Macroprudential capital requirements and systemic risk [*]

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Abstract

In the aftermath of the financial crisis, there is interest in reforming bank regulation such that capital requirements are more closely linked to a bank's contribution to the overall risk of the financial system. In our paper we compare alternative mechanisms for allocating the overall risk of a banking system to its member banks. Overall risk is estimated using a model that explicitly incorporates contagion externalities present in the financial system. We have access to a unique data set of the Canadian banking system, which includes individual banks' risk exposures as well as detailed information on interbank linkages including OTC derivatives. We find that macroprudential capital allocations can differ by as much as 50% from observed capital levels and are not trivially related to bank size or individual bank default probability. Macroprudential capital allocation mechanisms reduce default probabilities of individual banks as well as the probability of a systemic crisis by about 25%. Our results suggest that financial stability can be enhanced substantially by implementing a systemic perspective on bank regulation.

Keywords: Systemic Risk, Financial Stability, Bank regulation, Risk Management, Interbank Market

JEL-Classification Numbers: G21, C15, C81, E44

1 Introduction

> *Under our plan ... financial firms will be required to follow the example of millions of families across the country that are saving more money as a precaution against bad times. They will be required to keep more capital and liquid assets on hand and, importantly, the biggest, most interconnected firms will be required to keep even bigger cushions.*

<div align="right">

US Treasury Secretary Geithner (2009)

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The recent financial crisis has demonstrated the adverse effects of a large scale breakdown of financial intermediation for other banks as well as the rest of the economy. The concern that a failure of a large bank would trigger a chain reaction of insolvencies in the financial sector was often brought forward to justify large scale government intervention and bailouts of failed institutions. The failure of banks causes these externalities because financial institutions are directly linked through interbank loans and derivatives. However, the adverse consequences that a bank failure brings for other banks as well as the economy as a whole is not considered in current bank regulation. While academics, international institutions, and central bankers have argued for some time that bank regulation should be designed from a system perspective (Borio (2002)), bank regulation is currently aimed at the level of the individual bank. In the aftermath of the financial crisis, there is a growing consensus amongst regulators to bring a macroprudential perspective into bank regulation. One proposal is to require financial institutions to internalize the externalities they impose on the system by adjusting capital requirements so that they better reflect an individual bank's contribution to the overall risk of the financial system. We refer to these adjusted capital requirements as macroprudential capital requirements.

In this paper, we derive macroprudential capital requirements based on five approaches to measure each bank's contribution to overall risk: component and incremental value-at-risk from the risk management literature (Jorion (2007)), two allocation strategies using Shapley values, and the ΔCoVaR measure introduced by Adrian and Brunnermeier (2010). Computing macroprudential capital requirements, however, is more complex than computing risk contributions because once new capital requirements are implemented, bank probabilities of default (PDs), default correlations, the total risk in the banking system, and thus each bank's risk contribution will change. We therefore have to follow an iterative procedure to solve for a fixed point for which capital allocations to each bank are consistent with the contributions of each bank to the

total risk of the banking system under the proposed capital allocations. To our knowledge, we are the first to do so. In our sample we find that risk attributions can differ from macroprudential capital allocations by a factor of up to three.

To find the fixed point at which risk contributions equal capital requirements we need a model to analyze how overall risk in the banking system changes when capital requirements change. We build a structural model fitted to our data that includes spillover and contagion effects through network and asset fire sale externalities similar to the models used by central banks[1]. We generate a macro stress scenario which causes varying increases in PDs for different economic sectors affecting all banks' loan portfolios. Following Elsinger, Lehar, and Summer (2006a), we simulate loan losses for each bank using a portfolio credit risk model. Using a variant of Cifuentes, Shin, and Ferrucci (2005) that differentiates banks according to the riskiness of their assets, banks that fall short of regulatory capital requirements start selling assets to a market with inelastic demand and the resulting drop in prices forces other banks to sell assets as well. Banks that default either because of loan losses or decreasing asset valuations are not able to fully honor their interbank promises, potentially causing the contagious default of other banks. Clearing in the interbank market is captured using a network model as in Eisenberg and Noe (2001). The spillover effects from asset fire sales and contagious defaults make the correlation of banks' asset values dependent on the health of the overall financial system. For a given set of capital requirements the model computes the joint distributions of bank losses and defaults which will then be used to compute each bank's risk contribution.

We use a unique data set of the six largest Canadian banks as a representation of the whole Canadian banking system since they hold 90.3% of all banking assets. Our sample contains detailed information on the composition of the loan book, including the largest loan exposures of individual banks. We are the first to analyze a dataset that includes the full network of exposures from OTC derivatives as well as exposures between banks arising from traditional inter-bank lending and cross-shareholdings. While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify. Our expanded dataset enables us to better capture linkages among banks and contagious bank defaults.

With the same amount of overall capital in the banking system, we find that macroprudential capital requirements can reduce the default risk of the individual bank as well as the risk

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[1]See for example Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) for the model that is used by the bank of England.

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of a systemic crisis by about 25%. Macroprudential capital allocations differ from current observed capital levels by up to 50% for individual banks, and the reallocation of capital that the macroprudential rules suggest are not trivially related to bank size, bank PD, or risk weighted assets. We also find that ignoring our information on derivatives and cross shareholdings gives us a very different picture of individual bank risk, with potentially important effects on systemic risk. This speaks to the importance of obtaining better information about exposures between financial institutions.

In the literature we find two main approaches to measure and allocate systemic risk. One stream uses stock market data to get information on banks' correlation structure and potential spillovers. Adrian and Brunnermeier (2010) propose the ΔCoVaR measure, which they compute for a panel of financial institutions and regress on bank characteristics. This literature is related to existing studies of contagion in financial markets (see among others Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003)). Another stream of research builds on a network model in conjunction with an interbank clearing algorithm introduced by Eisenberg and Noe (2001). Elsinger, Lehar, and Summer (2006a) and Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) use a dataset of interbank linkages for the Austrian and British banking system, respectively, and compute measures of systemic risk and systemic importance for individual banks, conditional on a forward-looking stress scenario. This is the approach we follow. Upper (2007) reviews that literature.

These two complementary approaches (Acharya, Pedersen, Philippon, and Richardson (2009)) can be interpreted in light of economic theories of financial amplification mechanisms at work during a financial crisis. For example, the seminal paper by Allen and Gale (1994) shows how asset prices can be optimally determined by cash-in-the-market pricing in a crisis period. Allen and Gale (2000) propose a model of contagion through a network of interbank exposures. Shin (2008) develops a theory of liquidity spillover across a network of financial institutions resulting from expansions and contractions of balance sheets over the credit cycle. Krishnamurthy (forthcoming) reviews the literature on the mechanisms involving balance-sheet and asset prices, and those involving investors' Knightian uncertainty. Tarashev, Borio, and Tsatsaronis (2009) conduct a simulation study of a stylized banking system and find that the systemic importance of an institution increases in its size as well as its exposure to common risk factors. They use Shapley values to allocate risk measured by value-at-risk as well as expected loss.

We extend previous research in two ways: first we highlight that changing capital require-

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ments change the risk and correlation structure in the banking system and that macroprudential capital requirements have to be seen as a fixed point problem. Second, we provide empirical evidence that macroprudential capital requirements can reduce individual as well as systemic risk using actual data for a whole banking system.

The paper is organized as follows. Section 2 describes the approaches to assign macroprudential capital requirements, the model for assessing systemic risk is described in Section 3, and Section 4 details credit loss scenario generation and the data. We present the results in Section 5 and conclude in Section 6.

2   Macroprudential capital requirements

Setting macroprudential capital requirements raises two fundamental questions. First, what is the total level of capital required in the banking system, which determines the overall magnitude of the shock that a banking system can withstand? Second, how to break down the overall risk of the banking system and set capital requirements equal to each banks' contribution to systemic risk? The first question is a policy decision balancing efficiency of financial intermediation with overall stability of the system which we do not address in this paper. We focus on the second question by comparing alternative mechanisms to reallocate capital among banks, for a given level of total capital in the system.

Macroprudential capital requirements differ from risk contribution analysis as it is used in portfolio or risk management. In a risk management or portfolio management setting we want to compute risk contributions for a given portfolio with an exogenous level of overall risk. Using the same approach to compute macroprudential capital requirements in a banking system would be incorrect because both the overall risk and each bank's contribution depend on the capital allocation. As banks hold more capital they are less likely to default through either direct losses or contagion. Reallocating bank capital changes the overall risk of the banking system and thus each bank's risk contribution.[2]

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[2]Changing bank capital requirements might also change individual bank risk through another channel: Capital requirements might make certain assets more or less attractive and thus can create a long-term incentive to change banks asset portfolios. We do not consider this channel in our analysis for two reasons: First, macroprudential capital requirements can be continuously adjusted as banks' asset portfolios change. Second, we believe that the direct effect that changes in capital have on bank solvency risk outweigh the indirect incentive effects on banks' optimal asset choice.

