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Mathematics Content Knowledge, Mathematics Teacher Efficacy, and Pedagogy: An Examination of the Three Constructs in Mathematics Preservice Elementary Education

by

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Abstract

An important goal in elementary mathematics classrooms is for teachers to employ instructional practices that engage students to develop sustainable and deep conceptual mathematics understanding. In preservice education, this often requires transforming student teachers' beliefs about teaching mathematics, and their knowledge of mathematics content and pedagogy. This study involved preservice teachers from two mathematics methods courses and specifically examined teaching and learning experiences that contributed to their development in: 1) content knowledge, 2) teacher efficacy, and 3) pedagogy. Building capacity across these three constructs was considered significant in the preservice mathematics courses. This research focused on identifying the factors which influenced students' development in these three constructs with the view to enhance preservice mathematics programs.

The theoretical framework for this research was underpinned by several theories: mathematical knowledge for teaching theory, self-efficacy theory, constructivist learning theory, and adult learning theory. These selected theories were deemed to be highly pertinent to the phenomena of preservice mathematics teacher development and drew upon a range of theorists such as: Ball, Thames, and Phelps, 2008; Bandura, 1986; Dewey, 1938; Knowles, 1984; Mezirow, 1991; Piaget, 1952; and Vygotsky, 1978.

Through a mixed methods approach, this study utilized quantitative data related to student teachers' mathematics content knowledge, teacher efficacy, and anxiety. The qualitative data included student teachers' journals, interviews, and the researcher-instructor's journals. The five broad themes that emerged from the converged data were: 1) importance of the instructor's role in mathematics teacher development; 2) problem solving to support conceptual understanding; 3) building confidence as a mathematics teacher; 4) working towards constructivist pedagogy; and 5) classroom management.

Two models were developed from the findings in this study. Model one highlights the differentiating mathematics needs of preservice teachers and ways to support their needs prior to the start of their teacher education program; and model two addresses conceptualizations of coherence and how this can be achieved across the dimensions of mathematics preservice education. These models and this study's significant findings will be of interest to those seeking to enhance preservice elementary

mathematics teacher education through building coherence and implementing strategies to meet the varying backgrounds of student teachers.

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I wish to express my most sincere gratitude to my colleagues and friends: Nancy Steele, Dr. Larry Swartz, Beverley Strachan, Dr. Cathy Marks Krpan, and Dr. David Booth. Nancy, your unwavering support and kind-heartedness over the years have taught me about humility. Larry, thank you for always making me laugh especially during times when I thought finishing was not possible. Bev, you are a genuine and caring friend who is always there to reassure and encourage. Cathy, your leadership in mathematics education inspires me to grow as a learner and educator. David, I will forever remember being in your graduate course - you challenged my views about teaching and learning in powerful ways. My world is enriched by each of you and I am blessed to have you all as my life-long friends.

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Heartfelt acknowledgements and thank you go out to my family. To my brother Kelvin Seow, I am grateful that Kiana and Asia have the best “Dunki” ever. I sincerely appreciate my “Baba” for instilling the value of education from a very young age. Thank you to “Papa” and “Gampy” for your patience and time spent with the girls. To my mother, who sadly lost her battle to cancer when I was only 8 years old. I know she is with me in spirit and her memory lives on in my heart. Acknowledgements go out to Kiana and Asia – nothing is more beautiful than when you dance – I am honoured to be your mommy. This dissertation is not only dedicated to my husband Dr. Steven M. Reid, but I must acknowledge him as well. Thank you Steven for reading my thesis, chapter by chapter, and for guiding me in the right direction. You were instrumental in helping me achieve my goal of earning my doctorate degree.

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Dedication

In all the world, there is no heart for me like yours.
In all the world, there is no love for you like mine.
Maya Angelou

To my soul mate, my husband, my partner in life, Steven. Thank you for your unconditional love and support throughout this journey. Forever.

To my beautiful daughters, Kiana and Asia. Every day brings an opportunity for you to dance like there is no one watching.

To J.J. and Moy Moy, for always keeping my feet warm while at the computer.

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Chapter 1: INTRODUCTION

Introduction

An important goal for every teacher education mathematics program is to build capacity in preservice teachers so they become key agents in successfully teaching mathematics. This entails supporting prospective teachers so they attain the content knowledge, teacher efficacy, and pedagogical skills, to teach mathematics for deep understanding (Ball, Sleep, Boerst, & Bass, 2009; Ball, Thames, & Phelps, 2008; Sowder, 2007). In order to achieve this goal, student teachers must have the opportunity in their preservice education mathematics course to experience the kinds of university teaching that reflect active participation, exploration of problems, hands-on learning, and discovery of ideas (Karp, 2010; Sowder, 2007). Hence, preservice teachers require rich mathematical experiences that are envisioned in the standards of the National Council of Teachers of Mathematics (NCTM).

In 1989 and also in 2000, the NCTM established standards that called for greater emphasis on learning mathematics for deep understanding through meaningful problem solving contexts. NCTM's standards are often referred to as reform mathematics in comparison to traditional mathematics instruction or low-level mathematics teaching. The reform movement in mathematics teaching promotes the philosophy that teachers should engage learners to conceptually anchor mathematical ideas and support their ability to reason, communicate mathematically, and solve problems based on learners' own thinking and experiences.

Unfortunately, despite NCTM's mandate for mathematical reform over two decades ago, the literature reveals that insufficient change has transpired in mathematics instruction (Battista, 1999; Frykholm, 1999; Hiebert et al., 2003; Hiebert et al., 2005; Marshall, 2003, 2006; McLeod et al., 1997). Classroom practice remains dominated by teacher explanations, rule memorization, and reliance on text books and work sheets. Such studies illustrate how reform recommendations are often implemented in superficial ways. Hiebert et al. (2005) examined the teaching practices across seven countries involved in the Trends in International Mathematics and Science Study (TIMSS) and discovered that U.S. teachers focused more on low-level mathematics skills compared with higher achieving countries that executed a more balanced approach in mathematics conceptual understanding, procedural development, and challenging content. In another study that analyzed video recordings of mathematics lessons taught throughout classrooms in three countries, researchers observed that for 96% of time in mathematics periods, students in American classrooms did worksheets that consisted of computational skills and practising procedures, with limited time devoted to authentic problem solving (Stigler & Hiebert, 1997). Furthermore, Hiebert et al. (2003) found that teachers talked on average 90% of the time when teaching mathematics, thus promoting more teacher-centered classrooms.

Silver, Ghousseini, Gosen, Charalambous, and Strawhun, (2005) discuss the importance of problem solving in reform mathematics teaching, specifically the promotion of multiple solutions to a given problem. The researchers studied 12 veteran middle school mathematics teachers and speculated that a variety of perceived obstacles "blocked them from implementing an aspect of reform teaching that they endorsed" (p. 298). The actual or perceived limitations included teachers' mathematical knowledge, knowledge of students' mathematics understanding, and "a lack of opportunity to develop instructional routines related to innovative teaching practices" (p. 298). Other studies have raised the concern of teachers' deeply embedded conceptions of teaching and learning mathematics. The case studies of five California teachers highlight the overreliance on mathematical teaching techniques that are mechanical and by rote repetition (Cohen & Ball, 1990). Implications derived from these case studies raise the issue for teachers who never learned mathematics through reform methods. If teachers lack rich experiences as mathematics learners, then how are they expected to teach mathematics through meaningful problem solving approaches? Therefore, in preservice mathematics programs, it is critical to emphasize deep understanding over the mere recall of facts. Prospective teachers require university course experiences that enable them to 'relearn' mathematics so they focus on understanding concepts. This in turn will support effective teaching approaches.

Arriving at the Problem

The context of this study took place in an accredited Ontario university Bachelor of Education program. In Ontario, provincial assessment results have revealed a steady decline in grades three and six students' mathematics achievement over the previous five years. During the five year period between 2009 and 2013, the percentage of grade six students who achieved at or above the provincial standard in mathematics decreased by six percentage points (63% to 57%); while the percentage of grade three students at or above the provincial level declined by three percentage points (70% to 67%) (Education, Quality and Accountability Office [EQAO], 2013). Yet, there have been marked improvements in EQAO reading and writing scores across grades three and six students. On an international level, Canada's mathematics achievement of grade 10 students has dropped steadily since 2006. The latest results from the Program for International Student Assessment (PISA) show Canada placed 13th overall in mathematics out of 65 countries; this placement is down three spots from 2009, and down six spots from 2006 (Brochu, Deussing, Houme, & Chuy, 2013). Furthermore, a recent report conducted by Amgen Canada Incorporated and Let's Talk Science (2013) indicated that more than 50% of Canadian high school students drop mathematics and science as soon as they can, thereby only taking the minimal compulsory courses to grade 10 or 11. These recent findings reveal a current situation of major concern in mathematics performance across the province and nation. As a consequence of this concern, this research aimed to investigate preservice teachers' development of their mathematics capacities for effective teaching, with hopes that prospective teachers will be change agents in mathematics teaching and learning.

The Vicious Cycle

A number of researchers have pointed out that teachers teach much in the same way they were taught (Ball, Lubienski, & Mewborn, 2001; Kagan, 1992; Tabachnik & Zeichner, 1984). Tabachnik and Zeichner (1984) assert that constructivist modes of teaching (i.e., the facilitation of student learning rather than direct instruction or mastery learning) tend to conflict with student teachers' previous ideas about good teaching, and that they are inclined to maintain old conceptions. Specifically in the discipline of mathematics, there is research that suggests preservice teachers enter teacher education programs with predetermined ideas on how to teach mathematics based on the way they were instructed (Ball, 1988, 1996; Ball et al., 2009; Cohen & Ball, 1990; Frye, 1991; Hill & Ball, 2004, 2009; Knapp & Peterson, 1995). Overall findings from these studies revealed how prospective teachers are more likely to replicate teaching approaches that were modeled to them as students of mathematics. Moreover, Hill and Ball (2004) state that these approaches derive from years of personal experiences of traditional mathematics teaching, where the teacher is holder of all knowledge, with an emphasis on memorization of facts and procedures.

Similarly, in Kagan's (1992) review of the literature on prospective teachers, she examined how student teachers commence their programs with previously formed beliefs about classrooms and images of themselves as teachers. These beliefs tended to be oversimplified views of classroom practice, and did not take into account the complexities of teaching and learning. Kagan's research demonstrated how student teachers used information they learned in teacher education classes to confirm their pre-existing beliefs, rather than confront or challenge these notions. "Despite course work and field experiences, the candidates' beliefs about teaching and themselves as teachers remained unchanged throughout the semester" (Kagan, 1992, p. 140). Kagan discussed how preservice teachers would enter practicum inadequately prepared, due to their oversimplified views of classroom practice. Consequently, prospective teachers devoted less time and efforts to reform ideals in mathematics because they … "become obsessed with class control, designing instruction, not to promote pupil learning, but to discourage disruptive behavior … their attitudes toward pupils grow more custodial and controlling" (Kagan, 1992, p. 156).

It appears that mathematics education is caught in a vicious cycle. Elementary mathematics programs remain plagued by rules, procedures, and teacher directed instruction (Battista, 1999; Frykholm, 1999; Marshall, 2003, 2006; McLeod et al., 1997). University students entering teacher education programs are products of such mathematics instruction. Prospective teachers tend to hold oversimplified beliefs about classroom practice and pre-existing ideas of how to teach mathematics based on their experiences in traditional mathematics classrooms (Ball, Lubienski, & Mewborn, 2001; Ball, 1988, 1996; Cohen & Ball, 1990; Frye, 1991; Knapp & Peterson, 1995). In order to break this cycle, preservice mathematics courses must give meaning to the content and pedagogy teachers need to know (Thames & Ball, 2010). This requires considerable mathematical content knowledge, strong levels of teacher efficacy, and a wide range of pedagogical skills to implement mathematics programs that promote authentic problem solving, reasoning, and communication.

Reform Must Begin In Preservice Mathematics Programs

Preservice teachers’ notions in teaching mathematics are critical and warrant further research into the teaching and learning experiences that occur in preservice mathematics programs. Prospective teachers are the future of the reform movement in mathematics education, and their implementation of the movement depends largely on their beliefs about mathematics and how much they understand mathematical content and pedagogy (Blanton, 2002). Wilson and Cooney (2002) claim that the teaching and learning experiences that occur in preservice education programs can have a positive impact on prospective teachers’ beliefs about mathematics. Ma (1999) promotes preservice mathematics preparation as the medium to break the “vicious cycle formed by low-quality mathematics education and low-quality teacher knowledge of school mathematics” (p. 149). Morris, Hiebert, and Spitzer (2009) examined how preservice instruction that was supportive, explicit, and deliberate in its goals, led to relevant mathematical knowledge and disposition in student teachers. Chapman (2007) studied how preservice course tasks that are “framed in an inquiry, collaborative learning context can provide a meaningful basis to help preservice teachers to learn about, and develop mathematical knowledge for teaching” (p. 348). Further studies also support how preservice programs should be the target for instituting positive change by developing mathematics teachers’ skills so they can make effective pedagogical choices that positively impact on student learning in mathematics (Philipp et al., 2007; Thompson, 1992). Therefore, the mandate of the NCTM standards must begin in teacher preparation programs to support improved mathematics instruction. Based on these studies, it is important to examine the effectiveness of teacher education mathematics programs.

The Gap in the Research Literature

Identifying reasons for the vicious cycle of low-level mathematics teaching has been the focus of much of the literature since the inception of NCTM’s reform standards in 1989. Many studies have considered possible rationales for the limited progress in mathematics classroom reform. For example, significant research has been centred on the content knowledge of teachers, while other studies have examined the mathematics pedagogical skills, and yet other researchers choose to observe the effects of mathematics teacher efficacy. There are various research studies concerning preservice teachers' mathematics development of 1) content knowledge, 2) teacher efficacy, and 3) pedagogical skills. This study however went beyond the single construct focus and examined how all three constructs simultaneously interrelated with one another in the context of teaching and learning in a preservice mathematics course. Throughout this dissertation, the term *content knowledge* is used to describe both common and specialized content knowledge (CCK and SCK). CCK refers to the kinds of mathematical knowledge applied in a variety of settings that is not exclusive to teaching; while SCK constitutes the knowledge and skills that are unique to mathematics teaching, such as determining students' misconceptions and responding to students' questions.

All three constructs, that is, content knowledge, teacher efficacy, and pedagogical skills, are essential to the development of effective mathematics teachers. These fundamental domains often go hand-in-hand and are difficult to differentiate. For example, according to Ball (1988), elementary teachers hesitate or even avoid teaching abstract mathematics concepts that they themselves find confusing or unclear. This important observation necessitates more research to explore if, and how, poor content knowledge of mathematics contributes to feelings of low mathematics teacher efficacy, which may further be indicative of teacher-directed rote learning strategies. Alternatively, it may be assumed that prospective teachers, who have high content knowledge, may feel more confident about their teaching skills and therefore possess high mathematics teacher efficacy. This in turn, encourages them to implement effective mathematics pedagogical practices. Although these presumptions are logical, research is necessary to examine the tri-component notion of mathematics teacher preparation, in which all three constructs, that is, content knowledge, mathematics teacher efficacy, and pedagogy, are monitored simultaneously in preservice mathematics education.

While research documenting elementary teachers' deficiency in mathematical content knowledge, efficacy, and pedagogical skills is helpful, teacher education requires more than the documentation of the field’s status quo (Mewborn, 2000). There exists a significant need to study what learning opportunities best contribute to gains in mathematics teacher education programs (Mewborn, 2000; RAND Mathematics Study Panel, 2003). Therefore, focused attention must be given to the critical experiences that effectively and sustainably improve the mathematics development of preservice teachers (Mewborn, 2000; RAND Mathematics Study Panel, 2003).

A major goal for this research study was to inform scholars’ and practitioners’ understandings about how the teaching and learning experiences in preservice education impact the development of student teachers, and specifically investigate the relationship among 1) content knowledge of mathematics, 2) mathematics teacher efficacy, and 3) pedagogy. These three areas of teacher development are imperative in effective implementation of mathematics reform. For this reason, this research aimed to examine the mathematics development of prospective teachers from both the academic and student teachers’ perspective in order to reveal a more complete understanding of teaching mathematics to preservice teachers.

This study focused on two elementary preservice mathematics classes in a large urban southern Ontario university. The study occurred over an eight month period. A mixed methods design using both quantitative and qualitative data collection methods was utilized. The quantitative components included instruments that measured participants’ mathematics common content knowledge (CCK), mathematics anxiety, and mathematics teacher efficacy. These assessments were administered at the beginning and at the end of the academic year in order to determine if levels of mathematics anxiety, teacher efficacy, and content knowledge changed across the year. The quantitative results were used in conjunction with the qualitative data to support overall findings.

Qualitative data involved interviews with six student teachers based on the mathematics anxiety results - three student teachers who scored high in mathematics anxiety and three who scored low in mathematics anxiety. The six student teachers were interviewed twice in the year; at the beginning and end of the academic year, providing a total of 12 interviews. These qualitative data focused on the critical incidents that impacted student teachers’ mathematics development of content knowledge, efficacy, and pedagogy. Another qualitative data source included reflective journals written by all the student teachers. The journals examined the course experiences that supported mathematics development and investigated the intertwining relationships in the three areas being studied: 1) content knowledge of mathematics, 2) mathematics teacher efficacy, and 3) pedagogy. In addition, the researcher herself kept a journal to further examine the academic perspective of supporting student teachers' mathematics development.

Research Questions

The three constructs under study (i.e., content knowledge of mathematics, mathematics teacher efficacy, and pedagogy) play a vital role in student teachers' development of mathematics teaching abilities. Based on the research of effective mathematics preservice practice, the author posed questions that considered the development of the three constructs that student teachers must attain for successful mathematics teaching and learning. These questions highlight the importance of each construct which are emphasized by several prominent researchers in the field, such as: Ball, Thames, and Phelps (2008); Enochs, Smith, and Huinker (2000); Gresham (2008); Hoy and Spero (2005); Sowder (2007); Swars, Danne, and Giesen, (2006); Swars, Smith, Smith, and Hart (2009); and Thames and Ball (2010). Most importantly, the research questions in this study explore the *inter-relatedness* that exists between and among the constructs from both an academic and student teachers' perspective. Hence, as student teachers strive toward becoming successful mathematics teachers, it is vital to understand the specific course experiences that contribute to their development of mathematics content knowledge, teacher efficacy, and pedagogy. The following questions provide the focus and context of this dissertation.

Primary Research Questions:

How do specific teaching and learning experiences in an elementary preservice mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy, and 3) pedagogy?

How does mathematics anxiety relate to these constructs?

How do these constructs relate to and/or influence one another?

Academic perspective:

a. How can academics support student teachers' development to positively influence:

1) mathematics content knowledge;
2) mathematics teacher efficacy; and
3) pedagogy?

Student teachers' perspective:

b. What learning experiences impact student teachers' development of:

1) mathematics content knowledge;
2) mathematics teacher efficacy; and
3) pedagogy?

Theoretical Framework

This study's theoretical framework provided specific lenses through which to examine what was considered relevant within student teachers' mathematics development. The particular perspectives in the theoretical framework served as a foundation for the parameters of this study. The main focus of this research was to investigate specific program experiences that influenced prospective teachers' mathematics content knowledge, teacher efficacy, and pedagogy. With an established theoretical framework, this study's trajectory stayed on course and the research remained within its boundaries. Hence, the theoretical framework confined the scope of the relevant data by providing the researcher with explicit constructs and viewpoints to focus on. Without such an established framework, this research risked becoming too broad and wide-ranging. The theoretical framework of this research was rooted in several theories. These selected theories are considered pertinent to the phenomena of preservice mathematics teacher development. By applying such theories to form this study's theoretical framework, it supported the researcher in determining decisions about research methods, data collection, data analysis, and interpretation. The following theories were used as a means to guide and inform this study: mathematical knowledge for teaching (MKT) theory, self-efficacy theory, constructivist learning theory, and adult learning theory (see Figure 1.1).

Mathematical Knowledge for Teaching Theory

MKT is a practice-based theory formulated by extensive qualitative analysis of mathematics teaching practice and then designing measures of MKT based on those qualitative results (Ball et al., 2008; Hill, Schilling, & Ball, 2005; Thames & Ball, 2010). From these empirical findings, the theory of MKT was developed to play a fundamental role in improving the teaching and learning in teacher content preparation. Essentially, MKT is the specialized knowledge that teachers require in order to teach mathematics successfully, so that children attain mathematical concepts with deep and sustainable understanding. Ball et al. (2008) and Hill, Ball, and Schilling (2008) build on Shulman's theory (1986) of pedagogical content knowledge (PCK). PCK describes how teaching is complex and multifaceted, encompassing teachers' abilities to model, represent, deliver, and formulate subject matter so it is accessible to learners. The researchers behind MKT draw from PCK's categories and empirically generate four MKT domains through factor analysis. The four domains of MKT comprise of: 1) common content knowledge (CCK); 2) specialized content knowledge (SCK); 3) knowledge of content and students (KCS); and 4) knowledge of content and teaching (KCT). Furthermore, two categories are included in MKT, 1) horizon content knowledge, and 2) knowledge of content and curriculum. In this study, the term *content knowledge* encompasses both the CCK and SCK of prospective teachers. Essentially the mathematical knowledge and skills that are unique to teaching is referred to SCK; whereas CCK involves mathematical proficiency that is not unique to teaching, such as the ability "to do particular calculations, knowing the definition of a concept, or making a simple representation" (Thames & Ball, 2010, p .223).

Self-efficacy Theory

The theory of self-efficacy originated from Bandura's social cognitive theory (1986). Self-efficacy is an individual's perception or belief of his or her ability to perform a goal. With this belief intact, individuals are capable of performing in a way to attain specific goals. This study upholds that the development of self-efficacy in preservice mathematics teachers is paramount to their success in teaching.

Specifically for this research study, the theory of self-efficacy was applied through the lens of teacher efficacy (Enochs et al., 2000). Teacher efficacy is understood in two dimensions. The first dimension is ‘personal teaching efficacy’ which represents a teacher’s belief in his or her abilities to be an effective teacher. The second dimension is called ‘teaching outcome expectancy,’ which corresponds to a teacher’s beliefs that his or her teaching can impact on student learning regardless of external factors. Bandura (1997) and other researchers (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998) suggest that highly efficacious teachers are more likely to implement successful teaching techniques, tend to rebound from obstacles, and are more willing to experiment with new ideas or techniques.

According to Bandura (1986) there are four factors that influence efficacy development. 1) *Mastery experiences* refer to when an individual performs a task successfully which directly strengthens self-efficacy. 2) *Vicarious experiences* occur when an individual observes others complete or model a task successfully. 3) *Social* and/or *verbal persuasion* can support individuals’ beliefs about their abilities; this often occurs in the form of positive encouragement about one’s performance. 4) *Psychological* factors also play a part in self-efficacy. Negative emotions such as anxiety can undermine self-efficacy beliefs. Hence, in this current research, student teachers’ emotional feelings of mathematics anxiety were a significant area of investigation.

Constructivist Learning Theory

Fundamental to constructivist learning theory is the notion that learners construct knowledge for themselves. As learners build knowledge, they undergo the process of learning individually and socially by actively constructing meaning. Constructivist theory’s core ideas stem from the work of several foundational theorists, including Dewey (1916, 1938, 1964), Piaget (1952, 1963), and Vygotsky (1978, 1986). Constructivist learning theory is applied to this research study in two ways: 1) student teachers as *learners* of mathematics, and 2) student teachers as *teachers* of mathematics. This research upheld the assumptions that student teachers learn and teach mathematics most effectively when the pedagogical experiences involve the following practices: 1) student-centeredness, 2) facilitation of group dialogue and shared understanding, 3) direct instruction, exploration, and discovery of concepts, 4) opportunities for students to determine, dispute, modify, or add to existing beliefs and understandings, and 5) development of students’ meta-cognition of their learning process (Richardson, 2003).

Hence, to shape potential teachers' skills to teach mathematics successfully, preservice mathematics classes should exemplify a constructivist stance that focuses on learners actively constructing their own knowledge and developing deep understanding in the subject matter.

Another important aspect of constructivism is cooperative learning theory. Cooperative learning is defined as learning that occurs in formal groups of two or more students. Groups work collaboratively towards a shared goal and the emphasis is on teamwork and meaningful participation. Cooperative learning is considered an effective strategy in improving students' academic achievement (Slavin, 1995; Johnson, Johnson, & Smith, 2007; Wlodkowski, 2004). Furthermore, the term 'collaborative group work' is sometimes used throughout this study when examining cooperative learning principles.

Adult Learning Theory

Adult learning theory is comprised of a set of tenets about how adults learn new skills or information. Knowles (1968) popularized the term 'andragogy' to explain his theory of adult education. Fundamental to Knowles' andragogy are six assumptions of adult learning: 1) need to know 2) self-concept, 3) experience, 4) readiness to learn, 5) orientation to learning, and 6) motivation. These assumptions guided and informed this research study's investigation about the course experiences that influenced student teachers' mathematics development of content knowledge, teacher efficacy, and pedagogy.

A second adult learning theory that was utilized in this research was Mezirow's transformative learning theory (1991). Core principles of transformative learning theory involve a deep learning that goes beyond surface level acquisition of content. Mezirow (1991, 2000) suggests that deep learning in adults transpire in several ways such as transforming individual's points of view and transforming habits of the mind. These kinds of transformations strongly connected to this study's focus on challenging, changing, and extending student teachers' transmissive oriented knowledge of mathematics teaching. Furthermore, Mezirow discusses how deep transformative learning is attained when adults take ownership of their social and personal roles to make society a better place. Hence, for this research study, transformative learning theory provided a lens for studying how student teachers took ownership of their role as mathematics teachers.

Figure 1.1: Preservice Mathematics Teacher Development: Theoretical Framework

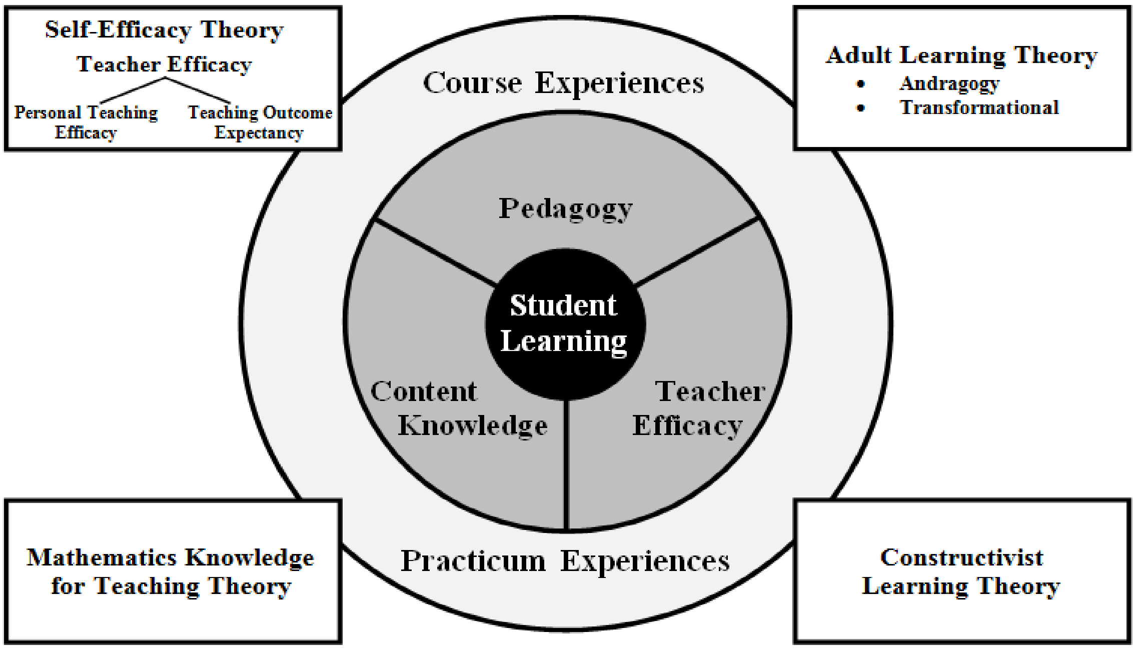

Figure 1.1. The theories surrounding the circular model provide a framework to understand the development of student teachers' mathematics content knowledge, teacher efficacy, and pedagogy. In this study, content knowledge refers to both common content knowledge (CCK) and specialized content knowledge (SCK). The full circle represents the various constructs and roles within preservice mathematics education. The most inner circle depicts student learning which represents the student teachers themselves and/or the children that are taught by student teachers. This inner circle signifies the central core purpose of mathematics teacher education. The middle circle represents the various constructs of preservice teachers as they develop their capacities in mathematics teaching. The outer circle signifies the experiences within the preservice mathematics program, which include both university classes and field placements. This outer circle also encompasses the roles of mathematics instructor and mentor teachers.

Definitions of Terms

Term	**Definition**
Conceptual Knowledge	Knowledge that is deep in understanding relationships among concepts within a discipline. Conceptual knowledge is a web of knowledge in which connected relationships among concepts are as prominent as the separate pieces of information.
Constructivism	A paradigm (a philosophical orientation that relates to belief systems) related to how individuals learn. In this paradigm, learning is believed to be a social construction of knowledge gained through the interaction of the learner, with the materials, other individuals, experts, and in relation to their prior schema.
Cooperative Learning	An instructional strategy which has students working collaboratively in groups. Students collectively work together towards a specific task.
Differentiated Instruction	Based on the belief that learners possess different needs and learning styles, differentiated instruction applies instructional approaches that are varied and adapted to match learners' level of readiness, preferred learning styles, and individual interests (Tomlinson, 2001). This approach to teaching requires flexibility in instructional approaches that are tailored to learners, rather than expecting students to modify themselves to fit only one form of pedagogical approach.
Manipulatives	Concrete materials/resources that promote students' exploration of mathematical ideas in an active, hands-on approach. Examples of mathematics

	manipulatives include blocks, cubes, geometrical shapes, and spinners. Manipulatives enable learners to connect mathematical ideas and symbols to physical objects so they can deepen their understanding of abstract concepts. Manipulatives also support students' problem-solving skills by providing concrete models to support abstract thinking. For example, in grade five mathematics, students learn about decimal numbers to hundredths. A common misconception of decimals is the longer the number, the larger its value. When comparing 0.7 and 0.56, many learners think that 56 hundredths is larger than 7 tenths because they treat the portion of the number to the right of the decimal as a whole number. However, when students use manipulatives such as base ten blocks to build and represent decimal numbers to tenths and hundredths, they are able to connect the model to the value of each number.
Mathematics Anxiety	An emotional feeling of nervousness, worry and/or apprehension that individuals may have about their ability to understand, perform mathematical functions, and/or explain problems.
Mathematics Content Knowledge	The breadth and depth of the mathematics knowledge possessed by individuals. This study examined both common content knowledge (CCK) and specialized content knowledge (SCK) of student teachers.
Mathematics Pedagogy	For the purpose of this research study, pedagogy refers to strategies of instruction. This includes instructional strategies, materials, resources, and

	means to promote mathematical thinking and assessment of students' mathematical understanding.
Mathematics Teacher Efficacy	This type of self-efficacy refers to teachers' overall beliefs about themselves as mathematics teachers. There are two dimensions of mathematics teacher efficacy. The first dimension is personal teaching efficacy and this corresponds to teachers' perceived beliefs about their ability to teach effectively. The second dimension is teaching outcome expectancy; defined as teachers' perceived beliefs that they can affect students learning through their effective teaching.
Preservice/Prospective/Student Teachers	Individuals enrolled in an accredited teacher education or preparation program (e.g., a Bachelor of Education) within a university studying to become teachers.
Procedural Knowledge	The application of a sequence of actions or steps to answer discipline-related questions. In mathematics, procedural knowledge is based on rules, algorithms, and step-by-step procedures.
Reform Mathematics	Based on NCTM's (1989, 2000) standards, reform mathematics is a change from traditional mathematics instruction. The reform movement in mathematics teaching emphasizes the need for students' deep understanding of mathematics concepts and their ability to reason, communicate mathematically, and solve problems based on their own thinking and experiences.
Transmissive/Transmission Mode of Teaching	The traditional model of teaching mathematics that transmits information from teacher to students

through demonstration and practice of procedures. For example, the teacher demonstrates the step-by-step procedure of solving two digit multiplication problems. The students follow the steps and repeatedly practise this procedure with similar problems. Attainment of deep conceptual understanding is difficult to achieve when students only focus on memorizing procedures and not on understanding why and how algorithms work.

Organization of Dissertation

The organization of this dissertation is in six chapters, followed by references, and then appendices. Chapter One provides an overview of the goals in elementary mathematics teaching and discusses the issues and challenges that elementary preservice teachers and university instructors face. This chapter further explains the significance of this current study, the developed research questions, and the theoretical framework used to guide the study's data collection and analysis. Chapter Two presents a review of literature pertinent to this study's focus on mathematics content knowledge, teacher efficacy, pedagogy, and preservice teachers as adult learners. The theory of mathematical knowledge for teaching, self-efficacy theory, constructivist learning theory, and adult learning theory are embedded throughout the literature review. Chapter Three explains this study's mixed methods research design. Throughout this chapter, the researcher describes the procedures for this study, data collection methods, instrumentation, data analysis, interpretation, and ethical considerations in working with preservice teachers as participants. Chapter Three also gives details about the data collection methods of quantitative data instrumentation tools (i.e., mathematics anxiety scale, mathematics teacher efficacy survey, and mathematics content test), and qualitative measures such as document reviews of journals and interviews. Chapter Four reports on the major results of this study. Five major themes emerged based on the analysis of the quantitative and qualitative data: 1) importance of the instructor's role in mathematics teacher development; 2) problem solving to support conceptual understanding; 3) building confidence as a mathematics teacher; 4) working towards constructivist pedagogy; and 5) classroom management. Chapter Five presents a further analysis of the key findings from the results chapter in relation to the theoretical framework. Three significant themes are discussed: 1) anxiety as barrier; 2) dimensions in mathematics efficacy; and 3) coherence between theory and practice. Chapter Six discusses the implications of this study's findings and presents two models to support the improvement of preservice mathematics teacher education courses, *Preservice Elementary Mathematics Education Structure (Macro Level)* and *Preservice Elementary Mathematics Program (Micro level)*.

Chapter 2: LITERATURE REVIEW

Introduction

The purpose of this study was to examine how the constructs of mathematics content knowledge, teacher efficacy, and pedagogical skills were developed and enhanced in elementary student teachers during a preservice mathematics course. Attending to the three constructs in elementary preservice teachers involves timely research in knowing how teachers improve their teaching ability of mathematics. The development of these constructs is significant in preservice mathematics courses, and by identifying factors which either contribute to or stifle such growth, will support the improvement of mathematics teacher preparation programs (Ball, Thames, & Phelps, 2008; Ball, Sleep, Boerst, & Bass, 2009; Hart, 2002). This literature review focuses on the theoretical framework that informs the research pertaining to the three constructs of mathematics development (see Figure 1.1 in Chapter One). In addition, the literature review discusses how each construct influences each other and where they intersect. This chapter is divided into four major sections: 1) mathematics content knowledge, 2) mathematics teacher efficacy, 3) mathematics pedagogy, and 4) preservice mathematics and the adult learner.

Mathematics Content Knowledge

Mathematics content knowledge refers to the breadth and depth of mathematics knowledge possessed by individuals. A number of research studies have raised serious concerns about the depth of content knowledge in preservice elementary mathematics teachers (Ball, 1990b; Grover & Connor, 2000; Hill & Ball, 2004, 2009; Ma, 1999; Philipp et al., 2007; Thames & Ball, 2010). In general, the literature pertaining to mathematics content knowledge of preservice teachers overwhelmingly supports the need for conceptual understanding of the subject matter, and specialized mathematics knowledge for teaching in order to implement solid teaching strategies. This section explores research about teachers' mathematics content knowledge and its implications for pedagogical instruction and ultimately student achievement.

Conceptual and Procedural Knowledge of Mathematics Content

Both Hiebert (1992) and McCormick (1997) describe procedural knowledge as applying a sequence of actions to find answers. These actions, also known as algorithms, follow a set of rules and students repeatedly practise these procedures to a set of questions to reinforce the algorithm. Conceptual knowledge is defined as a rich understanding of the relationships among mathematics concepts (NCTM, 2000). This involves solving problems through reasoning, communicating, and justifying. Merely learning computational procedures without understanding them will not develop the capacity to reason about what sort of calculations are needed. Thus, procedural skills that are not accompanied by some form of conceptual understanding are weak and easily forgotten (Hiebert et al., 2003). A major aspect of the National Council of Teachers of Mathematics' (NCTM) (2000) standards calls for a balance between conceptual and procedural knowledge of mathematics.

> One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility. … Students who memorize facts or procedures without understanding often are not sure when or how to use what they know and such learning is often quite fragile. (p. 20)

Research reports mathematics instruction typically observed in North American classrooms focuses mainly on procedural knowledge, in which teachers deliver curriculum in a repetitive, undemanding, and non-interactive fashion (Frykholm, 1999; Hiebert et al., 2003; Stigler & Hiebert 1997; Thames & Ball, 2010). Little attention is given to the development of conceptual ideas or making connections between procedures and mathematical concepts. The over emphasis of procedural knowledge in schools is most likely the type of mathematical preparation experienced by preservice teachers (Hill & Ball, 2004, 2009; Thames & Ball, 2010). Consequently, the challenge for preservice programs is to unpack the mathematical knowledge of prospective teachers in order to develop deeper conceptual understanding (Adler & Davis, 2006).

The intersection of procedural and conceptual knowledge is of utmost importance when examining the content knowledge of mathematics teachers (Ambrose, 2004; Hiebert, 1999; Hill & Ball, 2004, 2009; Lloyd & Wilson, 1998; McCormick, 1997;

Rittle-Johnson & Kroedinger, 2002). It is imperative that preservice education courses teach procedural skills *and* conceptual understanding as interconnected, so students have the capacity to understand why and how algorithms work and thereby grasp the underlying mathematical concepts (Ambrose, 2004; Eisenhart et al., 1993). The NCTM standards stipulate a balance of procedural skills and conceptual knowledge as essential in the development of content knowledge. Instructional practice that over emphasizes only one domain, either conceptual or procedural, will result in limited mathematical understanding. For example, research has demonstrated if students repeatedly practice algorithms before understanding them, they have more difficulty making sense of why and how the formula works (Hiebert et al., 2005; Mack, 1990; Wearne & Hiebert, 1988). Consequently, if students are required to memorize computational procedures and practice them, they face more challenges to go back and conceptually understand the formulas. This evidence sheds light on the on-going debate over whether procedural practice should come first before conceptual teaching or vice versa.

On the other hand, when conceptual understanding is the sole focus of instruction, then learners are likely to struggle with procedural competency (Alsup & Sprigler, 2003; Ross, 1996). Alsup and Sprigler (2003) studied 335 eighth grade students who were in either a traditional mathematics class that emphasized procedural skills or a non-traditional mathematics class that focused on developing conceptual understandings. According to the traditional mathematics teacher who participated in this study, he believed his practice appeared superior with skill development in procedural competency and therefore helped prepare students for high school level mathematics. Without any emphasis on computational algorithms, procedural knowledge can be negatively impacted (Alsup & Sprigler, 2003).

Wu (1999) argues that teaching algorithms and conceptual understanding should not be viewed as dichotomous extremes. Rather, he claims that procedural skills and problem solving skills are intertwined. Kamii (2004) professes that no algorithms should be taught to young children. Similarly, Wu (1999) agrees with immersing young learners in problems that they can visualize so they can grasp concepts firmly. However, Wu (1999) argues that algorithms can support abstract and sophisticated mathematics due to their generalizability and accuracy, especially for older grades. Similarly, Ross (1996) also discussed that a heavy focus on conceptual understanding can lead to some decline in procedural skill. Nevertheless, it is important to note that in another study, Alsup (2004) examined constructivist practices, and when implemented effectively with a balance of conceptual and procedural knowledge, findings showed high mathematics achievement and productivity.

Mathematics Content Knowledge of Preservice Teachers

The content knowledge of preservice teachers has received much attention in the last two decades. Content knowledge in mathematics is an important construct that can either support or hinder progress in mathematics reform (Ball, 1988). Ponte and Chapman (2008) stated that "while having strong knowledge of mathematics does not guarantee that one will be an effective mathematics teacher, teachers who do not have such knowledge are likely to be limited in their ability to help students develop relational and conceptual understanding" (p. 226). Ball et al. (2008) suggest that the absence of reform mathematics is resultant from teachers' lack of content knowledge within this subject area. "Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content" (Ball et al., 2008, p. 404). Philipp et al. (2007), Thames and Ball (2010), as well as Ball and Wilson (1990) strongly suggest it is necessary for teachers to possess conceptual mathematics knowledge in order to effectively explain algorithms, and describe and make connections between concepts.

In her studies, Ball (1990a, 1990b) examined mathematical conceptual content knowledge through responses to questionnaires and interviews by 252 prospective teachers. Findings revealed that the subject knowledge held by prospective teachers remains inadequate for teaching mathematics successfully. Part of Ball's (1990a, 1990b) studies focused on student teachers' specific knowledge of the division of fractions. During interviews as well as in questionnaires, student teachers were asked to solve the following problem: $1\frac{3}{4} \div \frac{1}{2}$. The majority demonstrated procedural knowledge of the invert and multiply rule, although some preservice teachers forgot this algorithm. The invert and multiply rule is perhaps the most prevalently misunderstood procedure in the elementary curriculum, yet it continues to be a staple strategy to teach the division of fractions in mathematics classrooms (NCTM, 2000). After arriving at an answer, preservice teachers were then asked to think of a story, model, and/or visualization to represent the 1 ¾ ÷ ½ problem statement. For example, an appropriate representation would be looking at 1 ¾ pizzas, and figuring out how many ½ pizzas are in 1 ¾ pizzas. Only a few secondary student teachers and none of their elementary counterparts were able to accurately generate representations of the division problem. The majority (~70%) of student teachers gave inaccurate or no representations of the division model. The most common error was to represent the division by two instead of one half. An example of an inappropriate model was looking at 1 ¾ pizzas and splitting the pizza into two. Ball (1990b) discussed that these findings indicate prospective teachers critically lack conceptual understanding of mathematics structures and principles even when they were able to perform the procedural calculations involved.

