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Essays on the Insurance Pricing of Distinct Business Lines in Multiple-line Property and Casualty Insurance Company

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Abstract

This thesis consists of two essays on financial models of insurance pricing that are able to price insurance by line in a multi-line property & casualty insurance company. Essay 1 is based on the Full Information Underwriting Beta Methodology while essay 2 uses a Contingent Claims Approach. The thesis extends the existing literature in insurance pricing by developing insurance pricing models that reflect the risk characteristics of different business lines.

Essay 1 applies the full information beta methodology to estimate the underwriting betas of distinct business lines, which are then applied to estimate the fair underwriting profit margins by lines of business. The full information underwriting betas of distinct business lines contain more information and measure the risks of business lines more reliably, i.e., the risk of underwriting varies among business lines in more regards than simply the length of the period over which premium can be kept for investment. Based on Canadian Property & Casualty insurance industry data, the primary empirical findings in essay 1 strongly support the argument that underwriting betas of distinct lines do not vary in proportion to the length of the period that the premium of the corresponding line can be kept for investment. The results of essay 1 also show that the expected underwriting profit margin of liability insurance is the lowest of the three distinct business lines: auto insurance, property insurance, and liability insurance.

Essay 2 elaborates upon the Doherty and Garven (1986) option pricing model and

develops a financial insurance pricing model that is able to price insurance by line in a multi-line insurer. The model developed in essay 2 improves the full information underwriting beta method developed in essay 1 by incorporating default risk and underutilized tax shields. The results of essay 2 are consistent with the findings and arguments in essay 1 and in prior studies. The results show that the expected underwriting profit margins (UPM) vary across different insurance business lines with the expected UPM of liability insurance being the lowest, followed by that of auto insurance and then property insurance. The results reconfirm that the high leverage factor, k, of liability insurance produces a larger contribution to investment income, which offsets the demand for profit from the underwriting activity. The relationship between expected UPM and leverage factor, k, is shown to be non-linear.

The sensitivity analyses in essay 2 and 3 show that under both the Full Information Underwriting Beta Methodology and the Contingent Claims Approach, corporate income tax (CIT) is positively related to expected premium and to expected UPM. Furthermore, the effect of CIT on expected premium and on expected UPM are quite stable over time and across firms. The premium-based tax (PBT) is positively related to the expected premium and is stable as well.

DEDICATIONS

To my parents, Yuxiang Deng and Shouhai Zhang

and my daughter, Yun

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1 Introduction

Setting a fair or competitive premium plays an important role in the insurance industry because capital is invested or retained in the insurance industry only if the return provided by the insurance industry is comparable to those offered by other industries. Determining the appropriate insurance premium has become the subject of extensive scrutiny over the last several decades among both academia and industry practitioners. Starting from the earliest attempt to determine the fair premium–the Target Underwriting Profit Margin promulgated by the National Convention of Insurance Commissioners in 1921– a variety of insurance pricing models have been proposed and applied, including the Capital Asset Pricing Model (e.g., Fairley, 1979; Hill, 1979; Hill and Modigliani, 1987), the Internal Rate of Return Approach (e.g., Cummins, 1990), the Discounted Cash Flow Approach (e.g., Myers and Cohn, 1987; D'Arcy and Garven, 1990; Cummins, 1990), the Arbitrage Pricing Model (e.g., Kraus and Ross, 1987; Urrutia, 1987a), and the Option Pricing Model (e.g., Doherty and Garven, 1986; D'Arcy and Garven, 1990; Phillips et al., 1998). Such financial insurance pricing models have the strength that they incorporate the capital market into insurance pricing and could provide non-arbitrage insurance pricing.

Many existing studies in insurance pricing implicitly or explicitly assume that insurers provide only one line of business (or assume the total business is one single line). Examples include Fairley (1979), Hill (1979), Cummins and Harrington (1985), Cummins (1988), Sommer (1996), and Chen et al. (2003). Few of the prior studies investigated the pricing of distinct lines of insurance business within a multi-line insurer. Although most studies focused on estimating the aggregate underwriting profit margin for all lines, insurers need to set appropriate rates for distinct lines of business because of the risk differences that exist across lines. If the weights of different lines of business change over time, using a single combined underwriting profit margin for all business lines may cause positive or negative abnormal profit, especially when the combination of the distinct lines of business changes sharply.

Fairley (1979) and D'Arcy and Garven (1990) pointed out that future research on various pricing models for distinct lines of business could provide more accurate results and avoid the aforementioned problem. Although significant variations exist in the risk characteristics across distinct business lines, e.g., liability insurance versus property insurance, little progress has been made in estimating the fair underwriting profit margin by line of business. Only a few studies have been conducted in an attempt to remedy this deficiency. For example, Urrutia (1987b) estimated distinct line underwriting betas by performing factor analysis on the combined ratios; however, the rationale for using factor analysis and the details of the methodology used were not fully discussed in his paper. Phillips et al. (1998) developed an insurance pricing framework for distinct lines of business based on OPM and conducted an hypothesis test. Their findings supported a conclusion that prices varied across firms depending upon overall firm default risk and the concentration of business among subsidiaries; but within a given firm, after controlling for line-specific liability growth rates, they concluded that the prices did not vary by line.

The purpose of this thesis is to remedy the deficiency in existing literature by developing insurance pricing models that reflect the risk characteristics of different business lines. The thesis develops financial insurance pricing models that are able to price insurance by line in a multi-line insurer based on Full Information Underwriting Beta (essay 1) and a Contingent Claims Approach (essay 2).

The first essay presents new evidence on insurance pricing by line of Property & Casualty insurance in that it uses the full information beta methodology to estimate the fair underwriting profit margin by line. The firms' underwriting betas are first estimated and then used to derive the full-information underwriting betas for distinct business lines, which are then used to estimate the underwriting profit margin by line of insurance. In addition to the corporate income tax, essay 1 also includes other taxes in the model, which allows further study of the taxes' impacts on insurance pricing. Essay 1 also conducts a comparative statics analysis with respect to the important parameters in the Insurance Capital Asset Pricing Model with the quantitative results of their impacts on insurance pricing presented. Based on Canadian property & casualty insurance industry data, the empirical findings of essay 1 strongly disprove the assumption in prior studies that underwriting betas of distinct lines vary in proportion to the length of the period that the premium of the corresponding line can be kept for investment. The results also show that fair underwriting profit margins vary across insurance business lines and are closely related to the equity risk premium, which itself varies over time. The findings imply that setting a single target underwriting profit margin rate for all distinct insurance business lines and over multiple years is inappropriate and could be dangerous. The comparative statics analysis shows that fair underwriting profit margin and fair net premium are positively related to effective corporate income tax rate, and are negatively related to premium-to-equity ratio and leverage factor. Also, fair net premium is positively related to effective expense-and-other-taxes rate.

The second essay elaborates upon the Doherty and Garven (1986) option pricing model and develops a financial insurance pricing model that is able to price insurance by line in a multi-line insurer. The contingent claims approach is adopted to model the financial claims of shareholders, policyholders, and tax authorities, which could be modelled as European options written on the income generated by insurer's asset and liability portfolio. The model developed in the second essay has the potential to yield significant improvements in insurance pricing techniques in several ways. First, the model extends the Doherty and Garven (1986) single-line model to a model that is suitable for insurance pricing by distinct line in a multi-line insurer subject to default risk and underutilized tax shields. Second, essay 2 provides numerical results of the expected underwriting profit margins (UPM) and the expected insurance premiums by major business line for ten Canadian Property & Casualty insurers during the period from 1999 through 2005, which augments the hypothesis tests used in the prior studies. Essay 2 finds that the expected UPM of liability insurance is the lowest, followed by that of auto insurance and then property insurance. The results reconfirm that the high leverage factor, k, of liability insurance results in a larger contribution of investment income, which offsets the demand for profit from the underwriting activity. The relationship between expected UPM and leverage factor, k, is shown to be non-linear. The second essay also demonstrates, both analytically and numerically, the impacts of taxes on the expected premium and the expected underwriting profit margin. The results from essay 2 are consistent with the findings in essay 1.

The remainder of the thesis is organized as follows: The first essay on the insurance pricing of distinct business lines in multiple-line property and casualty insurance company based on the full information underwriting beta is presented in chapter 2. The second essay on the insurance pricing of distinct business lines in multiple-line property and casualty insurance company based on a contingent claims approach follows in chapter 3. Chapter 4 concludes the thesis with a summary and conclusion.

2 Essay 1: The Pricing of Multiple-Line Property & Casualty Insurance Based on the Full Information Underwriting Beta

2.1 Introduction

The Capital Asset Pricing Model (CAPM) developed in the mid-1960s by Sharpe (1964), Lintner (1965) and Mossin (1966) has been widely applied in insurance to estimate both the fair total rate of return and the fair underwriting profit margin[1] (e.g., Cooper, 1974; Biger and Kahane, 1978; Fairley, 1979; Hill, 1979; Hill and Modigliani, 1987; Urrutia, 1986; D'Arcy and Garven, 1990; Derrig, 1994; D'Arcy and Gorvett, 1998; Cummins and Phillips, 2005). The "fair" rate is interpreted as the rate derived from equilibrium relationships in the competitive capital markets. In the early applications of CAPM to estimate fair underwriting profit margin, Cooper (1974) and Biger and Kahane (1978) assumed no taxes, and that each dollar of premium can be invested for a whole year before it was paid out as claim or expense. Systematic risk was measured by the underwriting beta that reflects the correlation between an insurer's underwriting portfolio and the market portfolio.

The initial model was extended in three ways by later studies. First, the assumption that the premium received could be invested for the whole year was relaxed in most of the later studies to allow premiums to be retained for a fraction of a year, k (e.g., Fairley, 1979; Kahane, 1979; Urrutia, 1986; Hill and Modigliani, 1987). Second, taxes were included in later refinements of the models. Fairley (1979), Hill (1979), Urrutia (1986), Hill and Modigliani (1987), and Derrig (1994) used different tax rates for underwriting incomes and investment incomes to reflect the special tax treatment on investment incomes.

[1] In this thesis, the fair underwriting profit margin and fair net premium are also called the expected underwriting profit margin and the expected net premium.

Third, the mono-line models began to be applied to multi-line companies. Biger and Kahane (1978) described an indirect method to derive the distinct line underwriting beta from the betas of individual assets and the beta of the company's stock. This method involved a heavy task of data collection for investment assets and "it is quite possible that noise in the data will cause the results to be obscure" (pp. 129). Their model was insufficient to determine the fair underwriting profit margin for each distinct line and had aforementioned estimation problems with respect to estimating the underwriting betas for distinct lines. Fairley (1979) argued that the underwriting beta of a distinct line was the product of the leverage factor, k, of the distinct line and the liability beta that was assumed constant over all business lines; where, the liability beta was defined as the covariance of the return on liabilities and market return divided by the variance of the market return. That is, the underwriting betas of distinct lines vary in proportion to the length of the period that the premium of the corresponding line can be kept for investment. "In the insurance industry, where 'risk' is generally conceived of in terms of total variability, the lines with the longer cash flows are viewed as the 'riskiest'" (Fairley, 1979, pp. 200). However, the length of the period over which claims are paid is only one of the risk factors. Other characteristics of business lines, such as the interdependence of the accident events, and the sizes and frequencies of the claims, are all important risk factors and should be considered. Fairley (1979) pointed out explicitly his assumption that loss beta "is constant by line is important and strong, and future work should be directed at relaxing it." Urrutia (1987b) estimated distinct line underwriting betas by performing factor analysis on the combined ratios; however, the rationale for using factor analysis and the details of the methodology were not fully discussed in his paper.

The ability to accurately measure the underwriting profit margin is very important in both the profit/premium-rate regulation and pricing in P&C insurance industry and in maintaining firms' healthy financial status. A firm may experience an underwriting loss if the underwriting profit margin is underestimated; it may also lose market share if the underwriting profit margin is overestimated. Using a single combined underwriting profit margin for all business lines may cause positive or negative abnormal profit depending on the business-participation weight of each distinct business line. Especially if there exists a systematic distortion caused by the combination of the distinct lines of business, the aggregate results may be misleading. Cummins and Harrington (1985) argued that betas differed across insurers and that this variation may be attributable to the inter-insurer differences in product-line mix. D'Arcy and Garven (1990) pointed out that future research on various pricing models for distinct lines of business could provide more accurate results and avoid the aforementioned problem. Although significant variations exist in the risk characteristics across distinct business lines, e.g., liability insurance versus property insurance, little progress has been made in estimating the fair underwriting profit margin by line of business. The purpose of this chapter is to remedy the deficiency in existing literature by developing an Insurance Capital Asset Pricing Model (ICAPM) that reflects the risk characteristics of different business lines by applying the Full Information Underwriting Beta Methodology.

The chapter uses the full information underwriting beta methodology to decompose the fair underwriting profit margin by line. The firms' underwriting betas are first estimated and then used to derive the full-information underwriting betas for distinct business lines, which are then used to estimate the fair underwriting profit margins and fair net premiums by lines of insurance business. In addition to the corporate income tax rate, the effective other-taxes rate is included in the model to allow further study of the impacts of taxes on the fair insurance pricing. Finally, the chapter provides comparative statics analysis with respect to the important parameters in the Insurance Capital Asset Pricing Model with the quantitative results of their impacts on insurance pricing presented. The chapter develops a model to extend the mono-line Insurance Capital Asset Pricing Model into a multi-line insurance pricing model.

This chapter is organized as follows. The theoretical model and the comparative statics analysis with respect to the important parameters in the ICAPM are developed in subsection 2.2. Data and variables are discussed in subsection 2.3. This is followed by the empirical results and discussion in subsection 2.4. Conclusions appear in subsection 2.5.

2.2 Theoretical Model and Comparative Statics Analysis

Insurer's total return is comprised by the return from its investment activity and the return from its underwriting section, which interact with each other. This essay applies the Insurance Capital Asset Pricing Model (ICAPM) to derive the fair underwriting profit margin that reflects the risk premium to insurer's underwriting activities, taking account of the investment income arising during the period between when premiums are received and when claims are paid. "(T)he 'fair' rate of return usually is interpreted as that which would prevail under competitive conditions" (Doherty and Garven, 1986, pp. 1031). Based on the assumption of the efficient market, the insurer's equity return and the equilibrium fair underwriting profit margin can be derived.

The chapter applies the Full Information Beta (FIB) Methodology to decompose the insurers' underwriting beta by insurance line. The full information beta methodology was applied by Cummins and Phillips (2005) in insurance to decompose the cost of equity by line of insurance for the property-liability insurance industry. Fairley (1979) conceptually described the similar idea of the FIB methodology in estimating the distinct business line underwriting betas. The underlying idea of FIB methodology is that the observable conglomerate firm beta is a weighted average of the unobservable betas of the firm's distinct business lines. The approach is to conduct a cross-section regression over firms, with the conglomerate firm betas as the dependent variables and the weights of the distinct business lines (which measure the firms' participation in distinct business lines) as the independent variables. The coefficients of the weights of business lines are interpreted as the full information betas for the corresponding business lines. The conglomerate firm underwriting betas used as the dependent variables in this chapter are derived from the traditional one-factor CAPM and are estimated by conducting regressions over time for each firm.

The current model extends prior ICAPM models (e.g., Fairley, 1979; Hill and Modigliani, 1987) in two ways: first, in addition to the corporate income tax rate, other taxes are considered in the model; second, the mono-line model is extended into a multi-line insurance pricing model. Specifically, Full Information Beta methodology is used to estimate the underwriting betas of property insurance, auto insurance, and liability insurance; and then the full information underwriting beta of each distinct business line is applied to estimate the fair underwriting profit margin and fair premium of each business line. In this subsection, the fair underwriting profit margin is derived, followed by an estimation of the underwriting betas for both the combined-line and each distinct line. The subsection concludes with a comparative statics analysis.

2.2.1 Fair Underwriting Profit Margin

The derivation of the fair underwriting profit margin (FUPM) follows the traditional ICAPM. The fair underwriting profit margin is derived based on the condition that at equilibrium the expected actual return should be equal to the expected required return by the capital market. An insurer's after-tax actual return can be expressed as the sum of the returns from its underwriting operation and its investment activities. For simplicity, the insurer is assumed to have initial equity investment, V_e, and to have written insurance for an expected premium income, P (the net premium earned). The actual return to shareholders, r_e, equals the after-tax profits from investment and underwriting activity divided by the insurer's initial equity. The relationship can be expressed as:

$$r_e = \frac{(r_a \cdot V_a + r_u \cdot P) \cdot (1 - t_{CI})}{V_e} \tag{2.1}$$

$$where, V_a = V_l + V_e \tag{2.2}$$

$$V_l = k \cdot P \tag{2.3}$$

where,

r_e : is the return on insurers' equity;

r_a : is the return on insurer's investment portfolio;

r_u : is the insurer's underwriting profit margin, expressed as a percentage of net premium earned;

t_{CI}: is the effective corporate income tax rate;

V_a : is the value of insurer's assets;

V_e : is the value of insurer's equity;

V_l : is the value of claims reserve;

P : is the annual net premium earned (net of reinsurance);

k : is a leverage factor reflecting the average holding period of a dollar of premium[2] (Fairley (1979), pp.198).

Substituting equations (2.2) and (2.3) into equation (2.1) and letting $b = \frac{P}{V_e}$, the ratio of the net premium earned to the value of insurer's equity, r_e can be expressed as:

$$r_e = (r_a \cdot (1 + k \cdot b) + r_u \cdot b) \cdot (1 - t_{CI}) \tag{2.4}$$

According to the CAPM model, the expected return on an insurer's asset portfolio equals the sum of the risk-free rate and compensation for the systematic risk of the insurer's

[2]For example, if the premium is retained within a firm for half a year before it is paid for claims, then k equals 0.5. Please see section 2.3.1 for the detailed description.

investment activity, i.e.,

$$E[r_a] = r_f + \beta_a \cdot (E[r_m] - r_f) \tag{2.5}$$

where,

r_f : is the risk-free rate;

β_a : is the beta of insurer's investment portfolio;

r_m : is the return on market portfolio;

$E[*]$: is the expectation operator.

Taking the expectation of equation (2.4), and combining with equation (2.5), the expected actual return of insurer can be expressed as:

$$E[r_e] = \{(r_f + \beta_a \cdot (E[r_m] - r_f)) \cdot (1 + k \cdot b) + E[r_u] \cdot b\} \cdot (1 - t_{CI}) \tag{2.6}$$

An insurer's required return on equity by the capital market can be derived from the CAPM also. The expected required return to insurer's equity can be measured by the sum of the risk-free rate and the compensation for the systematic risk of investing in the insurer, i.e.,

$$E[r_e] = r_f + \beta_e \cdot (E[r_m] - r_f) \tag{2.7}$$

where, $\beta_e = cov(r_m, r_e)/\sigma_m^2$ is the beta of the insurer's equity. Combining this with equation (2.4) and the linear property of covariance produces:

$$\beta_e = (\beta_a \cdot (1 + k \cdot b) + \beta_u \cdot b) \cdot (1 - t_{CI}) \tag{2.8}$$

Substituting equation (2.8) into equation (2.7) produces:

$$E[r_e] = r_f + \{(\beta_a \cdot (1 + k \cdot b) + \beta_u \cdot b) \cdot (1 - t_{CI})\} \cdot (E[r_m] - r_f) \tag{2.9}$$

where,

β_u : is the underwriting beta.