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Estimating macroprudential capital requirements is therefore a fixed point problem. We have to reallocate bank capital such that the risk contribution of each of the n banks to total risk equals the allocated capital. To measure by how much the risk of the system changes once we reallocate capital we need a model. Computing macroprudential capital requirements cannot be done as a purely empirical exercise. Assume that there is a model, like the one in this paper, that estimates a banking systems' joint risk distribution $\Sigma(C)$ for a given vector of bank capital endowments $C = (C_1, ..., C_n)$. A risk sharing rule $f(\Sigma)$ then allocates the overall risk $\Sigma(C)$ to individual banks. A consistent capital allocation C^* must then satisfy

$$C^* = f(\Sigma(C^*)). \tag{1}$$

The difference between performing a risk contribution analysis and the fixed point can be substantial. In Appendix A we document that capital adjustments based on risk contribution analysis can differ from adjustments based on the fixed point by a factor of three.

Because of the high degree of non-linearity, the fixed point in equation (1) can only be found numerically.[3] From the discussion above it also becomes clear that we cannot find macroprudential capital requirements without a model $\Sigma(C)$ of the banking system's risk. Our model, which we describe in detail in Section 3, is simulation based. For each of our m simulated scenarios we record the profit or loss for each bank to get the joint loss distribution for all banks, i.e. we get an $n \times m$ matrix of losses, which we call L. We then allocate the risk of the whole system to each individual bank using different risk allocation methodologies $f(.)$.

We now review the risk sharing methodologies that we use to compute macroprudential capital requirements.

2.1 Component value-at-risk (beta)

Following Jorion (2007) we compute the contribution of each bank to overall risk as the beta of the losses of each bank with respect to the losses of a portfolio of all banks. Let $l_{i,s}$ be the loss of bank i in scenario s and $l_{p,s} = \sum_i l_{i,s}$, then $\beta_i = \frac{cov(l_i, l_p)}{\sigma^2(l_p)}$. Furthermore let C_i be the preexisting tier 1 capital of bank i. We reallocate the total capital in the banking system according to the

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[3]We find that it takes on average 20 iterations until the norm of the changes in capital requirements from one iteration to the next is less that $500,000. Due to the non-linearity of the problem we cannot prove that the fixed point is unique, but as detailed in Appendix A we check for robustness by using alternative starting values and find convergence to the same point for our data.

following risk sharing rule

$$C_i^\beta = \beta_i \sum_{i=1}^{n} C_i.$$ (2)

where C_i^β is the reallocated capital of bank i. A nice property of this rule is that the sum of the betas equals one, so a redistribution of total capital amongst the banks is straightforward.

2.2 Incremental value-at-risk

We first compute the value-at-risk (VaR) of the joint loss distribution of the whole banking system, which we get by adding the individual losses across banks in each simulated scenario. We chose a confidence level of 99.5% and run 1,000,000 scenarios. The portfolio VaR, VaR_p, is therefore the $5,000^{th}$ largest loss of the aggregate losses l_p. Next we compute the VaR of the joint distribution of all banks except bank i, VaR^{-i}, as the $5,000^{th}$ largest value of the $l_s^{-i} = \sum_{j=1, j\neq i}^{n} l_{j,s}$. The incremental VaR for bank i, $iVaR_i$, is then defined as

$$iVaR_i = VaR_p - VaR^{-i}.$$ (3)

The incremental VaR therefore can be interpreted as the increase in risk that is generated by adding bank i to the system.[4]

While component VaR computes the marginal impact of an increase in a bank's size, incremental VaR captures the full difference in risk that one bank will bring to the system. The disadvantage of the second risk decomposition is that the sum of the incremental VaRs does not add up to the VaR of the banking system. In our analysis, however, we found that difference to be small (below 5%) and thus scale iVaR capital requirements as follows:

$$C_i^{iVaR} = \frac{iVaR_i}{\sum_i iVar_i} \sum_i C_i.$$ (4)

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[4]We calculate VaR^{-i} by adding the losses from all banks except bank i. Another way would be to remove bank i from the banking system and then compute the loss distribution of the reduced system. We decided against the latter approach, because removing a bank would leave holes in the remaining banks' balance sheets when claims on bank i do not equal liabilities to bank i as it is the case in our sample.

2.3 Shapley values

Shapley values can be seen as efficient outcomes of multi player allocation problems in which each player holds resources that can be combined with others to create value. The Shapley value then allocates a fair amount to each player based on the average marginal value that the player's resource contributes to the total.[5] In this context, one can argue that a certain level of capital has to be provided by all banks as a buffer for the banking system and that Shapley values help to determine how much capital each bank should provide according to its relative contribution to overall risk.[6]

To compute Shapley values we have to define the characteristic function $v(\mathcal{B})$ for a set $\mathcal{B} \subseteq \mathcal{N}$ of banks, which assigns a capital requirement to each possible combination of banks.[7] In our analysis we use two risk measures to define capital requirements: expected tail loss (EL) and value-at-risk. To compute $v(\mathcal{B})$ we add the profits and losses for all the banks in $\mathcal{B}$ across scenarios to get the joint loss distribution for $\mathcal{B}$, i.e. $l_{\mathcal{B},s} = \sum_{i \in \mathcal{B}} l_{i,s}$. We assume a confidence level of 99.5% and then assign to $v(\mathcal{B})$ either the corresponding VaR, which is the $5,000^{th}$ largest loss of $l_{\mathcal{B}}$, or the expected tail loss, i.e. the arithmetic average of the 5,000 biggest losses. Define furthermore $v(\emptyset) = 0$, then the Shapley value for bank i, equal to its capital requirement, can be computed as:

$$\phi_i(v) = \sum_{\mathcal{B} \subseteq \mathcal{N}} \frac{|\mathcal{B}|!(|\mathcal{N}| - |\mathcal{B}| - 1)!}{|\mathcal{N}|!} \left(v(\mathcal{B} \cup i) - v(\mathcal{B}) \right) \tag{5}$$

Because the sum of the Shapley values will in general not add up to the total capital that is currently employed in the banking system, we scale the Shapley values similar to Equation (4):

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[5]While Shapley values were originally developed as a concept of cooperative game theory, they are also equilibrium outcomes of noncooperative multi-party bargaining problems (see e.g. Gul (1989)).

[6]Shapley values are commonly used in the literature on risk allocation. Denault (2001) reviews some of the risk allocation mechanisms used in this paper, including the Shapley value. See also Kalkbrener (2005). In a recent paper, Tarashev, Borio, and Tsatsaronis (2009) propose to use Shapley values to allocate capital requirements to individual banks.

[7]A potential caveat of this methodology is that all characteristic function games assume that the value $v(\mathcal{B})$, which a group of banks $\mathcal{B}$ can achieve, is independent of how the other banks that are not in $\mathcal{B}$ group together. David and Lehar (2009) analyze under what conditions banks find it optimal to merge to avoid bankruptcy costs using a bargaining game in partition form, which defines the value of a coalition conditional on the coalition structure of the remaining banks. They find that under certain conditions bargaining can break down and inefficient liquidations occur.

$$C_i^{\text{SV}} = \frac{\phi_i}{\sum_i \phi_i} \sum_i C_i. \tag{6}$$

One potential caveat of all macroprudential capital requirements is that capital allocations can be negative, for example if a bank is negatively correlated with the other banks and therefore reduces the risk of the system. This problem also applies to the Shapley value procedure. Unless we assume monotonicity, i.e. $v(S \cup T) \geq v(S) + v(T)$, the core can be empty and negative Shapley values can be obtained. For our sample this problem did not occur since bank correlations were sufficiently high.

2.4 ΔCoVaR

Following Adrian and Brunnermeier (2010) we define CoVaR if bank i as the value-at-risk of the banking system conditional on bank i realizing a loss corresponding to its VaR. However, since we have to compute the loss distribution by simulation, we observe cases for which a bank realizes a loss exactly equal the VaR with measure zero. We therefore define $CoVaR_i$ for bank i as

$$Pr\left(l_p < CoVaR_i \mid l_i \in [VaR_i(1-\epsilon), VaR_i(1+\epsilon)]\right) = 0.5\% \tag{7}$$

where we set $\epsilon = 0.1$.[8] We then calculate

$$\Delta CoVaR_i = CoVaR_i - VaR_p \tag{8}$$

To get the overall capital requirements we scale the results with total capital

$$C_i^{\Delta CoVaR} = \frac{\Delta CoVaR_i}{\sum_i \Delta CoVaR_i} \sum_i C_i. \tag{9}$$

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[8]We found that the capital requirements and the overall results are not significatly different for $\epsilon = 0.15$ or $\epsilon = 0.05$.

