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Optimization of Demand-Responsive Transit Systems Using Zonal Strategy

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Optimization of Demand-Responsive Transit Systems Using Zonal Strategy

by

Lin Wang

A THESIS

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Abstract

In this thesis, two mathematical models were developed and applied to optimize demand-responsive transit systems using zonal strategies. Demand-responsive transit systems provide services between a residential area and a terminal. The first model considers a scenario where the terminal is located outside the service area, which optimizes the area of zones and bus capacity by minimizing the costs occurring within the entire service area; while the second model designs the transit system with its terminal sitting within the service area by minimizing the costs generated in service zones separately. Case studies for the application of both models were conducted for a presumptive demand-responsive transit system operating in northwest Calgary, showing that the models can successfully assist in designing demand-responsive transit systems. Furthermore, sensitivity analysis was conducted to identify the sensitivity of the model output to each model parameter.

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Table of Contents

Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	vii
List of Figures and Illustrations	viii
List of Symbols, Abbreviations and Nomenclature	x
 CHAPTER ONE: INTRODUCTION	 1
1.1 Background	1
1.2 Zonal strategy	2
1.3 Research motivation	3
1.4 Research objectives and scope	3
1.5 Contributions	5
1.6 Thesis outline	6
 CHAPTER TWO: LITERATURE REVIEW	 7
2.1 Public transit systems	7
2.1.1 FRT	8
2.1.2 DRT	8
2.1.3 HT	11
2.1.4 FRT and DRT costs	14
2.1.5 Selection of FRT, DRT and HT based on demand density	14
2.2 Advanced public transportation systems	15
2.3 Zonal transit	18
2.4 Shape of service area	20
2.5 Travel distance	21
2.6 Optimization models	22
2.6.1 Chang and Schonfeld (1991a) model	24
2.6.2 Clarens and Hurdle (1975) model	28
2.6.3 Comparison between the Chang and Schonfeld (1991a) and Clarens and Hurdle (1975) models	30
2.7 Discussion	32
 CHAPTER THREE: OPTIMIZATION OF DEMAND-RESPONSIVE TRANSIT SYSTEM: APPROACH ONE	 33
3.1 Model development	33
3.1.1 Operation policy	33
3.1.2 System description	34
3.1.3 Assumptions for model development	35
3.1.4 Model development procedure	36
3.1.5 Development of objective function	38
3.1.5.1 Cost on the line-haul route	38
3.1.5.2 Cost on the non-stop local route	40
3.1.5.3 Cost on the collection tour	40
3.1.5.4 Cost during the layover	41

3.1.5.5 Total cost.....	42
3.1.5.6 Average travel distances	42
3.1.5.7 Average cost per passenger.....	45
3.1.6 Decision variables	46
3.1.6.1 Manhattan measurement	46
3.1.6.2 Euclidean method	47
3.2 Case Study	48
3.2.1 Input data	51
3.2.1.1 Coefficient of hourly bus operation time cost	52
3.2.1.2 Demand density	53
3.2.1.3 Bus speeds.....	55
3.2.1.4 Hourly passenger riding and waiting time cost	56
3.2.2 Impact of bus entry point location on travel distances	56
3.2.3 Optimization of bus entry point location.....	59
3.2.4 System optimization with optimal bus entry point.....	61
3.2.5 Multi bus entry points system.....	64
3.2.6 Sensitivity analysis	66
3.2.6.1 Demand density	68
3.2.6.2 Hourly passenger waiting time cost.....	69
3.2.6.3 Bus local speed	70
3.2.6.4 Fixed coefficient of bus operation time cost.....	71
3.3 Conclusion	73
CHAPTER FOUR: OPTIMIZATION OF DEMAND-RESPONSIVE TRANSIT	
SYSTEM: APPROACH TWO	75
4.1 Model formulation	75
4.1.1 System description.....	75
4.1.2 Assumptions for model development.....	76
4.1.3 Model parameters	77
4.1.4 Development of objective function	78
4.1.4.1 Cost at terminal	78
4.1.4.2 Cost on the line-haul path	79
4.1.4.3 Cost on the local path.....	80
4.1.4.4 Cost on the back-haul path.....	82
4.1.4.5 Total cost.....	82
4.1.4.6 Average cost per passenger.....	83
4.1.4.7 Optimal area of service zone and headway	84
4.1.5 Determination of Zone Division.....	86
4.2 Case study	88
4.2.1 System description.....	88
4.2.2 Input parameters	89
4.2.2.1 Estimation of demand density.....	91
4.2.2.2 Estimation of τ	93
4.2.3 Impact of line-haul travel time	94
4.2.3.1 Daganzo's method	94
4.2.3.2 Clarens and Hurdle's method	95
4.2.4 Determination of Zone Division.....	96

4.2.4.1 Daganzo's method	97
4.2.4.2 Clarens and Hurdle's method	101
4.2.5 Sensitivity Analysis	104
4.2.5.1 Demand density	106
4.2.5.2 Hourly passenger waiting time cost.....	107
4.2.5.3 Hourly passenger riding time cost	108
4.2.5.4 Hourly bus operation cost	109
4.2.5.5 Bus local speed	110
4.2.5.6 Average time required for one vehicle serving one unit area of local service zones	111
4.3 Conclusion	112
CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH.....	
5.1 Conclusions.....	114
5.2 Recommendations for future research	117
APPENDIX A: DAGANZO'S MODEL	119
APPENDIX B: COMMUNITY INFORMATION.....	120
REFERENCES	124

List of Tables

Table 2.1 Comparison between Chang and Schonfeld (1991a) and Clarens and Hurdle (1975).	31
Table 3.1 Locations of all possible bus entry points.....	50
Table 3.2 Values of input parameters	51
Table 3.3 Bus Operation Data (Source: Calgary Transit).....	52
Table 3.4 Number of passengers disembarking at bus stops in the influential area (Source: Calgary Transit)	54
Table 3.5 Values of optimal variables	62
Table 3.7 Round trip time and entry points of each zone	66
Table 3.8 Summary of sensitivity analysis results.....	67
Table 3.9 Percentage changes of optimal bus capacity, optimal service zone area and minimum average cost per passenger vs deviations in demand density	69
Table 3.10 Percentage changes of optimal bus capacity, optimal service zone area and minimum average cost per passenger vs deviations in hourly passenger waiting time cost	70
Table 3.11 Percentage changes in optimal bus capacity, optimal area of service zone and minimum average cost per passenger vs deviations in bus local speed.....	71
Table 3.12 Percentage changes of optimal bus capacity, optimal area of service zone and minimum average cost per passenger vs deviations in fixed coefficient of bus operation ...	72
Table 4.1 Model parameters	77
Table 4.2 Values of input parameters	90
Table 4.3 Optimization results for each zone with Daganzo's method	100
Table 4.4 Optimization results for each zone with Clarens and Hurdle's method	103
Table 4.5 Summary of sensitivity	106
Table 5.1 Case study results of the two approaches	117

List of Figures and Illustrations

Figure 2.1 HT system types (Source: Errico et al. 2013).....	13
Figure 2.2 Terminal located on rectangular grids (Source: Hall 1984)	22
Figure 2.3 A subscription bus system in Chang and Schonfeld (1991a).....	24
Figure 3.1 Bus routes on a demand-responsive transit system	35
Figure 3.2 Flow chart of model development.....	37
Figure 3.3 Service area, possible bus entry points and terminal (modified from Bing map)	49
Figure 3.4 Impact of bus entry point location on distances under (a) the Manhattan method (b) Euclidean method	57
Figure 3.5 Extreme points on distances on QGIS map	58
Figure 3.6 Impact of bus entry point location on costs under (a) Manhattan method, (b) Euclidean method.....	60
Figure 3.7 Extreme Points on Costs on QGIS Map under Manhattan distance measurement	61
Figure 3.8 Zone division with optimal bus entry point.....	63
Figure 4.1 Service area and travel paths	76
Figure 4.2 Procedures of zone division determination	87
Figure 4.3 Service area and terminal (from QGIS map).....	88
Figure 4.4 Impact of line-haul travel time on (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger using Daganzo's method.....	95
Figure 4.5 Impact of line-haul travel time on (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger using Clarens and Hurdle's method.....	96
Figure 4.6 Zone division with Daganzo's model.....	99
Figure 4.7 Zone division with Clarens and Hurdle's method.....	102
Figure 4.8 Percentage change of (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger vs. deviations in demand density for different line- haul travel time	107
Figure 4.9 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly passenger waiting time cost for different line-haul travel time	108

Figure 4.10 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly passenger riding time cost for different line-haul travel time.....	109
Figure 4.11 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly bus operation cost for different line-haul travel time	110
Figure 4.12 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in bus local speed for different line-haul travel time	111
Figure 4.13 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in average time required for one vehicle serving one unit area of the local service zone for different line-haul travel time.	112

List of Symbols, Abbreviations and Nomenclature

Symbol	Definition
a	Fixed coefficient of bus operation time cost per vehicle per hour
A	Area of service zone
A_c	Area of service zone when buses filled to capacity
A_m	Area of community
A_σ	Size of service area
A_∞	Area of service zone when buses not filled to capacity
b	Variable operation cost coefficient
c	Bus capacity
C	Average cost per passenger
$C_{AverageO}$	Average operation cost per passenger
$C_{AverageR}$	Average riding cost per passenger
$C_{AverageW}$	Average waiting cost per passenger
$C_{backhaul}$	Cost on the back-haul path
$C_{backhaulO}$	Operation cost on the back-haul path
$C_{backhaulR}$	Passenger riding cost on the back-haul path
C_C	Cost on collection tour
C_{CO}	Operation cost on collection tour
C_{CR}	Passenger riding cost on collection tour
C_{CW}	Passenger waiting cost on the collection tour
C_H	Cost on the line-haul route
C_{HO}	Operation cost on the line-haul route
C_{HR}	Passenger riding cost on the line-haul route
C_{NSL}	Round trip cost on the non-stop local route
C_{NSLO}	Round trip operation cost on the non-stop local route
C_{NSLR}	Round trip passenger riding cost on the non-stop local route
C_L	Cost during the layover
$C_{linehaul}$	Cost on the line-haul path
$C_{linehaulO}$	Operation cost on the line-haul path

$C_{linehaulR}$	Passenger riding cost on the line-haul path
C_O	Bus operation cost
C_R	Passenger riding time cost
C_{Total}	Total cost
C_W	Passenger waiting time cost
d	Distance between bus entry point and service zone center
d_w	Expected distance between two consecutive points
D	Expected tour distance
D_c	Local collection tour distance
D_H	Line-haul distance
D_L	Equivalent line-haul distance between service zone and the service zone
D_{NSL}	Average non-stop local distance
F	Fleet size
$g(x)$	Symbol in Equation (A-2)
h	Bus headway
h_c	Bus headway when buses filled to capacity
h_∞	Bus headway when buses not filled to capacity
H	Market Mall annual open time
k	Constant factor
L	Length of the service area in Chang and Schonfeld (1991a)
M	Term defined by Equation (4.35)
n	Number of passenger in one travel cycle
n_a	Number of passenger alighting from the terminal to service zone
n_b	Number of passenger boarding from service zone to the terminal
n_p	Number of pickup location in one travel cycle
N	Number of service area zone
p	Demand density
p_a	Alighting passenger demand density
p_b	Boarding passenger demand density
$\overline{p_{\sigma a}}$	Average alighting demand density over service area

$\overline{p_{\sigma b}}$	Average boarding demand density over service area
$\overline{p_{ma}}$	Average alighting demand density over community
$\overline{p_{mb}}$	Average boarding demand density over community
P_m	Population in community
P_σ	Population in service area
S	Term defined by Equation (4.41)
t	Average round trip time on line-haul route and non-stop local route
t_E	Round trip time on line-haul route
t_C	Local travel time
t_L	Average bus layover time at the terminal
t_{NSL}	Round trip time on non-stop local route
t_r	Average vehicle round trip time
T	Travel time between terminal and service zone
T_p	Peak period duration
V_C	Local tour speed
V'_C	Local tour speed in case study of Clarens and Hurdle (1975)
V_H	Line-haul route speed
V_{NSL}	Non-stop local route speed
w	Width of the service zone in Appendix A
W	Width of the service area in Chang and Schonfeld (1991a)
x	Horizontal coordinate of the service area interior points
x_i	Horizontal coordinate of the i -th service zone center
x_E	Set of horizontal coordinate of bus entry point
x_T	Horizontal coordinate of the terminal
y	Vertical coordinate of the service area interior points
y_i	Vertical coordinate of the i -th service zone center
y_E	Set of vertical coordinate of bus entry point
y_T	Vertical coordinate of the terminal
Y	Term defined by Equation (2.18)
α	Ratio of community area to optimal zone area

γ_o	Bus operating cost per vehicle per hour
γ_R	Passenger riding time cost per passenger per hour
γ_w	Passenger waiting time cost per passenger per hour
δ	Points density over the service area in Appendix A
λ	Average number of passenger at one pickup location
τ	Average time required for one vehicle serving one unit area of the local service zone
τ'	Average time required for one vehicle serving one unit area of the local service zone in the case study of Clarens and Hurdle (1975)
φ	Proportion of trips destined to Calgary tower

Abbreviations Definition

ADA	Americans with disabilities act
AFT	Automated fare payment
APTS	Advanced public transportation system
ATIS	Automatic transit information system
AVL	Automatic vehicle location
CBD	Central business district
DRT	Demand responsive transit
FRT	Fixed-route transit
FMS	Fleet management systems
HT	Hybrid transit
PCAD	Paratransit computer aided dispatching
PDRT	Personalized demand responsive transit
QGIS	Quantum geographical information system
TDMS	Transportation demand management system
TSP	Traffic signal priority

Chapter One: Introduction

1.1 Background

As a result of urban sprawl, in the past number of decades the world's population has expanded from central urban areas to suburban areas, leading to low population density in urban areas (Sorensen 2000, Ewing et al. 2003, Deng et al. 2004, Kasanko et al. 2006, Catalán et al. 2008). The increasing dispersion of the population makes conventional fixed route transit (FRT) systems less efficient. FRT can provide reliable and cost-effective services to populated urban areas due to pre-determined schedules, large loading capacity of buses and consolidation of a substantial number of passengers onto one bus (ridesharing) (Quadrifoglio & Li 2009, Qiu et.al 2015). However, in areas with low population (demand) densities, people might need to walk long distances to access transit or might have less flexibility in scheduling their travel time due to long distances in bus stop spacing or low bus frequency. Therefore, an increasing number of passenger trips relied almost exclusively on private automobiles for citizens' transportation requirements, resulting in increased congestion, resource consumption and pollution problems (Wootton 1999, Quadrifoglio & Li 2009).

More effective and convenient forms of transit systems are required to overcome the disadvantages of FRT. Demand-responsive transit (DRT) systems have shown promising potential to effectively serve low demand situations, including suburban areas, rural areas, small towns and urban areas during off-peak periods (Koffman 2004). DRT is a demand-responsive, share-ride transit system characterized by the use of vans or small buses operating on flexible routes and/or flexible scheduling to accommodate passenger needs (Diana et al. 2007, Scott

2010). It is designed to offer user-specified service by transporting passengers from user-specified origins to user-specified destinations, within time windows to meet user-specified time. Modern communication and Internet technologies have broadly enhanced the operation and management of DRT systems (Goeddel 1996, Palmer et al. 2004). DRT-similar operation policies have been applied to a wide spectrum of services, such as paratransit services for people with disabilities in the U.S., required by Americans with Disabilities Act (ADA) (Koffman 2004); airport shuttle service worldwide, offering a shared-ride service to and from the airport; Zonetaxi service in Netherlands; Telebus operating in suburban areas of Melbourne (Australia); Connecting Wiltshire in rural Wiltshire, and Cango in Hampshire County (UK) (Scott 2010), etc. The popularity and promising future of shared-ride autonomous vehicle systems (Fagnant & Kockelman 2014) might also be a signal that user-specified transit alternatives are becoming more attractive to commuters. Similar to DRT systems, shared-ride autonomous vehicle systems seek to shift personal trip choices from private automobiles to services provided on demand.

1.2 Zonal strategy

The DRT systems studied in this thesis operate using a zonal strategy. In order to design transit systems for a large service area, a zonal transit strategy has been applied to achieve better system performance by separating the entire service area into smaller zones. The service zones are served by separate bus local services, and then connected with a common terminal by express services. Each vehicle circulates in its designated zone, picking up and dropping off passengers and then running non-stop between the service zone and the destination, which could be, for example a terminal, a shopping centre or a railway station. Travel distance using a zonal strategy

could be significantly reduced, compared to a non-zoning strategy. A zonal strategy could also provide easier management, reduced operate costs and passenger ride time, as well as a better level of service (Li & Quadrifoglio 2011, Smith et al. 2003, Krizek & El-Geneid 2007).

1.3 Research motivation

Existing DRT systems are generally costly to deploy (Currie 2007). To more effectively operate DRT systems, a number of mathematical models have been developed in order to optimize these systems (Daganzo et al. 1977, Chang & Schonfeld 1991a, Chang & Lee 1993, Chien et al. 2001, Li & Quadrifoglio 2009, Nourbakhsh & Ouyang 2012, Chandra & Quadrifoglio 2013). Most of these models were designed to operate in service areas with a regular shape (rectangular, square and circular). No mathematical model for a DRT system design in an irregularly shaped service area has been developed, yet most service areas for transit systems are in reality irregularly shaped. In addition, none of these mathematical models has considered the impact of the non-stop portion of bus travel time (line-haul travel time between terminal and service zones) on a DRT system optimization. However, Clarens and Hurdle (1975) showed that the non-stop portion of bus travel time played an important role in regular commuter bus system design.

1.4 Research objectives and scope

The overall goal of this research is to develop and apply two approaches (mathematical models) to optimize DRT systems using a zonal transit strategy. To achieve this goal, the objectives of the two approaches are defined below:

1) Develop an optimization model for a DRT system with a terminal located outside the service area:

- Extend the Chang and Schonfeld (1991a) model from a rectangular to an irregularly-shaped service area.
- Minimize average cost per passenger over the entire service area to achieve the expressions of optimal bus capacity and service zone area.
- Calculate travel distances on bus routes with two different measurements, including Manhattan and Euclidean grids.
- Conduct a case study on northwest Calgary, with the terminal located at the Calgary Tower, representing the central business district (CBD).
- Explore the impact of multi bus entry points on optimal system design.
- Divide the service area into zones with an optimal area.

2) Develop an optimization model for a DRT system with a terminal located inside the service area:

- Extend the Clarens and Hurdle (1975) model from a regular commuter bus system to a DRT system.
- Minimize the average cost per passenger in each service zone to achieve the expressions of optimal bus headway and service zone area.
- Establish procedures of zone division.
- Conduct a case study in northwest Calgary with the terminal located at Market Mall.
- Explore the impact of line-haul travel time on optimal system design.

- Divide the service area into zones with optimal areas.

1.5 Contributions

In this thesis, two approaches were successfully developed and applied to optimize DRT systems operating using a zonal strategy. The contributions from the two approaches in this thesis are summarized below:

1) Contributions from approach one

- Optimize DRT systems operating in an irregularly-shaped service area, with a terminal located outside the service area.
- Optimize a presumptive DRT system operating in northwest Calgary with its terminal located at the Calgary Tower, and estimate passenger demand density visiting the Calgary Tower from the service area.
- Compare a single bus entry point system with a multiple bus entry points system.

