since the Granger test for causality requires an autoregressive model there become two reasons why it is necessary to use the yield spread. A condition underlying tests based on an autoregressive model is that the series are stationary. Since, the stochastic process of the interest rate series are very near unit roots, this autoregressive model will be mis-specified unless this is taken into consideration.[23] Moreover, since it is well known that macroeconomic time series are typically non-stationary researchers have resorted to first differencing. This procedure is inefficient. First, it does not allow the full set of restrictions implied by rational expectations to be imposed.[24] Second, if two variables such as the short rate and long rate are cointegrated as shown in the previous chapter then an estimated autoregressive model containing their first differences will be mis-specified. To construct a well behaved model the spread term must be used.

The theory from chapter two is further developed here to employ the use of the yield spread. Heretofore, the implications of the pure expectations hypothesis for the levels of interest rates has been examined. Recall equation (2.14),

$$y_{nt} = (1/n)\sum_{i=0}^{n-1} E_t\left(y_{1,t-i}\right)$$

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[23] See Campbell et al. (1997) for a discussion of highly persistent unit root processes.
[24] See Shiller (1979) for the reasons behind this.

where the long rate is a weighted average of expected future short rates. In the previous chapter the implication of cointegration in this equation was tested and shown to exist. Now, recall the yield spread between the n-period yield and the one-period yield,

$$s_{nt} = y_{nt} - y_{1t}. \qquad \ldots(5.1)$$

When y_{1t} is subtracted from both sides of equation (2.14) and s_{nt} is substituted into the left hand side, the following equality is created

$$s_{nt} = E_t\left[\sum_{i=1}^{n-1}(1-i/n)\Delta y_{1,t+i}\right]. \qquad \ldots(5.2)$$

Where $\Delta y_{1,t+i} = y_{1,t+i} - y_{1,t+i-1}$. Equation (5.2) says that the yield spread equals a weighted average of expected future short-term interest rate changes. When the yield spread is high, for example, the logarithm of the short rate is expected to rise. Most importantly, equation (5.2) says that if changes in short rates are stationary then the yield spread must also be stationary. This means that yields of different maturities must be cointegrated, which has important implications for the relation between the yield spread and future short rate changes. It means that the yield spread is the optimal forecaster of the change in the short rate.

In particular, this implication suggests Granger causality from the long rate to the short rate as well as from the short rate to the long rate. Granger causality applies, for instance, when lagged values of both the short rate and the long rate result in better forecasts of the short rate than estimates obtained only including lagged values of the short rate.

An autoregressive model is outlined to test the implication of Granger causality imposed by the pure expectations hypothesis of the term structure from equation (5.2).

This approach models the short rate and the spread[25] as an autoregressive model. The model of the pure expectations hypothesis in equation (2.14) imposes cointegration, therefore, since the data support this, the residuals from the cointegrating regression along with the first differenced short rates will form the autoregressive model. From this a causal relationship between the long rate and the short rate is tested. This is possible because the spread contains the information on the long rate.

Thus, the residuals from the cointegrating regression equal the spread. Therefore, causality testing can tell us whether forecasts involving the short rate can be improved by using the additional information contained in the "spread." There has been extensive research on the predictive power of the "spread" between the long rate and short rate in forecasting future short-term interest rate changes. The research has found forecasting including the spread to be superior to using only historical information. Many studies, including those by Campbell and Shiller (1984) and Fama and Bliss (1987), have found the spread to have some positive predicative power, particularly for short rate changes further in the future. These studies typically involve testing whether the spread Granger causes changes in the short rate. The main reason for this is that the spread must be used in forecasting, otherwise the model will be mis-specified.[26] If causality is found between the spread and the short rate, this constitutes evidence in support of the pure expectations theory of the term structure.[27]

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[25] In MacDonald and Speight (1988) and McFadyen, Pickerill and Devaney (1991), the spread has been given the interpretation of an error correction term.
[26] As argued above. See MacDonald and Speight (1988) for a further discussion of the reasons for doing this.
[27] Future interest rates are not the only thing the term structure can forecast. A term structure may also contain expectations about future inflation or future economic activity,

The segmented markets theory of the term structure of interest rates implies the absence of causality, while the expectations (or liquidity premium) theory suggests that the long rate causes the short rate. The pure expectations theory requires both a long to short and short to long causal relationship. Hence, if cointegration is established and bi-directional causality can be shown then the pure expectations theory is supported.

