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Cultivating the Growth of Mathematical Images

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Cultivating the Growth of Mathematical Images

by

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Abstract

In this document, I present some of the findings of the study, Cultivating the Growth of Mathematical Images, in which I explore the role that spatial reasoning plays in the growth of mathematical images. This study involved Grade Five students labelled with learning disabilities, and their teacher. This study was a micro analysis of a participatory action research study, as it looked at the beginning stages of exploration with students and the impact that a more spatial approach to fractions might have on their growth. Information was gathered from students' psycho educational assessment, an informal assessment of spatial reasoning ability, and a pre-assessment task that looked at their understanding of basic number and fractions.

There were many layers of complexity surrounding each student's psycho educational profile, their performance in the classroom, and the pre-assessment task offered to the students. Certain aspects of these experiences seemed consistent with each other, others contradictory, and still others contained much variability, such as in the area of working memory.

During the second week of the study, video data were collected while students engaged with a task that created an interplay between visualizing and building fractions. In analysis of the data, close attention was paid to what sort of offerings the students were given such as signitive (written and oral), imaginative (visualizing), and perceptual (sensory), which are somewhat modified from Husserl (1970). During this task, a pattern of growth began to emerge which is discussed and connected to the Pirie-Kieren (1994) Dynamical Theory for the Growth of Mathematical Understanding. Generally, with the introduction of the signitive only, there was *No appearance of movement.* Then as the student began either producing or receiving perceptual experiences they progressed into the *Image Making* phase. The continued engagement with perceptual experiences appeared to create the beginnings of the imaginative, *Image Making* *emergent Image Having* phase, and some students were even able to reach the point of being predominately in the *Image Having* phase.

Each participant went through the various phases at varying speeds. Within these various phases there was found to be much complexity in terms of contributing factors towards growth. The fact that some participants built more than others and therefore had more perceptual offerings seemed a strong contributing factor but other aspects such as their own personal commitment to sense-making as they built, their social interactions, and their own self-belief seemed to also impact growth.

Keywords: visualization, images, embodied cognition, learning disabilities, dyslexia, mathematics education

Preface

This thesis is original, unpublished, independent work by the author, J. A. Plosz The experiments reported in Chapters 5-10 were covered by Ethics Certificate number REB16-1205, issued by the University of Calgary Conjoint Health Ethics Board for the project "Cultivating the Growth of Mathematical Images" on January 18, 2017.

Acknowledgments

I want to express my deep appreciation to the children who offered so willingly and enthusiastically to be part of my study even when school has caused them so much scarring; to the philosophers, researchers and theorists whose shoulders I have stood upon to try to make sense of all this complexity; to my supervisor, Jo Towers, whose patience and gentle guidance has allowed me to stretch my own understandings and move in unexpected directions as a learner; and finally, to my family, to my father who was often my sounding board as I would often need to verbally organize my thoughts; to my kids, Roman and Kelly, who patiently understood as mom would once again be spending the day at the library; and last but definitely not least, my husband, who supported and encouraged me every step of the way, who took on much more of the domestic responsibility in support of my endeavor. Thank you to all of these people who have made this document a reality.

Dedication

To my fellow dyslexics,

may our future path be smoother.

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Entering into the topic

"The opportunity to engage in hermeneutic understanding is likely to arise when individuals undergo any experience that serves to disrupt the ordinary, taken for granted aspects of existence" (McManus Holroyd, 2007, p. 2). This statement corresponds well with the stimulus which began my own particular turn towards meaning in the area of mathematical understanding. As a high school teacher of mathematics, I had a reputation for being a *good* math teacher. I considered myself to be relatively *in the know* in regards to the teaching of mathematics. I was a very dedicated teacher. I spent many Saturdays at the school preparing my lessons, and seeking out innovative ways of explaining mathematics, for this is what I believed my job to be—explaining mathematics.

My math lessons consisted of initially attempting to connect with students' previous knowledge, and then progressing to the new content from the next section in the textbook. Afterwards the students would work individually on questions from that same section within their textbook. I put a lot of time and energy into presenting this new material in a well thought out sequential manner. I developed creative approaches to breaking things down into simpler terms and coming up with mnemonics or engaging them with the adventures of invented characters such as the trig squad. Students seemed to enjoy my class, they achieved at an expected level, or even slightly higher in comparison to my colleagues. Yet, there was a distinct range of achievement. There were students who were "math people" and those who were not. This cultural description of the classroom seemed a perfectly acceptable outcome. My own mathematics education was situated in a similar environment, and I did not question its merit.

Upon becoming a parent, I underwent an experience that served to disrupt my dominant ideologies regarding mathematics education. In beginning to teach my son basic number skills through what I understood to be productive teaching methods, we reached what seemed like an impasse before even getting to what I believed to be the starting gate—symbols. My son and I were at a four-year-old birthday party, as the birthday cake entered the room ornately displaying a big number four, I noticed that all the kids seemed to know and recognized the symbol. My four-year-old son and I had been working on this for some time, yet he still could not recognize it. This perplexed me greatly for up to this point my son fit beautifully into the mold of my understanding of an intelligent child. His language skills were excellent, his building and reasoning skills were exceptional, and social sensibilities came to him with ease. How could he struggle with such a basic symbol like the number four? His struggle with symbols persisted as the years progressed. My understanding of intelligence was deeply linked to the ability to learn in our cultural style of—I do, we do, you do; yet my son was not connecting with this approach.

It was in this space and time that my mind began to notice the fork in the road. Do I accept the imposed rhetoric that my son is not a “math person” or could there be a different understanding that needs to be sought out? My dominant ideology about mathematics education offered minimal help, and I began to question my own knowing. I began to realize that my understanding was not wholly what I imagined it to be. This led me down a path towards a different understanding of what promotes the growth of mathematical thought. As I began to engage with my son through a more visual-intuitive mode, movement began, and not along a slow linear path, but more like leaps and bounds across sporadic intervals. I began tutoring other elementary students who struggled with learning number concepts and guest teaching in classrooms. Through these experiences I encountered many students who were having similar outcomes. My desire to understand this phenomenon deepened.

The opening of my mind to the fork in the road is where my hermeneutic thought within mathematics education was initiated. I began to see that our school culture has a history founded on establishing a dichotomy between the "intelligent" and "unintelligent", where this distinction to a large degree is bourne out through test taking and aptitude for mimicry. It is this *disclosure of the fore-structure* (Heidegger, 1927/1962) that opens up a space for the relinquishing of attachments to how a person currently knows and understands the world—in order to begin to understand differently. There is an opening to begin questioning what meaning has been imposed rather than interpreted. This realization allows us to push aside the distractions of incessant habits of thought and attempt to see the meaning before us. It is in this pushing aside that we enter into the *hermeneutic circle* (Gadamer, 1960/1989). Within this circle we can see the back-and-forth or movement between tradition and interpretation. By reflecting on our own fore-projections or pre-understandings and the meanings that exist within them, there lies the possibility of containing their influence on our transforming understanding.

These hermeneutic understandings that I began my journey with, I now believe, have deeply enriched this study. Although my final destination in regards to methodology is participatory action research (PAR), I am very thankful for my beginnings along the path of a hermeneutic phenomenological study.

It is interesting to note that my initial focus upon entering graduate school was centered around change. Yet, my graduate experience drew me more towards understanding and meaning. The pathway towards a hermeneutic phenomenological study was productive on many levels, but when approached by a school for students labelled with learning disabilities to work with their teachers, this was too tempting of an offer to look past. Yet, hermeneutic phenomenology is not focused around change but rather exploration of meaning within that particular situation. So, rather than give up this opportunity, my focus shifted towards a PAR methodology, which brought me full circle back to a focus around change.

The initial spiraling aspect within PAR of questioning, reflecting, and investigating all contain very strong links to a hermeneutic mindset, however, there is then a continuation from this point onto developing, implementing and then refining in order to bring about change, which is not the final goal within a hermeneutic study. For this reason, I believe that my initial focus in hermeneutic phenomenology has offered this study a depth that it may not have been attained had my journey begun with PAR. I have retained many of my initial elements from these beginnings as I feel they have been a strong influence on this journey and provide enrichment to the study.

Chapter One: Three horizons that surround this topic

Gadamer (1960/1989) emphasized the importance of looking at our historical horizon so as to understand something in its true dimensions. Where have we come from, is such an insightful way to begin any journey. As our educational system is mainly founded on a Western philosophical tradition, my discussion on historical horizons will also be situated within this tradition. As with any topic there seems to exist a mixture; there is often not just one horizon overlooking and influencing a topic, rather multiple overlapping horizons. In regard to the current discussion within the confines of this document, I see three dominant horizons that have influence on each other and our understanding of this phenomenon of growth in understanding within mathematics education, (a) the history of mathematics and its impact on mathematics education, (b) cognition related to imagery, and (c) learning disability labels within education.

1.1 Horizon of mathematics

As with any topic, mathematics has a history; and this history reflects itself in the present and lives on into the future. The relationship between the ideas of internal and external within mathematics has had its ebbs and flows: internal in the sense that mathematics is a human creation, whereas those who view it as external believe that mathematics is as something separate from us and exists out in the world waiting to be discovered. The struggle between rationalists and intuitionists has been a long tug-of-war that is as of yet unresolved (Dossey, 1992). This tug-of-war has not been so much of a win or lose kind of result but more of a swaying back and forth with the dominant ideas in certain time periods either leaning more towards the intuitionist or the rationalists. It began with Plato on one end, and Aristotle on the other; Descartes and Kant, while both on the Aristotelian side, were on either end of the intuitionist spectrum; Frege resided on the Platonic side, where Brouwer remained distinctly on the Aristotelian side; and at the beginning of the 20^{th} century Hilbert, while pulling for the intuitionist, was out shone by his teammate Gödel, and so it continues into modern times. Throughout our mathematical history the conceptions of mathematics seemed centered around an attachment to whether the nature of mathematics is externally or internally developed (Dossey, 1992). In the sections that follow, we discuss this internal-external debate which has had a profound influence on mathematics education over the centuries (Dossey, 1992; Mancosu, 2005; Zimmermann & Cunningham, 1991).

1.1.1 Plato and Aristotle

Debates on the nature of mathematics can be traced back as far as the fourth century BC, the first major contributors being Plato and his student, Aristotle. Plato believed that there are abstract mathematical objects, *Forms*, whose existence is independent of us and our language, thoughts, and practices—this way of thinking posits that just as electrons and planets exist independently of us, so do numbers, sets, points, lines, and circles (Bell, 2007). Mathematical truths are therefore discovered, not invented. These beliefs caused Plato to draw a distinction between the ideas of the mind and perceptions of our environment. In the *Republic* (1952), Plato distinguished between the theory of arithmetic, which he describes as the theory of numbers, and practical arithmetic, described as computations used by businessmen. He argued that the theoretical form of arithmetic has a positive effect on individuals, urging them to reason about abstract numbers. He saw no value in those that used physical arguments to "prove" results in applied settings (Dossey, 1992). Plato considered mathematics not as an idealization of aspects of the empirical world, but rather as a direct description of reality, that is, the world of *Forms*, which is captured through reason (Bell, 2007). Plato's rationalist views elevated the position of mathematics to an abstract mental activity based on externally existing mathematical objects (Dossey, 1992).

Aristotle, Plato's student, also drew distinctions, but his were between the physical, the mathematical, and the theological (Dossey, 1992).

> [Mathematics is the one] which shows up quality with respect to forms and local motions, seeking figure, number, and magnitude, and also place, time, and similar things. . .. Such an essence falls, as it were, between the other two, not only because it can be conceived both through the senses and without the senses, but also because it is an accident in absolutely all beings both mortal and immortal. (Ptolemy, 1952, p. 5)

The recognition of the senses' involvement in the abstraction of ideas concerning mathematics was different from Plato's views. Aristotle's view of mathematics was not based on a theory of an external, independent, observable body of knowledge. Rather mathematical knowledge was obtained through experimentation, observation, and abstraction. It was a constructed idea through idealizations performed by the mathematician as a result of experience with objects. By the Middle Ages, Aristotle's work became known for its contributions to logic and its use in authenticating scientific claims (Dossey, 1992).

1.1.2 Descartes and Kant

In the 17th century, Descartes entered the debate on the nature of mathematics. He worked to move mathematics back to the path of deduction from accepted axioms. In 1637, he published his ground-breaking philosophical and mathematical work *La géométrie*. This piece of work introduced what has become known as the standard algebraic notation, using lowercase *a*, *b,* and *c* for known quantities and *x*, *y*, and *z* for unknown quantities. It was perhaps the first book to look like a modern mathematics textbook, full of *a*'s, *b*'s, and *x*'s (Sorell, 2000). Descartes rejected input from experimentation and the senses in reference to mathematics because it might possibly delude the perceiver (Dossey, 1992). Descartes' view was as a rationalist following in Platonic thought of an external mathematics. As a result, he hoped to separate mathematics from the senses.

However, in the 18th century the senses and intuition were once again brought to an exalted status by a German philosopher, Immanuel Kant, in his *Critique of pure reason* (Bell, 2007). Kant positioned intuition as central to the nature of mathematics (Mancosu, 2005). He argued that the contents of Euclidean geometry were *a priori* understanding of the human mind, not externally existing objects (Dossey, 1992). This was in direct contradiction to the evolving understandings of non-Euclidean geometry, at the time. Kant's ideas had a profound impact. It was these philosophical views of the nature of mathematics that allowed the mathematicians of the time to be freed from the concept of a single set of restrictive axioms thought to be the only model for the external world (Dossey, 1992). Kant brought to mathematics a freedom of ideas.

The establishment of the consistency of non-Euclidean geometry in the mid-1800s showed the power of the human mind to construct new mathematical structures, free from an externally existing world (Körner, 1962). It was within this new freedom that a new set of perceived problems arose—the appearance of paradoxes in the real number system and the theory of sets (Dossey, 1992). This brought about three new views of mathematics to deal with these perceived issues.

1.1.3 Frege and Brouwer

The first school of thought formed was logicism by German mathematician Gottlob Frege in 1884 (Dossey, 1992). Frege set out to prove in true Platonic style that the concepts and properties of number are “logical” in the sense of being independent of spatiotemporal intuition. This approach was built on the acceptance of an externally existing mathematics (Dossey, 1992). Frege was concerned about supplying mathematics with rigorous definitions, based on logic (Bell, 2007). He undertook the task of fashioning in exacting detail the symbolic language within which his analysis was to be presented. In his writings he presented in full formal detail, the analysis of the concept of number, and the derivation of the fundamental laws of arithmetic (Bell, 2007).

A second school of thought to emerge was intuitionism. The followers of the Dutch mathematician Brouwer, like Kant, believed that mathematical concepts are admissible only if they are adequately grounded in intuition, that mathematical definitions must always be constructive. For both Kant and Brouwer, intuition meant “the mind’s apprehension of what it has itself constructed” (Bell, 2007, p. 16); based on this view, the only acceptable mathematical proofs are constructive—“a kind of ‘thought experiment’—the performance, that is, of an experiment in imagination” (Bell, 2007, p. 10). Brouwer did not call for the “inspection of external objects, but [for] close introspection” (Körner, 1962, p. 120). Mathematical ideas existed only insofar as they were constructible by the human mind. This insistence on construction placed intuitionism within the Aristotelian tradition (Dossey, 1992).

1.1.4 Hilbert and Gödel

The third school of thought to emerge near the beginning of the 20th century was that of formalism. This school was shaped by the German mathematician David Hilbert. Although in line with an Aristotelian tradition as he believed mathematics stemmed from intuition, Hilbert did not accept the Kantian notion that the structure of arithmetic and geometry are of *a priori* knowledge to the same extent that Brouwer did (Dossey, 1992). Hilbert’s focus was on securing what he hoped to be perfect rigour for all of mathematics. He wanted to ground mathematics on the description of concrete spatiotemporal configurations, only Hilbert restricts these configurations to concrete signs—such as inscriptions on paper (Bell, 2007). *Hilbert's program* (Bell, 2007), as it came to be called, intended to establish a new foundation for mathematics not by reducing it to logic, but instead by representing its essential form within the realm of concrete symbols, thereby attempting to free mathematics from contradictions (Dossey, 1992).

Much progress was made before Hilbert's program was shown to be unattainable in 1931, as a result of Kurt Gödel's work (Bell, 2007). Gödel demonstrated that it is impossible to establish consistency of a system by using only the major concepts and methods from traditional number theory (Dossey, 1992). These findings ended the attempt to formalize all of mathematics; however, Gödel did go on to show that Hilbert's program could be carried out in a weakened form by allowing the use of suitably chosen abstract objects (Bell, 2007). The formalist school of thought, however, continued to have a strong impact on the development of mathematics (Bell, 2007).

The three major schools of thought that emerged in the early 1900s to deal with the paradoxes discovered in the late 19th century advanced the discussion of the nature of mathematics, yet none of them provided a widely adopted philosophy. All three of them tended to view the contents of mathematics as products (Dossey, 1992). In logicism, the products were the elements of classical mathematics— its definitions, postulates, and theorems. In intuitionism, the contents were the theorems that could be constructed through patterns of reasoning. In formalism, mathematics was made up of the formal axiomatic structures developed to rid classical mathematics of its shortcomings. The influence of the Platonic and Aristotelian notions continued to be a strong undercurrent through each of these theories. The concept of "product"

—either as an externally existing object or as an object constructed through experience—remained the focus (Dossey, 1992).

1.1.5 Modern times

If we look to the present day there still remains a tug-a-war between the external and internal, which has profoundly impacted mathematics education. There remains a strong thread of Platonic and Aristotelian views. If we were to summarize modern day conceptions of mathematics, five different groupings are considered to be reflected in the literature of mathematics education (Sowder, 1989). These conceptions include two groups of educational research from the external (Platonic) view of mathematics; and three groups of research areas which take a more internal (Aristotelian) view; yet I use the term *internal* with some caution, for an emerging notion within cognition is a rejection of the concept of internal and external (Maturana & Varela, 1987), and within these groupings there are researchers who align themselves with these theories. I count myself among them. However, the distinction in lineage for Plato versus Aristotle remains arguably true.

In the external trending within mathematics education, research focuses on an established body of concepts, facts, principles, and skills, which are available in curricular materials. The first category of externally focused researchers situates their studies in assisting teachers and schools to be more successful in imparting mathematical knowledge to children. Their work takes a relatively fixed, static view of mathematics (Dossey, 1992). The second group of researchers adopting the external view promotes a more dynamic style of mathematics with a focus on adjusting the curriculum to reflect this growth within the discipline. The underlying focus is, however, still on student mastery of the curriculum or on the application of recent advances in technology or instructional technology to assist at furthering mathematical instruction (Dossey, 1992).

The remaining three conceptions found in mathematics educational research focus on mathematics as personally or “internally” constructed knowledge. One such grouping considers mathematics to be a process. Knowing mathematics is equated with doing mathematics. Research in this tradition focuses on examining the features of a given context that promotes the *doing* of mathematics. It is considered a personal matter in which learners develop their own personalized notions of mathematics as a result of engaging in activities (von Glasersfeld, 1987). A second conceptualization of mathematics within this internal group is based on a cognitive science approach to the study of mathematics. The main goals within this type of research are toward “the identification of representations for mathematical knowledge, of operations individuals perform on that knowledge, and of the manner in which the human mind stores, transforms, and amalgamates that knowledge” (Dossey, 1992, p. 45). The third conception of mathematics is one that views knowledge as resulting from social interactions. Within this grouping, the learning of mathematics stems from the acquiring of relevant facts, concepts, principles, and skills as a result of social interactions that rely heavily on context. This “sense making” in the learning of mathematics requires students to participate actively in “doing mathematics” to learn the skills of the discipline. These activities allow students to see how the results of such activities relate to the solution of problems in the social setting from which the problems originated (Dossey, 1992). And so, the debate continues.

All of these present-day conceptions of mathematics have strong historical roots. The mathematical horizon of the past, intertwining with the present, and living on into the future contains an influential current of struggle between the ideas of internal and external, dating all the way back to the fourth century B.C. These strong foundations and conceptions have had and continue to have profound influences on our views of the teaching and learning of mathematics. I define and highlight these cultural biases as it is only within the naming and defining of these currents that we can hope to contain their influence on our future views of understanding within mathematics education (Gadamer, 1960/1989). The secondary reason for defining these historical lineages is to situate myself and this study within them and show its connection to the more recent conceptualization of mathematics within the cognitive science realm, which leads us to a secondary historical lineage, that this study must situate itself within, that of cognitive science.

1.2 Horizon of images

The study of thought, the human mind, and the elements that are considered to play a role in this process or ability, is mired in differing opinions and controversy, which speaks to its complexity as a phenomenon but also the methodological ability to study it. Looking back, all the way to the fourth century B.C., the importance of mental images has generally had strong support for playing a central role in visual-spatial reasoning and inventive or creative thought. According to a long dominant philosophical tradition, mental images were believed to play a crucial role in all thought processes and are considered to provide the semantic grounding for language. How a mental image was defined, however, was in much debate, as it continues to be in the present day, but its importance in thought was generally not questioned. This long history of import was, in the 20th century, vigorously challenged, and its study was to a great extent abandoned. More recently, however, it has once again begun to find some defenders.

1.2.1 The philosophical tradition

The discussion of imagery has been in play for as long as humans have attempted to understand their own cognition. It can be found in some of the earliest writings of Greek thinkers and is extensively written about by both Plato and Aristotle (Arnheim, 1969). Its existence was not so much debated as was its value in rational thought. Once again Plato and Aristotle held differing opinions. Plato believed imagery to be irremediably deceptive, whereas Aristotle saw it as playing an essential and central role in human cognition (Arnheim, 1969). He asserts that “the soul never thinks without a mental image” (Aristotle, 1907, 431a), and considers the representational power of language being derived from imagery, spoken words being the symbols of the inner images (Aristotle, 1907, 420b). The concept of images in philosophy continued to be a universally accepted element of thought and was often connected to the concept of idea (Thomas, 1999).

In his writings, Descartes’ discusses *corporeal images* as representational, or as effects carrying information about perceptions in a form decipherable by the mind and connects these descriptions to the term ideas (Vinci, 1998). Another strong voice with respect to images during this era was Hobbes, who equated image with imagination and defined it as a *decaying sense* (Bursen, 2012). In the time of Kant, the term idea was replaced by concepts. However, images still played a significant role as a connector between our concepts and empirical reality. For Kant, it is the imagination that must synthesize the undeveloped deliverance of the senses, into a coherent, meaningful image (Arnheim, 1969). In the pre-scientific era, there was much debate as to whether images were pictorial, representational, or fleeting remnants of our senses, however there seemed to be a strong acceptance of their existence.

1.2.2 Age of experimental psychology

It was with the emergence of psychology as an experimental science that imagery came into question. This era began with experimentalists, Wilhelm Wundt, William James, Edward Titchener, and C.W. Perky. These researchers, as was true of their predecessors, took image as being central to mental life. Things changed when a former student of Wundt's, Oswald Külpe, challenged this idea. He claimed to have found empirical evidence (often based on Külpe himself or other members of his research team) that certain conscious thoughts are neither imaginal nor perceptual in nature (Holt, 1964; Thomas, 1999). He essentially was challenging the whole idea of imagery being central to thought. Külpe's results were challenged on several grounds by Wundt, Titchener and others, and questions around reproducible findings and method were fiercely debated, and a bitter dispute ensued. This *imageless thought* controversy, had a profound effect on the development of scientific psychology and philosophy (Holt, 1964). Questions surrounding productive forms of method were largely criticized for such an introspective act. Trust in the observers' ability to analyze and report their conscious experience was denounced by John Watson, who proposed a new methodological approach based on observable behavior and performance. Many psychologists became essentially disillusioned with the whole notion of mental imagery and our ability to study it, which led many scientists of the time to either avoid seriously considering the topic, treating it dismissively, or in some extreme cases denying the existence of the phenomenon all together (Ericsson, 2003; Holt, 1964).

1.2.3 Cognitive science

In the early twentieth-century, the analytical philosophy movement, which continues to deeply influence large numbers of English-speaking philosophers, originated from the hope that philosophical problems could be definitively solved through the analysis of language, using the tools of formal logic. As a result, it treated language as the fundamental medium of thought (Thomas, 1999), and argued against the traditional view of linguistic meaning deriving from images in the mind (notably Frege, Wittgenstein, and Schlick). These arguments were widely accepted, and imagery was largely pushed to the side during this time.

With the downgrading of the Behaviorist movement in the 1960s and early 1970s, a revival of interest in imagery took hold as the cognitive revolution in psychology began. In an article entitled *Imagery: the return of the ostracized* (1964), Holt outlines a number of impetus' for its return, some of which were:

> Radar operators who have to monitor a scope for long periods; long-distance truck drivers in night runs over turnpikes, but also other victims of "highway hypnosis"; jet pilots flying straight and level at high altitudes; operators of snowcats and other such vehicles of polar exploration, when surrounded by snowstorms—all of these persons have been troubled by vivid imagery, largely visual but often kinesthetic or auditory, which they may take momentarily for reality. In such a situation, when serious accidents can occur on its account, practical people are not likely to be impressed by the argument that imagery is unworthy of study because it is "mentalistic". (p. 257)

As a result of this renewed interest in image, there emerged a number of associations and journals during the 1970s with imagery as the central theme, such as the *International Imagery Association*, *American Association for the Study of Mental Imagery*, peer reviewed *Journal of Mental Imagery,* along with the journal *Imagination, Cognition and Personality.* During this time, two areas of study became prominent—mnemonics (Paivio, 1971/2013) and the spatial properties of imagery (Shepard & Cooper, 1986).

The study of spatial imagery done by Shepard, Kosslyn, Neisser and others argued that visual mental imagery has inherently spatial properties, and represents in an "analog" fashion that is quite different from the way in which language and other symbolic systems are represented (Kosslyn, Thompson, & Ganis, 2006). However, many who were strongly committed to a computational view of the mind, firmly rejected this conception of imagery (Kosslyn, Thompson, & Ganis, 2006). A very influential article written by Pylyshyn (1973) describes his critique of pictorial or analog theories of imagery in favour of images being descriptive in nature. So ensued another lively and high-profile theoretical debate—*the analog-propositional debate* or what is sometimes called the *picture-description debate*—challenging the nature of mental imagery and of mental representation in general (Thomas, 1999). Yet, few of the best-known imagery researchers were committed to the straightforward picture theory of imagery that Pylyshyn seemed to be criticising. Paivio, in response to Pylyshyn, quite explicitly rejected the picture metaphor and suggested, instead, that imagery is "a dynamic process more like active perception than a passive recorder of experience" (1977, p. 51). Shepard (1978) also rejected the picture metaphor and related it more to perceptual anticipation or readiness to recognize. Shepard aligned his ideas with Gestalt field theory and Neisser, also a critic of Pylyshyn's, aligned himself with the *ecological psychology* of Gibson (1979). Yet the most prominent figure on this side of the debate became Kosslyn, who followed a computational functionalist theory, which surprisingly enough was the same theory Pylyshyn was aligned with—Kosslyn defending computational pictoralism, and Pylyshyn, computational description theory. During this time of heated debate in the 1970s, cognitive theories based on symbolic computation, dominated the scene, but were not the only theories put forth. Taylor and Skinner looked for ways to assimilate imagery into Behaviorism, and another theory supported by several researchers, although it had little impact at the time, was *enactivism*—which has more recently been spurred on by developments in perceptual theory (Thomas, 1999). This theory depends on the idea that perception is not mere passive receptivity but a form of action. Imagery is then experienced when someone persists in acting out the seeking of some particular information in its known absence (Thomas, 1999). These ideas of Gibson's (1979) and later Maturana and Varela (1987) are, relatively speaking, just beginning to find a grounding in this debate of thought and cognition, yet there has been very little study of imagery in connection to the enactivist theory (Thomas, 1999).

There is still much debate surrounding imagery and its nature. Yet, what has come full circle and is continuing to grow is its importance in thought (Mancosu, 2005). The dominant presence of images associated with thought throughout history speaks to its relevance (see Figure 1). It is essential and extremely enlightening to look at where we have come from and realize that many of the same theories from the time of Aristotle and Plato are still in play today. It is important to relinquish our attachment to how we currently know and understand the world—in order to begin the act of interpreting. Through these actions, there exists the hope that we can begin to question what meaning has been imposed rather than interpreted. Through this pushing aside we enter into the *hermeneutic circle* (Gadamer, 1960/1989), where we begin to move back-and-forth between tradition and interpretation. It is reflecting on our own histories and the

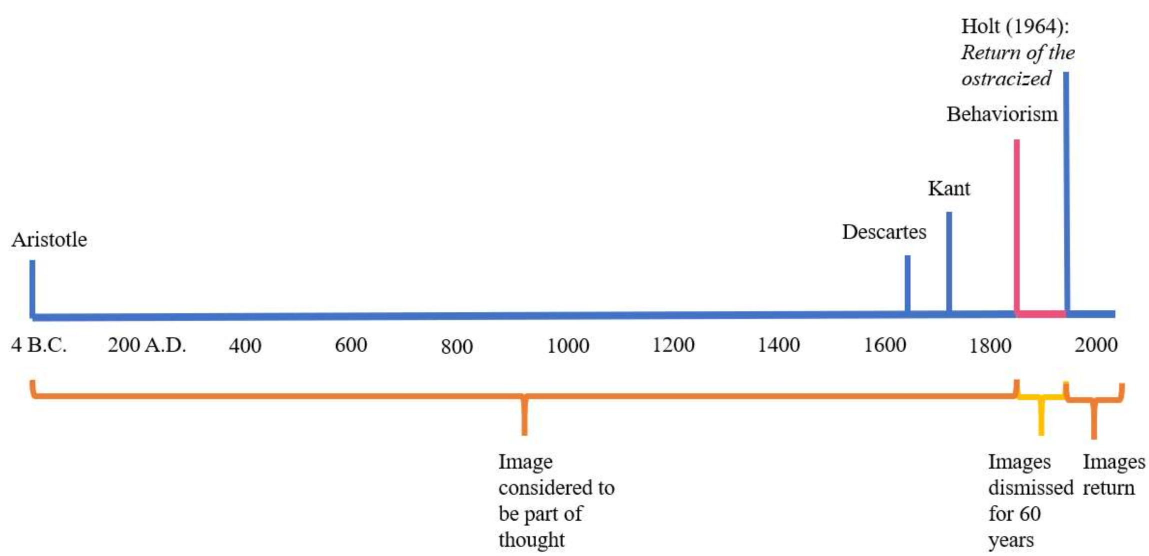


Figure 1: Imagery as a mode of thought timeline

meanings that exist within them, that allows for the possibility of containing their influence, as we attempt to deepen our understanding.

1.3 Horizon of dyslexia.

As with all the other horizons discussed thus far, this one is also filled with complexity. There are so many different twisted and winding pathways that I could go down in order to discuss this topic. As a result, I must reiterate that this is by no means exhaustive. My area of focus for this section will be dyslexia which is currently in Canada categorized as a learning disability (LD) or disorder. The reason for this focus on the learning disability subgroup dyslexia is because this subgroup is thought to make up about 80 percent of the LD community (Shaywitz S. , 2017). Typically, in a paper discussing mathematics one would not expect there to be much discussion about reading; however, as reading is the dyslexic's Achilles heel, I feel it is rather unavoidable.

The most up-to-date definition of dyslexia is written in the 2015 bipartisan Cassidy-Mikulski Senate Resolution 275: "Dyslexia is defined as an unexpected difficulty in reading in an individual who has the intelligence to be a much better reader; dyslexia reflects a difficulty in getting to the individual sounds of spoken language which typically impacts speaking (word retrieval), reading (accuracy and fluency), spelling, and often, learning a second language." Mathematics may not be included in this definition, but it is well known by those working or studying in the field of dyslexia (Eide & Eide, 2012; West, 1991) that there often exists a comorbidity with dyscalculia. Dyscalculia is a severe difficulty with both mental and written forms of arithmetic calculations. As calculating is such a large area of focus in the early years (Alberta Learning, 2016) this puts many dyslexics on a path towards believing they 'can't do math.' Yet, their area of mind strengths are in visual thinking, reasoning, conceptual thought, and problem solving (Eide & Eide, 2012; Shaywitz S. , 2017; West, 1991; Wolf, 2007), which seems to me to fit rather well with the subject of mathematics.

This cultural view of dyslexia being a disorder has profoundly affected the trajectory of many individual’s lives. In my opinion, dyslexia has a history riddled with social justice issues regarding a portion of the population that some estimate to be as high as twenty percent (Coltheart & Prior, 2006). This is quite a significant portion of the population to label as having a disorder. It is partially for this reason that there is actually much debate about whether dyslexia should be categorized as LD. Andrew Ellis, a British neuropsychologist, made this provocative statement some thirty years ago; “Whatever dyslexia may ultimately turn out to be, it is not a reading disorder” (1985, p. 170). These remarks do not seem so farfetched once you begin to look at it through evolutionary logic and the neuroimaging research of recent years.

1.3.1 The reading brain. The human brain was not prewired to read, as such; there are no ‘reading centers’ in the same way that there are cortical centers or speech and language comprehension centers. Rather, the act of reading uses cortical, subcortical, mid-brain, and cerebellar parallel-processing. In fact, the reading brain makes biologically innovative use of no fewer than seventeen regions in the brain and integrates them in milliseconds (Shaywitz, et al., 1998).

So essentially the brain has created an evolutionarily new function in which it virtually instantaneously synchronizes the abilities to: see small visual features, hear discrete sounds, and retrieve names for things. For some years now, a number of researchers like Wolf have been arguing that “the failure to acquire reading in developmental dyslexia and less severe reading disabilities can be based either on an impediment in one of the regions responsible for doing these ‘other things’, and/or on the ability of these regions to work automatically and in precisely timed synchrony" (Wolf & O'Brien, 2001, p. 124; Wolf & Bowers, 1999). Reading is a very hard complex task, in which we use multiple areas that were not originally intended for this use. Is it fair then to label a much valued and productive brain organization as disabled when our brains were never designed to even do the task of reading? After all, it is our own somewhat recent human invention that is the source of this apparent 'disability.' So, this begs the question, is the individual disabled or the invention poorly designed? If we look to our evolutionary past, this invention is not exactly optimally designed to fit with our neurological tendencies, as we clearly did not possess this type of knowledge at the time.

1.3.2 Oral to written culture. Human life is thought to have begun around 200 000 B.C. And with this new life, soon came language of varying forms. It was not until much later that the written word came on the scene. Socrates' era is very much connected to this time of pivotal change – a crossing over from an oral tradition of knowledge to a written tradition.

1.3.2.1 Oral culture. During the oral tradition, knowledge was passed on through experience and the spoken word. Many lively conversations surrounding philosophical ideas were held in the public square where anyone interested in philosophy could go to participate and be enlightened by *the other's* ideas. I imagine it to be that the judgement of one's words were based on depth of thought, not position in society – an inclusive rallying of like-minded people. Socrates spoke of words – as teeming, living things that can, with guidance, be linked to a search for truth, goodness, and virtue (Plato, The collected dialogues, 1961). For Socrates, "'living speech,' represented dynamic entities – full of meanings, sounds, melody, stress, intonation, and rhythms – ready to be uncovered layer by layer through examination and dialogue" (Wolf, 2007, p. 73).

If they could have been present, I believe that both Vygotsky and Gadamer would have sat nodding their heads as Socrates shared and expressed his thoughts on “living speech” and the value of dialogue. In Vygotsky’s classic work, *Thought and language* (Vygotsky, 1934/2012), he described the intensely generative relationship between word and thought and between teacher and learner. Vygotsky (1934/2012) held that social interaction plays a pivotal role in developing a child’s ever deepening relationship between words and concepts. Likewise, Gadamer places conversation at the center of new understanding, “as each one speaks – and more importantly, listens – to the other” (Moules, McCaffrey, Field, & Laing, 2015, p. 41).

1.3.2.2 Written culture. However, Socrates also argued that the written word was a form of dead discourse, that it doomed the dialogic process which he saw as the heart of education (Plato, 1961). In *Phaedrus* 274 Socrates says that, “writing will create forgetfulness in the learners' souls, because they will not use their memories; they will trust to the external written characters and not remember of themselves.” He stated that writing represented, “not truth but only the semblance of truth.” Written words, “seem to talk to you as though they were intelligent, but if you ask them anything about what they say, from a desire to be instructed, they go on telling you the same thing for ever.” The moment Socrates’ discussion switches to discussing the written word, I would think that both Vygotsky and Gadamer would very respectfully hold a different view point, as would most people within our current culture. Not that they would be wrong. It would seem preposterous to dispute the value that the written word has brought and continues to bring to our society – including Socrates’ own thoughts, which would never have been known if it were not for the written word. However, based on Socrates’ description, clearly something is lost in this transition from oral to written.

Before this transition, a struggle to read and write would have not been of any consequence. Dyslexics, known for their visual and creative modes of thought, would not have been considered learning disabled prior to this, they would have likely been the builders, inventors, story tellers, and business people of their communities (Eide & Eide, 2012; West, 1991; Wolf, 2007); as according to Eide and Eide (2012), there are specific MIND strengths within this profile, for which each dyslexic has varying levels: M-Strength being Material Reasoning, the ability to manipulate images or models in the mind's eye; I-Strengths being Interconnected Reasoning, which is the ability to spot, understand, and reason about connections and relationships (e.g., analogies, metaphors, systems, patterns); N-Strengths being Narrative Reasoning, which is a strong episodic memory and can reason using fragments of memory formed from past personal experience (i.e., using cases, examples, and simulations rather than abstract reasoning from principles); D-Strengths being Dynamic Reasoning, which is the ability of prediction, a strength with insight based problem solving when given incomplete information (Eide & Eide, 2012). I imagine with these skills and abilities that they would have been considered very valuable and productive members of society. Yet, as the value of the written word grew, the dyslexic mind organization became more and more devalued.

History of communication technology. Although writing can be found as early as 3500 B.C. (see Figure 2) it did not become such a dominant part of our culture until the invention of the printing press in the 15th century. Only 500 years ago, this introduction of mass printing news and other information gave access to the majority of the population. Printing helped increase literacy among lay people, this allowed many more people access to new ideas and modes of thought with the exception of the dyslexics and those without access to education. Increasingly this ability to read and write became a social filter between the white- and blue-collar workers (Bishop, 1993).

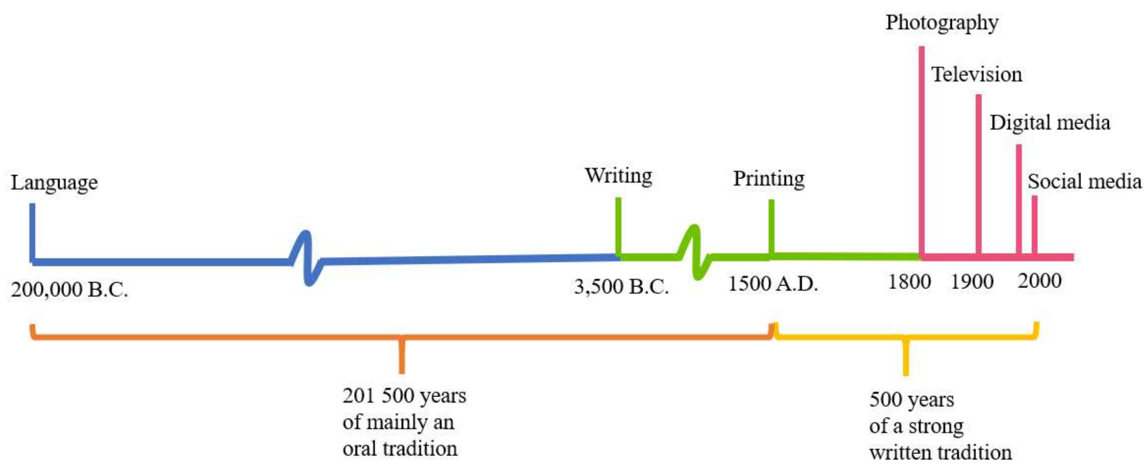


*Figure 2:*Information for this timeline was collected from *A history of mass communication (Fang, 1997).*

After approximately 200,000 years of being considered ‘typical’ and valuable members of society, dyslexics began to be pushed aside – marginalized. As educating the masses became more and more focused around the written word, dyslexics became lesser. Their strong reasoning skills were no match for this symbolic language which was extremely challenging to reason your way through.

Twenty-six letters that look nothing like a sound and then consider putting those letters in a sequence, such as ‘ough’, which in English presents with seven different sounds (ie. through, cough, enough, dough, bough, borough, bought) or the sound ‘oo’ that has five different spellings (ie. root, ruin, rude, new, through). With all this variability it makes reading extremely hard, but spelling is then a whole other challenge unto itself. For example, let us consider writing the sentence, “The cliff was extremely high.” If a dyslexic student is fortunate enough to receive intense phonemic instruction, which is currently still the most widely accepted pathway, then they are expected to use their reasoning skills in order to spell. Below is a description, from the perspective of an older student who can just simply remember the spelling, compared to a dyslexic of the same age who struggles to visually remember the spelling of words, and must rely on their knowledge of phonemes, graphemes, and spelling rules.

Table 1: Non-dyslexic versus a dyslexic who struggles to visually remember the spelling for words.

Non-dyslexic	Dyslexic
The cliff was extremely high.	*the*
This child then continues on to write the rest of their story.	Oh, forgot to capitalize.
	Erase. Rewrite – *The*
	space
	'cliff': /k/ sound which could be the letter 'c' or a 'k'. I do not know the rule for that. So, I will just pick one.
	k
	/l/ sound- *kl*
	/i/ sound - *kli*
	/f/ sound. Oh, this word has one of those special letters f, l, s, or z, at the end. Okay, was the previous vowel short or long? 'cliff'. Okay, short. Now, is it a one syllable word? 'cliff'. Yes, it is. So, I need to double the /f/
	kliff
	space
	'was': /w/ sound – *w*
	The /u/ sound could be spelt with a 'u' or 'ou'. I do not know. So, I will just guess.
	wu
	The /z/ sound could be 's', or 'z'. Hm. I will just guess.
	wuz
	space
	'extremely': /x/ sound – *x*
	/s/, /t/, /r/ sound – *xstr*
	/e/ sound. Oh, this is a long vowel sound. There are seven different ways. Could be 'e' with a silent 'e,' or 'ee,' or 'ea,' or 'i before e except after c'. Since there is no 'c,' it might be 'ie' but it couldn't be 'ei'. It could be 'y' but since it is not at the end of the word probably not. Or is this vowel at the end of the syllable in which case it would be just an 'e'. 'ex-treme-ly'. Okay, so it is not at the end of the syllable. So, I will just pick one.

	xstree
	/m/ sound – *xstreem*
	/l/ sound – *xstreeml*
	/e/ sound. Ugh, long /e/ sound again, because it is at the end of the word, I'll go with a 'y'.
	xstreemly
	space
	'high': /h/ sound – *h*
	/i/ sound. Could be 'y', or 'i' with a silent 'e', or /igh/. Since it is at the end, I'll go with 'y'.
	hy
	The kliff wuz xstreemly hy
	Oh, I have to remember to put in a period.
	The kliff wuz xstreemly hy.

By this point the student is totally exhausted and no doubt has lost interesting in telling their story. Clearly, they have demonstrated a much deeper understanding of the logic behind the English language than their counterpart in the column beside; who has just rote memorized how to spell those words. Yet, they got it all right and the student with all this insight got it wrong.

When the visual memory of the word is missing, the logic of the written code is then supposedly their entry point, however, the "logic" is so convoluted, that this, too, is unmanageable. It is not surprising that many dyslexics give up even wanting to try and often drop out of school. For much of history, this inability to read and spell would have been interpreted and judged as a lack of intelligence.

1.3.4 Birth of dyslexia as a disorder. During this time frame, dyslexics, generally, would have been pushed to the working class as their inability to read would have in many cases prevented extended schooling. It was not until about 120 years ago, November 1896, that a doctor in Sussex, England, published the first description of a phenomenon that would come to be known as developmental dyslexia. "Percy F., ... aged 14, ... has always been a bright and intelligent boy," wrote W. Pringle Morgan in the British Medical Journal, "quick at games, and in no way inferior to others of his age. His great difficulty has been—and is now—his inability to learn to read" (Shaywitz S. E., Dyslexia, 1996). In that brief introduction, Morgan captured the paradox of the profound and persistent difficulties some very bright people face in learning to read. It was as true today as it was in 1896, that reading ability is considered synonymous with intelligence; most people assume that if someone is smart, motivated and schooled, they will learn to read.

Everyone has areas of strength and weakness, yet some are considered more acceptable in our culture than others. If you are unathletic or struggle to learn music, you are not considered to be disabled or have a disorder, yet the struggle to read often causes a child's life to take a dramatic turn.

- Study of prevalence of dyslexia with young offenders in Scotland, 50% were found to be dyslexic to some degree (Kirk & Reid, 2001).
- Study of prevalence of dyslexia in Swedish prison inmates found 41% dyslexic (Jensen, Lindgren, Meurling, Ingvar, & Levander, 1999).
- Study of 253 subjects selected randomly from more than 130,000 Texas prison inmates found that 47.8% of the inmates were dyslexic (Moody, et al., 2000).

This creates a bleak picture; yet, there are many who have this profile that once they leave school behind are able to thrive based on their patterns of exceptionality (Eide & Eide, 2012). There are too many to name, but many exist as scientists (Ann Bancroft), engineers (Thomas Edison), entrepreneurs (Steve Jobs), filmmakers (Stephen Spielberg), artists (Andy Warhol), actors (Tom Cruise), politicians (Winston Churchill), authors (Scott F. Fitzgerald), poets (William Butler Yeats) (Eide & Eide, 2012; West, 1991), and athletes; one such example is Jackie Stewart, a Scottish racing driver, who won twenty-seven Grand Prix titles, was knighted by Prince Charles, and had one of the world's most successful racing careers before he retired. He speaks often of his lived experience growing up in a system that is centered around our human invention of the written word and how his early school years had a profound impact on him. At an international scientific conference on dyslexia he concluded his speech with, "You will never understand what it feels like to be dyslexic. No matter how long you have worked in this area, no matter if your own children are dyslexic, you will never understand what it feels like to be humiliated your entire childhood and taught every day to believe that you will never succeed at anything" (Stewart J. , 2001).

As a dyslexic and a parent of a child with dyslexia, I know exactly what Jackie Stewart is talking about. This same dyslexia story is replayed over and over again with minor variations all around the world, and is beautifully and accurately described by another researcher and parent of a dyslexic child, Maryanne Wolf (2007), in her book:

> A bright child, let's say a boy, arrives at school full of life and enthusiasm; he tries hard to learn to read like everyone else, but unlike everyone else he can't seem to learn how; he's told by his parents to try harder; he's told by his teachers that he's 'not working to potential'; he's told by other children that he's a 'retard' and a 'moron'; he gets a resounding message that he's not going to amount to much; and he leaves school bearing little resemblance to the enthusiastic child he was when he entered. One can only wonder how many times this tragic story has been repeated, just because of failure at learning to read. (Wolf, 2007, p. 166).

These same feelings of despair and failure can be used to describe a student in math class who tries and tries but cannot find success and looks around in disbelief at what others are able to do with what seems like minimal effort. Struggle with math on top of struggling to read and write is a recipe for disaster in our current system. It is not surprising that many give up on school altogether. As is stated by Statistics Canada, "Adults with a developmental disability were four times more likely to have not completed high school compared to those without disabilities (53.6% versus 13.1%). As well, those without any disability were three times more likely than those with a developmental disability to have completed postsecondary credentials (61.1% versus 18.9%)" (Statistics Canada, 2012).

So, can a disorder be based on a skill our brains were never intended for? A brain that has for hundreds of thousands of years thrived, but is now disabled? The term learning disability, in fact, was only first written about in the last 50 or so years and described here as:

> a retardation, disorder or delayed development in one or more of the processes of speech, language, reading, writing, arithmetic, or other school subject resulting from a psychological handicap caused by a possible cerebral dysfunction and/or emotional or behavioral disturbances. It is not the result of mental retardation, sensory deprivation, or cultural and instructional factors. (Kirk S. A., p. 263)

In this description, notice the absence of art, sport, or music. If you struggle in these areas you are not considered disabled. These are also areas of school learning, but we have constructed our own societal views of what are acceptable areas of struggle and what are unacceptable.

The brain up until quite recently in our evolutionary history was not expected to be able to read. Yet, societally it has basically become synonymous with intelligence, and those who struggle to become proficient with it we label as disabled or having a disorder.

Here we are again at a pivotal moment in history, when our written tradition is transitioning to a visual tradition. As Kirrane (1992) states, "the potential for 'visual culture' to displace 'print culture' is an idea with implications as profound as the shift from oral culture to print culture” (p. 58). Within this transition, there is inevitably going to be value lost, just as Socrates feared with the written word, but there is also the possibility of value gained. There is the possibility of a balancing upswing in the realm of visual and spatial reasoning. My hope is that our western society would place their emphasis on thought as being central towards the path to wisdom, with less emphasis on any one particular medium through which thought is expressed. And that we would proceed with caution into this next era so as to not marginalize another portion of our population due to their weakness with visualizing, such as those with aphantasia, which is a struggle or inability to see in the mind’s eye and voluntarily visualize imagery (Marks, 1973).

So then let us turn our attention to this idea of thought rather than reading and writing and look at what are some possible elements that make up thought and how can we better understand and then strengthen them. In this next section, I will delve more deeply into thought and the role that images might play in its growth and development.

Chapter Two: The Phenomenon

Stepping outside my door one spring morning, my senses are struck; my nose with a subtle promise of rain, my eyes with a white thick mist hovering over the contrasting vibrant green grass, as the cool crisp air draws my skin to attention. This stimulus involuntarily calls to my consciousness a scene from my childhood of walking along a smooth yellowish stained stone walkway in Ireland with my cousins on our way to the nearby sweet shop. These entities that have entered my thoughts have not transported me to another place, but rather they have shifted my focus. My front lawn has become my background and my thought has become my foreground, not as distinct entities, but as both existing in the same time and place.

My thought is a mixture of smell, touch, and very specific visual details with other non-specific visual details. I do not see the faces of my cousins, but I know they are my cousins, yet I am unsure how many are present. Similarly, in this scene, the grass is also a vibrant green, the air smells of moisture and is crisp and thick with mist. I sense the movement of jumping over a shiny single black chained fence connected by shiny black cylindrical posts lining the walkway. I cannot be sure who jumped only that the movement is there. The scene is short lived but the pleasant sense of Ireland lingers in my thoughts, and my foreground returns to my movement towards the car with my thoughts of Ireland in the background. All of this occurring in a brief flash of time.

In the moments before walking out the door, my thoughts contained no version of Ireland or sweet shops. The stimulus of a cool misty morning hitting my senses brought something to the foreground of my mind. These presences had a strong visual component, but it was not just visual, there seemed to be a mixture of many things. What can these presences be? A memory? A concept? An idea? A mental image? Or some combination thereof? And this idea of presence carries with it the notion of a spatial component; as though it entered something or somewhere. What is that something or somewhere? A space of visualizing? A space of consciousness? Or something else?

What is this entity that is swept into the foreground as a result of a stimulus? How are these entities formed? How do we come to “have” them? To where are they summoned? These are the subjects I am interested in exploring further; not with the hope of conclusion, but more with the hope of movement towards a deeper understanding. The area of understanding I am interested in deepening is specifically within the subject of mathematics. What I am interested in exploring are the foundational questions of thought and cognition as a starting point toward studying the growth of mathematical understanding. I desire to look more closely at the initial stages of growth in our knowledge structures: *Image Making* and *Image Having* (Pirie & Kieren, 1994). This journey for me began by focusing on the role of visualization in mathematical understanding, however, as with any endeavor our initial understandings grow and evolve the more we engage with topic.

When I wrote my proposal for this study my question was: What role might visualization play in the growth of mathematical images? I believed visualization to be the key factor, which I still believe to be true. Yet, once I had data in hand and I began exploring what factors were at play towards the growth of mathematical images, the more spatial reasoning seemed to emerge as a dominant factor. As a result, I broadened my focus from purely visualization to spatial reasoning. The main question this study has grown to explore is: what role might spatial reasoning play in the growth of mathematical images? In order to deepen my understanding of spatial reasoning, I explore some topics that I believe are intimately connected to spatial reasoning such as: perception, image, language, and gesture (see Figure 3). Throughout the next few sections, I will address my current understanding of each of these players in order to situate myself within the study of this phenomenon. I do this with the recognition that each of these players cannot be addressed with any sense of completeness as each is a subject of study on its own. I also do not presume to suggest that these are the only ones a play, rather these are the foundational ideas towards spatial reasoning that I have explored.

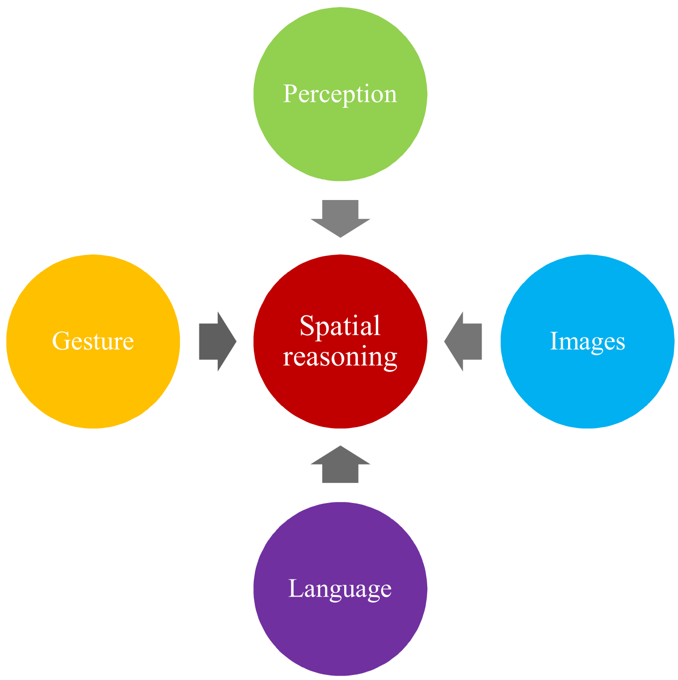


Figure 3: Elements that are connected to spatial reasoning.

2.1 Perception

"All perceiving is also thinking, all reasoning is also intuition, all observation is also invention" (Arnheim, 1954, p. 5). Referring back to the opening scene of my front lawn, as my thoughts of Ireland gradually dissipate, I move towards my car. I see my car, but do I actually see the whole car? Is seeing a purely sensory experience? If it was purely sensory I would only see maybe the front and one side of my car. Yet I do not perceive it as such. I perceive it as a whole car. I may not see the other side and rear of the car or the inside seats and the steering wheel yet I perceive them as the continuation of the car (Gibson, 1979). When I perceive an object, the object is never given in its totality but always incompletely, with a restricted profile. What I see before me is not the sum of all its parts, but rather more than what I can actually take in through my senses.