2.5 Benchmarks

One benchmark that we use against systemic capital requirements is banks' current capital levels. These might differ from minimum capital requirements as banks want to hold reserves against unexpected losses from risks that are not included in current regulation. Capital levels might also differ due to lumpiness in capital issuance. Most banks have issued new capital before our sample period and individual banks could not have found adequate investment projects for all the funds that they have raised and thus show excessive capital levels. To address the latter problem, we create a second benchmark, for which we redistribute the existing capital such that each bank has the same regulatory capital ratio, which is defined as tier 1 capital over risk weighted assets (RWA).[9] We refer to this benchmark as the "Basel equal" approach for the rest of the paper:

$$C_i^{Basel\ equal} = \frac{\text{RWA}_i}{\sum_i \text{RWA}_i} \sum_i C_i. \tag{10}$$

We now turn to a description of the model used to generate the system loss distribution.

3 A Model of the Banking System

3.1 Description of the model

Our model is designed to include externalities that are caused by a bank's distress or default. When a bank suffers an adverse shock to its asset portfolio and is not fulfilling its capital requirements it will start selling assets to improve its capital ratio. With inelastic demand asset prices will drop causing mark-to-market losses for other banks that hold the same assets. When these losses are high enough, the other banks will start selling, too, initiating a downward spiral in asset prices. We explicitly model these asset fire sale (AFS) externalities using an extension of Cifuentes, Shin, and Ferrucci (2005). When banks default and are therefore not able to pay their obligations in the interbank market they can cause other banks to default as well. We

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[9]Under current Basel capital requirements, banks have to assign a risk weight to each asset that ranges from zero for government backed assets to one for commercial loans. The RWA are the sum of asset values multiplies by their respective risk weight. The Basel accord requires at least 4% tier 1 capital, but countries are free to set higher limits. Canada requires 7%.

model these network externalities explicitly by clearing the interbank market and identifying banks that are in contagious default.[10]

To model the network of interbank obligations we extend the model of Eisenberg and Noe (2001) to include bankruptcy costs and uncertainty as done by Elsinger, Lehar, and Summer (2006a). Consider a set $\mathcal{N} = \{1, ..., N\}$ of banks. Each bank $i \in \mathcal{N}$ has a claim on specific assets A_i outside of the banking system, which we can interpret as the bank's portfolio of non-bank loans and securities. Each bank is partially funded by issuing senior debt or deposits D_i to outside investors. Bank i's obligations against other banks $j \in \mathcal{N}$ are characterized by the nominal liabilities x_{ij} .

The total value of a bank is the value of its assets minus the outside liabilities $A_i - D_i$ plus the value of all net payments to and from counterparties in the banking system. If the total value of a given bank becomes negative, the bank is insolvent. In this case we assume that its assets are reduced by a proportional bankruptcy cost Φ. After outside debtholders are paid off, any remaining value is distributed proportionally to creditor banks. We denote by $d \in \mathbb{R}_+^N$ the vector of total obligations of banks toward the rest of the system, i.e. $d_i = \sum_{j \in \mathcal{N}} x_{ij}$. We define a new matrix $\Pi \in [0, 1]^{N \times N}$ which is derived by normalizing x_{ij} by total obligations.

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

When an institution is unable to meet its obligations, it may be forced to sell assets at prices well below their fair value to achieve a quick sale. We integrate the impact of such AFS of a distressed institution on both its own mark-to-market balance sheet and those of other institutions holding the same class of assets. For this purpose, we extend the work done by Cifuentes, Shin, and Ferrucci (2005), in which banks were assumed equally risky, by differentiating banks according to the riskiness of their assets. We assume that the equilibrium market price of the illiquid assets of a bank is a decreasing function of their riskiness.

For each bank, the stock of outside assets, A_i, is divided into liquid and illiquid assets. Bank i's stock of liquid assets is given by λ_i and includes cash, government's securities and government insured mortgages.[11] Exposures between banks are also assumed liquid for simplicity.

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[10]Jorion and Zhang (2009) provide empirical evidence for contagion from counterparty risk for a sample of US banks and corporations.

[11]We include insured mortgages because they also carry a zero risk-weight under Basel II capital requirements

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The remainder of the bank's assets, e_i, are considered illiquid. The price of the illiquid asset of bank i, p_i, is determined in equilibrium, and the liquid asset has a constant price of 1. Thus the net worth of bank i assuming all interbank claims get paid in full is

$$p_i e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} d_j - d_i - D_i \tag{12}$$

We model capital requirements in the spirit of the Basel II capital accord. Since all liquid assets are backed by the government, they carry a zero risk-weight. Illiquid assets of bank i are assumed to attract a risk-weight equal to the average risk-weight of the bank's balance-sheet, w_i, and the average risk-weight of the banking sector assets is $\overline{w}$. For the mark-to-market value of the banks' illiquid assets to reflect their riskiness, we assume that p_i is a linear function of the equilibrium average price p, and the deviation of the bank risk-weight from the banking sector mean,

$$p_i = \min(1, p + (\overline{w} - w_i)\kappa) \tag{13}$$

where $\kappa > 0$ to ensure that assets sold by a riskier bank have lower mark-to-market value.

We describe a banking system as a tuple $(\Pi, e, \lambda, D, d, p)$ for which we define a *clearing payment vector* X^*. The clearing payment vector has to respect limited liability of banks and proportional sharing in case of default and denotes the total payments made by the banks under the clearing mechanism. We follow the formulation in David and Lehar (2009) to incorporate liquidation costs and define each component of X^* as

$$x_i^* = \min \left[d_i, \max \left((p_i e_i + \lambda_i) \left(1 - \Phi \mathbf{1}_{[p_i e_i + \lambda_i + \sum_j \pi_{ji} x_j^* - D_i < d_i]} \right) + \sum_j \pi_{ji} x_j^* - D_i, 0 \right) \right] \tag{14}$$

To find a clearing payment vector, we employ a variant of the fictitious default algorithm developed by Eisenberg and Noe (2001).

Banks must satisfy a minimum capital ratio which stipulates that the ratio of the bank's Tier 1 capital to the mark-to-market risk-weighted value of its assets must be above some prespecified minimum r^*.[12] When a bank violates this constraint, we assume that it has to sell assets to

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and, as will be explained further below, selling zero risk-weight assets does not improve regulatory capital requirements.

[12]In the numerical exercise, the required minimum is set at 7 percent, as imposed by the Canadian regulator.

reduce the size of its balance-sheet.[13] We denote by s_i the units of illiquid assets sold by bank i.[14] Whereas Cifuentes, Shin, and Ferrucci (2005) used a simple (non risk-weighted) leverage ratio, our constraint is closer in spirit to the Basel II accord in which banks have to hold capital commensurate with the risk on their balance sheet.[15] Our minimum capital requirement is therefore given by

$$\frac{p_i e_i + \lambda_i + \sum_j \pi_{ji} x_j - x_i - D_i}{w_i p_i (e_i - s_i)} \geq r^*. \tag{15}$$

The numerator is the equity value of the bank where the interbank claims and liabilities are calculated in terms of the realized payments. The denominator is the marked-to-market risk-weighted value of the bank's assets after the sale of s_i units of the illiquid assets. The underlying assumption is that assets are sold for cash and cash does not have a capital requirement. Thus if the bank sells s_i units of the illiquid assets, the value of the numerator is unchanged since this involves only a transformation of assets into cash, while the denominator is decreased since cash has zero risk-weight. Thus, by selling some illiquid assets, the bank can increase the regulatory capital ratio.[16]

We assume that banks cannot short-sell assets, i.e. $s_i \in [0, e_i]$. An equilibrium is the triple (X^*, S^*, p^*) consisting of a vector of payments, vector of sales of illiquid assets , and vector of prices p of the illiquid assets such that:

- For all banks $i \in \mathcal{N}$, x_i^* is determined according to equation (14).

- For all banks i, s_i^* is the smallest sale that ensures that the capital adequacy condition is satisfied. If there is no value of $s_i \in [0, e_i]$ for which the capital condition is satisfied then $s_i^* = e_i$.

- There is a downward sloping demand curve $q^{-1}(.)$ such that $p^* = q^{-1}(\sum_i s_i^*)$.

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[13]We do not consider the possibility of raising fresh capital nor the need to sell assets because of a loss of funding. The consequences of the latter would be similar to those described here, assuming the assets would have to be sold at a discount.

[14]Selling liquid assets does not help to reduce the size of the balance-sheet because of their zero-risk weight. Note however, that holding more liquid assets reduces the size of the balance-sheet ex-ante.

[15]In addition to the distinction between liquid and illiquid assets done in Cifuentes, Shin, and Ferrucci (2005), we differentiate banks according to the riskiness of their illiquid assets. Without doing so, riskier banks would unrealistically find it easier to reduce the size of their balance-sheets.

