THE UNIVERSITY OF CALGARY

Searching for Irving Fisher:

Unit-Roots, Trend Breaks, and the Fisher Hypothesis

by

Milanda Chan

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF ARTS

DEPARTMENT OF ECONOMICS

CALGARY, ALBERTA

NOVEMBER, 2000

0-612-64904-0

Abstract

The empirical validity of the long-run Fisher Effect is investigated using recent advances in time-series methodologies. Evidence is found in support of the hypothesis that the nominal interest rate and the rate of inflation in both Canada and the United States are trend-stationary series with two breaks. Tests for the existence of the Fisher Effect are conducted under a stationary VAR framework using detrended variables; and although findings appear to be consistent with partial long-run adjustment of the nominal interest rate to an innovation in the rate of inflation, they are not of the theoretical magnitude predicted by the Fisher Hypothesis.

Acknowledgements

A great deal of gratitude and thanks goes to Dr. Frank Atkins. Without his help and dogged persistence, I would not have been able to finish this thesis so relatively "quickly". The constant barrage of phone calls from "my conscience" encouraged me and kept me on track throughout this onerous, and sometimes painful, process of thesis writing. His guidance and willingness to help meant and still means a lot to me. Once again Frank, THANK YOU!!!

A special word of thanks must also be given to my fellow class of graduate students. Thank you all for your love, support, and sex talks during the numerous periods of suffering we all went through together during our time here in the M.A. program. Without those fun, drunken Friday nights at the Den, I don't know how else I would have made it through till the end.

One final thank you goes out to the people at the Economics department. Thank you every one for all your help – especially for those times when I would lock myself out of my office and the kind, kind ladies of the general office would lend me a key. Thank you, thank you, thank you! Also, I would really like to express my deepest of thanks to Dr. Bruce, Dr. Mansell, and Dr. McRae. It was the three of you combined who managed to convince me to enter grad school. If not for you three, I would not be at this stage today. Many thanks!!!

Dedication

This is for all those special people in my life whom I hold dear to my heart. I won't name names, because if you happen to be one of those who think they're in the category of "special people" but I fail to mention your name, you'd get mad at me and resent me for not doing so, and I don't want that so I'll just let you think that all of you are special – which you are – and leave it at that. Anyways, without all of your love and support and patience for putting up with me during my times of utter frustration, I would not be at this bliss point of utility! I love you lots, and remember that I'll always be there for you too!!!

Table of Contents

List of Tables

List of Figures

Chapter 1

INTRODUCTION

Underlying Classical macroeconomic theory is the key proposition that permanent movements in nominal variables have no effect on real economic variables in the long run. In particular, the classical hypothesis that a fully perceived change in the rate of inflation has no impact on real interest rates – the Fisher Effect – has been one of the most strongly held propositions in economics.

Attributed to Irving Fisher (1930), this seemingly simple and intuitive hypothesis of one of the most basic equilibrium relationships has been generally accepted in theory, yet continues to receive considerable attention empirically. This vast volume of research has been keenly pursued mainly due to the fact that a stable one-for-one relationship between nominal interest rates and inflation has found limited empirical support. Typically, the estimated slope coefficients in regressions of nominal interest rates on inflation are substantially less than the hypothesized value of one.

Recent econometric developments in the use of time series methodologies, due to King & Watson (1997), Fisher & Seater (1993), Johansen (1988), and Engle & Granger (1987), have contributed to the Fisher Effect literature by showing that meaningful neutrality tests of inflation on real interest rates can only be constructed if both series satisfy certain non-stationary conditions. This suggests that much of the older empirical literature on the Fisher hypothesis violates these requirements, and hence is not valid. Several recent empirical studies attempt to correct for these mis-specification problems in earlier literature by focusing on the (non)stationary properties of inflation and interest rate series, and then testing the Fisher Effect as a long-run equilibrium relationship through the application of co-integration techniques. Examples of such research in the literature include Koustas & Serletis (1999), Crowder (1997), Daniels et al (1996), Crowder & Hoffman (1996), Mishkin (1992), Atkins (1989), and MacDonald & Murphy (1989).

The use of co-integration techniques in the Fisher Effect literature is not surprising since many empirical studies present evidence showing that the inflation rate and the nominal interest rate each contain a unit-root – a necessary condition in co-integration theory. Given the assumed non-stationary properties in both the nominal interest rate and the inflation rate, the Fisher Effect appears to be an ideal candidate for the application of co-integration techniques in order to establish that the real interest rate is a stationary series, thereby providing empirical support for the long-run Fisher neutrality proposition.

Underlying the statistical notion of co-integration is the concept of non-stationarity. The vast majority of unit-root tests that are employed to detect this property within the literature are primarily Dickey-Fuller type tests. These tests, classified as Wald tests, are implemented under the alternative hypothesis that a variable is stationary with drift, or stationary around a deterministic trend. However, a potential difficulty in evaluating the time series properties of a variable arises with the existence of structural breaks. Perron (1989) suggested that standard stationarity tests are biased towards finding a unit-root since they fail to appropriately account for structural breaks in the data. Instead, these tests misinterpret structural breaks as permanent stochastic disturbances, which give rise to random walks. In particular, Perron focused on the Great Depression (Crash) of 1929 and the Oil Price Shock of 1973, citing only those two events as having had a permanent effect on various macroeconomic variables. Based on this, Perron argued that if the observations corresponding to these events are removed from the data, then one could reject the unit-root hypothesis in favour of a trend-stationary hypothesis for most macroeconomic time series. Many researchers have taken issue with this approach, arguing that Perron's choice of breakpoints is based on prior observation of the data, and hence the problems associated with data mining are encountered. Following the motivation of Perron's unit-root null against a one- exogenous-break-alternative testing procedure, subsequent literature, including Zivot & Andrews (1992) and Perron (1997), developed variations of Perron's test in which the break point is endogenously estimated, thus transforming Perron's unit-root test that is conditional on structural change at a pre-determined point in time into an unconditional unit-root test. This methodology was applied to the investigations of the Fisher Effect by Malliaropulos (2000), who found that the trend stationary with one break hypothesis was consistent with the inflation and nominal interest rates in the United States over the period 1960 - 1995.

Considering the focus which testing the unit-root null against a one-break alternative has received in the break literature, a natural extension of endogenous break methodology is to expand towards allowing for a two-break alternative. This is the focus taken in papers by Lumsdaine & Papell (1997) using the Nelson-Plosser data, and Arestis & Mariscal (1999) using OECD unemployment rates.

In this thesis, my objective is to extend the empirical literature on the Fisher hypothesis by investigating the time series properties of the inflation and nominal interest rates for Canada and the United States allowing for two structural breaks of unknown timing in the series. Although Malliaropulos' study found evidence which suggests that inflation and nominal interest rates in the United States are trend-stationary with one structural break in both the intercept and slope of the trend function in the early 1980's, it is far from obvious that only one break is a good characterization of the two Fisher series. Upon examination of plottings of inflation and nominal interest rates for both Canada and the United States from 1954 to 2000, it is conceivable, without any statistical examination, that there are at least two breaks within each series.

The importance of correctly determining the stationarity properties of inflation and nominal interest rates is essential for choosing the appropriate test strategy with which to test the Fisher Effect. Suppose that we were to assume that nominal interest rates and inflation are trend-stationary with a common structural break in their deterministic components. Standard stationarity tests would misinterpret this structural break and conclude that the two series are non-stationary with a unit-root. Proceeding with a test of the Fisher Effect based on a co-integration regression would imply, in this situation, that the inflation rate and nominal interest rate are co-integrated and share a common stochastic trend. This finding of co-integration, however, is found not because there is an underlying long-run relationship between the stochastic components of the series, but rather it is due to the fact that both series share a common deterministic shock.

If it is true that employing co-integration methodology to the testing of the Fisher relation is inaccurate in the presence of trend-stationary with structural break(s) inflation and interest rate series, then an appropriate procedure based in a stationary framework is needed. One of the most straightforward procedures to investigate the dynamic relation between the nominal interest rate and inflation is to estimate a Vector Auto Regression (VAR) in the filtered (detrended) levels of the variables. From the VAR, we can obtain the impulse response pattern of nominal interest rates and inflation, which in turn, will go into the derivation of a dynamic correlation coefficient. This correlation coefficient will act as a "diagnostic" of the Fisher Effect's dynamic relationship at various horizons.

In the following chapter, we provide a brief review of the pertinent Fisher literature, surveying previous empirical work that has been conducted in the examination of the Fisher hypothesis. Chapter 3 outlines and describes the econometric methodology we employ, in addition to presenting the results from these tests, while in Chapter 4, policy implications and concluding remarks are discussed.

Chapter 2

LITERATURE REVIEW

This chapter discusses previous efforts other researchers have taken in their examination of the Fisher Effect. Starting with Atkins (1989) and extending to the latest study by Malliaropulos (2000), this chapter contains a very selective review of some of the more recent empirical work that has been done with the Fisher Effect. The particular areas of coverage here include some relatively earlier studies which examined the stationarity properties of the data, co-integration tests of inflation and nominal interest rates, up to the most recent endogenous-break-unit-root testing methodologies and Vector Auto Regression (VAR) modeling procedures.

2.1 Non-stationarity and Co-integration

Atkins (1989) was one of the first to suggest the reason why early empirical evidence of the Fisher Effect is so weak was that the issue may be one of improper econometric methodology. Conducting a series of bivariate ordinary least squares regressions on United States 90 day Treasury Bill rate, a marginal tax rate on interest income, and inflation data spanning the period of 1953 to 1971, Atkins observes regression results which, although do not reject the Fisher Effect, yield low Durbin-Watson and large Box-Pierce Q statistics. These test statistic magnitudes, respectively, point to possible model mis-specification and rejection of white-noise residuals, thereby identifying these regressions as likely candidates for the class of spurious regressions having no classical econometric properties.

Atkins then repeats the above exercise with all variables expressed in first-difference form. The results obtained from this set of regressions resoundly reject the Fisher Effect; the estimated coefficients on inflation are found to not ever be significantly different from 0. Based on these outcomes, Atkins concludes that nominal interest rates and inflation are possible integrated series, and could therefore likely form a co-integrated relationship.

Demonstrating that each individual series is indeed a non-stationary I(1) process through the application of the augmented Dickey-Fuller test (ADF), Atkins subsequently tests for co-integration and finds that for both the United States and Australia, the null hypothesis of no co-integration is rejected, regardless of whichever variable is chosen as the dependent variable. Thus lending support to Atkin's argument that much of the early empirical work examining the existence of the Fisher Effect represents a mis-specification, and that a joint error-correction representation of the Fisher hypothesis is the more appropriate approach.

Falling in line with Atkins' argument that past empirical work may be mis-specified is Mishkin (1991). Taking into consideration the seminal research of Nelson & Plosser (1982), who provided evidence that many macroeconomic variables are characterized by a univariate time series structure with a unit-root, and recognizing the potential for misleading spurious regressions when non-stationary variables are examined via standard econometric methodologies[1], Mishkin conducts a reexamination of the Fisher Effect in postwar United States data.

---

[1] See Granger & Newbold (1974) and Phillips (1986) for a more detailed review of this topic.

