SOLUTIONS FOR THE CONSTRAINED DYNAMIC PLANT LAYOUT PROBLEM

Jaydeep Balakrishnan
Finance and Operations Management Area
Faculty of Management
University of Calgary
Calgary, Alberta T2N 1N4
CANADA

F. Robert Jacobs
Department of Operations Management

Munirpallam A. Venkataramanan
Department of Decision and Information Systems

School of Business
Indiana University
Bloomington, IN 47405
U.S.A.

Published in the European Journal of Operational Research.

1992, 57, pp 280-286

SOLUTIONS FOR THE CONSTRAINED DYNAMIC PLANT LAYOUT PROBLEM

Abstract

Much of the research in facility layout has focused on static layouts where the material handling flow is assumed to be constant over the planning horizon. But in today's market based, dynamic environment, this assumption may no longer be true. Layout rearrangement may be required, for which available funds may be limited. This research investigates the facility layout problem under the two assumptions of changing demand and a constraint on the layout rearrangement funds.

The problem is formulated, a new algorithm is proposed to solve the problem and it is compared to an extension of the old algorithm that has been used to solve the problem thus far. In addition, different factors that affect dynamic facility layout design and operation are statistically examined. The results indicate that the proposed algorithm has advantages over the old one and that some of the factors and their combinations can have significant effects on layout design and operation.

Keywords: Facility layout, Two-State Variable Dynamic Programming, Constrained Shortest Path

SOLUTIONS FOR THE CONSTRAINED DYNAMIC PLANT LAYOUT PROBLEM[1]

1.0 Introduction

Historically, most of the research in facility layout has focused on the static plant layout problem (SPLP). Various heuristic solution procedures have been used to solve these problems since optimal solutions can be obtained only for very small problems. (For a detailed review of the SPLP literature see Kusiak and Heragu, 1987.)

The dynamic plant layout problem (DPLP), introduced by Rosenblatt (1986), extends the SPLP by assuming that it may be desirable to make changes in the layout over time. Further complicating the problem are layout rearrangement costs which would be incurred if departments are actually shifted during the time horizon.

In this paper the DPLP is considered under an additional relevant present day constraint: a cap on the amount of funds that can be spent on layout rearrangement. We model the constrained dynamic

[1] The authors wish to thank Profs. John F. Muth and James H. Patterson of Indiana University, Bloomington, USA and Prof. Ishwar Murthy of the Louisiana State University, Baton Rouge, USA for their assistance in conducting this research. We also wish to acknowledge the reviewers of this paper for their suggestions.

plant layout problem (CDPLP) as a singly constrained shortest path problem (CSP) and compare it to dynamic programming (DP), which Rosenblatt uses to solve the DPLP.

The importance of the budget constraint has been stressed in studies of plant layout practices conducted by the Department of Industry in the U.K. (1976) and by Nicol and Hollier (1983). The studies observed that layout planning had low priority in most companies. Therefore, ad hoc layout planning was common and in some cases lack of funds had led to poor layouts. Budgeting the funds allocated for layout rearrangement is one method for dealing with the lack of financial resources.

This paper makes three contributions: (1) it extends the unconstrained dynamic formulation to the case where funds for layout rearrangement are limited, (2) it identifies a new approach to solve the constrained version of the dynamic phase of the DPLP, and (3) the paper investigates the effect of the different problem characteristics on the new algorithm in terms of the quality and speed of the solution.

2.0 Problem Formulation

The constrained dynamic plant layout problem (CDPLP) can be formulated as an extension of the SPLP.