The equilibrium fair underwriting profit margin is derived based on the non-arbitrage condition that, in equilibrium, the actual expected return should equal the required expected return by the capital market, i.e., equation (2.6) should be equal to equation (2.9). That equality becomes:

$$\begin{aligned} &\{(r_f + \beta_a \cdot (E[r_m] - r_f)) \cdot (1 + k \cdot b) + E[r_u] \cdot b\} \cdot (1 - t_{CI}) \\ &= r_f + \{(\beta_a \cdot (1 + k \cdot b) + \beta_u \cdot b) \cdot (1 - t_{CI})\} \cdot (E[r_m] - r_f) \end{aligned} \tag{2.10}$$

Solving equation (2.10), the equilibrium fair underwriting profit margin is derived as:

$$E[r_u] = -r_f \cdot k + \frac{r_f \cdot t_{CI}}{b \cdot (1 - t_{CI})} + \beta_u \cdot (E[r_m] - r_f) \tag{2.11}$$

From equation (2.11), it is observed that the fair underwriting profit margin (FUPM) depends on the risk-free rate, cash flows (i.e., leverage factor k), effective corporate income tax rate, premium-to-equity ratio, the systematic underwriting risk (i.e., the underwriting beta), and the equity risk premium. The FUPM does not depend on the insurer's actual investment performance. The first term of equation (2.11) means the investment return generated from the held policyholder's fund and used to lower the expected underwriting profit margin. The second term is an adjustment for corporate income tax; and the third term indicates the risk premium for the insurer's underwriting section. Although variables for expense rate and other-taxes rate (except corporate income tax) do not appear in the fair underwriting profit margin formula, the value of the leverage factor k, is directly related to the expense rate and other-taxes rate. The higher these rates of cash outflow, the larger the share of premium that is used to cover expense and other taxes up front, and the smaller is the amount kept for investment; i.e., high expenses and other taxes directly lower the value of k, which results in a higher fair underwriting profit margin. Further discussion about taxes is presented in subsection 2.3.

The fair underwriting profit margin is defined as one minus the sum of the expense ratio (including all taxes except corporate income tax) and loss ratio. It can be expressed as:

$$E(r_u) = \frac{E[P] \cdot (1 - t_{prem} - e) - E(L)}{E[P]}.$$

The above equation can be rearranged to produce the following expression for expected premium:

$$E[P] = \frac{E(L)}{1 - t_{prem} - e - E(r_u)} \tag{2.12}$$

where,

e: is the expense rate, measured as a percentage of premium;

t_{prem}: is the other-taxes rate, i.e., the total taxes paid except corporate income tax as a percentage of premium (e.g., premium tax, fire tax, property tax, payroll tax, etc.);

L: is the net loss incurred including expenses related to claims.

2.2.2 Estimating Underwriting Beta–A Full Information Beta Approach

The underlying premise of Full Information Beta (FIB) Methodology is that a firm can be viewed as a combination of distinct business sections/lines. "The rationale for the FIB decomposition is the value-additivity property of arbitrage-free capital markets, which holds that the arbitrage-free market value of the firm is the sum of the values of its individual projects. This conceptualization implies that the firms' overall market beta coefficient is a weighted average of the beta coefficients of the separate divisions or business lines" (Cummins and Phillips, 2005, pp. 447). In this subsubsection, the firms' underwriting betas are first estimated by conducting a time series regression for each firm, and then the firms' betas are used to derive the full-information underwriting betas for distinct business lines by conducting a cross-section regression.

Consistent with prior research (e.g., Hill, 1979; Cummins and Harrington, 1985), the underwriting profit margin is regressed against the current market return; if auto-correlation exists in the error terms, the model will be adjusted by including the prior year market return in the independent variables. Without the presence of auto-correlation, underwriting profit margin at time t can be expressed as:

$$r_{uit} = \alpha_i + \beta_{ui} \cdot r_{mt} + e_{it} \tag{2.13}$$

where,

α_i: is the constant in the regression model;

r_{uit}: is the i^{th} insurer's underwriting profit margin in period t;

r_{mt}: is the return on market portfolio in period t;

β_{ui}: is the i^{th} insurers' underwriting beta;

e_{it}: is the random error term in period t for the i^{th} insurer.

The information content of underwriting profit margin reported in financial statements may have been known by the market and been reflected in the market performance before the information about the underwriting profit margin is reported, i.e., auto-correlation may exist. Thereafter regressing underwriting profit margin on current market return may bias the β_u, the correlation between market return and the underwriting profit margin, towards zero. One way to mitigate this potential bias is to include the lagged market returns in the regression model. The equation (2.13) is adjusted by regressing the underwriting profit margin on both the current year and previous year's market return, i.e.,

$$r_{uit} = \alpha_i + \beta 1_{ui} \cdot r_{mt} + \beta 2_{ui} \cdot r_{m(t-1)} + e_{it} \tag{2.14}$$

and

$$\beta_{ui} = \beta 1_{ui} + \beta 2_{ui} \tag{2.15}$$

The β_{ui} in equation (2.15) is called the sumbeta hereafter.

Once the underwriting beta/sumbeta, β_{ui}, for each firm is known, the underwriting beta for each business line can be derived using the full information beta approach. Under this approach the underwriting beta of an insurer is the weighted average of the betas of its distinct business lines, i.e.,

$$\beta_{ui} = \sum_{j=1}^{J} \beta_{fuj} \cdot \omega_{ij} + \nu_{ui} \tag{2.16}$$

where,

the subscript i denotes the i^{th} firm; and the subscript j denotes the j^{th} business line;

β_{fuj}: is the full information underwriting beta for the j^{th} business line;

ω_{ij}: is firm $i's$ business-participation weight for the j^{th} business line, using the premium written as the weight;

ν_{ui}: the random error term for firm i.

2.2.3 Comparative Statics Analysis

In this subsubsection, a comparative statics analysis is conducted with respect to several important parameters in the ICAPM: effective corporate income tax rate, expense-and-other-taxes rate[3], premium-to-equity ratio, and leverage factor. For the fair underwriting profit margin (FUPM), results of the comparative statics analysis are as follows:

$$\frac{\partial E[r_u]}{\partial t_{CI}} = \frac{r_f}{b \cdot (1 - t_{CI})^2} > 0 \tag{2.17}$$

[3]Expense and other taxes are combined into a single parameter here because all taxes except corporate income tax are categorized into the expenses items in the annual financial reports. Please see subsection 2.3 for details.

$$\frac{\partial E[r_u]}{\partial(t_{prem}+e)}=0 \tag{2.18}$$

$$\frac{\partial E[r_u]}{\partial b}=\frac{-r_f}{b^2\cdot(1-t_{CI})}<0 \tag{2.19}$$

$$\frac{\partial E[r_u]}{\partial k}=-r_f<0 \tag{2.20}$$

From equations (2.17) through (2.20), it is found that in a normal situation (i.e., where the risk-free rate and net claims incurred are greater than zero; and the effective corporate income tax rate is less than one), a higher effective corporate income tax rate leads to a higher FUPM; expense-and-other-taxes rate is not directly related to FUPM; and a higher premium-to-equity ratio, b, as well as a higher leverage factor, k, projects a lower FUPM. The corporate income tax imposed upon insurers reduces their post-tax profitability; to provide competitive return to equityholders, insurers will have to raise their target underwriting profit. At the same given level of total profit and the same volume of underwriting business, firms with high b and k achieve higher *rates of total equity return* than those with low b and k,(since in firms with high b and k, the same level of total profit is distributed to a smaller amount of equity) That means, to obtain the same level of *the rates of total return,* the target underwriting profits for firms with high b and k need not be as high as those for firms with low b and k. Also, *ceteris paribus*, high b and k indicate a higher default risk; because of the extra default risk, insurers may not be able be charge as high an underwriting profit margin as they could otherwise. Furthermore, high leverage factor, k, indicates that the policyholders' fund is retained for a longer period of time, thus the higher contribution from the investment activity to the total profit reduce the expected required profit form underwriting operation; thus, high leverage factor, k, results in a low expected underwriting profit margin.

For the fair net premium (FNP), results of the comparative statics analysis are as follows:

$$\frac{\partial E[P]}{\partial t_{CI}} = \frac{\partial E[P]}{\partial E[r_u]} \cdot \frac{\partial E[r_u]}{\partial t_{CI}} = \frac{E(L)}{(1 - t_{prem} - e - E(r_u))^2} \cdot \frac{r_f}{b \cdot (1 - t_{CI})^2}) > 0 \tag{2.21}$$

$$\frac{\partial E[P]}{\partial (t_{prem} + e)} = \frac{E(L)}{(1 - t_{prem} - e - E(r_u))^2} > 0 \tag{2.22}$$

$$\frac{\partial E[P]}{\partial b} = \frac{\partial E[P]}{\partial E[r_u]} \cdot \frac{\partial E[r_u]}{\partial b} = \frac{E(L)}{(1 - t_{prem} - e - E(r_u))^2} \cdot \frac{-r_f}{b^2 \cdot (1 - t_{CI})} < 0 \tag{2.23}$$

$$\frac{\partial E[P]}{\partial k} = \frac{\partial E[P]}{\partial E[r_u]} \cdot \frac{\partial E[r_u]}{\partial k} = \frac{E(L)}{(1 - t_{prem} - e - E(r_u))^2} \cdot -r_f < 0 \tag{2.24}$$

From equations (2.21) through (2.24), it is shown that in a normal situation, both a higher effective corporate income tax rate and a higher expense-and-other-taxes rate result in a higher fair net premium; and both a higher premium-to-equity ratio and a higher leverage factor cause a lower fair net premium. The high expenses and others taxes (except corporate income taxes) directly increase insurers' general expense, which in turn increases the expected premium. The intuitive explanation about the relationships between expected premium, and t_{CI}, b and k are the same as that about the relationships between expected underwriting profit margin, and t_{CI}, b and k,

2.3 Variable Definition and Sample Selection

2.3.1 Variable Definition

The variables needed to empirically estimate the FUPMs for the property-casualty insurance industry include market data and insurers' operating data. The market data needed include the risk-free rate, equity market return and the equity risk premium. The equity risk premium is the difference between the equity market return and the risk-free rate. In this chapter, the risk-free rate is measured by the yield on the 91-day Government of

Canada T-bill. The equity market return is derived from the S&P/TSX composite total return index and calculated as the ratio of the difference between the year-end index and the year-beginning index to the year-beginning index.

Operating data are collected for each insurer and involve two levels of insurance business. The first level is the aggregate data for the overall business; the second level is for each of the following distinct insurance business lines: auto insurance, property insurance, liability insurance, and others. All the variables are either obtained or derived from *MSA Researcher P&C 2006* database published by MSA Research Inc. of Toronto. The insurers' operating variables include underwriting profit margin, premium-to-equity ratio, leverage factor, net premium earned, net losses incurred, effective corporate income tax rate, and effective expense-and-other-taxes rate.

- The underwriting profit margin is defined as 1 minus the combined ratio, and is expressed as a percentage of net premium earned. Combined ratio, as reported in *MSA Researcher P&C 2006* database, is the ratio of total underwriting expenses as a percentage of net premiums earned. The total underwriting expense includes the incurred losses and expenses. Investment income and capital gains are not taken into account.

- The premium-to-equity ratio is estimated as the ratio of the net premium earned to the value of an insurer's equity with that denominator defined as GAAP capital and surplus at the beginning of the year.

- The leverage factor, k, reflects the average holding period of a dollar of premium before it is used to pay losses and some other expenses (such as claim related expense), e.g., if the premium is retained within a firm for half a year before it is paid for underwriting claims, then k equals 0.5. It is measured as the ratio of the net unpaid claims as a proportion of net premium earned.

- The net premium earned is defined as direct premium earned plus any reinsurance premium received minus reinsurance premium ceded.
- The net loss incurred is defined as the net claims and adjustment expenses incurred, including any unpaid claims and corresponding expenses.
- The effective corporate income tax rate is calculated as the difference between the net income before tax and the net income after tax divided by the net income before tax.
- The expense-and-other-taxes rate is measured as a proportion of net premium earned. (in the thesis, it is also called the Premium-Based Tax). Because all the other taxes are reported within expense items in the annual financial reports, the data requires that, for analytical purposes, expense be combined with other taxes.[4]

[4]The Canadian Property and Casualty (P&C) Insurance industry pays both federal tax and provincial tax. At the federal level, the major taxes imposed upon the Canadian P&C Insurance industry include the Federal Corporate Income tax (which is charged on the amount of corporate net income) and the Federal Capital tax. At the Provincial level, the major taxes include Provincial Corporate Income tax, Provincial Capital tax (in Manitoba and Nova Scotia only), premium tax, fire tax (expect Alberta, Newfoundland, and Quebec), and sales tax (in Quebec, Ontario and Newfoundland only). All together the major taxes include corporate income tax, capital tax, premium tax, fire tax, and sales tax. (*Taxation of P&C Insurance: A Comparison between Canada and other G-7 Countries*, 2003). P&C insurers also are obligated for payroll tax, business tax, and property taxes in the same manner as other employers and property owners. Sales tax is not included in the expense-and-other-taxes estimate in this thesis because, where imposed, that sales tax is paid by the insured rather than the insurers and does not directly affect premium rates. So, while sales tax may affect the consumer perception of the price of insurance, insurers do not take sales tax into account when setting premium rates.

In P&C insurers' income statement, these taxes other than corporate income tax are categorized as follows: premium tax and fire tax are included in acquisition expenses; payroll tax, capital tax and business tax are included in general expenses; and property tax is included in investment expenses. Some taxes are interrelated, e.g., although the capital tax is not deductible in computing income for income tax purposes, it is reduced by the corporation's federal income tax liability, net of any federal surtax claimed against the Part 1.3 tax liability. (for more information see: *General Accepted Accounting Principles* and *the Annual Return Instruction of the Office of the Superintendent Financial Institutions*) The interaction of different types of taxes makes determining and using the marginal taxes rates very difficult and even meaningless. For this reason, unlike some prior research, e.g., D'Arcy and Garven (1990), this thesis adopts the effective tax rates as a variable for examination rather than the marginal rates.

Net premium earned, net claims incurred, and k for each distinct line are derived from annual report data. In the estimation of the fair underwriting profit margins and fair net premiums for distinct business lines, it is assumed that the effective corporate income tax rate, expense-and-other-taxes rate, premium-to-equity ratio are the same across insurance business lines as the aggregate rates for each insurer. While still somewhat restrictive, the assumptions that have appeared in earlier work are still considerably relaxed. The firm's business-participation weight for business line j is measured by the proportion of the j^{th} line's premium written as a proportion of the insurer's aggregate premium written.

2.3.2 Sample Selection

This subsubsection describes the data sources, sample selection procedures, and data screens employed to construct the samples. Estimating the fair underwriting profit margin and fair net premium from the ICAPM for distinct insurance business lines involves three steps: first, the CAPM betas are estimated for each firm using time series regression; second, Full Information Underwriting Betas (FIUB) of distinct business lines are estimated using cross-section regression; finally, equations (2.11) and (2.12) are applied to estimate the Fair Underwriting Profit Margin (FUPM) and Fair Net Premium (FNP) for each distinct business line. Each of these steps requires more parameters than the one before. Some insurers' annual reports did not include complete and accurate information needed for all three steps. For this reason, a reduction in the sample size is observed at each of the three steps.

The sample adopted in the estimation of CAPM underwriting betas includes all active Canadian P&C insurers in the *MSA Research P&C 2006* database that satisfy the following criteria: 1) have at least 8 years of reported underwriting profit margin; 2) have an absolute underwriting profit margin less than or equal to 200; 3) have net premium earned greater than CAN$10,000. In total, 132 insurers satisfy the aforementioned criteria and are included in the CAPM underwriting beta estimation sample. The underwriting profit margins and market return during the period from 1991 through 2005 are collected with the first sample–CAPM underwriting beta estimation sample– containing 1724 company-year observations. Market data are collected from the Toronto Stock Exchange (TSX) Database. Summary statistics for the CAPM underwriting beta estimation sample are presented in table 2.1.

Table 2.1: Descriptive Statistics of Variables in the CAPM Underwriting Beta Estimation Sample

Variable	Obs	Mean	Std. Dev.	Min	Max
rm	15	11.91	15.10	-12.57	32.55
rf	15	4.61	1.83	2.22	8.73
riskprem	15	7.30	15.33	-16.36	27.70
upm	1724	-3.84	30.91	-180.80	188.80

where:

rm	Market return, in %, derived from the S&P/TSX composite total return index and calculated as the ratio of the difference between the year-end index and the year-beginning index to the year-beginning index.
rf	Risk-free rate, in %, measured by the yield on 91-day T-bill.
riskprem	Risk premium, in %, measured by the differnence between market return and the risk-free rate.
upm	Underwriting profit margin, in %, defined as 1 minus the combined ratio, which is the ratio of the total underwriting expenses as a percentage of net premiums earned.

After the CAPM underwriting betas are estimated for each insurer, the model requires every insurer's business-participation weight for each business line in order to estimate the Full Information Underwriting Beta (FIUB) of each distinct line. That information is, thereafter, used to estimate the fair underwriting profit margin (FUPM) and fair net premium (FNP) using equations (2.11) and (2.12). Because the MSA database contains data for distinct lines starting only from 1999 (with some by-line data not available, incomplete or inaccurate, especially at the beginning), the sample size used to estimate the FIUBs is smaller than what was available for the CAPM underwriting beta estimation. The FIUB estimation sample includes 804 company-year observations.

Estimation of the fair underwriting profit margin (FUPM) and fair net premium (FNP) using equations (2.11) and (2.12), in addition to the by-line participation weights, requires further information. Limitations on the availability of these additional variables further shrinks the sample size for the FUPM and FNP estimation sample. The sample used in the estimation of the FUPM and FNP are the insurers that 1) are in the FIUB estimation sample; 2) have complete and accurate by-line data for equations (2.11) and (2.12); 3) have positive leverage factors that reflects the average holding period of a dollar of premium before it is used to pay losses. All these insurers' operating variables needed during the period from 1999 through 2005 are collected from *MSA Researcher P&C 2006* database. The resulting sample contains 393 company-year observations. Summary statistics of the FUPM & FNP estimation sample are shown in table 2.2. The net premium earned and the net claims incurred presented in table 2.2 are the sum across all the insurers in the FUPM and FNP estimation sample; other parameters listed are the average across all the insurers in the FUPM and FNP estimation sample

From table 2.2, it can be seen that, except in years 2000 and 2003, the number of insurers that reported complete and accurate distinct business lines was increasing. On average, over the period from 1999 to 2005 the sample comprised of about 70% of the total Canadian P&C insurance market and should be representative to the Canadian insurance market.[5] As expected, the leverage factor is highest for liability insurance, followed by auto insurance (including auto liability insurance), and then property insurance. This means the liability insurance has longest claim tail, followed by auto insurance and then property insurance. This ordering remains the same whether measured by the annual

[5]In the smallest sample–the FUPM & FNP estimation sample, the total net premium earned in the FUPM and NFNP estimation sample in year 1999 to 2005 made up 46%, 56%, 74%, 47%, 84%, 86% of the Canadian P&C insurance industry's total net premium earned in that year respectively. Except in year 2003, the percetage was increasing. The trend shows that more and more insurers and also larger insurers reported detailed data for distinct business lines over the sample period.