> Few prospective teachers, including math majors, seemed to have this kind of explicit conceptual understanding. As Ben, one of the math majors, reflected about the division by zero, 'I just know that . . . I don't really know why . . . It's almost become a fact . . . something that it's just there.' (p. 459)

Ball's (1990a, 1990b) findings are congruent with those of other studies. For example, Simon (1990) investigated prospective teachers' content knowledge of division and found that his participants were deficient in several areas. They revealed a conceptual weakness in explaining how algorithms work, understanding the relationship between partitive and quotitive division, and handling remainders (Simon, 1990). Other studies conducted by Zazkis and Campbell (1996), as well as Tirosh (2000) demonstrated how preservice elementary teachers were overly dependent on procedural knowledge. The authors of both studies examined participants' understanding of divisibility and multiplicative structures and found that they were compelled to apply a computational algorithm, resulting in procedural dependency and limited conceptual guidance (Tirosh, 2000; Zazkis & Campbell, 1996). Tirosh, Fischbein, Graeber, and Wilson (1999) researched student teachers' knowledge of rational numbers and found that participants over generalized their knowledge of whole numbers which led to misconceptions about rational numbers. For example, student teachers demonstrated the misconception that multiplying two numbers results in a larger product and failed to recognize that this claim is only applicable to whole numbers, and false for rational numbers. Moreover, several other studies have exposed teachers' impoverished understanding of the concepts of area and perimeter (Baturo & Nason, 1996; Fuller, 1997; Heaton, 1995; Ma, 1999). In these studies preservice and inservice teachers demonstrated the misconception that there is a constant relationship between area and perimeter and further failed to use square units when solving area measurement questions. Furthermore, Bartell, Webel, Bowen, and Dyson (2013) concluded that basic mathematics content knowledge is necessary but insufficient in supporting the assessment of children's conceptual understanding of mathematics. More specialized understanding of mathematics is required to understand the complexities behind children's mathematical thinking.

Perhaps Ma's (1999) comparative study of American and Chinese mathematics teachers revealed the most disconcerting evidence of deficiencies in American teachers' mathematics content knowledge for teaching. Her study involved interviewing teachers (23 U.S. and 72 Chinese teachers) and analyzing their responses to four problems that were in order of increasing difficulty. These questions included: 1) teaching subtraction and regrouping; 2) addressing students' mistakes in multi-digit multiplication; 3) generating representations for division of fractions; and 4) examining the relationship between perimeter and area. Overall results demonstrated that Chinese teachers overwhelming understood and could explain the mathematical concepts better than their American counterparts. For example, in the first question, fewer than 20% of the American teachers were able to explain the conceptual understanding of regrouping which is essentially decomposing a group of 10 into 10 ones, while 86% of Chinese teachers demonstrated a firm grasp of this concept. On the third problem, similar to Ball's (1990b) research study, a gap appeared in U.S. teachers' procedural computation of the division of fractions, let alone the conceptual understanding of the problem. Less than half of the American teachers calculated the division of fractions correctly, while 100% of the Chinese teachers accurately calculated the question, and 90% were able to explain the reasoning behind the mathematics.

Ma (1999) discusses in great detail how Chinese teachers' profound understanding of mathematics contributes to their students' success in the discipline. She speculates that a salient difference between the two groups begins at an early age, where Chinese elementary mathematics teachers possess specialized understanding of the discipline allowing them to teach concepts in a comprehensive way.

> In the United States, it is widely accepted that elementary mathematics is 'basic,' superficial, and commonly understood. The data in this book explode this myth. Elementary mathematics is not superficial at all, and anyone who teaches it has to study it hard in order to understand it in a comprehensive way. (Ma, 1999, p. 146)

Hence, the implications of Ma's study pinpoint the systemic deficiencies of American mathematics education, namely the limited understanding of mathematics held by teachers. This review of a range of studies gives substantial evidence that the majority of preservice elementary teachers lack conceptual understanding of the mathematics they are expected to teach.

Mathematics Content Knowledge and Pedagogical Instruction

Grover and Connor (2000) argue for content knowledge as a key characteristic of effective pedagogical instruction, and this should be a central focus in preservice mathematics courses. During their study of preservice education courses that prepared student teachers in secondary mathematics geometry, they examined several teacher preparation classes. They focused on course objectives in developing student teachers' 1) deep and broad knowledge of geometry, 2) instructional techniques, and 3) assessment practices that measure conceptual understanding. They found that a critical aspect of reaching these course objectives is to recognize the important interaction between teaching and subject content knowledge. The authors discussed the need for prospective teachers to not only understand mathematics, but to understand the concepts in ways that will support effective instruction and assessment of the discipline. These claims suggest that mathematics content knowledge for teaching is directly connected to pedagogical styles of teaching. Correspondingly, other researchers such as Darling-Hammond (1999), Hill and Ball (2004), Ma (1999), Shulman (1987), and Thames and Ball (2010), also advocate for deep subject matter knowledge and its subsequent positive influence on instructional techniques. Philipp et al. (2007) and Brown and Baird (1993) similarly, confirm that teachers who achieve greater mathematical knowledge are more capable of the conceptual teaching than their counterparts, who implement procedural based instruction. Both studies propose that teachers' decisions rely on their understanding of mathematical subject matter. Their research concludes that the deeper content knowledge that a teacher holds, the better equipped they are to communicate with students about mathematical concepts, models, and representations.

In her study of preservice elementary teachers, Kajander (2010) observed how prospective teachers' conceptual knowledge and beliefs about reform mathematics teaching changed over the progression of a mathematics methods course. Similar to Ma (1999), and Ball (1990b), Kajander's (2010) findings illustrated preservice teachers' inadequate understandings of mathematics for teaching. Most student teachers entered their teacher preparation programs with limited conceptual proficiency in how to represent mathematical concepts, explain their thinking, and justify mathematical procedures. However, after completing a mathematics methods course that focused on developing conceptual knowledge for teaching, improvements in conceptual knowledge were examined based on the comparisons of pre- and post-tests. This study endorses Hill and Ball's (2004) findings that content knowledge can be positively increased by a single course experience. Kajander (2007) conjectured that due to the increase in conceptual knowledge of mathematics, student teachers shifted their pedagogical beliefs about teaching mathematics … "At the end of the program, interviewees claimed to be less concerned with classroom practices typically associated with traditional learning, and more interested in the kinds of practices to promote deep learning and problem solving" (p. 248). Hence, these findings suggest how content knowledge and pedagogical beliefs are linked.

Understanding Mathematics for Teaching

Shulman's (1986, 1987) notion of pedagogical content knowledge gives attention to the role of content when teachers make pedagogical decisions. Pedagogical content knowledge recognizes that teaching requires a unique specialized knowledge of content. Over the last two decades, considerable research has gone into developing Shulman's notion of pedagogical content knowledge through the lens of mathematics teaching. Drawing upon Shulman's conceptualizations of pedagogical content knowledge, several researchers have identified and described a unique understanding of mathematical knowledge required for teaching (Ball, 1990a; Ball, 1996; Ball, Hill, & Bass, 2005; Ball et al., 2008; Hill, Rowan & Ball, 2005; Thames & Ball, 2010). Subsequently, Ball et al.

(2008) developed a practice-based theory of mathematical knowledge for teaching (MKT) (see Table 2.1 and Figure 2.1 for MKT overview).

Table 2.1:
Domains of Mathematical Knowledge for Teaching (MKT)

Domain	Definition
Domain One: Common Content Knowledge (CCK)	CCK is the mathematical knowledge used in a wide variety of settings that is not exclusive to teaching.
Domain Two: Specialized Content Knowledge (SCK)	SCK involves knowledge that goes beyond a conceptual understanding of mathematical ideas. It constitutes the knowledge and skills that are unique to mathematics teaching as it requires teachers to understand mathematics content with “pedagogically strategic intent” (Ball et al., 2008, p. 401).
Domain Three: Knowledge of Content and Students (KCS)	KCS comprises of teachers’ knowledge about students as well as mathematics content. Understanding common errors and misconceptions made by students, and interpreting students’ mathematical thinking are all key aspects of KCS.
Domain Four: Knowledge of Content and Teaching (KCT)	KCT involves the combination of pedagogical knowledge and mathematics content. This requires teachers to understand instructional design, such as how to represent mathematical concepts, sequence content, select examples, and explain methods and procedures.

Note. Ball et al. (2008) describe four domains that comprise their theory on mathematical knowledge for teaching (MKT). The four domains were empirically generated through factor analysis. Two categories are also included in MKT, but not shown in this table. The two categories include Horizon Content Knowledge, and Knowledge of Content and Curriculum, and these categories evolved from the researchers’ experiences related to teaching elementary mathematics.

Figure 2.1 Domains of Mathematical Knowledge for Teaching (MKT)

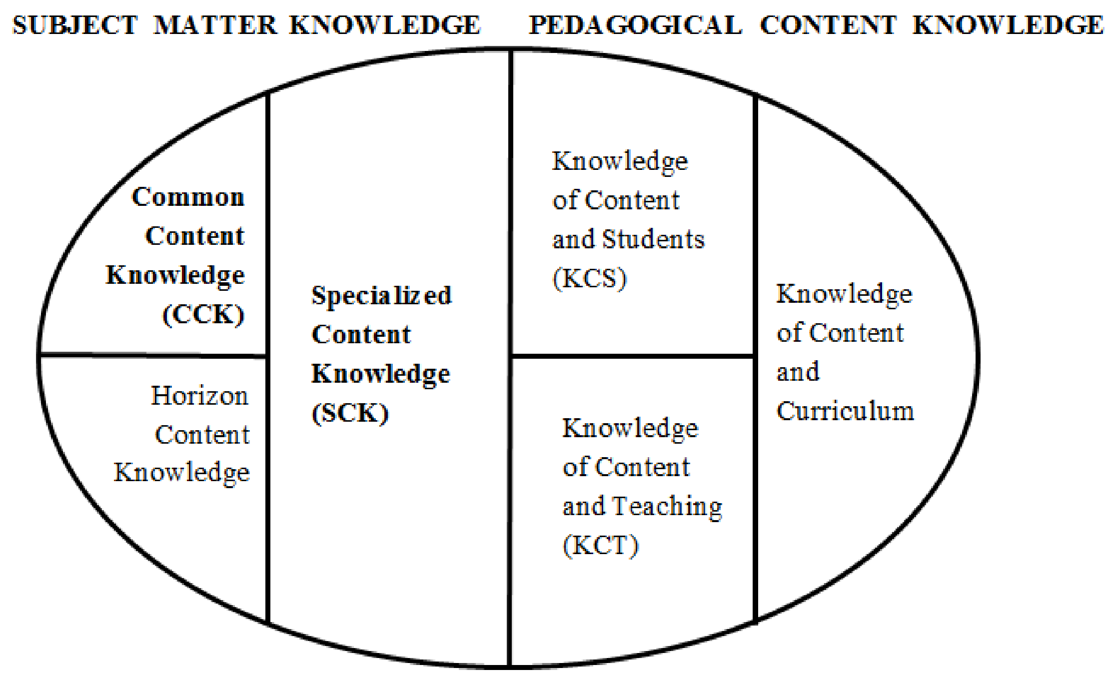

Figure 2.1. Ball et al. (2008) developed a model to represent the various domains and categories of MKT (p. 403). This model shows two categorizations of MKT: subject matter knowledge and pedagogical content knowledge. In this current study, the term content knowledge is used to describe both CCK and SCK within subject matter knowledge.

An underpinning of MKT is the notion that teachers require a specialized kind of knowledge to teach mathematics successfully. The theory of MKT describes four domains which were empirically generated through factor analysis. The first domain of MKT is identified as common content knowledge, and is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p. 399). Essentially, common content knowledge involves the ability to perform calculations and solve mathematical problems in various settings that are not exclusive to teaching. The second domain within MKT is identified as specialized content knowledge. This includes knowledge that teachers must have beyond the information they teach to their students, and it involves working through mathematics with "pedagogically strategic intent" (Ball et al., 2008, p. 401). This is the mathematical knowledge and skill that is exclusive to teaching.

> Teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed – or even desirable – in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work. (Ball et al., 2008, p. 400)

The third domain of MKT is identified as knowledge of content and students. This domain comprises teachers' knowledge about students as well as mathematics content. Understanding common errors and misconceptions made by students, and interpreting students' mathematical thinking are all aspects of knowledge of content and students. Examples of this domain include a teacher's ability to anticipate how students will respond to mathematics, for example, understanding students' conceptions, misconceptions, confusions, and interests. Finally, the fourth domain of MKT is identified as knowledge of content and teaching. This fourth domain entails instructional design tasks, such as how to sequence mathematical content, select powerful ways of representing mathematics concepts, and facilitate mathematical discussion. A teacher's MKT influences pedagogical decisions such as when to interject and redirect students, when to pose questions to further students' learning, and how to respond to students' mathematical remarks. Correspondingly, Morris, Hiebert, and Spitzer (2009) build on the theory of MKT by stating mathematics content knowledge for teaching is:

> … content knowledge of a particular kind. It is implicated in common teaching tasks such as choosing representations of mathematical ideas that reveal key subconcepts of the ideas, evaluating whether student responses show an understanding of key subconcepts, and justifying why arithmetic algorithms work. It involves unpacking or decompressing mathematical knowledge in order to make particular aspects of it visible for students or to identify the source of students' difficulties. (p. 494)

Ball et al. (2008) further assert that solid MKT enables teachers to develop and demonstrate mathematical models based on students' levels of understanding, and explain why a method works and whether it is generalizable to other problems. Furthermore, the authors claim that MKT is uniquely different from being a student of mathematics. Specifically, MKT requires a conceptual knowledge base to promote discussions about mathematical models and connections between mathematical concepts and procedures. These types of interactions are often done immediately on the spot, during teachable moments in response to students' questions.

Based on Ball et al.'s (2008) concept of specialized content knowledge as part of their theory of MKT, it is evident that mathematics for teaching requires unique knowledge. This involves the ability to understand students' misconceptions of mathematical ideas and determine next steps to support learning. Lo and Luo (2012) examined preservice teachers' CCK and SKC on the division of fractions and found that "representing fraction division, through either word problems or pictorial diagrams were challenging even for those highly proficient in elementary and middle school mathematics" (p. 481). Consequently, CCK alone is insufficient and teachers who possess only basic mathematics subject knowledge are less likely to teach students on how to conceptualize mathematical ideas (Ball, Hill, & Bass, 2005; Ball & Wilson, 1990). Many studies make a strong case for SKC and demonstrate that without this, teachers will face challenges when attempting to teach and uncover the concepts underlying the mathematical ideas and to make sense of their students' solutions. Unfortunately, research suggests that mathematics teachers are limited in their MKT, which poses many challenges for the implementation of reform mathematics (Lo & Luo, 2012).

Mathematics Content Knowledge and Its Impact on Student Achievement

There has been much attention given to the content knowledge of mathematics teachers and its relationship with student success in mathematics. Sowder (2007) stated "the key to increasing students' mathematical knowledge and to closing the achievement gap is to put knowledgeable teachers in every classroom" (p. 157). Rowan, Chiang, and Miller (1997) identified teachers' mathematics content knowledge as a predictor of student achievement in grade 10 mathematics. In their quantitative study, they completed a statistical analysis of mathematics teachers' quiz results and its relationship to students' achievement in test scores. Findings indicated that students produced higher levels of achievement if they were taught by teachers with higher quiz results. Furthermore, students who were taught by teachers who held a mathematics degree also earned higher levels of test scores (Rowan et al., 1997). The authors discussed the research behind their findings and speculate that:

> [a] deep knowledge of the subject being taught can support teachers in both the planning and interactive phases of teaching . . . subject-matter knowledge supports the interactive phases of instruction, acting as a resource as teachers formulate explanations and examples, both in lectures and in response to students' questions. (Rowan et al., 1997, p. 258)

It is important to note that higher levels of mathematics courses do not automatically equate to better mathematics teaching. Thames and Ball (2010) indicate "although it seems that majoring in math should provide an edge in teachers' capacity, it simply does not at the grades K-8 level, and it is an uneven predictor at the high school level" (p. 221). In order to make mathematics meaningful, Sowder (2007) argued for preservice teachers to become immersed in learning mathematical concepts and have opportunities in class to make connections between representations and applications, algorithms and procedures. Unfortunately, considerable evidence suggests that many mathematics teachers can apply the rules and procedures required to do mathematics but lack conceptual knowledge and reasoning skills to teach for deep understanding (Borko & Putnam, 1995; Ma, 1999).

Mathematics Content Knowledge Summary

In conclusion, this section provided substantial evidence demonstrating the critical need for improved mathematics content knowledge in preservice teachers. It is important to note that this entails a specialized understanding of content knowledge that goes beyond a common content knowledge of mathematics for everyday tasks. As illustrated in the research by Ball (1990a, 1990b) and Ma (1999), their interest was not simply to find out if teachers could calculate a particular problem (common content knowledge and/or procedural knowledge), but whether they could explain what the problem meant, and could generate concrete representations of the problem. This specialized content knowledge in mathematics is described in Ball et al.'s (2008) theory of MKT and it enables prospective teachers to make informed decisions about students' mathematics skills and develop mathematical models that are accessible by their students. As discussed throughout this literature review, there are considerable research studies documenting the significance of mathematics content knowledge in which researchers have spent a great deal of time studying what teachers know and do not know about mathematics content, and its impact on teaching students. It would be valuable to further examine any relationships mathematics content knowledge has with student teacher's efficacy and pedagogical decisions.

Mathematics Teacher Efficacy

Over the last 30 years, a great deal of the literature on teacher efficacy has been based on Bandura's (1977, 1986, 1997) self-efficacy research. Bandura first established social learning theory (1977), which he later entitled social cognitive theory (1986), and then revised to situated cognitive learning theory (1997). A theoretical underpinning of Bandura's work is his notion of self-efficacy; defined as one's judgement of his/her capabilities to complete a task at a certain level of performance. Bandura (1977) distinguishes teacher efficacy as a type of self-efficacy, in which teachers construct their beliefs about their ability to perform and affect students. Of most importance is that these beliefs play an influential role because they have an impact on teachers' outcomes of efforts, persistence during difficult tasks, resilience in dealing with failures, and the amount of stress experienced (Bandura, 1977, 1986).

Based on Bandura's (1995, 1997) theory of self-efficacy, teacher efficacy is comprised of two dimensions. Enochs, Smith, and Huinker (2000) describe the first dimension as 'personal teaching efficacy' which represents a teachers' belief in his or her skills and abilities to be an effective teacher. The second dimension is called 'teaching outcome expectancy,' which corresponds to a teacher's beliefs that effective teaching can impact on student learning regardless of external factors such as parental influences or home environment. Teacher outcome expectancy is also described by Guskey and Passaro (1994) as "teachers' belief or conviction that they can influence how well students learn, even those who may be difficult or unmotivated" (p. 4). Although the two dimensions of teacher efficacy are inter-related, Bandura argued that they are conceptually distinct. Personal teaching efficacy and teacher outcome expectancy "are differentiated because individuals can believe that a particular course of action will produce certain outcomes, but they do not act on that outcome belief because they question whether they can actually execute the necessary activities" (Bandura, 1986, p 392).

There are many studies about teacher efficacy in general; however, there remains less research on preservice teacher efficacy and its relationship with both the content and pedagogy of mathematics. According to Bandura's (1997) work, efficacy is largely situational and context specific. Pajares (1996) stated that efficacy cannot be generalized, but rather it is sensitive to contextual and situational factors. Based on Bandura's (1997) and Pajares' (1996) claims, it is likely for student teachers to possess high teacher efficacy in subject areas where confidence exists, but simultaneously possess low teacher efficacy in another subject area such as mathematics. Therefore, in an attempt to understand the interconnectedness of efficacy and associated constructs of mathematics capacity, this section examines the literature pertaining to mathematics teacher efficacy and its relationship with pedagogical choices and mathematics content knowledge, within the context of preservice education.

Teacher Efficacy and Pedagogical Choices

The literature in the area of teacher efficacy and pedagogical choices is substantial. Teacher efficacy has been correlated to significant attributes such as pedagogical strategies and willingness to embrace new ideas. Such studies note that preservice teachers who have high teacher efficacy use a greater variety of instructional strategies (Riggs & Enochs, 1990; Wenta, 2000). Therefore the benefits of having high levels of self-efficacy have been well established throughout the literature.

One such study which illustrates this claim was conducted by Wertheim and Leyser (2002). The authors worked with preservice teachers in Israel and investigated teacher efficacy beliefs and their choices of instructional practices. Preservice teachers completed a Likert scale to rate their intended use of 59 instructional approaches. The instructional approaches were organized into seven categories (a) individualized differentiated instruction, (b) assessment for instruction, (c) behavior management, (d) communication with parents, (e) communication with school professionals, (f) communication with principal, and (g) communication with students, (Wertheim & Leyser, 2002). Based on an analysis of questionnaire results and self-efficacy rating scales, the authors established a correlation between efficacy beliefs and pedagogical approaches. Wertheim and Leyser (2002) obtained small yet statistical significance that demonstrated a positive correlation between personal teaching efficacy and willingness to use each of the instructional categories. However, the authors found mixed results between teacher outcome expectancy and readiness to implement varied instructional strategies. In general, participants who were more efficacious revealed higher scores on their intentions to use differentiated teaching strategies; implement various behaviour management techniques; and communicate with all stakeholders, compared with participants with lower efficacy scores. Wertheim and Leyser concluded that the higher the sense of personal teaching efficacy in preservice teachers, the more willing they were to use a variety of pedagogical strategies to support students' learning.

Wertheim and Leyser's (2002) findings substantiate the research of Minke, Bear, Deemer, and Griffen (1996), who demonstrated in their study that teachers with high self-efficacy focused more on differentiated instruction, individualized programming, and adaptive teaching practices for students with special needs. Brownell and Pajares (1999) also sustain this correlation. They found teachers, who possessed supportive attitudes and perceived themselves as successful in integrating special education students into the mainstream classroom, were more involved in positive behavior management approaches and collaborative activities with others. These studies are further supported by Haney, Czerniak, and Lumpe (1996). They found science teachers with a high sense of self-efficacy were more likely to try out new instructional strategies that involved risks, such as implementing difficult techniques and giving students more control and ownership of the learning process. Similarly, Czernaik (1990) reported that highly efficacious teachers were more likely to put into practice inquiry based pedagogy and student-centered teaching strategies, such as using manipulatives, exploring new approaches, and sharing the control of learning with their students. On the contrary, Czernaik (1990) found those teachers with lower efficacy beliefs employed more teacher directed methods such as lecture, straight text reading, and very little, if any, problem solving strategies in the classroom. In a study conducted by Riggs and Enochs (1990), the authors demonstrated how science teachers with high efficacy used strategies in which teachers shared control with students. Specifically, lessons were opened-ended and students were encouraged to use their own strategies for solving problems. Therefore, the literature demonstrates that pedagogical choices are more reform-based and student-centred when teachers hold higher levels of self-efficacy.

In the area of mathematics, research confirms how a teacher's perceived self-efficacy can directly impact the decisions in using innovative mathematics practices (De Mesquita & Drake, 1994). Stipek, Givvin, Salmon, and MacGyvers (2001) examined 21 junior mathematics teachers (grades four to six) over the course of a school year. Their results were congruent with those of other studies, in which they found a substantial coherence between high teacher efficacy beliefs and reform based practices in mathematics. Alternatively, Stipek et al. (2001) observed teachers who had lower efficacy were drawn towards traditional instructional styles such as prescribed procedures, correct answers, and an over reliance on answer sheets. Subsequently, based on these cited studies, it can be posited that there is a link between teacher efficacy and choice of pedagogical strategies, essentially high levels of mathematics teacher efficacy can have a positive impact on reform based pedagogical choices teachers make.

Mathematics Teacher Efficacy as Content Knowledge Specific

Of growing importance is the understanding that teacher efficacy beliefs are content-matter and context-specific (Tschannen-Moran & Woolfolk Hoy, 2001).

> Teacher efficacy has been defined as both context and subject-matter specific. A teacher may feel very competent in one area of study or when working with one kind of student and feel less able in other subjects or with different students. (p. 790)

For example, an elementary teacher may feel secure in her ability to impact students' reading achievement, but feel ineffective in teaching mathematics. Therefore, teacher efficacy cannot be generalized. Consideration of the teaching context and the teacher's strengths and weaknesses in relation to the task must be examined (Ross, 1992). It is expected that teachers' self-efficacy levels are different depending on the teaching task, the student make-up of the class, and the specific subject matter being taught … "Even from one class period to another, teachers' level of efficacy may change" (Goddard, Hoy & Woolfolk Hoy, 2000, p. 482).

In considering the context of student teachers with poorer levels of mathematics content knowledge, it is speculated that preservice mathematics courses which focus on developing specialized mathematics knowledge for teaching can support teacher efficacy. This supposition is supported through Ma's (1999) research and her concept of profound understanding of fundamental mathematics, whereby she posits teachers need to have mathematical knowledge that is well connected, grounded in curriculum, and sequentially coherent. Further to Ma's research, Hill and Ball (2004) and Ball et al. (2005) assert teachers' specialized mathematics knowledge for teaching is central to how well they can present mathematical concepts, utilize curriculum materials, assess students' skills, and sequence lessons. Results of a study that examined teachers' self-efficacy and their involvement in mathematics content courses demonstrated that inservice teachers' efficacy in outcome expectancy was higher in teachers who had taken four or more mathematics content courses, (Swackhamer, Koellner, Basile, & Kimbrough, 2009). Participants in this study were mainly experienced teachers who lacked content knowledge in mathematics. These inservice teachers already exhibited high levels of personal teaching efficacy; however, they had low beliefs in their ability to impact student achievement due to their limited content knowledge. The authors hypothesized that the mathematics courses increased their content knowledge which contributed to the major increases in teaching outcome expectancy. In addition, findings derived from Swackhamer et al. (2009) are consistent with other studies that demonstrated content courses that focus on how to teach the content have been successful in raising preservice teachers' efficacy levels (Appleton, 1995; Palmer, 2001; Smith, 1996).

There is also literature that reveals how content knowledge directly impacts overall teacher efficacy beliefs. For example, in Czerniak's (1990) study, she found preservice elementary teachers' scores on a science content test were small but significant predictors of science teacher efficacy. Essentially, the greater the content knowledge, the more confident teachers felt about their teaching abilities and their impact on student learning. Similarly, Bates, Latham, and Kim (2011) examined how mathematics personal teaching efficacy influences the mathematical performance of preservice teachers. The authors discovered …"in regard to mathematics teaching efficacy, the group of preservice teachers that rated themselves higher had significantly higher Basic Skills Test mathematics scores" (p. 332). Therefore, these research results give evidence that teachers' overall efficacy beliefs can be positively impacted by an increase in content knowledge.

Mathematics Teacher Efficacy and Mathematics Anxiety

Research reveals that mathematics teacher efficacy is negatively related to mathematics anxiety (Gresham, 2008; Swars, Danne, & Giesen, 2006; Swars, Smith, Smith, & Hart, 2009). The significance of subject specific and contextual factors is illustrated in Gresham's (2008) research in which mathematics teacher efficacy in elementary preservice teachers was negatively influenced by fear and anxiety with mathematics. Gresham closely examined how student teachers' past experiences as mathematics learners potentially brought on fear and negative attitudes towards the subject. She maintains that detrimental experiences with mathematics are the basis for low efficacy. Furthermore, Wenta (2000) links low mathematics teacher efficacy and high mathematics anxiety to ineffective K-12 mathematics schooling with student teachers. Discussion from both Wenta's and Gresham's research indicates the importance of preservice mathematics courses. University mathematics classes need to encourage student teachers to acknowledge and overcome any detrimental experiences as mathematics learners. Other studies also advocate that university mathematics courses can have a positive effect in reducing mathematics anxiety and building teacher efficacy among preservice teachers (Huinker & Madison, 1997; McCulloch Vinson, 2001; Tooke & Lindstrom, 1998).

Results from Swars et al. (2006) stress that preservice teachers need opportunities to talk and reflect upon their past experiences as mathematics learners. Accordingly, Furner and Duffy (2002), as well as Sloan (2010), propose to reduce mathematics anxiety by allowing student teachers to openly address their previous mathematics experiences through discussion and written reflections. As a result, heightened self-awareness of negative mathematics experiences may bring student teachers closer towards reducing mathematics anxiety and increasing mathematics teacher efficacy. Additionally, Bandura (1986) stressed that efficacy beliefs were largely influenced by emotional states such as anxiety. He further maintained that individuals needed to confront these emotional states.

For that reason, raising self-awareness of mathematics anxiety during preservice education is imperative for building mathematics teacher efficacy.

Mathematics Teacher Efficacy - Significant in Preservice Education

Bandura's theory (1977) on self-efficacy suggests that teacher-development receptivity in relation to efficacy may be most impressionable in the early phases of teacher development. Therefore, it is essential to focus on preservice teachers; this is a significant time of training that is situated at the beginning phases of capacity building when they are most malleable. Hoy (2000), and Hoy and Spero (2005) support the notion of this important timeframe and argue that the first few years of teaching are critical to the long term development of teacher efficacy. In addition, Hoy (2000) notes that the teacher efficacy beliefs of preservice teachers are of particular interest because once “efficacy beliefs are established, they appear to be somewhat resistant to change” (p. 5). Further research suggests preservice teachers’ mathematics courses and practicum experiences can be positive factors in mathematics teacher efficacy levels (Swars, 2005; Utley, Moseley, & Bryant, 2005; Wenner, 2001). Hence the successful completion of reform oriented mathematics courses may lead to stronger commitments by preservice teachers to implement reform oriented strategies.

In their longitudinal case study, Mulholland and Wallace (2001) examined the experiences of one student teacher as she transitioned from her preservice year into her first year of teaching. The authors propose that some of the most paramount influences on the development of teachers’ sense of efficacy are experienced during the initial years of preservice and beginning teaching. In their study they describe that mastery experiences have the most powerful influence on overall self-efficacy beliefs. In general, mastery experience occurs when teachers perceive their teaching as successful which directly increases efficacy expectations. Whereas, when teachers’ perceive that their teaching is ineffective, it more often decreases efficacy beliefs which further contribute to an expectation that future performances will also be a failure. According to Mulholland and Wallace (2001), for novice teachers, mastery experience is an important source of efficacy beliefs. Hence, the repercussions of mastery experiences in developing high mathematics teacher efficacy are significant in preservice mathematics courses.

Other research studies suggest that the course work in teacher education programs and the practicum experiences have divergent effects on personal teaching efficacy and teaching outcome expectancy. Hoy and Woolfolk (1990) as well as Plourde (2002) studied how personal teaching efficacy of student teachers increased throughout the program and continued to increase during practice teaching. Contrastingly, preservice teachers' sense of teaching outcome expectancy beliefs declined during practicum teaching. Hoy and Woolfolk (1990) speculate that the decline in teaching outcome expectancy beliefs was derived from idealistic and unrealistic optimism of prospective teachers when they enter preservice programs. This study supports Kagan's (1992) views that student teachers enter their teaching programs with oversimplified and overzealous beliefs about teaching. Consequently, when student teachers are immersed within the complexities and realities of the classroom, their sense of teaching outcome expectancy is challenged in significant ways (Hoy & Woolfolk, 1990; Woolfolk, Rosoff & Hoy, 1990). Implications from these studies demonstrate the critical need for mathematics preservice course work to be strongly connected to the realities and authenticity of classrooms in order to develop greater beliefs in teaching outcome expectancy.

Huinker and Madison (1997) examined the efficacy beliefs of student teachers enrolled in university elementary mathematics and science methods courses. Each mathematics course focused on developing prospective teachers' capacity to teach mathematics for understanding. Their results corresponded to significant increases in both personal teaching efficacy and teaching outcome efficacy. The authors identified participants who had the greatest increase in overall self-efficacy beliefs often entered the program with negative experiences in learning mathematics. Throughout the academic year, these prospective teachers experienced course work that transformed their attitudes and consequently increased their mathematics teacher efficacy. Huinker and Madison (1997) also noted that mastery experiences play a significant role in overall efficacy growth. Similar to Mulholland and Wallace (2001), they discovered that successful performance in teaching mathematics during field placements strongly influenced mathematics efficacy. Of equal importance, they identified a second type of mastery experience in student teachers which occurred when participants experienced for themselves the successful attainment of learning mathematics during course work. At the beginning of the year, many of the student teachers doubted their own skills in mathematics and felt insecure about their abilities to learn mathematics. Nonetheless, these doubts diminished as participants gained a deeper understanding of mathematics content by struggling with and unravelling mathematics problems during class. These results are significant, as they substantiate the research that upholds the importance of student teachers as both learners and teachers of mathematics.

Borko and Putman (1995) noted that teacher educators must make a concerted effort to develop the self-efficacy beliefs of preservice teachers. It would be insufficient if preservice mathematics programs limited their focus only on content knowledge and pedagogy. The authors argue that self-efficacy beliefs held by prospective teachers are equally imperative as other constructs (i.e., subject knowledge and pedagogical knowledge) for successful development in teacher preparation. Borko and Putman (1995) stated that "[preservice teachers] must acquire richer knowledge of subject matter, pedagogy, and subject-specific pedagogy; and they must come to hold new beliefs in these domains" (p. 60). Consequently, teacher efficacy plays a major role in the teaching and learning experiences in teacher preparation programs.

Mathematics Teacher Efficacy Summary

In summary, evidence from this subsection on mathematics teacher efficacy purports that efficacy development must begin early in teachers’ careers, and for that reason, preservice education is a critical time (Hoy, 2000; Hoy & Spero, 2005). There is substantial literature that indicates how mathematics teacher efficacy can have a positive impact on pedagogical choices teachers make (Riggs & Enochs, 1990; Stipek et al., 2001; Wenta, 2000). The research illustrates that mathematics teacher efficacy is context and subject content specific, and necessitates the reduction of mathematics anxiety, and the development of mathematics content knowledge as well as mastery experiences during practice teaching and university course work (Gresham, 2008; Swars et al., 2006; Sloan, 2010).

Pedagogy

For the purpose of this study pedagogy refers to strategies of instruction. Specifically in mathematics, pedagogy is represented by instructional strategies that engage students in conceptualizing mathematical ideas and approaches. Such strategies may include non-routine, open-ended problem solving, the use of manipulatives, and cooperative learning. For teacher preparation programs in mathematics, this involves a shift away from delivering content through a transmission mode and moving towards instruction that helps learners construct their own knowledge of mathematics (NCTM, 2000). This subchapter will compare and contrast two major pedagogical approaches to mathematics teaching: constructivist approaches and the traditional transmissive mode. These approaches will be examined from the perspectives of preservice programs and classroom practice. Additionally, a discussion about how these pedagogical approaches influence and connect with teacher efficacy as well as content knowledge will be included.

Constructivism and Prominent Educators of the Twentieth Century

Constructivism is a philosophy that upholds individuals learn best by actively constructing their own knowledge (Cobb, Perlwitz, & Underwood-Gregg, 1998; Noddings, 1993; Zazkis, 1999). This knowledge is shaped through learners' interactions with their environment, materials, and with others. Moreover, learning is constructed by the student, and not transmitted directly by the teacher. Current constructivist pedagogies draw on the work from prominent educators of the twentieth century. These include Piaget, Dewey, and Vygotsky (Bhargava & Kirova, 2002; Gales & Yan, 2001; Kirova & Bhargava, 2002; Livingston, 2003).

Bhargava and Kirova (2002) discuss how Piaget (1952, 1963) heavily influenced the teaching of mathematics to pre-schoolers based on his theory of cognitive development in which learners interact with, and explore their surroundings in order to construct knowledge. Other researchers claim that Dewey's (1916, 1938, 1964) child-centred description of education resonates with constructivist ideals, therefore laying essential groundwork for constructivist philosophy (Brooks & Brooks, 1999; Livingston, 2003; Popkewitz, 1998). Likewise, much literature has been devoted to Vygotsky's (1978, 1986) work and how it is situated firmly in constructivist theory. Specifically, Vygotsky asserted that an individual's learning is maximized when subject matter is presented just slightly beyond his/her reach; he described this notion as 'the zone of proximal development' (Kirova & Bhargava, 2002). Hence, Piaget, Dewey, and

Vygotsky are considered influential in the foundations of constructivist philosophy in education.

Constructivism versus Transmissive Modes of Mathematics Instruction

In mathematics education, the constructivist approach reflects a very different paradigm from the transmissive mode, where knowledge is viewed as hierarchical, sequential, and transmittable from one person to another relatively intact and unchanged. The National Council for Teachers of Mathematics (NCTM) (2000) standards provide specific suggestions for mathematics instruction that reflect constructivist pedagogies. Grounded in the cognitive development literature, NCTM (2000) calls for instruction that is more student-centred and de-emphasizes rote memorization of isolated skills and facts. Its recommendations also emphasize instructional approaches that enable students to solve mathematical problems by generating their personal solution strategies and thereby making mathematical knowledge their own (NCTM, 2000). Ultimately students' mathematical understanding should stem from their own experiences formed through discovery, exploration, and interaction with other learners.

Research demonstrates however, that the typical mathematics pedagogy practiced across North American classrooms emphasizes procedural knowledge and relies heavily on whole-class instruction; with the teacher explaining step-by-step procedures on how to produce a correct answer, while the students listen passively. Students are then instructed to independently solve a set of problems that are similar to the example demonstrated during the whole class lecture (Marshall, 2003, 2006; Stigler & Hiebert 1997). The emerging goal for the teacher-directed instructional approach is for students to apply the procedure to a set of narrowly defined problems. The teacher and the textbook are the authorities during the lesson. Thus, the structure of classroom interactions is based on transmitting information from teacher to students. Further, the traditional model of teaching mathematics has also been referred to as the "telling pedagogy" (O'Brien et al., 1995, p. 450) , the "pedagogy of control" (O'Brien, Stewart, & Moje, 1995, p. 451), "parrot mathematics" (O'Brien, 1999, p. 434), or the "transmission model of instruction" (O'Brien et al., 1995, p. 451).

Unfortunately, researchers have purported that this type of top-down pedagogy has made it difficult for learners to gain meaningful and sustainable mathematical knowledge (Cobb et al., 1998). These researchers claim students who experience mathematics instruction through the traditional ‘telling pedagogy’ often enter adulthood with poor conceptual understandings. Their once memorized formulas are easily forgotten, resulting in limited content knowledge of mathematical concepts (Cobb et al., 1998). To further complicate matters, these adults enter preservice education programs facing the challenge to ‘relearn’ mathematics in ways that they can conceptualize ideas for deep understanding in order to embrace effective teaching approaches (Cohen & Ball 1990). It is worth reiterating though that procedural proficiency is significant and must not be abandoned altogether. Pedagogical instruction that balances procedural skills and conceptual knowledge is essential in the development of successful mathematical learning (NCTM, 2000).

Resistance to Constructivist Pedagogy

The resistance against constructivist instruction is voiced by various parent groups such as Honest Open Logical Debate (HOLD) on mathematics reform, Save Our Children from Mediocre Math (SOCMM), and the Mathematically Correct Organization of Concerned Parents. Such mathematical reform opponents have major issues against NCTM’s (2000) revised standards, specifically: problem solving that relies heavily on reading skills; the lack of textbooks; the notion of discovery learning and open-ended performance tasks; and limited focus on practice and repetition (HOLD, 1995). However, there has been virtually no academic research evidence to support the criticisms of what they label as ‘fuzzy’ mathematics. O’Brien (1999) stated that critics’ major arguments are mainly anecdotal.

Parental resistance to constructivist mathematics was illustrated in studies conducted by Lehrer and Shumow (1997) and Peressini (1998). These authors found a lack of consensus between parents and teachers about constructivist approaches in teaching mathematics. In the parental surveys that Lehrer and Shumow (1997) examined, they found about one third of parents strongly believed in step-by-step procedural demonstration. In another study with similar findings, Peressini (1998) attributed parents’ opinions to their previous mathematics experiences which further informed their expectations for their children’s mathematics achievement. Consequently, the vicious cycle continues in homes with parents influencing their children's mathematics development based largely on traditional approaches to teaching.

Another source of resistance to constructivist pedagogy is the culture shock felt by students. Cooney's (1985) research of beginning teachers endeavoring to implement a problem solving, constructivist approach identified students' reaction as counterproductive. In general, students rejected the open ended, constructivist methods due to culture shock. The problem solving style of teaching mathematics was vastly different from anything the students had previously experienced. Cooney (1985) suggested that these new experiences made students feel ill prepared to respond in an unstructured learning environment. Building on Cooney's research about student resistance, Johnson et al. (2009) discussed how students believe there is only one way to solve mathematics problems, and the solution should be delineated and explained by the teacher. The authors stated that students often have a strong desire for efficiency and direct teaching, and oppose time consuming problems that require complex thinking. Further, risk-taking in the mathematics classroom has also been identified as an impediment to constructivist instruction. Consequently, teachers must invest more time and support to build mathematics confidence and reassure students that their learning efforts within constructivist pedagogies are worthwhile (Johnson et al., 2009).

Research on Constructivist Pedagogy in Elementary Mathematics

Constructivist learning theory has been broadly addressed in a number of research studies in mathematics education (Kamii & Dominick, 1998; Katic, Hmelo-Silver, & Weber, 2009; Hmelo-Silver, Duncan, & Chinn, 2007). Such studies have demonstrated the effectiveness of constructivist pedagogy in contrast to the traditional approach to teaching mathematics. Hmelo-Silver et al. (2007) suggest that the use of tools and hands-on materials during collaborative problem solving scaffold the construction of solutions. Kamii and Dominick (1998) claim that elementary mathematics instruction ought to promote children's inventions of their own procedures to solve problems. Invoking Piaget's theory of cognitive development and constructivist pedagogy, Kamii and Dominick (1998) argue that direct teaching of procedural algorithms are not only unhelpful, but are also detrimental and hinder children's development of mathematical reasoning. Their main premise for purporting that algorithms are harmful is based on two reasons. First, traditional rote strategies lead children to give up their own thinking; and second, they impede the development of understandings of concepts, thereby preventing children from developing number sense.

Kamii and Dominick (1998) examined students' invented strategies for multi-digit addition and subtraction and they discovered that students naturally solve problems from left to right, beginning with the larger place values. Students' personal procedures began with the largest units, and then worked towards the smallest. However, when observing students who were taught the traditional algorithm, they were forced to calculate digits from right to left, working with the smallest units first. Kamii and Dominick (1998) concluded that children who relied on algorithms only saw numbers as mere digits and developed no conceptualization of place value. In other words, the transmission of traditional algorithms confused students about place value and ultimately made them dependent on the spatial arrangement of digits and not on their own mathematical thinking.

Kamii's (1993, 1994, 2004) subsequent research studies explored students' understanding of mathematics taught in constructivist classrooms compared to traditional modes. Repeatedly, her findings demonstrated how constructivist groups outperform the traditional group in both accuracy and speed when solving problems. For example, Kamii (2004) studied two groups of low socio-economic second graders in California. One group was immersed in constructivist instruction and was not taught any algorithms. The second group of similar socio-economic status was taught mathematics through traditional methods of direct teaching, and the use of a textbook and workbook. In response to a two-digit addition question that required regrouping, Kamii's findings revealed that "significantly more students in the constructivist group wrote the correct answer … (86% vs. 52%)" (2004, p. 30). Significantly however, when asked to explain their procedures with counters, none of the students who were taught traditional algorithms were able to explain the concept of regrouping, whereas most of the constructivist group explained their procedures with tens and ones (Kamii, 2004).