V.3. Granger Causality

The concept of causality testing is to test for model mis-specification by running a regression with and without the other variable(s) and compare the explanatory power of the model. The first consideration is that of formulating the appropriate autoregressive model. As mentioned in the previous section the yield spread from the cointegrating regression is used as well as the first difference of the short rate. Since the first difference of the short rate is stationary and the spread is stationary by definition, the autoregressive model will have well behaved properties. The two models used are

$$\Delta S_t = \alpha + \sum_{e=1}^{E} \beta_e z_{t-e,k} + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t \qquad \ldots(5.3)$$

$$z_{t,k} = a + \sum_{g=1}^{G} b_g z_{t-g,k} + \sum_{h=1}^{H} c_h \Delta S_{t-h} + e_t \qquad \ldots(5.4)$$

where $z_{t,k}$ is the spread term from the residuals in the cointegrating regression. Before the tests can be applied to the autoregressive models the optimal lag lengths need to be

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including growth. Fama (1975) suggests that the term structure of interest rates provides the best estimate of expected inflation. In addition, see Harvey (1997) for an excellent discussion on the relation between the term structure of interest rates and Canadian economic growth. The term structure and future inflation or economic growth are not discussed here.

found. A regression of every combination up to a total of fifteen lags on each of the change in the short rate and the spread was conducted for both models and evaluated by the Akaike information criterion. That is, the optimal lag length was evaluated from the 225 regression equations for equation (5.3) and 225 regression equations for equation (5.4) using the Akaike information criterion.[28] Test results are reported for only the autoregressive models at the optimal lag length.

The test is conducted as follows. First, to test the null hypothesis that the spread ($z_{t,k}$) does not Granger cause the short rate (ΔS_t), the restricted regression of ΔS_t regressed on the lagged ΔS_t's is run

$$\Delta S_t = \alpha + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$$

and the restricted sum of squares (RSS_r) is obtained. Next the unrestricted regression of ΔS_t on the lagged ΔS_t's and the lagged $z_{t,k}$ terms is run

$$\Delta S_t = \alpha + \sum_{e=1}^{E} \beta_e z_{t-e,k} + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$$

and the unrestricted sum of squares, RSS_u, is obtained. The test statistic

$$F = \frac{(RSS_r - RSS_u)/q}{RSS_u/(n-r)}$$

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[28] The Schwartz criterion was applied as well. The optimal lag length by the Schwartz criterion for the 30-day T-bill rate is AR(1,1), when the change in the short rate is the dependant variable and AR(4,2), when the spread is the dependant variable. The optimal lag length for the 90-day T-bill rate is AR(1,2), when the change in the short rate is the dependant variable and AR(1,2), when the spread is the dependant variable. The results of causality tests at the optimal lag length chosen by the Schwartz criterion are not shown here.

is distributed as an $F(q,n\text{-}r)$. Where n is the number of observations and r is the number of estimated parameters in the restricted equation. The number of parameter restrictions is q, in this case the number of lagged $z_{t,k}$ terms. Therefore, the degrees of freedom are $(n\text{-}r)$. If the computed F exceeds the critical F then the null is rejected, the $z_{t,k}$ terms belong in the equation and the spread causes the short rate. The same procedure is followed for the null hypothesis that the short rate does not Granger cause the spread. From this there can exist four possible relationships:

1. Uni-directional Granger causality exists from the spread to the short rate if the estimated coefficients on the lagged spread series, β_e, are significantly different from zero as a group and the set of estimated coefficients on the lagged short rate, c_h, are not statistically different from zero.
2. Uni-directional Granger causality exists from the short rate to the spread if the estimated coefficients on the lagged short rate series, c_h, are significantly different from zero as a group and the set of estimated coefficients on the lagged spread, β_e, are not statistically different from zero.
3. Feedback, or bi-directional Granger causality, is suggested when the sets of spread and short rate coefficients are significantly different from zero in both regressions.
4. Independence is suggested when the sets of spread and short rate coefficients are not statistically significant in both regressions.

If bi-directional causality can be established between the short rate and the long rate then the pure expectations hypothesis is supported.

V.4. Empirical Results

The results from the causality testing are presented in Table 5.1 and Table 5.2. The tests were conducted at the optimal lag length chosen by the Akaike information criterion. The critical F-values are reported and indicate at what confidence level the null can be rejected.