$$Min \sum_{t=2}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} A_{tijl}Y_{tijl} + \sum_{t=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} C_{tijkl}X_{tij}X_{tkl}$$

subject to

$$\sum_{i=1}^{N} X_{tij} = 1 \qquad j = 1,..N \quad t = 1,...P \qquad (1)$$

$$\sum_{j=1}^{N} X_{tij} = 1 \qquad i = 1,..N \quad t = 1,...P \qquad (2)$$

$$Y_{tijl} = X_{(t-1)ij} \times X_{til} \qquad i,j,l = 1,..N \quad t = 2,..P \qquad (3)$$

$$\sum_{t=2}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} A_{tijl}Y_{tijl} \le B \qquad (4)$$

i,k : *Departments in the layout*
j,l : *Locations in the layout*
Y_{tijl} : 0,1 *variable for shifting i from j to l in period t*
A_{tijl} : *Fixed cost of shifting i from j to l in period t* (*where* $A_{tijj} = 0$)
X_{tij} : 0,1 *variable for locating i at j in period t*
C_{tijkl} : *Cost of material flow between i located at j and k located at l in period t*
P : *Number of periods in the planning horizon*
N : *Number of departments in the layout*

The objective function minimizes the sum of the cost of layout rearrangement and the cost of material flow between departments during the planning horizon. Constraint set (1) requires every department to be assigned to a location in every period and (2) requires every location to have a department assigned to it in every period. (3) adds shifting costs to the material flow cost if a department is shifted between locations in a period. The total amount of funds used for layout rearrangement is constrained to be less than the budgeted amount by (4).

3.0 Solving the CDPLP

The first procedure is a DP algorithm based on Bellman & Dreyfus, (1962) which is the most commonly used method in previous studies. The DP algorithm used here has two state variables, one for the layouts and the other for the amount of the constrained resource available.

An alternate algorithm based on a singly constrained network model is also used to solve the problem. The underlying shortest path model is represented in the following diagram.