The research cited above clearly indicates that conceptualization of place value was limited in traditional classrooms. Kamii's (1986, 2004) findings suggest that when children are only taught the step-by-step procedures, their thinking about numbers come to a halt, and they are merely going through the motions of low-level procedures. In contrast, children who are encouraged to think about solving problems using their own strategies become deeply engaged at decomposing numbers and making sense of the relationships among numbers.

Although Kamii (1993, 1994, 2004) advocates the banishment of algorithms on the whole with young children, many would dispute this and argue for a balance of conceptual understanding and procedural algorithms, especially with students in higher grades (Alsup, 2004; Ross, 1996; Wu, 1999). These authors understand the justification for invented procedures based on constructivist paradigms, but also claim that procedural skills are helpful when working with very large numbers and sophisticated problems, as long as students can justify their use of the procedure and make sense of their problem solving strategies. Therefore, it can be posited that in a constructivist classroom, learning procedural skills and understanding conceptually can support mathematical thinking.

Constructivist Pedagogy in the Mathematics Classroom – Complex and Multifaceted

If studies support the principles underlying constructivist theory, then the question arises as to why research does not reflect more constructivist models of teaching in elementary mathematics classes? One possible reason for the lack of constructivist pedagogy is based on the misconception that constructivism exemplifies one conceptualisation of how to teach. On the contrary, constructivism cannot be thought of as a monolithic concept. There is no direct function that translates constructivist principles into teaching methodology (Bauersfeld, 1995). Essentially, the theory of constructivist pedagogy describes how learners develop knowledge, which may inform teaching methods, but does not prescribe them. Richardson (1997) explains that "constructivism is a descriptive theory of learning . . . it is not a prescriptive theory of learning" (p. 3). Hence, constructivism provides teachers with a description of how learners come to know and understand, but it does not prescribe what should be taught or specific pedagogical techniques for helping learners construct knowledge.

Constructivist teaching in mathematics classrooms is therefore not as delineated as traditional modes of teaching. For example, it would be unproductive to interpret constructivist teaching with overly simplistic ideas, such as, leaving the students alone to construct mathematical knowledge, or placing pupils in cooperative learning groups to generate solution strategies. These notions are unhelpful and can lead to vague understandings of constructivist teaching in the mathematics classroom (Richardson, 1999).

Constructivism poses a challenge to preservice teachers due to the complexities in developing models of teaching that build on fostering explorations of mathematical concepts (Frid, 2000). Pedagogical strategies such as cooperative learning, non-routine problem solving, and the use of manipulatives are often associated with the kinds of reform mathematics that are founded on constructivist principles. However, the ability to implement these strategies does not automatically equate to learning situations that result in conceptual growth in mathematics (Fosnot & Dolk, 2001). The following quote from Frid's (2000) study illustrates how preservice teachers were able implement such activities but seriously lacked the constructivist underpinnings of mathematics instruction:

> Children were allowed to talk to one another while completing individual work, but there was little evidence to indicate this was aimed at fostering or monitoring students' thinking or development of mathematical meanings. Use of concrete materials was interwoven into activities, but it did not appear, at least at a preliminary observational level, that children were learning much of the underlying mathematics. They were actively doing things and keeping busy, but it was not clear if mathematical connections were being made. There appeared to be an assumption by the student teachers that 'doing things' automatically leads to appropriate mathematical learning. They did not seem to see that for a teacher to act as a facilitator for children's construction of knowledge requires one to focus on what students already know and are able to do, and what they say and do as they are engaged in new activities. (p. 24)

In constructivist pedagogy, the teacher must carefully design activities that allow children to build their own meanings and understandings and provoke "periods of conflict, confusion, surprise, over long periods of time, during social interaction" (Wood, 1995, p. 337). Studies have shown that cooperative learning strategies using mixed ability groups have the potential to close the achievement gap. Specifically, low and middle achieving students make significant mathematics gains in such diverse ability groups, while achievement for high ability students is uncompromised (Adams & Hamm, 1996; Hooper & Hannafin, 1988; Linchevski & Kutscher, 1998). It is also imperative that children's prior knowledge is explicitly recognized as it directly impacts on how they learn. Therefore, when compared to the traditional modes of teaching mathematics, constructivist pedagogy is more complex and multifaceted. Constructivist teachers may find themselves in unpredictable and tenuous situations as they facilitate students toward desired knowledge (Bobis, Mulligan, Lowrie, & Taplin, 1999; Reys, Suydam, Lindquist, & Smith, 1998).

Constructivist Pedagogy in Preservice Mathematics Programs

Many research studies exist that support the value of constructivist modelling and teaching in preservice mathematics courses (Castle, 1997; Chen, 2001; Gales & Yan, 2001; Hart, 2002, 2004). Dangel and Guyton (2003) conducted a meta-analysis of constructivist literature surrounding preservice education from 1990 to 2003. They discovered that preservice education programs that exemplified constructivist pedagogy revealed eight common elements within the program's course structure. These elements include: learner-centered instruction, cohort groups, reflection, field placements, cooperative learning, problem solving, authentic assessment, and action research. In addition to the eight common constructs, the authors emphasized the value of social interaction infused throughout student teachers' experiences.

Dangel and Guyton (2003) concluded that the efforts toward constructivist teacher education programs had two major effects on preservice teachers, "change in teacher-learners' beliefs about teaching and learning and/or change in their pedagogy" (p. 17). For example, preservice teachers immersed in constructivist pedagogy developed a deeper understanding of mathematics content; valued other classmates' ideas; expanded their views about in-depth learning and student ownership of learning; demonstrated stronger beliefs about the importance of manipulatives, cooperative learning, student interaction, and solving problems in a variety of ways (Anderson & Piazza, 1996; Chen, 2001; Dangel & Guyton, 2003). Dangel and Guyton further described how these eight common elements positively influence teachers' understanding and practice of constructivist pedagogy. For this reason, constructivist pedagogy plays a significant role in preservice mathematics programs as prospective teachers cultivate their pedagogical skills and establish their mathematics efficacy.

Cooperative learning theory in higher education.

A major part of constructivism in higher education classrooms includes principles of cooperative learning theory. Johnson, Johnson, and Smith, (2007) discuss how cooperative learning strategies in postsecondary settings generate meaningful learning opportunities that would not occur if students worked in competitive environments or individually. However, course instructors must cultivate the appropriate conditions for cooperative learning to be successful. These conditions include five essential elements: "positive interdependence, individual accountability, promotive interaction, social skills, and group processing" (Johnson, Johnson, & Smith, 2007, p. 23). Johnson and Johnson (1996) explain that the first element of positive interdependence fosters environments in which students maximize the learning of all group members. This involves students' willingness to share resources, reciprocal support among group members, and celebrating the group's collective achievements. The second basic condition is individual accountability, in which individual group members are assessed and given feedback on their contributions to the group. Promotive interaction is the third essential element described by Johnson, Johnson, and Smith (2007), and this occurs when students encourage each other's contributions to the group's desired goals. A by-product of promotive interaction is the social relationship building, in that group members come to know each other on a personal level. The fourth element is the appropriate use of social skills. Students must understand classroom norms and interact professionally with one another. The final element is group processing. This condition is fostered when students are encouraged to reflect on how effectively the group is working and improvements for future learning. Hence, when university instructors have a thorough understanding of the five essential elements, they are able to deliver lessons that maximize students' achievement through cooperative learning strategies, as well as intervene when groups are not functioning effectively.

Constructivist Pedagogy from Preservice Course into Classroom Practice

If the literature suggests that constructivist teaching strategies have the potential to positively frame mathematics preservice education practices, then why has it not made a substantial impact on classroom practice? Wilson, Cooney, and Stinson (2005) and Frid (2000) examined the impact of field experiences on preservice teachers' mathematics pedagogy development. In her study, Frid (2000) identified that one of the most serious problems in mathematics preservice courses is the incoherence between constructivist pedagogy taught at the university and the realities of classroom practices.

> Many student teachers were using fairly traditional approaches to mathematics planning, teaching and assessment. Much of their mathematics planning and teaching appeared to be derived from textbooks and worksheets, with much emphasis placed on performance of basic arithmetical skills or recognition and recall of basic information (e.g., naming things). (p. 24)

Subsequently, the incoherence between field placement and preservice mathematics classes poses major challenges for preservice teachers to acquire constructivist pedagogy (Wilson et al., 2005).

Endorsing Frid's findings, Ensor (2001) in his two year study of seven preservice teachers transitioning into their first year of teaching found they reproduced only a small number of constructivist activities they learned during their preservice program. Ensor (2001) observed that the teachers seldom designed other similar lessons for their students. Consequently, the evidence illustrated in these research studies question the effectiveness of preservice programs in developing constructivist-minded mathematics teachers.

Wilson et al. (2005) sought to discover why the efforts of constructivist mathematics teaching at the university rarely transferred into classroom practice. They found that preservice teachers developed positive changes in their pedagogical beliefs about reform mathematics, but these failed to materialize in practicum teaching. In their study, the authors obtained interview results from teachers who were mentoring student teachers. The mentor teachers claimed that their primary source for effective mathematics instruction came from teaching experiences and interactions with colleagues. Mentor teachers were less inclined to refer to theoretical research such as those outlined in the NCTM standards. Essentially, mentor teachers undervalued the theory and research of mathematics teaching, whereas universities placed great importance on the theoretical concepts of mathematics pedagogy. Wilson et al. (2005) describe the dichotomy between preservice education and schools.

> Teacher educators take their cues from scholars of years past or from recent proclamations such as the NCTM documents, which, it could be argued, are essentially a reflection of what those scholars advocated. Teachers, on the other hand, point directly to experience and interaction with colleagues as the primary sources for their becoming good teachers. Although these two perspectives are not necessarily inconsistent, there is a certain tension that exists between them, a tension that often leads to limited change. (p. 107)

In short, Wilson et al. (2005) found that the support for constructivist pedagogy in preservice education led to changes in beliefs and understanding by student teachers, but only superficially. Comparable to Frid (2000), Wilson et al. (2005) suggest when prospective teachers enter their field placements the realities of the classroom present a second perspective that may be inconsistent with the theoretical underpinnings of constructivist pedagogy.

Traditional Models of Pedagogy and Mathematics Teacher Efficacy

Smith (1996) claimed that the traditional approach to teaching mathematics is one that is straight forward and simpler to implement when compared to a constructivist style of instruction. This in turn enables teachers to build a stronger sense of self-efficacy. The 'telling' pedagogy is prescriptive, teacher-centred, and teacher-controlled; therefore "many teachers are disposed to teach mathematics by 'telling'; by stating facts and demonstrating procedures to their students. Clear and accurate telling provides a foundation for teachers' sense of efficacy" (Smith, 1996, p. 387). As a result, it appears that constructivist teaching in mathematics classrooms is hindered by traditional models and undermines teachers' sense of self-efficacy.

Smith's (1996) research was the first of its kind to examine the connection between student teachers' mathematics efficacy and changes in teaching practice within the context of constructivist pedagogy. Smith's review of the literature proposed several issues in preservice teachers' reactions to reform mathematics causing them to retreat back to the comforts of the 'telling' pedagogy. One specific area pertains to student teachers' prior experiences which involve limited mathematical experiences as conceptual learners. Smith (1996) argued that a shift in teaching and learning mathematics through constructivist pedagogy can lead them to avoid teaching concepts due to limited understanding. Wilson (1994) confirmed this notion and discovered that even when preservice teachers changed their beliefs to emphasize constructivist pedagogy in mathematics, they did not always implement these practices into their own classrooms. In other words, mathematics teachers may feel less sure of what it is they should teach and therefore less capable of employing constructivist methods to teach it, resulting in low mathematics teacher efficacy.

Pedagogical Content Knowledge – Where Pedagogy and Content Intersect

Shulman (1986) devoted much work to pedagogy and ignited major discussion about teacher pedagogy. He coined the term 'pedagogical content knowledge,' namely, where pedagogy and content intersect. Shulman (1986) defined pedagogical content knowledge as "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations . . . the most useful ways of representing and formulating the subject that make it comprehensible to others" (p. 9). In other words, this knowledge represents the merging of teachers' content knowledge and pedagogical practices, into an understanding of how subject matter is organized and tailored for sound instructional delivery. Essentially, Shulman's theory claims that pedagogical content knowledge is primarily how teachers transform subject matter into teaching and learning experiences; when the teacher interprets the subject matter, and finds interesting and engaging ways to present it, thereby making it accessible to all learners. Pedagogical content knowledge has received a great deal of attention over the past 20 years in various disciplines (Ball et al., 2008).

Ball et al. (2008) built upon Shulman's (1986) theory of pedagogical content knowledge through their theory of mathematical knowledge for teaching (MKT). As mentioned previously in this chapter, MKT explores the need for teachers to possess specialized content knowledge for teaching, in which they understand mathematics in ways that make sense of students' understandings, challenges, misconceptions, and attainments. Transforming mathematical content knowledge into effective pedagogical practice is paramount. For example, when teaching about decimals, understanding the concept of decimals and how to place them in ascending order is part of common content knowledge. Mathematical knowledge for teaching transpires when the teacher strategically generates a problem involving a list of decimals to be ordered for a lesson;

thereby building a representation of place value concepts. Of utmost importance is recognizing which decimals students stumble over due to their misconceptions and the teacher deciding how to address these potential challenges. Ball et al. (2008) explain that this requires knowledge of content and students' needs that inform important instructional practice. Similarly, Shulman (1986) states:

> pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Traditional approaches to teaching mathematics, therefore, do not work due to the varying levels of understanding students bring to each lesson. Mathematical knowledge for teaching (MKT) uses instructional strategies that are differentiated based on students' understanding and these decisions are often made on the spot in response to students' work or discussion.

> A teacher must decide when to pause for more clarification, when to use a student's remark to make a mathematical point, and when to ask a new question or pose a new task to further students' learning. Each of these decisions requires coordination between the mathematics at stake and the instructional options and purposes at play. (Ball et al., 2008, p. 401)

Pedagogy Summary

In summary, the research surrounding pedagogy from this subchapter's review of literature strongly supports constructivist instruction and negates traditional modes of mathematics teaching (Kamii, 1994, 2004; Kamii & Dominick, 1998). However, the mathematics literature continues to reveal the 'telling pedagogy' as the dominant practice across North American classrooms (Hiebert et al., 2005; Stigler & Hiebert, 1997). A possible rationale involves the complexities of constructivist pedagogy. It is not prescriptive nor as delineated and straight-forward as its traditional transmissive counterpart. Essentially, constructivist pedagogy is a theory of learning and describes how students learn, but does not stipulate specific teaching techniques (Richardson, 1999). Smith (1996) noted that the convoluted nature of constructivism can influence teacher efficacy. He claimed that teachers may feel more in control and confident with the prescribed and simpler models of teaching. Other researchers consider the disequilibrium between preservice teachers' field experiences and constructivist pedagogy. Ensor (2001), Frid (2000), and Wilson et al. (2005) discuss the tension student teachers face when immersed in classrooms that do not practice the constructivist underpinnings they learn in their preservice mathematics programs. Finally, Ball et al. (2008) build upon Shulman's (1986) pedagogical content knowledge theory and examine the necessity of MKT for sound instructional pedagogy.

Preservice Mathematics and the Adult Learner

It is important that preservice mathematics programs take into consideration the development of adult learners in order to build capacity and develop successful mathematics teachers (Lerman, 2001). This subsection presents an overview of two adult learning theories relevant to understanding the experiences and development of preservice mathematics teachers. These theories are Knowles' concept of andragogy (1984) and Mezirow's (1991) transformative adult learning theory. Current research on adult learning theories pertaining to higher education across North America has drawn from the works of Knowles and Mezirow (Barker, Sturdivant, & Smith, 1999; Brown, 2006; Kiely, Sandmann, & Truluck, 2004; Paul, 2003; Yorks, 2008). This subchapter will examine how these theories apply to student teachers' development in preservice mathematics programs.

Knowles' Six Assumptions of Andragogy

Knowles' (1984) theory of andragogy has been widely studied in adult learning research since the 1960s. The term andragogy is derived from the Greek language for 'adult-leading,' just as pedagogy comes from the Greek for 'child-leading.' Knowles rejected the term 'pedagogy' and claimed its inadequacy was based on its teacher-centred approach instead of student-centeredness (1968). Throughout the 1960s, Knowles conceptualized the notion of adult education where he defined a functional adult learning theory that focused on learners' self-directedness, experiences, and problem centred strategies (Knowles, 1984). A decade later, in the 1970s, Knowles shifted his theory of 'andragogy versus pedagogy' toward a continuum that ranged from 'teacher-directed' to 'student-centred' learning (Merriam, 2001). With this new representation of andragogy, Knowles (1984) claimed that both approaches of teacher-directed and student-directed are applicable with children and adult learners depending on the needs of the learner and situation.

Merriam (2001) discusses Knowles' conception of andragogy as an attempt to build a comprehensive theory of adult learning that is anchored in the characteristics of adult learners. Knowles' (1984) andragogy refers to six assumptions or characteristics of how adults learn. These are: 1) need to know 2) self concept, 3) experience, 4) readiness to learn, 5) orientation to learning, and 6) motivation. In the field of adult education, it is widely maintained that effective teaching of adults requires a good understanding of the six basic assumptions (Knowles, Holton, & Swanson, 1998, 2005). The following are brief descriptions of each assumption and its implications in preservice mathematics programs.

1) Need to know.

This assumes that adults need to understand how their learning is relevant and beneficial to their lives. There exists a need in adults to identify the rationale behind what they are learning and its importance to them. If adults do not see the value of the learning, then they withdraw and disengage from the learning situation. In preservice mathematics, it is imperative that the instructor is transparent about the purpose of learning activities, and he/she must articulate how gaining the new knowledge and skill will positively impact preservice students' development as mathematics teachers.

2) Self-concept.

Adults as learners have a self-concept of taking responsibility for their own initiatives, decisions, and behaviors. Knowles (1984) claims that adults have a deep psychological need to be treated as independent and capable of self-direction. The implications of this assumption necessitate university mathematics programs to cultivate relationships where preservice students take responsibility for their own actions. If autocratic situations are imposed, then the adult-learners' self-concept will be undermined (Knowles, 1984). Therefore, mathematics instructors must develop relationships with student teachers that promote a relational trust and foster self direction, initiative, and responsibility.

3) Experience.

As individuals mature they accumulate an increasing reservoir of experiences that become a growing resource for learning. Knowles (1984) cautioned that predetermined ideas can also pose challenges for adult learners when adopting different ways of thinking. Knowles et al. (1998, 2005) propose specific strategies that incorporate learners' rich experiences and support learning. These strategies include group discussion, case studies, simulations, and problem solving activities. Consequently, in preservice mathematics programs, it is imperative that instructors do not ignore prospective teachers' experiences, but rather develop task-oriented strategies that encourage new attitudes and beliefs about reform mathematics. Preservice teachers' deeply ingrained habits and thought patterns of how to teach mathematics are most often derived from years of traditional learning experiences (Cohen & Ball, 1990; Knapp & Peterson, 1995). As a result, instructors are faced with the challenge to unpack prospective teachers' preconceived notions of traditional mathematics procedures, and shift their thought patterns towards constructivist learning and attainment of conceptual understanding.

4) Readiness to learn.

As adults undergo different stages in life and take on different roles, (e.g., spouse, parent, teacher, retiree) they experience a readiness to learn when their current experience or knowledge base is not adequate for some aspect of their lives. Adults become ready to engage in learning that is pertinent to their stages and roles in life (Knowles et al, 1998, 2005). In preservice mathematics education, it can be posited that student teachers are characterized by a readiness to learn as they prepare for their new roles and responsibilities in the teaching profession. Knowles (1990) maintains that readiness to learn is closely linked to need to know. Therefore, preservice teachers must understand why it is important to achieve the learning task, and how it will impact their teaching development before they begin learning it.

5) Orientation to learning.

This assumption acknowledges the immediacy and practicality of the learning in which the adult is engaged. During childhood, learning is often perceived as acquiring subject matter to be used at a future date. As learners enter adulthood, learning transitions from postponed application of knowledge to immediacy of application, and therefore orientation to learning shifts from one of subject-centeredness to one of problem-centeredness (Knowles, 1984, 1990). In preservice mathematics courses, task oriented activities that offer practicality in skill development is important. Student teachers will be more willing to engage in the acquisition of new skills, knowledge, beliefs, and values, when they distinguish how the learning could help enhance their role as mathematics teachers in an immediate and pragmatic way.

6) Motivation.

Knowles et al. (1998, 2005) maintain that as individuals mature into adulthood, motivation to learn becomes internal. Internal forces such as the desire to pursue personal self-development or job satisfaction are stronger motivators than extrinsic rewards of salaries and promotions. However, the authors also recognize the significance of external motivations, but maintain that adults' internal forces are most compelling. This assumption in preservice mathematics education highlights the critical need to focus on prospective teachers' primary purpose, that is, to positively affect students. Furthermore, McLaughlin (1991) and Sederberg and Clark (1990) also maintain that the desire to impact students is central to most teachers' motivation to teach.

Mezirow's Transformative Adult Learning Theory

Mezirow's (1991) theory of transformative learning focuses on how adults experience transformational change in their perspectives and behavior based on how they make meaning from their experiences (Anfara & Mertz, 2006). "Transformational learning theory as presented by its chief architect, Mezirow (1991), posits that significant learning in our lives involves meaning-making that can lead to a transformation of our personality or world view" (Anfara & Mertz, 2006, p. 24). Therefore, based on this theory of adult learning, the key to this research study was to uncover how preservice teachers made sense of their experiences in their mathematics course and how this process transformed their development as prospective teachers. For this current study, this process required student teachers to undergo critical reflection.

The need for critical reflection.

The ability and practice of reflection is fundamental to transformative learning theory. "Reflective learning involves assessment or reassessment of assumptions.

Reflective learning becomes transformative whenever assumptions or premises are found to be distorting, inauthentic, or otherwise invalid" (Mezirow, 1991, p. 6). In other words, reflection must involve a critical analysis of current understandings that challenges the validity of presuppositions in prior learning (Mezirow & Associates, 1990, 2000). This directly connects to preservice teachers' preconceived notions of mathematics education, and signifies the need for on-going reflection in university courses. The key to transformative learning is encouraging prospective teachers to critically examine their deeply ingrained thought patterns, habits, and underlying assumptions, and strive towards "assessing reason and reviewing the evidence and arguments for and against the problematic assertion to arrive at a consensus" (Mezirow, 1995, p. 53).

Merriam, Caffarella, and Baumgartner (2007) note that transformative learning is about how people change the way they see themselves and the world in which they live. Hence, this research study attempted to examine how preservice teachers undergo changes in their mathematics content knowledge, teacher efficacy, and pedagogical skills, and how this further transformed their view of themselves as mathematics teachers.

Chapter Summary

Chapter two contains a review of the literature in three major constructs for mathematics teacher development: 1) mathematics content knowledge, 2) mathematics teacher efficacy, and 3) pedagogy. In addition, an overview of two adult learning theories based on the works of Knowles (1984) and Mezirow (1991) are examined through the lens of mathematics preservice teacher development. The literature strongly suggests teachers who possess only basic mathematics subject matter are unable to teach and support students with conceptualizing mathematical ideas (Ball et al., 2008; Ball & Wilson, 1990). However, teachers' content knowledge of mathematics in itself does not translate to effective mathematics teaching. Ball et al. (2008) argue that teachers must develop mathematical knowledge for teaching (MKT) in order to effectively explain algorithms, make connections, describe concepts, assess students' misconceptions, and plan next steps for improved learning. Simultaneously, a sound grasp of pedagogical understanding is imperative in mathematics teaching. The mathematics literature supports constructivist instruction for sustainable and deep learning of concepts (Kamii, 1993, 1994, 2004; Kamii & Dominick, 1998). However, constructivism is a complex theory of learning, and is not as easy to implement as its traditional counterpart (Richardson, 1999; Smith, 1996). Hence, teachers with weak mathematics content knowledge or low teacher efficacy may gravitate towards a delineated, procedural-based, and teacher-centred approach (Smith, 1996).

The construct of teacher efficacy has been studied widely over the years (Gresham, 2008; Riggs & Enochs, 1990; Stipek et al., 2001; Wenta, 2000). Researchers have shown correlations between efficacy and pedagogical instruction; namely, high teacher efficacy is connected to innovative and learner-centred teaching (Wertheim & Leyser, 2002). Further, content knowledge and teacher efficacy overlap in various ways as well, in which increased proficiency in mathematics will likely lead to highly efficacious teachers (Gresham, 2008; Swackhamer et al., 2009).

The literature strongly reveals it is imperative that all three constructs (content knowledge, efficacy, and pedagogy) are developed in preservice teachers; however, more research is required in how these domains are simultaneously connected and overlap within the context of university mathematics preservice courses. This research study sought to identify the specific experiences that supported the development of each construct and further examined how each construct intersected and influenced one another. The effectiveness of preservice mathematics programs is critical when moving teachers towards a reform mathematics paradigm. Therefore, this study aimed to examine how student teachers develop content knowledge, efficacy, and pedagogical skills during their mathematics course, and further investigated how this growth transformed their capacity as mathematics teacher.

Chapter 3: RESEARCH DESIGN

Introduction

The aim of this study was to explore the impact of specific learning and teaching experiences that occurred within two elementary preservice mathematics classes. The experiences that were examined were specifically those that were designed to build capacity in student teachers' mathematics development of content knowledge, teacher efficacy, and pedagogical skills. In this study, *content knowledge* denotes both common content knowledge (CCK) and specialized content knowledge (SCK). To best understand the research problem, this study utilized a mixed methods design "to obtain different but complementary data on the same topic" (Morse, 1991, p. 122.) Within the pragmatic paradigm of mixed methodology, this study drew upon the strengths and sought to minimize the weaknesses of both qualitative and quantitative research approaches (Johnson & Onwuegbuzie, 2004). This chapter begins with an overview of paradigmatic orientation and then specifically describes this study's pragmatic paradigm, mixed methods methodology, and methods. A description of validity and reliability include applications of inter-rater reliability and triangulation. The sampling frame of this study is detailed to explain the study's target population, study population, and purposeful sampling. Descriptions of this study's instruments and procedures are also outlined throughout this chapter. The data analysis and interpretation section articulates how the researcher deconstructed the data sets, applied coding techniques and statistical descriptions, and converged the data for overall interpretations. Lastly, ethical considerations, limitations, delimitations, and the chapter summary are delineated.

Paradigmatic Orientation

The term paradigm was first utilized by Kuhn (1972), and refers to patterns of beliefs that guide inquiry. Essentially, a paradigm is the worldview or perspective that is held by a researcher, and it impacts how research is guided. A study's paradigm provides an overall theoretical framework which influences decisions about the research design and data collection techniques. There are a variety of research paradigms, including postpositivist, constructivist, advocacy/participatory, and pragmatic (Creswell, 2009).

The postpositivist paradigm reflects a “deterministic philosophy in which causes probably determine effects or outcomes” (Creswell, 2013, p. 7). Postpositivists hold the assumption that the absolute truth can never be found, and therefore their goal is to support or refute claims by gathering objective data through quantitative research. A constructivist paradigm, also referred to as interpretivism, is based on beliefs that the world is understood through individuals’ subjective meanings of experiences. Hence, constructivist researchers seek out the complexity of these meanings through qualitative research (Crotty, 1998; Mertens, 2010). The advocacy/participatory paradigm evolved in the 1980s when emancipatory researchers felt that the constructivist stance did not advocate strongly enough for those individuals who were part of marginalized groups (Kemmis & Wilkinson, 1998). Advocacy/participatory researchers uphold a political agenda with the view to drive social action and empower individuals. Furthermore, advocacy/participatory researchers involve their participants in designing the study and the research is not exclusive to qualitative methods (Creswell, 2009). Within a pragmatic paradigm, research is not restricted to a single method approach, rather pragmatist researchers have a “freedom of choice” about their methods, techniques, and procedures which best fit the study’s needs and purposes (Creswell, 2003, p. 12). Hence pragmatists draw upon quantitative and qualitative approaches in a single study. According to Cherryholmes (1992), a pragmatist researcher is concerned with conceivable and practical consequences. Patton (2002) discussed how pragmatists make decisions based on “methodological appropriateness” (p. 72) and he refers to the term “situational responsiveness” (p. 72) which occurs when researchers design their studies based on what is most suitable for a specific inquiry context.

Pragmatic Paradigm

The author of this study upheld the belief that the pragmatic paradigm was the most appropriate stance for this study’s intentions and goals. Both qualitative and quantitative data were required to provide a comprehensive understanding of student teachers’ development in mathematics content knowledge, teacher efficacy, and pedagogy. On one hand, the researcher valued the student teachers’ personal mathematical experiences. She recognized the complexity of the meanings behind each experience and appreciated the richness of the qualitative data. However, on the other hand, the researcher felt it was necessary to gather quantifiable evidence of student teachers' mathematical constructs, such as content knowledge, anxiety, and teacher efficacy, in order to gain a more complete picture of how these phenomena intersect. By using diverse approaches, this study gave primacy to the importance of the research problem and questions, and valued both quantitative and subjective knowledge (Morgan, 2007). Multiple sources and approaches were needed as a means to substantiate the overall data. For this reason, the pragmatic paradigm was utilized and the researcher was able to examine the socially constructed realities of student teachers' mathematics development through a methodology that encompassed both qualitative and quantitative approaches.

Mixed Methods Methodology

Each of the paradigms discussed previously are deeply rooted in methodology. According to Willis (2007), methodology refers to the researcher's conceptualizations in designing a study based on theoretical underpinnings. This includes "the design, the procedures for data collection, methods for data analysis, selection of subjects, and details of the specific treatments" (p. 14). The methodological approach in this current study is mixed methodology due to the need for both quantitative and qualitative data. Creswell and Plano Clark (2007) define mixed methods research as "a research design with philosophical assumptions as well as methods of inquiry. Its central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone" (p. 5). Over the years, mixed methods research has gained legitimacy and recognition as a distinct research methodology (Creswell & Plano Clark, 2007). Tashakkori and Teddlie (2003) maintain that the origins of mixed methods research emerged from the notion of triangulating different data sources. This technique was first observed in the field of psychology by Campbell and Fiske's (1959) creation of a multitrait-multimethod matrix. Currently, several prominent researchers, (see for example: Collins, Onwuegbuzie, & Sutton, 2006; Creswell, 2003; Creswell & Plano Clark, 2007; Johnson & Onwuegbuzie, 2004; Tashakkori, 2009; Tashakkori & Teddlie, 2003, 2010) purport that mixed methodology has the potential to access comprehensive evidence that cannot be attained by qualitative or quantitative approaches alone. More recently, Tashakkori and Teddlie (2010) describe mixed methods research as the “third methodological community” (p. 11). The authors claim that this distinct third paradigm encompasses a wide range of approaches to and interpretations of mixed methods research. They make a strong case for a new paradigm by proposing a set of characteristics that define this third paradigm. In their work, Tashakkori and Teddlie (2010) encourage the mixed methods research community to put forth efforts in generating commonalities across the field. Hence, with this continual effort, mixed methods research will strengthen its identity as the third methodological movement.

Mixed methods researchers understand when only one approach (qualitative or quantitative) is used to study a problem; it may be “inadequate by itself to address the research problem. . . . The combination of qualitative and quantitative data provides a more complete picture by noting trends and generalizations as well as in-depth knowledge of participants’ perspectives” (Creswell & Plano Clark, 2007, pp. 32-33). Consequently, researchers employ mixed methods for several reasons. Creswell (2003), Creswell and Plano Clark (2007), and Tashakkori and Teddlie (2003) discuss the types of research problems that are well suited for mixed methods studies. First, mixed methods designs are often required when a need exists for both quantitative and qualitative approaches because one type of evidence cannot provide comprehensive evidence. In this case, both quantitative and qualitative data are collected concurrently and given equal weight. Second, researchers employ mixed methods when there is a need to enhance the study with a secondary source of data. For example, when quantitative results are insufficient in itself, the researcher collects qualitative data to explain and substantiate quantitative results. In this situation, quantitative data need further interpretation of the research problem, and the researcher implements a second phase of qualitative data collection. Last, mixed methods are required when the researcher initially explores the problem qualitatively, before developing a quantitative study. This occurs when qualitative data allows for the researcher to gain an adequate exploration of the problem.

Specifically within mathematics teaching and learning, many researchers recognize the complexity of such educational phenomena and therefore warrant a multifaceted design as the most appropriate methodology (Caruth, 2013; Hart, Smith, Swars, & Smith, 2009). Mixed methodology is slowly gaining acceptance in the field of mathematics education. Ross and Onwuegbuzie (2010) reviewed the methodology designs of published articles in the Journal for Research in Mathematics Education (JRME) between 1999 and 2008. They found mixed methods research was utilized in about one third of all published articles in the JRME. Similarly, Hart et al. (2009) reviewed the methodology designs of 710 mathematics education studies published across six prominent journals, from 1995 to 2005. They found approximately 29% of the articles conducted mixed methods studies. Furthermore, of the 29% that were deemed mixed methodology, Hart et al. (2009) discuss how the manuscripts lacked clarity in the research design which posed categorization issues in the initial decision making process. Subsequently, Hart et al. advocate that explicit disclosure of design methodology should be described up front. The importance of clarity in quantitative and qualitative approaches is also suggested by Creswell (2003) and Bryman (2007). Moreover, Hart et al. (2009) concluded that mixed methods research may very well be the most effective design for mathematics education:

> As an applied discipline where research should be improving education, this integration is needed to not only know *if* particular educational experiments improve learning with understanding but also *how* those results are achieved and *why* we can expect them to be replicated elsewhere. (p. 39, italics in original)

Challenges in Mixed Methods Research

Although mixed methods designs are considered a practical and well-rounded approach for conducting research, there exists several challenges mixed methods researchers face. Researchers who choose to mix methods must endorse a pragmatic stance and embrace multiple paradigms in their study. This philosophical worldview goes against the historical perspective that upholds an adversarial relationship between quantitative and qualitative research, often described as the purist stance (Rossman & Wilson, 1985). Therefore, pragmatic researchers may have difficulty finding acceptance and support by purist researchers who uphold the dichotomy of quantitative and qualitative worldviews (Johnson & Onwuegbuzie, 2004). Another challenge is the amount of effort and resources required to implement a mixed methods approach because researchers must be familiar with both quantitative and qualitative methods. Hence, mixed methods research is time consuming due to the extensive data collection and interpretation (Creswell, 2003, 2009; Creswell & Plano Clark, 2007).

Methods

Methods are the modes of data collection (Creswell, 2003). In this mixed methodology study, the modes of data collection involved both quantitative and qualitative approaches. Each mode of data collection was carefully designed and/or selected to support the study's objective, i.e., to understand the course experiences that influenced student teachers' mathematics development of content knowledge, efficacy, and pedagogy. According to several prominent mixed methods researchers, there are two main factors that determine the type of mixed methods design: 1) priority and/or weight given to each method and 2) the order of data collection (Morse, 1991; Morgan, 2007; Tashakkori & Teddlie, 2010; Creswell, 2003). In this design study, the quantitative and qualitative methods of data were weighted equally, which means that the priority and/or emphasis of the two approaches were given equal consideration when data was converged (Ross & Onwuegbuzie, 2010). In addition, the implementation of the data collection occurred concurrently (within the academic year), and the results were integrated during the interpretation phase.

Another important construct in mixing the methods relates to expansion. This involves analyzing multiple dimensions of a phenomenon through mixing the methods of qualitative and quantitative data collection, thereby resulting in a deeper and more detailed understanding of the phenomenon (Caracelli & Greene, 1993). In this current research, expansion was applied because the quantitative part of this study focused on measuring the characteristics of mathematics anxiety, teacher efficacy, and content knowledge, while the qualitative approaches addressed the rich mathematics learning experiences student teachers shared through interviews and journals.

Data Collection Methods

The study used a total of four distinct methods to gather data: 1) qualitative data from document review of participants' culminating reflective journal assignments; 2) qualitative data from interview transcripts; 3) quantitative data from two established survey scale instruments; and 4) quantitative data from pre- and post-grade six mathematics tests which included open-ended response and multiple choice questions. The following subsections describe the data collection methods and provide an overview of its advantages and disadvantages from the perspective of this study's focus.

Document review.

In an attempt to uncover 'how do specific teaching and learning experiences in the mathematics course contribute to student teachers' a) mathematics content knowledge, b) teacher efficacy, and c) pedagogy?' participants from both classes were asked to keep a journal and record their learning processes that they experienced during the mathematics course. The culminating journals provided important data to the research study as the focus involved reflection on the significant activities and events experienced during the course. Reflective thinking through journal writing enabled student teachers and the researcher to be immersed in inquiry and self-examination. Journaling is an effective method that promotes deep reflection of learning experiences (Boline, 1998; Han, 1995; Hoover, 1994). The literature suggests that there is no correct way to record thoughts in a journal; ideas may be stimulated by guiding questions or it may be completely open-ended for the participants to choose what to write about (Houston & Clift, 1990). Journaling as a method of data collection has its disadvantages. Reflections may be incomplete and authenticity may be questioned due to participants' bias to impress the researcher (Hoover, 1994). Nonetheless, this researcher stressed to student teachers that there were no "correct" thoughts, and encouraged only honest and open reflection about their mathematics experiences in the program. In this research study, the researcher was specifically interested in the critical incidents, that is, the specific moments and activities that supported participants' capacity in mathematics teaching. These critical incidents were examined through student teachers' reflective thoughts about their mathematics class and field experiences which they recorded in their culminating journals.

In the journal, preservice teachers wrote their reflections, thoughts, and ideas after each mathematics class. Participants were encouraged to write about how they made sense of mathematics concepts, pedagogical practices, as well as difficulties they might have encountered. Mid-way through the year, student teachers wrote a culminating reflective journal that synthesized the significant experiences from their regular journal entries. At the end of the year, student teachers again completed a culminating reflective journal that synthesized their mathematics development of the second half of the year. The culminating reflective journals were submitted to the researcher in electronic format (see Appendix A.1 for journal assignment description). In addition to student teachers' journals, the author herself reflected on her teaching experiences through a journal. This journal discussed the author's perspective about the contents and structure of the course, as well as her beliefs about preservice teachers' mathematics teaching capacities.

This qualitative data provided depth and possible explanations to the quantitative survey results. For example, the quantitative results revealed a strong correlation between high mathematics anxiety and low mathematics content knowledge. The journaling process further demonstrated pertinent information about preservice teachers' specific course experiences that reduced anxiety and supported their understanding of mathematics content. Therefore, collecting different but complementary data (quantitative and qualitative) certainly enhanced this study's evidence, thus making the triangulation mixed methods approach a well suited design for this research.

Individual interviews.

Semi-structured individual interviews provided optimal opportunities to gain an in-depth understanding of student teachers' experiences of their mathematics development (Willis, 2007). However, interviews can pose disadvantages due to its qualitative nature. The understandings and meanings gleaned from the interviews may not be easily generalizable to other populations, and bias and idiosyncrasies of the researcher must be considered during the analysis phase (Johnson & Onwuegbuzie, 2004).

In this study, interviews were conducted with six student teachers who possessed high or low mathematics anxiety levels. Participants were selected based on quantitative pre-survey results of the RMAS. An external third party individual conducted two face-to-face interviews based on the central research question, 'how do specific teaching and learning experiences in the mathematics course contribute to student teachers' a) mathematics content knowledge, b) mathematics teacher efficacy, and c) pedagogy?' Student teachers' interviews focused on the activities they experienced during the course.

The interviews were audio-recorded and transcribed by the researcher after the submission of final grades. See Appendix A.2 for pre-interview script and questions, and Appendix A.3 for post-interview script and questions.

The initial interviews with student teachers occurred at the beginning of the course (middle of September). The major goals of these interviews were: to ascertain their beliefs about teaching elementary mathematics; understand students' anxiety levels (high or low); and discuss their mathematics goals for the year. Near the end of the year (middle of April), a second interview was conducted in which participants were asked about the course experiences throughout the year that influenced their development as mathematics teachers. Questions involved: the role of instructor; changes in mathematics teacher efficacy; pedagogical understanding; and development of mathematics content knowledge. Interviews were audio-recorded and transcriptions were completed and analyzed *after* the submission of grades (end of April). This timeframe ensured that there was no potential for or perception of punitive actions toward student teachers and influence on their grades as a result of their participation. Essentially, this timeline eliminated student teachers feeling at risk of retributive actions from the instructor. As mentioned earlier, an external interviewer conducted the interviews with student teacher participants to reduce potential bias by the researcher-instructor. Reporting to a third independent party promoted greater researcher objectivity as well as increased student comfort and openness. The researcher trained the external interviewer on how to conduct the interviews in order to ensure consistency in questioning and prompting. There were a total of 12 interviews - two one hour individual interviews with six student teachers.

Instrumentation data collection.

There were two established instruments used in this study: the Revised Math Anxiety Scale (RMAS) designed by Betz (1978) and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) developed by Enochs, Smith, and Huinker (2000). These instrumentation scales were administered at the beginning and at the end of the academic year in order to determine if levels of mathematics anxiety and mathematics teacher efficacy changed during the course. This study also measured CCK of preservice teachers through a grade six mathematics content pre- and post-test. Both mathematics tests included five open-ended questions and 10 multiple choice questions.

Advantages of using these instrumentation methods included the practicality of collecting information from all 99 student teachers in a short period of time. Two of the instruments had already established reliability and validity factors which promoted objectivity. In addition, the data was easily quantified so that comparisons were calculated between pre- and post-tests. Comparing the pre- and post-results enabled the researcher to quantifiably measure any changes throughout the academic year, as well as measure any existing correlations between and among mathematics anxiety, mathematics teacher efficacy, and CCK. Unfortunately, the instrumentation data on its own would not have been adequate in understanding this study's research focus on student teachers' mathematics development. Conceptualizations of emotions, behaviours, feelings, and meanings of experiences, could only be captured through qualitative data methods. For this reason, the mixing of the two approaches maximized this study's objectives. The qualitative findings from the interviews and journals supported the findings in the quantitative data. Later on in this chapter, the 'Instruments' section describes in detail the specific instrumentation tools used for this study.