As defined in Gestalt psychology, “we do not perceive in sums of sensations, but in meaningful, structured and organized wholes, and that most basic perception is always a figure against a ground” (Nilsen, 2008, p. 1). We intuitively invent or fill in the missing pieces. What is taken in through our eyes can never be fully seen, our past stored sensations and perceptions must bring to light the sensitivities for our present perceptions. These stored perceptions are built up over time. They are thickened by “the most recent phase of a stream of innumerable similar acts, performed in the past and surviving in memory. Similarly, the experiences of the present, stored and amalgamated with the yield of the past, precondition the percepts of the future” (Arnheim, 1969, p. 80). The influence of the past on the perception of the present is intensely powerful.

Virgil is a man who was blinded as a very young child and acquired sight in adulthood through surgery. The description Oliver Sacks (1996) gives of Virgil’s experience with his fiancée standing before him as he opens his eyes for the first time after surgery, is one of disappointment for both Virgil and his fiancée. They had expected a moment of jubilation but instead there was no reaction just silence. Virgil described seeing a mixture of light and color. Yet, his fiancée stood right before his eyes. Virgil could not instantly see for he had no past experience with seeing. He could not see the face of his fiancée who was standing right in front of him, for he had no visual knowledge of a face. In fact, Virgil’s experience with his new found sight was far from positive. It did not fill this massive void in his thoughts and understandings, for his thoughts and understandings were not lacking in structure. Through Virgil's personal past experiences, he had grown well-formed knowledge structures for the world around him. In fact, Virgil continued to interact with the world in the same way he did before. He spatially timed how far it was from his porch steps to the front door. He did not turn to look at people's faces when they spoke to him. He could not visually distinguish between his cat and dog but continued to reach down and touch them in order to identify which pet was in front of him. Vision was not part of his current knowledge structures for the world. To see, a person must have visual impressions that they understand. This new ability of Virgil's did not compute with his previous understandings and perceptions. Virgil went on to struggle with major bouts of depression and episodes of blindness, yet there was nothing wrong with his vision. It was as though his mind just stopped coping and he needed to turn off the visual input.

Perception is a skilled act (Noë, 2004). In those sighted from birth, visual sensation is smoothly integrated with capacities for thought, and for movement; we naturally turn our eyes to objects of interest. A distinct sound makes us turn in that direction. A ball is thrown toward us and we automatically duck. A person speaks to us, we turn toward him or her. In this sort of way, and in countless ways like this, sensory impressions are immediately coupled with spontaneous movement (Noë, 2004). This coupling is missing for Virgil. Virgil lacks understanding of the sensori-motor significance of his impressions; he lacks knowledge of the way the stimulation varies as he moves in his new visual world. He is still, to a substantial degree, blind, as his visual impressions are without structural knowledge.

Vision and perception are not processes that passively register or reproduce what happens in reality. Vision and perception are active, creative, and imaginative understanding. The act of perceiving is what structures and orders the information given by the world around us into determinable forms. We understand because this structuring and ordering is a part of our relationship with reality. Without our ability to order we could not understand at all (Arnheim, 1969). The imaginative process of perceiving is enabled through a skilled process developed over time. It is anticipatory. Our perceptions do not lag behind our sensory input. I do not see the front and side of the car and then fill in the rest afterward. There is no lag, my mind skillfully anticipates, creates, and imagines into understanding. These mundane everyday occurrences are actually amazing acts of skillful invention.

2.2 Images: the work of art

Järvilehto (1998) uses a beautiful analogy to describe this process of mental activity.

> Let's look at the action of an artist when he [sic] is preparing a piece of art. Where is "painting" located when the fine movements of the hand and fingers create a picture on the canvas—in the brain, in the hands, in the paintbrush, or on the canvas? If we destroy some of these elements, it becomes more difficult to create this piece of art. Some of these elements may be more easily substituted than some other, but in the act of painting they all are necessary. Can we say that the process of painting is located in the part which seems to be most active or important?
>
> No, of course not, because painting is a process which is realized as a whole organization of elements which are located in different parts of the world. This organization is realized as a totality in the painting. If some element, even a very tiny one, was missing the painting would not be the same or it would not be ready at all. Therefore, all elements are active in relation to the result of action; none of them is passively participating in the result. (p. 336)

This is similar to my understanding for the process of perception and thought. It is challenging to define what all the elements are in this process, rather there is just an acknowledgment that its "organization is realized as a totality" (Järvilehto, 1998, p. 336) in the resultant image or knowledge structure produced. And we understand the process to be rather cyclical and ever thickening; I perceive the present based on my past which influences my future perceptions. This system of growth develops images that are an accumulation of our personal knowledge about our understanding of the environment. These images are continually changing and growing based on a stream of perceptual encounters and conceptual revelations (Arnheim, 1969; von Glasersfeld, 1987). They have a strong visual component (Arnheim, 1969) and sense of structure (Lakoff & Nunez, 2000) which the term image implies yet they are not solely visual nor rigidly structured.

In the literature there seems to be much variation on what term to use when referring to these knowledge structures. It is challenging to determine if each piece of literature is discussing the same thing. For when reading articles on images of thought even the act of reading is dependent upon the researcher's ability to explain their ideas in words and what my background as the reader is on the subject in that particular time and place (Block, 1995; von Glasersfeld, 1987). Putting this aside, the terms I have most commonly encountered are mental image (Rosch, 1999; Zimmermann, W., & Cunningham, S., 1991), image (Pirie & Kieren, 1994), image schemas (Johnson, 2005; Lakoff, G. & Nuñez, R.E., 2001), concept image (Tall, & Vinner, 1981), visual representations (Arcavi, 2003), mental representations (Davis & Maher, 1990), and conceptual structures (von Glasersfeld, 1987). I have chosen to align myself with the more general term *image* and I will attempt to explain why.

2.2.1 Why not *representation*?

Firstly, the term representation does not sit well with my understanding, for the term presupposes that we can delineate objects within our environment and then represent them in our mind. My understanding is that things are much more complex than this. If images are grown from our perceptual experiences, yet these perceptions are only ever partial, and always seen in the middle of something else, as part of a "field" (Merleau-Ponty, 1962, p. 4), then we can never claim to experience an object or situation in its totality or as entirely distinct from its context. Our experience of our environment is a complex mixture of our anticipations, sensations, context, and interactions with others.

The experience of our senses is an example of this complex often unrecognized mixture. What we believe to be distinct is often not, but rather our senses often converge without our awareness. An example would be the McGurk effect (McGurk & MacDonald, 1976), which is challenging to describe in words, and I suggest rather that you experience it— https://www.youtube.com/watch?v=G-lN8vWm3m0. However, I will attempt it linguistically. It is an auditory speech utterance (e.g., a syllable or word) dubbed synchronously with a video of a face articulating a different speech pattern that induces subjects to report "hearing" the sound of the visual pattern rather than the actual sound that is produced. Hearers are influenced by the mismatched visual component. In other words, in this instance the visual information overrides the auditory. For the sound that is "ba", the face is motioning the sound "fa", and when audio is overlaid on the visual, the visual wins; you hear "fa" even though the audio is still "ba". How much of what we perceive is a mixture of senses is hard to say. Therefore, I do not separate the senses but define image as being based on a mixture of senses.

If we then look to the concept of "field" (Merleau-Ponty, 1962, p. 4) there are many illusions that I could draw upon in order to explicate my point. I have chosen the Ebbinghaus illusion (see Figure 4), where the center circle to the left is perceived as smaller than the center circle to the right. They are, however, of equal diameters. How should we describe and interpret this? Are we perceiving it incorrectly or is this just an illustration of the classical gestalt principle according to which the context influences the parts, in which case seeing the center circles as being of equal size would be a misperception, as you would fail to see the available perceptual gestalt. As Husserl (cited in Gallagher, & Zahavi, 2013) describes:

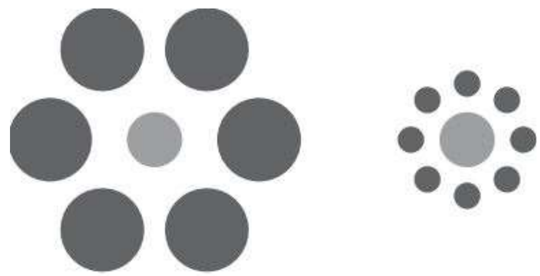


Figure 4: Ebbinghaus Illusion

> Every spatiotemporal perception (ordinarily termed 'external perception') can be deceptive, although it is perception that, according to its own meaning, is a direct apprehension of the thing itself. According to its own meaning it is anticipatory—the anticipation [*Vorgriff*] concerns something cointended—and, in such a radical fashion, that even in the content of that which is perceptually given as itself, there is, on closer inspection, an element of anticipation. In fact, nothing in perception is purely and adequately perceived. (p. 97)

Our experience of our environment is a complex mixture of our anticipations, sensations, context, and interactions with others. Due to our lack of ability to even access our environment in a distinct, direct way, the term representation does not connect with my understanding of this entity of thought.

2.2.2 Why not *mental*?

There are many preconceived ideas connected to the word *mental.* Mental carries with it the implication of being internal, something that occurs in the 'inner mind'. My understandings of the process of thought connect with an enactivist view of cognition. Enactivism (Maturana, & Varela, 1987) engages in sense-making through a dynamic coupling with the environment. We 'enact' or 'bring forth' a world of significance mutually–the organism with its enacted world. We arrive at an experiential awareness through lived embodiment in the world. Our "cognition is not tied into the workings of an 'inner mind', some cognitive core, but occurs in directed interaction between the body and the world it inhabits" (McGann & Torrance, 2005, p. 184). The mind is not located in the brain or the hand or the 'outside' world, but rather is a product of a lived embodied dynamic coupling with our environment. Just as in the painting analogy, you cannot say that it is occuring or dependent on any one element that is contained in the process, but rather this knowledge structure occurs because of all the elements in the process working together.

Merleau-Ponty (1968) uses the Mobius strip as a metaphor to help explain his phenomenological understanding of the body as *Flesh*. The Mobius strip is a continuous surface of interconnected sides. Rather than a loop, that has an inside surface and an outside surface, as a result of a kind of twisting, one side becomes the other. Inside and outside are no longer separate but intertwined and interconnected. Merleau-Ponty uses the term *Flesh* to support his idea that the body is neither material substance nor the container in which the mind is stored and hence separated from the world. Rather, *Flesh* is the intertwining of the physical body and the world or experience in mutual relation. *Flesh* denotes a body that is integrated with the mind, and entangled within experience. We as beings (organisms) are not objects, but are made up of meaning, which is grown through our embodied existence. For these reasons, I prefer not to engage with the term *mental image* more for reasons of association with the internal mind rather than definition, by using a more general term like *image*, I attempt to avoid these cultural assumptions.

2.2.3 Why *image*?

The terms *concept image, image schema* or *knowledge structure* fit quite well with my understanding, as the focus is on the idea or meaning that is being developed. However, I have chosen to mainly use the more general term *image* as I am aligning my study with the Pirie-Kieren (1994) Dynamical theory for the Growth of Mathematical Understanding. In this theory Pirie and Kieren use the term image, but also idea, and describe it as an elaboration on the constructivist definition of understanding as a continuing process of organizing one's knowledge structures (von Glasersfeld, 1987).

Finally, there is an additional reason that I prefer image; it contains a linguistic connection to imagining. If we return to the painting analogy and accept the conjecture that our experience of our environment is one of intuition and creativity, then just as painting requires a complex system of brain, muscles, nerves, fingers, paint brush, and canvas, so it is with images. Our personal images are used and produced as "a form of action of a living system . . . a system which consists of neurons, of many body parts, and parts of the environment, including other human beings" (Järvilehto, 1998, pp. 337, 340). Our ability to form and grow images is a complex creative whole that cannot be understood by any one part within the system, rather it is produced through an organic dynamic process that is continually changing and growing based on a stream of perceptual encounters and conceptual revelations (Arnheim, 1969; von Glasersfeld, 1987). The reason the term *image* fits my understanding is because I believe this presence to be imaginative, but also somewhat biased toward the visual.

The term *image* is often connected to a visual experience and this association is not an issue for my understanding of this phenomenon, so long as the visual component is not viewed as essential. For example, when we hear Tommy Edison, who was blind from birth, describe his dreams (Edison, 2012); one minute he will be in his experiences of a baseball game and the next he will be in his experiences of his seventh birthday. Just as erratic, but no visual detail. Images are not essentially visual. When I wake up in the morning and the room is dark, I cannot see very well. I feel along the ground for my slippers. Using touch, I can find my slippers, for how they feel is part of my image for my slippers. I open my bedroom door and I can smell coffee brewing. I know it is coffee, for how it smells is part of my image for coffee. This term image does not exclude Tommy Edison from having images. Tommy Edison and Virgil imaginatively anticipate their world just as reflexively as we do, but without vision. I would suppose that their images would have some significant differences, just as a painting produced without a paint brush. The painting still exists, but in a different form. Even within those who are sighted, the extent to which they incorporate and utilize their visual sensations or auditory sensations varies from person to person.

Yet, for most, dreaming and our continual interaction with the world around us are profoundly affected by our visual experience, and, just as in the McGurk effect, the visual overrides the auditory so that no matter how hard we endeavor to hear 'ba' we can only hear 'fa'. I believe for most vision is a fairly dominant sensory input to our biological and socio-cultural being, which is why this implied association of visual with the term image does not present as an issue for me.

> The largest part of the cerebrum is involved in vision and the visual control of movement, the perception and elaboration of words, and the form and color of

> objects. The optic nerve contains over 1 million fibers, compared to 50,000 in the auditory nerve. (Adams, & Victor, 1993, p. 207)

As for the socio-cultural aspect, I think it would not be shocking to state that we live in a world where information is often conveyed in a visual way; and that our culture positions vision as king of the senses. Our common expressions betray our fascination with appearances, for we say: “I see” when we understand, and “I see what you mean,” as if we could, or “I can see where you're coming from,” or “I see where you’re going,” or “I can see through your argument.” We even say: “I just wanted to see what it felt/tasted/smelled/sounded like,” as if we could see feels, tastes, smells, and sounds. Contained in these expressions are not just colloquialisms, but semantically marked preferences for seeing. My point here is merely to highlight what we culturally profess on a daily basis: “a picture is worth a thousand words.”

This primacy of the visual is not altogether a recent phenomenon, for the evidence of Indo-European etymology suggests that it is an ancient pattern, reflected most clearly in the equation seeing = knowing.

> This is from the productive PIE root weid- “to know, to see” (cognates: Sanskrit veda “I know;” Avestan vaeda “I know;” Greek oida, Doric woida “I know,” idein “to see;” Old Irish fis “vision,” find “white,” i.e. “clearly seen,” fiuss “knowledge;” Welsh gwyn, Gaulish vindos, Breton gwenn “white;” Gothic, Old Swedish, Old English witan “to know;” Gothic weitan “to see;” English wise, German wissen “to know;” Lithuanian vysti “to see;” Bulgarian vidya “I see;” Polish widzieć “to see,” wiedzieć “to know;” Russian videt’ “to see,” vest’ “news,” Old Russian vedat’ “to know”). (Harper, 2001)

From this root we have, among others, in English "idea," "vision," "view," "evidence," "wit," "wisdom," "witness," "wise," and "visible." Our legal system confirms this preference in its "evidence of eye witnesses." Yet, the dominance of our preference for vision is not one to be idly dismissed, for as Kirrane (1992) states, "the potential for 'visual culture' to displace 'print culture' is an idea with implications as profound as the shift from oral culture to print culture" (p. 58). Did our biology create this socio-cultural emphasis, or did the socio-culture over time influence the biology, or both? It is hard to say. It is my belief however, that this resultant emphasis exists.

For reasons of the environment-organism relationship, anticipatory-imaginative thought, along with our biological and socio-cultural primacy of the visual, the term image strikes me as the most fitting for my understanding of this resultant 'work of art' that is produced through perception and thought.

2.3 Cultivating growth in mathematical images

In accepting this 'work of art' as an element of thought, our next steps would seem to be: How are these images created and what causes them to grow? I cannot speak to all elements that are contained within this process, for I do not claim to know, but I can turn our attention to a few elements we have already discussed as being involved. Let us reflect on the sentence—*The black dog chased the cat up a tree*. How do you come to comprehend this sentence? What enters your thoughts? When you read the word *dog*, does a dog appear in your mind's eye? If so, is it a specific dog? Or is it just a general sense of dog. A sense that may have very little specific visual detail, but yet somehow you know it to be a dog. Is there a sense of movement toward a non-specific cat that goes up a non-specific tree? How were these images formed? If presented with the verbal stimulus of a neighbor's dog, a very specific image comes to the forefront not the previous generic one. Both were signaled by a stimulus of the word dog, yet they have distinctions. How much detail was attached to your image of the neighbor's dog? I would imagine it would depend on how often and closely you have interacted in the past with that dog (Rosch, 1999).

Imagine a child's first encounter with a dog. The animal bounds toward her with great enthusiasm; her ever improving skill of distinguishing between the foreground and background causes her to notice the dog. The dog becomes the foreground of her attention. That child sees the dog, hears the dog, and maybe feels the dog and is told that it is a dog. She may feel and see that it has fur, the dog may lick, bark, and wag its tail. This first experience stores something received through sensation and further develops her skill of distinction within her environment. Following this experience, the thought of the dog is no longer at the forefront of her thoughts, as something else in her environment has caught her attention. Yet, the information is stored in some form somewhere in the background of her thoughts. Sometime later, she encounters another dog and is told again that it is a dog. It does not look exactly the same. How do her thoughts handle this? Her present experience is stored and amalgamated with the "yield of the past" (Arnheim, 1969, p. 80).

From the first encounter, the gestalt principle of foreground and background is at work (Dreyfus & Dreyfus, 1992), initially with the dog itself, the dog being in the foreground, and the surroundings the background. This principle is then applied to the object itself for mental categorization purposes, certain aspects of the dog are simplified and others emphasized based on the skill of recognition developed from past patterning experiences (Arnheim, 1969; Rosch, 1999). What is stored in memory can only ever be partial, for it is largely dependent on what is deemed relevant in the past. The child may initially retain a very simplistic view of the dog by emphasizing only that it walks on four legs; upon another encounter with a cat, horse or cow she may think it is a dog (von Glasersfeld, 1987). If she is then told it is a cat not a dog she must reorganize and make further distinctions, and through multiple encounters and perceptions of various dogs and cats continue to refine her categorizations and form her conception of a generic dog (Rosch, 1999). Conceptions are the formation and skilled organization of these perceptions into broader categories through our recognition of patterns (Rosch, 1999). To the extent that our thoughts are not entirely single minded, the sensational impinges on the perceptual which impinges on the conceptual, but not necessarily in an organized or linear fashion. These are not separate and distinct processes, but woven together in a multitude of ways during the course of our awakened state. Nor is their nature static and rigid, but rather our images are in a constant state of thickening, strengthening, and restructuring.

2.3.1 Nature of images.

Von Glasersfeld (1987) in his article, *Learning as a constructive activity,* describes how we are explorers and builders. We explore and then build up a "picture" (p. 36) of the world. In reading this article, I found myself nodding and connecting to almost every aspect. The one area of disconnect for me was in his use of the term *build*. My understanding of images is more like an organic entity; the term builder does not seem to fit. Building conjures up the concept of taking a separate piece of material and attaching it to another separate piece. They do not meld into one another and become one, but remain as attached distinct pieces, and the growth that has occurred to the building is only that which is in proportion to the size of the piece added. I believe our images to be more organic and in constant motion. Our images stretch out to absorb new perceptions, which stimulates a growth that is not necessarily in proportion with the new material that is received. The perception may stimulate a growth well beyond the mass that has been absorbed. This growth may cause the image to attach or be intertwined with another image that has already been established, causing a massive growth. Or alternatively, this perception may just float to the base of the structure only to help in solidifying or thickening the foundation.

2.3.2 Pirie-Kieren Dynamical Theory.

It is this organic process of growth within images that is foundational to the Pirie-Kieren (1994) Dynamical Theory for the Growth of Mathematical Understanding. Pirie and Kieren (1994) have developed a model for describing this growth of mathematical understanding as a "whole, dynamic, leveled but non-linear, transcendently recursive process" (p. 166). It involves moving back and forth between different ways of knowing, represented within the model as a series of nested layers or levels (see Figure 5).

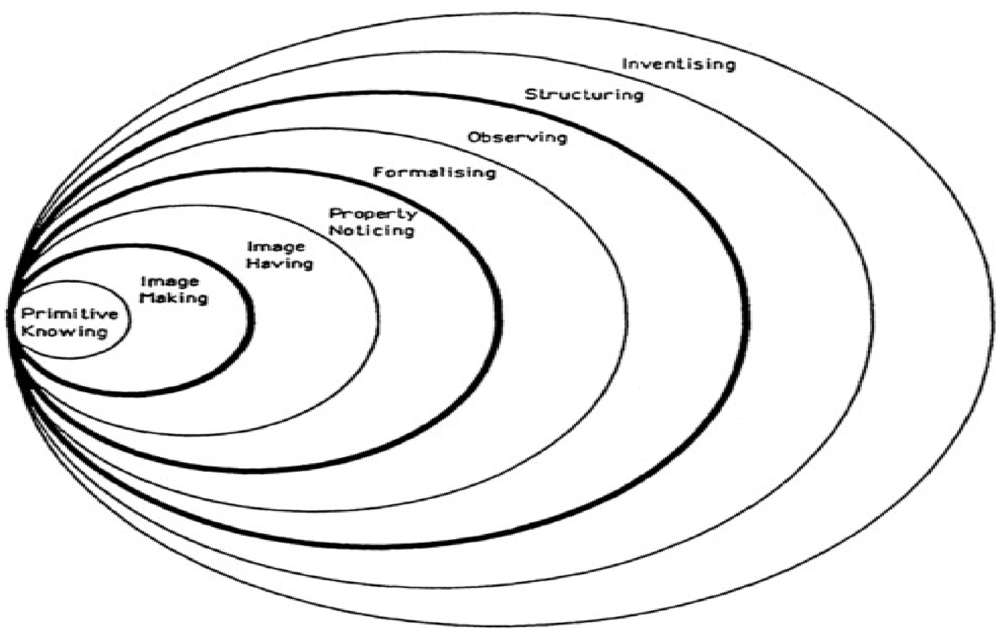


Figure 5: Pirie-Kieren (1994) Dynamical Theory for the Growth of Mathematical Understanding.

Moving from the innermost to the outermost layer, the levels of mathematical understanding are:

- *Primitive Knowing*: the term *Primitive* is not meant to imply low level mathematics, but rather the starting place for your growth within a particular mathematical understanding;

- *Image Making*: involves making distinctions in your previous knowing and then using it in new ways;
- *Image Having*: using a “mental construct” (p. 170) about a topic without having to do the particular activity which created it;
- *Property Noticing*: manipulating or combining certain aspects of one’s images to develop context-specific, relevant properties;
- *Formalising*: abstracting a method or common quality from one’s images;
- *Observing*: reflecting upon or coordinating your formal activities in order to develop theorems;
- *Structuring*: a person considers their formal observation a theory and therefore requires justification or verification of statements through logical or meta-mathematical argument; and
- *Inventising*: a person has a fully structured understanding and can therefore break away from their preconceptions and possibly create new questions which could have the potential to grow into totally new concepts.

(Pirie & Kieren, 1994, pp. 170-171)

Within their model, there is movement that occurs within the structure, a continuous restructuring—a process of returning to earlier levels of understandings in order to thicken and grow the foundations of understanding. This process is neither linear nor fixed—an individual’s growth in understanding may skip over levels, dance back and forth between levels, stay for long periods at one level and then in a moment of insightful intuition, leap past several levels all at once. There is no one path; instead there is the recognition of movement in the process, which is described as “folding back” and “moving out”; it is dynamic. This movement allows for the thickening, strengthening, and reorganizing of increasingly sophisticated and complex understandings (Pirie & Kieren, 1994).

2.3.3 Husserlian distinctions.

The area of my particular interest within this framework is the act of transitioning between *Image Making* and *Image Having* (Pirie, & Kieren, 1994). What elements might play a role in the growth of images to the point of *having.* Yet, I do not refer to *having* in a static state of completeness, for there is always movement, but rather one of usefulness in the absence of the perceived act. For the purpose of further discussions on the ideas of growth for mathematical images, I refer to a set of distinctions made by Husserl (1970). He refers to three ways that an object can be presented—*signitively*, *imaginatively*, and *perceptually*. For instance, I can talk about a withering oak which I have never seen, but have heard is standing in a nearby field; I can view a detailed drawing of the oak tree; or I can perceive the oak tree myself by standing in front of it, hearing the rustle of its leaves, feeling the roughness of its bark, and seeing its arching branches overhead. For Husserl, these different ways of intending are not unrelated. On the contrary, there is a strict hierarchical relation between them, in the sense that the modes can be ranked according to their ability to give us the object as directly, originally, and optimally as possible. The perceptual offerings of an object can be more or less direct according to Husserl.

The lowest and most empty way in which the object can be proposed is in the signitive act. These linguistic acts, whether given orally or in print, certainly have a reference, but apart from that, do not offer any perceptions of the object itself. The imaginative act, which is meant by Husserl to be pictorial in nature, has a certain intuitive content, but it presents the object indirectly. Pictorial acts present the object based on a representation which bears a certain resemblance to the object as seen from a certain perspective. It is only the actual bodily presence, or interaction between us and our environment which gives us the object most directly. This form of intending contains the most in the way of offerings for the object.

All this discussion about objects and representation no doubt seems counter to my earlier discussion of image. However, I describe these Husserlian ideas for I am interested in morphing them somewhat as a way of considering and relating them to meaning rather than object. For Husserl seems to suggest in this perspective that there exists a distinction between us and the external objects that surround us—that they exist in some pure state or form that is unchanging and we are expected to simply discover them—rather than, as discussed earlier, bringing them into existence through the creative act of perception. Husserl (1970), himself, concludes, “nothing in perception is purely and adequately perceived” (p. 97). How then can he claim that we are experiencing them in a ‘direct’ way?

It is for these reasons that I am choosing to morph Husserl’s ideas. If we were to contemplate our own bodies for example, which includes the objective body that can be perceived as an object and the lived body that is the non-object involved in the perceptual process, it might appear at first glance that there are two distinct bodies under consideration. Yet as Gallagher (1986) states,

> A human being neither ‘has’ nor ‘is’ two bodies; the body as it is lived and the body as it appears in objective observation are one and the same body. The lived body is the physiological body. The distinction between lived and objective is a perceptual distinction. The objective body is a perceived body; it is the objectification of a body that is also lived.

This distinction is discussed by Merleau-Ponty (1962) in the following way: “the objective body is not the truth of the phenomenal body, that is, the truth of the body such as we experience it.

The objective body is merely an impoverished image of the phenomenal body" (p. 456). The lived body is understood to have a physiological basis. From this viewpoint the lived body could be defined as "a certain power of action within the frame of the anatomical apparatus" (Merleau-Ponty, 1962, p. 111). For this reason, it is the action of a mathematical idea that holds my interest, and this may or may not be held within the frame of an "anatomical apparatus", which is why I am interested in morphing this Husserlian idea into a way of considering meaning rather than object. Meaning is grown through our embodied existence (Merleau-Ponty, 1962); it is the lived rather than the impoverished image that I am interested in pursuing.

With the hope of a sufficient level of clarification for this apparent contradiction, I turn back to the details of this proposed alteration. Within this reorientation, I suggest the signitive as being the linguistic form of mathematics, the imaginative as the utilization of our created images—visualization, and the perceptual as the enactment of mathematical ideas. The first mode of offering is unchanged from Husserl's original intention, the oral discussion of mathematics and the written symbolic form, which is an indirect and the emptiest way of presenting the meaning within the mathematical idea.

The second form, the imaginative, I would like to alter and discuss this level as the act of visualizing. Husserl describes his version as a pictorial representation of the object, which has a certain level of intuitive content. I propose this is true of visualization as well. It has been well documented in both sports and music that growth that occurs through the act of visualizing. When he was a teenager, my husband's swim coach would have all the swimmers lie on deck with their eyes closed and he would orally describe a setting for a swim race and guide the swimmers through a race, while they lay on their backs imagining themselves in the race. The oral description by the coach would be a signitive offering, but where each swimmer took those oral descriptions imaginatively, I propose, has a different level of offering. In a meta-analysis done to answer the question *Does mental practice enhance performance?*, Driskell, Copper, and Moran (1994) concluded that "mental practice is effective for both cognitive and physical tasks; however, the effect of mental practice is significantly stronger the more a task involves cognitive elements" (p. 485). In this analysis, it is noteworthy to mention that physical practice was the most effective form, and that those who were experienced in physical practice benefited more from mental practice than those who were novices in the task. The article suggests that the reason for this is that "the novices who mentally practiced a physical task may not have sufficient schematic knowledge about successful task performance and may be spending their effort imagining task behaviors that could turn out to be somewhat counterproductive" (p. 490). This supports the idea of the importance of the perceptual level of activity, while also supporting the idea of meaningful offerings through imagining an act—visualizing.

For the final Husserlian category, I refer to Gibson's book (1979), *The ecological approach to visual perception*, in which he introduces his Affordance theory, which states that the world is perceived not only in terms of object shapes and spatial relationships, but also in terms of environmental possibilities for action. Within this description, I feel there is fair justification for claiming the final category of perception to focus on the enactment of a mathematical idea rather than an object. Within these categories my discussion of the offerings is not the bodily presence with an object, but the offering of active or spatial meaning. It is within the spatial enactment of a mathematical idea that meaning is offered or 'lived' in the most direct, original, and optimal way, for we are embodying and experiencing the mathematical idea (Maturana, & Varela, 1987).

It should be noted that these final two Husserlian categories, imaginative and perceptual, are closely connect to an often-ignored aspect within mathematics education—spatial reasoning. Spatial reasoning is essentially our ability to reason our way through space. If we were to attempt to categorize the different types of reasoning, this would be quite challenging, for there are a multitude of actions which could be considered spatial reasoning. Davis, Okamoto, and Whiteley (2015, p. 141) created a wheeled diagram in an attempt to represent the emergent complexity of these spatial reasoning skills. It is through these skills that we can access mathematical ideas. These skills are important as they have been shown to be quite foundational; Mix and Cheng (2012) report that, "the relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related" (p. 206).

Spatial reasoning is more than just passively receiving sensations, but the intentional act of perceiving and then engaging our bodies purposefully (Khan, Francis, & Davis, 2015). Through acting out a mathematical idea there is a co-evolving which occurs in both our mental and physical skills. It is important to note that the actions being discussed are not only physical actions; spatial reasoning encapsulates mental actions as well. To rotate a cube mentally (visualizing) is just as much spatial reasoning as rotating a cube in your hand—both are spatial acts (Linn & Petersen, 1985; Uttal, et al., 2013). As discussed throughout this document, visualizing can be very productive within mathematics. Zimmerman and Cunningham (1991) state that the intuition that mathematical visualization affords is not a vague kind of intuition; rather, it is what gives depth and meaning to understanding. Mental interactive playing and the exercising of our images can stimulate growth in and of themselves without actually engaging our environment (Driskell, Copper, & Moran, 1994).

For our modified Husserlian offerings, I refer to this third category as a perceptual experience for it is a sensory act. It must include some level of active interaction with the environment. The mental rotation of a cube is not a sensory experience and would therefore not be a perceptual experience but rather an imaginative one. No new information from our environment was gathered; rather, information that had already been acquired was used to perform the spatial reasoning act of rotating the imagined cube. The perceptual experience would then only be the rotation of the cube in our hand, which is a sensory experience. Within this study, perceptual experiences are those in which people act out a mathematical idea through engagement with their environment; imaginative experiences are those in which people mentally act out a mathematical idea through the use of their personally grown images. Essentially, spatial reasoning includes both perceptual and imaginative experiences.

Our historical shift in the 19th century towards the symbolic, stripped the mathematics classroom of perceptual offerings for mathematical ideas (Mancosu, 2005). In recent years, the growing shift toward the perceptual ways of intending mathematics in the classroom is by far the most vital shift to take place, but I suggest there is an aspect of meaningful offerings that is often overlooked. In my observations and discussion with classroom teachers as they are beginning to engage with perceptual activities for mathematics, when they attempt to make the shift to the signitive, there are still groups of students who struggle with this transition. The reasons for this are no doubt complex; maybe they did not spend enough time in the perceptual activity; maybe the teacher did not provide an opportunity for the students to 'fold back' (Pirie & Kieren, 1994). I am interested in exploring this disconnect through looking at what factors contribute to students transitioning from Image Making to Image Having and the possible role that spatial reasoning in combination with the signitive might play as a contributing factor.

The growth of mathematical understanding is a complex process, yet I believe that the transition from Image Making to Image Having is a key element. What factors influence the growth of these images? If we accept the idea that images are grown through a dynamic process of restructuring based on a stream of perceptual encounters and conceptual revelations (Arnheim, 1969; von Glasersfeld, 1987), then is it possible that the playing, utilizing, and exercising of these images may help them to strengthen or grow? I consider this act of playing, utilizing, and exercising images to be the act of visualizing. As of yet, I have not come across a description or definition of visualization within the enactivist literature; however, Thomas (1999), in his article entitled *Are theories of imagery theories of imagination?,* discusses in connection to imagery, "an exploration in the absence of the appropriate object" (p. 224). This type of "exploration", although it does not refer to visualization specifically, I feel fits well as a description for visualization or imagination.

2.4 Elements of thought: images, language, and gesture

To resituate ourselves in this discussion and making presumptions on areas of acceptance, let us briefly recap. Perception of the world around us is a skilled act that is built up over time, through our dynamic coupling with the world around us. We creatively anticipate and enact our world based on this learned skill. We have developed patterns of spatially active engagement with fluid boundaries and changing components (Noë, 2009). We creatively interact and explore the three-dimensional world around us. It is through this exploration as embodied beings that we grow knowledge structures or rather images to help us understand our world. These images are an amazing resultant of this coupling between organism and environment, and they function because of this coupling.

Cognition is not something that occurs solely within the brain, rather our mind is embodied. Roth and Thom (2011) contend that "cognition— at least in part—exists in bodily rather than in mental form" (p. 209). Human cognition is embodied because bodily forms of experiences are extended to images and from there, by means of metaphorical and metonymical processes, to signitive or linguistic forms (Lakoff & Johnson, 1999). These signitive forms of mathematics are meaningful only because they activate the bodily forms from which they derive and together with which they form the conception as a whole (Roth & Thom, 2009).

What does all this practically mean? In order to make sense of this, I am reminded of a recent struggle my daughter had with subtraction. Up to this point, she and I have spent a lot of time playing with numbers, and she has demonstrated some good understandings for numbers between zero and one hundred. In the past, we had also worked on positioning numbers on the number line, so with an understanding of her background experience I put some painters' tape on the floor and had her build a number line from zero to thirty. For the past few days we have been progressing through questions on addition and discussing how one part plus another part combines to build a new number, which we labeled as the whole. As she became more comfortable with this idea, I gave her another question, in which the whole was fifteen and one of the parts was five. What could the other part be? We continued with similar questions until she began working quite flexibly between the idea of part-part and part-whole. The majority of our time was spent working with the number line in front of us—as Towers and Davis (2002) note, "it is these repeated Image Making activities which are significant in providing the ground for the deeper understandings" (p. 331). This mathematical activity had much to offer in the way of enacting a mathematical idea.

Based on my presumptions of the expanse of her understanding, I purposefully moved away from the number line and gave her another question. The number line was now not so conveniently located, and we used gestures that related back to the number line. We played with this mental space surrounding us through discussion and gestures that related to our previous enacting. She could choose to go back and look at the number line if she needed to, but our interactive playing with the space seemed to suffice and the flexible use of her images gradually became more fluid. Over the next few days that we worked on this mathematical idea she did a mixture of choosing to go back to the number line and just visualizing, but often played within this space through verbal discussion and gesturing.

This description of my daughter's mathematizing nicely illustrates my understanding of spatial reasoning—which is a combination of movement and visualizing, each feeding off each other, leading to more of a focus around visualizing, but then going back to movement if the visualizing is insufficient for understanding. Spatial reasoning is an act of exploring and utilizing our personally grown images within a space, not a distinctly located space that is inside you or inside the environment, but rather a space that is opened up as a resultant of an interaction between an organism and its environment. We will come back to these ideas in the findings and discussion sections as look at how the participants engaged with gesture and space.

My interpretation of these events contains elements of Image Making, Image Having, folding back, visualizing, looking up, gesturing, language, perceptual, imaginative, and signitive all within a space. These events urge me to question: How might the interaction between these elements be understood? What might cause images to grow and then be stabilized in order to make use of them without having to do the particular activity which created them?

2.4.1 A space.

A physics question posed in a study on the use of images done by Rosenfeld and Kaniel (2011) is:

> A person is standing on the roof of a building holding two balls at the same level. He throws one ball at any speed horizontally to the earth and at the same time he lets the other ball fall freely downward without any initial speed. Which ball will hit the ground first? Explain. (p. 372)

What did you use in your attempt to understand this question? Images? Was it a generic image of a person involved or a specific image? Did you see movement? Was there a ball actually present? Did you use abstract symbols? Upon first reading it did your mind feel blank, which caused you to have to read it again? Did you look up?

The first thing that struck me about the Rosenfeld and Kaniel study (2011), was that in their analysis no grouping was made for participants who did not use visualizing in order to comprehend and solve this question. Rather, all participants were grouped into those who used mainly specific images, general images, abstract, or combined; nor have I in my own poising of this question to friends and family found someone who in describing their thought process does not give a description that would seem to be based on personal images. It seems noteworthy that not a single person responded with, “I do not see anything in my thoughts.” I did notice a lot of variability in the retelling of their thoughts, and varying levels of detail as was also discussed by Rosenfeld and Kaniel (2011), ranging from very specific to general to abstract lines and shapes (Rosch, 1999). I also noticed lots of looking up and gesturing. I was intrigued by these embodied descriptions of how people used their personal images for comprehension. Yet, before asking them to explain their thoughts, there seems just an interaction between text and thought—no gesturing. The lack of gesturing could be a result of there being no need for gesturing, or maybe as a result of gesturing being suppressed due to cultural norms; either way gesturing was not a major participant during the thinking through of the question. When asked to give a verbal description, things changed—people often looked up, and gesturing came alive in the retelling of their thoughts. I interpreted this behavior as what seemed like an interaction occurring between language, gesturing, and their images.

In my own experiments with friends and family using this question, almost every person, save the ones I asked over the phone, looked up at some point in thought or description (Van Rosendaal, 2015). Why is that? Surrounding the concept of visualizing, there seems to be a spatial component. People say things like "My mind is blank". Obviously, they have not lost all their knowledge structures, but yet there is this feeling of emptiness. Where is it empty? Empty is a spatial idea, looking up is a spatial action, and the concept of images entering or "coming to mind" is spatial. What is this spatial element that is going on as we visualize?

Upon reading the physics question a selective stream of relevant images enter, let's call it a space. What is that space? Just as images are a resultant of our organism-environment relationship, could there also be a resultant space for the spatial reasoning work of visualizing. A space of attention for your images. A space that allows you to process, comprehend, organize, and grow your understanding. I conceptualize image and visualizing/spatial reasoning as two distinct components of thought that are bound to one another. One is the presence, the other is the exploration and interaction of these presences. The space in which this occurs does not exist in the brain, as discussed earlier regarding images. Gallagher (2015) also discusses the idea of space in his article *Doing the math*. He speaks of an affordance space, an area that is around our body which has significance for movement, action, attention, accomplishing tasks, and, as he argues, for higher-order conceptual and mathematical cognition. I consider the mind not to be located in the brain or the hand or the ‘outside’ world, but rather it is a space that includes our body and the area around our body, a space opened up as a product of a lived embodied dynamic coupling with the environment around us.

In describing the mind as an opening of a space, where our bodies are a part of that space, this does not give an indication of all the elements contained in this space for I do not claim to know. I have many questions surrounding this concept of mind, yet I feel strongly that one aspect within this conception is the spatial reasoning act of visualizing or mind’s eye—a space where our images grow, play, work, and interact. This notion of images being spatial is also not a new idea for as discussed earlier in the history of horzion section, Shepard, Kosslyn, Neisser and others argue that visual mental imagery has inherently spatial properties (Kosslyn, Thompson, & Ganis, 2006). To imagine then that they exist within a space seems only fitting. Originally, my own imaginings or metaphors surrounding this idea of space were more like that of a playground or workroom for your personal images. However, I became drawn to a different metaphor upon reading a book by a senior fellow at the Neurosciences Institue, Bernard Baars (1997), called, *In the theater of consciousness: The workspace of the mind*. In this book, he uses the metaphor of a theatre with the stage being the space that our images enter and then exit as our attention shifts within our thoughts. This metaphor connects well with my thoughts around images being a work of art, and then visualizing being the interaction or the script that is being played out by our images, often directed by our own narrative voice. Within this conceptual theatre, images are put together for the purpose of comprehension, invention, and reasoning.

Images used and put together in space is further developed by Fauconnier (1994), in his highly influential *mental-spaces* framework, in which he discusses how the mind creates multiple cognitive spaces to mediate its understanding of relations and activities in the world, and to engage in creative thought. He states that,

> Language, as we use it, is but the tip of the iceberg of cognitive construction. As discourse unfolds, much is going on behind the scenes: New domains appear, links are forged, abstract mappings operate, internal structure emerges and spreads, viewpoint and focus keep shifting. Everyday talk and commonsense reasoning are supported by invisible, highly abstract, mental creations, which grammar helps to guide, but does not by itself define. (Fauconnier, 1994, p. xxii)

In a later piece of work called *The way we think,* Fauconnier and Turner (2008) argue that all learning and thinking consists of blends of metaphors based on simple bodily experiences. These blends are then themselves continually blended together into an increasingly rich structure that make up our mental functioning in modern society. A child's entire development consists of learning and navigating these blends. This theory of *conceptual blending,* describes structure-mapping as being inherent in all of our thought processes, and especially in the construction of meaning that "we engage in effortlessly as we conceive the world around us, act upon it, talk about it, and stray beyond it in wild leaps of imagination, fantasy, and creativity" (Fauconnier, 2001, p. 255). A mathematical example of this might be the number line, one mental input space may be our concept of number, a second mental input space may be our concept of line, these blend together and an emerging mental space begins to form which contains our understanding of a number line. Fauconnier and Turner state that "the products of conceptual blending are always imaginative and creative" (2008). These ideas go toward further strengthening Einstein's statement of "Imagination is more important than knowledge" (Einstein, 1929), as knowledge is only possible with imagination.

2.4.2 Interweaving of elements.

Could the spatial use of gesturing in our explanation of our thoughts be a result of our minds creating these multiple cognitive spaces, causing a spatial act like gesturing to become useful? Are we describing a spatial play that is going on in our minds? My son came downstairs this morning and was explaining to me why Halo megabloks is more realistic than Lego. In his defense, there was hand waving, squatting, robotic walking, reaching, grabbing for and then shooting a massive gun. It made me wonder how much of our gesturing has been culturally suppressed. How useful would it be to explain something with our whole bodies like kids do? Not just with our hands, but leaping and bounding around the room connecting with this creative act of visualizing.

Our everyday conversations, gestures, and imaginings are part of the interweaving of a complex cognitive ability. As suggested by Clark (2008), could gesturing be an instance of extended cognition that allows us to off-load some of the cognitive load? Can it add to or supplement processes of mathematical cognition? Studies by Goldin-Meadow and others show that children perform faster and more accurately on math problems when they are allowed to use gestures, in comparison with when they are asked to sit on their hands (Alibali & DiRusso, 1999; Goldin-Meadow et al., 2001). As Roth and Thom (2009) show, “gestures, bodily experiences, and words are but different, one-sided expressions of the mathematical concepts that they metonymically denote” (p. 211). If these “invisible, highly abstract, mental creations” (Fauconnier, 1994, p. xxii) are at play, how might we engage and exercise them through our teaching? As with many of the things already discussed there is interweaving that occurs between the perceptual, imaginative, and signitive, but also between, language, gesturing, visualizing, and our images. These are all abilities that develop and work together, yet how much has this interactive space within a classroom setting been purposefully exercised?

Suppose you and a partner are presented with the question: Are a stack of nickels the height of a person worth more than a row of quarters the height of that same person? Initially, there is the job of comprehending what the question is asking. Each of your personal images of *stack, nickel, height, person, more, row,* and *quarters* are swept into what I propose to be your space for visualizing; as both you and your partner attempt to comprehend by visualizing the scene described in the question—imaginative. You discuss your understandings using gestures, objects, and language—signitive grounded in both the imaginative and the perceptual. You then attempt to manipulate and process the solution to this question. In each of your personal spaces for visualizing, you take your Image Having (Pirie, & Kieren, 1994) and you may attempt to conceptually blend your image of stack with your image of a nickel, and compare it to the height of a quarter. However, as you attempt to compare these two processed images, you are struck with the realization that you do not have enough information through visualizing alone. So, either (a) you need to provide growth to existing images or (b) grow a new image. To grow you seek out perceptions—spatial reasoning.

So, you take a stack of physical nickels and line them up against a single upright quarter. This perception offers the knowledge of thirteen nickels being the height of a single quarter—Image Making (Pirie & Kieren, 1994). Through this act you are provided with the necessary element of growth to your images to aid in the solution of this particular conundrum. You show this to your partner. In this space, you might discuss with your partner using gestures, objects, and language that for each quarter lined up against the height of a person there will be twenty-five cents added to your total, yet for each length of a quarter there will be a comparative thirteen nickels which will add sixty-five cents to your total. Both of you conclude that a stack of nickels the height of a person will be worth more than a row of quarters. I suggest that a good portion of this is occurring within an interactive space of visualizing and spatial reasoning, even with our perceptions there is an element of the imaginative, as we only ever see an object partially, our imagination provides the totality (Arnheim, 1969; Gibson, 1979); and at the points of discussion or writing something down, there is the signitive. Interweaving is continually occuring between the perceptual, imaginative, and signitive, but also between, language, gesture, visualizing, and our images. They are all at play influencing, connecting, and relating within space.

This interplay is a skilled act. Some teachers are beginning to engage with the more perceptual experiences in the classroom through the use of manipulatives, and they have for many years now engaged with the signitive. Would a purposeful emphasis and practise with visualizing further benefit a student's development of mathematical images? When I work with my children or in a classroom, I play with this idea. I do an activity that I call *moving things with your mind*. When my daughter needs to add numbers, say nine and six, we exercise our visualizing skills by using gestures within space. We throw an imagined nine over to the left and six over to the right, then with our hands scoop an imagined one from six and throw it over to the nine to make ten. Together we often discuss and play with ideas through acting out the mathematical idea with the use of our imagination.

In this study, I am interested in further exploring this idea of spatial reasoning which includes visualizing and the role it might play as students grow their mathematical images. At this stage of my understanding and interpreting data, limiting the focus to solely visualizing has proved to be challenging. However, if the concept is broadened to spatial reasoning, there is much that can be noticed and interpreted. At this point, I struggle to separate visualizing and spatial reasoning in order to discuss them in a distinct way. Many of the critical events in the data include visualizing but in connection to the broader concept of spatial reasoning. Is there evidence of visualizing as the student enacts a mathematical idea? How can this be observed? As I will discuss further in the methodology section, I interpret this act through students starring off into the distance, students gesturing in what seems like an interactive way with their images, and students' lingusitic explanations—as containing offerings of evidence for visualizing. I will also look to interpret what new ideas or elements regarding, spatial reasoning connected to visualizing, and the transition from Image Making to Image Having, can be gleaned from the observation of these students as they act out a particular mathematical idea. These are the questions I am interested in exploring within the confines of this study. As though this study was not complex enough, circumstances part way through the study, as I have already outlined in the introduction, presented themselves in which another layer of complexity was added—LD students as participants.

Chapter Three: Neurodiversity and the cultural construct of learning disabilities.

This study did not need to be made up of participants labelled with a LD. This study could have been more easily done with 'typical' students. However, I have a strong interest in this group and its place in society. I would like to put this area of interest into context and describe some recent research in the area of cognitive neuroscience, as well as reveal some of my biases that I hold towards these students and our cultural assessment practices within our school system.

I must admit to struggling with the label learning disability for I do not view these children as disabled. I hold more of a Vygotskyian perspective that a child labelled with a disability "is not simply a child less developed than his peers, but is a child who has developed differently" (Vygotsky, Knox, Stevens, Rieber, & Carton, 1929/1993, p. 30). I believe students labelled with disabilities develop differently in part due to our sociocultural ways of schooling which have evolved over the course of human history which may be incompatible with the student's biological development. The neurodiversity movement connects well with this perspective. The concept of neurodiversity, in its broadest sense, defines all atypical neurological development as normal human variation that should be tolerated and respected in the same way as other human differences (Armstrong, 2011). In

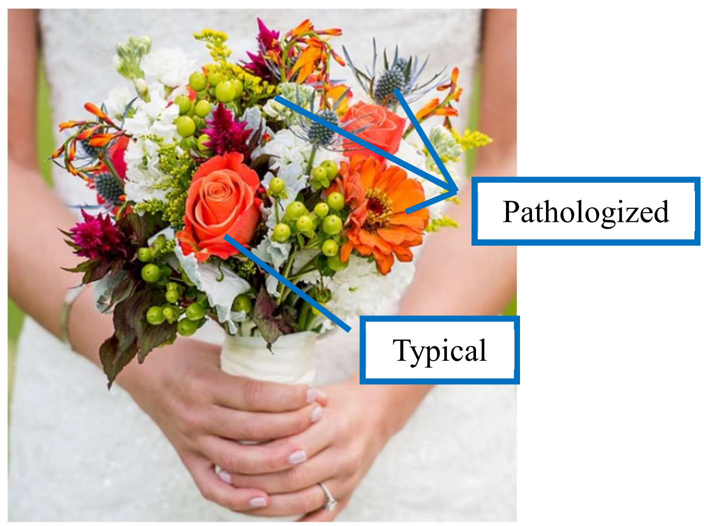


Figure 6: (Pexel, 2015) Bouquet of neurodiversity

Figure 7: (Tsaisir, 2014) Petal deficiency disorder

slightly different ways a number of authors (Baker, 2006; Broderick, 2008; Fenton & Krahn, 2007) suggest that people with different neurological conditions are just different, not handicapped or pathological. Armstrong (2011) likens this to a psychologist being a rose (see Figure 6 and 7) and every other type of flower that enters their office that does not fit their testing norms is therefore pathologized such as a calla lily: "You have PDD, or *petal deficiency disorder*" (p. 2).

The term 'neurodiversity' is generally credited to Judy Singer (1999), a sociologist diagnosed with Asperger Syndrome. The neurodiversity movement emerged in the 1990s by online groups of (high-functioning) autistic persons (Silberman, 2015). It is now associated with the struggle for the civil rights of all those diagnosed with neurological or neurodevelopmental disorders, such as attention deficit hyperactivity disorder, bipolar disorder, developmental dyspraxia, dyslexia, epilepsy, and Tourette's syndrome (Fenton & Krahn, 2007). Neurodiversity has remained a controversial concept over the last decade, however, as we learn more about how these 'disabled' brains are structured and the ideas around *dual exceptionalities* (Brody & Mills, 1997), extreme strength in conjunction with extreme weakness, many questions are arising. It is my hope that as we understand more our cultural views and beliefs around the ability of this population will change.

3.1 Our Cognitive Fingerprint: A structural look at autism and dyslexia.

I believe, most would agree with the statement – no two brains are alike, yet within diversity there are also trends and similarities. Each of us has a unique brain fingerprint. Just as fingerprints have their own unique pattern made up of grooves and ridges, so to do our brains. However, even our very unique fingerprints can be categorized based on similar patterns, such as the three basic overarching fingerprint designs: Whorl, Arch, and Loop (Karu & Jain, 1996).

Another example is if we look at the physiology of our hands, no two hands are exactly alike. Yet, if we focus attention on the width and length of our fingers, some of us have quite short stubby fingers, some have very long slender fingers, and the rest of us lie somewhere in between. The structure of our hands can have an impact on the types of tasks we are more suited towards. For example, those who have sausage-like fingers may struggle to do certain fine delicate detail work but have an advantage in tasks requiring strength, whereas those with slender fingers may struggle with tasks that require more strength but find delicate precision type tasks easy. In the study of brain physiology, we can likewise make similar categorizations to help us notice some broader trends. Just as some fingers are wide and some narrow, so it is with our brains, some brains are made up of wider folds, some have more narrow folds, and the rest fall somewhere in between (see Figure 8). Could our structural brain distinctions impact our cognitive areas of strength and weakness as well?

