UNIVERSITY OF CALGARY

Solving Multi-objective Optimization Problems in Power Systems Based on Extended Goal Programming Method

by

Syed Sabbir Ahmed

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

CALGARY, ALBERTA

SEPTEMBER, 2013

Abstract

This thesis proposes an approach to solve multi-objective optimization problems in power systems based on the Extended Goal Programming (EGP) method. In the first part, the EGP method is applied to deterministic multi-objective optimal power flow (MOOPF) problem. The results are compared with classical methods and the efficiency of the EGP method is evaluated. A method for ranking the solutions is introduced to help decision makers choose their preferred solution. In the second part, Taguchi's Orthogonal Array Technique (TOAT) and EGP method are jointly applied to solve probabilistic MOOPF problem with load and renewable generation uncertainties. This approach finds a solution that is robust to uncertain variations in load and renewable power generations. An analysis of the significance of generators ramp rate variation with the degree of robustness of the solution is shown. The results are compared with that of deterministic model and the robustness of the solution is evaluated.

Acknowledgements

At first, I would like to take the opportunity to convey my sincere gratitude to Dr. William D. Rosehart for his invaluable guidance and supervision throughout my days in The University of Calgary. Dr. Rosehart, being my mentor, not only guided my research work smoothly, but also helped me to develop myself as a research professional. I feel really very grateful for all of his support that I got here in Calgary.

I am equally thankful to my co-supervisor, Dr. Hamidreza Zareipour for his relentless support. I learned so many things from him that my words are not enough to explain. Whenever I faced troubles, I rushed to him and he made things easy for me. I found him as a true guide who guided me in the right path in terms of my research and professional development. He is the first person who introduced me to this great research community.

I am also thankful to my peers in the university. Yue, Hamid and Greg helped me always through my bad and good times. Pamela and Vivek, being my first two research mates in Calgary, helped me a lot during my course works. Mahdi, Ali, David, Han and Mo were always there to guide me in the right direction and shared their important research experiences.

I am greatly thankful to my Bangladeshi friends in Calgary. Tangim, Subashis, Riad, Shubhrajit, Imran, Ibrahim, Tauhid, Shawn, Hasib, Samiul and Ovy, you guys always gave me a feel that I am in my home in Bangladesh. There are so many loving memories of our friendship, that I cannot forget in my life time.

Last but certainly not the least, my family was always there with their huge support from thousands of miles away from Canada. My parents, my younger brother and my grandmother showered me so much love and care that my words are finding no bounds to express Thanks to almighty for gifting me such an awesome family. My research work in Calgary would never be easy without their endless love and care.

Dedication

"To my beloved parents, Prof. Rashida Begum and Dr. Enayet Hossain who shaped my life with all the hardest efforts possible"

Table of Contents

List of Tables

List of Figures and Illustrations

List of Symbols, Abbreviations and Nomenclature

Symbol	Definition
P_{Gi}, P_{Gj}	Real power generation of ith and jth units
Q_{Gi}, Q_{Gj}	Reactive power generation of ith and jth units
a_i, b_i, c_i, d_i, e_i	Generation cost coefficients of ith unit
$C(P)$	Total generation cost in \$/h
$E(P)$	Total emission in tons/h
P_{loss}	Total transmission loss of system in MW
α_i,β_i,γi,ζ_i,λ_i	Emission cost coefficients
N	Total number of units
B_{ij}	Susceptance of line between ith and jth bus
G_{ij}	Conductance of line between ith and jth bus
P_i	Real power input at the ith bus
Q_i	Reactive power input at the ith bus
P_{Di}	Real power demand at ith bus
Q_{Di}	Reactive power demand at ith bus
V_i, V_j	Voltage at ith and jth buses
θ_{ij}	Current and voltage angle between ith and jth bus
P_{Gi}^{min},P_{Gi}^{max}	Minimum and maximum real power generation limits of ith unit
Q_{Gi}^{min},Q_{Gi}^{max}	Minimum and maximum reactive power generation limits of ith unit
V_i^{min},V_i^{max}	Minimum and maximum voltage limits at ith bus
θ_i^{min},θ_i^{max}	Limits of angle between voltage and current at ith bus

Chapter 1

Introduction

1.1 Overview

Given the increasing complexity of power systems, optimization problems in this field, are mostly multi-objective by nature. Power system planning problems [1],[2], reactive power compensation schemes [3], transmission line expansion problems [4], economic-emission load dispatch problems [5], hydrothermal scheduling problems [6], and multi-objective optimal power flow (OPF) problems [7] are well known multi-objective optimization problems in power systems.

Many algorithms have been developed for multi-objective optimization problems, including the weighting method [8], the min-max optimum method [9], the weighted minmax method [10], the ϵ-constraint method [8], the utility method [11], the global criterion method [9], and the goal programming method [12]. However, solving multi-objective power systems planning is challenging because of the high complexity of modern power systems with variable loads and intermittent renewable power generators. Thus, developing efficient algorithms to model and solve multi-objective optimization problems in power systems are of interest.

The focus of the thesis is to propose an approach to solve multi-objective optimization problems in power systems based on extended goal programming method. In the first part of the thesis, extended goal programming method is applied to a deterministic multi-objective optimal power flow problem.The results are compared with classical goal programming methods and the efficiency of the extended goal programming method is shown. In the second part, Taguchi's Orthogonal Array Technique (TOAT) and extended goal programming method are jointly applied to a probabilistic multi-objective optimal power flow problem with load and renewable generation uncertainties. The aim of this approach is to find a solution which is robust to uncertain variations in load and renewable power generations. The results are compared with previously mentioned deterministic problem and the robustness of the solution is shown.

1.2 Literature Review

1.2.1 Review of Previous Work on Multi-objective Optimization Problems in Power System

Multi-objective optimization problems in power systems are of importance now-a-days. Planning and control for optimal operation of power systems are complex and with increasing adoption of new technologies (e.g. wind power generators, solar panel etc.) in this arena, making it even more complex day by day. As a result, there arises many conflicting issues and objectives in power system optimization to achieve which is difficult to model as classical single objective optimization problem. Therefore, power system optimization problems are essentially modeled as multi-objective optimization problem in current research.

Power system planning problem is often considered as a well known optimization problem with multiple objectives such as providing all instantaneous electricity demands, fulfilling every system security criterion[2]. Power system planning problems also includes other multiple objectives such as minimizing long term network operation costs, minimizing total investment cost for transmission and generation processes, minimizing negative environmental impact by electricity generation and transportation, etc[1]. Reactive power compensation schemes are also modeled as multi-objective optimization problem where minimizing transmission system power losses and maximizing active power transfer capacity are two conflicting objectives to optimize[3][7]. From the system operators point of view, it is also reasonable to model the power system optimization problem as a multi-objective optimization problem since there are two conflicting objectives such as minimization of investment and operation cost and maximization of benefits for all agents participating in the market. Power system expansion problem is also considered as a multi-objective optimization problem where the three conflicting objectives are such as minimization of network expansion cost, maximization of power system reliability, maximization of effects on energy prices for individual agents [4]. Some other examples of multi-objective optimization problems in power system include environmentally constrained economic dispatch [5], hydrothermal scheduling [6], optimal power flow[7].

1.2.2 Review of Previous Works on Goal Programming Theory

There are solution methods and algorithms proposed in the literature to solve multi-objective optimization problems. Among them, goal programming[12], [13] is a tool to solve multi-objective optimization problems. It was first proposed by Abraham Charnes and William Cooper in 1961. At the beginning stage, goal programming was limited to linear multiple-objective problems. The basic theory of goal programming was extended and modified by Ijiri et al.[14]. With these extension, goal programming was made capable to cope with nonlinear and integer problems and also provided an effective economic interpretation of its result by introducing the concept of multi-dimensional dual. However, there are three major goal programming variants in the literature namely, lexicographic/preemptive,weighted and min-max/Chebychev goal programming[15]. In the lexicographic goal programming method, solutions are achieved in a number of priority levels. Each priority level contains a number of unwanted deviations to be minimized. Preferential weights are assigned to design the relative importance of the minimization of the associated deviation variable. If there are any unimportant deviation variables which's minimization is neglected, a preferential weight of zero is assigned. Minimization of deviation variables at a higher priority level is considered more important than that of deviation variables at a lower priority level. This produces a series of sequential optimizations. As the minimal values of the higher priority level optimizations must be maintained, each new optimization has a reduced feasible region than the previous one. Lexicographic goal programming technique is well suited with the problems where decision maker has a pre-defined ordering of the goals in mind and does not want any direct trade-off comparisons between goals. On the other hand, direct trade off between all unwanted deviation variables are allowed in weighted goal programming variant. It is also known as non pre-emptive goal programming. Hence weighted goal programming variant gives the decision maker more flexibility than pre-emptive goal programming approach. Chebychev/min-max goal programming, the third major goal programming variant is first proposed by Flavell in 1976[16]. This method uses underlying Chebychev means (L_∞) of measuring distance. Here, the maximal deviation from any goal is minimized. Therefore, this variant is often regarded as Min-max goal programming. Measuring the Chebychev (L_∞) distance actually points out the balance between the solutions. In min-max method, the decision maker tries to achieve a balance between achievement of different set of goals where in lexicographic method, some goals are given priorities over others deliberately or in weighted goal programming, where a set of decision variables is chosen, regardless of maintaining any balance, just to make the achievement function lowest [14],[15].

A new goal programming variant is proposed by Carlos Romero in 2001 which is known as extended goal programming[14][17]. The importance and potential of this new method is justified from the utility representation of goal programming. From the utility theory point of view, the weighted and min-max solutions represent two opposite poles. The utility interpretation of weighted goal programming indicates the maximization of a separable and additive utility function in the number of attributes/objectives considered. That also states that weighted goal programming solution provides maximum achievement which means maximum efficiency. On the other hand, the utility interpretation of min-max method indicates the minimization of the maximum deviation from the goal. It states that min-max method provides the a balanced solution between achievements of different goals which means maximum equity. It also shows that the weighted goal programming solution could be extremely biased towards some of the goals while min-max solution could provide poor aggregated performance between different goals [14]. In extended goal programming model, the achievement function is formed as such that it encompasses the weighted and min-max goal programming variants in a unified format. Since the weighted and min-max formulation gives solution which resides in two completely different poles, extended goal programming formulation provides a compromise solution between these two opposite concepts of optimizing efficiency and equity which is very much efficient while dealing with conflicting goals [17].

1.2.3 Review of Previous Works on Goal Programming to Solve Multi-objective Optimization Problems in Power Systems

Goal programming algorithms have been applied to solve multi-objective problems in power system research. Among them, weighted goal programming and min-max goal programming are common and used frequently by the researchers in the literature [18].

In [7], a multi-objective optimal power flow model is proposed to optimize active and reactive power dispatch while maximizing voltage security at the same time. The proposed model is solved using interior point method via goal programming and linearly combined objective functions.

In [19], a fuzzy multi-objective mixed integer linear programming model is proposed for secondary voltage control method. Three control objectives have been taken into account: voltage of pilot node should be close enough to the reference value updated by the tertiary voltage control, deviation of the important bus voltage should be least and the reactive power output of the control generators should be adjusted according to the Mvar capabilities. Fuzzy goals are adopted for each objective and ranking of the priority of each objective is proposed based on fuzzy logic method. The entire problem is solved by fuzzy goal programming method.

A weighted goal mixed integer programming model for rescheduling of generation power in deregulated markets is proposed in [20]. There are several objectives in the optimization model which includes to achieve the energy and reserve programs based on market bidding, to minimize the total cost and to schedule smooth power changes in the plants. The problem is solved using mixed integer goal programming method and the real size application of the model for a Spanish utility has been presented successfully. The computational time is short and hence it is suitable for use in real time by a company in different steps of the sequential process of markets as decision support for obtaining schedules.

A fuzzy goal programming formulation of multi-objective optimal power flow is introduced in [21].The membership functions of the defined fuzzy goals are characterized first for measuring the degree of achievement of the aspiration levels of the goals specified in the decision making context. In the solution process, the genetic algorithm is employed to the fuzzy goal programming formulation of the problem for achievement of the highest membership value of the defined membership functions to the extent possible in the decision-making environment. In the GA based solution search process, the conventional Roulette wheel selection scheme [22], arithmetic crossover and random mutation are taken into consideration to reach a satisfactory decision.

In [23], a multi-objective problem of optimal planning of distributed generator units in the distribution system is formulated and evaluated using goal programming method along with genetic algorithm.

In [24], Genetic Algorithm (GA) based fuzzy goal programming (FGP) technique to multi-objective optimal planning of electric power generation and dispatch problem in power system operation and planning phases is presented. This method is similar to that of [21] but the problem formulation is slightly different since there are three objectives, minimum fuel cost, minimum emission and minimum voltage deviation. The main advantage of the proposed approach is that the use of FGP incorporates the major source of uncertainty in optimal dispatch problem.

A multi-objective fuzzy nonlinear goal programming approach is proposed in [25] to minimize wastage of electricity at the source of generation and supply line and maximize sales and profit. The model is implemented in the coal based thermal power plant in India in order to generate electricity at a controlled cost which in turn can maximize its sales and profit. Each objective function is optimized separately subject to the constraints of the problem. Optimum values of each objective are calculated. The value of remaining objective functions at each cases are calculated and a pay off matrix is constructed. From the pay off matrix, lower and upper bounds of the objectives are calculated. The membership functions of the maximization of the objective functions and the minimization of the objective functions are constructed and zimmermann model of nonlinear programming is formulated and solved.

In [26], a new model based on goal programming is proposed as constant voltage PQ model to solve the optimal reactive power flow in wind generation integrated system. In the CVPQ model, the wind farm bus is considered as PQ node with constant voltage to isolate the wind farms from system influences. Finally the goal programming model of optimal reactive power flow is solved by using a genetic algorithm since it can cope with complex nonlinear goal programming.

Artificial intelligence technique is integrated with sensitivity analysis for the formulation and resolution of the optimal reactive power flow problem in [27]. The objectives and constraints are transformed into fuzzy sets and the problem is solved by a fuzzy goal programming algorithm.

In [28], a maintenance scheduling problem of thermal generating units under economic and reliability criteria is solved by goal programming model. The problem is formulated as a large scale mixed integer programming problem implemented in the mathematical programming language GAMS and solved by using Optimization Subroutine Library (OSL). A sequential goal programming is used to solve the problem where economic criterion optimization is taken care of in the first place and then the reliability criterion optimization is done.

In [29], economic-emission load dispatch (EELD) problem is solved through linear and non-linear goal programming algorithms. For nonlinear goal programming formulation, Box complex method is used to minimize the achievement functions. The advantage is the operating limits on the decision variables will be taken care of while generating the feasible points. Equality constraints are transformed into inequality constraints since box complex method cannot handle them. Linear goal programming formulation is solved using sequential or multiphase linear goal priming method. A new algorithm is proposed for nonlinear formulation which overcomes the drawback of selection of step size or starting feasible point.

In [30], Fuzzy Goal Programming (FGP) is adopted to handle the multi-objective distributed generator (DG) placement problem incorporating the voltage characteristics of each individual load component. The original objective functions and constraints are transformed into the multi-objective function with fuzzy sets by FGP. The solution of the transformed multi-objective function with fuzzy sets is searched by Genetic Algorithm (GA).

Three objectives of Multi-objective Optimal Power Flow (MOPF) problem: cost of generation, system transmission losses, environmental pollution are considered and MOPF problem is attempted sequentially using sequential goal programming (SGP) in [31], [32]. Zangwill transformation has been used for transforming a constrained multi-objective optimal power flow problem into a sequence of unconstrained problems and a standard algorithm of quasi Newton variable metric method has been used for unconstrained minimization. Regret Analysis has been carried out to determine the optimal strategy. The optimal strategy is one for which the regret is minimum.

In [33], a goal programming model for the optimal mix and location of renewable energy plants in the north of Spain is proposed. As different types of plants can be placed in each location, the goal is to locate one plant in each place, maximizing the number of plants that are matched with comparable locations, in a way that the total deviations from goals are minimized. The problem was solved by using Lingo.

A fuzzy mixed integer goal programming approach for cooking and heating energy planning in rural India is introduced in [34]. The solutions provide energy resource allocations at micro level with minimized cost, minimized emission, maximized social acceptance and maximized use of local resources.

A fuzzy goal programming model is introduced to develop Oregon's renewable energy portfolio in [35].Portfolio analysis indicates to minimize the costs and maximize the benefits, therefore it has a multi-objective character here.

In [36], the energy allocation process is looked at from two points of view: economy and environment. The economic objectives include costs, efficiency, energy conservation, and employment generation. The environmental objectives consider environmental friendliness factors. The objective functions are first quantified and then transformed into mathematical language to obtain a multi-objective allocation model based upon pre-emptive goal programming techniques. The proposed method allows decision-makers to encourage or discourage specific energy resources for the various household end-uses.

In [37], a modified extended goal programming model is used with interval programming to model the Economic Emission Load Dispatch (EELD) problem. In this paper, the target goals are considered as interval-valued numbers. The solution is sought then using genetic algorithm.

A multi-objective electricity planning problem in Spain is formulated using compromise programming model in [38], . At first, the objective functions are converted into an equivalent compromise programming model. After, the compromise programming model is modified and converted into the extended goal programming model to achieve the set of best-compromise solutions.

1.2.4 Review of Previous Work on Solving Multi-objective Optimization Problems in Power System under Uncertainties

Most of the works discussed are developed to solve deterministic multi-objective optimization problems. Extensions of these approaches have been proposed in the literature to handle the uncertain factors in the optimization problem. There are a number of methods proposed in the literature to handle uncertainty in goal programming models. In [39], three approaches are proposed including the uncertain random expected value goal programming, the min-max chance-constrained goal programming and dependent-chance goal programming. In [40], a chance-constrained integer goal programming model for capital budgeting considering the uncertainty in product demand is proposed. In [41], a stochastic chance-constrained goal programming model and algorithm is proposed for oilfield measures. A chance constrained fuzzy goal programming model has been proposed in [42]. A scenario based goal programming model has been proposed in [43] to handle the uncertain factors. In [44], a stochastic goal programming approach based on fuzzy beta is proposed for portfolio selection problem under uncertainty. In [45], a transportation network design problem (NDP) with multiple objectives and demand uncertainty is modeled using stochastic goal programming. A stochastic goal programming model based on mean-variance minimization is proposed in [46]. In [47], a fuzzy goal programming model based on measuring goal attainment value is proposed. A decision support model to help public water agencies allocate surface water among farmers based on stochastic goal programming is proposed in [48].In [49], a location allocation problem under demand and supply uncertainty is modeled based on stochastic chance constrained goal programming theory. An uncertain multi-objective job shop problem is modeled based on fuzzy goal programming in [50].

In [51], a short-term unit commitment problem in a deregulated market environment has been modeled and solved using fuzzy mixed integer goal programming. In this paper, the goals for various objectives are assumed as uncertain due to their characteristics and modeled as fuzzy numbers. Finally, the proposed fuzzy mixed-integer goal programming model is converted into the equivalent crisp model and solved. In [52], a multi-objective optimal power flow problem is modeled based on fuzzy goal programming and then solved using genetic algorithm. Here also the goals are assumed as uncertain and thus modeled as fuzzy numbers. In [53], a power generation and dispatch problem is modeled and solved based on fuzzy goal programming theory. In these above research, uncertainties associated with load and renewable generations are not considered. In [54], a single objective DC optimal power flow is solved. Taguchi's Orthogonal Array Technique is used to select the optimal scenarios and finall the problem is solved by interior point method.

1.2.5 Summary of Reviews

Most of the previous works reviewed in earlier sections on solving multi-objective optimization problems, use classical goal programming methods such as weighted goal programming, min-max goal programming etc. However, extended goal programming has not been widely used for solving multi-objective optimization problems in power systems. The scope of this thesis is to implement extended goal programming method to solve multi-objective optimization problems in power systems and show the effectiveness of the method compared to other major goal programming variants.

1.3 Research Objectives and Motivation

In the modern power systems with variable loads and uncertain renewable power generators, it is a key issue to obtain an optimal dispatch schedule which can satisfy multiple system objectives (e.g. minimizing generation cost, emission, transmission loss etc.) simultaneously and also robust to the variations in load and renewable power generations. In this thesis, extended goal programming is applied to solve a multi-objective AC optimal power flow problem. In addition, with the integration of wind and solar power in power systems, methods to address the uncertainty of these resources in system operation and planning is becoming important. The later part of this thesis is dedicated to solve a multi-objective AC optimal power flow problem in power systems under uncertainties.

1.3.1 Solving multi-objective optimization problems in power systems

Solving multi-objective optimization problems in power system is important. Multi-objective optimization techniques in power system have number of advantages such as allowing the management of different objectives, simplifying the decision making process by trading off among conflicting objectives, providing information on the consequences of the decision considering all the objectives considered. This thesis considers a multi-objective AC optimal power flow problem to minimize generation cost, emission and transmission power loss simultaneously. Recent study shows that total reported green house gas emissions from 165 Alberta facilities across 15 industrial sectors equaled 122.5 mega tonnes of carbon dioxide equivalent (Mt CO_2) [56]. The consequences of this emission are concerning. Another study reports that coal-fired power generation is likely to cause thousands of early deaths in Alberta and cost the province hundreds of millions of dollars [57] [58]. The total transmission loss in Alberta power systems is estimated 2759 GWhr/ year. This loss costs 240 million CAD annually [59]. Minimizing emission and transmission loss along with generation cost is desired. However, since these objectives are conflicting in nature, it is difficult to find one single solution to the problem. Thus, compromise solution is necessary.

Chapter 3 of this thesis proposes an approach to solve multiple conflicting objectives of an AC optimal power flow problem based on extended goal programming theory. Extended goal programming is a method that can provide a compromise solution which is a trade off between maximum achievement of goals and maximum deviation from any of the goal. In this chapter, first a multi-objective AC optimal power flow problem has been transformed to an extended goal programming formulation. After that, target goals for each of the objectives are set using the solution of single objective AC optimal power flow problem for each of the objectives and optimal weights are calculated using analytic hierarchy process. Finally the problem is solved finding a compromise solution and results are compared with other classical goal programming methods.

A contribution of the work in this chapter is an extended goal programming formulation of multi-objective AC optimal power flow problem. Another contribution is an analysis of significance of the controlling parameter, Z, showing the efficiency of the model comparing to other classical goal programming models. The last contribution of this chapter is proposing a ranking strategy to choose the best routine based on decision makers' priorities.

1.3.2 Solving multi-objective optimization problems in power system under uncertainties

A recent report says globally over 27,000 MW of new wind generation capacity was added in 2008 which was 36% more than in 2007 [60]. This growing number of intermittent renewable generator penetration may potentially impact the stability of power system operation.In addition, the impact of uncertain load in the system makes the optimal operation of the system more difficult. Failing to handle these uncertainties, may cause system outage and anticipate huge loss to the government. Thus solving multi-objective optimization problems in power systems under uncertainties is important.

Chapter 4 of this thesis considers the deterministic multi-objective optimal power flow problem in Chapter 3 with added uncertainties. This chapter proposes an approach to solve multi-objective optimal power flow problem under load and renewable generation uncertainties. In this chapter, Taguchi's Orthogonal Array Testing technique is used to select a minimum number of testing scenarios with good statistical information in the uncertain space. After selecting the optimal scenarios, the uncertain multi-objective optimal power flow problem is transformed into a robust multi-objective optimization problem. Finally, the problem is modeled based on extended goal programming theory and solved. It is shown that the proposed method can provide a solution that is robust uncertain variations in load and renewable generations and also can satisfy all the conflicting objectives.

