THE UNIVERSITY OF CALGARY

OPTIMIZING CRUDE OIL PRODUCTION:
A LINEAR PROGRAMMING APPROACH

By

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Optimizing Crude Oil Production: A Linear Programming Approach", submitted by Brian Schellenberg in partial fulfilment of the requirements for the degree of Master of Arts.

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ABSTRACT

The objectives of this study are to survey analytical and computational exhaustible resource models and then to develop a hybrid reservoir production model encompassing desirable properties of both approaches.

Analytical models tend to give only a limited role to geological and reservoir engineering considerations in oil production. From these considerations it is evident that the physical properties contribute to the individuality of reservoir production. Any attempt to apply theoretical findings to a specific exhaustible resource is virtually destined to fail because of the complications omitted.

Computational models tend to be highly specialized and only through significant simplification and expansion can they be adjusted to handle economic relationships.

Both analytical and computational models have strengths and limitations. For economic analysis there appears to be considerable scope for developing hybrid models which incorporate the desirable features of both. The linear programming model developed in this study incorporates the physical properties

of a specific reservoir and the economic relationships important to decision making. Results of the after-tax model differ from what is expected by economic intuition and finding the cause of this deviation reveals the power of the computational approach.

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I would also like to express my thanks to John Hayward who supplied the reservoir production and cost data necessary for my reservoir production model.

DEDICATION

In memory of my Mom, Mary Schellenberg, who greatly encouraged my pursuit of wisdom and knowledge.

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Chapter 1

Introduction

1.1 General Remarks

How can exhaustible resources be allocated efficiently over time? Extensive progress has been made in answering this question since the OPEC-induced crude oil shortage of 1973. Even before this time, there was concern over optimal resource use; see Hotelling (1931). To answer the question of efficient allocation of resources and to understand the performance of the resource industries, theoretical models have been constructed, and these models have largely succeeded in revealing general efficiency patterns or relationships common to all exhaustible resources.

To a greater extent than any other exhaustible resource, petroleum has come to dominate world concern. Thus, oil production models have been extensively developed and widely applied. Through the use of oil production models analysis has been made of key relationships among capital flows, costs, revenue flows, rates of return, aggregate production, rates of production, and timing of production. See, for example, Kuller and Cummings (1974).

The conclusions that follow from these analytical models suggest how exhaustible resources should optimally be depleted. The decision concerning the amount of production is usually based on the relationship between marginal revenue, marginal cost, and the user cost. This relationship is discussed in greater detail in Chapter 3. Associated key elements for the decision include: expected price trends, the existence of a backstop technology, and the influence of a tax structure. All of these factors contribute to an understanding of the efficient allocation of an exhaustible resource over time.

However, the relationships embodied in such analytical models give only a limited role to geological and reservoir engineering considerations in oil production. For making real-world decisions, adequate attention must be given to these disciplines and their contribution to production decisions. For example, the geological makeup of the reservoir rock in terms of structure and placement of fluids affects the amount of recoverable reserves and rate of recovery. Porosity, permeability, and water saturation levels differ among reservoirs. Through reservoir engineering the size of the reservoir and feasible rate of recovery combine to show the individuality of reservoir production. All these reservoir properties contribute to the uniqueness of reservoirs as detailed in Chapter 2. Thus, to group all reservoirs into one stylized framework exposes one limitation of the theoretical models. Any attempt to apply theoretical findings to a specific exhaustible resource is virtually destined to fail because of the complications omitted.

Outside the mainstream economics literature exists a host of modelling approaches incorporating details of the technology of recovery for specific exhaustible resources. Computational approaches allow the assembly of models incorporating the specific characteristics of an actual reservoir. However, to capture all the significant properties and relationships, a computational model can easily become highly complex. For example, the Lasdon et al. (1986) gas model employs a complex equation to properly define the quantity of gas produced as a non-linear function of the gas pressure on a monthly basis. To expand this model to the life of the gas reservoir rather than the five months that are used would complicate the model considerably. Nevertheless, it is these computational models as formulated by petroleum geologists, reservoir engineers, and operation researchers which have elements that economists might want to borrow to breathe more realism into economic models of depletion.

From an economic standpoint, however, the prime disadvantage of such models lies in their highly specialized nature. Usually, they cannot be used directly to analyze questions of economic interest without significant simplification and expansion to include economic components. Again an example of this is the gas model of Lasdon et al. (1986). The model shows that through "proper" control of declining pressure, cumulative production can be increased, but there are very few economic factors in this model. No prices or costs are incorporated, and the time horizon is only five months. Major changes would be necessary to allow this model to be used for economic analysis.

Both analytical and computational models have strengths and limitations. For economic analysis there appears to be considerable scope for developing hybrid models which incorporate the desirable features of both.

1.2 Objectives of the Thesis

Several specific objectives are pursued in this work. One principal objective is to survey theoretical approaches to exhaustible resource depletion. This survey identifies the most important findings, the elements most critical for economic models of resource depletion, and the principal shortcomings. Since the literature is so vast, only selected approaches are surveyed.

A second objective is to survey computational approaches to resource depletion, some from outside the mainstream economics literature. In this survey certain components are identified that might best be included in modified economic models. Again this survey is selective.

A third objective is to develop a computational economic model embracing elements drawn from both theoretical models and computational models. A linear programming model is constructed to analyze the optimal depletion of a petroleum reservoir, taking account of economic factors and geological and technological considerations.

The final objective is to show, using the programming model, how the inclusion of taxes and changes in crude oil prices, discount rates, and tax rates may alter the optimal depletion path. Various scenarios are examined to determine the sensitivity of optimal decisions to these changes.

1.3 Format of the Thesis

The rest of the thesis is presented in six chapters. The content of these chapters is as follows.

Key geological and reservoir engineering considerations are discussed in Chapter 2. These are foundational elements necessary for a discussion of reservoir depletion. They are used later to pinpoint some of the limitations of the analytical models.

Analytical models of exhaustible resource production are then discussed in Chapter 3. These types of models form the basis for economic research on the efficient allocation of resources over time.

Computational models of resource depletion are surveyed in Chapter 4. These models range from petroleum reservoir production models to aggregate energy supply and demand models. Their inclusion of real world conditions and constraining factors produce model results that add to the conclusions of theoretical models.

The linear programming model of oil production is formulated in Chapter 5. The production profiles included in the model are described in the context of Leontief input-output production functions. Also discussed are the constraints on production, EOR techniques, and a tax structure.

Model results for different scenarios are presented and discussed in Chapter 6. These scenarios include the representation of a tax structure, different discount rates, changes in the price, changes in the tax rate, and deficit restrictions. Results of the after-tax model deviate from what is expected by economic intuition and finding the cause of this deviation reveals the power of the computational approach.

Concluding remarks are provided in Chapter 7. In particular, the success of incorporating ideas from computational models to enhance the findings of theoretical models is discussed. Further modifications and areas of research are also suggested.

Chapter 2

Geology, Reservoir Engineering, and Maximum Efficient Rate

2.1 Introduction

Much work in the economic literature on petroleum use has ignored basic geology and the technology of production, even though geology and reservoir engineering are fundamental to the analysis of reservoir depletion. The amount and duration of production is determined mainly by physical factors such as the size of the reservoir, the type of source rock, permeability, porosity, fluid saturation, and the pressure differential within the reservoir and at the well bore. These factors interact to determine the natural drive associated with a particular reservoir. The rate of production is set by the original natural drive pressure, the maximum efficient rate (MER), or the operator's decision to produce at a lower rate than the two previous limits on production.

The purpose of this chapter is to explain geological, petroleum engineering, and MER considerations which bear on reservoir depletion. The duration and rate of production is reservoir specific, a fact which can be explained by geology and reservoir engineering. Depending on the physical characteristics of the reservoir, there exists an MER, where a higher rate of extraction will lead to reservoir damage. The three sections of this chapter focus on the geology of reservoir production, reservoir engineering considerations, and the factors determining the MER. Several reasons are given for the differences between reservoirs and different reservoir performances. The geological factors mentioned above, such as porosity, all affect the quantity of recoverable oil-in-place and the rate of recovery. There is a high correlation between the type of trap and the type of drive mechanism available for petroleum production. Regarding engineering factors, the type of drive mechanism is also shown to affect the quantity of recoverable oil-in-place and the rate of recovery. Since each of these characteristics has a range of possibilities, the chance of two reservoirs being identical is very remote. Thus an economic-petroleum model which classifies all reservoirs into one group limits the application of conclusions from this model to a specific reservoir.

2.2 Geological Considerations

Several topics are discussed briefly in this section, including reservoir rock properties, types of traps, and how these geological elements affect reservoir performance.

2.2.1 Physical Properties of Reservoir Rock

Four properties that make reservoirs heterogeneous and affect the level of production are porosity, permeability, fluid saturation and fluid composition. In the following paragraphs each property is defined, elements influencing the property are identified, and the property's effect on reservoir performance or reservoir heterogeneity is discussed.

Porosity (∅) is the measure of the space in the reservoir rock occupied by pores which has the potential to contain petroleum. This decimal fraction of the rock volume is determined by the shape and size of the sediment, the degree of compaction, and the cementation.[1] If the particles are large and have the same shape, the porosity level can be high, thereby allowing for a greater potential volume of petroleum. If the pressure from overlying sediment is great, then compaction occurs and the pore volume decreases. Cementation is a process occurring during reservoir formation which can limit the amount of compaction. Certain mineral crystals act as a cement between sand grains. Once the sand grains placement is cemented, the pore space should not decrease due to compaction.

Permeability (K) is the ability of fluid to flow through a porous medium. A volume of rock may be porous, but if the pores are poorly interconnected, the rock is not very permeable. Usually, porosity and permeability are highly correlated so a high porosity value is associated with good permeability. The geological elements that determine porosity also determine permeability: size and shape of the sediment, compaction, and cementation. The permeability within a reservoir is also affected by the different viscosities of the fluids involved.[2] The chemical makeup determines the flow resistance (viscosity) of a fluid. A high degree of permeability in a reservoir is desirable since higher rates of production can be maintained without potential reserve loss.

Fluid saturation (ỏ) is the mixture of oil, water, and gas that exists in the reservoir. A low water saturation level indicates that a larger portion of the pore volume is occupied by petroleum. The water saturation level measures the amount of connate water present, or the water intrinsic to the reservoir. One cause of heterogeneity among reservoirs is that oil, water, and gas can be present in different proportions. Water saturation can also affect reservoir performance. As the oil is drawn from the reservoir, the percentage of water production increases over time. A high water saturation level indicates that reservoir performance will deteriorate much sooner.[3]

The last physical property considered in this section is the composition of the petroleum in the reservoir. This really is an area of petroleum chemistry, but is mentioned since it contributes to reservoir heterogeneity. Petroleum is a natural substance that can occur as a solid, liquid or gas. The terms "oil" and "gas" are used in reference to the liquid and gaseous states. An oil and gas solution is a mixture of hydrocarbons, usually containing impurities. Common examples of hydrocarbon compositions are: methane, ethane, propane, butane, pentane, and benzene. Since the mixture of hydrocarbons is variable, it is highly unusual for reservoirs to be identical.

All the properties mentioned influence reservoir performance or contribute to heterogeneity among reservoirs. The type of trap can also affect reservoir performance and cause heterogeneity among reservoirs.

2.2.2 Entrapment of Petroleum

Through time the petroleum migrates from the source rock to the permeable rock, and then may be trapped in this formation if the necessary boundaries exist. There are three main forms of traps: anticlinal traps, fault traps, and stratigraphic traps.[4]

An anticlinal trap is identified in Figure 2-1.[5] The features of this type of trap are: geometric closure (the structural contours form closed rings), a reservoir rock that is permeable, and a fine-grained or relatively impermeable cap rock that overlies the reservoir and seals it.

This type of trap is highly desirable since it has the potential to have a gas cap drive, a water drive, and a gravitational segregation drive. The advantages of these drive mechanisms are discussed later. Due to geometric closure and high structural relief, the existence of a gas cap greatly improves recoverability. There is also the potential for extensive lateral continuity (porosity over large areas) which can allow an aquifer to be in contact with the reservoir. If an aquifer is in contact with the reservoir, then water displaces the petroleum during production and reservoir pressure is maintained.

A fault trap is illustrated in Figure 2-2.[6] The characteristics of this type of trap are: an inclined reservoir, a cap rock, a fault that forms an up-dip barrier across the reservoir, and a barrier in the reservoir along the fault which prevents lateral movement. This trap can also be as productive as the anticline trap. The fault and the inclined reservoir act the same way as the geometric closure. Thus, there is the potential for gas cap drive and water drive with this type of trap.

A stratigraphic trap is depicted in Figure 2-3.[7] In this case migration is limited by stratigraphic causes such as permeability or porosity changes in the reservoir rock, convergence of rock units, isolated rock units of porosity and permeability different from adjacent units, and an unconformity providing the barrier. Stratigraphic traps usually do not have extensive lateral continuity and thus a water drive caused by an aquifer is highly unlikely or it has very limited influence. As well, stratigraphic traps are associated with small formation dips (limited vertical movement of the petroleum).

Figure 2-1

Anticlinal Trap

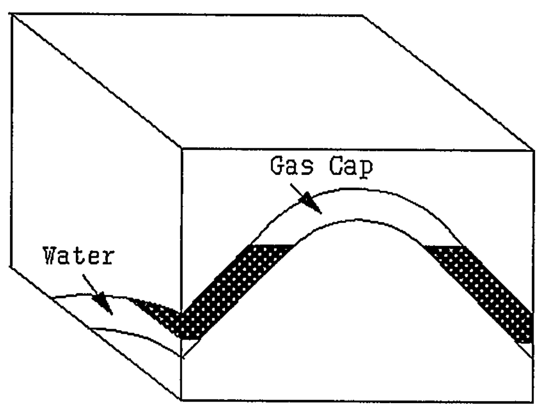


Figure 2-2

Fault Trap

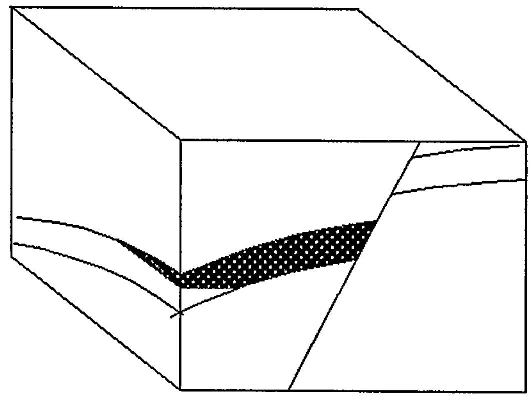
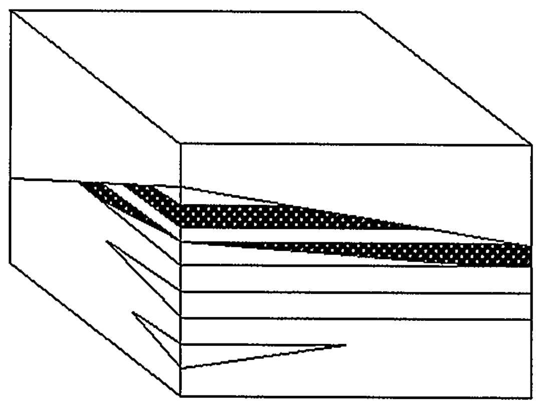


Figure 2-3

Stratigraphic Trap



Thus a gas cap has little influence on the whole reservoir, and gravitational segregation is severely limited as a type of drive mechanism. This leaves only solution gas drive as a force to push petroleum to the surface.

Clearly, the type of trap and its physical properties all play a role in determining reservoir performance.

2.3 Petroleum Reservoir Engineering

Of prime importance for reservoir engineering is precise information concerning physical conditions that exist in wells and underground reservoirs. With this information, estimating a relationship such as a gas-oil ratio, a volumetric measure, or a material balance equation can be made.

The most important basic information for reservoir performance calculations is the reservoir pressure. To determine the volume of oil and gas in place, it is necessary to know the physical properties of bottom-hole samples under various pressures. These properties can be of the rock or of the fluids in the reservoir. The important properties are: porosity, permeability, viscosity, and fluid saturations and distributions. Before describing the quantitative relationships of these properties, it is necessary to examine the forces that move the petroleum in the reservoir towards the well.

2.3.1 Petroleum Drive Mechanisms

As described earlier, petroleum occurs in traps where the portion of the trap that holds the petroleum is called a reservoir. Since different drive mechanisms can have different rates of recovery and different cumulative amounts of recovery, it is essential to know what type of drive is the major mechanism for petroleum production. Four possible ways that oil and gas can be displaced to the well are: fluid expansion, fluid displacement - natural or artificial, gravitational drainage, and capillary expulsion.[8]

Fluid expansion is a drive mechanism common to all reservoirs. It may not be the main driving force, but it does contribute to oil and gas production. This drive mechanism results from the pressure differential. In a reservoir the petroleum is contained under pressure in the trap which is greater than the atmospheric pressure at the earth's surface. When a well is drilled, the reservoir pressure declines to match the pressure at the earth's surface through fluid expansion forcing oil to the surface.

A fluid displacement drive forces petroleum to the surface by means of a gas cap or water from an aquifer. A gas cap drive has a pocket of gas which exerts downward pressure on the oil, and displaces it by expansion, thereby forcing it to the well and then to the surface. The water displacement drive pushes petroleum to the well by the upward pressure of incoming water. This aquifer exerts pressure on the oil, and as the oil leaves, more water enters the reservoir. Water displacement of oil can also be artificially induced by pumping water down to the bottom of the reservoir (waterflooding). This type of drive mechanism maintains pressure and allows greater oil recovery.

The gravitational drainage drive mechanism operates at low production rates. As petroleum is pumped to the surface from the lower region of the reservoir, gravity pulls petroleum from the higher regions down depending on the dip of the reservoir. If the rate of extraction is faster than the rate gravity pulls petroleum to the lower regions, the petroleum can become segregated by other fluids such as water. Thus, the effectiveness of the gravitational drainage is reduced and less petroleum is recovered.

The capillary expulsion drive mechanism contributes only a minimal amount to the potential production. Capillary forces are the result of surface and interfacial tensions in an oil and gas reservoir. The surface tension is derived from the molecular property of a liquid to expose a minimum amount of free surface. Interfacial tension is similar where the tendency is to achieve the minimum surface contact between two liquids. When the velocity of fluid flow is low and the capillary forces are stronger than the gravitational pull, the capillary forces pull petroleum to the well and less petroleum is lost by water entrapment.

When there is no aquifer and fluids are not artificially injected, fluid expansion and gravitational drainage are the mechanisms of recovery for an oil reservoir. It is possible for all four types of recovery mechanisms to operate, but usually only one or two dominate.

An example of how recovery mechanisms may change is the following. A reservoir having no aquifer will produce initially by fluid expansion. When the pressure from this type of displacement has substantially decreased, there can be a switch to gravitational drainage. Even later, water can be injected to force additional petroleum to other wells by fluid displacement. Reservoir engineering typically plans these cycles of displacement to maximize recovery. The amount of oil or gas recoverable is called the reserves which can occur in four different ways: free gas, dissolved gas, oil in the oil zone, and recoverable liquid from the gas cap. The sizes of these reserves are estimated through formulas provided in the next section.

2.3.2 Oil in Place by the Volumetric Method

Due to the physical properties of a reservoir, the size of reserves is not a simple calculation such as length by width by height. For the following volumetric equation, the oil that is produced equals the initial oil in place less the oil remaining after production:[9]

2.1 $$\text{Recovery} = 7758\ V_b\ \emptyset\ \left[\frac{(1 - S_w)}{B_{oi}} - \frac{(1 - S_w - S_g)}{B_o} \right],$$

where:

7758 - number of barrels per acre-foot

V_b - reservoir volume, in acre-feet

$\emptyset$ - porosity, as a percent

B_{oi} - initial formation volume factor

B_o - abandonment oil formation volume factor

S_w - water saturation, as a percent

S_g - gas saturation, as a percent.

The initial oil in the reservoir is the percentage of the pore space not occupied by water while the final amount of oil in the reservoir is the percentage not occupied by water or gas. The oil that is produced is replaced by the expansion of the remaining oil and gas. Since the volumes vary with temperature and pressure changes, the formation volume factors convert these subsurface volumes to surface volumes at a standardized temperature and pressure.

Another method that reservoir engineers use to calculate the recoverable oil is the material balance equation.

2.3.3 Material Balance Equations for Oil Reservoirs

It is not always possible to acquire all the information necessary for the volumetric method. In this situation the material balance method may be used to calculate the initial oil in place. Since the oil and gas are in solution (the gas is dissolved), initial production is through fluid expansion. As fluid expansion takes place the pressure declines and eventually the dissolved gas reaches its bubble point. At the bubble point the dissolved gas becomes free gas and can act like a gas cap. So the oil material balance equation can be viewed in two parts - before and after the dissolved gas reaches its bubble point. Initially production is given by the following equation:[10]

2.2 $$NB_{oi} = (N - N_p)B_o,$$

where:

N - the initial reservoir oil in stock tank barrels

N_p - the oil produced in stock tank barrels

B_{oi} - the oil volume factor at the initial pressure

B_o - the oil volume factor at the final pressure.

As the pressure decreases with continued production, the bubble-point pressure is reached and a free gas phase begins. The equation then becomes:

2.3 $$NB_{oi} = (N - N_p)B_o + G_fB_g,$$

where:

G_f - the free gas

B_g - the gas volume factor at the lower pressure.

The amount of free gas that will exist is equal to the initial gas less both the solution gas and the produced gas. Thus the free gas can be found from the following equation:

free gas = initial gas - solution gas - produced gas

2.4 $$G_f = NR_{si} - (N - N_p)R_s - N_pR_p,$$

where:

R_{si} - the initial solution gas-oil ratio

R_s - the final solution gas-oil ratio

R_p - net cumulative produced gas-oil ratio.

This equation for free gas can be substituted into the material balance equation to yield:

2.5 $$NB_{oi} = (N - N_p)B_o + (NR_{si} - (N - N_p)R_s - N_pR_p)B_g.$$

This gives the material balance equation for an oil reservoir without water drive. The equation can be solved for N to measure the initial stock tank oil in place.

To solve the material balance equation it is necessary to have information from two core samples with a significant production time between them. This limits the usefulness of the material balance equation until well into the production life of the reservoir.

2.3.4 Darcy's Law

Darcy's Law describes the movement of fluids through a porous medium. The velocity of the fluid is directly proportional to the pressure gradient and inversely proportional to the fluid viscosity. A representative equation is given below.[11]

$$2.6 \qquad v = \frac{k}{u} \times \frac{dp}{ds},$$

where:

v - the apparent velocity in centimetres per second (the word apparent is used since the fluid is moving through a porous medium)

u - the fluid viscosity expressed in centipoise units

dp/ds - the pressure gradient in atmospheres per centimetre

k - the proportionality constant showing the permeability of the rock expressed in darcy units.

Although this representation of the movement of fluids is in its simplest form, this equation estimates the movement of petroleum within a reservoir.

With these equations and core samples taken from exploratory wells the drive mechanisms in use can be ascertained and their effects on reservoir performance estimated.

2.4 Maximum Efficient Rate

For most pools the ultimate recovery depends on the rate of production, so at each instant of time a rate exists where production above this limit damages the reservoir. This limit is defined as the maximum efficient rate (MER).

The history of the MER shows a variety of possible definitions for this rate; see Kraus (1947). The following discussion is of general principles involved in determining the MER and a specific case of MER determination, namely how the Alberta government defines the MER for various Alberta reservoirs.

2.4.1 General Principles for Determining the MER

The MER depends on the type of recovery mechanism employed and the physical nature of the reservoir. The conditions that determine the MER are: the rate must not exceed the capabilities of the reservoir, the individual well rate must not be excessive, and the individual well rate must not be so low as to prohibit profitable operation.[12] The first two conditions are limitations imposed by physical properties and the third condition is a limitation imposed by an economic consideration. The types of recovery mechanisms that the MER is based on are dissolved-gas drive, gas-cap drive, and water drive.

As previously stated, a dissolved-gas driven reservoir has a very low recovery of the oil in place. The production rate is so low that the reservoir is not rate sensitive. Thus, there is no production rate above which reservoir damage can occur.