2) Contributions from approach two

- Optimize DRT systems by taking the travel time between terminal and service zones into consideration, with a terminal located inside the service area.
- Optimize a presumptive DRT system operating in northwest Calgary with Market Mall as the terminal, and estimate the number of passenger travel trips between the service area and Market Mall.
- Describe procedures in detail for dividing the service area into service zones with optimal areas.

1.6 Thesis outline

This dissertation is organized into five chapters as below.

Chapter One provides general background information, research motivation, research scope and objectives, research contributions, as well as a thesis outline.

Chapter Two presents a comprehensive literature review of public transit systems, DRT, performance comparison of public transit systems, advanced public transportation systems, zonal transit, optimization models and a discussion of the literature review.

Chapter Three describes the development and application of an approach (mathematical model) to optimize a DRT system operating in an irregularly shaped area, with a common terminal located outside the service area. A case study was conducted on a presumptive DRT system operating in northwest Calgary, with its terminal located at the Calgary Tower.

Chapter Four describes the development and application of an approach (mathematical model) to optimize a DRT system with a terminal sitting inside the service area. The optimal area of the service zone, as well as optimal bus headway for different locations, was determined. Procedures to partition the service area into service zones with optimal areas was illustrated in this chapter.

Chapter Five presents the summary of research results, as well as recommendations for future work.

Chapter Two: Literature Review

2.1 Public transit systems

Public transit systems provide shared transit services for the general public. They are considered to be viable options for sustainable transportation in urban areas, which can alleviate traffic congestion, reduce energy consumption and air pollution, enhance mobility and preserve social equity (Kepaptsoglou & Karlaftis 2009). In terms of operational policy, public transit can be grouped into three major categories: fixed route transit (FRT), demand-responsive transit (DRT) and hybrid transit (HT). FRT is a conventional transit service operating on fixed routes and schedules. Passengers must walk a certain distance (< 400 meters) to pre-determined bus stops in order to access the FRT services. Generally, FRT is considered to be the most cost-effective transit system to serve passengers in relatively dense urban areas (Chang & Schonfeld 1991a, Kim & Schonfeld 2011). DRT is a flexible public transit providing “on-demand” service. It picks up and drops off passengers in accordance with their needs. DRT operates on flexible routes and flexible schedules providing higher productivity when passenger demand density is low (Batchelder & Kullman 1977, Chien et al. 2001). HT is a semi-flexible transit system that integrates a pure DRT with a FRT using different operational policies (Hickman & Blume 2001, Aldaihani & Dessouky 2003, Qiu et al. 2014). Both DRT and HT systems are considered to be promising solutions for widespread public transportation in rural areas (Mulley 2010).

2.1.1 FRT

FRT is the most common form of public transit, characterized by predetermined routes and schedules, large loading capacity of vehicles, shared rides, as well as dense stops, frequency and routes in urban areas, which in turn reduces passenger waiting time and access time (Quadrifoglio & Li 2009, Kim & Schonfeld 2011). It is a safe, economical and energy-efficient transit mode in densely populated cities and metropolitan areas. FRT produces fewer carbon dioxide emissions, and uses far fewer resources than private automobiles (Vuchic 2002, Nourbakhsh & Ouyang 2012). However, FRT costs passengers in terms of longer riding time and more transfers, due to its inflexibility in pick-up/drop-off locations and scheduling, especially in areas with low population density (Quadrifoglio et al. 2007). Consequently, an increasing portion of people is inclined to travel by private automobiles, resulting in severer congestion and pollution issues (Quadrifoglio & Li 2009).

2.1.2 DRT

DRT is an advanced, user-oriented form of public transit characterized by flexible routes and schedules (Palmer et al. 2004, Quadrifoglio et al. 2008). It provides door-to-door services to accommodate passengers' needs. Passengers are picked up at and transported to their desired locations. Since the 1970s, DRT has been used as an alternative transport solution in circumstances where FRT is not economically viable (Davison et al. 2014). DRT systems are also termed as subscription bus systems (Chang & Schonfeld 1991a, Chien et al. 2001), dial-a-ride (Desrosiers et al. 1986, Cordeau & Laporte 2003, Luo & Schonfeld 2007), paratransit (Shen

2012), call-n-ride (Baumgartner & Schofer 2011), high coverage point-to-point transit (Jung & Jayakrishnan 2011), and demand adaptive systems (Crainic et al. 2012).

In the late 1970s and early 1980s, large DRT demonstration projects were developed in Santa Clara County (California), Haddonfield (New Jersey), Orange County (California), Rochester (New York), and among others (Lave et al. 1996). However, in the early stages the wider usage of DRT was prevented by economic, social and technical barriers (Behnke et al. 1982). These early systems were discontinued for two primary reasons: (1) Demand requests in the demonstrated areas were very low; (2) They could not be effectively managed, due to a deficiency in communication and computer technologies (Lave et al. 1996).

In recent decades, DRTs have been successfully applied in Winnipeg (Canada), Canberra (Australia), Melbourne (Australia), Adelaide (Australia), Tyne and Wear (England), Gothenburg (Sweden), Kashiwa (Japan), Sakai (Japan), Moriyama (Japan) and other cities (Westerlund et al. 2000, Scott 2010, Tsubouchi et al. 2010, Nelson & Phonphitakchai 2012). The widespread application of DRTs is driven by improvements in communication technologies and government policies, along with passenger's willingness to use DRT.

- Improvement in technologies

The development of advanced intelligent technologies and computers has improved the productivity of public transit systems. Advanced routing and scheduling algorithms in particular have enhanced the efficiency of real-time dispatching and scheduling for shared-ride systems (Stone et al. 1994). The implementation of automatic vehicle location technology and advanced bus scheduling has been considered to be major factors in improving the efficiency of paratransit services (Higgins et al. 2000).

- Governments policy

In the U.S., ADA came into effect in 1990, through which municipal public transit system operators were obligated to provide complementary paratransit services for mobility impaired individuals who were unable to use FRT services. ADA played a substantial role in reshaping public transportation and paratransit in the U.S., which has led to an expansion of DRT usage. The total number of DRT passenger trips in 2000 was 73.2 million, which was 75% more than in 1991 (Palmer et al. 2004). It is noted that the U.S. population in the same period only increased by 13%. In 1992, Australia passed the Disability Discrimination Act to prohibit discrimination against person with disabilities in different areas of life including transport options. In the UK, the government's Transport 10 Year Plan 2010 was intended to provide more support for flexible and innovative transit services, such as an extension of a fuel duty rebate, and cheaper fares for seniors and people with disabilities (Transport 2010: The 10 Year Plan). Substantial funding has been granted to local authorities under the Rural and Urban Bus Challenge programmes for the implementation of DRT (Mageean & Nelson 2003). In 2001, the European Conference of Ministers of Transport endorsed the potential of DRT in its resolution on accessible transport service (Nelson & Mageean 1999). The successful demonstration of DRT research and development (R&D) projects, along with the use of transportation telematics has increasingly enhanced the concept that DRT can provide an efficient and viable transportation alternative.

- People's willingness to use DRT

Khattak and Yim (2004) conducted a traveller response survey of innovative personalized demand responsive transit (PDRT) system in the San Francisco Bay Area. It was reported

that about 60% of respondents stated their willingness to consider using PDRT as a trip option, and about 12% were very likely to use PDRT.

2.1.3 HT

HT system is a public transit system that incorporates the regularity of FRT with the flexibility of DRT (Alshalalfah 2009). A well-designed HT is more user-oriented, comfortable and cost-effective. The introduction of HT systems has been demonstrated to overcome the shortcomings of pure DRT, such as relatively high operational cost and lack of flexibility in route design with long pre-booking time (Westerlund et al. 2000). HT operation types can be classified into six major groups: route deviation, point deviation, demand-responsive connector, request stops, flexible-route segments and zone route (Errico et al. 2013). They are depicted in Figure 2.1 and are briefly described as follows:

- Route deviation: Vehicles are operated on a well-defined baseline route, with one or multiple mandatory checkpoints located at major trip generators for buses deviating from the baseline to serve optional requests.
- Point deviation: DRT operates on most bus routes, with the exception of a few bus stops that are pre-scheduled.
- Demand-responsive connector: DRT operates within a given zone, connecting with a FRT via at least one transfer point.

- Request stops: In addition to providing conventional FRT service, vehicles also serve requests assigned near the route for a limited number of stops. Usually, the optional stops are very few and located near the main path.
- Flexible route segments: DRT services are provided along a limited portion of the route, while FRT services are provided for the rest of the route.
- Zone routes: DRT is operated over a given zone, which has scheduled departures or destinations at one or more end points.

According to Potts et al. (2010), the percentage of North American transit agencies adopting route deviation, point deviation, demand-responsive connector, requests stops, flexible-route segments, and zone route was 51.6%, 19.5%, 30.5%, 30.9%, 19.5% and 32.9%, respectively.

Figure 2.1 HT system types (Source: Errico et al. 2013)

2.1.4 FRT and DRT costs

DRT systems are generally more expensive than FRT. The U.S. National Transit Summary and Trends report for 2015 indicated that the average cost per unlinked passenger trip for DRT (\$35.55) was almost eight times higher than for FRT (\$4.04). The cost per passenger mile for DRT and FRT was \$4.21 and \$1.09. The unlinked passenger trips per vehicle revenue hour were 2.0 and 32.4 for DRT and FRT. The passenger fares for DRT were 7.7% of operating cost, while the passenger fares for FRT were 25.7% of operating cost. A low fare/operating cost ratio for DRT indicated that it was a highly subsidized service. Although DRT is generally more costly to deploy, it has been executed as a complement to traditional public transit systems in many cities, such as Chicago, San Francisco, Denver, Winnipeg and Florence since the 1970s, especially after the introduction of the ADA of 1990.

2.1.5 Selection of FRT, DRT and HT based on demand density

The selection of transit operation policy strategy based on demand density has been proposed by many researchers (Chien et al. 2001, Luo & Schonfeld 2007, Alshalalfah & Shalaby 2011, Nourbakhsh & Ouyang 2012, Edwards & Watkins 2013, Qiu et al. 2015 b). In these models, critical demand density was determined to select the best operational policy. Critical demand density is considered to be the switching point between the two operational policies (Qiu et al. 2015 b). Critical demand density is influenced by system parameters, such as the shape of the service area, demand density, vehicle speed, number of service zones, dwell time at stops, time value of operation, and time value of passengers waiting and riding.

Generally, if demand level is lower than critical demand level, DRT could be offered at a lower price, and could be preferred; while if demand level was higher than critical demand level, then fixed route buses became cost-effective and would be preferred. Critical demand density varied from different system settings (such as shape of the service area, route network pattern, demand pattern, operating policy etc.). Li et al. (2007) modelled the fixed route operation over an idealized grid network, and found that the optimal cost per passenger was inversely proportional to a cube root of demand density. Li and Quadrioglio (2010) found that DRT was preferable when critical demand ranged from 10 to 50 passengers/mile²/h. Nourbakhsh and Ouyang (2012) compared fixed-route transit, flexible-route transit and taxi systems in terms of optimal system costs. A taxi system was desirable when demand density was less than 4 passengers per hour per square kilometre; the fixed-route transit system was favourable when demand density was more than 40 passengers per hour per square kilometre; the flexible-route transit system was desirable when demand density was between 4 and 40 passengers per hour per square kilometre. Edwards and Watkins (2013) compared the performance of DRT and traditional FRT by utilizing open source data and software in a variety of street and transit layouts. The results showed that DRT was more cost-effective than FRT systems when demand density was less than 6 passengers per minute. Jacobson (1980) compared a pure flexible service with a hybrid service at very low demand density, and found that the flexible service was less costly.

2.2 Advanced public transportation systems

Advanced public transportation systems (APTSs) are the management techniques and technologies used to improve the efficiency and safety of public transportation systems (FTA

2000). Through APTSs, users have greater access to information on system operations and can reduce their travelling time. Examples of APTS applications include automatic vehicle location systems, fleet management systems, transportation demand management systems, automatic fare systems, automatic transit information systems, paratransit computer-aided dispatching systems and transit signal priority systems.

- Automatic vehicle location (AVL)

AVL is a computer-based vehicle location tracking system that includes determination of vehicle location and transmission data from vehicles to the dispatch centre using a global positioning system, active signposts and ground-based radio. AVL can increase the performance of transit systems and reduce their costs. It has been applied in the Hamilton Street Railway, Kansas City Area Transit Authority, Maryland Mass Transit Administration, Milwaukee County Transportation Division, Winston-Salem Transit Authority and others (Transit 1996).

- Fleet management system (FMS)

FMS is a standard interface integrating fleet communication, automatic passenger count, vehicle location and vehicle control technologies (Goeddel 2000). An FMS can improve both operational efficiency and transit safety. Emergency response time has been reported to be reduced up to 40% when FMS is used (Goeddel 1996).

- Transportation demand management system (TDMS)

TDMS is the art of modifying travel behaviours and improving existing transportation infrastructure implementation to accommodate increasing demand for transit (Ferguson 1990). The technologies of TDMS included dynamic ridesharing, automated service

coordination, transportation management centres and high occupancy vehicle facility monitoring (Pagano et al. 2000).

- Automated fare payment (AFP)

AFP is a fare payment system that allows passengers to pay their fare with magnetic stripe cards, smart cards, credit cards or debit cards. It makes fare collection more efficient and more flexible for passengers and transit operators. AFP can improve the security of transit revenues, increase customer convenience and reduce fare collection and processing costs (Goeddel 1996).

- Automated transit information system (ATIS)

ATIS is a computer-based system providing the public with real-time transit information (such as vehicle location or anticipated arrival time). With this system, passengers can make better decisions with respect to their travel modes and plans for choosing their own travel route and time. ATIS distributes information using several communication techniques, such as wireless broadcast, TV, Internet, sidewalk page, cell phones and vehicle screens (Test 1998, Pagano et al. 2000).

- Paratransit computer-aided dispatching (PCAD)

PCAD systems utilize computerized software for automated scheduling and dispatching. PCADs have the potential to improve system performances, through interagency connections, vehicle load ratio, dramatically streamlined billing operations and interactive voice driven reservation systems (Transit 1996, Goeddel 1996, Khattak & Hickman 1998, Pagano et al. 2000).

- Traffic signal priority treatment (TSP)

TSP is a technology in which traffic signals remain green, or lights turn green earlier than scheduled, so that a particular vehicle can pass through intersections more quickly. This technology is specifically designed for emergency vehicles.

2.3 Zonal transit

Zonal transit is an operating strategy used to improve both service quality and productivity of public transit systems (Turnquist 1974, Jordan & Turnquist 1979, Tsao & Schonfeld 1983, Tsao & Schonfeld 1984, Furth 1986, Li & Quadrifoglio, 2009, Chen & Nozick 2016), which is especially useful in a many-to-one service (Jordan & Turnquist, 1979).

In zonal transit, long bus service routes or large service areas are divided into shorter service routes or smaller service zones. Each service zone is served by a separate sub-route bus service. Each sub-route bus service is a combination of local feeder and non-stop line-haul services. The buses collect passengers at bus stops within service zones, and then travel back to a same terminal at non-stop line-haul speed. Tsao and Schonfeld (1983) summarized the advantages of zonal transit as follows:

- The vehicle miles required to satisfy demand could be reduced by using a zonal transit strategy, since vehicles filled up relatively quickly during the local part of their sub-routes, and returned back before the end of the bus route.
- Average travel speeds were increased, as buses were driven at non-stop line-haul speeds outside zones, thus it can improve vehicle productivity, resulting in a substantial decrease in fleet size.

- It reduced trip time for most passengers, as each bus only served one zone, and passengers travelled at non-stop line-haul speed for part of their trips.
- Operating problems, such as delays and vehicle breakdowns, were localized, leading to an improvement in service reliability.
- It facilitated the collection of different fares for zones with different trip lengths, which can enhance equity as well as revenues.

Zonal strategies have been used successfully for both bus and rail transit services. Through minimizing the total cost function, a number of models have been developed to design transit systems with zonal strategies (Clarens & Hurdle 1975, Banks et al. 1982, Burns et al. 1985, Furth 1986, Erlenkotter 1989, Chang & Schonfeld 1991a, Aldaihani et al. 2004, Li & Quadrifoglio 2009). Chang and Schonfeld (1991a) developed optimization models for zonal service operation, which included service zone area and bus capacity as decision variables. Li and Quadrifoglio (2011) developed an analytical model to determine the optimal number of service zones in a residential service area. Chien and Schonfeld (1997) proposed a modelling approach to optimize the design and operation of bus transit systems in a realistic geographic environment, while Chien and Spasovic (2002) developed an optimization approach to deal with practical geographic variability.

However, zonal transit was likely to increase headway and passenger waiting time, and also led to more transfers if passengers were seeking inter-zone trips (Tsao & Schonfeld, 1983).

2.4 Shape of service area

The shape of the service area influences the mode of operation and bus travelling distances (Daganzo, 1984a), which is an important factor in transit system design. Typically, actual service area shapes are irregular. However, they can be approximated to a similar regular shape, such as a rectangle, circle and so on. Shape approximation can simplify the development and solution process of models, while tolerating some inaccuracy. Varying regular shapes of service areas have been adopted in transit design research, such as rectangular (Hurdle 1973, Aly & White 1978, Newell 1979, Koning & Eizenberg 1981, Daganzo 1984 b, Chang & Schonfeld 1991b, Chien & Schonfeld 1998, Koester 2000, Wang et al. 2006), square (Aldaihani et al. 2004, Diana et al. 2006, Daganzo 2010, Nourbakhsh & Ouyang 2012), radical (Capozza and Van Order 1978, Langevin & Soumis 1989, Diana et al. 2006). Erlenkotter (1989) developed a general optimal market area model considering different market shapes (circle, regular hexagon, square, diamond) and distance norms (Manhattan distance, Euclidean distance and extended form). The model gave values of a “configuration factor” to reflect the shape of the service market, distance metrics and transport cost exponent. So far, only a limited number of researchers have focused on non-regular shapes of service areas. In most cases, the irregularly shaped service area was divided into rectangular zones according to street spacing and demand distribution, to simplify computation (Chang & Schonfeld 1993, Chien & Spasovic 2002, Chien et al. 2003, Chien & Yang 2000).