Validity and Reliability

Mixed methods research presents advantages as well as challenges in terms of validity and reliability considerations. A fundamental principle of mixed methods research is recognizing the limitations and strengths of all methods, and mixing the methods in ways that strengthens the overall interpretation of the data (Tashakkori & Teddlie, 2010). However, one challenge is that mixed methods research is considered fairly new in the methodological community; hence debates and questions continue to arise about the complexities behind validity and reliability. According to Johnson (1997), valid research refers to "plausible, credible, trustworthy, and, therefore, defensible" (p. 282). There are several validity types such as: internal validity, external validity, and statistical conclusion validity (Tashakkori & Teddlie, 2003). Internal validity involves the strength of cause-effect relationships, whereas external validity refers to the generalizability of the results to other contexts beyond the study (Ross & Onweugbuzie, 2010). According to Cook and Campbell (1979), statistical conclusion validity involves the appropriate use of statistics for the study's purpose. The concept of reliability refers to the study's capacity to replicate the results of a study (Golafshani, 2003).

Historically, quantitative research has embodied notions of validity and reliability. The concepts of validity and reliability in qualitative research are applicable and hold high importance. However, many qualitative researchers felt that the terms validity and reliability were not suitable to this type of research (Morse, Barrett, Mayan, Olson, & Spiers, 2002). The terms trustworthiness and credibility are used by Lincoln and Guba (1985), and denote similar characteristics to validity and reliability, but are more suitable terms for qualitative approaches. As a result, the terms trustworthiness and credibility have emerged over the years in qualitative research. For this current mixed methods study, the terms validity and reliability are used, and are considered synonymous to trustworthiness and credibility.

Inter-rater Reliability

Inter-rater reliability was applied throughout this study. Simply put, inter-rater reliability refers to the degree of consensus or agreement among individual raters (Kurasaki, 2000). Inter-rater reliability was applied to the content assessment instrument as well as to the coding of the qualitative journals and interviews. For the quantitative content assessment instrument, inter-rater reliability was obtained on each of the open-ended questions. The five open response questions were scored independently by the researcher and her colleague. The scoring rubric was used to determine the level for each question (see Appendix A.4). Inter-rater reliability was calculated in each of the open-ended questions for the pre- and post-content test, and ranged from 86% to 94% (see Table 3.1). The assessments were further reviewed together by the researcher and her colleague and reassessed until 100% agreement was attained for each open response question. Details of the scoring process are explained in the 'Data Analysis and Interpretation' section, later in this chapter.

Table 3.1:
Inter-rater Reliability Measures for Open-ended Questions in Pre- and Post-content Test

	Open-Ended Question #1	Open-Ended Question #2	Open-Ended Question #3	Open-Ended Question #4	Open-Ended Question #5
Pre-test	88%	90%	88%	90%	86%
Post-test	90%	92%	90%	94%	88%

Note. On the pre- and post-content tests, the five open response questions were scored independently by the researcher and her colleague. The scoring rubric was used to determine the level for each question (see Appendix A.4). Inter-rater reliability was calculated in each of the open-ended questions for the pre- and post-content test. The assessments were further reviewed together by the researcher and her colleague and reassessed until 100% agreement was attained for each open response question.

For the qualitative data, inter-rater reliability was applied during the coding process of the interviews and journals. The researcher's colleague independently reviewed four student teachers' journals and two interview transcripts and arrived with an initial set of codes. The inter-rater reliability of the coding resulted in an 88% agreement for both data sets, between the researcher and her colleague. The researcher compared notes to that of her colleague's and reconciled any differences that showed up on their initial codes. This ensured inter-rater reliability of the coding system (Kurasaki, 2000).

Triangulation

Various authors have noted that mixed designs allow for triangulation which serves as an advantage over other design methods (Creswell, 2003). Triangulation occurs when the convergence of the quantitative and qualitative approaches are integrated in ways that make the results more reliable. According to Creswell and Plano Clark (2007), there are four major types of mixed methods designs: 1) triangulation (e.g., merging of qualitative and quantitative data); 2) embedded (e.g., quantitative data embedded within a largely qualitative study); 3) explanatory (e.g., emphasis on quantitative data and qualitative results explain and elaborate); and 4) exploratory (e.g., develop quantitative instrument based on qualitative data to test quantitatively). This study employed the triangulation design, the most common and well-known of the four major mixed methods models. The researcher felt that the triangulation design was the most beneficial way to uncover the study questions and it allowed for the data sets to be compared and analyzed into one overall interpretation.

The convergence of the quantitative and qualitative data sets supported the triangulation process of this study. However, as delineated by Denzin (1978, 2010), other essential aspects of triangulation were also considered in this current research. Namely, triangulation of methods was attained, in that data were collected using various techniques, i.e., individual interviews, written journals, surveys, and tests. In this study, when a data set was analyzed, for example, the research-instructor's journal data, the researcher would then apply triangulation strategies by crosschecking the codes with other data sets, such as student teachers' journal and interview data, as well as the quantitative data to ensure corroboration. A set of categories were then developed which were further condensed to main themes (Creswell, 2008). Triangulation of theory was also attained in this study, in which different theoretical perspectives on the data uncovered a more complete picture. The theories that informed this study were: self-efficacy theory (Bandura, 1977, 1986, 1997); mathematical knowledge for teaching theory (Ball et al., 2008); constructivist learning theory (Dewey, 1916, 1938, 1964; Piaget, 1952, 1963; Vygotsky, 1978, 1986); and adult learning theory (Knowles, 1984; Mezirow 1991; Mezirow & Associates, 2000). Lastly, triangulation of participants incorporated both the student teachers' and the academic instructor's perspective. ''By combining multiple observers, theories, methods and data sources, sociologists can hope to overcome the intrinsic bias that comes from single-method, single-observer, single-theory studies'' (Denzin, 1978, p. 307).

Sampling Frame

The source of participants in this study was drawn from two university mathematics classes taught by the researcher herself. This specific population represented undergraduate students who were completing a one-year Bachelor of Education degree in a large urban university in southern Ontario, Canada. Mathematics classes were held at the main campus located downtown in the heart of the city. The total amount of hours spent in class was 36 hours from September to mid-April. Each mathematics class focused on elementary mathematics (kindergarten to grade six) with a total of 99 student teachers enrolled; 60 student teachers in class A, and 39 in class B (see Figure 3.1). This study's 99 preservice teacher participants were purposefully selected and considered the study population as both classes were entirely involved and all 99 participants shared common characteristics based on acceptance criterion for the degree program. The criterion for entrance to preservice education programs are consistent across the nation in that applicants require: an undergraduate degree with a minimum of a B standing; must provide evidence for their deep commitment to social justice issues; and have some teaching-related or other life experiences that has prepared them for a career in teaching. For this reason, the student teacher participants in this study fit a typical profile of student teachers enrolled across other one-year elementary preservice programs, i.e., this study's target population. Figure 3.2 illustrates the sampling frame of the study.

Figure 3.1: Overview of Participants, Sampling, and Data Collection

Mathematics Class A	**Mathematics Class B**	**Data Collection**
• 60 student teachers • researcher-instructor	• 39 student teachers • researcher-instructor	
Pre- and post-test: • RMAS • MTEBI • Content Assessment	Pre- and post-test: • RMAS • MTEBI • Content Assessment	Pre-test – 99 student teachers completed instrumentation data Post-test – 97 student teachers completed instrumentation data (two student teachers were absent in class A)
• Researcher-instructor's journal	• Researcher-instructor's journal	Researcher-instructor's journal • 13 entries throughout academic year
• Reflective journal assignment 60 student teachers x 2 journals (fall and spring) =120 journals	• Reflective journal assignment 39 student teachers x 2 journals (fall and spring) =78 journals	192 journals were used as research data in this study Three student teachers' journals were not used - ethics permission for journal entries were not granted
• Individual interviews 4 student teachers x 2 interviews = 8 Conducted by third party external interviewer	• Individual interviews 2 student teachers x 2 interviews= 4 Conducted by third party external interviewer	Purposeful Sampling of 6 student teachers (two interviews each) • 3 with high anxiety • 3 with low anxiety =12 interviews in total

Figure 3.1. Participants in the study consisted of student teachers from two mathematics classes and the researcher as participant observer. Data collection from the researcher-instructor included journal reflections written throughout the academic year. Data collection from student teacher participants included instrumentation measures, journals, and interview transcripts. There was purposeful sampling of student teachers with high and low mathematics anxiety for individual interviews. Note that a total of 192 student teacher journal entries were collected, not 198 due to three student teachers who did not submit permission to use their journal entries as data.

Figure 3.2: Sampling Frame

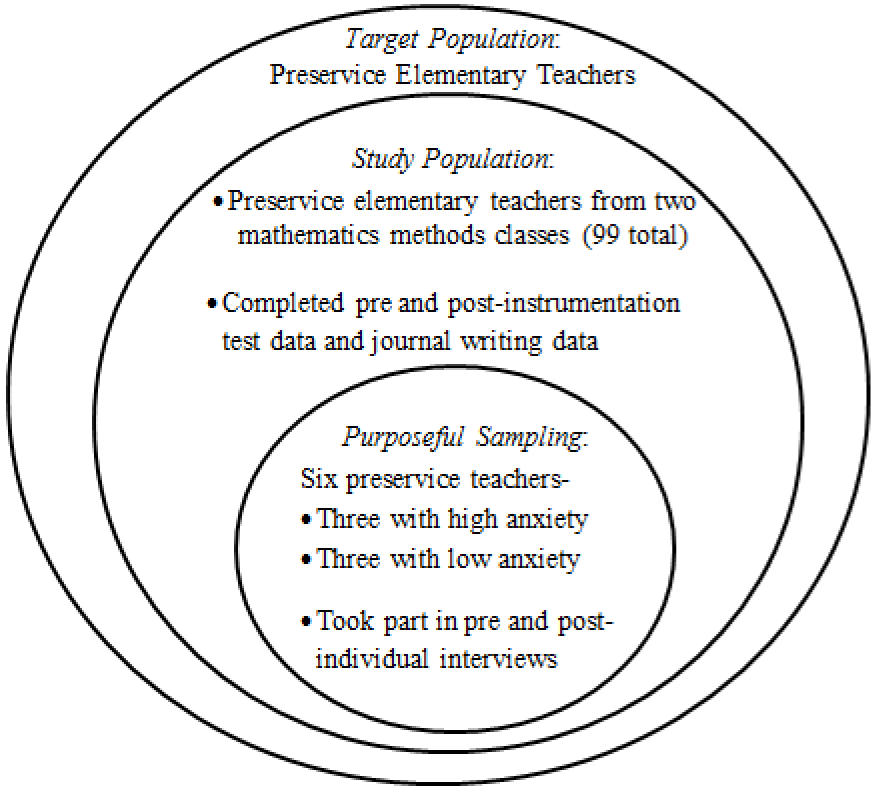

Figure 3.2. In this figure, the target population refers to preservice elementary teachers to which this study is interested in generalizing the conclusions. The study population is the researcher's student teachers from her two mathematics methods course. This is a subset of the target population and provides the source of participants due to its accessibility to the researcher. Purposeful sampling is a subset of the study population, and this includes six student teachers with varied mathematics anxiety levels.

Sampling

In order to gain a deeper understanding of how mathematics anxiety connected with student teachers' mathematics teacher development, "purposeful sampling" from the study's participant population of 99 student teachers was used (Creswell & Plano Clark, 2007, p. 112). The researcher intentionally selected student teachers with either high mathematics anxiety or low mathematics anxiety in order to explore both ends of the anxiety spectrum. Individual interviews of six student teachers provided this study with detailed data about the influence anxiety had on mathematics teacher development. Based on the Revised Math Anxiety Scale (RMAS) results on the first day of mathematics class, six student teachers were invited to participate in two in-depth interviews throughout the year. The six participants were comprised of three student teachers with low anxiety (scoring more than 45), and three student teachers with high anxiety (scoring less than 15).

A third party who was external to both classes conducted the interviews to ensure objectivity of data collection and to promote greater student comfort and honesty in reporting to a third independent party. This external interviewer underwent interviewer training with the researcher prior to interviewing to ensure consistency in questioning and prompting. Participants were reminded of their RMAS scores during the interview. The main premise for working with student teachers with high mathematics anxiety and low anxiety participants was to compare if and how teaching and learning experiences impacted differently on the two groups. In addition, the RMAS was also used as a diagnostic tool to identify student teachers with high mathematics anxiety, so more support was given to those preservice teachers. Examples of additional support structures included: offering small group tutoring to further develop mathematics content knowledge; forming mixed ability groupings during mathematics classes and collaborative projects; and providing more time and assistance during problem solving activities.

The entire population from both mathematics classes were invited to participate in this study through their journals. A usual component of the coursework was for all student teachers to complete a reflective journal as part of their course assignments. The study population of 99 student teachers submitted two culminating journals to the researcher instructor as part of the course assignment requirements; this resulted in 198 journals. Three student teachers did not grant permission to use their reflective journals as part of this study's data. Therefore, 96 student teachers participated in the journal reflection data, with a total of 192 journals. In addition, all 99 student teachers completed the quantitative instrumentation tools (i.e., MTEBI, RMAS, mathematics content assessment) in the fall, and 97 student teachers completed the post-tests in the spring. Two student teachers were absent during the post-tests and therefore were unable to complete the spring surveys and content assessment.

Researcher-instructor as Participant Observer

This study also included the researcher-instructor as participant observer. Throughout this research, the author took on the role as a participant observer. Participant observation is a method that is employed in qualitative research to achieve an understanding of a specific phenomenon. The method of participant observation requires the researcher to use information gained from participating and observing the people in the study (DeWalt & DeWalt, 2002). Patton (2002) describes participant observation as follows:

> During fieldwork, the researcher spends time in the setting under study … The researcher makes firsthand observations of activities and interactions, sometimes engaging personally in those activities as a *participant observer*. …The qualitative researcher talks with people about their experiences and perceptions. More formal individual or group interviews may be conducted. Relevant records or documents are examined. Extensive field notes are collected through these observations, interviews, and document reviews. (p. 4, italics in original)

The author's participation in this study supported the understanding of the academic perspective of preservice experiences that occurred within the mathematics course. The author's observations and reflections were recorded in a journal after each mathematics class.

Instruments

At the beginning of the school year, preservice teachers from the two mathematics classes completed three instrumentation tools: Revised Mathematics Anxiety Scale (RMAS) (Betz, 1978), Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs et al., 2000), and a mathematics content test at a grade six level. Completing these survey instruments and the mathematics content assessment were part of the regular activities of the mathematics course. The author sought out written permission to use survey and assessment results for this study. The following subsections detail the instrumentation design, rationale for the use of tools, and the method of administration.

Revised Mathematics Anxiety Scale

The RMAS is a 10 item scale that was designed by Betz (1978). Betz modelled the RMAS after the Math Anxiety Response Scale (MARS), originally developed by Fennema and Sherman (1976). The Fennema-Sherman MARS is composed of 12 items and was designed for administration to high school students to assess the "feelings of anxiety, dread, nervousness, and associated bodily symptoms related to doing mathematics" (Fennema & Sherman, 1976, p. 4). Betz (1978) modified several of the Fennema-Sherman MARS items to be more appropriate for college students. Of the 12 Fennema-Sherman MARS items, Betz established 10 to assess mathematics anxiety of college level students. Permission to use the RMAS was obtained by its author through email.

The RMAS uses a five-point Likert scale ranging from 1= strongly agree, 2 = agree, 3 = undecided, 4 = disagree, and 5 = strongly disagree. Reverse scoring for the positively worded statements means that low item responses represent high anxiety. Total scores could range from 10 to 50, in which 10 represents high anxiety, and 50 represents low anxiety. Betz (1978) reported the RMAS to have high reliability, coefficient of .92 based on 652 undergraduate students in the United States. See Appendix A.5 for the RMAS and Appendix A.6 for RMAS scoring instructions.

For this study, the RMAS was the most suitable instrumentation tool to gather data on student teachers' mathematics anxiety levels. This anxiety scale was specifically designed for college or university level students which fit the profile of this study's participants. The RMAS was not an arduous scale because it is only comprised of 10 items, which made for completion of the scale convenient and less time consuming; it took approximately six minutes for student teachers to complete the RMAS. Another important rationale for the use of the RMAS is its high reliability factor of .92 (Betz, 1978). During the first mathematics class, student teachers completed the RMAS and submitted to the researcher-instructor. Student teachers were informed of their RMAS scores in the second mathematics class and they were encouraged to reflect on their mathematics anxiety levels. Of most importance, the RMAS results were examined by the instructor as a diagnostic tool to identify students with high mathematics anxiety, so more support was given to those student teachers. The RMAS was administered for the second time near the end of the course, in which pre- and post-results were compared and examined.

Mathematics Teaching Efficacy Beliefs Instrument

The MTEBI (Enochs et al., 2000) was designed to ascertain preservice teachers' perceived efficacy in teaching mathematics. Permission to use the MTEBI was obtained by two of its authors through email. The MTEBI consists of 21 items, in which 13 items focus on the Personal Mathematics Teaching Efficacy (PMTE) subscale. PMTE has been defined as the belief in one's own ability to teach mathematics effectively (Woolfolk, Rossoff, & Hoy, 1990). Possible scores on the PMTE scale range from 13 to 65. The remaining eight items on the MTEBI are the Mathematics Teaching Outcome Expectancy (MTOE) subscale. MTOE refers to the belief that effective teaching has a positive impact on students' mathematics learning (Woolfolk et al., 1990). MTOE scores range from 8 to 40. Similar to the RMAS, the MTEBI uses a 5 point Likert scale ranging from 1= strongly agree, 2 = agree, 3 = undecided, 4 = disagree, and 5 = strongly disagree. The PMTE and MTOE items are scattered randomly, with some items requiring reverse scoring. The MTEBI is a self-administered questionnaire and takes approximately ten minutes to complete (see Appendix A.7 for the MTEBI).

The MTEBI was developed through the modification of past teacher efficacy scales. Bandura (2001) recommended that self-efficacy scales must be customized to the particular domain, relative to the action or task to be performed. Therefore, measures used to determine efficacy levels need to be content specific. Modeled after Gibson and Dembo's (1984) Teacher Efficacy Scale (TES), Riggs and Enochs (1990) developed the Science Teaching Efficacy Beliefs Instrument (STEBI). STEBI was specifically designed to measure elementary inservice teachers' efficacy beliefs in science. Similar to TES, STEBI measures the constructs of outcome expectancy and personal teaching efficacy separately. Riggs and Enochs (1990) administered the STEBI to 331 elementary inservice teachers. Results from this study showed that the Personal Science Teaching Efficacy Belief (PSTEB) sub-scale of the STEBI generated a coefficient alpha of .91. The Science Teaching Outcome Expectancy (STOE) subscale of the STEBI produced a coefficient alpha of .76. The authors established factorial validity with the STEBI for studying the beliefs towards teaching science in elementary teachers.

After the development of the STEBI, Enochs and Riggs (1990) modified the instrument for use with preservice teachers. The Science Teaching Efficacy Beliefs

Instrument form B (STEBI B) was designed by revising the original STEBI to measure the beliefs of elementary preservice teachers, as opposed to inservice teachers. STEBI B was administered to 212 preservice elementary teachers and results produced a coefficient alpha of .90 on the Personal Science Teaching Efficacy Belief (PSTEB) subscale and .76 on the Science Teaching Outcome Expectancy (STOE) subscale. The authors concluded that the STEBI B is a valid and reliable instrument (Enochs & Riggs, 1990).

Years later, Enochs et al. (2000) developed the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) based on modifications of the STEBI B. The authors performed a study to establish formal factorial validity of the MTEBI for preservice elementary teachers. The study's sample included 324 preservice elementary teachers in mathematics methods courses. The MTEBI was given at the end of each methods course. Item analysis was performed on both the PMTE and the MTOE subscales. The authors found low correlations of .30 on two specific items; therefore these items were dropped from the questionnaire. As a result, the final MTEBI contains 21 items. The PMTE subscale consists of 13 items and the MTOE subscale consists of eight items. Enochs et al. (2000) found the reliability analysis of the MTEBI to have an alpha coefficient of .88 on the PMTE subsection and a .75 on the MTOE subsection. Overall, the researchers claim the MTEBI to be valid and reliable (see Appendix A.8 for MTEBI scoring instructions).

The researcher of this study used the MTEBI due to its prominence in the field of preservice mathematics education. In reviewing mathematics education literature published over the last decade the researcher found the MTEBI to be the most widely used survey instrument across mathematics teacher efficacy research studies in North America. For this current study, it was critical to measure student teachers' mathematics teacher efficacy levels using an instrumentation tool with established validity and reliability. Therefore the MTEBI was the selected instrument. In addition, the ease of completion was also an important rationale; student teachers took approximately ten minutes to complete the 21 item scale. The MTEBI was administered during the first day of mathematics class to determine diagnostic entry points of student teachers' efficacy levels in mathematics teaching. The student teachers were informed of their efficacy levels in the following class and were encouraged to reflect on how their efficacy levels influenced their mathematics teacher development. The post-survey MTEBI was completed near the end of the academic year and the researcher-instructor compared any changes between pre- and post-efficacy levels.

Mathematics Content Assessment

In order to gain an understanding of student teachers' CCK, a mathematics test was administered at the beginning of the year and a different mathematics test was given at the end of the academic year. Items for the mathematics test were obtained from the Education Quality Accountability Office (EQAO). The EQAO produces the annual grade six Ontario provincial mathematics tests; its mathematics items are based on the Ontario mathematics curriculum and include both open-ended problems and multiple choice questions.

The author selected questions from various grade six EQAO tests and compiled them to form a pre- and post-test administered in the fall and in the spring (see Appendix A.9 for the fall content assessment and Appendix A.10 for the spring content assessment). Each test included questions that assessed a balance of all five mathematics strands; that is 1) number sense and numeration, 2) patterning and algebra, 3) data management and probability, 4) geometry and spatial sense, and 5) measurement.

The pre- and post-mathematics test comprised of 15 questions. There were five open-ended and 10 multiple-choice questions. The 10 multiple choice items were scored as correct or incorrect, giving a maximum score of 10. The five open-ended items required the researcher to use EQAO's four-level scoring rubric (see Appendix A.4), with level three and above as meeting the provincial standard. The scoring criteria included preservice teachers' understanding of concepts and accurate application of the procedures. In addition, the author looked for responses that demonstrated effective problem solving processes based on understanding the relationships between concepts, identification of important elements, and appropriate solutions to the problem. Responses that were blank or irrelevant (e.g., "I don't know how to do this") were scored as zero. Therefore the highest level that was given to an open-ended question was four, and the lowest level was zero, with the highest total score attainable of 20.

The rationale for using EQAO grade six mathematics questions resides in the fact that these items were developed based on the Ontario mathematics curriculum. As this research took place in an accredited Ontario university preservice program, the Ontario curriculum was a foundational component of the researcher's mathematics elementary course content. Furthermore, the questions in EQAO mathematics assessments are often used as models for reform oriented problem solving in which inservice teachers utilize them as instructional tools or for classroom assessment of students. Therefore, EQAO mathematics questions provided an effective way to measure student teachers' basic content knowledge through an open-ended and reform based approach.

Procedures

The mixed methods design for this study involved a variety of qualitative and quantitative data sources to better understand how mathematics course experiences influenced the development of student teachers' mathematics content knowledge (CCK and SCK), teacher efficacy, and pedagogical skills. See Figure 3.3 for the overall design study and details of timelines. This section describes the procedures and the timeframe of when data were collected throughout the one-year academic program.

At the onset of this study, a ten minute presentation of this study's goals and procedures were orally communicated to both classes by an external third party individual. The script for the presentation is outlined in Appendix A.11, as well the consent form for use of instrumentation results and journal data were distributed at this time (see Appendix A.12). All student teachers received an invitation to participate through the use of their results derived from the Revised Mathematics Anxiety Scale (RMAS), Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), the mathematics content test, and journal data. This presentation was conducted at the end of the first mathematics class shortly after student teachers completed the RMAS, MTEBI, and the fall mathematics content test. All student teachers granted permission to use their pre- and post-instrument results of their RMAS, MTEBI and the mathematics content test.

Figure 3.3: Timeline of the Mixed Methods Data Collection

September Pre-test data	**October to March**		**Mid-April** Post-test data
Quantitative Data	**Qualitative Data**		**Quantitative Data**
Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)	Reflective journals Semester one Sept. to Dec. Written by student teachers and researcher/instructor	Reflective journals Semester two Jan. to Apr. Written by student teachers and researcher/instructor	Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)
Revised Math Anxiety Scale (RMAS)	Semi-structured individual interviews with six student teachers conducted in September and April • Three with high mathematics anxiety • Three with low mathematics anxiety		Revised Math Anxiety Scale (RMAS)
Fall Mathematics Content Test			Spring Mathematics Content Test

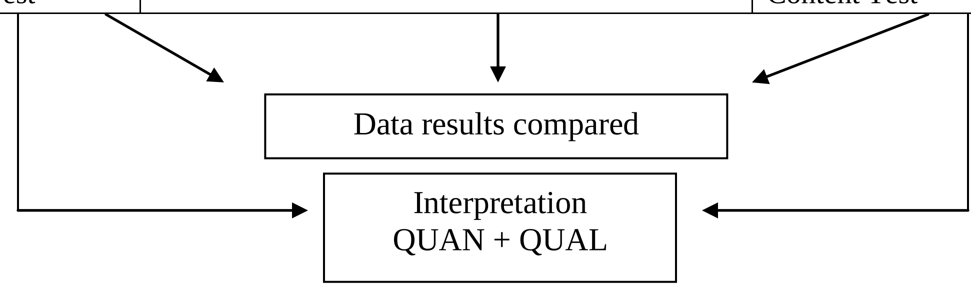

Figure 3.3. Timeline of the mixed methods data collection during the one-year mathematics preservice course. In the fall, quantitative pre-test data were collected followed by qualitative data collection during the year, then post-test quantitative data were gathered to measure changes from initial findings. The qualitative and quantitative data were converged to understand the university course experiences that supported preservice teachers' mathematics development. The notations for QUAN + QUAL are shorthand labels for *quantitative* and *qualitative*. The equal number of letters and capitalization in both notations indicate that both sets of data were equally weighted. The plus sign "+" denotes that both types of data were collected concurrently (within the preservice academic year).

Qualitative sources included student teachers' and the researcher's journal reflections. The journal data involved student teachers' written reflections about their mathematics teaching development. At the end of every mathematics class, student teachers were given 15 minutes to write in their journals about their ideas, thoughts, and challenges in mathematics teaching and learning. At the end of both semesters (December and April), student teachers submitted one comprehensive journal that synthesized their mathematics reflections for the entire semester. The researcher instructor also maintained a journal which comprised of her thoughts about her instructional strategies and course content. In regards to the journal reflection assignment, three student teachers did not grant their permission to use this data as part of the study.

Another source of qualitative data collection were individual interviews with student teachers – three with high mathematics anxiety and three with low mathematics anxiety. Invitations to participate in individual interviews were sent out to six student teachers via email by an external third party individual (see Appendix A.13 for email script). All six individuals agreed to participate and each granted permission to use their interview results as part of this study's data (see Appendix A.14 for interview consent form). Interviews were conducted in mid-September and in mid-April by an external third party interviewer. Furthermore, the external third party individual who conducted the interviews signed a confidentiality agreement to ensure that the interview contents and interviewees' names remained concealed until after the submission of final grades in April (see Appendix A.15).

The final stages of data collection involved the quantitative post-test data which were used to measure changes from initial findings. Student teachers completed these post-tests in mid-April during the final mathematics class. Once all the data sources were collected the researcher attempted to bring the separate data sets together and converged the results by comparing and contrasting the data sets. It is important to note that the interview transcripts and the journal reflection data were not analyzed for this study until *after* the submission of final grades (end of April).

Data Analysis and Interpretation

As part of a mixed methods study, qualitative and quantitative data were analyzed to develop themes based on student teachers' mathematics teacher development in content knowledge, teacher efficacy, and pedagogy. The data sets were analyzed independently prior to the convergence of results (Creswell, 2003). The scores from the pre- and post-quantitative instrument tools (i.e., RMAS, MTEBI, and mathematics content assessment) were analyzed, compared, and summarized using descriptive statistics. The qualitative data derived from student teachers' journals, researcher-instructor's journal, and student teacher interviews were analyzed through an iterative process as described by Anfara, Brown, & Mangione (2002). For this study, iterations of analysis involved: first the coding of the qualitative data by listing the initial coded items; second, reorganizing the full set of initial codes into categories; and then revisiting the list of categories in conjunction with the quantitative data to examine how each data set substantiated one another. As the researcher brought the qualitative and quantitative data together and compared, contrasted, and analyzed all the data sets, the list of categories were further condensed into the current study's main themes (Creswell, 2008). The following subsections describe in detail the analysis and interpretation of data.

Quantitative Data Analysis

The pre- and post-quantitative data results were summarized through descriptive statistics. The authors of the RMAS (Betz, 1978) and MTEBI (Enochs et al., 2000) had already established reliability and validity factors for these instruments as described previously in this chapter. On the pre- and post-content tests, the five open response questions were scored independently by the researcher and her colleague. The scoring rubric was used to determine the level for each question (see Appendix A.4). Inter-rater reliability was calculated in each of the open-ended questions for the pre- and post-content test, and ranged from 86% to 94% (see Table 3.1 earlier in this chapter in 'Validity and Reliability' section). The assessments were further reviewed together by the researcher and her colleague and reassessed until 100% agreement was attained for each open response question. Student teachers' responses that were both procedurally correct and conceptually accurate were scored level four because student teachers demonstrated a thorough understanding of concepts and procedures. When answers included either an accurate procedure or conceptually correct response, they were scored at a level three due to the demonstration of either procedural or conceptual understanding. Student teachers who made an attempt at a solution process, either procedurally and/or conceptually, with some errors and/or omissions were given a level two. Answers with misunderstood concepts or incorrect use of procedures were scored at level one. Finally, irrelevant or blank responses were given a score of zero. The 10 multiple choice items were scored either correct or incorrect. A total possible score out of 30 was calculated for each content assessment (20 for open-ended items and 10 for multiple choice items). The individual open ended items and the multiple choice questions were compared and descriptive statistics were calculated to look for changes and patterns across the year.

The researcher compared the pre- and post-results of all three instruments (RMAS, MTEBI, and content test). Separate results on the two subscales of the MTEBI and the instrument as a whole were analyzed with descriptive statistics. The researcher calculated the mean and standard deviation for each test result to determine if there were any significant changes in student teachers' mathematics anxiety, mathematics teacher efficacy, and CCK. Pearson product-moment correlation coefficient was calculated to determine existing correlations between the measured constructs. The results of the quantitative data were integrated with student teachers' interview and journal data results for further analysis of emergent themes.

Qualitative Data Analysis: Student Teachers' Perspective

In order to understand what course experiences the student teachers felt had the most impact on their mathematics teacher development, student teachers' journals and student teachers' interview transcripts were analyzed separately using the process of content analysis. Content analysis is defined as "a research method for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns" (Hsieh & Shannon, 2005, p. 1278). A total of 192 student teachers' journals and 12 interview transcripts were reviewed in separate data sets, as the researcher considered student teachers' perspectives on the mathematics course experience. NVivo 9 data analysis software was utilized in this study for the coding process, organization, storage, and display of data. The journals were submitted to the researcher in Microsoft Word or PDF files via the course's electronic drop box. Interviews were transcribed in Microsoft Word. All the journals and interview transcripts were then imported into NVivo 9. Considering the large volume of qualitative data (192 journals, each journal contained about 750 words, and 12 one-hour interview transcripts), NVivo 9 allowed for the coding process of these data sets to be efficient and easily manageable.

This study's theoretical framework (see Figure 1.1 in Chapter One) provided specific parameters for the researcher when she considered various codes in the qualitative data (Creswell, 2009; Denzin & Lincoln, 2000). During the initial phases of analysis, the researcher read through several journals from semester one and two, as well as pre- and post-interview transcripts, and manually coded items that were aligned to theoretical perspectives embedded in this research study. For example, the self-efficacy theory and research about mathematics anxiety involves the concept of previous negative mathematics experiences. This resulted in 8 codes such as “negative experiences in K-12 schooling.” The researcher continued to code the data sets until she reached a point of saturation, in which no new codes were found. Saturation occurred after nine journals and four interviews. An initial list of about 58 codes was identified for the journals, and 27 codes for the interview data. At this point, the researcher’s colleague independently reviewed four student teachers’ journals and two interview transcripts and arrived with an initial set of codes. An initial inter-rater reliability agreement of 88% was obtained. For more details of the inter-rater reliability process, refer to the ‘Validity and Reliability’ section earlier in this chapter.

Once the inter-rater reliability was established, the researcher revisited the initial coding list, integrated both journal and interview data sets, collapsed many of the codes and categorized them into categories. The use of categories enabled the researcher to cluster the content of codes based on common characteristics. This study’s categories were established based on the common experiences of the student teachers. These shared experiences found in the journal data and interview transcripts strengthened the identification of the patterns across the codes (Krippendorff, 1980). The finalized coding was then applied to all the journals and interviews using Nvivo 9’s advanced coding methods. At this point, the researcher examined each of the categories, further examined the results of the quantitative data, and compared all data to further examine how each data set substantiated one another through triangulation. Once the comparison was completed, convergence was determined, and broad themes were identified.

Qualitative Data Analysis: Academic Perspective

The researcher-instructor’s journal entries were analyzed to capture the academic perspective in the development of student teachers’ mathematics common and specialized content knowledge, teacher efficacy, and pedagogy. The researcher-instructor taught 12 three-hour classes to each cohort. There were about 13 journal entries which were created in Microsoft Word software. This journal data was imported into NVivo 9, and analyzed as a separate data set because it reflected the academic perspective. Once again, the process of content analysis was applied to the coding of the journal data as described previously. As the researcher-instructor analyzed specific statements, words, and/or phrases for all possible meanings, the theory and research on mathematical knowledge for teaching, adult learning, self-efficacy, and constructivism guided the coding process. Once the initial set of codes was developed, the researcher applied triangulation strategies by crosschecking the codes with student teachers' journal and interview data, as well as the quantitative data to ensure corroboration. A set of categories were then developed which were further condensed to main themes (Creswell, 2008).

The researcher-instructor acknowledged bias as an issue of self-reporting and participant observer. Therefore, by seeking feedback from her mathematics instructor colleague and student teachers, the researcher attempted to ensure an accurate and complete record of events was presented. The researcher shared her journal entries with her teaching colleague and discussed goals and observations found throughout the reflections. In addition, the researcher clarified any incidents with her student teachers in regards to significant events that occurred during class.

Interpretation

As described earlier in this chapter, this study used a triangulation design model which collected quantitative and qualitative data in order to best understand course experiences that influenced preservice teachers' mathematics teaching development. Throughout the interpretation phase of this study, the researcher merged all the quantitative and qualitative data sets, by bringing the separate results together into an overall interpretation. Creswell (2007) describes this as the convergence model in which the researcher "collects and analyzes quantitative and qualitative data separately on the same phenomenon and then the different results are converged (by comparing and contrasting the different results) during the interpretation" (p. 64). The convergence model allowed for comparison of results which further validated, confirmed, and substantiated the quantitative results with qualitative findings. Furthermore, the coded qualitative data was also quantified to strengthen the overall interpretation of this study. The interpretation of the qualitative data corroborated the quantitative results in a multitude of ways. For example, the quantitative data revealed that student teachers' content knowledge improved significantly over the academic year; while the qualitative journal data reflected on how problem solving activities impacted on student teachers' mathematics understanding. Therefore, through the interpretation of the qualitative and quantitative data sets, well-substantiated conclusions about student teachers' mathematics development in content knowledge, teacher efficacy, and pedagogy were better understood.

Ethical Considerations

Throughout all phases of this research study, ethical considerations were brought to the forefront. In this section, ethical considerations are addressed in regards to student teachers' involvement in the study and with each data collection method. The following subsections outline the measures taken to ensure student teachers' safety at each juncture.

Ethical Considerations with Student Teacher Participants

During the first mathematics class of the academic year, this proposed research was discussed with both cohorts. The goals of the study and the potential for more effective learning were outlined in a short presentation by a third party individual, and in written form. Written consent was obtained from interested student teachers prior to commencing the study. The research maintained the confidentiality of all participants, through aggregation of data. Also, the anonymity of individuals involved in the research was protected, in that no real names were published in the final or subsequent documents. Student teachers had the opportunity to withdraw from participating at any time during the year, without penalty. This was stated in the information letter and the consent form. If participants choose to withdraw; only the data prior to the withdrawal would have been included in the project. In this current study, no student teachers withdrew their participation.

This study involved in part working with preservice teachers who had high mathematics anxiety, and therefore, may have felt vulnerable during the research process. By contrast however, there were potential advantages to this subgroup because student teachers with high anxiety were the most likely to benefit from this study due to increased opportunities they had to openly discuss and address their issues. The literature demonstrates how preservice teachers need time to talk and reflect upon their anxiety to support heightened self-awareness of negative mathematics experiences. This in turn will reduce mathematics anxiety, and increase mathematics teacher efficacy (Furner & Duffy, 2002; Swars, Daane, & Giesen, 2006). Hence, participation for these more vulnerable individuals had actually provided them with effective and supportive learning experiences as a result of their involvement in this research.

Ethical Considerations with Culminating Reflective Journals

A regular component of the coursework was for all student teachers to complete a reflective journal as part of their course assignments. Permission to analyze journals as part of the research data was obtained from almost all student teachers, except for three. The researcher-instructor was not aware of who granted permission until after the submission of final grades. When written permission was not obtained by those three student teachers, their journals were not part of the data set. The researcher collected culminating journals in December and April to consider emerging themes about preservice mathematics course experiences.

Research demonstrates that journal writing is an effective practice that enables teachers to be immersed in inquiry, self-examination, and further promotes deep reflection of learning experiences (Boline, 1998; Han, 1995; Hoover, 1994). Specifically, for student teachers who are weak in mathematics content knowledge, have low mathematics teacher efficacy, or feel high mathematics anxiety, the reflective process of journal writing can reduce mathematics anxiety and build confidence as mathematics learners (Furner & Duffy, 2002). Therefore, the potential for this data collection had favourable outcomes to student teachers' development as mathematics teachers.

Ethical Considerations with Individual Interviews

A total of six preservice teachers, who possessed either high or low mathematics anxiety levels, were invited to participate in two one-hour semi-structured interviews. In order to ensure that there were no detrimental implications for student teacher interviewees and their grades as a result of their participation; interviews were not transcribed or analyzed until after final grades were submitted in late April. This approach and its rationale were clearly stated in the consent form.

The author was fully aware of the power imbalance between herself as instructor and her student teachers. Therefore, in an attempt to ensure that participants felt safe to dialogue openly and honestly without fear of retribution by the instructor, a third party interviewer who was external to both classes invited the student teachers to participate and conducted all student teacher interviews. The researcher trained the external interviewer on how to conduct the interviews to ensure consistency in questioning and prompting.

Ethical Considerations with Instrumentation Data

At the beginning and at the end of the academic year, three quantitative instruments were administered to preservice mathematics teachers. These were the RMAS, MTEBI, and the mathematics content assessment. The use of these instruments were a usual component of the mathematics course and the results helped the instructor determine student teachers' levels of mathematics anxiety, mathematics teacher efficacy, and CCK, and whether or not any changes in these levels occurred over the course of the year. Permission to analyze instrumentation results as part of the research data was obtained. The research-instructor was unaware of who granted permission to use this data until after the submission of final grades. If written permission was not obtained, then those results were not used in the data analysis. In this current study, permission to use quantitative instrumentation data was granted by all 99 student teachers. However, two student teachers from class A were absent during the administration of the post-surveys and post-content assessment and therefore they were unable to submit post-data results.

These various instruments were important diagnostic tools because the results informed the differentiated instruction of preservice teachers. The researcher-instructor provided more support to those preservice teachers who were in need. This was a usual aspect of the course. Examples of additional support structures included: small group tutoring to further develop mathematics content knowledge; mixed ability groupings for collaborative projects; offering course content material to review ahead of time prior to class; and providing more time and assistance during problem solving activities to alleviate anxiety and promote learning.

Delimitations

Delimitations define the boundaries of the study and address how a study may be narrowed and confined (Leedy & Ormrod, 2005). Delimitations are often imposed by the researcher prior to the study and refer to what the researcher does *not* do in order to make the study's workload manageable. It is important to consider delimitations in order to understand the boundaries of the research. Typically, delimitations involve the sampling size and the unit of analysis that is the primary focus of the study. Preservice mathematics education was the unit of analysis in this study with elementary student teachers as the primary focus. This researcher made exclusionary and inclusionary decisions regarding the sample of participants. This research study was delimited to student teachers. Specifically, participation in this study was delimited to individuals who (a) held an undergraduate degree (b) enrolled in a one-year Bachelor of Education elementary program in a large urban university located in southern Ontario and (c) were students in the elementary mathematics course taught by the researcher herself. Nevertheless, the profile of this study's participants is transferable to other student teachers due to consistent university program entry prerequisites across the nation. As a result, this study's findings will be "useful to others in similar situations" and therefore, generalizations may still be warranted across mathematics preservice classrooms (Marshall & Rossman, 2006, p. 201).

The researcher's role as participant-observer could have been a further delimitation in this study. The researcher-instructor's reflective journal provided this study's data on the academic perspective. This may be considered narrowed as it only entailed one instructor. However, the researcher's course work and class activities were similar to other instructors' who taught elementary preservice mathematics at the same university due to co-planning and collaborative teaching.

Limitations

According to Creswell (2003), limitations refer to the potential weaknesses of the study in which the matters are not within the researcher's control. In other words, limitations are conditions that confine the scope of the study and cannot be controlled by the researcher. In most instances, the researcher's assumptions about the study are often considered limitations.

A limitation in this study was based on the nature of the qualitative data collection. These included self-reporting methods through reflective journal and individual interviews. The researcher held the assumption that responses to interview questions and reflections written in journals were accurate representations of student teachers' beliefs, values, and actual course experiences in their mathematics teaching development. She felt that this limitation was not a concern because of implemented strategies that ensured student teachers felt safe to participate honestly. This included an external third party individual who presented the research study and conducted the interviews.

Another potential limitation to this study involved the quantitative instrumentation tools utilized. Three separate instruments were used to ascertain the levels of mathematics 1) teacher efficacy, 2) anxiety, and 3) CCK of preservice teachers. These measurements were based on the assumption that student teachers responded to the mathematics instrumentation tools in a thoughtful and serious manner. The researcher believed that this was not an issue because student teachers continually expressed their desire and keen interest to improve their mathematics development and reflected on their instrumentation results with care.

Lastly, the duration of this study may have posed as a limitation in this research. The program under study was only one academic year or approximately eight months (from September to April). The literature indicates the need for longitudinal studies in this area; however a longitudinal timeline was beyond the scope of this study.