A gas-cap driven reservoir does have an MER since recovery efficiency may be very sensitive because gas is not an effective oil displacement mechanism. Given that the encroachment of free gas through the oil zone occurs through the most permeable channels, if the rate of production is sufficiently high, oil is left behind. The gas that would otherwise displace the oil escapes through the more permeable channels which become accessible at the higher production rate.

With this type of drive the MER depends on the formation permeability, the permeability distribution, the relative permeability-saturation relationship between gas and oil, the angle of formation dip, the viscosity of the oil, and the size of the gas cap.[13] In the section on Alberta's MER calculation, it is shown how these characteristics directly determine the MER.

A water driven reservoir requires estimates of the oil, gas and water production to determine the MER. These estimates are used to find a sustainable pressure level that will not cause reservoir damage.

The relation between reservoir pressure and the water influx is determined by the permeability of the formation, the uniformity of the productive horizon, the reservoir structure and zone of water entry, the areal extent of the reservoir and formation thickness, the stage of reservoir depletion, and the pressure decline.[14]

2.4.2 Alberta's MER Regulations and Formula

The acronym used by the Alberta Government for the MER is MRL - maximum rate limitation.[15] The MRL is determined by the greater of: the basic well rate (BWR), the rate determined using the Preliminary Rate Limitation (PRL) surveillance formula, and the rate established by the Energy Resources Conservation Board on the basis of MRL studies submitted by the operator. The three rates are described next.

The basic well rate (BWR) or well minimum allowance (MA) has been set at $150m^3$/month ($5m^3$/day). This is the rate of production that the Board has set that will cover well operating and completion costs. This minimum allowance has been set to prevent premature abandonment of wells.

The Preliminary Rate Limitation (PRL) in m^3/day is found using the following formula:

2.7 PRL $= 9000 \times 10^{-6} (U) \times 12/365$.

U equals the initial recoverable reserves of oil in thousand m^3 and is obtained from the following volumetric equation:

2.8 $U = R_i \times h \times \emptyset \times (1 - S_w) \times 1/B_{oi} \times AA \times 10000/1000$,

where:

R_i - expected initial recovery of oil in place, a decimal fraction

h - net oil pay thickness, in meters

$\emptyset$ - porosity, a decimal fraction

S_w - water saturation, a decimal fraction

B_{oi} - oil volume factor at the bubble point, a decimal fraction

AA - assigned area, in hectares.

The information in this volumetric equation is provided by the operator to the Board with an application for a New Well Base Allowable (production rate) or Base MRL (form O-38).

The MRL may also be determined by the Board on the basis of MRL studies submitted by the operator.

The production rate that a well can be operated at is known as the base allowable. The base allowable for the month is the greater of the minimum allowance (MA) multiplied by the number of days in the month, and the proratable reserves multiplied by the monthly proratable reserves allocation factor. If the proratable reserve (U-1/2P) is not stated on the O-38 form, then the established reserve figure (U) is used instead. The base allowable is the greater of the above two; however, it must not exceed the base MRL multiplied by any applicable penalty factors multiplied by the number of days in the month. It is in the penalty factors that water and gas production are taken into account.

The penalty factors allow a certain amount of gas and water production. If this level is exceeded, then a penalty is imposed by lowering the MRL. The amount of gas and water produced is measured by the gas-oil ratio (GOR) and the water-oil ratio (WOR). For gas production a penalty is imposed if the produced GOR exceeds the base GOR. For water production a penalty is imposed if the produced WOR is greater than zero, and the water drive index (WDI) is zero. (A WDI of zero implies that there is no water drive.) The penalty factors are given below in equation form.

2.9 Compound penalty factor

$$= \text{GOR penalty factor} \times \text{WOR penalty factor}$$

$$= \frac{\text{Base GOR}}{\text{Produced GOR}} \times \frac{2}{2 + (\text{Produced WOR})(1 - \text{WDI})}$$

These penalty factors can also increase the MRL if the produced GOR is less than the base GOR or if the WDI has a more significant effect on the WOR penalty factor than the produced WOR. A numerical example is given in the Appendix.

The base allowable is determined by three elements. First, there is the MA, which ensures that production will cover operating costs. Second, there is the reserves allocation, which partitions production between the production space units (PSU - wells or fields). Third, the base allowable is limited from above by the penalized MRL. The penalized MRL is determined by the BWR (MA), or the PRL (physical characteristics of the PSU), or on the basis of MRL studies submitted by the operator, and gas and water production.

2.5 Conclusion

From the discussion of geology, petroleum engineering, and MER considerations, it is evident that certain physical factors greatly influence the size, composition, and performance of a reservoir. These factors must somehow be accounted for in a reservoir production model. Computational models, as discussed in Chapter 4, are able to include this type of behaviour. Due to the heterogeneity of reservoirs caused by differences in porosity, permeability, water saturation, fluid composition, and fault type, aggregation of reservoirs weakens the ability of a model to "portray reality." Performance patterns are also reservoir dependent due to variation in the operating drive mechanisms and variation in certain physical properties (MER and degree of porosity and permeability). Thus, costs and production paths are different for each reservoir.

NOTES

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[10] Craft and Hawkins, 110-112.

[11] Craft and Hawkins, 259.

[12] Thomas C. Frick, ed., Reservoir Engineering, Vol. 2 of Petroleum Production Handbook (Dallas: Millet the Printer, 1962), 33-23.

[13] Thomas C. Frick, ed., 2: 33-25.

[14] Thomas C. Frick, ed., 2: 33-26.

[15] Energy Resources Conservation Board, Production Accounting Handbook, 1979, sec. 040 p. 5. All the information for the MRL is obtained from this book.

Chapter 3

Analytical Models of Exhaustible Resources

3.1 Introduction

With the physical nature of reservoir behaviour as background, several analytical models of exhaustible resource allocation are surveyed in this chapter. Since the literature involving analytical models of exhaustible resource extraction is vast, only selected articles are discussed. In this survey the models are formulated, the underlying assumptions delineated, and the associated conclusions presented. When a model involves the depletion of a petroleum reservoir, how it captures the physical nature of reservoir performance is emphasized.

The majority of the analytical models of this chapter yield the outcome that optimal production occurs where marginal revenue equals marginal cost plus user costs. This condition values the exhaustible resource differently from renewable resources through the user cost elements.

To begin this survey the model of Kuller and Cummings (1974) is presented in detail and henceforth is referred to as the KC model. Their model has many appealing features and other models can be viewed as representing only a specific part of their model or using a simplified version of their model and extending their analysis in a slightly different direction.

3.2 Production, Investment and User Cost Relationships

The KC model provides an excellent foundation for analytical models by incorporating the technical nature of reservoir behaviour and several economic considerations.[1] The optimal production path is determined for a reservoir through the interaction among current production rates, investment and the dependence of recoverable reserves on the time-path of production.

Due to the physical nature of a reservoir, the annual extraction rate (u) is directly related to pressure. Under a water drive mechanism, the reservoir pressure is maintained by the incoming water displacing the exiting oil, and this displacement rate is dependent on the reservoir properties as mentioned in Chapter 2 (permeability and porosity). However, faster extraction allows water to infiltrate the reservoir more rapidly, and since water moves through the porous channels of a reservoir easier than oil, the water bypasses oil in place causing permanent loss. Thus, a faster extraction rate resulting in a quicker pressure decline lowers ultimate recoverable reserves. This relationship establishes the rate sensitivity of the reservoir and the need for MER restrictions as described in

Chapter 2. However, the KC model also provides an opportunity to regain these lost reserves through investment (v). By implementing artificial lifting and enhanced oil recovery (EOR) techniques, the pressure decline and lost reserves of faster extraction can be restored.

Since this is a single reservoir model, and thereby only a small portion of total industry production, the firms are assumed to be price takers, and the prices are exogenously determined. The model maximizes discounted profits subject to the physical constraints on reservoir production.

The model variables are:

u_{jt} - volume of petroleum extracted by firm j in period t, for j = 1 to J

U_t - annual production rates by all firms during period t; $U_t = (u_{1t}, u_{2t}, ..., u_{Jt})$ for t = 1 to T

v_{jkt} - gross investment by firm j in capital type k in period t; $v_{jt} = (v_{j1t}, v_{j2t}, ..., v_{jKt})$ for t = 1 to T and j = 1 to J

V_t - gross investment for all types of capital for all firms in period t; $V_t = (v_{1t}, v_{2t}, ..., v_{Jt})$ for t = 1 to T

K_{jkt} - firm j's physical capital stock of type k at the beginning of period t; $K_{jt} = (K_{j1t}, K_{j2t}, ..., K_{jKt})$ for t = 1 to T and j =1 to J

D_{jkt} - net depreciation of firm j's capital stock k during period t

F_{jt} - upper bound on firm j's capacity to produce petroleum during period t

C_{jt} - firm j's cost function during period t

β_t - discount factor, $1/(1 + r)^t$, where r is the discount rate

p_t - price at time t; all prices are exogenously specified

X - the total recoverable quantity of petroleum in the reservoir.

The model is:

3.1 $$\text{Max} \sum_{t=1}^{T} \sum_{j=1}^{J} (p_t u_{jt} - C_{jt}(U_t, V_t, K_{jt}))\beta_t$$

subject to:

3.2 $$K_{jk,t+1} = K_{jkt} - D_{jkt}(u_{jt}, v_{jkt}, K_{jkt}) \quad \text{for all j, k and t}$$

3.3 $$u_{jt} \leq F_{jt}(U_t, V_t, K_{jt}) \quad \text{for all j and t}$$

3.4 $$\sum_{t=1}^{T} \sum_{j=1}^{J} u_{jt} \leq X(U_T, V_T)$$

3.5 $$u_{jt} \geq 0, \; v_{jkt} \geq 0 \quad \text{for all j, k and t.}$$

The objective function is the present value of revenues minus costs. Costs are a function of production, investment and existing capital stocks. If current production and investment increase, then current costs will also increase. However, current investment lowers future costs. It is also assumed that an increase in capital stocks will cause more efficient production by lowering current unit production costs.

Equation 3.2 defines the capital stock in each period as the capital stock in the previous period less net depreciation. The amount of depreciation in a period depends on the rate of production, the size of existing capital stocks and the amount of gross investment. If the production rate increases or if the size of existing capital stock increases, then the amount of depreciation increases. Depreciation is only offset by gross investment which is the addition of new capital stock. Equation 3.3 sets an upper bound on current production. It is the current reservoir pressure which is influenced by past production, current production, and investment rates from all firms producing from the reservoir that sets this boundary on current production. Thus, the physical behaviour of the reservoir, the key element of Chapter 2, is implicitly represented in these constraints. The limit on recoverable stock is determined by the time-path of production and investment during the life of the reservoir in equation 3.4. Finally, inequalities 3.5 require all variables to be non-negative.

The necessary conditions for maximization characterize the optimal production and investment paths:

Production Path

3.6 $$(p_t - \frac{\delta C_{jt}}{\delta u_{jt}})\beta_t = \sigma\beta_T(1 - \frac{\delta X}{\delta u_{jt}}) + \Phi_{jt}\beta_t - \sum_{\tau=t}^{T}\sum_{i=1}^{J}\Phi_{i\tau}\frac{\delta F_{i\tau}}{\delta u_{jt}}\beta_\tau$$

$$+ \sum_{k=1}^{K}\mu_{jk,t+1}\beta_{t+1}\frac{\delta D_{jkt}}{\delta u_{jt}} + \sum_{\tau=t+1}^{T}\frac{\delta C_{j\tau}}{\delta u_{jt}}\beta_\tau + \sum_{\tau=t}^{T}\sum_{\substack{i=1\\ i\neq j}}^{J}\frac{\delta C_{i\tau}}{\delta u_{jt}}\beta_\tau$$

$i, j = 1, ..., J; 1 \leq t \leq T$

Investment Path

3.7 $$\frac{\delta C_{jt}}{\delta v_{jkt}}\beta_t = - \mu_{jk,t+1}\beta_{t+1}\frac{\delta D_{jkt}}{\delta v_{jkt}} + \sigma\beta_T\frac{\delta X}{\delta v_{jkt}}$$

$$+ \sum_{\tau=t}^{T}\sum_{i=1}^{J}\Phi_{i\tau}\beta_\tau\frac{\delta F_{i\tau}}{\delta v_{jkt}} - \sum_{\substack{i=1\\ i\neq j}}^{J}\frac{\delta C_{it}}{\delta v_{jkt}}\beta_t - \sum_{\tau=t+1}^{T}\sum_{i=1}^{J}\frac{\delta C_{i\tau}}{\delta v_{jkt}}\beta_\tau$$

$i, j = 1, ..., J; k = 1, ..., K; 1 \leq t \leq T$

In 3.6 the present value of marginal net income is equated to six terms which are defined as user costs. User costs are the discounted future profit foregone by producing one more unit now instead of in the future.

The first user cost term values the limited stock of the resource. When one unit of crude is produced now instead of in the future, the value, $\sigma\beta_T$, is the future profit foregone. Also, since faster current rates of production cause a decline in total recoverable reserves, producing this one unit today lowers total reserves. Thus, future production falls by the amount $(1 - \delta X/\delta u_{jt})$, where $\delta X/\delta u_{jt} < 0$. The stock user cost is the discounted shadow price (value of an additional unit of crude oil) multiplied by the lost future production.

The second and third terms are defined as "boundary user costs". The term $(\Phi_{jt}\beta_t)$ gives the discounted profit foregone from an additional unit of crude oil which cannot be produced once the maximum production rate is reached. If another firm j increases production in an earlier period t, then the maximum production level for firm i in period τ declines due to the rate sensitivity of production described earlier. Thus, through the summation of the components of this user cost, current production is influenced by production in other periods. It is the profit foregone due to this lowering of the maximum extraction rate which is captured by the third term.

The fourth term in 3.6 is called "capital consumption user costs". The shadow price, $\mu_{jk,t+1}\beta_{t+1}$, is a measure of the productivity from an additional unit of capital stock. If production increases, the capital equipment runs down sooner and therefore depreciates faster. Thus, the capital consumption user cost shows the discounted profits foregone because capital is used (consumed) now instead of in the future.

The last type of user cost, involving the last two terms, comprises "production user costs". Increasing current production causes a faster decline in reservoir pressure and future production to have higher costs; i.e. alternative recovery methods are developed sooner since the natural drive is no longer sufficient to bring the oil to the surface. The other production user cost term is identical except that it deals with the relationship among firms. If firm j extracts oil at a faster rate today, then firm i has higher costs in the future and thereby foregoes future profits. The summation that is involved in the user costs is important because it links production, limits on production, investment, and costs between periods so that any current decision affects future periods as well.

Characterizing the optimal level of firm j's investment in capital type k during each period, equation 3.7 equates the discounted marginal costs of investment with the discounted marginal benefits. The first term on the right hand side of 3.7 measures the impact on marginal productivity of increasing the capital stock, the second term measures the increase in total recoverable stocks, the third term measures the increase in the upper bounds of production, the fourth term measures the decline in variable costs for other firms, and the fifth term measures the decline in future variable costs of an increase in investment.

These two sets of equations determine the optimal path of operation for the firm where reservoir properties contribute to defining rates and limitations on production. In each period production should be at the rate where discounted marginal net income equals discounted marginal user costs, and investment should be at the rate where the discounted marginal cost of investment equals the discounted marginal benefit. These conclusions show that analytical models provide results useful for understanding optimal exhaustible resource behaviour.

However, a possible shortcoming of this approach in practise is the generality of the KC model. In constraint set 3.3 the extraction rate is a function of previous production rates, investment and capital stocks, but what is the functional form? Are these variables connected in a linear, exponential or quadratic relationship? These forms are not specified, and based on Chapter 2 and the introduction to the KC model, the nature of these equations is unique to each reservoir. Similarly, the optimal production rates and user costs are unique.

Yet the KC model is not intended to yield specific solutions, but rather to show the relationships among the key variables. The model successfully captures the relationships governing the physical behaviour of production and the economic considerations in the production and investment of a petroleum reservoir. Thus, it is useful for comparison with other analytical models.

3.3 Hotelling's Economics of Exhaustible Resources

The key seminal paper in the theory of exhaustible resources is Hotelling (1931). A summary of this paper is presented in Levhari and Liviatan (1977), who also provide some extensions. In their summary, Levhari and Liviatan discuss the important assumptions of and conclusions found by Hotelling. The assumptions are: output of the resource shrinks to zero at the terminal time, firms produce up to the point where the resource is completely exhausted, costs are directly proportional to output, and the firm maximizes the present value of profits. Model variables, functions and relations are:

$x(t)$ - cumulative output by the firm at time t; $x(t) \leq a$, cumulative output is constrained by resource availability

$q(t)$ - output at time t; $q(t) = dx(t)/dt$

$R(q)$ - revenues as a function of output; prices are exogenous

$C(q)$ - costs as a function of output

$\pi(q)$ - profits as a function of output, and prices are exogenous;

$\pi(q) = R(q) - C(q)$.

The model is:

3.8 $\quad \text{Max} \int_0^T e^{-rt} \, \pi(q)dt$, subject to:

3.9 $\quad x(t) \leq a$

3.10 $\quad q \geq 0$.

The condition for optimal extraction is:

3.11 $e^{-rt}\,\delta\pi/\delta q = \Phi$ or $\delta\pi/\delta q = \Phi e^{rt}$, for $0 \leq t \leq \infty$.

Optimal extraction occurs when marginal profit rises exponentially over time at the rate of interest r. This condition is widely known as the Hotelling R-percent rule and is a highly simplified version of equation 3.6 in the KC model. The only user cost in the Hotelling model is associated with the limited availability of the resource.

The Levhari and Liviatan extension to the Hotelling model allows costs to be an increasing function of output, which also allows costs to be a function of current and cumulative output: C(q,x). This extension allows for incomplete exhaustion: since costs increase with cumulative output, the termination time can be reached before complete exhaustion occurs. The condition for optimal extraction for the new model is:

3.12
$$R'(q) = \delta C(q,x)/\delta q + e^{-r(T-t)}(R'(q(T)) - \delta C(q(T),x(T))/\delta q(T))$$
$$- \int_t^T e^{-r(s-t)}(\delta e^{-rt}\pi(q,x)/\delta x)ds, \qquad \text{for all } t.$$

Using MR to denote marginal revenue and MC to denote marginal cost, the condition can be rewritten as:

3.13
$$MR_t = MC_t + e^{-r(T-t)}(MR_T - MC_T) + \int_t^T e^{-r(s-t)}(\delta C(q,x)/\delta x)ds.$$

Profit is maximized when marginal revenue equals the marginal cost of current production plus two user cost terms. The first term is the profit foregone at time T when an additional unit is produced at t, which is similar to the stock user cost term in equation 3.6 of the KC model. The second user cost term is the present value of all future additional costs incurred from producing an additional unit now (time t), which is similar to the production user cost term (the fifth term in equation 3.6) of the KC model.

The presence of the second user cost term means that the R-percent rule no longer applies. If complete exhaustion does not occur, then the first user cost term vanishes leaving only the second user cost term as the difference between marginal revenue and marginal cost. If we assume that complete exhaustion occurs in the Levhari and Liviatan model, then marginal profit increases but at a rate slower than the interest rate.

Altering the interest rate, the demand, or adding a severance tax changes the optimal output path for both scenarios of complete and incomplete exhaustion. For both cases an increase in the interest rate accelerates production, provided that production cost does not depend on the interest rate. The cumulative output is the same, but it is recovered sooner. With an increase in demand the results differ between scenarios. Since a competitive firm is modelled, an increase in demand can be viewed as an increase in price. For both cases, an increase in price accelerates production. However, for incomplete exhaustion, output that was previously uneconomical is now profitable and produced, resulting in a higher level of cumulative production. A severance tax per unit of resource or ad valorem tax has a similar impact as a decline in price or decline in the demand for the resource. In the case of complete exhaustion, an ad valorem tax decreases the rate of production and prolongs production. In the case of incomplete exhaustion the effects are ambiguous. The rate of production declines, lengthening the duration of production, but cumulative production also declines, shortening the duration of production. Thus, the net change in the duration of production can be either positive or negative.

Since this model is intended to apply to all exhaustible resources, no reference is made to the influence of the physical characteristics of resource extraction on the output path.

3.4 Rule of Capture

Davidson (1963) also considers the short run question of the optimal level of production. His optimal production condition is similar to the one for the Kuller and Cummings model - marginal revenue equals marginal cost plus marginal user costs. Marginal revenue is equal to the price under the assumption that the producer faces a perfectly elastic demand curve (prices are exogenous). Davidson argues that physical factors distinguish reservoirs in terms of operating costs and that MER should be imposed for optimal operation. However, Davidson does not specify an explicit model of reservoir depletion but just the conditions for the optimal allocation of crude oil.

Similar to the KC model, Davidson's optimality condition contains user costs: a raw material user cost, an ultimate recovery user cost, and a rule of capture user cost. The raw material user cost (U_m) is the profit foregone when production occurs today rather than in the future at a possibly higher price (stock user cost). The ultimate recovery user cost (U_u) is the profit foregone when production exceeds the maximum efficient rate (MER) and output is lost (boundary user cost). The rule of capture user cost (U_c), is the profit foregone when a competitor produces from the same reservoir and captures potential production and its profit (producer user cost - the sixth term in equation 3.6). Denoting marginal revenue and marginal cost by MR and MC, the optimal output condition can be formulated as:

3.14 $\quad MR = \text{price} = MC + dU_m/dQ + dU_u/dQ + dU_c/dQ.$

From this condition the rule of capture user cost lowers the value of crude oil and hastens production. Davidson suggests that the optimal allocation of resources over time occurs when the rule of capture user cost is eliminated and free competition exists within the market.

3.5 Conservation and the Theory of Exhaustible Resources

Gordon (1966) argues that a lower discount rate does not necessarily lead to resource conservation. Starting with Hotelling's R-percent rule, Gordon discusses how interest rate changes bear on the present value of marginal profits.

A discount (interest) rate increase has two different effects. First, future marginal profits have lower present values, causing production to be shifted towards the present. This is the traditional conservationist argument where the resource is exhausted too soon because of the high discount rate. The second effect of higher interest rates is an increase in costs. With higher interest rates, less is invested in cost-reducing machinery and equipment rental costs rise. "The longer into the future this cost increase is discounted, the less impact it has on present value; this effect therefore increases the attractiveness of the future."[2] However, higher costs also make some deposits unprofitable, reducing the total recoverable stock of resources. Technical progress may lead to cost reductions in the future and the previously uneconomical recoverable stock can then be produced.

With the marginal cost constant for all periods and defined as the rental cost of capital equipment based on the interest rate, Gordon shows that increasing the interest rate lengthens the lifetime of the resource industry. This result only occurs over a certain range of interest rates, however, and then for higher rates the traditional effect dominates, shortening the resource industry lifetime. Prices in this model are endogenously determined through a linear demand function.

3.6 Supply Price and Initial Capital Outlay

Bradley (1967) examines the question: "Should this reservoir be developed?" where the production rates are modelled by an exponential decline curve. The key parameters of this function are the initial production rate and the decline rate, both of which are determined by the physical characteristics of the reservoir. The production rates along with the initial capital costs are used to determine the unit cost or supply price of developing the reservoir. Production occurs if current prices exceed this supply price.

The equation for finding the supply price is given below. Z is the supply price, I is investment, q(t) is the production in period t, and the integral represents cumulative production. Operating costs are very small relative to investment costs, and therefore are not included in the supply price formulation.

3.15 $$Z = \frac{I}{\int_0^T q(t)e^{-rt}\, dt}$$

The supply price (z) shows the investment cost per barrel that must be met by current and future prices before production occurs. Assuming that production follows an exponential decline path, $q(t) = q_0e^{-Dt}$, the equation can be rewritten as:

3.16 $$Z = \frac{I}{\int_0^T q_0e^{-(D+r)t}\, dt}.$$

Integrating and assuming that the T is large (T approaches ∞), the relation is approximated by:

3.17 $$Z = \frac{I}{q_0}(D + r).$$

Estimates of I, D, r, and q_0 can be obtained and thus an estimate of the supply price can be made.

Bradley's approach differs from the KC approach but adopts some of their assumptions, namely that the unit cost for a reservoir varies with the output rate and the total volume of production.