Table 2.2: Descriptive Statistics of Insurers' Operating Variables in the FUPM & FNP Estimation Sample

	1999		2000		2001		2002		2003		2004		2005		total		
Variable	Obs	Mean	Obs	Mean	Obs	Mean	Obs	Mean	Obs	Mean	Obs	Mean	Obs	Mean	Obs	Mean	Std. Dev.
upm	41	1.06	28	-6.35	61	-9.47	60	-2.07	31	0.01	90	11.90	82	9.80	393	2.64	19.13
incometax	41	0.25	28	0.33	61	0.47	60	0.29	31	0.30	90	0.42	82	0.26	393	0.34	0.78
expe_tax	41	0.33	28	0.33	61	0.33	60	0.30	31	0.29	90	0.34	82	0.34	393	0.33	0.13
b	41	1.37	28	1.54	61	1.40	60	1.53	31	1.85	90	1.52	82	1.40	393	1.49	0.82
k	41	0.93	28	0.97	61	1.06	60	0.98	31	0.95	90	0.86	82	1.00	393	0.96	0.56
auto_k	37	1.16	25	1.13	51	1.21	51	1.23	28	1.10	65	1.33	61	1.60	318	1.29	0.95
prop_k	39	0.36	26	0.41	57	0.45	57	0.39	28	0.40	78	0.39	70	0.42	355	0.40	0.31
liab_k	33	2.42	21	2.61	51	2.70	50	2.51	26	2.26	65	2.14	59	2.33	305	2.40	1.90
other_k	27	0.77	17	0.83	44	0.89	45	0.74	22	0.47	63	0.72	57	0.72	275	0.74	0.82
npe	41	165572	28	179441	61	205192	60	250661	31	260585	90	248276	82	280881	393	236194	319914
auto_npe	41	93351	28	105372	61	112673	60	132972	31	149670	90	122542	82	141828	393	124498	213585
prop_npe	41	53644	28	51244	61	61257	60	77213	31	71845	90	74953	82	83768	393	70854	101524
liab_npe	41	13169	28	16481	61	18964	60	26511	31	29279	90	31752	82	35206	393	26466	45918
other_npe	41	5492	28	6275	61	12250	60	13972	31	16825	90	19063	82	20095	393	14940	38681
nci	41	120264	28	140908	61	164646	60	194395	31	193085	90	152840	82	175408	393	164651	229097
auto_nci	41	72722	28	88642	61	97743	60	116385	31	125343	90	81297	82	90864	393	94306	165154
prop_nci	41	35216	28	36677	61	44046	60	48779	31	43390	90	40372	82	54566	393	44624	65853
liab_nci	41	9995	28	11906	61	14517	60	20102	31	21286	90	23338	82	22858	393	19006	34447
other_nci	41	2330	28	3577	61	8298	60	9128	31	9979	90	7784	82	7120	393	7235	23653

where

upm	Underwriting profit margin, in %, defined as 1 minus the combined ratio--the ratio of the total underwriting expenses as a percentage of net premiums earned
incometax	Effective corporate income tax, calculated as the ratio of the difference between the net income before tax and the net income after tax to the net income before tax
expe_tax	Effective expense-and-other-taxes rate (EOT), measured by the ratio of aggregate expenses and other taxes to the net premium earned
b	Ratio of the net premium earned to the value of insurer's equity
k	Leverage factor, reflecting the average holding period of a dollar of premium before it is used to pay expenses and losses. It is measured by the ratio of the net unpaid claims to the net premium earned.
auto_k	Leverage factor of auto insurance
prop_k	Leverage factor of property insurance
liab_k	Leverage factor of liability insurance
other_k	Leverage factor of all other insurance business lines
npe	Total net premium earned, net of reinsurance, in $,000
auto_npe	Net premium earned of auto insurance, in $,000
prop_npe	Net premium earned of property insurance, in $,000
liab_npe	Net premium earned of liability insurance, in $,000
other_npe	Net premium earned of all other insurance business lines, in $,000
nci	Net claims and adjustment expenses incurred, including unpaid claims and corresponding expenses, in $,000
auto_nci	Net claims and adjustment expenses incurred of auto insurance, in $,000
prop_nci	Net claims and adjustment expenses incurred of property insurance, in $,000
liab_nci	Net claims and adjustment expenses incurred of liability insurance, in $,000
other_nci	Net claims and adjustment expenses incurred of all other insurance business lines, in $,000

means or by the grand means. The results related to the "other" business lines group is not discussed in any detail here, because the combination of lines within this group could be very different, thus making any comparison of results potentially meaningless.

2.4 Empirical Results

The empirical results from time series regression (CAPM underwriting betas) and cross-section regression (Full Information Underwriting Beta of each distinct line) are presented. The fair underwriting profit margins and fair net premium are estimated and then followed by the comparative statics analysis.

2.4.1 CAPM Underwriting Betas and Full Information Underwriting Betas

The CAPM underwriting beta and sumbeta estimates for Property & Casualty insurers are summarized in Table 2.3. The betas and sumbetas are estimated using equations (2.13) and (2.14) for each of the 132 insurers. The means, standard deviations, minimums and maximums of the beta and sumbeta estimates in the different size quantiles are provided with size measured by the average of an insurer's equity over the sample period. The sumbeta is derived here because the insurer's annual reports are usually perceived to have some time lag; thus, the underwriting profit margin from that accounting document may relate to market returns in the prior year. The author conducted a Durbin-Watson test to examine the first-order auto-correlation of the residuals of the model based on equation (2.13). Almost half (57 out of 132) of the regressions indicate either the existence of first-order serial correlation or are inconclusive about the existence of first-order serial correlation. In order to examine the significance of this factor, the model was re-run including the lagged variable. As a result of taking into account the prior year's market return in the regression, the average R-square almost doubles. Thus, the chapter includes both models.

Table 2.3: CAPM Underwriting Betas for P&C Insurers

The betas and sumbetas are estimated based on time series regressions using data from 1991 through 2005, and conducted for every firm based on the following equations.

$$r_{uit} = \alpha_i + \beta_{ui} \cdot r_{mt} + e_{it}$$

$$r_{uit} = \alpha_i + \beta 1_{ui} \cdot r_{mt} + \beta 2_{ui} \cdot r_{m(t-1)} + e_{it}$$

$$sumbeta = \beta 1_u + \beta 2_u$$

where,

r_{mt} Market return, in %, derived from the S&P/TSX composite total return index and calculated as the ratio of the difference between the year-end index and the year-beginning index to the year-beginning index

r_{ut} Underwriting profit margin, in %, defined as 1 minus the combined ratio, which is the ratio of the total underwriting expenses as a percentage of net premiums earned

size The average of the GAAP capital and surplus over the sample period for each firm, in $,000

	Variable	Obs	Sample Mean	Sample Std. Dev.	Sample Min	Sample Max
total	beta	132	0.2405	0.5576	-1.1750	2.9020
	sumbeta	132	0.2911	0.8121	-1.9180	3.8370
	size	132	136396	193183	1187	1199408
small	beta	33	0.2883	0.8760	-1.1750	2.9020
	sumbeta	33	0.2657	1.1916	-1.9180	3.8370
	size	33	13075	6881	1187	25008
2	beta	33	0.2299	0.5028	-0.6550	1.8870
	sumbeta	33	0.2629	0.6839	-1.4150	2.4620
	size	33	40164	8499	25717	55893
3	beta	33	0.2449	0.4327	-0.4170	1.8450
	sumbeta	33	0.3325	0.7043	-0.9460	2.7320
	size	33	91418	28137	56220	151591
large	beta	33	0.1990	0.2478	-0.3110	0.8290
	sumbeta	33	0.3032	0.5590	-1.6330	2.1220
	size	33	400925	228877	163806	1199408

From the results shown in Table 2.3, the author finds that small insurers on average have higher underwriting betas and exhibit larger standard deviations. Large insurers on average have lower underwriting betas and have smaller standard deviations. The means and standard deviations of medium size insurers (the second and third quantile) fall in between. The findings are consistent with the arguments in the prior research that beta is negatively related to size. However, that same relationship does not hold for sumbetas across firms with different sizes. Also, the difference observed between betas and sumbetas is larger for bigger firms. This observation indicates that, compared to smaller firms, larger firms exhibit a larger degree of early information release. The result could occur because larger firms are more heavily publicized; more financial information is released before the annual financial statements are officially reported. The results indicate that, after taking into account the early release of information, little difference in underwriting betas remains among firms with different sizes.

The Full Information Underwriting Betas for auto insurance, property insurance, liability insurance, and all other insurance lines are presented in Table 2.4. These results are based on a cross-section regression with the CAPM underwriting betas as the dependent variables and each firm's participation weights of business lines as the independent variables. In panel A, the dependent variables are the betas; and in panel B, the dependent variables are the sumbetas. The coefficients of the business participation weights are the estimates of the Full Information Underwriting Betas of the corresponding business lines. In the prior studies, the underwriting betas were assumed and argued to be proportional to the leverage factors (e.g., Fairley, 1979; Michel and Norris, 1982; Hill and Modigliani, 1987). Specifically, it was assumed that underwriting betas across business lines varied in proportion to the leverage factors of these business lines. Based on the Canadian property & casualty insurance industry data, the empirical findings disprove this assumption. F-tests were conducted to test the null hypothesis that the by-line underwriting betas

Table 2.4: Regression Results of Full Information Underwriting Beta by Line

The table displays full information underwriting beta estimates for auto insurance, property insurance, liability insurance, and all other insurance business lines. The full information underwriting beta is estimated from the following cross-section regression.

$$\beta_{ui} = \sum_{j=1}^{J} \beta_{fuj} \cdot \omega_{ij} + \upsilon_{ui}$$

where,

β_{ui} Insurer i's underwriting beta

β_{fuj} The full information underwriting beta for business line j

ω_{ij} Firm i's business-participation weight for business line j, with the premium written as the weight

υ_{ui} The random error term for firm i

The regression is estimated by OLS. The full information regression conducted separatedly for each calendar year and for the pooled 7-year data set.

	1999	2000	2001	2002	2003	2004	2005	panel estimate
Panel A: The full information underwriting betas estimated from betas								
auto	0.381	0.321	0.37	0.287	0.354	0.139	0.203	0.293
	(3.46)**	(2.74)**	(3.59)**	(2.66)**	(3.11)**	-1.4	-1.86	(7.18)**
property	0.298	0.02	0.166	0.195	0.065	0.286	0.283	0.199
	(2.35)*	-0.13	-1.45	-1.62	-0.48	(2.55)*	(2.37)*	(4.26)**
liability	-0.257	0.171	0.142	0.273	0.249	0.089	0.097	0.113
	-1.15	-0.85	-0.76	-1.46	-1.24	-0.49	-0.56	-1.56
other	0.164	0.118	0.128	0.084	0.262	0.396	0.214	0.188
	-1.2	-0.78	-1	-0.68	-1.73	(3.15)**	-1.63	(3.71)**
Observations	129	92	130	127	96	120	110	804
R-squared	0.19	0.12	0.18	0.16	0.2	0.21	0.18	0.16
F-test for panel estimates:		$\beta_{fu(auto)}/k_{auto} = \beta_{fu(prop)}/k_{prop}$				F(1,800)=4.25		Prob>F=0.0396
		$\beta_{fu(auto)}/k_{auto} = \beta_{fu(liab)}/k_{liab}$				F(1,800)=16.04		Prob>F=0.0001
		$\beta_{fu(prop)}/k_{prop} = \beta_{fu(liab)}/k_{liab}$				F(1,800)=12.09		Prob>F=0.0005
Panel B: The full information underwriting betas estimated from sumbetas								
auto	0.589	0.353	0.531	0.447	0.478	0.216	0.314	0.42
	(3.57)**	(2.19)*	(3.53)**	(2.94)**	(2.77)**	-1.5	(2.12)*	(7.15)**
property	0.309	0.072	0.149	0.321	0.17	0.465	0.515	0.3
	-1.63	-0.34	-0.89	-1.89	-0.83	(2.85)**	(3.17)**	(4.45)**
liability	0.064	0.009	0.369	0.32	0.4	0.191	0.217	0.23
	-0.19	-0.03	-1.34	-1.21	-1.32	-0.72	-0.92	(2.19)*
other	-0.07	-0.095	-0.112	-0.114	-0.08	0.171	-0.207	-0.075
	-0.34	-0.45	-0.6	-0.66	-0.35	-0.94	-1.16	-1.03
Observations	129	92	130	127	96	120	110	804
R-squared	0.17	0.07	0.16	0.17	0.16	0.17	0.23	0.15
F-test for panel estimates:		$\beta_{fu(auto)}/k_{auto} = \beta_{fu(prop)}/k_{prop}$				F(1,800)=5.00		Prob>F=0.0256
		$\beta_{fu(auto)}/k_{auto} = \beta_{fu(liab)}/k_{liab}$				F(1,800)=12.64		Prob>F=0.0004
		$\beta_{fu(prop)}/k_{prop} = \beta_{fu(liab)}/k_{liab}$				F(1,800)=12.25		Prob>F=0.0005

Absolute values of the significant t-statistics are shown in parentheses

* significant at 5%; ** significant at 1%

are proportional to the leverage factors, k, for the panel estimates. The findings rejecting the null hypothesis are significant at the 95% confidence level. Therefore, the analysis presented in this chapter strongly supports the conclusion that underwriting betas are not proportional to the leverage factors.

Other line-specific risk factors combined with k may influence the by-line underwriting betas together. To examine what the extant data indicates regarding this possibility, consider the average leverage factor, k, for auto insurance, property insurance and liability insurance. These leverage factors have been measured at 1.29, 0.4 and 2.4 respectively. If the null hypothesis–the by-line underwriting betas are proportional to the leverage factors– were true, then the underwriting beta of liability insurance should be higher than those of auto insurance and property insurance; however, the underwriting beta for liability insurance, at 0.113 (for beta, or 0.23 for sumbeta), is the lowest of the three. Therefore, the null hypothesis is not supported by the empirical results of the present study. The positive underwriting betas of auto insurance, property insurance and liability insurance suggest that the underwriting profits of these insurance business lines are positively correlated with the financial market (i.e., generally follows the financial market). Liability insurance with the lowest underwriting beta has the lowest volatility in relation to the financial market.

2.4.2 The Results of Fair Underwriting Profit Margin and Fair Net Premium

In Table 2.5, the actual and the estimated fair underwriting profit margins and net premiums for distinct business lines are provided. The risk-free rate and the equity risk premium for systematic risk are listed in the first two rows of the table for the convenience of later discussion. The by-line FUPM and FNP estimates are calculated based on equations (2.11) and (2.12), and on the by-line Full Information Underwriting Betas as presented in Table 2.4. Panel A shows the actual and the estimated fair underwriting profit margins; panel B shows the actual and the estimated fair net premiums.

Fair underwriting profit margins depend on the risk-free rate, leverage factor, effective corporate income tax rate, premium-to-equity ratio, underwriting beta, and equity risk premium. Comparing the results across different years, the author finds that FUPMs for all lines are closely related to the equity risk premium. When the equity risk premium is high, the FUPM is relatively high and tends to be positive; when the equity risk premium is low or negative, the FUPM is relatively low and may be negative. This phenomenon can be easily observed during the sample period because the risk-free rate and leverage factor k for all lines are relatively stable over the sample period and also the beta and sumbeta for all lines are positive. Comparing the results across different business lines within the same year, it is found that the FUPMs of liability insurance are always the lowest and are consistently considerably lower than the FUPM of the combined lines. This means that the effect of the high leverage factor, k, of liability insurance heavily influences the FUPM in a negative direction. For property and auto insurance, the effects of k interact with the effects caused by the underwriting beta and equity risk premium. Sometimes the FUPMs of auto insurance are higher than those of property insurance, and sometimes lower. Comparing the results based on betas and sumbetas, the author finds that during times when equity risk premium is positive, most of the time the FUPM based on sumbetas are higher than those based on betas, since both betas and sumbetas are positive and most of the time sumbetas are higher than betas for both combined lines and for the distinct lines. When the equity risk premium is negative, the relationship is reversed. For FNP, since the amount of FNP depends on the amount of business that insurers assume, only the expected premium based on betas and on sumbetas in the same year can be compared. The author finds that when equity risk premium is positive, the FNPs based on sumbetas are higher than those based on betas.

Overall, the empirical findings in Table 2.5 strongly support the statements that 1)

Table 2.5: Estimates of Fair Underwriting Profit Margin and Fair Net Premium

This table displays the estimates of the fair underwriting profit margin (FUPM) and fair net premium (FNP) for each year and for the aggregated 7-year data set. The fair underwriting profit margin and fair net premiums were estimated based on the following formulas for combined lines and for each distinct line (i.e., auto insurance, property insurance, liability insurance, and other business lines). Both the estimates calculated based on betas and on sumbetas are provided.

$$E[r_u] = -r_f \cdot k + \frac{r_f \cdot t_{CI}}{b \cdot (1 - t_{CI})} + \beta_u \cdot (E[r_m] - r_f) \qquad E[P] = \frac{E(L)}{1 - t_{prem} - e - E(r_u)}$$

where

$E[r_u]$	Fair underwriting profit margin, in %.
$E[r_m]$	Market return, in %, derived from the S&P/TSX composite total return index and calculated as the ratio of the difference between the year-end index and the year-beginning index to the year-beginning index.
r_f	Risk-free rate, in %, measured by the yield on 91-day T-bill.
k	The aggregate leverage factor, reflecting the average holding period of a dollar of premium before it is used to pay expenses and losses. It is measured by the ratio of the net unpaid claims to the net premium earned.
b	Ratio of the net premium earned to the value of insurer's equity
t_{CI}	Effective corporate income tax, calculated as the ratio of the difference between the net income before tax and the net income after tax to the net income before tax
β_u	Underwriting beta
$t_{prem} + e$	Effective expense-and-other-tax rate, measured by the ratio of aggregate expenses and other taxes to the net premium earned
E[P]	Fair net premium, in $,000
L	Net claims and adjustment expenses incurred, including unpaid claims and corresponding expenses.

Data year	1999	2000	2001	2002	2003	2004	2005	all years
rf	4.72	5.49	3.79	2.58	2.87	2.22	2.73	3.48
rm-rf	26.87	2.02	-16.36	-15.02	23.85	12.26	21.40	7.86
Panel A: Actual and estimated fair underwriting profit margin (FUPM)								
1. Actual underwriting profit margin								
upm	1.06	-6.35	-9.47	-2.07	0.01	11.90	9.80	2.64
2. Estimated fair underwriting profit margin (FUPM) based on betas								
FUPM	2.96	-3.08	-5.39	-5.29	3.46	1.96	2.88	-0.33
auto FUPM	5.91	-3.80	-8.25	-6.80	5.97	-0.19	0.67	-0.99
prop FUPM	7.43	-0.47	-2.01	-3.23	1.08	3.68	5.60	1.36
liab FUPM	-17.18	-12.22	-10.15	-9.87	0.12	-2.60	-3.60	-6.28
other FUPM	1.93	-2.54	-3.08	-2.47	5.58	4.31	3.28	0.10
3. Estimated fair underwriting profit margin (FUPM) based on sumbetas								
FUPM	4.36	-2.98	-6.25	-6.07	4.71	2.60	4.00	0.08
auto FUPM	11.50	-3.74	-10.89	-9.20	8.93	0.75	3.05	0.01
prop FUPM	7.72	-0.36	-1.73	-5.12	3.58	5.88	10.57	2.16
liab FUPM	-8.55	-12.55	-13.87	-10.58	3.72	-1.35	-1.04	-5.36
other FUPM	-4.36	-2.97	0.85	0.50	-2.58	1.55	-5.73	-1.97

Data year	1999	2000	2001	2002	2003	2004	2005	all years
Panel B: Actual and estimated fair net premium (FNP)								
1. Actual net premium								
P	6,788,453	5,024,333	12,516,686	15,039,672	8,078,119	22,344,835	23,032,203	92,824,301
auto P	3,827,393	2,950,420	6,873,022	7,978,289	4,639,771	11,028,758	11,629,881	48,927,534
prop P	2,199,407	1,434,826	3,736,667	4,632,758	2,227,209	6,745,810	6,868,961	27,845,638
liab P	539,913	461,459	1,156,775	1,590,684	907,659	2,857,689	2,886,865	10,401,044
other P	225,192	175,702	747,247	838,344	521,584	1,715,663	1,647,756	5,871,488
2. Estimated fair net premium (FNP) based on betas								
FNP	7,741,271	5,660,361	13,848,043	15,458,228	8,888,535	21,342,220	22,887,123	95,625,693
auto FNP	4,908,767	3,524,443	7,909,046	9,073,173	5,993,527	10,985,533	11,452,489	54,243,382
prop FNP	2,437,904	1,530,839	3,885,856	3,987,570	1,929,205	5,792,223	7,441,381	26,581,140
liab FNP	488,832	422,838	1,145,849	1,507,025	933,576	3,043,453	2,703,338	10,146,257
other FNP	147,634	144,820	720,940	754,014	474,285	1,128,059	934,986	4,228,553
3. Estimated net premium based (FNP) on sumbetas								
FNP	7,915,757	5,668,933	13,686,717	15,299,156	9,056,140	21,556,442	23,301,688	96,209,631
auto FNP	5,406,293	3,527,676	7,641,995	8,798,514	6,280,064	11,143,474	11,886,503	55,047,731
prop FNP	2,450,134	1,533,237	3,901,551	3,887,361	2,001,093	6,002,198	8,111,139	26,904,919
liab FNP	544,905	421,092	1,093,305	1,493,852	983,709	3,099,615	2,807,320	10,274,624
other FNP	134,560	143,926	763,650	786,193	421,555	1,080,086	817,094	4,102,417

using betas instead of sumbetas in insurance pricing may underestimate both the fair underwriting profit margin and the fair net premium when the market's equity risk premium is positive; 2) FUPMs are closely related to the equity risk premium, which varies across years; 3) FUPM varies across business line, with the lowest for liability insurance. These findings imply that setting a single target underwriting profit margin rate for distinct business lines and across years is inappropriate and could be dangerous.