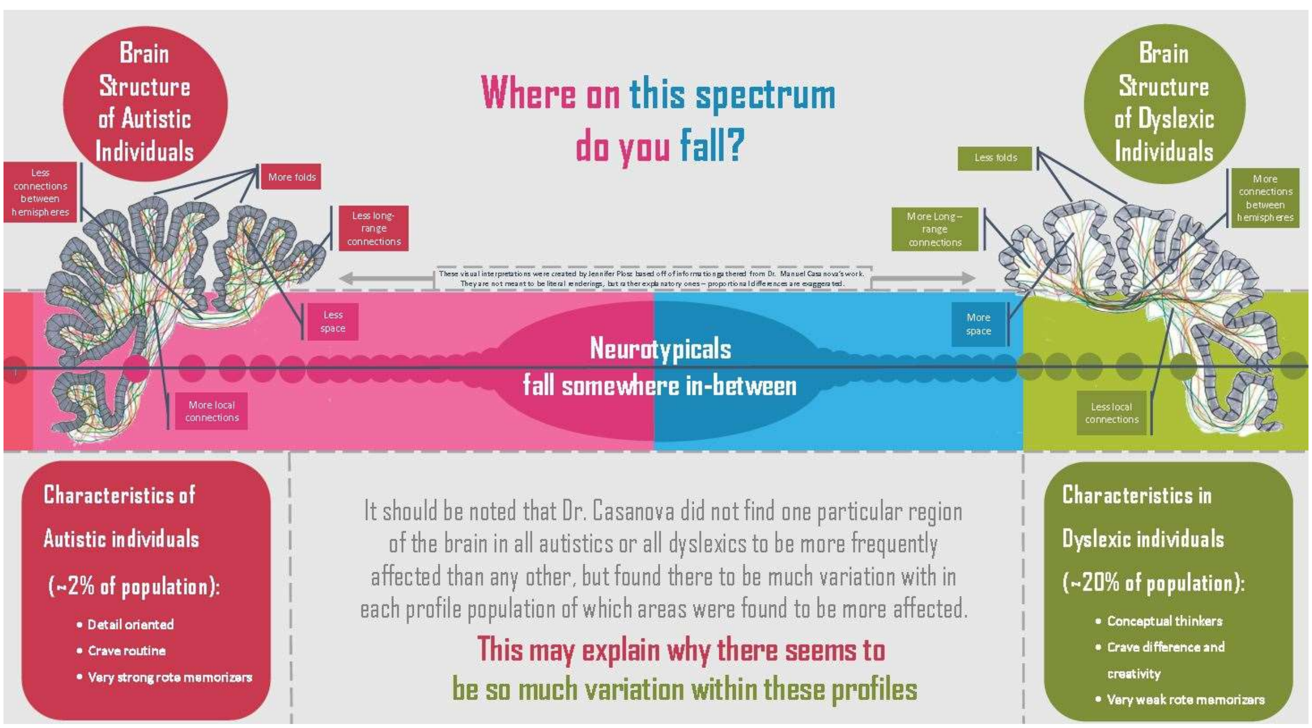
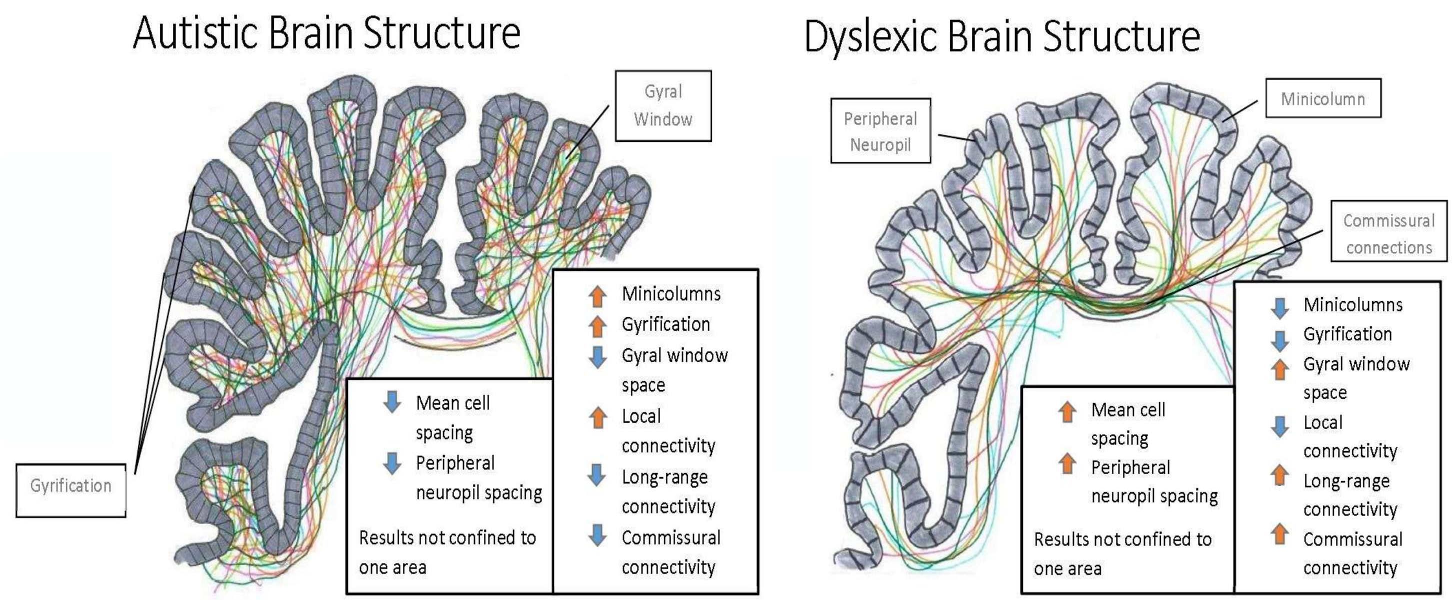


Figure 8: (Plosz J. , 2017) Cognitive brain differences between dyslexia and Autism

Dr. Casanova's work is in studying the brain physiology of different cognitive profiles. Over his many years of research, he has come to believe that there are two cognitive profiles that stand at either end of a spectrum from each other (Williams & Casanova, 2010); Autism is at one end and dyslexia at the other end, with what may be considered "neurotypicals" falling somewhere in-between. This implies that some people may present closer to the autistic end of the spectrum, as they tend towards being more detail oriented, craving routine, and excellent rote memorizers, while others may tend towards the dyslexic end of the spectrum and be more conceptually oriented, crave difference and struggle to rote memorize (Perrachione, et al., 2016).

3.1.1 Autism. The structural differences in these two brain physiologies appear to have opposite characteristics (See table 1). The brain of a person with autism has more folds in the brain than a 'typical' brain (See Figure 9). This increase in folds causes the folds to be narrower, which makes the white matter area held within the fold to be denser than a 'typical' brain. This affects how their neural connections are created. Autistic brain structures create more local connections than a 'typical' brain structure and fewer long-range connections, which also causes them to have fewer connections between the two hemispheres of the brain than 'typicals'. This affects how people with autism think and learn. They tend to be very detail oriented and can develop very specialized skills but struggle to make some of the larger or big picture connections. A person with autism often struggles to see the forest for the trees.

This tendency can play out well in school for a period of time as students with autism are great at picking up detail and spitting back the information exactly how it was presented to them. The teacher can then believe that the student is understanding. The struggle for the student with autism in school is checking for a deep understanding. Are they making the connections that they should be making? Often the struggle is highlighted when given a bigger project; they may focus too intently on one aspect of the assignment or veer off track on something very loosely connected, missing the main overarching point to the assignment.



These visual interpretations (Plosz, 2017) were created based off information gathered from Manuel Casanova's work (Williams & Casanova, 2010). It is not meant to be a literal rendering, but rather an explanatory one – proportional differences are exaggerated.

Figure 9: Visual interpretation of Dyslexic and Autistic Brain structures.

3.1.2 Dyslexia. The dyslexic brain, on the other hand, has fewer folds than the 'typical' brain (see Figure 9). This decrease in folds causes the folds to be wider, which makes the white matter area held within the fold more spacious. This affects how neural connections are created. Dyslexic brain structures create more long-range connections than a 'typical' brain structure and fewer local connections. More long-range connections also cause them to have more connections between the two hemispheres of the brain than 'typicals'. This affects how dyslexics think and learn; they tend to be very conceptual, big-picture thinkers, who often make connections between ideas that others do not, but they struggle to learn the details. They also tend to crave difference and creativity but are very weak rote memorizers (Perrachione, et al., 2016). This group struggles to see the trees for the forest, but if they are not given the forest/big picture/conceptual aspect of a topic they are often left without a starting point, which is the root of a lot of their struggle with school.

Mimicry is alive and well in the classroom. The teacher often presents material and the student is expected to give that information back to the teacher in exactly the same form as it was presented. Students who can do this successfully often receive top marks. Dyslexics struggle to retain the details and specifics when presented with material in this way (Perrachione, et al., 2016). However, they are often quite adept at taking that information and then synthesizing it or connecting it to something else (Eide & Eide, 2012). Yet, many classrooms do not focus on the critical thinking aspects of a subject. In mathematics for example, when the material is presented as a procedure that should be mimicked, the dyslexic will struggle; yet if that same topic was taught through relational understanding, the dyslexic would often excel. The dyslexic's mind must understand first, and then they gradually learn the details. Yet, often in school when students struggle with a topic, there is a belief that because the student is struggling, they are not intelligent. This in combination with the cultural belief that memorization is easier than conceptual understanding, often prompts the teacher to breaks down the topic into little steps that the student is to memorize. This assumption of lack of intelligence and cultural belief in the ease of memorization creates an environment for dyslexics that is a downward spiral of constant challenge in school and contributes to their own belief in their lack of intelligence and ability.

Table 2: Autism and dyslexia at opposite ends of a physiological spectrum *(Williams & Casanova, 2010).*

Autism	Dyslexia
Decrease in the spacing of cells within each minicolumn. The minicolumns are thinner, yet they contain the same average number of cells 80 – 100.	Increase in the spacing of cells within each minicolumn. The minicolumns are wider, yet they contain the same average number of cells 80 – 100.
Decrease of neuropil spacing within each minicolumn.	Increase in neuropil spacing within each minicolumn.
Increase in the number of minicolumns contained in the cerebral cortex.	Decrease in the number of minicolumns contained in the cerebral cortex.
Increase in gyrification.	Decrease in gyrification.
Decrease in gyral window space. As there is greater gyrification, the gyri are thinner, and therefore the gyral window beneath is less spacious.	Increase in gyral window space. As there is less gyrification, the gyri are wider, and therefore the gyral window beneath is more spacious.
Increase in local connectivity, which is theorized to be due to a thinner gyral window, along with more minicolumns, increasing the amount of fibers in the area – more dense.	Decrease in local connectivity, which is theorized to be due to a wider gyral window, along with fewer minicolumns, decreasing the number of fibers in the area – more spacious.
Decrease in long-range connectivity, which is theorized to be as a result of denser white matter.	Increase in long-range connectivity which is theorized to be a result of a less dense white matter
Decrease in commissural connectivity, theorized to be a resultant of less long-range connections	Increase in commissural connectivity, theorized to be a resultant of more long-range connections

It should be noted that Dr. Casanova (2010) did not find one particular region of the brain in all people with autism or all people with dyslexia to be more frequently affected than any other, hence the fingerprint analogy. This has very interesting ramifications for some commonly held beliefs regarding these profiles. For example, it is a commonly held belief that all dyslexics struggle with phonemic awareness. One would then expect that this area of the brain associated with phonemic awareness would be affected by wider brain folds in all dyslexics. Yet, Dr. Casanova did not find this to be the case. The area of the brain that deals with phonemic awareness was not more or less affected in all the dyslexic participants. Rather, he found there to be no pattern when it came to any one area as being more affected by wider folds than others; he just found variation throughout the participants. So, some areas within the dyslexic brains were more affected than others, but as a whole group, one area was not found to be consistently affected. This seems to support the extreme level of variation that is seen in how each student with these profiles presents areas of strength and weakness. Yet, there are still overarching characteristics that can be found within each of these groups (Eide & Eide, 2012; Armstrong, 2011; Silberman, 2015).

These ideas of spectrum and variation create an interesting discussion in many areas of the dyslexia and autism debate. Does this influence commonly held beliefs in right brain dominance for dyslexics? A University of Utah study (Nielsen, Zielinski, Ferguson, Lainhart, & Anderson, 2013) looked at over 1,000 brains and discovered similar findings of variation, but in areas of brain activation rather than physiology. They found that none of their participants demonstrated preferential activation in one hemisphere over the other. Yet, there did seem to be evidence of left-handed people having more brain symmetry which would lead to better communication between the right and left sides.

Ideas of phonemic awareness and dyslexia may also spark discussion in the context of this found variation. Is phonemic awareness the root of dyslexia, or is it weak neural adaptation (struggle to automatize/rote memorize) (Perrachione, et al., 2016)? Is the weakness for phonemic awareness attached to a weakness in that portion of the brain? Maybe for some. However, for others the issue may be more related to rote memorizing—which sounds go with which letters. As discussed earlier, reading is a complex task for our brains, and English is a complex language containing many variations and exceptions. Think about the English language having 26 letters that look nothing like a sound and then put those letters in a sequence, such as ‘ough’, which in English presents with seven different sounds (i.e., through, cough, enough, dough, bough, borough, bought) or look at the sound 'oo' that has five different spellings (i.e., root, ruin, rude, new, through). Retaining the look of a word attached to its pronunciation or meaning is, therefore, an extreme task in rote memorization.

And finally, how does this idea of spectrum fit with those who claim to have both autism and dyslexia? Do these individuals have both narrow and wide brain folds affecting different parts of the brain? Or does their autistic brain physiology present with some dyslexic tendencies with the root really being autism? Or vice versa?

There are many aspects of these profiles left to be discovered and re-examined. Dr. Casanova's (Williams & Casanova, 2010) research opens the discussion connecting learning patterns with brain physiology allowing us to better understand how different cognitive profiles make neural connections. As we learn more, this will hopefully have an impact on how we can more effectively engage these students and maybe allow us to revisit our cultural views of their abilities. Every brain organization has strengths and weaknesses; however, it seems that some weaknesses are more harshly viewed by society. Is learning disabled a fair label? Can a student with more neural connections, either long-range or local, than 'typicals' be considered disabled, or would another term seem more fitting? Maybe our society will come to a recognition of the need we have as a society for the areas of strength that these brain organizations have (Eide & Eide, 2012; Silberman, 2015) and look beyond their weakness. How can we support them in their weakness, but more importantly tap into their strengths? As it seems there is still so much to learn and understand about these profiles, how confident can we be in both our diagnosis and intervention practices? It is my belief that it is a social justice issue to be careful how we label and assess them. If a significant portion of our population share a particular pattern of characteristics, it seems likely that these characteristics present us as a society with some kind of evolutionary advantage that perhaps we would be wise to consider more closely...both for the sake of the individual and for the sake of society.

3.2 Psychoeducational assessments.

This concept of labeling is a difficult one. In a perfect world, I believe it would be better to not need a label, as the classroom would be set up for multiple brain organizations to thrive, and the students could pick and choose the resources that they needed for learning. Universal design for learning (Rose, 2000) takes this approach to the classroom, and these ideas are just beginning to trickle into certain classrooms. However, we are not there yet, so what do we do in the mean time?

Currently students who are struggling in school are offered a psychoeducational assessment. This assessment is typically done over the course of two or more days. The first portion is the Wechsler Intelligence Scale for Children (WISC). It is an individually administered clinical instrument for assessing the cognitive ability of children aged 6 years 0 months through 16 years 11 months. This first assessment is not based on typical school work but it looks at different modes of ability. The typical battery of tests delves into five main areas with two subtests for each area. The five areas include Verbal Comprehension Index (VCI), Fluid reasoning Index (FCI), Working Memory Index (WMI), Processing Speed Index (PSI), and Visual-Spatial Index (VSI). These are meant to give a broad overview of possible areas of weakness and strength that will allow for a deeper understanding of the student and their abilities and inabilities. Through this testing the student is given an IQ score. The second part of the assessment is the Wechsler Individual Achievement Test (WIAT) assesses how the student performs with typical classroom type tasks, including reading, written language, oral language, and mathematics. From these assessments, exceptional students are then labelled with any number of terms from gifted to learning disabled to intellectually disabled.

Embedded in these assessments, depending on who you talk to, is this underlying idea that how we assess and how we currently have school set up is correct and those that do not fit or struggle to perform are incorrect. These labels and students' school experiences reinforce this idea that certain students are incorrect or wrong somehow. Yet, can we really be that confident that it is the student and not the teaching or assessing? These are things we need to be constantly reevaluating, for currently we are labelling children as young as six with some very harsh labels, like learning disability or intellectual disability. How students view themselves and their abilities has a profound impact on their learning as Dweck's (2006) research on mindset has shown. Disability is not a very growth mindset term. Could it be our teaching approaches or classroom environments that are causing the disability, because we are not taking into account different brain organizations in how we set things up? And how accurate are our assessment tools that require the administrator to spend at most seven hours with the student and then the label is with them for life either in their psych or school file?

In my mind, there are so many questions surrounding these assessments and the influence of other factors on them, that I tend to view them in a very general way. I believe they are valuable and help as a starting point for delving deeper into a better understanding of the child's complex profile, but that they should be viewed critically. Things to consider are that these tests are given during a specific time and place in this student's life. I feel strongly that time and place can be very strong influencers as to the outcome of testing. How was the student feeling at the time? Is the student confident or self-critical when approaching an assessment? Was the testing done at school where they maybe do not feel confident, or was it done at home where maybe they feel more supported?

Many of my views and opinions have developed over time from my own personal experiences with these assessments. To have a better understanding of why and how I am using these assessments in this study, I believe it is important to share some of these experiences, as they provide context for my understandings. I was assessed in both Grade 4 and Grade 12. We have come a long way over the years in regard to these assessments in both some positive and negative directions. On my Grade 12 assessment, I was given eleven subtests. While it was written that I had scored "in the gifted range on three of the subtests, at the very high range on one, at the high average range on three, at the average on two" (Stewart E. M., 1989, p. 2), none of these were expanded on beyond this one sentence. To this day, I have no idea what I scored high on. The remainder of the four-page document was in reference to the last two subtests, which were in the "low average to limiting range on one, and at the limiting to disabling range on one of the subtests" (Stewart E. M., 1989). This experience of only focusing on areas of weakness, rather than developing strengths has greatly impacted my view towards working with exceptional learners.

My report goes on to describe how students' ability – in my case inability in these subtests affect "basic conceptualization and decoding skills, and is necessary for proficient performance in advanced reading and mathematics, and related to critical thinking ability exhibited in deductive reasoning and problem solving" (Stewart E. M., 1989, p. 2). This particular subtest covers a lot of skills and my struggle to perform well at it offered me many areas of weakness. Yet, what was not taken into account was that I was already attaining top marks in high school mathematics. Not to mention that the year after this assessment was done, I went on to do quite well in university mathematics, which seems inconsistent with my areas of apparent struggle with “advanced reading and mathematics . . . related to critical thinking.”

The other event that has impacted my view on these reports is my son’s history with psychoeducational assessments. He was assessed at the beginning of Grade 3 and then again in Grade 6. The woman administering this initial assessment had very strong opinions about what was best for my son. I had explained to her that I would like to let my son know what day his assessment would be on. She refused to tell me what day she would be coming into the school, as it was ‘better’ for him not know. I attempted to explain that I know my son well and that he handles things much better when he can mentally prepare himself. She still refused.

She came to my son’s classroom one day, took him and another student to a separate room and began the assessment. She wanted to complete the assessment in two days so kept my son in over recess to give them more time. Imagine a nine-year-old boy hearing the recess bell ring and hearing everyone in the school leave to go outside while he had to stay in with a woman he had just met and do testing. Recess, as is true for many kids, is the one part of the day he looked forward to. When I picked my son up at the end of the day, he burst into tears, and I had to inform him that he would have one more day of testing to endure. In the end, Roman was assessed as having a low average IQ. Three years later and much the wiser, we found a psychologist who could come to our home where Roman feels successful and intelligent. The psychologist took many breaks throughout the assessment in which Roman could go outside and jump on our trampoline for a bit. In this assessment, Roman was scored as having a very high IQ. Quite a discrepancy in the two scores.

These are the two major events which have formed my biases towards the psychoeducational assessments and my conviction that low scores could be a result of the child’s anxiety towards testing, years of a low self-image, not getting the same level of content or opportunity to engage with learning due to a struggle with reading and writing, reaction to the person administering the test, starting the day off wrong, or just hating the whole process and deciding to disengage. Strong scores, however, I view as unrefutable, though narrowly explored. The student was clearly able to do the task at that moment and time when the test was administered.

The combination of having a deeper understanding of how different brain organizations can influence a learner, along with ideas of dual exceptionalities, and my own personal experiences with psychoeducational assessments have greatly impacted how I view the LD community of learners and the high level of expectations that I have for them. It was for these reasons that I embraced the idea of making this population of learners the participants for my study. Through this study it is my hope that the teachers and the students themselves would begin to question our cultural norms and beliefs around what students labelled with LDs are capable of. One area of commonality that I believe we all have, both LD and ‘typical’, is that we are all spatial creatures and I believe that engaging spatially with the subject of mathematics would be a better universal design for teaching and learning mathematics.

Chapter Four: Methodology

4.1 Participatory Action Research

The aim of this study was to investigate the role that spatial reasoning plays in the growth of mathematical images for student labelled with a learning disability. The more I engaged with the topic, the more complex and expansive I came to realize it is. This has led me to address a number of overlapping topics, which is reflected in my discussion of horizons at the beginning of this document—mathematics, mathematics education, images, and learning disabilities. We have also delved deeply into the interweaving of perception, images, visualizing, gesturing, affordance space, conceptual blending, and language. We then further layer on top of all this our cultural views and treatment of students labelled as having a learning disability. All of this was preparation for entering into the mathematics classroom at a school for LD students and engaging with spatial exploration to promote growth with mathematical images.

4.1.1 Why Participatory Action Research?

As I stated earlier, my initial engagement with this topic was intended to be through a hermeneutic phenomenological approach. While waiting for ethics approval, I continued to engage in professional development (PD) at various schools. One such PD session was at a school for students with learning disabilities. Soon after this session, I was contacted by the school to do a math residency with their students. This was a productive month of exploration by both myself and the teachers. Although, this residency occurred over a month's time, I was actually teaching in the classroom for only 10 days. The school decided they would like to continue this relationship but in a math coaching role. Through our discussions we found ourselves to have aligned goals, and the principal expressed interest in me collecting data from their student population. Both parties, myself and the school (principal and teachers) were interested in exploring a more spatial approach to mathematics education with this specific population and seeing what effect it might have. This was an exciting opportunity for me to explore the aspect of change within this topic of spatial reasoning and growth of mathematical images.

However, hermeneutic phenomenology does not pursue change but rather is an exploration of what is currently at play within an experience. This was not the objective of the school, they were interested in change. As a result, I began exploring other methodologies that would allow for this objective to be pursued. Participatory action research (PAR) is where I was able to align these goals. The underlying tenets that are specific to the field of PAR and that inform the majority of PAR projects are:

- a collective commitment to investigate an issue or problem,
- a desire to engage in self- and collective reflection to gain clarity about the issue under investigation,
- a joint decision to engage in individual and/or collective action that leads to a useful solution that benefits the people involved, and
- the building of alliances between researchers and participants in the planning, implementation, and dissemination of the research process.

(McIntyre, 2008, p. 1)

To offer a deeper understanding as to why PAR connects with this particular population and a research question, we will explore these ideas in a bit more detail.

"Collective commitment to investigate an issue or problem." Through my PD sessions and residency with the teachers they were excited to explore this idea of learning differently. Up to this point, their approach to mathematics was centered around a concept of working harder and providing students with tricks and procedures for answer-getting. During these sessions, I shared with them dyslexic students' struggle to memorize and their areas of strength and ability. These ideas connected with their observation of students mixing up procedures and struggling to memorize basic facts. They were intrigued by the idea of teaching math to fit their populations' mind organization and strengths; I would also argue, the high majority of mind organizations, not just LD students. For my part, I was interested to explore the more practical classroom side of things. Ultimately, both parties were interested in growth and change.

What is so powerful about PAR towards creating change is that it works on the basis that there is no singular or fixed version of reality awaiting detection. It recognizes and embraces complexity. There is an emphasis on collaborative knowledge, on ideas of becoming, not so much arriving at some determined final destination, but being situated somewhere between knowledge, analysis, and action (Pain, Kindon, & Kesby, 2007).

The teachers at this school were coming to this issue of mathematics pedagogy with the practical experience of being in the classroom but also cultural views about learning disabilities and their students' deficiencies. I was entering this collaborative process with theories and a more one-on-one practical experience with students who were labelled with as LD, but a belief and focus on their abilities. I had much less experience in a whole classroom approach to teaching these students. This coming together to grow and learn from each other and the students seemed a rather intriguing and hopefully symbiotic match-up, the hope being that together we could expand our ideas around not just growth of mathematical images for students labelled with as LD, but also facing our cultural views of LD students and what they are capable of.

"A desire to engage in self- and collective reflection to gain clarity about the issue under investigation." An important aspect of PAR is that it encourages cyclical improvement and empowers teachers to use their professional knowledge for the betterment of their school. Importantly, the reflective component of PAR challenges practitioners to look introspectively, interpersonally, and globally at their beliefs and practices. In PAR, reflection is tied to action. At each stage of the process, participants were asked to reflect on what they believe, how they view themselves, how they view others, and how they understand the chosen problem, and not just at the school level but within a larger discussion of education itself. One framework for reflection that seemed relevant to the goals of this study, was in connection with the idea of multiple levels of reflecting on oneself either in a *mirror*, through a *microscope*, or through *binoculars* (Pine, 2009). In the mirror, reflection occurs on one's own beliefs, values, assumptions, and biases in order to learn more about a contextual problem. A microscope can be used to reflect on interpersonal experiences towards outcomes in a specific context. And through binoculars, one can look at things more globally and how those views impact the local and vice versa.

My hope with this study is that we all would challenge our personal, community, and global assumptions about the particular population of students labelled as LD, that we would look beyond what the psychoeducational report tells us, what the label tells us, and continually reflect on what progresses student understanding and why, as we move forward in our work with students.

"A joint decision to engage in individual and/or collective action that leads to a useful solution that benefits the people involved." Benefiting the people who are involved is central to all of this. I am reminded of a quote by Allen, Michalove, and Shockley (1993) when they wrote:

> When you teach . . . and half the class gives you a blank look, you ask yourself, "How else can I teach this concept?" That's research. You observe, and respond to what you

> observed. You begin to be aware of the intricate teaching and learning dance with your students. (p. 33)

When you have a classroom filled with students who are labelled as LD, this idea can get lost somewhat. There can be an underlying excuse for them giving you a "blank look." PAR affords an opportunity to influence each other's perspectives through action. Together we accept the challenge of that "blank" stare and the barriers those students face with the assumption that together we can explore our way through. This action piece is so essential in broader educational research within the classroom that it should be both practice and theory. PAR allows us to theorize from our own practices and those around us and, in turn, transform them. By taking this action together we create an environment where there exists an opportunity to open ourselves to a socially constructed reality within which multiple interpretations of a single phenomenon are possible (Greenwood & Levin, 1998). We open ourselves to a scenario where my personal experience as a student labelled as LD is present and interpreting the classroom's interactions in conjunction with a person who may not have the lived experience of being labelled as LD but has taught LD students for a number of years. This same experience may then be interpreted and understood differently as we are coming from different lived experiences. This cyclical process of reflection and action within PAR allows for this socially constructed reality to emerge, and together we begin the process of questioning, which can lead us towards a pathway of change.

"The building of alliances between researchers and participants in the planning, implementation, and dissemination of the research process." In attending multiple conferences both on the educational research side and teacher side, I am often struck by the lack of respect I sometimes encounter on both sides toward each other. At teacher conferences, a researcher may stand at the podium as teachers' attention is fully focused on them; or the other possible response is a polite level of busyness which begins to erupt as no one is listening. All too often it is one or the other and it completely hinges on whether that researcher has established ‘street cred’. I have also seen the other side at education research conferences where there is a superiority of theory and eye-rolling at the unsophisticated practices of classroom teachers. If we truly desire change then we need to listen to and engage with each other. Educational researchers need to spend time in classrooms experiencing the complexity that teachers face every day and with this notion of where theory meets practice, but on the other side, it is equally important to recognize that as teachers that “our teaching is transformed in important ways because we become theorists who articulate our intentions, test assumptions, and find connections with practice” (Goswami & Rutherford, 2009, p. 3). Simply stated, we become better teachers through teacher research.

It is for all of these reasons that PAR seemed a beautiful fit with this situation that had developed. I applied for a modification to my ethics application and we as a team moved forward.

4.1.2 Design

Where life, practicality, and methodology intersect has had a profound impact on how this study has been designed. It is complicated. My time at the school has been split into two sections. One team consists of four teachers Grades 3 to 6 and the principal, along with myself and the students. This portion of my time at the school was framed as a math coach. However, initially my time was dedicated to a smaller team which had been devised for the purpose of collecting data. The two portions consisted of the study (two-week period) and then the remainder as a math coach (six-month period). The smaller team is the portion that will be discussed within this document. The reason for this distinction was due to the length of time it took to gain ethics approval for video recording students and then apply for the modification of methodology (Ethics submitted: October 5, 2016 – Approval: December 13, 2017). The more extensive time frame of math coaching continued until June 2018 and was not included, as it fell beyond the date that would be required to finish my Masters degree on time.

This two-week study will look at the beginning stages of the PAR study and investigate whether spatially designed tasks can be shown to impact students' mathematical images, which is the premise for entering into this longer term of six months with the teachers and students. The goal is to go through one cycle of a PAR spiral in order to establish some initial ideas about how this longer study might be engaged with. It will also provide teachers with an experiential understanding of what a spatially designed lesson might look like. As this study is only focusing on the beginning stages, the methodology must be reframed as typically a PAR study is over a much longer period. I have reframed the methodology to be a micro-analysis of PAR, for which there is precedence as demonstrated in a study done by a group at the University of Queensland (Tsey, Patterson, Whiteside, Baird, & Baird, 2002). In this study, they analysed the formative stages of a participatory action research (PAR) process which aims to engage and support the members of an indigenous men's group in Australia.

My desire was to set up a study that would allow for participation in self- and collective reflection as well as action with the hope that this would allow for the group to learn in and through the processes that are being put into place in order to benefit all those involved. Towards this end, I began by organizing an initial group meeting. The Principal and two of the four teachers had worked with me before. Overall, my objective was to hear from them and allow them to discuss what was meaningful for them. The focus quickly went towards all the changes they wanted to see happen. Such as,

- Tailoring their mathematics teaching with their population of students in mind,

- Getting students to stay focused on tasks with manipulatives,
- Remediating number sense,
- Overwhelmed by the number of curriculum topics to cover
- (I suggested looking at: How can we integrate topics differently?), and
- How can we get students to perform better on pencil paper?

These were all good discussion points for us to orient ourselves to within the study. Many of the topics discussed are more related to the longer study over the semester. Generally, I sensed that the teachers were interested and somewhat skeptical as to what impact a week of spatial fraction work would have. As a result, the main topic we decided to engage with during the micro PAR was what impact does a spatially designed lesson have on the population. I have worked one-on-one with this population for a numbers of years and have been amazed by their progress in a short space of time when engaged spatially. I shared this with the group but I also shared that I was unsure of how much impact this approach would have in a whole classroom setting as opposed to one-on-one. The meeting finished with two teachers and the Principal completely energized and onboard, one teacher going with the flow, and one teacher mainly skeptical. My co-researchers for the smaller scale study, the Principal and Molly (pseudonym), were energized and onboard, so with much anticipation we moved forward with the work involved in the collection of data.

In PAR, McTaggart (1997b) highlights the distinction between "involvement" and "participation". He states that authentic participation means that the participants share "in the way research is conceptualized, practiced, and brought to bear on the life-world" (p. 28). This is different from being merely "involved" in PAR, where there is no ownership over the project. Yet, what does "involved" look like? In my mind, it seemed artificial and unnecessarily complex to have every team member “involved” in every decision. It seemed more natural to adopt the approach of McIntyre (2008): “what is important to and in a PAR project is the quality of the participation that people engage in, not the proportionality of that participation. It is my experience that the most effective strategy for engaging people in PAR projects is for the participants and the researchers to make use of ‘commonsense’ participation” (p. 15). However, as this was my first attempt in this approach, it was challenging to determine what commonsense looked like. My co-researchers for the smaller study were the principal and the classroom teacher. There was also an educational classroom assistant; however, the culture of the school did not involve them in meetings outside the classroom. This aspect was not one that I had influence over, yet, I did attempt to involve her as much as possible in our after-class times of reflection. The participants were the students from a Grade Five class. A source of struggle with this small group of co-researchers was to involve yet not overburden.

During the week that the consent forms were being collected, the principal and I had a number of meetings just the two of us, as the teacher was unavailable. We discussed aspects of the assessment in which the principal expressed a desire for the assessment to have some connection to the Wechsler assessment. I received a copy of this assessment from him and got to work developing a new assessment. The assessment was a compromise of ideas, merging aspects that I was interested in exploring as well as keeping to the main themes within the Wechsler assessment. Once we had the assessment (see Appendices A and B), we looked over it with the teacher and she had concerns about some of the advanced topics within the assessment. The school’s Wechsler assessment does not necessarily keep to grade specific curricular themes, and a number of questions are well beyond that grade’s curricular expectations. In the latter part of the study, I matched up the program of studies with the topics in the Weschler assessment to give a better understanding of how out of range these topics are for the specific grade levels that it is administered to (See Figure 10). I attempted to create a balance between curricular topic and the Wechsler topics as requested. In the end, we all agreed that it would provide us with valuable information.

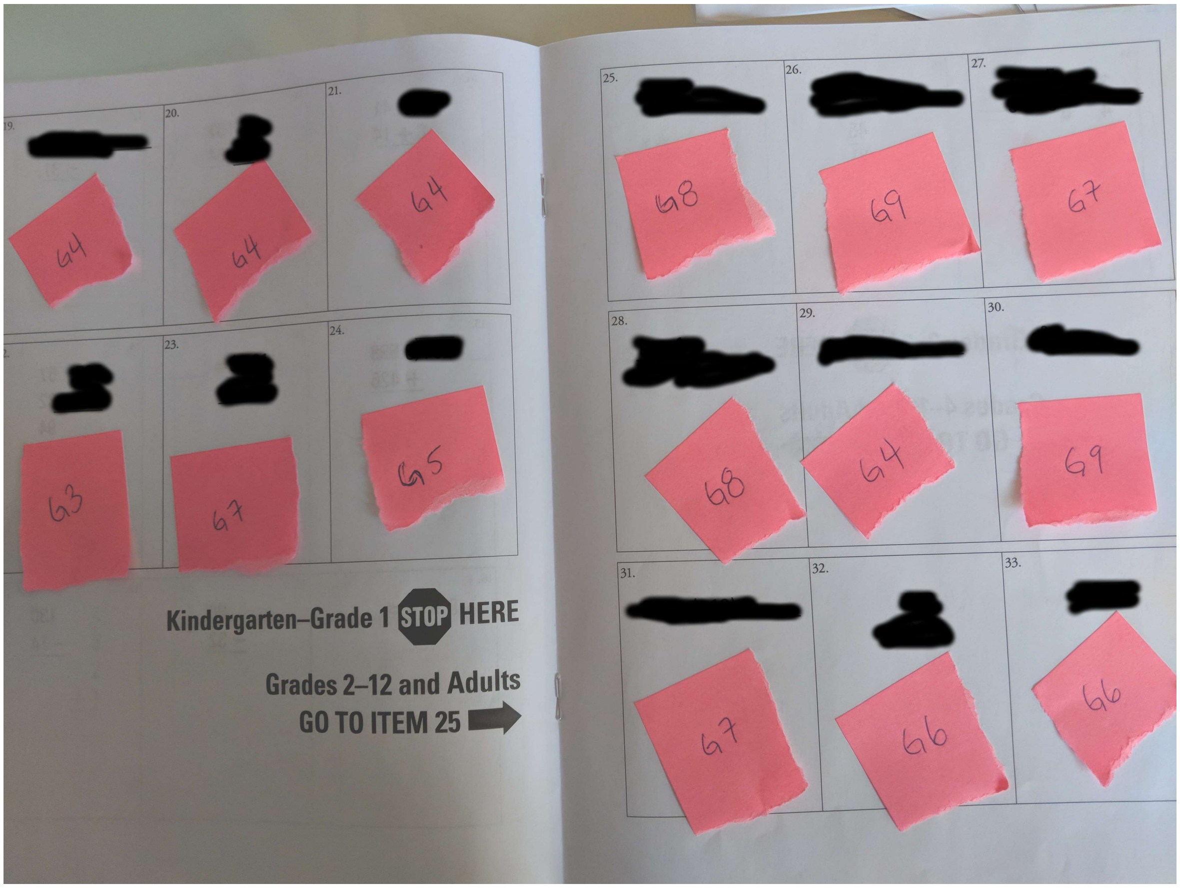


Figure 10: Wechsler fundamentals academic skills: Spelling and Numerical Operations. Stick-notes display the grade level that the particular question is connected to in the Alberta program of studies. G-denotes grade. The actual questions have been redacted.

The principal and I planned the parent night to allow for questions and a more detailed description of the study and who I was as the researcher. Two sets of parents attended, as did the teacher and the principal. The general impression of the evening was supportive and the parents seemed excited about having their children participate. There were some questions surrounding why there was no control group, and so it was a productive conversation discussing what a qualitative study looks like and what the goals were. Once the parent night was finished, our focus turned to the collection of data and working in the classroom with the teacher, educational assistant, and the students.

4.2 Ethical considerations

In working with a population of students who have been labeled with a dis-ability on or before the age of 10, there are many ethical considerations to consider. The term *dis-* meaning "lack of, not" (Harper, 2001) applied to ability to learn is impacting to a student's belief in themselves and their own abilities. These students' experiences with school has for the first five years mainly been six hours a day of you-are-not-good-at-this, while large portions of those around them were. This takes its toll on a student's self-image and willingness to risk. Also, this group of students is assessed more than their 'typical' counterparts. They often feel more anxiety as a result in regards to assessment based on a history of performing poorly. All these aspects of this population's experiences were front and center for me as I designed and carried out this study, as I too, along with my children, have had the lived experience of growing up in school being labeled with this "lack of" (Harper, 2001) ability to learn.

I agonized over ethical questions before the study began. How will they feel being video-recorded as they engage with tasks that they are not confident in? How can I add to their long list of assessments at the beginning and then at the end with paper-pencil assessment, which I knew they would hate? Am I just taking from them without giving anything back in return?

These were upsetting questions that I faced and still wonder about. Yet, there was a balancing that occurred in my mind as I moved forward with this research. Currently, in their classroom they were engaging with mathematics daily through working the majority of time with pencil-paper through questions from the textbook. During the initial assessment, I would listen to the math lesson, while the teacher would explain how to do the questions and the students would then sit at their desks and do Question #1, then #2, then #3, then #4 while the teacher and educational assistant would keep reminding them to work silently. For me, the balancing was the opportunity to change the everyday and bring awareness to the school of the gifts and abilities these exceptional learners have. My motivation was to give back to these students and help them understand what capable learners they are.

Although parents had consented to the study, I wanted to be clear with each student, what the study involved and that they were not required to participate even if their parents consented. For this reason, I took each individual student aside, away from any peer pressure, and asked if they were comfortable with giving their assent. All then ten students seemed to understand and seemed excited to work with me again, as many of them knew me from last year.

I was pleasantly surprised how many students were excited to participate in the study even after I told them I would be assessing them before and after. I was so impressed with how they seemed to take it all in stride, being video-recorded while doing a pencil-paper assessment that the majority struggled with immensely. They would openly comment, "I have no idea what I'm doing," or "I'm totally guessing." The students did not seem phased by the pre-assessment at all, other than one student, Walter. He is very hard on himself, but typically does very well in math. When he began the fraction section of the assessment and it was beyond his understanding, he became visibly unhappy. When I told him he did not have to finish he attempted to keep going for a bit but then stopped early. I could see this was upsetting to him. It broke my heart. I knew this student from last year and having now worked with him for a few more months in the classroom this year, I have seen this student repeatedly getting upset if he struggles, this behavior is a common occurrence for this student. However, it felt awful to contribute another experience of this nature.

The classroom tasks were definitely a more positive experience of engagement and excitement, especially the final two days of the *imagine-build-steal* game. The *imagine-build-steal* game was really quite basic. The students were put into teams of two or three and then offered an addition of fractions equation on a white board. They were then asked to *imagine* the solution; once they had indicated the completion of this and verbalized a solution, they were given an opportunity to *build* it with fraction pieces. If the other team believed their solution to be incorrect, they could offer their own solution as an opportunity to *steal*. The team with the correct solution was asked to pick a number and behind that number were a multitude of possible score adjustments—such as add 20, subtract 6, switch scores with the other team, change a positive score to a negative or vice versa. The majority of students experienced a high level of success. However, on the fifth and final day near the end of the class, the teacher and I announced a summative assessment with paper and pencil. I included this in the study even though I was aware of the high potential for negative impact on the students because this is often how student achievement is measured in schools, and I considered it an interesting comparison to the video data.

I had left the arrangement of the final task to the teacher, as I wanted it to proceed in a way that was familiar to the students. Although well intentioned in this decision, given to do over, I may have chosen differently. After four days of students enjoying the tasks I had planned for them and having just finished a very engaging and successful task of the *imagine-build-steal* game for comparing fractions, the announcement of separating your desks and completing this written work produced a mood in the classroom that felt as though a balloon had just deflated.

The physical stature of students also deflated. Students went from standing up out of their chairs due to excitement to flopping on their desks as though they had just run a marathon. A number of students only did a couple of the questions and then quit. This task felt like a betrayal after a week of engaging mathematics and a horrible way to end my time with them before Christmas. Thankfully, I would have more time after the holidays to regain their trust, as I feel like I damaged that. I felt their judgment. Whether it was invented by my own guilt or not, I do not know. I felt like I was now in the category, “You are just like everyone else defining us by a piece of paper.” Although these students are assessed on an ongoing basis through summative pencil-paper testing; it just felt wrong.

I will never be happy with how this study ended on such a sour note; however, I am pleased to have collected data that does demonstrate that they are very capable learners and can not be defined by a piece of paper. I have hours of footage with students demonstrating their ability to learn complex material and even some being able to solve complex questions beyond their grade level in their head after only four days of classroom activities. It is my hope that this document is able to express these views adequately. However, most importantly, I was thrilled to hear a comment from the educational assistant in the study, as she entered the room one day: “I know what you’re trying to do here. You’re trying to show us how these students are so much more capable than we give them credit for and we should expect more from them.” I beamed from ear to ear and said, “Well, I do think they are very capable.”

Chapter Five: Methods

5.1 Data collection

In this study, data consisted of field notes, drawings, photographs, audio recordings, and video recordings. The intent was to retain as much as possible a highly contextualized form of data. As the study seeks to analyze a *part* of a complex *whole*—observing spatial reasoning, Image Making, and Image Having through words and gesture as the participants engage in mathematics—video recording was determined to be the ideal method for data collection. Powell et al. (2003) discuss the potential of video to collect data that shows rich behavior and complex interactions, in a way that is "the least intrusive, yet most inclusive, way of studying the phenomenon" (Pirie, 1996, p. 4). Video recordings make it possible to capture large amounts of data that preserve, in detail, many of the verbal, nonverbal, and social elements of student learning as it occurs.

I did not transcribe the audio in the traditional sense as my goal is to observe the integration of language with gesture in the study of this phenomenon. The decision to work mainly with audio and video recordings rather than with the addition of transcriptions is an approach supported by many investigators (Powell, Francisco, & Maher, 2003, p. 411). It is the interaction of words, gestures, movement, and facial expressions that I have viewed as my evidence not the isolation of any one of these to inform my study. Having said this, I did in the end find it productive to create a type of transcription that played out more like a story of my observations as I watched the video attempting not to interpret the behavior but merely giving an *accounting of* rather than giving an *accounting for* (Mason, 2002).

The study was situated in a private school for students labelled with learning disabilities. Data was collected from a Grade Five classroom. This particular grade was chosen as the majority of students already knew me from last year, and the Principal felt that both parents and students would be more comfortable with my presence in the classroom as I was known to them.

Consent forms where given to the students in their agendas for take home. They had one week to return the forms to their classroom teacher. We had eleven out of thirteen students in the Grade Five classroom return the forms fully signed. One parent contacted me, saying she did not want her daughter to participate, but she still wanted her to have access to the lessons being given to the participating students. I assured her that her daughter would still be in the classroom and fully involved with the activities; the only aspect she would miss out on was the initial assessment.

As I began this time in the classroom, I strongly connected with McTaggart's (2001) thoughts on the distinction between academics and participants of PAR projects, that there not be an implied "theory resid[ing] in one place and its implementation in another. Such a view is the antithesis of the commitment of participatory action research that seeks the development of theoretically informed practice for all parties involved" (p. 266). One aspect of success in working with different teachers in their classrooms is that I feel classroom teaching is a very skilled art which requires the teacher to understand and implement many theories and many different aspects of complexities all at once. As I was a classroom teacher for many years, I feel a strong connection and level of respect for their expertise. I do not enter a teacher's classroom with the air of having expertise that is superior to that of the teacher, rather that we may have some different and some overlapping areas of expertise. I have profound respect for the fact that that teacher enters her classroom every day with a deep understanding of psychological and pedagogical theories intertwining with student and teacher needs. I approach each classroom in hopes of honing my own abilities and deepening my understand of the theories behind student learning.

The teacher, Molly, the educational assistant, Tara (pseudonym), and I met up the week before the data collection to discuss the students and some of the logistics of the study. Molly had expressed concern about her mathematical knowledge and explained that her background was in drama. She clearly had a strong affection for her students and this affection was visibly reciprocated by the students. Molly had a desire to serve the special needs of her students in a more effective way and was very open to trying a different approach. I expressed that I would need her expertise in the area of understanding with regards to her students and in the description of tasks. As a previous high school teacher, I often struggle to provide an adequate explanation to younger aged students. I expressed a need for her to provide me with some support in this area. Together we discussed and planned the approach we would put into effect the following week.

Molly, Tara, and I seemed to work well together. The teacher and I especially had a strong connection in regards to how we interacted with students—relaxed familiar style. The educational assistant had a more authoritarian style, which was more distinct from Molly's and mine. However, this was not an issue as the classroom was mainly directed by Molly, so her and I having similar styles worked beautifully. I leaned on her with regards to her knowledge of students and how best to set things up, and she relied on me in terms of lesson planning for the students' spatial experience. During the assessment piece, the classroom teacher had very little involvement other than the logistical piece of deciding who should be partnered with whom, timing, and location. We did the pre-assessment interview in an adjoining room that students had frequent access to and were very comfortable and familiar with.

The students were assessed in pairs. During this pre-assessment interview there was a steep learning curve. With the initial participant, I did not probe their thinking too much but let them take the lead as far as their approach to the task. I let them know they could ask me any questions they wanted, and do as much or as little as they desired. This minimal interventional approach to our time together seemed to prompt the students to treat the paper-pencil task similar to that of a test and there was very little interaction between myself and the two participants. During this first interview, I had time to question and reflect whether this was the atmosphere I wanted to create. As the second group entered I decided to intervene more and attempt to establish a more interactive feel to our time together and began to intermittently ask students questions, in an attempt to disassociate this pencil-paper task from a test. This was a much more productive approach and seemed to ease the students' feelings of intensity toward the task. With each interview, I honed my own skills of noticing and asking probing questions.

After a week of the assessment interviews, we moved on to the classroom teaching phase. Both the Principal and the classroom teacher were quite open to any topic I was interested in teaching so long as it was in the curriculum. This was great, as I was interested in choosing a topic that the students had spent no time on this year and minimal time in years past. I knew from working with both Grade 4 teachers from last year that fractions had not been a big focus. Although they would have had many life experiences that come to bear on the topic of fractions, I felt that this topic would be a good one to choose as they would have had minimal exposure to formal knowledge of fractions.

Before the first lesson, Molly, Tara, and myself met to plan and set up expectations. As the majority of students were involved in the study (only two were not), we set up two cameras in the classroom and grouped the participants into two desk groups. In general, the students did not seem to notice the cameras, other than the odd wave or small performance during a transition time. In my discussion Molly and Tara spoke about the idea of allowing the kids to come to the ideas on their own, yet we also must paying attention to flow (Liljedahl, 2016), offering hints when needed but mainly asking questions rather than saving them from struggle. I did the majority of the teaching and planning of tasks, as the goal was to offer both the students and the teacher an experience in a spatially designed mathematics lesson to promote the growth of mathematical images.

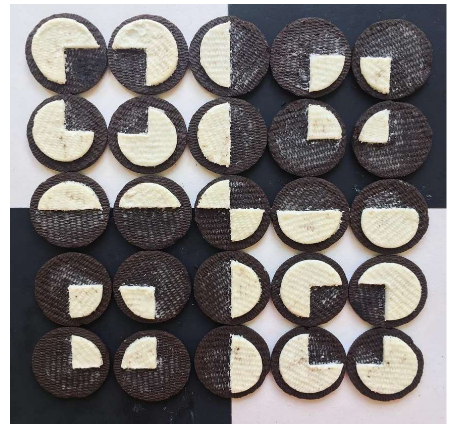


Figure 11: Cream of the Crop *(Hillman, 2017)*. What is the minimum number of Oreos you would need to create this image?

The structure of each lesson was 3-fold:

- Number talk
- Curricular spatial task
- Spatial task or puzzle

We started out every day with a number talk, which was often an image (e.g., Figure 11), in which I would ask the students to, "Turn to the person beside you and discuss what math you see in this image." We would then come together and discuss some ideas; then I would often pose a question, such as: "What is the minimum number of Oreos you would need to create this image?"

Classroom discussion is a complex activity. Generally, how to set this up is challenging, but there is another layer of complexity when you come in as a guest. What are the current classroom routines around group discussion? Who are the students that hide? Who are the students who have anxiety about being called upon? What background knowledge do they bring to bear on the task? For this reason, I chose to have them turn to their neighbors first. Also, this particular class is filled with a number of students who are slow processors (as measured by the WISC V), I felt that giving everyone an opportunity to talk or listen before the group discussion would be productive. I believe students develop their ability to use language as a tool for thinking about mathematics, both individually and in collaboration with other students. As Mercer, Dawes, Wegerif, & Sams (2004) discuss, the effect of talk-based activities and discussion as a tool for reasoning was found to be quite productive. However, as we will refer to later in this text, discussion with classmates is not always productive as we will see later with Alex and how conversations with his classmates did more damage than good to his sense-making. Part of this work of stepping into a new classroom was also to balance the classroom teacher's routines with my own. This was a continual point of navigation throughout the study with varying degrees of success.

After the number talk, there was usually a brief explanation of the curricular spatial task and we would then allow the students to actively engage with the mathematics. I, the teacher, and the educational assistant would circulate and support the students. The lesson would then end with a spatial task which was focused around critical thinking or puzzle solving. As the majority of the class have an ADHD diagnosis, ending the class in this way was a very positive addition to the lesson. The timing of this final task was typically decided by the teacher and me looking at one another and saying, “Yep, I think they’re done!” This final task was designed with the objective of acknowledging the importance of improving spatial reasoning and promoting other forms of critical thinking within mathematics that are enjoyable in an attempt to further alter the students’ views of mathematics from drudgery towards playful curiosity and struggle. Some examples, of activities that meet this criterion are the skyscraper puzzle using connecting cubes as your towers, or Cuisenaire rod picture puzzles (see Figure 12). One study that informed this decision was Grissmer et al. (2013) in which the authors designed a seven-month study for an underserved at-risk population in which they carried out an extremely intensive spatial intervention for preschoolers. Compared to the control group, those in the spatial intervention group had multiple areas of improvement including spatial reasoning, self-regulation, and overall mathematics performance. The tasks given to the

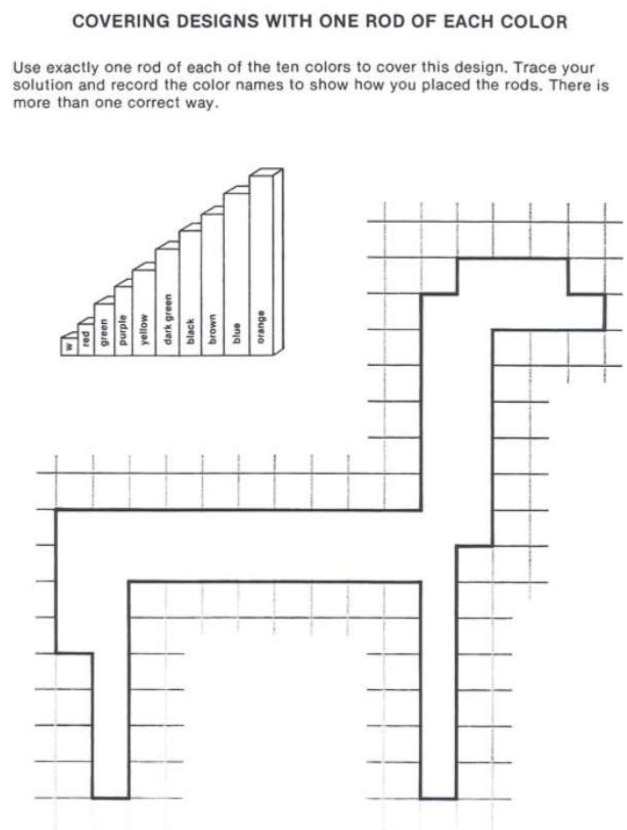


Figure 12: Cuisenaire rod picture puzzle *(Davidson & Willcutt, 1983)*

students were not number-based tasks and yet there was an impressive seventeen percentile point gain in numeracy and problem solving over the control group—time well spent!

Each day of the lessons ended with Molly, Tara and myself spending time reflecting on how the lesson went. What went well? What did they struggle with? Where do you think we need to go next? Where do you feel they are with Image Making? Have you seen evidence of Image Having? For the first few days, we generally stuck to the plan laid out with a few adjustments here and there, but at the end of the third day, we felt the students struggled to remain on task. A number of factors contributed to this—one was a particular student and their mood that day, another was the design of the task with needing to share manipulatives. The students were instructed to take turns being the controller of the manipulatives, and we noticed that when they were not the ones controlling the manipulatives many seemed to disengage with the task, and we felt there was very little discussion and collaboration occurring between group members. A third factor that could contribute to this struggle is that the typical structure of the classroom to this point had not been open-ended tasks but rather teach and then do questions out of the textbook. A culture of collaboration and working together had never been established for the math classroom; students mainly worked individually during this time. A final contributing factor could also have been that it was the week before Christmas holidays. With all these factors in mind, we decided to readjust our plan and incorporate more of a game-style lesson to keep engagement up. Although, there was not a strong impression of Image Having by either myself, the teacher, or the ed. assistant, we decided to assess where they were at in terms of Image Having as we only had two lessons left. That evening, I developed a game that I called *Imagine-Build-Steal*, that we implemented the next day with what felt like a strong level of success by all three of us.

Day Four, students were video-recorded playing Imagine-build-steal with adding fractions, then Day Five we decided to play the same game as it was so successful the first day but surrounding the topic of comparing fractions. Although the central focus of engagement was the game as opposed to the style of math questions, this was a compromise that we felt we needed to make as there was not enough time to establish a culture around group work and persistence in struggle during the week I was offered for the video-recording. In the end, the structure of the game allowed for some interesting tracking of the development of thought surrounding Image Making and Image Having (Pirie & Kieren, 1994).

Class time ended with the recess bell, which often allowed Molly, Tara, and me time to reflect on the lesson. I found our process on a micro level to be somewhat different from the typical PAR spiral (see Figure 13). During this time after class, we would engage with the process of reflecting and questioning how the lesson went. We would then alter or readjust the plan and investigate these changes the next day and then further refine from there. I also found that during this time, I learned about the views and background of the teachers I was working with, as I also shared with them what my motivations were in pursuing this research. Other than this brief amount of time after the lesson, we as a team were not able to meet during the two-week period that we collected data. This was unfortunate as more time, I believe, would have been productive. However, this is the life of a teacher. Lots of other events and meetings occur, especially in a school with specialized learners.