A contribution of the work in this chapter is modeling a multi-objective AC optimal power flow problem under uncertainties using Taguch's Orthogonal Array Technique and extended goal programming method. The second contribution is an analysis of the significance of ramp rate variation with the degree of robustness of the solution. The final contribution of this chapter is a comparison between robust and non-robust solution from the extended goal programming point of view showing the efficiency of the proposed approach.

1.4 Structure of the Thesis

Chapter 2: A detail background discussion on modeling single and multi-objective optimization problems from power systems point of view is given in this chapter. Background information is also given on basics of goal programming and its different variants. A discussion on Analytic Hierarchy Process (AHP) is given and detail methodology is shown step by step. Finally, a brief discussion is given on Taguchi's Orthogonal Array Technique (TOAT).

Chapter 3: An approach based on extended goal programming to solve multi-objective optimal power flow problem is proposed in this chapter. Analytic Hierarchy Process has been applied to choose optimal weights. A ranking strategy is also presented to choose the best routine based on the decision makers' priorities. The model is tested using the standard IEEE-30 and IEEE-118 bus systems. Finally the results are shown and the efficiency of the approach is verified comparing with other classical methods.

Chapter 4: In this chapter, an approach to solve multi-objective optimal power flow problem under load and renewable generation uncertainties are presented. Taguchi's Orthogonal Array Technique (TOAT) is used to select optimal scenarios. After that the uncertain multi-objective optimal power flow problem has been transformed into a robust multi-objective optimal power flow problem. Finally the problem is modeled using extended goal programming and solved. The results are shown and the efficiency of the algorithm is verified.

Chapter 5: This chapter summarizes the main contributions and conclusions of this thesis.

Chapter 2

Background Review

2.1 Introduction

In this thesis, different methods and techniques are used to model multi-objective optimization problem in the context of power systems, to evaluate the solution quality and to handle the uncertainties in the system for making the model robust.

In this chapter, relevant background information is provided on the the basics of modeling different types of optimization problems and optimal power flow problems, basics of goal programming, analytic hierarchy process and Taguchi's Orthogonal Array Technique.

2.2 Basics of Modeling Simple Optimization Problem

An optimization problem has been well defined in the literature. An optimization is the problem of finding the best solution out of a set of feasible solution. The solution is called optimal solution for that specific problem. In this section, basic modeling of simple optimization problems are discussed. Optimization problems can be divided into two categories based on the number of objectives. They are single objective and multi-objective. The modeling of these two types of problems is as follows:

2.2.1 Single Objective Optimization Problem

Single objective optimization problems has one objective to minimize or maximize. Thus the name Single Objective Optimization Problem (SOP) has come. The general form of single objective optimization problem is written as follows:

$$max \quad f(x) \tag{2.1}$$

$$subject \quad to :$$

$$v_j(x) = 0, \quad j = 1, 2, 3, ..., p \tag{2.2}$$

$$g_j(x) \leq 0, \quad j = 1, 2, 3, ..., p \tag{2.3}$$

which maximizes a real valued function f of $x = (x_1, x_2, x_3, ..., x_n)$ subject to sets of constraints $g_j(x) \leq 0, j = 1, 2, 3, ..., p$ and $v_j(x) = 0, j = 1, 2, 3, ..., p$. In this formulation, x is decision vector, and $x_1, x_2, ..., x_n$ are decision variables. The function f is called the objective function. The set S,

$$S = \{x \quad \varepsilon \quad \Re^n \mid g_j(x) \leq 0, j = 1, 2, ..., p\} \tag{2.4}$$

is called feasible set. An element x in S is called a feasible solution.

A feasible solution x^* is called the optimal solution of single objective optimization problem if and only if,

$$f(x^*) \geq f(x) \tag{2.5}$$

2.2.2 Multi-objective Optimization Problem

Single objective optimization problem is related to maximizing or minimizing a single function subject to a number of constraints. However, it has been increasingly recognized that many real-world decision making problems involve multiple, non-commensurable, and conflicting objectives which should be considered simultaneously. As an extension, multi-objective optimization problem is defined as a means of optimizing multiple objective functions subject to a number of constraints, i.e.,

$$max \quad [f_1(x), f_2(x), ..., f_m(x)] \tag{2.6}$$

$$subject \quad to:$$

$$v_j(x) = 0, \quad j = 1, 2, 3, ..., p \tag{2.7}$$

$$g_j(x) \leq 0, \quad j = 1, 2, 3, ..., p \tag{2.8}$$

where $f_i(x)$ are objective functions, $i = 1, 2, ..., m$, and $g_j(x) \leq 0$ and $v_j(x) = 0$ are system constraints, $j = 1, 2, 3, ..., p$.

When the objectives are in conflict, there is no optimal solution that simultaneously maximizes all the objective functions. For this case, a concept of pareto solution is employed, which means that it is impossible to improve any one objective without sacrificing on one or more of the other objectives.

A feasible solution x^* is said to be a Pareto solution if there is no feasible solution z such that,

$$f_i(x) \geq f_i(x^*), \quad i = 1, 2, 3, ..., m \tag{2.9}$$

and $f_j(x) \geq f_j(x)$ for at least one index j.

If the decision maker has a real-valued preference function aggregating the m objective functions, then the aggregating preference function subject to the same set of constraints can be maximized. This model refers to as a compromise model whose solution is called a compromise solution.

A well known compromise model is set up by weighting the objective functions, i.e.,

$$max \quad \sum_{i=1}^{m} \lambda_i f_i(x) \tag{2.10}$$

$$subject \quad to:$$

$$g_j(x) \leq 0, \quad j = 1, 2, 3, ..., p \tag{2.11}$$

where the weights $\lambda_1, \lambda_2, ..., \lambda_m$ are non-negative numbers with $\lambda_1 + \lambda_2 + ... + \lambda_m = 1$. Note that the solution of (2.8) must be a pareto solution of the original problem.

2.3 Optimal Power Flow Problem

Optimal Power Flow (OPF) is considered one of the most efficient and powerful analyzing tool for the economic operation of power system[61]. Optimal power flow (OPF) in electric power systems is a method of determining the optimal settings of various control variables for minimizing generation cost, voltage deviations, emission cost, transmission losses etc. while satisfying power flow constraints. OPF problem is normally static, non-linear, multi-objective in nature.

2.3.1 Single Objective Optimal Power Flow Problem

A general single objective OPF problem can be expressed as follows,

$$min \quad f(x_1, x_2, ..., x_n) \tag{2.12a}$$

$$St. \quad g_m(x_1, x_2, ..., x_n) = a \tag{2.12b}$$

$$b_n^{min} \leq x_n \leq b_n^{max} \tag{2.12c}$$

where $f(x_1, x_2, ..., x_n)$ is the function to be minimized, $x_1, x_2, ..., x_n$ are the control variables associated with the system, $g_m(x_1, x_2, ..., x_n)$ are the power balance equations of the system, b_n^{min} and b_n^{max} are the lower and upper limits of the control variables respectively.

A sample single objective OPF problem is as follows,

$$min \quad C(P) = \sum_{i=1}^{N}(a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \left|d_i sin(e_i(P_{Gi}^{min} - P_{Gi}))\right| \tag{2.13a}$$

$$s.t. \quad P_{Gi} - P_{Di} - V_i \sum_{j \in i} V_j(G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{2.13b}$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j \in i} V_j(G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{2.13c}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{2.13d}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{2.13e}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{2.13f}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{2.13g}$$

where P_{Gi}, P_{Gj} are real power generation of ith and jth units, Q_{Gi}, Q_{Gj} are reactive power generation of ith and jth units, a_i, b_i, c_i are generation cost coefficients of ith unit, $C(P)$ is total generation cost in \$/h, N is total number of units, B_{ij} is susceptance of line between ith and jth bus, G_{ij} conductance of line between ith and jth bus, P_{Di} is real power demand at ith bus, Q_{Di} is reactive power demand at ith bus, V_i, V_j are voltage at ith and jth buses, θ_{ij} are current and voltage angle between ith and jth bus, $P_{Gi}^{min}, P_{Gi}^{max}$ are minimum and maximum real power generation limits of ith unit, $Q_{Gi}^{min}, Q_{Gi}^{max}$ are minimum and maximum reactive power generation limits of ith unit, V_i^{min}, V_i^{max} are minimum and maximum voltage limits at ith bus, $\theta_i^{min}, \theta_i^{max}$ are minimum and maximum limits of angle between voltage and current at ith bus.

2.3.2 Multi-objective Optimal Power Flow Problem

Due to the complex operational perspective of the power system, it is obvious that there are objectives which are to be optimized jointly. Here lies the importance of the multi-objective optimal power flow in which two or more objectives are jointly minimized satisfying the system constraints. A general multi-objective OPF problem can be expressed as follows,

$$min \quad f_i(x_1, x_2, ..., x_n) \qquad i = 1, 2, 3, ...p \tag{2.14a}$$

$$St. \quad g_m(x_1, x_2, ..., x_n) = a \quad m = 1, 2, 3, ...q \tag{2.14b}$$

$$b_n^{min} \leq x_n \leq b_n^{max} \tag{2.14c}$$

where $f_i(x_1, x_2, ..., x_n)$ are the objective functions to be minimized, p is the total number of objectives to be minimized, $x_1, x_2, ..., x_n$ are the control variables associated with the system, $g_m(x_1, x_2, ..., x_n)$ are the power balance equations of the system, q is the total number of power balance constraints, b_n^{min} and b_n^{max} are the lower and upper limits of the control variables respectively.

A sample multi-objective optimal power flow problem is as follows,

$$min \quad C(P) = \sum_{i=1}^{N}(a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \left|d_i sin(e_i(P_{Gi}^{min} - P_{Gi}))\right| \tag{2.15a}$$

$$min \quad E(P) = 10^{-2}(\sum_{i=1}^{N} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \zeta_i exp(\lambda_i P_{Gi}) \tag{2.15b}$$

$$min \quad P_{loss} = \sum_{k=1}^{N_L} g_k[V_i^2 + V_j^2 - 2V_i V_j cos(\delta_i - \delta j)] \tag{2.15c}$$

$$s.t. \quad P_{Gi} - P_{Di} - V_i \sum_{j \in i} V_j(G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{2.15d}$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j \in i} V_j(G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{2.15e}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{2.15f}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{2.15g}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{2.15h}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{2.15i}$$

where P_{Gi}, P_{Gj} are real power generation of ith and jth units, Q_{Gi}, Q_{Gj} are reactive power generation of ith and jth units, a_i, b_i, c_i are generation cost coefficients of ith unit, $C(P)$ is total generation cost in \$/h, $E(P)$ is total emission in tons/h, P_{loss} is total transmission loss of system in MW, $Bk_{ij}, Bk_{0i}, Bk_{00}$ are Kron's loss coefficients, $\alpha_i, \beta_i, \gamma i, \zeta_i, \lambda_i$ are emission cost coefficients, N is total number of units, B_{ij} is susceptance of line between ith and jth bus, G_{ij} conductance of line between ith and jth bus, P_{Di} is real power demand at ith bus, Q_{Di} is reactive power demand at ith bus, V_i, V_j are voltage at ith and jth buses, θ_{ij} are current and voltage angle between ith and jth bus, $P_{Gi}^{min}, P_{Gi}^{max}$ are minimum and maximum real power generation limits of ith unit, $Q_{Gi}^{min}, Q_{Gi}^{max}$ are minimum and maximum reactive power generation limits of ith unit, V_i^{min}, V_i^{max} are minimum and maximum voltage limits at ith bus, $\theta_i^{min}, \theta_i^{max}$ are minimum and maximum limits of angle between voltage and current at ith bus.

2.3.3 Probabilistic Single Objective Optimal Power Flow Problem

The optimal power flow problems discussed is Section 2.3.1 and 2.3.2 are deterministic in nature. But in a real-life power system optimization problems, there are a number of uncertain parameters exist. Two of the most common uncertain parameters are load and renewable power generation uncertainties. With the increasing integration of renewable generators into the grid such as wind, solar etc., the optimal power flow problems turn out to be a probabilistic optimization problem. A general probabilistic single objective optimal power flow problem can be expressed as,

$$min \quad f(x_1, x_2, ..., x_n) \tag{2.16a}$$

$$St. \quad g_m(x_1, x_2, \widetilde{x}_3, \widetilde{x}_4 ..., x_n) = a \tag{2.16b}$$

$$b_n^{min} \leq x_n \leq b_n^{max} \tag{2.16c}$$

where $f(x_1, x_2, ..., x_n)$ is the function to be minimized, x_1, x_2 are the controllable variables associated with the system, $\widetilde{x}_3, \widetilde{x}_4$ are the uncertain variables associated with the system, $g_m(x_1, x_2, ..., x_n)$ are the power balance equations of the system, b_n^{min} and b_n^{max} are the lower and upper limits of the control variables respectively.

A sample probabilistic single objective optimization problem is as follows,

$$min \quad C(P) = \sum_{i=1}^{N}(a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \left|d_i sin(e_i(P_{Gi}^{min} - P_{Gi}))\right| \tag{2.17a}$$

$$St. \quad P_{Gi} + \widetilde{P}_{Ri} - \widetilde{P}_{Di} - V_i \sum_{j \epsilon i} V_j (G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{2.17b}$$

$$Q_{Gi} + \widetilde{Q}_{Ri} - \widetilde{Q}_{Di} - V_i \sum_{j \epsilon i} V_j (G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{2.17c}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{2.17d}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{2.17e}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{2.17f}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{2.17g}$$

where P_{Gi}, P_{Gj} are real power generation of ith and jth units, Q_{Gi}, Q_{Gj} are reactive power generation of ith and jth units, a_i, b_i, c_i are generation cost coefficients of ith unit, $C(P)$ is total generation cost in \$/h, N is total number of units, B_{ij} is susceptance of line between ith and jth bus, G_{ij} conductance of line between ith and jth bus, $\widetilde{P}_{Di}$ is uncertain real power demand at ith bus, $\widetilde{Q}_{Di}$ is uncertain reactive power demand at ith bus, $\widetilde{P}_{Ri}$ is uncertain renewable real power generation, $\widetilde{Q}_{Ri}$ is uncertain renewable reactive power generation, V_i, V_j are voltage at ith and jth buses, θ_{ij} are current and voltage angle between ith and jth bus, $P_{Gi}^{min}, P_{Gi}^{max}$ are minimum and maximum real power generation limits of ith unit, $Q_{Gi}^{min}, Q_{Gi}^{max}$ are minimum and maximum reactive power generation limits of ith unit, V_i^{min}, V_i^{max} are minimum and maximum voltage limits at ith bus, $\theta_i^{min}, \theta_i^{max}$ are minimum and maximum limits of angle between voltage and current at ith bus.

2.3.4 Probabilistic Multi-objective Optimal Power Flow Problem

A general probabilistic multi-objective OPF problem can be expressed as follows,

$$min \quad f_i(x_1, x_2, ..., x_n) \qquad i = 1, 2, 3, ...p \tag{2.18a}$$

$$St. \quad g_m(x_1, x_2, \widetilde{x}_3, \widetilde{x}_4 ..., x_n) = a \tag{2.18b}$$

$$b_n^{min} \leq x_n \leq b_n^{max} \tag{2.18c}$$

where $f_i(x_1, x_2, ..., x_n)$ is the function to be minimized, p is the total number of objectives to be minimized, x_1, x_2 are the controllable variables associated with the system, $\widetilde{x}_3, \widetilde{x}_4$ are the uncertain variables associated with the system, $g_m(x_1, x_2, ..., x_n)$ are the power balance equations of the system, b_n^{min} and b_n^{max} are the lower and upper limits of the control variables respectively.

A sample probabilistic multi-objective optimal power flow problem is as follows,

$$min \quad C(P) = \sum_{i=1}^{N}(a_i + b_iP_{Gi} + c_iP_{Gi}^2) + \left|d_isin(e_i(P_{Gi}^{min} - P_{Gi}))\right| \tag{2.19a}$$

$$min \quad E(P) = 10^{-2}(\sum_{i=1}^{N} \alpha_i + \beta_iP_{Gi} + \gamma_iP_{Gi}^2) + \zeta_iexp(\lambda_iP_{Gi}) \quad Tons/hr \tag{2.19b}$$

$$min \quad P_{loss} = \sum_{k=1}^{N_L} g_k[V_i^2 + V_j^2 - 2V_iV_jcos(\delta_i - \delta j)] \tag{2.19c}$$

$$St. \quad P_{Gi} + \widetilde{P}_{Ri} - \widetilde{P}_{Di} - V_i\sum_{j\epsilon i} V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{2.19d}$$

$$Q_{Gi} + \widetilde{Q}_{Ri} - \widetilde{Q}_{Di} - V_i\sum_{j\epsilon i} V_j(G_{ij}sin\theta ij - B_{ij}cos\theta_{ij}) = 0 \tag{2.19e}$$

$$V_i^{min} \le V_i \le V_i^{max} \tag{2.19f}$$

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \tag{2.19g}$$

$$Q_{Gi}^{min} \le Q_{Gi} \le Q_{Gi}^{max} \tag{2.19h}$$

$$\theta_i^{min} \le \theta_i \le \theta_i^{max} \tag{2.19i}$$

where P_{Gi}, P_{Gj} are real power generation of ith and jth units, Q_{Gi}, Q_{Gj} are reactive power generation of ith and jth units, a_i, b_i, c_i are generation cost coefficients of ith unit, $C(P)$ is total generation cost in \$/h, $E(P)$ is total emission in tons/h, P_{loss} is total transmission loss of system in MW, $Bk_{ij}, Bk_{0i}, Bk_{00}$ are Kron's loss coefficients, $\alpha_i, \beta_i, \gamma i, \zeta_i, \lambda_i$ are emission cost coefficients, N is total number of units, B_{ij} is susceptance of line between ith and jth bus, G_{ij} conductance of line between ith and jth bus, $\widetilde{P}_{Di}$ is uncertain real power demand at ith bus, $\widetilde{Q}_{Di}$ is uncertain reactive power demand at ith bus, $\widetilde{P}_{Ri}$ is uncertain renewable real power generation, $\widetilde{Q}_{Ri}$ is uncertain renewable reactive power generation, V_i, V_j are voltage at ith and jth buses, θ_{ij} are current and voltage angle between ith and jth bus, $P_{Gi}^{min}, P_{Gi}^{max}$ are minimum and maximum real power generation limits of ith unit, $Q_{Gi}^{min}, Q_{Gi}^{max}$ are minimum and maximum reactive power generation limits of ith unit, V_i^{min}, V_i^{max} are minimum and maximum voltage limits at ith bus, $\theta_i^{min}, \theta_i^{max}$ are minimum and maximum limits of angle between voltage and current at ith bus.

2.4 Goal Programming

Goal programming, a methodology for the modeling, solution, and analysis of problems having multiple and conflicting goals and objectives, has often been cited as being the workhorse of multiple objective optimization (i.e., the solution to problems having multiple, conflicting goals and objectives) as based on its extensive list of successful applications in actual practice.

2.4.1 Multiplex Model

Multiplex model is the backbone of goal programming formulation. Any multi-objective optimization problem can be transformed into a multiplex model to fit with goal program method. Suppose a optimization problem is given as follows,

$$max \quad f(x_1, x_2) \tag{2.20a}$$

$$St. \quad g_1(x_1, x_2) \leq a \tag{2.20b}$$

$$g_2(x_2) \geq b \tag{2.20c}$$

$$x \geq 0 \tag{2.20d}$$

The multiplex model of the above problem is as follows,

$$min \quad U = (p_1 + n_2), -f(x_1, x_2) \tag{2.21a}$$

$$St. \quad g_1(x_1, x_2) + \eta_1 - \rho_1 = a \tag{2.21b}$$

$$g_2(x_2) + \eta_2 - \rho_2 = b \tag{2.21c}$$

$$x, \eta, \rho \geq 0 \tag{2.21d}$$

Here, a negative deviation variable is added to, and a positive deviation variable is subtracted from, each constraint. In addition,the maximizing objective function is transformed into a minimizing form by simply multiplying the original objective function by a negative one. The new variables (i.e., the negative and positive deviation variables, that have been added to the constraints) indicate that a solution to the problem may result, for a given constraint i, in a negative deviation or a positive deviation or no deviation. That is to say that a goal (be it a hard or soft constraint) can be underachieved, overachieved , or precisely satisfied. In the multiplex formulation, the deviation variables that are to be minimized is appeared in the first (highest priority) term of the achievement function. Once the first term has been minimized, the next term the second term can be dealt with. The algorithm will seek a solution that minimizes the value of this second term, but this must be accomplished without degrading the value already achieved in the higher priority term.

A simple multiobjective problem is given below,

$$max \quad Z_1(x_1, x_2) \tag{2.22a}$$

$$max \quad Z_2(x_1, x_2) \tag{2.22b}$$

$$St. \quad G_1(x_1, x_2) \leq a \tag{2.22c}$$

$$G_2(x_1) \leq b \tag{2.22d}$$

$$G_3(x_2) \leq c \tag{2.22e}$$

$$x \geq 0 \tag{2.22f}$$

Multiplex form of the above problem is given below,

$$min \quad U = (\rho_1 + \rho_2 + \rho_3), (\eta_4, \eta_5) \tag{2.23a}$$

$$St. \quad G_1(x_1, x_2) + \eta_1 - \rho_1 = a \tag{2.23b}$$

$$G_2(x_1) + \eta_2 - \rho_2 = b \tag{2.23c}$$

$$G_3(x_2) + \eta_3 - \rho_3 = c \tag{2.23d}$$

$$Z_1(x_1, x_2) + \eta_4 - \rho_4 = g_1 \tag{2.23e}$$

$$Z_2(x_1, x_2) + \eta_5 - \rho_5 = g_2 \tag{2.23f}$$

$$x, \eta, \rho \geq 0 \tag{2.23g}$$

To transform an objective into a goal, one must assign some estimate (usually the decision makers preliminary estimate) of the aspired level for that goal. It is assumed here that the aspiration level for $Z_1(x_1, x_2)$ is g_1 units while that of $Z_2(x_1, x_2)$ is g_2 units. In the achievement function in eqn. (2.23a), minimization of negative deviation variables η_1, η_2 and η_3 are ignored since function $G_1(x_1, x_2)$, $G_2(x_1)$, $G_3(x_2)$ must be less than of equal to a, b and c, respectively. Similarly, minimization positive deviation variables ρ_4 and ρ_5 are also ignored since function $Z_1(x_1, x_2)$ and $Z_2(x_1, x_2)$ must be greater than target goals g_1 and g_2.

2.4.2 Lexicographic Goal Programming Model

In the lexicographic goal programming model, decision makers prioritize their goals into different priority levels such as 1, 2, 3 etc. Each of this priority level may contain one or more goals. If a priority level contains two or more goals, these goals should be weighted as same. The main idea behind this model is that a lower priority level goals must not be achieved at the expense of higher priority goals. That indicates, if the minimum total weighted deviation of priority 1 goals has a value of N_1, then it must be ensured that this value remains same while looking for to minimize the total weighted deviations of priority 2 goals. Similarly, if the weighted deviation of the priority 2 goals are valued as N_2, then this value must remain same while seeking to minimize the total weighted deviation of priority 3 goals.