2.4.3 The Results of the Comparative Statics Analysis

This subsubsection examines the impacts of several parameters on insurance pricing, by changing the value of particular parameters respectively and holding the others constant at the same time. Through these sensitivity tests, the impact of each variable in insurance pricing can be discerned. This allows insurer management to put more effort into monitoring and improving the estimation accuracy of those parameters that are most important in insurance pricing. The results also allow insurer management and regulators to better evaluate the potential impact on premium rates that may be implied by alternative management decisions (e.g., new business plan or new financing that will influence the premium-to-equity ratio) or regulatory activities (e.g., changes in taxation policy).

Table 2.6 presents the results of the comparative statics analysis. The sensitivity results based on the beta estimates and the sumbeta estimates are consistent, even though the sumbetas are usually higher than betas. Also the results for the aggregated combined lines and for every distinct line are consistent. Panel A and panel B show the sensitivity of the fair underwriting profit margin and of the fair net premium respectively when the values of effective corporate income tax rate, effective expense-and-other-taxes rate, premium-to-equity ratio, and leverage factor change respectively.

These results show that a 1% increase in the effective corporate income tax (CIT) rate (i.e., CIT increases from 34% to 35%)[6] will lead to a 0.05% increase in fair underwriting profit margin (i.e., FUPM increases from -0.33% to -0.28%) and a 0.08% increase in fair net premium (i.e., 95,625,693,000*0.08%=$76,500,554 increase in the premium charged on the full market of the sample firms). The decrease in CIT rate implied that in order to achieve the same level of post-tax profit level the required pre-tax profit level does not need to be as high as before. The results show that ICAPM is not highly sensitive to the change in CIT rate.

Change in the effective expense-and-other-taxes rate (EOT) does not influence the fair underwriting profit margin directly; however, it does directly influence the leverage factor, k. A higher EOT results in a lower k. As discussed earlier and shown in Table 2.6, a lower k produces a higher FUPM; i.e., higher effective expense-and-other-taxes rate indirectly results in a higher fair underwriting profit margin. The results above also indicate that a 1% increase in EOT (i.e., EOT increases from 33% to 34%) estimates a 1.5% increase in fair net premium (i.e., 95,625,693,000*1.5%=$1,434,385,395 increase in the premium

[6]The parameters' values in the examples in the parentheses in this paragraph are based on the aggregated values over year 1999-2005 of all combined lines of the FUPM and FNP estimation sample.

charged on the full market of the sample firms). Economically, an increase (or decrease) in premium of more than 1% is reasonable, since administrative issues ensure that it costs more than \$1 to collect, report, and remit \$1 of taxes to the government. The change in an economic parameter (e.g., tax rate) applies to all the insurers in the same market, insurers could pass all or at least part of these costs along to customers.

The table shows the results of the comparative statics analysis of fair underwriting profit margin (FUPM) and fair net premium (FNP) with respect to the parameters that reflect the insurers operation. The estimates of FUPM and FNP for combined-lines and each distinct line based on the average values of the parameters over the period from 1999 through 2005 are listed under the average estimates column. Keeping all other parameters unchanged, the values of corporate income tax (CIT), expense-and-other-taxes rate (EOT), premium-to-equity ratio (b), and leverage factor (k) varies in turn; the **difference changes in FUPM** (the FUPM is expressed in %) are listed in panel A and the **percentage changes in FNP** are listed in panel B. For example, under "CIT+1%" column the results show the difference change in FUPM and percentage change in FNP for combined-line and each distinct line if the corparate income tax is increased by 1% point (i.e. CIT increases from 34% to 35%).

	average estimates	CIT-2%	CIT-1%	CIT+1%	CIT+2%	EOT-2%	EOT-1%	EOT+1%	EOT+2%	b-0.2	b-0.1	b+0.1	b+0.2	k-0.1	k+0.1
Panel A: sensitivity analysis of fair underwriting profit margin															
Estimates based on beta															
FUPM	-0.33	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
auto FUPM	-0.99	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
prop FUPM	1.36	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
liab FUPM	-6.28	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
other FUPM	0.10	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
Estimates based on sumbeta															
FUPM	0.08	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
auto FUPM	0.01	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
prop FUPM	2.16	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
liab FUPM	-5.36	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
other FUPM	-1.97	-0.10	-0.05	0.05	0.11	0.00	0.00	0.00	0.00	0.19	0.09	-0.08	-0.14	0.35	-0.35
Panel B: sensitivity analysis of fair net premium															
Estimates based on beta															
FNP	95,625,693	-0.15%	-0.08%	0.08%	0.16%	-2.87%	-1.46%	1.50%	3.05%	0.28%	0.13%	-0.11%	-0.21%	0.52%	-0.51%
auto FNP	54,243,382	-0.15%	-0.08%	0.08%	0.16%	-2.84%	-1.44%	1.49%	3.02%	0.27%	0.13%	-0.11%	-0.21%	0.51%	-0.51%
prop FNP	26,581,140	-0.16%	-0.08%	0.08%	0.17%	-2.94%	-1.49%	1.54%	3.13%	0.28%	0.13%	-0.11%	-0.22%	0.53%	-0.53%
liab FNP	10,146,257	-0.14%	-0.07%	0.07%	0.15%	-2.64%	-1.34%	1.38%	2.79%	0.25%	0.12%	-0.10%	-0.19%	0.48%	-0.47%
other FNP	4,228,553	-0.15%	-0.08%	0.08%	0.17%	-2.89%	-1.47%	1.51%	3.07%	0.28%	0.13%	-0.11%	-0.21%	0.52%	-0.52%
Estimates based on sumbeta															
FNP	96,209,631	-0.15%	-0.08%	0.08%	0.17%	-2.89%	-1.47%	1.51%	3.06%	0.28%	0.13%	-0.11%	-0.21%	0.52%	-0.52%
auto FNP	55,047,731	-0.15%	-0.08%	0.08%	0.16%	-2.88%	-1.46%	1.51%	3.06%	0.28%	0.13%	-0.11%	-0.21%	0.52%	-0.51%
prop FNP	26,904,919	-0.16%	-0.08%	0.08%	0.17%	-2.98%	-1.51%	1.56%	3.17%	0.29%	0.13%	-0.12%	-0.22%	0.54%	-0.53%
liab FNP	10,274,624	-0.14%	-0.07%	0.08%	0.15%	-2.68%	-1.36%	1.39%	2.83%	0.26%	0.12%	-0.10%	-0.20%	0.48%	-0.48%
other FNP	4,102,417	-0.15%	-0.08%	0.08%	0.16%	-2.80%	-1.42%	1.46%	2.97%	0.27%	0.13%	-0.11%	-0.21%	0.51%	-0.50%

Table 2.6: Results of the Comparative Statics Analysis

It is expected that, *ceteris paribus*, high premium-to-equity ratio may indicate a higher default risk (e.g., Phillips et al., 1998). Because of the extra default risk, insurers may not be able to charge as high a premium as they could otherwise. The fair underwriting profit margin and fair net premium are expected to decrease with an increase in the premium-to-equity ratio. The results confirm this relationship and show that an increase of 0.1 in the premium-to-equity ratio (i.e., b increases from 1.49 to 1.59) causes the fair underwriting profit margin to decrease by 0.08% (i.e. FUPM falls from -0.33% to -0.41%) and the fair net premium to decrease by 0.11%.

A high leverage factor, k, results in a relatively low FUPM and low FNP, since insurers with higher k retain the policyholders' fund for a longer period of time and need to compensate policyholders for investing their funds with a lower premium. As expected, the results show that 0.1 increase in k (i.e., k increases from 0.96 to 1.06) leads to a 0.35% decrease in fair underwriting profit margin (i.e., FUPM decrease from -0.33% to -0.65%) and a 0.51% decrease in fair net premium.

The results of the comparative statics sensitivity analysis show that the effective expense-and-other-taxes rate has the greatest impact on insurance premium. The corporate income tax rate, premium-to-equity ratio and leverage factors also are important parameters influencing the insurance premium.

2.5 Conclusion

Chapter 2 presents new evidence on insurance pricing by line of Property & Casualty insurance. The analysis in the present chapter differs from previous research in that it uses the full information beta methodology to estimate underwriting betas of distinct business lines, which are then applied to estimate the fair underwriting profit margin by line. It is expected that the full information underwriting betas of distinct business lines contain more information and measure the risks of business lines more reliably, i.e., the risk of underwriting varies among business lines in more regards than simply the length of the period over which premium dollars can be held.

The firms' underwriting betas are first estimated and then used to derive the full-information underwriting betas for distinct business lines, which are then used to estimate the fair underwriting profit margin by line of insurance. In addition to the corporate income tax, the chapter also includes other taxes in the model, thus allowing further study of the impacts of taxes on insurance pricing. Finally, the chapter conducts comparative statics analysis with respect to the important parameters in the Insurance Capital Asset Pricing Model with the quantitative results of their impacts on insurance pricing presented.

The samples include all the Canadian Property & Casualty insurers in the *MSA Researcher P&C 2006* database with complete and accurate values of the parameters needed in the models. Despite limitations of data availability, the smallest sample–the FUPM and FNP estimation sample–comprises about 70% of the Canadian Property & Casualty insurance market. Based on Canadian P&C insurance industry data, the primary empirical finding strongly supports the argument that underwriting betas of distinct lines do not vary in proportion to the length of the period that the premium of the corresponding line can be kept for investment. The other important findings include:

- Compared to smaller firms, larger firms appear to release a larger degree of information earlier than the annual financial statements are reported. After taking into account the early information release, the difference in underwriting betas among firms with different sizes is not obvious. Also, smaller insurance firms exhibit a higher standard deviation of underwriting beta.

- The fair underwriting profit margin depends on risk-free rate, leverage factor, effective corporate income tax rate, premium-to-equity ratio, underwriting beta, and market equity risk premium. In addition to these parameters, fair net premium is also related to expense-and-other-taxes rate. The empirical findings highly support the statements that 1) using betas instead of sumbetas in insurance pricing may underestimate the fair underwriting profit margin and fair net premium when the equity risk premium is positive; 2) FUPMs are closely related to market equity risk premium, which varies over years; 3) FUPMs vary across business lines with liability insurance having the lowest FUPM. These findings imply that setting a single target underwriting profit margin rate for distinct business lines and over time is inappropriate and could be dangerous.

- The results of the comparative statics analysis show that fair underwriting profit margin and fair net premium are positively related to effective corporate income tax rate, and are negatively related to premium-to-equity ratio and leverage factor. Also, fair net premium is positively related to effective expense-and-other-taxes rate. The empirical results confirm the results predicted by the comparative statics analysis and show that effective expense-and-other-taxes rate has a largest impact on insurance premium and that effective corporate income tax rate, premium-to-equity ratio, and leverage factor are all important parameters influencing the insurance premium.

The underwriting beta derived based on the Capital Asset Pricing Model measures the systematic risk of insurance underwriting activity related to the financial market, i.e. the underwriting beta is a measure of volatility of the insurance underwriting activity in relation to the financial market. A positive beta means that insurance underwriting performance generally follows the financial market; the underwriting part of the business increases in value if the financial market goes up. Thus, in the Insurance Capital Asset Pricing Model, only systematic risk is taken into account, while the firm-specific risks are not considered in pricing. This characteristic makes this model a suitable candidate for setting target underwriting profit margin by regulators attempting to regulate insurance pricing since in such a setting the regulator would not normally set different rates for different companies based on individual companies' risk profiles. However, this model may not provide individual insurers enough information for accurate pricing because that insurer needs to consider the company's total risk rather than just the systematic risk of the company.

As in all other studies, there are some limitations inherent in this approach. First, due to the limitations of the ICAPM, default risk could not be fully considered in the present model. A model that takes into account default risk and under-utilized tax shield is developed in chapter 3. Second, only yearly data is available in *MSA Researcher P&C 2006* database for this study. Further research based on quarterly data may increase the accuracy and power of the current results.

Furthermore, future study can be conducted to explore the risk factors that cause the differences in the fair underwriting profit margin across business lines. In general, the full information underwriting betas derived in this chapter enable more accurate estimation of the fair underwriting profit margin and the fair net premium by distinct insurance business line. In turn, the results from the full information beta methodology can provide better guidance for decisions by both regulators and management.

3 Essay 2: The Insurance Pricing of Distinct Business Lines in Multiple-line Property & Casualty Insurance Company: A Contingent Claims Approach

3.1 Introduction

Setting a fair or competitive premium plays an important role in the insurance industry because capital is invested or retained in the insurance industry only if the return provided by the insurance industry is comparable to that offered by other industries. Determining an appropriate insurance premium has become the subject of extensive scrutiny over the last several decades among both academia and industry practitioners. Starting from the earliest attempt to determine the fair premium–the Target Underwriting Profit Margin promulgated by the National Convention of Insurance Commissioners in 1921– a variety of insurance pricing models have been proposed and applied, including the Capital Asset Pricing Model (e.g., Fairley, 1979; Hill, 1979; Hill and Modigliani, 1987), the Internal Rate of Return Approach (e.g., Cummins, 1990), the Discounted Cash Flow Approach (e.g., Myers and Cohn, 1987; Cummins, 1990; D'Arcy and Garven, 1990), the Arbitrage Pricing Model (e.g., Kraus and Ross, 1987; Urrutia, 1987a), and the Option Pricing Model (e.g., Doherty and Garven, 1986; D'Arcy and Garven, 1990; Phillips et al., 1998). Such financial insurance pricing models have the strength that they incorporate the capital market into insurance pricing and could provide non-arbitrage insurance pricing.

D'Arcy and Garven (1990) compared the major property-liability insurance pricing models, including Target Underwriting Profit Margin Method, Total Rate of Return Model, Capital Asset Pricing Model and Option Pricing Model, over the 60-year period from 1926 through 1985. Their results showed that usually the Total Rate of Return Model and Option Pricing Model produced a better fit, but the relative goodness-of-fit of the these models was not stable over time. Also their results found that the option pricing model was particularly sensitive to changes in the tax-related parameters, which makes it a good tool to conduct a careful examination on the effects of taxation on underwriting profit margin and insurance premium. Garven (1992) concluded several important practical advantages of Option Pricing Model (OPM). First, OPM gets around some peculiar difficulties related to parameter estimation; second, OPM can explicitly quantify the value of insolvency risk; and third, the effects of under-utilized tax shield can be explicitly modelled.

Since the 1970's, the financial field has witnessed tremendous growth in the application of the option pricing model. Unexceptionally OPM also has received increasing attention among both insurance academia and industry practitioners (e.g., Doherty and Garven, 1986; Cummins, 1988; Derrig, 1989; D'Arcy and Garven, 1990; Garven, 1992; Wang, 2000). The rationale for applying OPM in insurance pricing is that insurance policies can be viewed as a package of contingent payments depending on the insurer's underwriting and investment performance; and the value of the contingent payments can be estimated by the framework of OPM.

Related work in insurance pricing developed from conventional Black-Scholes option pricing model is briefly discussed here as background. In the early insurance application of the Black-Scholes model, Merton (1977) applied the OPM to estimate the pricing of loan guarantees and deposit insurance. Doherty and Garven (1986) modelled the contingent claims to shareholders, policyholders, and tax authorities by the use of European options. To estimate the insurance premium and underwriting profit margin, Doherty and Garven assumed two alternative valuation frameworks: 1) jointly normal distribution of assets and liabilities and Constant Absolute Risk Aversion (CARA) investors and 2) jointly log-normal distribution of assets and liabilities and Constant Relative Risk Aversion (CRRA) investors. Although closed-form formulae were not derived, the numerical results derived from both frameworks were illustrated using values for the various parameters as selected by the authors.

Many existing studies in insurance pricing implicitly or explicitly assumed that insurers provide only one line of business (or assumed that the total business is one single line). For example, Sommer (1996) applied the OPM framework to measure insolvency risk and derived that insurance price was the present value of loss claims minus the value of an insolvency put option that captured the insolvency risk of insurer. The results from the empirical regression supported the hypotheses derived from the theoretical framework that 1) insolvency risk was negatively related to the insurance price, 2) insurance purchasers recognized that the existence of guaranty funds did not provide them with full protection, and 3) flat rate guarantee fund premiums gave insurers the incentive to undertake more risky strategies. Motivated by the problems caused by the flat rate guarantee fund premium scheme, Cummins (1988) developed a risk-based premium estimation technique for insurance guaranty funds. The models developed in his paper assumed the lognormal distributions of insurer's asset and liability, and explicitly allowed for discrete jumps in liability. The value of the guaranty fund insurance was modelled by a put option with the value of the insurer's total liability being the exercise price and the insurer's total asset being the underlying security.

The importance of default risk in insurance pricing encouraged a series of studies on this topic. Chen et al. (2003) extended the traditional Insurance Capital Assets Pricing Model (ICAPM) (e.g., Fairley, 1979; Hill, 1979; Cummins and Harrington, 1985) by incorporating the insolvency risk of insurers measured through the OPM framework into the ICAPM. The result from Chen et al.'s (2003) empirical regression model confirmed the hypothesis derived from their theoretical model and was consistent with the prior studies that for a given risk exposure the insurance premium was reduced by insurer's risk of insolvency. Without accounting for the insolvency risk of insurers, the traditional ICAPM overestimated the insurance premium.

Few of the prior studies investigated the pricing of distinct lines of insurance business within a multi-line insurer. Although most studies focused on estimating the aggregate underwriting profit margin for all lines, insurers need to set appropriate rates for distinct lines of business because of the risk differences that exist across lines. If the weights of different lines of business change over time or across firms, the aggregate results may be misleading. D'Arcy and Garven (1990) pointed out that future research on various pricing models for distinct lines of business could provide more accurate results and avoid the aforementioned problem. In an attempt to remedy the deficiency, Phillips et al. (1998) developed an insurance pricing framework for distinct lines of business based on OPM and conducted an hypothesis test. They extended Sommer's (1996) single-line model to the multi-line insurance business. Their regression results supported that prices varied across firms depending upon overall firm default risk and the concentration of business among subsidiaries; but within a given firm, after controlling for line-specific liability growth rates, the prices did not vary by line.

The purpose of this chapter is to propose an insurance pricing model for distinct business lines in the setting of a multi-line business. The methodology developed in this chapter is an extension of the work on insurance pricing by Doherty and Garven (1986). The contingent claims approach is adopted to model the financial claims of shareholders, policyholders, and tax authorities, which could be modelled as European options that are written on the income generated by the insurer's asset and liability portfolio. First, the chapter extends the work by Doherty and Garven (1986) from a single-line model to a multi-line model. Second, the chapter conducts a comparative statics analysis of the insurance premium and underwriting profit margin with respect to taxes and provides the numerical results of the sensitivity analysis. Third, this chapter provides numerical illustrations of expected underwriting profit margins by business lines that augment the hypothesis tests used in the prior studies(e.g., Sommer, 1996; Phillips et al., 1998; Chen et al., 2003).