[16]A decrease in price should be seen as the average price decrease of all the illiquid assets on the balance-sheet, some assets' price potentially being unaffected while others suffering from huge mark-to-market losses.

The first condition reiterates the limited liability of equity holders, and the priority of debt holders over interbank liabilities. The second condition says that either the bank is liquidated altogether, or its sales of illiquid assets reduces its assets sufficiently to comply with the capital adequacy ratio. Finally, the third condition stipulates that the price of the illiquid assets is determined by the intersection of a downward-sloping demand curve and the aggregate supply curve.

The inverse demand curve for the illiquid asset is assumed to be

$$p = e^{-\alpha\left(\sum_i s_i\right)} \tag{16}$$

where is α a positive constant. We define $p_{min} = p(\sum_i e_i)$ as the lowest average price for the illiquid assets when all assets are sold.[17]

By rearranging the capital condition in Equation (15), we can write the asset fire sale s_i as a function of p_i, where $s_i = 0$ if capital is already above the minimum required,

$$s_i = \min\left(e_i, \max\left(0, \frac{(1 - r^* w_i)p_i e_i + \lambda_i + \sum_j x_i \pi_{ji} - D_i - x_i}{-r^* w_i p_i}\right)\right) \tag{17}$$

Since each $s_i(p)$ is decreasing in p, the aggregate sale function is decreasing in p. The lower the price, the lower the mark-to-market value of banks' assets, and the bigger the need to sell assets to bring capital ratios in line with the required regulatory minimum. The price adjustment process is illustrated in Cifuentes, Shin, and Ferrucci (2005).

An equilibrium price of the illiquid asset is a price p^* such that aggregate supply is equal to demand

$$p^* = q^{-1}\left(\sum_i s_i\left(p^* + (\overline{w} - w_i)\kappa\right)\right) \tag{18}$$

From the solution of the clearing problem, we can gain additional economically important information with respect to systemic stability. A bank is in default whenever it cannot meet its interbank obligations, $(x_i^* < d_i)$. We refer to the default of bank i as fundamental if bank i is

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[17]The demand curve (parameter α) and the asset price function (parameter κ) need to be calibrated such that an equilibrium price exists for all potential positive levels of aggregate supply. We assume an exogenously fixed lower bound on the asset price and then calibrate α accordingly.

not able to honour its promises under the assumptions that all other banks honor their promises and that prices are not affected by AFS ($p = 1$)

$$e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} d_j - D_i < d_i \tag{19}$$

We define a bank to default because of AFS, whenever the bank is not in fundamental default but cannot honour its interbank obligations at the equilibrium price of the illiquid assets, even when all other banks meet their interbank obligations. An AFS default occurs when

$$e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} d_j - D_i > d_i \quad \text{and}$$

$$p_i^* e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} d_j - D_i < d_i \tag{20}$$

A contagious default occurs, when bank i defaults only because other banks are not able to keep their promises, i.e.,

$$p_i^* e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} d_j - D_i > d_i \quad \text{but}$$

$$p_i^* e_i + \lambda_i + \sum_{j=1}^{N} \pi_{ji} x_j^* - D_i < d_i \tag{21}$$

To use the model for risk analysis, we model shocks to banks' asset values by introducing a distribution of banks' credit losses as described in Section 4. As there is no closed form solution for the distribution of the clearing vector X^*, we have to resort to a simulation approach where each draw from the credit loss distributions, which we refer to as a scenario, maps into new asset values for each bank. We solve the clearing problem for each scenario numerically. Thus from an ex-ante perspective we can assess expected default frequencies, and decompose insolvencies across scenarios into fundamental, AFS, and contagious defaults.

3.2 Model properties

Our network model is able to capture two properties that we believe are important in modeling systemic risk: spillover effects and feedback loops. When a bank gets in distress it sells assets or defaults on its interbank claims, causing externalities for other banks. An increase in a bank's PD will therefore make asset fire sales and default in the inter-bank market more likely, and therefore increase the PDs of the other banks in the system. This spillover effect makes the correlation of bank asset values dependent of the health of the overall system. When all banks are well capitalized, asset fire sales and interbank defaults are unlikely and correlation of banks asset portfolios is driven by the correlation in the outside assets, i.e. the loan portfolios, alone. In an asset fire sale or contagion scenario, all asset values fall, exhibiting a correlation close to one. As the capitalization of the whole banking system decreases, the probability of realizing the asset fire sale or contagion scenario increases and thus the ex-ante asset and default correlation.

The feedback effect is driven by the asset fire sale externalities and is described in detail in Cifuentes, Shin, and Ferrucci (2005). When a bank starts selling assets, prices drop, which causes other banks to violate their capital requirements and forces them to sell assets as well, causing all banks' asset values to drop even further. This feedback effect accelerates bank defaults as bank capitalization decreases.

To illustrate these two effects we compare our model to a Merton model. Figure 1 shows the PDs of banks 2 and 3 in our sample for different levels of bank three's capitalization while leaving the other banks' capital levels unchanged. The solid line represents the PD for bank 3 under the network model. As a benchmark we calibrate a Merton model such that the one year PD and its first derivative of the Merton model match the PD and its derivative of the network model.[18] The resulting PD from the Merton model (dashed line) is below the PD from the

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[18]Given that we compute all values under a macro stress test scenario, we cannot use stock market data to calibrate the Merton model. The Merton model gives us a bank's PD given the market value of the assets V, the face value of debt, the asset growth rate μ and volatility σ, and a time horizon. We assume a time horizon of one year, set the face value of debt equal to $D_i + d_i$, and assume $\mu = 0.05$. The bank's PD is then given as $PD = N(d_2)$ with $d_2 = (ln(V/(D_i + d_i)) + \mu - \sigma^2/2)/\sigma$. We then solve for the two unknowns V and σ by solving the following two conditions: First, we require the Merton-PD to match the PD from our model. Second the partial derivative of the PD with respect to changes in book value of equity E have to be equal for both models. Specifically we define book value of equity as $E = V - D_i - d_i$ and then match $\partial PD/\partial E$ from the Merton model with the corresponding value from the network model. We use book value of equity because for regulatory purposes bank equity is defined in book value terms. Our conditions are chosen to enable a fair comparison between the models since we want to analyze how the PD changes with bank capital. The graphs look similar for the other banks and the results are robust with respect to different asset value growth rates μ.

Figure 1. Sensitivity of bank PDs with respect to changes in bank capital: The solid and dashed lines show bank three's probability of default (PD) for different levels of tier 1 capital in the Macro-stress scenario under the network model and the Merton Model, respectively. The Merton model is calibrated to match the PD of the network model for bank three's observed level of tier 1 capital. The dotted line shows how bank two's PD changes with changes in bank three's capital under the network model.
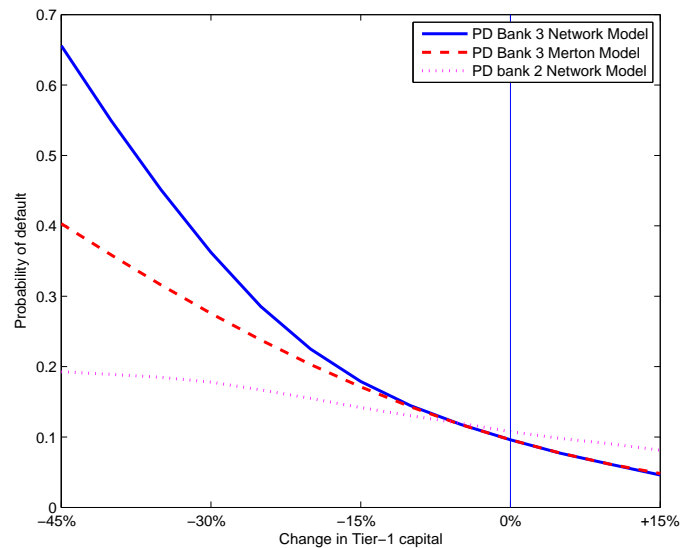
network model. The sharp increase in the PD of the network model is driven by the feedback effect.

The dotted line in Figure 1 illustrates the spillover effect by depicting bank 2's PD as a function of bank 3's capital. In a Merton framework correlations in bank defaults can only be driven by asset correlations but changing one bank's capitalization has no effect on other banks' PDs. Asset fire sales and contagion externalities cause an increase in bank 2's PD as bank 3 reduces its capital.

4 Data and credit loss distributions

4.1 Simulation of credit losses

For the modeling of credit losses, we first generate a stress scenario in which average default rates in each sector are specified. Depending on composition of their loan portfolio, all banks are affected by this shock to a certain extent. We then use an extended CreditRisk+ model to simulate individual loan losses for each bank.