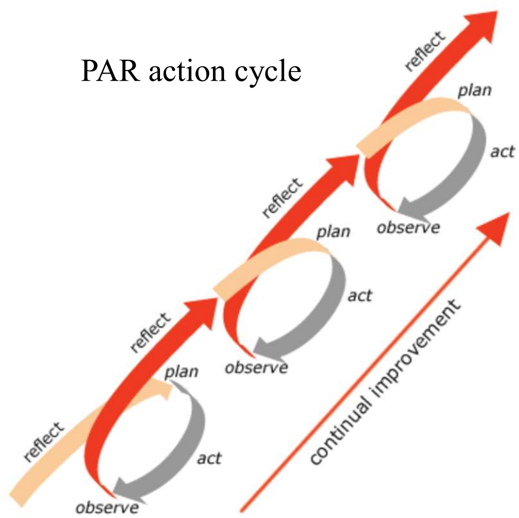


Figure 13: PAR Action cycle *(Queensland Government, 2012)*

Overall the design of the study allowed us to gain a starting point for some aspects that we needed to work on, especially in reference to lesson design. Some things we learned were to focus on designing lessons that were:

- engaging for a large population of students who on average struggle to focus;
- developing their acceptance of struggle within a task—participants were quick to ask for help or just give up;
- less reliant on whole-group discussion time that moved more quickly to an action-based activity, as the group would get restless;
- action-based, especially with manipulatives rather than drawing, as many of the students struggle with fine-motor skills.

It would have been productive to have more time to reflect with the teacher, yet this was just not possible during this time frame. The response from the teacher to these two-weeks in the classroom was positive and she expressed a desire to continue with the study. After Christmas, I began work in another classroom, but Molly informed me that over the holidays she worked on her lessons, so they were no longer focused around just a list of questions on a piece of paper, but that her lessons would incorporate more visual and action-based tasks.

In future planning of a study there are a few things I found myself questioning; one such dilemma was my role as both the facilitator of the lesson and the collector of data. My attention to student thinking was much more analytic when I was doing the pre-assessment interviews. Being responsible for maintaining the flow of the classroom and keeping the engagement going seemed to interfere with my probing deeper into student thinking. I felt them losing focus when I would continually ask students to explain their thinking; this could be done only so much before other students would get too off task. Doing a task-based interview would allow for a more in-depth probing. I would also not have the classroom teacher lead a group of students that are being video-recorded for data purposes, as I noticed there was some inconsistency in the process and depth of probing when it was being led by the classroom teacher.

5.2 Analysis

The question that this study set about to explore was: What role might spatial reasoning play in the growth of mathematical images? As this study initially began with a focus on hermeneutic phenomenology, which then switched to a focus of on the impact of change within an environment, this creates an interesting meld of the two. How I conceptualized this was to seek out meaning within the change in structure. Prior to me joining this classroom, mathematics was done in a fairly traditional manner. I focused on exploring aspects of growth in the students' understanding. How were they being offered information? And what impact did these different types of offerings afford? In my analysis, I generally followed the process laid out by Powell, Francisco, and Maher (2003). In this process they define these steps for approaching data:

1. Viewing attentively the video data
2. Describing the video data
3. Identifying critical events
4. Transcribing
5. Coding
6. Constructing storyline
7. Composing narrative.

My only point of deviation from this process was in the transcribing—this was more descriptive than what is typical of a transcription as I included some *accounting of* (Mason, 2002) interactions between people and actions significant to the activity. I found it a struggle to not focus on assessing patterns in outcomes. This focus diverted me for a while, until I came back to what was the story here? After making a number of assessment focused tables and charts, I decided to sit down and write the videos as a story—what was happening? —writing this story as an *accounting of* and attempting not to *account for* (Mason, 2002). As Davey (2006) so eloquently states, I am interested in "what happens as a consequence of embarking upon such a quest for knowledge" (p. 38). What is the story occurring within the data? As Davey (2006) continued on to describe:

> The process of "becoming cultured" [in your topic] does not involve the acquisition of predictable responses to known problems but the accumulation of sufficient practical experience within a discipline so as to offer a spontaneous and yet informed response to a question permitting it to be grasped in a new and unanticipated way. (p. 39)

In writing and describing this video in words, a story began to emerge between the offerings and the growth of mental images. I began to rely more heavily on the work done by Pirie-Kieren (1994) in their Dynamical Theory for the Growth of Mathematical Understanding, and Husserl's (1970) description of offerings but in their altered state as I described earlier of signitive (written or oral), imaginative (visualizing), and perceptual (sensory). During this process of analysis, I often went back and forth between, the video, the written story, and the students' written work. This triangulation was necessary to ensure as accurate a picture/interpretation as possible.

In taking on such a topic as visualization and images, there exists a struggle of the unseen. How is it that one is to observe visualization and images? This limitation is significant. I can interpret that which can be expressed in language—both verbally and bodily—and I can collect drawings of their interpretations of their images. I can never know precisely what is occurring in their minds or verify what they are visualizing, so I paid close attention to my participants' speech in connection to their bodily movement. I observed instances of Image

Making; participants making “distinctions in previous knowing and us[ing] it in new ways” (Pirie & Kieren, 1994, p. 66). I also observed instances of Image Having; students who seemed to use “a mental construct about a topic without having to do the particular activities which brought it about” (Pirie & Kieren, 1994, p. 66). For this aspect of Image Having or visualizing, I observed how they either stared off in the distance almost like a trance, looked up, gestured into the space in front of them to help make sense of the question; some students seemed to exhibit no physical indicators but would just blurt out the answer. I keep at the forefront these sorts of questions: Is the transition from Image Making to Image Having a gradual transition or instantaneous? How do they present this ability as observable? How are they using their Image Having? Do they go back to an Image Making activity or are they determined to use Image Having from this point on? How is gesturing and language being used during this transition and afterwards?

Chapter Six: Student Profiles

This section is meant to offer additional information on each student, as it was important for me to honor the complexity of each individual within this study. Their profiles offer a more holistic look at the students. Each profile offers information on their psychoeducational assessments, their pre- and post-assessments, an overall look at their tendencies toward spatial reasoning, and some observations of the student within the classroom during the two weeks. These students have faced many challenges within the school environment in which there is much labelling occurring, and it was important to spend time delving into each profile showing the multidimensionality of these students and what the barriers to their learning might be.

My data were collected at a school for students who are labelled with learning disabilities. Each of these students must have a psychoeducational assessment in order to gain admittance to the school. All students were given the Wechsler Intelligence Scale for Children–Fourth or fifth Edition (WISC–IV or V). It is an individually administered clinical instrument for assessing the cognitive ability of children aged 6 years 0 months through 16 years 11 months. Below are the profiles of each student with an interpretive analysis of possible struggles associated with each profile that are generally understood to translate to the mathematics classroom. I have drawn my understanding of these profiles in a very general way from the work of Dr. Gloria Maccow, who is an assessment training consultant with the Pearson Clinical Assessment company. In her training workshops she describes the interpretation practices of the WISC assessment tool. I have focused on four areas related to mathematics: abstract conceptual thinking ability, reasoning ability, part to whole and whole to part thinking, and mental manipulation ability. I will base my interpretations of the students' profiles on Maccow's (2015) advanced interpretation training materials.

Verbal comprehension index (VCI). The VCI consists of mainly two subtest, Similarities, and Vocabulary. The Similarities subtest measures a child's conceptual thinking, asking them to explain essential similarities between pairs of concepts. As the items progress in difficulty, the demands of the task require strong abstract thinking skills and the ability to make connections or classify.

Fluid reasoning index (FRI). The FRI is made up of two subtests. The first is the Matrix Reasoning subtest in which participants study a succession of visual pattern arrangements and reason by analogy in order to select an appropriate picture or form that would fit into a missing part of the pattern. Doing well on this test shows strong logical thinking and deductive reasoning skills. Figure Weights is the other subtest in the FRI, which measures the reasoning skills of the child as they look at pictures of a scale representing equivalent weights. The child must choose which additional weight will balance the scale. Children will need to understand concepts of equality and apply quantitative concepts of matching, addition, and/or multiplication in order to identify the correct response. This involves inductive and deductive logic as well as quantitive fluid reasoning. Overall, the FRI measures the natural ability to engage in fluid reasoning tasks that require the child to solve new problems, to think about logical relationships, to uncover rules or generalizations, and to think in abstract terms.

Visual-spatial reasoning index (VSI). The VSI is also measured through the use of two subtests, Block Design and Visual Puzzles. Block Design uses geometric pattern designs to have the student reconstruct these using patterned blocks. This requires the student to work from the whole down to parts. The Visual Puzzles task asks them to look at the pieces and construct a whole, often complex, geometric shape. This task requires considerable mental manipulation and the ability to work from part to whole.

Working memory index (WMI). The WMI is made up of the Digit Span and the Picture Span subtests. Digit Span looks at short-term auditory memory span, attention/concentration, and mental manipulation. The Visual Puzzle subtest requires students to remember a number of images and then sequence them. This looks at a student's visual memory span and sequencing ability.

Processing Speed Index (PSI). The PSI will not be a big focus within this study other than to recognize whether they are slow processors or not. In my mind, slow processing is not a defect. I believe slow can often mean deep and is not a reflection of the student's mathematical ability or inability (Nadolny, 1987).

Table 3: Core battery of tests within a psychoeducational assessment and how these might relate to mathematical skills and concepts.

Index	Subtest	Application to mathematics
Verbal comprehension	Vocabulary	
	Similarities	Abstract-conceptual thinking
Fluid Reasoning	Matrix reasoning	Reasoning ability
	Figure weights	
Visual-spatial reasoning	Block design	Whole to part reasoning
	Picture span	Part to whole reasoning
Working memory	Digit span	Maintain and manipulate symbols
	Picture span	Maintain and manipulate visuals
Processing speed		

Using these indexes, each student's profile will be assessed to provide a prediction of their abilities in the areas of conceptual abstract thinking, reasoning, part to whole or whole to part thinking, mental manipulation, and working memory. Information for the students' expected abstract thinking ability will be connected to their VCI and in particular the Similarities subtest as it has more of a connection to conceptual abstract thinking than the Vocabulary subtest, and the FRI scoring. Their expected reasoning skills will be interpreted through their FRI scoring. Their potential with mental calculation or mental manipulation ability will be interpreted through their working memory (WMI) score based on Maccow's (2015) work in which she states, "the WMI measures the child's ability to register, maintain, and manipulate visual and auditory information in conscious awareness" (p. 11). Their part to whole and whole to part reasoning will be based on their visual spatial reasoning (VSR) score. It is important to keep in mind that there are many other types of index scores that can be compiled, such as cognitive proficiency index, non-verbal index, general ability index, etc. However, for the purpose of this study, I am wanting to pay attention to more of the specific indexes. In the two tasks that I interpret during this study, adding fractions and comparing fractions through visual representation and use of manipulatives, many different abilities are required such as general reasoning ability, part-whole reasoning, abstract thinking, and mental manipulation.

There were eleven students who consented to being in the study. One of the eleven participants, however, was away the whole week during the intervention. They are not included in the data presented.

Each student profile will begin with

- a prediction of their math ability based on their psychoeducational assessment,
- a description of their performance on the pre-assessment (see Appendices A and B) which contains the topics of
    - general number,
    - addition/subtraction,
    - multiplication/division, and then
    - fractions.

- some classroom observation,
- some observed spatial tendencies,
- observations during the *Imagine-build-steal* game, and then
- their post-assessment, which is a repeat of the initial assessment's fraction section,
- and I conclude each profile with a brief summary.

A couple things to note: the fraction assessment is beyond their grade level as I was mimicking what was expected of them in the Wechsler assessment for this grade. I was interested in seeing how much these students could achieve if engaged spatially. As the study progressed, due to some classroom norms and a very ambitious goal on my part, the students never actually attempted subtraction of fractions before being given the post-assessment yet, I believe, due to their visual understanding, there were students able to be successful with it. The style of the addition and subtraction questions in the assessment were not in the typical symbolic form but in a more visually organized form to help students engage with the idea of addition being the sum of parts and subtraction being a missing part (See Figure 14). Question 2 is an addition question with the solution in the center. Question 3 is a subtraction question with the missing part to be written in the blank space. This format was inspired by Wheatley and Abshire (2002).

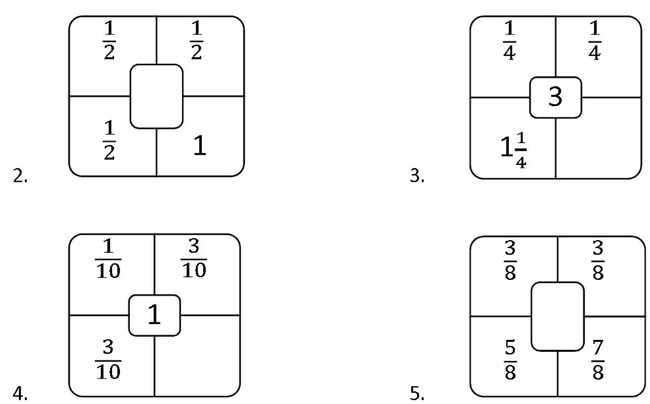


Figure 14: Pre-assessment addition/subtraction questions.

It should be noted that I continued on at the school for another six months after the research period and during this time, I spent two months back in this classroom with these students. What follows are their psychoeducational profiles in regard to their IQ testing and my observations of each student during the study. The language I use for their psychoeducational assessment is from the WISC V (see Figure 15).

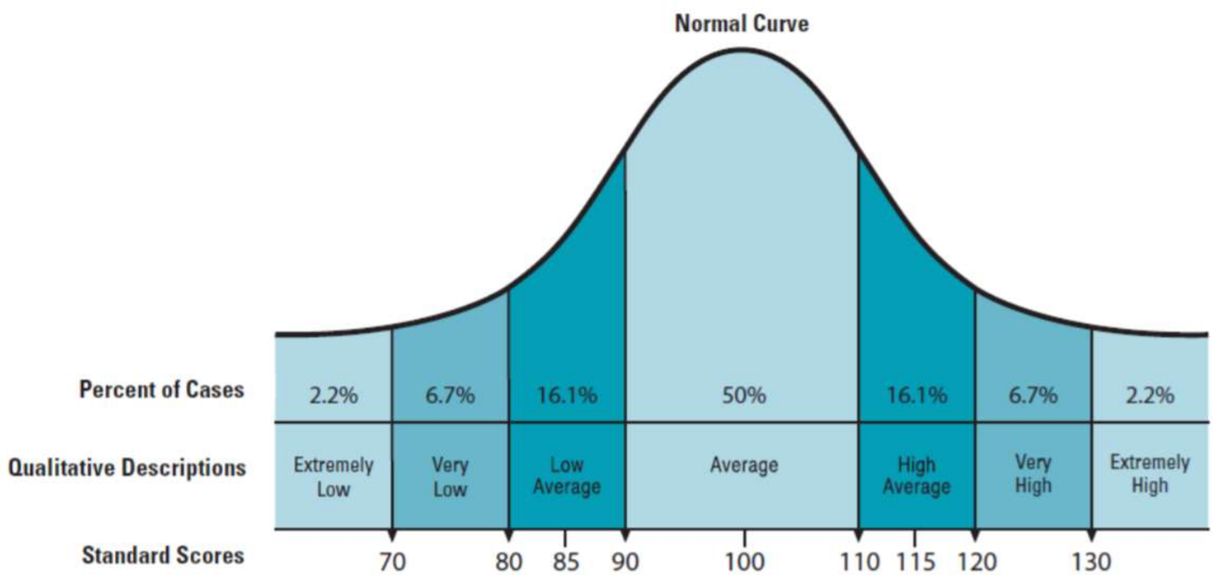


Figure 15: WISC V scoring distribution.

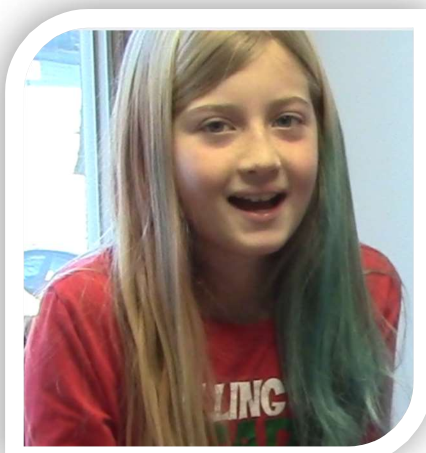
It is my pleasure to introduce my participants:

Figure 16: Participant profile pictures

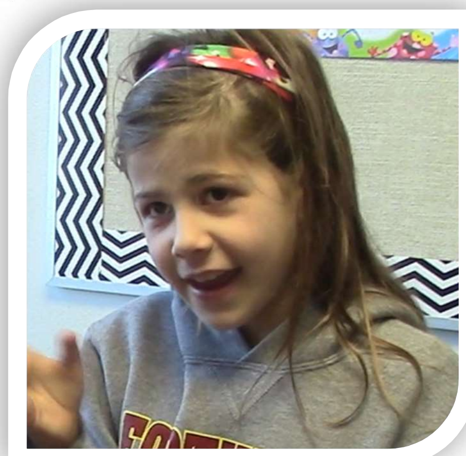
Iris

Elliot

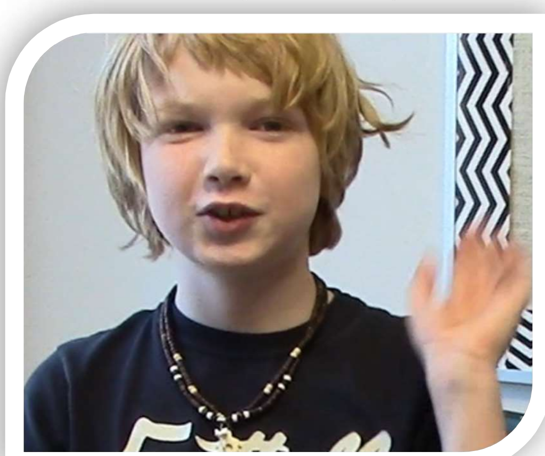
Henry



Joanne



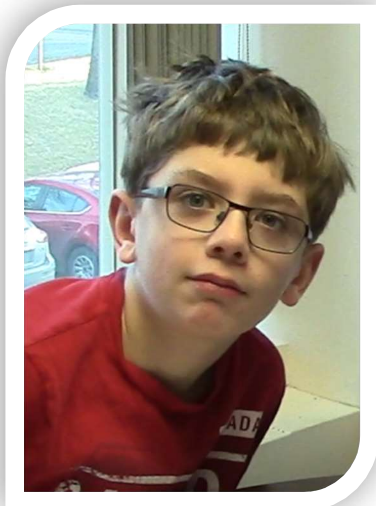
Melinda



Sean

Walter

6.1 Alex



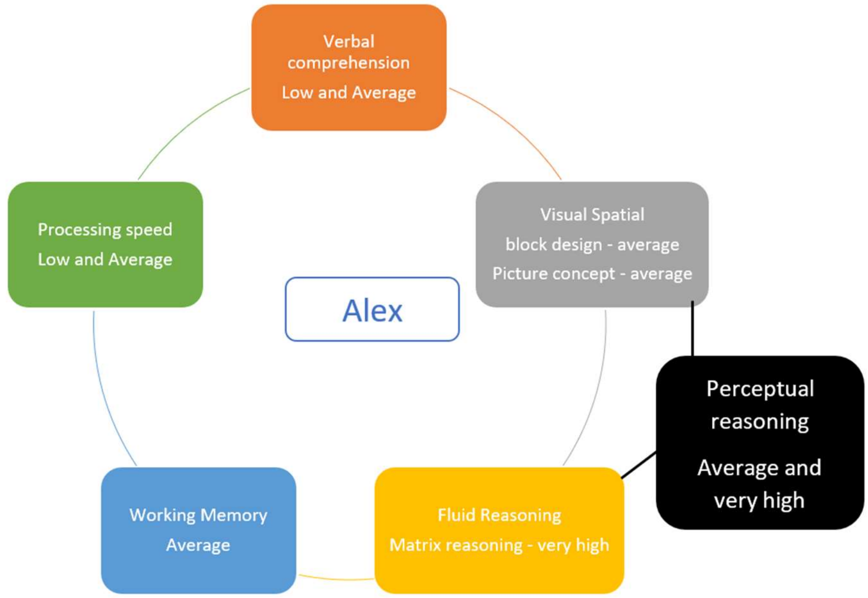


Figure 17: Alex's picture and psychoeducational profile.

6.1.1 Psychoeducational assessment.

- Reasoning skills—based on FRI presented as very high average.
- Abstract thinking ability—based on Alex's VCI (Similarities subtest) he would be in the average range. Alex's FRI scores presented in the very high range.
- Mental manipulation ability—based his WMI he presented in the average range.
- Part to whole and whole to part reasoning—based on his VSR score he presents in the average range.

Overall (See Figure 17), based on Alex's intellectual profile, he should present in the average or above average range with mathematics. He is also a slow processor.

6.1.2 Pre-assessment.

During the pre-assessment (31%), probably the aspect of Alex that stood out the most was his struggle with numbers. Alex would count by ones on virtually every addition question of the pre-assessment using what he calls, Chisenbop. Alex was taught to use this approach at this school to help him get the right answers to calculating questions. This approach was an immense amount of work that did not result in many accurate answers. Alex attempted six questions but only got one correct. He quit when the numbers got bigger. Alex stated, “Its hard cuz, you kinda have to remember the numbers in Chisenbop…and it will take a long time…and I don’t know past one hundred.”

In the fraction portion of the pre-assessment (see Figure 18), he wrote solutions 48 and 10 for these questions. The majority of the other questions he simply put question marks. He got two out of ten on the fraction pre-assessment. Yet even his score of two seemed mainly by chance. He circled two correctly in the comparing fraction section, yet, while choosing, he commented, “I’m just circling random ones, cuz I don’t know.”

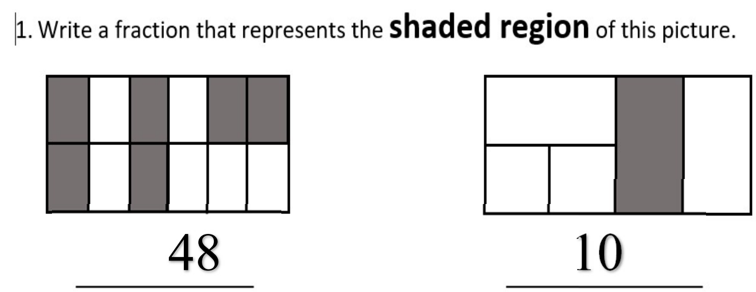


Figure 18: Pre-assessment response by Alex.

6.1.3 Classroom task.

Alex participated in group discussion but was often not very confident in his responses, yet he still offered them freely.

Imagine-build-steal game: addition of fractions. Alex struggled probably the most compared with other students in the study. Alex was very open to suggestions from his classmates who often offered him confusing explanations. This seemed to have a profound impact on his sense making. There were many conflicting messages that he received and as a result he struggled to grasp onto many of the concepts. He openly would admit, “I don’t get this." Although, there were some glimmers of understanding, they were intermixed with statements like, "Would it be smaller if there was one more piece after the whole?" Alex's confidence in regards to which information that he received was correct and which was incorrect was quite shaky; he seemed to just assume that whatever information was offered to him was correct. The underlying assumption was that whatever he did not understand was his fault never the person providing the explanation. He did not get to a place of strong visualizing but did gain a fairly strong level of confidence with building correctly near the end of the task.

Imagine-build-steal game: comparing fractions. In regard to comparing fractions, Alex was able to explain that the bottom number was, "how many pieces", and the top number was "how many are shaded in." Yet, when presented with a comparing fraction question on the board, and asked:

Researcher: If you have three of the same size pieces compared to five of the same size pieces which is bigger?

Alex: uh, five pieces?

Researcher: You don't sound very confident about that. Are you confident or no?

Alex: Kind of

Alex overall remained uncertain or easily swayed with only glimmers of confidence peeking through every now and then.

6.1.4 Spatial tendency

In the Q-bitz[1] spatial activity, Alex seemed to find it quite easy. He kept commenting, "I love these puzzles." He completed an average number of cards. This would speak to his ability for part-whole reasoning as well as particular aspects of spatial ability.

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[1] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

Alex did engage a bit with the manipulatives to help him calculate during the pre-assessment, although he mainly used Chisenbop. He did engage with a play build briefly during the pre-assessment after his partner began building.

During classtime, I gave the students many opportunities for free builds, where they could build whatever they wanted. Alex seemed somewhat interested in building. Although, he did not build in every spare moment as some students did but was happy to engage when given the option. During the free build time Alex's builds (see Figure 19) were somewhat distinct from the other students' in that he would build by placing rods in a way that seemed rather random. Inevitably the structure would fall and he would simply start again.

Figure 19: Alex's free build

6.1.5 Post-assessment:

Alex once again wrote mainly question marks but answered the first two questions with 6/6 and ¼ respectively (see Figure 20). This answer of ¼ was the only one he got correct, which many of his classmates actually got incorrect. He was the only student to have not improved.

Overall, based on Alex's intellectual profile he should have presented in the average or above range in mathematics, yet due to a multitude of reasons, Alex probably struggled the most compared to the other participants. While Alex had some glimmers of solid mathematical thinking this was often overshadowed by some major conceptual

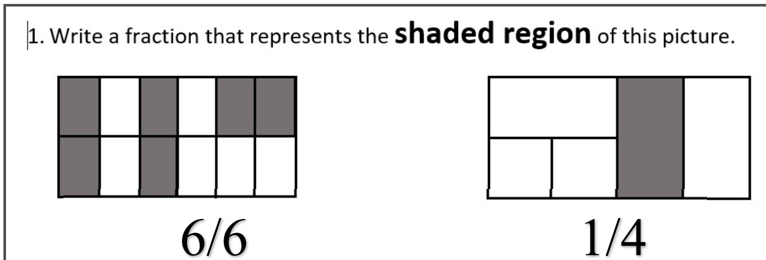


Figure 20: Alex's post-assessment response.

misunderstandings that he had adopted over time. I feel strongly that Alex's biggest issue was not his lack of ability to do mathematics but that he had very little grounding in his own thinking. It was as though he was being tossed to and fro by the opinions of those around him. Alex was very quick to abandon his connection to sense making and go with a classmate's suggestion, always assuming he was wrong. This lack of confidence no doubt stemmed from his lack of background knowledge. As I spent more time with him following the study, I attempted to get him to continually go back to "What do YOU think makes sense?" I hoped to enable him to find his way back to his own integrity of thought. By the end of this two-week period, Alex did not demonstrate a solid grasp on visualizing fractions or compare fractions but was beginning to show understanding of how to build when offered a signitive question.

6.2 Andrew

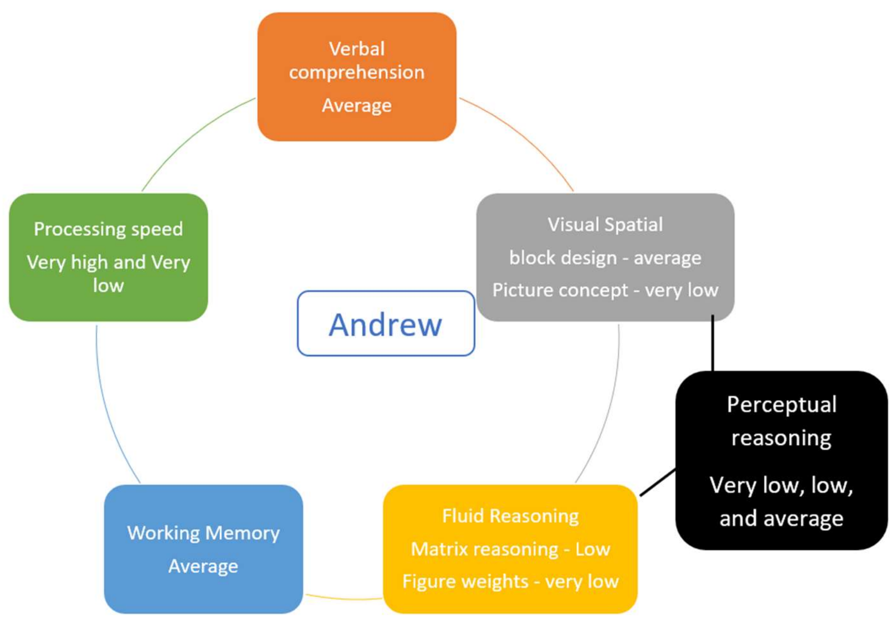


Figure 21: Andrew's profile picture and psychoeducational profile.

6.2.1 Psychoeducational assessment.

- Reasoning skills—based on FRI presented within the low to very low range.

- Abstract thinking ability—based on VCI (similarities test) and FRI scores he presented as average and low to very low range respectively.
- Mental manipulation ability—based on his WMI he presented in the average range.
- Part to whole and whole to part reasoning—based on his VSR score as average for whole to part and very low for part to whole reasoning.

An interesting aspect to Andrew's profile (see Figure 21) is that he has a significant split in his processing speed with his visual processing speed being in the high average range and time sequencing speed in the very low range. Overall, based on these tests Andrew's abstract thinking and low reasoning abilities would suggest that he would struggle in mathematics. His part to whole reasoning was also very weak, yet his whole to part was average. Andrew's mental manipulation abilities should present in the average range.

6.2.2 Pre-assessment.

In the written assessment (55%), Andrew used his fingers to count by ones, such as "13, 14, 15, 16, 17, 18, 19, 20, 21". During the assessment, he made comments like, "I'm just guessing this. I just guessed it. I probably did it wrong." Andrew used a lot of procedure-based mathematics strategies, such as vertically stacking the numbers. He ended the addition and subtraction page by stating, "Can I just stop here cuz I just—I kinda have brain block right now."

For the fraction pre-assessment, he answered 6/6 and ¼ for the first two questions (see Figure 22). In the first addition question, ½ + ½ + ½ + 1, he gave a solution of 10. Andrew got three of the four comparing fraction questions correct, and made the comment, "I know how to do this." He got four out of ten correct for the fraction assessment piece.

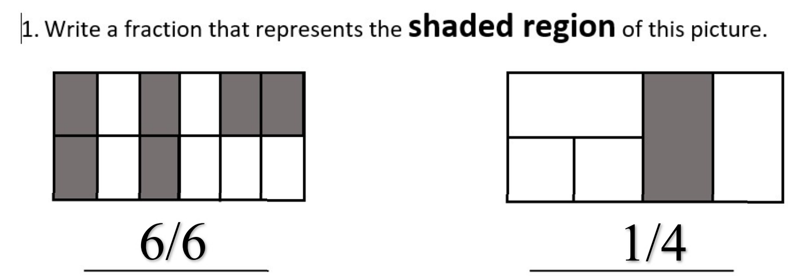


Figure 22: Andrew's pre-assessment response

6.2.3 Classroom tasks.

Andrew was quite comfortable with group discussion. In one particular discussion, after a period of time of struggle from his classmates, Andrew offered a beautiful solution to the question below (see Figure 23). He described his solution through a verbal and gestural description (air drawing). I found this interesting as he presents as quite weak for spatial reasoning in his psychoeducational profile as well as in the Q-bitz[2] task yet seemed able to see this solution.

Figure 23: Slice it demonstration question.

[2] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

Imagine-build-steal game: addition of fractions. Andrew was the first student in his group to make the connection from an addition of fractions expression to the manipulatives and was able to share his understanding with his classmate, Walter. Andrew continued to develop his understanding as he consistently engaged with the perceptual. He demonstrated a solid understanding of how to build the fraction expressions, yet he did not seem to reach the same level of competency with visualizing as some other students did; continued to rely on building it. Yet, he would often claim that he was visualizing the solution. As the teacher did not require a solution based on their visualizing, his visualizing solutions were not confirmed.

Imagine-build-steal game: comparing fractions. During comparing fractions, Andrew did not build at all. This seemed the general trend of all the students. Yet, Andrew, while often claiming to "know the answer," expressed some contradictory verbal responses that would imply he did not "know the answer." Overall, Andrew sporadically was able to express correct answers on his own. My impression was that there was a bit of confusion as the questions sometimes asked, "Which is the biggest fraction?" and at other times, "Which is the smallest?" He would make comments like, "Oh, the smallest". It is unclear whether he was making up excuses for his incorrect response or if he was actually mistakenly looking for the opposite. My impression was that he would mix up the questions and look for the biggest instead of the smallest. Generally, Andrew seemed to understand the logic and explanation of others when they explained their choice.

6.2.4 Spatial tendency.

In the Q-bitz[3] spatial activity, Andrew completed the least number of cards, and commented often, "This is hard." This could speak to his ability for whole-to-part reasoning as well as spatial ability.

During the pre-assessment interview, Andrew did not engage at all with the blocks to assist in solving questions, nor did he demonstrate much mental calculating ability.

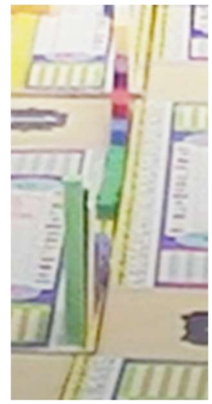


Figure 24: Andrew's free build

Andrew's builds were not elaborate structures but more of a sequencing and lining up of the rods (see Figure 24). Andrew did not demonstrate a need to always be building but rather used manipulatives mainly as tools that he could use to aid him in finding solutions.

6.2.5 Post-assessment.

Andrew correctly answered eight out of the ten questions, getting all the comparing fraction questions correct and three out of the four very challenging addition questions correct. Two of the questions were subtraction questions, which he got correct. For the final addition question, he gave a solution of $2\frac{1}{8}$, when the solution was $2\frac{2}{8}$. He shared the highest score for the assessment with one other student.

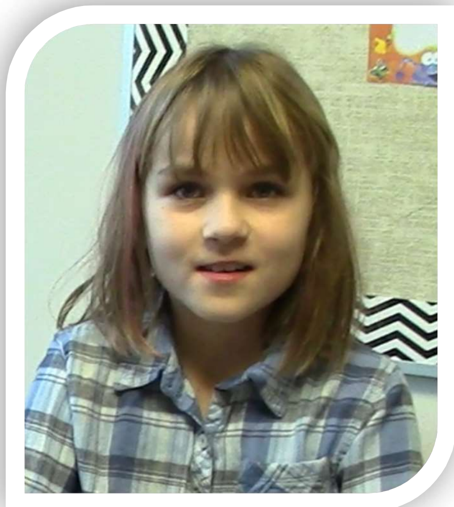
Overall, based on Andrew's psychoeducational profile it would be very plausible for him to struggle with mathematics. In fact, in the pre-assessment interview, Andrew stated that he was at this school because of his struggles with mathematics, although he did not have a MLD label. Based on his profile, his mental manipulation abilities should present in the average range. In

[3] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

practice, Andrew's understanding of number was quite weak for a Grade Five student and he did not demonstrate the use of mental calculations but rather finger counting by ones and procedural strategies for two-digit addition. Andrew showed an interest in building, but he did not engage in very elaborate builds, more of a lining up style than structural builds. When asked to describe his build, he could give no description of his reasoning. He did not build with the manipulatives at all during the pre-assessment but relied heavily on them during the *Imagine-Build-Steal* activity for the purpose of finding a solution. Andrew showed some spatial reasoning ability in his solution to the slice it[4] activity, and although he demonstrated what seemed like a general slipperiness with his visualizing during the adding fractions activity, he was able to demonstrate a very solid level of understand in the post-assessment (80%) for this material that is beyond his grade level.

[4] An activity modelled after the mobile game called Slice it, where you are given a geometric shape and asked to create equal parts by drawing a given number of lines. For example, given a square draw 4 lines to create 9 equal parts.

6.3 Aven



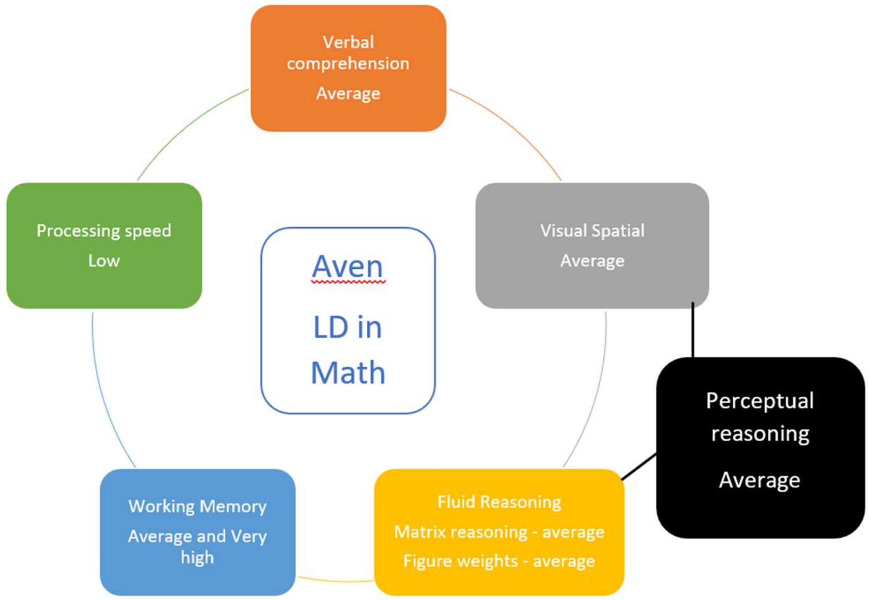


Figure 25: Aven's profile picture and psychoeducational profile.

6.3.1 Psychoeducational assessment.

- Reasoning skills—based on FRI presented as average.
- Abstract thinking ability—based on the VCI (similarities test) and FRI scores she presented as average.
- Mental manipulation ability—based on her WMI she presented in the average to very high range.
- Part to whole and whole to part reasoning—based on her VSR score she is within the average range.

Aven's profile (see Figure 25) is interesting, as she is labelled with a learning disability in math. I can only presume it is due to her academic testing. Other than being a slow processor, based on this profile she should perform at an average level in mathematics.

6.3.2 Pre-assessment.

In the written assessment (63%), Aven demonstrated a good understanding of our number system, although she demonstrated a struggle in the area of addition/subtraction. She did demonstrate thinking in 10's with the use of fingers for the first three addition/subtraction questions. By the third question she needed to use base 10 blocks and counted out the ones. The three questions she answered in this section she got correct and put question marks for all the remaining questions. These three questions took a fair amount of time to complete.

In the fraction section, she said, "I'm really bad at this," then wrote question marks for everything.

6.3.3 Classroom tasks.

During group discussion Aven looked disengaged and seemed to struggle to pay attention. As a result, she did not contribute much to group discussion. Yet, when given a task, she took what was given to her and often went the extra mile. For example, during the Slice it[5] activity both her and Melinda came up with a story to go with each image they were asked to slice (see Figure 26).

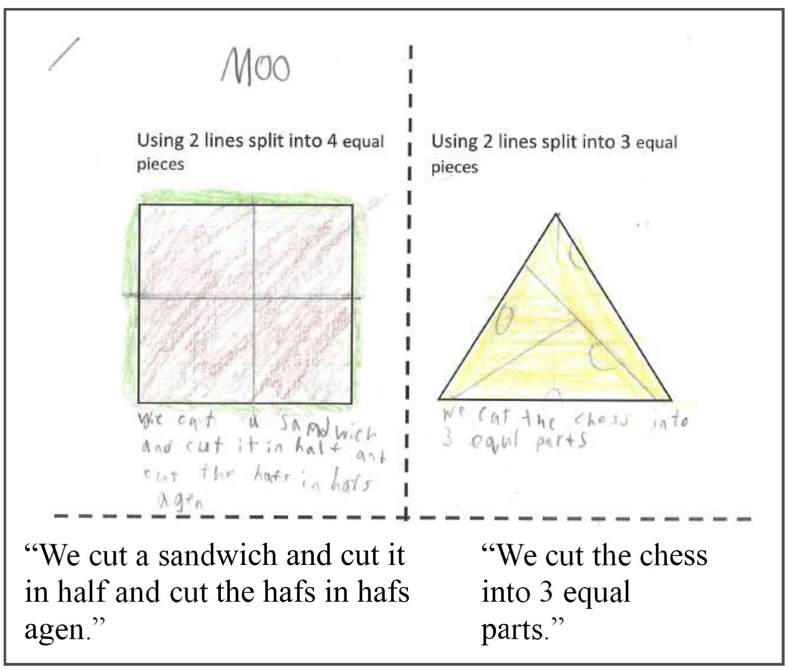


Figure 26: Slice it activity response by Aven and Melinda

[5] An activity modelled after the mobile game called Slice it, where you are given a geometric shape and asked to create equal parts by drawing a given number of lines. For example, given a square draw 4 lines to create 9 equal parts.

Imagine-build-steal game: adding fractions. During the imagine-build-steal game, Aven was very engaged and took the opportunity to build with the manipulatives any chance she could get. Aven started out with some incorrect reasoning but grew more and more confident in her visualization skills to the point where at the end she was able to correctly answer the addition of three fractions with different denominators in her head.

Imagine-build-steal game: comparing fractions. During this activity Aven did not build at all. Yet, she confidently gave her answers based on sound reasoning. When asked which fraction is the smallest? $\frac{7}{12}$ $\frac{5}{12}$ $\frac{4}{12}$ she referred to the numerators by saying, "because 4 is the smallest between 5 and 7, and all the pieces are the same size."

6.3.4 Spatial tendency.

Aven was one of the highest scoring as far as number of completed Q-bitz[6] images. Upon beginning the task, she expressed how bad she was at it. I responded to her that my impression of her to be quite spatial and believed she would enjoy the task. She demonstrated a relaxed nature and ease with the task. This would speak to her ability for part-whole reasoning as well as spatial ability.

Aven demonstrated a bit of mental calculation for the first two questions she did in the pre-assessment addition/subtraction section. She also engaged with the manipulatives on the third and last question she did in this section.

[6] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

One thing that stood out during the pre-assessment was that Aven spent every spare moment building structures and even created a small catapult with the pieces and was excited to share how she could launch the pieces. During the free builds with Cuisenaire rods, it was clear that Aven was in her element (see Figure 27). Her builds were quite elaborate compared to her peers.


Figure 27: Aven's free build

6.3.5 Post-assessment.

Aven got three out of ten. During the post assessment, Aven did the first two questions and then pushed her paper away and started drawing on the paper. When I came around to her and encouraged her to try the next question she said she couldn't. I then read the question to her out loud, "a half plus a half, you can do that?" She completed the question correctly with me standing beside her but as I moved away from her desk she once again pushed the paper aside and started drawing. The rest of the questions were left unanswered. Overall the mood in the room plummeted with the introduction of a paper-pencil assessment. It went from students standing up excited to give their answers to slumping over in their chairs. Many students did not engage or seem to care about completing the written task, Aven was not the only one.

Overall, based on Aven's psychoeducational assessment she should present within an average ability range in mathematics, with a strong working memory which translates to her mental manipulation ability. In practice during the two-week study, Aven demonstrated a struggle with understanding and working flexibly with numbers in the pre-assessment. Yet, she demonstrated strong visualizing ability. Aven loved to build and engage in the spatial tasks offered to her. Often during group discussions, she seemed to not be paying attention, but it is hard to know what she was actually absorbing or not absorbing. There were times during the

Imagine-build-steal game where she looked like she was disengaged and then she would suddenly turn to you with the answer. Aven was eager to engage with many of the tasks but was quick to believe she was bad at something if she struggled or got an incorrect answer. In the end, although Aven was weak in her calculating skills, her ability to add and compare fractions in her head was very strong even though she seemed unwilling to demonstrate it on paper.

6.4 Iris

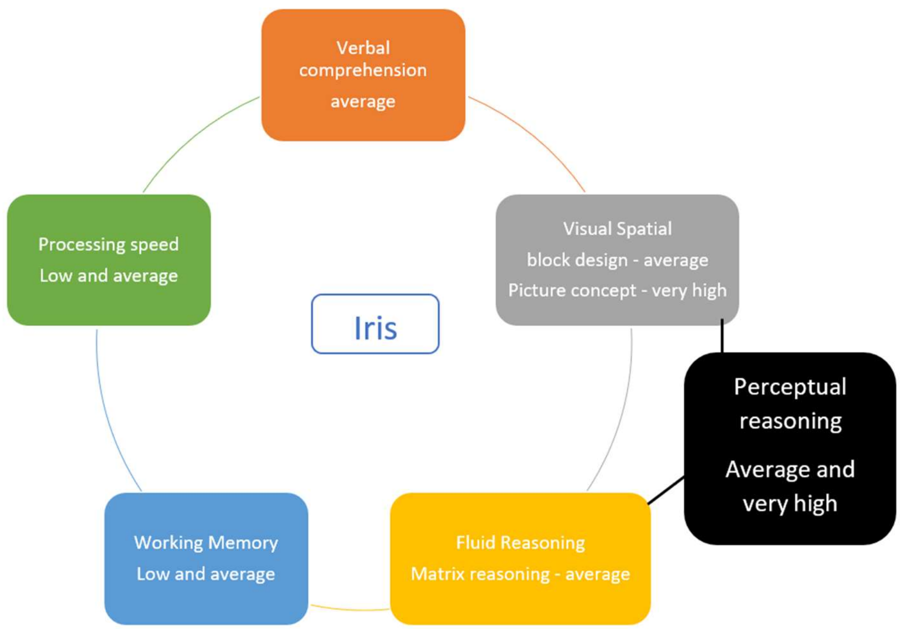


Figure 28: Iris' picture and psychoeducational profile

6.4.1 Psychoeducational assessment.

- Reasoning skills—based on FRI presented as average.
- Abstract thinking ability—based on VCI (similarities test) and FRI scores she presented within the average range.
- Mental manipulation ability—based on her WMI she presented in the low to average range.

- Part to whole reasoning—based on her VSR score is within the average to very high range.

Based on this assessment (see Figure 28), Iris is a slow processor with an average to low ability at mental manipulation. Overall, Iris should present as a person with average ability in mathematics who may struggle in the area of mental manipulation.

6.4.2 Pre-assessment.

During the written assessment (47%), Iris used her fingers a lot and regularly counted by ones, even for, “1, 2, 3, 4, 5.” She used some vertical stacking of numbers for subtraction. She made comments like, “I’m going to guess it,” and “these numbers are getting pretty big.”

For the fraction assessment, she had written 6/6 and ¼ for the first question (see Figure 29). For the addition/subtraction of fractions section, on the first question $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1$, she gave a solution of 16. The second question, $\frac{1}{4}+\frac{1}{4}+1\frac{1}{4}+?=3$, as $\frac{1}{3}$. The third, $\frac{1}{10}+\frac{3}{10}+\frac{3}{10}+?=1$, as $\frac{1}{10}$. And the fifth, $\frac{3}{8}+\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$, as 20. She also got two correct in the comparing fractions section, but she stated, “I am just full on guessing.”

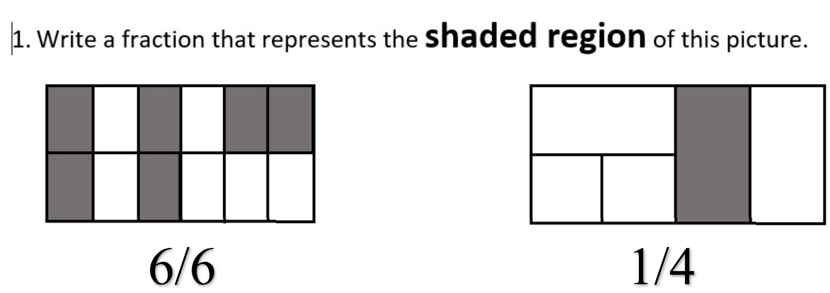


Figure 29: Iris' response in the pre-assessment

6.4.3 Classroom tasks.

Iris was generally a hard-working student who would occasionally volunteer an answer during group discussions. Iris enjoyed the spatial tasks and engaged quite enthusiastically with them. Overall, she is a quiet student, one who could easily slip through the cracks.

Imagine-build-steal game: adding fractions. During this activity, Iris was part of the classroom teacher's group. As with all of the students, except Andrew and then Walter, she was confused by the purely signitive offering of the questions and initially struggled to understand even what she was expected to build. Once Iris was shown a perceptual offering by the teacher she seemed to begin to understand and from this point was mainly able to build given purely a signitive offering. After the fourth question, Iris claimed to be able to visualize the answers for almost every question but as the teacher did not require them to give a solution, this was never confirmed.

Imagine-build-steal game: comparing fractions. Iris was away during the majority of this task and did not come to class until near the end. She watched and listened to the other students' responses to the lesson. And she participated in the previous day's number talk on comparing fractions.

6.4.4 Spatial tendency.

Iris was one of the highest ranking in the Q-bitz spatial task; she completed the second most number of cards. She demonstrated a clear enjoyment of the task. This would speak to her ability for part-whole reasoning as well as spatial ability.

Iris did not use either manipulatives or mental manipulation to help her find solutions to the addition/subtraction questions in the pre-assessment. She also did not play with the manipulatives in the pre-assessment and during free play she mainly lined up the rods (see Figure 30). She gave no evidence of creating a story line or developing anything structural.

Figure 30: Iris' free build

6.4.5 Post-assessment.

During the post assessment, Iris answered all the questions, getting five out of ten correct. All of the comparing fraction questions were correct, even though she missed most of this activity. The addition/subtraction solutions were also very close to the actual, other than the last one. In the first question she put a solution of $3\frac{1}{2}$ when the solution was $2\frac{1}{2}$. The second question she gave the solution $\frac{1}{4}$, when it should have been $1\frac{1}{4}$. In the third she gave a solution of $\frac{2}{10}$, when it should have been $\frac{3}{10}$. There seemed to be a good beginning level of understanding especially compared to her initial testing. These struggles of being off one, whether it was a whole or not, could have been due to a weaker working memory score than was present in her profile, this idea of maybe trying to hold things in her head. I believe she would have done well had she been offered manipulatives.

6.4.6 Summary.

According to Iris' psychoeducational assessment, she should present with average ability in mathematics with a bit of a struggle in the area of mental manipulation. Iris demonstrated a very weak understanding of number structure for a Grade Five student as she mainly used her fingers to count by ones. Adding and subtracting seemed a very arduous task. Iris responded positively when provided perceptual offerings and she seemed eager to engage with any spatial task offered to her. In the end, she claimed to be visualizing the addition of fractions but this was not verified. She could confidently build solutions from just being offered the signitive fraction expression by the end of the *imagine-build-steal* task. Also, although she missed most of the comparing fraction task, she was able by the end of that period to demonstrate a solid understanding of the task by answering all four questions correctly on the post-assessment. She persevered through the whole written task and demonstrated some promising understandings of the addition/subtraction of fraction questions, even though none ended up being correct.

6.5 Elliot

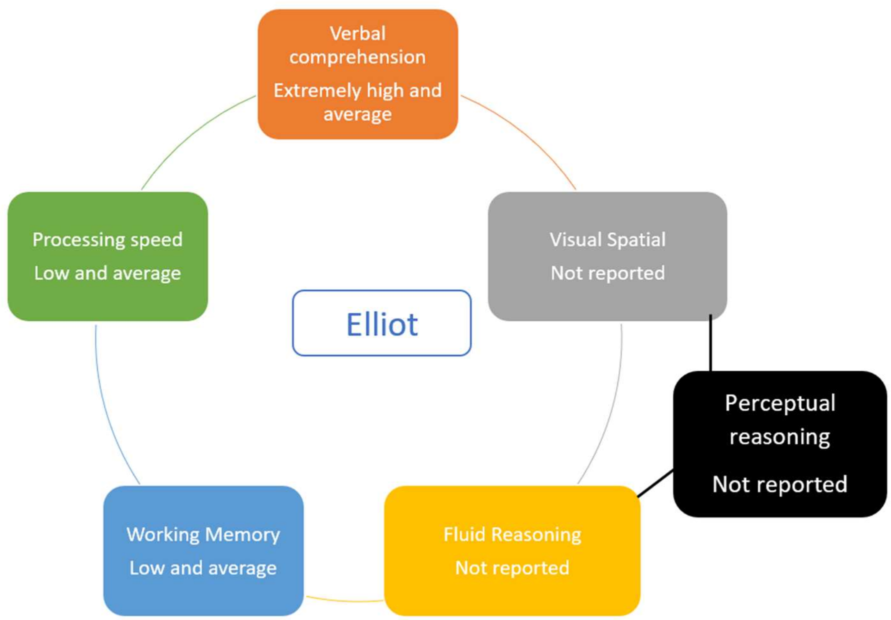


Figure 31: Elliot's picture and psychoeducational profile

6.5.1 Psychoeducational assessment

- Reasoning skills—not reported.
- Abstract thinking ability—based on VCI (similarities test) scores in the average range. This can only be partially commented on as there was no reported testing on the FRI scores.

- Mental manipulation ability—based his WMI he presented in the low to average range.
- Part to whole reasoning—not reported.

It is tough to give an overall summary of Elliot's profile as it is incomplete (see Figure 31), but it generally seems within the average range with a possible area of struggle being mental manipulation.

6.5.2 Pre-assessment.

During the written assessment (78%), Elliot had the highest score in the class. On the addition/subtraction section he answered every question correctly doing all the calculations in his head. With an apparent low score for working memory, Elliot did not use his fingers or any manipulatives. He also did not build or play with the manipulatives during the session. Interestingly enough, one question that he could not answer was how to draw a visual representation of 4x6.

For the fraction pre-assessment, Elliot got eight out of ten. He was definitely the strongest student on this section, when asked about his solid understanding of fractions he explained that he did half a year at his old school in Grade 3. Elliot got three out of four of the addition and subtraction questions along with all four comparing questions. Elliot had a solid understanding of fractions and was able to add and subtract fractions even though he would not have been taught this in school up to this point.

6.5.3 Classroom tasks.

Elliot was very involved during classroom discussions. He frequently had insightful comments to make and had a strong level of confidence in his mathematical ability.

Imagine-build-steal game: adding fractions. Elliot caught on quickly in this task and was able to visualize the easier questions quite quickly. However, it turns out that his skin was reacting to the foam that the manipulatives were made of. This started to impact his performance or maybe just his engagement as the task continued. Those students who continued to engage with the building of the solutions seemed to surpass Elliot's ability. It is hard to know what the contributing factor was to Elliot's eventual disengagement with the task; it could have been boredom as he was not able to build; it could have been harder for him to visualize so he gave up; it could have been that others were surpassing him and so he disengaged to save face. It is hard to know, but by the end of this task, it was clear that Elliot was not putting in much effort on the final questions. For what seemed like lack of effort, Elliot did not demonstrate a strong ability to visualize the solutions to the more complex fraction questions.

Imagine-build-steal game: comparing fractions. Elliot was able to easily engage with the comparing fractions questions with what seemed like minimal think time and no manipulatives. He provided good reasoning when asked to give justification. In one question, Elliot explains his reasoning by holding up the twelfth's pieces and states, "These are smaller than eighths and fifths."

6.5.4 Spatial tendency.

In the Q-bitz[7] spatial task, Elliot presented in the average range. He seemed very aware of how everyone else was doing at the table and seemed somewhat stressed to make sure he was faster.

[7] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

Elliot used lots of mental manipulation to solve all the addition/subtraction questions. He did not use or seem to need the manipulatives.

Figure 32: Elliot's free build

He also did not play with the manipulatives during the pre-assessment. Elliot enjoyed the free building time with the Cuisenaire rods and built what he described as a skate park (see Figure 32).

6.5.5 Post-assessment.

During the post assessment, Elliot actually got one point less than his pre-assessment score. He only got two of the four addition/subtraction questions correct but was very close on the final one with an answer of $2\frac{1}{8}$, instead of $2\frac{2}{8}$. There was one subtraction question he got correct in the pre-assessment that he did not even answer in the post-assessment. All of the comparing questions were correct.