Chapter Summary

This chapter provided an overview of mixed methods methodology as part of the pragmatic paradigm. The researcher selected a mixed methods approach for this study because both quantitative and qualitative data were necessary to obtain a comprehensive understanding of preservice teacher development in mathematics common and specialized content knowledge, teacher efficacy, and pedagogy. Quantitative and qualitative data were collected throughout the study and given equal weight. Various data collection tools were used to uncover the study's research questions. These included document review of journals, individual interviews, and quantifiable measures of mathematics scales and mathematics assessments. Participants in this study were selected based on their enrollment in the researcher's mathematics class. Triangulation of the qualitative and quantitative data was attained, as the researcher analyzed each data set separately and the data were then converged for comparison and overall interpretation. In regards to ethical considerations in this study, the researcher was fully aware of the power imbalance between herself as instructor and her student teachers. Therefore, actions were taken to ensure that participants felt safe to participate honestly in the study without fear of retribution by the instructor. These actions included an external third party individual who presented the study's overview, collected consent forms, invited student teachers for interviewing, as well as conducted the interviews. In addition, the researcher did not analyze student teachers' interview transcripts and journal data for her study until after the submission of final grades. Finally, delimitations and limitations of this study were outlined to describe the extent to which generalizations can be made for preservice mathematics teachers in elementary programs.

Chapter 4: RESULTS

Introduction

The purpose of this study was to examine how the teaching and learning experiences in two preservice education mathematics courses impacted the professional growth of student teachers; and specifically to investigate the relationship among preservice teachers' mathematics development of: 1) content knowledge, 2) teacher efficacy, and 3) pedagogical knowledge. In this study, the term content knowledge refers to both common and specialized content knowledge (CCK and SCK). The three constructs of teacher development are imperative in effective implementation of mathematics reform. It may be expected that student teachers who possess low content knowledge may feel low efficacy about their teaching skills, and therefore their pedagogical knowledge may be limited. Although these presumptions seem logical, further research is required to examine the tri-component notion of mathematics teacher preparation. Therefore, in this study, student teachers' mathematical constructs (content knowledge, efficacy, and pedagogy) were studied simultaneously in their preservice mathematics course. Furthermore, this study aimed to examine the mathematics development of prospective teachers from both the academic and student teachers' perspective in order to reveal a more complete understanding of the teaching of mathematics to preservice teachers as well as their learning process within this course. For this reason, data sources from both the students' and academic perspectives were included.

Qualitative and Quantitative Data Presentation

Data analysis for this study included various quantitative instrumentation results, written journals, and interviews. This chapter reports the results of the following quantitative instruments: pre- and post-mathematics content assessments (Appendices A.9 and A.10), the Revised Mathematics Anxiety Scale (RMAS) (Appendix A.5), and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Appendix A.7). The MTEBI measures mathematics efficacy beliefs in preservice teachers which includes two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE). PMTE represents a teacher's belief in his or her skills and abilities to be an effective teacher. The second subscale, MTOE corresponds to a teacher's beliefs that effective teaching can impact on student learning regardless of external factors. The instruments used in the study were analyzed to determine any changes that occurred in student teachers' CCK, mathematics anxiety, and efficacy levels, as well as to measure the relationships between and among mathematics anxiety, CCK, and mathematics teacher efficacy dimensions.

The qualitative findings of journals written by student teachers and the researcher-instructor, as well as student teacher interviews are outlined in this chapter. As part of the compulsory coursework, student teachers submitted two journals, one at the end of semester one, and the second at the end of the academic year. A total of 192 journals were submitted - 96 journals in semester one, and 96 journals in semester two. Interviews were also conducted with six student teachers, based on RMAS results on the first day of mathematics class. Interview participants were comprised of three student teachers with low anxiety (scoring more than 45), and three student teachers with high anxiety (scoring less than 15). There were 12 interviews conducted in total, six pre- and six post-interviews.

The researcher utilized the process of content analysis and applied coding techniques while examining the qualitative data. The researcher used NVivo 9 qualitative software analysis to code the written journals and interview transcripts accordingly to qualitative research protocol. Coding is a process in which labels, or names, are given to chunks of information. Open coding was used initially, that is; data were broken down by comparing similar incidents which were grouped together and given the same category.

Emergent Themes

In this chapter the results are reported by way of five major themes and the corresponding sub-themes. The five broad themes that emerged from the data were: 1) importance of the instructor's role in mathematics teacher development 2) problem solving to support conceptual understanding; 3) building confidence as a mathematics teacher; 4) working towards constructivist pedagogy; and 5) classroom management (see Figure 4.1). Furthermore this chapter concludes with a subsection that presents a summary of the key findings.

Within each theme, the researcher analyzed the codes derived from the qualitative data and the quantitative data results, and further identified sub-themes to provide detailed descriptions of student teachers' and the researcher-instructor's perspectives when considering how the three mathematics constructs (i.e., content knowledge, efficacy, and pedagogy) were influenced during the preservice mathematics course. Quotes from written journals and interview transcriptions were cited in this chapter when these illustrated the essence of the themes and sub-themes.

The first theme, the importance of the instructor's role in mathematics teacher development, captured the researcher-instructor's perspective, specifically how academics can support student teachers in their mathematics teaching development. The main data source for this theme came from the researcher's journal entries. The researcher-instructor kept a journal that focused on her reflections about teaching the two mathematics courses. Her journals were ongoing throughout the period of teaching and enabled her to track observations, perceptions, and other relevant data related to the development of the three mathematics constructs (i.e., content knowledge, efficacy, and pedagogy). Approximately 13 journal entries were completed throughout the academic year by the researcher-instructor.

The remaining four themes pertain to student teachers' perspectives. The order of these themes was largely determined by the frequency of the codes found in the journals and interview transcripts. The researcher also took into account the measurements of the pre- and post-results of the quantitative data. For example, the greatest quantitative gains that occurred over the academic year were in the mathematics content test. This specific data was considered when reviewing the codes related to the theme of problem solving, as this was directly connected to the results of mathematics content knowledge. The second greatest gains occurred in the PMTE scale. The PMTE is a subscale of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), and it specifically measures preservice teachers' efficacious beliefs in their abilities to be an effective teacher. Therefore the PMTE quantitative results were reflected upon when examining the codes under the theme of confidence, since efficacy and confidence are closely connected. In addition, mathematics anxiety results were examined upon consideration of all the codes. Table 4.1 displays the frequency of codes found under each sub-theme in the journals and interview transcripts. Based on the frequency of use for each theme and sub-theme, the researcher determined the ranking of each sub-theme, which thereby delineated the priority and order of the themes and sub-themes.

Figure 4.1: Course Experiences Contributing to Student Teachers' Mathematics Content Knowledge, Teacher Efficacy, and Pedagogy: Emergent Themes and Sub-themes from Study Data

Problem Solving to Support Conceptual Understanding

Sub-themes:

- Developing various strategies for problem solving;
- Solving problems independently; and
- Moving away from algorithm dependency.

Building Confidence as a Mathematics Teacher

Sub-themes:

- Teaching and assessing content; and
- Building capacity efforts.

Importance of the Instructor's Role in Mathematics Teacher Development

Sub-themes:

- Establishing a safe learning environment;
- Gradually increasing difficulty; and
- Modelling effective strategies and enacting them in field experiences.

Working towards Constructivist Pedagogy

Sub-themes:

- Reflecting on preconceived notions;
- Becoming a constructivist teacher; and
- Understanding constructivist teaching.

Classroom Management

Sub-themes:

- Establishing classroom management systems; and
- Taking risks in a transmissive mathematics classroom placement

Figure 4.1. Four broad themes emerged from the qualitative and quantitative study data. The themes were determined through content analysis of written journals, interviews, and quantitative measurements of participants' mathematics content knowledge, mathematics anxiety, and mathematics teacher efficacy. Content knowledge refers to both common and specialized content knowledge. The 'Importance of the Instructor's Role in Mathematics Teacher Development' theme is shaded because this theme was based on different data to that of the student teachers. The instructor's ongoing journal provided the main data source which captured how academics can support student teachers. Sub-themes provide detailed descriptions of student teachers' and instructor's perspectives when considering how the three constructs were influenced during the preservice mathematics course.

Table 4.1:
Coding Based on Primary and Sub-Themes and Data Source: Frequency and Ranking

		Frequency of Codes			
Theme	Sub-Theme	Journals	Interviews	Total	Overall Ranking
Problem Solving to Support Conceptual Understanding	Developing various strategies for problem solving	192	11	203	1
	Solving problems independently	169	9	178	2
	Moving away from algorithm dependency	167	11	178	2
Building Confidence as a Mathematics Teacher	Teaching and assessing content	141	9	150	4
	Building capacity efforts	138	11	149	5
Working Towards Constructivist Pedagogy	Reflecting on preconceived notions	160	9	169	3
	Becoming a constructivist teacher	113	10	123	6
	Understanding constructivist teaching	52	7	59	7
Classroom Management	Establishing classroom management systems for a constructivist learning environment	139	11	150	4
	Taking risks in a transmissive mathematics classroom placement	27	2	29	8

Note. This table displays the frequency of codes found in each theme and sub-theme across participants' journals and interview transcripts. Based on the frequency of use for each theme and sub-theme, the researcher determined the ranking of each sub-theme, which thereby delineated the priority and order of the themes and sub-themes.
Journals – N = 192. Interviews – N = 12.

Importance of the Instructor's Role in Mathematics Teacher Development

The purpose of this theme was to examine course experiences from the academic instructor's perspective. The data source for this theme was mainly the instructor's journals and observations made during class and practicum supervision. While the qualitative analysis of the instructor's journals cannot claim a causal relationship, or a correlational relationship; the researcher as the instructor considered she was most knowledgeable on what happened during classes that may have helped understanding, and therefore was in the best position to theorize on what teaching experiences influenced preservice teachers' mathematics development. As discussed previously in chapter three, the researcher acknowledged the issue of bias in self-reporting and participant as observer. Therefore, the researcher sought feedback from her mathematics instructor colleague and student teachers, as well as crosschecked the journal data source with student teachers' data results to interpret the meaning of the context and experiences.

The researcher-instructor had three major goals for the mathematics course she taught. The first of these goals was for her student teachers to make sense of mathematics. Throughout the entire year, the course emphasized multiple ways of making sense of concepts, not just one way. The second objective involved the beliefs and attitudes about mathematics. Preservice teachers often entered the program not only afraid of mathematics, but feeling anxious about how their attitudes toward mathematics could negatively influence the children they teach. Therefore, the instructor aimed to present mathematics as meaningful, engaging, and something to look forward to. This largely involved increasing her student teachers' confidence to learn and teach mathematics. The instructor's third objective was to improve the pedagogical skills of her student teachers. She devoted much time to modelling effective techniques that exemplified constructivist approaches. This goal was best attained when preservice teachers linked course experiences to their field placements. After analysis of the instructor's journal entries using Nvivo 9 qualitative software, three sub-themes emerged: establishing a safe learning environment; gradually increasing difficulty; and modelling effective practices and enacting them in field placements. The following sections describe the three sub-themes from the perspective of the instructor, thereby focusing on the teaching experiences of the mathematics course.

Establishing a Safe Learning Environment

As part of developing a psychologically safe learning environment, the instructor wanted to ensure that her student teachers felt supported in their mathematics problem solving, and that they wouldn't feel self-conscious or humiliated if they had questions, confusions, or misconceptions about the mathematics content. Therefore, the instructor introduced a set of norms for the mathematics class at the beginning of the year. These included: mutual respect, attentive listening, appreciation, and the right to pass. Additionally these norms were practiced regularly throughout the entire program in other subject areas.

It did not take long for the instructor to have to revise the norms to be more explicit, as they were too broad and required specific direction, especially in a mathematics class where anxiety and fear of content knowledge easily manifested. During the first class, the instructor invited participants to solve the problem, "317 ÷3." Not surprisingly, almost all of the participants used the long division algorithm to solve the question. However, there were several student teachers who forgot the procedures of the division algorithm and they became devastated by it. "*One of the students approached me after class and told me that they never felt so embarrassed by the fact that they couldn't complete a simple division question*." As the conversation continued with the student teacher, the instructor realized that part of the student teacher's anxiety stemmed from what was said by her classmates. "*I realized that her anxiety heightened immediately when her table members were saying things out loud, like 'I'm done,' 'To what decimal point should we solve the answer?' and 'All finished.*'" Based on this incident, the instructor endeavoured to nurture a supportive mathematics class, where the community of learners respected all learning styles and entry points. At that critical moment in the instructor's reflection, she decided to devote the next class on mathematics anxiety. She had student teachers think about why individuals would be hesitant to take risks in showing their mathematical thinking and had the class develop more specified norms to ensure a safe learning environment. "*Merely giving out a set of norms without* *the input of the student teachers did not give them ownership of the culture we were trying to cultivate."*

The new and revised norms for the mathematics class reflected an empathetic and caring environment. It not only supported the needs of all learners, but it recognized that class members were at varying stages of mathematics teacher development. The instructor had participants discuss expectations for behaviours before, during, and after problem solving activities. As a result, the newly developed norms were:

- during problem solving activities, be cognizant about how your comments might make others feel, for example, refrain from comments that boast about easiness, such as "that was easy" or "pretty simple," and refrain from comments that boast about timing, "I'm all done" or "that was quick to solve";
- everyone be accountable for a mathematical opinion;
- listen to and try to make sense of one another's mathematics reasoning; and
- always feel safe to ask questions.

It did not take long for the new norms to infiltrate throughout the mathematics classes. Student teachers who had high anxiety and low content knowledge were most appreciative of the newly developed norms.

> *I had two emails from student teachers saying how relieved they were with mathematics class today. They felt secure as they explained their mathematical thinking to others. Their vulnerability during math class was diminished due to a mutual respect for all learners. These student teachers used to conceal their insecurities about mathematics through behaviours of avoidance and silence. They now feel safe to demonstrate their mathematical thinking and they are building more self-efficacy as they do this.*

Therefore, a caring learning environment especially supported student teachers with low content knowledge and high anxiety. This result is further substantiated in the quantitative data discussed later in this chapter. As these student teachers continually worked on mathematical problems, they gained more content knowledge over the course, and thereby increased their efficacy levels and decreased their anxiety.

__Gradually Increasing Difficulty__

A major goal for the instructor was for her student teachers to make sense of mathematics. This goal was not simply achieved by giving students a string of mathematics problems to solve. It required careful strategic planning on the part of the instructor.

> *I couldn't just give them a litany of geometry problems and say 'OK go and solve it.' I needed to 'warm up' the class, get them hooked into a rich problem, stimulate background knowledge, and set the stage for accountable talk for each table group.*

Of most importance was the warm-up activity. This activity was presented prior to the main problem and involved activating background knowledge of key mathematics concepts. It was usually a less difficult problem that student teachers were able to solve. The instructor recognized that when her student teachers were successful with the warm-up problem, they gained immediate self-efficacy to continue with a more difficult and complex problem.

> *I noticed today in class how the first warm up activity influenced student teachers' confidence and attitudes for the remainder of the class. I witnessed a couple of my student teachers feeling so relieved that they were able to solve the geometric puzzle. They were actually surprised that they were successful, and were eager to move into the meat of the lesson.*

Because the instructor was highly sensitive to the needs of student teachers who were reluctant and hesitant to engage in mathematics activities, she ensured that warm-up activities were implemented for each class. This pre-activity allowed for a gradual increase in difficulty level of mathematics problems. By starting the lesson with an activity in which student teachers felt successful, it gave them confidence to move on to the more challenging mathematics content. Therefore, a gradual increase in difficulty level was an important strategy in supporting students' efficacy levels, content knowledge development, and anxiety relief. This coincided with the quantitative improvements of CCK, personal teaching efficacy, and decrease in anxiety. Not only did these constructs progress over the long-term course, but there also appeared to be short-term gains because the gradual increase in difficulty level afforded immediate gains in efficacy and content, and anxiety relief during each mathematics class. In addition, this strategic sequence supported student teachers' development in pedagogical knowledge. With warm-up activities, participants recognized the importance of a three-part lesson and were convinced that all three parts (i.e., warm-up, rich problem, reflection) were necessary in teaching mathematics effectively.

Modelling Effective Strategies and Enacting Them in Field Placements

As part of the instructor's goal to improve the pedagogical knowledge of her student teachers, it was important to model effective practices. This enabled student teachers to experience a different way of learning mathematics. The instructor would often focus on a specific strategy in each class, such as questioning, use of manipulatives, or three-part lessons. At the end of the class, the instructor made explicit what technique was being modelled and discussed how the technique could be modified for other grade levels or mathematics strands. It was most beneficial when these strategies were implemented in student teachers' field placements, so they would have opportunities to enact such constructivist principles.

> *I had preservice teachers who were in practicum where the mentor teacher was actually teaching through problem solving. This was the best case scenario. I had one student teacher in second practicum, who said to me, 'OK, Mary, I'll be honest with you. I thought what you were saying didn't really happen in the real world until I saw it in this class.' Then, there were other placements, where the mentor teacher was unfamiliar with constructivist strategies, such as bansho or congress; but somehow the student teacher and mentor teacher were able to negotiate the implementation of open-ended math lessons. This scenario also proved to be positive because the learning was reciprocal, in that both the mentor and student teacher worked together to improve instruction.*

However, for some preservice teachers, particularly those who had high anxiety and low content knowledge, the modelling of effective strategies did not transfer into their practice teaching if they were placed in a transimissive mathematics classroom. Prior to practicum, the instructor had several student teachers confidentially ask her for advice about their conflicting placement. At this point, the instructor recognized participants' desire to become constructivist teachers, but were unable to realize this goal in transmissive field placements. The instructor could not give a clear cut answer to her preservice teachers because the risk involved in implementing constructivist methods in a transmissive classroom depended on a multitude of variables such as, classroom management routines, needs of the learners, prior knowledge, parental community, mentor teacher, etc.

> *Unfortunately there were many practicum classrooms where mentor teachers did not allow for their student teachers to link any course experiences to practicum. Essentially, these classrooms relied solely on the textbook. These student teachers suffered the most as they had limited opportunity to enact forms of constructivist teaching. If they did, they were taking a major risk in classroom management issues and/or a poor evaluation.*

Even with the modelling of effective strategies, it was challenging for student teachers to link this to their practice teaching if their placements exemplified transmissive modes. It appeared that many student teachers taught in the same manner as their mentor teachers, regardless of whether they agreed with their philosophies.

> *Today I observed a math lesson on the multiplication of decimals. I was forewarned via email by my student teacher* [G]. *G informed me that he was following his mentor teacher's mathematics program, and I probably wouldn't be pleased with the lesson. He was right, I wasn't pleased. I saw nothing but rote learning. G had students take up the math homework, and then they moved on to the next page. The students followed the steps in multiplying decimal numbers, and at the end of the procedure, they had to count over the number of decimal points. When one student asked, why does this work. G responded 'because that's the way this algorithm works.' G had no lesson plan, just page numbers from the text book. I was not impressed. It seemed like nothing came to fruition from my mathematics classes.*

This left the instructor wondering about the long-term repercussions for student teachers who only experienced transmissive models of teaching mathematics during practicum. The instructor hoped that constructivist pedagogy development was only stalled temporarily until student teachers could embrace constructivism in their own classrooms. However, the instructor also wondered if this experience of teaching in a transmissive classroom could very well imprint onto student teachers' practices.

> *If my student teachers never get the chance to observe first hand or try out any form of constructivist teaching, then how will their pedagogical skills develop? Will the transmissive ways of teaching bond with their teaching styles – as this is the only format they've been encouraged to enact in practicum? My only hope is that their beliefs and values will prevail, and their moral compass will direct them towards effective pedagogical practices.*

Problem Solving to Support Conceptual Understanding

Based on student teachers' perspectives of the mathematics course, 'problem solving to support conceptual understanding' emerged as the most dominant theme, when analysis of student teachers' written journals, interviews, content assessments, and mathematics anxiety scales were conducted. In order to examine what changes occurred in student teachers' problem solving skills, the researcher initially analyzed the pre- and post-test results for the grade six assessments. The quantitative data from the pre- and post-mathematics test comprised of 15 problem solving items. There were five open-ended and 10 multiple-choice questions selected from Ontario's Education Quality Assessment Office (EQAO) grade six assessments (see appendices A.9 and A.10). The 10 multiple choice items were scored as correct or incorrect, giving a maximum score of 10. The five open-ended items required the researcher to use EQAO's scoring rubric. The open ended questions were assessed based on a four-level rubric (see Appendix A.4), with level three and above as meeting the provincial standard. Responses that were blank or irrelevant (e.g., "*I don't know how to do this*") were scored as zero. Therefore the highest level that was given to an open-ended question was four, and the lowest level was zero, with the highest total score attainable of 20.

Tables 4.2 and 4.3 reveal student teachers' significant improvements in the open-ended problems from pre- to post-test, with greatest gains in the categories of patterning and algebra, and measurement. In the pre-test, more than half (54%) of the student teachers were unable to solve the measurement problem and almost half (48%) did not solve the patterning and algebra problem. Several preservice teachers responded to these questions indicating they did not know how to solve the question. For example, participants wrote on their test paper . . . "*I don't remember what formula to use,*" "*I don't know where to begin with this question,"* and *"I am so confused on how to solve this problem*." The multiple choice items proved to be less challenging as results were generally higher than the open-ended items (see Table 4.4).

Table 4.2:

Percentage of Student Teachers at All Levels for Open Ended Items of Content Assessment

Results in %		Number Sense & Numeration		Measurement		Patterning & Algebra		Data Management & Probability		Geometry & Spatial Sense	
Pre N=99	Post N=97	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Level 4		52	80	5	52	5	84	10	98	15	69
Level 3		23	16	7	16	15	4	55	2	18	9
Level 2		12	4	14	29	13	9	6	0	20	6
Level 1		7	0	20	0	19	3	20	0	19	6
Blank or irrelevant content		6	0	54	3	48	0	8	0	28	10
At or Above Ontario Provincial Standard (Levels 3 & 4)		75	96	12	68	20	88	65	100	33	78

Note. Results are reported in percentage. Each of the five open-ended questions was scored on a four level rubric, with level three and four as meeting provincial standard. Greatest gains occurred in patterning and algebra, and measurement. Provincial standard scores (levels three and four) increased by 68% for patterning and algebra, and 56% for measurement.

Table 4.3:
Content Test Open Ended Items Based on Four Level Scoring Rubric

		Number Sense & Numeration		Measurement		Patterning & Algebra		Data Management & Probability		Geometry & Spatial Sense	
Pre N=99	Post N=97	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Mean		3.08	3.75	0.89	3.14	1.1	3.70	2.38	3.99	1.73	3.22
Standard Deviation		1.21	0.53	1.19	1.03	1.29	0.75	1.15	0.12	1.43	1.37

Note. Each of the five open-ended questions was scored on a four level rubric, with level three and four as meeting provincial standard. The lowest mean of 0.89 occurred in the pre-test measurement question and the highest mean of 3.99 occurred in the post-test data management and probability question. Greatest mean gains were in patterning and algebra, and measurement, with a mean increase of 2.6 and 2.25, respectively.

Overall, Table 4.4 illustrates that student teachers substantively increased their CCK scores over the course of the academic year, signifying improved problem solving skills. Total mean scores increased by 37.14 percent. When comparing the standard deviations between the pre- and post-test results, the pre-test reveals a larger variance from the mean in both the multiple choice and open-ended items. This larger standard deviation indicates that there were relatively more preservice teachers scoring toward one extreme or the other, suggesting that student teachers either solved the problem or did not, with limited scores in the mid-range. This was not the case in the post-test results, as the standard deviation decreased significantly, indicating that participants scored more consistently in each of the problem solving items.

Table 4.4:
Content Assessment Total: Multiple Choice (out of 10) and Open Ended Items (out of 20)

Pre N=99	Mean		Standard Deviation	
Post N=97	Pre	Post	Pre	Post
Multiple Choice	7.01	9.53	2.08	0.72
Open Ended	9.18	17.8	6.27	3.8
Total	16.19 (53.96%)	27.33 (91.1%)	8.35	4.52

Note. Student teachers increased their common content knowledge scores over the course of the academic year, signifying improved problem solving skills. Total mean scores increased by 37.14 percent. Multiple choice items were out of 10, and the open ended questions were out of 20, with a total attainable score of 30.

Student teachers who possessed higher levels of mathematics anxiety performed less favourably in their problem solving abilities. This was manifested in the comparison analysis of the Revised Mathematics Anxiety Scale (RMAS) and content assessment results. Descriptive data for RMAS are provided (Table 4.5). The highest possible score on the RMAS is 50, which indicates the least amount of anxiety. The lowest possible RMAS score is 10, which signifies the highest level of anxiety. A mean of 28.76 in semester one, then 33.8 in semester two provides a mean difference of 5.04 or 10.1 percent. This difference suggests that over the course of the academic year student teachers experienced a moderate decline in their mathematics anxiety. RMAS was administered at the beginning of class and the content assessment was completed near the end of the same session.

Table 4.5:
Mathematics Anxiety Results Using the RMAS Instrument

	Mean	Standard Deviation
Pre N=99	28.76	10.83
Post N=97	33.8	9.49

Note. The highest score for the RMAS is 50 indicating low anxiety, while the lowest score is 10 indicating high anxiety. A mean increase of 5.04 suggests a moderate decrease in mathematics anxiety over the course.

The RMAS and the mathematics content assessments results were analyzed to determine the relationship between student teachers' mathematics anxiety and CCK by calculating the Pearson product-moment correlation coefficient (Table 4.6). Results revealed statistically significant correlations. Overall, scores showed that student teachers with higher levels of mathematics anxiety had lower mathematics CCK and student teachers with low levels of mathematics anxiety scored higher in their mathematics content knowledge assessments. According to the correlation coefficients computed, student teachers' mathematics anxiety was strongly related to their CCK ($r^2= 0.7981$, semester one, and $r^2=0.6135$, semester two). Therefore preservice teachers who possessed extensive content knowledge to solve mathematics tasks were more likely to feel less anxiety in their ability to problem solve in mathematics. Conversely, potential teachers who felt high mathematics anxiety were more likely to perform poorly when solving mathematics problems.

Table 4.6:
Correlation between Mathematics Content Test and RMAS Results

	Pre-Content Knowledge Test (N=99)		Post-Content Knowledge Test (N=97)
Pre-RMAS	$r=0.89$ $r^2=0.798$	Post-RMAS	$r=0.78$ $r^2=0.6135$

Note. Pearson product-moment correlation coefficient was calculated to determine the correlation between mathematics anxiety and common content knowledge. Results suggest a statistically strong correlation in the pre-test $r=0.89$, $r^2=0.798$ and continued strong correlation in the post-test, $r=0.78$, $r^2=0.6135$.

Due to the statistically strong relationship between CCK and anxiety, it is worth discussing the coefficient of determination, or r^2, as this demonstrated statistically significant linear regression lines (see Figures 4.2 and 4.3). A stronger correlation occurred in semester one than in semester two. The coefficient of determination represents the percent of the data that is closest to the regression line. For the pre-test results, $r = 0.89$, then $r^2= 0.7981$, which means that almost 80% of the total Y axis (CCK results) can be directly attributable to the RMAS results, while the remaining 20% in Y remains unexplained. For the post-test results, $r=0.78$, then $r^2=0.6135$, so 61% of the total variations in the post-content knowledge results can be explained by mathematics anxiety. This illustrated the very strong correlation between the two variables of mathematics anxiety and CCK, in which one was attributable to the other by as much as 80 percent. Therefore addressing the shortcomings of student teachers' mathematics content had directly relieved anxiety. It was imperative that the strength of the correlation between mathematics anxiety and CCK be taken into consideration when teaching preservice mathematics courses.

Figure 4.2: Coefficient of Determination, or r^2, between Pre-mathematics Content Assessment and Pre-RMAS Results

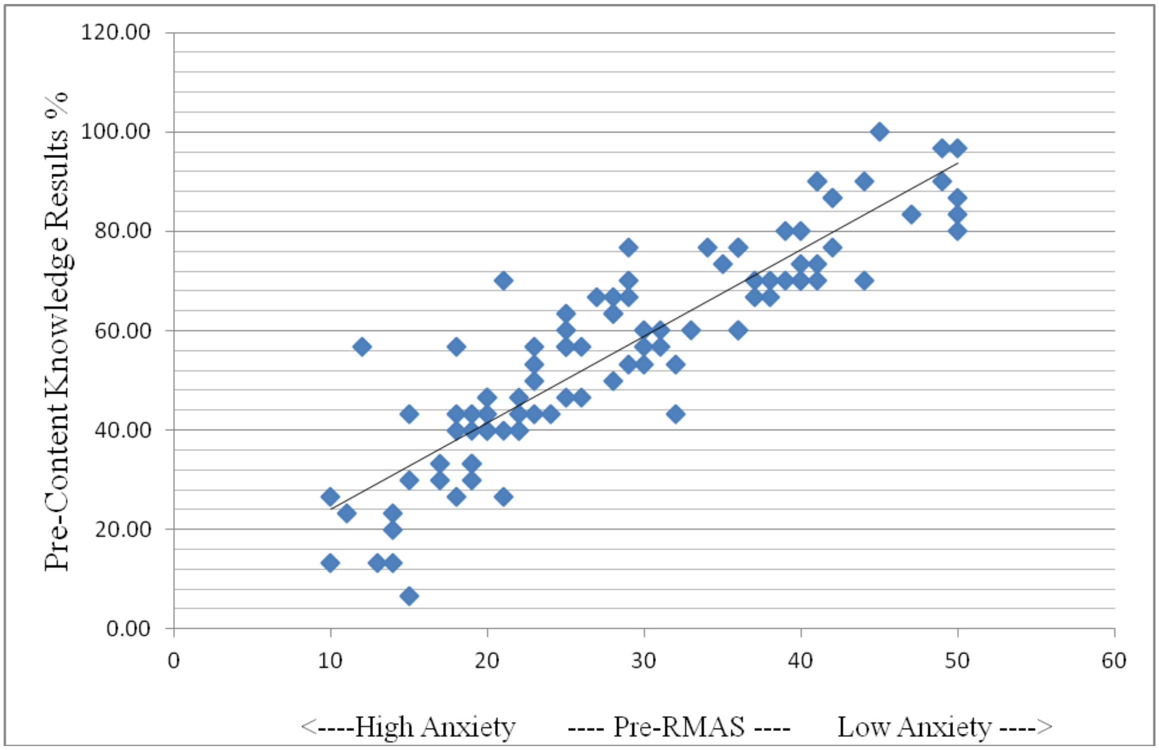

Figure 4.2. Linear regression line demonstrates a strong coefficient of determination (r^2=0.7981) between pre-mathematics test and pre-RMAS scores. Pearson product-moment correlation coefficient was used to calculate the coefficient of determination, or r^2, which represents the percent of the data that is closest to the line; therefore almost 80% of the total variations of the common content knowledge results were directly attributable to mathematics anxiety.

Figure 4.3: **Coefficient of Determination, or r^2, between Post-Mathematics Content Assessment and Post-RMAS Results**

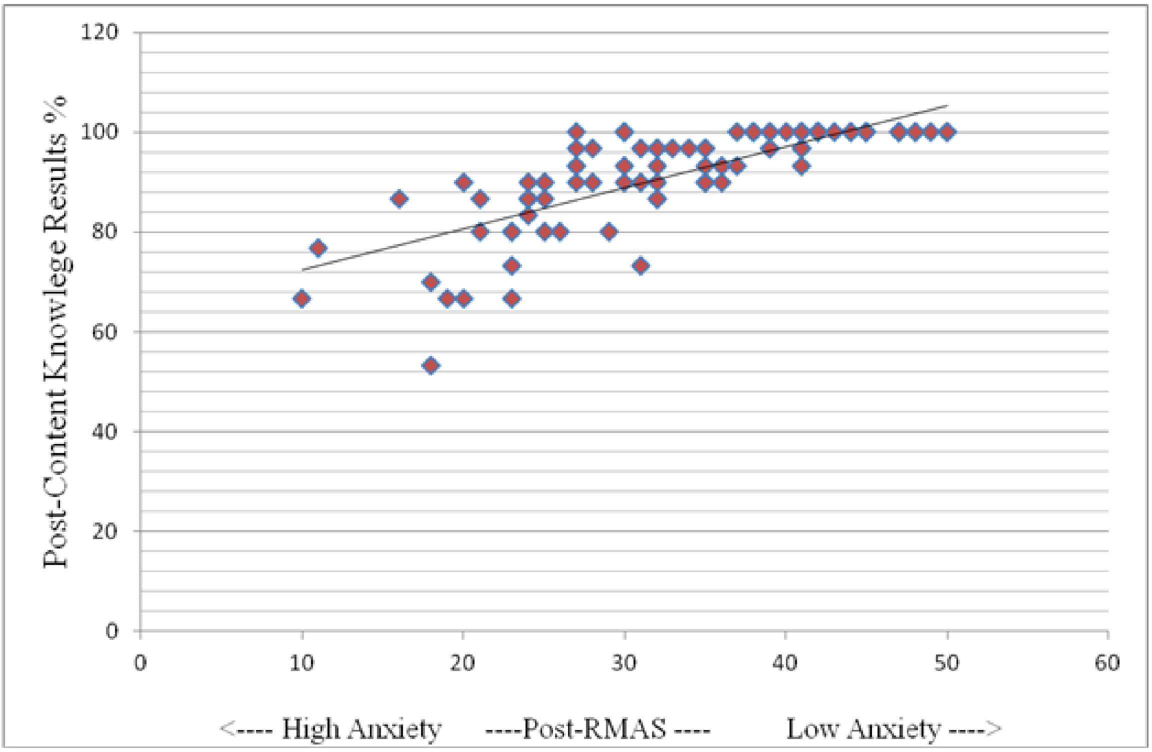

Figure 4.3. Linear regression line demonstrates a strong coefficient of determination (r^2=0.6135) between post-mathematics test and post-RMAS scores. Pearson product-moment correlation coefficient was used to calculate the coefficient of determination, or r^2, which represents the percent of the data that is closest to the line; therefore almost 61% of the total variations of the common content knowledge results were directly attributable to mathematics anxiety.

Qualitative analysis of student teachers' mathematics journals and interviews substantiated the quantitative data and further revealed contextual details of *why* and *how* problem solving emerged as an important aspect of their development as mathematics teachers and learners. Three sub-themes materialized: developing strategies for problem solving; solving problems independently; and moving away from algorithm dependency. Table 4.7 gives examples of the three sub-themes found in student teachers' journals or interview transcripts regarding problem solving.

Table 4.7:
Description of Three Sub-themes Related to the Overarching Theme of Problem Solving to Support Conceptual Understanding

Sub-theme	Journals Semester One	Journals Semester Two	Interviews Semester One	Interviews Semester Two	Example of Code
Developing Various Strategies for Problem Solving	96 (100%)	96 (100%)	6 (100%)	5 (83%)	*Today we worked on non-standard multiplication strategies. I understood them, and am excited by their intelligibility, and how I might utilize them. I especially like the multiplication array (open array). This visualization of the multiplication problem brings into focus exactly what processes are at work.*
Solving Problems Independently	91 (95%)	78 (81%)	6 (100%)	3 (50%)	*I find particularly helpful are the in-class problem solving activities when we are given time to solve it on our own, and then go through the problem step by step with our peers to find out how we all solved it.*
Moving Away from Algorithm Dependency	85 (89%)	82 (85%)	6 (100%)	5 (83%)	*This challenged my assumption that the traditional algorithm was the only or best method of solving math problems. For example, using halves and doubles is a simple and reliable way to break down a math problem in my head without using a traditional algorithm.*

Note. Within the theme of 'problem solving to support conceptual understanding,' three sub-themes emerged 1) developing various strategies for problem solving; 2) solving problems independently; and 3) moving away from algorithm dependency. The table displays frequency of codes found in the journals and interviews within the sub-themes. Journals – N = 96. Interviews – N = 6.

Developing Various Strategies for Problem Solving

All 96 journal entries (100%) for semester one and two, as well as six pre-interviews and five post-interviews (100% and 83%, respectively), included discussions about the different strategies used to solve mathematics problems. The various strategies that were examined during class did not use algorithms, or any other traditional mathematics technique that required memorization or rote learning. Rather, the strategies were based on making sense of the problem and working through solutions based on current understandings. For example, the 'moving strategy' involves rounding numbers to friendly numbers by moving quantities from one addend (i.e., the numbers that are being added) to the other addend in order to create numbers that are easier to work with (see Figure 4.4).

Figure 4.4: Example of the 'Moving Strategy'

4

196 + 378

200 + 374 = 574

Figure 4.4. The 'moving strategy' is effective when one addend is close to a friendly number (e.g., a multiple of 10). Here the addend 196 is close to 200. By 'moving' 4 from 378 to 196, the question is changed to 200 + 374.

Remarkably, when student teachers analyzed and experimented with non-traditional strategies, these experiences were reflected upon as defining moments in their mathematics content development. Many student teachers described the various strategies as "*a paradigm shift*," "*an ah-ha moment*," "*transformational*," "*eye opening*," and "*life changing*." Several preservice teachers noted how these strategies suddenly improved mental mathematics skills and further developed their understanding about the relationship among numbers.

> *This change was ignited through the learning of some fantastic strategies. For example, 'Partitioning' (27x4 can also be 20x4 plus 7x4 … why didn't I ever see*

this before? I didn't realize that you can split numbers for multiplication too. This really deepened my understanding of multiplying larger numbers.

Student teachers also commented on how confident they felt to introduce these types of mathematical strategies into their teaching. This suggests that personal teaching efficacy increased due to improved understanding of mathematical concepts… "*I had an 'ah-ha' moment during open arrays and 2-didgit multiplication. I had forgotten how to use the algorithm, and looking at it through arrays made me feel very confident to bring this into the classroom.*"

One other finding showed that many student teachers were already familiar with some of the strategies because they themselves naturally employed these strategies without knowing that they were mathematics techniques.

> *I frequently make numbers friendly in my head when doing addition, subtraction, multiplication, or division, but I never knew what it was that I was doing. I'm so glad that my ways of thinking is identified as a legitimate way of solving problems…I really feel that using strategies based on your own learning … shows a deeper understanding of mathematics …compared to just performing the standard algorithm.*

Unfortunately, some student teachers described how the prominence and repeated practice of algorithms during their K – 12 schooling eradicated these strategies from their mathematical repertoire.

> *I used some of these strategies on my own in elementary school. At that time, I didn't know it was a strategy and was not aware that I was just making sense of numbers. I remember in grade two adding large numbers and making 'jumps of tens' in my head to solve the problem. I have not used this strategy since grade two because the addition algorithm has totally wiped it out of my mathematical thinking.*

Solving Problems Independently

It was evident from student teachers' written journals and the interviews that devoted class time to solving open-ended mathematics problems supported their content knowledge. Almost all participants reflected on at least one mathematics problem conducted during class and discussed its benefits to their content knowledge. Problem solving activities were mentioned more often in semester one compared to semester two journals (95% and 81%, respectively). Additionally, problem solving was discussed in all of the pre-interviews, and only in half of the post-interviews. This suggests that content knowledge development was more of a priority for student teachers during the first half of the year, and as content knowledge improved, other mathematics foci became the source of reflection. This result triangulated the quantitative component that mathematics content knowledge improved over the course.

One interesting finding revealed how student teachers were actually surprised when they were able to solve the mathematics problems during class.

> *Doing these questions allowed me to see that I do remember some math, and that I am capable of working through math problems. My math content knowledge is not as bad as I thought it was because I am solving these in-class math questions on my own!*

This outcome proposes that many student teachers entered the program under-estimating their mathematics problem solving skills and possessed low mathematics self-efficacy. However, when student teachers felt successful at a problem, they gained immediate self-efficacy about their mathematics content knowledge. "*The toothpick problem activity was a task I excelled in and it gave me the confidence to feel capable of the rest of the math we did. I felt so relieved.*"

In general, independent problem solving promoted students' mathematical thinking beyond the application of the algorithm . . . "*I thought I would fail without knowing what algorithms to apply. I didn't realize there were so many different avenues to solving a problem*." This suggests that many student teachers who experienced rote memorization of mechanical processes ultimately became dependent on algorithms. As a result, participants showed limited ownership of their mathematical reasoning, but rather depended on traditional algorithms to solve mathematics problems.

Moving Away From Algorithm Dependency

Moving away from algorithm dependency was a recurring sub-theme in most student teachers' journals for both first and second semester (89% and 85%, respectively), as well as pre-and post-interview transcripts (100% and 83%, respectively). For many student teachers, this was a challenging shift in mathematical thinking. The main message that was conveyed to student teachers was that an algorithm was simply one way to solve a mathematics problem, but it was not the only way. Student teachers were challenged to solve problems by making sense of the mathematical concepts. Many of the student teachers felt it was demanding to refrain from using traditional algorithms, thus creating some anxiety, as mentioned in this interview. "*I experienced a bit of nervousness and math anxiety because my safety net of algorithms was now thrown out the window, and practicum was approaching quickly."*

In the second semester journals and post-interview transcripts, it was evident that mathematics content knowledge of student teachers improved significantly due to decreased incidents about dependency on algorithms. This was illustrated by many reflections that noted how solving problems without algorithms actually improved their conceptual understanding of the mathematics task… "*What helped me the most has been unlearning what is automatic for me – to jump straight to an algorithm to solve a problem. This has really forced me to understand the concepts in the math and not just follow procedures.* "

The sub-theme of algorithm dependency triangulated the quantitative results of the grade six content assessments. In the pre-test, more than half (54%) of the student teachers were unable to complete the measurement question and almost half (48%) did not complete the patterning and algebra problem. Several participants responded to these questions by writing on their test paper, "*I don't remember what formula to use.*" This was further documented in many preservice teachers' journals. *"I then thought about my own experience of math in elementary school. I was taught and always used traditional algorithms, however at the time they did not really make sense to me."*

The following interview excerpt also illustrates algorithm dependency that proved to be unsuccessful:

> *I don't have any content knowledge. When we were doing the questions on the assessment test I could see kind of what I'm supposed to remember in terms of formulas but I couldn't put it together. And again I think it's because a lot of it was memorization so I was like, I'm supposed to remember something, but I don't understand what.*

Many participants openly admitted they were unable to remember much of the mathematics they learned in school because much was taught as a collection of algorithms and skills to be memorized… "*I have no number sense, especially when it comes to double digit adding. I don't take place value into consideration. I instantly stack one number on top of the other and carry the numbers. It is purely procedural knowledge."* Hence, moving away from algorithm dependency forced student teachers to grasp the conceptual structure of the mathematics ideas and make sense of the problem at hand, thereby improving common and specialized content knowledge.

Building Confidence as a Mathematics Teacher

The second theme of 'building confidence as a mathematics teacher' represented student teachers' realistic beliefs about achieving success as a learner and teacher of mathematics. In order to fully understand the theme of confidence, it is important to examine preservice teachers' mathematics teacher efficacy levels because efficacy is closely connected to confidence levels (Bandura, 1977). Mathematics teacher efficacy is a type of self-efficacy, in which teachers construct their beliefs about their ability to perform and affect students in relation to mathematics.