3.7 The Relationship Between Price and Extraction Cost for a Resource with a Backstop Technology

The influence of a backstop technology and increasing extraction costs on the optimal production conditions (marginal revenue equals marginal costs plus user costs) is determined by Heal (1976). Since the backstop technology, an inexhaustible resource which forms a perfect substitute, provides an unlimited stock of resources, the stock user costs of the KC model vanish. This also implies that the Hotelling R-percent rule no longer holds for this model. However, the production user costs caused by increasing extraction costs through time are included.

Initially, production occurs from the lower cost exhaustible resource and then at exhaustion switches to the backstop technology. Assuming perfect competition, the optimal rate of production equates price to the marginal extraction cost plus the production user costs. Once production starts from the backstop technology, there are no user costs since production costs are constant, and prices equal the cost of the backstop technology. Thus, the cost of the backstop technology sets an upper bound for prices which are determined endogenously and also an upper bound for the marginal extraction cost plus production user cost of the exhaustible resource.

Heal describes the production user costs as social costs reflecting the effect of present extraction pushing up the costs of future extraction. As extraction costs approach the cost of the backstop technology, the potential increases in extraction costs shrink, and therefore, the associated user costs decline. This conclusion of decreasing user costs is completely opposite to the conclusions of the previous analytical models, exemplifying the significance of the assumptions used to frame the model.

3.8 Economic Theory and Exhaustible Resources

Dasgupta and Heal (1979) argue that under competitive conditions an individual is indifferent between owning a unit of an exhaustible resource whose price rises at the interest rate and a unit of a commodity that earns interest through time. Assuming that there is no extraction cost, the price of the extracted resource (p_t) is the same as the stock price (q_t). If the individual maximizes the present value of profits through his choice of the extraction rates, it follows that

$$3.30 \quad q_t S_t = \max \int_t^{\infty} p_t R_i \, e^{(-r(i-t))} \, dt, \qquad (i \geq t).$$

subject to:

$$3.31 \quad \int_t^{\infty} R_i di = S_t, \qquad (i \geq t).$$

In equation 3.30 the value of owning the stock (S_t) equals the present value of selling the extracted resource (R_i), and equation 3.31 limits the aggregate extraction to the size of the stock.

Differentiating equation 3.30 leads to the following optimality condition:

$$3.32 \quad (\dot{q}_t - rq_t) = (q_t - p_t) \frac{R_t}{\dot{S}_t},$$

where $\dot{q}_t$ = dq/dt and $\dot{S}_t$ = dS/dt.

But with the extracted resource price equal to the stock price, this equation becomes:

$$3.33 \quad \frac{\dot{q}}{q} = r \quad \text{or} \quad q_t = q_0\, e^{rt}.$$

This equation states that the stock price must increase at the interest rate which is just the Hotelling rule.

Dasgupta and Heal then change their focus to a socially managed exhaustible resource where the discounted consumer surplus is maximized. This also yields the optimal condition of equation 3.33; however, the initial price and extraction rate are determined which did not happen in the previous example. Thus, in the absence of forward-looking planners or a complete set of future markets, the producers are myopic decision makers and could set the wrong initial price and extraction rate for optimal depletion.

Relaxing the assumption of no extraction costs places a wedge between the stock price and the resource flow price. This difference is the marginal cost of extraction.

3.34 $p_t = q_t + \delta C/\delta R_t$

Equation 3.33 now becomes

3.35 $\frac{\dot{q}}{q} - \frac{\delta C/\delta S_t}{q} = r.$

Consequently, when the resource stock is abundant, the stock price is small and so the resource flow price is almost entirely determined by the marginal extraction cost. As the resource nears exhaustion the stock price and the resource price rise, with a large percentage of the resource price determined by the stock price.

In their analysis of taxation, Dasgupta and Heal consider a sales tax, profits tax, royalties, and a capital gains tax. A sales tax that increases at the rate of interest identical to the nonextracted resource price does not alter the pattern of extraction. A constant profits tax has the same impact and so in both cases the tax is absorbed by the resource owner. If the sales tax is constant over time, production is lower initially and prices are higher. Thus, the pattern of extraction is distorted and the consumer bears a portion of the tax. Generating the same type of distortion, a royalty tax acts like an increase in average extraction costs. A capital gains tax causes no distortion if the tax on capital gains is the same as the tax on interest income.

3.9 Analyzing Nonrenewable Resource Supply

Several different exhaustible resource models are considered by Bohi and Toman (1984). They start with a simple case, then add complexities to show how the optimal production path changes as extensions are made. In their base model firms make decisions independent of other firms' behaviour, each firm is a price taker, and each firm is fully integrated.[3] The decision to produce is based on maximizing profits (equation 3.36) subject to the stock change equation ($R_{t+1} = R_t - q_t$) and the firm's knowledge of prices, extraction costs and initial reserves. The extensions made to this model are the addition of the development phase to the extraction phase, capacity constraints on extraction, joint products, common property externalities, and technical progress. Each of these extensions is made separately; no single model incorporates all of them simultaneously.

The variables of the base model are:

V - the present value

q_t - extraction rate in period t

T - terminal date (model determined)

d - discount factor $d = 1/(1+r)$

R - stock of reserves; $R_{t+1} = R_t - q_t$, $t = 0, 1, 2, ..., T$

p - price

E - cost of extraction; a function of q_t and R.

The firm maximizes the net present value of extraction, namely

3.36 $$V(q_0, q_1, ..., q_T, T) = \sum_{t=0}^{T} d^t(p_t q_t - E(q_t, R_t)).$$

The marginal profit rule to maximize V is:

3.37 $$p_t = E_q(q_t, R_t) + \sum_{s=t+1}^{T} d^{s-t}(-E_R(q_s, R_s)), \quad t = 0, ..., T.$$

In the preceding equations and the ones following, subscripts other than s or t represent derivatives.

The optimal production occurs where the price is equal to the marginal extraction cost plus a degradation charge (user cost). These equations imply that:

$$3.38 \quad \frac{(p_{t+1} - E_q^{t+1}) - (p_t - E_q^t)}{(p_t - E_q^t)} + \frac{-E_R^{t+1}}{(p_t - E_q^t)} = r, \quad t = 0, ..., T.$$

This is equivalent to equation 3.35 that Dasgupta and Heal derived. The first term represents the rate of capital gain from holding the reserves in the ground, or it could be interpreted as the rate of change in the value foregone by extracting one unit of reserves now instead of in the future. The second term shows the rate at which future extraction costs change due to current extraction. Thus, the second term is the rate at which future costs increase if one more unit of reserves is produced today and can be regarded as a user cost. The term on the right-hand side is the interest rate, and it shows the rate of return the investment could have made in the market instead of in the development of the reserve. If the left-hand side were greater then the discount rate, then the firm would not produce; a higher rate of return would be earned by leaving the reserves in the ground.

The extensions of the base model alter this marginal profit rule. Including the development phase allows for the possibility of reserve additions. The marginal profit rule for extraction remains almost the same except that the user cost component also depends on the future levels of reserves. The future levels of reserves are optimized through a marginal profit rule for reserve additions similar to the marginal profit rule for resource extraction. The marginal benefit from adding to reserves is equated to the marginal cost of the reserve addition plus a user cost (the depletion of undiscovered reserves is similar to the depletion of a reserve). The development marginal profit rule sets the level of marginal benefits from reserve additions equal to the extraction user cost. Since the extraction user cost occurs in both marginal profit rules, the decisions to extract and develop are simultaneously determined.

Capacity constraints lead to another user cost term in the extraction marginal profit rule. "If extraction reduces capacity, an incremental increase in current output reduces the present value of future capacity."[4] This type of user cost is similar to the boundary user cost of the KC model.

When joint products are present, the quantities and reserves of all joint products are included in the model, and there is a marginal profit rule for each joint product. The rate of extraction of each resource depends on the price of that particular resource, the prices of all the joint products and the quantity of reserves.

The common property externality modifies the extraction marginal profit rule such that as the number of firms depleting the reserve increases, the value of the user cost of extraction declines. This externality leads to higher initial extraction rates and lower prices. So this optimality is seen from the standpoint of the individual producer and not the industry.

By adding a marginal investment rule, technical progress is incorporated into the model. Investment in knowledge can result in a decrease in extraction costs. The marginal investment rule equates the marginal cost of investment to the marginal benefits (the present value of future decreases in extraction costs).

3.10 Intertemporal Extraction of Mineral Resources Under Variable Rate Taxes

Conrad and Hool (1984) examine the effects of three different types of taxes on the optimal production path which, in the absence of taxes, equates marginal revenue to marginal extraction cost plus a user cost in each period. The user cost arises from the constrained availability of the resource - the stock user cost of the KC model. Their model distinguishes between the grades of the ore extracted. Applying the Kuhn-Tucker conditions to the Lagrangian function of their model yields in the following profit maximizing equation:

3.39 $$\frac{1}{(1 + r)^{t-1}} \left(P_t\alpha_g - C_t'(X_t)\right) \leq \sigma_g, \quad g = 1, ..., G,$$

where:

P_t - the exogenously determined price of metal in period t

X_t - quantity of ore extracted in period t; $X_t = \sum_{g=1}^{G} X_{tg}$

α_g - proportion of metal in ore of grade g, $0 < \alpha < 1$

C_t' - marginal cost in period t; where $C_t' > 0$ and $C_t'' > 0$

σ_g - shadow price or user cost of reserves of grade g

r - discount rate.

The inequality in equation 3.39 allows the discounted marginal revenues less the marginal costs in a period to be less than the shadow price. If production is zero in a particular period, then the marginal revenue less the marginal cost is zero even though the shadow price is positive.

The three types of taxes are: variable per-unit severance tax, variable ad valorem severance tax, and progressive profits tax.

The per-unit tax is included by substituting $P_t - \tau_t$ for P_t. If this is a constant per-unit tax, there occurs a reallocation of extraction from the present to the future and a decline in total extraction. If the tax rate is not constant but has a sufficiently high growth rate, the effects are reversed.

The ad valorem tax is included by substituting $(1 - \beta_t)P_t$ for P_t. If this is a constant ad valorem tax, there will be a reallocation of extraction towards the periods with lower discounted prices and a decline in total extraction. If the tax rate is variable, the grade selection may be changed, the intertemporal profile may be altered in either direction, and the total recovery may be increased.

The progressive profit tax is not as easily incorporated into the model. To show the tax rate as being progressive it is given in a quadratic form.

3.40 $T_t = \Phi\pi_t + \mu\pi_t^2/2$, where $\Phi, \mu > 0$

Inserting T_t into the Lagrangian function of the original model yields 3.41:

3.41 $L = \sum_{t=1}^{T} (1+r)^{-(t-1)} (1- \Phi - \mu\pi_t/2)\pi_t + \sum_{g=1}^{G} \sigma_g(R_g - \sum_{t=1}^{T} X_{tg})$

With this type of tax the grade selection, the extraction rate, and the recovery path are all modified depending on the past path of profits.

The different types of variable tax rates included in the model all change the optimal production path. Similar changes occur in the optimal production path of the reservoir production model described in Chapter 5. These changes are shown in the results presented in Chapter 6.

3.11 The Economics of Exhaustible Resources and The Economics of Mining

Bradley (1985) considers whether or not advances in the theory of exhaustible resources have contributed to the understanding of observed depletion behaviour in the context of capital investment and the sequential depletion of ore deposits.

He formulates a model yielding an optimality condition which is similar to that of the KC model except that investment is directly included in the marginal profit condition. The condition equates net present value of profits to marginal investment cost plus a user cost:

3.42 $(P - C)a = dI/dQ_o + (P - C)Tv,$

where:

P-C	- price less marginal costs; net profits
a	- annuity factor evaluated for T periods
dI/dQ_o	- incremental investment per unit of capacity
(P-C)Tv	- user cost
Tv	- discount factor associated with the user cost.

Since extractive industries are highly capital intensive, an examination of changing capital cost conditions is warranted. Assuming a knowledge of future prices and a perfectly elastic supply of capital, the optimal solutions given by most analytical models are acceptable. However, in reality producers compete for capital with other industries and face price risks. These conditions of possible rising capital costs, risk associated with future prices, and variation in estimation of reserves may alter the decision process for the producer. In Bradley's study the present value of profits is not very sensitive to changes in the rate of extraction near the optimum. Bradley shows this by using an example of a hypothetical petroleum reservoir. When output is set at half the rate suggested by the optimal solution the loss in present value is only five per cent. If a firm is operating at this sub-optimal position, a large increase in investment would only bring a slight increase in profits. "With small and falling incremental return to additional investment, a risk-averse producer will probably opt to commit less capital and to operate at a lower rate than dictated by present value maximization."[5] The above statement by Bradley readily explains a large portion of any divergence between observed rates of use and calculated optimal ones.

Another assumption common to analytical models is that resources are depleted from highest to lowest grades. However, sequential depletion of varying grade deposits is highly unlikely since nature does not arrange them in this order nor are they necessarily discovered in this order. Thus, high cost resources are developed while low cost resources are still undiscovered. When these low cost resources are discovered, they have a cost advantage which is defined as a differential rent. Firms have an added incentive to explore for and find low cost deposits to take advantage of these differential rents.

Bradley argues that the marginal benefit from modelling more realistically the geological, engineering, and institutional characteristics is larger than the marginal benefit from refinements in the optimizing models.

3.12 Switching From Primary to Secondary Recovery

Amit (1986) examines the switch from primary to secondary recovery. Two types of investment are possible: capacity investment which improves the productive capacity of the reservoir but does not affect cumulative production (adding wells), and reserve investment which is aimed at increasing the cumulative recovery (switching to secondary recovery). For primary recovery, the natural drive moves the oil to the wellhead, there are no lifting costs, and the recovery rate is not controlled by the producer. For secondary recovery artificial displacement mechanisms are used, there are lifting costs, and the recovery rate is controlled by the producer.

From the optimality conditions of Amit's model, all of the capacity investment (number of wells drilled) should take place before production commences. Production should occur at the highest allowable rate. If secondary recovery is inexpensive and the natural drive is weak, then secondary recovery should be started immediately. If secondary recovery is expensive and the natural drive is strong, then only primary recovery is necessary. A third possibility exists where reservoir production occurs initially by primary recovery and then switches to secondary recovery when the reward from secondary recovery just equals the reward foregone by ceasing primary recovery.

Sensitivity analysis yields two other results. If price increases, secondary recovery should occur sooner, allowing faster extraction to take advantage of the price increase. Second, if the capital outlay required to commence secondary recovery increases, then the primary phase should last longer.

This model has a parallel framework to the KC model where production is categorized into two types: unaided natural production or primary recovery and artificially increased ultimate recovery through investment or secondary recovery. However, in the Amit model production relationships are developed to a more detailed extent (eg. production will follow a natural decline path) which allows for the direct conclusions already presented, but some strong assumptions are necessary to determine this result. These assumptions are that a single fixed price represents future prices, a single firm produces the reservoir, secondary recovery can only be initiated once, and the capital required to start secondary recovery is fixed. Due to these assumptions the optimality condition for the Amit model is to produce as long as the price is greater than the marginal lifting costs plus the stock and production user costs. Furthermore, the condition for initiating secondary recovery in the Amit model is similar to the optimal condition for capital in the KC model.

3.13 Rate Sensitivity and the Optimal Choice of Production Capacity of Petroleum Reservoirs

Nystad (1987) investigates an oil company's optimal choice of production capacity (the level of initial production that will maximize profits) taking into account the production decline rate, up-front capital costs, and the rate sensitivity (if any) of the reservoir. A rate sensitive reservoir is defined as a reservoir where an increase in the depletion rate leaves oil trapped underground that cannot be recovered.

Increasing the discount rate in a simple model (which excludes capital costs and follows the Hotelling rule) always shifts production from the future to the present. This result does not hold for Nystad's model due to the inclusion of capital costs. In both models an increase in the discount rate attaches a higher value to discounted current production revenues as compared to future production revenues. But, in Nystad's model, it also raises the up-front investment costs relative to the discounted future revenues. Nystad demonstrates that there is a discount rate which maximizes the initial production rate. As the discount rate approaches this rate, the initial production rate rises since the increased discounted revenues are greater than the increase in the discounted investment costs. The reverse holds for discount rates above this rate. The initial-output maximizing discount rate changes depending on the degree to which a reservoir is rate sensitive. The discount rate that maximizes the initial production rate is lower for a reservoir that loses cumulative production when the initial production rate is increased.

When a reservoir is rate sensitive, a change in prices or the addition of a tax rate changes the optimal output rate. Comparing the results under two different price paths shows the higher price path reaching its optimal initial production rate at a higher discount rate. When taxes are included, the initial production rate reaches its maximum level at a lower discount rate than the before-tax initial production rate.

3.14 Conclusions

The analytical models presented in this chapter suggest that optimal production occurs where marginal revenue equals marginal cost plus user costs. The most elaborate user cost configuration is presented in the KC model. When prices are assumed exogenous, the condition is price equals marginal cost plus user costs.

There are many types of user costs as discussed in relation to the analytical models. Stock user costs reflect the value of having only a limited amount of the resource. The stock user cost measures the profit foregone from producing an additional unit now instead of in the future. A variant of this user cost is quantified in Chapter 6. Boundary user costs reflect the profit of an additional unit of production foregone due to the maximum production rate limitations. Capital consumption user costs reflect the profits foregone because capital is used now instead of in the future. Production user costs reflect the profit foregone due to increasing extraction costs. This type of user cost also occurs due to the rule of capture or common property problem. Heal also introduces a backstop technology which dramatically affects the path of user costs.

Few of the analytical models consider physical characteristics. Exceptions to this assertion are the KC model, the Amit model and the Nystad model. The KC model, however, does not capture explicitly the influence of these factors on production. In the Amit model the switch from primary to secondary recovery mechanisms is emphasized. In the Amit and Nystad models the importance of the MER is included in the determination of production.

Most analytical models ignore investment. Bradley emphasizes this limitation and argues that it can greatly influence the production decision. The models which have incorporated the investment decision are the KC model, the

Bohi and Toman model, the Amit model, and the Nystad model.

The next chapter focuses on computational models which successfully incorporate the physical characteristics of reservoir depletion and investment costs.

NOTES

[1] Two models very similar to the KC model are those of Cummings and Burt (1969) and Burt and Cummings (1970).

[2] Richard L. Gordon, "Conservation and the Theory of Exhaustible Resources," Canadian Journal of Economics, 32, no. 3 (1966): 322.

[3] A vertically integrated company is involved in all aspects of the industry, namely exploration, development, production and marketing.

[4] Douglas R. Bohi and Michael A. Toman, Analyzing Nonrenewable Resource Supply (Washington: Resources for the Future, 1984), 31.

[5] Paul G. Bradley, "Has the 'Economics of Exhaustible Resources' Advanced the Economics of Mining?" Progress in Natural Resource Economics, ed. Anthony Scott (Oxford: Oxford Univ. Press, 1985), 322.

Chapter 4

Computational Models of Exhaustible Resources

4.1 Introduction

This chapter surveys computational models of exhaustible resources and identifies their contributions to exhaustible resource economics. These models attempt to capture the physical and market conditions of exhaustible resources, which can be resource and even resource-deposit specific. Examples of these conditions are the reservoir pressure used in determining the production for an oil reservoir, the investment associated with platforms, rigs, and wells, the reservoir formation and pressure maintenance used to improve reservoir performance, interfuel substitution, and the load duration curve associated with electricity generation. In representing these conditions, computational models may be able to contribute to areas of exhaustible resource economics where analytical models can not.

Bradley (1985) argues that the proper inclusion of physical and investment considerations may be able to extend exhaustible resource theory. This chapter examines computational models to show how they have handled these considerations. At the same time this chapter notes the exclusion of economic analysis by some of these models. The scope of the work surveyed is very broad, ranging from a single reservoir depletion model to a global energy supply and demand model, and thus the survey is selective. Most models adopt a mathematical programming approach.

4.2 Computational Models of Petroleum Reservoir Production and Development

4.2.1 Pressure as a Key Determinant of Reservoir Production

The computational model of Garvin, Crandall, John, and Spellman (1957), shows how pressure affects the production rate. By focusing on pressure, the model includes a very important physical characteristic as discussed in Chapter 2 and only mentioned by a few of the analytical models in Chapter 3. The model is presented below, with the variables and parameters defined as follows:

Q_{ij} - production from reservoir i in period j

Q_j - oil purchased from an outside source in period j

Q_{cj} - production commitment that must be met in period j

Q_{max} - production limitation

P_{i0} - exogenously determined initial pressure in reservoir i

$P_{i,min}$ - exogenously determined minimum pressure in reservoir i

c_{ij} - profit/barrel for oil from reservoir i in period j

c_j - profit/barrel for oil from an outside source

f_{ij} - a function relating production to pressure.

4.1 $$\text{Max } Z = \sum_{j=1}^{K} \sum_{i=1}^{N} c_{ij}Q_{ij} + \sum_{j=1}^{K} c_jQ_j$$

4.2 $$\sum_{j=1}^{k} (f_{i,k-j+1} - f_{i,k-j})Q_{ij} \leq P_{i0} - P_{i,min}, \quad \text{for all } i$$

4.3 $$\sum_{i=1}^{N} Q_{ij} + Q_j = Q_{cj}, \quad \text{for all } j$$

4.4 $$Q_{ij} \leq Q_{ijmax}, \quad \text{for all } i \text{ and } j$$

The objective function maximizes profits from production from the reservoirs and from oil imported. Constraint 4.2 defines production as a function of the exogenously-specified change in pressure. Equation 4.3 requires production from the reservoirs and from the outside source to meet demand or the resource commitment. Finally, there is a limit on production from a given reservoir in a given period, and all variables are non-negative.

4.2.2 Well Placement and Decline Curve Production

Following the same approach in one of their models, Aronofsky and Williams (1962) assume a completely developed field and optimize profits with pressure being the main determinant of production. In a second model production follows a specified production decline curve and profits are maximized by the optimal number and placement of wells.

The variables and parameters of the second model are:

$h(t)$ - production at period t based on a production rate decline curve

n_k - number of wells drilled at the end of period k

r_k - number of rigs purchased at the beginning of period k

$\bar{r}_k$ - number of wells drilled by a rig in one period

v_k - number of rigs in operation during period k

a - maximum number of wells drilled by a rig

c - revenue/unit at pipeline terminal

d - the cost of a rig and installation

e - operating expenses to lift and transport one unit of oil

w - the cost of operating a rig

x - completion costs for a well

β - the discount rate

N - the number of cells (blocks of land) for the reservoir; only one well can be drilled per cell

R_i - pipeline capacity

t_s - time period s

B - maximum reserves available from a single cell

I - final time period of production

The model is as follows.

4.5 $$\max Z = \sum_{s=1}^{I} \frac{(c_s - e_s) \sum_{k=1}^{s} (n_k h(t_s - t_k)) - dr_s - wv_s - xn_s}{(1 + \beta)^s}$$

4.6 $$v_k \leq \sum_{s=1}^{k} r_s, \qquad k = 1 \ldots I$$

4.7 $$n_k \leq v_k \bar{r}_k, \qquad k = 1 \ldots I$$

4.8 $$\sum_{s=1}^{k} n_s \leq a \sum_{s=1}^{k} v_s, \qquad k = 1 \ldots I$$

4.9 $$\sum_{s=1}^{k} n_s \leq N, \qquad k = 1 \ldots I$$

4.10 $$\sum_{k=1}^{i} n_k h(t_i - t_k) \leq R_i, \qquad i = 1 \ldots I$$

4.11 $$\sum_{s=1}^{i} \sum_{k=1}^{s} n_s h(t_s - t_k) \leq B \sum_{s=1}^{i} n_s, \qquad i = 1 \ldots I$$

The objective (4.5) is to maximize profits, where the profits are the revenues minus operating costs minus fixed costs. Fixed costs are: the purchase price of a rig, the operation of a rig, and the completion costs of a well. Constraints of the model are: the number of rigs in operation must not exceed the number of rigs purchased (4.6), the number of wells drilled can not exceed the number allowed by the number of rigs in operation in a period (4.7), the cumulative number of wells drilled can not exceed the cumulative number of rigs in operation multiplied by the maximum number of wells each rig can drill (4.8), the cumulative number of wells drilled can not be greater than the number of cells (4.9), the well flow can not exceed the pipeline capacity (4.10), and cumulative production can not exceed the reserves available from the cells (4.11).