Considering a simple problem, where there are 6 goals such as one goal in priority level 1, two goals in priority level 2 and three goals in priority levels 3. Assuming the detrimental deviations from the goals are listed as, D_1, D_2, D_3, D_4, D_5 and D_6, respectively. Weights are assigned as W_1 for priority level 1, W_2, W_3 for priority level 2 and W_4, W_5, W_6 for priority level 3. Based on the concept of lexicographic goal programming model, it will first solve the priority level 1 problem as follows:

$$\begin{aligned} Min \quad W_1D_1 \\ Subject \quad to \quad Goal \quad Equations \\ Functional \quad Constraints \\ Non-negativity \quad Constraints \end{aligned} \tag{2.24}$$

Assuming the solution yields a minimum objective function value, N_1. Then the following priority level 2 problem is solved as:

$$\begin{aligned} Min \quad W_2D_2 + W_3D_3 \\ Subject \quad to \quad Goal \quad Equations \\ Functional \quad Constraints \\ W_1D_1 = N_1 \\ Non-negativity \quad Constraints \end{aligned} \tag{2.25}$$

Assuming the solution yields a minimum objective function value, N_2. Then the priority level 3 problem is solved as:

$$\begin{aligned} Min \quad W_3D_3 + W_4D_4 + W_5D_5 \\ Subject \quad to \quad Goal \quad Equations \\ Functional \quad Constraints \\ W_1D_1 = N_1 \end{aligned} \tag{2.26}$$

$$W_2D_2 + W_3D_3 = N_2$$

$$Non - negativity \quad Constraints$$

The solution to the priority level 3 problem is the final optimal solution to the main goal programming problem.

2.4.3 Weighted Goal Programming Model

Weighted Goal Programming is important when decision makers do not have any pre-emptive ordering of the objective functions. Instead of prioritizing the objective functions in different levels, they assign different weights for deviation variable of each objective function in the single priority level.

Considering a simple problem, where there are 6 goals, all in a single priority level. Assuming the detrimental deviations from the goals are listed as, D_1, D_2, D_3, D_4, D_5 and D_6, respectively. Weights are assigned as W_1, W_2, W_3, W_4, W_5, W_6, respectively. Based on the concept of weighted goal programming model, the problem is modeled as follows:

$$Min \quad W_1D_1 + W_2D_2 + W_3D_3 + W_4D_4 + W_5D_5 + W_6D_6 \tag{2.27}$$

$$Subject \quad to \quad Goal \quad Equations$$

$$Functional \quad Constraints$$

$$Non - negativity \quad Constraints$$

The solution to this problem is the optimal solution of this weighted goal programming formulation. This solution provides the maximum achievement between the different goals based on the weights assigned to each objective function. Realizing the weighted goal programming, makes sense only if the numerical weights can be assigned to the non-achievement of each goal. If the goals are non-commensurable, weighted goal programming model should not be used since it unifies all the weights in the same achievement function.

2.4.4 Min-max Goal Programming Model

The notion of min-max goal programming method is that the solution sought is the one that minimizes the maximum deviation from any single goal. Considering a simple problem, where there are 3 goals, all in a single priority level. Assuming the detrimental deviations from the goals are listed as, D_1, D_2 and D_3, respectively. Weights are assigned as W_1, W_2 and W_3, respectively. A dummy variable D^{max} is used to measure the maximum deviation from any of the goal. Based on the concept of min-max goal programming, the problem is modeled as follows:

$$Min \quad D^{max} \tag{2.28a}$$

$$W_i D_i - D^{max} \leq 0, \quad i = 1, 2, 3 \tag{2.28b}$$

$$Subject \quad to. \quad Goal \quad Equations$$

$$Functional \quad Constraints$$

$$Non-negativity \quad Constraints$$

The constraint in eqn. (2.28b) satisfies that deviation $W_i D_i$ for each objective function i where $i = 1, 2, 3$, must not be greater than the maximum deviation D^{max}. It is clear that the min-max model, as shown, is simply a single objective optimization problem in which we seek to minimize a single variable. In other words, we seek to minimize the single worst deviation from any one of the goals. It also indicates that min-max goal programming method provides a balanced solution where the maximum deviation from each goal is minimized.

2.4.5 Extended Goal Programming Method

According to the arguments developed earlier sections, Section 2.4.3 and 2.4.4, from a preferential point of view the weighted and the Chebyshev goal programming solutions represent two opposite poles. Since the weighted option maximizes the aggregate achievement among the goals considered, the results obtained with this option can be biased against the performance achieved by one particular goal [17]. On the other hand, because of the preponderance of just one of the goals, the min-max model can provide results with poor aggregate performance between different goals. The extreme character of both solutions can lead to some cases to possibly unacceptable solutions by the decision maker. A possible modeling solution for this type of problem consists of compromising the maximum achievement of the weighted goal programming model with the maximum deviation from the goal of the min-max model. Thus, the example of the earlier section, Section 2.4.1, can be reformulated with the help of the following multiplex extended goal programming model:

$$min \quad U = (\rho_1 + \rho_2 + \rho_3), [(1 - Z)\delta + Z(\eta_4 + \eta_5)] \tag{2.29a}$$

$$St. \quad G_1(x_1, x_2) + \eta_1 - \rho_1 = a \tag{2.29b}$$

$$G_2(x_1) + \eta_2 - \rho_2 = b \tag{2.29c}$$

$$G_3(x_2) + \eta_3 - \rho_3 = c \tag{2.29d}$$

$$Z_1(x_1, x_2) + \eta_4 - \rho_4 = g_1 \tag{2.29e}$$

$$Z_2(x_1, x_2) + \eta_5 - \rho_5 = g_2 \tag{2.29f}$$

$$(1 - Z)\eta_4 - \delta \leq 0 \tag{2.29g}$$

$$(1 - Z)\eta_5 - \delta \leq 0 \tag{2.29h}$$

$$\delta, x, \eta, \rho \geq 0 \tag{2.29i}$$

where, the parameter Z weights the importance attached to the minimization of the sum of unwanted deviation variables. For $Z = 0$, we have a min-max goal programming model. For $Z = 1$ the result is a weighted goal programming model, and for other values of parameter Z belonging to the interval (0, 1) intermediate solutions between the solutions provided by the two goal programming options are considered. Hence, through variations in the value of parameter Z, compromises between the solution of the maximum aggregate achievement and the min-max solution can be obtained. In this sense, this extended formulation allows for a combination of goal programming variants that, in some cases, can reflect a decision makers actual preferences with more accuracy than any single variant.

2.4.6 Theory of Extended Goal Programming

The importance of the extended goal programming model arises from the utility representation of goal programming. From the utility point of view, the weighted and min-max models represent two opposite poles. The weighted method provides maximum achievement of the target goals, which means maximum efficiency. The min-max method provides the most balanced solution: it minimizes the maximum deviation from the goals, which means maximum equity. The weighted solution can be extremely biased toward some of the goals, whereas the min-max solution can provide poor aggregate achievement. These two opposite phenomena are undesirable. The extended goal programming model provides a compromise between these two models. Its achievement function unifies the weighted and min-max models. Thus, it provides a better compromise solution that is efficient for optimization problems with conflicting goals [17].

In this section, a basic multi-objective optimization problem is formed from utility point of view. After that, the formulation of the weighted and the min-max goal programming is shown successively. Finally, the extended goal programming model is formulated combining these two classical models. A basic multi-objective optimization problem is as follows:

$$min \quad f_i(\mathbf{x}), \qquad i = 1, 2, 3, ..., q \tag{2.30a}$$

$$s.t. \quad \mathbf{x} \in \mathbf{F} \tag{2.30b}$$

where, $f_i(x)$ is the function to minimize, F is the feasible set for variable x, and q is the total number of objectives to optimize. We can define $f_i(x)$ as follows:

$$f_i(\mathbf{x}) + n_i - p_i = t_i, \qquad i = 1, 2, 3, ..., q \tag{2.30c}$$

where, n_i is the negative deviation variable, p_i is the positive deviation variable, and t_i is the target value or goal. The deviation variables n_i and p_i can be expressed as follows [13]:

$$n_i = \frac{1}{2}[|t_i - f_i(\mathbf{x})| + (t_i - f_i(\mathbf{x}))], \qquad i = 1, 2, 3, ..., q \tag{2.30d}$$

$$p_i = \frac{1}{2}[|t_i - f_i(\mathbf{x})| - (t_i - f_i(\mathbf{x}))], \qquad i = 1, 2, 3, ..., q \tag{2.30e}$$

Adding eqn. (2.30d) and (2.30e) gives:

$$n_i + p_i = |t_i - f_i(\mathbf{x})|, \qquad i = 1, 2, 3, ..., q \tag{2.30f}$$

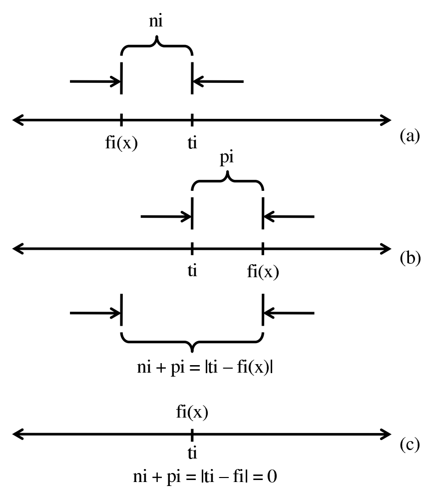

Figure 2.1: Significance of Deviation Variables

Fig. 2.1(a) and 2.1(b) show that n_i and p_i are the difference between the optimal solution, $f_i(x)$ and the target goal, t_i for the maximization and the minimization problem, respectively. Based on the type of the problem, either n_i or, p_i will be zero. For example, for a maximization problem, p_i is always zero. That means, eqn. (2.30f) provides the actual distance from the goal (see fig. 2.1). If the distance is minimized to zero, the optimal solution actually reaches to the goal (see fig. 2.1(c)). Goal programming algorithms basically try to minimize this distance. Eqn. (2.30c) could be written as a utility function [17]:

$$max \quad -\sum_{i=1}^{q} W_i^p \left| t_i - f_i(\mathbf{x}) \right|^p, \qquad i = 1, 2, 3, ..., q \tag{2.31a}$$

$$s.t. \quad \mathbf{x} \in \mathbf{F} \tag{2.31b}$$

Here, W_i is the weight attached to the difference between the achievement of the ith goal and its aspiration level, p is a real number in the interval $[1,\infty)$, and $\mathbf{F}$ is the feasible set.

Substituting eqn. (2.30f) into eqn. (2.31a), we get:

$$min \quad \sum_{i=1}^{q} W_i^p (n_i + p_i)^p \qquad i = 1, 2, 3, ..., q \tag{2.32a}$$

$$s.t. \quad f_i(\mathbf{x}) + n_i - pi = t_i \tag{2.32b}$$

$$n_i \geq 0 \quad p_i \geq 0 \tag{2.32c}$$

$$\mathbf{x} \in \mathbf{F}. \tag{2.32d}$$

This is the utility equivalent of weighted goal programming [17]. The utility equivalent min-max goal programming model can be written as:

$$min \left[max \quad W_i \left| t_i - f_i(\mathbf{x}) \right| \right], \qquad i = 1, 2, 3, ..., q \tag{2.33a}$$

$$s.t. \quad \mathbf{x} \in \mathbf{F} \tag{2.33b}$$

A more clear interpretation of the optimization model described from eqn. (2.33a)-(2.33b) is as follows:

$$min \quad \delta \tag{2.34a}$$

$$s.t. \quad W_i(n_i + p_i) \leq \delta, \qquad i = 1, 2, 3, ..., q \tag{2.34b}$$

$$f_i(\mathbf{x}) + n_i - p_i = t_i \tag{2.34c}$$

$$n_i \geq 0 \quad p_i \geq 0 \tag{2.34d}$$

$$\mathbf{x} \in \mathbf{F} \tag{2.34e}$$

where, δ is the maximum deviation from any goal. By combining the weighted goal programming model described from eqn. (2.32a)-(2.32d) and the min-max goal programming model described from eqn. (2.34a)-(2.34e), we can form the extended goal programming model:

$$min \quad (1 - Z)\delta + Z \sum_{i=1}^{q} (\alpha_i n_i + \beta_i p_i)^p \tag{2.35a}$$

$$i = 1, 2, 3, ..., q$$

$$s.t. \quad (1 - Z)(\alpha_i n_i + \beta i p_i) \leq \delta \tag{2.35b}$$

$$f_i(\mathbf{x}) + n_i - p_i = t_i \tag{2.35c}$$

$$n_i \geq 0 \quad p_i \geq 0 \tag{2.35d}$$

$$\mathbf{x} \in \mathbf{F} \tag{2.35e}$$

where, α_i and β_i are the weights attached to the negative and positive deviation variables, respectively and parameter Z weights the importance attached to the minimization of the weighted sum of unwanted deviation variables. Parameter p indicates the importance of maximum deviation, δ compared to deviation variables. For balanced comparison, the value of p is normally chosen as 1. $Z = 0$ gives the min-max formulation, and $Z = 1$ gives the weighted formulation. Values of Z between 0 and 1 give a compromise between these two formulations.

2.5 Pareto Efficiency in Multi-objective Optimization

In multi-objective optimization, it is difficult to find a single solution which satisfies all the objectives. Rather, it provides a set of candidate solutions. For comparison among the candidate solutions, Pareto dominance and Pareto optimality are widely used. A solution is in the Pareto set if there is no single solution exists that improves at least one of the objectives without degrading any other objectives.

Mathematically, a decision vector $x = [x_1, x_2, x_3, x_4, ...x_n]$ is considered to Pareto-dominate the decision vector $y = [y_1, y_2, y_3, y_4, ...y_n]$, in a minimization problem, if and only if:

$$\forall i \epsilon \{1, 2, 3, ...N\}, f_i(x) \leq f_i(y), \tag{2.36a}$$

$$\exists j \epsilon \{1, 2, 3, ...N\} : f_j(x) < f_j(y) \tag{2.36b}$$

Pareto dominance is used to compare and rank decision vectors. x dominates y indicates that $f_i(x)$ is better than $f_i(y)$ for all i. A solution m is considered as Pareto optimal if and only if there does not exist another solution that dominates it. It also means that solution m can not be improved in one of t he objectives without affecting at least one of the objectives. The corresponding vector $f(m)$ is called the Pareto dominant vector or non-dominated vector. The set of all Pareto optimal solutions is called Pareto optimal set. The corresponding objective vectors are said to be on the Pareto front. Figures 2.2 and 2.3 illustrate two examples of Pareto front in case a biobjective minimization and maximization problem.

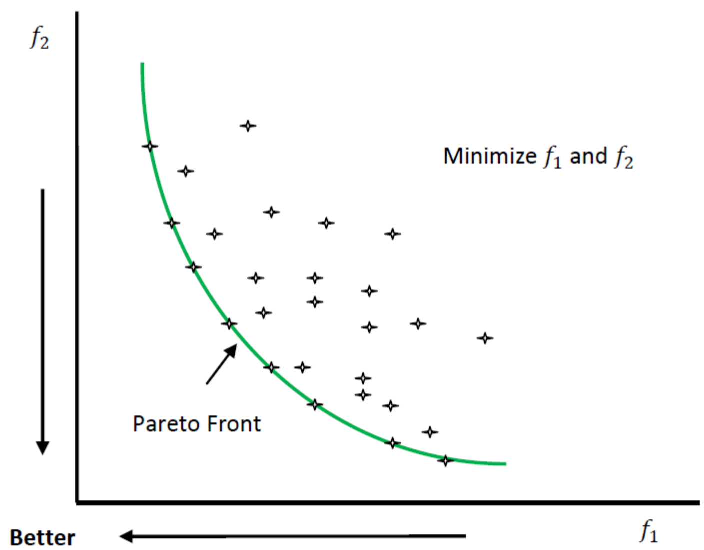

Figure 2.2: Pareto Front in a Minimization Problem

Pareto-optimal sets can be obtained by different multi-objective approaches and other methods. After that, the decision-maker has to select one unique solution from these sets for system implementation. However, selecting a single solution from the Pareto optimal set is difficult, specially when the number of objectives are more than two. Therefore, meaningful research has been carried out to support the decision maker during this post-Pareto analysis

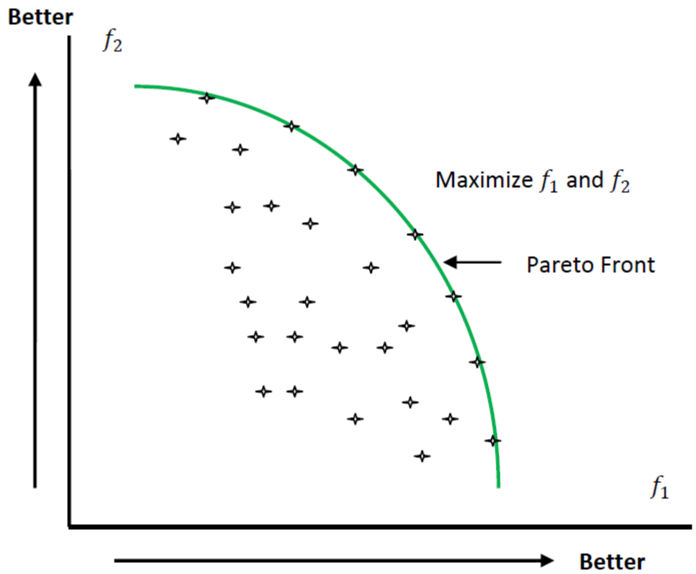

Figure 2.3: Pareto Front in a Maximization Problem

phase[62]. Also, solution methods that provide necessary information on the trade offs between different objective functions are of importance.

2.6 Analytic Hierarchy Process

The analytic hierarchy process (AHP) proposed by Saaty [63] is widely used for multi-criteria decision support. In the AHP method, a complex decision-making problem is modeled as a hierarchical structure of goals, primary criteria, alternatives, and subcriteria. It uses the decision maker's pairwise comparison to provide the order in which the factors affect a decision, the consistency of the pairwise comparison matrix, and finally a prioritized list of the decisions to be taken.

The main steps of the AHP method [64] are briefly discussed below:

1) Step 1: Break down the problem into a hierarchical structure including goals, primary criteria, subcriteria, and alternatives. The overall goal is placed at the top and then primary

criteria, subcriteria, and the set of alternatives are placed in the hierarchy levels. The detail of this step is discussed below:

The first step in the analytic hierarchy process is to model the problem as a hierarchy. An AHP hierarchy is a structured means of modeling the decision at hand. It consists of an overall goal, a group of options or alternatives for reaching the goal, and a group of factors or criteria that relate the alternatives to the goal. The criteria can be further broken down into subcriteria, sub-subcriteria, and so on, in as many levels as the problem requires. A criterion may not apply uniformly. In that case the criterion is divided into subcriteria indicating different intensities of the criterion and these intensities are prioritized through comparisons under the parent criterion. To better understand AHP hierarchies, consider a decision problem with a goal to be reached, three alternative ways of reaching the goal, and four criteria against which the alternatives need to be measured. Such a hierarchy can be visualized as a diagram like the one immediately below, with the goal at the top, the three alternatives at the bottom, and the four criteria in between. There are useful terms for describing the parts of such diagrams: Each box is called a node. A node that is connected to one or more nodes in a level below it is called a parent node. The nodes to which it is so connected are called its children.

Applying these definitions to the figure 2.4 below, the goal is the parent of the four criteria, and the three criteria are children of the goal. Each criterion is a parent of the two Alternatives. Note that there are only two Alternatives, but in the figure, each of them is repeated under each of its parents.

2) Step 2: Collect input from the decision makers and form the pairwise comparison matrix. The matrix is formed for each element with respect to the level immediately above in the hierarchy. Thus, primary criteria are evaluated based on their importance to the goal, subcriteria are evaluated based on their importance to the primary criteria, and alternatives are evaluated based on their importance to their parent subcriteria. The detail of this step

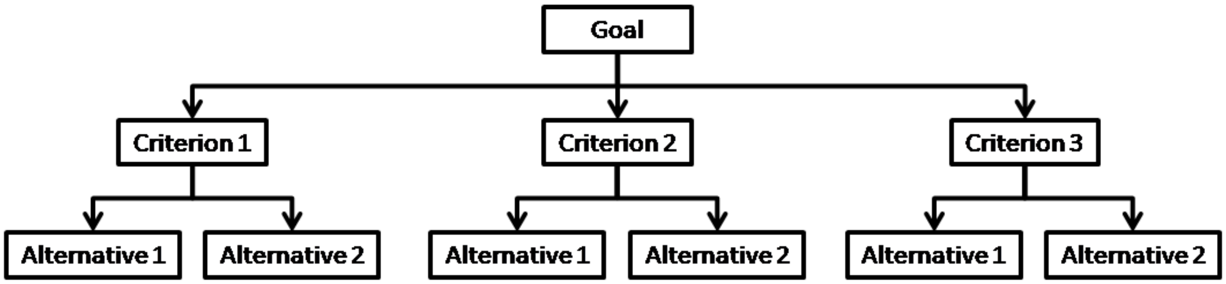

Figure 2.4: Hierarchy Tree

is discussed below:

AHP uses decision maker's pairwise comparison. For quantifying pairwise comparison, a scale of related importance should be used. By using this scale, decision makers can easily provide their preferences. A standard scale of relative importance is shown in Table 2.1 below:

Table 2.1: Scale of Relative Importance

Intensity	Definition
1	Equally important
3	Somewhat more important
5	Much more important
7	Very much more important
9	Absolutely more important
2,4,6,8	Intermediate values

The next, is to forming pairwise comparison matrix for each element in the hierarchy tree. The matrix is formed for each of the element in the hierarchy tree with respect to the level immediately above. For example, in Figure 2.4, Criterion 1, 2 and 3 should be evaluated based on their importance to the Goal. Similarly Alternatives should be evaluated based on their importance to the Criterion 1, 2 and 3. A sample pairwise comparison matrix criterion 1 is shown in figure 2.5 below:

This figure shows the pairwise comparison between Alternative 1 and 2. 5/1 in the second element of the first row indicates that Alternative 2 in 5 times more important than

	Alternative 1	Alternative 2
Alternative 1	1	5/1
Alternative 2	1/5	1

Figure 2.5: Pairwise Comparison Matrix for *Criterion*1

Alternative 1. 1/5 in the first element of the second row indicates the vice versa.

3) Step 3: Find the maximal eigenvalue and the associated eigenvector [65], [66] for each matrix to get the relative weights of the primary criteria, sub criteria, and alternatives.

4) Step 4: Aggregate the relative weights of the primary criteria and sub criteria to obtain a composite priority for each criterion and each level. This process gives an overall priority vector for all the alternatives that helps the decision maker to select the best option.

2.6.1 Taguchi Orthogonal Array Testing (TOAT) Method

TOAT is a method to select minimum number of testing scenarios with good statistical information in the uncertain space. It has been proven that TOAT is able to select optimal representative testing scenarios from the possible combinations in additive and quadratic models. Compared with Monte Carlo simulation, the number of testing scenarios of TOAT are much less, therefore, the computational burden is also less. Additionally, achieving scenarios with TOAT is much more simple than other scenario reduction methods. Below is a tutorial where the methodology of applying TOAT is discussed:

Assume a system z, depicted by $z = Z(x_1, x_2, ..., x_K, \widetilde{u}_1, \widetilde{u}_2, ..., \widetilde{u}_M)$ where $x_1, x_2, ..., x_K$ are controllable factors and $\widetilde{u}_1, \widetilde{u}_2, ..., \widetilde{u}_M$ are uncertain factors. To make the system robust, the uncertain factors $\widetilde{u}_1, \widetilde{u}_2, ..., \widetilde{u}_M$ are considered by a series of scenarios. If the total number of uncertain factors are very big, it is not feasible to consider all the scenarios all together. Rather, some representative scenarios are selected to ease the computational burden. For the simplicity of the problem, for each uncertain variable $\widetilde{u}_1, \widetilde{u}_2, ..., \widetilde{u}_M$, a total of B representative levels are selected. Thus, the total number of operating states of $\widetilde{u}_1, \widetilde{u}_2, ..., \widetilde{u}_M$ will be B^M. But B^M could be a very big number if M is large.