Using Monte Carlo simulation experiments as well as augmented Dickey-Fuller tests (ADF), Mishkin demonstrates that the level of inflation and interest rates contain stochastic trends (i.e., a unit-root). Based on these results, Mishkin proposes that the ability of short-term interest rates to forecast inflation is spurious; as well as explaining why the finding of inflation's ability to forecast short-term interest rates is not robust. Due to this, Mishkin suggests that there might be no short-run Fisher relationship. However, Mishkin does make it a point to note that the absence of a short-run Fisher Effect does not effectively rule out the possibility of a long-run Fisher Effect in which inflation and nominal interest rates trend together in the long run when they exhibit trends. Applying Engle & Granger's (1987) co-integration methodology, Mishkin is able to form a co-integrated system between inflation and nominal interest rates, thereby reconciling a stationary real interest rate[2], and essentially providing support for the existence of a long-run Fisher relationship.

Crowder & Hoffman (1996), like Mishkin (1991), recognize that the nominal interest rate and expected inflation rate can be modeled as distinct non-stationary series that share a common stochastic trend. In their examination of the Fisher Effect, Crowder & Hoffman use Johansen's (1988) maximum likelihood estimator (MLE) co-integration technique on United States inflation and three-month T-Bill rate quarterly data running from 1952:1 to 1991:4. The authors argue that prior work[3], which utilized a DOLS specification (a least squares projection of nominal interest rates on inflation in a model augmented with lead and lag changes in inflation) to estimate the long-run equilibrium

---

[2] It has been generally predicted within economic theory that the real interest rate follows a stationary process.

[3] See Evans & Lewis (1995). "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance* 50(1), pp. 225-53.

relationship between interest rates and inflation suffers from small sample bias, and effectively has important implications in testing the validity of the Fisher hypothesis; those implications primarily being the observation of less-than-unity expected inflation coefficients. Through the maximum likelihood approach that they take, Crowder & Hoffman yield estimates that are consistent with evidence of an underlying long-run equilibrium Fisher relationship.

Having found evidence of a co-integrating relationship that is consistent with the theoretical implications of the Fisher hypothesis, Crowder & Hoffman then proceed with an examination of the vector error correction model (VECM) which reveals a "causal" ordering that suggests that inflation is strongly exogenous and Granger-causes interest rates. Further analysis of the dynamic response pattern of nominal interest rates and inflation provided additional support for the previously mentioned finding of Granger causality – innovations to the rate of inflation are exclusively responsible for all of the long-run variance in both inflation and interest rates.

Koustas & Serletis (1999) continue the volume of non-stationarity and co-integration empirical work done on the Fisher Effect with their application of King & Watson's (1997) nonstructural bivariate autoregressive methodology. Using post-war quarterly data for eleven countries – Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Japan, the Netherlands, the United Kingdom, and the United States – Koustas & Serletis find evidence which generally rejects the Fisher Effect.

In contrast to the methodologies proposed by Engle & Granger (1987) and Johansen (1988), the King & Watson (1997) methodology claims that meaningful Fisher Effect tests are possible if the inflation and interest rate series are non-stationary (i.e., contain a unit-root, or are integrated of order one) and do not co-integrate. All three methodologies call for an initial investigation into the integration properties of the variables, and Koustas & Serletis do this by testing for unit-roots in the inflation rate and interest rate series using three alternative testing procedures. The Weighted Symmetric (WS) unit root test and the augmented Dickey-Fuller (ADF) test are both classified as Wald tests where the unit-root is the null hypothesis to be tested. For both these tests, the null hypothesis of a unit-root cannot be rejected in both series for all countries except for Denmark's inflation rate and Japan's interest rate. The third test procedure, owing to Kwaitkowski et al. (1992) and known as the KPSS test, takes the null hypothesis of stationarity against the alternative of a unit-root. Kwaitkowski et al. (1992) argue that this method is a more powerful test than those of the WS and the ADF tests. The results from this stationarity test combined with those of the two unit-root tests conclude that all the inflation rate and interest rate series, except for Denmark's inflation rate, contain a unit-root.

Having established the unit-root properties of the two Fisher series, Koustas & Serletis conduct co-integration tests using the methodology of Engle & Granger (1987) and find that the null hypothesis of no co-integration between the inflation rate and the interest rate cannot be rejected. These results then lead to King & Watson's (1997) procedure where inflation and interest rates are modeled as a bivariate vector-autoregressive model, and the focus centres on the dynamic effects of an inflation shock on the interest rate. The empirical evidence that Koustas & Serletis arrive at suggests that there is considerable evidence against the long-run Fisher Effect.

2.2 *Co-integration and Structural Breaks*

Initial consideration of structural breaks within Fisherian inflation and interest rate series centered upon the fact that most empirical studies had sample periods that encompassed an important regime change, namely the early 1970s collapse of the Bretton Woods system. MacDonald & Murphy (1989) take this event into consideration in their study when they examine the long-run Fisher hypothesis for the United States, Belgium, Canada, and the United Kingdom over the period from 1955 to 1986. Using standard Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests, the authors first conduct their analysis over the entire data period and find evidence showing that for all countries, the inflation rate and three-month treasury bill rate[4] are integrated of order one, and hence meet the necessary condition for possible co-integration. Proceeding with the co-integration technique of Engle & Granger (1987), MacDonald & Murphy find that the ADF statistics on the residuals from the co-integrating regression reject the null hypothesis of co-integration at any reasonable level of significance.

MacDonald & Murphy then go on to split their data sample in two in order to ascertain whether or not the fixed-floating exchange rate regime shift altered the inflation-interest rate relationship.[5] After re-estimating the relationships, it is clear that the results MacDonald & Murphy obtain indicate that the two sub-periods are significantly different. In the fixed exchange rate periods, the null of no co-integration can be rejected for the United States and Canada. However, in the floating rate period, there is no evidence of co-integration for any of the countries analyzed.

---

[4] The data frequency MacDonald & Murphy use is quarterly, running from 1955Q1 to 1986Q4 for all countries except Belgium, whose data set runs from 1957Q1 to 1986Q4 (due to constraints on data availability).

Much in the same manner as MacDonald & Murphy (1989) conducted their empirical study, Daniels et al (1996) also conduct a co-integration analysis of the long-run Fisher relation and later examine the sensitivity of their findings to possible structural breaks in the data. However, unlike MacDonald & Murphy (1989), Daniels et al consider the possibility of two events – the collapse of the Bretton Woods Agreement, as well as the 1979 change in the United States Federal Reserve's targetting procedures.

After initially identifying the unit-root properties of their data, Daniels et al apply Johansen's (1990) co-integration tests on the two I(1) series and are able to find evidence of a long-run equilibrium relationship between the United States rate of inflation and three-month Treasury bill rates over the period 1957Q1 to 1992Q4. The authors are also able to demonstrate, through the uniqueness of the co-integrating relation, that inflation Granger-causes the rate of interest.

In conducting their examination of the possible effects that structural breaks could impose upon the Fisher relation, Daniels et al include two dummy variables in their model in an attempt to capture the impact of the Bretton Woods collapse and the change in the Fed's monetary policy regime. The dummy variable for the Bretton Woods event was set to take on a value of 1 from 1971Q4 to 1973Q3, to account for the transition period from fixed exchange rates to floating rates. While the dummy variable for the change in targetting policy assumed a value of 1 from 1979Q4 to 1982Q3, representing the period during which the Federal Reserve primarily targetted various reserve aggregates instead of interest rates.

---

[5] For Belgium, Canada, and the United States the split was chosen as 1955Q1-1973Q1 / 1973Q2-1986Q4, while for the United Kingdom the split was 1955Q1-1972Q2 / 1972Q3-1986Q4.

The results obtained from this set of estimations were found to be virtually identical to those Daniels et al observed in their initial full period estimation. Inflation and the nominal rate of interest were found to be co-integrated. The estimated long-run coefficient on inflation was not significantly different from one. And inflation was found to Granger-cause interest rates.

Crowder (1997) addresses many of the same issues that MacDonald & Murphy (1989) and Daniels et al (1996) deal with. However, in examining the matter of structural breaks within the data, Crowder takes the view where the choice of break points is made endogenously, rather than being fixed a priori as in the aforementioned studies.

Conducting the analysis on a quarterly data sample consisting of the three-month commercial paper rate and the GDP implicit price deflator spanning from 1960Q1 to 1991Q4, Crowder finds that the ADF statistics for inflation and nominal interest rates cannot reject the null hypothesis of a unit-root. Recognizing that both the Canadian inflation rate and nominal interest have undergone changes in money policy regimes during the period of study, Crowder then applies Zivot & Andrews' (1992) unit-root test that makes allowances for these structural changes and determines the respective break point endogenously, if such exists. The results Crowder obtains with the Zivot-Andrews unit-root test do not reject the unit-root null, and therefore allow Crowder to proceed with co-integration analysis.

Using Johansen's (1988) test for co-integration, the trace test, Crowder concludes that the two series are co-integrated. However, Crowder is quick to point out that these results are obtained under the assumption that the long-run Fisher equilibrium is stable over the entire sample period, and that this may be an unrealistic assumption. To account for this, Crowder performs recursive likelihood ratio tests of the trace statistics and finds that, although the results imply that there has been only one co-integrating vector for the whole sample, at least two periods exist during which the Fisher Effect was not stable. The first break appears to have occurred in early 1971 and Crowder attributes this to the breakdown of the Bretton Woods system. The second break appears to occur in late 1981, early 1982, and is thought to be a consequence of the Bank of Canada's changing its monetary policy targetting procedure, or quite possibly be a spillover effect from an analogous change that occurred in the United States.

Assuming that these events represent exogenous changes in the economy, Crowder introduces dummy variables in the Vector Error Correction Model (VECM) to capture their effects. The first dummy variable assumes a value of 0 up until 1971Q2 when it takes on a value of 1 for the rest of the sample. The second dummy variable assumes a value of 0 up until 1982Q2 and a value of 1 thereafter. Re-estimating the model with the regime shift dummies results in an outcome that is supportive of a stable real interest rate, with estimates that exhibit considerable overlap with those that do not include regime shift dummies. As was done with the first set of estimates, Crowder performs recursive estimates of the trace statistics and finds no evidence of significant instability in the Fisher relationship with the inclusion of regime shift dummies in the estimation. Having established that the co-integration vector is consistent with the theoretical Fisher long-run equilibrium relationship, Crowder examines the response of the two Fisher variables to discernible shocks to the system in order to gain more insight into the relationship between inflation and nominal interest rates. Looking at the cumulative impulse response functions of each series, Crowder finds that innovations to the nominal interest rate are what accumulate to form the common stochastic trend between interest rates and inflation, thus implying that nominal interest rates help to predict inflation rates, and not vice versa.

In the studies discussed above, findings of co-integration between inflation and interest rates were mixed. MacDonald & Murphy (1989), after splitting their data set in two in order to account for the shift in regime associated with the breakdown of the Bretton Woods Agreement, found evidence in support of co-integration during the period of fixed exchange rates for Canada and the United States, but no co-integration for the period of floating rates. Daniels et al (1996) and Crowder (1997), on the other hand, introduce dummy variables into their estimations in order to represent regime changes. The findings from these two studies contradict the results of MacDonald & Murphy (1989) in that even with regime changes accounted for, a co-integrating relationship between inflation and interest rates is found.

Recognizing the important implications that structural breaks in inflation and interest rates may have for tests of the Fisher Effect, Malliaropulos (2000) conducts a re-examination of the empirical evidence of the Fisher hypothesis in United States 1960-1995 quarterly data. Malliaropulos (2000) questions the results arrived at by Daniels et al (1996) and Crowder (1997) by noting that "if a broken linear trend is common in both series, co-integration analysis may lead to the wrong inference of a common stochastic trend, although the series are trend-stationary".