6.5.6 Summary.

Based on Elliot's partial psychoeducational assessment, he should present as average in his abstract thinking ability but may struggle with mental manipulations, the other aspects were not reported on. Overall, for a student with apparently weak working memory, Elliot showed a strong ability at calculating in his head and seemed to have a strong understanding of number. Elliot was confident in mathematics and seemed to enjoy the position of strength among his classmates. He enjoyed building during free play with the Cuisenaire rods in which he built a structure for a skate park. Elliot showed an ease with visualizing initially, but as the addition fraction questions became more difficult he ended up falling behind some of the other students for various reasons.

6.6 Henry

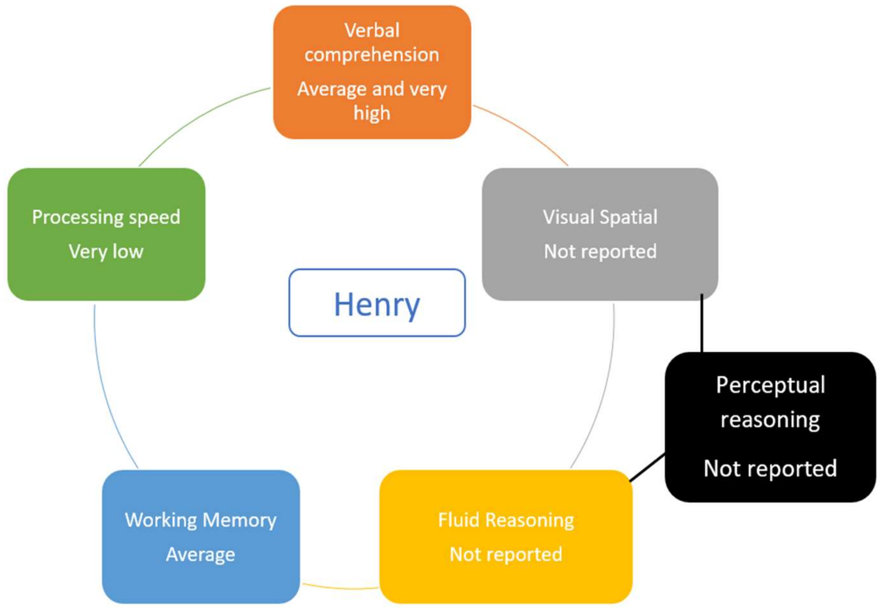


Figure 33: Henry's picture and psychoeducational profile

6.6.1 Psychoeducational assessment.

- Reasoning skills—not reported.
- Abstract thinking ability—based on VCI (similarities test) he is in the very high range, but this is only partial as his FRI scores were not assessed.
- Mental manipulation ability—based on his WMI he presented in the average range.
- Part to whole reasoning—not reported.

Henry presented as a slow processor. It is tough to give an overall summary of his profile as it is incomplete (see Figure 33), but he generally presented in the average range with strong abstract thinking ability.

6.6.2 Pre-assessment.

In the written assessment (42%), Henry started building right way, and took every opportunity to build. Henry did not use his fingers to add but rather seemed to prefer the base ten blocks for every question. He did three out of the eight addition and subtraction questions with two that were answered correctly. He demonstrated some great spatial reasoning ability with the blocks for subtraction. He built twenty then built 9 + 2 + 8 and measured it up against 20, and noticed it was 1 less. So, he wrote a solution of 1 which was correct. The one question of the three that he got incorrect he only partially built, which caused him to be a bit confused and off by 5 on his solution.

> Henry: Okay, 35 plus another 10 [He grabbed the block] plus 5 [He did not grab 5 blocks. He then looked up.] That would make 45, no. [Paused. Shifted in his chair.] Yeah, I think I did that right. [Laughed.] I think, okay. [Looked up again. Rubbed his head.] Um.

Also, when ask to write an example of a multiplying question, he wrote 3 x 2 + 4 + 8 then said,

> Henry: So, subtracting?
>
> Researcher: No timesing.

Henry crosses his arms to form an x.

> Henry: 3 times 2 is like 10. No, that would be 12 plus…okay, 3 times 2 is 6 plus 4 plus 8.

Henry would often think things through out loud. It would often occur that I would have no idea what he was referring to because he was not talking to me but rather himself. Henry struggled a lot with basic number skills for a Grade Five student. With addition and subtraction, it struck me as a lot of effort for him. He did demonstrate some understanding of the structure of number but was definitely still in the Image Making phase (Pirie & Kieren, 1994), which is well below grade level based on the questions he was given.

In the fraction section, Henry got four out of ten correct. He was the only student to write one half for the first question (see Figure 34). He then answered the question, $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1$, as being equal to 9, and $\frac{1}{4}+\frac{1}{4}+1\frac{1}{4}+?=3$, as $\frac{1}{2}$. The rest were blank. He also got two comparing fraction questions correct. In this section, he described using two different strategies for finding the biggest fraction, in one grouping he chose the one with the biggest number on the bottom and in the other grouping he chose based on which had the biggest number on top. He figured one of the two strategies would end up being correct. He showed some good reasoning skills although not always conceptually correct. His base knowledge was extremely weak which was demonstrated by his initial uncertainty of even how to write in fraction form.

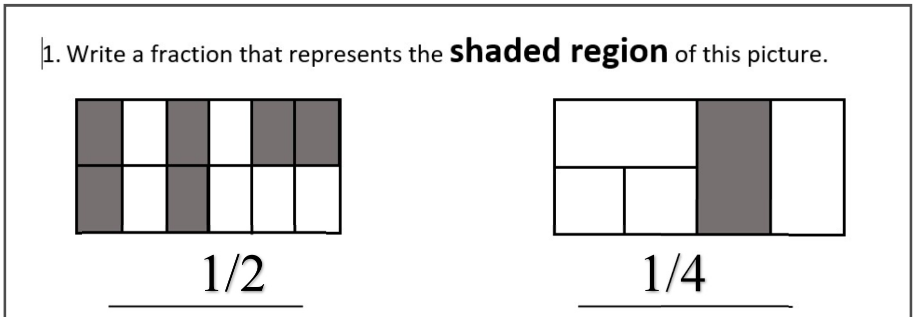


Figure 34: Henry's pre-assessment response

6.6.3 Classroom tasks.

Henry loved to participate in class discussion. He often would express his ideas and logic behind his solutions. Henry seemed to have a vivid image going on in his head often, but it was sometimes hard to follow his descriptions.

Imagine-build-steal game: adding fractions. During the imagine-build-steal game, Henry was very engaged and took every opportunity to build with the manipulatives. Like Aven, Henry started out with some incorrect reasoning, but grew more and more confident in his visualization skills to where at the end he too was able to correctly answer the addition of three fractions with different denominators in his head.

Imagine-build-steal game: comparing fractions. During this activity Henry only did a partial build once at the beginning and then another near the end. Throughout the rest of the activity he expressed answers confidently and gave sound reasoning to back them up. When given the question, $\frac{2}{12}\ \frac{2}{8}\ \frac{2}{4}$, and asked to find which the smallest is he offers the explanation: "12 is like the highest number but like, but if you cut up—if you get a sandwich cut it up 12 times you have the smallest pieces of sandwiches."

6.6.4 Spatial Tendency.

Henry presented within the average range for the Q-bitz[8] activity. He did not especially stand out in finding this task either too easy or too hard.

Henry used the manipulatives a lot during the pre-assessment as his main tool towards finding solutions. He did not demonstrate visualizing while performing mental calculations.

Henry seemed to enjoy building with blocks and during the pre-assessment he would grab for the blocks and start building any chance he could get. During free play with the blocks Henry did more of a lining up style interaction with the rods (see Figure 35), although more elaborately than the others who also were mainly lining up.

Figure 35: Henry's free build

[8] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

6.6.5 Post-assessment.

During the post assessment, Henry did the first four questions and then stopped. He got three out of four correct. Just as with some other students he did not seem motivated by the task and did not even attempt the rest of the questions. As I stated earlier, the mood in the room plummeted with the introduction of a paper-pencil assessment. There were a number of students who did not engage or seem to care about completing the task. Henry was one of those, even though orally he demonstrated a strong understanding in the both of the imagine-build-steal tasks.

6.6.6. Summary.

Based on Henry's partial psychoeducational assessment, he presented with strong abstract thinking ability and an average ability to perform mental manipulations. In practice, Henry loved to build and engage in the spatial tasks offered to him. He did not demonstrate a strong understanding of number for his grade level, but strong reasoning skills. He engaged with the tasks enthusiastically and demonstrated a strong ability to visualize. Henry would not be easily swayed by other students' opinions but rather he would only change his views if he could be convinced. Henry especially enjoyed engaging with group discussion and sharing his thinking.

6.7 Joanne

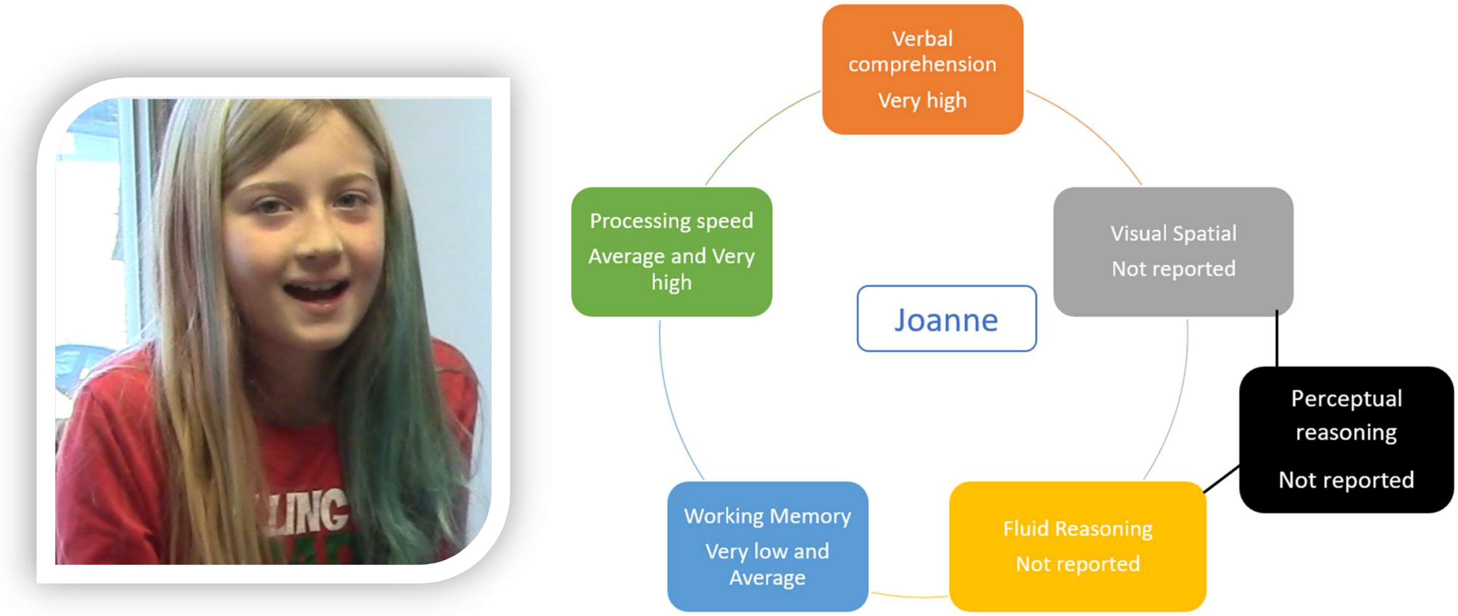


Figure 36: Joanne’s picture and psychoeducational profile

6.7.1 Psychoeducational assessment.

- Reasoning skills—not reported.
- Abstract thinking ability—based on VCI (similarities test) she is within the very high range, but this can only be partial commented on as her FRI scores were not reported.
- Mental manipulation ability—based her WMI she presented in the very low to average range.
- Part to whole reasoning—not reported.

Joanne’s report is only partial (see Figure 36). Joanne’s processing speed is strong, but her mental manipulation ability presented in the very low to average range which could impact her visualizing skills. It is tough to give an overall summary of her profile as it is incomplete, but it generally seems in terms of abstract thinking ability Joanne should be strong but with a struggle to mentally manipulate.

6.7.2 Pre-assessment.

During the written assessment (46%), Joanne used her fingers but she also used the vertical stacking of numbers a lot. She even wrote 2 + 3 vertically and put a solution of 5 underneath. She also wrote 5 + 7 and 5 + 12 using this same strategy. She only attempted the first three addition/subtraction questions and only got the first correct. These questions seemed to be quite effortful for her. Joanne demonstrated a strong level of persistence in many areas of the assessment. During the multiplication section she attempted the larger multiplication 8 x 12 by writing out: 8, 16, 24, 32, 40, 48, 46 [error], 54, 62, 70, 78, 85, 92. This was a painful amount of work as I watched her count up by ones to eight on her fingers, then write the number, and repeat. She also attempted 6 x 30 and wrote a solution of 30 using the logic that, "because if you times anything by 0 it is just 0."

For the fraction assessment, in the initial question she demonstrated what seemed like an uncertainty with how to write a fraction (see Figure 37). She also attempted the first addition question, $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1$, and gave a solution of $\frac{3}{7}$. She seems to have attempted to add up denominators 2 + 2 + 2 + 1 for the bottom and then 1 + 1 + 1 for the numerator. The remainder of the questions she left blank and stated, "I am really confused."

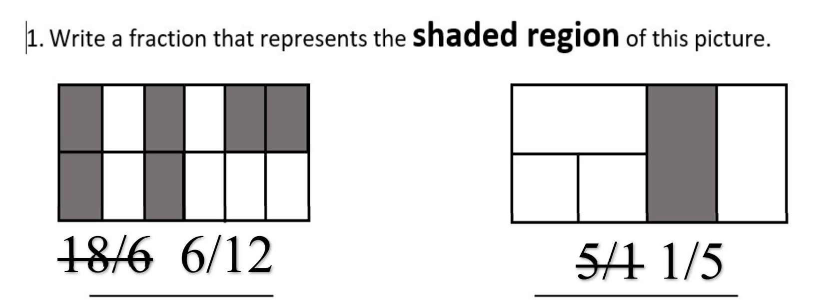


Figure 37: Joanne's pre-assessment response

6.7.3 Classroom tasks.

Joanne presented as a hard-working student who often volunteered her thinking during group discussions. Joanne seemed to enjoy the spatial tasks and engaged quite enthusiastically with them. Overall, Joanne worked diligently on the task she was given and seemed eager to please.

Imagine-build-steal game: adding fractions. During this activity, Joanne was part of the classroom teacher's group. As with all of the students, other than Andrew and later Walter, she was very confused by the purely signitive offering of the questions and initially struggled to understand even what she was expected to build. By the fourth question when Joanne was shown a perceptual offering by the teacher she began to understand and from this point on claimed to be able to visualize the solutions before the build. Once again, as a verbal response was not a required, it was hard to know. Joanne did verbally demonstrate on a couple of questions that she had the solution before she built it. However, she always asked if she could verify with the manipulatives.

Imagine-build-steal game: comparing fractions. Joanne demonstrate a strong level of competency with comparing fractions. It was especially interesting how she justified her solutions, "If you fold a blanket 6 times you still get quite a bit of blanket, or if you cut a sandwich 6 times you still get quite a bit of sandwich not like if you had 14. If you fold a blanket 14 times it will have little, little, little pieces."

6.7.4 Spatial reasoning.

Joanne finished an average number of Q-bitz[9] cards, but expressed that it, "hurt her brain." This could relate back to her WMI score being low and as a result struggling to mentally manipulate the pieces.

Joanne did not demonstrate the use of visualizing to attain solutions in the addition/subtraction session, however, she did use manipulatives.

She did not use the blocks to fidget or play with during the pre-assessment, nor did she seem very engaged with the Cuisenaire rods during the free play (see Figure 38). She briefly lined up some blocks but otherwise did very little with them.

Figure 38:
Joanne's free build

6.7.5 Post-assessment.

During the post assessment, Joanne got six out of ten correct. All of the comparing fraction questions were correct. With the addition/subtraction questions, she attempted three and got the first correct and was close on the other two. The second subtraction question, she gave the solution $\frac{1}{4}$, when it should have been $1\frac{1}{4}$. In the third subtraction question, she gave a solution of $\frac{1}{10}$, when it should have been $\frac{3}{10}$. There seemed to be a much better level of understanding compared to her initial testing. I believe she would have done well had she been offered manipulatives.

6.7.6 Summary.

Overall, based on Joanne's psychoeducational assessment she was partially reported to have very strong abstract thinking skills, with a possible struggle in the area of mental

[9] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

manipulation. In the pre-assessment and classroom tasks, Joanne demonstrated a very weak understanding of number structure for a Grade Five student as she mainly used her fingers to count by ones and used vertical stacking of number procedures. Adding and subtracting seemed very challenging for her. For multiplication she had a much better understanding but could not remember her times tables and then was left to adding, but as her addition skills are so weak she would count up by ones. Joanne responded positively when provided perceptual offerings and she seemed eager to engage with any spatial task offered to her. In the end, she claimed to be visualizing the addition of fractions and was able to demonstrate this ability on a couple of questions. She could confidently build solutions from just being offered the signitive fraction expression by the end of the imagine-build-steal task. She had a strong level of reasoning skills for comparing fractions and was able to demonstrate this by answering all four questions correctly on the written task at the end of period. She did three of the four addition/subtraction fraction questions and demonstrated some promising understanding, even though only one ended up being correct. This was a strong distinction from her initial fraction work of adding the numerators and then adding the denominators as she demonstrated in the pre-assessment.

6.8 Melinda

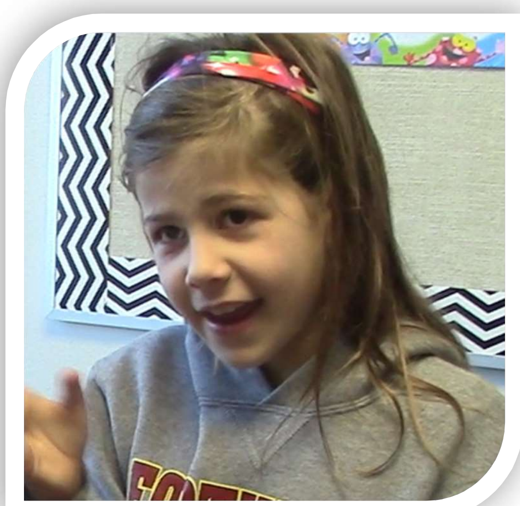
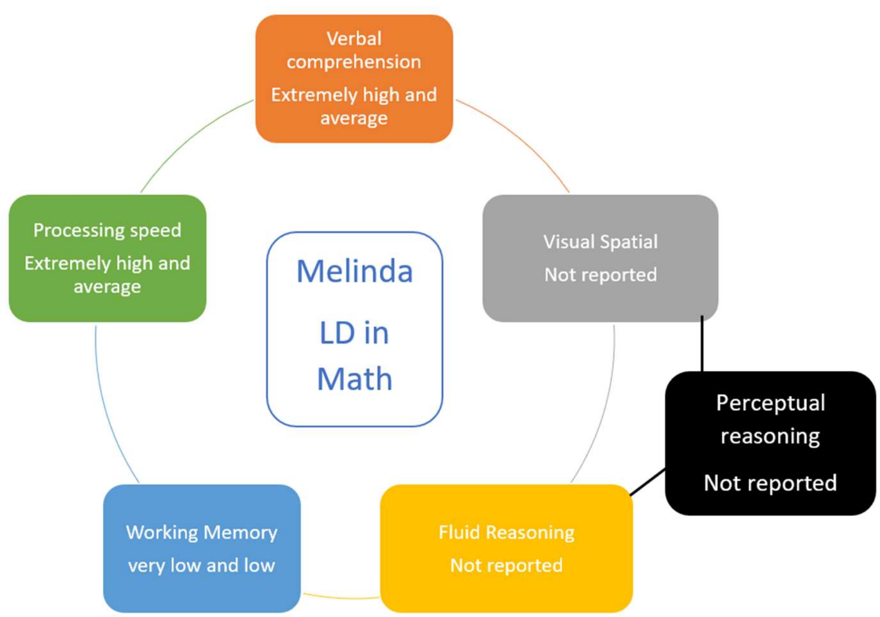


Figure 39: Melinda's picture and psychoeducational profile



6.8.1 Psychoeducational assessment.

- Reasoning skills—her FRI was not reported.
- Abstract thinking ability—based on VCI (similarities test) she scored in the very high range, but this is partial as her FRI scores were not reported.
- Mental manipulation ability—based on her WMI she presented in the very low to low range.
- Part to whole reasoning—not reported.

Melinda’s report is also partial (see Figure 39). She presented as very strong in her processing speed but could present quite weak in the area of mental manipulation as her WMI was scored as very low to low average, otherwise she should present in the very high to average range for abstract thinking ability.

6.8.2 Pre-assessment.

During the written assessment (48%), Melinda used her fingers for finding solutions to addition/subtraction questions. She also vertically stacked numbers for questions like 3 + 2 and 5 + 7 and 12 + 5. She only attempted the first three addition/subtraction questions and got the two addition ones correct. At the end of these three questions she stated, “Yeah, I’m done. I feel done.” She then began building. Melinda did not produce a drawing for a representation of multiplication but rather wrote “number 24”. She produced the solutions to basic multiplying question quickly but did not attempt the ones with bigger numbers.

For the fraction assessment, she was able to correctly answer the first one and made a common error of 1/5 for the second (see Figure 40). She attempted the first addition question, $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1$, and like Joanne gave a solution of $\frac{3}{7}$, but did not attempt any more. Melinda got one of the comparing fraction questions correct.

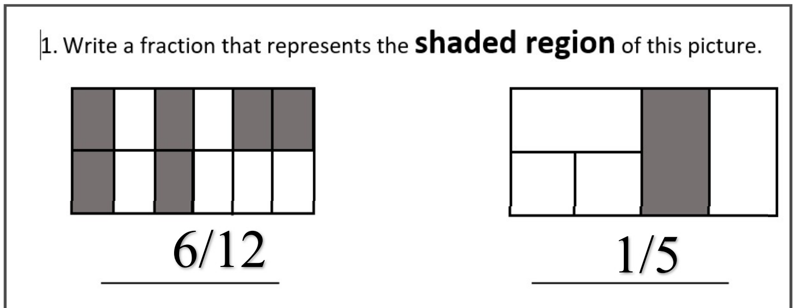


Figure 40: Melinda's pre-assessment response

6.8.3 Classroom tasks.

Melinda was an extremely friendly and helpful student to her classmates. She often demonstrated a lack of confidence in her mathematical ability as she tended to rely on help from classmates even when her thinking was correct. However, Melinda loved to engage with story as seen by her contribution to the creation of a story with the slice it[10] activity (see Figure 41). Both Aven and Melinda were very excited about what they had produced.

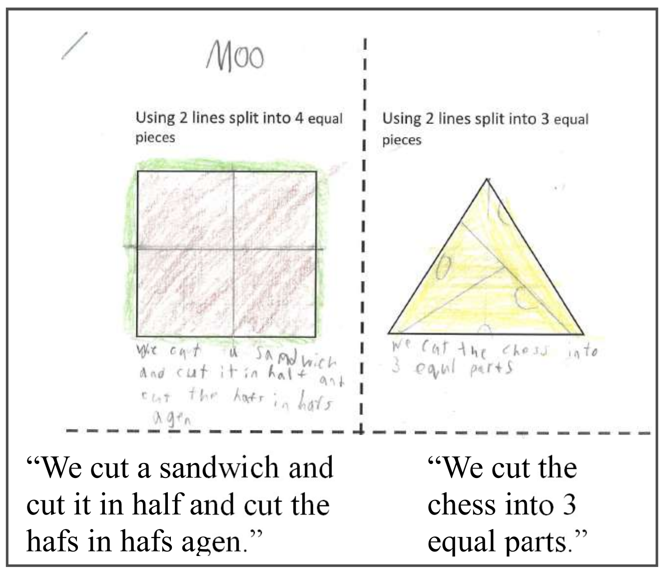


Figure 41: Slice it activity response by Aven and Melinda

Imagine-build-steal game: adding fractions. During this activity, Melinda was part of the researcher's group. Melinda's confidence

[10] An activity modelled after the mobile game called Slice it, where you are given a geometric shape and asked to create equal parts by drawing a given number of lines. For example, given a square draw 4 lines to create 9 equal parts.

was generally quite low. This was demonstrated by her repeated verbal responses of "I don't know" and "I'm not good at this." Melinda showed little confidence in her own sense making as in this situation, $\frac{2}{3}+\frac{1}{3}$, when she had an idea about the solution being a whole but did not speak to it and seemed to just assume she was wrong and her partner was right.

Aven: A whole and a half.

Researcher: What do you have there Melinda?

Melinda holds up her circle with three thirds that she's built. Aven takes one of her thirds away.

Melinda: I'm so confused. This is so hard. I feel like I'm in court.

Researcher: So, what is your answer?

Aven: [pauses for a long time then says] Three sixths.

Researcher: Three sixths? [researcher turns to the boys]

Melinda: I have an idea. [speaks very quietly]

Researcher does not hear her.

Henry: A whole. A whole. One whole.

Elliot: A whole.

Melinda: Yeah, that is what I was kinda thinking.

Researcher: [turning to the girls] Okay, can you build it for me? So, two thirds [pointing to the fraction]. Okay so you guys build two thirds, and then another third.

Melinda: That's what I was thinking.

By the end of the activity, Melinda was able to confidently build the solutions with her manipulatives but presented as struggling to visualize. Melinda often used gesturing to help construct the image (see Figure 42), which seemed to aid her visualizing. In fact, for the question, $\frac{3}{6} + \frac{2}{6} + \frac{1}{6}$, she used this gesturing technique and verbally produced the solution of a whole before she built with the manipulatives. Overall, Melinda seemed to rely heavily on building the solutions with manipulatives which she was able to do well. Her confidence remained low with visualizing, which actually fits with her psychoeducational assessment, but once again it is hard to know what images she saw in her mind but was not confident enough to share.

Figure 42: Melinda's hand gestures

Imagine-build-steal game: comparing fractions. Melinda was away for most of this activity. She did not answer any of the questions for her team, so I cannot comment on her understanding during this task.

6.8.4 Spatial tendency.

Melinda completed the most cards in the Q-bitz[11] task along with Sean. She clearly enjoyed the task and completed the builds with ease.

Melinda did not demonstrate the use of mental manipulation or manipulatives for the purpose of finding solutions during the pre-assessment.

[11] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

Her building during the pre-assessment was not so much creative as making a longer rod or building a bigger cube, but she seemed quite engaged by the activity. Melinda seemed to struggle with the free build activities (see Figure 43). Initially, she just mimicked Aven's structure and then decided she wanted to build a bed. She attempted to build a bed a number of times but seemed frustrated by the task, so turned to Aven and said, "I need help making a bed." Aven quickly built one for her. Melinda did not produce much in the way of building during this activity.

Figure 43: Melinda's free play

6.8.5 Post-assessment.

Melinda attempted all the questions on the assessment with a score of six out of ten. She got the first addition question correct without the use of manipulatives. Although, Melinda was away for most of the comparing fractions activity she was able to answer all four questions correctly.

6.8.6 Summary.

Overall in Melinda's psychoeducational assessment, she was partially reported as being very strong in abstract thinking. Her mental manipulation ability presented as quite weak in this assessment. In practice, Melinda seemed to enjoy coming up with stories and was able to reason through tasks quite well despite having missed most of the task with comparing fractions. Melinda's confidence in mathematical thinking presented as quite low which seemed to affect her persistence with the tasks. She did seem to struggle with visualizing, but, as always, it is hard to say if this is due to her struggle to produce an image or her lack of confidence in her image and her reluctance to share as a result.

6.9 Sean

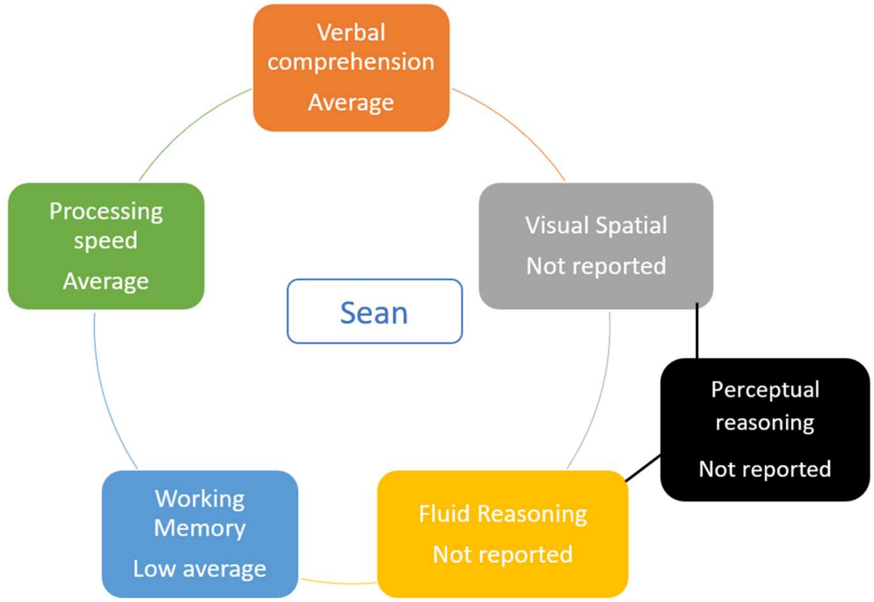


Figure 44: Sean's picture and psychoeducational profile

6.9.1 Psychoeducational assessment.

- Reasoning skills—his FRI was not reported.
- Abstract thinking ability—based on VCI (similarities test) he is within the average range, but this is only partial as his FRI scores were not reported.
- Mental manipulation ability—based on his WMI he presented in the low range.
- Part to whole reasoning—not reported.

Once again this is a partial assessment as his report did not include either a visual spatial or fluid reasoning report (see Figure 44). He presented with an average processing speed. Overall, based on this profile he should present as average in mathematics with a possible struggle to mentally manipulate.

6.9.2 Pre-assessment.

It should be noted that Sean has one of the worst cases of dysgraphia that I have ever come across. His struggle and the enormous effort that is required for him to write I am sure influences his willingness to demonstrate his knowledge. Yet, he seemed motivated during the pre-assessment.

Unlike what was expressed by his psychoeducational assessment, Sean demonstrated a strong ability to perform mental manipulations. His understanding of number presented as very strong. He answered all the addition/subtraction questions without writing anything down or using any of the manipulatives. He only got one wrong and was the only student to demonstrate an understanding of the difference between addition and subtraction based on the questions given. He did not however produce a drawing to represent the multiplication question 4x6 but instead wrote a question mark. Yet, he was able to multiply 4 x 24 in his head.

For the fraction pre-assessment, he simply stated, “I don’t know what a fraction is,” and then proceeded to draw one big question mark over the whole page.

6.9.3 Classroom tasks.

During classroom discussions, Sean would often raise his hand, but when called upon would say something like, “I thought I had something but I don’t.” Sean loved to make up silly names and call them out for different things we were doing in the classroom. Yet, this did not present as disruptive. He was sporadically engaged with the tasks provided he was able to be successful near the beginning; if he was not finding much success he would drift off into what seemed like his own thoughts.

Imagine-build-steal game: adding fractions. Sean was in the classroom teacher’s group for this activity. At the beginning, Sean did not say much and seemed generally disengaged. By the third question he stated, “I don’t know how this works.” He built the solution correctly but did not know how to say it. When he received no explanation to help him better understand, he once again seemed to disengage. Sean did not engage during most of this activity, he did not seem to watch as others built but rather just looked off into the distance in what presented as being deep in his own thoughts. Near the end, Sean was moved closer to the teacher by the eighth question. Sean presented as disengaged until the last question when he was asked to give a solution. On this very challenging final question, $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$, he built it correctly and gave the answer:

Sean: One and then four more.

Teacher: Four more what?

Sean: Sixths.

Teacher: So, one and four of six?

Sean nods.

For having seemed to be disengaged through the whole activity, Sean showed great reasoning ability. Although, he struggled to know how to say it correctly, he was able to build and reason through a very challenging question and use the understanding that 2/12 is equal to 1/6th.

Imagine-build-steal game: comparing fractions. In the comparing fractions activity, Sean did not build any of the questions, nor did he need to. Sean was quick to respond, “Already know it,” when a question was offered. When Sean was asked to give an explanation, he stated his response often in a tone that suggested this is so obvious, “cause that one’s a half and those two are a whole and a half.” One trend I have noticed with Sean is that if he can get a good grasp early on, he is engaged and eager. If he is initially confused by a task, then he is quick to disengage. Throughout the task, Sean was able to easily reason through each question offered to him.

6.9.4 Spatial tendency.

Sean completed the most cards in the Q-bitz[12] task along with Melinda. He seemed to put a lot of pressure on himself to be successful. Sean may have been able to go even further than Melinda, but near the end I could not score his images because he would produce it and then destroy it without showing me, yet with all the previous builds he had shown me. He began doing this on his second last build, when he showed me his build I made the statement that it was not quite correct. He immediately took it apart and said, “Yes, it was.” He then moved on to the next build which he also took apart before I had a chance to check. I was able to monitor him as he built and they seemed slightly off. I believe, he knew they were slightly wrong but wanted to present to both myself and his classmates who were also doing the task that it was correct. This speaks to Sean’s ability/need to be successful in mathematics. Regardless, he was able to show a strong aptitude for part-whole reasoning.

Sean, on a number of occasions, demonstrated that he can use mental manipulation as a tool to solve calculating questions. He did not use, nor did he seem to need, manipulatives.

6.9.5 Post-assessment.

Sean looked instantly deflated with the mention of the paper-pencil assessment. This was not surprising based on his struggle to write. Once given the sheet he looked at it closely for a short time and then physically turned his chair away from it. No attempt was made to reach for a pencil or engage with the assessment in any way. The teacher handed him a pencil, he just fidgeted with it while turning in his chair, then after about two minutes he drew a question mark over the first two questions. The researcher came over,

Sean: I still don’t know how to do this.

[12] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

Researcher: Okay, what is your question?

Sean shrugs his shoulders.

Researcher: Okay, for the first question, write what fraction of the region is shaded.

Sean does not respond.

Researcher: This section is just adding up all the fractions and putting the solution in the center.

Sean does not respond.

Researcher: Like we did yesterday. Then comparing fractions, you just did a whole bunch of those questions a few minutes ago.

Sean does not respond.

Researcher: Or you could just put a question mark and claim you don't know.

Sean leans forward and starts drawing a big question mark over the second section, but then does go on to answer the comparing fraction questions, of which, he answers all of them correctly.

6.9.6 Summary.

Overall, Sean presented as average within most areas of his psychoeducational assessment with a low average score in working memory which could impact his mental manipulation ability. However, Sean presented as very strong for mental manipulation in his pre-assessment. Sean has extreme issues with writing and seems to avoid writing as much as possible. He did all the mental calculation questions with what seemed like minimal effort. Sean presents as having fairly strong reasoning and abstract thinking skills but seems to put forth minimal effort if he does not pick up on the task near the beginning. If Sean is not finding success he will disengage from the task quite quickly. Based on his Q-bitz behavior and raising his hand but not giving an answer, I believe Sean cares a lot about his status as a strong math student and is not willing to risk too much out of fear of losing it.

6.10 Walter

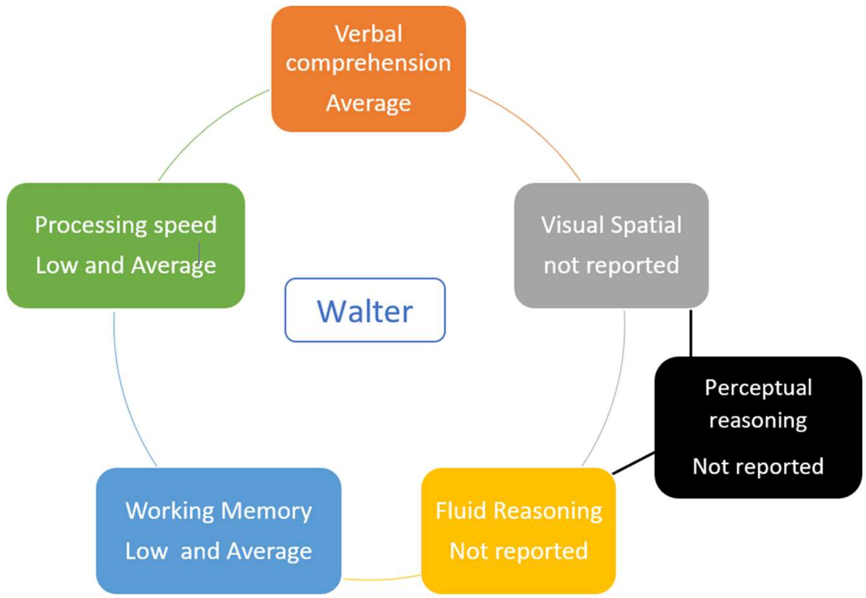


Figure 45: Walter's picture and psychoeducational profile

6.10.1 Psychoeducational assessment.

- Reasoning skills—not reported.
- Abstract thinking ability—based on VCI (similarities test) he is within the average range, but this is only partial as his FRI scores were not reported.
- Mental manipulation ability—based on his WMI he presented in the low to average range.
- Part to whole reasoning—not reported.

Once again this is a partial assessment as Walter's report did not include either visual spatial or fluid reasoning (see Figure 45). He presented as having a low to average processing speed and

WMI score which could present as a struggle with mental manipulation, but average abstract thinking ability.

6.10.2 Pre-assessment.

During the Q-bitz[13] activity, Walter approached the table with a high level of confidence, saying, "Oh, I'm good at this." Aven did the task at the same time as Walter. This definitely impacted Walter's performance. Aven took to the task with ease, enjoyed it, and I do not think she looked once at where Walter was in comparison to her. Walter on the other hand, everytime Aven completed a card he would look over at her. You could sense the pressure he was placing on himself to stay ahead of her. Once Aven surpassed him, he started making comments about how bad he was doing. I told him he was doing great and that he puts too much pressure on himself and to just enjoy the task. This was clearly challenging for him. As he became more visibly panicked, his performance slowed. Walter scored well completing only one less card than Aven, but he left the table all sunken and deflated, as though he failed the task. This is a strong pattern of behavior affecting Walter's learning.

Unlike what was expressed by his psychoeducational assessment (a low to average WMI), Walter demonstrated a strong ability to perform mental manipulations. In his pre-assessment (69%), Walter demonstrated a fairly strong understanding of number structure. He answered all but one of the addition/subtraction questions without writing anything down or using any of the manipulatives. Of the seven that he answered, he got 5 correct. He did not demonstrate an understanding of the conceptual difference between addition and subtraction. He also did not draw an image demonstrating an understanding of 4x6. He answered all the basic multiplication questions but did not attempt the larger ones. It should be noted that I gave him

[13] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

the fraction page before the multiplication one, which seemed to impact his effort level. Walter was very positive up until he reached the fraction page, which is a really challenging assessment covering topics that he would not have encountered before. As a result, he did not do well. This caused him to become visibly upset with himself, which may have impacted his effort level on the next multiplication/division page.

For the fraction pre-assessment, he got four out of 10 correct. In the addition/subtraction section, he got one out of four correct. He gave a solution of 3/10 to this subtraction question (see Figure 46), the content of which he would not have seen before, demonstrating an understanding of the numerator needing to add to 10 in order to produce a solution of 1. He also got two comparing fraction questions correct. By the end of this page, Walter was looking visibly upset.

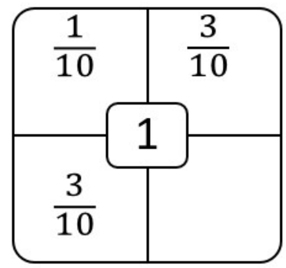


Figure 46: Fraction assessment question

6.10.3 Classroom tasks.

Walter, when he was feeling positive about his ability to perform, was quite engaged in group discussions. During open-ended group work activities on the first three days of the study, it was challenging to keep Walter engaged. He was often off topic discussing things about his weekend or stories about his cat. Walter seemed a strong instigator of off-task behavior for those around him. He was generally quite respectful though and would apologize for being off task when asked to refocus.

Imagine-build-steal game: adding fractions. Walter was an extremely interesting participant. For this activity, he was with the classroom teacher. As with the majority in this group, Walter started out very confused. When offered the signitive expression, $\frac{2}{4} + \frac{1}{2}$, Walter made statements like “What are we supposed to do?”; “Is that the answer or what?”; “I don’t know how to do this." With Walter's high need to be successful it was important that he come to a solid understanding quickly or it would have been game over for him. The crucial piece of information came from Andrew, his classmate. As the first question came to a close and the teacher accepts Iris' answer of "2 out of 4?", there was no discussion or information offered of how to make sense of these signitive offerings and the actual solution was never discussed either orally or with the pieces. Yet, as the teacher moved on to the next question, Andrew leaned over and talked to Walter, showing him the correct build for the signitive expression. Walter responds to seeing this by saying, "Ooooh. [as in a eureka moment] [pause] Thank you so much Andrew. No seriously, thank you Andrew, you may have saved my life. Literally." Walter then goes on to be extremely successful at building from the signitive offerings and later into visualizing the solutions without the manipulatives.

Imagine-build-steal game: comparing fractions. In the comparing fractions activity, Walter did not build any of the questions, nor did he need to. He was quick to respond when a question was offered and gave solid reasoning for his choices such as, "They have the least amount of pieces that are the same size."

6.10.5 Post-assessment.

Walter did not seem as discouraged when presented with the paper-pencil assessment. He seemed almost pleased to be able to demonstrate his understanding. Performing well seems a cornerstone to Walter's engagement with mathematics. He got eight out of ten on this assessment with all but the last of the addition questions correct, and all the comparing fractions correct.

6.10.6. Summary.

Overall, Walter presented as average within most areas of his partial psychoeducational assessment with a low to average score in working memory which could impact his mental manipulation ability. However, Walter presented as very strong for mental manipulation in his pre-assessment. Walter's biggest struggle was his high need for success. Walter showed a strong ability in all the tasks, but this could have easily gone astray if his early misunderstandings had not been addressed by another student. Walter did not show an especially strong need to build for pleasure, but definitely relied on building to help him connect to the meaning of the signitive expressions. He seemed quite motivated to be able to visualize without building near the end of the addition of fractions activity. During both days of imagine-build-steal, Walter presented with strong reasoning skills along with a strong need for a successful performance.

6.11 Overall summary

I thoroughly enjoyed getting know all the unique details of each student and found the combination of seeing their psychoeducational profile, along with getting to spend time with them in the classroom, as very enlightening. In some cases, their psychoeducational profiles fit well with their behavior in the classroom, but in many cases there were deviations from these profiles. This is not surprising as no learner can be summed up in a few hours' worth of assessment. Many factors could have influenced their scoring—their attitude that day, their cycle of failure in school which impacts belief in themselves, how much sleep they had the night before, who administered the testing, etc.

Chapter Seven: Findings

In the profile section, I discussed some of the complexities of each individual participant. Two areas discussed in their profiles were number sense and tendency towards spatial reasoning. In this section, I will compare these two areas as this study is looking at the role of spatial reasoning in the growth of mathematical images with our chosen area focus being within the curricular strand of number. I will share the findings that came from looking at how these particular students engaged spatially during our pre-assessment and the classroom tasks and compare that to their pre-assessment score on number. From here we will move on to outlining the experiences of the students which have led to my understanding of their progression from Image Making to Image Having as outlined by Pirie and Kieren (1994) and the impact of each style of offering, either signitive, imaginative or perceptual. As described earlier, both the imaginative and perceptual are connected to visual spatial reasoning, which is the question I am attempting to delve into: What role might spatial reasoning play in the growth of mathematical images? We will look at the students' varied beginning points, then look at the evidence of growth, where in the progression they seemed to land on that final day, and what types of language and gestures they used as they performed the tasks.

7.1 Student spatial and number concept comparisons.

This study looks at the number concept of fractions but from a spatial perspective. Also, in the literature there is precedence from a variety of sources that suggests that humans are predisposed to associate numbers with space (Dehaene, Bossini, & Giraux, 1993; De Hevia & Spelke, 2010; Pinel, Piazza, Le Bihan, & Dehaene, 2004). Yet, through my experience in working with struggling students, many of them have not learned to engage spatially with numbers, and it is my belief that this could be a source of their struggle, rather than an inate inability. I wanted to have a general sense of where these students' knowledge of number concepts was to begin with and what spatial tendencies they showed as they engaged with tasks.

This spatial tendency score was compiled through looking at tasks that each student had an opportunity to engage with. Did they engage in these tasks with a tendency towards thinking spatially or not? I looked at things like:

- During the pre-assessment, did the student engage with the blocks that were on table either through free play build or as tools for the math?
- Was the student able to demonstrate the use of mental manipulation for calculating? Was their verbal description connected to ideas of grouping in tens or were they engaging more with a procedure?
- When they did use the manipulatives, did they use them for more than just counting?
- How many cards did they complete of the Q-bitz[14] game?
- And finally, how did they engage with the Cuisenaire rods when given the opportunity for free play? Did they just line them up or did they develop a pattern, build a structure, or build a representation based on story or some known physical object?

I used these experiences to get a general feel for whether the student had more of a natural tendency to engage spatially with the objects around them or not. Some students with every spare moment afforded to them picked up the objects around them and began building, while other students seemed to not even notice they were there. Obviously, this is not a

[14] This game is very similar to the Block design subtest within the core battery of a psychoeducational assessment. The game offers the student a visual design on a card and they must recreate it with the provided blocks.

standardized way of measuring, but rather an accumulation of experiences that relate to this topic and are noteworthy. These observances were then compared to their pre-assessment score for number concepts; as an interesting comparison (see Figure 47).

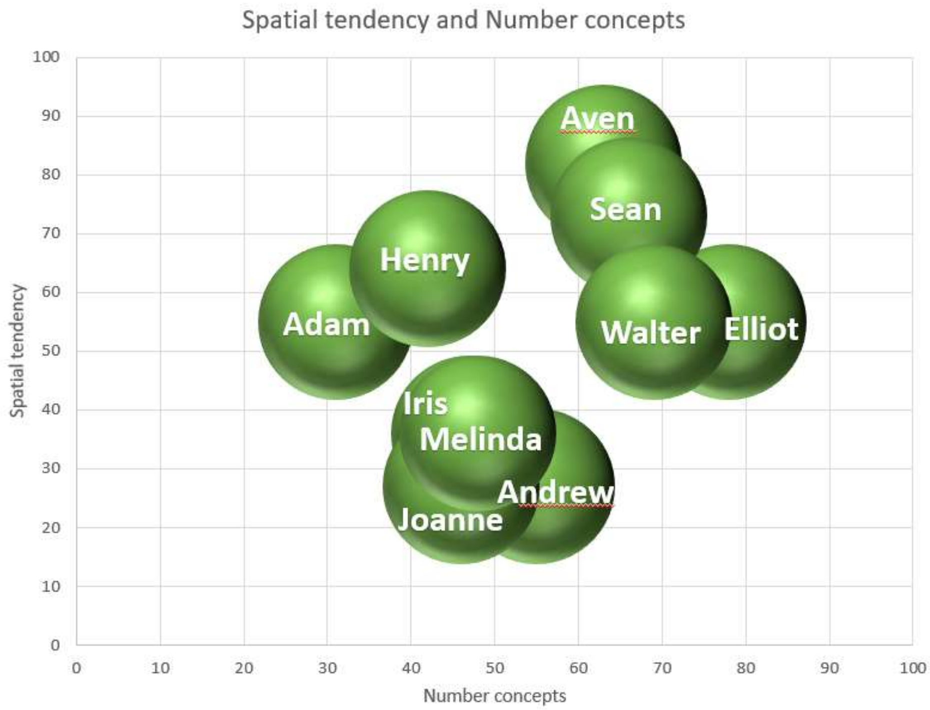


Figure 47: A comparison of each participants' spatial tendency and understanding of number concepts based on the pre-assessment.

These scores are in reference to students' spatial tendencies and where students present in their number sense; they are interesting to keep in mind as we move into the analysis of the classroom tasks.

7.2 Analysis of the addition of fraction task.

The point at which this analysis begins is Day Four of the classroom portion. After the pre-assessment was done, the progression of activities leading up to Day Four were initially quite open-ended; however, plans changed as the week progressed, and my understanding of the complexities that exist when entering a classroom that has its own history deepened. As I do not believe my confidence in the students' ability to engage with open-ended tasks was misplaced, I may have not accounted for their struggle with attention and the lack of established culture around a more open-ended style of task. During the week of classroom instruction, originally the majority of the lessons planned were of an open-ended style, but the final two days, days four and five, were adjusted for data collection purposes. What follows is a brief outline of these initial tasks and then we progress into the Day Four and five analysis.

On the first day, the students were asked to work in partners on a spatial puzzle task modelled after a mobile phone game called Slice it. Students were asked to create equal parts based on a given number of lines that could be drawn. For example, they were given a variety of different shapes with instructions that described how many pieces you need to make and how many lines you are allowed to draw (see Figure 48).

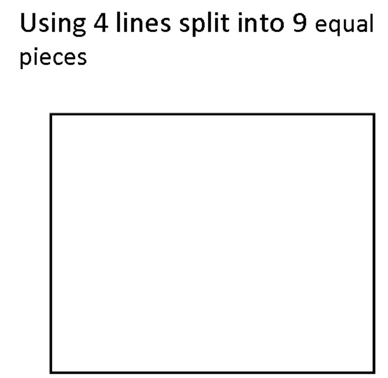


Figure 48: Slice it question

On Day Two, we gave them a pile of hexagons and asked them to cut them into equal parts and then label them using the language, one-of-three. We then did some work with gesturing and asking kids to hold up different variations of fractions such as, two-of-three.

On Day Three, they were to work in partners again and were given similar pieces to what they had cut the day before, but ones that were more precisely sized. They were asked to use the one circle or whole circle in their set and build on top of it another complete circle with fraction pieces that were at least two different sizes. They were then to write an addition statement that described the build (see Figure 49). They were tasked to do as

$$\frac{2}{3} + \frac{4}{12} = 1$$

Figure 49: Example from the activity on Day Three.

many variations as they could think of. Through this activity it was hoped that they would begin to notice certain relationships between the different sized pieces. Some groups had many different examples, but others only did three or four in the time given.

I reviewed the video that night after sensing a fair amount of off-task behavior. Students were asked to share a set of manipulatives and work together to build a circle and then write the equation out. However, what occurred was one person built and the other person seemed to disengage. So, we began asking them to alternate who was doing the build and explain that both should be engaged in figuring out how to write it. However, we noticed that whoever was not building became disengaged and then just copied the other partner's answer; then they switched and the other partner became disengaged. They were not working together. Some possible reasons for this may have been a lack of established norms around how to engage in partnered open-ended style tasks, it was a week before Christmas break, and a large percentage of the class were coded as having ADHD. We realized we needed to make some adjustments to our plans.

This group of students in the past struggled in the classroom, and many have learned to expect that they will be unable to do certain things. As a result, when faced with struggle they are quick to give up or wait to get help; many have developed the behavior of learned helplessness. This, along with not having a history of exposure to open-ended tasks, I believe, contributed to the amount of off task behaviors when offered an open-ended task as seen in the video data. With some groups, we learned a lot about the students' cats and flu shots and that Walter does not want to have a pajama day because his pajamas are embarrassing—they have golden toilets on them. Focus was a bit of an issue and we only had one week to video record. As a result, on Day Three, after discussion with the classroom teacher, we decided to switch gears and I came up with a game, which I called, *Imagine-build-steal,* for Day Four. I was still quite confident that they could get to a place of complexity with addition of fractions it just ended up taking a more direct course than we had originally planned.

Leading up to Day Three, we felt the students had engaged well enough with the activities to continue onto the next topic. Distraction does not necessarily represent a lack of learning (Carey, 2015), and we were curious to understand where they were at. In my work with differently-abled students in the past, they were quite quick at picking up addition of fractions when approached in this manner, which is why I continued to pursue this topic on Day Four.

In the Grade Five program of study the students are expected to:

Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:

- create sets of equivalent fractions
- compare fractions with like and unlike denominators. [C, CN, PS, R, V] (Alberta Learning, 2016)

The questions in this activity included many opportunities for students to engage and make connections to equivalent fractions; however it required them to go a bit further. Although addition of fractions is not a specific outcome in the Grade Five program of studies, the topic does appear in the Wechsler Fundamental academic skills testing that the students at this school are given at the beginning and end of each year. In the section prior to where the Grade three students are told to stop, there are addition, subtraction, and multiplication of fractions with mixed denominators. The Grade Five students are expected to complete this section and move on to the next where there are more operations with fractions – simplifying, addition, multiplication, and division of fractions, none of which appear in the outcomes for Grade Five or below. After this section, the Grade Four and Five students are asked to stop. For these reasons, we were curious to explore the topic more in-depth. Now, part of what needs to be acknowledged is that in regard to the pre- and post-assessment comparisons these students may be influenced simply by exposure to the topic of fractions. It is hard to definitively say that a spatial approach accounts for improved scores as parts of the intervention involved teaching things on the test that the students had not been previously exposed to, such as adding fractions. Up to this point, the students had been exposed to:

Grade 3: Demonstrate an understanding of fractions by:

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole that have like denominators.

Grade 4: Demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial and symbolic representations to:

- name and record fractions for the parts of a whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity

They would in Grade Five have been expected to cover:

Grade 5: Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:

- create sets of equivalent fractions
- compare fractions with like and unlike denominators.

We, as a team, were curious to see how well these students, many of whom (50%) tested very low or low on working memory, could engage with a visualization task. The students were split randomly into two groups. One group was with me, and the other group was with the classroom teacher. There were some differences in interpretation of how the game was meant to be played, which was most likely due to a lack of communication on my part.

The game was set up such that students were given an addition question and then asked to imagine the solution. They each had their own set of manipulatives, foam fraction circles, which they had cut out and then used to build fractions on the second and third day. In my group, the students could not build a solution until they had orally given me their imagined solution, at which point they were allowed to build and either keep their orally stated solution or change it based on what they had built. In the classroom teacher's group, they were asked to imagine a solution but were not expected to express it orally. This difference was due to my lack of communication with the teacher. As a result, in the teacher's group they gave their solution after the build. In both groups the other team was then asked if they agreed with the offered solution and if not had a chance to steal while providing a different solution. Whichever team had a solution that could be justified was asked to pick a number. Once the number was picked the teacher would then refer to her cheat sheet and attached to that chosen number could have been any number of actions: +20 points, -6 points, +/-, switch scores etc. This unpredictability was very motivating for the students.