The Aronofsky and Williams linear programming model captures the influence of investment and physical factors such as pressure and rig constraints on production decisions. It is not obvious if the model follows the optimizing rule of analytical models where marginal revenues equal marginal costs plus user costs. However, production continues as long as marginal revenues are greater than marginal costs, and production is limited by capacity constraints. Production is terminated when the physically defined reservoir capacity is exhausted.

4.2.3 Platform Placement, Well Location, and Production Scheduling Models

Frair and Devine (1975) employ a model similar to the second model of Aronofsky and Williams. They determine the strategy of platform and well placement for an offshore oil reservoir that maximizes discounted after-tax cashflow. Production is assumed to follow an exponential decline curve, and prices are exogenous. The development strategy determines: the number of fixed platforms, the size and location of each platform, the assignment of wells to platforms, when platforms are set up, when wells are drilled, and the production rates in each period for the oil reservoirs.

There are the normal capacity constraints on production, wells and platforms, but the production constraints also allow for shut in production. Shut in production may be desirable under certain conditions such as very low prices for a certain period of time followed by higher expected prices. In this case, during the low price period, the model sets production levels to zero and then returns them to the pre-shut in levels at the higher prices. No production is lost, and the production strategy is able to take advantage of the high prices. Few if any analytical models of exhaustible resources appear to allow for shut in production.

The model also incorporates discrete (integer) variables - rarely seen in analytical models - for platforms and for connecting platform wells to the target areas of a reservoir. Drilling a fraction of a well or using only a fraction of a platform are not viable alternatives.

Investment expenditures in the form of platform, well and drilling costs are incorporated into the model. Thus, by varying the number of wells and platforms used in production, Frair and Devine can determine the sensitivity of the optimal solution to changes in the amount of investment.

This model is developed from an earlier model by Devine and Lesso (1972) in which only the platform placement and the assignment of targets to platforms is modeled. Devine (1973) extends the latter model to allow for dual completion wells. For this extension each well is allowed to extract oil from one or two targets.

4.2.4 Pressure and Investment for Reservoir Development

McFarland, Lasdon, and Loose (1984) model a gas reservoir where pressure and investment are allowed to influence production. They employ an optimal control approach and show numerically how sensitive the objective function value, initial investment, production horizon, and ultimate recovery are to changes in price, cost, discount rate, initial pressure, size of the reservoir, and the strength of the water drive.

Their objective is to:

4.12 $\text{Max } J = \int_0^{t1} (\pi_g(t)(1 - \sigma)q_g(t) - \Phi(t)I(t) - \Theta(t)K(t))(1 - \beta)e^{-it}dt - C(u),$

where:

$K(t)$ - number of producing wells at time t

$I(t)$ - rate of well drilling at time t (wells per year)

$q_g(t)$ - reservoir production rate at time t

$\pi_g(t)$ - wellhead price of gas, exogenously determined

σ - royalty rate

$\Phi(t)$ - drilling cost per well

$\Theta(t)$ - operating, maintenance, and overhead cost per well

β - tax rate

i - discount rate

u - maximum number of wells that can be drilled

$C(u)$ - platform cost.

In this model the discounted profits from a gas reservoir are maximized subject to the following constraints.

4.13 $dV/dt = -\mu(P_0 - P),\ V(0) = V_0$

4.14 $dP/dt = (-RTq_g + \mu P(P_0 - P))/V, \quad P(0) = P_0$

4.15 $dK/dt = I(t) - (1 - V/V_0)K(t),\ K(0) = K_0$

4.16 $q_g(t) = \alpha_g(P(t))^2 - ((P_b)^2)^n K(t),$

where:

V - volume

P - pressure

P_0 - initial pressure

μ - water drive constant

R - gas constant

T - absolute temperature

α_g - gas well flow constant

n - gas well flow constant

P_b - bottom-hole pressure.

Equation 4.13, the change in reservoir volume, shows that as pressure declines water moves into the space originally occupied by gas. Thus the reservoir volume decreases through time. Equation 4.14, the change in pressure, is derived from a material balance equation (Chapter 2) and the gas law. Equation 4.15, the change in the number of producing wells, is the difference between the number of wells drilled and the number of wells flooded-out (produce only water). Finally, equation 4.16 shows production as the flow per well determined by the pressure multiplied by the number of producing wells.

Similar to the investment optimality condition of the KC model, wells are only drilled when the marginal benefit is greater than or equal to the marginal cost of drilling the well. With a constant price for all periods, following this decision rule leads to drilling only in the first period.

Sensitivity analysis shows how economic and physical parameters can change the production schedule, production horizon, net present value (NPV), final pressure, and ultimate recovery. The effects of an increase in price, decrease in royalty, and decrease in cost are similar. Production is shifted toward the present, the exploitation period is shortened, the final pressure is lowered, the NPV is increased, and the ultimate recovery is increased. The decision to start production is delayed only when future prices are significantly higher than current prices (future prices are three times the size of current prices). A decrease in the discount rate lengthens the production horizon, decreases the initial investment (fewer wells drilled), slightly decreases ultimate recovery, and increases NPV. If the water drive is stronger, then production occurs faster, ultimate recovery is lower, and the NPV shrinks. If the initial pressure is higher, then the reservoir operates fewer years, ultimate recovery is higher, and NPV is higher. Finally, if the size of the reservoir is smaller, then there are fewer wells drilled and NPV is lower.

This model follows very closely the model of McFarland, Aggarwal, Parks, and Lasdon (1982). However, the latter model also has an oil/gas example. The sensitivity analysis follows the same pattern as the gas example.

Although the sensitivity analysis shows changes in production with variations in the economic factors, these changes are very small, especially for ultimate recovery. For the gas and oil/gas models ultimate recovery is determined mainly by the physical nature of the reservoir, and only very small changes in ultimate recovery result from changes in economic parameters. This conclusion is not stressed by analytical models but becomes very evident in computational models. By showing how a change in the cost of investment changes the optimal production strategy, the model reinforces a conclusion of Bradley (1985) that investment plays an important role in production decisions.

4.2.5 Pressure Maintenance Model

Lasdon, Coffman, MacDonald, McFarland, and Sepehrnoori (1986) seek the production strategy to maximize gas deliverability subject to the properties of a gas storage reservoir. This production strategy involves well spacing and the production rate schedule for each well. Different objectives are maximized. One objective is the deliverability potential for the last period. To maximize this deliverability, production in each period is determined so that it just meets demand in each period, and then the flow rate can be at its highest level in the last period. The other objective is the excess of production over demand. Model constraints consist of the deliverability limits of each well in standard back pressure form (4.20), the daily demands for production (4.21), and flow rates which must lie within specified ranges (4.22), and nonnegativity restrictions (4.23). This is a reduced form of the model. In the original model the material balance equation and Darcy's law for gases are included in a system of difference equations. These equations incorporate the physical properties of the reservoir which influence pressure such as pore volume, compressibility, viscosity, and the amount of underground migration. It is assumed that the daily well flows satisfy this set of difference equations.

Key variables and parameters of the model are:

W - set of indices of grid blocks which contain a well

i, j - horizontal and vertical grid block indices

t - time period index

q_{ijt} - flow rate from the well in block W(i, j) during time period t

p_{ijt} - pressure in block W(i, j) at the end of period t

c_{ij} - well constant for well in block W(i, j)

$(pb)_{ij}$ - back pressure of well in block W(i, j)

n_{ij} - well slope for well in block W(i, j)

d_t - specified demand field flow rate in period t; an underline indicates a minimum and an over bar indicates a maximum

e_t - excess production in period t

IMP - a parameter between 0 and 1 showing the importance attached to the deliverability or excess objective

DEL - the deliverability objective

EXCESS - the excess objective

The objective is to:

4.17 Max Z = IMP(DEL) + (1 - IMP)(EXCESS)

where:

4.18 $$DEL = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}((p_{ijT})^2 - ((pb)_{ij})^2)^{nij}$$

4.19 $$EXCESS = \sum_{t=1}^{T} e_t$$

subject to

4.20 $q_{ijt} \leq c_{ij}((p_{ijt})^2 - ((pb)_{ij})^2)^{nij}$ for all i and j, t=1...T

4.21 $\sum_{i=1}^{I} \sum_{j=1}^{J} q_{ijt} - e_t = \underline{d}_t,$ t=1...T

4.22 $0 \leq e_t \leq \bar{d}_t - \underline{d}_t,$ t=1...T

4.23 $q_{ijt} \geq 0,$ for all i and j, t=1...T.

The model maximizes a convex combination of the well flow rate and excess production taking into account the loss of pressure caused by other operating wells. It is solved by nonlinear programming techniques.

With the emphasis on the physical nature of the reservoir and its impact on optimal production, few economic considerations are included. Indeed, no prices, costs or investment outlays are incorporated into the model. The suggested time duration for analysis is one to three years. For an economic analysis a much longer time is preferable (20-30 yrs). In ignoring economic considerations a potential deficiency of this and other computational models is revealed. However the complexity of the model does not readily allow for these extensions.

This numerical model shows that pressure maintenance increases the amount of recoverable gas for a reservoir consisting of two connected deposits. If the production rate is substantially higher for one of the areas, gas will migrate toward this area and from the other deposit. An undesirable consequence of this migration is trapped gas between the two areas. Pressure maintenance at the wellhead controls this migration by creating a pressure barrier between the two areas, and the potential loss does not occur. This type of optimal production strategy is found because the physical nature of the reservoir is so well detailed by the computational model. An analytic economic model is unlikely to give similar results.

4.2.6 Optimizing Gas Production Using the Decomposition Method

Dougherty, Dare, Hutchison, and Lombardino (1987) develop a multi-reservoir production model to determine how best to exploit the gas and oil reservoirs in Australia's Cooper Basin. The need to evaluate the economic options arises from the complexity of dealing with 60 small to medium size reservoirs. For each reservoir the questions are: how many wells should be drilled and when, how much compressor capacity should be installed and when, and at what rate should each reservoir produce?

Because of the complexity of the problem addressed, the authors employ decomposition. Their decomposition method involves solving a series of models where the outputs of one model become the inputs of another model. Diagram 4.1 shows the path for determining a solution. First, the whole system is solved in the Systems Integration Module (SIM). The shadow prices on the sales and spare capacity constraints from the SIM result are used as pointers for changing the production rates and capacities in the Linearized Field Development Module. The Trunkline Optimization Module (TOM) then accepts these modified production capacities and finds the least cost trunkline expansion to meet them. This information is part of the input into the Field Development Module/Simple Reservoir Model (FDM/SRM) which determines the optimal schedule for drilling wells and installing compressors. With more than one reservoir in a field, only the necessary development is performed in this module and additional costs added to the NPV calculation. Then the TOM is run again. All four of these modules provide data that can be used in the next iteration of SIM. This model incorporates both simulation and optimization approaches to reservoir development.

According to the optimal solution, NPV improves by $40 million (Australian) above the NPV from the production plan that existed before this model was built.

Figure 4.1

Flow Diagram of SIPS Optimization[1]

```
Start                                                          Finish
 ----->------->-------------SIM  Systems Integration Module--->----->----
 |                            All Reservoirs                           |
 |                            All Demands                              |
 |                            All Years                                |
 |                    Selected Pipeline Constraints                    |
 |                          Selected Years                             |
 |                                                                     |
 |                                                                     |
TOM---<------<---FDM/SRM----<-----<-----TOM-------<------<-----LFDM
                 Field
                 Development
Trunkline        Module            Trunkline          Linearized
Optimization     Simple            Optimization       Field
Module           Reservoir         Module             Development
                 Model
___________      _________         ___________        ___________
Complete                           Complete           Each Gas
Trunkline        Each Field        Trunkline          Reservoir
All Years        All Years         All Years          All Years
```

This model is so large that it can only be solved by dividing the problem into submodels. This feature illustrates two unfavourable characteristics of large scale computational models: in detailing the physical characteristics, the model expands so much that it cannot be solved as a single model, nor is it easy to comprehend.

4.2.7 Conclusion to Petroleum Reservoir Production and Development Models

In these computational models certain factors are captured that are either only mentioned or glossed over by analytical models. Constraining physical factors such as pressure in determining production, pipeline capacity limiting flows, the limitation of wells per platform for offshore oil reservoirs, and the effect of reservoir configuration and pressure maintenance definitely influence the optimal production strategy. Investments such as rig construction, well drilling, and platform construction are also included in numerical models. Thus, several computational models incorporate elements that Bradley (1985) considers important.

Discreteness is also represented in several computational models. In determining the number of platforms, the number of rigs used, or the number of wells drilled, the models allow only for integer values. The case of only a fraction of a well being drilled is not possible.

The computational models contrast with the analytical models of the previous chapter. The analytical models simplify the production process, emphasize economic factors such as economic rents and user costs, and employ a much higher level of aggregation in production. User costs form part of the optimal solutions for computational models but are rarely discussed.

4.3 Aggregated Energy and Exhaustible Resource Computational Models

The computational models surveyed in the rest of this chapter employ a much higher level of aggregation. In achieving this level of aggregation some of these models make simplifying assumptions about the physical characteristics similar to the analytical models. At the same time these computational models place more emphasis on economic factors such as economic rents and user costs than the previous computational models. However, these computational models are able to incorporate complexities such as inter-fuel substitution, terminal conditions, backstop technologies, exports, and the cost of incorrect speculative information about the future which are not represented in analytical models.

4.3.1 The Allocation of Energy Resources

Nordhaus (1973) seeks the optimal allocation of the world's resources to meet energy demands over a 200-year time horizon (1970-2170). He employs five demand categories which are met from seventeen resource categories. The model is a linear programming problem which minimizes the extraction, transport and processing costs subject to supply and demand constraints. Thus, the Nordhaus model is a modified transportation problem which minimizes the transportation costs between energy sources and demand destinations.

Nordhaus shows that the least cost fossil fuels are used first, followed by the rest of the fossil fuels in the order of increasing costs. Nuclear power is provided by the light-water reactor and then the breeder reactor as the technology is developed. Eventually all the fossil fuels are exhausted by 2120, and the breeder reactor supplies all energy demand. Since perfect knowledge of future costs and availabilities is not possible, the dates of changing technologies are only suggestive; however, the pattern of switching appears plausible.

In his theoretical discussion Nordhaus argues that prices are the sum of the marginal extraction cost plus a royalty or user cost reflecting the scarcity of the exhaustible resource. Before including the effect of a backstop technology and assuming extraction costs are zero, the optimality condition is that efficient prices rise at the rate of interest, which is the same as the Hotelling rule. Given the long run nature of the problem, the terminal conditions are critical and influence the optimizing condition. Nordhaus postulates the existence of a backstop technology - a virtually infinite supply of a specific resource which is able to meet the demand requirements. This is similar to the backstop technology included by Heal (1976). The example given by Nordhaus is nuclear energy from a breeder reactor. After switching from the exhaustible resource to the backstop technology, the efficient price of energy is the cost of the backstop technology. The price and royalty of the exhaustible resource at this date is also determined by the backstop cost. Assuming that there is only one exhaustible resource, the efficient price for the exhaustible resource in each previous period is the backstop cost discounted to that period. The user cost (royalty) in each period is the difference between the price and the marginal extraction cost. Thus, at the switch date user costs are reduced to zero.

The optimal solution provides the least cost combination of supplies to meet demands. It also provides shadow prices with the demand and supply constraints which can be interpreted as efficient delivered prices or royalties. The royalties are very small so that the delivered prices for 1970 as calculated by the model are close to the actual market prices for 1970. The main exception is the petroleum price which is higher than the efficient delivered price, but this is explained by import restrictions and prorationing.

Nordhaus also displays results relating to free trade and self-sufficiency policies. In the optimal solution the U.S. incurs large trade deficits and is highly dependent on foreign petroleum in the 1990's. However, this trend reverses between 2020 and 2070 with high exports of coal and shale oil. If the U.S. were to avoid the periods of foreign dependence for energy requirements, costs to the U.S. would increase significantly.

Solving the model extensions such as free trade, self-sufficiency and limit on imports shows one of the major advantages of employing computational models. Analytical models would have difficulty incorporating these extensions.

4.3.2 Waiting for the Breeder - Electricity in the United States

In a long term model of electricity supply and demand, Manne (1974) focuses on the race between the development of breeder fission and the exhaustion of low cost natural uranium. The linear programming model also represents uncertainty as to the date of a functional breeder fission technology. Manne finds that near term decisions are insensitive to the future uncertainties.

The model time frame is 1983 to 2027. Capacities that come on stream during this time are from decisions made five to ten years earlier. Uncertainty is incorporated through the breeder capacity becoming available in 1990, 1995, or not until after 2025, each of these events occurring with exogenously specified probabilities. The electricity supply is from hydro plants, three types of fossil units, peak storage, light water reactors (LWR), and breeders. Uranium supply becomes important because uranium is used extensively in LWR plants. These plants have a comparative advantage in the production of base-load, intermediate-load and peak-load electricity as characterized by the load-duration curve. The load-duration curve illustrates how electricity demand can be allocated into blocks over time. The peak-load block requires the highest electricity load but only for a very short period of time. The base-load block requires a lower level of electricity generation, but this level must be maintained throughout the period.

The demand projections are included in two different ways: independent of future prices and dependent on future prices. The exogenously determined demands follow forecasted expectations and after 1990 a growth rate of 3 per cent per year. The endogenously determined demands allow rising future prices to decrease demands below the exogenously specified reference levels.

The model assumes only a limited amount of hydro and current technology coal-fired thermal potential for electricity generation. Coal is viewed as a backstop technology to be used if a safe and competitive breeder reactor is not developed. Thus, the LWR using uranium must supply electricity if the breeder reactor is not a feasible technology during the time frame considered. Uranium extraction costs are assumed to increase with cumulative production and the supply of uranium is modelled by a step function. The model divides the uranium supply into five categories with the costs increasing as each category is exhausted.

Included in the model is a terminal value credit to reduce horizon effects. Since the capacity additions have different life spans, if no correction is made a capacity addition with a short life span and a high unit cost would be selected instead of a capacity addition with a longer life and a lower unit cost.

The numerical results show that the value of perfect information concerning the breeder implementation date is relatively low. Even if the breeder is developed as soon as possible, there will be no significant changes in the pattern of capacity expansions in the first two periods or 15 years. It is only in the final periods that the pattern is changed, but these alterations have only a very small impact because of the high discount factor based on a 10% discount rate.

Results concerning the use of uranium show that the price path is directly linked to the implementation of the breeder reactor. If the breeder reactor is not implemented before 2025, uranium is used in greater quantities and at higher prices. The shadow prices associated with the constraints on uranium supply are equal to or greater than the uranium supply costs. The differences between the shadow prices and supply costs are interpreted as scarcity rents by Manne, which are the same as the royalty values defined by Nordhaus. In the Manne model, the backstop technology is not reached but for the last category of uranium ore which is produced and not exhausted, the shadow price equals the cost of extraction. Thus, the Manne model determines efficient prices in a similar fashion as the Nordhaus model. However, unlike the Nordhaus model, Manne allows for price responsive demand.

4.3.3 Strategies for a Transition From Fossil to Nuclear Fuels

Häfele and Manne (1975) examine the transition of fuel use in a manner similar to Nordhaus. In their model there is more than one type of breeder reactor, secondary forms of energy are included and three different demand projections are used. The third demand scenario allows demands to be responsive to prices. The reserves of oil and gas are assumed to last 40, 60 and 80 years and thus the base results include nine scenarios.

The model minimizes the discounted costs of capacity expansion and operating these plants to meet intermediate and final demands. Before stating their results, Häfele and Manne stress the ambiguities in the estimates of resource availabilities. The exact dates of change from one resource to another are highly suspect, but the pattern of change seems plausible.

Model results show that the cost of meeting energy demands increases with faster demand growth rates and with shorter years-of-reserves. By comparing the 40, 60 and 80 years-of-reserves scenarios, a value is attached to having additional reserves. The model is also solved when one type of reactor is excluded.

Comparing the costs for this scenario with the original results shows the benefits of implementing this technology.

The results of the model also show shadow prices in each period for electricity and non-electricity demand constraints. These shadow prices are dependent on the growth rate of energy demand. If the demand growth rate is high in the near term and then declines, the shadow prices increase rapidly through time. If the demand growth rate starts small and increases exponentially through time, the shadow prices increase gradually. When demand is endogenous, the shadow prices change even less.

4.3.4 ETA: A Model for Energy Technology Assessment

Another energy model by Manne (1976) follows a similar framework but incorporates both own- and cross-price elasticities of demand which allow for price-induced interfuel substitution and fuel conservation. The objective of his non-linear programming model can be viewed as maximizing consumers' plus producers' surplus or minimizing the sum of costs of conservation plus interfuel substitution plus the costs of energy supply. This objective is subject to constraints on extraction of exhaustible resources and expanding capacities for electricity generating plants and reactors and the demand for electric and non-electric energy. Manne's contribution lies in the specification of his objective function, which captures the following behaviour: the higher the price of a fuel, the less that is consumed and the greater the demand for a competing fuel. That is, interfuel substitution occurs which is not incorporated in the Häfele and Manne model. Moreover, the higher the price of energy in general, the less electric and non-electric energy that are consumed. Manne's results show patterns similar to the Nordhaus model and his own previous work.

The model incorporates increasing costs for the supply of uranium. There is a step function representing the various quantities of ores at different costs similar to Manne (1974). Given the equilibrium price path and that uranium is exploited along the step function from the low cost to the high cost deposits, the user costs associated with uranium production are declining through time. This result of declining user costs is derived analytically by Heal (1976).

4.3.5 A Dynamic Optimization Model of Depletable Resources

Modiano and Shapiro (1980) construct a dynamic optimization model of the U.S. coal industry. Their model minimizes the present value of total costs of meeting exogenous demands for the energy sector and is solved using mathematical programming decomposition methods.

Model constraints embody the conversion of the depletable resource to end-use products, the conversion of other resources to end-use products, the demands for the end-use products, and limit the depletable resource use to no more than its ultimate recovery. The objective function minimizes the costs of conversion, the costs of extraction (which increase with cumulative production), and the costs of the other primary commodities. The model time frame is 1979 to 2000. But the future beyond 2000 must be represented somehow since resource depletion cannot continue forever. Thus, a salvage value for the unused exhaustible resource (coal) is included and subtracted from these costs.

The model is subdivided into two parts. First, there is a model for each period which minimizes the costs of meeting the exogenous demands. These temporal models are linked through the supply model which maximizes the cost savings of using the exhaustible resource as opposed to a higher cost alternative less the costs of extraction plus the salvage value.

Modiano and Shapiro then focus on the dual program of the temporal and supply models. A demand curve for the depletable resource is derived by examining how the dual model responds to various prices. The optimality condition for the temporal and supply models is for the marginal benefits to equal the marginal costs. Marginal costs consist of marginal extraction costs plus three user costs. These user costs are the discounted effect of current extraction increasing future extraction costs, the stock user cost, and the loss in salvage value due to current extraction.

Modiano and Shapiro show that the growth rate of coal consumption is very low due to increasing reliance on nuclear power as an energy source. The price of coal rises until 1991 and remains constant thereafter. The user costs increase from 1979 to 1982 and then shrink to zero by 1991. By that year cumulative extraction reaches the point where costs remain constant. The results also show that coal usage is much more sensitive to changes in nuclear power supply than to changes in oil and gas supply.

4.3.6 Dynamic Equilibrium Energy Modelling: The Canadian BALANCE Model

Daniel and Goldberg (1981) model the energy sector in Canada in a framework similar to the Nordhaus model but make prices endogenous. The model consists of three parts: a linear programming supply model, an econometric demand model, and a mechanism to equilibrate prices between the two models. Their work is an extension of the PIES framework (see Hogan (1975)) to many time periods. This model goes a step beyond the Nordhaus model which obtains an efficient set of prices from exogenously specified demands. Hence Nordhaus's demands are not based on the "efficient prices." The objective of the equilibrium mechanism is to derive a set of prices which are inputs for the demand model and are simultaneously consistent with the shadow prices from the supply model. Given these prices, demands are generated in the demand model. Then, these demands are used as inputs for the supply model which generates shadow prices of supply. These shadow prices are subsequently used in the equilibrium mechanism to derive the prices for the demand model.