Applying Zivot & Andrews' (1992) sequential ADF test that accounts for structural breaks within the data with endogenous timing, Malliaropulos assesses the unit-root properties of inflation and interest rates and finds that the two series are trend-stationary (i.e., do not possess a unit-root) with common structural breaks. The break points estimated by Malliaropulos occur at 1980Q3 for the nominal interest rate and 1981Q3 for inflation. This finding contradicts the unit-root findings that Crowder (1997) obtains from his application of the Zivot-Andrews unit-root test.

Given the existence of a common structural break in the two series, Malliaropulos notes that testing for co-integration would erroneously not reject the existence of a long-run relationship. A more appropriate test of the Fisher relationship that Malliaropulos proposes is based on a Vector Auto Regressive (VAR) model representation that is carried out in a stationary framework. Malliaropulos detrends the two variables first by filtering out the structural break from the series, rendering them zero-mean stationary, and then uses these filtered series in the VAR analysis. To test the validity of the Fisher Effect, Malliaropulos computes dynamic correlation coefficients between inflation and nominal interest rates in order to examine the response of interest rates to an orthogonal shock in inflation relative to the response in inflation to its own contemporaneous shocks. If the Fisher Effect holds at every point in time, then the dynamic correlation coefficient should equal unity over the entire period; if the Fisher Effect holds only in the long-run, then the correlation coefficient should converge to unity over longer time horizons. The correlation results Malliaropulos obtains are not significantly different from unity for periods longer than five quarters after the initial shock, leading the author to conclude that the Fisher Effect holds in the medium- to long-run.

Chapter 3

ECONOMETRIC METHODOLOGY & ESTIMATION RESULTS

This chapter describes the methodology and results that form the basis of this thesis. The main emphasis will be on the development of appropriate tests to uncover the stationarity properties of the Fisher variables – rate of inflation and nominal interest rate – which to that end, will lead to an examination of the existence of the Fisher Effect. The chapter is comprised of five sections. Section 3.1 gives a brief description of the data used in the study. Section 3.2 reviews the theory and statistical procedures used to test for a unit-root allowing for one break. The third section presents an extension of the endogenous one-break model to the case of two breaks. Section 4 implements the empirical results from the above mentioned unit-root tests and outlines a procedure to remove the structural breaks from the series, while the final section presents the results from tests of the Fisher Effect.

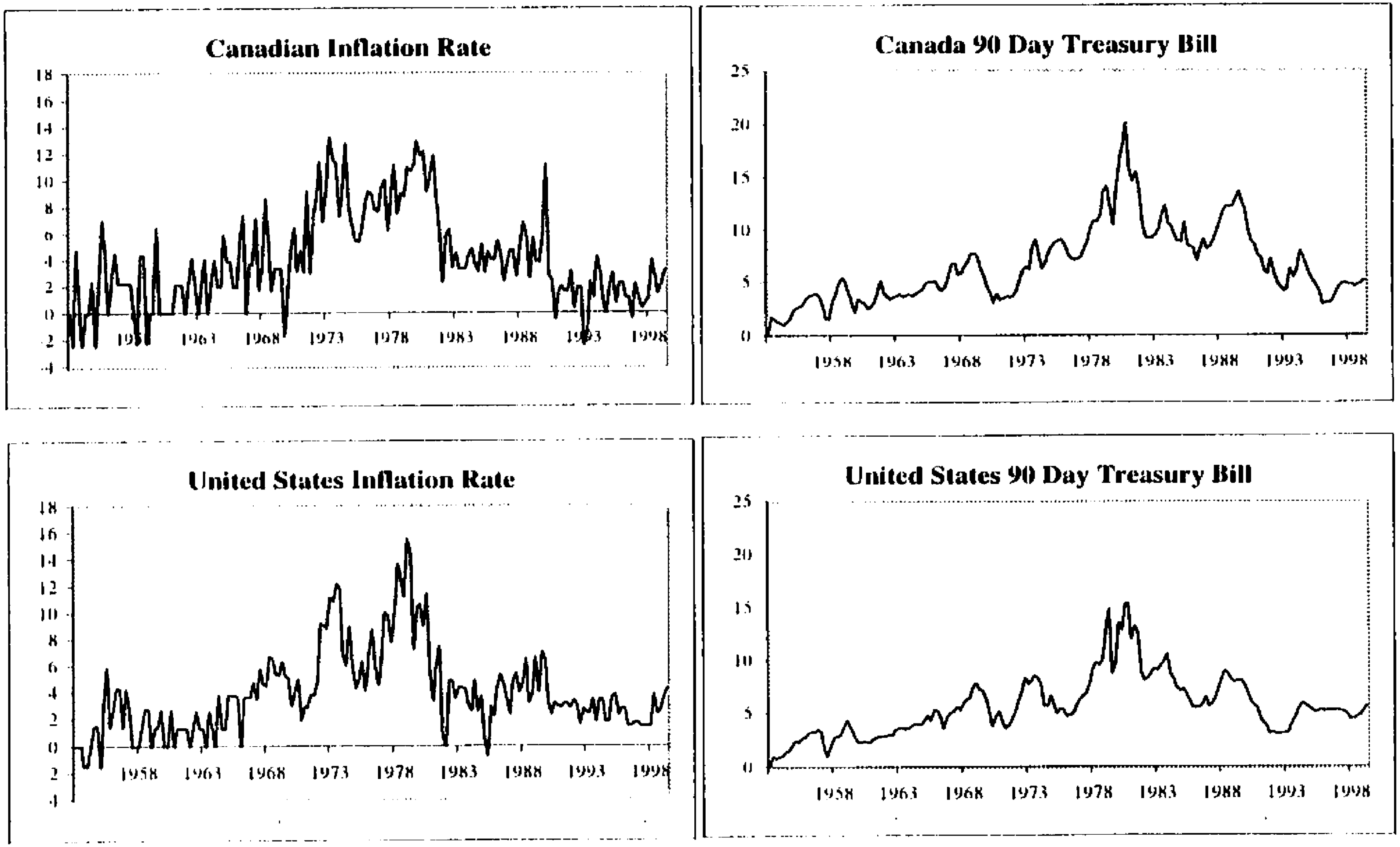
3.1 The Data and Their Unit-Root Properties Under Standard Alternative Hypotheses

In this study we investigate the period 1954Q1 to 2000Q1 using quarterly data for both Canada and the United States. Following common practice, the yield on the 90-day Treasury Bill for both countries is used as the nominal interest rate (R), and the Consumer Price Index (CPI) is used to construct the rate of inflation (π). Plots of the series appear in Figure 1. Note that for the purposes of this study, inflation is calculated as

$$\pi_t = \left(\ln P_{t+1} - \ln P_t\right) * 400 \qquad (1)$$

This represents inflation as the one-quarter ahead change in the log of the CPI,

FIGURE 1



Canadian Inflation Rate
Canada 90 Day Treasury Bill
United States Inflation Rate
United States 90 Day Treasury Bill
1958
1963
1968
1973
1978
1983
1988
1993
1998

annualized by multiplying by 400. It is also assumed that inflation expectations are formed in a "rational" manner, whereby individuals are assumed to have perfect foresight and are able to predict the rate of inflation correctly.

As a precursor to our later examination of the structural break properties of the inflation and interest rate series, we use standard augmented Dickey-Fuller (ADF) tests and the modified ADF tests due to Elliot, Rothenberg and Stock (1996) and Elliot (1999) – for the remainder of this section, these latter tests will be collectively referred to as ERS tests – to identify the unit-root properties of π and R without allowances for breaks in the trend function. These results will provide a standard against which we will be able to compare the results from the following sections with.

In their seminal paper, Granger & Newbold (1974) presented the first arguments that past empirical research had not paid sufficient attention to the stationarity properties of macroeconomic data, and that in regressions involving the levels of such data, the standard significance tests were usually misleading. At the centre of this issue is that the regression of one random walk (non-stationary variable) on another is almost certain to produce a significant relationship, even if the two series are, in fact, independent.

The profound implications of unit-roots in macroeconomic data were further confirmed in Nelson & Plosser's (1982) influential study that examined 14 long-term annual macroeconomic series, and found evidence in support of a univariate time series structure (i.e., a unit-root) for 13 of the series studied. This observation catalyzed some serious reconsideration of the analysis into the degree of persistence in the economy.

At the focal point of the vast volume of literature examining the unit-root properties of economic series, is the statistical methodology of Dickey & Fuller (1979). The procedure formulated by Dickey & Fuller (1979) tests for unit-roots in the autoregressive polynomial of time series processes with a lag-augmented regression model.

In implementing these tests, two alternative hypotheses are usually adopted: stationarity with drift (t_μ) and stationarity around a deterministic trend (t_τ). Therefore, the ADF testing equations[1] respectively are:

$$\Delta X_t = \alpha + \rho X_{t-1} + \sum_j^k \delta_j \Delta X_{t-j} + \varepsilon_t \qquad (2)$$

$$\Delta X_t = \alpha + \beta t + \rho X_{t-1} + \sum_j^k \delta_j \Delta X_{t-j} + \varepsilon_t \qquad (3)$$

where j = 0 to k. Determination of the number of lags, k, to use is important since allowing for too few lags will change the size of the test in an unpredictable manner, and allowing for too many lags will reduce the power of the test. The choice of the number of lags to use in this study was determined using the procedure suggested by Perron (1989). Doing so, we first we start with an upper bound k_{max} for k, here $k_{max} = 6$. If the last included lag is significant, then we choose $k = k_{max}$. If not, reduce the order by 1 until the last included lag is significant or none are included. The number of lags, k, used in our ADF regressions are shown in Table 1; the lag length chosen for the ADF was also used as the optimal lag length for the ERS tests.

Despite its wide usage, the ADF procedure has been criticized for its lack of power and estimation efficiency, primarily due to its dependence on the order k of the approximating autoregression (which we mentioned above). Numerous econometricians have developed alternative procedures to address this issue. One such study conducted by Elliott et al (1996) proposes a modified version of the Dickey-Fuller t test that the

---

[1] See Elliot, Rothenberg and Stock (1996) for the DFGLS testing equations.

TABLE 1
Tests for Unit-Roots in Levels

ADF tests	t_{μ}	lags	t_{τ}	lags
Canada				
Inflation - π	-2.24	3	-2.08	3
Interest rate - R	-2.53	1	-2.53	1
U.S.				
Inflation - π	-2.85	5	-2.87	5
Interest rate - R	-2.56	5	-2.70	5
ERS tests (with a constant only)	**DFGLS**	lags	**DFGLSu**	lags
Canada				
Inflation - π	-0.62	3	-2.08	3
Interest rate - R	-1.44	1	-2.47	1
U.S.				
Inflation - π	-1.82	5	-2.82	5
Interest rate - R	-1.48	5	-2.51	5
ERS tests (with a constant & trend)	**DFGLS**	lags	**DFGLSu**	lags
Canada				
Inflation - π	-1.15	3	-1.75	3
Interest rate - R	-2.39	1	-2.57	1
U.S.				
Inflation - π	-2.65	5	-2.84	5
Interest rate - R	-2.59	5	-2.71	5

authors show as having substantially improved power when an unknown mean or trend is present. The DFGLS statistic that Elliott et al develop uses Generalized Least Squares (GLS) to estimate trend coefficients under a local alternative. This methodology, according to the authors, provides more efficient estimates of the constant and trend terms in ADF-type regression.