2.5 Travel distance

Travel distance in a transit system is one of the most important determinants of transportation cost. An understanding of the relationship between travel distances and the characteristic of the system parameters (dimension of service area, distance metrics, distribution of demand) allows one to address strategic issues such as zone division, vehicle capacity and headway. Daganzo (1984a) developed a formula to estimate the length of a tour in regions of different shapes where demand was homogeneously distributed. The length of tour was equal to a constant factor multiplied by the square root of the number of pickup points and the square root of the area of the service zone. With the Manhattan metric, the constant factor was equal to 1.15, while for the Euclidean metric, it was 0.9. Larson and Odoni (1981) found that the expected Euclidean distance between two random points in a square zone was equal to 0.52 of its side length, and the expected Euclidean distance from the centre of the zone to other random points was equal to 0.383 of its side length. Diana et al. (2009) estimated the expected travel distance for a square area with homogeneous demand distribution to be 0.667 of its side length. Quadrifoglio and Li (2009) derived the expected total travel distance by incorporating the maximum expected horizontal distance in a rectangular service area. The expected total rectilinear distance per cycle for a no-backtracking policy was $2L \frac{n}{n+1} + \frac{2W}{3} + \frac{W}{6}n$, where L and W were the length and width of the service area, n was the number of customers served per cycle. Hall (1984) developed formulas for average travel distances between randomly located origins and randomly located destinations through terminals. The rectangular grid region with I columns and J rows of terminals is shown in Figure 2.2. For one-dimensional models, the average travel distance

between any origin at point x_0 and randomly located destinations was $x/2 - x_0(1 - x_0/x)$, where x was the length of the line segment. For two-dimensional models, the average travel distance between origins and destinations over a rectangular grid was $(x/3 + x/4I - x/12I^2) + (y/3 + y/4J - y/12J^2)$, where x was the horizontal dimension of the rectangular region and y was the vertical dimension of the rectangular region.

Figure 2.2 Terminal located on rectangular grids (Source: Hall 1984)

2.6 Optimization models

Since the early 1970s, different mathematical models have been developed to optimize and evaluate the performance of DRT systems (Chang & Schonfeld 1991a, Clarens & Hurdle 1975, Chien & Schonfeld 1998, Chien & Yang 2000, Dessouky et al. 2003, Diana et al. 2006). Most optimization models were established based on minimization of total system cost. Clarens and

Hurdle (1975) developed an analytical model to optimize the size of service zone based on travelling time, demand density and other impact parameters. Chang and Schonfeld (1991a) constructed mathematical models to optimize a DRT system providing feeder services to a single point (many-to-one service). Chien and Schonfeld (1998) presented an optimization model for an integrated rail-bus transit system operating in a service area with heterogeneous demand. The integrated transit system was analytically optimized by minimizing the total cost function with an iterative method. The model can optimize design variables such as railway length, rail station spacing, bus headway, bus stop and route spacing. Chien and Yang (2000) presented an optimization model for feeder bus systems in a heterogeneous service area by minimizing the total cost. The model was able to determine the optimal locations of feeder bus routes operating in an irregularly shaped service area. Dessouky et al. (2003) presented a methodology to joint optimize of service, cost, and life-cycle environmental consequences for vehicle routing and scheduling of a DRT system. This methodology substantially reduced environmental impacts, while only slightly increasing operating costs and service delays. Diana et al. (2006) proposed an analytical modelling approach to forecast the optimal fleet size for a DRT system with pre-determined quality for users in terms of passenger waiting time and maximum allowed detours. The details of the Chang and Schonfeld (1991a) and Clarens and Hurdle (1975) models are described below, as the two models developed in this thesis are based on the extension of these two models. Chang and Schonfeld's model optimized the transit system by minimizing the total cost occurring in an entire service area, while Clarens and Hurdle's model optimized the system by minimizing the cost occurring in service zones.

2.6.1 Chang and Schonfeld (1991a) model

Chang and Schonfeld (1991a) developed an analytical model for a flexible route subscription bus system providing many-to-one feeder services. The studied system operated in a rectangular service area with a single transportation terminal located outside, shown in Figure 2.3.

Figure 2.3 A subscription bus system in Chang and Schonfeld (1991a)

The service area was divided into equal service zones. Buses travelled a line-haul distance from the terminal to the service area, and then travelled a non-stop local distance to the centre of its assigned service zone. Within the service zone, buses collected passengers via door-to-door service, and then returned to the terminal.

A few assumptions were made for model development: (1) Demand density was uniformly distributed over the service area; (2) Buses were operated on a pre-set schedule and rectangular street network; (3) Average passenger waiting time was half of the headway; (4) Buses were fully filled after the collection tour.

This model was intended to optimize vehicle capacity and service zone area by minimizing the average cost per trip. The average cost per trip function (C) was the total cost (C_{Total}) divided by the number of passengers n ,

$$C = \frac{C_{Total}}{n} \quad (2.1)$$

Total cost (C_{Total}) is the sum of bus operator cost (C_O), user in-vehicle cost (C_R), and waiting cost (C_W),

$$C_{Total} = C_O + C_R + C_W \quad (2.2)$$

The bus operator cost (C_O) was expressed as:

$$C_O = \gamma_O F \quad (2.3)$$

where γ_O was the hourly bus operation time cost and F was the fleet size.

Hourly bus operation time cost (γ_O) can be calculated by,

$$\gamma_O = a + bc \quad (2.4)$$

where a was the fixed coefficient of operation cost; b was the variable coefficient of operation cost; c was vehicle capacity.

Fleet size (F) can be calculated by,

$$F = \frac{LWt_r}{Ah} \quad (2.5)$$

Where L was the length of rectangular area; W was the width of rectangular area; t_r was the average vehicle round trip time; A was the area of service zone; h was headway.

The average vehicle round trip time (t_r) was calculated by,

$$t_r = 2 \left(\frac{L + W}{2V_{NSL}} + \frac{D_H}{V_H} \right) + \frac{D_C}{V_C} = \frac{D_L + D_C}{V_C} \quad (2.6)$$

where V_{NSL} was the non-stop local speed; D_H was the line-haul distance; V_H was the line-haul speed; D_C was the collection tour distance; V_C was the local collection speed; D_L was the equivalent round trip distance between the terminal and the service zone.

The equivalent round trip distance D_L was formulated as,

$$D_L = \frac{L + W}{V_{NSL}} + \frac{2D_H}{V_H} \quad (2.7)$$

Collection tour distance D_C was approximately estimated following Daganzo (1984a)'s method,

$$D_C = kA\sqrt{ph/\lambda} \quad (2.8)$$

where k was the constant with a value of 1.15 for Manhattan distance measurement and 0.90 for Euclidean distance measurement; p was the demand density throughout the service area; λ was the average number of passengers per pick-up location.

Substituting Equation (2.4 ~ 2.8) into Equation (2.3) yielded

$$C_o = \frac{LW(a + bc)(D_L + kA\sqrt{\frac{ph}{\lambda}})}{V_C Ah} \quad (2.9)$$

The user in-vehicle cost (C_R) was formulated as:

$$C_R = \frac{\gamma_R LW p t}{2} = \frac{\gamma_R LW p (D_L + kA\sqrt{\frac{ph}{\lambda}})}{2V_C} \quad (2.10)$$

where γ_R was the riding time cost per passenger per hour.

The user waiting cost (C_W) was formulated as:

$$C_W = \frac{\gamma_W h L W p}{2} \quad (2.11)$$

where γ_W was the passenger waiting time cost.

Then the total cost (C_{Total}) was,

$$C_{Total} = \frac{LW(a + bc)(D_L + kA\sqrt{\frac{ph}{\lambda}})}{V_C Ah} + \frac{\gamma_R LWp(D_L + kA\sqrt{\frac{ph}{\lambda}})}{2V_C} + \frac{\gamma_W h L W p}{2} \quad (2.12)$$

Since vehicles were filled to capacity after one collection tour, the vehicle headway (h) can be formulated as,

$$h = \frac{c}{Ap} = \frac{n}{Ap} \quad (2.13)$$

Then the total cost (C_{Total}) can be obtained as a function of decision variables, including service zone area (A) and vehicle size (c):

$$C_{Total} = \frac{LWpD_L(a + bc)}{V_C c} + \frac{LWpk\sqrt{A/\lambda c}(a + bc)}{V_C} + \frac{\gamma_R LWpD_L}{2V_C} + \frac{\gamma_R LWpk\sqrt{Ac/\lambda}}{2V_C} + \frac{\gamma_W c L W}{2A} \quad (2.14)$$

The average cost per trip (C) was:

$$C = \frac{D_L(a + bc)}{cV_C} + \frac{k\sqrt{A/\lambda c}}{V_C}(a + bc) + \frac{\gamma_R D_L}{2V_C} + \frac{k\gamma_R\sqrt{cA/\lambda}}{2V_C} + \frac{\gamma_W c}{2pA} \quad (2.15)$$

Optimal vehicle capacity and area of service zone were obtained by taking partial derivatives of Equation (2.15),

$$c^* = \left[\frac{\lambda p a^3 D_L^3}{\gamma_W k^2 V_C (b + \gamma_R/2)^2} \right]^{1/5} \quad (2.16)$$

$$A^* = \left(\frac{a \gamma_W^3 V_C^3 D_L^3 \lambda^{\frac{8}{3}}}{k^4 p^{\frac{7}{3}} Y^{\frac{10}{3}} (b + \gamma_R/2)^2} \right)^{1/5} \quad (2.17)$$

$$Y = (\gamma_W a^2 k^2)^{1/5} + (\lambda p D_L^3 (b + \gamma_R/2)^3)^{1/5} \quad (2.18)$$

Then, the optimal expression of total cost (C_{Total}^*), average cost per trip (C^*), fleet size (F^*), and headway (h^*) can also be obtained.

2.6.2 Clarens and Hurdle (1975) model

Clarens and Hurdle (1975) established a model for a commuter bus system to obtain the optimal area of service zone for every point in the city. Buses carried passengers from a central terminal to different destinations (i.e. one-to-many service) throughout the city and its suburbs. Services and costs occurring in different zones were assumed to be independent of each other. Optimal zone area for different locations was obtained by minimizing daily average cost per passenger within the zone. The average cost per passenger function (C) was the sum of bus operation cost ($C_{AverageO}$), passenger waiting cost ($C_{AverageW}$) and passenger riding cost ($C_{AverageR}$).

$$C = C_{AverageO} + C_{AverageW} + C_{AverageR} \quad (2.19)$$

The average cost per passenger for bus operation can be obtained by,

$$C_{AverageO}(x, y) = c \gamma_o [T(x, y) + \tau(x, y) A(x, y)] / p(x, y) A(x, y) h(x, y) \quad (2.20)$$

Where c was the vehicle capacity; γ_o was the cost per seat per unit time to operate vehicles capable of carrying c passengers; $T(x, y)$ was the travel time for the line-haul trip from terminal to point (x, y) ; $\tau(x, y)$ was the average time required for a vehicle to service one unit area of

service zone that included point (x, y) ; $A(x, y)$ was the area of service zone that included point (x, y) ; $p(x, y)$ was the number of passengers per unit area seeking service to destinations in the vicinity of point (x, y) ; $h(x, y)$ was the headway between vehicles serving point (x, y) .

Average cost per passenger for passenger waiting cost can be obtained by,

$$C_{AverageW}(x, y) = \frac{1}{2} h(x, y) \gamma_W \quad (2.21)$$

where γ_W was the time value per passenger per unit time waiting for a bus.

The average cost per passenger for passenger riding cost can be obtained by,

$$C_{AverageR}(x, y) = \frac{1}{2} \tau(x, y) A(x, y) \gamma_R \quad (2.22)$$

where γ_R was the time value per passenger per unit time riding a bus.

Substituting Equations (2.20 ~ 2.22) into Equation (2.19) yielded

$$C(x, y) = \frac{C \gamma_o [T(x, y) + \tau(x, y) A(x, y)]}{p(x, y) A(x, y) h(x, y)} + \frac{1}{2} [\gamma_w h(x, y) + \gamma_R \tau(x, y) A(x, y)] \quad (2.23)$$

The optimal area of service zone and headway were obtained by taking partial derivatives of Equation (2.23).

When vehicles were not filled to capacity [$p(x, y) A_\infty(x, y) h_\infty(x, y) \leq C$], the optimal service zone area A_∞^* and the optimal headway h_∞^* were obtained from the following equations,

$$A_\infty^* \left[1 + \frac{\tau(x, y) A_\infty(x, y)}{T(x, y)} \right] = \left[\frac{2C \gamma_w \gamma_o}{\gamma_\tau^2} \right] \left[\frac{T(x, y)}{\tau(x, y)^2 p(x, y)} \right] \quad (2.24)$$

$$h_\infty^* = \left[\frac{2C \gamma_o}{\gamma_\tau} \right] \left[\frac{T(x, y)}{\tau(x, y) p(x, y) A_\infty(x, y)^2} \right] \quad (2.25)$$

When vehicles were filled to capacity, the optimal service zone area A_c^* and the optimal headway h_c^* were obtained from the following equations,

$$A_c(x, y) = [C\gamma_w/(\gamma_\tau + 2\gamma_o)\tau(x, y)p(x, y)]^{\frac{1}{2}} \quad (2.26)$$

$$h_c(x, y) = C/p(x, y)A_c(x, y) \quad (2.27)$$

Hence, the optimal zone size $A(x, y)^*$ at any point (x, y) was

$$A(x, y)^* = \begin{cases} A_\infty(x, y), & p(x, y)A_\infty(x, y)h_\infty(x, y) \leq C \\ A_c(x, y), & \text{otherwise} \end{cases} \quad (2.28)$$

2.6.3 Comparison between the Chang and Schonfeld (1991a) and Clarens and Hurdle (1975) models

Both mathematical models in Chang and Schonfeld (1991a) and Clarens and Hurdle (1975) were developed to obtain optimal area of service zones using a zonal strategy. The similarities and differences between the two models are summarized in Table 2.1.

The average passenger waiting time was estimated as half of the bus headway in both models. Additionally, the objective function in the two models was expressed as a function of average cost per passenger. The demand was assumed to be known and not affected by the level of service.

Chang and Schonfeld (1991a) developed a model for subscription bus systems operating on a rectangular route network with assumed uniformly distributed demand, while Clarens and Hurdle (1975) developed their model for regular commuter bus service operating on a non-specified route network with heterogeneous demand. The subscription system in the Chang and Schonfeld (1991a) model was designed to provide many-to-one service between a rectangular service area and terminal that was located outside the service area, while the regular commuter bus system in the Clarens and Hurdle (1975) model was designed to provide one-to-many service from the

terminal to a service area with no specified location or shape. Local travel time within the service zones in Chang and Schonfeld's model was estimated based on the results in Daganzo (1984a), while in Clarens and Hurdle's model it was estimated based on an actual bus route network.

Table 2.1 Comparison between Chang and Schonfeld (1991a) and Clarens and Hurdle (1975)

	Chang and Schonfeld (1991a)	Clarens and Hurdle (1975)
Average		
passenger	Half of bus headway	Half of bus headway
waiting time		
Objective	Expression of average cost per	Expression of average cost per
function	passenger	passenger
Demand pattern	Not affected by service level	Not affected by service level
Service type	Subscription bus service	Regular commuter bus service
Service area	Rectangular	Not specified
Demand density	Uniformly distributed	Varied from locations
Terminal location	Outside service area	Not specified
Travel pattern	Many-to-one	One-to-many
Optimal variables	Bus capacity & service zone area	Bus headway & service zone area
Routing network	Rectangular	Not specified
Local travel time	Adopted Daganzo (1984a)'s method	Acquired from real route network
Vehicle capacity	Optimal variable	Fixed

2.7 Discussion

Shape of service area plays an important role in transit system design and system optimization. Most optimization models have simplified the service shape, and use ideal service shapes (such as rectangular, square or radial) or have solved the problem of the irregularly shaped service areas by transforming the service area into regular service zones for simplification. Shape approximation can significantly reduce the computation effort and simplify the model. However, it causes model inaccuracy, and might make the model impractical for a real service area with an irregular shape, which is the case in the majority of suburban areas that rely on DRT. Based on our knowledge, no zonal transit optimization study for DRT has been conducted on an irregular service area without shape simplification.

A number of mathematical cost models have been successfully developed to optimize regular public transit systems by implementing a zonal strategy. These models determine the optimal area of zones leading to a minimal total system cost or average cost per passenger. The optimal area of service zone has been reported to be dependent on its line-haul travel time (travel time between service zone and terminal) when the vehicles were not fully filled (Clarens & Hurdle 1975, Banks et al. 1982, Burns et al. 1985). However, when the vehicles were filled to capacity, the optimal area of service zones was independent on line-haul route travel time (Clarens & Hurdle 1975, Burns et al. 1985). So far, models studying the impact of line-haul route travel time on optimal zone area have been limited to traditional bus transit systems. No study has reported the impact of line-haul travel time on the optimal area of service zone for a DRT system.

Chapter Three: Optimization of Demand-Responsive Transit System: Approach One

3.1 Model development

The DRT system studied in this research provided transit services between an irregularly shaped area and a terminal through a many-to-one travel pattern during peak periods. The terminal was located outside the service area. The service area was divided into smaller service zones served separately by its bus route. Those service zones had identical areas and demand density, served by buses with the same capacity and headway. An optimization model was developed to optimize the service zone area, bus capacity and bus entry point for the DRT system following a similar method to the one proposed by Chang and Schonfeld (1991a). However, while the model developed in this thesis considers irregularly shaped areas with optimizable bus entry point, the Chang and Schonfeld (1991a) analytical model for a subscription feeder bus system considered an ideal rectangular shaped service area with one fixed bus entry point closest to the terminal. Through minimizing the average cost per passenger, the model can achieve optimal decision variables, such as area of service zones and bus capacity. The proposed optimization model in this chapter was developed for a DRT system operating in an irregularly shaped service area.

3.1.1 Operation policy

Within each service zone, DRT systems provide door-to-terminal service for passengers by picking up them at their desired location and dropping off them at the terminal. Buses began and ended trips at the terminal. Buses were operated according to the following steps;

1. Passengers contacted the travel agency requesting service in advance by email, cell phone, or online booking system, providing information such as desired pick-up location and time, desired drop-off location and time, accepted time deviation between actual pick-up time and desired pick-up time, etc.
2. The travel agency organized the request information to generate a bus schedule and route. Passengers were not accepted after the buses were dispatched, and could be scheduled for the next trip.
3. Empty buses were dispatched from the terminal to its designated service zone to collect passengers, and then fully filled buses deliver the passengers to the terminal. Buses are designed to travel on a Manhattan (grid) or Euclidean (regular) route network. Travelling on the routes between the terminal and service zones, buses do not stop for passengers alighting or boarding.
4. Buses reached the terminal to allow all of the passengers to disembark.
5. Buses remain at the terminal for a layover, and then leave for the next trip.

To operate the buses efficiently throughout the service area, the determination of optimal area of service zone, bus capacity and headway should be discussed.

3.1.2 System description

Service area, bus entry point, terminal and bus routes for the proposed DRT system are shown in Figure 3.1. A service area was equally partitioned into N smaller service zones with an area of A . All zones shared the same bus terminal located at (x_T, y_T) and the same bus entry point (x_E, y_E) on the service area boundary. $[(x_E, y_E)]$ is the set of all the possible bus entry points. The

terminal can be any major trip generators such as railway stations, major business or shopping centres. Buses from the terminal travelled a line-haul distance of D_H toward the bus entry point at a line-haul speed of V_H , and then travelled a non-stop local distance d to the assigned service zone at a non-stop local speed of V_{NSL} . Within the service zone, the buses travelled a local collection distance of D_C at a local collection speed of V_C to pick up passengers at their desired locations. Afterwards, the buses returned to the bus entry point at a speed of V_{NSL} , and then headed to the terminal at a speed of V_H .

Figure 3.1 Bus routes on a demand-responsive transit system

3.1.3 Assumptions for model development

The following assumptions were made for model development.

- Demand density over the service area was insensitive to the level of service, and was uniformly distributed with p passenger per square kilometre per hour.