To handle this difficulty, Taguchi's orthogonal array technique (TOAT) is used. In this technique, scenarios are selected by orthogonal arrays. An orthogonal array matrix can be denoted by $L_H(B^M)$ where H and M are the number of rows and columns respectively and B is the number of matrix element levels. An OA $L_9(3^4)$ can be shown as follows:

$$L_9(B^4) = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 3 & 3 & 3 \\ 2 & 1 & 2 & 3 \\ 2 & 2 & 3 & 1 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 3 & 3 & 2 & 1 \end{vmatrix}.$$

Based on the system z, an appropriate OA for that particular system can be obtained from OA libraries[74]. OA is selected based on the following considerations.

2.6.1.1 Value of B

The number of element levels $1, 2, ..., B$ in OA matrix indicated the number of representative levels of uncertain factors in the system. Based on Taguchi's theory, if an uncertain factor $\widetilde{u}_M$ has a linear effect on the system z, then $\widetilde{u}_M$ should have two testing levels. For symmetrically distributed $\widetilde{u}_M$, then $\mu(\widetilde{u}_M) - \sigma(\widetilde{u}_M)$ and $\mu(\widetilde{u}_M) + \sigma(\widetilde{u}_M)$ should be chosen. If an uncertain factor $\widetilde{u}_M$ has a quadratic effect on the system z, then $\widetilde{u}_M$ should have three testing levels. For symmetrically distributed $\widetilde{u}_M$, $\mu(\widetilde{u}_M) - \sqrt{3/2\sigma}(\widetilde{u}_M)$, $\mu(\widetilde{u}_M)$ and $\mu(\widetilde{u}_M) + \sqrt{3/2\sigma}(\widetilde{u}_M)$ should be chosen[54].

2.6.1.2 Value of M

Value of M is directly related to the number of uncertain factors of the system. If the number of uncertain factors in the system is M, then the number of columns in OA is also chosen as M. In this problem, the value of M is chosen based on the number of uncertain loads and generations.

After selecting the appropriate values of B and M, the OA is formed for the system Z. The rows of OA indicates the number of scenarios selected. For system Z, determined by OA $L_H(B^M)$, a total of H scenarios are formed and H is much smaller than B^M. For example, assume that there are four uncertain variables $\widetilde{u}_1, \widetilde{u}_2, ...\widetilde{u}_4$ in the system and two levels are selected for each uncertain variable. Determined by the number of variables and the number of variable levels, OA $L_9(3^4)$ is selected to form the testing scenarios. The way of forming nine testing scenarios according to $L_9(3^4)$ is shown in Table 2.2. In this case, the total of nine testing scenarios are formed, which is less than the number of full combinations 3^4. Therefore, by applying TOAT, the number of testing scenarios are minimized.

Table 2.2: Generation scenarios for system Z based on Orthogonal Array $L_9(3^4)$

No. testing scenarios	Variable levels			
	$\widetilde{u}_1$	$\widetilde{u}_2$	$\widetilde{u}_3$	$\widetilde{u}_4$
1	$u_1(1)$	$u_2(1)$	$u_3(1)$	$u_4(1)$
2	$u_1(1)$	$u_2(2)$	$u_3(2)$	$u_4(2)$
3	$u_1(1)$	$u_2(3)$	$u_3(3)$	$u_4(3)$
4	$u_1(2)$	$u_2(1)$	$u_3(2)$	$u_4(3)$
5	$u_1(2)$	$u_2(2)$	$u_3(3)$	$u_4(1)$
6	$u_1(2)$	$u_2(3)$	$u_3(1)$	$u_4(2)$
7	$u_1(3)$	$u_2(1)$	$u_3(3)$	$u_4(2)$
8	$u_1(3)$	$u_2(2)$	$u_3(1)$	$u_4(3)$
9	$u_1(3)$	$u_2(3)$	$u_3(2)$	$u_4(1)$

The following features of an OA ensure that TOAT achieves representative testing scenarios which are uniformly distributed over the uncertain operating space.

1) In each OA column, every level occurs H/B times. For example, in $L_9(3^4)$, 1,2 and 3 occur $H/B = 9/3 = 3$ times.

2) In any two columns, the level combinations appear the same number of times. In $L_9(3^4)$, 1 1, 1 2, 1 3, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3 occur once in any two columns.

2.7 Summary

In this chapter, background information needed for this research is presented. A general overview of forming simple optimization problem is provided. Optimal power flow problem is discussed in detail from classical formulation to the probabilistic multi-objective formulation. A detailed discussion on different variants of goal programming is given. Difference between various methods and importance of using extended goal programming is shown. The use of analytical hierarchy process for choosing optimal weight combination is discussed in detail. Finally, taguchi method of orthogonal arrays is discussed.

Chapter 3

Extended Goal Programming Approach to Solve Multi-objective AC Optimal Power Flow Problem

3.1 Introduction

In this chapter, an extended goal programming approach to solve multi-objective AC optimal power flow problem is proposed. A ranking strategy is also presented to choose the best routine based on the decision makers' priorities. The model is tested using the standard IEEE-30 and IEEE-118 bus systems.

Goal programming is an efficient tool for modeling multi-objective optimization problems. However, extended goal programming has not been widely used in modeling multi-objective optimization problems in power systems. Most of the works use classical goal programming methods such as weighted goal programming, min-max goal programming, fuzzy goal programming etc. Extended goal programming is a relatively new variant of goal programming where the achievement function is a convex combination of weighted and min-max goal programming. Thus, it can provide a better compromise solution than the classical goal programming models.

In this chapter, a multi-objective AC optimal power flow problem has been modeled and solved based on extended goal programming method to achieve a compromise solution. Extended goal programming is chosen because it can provide an efficient compromise solution which is a trade off between two opposite characteristics of the model, maximum achievement of goal and maximum deviation from goal. Unlike [37], [38], a multi-objective AC optimal power flow problem has been formulated directly into the extended goal programming model. Also, a different strategy for analytic hierarchy process (AHP) to select weight considering different groups of decision makers is presented and significance of Z parameter in the extended goal programming model is demonstrated.

The first contribution of the work in this chapter is an extended goal programming formulation of multi-objective AC optimal power flow problem.The second contribution is an analysis of significance of the Z parameter, showing the efficiency of the model comparing to other classical goal programming models. The last contribution of this chapter is proposing a ranking strategy to choose the best routine based on decision makers' priorities.

The rest of this chapter is organized as follows: Section 3.2 describes the methodology and modeling used in this chapter. Section 3.3 describes the numerical results obtained from different IEEE case study models. Finally, Section 3.4 provides summary and conclusion.

3.2 Methodology

In this section, the methodology and the modeling of the proposed approach are discussed in detail. The considered multi-objective AC optimal power flow has three objectives to minimize. They are generation cost, emission and transmission power loss. This problem is a deterministic optimization problem since the uncertainties in the power systems such as load variation, renewable generation variation is not considered. An extended goal programming formulation is formulated and finally the solution is achieved step by step. The methodology and modeling can be summarized as follows:

Step 1) First a multi-objective AC optimal power flow problem is formulated considering three objectives, namely generation cost, emission and transmission loss, respectively.

Step 2) Extended goal programming theory has been applied. Then, the considered traditional multi-objective AC optimal power flow problem has been transformed to an extended goal programming formulation.

Step 3) Target goals for each of the objectives are calculated. Analytic hierarchy process has been employed to calculated the optimal weights for each of the objective function based on decision makers choice. Then the extended goal programming model is solved for different values of Z.

Step 4) Achievement and deviation level for each of the solution for different values of Z is calculated and the compromise solution, which is a trade off between the maximum achievement and deviation level, is selected. Comparison with other classical goal programming model is made and hence, it shown that extended goal programming formulation has advantages over these models.

Step 5) Finally the model is solved for different weight combinations, calculated by analytic hierarchy process and a ranking is proposed to choose the best compromise solution based on decision makers choice.

3.2.1 Extended Goal Programming Formulation of Multi-objective Optimal Power Flow

In this section, an extended goal programming formulation of muti-objective optimal power flow problem has been proposed. The model can be written as below,

$$min \quad (1-Z)\delta + Z(w_1p_1 + w_2p_2 + w_3p_3)^p \tag{3.1a}$$

$$St. \quad (1-Z)(u_in_i + w_ip_i) \leq \delta \tag{3.1b}$$

$$\sum_{i=1}^{N}(a_i + b_iP_{Gi} + c_iP_{Gi}^2) + \left|d_isin(e_i(P_{Gi}^{min} - P_{Gi}))\right| + n_1 - p_1 = t_1 \tag{3.1c}$$

$$\sum_{i=1}^{N} 10^{-2}(\alpha_i + \beta_iP_{Gi} + \gamma_iP_{Gi}^2 + d_iexp(e_iP_{Gi})) + n_2 - p_2 = t_2 \tag{3.1d}$$

$$\sum_{k=1}^{N_L} g_k[V_i^2 + V_j^2 - 2V_iV_jcos(\delta_i - \delta j)] + n_3 - p_3 = t_3 \tag{3.1e}$$

$$P_{Gi} - P_{Di} - V_i\sum_{j\epsilon i} V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{3.1f}$$

$$Q_{Gi} - Q_{Di} - V_i\sum_{j\epsilon i} V_j(G_{ij}sin\theta ij - B_{ij}cos\theta_{ij}) = 0 \tag{3.1g}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{3.1h}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{3.1i}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{3.1j}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{3.1k}$$

where, Z is the controlling parameter, w_1,w_2 and w_3 are the weights attached to each positive deviation variable p_1, p_2 and p_3, respectively, u_i is the weight attached to the ith negative variable n_i, δ is the maximum deviation from the goal,P_{Gi}, P_{Gj} are real power generations of ith and jth units, Q_{Gi}, Q_{Gj} are reactive power generations of ith and jth units, a_i, b_i, c_i are generation cost coefficients of ith unit, Bk_{ij}, Bk_{0i}, Bk_{00} are Kron's loss coefficients, α_i, β_i, γi, ζ_i,λ_i are emission cost coefficients, N is total number of units, B_{ij} is susceptance of line between ith and jth bus, G_{ij} is conductance of line between ith and jth bus, P_{Di} is real power demand at ith bus, Q_{Di} is reactive power demand at ith bus, V_i, V_j are voltages at ith and jth buses, θ_{ij} is current and voltage angle between ith and jth bus, P_{Gi}^{min}, P_{Gi}^{max} are minimum and maximum real power generation limits of ith unit Q_{Gi}^{min}, Q_{Gi}^{max} are minimum and maximum reactive power generation limits of ith unit, V_i^{min}, V_i^{max} are minimum and maximum voltage limits at ith bus, θ_i^{min}, θ_i^{max} are minimum and maximum limits of angle between voltage and current at ith bus.

3.2.2 Selection of Target Goals

One objective at a time is considered. First, the minimum generation cost is considered. To calculate the generation cost goal, a single objective OPF problem is formulated. The solution of this problem gives the minimum generation cost. The emission and transmission loss for the minimum generation cost are also found. In the same way, the minimum emission and the minimum transmission loss are calculated. These minimum values are lower bounds on the solution of the multi-objective optimization problem, so they are selected as the target goals i.e. t_i^{best}. The worst-case values of these three objectives are recorded, and these give the worst possible value of the objective functions i.e. t_i^{worst}. t_i^{best} is the best possible value for the ith objective and t_i^{worst} is the worst possible value for the ith objective.

3.2.3 Achievement and Deviation Level

The three objectives are expressed in different functions and units, so the optimal values need to be normalized for a fair comparison. A well-known method is used to normalize the achievement level for each objective [67]. The achievement of an objective is

$$t_i^{achieve} = 1 - \frac{t_i^{best} - t_i^{optimum}}{t_i^{best} - t_i^{worst}} = \frac{t_i^{optimum} - t_i^{worst}}{t_i^{best} - t_i^{worst}} \tag{3.2a}$$

where $t_i^{achieve}$ is the normalized achievement level of the ith objective,, and $t_i^{optimum}$ is the actual optimized value for the ith objective.

The value of $t_i^{achieve}$ is bounded between 0 and 1. When $t_i^{optimum} = t_i^{best}$, the normalized achievement level is 1. When $t_i^{optimum} = t_i^{worst}$, the normalized achievement level is 0. When an objective has an achievement level of 1, it fully satisfies the desired goal. When the level is 0, it completely fails to satisfy the goal.

The normalized deviation value of the ith objective is

$$t_i^{deviation} = 1 - t_i^{achieve}. \tag{3.2b}$$

The value of $t_i^{deviation}$ is also bounded between 0 and 1. A value of 0 indicates that the objective achieves the goal and there is no deviation. A value of 1 indicates that it completely fails to achieve the goal.

3.2.4 Difference Level

To compare the solution quality based on these two phenomena, maximum achievement and maximum deviation, an unique measure is taken. It is defined as difference level. The difference level provides the absolute difference between maximum deviation and maximum achievement. The difference level of a solution is calculated as follows:

$$L_{diff} = \left| t_{max}^{achieve} - t_{max}^{deviation} \right| \tag{3.3}$$

For an ideal solution, maximum achievement, $t_{max}^{achieve}$ should be 1 and maximum deviation, $t_{max}^{deviation}$ should be 0. Thus, the ideal value for difference level of a solution L_{diff} should be 1. Similarly, the worst value for difference level of a solution L_{diff} should be 0. Thus, difference level is an unified approach to measure the solution quality based on maximum achievement and maximum deviation of/from any of the goals.

3.2.5 Compromise Solution

The extended goal programming formulation is a convex combination of weighted and min-max goal programming [14], [17]. The extended goal programming model of the multi-objective OPF problem is solved for different values of Z bounded between 0 and 1. The maximum achievement and deviation levels are then calculated for a particular value of Z. The weighted model provides a maximum achievement level by worsening the maximum deviation from the goals for some objectives. The min-max model provides a more balanced solution by minimizing the maximum deviation from the goals while lowering the maximum achievement level. In the extended formulation intermediate values of Z can provide a set of compromise solutions to the decision makers. Extended goal programming hence finds a compromise between maximum achievement and minimized maximum deviation.

This characteristic can be explained from the Figure 3.1. The main target for a multi-objective optimization problem is to find a solution which satisfies the target goals. Hence, the maximum achievement level of 1 and maximum deviation level of 0 are desirable. But for the conflicting nature of different objectives, it is actually unachievable. Thus, solutions that provide a high maximum achievement level and low maximum deviation are chosen. In the figure, P, Q are R denote min-max, weighted and extended goal programming solutions, respectively. It is seen that $PD < QE < RF$ and $AR > BQ > PC$. It explains that min-max solution at P is providing the lowest maximum deviation and weighted solution at R is providing the highest maximum achievement which is good. However, min-max solution at P has the lowest maximum achievement level and weighted solution at R has the

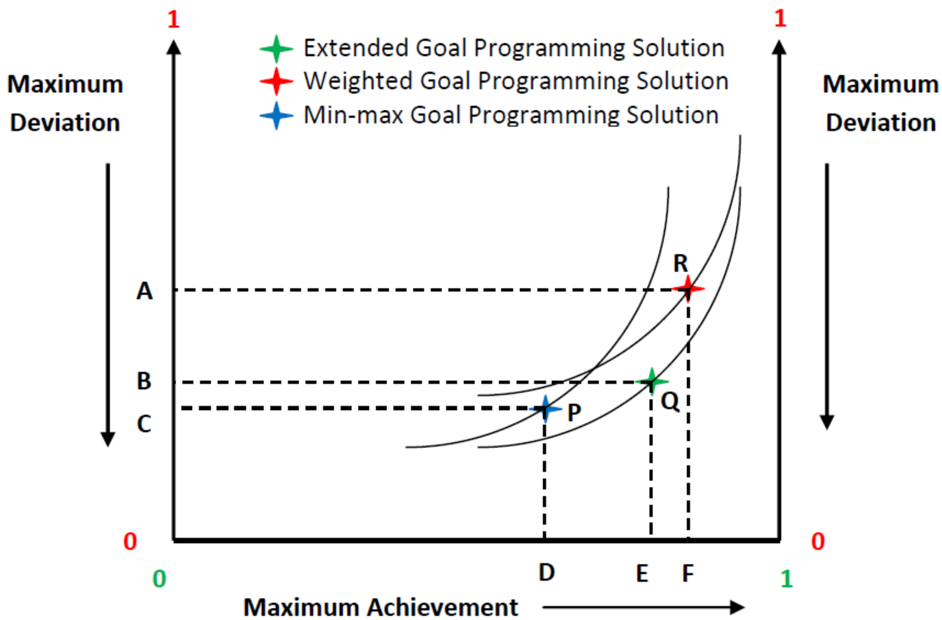

Figure 3.1: Compromise Solution in Extended Goal Programming

highest maximum deviation. Hence, these solutions are not desirable. On the other hand, extended solution at Q provides a lower maximum deviation than weighted solution and higher maximum achievement than min-max solution. Thus, extended goal programming solution provides a compromise between weighted and min-max solutions.

3.2.6 Non-dominated Solutions of Extended Goal Programming Method

Goal Programming solutions can provide Pareto dominant solution which is in-efficient. Research has been carried out to overcome this draw back. In [68], the methods and techniques to achieve non-dominated solutions from goal programming has been discussed. It is suggested in this paper that goal programming method can provide non-dominated solutions if the target goals are set too conservatively. Thus, selecting the target goals strictly is necessary. In our research, we set the target goals conservatively by solving single optimal power flow problem for each of the objectives. Hence, each of the target goals are the solution to the single objective optimal power flow and no better solution can be achieved for multi-objective problem beyond this. Because the other objectives are considered as relaxed during selecting the target goals.

Although a extended goal programming can find all the Pareto optimal solutions by changing weights of the objective function for a multi-objective optimization problem, we are interested in one unique solution that is the best compromise depending upon the weights assigned. Unlike, classical multi-objective optimization methods, extended goal programming method finds a single solution which is a trade off between maximum achievement level and maximum deviation level. For classical multi-objective approach, it is often very difficult to find a single solution from the Pareto curve. Hence, extended goal programming is useful for finding a single solution compromising different conflicting objectives and considering the particular weights assigned.

3.2.7 Ranking Strategy

A ranking system is used to find the best solution based on two parameters, maximum achievement and maximum deviation. The decision maker may be uncertain about the best set of weights. The AHP method [63] is used to select different sets of weights and calculate the maximum achievement and maximum deviation. Based on these two parameters, rankings for different values for Z are obtained. These help the decision makers to choose the best solution given their preferences.

3.3 Numerical Results

The multi-objective extended goal programming model is solved for IEEE-30 bus and IEEE-118 bus systems. GAMS and a PC with an Intel Core i3 processor of 2.53 GHz and 6 GB of Ram are used. The IEEE-30 bus data is used from Table 3.1 and the other data is acquired from [69]. The IEEE-118 bus data are acquired from [69],[70]. The cost and emission coefficients of the generators for the IEEE-118 bus system are taken from [71].

3.3.1 Target Goals

Table 3.2 lists the target goals and worst-case values for the IEEE-30 bus and the IEEE-118 bus. The target goals are obtained by solving a single-objective optimization for each objective.

Table 3.1: Generation cost, Emission Coefficients, and Real and Reactive Power Limits of Generators for IEEE-30 bus system

Generator		G1	G2	G3	G4	G5	G6
Real Power Limit (MW)	P_G^{min}	50	50	50	50	50	50
	P_G^{max}	50	60	100	120	100	60
Reactive Power Limit (MVar)	Q_G^{min}	0	-20	-15	-15	-10	-15
	Q_G^{max}	0	100	80	60	50	60
Generation Cost Coefficient	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
	d	300	200	200	150	200	100
	e	0.2	0.22	0.35	0.42	0.35	0.30
Emission Cost Coefficient	α	4.091	2.543	4.258	5.326	4.258	6.131
	β	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	γ	6.49	5.638	4.586	3.380	4.586	5.151
	ζ	0.0002	0.005	0.000001	0.002	0.000001	0.00001
	λ	2.857	3.333	8.000	2.000	8.000	6.667

Table 3.2: Best and Worst Values of Each Objective

	IEEE-30 Bus		IEEE-118 Bus	
	t_i^{best}	t_i^{worst}	t_i^{best}	t_i^{worst}
Generation Cost	609.39 \$/h	650.86 \$/h	214004.47 \$/h	238789.56 \$/h
Emission Cost	194.1 kg/h	228.3 kg/h	5306.3 kg/h	5510 kg/h
Transmission Loss	1.72 MW	3.58 MW	32.52 MW	151.6 MW

3.3.2 Selection of Weights

To show the efficiency of the extended goal programming, weights can be chosen arbitrarily. However, to address the multi-objective problem from system operator's point of view analytic hierarchy process (AHP) has been used. Analytic hierarchy process is widely used for decision making in various research field [63], [64], [65], [66]. The AHP is used to select the weights. For the multi-objective OPF problem, the decision makers can be divided into four major groups:

1) Group A: Emphasize generation cost objective

2) Group B: Emphasize emission cost objective

3) Group C: Emphasize power loss objective

4) Group D: Emphasize all three objectives

Table 3.3: Scale of Relative Importance

Intensity	Definition
1	Equally important
3	Somewhat more important
5	Much more important
7	Very much more important
9	Absolutely more important
2,4,6,8	Intermediate values

To focus the contribution and analyze the significance of the extended goal programing method, any of the groups can be considered. In this case, Group A is selected randomly. For the pairwise comparison, a standard scale [63] as defined in Table 3.3 is used. Depending on the types of the decision makers, different weight combinations can be calculated; three possibilities are considered, each with a different emphasis on the generation cost. They are:

1) Set A_1: Extreme emphasis on generation cost, very low emphasis on other objectives

2) Set A_2: Strong emphasis on generation cost, moderate emphasis on emission cost, and very low emphasis on power loss

3) Set A_3: Moderate emphasis on generation cost, low emphasis on emission cost, and very low emphasis on power loss

The hierarchy tree is then formed as shown in Fig. 3.2. Since the same decision-maker group is providing three different sets of weights, it can be assume that they are equally

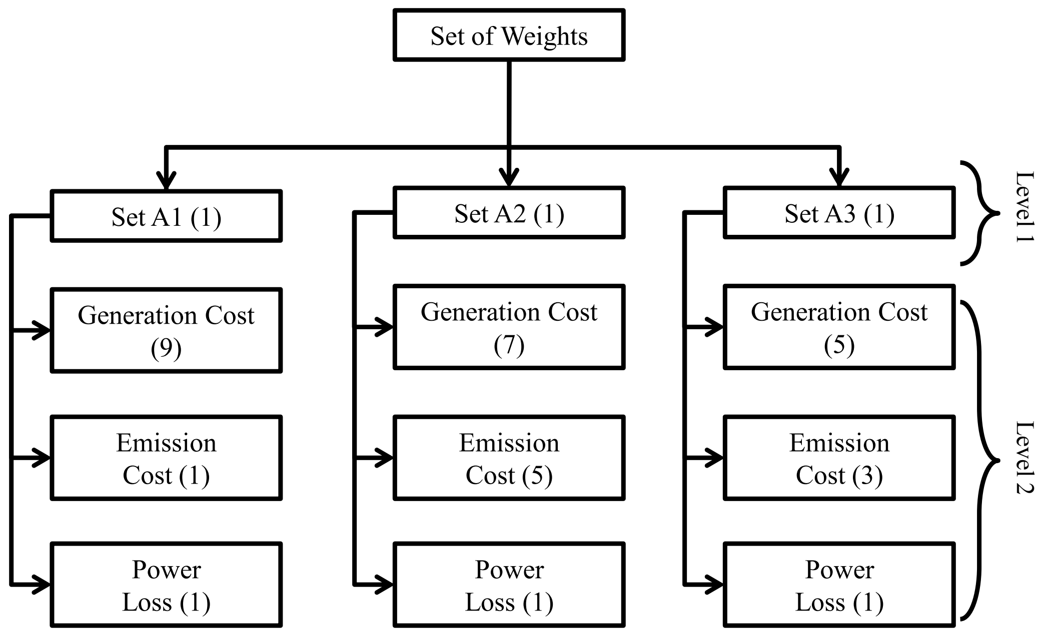

Figure 3.2: Hierarchy Tree for Group A

important. The value is set to 1 for each element in level 1. For the three sets, based on the definition given above, the value for each objective is selected from Table 3.3, as shown in Fig. 3.2. Then three 3×3 pairwise comparison matrices are formed for the three sets. The principal eigenvector for each matrix is calculated [65] based on Step 3 discussed in Section 2.5 in Chapter 2. For example, Fig. 3.3 shows the pairwise comparison matrix for Set A_1. Each element of this matrix shows the relative importance of one objective over another. For example, the value 9/1 at the 2nd element of the 1st row means, the generation cost is 9 times more important than emission cost in this case. Finally, the vector of priorities based on Step 4 discussed in Section 2.5 in Chapter 2 is calculated [72]. In this case, it is $P = [1.91, 0.81, 0.28]$. After normalization, the vector of priorities is $P_{norm} = [0.64, 0.27, 0.09]$. This indicates the relative weights for the generation cost, emission cost, and power loss.