With one exception, the results of the unit-root tests reported in Table 1 are consistent with most of the Fisher Effect literature: in both Canada and the United States, R and π cannot reject the I(1) null. The only exception to this occurs with the United States inflation rate. Estimated under the alternative of stationarity with drift, the null hypothesis that π_{US} is I(1) can be rejected. However, this result is sensitive to lag length and does not extend to the alternative of stationarity around a deterministic trend. Therefore, this result notwithstanding, we conclude that the evidence from our standard unit-root tests is consistent with π and R being I(1) for Canada and the United States[2].

3.2 Unit-Root Tests with a One-Break Alternative

Acknowledging Perron's (1989) argument that many macroeconomic time series could be modelled as stationary fluctuations around a deterministic trend function if allowances are made for a structural break in the series, we now proceed with an examination of the stationarity properties of π and R with this allowance taken into consideration.

In Perron's (1989) seminal work, he argued that if the years associated with the Great Depression and the 1973 Oil Crisis are treated as points of structural change, then

---

[2] To ensure completeness, a test of the null hypothesis of I(2) was performed. All four variables rejected the null, thus confirming our conclusion from above. See Appendix A for results.

removal of observations corresponding to those break points would allow one to reject the unit-root hypothesis in favour of a trend-stationary hypothesis for most macroeconomic aggregates. This essentially implied that the only observations that had had a permanent effect on the long-run response of a trending data series were those associated with the Great Depression and the first oil-price crisis. However, Perron's choice of these particular breakpoints was based on prior, ex-ante observation of the data, and many researchers took issue with this, arguing that the choice of breakpoints within a series should instead be viewed as being correlated with the data. The ex-ante determination of the breakpoints Perron chose was related to exogenous occurrences for which economic theory would have predicted as happening. In other words, these pre-determined breakpoints were correlated with theory, independent of the data. Zivot & Andrews (1992), and later Perron (1997), addressed this issue by proposing that a more appropriate procedure would take the view where the choice of breakpoints is determined endogenously. It is only then after looking at the data, ex-post, that it can be said that the determined breaks that followed these respective exogenous changes actually occurred as theory predicted.

Taking the unit-root test that Perron (1989) developed and proxying it with a data-dependent algorithm, Zivot & Andrews (1992) transform Perron's conditional-on-a-known-breakpoint procedure into an unconditional procedure. Thus, in contrast to Perron's alternative hypothesis of a trend-stationary system allowing for a fixed, one-time change in the trend function, the alternative hypothesis assumed by Zivot & Andrews specifies that the series being examined can be represented by a trend-stationary process with a one-time trend break occurring at an unknown point in time. Therefore, the regression equation used to test for a unit-root under this assumption can be represented as follows:

$$\Delta X_t = \alpha + \beta t + \gamma_1 DU_t + \theta_1 DT_t + \rho X_{t-1} + \sum_{i=1}^{k} \delta_i \Delta X_{t-i} + \varepsilon_t \qquad (4)$$

where,

$DU_t \quad = 1 \qquad t > TB$
$DU_t \quad = 0 \qquad t \leq TB$

$DT_t \quad = (t\text{-}TB)^* DU_t$

DU_t is a dummy variable for a mean shift occurring at time TB, while DT_t is the corresponding trend shift variable. TB is the date of the endogenously determined break.

For this test, the unit-root hypothesis considered is that $\rho = 1$ and the goal is to estimate the breakpoint that gives the most weight to the trend stationarity alternative. Therefore, we are essentially choosing the breakpoint that yields the least favourable result for the null hypothesis; that being the break years corresponding to the minimum t statistic for testing $\rho = 1$. To ensure that the range of possible values for the breakpoint falls within some subset that excludes values at the endpoints of the sample, we adopt a 'trimming' value of 0.15. This restricts us from choosing breakpoints that correspond with the beginning and end of a series. Based on these minimum t values, the break points for each series are shown in Table 2. These results are generally supportive of the one-break hypothesis, with the unit-root null being rejected at 95% for all series except one. Canadian inflation rejects the unit-root hypothesis at 90%. The critical values used are those of Zivot & Andrews (1992): -5.08(95%) and –4.82(90%).

To aid in the discussion and interpretation of the timing of these break points we compare the estimated breaks with the plotted data for each country in Figure 1 as

TABLE 2
Unit-Root Test with a One-Break Alternative

	TB1	t	lags (k)	5%	10%
Canada					
Inflation - π	1972Q3	-5.04	5	I(1)	I(0)
Interest rate - R	1980Q3	-6.03	6	I(0)	I(0)
U.S.					
Inflation - π	1981Q2	-6.01	5	I(0)	I(0)
Interest rate - R	1978Q2	-5.58	5	I(0)	I(0)

follows. According to Figure 1, both the Canadian interest rate and inflation rate exhibit a general upward trend from the beginning of the sample period (1954) until the early 1980s. For the Canadian interest rate, the break point is estimated at 1980Q3. This is close to the peak of the upward trend that is observed in Figure 1 – slightly after the introduction of money supply targeting by the Federal Reserve Board and the Bank of Canada in late 1979. Meanwhile, the break point for the Canadian inflation rate is estimated at 1972Q3. This break is apparently picking up the change in the slope owing to the rapid increase in the rate of inflation in the early 1970s. Thus the breaks, which our test is endogenously determining for the Canadian data, appear to be reasonable and consistent with what has been observed in the series.

These results are similar to those of Crowder (1997), who found two breaks in the Canadian Fisher relation at approximately the same periods to those estimated above. The methodology by which Crowder estimates his study's breaks however, is dependent upon the maintained hypothesis that the rates of interest and inflation each have a unit-root and are co-integrated. Thus the breaks Crowder finds are not attributed to the stochastic properties of the individual variables, but rather to a change in the nature of the co-integrating relationship between the variables. In contrast, the methodology used here attributes the first break to a change in the behaviour of the inflation rate and the second break to a change in the behaviour of the interest rate.

For the United States, the estimated 1978Q2 break point in the interest rate is consistent with the rapid rise in interest rates that that country experienced during the mid 1970s, but from a strict timing perspective, it is somewhat early to attribute to the change in monetary policy discussed above. Also, it appears that the one-break model is not picking up the rapid decline in the United States interest rate witnessed during the early

1980s. However, for the United States rate of inflation, it can be seen that according to Figure 1 the estimated break point at 1981Q2 corresponds roughly with the beginning of the rapid drop in inflation experienced during the early 1980s recession. This drop in inflation occurs almost two years after the introduction of monetary supply targeting by the Federal Reserve in late 1979.

In a recent study conducted by Malliaropulos (2000), the author found evidence in support of the one-break model using United States data from the period 1965 to 1995. The break point Malliaropulos estimates for the inflation rate occurs at 1981Q3, almost identical to our estimated break point at 1981Q2. Yet, the break point that Malliaropulos estimates for the interest rate is not consistent with our findings. His break point is estimated at 1980Q3, considerably after our estimation of 1978Q2. Yet, when using a two-break model, which will be discussed in the following section, we find that one of the two break points estimated for the United States interest rate is identical to the 1980Q3 break point that Malliaropulos uncovers.

3.3 Unit-Root Tests with a Two-Break Alternative

As we have shown in the previous section, the one-break alternative does appear to be a consistent characterization of the Fisher data. However, there are reasons that lead us to believe that there may be more than one break within each of the series. For example, the sequential testing procedure that was employed in the previous section calls for the estimation of the minimum t statistic over the selected grid of dates; but t statistics other than the minimum also reject the unit-root hypothesis. This suggests that there may be more than one break present within the series. Also, from simple visual inspection of Figure 1 and without any statistical examination, it is plausible that there may be more than one break. For instance, the one-break model estimates a breakpoint in the Canadian interest rate in 1980, but it is apparent from the plot of the series that there is another sharp drop in the interest rate after 1987, which the model cannot pick up. As well, consider the early 1980s rapid decline in the United States interest rate, which is evident in Figure 1 but is not identified by the model.

Lumsdaine & Papell (1997) were one of the first to extend the endogenous break methodology to allow for a two-break alternative. The multiple trend break test that the authors considered was based on the sequential tests for changes in coefficients proposed by Banerjee et al (1992)[3] which allowed for only one structural break.

In order to test for the possibility of a second break, a sequential testing methodology implementing two breakpoints is employed. This testing regression is as follows:

$$\Delta X_t = \alpha + \beta t + \gamma_1 DU1_t + \theta_1 DT1_t + \gamma_2 DU2_t + \theta_2 DT2_t + \rho X_{t-1} + \sum_{i=1}^{k} \lambda_i \Delta X_{t-i} + \varepsilon_t \qquad (5)$$

where,

$$\begin{aligned} DU1_t &= 1 \; t > TB1 \\ &= 0 \; t \leq TB1 \\ DT1_t &= (t - TB1) * DU1_t \end{aligned}$$

$$\begin{aligned} DU2_t &= 1 \; t > TB2 \\ &= 0 \; t \leq TB2 \\ DT2_t &= (t - TB2) * DU2_t \end{aligned}$$

As there is little justification for the assumption that the two breaks are of the same form, 4 different models are estimated – AA, CC, CA(1), and CA(2)[4]. Model CC allows for two breaks in both the intercept and slope of the trend function; this is equation (5)

[3] For a more detailed discussion, see Banerjee et al. (1992) "Recursive and Sequential Tests of the Unit-Root and Trend Break Hypotheses: Theory and International Evidence" *Journal of Business and Economic Statistics* 10(3), pp. 271-87.

[4] Following notation in the literature – see Zivot & Andrews (1992) and Lumsdaine & Papell (1997).

from above. Model AA corresponds to equation (5) with *DT1* and *DT2* omitted, thereby restricting both breaks to be in the intercept. Model CA(1) allows for one break in both the intercept and slope, and the second in just the intercept of the trend function. This is equation (5) with *DT2* omitted. Model CA(2) is similar to model CA(1) except that *DT1* is omitted instead. The critical values associated with each model are taken from Lumsdaine & Papell (1997) and are presented beneath Table 3.

The results for estimating the two-break model are reported in Table 3. For the Canadian interest rate and the United States interest rate and inflation rate, the unit-root null is rejected in favour of the two-break alternative at 95% for every model except model AA. The Canadian inflation rate rejects the null at 90% for models CA(2) and AA only. For this particular series, the inability to reject the unit-root hypothesis with models CC and CA(1) can be attributed to a failure to model breaks in the trend function properly.