- Only one passenger waited at one pick-up location within a service area.
- All buses entered the service area via the same bus entry point, indicating the line-haul route distance was the same for all service zones. A method including multiple bus entry points will be explained in the case study.
- Buses were dispatched with uniform headway and the same capacity.
- Buses travelled on grid or regular street network respectively, thus the travel distances were measured using Manhattan or Euclidean distance measurements.
- Hourly bus operation time cost, hourly passenger waiting and riding time cost were the same.
- Buses were fully filled after one local collection tour (Chang and Schonfeld 1991a).
- External costs, such as bus delay cost, were not considered here.

3.1.4 Model development procedure

A stepwise model development is shown in Figure 3.2 and described below,

- 1) Identify all possible bus entry points on the service area boundary. Usually, all intersections located on the service area boundary were possible bus entry points.
- 2) Develop an average cost per passenger function, which includes bus capacity, area of service zone and bus entry point as decision variables. The average cost per passenger function was defined as the objective function here.
- 3) Optimize the area of service zone and bus capacity by minimizing the average cost per passenger.

- 4) Identify the optimal bus entry point leading to the minimal average cost per passenger.
- 5) Identify the optimal area of service zone and bus capacity corresponding to the optimal bus entry point.
- 6) Divide the whole service area into smaller zones based on the route network, land use and community boundaries.

Figure 3.2 Flow chart of model development

3.1.5 Development of objective function

The objective function is a function of average cost per passenger (C). The development of objective function starts with the formulation of a total cost equation. Then the average cost per passenger would be the total cost (C_{Total}) divided by the number of passengers during one collection tour (n). The total cost (C_{Total}) shown in Equation (3.1) encompasses cost occurring on a round trip line-haul route (C_H), on a round trip non-stop local route (C_{NSL}), on a local collection tour (C_C) and during a layover (C_L).

$$C_{Total} = C_H + C_{NSL} + C_C + C_L \quad (3.1)$$

3.1.5.1 Cost on the line-haul route

The cost on the line-haul route (C_H) is the sum of round trip operation cost (C_{HO}) and passenger riding cost (C_{HR}) of the return trip on the line-haul route.

$$C_H = C_{HO} + C_{HR} \quad (3.2)$$

The operational cost of the round trip on the line-haul route (C_{HO}) is obtained by

$$C_{HO} = t_H \gamma_O \quad (3.3)$$

where t_H is round trip travel time on the line-haul route; γ_O is operating time cost per bus per hour.

$$t_H = 2D_H/V_H \quad (3.4)$$

where D_H is the line-haul route distance; V_H is the bus line-haul speed.

$$\gamma_O = a + bc \quad (3.5)$$

where a is the fixed coefficient of hourly bus operation time cost; b is the variable coefficient of hourly bus operation time cost; c is the bus capacity.

Substitute Equations (3.4) and (3.5) into Equation (3.3), the operation cost of round trip on line-haul route (C_{HO}) becomes

$$C_{HO} = 2D_H(a + bc)/V_H \quad (3.6)$$

The riding time cost of a return trip on a line-haul route (C_{HR}) is obtained by

$$C_{HR} = n * \gamma_R t_H / 2 \quad (3.7)$$

where γ_R is the riding time cost per passenger per hour; n is the number of passengers during one collection tour.

Since buses are assumed to be fully filled after one collection tour, the number of passengers (n) during one collection tour is equal to the bus capacity (c). Then n can be estimated by Equation (3.8),

$$n = c = pAh \quad (3.8)$$

where p is the demand density; A is the area of service zone; h is the bus headway.

Bus headway h is obtained by

$$h = c/pA \quad (3.9)$$

Thus, the cost on the line-haul route (C_H) becomes

$$C_H = 2D_H(a + bc)/V_H + pAh * \gamma_R D_H / V_H \quad (3.10)$$

3.1.5.2 Cost on the non-stop local route

The cost on the non-stop local route (C_{NSL}) is the sum of round trip operation cost (C_{NSLO}) and passenger riding cost of the return trip (C_{NSLR}) on the non-stop local route.

$$C_{NSL} = C_{NSLO} + C_{NSLR} \quad (3.11)$$

The operation cost of a round trip on the non-stop local route (C_{NSLO}) is obtained by

$$C_{NSLO} = t_{NSL}\gamma_o \quad (3.12)$$

where t_{NSL} is the round trip travel time on the non-stop local route.

$$t_{NSL} = 2D_{NSL}/V_{NSL} \quad (3.13)$$

where D_{NSL} is the non-stop local distance; V_{NSL} is the bus non-stop local speed.

The operation cost of the round trip on the non-stop local route (C_{NSLO}) becomes

$$C_{NSLO} = 2D_{NSL}(a + bc)/V_{NSL} \quad (3.14)$$

The passenger riding cost (C_{NSLR}) on the return trip of the non-stop local route is obtained by:

$$C_{NSLR} = n * \gamma_R t_{NSL}/2 = pAh * \gamma_R D_{NSL}/V_{NSL} \quad (3.15)$$

The cost on the non-stop local route (C_{NSL}) becomes

$$C_{NSL} = 2D_{NSL}(a + bc)/V_{NSL} + pAh * \gamma_R D_{NSL}/V_{NSL} \quad (3.16)$$

3.1.5.3 Cost on the collection tour

The cost on collection tour (C_C) is the sum of the operation cost (C_{CO}), passenger riding cost (C_{CR}) and waiting cost (C_{CW}) on the collection tour,

$$C_C = C_{CO} + C_{CR} + C_{CW} \quad (3.17)$$

The operation cost on collection tour (C_{CO}) is obtained by

$$C_{CO} = t_c \gamma_o \quad (3.18)$$

where t_c is the round trip time on a local collection tour.

$$t_c = D_C / V_C \quad (3.19)$$

where D_C is the local collection distance; V_C is the bus local collection local speed.

The operation cost on collection tour (C_{CO}) becomes

$$C_{CO} = D_C(a + bc)/V_C \quad (3.20)$$

Passenger riding cost on collection tour (C_{CR}) is obtained by

$$C_{CR} = n * \gamma_R t_c / 2 = pAh * \gamma_R D_C / 2V_C \quad (3.21)$$

Passenger waiting cost on collection tour (C_{CW}) is obtained by

$$C_{CW} = n * h\gamma_W / 2 \quad (3.22)$$

where γ_W is the passenger waiting time cost.

The cost per passenger on collection tour (C_C) becomes

$$C_C = D_C(a + bc)/V_C + pAh * \gamma_R D_C / 2V_C + pAh * h\gamma_W / 2 \quad (3.23)$$

3.1.5.4 Cost during the layover

The average cost per passenger during the layover (C_L) is the operation cost during layover time

t_L .

$$C_L = \gamma_o t_L = (a + bc)t_L \quad (3.24)$$

where t_L is the bus layover time.

3.1.5.5 Total cost

Substituting Equations (3.10, 3.16, 3.23 and 3.24) into Equation (3.1), the total cost (C_{Total}) becomes

$$C_{Total} = \frac{2D_H(a + bc)}{V_H} + \frac{pAh * \gamma_R D_H}{V_H} + \frac{2D_{NSL}(a + bc)}{V_{NSL}} + \frac{pAh * \gamma_R D_{NSL}}{V_{NSL}} + \frac{D_C(a + bc)}{V_C} + \frac{pAh * \gamma_R D_C}{2V_C} + \frac{pAh * h\gamma_W}{2} + (a + bc)t_L \quad (3.25)$$

3.1.5.6 Average travel distances

Average travel distances on the three path sections (line-haul route, non-stop local route and local collection tour) in Equations (3.25) can be calculated by Manhattan or Euclidean distance measurement respectively. Manhattan distance measurement is applied when buses operate on a grid street network, and Euclidean distance measurement is applied when buses operate on a regular street network.

3.1.5.6.1 Manhattan measurement

The line-haul route distance (D_H^M) is the distance between the terminal (x_T, y_T) and bus entry point (x_E, y_E) under Manhattan distance measurement.

$$D_H^M = |x_T - x_E| + |y_T - y_E| \quad (3.26)$$

where x_T is the horizontal coordinate of the terminal; y_T is the vertical coordinate of the terminal; x_E is the set of horizontal coordinates of bus entry points; y_E is the set of vertical coordinates of bus entry points.

The average non-stop local distance is the mean Manhattan distance (d_i^M) between the bus entry point (x_E, y_E) and the i -th ($1 \leq i \leq N$) service zone centroid (x_i, y_i) under Manhattan distance measurement (D_{NSL}^M).

$$D_{NSL}^M = \frac{\sum_{i=1}^N d_i^M}{N} \quad (3.27)$$

$$d_i^M = |x_i - x_E| + |y_i - y_E| \quad (3.28)$$

where N is the number of service zone; x_i is the horizontal coordinate of the i -th service zone centroid; y_i is the vertical coordinate of the i -th service zone centroid.

Then,

$$D_{NSL}^M = \frac{\sum_{i=1}^N |x_i - x_E| + |y_i - y_E|}{A_\sigma / A} \quad (3.29)$$

where A_σ is the area of the service area; A is the area of the service zone.

When N is infinite,

$$D_{NSL}^M = \frac{\iint_{(x,y) \in \sigma} (|x - x_E| + |y - y_E|) dx dy}{A_\sigma} \quad (3.30)$$

where $(x, y) \in \sigma$, σ is the service area.

The expected collection tour distance (D_C^M) under Manhattan distance measurement can be approximated by Equation (3.31) (Daganzo 1984a), Equation (3.31) was introduced in detail in Appendix A.

$$D_C^M = 1.15 \sqrt{n_p A} \quad (3.31)$$

where A is the area of service zone, n_p is the number of pickup stops in one collection tour.

Assume there is only one passenger waiting at one pickup point,

$$n_p = n \quad (3.32)$$

Substitute Equation (3.8 & 3.32) into Equation (3.31), then the local collection tour distance

(D_C^M) becomes

$$D_C^M = 1.15\sqrt{cA} \quad (3.33)$$

3.1.5.6.2 Euclidean measurement

The line-haul route distance under Euclidean distance measurement (D_H^E) is

$$D_H^E = \sqrt{(x_T - x_E)^2 + (y_T - y_E)^2} \quad (3.34)$$

The average non-stop local distance under Euclidean distance measurement (D_{NSL}^E) is

$$D_{NSL}^E = \frac{\sum_{i=1}^{i=N} d_i^E}{N} = \frac{\sum_{i=1}^{i=N} (\sqrt{(x_i - x_E)^2 + (y_i - y_E)^2})}{A_\sigma / A} \quad (3.35)$$

where d_i^E is the Euclidean distance between the bus entry point (x_E, y_E) and the i -th service zone centroid (x_i, y_i) .

$$d_i^E = \sqrt{(x_i - x_E)^2 + (y_i - y_E)^2} \quad (3.36)$$

When N is infinite,

$$D_{NSL}^E = \frac{\iint_{(x,y) \in \sigma} \sqrt{(x_i - x_E)^2 + (y_i - y_E)^2} dx dy}{A_\sigma} \quad (3.37)$$

where $(x, y) \in \sigma$, σ is the service area.

The expected collection tour distance under Euclidean distance measurement D_C^E can be approximated by Equation (3.38) (Daganzo 1984a), which was introduced in detail in Appendix A.

$$D_C^E = 0.9\sqrt{nA} \quad (3.38)$$

Substitute Equation (3.8 & 3.32) into Equation (3.38), the collection tour distance D_C^E becomes

$$D_C^E = 0.9\sqrt{cA} \quad (3.39)$$

3.1.5.7 Average cost per passenger

The average cost per passenger (C) is the total cost (C_{Total}) divided by the number of passengers (n) during one collection tour.

$$C = C_{Total}/n = C_{Total}/c \quad (3.40)$$

3.1.5.7.1 Average cost per passenger using Manhattan measurement

Combine Equations (3.9, 3.25, 3.33 and 3.40), the average cost per passenger (C^M) with Manhattan method becomes

$$\begin{aligned} C^M = & \left(\frac{2D_H^M}{V_H} + \frac{2D_{NSL}^M}{V_{NSL}} + t_L \right) \frac{a + bc}{c} + \gamma_R \left(\frac{D_H^M}{V_H} + \frac{D_{NSL}^M}{V_{NSL}} \right) + \frac{1.15\sqrt{A/c}(a + bc)}{V_C} \\ & + \frac{1.15\gamma_R\sqrt{cA}}{2V_C} + \frac{\gamma_W c}{2Ap} \end{aligned} \quad (3.41)$$

3.1.5.7.2 Average cost per passenger using Euclidean measurement

Combine Equations (3.9, 3.25, 3.39 and 3.40), the average cost per passenger (C^E) with Euclidean method becomes

$$C^E = \left(\frac{2D_H^E}{V_H} + \frac{2D_{NSL}^E}{V_{NSL}} + t_L \right) \frac{a + bc}{c} + \frac{\gamma_R}{c} \left(\frac{2D_H^E}{V_H} + \frac{2D_{NSL}^E}{V_{NSL}} \right) + \frac{0.9\sqrt{A/c} (a + bc)}{V_C} + \frac{0.9\gamma_R\sqrt{cA}}{2V_C} + \frac{\gamma_W c}{2Ap} \quad (3.42)$$

3.1.6 Decision variables

The decision variables in this model include the area of service zone and bus capacity. They can be optimized by minimizing the objective functions [Equations (3.41 & 3.42)] using partial derivatives.

3.1.6.1 Manhattan measurement

Set partial derivatives of C^M in Equation (3.41) with respect to variables (A and c) to 0.

$$\frac{\partial C^M}{\partial A} = \frac{1.15}{2V_C} (a + bc) \sqrt{\frac{1}{Ac}} + \frac{1.15\gamma_R}{4V_C} \sqrt{\frac{c}{A}} - \frac{\gamma_W}{2pA^2} = 0, \quad (3.43)$$

$$\frac{\partial C^M}{\partial c} = \frac{-a}{c^2} \left(\frac{2D_H^M}{V_H} + \frac{2D_{NSL}^M}{V_{NSL}} + t_L \right) - \frac{1.15a}{2V_C} \sqrt{\frac{A}{c^3}} + \frac{1.15b}{2V_C} \sqrt{\frac{A}{c}} + \frac{1.15\gamma_R}{4V_C} \sqrt{\frac{A}{c}} + \frac{\gamma_W}{2pA} = 0. \quad (3.44)$$

From Equation (3.43), the optimal service zone area A^{M*} is

$$A^{M*} = c \left[\frac{\gamma_W V_C}{1.15p(a + bc + c\gamma_R/2)} \right]^{\frac{2}{3}} \quad (3.45)$$

The optimal service zone number N^{M*} is

$$N^{M*} = A_0 / A^{M*} \quad (3.46)$$

Substitute Equation (3.45) to Equation (3.44)

$$c^{M^*} = \left[\frac{pa^3 V_C^2}{1.15^2 \gamma_W (b + \gamma_R/2)^2} \left(\frac{2D_H^M}{V_H} + \frac{2D_{NSL}^M}{V_{NSL}} + t_L \right)^3 * \left(\frac{a}{c^{M^*} \left(b + \frac{\gamma_R}{2} \right)} + 1 \right) \right]^{\frac{1}{5}} \quad (3.47)$$

The optimal bus capacity (c^{M^*}) can be obtained from Equation (3.47).

Then average cost per passenger C^{M^*} can be obtained by substituting A^{M^*} and c^{M^*} into Equation (3.41).

Then minimum average cost per passenger $C_{min}^{M^*}$ can be obtained by

$$C_{min}^{M^*} = \min(C^{M^*}) \quad (3.48)$$

Then, optimal bus entry point ($x_E^{M^*}, y_E^{M^*}$), optimal area of service zone A^{M^*} , bus capacity c^{M^*} , and optimal headway h^{M^*} corresponding to the minimum average cost per passenger ($C_{min}^{M^*}$) can easily be obtained.

3.1.6.2 Euclidean method

Set partial derivatives of C^E in Equation (3.42) with respect to variables (A and c) to 0.

$$\frac{\partial C^E}{\partial A} = \frac{0.9}{2V_C} (a + bc) \sqrt{\frac{1}{Ac}} + \frac{0.9\gamma_R}{4V_C} \sqrt{\frac{c}{A}} - \frac{\gamma_W}{2pA^2} = 0, \quad (3.49)$$

$$\frac{\partial C^E}{\partial c} = \frac{-a}{c^2} \left(\frac{2D_H^E}{V_H} + \frac{2D_{NSL}^E}{V_{NSL}} + t_L \right) - \frac{0.9a}{2V_C} \sqrt{\frac{A}{c^3}} + \frac{0.9b}{2V_C} \sqrt{\frac{A}{c}} + \frac{0.9\gamma_R}{4V_C} \sqrt{\frac{A}{c}} + \frac{\gamma_W}{2pA} = 0. \quad (3.50)$$

From Equation (3.49), the optimal area of service zone A^{E^*} is

$$A^{E^*} = c \left[\frac{\gamma_W V_C}{0.9p(a + b + c\gamma_R/2)} \right]^{\frac{2}{3}} \quad (3.51)$$

The optimal service zone number N^{E^*} is

$$N^{E^*} = A_0 / A^{E^*} \quad (3.52)$$

Substitute Equation (3.51) to Equation (3.50)

$$c^{E*} = \left[\frac{pa^3V_C^2}{0.9^2\gamma_W(b + \gamma_R/2)^2} \left(\frac{2D_H^E}{V_H} + \frac{2D_{NSL}^E}{V_{NSL}} + t_L \right)^3 * \left(\frac{a}{c^{E*}(b + \gamma_R/2)} + 1 \right) \right]^{\frac{1}{5}} \quad (3.53)$$

The optimal bus capacity (c^{E*}) can be obtained from Equation (3.53).

The average cost per passenger C^{E*} can be obtained by substituting A^{E*} and c^{E*} into Equation (3.42).

The minimum average cost per passenger C_{min}^{E*} can be obtained by

$$C_{min}^{E*} = \min(C^{E*}) \quad (3.54)$$

Thus, optimal bus entry point (x_E^{E*}, y_E^{E*}), optimal area of service zone A^{E*} , bus capacity c^{E*} , and optimal headway h^{E*} corresponding to the minimum average cost per passenger (C_{min}^{E*}) can easily be obtained.

3.2 Case Study

A region in northwest Calgary was selected as a service area. The service area is bounded by Stoney Trail, Deerfoot Trail, the Bow River, Sarcee Trail S.W., and the Trans-Canada Highway as shown in Figure 3.3. Nose Hill Park was excluded from the service area, as there are no residences there. Eighteen main intersections on the boundary of the service area were assigned as possible bus entry points. The location of each bus entry point was labelled with a number and listed in Table 3.1. The Calgary Tower, a centre of the CBD, was selected as the terminal. A hypothetical DRT system serves passengers between the service area and the Calgary Tower during the morning peak period (6 a.m. to 8:59 a.m.). The service area was assumed to have the

same demographic and geographic characteristics, such as demand density, land use value, bus operation and passenger hourly cost.