Table 5.1 conducts the tests using the 30-day T-bill rate and the spread between the 30-day T-bill rate and the 20-year bond rate from the cointegrating regression equation (4.5). Table 5.2 conducts the tests using the 90-day T-bill rate and the spread between the 90-day T-bill rate and the 20-year bond rate from the cointegrating regression equation (4.5).

The results from the causality tests provide evidence in support of the pure expectations hypothesis. Causality testing in Table 5.1, using the 30-day T-bill rate, suggests uni-directional causality from the spread onto the short rate at the 5% level of significance. Causality testing in Table 5.2, using the 90-day T-bill rate, suggests bi-directional causality between the spread and the short rate at the 5% level of significance. Uni-directional causality is shown using the 30-day T-bill rate and bi-directional causality is shown using the 90-day T-bill rate.

TABLE 5.1
CAUSALITY TESTS BETWEEN THE SPREAD AND THE CHANGE IN THE 30-DAY T-BILL RATE

Restricted regression: $\Delta S_t = \alpha + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$

Unrestricted regression: $\Delta S_t = \alpha + \sum_{e=1}^{E} \beta_e z_{t-e,k} + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$

$$F = \frac{(RSS_r - RSS_u)/q}{RSS_u/(n-r)}$$

H_o: Spread does not Granger cause the 30-day rate

RSS_r **AR(12,0)**	RSS_u **AR(12,8)**	Observations	F critical (5%)	F statistic
1.254363	1.315843	948	2.93	5.6793727•

(r=21, q=8)

Restricted regression: $z_{t,k} = a + \sum_{g=1}^{G} b_g z_{t-g,k} + e_t$

Unrestricted regression: $z_{t,k} = a + \sum_{g=1}^{G} b_g z_{t-g,k} + \sum_{h=1}^{H} c_h \Delta S_{t-h} + e_t$

$$F = \frac{(RSS_r - RSS_u)/q}{RSS_u/(n-r)}$$

H_o: 30-day rate does not Granger cause the Spread

RSS_r **AR(8,0)**	RSS_u **AR(8,14)**	Observations	F critical (5%)	F statistic
1.746497	1.706437	946	2.13	1.547727

(r=23, q=14)

Note: Spread term is taken from the residuals in the cointegrating equation (4.5).
(•) indicates rejection of the null at the 5% significance level.

TABLE 5.2
CAUSALITY TESTS BETWEEN THE SPREAD AND THE CHANGE IN THE 90-DAY T-BILL RATE

Restricted regression: $\Delta S_t = \alpha + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$

Unrestricted regression: $\Delta S_t = \alpha + \sum_{e=1}^{E} \beta_e z_{t-e,k} + \sum_{f=1}^{F} \gamma_f \Delta S_{t-f} + \varepsilon_t$

$$F = \frac{(RSS_r - RSS_u)/q}{RSS_u/(n-r)}$$

H_0: Spread does not Granger cause the 90-day rate

RSS$_r$ **AR(6,0)**	*RSS$_u$* **AR(6,8)**	Observations	F critical (5%)	F statistic
.923582	.891128	953	2.93	4.270129•

(r=15, q=8)

Restricted regression: $z_{t,k} = a + \sum_{g=1}^{G} b_g z_{t-g,k} + e_t$

Unrestricted regression: $z_{t,k} = a + \sum_{g=1}^{G} b_g z_{t-g,k} + \sum_{h=1}^{H} c_h \Delta S_{t-h} + e_t$

$$F = \frac{(RSS_r - RSS_u)/q}{RSS_u/(n-r)}$$

H_0: 90-day rate does not Granger cause the Spread

RSS$_r$ **AR(9,0)**	*RSS$_u$* **AR(9,19)**	Observations	F critical (5%)	F statistic
.991588	.949355	941	1.88	2.1353277•

(r=29, q=19)

Note: Spread term is taken from the residuals in the cointegrating equation (4.5).
(•) indicates rejection of the null at the 5% significance level.

V.5. Summary

Evidence of the pure expectations hypothesis of the term structure of interest rates was tested using the Granger (1969) test for causality. The pure expectations hypothesis demanded a rejection of the conventional understanding of the relationship between the long and short rate. Econometrically, the methodology required the use of the interest rate spread. As a result, equation (5.2) manipulated the pure expectations theory to show a theoretical relationship between the changes in the short rate and the interest rate spread. Bi-directional causality was tested for and found between the changes in the short rate and the interest rate spread when the 90-day T-bill rate is used as a proxy for the short rate. When the 30-day T-bill rate was used as a proxy for the short rate, uni-directional causality was found from the spread onto the change in the short rate.