The first model generates sectoral default rates that capture systematic factors affecting all banks' loans simultaneously. It relates the default rates of bank loans in different sectors to the overall performance of the economy as captured by a selected set of macroeconomic variables. The specification adopted for the model allows for non-linearities. Historical proxies for sectoral default rates were constructed based on sectoral bankruptcy rates.[19] The included macroeconomic variables are GDP growth, unemployment rate, interest rate (medium-term business loan rate), and the credit/GDP ratio. We simulate sectoral distributions of 10,000 default rates for 2009Q2 under a severe recession macro scenario.[20] Descriptive statistics of these distributions as well as historic peaks over the 1988-2006 period are presented in Table 1. Consistent with the severity of the macro scenario, mean default rates are much higher than historic peaks.[21]

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[19]The sectoral classification used in constructing the default rates is the one used by banks in reporting their balance sheet loan exposures. The seven sectors included were accommodation, agriculture, construction, manufacturing, retail, wholesale, and mortgages in the household sector. For more details on the construction of historical default rates, see Misina and Tessier (2007).

[20]We use the macroeconomic scenario that was designed for the macro stress-testing exercise conducted as a part of Canada's Financial Sector Assessment Program (FSAP) update with the IMF in 2007. With a recession that is about one-third larger than experienced in the early 1990s, the stress-test scenario is plausible but extreme. See Lalonde, Misina, Muir, St-Amant, and Tessier (2008) for a detailed description of the scenario. Sectoral distributions of default rates are centered on fitted values from sectoral regressions, and are generated using the correlation structure of historical default rates. See Misina, Tessier, and Dey (2006) for more details on the simulation of default rates.

[21]A key component in modeling credit losses is banks' sectoral Exposure-at default (EAD). Since the sectoral classification used for reporting EADs by banks under Basel II is more aggregated than the one used in constructing default rates, we simulated for each Basel II sector a distribution of weighted average default rate (PD_w) according to:

$$PD_w = \sum_{i=1}^{k} \frac{BSE_i}{\sum\limits_{i=1}^{k} BSE_i} PD_i$$

where k represents the number of balance sheet sub-sectors that can be subsumed in one of the Basel II sectors, PD_i represents our model's default rates of sector i, and BSE_i the corresponding balance sheet exposure.

Table 1. Summary statistics of simulated default rate distributions. Columns two to four show the minimum, maximum and average default rates generated for each sector. Column five gives the historic peak over the 1988-2006 period.

	Minimum	Average	Maximum	Historic Peaks
Accommodation	3.0	11.7	21.0	7.6
Agriculture	1.0	1.7	2.0	0.8
Construction	2.0	6.4	10.0	3.3
Manufacturing	5.0	12.2	20.0	8.3
Retail	0.0	4.3	8.0	5.3
Wholesale	2.0	7.0	12.0	4.6
Mortgage	0.0	0.6	1.0	0.6

In order to capture idiosyncratic risk factors arising from the size distribution of banks' exposures, we use an extended CreditRisk+ model as in Elsinger, Lehar, and Summer (2006a).[22] A key input in this model is banks' loan portfolios compositions by sector, which are estimated using the Bank of Canada Banking and Financial Statistics, and private data on banks' largest exposures towards non-banks obtained from the Office of the Superintendent of Financial Institutions (OSFI). For each bank, we draw 100 independent loan loss scenarios for each of the $10,000$ sectoral default rates simulated previously, yielding a total of $1,000,000$ sectoral loan loss scenarios. The individual distributions of total expected losses are derived by adding expected sectoral loan losses. Other key inputs include: the Loss-given-default (LGD) assumed to be 50 percent[23] and the sectoral Exposure-at-default (EAD) as reported by banks.[24]

Table 2 shows the importance of considering both sources of uncertainty. When considering systematic factors only, expected losses to the 6 big banks average $45.7 Billion or 47.7 percent of total Tier 1 Capital, with a standard deviation of $7.9 Billion. Taking both systematic and idiosyncratic factors into account, the expected losses are approximately the same ($46.4 Billion on average), and, not surprisingly, are larger in the tail of the distribution (the 99 percent VaR

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[22]A recent overview on different standard approaches to model credit risk is Crouhy, Galai, and Mark (2000). CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in Credit Suisse (1997).

[23]There is little information on loss-given-default in Canada. Based on available information from the Office of the Superintendent of Bankruptcy, Misina, Tessier, and Dey (2006) estimated an average loss-given-bankruptcy over the 1988-2006 period of 65%. This overstates losses in case of default because bankruptcy is the last stage of distress, and includes more than losses related to missed interest payments.

[24]Under Basel II, banks are required to provide an estimate of the credit exposure of a facility, should that facility go into default at the risk horizon (typically one year).

is $68.7 Billion as compared to $63.7 Billion in the first distribution). As a consequence, the frequency of bank defaults varies from zero when only systematic factors are considered, to 0.06 percent on average (ranging between 0 and 0.19 percent) with both sources of uncertainty.

Table 2. Aggregate losses and probabilities of default due to credit risk. Panel A provides descriptive statistics of aggregate losses considering systematic factors only (Column 1) and both systematic and idiosyncratic factors (Column 2). Panel B gives frequencies of bank defaults.

	Systematic factors		Systematic and idiosyncratic factors	
	$Billion	%of Tier1 capital	$Billion	%of Tier1capital
Panel A: Descriptive statistics of aggregate loss distributions				
Mean	-45.7	47.7	-46.4	48.5
Standard Deviation	7.9	8.38	9.5	9.9
Quantiles:				
99%	-27.3	28.5	-25.7	26.9
10%	-55.8	58.4	-58.8	61.4
1%	-63.7	66.6	-68.7	71.8
Panel B: Frequencies of bank defaults (%)				
Minimum	0		0.00	
Average	0		0.06	
Maximum	0		0.19	

4.2 Data on exposures between banks

As in previous studies of systemic risk in foreign banking systems [see among others Sheldon and Maurer (1998), Wells (2002) and Upper and Worms (2004)], our data cover exposures between banks that arise from traditional lending (unsecured loans and deposits). We improve on previous literature by expanding this set of exposures to also cover another on-balance sheet item, cross-shareholdings, and off-balance sheet instruments such as OTC derivatives. While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify. Our expanded dataset enables us to better capture linkages among banks and contagious bank defaults.[25] Of course, other types of exposures

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[25]Zero-risk exposures, mainly repo style transactions, were excluded despite their large size. They account for more than 80% of total exposures between the Big Six Canadian banks in our sample. These exposures may represent a contagion channel in time of liquidity crisis, but this is left for future work.

between banks exist - most notably those coming from intraday payment and settlement, from bank holdings of preferred banks' shares (and other forms of capital), and from holdings of debt instruments issued by banks like debentures and subordinated debt. Owing to data limitations, however, the latter are not considered in this work.

Data on the exposures are collected on a consolidated basis and come from different sources as described below. Available data are collected for May 2008 with the exception of exposures related to derivatives which are recorded as of April 2008. We present descriptive statistics in Table 3. Data on deposits and unsecured loans come from the banks' monthly balance-sheet reports to OSFI. These monthly reports reflect the aggregate asset and liability exposures of a bank for deposits, and only aggregate asset exposures for unsecured loans. Data on exposures related to derivatives come from a survey initiated by OSFI at the end of 2007. In that survey, banks are asked to report their 100 largest mark-to-market counterparty exposures that were larger than $25 million. These exposures were related to OTC derivatives. They are reported after netting and before collateral and guarantees.[26] Data on cross-shareholdings exposures were collected from Bank of Canada's quarterly securities returns.[27]

The aggregate size of interbank exposures was approximately $21.6 billion for the six major Canadian banks. As summarized in Table 3, total exposures between banks accounted for around 25% of bank capital on average. The available data suggest that exposures related to traditional lending (deposits and unsecured loans) were the largest ones compared with mark-to-market derivatives and cross-shareholdings exposures. Indeed, in May 2008, exposures related to traditional lending represented around $12.7 billion on aggregate, and 16.3% of banks' Tier 1 capital on average. Together, mark-to-market derivatives and cross-shareholdings represented 10% of banks' Tier 1 capital on average.

A complete description of linkages between banks requires a complete matrix of the bilateral exposures. Such a complete matrix was available only for exposures related to derivatives. Unavailable bilateral exposures were estimated under the assumption that banks spread their lending and borrowing as widely as possible across all other banks using an entropy maxi-

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[26]Anecdotal evidence suggests that the major Canadian banks often rely on collateral to mitigate their exposures to OTC derivatives. However, as Stulz (2009) points out, even full collateralization can leave a bank with counterparty risk.

[27]These returns provide for each bank aggregate holdings of all domestic financial institutions' shares. Due to data limitations, cross-shareholdings among the Big Six banks were estimated by (i) distributing the aggregate holdings of a given bank according to the ratio of its assets to total assets of domestic financial institutions, and (ii) excluding shares that were held for trading (assuming that they are perfectly hedged).