Figure 3.3 Service area, possible bus entry points and terminal (modified from Bing map)

Table 3.1 Locations of all possible bus entry points

Bus entry point No.	Location of bus entry point
1	Intersection of Crowchild Trail N.W. and Stoney Trail
2	Intersection of Harvest hill Blvd. N.W. and Stoney Trail
3	Intersection of Country hills Blvd. N.E. and Deerfoot Trail
4	Intersection of 96 Ave N.E. and Deerfoot Trail
5	Intersection of Beddington Trail N.W. and Deerfoot Trail
6	Intersection of 64 Ave N.E. and Deerfoot Trail
7	Intersection of McKnight Blvd. N.E. and Deerfoot Trail
8	Intersection of 32 Ave NE and Deerfoot Trail
9	Intersection of Trans-Canada Hwy and Deerfoot Trail
10	Intersection of Edmonton Trail and Memorial Dr. N.E.
11	Intersection of Centre St N. and Memorial Dr. N.W.
12	Intersection of 10 St. S.W. and Memorial Dr. N.W.
13	Intersection of 14 St S.W. and Memorial Dr. N.W.
14	Intersection of Crowchild Trail N.W. and Memorial Dr. N.W.
15	Intersection of Shaganappi Trail N.W. and Bowness Rd. N.W.
16	Intersection of 16 Ave N.W. and Bowness Rd. N.W.
17	Intersection of Sarcee Trail S.W. and Trans-Canada Hwy
18	Intersection of Nose Hill Dr. N.W. and Stoney Trail

3.2.1 Input data

In this case study, the value of input parameters was obtained from different sources, as summarized in Table 3.2 and explained below.

Table 3.2 Values of input parameters

Symbol	Definition	Value	Unit
a	Fixed coefficient of bus operation time cost	88	\$/veh/hour
A_{σ}	Service area	139.76	km ²
b	Variable coefficient of bus operation time cost	0.92	\$/seat/hour
p	Demand density	4.82	passenger/km ² /hour
t_L	Bus layover time	0.1	hour
V_C	Local collection speed	22.4	km/hour
V_H^*	line-haul speeds	12 ~71.8	km/hour
V_{NSL}	Non-stop local speed	33.6	km/hour
γ_R	Hourly passenger riding time cost	5.03	\$/passenger/hour
γ_W	Hourly passenger waiting time cost	11.47	\$/passenger/hour

* The line-haul route speeds for different bus entry points were different.

The coefficients of hourly bus operation time cost were estimated based on the data obtained from Calgary Transit; demand density was estimated based on Khattak and Yim's (2004) study; the local bus collection speed was obtained from Calgary Transit; the bus line-haul speeds were extracted from Google maps; the non-stop local speeds were estimated to be 1.5 times local collection speed (Chang & Schonfeld 1991a); the hourly passenger riding and waiting time cost

were estimated based on Hossain's (2011) investigation; the size of service area, coordinates of interior points, possible bus entry points and terminal were extracted from maps in Quantum Geographical Information System (QGIS). The estimation details for these input parameters are described as follows.

3.2.1.1 Coefficient of hourly bus operation time cost

The hourly bus operation time cost data with different bus capacities were obtained from Calgary Transit (Table 3.3). The relationship between the bus hourly operation cost (γ_O) and bus capacity (c) was $\gamma_O = a + bc$. The fixed coefficient of bus operation time cost a and variable coefficient b were estimated to be 88 and 0.92 based on linear regression. The R squared value of this regression was 0.9998. It is noted there were only three points for this linear regression process due to limited available data.

Table 3.3 Bus Operation Data (Source: Calgary Transit)

Bus type	Capacity		Bus operating cost (\$/hour)	Average travel speed (km/hour)
	Seated	Standing		
Community Shuttle	20	10	116	22.4
Regular	40	25	147	22
Articulated	60	65	203	22

3.2.1.2 Demand density

The DRT demand density for the morning peak period was used for the model application. It was estimated based on current Calgary bus services data and the following assumptions.

- The number of passengers travelling to the terminal using the current bus service was equal to the number of passengers disembarking from all bus stops in the influential area. The influential area is defined as a circular area with its centre located at the Calgary Tower and a radius of 450 m. There are 17 bus stops in the influential area, whose transit information is shown in Table 3.4.
- The percentage of citizens who are likely to use DRT services is equal to 12%, which was adapted from a survey in San Francisco (Khattak & Yim 2004).
- Each person in the service area generated one travel trip during the morning peak period (6 a.m. ~9 a.m.). Therefore, the total number of travel trips generated from the service area was equal to the population of the service area, which was 338,484 according to 2015 Calgary census results.

Based on the above assumptions, the number of travel trips preferring DRT (p_{DRT}) during the morning peak period in the service area is

$$p_{DRT} = 12\% * P_{\sigma} \quad (3.55)$$

where P_{σ} is the population in the service area.

Table 3.4 Number of passengers disembarking at bus stops in the influential area (Source:
Calgary Transit)

Stop ID	Stopway	Number of passengers getting off during morning peak period
N6207	SB Centre St. S. @ 5 Ave. S.	1456.8
N6539	WB 6 Ave. S.W. @ E. of 1 St. S.W.	0
2234	WB 6 Ave. S.E. @ 1 St. S.E.	130.1
8343	EB 7 Ave. S. @ Centre St. S.	0
N6516	SB 1 St. S.E. @ 7 Ave. S.E.	92.1
N6512	EB 7 Ave. S.W.@ 1 St. S.W.	102.7
N6528	NB 1 St. S.W.@ 8 Ave. S.W.	62
N1200	SB 1 St. S.E. @ 8 Ave. S.E.	7.2
N6525	SB 1 St. S.W. @ 8 Ave. S.W.	25.4
N6528	NB 1 St. S.W.@ 8 Ave. S.W.	62
7380	EB 9 Ave. S. @ Centre St. S.	0.4
N6202	EB 9 Ave. S.E. @ Macleod Tr.	16.8
5284	SB 1 St. S.E.@ 10 Ave. S.E.	26.4
7840	SB 1 St. S.W. @ 10 Ave. S.W.	7.2
6509	EB 9 Ave. S.W. @ 2 St. S.W.	71.8
6669	SB Centre St. S. @ 4 Ave. S.	391.6
N6503	WB 6 Ave. S.W. @ 1 St. S.W.	128.6

Therefore, the potential DRT demand density (p) destined for the Calgary Tower from the service area can be calculated using Equation (3.56).

$$p = \frac{P_{DRT} * \varphi}{A_{\sigma} * T_p} \quad (3.56)$$

where φ is the proportion of trips from the service area destined to the Calgary Tower over the total number of trips, and T_p is the peak period duration.

The size of service area is 139.76 km^2 , based on map information obtained from City of Calgary. The morning peak period is from 6 a.m. to 8:59 a.m., therefore the morning peak period duration is 3 hours. The proportion of trips destined for the Calgary Tower over the total trips was estimated based on current Calgary bus service data (seen in Table 3.4). During the morning peak period, the average total number of passengers disembarking at all bus stops in the City of Calgary was calculated to be 51,790.2; while the average total number of passenger disembarking at the bus stops in the influential area was 2,581.1. Therefore, φ was calculated to be 4.98%.

Based on Equations (3.55 & 3.56), the potential DRT demand density (p) destined for the Calgary Tower from the service area was calculated to be 4.82 passengers/ km^2 /hour.

3.2.1.3 Bus speeds

The average travel speed of a community shuttle in Calgary was used as the local collection speed, which was 22.4 km/hour. The line-haul speeds between the Calgary Tower and each possible bus entry point during the morning peak on weekdays were extracted from Google maps. Certainly, line-haul speeds were different for different entry points, ranging from 12~71.8

km/hour. The non-stop local speed was estimated to be 1.5 times of local collection speed (Chang & Schonfeld 1991), which was 33.6 km/hour.

3.2.1.4 Hourly passenger riding and waiting time cost

Hossain (2011) estimated that hourly passenger riding time cost in Calgary was \$4.73 per passenger per hour in 2011 based on investigation data. According to the Bank of Canada, the average annual inflation rate was 1.24% from 2011 to 2016; the riding time cost in 2016 was then estimated to be \$5.03 per passenger per hour after taking inflation into consideration. Hossain (2011) also found that the average cost of out-of-vehicle time was 2.28 of the average cost of in-vehicle time. Thus, the passenger waiting time cost could be estimated at \$11.47 per passenger per hour, taking the inflation rate into consideration.

3.2.2 Impact of bus entry point location on travel distances

Line-haul route distance and non-stop local route distance corresponding to each bus entry point were calculated based on Manhattan and Euclidean distance measurements respectively, as shown in Figure 3.4.

Figure 3.4 Impact of bus entry point location on distances under (a) the Manhattan method (b) Euclidean method

As shown in Figure 3.4, line-haul route distances and non-stop local route distance varied as the bus entry point locations. Using the Manhattan method, the maximum line-haul route distance (D_H) and average non-stop local distance (D_{NSL}) were 21.17 km and 11.99 km, while the minimum D_H and D_{NSL} were 1.17 km and 8.29 km. Using the Euclidean method, the maximum D_H and D_{NSL} were 15.02 km and 10.32 km, while the minimum D_H and D_{NSL} were 1.13 km and 6.71 km. The bus entry points leading to the extreme values of D_H and D_{NSL} were the same for both the Manhattan method and the Euclidean method, labelled in Figure 3.5.

Figure 3.5 Extreme points on distances on QGIS map

The entry point leading to the minimum D_H was located at the intersection of Centre St. N. and Memorial Dr. N.E.; the entry point leading to the minimum D_{NSL} was located at the intersection of 64 Ave. N.E. and Deerfoot Trail; the entry point leading to the maximum D_H was located at the intersection of Crowchild Trail N.W. and Stoney Trail; and the entry point leading to the maximum D_{NSL} was located at the intersection of Edmonton Trail and Memorial Dr. N.E.

3.2.3 Optimization of bus entry point location

To investigate the impact of bus entry point location on the bus transit system optimization, the costs corresponding to each bus entry point were calculated based on Manhattan and Euclidean measurements respectively, shown in Figure 3.6. The average costs per passenger on line-haul route, non-stop local route, local collection tour, and during a layover, as well as total cost, varied with the bus entry point locations for both distance measurements. The average cost per passenger on the line-haul route and non-stop local route were more sensitive to the entry point locations, compared to the average cost per passenger on a local collection tour and during a layover. Using the Manhattan method, the average cost per passenger varied from \$14.52 to \$19.43. Using the Euclidean method, the average cost per passenger varied from \$11.99 to \$15.56. Using the Manhattan method meant a higher cost, as the distances measured by the Manhattan method are generally larger than the distances measured using the Euclidean method. Although the Euclidean method gave a more economical solution, it was not applicable in most cases, as the street network was not made up of a Euclidean grid.

Figure 3.6 Impact of bus entry point location on costs under (a) Manhattan method, (b) Euclidean method.

The bus entry points leading to minimum/maximum average cost per passenger for the Manhattan method are labelled in Figure 3.7. The entry point leading to the minimum average cost per passenger was located at the intersection of 10 St. S.W. and Memorial Dr. N.W., while the entry point leading to the maximum average cost per passenger was located at the intersection of Crowchild Trail N.W. and Stoney Trail. The entry point leading to the minimum average cost per passenger was not the one leading to the minimum line-haul route distance. This can be explained by the following two reasons: (1) Bus entry points leading to the shortest line-haul route distance were located in the core of downtown Calgary, where the traffic is usually heavy during the morning peak; (2) The average cost per passenger was also affected by the non-

stop local route distance. As shown in Figure 3.4, the bus entry point leading to the minimum line-haul route distance did not generate a short non-stop local route distance.

Figure 3.7 Extreme Points on Costs on QGIS Map under Manhattan distance measurement

3.2.4 System optimization with optimal bus entry point

The major arterial roads in the service area are more like a grid street network, except for Crowchild Trail, Beddington Trail and John Laurie Blvd. The buses are more likely to travel on a

grid road street network within communities. Hence, in this section the Manhattan method was adopted. From section 3.3.3, the intersection of 10 St. S.W. and Memorial Dr. N.W. was determined to be the optimal bus entry point. The optimized variables corresponding to the optimal bus entry point can easily be obtained. Optimal values of variables were rounded to ensure that zone number and bus capacity were integers. The calculated and rounded values are summarized in Table 3.5. The rounded zone number was 13, and then the updated service zone area was 10.75 km^2 . The optimal bus capacity was chosen to be 27 seats per vehicle, as no passenger should be left waiting for the next bus. The optimal headways were updated to 0.52 hour based on rounding zone number and bus capacity. Although the optimal headway was greater than the headway of regular bus service, which in the morning peak period was generally approximately 5~15 minute, there was no access time for DRT services. With the updated parameters, the optimal average cost per passenger C^* was \$14.52.

Table 3.5 Values of optimal variables

Variables	Original data	Rounding data
$A^*(\text{km}^2)$	10.9	10.75
N^*	12.8	13
$c^*(\text{seats/veh})$	26.3	27
$h^*(\text{hr})$	0.51	0.52
$C^*(\$)$	14.52	14.52

The service area was then divided into 13 smaller equal zones referring to the community boundaries. Initial zone division started from one of the edge communities, Arbour Lake. The

area of Arbour Lake is 4.42 km^2 , far smaller than the optimal area of 10.75 km^2 . Because of this, Arbour Lake was combined with adjacent communities, Citadel and Hawkwood, as one single service zone. The actual area of the zone was 10.37 km^2 , which was close to the optimal service zone area. Therefore, the combination of the communities of Arbour Lake, Citadel and Hawkwood was determined to be a service zone, denoted as zone 1. The rest of the service zones were determined following a similar manner. The zone division map is shown in Figure 3.8.

Figure 3.8 Zone division with optimal bus entry point

The actual area of the service zones and the differences from the optimal area are shown in Table 3.6. The actual area of the service zones deviating from the optimal value by more than 10% is three out of 13 service zones.

Table 3.6 Deviation of actual area from optimal area of each service zone

	Actual area (km ²)	Difference from the optimal area
Zone 1	10.37	-3.7%
Zone 2	10.12	-6.2%
Zone 3	10.72	-0.3%
Zone 4	11.37	5.5%
Zone 5	12.71	15.4%
Zone 6	10.43	-3.1%
Zone 7	10.86	1.0%
Zone 8	10.76	0.1%
Zone 9	11.07	2.9%
Zone 10	9.62	-11.7%
Zone 11	10.32	-4.2%
Zone 12	9.72	-10.6%
Zone 13	11.7	8.1%

3.2.5 Multi bus entry points system

In reality, different buses more likely to enter the service area through different bus entry points instead of one bus entry point, especially for a large service area. This section shows whether a multiple entry point bus system can be more economical than a single entry point bus system. The multiple entry point scenario is established based on the results of zone division in section

3.3.4. The multiple entry points were assigned to each service zone based on the following two steps:

- 1) Calculate the average round trip time (t) between each zone and the Calgary Tower through each bus entry point. The average round trip time was calculated by Equation (3.26, 3.30, and 3.57).

$$t = 2D_H/V_H + 2D_{NSL}/V_{NSL} \quad (3.57)$$

- 2) The entry point leading the minimum average round trip time for each zone was assigned as its bus entry point.

The average cost per passenger for each service zone was calculated based on Equation (3.26, 3.30, 3.33 and 3.41) and is summarized in Table 3.7.

The average cost per passenger for the service area using a multiple entry point scenario was then calculated to be \$12.4, which was 14.6% less than the cost associated with one bus entry point system. This result indicates that a multiple entry points system is more cost-effective than a single entry point system.

Table 3.7 Round trip time and entry points of each zone

Zone No.	Entry point location	Average round trip time (min)	Average cost per passenger (\$)
1	5-th	53	15.2
2	5-th	43	14.1
3	5-th	23	11.8
4	2-th	26	12.8
5	4-th	27	12.2
6	6-th	20	11.5
7	9-th	16	11
8	11-th	12	10.6
9	13-th	13	10.7
10	12-th	26	12.1
11	12-th	27	12.2
12	15-th	31	12.7
13	18-th	48	14.7

3.2.6 Sensitivity analysis

In this section, sensitivity analysis was conducted for input parameters, including demand density, hourly passenger waiting time cost, bus local speed and fixed coefficient of hourly bus operation cost. The degree of sensitivity is defined as follows:

- Non-sensitive: percentage change of decision variable is $<5\%$ when the value of input parameter changes by 20%.
- Slightly Sensitive: percentage change of decision variable is $5 \sim 20\%$ when the value of input parameter is changed by 20%.
- Sensitive: percentage change of decision variable is $20 \sim 100\%$ when the value of input parameter changes by 20%.
- Very sensitive: percentage change of decision variable is $>100\%$ when the value of input parameter changes by 20%.

The sensitivity results for different input parameters are summarized in Table 3.8.

Table 3.8 Summary of sensitivity analysis results

Input parameters	Degree of sensitivity		
	Optimal bus capacity	Optimal area of service zone	Minimum average cost per passenger
Demand density	Non-sensitive	Slightly-Sensitive	Non-sensitive
Hourly passenger waiting time cost	Non-sensitive	Slightly-Sensitive	Non-sensitive
Bus local speed	Non-sensitive	Slightly-Sensitive	Slightly-Sensitive
Fixed coefficient of bus operation time cost	Slightly-Sensitive	Non-sensitive	Slightly-Sensitive

Optimal bus capacity was non-sensitive to changes in demand density, hourly passenger waiting time cost and bus local speed, but slightly sensitive to change in the fixed coefficient of bus

operation cost. Optimal area of the service zone was slightly sensitive to changes in demand density, passenger waiting time cost and bus local speed, but non-sensitive to change in fixed coefficient of bus operation cost. Minimum average cost per passenger was non-sensitive to changes in demand density and hourly passenger waiting time, but slightly sensitive to changes in bus local speed and fixed coefficient of bus operation cost. The details of sensitivity analysis results for each parameter are discussed below.

3.2.6.1 Demand density

Demand density in this case study was estimated to be 4.82 passengers per hour per square kilometre. The sensitivity analysis chose $\pm 5\%$, $\pm 10\%$, and $\pm 20\%$ of deviations in demand density to evaluate its impact on optimal bus capacity, optimal area of service zone and minimum average cost per passenger. The sensitivity analysis results are listed in Table 3.9. When demand density increased/decreased by 5%, the maximum changes in terms of optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 0.9%, 2.8% and 1.0% respectively. As the percentage change of demand density increased to 20%, the maximum changes on optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 4.0%, 12.9% and 4.6% respectively. The results indicate that both optimal bus capacity and minimum average cost per passenger are not sensitive to change in demand density, and the optimal area of a service zone is slightly sensitive.

Table 3.9 Percentage changes of optimal bus capacity, optimal service zone area and minimum average cost per passenger vs deviations in demand density

Deviations in demand density	Percentage change		
	Optimal bus capacity	Optimal area of service zone	Minimum average cost per passenger
-20%	-4.0%*	12.9%	4.6%
-10%	-1.9%	5.9%	2.1%
-5%	0.9%	2.8%	1.0%
5%	0.9%	-2.6%	-1.0%
10%	1.7%	-5.1%	-1.9%
20%	3.7%	-9.4%	-3.6%

*The negative sign indicates that the value is decreased.