When the 90-day T-bill rate or the 30-day T-bill rate is used as a proxy for the short rate, causality from the spread onto the short rate was found. In either case this causal relationship implies models of the change in the short rate can be improved by including information on the yield spread. This suggests that forecasts of the short rate using historical short rates can be improved by including the information from the interest rate spread.

These results along with the results from chapter IV will be brought together in the next chapter. With testing completed, the next chapter will, among other things, seek some resolution based on these results.

VI. CONCLUSIONS

As a whole, the previous chapters tested the relationships between the long rate and short rate implied by the pure expectations hypothesis of the term structure of interest rates. As stated in the introduction an accurate theory of the term structure of interest rates offers valuable information to any economic agent with exposure to interest rate risk. The results of these tests and how they affect policy makers, financial institutions and corporations are discussed next. Following this, criticisms regarding this and other studies will be addressed as well as directions for further research.

Prior to cointegration tests, the Augmented Dickey-Fuller unit root test, Dickey-Fuller (1981) was used to test for stationarity. This procedure was shown in Section IV.3 to be necessary because of the nature of macroeconomic time series properties. The interest rate series in this study were found to be integrated of order one or first difference stationary.

Once the stationarity and integration of the series were considered, an implication of the pure expectations hypothesis in Chapter II could be tested for in the term structure of interest rates. That is, the log n-period interest rate equals an average of expected log one-period interest rates for n-periods. This was done by employing the Engle and Granger (1987) test for cointegration. In summary, testing found the 30-day T-bill rate and the 20-year bond rate as well as the 90-day T-bill rate and the 20-year bond rate to be cointegrated. This suggests that short-term rates and long-term rates move together over time, hence supporting the pure expectations theory of the term structure.

Causality testing sought evidence of the relation proposed in Chapter V. That is, the spread equals a weighted average of expected future short-term interest rate changes. Consequently, this implied that the spread is the optimal forecaster of the change in the short rate. From this the Granger (1969) test for causality attempted to resolve the issues of causality and forecasting. The results strongly support the pure expectations hypothesis. When the 30-day T-bill rate was used as a proxy for the short rate, uni-directional causality was found, and when the 90-day T-bill rate was used as a proxy for the short rate, bi-directional causality was found.

The only unsupportive result was that the 30-day T-bill rate does not Granger cause the spread. This means that a model of the spread is not improved by including past values of the change in the short rate. This is inconsistent with the conventional analysis in macroeconomics, that the short rate causes the long rate. Yet, it is also inconsistent with the pure expectations hypothesis, that bi-directional causality exists between the short rate and the long rate. This is a third case were the long rate causes the short rate only. The only useful information from causality testing using the 30-day T-bill rate is that a model of the change in the 30-day T-bill rate can be improved by including the interest rate spread between the 20-year Government of Canada bond and the 30-day T-bill rate.

The results using the 90-day T-bill rate, on the other hand, revealed bi-directional causality. These results are similar to Hall et al. (1992), McFadyen et al. (1991), and MacDonald and Speight (1988). Such that these authors used only the 90-day T-bill rate and they all found similar results for unit root, cointegration, and causality testing.

Other maturities were not used as proxies for either the short rate or the long rate. This study focused on the relationship between the long rate and the short rate only. An argument can be made that it becomes more likely to find the pure expectations hypothesis hold as we use closer and closer maturities. This is because we would assume near term maturity bonds would be closer substitutes than maturities farther apart. We would expect to find the one-year rate, for example, to be supportive if support was found for the 90-day T-bill rate.

The results strongly support the pure expectations hypothesis of the term structure of interest rates, thus, the short rate and long rate were found to be cointegrated and causal. This rejects the orthodox view of the monetary transmission mechanism. Recall, the orthodox view says that an expansionary monetary policy increases the money supply and the interest rate falls. In the short run, we would expect to see in the term structure of interest rates a reduction in short-term rates and an increase in long-term rates to accommodate higher expected inflation in the long run. This has been referred to as "twisting" in the term structure, [Malkiel (1966)]. The results in this thesis show that such a policy would cause interest rates at all horizons to move in the same direction.

As stated in the introduction, the conventional analysis in macroeconomics implied causality from the short rate to the long rate. This analysis is said to be a result of the central bank implementing policy at the short end of the maturity spectrum. Causality tests found the long rate cause the short rate and the short rate causes the long rate for the 90-day T-bill rate only.