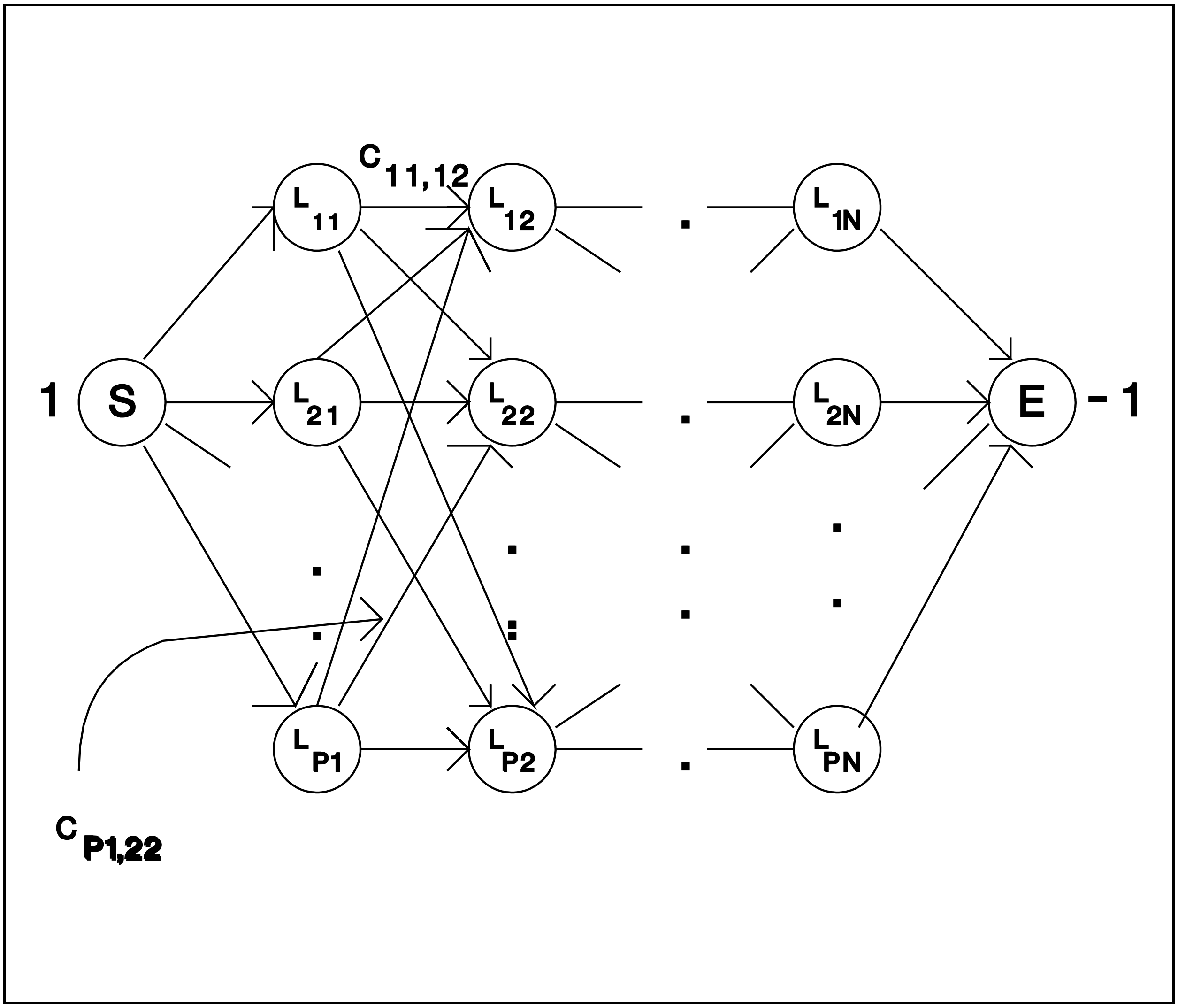

Figure 1   The Constrained Dynamic Plant Layout Problem

L_{it}	Layout i in time period t
S,E	Source and end nodes respectively (dummy layouts)

$C_{it,k(t+1)}$	The sum of the material handling cost in layout i in period t and the cost of rearranging layout i in period t to layout k in period (t+1)
P	Possible number of layouts in each period which is constant across periods.
N	Number of time periods in the planning horizon

Mathematically the model can be represented as

$$Min \sum_{it} \sum_{k(t+1)} C_{it,k(t+1)} x_{it,k(t+1)}$$

subject to

$$\sum_{i1} x_{S,i1} = 1$$

$$\sum_{iN} x_{iN,E} = 1$$

$$\sum_{it} x_{it,k(t+1)} - \sum_{m(t+2)} x_{k(t+1),m(t+2)} = 0 \quad (for\ all\ k,t) \qquad (7)$$

$$\sum_{it} \sum_{k(t+1)} b_{it,k(t+1)} x_{it,k(t+1)} \leq B \qquad (8)$$

$x_{it,k(t+1)}$	The arc representing the material handling cost in layout i in period t and the cost of rearranging layout i in period t to layout k in period (t+1)

$C_{it,k(t+1)}$	The sum of the material handling cost in layout i in period t and the cost of rearranging layout i in period t to layout k in period (t+1)
$b_{it,k(t+1)}$	Resource used in rearranging layout i in period t to layout k in period (t+1)
$x_{S,i1}$	Arc connecting source to layout i in period t
$x_{iN,E}$	Arc connecting layout i in period t to end
B	Available budget for resource

Constraint sets 5, 6 and 7 represent flow conservation and constraint 8 represents the budget constraint. Nodes S and E are dummy nodes and $x_{S,i1}$ and $x_{iN,E}$ are dummy arcs to facilitate the shortest path formulation described below. All other nodes represent layouts. Each arc has two parameters associated with it: $C_{it,k(t+1)}$ and $b_{it,k(t+1)}$. For example, in Figure 1 node L_{11} represents a layout in period 1 during which some material flow costs are incurred. At the end of this period we might decide to rearrange the layout. Node L_{22} represents the rearranged layout in period 2. The arc L_{11} - L_{22} therefore represents the sum of the material handling cost and the cost of layout rearrangement ($b_{it,k(t+1)}$) and is denoted by $C_{it,k(t+1)}$. Over the planning horizon, therefore, the shortest path through the network will provide the lowest cost constrained dynamic layout plan.

The motivation for this approach is the efficiency of the network based algorithms. Though the CSP is NP-complete (Garey and Johnson, 1979), Mote et al. (1988) report impressive computational results with their parametric programming based algorithm to solve the CSP.