For the Canadian interest rate and inflation rate, estimates of the first break point are identical to the break points estimated from the one-break model, with the second break points being estimated at 1988Q1 and 1982Q2 respectively. Hence, it appears to be the case that there are essentially three regimes for the interest rate and inflation rate. The first regime is characterized by a steady, gradual increase in both variables through the 1950s and 1960s. By the early 1970s, there is an accelerated rise in the rate of inflation that is picked up by the first inflation break dummy variable. The second inflation regime is one where inflation generally increases and remains high up until the early 1980s. In the years following, the inflation trend is negative, eventually resulting in an inflation rate that hovered around its 1960s level. For the interest rate series, the first break is not picked up until early 1980. This break point coincides with a tightening of

TABLE 3
Unit-Root Tests with a Two-Break Alternative Hypothesis

Canada	TB1	TB2	t	lags (k)	5%	10%
Inflation - π						
Model CC	1972Q3	1982Q2	-6.19	5	I(1)	I(1)
Model CA(1)	1972Q3	1982Q3	-6.17	5	I(1)	I(1)
Model CA(2)	1972Q3	1982Q3	-6.34	5	I(1)	I(0)
Model AA	1982Q1	1990Q4	-6.02	3	I(1)	I(0)
Interest rate - R						
Model CC	1980Q3	1988Q1	-7.16	6	I(0)	I(0)
Model CA(1)	1980Q3	1988Q2	-6.85	6	I(0)	I(0)
Model CA(2)	1975Q1	1980Q3	-6.69	6	I(0)	I(0)
Model AA	1974Q1	1990Q4	-5.04	1	I(1)	I(1)

U.S.	TB1	TB2	t	lags (k)	5%	10%
Inflation - π						
Model CC	1966Q4	1981Q2	-6.93	5	I(0)	I(0)
Model CA(1)	1977Q4	1981Q2	-6.88	5	I(0)	I(0)
Model CA(2)	1972Q3	1981Q2	-6.83	5	I(0)	I(0)
Model AA	1981Q2	1990Q3	-6.22	5	I(1)	I(0)
Interest rate - R						
Model CC	1980Q2	1984Q3	-7.51	5	I(0)	I(0)
Model CA(1)	1980Q2	1984Q3	-7.34	6	I(0)	I(0)
Model CA(2)	1980Q2	1984Q3	-6.78	6	I(0)	I(0)
Model AA	1984Q3	1990Q4	-5.87	5	I(1)	I(1)

the monetary policy stance adopted by the Bank of Canada (and the Federal Reserve). The interest rate then remains high for the remainder of most of the decade, until in 1988, when interest rates fell back to their 1960s level.

A similar pattern is also observed in the United States inflation and interest rate series. The first break point estimated for the interest rate, 1980Q2 is nearly identical to that estimated in Canada's interest rate, 1980Q3. However, the second break that is uncovered occurs much sooner at 1984Q3, reflecting the fact that the United States interest rate decreased quicker and more smoothly than the Canadian rate. For the United States inflation rate, the first break, 1966Q4, is estimated to occur much sooner than that of Canada's 1972Q3. This is consistent with the observation that inflation in the United States accelerated at a more rapid rate that in Canada. The second estimated United States inflation break point occurs one year prior to Canada's at 1981Q2, reflecting a continuation of this relative accelerated rate.

3.4 Structural Breaks and the Data

Having found evidence in support of the view that the presence of structural breaks in the inflation and interest rate series are what leads to mis-specification of unit-root behaviour in standard unit-root tests, implementation of co-integration techniques will yield misleading results. Previous co-integration tests of the Fisher relationship were based on the assumption that inflation and nominal interest rates are both non-stationary processes. However, having demonstrated that inflation and interest rates are trend-stationary with common structural breaks, a more appropriate test of the Fisher hypothesis should be conducted within a stationary framework. The Vector Auto Regression (VAR) procedure proposed in the following section is one such test.

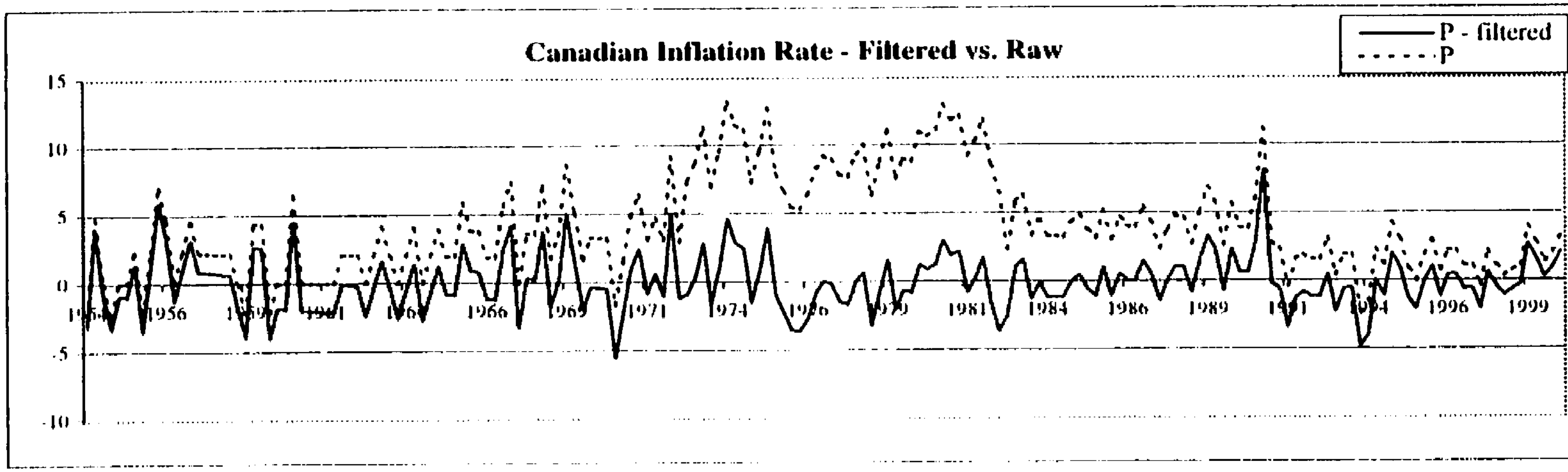
Before we can proceed with fitting the VAR model, it is necessary to appropriately detrend the variables. This is carried out by removing the structural break from each series according to its estimated break points through the OLS regression of the following filtering equation:

$$y_t = \alpha + \beta t + \gamma_1 DU1_t + \theta_1 DT1_t + \gamma_2 DU2_t + \theta_2 DT2_t + \varepsilon_t \qquad (6)$$

where $y_t = (\pi_t, R_t)$ is the unfiltered data series, and the dummy variable dates take on their respective values determined in Table 3[5]. The residuals of these regressions are zero-mean stationary by construction and are what we use as our inflation and interest rate series in the VAR analysis. Figures 2a & 2b plots the filtered series against their respective unfiltered (raw) counterparts. From this we can see that the process of filtering has effectively removed the trend from the data series, rendering us with trend-stationary processes.

[5] For the Canadian inflation rate, $\theta_1 = 0$.

FIGURE 2a


Canadian Inflation Rate - Filtered vs. Raw
P - filtered
P
15
10
5
0
-5
-10
1956
1966
1971
1974
1981
1984
1986
1989
1996
1999

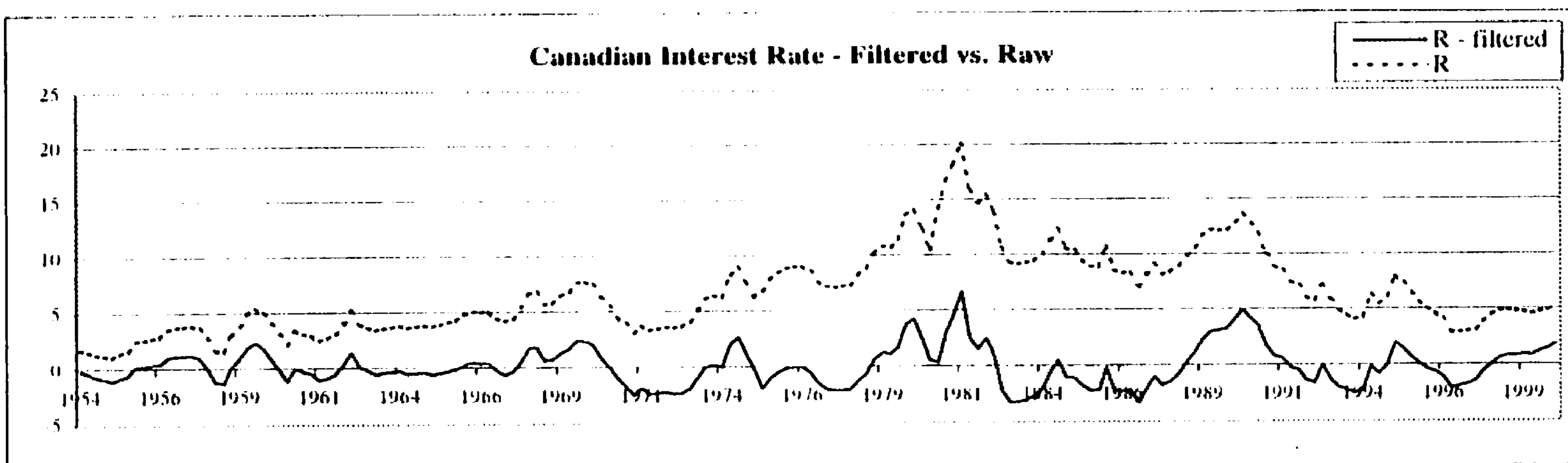
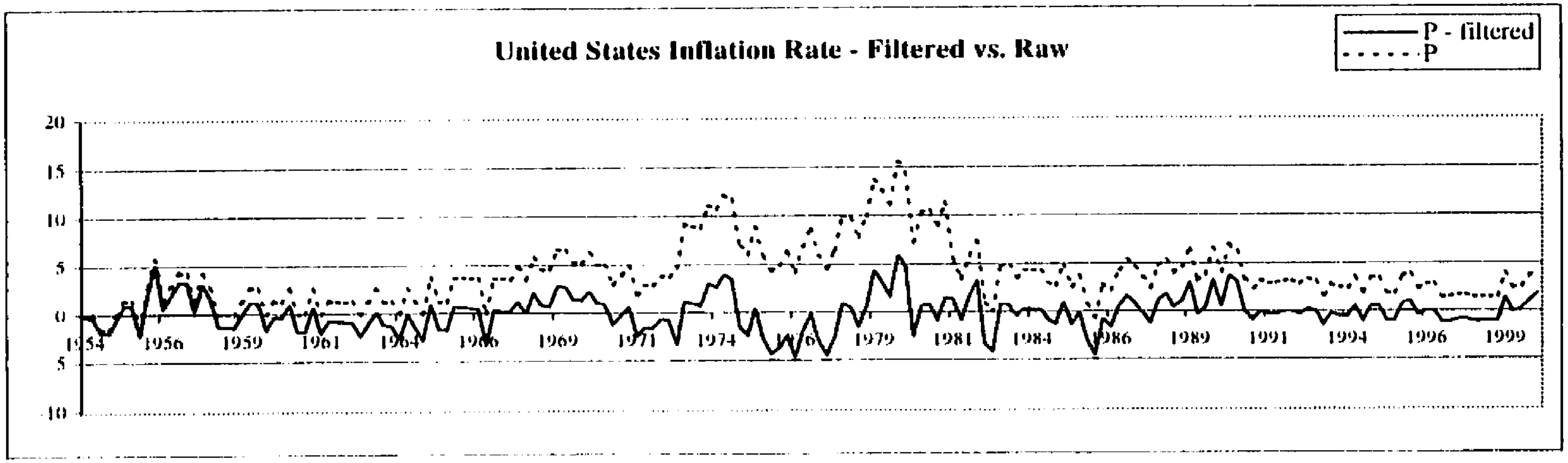
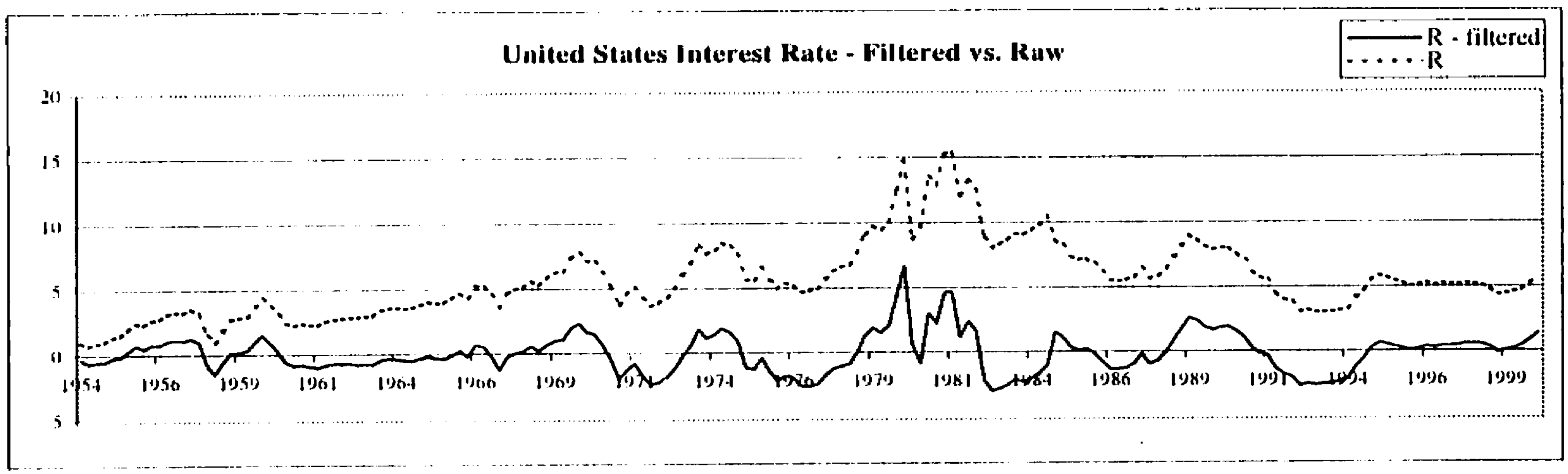