There are 20 supply sources and 3 primary demands for energy - crude oil, natural gas, and coal. Coal demand is assumed not to be price sensitive, so cross-price and own-price elasticities are included only for oil and gas. The model is expanded by allowing gas to be exported to the U.S. The time frame of the model is 1980 to 2000. Terminal conditions for the year 2000 are important because Canadian reserves are projected to last until just after 2000. Thus, a model which ignores terminal conditions would show a problem-free situation, low user costs or rents on scarce resources, and little investment in long term energy technology. The terminal conditions for this model aggregate all future periods beyond the time frame into one final period.

The results of the model show that as the demand prices and the shadow prices of supply converge to equilibrium, oil and gas demand prices increase and demand declines between the first and second iteration, and the same occurs for oil prices and demand between the second and third iteration, but gas prices fall and gas demand increases. These results exhibit both own-price and cross-price elasticity effects. In moving to the third iteration where gas prices fall, the demand for gas increases because gas becomes more attractive relative to oil.

Given the equilibrium shadow prices and the supply costs which increase with cumulative production, the user costs must decline through time, a result similar to the ETA model of Manne (1976). Since ETA is a nonlinear programming model and BALANCE is an equilibrium model, it is unlikely that an optimizing model can be formulated for BALANCE because of non-symmetric cross-price effects.

4.3.7 Allocation of Canadian Natural Gas to Domestic and Export Markets

Rowse (1986a) models the allocation of Canadian natural gas to domestic and export markets. Gas is supplied from four producing areas to seven domestic and five export markets. The nonlinear program consists of twenty-five 3-year time periods. The objective function maximizes the sum of discounted consumer surplus plus revenues from domestic and export gas consumption less discounted costs of gas production and transportation. There is a salvage value determined by the backstop cost for all conventional gas left in the ground at the horizon (terminal conditions). Rising unit supply costs are also captured in the objective function. The constraints for the model are: mass balance constraints equating supply and demand, constraints on production from current supply, constraints on production from new supplies, constraints limiting supplies of gas from new sources to the size of estimated reserves, and limits to the size of exports.

The results of the model show that efficient gas prices do not follow the standard Hotelling rule because of the recovery profile, allowance for shut in production and technology transition constraints. Due to the size of Canadian gas supplies relative to domestic and export demands, the discount rate and the natural gas recovery profile, near term prices are low. The model also shows many near-optimal depletion paths so that the cost of wrong assumptions about future demands is small.

Rowse (1986b) also examines the user costs and efficient prices for a British Columbia gas model. Prices rise monotonically over time to the backstop technology. User costs of existing supplies of natural gas rise in a similar manner; however, user costs of new supplies at the margin first rise, reach a maximum, and then decline to zero as the backstop technology is implemented.

4.4 Conclusion

Computational models include details of behaviour or complexities (self-sufficiency or limited imports) that are only mentioned or glossed over by analytical models. As already mentioned, computational models are able to capture the physical behaviour of resource depletion through constraints not easily represented in analytical models. The highly aggregated models surveyed in the latter half of this chapter also show that computational models can provide information about user costs and can incorporate extensions such as export markets, interfuel substitution, and comparisons of optimal solutions under different circumstances (different price paths, different reserve estimates, and different export prices). They are thus able to address resource allocation problems that analytical models cannot.

The next chapter presents a computational model which maximizes profits of producing petroleum from a crude oil reservoir. This model includes the physical characteristics of the reservoir production in the cost and production data.

Notes

[1] Elmer L. Dougherty et al., "Optimizing SANTO'S Gas Production and Processing Operations in Central Australia Using the Decomposition Method," Interfaces 17, no. 1 (1987): 75.

Chapter 5

The Reservoir Production Model

5.1 Introduction

Reservoir development and production are modelled in this chapter by incorporating some of the characteristics of analytical and computational models into a linear programming model maximizing the profits of producing oil from a single reservoir. Production is allowed to occur according to three different production profiles through primary, water flooding and enhanced oil recovery (EOR). The physical characteristics unique to this reservoir are captured in the cost and production data, and economic factors are captured by the inclusion of a royalty, tax and pricing regime. The physical and economic characteristics are discussed next, and then presented in equation form.

5.2 Production Profiles

Production profiles show the allowable pattern of production. Each production profile is represented by a Leontief production function capturing a fixed-proportions technology. Several inputs are required to produce one or more outputs, and the increase of each output requires all inputs to expand proportionately. The intertemporal nature of oil production is embodied in the Leontief production technology. The inputs to the Leontief production technology are the initial capital costs and the operating costs which depend on the number of operating wells and the production levels which decline through time. The outputs are the production levels and revenues. Each Leontief process specifies: the number of producing wells in each period, the time duration of reservoir production in number of periods, the amount of production in each period, the size of the capital outlay, and the operating costs associated with each period.[1] Thus, for each process, the number of wells drilled, production in each period, and costs in each period are exogenously specified.

Leontief processes have several advantages. Costs are matched precisely to the corresponding profile of production. Each profile incorporates variable and fixed costs which are production specific, well specific, and facilities specific as shown in the cost data section. The Leontief production functions also allow for limited substitution. If half of the cumulative reserves are produced using the 8-well profile and half are produced using the 10-well profile, then this combination can be viewed as the 9-well profile result.

For computational ease, only three production profiles are available. Each profile is feasible for production by MER restrictions and constitutes a plausible alternative to the producer. The reservoir modelled is assumed to have original oil in place (OOIP) of 1,250,000 m^3.[2] Recovery from primary production is approximately 20% or 242,725 m^3 of oil. The production technologies 1 (long profile), 2 (medium profile), and 3 (short profile), require drilling 8, 10, and 12 wells, respectively. The placement of the wells and connecting pipeline facilities is shown in Figures 5-1, 5-2, and 5-3.

Figure 5-1

Well and Pipeline Placement for Profile 1 (Long)

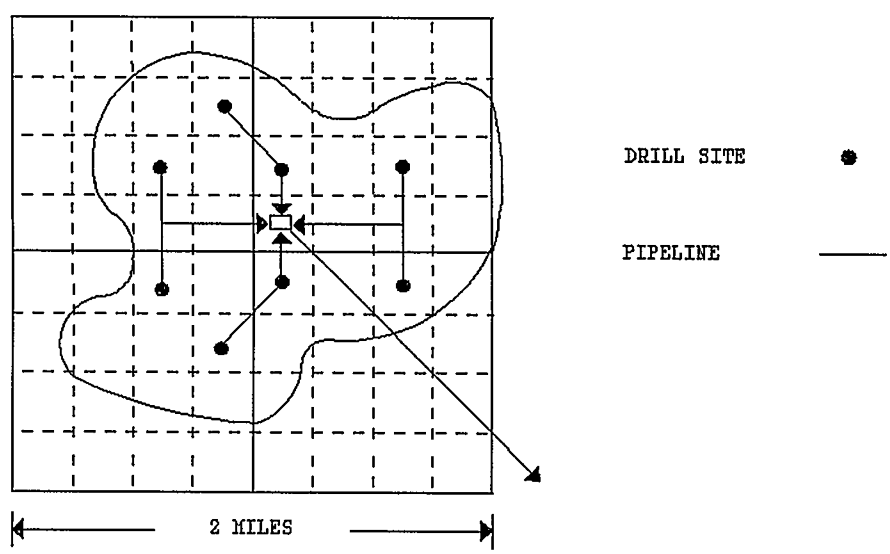


Figure 5-2

Well and Pipeline Placement for Profile 2 (Intermediate)

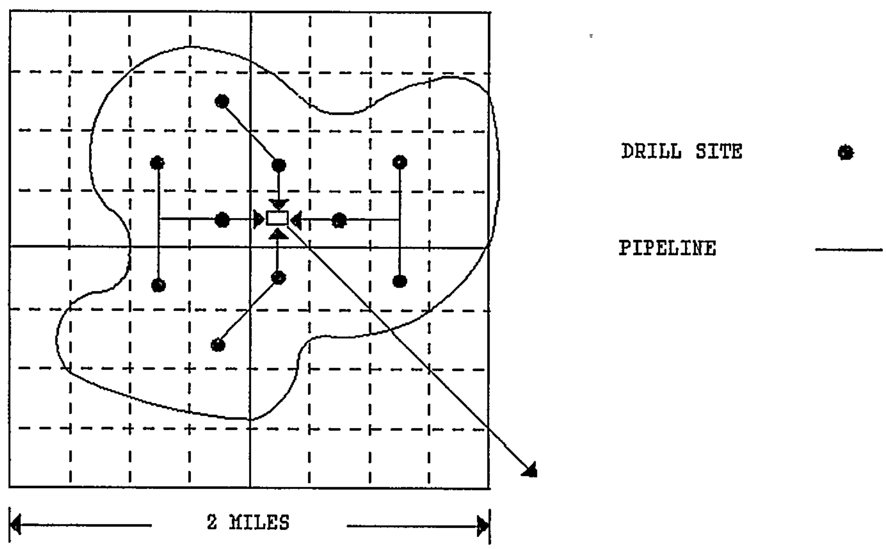
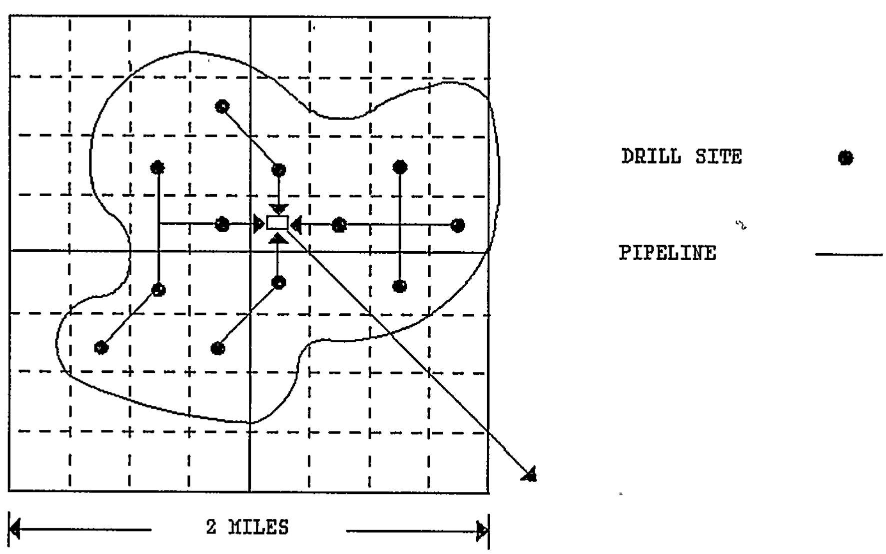


Figure 5-3

Well and Pipeline Placement for Profile 3 (Short)



The daily production rates and well count for each year are given in Table 5-1. These production rates are reservoir specific since they are determined by the physical nature of the reservoir; see Chapter 2. The profile drilling 12 wells incurs higher costs, higher initial production rates, a higher decline rate, and therefore depletes the reservoir faster. The annual production decline rates are approximately 10, 14, and 18% for profiles 1, 2, and 3, respectively. The years specified in Table 5-1 are for production profiles beginning in period one. Each period is three years long. Since the model allows profiles to be initiated in each of the first ten periods, this greatly expands the choices and time frame of the model. Furthermore, due to the inclusion of future production the model must include not just one or ten periods but twenty-four periods (72 years). This is explained in a later section.

A rule of thumb for production termination is to stop producing when the level of production reaches $1m^3$/well/day. This is similar to the shutdown condition of analytical models where MR = MC. If the price is \$113/m^3 and given the costs as defined in the Cost Data section, an additional year of production will incur operating costs slightly higher than production revenues. Due to these losses, no production occurs in subsequent years.[3]

Waterflooding is a secondary recovery technique that is assumed to be possible for this reservoir. The decision to produce using waterflooding depends

Table 5-1

Primary Production

Year	Profile 1 Output (m^3/day)	Profile 1 Well Count	Profile 2 Output (m^3/day)	Profile 2 Well Count	Profile 3 Output (m^3/day)	Profile 3 Well Count
1988	0	0	0	0	0	0
1989	6	1	12	2	18	3
1990	20	3	40	6	60	9
1991	48	8	64	10	80	12
1992	64	8	80	10	96	12
1993	60	8	75	10	90	12
1994	54	8	65	10	74	12
1995	49	8	56	10	61	12
1996	44	8	48	10	50	12
1997	39	8	41	10	41	12
1998	35	8	35	10	34	12
1999	32	8	30	10	28	12
2000	29	8	26	10	23	12
2001	26	8	22	10	10*	12
2002	23	8	19	10		
2003	21	8	16	10		
2004	19	8	14	10		
2005	17	8	12	10		
2006	15	8	10	10		
2007	14	8				
2008	12	8				
2009	11	8				
2010	10	8				
2011	9	8				
2012	8	8				

* only a half year of production

mainly on the physical characteristics of the reservoir. Water is only injected if it does not destroy the pore structure of the reservoir. There must also be significant lateral continuity for the injected water to push the oil to the producing wells. Thus the reservoir's physical properties must be analyzed to find if water injection is possible. Also a water supply must be accessible before water injection can occur.

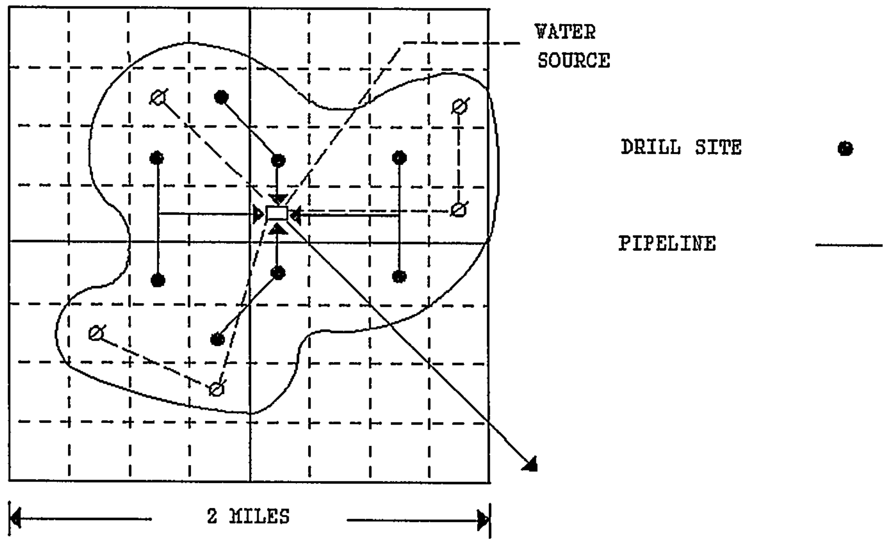
If waterflooding is feasible, it will be initiated quickly to take advantage of the improved production due to pressure upkeep. Production declines when water breakthrough occurs; water from the injection wells comes out in the production wells.

For each of the production profiles 5 more wells are assumed to be drilled for water injection. The pattern of well drilling for profile 1 with waterflooding is shown in Figure 5-4. Cumulative reserves are now increased by an additional 10% to 379,965 m^3. The daily production values and well count for each year are given in Table 5-2 for both primary and waterflooding techniques.

A similar shutdown condition applies to primary and waterflooding recovery as to primary recovery alone. Costs are higher under secondary recovery, so termination occurs at a slightly higher level than $1m^3$/well/day.

Figure 5-4

Well Placement for Water Injection for Profile 1



For computational ease each time "period" of the model consists of three years. Using the daily production rates from Table 5-2, production is found for each period and divided by the total production. The resulting decimal fractions comprise the physical production components of each Leontief process and are shown in Table 5-3. Since the first year incurs only capital costs and no production, it has been defined as a separate period.

Table 5-2

Primary and Waterflooding Production

Year	Profile 1 Output (m^3/day)	Profile 1 Well Count	Profile 2 Output (m^3/day)	Profile 2 Well Count	Profile 3 Output (m^3/day)	Profile 3 Well Count
1988	0	0	0	0	0	0
1989	6	1	12	2	18	3
1990	20	3	40	6	60	9
1991	48	8	64	10	80	12
1992	64	13	80	15	96	17
1993	64	13	80	15	96	17
1994	64	13	80	15	96	17
1995	64	13	80	15	96	17
1996	64	13	80	15	96	17
1997	64	13	75	15	83	17
1998	60	13	64	15	68	17
1999	55	13	58	15	56	17
2000	51	13	51	15	46	17
2001	47	13	45	15	37	17
2002	43	13	40	15	30	17
2003	40	13	36	15	25	17
2004	36	13	31	15	21	17
2005	33	13	27	15	19	17
2006	31	13	24	15	18	17
2007	28	13	22	15		
2008	26	13	19	15		
2009	24	13	17	15		
2010	22	13	16	15		
2011	20	13				
2012	19	13				
2013	17	13				
2014	16	13				
2015	15	13				

Table 5-3

Output per Period as a Fraction of Total Production

Periods	1	2	3	4	5	6	7	8	9	10
Profile 1 Production	0.000	0.071	0.184	0.184	0.160	0.125	0.096	0.075	0.059	0.046
Profile 2 Production	0.000	0.111	0.231	0.226	0.166	0.116	0.079	0.056	0.015*	
Profile 3 Production	0.000	0.152	0.277	0.264	0.163	0.084	0.056			

* This last coefficient is for only one year of production.

The sum of all of the components of each of these normalized production vectors is unity. If the reserve consists of one cubic meter of crude, then each coefficient shows the fraction of the cubic meter that is produced during that particular time period. Each vector is multiplied by a decision variable representing resource commitment.[4] Each resource commitment variable can range in size from zero to the capacity of the reservoir (379,965 m3). Thus the resource commitment variable indicates the number of cubic meters of the reservoir that are used with a particular production technology. Therefore, the products of the resource commitment variable and the vectors defining a production technology are vectors representing the physical production, capital and operating costs that occur over all periods. The vectors defining the production technologies for primary and waterflooding recovery are given in Table 5-6 at the end of the next section.

5.3 Cost Data

The capital and operating cost data for each primary recovery profile are given in Table 5-4. Once again these data pertain to a production decision occurring in period 1. The capital costs are for drilling and pipeline construction. Comparing the drilling costs with the number of wells completed (Table 5-1) shows that each well costs $600,000.00. The facilities costs are not proportional but show economies of scale. The total cost of facilities per well declines when comparing the three profiles as more wells are drilled. These facilities include the pipelines from the wells to a collection centre and then to a major pipeline, road construction, and a production plant to handle and separate the fluid volumes.

There are two types of operating costs, fixed and variable, which are based on the number of wells and production levels respectively. The monthly well cost covers labour, road maintenance, and well upkeep costs. The production costs include transportation and processing costs. Initially the oil produced is transported by truck at \$9.00/m^3. After the pipeline facilities have come on stream, oil can be transported through the pipelines at a lower cost of \$4.00/m^3. Included in this cost is oil processing, water separation and disposal, and solution gas separation. It is assumed that the solution gas produced is minimal and therefore can be consumed on site as fuel.

Table 5-4

Capital and Operating Costs for Primary Recovery

Capital Costs (000's of 1987 dollars)

Year	Profile 1 Drilling	Profile 2 Drilling	Profile 3 Drilling
1988	600	1200	1800
1989	1200	2400	3600
1990	3000	2400	1800
Total	4,800	6,000	7,200
	Facilities	Facilities	Facilities
1988	60	120	180
1989	770	900	1000
1990	600	700	750
Total	1,430	1,720	1,930

Operating Costs - common to all scenarios (1987 dollars)

Fixed Costs $3000/well/month

Variable Costs

1989	$9.00/m^3 of oil
1990	$9.00/m^3 of oil
1991	$4.00/m^3 of oil
1992	$4.00/m^3 of oil
1993	$4.00/m^3 of oil
1994	$4.00/m^3 of oil
1995	$4.00/m^3 of oil
1996-2012	for remaining production life there is a real growth rate of 5% on this $4.00/m^3 cost of oil

The 5% growth rate in the variable production cost is caused by the increase in water production. Higher costs are incurred because more water has to be separated from the oil and disposed of.

Waterflooding requires increased capital and operating costs. The capital and operating costs of both primary and waterflooding recovery are presented in Table 5-5. The drilling costs increase by the same amount since five additional wells are drilled for each profile. The facility costs also increase because a pipeline from the water source, water treating facilities, injection pumps, enlarged production facilities, and lines to injection wells are needed. Larger production facilities are required to handle and separate higher fluid volumes.

Operating costs climb from \$4.00/m^3 to \$10.00/m^3 due to the increase in water production and the treatment of the water injected.

It should be noted that these cost conditions are not readily incorporated into an analytical model but are easily handled by a computational model. The years specified in this table and the previous ones are for the profiles beginning in the first period, and profiles initiated in subsequent periods would have the same costs but a different time frame.

Table 5-5

Capital and Operating Costs for Primary and Waterflooding

Capital Costs (000's of 1987 dollars)

Year	Profile 1 Drilling	Profile 2 Drilling	Profile 3 Drilling
1988	600	1200	1800
1989	1200	2400	3600
1990	3000	2400	1800
1991	3000	3000	3000
Total	7,800	9,000	10,200
	Facilities	Facilities	Facilities
1988	60	120	180
1989	770	900	1000
1990	830	1200	1100
1991	700	775	830
Total	2,360	2,995	3,110

Operating Costs - common to all scenarios (1987 dollars)

Fixed Costs	\$3000/well/month
Variable Costs	
1989	\$9.00/m^3 of oil
1990	\$9.00/m^3 of oil
1991	\$4.00/m^3 of oil
1992	\$10.00/m^3 of oil
1993	\$10.00/m^3 of oil
1994	\$10.00/m^3 of oil
1995	\$10.00/m^3 of oil
1996-2015	for remaining production life there is a real growth rate of 5% on this \$10.00/m^3 cost of oil

The normalized cost coefficients of the production profiles are presented in Table 5-6 along with the production coefficients. The unit capital cost coefficients are found by dividing the capital cost for a period by total production. The operating cost coefficients are found by dividing the operating cost for a period by the production for that period and then multiplying by the production coefficients for that period. These capital and operating cost coefficients, when multiplied by the resource commitment variable, yield the capital and operating costs in each period for the production profile assigned to the particular resource commitment variable. The production rates are determined by multiplying the resource commitment variable by the production profile coefficients.

5.4 Enhanced Oil Recovery

Enhanced oil recovery (EOR) is very uncertain at the beginning of reservoir depletion and only appraised for large sized pools. Since the original oil in place shows that a small to medium sized pool is developed, EOR would not normally be considered. At existing prices, EOR is only economically feasible for large pools. However, since EOR production may be highly sensitive to price and tax changes, it is an option included in the model and becomes economically feasible with a substantial increase in oil prices.

Table 5-6

Coefficients for the Leontief Processes for Primary and Waterflooding Recovery

Periods	1	2	3	4	5	6	7	8	9	10
Profile 1										
Production	0.000	0.071	0.184	0.184	0.160	0.125	0.096	0.075	0.059	0.046
Operating Costs	0.000	1.546	5.540	5.633	5.630	5.449	5.257	5.106	4.972	4.859
Capital Costs	1.737	25.002								
Profile 2										
Production	0.000	0.111	0.231	0.226	0.166	0.116	0.079	0.056	0.015*	
Operating Costs	0.000	2.401	6.569	6.633	6.278	5.895	5.542	5.311	1.741*	
Capital Costs	3.474	27.568								
Profile 3										
Production	0.000	0.152	0.277	0.264	0.163	0.084	0.056			
Operating Costs	0.000	3.256	7.599	7.602	6.806	6.069	5.738			
Capital Costs	5.211	29.819								

* This last coefficient is for only one year of production.

EOR requires that the physical properties such as porosity, permeability, and continuity between wells not deteriorate with the implementation of this technique. EOR occurs only after waterflooding. Unlike waterflooding which is an immiscible flooding technique, EOR is a miscible flooding technique. Miscible flooding is the injecting of a solvent into the reservoir to bond with the oil. This bonding also occurs with oil that would have been left behind by waterflooding. Injecting this solvent, likely consisting of natural gas liquids, lowers the viscosity of the oil and improves the recovery from the reservoir.