By relaxing the constraint and considering it as another objective, the CSP model is converted into a bi-criterion shortest path (BSP) model. Solving the BSP provides all the pareto-optimal paths to the model. A pareto-optimal path is one which cannot be dominated by any other path. As an illustration, in Figure 1, let path S - L_{11} - L_{22} - ... - L_{PN} - E be a pareto-optimal path in the network with the total cost (sum of material handling and layout rearrangement) C and associated layout rearrangement costs R (less than B). This means that there are no other paths in the network with total cost less than C and at the same time having layout rearrangement costs less than R. The other paths will have higher C and lower R or vice versa. Mote et al. (1987) report that such (pareto-optimal) paths are small in number. This pareto-optimal path also provides an upper bound. The upper bound fathoms other potential pareto-optimal paths with higher length, improving the efficiency of the algorithm. The FORTRAN implementation is a specialized primal shortest path simplex algorithm and it outperformed other CSP algorithms (Mote et al., 1988)

4.0 Experimental Design and Computational Analysis

A study was conducted in order to evaluate the effect of different factors on the quality of solution and the CPU time required for solving the CDPLP.

4.1 Problem Set

Eight different problems were used. Each problem was a computer generated six department, five period problem similar to the one in Rosenblatt. The material handling flows and the department shifting costs were generated from the uniform distribution (see Table 1 for the ranges). The individual material handling flows generated (from one department to another) were proportionally adjusted so that their sum equalled the total material flow in the layout during each period which remained constant during the horizon. This was done in order to prevent any period from dominating the others. The generated shifting costs were also proportionally adjusted so that the average department shift cost was 15% of the average department flow cost. Flow dominance was introduced by randomly selecting between 1 and 3 departments from each period and increasing the flow to these departments by a factor of 5 to 7. The shape of the facility and the cost to move a unit distance were held constant over the horizon.

(Insert Table 1 here)

4.2 Performance Measures and Computational Factors

The performance measures used in the study were the solution quality (sum of material handling and layout rearrangement costs) over the planning horizon, and the CPU time required for the solution. Four different factors were used in the study. Details for each factor are given next.

Solution Technique

As described earlier, two techniques were compared, dynamic programming (DP) and the simplex based constrained shortest path (CSP). Both algorithms obtain optimal solutions and were implemented in FORTRAN.

Number of Layouts Included (SIZE)

Though it was expected that including more layouts (larger problem size) would result in higher solution times and better solutions, we were also interested in determining whether the increase in time was different for the two procedures. Two settings of fifty layouts (small) and one hundred layouts (large) in each period were used.

Method for Generating Static Layouts (LAY)

Three methods were employed in order to select the static layouts to be included: a) select the best static layouts in each period (referred as the 'best layouts'), b) select the layouts randomly (referred to as the 'random layouts') and, c) use a combination of random layouts as well as the best layouts, each contributing half of the layouts selected (these are referred to as the 'mixed layouts'). The times required for generating these candidate layouts was not included in the computer time measure described above.

Tightness of Constraint (CONSTR)

The budget constraint was either tight or loose. To determine the levels, the unconstrained problem was first solved. Then the funds available for layout rearrangement was set to be loosely constrained (90% of unconstrained usage) or tightly constrained (50% of the unconstrained usage).

4.3 Experiments

Three different experiments were conducted. The first two used the six department problems described earlier. The third consisted of three problems with twelve, fifteen and twenty departments respectively, each with a four period horizon. This problem was used to test whether the results of the six department experiments could be generalized.

In the first experiment, the effect of the SIZE (2 levels) and LAY (3 levels) factors on solution quality was investigated. The ALG and CONSTR factors were not included (both the algorithms are optimizing and tighter constraints were designed to result in poorer solutions). Thus there were 3*2 or 6 factor combinations in the full factorial design. Each factor combination was run with all the eight problems resulting in 48 computer runs. In the second, the effect of the four factors on solution time was investigated.

In the full factorial design employed, there were 3 levels of the LAY factor and 2 levels each of the ALG, SIZE and CONSTR factors. This resulted in 3*2*2*2 or 24 factor combinations. With eight problems used in each, 192 computer runs were made.