The remainder of the chapter is organized as follows: The next subsection below develops the theoretical model. The comparative statics analysis is conducted in subsection 3.3. Descriptions of the data sample and definitions of variables and parameters needed in the model follow in subsection 3.4. The numerical results are presented in subsection 3.5. Subsection 3.6 concludes the chapter.

3.2 Theoretical Model

In this subsection, the author extends Doherty and Garven's (1986) model by taking into account expenses and taxes as well as extending the single-line model into a multi-line model. The model for aggregate line is first derived and then followed by the derivation for the model for distinct lines.

3.2.1 For Aggregate Line

In the single-period model, S_0 denotes shareholders' initial capital investment, and P_0 denotes the premium received from the policyholders. Net of taxes (except corporate income tax)[7] and expenses, the initial cash flow, Y_0, is expressed as,

$$Y_0 = S_0 + P_0 \cdot (1 - t_{prem} - e) \tag{3.1}$$

where,

t_{prem} is the tax rate of all other taxes except corporate income tax, expressed as a proportion of premium;

e is the expense rate and also is expressed as a proportion of premium.

Assume claims and corporate income tax are paid at the end of period, and investment

[7] See subsubsection 2.3.1 for the detailed discussion of tax-related issues.

income is generated at rate $\tilde{r}_a$. Before claims and corporate income tax, the terminal cash flow, $\tilde{Y}_1$, is:

$$\tilde{Y}_1 = S_0 + P_0 \cdot (1 - t_{prem} - e) + (S_0 + k \cdot P_0 \cdot (1 - t_{prem} - e)) \cdot \tilde{r}_a \tag{3.2}$$

where,

k is the leverage factor reflecting the average holding period of a dollar of premium[8] (Fairley, 1979, pp.198).

The value of $\tilde{Y}_1$ is allocated to different claimholders, i.e., policyholders, governments and shareholders, in a set of payments having the characteristics of call options. Under the usual bankruptcy constraint (i.e., limited liability for shareholders), the aggregate payment to policyholders is the minimum of the total claims and the insurer's total assets. Assuming $\tilde{Y}_1$ will not be negative, the payment to policyholders $\tilde{H}_1$ is:

$$\tilde{H}_1 = \min(\tilde{Y}_1, \tilde{L}) = \tilde{Y}_1 - \max(\tilde{Y}_1 - \tilde{L}, 0) \tag{3.3}$$

where, $\tilde{L}$ is the insurer's underwriting claims cost including the claim adjustment expenses.

The corporate income tax is assumed to be paid to the government if the insurer has positive profit with the insurer paying zero corporate income tax if its profit is zero or negative. The corporate income tax paid to the government, $\tilde{T}_1$, can be expressed as:

$$\tilde{T}_1 = \max(t_{CI} \cdot (\tilde{Y}_1 - S_0 - \tilde{L}), 0) \tag{3.4}$$

where, t_{CI} is the corporate income tax rate.

[8] For example, if the premium is retained within a firm for half a year before it is paid for underwriting claims, then k equals 0.5. Please see subsection 3.4 for the detailed description.

The present values of $\tilde{H}_1$ and $\tilde{T}_1$ at the beginning of the period, H_0 and T_0, can be expressed by the values of call options:

$$H_0 = V(\tilde{Y}_1) - C[\tilde{Y}_1; \tilde{L}_1] \tag{3.5}$$

$$T_0 = t_{CI} \cdot C[\tilde{Y}_1 - \tilde{S}_0; \tilde{L}] \tag{3.6}$$

where, V(·) means the present value; C[A;B] is the current market value of a European call option based on underlying assets having a terminal value of A and exercise price of B.

Shareholders own the residual claim, i.e., the difference between the market value of insurer's total assets $V(\tilde{Y}_1)$, and the values of claims to policyholders H_0 and governments T_0. Shareholders' value V_e can be expressed as:

$$V_e = V(\tilde{Y}_1) - H_0 - T_0 \tag{3.7}$$

$$= C[\tilde{Y}_1; \tilde{L}] - t_{CI} \cdot C[(\tilde{Y}_1 - \tilde{S}_0; \tilde{L})]$$

$$= C_1 - t_{CI} \cdot C_2 \tag{3.8}$$

If the insurance premium is set at a level such that a "fair" return is delivered to shareholders, then the current market value of the shareholders' value V_e must be equal to the initial capital investment S_0. The "fair" return is defined as the investment return derived from equilibrium relationship in the competitive capital markets. $\tilde{Y}_1$ is a function of P_0. The fair rate of return is implied in equation (3.9) shown below, by solving for the fair premium, P_0^*, that satisfies equation (3.9).

$$V_e = C_1^* - t_{CI} \cdot C_2^* = S_0 \tag{3.9}$$

C_1^* and C_2^* depend on the random variables $\tilde{r}_a$ and $\tilde{L}$; so does P_0^*.

In order to solve equation (3.9), the model incorporates the following formal assumptions:

1. The aggregation assumption: securities are priced as if all investors have the same characteristics as a representative investor.

2. Jointly lognormal distribution assumption: assuming the wealth of the representative investor, the return of insurers' asset portfolios, and the aggregate claim costs to insurers are jointly lognormally distributed.

3. Investors' utility function assumption: Assuming the representative investor has a utility function that exhibits Constant Relative Risk Aversion (CRRA).

Brennan (1979) showed that CRRA was a necessary and sufficient condition for the Risk Neutral Valuation Relationship to hold for pricing a bivariate contingent claim in discrete time when the returns on the underlying asset and on aggregate wealth were bivariate lognormally distributed. Stapleton and Subrahmanyam (1984) had shown that Brennan's results could be applied to a multivariate contingent claim. Based on these three assumptions, C_1 can be expressed by the following discounted certainty-equivalent expectation:

$$\begin{aligned} C_1 &= C[\tilde{Y}_1; \tilde{L}] \\ &= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max[(\tilde{Y}_1 - \tilde{L}); 0] \cdot \hat{g}(\tilde{Y}_1, \tilde{L}) \mathrm{d}\tilde{Y}_1 \mathrm{d}\tilde{L} \end{aligned} \tag{3.10}$$

where,

$\tilde{g}(\tilde{Y}_1, \tilde{L})$ is the bivariate risk-neutral density function showing the relation of the lognormal variates $\tilde{Y}_1$ and $\tilde{L}$;

$R_f = 1 + r_f$, and r_f is the risk free rate.

By letting $\tilde{U} = \tilde{Y}_1 - \tilde{L} + P_0 \cdot (1 - t_{prem} - e)$, C_1 is expressed as follow,

$$C_1 = R_f^{-1} \int_{P_0 \cdot (1 - t_{prem} - e)}^{\infty} (\tilde{U} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{g}(\tilde{U}) \mathrm{d}\tilde{U} \tag{3.11}$$

where,

$\hat{g}(\tilde{U})$, the risk-neutral density function of $\tilde{U}$, is lognormally distributed.

The certainty equivalent expectation of $\ln(\tilde{U}), a$, is $a = \ln(\hat{E}(\tilde{U})) - \frac{1}{2}\ln(1 + \frac{\mathrm{Var}(\tilde{U})}{(\hat{E}(\tilde{U}))^2})$; the certainty equivalent variance of the $\ln(\tilde{U})$, σ_u^2 , is $\sigma_u^2 = \ln(1 + \frac{\mathrm{Var}(\tilde{U})}{(\hat{E}(\tilde{U}))^2})$.
where[9],

$$\hat{E}(\tilde{U}) = R_f \cdot \hat{E}(U_0) = R_f \cdot (S_0 + R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) - V_0^L) = R_f \cdot V_0^U \tag{3.12}$$

and,

$$V_0^U = S_0 + R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) - V_0^L$$
$$V_0^L = R_f^{-1} \cdot \hat{E}(\tilde{L}) = R_f^{-1} \cdot E(\tilde{L}) \cdot \mathrm{e}^{-\Psi \mathrm{cov}(\ln \tilde{L}, \ln \tilde{R}_m)}$$

where, Ψ, the representative investor's relative risk aversion parameter, is

$$\Psi = \frac{E(\ln \tilde{R}_m) - \ln R_f}{\mathrm{Var}(\ln \tilde{R}_m)} + \frac{1}{2}$$

Let $\tilde{z} = \frac{\ln(\tilde{U}) - a}{\sigma_u}$, i.e., $\tilde{U} = \mathrm{e}^{\tilde{z}\sigma_u + a}$. Substituting this expression into equation (3.11) results

[9]The certainty-equivalent expectation of the rate of return on an insurer's asset portfolio is

$$\hat{\mathrm{E}}(\tilde{r}_a) = \mathrm{E}(\tilde{r}_a) - \beta_a \cdot (\mathrm{E}(\tilde{r}_m) - r_f) = \mathrm{E}(\tilde{r}_a) - \frac{\mathrm{cov}(\tilde{r}_a, \tilde{r}_m)}{\sigma_m^2}(\mathrm{E}(\tilde{r}_m) - r_f) = r_f$$

in the following expression for C_1,

$$C_1 = R_f^{-1} \int_{\frac{\ln(P_0 \cdot (1-t_{prem}-e))-a}{\sigma_u}}^{\infty} (\mathrm{e}^{\tilde{z}\sigma_u + a} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \tag{3.13}$$

where,

$f(\tilde{z})$ is the probability function of normal distribution.

By substituting a into $\mathrm{e}^{\tilde{z}\sigma_u + a}$ and $\frac{\ln(P_0 \cdot (1-t_{prem}-e))-a}{\sigma_u}$, the author gets,

$$\mathrm{e}^{\tilde{z}\sigma_u + a} = \mathrm{e}^{\tilde{z}\sigma_u + \ln(\hat{E}(\tilde{U})) - \frac{1}{2}\sigma_u^2} = \hat{E}(\tilde{U}) \cdot \mathrm{e}^{\tilde{z}\sigma_u - \frac{1}{2}\sigma_u^2} \tag{3.14}$$

$$\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - a}{\sigma_u} = \frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \ln(\hat{E}(\tilde{U})) + \frac{1}{2}\sigma_u^2}{\sigma_u} \tag{3.15}$$

Substituting equation (3.12) into equations (3.14) and (3.15), these two equations can be expressed as,

$$\mathrm{e}^{\tilde{z}\sigma_u + a} = R_f \cdot V_0^U \cdot \mathrm{e}^{\tilde{z}\sigma_u - \frac{1}{2}\sigma_u^2} \tag{3.16}$$

$$\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - a}{\sigma_u} = -\frac{\ln \frac{V_0^U}{P_0 \cdot (1 - t_{prem} - e)} + \ln R_f - \frac{1}{2}\sigma_u^2}{\sigma_u} = -d_2^U \tag{3.17}$$

Substituting equations (3.16) and (3.17) into equation (3.13), C_1 is derived as,

$$\begin{aligned}
C_1 &= R_f^{-1} \int_{-d_2^U}^{\infty} (R_f \cdot V_0^U \cdot \mathrm{e}^{\tilde{z}\sigma_u - \frac{1}{2}\sigma_u^2} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \\
&= V_0^U \cdot \int_{-d_2^U}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{\tilde{z}\sigma_u - \frac{1}{2}\sigma_u^2 - \frac{1}{2}\tilde{z}^2} \mathrm{d}\tilde{z} - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \int_{-d_2^U}^{\infty} \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \\
&= V_0^U \cdot \int_{-d_2^U}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}(\tilde{z} - \sigma_u)^2} \mathrm{d}(\tilde{z} - \sigma_u) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \\
&= V_0^U \cdot N(d_1^U) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U)
\end{aligned} \tag{3.18}$$

where,

$$d_1^U = d_2^U + \sigma_u;$$
$$d_2^U = \frac{\ln \frac{V_0^U}{P_0 \cdot (1 - t_{prem} - e)} + \ln R_f - \frac{1}{2}\sigma_u^2}{\sigma_u}$$

and, $N(*)$ is the standard normal distribution function.

The value of C_2 can be derived similarly.

$$\begin{aligned} C_2 &= C[(\tilde{Y} - S_0); \tilde{L}] \\ &= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max[(\tilde{Y} - S_0 - \tilde{L}); 0] \cdot \hat{g}(\tilde{Y}_1, \tilde{L}) \mathrm{d}\tilde{Y}_1 \mathrm{d}\tilde{L} \end{aligned} \tag{3.19}$$

where,

$\hat{g}(\tilde{Y}_1, \tilde{L})$ is the bivariate risk-neutral density function showing the relation of the variates $\tilde{Y}_1$ and $\tilde{L}$;

$R_f = 1 + r_f$, and r_f is the risk free rate.

By letting $\tilde{M} = \tilde{Y}_1 - S_0 - \tilde{L} + P_0 \cdot (1 - t_{prem} - e)$, C_2 can be expressed as follows,

$$C_2 = R_f^{-1} \int_{P_0 \cdot (1 - t_{prem} - e)}^{\infty} (\tilde{M} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{g}(\tilde{M}) \mathrm{d}\tilde{M} \tag{3.20}$$

where,

$\hat{g}(\tilde{M})$, the risk-neutral density function of $\tilde{M}$, is lognormally distributed.

The certainty equivalent expectation of $\ln(\tilde{M}), \chi$, is $\chi = \ln(\hat{E}(\tilde{M})) - \frac{1}{2}\ln(1 + \frac{\mathrm{Var}(\tilde{M})}{(\hat{E}(\tilde{M}))^2})$, the certainty equivalent variance of the $\ln(\tilde{M}), \sigma_M^2$, is $\sigma_M^2 = \ln(1 + \frac{\mathrm{Var}(\tilde{M})}{(\hat{E}(\tilde{M}))^2})$;

where,

$$\hat{E}(\tilde{M}) = R_f \cdot \hat{E}(M_0) = R_f \cdot \{R_f^{-1} \cdot (S_0 \cdot r_f + P_0 \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f)) - V_0^L\} = R_f \cdot V_0^M \tag{3.21}$$

$$V_0^M = R_f^{-1} \cdot (S_0 \cdot r_f + P_0 \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f)) - V_0^L$$

Similarly, let $\tilde{z} = \frac{\ln(\tilde{M}) - \chi}{\sigma_M}$, i.e., $\tilde{M} = \mathrm{e}^{\tilde{z}\sigma_M + \chi}$, and substitute it into equation (3.20), C_2 becomes,

$$C_2 = R_f^{-1} \int_{\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \chi}{\sigma_M}}^{\infty} (\mathrm{e}^{\tilde{z}\sigma_M + \chi} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \tag{3.22}$$

where, $\hat{f}(\tilde{z})$ is the probability function of normal distribution.

By substituting χ into $\mathrm{e}^{\tilde{z}\sigma_M + \chi}$ and $\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \chi}{\sigma_M}$, the author gets,

$$\mathrm{e}^{\tilde{z}\sigma_M + \chi} = \mathrm{e}^{\tilde{z}\sigma_M + \ln(\hat{E}(\tilde{M})) - \frac{1}{2}\sigma_M^2} = \hat{E}(\tilde{M}) \cdot \mathrm{e}^{\tilde{z}\sigma_M - \frac{1}{2}\sigma_M^2} \tag{3.23}$$

$$\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \chi}{\sigma_M} = \frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \ln(\hat{E}(\tilde{M})) + \frac{1}{2}\sigma_M^2}{\sigma_M} \tag{3.24}$$

Substituting equation (3.21) into equations (3.23) and (3.24), these two equations can be expressed as,

$$\mathrm{e}^{\tilde{z}\sigma_M + \chi} = R_f \cdot V_0^M \cdot \mathrm{e}^{\tilde{z}\sigma_M - \frac{1}{2}\sigma_M^2} \tag{3.25}$$

$$\frac{\ln(P_0 \cdot (1 - t_{prem} - e)) - \chi}{\sigma_M} = -\frac{\ln \frac{V_0^M}{P_0 \cdot (1 - t_{prem} - e)} + \ln R_f - \frac{1}{2}\sigma_M^2}{\sigma_M} = -d_2^M \tag{3.26}$$

Substituting equations (3.25) and (3.26) into equation (3.22), C_2 is derived as follow:

$$\begin{aligned} C_2 &= R_f^{-1} \int_{-d_2^M}^{\infty} (R_f \cdot V_0^M \cdot \mathrm{e}^{\tilde{z}\sigma_M - \frac{1}{2}\sigma_M^2} - P_0 \cdot (1 - t_{prem} - e)) \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \\ &= V_0^M \cdot \int_{-d_2^M}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{\tilde{z}\sigma_M - \frac{1}{2}\sigma_M^2 - \frac{1}{2}\tilde{z}^2} \mathrm{d}\tilde{z} - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \int_{-d_2^M}^{\infty} \cdot \hat{f}(\tilde{z}) \mathrm{d}\tilde{z} \\ &= V_0^M \cdot \int_{-d_2^M}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}(\tilde{z}-\sigma_M)^2} \mathrm{d}(\tilde{z} - \sigma_M) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \\ &= V_0^M \cdot N(d_1^M) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \end{aligned} \tag{3.27}$$

where,

$$\begin{aligned} d_1^M &= d_2^M + \sigma_M; \\ d_2^M &= \frac{\ln \frac{V_0^M}{P_0 \cdot (1 - t_{prem} - e)} + \ln R_f - \frac{1}{2}\sigma_M^2}{\sigma_M} \end{aligned}$$

The "fair" premium P_0^* is implied in the equation $V_e = S_0$. By solving this equation, the "fair" premium P_0^* can be estimated. Due to the complexity of the equation, the iteration method must be used to solve it. Based upon the relationship between underwriting profit margin and premium, the fair underwriting profit margin can be derived through the following formula:

$$E(\tilde{r}_u^*) = \frac{P_0^* \cdot (1 - t_{prem} - e) - E(\tilde{L})}{P_0^*} \tag{3.28}$$

3.2.2 For Distinct Lines of Insurance

Except for the historical model used at Lloyd's of London, insurers globally are organized as corporations that are subject to limited liability. (See National Association of Insurance Commissioners, 1993) For multi-line insurers, each line of business has equal priority in the event of default. If the premium and accumulated investment income of one particular business line is insufficient to cover the liabilities/claims from this business line, part or all of the firm's equity may be used to make up the deficiency. However, if the total equity is not sufficient to cover the total shortfall, the equityholders have to declare bankruptcy and default on the remaining loss payments. In the event of default, the liabilities to policyholders of all business lines are ranked equally; and the amounts that the policyholders can expect to receive are proportional to the value of the claims that they hold against the insurer, i.e., policyholders in j^{th} business line become entitled to a share of L_j/L of the total assets of the insurer; where, L_j is the outstanding claim amount of policyholders in j^{th} business line and L is the total outstanding claim of all business lines, i.e., $L = \sum_{j=1}^{J}(L_j)$.