There are many possible reasons for bi-directional causality between the 90-day T-bill rate and the interest rate spread apart from the ones given by the pure expectations hypothesis. One reason, offered by Romer (1996), is that the central bank may be implementing policy based on future inflation information that it has and the market doesn't have. Therefore, economic agents would revise their expectations of inflation after observing the policy. Another reason, as stated by Hsu and Kugler (1997), could be a result of the central bank adopting a policy response function that models the spread as a function of the change in the short rate. This means the central bank is implementing policy to keep long-term and short-term yields competitive. This will maintain the relationship that the spread equals a weighted average of expected future short-term interest rate changes.

The interest rate spread Granger caused the short rate when both the 30-day T-bill rate and 90-day T-bill rate were used. The results of the causality testing found that a model of the changes in the short rate can be improved by including the past values of the interest rate spread. Therefore, the slope of the yield curve can be used to improve interest rate forecasting. This has important implications for project analysis. For example, if a project is being evaluated today but investment will take place some time in the future, forecasts for discounting cash flows can be made using the forward rates implied in the term structure. Thus, a corporations management have a theory to form expectations of future interest rates for project analysis.

If the pure expectations hypothesis holds, not only can forecasts be improved but these results can be used for pricing securities. Financial institutions can use the term structure of interest rates for pricing their derivative securities. Arbitrage pricing sets the underlying rate implied in the price of a futures contract equal to its equivalent implied forward rate. Similarly, for project analysis, management can use forward rates implied in the term structure for pricing their corporate bonds and determining project financing costs.

A number of criticisms arise from the study conducted here. Firstly, the literature only offers ad hoc methods to determine the lag length in the autocorrelation function. For example, the model selection criteria in this study arrived at quite different lag length specifications. The acceptance or rejection of a test will critically depend on which model selection criteria is used. For instance, it has been shown in Gordon and Hannesson (1996) that model selection criteria often give conflicting results and that the lag length can be increased (or decreased) until the test is rejected (or not rejected).

The second criticism pertains to sample size. The testing done here was on the full sample period and made no allowances for structural breaks. Failure to accommodate exogenous structural breaks can lead to integration testing being biased towards non-rejection of the null hypothesis of a unit root. The integration testing here may be more robust if structural breaks are accounted for. On the other hand though, testing in sub-samples substantially reduces the power of the cointegration testing, since cointegration is a long run phenomenon. Therefore, accounting for structural breaks can be a trade-off in the analysis as a whole. However, there is no question that a researcher should use as large a sample as possible when conducting econometric testing. Of the literature surveyed in this study, none had a sample size comparable. Not only did this study use a very large sample but a higher frequency as well. The data was weekly and spanned from 1980 to 1998. Testing at a higher frequency makes unit root, cointegration and causality testing more robust.

The last criticism is the use in other studies of only the 90-day T-bill rate as a proxy for the short rate. It was argued here that attempting to use the 30-day T-bill rate as a proxy for the short rate would increase the robustness of the test results. Testing for cointegration and causality using the interest rates at either end of the maturity spectrum would be the strictest form of the pure expectations hypothesis in the term structure, whereas it is likely that maturities close together would be closer substitutes.

Of course there are many possible extensions to this study. The next most logical direction of additional research in keeping with the spirit of this study would be to decompose the yield spread. To do this requires identifying its two parts: the change in the long bond yield and the excess return, if any. This is because, the excess returns would equal the yield spread minus the change in the long bond yield, by definition. Since the yield spread was found to be stationary the only conclusion regarding term premiums in this study is that they must be stationary as well.

Since the data are weekly, the effect of seasonal variation could be examined. Smoothing the data by using annualised observations is an alternative. Smoothing has been argued by Shiller and Perron (1985) to be effective in cointegration testing because the power of the test is associated with the time span not the number of observations. Removing exogenous fluctuations, such as seasonality, may be fruitful for analysing policy effects in isolation.

Two additional extensions to future research would be to incorporate recent advancements in econometrics and to accurately specify a forecasting model. Such econometric advancements are the Leybourne and McCabe opposite hypothesis test, Leybourne and McCabe (1994), the KPSS unit root test, Shin and Schmidt (1992), or fractional unit root testing, Sowell (1990). Moreover, an accurate forecasting model could be directly applied to any number of applications.

In conclusion, evidence that the term structure of interest rates can be explained by the pure expectations hypothesis was found. This has important implications for any economic agent with exposure to interest rate risk.

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