The researcher analyzed the pre and post-data results from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). The MTEBI contains two subscales: personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE) with 13 and eight items, respectively. The range of scores for PMTE is from 13 to 65 and for MTOE the scores range from eight to 40. Descriptive data are provided (Table 4.8) for the MTEBI results. PMTE mean scores increased substantively over the course (from 45.88 to 53.89), while MTOE scores remained relatively stable (from 29.5 to 30.41). The increase in PMTE mean scores (8.01 or 12.32%) indicates that the student teachers gained confidence in their capacity to teach mathematics. Whereas, the minor increase in MTOE scores (0.91 or 2.25%) reveals limited change in student teachers' beliefs that children can learn from their teaching, regardless of external factors.

Table 4.8:
Mathematics Teacher Efficacy Results Using the MTEBI Survey Instrument

	PMTE		MTOE	
	Pre (N=99)	Post (N=97)	Pre (N=99)	Post (N=97)
Mean	45.88 (70.58%)	53.89 (82.9%)	29.5 (73.75%)	30.41 (76%)
Standard Deviation	7.68	6.92	4.67	4.09

Note. The scores can range from 13 to 65 for PMTE, and eight to 40 for the MTOE. PMTE mean scores increased by 8.01 or 12.32%, indicating increased personal teaching efficacy levels. An increased mean in MTOE of 0.91 or 2.25% suggests limited change in MTOE, i.e., student teachers' beliefs that children can learn from their teaching, regardless of external factors.

Qualitative analysis of student teachers' mathematics journals and interview transcripts revealed that student teachers' confidence to teach and learn mathematics was an important aspect of their professional development over the course. As part of the written journal assignment, student teachers were invited to discuss their development of mathematics confidence and/or efficacy – both personal teaching efficacy and outcome expectancy. Analysis of the written journals revealed that the student teachers rarely commented on their outcome expectancy (beliefs that effective teaching can impact on student learning), but rather discussed their personal teaching efficacy (beliefs in skills and abilities to be an effective teacher). Therefore, this suggests that participants were mainly focused on their skills to teach mathematics.

After reviewing all the codes that were classified under the confidence theme, two sub-themes materialized. These sub-themes were: teaching and assessing content; and building capacity efforts. Table 4.9 provides a description of the two sub-themes found in student teachers' journals and interviews regarding their mathematics confidence.

Table 4.9:

Description of Two Sub-themes Related to the Overarching Theme of Building Confidence as a Mathematics Teacher

Sub-theme	Journals Semester One	Journals Semester Two	Interviews Semester One	Interviews Semester Two	Example of Code
Teaching and Assessing Content	82 (85%)	59 (61%)	5 (83%)	4 (67%)	*I still worry and feel anxious about teaching concepts in mathematics when I don't have a deep understanding myself. What will I do when students ask me a mathematics question and I can't answer?*
Building Capacity Efforts	70 (73%)	68 (71%)	6 (100%)	5 (83%)	*I know that I have to work harder at preparing math lessons because I must learn the math concepts first, and then come up with meaningful problem solving activities. I am confident that I will work hard, but still worry if my lessons will go well.*

Note. Within the theme of 'building confidence as a mathematics teacher,' two sub-themes emerged 1) teaching and assessing content; and 2) building capacity efforts. The table displays frequency of codes found in the journals and interviews within the sub-themes. Journals – N = 96. Interviews – N = 6.

Teaching and Assessing Content

Analysis of the student journals and interview transcripts revealed that the most commonly addressed topics regarding preservice teachers' mathematics confidence was their ability to effectively teach and assess content. These types of reflections occurred more often in first semester compared to second semester journals (85% and 61%, respectively). This finding suggests that student teachers were more concerned about teaching the content of mathematics earlier in the course, and this became less of an issue towards the end of the year. A major reason for this finding can be attributed to the improved content knowledge, increased efficacy levels, and reduced anxiety levels. The quantitative results demonstrated marked improvements in CCK and efficacy, and a moderate decrease in mathematics anxiety. Therefore, as preservice teachers gained content knowledge and efficacy levels, anxiety subsided, and their confidence in their abilities to teach and assess mathematics content correspondingly increased.

Confidence varied for each preservice teacher, as it was dependent on factors such as content knowledge and comfort with constructivist models… "*I have inadequate content knowledge of Grade five and six math, so I feel totally unconfident about teaching math to junior grades.*" Hence, it was important to examine the quantitative relationship between mathematics efficacy levels and CCK assessment results. The researcher conducted an analysis of the PMTE and the content assessment scores using the Pearson product-moment correlation coefficient (see Table 4.10). Results demonstrated a moderate and consistent correlation between PMTE and CCK in pre- and post-tests, $r=0.46$, $r^2=0.21$, and $r=0.42$, $r^2=0.18$, respectively. Furthermore, no significant correlation was found between MTOE and CCK in pre- and post-results.

Table 4.10:
Correlation between PMTE and Content Assessment

N=99	Pre-PMTE	N=97	Post-PMTE
Pre-content Assessment	$r=0.46$ $r^2=0.21$	Post- content Assessment	$r=0.42$ $r^2=0.18$

Note. Pearson product-moment correlation coefficient was calculated to determine the correlation between PMTE and content assessment results. Results show a moderate correlation between common content knowledge and PMTE pre-test and post-test, $r=0.46$, $r^2=0.21$, and $r=0.42$, $r^2=0.18$, respectively.

Results outlined in Table 4.10 reveal that student teachers with lower levels of CCK were more likely to possess lower levels of mathematics teacher efficacy. When student teachers reflected about their beliefs and abilities to teach mathematics content effectively, the notion of teaching older students was often discussed. In semester one journals and interviews, several student teachers noted their lack of confidence in teaching older grades, due to more demanding mathematics content… "*I was not so worried about grade one but I am concerned for when I am in a junior classroom. Math at that age level would be way over my head at this point.*"

The data not only revealed that student teachers were concerned about their abilities to teach content, but more specifically about assessing students' learning in mathematics. Many preservice teachers exhibited low confidence in determining and making judgements about students' knowledge and understanding of mathematical concepts.

> *I am excited about looking for understanding in my students' answers, but worry that my own understanding is so tenuous that unless the answer were to somehow reflect my own understanding, I wouldn't be able to recognize the understanding of the student.*

In addition to low content knowledge, another major obstacle in gaining mathematics confidence was anxiety that student teachers experienced with mathematics teaching and learning. Student teachers who possessed higher levels of mathematics anxiety were also likely to have lower levels of overall teacher efficacy. This was manifested in the comparison analysis of the Revised Mathematics Anxiety Scale (RMAS) and MTEBI results.

The two subscales of the MTEBI, (i.e., PMTE and MTOE) were analyzed separately. The results of the correlation coefficient using Pearson product-moment revealed a statistically moderate relationship between RMAS and PMTE results (Table 4.11), with a slightly stronger correlation in semester one ($r=0.56$, $r^2=0.31$) compared to semester two ($r=0.51$, $r^2=0.26$).

Table 4.11:
Correlations between RMAS and PMTE Results, and RMAS and MTOE Results

N=99	Pre-PMTE	Pre-MTOE	N=97	Post-PMTE	Post-MTOE
Pre-RMAS	r=0.56	r=0.13	Post-RMAS	r=0.51	r=0.18
	r^2=0.31	r^2=0.02		r^2=0.26	r^2=0.03

Note. Pearson product-moment correlation coefficient was calculated to determine the correlation between mathematics anxiety and mathematics teacher efficacy. Results show a moderate correlation between mathematics anxiety and PMTE pre-test and post-test, r=0.56, r^2=0.31, and r=0.51, r^2=0.26, respectively. No significant correlation occurred between mathematics anxiety and MTOE pre-test r=0.13, r^2=0.02, and post-test r=0.18, r^2=0.03.

From these results, student teachers who had confidence in their skills to teach mathematics (PMTE) had lower mathematics anxiety levels, while student teachers who had lower PMTE levels possessed higher mathematics anxiety. Of particular note is the correlation results between RMAS and PMTE and how it remained relatively stable with only a slight decrease over the year (pre r=0.56, post r=0.51). This finding signifies a reasonably consistent relationship between personal teaching efficacy and mathematics anxiety. The RMAS and the MTOE revealed no significant correlation in both the pre- and post-test results (table 4.11). This suggests that no significant relationship occurred between student teachers' levels of mathematics anxiety and their beliefs that children can learn from their effective teaching.

The moderate correlation calculated between PMTE and anxiety (pre r=0.56, r^2=0.31 post r=0.51, r^2=0.26) was substantiated by the qualitative data throughout the theme of confidence. Essentially, preservice teachers with higher levels of mathematics anxiety were more likely to worry about their teaching abilities and have less confidence, specifically in two areas: teaching mathematics content to older grades and how to assess students' mathematical understandings.

The quantitative correlations discovered among mathematics anxiety, CCK and personal teaching efficacy levels are substantiated in the qualitative data. The following student teacher's interview excerpt demonstrates her low confidence levels and the interconnectedness among her anxiety, content knowledge, and PMTE levels. This student teacher's instrument results showed she scored high in anxiety, low in teacher efficacy, and low in CCK.

> *My efficacy would be low, why, because I don't know how to do it. And along with the trying to learn it as soon as you put numbers in front of me I start to sweat; like I panic. Oh here it is, in our class I can hear some people going, 'Oh that was easy! Oh I got it' and I'm sitting there going* [gasp]. *The numbers are blurry. I can't figure it out. So in terms of efficacy, no, I don't know the content, I don't know how to do it, and then there's the emotional attachment to it that gets everything flustered.*

A specific course experience that supported student teachers' confidence in teaching mathematics was the opportunity to examine students' mathematics work. This supported mathematics teaching confidence because participants were able to position themselves in the role of mathematics teacher and gained ideas on how to respond to students, ask pertinent questions, and plan next steps. Journal entries and interviews revealed how experiencing students' work samples during class supported student teachers in their teaching of the mathematics content.

> *The connections and conversations we made in class about a child's answer to a mathematical problem were essential for my own understanding about mathematics teaching. It gave me confidence on how to give feedback to students and plan next steps.*

Examining the provincial mathematics curriculum was also commonly cited as a positive experience that supported student teachers' confidence in teaching and learning mathematics. For most preservice teachers, they were unaware of when to introduce specific mathematical concepts. Many pondered about what grade levels children developed certain skills, and several participants had limited understanding of the five mathematics strands outlined in the provincial curriculum. Therefore, having in-class opportunities to study the curriculum document was beneficial as it gave preservice teachers more confidence about the scope and sequence of what concepts to teach.

During some in-class curriculum activities, student teachers examined open-ended mathematics questions and then aligned them to specific mathematics expectations outlined in the provincial curriculum. The purpose of these tasks was to illustrate how meaningful mathematics problems can address a large amount of curriculum, while keeping pupils engaged in deep learning. In general, student teachers felt this was a valuable exercise.

> *It was incredibly helpful to be given different types of math questions and then given the opportunity to look through the* [curriculum] *document to find out which curriculum expectations could be covered in one math question. This was incredibly reassuring, given the size of the math curriculum, I was nervous about how to cover all the expectations with such limited time.*

Building Capacity Efforts

The second sub-theme that materialized from the data around confidence was that of teaching efforts to build capacity. This was described in most of the journals in both first and second semester (73% and 71%, respectively) and in all of the pre- and post-interview transcripts (100% and 100%, respectively). Many of the participants attributed their hard work and diligence to being an effective teacher which in turn supported student teachers' confidence in teaching and learning mathematics. Student teachers often discussed their ability to work hard, learn the mathematics materials, plan lessons, and seek out ideas for rich mathematics activities.

> *I am confident in my ability to seek out new teaching strategies, and ways of organizing the learning environment that I intend to create. I do not feel that I could go into a Grade six classroom and teach any strand of mathematics at the drop of a hat, however I do feel strongly that I would be able to develop a plan to do so successfully.*

Of particular note was the belief that student teachers must exert more effort in the planning and preparation of teaching mathematics compared to other subjects such as language. Mathematics teaching required student teachers to deeply understand how mathematical concepts are connected and to anticipate students' conceptions. Hence, this involved teaching for reasoning, developing representations, and encouraging various solution strategies. Unlike other subjects such as language, participants discussed how they needed to unlearn and relearn mathematics in order to gain confidence in their teaching abilities due to the dichotomy between how they learned mathematics and their goals for teaching mathematics.

> *At this point, teaching Grade six math is consuming all my time and energy. Not only do I have to plan and prepare lessons, but before starting this, I need to feel confident about the math concepts I am teaching and figure out the most engaging ways to present the material. This is NOT the way I learned math, so I feel like I have to relearn math again to teach for understanding. Fortunately, once I put the time into the preparation, I do feel more confident to teach.*

Course experiences that supported efforts to build capacity were activities devoted to lesson planning. Several preservice teachers believed they could plan a mathematics lesson using constructivist strategies and felt confident about implementing the lesson. This was reflected as important because lesson planning required concerted efforts on the part of student teachers. Detailed planning generally involved student teachers to: establish learning goals for their students; develop a rich mathematics problem; determine assessment strategies; organize materials; and strategize the procedure of the lesson. Having put forth all this effort in lesson planning, journal entries revealed how student teachers felt more confident and excited to deliver the mathematics lessons.

> *I've gained so much confidence and skill in developing math lessons. The resources at my practicum school, the internet, and the in-class sessions about developing three-part math lessons all gave me confidence and good ideas of what direction to take my lessons to ensure student engagement. I would never go into a class and teach a math lesson without meticulously planning my objectives, assessments, and step-by-step procedure.*

The qualitative data surrounding the sub-theme of building capacity efforts triangulated quantitative correlations found between and among mathematics anxiety, content knowledge, and PMTE levels. Essentially, when preservice teachers exerted efforts to build their capacities in content knowledge, pedagogical lesson planning, and understanding the curriculum documents, their confidence to teach and learn mathematics increased.

Working Towards Constructivist Pedagogy

The third major theme that emerged from the data was the transformative change that student teachers' experienced; moving from transmissive teaching beliefs to constructivism. This theme was evident when analyzing student teachers' journals and interviews, and comparing semester one and semester two. During semester one; the researcher discovered a set of codes that represented student teachers' assumptions about mathematics teaching. Prospective teachers described their preconceived notions of mathematics instruction, and were faced with conflicting paradigms when they learned about constructivism. Essentially, preservice teachers' assumptions about transmissive mathematics teaching were challenged by constructivist models. And this was reflected in the noticeable increase in the frequency of codes in semester one. In semester two, the codes shifted towards the enactment of constructivist pedagogy and less about the struggles of understanding constructivism (see Table 4.12).

The researcher coded items that reflected preservice students' understanding of mathematics pedagogy and/or instructional strategies. The codes in first semester were then compared to the second semester. Based on this analysis, several sub-themes emerged that reflected the changes in student teachers' pedagogical knowledge. Based on the frequency of codes, two sub-themes materialized from semester one journals and pre-interview transcripts: reflecting on preconceived notions; and understanding constructivist teaching. Semester one data included many more questions about constructivist pedagogy indicating that their assumptions about how to teach mathematics were challenged. As the researcher examined semester two journals and post-interview transcripts, she observed that student teachers had moved beyond struggling with the notion of constructivism due to their goals and reflections about constructivist pedagogy. Specifically, participants became more focused on enacting constructivist strategies in their practice teaching. These strategies included: understanding children's prior knowledge; using open-ended questions; and collaborating. Table 4.12 provides a description of the three sub-themes found in student teachers' journals and interviews regarding their shift from transmissive teaching models to constructivism. The sub-theme 'Becoming a Constructivist Teacher' is after the sub-theme 'Understanding Constructivist Teaching.' In this section the sub-themes are not presented in order of predominance of codes, rather, the order of the sub-themes is chronological with semester one data first, then following with semester two data.

Table 4.12:
Description of Three Sub-themes Related to the Overarching Theme of Working towards Constructivist Pedagogy

Sub-theme	Journals Semester One	Journals Semester Two	Interviews Semester One	Interviews Semester Two	Example of Code
Reflecting on Preconceived Notions	92 (96%)	68 (71%)	6 (100%)	3 (50%)	*The only way that I knew how to teach math was the way I was taught it…lecture and worksheets*
Understanding Constructivist Teaching	44 (46%)	8 (8%)	4 (67%)	3 (50%)	*I am still grappling with how to plan open-ended math problems that are rich and meaningful.*
Becoming a Constructivist Teacher	29 (30%)	84 (88%)	4 (67%)	6 (100%)	*I actually taught a Bansho lesson in my practicum. It was amazing to witness so many different ways the children solved the problem.*

Note. Within the theme of ‘working towards constructivist pedagogy,’ three sub-themes emerged. Two sub-themes were prominent in semester one journals: 1) reflecting on preconceived notions; and 2) understanding constructivist teaching. One sub-theme was prominent in semester two journals, 3) becoming a constructivist teacher. The shaded areas indicate the prominent sub-themes based on the frequency of codes. The table displays frequency of codes found in the journals and interviews within the sub-themes. Journals – N = 96. Interviews – N = 6.

Reflecting on Preconceived Notions

In semester one journals, almost all the student teachers (96%) and all pre-interview transcripts included descriptions of preconceived notions on how to teach mathematics. Before commencing their teacher education program, many preservice teachers assumed that mathematics instruction comprised of teaching procedures and following the text book.

Before this course, I assumed math was a subject that thrived on precision. I assumed "good" math was "accurate" math: collections of efficient algorithms whose sole purpose was to produce correct answers. I viewed math as a subject that required the text book, repetition, and memorization.

Preconceived notions were described in detail when student teachers revealed their schooling experiences in transmissive classrooms. This influenced participants' conceptions that mathematics instruction was largely about teaching procedures through repetitive practice. Participants considered the teacher as the holder of knowledge who provided students step-by-step ways to solve a specific problem, and then students were to duplicate the procedure.

Throughout my time as a math student in elementary and intermediate grades, I only experienced one style of math, teaching using traditional algorithms and rote memorization. My pedagogical knowledge before Mary's class, was that math is teacher-directed (lectures only), and then individual desk work.

The following interview excerpt also demonstrates the notion of how mathematics schooling memories influenced assumptions about teaching models:

I just remember always sitting in math class having no idea what was going on. I remember students having to go up to the chalkboard and write the answers and I'd be praying, Please don't pick on me. And I was a very shy child, so I didn't want to ask for help. I just tried to memorize the steps the teacher showed us and worked on the 50 or so questions we were given.

Understanding Constructivist Teaching

Almost half of semester one journals (46%) and 67% of the pre-interview transcripts demonstrated student teachers' realization that creating a constructivist classroom is not an explicit, prescriptive heuristic. When compared to transmissive modes of teaching mathematics, preservice teachers described constructivist pedagogy as more complex and multifaceted. Due to student teachers' inexperience with constructivist mathematics teaching methods, data reflected students' questions and concerns about how to: construct effective open-ended problems that were meaningful to children; respond to struggling pupils; and guide students towards conceptual attainment.

When we got our manipulatives kit, I was excited about using this in my practicum. As I have no memory of using manipulatives to learn math as a child, I had no idea how to use such tools to teach math to my students. I still don't know how to use them beyond the examples that were modelled in class. I know that using concrete materials in a math lesson does not automatically equate to constructivist teaching…How do I "guide" students through meaningful problems with these manipulatives?

Journal entries also revealed how content knowledge influenced student teachers' willingness to embrace constructivist teaching. Specifically, when content knowledge was stronger, student teachers appeared more ready to endorse constructivist methods. However, when participants' content knowledge was reliant on rote learning and algorithms, embracing constructivism did not transpire as easily.

Teaching math the traditional way is so much easier. I remember following the text book page by page as a student. Now teaching through a constructivist approach is not easy. I look forward to trying out some methods we learned in class, but I must confess that my content knowledge and the way I learned math does not align to the constructivist kind of teaching.

The above quote revealed that teaching through constructivist mathematics required a specialized understanding of mathematical concepts. In order to explain and represent concepts, and develop students' mathematical reasoning, constructivist teachers must have a deeper understanding of concepts and relationship among numbers. Whereas, this is rarely required in transmissive models of teaching mathematics as this largely depends on memorization of procedures and repeated practice of the procedures.

When semester two qualitative data were compared to semester one data, it was evident that student teachers' pedagogical knowledge about constructivism significantly increased. Preservice teachers began to recognize that constructivism was not a prescriptive formula for teaching, rather it was a philosophy on student learning . . . "*My math lessons can not be about just memorizing the right answers and regurgitating the teacher's meaning of a math concept. The learning must come from the students. It's got to be meaningful*."

Participants demonstrated their desire to create risk-free environments where teachers guided students toward desired learning goals, while pupils took ownership of their learning. Semester two journals (88%) and post-interview transcripts (100%) revealed preservice teachers' goals in becoming constructivist teachers. The data displayed participants' ideas about specific teaching strategies that exemplified constructivist pedagogy, such as understanding students' prior knowledge, asking deep open-ended questions, and cooperative learning. Due to this important shift in student teachers' pedagogical knowledge, it is important to make sense of the course experiences that contributed to this transformation. The remainder of this section describes the sub-theme of semester two data that reflected student teachers' shift towards constructivist teaching models and the course experiences that contributed to this critical turning point.

Becoming a Constructivist Teacher

As part of the beginning change process of becoming a constructivist teacher, participants attempted to implement various teaching strategies that optimized constructivist models. One of the teaching strategies that participants largely focused on was making connections to children's prior knowledge. Capitalizing on pupils' background experiences and interests to drive the lesson was a recurring theme for student teachers' goals in transforming into a constructivist teacher. Student teachers reflected on how children learned more effectively when they were able to connect new material with their prior schema. During the course, a variety of activities were modelled that focused on understanding students' prior knowledge; these included: diagnostic assessment, anticipation guides, preview vocabulary, link to personal experiences and interests, use of pop culture and multimedia, graphic organizers, etc. In the following journal entry, a student teacher describes how he capitalized on his kindergarteners' interest in Silly Bandz. These are rubber bands that form various shapes and are of different colours.

> *During second practicum my kindergarten students were part of the Silly Bandz craze. Every student in the school was collecting and wearing Silly Bandz. So I seized the opportunity and used the students' interest in Silly Bandz to teach about sorting and classification. Every student was so eager to participate in the*

> *sorting games. The children were able to develop sorting rules and guess others' sorting rules using the various attributes of the Silly Bandz.*

Other data that represented student teachers' goals in becoming more of a constructivist teacher centred on the use of open-ended questions. This was a teaching strategy that many preservice teachers struggled with at the beginning of the year . . . *"Before this course, I honestly thought that grade one students could not answer synthesis and evaluation questions. I thought that asking higher order questions would be way over their heads."* However, as the course proceeded beyond the mid-way point, the data revealed that open-ended questioning became an important part of preservice teachers' instructional repertoire. Student teachers described in detail the process of using more open-ended questions while planning and teaching mathematics lessons. The following journal entry demonstrates how participants shifted to using open-ended questions in order to enact constructivist methods:

> *The class that focused on questioning helped me to explore various types of questions that guide students toward concept attainment. I am cognizant about my questions now more than ever. I ensure that I don't just ask questions that only have one correct answer. Asking rich questions that enable children to think deeper will help me to understand where students are at and how to further guide them.*

Furthermore, the following interview quote describes how a student teacher changed her questioning style in order to elicit higher mathematical thinking from students:

> *I've been able to learn how to ask more open ended questions. So instead of saying, name this – geometry, for example – name this shape, which there's only one name for it – I would say, describe the shape. This open-ended question allowed all students to contribute their thoughts and not just look for the correct answer.*

Another major aspect of becoming a constructivist teacher was the notion of collaboration. Most of the student teachers reflected on the use of cooperative learning strategies that emphasized social learning, respect of individuals, and cultivating a nurturing community. Participants frequently noted the positive outcomes of collaborating. It is important to note that two types of collaboration experiences emerged from these data. The first type related to collaborative incidents that preservice teachers themselves experienced during the course, while the second was focused on participants' mathematics lessons that implemented collaborative learning techniques in their practicum placements. This suggests that having experienced positive collaboration as learners encouraged participants to implement such strategies as constructivist teachers.

When student teachers were given opportunities to interact and communicate their mathematical thinking with their peers, they readily reflected on productive learning experiences. *"I believe the greatest learning happened for me when we are able to collaborate, discuss and think out loud."* It appeared that building upon the social dimensions of learning increased participants' achievement in mathematics learning. Student teachers not only had to explain their problem solving strategies within a safe cooperative learning group, but were encouraged to positively respect all group members' thinking, thereby boosting relationship-building.

> *I had been used to doing mathematics independently throughout my school years. However, I have learned in this course that collaborative learning can take place in mathematics classes. The "mulling to music" activity was so much fun and great for community-building. During this activity I met a classmate that I never really got to know up until that point. Her way of problem solving was so different from mine. I was fascinated by her mathematical ways of thinking. We are now working together to plan a geometry unit since we both are teaching Grade three.*

The second type of collaboration was related to student teachers' mathematics lessons that implemented collaborative learning techniques in their practicum. During in-class activities, student teachers regularly reviewed the recommendations of The National Council of Teachers of Mathematics (NCTM); that children be provided with opportunities to work together cooperatively in large and small groups on significant problems. Much in-class time was devoted on how to set up collaborative learning in mathematics lessons. Preservice teachers were well aware of the strategic planning involved in setting up cooperative learning tasks, and that simply placing students together and giving them an assignment was not enough. Participants were taught about intentional heterogeneous groupings, and formal collaborative learning structures including designated student roles and specific steps for completing group goals. In addition, a variety of cooperative learning strategies were modelled regularly in-class. Much in-class time was devoted to collaborative models as an underpinning of constructivist teaching.

As a result, a major aspect of student teachers' goals in becoming constructivist teachers was to establish collaborative learning environments. Data revealed that preservice teachers reflected on how they themselves cultivated collaborative environments so that their pupils worked in cooperative groups which fostered creative thinking and productive problem solving.

> *In practicum I was also able to review math ideas with students using a gallery walk. The students worked in groups of three and took turns explaining their group's work to others during the gallery walk. They were so engaged and genuinely interested in how other groups solved the problem. The math period flew by and the students wanted more time.*

Classroom Management

Not surprisingly, an underlying theme across all three constructs (mathematics content knowledge, efficacy, and pedagogy) was classroom management. Although some participants mentioned that classroom management was not as much an issue as they thought it would be, "*preventative strategies and rich math problems diminished classroom management issues, even the students who usually acted out, were so engaged in solving the math problem,"* most student teachers believed that classroom management challenges were of their highest concern.

A total of two sub-themes emerged from the classroom management theme: establishing classroom management systems for a constructivist learning environment; and taking risks in a transmissive mathematics classroom placement. Although the second sub-theme about practice teaching in a transmissive mathematics practicum did not result with high quantitative numbers (16% and 13% for the pre- and post-journal entries; and two interviewees); nonetheless, when this sub-theme was reflected upon, particularly during interviews, it manifested as a most pressing issue. As a result, the researcher felt it necessary to further analyze the challenges of practice teaching in a transmissive mathematics program.

Data from journals and interviews reflected student teachers' questions and concerns about how to establish collaborative, hands-on mathematics learning, without losing control. Table 4.13 provides a description of the sub-themes found in the qualitative data regarding classroom management.

Table 4.13:
Description of the Sub-themes Related to the Overarching Theme of Classroom Management

Sub-theme	Journals Semester One	Journals Semester Two	Interviews Semester One	Interviews Semester Two	Example of Code
Establishing Classroom Management Systems for a Constructivist Learning Environment	77 (80%)	62 (65%)	6 (100%)	5 (83%)	*My kids in this practicum are very talkative and social. I felt anxious about teaching open-ended math with this group of children. I am still unsure how to re-direct my students if they are off-task without sounding like a 'nagging' teacher.*
Taking Risks in a Transmissive Mathematics Classroom Placement	15 (16%)	12 (13%)	1 (17%)	1 (17%)	*It was obvious that my students were not used to the open-ended style of learning mathematics. They knew math as controlled, step by step procedures. I wasn't confident to take the risk of trying out cooperative math groups.*

Note. Within the theme of 'classroom management,' two sub-themes emerged: 1) establishing classroom management systems for a constructivist learning environment; and 2) taking risks in a transmissive mathematics classroom placement. The table displays frequency of codes found in the journals and interviews within the sub-themes. Journals – N = 96. Interviews – N = 6.

Establishing Classroom Management Systems for a Constructivist Learning Environment

Data from semester one and semester two journals (80% and 65%, respectively), as well as pre-and post-interview transcripts (100% and 83%, respectively) illustrated that student teachers were concerned about how to effectively implement strategies to nurture community building, establish rules and norms, and guide children's learning. Many preservice teachers reflected on their mathematics lessons that did not unfold as well as they hoped, due to ineffective classroom management.

> *I experienced some challenges with a small group of boys who seemed to take advantage of the hands-on style of math teaching. They were preoccupied with the scented markers, and didn't seem interested in the math lesson. Obviously, they were not used to learning math this way… I needed to figure out how to collaboratively establish group norms that promoted on-task behaviour and respected all community members.*

It was evident that student teachers' pedagogical approaches shifted from teacher-controlled to an emphasis on student engagement, self-regulation, and community responsibility with teacher guidance. However, some participants experienced difficulties with the amount of teacher guidance required. In the following journal entry, this preservice teacher misjudged how much teacher guidance was necessary, as she strived toward a constructivist learning environment.

> *There was one thing I had to be very careful with. When using manipulatives, students usually wanted to play with them. I actually provided a little bit of extra time to play with the manipulatives (pattern blocks) however, I should've removed them right after their activities because when we gathered after the activity for reflection, some students were having a hard time focusing. They were distracted by the pattern blocks. I needed to build up more effective classroom management that had rules, clear indications, and preparations. I think I tried to ignore this because I wanted students to take ownership of their behaviours and learning. I really didn't think I needed to be on them so much.*

Of particular note were student teachers who possessed low CCKcoupled with their discomfort levels about managing a constructivist mathematics lesson. Some participants' journals reflected their low personal mathematics teaching efficacy and a desire for more time to learn mathematics content and classroom management techniques. As a result, some participants felt ill-equipped for teaching mathematics through problem solving.

> *Several of my lessons did not go very well… I have come to realize that the problems I experienced were primarily related to classroom management and*

fear of math content… I am still uneasy with teaching math as a result. I still feel more like a math student, than a math teacher. I believe this to be a somewhat realistic place to be at this stage of the program, particularly given my past experiences as a math learner. I know this is an area in which I will need support, even if much of it is psychological barriers I am putting up. I know this will change in time as I strengthen my math skills.

This illustrated the correlational relationship discovered in the quantitative results regarding the interconnectedness among CCK, personal teaching efficacy, and mathematics anxiety. These constructs played significant roles in student teachers' pedagogical skills in classroom management. Essentially, when these constructs were limited, then preservice teachers' readiness to teach through constructivism declined.

Taking Risks in a Transmissive Mathematics Classroom Placement

The researcher found that classroom management became a pressing issue particularly when student teachers were placed in a classroom that implemented transmissive teaching models. Two student teachers who were interviewed experienced field placements that exemplified transmissive mathematics teaching, in which their mentor teacher's program emphasized rote learning and teacher controlled lessons. One student teacher experienced this in practicum one, whereas the second student teacher experienced this in practicum two. As a result, these preservice teachers described the inconsistency between the philosophies of their mathematics course experiences and practicum placements

When mathematics course experiences were coherent with student teachers' practicum placements then preservice teachers' development as a mathematics teacher was impacted positively. Student teachers benefited considerably when they observed constructivist principles enacted in field placements as well as course activities. This promoted an integrated teacher education program as preservice teachers made links between course experiences and field placements.

*I watched my [*mentor*] teacher with the students working out problems using adding up, compensation with addition and subtraction, math strings, jumping, etc. Seeing those little students use those strategies so easily was eye-opening to me - what we learned in our course made sense finally.*

Furthermore, making links between course experiences and field placements was described in the following interview quote, which was in response to what was the most rewarding experience in mathematics.

> *I think being able to take what was taught in the class and truly apply it and then see it play out with children, because so often it's not quite the same. And you know, in principle of course, it works so much more smoothly in the class with our peers. But a lot of the things we learned in class need further examination and practice in the real classroom, so we can really solidify the theory and practice.*

However, if the practicum placement did not exemplify constructivist models, then student teachers were confronted with many learning challenges. Several of the journal entries subtly hinted about placements that were not aligned to constructivist philosophies. "*My students were not used to flexible math approaches," "The students usually worked independently during math," "My* [mentor] *teacher needs the noise level no louder than 'one,'" and "The text book was the main resource."*

Consequently, student teachers in transmissive placements were faced with the decision to either expend efforts to enact constructivist techniques in their practicum or comply with the mentor teacher's transmissive models. The highest concern in these situations was classroom management.

> *My students weren't taught in a constructivist math environment… I dreaded that I would lose total control. I had nightmares of the kids going like bouncing off the walls, and no learning was happening because they'd never worked in a productive problem solving format before.*

Accordingly, preservice teachers were well aware of the tremendous risk involved with implementing constructivist techniques… "*If my lesson fails, then I might be evaluated poorly. If my lesson goes well, then my* [mentor] *teacher might react with animosity*."

The following interview quote illustrated the classroom management struggles felt by a student teacher. She had unfavourable characteristics due to her high anxiety, low content knowledge, and low efficacy scores. Moreover, this mathematics experience occurred during her first practicum in the fall. Signs of trouble began early in semester one.

> *When I observed him* [mentor teacher] *for the first time teaching math, the first time, he basically showed them one way to solve each problem and never once suggested or invited the students to try out their own strategy. The entire lesson was about memorizing the two digits by two digits multiplication algorithm. He basically told the students the answers, if they were struggling; he told them the answers right away. At that point, I remember feeling so confused, like really conflicted, and even more anxious about teaching math. I said to myself, should I try out some constructivist methods, like teaching the students about open arrays for multiplication, or do I follow his math program? But I truly felt like I would bomb the lesson if I taught something like open arrays, or even putting the kids in groups to solve a problem.*

Not only were there stark contrasts between what the student teacher was learning in her mathematics course and her practicum, she was also uncertain about what to do about it. Her concerns about classroom management exacerbated her already high levels of anxiety. Hence, this situation caused her to feel unprepared to teach in constructivist ways. Since this student teacher also had low content knowledge and low efficacy, it added more challenges to her mathematics development during practice teaching.

> *I couldn't work through it* [math question from textbook]. *And we had a discussion where he* [mentor teacher] *told me I didn't have to be able to work through it; if I had the right answer from the text book I'd be fine. And I said, no, because if students aren't getting it right, I won't be able to see how. And he's like, no, no, no, no, no. … after giving me this lecture, and I finally was like, all right, I'll do what he says, it's his classroom, so fine, if the answer is 12, then I'll go with 12, even though it doesn't seem to make sense to me. I couldn't explain to the kids. I was just hoping the students who didn't understand wouldn't get frustrated and act out.*

This student teacher did not enact any constructivist methods, and followed her mentor teacher's mathematics program, which relied heavily on the text book.

The second student teacher interviewee who experienced a transmissive mathematics practicum placement had more favourable characteristics. She scored low on anxiety and possessed high levels of content and efficacy levels. In addition, this preservice teacher experienced a positive practicum placement during first semester so she knew that constructivist principles could work.

This student teacher managed to negotiate with her mentor teacher to implement a mathematics strategy called Bansho during practicum. Bansho is a constructivist strategy originated from Japan, in which students solve a rich problem, and their strategies are categorized, displayed on the board, shared, and compared with the entire class. The outcome of her Bansho lesson left much to be improved upon, due to some classroom management issues.

> *I planned a bansho lesson with my class . . . we didn't finish . . . they needed more time to do their work . . . cause I wanted them finished and the work could be displayed, but some of them got stuck. I had a couple students that were stuck on making it look perfect and not really solving the problem, or I had students that fooled around . . . I wasn't on them as much as I should have been monitoring whether they were answering the question. Also because some of them tried strategies that really weren't working for them, so given more time I would have revisited that. But as it was – they weren't used to working in groups and recording their answers on blank chart paper. It was too much independence…I guess freedom for them, and the lesson didn't go as well. I should have given more structure.*

This student teacher believed that if the pupils had been learning in a flexible learning environment where strategies such as Bansho were a regular part of the mathematics program, she would have been more successful. Nevertheless, the risk she took with the Bansho lesson was a decision she did not regret. "*I am really pleased that I tried* [Bansho] *in my practicum. I know I made some mistakes with classroom management, but I think the children overall enjoyed the rich learning."*

It appears from comparing the above two practicum experiences that placement order may be of significance. For the preservice teacher who experienced constructivist mathematics in her first practicum, when she was faced with learning challenges in second practicum, at least she had her first placement to positively reflect upon. Whereas, the student teacher who taught in a traditional mathematics class during first practicum felt more challenged and did not have a positive placement previously to fall back on.

The qualitative data once again triangulated the quantitative relationships among mathematics anxiety, content knowledge, and efficacy levels. These constructs were important influences on pedagogical challenges of classroom management. The student teacher who had well developed constructs of content and efficacy levels, and low anxiety took the risk and tried out constructivist approaches, regardless of classroom management apprehensions. While the other student teacher who had high anxiety, low content knowledge, and low efficacy levels did not take the risk of implementing constructivist methods, due to distress over classroom management. As a result, she ended up following the transmissive modes that were already in place.

Chapter Summary

This section provides a summary of the quantitative and qualitative data and the emergence of the main themes. The quantitative data reported on the findings of instruments and statistical correlations between the various instrument results. These instruments included the Revised Mathematics Anxiety Scale (RMAS), mathematics tests, and the Personal Mathematics Teaching Efficacy (PMTE) subscale of the Mathematics Teacher Efficacy Beliefs Instrument (MTEBI). The qualitative data was derived from student teachers' and the instructor's written journals and student teacher interview transcripts. The qualitative data provided detailed descriptions of *how* and *why* the quantitative results might have occurred. The remainder of this chapter summarizes the key findings of each of the five themes: importance of the instructor's role in mathematics teacher development; problem solving to support conceptual understanding; building confidence as a mathematics teachers; working towards constructivist pedagogy; and classroom management.

Importance of the Instructor's Role in Mathematics Teacher Development

The purpose of this theme was to describe course experiences from the researcher-instructor's perspective based on the instructor's journals. The first sub-theme involved establishing a safe learning environment so that student teachers would: be able to work through mathematics problems, feel supported in asking questions, and be encouraged to express misconceptions and confusions. As part of establishing a nurturing community of learners, collaborative norms and expectations for behaviours were developed with student teachers' input. The norms especially benefited preservice teachers who possessed high anxiety, low content knowledge, and low self-efficacy. The second sub-theme emerged, as the instructor observed the positive ramifications of the warm-up activities during class. The instructor recognized that when her student teachers were successful with the warm-up problem, they gained immediate self-efficacy to continue with a more difficult and complex problem. Therefore, a gradual increase in difficulty level was an important strategy in supporting students' efficacy levels, content knowledge development, and anxiety relief. The third sub-theme, modelling effective strategies and enacting them in field experiences, involved the instructor`s goal to improve the pedagogical skills of her student teachers. Unfortunately, for preservice teachers who were placed in transmissive classrooms during practicum, it was difficult for them to practice pedagogical strategies that were of constructivist in nature. The instructor observed first-hand how student teachers in transmissive placements taught in the same manner as their mentor teachers, regardless of whether they agreed with their philosophies.

Problem Solving to Support Conceptual Understanding

In the theme of 'problem solving to support conceptual understanding,' the quantitative data revealed major improvements in student teachers' problem solving skills through the results of the content tests. The largest gains observed were in student teachers' CCK as indicated by a quantitative mean increase of 37.14% between pre- and post-content tests. This was triangulated throughout the qualitative data, in that participants described how their common and specialized content knowledge was positively developed by in-class opportunities to examine various strategies to solve problems and to work through mathematics tasks independently. Furthermore, a strong correlation was discovered between mathematics anxiety and CCK (pre, $r=0.89$, post, $r=0.78$). This was triangulated in the qualitative data where mathematics anxiety was noticeably debilitating for students that possessed limited problem solving abilities. Journal data revealed how student teachers were actually surprised when they were able to solve the mathematics problems during class. This suggests that participants entered the program with low self-efficacy levels in mathematics, as they under-estimated their problem solving skills. Another major aspect of improved problem solving was when student teachers moved away from algorithm dependency. This forced student teachers to grasp the conceptual structure of the mathematics ideas and make sense of the problem at hand, thereby improving content knowledge.

Building Confidence as a Mathematics Teacher

The theme of ‘building confidence as a mathematics teacher’ exemplified how mathematics teacher efficacy played a key role in student teachers’ mathematics development. The quantitative data revealed improvements in student teachers’ PMTE over the course of the year as well as a moderate correlation between mathematics anxiety and PMTE, (pre, $r=0.56$, and post, $r=0.51$), and between CCK and PMTE, (pre, $r=0.46$, and post, $r=0.42$). In addition, these correlation results remained relatively stable from pre- to post-tests, signifying a reasonably consistent influence mathematics anxiety and CCK had on PMTE.

In the qualitative data, student teachers reflected on their confidence (or lack thereof) in teaching and assessing content. Many preservice teachers with low content knowledge focused their efforts on *relearning* mathematics content, in order to gain efficacy, especially for teaching older students in grades five or six. Student teachers who possessed higher content knowledge tended to devote efficacy foci on instructional strategies and/or assessing students’ mathematics understanding. This finding further illustrates the moderate correlation between mathematics content and PMTE. Therefore, it appears that content knowledge levels may be indicative of where preservice teachers focus their efforts to build efficacy. In addition, journal data revealed that when preservice teachers exerted major efforts to build their capacities in common and specialized content knowledge, pedagogical lesson planning, and understanding the curriculum documents, their confidence to teach and learn mathematics increased and their anxiety was relieved.

Working towards Constructivist Pedagogy

The qualitative data illustrated the journey student teachers embarked on as they pursued their goals in becoming constructivist teachers. The first sub-theme of ‘working towards constructivist pedagogy’ involved student teachers reflecting on preconceived notions. When participants entered the program, their assumptions about how to teach mathematics were often challenged immediately. These assumptions were largely based on the way they were taught mathematics during their own schooling. The second sub-theme involved student teachers` realization that creating a constructivist classroom was not prescriptive and clear-cut, but rather a complex and multifaceted pedagogy that centred on learners' needs and interests. As the course progressed beyond the mid-way point, semester two journal results and post-interview transcripts showed how student teachers became focused on enacting constructivist strategies in their practice teaching. These strategies included: understanding children's prior knowledge; using open-ended questions; and cooperative learning. This appeared more prominent with participants who had low mathematics anxiety, high CCK, and high efficacy. It appears that these constructs impacted student teachers' readiness to embrace constructivism.