Canadian Interest Rate - Filtered vs. Raw
R - filtered
R
25
20
15
10
5
0
-5
1956
1959
1961
1964
1966
1969
1974
1976
1979
1981
1989
1991
1999

FIGURE 2b


United States Inflation Rate - Filtered vs. Raw
P - filtered
P
20
15
10
5
0
5
10
1954
1956
1959
1961
1964
1966
1969
1971
1974
1976
1979
1981
1984
1986
1989
1991
1994
1996
1999


United States Interest Rate - Filtered vs. Raw
R - filtered
R
20
15
10
5
0
5
1954
1956
1959
1961
1964
1966
1969
1971
1974
1976
1979
1981
1984
1986
1989
1991
1994
1996
1999

3.5 Tests of the Fisher Effect

Using standard notation, we begin with a VAR representation of each series of the form:

$$B(L)\tilde{y}_t = u_t \tag{7}$$

where $\tilde{y}_t = (\tilde{\pi}_t, \tilde{R}_t)$, $B(L)$ is a (2x j) matrix of polynomials in L, the lag operator, u_t is the residual vector, and the tilde denotes a filtered variable. Manipulation of equation (7) allows the VAR model to be rewritten as the vector moving average representation of this system with orthogonalized innovations. This representation is given by:

$$\tilde{y}_t = B(L)^{-1} De_t = C(L)e_t \tag{8}$$

where e_t = $(e_{1,t}, e_{2,t})$ = $D^{-1}u_t$ and D is the lower triangular Cholesky matrix satisfying $\Sigma DD'$.

VARs are a major methodological approach to macroeconometrics, often used for forecasting, describing various characteristics of data, and testing some types of theories. Essentially calling for little theoretical formulation in the design of a particular model's structure, the VAR methodology assumes that all the variables in the system are endogenous and that each can be written as a linear function of its own lagged values and the lagged values of all the other variables within the system, where the number of lags used may be determined by a variety of choice mechanisms. Hence, the dynamics of the system is determined by the actual data, rather than by prior theoretical assumptions.

For Canada a VAR(2) model is estimated, while for the United States, a VAR(1) model is estimated in the filtered variables $\tilde{\pi}_t, \tilde{R}_t$. The order of each VAR was determined by performing likelihood ratio tests, where we started with a VAR(4) and descended by one lag each time. With each reduction, we tested the restriction to drop a lag against the previous lag's "unrestricted" model successively until the restriction could be rejected. The results of this test are presented in Table 4, where for each respective lag the p-value associated with the Chi-squared statistic is given. The p-value is the degree of confidence associated with the null hypothesis that imposition of the restriction has not changed the value of the determinant ratio. Thus, the lower the p-value, the greater is the degree of confidence with which one can reject the hypothesis that the imposed restriction is not binding.

In addition to the primary functions that VARs have been used for, as was outlined above, one of the most interesting characteristics that a VAR analysis uncovers is a system's response to shocks, or innovations in its variables. By introducing an innovation in an element of e_t, y_t will move away from, and then return to, its equilibrium. The path of the resulting impact over time on y_t produces the orthogonalized impulse response function. However, proper identification of these impulse response functions requires specific assumptions of the method of orthogonalization of e_t. Ideally, we would like to be able to associate shocks with specific endogenous variables in order to determine how an unanticipated change in one variable filters through the model to affect all variables within the system over time. Yet, as is often the case, the innovation processes (e_t) are correlated and there is no simple way of unambiguously identifying shocks with specific variables. This is primarily due to the fact that the e_t terms have common components that filter through to more than one variable within the system. Typically, the usual procedure implemented to address this issue is to arbitrarily assign all of the effects of such common components to one variable. The method employed here is the standard Cholesky decomposition of the variance-covariance matrix derived

TABLE 4
Likelihood Ratio Test Results on Optimal VAR Lag Length

	Lags	Chi-Squared Statistic	p-value	Decision
Canada	4 vs. 3	13.53	0.0090	Reject 3
	3 vs. 2	2.91	0.5736	Accept 2
	2 vs. 1	29.45	0.0000	Reject 1
U.S.	4 vs. 3	19.00	0.0008	Reject 3
	3 vs. 2	28.04	0.0000	Reject 2
	2 vs. 1	7.37	0.1177	Accept 1

from estimation of the VAR, where a unit innovation in the orthogonal error corresponds to a change of one standard deviation. This method imposes a causality structure into the VAR in which the first variable of the VAR, in our case π_t, is attributed the entire common component of a shock to inflation. Thus π_t is only affected contemporaneously by the shock to itself, while the second variable, R_t, is affected contemporaneously by the shocks to the first variable and the shock to itself.

Depicted in Figures 3a & 3b are the patterns of impulse responses of each variable in the system to innovations in itself and the other variable. Also shown in the graphs are estimated confidence bands of one standard deviation. Once these bands consistently include zero, the response cannot be considered statistically significant – as can be seen in the figures, this is reflected in our assumption of a Cholesky decomposition with respect to the response functions associated with the inflation rate. For both Canada and the United States, it can be clearly seen that an inflation shock causes the inflation rate to exhibit a sharp and immediate response, whereby it gradually recovers and stablizes at its equilibrium state after a period of approximately 15 quarters for the United States and 10 quarters for Canada. Interest rates, on the other hand, appear to have a much smaller response to the shock in inflation, exhibiting a relatively more dampened immediate response than that of inflation. Ultimately, the interest response functions of both countries fall back to zero within an adjustment period of 15 to 20 quarters.

The response functions of each country's nominal interest rate both exhibit a large initial response that gradually falls back to its equilibrium level over a period of 15 quarters for Canada, and 22 quarters for the United States. Inflation rates, in contrast, show little initial movement, with both countries' inflation rates readjusting from the interest rate shock in about 13 quarters for Canada and 20 quarters for the United States.