For the remainder of this section, I will outline some of the different offerings students received throughout the task and discuss the progression of growth which was demonstrated by the participants. As I describe this progression I use the word movement. I chose this term as the act of growth is a movement and, in these interactions, certain elements caused what appeared to me to be movement or growth to occur in either the Image Making or Image Having of their mathematical ideas. Within each group and for most of the individuals there was a similar progression of growth observed. This progression (see Figure 50) is a combining of the Pirie & Kieren Theory with the modified Husserlian ideas of signitive, imaginative, and perceptual offerings defined earlier, that looks at the impact of these offerings on the growth of mathematical understanding.

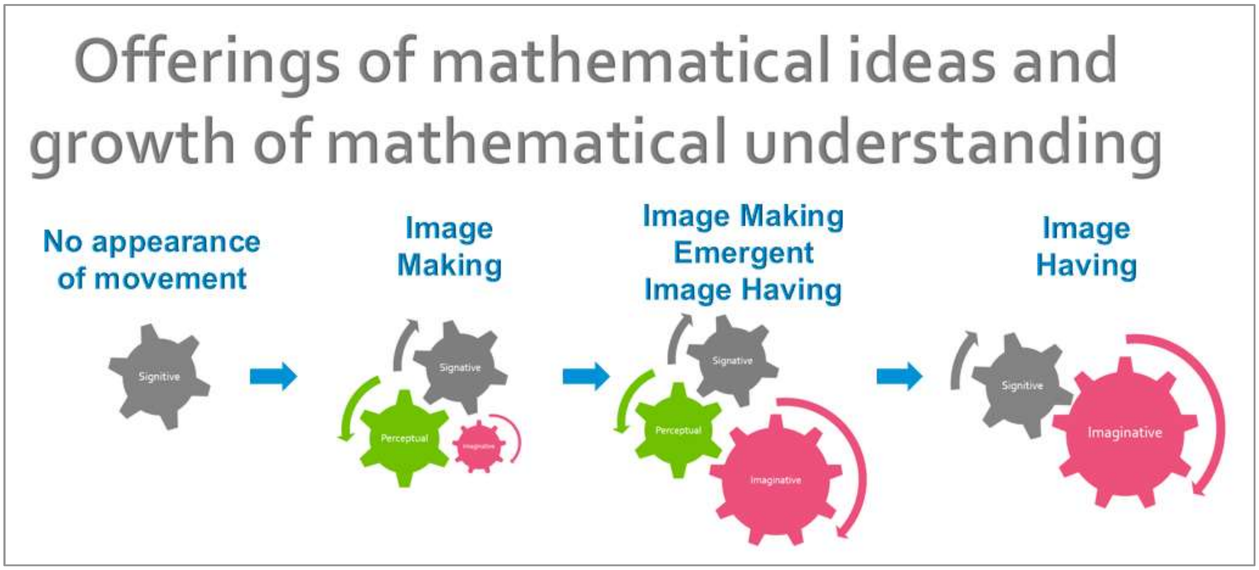


Figure 50: Progression of growth connected to the beginning stages of the Pirie-Kieren theory (1994) and the offering of signitive then perceptual with the eventual growth of imaginative.

As discussed earlier, the challenge is what sort of evidence will be considered when categorizing students as having reached a particular phase. Below I offer a brief description of the types of offerings being considered for categorization.

- *No appearance of movement*: The student is generally confused by what this signitive offering means and seems at a standstill.
- *Image Making*: With the introduction of a perceptual experience, the student is able to make connections between the signitive and the perceptual which contributes to the growth of the imaginative. Image Making can be observed

through the student building with manipulatives in such a way that connects to the signitive.

- *Image Making emergent Image Having*: A student in this phase can consistently build correctly and may claim that they can see the solution without having to build, but still want to build to confirm. The student may struggle to find the solution based solely on their Imaginative; as a result, they need to do a partial build,
- *Image Having*: The student is offered only the signitive and is able to produce a solution confidently without building.

7.2.1 Groupings:

The two groups began slightly differently. There were a few distinctions, one being the group size. There was a group of four and a group of six as it was important to the students to have equal sized teams. However, probably the most significant distinction was due to a lack of communication on my part. I required my group to give their imagined solutions before they were allowed to build. The classroom teacher did not require the students to give a solution until after they built. Also, it should be noted that Elliot, in the researcher's group, seemed to have a bit of an allergic reaction to the foam pieces that we were using; he commented that he hated touching them, they made his "fingers feel funny", so he never built but only watched as Henry (his partner) built. This was interesting as he then did not receive as multisensory of an experience as the other students.

Classroom teacher's group (CTG): (see Figure 51 and 52)

Figure 51: Grade Five students in the classroom teacher's group.

Walter	Iris
Andrew	Joanne
Alex	Sean

Figure 52: Student seating arrangement as seen in figure 52.

Researcher's group (RG): (see Figure 54 and 55)

Figure 54: Grade Five students in researcher's group.

Figure 53: Seating arrange of students from figure 55.

List of the progression of questions students were offered signitively on a small white board one at a time (see table 4).

Table 4: Progression of questions offered to students.

1. $\frac{2}{4}+\frac{1}{2}$	8. $\frac{2}{6}+\frac{1}{6}$
2. $\frac{2}{4}+\frac{1}{4}$	9. $\frac{2}{6}+\frac{2}{3}$

3. $\frac{1}{2}+\frac{1}{4}$	10. $\frac{3}{4}+\frac{1}{4}+\frac{1}{2}$
4. $\frac{4}{4}+\frac{3}{6}$	11. $\frac{3}{6}+\frac{2}{6}+\frac{1}{6}$
5. $\frac{2}{3}+\frac{1}{3}$	12. $\frac{3}{8}+\frac{1}{4}+\frac{1}{2}$ (CTG); $\frac{4}{8}+\frac{1}{4}+\frac{1}{2}$ (RG)
6. $\frac{2}{8}+\frac{2}{8}$	13. $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$ (CTG); $\frac{3}{6}+\frac{1}{2}+\frac{2}{4}$ (RG)
7. $\frac{2}{8}+\frac{1}{2}$	14. $\frac{2}{3}+\frac{2}{6}+\frac{2}{8}$ (RG only)

7.2.2 Primitive Knowing – various beginnings.

In the Pirie & Kieren Dynamical Theory for the Growth of Mathematical Understanding, the authors describe the initial starting point for growth as *Primitive Knowing* where the term *Primitive* is not meant to imply a low level of mathematics, but rather the starting place for one's growth within a particular mathematical understanding. In these initial interactions we look at categorizing in a general sense based on the progression described above, where each participant may have begun their journey for this task.

CT Group. This group's time together began with an offering of only the signitive, $\frac{2}{4}+\frac{1}{2}$, and they were told, "You're going to imagine and try and figure out what the answer is". What ensued were statements like: "What are we supposed to do?"; "Is that the answer or what?"; "So, is this 2 plus or . . .?"; and "I don't get this." The responses from the teacher included only an oral restating of the question.

Teacher: Two over four plus one over two.

The only student who may have understood the signitive offering was Andrew. He stated after the reveal of the question, "Oh, I can see it. I just need a piece of paper to write it down." He then draws a plus sign, like he is splitting something into quarters in the air in front of his face with his finger; after a few seconds of this he puts his hand up, as if to answer the question.

Generally, at the beginning of the task, there was just confusion as the students were asked to imagine the solution. After about 30 seconds, Sean and Alex both seemed to disengage, as they began moving their fraction piece bags around on their desks. When Iris was asked to give an answer, she built two quarters which she held up.

Iris: Two out of four?

Teacher: [Turning to the other team.] Do you guys want to steal?

Walter: I don't know how to do this.

The teacher then informs Iris that her answer is incorrect. There is no discussion of how to make sense of these signitive offerings, and the actual solution is never discussed either orally or with the pieces. The teacher moves on to the next question.

However, as she is writing the next question on the white board, Andrew talks to Walter and builds the solution correctly, two quarters and one-half piece to create a full circle, in front of Walter, not in such a way that he is trying to figure out the solution, but as though he knows the solution and is just wanting to explain it. Walter has a very dramatic response to this offering, which fits with his profile of intense need to perform well or he shuts down.

Walter: Ooooh. [as if having a eureka moment] [pause] Thank you so much Andrew. No seriously, thank you Andrew, you may have saved my life. Literally.

The offering by the teacher in this section was purely signitive. In response to this offering, the students expressed confusion. There was little or no evidence that an imaginative (Image Having) offering was present in students' minds. The only evidence present was from Andrew, who may have had some level of Image Having, as he stated just after the question (signitive) was given to him, "I think I see it." He then did some air drawing in front of his face connected to the idea of fourths, and at the very end, he built it correctly with pieces for Walter. This seemed to be a perceptual offering for both Walter and Andrew, or it may have possibly been merely a confirming perceptual offering for Andrew which Walter clearly appreciated and received information from. In the end, only Walter and Andrew saw the correct build.

Joanne and Iris attempted to build, $\frac{1}{2} + \frac{2}{4}$, but only ever built two quarters. However, during this first question, they set up all their pieces into completed circles in front of them; this is a perceptual offering although not necessarily connecting to a specific question for them. Neither Sean nor Alex attempted to build or set up their circles and seemed to disengage from both the question and their classmates.

Figure 55: First phase of progression.

All but one participant in the CTG began with confusion when offered the question in a signitive form. In the progression observed, I categorized these responses as *No appearance of movement* (see Figure 55), with one cog representing the signitive. Their responses seemed to offer no movement within the system of growth.

Andrew seemed to be connecting to some level of Image Having, I would categorize it as at the *Image Making emergent Image Having* phase possibly the third phase. Andrew struggled in future tasks to provide solutions without building. He also showed a certain level of perceptual offering by drawing in the air but did not verbalize that he had the solution, making it unclear to what level he was Image Having. Yet, he claimed to see the solution and was able to quickly demonstrate the correct build to his classmate. For these reasons, he could have been at the

Image Making emergent Image Having phase. As this was the first question, it is also clear that Andrew gained even this level of understanding from the past few days of work. Andrew's starting point in his pre-assessment was very disconnected from the signitive as seen by such answers on the assessment as, ½ + ½ + ½ + 1 = 10. He may have been engaging with signitive representation by adding up all the numbers, interpreting ½ as 1+2=3, which would then produce a solution of 10 for this equation. As his pre-assessment solutions do not demonstrate a strong understanding, I have made the assumption that these emergent Image Having abilities were established during the perceptually based activities of the previous three days, or maybe an awareness of what the signs were pointing to.

Walter began at the initial stage, as he made comments like "What are we supposed to do?" and "I don't know how to do this." Once Walter engaged with a perceptual offering, movement began, demonstrated in both his comment, "Ooooh. Thank you so much Andrew. No seriously, thank you Andrew, you may have saved my life. Literally." His behavior in response to the next question demonstrates that growth occurred. So, Walter would have begun at the first stage but ended this question at the second phase (see Figure 56) after being offered a perceptual experience.

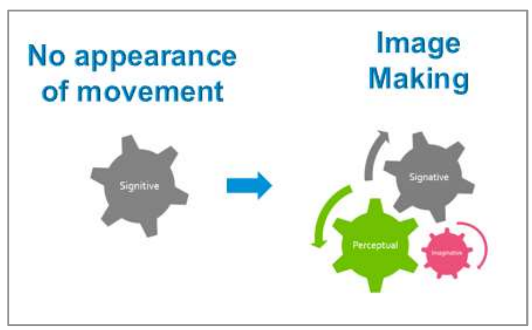


Figure 56: Second phase of the progression.

R Group. $\frac{2}{4} + \frac{1}{4}$: In this group, the activity started out with me explaining the task to the students, then instructed the students to pull out their one-half and one-fourth pieces. This request offered the students some connection to the previous day's task. I also suggested that these pieces would have a connection to the signitive about to be offered. The signitive form of the question was then offered on the white board and they were asked to, "Imagine what the answer is. See if you can build it in your mind first." All the students seemed to understand what was expected and this was the interaction that ensued.

Researcher: Imagine what the answer is. See if you can build it in your mind.

Melinda: Wait, I might need to build . . .

Henry: I think I know. I think I know.

Melinda: I don't [she replies in a pretend crying tone]

Henry: [turns to Melinda taunting her] I know. I think I know.

Researcher: [to Melinda] Okay, two of four. Do you have two of four in your mind?

Melinda continues to squirm.

Melinda: I don't know.

Aven's head all of a sudden pops up.

Aven: Oh, I know what it makes. It makes a whole.

Researcher: [laughs] Oh, you weren't suppose to say it right away. You're suppose to wait. But you are correct. So, I'll give it to you.

Aven: Yay!

Researcher: Were you confident about that?

Aven: Yes.

Melinda: I wasn't.

Researcher: Okay.

Aven: I just basically added it together.

In this interaction, there is some variation in *Primitive Knowing*. It is hard to know what Henry visualized, but Aven was definitely able to produce a solution without having to build it first. However, Melinda definitely struggled. There was no indication either way from Elliot. No one built the solution to this question, as the researcher had already blurted out that Aven's solution was correct. As a result, the students saw no need to build.

Based on these interactions and future interaction, I categorized Henry and Aven at the *Image Making emergent Image Having* phase (see Figure 57) as they both seemed to be able to visualize something, and neither used any perceptual offerings such as hand movements to help produce an image. This is obviously a shaky placement in regards to Henry, as he never actually described what he visualized, based on his behavior throughout the task, Henry was generally quite forthcoming with his statements of being able to visualize or not being able to visualize, whereas Andrew in the CT group seemed to always claim he was visualizing but was not always able to produce a description that supported this. Based on future interactions, it became clear that neither Aven nor Henry were at the full *Image Having* phase. Melinda, seems to be at the first phase, *No appearance of movement*, as she claims over and over again that she needs to build and cannot seem to build it in her mind.

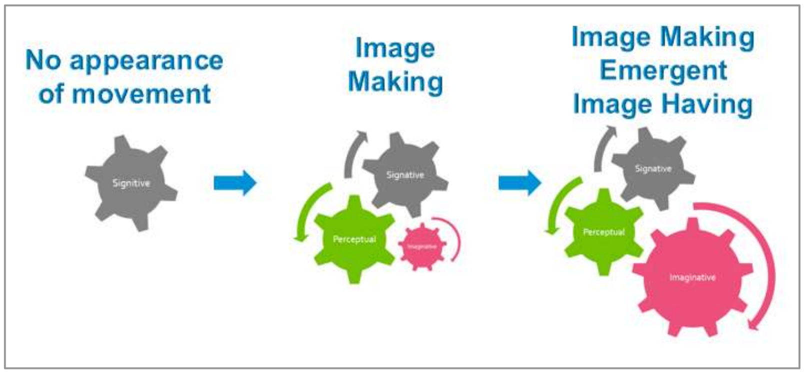


Figure 57: Third phase of the progression.

Elliot does not indicate where he is at until the fourth question in which he see the question $\frac{4}{4}+\frac{3}{6}$ and states:

Researcher: Here is the next question. Try to visualize this.

Elliot: [Whispers to Henry] A whole and half. You see that's a whole [pointing to 4/4 and that's a half [points to 3/6].

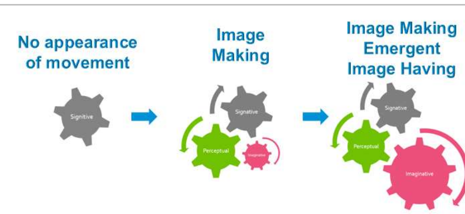
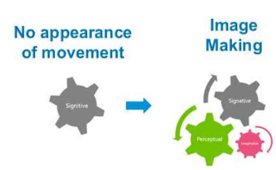
Researcher: What do you guys think?

Elliot: It's a whole and a half.

Based on the fact that Elliot gave no indication until he had watched as others built the previous questions, I have not placed his beginning stage. This does fit with his profile; Elliot has not shown himself to be a risk taker, as he has demonstrated on multiple occasions that his reputation as a strong math student is very important to him. I find it interesting that he said nothing up to this point and then without being prompted to respond to this question volunteered a solution. This makes me believe that he was not confident until this question, but I also cannot claim that he had any level of Image Having prior to this, as he gave no indication.

Below is a summary of the initial phases of each student (see table 5).

Table 5: Summary of phases reached by students after the first few questions.

Where particpants presented in the first few questions:	
Aven, Henry	
Andrew	
Melinda, Joanne, Iris, Walter, Sean, and Alex	
Elliot	?

7.2.3 The production of movement.

As this task progressed both groups had a growing number of students claiming to be able to see a solution. The starting point for this growth seemed to be brought on by the offering of a perceptual experience.

CT group. In the CT group, Andrew seemed to hit the ground running as he was the first student to claim to know the answer, “Oh, I can see it.” He then seemed to try and further define what he was “seeing” by offering himself a perceptual experience of drawing in the air. He seemed to use this perceptual offering to deepen his understanding. Later, he built the solution for Walter. Walter seemed to be completely stagnant at the beginning, but once he was offered a perceptual experience he seemed to have a leap of growth. Joanne, Iris, Alex, Sean and Melinda all started out as stagnant as well. No appearance of growth. Yet, once they began to be offered perceptual experiences movement in their growth of mathematical understanding began. The following is a description of students’ initial perceptual experiences during this task and how this seemed to impact their growth.

Team one – Walter, Andrew, and Alex: In the second question, $\frac{2}{4}+\frac{1}{4}$, Walter and Andrew seemed to immediately crave a perceptual experience. Once offered the signitive question, Walter and Andrew immediately tried to build on the sly. As Walter attempted to put the third quarter down, he was stopped by the teacher and told to visualize. Andrew gestures in the space in front of his face, in a fashion to mimic cutting into fourths with his finger, then picks up a pencil and pretends to draw on the desk making fourths. Andrew is asked to give an answer.

Andrew: I’d like to build.

The teacher gives permission, and both Andrew and Walter build. Andrew builds two-fourths.

Andrew: A whole.

Walter then physically reacts by jumping in his chair and urgently whispering and poking at Andrew.

Walter: Three-fourths. It's three-fourths.

Andrew then says to the teacher, "three-fourths." Andrew does not complete the build. He never sees the actual solution built, only Walter did the correct build.

Walter does not make any claims of knowing the solution before he built but demonstrates a strong need to build as he tries to do it on the sly without the teacher noticing. As I stated earlier, that places him at the *Image Making* phase. Andrew seems to behave in a similar fashion.

By the third question, $\frac{1}{2}+\frac{1}{4}$, Walter seems able to visualize and provide himself with an imaginative offering, as he states, "I got it", before he builds and whispers "Three-of-four" to Andrew. This shows him progressing to the next phase of *Image Making emergent Image Having*, as he continues to want to build in order to verify his solution.

In the fourth question, $\frac{4}{4}+\frac{3}{6}$, when offered the signitive:

Walter: [pauses then does a little jump in his chair and claps] Got it.

Andrew: Oh, oh yeah.

Walter shows Andrew a whole circle and then picks up the bag with halves in it. Notice here he does not build with the fourths or the sixths but goes directly to a whole circle and then picks up a half. Walter seemed able to put the ideas together without using the pieces. Up to this point, Walter has been very quick to offer himself perceptual experiences even if he must hide it from the teacher, yet in this question he does not seem to need to, rather he uses the pieces to show the solution to his teammate. On the other hand, upon hearing Walter's solution, Andrew picks up the quarter pieces and starts building four-of-four.

From this point on, Walter is generally able to give correct solutions without building first, and he grows quite confident in his abilities, as seen in question ten, $\frac{3}{4}+\frac{1}{4}+\frac{1}{2}$.

Teacher: Okay guys, this is going to get hard.

Walter: I don't mind. I understand all of this.

Andrew: Yeah, me too.

Walter: My mind is a calculator right now.

Walter is practically vibrating at this point.

Teacher: Okay, visualize.

No one builds secretly.

Joanne: I've got it

Walter: Got it!

Walter holds up his hands in victory. He then starts bouncing, waving his hands and then clapping in his chair excitedly. "Got it. I've got it."

Andrew: I think I got. I think I got. [whispers to Walter] It's a whole and a half.

Walter: Stop it. I already know. Andrew, I know how to do this stuff now.

Teacher: You guys are too competitive.

Walter: It's a whole and a half.

Teacher: So, one and a half?

Walter: Yes, sorry.

Turning to Andrew.

Walter: I didn't need your help, I have this stuff.

As this is fairly consistent behavior for him at this point, Walter, may have progressed into the *Image Having* phase (see Figure 58). We also see that by this question

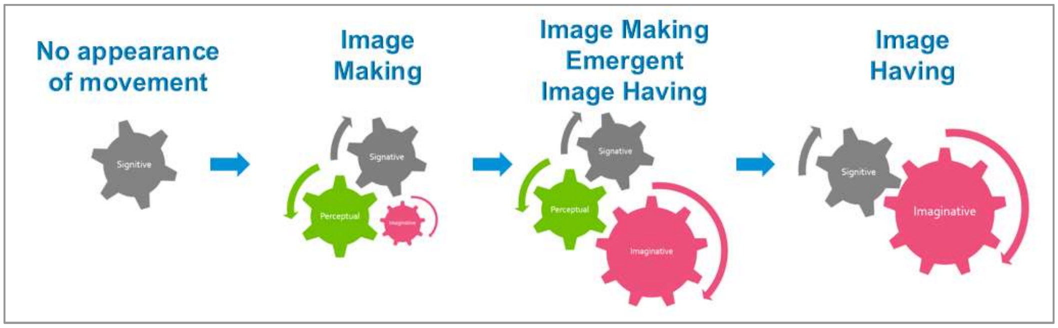


Figure 58: Fourth phase of the progression.

Andrew may have also reached this phase, although he does not present as consistently or confidently as Walter. Both are still quick to fold back to Image Making the instant they are unsure of the solution, as seen in this fairly challenging question, $\frac{3}{8}+\frac{1}{4}+\frac{1}{2}$.

Teacher: Do not build. Visualize.

Walter puts the pieces back down.

Walter: Okay, I'm sorry.

Walter continues to visually search through the pieces. It is almost like he can't help himself as he seems to crave the perceptual.

Andrew: I know. I know. I got the answer.

Iris: [Sheepishly raises her hand and twists it in a maybe kinda way] I think I might have it.

She does not sound very confident. Walter continues to search then in an instant turns away from the pieces and claps his hands.

Teacher: Whose turn is it?

Walter: It's Andrew's turn.

Andrew: I'm pretty sure . . . Can I check?

Teacher: Sure.

Andrew builds one quarter, then attaches three-eights. He pauses and grabs the one-half, doesn't seem to know what to do with it. Walter takes the one-half out of Andrew's hand, pauses while studying the pieces. Takes away one of the one-eighth piece with one hand and attaches the one-half piece to make a full circle. Andrew grabs the one-eighth piece out of Walter's hand and says, "And then, this." And puts it beside the full circle.

Teacher: So what do you say?

Andrew without prompting from Walter responds.

Andrew: A whole and one eighth.

Walter: Yeah.

Since question four, Walter has had minimal engagement with the manipulatives in front of him. He has mainly used them to explain solutions to his teammates. However, this question did not fit into the typical visual shapes of one-quarter, one-half, or a whole, so when offered the question Walter immediately attempts to build even when he is expected to wait. The teacher reminds him to not build, and so he visually searches through the pieces in front of him looking for some perceptual help. Through these actions we see Walter needing to fold back to an Image Making phase in order to help clarify his thinking.

Andrew, throughout these tasks, is not as consistent as Walter at offering himself correct perceptual experiences but seems rather focused on being able to speak to the solution as opposed to understanding. This behavior was described earlier in the second question, $\frac{2}{4} + \frac{1}{4}$, when Andrew is asked to give a solution.

Andrew: I'd like to build.

The teacher gives permission, and both Andrew and Walter build. Andrew builds two-fourths.

Andrew: A whole.

Walter then physically reacts by jumping in his chair and urgently whispering and poking at Andrew.

Walter: Three-fourths. It's three-fourths.

Andrew then says to the teacher, "three-fourths." Andrew does not complete the build to try and understand why the solution is three-fourths. He never sees the actual solution built, only Walter did the correct build. Andrew almost always attempted a build but was inconsistent at completing the builds when offered an oral solution by Walter. Andrew seemed to remain at the *Image Making* phase for much longer than Walter did. He did not seem able to produce an image without offering himself a perceptual experience either through hand gestures or building with the pieces until question ten, whereas Walter had begun this behavior much earlier. Andrew's focus on answer-getting through Walter's verbal suggestions seems a point of hinderance both for Andrew and also Alex as seen in this interaction.

Alex at this point, question four, $\frac{4}{4}+\frac{3}{6}$, has still not seen a correct perceptual solution.

Alex: I don't have it.

Teacher: "Alex, this is your question."

Alex: I don't know how to build it.

Teacher: You need to use your pieces and build four-of-four and three-of-six.

Alex grabs a one quarter piece, but almost immediately Andrew takes over for him and starts building.

Andrew: Look, look, look, you put one [laying a quarter down], two [lays a second down]. Right, Walter?

Walter nods. Alex goes to grab another quarter and begins to add it.

Walter: Yeah, but it's out of six, so you have to use the six pieces.

Andrew turns to Alex and takes the quarter piece out of his hand then removes all the other quarter pieces and starts building three-of-six with the sixths pieces.

Walter: [Whispering to Alex] It's a whole and a half.

Andrew stops building and takes the pieces apart. Alex never sees a complete build.

Alex: [says to the teacher] Um, one and a half?

Teacher: One and a half? That's correct. [to the group] Let's just go over this one quickly.

She then turns to Iris and uses her pieces to build. Only Iris and Joanne continue to watch her as she describes and builds the solution in front of them.

Teacher: [turns to Alex who had been talking to Sean] You got the answer right, so you knew this. Right?

Alex nods his head in agreement. Yet, it was clear from the interaction that he had not.

In this exchange both Alex and Andrew received confusing perceptual messages. Alex picks up the correct pieces to start building, Andrew takes them out of his hands behaving as though Alex is not doing it right. He starts showing him a build with the same pieces. Then Walter makes a comment about one-sixth pieces and Alex watches as now Andrew takes that first build apart sending a message that this is not right either. Andrew starts building with the one-sixth pieces but does not finish as Walter whispers the answer to Alex. Alex then repeats Walter's solution to the teacher, and when the teacher claims that he understood, Alex nods in agreement.

Both Alex and Andrew seem content to not see what the actual build looks like, rather they just seemed interested in having an answer to offer up. Both Alex and Andrew seemed to remain stagnant, Andrew at the *Image Making* phase and Alex at the *No appearance of* *movement* phase, for quite some time. Alex makes a very slow progression to the *Image Making* phase in part because of his teammates, until he reaches a tipping point in question eight, $\frac{2}{6}+\frac{1}{6}$:

Teacher: Okay, everyone visualize.

Andrew, Iris, Joanne and Walter all claim to have it. Andrew builds it with Walter's pieces without the teacher noticing, still needing that perceptual experience.

Iris: [sings] I know what it is, I know what it is.

Teacher: Alex, this is for you.

Alex: I need to build.

He begins building with the one-sixth pieces. Walter and Andrew also build it, and then start whispering to Alex.

Walter: [whispers to Alex] It's a half.

Andrew: [Whispers to Alex] It's a half.

Teacher: Don't talk to him. Let him build it.

Andrew: Please Alex, I hope you get it right.

Walter starts waving the one-half piece around.

Teacher: Walter stop. [Alex does not notice this exchange.] K, what do you think?

Alex has built it correctly, but with all the whisperings of one-half he decides to pick up the one-half piece and adds it to his build.

Andrew: No, no, no, no, no.

Teacher: Guys stop. Okay Alex, do you know the answer.

Alex looks at his teammates.

Alex: No.

Teacher: Do you want to take a guess?

Alex sits back down looking very remorseful.

Alex: No.

Walter mouths the words “a half” to him. Alex sees him.

Teacher: Look at your pie.

Alex: Three and a half?

Teacher: [Turns to the other team] Do you agree?

Joanne: I think it’s a half.

Alex: [mumbles to his teammates, who have now turned their back to him] Sorry.

Walter leans over and talks to Alex.

Walter: Okay, so when it’s like this [points to the three-sixths that has been built] it’s a half.

Alex: [in a frustrated tone] I don’t get this.

Walter: Alex, do you see that this is the same shape as a half? It’s a half.

By this point, it is evident that Alex is very upset. The teacher asks Alex and Sean to come sit by her and moves Walter and Iris to the back seats where Alex and Sean had been sitting.

From this point on the teacher begins controlling the perceptual experiences that Alex receives, and Alex begins to pay more attention to what Andrew is building beside him whereas prior to this Alex had generally disengaged himself. Alex watches Andrew’s correct build in question nine. Then in question ten, the teacher offers Alex another correct build.

Teacher: [Turning to Alex] Alex, while she’s building I’ll show you this. So, three of four pieces. One, two, three, [the teacher collects three quarters together] so I put them in my circle. Plus, one of four, here. [the teacher puts another down], then add one of two [the teacher puts the half piece beside]. So, one [she finger-traces around the circle] and a half [points to the half]. Got it?

Alex nods. Although she does not have him build, he begins to grow in confidence, *Image Making.* By question eleven, $\frac{3}{6}+\frac{2}{6}+\frac{1}{6}$, he is able to make correct builds on his own and the teacher just checks in on him.

Iris: A whole.

Teacher: [turning to Alex] Is she correct?

Alex: Yep, a whole.

Teacher: Alex says yes, and he is very confident. And so, they are correct.

Before Alex was offered a correct perceptual experience by the teacher, he expressed his confusion, but was offered no explanation, so he disengaged. When asked to provide a solution, he would attempt to build but his classmates would take over and offer him some very confusing and incomplete perceptual experiences. So, when offered an oral solution, Alex, seemed all too happy to offer this solution to the teacher as his own. He even would claim to the teacher that he understood as in question four after offering Walter's solution of one and a half:

Teacher: [to Alex who had been talking to Sean] You got the answer right, so you knew this. Right?

Alex nods his head in agreement. Yet, it was clear from the interaction that he had not. Yet, once the teacher began controlling the perceptual experiences that Alex was receiving by placing him next to her and offering him a correct build, he quickly, the very next question, was able to produce a correct build.

Each member of this team has gone through the various phases at varying speeds. Walter was very consistent with his perceptual offerings and ensuring that he understood. Andrew definitely pursued perceptual offerings but was also quite content to just be given the answer by Walter without taking the time to build himself in order to ensure his understanding. Alex expressed his confusion early on, but when offered no help chose to disengage. When asked to offer a solution, his teammates would continually intervene and then verbally give him the solution, which he seemed content to claim as his own; he did not push for clarification or understanding. Alex never saw a correct perceptual offering until question nine and ten. After this point, he was able to build a correct solution. The correct perceptual experiences seemed key to each of these students' growth, but other factors such as personal motivation clearly came into play.

Team two – Joanne, Iris, and Sean: In the second question, $\frac{2}{4}+\frac{1}{4}$, after the other team answered, the teacher turned to team two. Joanne and Iris had both built two-fourths.

Joanne: I don't get this.

Teacher: Do you think that's right?

Joanne: No, it's two-fourths.

Teacher: Well, I'll tell you guys that three-fourths is correct. [to all students] Look guys, if you have the same bottom, you can just add the tops together.

As the teacher is explaining this to the group, Joanne confirms the solution by building it correctly with her pieces.

Both Iris and Joanne struggled to build initially. Once the solution was offered orally (signitive) Joanne then tried again and built it correctly making the connection of the perceptual to the signitive. At this point, Joanne seems to have entered the *Image Making* phase, as she is able to build a correct solution after hearing the oral solution (signitive). This is her first correct perceptual offering. She then continues to receive perceptual offerings in question three and four but does not make any comments nor is she asked to. By the fifth question, $\frac{2}{3}+\frac{1}{3}$, when asked, she was able to produce it without building. Upon being asked for a response she:

Joanne: Yay, finally. I think one whole. Is it one whole?

She then asks:

Joanne: Can I build?

Teacher: Sure.

This demonstrates a progression of growth for Joanne. Based on her ability to produce a solution without building, yet still wanting to build, she seems to be at the *Image Making emergent Image Having* phase.

Iris also initially struggles but does not see a correct perceptual offering until question four, $\frac{4}{4}+\frac{3}{6}$, when the teacher offers a correct build to Iris at the end.

Teacher: [to the group] Let's just go over this one quickly.

It is in this fourth question that the first perceptual offering is given by the teacher. Although, she addresses the group initially, she then turns toward Iris, who is sitting beside her, and only addresses her and uses her pieces to build. Only Iris and Joanne continue to watch her. She refers to the signitive four-of-four and counts out the quarter pieces.

Teacher: One, two, three, four, which is four out of four pieces, [points to the written fraction] and one, two, three of six pieces, [points to the written fraction] which makes one, [puts her hand on the whole circle] and a half, [puts her hand on the half circle].

Iris: Ooooh.

Teacher: [turns to Alex, who was not watching and instead talking to Sean] You got the answer right, so you knew this. Right?

Alex nods his head in agreement.

Iris: I didn't know you could do that.

This is the first correct perceptual offering that Iris receives. It is by this point in the activity that Walter, Andrew, Joanne and Iris have all been offered at least one correct perceptual experience, either one they created for themselves or from the teacher. From this point on, for questions six through to eleven, Andrew, Walter, Joanne, and Iris generally claim to know the solutions before they build. Keep in mind these are not verified by the teacher, as this was not a requirement in this grouping. However, Joanne, Iris, and Andrew, throughout the game, are never willing to just trust their images and not build; they always ask to build just to verify. This places them in the *Image Making emergent Image Having* phase.

Sean is another story. As described in his profile, Sean is quick to disengage if he does not experience success early on in an activity. Sean does attempt to ask for help a couple times early on.

Question 3, $\frac{1}{2}+\frac{1}{4}$:

Sean: I don't know how this works.

Teacher: Take the pieces out the bag and build this equation.

Question 7, $\frac{2}{8}+\frac{1}{2}$:

Sean: But I'm confused.

Teacher: What's confusing?

Sean: Everything.

Teacher: Do you think you can visualize the answer in your head or are you going to need to build it? [Sean shrugs. Teacher turns to Iris.]

Overall, Sean spent the majority of the time looking off into the distance. His behavior presented as being very disengaged with the task and those around him. By the end of question eight, Sean along with Alex are moved to the seat beside the teacher, and she builds the solution for question ten in front of them, mainly directing this interaction at Alex. This is Sean's first correct perceptual offering connected to the signitive that he had seemed to somewhat engage with—*Image Making.* By the eleventh question, $\frac{3}{6}+\frac{2}{6}+\frac{1}{6}$, he leans forward, looks at the question and says, "I know," then leans back but once again does not build—possibly *Image Making emergent Image Having*. Then in the last question, question thirteen, $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$, which is a fairly challenging question, Sean is asked to give a solution.

Teacher: Sean it's your question.

Walter starts clapping.

Walter: Okay, I got it. I got it.

Sean starts building. He builds two-sixths then two-twelfths and then another one-sixth (see Figure 59).

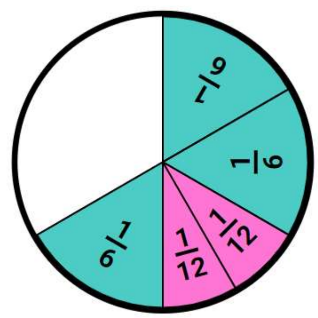


Figure 59: Sean's build for question 13.

Sean: A whole and half-ish. It's a whole and a bit over half.

Teacher: Okay. [pauses and then she points to the two-twelfths] Is this equal to anything else?

Sean swings back and forth in his chair and stares down.

Sean: One and one-fourth?

Teacher: Okay, do you guys agree?

Walter: One-fourth? No, we do not agree.

Sean interrupts.

Sean: One and then four more.

Teacher: Four more what?

Sean: Sixths

Teacher: So, one and four of six?

Sean nods.

Teacher: Okay, so he says one and four-of-six. Do you guys agree?

Andrew points to his build.

Andrew: This is the answer.

Walter: Yeah, but we have to say it. Okay, let me say it. Okay, one whole . . .

Teacher interrupts.

Teacher: Sean says one and four-sixths. Do you agree?

Walter: Um. No.

Teacher: Okay what do you think is the answer?

Andrew: One whole.

Walter interrupts.

Walter: Okay let me say this. Okay we think the answer is one whole . . . plus one ha. . . plus one half . . . and ha . . . and . . . and three-quarters of a quarter.

Teacher: No, Sean was right. It is one and four-sixths or one and two-thirds.

Walter: What!

In this question, Sean only does a partial build. He builds the first two fractions, $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$, of the question and then gives a solution of one and four-sixths after some clarifying help from the teacher—*Image Making emergent Image Having,* as he did not need to make a complete build.

Walter, who presented as the strongest in the group, was not able to come up with this solution. Up until this question, other than one question at the beginning, $\frac{1}{2}+\frac{1}{4}$, which he was not able to produce a verbal solution to, Sean had not built any of the solutions himself. He was offered one build in question ten by the teacher, although the build was mainly directed at Alex. After being offered one build, by the next question he claimed to be able to visualize the solution to, $\frac{3}{6}+\frac{2}{6}+\frac{1}{6}$, although this was not confirmed. Then in question thirteen, $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$, he did a correct partial build and was, with minimal support, able to produce a correct solution.

Both Iris and Joanne seemed able to progress after their first perceptual offering and then continued to offer themselves builds for most questions. By the end, they were both able to offer solutions without builds, but were never at a point of confidence with them. They ended the task at the *Image Making emergent Image Having* phase. Sean also ended the activity at this same phase even though he was mainly disengaged throughout the activity; then near the end is able to make solid progress after he is offered a perceptual offering in question ten by the teacher. Once again, each of the students demonstrated the initiation of growth once they were offered a perceptual experience.

R group. In the R group, the students were all expected by the researcher to build perceptually on the second and subsequent questions, other than Elliot who was allergic to the foam pieces. Henry and Aven seemed able to produce sporadic images. Elliot was able by the fourth question to produce some correct solutions based solely on being offered the signitive, although there is some question as to whether he was using number logic or visual spatial thinking. Melinda made slow gradual progress but remained very unsure.

Team one—Elliot and Henry: Elliot was the only student to do extremely well on his pre-assessment for fractions. Previous to this question four, $\frac{4}{4}+\frac{3}{6}$, Elliot had made no verbal indications about his thoughts on the questions, and since he seemed allergic to the foam pieces he never built. It is hard to say in this interaction whether he was visualizing or just using number logic. I refer to number logic in the sense of whether he just remembered that the same number in the numerator and denominator is equal to one and that when the numerator is half the denominator it is equal to one-half. Is he just looking at symbols and reasoning his way to a solution or does he visualize things spatially in his mind and through visual reasoning come to the understanding that four-of-four is one and three-of-six is a half?

Researcher: Here is the next question. Try to visualize this.

Elliot: [Whispers to Henry] A whole and half. You see that's a whole [pointing to $\frac{4}{4}$] and that's a half [points to $\frac{3}{6}$].

Researcher: What do you guys think?

Elliot: It's a whole and a half.

Elliot here was able to interpret the signitive offering. Henry, however, leans back in his chair shrugging and laughing.

Henry: Yeah, I don't know.

Researcher: Henry, you're not sure? Okay, so build it.

Henry: Okay, so two . . . so like four fours.

Elliot: So, one whole.

Researcher: [To the other team] So, you guys are going to want to check this. So, Melinda you should be building. So, you have to decide if they're right or wrong. Right?

Both girls begin building. Aven builds three-sixths then raises her hand.

Aven: I know what it is.

Researcher: Just wait a sec. They have to give me an answer. [Turning to the boys] Okay, so what do you have there, four-of-four and three-of-six. So, what are you guys saying the solution is?

Elliot: A whole and a half

Henry: Yeah, a whole and half.

Researcher: Okay, so a whole and a half.

Aven: I got that too.

Researcher: You guys agree with that?

Aven nods.

Elliot gave the answer from purely looking at the signitive, but later watches Henry build. Henry did not seem to be able to visualize the solution, but had to build it, which he does correctly. Aven claims to know the solution after building only the three-sixths. Melinda only collected her pieces but never actually builds the solution and doesn't indicate her agreement or disagreement.

In this interaction, it is possible that Elliot is at the *Image Making emergent Image Having* phase, yet it is hard to say, as Elliot seems to possibly rely on number logic. When Elliot is faced with a question for which he is not able to use number logic, his offered solution could be quite off the mark, as seen here in question eight, $\frac{2}{8}+\frac{1}{2}$.

Elliot: I think I got an idea. Can we give the answer?

Researcher: Okay, well talk about it with Henry, and decide on an answer.

Elliot discusses with Henry (unfortunately, it is undiscernible) while pointing to the expression on the whiteboard.

Elliot: Three-eighths.

Researcher: Okay, so you say three-eighths.

Henry: But we want to build.

Researcher: Do you want to build?

Henry: Yeah, [turning to Elliot] should we try?

Elliot: Yeah, just to make . . . just to confirm

Researcher: To confirm. So, you're saying three-eighths.

Henry: But we want to build.

Researcher: But you want to build. Yep, that's totally fine.

Elliot: I don't like touching these, because it makes my skin feel weird.

Researcher: Oh, maybe you're allergic to them.

Henry: Oh, one half.

Henry puts the last piece down and then stares at it for a while blinking his eyes. [It seems as though he is processing this new image as it does not connect with Elliot's answer of three-eighths. He looks at Elliot as though to say, "Do you see what I'm seeing?] Elliot seems to take no notice. Henry looks back at the pieces.

Researcher: So do you want to ch . . . So, how would you say that?

Henry stares at the desk just beyond his build as though processing.

Researcher: How much of the circle is that? We've seen that before, haven't we?

Elliot puts his head down.

Henry leans back in his chair.

Henry: k. um so like, it's like.

Melinda seemingly taunts him gesturing, making noises, and smiling confidently, as though she knows.

Henry: Shhhh. I'm thinking.

Researcher: Are you guys going to stick with your answer of three-eighths?

Elliot shakes his head.

Elliot: No. Wait, wait, wait, where's the . . .

Elliot grabs for the bag of eighth pieces.

Researcher: Henry, you do it for him, cause he doesn't like touch the pieces.

Henry: Two-eighths and a half.

Elliot: Put these over the half.

Henry starts to build, gets two-eighths on top of the half pieces. Aven after seeing only two-eighths laid down leans over to whisper to Melinda.

Aven: Six-eighths. [then turns to the researcher] We know the answer.

Researcher: Okay, just wait they're still . . .

Elliot: No, just put these [the eighths pieces] over the whole thing, and then two there [the two-eighth pieces from the question].

Henry covers the one-half piece with four-eighths and then places the two-eighths beside that. Elliot counts up all the eighths pieces.

Elliot: Six-eighths.

Aven: Darn it! We . . . I made six-eighths too.

Aven never actually builds six-eighths with her pieces, just in her mind.

In this question, Elliot is not able to use number logic in order to figure out the solution, however, he uses a strategy discussed earlier in the task of creating equal parts to find the solution. Yet, as the questions get more complicated, Henry begins to surpass Elliot in his visualizing of solutions, as in question fourteen, $\frac{2}{3}+\frac{2}{6}+\frac{2}{8}$.

Researcher: Do you have the answer Elliot?

Elliot shakes his head ‘no’

Researcher: Remember this is your last question.

Elliot: We want to build, but we have to give you the answer first.

Henry: [staring off into the distance] We are going to say a whole and two of these [holds up the eighths bag]

Researcher: Which is how much?

Elliot: one-eighth

Researcher: [speaking to Henry] How much is two-eighths?

Henry: [laughs] Two-eighths?

Researcher: Is there another shape that that’s equal to?

Henry points to the quarter bag.

Researcher: Okay, so what are you saying then? You want a whole and . . .

Elliot: We would like to build. We would definitely like to build.

Henry: A whole and one-fourth.

Researcher: A whole and one fourth. Do you agree with that Elliot?

Elliot: Yeah, but we would like to build.

Researcher: Yep, you can build.

Henry builds the two-thirds and the two-sixths.

Henry: Okay, so right on the whole. Yeah, so that’s the answer.

He puts down the pieces and stops building.

Elliot at no point in the task actually builds for himself but watches Henry build. I struggled to be confident about Elliot’s visualizing. There seemed to be an element of number logic for him. The majority of the participants were extremely weak in their concept of fractions and the interaction offered to them in the previous days were all very spatial, the majority of students seemed to continue to engage spatially. However with Elliot, I was less confident of how he was engaging with the signitive offerings in his mind. Yet, in question six, $\frac{2}{8}+\frac{2}{8}$, Elliot offers a solution of four-eighths. Henry agrees but wants to build. It is only once they build that they seem to make an association with one-half. Elliot then describes his visual thinking, "I just added the pieces together." He says nothing about adding the numbers or numerators but gives his description in pieces. However, in comparison, Henry's description is extremely visual.

Henry: So, I like, wait, I went two of these [lifts up a one-eighth] kinda make something like one of these [lifts up a quarter] and like another two of those [points to one-eighths again] make something another one of these [points to quarters] and that looks like a half. And I'm like that's a half.

Henry, Elliot, and Aven after building give the solution as a half associating the shape of four-eighths with one-half. I do believe there was an element of the imaginative for Elliot but I cannot be sure.

There was definite growth in how Elliot was able to use equal parts to help produce a solution, as initially this was a point of struggle. In question 3, $\frac{1}{2}+\frac{1}{4}$, neither group could offer me a solution as they did not know how to convert to equal parts. I went on to offer both groups a visual of laying two quarters onto the one-half so they could see it created the same shape as three-quarters. It is clear from the above interaction that Elliot was able to learn from our earlier discussion of describing fractions in equal parts. Elliot was also able to sporadically offer solutions purely from the signitive by the end of the task, but he always wanted to confirm his solutions with a build. I placed him at the *Image Making emergent Image Having* phase. This placement is rather a shaky one as his visual thinking was less descriptive.

Henry, who was extremely weak in both number and specifically fraction concepts in his pre-assessment, was always eager to build. He would build the solution whether it was his question or not. He was quite open about whether he understood or whether he was totally confused. He definitely craved a clarity of understanding through the builds. As we can see in this last question, Henry was able to produce an imaginative solution from the signitive with a small amount of perceptual input from holding the bags containing the pieces. He still wanted to always verify with a build, but near the end they were partial builds. This places him at the *Image Making emergent Image Having* phase.

Although Elliot started out by far the strongest student in the category of Primitive Knowing for this topic, he did not seem to progress as far as Henry in the end. These two students are an interesting contrast, as Henry, with very limited Primitive Knowing, took every opportunity available to build, whereas Elliot only watched others build. In the end, Henry seemed to progress farther than Elliot.

Team two—Aven and Melinda. Aven presented as very similar to Henry, but possibly a bit stronger as she was more frequently able to produce correct solutions without building. By contrast, Melinda struggled to see images, and it was often a struggle to get her to consistently build. Here in question three, $\frac{1}{2}+\frac{1}{4}$, we see evidence of this.

Melinda: I can do that.

Researcher: Imagine this in your head, and then see if you can give me an answer.

Melinda: I might have to build.

Researcher: Just to confirm it?

Melinda: Yeah, just to confirm it.

Researcher: What do you think it is? Can you tell me what you think it is?

Melinda: I think it's a whole.

Researcher: Okay, then build it, because you said you wanted to build.

Melinda: I need one of these. [She takes a one-half and places it on the table] and one of these [grabs the bag of quarters and starts to open it.]

Researcher: [before she opens the bag] Now do you still think it is a whole?

Melinda: No.

Researcher: Okay.

Melinda: It's a half

Researcher: Well, [pointing to the question] it's a half plus a quarter, so could it be just a half?

Melinda throws her head back, then starts waving her arms.

Melinda: Aven, help me.

Melinda stops building and starts pulling out her one-twelfth pieces from the bag.

Researcher: No, no, no Melinda. Keep building. You have to build the question.

Melinda: Aw.

She then builds it correctly. As this question progressed it was evident that there was just general confusion about how to say the solution to this question, so the researcher explained equal parts to the students.

Researcher: You ready? [researcher waits for everyone's attention] Henry built this last time. Do you remember how Henry built this last time?

Aven: Oh, three-fourths.

Researcher: Three-fourths. Do you see how that is the same shape but in equal parts?

Aven nods.

Researcher: How about I let you both pick a number. Okay, but I want to make sure you guys understand that. Melinda can you show everyone what you've built [half and a quarter]. Do you see that guys? Henry and Elliot look over there? What shape is that? Do you see that a half and quarter is the same shape as this?

All students say: Yeah.

Researcher: So when we describe fractions we describe them with equal parts, so we would say three-quarters, because there are three of the same pieces.

In this interaction all four students engaged with the perceptual representation of the question. There was a lot of confusion about how to describe fractions. Melinda could obviously build the solution, but found naming it hard, as did all the other students. It should be noted that Melinda was very quick to give up and ask Aven for help. This is a pattern I saw often from her surrounding the subject of mathematics. This interaction was mainly an Image Making experience for the students, as there was general confusion, not around building it, but around naming.

It was Melinda's lack of confidence that often caused her confusion as she would just assume she was incorrect and Aven was always correct. In question five, $\frac{2}{3}+\frac{1}{3}$, we can see this play out.

Melinda: I'm not sure.

Elliot, without building and only looking at the signitive, whispers to Henry.

Elliot: It's a whole.

Henry just looks at him, and then searches on his desk for the one third pieces and starts building.

Aven: I know the answer.

Researcher: Melinda, hold your one-third pieces. Where are your one third pieces? And so, imagine what this is saying.

Melinda, Aven, and Henry have picked up their bags of one third pieces.

Aven: I know what the answer is.

Aven: [whispers to Melinda] It's a whole and a half. [Turns to researcher] I told her.

Researcher: What do you guys think it is?

Melinda: It's a whole and a half.

Researcher: A whole and a half. Okay, did you guys want to build or are you confident?

Aven: Let's build it.

They start building. Melinda gets two one-third pieces down.

Melinda: No, I don't think it's that anymore.

Melinda builds three-of-three. Aven builds only two-of-three.

Aven: A whole and a half.

Researcher: What do you have there, Melinda?

Melinda holds up her circle that she's built. Aven takes one of her thirds away.

Melinda: I'm so confused. This is so hard. I feel like I'm in court.

Researcher: So, what is your answer?

Aven: Three-sixths.

Researcher: Three-sixths? [Turning to the boys]

Melinda: I have an idea.

Melinda is speaking very quietly and the researcher does not hear her as the boys are yelling. In this interaction Melinda was along the right track, but Aven seemed to confuse her by taking away one of her third pieces. Melinda is typically quick to lose confidence, which is why I think she did not push to be heard more, even though her thinking was correct. The boys, on the other hand, started yelling, and the researcher's attention was drawn away from Melinda without hearing that she had an idea.

Henry: A whole. A whole. One whole.

Elliot: A whole.

Melinda: [very quietly] Yeah, that is what I was kinda thinking.

Researcher: [Turning to the girls] Okay, can you build it for me? So, two-thirds [pointing to the fraction]. Okay, so you guys build two thirds, and then another third.

Melinda: That's what I was thinking.

Aven: Is it a whole?

Researcher: Yeah, it is a whole.

Then Henry goes on to explain it to the girls.

Henry: Two-threes [puts down two-thirds] plus the one-third [puts down another third]. See whole.

Researcher: [Writing on the white board] So in total you get three-of-three. Does that make sense? Does that make sense to you, Aven?

Aven: Nope.

Researcher: Okay, Aven take two-of-three. [pointing to the fraction two-thirds] Put two-of-three together.

Melinda: It makes sense.

Researcher: And then it says [points to the one-third fraction] add one-third. What does that create?

Aven puts the pieces together.

Aven: A whole.

Researcher: Yeah.

Although, Aven was incorrect, she seemed very confident about her visualizing with a solution of a whole and a half. She then built two-thirds, but gave an answer of three-sixths. In the early stages of this task, Aven was sporadic in her correct solutions, but became much more consistent later on. Melinda claimed to not be able to visualize but built correctly. She mentioned the answer of a whole but was not confident enough to push her solution with Aven, and assumed she was incorrect. Aven continued to be unconvinced of the solution when others just showed her their builds; it was not until she actually built it herself that she could agree to the solution.

Melinda up to this point has not claimed to be able to visualize any solutions but is able to produce correct builds albeit with little confidence. Melinda throughout these questions when asked to visualize would need to offer herself some perceptual experience through gesturing or holding and comparing the size of the pieces to a whole.

In question eleven, $\frac{3}{6}+\frac{2}{6}+\frac{1}{6}$, Aven was able to visualize a whole with only holding a single one-sixth in her hand but was not confident.

Melinda used a lot more perceptual clues by comparing the two-sixths to a full circle and then moving two-sixths around the circle (see Figure 60).

Figure 60: Melinda attempts to aid her imagining.

Researcher: You have to give an answer first, then you can build.

Aven: I think a whole.

Melinda says at the same time.

Melinda: I guess a whole.

Researcher: Aven says a whole do you agree Melinda?

Melinda: Yes, I agree.

Henry whispers to Elliot.

Henry: They got it.

Then in question thirteen, $\frac{3}{6}+\frac{1}{2}+\frac{2}{4}$, when given the question Melinda immediately starts to estimate the sizes by measuring the size of pieces using her fingers.

Melinda: How big is it? Like yeh big.

Melinda measures the piece with her fingers and then lifts her fingers into the air in front of her face and starts constructing it (see Figure 61).

Figure 61: Another attempt by Melinda to help her imagine.

Melinda: One, two, three. Plus, how big . . . okay.

She measures the one-half piece with her fingers. Melinda then looks around for the quarter pieces.

Melinda: I think I know what it is, but I still want to build.

Aven just sits with hands on her chin and stares at the question. The first thing she says is:

Aven: A whole and a half.

Researcher: You think a whole and a half. Melinda, do you agree with that?

Melinda: I think a whole and a half.

Researcher: Do you guys want to build?

Melinda: Yes, just to double check.

Aven builds three-sixths then reaches for the quarters bag takes two out but does not add them to the circle.

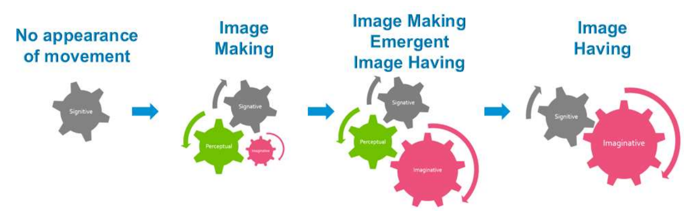
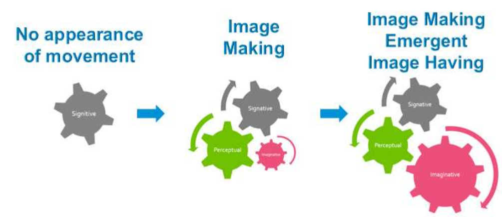
Aven: Oh, I know what it is.