3.2.6.2 Hourly passenger waiting time cost

The sensitivity analysis results of hourly passenger waiting time cost value are showed in Table 3.10. When hourly passenger waiting time cost increased/decreased by 5%, the maximum changes on optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 0.9%, 2.7% and 1.0% respectively. As the percentage changes of hourly passenger waiting time cost increased to 20%, the maximum changes in optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 4.1%, 11.5% and 4.4% respectively. The results indicate that both optimal bus capacity and minimum average cost

per passenger are not sensitive to change in hourly passenger waiting time, while the optimal area of service zone is slightly sensitive.

Table 3.10 Percentage changes of optimal bus capacity, optimal service zone area and minimum average cost per passenger vs deviations in hourly passenger waiting time cost

Deviations in hourly passenger waiting time cost	Percentage change		
	Optimal bus capacity	Optimal area of service zone	Minimum average cost per passenger
-20%	4.1%	-11.5%*	-4.4%
-10%	1.9%	-5.6%	-2.1%
-5%	0.9%	-2.7%	-1.0%
5%	-0.9%	2.7%	1.0%
10%	-1.7%	5.4%	1.9%
20%	-3.3%	10.4%	3.7%

*The negative sign indicates that the value is decreased.

3.2.6.3 Bus local speed

The sensitivity analysis results on local speed are shown in Table 3.11. When bus local speed increased/decreased by 5%, the maximum changes in optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 0.6%, 2.9% and 3.9% respectively. As the percentage change in bus local speed increased to 20%, the maximum changes in optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 2.9%,

12.3% and 18.9% respectively. The results indicate that both the optimal area of a service zone and minimum average cost per passenger are slightly sensitive to change in bus local speed, while optimal bus capacity is not sensitive.

Table 3.11 Percentage changes in optimal bus capacity, optimal area of service zone and minimum average cost per passenger vs deviations in bus local speed

Deviations in bus local speed	Percentage change		
	Optimal bus capacity	Optimal area of service zone	Minimum average cost per passenger
-20%	2.9%	-12.3%*	18.9%
-10%	1.3%	-5.9%	8.3%
-5%	0.6%	-2.9%	3.9%
5%	-0.6%	2.9%	-3.7%
10%	-1.1 %	5.7%	-7.0%
20%	-2.1%	11.4%	-13.1%

*The negative sign indicates that the value is decreased.

3.2.6.4 Fixed coefficient of bus operation time cost

Fixed coefficient of bus operation cost in this case study is estimated to be \$88 per vehicle per hour. The sensitivity analysis results on fixed coefficient of bus operation time cost are presented in Table 3.12. When the fixed coefficient of bus operation time cost increased/decreased by 5%, the maximum changes in optimal bus capacity, optimal area of service zone and minimum

average cost per passenger were 3.9%, 0.6% and 2.4%, respectively. As the percentage change of fixed coefficient of bus operation time cost increased to 20%, the maximum changes on optimal bus capacity, optimal area of service zone and minimum average cost per passenger were 13.1%, 2.0% and 8.2% respectively. The results indicate that both optimal bus capacity and minimum average cost per passenger are slightly sensitive to fixed coefficient of bus operation time cost, while the optimal area of service zone is not sensitive.

Table 3.12 Percentage changes of optimal bus capacity, optimal area of service zone and minimum average cost per passenger vs deviations in fixed coefficient of bus operation

Deviations in fixed coefficient of bus operation cost	Percentage change		
	Optimal bus capacity	Optimal area of service zone	Minimum average cost per passenger
-20%	-12.6%	-2.0%*	-8.1%
-10%	-5.8%	-0.9%	-3.7%
-5%	-2.5%	-0.4%	-1.7%
5%	3.9%	0.6%	2.4%
10%	7.0%	1.0%	4.4%
20%	13.1%	1.8%	8.2%

*The negative sign indicates that the value is decreased.

3.3 Conclusion

In this chapter, a mathematical model was developed for optimizing DRT systems serving an irregularly shaped area. The proposed model was designed for a many-to-one travel pattern, but is easily expanded to a one-to-many pattern. The model was established based on minimization of average cost per passenger, where area of service zone, bus capacity and entry point were decision variables. The distance in this model considered both Manhattan and Euclidean distance measurements respectively. A case study was conducted in northwest Calgary to demonstrate the applicability of the proposed model. The results of the case study are summarized below:

- The location of entry points impact the average cost per passenger by influencing the line-haul route distance and non-stop local route distance. The best entry point was determined to be the intersection of 10 St. S.W. and Memorial Dr. N.W.;
- With the best entry point, the optimal value of service zone area, bus capacity and headway are 10.75 km², 27 and 0.52 hour respectively. The average cost per passenger for optimal conditions was determined to be \$14.52.
- A multiple entry point scenario was proposed based on the one entry point scenario. The average cost per passenger for the multiple entry point was 14.6% less expensive than the cost of the one entry point scenario.
- Optimal bus capacity is non-sensitive to changes in demand density, hourly passenger waiting time cost and bus local speed, but slightly sensitive to change in fixed coefficient of bus operation time cost.

- Optimal area of service zone is slightly sensitive to changes in demand density, hourly passenger waiting time cost and bus local speed, but non-sensitive to changes in fixed coefficient of bus operation time cost.
- Minimum average cost per passenger is non-sensitive to changes in demand density and hourly passenger waiting time cost, while it is slightly sensitive to change in bus local speed and fixed coefficient of bus operation time cost.

Chapter Four: Optimization of Demand-Responsive Transit System: Approach Two

4.1 Model formulation

The model to be developed is for optimizing a DRT system operating over a large area, with a terminal located inside. The terminal can be a shopping mall, a rail or bus terminal, or other large trip generators. The large service area is divided into smaller zones. Each zone is served by one bus route. Buses provide passengers one-to-many service from terminal to service zones, and many-to-one service from service zones to terminal.

The model in this paper will be developed using a similar method proposed by Clarens and Hurdle (1975). The Clarens and Hurdle model was designed for a regular bus service, while our model was developed for a DRT system for any shape of service area using a zonal transit strategy. The objective of the proposed model is to optimize the DRT system by minimizing the average cost per passenger of each service zone.

4.1.1 System description

A conceptual service area with a terminal is shown in Figure 4.1. Buses travelled on three route segments: line-haul path, local path and back-haul path. Passengers at the terminal who sought to alight at service zones, defined as alighting passengers, were collected, and then buses travelled on a line-haul path to the assigned service zone. Within the zone, the alighting passengers were dropped off at their desired locations. In the meantime, passengers who sought delivery to the terminal, defined as boarding passengers, were also picked up at their requested locations. Buses may be overloaded within service zones, as the number of pickups may be temporarily larger

than the number of drop-offs during a local tour. Thus the bus capacity was carefully chosen (for example, number of seats) to allow for random fluctuation. After the local tour, buses left the zone and retraced back to the terminal on a back-haul path, to drop off the passengers who had boarded within service zones. No passenger can board or alight on a line-haul or back-haul path.

Figure 4.1 Service area and travel paths

4.1.2 Assumptions for model development

The assumptions made for model development are listed below:

- The service provided in one zone did not affect the services in other zones.
- For each service zone, travel time on a line-haul and back-haul path were approximately the same, which was defined as the shortest travel time between service zones and the terminal.
- Demand density was assumed to be homogeneous within one service zone.

- Buses cannot pick up or drop off passengers simultaneously at one location within service zones.
- Hourly bus operation time cost, hourly passenger riding and waiting time cost were constant over the service area.
- Passenger average waiting time at the terminal or their pickup locations in service zones were equal to half of the bus headway.

4.1.3 Model parameters

The input parameters used for model development are shown in Table 4.1. These input parameters can be obtained from surveys or estimated from existing literature.

Table 4.1 Model parameters

Symbol	Definition	Unit
p_a	Alighting passenger demand density from terminal to service zone	passenger/km ² /hr
p_b	Boarding passenger demand density from service zone to terminal	passenger/km ² /hr
T	Line-haul route travel time between terminal and service zone	hour
V_C	Vehicle local speed	km/hr
γ_O	Hourly operation time cost	\$/hr
γ_R	Hourly passenger riding time cost	\$/passenger/hr
γ_W	Hourly passenger waiting time cost	\$/passenger/hr
τ	Average time required for one vehicle serving one unit area of local service zone	hour/km ²

4.1.4 Development of objective function

The objective function is an expression of average cost per passenger (C), which is equal to the total cost (C_{Total}) divided by the number of passengers during one travel cycle (n). The total cost during one travel cycle consists of the cost in four phases:

- Cost at terminal (C_T)
- Cost on the line-haul path ($C_{linehaul}$)
- Cost on the local path (C_C)
- Cost on the backhaul path ($C_{backhaul}$)

Therefore, the total cost is expressed as

$$C_{Total} = C_T + C_{linehaul} + C_C + C_{backhaul} \quad (4.1)$$

4.1.4.1 Cost at terminal

Cost occurring at terminal (C_T) is the sum of waiting cost of alighting passengers at terminal. Passenger average waiting time is assumed to be half of the headway. So cost at terminal C_T is formulated as

$$C_T = n_a * h\gamma_W/2 \quad (4.2)$$

where n_a is the number of passenger from the terminal to the service zone during one cycle, h is the bus headway, γ_W is the hourly passenger waiting time cost.

n_a can be estimated by Equation (4.3),

$$n_a = p_a Ah \quad (4.3)$$

where p_a is alighting passenger demand density from the terminal to the service zone, A is the area of service zone,

Then the cost occurring at terminal (C_T) becomes

$$C_T = p_a Ah * h\gamma_w/2 \quad (4.4)$$

4.1.4.2 Cost on the line-haul path

Cost on the line-haul path ($C_{linehaul}$) is the sum of operation cost ($C_{linehaulO}$) and passenger riding cost ($C_{linehaulR}$) on the line-haul path.

$$C_{linehaul} = C_{linehaulO} + C_{linehaulR} \quad (4.5)$$

The operation cost on the line-haul path $C_{linehaulO}$ is

$$C_{linehaulO} = \gamma_O T \quad (4.6)$$

where γ_O is the hourly bus operation time cost, T is the line-haul path travel time defined as the shortest travel time between terminal and service zone.

The passenger riding cost on the line-haul path $C_{linehaulR}$ is

$$C_{linehaulR} = p_a Ah * \gamma_R T \quad (4.7)$$

where γ_R is the hourly passenger riding time cost.

Then the cost on the line-haul path $C_{linehaul}$ becomes

$$C_{linehaul} = \gamma_O T + p_a Ah * \gamma_R T \quad (4.8)$$

4.1.4.3 Cost on the local path

Cost on the local path (C_C) is the sum of operation cost (C_{CO}), passenger waiting cost (C_{CW}) and passenger riding cost on local tour (C_{CR}). Because passengers are picked up and dropped off at their desired locations, there is no passenger access cost.

$$C_C = C_{CO} + C_{CW} + C_{CR} \quad (4.9)$$

The operation cost on the local path C_{CO} is

$$C_{CO} = \gamma_o T_C \quad (4.10)$$

where T_C is the bus travel time on local path.

The passenger waiting cost C_{CW} on local path is

$$C_{CW} = n_b * h\gamma_W/2 \quad (4.11)$$

where n_b is the number of boarding passenger from the service zone to the terminal.

n_b can be estimated by Equation (4.12),

$$n_b = p_b Ah \quad (4.12)$$

where p_b is boarding passenger demand density from the terminal to the service zone.

Then the passenger waiting cost C_{CW} on the local path becomes

$$C_{CW} = p_b Ah * h\gamma_W/2 \quad (4.13)$$

The average local riding time for alighting passengers and boarding passengers is the same, which is equal to half of T_C . Passenger riding cost C_{CR} on the local path can then be formulated as,

$$C_{CR} = p_a Ah\gamma_R * T_C/2 + p_b Ah\gamma_R * T_C/2 \quad (4.14)$$

The cost on the local path C_C becomes

$$C_C = \gamma_0 T_C + p_b A h \gamma_W h/2 + p_a A h \gamma_R * T_C/2 + p_b A h \gamma_R * T_C/2 \quad (4.15)$$

Local travel time T_C can be estimated based on Daganzo's (1984a) method or Clarens and Hurdle's (1975) method as follows:

4.1.4.3.1 Local travel time estimated based on the method in Daganzo (1984a)

Daganzo (1984a) estimated the expected local path distance D_C as

$$D_C = k\sqrt{nA} \quad (4.16)$$

where k was the constant factor with a value of 1.15 for Manhattan distance measurement, and 0.9 for Euclidean distance measurement, n was the number of stops within the zone.

Then the expected travel time on local path T_C should be

$$T_C = D_C/V_C \quad (4.17)$$

where V_C is bus local speed travelling within the service zone.

As buses cannot pick up and drop off passengers simultaneously at one location, n can be estimated as,

$$n = A(p_a + p_b)h \quad (4.18)$$

So, the expected local path distance D_C becomes

$$D_C = kA\sqrt{(p_a + p_b)h} \quad (4.19)$$

The bus local travel time T_C becomes

$$T_C = kA\sqrt{(p_a + p_b)h}/V_C \quad (4.20)$$

4.1.4.3.2 Local travel time estimated based on the method in Clarens and Hurdle (1975)

Clarens and Hurdle (1975) estimated that local travel time T_C was,

$$T_C = \tau A \quad (4.21)$$

where A was the area of the local service zone, τ was average travel time required for a vehicle serving a unit area of the local service zone.

4.1.4.4 Cost on the back-haul path

Cost on the back-haul path $C_{backhaul}$ is the sum of operation cost ($C_{backhaulO}$) and passenger riding cost on the back-haul path ($C_{backhaulR}$).

$$C_{backhaul} = C_{backhaulO} + C_{backhaulR} \quad (4.22)$$

The travel time on the back-haul path as assumed is the same as the travel time on the line-haul path T , so the operation cost on the back-haul path $C_{backhaulO}$ is

$$C_{backhaulO} = \gamma_O T \quad (4.23)$$

The passenger riding cost on back-haul path $C_{backhaulR}$ is

$$C_{backhaulR} = p_b A h * \gamma_R T \quad (4.24)$$

The cost on the back-haul path $C_{backhaul}$ becomes

$$C_{backhaul} = \gamma_O T + p_b A h * \gamma_R T \quad (4.25)$$

4.1.4.5 Total cost

Substitute Equations (4.4, 4.8, 4.15 and 4.25) to Equation (4.1), total cost C_{Total} becomes

$$C_{Total} = \frac{(p_a + p_b)Ah * h\gamma_w}{2} + 2\gamma_o T + T\gamma_R(p_a + p_b)Ah + [\gamma_o + \frac{(p_a + p_b)Ah\gamma_R}{2}] * T_C \quad (4.26)$$

1) Daganzo's method

When the local travel time is estimated by Daganzo's method, total cost C_{Total} becomes

$$C_{Total} = \frac{(p_a + p_b)Ah * h\gamma_w}{2} + 2\gamma_o T + T\gamma_R(p_a + p_d)Ah + [\gamma_o + \frac{(p_a + p_b)Ah\gamma_R}{2}] * \frac{kA\sqrt{(p_a + p_b)h}}{V_C} \quad (4.27)$$

2) Clarens and Hurdle's method

When the local travel time is estimated by Clarens and Hurdle's method, total cost C_{Total} becomes

$$C_{Total} = \frac{(p_a + p_b)Ah * h\gamma_w}{2} + 2\gamma_o T + T\gamma_R(p_a + p_b)Ah + [\gamma_o + \frac{(p_a + p_b)Ah\gamma_R}{2}] * \tau A \quad (4.28)$$

4.1.4.6 Average cost per passenger

Average cost per passenger C is the total cost C_{Total} divided by the number of passenger trips generated within the service zone during one travel cycle.

$$C = C_{Total}/n \quad (4.29)$$

4.1.4.6.1 Daganzo's method

Substituting Equation (4.18 & 4.27) into Equation (4.29), the average cost per passenger (C) becomes,

$$C = \frac{2T\gamma_O}{(p_a + p_b)Ah} + \frac{k\gamma_O}{V_L\sqrt{(p_a + p_b)h}} + \frac{1}{2}\gamma_W h + \gamma_R T + \frac{k\gamma_R A\sqrt{(p_a + p_b)h}}{2V_C} \quad (4.30)$$

4.1.4.6.2 Clarens and Hurdle's method

Substituting Equation (4.18 & 4.28) into Equation (4.29), the average cost per passenger (C) becomes,

$$C = \frac{2T\gamma_O}{(p_a + p_b)Ah} + \frac{\tau\gamma_O}{(p_a + p_b)h} + \frac{1}{2}\gamma_W h + \gamma_R T + \frac{\tau A\gamma_R}{2} \quad (4.31)$$

4.1.4.7 Optimal area of service zone and headway

4.1.4.7.1 Daganzo' method

The optimal area of service zone and headway is obtained by setting the partial derivatives of Equation (4.30) with respect to the area of service zone (A) and headway (h) to 0.

$$\frac{\partial C}{\partial A} = -\frac{2T\gamma_O}{(p_a + p_b)hA^2} + \frac{k\gamma_R\sqrt{(p_a + p_b)h}}{2V_C} = 0 \quad (4.32)$$

$$\frac{\partial C}{\partial h} = -\frac{k\gamma_O}{2V_C}\sqrt{\frac{1}{(p_a + p_b)h^3}} - \frac{2T\gamma_O}{(p_a + p_b)Ah^2} + \frac{kA\gamma_R}{4V_C}\sqrt{\frac{(p_a + p_b)}{h}} + \frac{1}{2}\gamma_W = 0 \quad (4.33)$$

The optimal area of service zone A^* is

$$A^* = \left[\frac{4T\gamma_O V_C}{k\gamma_R(p_a + p_b)^2 h^2} \right]^{\frac{1}{2}} \quad (4.34)$$

Substitute A^* into Equation (4.33),

$$\frac{M^2}{h^{*\frac{3}{2}}} + \frac{M(T\gamma_R)^{\frac{1}{2}}}{h^{*\frac{5}{4}}} = \gamma_W \quad (4.35)$$

where

$$M = \left[\frac{k\gamma_O}{V_C(p_a + p_b)^2} \right]^{\frac{1}{2}} \quad (4.36)$$

The optimal headway h^* can be numerically solved based on Equation (4.35 & 4.36).

4.1.4.7.2 Clarens and Hurdle's method

Set the partial derivatives of Equation (4.31) with respect to the area of service zone (A) and headway (h) to 0.

$$\frac{\partial C}{\partial A} = -\frac{2T\gamma_O}{(p_a + p_b)hA^2} + \frac{\tau\gamma_R}{2} = 0 \quad (4.37)$$

$$\frac{\partial C}{\partial h} = -\frac{2T\gamma_O}{(p_a + p_b)Ah^2} - \frac{\tau\gamma_O}{(p_a + p_b)h^2} + \frac{1}{2}\gamma_W = 0 \quad (4.38)$$

The optimal area of service zone A^* is

$$A^* = \left[\frac{4T\gamma_O}{\tau\gamma_R(p_a + p_b)h} \right]^{\frac{1}{2}} \quad (4.39)$$

Substitute A^* into Equation (4.38),

$$\frac{S^2}{h^{*2}} + \frac{S(T\gamma_R)^{\frac{1}{2}}}{h^{*\frac{3}{2}}} = \frac{\gamma_W}{2} \quad (4.40)$$

where

$$S = \left[\frac{\tau \gamma_o}{p_a + p_b} \right]^{\frac{1}{2}} \quad (4.41)$$

The optimal headway h^* is obtained numerically from Equations (4.40 & 4.41).