The decision to start EOR occurs after considerable production takes place. A rule of thumb to start EOR studies is when water production reaches 66% of total fluid production. These studies determine if EOR is physically feasible. If EOR is found to be technically feasible, then sensitivity studies are undertaken at different percentages of recovery (3% to 10%) to find the necessary level for economic feasibility. Given expected prices and costs these studies determine what level of recovery is necessary for a company to achieve its profit target. A rule of thumb for the commencement of EOR is that the level of water production is 90% of fluid production.

If the incremental recovery factor is 10% for EOR, then the daily production rates for primary, waterflooding, and EOR together and for EOR alone are those shown in Table 5-7. The solvent, natural gas liquids, has a higher cost than the price of the oil produced. Of the solvent injected, 80% is recovered during the production using EOR. The solvent is injected during the first five years of EOR followed by renewed water injection.

The years presented in Table 5-7 correspond to the primary and waterflooding profiles starting in the first period. If one of the profiles chosen starts in a later period, then EOR would also be correspondingly delayed.

Figure 5-5 shows annual production rates using primary, waterflooding, and EOR for the longest profile (1) starting in the first period utilizing the data from Table 5-7. The EOR production, operating cost and capital cost coefficients are determined in the same way as the primary and waterflooding coefficients.

The additional costs incurred for EOR production are presented in Table 5-8 and the coefficients for each profile in Table 5-9. Again it should be noted that these EOR coefficients correspond to the specific profiles initiated in period 1. If a production profile is started in later period, then there will be a corresponding delay in the EOR production. The model is constructed so that the decision to begin an EOR Leontief process can only occur if the corresponding primary and waterflooding Leontief process has been implemented.

Table 5-7

Coefficients for Primary, Waterflooding, and EOR Production

Year	Profile 1 (m³/day) O[1]	Profile 1 (m³/day) IO[2]	Profile 2 (m³/day) O[1]	Profile 2 (m³/day) IO[2]	Profile 3 (m³/day) O[1]	Profile 3 (m³/day) IO[2]
2001	47		45		37	
2002	43		40		30	
2003	40		36		29*	4
2004	36		31		31	10
2005	33		27		33	14
2006	31		28*	4	33	15
2007	28		30	8	32	32
2008	26		34	15	31	31
2009	24		33	16	30	30
2010	27*	5	32	16	29	29
2011	29	9	31	31	28	28
2012	31	12	30	30	28	28
2013	34	17	29	29	27	27
2014	34	18	28	28	27	27
2015	32	17	27	27	26	26
2016	31	31	27	27	26	26
2017	31	31	26	26	25	25
2018	30	30	26	26	25	25
2019	29	29	25	25	25	25
2020	28	28	25	25	24	24
2021	27	27	24	24	24	24
2022	27	27	24	24	24	24
2023	26	26	24	24		
2024	25	25	23	23		
2025	25	25	23	23		
2026	24	24	23	23		
2027	24	24				
2028	23	23				
2029	24	24				
2030	24	24				

[1] Output
[2] Incremental Output
* The start of EOR.

Table 5-8

Costs During EOR

Capital Costs		
	Solvent	$60{,}000m^3$ x $\$150/m^3$ =\$9,000,000
	Facilities	\$3,000,000
Operating Cost		
	Fixed	\$3000/well/mos.[1]
	Variable	Continue with waterflooding costs of $\$10/m^3$ which grows at rate of 5% starting in the eighth year of production.

1 This is the same as during waterflooding so this cost is only incurred in the production years extending beyond waterflooding.

Figure 5-5

Annual Production Rates for Profile 1 (Long)

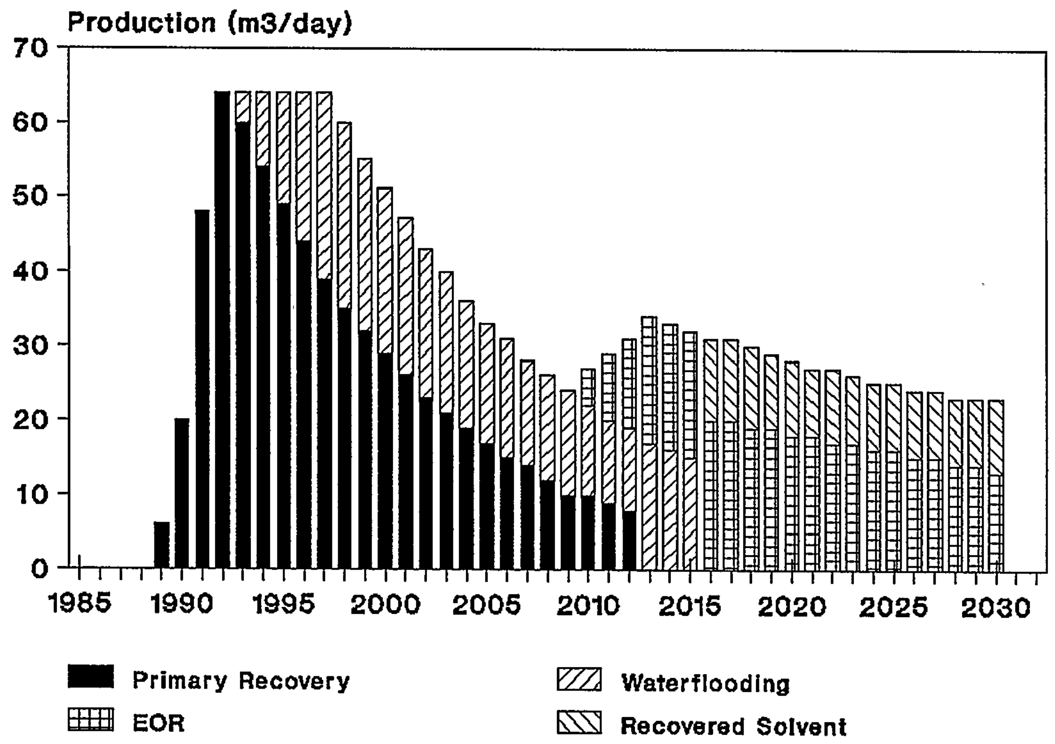


Table 5-9

Coefficients for the EOR Leontief Processes

Profile 1

Periods	9	10	11	12	13	14	15
Production	0.055	0.110	0.194	0.177	0.165	0.154	0.146
Operating Costs	1.214	2.774	13.794	14.114	14.563	15.105	15.769
Capital Cost	75.140						

Profile 2

Periods	7	8	9	10	11	12	13	14
Production	0.008*	0.082	0.163	0.177	0.167	0.156	0.150	0.097**
Operating Costs	0.144	1.568	9.822	13.840	14.240	14.651	15.236	10.542
Capital Costs	25.047	50.094						

Profile 3

Periods	6	7	8	9	10	11	12	13
Production	0.008*	0.082	0.196	0.179	0.169	0.160	0.154	0.051*
Operating Costs	0.125	1.350	14.311	14.527	14.878	15.303	15.828	5.428
Capital Costs	25.047	50.094						

* One Year of Production

** Two Years of Production

5.5 Royalties, Taxes and Prices

5.5.1 Royalties

Royalties and taxes can exert a major influence on the profitability of production. The Alberta royalty formula is shown in equation 5.1 and is adjusted from time to time. The latest parameter values are used.

$$R = S + yS\frac{(X - B)}{X} \tag{5.1}$$

R - royalty payable in m^3

S - for production of 0.1 to 190.7 m^3, $S = p^2/1271.28$; for production of 190.7 m^3 and over, $S = 28.6 + .25(p - 190.7)$, where p = produced m^3 for the month.

B - new oil select price for the month, \$40.90/m^3

X - new oil par price for the month, \$138.00/m^3

y - new oil royalty factor, 0.246154

This formula specifies the royalty that the Alberta Government collects per month per well. A vector of royalty coefficients can be determined from this formula for each profile using the production coefficients, total production, and the well count. The royalty coefficients are presented in Table 5-10 for primary recovery, waterflooding recovery, and incremental EOR production. The royalty for EOR is 5% of the incremental production. These royalty coefficients are determined under the assumption that total production occurs using each of the technologies separately.[5]

Table 5-10

Royalty Coefficients

Primary and Waterflooding

Periods	1	2	3	4	5	6	7	8	9	10
Profile 1	.0000	.0057	.0255	.0255	.0191	.0117	.0069	.0042	.0026	.0016
Profile 2	.0000	.0105	.0345	.0331	.0181	.0089	.0041	.0020	.0005*	
Profile 3	.0000	.0164	.0439	.0402	.0157	.0046	.0018			

EOR

Periods	9	10	11	12	13	14	15
Profile 1	.0027	.0055	.0097	.0089	.0082	.0077	.0073

Periods	7	8	9	10	11	12	13	14
Profile 2	.0004*	.0041	.0081	.0089	.0083	.0078	.0075	.0049**

Periods	6	7	8	9	10	11	12	13
Profile 3	.0004*	.0041	.0098	.0090	.0085	.0080	.0077	.0025*

* one year of production
** two years of production

5.5.2 Income Taxes

Federal corporate income tax is calculated at 33% of taxable income, and Alberta provincial corporate income tax is calculated at 15%. To calculate taxable income requires that certain allowances, credits, and deductions must be defined. The relevant items are: capital consumption allowance (CCA), resource allowance, Canadian Development Expense (CDE), Canadian Exploration and Development Incentive Program (CEDIP), Alberta Royalty Tax Credit (ARTC) - this credit will be terminated in 1991, and an Alberta royalty deduction. Since some of the tax items are very similar (CCA and CDE) or have no influence on this reservoir's evaluation (Alberta royalty deduction), the tax structure has been simplified. The tax system modelled includes the CCA, the resource allowance, and the ARTC. As well the tax rates have been "grouped together."

All of the capital costs are assumed to qualify for the capital consumption allowance (CCA). The oil and gas equipment is designated as Class 10 which has a 30% write-down on a declining balance basis. In the initial year of the capital equipment only 1/2 of the 30% can be claimed under CCA.

Another allowance available to the producer is the resource allowance. The resource allowance is 25% of resource profits where resource profits are defined as revenues less operating costs and CCA.

The Alberta Government allows for an Alberta Royalty Tax Credit (ARTC). The ARTC is 50% of crown royalties with a ceiling of $2 million which will be paid to producers.

Taxable income is defined as resource profits less the resource allowance plus the ARTC. Net revenues are production revenues plus the ARTC less operating costs, capital costs, royalties, and income taxes. The income tax rate used is 46% which is based on: a 33% federal income tax, a 3% federal surtax, and a 15% provincial tax on the remaining taxable income.

5.5.3 Prices

Two price scenarios are examined. The first scenario assumes that real prices are constant at $125.00/m^3$ ($20.00/bbl) over the life of the reservoir. The second scenario assumes that real prices start in 1987 at $125.00/m^3$ and grow at a rate of 2% per year.

5.6 The Reservoir Development and Production Model

5.6.1 Production From Primary and Waterflooding Recovery

The reservoir production model selects among three decision variables which correspond to each of the three production profiles in each of ten time periods. These resource commitment variables select the portion of the total reservoir capacity that will be developed through a particular Leontief process. The production decision variables are:

RCG_i - resource commitment variable for the longest profile initiated in period i, i = 1 to 10

RCM_i - resource commitment variable for the medium profile initiated in period i, i = 1 to 10

RCS_i - resource commitment variable for the shortest profile initiated in period i, i = 1 to 10

TP_t - total production in period t, t = 1 to 19

The production coefficients from the Leontief processes (Table 5-6) are represented by:

pdg_g - coefficients for the long profile, g = 1 to 10

pdm_m - coefficients for the medium profile, m = 1 to 9

pds_s - coefficients for the short profile, s = 1 to 7

The objective function consists of net revenues to the producer. To simplify the understanding of the model and allow easy modification, the objective function is defined as the sum and/or difference of several elements, each defined in separate equations. The first set of equations discounts future net revenues to the present. The next set of equations defines net revenues as production revenues less capital and operating costs. The remaining equations define production revenues, capital costs and operating costs which are based on production and the resource commitment variables. By decomposing the objective function into these components, changes in prices, costs or the discount rate can easily be made.[6] The single constraint of the model is the stock constraint.

The remaining decision variables and coefficients of the model are:

R_t - the production revenue in period t

OC_t - the operating costs in period t

KC_t - the capital costs in period t

NR_t - the net revenue in period t (total revenue less operating and capital costs)

VP_t - the value in period t of the sum of net revenue in period t and the present value of VP_{t+1}

coefficients:

β^3 - the time-period discount factor; $\beta = (1/1+i)$, where i is the discount rate and the power of three causes the discounting to be for the length of the period (three years)

kcg_g, kcm_m, kcs_s - the capital cost coefficients for the corresponding Leontief production vector

ocg_g, ocm_m, ocs_s - the operating cost coefficients for the corresponding Leontief production vector

p_t - the price in period t

RC - reservoir capacity

The objective function is defined as follows:

$$\text{Max obj} = \beta^3 VP_1$$

5.1 $$VP_t = NR_t + \beta^3 VP_{t+1} \qquad \text{for } t = 1 \text{ to } 18$$

$$VP_{19} = NR_{19}.$$

The equations for t > 1 discount all net revenues to the present. To show that this format discounts properly, it is briefly described starting in period 18. The value for period 18 (VP_{18}) is the net revenue in period 18 plus the net revenue in period 19 discounted to period 18. Likewise, the value in period 17 (VP_{17}) is the net revenue from period 17 plus the discounted value from period 18 (VP_{18}). In the row or equation for period 17 the net revenue in period 18 is discounted for one period and net revenue in period 19 is discounted for two periods. Thus, each row has all future period values discounted to the time period of that particular row.

Equation 5.2 defines net revenue in each period as the difference between production revenues and total costs.

$$5.2 \qquad NR_t = R_t - OC_t - KC_t \qquad \text{for } t = 1 \text{ to } 19$$

Equations 5.3 define total revenue as the total production in period t multiplied by the price.

$$5.3 \qquad R_t = p_t TP_t \qquad \text{for } t = 1 \text{ to } 19$$

Production in each period is defined in equations 5.4. There are thirty resource commitment variables corresponding to the three types of profiles which can be initiated in any of the first ten periods. Each resource commitment variable has a set of production coefficients representing a Leontief technology. Production in each period consists of crude production during that period from current and past resource commitments. The subscript values for the production coefficients are presented in more detail below equation 5.4 in order to show that production is initiated and terminated in the proper periods depending on the type of profile and the starting period of the profile.

5.4 $$TP_t = \sum_{i=1}^{10} (pdg_{t-i+1}RCG_i + pdm_{t-i+1}RCM_i + pds_{t-i+1}RCS_i)$$

for $t = 1$ to 19

pdg_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 10$

pdm_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 9$

pds_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 7$

The equation system is expanded below for the medium length technology to show how the production coefficients relate to each resource commitment variable.

$$\begin{aligned}
TP_1 &= pdm_1RCM_1 + 0 * RCM_2 + 0 * RCM_3 + \qquad + 0 * RCM_{10} \\
TP_2 &= pdm_2RCM_1 + pdm_1RCM_2 + 0 * RCM_3 + \ldots \quad + 0 * RCM_{10} \\
TP_3 &= pdm_3RCM_1 + pdm_2RCM_2 + pdm_1RCM_3 + \qquad + 0 * RCM_{10} \\
&\vdots \\
TP_9 &= pdm_9RCM_1 + pdm_8RCM_2 + \ldots + pdm_1RCM_9 + 0 * RCM_{10} \\
TP_{10} &= 0 * RCM_1 + pdm_9RCM_2 + \ldots + pdm_2RCM_9 + pdm_1RCM_{10} \\
&\vdots \\
TP_{17} &= 0 * RCM_1 + 0 * RCM_2 + \qquad + pdm_9RCM_9 + pdm_8RCM_{10} \\
TP_{18} &= 0 * RCM_1 + 0 * RCM_2 + \ldots + 0 * RCM_9 + pdm_9RCM_{10} \\
TP_{19} &= 0 * RCM_1 + 0 * RCM_2 + \qquad + 0 * RCM_9 + 0 * RCM_{10}
\end{aligned}$$

Following the same form as equation 5.5, the operating and capital costs are defined in equations 5.5 and 5.6, respectively.

5.5 $$OC_t = \sum_{i=1}^{10} (ocg_{t-i+1}RCG_i + ocm_{t-i+1}RCM_i + ocs_{t-i+1}RCS_i)$$

for $t = 1$ to 19

ocg_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 10$

ocm_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 9$

ocs_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 7$

Capital costs for a particular profile are not incurred in every period of the profile. In fact, there are only two non zero capital cost coefficients, as indicated in the notes to equation 5.6.

5.6 $$KC_t = \sum_{i=1}^{10} (kcg_{t-i+1}RCG_i + kcm_{t-i+1}RCM_i + kcs_{t-i+1}RCS_i)$$

for $t = 1$ to 19

kcg_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 2$

kcm_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 2$

kcs_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 2$

In constraint set 5.7, the sum of the production from all of the technologies is limited to the size of the reservoir capacity (RC). These are mutual exclusivity-type constraints.

5.7 $$\sum_{i=1}^{10} (RCG_i + RCM_i + RCS_i) \leq RC$$

Non-negativity restrictions apply to all variables except for net revenues.

$$RCG_i,\ RCM_i,\ RCS_i,\ VY_t,\ R_t,\ C_t, \text{ and } TP_t \geq 0 \quad \text{for } i = 1 \text{ to } 10, \text{ and } t = 1 \text{ to } 19 \tag{5.8}$$

5.6.2 Production From EOR

Representation of EOR production is similar to the above. There are equalities or constraints for production (5.9), operating cost (5.10), capital cost (5.11), and reservoir EOR capacity (5.12). The decision variables for EOR determine the portion of the total incremental production that is produced using a particular technology. The EOR Leontief processes are viewed as extensions to the existing technologies from primary and waterflooding recovery. Thus, there must also be constraints linking EOR production to the relevant primary and waterflooding profiles. This linkage is provided in constraints 5.13. The decision variables are:

ECG_i - EOR commitment variable for the corresponding RCG_i variable; EOR commences in period i + 8

ECM_i - EOR commitment variable for the corresponding RCM_i variable; EOR commences in period i + 6

ECS_i - EOR commitment variable for the corresponding RCS_i variable; EOR commences in period i + 5

The coefficients are:

$ekcg_g$, $ekcm_m$, $ekcs_s$ - capital cost coefficients for EOR

$eocg_g$, $eocm_m$, $eocs_s$ - operating cost coefficients for EOR

$epdg_g$, $epdm_m$, $epds_s$ - production coefficients for EOR

EORC - EOR incremental-cumulative capacity

New elements or equations to be added to existing constraints are:

5.9 $$TP_t = \sum_{i=1}^{10} (epdg_{t-i-7}ECG_i + epdm_{t-i-5}ECM_i + epds_{t-i-4}ECS_i)$$

for $t = 1$ to 24 ;

$epdg_{t-i-7}$ is defined to equal 0 when $t-i-7 < 1$ or $t-i-7 > 7$

$epdm_{t-i-5}$ is defined to equal 0 when $t-i-5 < 1$ or $t-i-5 > 8$

$epds_{t-i-4}$ is defined to equal 0 when $t-i-4 < 1$ or $t-i-4 > 8$

The coefficients and variables for the medium length profile are presented on the next page to show the production equations in expanded form with the addition of EOR.

$$
\begin{array}{llllllll}
TP_7 & = epdm_1ECM_1 + & 0 * ECM_2 + & 0 * ECM_3 & & & & \\
TP_8 & = epdm_2ECM_1 + & epdm_1ECM_2 + & 0 * ECM_3 + \ldots & & & & \\
TP_9 & = epdm_3ECM_1 + & epdm_2ECM_2 + & epdm_1ECM_3 & & & & \\
 & \quad \vdots & & & \vdots & & & \\
TP_{14} & = epdm_8ECM_1 + & epdm_7ECM_2 + & \ldots & & & & \\
TP_{15} & = \;\; 0 * ECM_1 + & epdm_8ECM_2 + & \ldots & + \; epdm_1ECM_9 + & 0 * ECM_{10} & & \\
TP_{16} & = \;\; 0 * ECM_1 + & 0 * ECM_2 + & \ldots & + \; epdm_2ECM_9 + & epdm_1ECM_{10} & & \\
 & \quad \vdots & & \vdots & & \vdots & & \\
TP_{22} & = & & & + \; epdm_8ECM_9 + & epdm_7ECM_{10} & & \\
TP_{23} & = & & & + \;\; 0 * ECM_9 + & epdm_8ECM_{10} & & \\
TP_{24} & = \;\; 0 * ECM_1 + & 0 * ECM_2 + & \ldots & + \;\; 0 * ECM_9 + & 0 * ECM_{10} & &
\end{array}
$$

The other additions to the equations from EOR are:

5.10 $$OC_t = \sum_{i=1}^{10} (eocg_{t-i-7}ECG_i + eocm_{t-i-5}ECM_i + eocs_{t-i-4}ECS_i)$$

for $t = 1$ to 24

$eocg_{t-i-7}$ is defined to equal 0 when $t\text{-}i\text{-}7 < 1$ or $t\text{-}i\text{-}7 > 7$

$eocm_{t-i-5}$ is defined to equal 0 when $t\text{-}i\text{-}5 < 1$ or $t\text{-}i\text{-}5 > 8$

$eocs_{t-i-4}$ is defined to equal 0 when $t\text{-}i\text{-}4 < 1$ or $t\text{-}i\text{-}4 > 8$

5.11 $$KC_t = \sum_{i=1}^{10} (ekcg_{t-i-7}ECG_i + ekcm_{t-i-5}ECM_i + ekcs_{t-i-4}ECS_i)$$

for $t = 1$ to 24

$ekcg_{t-i-7}$ is defined to equal 0 when $t\text{-}i\text{-}7 < 1$ or $t\text{-}i\text{-}7 > 1$

$ekcm_{t-i-5}$ is defined to equal 0 when $t\text{-}i\text{-}5 < 1$ or $t\text{-}i\text{-}5 > 2$

$ekcs_{t-i-4}$ is defined to equal 0 when t-i-4 < 1 or t-i-4 > 2

5.12 $\sum_{i=1}^{10} (ECG_i + ECM_i + ECS_i) \leq EORC$

Equation 5.12 limits the amount of production from the EOR profiles to the capacity for EOR production. Constraints 5.13 allow EOR to be initiated only when the corresponding technology for primary and waterflooding has been selected.

$ECG_i \leq EORC/RC \times RCG_i$

5.13 $ECM_i \leq EORC/RC \times RCM_i$ for i = 1 to 10

$ECS_i \leq EORC/RC \times RCS_i$

For example, if the RCG_1 variable is made positive, it is then possible for the ECG_1 variable to be made positive, and EOR may occur starting in period 8. With the addition of EOR, production can occur even further into the future. Thus, more equations must be added to equations 5.1, 5.2, and 5.3 so that the model can handle twenty-four time periods (t = 1 to 24) or 72 years.