Table 3. Summary statistics on exposures between Banks. Panel A gives the aggregate size of interbank exposures related to traditional lending, derivatives and cross-shareholdings (reported in $billion and as percentage of banks' Tier 1 capital). Panel B gives banks' bilateral exposures as percentage of Tier 1 capital under two assumptions: entropy maximization and relationship banking.

Panel A: Aggregate exposures between banks				
	Agregate exposure ($Billion)	As percentage of Tier1 capital		
		Minimum	Average	Maximum
Traditional lending	12.7	5.25	16.3	38.6
Derivatives exposures	5.4	0.0	5.9	21.1
Cross-shareholdings	3.5	0.3	4.1	8.8
Total exposures	21.6	5.5	26.4	51.2
Panel B: Banks' bilateral exposures as percentage of Tier 1 capital				
Assumption:		Minimum	Average	Maximum
Entropy maximization		0.6	4.4	15.6
Proportional to asset size		0.5	4.4	16.2

mization algorithm (see e.g. Blien and Graef (1997)). A difficulty with this solution is that it assumes that all lending and borrowing activities between banks are completely diversified. As a robustness check, banks' bilateral exposures were also estimated under the assumption that concentrations of exposures between banks are broadly consistent with their asset sizes. As shown in Table 3, banks' bilateral exposures are comparable under these two assumptions. Indeed, they are consistent with the concentration of bilateral exposures related to derivatives (not shown).

5   Results

In this section, we first present the results from simulations considering the two contagion channels described above. Credit losses were generated using systematic and idiosyncratic factors. We then address the probability of a systemic crisis and move towards identifying systemic banks. Finally, we compare the outcome of different mechanisms of capital allocation according to individual contribution to systemic risk.

Table 4. Bank defaults in the base scenario (P_{min}=0.98, N = 1 million): A bank is in fundamental default, when it defaults given that all interbank claims are paid in full. Contagious defaults are recorded when a bank defaults just because other banks do not honour their interbank commitments assuming that no asset fire sales occur. AFS PD refers to the probability of default stemming from write downs due to asset fire sales. Contagious PD (AFS) includes the defaults that arise because of contagion after writedowns have been realized in banks' balance sheets. Total PD (AFS) is the sum of columns 2,4, and 5.

Bank	Fundamental PD (%)	Contagious PD (%)	AFS PD (%)	Contagious PD (AFS) (%)	Total PD (AFS) (%)
1	0.00	0.00	2.96	3.47	6.43
2	0.15	0.00	9.09	0.94	10.19
3	0.00	0.00	2.99	6.33	9.31
4	0.01	0.00	6.50	3.93	10.45
5	0.00	0.00	1.61	6.09	7.70
6	0.19	0.01	4.53	6.93	11.65

5.1 The impact of contagion channels

Table 4 reports the results obtained from simulations considering potential contagion from network and asset fire-sale effects for 1 million generated credit losses assuming that the minimum price of the illiquid asset $P_{min} = 0.98$, i.e. the stock of illiquid assets at each bank can at most lose 2% in value, even when banks sell all their holdings.[28] Two percent write-down of all banks' illiquid assets (defined as all balance-sheet assets but cash, government securities and insured mortgages) represent roughly $40 billion. As a comparison, Canadian banks recorded $22 billion losses in the trading book alone from the onset of the current crisis to the third quarter of 2009.

A default is called fundamental when credit losses are sufficient to wipe out all the capital of the bank as defined in Equation (19). Column three in Table 4 summarizes the PD from defaults due to interbank contagion without asset fire sales, i.e. when a bank has sufficient capital to absorb the credit losses in the non-bank sectors but is pushed to bankruptcy because of losses

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[28]Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) assume maximum declines in asset values between 2% and 5%, depending on the asset class. Still, even a 2% drop in asset value is quite severe given the high leverage of banks. For example, a bank with 5 units of capital, an average risk-weight of 0.5 and 200 units of illiquid assets would see its capital ratio going from 5% to a little less than 1% after a 2% marked-down of its illiquid assets.

on its exposures to other banks (as defined in Equation (21) with $p^* = 1$). The third category of default is the contagion due to asset fire sales as defined in Equation (20). In these cases a bank has enough capital to withstand both the credit losses in the non-banking sector and the write-down on other banks' exposures but not enough to withstand the mark-to-market losses due to its own asset fire sale and/or the asset fire sale of the other banks. Asset fire sales will also weaken the surviving banks and thus make them more prone to contagion as reflected in column five (Contagious PD (AFS)), which records the defaults defined in Equation (21) with $p^* < 1$. Total PD is the sum of columns two, four, and five and summarizes the total PD of a bank including the effect of asset fire sales.[29]

The Canadian banking system is very stable without the consideration of asset fire sales. Both fundamental and contagious PDs are well below 20 basis points, even though we generate high loan losses due to an extreme macro scenario throughout the paper. In the asset fire sale scenario, troubled banks want to maintain regulatory capital requirements by selling off assets, which causes externalities for all other banks as asset prices fall. PDs increase and as banks get weaker because of writedowns they also become more susceptible for contagion. We find that banks 2, 4 and 6 have a higher risk of defaulting due to asset writedowns. While banks 1, 3 and 5 are more likely to survive writedowns, losses to bank 3 and 5 weaken them substantially so that they are more likely to default due to second round interbank contagion. Overall, banks have PDs ranging from 6.43 to 11.65. Bank 1 and 5 stand out with the lowest PDs while bank 6 has the highest one. The bankruptcy probability jumps dramatically as market liquidity decreases. In columns two to four of table 5 we allow asset prices to drop 50% more than in the base scenario. We immediately see that our analysis is very sensitive to the minimum asset price, which defines our demand function for the illiquid asset. Allowing fire sale discounts of three percent increases banks' PDs significantly. All banks default almost in two out of three cases. Default correlation is almost one (not shown), which explains why the total PDs are almost identical. While banks 2 and 4 are very likely to default because of writedowns in the value of their illiquid assets, banks 3, 6 and especially bank 5 are more likely to be affected by contagion. Two possible reasons could explain why these results are so sensitive to asset fire sale discounts. First, high Tier 1 capital requirements of 7%, compared to the 4% under Basel rules, trigger asset fire sales early, causing other banks to follow. Second the small number of

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[29]Following the insight of Morris and Shin (2009), one can interpret the total PD as total credit risk or the unconditional probability of default, and see total credit risk as the sum of the fundamental PD (the PD conditional on not having liquidity problems because of an AFS or interbank funding problems) and the PD due to liquidity problems.

Table 5. Severe asset fire sales and partial interbank data, N = 1 million: AFS PD refers to the probability of default stemming from writedowns due to asset fire sales. Contagious PD (AFS) includes the defaults that arise because of contagion after writedowns have been realized in banks' balance sheets. Total PD (AFS) is the sum of fundamental PD (from Table 4), AFS PD, and contagious PD (AFS). Columns 2 to 4 show PDs when minimum price of the illiquid asset is reduced to $P_{min} = 0.97$. Columns 5 to 7 repeat the base scenario considering only exposures from interbank deposits.

Bank	AFS PD	Contagious PD (AFS)	Total PD	AFS PD	Contagious PD (AFS)	Total PD
	(All data, P_{min}=0.97, in %)			(IB deposits only, P_{min}=0.98, in %)		
1	33.58	24.84	58.42	2.44	2.44	4.86
2	57.64	3.92	61.74	8.85	0.37	9.37
3	29.05	34.23	63.29	2.51	0.68	3.19
4	55.06	8.63	63.71	6.50	2.22	8.73
5	17.17	44.67	61.84	1.12	1.57	2.69
6	26.88	36.86	63.94	4.28	3.63	8.10

banks causes each bank to have a huge price impact when selling off illiquid securities, creating negative externalities for the whole system.[30]

Two policy insights stand out from the latter results. First, in the last financial crisis, regulators were criticized for helping banks to offload assets from their balance sheets at subsidized prices and for relaxing accounting rules, which allowed banks to avoid mark-to-market writedowns of their assets. While our analysis cannot show the long-term costs associated with these measures, we can at least document that there is a significant immediate benefit for financial stability by preventing asset fire sale induced writedowns. Second, a countercyclical reduction in the minimum Tier 1 capital requirements would reduce the risk of default triggered by AFS dramatically.

The right part of Table 5 presents the results when the matrix of exposures between banks is restricted to interbank deposits. Since most of the previous literature analyzing exposures between banks focuses on the interbank deposit market, these results serve as a good benchmark of the models in previous papers. Without cross-shareholdings and derivatives exposures we get substantially lower PDs for some banks as well as a very different ranking of banks' PDs. Given that most regulators around the world do not have access to data for derivatives exposure

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[30]Besides the systemic aspect of the individual banks, the calibration of the demand curve is a key factor, and requires judgment about the health and willingness to buy of other market players under the scenario.