FIGURE 3a
Canada

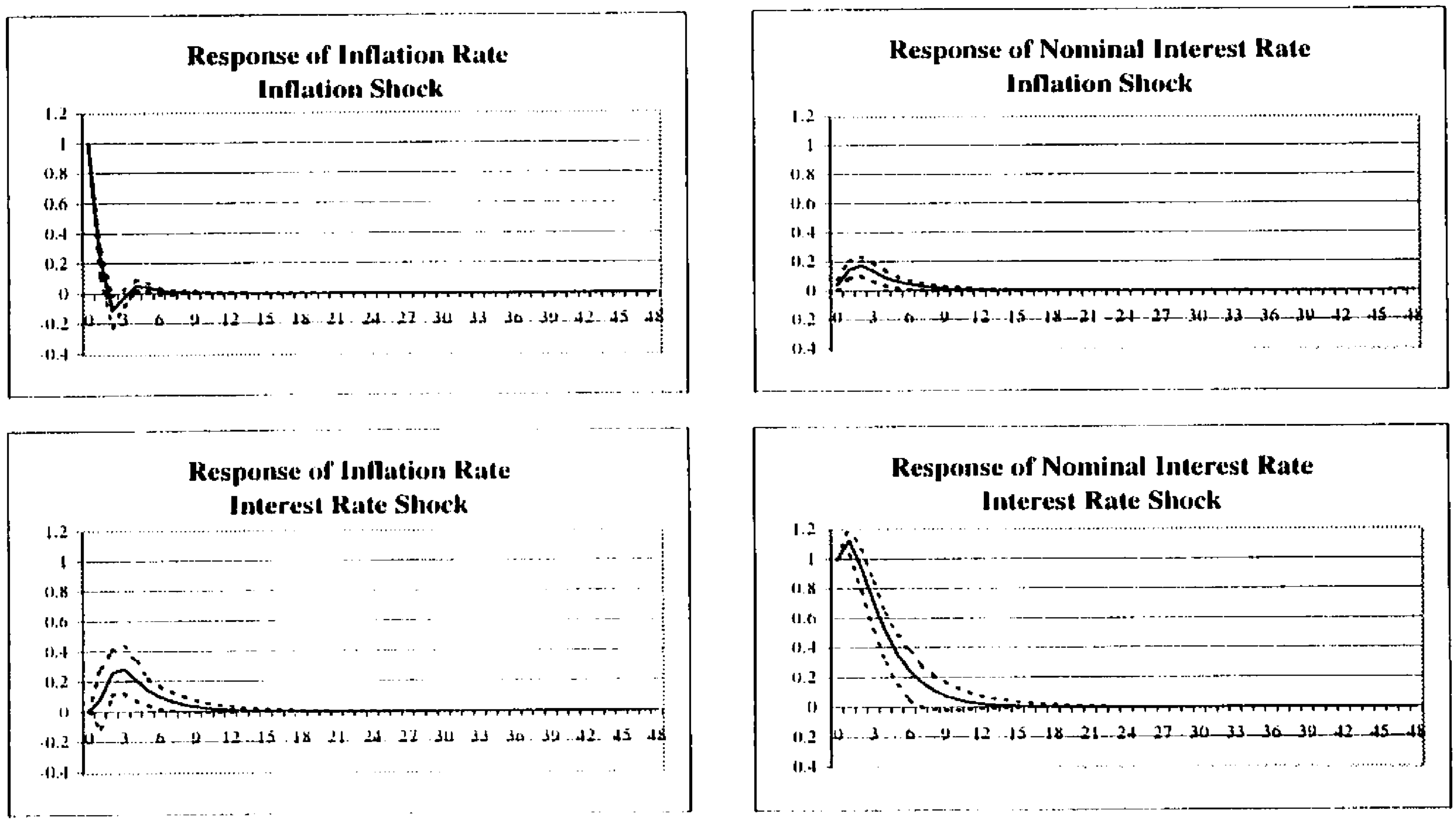
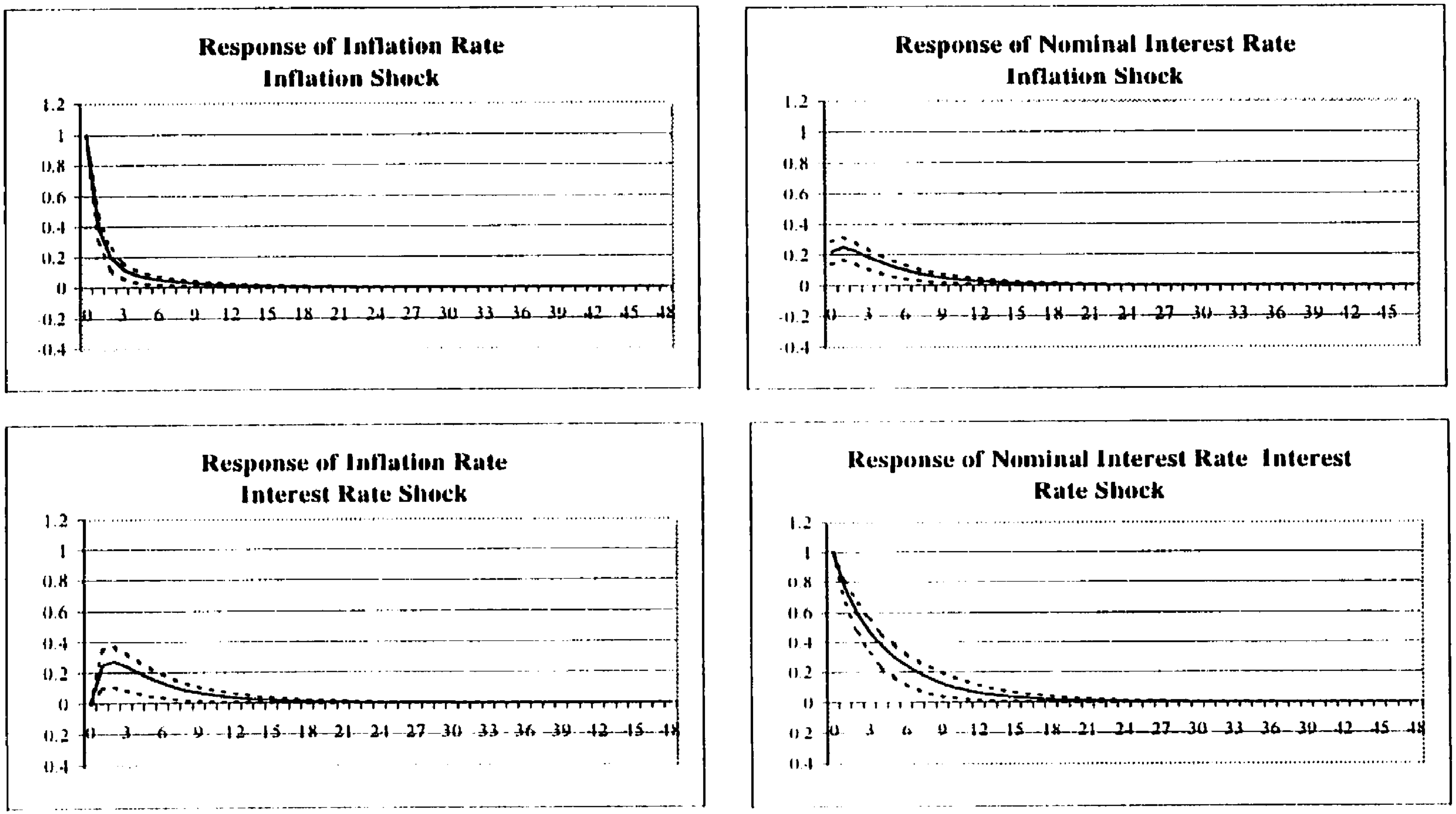


FIGURE 3b
United States



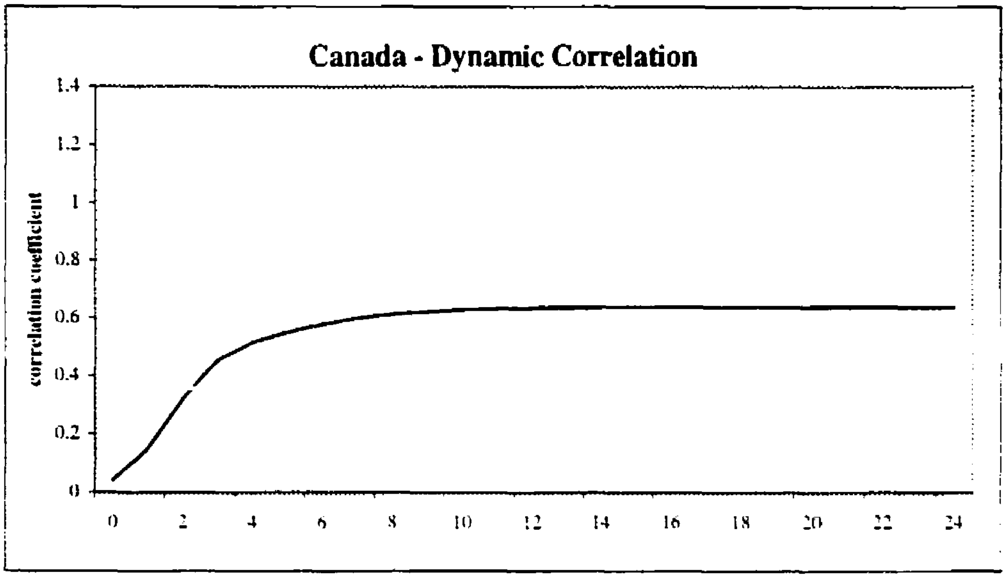
In order to test the Fisher Effect, dynamic correlations between the interest rate and the inflation rate in response to an orthogonal shock in inflation are computed. The dynamic correlation is the ratio of the cumulated impulse-response functions of the interest rate to that of the inflation rate, i.e.:

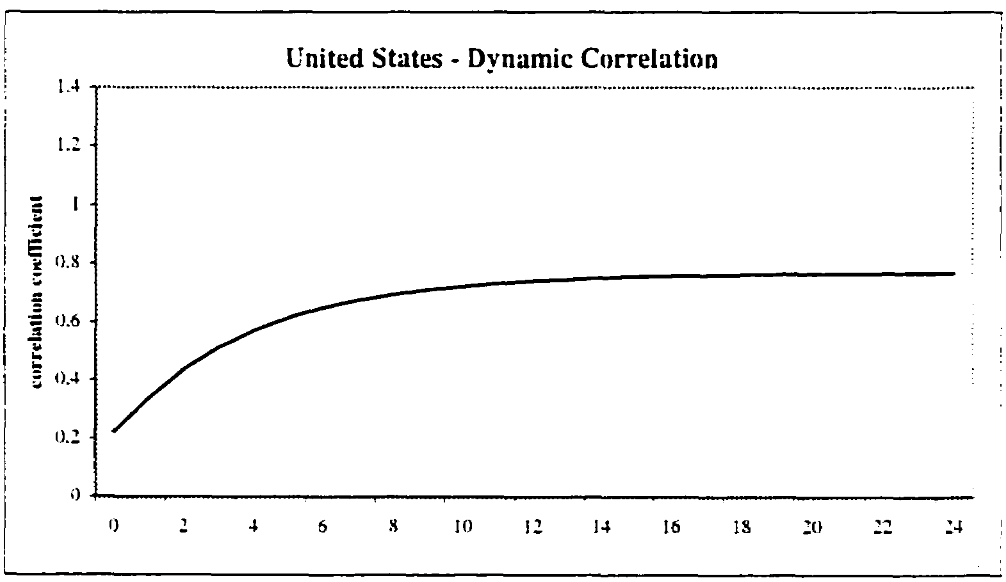
$$\rho_s = \frac{\sum_{i=0}^{s} C_{21,i}}{\sum_{i=0}^{s} C_{11,i}} \quad , for\ s = 1,2\ldots \tag{9}$$

These correlation coefficients express the response of R to an orthogonal shock in π relative to the response in π to its own contemporaneous shock. Thus, if the Fisher equilibrium relationship does hold in the long run, then ρ_s should converge to unity for large s. Figure 4 plots the dynamic correlations between R and π for each country over a horizon of 24 quarters. According to these figures, it can be seen that for both Canada and the United States, at low s, the correlations are quite small, but gradually increase as s increases. For Canada, the maximum correlation value ρ_s reaches is just above 0.6, after approximately a period of 12 quarters. While for the United States, ρ_s reaches a value just under 0.8 at approximately 16 quarters. These results suggest that all of the adjustments of interest rates to inflation shocks appear to be achieved after a period of 3 to 4 years.

Using a similar methodology, Malliaropulos (2000) reports point estimates of the dynamic correlation coefficient not significantly different from unity after a period of adjustment of only 4 quarters. Crowder & Hoffman (1996), who also examined the speed of adjustment of nominal interest rates to shocks in inflation, found evidence of adjustment lags of 6 to 8 years. These contrasting results may be attributed to the fact

FIGURE 4


Canada - Dynamic Correlation
correlation coefficient
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20 22 24


United States - Dynamic Correlation
correlation coefficient
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20 22 24

that: in Crowder & Hoffman's (1996) study, not only are they implementing variables that have not be filtered of their structural breaks – of which we have shown there is evidence of – they estimate a VAR in first differences that, according to Malliaropulos (2000) contains per definition a much higher proportion of permanent shocks which may lead to substantial, spurious lags in adjustment; and Malliaropulos (2000) estimates his study's VAR in detrended, stationary variables accounting for only one structural break.

Chapter 4

SUMMARY & CONCLUSIONS

Ever since Irving Fisher (1930) first proposed that nominal interest rates should adjust in such a manner that real interest rates are unaffected by inflation, the Fisher Effect has been the focus of a considerable volume of empirical research. In an attempt to extend this literature, the objective of this thesis was to re-examine the empirical evidence on the Fisher Effect using recent advances in time-series analysis. In doing so, we analyzed the unit-root properties of the Fisher variables – rate of inflation and nominal interest rate – under a framework that tested the null of a unit-root against the alternative of trend-stationarity with two breaks. The results from these tests then allowed us to proceed with an investigation into the existence of the long-run Fisher relationship within a stationary VAR construct.

Finding evidence in support of the hypothesis that the nominal interest rate and rate of inflation for both Canada and the United States are trend-stationary with two structural breaks, we are able to infer that there were essentially three regimes in the inflation-interest rate relation in the two countries over the period examined. The first regime, spanning through the 1950s and 1960s, exhibits a steady, gradual increase in both the inflation rate and nominal rate of interest. This is then followed by a strong, general upward surge in both variables through the 1970s and into the early 1980s, which eventually declined subsequent to the sample's second regime shift.

To investigate the dynamic relationship between interest rates and inflation, we conducted a VAR analysis in appropriately detrended, stationary variables. For this, each series was passed through a filtering regression that removed their respective trend breaks according to the previously estimated breakpoints. Then, based on the impulse response functions of the VAR, dynamic correlation coefficients were derived. These correlation coefficients provide a means by which the dynamic Fisher relationship may be tested. Our results suggest that the data is consistent with a medium to long-run co-movement of the interest rate and inflation rate in both Canada and the United States. However, the magnitude of these correlations are not of the level predicted by the Fisher Effect: Canada reaches a maximum correlation of approximately 0.6, while the United States attains a correlation slightly under 0.8; both correlation coefficients fall short of the hypothesized Fisher correlation of 1.0.