5.6.3 Royalties and Taxes

The model allows for taxes and royalties through equalities defining taxes and royalties which are then deducted from profits. The royalty equations have the same form as the production equations and are presented in 5.14. The additional variables and coefficients are:

ROY_t - the crown royalty in year t, 10^3 m^3

RY_t - the crown royalty in year t, 10^3 dollars

rg_g, rm_m, rs_s - the royalty coefficients for primary and waterflooding production

reg_g, rem_m, res_s - the royalty coefficients for EOR production

5.14 $$ROY_t = \sum_{i=1}^{10} (rg_{t-i+1}RCG_i + rm_{t-i+1}RCM_i + rs_{t-i+1}RCS_i$$
$$+ reg_{t-i-7}ECG_i + rem_{t-i-5}ECM_i + res_{t-i-4}ECS_i)$$

for $t = 1$ to 24

rg_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 10$

rm_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 9$

rs_{t-i+1} is defined to equal 0 when $t-i+1 < 1$ or $t-i+1 > 7$

reg_{t-i-7} is defined to equal 0 when $t-i-7 < 1$ or $t-i-7 > 7$

rem_{t-i-5} is defined to equal 0 when $t-i-5 < 1$ or $t-i-5 > 8$

res_{t-i-4} is defined to equal 0 when $t-i-4 < 1$ or $t-i-4 > 8$

5.15 $RY_t = p_t ROY_t$ for $t = 1$ to 24

To determine the federal and provincial income taxes, equations are included to define: federal and provincial taxable income; federal income tax, provincial income tax and federal surtax; CCA balance, CCA, resource allowance, resource profits; and the ARTC. The variables for these constraints are as follows:

$CCAB_t$ - the CCA balance in period t

CCA_t - the CCA in period t

RP_t - the resource profits in period t

RA_t - the resource allowance in period t

TI_t - positive federal taxable income in period t

TAX_t - the federal income tax, federal surtax, and provincial income tax

$ARTC_t$ - the ARTC in period t

The equations are:

$$CCAB_1 = KC_1$$

5.16 $$CCAB_t = KC_t + CCAB_{t-1} - CCA_{t-1} \quad \text{for } t = 2 \text{ to } 24$$

$$CCA_t \leq 0.3CCAB_t \quad \text{for } t = 1 \text{ to } 24$$

The above constraints create a capital consumption allowance balance. Capital costs are stored in this balance and can be deducted from resource profits at a maximum 30% rate. This deduction is defined as a capital consumption allowance which lowers taxable income by decreasing the size of resource profits. The inequality allows the use of the CCA to be deferred to future periods.

5.17
$$RP_t = R_t - OC_t - CCA_t \qquad \text{for } t = 1 \text{ to } 24$$
$$RA_t = 0.25RP_t \qquad \text{for } t = 1 \text{ to } 24$$

5.18
$$TI_t = RP_t - RA_t + ARTC_t \qquad \text{for } t = 1 \text{ to } 24$$
$$TAX_t = 0.46TI_t \qquad \text{for } t = 1 \text{ to } 24$$

5.19
$$ARTC_t = 0.5RY_t \qquad \text{for } t = 1 \text{ to } 24$$

Inclusion of these taxes and royalties changes the net revenue equations. Equations 5.2 are thus modified to 5.20:

5.20
$$NR_t = R_t - OC_t - KC_t - RY_t - TAX_t + ARTC_t \qquad \text{for } t=1 \text{ to } 24$$

The additional variables are also constrained to be nonnegative:

$ECG_i, ECM_i, ECS_i \geq 0$ for $i = 1$ to 10 and

5.21 $ROY_t, RY_t, CCAB_t, CCA_t, RP_t, RA_t, TI_t, TAX_t,$

$ARTC_t \geq 0$ for $t = 1$ to 24

5.7 Concluding Remarks

The linear programming model is designed to select from among three production profiles and can initiate these profiles in any of the first ten periods. With each of these thirty possible production strategies exists a compatible EOR production profile which can also be initiated if it is profitable. In representing the future production from these reservoir development strategies, the model includes 24 time periods or 72 years. The model also includes a tax system consisting of royalties and corporate income taxes. This model thus effectively includes the physical behaviour of a reservoir and a complex tax system.

The results of this model as presented in Chapter 6 include profits, revenues, operating and capital costs, royalties, and taxes. Simulation models such as POGO (Profitability of Oil and Gas Opportunities), and other similar industry models, are able to produce this type of analysis. POGO can represent different types of production strategies and complex royalty and taxation formulas. Unlike POGO this model has optimization capabilities and searches out the best depletion strategy, namely the one with the highest discounted profits. Only one production strategy at a time can be analyzed by POGO. Any scenario represented in POGO may have greater detail but really presents the same information as a single Leontief process. Advantages of POGO are its ease of data input and allowing for changing the duration of production given different price and cost schedules. Although this last capability is not represented in the current model, it could be. This numerical model is able to optimize, and therefore, choose among the different technologies and whether EOR is a viable alternative. With this method the effects of changing taxes, royalties, prices, and discount rates on the choice of technology can readily be determined. Constraints limiting the size of negative net revenues (debt) can also be imposed to measure the loss of profits as debt restrictions are tightened. This type of analysis is not easily accomplished by POGO.

The next chapter shows the results that are obtained from the reservoir production model.

NOTES

[1] Hung-po Chao, "Exhaustible Resource Models: The Value of Information," Operations Research, 29 (1981) 907-908.

[2] Source of Data: Petro-Canada

[3] Simplification of course incurs costs, but it also gains benefits. (The shutdown condition is not allowed to vary when prices are increased.)

[4] The term "resource commitment" is borrowed from Hung-po Chao, "Exhaustible Resource Models: The Value of Information," Operations Research, 29 (1981) 907.

[5] Due to the nature of the royalty formula (non-linear and well dependent), if the model selects a combination of profiles as optimal, the royalties calculated will be slightly biased.

[6] Since all of these equations are equalities the model (in the absence of EOR and taxes) could be reduced to two equations: the objective function and the reservoir capacity constraint. However, any adjustments to this model would be extremely complicated. Such a reduction would also rule out the imposition of constraints on the sizes of cashflows or certain elements entering the objective function, such as capital expenditures in one or more periods.

Chapter 6

Results From the Reservoir Production Model

6.1 Introduction

Results from solving the reservoir production model are presented in this chapter. The discussion is divided into three sections: the before-tax model results, the after-tax model results, and user costs.

A high discount rate penalizes future net revenues relative to current net revenues, implying that the earliest production generates the highest profits. Profits are maximized for the before-tax model by developing the reservoir as soon as possible (drilling commences in the first period) and by producing the crude as quickly as possible (the short profile). This outcome is highly insensitive to changes in the price or the discount rate. Only extreme levels of change, such as doubling the price, alter the choice of production profile. Production from EOR does not enter the optimal solution since enhanced recovery is not profitable at the existing prices of $125.00/m3.

However, including a tax structure changes the optimal solution to include both the short and medium length profiles, not just the short profile. The reason is that, with the tax structure represented, the interaction between the short and medium profile capital consumption allowances (CCAs) causes a combination of the profiles to be optimal. Comparing this solution with the before-tax optimal solution shows the inefficiency of the tax system. The surplus or profits from the reservoir are now divided between the producer and the government. For the producer to maximize profits under the tax regime, the production path changes and total surplus declines. Other results are obtained by varying the discount rate, prices, tax rate, and CCA rate. The optimal production path is much more sensitive to these parameter changes with the inclusion of the tax structure. This sensitivity occurs because the sizes of the individual short and medium profiles' after-tax profits are very close and gains are available from pursuing a convex combination of these two production profiles. An additional constraint, on the size of the deficit associated with high initial costs, shows the trade-off between the reduction in the maximum deficit and the loss of profits.

6.2 Before-Tax Model Results

The optimal solution of the reservoir production model includes only the short profile (RCS_1 = 379.965). Intuition suggests that production should occur as quickly as possible because the high discount rate causes discounted current production revenue to be larger than discounted future production revenue. Net profits increase if the higher discounted cost of present production does not exceed the higher discounted revenues.

Table 6-1 shows the production profiles before and after discounting when each profile is used individually to develop the reservoir and drilling commences in the first period. The entries in the table are based on the cost and production coefficients of Chapter 5 as well as a price of $125.00/m3 for all years and a discount rate of 10% per year.

Table 6-1

Revenues, Costs, and Profits Before and After Discounting for the Before-Tax Model ($1,000s)

	Short	Medium	Long
Undiscounted Revenues	+47,821	+47,821	+47,821
Undiscounted Capital Costs	-13,310	-11,795	-10,160
Undiscounted Operating Costs	-14,085	-15,339	-16,715
	----------	----------	----------
Undiscounted Profits	20,426	20,687	20,946
Discounted Revenues	+24,485	+21,758	+18,585
Discounted Capital Costs	-11,344	- 9,977	- 8,512
Discounted Operating Costs	- 6,107	- 5,696	- 5,135
	----------	----------	----------
Discounted Profits	7,034	6,085	4,938

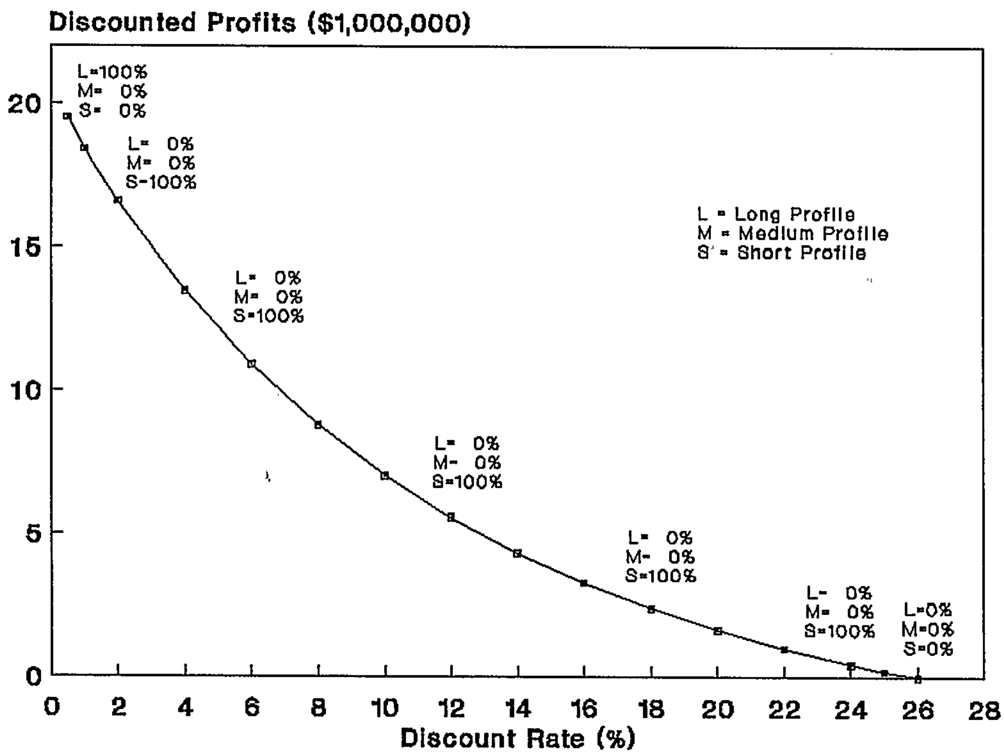
Since all profiles produce the same amount of crude oil, undiscounted revenues are the same for each profile. However, the differences between discounted revenues are due to the differences in each profile's production over time. With the short profile lasting for fewer periods and having a higher initial production, the short profile's revenue is not discounted as heavily as the revenue for the other two profiles. Even though the short profile has lower undiscounted operating costs, discounted operating costs are higher for the short profile because the operating costs are incurred sooner (similar to discounted revenues). The ranking of capital costs among the three profiles does not change when discounting occurs. However, since capital costs occur as a lump sum at the beginning of production, the decline in value after discounting is not as much as compared to operating costs and revenues. Thus, the magnitude of the differences between revenues and operating and capital costs is also dependent on the discount rate which, as Nystad (1987) notes, can affect the optimal production strategy (this is much more evident in the after-tax model). Before taxes, discounted profits are maximized when production occurs using the short profile.

EOR production is not included in the optimal production strategy since the discounted costs from this production technique exceed the discounted revenues at the assumed prices. This result is similar to one of the possibilities of the Amit (1986) model, where EOR production occurs depending on the prices and the costs of primary and EOR production. For the reservoir production model, EOR production occurs when prices rise far enough for this type of production to be profitable. Also for the reservoir production model, the allocation of EOR production over time is determined by the choice of primary-waterflooding production profiles.[1] Thus, by varying the prices and the primary-waterflooding profiles selected, different EOR results can be found. This shows how the computational model can go well beyond the type of analysis that is possible with an analytical model, such as that of Amit (1986).

Considerable sensitivity analysis was carried out for the before-tax model. In the following diagrams, sensitivity results are represented showing the discounted profits as various parameters are changed. In each diagram the allocation of production to each profile as a percentage of the total reservoir capacity is also provided as a parameter adjusts. In Figure 6-1, the short profile dominates other profiles as the optimal choice when the discount rate is varied from 1% to 25%. Only at a 0.5% discount rate does the model select an alternative profile (the long profile) as optimal. At a 26% discount rate, the discounted capital and operating costs exceed the discounted revenues, and thus no production occurs.

Figure 6-1

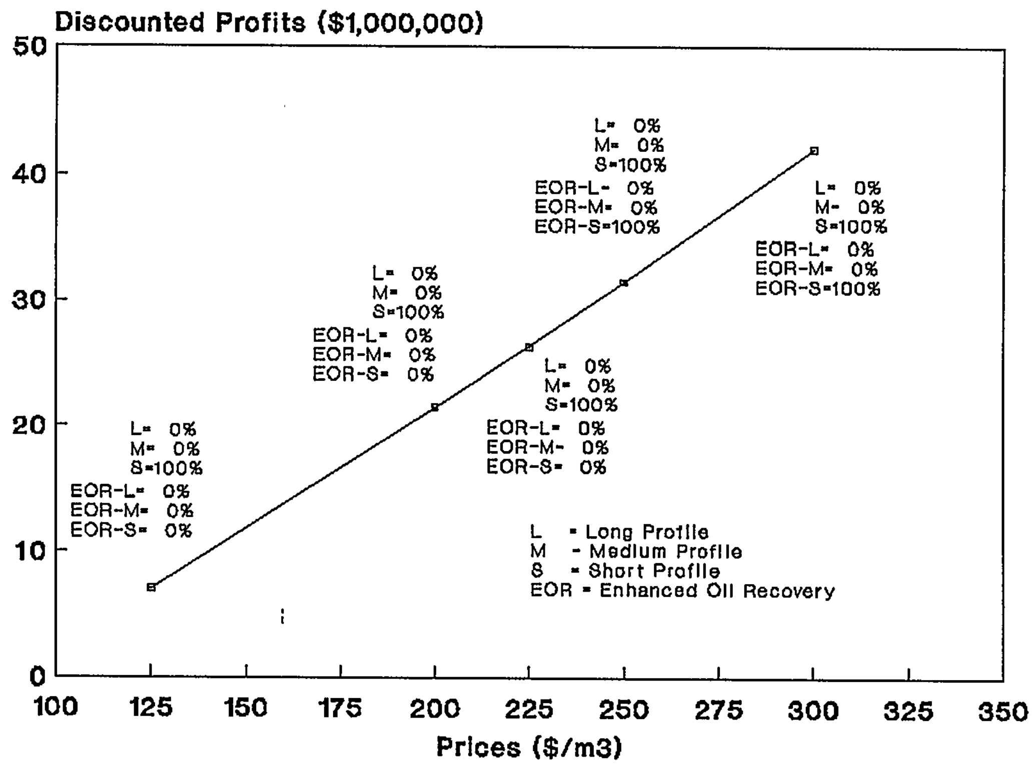
Before-Tax Choice of Profile and Discounted Profits
For Different Discount Rates



A second set of analyses involves varying the price of crude oil, with the discount rate returned to 10%. Figure 6-2 shows that the short profile is selected at all prices, and it is not until the price reaches $250.00/m3 for all periods that EOR is initiated. For the case where prices are allowed to rise at 2% per year there is no significant change since the short profile is still optimal, and discounted profits are $8,709,000.

Figure 6-2

Before-Tax Choice of Profile and Discounted Profits
For Different Prices



These figures show that the optimal production strategy for the before-tax model is highly insensitive to changes in the discount rate and prices.

6.3 After-Tax Model Results

After-tax model results are generated by a model which includes a tax structure composed of royalties, an Alberta Royalty Tax Credit (ARTC), a capital consumption allowance (CCA), and income taxes. If the tax structure causes no major bias, the short profile will likely continue to be the most profitable. However, the optimal production path for the after-tax model is a combination of the medium and short profiles. In particular the variables chosen are RCS_1 = 273.619, RCS_6 = 13.937 and RCM_1 = 92.409.

The optimal solution produces 287,556 m3 using the short profile and 92,409 m3 using the medium profile for a total of 379,965 m3. Table 6-2 shows the revenues, costs and taxes for the short profile, medium profile, and the optimal combination of profiles before and after discounting. From these tabular entries it is not immediately obvious why the optimal solution includes the short and medium profiles. However, the discounted profits for the medium profile are greater than the discounted profits for the short profile. This is opposite to the before-tax results and is due to the tax structure penalizing faster production to a greater extent. If any gains are available to one of these profiles from unused tax-reducing capital cost allowance of the other, a combination of the two profiles could provide higher profits. This is exactly what occurs. Even though the medium profile produces higher discounted profits, a cubic meter of production from the short profile has unused CCAs which can be used by the medium profile to reduce taxes paid. Thus, a combination of the medium and short profiles increases discounted profits.

As defined in Chapter 5, taxes are a percentage of Alberta Royalty Tax Credits and resource profits. The latter consist of production revenues less operating costs and CCAs. Consequently, taxes can be reduced by lowering resource profits by maximizing the CCA used for a given period. Once the resource profits have been reduced to zero, any unused CCA will be left in the CCA balance and carried forward to future periods.

Table 6-2

Revenues, Costs, and Profits Before and After Discounting for the After-Tax Model ($1,000s)

	Short	Optimal	Medium
Undiscounted Revenues	+47,821	+47,821	+47,821
Undiscounted Capital Costs	-13,310	-12,935	-11,795
Undiscounted Operating Costs	-14,085	-14,390	-15,339
Undiscounted Royalties	- 5,863	- 5,736	- 5,342
Undiscounted Taxes	- 9,001	- 8,596	- 8,675
Undiscounted ARTC	+ 2,932	+ 2,868	+ 2,671
	----------	----------	----------
Undiscounted Profits	8,494	9,032	9,341
Discounted Revenues	+24,485	+23,139	+21,758
Discounted Capital Costs	-11,344	-10,695	- 9,977
Discounted Operating Costs	- 6,107	- 5,837	- 5,696
Discounted Royalties	- 3,187	- 2,967	- 2,649
Discounted Taxes	- 4,735	- 4,392	- 4,043
Discounted ARTC	+ 1,593	+ 1,484	+ 1,324
	----------	----------	----------
Discounted Profits	705	732	717

Production using only the short profile occurs for 7 periods, resulting in production revenues and operating costs for only 7 periods. The CCAs available for the short profile reduce the resource profits to zero in period 7, and leave unused CCA in period 7 and for all future periods. For the medium profile, production occurs for 9 periods with unused CCA in period 9 and all future periods. For this profile, resource profits in period 9 are reduced to zero but are still positive in periods 7 and 8. Thus the unused CCA in periods 7 and 8 of the short profile can be used by the medium profile to reduce its resource profits for tax purposes, yielding higher profits. Therefore, medium profile production is exchanged for short profile production to the point where the gains from the unused CCA vanish. Medium profile production is exchanged for short profile production until the resource profits in period 8 are reduced to zero. This occurs at 92,409m3 of medium profile production and 287,556m3 of short profile production.[2]

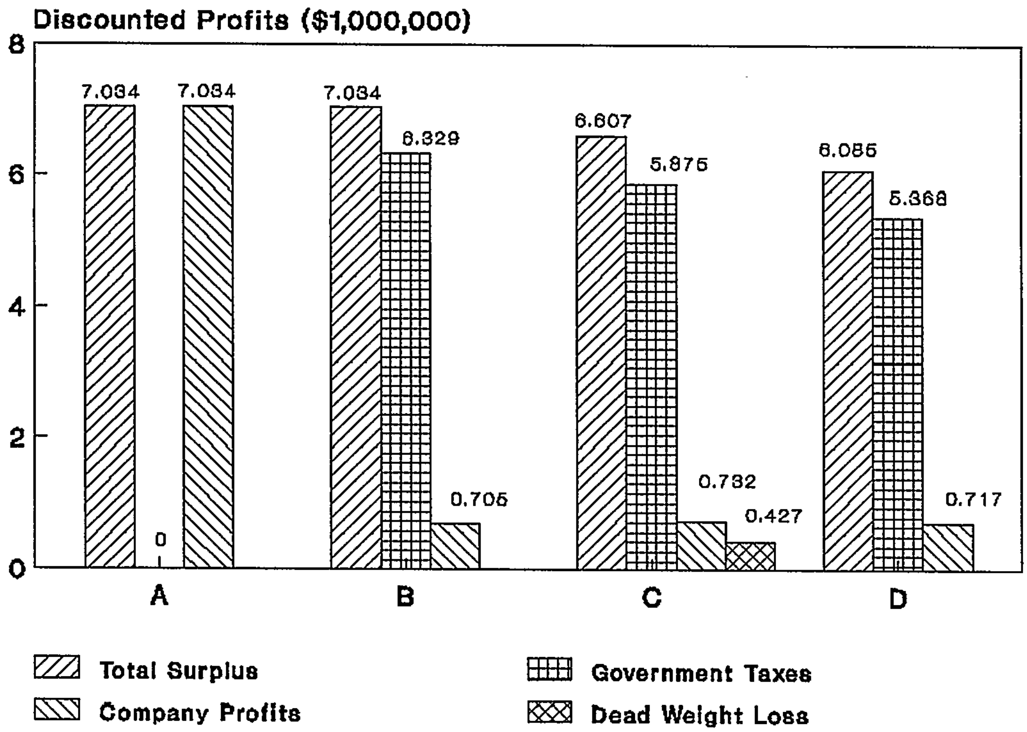
The inclusion of the tax structure alters the optimal production profile, the amount of surplus (discounted profits) available and the distribution of this surplus between the private producer and the government. From the data of Tables 6-1 and 6-2, Figure 6-3 shows that the dead weight loss or loss of surplus from including the tax structure is $427,000 or 6% of the before-tax discounted profits. The dead weight loss is calculated as the difference between the total surplus before and after tax ($7,034,000 - $6,607,000). This is a small loss of surplus, but the results show the inefficiency of the tax system.

Although this tax system, by including CCA as a tax deduction, is not similar to any of those employed by Conrad and Hool (1984), both here and for the Conrad and Hool results, distortions occur in the optimal production strategy. Results of the Conrad and Hool model show that a tax structure usually delays production from the present to the future and causes a decline in total production.

Applying the before-tax model optimal production strategy of using only the short profile, to the after-tax model and comparing this result to the after-tax optimal result shows a gain to the private producer in the after-tax results of $27,000 and a loss to the government of $454,000. Comparing these two findings shows that, if the private producer alters his production strategy slightly, there is a very small percentage change in profits and thus there are multiple near-optimal solutions.

Figure 6-3

Distribution of Surplus (Profits)



A - The before-tax optimal solution where production occurs using only the short profile.

B - The after-tax solution when producing only from the short profile.

C - The after-tax optimal solution where production occurs using both the short and medium profiles.

D - The after-tax solution when producing only from the medium profile.

For the next set of after-tax analyses the discount rate is varied from 1% to 12%, with all prices held at $125.00/m3. In Figure 6-4, depending on the discount rate, each of the profiles enters the optimal solution. Comparing Figures 6-4 and 6-1 indicates how the tax structure changes the optimal choice of profiles at the various discount rates. The short profile no longer dominates the other profiles as it did before taxes, and the optimal production strategy is very sensitive to changes in the discount rate.

The next three figures (Figures 6-5 to 6-7) show how discounted profits and the choice of profiles change as the price (increase), tax rate (decrease) and CCA rate (increase) change. From these three figures it can be seen that, once the model moves away from the initial values of these parameters (price - $125.00/m3, tax rate - 46%, CCA rate - 30%), the short profile is chosen as the optimal production path and again dominates the other profiles. When these parameters are changed, the difference between the discounted profits of the individual short and medium profiles increases such that the gain from sharing CCA is no longer greater than the loss of profits through switching production from the short profile to the medium profile.