Table 6. Probability of multiple bank defaults: column two lists the probabilities that one to six banks default simultaneously in the given macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$. Columns three to eight show for each row n the probability that a particular bank defaults given that a total of n banks default.

Number defaults	Probability (in %)	probability of involvement of bank					
		1	2	3	4	5	6
1	3.53	4.64	30.86	6.97	15.49	0.75	41.28
2	1.16	9.07	47.48	20.70	49.03	3.44	70.29
3	0.84	14.60	57.19	45.28	83.50	12.40	87.03
4	1.11	22.21	69.86	81.68	95.25	34.96	96.04
5	2.66	32.17	88.55	97.95	99.15	82.69	99.48
6	4.94	100.00	100.00	100.00	100.00	100.00	100.00

between banks, our analysis shows that inaccurate data can lead to severe underestimation of systemic risk by regulators' offsite analysis models.

5.2 The probability of a financial crisis

So far we examined the impact of contagion through linkages among banks and asset fire sales on the individual bank's riskiness. In this section we want to address the probability of a systemic crisis and move towards identifying systemically relevant banks.

Table 6 shows the probability that one to six banks default simultaneously in our stress scenario. In almost 25% of the cases in which a default is observed, contagion is contained as only one bank defaults. However, default correlation is relatively high as five or six banks would default in more than fifty percent of the cases. For each row n, Columns three to eight show the PD of each bank conditional on the default of n banks. Bank 1 and 5 have a low default correlation with other banks as they are less likely to default in scenarios of multiple defaults. In contrast, banks three, four and six are almost always involved when three or more banks default.

Identifying which banks contribute most to the probability of a systemic crisis is not straightforward. One approach is to see which banks are most often involved in multiple bank defaults. Table 7 shows the likelihood that one to six banks will default conditional on the default of a specific bank, i.e. row 2 column 6 shows the probability that exactly four banks default, con-

Table 7. Probability of multiple bank defaults conditional on a bank's default: column two contains for each bank one to six the PD in the given macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$. Columns three to eight show for each row n the probability of one to six total defaults conditional on bank n's default.

Bank	PD	Cond. probability of multiple defaults					
	(in %)	1	2	3	4	5	6
1	6.43	2.55	1.64	1.91	3.82	13.28	76.80
2	10.19	10.70	5.41	4.72	7.60	23.08	48.50
3	9.31	2.64	2.58	4.09	9.71	27.93	53.05
4	10.45	5.24	5.45	6.72	10.10	25.21	47.30
5	7.69	0.35	0.52	1.35	5.03	28.54	64.21
6	11.65	12.51	7.00	6.28	9.13	22.68	42.41

ditional on the default of bank 2. Again, mainly because of contagion due to AFS, we see that banks' defaults are correlated as the default of any bank is associated with the wipeout of at least four other banks with close to 80% probability on average. Based on this approach, banks 2 and 6 are less involved in a systemic crisis, as there is more than a 10% chance that their default will be contained and not associated with a domino effect. They are indeed the only two banks with a lower than 80% probability of being involved when more than 3 banks default. On the opposite, bank 5 is most likely to be involved in more than three defaults.

In the recent crisis there has been a lot of discussion on the topic of systemically relevant banks. Similar to Elsinger, Lehar, and Summer (2006b) we compute in Table 8 for each bank the probability of default conditional on one other bank's default. We can see again that banks in our sample are interdependent. The default of any one bank is associated with the default of any other bank with a probability of more than 50%. Consistent with the results in table 7, the defaults of banks one and five (two and six) are correlated the most (least) with other banks' defaults.

5.3 Macroprudential capital requirements

Macroprudential capital requirements attempt to internalize the externalities in the financial system. The results presented above show that network and asset fire sales externalities are important and should be taken into account in designing capital requirements policy. Table 9

Table 8. Individual bank default probability conditional on the default of a specific bank in the given macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$: Column 2 shows the PD for each bank. Columns 3 to 8 of row n show each bank's PD conditional of bank n's default.

Bank	PD	Cond. Default Probability (%)					
		1	2	3	4	5	6
1	6.43	-	87.57	92.99	94.72	83.59	95.20
2	10.19	55.30	-	78.31	83.02	69.32	86.51
3	9.31	64.23	85.66	-	94.06	78.97	93.94
4	10.45	58.33	80.97	83.86	-	72.33	91.00
5	7.69	69.88	91.78	95.59	98.19	-	98.09
6	11.65	52.56	75.64	75.09	81.59	64.78	-

shows the results of the five capital allocation procedures presented in Section 2 in percent of actual observed capital requirements. All allocation rules (except for the $\Delta CoVaR$ in the case of Bank 4) suggest that Banks 2, 4 and 6 are undercapitalized, while banks 3, and 5 are over-capitalized, from a macroprudential viewpoint. This is consistent with the results of Table 5 showing the relatively higher (lower) default probabilities of undercapitalized (overcapitalized) banks. Bank 6 for example should increase capital by as much as 50% according to the $\Delta CoVaR$ methodology. Results are less clear for Bank 1. While three procedures find it to be undercapitalized, the other two suggest the reverse. This is in line with mixed indications on Bank 1's systemic importance as shown in Tables 6 to 8.

Table 10 shows correlations between macroprudential capital ratio, defined as capital over total assets, and some bank's characteristics. Macroprudential capital requirements are not trivially related to individual bank PDs or size. The correlation of adjusted capital ratios and total assets are between -0.72 and -0.25 with four out of five not being significantly different from zero. This finding is not consistent with the suggestion that larger banks pose a greater threat to systemic risk and should be required to hold more capital.[31] The lack of correlation with PDs as well, suggest that macroprudential capital requirements is likely based on a combination of factors. Interestingly, holdings of interbank assets as a proportion of total assets are significantly and positively correlated with macroprudential capital requirements, whereas that correlation is negative and only marginally significant for interbank liabilities. Our results are therefore con-

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[31]This may be specific to Canada, given the relatively similar size of the major banks. The smallest bank in our sample holds 5% of total assets, the largest bank has 26%.

Table 9. Capital requirements for selected macroprudential capital allocation rules in percent of observed tier 1 capital (in %): Capital requirements are computed such that they match the risk contributions under the five risk allocation mechanisms. Loss distributions are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$

Bank	Component VaR	Incremental VaR	Shapley value		$\Delta CoVaR$
			Expected loss	VaR	
1	95.33	104.56	105.18	105.25	112.39
2	103.95	101.91	102.52	102.34	92.98
3	96.17	92.95	92.62	92.54	89.71
4	110.44	114.07	113.11	113.47	107.57
5	91.74	89.23	89.20	89.14	76.31
6	106.66	104.26	104.31	104.38	212.88

Table 10. Correlation between macroprudential capital ratio, defined as capital over total assets, and bank characteristics. One, two, and three starts correspond to significance at the ten, five, and one percent level, respectively. Loss distributions are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$

Bank characteristic	Component VaR	Incremental VaR	Shapley value		$\Delta CoVaR$
			Expected loss	VaR	
Total Assets (TA)	-0.25	-0.41	-0.41	-0.42	-0.86**
Interbank assets/TA	0.74**	0.83**	0.82**	0.83**	0.73*
Interbank Liabilities/TA	-0.57*	-0.71*	-0.73*	-0.72*	-0.02
Total PD	0.35	0.22	0.20	0.20	0.55
Tier 1/Risk weighted assets	-0.49	-0.36	-0.31	-0.32	-0.26
Tier 1/TA	0.88**	0.91***	0.92***	0.92***	0.07

Table 11. Individual bank default probability under selected macroprudential capital allocation rules (in %). Default probabilities are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
1	6.43	9.05	6.60	3.91	3.75	3.73	1.04
2	10.19	9.97	7.68	8.15	7.91	7.97	15.46
3	9.31	8.91	8.34	8.82	8.87	8.91	10.53
4	10.45	9.04	6.72	5.77	5.91	5.85	6.28
5	7.70	7.73	7.55	7.76	7.73	7.74	13.93
6	11.65	10.53	8.28	8.49	8.44	8.43	0.00
Average	9.29	9.21	7.53	7.15	7.10	7.11	7.87

sistent with a bank's macroprudential requirement being an insurance against potential losses caused by its interbank counterparties, and not against losses it may cause to them. Finally, macroprudential capital requirements are not significantly related to regulatory capital ratios, but strongly correlated with a leverage ratio measured as Tier 1/TA. This supports the current international initiatives to regulate leverage ratios.