The ANOVA procedure was employed. F-Tests were used to test the null hypotheses for the main effects and interactions. Specific differences between the factor levels were identified using Tukey's method of pairwise comparisons (Neter et al., 1985). No significant problems were found with deviations from normality or constancy of variance which might make the use of these tests suspect. The experiments were conducted using an IBM 3090 computer. The SAS package was used to perform the ANOVA.

In the large layouts experiment, it was not possible to obtain the static optimal solutions because of the layout size. CRAFT was used to obtain heuristic solutions. Using different initial layouts, five final layouts were obtained for each of the four periods. The objective of this experiment was to test whether using heuristic layouts is better than using random layouts. The other factors like the type of algorithm, the problem size and the constraint tightness are related to the dynamic part of the CDPLP and were not relevant in this experiment. Two different shift cost levels (15% and 10% of the material flow levels) and three constraint tightness levels were used along with each layout type giving 12 cells. With the 12, 15 and 20 department problems this resulted in 96 computer runs.

5.0 Results

Table 2 gives a summary of the results for the solution quality experiment. The values are percentages over or below (negative) a base cost solution and are averages from the eight replications. The factor combination of the large problem size and best layout was designated as the base solution value. To determine whether these differences were significant the F-Test and Tukey's Test were used.

(Insert Table 2 here)

Both the number of layouts included and the method of layout selection have significant effects on the quality of the solution. The interaction between the two factors is not significant. Using a larger number of layouts resulted in a lower cost solution which was expected. The results also show based on Tukey's Test that there is no significant difference in solution quality between the best layout selection method and the mixed layout selection method. Both these however, performed significantly better than the random selection method.

Table 3 gives a summary of the results for the solution time experiment. All values are in CPU seconds and are eight replication averages. It appears from this table that the CSP approach is better overall than the DP approach. In order to further analyze the results, F-Tests and Tukey's Tests were employed. All the main effects were significant. In addition five two-way and two three-way interactions were found.

(Insert Table 3 here.)

The most important effect in the table is that of the type of algorithm. Overall the CSP algorithm performed much better than the DP algorithm. The solution times for the CSP varied from 0.30 CPU seconds for the small problems to 11.0 CPU seconds for the large problems. For the DP procedure it varied between 0.15 and 850.0 CPU seconds. As expected larger problem sizes required more solution time. But more important are the different effects the problem size had on the algorithms.

The solution time increase was much greater for the DP algorithm when going from small to large problems, than for the CSP. This difference is due to the fact that the DP algorithm is a pure enumeration algorithm whereas the CSP is a combination of the simplex method and enumeration. Enumeration methods are sensitive to both the number of constraints and the number of arcs while the simplex method is sensitive mainly to the number of constraints. Since the CDPLP has many more arcs as compared to constraints (nodes or layouts), the effect of increased problem size is felt more by DP.

The constraint level also had an effect on solution time. Tighter constraints lead to more fathoming of arcs and thus reduced solution time. This effect is also greater on the DP approach.

Based on Tukey's Test there is statistically no difference in the solution times between the best and the mixed layout methods or between the random and mixed layout methods. However, there is a significant difference between the random and best layout methods. This is due to the fact that the random layout method used higher absolute constraint values, due to poorer candidate solutions (static layouts). This resulted in less fathoming of the arcs. As before, the effect was higher on DP.

Table 4 gives the results from the large layout experiment. No statistical analysis was employed. Each cell value is the average of the three large layouts and is the percentage above the mixed layout cost solution. The mixed layouts had 20 heuristic layouts and 30 random layouts for a total of 50 layouts in each period. In fact, in every computer run the mixed layout method performed better than the random layout method which had 100 layouts in each period. The difference ranged from 0.5% to 7.5%.

(Insert Table 4 here.)

6.0 Conclusion

Three important results are observed from this study. First, when the DPLP is extended to include a constraint on the amount of money that can be spent on layout rearrangement, the CSP performs better than DP in most of the situations tested. The only conditions under which the DP algorithm outperforms the CSP algorithm is if the problem size is small, or the constraints are very tight. Combinations of these also favour the DP algorithm. But in practice, the above conditions are not likely to occur and so the CSP approach seems to be a better alternative for the CDPLP.