The growth for Melinda did not seem to get past the second phase of *Image Making*, as she with every question either could not visualize anything or she needed to offer herself some level of a perceptual offering, albeit less concrete over time, as she progressed to only gestures by question thirteen, which was quite an involved question.

Aven, on the other hand, was able to produce correct solutions with minimal perceptual input by the end. Aven's starting point was at the third phase and she definitely grew in confidence and began engaging in only partial builds to confirm her imagined solutions, but she never became confident enough to not request a build. For this reason, she remained at the third phase, *Image Making emergent Image Having*, throughout the task.

Overall the perceptual experience of the participants was often the stimulus to growth in their imaginative offerings. Yet, each student moved through these pathways at different speeds. Walter, Aven, and Henry seemed to progress the farthest; however, all the participants were, other than Melinda and Alex, able to reach the phase of *Image Making emergent Image Having* by the end of Day Four (see table 6).

Table 6: Summary each participants' final phase destination at the end of Day Four.

End of Day Four:	
Walter, reached a fairly confident level of Image Having, yet he would still fold back to building the moment he was uncertain.	
Aven, Henry, Andrew, Joanne, Iris, Sean, Elliot	
Alex and Melinda	

7.2.4 Types of visualizing behaviors.

One aspect of this I found very fascinating was the behaviors of the various students as they attempted to visualize the solutions. Some used gestures or objects, such as Melinda and Andrew (see Figures 62 and 63), but oddly enough they consistently held pieces out

Figure 62: Andrew's gesturing when asked to visualize.

vertically or made gestures in front of their face rather than on the table. They rarely performed this movement on the table, which might seem more natural as their experience with drawing or building has

*Figure 63:*Melinda's gesturing when asked to visualize.

typically been done on a flat surface. Yet while gesturing, the participant almost always gestured or held objects in the air in front of their face.

Andrew was able to eventually offer himself imaginative solutions without gesturing

Figure 64: Andrew after being asked to visualize.

(see Figure 64), but Melinda right up until the last question continued to use gesturing.

Aven, Henry, Walter, Iris, and Joanne would lay their hands on the bag or pull one piece out the bag and stare off into nothingness, as they attempted to offer themselves an imagined solution (see Figure 65).

Figure 65: Henry, Walter, and Aven interacting with the fraction pieces after being asked to visualize.

Other students would more often just look straight ahead at nothing or at the signitive question, with a hand on their chin or their temple (see Figures 66 and 67).

Figure 66: Aven starring off into space as she imagines.

Figure 67: Walter after being asked to imagine.

By the end, some students even felt the need to stand (see Figure 68).

Overall, every one of them seemed to enter what looked like a trance like state, and many would physically jump out of this state and blurt out the answer or some acknowledgement that they had arrived at an imagined solution, such as: “Got it”; “I know the answer”; or “I know what it is, I know what it is.”

Figure 68: Walter and Andrew stand as they imagine.

7.2.5 Complex mixture of factors towards growth.

Each participant went through the various phases at varying speeds. Within these levels for evidence of growth there was found to be much complexity in terms of contributing factors. The fact that some participants built more than others and therefore had more perceptual offerings seemed a strong contributing factor, but other aspects, such as their own personal commitment to sense-making as they built, their social interactions, and their own self-belief seemed to also impact growth.

Within the following paragraphs, we will look at participants Walter, Aven, Henry, Andrew, and Elliot, outlining some of the contributing factors to growth as well as bringing in the complexity with their personal profiles.

Strong builders: As shown in the findings above, the students who seemed to, by the end of the task, start to reach the Image Having phase were the participants who were the most committed to building solutions as well as committed to sense-making. Walter, Aven, and Henry built more consistently than the other participants. Andrew did build often, but his intent seemed more focused on answer-getting than sense-making. Elliot, on the other hand, did not build at all; rather, he watched others build. Walter, Aven, Henry, and Andrew all engaged in partial builds. Yet, Walter, Aven, and Henry's builds were often abandoned due to what seemed like having reached a sufficient level of sense-making whereas Andrew was a mixture of sometimes completing and verifying a solution through building but other times his builds would be abandoned after someone else offered the solution. Elliot did not build at all, and it was unclear if his intent was answer-getting or sense-making.

Walter, as I described in his profile, has a strong need for sense-making or he will disengage from the task. In this activity, Walter very openly stated that he did not understand, with such comments as "What are we supposed to do?", "I don't know how to do this." Once he understood the general intent of the task, Walter would need to be constantly reminded not to build immediately, and he still on many occasions attempted to build immediately. If the teacher was distracted, he would often get quite a way into the build until he was caught and would then apologize. As the task progressed he began to build less and less, as he did not seem to find the need.

Question 6. $\frac{2}{8} + \frac{2}{8}$

The next question goes up. The teacher asks Walter if he wants to build first.

Walter: No, I just want to say the answer—four eighths.

Teacher: Is that equal to anything?

Walter: [pauses for 5 seconds and looks straight ahead.] A half.

However, when Walter was presented with a question that he struggled to visualize he would immediately attempt to go back to building.

Question 12. $\frac{3}{8}+\frac{1}{4}+\frac{1}{2}$

Walter attempts to go immediately to the pieces.

Teacher: Do not build. Visualize.

Walter puts the pieces back down.

Walter: Okay, I'm sorry.

Walter continues to visually search through the pieces.

As seen in this interaction, Walter was always willing to go back to the build in order to deepen his understanding, as sense-making seemed to be his focus. The reason I say that sense-making was his focus as oppose to answer-getting, is that Walter showed a strong pattern of getting quite upset when something did not make sense. He would verbally state that he does not understand and then either give up all together or keep building and asking questions until he was at a sufficient level of sense-making. Other students who seemed more focused on answer-getting rather than sense-making would often not finish building a question once an answer was given or claim to understand but it would be clear through further discussion that they did not. Walter did not display this behavior. By the end of this day's task, Walter displayed a strong level of confidence.

Teacher: Okay guys, this is going to get hard.

Walter: I don't mind. I understand all of this.

Andrew: Yeah, me too.

Walter: My mind is a calculator right now.

Walter is practically vibrating at this point.

Teacher: Okay, visualize.

No one builds secretly.

Walter: Got it!

Walter holds up his hands in victory. He then starts bouncing, waving his hands and then clapping in his chair excitedly.

Walter: Got it. I've got.

Walter whispers something to Andrew.

Andrew: I think I got. I think I got. [whispers to Walter] It's a whole and a half.

Walter: Stop it. I already know. Andrew, I know how to do this stuff now.

Teacher: You guys are too competitive.

Walter: It's a whole and a half.

Teacher: So, one and a half?

Walter: Yes, sorry.

Walter: [Turning to Andrew] I didn't need your help, I have this stuff.

Walter had both a strong need for sense-making but also a strong need to show those around him that he understood. In this example, Andrew was also able to offer a correct solution, but he was often very uncertain with his responses and he was not as consistent with correct solutions as Walter was, yet he would always claim to have the solution.

Aven is another student who reached a strong level of visualizing although not to the same degree as Walter. Like Walter, she had a strong level of commitment to both building and her own sense-making. In this interaction, Aven's team has given an incorrect solution and the other team has in response offered a different solution which the researcher has accepted and orally discussed as being correct.

Researcher: [writing on the white board] So in total you get three of three. Does that make sense? Does that make sense to you, Aven?

Aven: Nope.

Researcher: Okay, Aven take two of three. [pointing to the two-thirds fraction on the whiteboard]. Put two of three together.

Researcher: And then it says [points to the one-third fraction on the board] add one third. What does that create?

Aven puts the pieces together.

Aven: A whole.

Aven would not just accept the solution as correct until she had built it for herself. Aven throughout the task was always eager to build and reached a fairly solid level of confidence in her imagined responses, although she would always build a portion of the question.

Question 13, $\frac{3}{6}+\frac{1}{2}+\frac{2}{4}$

Aven is sitting staring down and a little in front of herself. Then Aven's head pops up.

Aven: A whole and a half.

Researcher: You think a whole and a half. Melinda, do you agree with that?

Melinda: I think a whole and a half.

Researcher: Do you guys want to build?

Melinda: Yes, just to double check.

Aven builds three-sixths then reaches for the quarters bag takes two out but does not add them to the circle.

Aven: Oh, I know what it is.

Researcher: Talk to your teammate.

Aven: A whole and a half, a whole and a half, a whole and a half, a whole and a half.

Aven often spoke with confidence when she offered her solutions to the questions, yet she would still build at least a portion of the question. In this context, her reason for not completing the build seems attached to there no longer being a need for her to continue as she is confident of her imagined solution. Andrew, on the other hand, would often stop when someone else had given a solution rather than him continuing to build in order to confirm his own understanding.

Walter: Yeah, but it's out of six, so you have to use the six pieces.

Andrew turns to Alex and takes the quarter piece out of his hand then removes all the other quarter pieces and builds three-of-six with the sixth's pieces.

Walter: [Whispering to Alex] It's a whole and a half."

Andrew stops building and takes the pieces apart.

Andrew with almost every question claimed to "know" the answer but he was inconsistent at being able to then actually produce the solution.

Henry was also eager to build in order to ensure his understanding. Early on in the task, Elliot, Henry's partner, would whisper to him the solution. Rather than just accepting what Elliot is telling him as correct he would always make a point of building it for himself.

Elliot: [whispers to Henry] It's a whole.

Henry just looks at Elliot, and then searches on his desk for the one third pieces and starts building.

Henry was continually asking to build. If there was no question to build he would just build anything for the sake of building, this was true in both the classroom task and the pre-assessment.

Elliot was an interesting case, as he had by far the strongest primitive knowing (eight out of ten on the pre-assessment), but he ended up not building for himself at all as the foam pieces made his fingers feel funny. His partner Henry who had a much lower level of primitive knowing (four out of ten on the pre-assessment) built for him. However, by the end of the task, Henry seemed to display more growth than Elliot.

Question 14. $\frac{2}{3}+\frac{2}{6}+\frac{2}{8}$

Researcher: Do you have the answer Elliot?

Elliot: [shakes his head] No.

Researcher: Remember this is your last question.

Elliot: We want to build, but we have to give you the answer first.

Henry: [staring off into the distance] We are going to say a whole and two of these [holding up the eighths bag].

Researcher: Which is how much?

Elliot: One-eighths.

Researcher: How much is two-eighths?

Henry: Two eighths?

Researcher: Is there another shape that that's equal to?

Henry points to the quarter bag.

Researcher: Okay. So, what are you saying then? You want a whole and . . .?

Elliot: We would like to build. We would definitely like to build.

Henry: A whole and one-fourth.

Researcher: A whole and one-fourth. Do you agree with that Elliot?

Elliot: Yeah, but we would like to build.

Researcher: Yep, you can build.

Henry builds the two-thirds and the two-sixths.

Henry: Okay. So, right on the whole. Yeah. So that's the answer.

Henry puts down the pieces and stops building.

Elliot did not present as having the visual solution at all. He just kept asking to build, whereas Henry was visualizing the solution. It could have been that Elliot had disengaged from the task yet there was evidence earlier on that his knowledge of fractions was not as spatially formed as Aven's and Henry's.

Question 8. $\frac{2}{8}+\frac{1}{2}$

Elliot: I think I got an idea. Can we give the answer?

Researcher: Okay. Well, talk about it with Henry and decide on an answer.

Elliot discusses with Henry (unfortunately, it is inaudible) while pointing to the expression on the whiteboard.

Elliot: Three-eighths.

Researcher: Okay, so you say three-eighths.

Henry: But we want to build.

Researcher: Do you want to build?

Henry: Yeah, [turning to Elliot] should we try?

Elliot: Yeah, just to make . . . just to confirm.

Researcher: To confirm. So, you're saying three-eighths.

Henry: But we want to build.

Researcher: But you want to build. Yep, that's totally fine.

Henry: Oh, one half. [Prior to this he could have assumed it was one-eighth based on Elliot's answer. He seems surprised by the one-half as he looked again at the question in order to start building]

Henry builds correctly. He puts the last piece down and then stares at it for a while blinking his eyes. It seems as though he is processing this new image as it does not connect with Elliot's answer of three-eighths. He looks back and forth between Elliot and the image. What adds to this context are the social norms of hierarchy in the classroom—Elliot is viewed as a strong math student and Henry is not. He seems hesitant to question Elliot's solution. Elliot seems to take no notice.

Researcher: So, do you want to ch . . . So, how would you say that?

Henry stares at the desk just beyond his build.

Researcher: How much of the circle is that? We've seen that before, haven't we?

Elliot puts his head down.

Henry leans back in his chair.

Henry: K. um so like, it's like.

Melinda seemingly taunts him gesturing, making noises, and smiling confidently, as though she knows.

Henry: Shhhh. I'm thinking.

Researcher: Are you guys going to stick with your answer of three-eighths?

Elliot shakes his head.

Elliot: No. Wait, wait, wait, where's the . . .

Elliot grabs for the bag of eighth pieces.

Researcher: Henry you do it for him, cause he doesn't like to touch the pieces.

Henry: Two-eighths and a half.

Elliot: Put these over the half.

Henry starts to build gets two-eighths on top of the half pieces.

Aven: [leans over and whispers to Melinda] Six-eighths. [Then turns to the researcher] We know the answer.

Researcher: Okay, just wait they're still . . .

Elliot: No, just put these over the whole thing, and then two there.

Elliot counts out the pieces.

Elliot: Six-eighths.

Aven: Darn it! We . . . I made six-eighths too.

Aven never actually builds six-eighths with her pieces, just in her mind.

Researcher: Cool. Okay. Now, what is another way of saying that? Remember we came across this shape before.

Elliot: One-third.

Here is a situation where Elliot gives a solution of three-eighths which does not connect well with the signitive expression on the board. It is unclear how he was seeing it, he may have misread one-half as another one-eighth. Henry seemed less certain of this solution and kept asking to build. Once they built, Henry seemed to then know this solution of three-eighths could not be correct. Once Elliot looked at the solution, he showed a strong level of understand of fractions as equal parts but needed to completely cover the image and then count all the pieces whereas just after two pieces were laid down over the half Aven could immediately see that the solution would be six-eighths. Finally, when asked for another way of saying the solution, Elliot replies with one-third. Elliot was definitely engaged in this task but struggled to see it spatially.

In this question, he did not show much of an aptitude for visualizing a solution, both at the beginning and while predicting how many pieces would cover the image. Yet, his knowledge of equal parts and fractions allowed him to find a solution. Elliot's growth for imagined solutions, although it seemed strong at the beginning, stagnated as the questions became harder.

Progression of growth seemed to be influenced strongly by the introduction of a perceptual experience, however, getting to the point of building and the intention during the build were definitely complicating factors. In this next grouping of students, there was not a strong level of commitment to building.

Sporadic builders: Iris, Joanne, and Melinda were all fairly agreeable and willing participants however seemed less enthusiastic builders. As a result, they did not have as many perceptual experiences as some of the others. Iris and Joanne were in the classroom teacher's group so were not expected to build unless it was their turn. As a result, they only built sporadically. They often did not choose to build outside of their turn. Had they built more frequently and been offered an earlier connection to the perceptual, it would have been interesting to see if they would have made more progress in the time given.

Melinda was in the researcher's group, where it was expected that everyone build for every question, but Melinda often needed to be reminded or encouraged to build. If not specifically asked she would avoid the task.

Melinda: I'll be the imaginer and you'll be the creator. I'm the imaginer.

Henry: Where's the two threes?

Henry is searching his desk and picks up the one-third pieces. He lays his hands on them.

Aven: [Takes the relevant pieces to another table] I'm going to go to the building table.

Melinda: I will keep imagining unicorns. [To Aven] How's the building going? Do you think you have the answer?

Melinda struggled the most in the researcher's group and would often need to use gesturing to aid her (see Figure 69) as she struggled to visualize, or she would hold pieces up against each other as a comparison.

Melinda also struggled with her belief in herself as was demonstrated by statements like: "I don't know"; "Aven, help me"; "I'm not good at this," along with speech being often very soft and timid when expressing her answers. This sometimes caused her ideas to be missed completely even though they were on track.

Figure 69: Melinda as she struggles to visualize.

Melinda struggled to visualize, but when she did engage in a build her connection to sense-making with the signitive expression was usually accurate.

Alex and Sean struggled the most throughout this task. They had the lowest tendency towards building. Alex seemed unsure of what to do with pieces, mostly due to his classmates' misdirection; whereas, Sean seemed to know how to build but not know how to say it. Both Sean and Alex attempted to build only twice during the session. Sean's first build was correct, but he could not give a verbal solution.

Question 3. $\frac{1}{2} + \frac{1}{4}$

Sean: I don't know how this works.

Teacher: Take the pieces out of the bag and build this equation.

He builds correctly one quarter and one half. Points to what he built.

Sean: It's this.

Teacher: Can you tell me what that is?

Sean: Two out of three?

The teacher asks the other team if they agree.

Andrew: Yeah. [hesitant tone]

Walter: No, three out of four.

Teacher: You're correct.

Iris: WHAT?

No further explanation is offered to either team, and the other team gets to pick a number. The teacher moves on to the next question.

This experience seemed to solidify Sean's decision to disengage. He does not build or seem to engage with any of the questions again until Question 13 when he is asked again to give a solution but this time to a fairly complex question.

Question 13. $\frac{3}{6}+\frac{2}{12}+\frac{4}{4}$

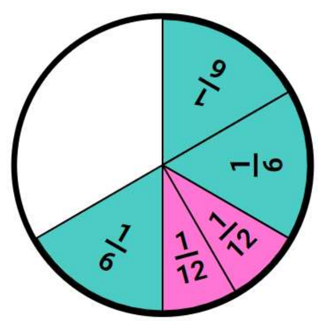


Figure 70: Sean's build for question 13.

Teacher: Sean, it's your question.

Walter starts clapping.

Walter: Okay, I got it. I got it.

Sean starts building. He builds two-sixths then two-twelfths and then another one-sixths (see Figure 70).

Sean: A whole and half-ish. It's a whole and a bit over half.

Teacher: Okay. [pauses and then points to the two-twelfths] Is this equal to anything else?

Sean swings back and forth in his chair and stares down.

Sean: One and one-fourth?

Teacher: Okay, do you guys agree?

Walter: One-fourth? No, we do not agree.

Sean interrupts.

Sean: One and then four more.

Teacher: Four more what?

Sean: Sixths.

Teacher: So, one and four of six?

Sean nods.

Teacher: Okay, so he says one and four of six. Do you guys agree?

Andrew: [Points to his build] This is the answer.

Walter: Yeah, but we have to say it. Okay, let me say it. Okay, one whole . . .

Teacher interrupts.

Teacher: Sean says one and four sixths. Do you agree?

Walter: Um. No.

Teacher: Okay what do you think is the answer?

Andrew: One whole.

Walter interrupts.

Walter: Okay let me say this. Okay we think the answer is one whole . . . plus one ha. . . plus one half . . . and ha . . . and . . . and three quarters of a quarter.

Teacher: No, Sean was right. It is one and four sixths or one and two thirds.

Walter: What!

The interesting part of this is how well Sean does in this very challenging question after seeming not to be engaged. As is Sean's style, he stays true to his sense-making, refining it slightly as he is offered more information from the teacher. Also, he does not actually build the whole but seemed to know that four-fourths was a whole, this Image Having stemming from years past or the last few days. He was also able to make the connection that two-twelfths is the same as one-sixth after being prompted by the teacher to find its equal shape. In both questions that Sean did he struggled to know how to say the solution that he was visually reasoning through—"One and one-fourth," then "One and then four more."

Sean has both strong number sense and strong spatial tendency. Throughout the activity, Sean frequently demonstrated either a strong connection to sense-making or total disengagement; rarely was he anywhere in between. This strong connection to sense-making definitely helped him to solve this question, but his lack of tolerance for confusion also prevented him from coming to a point of understanding earlier on.

Alex was similar to Sean in the number of builds he engaged with, but the one major difference was his lack of connection to sense-making. Alex in the pre-assessment engaged only with procedures or tricks for answer-getting and then in the classroom work was quick to abandon sense-making. I believe this was due to lack of confidence. He seemed quick to believe he was most often wrong and would as a result follow another's idea. Within the classroom culture he was not seen as a strong math student, but rather one of the weakest students. The following interaction was probably the most telling and upsetting one in reference to where Alex was at in his sense-making and treatment by classmates.

Question 8. $\frac{2}{6}+\frac{1}{6}$

Teacher: Okay, everyone visualize.

Iris: [sings] I know what it is, I know what it is.

Teacher: Alex, this is for you.

Alex: I need to build.

He begins building with the one-sixth pieces.

Walter: [whispers to Alex] It's a half.

Andrew: [whispers to Alex] It's a half.

Teacher: Don't talk to him. Let him build it.

Andrew: Please Alex, I hope you get it right.

Walter starts waving the one-half piece around.

Teacher: Walter stop. [Alex does not seem to notice.] Okay, what do you think?

Alex has built it correctly, but with all the whisperings of one-half decides to pick up the one-half piece and adds it to his build.

Andrew: No, no, no, no, no.

Teacher: Guys stop. Okay Alex, do you know the answer.

Alex looks at his teammates.

Alex: No.

Teacher: Do you want to take a guess?

Alex sits back down looking very remorseful.

Alex: No.

Walter mouths the words "a half" to him. Alex sees him.

Teacher: Look at your pie.

Alex: Three and a half?

His teammates turn their back to him with frustrated mumblings.

Teacher: [Turns to the other team] Do you agree?

Joanne: I think it's a half.

Alex: [mumbles to his teammates who have turned their backs on him] Sorry.

Walter leans over and talks to Alex.

Walter: Okay, so when it's like this [points to the three-sixths that has been built]. It's a half.

Alex: [in a frustrated tone] I don't get this.

Walter: Alex, do you see that this is the same shape as a half? It's a half.

Poor Alex knew up to this point that he was struggling to understand and received so many mixed messages from his own build in connection to his teammates' whisperings along with a pressure to give the right answer or let his teammates down. This struggle between lack of confidence and lack of trust in his own sensemaking, along with social pressure and mixed messages created an atmosphere that the perceptual experience could not seem to permeate.

Although Iris and Joanne did not build to the same extent as some others, they were progressing with this smaller proportion of builds and were generating fleeting moments of Image Having—*Image Making emergent Image Having*. Sean also seemed on the cusp of this progression as he was visualizing a whole and able to make a visual comparison of two-twelfths and one-sixth without manipulatives. Yet, Melinda and Alex continued to struggle. Melinda is seemingly more capable than Alex, as she could offer correct solutions once she built and seemed to be able to connect to sense-making. However, Alex continued to struggle throughout. Near the end he did have moments of confidence in the connection of his verbal solution to his build, but generally there was a major lack of building, lack of confidence, lack of connection to sense-making, and overall frustration which resulted in disengagement.

It's complicated. Although the type and amount of perceptual experiences that students had seemed to impact their growth, to address the perceptual experience that students were having as the only factor/influencer on growth would have not been a fair representation of the experience. Classrooms are complex places. Factors of individual history, social influencing by peers, the management of flow by the educator, and the perceptual experiences offered, all seemed to present as factors towards student growth.

7.3 Comparing fractions—Day Five

Below is a list of the progression of questions students were offered signatively on a small white board one at a time.

Table 7: Progression of comparison questions.

1. $\frac{2}{12}\ \frac{2}{8}\ \frac{2}{4} \rightarrow$ biggest	9. $\frac{2}{4}\ \frac{3}{4}\ \frac{5}{4} \rightarrow$ biggest
2. $\frac{6}{12}\ \frac{6}{14}\ \frac{6}{8} \rightarrow$ smallest	10. $\frac{5}{8}\ \frac{2}{8}\ \frac{9}{8} \rightarrow$ smallest
3. $\frac{5}{12}\ \frac{5}{8}\ \frac{5}{5} \rightarrow$ smallest	11. $\frac{4}{12}\ \frac{8}{12}\ \frac{11}{12} \rightarrow$ smallest
4. $\frac{3}{12}\ \frac{3}{14}\ \frac{3}{6} \rightarrow$ biggest	12. $\frac{1}{12}\ \frac{3}{4}\ \frac{2}{6} \rightarrow$ biggest
5. $\frac{9}{15}\ \frac{9}{8}\ \frac{9}{10} \rightarrow$ biggest	13. $\frac{3}{10}\ \frac{8}{4}\ \frac{6}{8} \rightarrow$ biggest
6. $\frac{7}{12}\ \frac{5}{12}\ \frac{4}{12} \rightarrow$ smallest	14. $\frac{1}{6}\ \frac{3}{4}\ \frac{2}{5} \rightarrow$ smallest
7. $\frac{9}{6}\ \frac{3}{6}\ \frac{7}{6} \rightarrow$ smallest	15. $\frac{3}{9}\ \frac{2}{12}\ \frac{5}{6} \rightarrow$ smallest
8. $\frac{2}{8}\ \frac{8}{8}\ \frac{4}{8} \rightarrow$ biggest	16. $\frac{4}{5}\ \frac{3}{10}\ \frac{1}{12} \rightarrow$ biggest

I will not be going into as much detail for this section, as generally it was found to be comparatively uneventful. All the students but one found this task of comparing three fractions to be easy and students rarely felt the need to build. I think a major distinction between the adding fractions and comparing fractions activities is that in the adding fractions task students were expected to produce an exact solution, whereas in the comparing fractions task they were asked to choose from three solutions.

Yet, it was clear compared to the pre-assessment that a fair amount of growth had occurred. Even though, there were some correct answers in the multiple choice comparing fractions section, these were the kind of comments made by different students, “I'm just circling random ones, cuz I don't know.”; “I don't know how to do this.”; “I'm really bad at this.”; “I am just full on guessing.”; “I am really confused.” Or they described their logic as using two different strategies for finding the biggest fraction; in one grouping the student described choosing the one with the biggest number on the bottom and in the other grouping he chose based on which had the biggest number on top. He stated that one of the two strategies would end up being correct. Other students just did not even attempt it with comments like, “I don't know what a fraction is.” So even the correct answers, other than by Elliot, are very suspect. Yet, by the Day Five activity and with basically no instructions for how to compare fractions, other than a brief discussion with students to collect ideas and reasoning for why one fraction is bigger than another, from the very first question, the students were able to easily give correct solutions and describe their thinking. The task seemed almost too easy for them. Also, on Day Five, a very distinct change in behaviour occurred from the adding fractions day to comparing fractions – very few students actually used their manipulatives to build (see Figure 71). The day before they consistently asked to verify through building, but now they were describing their thinking and seemed to express a strong level of confidence to the point where they did not even want to build. There decreased need for building might have been at least in part due to the fact that comparing does not require them to hold the image in their mind, which then reduces the need for physical objects. This would then reduce the load on their working memory

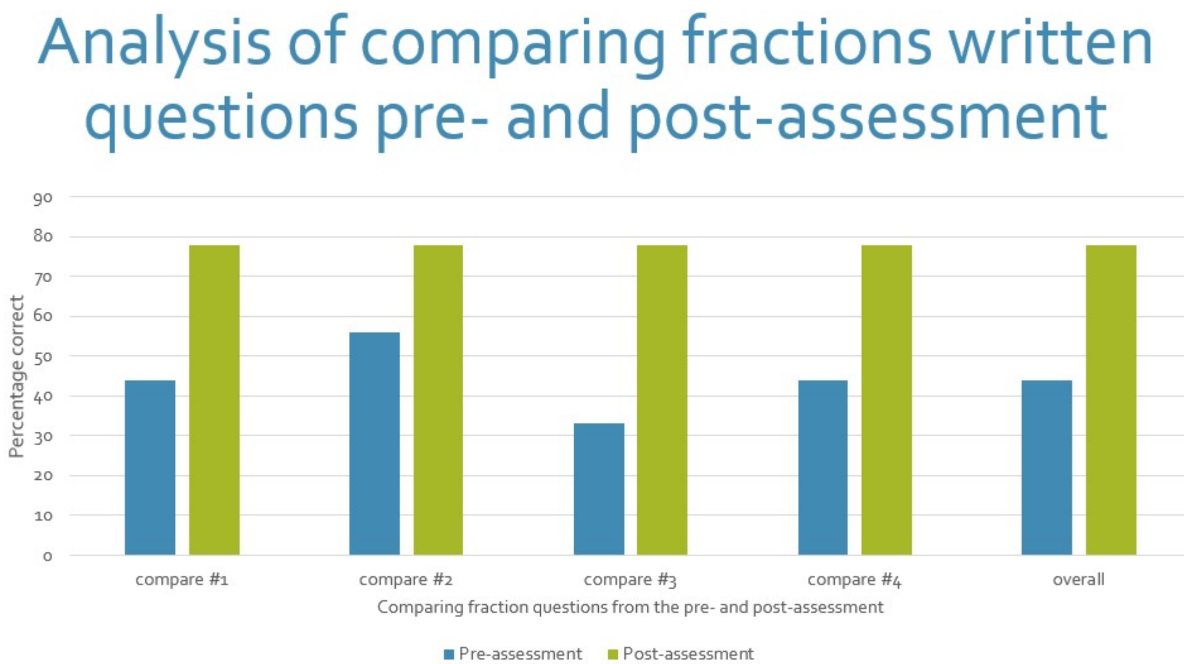


Figure 71: Comparing fraction questions 1 through 4 and then an overall comparison between the pre- and post-assessment.

The pre- and post-assessment that was given contained the same questions. At this point, it should be noted that while their experience over this five-day period was clearly impacting, as there was no control group it may be overstating things by attributing this growth solely to spatial engagement. In years past, they had been taught to compare fractions with the same denominator in Grades 3 and 4, however, comparing fractions with different denominators was a new topic for them. Although they did not receive explicit instruction for this topic they had been offered experiences which contributed to their understanding, so one could argue that a nonspatial experience could have made a similar difference. Overall, for the comparing fractions activity all but one student either demonstrated their understanding through getting all four questions correct on the written assessment at the end or through an oral description in the task. Seven out of the ten got all four correct in the post-assessment. Of the three that did not give correct solutions, two decided not to complete the post-assessment as is sometimes the case with students who dislike writing. However, these two students were able to demonstrate a solid level of understanding orally during the task. Ultimately only one student, Alex, struggled in both the task and the written assessment; otherwise nine out of the ten seemed to answer these questions with ease with only being offered the signitive. This growth could be attributed to them Image Having; it is hard to say what specifically they were imagining as they were not asked for specific answers. It is possible that they could have been using visual-spatial reasoning based on the size of the pieces. For example, one-third is bigger than one-fourth because they reason that cutting the circle less produces bigger pieces. This does not necessarily require them to visualize one-third specifically just knowing that one is bigger than the other. This would still be visual-spatial reasoning, but it is unclear if they were Image Having for one-third and one-fourth specifically.

In the graph below, we see the breakdown of each individual student's behavior, both during the Day Five task and the written assessment. It outlines what percent of questions they engaged with building, percentage of offered descriptions using spatial language, percentage of correct oral solutions, percentage of correct written solutions, who was absent for a portion of the

Day Five task, and who did not complete the written assessment. This offers a general overview for the findings, in this section (see Figure 72).

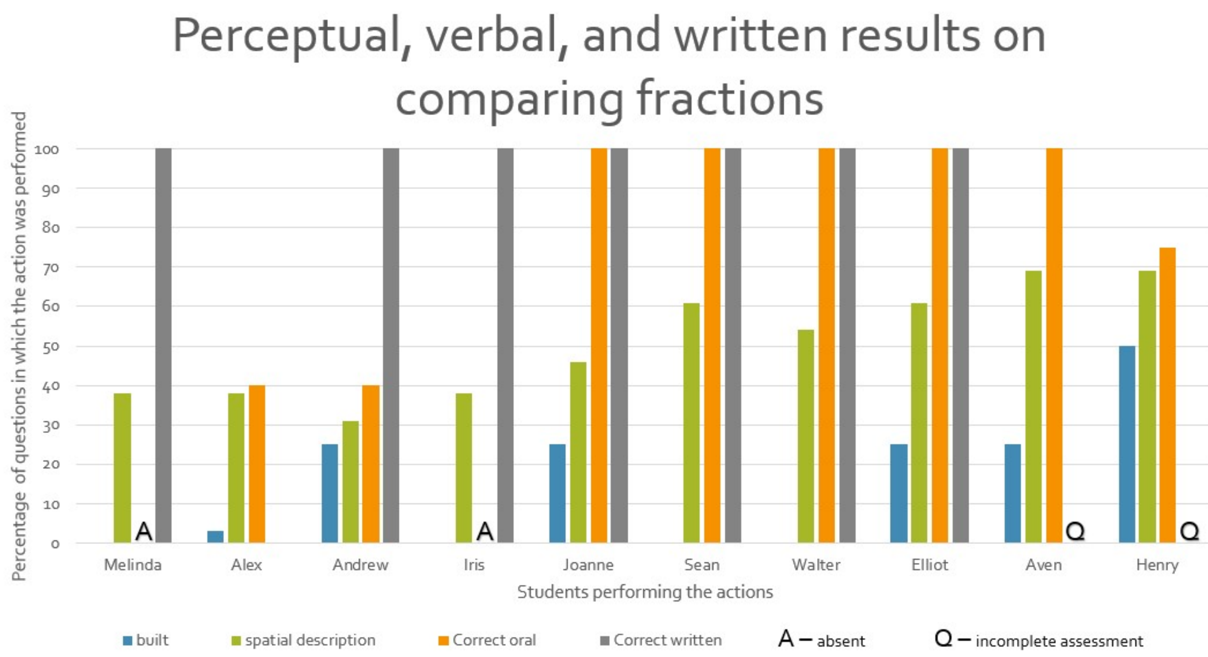


Figure 72: Percentage of questions that each student engaged with building, offering a spatial description, correctly gave an oral solution, and correctly gave a written solution.

7.3.1 Language used when describing their thinking.

Overwhelmingly, when students were asked to describe their thinking when comparing fractions, they used very spatial language. Some examples of the types of descriptions that were given are:

- "These pieces [referring to the quarter fraction] are bigger so they fill up the space more."
- "Twelve is like the highest number but like, but if you cut up—if you get a sandwich cut it up twelve times you have the smallest pieces of sandwiches."

- "If you fold a blanket six times you still get quite a bit of blanket, or if you cut a sandwich six times you still get quite a bit of sandwich not like if you had fourteen."
- "Because it has the smallest number on the bottom, which means it has less empty space than all the others."

When students were asked to describe their thinking, other than those who were absent—Melinda and Iris—they all used spatial language.

During this activity of comparing fractions, the students found it easy. There was the odd student who engaged in building, but generally they just used spatial logic to come to a solution. The only student who continued to struggle and then was unable to provide correct answers was Alex, who consistently struggled in the previous activity as well. Alex demonstrated a general uncertainty about quantity and spatial thinking. One comment that was made during this activity was when he was building a fraction that was greater than one, he built one whole circle then put another piece beside it and asked, "Would it be smaller if there was one more piece after the whole?" Throughout both activities Alex continued to have general confusions and lack of confidence about some very basic ideas and mainly remained in a perpetual state of uncertainty.

Overall, during the two days of activities there was much growth demonstrated by the students. There was definite variation in what students could do orally as opposed to written, as the population varied in their willingness or ability to express themselves in written form, but overall there was a strong level of demonstrated growth. However, we can still see a substantial level of growth when scoring them as a group (see Figure 73).

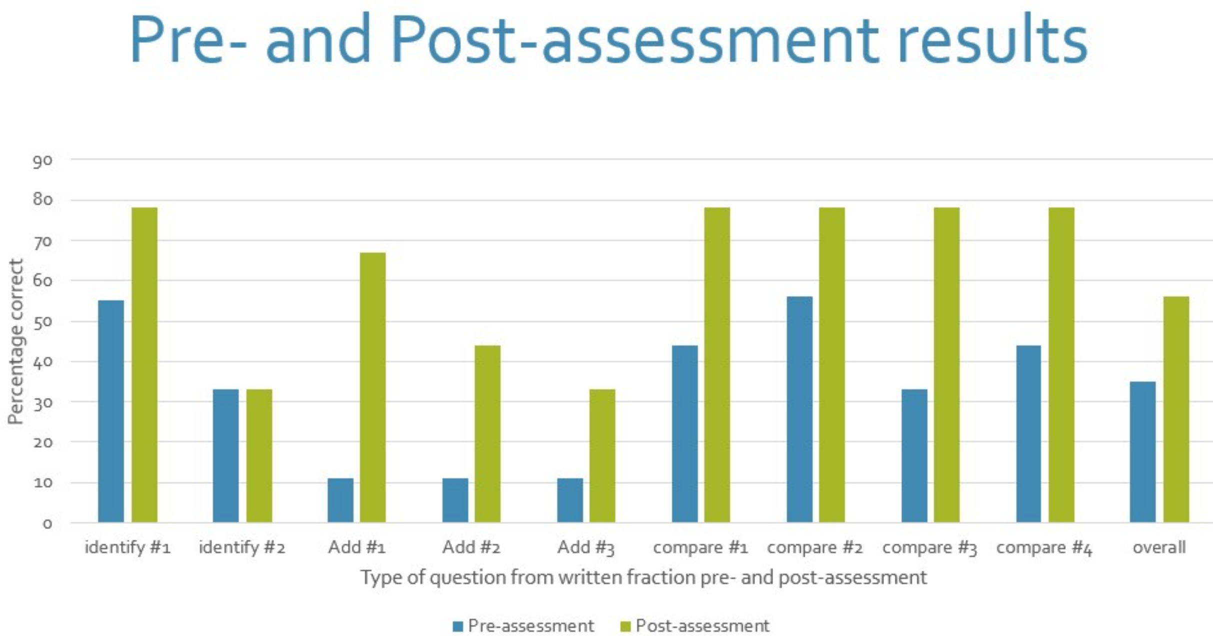


Figure 73: Comparison of pre- and post-assessment.

Chapter Eight: Discussion

The purpose of this study was two-fold. Broadly, it explored the role that spatial reasoning might play in the growth of mathematical images, looking at the transition from Image Making to Image Having described in the Pirie and Kieren model (1994). Specifically, we were looking at the different types of offerings that provided opportunities for growth in students' visualizing abilities. These explorations have been positioned within a community of learners, namely those labelled with a learning disability. This discussion section begins with looking at the variability within these labels, the connection between spatial reasoning and number sense, reasoning behind the choice of fractions as a topic, and why it was of particular interest for this group. Then we will return to the Pirie-Kieren theory and its connection to the Husserlian ideas, looking at the complexity of factors that promoted or hindered growth.

8.1 Variability in Psychoeducational assessment

My assumptions at the beginning of the study in regard to the students' assessments were very different from what I experienced. I did not look at student profiles before working with the students, as I did not want this to influence my observations. I was concerned about prejudging their abilities and struggles. Rather, I focused on getting to know the students through the multitude of experiences that we engaged with during the two-week period. Once I did engage with the profiles, I was often surprised by the areas of weakness and strength contained within, based on the experiences I had getting to know the learners through assessments and classroom work. I was also surprised by the MLD labelling and the absence of certain information.

8.1.1 Absent from the assessments. Not looking at the profiles first, ended up presenting as a bit of problem. I had anticipated that the profiles would include the visual spatial and the fluid reasoning index, as these are included in the core battery of tests required in a psychoeducational assessment. My plan was to then use these scores as a reference point for their spatial and mathematical reasoning ability; however only four student profiles contained both these areas. The fact that six out of ten students were not reported in these areas for their psychoeducational assessments seems quite significant. I am not sure of the reason for this. In my request for psychoeducational assessments, I was not given access to their whole profile but rather given a summary of their scoring for the subtests. There were clearly areas missing. When I asked the school, I was told this is all they had. This is interesting because, as stated earlier, spatial reasoning (VRI) and abstract thinking (FRI) are part of the standard battery of tests and foundational skills within mathematics. As a result of this absence in the majority of the profiles, I had to disregard any reference to their psychoeducational profile when categorizing students in these areas, as it could not be used as a baseline for all participants.

Instead, I based their tendency towards visual spatial thinking on my video footage noticings for the classroom tasks and by administering an assessment with the Q-bitz game as it is similar to one of the subtests within the psychoeducational battery of testing for the visual-spatial index. The visual-spatial index is determined through two subtests, one called block design and the other a timed visual puzzle task which I did not attempt to replicate. As with the block design subtest, I used the Q-bitz blocks and cards to have the student rotate the blocks in such a way to recreate the pattern given to them on a card. My aim was to have an informal assessment of every student on one of the subtests, even if it would be considered an unofficial scoring. I offered this task by leveling the cards from less abstract to more abstract and kept track of how many cards they could complete in a 10-minute timeframe. I also included some general classroom observations in this visual-spatial categorization, as even if I had had the visual-spatial index, it is important to also recognize that these assessments are not holistic in nature either.

The visual-spatial assessment piece, made up of the block design and visual puzzle tasks, do not encompass all aspects of visual-spatial reasoning but only a small portion. The task of even attempting to assess all the aspects of spatial skills would be a major undertaking. As Davis (2015) discusses there are multiple of different forms of reasoning that make up spatial reasoning, to name just a few: “locating, orienting, decomposing/recomposing, shifting dimensions, balancing, diagramming, symmetrizing, navigating, transforming, comparing, scaling, feeling, and visualizing” (p. 140). Overall, when referring to spatial ability, I use the phrase ‘tendency towards’ as a purposely vague statement, for to claim conclusively that someone is spatial or not spatial is too broad of an analysis to be holistically claimed within the constraints of this brief study.

8.1.2 Math learning disability (MLD) diagnosis. In this study, I made very little distinction between the terms MLD and LD as I found the diagnosis of MLD to be challenging in its distinction from low-achieving (LA). Butterworth (2010) categorized MLD as students having specific difficulties with quantitative information. In this study, the only students who were given a diagnosis of MLD were Aven and Melinda, yet seven out of ten of these Grade Five students had significant struggles with adding, well beyond what is typical for a Grade Five student. These seven students included Aven and Melinda, who did not stand out as any worse than the others. In fact, Aven seemed a bit stronger than most. Also, when asked to engage spatially with fractions, Aven was one of my strongest participants. Yet, none of the other five had an MLD diagnosis. The only student I found to have consistent struggles with quantity was Alex. During a comparing fractions question, Alex had built a whole circle and then needed to add another piece in order to build the fraction. He hesitated and asked, “Would it be smaller if there was one more piece after the whole?” My understanding of what he meant by this was that, if you complete the circle do you start over again; his ‘new’ circle was smaller. Alex did not have an MLD diagnosis. Yet my experience with Alex is that he was so detached from sense-making that he struggled to be certain of anything. So, is it an MLD or just a large dose of insecurity and uncertainty, due to knowledge gaps and deflating mathematical experiences? It would be interesting to see if he were able to find his way back to relying on sense-making whether he would still struggle with quantity.

In this study, I wanted to explore engaging with what is considered a challenging mathematics topic going beyond the students’ curricular requirement like adding and comparing fractions with different denominators and see where they could go with that. I chose the topic of fractions for a multitude of reasons: (a) fractions are shown to pose a significant challenge for many children, LD or not, (b) you can keep numbers low so as not to be a barrier for those students who have not developed number sense, (c) many studies have been done with LD students and fractions demonstrating their struggle with them, and (d) fractions can be engaged with as a visual-spatial topic.

I was also purposeful to include comparing fractions, as this should have then presented as a significant problem to students with an MLD based on their defined struggle with comparing quantity. Yet by Day Five with essentially no instruction on how to compare fractions, all but Alex were able to easily perform these comparisons with 100% accuracy, including Melinda and Aven (the two students who had the official MLD label). I found both the diagnosis and lack of diagnosis for MLD to seem quite arbitrary.

8.1.3 Working Memory (WM). Another area I found to have much variation between the students’ psychoeducational assessment and classroom work was in working memory. This indicator generally did not seem to be a predictor of either ability or struggle with mental calculations or mental manipulation. In the table below, I categorized the students based on their WMI and their ability or struggle with mental manipulation. The first two columns indicate their ability for mental addition and subtraction from the pre-assessment. The last four columns are their ability for mental manipulation when adding and comparing fractions based on their oral performance during the imagine-build-steal game and their final post-assessment.

I categorized them according to their lowest scoring Working Memory subtest, for example if they were very low and low in their two subtests, I put them in the very low category in this table. Ranking of Index scores from highest to lowest:

- Extremely high average
- Very high average
- High average
- Average
- Low average
- Very low average
- Extremely low average

Now keep in mind that although I categorized Melinda, Joanne, Sean, and Iris as struggling, they were able to produce solutions eventually with the use of manipulatives and occasionally through their emergent Image Having. I categorized them as struggling because they were not able to produce a solution with confidence by the end of task. All students, with the exception of Alex, did get to the level of *Image Making Emergent Image Having,* requesting verification with the manipulatives. By the end, however, Walter, Aven, and Henry were able to produce complex solutions of three fractions with different denominators when offered only a signitive equation.

Table 8: Categorizing students by their demonstration of ability and struggle within the area of working memory for various activities, alongside their psychoeducational profile status.

Student WMI rating	**Ability** to mentally calculate Pre-assessment	**Struggle** to mentally calculate Pre-assessment	**Ability** for mental manipulation when adding fractions	**Struggle** for mental manipulation when adding fractions	**Ability** for mental reasoning when comparing fractions	**Struggle** for mental reasoning when comparing fractions
Very low WMI		Melinda Joanne		Melinda Joanne	Melinda Joanne	
Low WMI	Sean Elliot Walter	Iris	Elliot Walter	Sean Iris	Sean Elliot Iris Walter	
Average WMI		Henry Alex Andrew Aven	Henry Andrew Aven	Alex	Henry Andrew Aven	Alex
Very high WMI						

Of the ten participants Sean, Walter, Elliot, Joanne, Iris, and Melinda were considered weak in their WMI scores (see table 8). Of these six, the first three were strong mental calculators and visualizers, with a question around Sean, who was a strong mental calculator and strong with the task of comparing fractions but disengaged from the addition of fractions activity due to early confusion, so it is unconfirmed whether he would have performed as well as the others if he had remained engaged.

By the end, Joanne, Iris, and Melinda all seemed to be starting to pick up the visualizing of the fractions but did struggle. They did not, however, seem to struggle any more than two of the other students, Alex and Andrew, who had average WMI scores. It would have been interesting to see how far Joanne, Iris, Alex, Andrew, and Sean would have been able to go, if they were offered more consistent perceptual representations earlier on in the addition of fractions task and expected to verbally offer their visual solutions before building.

Henry had an average WMI, and Aven had an average to high average WMI; these two along with Walter (low to average WMI) demonstrated a strong ability to solve complex addition of fraction questions in their heads.

Yet, Joanne, Iris, Andrew, Melinda, Aven, and Henry, four with low WMI scores, and two with average or high average scores, all had issues with basic adding and subtracting in their pre-assessment. I include Aven in this grouping as she demonstrated a struggle in the area of adding and subtracting (three out of eight) although her overall number concept pre-assessment score was at sixty percent. As described in her profile she was only able to do mental calculating for the first two questions in the addition and subtraction section of the pre-assessment but then used manipulatives for the third and did not attempt the remaining five claiming they were too difficult for her.

I believe these struggles with mental calculations, manipulations, and reasoning may not necessarily be related to the students' WMI score but rather their lack of understanding, development of images for these ideas, and struggle with memorization. Many students with an LD profile have weak neural adaptation (struggle with memorization) (Perrachione, et al., 2016), and as a result their working memory (ability to hold information and manipulate it in their mind) is more overloaded than another student who memorizes with ease, as they do not have access to the same amount of information in long-term memory.

Ease with memorizing: For example, a 'typical' student when asked to add 7 and 8 may just remember that 7 + 8 is 15. This student may not have a great understanding of number structure but can just remember. They have used very little working memory to hold the information, as they were just able to remember. This correct answer that they were able to produce could contribute to a belief that they are good at math and understand it. These ideas may possibly encourage them to persist with more challenging math which provides them with more opportunities for growth in the subject.

Struggle to memorize: Let us assume that the previous style of student and this LD student have a similar level of understanding for number structure. A student with weak neural adaptation (many LD students) would not have memorizing as an option in the same way. As a result, if they also do not have an understanding for number structure, they are left to count on or count all. This is much more challenging mentally in comparison to a student who can memorize. As a result, the margin for error is much higher for this student, as they are having to keep track of more in their head and will be much slower at producing an answer. If the answer is incorrect, as will be more often the case with this approach, they may be more likely to feel that they are not good at math. If they did get the answer correct, they will most likely be significantly slower and may feel less competent having used a culturally less sophisticated strategy than a memorizing peer. This experience does not offer the same opportunity for confidence building.

Understanding number structure: If however, both types of students had an understanding that 7 is made up of a 2 + 5 and that they could borrow the 2 and give it to the 8 to make 10 and be left with 5 to make 15, this approach would be more grounded in understanding, less in memorization, and would create less cognitive load than counting all or counting on, however, more cognitive load than memorizing. This approach may take longer than just memorizing, but it will contribute to their ability to add larger numbers. I find many LD students can engage at a similar ability level to 'typical' students when they do mental calculations in this way.

A point to consider when looking at the impact of confidence levels between the two groups discussed is that even for those students who can find success at getting the answer through memorized facts, the chance for these students of reaching that point of understanding, I believe, is higher than for the LD student. This memorization path may allow these students to remain in the math task with more feelings of success, which allows them more access overtime to develop a deeper understanding of the structure of number as they continue to engage with more ease in tasks that involve number. The LD student with weak neural adaptation, however, does not have the option of following this path. Generally, they do not have the same access to this memorization path and therefore need to access the path of deep understanding. If they do not have access to this and are given more procedure and steps the student continues to struggle to engage with tasks involving number, which does not allow them to continue to progress in the same way as the student who was able to memorize. The ability to memorize may translate into an actual advantage if these students begin to then engage conceptually with what they have memorized. However, if they continue to rely primarily on memory without understanding, they will soon reach a point where even a high capacity for memory is insufficient.

Often the perceived cause of students struggle to mentally calculate is working memory, when it may just be their lack of conceptual images for number. In this study, a possible reason for students' ability to mentally manipulate these fractions and then compare them was due to the typically smaller numbers involved with fractions. The students would have been more familiar with these values and the participants working memory would not have been as overloaded by their lack of memorized facts. As a result, their working memory would have been utilized or filled similar to that of a 'typical' student, resulting in less struggle with the mental calculation of fractions. Engaging students spatially with a focus on conceptual image-building is beneficial to all, both those who are memorizers as well as those who are not.

I believe many of these students are very capable of performing mental calculations and reasoning as demonstrated during the addition and comparing fractions tasks. It is my hypothesis that students' lack of images for concepts within mathematics causes tasks to present as extremely challenging and indicators such as working memory, presenting as inability to hold things in their head, are thought to be the culprit when really it is a result of their lack of understanding of our number system's structure and they do not have memory to fall back on like other students.

In general, I found the results of the WMI to be a shaky indicator of whether a student would be successful with mental calculations and mental manipulations. For this reason, in general I believe it is important to consistently challenge students with mathematical tasks that may seem outside their wheelhouse based on their psychoeducational profile, and then observe what are the actual barriers as to why this may present as a weakness. I find that often their inability to do a task lies in a conceptual misunderstanding or struggle to memorize rather than a true working memory disability. This idea of students struggling due to conceptually different understandings rather than disability, specifically in the area of fractional work, is supported in the literature through studies done by Lewis (2014) and Mazzocco, Myers, Lewis, Hanich, and Murphy (2013).

Through both my own personal experiences, along with my son's, and years of working with students labelled with LD's, my viewing of the psychoeducational outcomes is not as holistic in nature as some may think. Areas of strength for that student in the assessment is tough to question as they were clearly able to demonstrate ability in the task, albeit specific in nature. If the student is then not able to demonstrate this same area of strength in the classroom, it is important for the educator to look deeper into what barriers might exist for the student or other areas of weakness that may be related to the task. In the case of Alex, his lack of confidence in his own sense-making and past conceptual misunderstandings presented as a major barrier towards his learning. Yet in his psychoeducational profile he presented with average Visual-Spatial and very high Fluid Reasoning. Alex was clearly able to perform the reasoning and spatial tasks offered to him during his Psychoeducational assessment, yet in the classroom he struggled to perform in these areas. Whereas, areas of weakness in a student's psychoeducational profile, in my opinion, are a question mark, something to take note of but not to prejudge about. If an area of weakness presents itself in the classroom the task is still to dive in deep to determine what factors are possibly contributing to this weakness and not chalking it up to sheer inability.

8.2 Tricks for answer-getting.

The belief in inability can influence educators' decision-making process when it comes to what form of pedagogy they will choose for their students. Throughout the implementation of this study, I have approached these LD participants as students who are capable of doing high levels of mathematics but often struggle to rely on memorization as an entry point. This belief influences how I teach them. In the past, many of these students have been offered tricks and procedures to help them with "answer getting". This choice of pedagogy may be due to a possible underlying belief by the educator that these students are not capable of understanding or to the educator themselves a deep enough understanding to teach the conceptual ideas. In this interaction, we see the impact of these tricks and procedures on two LD students.

Andrew: Iris, do you know the bow tie method?

Iris: Yeah.

Andrew: We should use it on that one.

Iris: Maybe.

Researcher: What's the bow tie method?

Andrew: It's just like where you go [he starts writing on the paper beside him] say fifteen times . . . twelve. I don't know. It doesn't matter. And then you can go like make a bow tie.

Researcher: Oh, okay.

Iris: And then like you add the one to the on—[pause] like times – like add one to the one and . . .

Andrew: Yeah, like times this to this and then like five to one and then add the – and then [he shrugs].

Iris: And then it looks like this after.

Researcher: Oh, I see so then it looks like a bow tie. Do you understand what is happening and why you are getting the answer with that bow tie method? Or is it just like magic and you don't know why?

Andrew: Magic sorta to me.

Iris: [Laughs and shrugs her shoulders] I don't know.

Researcher: What about the window method [this strategy was mentioned earlier in the conversation]? Do you understand why it works?

Andrew: Not really. Not as much.

Iris: I think you [she leans over to point at the window method drawn on the paper.]

Andrew: There is another method called the dinosaur method. It takes so long. It takes like 52 hours.

Researcher: Like the dinosaur, ancient history a long long time.

Iris: It would be like 20 times 20 and you put the answer there [pointing to the paper.]

Researcher: Okay.

Iris: And then 20 times uh . . . like 20 times uh . . . [pause]

Researcher: I've seen this before, so you times this by this next.

Iris: Yeah and then you figure it out.

In all three strategies discussed, neither student could give a full explanation of how to come to a solution, and in the end, neither student used any of the methods discussed to solve the questions. This idea that you do not need to understand the math in order to get an answer is really problematic, as it can then be considered normal to not strive for understanding as it is only the solution that matters, or worse that math really is just a bunch of procedures. The other issue for students with weak neural adaption is that they will forget the steps to the answer-getting trick and then they will have neither the understanding nor the answer-getting capabilities. So, this is where we turn to some productive ways for students, not just LD students, to grow their images for mathematical ideas.