The value of the policyholders' claims of the j^{th} business line equals the amount of the j^{th} business line's claims if the insurer's total assets are greater than the total claims, or equals a proportion of the total assets if the insurer's total assets are less than the total claims. This relationship can be presented by the following formula:

$$H_{j1} = \{ {\tilde{L}_j \quad if\, \tilde{Y}_1 > \tilde{L} \atop \frac{\tilde{L}_j}{\tilde{L}} \cdot Y_1 \, if\, \tilde{Y}_1 < \tilde{L}} \tag{3.29}$$

$$\begin{aligned} &= \min(\tilde{L}_j, \frac{\tilde{L}_j}{\tilde{L}} \cdot \tilde{Y}_1) \\ &= \frac{\tilde{L}_j}{\tilde{L}} \cdot \tilde{Y}_1 - \max(\frac{\tilde{L}_j}{\tilde{L}} \cdot \tilde{Y}_1 - \tilde{L}_j, 0) \\ &= \frac{\tilde{L}_j}{\tilde{L}} \cdot \tilde{Y}_1 - \frac{\tilde{L}_j}{\tilde{L}} \cdot \max(\tilde{Y}_1 - \tilde{L}, 0) \\ &= \omega_j \cdot (\tilde{Y}_1 - \max(\tilde{Y}_1 - \tilde{L}, 0)) \end{aligned} \tag{3.30}$$

where,

$$\omega_j = \frac{\tilde{L}_j}{\tilde{L}}$$

Hence, the value of the policyholders' claim of the j^{th} business line at the beginning of the period is expressed as follow,

$$H_{j0} = \omega_j \cdot (V(\tilde{Y}_1) - C[\tilde{Y}_1, \tilde{L}]) \tag{3.31}$$

Similarly, the corporate income tax paid by each distinct line depends on the insurer's total profit. If the insurer's total profit is positive, no matter whether the profit of the j^{th} business line is positive or negative, its tax contribution is the tax rate times the j^{th} business line's profit. If the insurer's total profit is negative, no matter whether the profit of the j^{th} business line is positive or negative, no tax is paid. Thus, the value of the tax payments of the j^{th} business line to government is:

$$\begin{aligned} T_{j1} &= \{ \begin{matrix} t_{CI} \cdot (\tilde{Y}_{j1} - S_{j0} - \tilde{L}_j) & if\, \tilde{Y}_1 - S_0 - \tilde{L} > 0 \\ 0 & if\, \tilde{Y}_1 - S_0 - \tilde{L} \leq 0 \end{matrix} \\ &= \frac{\tilde{Y}_{j1} - S_{j0} - \tilde{L}_j}{\tilde{Y}_1 - S_0 - \tilde{L}} \max(t_{CI} \cdot (\tilde{Y}_1 - S_0 - \tilde{L}), 0) \\ &= \upsilon_j \cdot \max(t_{CI} \cdot (\tilde{Y}_1 - S_0 - \tilde{L}), 0) \end{aligned} \tag{3.32}$$

where,

$$\upsilon_j = \frac{\tilde{Y}_{j1} - S_{j0} - \tilde{L}_j}{\tilde{Y}_1 - S_0 - \tilde{L}}$$

So that, the present value of the tax payments of the j^{th} business line to government at the beginning of the period is:

$$T_{j0} = \upsilon_j \cdot t_{CI} \cdot C[(\tilde{Y}_1 - S_0), \tilde{L}] \tag{3.33}$$

The present value of the j^{th} business line to shareholders is the present value of the j^{th} line's assets minus the present value of the claims to the policyholders and the taxes paid to government, i.e., $\hat{V}(\tilde{Y}_{j1}) - H_{j0} - T_{j0}$.

Based on the equilibrium condition, the "fair" premium of the j^{th} business line, P^*_{j0}, should satisfy the condition that the market value of j^{th} business line to shareholders should equal the initial capital investment in it, i.e.,

$$S_{j0} = \hat{V}(\tilde{Y}_{j1}) - H_{j0} - T_{j0} \tag{3.34}$$

where, "$\hat{V}$" means the certainty-equivalent value.

In order to estimate the "fair" premium of the j^{th} business line, P^*_{j0}, the initial equity to each distinct line of business needs to be virtually allocated for the purpose of analysis. The virtual allocation of initial equity does not affect the solvency risk of each distinct line, which depends only on the insurer's total assets and liabilities. But the expected return on the distinct line will reflect the implied equity leverage by line of business resulting from the allocation of the initial equity.

The allocation of initial equity is not unique, but rather can be assumed to match any of a variety of possible allocations. For example, Merton and Perold (1993) proposed a framework for the allocation of capital based on the *marginal impact on the risk-based capital* by adding each line of business while assuming the other lines of business remain constant. Myers and Read's (2001) capital allocation model was based on the *marginal contribution to the option-based default value* for each line of business. Sherris (2006) supported the allocation of assets based on *solvency ratio.* This chapter adopts Sherris' (2006) surplus allocation approach, i.e., assumes the allocation that yields the same solvency ratio for each business line and as for the whole business. The logic behind the use of this approach is that if an insurer becomes insolvent, all lines become insolvent and the claims on all lines are ranked equally; if an insurer is solvent, none of the lines individually could be insolvent. Every line has the same insolvency probability, i.e., each line has the same solvency ratio with that ratio equal to the insurer's aggregate solvency ratio:

$$(Sol)_j = \frac{\alpha_j \cdot S_0 + P_{j0} \cdot (1 - t_{prem} - e) + (\alpha_j \cdot S_0 + k_j \cdot P_{j0} \cdot (1 - t_{prem} - e)) \cdot \tilde{r}_a - \tilde{L}_j}{\tilde{L}_j}$$
$$= \frac{S_0 + P_0 \cdot (1 - t_{prem} - e) + (S_0 + k \cdot P_0 \cdot (1 - t_{prem} - e)) \cdot \tilde{r}_a - \tilde{L}}{\tilde{L}} \tag{3.35}$$

and

$$\sum \alpha_j = 1 \tag{3.36}$$

After the proportion of initial equity allocated to the j^{th} business line, α_j, is determined, the value of $\tilde{Y}_{j1}$, $\hat{V}(Y_{j1})$ and $\hat{V}(Y_1)$ can be estimated; where,

$$\tilde{Y}_{j1} = \alpha_j \cdot S_0 + P_{j0} \cdot (1 - t_{prem} - e) + (\alpha_j \cdot S_0 + k_j \cdot P_{j0} \cdot (1 - t_{prem} - e)) \cdot \tilde{r}_a \tag{3.37}$$

$$\hat{V}(\tilde{Y}_{j1}) = \frac{\alpha_j \cdot S_0 + P_{j0} \cdot (1 - t_{prem} - e) + (\alpha_j \cdot S_0 + k_j \cdot P_{j0} \cdot (1 - t_{prem} - e)) \cdot r_f}{1 + r_f} \tag{3.38}$$

$$\hat{V}(\tilde{Y}_1) = \frac{S_0 + P_0 \cdot (1 - t_{prem} - e) + (S_0 + k \cdot P_0 \cdot (1 - t_{prem} - e)) \cdot r_f}{1 + r_f} \tag{3.39}$$

3.3 The Comparative Statics Analysis

In a 1997 study for the Insurance Bureau of Canada (IBC), management consultants Ernst & Young found that the average taxes paid by Canadian property and casualty insurers and their customers were more than three times those paid by banks, trusts, credit unions and their customers, and almost two times those paid by life insurers and their clients.[10] The imposition of taxes both increases the burden on insurers and customers, and results in either a lower rate of return to shareholders or a higher premium for P&C insurance. "Studies for these industries have shown the dramatic impact of high taxation on consumer purchase patterns. Ten per cent of drivers or more may be driving

[10] These taxes are measured as a percentage of the value-added, where the value-added refers to the additional value created at a particular stage of production or through image and marketing.

without insurance. The heavy taxation of insurance is one of the key factors behind this trend." (Taxation of Property and Casualty Insurance in Canada—Comparisons within the Financial Service Sector, Oct. 1997, p.2) Canadian insurance regulators started a corporate income tax reduction plan in 2000 (Taxation of P&C Insurance: A Comparison between Canada and other G-7 Countries 2003, hereafter TPCI 2003). TPCI 2003 reported that the average CIT rate is 41.4% for large firms and 19.9% for small firms in 2001, and projected 31.9% for large firms and 18.4% for small firms in 2006. The average provincial premium tax rate (excluding fire tax) is 3.2%.

In order to examine the effects of the taxes reduction on the expected insurance premium and expected underwriting profit margin numerically, this subsection conducts a comparative statics analysis. The empirical results will be presented in section 3.5. The comparative statics analysis of the expected premium with respect to corporate income tax and premium-based tax are conducted separately and presented in the next two subsubsections respectively, followed by the comparative statics analysis of the expected underwriting profit margin.

3.3.1 Premium versus Corporate Income Tax

First, a function of premium and corporate income tax, $\Lambda(P_0, t_{CI})$ is constructed, where $\Lambda(P_0, t_{CI}) = C_1 - t_{CI} \cdot C_2 - S_0$. The first derivative of premium with respect to corporate income tax can be derived as follow.

$$\Lambda_{P_0} \cdot \mathrm{d}P_0 + \Lambda_{t_{CI}} \cdot \mathrm{d}t_{CI} = 0 \tag{3.40}$$

$$i.e., \frac{\mathrm{d}P_0}{\mathrm{d}t_{CI}} = -\frac{\Lambda_{t_{CI}}}{\Lambda_{P_0}}$$

where,

$$\Lambda_{t_{CI}} = \frac{\partial C_1}{\partial t_{CI}} - C_2 - t_{CI} \cdot \frac{\partial C_2}{\partial t_{CI}} \tag{3.41}$$

Since $\frac{\partial C_1}{\partial t_{CI}} = \frac{\partial C_2}{\partial t_{CI}} = 0$, then

$$\Lambda_{t_{CI}} = -C_2 \leq 0 \tag{3.42}$$

and,

$$\Lambda_{P_0} = \frac{\partial C_1}{\partial P_0} - t_{CI} \cdot \frac{\partial C_2}{\partial P_0} \tag{3.43}$$

where, $\frac{\partial C_1}{\partial P_0}$ and $\frac{\partial C_2}{\partial P_0}$ can be derived as follow.

$$\begin{aligned} \frac{\partial C_1}{\partial P_0} = \frac{\partial V_0^U}{\partial P_0} \cdot N(d_1^U) + V_0^U \cdot N(d_1^U) \cdot \frac{\partial d_1^U}{\partial P_0} \\ - R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^U) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{d_2^U}{\partial P_0} \end{aligned} \tag{3.44}$$

$$\frac{\partial V_0^U}{\partial P_0} = R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \tag{3.45}$$

$$\frac{\partial \mathrm{d}_1^U}{\partial P_0} = \frac{\partial \mathrm{d}_2^U}{\partial P_0} = (\frac{1}{V_0^U} \cdot \frac{\partial V_0^U}{\partial P_0} - \frac{1}{P_0}) \cdot \frac{1}{\sigma_U} = -\frac{S_0 - V_0^L}{V_0^U \cdot P_0} \cdot \frac{1}{\sigma_U} \tag{3.46}$$

Substituting equations (3.45) and (3.46) into equation (3.44), $\frac{\partial C_1}{\partial P_0}$ is expressed as,

$$\begin{aligned} \frac{\partial C_1}{\partial P_0} = R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \cdot N(d_1^U) \\ - N(d_1^U) \cdot \frac{S_0 - V_0^L}{P_0} \cdot \frac{1}{\sigma_U} - R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \\ + R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{S_0 - V_0^L}{V_0^U \cdot P_0} \cdot \frac{1}{\sigma_U} \end{aligned} \tag{3.47}$$

Similarly,

$$\begin{aligned} \frac{\partial C_2}{\partial P_0} = \frac{\partial V_0^M}{\partial P_0} \cdot N(d_1^M) + V_0^M \cdot N(d_1^M) \cdot \frac{\partial d_1^M}{\partial P_0} \\ - R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^M) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \cdot \frac{\partial d_2^M}{\partial P_0} \end{aligned} \tag{3.48}$$

where,

$$\frac{\partial V_0^M}{\partial P_0} = R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \tag{3.49}$$

$$\frac{\partial d_1^M}{\partial P_0} = \frac{\partial d_2^M}{\partial P_0} = (\frac{1}{V_0^M} \cdot \frac{\partial V_0^M}{\partial P_0} - \frac{1}{P_0}) \cdot \frac{1}{\sigma_M} = -\frac{S_0 \cdot r_f - V_0^L}{V_0^M \cdot P_0} \cdot \frac{1}{\sigma_M} \tag{3.50}$$

Substituting equations (3.49) and (3.50) into equation (3.48), $\frac{\partial C_2}{\partial P_0}$ can be shown as,

$$\begin{aligned}\frac{\partial C_2}{\partial P_0} &= R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \cdot N(d_1^M) \\ &- N(d_1^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{P_0} \cdot \frac{1}{\sigma_M} \\ &- R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \\ &+ R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{P_0 \cdot V_0^M} \cdot \frac{1}{\sigma_M}\end{aligned} \tag{3.51}$$

Substituting equation (3.47) and equation (3.51) in equation (3.43), Λ_{P_0} is expressed as,

$$\begin{aligned}\Lambda_{P_0} &= \frac{\partial C_1}{\partial P_0} - t_{CI} \cdot \frac{\partial C_2}{\partial P_0} \\ &= R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \cdot N(d_1^U) \\ &- N(d_1^U) \cdot \frac{S_0 - V_0^L}{P_0} \cdot \frac{1}{\sigma_U} - R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \\ &+ R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{S_0 - V_0^L}{V_0^U \cdot P_0} \cdot \frac{1}{\sigma_U} \\ &- t_{CI} \cdot \{R_f^{-1} \cdot (1 - t_{prem} - e) \cdot (2 + k \cdot r_f) \cdot N(d_1^M) \\ &- N(d_1^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{P_0} \cdot \frac{1}{\sigma_M} - R_f^{-1} \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \\ &+ R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{P_0 \cdot V_0^M} \cdot \frac{1}{\sigma_M}\}\end{aligned} \tag{3.52}$$

Substituting equations (3.42) into equation (3.40), $\frac{\mathrm{d}P_0}{\mathrm{d}t_{CI}}$ is derived as,

$$\frac{\mathrm{d}P_0}{\mathrm{d}t_{CI}} = -\frac{\Lambda_{t_{CI}}}{\Lambda_{P_0}} = \frac{C_2}{\Lambda_{P_0}} \tag{3.53}$$

Combining with equation (3.52), the first derivative of the expected premium with respect to corporate income tax is derived. Because the expression of Λ_{P_0} is too complicated, it can not be analytically concluded whether Λ_{P_0} is greater than zero. It is known $C_2 \geqslant 0$, since the value of a call option will not be negative. If $\Lambda_{P_0} > 0$, then the expected premium is decreasing with the reduction of corporate income tax. The chapter will numerically examine this relationship in section 3.5.

3.3.2 Premium versus Premium-Based Tax

Similarly, a function of premium and premium-based tax, $\Lambda(P_0, t_{prem})$ is constructed, where $\Lambda(P_0, t_{prem}) = C_1 - t_{CI} \cdot C_2 - S_0$. The first derivative of premium with respect to premium-based tax can be derived as follows:

$$\begin{aligned} \Lambda_{P_0} \cdot \mathrm{d}P_0 + \Lambda_{t_{prem}} \cdot \mathrm{d}t_{prem} = 0 \\ \frac{\mathrm{d}P_0}{\mathrm{d}t_{prem}} = -\frac{\Lambda_{t_{prem}}}{\Lambda_{P_0}} \end{aligned} \tag{3.54}$$

where,

$$\Lambda_{t_{prem}} = \frac{\partial C_1}{\partial t_{prem}} - t_{CI} \cdot \frac{\partial C_2}{\partial t_{prem}} \tag{3.55}$$

$\frac{\partial C_1}{\partial t_{prem}}$ and $\frac{\partial C_2}{\partial t_{prem}}$ can be derived as follow.

$$\begin{aligned} \frac{\partial C_1}{\partial t_{prem}} = \frac{\partial V_0^U}{\partial t_{prem}} \cdot N(d_1^U) + V_0^U \cdot n(d_1^U) \cdot \frac{\partial d_1^U}{\partial t_{prem}} \\ + R_f^{-1} \cdot P_0 \cdot N(d_2^U) - R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{\partial d_2^U}{\partial t_{prem}} \end{aligned} \tag{3.56}$$

$$\frac{\partial V_0^U}{\partial t_{prem}} = -R_f^{-1} \cdot P_0 \cdot (2 + k \cdot r_f) \tag{3.57}$$

$$\frac{\partial d_1^U}{\partial t_{prem}} = \frac{\partial d_2^U}{\partial t_{prem}} = \left(\frac{1}{V_0^U} \cdot \frac{\partial V_0^U}{\partial t_{prem}} + \frac{1}{1 - t_{prem} - e}\right) \cdot \frac{1}{\sigma_U} = \frac{S_0 - V_0^L}{V_0^U \cdot (1 - t_{prem} - e)} \cdot \frac{1}{\sigma_U} \tag{3.58}$$

Substituting equations (3.57) and (3.58) into equation (3.56), $\frac{\partial C_1}{\partial t_{prem}}$ is expressed as,

$$\begin{aligned}\frac{\partial C_1}{\partial t_{prem}} &= -R_f^{-1} \cdot P_0 \cdot (2 + k \cdot r_f) \cdot N(d_1^U) \\ &+ N(d_1^U) \cdot \frac{S_0 - V_0^L}{(1 - t_{prem} - e)} \cdot \frac{1}{\sigma_U} \\ &+ R_f^{-1} \cdot P_0 \cdot N(d_2^U) \\ &- R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{S_0 - V_0^L}{V_0^U \cdot (1 - t_{prem} - e)} \cdot \frac{1}{\sigma_U}\end{aligned} \tag{3.59}$$

Similarly, $\frac{\partial C_2}{\partial t_{prem}}$ is derived as,

$$\begin{aligned}\frac{\partial C_2}{\partial t_{prem}} &= -R_f^{-1} \cdot P_0 \cdot (2 + k \cdot r_f) \cdot N(d_1^M) \\ &+ N(d_1^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{(1 - t_{prem} - e)} \cdot \frac{1}{\sigma_M} \\ &+ R_f^{-1} \cdot P_0 \cdot N(d_2^M) \\ &- R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{V_0^M \cdot (1 - t_{prem} - e)} \cdot \frac{1}{\sigma_M}\end{aligned} \tag{3.60}$$

Substituting equations (3.59) and (3.60) into equation (3.55), the expression of $\Lambda_{t_{prem}}$

can be shown in the following equation,

$$\begin{aligned}
\Lambda_{t_{prem}} &= \{-R_f^{-1} \cdot P_0 \cdot (2 + k \cdot r_f) \cdot N(d_1^U) \\
&+ N(d_1^U) \cdot \frac{S_0 - V_0^L}{(1 - t_{prem} - e)} \cdot \frac{1}{\sigma_U} \\
&+ R_f^{-1} \cdot P_0 \cdot N(d_2^U) \\
&- R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^U) \cdot \frac{S_0 - V_0^L}{V_0^U \cdot (1 - t_{prem} - e)} \cdot \frac{1}{\sigma_U}\} \\
&- t_{CI} \cdot \{-R_f^{-1} \cdot P_0 \cdot (2 + k \cdot r_f) \cdot N(d_1^M) \\
&+ N(d_1^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{(1 - t_{prem} - e)} \cdot \frac{1}{\sigma_M} \\
&+ R_f^{-1} \cdot P_0 \cdot N(d_2^M) \\
&- R_f^{-1} \cdot P_0 \cdot (1 - t_{prem} - e) \cdot N(d_2^M) \cdot \frac{S_0 \cdot r_f - V_0^L}{V_0^M \cdot (1 - t_{prem} - e)} \cdot \frac{1}{\sigma_M}\}
\end{aligned} \tag{3.61}$$

Substituting equation (3.61) and equation (3.52) into equation (3.54), $\frac{\mathrm{d}P_0}{\mathrm{d}t_{prem}}$ is simplified as,

$$\frac{P_0}{\mathrm{d}t_{prem}} = -\frac{\Lambda_{t_{prem}}}{\Lambda_{P_0}} = \frac{P_0}{1 - t_{prem} - e} > 0 \tag{3.62}$$

Consider the normal situation where the premium is always non-negative and the sum of the expense rate and premium-based tax rate is always less than 1. In that instance, the result from equation (3.62) shows that expected premium decreases with a decrease in the premium-based tax.