	Gen. Cost	Emission	Power Loss
Gen. Cost	1	9/1	9/1
Emission	1/9	1	1/1
Power Loss	1/9	1/1	1

Figure 3.3: Pairwise Comparison Matrix for A_1

3.3.3 Compromise Between Maximum Achievement and Deviation Level

Table 3.4 shows the results for the IEEE-30 bus system. For $Z = 1$ the extended formulation is a classical weighted model. The achievement level for each objective is calculated from eqn (3.2a), and the deviation level is calculated from eqn (3.2b). When $Z = 1$, the maximum achievement is 0.99 and the maximum deviation is 0.75. This model provides the highest achievement but also the highest deviation. For $Z = 0$ the extended formulation is a classical min-max model. When $Z = 0$, the maximum achievement is 0.94 and the maximum deviation is 0.54. This model gives a balanced solution by minimizing the maximum deviation, but the maximum achievement is lower. Although the change is small, it might not be acceptable to a decision maker who emphasizes the achievement of the goal. Also, the goals are measured in dollars per hour, so a small reduction in the achievement could add a large penalty. For example, an increase of 0.01 in the achievement level of the gen. cost objective for the IEEE-118 bus system saves 247.85 \$/hr. That means, it can save 5948.42 \$/day and 2171173.88 \$/yr. To find a compromise solution, intermediate values of Z such as 0.3, 0.5, and 0.8 are chosen. For $Z = 0.5$, the maximum achievement is 0.97, and the maximum deviation is 0.67, which is better than the worst-case value of 0.75. It shows that by compromising only a factor of 0.02 in the achievement level, we can improve the deviation level by 0.08. Practically, here we can reduce the emission by 62.4 kg/day by compensating only 0.72 MW/day. Hence, this intermediate value provides a better compromise solution.

Finally, the efficiency of the extended goal programming solution is shown by calculating the difference level of the solution based on maximum achievement and maximum deviation. The difference level is calculated from eqn. (3.3). It is found that for $Z = 0.5$, the difference level is calculated as 0.30 which is the highest among the other values of Z considered. For an ideal solution, the difference level should be 1. Since extended goal programming solution is providing the maximum difference level, it is considered as a better compromise solution. Difference level for different Z values for IEEE-30 bus are illustrated in Figure 3.4.

Table 3.4: Comparison of Maximum Achievement and Maximum Deviation for Different Z Values for IEEE-30 Bus System

Z		**0**	**0.3**	**0.5**	**0.8**	**1**
Cost	Generation ($\$/hr$)	630.8	629.8	631.68	632.93	633.1
	Emission (Kg/hr)	215.9	216.3	217.1	219.4	219.7
	Loss (MW)	2.01	1.82	1.77	1.75	1.74
Achievement Level (1-0)	Generation	0.48	0.51	0.46	0.43	0.42
	Emission	0.46	0.35	0.33	0.26	0.25
	Loss	0.84	0.93	0.97	0.98	0.99
	Maximum	**0.84**	0.93	**0.97**	0.98	**0.99**
Deviation Level (1-0)	Generation	0.52	0.49	0.54	0.57	0.58
	Emission	0.64	0.65	0.67	0.74	0.75
	Loss	0.06	0.55	0.03	0.02	0.01
	Maximum	**0.64**	0.65	**0.67**	0.74	**0.75**
Difference Level (1-0)		0.20	0.28	**0.30**	0.24	0.24

A similar phenomenon is observed for the IEEE-118 bus system in Table 3.5. For $Z = 1$ the extended formulation is a classical weighted model. When $Z = 1$, the maximum achievement is 0.94 and the maximum deviation is 0.44. This model provides the highest achievement but also the highest deviation. For $Z = 0$ the extended formulation is a classical min-max model. When $Z = 0$, the maximum achievement is 0.87 and the maximum deviation is 0.33. This model gives a balanced solution by minimizing the maximum deviation, but the maximum achievement is lower. For $Z = 0.5$, the maximum achievement is 0.93, and the maximum deviation is 0.34, which is better than the worst-case value of 0.44. It

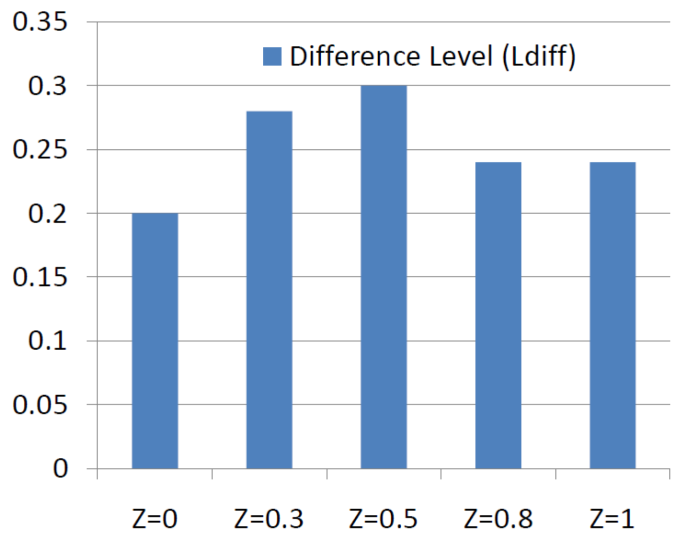

Figure 3.4: Difference Level for Different Z values for IEEE-30 Bus System

shows that by compromising only a factor of 0.01 in the achievement level, we can improve the deviation level by 0.10. Practically, here we can reduce the emission by 489.60 kg/day by compensating only 28.57 MW/day. Hence, this intermediate value provides a better compromise solution.

The efficiency of the extended goal programming solution is shown by calculating the difference level of the solution based on maximum achievement and maximum deviation. It is found that for $Z = 0.5$, the difference level is calculated as 0.59 which is the highest among the other values of Z considered. For an ideal solution, the difference level should be 1. Since extended goal programming solution is providing the maximum difference level, it is considered as a better compromise solution. Difference level for different Z values for IEEE-118 bus are illustrated in figure 3.5.

3.3.4 Comparison of Results from Different Models

To analyze the performance of our model, we must carefully check the value of the maximum achievement, maximum deviation and difference level. The desired values are 1, 0 and 1,

Table 3.5: Comparison of Maximum Achievement and Maximum Deviation for Different Z Values for IEEE-118 Bus System

Z		**0**	**0.3**	**0.5**	**0.8**	**1**
Cost	Generation ($\$/hr$)	220084.90	220129.99	220164.33	218625.18	218320.05
	Emission (Kg/hr)	5374.2	5374.80	5375.2	5391.9	5395.7
	Loss (MW)	47.41	41.95	40.47	40.32	40.01
Achievement Level (1-0)	Generation	0.76	0.75	0.75	0.81	0.83
	Emission	0.67	0.66	0.66	0.58	0.56
	Loss	0.87	0.90	0.93	0.93	0.94
	Maximum	**0.87**	0.90	0.93	0.93	**0.94**
Deviation Level (1-0)	Generation	0.24	0.25	0.25	0.19	0.17
	Emission	0.33	0.34	0.34	0.42	0.44
	Loss	0.13	0.08	0.07	0.07	0.06
	Maximum	**0.33**	0.34	0.34	0.42	**0.44**
Difference Level (1-0)		0.54	0.56	**0.59**	0.51	0.50

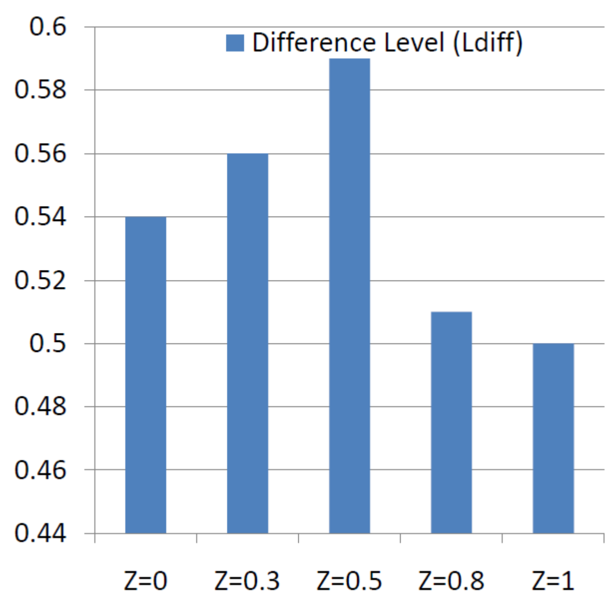

Figure 3.5: Difference Level for Different Z values for IEEE-118 Bus System

respectively. For the IEEE-30 bus system, Table 3.6 compares different single and multi-objective models. The single-objective models give the highest maximum achievement levels. However, the maximum deviation levels are 0.78, 1, and 1, respectively. This indicates that one or more objectives totally deviate from the goal. Thus, these simple models are not efficient; the goal programming models give better results. For the weighted method, the maximum achievement is 0.99 and the maximum deviation is 0.75. For the min-max method, these values are 0.84 and 0.64, respectively: the maximum deviation is improved, but the maximum achievement is lower. In the extended model, the maximum achievement is 0.97, which is very close to that of the weighted solution, and the maximum deviation is only 0.67. Thus, the method provides a compromise between the weighted and min-max models.

Table 3.6: Optimal Dispatch Schedule for Different Optimization Models for IEEE-30 Bus System

	Single			Multi-Objective		
	Gen Co OPF	Emission OPF	Loss OPF	Weighted	Min-max	Extended
Gen. Cost ($/h)	609.39	650.86	640.85	633.10	630.80	631.68
Emission (kg/h)	220.8	194.2	228.4	219.7	215.9	217.1
Loss (MW)	2.57	3.58	1.72	1.74	2.01	1.77
Max. Achievement	1	1	1	0.99	0.84	**0.97**
Max. Deviation	0.78	1	1	0.75	0.64	**0.67**
Difference Level	0.22	0	0	0.24	0.20	**0.30**

This characteristic of the solution can be better observed from the difference level values achieved by different methods. Single objective emission OPF and loss OPF provides a difference level of 0 which indicates the inefficiency of these solutions. Among the other methods, extended goal programming solution provides the highest value for difference level, 0.30, that verifies the efficiency of the solution. Difference level for different optimization methods for IEEE-30 bus are illustrated in figure 3.6.

Similarly for the IEEE-118 bus system, Table 3.7 compares different single and multi-objective models. The single-objective models give the highest maximum achievement levels. However, the maximum deviation levels are 1, 1, and 0.69, respectively. This indicates that

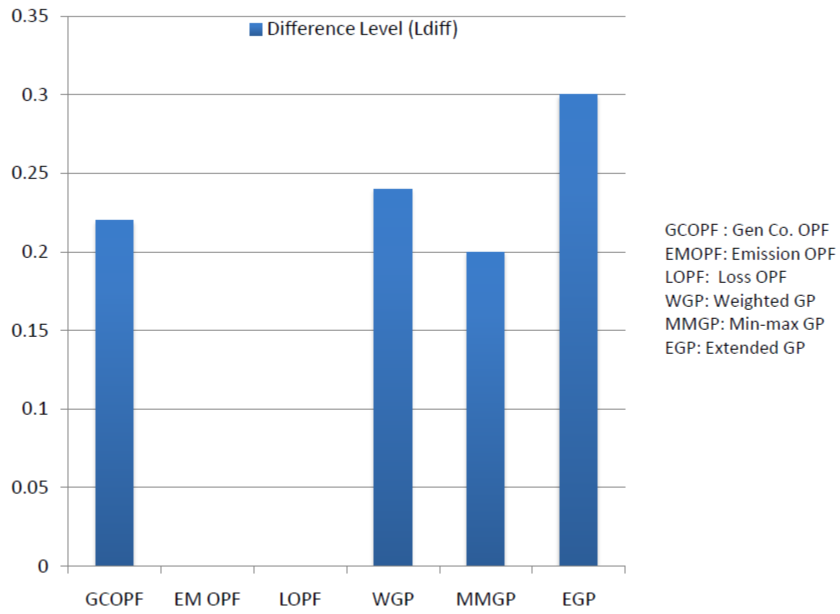

Figure 3.6: Difference Level for Different Optimization Methods for IEEE-30 Bus System

Table 3.7: Optimal Dispatch Schedule for Different Optimization Models for IEEE-118 Bus System

	Single			Multi-Objective		
	Gen Co OPF	Emission OPF	Loss OPF	Weighted	Min-max	Extended
Gen. Cost ($/h)	214004.47	238789.56	230990.23	218320.05	220084.90	220164.33
Emission (kg/h)	5510	5310	5380	5395.7	5374.2	5375.2
Loss (MW)	108.72	151.57	32.53	40.01	47.41	40.47
Max. Achievement	1	1	1	0.94	0.87	**0.93**
Max. Deviation	1	1	0.69	0.44	0.33	**0.34**
Difference Level	0	0	0.31	0.50	0.54	0.59

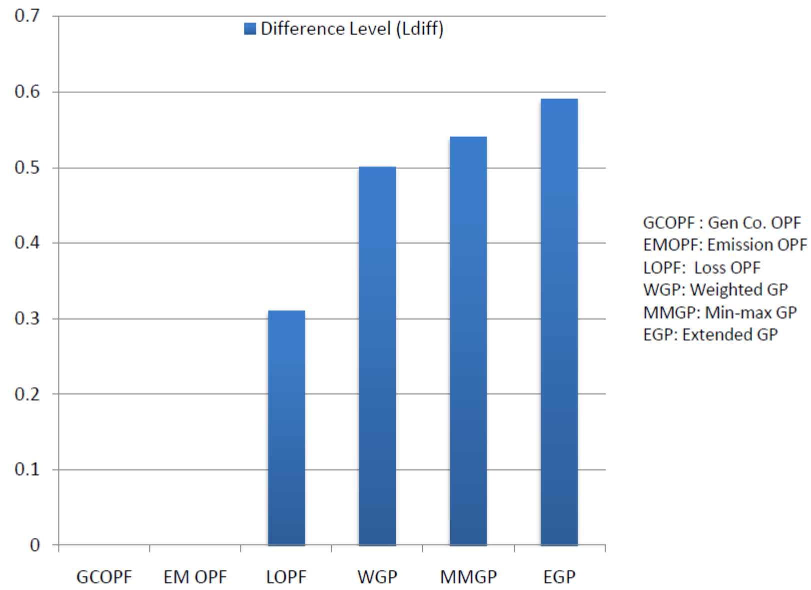

Figure 3.7: Difference Level for Different Optimization Methods for IEEE-118 Bus System

one or more objectives totally deviate from the goal. Thus, these simple models are not efficient; the goal programming models give better results. For the weighted method, the maximum achievement is 0.94 and the maximum deviation is 0.44. For the min-max method, these values are 0.87 and 0.33, respectively: the maximum deviation is improved, but the maximum achievement is lower. In the extended model, the maximum achievement is 0.93, which is very close to that of the weighted solution, and the maximum deviation is only 0.34. Thus, the method provides a compromise between the weighted and min-max models.

This characteristic of the solution can be better observed from the difference level values achieved by different methods. Single objective emission OPF and loss OPF provides a difference level of 0 which indicates the inefficiency of these solutions. Among the other methods, extended goal programming solution provides the highest value for difference level, 0.59, that verifies the efficiency of the solution. Difference level for different optimization methods for IEEE-118 bus are illustrated in figure 3.7.

3.3.5 Sensitivity of Weights and Ranking of Optimal Schedules

The other three groups of decision makers are considered now. The AHP is used to calculate the sets of weights for each group. Then the model is solved for the four groups and different values of Z to analyze the impact of the weights (see Table 3.8 and 3.9). A ranking is generated based on difference levels for different Z values. For the IEEE-30 bus system, when $Z = 0.5$, a ranking is generated based on the difference level. From Table 3.8, for $Z = 0.5$, set $S4$ provides the highest value,0.38, for the difference level and set $S3$ provides the lowest value,0.18, for difference level (ranking 1, Fig. 3.10). From Table 3.9, when $Z = 0.5$, set $S4$ provides the highest difference level, 0.64 and set $S2$ provides the lowest difference level, 0.35 (ranking 2, Fig. 3.10). As discussed in the previous section, for other intermediate values of Z, we could generate a ranking based on the maximum achievement and minimum deviation to help decision makers.

These phenomena is also illustrated in figure 3.8 and 3.9, showing the values of difference level for different set of weights for IEEE-30 and IEEE-118 bus systems respectively.

Table 3.8: Comparison of Different Weights for Different Values of Z for IEEE-30 Bus System

Sets		S1	S2	S3	S4
Weights	W1	0.64	0.16	0.10	0.33
	W2	0.27	0.67	0.18	0.33
	W3	0.09	0.17	0.72	0.33
Z = 0	Max Achieve	0.84	0.67	0.77	0.82
	Max Deviation	0.64	0.49	0.67	0.49
	L_{diff}	**0.20**	**0.18**	**0.10**	**0.33**
Z = 0.3	Max Achieve	0.93	0.73	0.80	0.85
	Max Deviation	0.65	0.51	0.69	0.51
	L_{diff}	**0.28**	**0.22**	**0.11**	**0.34**
Z = 0.5	Max Achieve	0.97	0.85	0.99	0.95
	Max Deviation	0.67	0.52	0.81	0.57
	L_{diff}	**0.30**	**0.33**	**0.18**	**0.38**
Z = 0.8	Max Achieve	0.98	0.85	0.99	0.97
	Max Deviation	0.74	0.58	0.88	0.65
	L_{diff}	**0.24**	**0.27**	**0.11**	**0.32**
Z = 1	Max Achieve	0.99	0.87	1.00	0.97
	Max Deviation	0.75	0.64	0.89	0.66
	L_{diff}	**0.24**	**0.23**	**0.11**	**0.31**

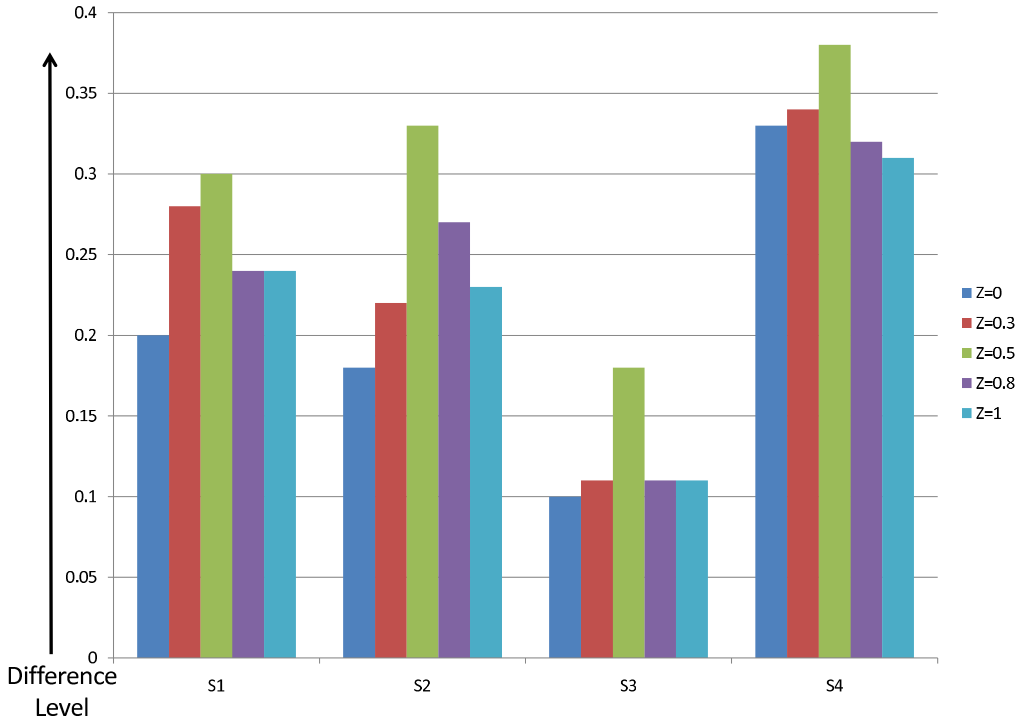

Figure 3.8: Difference Level for Different Set of Weights for IEEE-30 bus

Table 3.9: Comparison of Different Weights for Different Values of Z for IEEE-118 Bus System

Sets		S1	S2	S3	S4
Weights	W1	0.64	0.16	0.10	0.33
	W2	0.27	0.67	0.18	0.33
	W3	0.09	0.17	0.72	0.33
Z = 0	Max Achieve	0.87	0.87	0.89	0.87
	Max Deviation	0.33	0.56	0.30	0.31
	L_{diff}	**0.54**	**0.31**	**0.59**	**0.56**
Z = 0.3	Max Achieve	0.90	0.91	0.92	0.94
	Max Deviation	0.34	0.58	0.32	0.32
	L_{diff}	**0.56**	**0.33**	**0.60**	**0.62**
Z = 0.5	Max Achieve	0.93	0.93	0.97	0.96
	Max Deviation	0.34	0.58	0.34	0.32
	L_{diff}	**0.59**	**0.35**	**0.63**	**0.64**
Z = 0.8	Max Achieve	0.93	0.93	0.98	0.97
	Max Deviation	0.42	0.61	0.39	0.36
	L_{diff}	**0.51**	**0.32**	**0.59**	**0.61**
Z = 1	Max Achieve	0.94	0.94	0.98	0.97
	Max Deviation	0.44	0.62	0.42	0.37
hline	L_{diff}	**0.50**	**0.32**	**0.56**	**0.60**

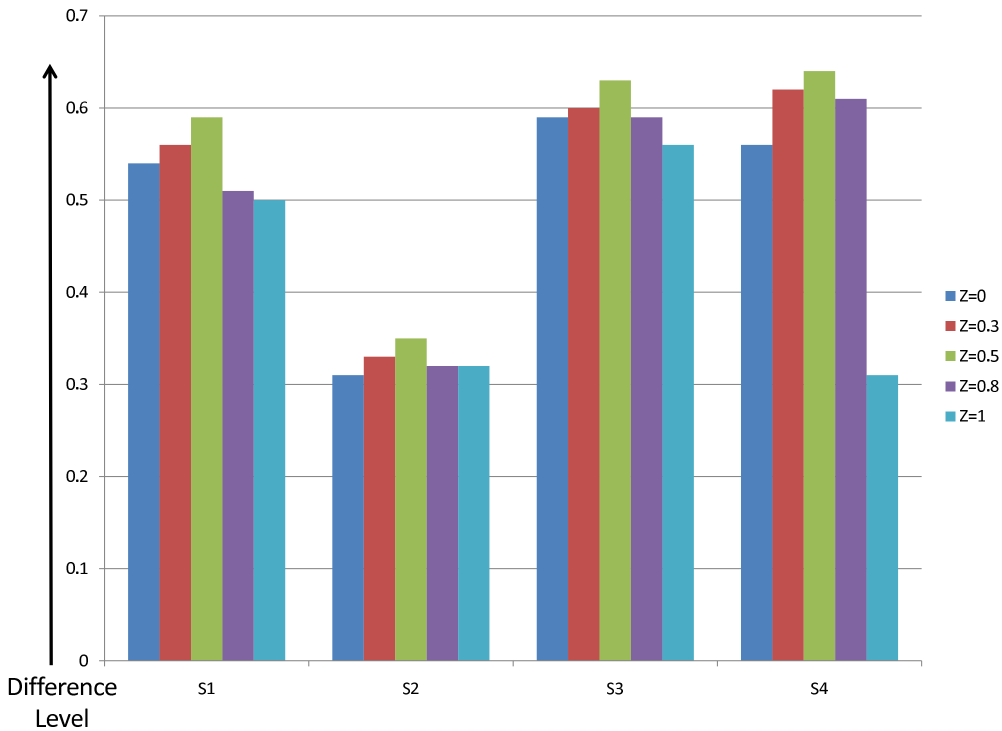

Figure 3.9: Difference Level for Different Set of Weights for IEEE-118 bus

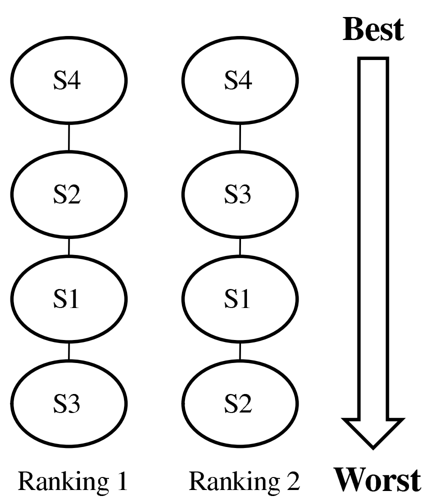

Figure 3.10: Ranking of optimal weight combinations based on based on difference level for IEEE-30 Bus and IEEE-118 Bus

3.4 Summary

In this chapter, an approach to solve multi-objective AC optimal power flow problem based on goal programming is proposed. This model can provide a compromise solution that is a trade off between the maximum achievement and deviation levels. A ranking is also presented to help the decision maker to choose the preferred solution. The model was implemented in the IEEE-30 and IEEE-118 bus systems.