4.1.5 Determination of Zone Division

In this section, stepwise procedures were proposed to divide the service area into a number of zones with optimal areas. Practically, zone division should cooperate with the existing street networks and geographic boundaries of communities. The procedures are described below and listed in Figure 4.2.

- 1) Select the community farthest away from the terminal to initialize the zone division.
- 2) Identify all possible entrances on this community boundary, and obtain line-haul travel time between each entrance and the terminal.
- 3) Identify the entrance leading to the shortest line-haul travel time as the “best entry point” for this community.
- 4) Calculate the optimal area of the service zones (A^*) for the “best entry point” based on Equations (4.34 ~ 4.36) or Equations (4.39 ~ 4.41).
- 5) Calculate the ratio (α) of the area of this community (A) to the optimal area of service zone (A^*) with respect to the shortest line-haul travel time;
- 6) If α was approximately equal to 1, then this community was considered to be one service zone. The adjacent community was selected for further iteration;
- 7) If α was smaller than 1, this community was combined with adjacent communities as a new pseudo-community for further iteration;

- 8) If α was larger than 1, then an area of A^* were assigned to be a service zone. The rest of the community was considered as a new pseudo-community for further iteration.
- 9) Repeat Step 2 ~ Step 8 until zone division is finalized.

Figure 4.2 Procedures of zone division determination

4.2 Case study

4.2.1 System description

A hypothetical DRT system was designed to operate between a service area and Market Mall in the City of Calgary. The study service area was the same as the one in Chapter Three, which is bounded by Stoney Trail, Deerfoot Trail, the Bow River, Sarcee Trail S.W., and the Trans-Canada Highway, with Nose Hill Park excluded as shown in Figure 4.3. The service area has 58 communities (3 industrial, 55 residential) with different areas and populations.

Figure 4.3 Service area and terminal (from QGIS map)

The terminal was selected to be Market Mall, the most productive shopping mall in northwest Calgary and the second largest shopping mall in terms of sales per square foot in the City of Calgary. The annual customer count of Market Mall in 2016 was 9,020,838 (RCC, 2016).

4.2.2 Input parameters

Input parameters, including service area, service area population, hourly passenger riding and waiting time cost, as well as bus local speed were the same as Chapter Three. The remaining input parameters were collected from different resources: (1) The estimation of alighting and boarding demand densities in the service area and communities will be discussed in section 4.3.2.1; (2) The average time required for one vehicle serving one unit area of local service zones was estimated from the case study in Clarens and Hurdle (1975), discussed in section 4.3.2.2; (3) line-haul travel time was extracted from Google maps; (4) The hourly bus operation time cost adopted the operation time value of Calgary commuter buses, which was \$116 per hour per vehicle; (5) The constant in Equation (4.16) was set as 1.15, since buses were more likely to travel on a grid road network, as mentioned in Chapter Three. Values of input parameters are summarized in Table 4.2.

Table 4.2 Values of input parameters

Symbol	Definition	Value	Unit
A_σ	Service area	139.76	km ²
A_m	Area of the communities	0.27~6.85	km ²
k	Constant in Equation (4.16)	1.15	
$\overline{p_{\sigma a}}$	Average alighting demand density of the service area	1.36	passenger/km ² /hr
$\overline{p_{\sigma a}}$	Average boarding demand density of the service area	1.36	passenger/km ² /hr
p_{ma}	Alighting demand density of a community	0~2.64	passenger/km ² /hr
p_{mb}	Boarding demand density of a community	0~2.64	passenger/km ² /hr
P_m	Population in a community	0~25,990	
P_p	Annual pedestrian count of Market Mall	9,020,838	
P_σ	Population in the service area	338,484	
T	Line-haul travel time	0.017~0.25	hour
V_C	Local speed	22.4	km/hr
γ_O	Operation time cost per bus per hour	116	\$/hr/veh
γ_R	Passenger riding time cost per passenger per hour	5.03	\$/passenger/hr
γ_W	Passenger waiting time cost per passenger per hour	11.47	\$/passenger/hr
τ	Average time required for one vehicle serving one unit area of the local service zone	0.055	Hour/km ²

4.2.2.1 Estimation of demand density

Currently, people do not prefer public transit while travelling to Market Mall. There are 10 bus stops with fewer than 400 metres walking distance to Market Mall. In 2015 the annual number of passengers boarding at these 10 stops was 226,276.8, with 260,721 alighting passengers (data from Calgary Transit), far less than the number of annual customers (9,020,838) of Market Mall (supposing the transit customer count did not vary much from 2015). Therefore, demand density was estimated based on the annual customer number at Market Mall, rather than the passenger travel data at bus stops around Market Mall.

The City of Calgary can be geographically divided into four districts: northwest, northeast, southeast and southwest. Each district has its own major shopping malls, such as Market Mall, in the northwest district, Sunridge Mall in the northeast district, Southcentre Mall in the southeast district, CF Chinook Centre in the southwest district, The CORE Shopping Centre in downtown Calgary, and CrossIron Mills north of Calgary. We assume that two-thirds of customers visiting Market Mall are from the northwest district, as people are likely to shop at the closest shopping malls to where they live.

Demand density levels tend to vary greatly among different periods of the day and days of week. Due to a lack of periodical demand data, average demand densities during opening hours are utilized in this case study. The following assumptions are made to estimate the average DRT demand densities during opening hours:

- Approximately two-thirds of the annual customers at Market Mall are from the service area.
- The percentage of customers preferring to use DRT services to Market Mall is 12%, which is adapted from a survey in the San Francisco Bay Area (Khattak & Yim 2004).

- Average alighting demand density is the same as average boarding demand density for each service zone.
- Within the service area, demand density is proportional to population density.

Based on assumptions, the average hourly number of customers preferring to use DRT service from the service area to Market Mall (p_{DRT}) is estimated to be,

$$p_{DRT} = 12\% * \frac{2}{3} * P_p / H = 0.08 P_p / H \quad (4.42)$$

where P_p is the annual customer count of Market Mall, H is the annual operation hours of Market Mall.

Open days at Market Mall in 2016 were 354, and average daily open time was 10.71 hours.

Hence, the annual operation hours H is 3,791.34 hours.

The potential average DRT demand density travelling from Market Mall to the service area was defined as the average alighting demand density of the service area ($\overline{p_{\sigma a}}$), while the average demand density from the service area to Market Mall was defined as the average boarding demand density of the service area ($\overline{p_{\sigma b}}$). They were assumed to be the same in general, and can be calculated by

$$\overline{p_{\sigma a}} = \overline{p_{\sigma b}} = \frac{p_{DRT}}{A_{\sigma}} = \frac{0.08 P_p}{A_{\sigma} H} \quad (4.43)$$

where A_{σ} is the size of the service area.

As demand density is proportional to population density within a service area, alighting demand density for a community ($\overline{p_{ma}}$) and boarding density for the community ($\overline{p_{mb}}$) were estimated by

$$\overline{p_{ma}} = \overline{p_{mb}} = p_{DRT} * \frac{P_m / A_m}{P_{\sigma} / A_{\sigma}} \quad (4.44)$$

where P_m is the population of a community, A_m is the area of the community, P_σ is the population of the service area.

Area, population, and alighting and boarding demand density of each community are shown in Appendix B. The demand densities for different communities range from 0 to 2.64 passengers per hour per square kilometre. The demand density for three industrial communities (Stoney 1, Aurora Business Park, Skyline West) and the University District community were calculated to be zero, as the number of residents is zero in those communities.

4.2.2.2 Estimation of τ

τ is the average time required for one vehicle serving one unit area of service zone, which is inversely proportional to bus local speed. In this chapter, τ was estimated based on the results in Clarens and Hurdle (1975), shown in Equation (4.45).

$$\tau = \frac{V'_C}{60 * 2.59 * V_C} \tau' \quad (4.45)$$

where τ' was the average time required for one vehicle serving one unit area of service zone in Clarens and Hurdle's model, which was approximately 10 min per square mile; V'_C was the bus local speed in Clarens and Hurdle's model, which was 12 miles per hour; V_C was the bus local speed in Calgary, which was 22.4 km per hour.

Thus, τ was calculated to be 0.055 hour per square kilometre.

4.2.3 Impact of line-haul travel time

We intend to explore the impact of line-haul travel time on optimal variables (i.e. area of service zone size and headway) and minimum average cost per passenger throughout the entire service area. For convenience, the line-haul travel time in the following analysis was measured in minutes. Hourly passenger waiting and riding cost, bus operation cost, vehicle local speed, as well as average time serving one unit area of the local service zone were assumed to be the same throughout the service area. The average demand density of the service area was adopted here. Expected local travel time was estimated using the Daganzo (1984a) method, and the Clarens and Hurdle (1975) method separately.

4.2.3.1 Daganzo's method

The optimal headway, optimal area of service zone and minimum average cost per passenger corresponding to different line-haul travel time were calculated based on Equations (4.34 ~ 4.36), and shown in Figure 4.4. Optimal headway, optimal area of service zone and minimum average cost per passenger all increased as line-haul travel time increased. When the line-haul travel time was 5 minutes, the optimal headway was 0.55 hour, optimal area of service zone was 9.05 km² and the associated minimum average cost per passenger was \$11.3. As the line-haul travel time increased to 20 minutes, optimal headway was 0.64 hour, optimal area of service zone was 16.24 km², and the associated cost increased to \$15.36.

Figure 4.4 Impact of line-haul travel time on (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger using Daganzo's method

4.2.3.2 Clarens and Hurdle's method

The optimal headway, optimal area of service zone and minimum average cost per passenger for different line-haul travel time were calculated based on Equations (4.39 ~ 4.41) and the results are shown in Figure 4.5. The optimal headway, optimal zone area and minimum average cost per passenger increased as the line-haul travel time increased. When the line-haul travel time was 5 minutes, optimal headway was 0.79 hour; optimal area of service zone was 7.6 km² and the associated minimum average cost per passenger was \$10.67. When line-haul travel time was 20

minutes, optimal headway increased to 0.9 hour; optimal area of service zone increased to 14.24 km^2 and the associated minimum average cost per passenger increased to \$14.22.

Figure 4.5 Impact of line-haul travel time on (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger using Clarens and Hurdle's method

4.2.4 Determination of Zone Division

This section demonstrates how to partition the service area into service zones following the procedures in section 4.2.6, with the local travel time estimated in Chang and Schonfeld (1991a) and Clarens and Hurdle (1975) respectively.

The area served by the DRT system consists of 58 communities with different areas and population. Since the communities differ from one another, a finer analysis was conducted by disaggregating the area-wide system information to obtain community-level data, especially line-haul travel time and demand densities. Average line-haul travel time for different locations in the service area was extracted from Google maps. In this case, the model was applied to every community or combined communities to obtain the optimal variables. Then the optimal area of community or combined communities was compared with the actual area of the community or combined communities, to decide whether the community or combined communities could be one service zone.

4.2.4.1 Daganzo's method

Initial zone division started from the farthest community (Coventry Hills). Possible bus entry points were the intersections of Harvest Hills Blvd. N. and Coventry Hills Dr. N.E., Country Hills Blvd. N.E. and Coventry Hills Blvd. N.E., Country Village Rd. N.E. and Coventry Hills Way N.E., Country Hills Blvd N.E. and Coventry Blvd N.E. The intersection of Harvest Hills Blvd. N. and Coventry Hills Dr. N.E. was the one leading to minimum average line-haul travel time. The optimal area for a service zone with respect to line-haul travel time was far larger than the actual area of the community of Coventry Hills. Thus, Coventry Hills was combined with adjacent communities (Stoney 1 and Country Hills Village) as a new pseudo-community for further iteration. Possible entry points for the pseudo-community were the intersections of Harvest Hills Blvd. N. and Coventry Hills Dr. N.E., Harvest Hills Blvd. N. and Country Village Rd N.E., Country Hills Blvd N.E. and Coventry Hills Blvd N.E., Country Hills Blvd. N.E. and

Coventry Blvd. N.E. The intersection of Country Hills Blvd. N.E. and Coventry Blvd. N.E. was the one leading to minimum average line-haul travel time. With the average demand density of the pseudo-community, the optimal area of the service zone with respect to the minimum line-haul travel time was still larger than the actual area of the pseudo-community. Thus, Harvest Hills, Aurora Business Park and Country Hills were added to the pseudo-community. The updated entry point leading to the minimum average travel time became the intersection of Country Hills Blvd. N.W. and Country Hills Dr. N.W. The optimal area of the service zone with respect to the updated line-haul travel time was calculated to be 17.29 km^2 . The total area of the communities of Stoney 1, Coventry Hills, Country Hills Village, Harvest Hills, Aurora Business Park and Country Hills was measured to be 17.74 km^2 , which was close to the calculated optimal service zone area. Therefore, zone 1 was determined to be the combination of the communities of Stoney 1, Coventry Hills, Country Hills Village, Harvest Hills, Aurora Business Park and Country Hills.

The above was the scenario for the optimal area of service zone larger than the area of a community. The following discussion showed how to assign service zones when the optimal area of service zone is smaller than the area of a community. The optimal area of a service zone corresponding to the community of Varsity with respect to its shortest line-haul travel time was smaller than the actual area of the community. The area of the western section of Varsity (4.67 km^2) was close to the optimal area of a service zone (4.97 km^2), therefore it was assigned as a service zone. The rest of Varsity was combined with the University District, University Heights, and the University of Calgary as a pseudo-community. The intersection leading to minimum line-haul travel time was 40 Ave N.W. and Shaganappi Trail N.W. The actual area of the

pseudo-community was 5.65 km^2 , which was close to the optimal area with respect to the minimum line-haul travel time (5.07 km^2).

The rest of the service area was assigned to service zones based on similar zone division procedures. Finally, the service area was partitioned into 13 zones shown as Figure 4.6.

Figure 4.6 Zone division with Daganzo's model

The results of the zone division are shown in Table 4.3. The actual area of the service zone deviating from the optimal value by more than 10% was only 2 out of 13 service zones.

Table 4.3 Optimization results for each zone with Daganzo's method

Zone	Service zone area (km ²)			Optimal headway	Minimum average cost per
No.	Actual	Optimal	Difference	(hour)	passenger (\$)
1	17.74	17.29	2.6%	0.67	15.48
2	10.64	11.18	-4.8%	0.54	12.49
3	11.54	11.56	-0.2%	0.57	12.54
4	10.36	10.4	-0.4%	0.54	11.9
5	13.75	13.11	4.9%	0.61	13.35
6	9.1	9.26	-1.7%	0.58	11.67
7	4.67	4.97	-6.0%	0.55	10.17
8	5.65	5.07	11.4%	0.55	10.3
9	9.51	8.96	6.1%	0.54	11.2
10	10.08	10.16	-0.8%	0.54	11.76
11	15.42	14.95	3.1%	0.63	14.33
12	11.63	11.73	-0.9%	0.58	12.63
13	9.77	11.7	-16.5%	0.55	12.76

Zone 1 was the farthest service zone from Market Mall. The minimum average cost per passenger (\$15.48) was the highest in the service area. Zone 7 was the nearest service zone from Market Mall. The minimum average cost per passenger (\$10.17) was the lowest in the service area.

4.2.4.2 Clarens and Hurdle's method

Zone division with local travel time estimated from Clarens and Hurdle's method also started from the farthest community (Coventry Hills). The optimal area of the service zone with respect to the minimum average line-haul travel time of Coventry Hills was far larger than the actual area of the community of Coventry Hills. Thus, Coventry Hills was combined with adjacent communities (Stoney 1 and Country Hills Village) as a new pseudo-community for further iteration. The optimal area of the service zone with respect to the minimum average line-haul travel time of the pseudo-community was still larger than the actual area. Thus, Harvest Hills, and Aurora Business Park were added to the pseudo-community. The possible intersections were Deerfoot Trail and Beddington Trail N.W., 96 Ave. N.E. and Harvest Hills Blvd. N., Country Village Rd. N.E. and Harvest Hills Blvd. N., Country Hills Blvd. N.E. and Harvest Hills Blvd N., Coventry Hill Dr. N.E. and Harvest Hills Blvd. N., 96 Ave. N.E. and Deerfoot Trail, Deerfoot Trail and Country Hills Blvd N.E. The intersection leading to the minimum average line-haul time became the intersection of Country Hills Blvd. N.E. and Harvest Hills Blvd. N. The optimal area of service zone with respect to the updated line-haul travel time was calculated to be 16.45 km², while the total area of the communities of Stoney 1, Coventry Hills, Country Hills Village, Harvest Hills, and Aurora Business Park was measured to be 15.85 km², close to the calculated optimal service zone area. Therefore, the combination of communities Stoney 1, Coventry Hills, Country Hills Village, Harvest Hills and Aurora Business Park were determined to be a service zone, denoted as zone 1.

The rest of the service area followed similar procedures as in section 4.3.4.1. The service area was divided into 14 service zones, shown in Figure 4.7. The results of the zone division are shown in Table 4.4.

Figure 4.7 Zone division with Clarens and Hurdle's method

The area of the service zone ranged from 4.67 km² to 15.85 km², the optimal headway ranged from 0.71 hour to 0.98 hour, and the minimum average cost per passenger ranged from \$9.67 to \$15.1. The actual zone area deviating from the optimal values more than 10% was only 2 out of 14.

Table 4.4 Optimization results for each zone with Clarens and Hurdle's method

Zone	Service zone area (km ²)			Optimal headway	Minimum average cost per
No.	Actual	Optimal	Difference	(hour)	passenger (\$)
1	15.85	16.45	-3.6%	0.98	15.1
2	12.53	12.12	3.4%	0.71	11.05
3	10.12	10.97	-7.7%	0.8	11.46
4	10.37	10.34	0.3%	0.71	10.4
5	11.7	11.47	2.0%	0.83	11.93
6	9.72	8.18	18.8%	0.87	11.52
7	4.67	4.12	13.3%	0.79	9.67
8	5.65	5.9	-4.2%	0.8	10.43
9	8.06	8.36	-3.6%	0.74	10.13
10	7.58	8.15	-7.0%	0.73	9.97
11	10.02	10.28	-2.5%	0.73	10.57
12	11.67	12.82	-9.0%	0.82	12.27
13	12.06	12.05	0.1%	0.86	12.41
14	11.19	11.79	-5.1%	0.71	10.9

Compared to the results of Daganzo's method, the optimal area of service zones and minimum average cost per passenger derived from Clarens and Hurdle's method was generally smaller, while the optimal headway was generally larger. This is because the average local travel time for

one unit area of service zone calculated using Daganzo's method was larger than for the one calculated using Clarens and Hurdle's method.

4.2.5 Sensitivity Analysis

In section 4.3.3, decision variables were optimized by minimizing average cost per passenger under different line-haul travel time. However, it is useful to see how optimal headway, optimal area of service zone and minimum average cost per passenger changed with deviations in system parameters. The parameters discussed in this sensitivity analysis were demand density, hourly passenger waiting and riding time cost, hourly operation time cost, local travel speed (for Daganzo's method) and average time required for one vehicle serving one unit area of the local service zone (for Clarens and Hurdle's method). Also, the sensitivity analysis was used to evaluate the robustness of the model. If the values of the decision variables are too sensitive to the deviation in a particular parameter, this parameter should be carefully estimated with good accuracy.