3.3.3 Underwriting Profit Margin versus Corporate Income Tax

The expected underwriting profit margin is presented by equation (3.63) as follows:

$$E(\tilde{r}_u) = \frac{P_0 \cdot (1 - t_{prem} - e) - E(\tilde{L})}{P_0} = (1 - t_{prem} - e) - \frac{E(\tilde{L})}{P_0} \tag{3.63}$$

By taking the first derivative of $E(\tilde{r}_u)$ with respect to corporate income tax, t_{CI}, the author gets,

$$\frac{\partial E(\tilde{r}_u)}{\partial t_{CI}} = \frac{E(\tilde{L})}{P_0^2} \cdot \frac{\partial P_0}{\partial t_{CI}} \tag{3.64}$$

If $\frac{\partial P_0}{\partial t_{CI}} > 0$, i.e., if the expected premium is an increasing function of corporate income tax, then the expected underwriting profit margin, $E(\tilde{r}_u)$, increases with corporate income tax as well. This relationship is checked numerically in section 3.5.

3.3.4 Underwriting Profit Margin versus Premium-Based Tax

Similarly, by taking the first derivative of $E(\tilde{r}_u)$ with respect to premium-based tax, t_{prem}, equation (3.65) is obtained:

$$\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} = \frac{E(\tilde{L}) - P_0 \cdot (1 - t_{prem} - e)}{P_0 \cdot (1 - t_{prem} - e)} \tag{3.65}$$

- If $E(\tilde{L}) > P_0 \cdot (1 - t_{prem} - e)$, then $\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} > 0$, i.e., if the claims incurred exceed the premium net of expense and other taxes (excluding corporate income tax), then the expected underwriting profit margin $E(\tilde{r}_u)$ is an increasing function of premium-based tax, t_{prem}.

- Otherwise, if $E(\tilde{L}) < P_0 \cdot (1 - t_{prem} - e)$, then $\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} < 0$, i.e., the expected fair underwriting profit margin $E(\tilde{r}_u)$ is a decreasing function of premium-based tax, t_{prem}.

- In sum, the relationship between the expected underwriting profit margin and premium-based tax is more complex and depends on the firm's premium, expenses, and other taxes.

3.3.5 Summary of the Comparative Statics Analysis

This subsubsection summarizes the results of the comparative statics analysis. In sum, the relationship between expected premium and corporate income tax can be expressed as:

$$\frac{\mathrm{d}P_0}{\mathrm{d}t_{CI}} = -\frac{\Lambda_{t_{CI}}}{\Lambda_{P_0}} = \frac{C_2}{\Lambda_{P_0}}$$

Since $C_2 > 0$, if $\Lambda_{P_0} > 0$, then $\frac{\mathrm{d}P_0}{\mathrm{d}t_{CI}} > 0$.

The relationship between expected premium and premium-based tax is shown as follow:

$$\frac{P_0}{\mathrm{d}t_{prem}} = -\frac{\Lambda_{t_{prem}}}{\Lambda_{P_0}} = \frac{P_0}{1 - t_{prem} - e} > 0$$

The relationship between expected underwriting profit margin and corporate income tax is presented in the following equation:

$$\frac{\partial E(\tilde{r}_u)}{\partial t_{CI}} = \frac{E(\tilde{L})}{P_0^2} \cdot \frac{\partial P_0}{\partial t_{CI}}$$

If $\frac{\partial P_0}{\partial t_{CI}} > 0$, then $\frac{\partial E(\tilde{r}_u)}{\partial t_{CI}} > 0$; i.e., if P_0 is an increasing function of t_{CI}, then $E(\tilde{r}_u)$ also is an increasing function of t_{CI}.

The relationship between expected underwriting profit margin and premium-based tax is shown as follow:

$$\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} = \frac{E(\tilde{L}) - P_0 \cdot (1 - t_{prem} - e)}{P_0 \cdot (1 - t_{prem} - e)}$$

If $E(\tilde{L}) > P_0 \cdot (1 - t_{prem} - e)$, then $\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} > 0$.
If $E(\tilde{L}) < P_0 \cdot (1 - t_{prem} - e)$, then $\frac{\partial E(\tilde{r}_u)}{\partial t_{prem}} < 0$.

3.4 Data and Variable Definition

Many of the prior applications of financial pricing models have either adopted the values of the parameters from previous research on a similar topic (e.g., D'Arcy and Garven, 1990; D'Arcy and Gorvett, 1998), simply assume particular values for certain variables (e.g., Doherty and Garven, 1986), determine the values of variables based on a fictitious insurer (e.g., D'Arcy and Gorvett, 1998), or assign artificial values to the required parameters (e.g., Sherris, 2006). These approaches provide little information about how well the particular pricing model fits the actual situation, since financial pricing models are sensitive to the values of the parameters. In order to facilitate the realistic application of OPM, the chapter collects insurers' data for the period of 1999 through 2005 from *MSA Researcher P&C 2006* database published by MSA Research Inc. of Toronto. 1999 is the earliest year that the distinct by-line data is collected in the database. In the end, only 10 insurers have complete distinct line data set for every year during the sample period. The parameters for these 10 insurers are derived.

The variables needed include both market data and insurers' operating data. The market data needed include risk-free rate and market return. The risk-free rate is measured by the yield on 91-day Government of Canada T-bills. Market return is derived from the S&P/TSX composite total return index and calculated as the ratio of the difference between the year-end index and the year-beginning index to the year-beginning index.

Operating data are collected for each insurer and involve two levels of insurance business. The first level is the aggregate data for the overall business; the second level is the data for each of the following distinct business lines: auto insurance, property insurance, and liability insurance. All the variables and parameters are either obtained or derived from *MSA Researcher P&C 2006* database. The insurers' operating variables are discussed as follows:

- The underwriting profit margin is defined as 1 minus the combined ratio, and is expressed as a percentage of net premium earned. The combined ratio as reported by MSA is the ratio of total underwriting expenses, which include the incurred losses and expenses, as a percentage of net premiums earned. Investment income and capital gains are not taken into account.

- The leverage factor, k, reflects the average holding period of a dollar of premium before it is used to pay expenses and losses, e.g., if the premium is retained within a firm for half a year before it is paid for underwriting claims, then k equals 0.5. It is measured as the ratio of the net unpaid claims as a proportion of net premium earned.

- The net premium earned is defined as the direct premium earned plus any reinsurance premium received minus reinsurance premium ceded.

- The net loss incurred is defined as the net claims and adjustment expenses incurred, including unpaid claims and corresponding expenses.

- Equity at the beginning of the year is the GAAP capital and surplus.

- Return on assets is derived from the total net investment return and total assets.

- The effective corporate income tax rate is calculated as the difference between the net income before tax and the net income after tax divided by the net income before tax.

- The effective expense-and-other-taxes rate is measured as a proportion of net premium earned. Because all the other taxes are reported within expense items in the annual financial reports, the data requires that, for analytical purposes, expense be combined with other taxes.

The variance of market return, variance of claims-incurred, covariance between return on assets and claims-incurred, covariance between claims-incurred and market return are estimated. Net premium earned, net claims incurred, and k for each distinct line are collected or derived from the annual report data. In the estimation of the fair underwriting profit margins and fair net premiums of distinct business lines, the chapter assumes the effective corporate income tax and the effective expense-and-other-taxes rate are the same across insurance business lines as the aggregate rates for each insurer. While still somewhat restrictive, the assumptions that have appeared in earlier work are still considerably relaxed. The variance of market returns during the sample period is estimated as 0.0289. The average of variables and other estimated parameters across firms for each year are listed in Table 3.1. The average over years from 1999-2005 for each firm are reported in Table 3.2. The values of the variables for each firm in each year is available upon request.

In Table 3.1, the average leverage factors across firms for each year show that the leverage factor of liability insurance is the highest, followed by auto insurance, and then property insurance. Also, it is shown that the average leverage factors remain quite stable over time. The investor's relative risk aversion parameters change dramatically over time. The average equity of these 10 firms is increasing over time.

Table 3.1: The Average Values of Parameters across Firms in Each Year

	1999	2000	2001	2002	2003	2004	2005
ti	0.27	0.38	0.26	0.11	0.23	0.16	0.10
te	0.34	0.34	0.33	0.33	0.30	0.30	0.32
s	251897	261125	261125	243333	289072	352313	414075
ra	0.0431	0.0499	0.0347	0.0286	0.0310	0.0329	0.0342
Rf	1.0472	1.0549	1.0379	1.0258	1.0287	1.0222	1.0273
lnRm	0.2745	0.0724	-0.1344	-0.1328	0.2368	0.1352	0.2161
Vra	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
VL	8.38E+09	8.38E+09	8.38E+09	8.38E+09	8.38E+09	8.38E+09	8.38E+09
CVraL	-130.21	-130.21	-130.21	-130.21	-130.21	-130.21	-130.21
CVLRm	0.0063	0.0063	0.0063	0.0063	0.0063	0.0063	0.0063
fai	8.3920	1.1545	-5.4272	-4.9683	7.7052	4.4134	7.0380
k	0.909	0.934	0.969	1.003	0.978	1.035	1.067
auto_k1	0.969	0.990	0.992	1.011	1.015	1.102	1.170
prop_k1	0.296	0.308	0.292	0.292	0.291	0.268	0.325
liab_k1	1.998	1.894	2.200	2.313	2.464	2.768	2.499
other_k1	0.445	0.497	0.585	0.497	0.331	0.270	0.269

where,

ti	is the effective corporate income tax rate
te	is the other-taxes-and-expense rate, as a proportion of net premium
s	is the initial equity, in $,000
ra	is the firm's investment return rate
Rf	is 1 plus the risk free rate
lnRm	is the natural log of the sum of 1 and market return rate
Vra	is the variance of the firm's investment return
VL	is the variance of the claims incurred
CVraL	is the covariance of the investment return and the claims incurred
CVLRm	is the covariance of the market return and the claims incurred
fai	is the investor's relative risk aversion parameter
k	is the leverage ratio
auto_k1	is the leverage ratio of auto insurance
prop_k1	is the leverage ratio of property insurance
liab_k1	is the leverage ratio of liability insurance
other_k1	is the leverage ratio of other business lines

	Royal & Sun Alliance Insurance (Canada)	Saskatchewan Mutual Insurance	SGI CANADA	Sovereign General Insurance	State Farm Fire and Casualty	State Farm Mutual Automobile Insurance	TD General Insurance Company	Trafalgar Insurance (Canada)	Wawanesa Mutual Insurance	Western Assurance
ti	0.34	0.23	0.00	0.39	-0.17	-0.04	0.26	0.35	0.32	0.47
te	0.33	0.42	0.38	0.39	0.37	0.24	0.21	0.31	0.24	0.33
s	617111	16434	106945	60235	187337	650222	27241	36814	1199408	59597
ra	0.0443	0.0391	0.0513	0.0291	0.0416	0.0487	0.0179	0.0228	0.0427	0.0257
Rf	1.0348	1.0348	1.0348	1.0348	1.0348	1.0348	1.0348	1.0348	1.0348	1.0348
lnRm	0.0954	0.0954	0.0954	0.0954	0.0954	0.0954	0.0954	0.0954	0.0954	0.0954
Vra	0.0003	0.0000	0.0004	0.0000	0.0000	0.0001	0.0000	0.0000	0.0005	0.0001
VL	20037331095	10076022.14	1.36E+08	336774806	544878079	3.4741E+10	735237405	105905983	2.5932E+10	1.22E+09
CVraL	518.50	1.46	-48.58	-25.47	-82.22	-1003.13	-60.69	-8.79	-609.94	16.74
CVLRm	-0.0225	0.0141	0.0059	0.0091	-0.0057	-0.0037	0.0300	-0.0153	0.0046	0.0470
fai	2.6154	2.6154	2.6154	2.6154	2.6154	2.6154	2.6154	2.6154	2.6154	2.6154
k	1.584	0.371	0.676	0.722	0.617	1.558	0.778	0.910	1.052	1.584
auto_k1	1.643	0.425	0.764	1.130	n/a	1.558	0.846	1.027	1.319	1.643
prop_k1	0.350	0.243	0.284	0.267	0.469	n/a	0.311	0.203	0.483	0.350
liab_k1	6.629	1.080	2.501	1.588	1.581	n/a	n/a	1.690	1.352	6.630
other_k1	0.439	1.102	0.439	0.257	1.168	n/a	n/a	0.012	0.276	0.439

where,

ti	is the effective corporate income tax rate
te	is the other-taxes-and-expense rate, as a proportion of net premium
s	is the initial equity, in $,000
ra	is the firm's investment return rate
Rf	is 1 plus the risk free rate
lnRm	is the natural log of the sum of 1 and market return rate
Vra	is the variance of the firm's investment return
VL	is the variance of the claims incurred
CVraL	is the covariance of the investment return and the claims incurred
CVLRm	is the covariance of the market return and the claims incurred
fai	is the investor's relative risk aversion parameter
k	is the leverage ratio
auto_k1	is the leverage ratio of auto insurance
prop_k1	is the leverage ratio of property insurance
liab_k1	is the leverage ratio of liability insurance
other_k1	is the leverage ratio of other business lines

Table 3.2: The Average Values of Parameters during Year 1999-2005 for Each Firm

As presented in Table 3.2, the average corporate income tax rate of insurers varies from a low that is negative to a high of 47%. The other-taxes-and-expense rate appears to be relatively stable. Some firms (e.g., State Farm Mutual Automobile Insurance Company) exhibit a negative correlation between the claims incurred and investment return while other insurers have a positive correlation. The correlations between the claims incurred and market return are close to zero with some showing as positive while others are negative. The leverage factors, k, of both the combined-lines and each distinct line vary significantly across firms; however, the relationship between the leverage factors of different business lines remains the same and holds for all firms. Specifically, the leverage factor of liability insurance is the highest, followed by auto insurance, and then property insurance.

3.5 Empirical Results

The empirical results are presented in Table 3.3 and Table 3.4. Table 3.3 shows the average of the seven years' underwriting profit margin and net premium for each firm. Table 3.4 presents the average of the ten firms' underwriting profit margin and net premium for each year. The results for each firm in each year during the period 1999 to 2005 are available upon request. The first column compares the actual premium of all combined lines with the expected premium derived from the OPM. The second column compares the actual underwriting profit margin (UPM) of the combined-lines and the expected combined-lines' underwriting profit margin derived from the OPM. The third column presents the expected underwriting profit margins of distinct lines.

company	Column 1		Column 2		column 3		
	A_npe	E_npe	A_upm	E_upm	auto_upm	prop_upm	liab_upm
Royal & Sun Alliance Insurance (Canada)	809651.1	950645.6	-7.73	2.60	2.60	4.95	-6.85
Saskatchewan Mutual Insurance	19843.0	19367.7	0.63	-0.74	-0.19	-0.73	-3.56
SGI CANADA	187276.9	180091.3	-0.64	-2.55	-2.75	-1.62	-6.58
Sovereign General Insurance	140514.6	141315.9	-1.21	0.21	0.33	-2.54	4.27
State Farm Fire and Casualty	261291.9	300807.4	-11.46	-0.06	n/a	0.64	-4.79
State Farm Mutual Automobile Insurance	803524.4	932140.7	-17.19	-3.20	-3.19	n/a	na
TD General Insurance Company	49416.0	43744.2	-0.80	-5.60	-5.71	-4.78	na
Trafalgar Insurance (Canada)	74020.1	84887.6	-7.03	2.79	1.79	23.19	9.94
Wawanesa Mutual Insurance	1007113.9	1005347.7	-0.66	0.24	-0.30	1.45	-3.71
Western Assurance	74084.3	72847.3	-8.14	-3.57	-3.65	-0.80	-14.21
Grand Average	342673.6	373119.5	-5.42	-0.99	-1.23	2.19	-3.19

where,

A_npe is the actual net premium for the combined lines, in $,000

E_npe is the expected net premium for the combined lines derived from the OPM, in $,000

A_upm is the actual underwriting profit margin for the combined lines, in %

E_upm is the expected underwriting profit margin for the combined lines derived from the OPM, in %

auto_upm is the expected underwriting profit margin for the auto insurance derived from the OPM, in %

prop_upm is the expected underwriting profit margin for the property insurance derived from the OPM, in %

liab_upm is the expected underwriting profit margin for the liability insurance derived from the OPM, in %

Table 3.3: The Average Underwriting Profit Margin and Net Premium during Year 199-2005 for Each Firm

Table 3.4: The Average Underwriting Profit Margin and Net Premium across the 10 Firms for Each Year

	Column 1		Column 2		column 3		
year	A_npe	E_npe	A_upm	E_upm	auto_upm	prop_upm	liab_upm
1999	277901	318434.4	-6.10	-6.47	-8.09	-5.64	-7.82
2000	283380.8	334165.8	-10.36	2.48	2.55	16.04	9.40
2001	300549.9	357140.9	-14.59	2.89	3.54	5.38	-2.93
2002	324943.4	395357.5	-13.32	3.55	4.67	6.30	-3.01
2003	381227.3	425135.8	-3.42	-3.90	-4.69	-3.03	-7.26
2004	408732.3	382384.3	5.85	-1.44	-1.26	-1.11	-4.42
2005	421980.6	399218.0	3.98	-4.03	-5.34	-2.58	-6.27
Average	342673.61	373119.5	-5.42	-0.99	-1.23	2.19	-3.19

where,

A_npe is the actual net premium for the combined lines, in $,000
E_npe is the expected net premium for the combined lines derived from the OPM, in $,000
A_upm is the actual underwriting profit margin for the combined lines, in %
E_upm is the expected underwriting profit margin for the combined lines derived from the OPM, in %
auto_upm is the expected underwriting profit margin for the auto insurance derived from the OPM, in %
prop_upm is the expected underwriting profit margin for the property insurance derived from the OPM, in %
liab_upm is the expected underwriting profit margin for the liability insurance derived from the OPM, in %

As shown in Table 3.3 and Table 3.4, on average the expected UPMs derived from OPM are higher than actual UPMs. This result is consistent with findings in prior studies and can be partially explained by the total tax redundancy assumption in the OPM. In reality, the unused negative profit can be used to offset the positive profit in the next few years. Also consistent with the findings in prior studies, underwriting profit margins are around zero and vary over time. On average, the expected UPM is negative.

It is also found that expected UPMs vary across different insurance business lines. Consistent with the findings in the Full Information Underwriting Beta Model (FIUBM), the results from the OPM show that the expected UPM of liability insurance is the lowest, followed by that of auto insurance and then property insurance. The results reconfirm the commonly held notion, consistent with the high leverage factor k of liability insurance, that such business is supported by a larger investment income which, in turn, offsets a portion of required profit from the underwriting activity. The relationship between expected UPM and k is not linear.

However, exceptions exist for Sovereign General Insurance Company and Trafalgar Insurance Company of Canada and for the year 2000. The high liability insurance underwriting profit margins of the year 2000 aggregate average, of Sovereign General Insurance Company average, and of the Trafalgar Insurance Company of Canada average are caused by the extraordinarily high expected UPM of liability insurance for these two firms in year 2000. A careful examination of the firm-specific-factors of these two firms, and comparison with those of other firms, finds that the claims incurred and claims-to-premium ratio of liability lines of Trafalgar Insurance Company of Canada in year 2000 are negative. This may cause the exceptional result. Other firm-specific-factors examined include the leverage factor k of combined lines and each distinct line, the investment return, the correlation between investment return and the claims incurred, the correlation between market return and the claims incurred.