These findings have important implications for policy makers. Considering the evidence suggests that a relatively weak, long-run Fisher Effect exists for both Canada and the United States, this implies that a substantial amount of adjustment in real interest rates occurs in response to changes in expected inflation. Hence, the level of interest rates is an inappropriate guide for monetary policy since a persistently high interest rate is an indication that inflation expectations are high, and not a reflection of a tight monetary stance. However, short-run changes in the short-term interest rate will reflect changes in the real interest rate. Thus, changes in short-term interest rates will reflect the stance of monetary policy.

Integral to our analysis was the finding that the two Fisher series were trend-stationary processes in the presence of structural breaks. The technique[6] by which this was determined is rooted in the belief that the timing of breakpoints is associated with

[6] See Zivot & Andrews (1992) and Perron (1997).

major economic events that have had permanent effects on the system. However, according to Kilian & Ohanian (1998) the fluctuations associated with most economic events are transitory, rather than permanent. Because of this, Kilian & Ohanian argue that unit-root tests against trend break alternatives will mistakenly identify these transitory fluctuations as trend breaks, and falsely reject the unit-root null.

Given the dependency of the analysis conducted here on the above-mentioned unit-root against trend break alternatives technique, this argument must be taken into careful consideration. If it is true that large temporary fluctuations within our data may be misinterpreted as a change in intercept or trend slope or both in a trend stationary process, then our conclusions about the stationarity properties of the inflation and interest rate series are invalid.

However, the break points estimated in our analysis can arguably be identified as being associated with economic events that have had a permanent effect on the two Fisher variables. The breakpoints estimated at 1972Q3 for Canada's rate of inflation and 1972Q3 for U.S. inflation corresponds closely to the early 1970s breakdown of the Bretton Woods fixed exchange rate system. Meanwhile, the early 1980s breakpoints estimated for Canada's inflation and interest rate, 1982Q3 and 1980Q3 respectively, and 1981Q2 and 1980Q2 for the United States' respective series, appear to be linked to the change in the Bank of Canada's monetary policy targeting strategy and the analogous change by the U.S. Federal Reserve Board occurring in the early part of that decade. As for the 1975Q1 breakpoint estimated for Canada's interest rate, around the mid-1970s when relatively high inflation became entrenched in the world economy, many central banks shifted their focus from operational targets for short-term interest rates to intermediate targets; the Bank of Canada established their intermediate target in terms of the narrowly defined monetary aggregate M1. With respect to the 1984Q3 break in the United States rate of interest, at that time the Federal Reserve had abandoned their commitment to monetary growth targets and adopted a monetary policy approach that centered more closely on controlling short-term interest rate movements. These events effectively introduced an exogenous structural change to the levels of economic activity, and hence can plausibly be interpreted as not being simply a realization of the underlying data-generating mechanism (i.e., a transitory fluctuation), but rather as exogenous events that have triggered permanent changes in economic growth. Therefore, the implementation of a unit-root test with a two-break alternative appears to be justified in this instance.

In future research, a number of important issues ought to be considered. One such direction is to address the possibility that even higher order break models are more appropriate. From the results here, there is little reason to expect that there have been exactly two breaks in the data; however, careful consideration must be given to extensions of these tests that allow for multiple trend breaks. As Kilian & Ohanian (1998) have argued, one must take into account the possibility that break tests could misinterpret large transitory movements in the data as trend breaks; as a consequence, careful thought must be given in determining the nature of exogenous events that may have resulted in permanent structural changes and to alternative economic explanations for possible trend breaks based on endogenous transitory fluctuations.

Another important direction for further research is to consider adopting an asymmetric vector autoregression (AVAR)[7] or a near-VAR[8] representation of the Fisher system. In a VAR, each variable is regressed against the same number of lags of every variable, i.e.; the lag specifications are symmetrical. However, it is often the case that VARs estimate a large number of coefficients that are statistically insignificant; therefore the corresponding impulse response functions derived from the system will often be imprecisely determined. On the other hand, in an AVAR system each equation consists of the same explanatory variables, but each variable may have a different number of lags. Thus, AVAR models permit more flexibility in modeling dynamic systems and as a result may yield more efficient estimates of the system's underlying relationship.

One final aspect, which should be considered, is, in this thesis our analysis was conducted under the generally accepted assumption that the rate of inflation was exogenous. In other words, it was assumed that inflation responds only to its own contemporaneous shocks while the nominal interest rate responds to contemporaneous shocks to both inflation and itself. Thus an interesting extension of this work would be to re-examine the inflation-interest rate relationship under the alternative hypothesis proposed by Fama (1975), where nominal interest rates predict inflation, but inflation does not predict nominal interest rates. This question of the direction of causality between inflation and interest rates has appealed to many researchers and it would be of great interest to see what the methodology used in this thesis would have to contribute to this question.

[7] See Keating (1998).
[8] See RATS 4.31 *User's Manual*, pages 8-2 to 8-3.

References

BOOKS & JOURNALS

Arestis, P. and I.B.F. Mariscal (1999). "Unit Roots and Structural Breaks in OECD Unemployment." *Economics Letters* 65(2), pp. 149-156.

Atkins, F. J. (1989). "Co-Integration, Error Correction and the Fisher Effect." *Applied Economics* 21, pp. 1611-1620.

Banerjee, A., R.L. Lumsdaine and J.H. Stock (1992). "Recursive and Sequential Tests of the Unit-Root and Trend-Break Hypotheses: Theory and International Evidence." *Journal of Business and Economic Statistics* 10(3), pp. 271-287.

Bank of Canada. (1996). "The Transmission of Monetary Policy in Canada." Online at: http://www.bankofcanada.ca/en/pdf/hermes.pdf

Crowder, W.J. (1997). "The Long-Run Fisher Relation in Canada." *Canadian Journal of Economics* 30(4b), pp. 1124-1142.

Crowder, W.J. and D. L. Hoffman (1996). "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited." *Journal of Money, Credit and Banking* 28(1), pp. 102-118.

Daniels, J.P., F. Nourzad and R.K. Toutkoushian (1996). "Testing the Fisher Effect as a Long-Run Equilibrium Relation" *Applied Financial Economics* 6(2), pp. 115-120.

Dickey, D.A. and W. Fuller (1979). "Distribution of the Estimators for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association* 74(366), pp. 427-431.

Elliott, G. (1999). "Efficient Tests for a Unit Root When the Initial Observation Is Drawn from Its Unconditional Distribution." *International Economic Review* 40(3), pp. 767-783.

Elliott, G., T.J. Rothenberg and J.H. Stock (1996). "Efficient Tests for an Autoregressive Unit Root." *Econometrica* 64(4), pp. 813-836.

Engle, R.F. and C.W.J. Granger (1987). "Co-integration and Error Correction: Representation, Estimation and Testing." *Econometrica* 55(2), pp. 251-276.

Evans, M.D.D. and K.K. Lewis (1995). "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance* 50(1), pp. 225-253.

Fama, E.F. (1975). "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review* 65(3), pp. 269-282.

Fisher, I. (1930). *The Theory of Interest as Determined by Impatience to Spend Income and Opportunity to Invest It.* New York: Kelly, 1961.

Fisher, M. and J. Seater (1993). "Long-Run Neutrality and Superneutrality in an ARIMA Framework." *American Economic Review* 83(3), pp. 402-415.

Friedman, B.M. (1988). "Lessons on Monetary Policy from the 1980s." Working Paper Series No. 2551, National Bureau of Economic Research.

Granger, C.W.J. and P. Newbold (1974). "Spurious Regression in Econometrics." *Journal of Econometrics* 2(2), pp. 111-120.

Greene, W.H. (2000). *Econometric Analysis* (4^{th} Edition). Upper Saddle River, N.J: Prentice Hall.

Johansen, S. (1988). "Statistical Analysis of Cointegration Vectors." *Journal of Economic Dynamics and Control* 12(2/3), pp. 231-254.

Keating, J.W. (1998). "Macroeconomic Modeling with Asymmetric Vector Autoregressions." Online at: http://www.econ.ukans.edu/workpaper/workp98.htm

Kennedy, P. (1998). *A Guide to Econometrics* (4th Edition). Cambridge, Mass: MIT Press.

Kilian, L. and L.E. Ohanian (1998). "Is There a Trend Break in U.S. GNP? A Macroeconomic Perspective." Federal Reserve Bank of Minneapolis, Research Department Staff Report 244.

King, R.G. and M.W. Watson (1997). "Testing Long-Run Neutrality." Federal Reserve Bank of Richmond, *Economic Quarterly* 83(3), pp. 69-101.

Koustas, Z. and A. Serletis (1999). "On the Fisher effect." *Journal of Monetary Economics* 44(1), pp. 105-130.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt, and Y. Shin (1992). "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root?" *Journal of Econometrics* 54(1-3), pp. 159-178.

Lumsdaine, R.L. and D.H. Papell (1997). "Multiple Trend Breaks and the Unit-Root Hypothesis." *The Review of Economics and Statistics* 79(2), pp. 212-218.

MacDonald, R. and P.D. Murphy (1989). "Testing for the Long Run Relationship between Nominal Interest Rates and Inflation Using Cointegration Techniques." *Applied Economics* 21(4), pp. 439-447.

Malliaropulos, D. (2000). "A Note on Nonstationarity, Structural Breaks, and the Fisher Effect." *Journal of Banking and Finance* 24(5), pp. 695-707.

Mishkin, F.S. (1991). "Is the Fisher Effect for Real? A Reexamination of the Relationship between Inflation and Interest Rates." Working Paper Series No. 3632, National Bureau of Economic Research.

Nelson, C.R. and C.I. Plosser (1982). "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications." *Journal of Monetary Economics* 10(2), pp. 139-162.

Perron, P. (1989). "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57(6), pp. 1361-1401.

_______. (1997). "Further Evidence on Breaking Trend Functions in Macroeconomic Variables." *Journal of Econometrics* 80(2), pp. 355-385.

Phillips, P.C.B. (1986). "Understanding Spurious Regressions in Econometrics." *Journal of Econometrics* 33(3), pp. 311-340.

Pindyck, R. and D. Rubinfeld (1998). *Econometric Models and Economic Forecasts* (4th Edition). Boston, Mass: Irwin/McGraw-Hill.

RATS Version 4.31 User's Manual

Zivot, E. and D.W.K. Andrews (1992). "Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis." *Journal of Business and Economic Statistics* 10(3), pp. 251-270.

DATA

All data series were obtained from the Statistics Canada computerized database, CANSIM (Canadian Socio-Economic Information Management System), available through the CHASS Data Center (http://datacenter.chass.utoronto.ca) at the University of Toronto.

Consumer Price Index (Canada): P100000

Consumer Price Index (United States): D139105

3-month Treasury Bill Tender Rate (Canada): B14007

3-month Treasury Bill at Monday Tender Rate (United States): B54409

APPENDIX A

Tests for Unit-Roots in First Differences (ADF tests)

	t_μ	lags	t_τ	lags
Canada				
Inflation - π	-16.20	2	-16.24	2
Interest rate - R	-6.18	6	-6.25	6
U.S.				
Inflation - π	-5.93	4	-5.91	4
Interest rate - R	-6.03	6	-6.07	6