Figure 6-4

After-Tax Choice of Profile and Discounted Profits
For Different Discount Rates

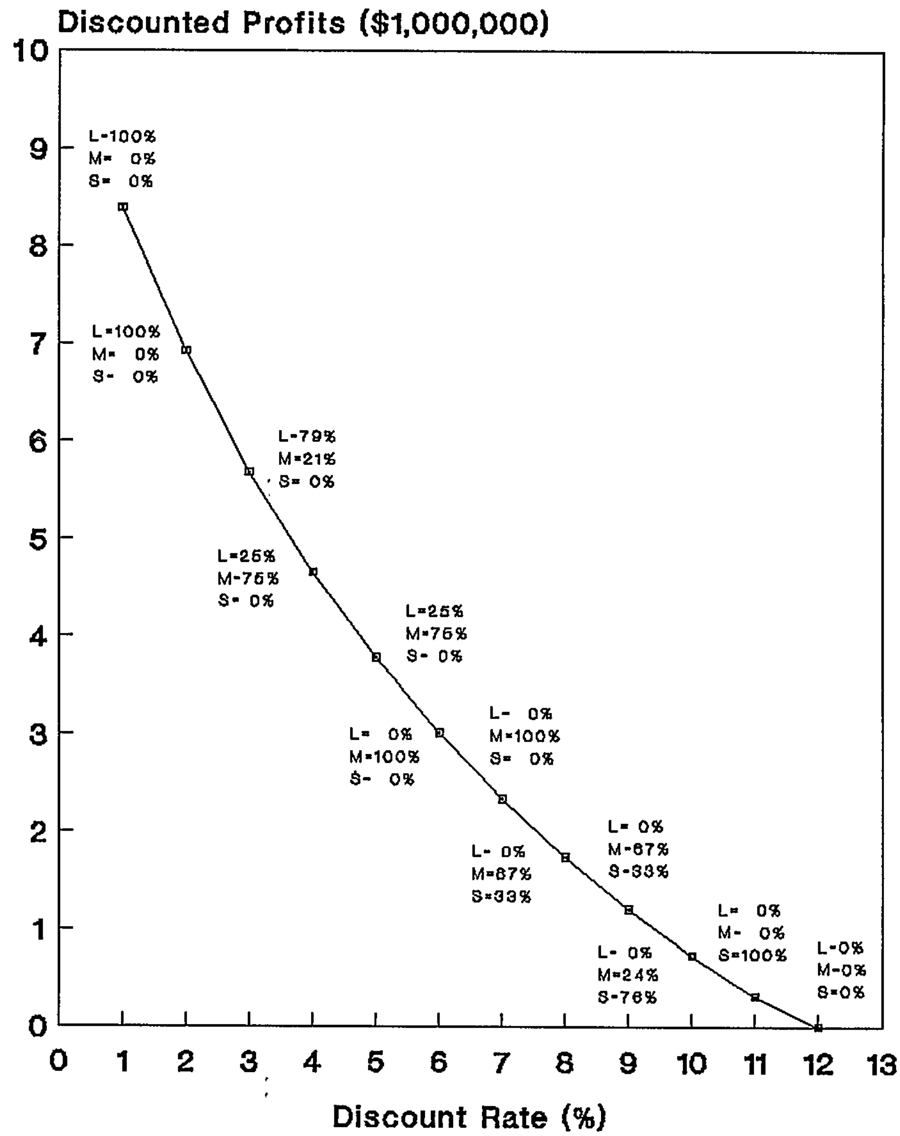
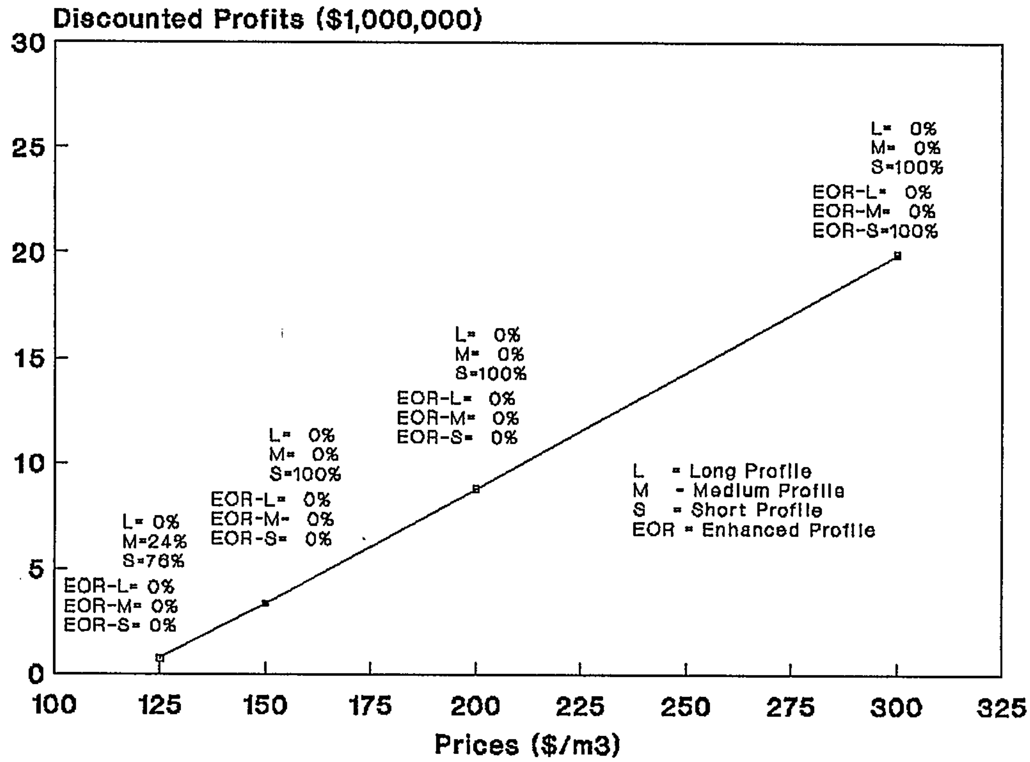


Figure 6-5

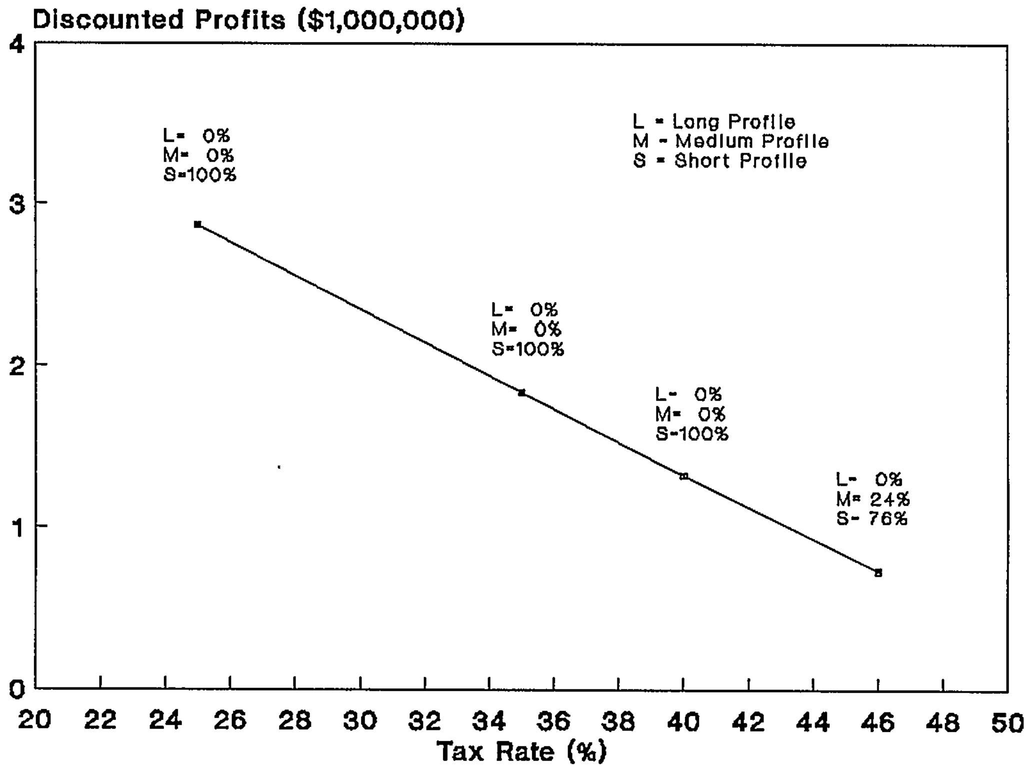
After-Tax Choice of Profile and Discounted Profits
For Different Prices



Allowing prices to rise at 2% per year yields discounted profits of $1,994,134 with 68% of production from the short profile and 32% from the medium profile. For the short profile, production is not initiated until the third period since the gains from higher future prices offset the losses from discounting the revenues from production which occurs further into the future.

Figure 6-6

After-Tax Choice of Profile and Discounted Profits
For Different Tax Rates



There are many near-optimal solutions for both the constant and rising price scenarios. If each of the profiles produced the reservoir individually, the percentages in Table 6-3 show how close discounted profits for these profiles would be to discounted profits for the optimal solution.

Figure 6-7

After-Tax Choice of Profile and Discounted Profits
For Different CCA Rates

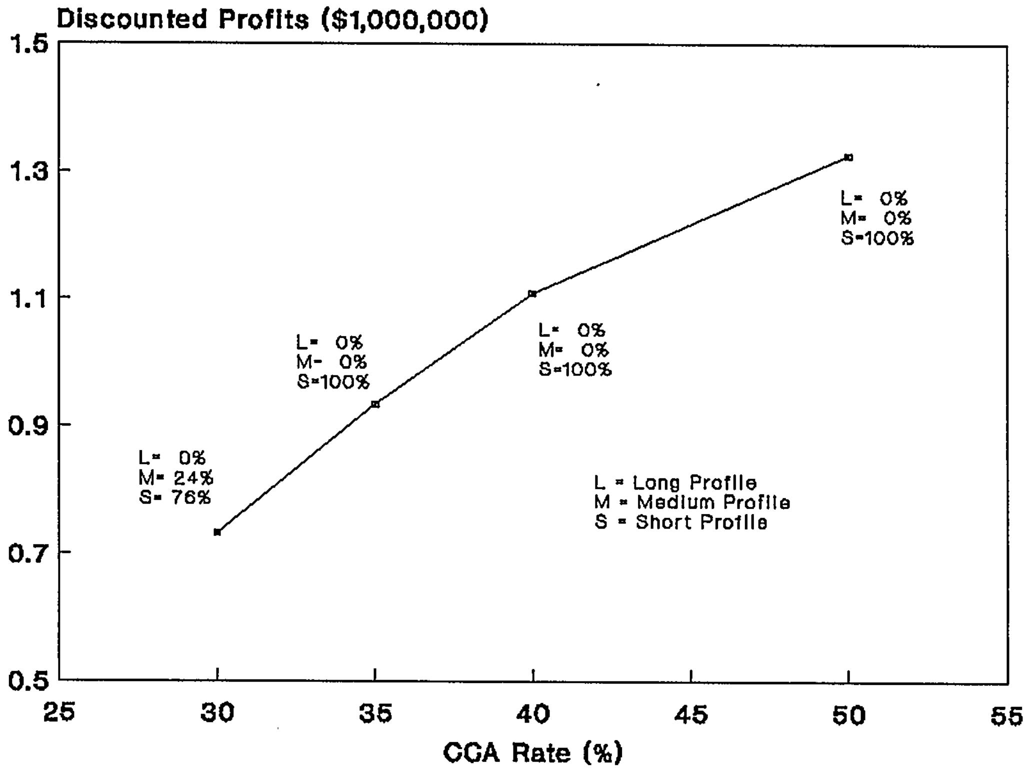


Table 6-3

Percentage of Optimal Solution's Discounted Profits

	Constant Prices	Rising Prices
Short Profile	96%	84%
Medium Profile	98%	92%
Long Profile	73%	88%

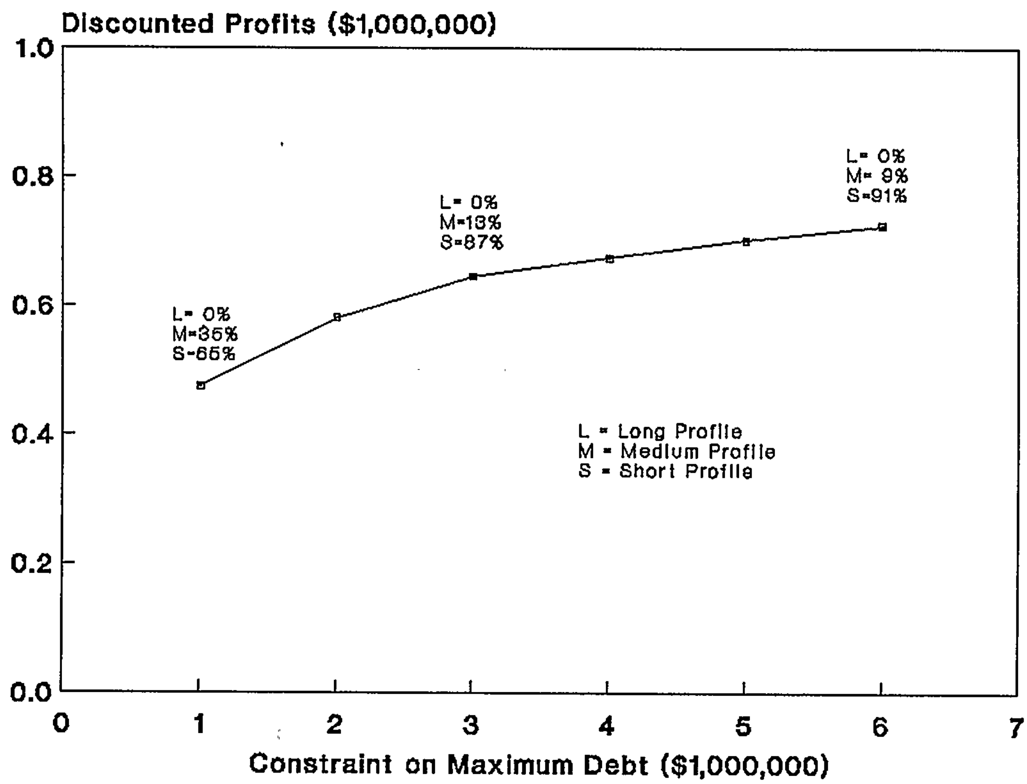
Thus, if a producer decided to minimize his capital outlay by using the longest profile to produce the reservoir, a loss of at most 12% to 27% of discounted profits would result.

As emphasized by Bradley (1985), capital costs play a significant role in determining production strategies. In general, investors prefer to minimize capital investment as part of a risk minimizing strategy. In the model the trade-off between decreasing the maximum deficit and reducing profits can be examined by imposing a cash flow constraint limiting the size of the deficit for each period.[3] This constraint is included in the model by setting a lower bound on the net revenue variables in equations 5.20 in Chapter 5. Thus, a limit is imposed on the size of negative net revenues which acts as a deficit restriction.

The results are shown in Figure 6-8. At first the deficit is restricted to be $6 million since the largest single period debt for the optimal solution is $6.45 million. As the deficit restriction is tightened, some of the reservoir production is delayed until later periods and increasing amounts of production occur from the medium profile. Comparing the results of a maximum single-period debt of $6 and $3 million shows that the producer loses only 11% of discounted profits when the maximum deficit is reduced by 50%. Furthermore, an 83% reduction in the maximum deficit leads to a 34% reduction in profits. This trade-off may well be preferred by a decision maker and illustrates Bradley's conclusion that a producer may choose a lower output rate than the unrestricted profit maximum in order to avoid debt or a high initial capital outlay. This type of comparison is not easily accomplished by analytical models if at all, and is very important to the private producer in evaluating production decisions.

Figure 6-8

After-Tax Choice of Profile and Discounted Profits
For Different Deficit Restrictions



6.4 User Costs

Although this model does not quantify all the user costs presented by the analytical models, there is a shadow price associated with each constraint of the model. The shadow price associated with the reservoir capacity is the stock user cost (Kuller and Cummings (1974)). This stock user cost is equivalent to the discounted unit net profit associated with developing, producing and selling crude from the reservoir. For the before-tax model the user cost is $18.51/m3 and for the after-tax model it is $1.93/m3. If an additional cubic meter of oil is available for the after-tax model, it is allocated over all of the time periods according to the optimal production profiles and yields a discounted profit of $1.93. The other shadow prices show the discounted gain or loss of profit if an additional unit is available for the constraints associated with net revenues, revenues, operating costs, capital costs, royalties, taxes, taxable income, CCA, and resource profits.

The other types of user costs as mentioned in Chapter 3 could have been calculated by the model with the addition of endogenous demands, endogenous capital costs, rising unit operating costs, and other possible companies' production profiles. However, including these complexities would cause a tremendous increase in model size and require solution by nonlinear programming techniques.

6.5 Summary

Although the reservoir production model provides many results that agree with intuition, the initial parameter settings (price - \$125.00/m3, discount rate - 10%, tax rate - 46%, CCA rate - 30%) provide a result that is not likely to be predicted by intuition alone. The tax structure represented, which is different than any used by the analytical models and arguably far more realistic, moves the optimal solution away from the before-tax model optimal solution. The tax structure causes a combination of the short and medium profiles to be optimal, rather than just the short profile in the before-tax model.

This result also shows the importance of computational models in analyzing the resource allocation problem. A first glance at the tax structure described in Chapter 5 does not show the tax structure significantly taxing one profile more than another. However, the tax system does impose greater taxes on the short profile, and allows tax reducing CCA trade-offs between profiles. These distortions are only revealed and quantified through the results of the computational model.

The results discussed must be regarded as reservoir specific, meaning that any conclusions reached apply only to very similar reservoirs and not to all reservoirs. This is due to the differences in physical characteristics between reservoirs, which for this particular reservoir are included in the production profiles and capital and operating costs. Nevertheless, the empirical results indicate the kinds of findings that may well extend to a large number of different types of reservoirs.

NOTES

[1] EOR production can only occur if the physical nature of the reservoir is suitable for the implementation of this recovery technique.

[2] Representing the tax structure leads to the inclusion of a number of inequality constraints in the model. Thus more than one variable enters the basis at the optimum, meaning that more than one profile can be selected for the optimal solution.

[3] Adding this maximum debt constraint causes computational difficulties for the linear programming package (APEX IV). On numerous occasions the system exited without giving an optimal solution but printing the message "convergence too slow" and providing a basis which could be used for a subsequent computer run. An optimal solution was only found when this basis was included in a later computer run of the model.

Chapter 7

Concluding Remarks

7.1 Summary and Conclusions

Substantial ground has been surveyed in the following areas: the physical nature of reservoirs, analytical models of exhaustible resources and computational models of specific energy commodities. Although significant differences exist among these areas, they all provide ingredients that are necessary for a realistic approach to determining the optimal allocation of exhaustible resources through time.

From the survey of geological, reservoir engineering and MER considerations, it is evident that the physical nature of a reservoir plays a vital role in the rate of production and in the size of total production. Such properties as porosity, permeability, water saturation, the type of trap, and the composition of the fluids in the reservoir all cause reservoirs to be unique. Thus, any model that groups all reservoirs together weakens the credibility of any conclusions concerning the optimal allocation of production for a specific reservoir.

Although most analytical models adopt highly simplifying assumptions, these models contribute substantially to the understanding of the resource allocation process. The general rule for optimization from the analytical models equates marginal revenue to the sum of marginal cost and user cost. User costs provide a wedge between prices and costs to recognize the exhaustibility of a nonrenewable resource, the fundamental difference with a renewable resource. Other considerations such as the rule of capture, increasing costs as production proceeds, a backstop technology, and investment are incorporated by a few of the analytical models, thereby changing the optimizing condition. It is through these additions and the changes in the optimizing condition that insight is gained into understanding optimal resource management.

Computational models provide a framework for including the physical nature of reservoir development, investment, terminal conditions, a backstop technology, a complex tax structure, and discrete variables. Representation of these complexities is not easily accomplished, if at all, by analytical models. Although some of the computational models surveyed do not include economic considerations (prices, demands, costs, or discount rate), these models provide examples of specific resource problems that computational models could solve. Such problems as the optimal production of an offshore reservoir, the optimal pressure maintenance of a gas reservoir, and the optimal distribution of exhaustible resources and energy supply from new technologies to meet world energy demands are readily modeled through the computational framework.

The reservoir production model presented in this study is a hybrid of the analytical and computational models. The model "takes into account" the physical nature of a reservoir in determining production rates and each profile has its own unique capital and operating costs. By including a set of equations representing the tax structure and the constraint on the maximum deficit, the advantage of the computational approach is shown. Model results are found that could not be attained by an analytical model. Moreover, the reservoir production model does focus on results such as user costs and the sensitivity of the optimal production path to changes in prices, discount rates and taxes which are considered by analytical models.

For a simple profit maximizing reservoir production model, intuition suggests that production should take place "as quickly as possible." But with real world conditions represented, intuition becomes clouded and the nature of the optimal results become less clear. The tax structure penalizes fast production, leading to the optimality of a combination of the short and medium length production profiles. Trade-offs are made to minimize the taxable income such that capital expenditures are written off through the capital consumption allowance (CCA). The tax structure modeled is inefficient in that it distorts the optimal allocation; different profiles are selected than for the before-tax model. Hence, this computational model shows a result that would be difficult to predict with any confidence and may even be impossible to obtain from an analytical model.

Investment costs also play an important role in the reservoir production model. Since the majority of the capital costs are incurred before production, they become much larger relative to operating costs and revenues after discounting. Thus, if a company pursues not just the single objective of maximizing profits but is also concerned with risk or the financing of capital expenditures, alternative production scenarios may be desirable. One set of results from the computational model suggests that the percentage loss of profits may be small when capital costs are reduced. Given the complexities of the tax and royalty systems, this result could have been found using an analytical model only with great difficulty or perhaps not at all.

This reservoir production model also compares favourably with POGO (Profitability of Oil and Gas Opportunities). POGO is a simulation model which allows the easy incorporation of different production rates, prices and tax structures. The reservoir production model in its present form does not have the ease of data input or ease of changing to different scenarios that POGO has, but all of these variations or extensions can be included. However, POGO does not have the optimizing capabilities of the reservoir production model.

7.2 Extensions of the Reservoir Production Model

Simple additions to the reservoir production model could be made by including more recovery profiles and changing to annual time periods. Complex extensions include discreteness in production strategies through mixed-integer programming, allowing production to switch from one type of profile to another in the middle of production, developing more than one reservoir, including exploration costs and demand constraints, modelling rising marginal costs of production, and representing byproducts. All of these changes would make the model larger but the only constraint on implementing these extensions would be the capabilities of the computer programming package. The package would have to handle a larger model, and be able to solve mixed-integer programming and nonlinear programming problems.

Overall, there appears to be considerable scope for constructing hybrid models for examining issues of exhaustible resource allocation which require incorporation of geological, engineering and economic factors which bear on resource extraction.

Appendix

The following numerical example determines the MRL (Maximum Rate Limitation) and Base Allowable for an Alberta PSU (production space unit) which were briefly described in section 2.4.2 of Chapter 2.

The Base MRL is the greater of the Basic Well Rate (BWR), the Preliminary Rate Limitation (PRL) or the MRL set by the Energy Resources Conservation Board (ERCB) on the basis of studies submitted by the operator.

BWR - set by the ERCB as the value of the Maximum Allowable (MA)

- 5.0 m^3/d

PRL = 9000×10^{-6} (U) x 12/365

- The value of U is determined from the O-38 form.
- for example if U = 11.0 10^3 m^3, then PRL = 3.25 m^3/d

- in this example there are no submissions

So for this example the Base MRL is 5.0 m^3/day.

The Base Allowable is the greater of the MA for the month and the Reserves Allocation.

MA - (from the O-38 form) x 30

- 5.0 x 30 = 150 m^3/month

Reserves Allocation

- U (from the O-38 form) x reserves allocation (from the MD Order form) - 11.0 x 4.7867 = 52.7 m3/month

Therefore the Base allowable is 150.0 m3/month, but it cannot be greater than the penalized MRL.

Penalized MRL

- MRL x days x combination penalty factor

- 5.0 x 30 x 0.80 = 120

The effective monthly allowable is 120 m^3/month.

Combination penalty factor (all values from the O-38 form)

$$= \frac{\text{Base GOR}}{\text{Produced GOR}} \times \frac{2}{2 + (\text{Produced WOR})(1 - \text{WDI})}$$

$$= \frac{130}{152} \times \frac{2}{2 + (0.14)(1 - 0)} = .80$$

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