Classroom Management

Classroom management transpired as a major theme because many student teachers felt anxious, ill-prepared, and/or nervous about *how-to* implement constructivist mathematics teaching without losing control. A pressing concern manifested when student teachers were placed in classrooms that exemplified transmissive modes of teaching mathematics. This reflected the stark contrasts between what student teachers were learning in their mathematics course and their practicum. Classroom management was of highest concern during these situations. Student teachers, who possessed low content knowledge, low efficacy levels, and high anxiety, experienced exacerbated learning challenges during practice teaching. Whereas, preservice teachers with more favourable circumstances of high content knowledge, high efficacy, and low anxiety, were more likely to take risks in enacting constructivist strategies in their discrepant placements, regardless of classroom management apprehensions. Overall, the theme of classroom management triangulated the quantitative data that demonstrated the interconnectedness among mathematics CCK, personal teaching efficacy, and anxiety. More importantly, it appears that these constructs had a direct impact on pedagogical decisions during field placements. When these dimensions of mathematical teacher development were nurtured, then constructivist pedagogical actions were more likely to be enacted in the classroom regardless of classroom management concerns.

Chapter 5: DISCUSSION

Introduction

In order to fully understand the purpose of the discussion chapter and its major sections, it is important to establish the current understandings about this research. Prior to the discussion of the major themes of this chapter, a brief overview of this study's research is presented. The following section serves as a preamble that will lead into the purpose of the discussion chapter and the significant themes within it.

Preamble

An important goal for reform-based mathematics is for teachers to employ instructional practices that engage students more fully in order to promote the development of thorough and deeper conceptual understandings of mathematics content (NCTM, 2000). In preservice education, such instructional reform often necessitates transforming student teachers' beliefs about teaching mathematics (pedagogy), and their understanding of mathematics content and pedagogy (their knowledge of mathematics and their understanding of how best to teach it) (Blanton, 2002). In this current study, the majority of student teachers entered the teacher education program with views that mathematics was linear, procedural, and fixed. These findings correspond with much of the literature that describes preservice teachers' assumptions about mathematics teaching (Ball, 1988, 1996; Cohen & Ball, 1990; Frye, 1991; Knapp & Peterson, 1995; Pajares, 1992). These assumptions are typically based on student teachers' personal experiences of K-12 schooling in which they understand mathematics lessons as teacher-directed rule memorization, followed by assignments of independent seatwork. For these reasons, it is critical that preservice mathematics curriculum courses embody constructivist philosophies and reform mathematics practices and instructional approaches. In mathematics instruction, the reform movement emphasizes students' deep understanding of mathematics concepts and promotes students' abilities to reason, communicate mathematically, and solve problems based on their own thinking and experiences. Reform mathematics requires prospective teachers to develop confidence in their ability to do mathematics so they can teach students to reason effectively and solve a variety of mathematical problems, therefore mathematics curriculum and pedagogy courses in preservice education programs must nurture and increase student teachers' mathematical content knowledge, high mathematics efficacy, and a wide range of pedagogical skills and strategies to implement mathematics programs that promote authentic problem solving approaches. The following sections delve into the purpose of the discussion chapter and the major findings as they relate to this study's theoretical framework and prior research in this field.

Significant Themes

This chapter presents a further analysis of the key findings from the previous results chapter. In the previous results chapter, five major themes surfaced from the analysis of the qualitative and quantitative data: 1) importance of the academic instructor's role in mathematics teacher development; 2) problem solving to support conceptual understanding; 3) building confidence as a mathematics teacher; 4) working towards constructivist pedagogy; and 5) classroom management. The purpose of this chapter is to examine the significant themes that emerged from the results findings through the lens of this study's theoretical framework. The theoretical framework included the following theories outlined earlier in chapters one and two: mathematical knowledge for teaching (MKT), constructivist learning theory, self-efficacy theory, and adult learning theory (see Figure 1.1 in Chapter One). When comparing this study's findings to the published literature and theories, a number of significant themes emerged, some of which aligned with prior research, while others contradicted or presented as divergent from previous studies. Three significant themes were identified from this analysis and these are presented in relation to the theoretical literature throughout this chapter. Furthermore, the themes are considered of equal importance, therefore the presentation order is of no relevance. The three significant themes of this chapter are as follows:

1. *Coherence Between Theory and Practice* – When the practicum classroom was aligned in significant ways with the philosophy of the mathematics course, it fostered a coherent learning context for student teachers. In contrast, when the practicum classroom reflected transmissive models of teaching, it posed serious challenges for student teachers' MKT, efficacy, and pedagogical development. These findings indicated that coherence between field and university experiences played a vital role in student teachers' preparedness for constructivist teaching.

2. *Dimensions in Mathematics Efficacy* – Using Bandura's (1977) theory and research on self-efficacy, this current study examined the two dimensions of mathematics teacher efficacy, that is, personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). These two dimensions disclosed different outcomes. As part of this study's major findings, sources of efficacy were largely connected to student teachers' levels of mathematics content knowledge, anxiety, and were influenced by specific course experiences.
3. *Anxiety as a Barrier* – Anxiety is considered an emotional dimension that can negatively impact student teachers' mathematics development (Hembree, 1990). In this study, preservice teachers who scored high in mathematics anxiety were most likely to score low in CCK results, and have lower efficacy scores. Hence, one of the major findings revealed how mathematics content knowledge and personal teaching efficacy were connected to mathematics anxiety.

The remaining sections of this chapter elaborate on the significant themes of this study. The themes focus on specific teaching and learning experiences across two elementary preservice mathematics courses, as they relate to this study's theoretical framework and prior research. The research specifically examined how the mathematics course experiences influenced student teachers' mathematics content knowledge, teacher efficacy, and pedagogy. In this study, content knowledge refers to student teachers' common and specialized content knowledge (CCK and SCK). Mathematics anxiety was also examined in relation to the constructs of content knowledge, efficacy, and pedagogy.

In an attempt to examine the teaching opportunities that preservice teachers experienced during practicum, it is important to draw upon the research on coherence between field experience and university course work. The following section presents the first major theme of the discussion chapter. The theme of 'coherence between theory and practice' and its sub-themes examine pedagogical decisions that were largely influenced by how strong the bridge was between the university's theoretical underpinnings of constructivism and the practical realities of the field placement.

Coherence Between Theory and Practice

Coherence between the university`s theoretical focus and field experiences can impact on how student teachers develop their teaching skills. Darling-Hammond and Hammerness (2005) noted field placements that are inconsistent in significant ways with the philosophy and practice of teacher education programs frequently pose challenges for student teachers. For example, when preservice teachers "encounter mutually reinforcing ideas and skills across learning experiences" they have opportunities to enact techniques and strategies learned in their teacher education courses (Darling-Hammond & Hammerness, 2005, p. 393), while the converse would present reality shock, i.e., "the collapse of the missionary ideals formed during teacher training by the harsh and rude reality of everyday classroom life" (Veenman, 1984, p. 143). Specifically in mathematics education, Wilson, Cooney, and Stinson (2005) and Frid (2000) identified that one of the most serious problems in mathematics preservice courses is the incoherence between constructivist pedagogy taught at the university and the transmissive practices in the field.

Based on this current study's findings, the lack of coherence between theory and practice was a significant reason why student teachers felt defeated in their goals to teach mathematics through a constructivist philosophy. Rather than drawing on the theoritical underpinnings of what they learned from their mathematics course, many preservice teachers resorted to their mentor teachers' traditional methods. Their struggle to enact constructivist pedagogy and to learn from the practicum classroom continued throughout their entire teaching practicum period. Similarly, Bullough and Draper (2004) studied practicum placements which were incongruent to the teachings of the university; their findings demonstrated that student teachers were more likely to align their behaviours and practices with the mentor teacher and largely ignore their university learning. Lortie (2002) also revealed that student teachers disregarded the theoretical teachings by the university because it was easier to revert back to the conceptions of teaching they had prior to entering their program; these conceptions were often based on transmissive oriented methods of teaching. The subsections below illustrate the critical importance of coherence between classroom practice and university courses, and further examine how this relationship effects the pedagogical development of preservice teachers.

Enactment of Constructivism in Traditional Classrooms

This study found that student teachers placed in traditional mathematics classrooms experienced limited opportunities to explore constructivist methods they had learned from their mathematics course; instead, many student teachers felt obligated to continue and maintain already existing routines and techniques even if these were promoting transmissive teaching practices . . . "*I felt like a hypocrite when I taught math. I knew fully well that the drills and worksheets were not teaching students to think deeply and problem solve."* This result confirms Hart's (2004) claims that even teachers who possess reform values and beliefs are frequently hesitant to implement them within the culture of a traditional school setting. Likewise, Stones and Morris (1972) stated that the typical approach to student teaching "inspires conformity and tends to penalize innovation so that its products conform to a bureaucratically structured stereotype" (p. 4). Hence, when this study's student teachers were confronted with conflicting views on mathematics teaching, they likely felt indebted to maintain the status quo of the classroom culture.

There is considerable research (Ball, 1993; Gresham, 2008; Ma, 1999; Smith, 1996) that attempts to give reasons for the prominence of traditional mathematics classrooms, even in classrooms that are selected for mentoring prospective teachers. One salient reason is noted by Smith (1996), suggesting that the traditional approach to teaching mathematics is predominant because it is straight-forward and simpler to implement when compared to a constructivist approach to instruction. Similarly, for student teachers in this current study, it appeared easier for students to implement the traditional routines established at the beginning of the school year due to classroom management concerns. The risk of losing control and receiving a poor evaluation was too high for student teachers to trial constructivist techniques. Smith's claim was confirmed in the current study's data… "*It was easier to follow the text book. Unfortunately, it wasn't the best way to teach fractions.*" It appeared that the teacher-controlled style of teaching mathematics through procedural emphasis and reliance on the text book was the easiest method to implement, but not necessarily the most effective.

Wilson et al. (2005) examined why the efforts of constructivist mathematics teaching at the university rarely manifested into practice teaching. In their study, preservice teachers developed positive changes in their pedagogical beliefs about constructivism and reform mathematics; however, those espoused beliefs failed to transfer into the practicum. Based on interview data, Wilson et al. (2005) demonstrated that mentor teachers generally undervalued theory and research of mathematics pedagogy, whereas universities emphasized theoretical frameworks behind mathematics teaching, thereby presenting contention between two prevailing ideologies – the pragmatist versus the theorists. Wilson et al.'s (2005) research may possibly explain the dichotomy observed in the current study between the ideologies and practices espoused in the mathematics course compared with those in the schools. In this current study, mentor teachers received no professional development about the university's constructivist philosophies in mathematics, in preparation for mentoring students within the practicum period. As a result, mentor teachers may have been uninformed of the constructivist underpinnings taught at the university, and perhaps they were oblivious to the fact that their traditional practices conflicted with those taught at the university.

Implications from this finding demonstrate the critical need for building stronger communication and coherence between the practicum classroom experience and theoretical coursework experiences. Kajander (2010) noted that the dichotomy between university and field placements may be a concerning factor in preservice teachers' mathematics development… "Distressingly, most interviewees indicated that their [mentor] teachers … did not use the kinds of teaching methodologies they had learned about and experienced in the methods course" (p. 244). The mentor teacher plays a vital role in nurturing the pedagogical skills of student teachers and this study illuminated the importance of this responsibility. Jenkins and Fortnam (2010) noted, "While [mentor] teachers may be comfortable guiding student teachers through day-to-day classroom events, they receive little, if any, training in how to lead student teachers beyond these events to analyze and reflect on their teaching and the profession" (p. 23). Therefore, rather than simply promoting direct imitation of instructional styles, effective mentor teachers should encourage their student teachers to construct their own knowledge, question relationships between theory and practice, and experiment with more innovative strategies, thereby offering a practicum placement that is consistent with the university's philosophy.

The Role of Socialization within an Incoherent Context

Bandura's (1986) concept of self-efficacy is deeply rooted in his work in social cognitive theory which supports the notion that observational learning and social experience are all part of a complex process contributing to how individuals learn. Bandura claims that self-efficacy is largely developed from external social factors and self-perception. Therefore, in the case of student teachers' learning in their practicum, behaviours of socialization can directly impact the decisions and actions made during practicum. Wilson, Floden, and Ferrini-Mundy (2002) examined how mentor teachers perceived their responsibility as a socializing agent in their student teachers' development. The authors state that mentor teachers believed their role was to "socialize student teachers into the status quo of schools or into the [mentor] teachers' own practices" (p. 196). According to Goodman (1985) and Beyer (2001), student teachers' conformity to mentor teachers' methods is the safest response in order to protect the investment they have made in their teaching career. Hence student teachers tend "not to rock the boat in the classrooms in which they are placed and thus do not always engage in critical conversations about their own teaching or their [mentor] teachers' practice" (Wilson et al., 2002, p. 195). The literature strongly coincides with the current study's findings in which prospective teachers felt it was necessary to adapt to the cultural norms of the classroom in order to feel socially accepted and be positively evaluated by the mentor teacher. This often meant compromising their beliefs and values about constructivist learning theory due to the lack of coherence between their field placement and university.

According to Bandura (1986), a major source of self-efficacy development is social persuasion; this is verbal communication such as coaching and feedback given on a specific performance. When individuals are persuaded that they are capable of specific tasks, their self-efficacy beliefs are strengthened. Conversely, direct discouragement through verbal feedback results in decreased self-efficacy. Research conducted by Puk and Haines (1999) found that mentor teachers upheld the transmissive oriented norms of the school by verbally discouraging student teachers from trialling innovative approaches because they deemed them inappropriate, too difficult to implement, or only suited for the university context:

> While the student teacher was explaining what she intended to do in her lesson, another teacher looked on and commented that "this was way too hard for the students, way too many steps." The student teacher explained that she had learned this format at the faculty of education and that she was in fact using a format designed for junior level students (grades 4-6). The other teacher replied that "a lot of what is taught at the Faculty of Education isn't realistic." (p. 548)

Similarly, in this current study, social persuasion was largely forms of discouragement. Student teachers were discouraged to trial reform oriented strategies in order to maintain the status quo… "*My* [mentor] *teacher wanted me to keep the math routines in place. She said that the students needed lots of structure and open-ended problem solving would cause chaos*."

Hence, the literature provides a strong rationale as to why participants in the current study found it challenging to implement reform oriented mathematics due to the fact that their practice teaching deeply involved a socialization of the classroom culture. This enculturation often challenged student teachers' idealistic views learned from the university`s theoretical stance. Belonging to a school's culture was deemed to be both essential and strenuous as student teachers had to work through conflicting views and incoherence between their practicum and university. This finding revealed that the role of socialization in an environment that was inconsistent with beliefs and values can be detrimental to self-efficacy development. It is important to note that verbal persuasion is only one source of self-efficacy as described by Bandura (1986). Other significant sources of self-efficacy include mastery and vicarious experiences. The following subsection describes how other sources of student teachers' self-efficacy in mathematics teaching were influenced by coherency between university and field experiences.

Mastery and Vicarious Experiences

Bandura (1977, 1986) suggested through his theory of self-efficacy that efficacy beliefs are malleable, and that mastery and vicarious experiences are powerful sources of efficacy development. Specifically in teaching, mastery experiences occur when teachers believe that their teaching actions were successful. According to Bandura (1977), the experience of mastery is the most significant factor in determining self-efficacy. When individuals believe their performance was successful, it raises self-efficacy, while failure lowers it. Mulholland and Wallace (2001) asserted that preservice and beginning teachers' mastery experiences have powerful influences on efficacy. Similarly, Charalambous, Philippou, and Kyriakides (2008) described how mastery experiences enabled student teachers to experiment with techniques learned from university coursework. In this current study, student teachers placed in incoherent practicums that espoused traditional orientated practices, had limited opportunities to gain mastery experiences of constructivist pedagogy mainly due to mentor teachers' discouragement of trialling innovative strategies. Therefore, the traditional classroom placements negatively influenced efficacy growth because enactment of constructivist pedagogy was almost non-existent.

Bandura's (1997) concept of vicarious experiences, that is observations of individuals performing activities successfully - often referred to as modeling, is also essential to student teachers' efficacy growth. Charalambous, et al. (2008) noted that student teachers described vicarious experiences as inspirational and helped them set high teaching standards. The current research revealed that student teachers were most likely to follow their mentor teachers' traditional mathematics. When student teachers' lacked vicarious experiences they were unable to observe first-hand how to implement constructivist strategies such as cooperative group work, open-ended problem solving, and/or use of mathematics manipulatives. This finding further corroborates Frid's (2000) claim that one of the most serious problems in mathematics teacher education is the incoherence between constructivist pedagogy taught at the university and the transmissive practices in the field. Goodlad (1991) went even further by stating… "[preservice instructors] felt that their teachings were undone by the student teachers' experiences in the schools" (p. 8). Correspondingly, this current study found that any stark contrast between the two experiences stifled opportunities for mastery and vicarious experiences which then negatively influenced pedagogical decisions.

Readiness for mastery and vicarious experiences.

One of this study's major findings suggests that student teachers with higher content knowledge had lower anxiety and displayed a positive confidence in their constructivist strategies. This group of student teachers demonstrated a higher personal teaching efficacy through journal reflections about their mastery experiences. Even when placed in traditional mathematics classrooms, these preservice teachers enacted small but significant steps towards constructivist principles, such as asking more open-ended questions, and using visuals to display mathematics concepts. This finding coincided with research conducted by Newton, Leonard, Evans, and Eastburn, (2012) where they found that preservice teachers with high mathematics content knowledge benefited most from mastery experiences as this group of student teachers "were more likely to make connections between these experiences and their future teaching experiences" (p. 298). Similarly in this current study, when coherence between field and course work was realized, this group of student teachers maximized their learning opportunities and felt confident to employ reform methods in their own classroom. By contrast however, preservice teachers in this study who had less extensive content knowledge were more likely to possess higher anxiety and therefore required more time to grasp the content first in order to gain personal teaching efficacy. It appeared that vicarious experiences were of most value for this group of student teachers who had less favourable characteristics. It was evident through the journals that this group of student teachers expressed great appreciation and need to observe effective models of constructivist teaching (vicarious experiences) in order to support their synthesis of constructivist theory and classroom practice. This corroborates with Newton et al.'s (2012) study, in which they found that vicarious experiences were of high importance to those preservice teachers with low mathematics content knowledge. Newton et al. (2012) speculated that lower content knowledge necessitated "a greater need to see effective models of teaching" (p. 298). In this current study, it appeared that readiness for mastery experiences and the need for vicarious experiences was dependent on student teachers' content knowledge, anxiety, and efficacy levels. Hence, the lack of coherence between practice teaching and the university`s theoretical underpinnings limited the necessary mastery and vicarious experiences essential for student teachers' mathematics teaching growth. The following section presents the second theme of the discussion chapter. This theme describes the dimensions of student teachers` efficacy levels and examines how the levels influence the development of student teachers' mathematical knowledge for teaching.

Dimensions of Mathematics Efficacy

This study examined two dimensions of mathematics teacher efficacy, personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE) with the goal to further understand how efficacy levels impacted the development of student teachers' mathematical knowledge for teaching (MKT) through reform strategies. These dimensions were measured using the Mathematics Teaching Efficacy Beliefs Instrument (Enochs, Smith, & Huinker, 2000). PMTE represents a teacher's belief in how effectively he or she can teach mathematics, while MTOE corresponds to a teacher's sense of how well a student can learn from his or her teaching. A discussion is presented based upon the current study's finding in which the dimensions in mathematics teacher efficacy were influenced by content knowledge and anxiety, in comparison to previous literature. The following subsections examine the different outcomes that materialized in PMTE and MTOE scores and possible influential factors involved.

Personal Mathematics Teaching Efficacy

In this study a moderate correlation was detected between preservice teachers' mathematics CCK and PMTE (pre-test $r=0.46$, and post-test $r=0.42$, see Table 4.10 in Chapter Four for details). Of particular note was the stable relationship discovered between mathematics CCK and PMTE, in that CCK and PMTE scores were correlated in both pre- and post-results indicating that this positive relationship remained consistent throughout the academic year, i.e., as content knowledge increased, so did personal mathematics teaching efficacy. This finding differed from those of Swars, Hart, Smith, Smith, and Tolar (2007), in which they found no relationship between their student teachers' mathematics PMTE and content scores. These apparently discrepant findings can perhaps be explained by how mathematics content knowledge was measured in each of the studies. Swars et al. (2007) used the Learning for Mathematics Teaching (LMT) instrument which … "assesses this knowledge by posing mathematical tasks that reflect what teachers encounter in the classroom such as assessing students' work, representing mathematics ideas and operations, and explaining mathematical rules or procedures" (p. 329). Essentially, the content knowledge that was measured by Swars et al. (2007) was the specialized mathematical knowledge for teaching (MKT); whereas the content knowledge measured in this current study was based on items from the Ontario provincial grade six assessments. The content knowledge test used in this study did not go beyond problem solving at a grade six level. Therefore, the correlation found in the current study reflects the common content knowledge for everyday problem solving. Additionally, it might be plausible that Swars et al. (2007) did not find any relationship between PMTE and MKT due to the fact that MKT requires more demands by teachers when compared to common content knowledge by itself; that is, the complexities of MKT exceeds common content knowledge as it requires teachers to facilitate students' sense making with mathematical concepts (Thames & Ball, 2010). Due to the higher demands and complexities involved in developing MKT, efficacy levels may be more difficult to increase; whereas efficacy levels associated with basic CCK may have been more easily developed.

The researcher-instructor planned learning opportunities with an initial emphasis placed on improving preservice teachers' conceptual understanding of mathematics, and as a result student teachers reported feelings of excitement after accomplishing mathematics tasks and understanding the concepts being taught. This was often noted as pleasurable and pivotal moments for preservice teachers. This finding was in alignment with Schmitt and Safford-Ramus (2001) findings where they proposed that adults learning mathematics must positively experience deepened conceptual understanding, as these moments have the potential to empower adult learners to be intrinsically motivated and to highly value mathematical learning. These types of 'ah ha' experiences were also evident in this current study in that preservice teachers felt encouraged to continue to extensively invest in deepening their conceptual knowledge in subsequent mathematics classes when they experienced new mathematical understandings. Hence, preservice teachers in this study attained a sense of accomplishment with successfully completing CCK tasks, which also allowed for their PMTE levels to increase more easily. This finding indicated that efficacy scores might be influenced differently depending on the type of content knowledge measured, that is, specialized MKT or common content knowledge.

Outcome Expectancy Beliefs

Mathematics teaching outcome expectancy (MTOE), that is, student teachers' sense that their teaching can bring about student learning, were unchanged for preservice teachers throughout the course. Unlike the findings in this current study, Swars et al. (2007) found significant mean increases in MTOE subscale scores, which largely occurred during the second methods course. In order to explain plausible reasons for the variant findings of MTOE across these two studies, it is worth examining the research by Hoy and Woolfolk (1990). In contrast to Swars et al. (2007), Hoy and Woolfolk indicated that preservice teachers' beliefs in their ability to have a positive impact on student learning (outcome expectancy beliefs) were more likely to decline after practicum. They attributed the decline in outcome expectancy beliefs to "student teachers' unrealistic optimism about overcoming difficulties" prior to student teaching experiences (1990, p. 294). In other words, the realities of the actual classroom dispel unrealistic expectations of preservice teachers leading them to doubt their capacity or abilities to effectively teach. Similar to Hoy and Woolfolk (1990), Kagan (1992) suggested that student teachers enter their teaching program with oversimplified views about teaching, without considering the complex intricacies involved in teaching. Swars et al. (2007) claimed that their preservice teachers had extensive field experiences prior to entering the program and this in itself prepared them for the cognitive challenges in the field, thereby contributing to the positive findings of increased MTOE levels. This extensive field practice suggested by Swars et al. (2007) might have included prior volunteer classroom experience. Unlike the findings of Swars et al. (2007) and Hoy and Woolfolk (1990), neither an increase nor decrease in MTOE was detected in this current study. The absence of increased MTOE levels indicated that the majority of the student teachers in this study may not have had extensive field experiences such as those in Swars et al.'s research (2007). In addition, the lack of declining MTOE results suggests that preservice teachers in this study had practicum expectations that were not unrealistically optimistic as proposed by Hoy and Woolfolk (1990). Rather, they were expecting challenges during their practicum teaching because the realities of the classroom were explicitly articulated and described by the instructor.

The mathematics course connected student teachers to the realities of the classroom and this may have been a major factor for the unchanged outcome expectancy beliefs (MTOE) in the current study. If preservice teachers possess an unrealistic optimism about teaching, as speculated by Hoy and Woolfolk (1990) and Kagan (1992), then it is important to challenge these assumptions prior to practicum. In this current study, the instructor engaged student teachers in course activities that focused on the complexities of teaching mathematics. These activities included assessing students' authentic mathematics products, planning next steps for student improvement, developing constructivist mathematics lessons, studying curriculum documents, and examining case studies about students with special needs. Therefore, it is critical that unrealistic or disillusioned assumptions about mathematics teaching be challenged through engaging activities that transform student teachers' simplistic assumptions about teaching towards more realistic and effective conceptions.

In another study about mathematics teaching outcome expectancy (MTOE), Swackhamer, Koellner, Basile, and Kimbrough (2009) discovered that inservice teachers' efficacy in outcome expectancy was higher in teachers who had taken four or more mathematics content courses. The authors explained the reason why teachers possessed low beliefs in their ability to impact student achievement (i.e., MTOE) was due to limited mathematics content knowledge. This is logical as a teacher is likely to feel less competent and confident in teaching content that they themselves do not understand thoroughly enough to explain it to students. Swackhamer et al. (2009) theorized that additional mathematics courses increased inservice teachers' content knowledge which contributed to the major increases in outcome expectancy beliefs. This does not align with the results of the current study which revealed that MTOE scores had no relationship with content knowledge improvement; however, there were improved PMTE results that correlated with content knowledge. This researcher speculated that the discrepancy between these two findings may be explained by the differences between preservice and inservice teachers; that is, inservice teachers have more autonomy and frequent opportunities to try out new strategies in their own classrooms compared to preservice teachers. Therefore inservice teachers would have greater chances to become more efficacious in their outcome expectancy due to the opportunities to experiment and hone new strategies within the relative safety of their own classroom. These findings highlighted the unique needs of preservice teachers, in that MTOE may be more difficult to nurture due to inexperience and limited opportunities to work directly with students.

The following section presents the third theme 'anxiety as a barrier' in the discussion chapter. It is focused on Bandura`s self-efficacy about the critical need to address emotional states such as anxiety. This theme examines how the phenomenon of mathematics anxiety impacts student teachers` development of mathematical knowledge for teaching, as well as specific course experiences that support the reduction of anxiety.

Anxiety as a Barrier

Mathematics anxiety, that is an emotional feeling of worrisome and/or apprehension when attempting to perform mathematical functions, serves as a barrier in mathematics teacher development (Hembree, 1990). Research suggests that teachers who suffer from mathematics anxiety may unintentionally pass on their negative feelings and attitudes to their students by leading students to believe that mathematics is overly complex, unpleasant, and/or something to be avoided if possible (Brett, Nason, & Woodruff, 2002; McCulloch Vinson, 2001). Furthermore, since mathematics anxiety is strongly associated with mathematics avoidance (Hembree, 1990), then mathematically anxious teachers are likely to spend less time on planning for mathematics activities and ultimately less time on teaching mathematics. Hembree's claims were endorsed in the current study whereby highly anxious student teachers were hesitant to teach mathematics lessons during their field placements, illustrated by this comment … "*I was so relieved I didn't have to teach the geometry unit*. *My* [mentor teacher] *sensed my nervousness around math*." Schmidt and Buchmann (1983) determined that teachers with mathematics anxiety engage in 50% less time teaching mathematics compared to teachers with positive mathematics attitudes. The following subsections examine how mathematics anxiety was directly linked to preservice teachers' content knowledge and personal mathematics teaching efficacy (PMTE), and specifically what course activities were found to support a reduction in mathematics anxiety.

Anxiety and its Relationship with Content Knowledge

This study revealed substantial improvements in preservice teachers' content knowledge were directly connected to reducing their mathematics anxiety. Common content knowledge and mathematics anxiety were strongly correlated (pre-test r=0.89, and post-test r=0.78, see Table 4.6 in Chapter Four for details). Correspondingly, Hembree's (1990) meta-analysis of 151 studies on mathematics anxiety indicated that poor performance in mathematics is directly correlated to anxiety. He concluded that effective reduction in mathematics anxiety can be realized through improved performance in mathematics concepts, problem solving, abstract reasoning, and spatial ability. Thus, in this current study, devoting time to improving preservice teachers' problem solving and conceptual understanding proved to be effective in decreasing their mathematics anxiety.

Similar results were found by Akinsola's study (2008) which involved 122 inservice teachers; his research measured four constructs and its relationship with problem solving ability. Of the four constructs, Akinsola discovered mathematics anxiety had the strongest correlation with problem solving ability. Specifically, teachers with lower mathematics anxiety demonstrated advanced problem solving skills, whereas those with higher mathematics anxiety had weaker problem solving abilities. Furthermore, the remaining three constructs in Akinsola's study (mathematics teacher efficacy, locus of control, and study habits) were also found to have correlations with mathematics problem solving skills, wherein efficacy came in as the second strongest factor, followed by locus of control, while study habits had the lowest correlation to problem solving ability. This current study's results about mathematics anxiety, mathematics teacher efficacy, and CCK endorsed the pattern found in Akinsola's research, where it was mathematics anxiety had the strongest correlation to content knowledge, followed by personal mathematics teaching efficacy (PMTE). These parallel findings demonstrated the strong influence anxiety has on mathematics content knowledge first and foremost, followed by personal teaching efficacy.

This study also supported other previous research about preservice elementary teachers' mathematics anxiety and its influence on content knowledge. Battista's (1986) results revealed that mathematics anxiety correlated significantly with mathematics content knowledge. Similarly, Cohen and Green (2002) found that teachers who had high levels of mathematics anxiety frequently lacked concept development. Comparatively in this current study, student teachers who lacked concept development in mathematics were most likely unable to answer the open-ended problems in the mathematics content test. Therefore, these findings coincided with Battista's (1986) and Cohen and Green's (2002) claims that conceptual understanding can be an important remedy for alleviating mathematics anxiety.

The corresponding results of this study and previous research suggested that mathematics anxiety can deleteriously impact mathematical performance and understanding. Sloan, Vinson, Haynes, and Gresham (1997) and Hembree (1990) both suggested that deepening mathematics content knowledge through mathematics course work can be effective in reducing mathematics anxiety. However, Ball (1988) warned that additional course work in higher levelled mathematics courses may not automatically improve knowledge, and therefore it should never be assumed that post-secondary mathematics courses automatically improve deep conceptual understanding. Ball's assertion is based on the fact that the structure of higher levelled courses in post-secondary institutions is typically procedurally oriented. This goes against the entire notion of developing mathematical knowledge for teaching (MKT), which demands teachers of a specialized knowledge that is unique to teaching and requires "pedagogically strategic intent" (Ball et al., 2008 p. 401). Coinciding with Ball's (1988) assertion, in this current study the preservice mathematics course emphasized developing deep conceptual understandings by devoting in-class time on problem solving, participating in constructivist oriented mathematics tasks, and deconstructing why and how mathematics procedures work. These types of course experiences contributed to the reduction of students' mathematics anxiety due to the emphasis on enhancing deeper conceptual understanding of mathematics. However, it is important to acknowledge that the focus on content knowledge alone was not sufficient in eradicating mathematics anxiety in preservice teachers. This study's results also demonstrated that personal teaching efficacy was correlated to mathematics anxiety. The following subsection examines the key findings about mathematics anxiety and its influence on personal teaching efficacy.

Anxiety and its Relationship with Efficacy

A moderate correlation was revealed between personal mathematics teaching efficacy (PMTE) and mathematics anxiety in this study (pre-test r=0.56, and post-test r=0.51, see Table 4.11 in Chapter Four for details). PMTE represents a teacher's belief in how effectively he or she can teach mathematics. The mathematics anxiety felt by student teachers decreased significantly while PMTE correspondingly increased throughout the course. The significant decrease in the mathematics anxiety aligns with Bandura's (1977) efficacy research. Bandura claims that a critical means of building efficacy beliefs is to address emotional states such as anxiety. Bandura's contention is confirmed in this current study because efficacy beliefs were strengthened when student teachers were encouraged to confront their sources of mathematics anxiety through reflection and rich discussion.

Much of the literature describes mathematics anxiety as a source of preservice teachers' low confidence in mathematics activities (Bursal & Paznokas, 2006; Harper & Daane, 1998 ; Hembree, 1990; Sloan, Daane & Giesen, 2002; Wenta, 2000). Swars, Danne, and Giesen's (2006) research coincides with this current study, which suggests that low mathematics anxious preservice teachers possess higher efficacy levels compared with those who are highly mathematics anxious with low personal mathematics teaching efficacy. In addition to this study's quantitative correlation calculated between anxiety and PMTE scores, themes from the qualitative data, namely, student journals and interviews also supported findings that there was a relationship between mathematics anxiety and personal teaching efficacy. These findings showed that those pre-service teachers with higher anxiety attitudes felt less confident in their constructivist teaching strategies. They were focused more on learning the mathematics content first before their attempts to implement constructivist pedagogy; whereas, those student teachers with lower mathematics anxiety levels demonstrated a keen readiness to implement constructivist methods and higher personal teaching efficacy. This significant finding suggests that reducing mathematics anxiety has an order of priority: first, the mathematics content knowledge of student teachers must be addressed; second, once student teachers have a firm conceptual understanding of the mathematics content, then anxiety can be reduced and they can then proceed to gain the necessary efficacy levels to embrace mathematics knowledge for teaching (MKT). The following subsection presents specific course experiences that reduced mathematics anxiety in the current study's participants.

Course Experiences that Reduced Mathematics Anxiety

Hembree (1990), and Kelly and Tomhave (1985) noted in their research that mathematics anxiety was most prevalent in preservice elementary teachers when compared to other university discipline majors, which was borne out in this study. Fortunately, prior research suggests that preservice elementary mathematics courses can effectively reduce the anxiety of student teachers (Harper & Daane, 1998; McCulloch Vinson, 2001; Tooke & Lindstrom, 1998). The literature suggest that some of the pivotal experiences discovered in such mathematics courses include: time spent on the development of conceptual knowledge before moving to procedural knowledge; the cultivation of an understanding and nurturing classroom environment; and successful problem solving experienced by student teachers. The current study's findings confirm this literature. For example, as part of developing a safe learning environment, the researcher-instructor ensured that her student teachers felt supported in their mathematics problem solving, and that they wouldn't feel self-conscious or humiliated if they had questions, or misconceptions and/or experienced confusion about the mathematics content. In addition, considerable time was devoted to open-ended problem solving throughout the mathematics course. This led to student teachers having a better understanding of the concepts and the reasoning behind procedures.

It was evident that the mathematics anxiety of student teachers in this study decreased as their content knowledge and PMTE increased over the course. Since content knowledge was strongly influenced by mathematics anxiety, more so than PMTE, then it is reasonable to claim that mathematics course experiences ought to initially focus on the mathematics content knowledge of student teachers. This type of content knowledge must go beyond the common knowledge used for everyday mathematics. Preservice instructors must transition from common content knowledge to specialized mathematics knowledge for teaching with the focus on constructivist pedagogy (Hill & Ball, 2009). In addition, it is important to note that personal teaching efficacy in mathematics must not be ignored. The correlation found between student teachers' mathematics anxiety and personal mathematics teaching efficacy rated second strongest, therefore course experiences should also foster efficacy levels of potential teachers.

Chapter Summary

The primary purpose of this study was to investigate what experiences contributed to gains in student teachers' mathematics development in the constructs of content knowledge, efficacy, and pedagogy. This included an examination of how mathematics anxiety influenced these various constructs and an analysis of course and field experiences that supported preservice teachers' preparedness to teach mathematics through a constructivist approach. The discussion chapter was based on this study's theoretical framework which was comprised of the following theories: mathematical knowledge for teaching (MKT), constructivist learning theory, self-efficacy theory, and adult learning theory. Based on this analysis, the significant findings of this chapter emerged through the following themes: coherence between theory and practice; dimensions of mathematics efficacy; and anxiety as a barrier. The following subsections provide a brief summary of each major theme.

Coherence Between Theory and Practice - Student teachers' development of mathematics teaching capacities improve when they experience "mutually reinforcing ideas and skills" across their teacher education program (Darling-Hammond & Hammerness, 2005, p. 393). Vital to coherence is building strong partnerships between the mentor teacher and university context. This current study revealed that most student teachers merely imitated the instructional styles of their mentor teachers in order to conform to the classroom norms. As student teachers underwent the socialization process during practicum, they faced challenges of conflicting views between their practice teaching and the university`s theoretical underpinnings. If these novices were placed in transmissive oriented mathematics classrooms, then attainment of mastery and vicarious experiences were generally not realized. Due to the fact that mastery and vicarious experiences are considered powerful sources of efficacy growth (Bandura, 1977); the lack of coherence between field and university posed serious challenges for efficacy development, and ultimately mathematical knowledge for teaching. Furthermore, vicarious experiences were of great importance to those preservice teachers with low mathematics content knowledge, high anxiety, and lower efficacy levels. This group of student teachers benefited greatly from observing constructivist models first hand. On the other hand, mastery experiences were more likely to occur with those who possessed high content knowledge, low anxiety, and greater efficacy. These preservice teachers were more likely to enact constructivist strategies even when placed in traditional mathematics classrooms. The implications from these findings are that coursework experiences need to promote deeper understandings, lower mathematics anxiety, and higher efficacy to ensure that maximum impact from practicum experiences can occur.

***Dimensions in Mathematics Efficacy* -** Previous research studies have reported that highly efficacious teachers are more likely to embrace constructivist techniques such as inquiry based instruction and the use of manipulatives (Czernaik, 1990; Swars et al., 2006), while teachers with low efficacy beliefs tend to use more teacher-controlled strategies such as lecture (Czernaik, 1990). Similarly, in this study, stronger personal mathematics teaching efficacy (PMTE) was correlated to high mathematics CCK, which further gave student teachers confidence to employ constructivist strategies. As for mathematics teaching outcome expectancy (MTOE), previous research would expect a decline in MTOE due to unrealistic optimism in practicum expectations (Hoy & Woolfolk, 1990). However in this study, MTOE results remained stable over the duration of the course. A major factor for the unchanged MTOE beliefs in the current study may be attributed to how the mathematics course connected student teachers to the realities of the classroom. This involved course activities such as assessing students' authentic mathematics work, planning next steps for improvement, developing mathematics lessons which focus on problem solving, studying curriculum documents, and examining case studies about students with special needs. Hence, student teachers in this study were expectant of the challenges and complexities likely to be encountered during practicum experiences.

***Anxiety as a Barrier* -** According to Hembree (1990) and McCulloch Vinson (2001), mathematics anxiety is an emotional feeling of nervousness that individuals may possess about their limited understanding of mathematics. This anxious state can cause debilitation when solving mathematical problems within a variety of contexts. In the current research, the strongest correlation was discovered between anxiety and CCK, followed by a moderate correlation between anxiety and personal teaching efficacy. Essentially, preservice teachers who scored as having high mathematics anxiety were most likely to score low in the content knowledge assessment, and have lower efficacy scores. This group of student teachers required time to first learn the mathematics content before proceeding to conceptualize constructivist pedagogy; whereas, those student teachers with lower mathematics anxiety levels felt more confident and prepared to experiment and implement constructivist methods, mainly due to having higher levels of mathematics content. This study discovered student teachers overcame mathematics anxiety when they did not feel embarrassed about their confusion or misconceptions during collaborative problem solving activities. Consequently, these significant findings suggest that reducing mathematics anxiety might have an order of priority. First, the mathematics content knowledge of student teachers should be addressed. Once conceptual understanding is in place, then anxiety is reduced, and efficacy levels develop. Furthermore, anxiety reduction is also attained through the cultivation of a psychologically safe learning environment wherein student teachers are supported in their mathematics development free from ridicule or censure.

Based on the significant findings of this study, the exploration of mathematics content knowledge, mathematics efficacy, pedagogical decisions, and mathematics anxiety can provide important considerations for preparing potential elementary mathematics teachers. In the final conclusion chapter, several implications are examined to better understand how to prepare preservice teachers to become highly effective mathematics teachers. These implications support the need for transforming potential teachers' beliefs about teaching mathematics so they can employ constructivist practices that promote students' mathematics skills in reasoning, communicating, and solving authentic problems.

Chapter 6: CONCLUSION

Introduction

In the previous discussion chapter, this study's significant findings were described under three sections: coherence between theory and practice; dimensions in mathematics efficacy; and anxiety as a barrier. The purpose of chapter six is to examine the implications derived from the major findings based on this study's research questions. The intent of this study was to identify university program experiences that contributed to student teachers' development in mathematics content knowledge (common and specialized content knowledge), efficacy, and pedagogy, as well as to understand how anxiety impacted these various constructs. As a result of further synthesizing this study's significant findings, two models were developed by the researcher to provide programing recommendations for preservice mathematics teacher education. Model one proposes structural and programming features at a macro level, while model two addresses the critical need of coherence across all program components at a micro level. The implications from these two models will be significant for elementary mathematics teacher educators and leaders as they consider how to support the varying backgrounds of student teachers entering the preservice program.

Model one, *Preservice Elementary Mathematics Education Structure (Macro Level)* outlines the importance of mathematics common content knowledge admission requirements into the preservice program. It is recommended that the university employs a mathematics content test to identify prospective student teachers who are underprepared in their mathematics understanding. Those who fail the test must enrol in a supplementary mathematics course *prior* to the commencement of the program. The supplementary course will provide the necessary mathematics upgrading. Hence, student teachers will enter the preservice program better equipped and prepared for mathematical knowledge for teaching (MKT) development.

Model two, *Preservice Elementary Mathematics Program (Micro Level)* illustrates how program components and stakeholders intersect with each other into a coherent trajectory of mathematics teaching and learning. Coherence across preservice mathematics teacher education entails that all program features and participants share a set of core beliefs and understanding, as well as norms and practices about constructivist mathematics. This model details preservice mathematics components (i.e., university mathematics course, practicum, student teachers' mathematics development) with a shared vision and set of goals that support student teachers' reflective and transformative learning.

To best address the research questions, the subsequent sections and subsections present an overview of major findings in relation to this study's research questions. Following the overview of major findings, this chapter concludes with sections on: models for elementary preservice mathematics education; implications for elementary preservice mathematics programming; implications for further research; and a conclusion.