Compared to the observed capital levels, all macroprudential capital allocations reduce the default probability of the average bank. Table 11 shows the default probabilities of the six banks under the observed capital ratio, the "Basel equal" benchmark, as well as under the five macroprudential allocations. The Component VaR method creates relatively homogeneous PDs for all banks, increasing the likelihood of default for bank 1, but making banks 2 to 6 less prone to default. The incremental VaR and Shapley allocations leave bank 3 and 5's PD relatively unchanged but reduce the PD for banks 1 and 4 and 6 substantially. The $\Delta CoVaR$ reduces Bank 6 PD dramatically.

All five macroprudential capital requirements reduce risk in the banking system. Table 12 presents the probability of multiple bank defaults for the five capital allocation rules. Especially incremental VaR and the Shapley value allocations reduce the probability of multiple bank failures significantly. Under Shapley value based capital allocations the probability of five or six banks defaulting can be reduced from 7.6%, which is based on current banks' capital levels, to 5.5%. This corresponds to a 27% reduction in the probability of a financial crisis.

Table 13 computes the expected loss under the different capital allocation mechanisms. The

Table 12. Probability of multiple bank defaults under selected macroprudential capital allocation rules (in %): The table shows the probabilities that one to six banks will default simultaneously. Default probabilities are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$. Column two is identical to column two in Table 6.

Number defaults	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
1	3.53	3.55	3.01	3.43	3.39	3.41	10.20
2	1.16	1.05	0.85	1.11	1.09	1.10	4.77
3	0.84	0.60	0.51	0.75	0.73	0.75	3.47
4	1.11	0.70	0.67	1.08	1.06	1.08	3.78
5	2.66	1.70	1.71	2.49	2.58	2.57	0.31
6	4.94	6.09	4.62	3.04	2.95	2.93	0.00
≥ 5	7.60	7.78	6.33	5.53	5.53	5.50	0.31
≥ 4	8.70	8.48	7.00	6.60	6.59	6.59	4.09

Table 13. Expected loss in percent of total assets under selected macroprudential capital allocation rules: Expected loss is defined as negative equity capital in the case of default and zero otherwise. Default probabilities are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$.

Number defaults	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
1	0.55	0.84	0.59	0.33	0.31	0.31	0.00
2	0.96	0.94	0.72	0.77	0.75	0.76	0.01
3	0.93	0.89	0.85	0.90	0.91	0.91	0.02
4	0.99	0.84	0.61	0.52	0.53	0.53	0.00
5	0.73	0.73	0.76	0.78	0.78	0.78	0.03
6	1.18	1.06	0.82	0.85	0.84	0.84	0.00
Total	0.22	0.22	0.18	0.17	0.17	0.17	0.00

expected loss equals the fair deposit insurance premium assuming that all bank liabilities are insured. We can again see that macroprudential capital requirements can substantially reduce bank risk compared to capital levels under the current regulatory regime.

6   Conclusions

One objective of macroprudential regulation is to internalize the externalities within the financial system. In this paper, we find that financial stability can be enhanced substantially by implementing a systemic perspective on bank regulation. Macroprudential allocations of capital reduce significantly both the default probabilities of individual institutions and the probability of multiple bank defaults. We compare alternative mechanisms of allocating systemic risk to individual institutions. Importantly, we follow an iterative procedure to solve for a fixed point for which capital allocations are consistent with the contributions of each bank to the total risk of the banking system, under the proposed capital allocations.

To derive the capital allocations, we use a model which explicitly considers contagion effects through network and asset fire sale externalities and sheds light on their importance. We generate an aggregate loss distribution of the banking system using a stress-testing model that integrates credit and market liquidity risk in a network of exposures between banks. To better capture the likelihood of contagion default, we use a unique sample of the Canadian banking system, which includes detailed information on interbank linkages including OTC derivatives.

Our findings have important policy implications for future bank regulation. For our sample, all of the analyzed macroprudential capital mechanisms brought a substantial reduction for bank risk. It is therefore probably more important for policymakers to implement a systemic perspective on bank regulation rather than to find the best risk allocation mechanism.

Implementing macroprudential capital requirements in practice will not be easy. Bank regulators will need to collect a large amount of data and especially information on the interbank exposures between banks. While we were lucky to have that information for the Canadian banks, in many countries around the world, this information is unavailable. Another hurdle will be to base each bank's capital requirements not solely on that bank's characteristics. Banks will complain to be treated unfairly as banks with a similar asset mix will be charged with different capital allocations based on systemic importance. One possible way to implement macropru-

dential capital requirements in to augment existing capital ratios with a charge that is based on regulatory assessment, similar to FDIC premiums, which are in part determined by the the primary regulator's composite rating.[32]

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[32]See also Acharya, Santos, and Yorulmazer (forthcoming) for a mechanism to determine deposit insurance premiums in the presence of systemic risk.

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A   Fixed point convergence

In this appendix we want to achive two objectives: first we will demonstrate that the fixed point can differ substantially from a simple risk attribution analysis in which banks' risk contributions are computed given the observed level of bank capital. Second, we want to provide some evidence that the fixed point is well defined. Because of the nonlinear nature of the model that we use for clearing the interbank claims and computing bank PDs, we cannot explicitly prove the uniqueness of the fixed point that determines the macroprudential capital requirements. To check for robustness we performed a Monte Carlo analysis by using alternative starting values and found that all of them converged to the same fixed point.

To show that the macroprudential capital requirements as they are computed throughout the paper using a fixed point can differ substantially from the capital that one would attribute to a bank using a simple risk attribution analysis, we compare fixed point macroprudential capital allocations $C^* = f(\Sigma(C^*))$ as defined in Equation (1) with the results of a simple risk allocation analysis $C^1 = f(\Sigma(C^0))$, where C^0 is the observed tier 1 capital for each bank. C^1 can also be interpreted as the first iteration of an iterative procedure $C^i = f(\Sigma(C^{i-1}))$ which can be used to find the fixed point $C^* = \lim_{i \to \infty} C^i$.

We compute the following statistic z, defined as

$$z = \frac{C^1 - C^0}{C^* - C^0} \tag{22}$$

For all the methods described in Section 2

We can interpret z as the fraction of the adjustment in bank capital that is realized in the first iteration. Ideally $z = 1$, in which case the risk attribution analysis yields a fixed point. A value of $z > 1$ indicates overadjustment in the sense that banks that should increase (decrease) capital according to the macroprudential perspective are allocated too much (too little) under the risk

Table 14. Capital adjustments under a risk attribution analysis as a multiple of macroprudential capital adjustments under the fixed point mechanism. Capital allocations are computed for the macro stress scenario including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$.

Bank	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
1	3.68	1.64	1.40	1.42	1.33
2	2.14	1.31	1.33	1.31	3.95
3	1.67	1.39	1.35	1.34	0.49
4	2.38	1.30	1.34	1.33	0.56
5	2.02	1.36	1.36	1.35	1.40
6	1.91	1.49	1.46	1.44	2.05

attribution analysis. A value of $z < 1$ can be interpreted as partial adjustment in the sense that banks that a risk allocation mechanism underestimates any required increases or decreases in bank capital.

Table 14 shows z for all risk allocation mechanisms. We find that four out of five mechanisms overadjust capital, sometimes substantially. Adjusting bank capital using component VaR risk contributions based on observed capital can be wrong by a factor of up to three. We thus can see the importance of considering the fact that overall risk as well as each bank's risk contribution changes when bank capital requirements change. While the first four risk allocation measures tend to overadjust, the $\Delta CoVaR$ measure has – at least for our sample – a tendency to under adjust. For two out of the six banks the simple risk attribution analysis using $\Delta CoVaR$ comes very close to the fixed point of a consistent capital allocation.

In our algorithm for finding the fixed point we compensate for the problem of overadjustment by changing capital from one iteration to the next only by a fraction $\gamma < 1$ of the adjustment that has been proposed by the risk allocation mechanism, i.e. we use $C^i = (1 - \gamma)C^{i-1} + \gamma f(\Sigma(C^{i-1}))$, where we found $\gamma = 0.5$ to yield numerically stable results.

To check for uniqueness of the fixed point we run a Monte Carlo simulation using alternative starting capital values C^0 and check that all of them converge to the same fixed point. Finding the fixed point is numerically intensive because we have to run a full Monte Carlo simulation in every iteration to get the joint loss and default distribution Σ. In this robustness check we draw 100 random capital endowments for the banks with the restriction that each bank has to be above the minimum capital requirement of 7% of risk weighted assets and that the total capital in the banking system stays the same. We find that our procedure converges to the same fixed point for all starting points. Figure 2 shows the norm of the distance to the fixed point $|C^i - C^*|$ over the first 15 iterations for the first 50 starting values and the Shapley Value-expected loss risk attribution model. While we cannot provide a formal proof, evidence from our simulations makes us confident that for our data macroprudential capital requirements are well defined.

Figure 2. Convergence to the fixed point or alternative starting values: The graph shows the norm of the distance to the fixed point for the different iterations for the Shapley values with expected losses.