Tables 3.5 and 3.6 report the effects of taxes on combined lines. Additional tests were conducted on the distinct lines. The results show a similar influence to those reported for combined lines; thus, only the taxes effects on the combined lines are reported. Table 3.5 shows the average percentage change of expected premium and average change of expected underwriting profit margin during the period from 1999 to 2005 for each firm. Table 3.6 presents the average percentage change of expected premium and average change of expected underwriting profit margin of the ten firms for each year. The percentage change of expected premium and change of expected underwriting profit margin of each firm for each year are available upon request.

company	percentage change of expected Premium				change of expected UPM			
	CIT-1%	CIT-2%	PBT-1%	PBT-2%	CIT-1%	CIT-2%	PBT-1%	PBT-2%
Royal & Sun Alliance Insurance (Canada)	-1.64%	-3.22%	-1.48%	-2.92%	-1.06	-2.12	0.04	0.08
Saskatchewan Mutual Insurance	-1.78%	-3.62%	-1.67%	-3.32%	-1.08	-2.21	-0.01	-0.04
SGI CANADA	-1.65%	-3.25%	-1.60%	-3.13%	-1.08	-2.17	-0.05	-0.08
Sovereign General Insurance	-1.81%	-3.54%	-1.62%	-3.21%	-1.11	-2.21	0.01	0.00
State Farm Fire and Casualty	-1.66%	-3.28%	-1.57%	-3.10%	-1.06	-2.12	0.00	0.00
State Farm Mutual Automobile Insurance	-1.42%	-2.81%	-1.30%	-2.56%	-1.14	-2.29	-0.04	-0.08
TD General Insurance Company	-1.72%	-3.32%	-1.36%	-2.39%	-1.47	-2.87	-0.20	-0.03
Trafalgar Insurance (Canada)	-1.55%	-3.05%	-1.43%	-2.81%	-1.04	-2.07	0.04	0.09
Wawanesa Mutual Insurance	-1.50%	-2.95%	-1.30%	-2.58%	-1.14	-2.29	0.00	0.01
Western Assurance	-2.09%	-4.09%	-1.40%	-2.90%	-1.47	-2.92	0.00	-0.10
Grand Average	-1.68%	-3.31%	-1.47%	-2.89%	-1.17	-2.33	-0.02	-0.02

where,

CIT-1%: means corporate income tax rate reduced by 1% point, e.g., reduced from 33% to 32%

CIT-2%: means corporate income tax rate reduced by 2% point

PBT-1%: means other taxes (except CIT) rate (as a percentage of premium) reduced by 1% point

PBT-2%: means other taxes (except CIT) rate (as a percentage of premium) reduced by 2% point

Table 3.5: The Average Percentage Change of Expected Premium and Average Change of Expected UPM during Year 1999-2005 for Each Firm

Table 3.6: The Average Percentage Change of Expected Premium and Average Change of Expected UPM across the 10 Firms for Each Year

	percentage change of expected Premium				change of expected UPM			
year	CIT-1%	CIT-2%	PBT-1%	PBT-2%	CIT-1%	CIT-2%	PBT-1%	PBT-2%
1999 Average	-1.84%	-3.51%	-1.51%	-2.91%	-1.38	-2.68	-0.12	-0.10
2000 Average	-1.83%	-3.62%	-1.50%	-2.99%	-1.17	-2.35	0.05	0.07
2001 Average	-1.69%	-3.40%	-1.49%	-2.94%	-1.08	-2.20	0.04	0.09
2002 Average	-1.66%	-3.36%	-1.45%	-2.88%	-1.04	-2.14	0.07	0.14
2003 Average	-1.59%	-3.09%	-1.45%	-2.82%	-1.19	-2.33	-0.07	-0.12
2004 Average	-1.53%	-3.03%	-1.42%	-2.78%	-1.11	-2.23	-0.02	-0.03
2005 Average	-1.64%	-3.19%	-1.50%	-2.92%	-1.20	-2.37	-0.09	-0.15
Grand Average	-1.68%	-3.31%	-1.47%	-2.89%	-1.17	-2.33	-0.02	-0.02

where,

CIT-1%: means corporate income tax rate reduced by 1% point, e.g., reduced from 33% to 32%

CIT-2%: means corporate income tax rate reduced by 2% point

PBT-1%: means other taxes (except CIT) rate (as a percentage of premium) reduced by 1% point

PBT-2%: means other taxes (except CIT) rate (as a percentage of premium) reduced by 2% point

The results in Tables 3.5 and 3.6 show the expected premium to be decreasing with a decrease in corporate income tax (CIT) rate. On average a 1% reduction in the effective corporate income tax rate is projected to produce premiums that are 1.68% lower. Consistent with the result of the comparative statics analysis, the expected underwriting profit margin also is projected to decrease with a reduction in the corporate income tax. On average a 1% deduction in the effective CIT rate results in an estimated 1.17 percentage point reduction in underwriting profit margin (e.g., from 2.5% to 1.33%). The decrease in CIT rate implied that in order to achieve the same level of post-tax profit level the required pre-tax profit level does not need to be as high as before.

As anticipated, the expected premium decreases with a reduction in the premium-based tax (PBT) rate. A 1% lower PBT rate on average leads to 1.47% lower premium. Economically, the higher than 1% increase (or decrease) in premium is reasonable, since it costs more than $1 to deliver $1 of taxes to the government because of the administration cost. Any change in this economic parameter–tax rate— applies to all the insurers in the same market. As such, insurers could pass all or at least part of these cost along to customers.

However, the effect of PBT rate on the expected underwriting profit margin is more complex. When the expected loss is higher than the premium net of expenses and other taxes (i.e., when insurer has negative expected underwriting profit), the PBT rate has positive effect on the expected UPM. In this situation, as we have already shown, a 1% increase in premium based tax leads to more than 1% increase in expected premium. The resulting increase in expected premium makes the expected underwriting profit less negative; moreover, when the less negative expected underwriting profit is expressed as a percentage of the increased expected premium, that expected underwriting profit margin becomes less negative. Meanwhile, when the insurer has a positive expected underwriting profit, a higher PBT rate results in a lower expected underwriting profit margin. The slightly increased expected underwriting profit when expressed as a percentage of the relative larger increased expected premium becomes smaller than before. That is, the expected UPM is reduced with the increase in the PBT. Overall the effect of PBT on expected underwriting profit margin demonstrates an ability to soften the underwriting cycle.

As shown in Tables 3.5 and 3.6, both the effect of CIT rate on expected premium and of PBT rate on expected premium are quite stable over time and across firms. Similar stability is found in the relationship between CIT rate and expected underwriting profit. However, the effect of PBT rate on expected underwriting profit margin changes depending on the firm-specific situation.

3.6 Conclusion

The chapter elaborates upon the Doherty and Garven (1986) option pricing model and develops a financial insurance pricing model that is able to price insurance by line in a multi-line property & casualty insurance company. The model developed in the chapter has the potential to yield significant improvement in insurance pricing techniques in several ways. First, the model extends the Doherty and Garven (1986) single-line model to a model that is suitable for insurance pricing by distinct line in a multi-line insurer subject to default risk and underutilized tax shields. Second, the chapter provides numerical results of the expected insurance premiums and the expected underwriting profit margins by major business line for ten Canadian Property & Casualty insurers during the period from 1999 through 2005, which augments the hypothesis tests used in the prior studies. The chapter also both analytically and numerically demonstrates the impact of taxes on the expected premium and the expected underwriting profit margin.

The results in the chapter are consistent with findings and arguments in prior studies. The underwriting profit margins are around zero and vary over time; on average, the expected UPM is negative. The results show that the expected UPMs vary across different insurance business lines. Consistent with the findings in chapter 2, the results from the OPM show that the expected UPM of liability insurance is the lowest, followed by that of auto insurance, and then property insurance. The results reconfirm that the high leverage factor k of liability insurance means a larger contribution from investment income, which offsets the demand for profit from the firm's underwriting activity. The relationship between expected UPM and leverage factor k is not linear.

The OPM considers the total risk of an insurer in the pricing, including both the systematic risk and the firm-specific risk, by adopting the variance of the insurer's total performance parameters (including variance of investment return, variance of incurred loss, covariance between the incurred loss and investment return, and covariance between the incurred loss and market equity return) as measures of the insurer's total risk. This characteristic makes the OPM an appropriate candidate for the pricing for each individual insurer's products because this model enable insurers to consider the total risk assumed in setting an appropriate price level for its products. But the OPM may not be a suitable model for the purpose of insurance pricing regulation, since it is inappropriate for the regulator to set different target underwriting profit margins for different insurers because of the different levels of risk assumed.

The sensitivity analysis shows that corporate income tax (CIT) rate is positively related to the expected premium and expected UPM and that the effect of CIT rate on expected premium and on expected UPM are quite stable over time and across firms. On average, a 1% reduction in the CIT rate projects 1.68% lower premium and produces an estimated 1.17 percentage point reduction in the expected underwriting profit margin (e.g., from 2.5% to 1.33%).

The effect of premium-based tax (PBT) rate on the expected premium is stable as well. However, the effect of PBT on expected underwriting profit margin changes with the firm-specific situation. A 1% lower PBT rate on average leads to a 1.47% lower premium. The effect of a change in the PBT rate on the expected underwriting profit margin is more complex. If claims incurred are higher than the premium net of expenses and other taxes, a higher PBT will generate a higher expected underwriting profit margin; otherwise, the higher PBT produces a lower expected underwriting profit margin.

Insurance is a very complex financial transaction. The premiums are collected over time in return for the promise to compensate possible future losses. The timings and amounts of the possible losses are unknown. The mathematical models about stochastic asset and liability diffusion processes have not yet been perfected. This complexity means that scrutiny on the asset and liability distributions certainly should be warranted before the model is actually used for insurance pricing purposes and that combining the actuarial consideration in the determination of an appropriate underwriting profit margin and insurance premium could be instructive.

4 Summary and Conclusion

This thesis extends the existing literature in insurance pricing by developing insurance pricing models that reflect the risk characteristics of different business lines. The thesis develops financial insurance pricing models that are able to price insurance by line in a multi-line insurer based on Full Information Underwriting Beta in chapter 2 and on a Contingent Claims Approach in chapter 3.

Chapter 2 presents new evidence on the insurance pricing by line of Property & Casualty insurance. Chapter 2 applies the full information methodology to estimate underwriting betas of distinct business lines, which are then applied to estimate the fair underwriting profit margin by line. The full information underwriting betas of distinct business lines contain more information and measure the risks of business lines more reliably, i.e., the risk of underwriting varies among business lines in more regards than simply the length of the period over which premium can be kept.

Based on Canadian Property & Casualty insurance industry data, the primary empirical findings in chapter 2 strongly support the argument that underwriting betas of distinct lines do not vary in proportion to the length of the period that the premium of the corresponding line can be kept for investment. The findings also show that the expected underwriting profit margin varies across business lines, with liability insurance having the lowest expected underwriting profit margin. These findings imply that setting a single target underwriting profit margin rate for distinct business lines and across years is inappropriate and could be dangerous. The results of the comparative statics analysis show that expected underwriting profit margin and expected net premium are positively related to the effective corporate income tax rate, and are negatively related to premium-to-equity ratio and leverage factor. Also, expected net premium is positively related to effective expense-and-other-taxes rate.

Because of the limitations of the ICAPM, default risk could not be fully considered in that model. Also, only yearly data is available in *MSA Researcher P&C 2006* database for the study. Further research based on quarterly data may increase the accuracy and power of the current results.

Chapter 3 elaborates upon the Doherty and Garven (1986) option pricing model and develops a financial insurance pricing model that is able to price insurance by line in a multi-line insurer. The model developed in chapter 3 improves the full information underwriting beta method in chapter 2 by incorporating default risk and underutilized tax shields.

The results in chapter 3 are consistent with findings and arguments in chapter 2 and in prior studies. The expected underwriting profit margins (UPM) are around zero and vary over time; on average, the expected UPM is negative. The results show that the expected UPMs vary across different insurance business lines. The results from the OPM show that the expected UPM of liability insurance is the lowest, followed by that of auto insurance and then property insurance. The results reconfirm that the high leverage factor k of liability insurance results in that line contributing a larger part of investment income which, in turn, offsets some demand for profit from the underwriting activity. The relationship between expected UPM and leverage factor k is not linear.

The sensitivity analysis in chapter 3 shows the corporate income tax (CIT) rate is positively related to the expected premium and expected UPM and that the effects of CIT on expected premium and on expected UPM are quite stable over time and across firms. The effect of premium-based tax (PBT) on the expected premium is stable as well. However, the effect of PBT on expected underwriting profit margin changes with the firm-specific situation. If claims incurred are higher than the premium net of expense and other taxes, a higher PBT will generate a higher expected underwriting profit margin; otherwise, a higher PBT produces a lower expected underwriting profit margin.

Compared to other studies in insurance pricing, the current thesis has the following strengths. First, many of the previous models and techniques (such as Target Total Rate of Return Model, Hill's (1979) and Hill and Modigliani's (1987) ICAPM, Cummins' (1990) Internal Rate of Return Model, and D'Arcy and Garven's (1990) OPM,) are best applied in a single line of business framework. The most significant contribution of the current study is extending the literature by developing models that are suitable for pricing in a multi-line framework.

Second, compared to other insurance pricing studies that have been conducted in a multi-line framework, 1) this thesis augments the hypothesis test used in prior studies, such as Sommer (1996), Phillips et al. (1998) and Chen et al. (2003) and 2) this thesis provide valuable information about the fair underwriting profit margin, which could be combined with the results of Cummins and Phillips (2005) to provide comprehensive information to both insurers and regulators. While Cummins and Phillips (2005) applied the full information beta methodology to extend the traditional CAPM and FF-3-Factors model to estimate the *fair total equity return* for distinct business line, the current thesis examines the contribution of underwriting to the insurer's profitability for each major distinct business line. While the fair total equity returns measure the insurer's company-level total performance, those were captured by Cummins and Phillips (2005) and were received more attention in high interest rate environment. Their model is more relevant for applications such as capital budgeting; the model(s) presented in this thesis relate more directly to the actuarial issue of insurance pricing.

Third, compared to other insurance pricing techniques (for example, Discounted Cash Flow Model, Target Total Rate of Return Model), the ICAPM and OPM have some relative strengths and weaknesses. These models differ widely in terms of underlying assumptions, parameter specifications, and methods of calculation. But "they are generally organized around the basic principle that certain targets must be met so as to justify continued or even further allocation of capital to a particular set of insurance activities." (D'Arcy and Garven, 1990, p.394) The relative strengths and weaknesses of each model are briefly discussed as follow.

Discounted Cash Flow (DCF) model considers the cash flows between the insurer and the policyholders. The expected premium is determined as the discounted value of future loss, expense, and tax liability; and then the expected premium is used to determine the expected underwriting profit margin. One of the keys to the use of the DCF is to properly determine a method of discounting each of the above components. The result is very sensitive to the discount rate used. On the other end, Target Total Rate of Return (TTRR) model, ICAPM and OPM all consider the cash flows between the investors and insurance company.

Target Total Rate of Return model combines the underwriting and investment returns of an insurance policy into a single target. In this model, the expected underwriting profit margin is determined based on a selected total rate of return and an estimation of the investment income on a policy. The primary problem involved in using this technique is determining the appropriate target for the total rate of return, which could be the weighted average cost of capital: debt and equity. The results from this model are also very sensitive to the investment return adopted in the model. If the investment return includes capital gain and capital loss, then the results may be volatile and tend to be biased as a function of the tax position of the insurer.

When using the Insurance Capital Asset Pricing Model, the expected underwriting profit margin is calculated as the risk premium associated with the systematic risk of the insurance underwriting activities, offset by investment income, which is credited at the risk-free rate of return, and adjusted for taxes. The rationale of the risk-free investment return in ICAPM is that the insurer can choose an aggressive investment strategy or can choose to invest prudently, but any excess return, or below risk-free rate achieved should be borne by the insurer. On the underwriting component, i.e., the fair underwriting profit margin, ICAPM only rewards systematic risk. This can lead to underpricing, because the method ignores insurance-specific risk. Furthermore, ICAPM implicitly assumes that either the probability of insolvency is negligible or that shareholders have unlimited liability which introduces a bias that may cause results to be too high. Another limiting assumptions is that there is no tax redundancy which again results in the fair underwriting profit margin being underestimated. "For a steady state insurer, this approach would be correct, if the company has changed premium or exposure volume, however, this calculation would need to be refined." (D'Arcy & Gorvett, 1998, p.9)

The Option Pricing Model considers the systematic and firm-specific risk, redundant tax shields, and an insurer's insolvency risk in its approach to pricing. This model avoids the need for estimating and using underwriting betas and considers the insurer's aggregate risks. However, assuming the unused tax shield is worthless overestimates the expected underwriting profit margin. As indicated by D'Arcy and Garven (1990), this model is well suited for periods with volatile input parameters, such as during periods of higher and more volatile interest rates and high k environment; both the ICAPM and OPM are capable of producing more accurate estimates of the underwriting profit margin than the simpler total rate of return models.

As to the application of the results from the two models developed in this thesis, ICAPM is more suitable for the purpose of insurance pricing regulation and the OPM is more appropriate for the purpose of the product pricing for each individual insurer. The underwriting beta derived based on the Capital Asset Pricing Model measures the systematic risk of insurance underwriting activity related to the financial market. More specifically, the underwriting beta is a measure of volatility of the insurance underwriting activity in relation to the financial market. Because only systematic risk is taken into account in the Insurance Capital Asset Pricing Model, while the firm-specific risks are not considered, this model is a suitable candidate for setting the target underwriting profit margin for use in insurance price regulation. In such a setting the regulator would not normally set different rates for different companies based on individual companies' risk profiles. However, this model may not provide individual insurers enough information for accurate pricing because that insurer needs to consider the company's total risk rather than just the systematic risk of the company.

The OPM considers the total risk of an insurer in the pricing, including both systematic risk and the firm-specific risk, by adopting the variance of the insurer's total performance parameters (including variance of investment return, variance of incurred loss, covariance between the incurred loss and investment return, and covariance between the incurred loss and market equity return) as measures of the insurer's total risk. This characteristic makes the OPM an appropriate candidate for the pricing of an individual insurer's products because this model enables insurers to consider the total risk assumed in setting an appropriate price level for its product. Conversely, the OPM may not be a suitable model of the purpose of insurance pricing regulation, since the regulator will find it inappropriate to set different target underwriting profit margins for different insurers because of the different levels of risk assumed.

Using a variety of financial pricing models to estimate expected underwriting profit margin is expected to generate different estimates." Selecting the appropriate profit margin requires actuarial judgment, including a thorough understanding of the reliability of the inputs used in the models and the strengths and weaknesses of the different techniques." (D'Arcy & Gorvett, 1998, p.31) "Insurance prices should not be set according to a given model unless that model accurately represents the pricing mechanism." (D'Arcy and

Garven, 1990, p.394) If certain variables cannot be accurately measured, then the results from a model that is highly sensitive to that variable should be given less weight; and the model that is less sensitive to this variable should be given higher weight. For example, Insurance CAPM could be more appropriately applied to a line of business that is considered to have little insurance-specific risk (for example, fidelity) than one with a high degree of insurance-specific risk (for example, homeowner insurance). If the value of an insurer's equity cannot be easily estimated (such as mutual firms), then models that are very sensitive to the equity and premium-to-equity ratio(as is this case with the Target Total Rate of Return) should be give less weight than they would if the amount of equity could be more accurately valued (D'Arcy & Gorvett, 1998).

Insurance is a very complex financial transaction. The premiums are collected over time in return for the promise to compensate possible future losses. The timing and amounts of the possible losses are unknown. The mathematical models of stochastic asset and liability diffusion processes have not yet been perfected. The complexity means that scrutiny of the asset and liability distributions certainly should be warranted before the model is actually used for insurance pricing purposes and that combining the actuarial consideration in the determination of an appropriate underwriting profit margin and insurance premium could be instructive. "Selecting an appropriate underwriting profit margin is as much of an actuarial art as selecting the appropriate loss reserve level." (D'Arcy & Gorvett, 1998, p32). Although none of the financial pricing models is perfect and none of them can be relied upon to provide the appropriate underwriting profit margin in all situations, they can still be used if they are properly applied and if used in conjunction with other models. Knowing the differences in the basic structure, the likely relationships among models, and the sensitivity of the estimate in different models to specific parameters can help the user to select reasonable underwriting profit margin.

In general, the full information underwriting betas derived in the chapter 2 and the OPM developed in chapter 3 enable us to conduct more accurate estimation for the fair underwriting profit margin and the fair net premium by distinct insurance business lines than did prior studies. In turn, the results from the distinct lines models can provide better guidance for decisions by both regulators and management.

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