Sensitivity analysis with demand density, hourly passenger waiting and riding time cost, as well as the hourly operation time cost were examined based on the model using local travel time estimated by Daganzo's method, since the solutions of the two methods have a similar model structure, with the exception of the estimation of bus local travel time. Therefore, there is no need to use both in performing sensitivity analysis. An analysis using local travel time estimated by Clarens and Hurdle's method produces similar general trends in sensitivity with respect to deviations in those system parameters. Besides those system parameters, for solutions with Daganzo's method, sensitivity analysis was also examined with bus local travel speed, and for

solutions with Clarens and Hurdle's method, sensitivity analysis was only examined on the average time for one vehicle serving one unit area of local service zone.

Listed input system parameters are deviated by $\pm 5\%$, $\pm 10\%$ and $\pm 20\%$ based on the baseline values in Table 4.2 to explore the sensitivity on optimal headway, optimal area of service zone and minimum average cost per passenger. Sensitivity analysis was also based on the average data of the service area, the same as in the study in section 4.3.3. Also, in order to explore the effect of line-haul travel time on sensitivities, three different line-haul travel time (5 min, 15 min and 30 min) were selected.

The degree of sensitivity was defined the same as in Chapter 3:

- Non-sensitive: deviation in decision variable is less than 5% when the value of input parameter is changed by 20%.
- Slightly sensitive: deviation in decision variable is 5%~20% when the value of input parameter is changed by 20%
- Sensitive: deviation in decision variable is 20%~100% when the value of input parameter is changed by 20%.
- Very sensitive: deviation in decision variable is larger than 100% when the value of input parameter is changed by 20%.

The sensitivity results for different input parameters are summarized in Table 4.5.

Table 4.5 Summary of sensitivity

Input parameters	Degree of sensitivity		
	Headway	Area of service zone	Average cost per passenger
Demand density	Slightly sensitive	Slightly sensitive	Slightly sensitive
Hourly passenger waiting time cost	Slightly sensitive	Slightly sensitive	Slightly sensitive
Hourly passenger riding time cost	Non-sensitive	Slightly sensitive	Slightly sensitive
Hourly bus operation cost	Slightly sensitive	Non-sensitive	Slightly sensitive
Bus local speed	Slightly sensitive	Slightly sensitive	Slightly sensitive
Average time required for one vehicle serving one unit area of local service zone	Slightly sensitive	Slightly sensitive	Slightly sensitive

4.2.5.1 Demand density

Here, demand density used the sum of alighting and boarding demand density throughout the service area, which was 2.72 passengers per unit area per hour. Figure 4.8 presents the sensitivity results of the optimal variables and minimum average cost per passenger on deviations in demand density for different line-haul travel time.

As shown in Figure 4.8, line-haul travel time had little effect on the sensitivity of the optimal variables and minimum average cost per passenger. Both optimal variables and minimum average cost per passenger were slightly sensitive to deviations in demand density. As line-haul

travel time increased, optimal headway and minimum average cost per passenger became less sensitive to deviations in demand density, while optimal area of service zone became more sensitive.

Figure 4.8 Percentage change of (a) optimal headway (b) optimal area of service zone (c) minimum average cost per passenger vs. deviations in demand density for different line-haul travel time

4.2.5.2 Hourly passenger waiting time cost

Sensitivity of optimal variables and minimum average cost per passenger on hourly passenger waiting time cost for different line-haul travel time are shown in Figure 4.9. As in section 4.3.5.1, line-haul travel time hardly impacted the sensitivity of optimal variables and the minimum average cost per passenger with deviations in hourly passenger waiting time cost. Optimal variables as well as minimum average cost per passenger were slightly sensitive to deviations in hourly passenger waiting time cost. Also, as line-haul travel time increased, minimum average cost per passenger became less sensitive to the hourly passenger waiting time cost, while optimal headway and area of service zone became more sensitive.

Figure 4.9 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly passenger waiting time cost for different line-haul travel time

4.2.5.3 Hourly passenger riding time cost

Results of sensitivity analysis with deviations in hourly passenger riding time cost for different line-haul travel time are listed in Figure 4.10. Optimal headway was non-sensitive about deviations in hourly riding time cost for different line-haul travel time, while the optimal area of the service zone was slightly sensitive. Minimum average cost per passenger was non-sensitive for short line-haul travel time (5 min), and slightly sensitive for medium and long travel time (15 and 30 min). As line-haul travel time increased, optimal headway, area of service zone and minimum average cost per passenger became more sensitive to the hourly passenger riding time cost,.

Figure 4.10 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly passenger riding time cost for different line-haul travel time

4.2.5.4 Hourly bus operation cost

Results of the sensitivity analysis with deviations in hourly bus operation cost are presented in Figure 4.11. As shown in Figure 4.11, the effect of line-haul travel time on the sensitivity of optimal variables and minimum average cost per passenger with deviations in hourly bus operation time cost was also negligible. Optimal headway and minimum average cost per passenger were slightly sensitive to deviations in hourly passenger riding time cost, while optimal area of service zone was non-sensitive. Additionally, as line-haul travel time increased, optimal headway and minimum average cost per passenger became less sensitive to deviations in hourly bus operation cost, while the optimal area of the service zone became more sensitive.

Figure 4.11 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in hourly bus operation cost for different line-haul travel time

4.2.5.5 Bus local speed

Bus local speed adopted the value of commuter shuttle buses in Calgary Transit. Under certain circumstances (e.g., adverse weather, night time) buses would be driven with extra caution, which might be addressed by a deduction of bus local speed. Sensitivity analysis results are listed in Figure 4.12. As shown in Figure 4.12, line-haul travel time also almost did not influence the sensitivity of optimal variables and the minimum average cost per passenger with deviations in bus local speed. Optimal variables and minimum average cost per passenger were slightly sensitive to deviations in hourly passenger riding time cost. As line-haul travel time increased, optimal headway, area of service zone and minimum average cost per passenger became less sensitive to deviations in bus local speed.

Figure 4.12 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in bus local speed for different line-haul travel time

4.2.5.6 Average time required for one vehicle serving one unit area of local service zones

Average time required for one vehicle serving one unit area of the local service zone was derived from the Clarens and Hurdle (1975) method. As shown in Figure 4.13, optimal variables and minimum average cost per passenger were not sensitive to different line-haul travel time with deviations in average time required for one vehicle serving one unit area of the local service zone. Optimal variables and minimum average cost per passenger were all slightly sensitive to deviations in average time required for one vehicle serving one unit area of the local service zone regardless of the line-haul travel time. Additionally, as line-haul travel time increased, optimal headway, optimal area of service zone and minimum average cost per passenger all became less sensitive to deviations in average time required for one vehicle serving one unit area of the local service zone.

Figure 4.13 Percentage change of (a) optimal headway, (b) optimal area of service zone, (c) minimum average cost per passenger vs. deviations in average time required for one vehicle serving one unit area of the local service zone for different line-haul travel time

4.3 Conclusion

This research developed a mathematical model for zone division of DRT systems. A stepwise method was developed to divide the entire service area into a number of service zones with optimal areas. A case study was conducted in northwest Calgary to show the applicability of the proposed model. The results of the case study are summarized below,

- The optimal headway, optimal area of service zone and minimum average cost per passenger increased as the line-haul travel time increased.
- The service area was divided into 13 service zones using the proposed model, with local travel time estimated by the model in Daganzo (1984a). The optimal headway of the service zones was from 0.54~0.67 hour, and the optimal area of service zones was from 4.97~17.29 km².

- The service area was divided into 14 service zones using the proposed model with local travel time estimated by the model in Clarens and Hurdle (1975). The optimal headway of the service zones ranged from 0.71 hour to 0.98 hour, and the optimal area of the service zone ranged from 4.12 km² to 16.45 km².
- Optimal vehicle headway was slightly sensitive to demand density, hourly passenger waiting time cost, hourly operation cost, and bus local speed, as well as average time serving one unit area of the service zone, and non-sensitive to hourly passenger riding time cost.
- Optimal area of the service zone was slightly sensitive to demand density, hourly passenger waiting time cost, hourly passenger riding time cost and bus local speed, as well as average time serving one unit area of the service zone, and non-sensitive to hourly bus operation cost.
- Minimum average cost per passenger was slightly sensitive to demand density, hourly passenger waiting time cost, hourly passenger riding time cost, hourly operation cost and bus local speed, as well as average time serving one unit area of the service zone.

Chapter Five: Conclusions and Recommendations for Future Research

5.1 Conclusions

Two approaches (mathematical models) were successfully developed and applied in this research to optimize DRT systems. With the first approach, a DRT system was optimized based on minimizing the average cost per passenger function over the entire service area, while with the second approach, a DRT system was optimized based on minimizing the average cost per passenger for each service zone. Furthermore, zone division procedures were developed successfully.

The main conclusions of approach one are:

- The model considered two distance measurements (Manhattan and Euclidean).
- Distance on a line-haul route and non-stop local route varied as the location of bus entry points changed.
- Average cost per passenger varied greatly from the location of entry points.
- In the studied area (northwest Calgary), the intersection of 10 St. S.W. and Memorial Dr. N.W. was the best entry point leading to the minimum average cost per passenger of \$14.52.
- The studied area was divided into 13 zones through the best entry point, with optimal service zone area of 10.75 km², bus capacity of 27 seats per vehicle and headway of 0.52 hour respectively.
- A multiple entry point scenario was considered, where each service zone was assigned its own best bus entry point by minimizing the average round trip time on its line-haul route

and non-stop local route. In a multiple entry point scenario, the minimum average cost per passenger was 14.6% less expensive than the one for the one entry point scenario.

- Optimal bus capacity was slightly sensitive to fixed coefficient of bus operation time cost, and non-sensitive to demand density and hourly passenger waiting time cost, as well as bus local speed.
- Optimal area of service zone was slightly sensitive to demand density and hourly passenger waiting time cost, as well as bus local speed, and non-sensitive to fixed coefficient of bus operation time cost.
- Minimum average cost per passenger was slightly sensitive to bus local speed and fixed coefficient of bus operation time cost, and non-sensitive to demand density and hourly passenger waiting time cost.

The main conclusions of approach two are:

- The study showed positive correlation between line-haul travel time and optimal headway/optimal area of service zone/minimum average cost per passenger.
- In the case study, with local travel time estimated based on Daganzo (1984a) the studied area was divided into 13 service zones with the optimal headway of service zones varying from 0.54 hour to 0.67 hour, the optimal area of service zones varying from 4.97 km² to 17.29 km² and the minimum average cost per passenger varying from \$10.17 to \$15.48.

With local travel time estimated based on Clarens and Hurdle (1975) , the studied area were partitioned into 14 zones with optimal headway ranging from 0.71 hour to 0.98 hour, the

optimal area of service zones ranging from 4.12 km² to 16.45 km², and minimum average cost per passenger ranging from \$9.67 to \$15.1.

Compared to the local travel time estimated by the Daganzo (1984a) method, the results with local travel time estimated by the Clarens and Hurdle (1975) method are relatively smaller in terms of optimal area of service zone and minimum average cost per passenger.

- Optimal vehicle headway was slightly sensitive to demand density, hourly passenger waiting time cost, hourly operation cost and bus local speed as well as average time serving one unit area of the service zone, and non-sensitive to hourly passenger riding time cost.
- The optimal area of service zone was slightly sensitive to demand density, hourly passenger waiting time cost, hourly passenger riding time cost and bus local speed, as well as average time serving one unit area of the service zone, and non-sensitive to hourly bus operation cost.
- Minimum average cost per passenger was slightly sensitive to all of the tested system parameters including demand density, hourly passenger waiting time cost, hourly passenger riding time cost, hourly operation cost and bus local speed, as well as average time serving one unit area of the service zone.

In order to have a better understanding of two approaches, some of the results in the two case studies are shown in Table 5.1.

Table 5.1 Case study results of the two approaches

		Approach 2	
Approach 1		With Daganzo (1984)'s method	With Clarens and Hurdle (1975)'s method
Zone number	13	13	14
Optimal zone area	10.75 km ²	4.97 ~ 17.29 km ²	4.12 ~ 16.45 km ²
Optimal headway	0.52 hour	0.54 ~ 0.67 hour	0.71 ~ 0.98 hour
Minimum average cost per passenger	One bus entry		
	point	\$14.52	
		\$10.17 ~ \$15.48	\$9.67 ~ \$15.1
	Multi bus entry		
	points	\$12.4	

5.2 Recommendations for future research

The following suggestions for future research are recommended:

- Conduct surveys for demand density prediction

Demand densities adopted in the two case studies were estimated based on existing public transit data and annual passenger counts at the terminal. However, prior to implementing a real-world transit system, demand density should be carefully predicted based on large-scale surveys, in order to understand how passengers respond to such

systems and what they want. Then transit planners could have a clear picture of the demand densities varying by geography and time periods.

- Expand the homogeneous environment to heterogeneous in the first model

System parameters such as demand density, passenger and bus operation time cost as well as bus local travel speed in the first model were assumed to be a constant over the service area, which could be extended to be heterogeneous. With real-world data from surveys or existing transit systems, the DRT system could be better optimized.

- Consider multiple bus entry points into an analytical solution in the first model

In the first model, a multiple bus entry points system was proposed in the case study to compare with a one bus entry point system in terms of minimum average cost per passenger. Further optimization of multiple bus entry points system could be explored in future work.

- Integrate DRT systems with existing mass transit systems

It may be advantageous to incorporate DRT systems with traditional FRT systems especially when the destinations of DRT users are centred around mass transit systems. This may offer a comprehensive solution for transit network gaps. Using this concept, DRT services could be expanded to more market scenarios, such as feeder bus services during peak hours.

- Extend the travel pattern from “many-to-one” to “many-to-few”

A few more terminals could increase the attractiveness of the transit system compared to one single terminal.

APPENDIX A: DAGANZO'S MODEL

For a service zone with an approximate width w containing a large number of uniformly, randomly scattered points n_p , the expected distance between two consecutive points can be calculated based on probability distribution function,

$$d_w = \frac{w}{3} + \frac{1}{\delta w} \text{ for Manhattan method} \quad (\text{A-1})$$

$$d_w = \frac{w}{3} + \frac{1}{\delta w} * g(\delta w^2) \text{ for Euclidean method} \quad (\text{A-2})$$

where the density of the service zone was δ points per unit area, $g(x) = \frac{2}{x^2} [(1+x) \log(1+x) - x]$.

The optimal expected distance between two consecutive points can be obtained by minimizing Equation (A-1 & A-2),

$$d_w^* = 1.15\delta^{-\frac{1}{2}} \text{ for Manhattan method} \quad (\text{A-3})$$

$$d_w^* = 0.90\delta^{-\frac{1}{2}} \text{ for Euclidean method} \quad (\text{A-4})$$

The total tour distance was the product of the number of collection points (n) and the optimal expected distance (d_w^*) between two consecutive points,

$$D_C = n_p d_w^* = n_p * 1.15\delta^{-\frac{1}{2}} = 1.15n_p \left(\frac{n_p}{A}\right)^{-\frac{1}{2}} = 1.15\sqrt{n_p A} \text{ Manhattan method} \quad (\text{A-5})$$

$$D_C = n_p d_w^* = n_p * 0.90\delta^{-\frac{1}{2}} = 0.9n_p \left(\frac{n_p}{A}\right)^{-\frac{1}{2}} = 0.9\sqrt{n_p A} \text{ Euclidean method} \quad (\text{A-6})$$

where A was the area of the service zone.

APPENDIX B: COMMUNITY INFORMATION

Table A-B-1 shows the population and areas for all communities in the service area (northwest Calgary), as well as the average alighting and boarding demand densities calculated based on Equation (4.44).

Table A-B-1 Population, area, alighting and boarding demand density for every community

Community	Population	Area (km ²)	Alighting and boarding demand density (passenger/km ² /hr)
ARBOUR LAKE	10947	4.42	1.39
AURORA BUSINESS PARK	0	2.4	0
BANFF TRAIL	4335	1.45	1.68
BEDDINGTON HEIGHTS	11973	3.2	2.11
BOWNESS	11235	5.61	1.13
BRENTWOOD	6941	2.89	1.35
BRIDGELAND/RIVERSIDE	6010	3.05	1.11
CAMBRIAN HEIGHTS	2150	0.86	1.41
CAPITOL HILL	4465	1.42	1.77
CHARLESWOOD	3669	1.83	1.13
CITADEL	10517	2.74	2.16
COLLINGWOOD	2300	1.56	0.83

Community	Population	Area (km ²)	Alighting and boarding demand density (passenger/km ² /hr)
COUNTRY HILLS	3835	1.89	1.14
COUNTRY HILLS VILLAGE	2691	0.95	1.59
COVENTRY HILLS	17543	4.08	2.42
CRESCENT HEIGHTS	6380	1.63	2.2
DALHOUSIE	9362	3.34	1.58
EDGEMONT	15986	6.62	1.36
GREENVIEW	2113	0.45	2.64
GREENVIEW INDUSTRIAL PARK	130	2.14	0.04
GREENWOOD/GREENBRIAR	957	1.16	0.47
HAMPTONS	7767	3.5	1.25
HARVEST HILLS	7593	2.18	1.96
HAWKWOOD	9828	3.21	1.72
HIDDEN VALLEY	11900	4.31	1.55
HIGHLAND PARK	4139	1.37	1.7
HIGHWOOD	2314	0.95	1.37
HILLHURST	6737	2.07	1.83
HOUNSFIELD HEIGHTS/BRIAR HILL	2972	1.18	1.42

Community	Population	Area (km ²)	Alighting and boarding demand density (passenger/km ² /hr)
HUNTINGTON HILLS	13781	4.77	1.63
MACEWAN GLEN	5029	1.42	1.99
MONTGOMERY	4246	2.95	0.81
MOUNT PLEASANT	5617	1.86	1.7
NORTH HAVEN	2473	0.85	1.64
NORTH HAVEN UPPER	624	0.27	1.3
PANORAMA HILLS	25990	6.33	2.31
PARKDALE	2393	1.05	1.28
POINT MCKAY	1334	0.38	1.98
QUEENS PARK VILLAGE	486	0.62	0.44
RANCHLANDS	7873	2.35	1.89
RENFREW	6550	2.66	1.39
ROSEDALE	1653	0.67	1.39
ROSEMONT	1276	0.45	1.6
SANDSTONE VALLEY	6103	1.8	1.91
SCENIC ACRES	8542	4.37	1.1
SILVER SPRINGS	8959	4.98	1.01
SKYLINE WEST	0	0.83	0
ST. ANDREWS HEIGHTS	1763	1.16	0.86

Community	Population	Area (km ²)	Alighting and boarding demand density (passenger/km ² /hr)
SUNNYSIDE	3852	1.01	2.15
THORNCLIFFE	8931	3.27	1.54
TUXEDO PARK	5119	1.32	2.18
UNIVERSITY DISTRICT	0	1.15	0
UNIVERSITY HEIGHTS	3020	0.82	2.07
UNIVERSITY OF CALGARY	2968	1.5	1.11
VARSITY	12669	6.85	1.04
WEST HILLHURST	6409	2.36	1.53
WINSTON HEIGHTS/MOUNTVIEW	4035	3.01	0.76
STONE 1	0	6.24	0

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