8.3 Student spatial and number concept comparisons.

A key aspect, I believe, is to engage students spatially in mathematics. In the literature there is precedence from a variety of sources that suggests that humans are predisposed to associate numbers with space (Dehaene, Bossini, & Giraux, 1993; De Hevia & Spelke, 2010; Pinel, Piazza, Le Bihan, & Dehaene, 2004). Yet, through my experience in working with struggling students, many of them have not learned to engage spatially with numbers, and it is my belief that this could be a source of their struggle rather than an inability. I wanted to have a general sense of where these students' knowledge of number concepts was to begin with and what spatial tendencies (refer to section 8.1 for a description of how this was determined) they showed as they engaged with tasks.

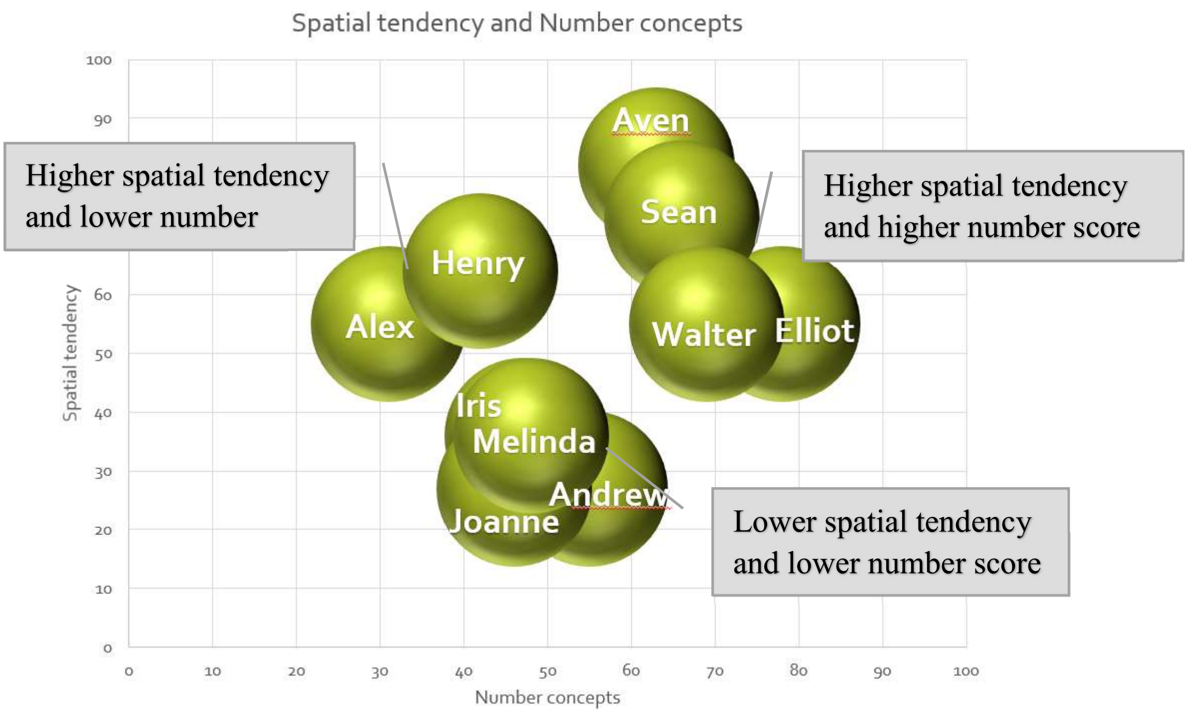


Figure 74: Percentage comparison of Grade Five students' spatial tendency versus written pre-assessment score.

As you can see there seem to be three general clusters (see Figure 74).

- Those who demonstrated both a tendency towards spatial reasoning and had a score above sixty percent in their pre-assessment for number concepts.
- Those who demonstrated a tendency towards spatial reasoning but were below sixty percent in the pre-assessment.
- Those who demonstrated a lower tendency for spatial reasoning with a below sixty conceptual number score.

What I found interesting was that there was no cluster for lower spatial tendency with a higher conceptual number score. There could be a few reasons for this, (1) my indicators for spatial tendency in the higher scoring students were not very good indicators and some students who demonstrated as spatial were not; (2) there is a correlation between students with higher conceptual number ability and spatial reasoning, this seems to be supported in the literature (Mix & Cheng, 2012); (3) there just did not happen to be anyone with this profile in the study, which is completely possible as there were only ten students.

Higher number with higher spatial tendency. Within these groupings there were some notable aspects. All four students who scored higher on the pre-assessment for number concepts demonstrated the ability to strategically mentally manipulate numbers in their heads for addition and subtraction using their understanding for the structure of number, such as Aven here who describes her thinking (question 3: 25 + 7 + 3 + 15), "There is two fives here…so that makes ten. So, one ten plus twenty is thirty plus another ten from the fives is forty plus seven is forty-seven plus three is fifty." Aven then goes on to use the base 10 blocks as the numbers got bigger. Three of the students were able to successfully complete a minimum of six out of eight questions in this way. The fourth student, Aven, although she beautifully describes her thinking in this the example above was actually the weakest of the four. She seemed emergent in her ability for mental calculations as she only completed three of the questions and in the first two she used a combination of manipulatives and visualizing.

Lower number score with both higher and lower spatial tendency. All the lower scoring students, those who scored below sixty percent on the assessment, depended heavily on counting by ones with their fingers or used the technique of vertically stacking the numbers. Both of these approaches do not engage the student in thinking spatially and structurally with numbers. These are "answer getting" (Daro, Mosher, & Corcoran, 2011, p. 50) techniques—methods that focus on finding answers rather than mathematical understanding. Henry was the one exception. He used mainly base 10 blocks in a very spatial way, although he did count the individual blocks out one by one rather than grouping them. Henry also just generally seemed to have a higher spatial tendency than the others in the lower pre-assessment grouping. Alex was the other student who seemed to have a higher spatial tendency based on the spatial assessment tasks used; however, when given addition/subtraction questions he almost exclusively used a method that he had learned from the school's tutoring wing called Chisenbop. This is just a slightly more sophisticated counting strategy which, according to Alex, only works for numbers up to one hundred. Alex was by far the weakest student in his number concepts, yet he demonstrated a spatial tendency similar to the two strongest number concept students, Walter and Elliot.

This was extremely interesting, for as we continued on to the fractional work, which was approached in a very spatial way, four of the students with a stronger spatial tendency, Aven, Henry, Walter, and Elliot, did progress more quickly than those who presented with a lower spatial tendency with the exception of Alex once again, who struggled in both tasks and Sean, who struggled in only the adding task. Yet, as I describe in their profiles, I believe other factors were hindering their progress, namely for Sean—early struggle—and for Alex—an eroded belief in himself. Sean, once he was offered a response to his questions, was able to make excellent progress in the adding fractions task and excelled at comparing fractions through visualizing. I believe he would have also excelled in the addition of fractions, had he been engaged, for he was able to produce the correct answer to the last question while everyone else was struggling and in this question he demonstrated strong spatial reasoning. However, Alex throughout both tasks held many misconceptions towards numbers and demonstrated a very weak spatial connection with them. He continued to struggle with some glimmers of understanding near the end. Alex's lack of connection to his own sense making seemed a major barrier for him.

In the lower spatial reasoning group—Iris, Melinda, Andrew, and Joanne—Iris, Melinda, and Joanne built inconsistently. Andrew, on the other hand, built a lot. Out of this grouping he definitely made the most gains, but he still did not reach the same level of consistency with his answers as some of the students with a higher spatial tendency. The reason for this is no doubt multifaceted.

It is hard to say what impact on growth their spatial tendency had. One thing that can be stated is that the less you build the less sensory input you receive. Moving forward, it would be productive to do a more thorough testing of each of these students' spatial ability and spend more time with a spatially designed curriculum and see the impact on the various spatial levels. Is it good for all? Or only good for some? If we embrace the Pirie & Kieren Dynamical Theory (1994) then it is important to look at what factors impact those initial stages of the growth for our mathematical images.

8.4 Progression of growth

Pirie, Kieren, and colleagues stress that their model for growth in mathematical understanding "is *not* meant to be used as a tool with which one can categorize, level, or sequence forms of mathematical knowledge;" rather it is meant to offer a way of describing "the complexities inherent in mathematical understanding" (Thom & Pirie , 2006, p. 187). This desire to describe complexity has also been my goal. Not only the complexity of students, but also classrooms, and the multitude of offerings (signitive, imaginative, and perceptual) for meaning that students engage with during an activity.

Now, while I have used this idea of the progression of growth, as described by Pirie and Kieren, there was much *folding back* in between levels. Students often *folded back*, based on the question and its complexity for the individual student. Another noteworthy observation was that I actually found it a struggle to place a student completely in the Image Having category as some students used a combination of mental images and manipulatives to attain their solutions. This interplay between the two lead me to create an in between phase called *Image Making emergent Image Having*. However, some students did seem to reach the level of Image Having and others did not progress beyond Image Making. All students did demonstrate some level of growth however.

It was paying attention to student understanding that urged me to look at what drives this growth. What experiences seemed the catalyst for this movement? Looking at two particular levels of mathematical understandings as described by Pirie and Kieren (1994) we explored the transition from Image Making to Image Having. In focusing on this transition, we paid attention to the different types of offerings the students were receiving for the mathematical ideas. This style of exploration brought in another realm of ideas from Husserl. These offerings were categorized into signitive, imaginative, and perceptual, which originated from Husserl's work with the various types of offerings for objects (Husserl, 1970). These ideas have been morphed somewhat to refer to the offering of a mathematical idea rather than an object. As described by

Husserl, these offerings have a certain leveling of meaning held within each. The lowest and most empty way in which the idea can be offered is in the signitive act. These linguistic acts, whether given orally or in print, certainly have a reference, but apart from that, do not offer any perceptions of the idea itself. The imaginative act, which is meant by Husserl to be pictorial in nature, I have adjusted to refer to visualizing which is evidence of students' Image Having (Pirie & Kieren, 1994). The visualizing of an image offers a level of information to the person that they have constructed themselves. This offering, however, is constructed by the person themselves so is therefore an interpretation from a specific perspective and as a result would be considered an indirect offering. The perceptual interaction, however, is between us and our environment, which offers an enactment of the concept, which allows for the most direct offering. This form of intending contains the most of the three in the way of direct offerings for the idea. In this activity, the perceptual offering is the fraction pieces with which the students have had previous experience on Day Two—cutting them into fraction pieces themselves—and Day Three—combining them and then writing a signitive representation for them.

During this data collection period, Image Making experiences could be seen as students used manipulatives, made gestures, and discussed their builds. Students' Image Having was often a combination of a sudden jerking up of the head and shouting the answer, other times just a staring off into the distance, and then other times it seemed completely uneventful just a statement of the solution with no need or attempt to build. These events would have not been captured with the use of written work or even just spoken language; it was the expression on the face, the glint in the eye, the jumping back in the chair or raising of hands in victory. These experiences were found in the combination of bodily movements, facial expressions, use of materials, the fragmented builds, ideas shared between group members. Their understanding was expressed through these various means that were often charged with energy, excitement, and sometimes much frustration.

However, through all this complexity a third phase seemed to emerge. As I was categorizing students in either Image Making or Image Having, I often found myself in a position of struggle. Certain situations could not be categorized as solely Image Making or solely Image Having, rather a combination of the two.

In the Pirie-Kieren theory (1994), there is a discussion of boundaries between levels, and what they refer to as certain boundaries being ‘don’t need’ boundaries. One of these boundaries exists between Image Making and Image Having. The purpose of this style of boundary is to convey the idea that, “these previous forms are embedded in the new level of understanding and readily accessible if needed” (p. 68). This means that once the student has the mental picture for the mathematical idea they no longer need the action that was used to help create it.

In my experience with this study, however, there seemed to exist a middle ground. Initially, students would not be able to offer a solution without building it first with their fraction pieces – solidly in the Image Making phase. Then as many of them progressed in the activity, they would begin to only produce partial builds. They might just build one of the fractions or a portion of the fraction, as the remaining portion they could mentally produce without building it—Image Making emergent Image Having. They seemed to have one foot in the Image Having phase but were not yet completely there. Or students could produce a solution but wanted to confirm their build, as they were not completely confident with their mentally produced solution. Neither of these behaviors seemed to fit the description of Image Having; rather they seemed to be in a state somewhere in between, or on the precipice of making that leap to the next level, but still need that further interaction with their environment to aid in the transition.

All of these goings on could be thought of as occurring within an interactive space of visualizing and spatial reasoning, where the imaginative is interacting with the perceptual and the signitive, as the imaginative provides the totality (Arnheim, 1969; Gibson, 1979). Interweaving is continually occuring between the perceptual, imaginative, and signitive, moving the student forward in their Image Making towards the point of Image Having. This was made visible through a mixture of language and gestures. However, there did exist what seemed to be some delination between they types of gestures and space that students engaged with during the different stages of the task. Students' usable space seemed to shift somewhat when they were imagining versus building. While imagining, students would either draw images in the air in front of them or make gestures in an attempt to recreate the fraction pieces. It was interesting that there was only once that a student actually drew on their desk; instead all the drawing and gesturing was done in the air in front of them, or they would simply look straight ahead. This use of space right in front of their face seemed almost necessary. Melinda even chose to awkwardly pick up her fraction pieces and held them in front of her face, which is a much more challenging task than keeping them on the desk (see Figure 59). Other times students would appear to search their desk for images, and then lay their hands on the pieces but rather than look at the pieces would look straight ahead with their hands merely resting on them (see Figure 64). Some students would hold a fraction piece up against their mouth and then stare off into the distance (see Figure 64). Generally, there seemed to be a strong preference for looking straight ahead, using the space in front of them as a location for thought while imagining. Interestingly, I did not notice many students looking up (Van Rosendaal, 2015), as discussed earler in this document, but rather more looking straight off into the distance.

Upon being given the freedom to build, this space then seemed to expand and include their desk and the fraction pieces on it. This affordance space expanded to the area around them and had significance for movement, action, attention, accomplishing tasks, and, as Gallagher (2015) may argue, engaging in higher-order conceptual and mathematical cognition. Within each question all of these elements were at play influencing, connecting, and relating within the environment or space immedately surrounding them.

Each participant made their way through the various phases. Generally, with the introduction of the signitive only there was *No appearance of movement.* Then as the student began either producing or receiving perceptual experiences they progressed into the *Image Making* phase. The continued engagement with perceptual experiences, appeared to create the beginnings of the imaginative, *Image Making emergent Image Having* phase, and some students were even able to enter fully into the *Image Having* phase. This progression occurred at varying speeds. Within these levels for evidence of growth, there was found to be much complexity in terms of contributing factors. The fact that some participants built more than others and therefore had more perceptual offerings seemed a strong contributing factor, but other aspects such as their own personal commitment to sense-making as they built, their social interactions, and their own self-belief seemed to also impact growth.

Part of what was revealed through the data was not only the interweaving of language, gesture, images, and offerings, but also the student's history, which has formed their self-belief and mindset. Throughout this task, there is a revealing complexity in the interactions of students with their own connection to sense-making, interactions with each other and with the teacher or researcher. The teacher/researcher seemed to play a role in maintaining engagement and managing flow (Liljedahl, 2016), which for some students seemed to have an impact on growth.

Even so, building with the fraction pieces seemed to be a strong catalyst towards movement or growth.

8.4.1 Brief summary of the participants and their progression of growth.

Participants who seemed the most engaged with building:

Demonstrated commitment to sense-making:

Walter, Aven and Henry reached the *Image Making Emergent Image Having* phase and seemed very close to the *Image Having* phase. Walter seemed at certain points during the activity to have reached the *Image Having* phase. These students built the most and seemed to focus their builds around their own understandings and would sometimes not complete due to having reached a sufficient level of confidence in their own thinking and understanding.

Demonstrated varied committment to sense-making:

Andrew—reached the *Image Making Emergent Image Having* phase but did not seem to reach the same level of confidence as the previous group. Andrew also built at a similar frequency to the previous group, but he seemed more focused on answer-getting as opposed to understanding, for he would often abandon his build once a solution was offered by another student, instead of continuing to build in order to prove the solution to himself.

Demonstrated commitment to sense-making but was not able to physically build: Elliot—also reached the *Image Making Emergent Image Having* phase. Elliot seemed interested in connecting to sense-making, and he had a strong *previous knowing* which seemed to help him come to solutions, possibly based on procedure rather than solely images from his mind. Elliot did not build himself, as he did not like the feel of the foam. However, he was generally engaged visually in each build other than near the end when the questions got harder. Elliot did not seem to develop as strong of an imaginative offering as those who built for themselves regularly.

The reasons for Elliot's stagnation are unclear. It could have been that his perceptual experiences were less direct than Aven's and Henry's or that Elliot was not as naturally spatial as Aven and Henry. He did score slightly lower than them on the spatial tendency assessment.

Discussion of the above groupings: Walter, Aven, and Henry were very consistent with their perceptual offerings and would take whatever measures were necessary to confirm their understandings. Although Andrew also built consistently and was definitely able to produce some correct solutions, he was often unsure of his solutions and did not engage in the same level of verification for the solutions as the other students. Elliot, however, only watched as others built and seemed to struggle to produce mental images as the questions became harder. It is not possible to categorize Elliot as a student who was not committed to building, as he seemed to have a physical barrier to building. Elliot was also generally engaged visually with each build even when it was the other team's question, he just never physically built himself. This made him an interesting case to compare, as he is a student who had typically been quite strong in the math classroom. Yet, as the questions became harder, Elliot did seem to disengage somewhat and struggles to visualize. Elliot, like Andrew, in general seemed to have a harder time admitting when he struggled.

Although Walter, Aven, and Henry were all willing to admit when they struggled, Walter had a short time-frame of tolerance for struggle before he would give up and disengage. Had Walter not been offered some level of understanding in that first question, I believe that his progression of growth would have ended up in a similar position to Sean's, having made very little progress during the majority of the task. This may speak towards implications for how initial tasks are offered. The offer of that perceptual experience that Andrew presented to Walter, could intentionally and pro-actively be fulfilled by a teacher early on in a task to keep engagement intact. The pedagogical decision to send students off to try the task and then early on in the activity, calling them back together to exchange ideas or present clarifying perceptual experiences.

Progression of growth seemed to be influenced strongly by the introduction of a perceptual experience; however, getting to the point of building and the intention during the build were definitely complicating factors.

Participants who seemed less engaged with building:

Built sporadically:

Joanne and Iris—reached the *Image Making Emergent Image Having* phase but they did not have the same level of commitment to building. Their builds were sporadic and overall they received fewer perceptual offerings than the previous cluster of students (Walter, Aven, Henry, and Andrew). Joanne and Iris' group was not expected to build, but they still did choose to do so sporadically. During the task, they demonstrated growth in their imaginative offerings to themselves.

Melinda—did not seem to reach the *Image Making Emergent Image Having* phase, but remained at the *Image Making* phase. She struggled to visualize solutions but would often need to offer herself a level of perceptual offerings through gesture or holding the pieces and moving them in front of her. Melinda did build and seemed to connect to sense-making in her builds but was not very committed to building and often needed to be reminded.

Only built when it was required:

Sean—surprisingly Sean, with very few builds, was able to demonstrate that he was in the *Image Making Emergent Image Having* phase, albeit in the last question. Sean spent the majority of his time seemly disengaged from the activity. He built twice during the activity. This offered a very interesting perspective for the data. Overall, Sean was shown to have strong number sense and strong spatial skills in his pre-assessment. Sean had a similar profile to Walter—he felt a strong need to connect to sense-making, but if he did not make that connection early on in a task he would just disengage. Unlike Walter, Sean was not provided with a perceptual offering that made sense to him near the beginning of the task. This caused him to disengage and, I believe, stunted his growth. I feel that Sean would have reached a similar level to Walter had he been engaged. Walter reached an *Image Having* phase fairly quickly (question 4), just as Sean reached the *Image Making Emergent Image Having* phase with relatively few builds and went on to the next day to compare fractions with ease without using any manipulatives.

Alex—like Melinda, did not demonstrate moving beyond the *Image Making* phase. Alex was generally disengaged during the task, much like Sean. Alex produced one correct build himself (question 11) during the task. When it was his question, Alex would often start out with some correct thinking but had very little confidence and was easily derailed from sense-making by his teammates. He did not attempt to build unless it was his question and would openly state that he had no idea what he was doing. He did not see a correct build until question 10, which the teacher built for him. He did not show evidence of visualizing and continued to struggle with comparing fractions, unlike all his other classmates who went on to compare fractions with ease.

Generally, the more a student built, the more growth seemed to occur, yet it was not this simple overall. The type of perceptual experience seemed important. Elliot was offered many visual representations of fractions but he never actually built them himself. This seems significant, as he started out with the most Primitive Knowing for fractions, yet he did not shine like some others did in the end. These ideas bring to mind a graphic by the physicist Tor

Nørretranders (1998), called *Bandwidth of our senses*. It compares the amount of information each of our senses perceives per second. This is compared to the white space in the bottom right hand corner which indicates how much we are consciously aware that we are processing (Norretranders, 1998). When engaging multiple senses and engaging in the physical act of building, deciding which pieces to pick up and put next to each other requires a higher level of engagement than just watching someone else do it and taking in that visual. That being said, having a visual would seem, based on this graphic, to be more productive than not. This was demonstrated by Alex and Sean who seemed to not engage visually or physically throughout most of the task and seemed to present as being at a disadvantage for it throughout much of the task.

The focus of this study was the role that spatial reasoning plays in the growth of mathematical images. The impact of the perceptual experience seemed quite evident as a necessary starting point for growth, along with a continual need for its presence to encourage further growth through to the *Image Having* phase. The interweaving of gesture, language, images, and space; along with signitive, imaginative, and perceptual offerings was also evident. These factors, along with the complexity of the individual and their historical experience with school appeared to be strong influencing factors. Student self-belief, along with a focus on answer-getting as opposed to understanding, impacted their levels of effort and as a result their growth. These factors, I would argue, are not unique to LD students, but may be somewhat amplified due to their higher level of struggle within a school environment. Although these were inhibiting factors, in order for growth to be initiated, a perceptual experience seemed necessary regardless of these other influences. This implies that if these other factors were not present as hinderances to growth, without a perceptual experience these students may have remained at a standstill. This further emphasizes the importance of offering perceptual experiences along with signitive to students, and not just visual but hands-on movement-based experiences that can then be attached to the symbols that represent them. As demonstrated by Elliot, offering only visual may not have as strong of an impact on growth as a more movement-based experience.

8.4.2 Why the shift on Day-five?

Day Four was full of complexity and excitement but then for some reason on Day Five the majority of this complication just seemed to melt away. On Day Five, comparing fractions, students seemed to breeze through the task. It seemed so basic to them, as they got almost all the questions correct and responded to questions in what seemed like an almost bored, “This is so obvious” kind of tone. There was not the same level of frustration or excitement as the previous day. Very few students engaged with the manipulatives. The only exception was Alex, who continued to struggle. Why the dramatic shift? One contributing factor was clearly that a strong level of growth (see Figure 75) over our time was achieved, as described in the findings.

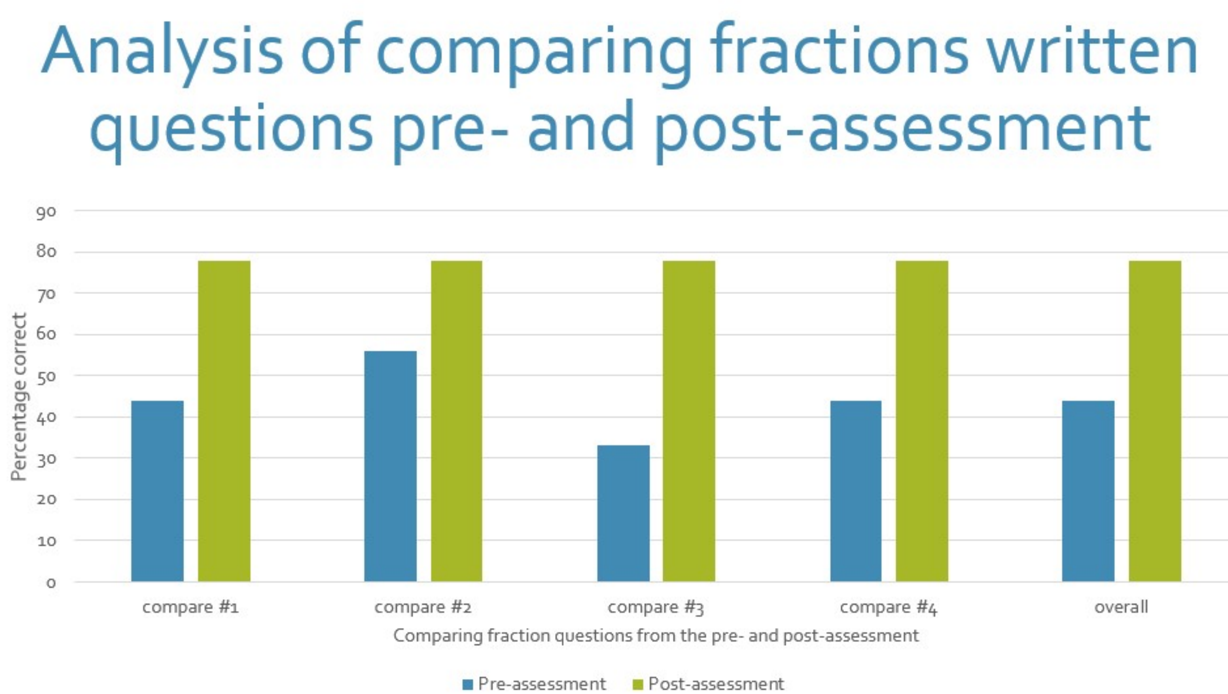


Figure 75: Comparing fraction question results compared from pre- and post-assessment.

I believe there are a few reasons for this leap in understanding. One reason may have been that this task did not ask the students to develop a specific solution but rather to choose one of the solutions offered, unlike Day-four. Now, they no longer had to create a very specific visual image in their mind but rather to spatially reason their way to the solution: “These pieces [referring to the quarter fraction] are bigger so they fill up the space more.”

Also, the students on the previous days had seemed to develop a solid understanding for the size of the different fraction pieces, and that when you cut a circle into more pieces that each individual piece will end up being smaller. This was evident in the first question offered at the beginning of Day Five: $\frac{2}{4}$ compared with $\frac{2}{6}$, which is bigger? The response was “Two-fourth” as a choral response by the class. When asked to defend their choice, a student responded with, “Well, the more fractions there is, the smaller it has to be” When asked to clarify what he meant by fractions, he explained that he was referring to pieces.

From the beginning, all the students seemed to make connections to the more you cut the smaller the pieces:

- “Twelve is like the highest number but like, but if you cut up—if you get a sandwich cut it up twelve times you have the smallest pieces of sandwiches.”
- “If you fold a blanket six times you still get quite a bit of blanket, or if you cut a sandwich six times you still get quite a bit of sandwich not like if you had fourteen.”

We also see here a natural connection with students using past experiences to describe their thinking or past concepts to new concepts as discussed earlier on with the idea of *conceptual blending* (Fauconnier & Turner, 2008). Even during the Slice it[15] activity (see Figure 76), Melinda and Aven made each shape into a known object that they were cutting, like cheese or a sandwich. This was again used by other students in offering oral descriptors for their fraction choices. The concepts of both cutting and folding were referred to. Once again, this connects back to the importance of our experiences. Not once were these students asked to make such comparisons to objects, yet multiple students did. These students had previously grown images for cutting, folding, and breaking into pieces. They naturally connected their current experience with those of their previously grown images—in a process known as *conceptual blending* (Fauconnier & Turner, 2008).

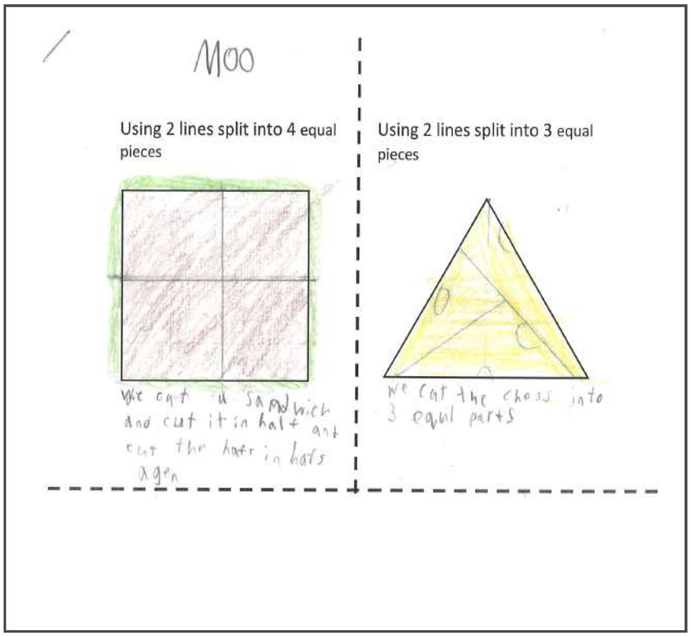


Figure 76: Melinda and Aven's written work from the Slice it activity

In summary, as we look at how each student engaged with spatial experiences, their spatial tendency and their self-belief within a safe social environment were important. It was also important that the teacher learning to maintain flow. However, as seen in the data, there was little growth until some type of perceptual experience was introduced, which set learners on a

[15] An activity modelled after the mobile game called Slice it, where you are given a geometric shape and asked to create equal parts by drawing a given number of lines. For example, given a square draw 4 lines to create 9 equal parts.

trajectory towards Image Making. It is important for educators to pay attention to when and how they are incorporating a perceptual experience into the development of a topic, for a well-designed experience opens connections to other experiences that students can then draw on to deepen and expand their understanding. It is important to create an environment where manipulatives, building, and drawing are not something for early elementary students only or only for students who struggle; they are a necessary tool for every student to deepen their understanding. It is also important to not remove these tools too quickly with the first signs of Image Having, as learners may just be in the transition phase—Image Making Emergent Image Having—and need these perceptual experiences to solidify their transition. The integration of perceptual experiences alongside signitive experiences within the classroom seems to be an important shift for educators.

Another unexpected aspect of this study was the inconsistency that I found in students' psychoeducational profiles. The gaps in their profiles, the diagnosis of an MLD, and a disconnect between working memory and the experiences of the students during the tasks highlighted the limiting nature of these assessments. This may be due in part to the specific time frame of only a few hours within which the student assessed, the impact of their mental states during that brief time with the psychologist, and the limitations of the specific tasks being offered to the students. As an overall takeaway, it is important to recognize that these are just snapshots of student ability and do not define the student as a whole; nor do they offer a solid prediction into future growth or stagnation. They do offer a window into areas of strength which need to be pursued and developed further and offer to the educator areas of possible weakness. I would definitely want to emphasize the word 'possible,' as there are many reasons that have nothing to with the task that a student might perform poorly. This has been clearly demonstrated even in this study's

Day-four task looking at the impact of confidence, interaction with the educator, peers, and tendency towards building. A gifted student who spends their six hours a day at school feeling confident will likely approach a psychoeducational assessment with a very different mindset than one who spends their six hours a day at school feeling slow and unsuccessful. These assessments are a snapshot in time within a particular context.

Teaching is so multifaceted. As an educator of LD students, and I would argue of ‘typical’ students as well, I know that teachers strive to scaffold their lessons to create Image Making experiences, but they must also be aware of a multitude of other factors as well—of not pre-judging but maintaining belief in the students’ ability to do challenging work while understanding their areas of struggle with memorization and lack of confidence after years of struggle of maintaining flow within the lesson, and of being observant and present to the individual. We need to consider ways in which we can support teachers, as all these factors are important but require much skill in order to hold in balance.

Chapter Nine: Conclusion

This study is an offering of a brief one-week experience with students who have been labelled as learning disabled and looked at the impact that a more spatial approach to the topic of fractions could have on them. A much longer study would be interesting to do with these students to see what level of understanding is retained, as their struggle is often memory. What is the impact of this more physical approach to long term retention? How effective is our current labelling of MLD? Are LD students' apparent issues with working memory really a factor with mental calculation, or is their working memory just bogged down due to a lack of a visual structure of number? All of these factors appear to have impacted student growth, yet, the main essential towards student growth of mathematical images seemed to be the offering of perceptual experiences. In this study, without this factor students seemed to struggle to even begin to engage with the questions.

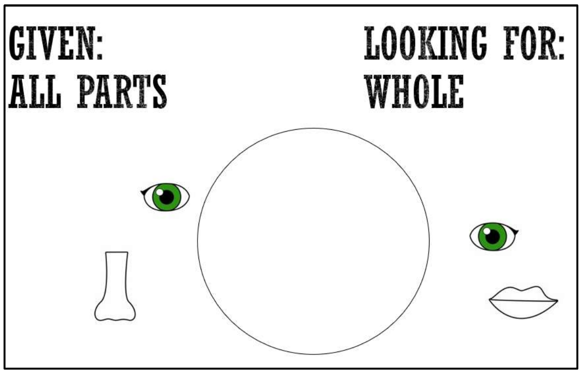
The act of teaching requires a lot of skill and comfort with complex situations. While in the classroom, there is a continual stream of tangled experiences, both present and past and from both student and teacher, that must be held in balance. The teacher must wade through all this murkiness with some preplanned ideas and others spontaneous. Spatial experiences have much to offer teachers as a way through. They have a unique offering, if done thoughtfully, in that they allow for rich, clarifying experiences that provide an easy entry point into a topic that can help to balance out the varying levels of Primitive Knowing (Pirie & Kieren, 1994).

But what does a thoughtful approach look like? As fractions was the main topic discussed in this paper, let us explore this idea in relation to fractions. In order to design a task for fractions thoughtfully, we would first need to look at the concepts surrounding the topic. Research has identified at least four basic constructive mechanisms for rational number knowledge-building:

whole number schemes, partitioning schemes; measuring schemes; and equivalencing schemes (Kieren, 1993). In designing the activities in this study, we paid attention to what actions connected with these schemes. The first two tasks were related to the idea of partitioning. We started with partitioning as it is thought to be central to the development of fractional number knowledge (Kieren, 1993). The students' action that they had to reason their way through, on day one, was to slice the shapes equally. On day two, they were given only the hexagon shape and asked to fold and cut them evenly. These first two days were focused on a partitioning action as a perceptual experience. Day three, they then used the partitioned circle pieces to look at different ways they would build a whole and then they would write their build in symbolic form. The action here was to lay pieces on top of a whole circle to recreate the complete circle but using multiple pieces, this engages the students with measuring schemes and equivalence schemes by performing the actions of laying over top and arranging the pieces to create a whole. This task also included the added action of writing in symbolic form. On day four, the task was to look at the signitive offering and arranging the pieces to match. The students were also asked to use the action of visualizing. The reasoning action on both the third and fourth day was arranging the appropriate pieces, then on only the fourth day, visualizing. Finally, on day five, they could use their partitioned fraction circles to arrange and then compare the fractions. The reasoning action that was mainly used by the students for this day's task transitioned to visualizing. The actions embedded in these tasks were not arbitrary actions but require reasoning that was connected to the mathematical concept that we wanted them to focus on. During each days' task, there was a perceptual offering that was available, even on day five for those that needed it as an entry point into the mathematical concept if they had not yet grown their own images. These entry points are a key aspect that must always be a point of consideration for the classroom teacher when designing a task.

For example, a student may enter the classroom with anxiety and stress due to their lack of Image Having for number, lack of confidence, or fixed mindset. Let us say that this student's teacher is embarking on the topic of subtraction, often a point of anxiety for students. A more traditional approach to the mathematics classroom in Canada would have been the, "I do, we do, you do" approach that is mainly symbol based. Some students in the class would have images for these symbols, others do not. Some students would be able to subtract by chunking and decomposing numbers, while others would need to use their fingers to count by ones backwards, resulting in a large margin of error. Although, this is a perceptual experience it is not a very productive one for those who already have one-to-one correspondence. The action of counting by ones is not an action that gives entry into the bigger concept of grouping within mathematics, if the child uses their fingers to count on, or consider different ways of chunking numbers such as 5 and 3 is 8, this would be a productive perceptual offering. It is important to consider: What conceptual experience is being addressed within the perceptual experience? And in what ways does it offer entry points into deeper conceptual ideas? Based on this study's findings, symbols on a page for students who have not grown images for the mathematical ideas will not offer opportunities of growth for those students.

Imagine that instead of a worksheet, the teacher offers images of the distinction between addition and subtraction (see Figure 77) that are not symbol based. Then as an activity, the teacher offers the students Cuisenaire rods, asking them to build a comparison and find the



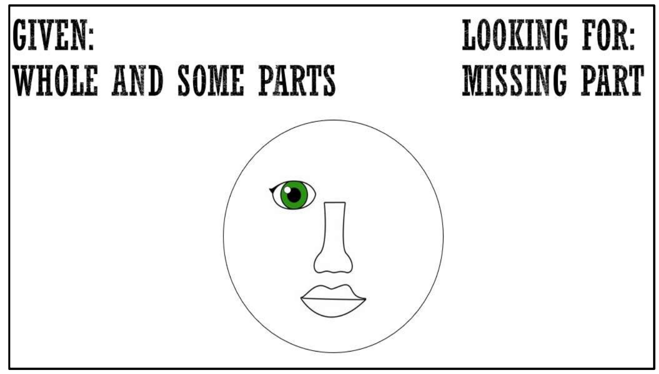


Figure 77: Visual describing addition as collection of parts looking for the whole (left) and visual describing subtraction as given the whole and missing a part.

missing part (see Figure 78). This activity does not assume that students are at the Image Having phase. It offers a point of engagement for students at varying levels. All students then can engage with the concept of subtraction as difference or finding the missing part. Those who are not yet at the point of Image Having for their numbers can both engage in the activity as well as use it as another opportunity for growth in this area. Without these visual and spatial offerings, a number of students would be at a standstill.

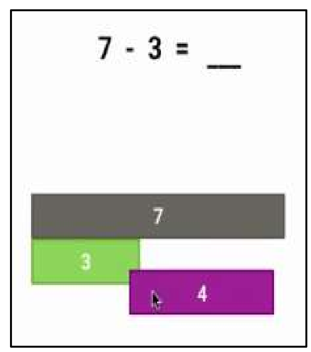


Figure 78: Cuisenaire rods being used to model subtraction as a missing part for the question 7 - 3.

Based on the findings of this study, it would seem important for teachers to pay attention to these ideas of different types of offerings and recognize the empty offering for the mathematical ideas contained in a purely signitive representation. The meaning for these symbols is only based on what the student already knows and can bring to the symbol themselves. As a result, with the varying levels of Primitive Knowing in the classroom, a certain portion of the population may be excluded from both entry into the topic and an opportunity for growth. As stated multiple times, there is much variety in the classroom this variety often includes dyslexic students. If teachers viewed themselves as cultivators of images within the classroom, I believe it would impact their practices and offer an opportunity for growth to all students.

The other aspect of this study I feel is an important take-away is in regard to the LD community of students. The need for more visual-spatial experiences, I believe, is productive for all, but especially for this population, as they do not have the fallback of memorizing that other students may be able to use as an entry point into a topic. It is important for teachers to understand that memory is not synonymous with intelligence or understanding, but it can allow a student to continue to engage with a topic, which may provide an opportunity for growth. LD students often do not have this option, and they can find themselves at a standstill without an entry point into the topic if it is presented only by way of memorized facts or procedure. Further research is needed in this area to help determine what other barriers exist for this population of student and others within the classroom.

In this study, I did not shy away from a challenging topic in regards to this community of students labelled with an LD. We tackled the much-feared topic of fractions and, in the week before Christmas break, went beyond that grade's curriculum into adding (Grade 7) and comparing fractions (Grade Five) with different denominators. They started out with a very limited level of Primitive Knowing in the subject of fractions, and at the beginning many commented on how they "don't get fractions." Yet, by the end of a week, they were all able to demonstrate adding fractions with different denominators concretely and some to the point of Image Having. All students, with the exception of one, were able to easily compare three fractions with different denominators. High expectations matched with a visual-spatial approach were shown to be very productive. It is important for teachers to understand what aspects of learning are grounded in memorization and how then to reduce that component and instead engage students' senses.

A student's label of disability with learning can become a reason for accepting a lack of progress in a topic. As teachers we should be placing high expectations on these students' conceptual understanding. It is the memorization piece that we need to have patience with and put things in place that offer students a visual-spatial tool for support, such as a visual representation for their times tables (see Figure 79) or manipulatives to support basic facts. When offering students a tool that addresses both support for daily use as well as further growth in understanding of the concept, moving towards automaticity is important. All too often, there is an acceptance of lack of ability, which is understandable as we have labelled these students as such—disabled. However, these students have lots of ability which is curtailed by multiple barriers; one that often goes unrecognized is memorization, and as educators we need to pay attention to how often it is that we expect mimicry within the classroom.

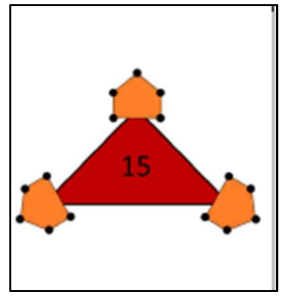


Figure 79: An example of the 3 x 5 as a visual representation. Three represented as a red triangle and five as an orange pentagon.

These ideas around memorization and dyslexia need to be studied further by researchers, as dyslexics make up about 80 percent of the LD community (Shaywitz S. , 2017). Many 'typical' students who may not have a strong conceptual understanding are able to continue to engage at some level, as those who are able to memorize their basic facts are not curtailed in the same way, at least not intially. However, all students would benefit from a more conceptual approach, which is beneficial to all if done through perceptual experiences. Mathematics is a very conceptual subject centered around pattern, logical thought, and creative critical thinking, not memorization.

Another area to be considered in response to this study is the treatment of psychoeducational assessments within schools and how they are engaged with by educators. For unknown reasons, many of the student profiles I received were incomplete. There is a standard battery of tests that psychologists give to a student they are assessing for the possibility of a LD, yet the teachers in this study did not seem to receive the full report. This is surprising at a school specifically dedicated to LD students. The reasons for this are unclear. It would be interesting to investigate how common this is and what level of focus is given to these assessments by teachers and administrators. How do teachers view these assessments and how much do they impact their engagement with the student?

In regard to the psychoeducational assessment, another aspect within the study was what seemed like an inconsistency of the MLD label and those students' subsequent performance within the study. Further research and/or education for psychologists surrounding this label seems necessary. It may be productive, based on the findings of this study, to look into how teachers and psychologists might work more closely in the assessment of a student to come to a more holistic picture of ability and struggle. A combination of those few hours of testing with a psychologist and then possibly a time of observation in the classroom with a trained eye as to the type of struggle, might be very enlightening. If MLD is defined as students having specific difficulties with quantitative information, is there an issue with the concept of quantity, or is there just a general lack of images for number? In this study, I found the latter to be true. It was not that the students in my study were not capable, but rather that they had not been engaged visually. If they were engaged more visually, their retention may have been much better. It is not that they could not understand quantity, but they may just have not developed images for these quantities.

Are these inconsistencies in assessment and classroom the reason for what seemed like a lack of engagement by teachers with the psychoeducational assessments? Is there a belief by teachers that they are generally not accurate? In my experience they are an important piece of the puzzle, they offer us possible areas of struggle and strength. If a student is labelled as twice exceptional, yet performs poorly in school, does the teacher disregard areas of strength and only focus on the weakness? Many LD students are twice exceptional, yet how often do schools and teachers engage with their areas of strength? Are we still mainly deficit focused?

Another aspect of the psychoeducational assessment that would seem in further need of investigating is working memory. When a student is assessed as having low working memory, how does that play out in the classroom? What other factors might be contributing to their struggles? And how do teachers interpret this weakness? Are these students automatically given calculators as it is assumed they are unable to perform mental calculations? Or does the student simply have inefficient or incorrect images for the mathematical idea? If so, then their struggle lies in the approach of counting by ones or trying to remember procedures and executing those in their mind rather than using a more efficient strategy. Is their working memory overloaded by their lack of memorization skill, thus requiring them to go through more steps? Whereas another student is able to just automatically recall? In this study, there seemed to be much variability between ability to mentally calculate and the students' working memory index.

One factor that may have helped the students in this study with working memory was the use of fractions with smaller numbers. These smaller numbers would have allowed for easily created visuals. For example, if we had used larger numbers, these students may not have had the images for these numbers readily available and then with each fraction introduced they may have struggled to create an image for the numbers. As a result, they would have had to spend valuable working memory space to interpret the numbers in the fraction in terms of an image, then build an image in their minds for the fraction and then combine the multiple fraction images. As we were using smaller numbers, these were quick to interpret and visualize, so combining them was the only major task within their working memory. More research is necessary to further tease out what is occurring here. How accurate are these working memory indexes when played out in the classroom? Is students' progress being restricted due to an assumption of lack of ability, rather than pushing that student for greater conceptual understanding to lighten the load of an overflowing working memory due to lack of automaticity?

This study also contributes to the Pirie-Kieren dynamical theory for the growth in mathematical understanding with the discussion of an intermediate phase between Image Making and Image Having—*Image Making Emergent Image Having*. We encountered students who seemed to not be fully in either phase but seemed to exist somewhere in between—able to produce a portion or the beginning stages of an image, but not yet confident enough in their images or not having complete enough images to be considered at the Image Having phase. It would be important for educators to not remove the manipulatives that are contributing to the students' growth while in this phase as they seem to be on the precipice of making a full transition into Image Having.

Further study would also be needed in the area of spatializing the mathematics classroom and its impact on both the LD community and a 'typical' classroom. How does the progression of growth play out? Is it similar to the findings of this study (see Figure 49)? Or is there a group of students able to find movement for their mathematics understanding with a purely signitive offering. If so, why is that? Which type of offering inspires more growth for those students—signitive or perceptual? It would also be interesting to look at a grouping of students who have aphantasia—a struggle or inability to see in the mind's eye (Marks, 1973)—and study how they engage with these different types of offerings. Are visual-spatial offerings more or less productive for them? Also, looking into a grouping of students who are strong in number concepts but weak in spatial reasoning would be interesting as this particular population was not involved in this study. It may be that in order to be strong in number concepts you must therefore also be strong spatially and as a result this population does not exist. I would suspect however, that within a population of strong memorizers, of which this study's population was not, it would be possible to find such a grouping of students. What type of offerings are then productive for them? Visual-spatial or not?

An aspect of this study that would also be worth exploring further is in the area of visualization in conjunction with the perceptual offering. Due to focus issues with the students, we set up the activity such that students were expected to try to visualize before being offered an opportunity to build. This emphasis on the practice of visualizing proved to be very interesting and I believe productive towards their growth in the imaginative. It would be interesting to take this idea and explore multiple activities focused around intentional visualizing and how engaging in this practice contributes to the students' growth.

As we look back through this document, we have journeyed through a lot of topics.

We discussed enactivism and how perception is not us passively receiving information but a form of action. Each person 'brings forth' a world of significance in connection to their environment (Maturana & Varela, 1987). We as educators can then thoughtfully put things into the environment in order to offer students possibilities for action—the Affordance theory (Gibson, 1979). This begs the question: what kinds of offerings contain the most meaning for a mathematical idea?

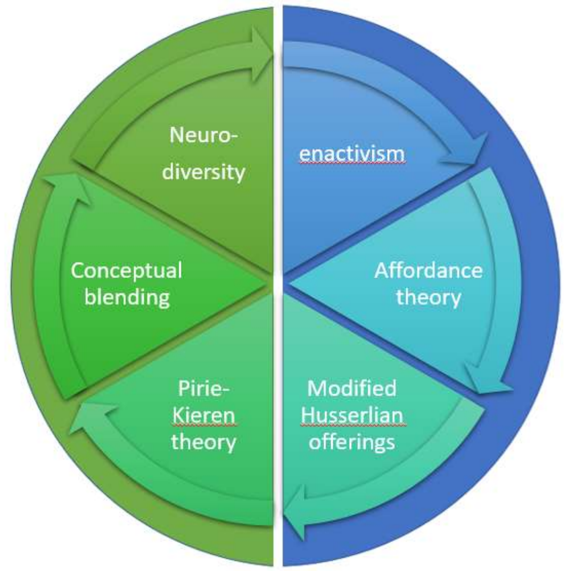


Figure 80: Interwoven topics

This triggered us to discuss a modified form of some Husserlian ideas—signitive, imaginative, and perceptual. We then began to transition a little bit more into the individual's realm of growth for personal images and spoke of the Pirie-Kieren Dynamical Theory, exploring the beginning stages of Image Making and Image Having (Pirie & Kieren, 1994). Once an individual transitions to Image Having, how are these images used? As our images interact with each other (visualizing) further growth can occur through conceptual blending (Fauconnier & Turner, 2008). Yet within this whole process there also exist our individual brain organizations and structures—neurodiversity. This continual cycle of perceiving, recalling, thickening, melding, and growing of images is an amazing resultant of our interaction with the world around us.

My attempt with this study was not to shy away from the complexity but embrace the journey and struggle along with it to see what might arise along the way in terms of lessons to be learned. With the findings that have bubbled to the surface, it is my hope now to do the work towards furthering these ideas within classrooms and our educational system as a whole. Already, there has been progress made toward making the classroom more spatial through the use of manipulatives, but there is much more yet to be done in regard to making this shift systemic and meaningful. Mathematics remains as one of those subjects which is socially acceptable to be not good at. Offering all students an entry point to growing mathematical images is essential. We are three dimensional creatures and we gather information through our senses.

In the area of science education, which Canada has a well-established reputation of achievement, there is a culture of experimentation and observation—three-dimensional experiences. Yet, all too often mathematics, remains bound to a two-dimensional world of symbols. It would be interesting to see the shift that occurs if mathematics teachers viewed themselves as cultivators of images. How would this change the pedagogy within the classroom? If our societal goal is for all students to have a deep conceptual understanding of mathematics, then we must spend time engaging our senses as we act out mathematical concepts. It is in this realm that we can offer students the most direct and rich experiences for the concepts, which will then contribute to their personal growth of mathematical images.

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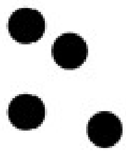
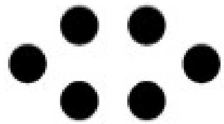
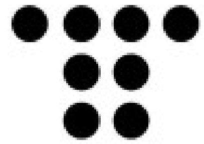
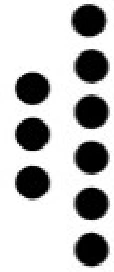
Appendix A – pre-assessment for Number

Circle how you see these dots combining when your figuring out the total?

1. a.  ______ b.  ______ c.  ______ d.  ______

2. How many 10's are in each number?
   a. 32 ________
   b. 78 ________
   c. 100 ________
   d. 132 ________
   e. 500 ________
   f. 741 ________
   g. 1020 ________

3. Using all the digits rearrange 8, 2, 1, and 5 to make:
   a. The **biggest** number

   b. The **smallest** number

   c. The **smallest odd** number

4. Estimate where these numbers go on the number line below:
   - 27
   - 82
   - 11
   - 45
   - 100
   - 135

Find the **missing number** (total goes in the center box) and **circle** whether it is an addition or subtraction question:

1.

3	2
5	7

+ / -

2.

9	2
8	

20

+ / -

3.

25	7
3	15

+ / -

4.

38	6
4	15

+ / -

5.

7	13
	24

50

+ / -

6.

17	
16	8

45

+ / -

9.

41	35
16	19

+ / -

10.

61	24
87	

203

+ / -

Draw something in the rectangle below that shows what **4 x 6** means:

1.

2. Write an equation for **18** using:

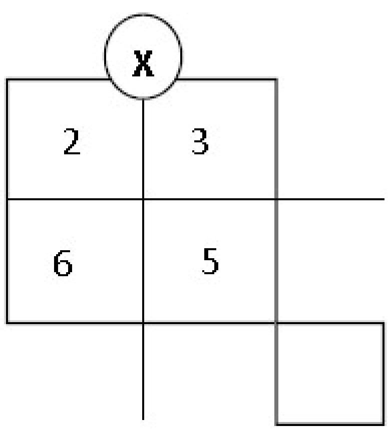
   a. Adding ____

   b. Multiplying ____

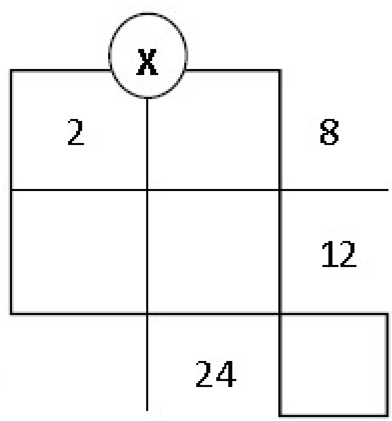
   c. Finding the difference ____________________________________

   d. Dividing ____

3.



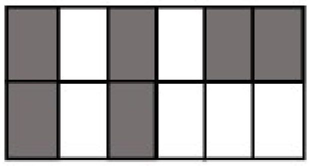
4.

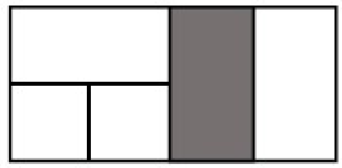


5. $438 \div 3 =$ ______________

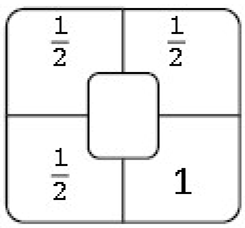
Appendix B – pre- and post- assessment for fractions

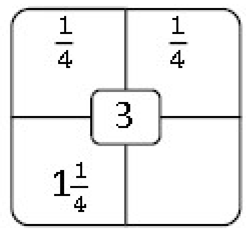
1. Write a fraction that represents the **shaded region** of this picture.

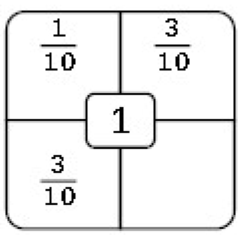


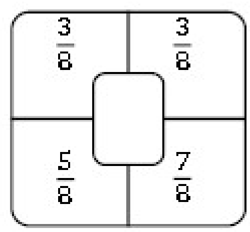


______________ ______________

2.


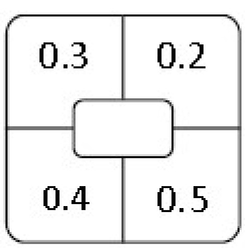
3.


4.


5.


6. Circle the **smallest** fraction: a) $\frac{9}{8}$ $\frac{9}{10}$ $\frac{9}{14}$ b) $\frac{4}{10}$ $\frac{3}{10}$ $\frac{1}{10}$

7. Circle the **biggest** fraction: a) $\frac{5}{10}$ $\frac{2}{11}$ $\frac{1}{15}$ b) $\frac{2}{6}$ $\frac{1}{8}$ $\frac{3}{5}$

8.


9.