The results show that the extended formulation has advantages over the classical weighted and min-max formulations. The extended formulation can find the weighted solution, the min-max solution, or a compromise of the two by changing the value of the Z parameter. Thus, it can offer a set of solutions for decision makers to consider. The proposed approach can also be used for other linear/nonlinear multi-objective problems in power systems. It can also be applied to, for example, security constrained multi-objective optimal power flow problems, multi-objective optimal power flow problems in the market environment, and multi-objective power system planning problems.

Chapter 4

An Approach to Solve Multi-objective Optimal Power Flow Problem under Load and Renewable Generation Uncertainties

4.1 Introduction

In this chapter, an approach to solve multi-objective AC optimal power flow problem under load and renewable generation uncertainties based on extended goal programming is proposed. Taguchi's Orthogonal Array Testing is used to select a minimum number of testing scenarios with good statistical information in the uncertain space. It is shown that the proposed method can provide a solution that is robust to uncertain variations in load and renewable generations. The results are compared with the deterministic model discussed in Chapter 3. The model is tested using the IEEE-14 and IEEE-30 bus systems.

Consideration of load and renewable generation uncertainties in multi-objective optimal power flow is increasingly important since more renewable generators, whose outputs are variable and intermittent, are connected into modern power systems. Therefore, approaches to satisfy multiple objectives of an optimal power flow with the consideration of load and renewable generation uncertainties are of importance. The same multi-objective AC optimal power flow problem in Chapter 3 is used here with added load and generation uncertainties in it. This consideration converts it to a probabilistic multi-objective power flow problem. The probabilistic formulation is then modeled and solved using proposed approach which is a combination of Taguchi's Orthogonal Array Technique (TOAT) and Extended Goal Programming Method.

The proposed approach differs significantly from other methods of multi-objective optimal power flow formulation under uncertainties. Unlike [54], an AC optimal power flow problem with multiple objectives under load and generation uncertainties is considered in this research. Unlike [51], [52], [53], a probabilistic multi-objective optimal power flow problem is formulated and then converted into a robust extended goal programming model. The method is different since it introduces a new approach using a combination of Taguchi's Orthogonal Array Technique and extended goal programming theory. Also, uncertainties in load and renewable generations are considered jointly rather than uncertainties in target goals.

A contribution of the work in this chapter is modeling a multi-objective AC optimal power flow problem under uncertainties using Taguch's Orthogonal Array Technique and extended goal programming method. The second contribution is an analysis of the significance of ramp rate variation with the degree of robustness of the solution. The final contribution of this chapter is a comparison between robust and non-robust solution from the extended goal programming point of view showing the efficiency of the proposed approach.

The rest of this chapter is organized as follows, Section 4.2 describes the detail methodology and modeling of the proposed algorithms, Section 4.3 describes an analysis of numerical results for different IEEE test cases and demonstrating the efficiency of the model comparing to the non-robust model proposed in Chapter 3. Finally, Section 4.4 summarizes this chapter.

4.2 Methodology and Modeling

In this section, the detailed methodology and modeling criteria of the proposed approach are discussed. In the considered probabilistic multi-objective AC optimal power flow problem, the non-renewable generations are considered as controllable factors and loads and renewable generations are considered as uncertain factors. For these uncertainties in loads and renewable generations, the future operating states are uncertain. Among the large number of possible uncertain scenarios, the scenario with largest probability has the high significance to appear in the future.Therefore, the objective of the proposed approach is to minimize generation cost, emission and power loss simultaneously, for the scenario which has the largest probability to appear in the future. In order to avoid the power flow becoming infeasible if other possible scenarios happen, Taguchi's Orthogonal Array Technique and probabilistic AC optimal power flow formulation are employed to derive the modified power flow constraints. In this research, the loads and renewable energies are modeled as independent. The loads are modeled as normally distributed and the power output variation ranges of the uncertain renewable energies are between zero and the capacity [54]. The details of modelling uncertain wind speed is discussed in Section 4.3.

The methodology and modeling can be summarized as follows:

Step 1) First a probabilistic multi-objective AC optimal power flow problem is formulated considering three objectives, namely generation cost, emission and transmission loss, respectively. Then the probabilistic constraints are identified.

Step 2) Probabilistic constraints are then converted into robust deterministic constraints using Taguchi's Orthogonal Array Testing (TOAT) method.

Step 3) After the conversion, the probabilistic constraints of the probabilistic multi-objective AC optimal power flow formulation are replaced by the robust deterministic constraints. Hence, the robust multi-objective AC optimal power flow formulation is done.

Step 4) Extended goal programming theory has been applied to the robust multi-objective AC optimal power flow model and thus, the robust extended goal programming formulation is done.

Step 5) Robust extended goal programming formulation is modified to find the single robust solution to the problem. Finally, the modified robust extended goal programming model is formulated.

Step 6) The steps 3 and 4 described in Section 3.2 are done accordingly. Target goals and optimal weights for each of the objectives are calculated. The modified robust extended goal programming model is then solved for different values of Z. Achievement and deviation level for each of the solution for different values of Z is calculated and the compromise solution is selected.

Step 7) The degree of robustness is calculated for different ramp rate. Finally the results are compared with the non-robust model of Chapter 3 to verify the significance of the research work.

4.2.1 Probabilistic MO-OPF Constraints Considering Load and Renewable Generation Uncertainties

This is the first step of the proposed approach as stated in step 1 in Section 4.2. There are two types of probabilistic constraints in the probabilistic multi-objective AC optimal power flow problem. They are probabilistic real power flow constraints and reactive power flow constraints.

1) Probabilistic real power flow constraints is as follows:

$$P_{Gi} + \widetilde{P}_{Ri} - \widetilde{P}_{Di} - V_i \sum_{j \epsilon i} V_j (G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{4.1a}$$

Here, $\widetilde{P}_{Ri}$ and $\widetilde{P}_{Di}$ are the uncertain real power generations and the uncertain real power demands at ith bus.

2) Probabilistic reactive power flow constraints is as follows:

$$Q_{Gi} + \widetilde{Q}_{Ri} - \widetilde{Q}_{Di} - V_i \sum_{j \epsilon i} V_j (G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{4.2a}$$

Here, $\widetilde{Q}_{Ri}$ and $\widetilde{Q}_{Di}$ are the uncertain reactive power generations and the uncertain reactive power demands at ith bus.

4.2.2 Converting Probabilistic Constraints into Deterministic Constraints by applying TOAT

This step is stated as step 2 in Section 4.2. The total number of uncertain parameters for the system will be the sum of the total number of renewable generations (n_R) and the total number of loads (n_D). Now, reducing the number of random variables is helpful since the computational burden will be less. Also more random variables eventually lead to more testing scenarios in Taguchi's Orthogonal Array Technique. In this algorithm, the loads are assumed as normally distributed. It is assumed that all the loads of the system are varied using one single random variable. By this the total number of random variables is reduced to n_R+1 from the previous case of n_R+n_D. Based on the characteristics of the optimization problem, three levels for each uncertain factors are chosen [55]. For normally distributed real loads (P_{Di}) and reactive loads (Q_{Di}), $\mu(*)-\sqrt{3/2\sigma}(*)$, $\mu(*)$ and $\mu(*)+\sqrt{3/2\sigma}(*)$ are adopted as representative values where $*$ indicates $\widetilde{P}_{Di}$ or $\widetilde{Q}_{Di}$. Since the real and reactive power outputs of the renewable generations are normally varied between zero and capacity, zero, mean of the wind power distribution and capacity values are adopted as representative values of the renewable generation outputs[54]. Then the scenarios are generated using TOAT. TOAT emphasizes a mean performance characteristic value close to the target value rather than a value within certain specification limits. Thus it improves uncertainty measure[74]. Also, it can be applied to experimental design involving a large number of design factors. This also suggests the efficiency of TOAT in case of large power systems with thousands of uncertain parameters[75]. TOAT method is successfully used in [54] to solve a single objective optimal DC power flow problem.

First, a three level OA $L_H(B^M)$ is chosen which satisfies $M = n_R+1$. Then, the uncertain factors $\widetilde{P}_{Ri}$, $\widetilde{P}_{Di}$, $\widetilde{Q}_{Ri}$ and $\widetilde{Q}_{Di}$ can be expressed as follows:

$$P_{Ri}(1) = 0 \tag{4.3a}$$

$$P_{Ri}(2) = \mu(\widetilde{P}_{Ri}) \tag{4.3b}$$

$$P_{Ri}(3) = C_{Pi} \tag{4.3c}$$

$$P_{Di}(1) = \mu(\widetilde{P}_{Di}) - \sqrt{3/2\sigma}(\widetilde{P}_{Di}) \tag{4.3d}$$

$$P_{Di}(2) = \mu(\widetilde{P}_{Di}) \tag{4.3e}$$

$$P_{Di}(3) = \mu(\widetilde{P}_{Di}) + \sqrt{3/2\sigma}(\widetilde{P}_{Di}) \tag{4.3f}$$

$$Q_{Ri}(1) = 0 \tag{4.3g}$$

$$Q_{Ri}(2) = \mu(\widetilde{Q}_{Ri}) \tag{4.3h}$$

$$Q_{Ri}(3) = C_{Ri} \tag{4.3i}$$

$$Q_{Di}(1) = \mu(\widetilde{Q}_{Di}) - \sqrt{3/2\sigma}(\widetilde{Q}_{Di}) \tag{4.3j}$$

$$Q_{Di}(2) = \mu(\widetilde{Q}_{Di}) \tag{4.3k}$$

$$Q_{Di}(3) = \mu(\widetilde{Q}_{Di}) + \sqrt{3/2\sigma}(\widetilde{Q}_{Di}) \tag{4.3l}$$

Here, C_{Pi} and C_{Ri} indicates the capacity of the renewable real and reactive power generations attached to the ith bus.

4.2.3 Robust Multi-objective Optimal Power Flow Formulation

In this step, the probabilistic constraints discussed in Section 4.2.1 are replaced by Eqn. (4.3a - 4.3l) and a robust multi-objective optimal power flow is formulated. This step is stated in Section 4.2 as Step 3. The robust multi-objective OPF for all H scenarios can be formed as follows:

$$min \quad C(P) = \sum_{i=1}^{N}(a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \left|d_i sin(e_i(P_{Gi}^{min} - P_{Gi}))\right| \tag{4.4a}$$

$$min \quad E(P) = 10^{-2}(\sum_{i=1}^{N} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \zeta_i exp(\lambda_i P_{Gi}) \tag{4.4b}$$

$$min \quad P_{loss} = \sum_{k=1}^{N_L} g_k[V_i^2 + V_j^2 - 2V_i V_j cos(\delta_i - \delta j)] \tag{4.4c}$$

$$St. \quad P_{Gi} + P_{Ri}^{(1)} - P_{Di}^{(1)} - V_i \sum_{j \epsilon i} V_j(G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{4.4d}$$

$$Q_{Gi} + Q_{Ri}^{(1)} - Q_{Di}^{(1)} - V_i \sum_{j \epsilon i} V_j(G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{4.4e}$$

.

.

.

$$P_{Gi} + P_{Ri}^{(H)} - P_{Di}^{(H)} - V_i \sum_{j\epsilon i} V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{4.4f}$$

$$Q_{Gi} + Q_{Ri}^{(H)} - Q_{Di}^{(H)} - V_i \sum_{j\epsilon i} V_j(G_{ij}sin\theta ij - B_{ij}cos\theta_{ij}) = 0 \tag{4.4g}$$

$$V_i^{min} \le V_i \le V_i^{max} \tag{4.4h}$$

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \tag{4.4i}$$

$$Q_{Gi}^{min} \le Q_{Gi} \le Q_{Gi}^{max} \tag{4.4j}$$

$$\theta_i^{min} \le \theta_i \le \theta_i^{max} \tag{4.4k}$$

Here, eqn. 4.4(d)- eqn. 4.4(g) are used for each scenario testing.

4.2.4 Robust Extended Goal Programming Formulation

This step is stated as Step 4 in Section 4.2. The simple robust multi-objective AC optimal power flow problem can be formulated as extended goal programming model. For this, first three simple single objective optimal power flows are run to select the target goals for the three objectives (Step 3 in Section 3.2). These goals are fed into the extended goal programming model. The extended goal programming formulation is as follows:

$$min \quad (1-Z)\delta + Z(w_1p_1 + w_2p_2 + w_3p_3) \tag{4.5a}$$

$$St. \quad (1-Z)(u_in_i + w_ip_i) \le \delta \tag{4.5b}$$

$$\sum_{i=1}^{N}(a_i + b_iP_{Gi} + c_iP_{Gi}^2) + \left|d_isin(e_i(P_{Gi}^{min} - P_{Gi}))\right| + n_1 - p_1 = t_1 \tag{4.5c}$$

$$10^{-2}(\sum_{i=1}^{N} \alpha_i + \beta_iP_{Gi} + \gamma_iP_{Gi}^2) + \zeta_iexp(\lambda_iP_{Gi}) + n_2 - p_2 = t_2 \tag{4.5d}$$

$$\sum_{k=1}^{N_L} g_k[V_i^2 + V_j^2 - 2V_iV_jcos(\delta_i - \delta j)] + n_3 - p_3 = t_3 \tag{4.5e}$$

$$St. \quad P_{Gi} + P_{Ri}^{(1)} - P_{Di}^{(1)} - V_i \sum_{j\epsilon i} V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{4.5f}$$

$$Q_{Gi} + Q_{Ri}^{(1)} - Q_{Di}^{(1)} - V_i \sum_{j \epsilon i} V_j (G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{4.5g}$$

.

.

.

$$P_{Gi} + P_{Ri}^{(H)} - P_{Di}^{(H)} - V_i \sum_{j \epsilon i} V_j (G_{ij} cos\theta ij + B_{ij} sin\theta_{ij}) = 0 \tag{4.5h}$$

$$Q_{Gi} + Q_{Ri}^{(H)} - Q_{Di}^{(H)} - V_i \sum_{j \epsilon i} V_j (G_{ij} sin\theta ij - B_{ij} cos\theta_{ij}) = 0 \tag{4.5i}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{4.5j}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{4.5k}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{4.5l}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{4.5m}$$

Here, t_1, t_2, t_3 are target goals obtained from three single objective OPF. $n_1, ..., n_3$ are negative deviation variables and $p_1, ...p_3$ are positive deviation variables.

This formulation is different than that of eqn 3.1 (a-k) in Chapter 3. The power flow constraints are robust. Eqn. 3.1(f-g) are the deterministic power flow constraints where the uncertainties associated with real and reactive power generations, and real and reactive demands are not considered. Eqn 4.5(f-i) are the robust power flow constraints. Here, uncertain real and reactive power generations, and real and reactive demands are replaced by the deterministic levels calculated by Taguchi's method in eqn. 4.3(a-l).

4.2.5 Modified Robust Extended Goal Programming Formulation

For each of the scenarios, the optimization problem can be run and solved. For a x number of testing scenarios, x different optimized solutions can be sought. But it is not desirable since the main objective of the proposed approach is to find a single robust solution that satisfies all the scenarios. Hence two dummy variables P_{Gi}^* and Q_{Gi}^* are used. P_{Gi}^* and Q_{Gi}^* are coupled with each of the scenarios from $P_{Gi}^{(1)}$ to $P_{Gi}^{(H)}$ and $Q_{Gi}^{(1)}$ to $Q_{Gi}^{(H)}$, respectively.

Two ramping constants R^{up} and R_{down} are used. New limiting constraints are introduced from eqn. (4.6j)- (4.6m). This step is stated as Step 5 in Section 4.2. The modified robust extended goal programming formulation is below:

$$min \quad (1-Z)\delta + Z(w_1p_1 + w_2p_2 + w_3p_3) \tag{4.6a}$$

$$St. \quad (1-Z)(u_in_i + w_ip_i) \leq \delta \tag{4.6b}$$

$$\sum_{i=1}^{N}(a_i + b_iP^*_{Gi} + c_iP^{*2}_{Gi}) + \left|d_isin(e_i(P^{min}_{Gi} - P^*_{Gi}))\right| + n_1 - p_1 = t_1 \tag{4.6c}$$

$$10^{-2}(\sum_{i=1}^{N}\alpha_i + \beta_iP^*_{Gi} + \gamma_iP^{*2}_{Gi}) + \zeta_iexp(\lambda_iP^*_{Gi}) + n_2 - p_2 = t_2 \tag{4.6d}$$

$$\sum_{k=1}^{N_L}g_k[V_i^2 + V_j^2 - 2V_iV_jcos(\delta_i - \delta j)] + n_3 - p_3 = t_3 \tag{4.6e}$$

$$St. \quad P^{(1)}_{Gi} + P^{(1)}_{Ri} - P^{(1)}_{Di} - V_i\sum_{j\epsilon i}V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{4.6f}$$

$$Q^{(1)}_{Gi} + Q^{(1)}_{Ri} - Q^{(1)}_{Di} - V_i\sum_{j\epsilon i}V_j(G_{ij}sin\theta ij - B_{ij}cos\theta_{ij}) = 0 \tag{4.6g}$$

.

.

.

$$P^{(H)}_{Gi} + P^{(H)}_{Ri} - P^{(H)}_{Di} - V_i\sum_{j\epsilon i}V_j(G_{ij}cos\theta ij + B_{ij}sin\theta_{ij}) = 0 \tag{4.6h}$$

$$Q^{(H)}_{Gi} + Q^{(H)}_{Ri} - Q^{(H)}_{Di} - V_i\sum_{j\epsilon i}V_j(G_{ij}sin\theta ij - B_{ij}cos\theta_{ij}) = 0 \tag{4.6i}$$

$$R_{down} * P^{(1)}_{Gi} \leq P^*_{Gi} \leq R^{up} * P^{(1)}_{Gi} \tag{4.6j}$$

$$R_{down} * Q^{(1)}_{Gi} \leq Q^*_{Gi} \leq R^{up} * Q^{(1)}_{Gi} \tag{4.6k}$$

.

.

.

$$R_{down} * P^{(H)}_{Gi} \leq P^*_{Gi} \leq R^{up} * P^{(H)}_{Gi} \tag{4.6l}$$

$$R_{down} * Q_{Gi}^{(H)} \leq Q_{Gi}^{*} \leq R^{up} * Q_{Gi}^{(H)} \tag{4.6m}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{4.6n}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{4.6o}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \tag{4.6p}$$

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \tag{4.6q}$$

Eqn. 4.6 (j-m) are the limiting constraints. In this modified robust extended formulation, dummy variables P_{Gi}^{*} and Q_{Gi}^{*} are used as such that its values will be bounded between the upper and lower values of $P_{Gi}^{(1)}$ to $P_{Gi}^{(H)}$ and $Q_{Gi}^{(1)}$ to $Q_{Gi}^{(H)}$, respectively. The upper and lower values of $P_{Gi}^{(1)}$ to $P_{Gi}^{(H)}$ and $Q_{Gi}^{(1)}$ to $Q_{Gi}^{(H)}$ are selected using ramping constraints R^{up} and R^{down}. Hence, P_{Gi}^{*} and Q_{Gi}^{*} are the robust solution to this formulation which satisfy all the scenarios.

4.2.6 Solution Methodology

The modified robust extended goal programming formulation can be solved in following steps:

4.2.6.1 Selection of Goals

For selecting target goals, the same methodology is used as discussed in Section 3.2.4 in Chapter 3. However, the power flow constraints for the single objective optimal power flow are different than Chapter 3. In chapter 3, since it is a non-robust model, only one scenario is considered for power flow constraints. In this case, all the scenarios obtained from Taguchi method, are considered for power flow constraints. Thus, the single objective optimal power flow problem is solved and target goals for each of the objectives are calculated.

4.2.6.2 Achievement and Deviation Level

Achievement and deviation levels are calculated using the method discussed in Section 3.2.5 in Chapter 3. The value of achievement level, $t_i^{achieve}$ is bounded between 0 and 1. When an objective has an achievement level of 1, it fully satisfies the desired goal. When the level is 0, it completely fails to satisfy the goal. The value of $t_i^{deviation}$ is also bounded between 0 and 1. A value of 0 indicates that the objective achieves the goal and there is no deviation. A value of 1 indicates that it completely fails to achieve the goal.

4.2.6.3 Compromise Solution

The philosophy behind finding a compromise solution using extended goal programming is discussed in Section 3.2.6 in Chapter 3. The extended goal programming formulation is a convex combination of weighted and min-max goal programming [14], [17]. The weighted model provides a maximum achievement level by worsening the maximum deviation from the goals for some objectives. The min-max model provides a more balanced solution by minimizing the maximum deviation from the goals while lowering the maximum achievement level. In the extended formulation intermediate values of Z can provide a set of compromise solutions to the decision makers. Extended goal programming hence finds a compromise between maximum achievement and minimized maximum deviation.

4.2.6.4 Calculating the Degree of Robustness

After the modified robust extended goal programming formulation explained from eqn. (4.6a)-(4.6q) is solved, the results of the proposed algorithm are tested to verify feasibility with uncertain loads and renewable energy outputs. In this experiment, a series of deterministic testing scenarios are generated by Monte Carlo simulation based on the given distributions of load and renewable energy source. The non renewable generator's limits are changed based on the optimized solution. Limits are varied by different ramp rate constants. For each scenario, a deterministic power flow calculation is performed. If the optimization converges, the testing scenario is defined as feasible, otherwise it is defined to be infeasible. The degree of robustness of the solution is calculated by the following equation:

$$R_{deg} = \frac{S_f}{S_{all}} \times 100\% \tag{4.7}$$

where S_{all} is the total number of testing and S_f is the total number of feasible scenarios. This measure of robustness is reasonable since Monte Carlo simulation is capable of generating a number a testing scenarios which can happen in uncertain space. Hence, verifying the robustness of the solution by these testing scenarios is logical.