Second, it appears that selecting the candidate static layouts by including some randomly generated static layouts as well as the best layouts is statistically just as good as using just the best layouts. It was observed in the study that the mixed layout method generally resulted is solutions close in quality to the best layout method and sometimes surpassed it. This was due to the random layouts included in the selection. Thus, using the mixed method is a good alternative to the best layout since the penalty is usually very small while there is always the chance that we could surpass the best layout selection method.

Third, even if we cannot obtain the optimal static solution which will be the case with most practical sized layouts, using heuristic static layout solutions (obtained using CRAFT in our study) is better than selecting the candidate static layouts randomly.

This study examined the DPLP under a financial constraint. This model can also be easily adapted to study the DPLP under other constraints such as a restriction on the amount of productive time spent on layout rearrangement. Other issues, such as the sensitivity of this model to forecast errors and the possibility of adding more constraints would be interesting topics for further studies. Further details regarding this paper can be found in Balakrishnan (1991).

References

1 Balakrishnan, J., "Solutions for the Constrained Dynamic Plant Layout Problem", Unpublished PhD Dissertation, Indiana University, Bloomington, Indiana, 1991.

1 Bellman, R.E. & Dreyfus, S.E., "**Applied Dynamic Programming**", Princeton, N.J.: Princeton University Press, 1962.

3 Dept. of Industry, "Material Handling Costs: A New Look at Manufacturing", Committee for Material Handling,H.M.S.O., London, U.K., 1976.

4 Garey, M., & Johnson, D.S., **Computers and Intractability: A Guide to the Theory of NP-Completeness**", San Francisco, Freeman, 1979.

5 Kusiak, A., & Heragu, S.S., "The Facility Layout Problem", **European Journal of Operational Research**", 29, pp. 229-251, 1987.

6 Mote, J., Murthy, I., & Olsen, D.L., "An Empirical Investigation of the Number of Pareto-optimal Paths Obtained for the Bicriterion Shortest Path Problem", Working Paper, Department of MSIS, University of Texas, Austin, 1987.

7 Mote, J., Murthy, I., & Olsen, D.L., "On Solving Singly Constrained Shortest Path Problems by Generating Pareto-optimal Paths Parametrically", Working Paper, Department of MSIS, University of Texas, Austin, 1988.

8 Neter, J., Wasserman, W., & Kutner, M.H., **Applied Linear Statistical Models**, Richard D. Irwin, Homewood, Illinois, 1985.

9 Rosenblatt, M.J., "The Dynamics of Plant Layout", **Management Science**, 32, pp. 76-86, 1986.

Problem	Individual Flow		Shifting Cost	
	Lower Limit	Upper Limit	Lower Limit	Upper Limit
1	100	200	100	500
2	200	400	100	500
3	300	600	400	1000
4	100	200	100	500
5	500	1000	500	1500
6	1000	2000	1000	3000
7	200	500	200	700
8	100	200	100	1000

Ranges for the Uniform Distributions

Table 1

SIZE LAY	Small %	Large %
Best	25.86	1.21
Random	26.07	1.83
Mixed	24.99	1.50

Solution Quality Results

Table 2

ALG	CSP				DP			
SIZE	Small		Large		Small		Large	
CON. LAY	Tight	Loose	Tight	Loose	Tight	Loose	Tight	Loose
Best	0.94	1.27	4.00	4.79	3.50	20.09	33.02	243
Mix.	1.27	1.67	5.49	6.43	6.89	45.53	103	425
Ran.	1.01	1.39	4.23	5.00	5.31	31.89	48.88	297

Solution Time Results

Table 3

Better Layout	High Shift %	Low Shift %
	Unconstrained	
Random	2.70	4.48
Mixed	0.00	0.00
	10% Constrained	
Random	2.35	3.10
Mixed	0.00	0.00
	50% Constrained	
Random	1.95	1.99
Mixed	0.00	0.00

Results From the Large Problems Experiments

Table 4