4.3 Numerical Results

The modified robust multi-objective extended goal programming model is solved for IEEE-14 and IEEE-30 bus systems. GAMS and a PC with an Intel Core i3 processor of 2.53 GHz and 6 GB of RAM are used. The IEEE-14 and the IEEE-30 bus data are acquired from [69]. The cost and emission coefficients of the generators for the IEEE-14 bus system and the IEEE-30 bus system are also taken from [69].

In these two systems, the mean of the load distribution are assumed to be the values of the base case, and the load standard deviations are assumed to be 5% of the mean. The renewable energy is assumed to be wind power. The distribution of wind speed is modeled as Weibull, and the scale parameter and the shape parameter are assigned to be 11.086 and 1.9622 m/s, respectively. The cut-in speed v_{ci}, cut-out speed v_{co} and rated wind speeds v_{rate} of wind power generators are assigned to be 4, 25 and 13.61 m/s [54]. After the wind speed v is simulated, the wind power output is calculated as follows[54]:

$$P_R = \begin{cases} P_{rate}(v - v_{ci})/(v_{rate} - v_{ci}) & v_{ci} \leqslant v \leqslant v_{rate} \\ P_{rate} & v_{rate} \leqslant v \leqslant v_{co} \\ 0 & otherwise \end{cases} \tag{4.8}$$

where, P_{rate} is the wind power rated capacity. In this research, the number of testing scenarios of Monte Carlo simulation is $f = 5000$.

4.3.1 Selection of Appropriate Orthogonal Array

The IEEE-14 bus systems consists of 14 buses, 5 generators and 20 lines. It is assumed that two wind farms are connected to bus-2 and bus-3, respectively, and the capacity of the wind farm is 40 MW. For robust modeling, the total number of uncertain variable is calculated. For this system, there are total 22 uncertain real and reactive loads. By the methodology discussed in Section 4.2, the number of uncertain real and reactive loads are assumed as one single random variables. It is assumed that all the bus loads go up or down at a same random rate. Also there are two uncertain real power and two uncertain reactive power in the system. The total number of uncertain variables is 5. Therefore a three level OA $L_{18}(3^5)$ with 5 uncertain parameters is selected to generate testing scenarios [74]. The robust constraints at eqn. 4.6(f)-4.6(i) are then calculated based on these 18 scenarios.

$$L_{18}(B^5) = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 3 & 1 \\ 1 & 3 & 1 & 2 & 1 \\ 1 & 3 & 2 & 3 & 2 \\ 1 & 3 & 3 & 1 & 3 \\ 2 & 1 & 1 & 3 & 3 \\ 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 2 & 2 \\ 2 & 2 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 & 1 \\ 2 & 2 & 3 & 1 & 2 \\ 2 & 3 & 1 & 3 & 2 \\ 2 & 3 & 2 & 1 & 3 \\ 2 & 3 & 3 & 2 & 1 \end{vmatrix}.$$

Similarly, the IEEE-30 bus systems consists of 30 buses, 6 generations and 41 lines. It is assumed that three wind firms are connected to bus-2, bus-5 and bus-11 respectively. For this system, there are 42 uncertain real and reactive loads. By the above mentioned methodology, the number of uncertain real and reactive loads are assumed as one single random variables. Also there are three uncertain real power outputs and three uncertain reactive power outputs in the system. So total number of uncertain variables is 7. Therefore a three level OA $L_{18}(3^5)$ with 7 uncertain parameters is selected to generate testing scenarios [74].

$$
L_{18}(B^7) = \begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 2 & 2 & 2 & 2 \\
1 & 1 & 3 & 3 & 3 & 3 & 3 \\
1 & 2 & 1 & 1 & 2 & 2 & 3 \\
1 & 2 & 2 & 2 & 3 & 3 & 1 \\
1 & 2 & 3 & 3 & 1 & 1 & 2 \\
1 & 3 & 1 & 2 & 1 & 3 & 2 \\
1 & 3 & 2 & 3 & 2 & 1 & 3 \\
1 & 3 & 3 & 1 & 3 & 2 & 1 \\
2 & 1 & 1 & 3 & 3 & 2 & 2 \\
2 & 1 & 2 & 1 & 1 & 3 & 3 \\
2 & 1 & 3 & 2 & 2 & 1 & 1 \\
2 & 2 & 1 & 2 & 3 & 1 & 3 \\
2 & 2 & 2 & 3 & 1 & 2 & 1 \\
2 & 2 & 3 & 1 & 2 & 3 & 2 \\
2 & 3 & 1 & 3 & 2 & 3 & 1 \\
2 & 3 & 2 & 1 & 3 & 1 & 2 \\
2 & 3 & 3 & 2 & 1 & 2 & 3
\end{vmatrix}.
$$

4.3.2 Target Goals

Table 4.1 lists the target goals and worst-case values for the IEEE-14 bus and the IEEE-30 bus. The target goals are obtained by the method discussed in Section 4.2.6.1. It is also observed that the target goals for IEEE-30 bus system in Table 4.1 are different than in Table 3.2 in Chapter 3. It is reasonable because the power flow constraints are different for this robust model.

Table 4.1: Best and Worst Values of Each Objective

	IEEE-14 Bus		IEEE-30 Bus	
	t_i^{best}	t_i^{worst}	t_i^{best}	t_i^{worst}
Generation Cost	52.77 $/h	95.50 $/h	788.7852 $/hr	1072.70 $/hr
Emission Cost	31.96 kg/h	442.62 kg/h	194.20 kg/h	289.62 kg/h
Transmission Loss	0.47 MW	10.11 MW	4.47 MW	45.18 MW

4.3.3 Selection of Weights

For selecting optimal weights for each of the objectives in the modified robust extended goal programming formulation, analytic hierarchy process based approached discussed in Section 3.3.2 is applied. The normalized relative weights for the generation cost, emission cost and power loss are selected as 0.64, 0.27 and 0.09 respectively.

4.3.4 Compromise Between Maximum Achievement and Deviation Level

Table 4.2 shows the results for the IEEE-14 bus system. For $Z = 1$ the extended formulation is a classical weighted model. The achievement level for each objective is calculated from eqn (3.2a), and the deviation level is calculated from eqn (3.2b). When $Z = 1$, the maximum achievement is 0.84 and the maximum deviation is 0.54. This model provides the highest achievement but also the highest deviation. For $Z = 0$ the extended formulation is a classical min-max model. When $Z = 0$, the maximum achievement is 0.77 and the maximum deviation is 0.43. This model gives a balanced solution by minimizing the maximum deviation, but the maximum achievement is lower. Although the change is small, it might not be acceptable to a decision maker who emphasizes the achievement of the goal. Also, the goals are measured in dollars per hour, thus a small reduction in the achievement could add a large penalty. To find a compromise solution we can choose intermediate values of Z. For this research, we choose the value of Z as 0.5. For $Z = 0.5$, the maximum achievement is 0.83, and the maximum deviation is 0.44, which is better than the worst-case value of 0.54. It shows that by compromising only a factor of 0.01 in the achievement level, we can improve the deviation level by 0.10. Practically, here we can reduce the emission by 229.08 kg/day by compensating only 9.77 MW/day. Hence, this intermediate value provides a better compromise solution.

Finally, the efficiency of the extended goal programming solution is shown by calculating the difference level of the solution based on maximum achievement and maximum deviation. The difference level is calculated from eqn. (3.3). It is found that for $Z = 0.5$, the difference level is calculated as 0.39 which is the highest among the other values of Z considered. For an ideal solution, the difference level should be 1. Since extended goal programming solution is providing the maximum difference level, it is considered as a better compromise solution. Difference level for different Z values for IEEE-14 bus are illustrated in figure 4.1.

Table 4.2: Comparison of Maximum Achievement and Maximum Penalty for Different Z Values for IEEE-14 Bus System

Z		**0**	**0.5**	**1**
Cost	Generation ($\$/hr$)	67.29	67.72	64.31
	Emission (Kg/hr)	208.54	212.65	253.71
	Loss (MW)	2.69	2.11	2.01
Achievement Level (1-0)	Generation	0.66	0.65	0.73
	Emission	0.57	0.56	0.46
	Loss	0.77	0.83	0.84
	Maximum	0.77	**0.83**	0.84
Deviation Level (1-0)	Generation	0.34	0.35	0.27
	Emission	0.43	0.44	0.54
	Loss	0.23	0.17	0.16
	Maximum	0.43	**0.44**	0.54
Difference Level (1-0)		0.34	**0.39**	0.30

A similar pattern is observed for the IEEE-30 bus system. From Table. 4.3, when $Z = 1$, the maximum achievement is 0.94 and the maximum deviation is 0.68. This provides the highest achievement but also the highest deviation. For $Z = 0$ the extended formulation is a classical min-max model. When $Z = 0$, the maximum achievement is 0.81 and the maximum deviation is 0.58. This model gives a balanced solution by minimizing the maximum deviation, but the maximum achievement is lower. Although the change is small, it might

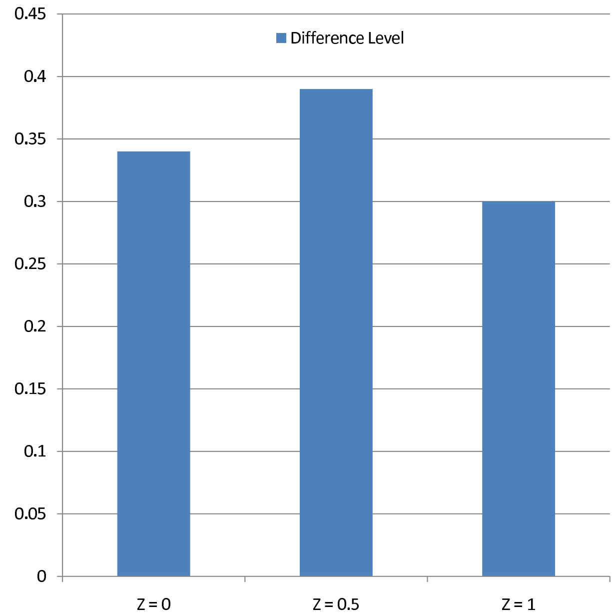

Figure 4.1: Difference Level for Different Set of Weights for IEEE-14 bus

not be acceptable to a decision maker who emphasizes the achievement of the goal. Also, the goals are measured in dollars per hour, thus a small reduction in the achievement could add a large penalty. To find a compromise solution we can choose intermediate values of Z. For this research, we choose the value of Z as 0.5. For $Z = 0.5$, the maximum achievement is 0.92, and the maximum deviation is 0.65, which is better than the worst-case value of 0.68. It shows that by compromising only a factor of 0.02 in the achievement level, we can improve the deviation level by 0.07. Practically, here we can reduce the generation cost by 476.98 $\$/day$ by compensating only 19.54 MW/day. Hence, this intermediate value provides a better compromise solution.

The efficiency of the extended goal programming solution is shown by calculating the difference level of the solution based on maximum achievement and maximum deviation. The difference level is calculated from eqn. (3.10). It is found that for $Z = 0.5$, the difference

Table 4.3: Comparison of Maximum Achievement and Maximum Penalty for Different Z Values for IEEE-30 Bus System

Z		**0**	**0.5**	**1**
Cost	Generation ($\$/hr$)	953.45	973.33	981.85
	Emission (Kg/hr)	240.00	240.01	243.81
	Loss (MW)	12.20	7.72	6.91
Achievement Level (1-0)	Generation	0.42	0.35	0.32
	Emission	0.52	0.52	0.48
	Loss	0.81	0.92	0.94
	Maximum	0.81	**0.92**	0.94
Deviation Level (1-0)	Generation	0.58	0.65	0.68
	Emission	0.48	0.48	0.52
	Loss	0.19	0.08	0.06
	Maximum	0.58	**0.65**	0.68
Difference Level (1-0)		0.23	**0.27**	0.26

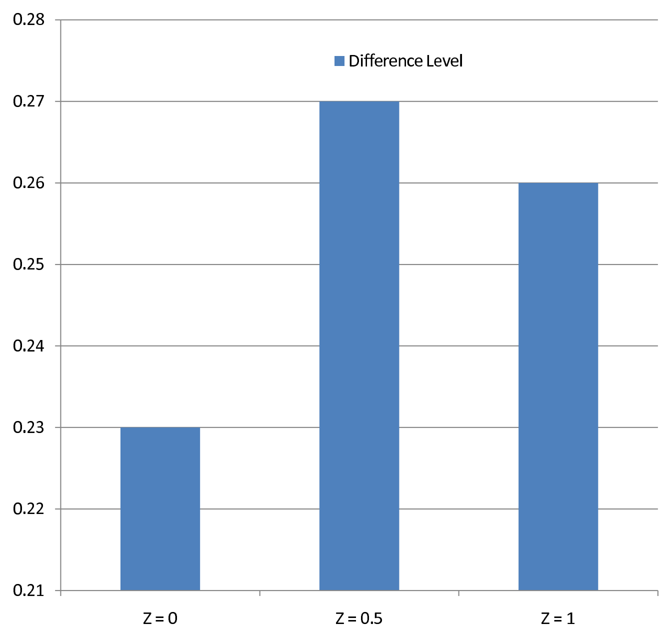

Figure 4.2: Difference Level for Different Set of Weights for IEEE-30 bus

level is calculated as 0.27 which is the highest among the other values of Z considered. For an ideal solution, the difference level should be 1. Since extended goal programming solution is providing the maximum difference level, it is considered as a better compromise solution. Difference level for different Z values for IEEE-30 bus are illustrated in figure 4.2.

Comparing Table 4.2 and 4.3 with Table 3.4 and 3.5, it is observed that extended goal programming formulation for robust multi-objective AC optimal power flow problem can still provide better compromise solution than classical goal programming models.

4.3.5 Degree of Robustness

Degree of robustness (R_{deg}) of the solution is calculated using eqn. (4.7). Table 4.4 shows the results for the robust model for the IEEE-14 bus systems. For only 5% ramp rate variation, the degree of robustness (R_{deg}) is 40.28%. It indicates that the number of feasible scenarios that satisfy the robust solution is 40.28%. The degree of robustness (R_{deg}) are 72.66% and 79.10% for 10% and 15% variation, respectively. This indicates that the more we allow the ramp rate variation, the better the degree of robustness is achieved. However, allowing more than 15% of ramp rate might not be feasible.

Table 4.5 shows the result for the non robust model for the IEEE-14 bus systems. The results for non-robust solution are calculated using the methodology discussed in Section 3.2 in Chapter 3. It is reasonable that the non robust solution can not satisfy most of the uncertain scenarios. For a 5% ramp rate variation, the degree of robustness(R_{deg}) is only 6.24% which is almost 7 times lower than that of the robust model. For a 15% variation in ramp rate, the non-robust model gets 40.42% degree of robustness(R_{deg}) which is almost two times lower than that of robust model. The results shows that the algorithm can handle uncertain scenarios generated by the Monte Carlo simulation effectively. For a higher ramp rate(15%), the robust solution can satisfy almost 80% of the uncertain scenarios of the power system operation. This results are illustrated in figure 4.3 showing high degree of robustness for robust model comparing to that of non-robust model.

A similar phenomenon is observed for the IEEE-30 bus systems. The degree of robustness (R_{deg}) for IEEE-30 bus systems for both the robust and non-robust models are shown in Table 4.6 and 4.7. From the Table 4.6, for only 5% ramp rate variation, the degree of robustness (R_{deg}) is 38.26% for the robust model. It indicates that the number of feasible scenarios that satisfy the robust solution is 40.28%. The degree of robustness (R_{deg}) are 76.90% and 82.54% for 10% and 15% variation, respectively. This indicates that the more we allow the ramp rate variation, the better the degree of robustness is achieved. However, allowing more than 15% of ramp rate might not be feasible.

Table 4.7 shows the result for the non robust model for the IEEE-30 bus systems. The results for non-robust solution are calculated using the methodology discussed in Section 3.2 in Chapter 3. It is reasonable that the non robust solution can not satisfy most of the uncertain scenarios. For a 5% ramp rate variation, the degree of robustness(R_{deg}) is only 5.56% which is almost 7 times lower than that of the robust model. For a 15% variation in ramp rate, the non-robust model gets 31.72% degree of robustness(R_{deg}) which is more than two times lower than that of robust model. The results shows that the algorithm can handle uncertain scenarios generated by the Monte Carlo simulation effectively. For a higher ramp rate(15%), the robust solution can satisfy almost 83% of the uncertain scenarios of the power system operation. This results are illustrated in figure 4.4 showing high degree of robustness for robust model comparing to that of non-robust model.

Table 4.4: Degree of Robustness (R_{deg}) for IEEE-14 bus system robust model

	Ramp Rate Variation Allowed (%)		
	5%	10%	15%
Total No. of Scenarios	5000	5000	5000
No. of Feasible Scenarios	2014	3633	3955
No. of Infeasible Scenarios	2986	1367	1045
Degree of Robustness(R_{deg}) (%)	40.28	72.66	79.10

Table 4.5: Degree of Robustness (R_{deg}) for IEEE-14 bus system non-robust model

	Ramp Rate Variation Allowed (%)		
	5%	10%	15%
Total No. of Scenarios	5000	5000	5000
No. of Feasible Scenarios	312	1235	2021
No. of Infeasible Scenarios	4688	3765	2979
Degree of Robustness(R_{deg}) (%)	6.24	24.70	40.42

Table 4.6: Degree of Robustness (R_{deg}) for IEEE-30 bus system robust model

	Ramp Rate Variation Allowed (%)		
	5%	10%	15%
Total No. of Scenarios	5000	5000	5000
No. of Feasible Scenarios	1913	3845	4127
No. of Infeasible Scenarios	3087	1155	873
Degree of Robustness(R_{deg}) (%)	38.26	76.90	82.54

Table 4.7: Degree of Robustness (R_{deg}) for IEEE-30 bus system non-robust model

	Ramp Rate Variation Allowed (%)		
	5%	10%	15%
Total No. of Scenarios	5000	5000	5000
No. of Feasible Scenarios	278	1586	2127
No. of Infeasible Scenarios	4722	3414	2873
Degree of Robustness(R_{deg}) (%)	5.56	31.72	42.54

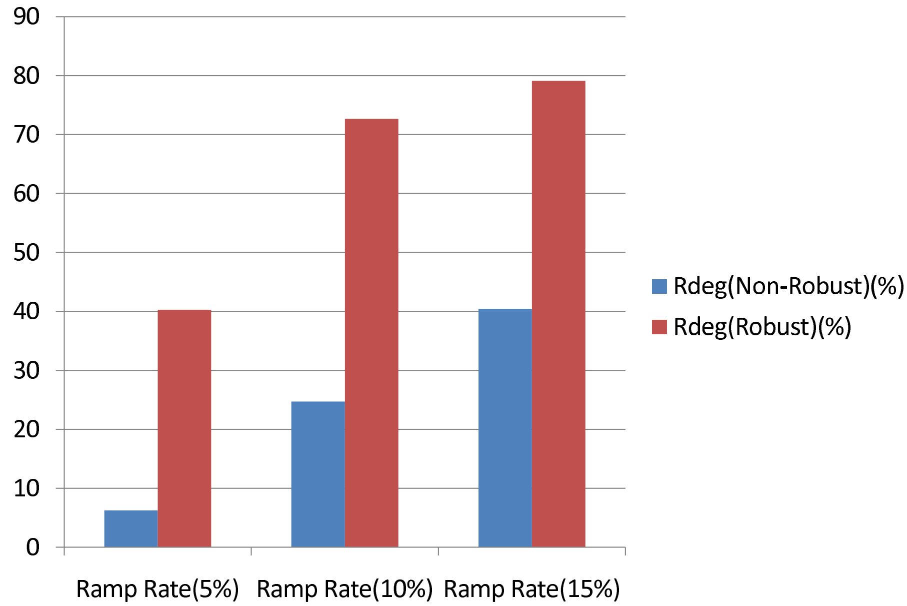

Figure 4.3: R_{deg} for Robust and Non-robust model for IEEE-14 bus

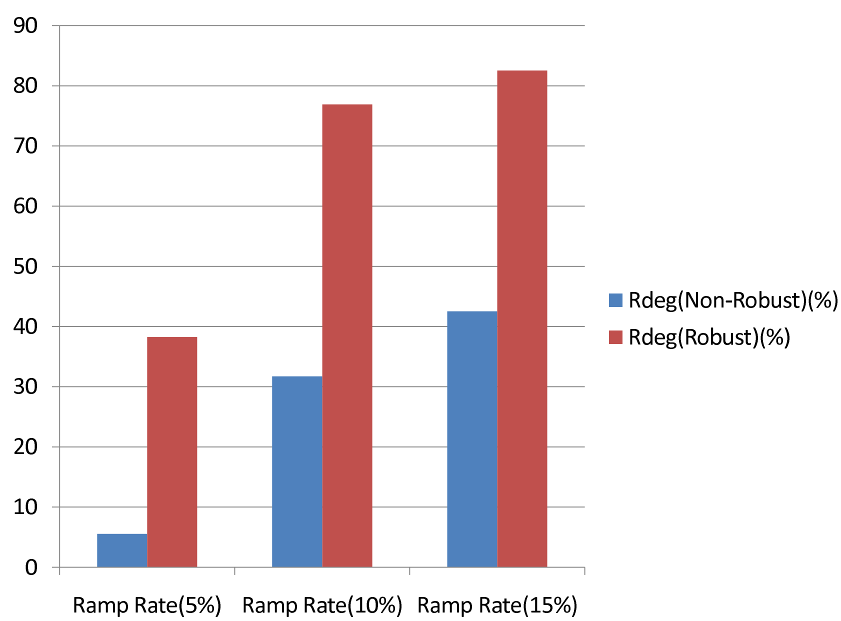

Figure 4.4: R_{deg} for Robust and Non-robust model for IEEE-30 bus

4.3.6 Comparison Between Robust and Non-robust Model

The robust solution can be analyzed from the extended goal programming point of view. For the IEEE-14 bus systems, from Table 4.8, it is shown that in case of robust solution both the maximum achievement level and the maximum deviation level are affected. For the non robust solution, the maximum achievement level was 0.93 and deviation level was 0.35. This non robust solution is obtained by the method discussed in Section 3.2 in Chapter 3. But for the robust solution, both the cost is increased. The achievement level is decreased to 0.83 while the maximum deviation is increased to 0.44. So 0.10 decrement in achieving the goal and 0.09 increment in deviation level are the robust cost of this algorithm. 0.10 decrement in achievement level refers to 4.27 $\$/hr$ increment of generation cost. On the other hand, 0.09 increment in deviation level refers to an increment of 36.96 kg/hr in emission. The annual robust cost of this algorithm are 37405.20 $\$/year$ and 323.77 $tons/year$.

The robust cost of the method can also be assumed from the difference level value. The value of difference level is 0.58 for the non-robust model while it is only 0.39 for the robust model. The difference value is decreased by a factor of 0.19.

Table 4.8: Comparison Between Robust and Non-robust Model for IEEE-14 bus system

	Goal	Robust Solution	Non-Robust Solution
Gen. Cost ($/hr)	52.77	75.36	68.91
Emission (Kg/hr)	31.96	45.34	33.74
Power Loss (MW)	0.47	4.85	2.56
Max Achievement	1	**0.83**	**0.93**
Max Deviation	0	**0.44**	**0.35**
Difference Level (L_{diff})	1	**0.39**	**0.58**
No. of Scenarios	-	18	1
No. of Uncertain Variables	-	5	0

Again, in case of IEEE-30 bus systems, from Table 4.9, the maximum achievement is 0.75 and the maximum deviation is 0.46 for the non-robust solution (Obtained by the method discussed in Section 3.2 of Chapter 3). On the other hand, for the robust solution, the maximum achievement is decreased to 0.72 and the maximum deviation is 0.65. It is observed that for robust solution for the IEEE-30 bus systems, the maximum achievement is decreased by 0.03 and the maximum deviation is increased by 0.19. 0.03 decrement in achievement level refers to 1.22 MW/hr increment of power loss. On the other hand, 0.19 increment in deviation level refers to an increment of 53.94 $\$/hr$ in emission. The annual robust cost of this algorithm are 10687.20 $MW/year$ and 472514.40 $\$/year$.

Table 4.9: Comparison Between Robust and Non-robust Model for IEEE-30 bus system

	Goal	Robust Solution	Non-Robust Solution
Gen. Cost ($/hr)	788.78	972.82	920.41
Emission (Kg/hr)	194.20	239.81	227.31
Power Loss (MW)	4.47	15.76	14.63
Max Achievement	1	**0.72**	**0.75**
Max Deviation	0	**0.65**	**0.46**
Difference Level (L_{diff})	1	**0.07**	**0.29**
No. of Scenarios	-	18	1
No. of Uncertain Variables	-	7	0

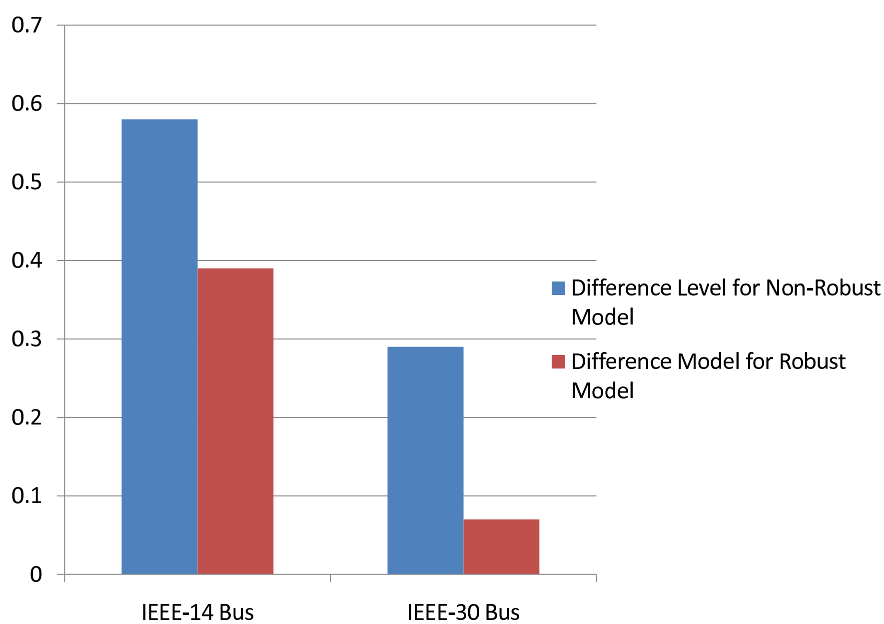

Figure 4.5: Difference Level for Robust and Non-robust model for IEEE-14 Bus and IEEE-30 bus

The robust cost of the method can also be assumed from the difference level value. The value of difference level is 0.29 for the non-robust model while it is only 0.07 for the robust model. The difference value is decreased by a factor of 0.22.

In conclusion for both the IEEE-14 bus and IEEE-30 bus system, although the robust model can efficiently handle the uncertainties associated with load and renewable generations, the solution quality from a multi-objective point of view is decreased. And this decrement in the quality of the solution incurs higher cost for robust model. This phenomena is illustrated in figure 4.5.

4.3.7 Summary

In this chapter, an approach to solve multi-objective optimal power flow problem under load and renewable generation uncertainties based on extended goal programming is proposed. The proposed approach is tested using the IEEE-14 and IEEE-30 bus systems. A multi-objective optimal power flow problem under load and renewable generations uncertainties is considered. Taguchi's orthogonal array testing (TOAT) is employed to select the optimal scenarios and hence, the probabilistic constraints are converted into robust deterministic constraints. Finally, a modified robust extended goal programming model of the multi-objective optimal power flow problem is formulated. The degree of robustness in calculated for both the non-robust and robust model.

It is shown that the proposed model provides a solution which is robust to uncertain variations in load and renewable generations. It is also observed that the degree of robustness can be improved by changing the ramp rate. For a higher ramp rate, the model can achieve a higher degree of robustness. The cost of robustness is also calculated from the extended goal programming point of view. It is shown that to achieve the robust solution, both the maximum achievement level and deviation level are affected. Maximum deviation is increased and maximum achievement is decreased comparing to that of the non-robust model discussed in Chapter 3. The proposed algorithm can be easily adopted to model and solve other types of uncertain multi-objective optimal power flow problems.

Chapter 5

Conclusions

In modern power systems, optimization problems are mostly multi-objective by nature such as power system planning problems [1],[2], reactive power compensation schemes [3], transmission line expansion problems [4], economic-emission load dispatch problems [5], hydrothermal scheduling problems [6], multi-objective optimal power flow (OPF) problems [7] etc. In this thesis, extended goal programming is applied to solve a multi-objective optimal power flow problem. In addition, with the integration of wind and solar power in power systems, methods to address the uncertainty of these resources in system operation and planning is becoming important. The later part of this thesis is dedicated to solve multi-objective optimization problems in power systems under uncertainties.

Chapter 3 of this thesis proposes an approach to solve multiple conflicting objectives of an optimal power flow problem based on extended goal programming theory. In this chapter, first a multi-objective optimal power flow problem has been transformed to an extended goal programming formulation. After that, target goals for each of the objectives are set using the solution of single objective optimal power flow problem for each of the objectives and optimal weights are calculated using analytic hierarchy process. Finally the problem is solved and results are compared with other classical goal programming methods.

The main contributions of Chapter 3 are:

1. An extended goal programming formulation of multi-objective AC optimal power flow problem.

2. An analysis of significance of the Z parameter, showing the efficiency of the model comparing to other classical goal programming models.

3. A ranking strategy to choose the best routine based on decision makers' priorities.

The significance of the above contributions is as follows:

1. The results show that the extended formulation has advantages over the classical weighted and min-max formulations. It can provide a optimal solution which is a trade off between maximum achievement of the goal and the maximum deviation from the goal.

2. The extended formulation can find the weighted solution, the min-max solution, or a compromise of the two by changing the value of the Z parameter. Thus, it can offer a set of solutions for decision makers to consider.

3. The formulation is simple, faster and hence it is easy to model complex problems.

Chapter 4 of this thesis proposes an approach to solve multi-objective AC optimal power flow problem under load and renewable generation uncertainties. In this chapter, Taguchi's Orthogonal Array Testing is used to select a minimum number of testing scenarios with good statistical information in the uncertain space. After selecting the optimal scenarios, the uncertain multi-objective AC optimal power flow problem is transformed into a robust multi-objective optimal power flow problem. Finally, the problem is modeled based on extended goal programming theory and solved. It is shown that the proposed method can provide a solution that is robust to uncertain variations in load and renewable generations and also can satisfy all the conflicting objectives.

The main contributions of Chapter 4 are:

1. Modeling a multi-objective AC optimal power flow problem under uncertainties using Taguch's Orthogonal Array Technique and extended goal programming method.

2. An analysis of the significance of ramp rate variation with the degree of robustness of the solution.

3. A comparison between robust and non-robust solution from the extended goal programming point of view showing the efficiency of the proposed approach.

The significance of the above contributions is as follows:

1. The proposed algorithm provides a solution that is robust to uncertain variations in

load and renewable generations.

2. It is also observed that the degree of robustness can be improved by changing the ramp rate. For a higher ramp rate, the model can achieve a higher degree of robustness.

3. The cost of robustness is also calculated from the extended goal programming point of view. It is shown that to achieve the robust solution, both the maximum achievement level and deviation level are affected. Maximum deviation is increased and maximum achievement is decreased comparing to that of the non-robust model.

4. The proposed algorithm can be easily adopted to model and solve other types of uncertain multi-objective optimal power flow problems.

Bibliography

[1] Rivas-Davalos, F., Moreno-Goytia, E., Gutierrez-Alcaraz, G., Tovar-Hernandez, H. *Multiobjective optimization challenges in power system: The next step forward*, Electronics, Robotics and Automotive Mechanics Conference, 2007.

[2] Stoll, H. G. *Least Cost Electric Utility Planning*. John Wiley Interscience, USA, 1989.

[3] CIGRE Task Force 380103, *Planning Against Voltage Collapse*, CIGRE, pp. 126, Paris, France, October 1986.

[4] Maghouli, P., Hosseini, S.H., Buygi, M.O., Shahidehpour, M. *A multi-objective framework for transmission expansion planning in deregulated environments*, IEEE Transactions on Power Systems, Vol. 24, No. 2, May 2009, pp. 1051-1061.

[5] Niknam, T., Narimani, M.R., Aghaei, J., Azizipanah-Abarghooee, R. *Improved particle swarm optimisation for multi-objective optimal power flow considering the cost, loss, emission and voltage stability index*, Generation, IET Transmission and Distribution, Vol. 6, No. 6, June 2012, pp. 515-527.

[6] Dhillon, J.S., Parti, S.C., Kothari, D.P. *Fuzzy decision-making in stochastic multiobjective short-term hydrothermal scheduling*, IEE Proceedings Generation, Transmission and Distribution, Vol. 149, No. 2, March 2002, pp. 191-200.

[7] Rosehart, W.D., Canizares, C.A., Quintana, V.H. *Multiobjective optimal power flows to evaluate voltage security costs in power networks*, IEEE Transactions on Power Systems, Vol. 18, No. 2, May 2003, pp. 578-587.

[8] Haimes, Y.Y. *Hierarchical Analysis of Water Resources Systems: Modeling and Optimization of Large-Scale Systems [M]*. New York: McGraw Hill, 1977.

[9] Osyczka, A., Davies, B.J. *Multicriterion Optimization Problems in Engineering with Fortran Programs*. Ellis Horwood Ltd., 1984.

[10] Charalambous, C. *A new approach to multicriterion optimization problem and its application to design of 1-D digital filters*, IEEE Trans. on Circuits and Systems, Vol. 36, No. 6, 1989, pp. 773-784.

[11] Rao, S.S. *Optimization: Theory and Applications*, 2nd edition. Wiley Eastern Limited, New Delhi, 1987.

[12] Charnes, A. and Cooper, W.W. *Management models and industrial applications of linear programming*, Management Science, Vol. 4, No. 1, October 1957, pp. 38-91.

[13] Charnes A., Cooper W.W. *Goal programming and multiple objective optimization, Part I*, European Journal of Operational Research, Vol. 1, 1977, pp. 39-54.

[14] Ignizio, J., Romero, C. *Goal programming*. In: H. Bidgoli, ed. Encyclopedia of Information Systems, Vol. 2. Academic Press, San Diego, pp. 489-500, 2003.

[15] Jones, D., Tamiz, M. *Practical Goal Programming*, International Series in Operations Research and Management Science, Vol. 141, 1st Edition, 2010.

[16] Flavell, R.B. *A new goal programming formulation*, Omega, Vol. 4, No. 6, 1976, pp. 731-732.

[17] Romero, C. *Extended lexicographic goal programming: A unifying approach*, Omega, Vol. 29, No. 1, February 2001, pp. 63-71.

[18] Belad Aounia,Ossama Kettanib, *Goal programming model: A glorious history and a promising future*, European Journal of Operational Research, Volume 133, Issue 2, 1 January 2001, Pages 225231

[19] Yun, Z., Liu, Y., Mu, H., Ma, X. *Fuzzy multi-objective mixed integer linear programming based secondary voltage control*, 7th IET International Conference on Advances in Power System Control, Operation and Management (APSCOM 2006), IET Conf. Pub., Vol. 2006, pp. 445.

[20] Centeno, E., Vitoriano, B., Campos, A., Muñoz, A., Villar, J., Sánchez-Úbeda, E.F. *A goal programming model for rescheduling of generation power in deregulated markets*, Annals of Operations Research, Vol. 120, No. 1–4, 2003, pp. 45-57.

[21] Biswas, P., Chakraborti, D., Paruya, S., Kar, S., Roy, S. *A fuzzy goal programming procedure for solving multiobjective load flow problems via genetic algorithm*, AIP Conference Proceedings, Vol. 1298, No. 1, 2010, pp. 415-420.

[22] Liming Zhang, Huiyou Chang, Ruitian Xu, *Equal-Width Partitioning Roulette Wheel Selection in Genetic Algorithm*, Conference on Technologies and Applications of Artificial Intelligence (TAAI), 2012, Page(s): 62 - 67

[23] Vinothkumar, K., Selvan, M.P. *Distributed generation planning: A new approach based on goal programming*, Electric Power Components and Systems, Vol. 40, No. 5, 2012, pp. 497-512.

[24] Chakraborti, D., Biswas, P., Mukhopadhyay, A. *Bio-inspired computational technique to multiobjective optimal planning of electric power generation and dispatch*, International Conference on Communication and Industrial Application (ICCIA), 2011, pp. 1-4.

[25] Ghosh, S., Basu, S.C., Sengupta, P.P. *Improvement of financial efficiency and cost effectiveness in energy sector: A case study from Indian thermal power plant*, International Conference on Education and Management Technology (ICEMT), 2010, pp. 512-516

[26] Qiao,Jiageng, Min,Yong, Lu,Zongxiang *Optimal Reactive Power Flow in Wind Generation Integrated Power System*, International Conference on Power System Technology, PowerCon 2006, pp.1-5

[27] Terra, L.D.B., Gouvea, M.M., Jr. *Fuzzy goal programming applied to the optimal reactive power flow problem*, International Conference on Electric Power Engineering, PowerTech Budapest 1999.

[28] Munoz Moro, L., Ramos, A. *Goal programming approach to maintenance scheduling of generating units in large scale power systems*, IEEE Transactions on Power Systems, Vol. 14, No. 3, 1999, pp. 1021-1028.

[29] Nanda, J., Kothari, D.P., Lingamurthy, K.S. *Economic-emission load dispatch through goal programming techniques*, IEEE Transactions on Energy Conversion, Vol. 3, No. 1, 1988, pp. 26-32.

[30] Kim, K.-H., Song, K.-B., Joo, S.-K., Lee, Y.-J., Kim, J.-O. *Multiobjective distributed generation placement using fuzzy goal programming with genetic algorithm*, European Transactions on Electrical Power, Vol. 18, No. 3, 2008, pp. 217-230.

[31] Nangia, U., Jain, N.K., Wadhwa, C.L. *Regret analysis of sequential goal programming for multiobjective optimal power flows*, 7th International Power Engineering Conference, IPEC 2005, Vol. 2, pp. 1200-1204.

[32] Wadhwa, C.L., Jain, N.K. *Multiple objective optimal load flow: A new perspective*, IEE Proceedings on Generation, Transmission and Distribution, Vol. 137, No. 1, 1990, pp. 13-18.

[33] Jos Ramn San Cristbal *A goal programming model for the optimal mix and location of renewable energy plants in the north of Spain*, Renewable and Sustainable Energy Reviews, Volume 16, Issue 7, September 2012, pp. 44614464

[34] Jinturkar,A. M., Deshmukh, S. S. *A fuzzy mixed integer goal programming approach for cooking and heating energy planning in rural India*, Expert Systems with Applications: An International Journal, Volume 38 Issue 9, September, 2011, pp. 11377-11381

[35] Daim,Tugrul U., Kayakutlub,Gulgun, Cowana, Kelly *Developing Oregons renewable energy portfolio using fuzzy goal programming model*, Computers and Industrial Engineering, Volume 59, Issue 4, November 2010, pp. 786793

[36] Mezher, T., Chedid, R., Zahabi, W. *Energy resource allocation using multi-objective goal programming: The case of Lebanon*, Applied Energy, Vol. 61, No. 4, 1998, pp. 175-192.

[37] Pal, B.B., Biswas, P., Mukhopadhyay, A. *Using genetic algorithm to goal programming model of solving economic-environmental electric power generation problem with interval-valued target goals*, Mathematical Modelling and Scientific Computation, Communications in Computer and Information Science, 2012, Vol. 283, Part 1, 156-169.

[38] Linares, P., Romero, C. *A multiple criteria decision making approach for electricity planning in Spain: Economic versus environmental objectives*, Journal of the Operational Research Society, Vol. 51, 2000, 736-743.

[39] Qin, Zhongfeng *Uncertain random goal programming*, Uncertainty Society of China.

[40] Keown, Arthur J.; Taylor III, Bernard W.; *A Chance-Constrained Integer Goal Programming Model for Capital Budgeting in the Production Area*, The Journal of the Operational Research Society, Vol. 31, No. 7 (Jul., 1980), pp. 579-589

[41] Jiekun Song, Tao Zhang, Zaixu Zhang *Stochastic Chance-constrained Goal Programming Model and Algorithm of Oilfield Measures*, International Conference on Information Management, Innovation Management and Industrial Engineering, 2008, pp. 129 - 132

[42] W. Mohammed, *Chance constrained fuzzy goal programming with right-hand side uniform random variable coefficients*, Fuzzy Sets and Systems, Volume 109, Issue 1, 1 January 2000, Pages 107110

[43] Ian Durbach, Theodor J. Stewart, *Integrating scenario planning and goal programming*, Journal of Multi-Criteria Decision Analysis, Volume 12, Issue 4-5, pages 261271, July - October 2003

[44] Enrique Ballestero, Blanca Prez-Gladish, Mar Arenas-Parra, Amelia Bilbao-Terol, *Selecting Portfolios Given Multiple Eurostoxx-Based Uncertainty Scenarios: A Stochastic Goal Programming Approach from Fuzzy Betas*, INFOR: Information Systems and Operational Research, University of Toronto Press, pp. 59-70, December 15, 2009

[45] Anthony Chen, Xiangdong Xua, *Goal programming approach to solving network design problem with multiple objectives and demand uncertainty*, Expert Systems with Applications, Volume 39, Issue 4, March 2012, Pages 41604170

[46] Enrique Ballestero, *Stochastic goal programming: A meanvariance approach*, European Journal of Operational Research Volume 131, Issue 3, 16 June 2001, Pages 476481

[47] Nguyen Van Hop, *Fuzzy stochastic goal programming problems*, European Journal of Operational Research Volume 176, Issue 1, 1 January 2007, Pages 7786

[48] Mila Bravo, Ignacio Gonzalez, *Applying stochastic goal programming: A case study on water use planning*, European Journal of Operational Research, Volume 196, Issue 3, 1 August 2009, Pages 11231129

[49] Sha-lei Zhan, Nan Liu, *A Multi-objective Stochastic Programming Model for Emergency Logistics Based on Goal Programming*, Fourth International Joint Conference on Computational Sciences and Optimization (CSO), 2011, pp. 640-644

[50] Ins Gonzlez-Rodrguez, Camino R. Vela, Jorge Puente, *A genetic solution based on lexicographical goal programming for a multiobjective job shop with uncertainty*, Journal of Intelligent Manufacturing, February 2010, Volume 21, Issue 1, pp 65-73

[51] M M Lotfi, S F Ghaderi, *A compromised multi-objective solution using fuzzy mixed integer goal programming for market-based short-term unit commitment problem*, Journal of the Operational Research Society, Volume 64 Issue 8, February, 2013

[52] Biswas, Papun; Chakraborti, Debjani*A Fuzzy Goal Programming Procedure for Solving Multiobjective Load Flow Problems via Genetic Algorithm*, International Conference on Modeling, Optimization, AND Computing (ICMOS 20110). AIP Conference Proceedings, Volume 1298, pp. 415-420.

[53] Bijay Baran Pal, Mousumi Kumar, *A linear Fuzzy Goal Programming Method for Solving Optimal Power Generation and Dispatch Problem*, International Journal of Advanced Computer Research, Volume.3, Number.1, Issue.8, March, 2013.

[54] Han, Yu; Rosehart, W. D. *An Optimal Power Flow Algorithm to Achieve Robust Operation Considering Load and Renewable Generation Uncertainties*, IEEE Transactions on Power Systems, Vol. 27, Issue: 4, 2012, pp: 1808-1817.

[55] R. N. Kackar, *Off-line quality control, parameter design, and the Taguchi method*, J. Quality Technol., vol. 17, no. 4, pp. 176188,1985.

[56] Alberta Energy: Facts and Statistics, http://www.energy.alberta.ca/OilSands/791.asp

[57] Kristi Anderson, Tim Weis,Ben Thibault, Farrah Khan, Beth Nanni, and Noah Farber, *A Costly Diagnosis: Subsidizing coal power with Albertans health*, March 2013, The Pembina Foundation

[58] Coal-fired emissions likely to cause thousands of deaths in Alberta: study,

http://www.ctvnews.ca/health/coal-fired-emissions-likely-to-cause-thousands-of-deaths-in-alberta-study-1.1211700

[59] Loss Factors - High Level Process Review, http://www.aeso.ca

[60] American Wind Energy Association (AWEA), Us and China race to top of wind energy , World Wide Web, 2009, accessed: Feb 2009. [Online]. Available: http://www.awea.org/newsroom/releases/us and china race to top of wind energy 02Fed09.html

[61] Dommel, H.W., Tinney, W.F. ,*Optimal Power Flow Solutions*,IEEE Transactions on Power Apparatus and Systems, Vol. PAS-87 , Issue: 10, Page(s): 1866- 1876, October 1968

[62] Heidi A. Taboada , David W. Coit, *Post-Pareto Optimality Analysis to Efficiently Identify Promising Solutions for Multi-Objective Problems*

[63] Saaty, T.L. *How to make a decision: The analytic hierarchy process*, European Journal of Operational Research, Vol. 48, No. 1, September 1990, pp. 9-26

[64] Zahedi, F. *The analytic hierarchy process: A survey of the method and its applications*, Interfaces, Vol. 16, No. 4, Jul/Aug 1986, pp. 96-108.

[65] Saaty, T.L. *Decision-making with the AHP: Why is the principal eigenvector necessary*, European Journal of Operational Research, Vol. 145, 2003, pp. 85-91.

[66] Farkas, A. *The analysis of the principal eigenvector of pairwise comparison matrices*, Acta Polytechnica Hungarica, Vol. 4, No. 2, 2007.

[67] Diaz-Balteiro, L., Romero, C. *Sustainability of forest management plans: A discrete goal programming approach*, Journal of Environmental Management, Vol. 71, 2004, 351-359.

[68] Edward L. Hannan *An assessment of some criticisms of goal programming*, Computers and Operations Research, Volume 12, Issue 6, 1985, Pages 525541

[69] Power Systems Test Case Archive, http://www.ee.washington.edu/research/pstca/

[70] IEEE 118 bus data, http://www.ece.ubc.ca/ hameda/downloadfiles/case118.m

[71] Hazra, J., Sinha, A.K. *A multi-objective optimal power flow using particle swarm optimization*, European Transactions on Electrical Power, Vol. 21, No. 1, January 2011, pp. 1028-1045.

[72] AHP (Analytic Hierarchy Process). Calculation software by CGI, http://www.isc.senshu-u.ac.jp/ thc0456/EAHP/AHPweb.html

[73] Christensen, G. *Optimal Economic Operation of Electric Power Systems*. Academic Press, 1979.

[74] Design of experiments via taguchi methods: orthogonal arrays, https://controls.engin.umich.edu/wiki/images/0/0b/Lecture.25.pdf

[75] Orthogonal Array, http://kernow.curtin.edu.au/www/Taguchi/SET7.HTM