Overview of Major Findings

This study examined two university elementary mathematics curriculum pedagogy classes in a large urban southern Ontario university, taught by the researcher with a total of 99 student teachers. A mixed methods design using both quantitative and qualitative data collection methods was utilized. The quantitative components included instruments that measured the constructs of mathematics CCK, teacher efficacy, and anxiety. These instrumentation tools were administered at the beginning and at the conclusion of the academic year to determine if preservice teachers' levels of mathematics anxiety, teacher efficacy, and CCK had changed, and to identify any correlations between these constructs. The quantitative results were used in conjunction with the qualitative data to support overall findings. The qualitative data involved pre- and post-interviews with six student teachers, three of whom possessed high anxiety, while the other three scored low in anxiety. Another qualitative data source included reflective journals written by all the student teachers. A total of 192 journals were submitted - 96 journals in semester one, and 96 journals in semester two. The qualitative data focused on three constructs (i.e., content knowledge, efficacy, and pedagogical decisions) and specifically asked questions about student teachers' levels of mathematics anxiety. Various software tools were used by the researcher to analyze the data. Quantitative data analysis involved the use of Microsoft Excel and reliability calculators, while Nvivo 9 software was used to code the qualitative data. Based on this study's research questions, an overview of this study's major findings is outlined in the following subsections: primary research questions - one, two, and three; the academic perspective; and student teachers' perspective.

Primary Research Question - One

How do specific teaching and learning experiences in an elementary preservice mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy, and 3) pedagogy?

There were a variety of teaching and learning experiences throughout the preservice course that contributed to student teachers' development of mathematics content knowledge, efficacy, and pedagogical skills. Content knowledge refers to both CCK and SCK. One area in particular was the focus on content knowledge through problem solving activities. Overall, this study's major findings suggest that student teachers entered the preservice mathematics course with weak CCK in mathematics. As part of Ball, Thames, and Phelps' (2008) theory on mathematical knowledge for teaching (MKT), CCK is one of four domains. CCK is the mathematical knowledge that is used in a wide variety of settings and it is not necessarily exclusive to teaching (see Table 2.1 in Chapter Two for overview of MKT domains).

Student teachers' CCK was measured by a content assessment based on grade six mathematics provincial assessment questions. The average overall score for the pre-content assessment was slightly under 54% (see Table 4.4 in Chapter Four for details). Poor results in the pre-test for CCK were further challenged by the strong correlation between high anxiety scores and low mathematics content. These findings appeared to be indicative of the K-12 schooling of the preservice teachers involved in this study, in which insufficient teaching and learning occurred for developing an understanding of the most basic mathematics concepts. In order to meaningfully teach mathematics, the CCK of mathematics must first be grasped. A strong conceptual mathematics understanding is essential in the implementation of mathematics reform pedagogy (National Council of Teachers of Mathematics, 2000). Therefore the university course experiences that nurtured student teachers' CCK through various problem solving activities benefited student teachers' conceptual understanding and transformed their outlook of mathematics beyond procedures and rule memorization.

Course experiences that revolved around problem solving activities involved three distinct phases: 1) a warm up activity, 2) working on the problem, and 3) reflection and consolidation. All three phases were deemed valuable to student teachers' development of content knowledge and efficacy. In phase one, the warm up activity was presented prior to the main problem and involved activating background knowledge of key mathematics concepts. It was usually a less difficult problem that student teachers were able to solve. When student teachers were successful with the warm-up problem, their anxiety eased and they gained immediate self-efficacy to continue with a more difficult and complex problem.

During the second phase, the main mathematics task was presented. It was important that the class was provided with an uninterrupted block of time to solve the main problem. This enabled student teachers to comprehensively think through the task. According to Schmitt and Safford-Ramus (2001) preservice teachers who experience success with mathematics tasks feel a sense of accomplishment and become intrinsically motivated to continue to invest in their mathematical learning. In this current study, experiences with solving major mathematics tasks were not only critical to student teacher's content knowledge development, but also self-efficacy due to the confidence they gained in their mathematics problem solving ability.

Finally, in phase three of reflection and consolidation, preservice teachers shared their reasoning behind the strategies they used to solve the problem, and they listened to others explain their solutions. These small group discussions positively influenced preservice teachers' content knowledge because they were exposed to multiple ways of making sense of mathematics concepts. During this phase, student teachers deepened conceptual understanding of the diverse strategies used to solve the problem. This phase helped student teachers appreciate various approaches to finding solutions, and this further validated the different ways of understanding mathematics concepts.

Mathematics course experiences that were meaningful to real-life situations and that could be replicated in a typical school classroom supported student teachers' pedagogical knowledge. This involved the instructor modelling constructivist techniques that reflected mathematics reform pedagogy. At the end of every mathematics class, student teachers engaged in discussions about the explicit techniques that were modelled.

Examples of instructional strategies included: building on students' mathematics strengths, higher-order questioning, collaborative problem solving, representing concepts through various models, validating student-generated strategies, and the use of manipulatives. It was important for student teachers to comprehend how the instructional strategies could be modified for other grade levels or mathematics strands. This course experience was vital because it afforded time for student teachers to reflect on their own pedagogical decisions and expand their instructional repertoire of mathematics teaching strategies. Most importantly, it was highly beneficial when these strategies were implemented in student teachers' field placements, so they would have opportunities to observe (vicarious experiences) and enact such constructivist principles (mastery experiences).

Primary Research Question - Two

How does mathematics anxiety relate to these constructs (mathematics content knowledge, mathematics teacher efficacy, and pedagogy)?

Due to the negative ramifications anxiety has on mathematics teacher development, it was important to investigate how mathematics anxiety related to the specific mathematics constructs of this study: student teachers' content knowledge, efficacy, and pedagogy (Brett, Nason, & Woodruff, 2002; Hembree, 1990; McCulloch Vinson, 2001). This study's major findings revealed that mathematics CCK and mathematics anxiety had a strong correlation, and this relationship was stable throughout the course. Essentially, poor performance in mathematics was directly correlated to high levels of anxiety. This research demonstrated that an effective reduction in mathematics anxiety was realized through improved understanding of mathematics concepts and problem solving ability. Therefore, university course experiences that fostered CCK and problem solving abilities reduced student teachers' anxiety.

A moderate correlation was revealed between personal mathematics teaching efficacy (PMTE) and mathematics anxiety in this study. PMTE represents a teacher's belief in how effectively he or she can teach mathematics. Significant findings in this study revealed that the mathematics anxiety felt by student teachers decreased significantly while PMTE increased throughout the academic year. This was partly attributed to course experiences that enabled student teachers to confront, discuss, and reflect on their anxiety. Bandura (1997) asserted that efficacy growth can be fostered when individuals confront their emotional states, such as anxiety. This claim was endorsed in this study due to the fact that a major part of journal reflections, as well as in-class discussions, were devoted to understanding sources of mathematics anxiety. When student teachers were made aware of their anxiety scores after they completed the Revised Mathematics Anxiety Scale (RMAS), they became more mindful of their anxiety levels especially during problem solving activities. As a result of reflection and discussion about anxiety, preservice teachers with high anxiety were able to take measures in combatting such emotions, through behaviours such as deep breathing, reviewing the mathematics tasks thoroughly before attempting to solve it, and asking questions to clarify confusions about concepts.

This study investigated how anxiety was connected to pedagogical decisions. Essentially, student teachers who possessed higher anxiety felt less efficacious to enact constructivist approaches. These highly anxious preservice teachers were immersed in learning the mathematics content as a priority before they felt ready to synthesize constructivist pedagogy with content knowledge. Conversely, those student teachers with lower mathematics anxiety levels demonstrated a keen readiness to attempt constructivist strategies, as well as, displaying higher personal teaching efficacy. Hence, reducing mathematics anxiety has an order of priority; mathematics content knowledge of student teachers must be established first. Firm grounding in CCK is required in order to decrease anxiety and increase PMTE, which further supports student teachers' SCK development and mathematics reform pedagogy. This is further substantiated by the fact that this study's strongest correlation was evident between CCK and anxiety, followed by PMTE and anxiety.

Primary Research Question – Three

How do these constructs (mathematics content knowledge, mathematics teacher efficacy, pedagogy, and anxiety) relate to and/or influence one another?

This study examined the inter-relatedness of content knowledge, efficacy, pedagogy, anxiety, and its influences on the development of student teachers' capacities as mathematics learners and teachers. In previous literature only one construct or the relationship between two have been researched. Previous studies also claim that all constructs are valuable in the development of mathematics teaching. This current study, however, went beyond the single construct focus by investigating the inter-relatedness between the constructs and their impact on student teachers' mathematics preparedness for teaching. Significant findings from this study revealed how the relationships among and between these constructs should be considered when making informed programming decisions for preservice university mathematics courses. Understanding the varying levels of student teachers' mathematics content knowledge, mathematics anxiety, efficacy levels, and pedagogical knowledge can support the differentiation of instruction in order to maximize prospective teachers' mathematics development.

Bandura (1997) argued that existing self-beliefs affect what people "look for, how they interpret and organize the efficacy information generated in dealing with the environment, and what they retrieve from their memory in making efficacy judgments" (p. 81). Bandura's claim is confirmed in this study wherein student teachers' efficacy sources were supported differently depending on their prior confidence in mathematics content knowledge and anxiety levels. This study revealed that preservice teachers with high content knowledge, most likely had low anxiety and greater efficacy; and they benefited most from mastery learning experiences. Bandura claimed that mastery experiences are actual performances of a task that are perceived to be successful and attributed to the individual's own efforts and abilities. This group of student teachers were engaged in mastery experiences of successful mathematics teaching. The researcher attributed the mastery experiences of these student teachers to strong confidence in mathematics content and constructivist models of pedagogy.

Conversely, student teachers in the group with less favourable characteristics of low content knowledge, high anxiety, and weaker efficacy, expressed the need for more vicarious experiences in order to support their pedagogical knowledge of constructivist teaching. Bandura asserts that vicarious experiences occur when exemplary models are observed and these observations contribute to efficacy development. In the case of mathematics teaching development, examples of vicarious experiences during this study included videos, course activities, and field placements that enacted effective models of constructivist pedagogy. It appeared that this group of student teachers required vicarious experiences due to low confidence levels to fully employ constructivist models themselves. Therefore, the inter-relatedness of the constructs demonstrated the diverse needs of student teachers in mathematics courses and has critical implications for professors' programming features, decisions, and approaches. In terms of the constructs of efficacy, content knowledge, and anxiety, it was evident that all of these strongly influenced student teachers' pedagogical decisions.

Academic perspective:

a. How can academics support student teachers' development to positively influence:

1) mathematics content knowledge;

2) mathematics teacher efficacy; and

3) pedagogy?

The academic perspective offered important insights into the critical role university mathematics educators play in developing preservice teachers' content knowledge, efficacy, and pedagogical skills. The instructor examined how her disposition was instrumental in supporting her preservice teachers' mathematics content knowledge, efficacy, and pedagogical skills. Due to the fact that anxiety negatively impacted the development of the various mathematics constructs, it was critical that the instructor was sensitive to the anxiety felt by her student teachers. This required the instructor to be understanding of the debilitating effects of mathematics anxiety. Through a warm, non-intimidating, and supportive delivery, she cultivated a nurturing and psychologically safe classroom environment. Specifically, a caring community was established through the co-creation of the following classroom norms:

- during problem solving activities, be cognizant about how your comments might make others feels, for example, refrain from comments that boast about easiness, such as "that was easy" or "pretty simple," and refrain from comments that boast about timing, "I'm all done" or "that was quick to solve";
- everyone be accountable for a mathematical opinion;
- listen and try to make sense of one another's mathematics reasoning; and
- always feel safe to ask questions.

These norms supported a non-competitive learning environment, where preservice teachers were respectful of the varying levels of mathematics knowledge. It was critical that the instructor demonstrated a nurturing demeanor so that student teachers did not feel embarrassed about mathematical misconceptions or confusions. As a result, anxiety dissipated over the course, while content knowledge and efficacy levels increased.

The quality of instruction proved to be an important factor in supporting student teachers' mathematics development. It was valuable when the instructor explained mathematical concepts comprehensively through various representations as this fostered student teachers to conceptualize the mathematics being taught. High quality instruction was also grounded in the modelling of effective constructivist methods that could easily be replicated in a typical school classroom. More importantly, it was critical that the instructor spent time debriefing how and why such reform strategies were effective, in order for student teachers to set their own pedagogical goals.

One other aspect that encompassed the quality of instruction was the pace of teaching. Student teachers with lower content knowledge, higher anxiety, and lower efficacy levels appreciated when the instructor presented the material at a slower pace. Unfortunately, attempts to remediate student teachers during regular class became challenging due to the lack of time to cover all the course content. As a remedy for this challenge, the instructor provided additional mathematics content to review prior to class, as well as, optional mathematics sessions during lunch, so student teachers had multiple opportunities to understand the content. Consequently, for many student teachers' self-efficacy levels were increased because they felt adequately prepared for the in-class mathematics activities. Hence supplementary support outside of class was very helpful and it was not confounded by time constraints that often occurred during in-class remediation. Nevertheless, the instructor was cognisant that the additional mathematics support beyond the class added to a heavy workload for student teachers.

Student teachers' perspective:

a. What learning experiences impacted student teachers' development of:

1) mathematics content knowledge;

2) mathematics teacher efficacy; and

3) pedagogy?

The student teachers' perspective provided an understanding of the course experiences that were motivational in improving their mathematical knowledge for teaching. A turning point for student teachers' mathematical development transpired when student teachers' felt they belonged to an emerging mathematics community in which they perceived themselves to be supported. Student teachers in this study revealed that working in small groups of heterogeneous mathematics abilities contributed to their development of content, efficacy, and pedagogical knowledge. These groupings were strategically planned based on the CCK results ascertained at the beginning of the year. Each group comprised five or six student teachers, with a minimum of two student teachers identified with medium to high common content knowledge per group. Groupings were changed three times over the course of the academic year, approximately every three months. Based on adult learning theories (Knowles, 1884; Mezirow, 1991) and cooperative learning principles (Slavin, 1995; Sollivan, 1984), the main goal of these mixed ability groups was to create opportunities for respecting the achievement of all group members. The expectation of the small groups was to enhance student teachers' mathematical achievement by means of working towards a common goal during problem solving activities. Participants were informed about the detriments of relying on the mathematical skill of one member, or if one or two members dominated the activities.

While working in small groups, student teachers freely explored mathematical activities and eagerly shared successful pedagogical strategies that they saw in practicum or other places. The small groups fostered a mathematics community because student teachers felt supported in three major ways. First, when student teachers with low CCK experienced difficulty with a mathematical problem, they appreciated that their group members gave them uninterrupted time to work through the question on their own. Student teachers did not feel that their knowledge was under evaluation which promoted the reduction of mathematics anxiety. Second, all group members shared their strategies for solving the problem. Student teachers asked questions about each other's problem solving methods which further demonstrated a sincere interest in each other's learning styles. Third, student teachers with low CCK did not feel embarrassed to ask their group members for clarification about mathematical concepts; while student teachers with stronger CCK gained confidence in their approaches in explaining and representing mathematical concepts. Therefore, working in small groups based on cooperative learning principles strengthened positive relationships due to the fact that student teachers supported one another with the collective goal of becoming effective mathematics teachers.

Models for Elementary Preservice Mathematics Education

Efforts to reform the teaching of mathematics have been a major focus of study over the last few decades (Ball, Sleep, Boerst, & Bass, 2009; National Council of Teachers of Mathematics, 2000). The literature has noted that mathematics teacher education may have a weak effect on preservice teachers' mathematical knowledge and beliefs (Ball, 1988; Ball, Lubienski, & Mewborn, 2001; Zeichner & Tabachnick, 1981). These researchers purport that by the time prospective teachers begin their professional careers, they often relapse back to the traditional ways of mathematics teaching; the methods that they themselves were exposed to during their own K-12 schooling experience. Student teachers have typically experienced transmissive oriented methods of mathematical learning throughout their educational career, "which not only has instilled traditional images of teaching and learning but also has shaped their understanding of mathematics" (Ball, Lubienski, & Mewborn, 2001, p. 437). Therefore, it is imperative that teacher preparation programs transform prospective teachers' understanding of mathematics.

Two models associated with elementary preservice mathematics education were created based on the significant findings of this study. The models propose ideal features, core ideas, and structures for elementary mathematics teacher education. The model for *Preservice Elementary Mathematics Education Structure (Macro Level)* (see Figure 6.1) necessitates admission requirements for prospective student teachers to pass a mathematics CCK test. The research evidence of this study indicates that the development of CCK is related to increased efficacy, anxiety reduction, and provides a reasonable level of preparedness for MKT. Therefore, a preservice university structure such as the one depicted in Figure 6.1 will identify student teachers who possess weak CCK prior to program commencement so that necessary CCK upgrading can be provided for such candidates. The second model, *Preservice Elementary Mathematics Program (Micro Level)* (see Figure 6.2) examines how coherence across all program components is foundational in student teachers' mathematics teaching development. This model demonstrates when theoretical underpinnings, core ideas, practices, and experiences are coherent across all program structures, it promotes learning MKT in ways that are suitable and meaningful.

Figure 6.1: Model for Preservice Elementary Mathematics Education Structure (Macro Level)

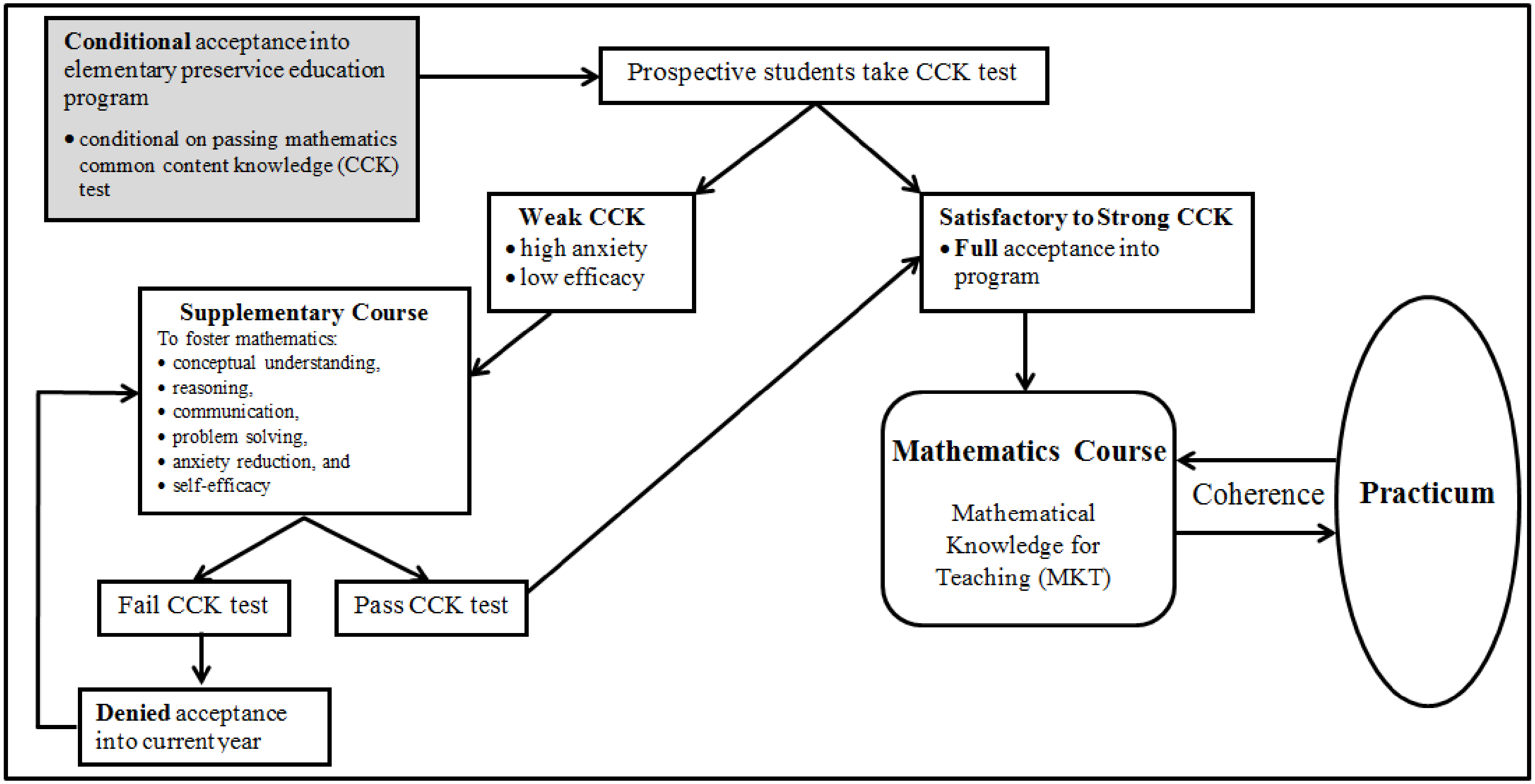

Figure 6.1. This model proposes entry requirements of mathematics common content knowledge (CCK) for elementary preservice education programs. University program acceptance is conditional on passing the mathematics CCK test. The research evidence of this study indicates that the development of CCK is related to increased efficacy, anxiety reduction, and provides a reasonable level of preparedness for MKT development. Therefore, a preservice university structure such as the one depicted in this model will identify student teachers who possess weak CCK prior to program commencement. Student teachers who fail the CCK test must take a supplementary course before the regular program begins. The supplementary course strives to provide the necessary mathematics development for program acceptance. If prospective students fail the CCK test again after completing the course, acceptance into the preservice program will be denied for the current year. Towards the right of the model demonstrates coherence between practicum and the mathematics course as essential for MKT to develop successfully through the application of constructivist teaching strategies.

Model for Elementary Preservice Mathematics Structure (Macro Level)

A central underpinning of this model involves the critical need to first assess prospective students' common content knowledge (CCK) *before* the academic year begins. Based on the findings of this study, CCK was an essential foundation in planning and implementing constructivist mathematics university lessons. With weak CCK, student teachers frequently have difficulty problem solving due to limited conceptual understanding, are unable to explain how and why mathematical procedures work, make calculation errors, and mispronounce mathematical terms. Essentially, CCK necessitates student teachers to do the mathematical problem solving that that is expected by their students. Without this basic knowledge, mathematics instruction suffers. Weak CCK is also connected to higher levels of mathematics anxiety and lower levels of personal teaching efficacy. Mathematics anxiety is associated with mathematics avoidance, which further exacerbates the problem of low content knowledge.

Based on the importance of student teachers' CCK, the model proposes that a satisfactory level of mathematics CCK is required for program acceptance. As part of the preservice program structure, it is recommended that prospective students who not pass the mathematics CCK test enrol in a supplementary mathematics course. It is important to note that this model does not represent a linear process in that CCK is developed and never addressed again. The process of achieving successful mathematical knowledge for teaching (MKT) domains is not a step-by-step sequence in that CCK is learned first, then the second MKT domain is next, etc. Instead, the model depicts that the trajectory for MKT attainment is best when CCK is strengthened. The CCK continually develops throughout mathematics teachers' careers at deeper levels, in conjunction with the other MKT domains. This model suggests that a minimal level of CCK is compulsory before enrolling in the university mathematics course. Therefore, student teachers with weak CCK require intensive additional support in their conceptual understanding of mathematics. Once stronger CCK is established, preservice teachers' development of the remaining three MKT domains will be more effectively attained. The following subsections provide details about the model's features: sequence of CCK test and supplementary course; content of CCK test and minimal pass requirements; addressing mathematics anxiety and low teacher efficacy; and mathematical knowledge for teaching and practicum.

Sequence of CCK Test and Supplementary Course

The issue of how to support student teachers with low mathematics CCK has been longstanding within preservice mathematics teacher education. Student teachers with weak CCK often become frustrated and even discouraged at the pace and content level of preservice mathematics courses. Attempts to remediate during class time is challenging due to large class sizes and the varying needs of learners. Implementing principles of cooperative learning (e.g., strategically placing student teachers in heterogeneous ability groups) and cultivating a safe learning environment can support the low CCK of student teachers. Additional assistance provided outside of regular class time is also effective (e.g., extra mathematics classes during lunch and giving student teachers mathematics material to review prior to class). However, intensive CCK supports to student teachers during the academic year can add stress to an already heavy workload. For this reason, this model recommends CCK testing *before* the start of the program in order to identify and immediately support prospective students who require remedial CCK preparation. With this timeline, students with weak CCK will have acquired some sufficient content knowledge and therefore feel more confident and prepared at the onset of the mathematics course.

It is suggested that the supplementary mathematics course runs for approximately three weeks before the regular program begins. The major goal of the course is to engage prospective students in intensive learning activities that foster their CCK growth. Supplementary courses should be available over the summer and at night at a number of locations. Furthermore, the courses must be delivered in ways that reflect the nature of adult learners of mathematics and class sizes should be capped at 20 students. The training of faculty who teach the supplementary courses should focus on understanding mathematics anxiety and ways to remedy such emotional states. Once prospective students complete the course, they must take the CCK test again. If they pass, then they are offered full acceptance into the university's preservice program. If prospective students fail, then they are denied acceptance for the current year and are given the option to take the test again the following year. Furthermore, for some student teachers who take the supplementary course and pass the CCK test, they may continue to feel they require supplementary CCK support throughout the academic year. Mathematics instructors should provide additional supports to these student teachers, though these supports will probably not be as intense due to the supplementary course experience previously taken.

Content of CCK Test and Minimal Pass Requirements

The CCK test requires careful development and ongoing review to ensure that it assesses what it is intended to test, that is, the CCK of prospective students. Because this study's location took place in an Ontario university, it is recommended that the CCK test questions are modelled on the Ontario provincial mathematics assessments, developed by the Education Quality and Accountability Office (EQAO). This research examined prospective teachers in the primary/junior division working towards qualifications to teach from kindergarten to grade six. Graduates of this program are not qualified to teach beyond grade six, therefore it is recommended that the CCK test assesses mathematics skills at the grade six level. The content of the CCK test should cover grade six provincial curriculum objectives. Prospective students should be offered online practice tests to review basic mathematical concepts.

It is proposed that the minimal pass requirement be 75% as this is considered within the Ontario provincial standard mid-range. In other words, when Ontario grade six pupils score between 70% to 79%, they have demonstrated most of the required knowledge and skills and their achievement meets the provincial standard (EQAO, 2013). An assessment score of 80% or higher indicates achievement beyond the provincial standard, but still within the grade six level. Consequently, a minimal pass score of 75% from prospective teachers will ensure that they understand basic mathematical concepts of the content they are required to teach.

Addressing Mathematics Anxiety and Low Teacher Efficacy

By determining the CCK levels of prospective students prior to the program, imperative information is also revealed about student teachers' anxiety and teaching efficacy towards mathematics. Low content knowledge is strongly correlated to mathematics anxiety and moderately correlated to mathematics teaching efficacy. Of most importance is that this correlational data offers significant insights into the pedagogical decisions student teachers make during practicum. Student teachers with lower CCK feel uncertain about their potential effectiveness in teaching mathematics. This is often illustrated by requests to teach only primary grades (kindergarten to three), or to avoid teaching mathematics altogether. Unfortunately, low CCK in student teachers implicates traditional teaching methods because it enables teacher-controlled methods. Furthermore, student teachers often believe that traditional rule-based teaching promotes the illusion of teacher expertise through rule memorization. This model shows that by strengthening student teachers' CCK, anxiety decreases and personal teaching efficacy improves due to the fact that confidence is gained through deepened understanding of the mathematics curriculum content.

Critics might question whether high-stakes testing in which the outcome determines acceptance into the program might cause further mathematics anxiety in those with low mathematics content knowledge. This model argues that those who fail the CCK test will benefit tremendously from the supplementary course. It is highly recommended that practice tests be made available and as adult learners of mathematics the onus must be left on prospective students to review, study, and work towards mathematics understanding. The supplementary course will prepare prospective students to achieve higher levels of CCK through activities that support mathematical reasoning, and solving problems in a variety of ways, rather than applying rote formulas that may not make sense. In the current preservice mathematics structure, the supplementary course is non-existent, and student teachers with low CCK enter the mathematics course with high anxiety. Great efforts and supports are put into place to remedy the anxiety; however, for most student teachers anxiety peaks at the beginning of the year and does not dissipate until after first practicum, despite all the efforts to diminish anxiety. Therefore, this model proposes that the supplementary course will reduce anxiety and provide prospective students with the necessary mathematics upgrading before entering the preservice program.

Mathematics Knowledge for Teaching and Practicum

This model demonstrates the importance of developing each of the four domains of mathematical knowledge for teacher (MKT) (see Table 2.1 in Chapter Two). The domains of MKT include: CCK, specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). It is worth noting that MKT development is not a linear progression in that each domain is taught sequentially. Rather, student teachers should experience learning opportunities within all four MKT domains throughout the academic year.

A vital component that is illustrated in this model is practicum experience. Goal setting and lesson planning for practicum teaching in schools will enable student teachers to apply their MKT in an authentic context. Coherence between practicum and course experiences amplifies MKT as student teachers employ this knowledge into practice, whereas incoherence can stifle MKT development. The concept of coherence is further outlined in Figure 6.2. The following section presents the second model for elementary preservice mathematics, which examines the details of coherence through a micro level.

Figue 6.2: Model for Preservice Elementary Mathematics Program (Micro Level)

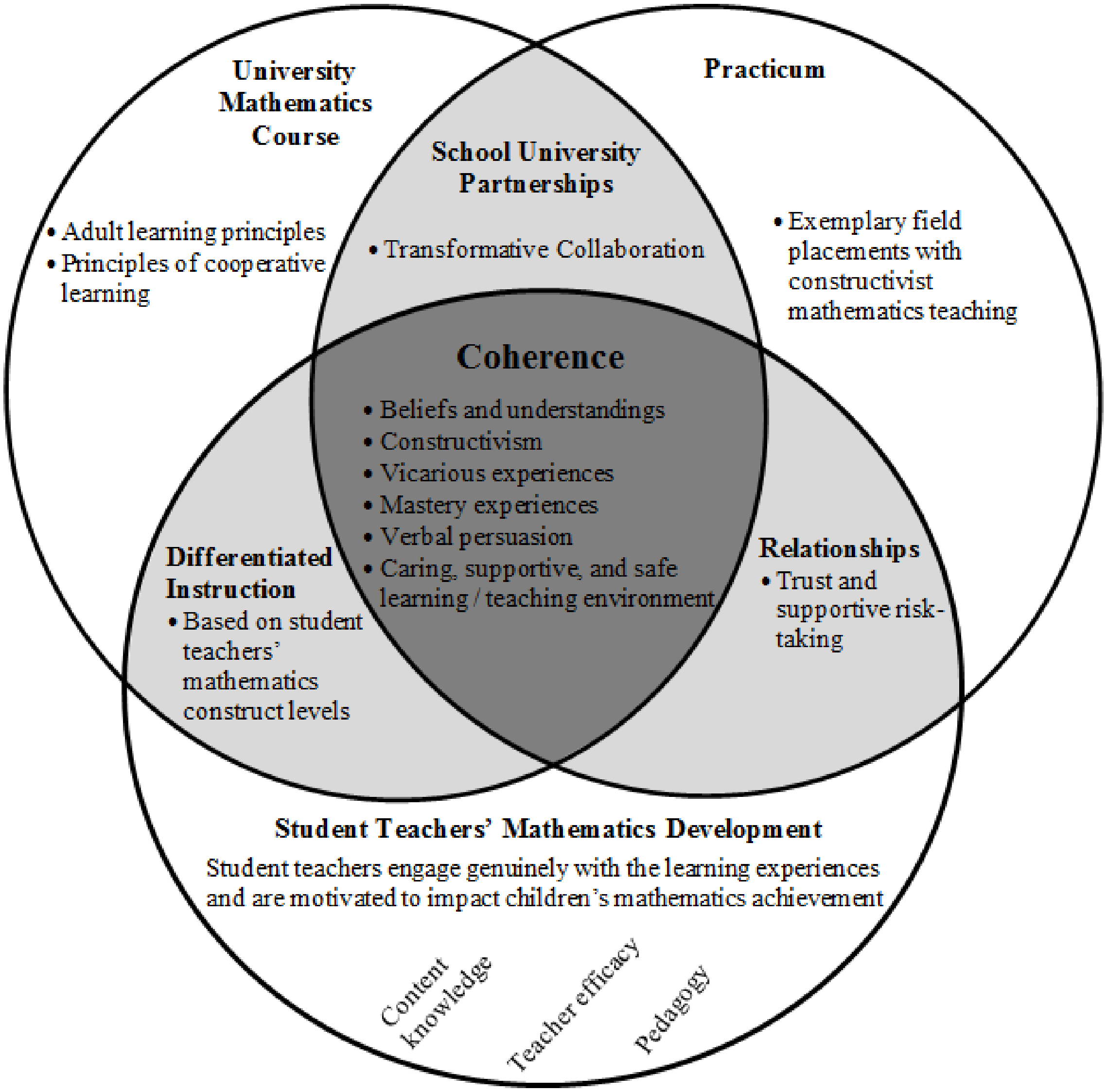

Figure 6.2. This model illustrates coherence across the mathematics teacher education program as a foundation upon which student teachers are enabled to engage in transformative learning and develop the necessary skills and beliefs to become effective mathematics teachers. There must be strong coherence between and among the university mathematics course, practice teaching in schools, and student teachers' goals and efforts in mathematics teaching. The model is a Venn diagram which illustrates important overlays that create seamless experiences of learning to teach mathematics effectively. Each section demonstrates the important responsibilities of all stakeholders involved in mathematics teacher education. At the centre of the model are coherent theoretical underpinnings, core ideas, and experiences that enable student teachers to synthesize their mathematics teaching and learning.

Model for Preservice Elementary Mathematics Program (Micro Level)

The model depicted in Figure 6.2 considers coherence across elementary mathematics teacher education as foundational to student teachers' MKT development. It is strongly recommended that coherence-building in preservice mathematics programs be a priority goal for all stakeholders involved. With coherence, prospective teachers have ongoing opportunities to synthesize their mathematics learning from many aspects of their preservice education program. Figure 6.2 illustrates coherence through a Venn diagram. It highlights three major components of preservice mathematics: university mathematics course, practicum, and student teachers' mathematics development. The important overlaps demonstrate how the overall mathematics preservice program is supported and strengthened due to a coherent purpose and shared understandings about mathematics teaching and learning. At the centre of the model are coherent theoretical underpinnings, core ideas, norms, and experiences that enable student teachers to engage in transformative learning as they strive towards becoming effective mathematics teachers. The subsections below describe in detail the various overlay segments of the model: school and university partnerships, relationships, differentiated instruction, and coherence attainment.

School University Partnerships

As schools and university come together to support preservice teachers' MKT development during practice teaching, it is important to form strong partnerships to reach coherence. However, stakeholders from both institutions come from different cultures which can pose fragmented and disjointed ideologies. Under the current preservice conditions, each institute dominantly works separately from one another because each is structured by different forms of governance, reward systems, and priorities; and this further exacerbates the great divide. Notwithstanding, one commonality is that both institutions have ultimate goals in student achievement. This model therefore proposes the critical need for strong partnerships between school and university through collaborative efforts in developing preservice teachers' MKT capacities.

Teitel (2008) uses the term transformative collaboration, where universities and schools "retain their identities but are willing to learn from and with each other. Partners approach common purposes. . . . more collaboratively and with a greater willingness to explore deeper changes in practice" (p. 28). A reconceptualization of ownership emerges from transformative collaboration, in which success or failure of student teachers become a united responsibility. Therefore this model recommends that stakeholders from schools (e.g., mentor teacher, principal, and school liaison) regularly meet with university officials (e.g., practicum supervisor, mathematics course instructor) to collectively build a common understanding of mentorship and constructivist teaching and learning in mathematics. In doing so, all stakeholders should feel a mutual desire to learn from each other, with the common goal to impact student teachers' MKT growth.

Relationships

Figure 6.2 emphasizes the critical relationship between the mentor teacher and the student teacher. Trust and supportive risk-taking are essential in building positive relationships. Mentor teachers play a vital role because they offer exemplary models of constructivist mathematics teaching in their classrooms. In this model, trust is realized when mentor teachers go beyond 'telling' student teachers what to do in mathematics lessons. Rather than simply promoting the imitation of instructional styles, effective mentor teachers encourage their student teachers to construct their own knowledge, and question relationships between theory and mathematics practice. Trusting relationships also involve mentor and student teachers sharing resources and engaging in relevant discourse about mathematics teaching. Mentor teachers nurture risk-taking by allowing student teachers to dialogue, question, and trial mathematics teaching strategies without fear of reproach. Mentor teachers strengthen the relationship through effective mentoring practices such as observing their student teachers teach mathematics, and giving them explicit and timely feedback to reflect upon and refine for future teaching. Furthermore, mentor teachers also learn from their student teachers as they collaboratively teach, assess, and plan mathematics lessons. Together, the mentor and student teachers' willingness and desire to improve mathematics instruction strengthen their relationship.

Differentiated Instruction

This model highlights the critical importance of differentiated instruction of the university mathematics course based on student teachers' prior understandings and attitudes towards mathematics. This study's significant findings demonstrate the inter-relatedness of mathematics content knowledge, efficacy, anxiety, and pedagogical decisions. When mathematics instructors take into account the varying construct levels of their student teachers, they can target instruction and thereby maximize prospective teachers' preparedness to teach mathematics for understanding. Furthermore, student teachers should be aware of their levels of content knowledge, anxiety, efficacy, and pedagogical beliefs, so they can critically self-reflect on how these constructs influence their MKT development.

Figure 6.2 highlights adult learning principles in that student teachers must engage genuinely in the learning experiences of the mathematics university course and be accountable for their mathematics growth. It is critical that student teachers are motivated by a deep desire to impact children's mathematics achievement. In addition, principles of cooperative learning theory should be employed during course activities, as this supports the cultivation of a caring and supportive mathematics learning community.

Coherence Attainment

One serious challenge for university preservice programs is the attainment of coherence, not only within its own institutional structures, but also building coherence with partners in the field. At the core of Figure 6.2 is a representation of coherence attainment, in which three major program components intersect (i.e., university mathematics course, practicum, and student teachers' mathematics development). This model suggests that coherence is ideally considered from a variety of perspectives, such as: beliefs and understandings, theoretical underpinnings, norms, practices, course and program structure, and curriculum. The outcome of coherence attainment creates a seamless experience of learning to teach mathematics effectively, in which each aspect of the preservice mathematics program reinforces and strengthens student teachers' transformative learning.

This model emphasizes the coherence of efficacy sources. Student teachers' efficacy development must include vicarious and mastery experiences, as well as verbal persuasion. Student teachers should be immersed in environments, whether it be in school or university mathematics course, in which they observe exemplary mathematics teaching (vicarious experiences); successfully employ constructivist mathematics strategies (mastery experiences); and are encouraged to trial innovative mathematics lessons (verbal persuasion). While these types of efficacy-building experiences are consistently practiced across the preservice program, student teachers can synthesize their understanding of MKT upon which they can build a reflective and transformative practice.

Implications for Preservice Elementary Mathematics Education

This study's findings offer thought provoking implications for mathematics teacher preparation programs. The implications from the two models, *Preservice Elementary Mathematics Education Structure (Macro Level)* and *Preservice Elementary Mathematics Program (Micro Level)* highlight the critical importance of university structure and program design to ensure a transformative learning experience for student teachers' MKT development.

In model one, the implications of the high-stakes entrance CCK test are substantial. The mathematics content knowledge admission requirements will change the landscape of mathematics teacher education. University structures should support compulsory basic mathematics knowledge for program acceptance because the mathematics CCK of elementary preservice teachers must be sufficient enough for successful development of MKT capacities. This will raise the significance of teaching mathematics for understanding and increase accountability for all mathematics educators. With this university program structure in place, mathematics teachers will be forced to critically re-evaluate and question their teaching practice. Furthermore, careful attention and research should be given to the CCK test design and the supplementary course design in that the content must be reviewed rigorously for potential bias and concerns of anxiety.

The implications from model two demonstrate the need for more resources and supports to focus on coherence building between and among three components of mathematics preservice programs (i.e., university mathematics course, practicum, and student teachers' mathematics achievement). This can only be attained when all stakeholders involved with mathematics preservice education collaboratively set goals and purposes specifically designed to improve student teachers' mathematics development as learners and teachers. Resources that support transformative collaboration between the university and school will provide student teachers opportunities to learn mathematical content and practice teaching mathematical content in authentic constructivist contexts.

Implications for Further Research

Further research is needed in field placement decisions that support the successful matching of student teachers and mentor teachers. Mentor teachers who have the willingness and desire to learn with their student teachers promote a positive practicum, and build coherence between university and classroom (Campbell & Brummett, 2007; Zeichner, 2002). The model for *Preservice Elementary Mathematics Program (Micro level)* (see Figure 6.2) recommends mentor teachers whose classrooms embody exemplary mathematics programs. Unfortunately, the current realities pose a lack of highly skilled mentor teachers willing to relinquish control of their classrooms to student teachers; as this might possibly disrupt well established routines or even jeopardize provincial/state test scores. (Darling-Hammond, Chung, & Frelow, 2002; Goddard, 2004; Parot-Juraska, 2009). Therefore, further studies should be conducted on how student teachers' MKT development is impacted by the varying degrees of constructivist teaching and differing mentoring styles and abilities. Hence, findings from this research would support preservice instructors' decisions on how to best pair up mentor teachers and student teachers.

The model for *Preservice Elementary Mathematics Education Structure (Macro Level)* requires further research in the area of admission requirements for CCK capacities. Kajander's (2010) research on preservice teachers' conceptual knowledge led to mathematics program changes at Lakehead University located in northern Ontario, Canada. Essentially, Lakehead student teachers must pass a mathematics exam (minimum of 50%) as a requisite for passing the methods course. A promising finding from Kajander`s research reveals that those preservice teachers who took part in an optional supplementary course during the academic year (approximately 20 hours) successfully passed the exam, whereas the few student teachers, who failed the exam, did not take the optional course. In other institutions, high stakes numeracy tests for student teachers were introduced years ago in England and Wales. Similar to Lakehead's program, the numeracy tests in England and Wales were *exit* requirements in that student teachers were required to pass during their teacher preparation year. However more recently, in September 2012, England's department for education tightened the parameters of numeracy requirements. The numeracy test is now an *entry* requirement. Students are denied acceptance into the program if they fail the basic numeracy test three times. Prior to 2012, candidates were allowed unlimited attempts while in their teacher preparation program. The research literature around England's and Wales' admissions testing is mixed. Some researchers warrant government requirements for numeracy testing due to the relationship between weak numeracy skills and poor planning and teaching (Goulding, Rowland, & Barber, 2002). Whereas, others have criticized the mathematics requisites and claim that the tests unfairly target middle-aged white women and individuals of colour (Hextall, Mahony, & Menter, 2001). Further research is needed in the effects of the actual test and how the test-taking experience impacts potential student teachers, especially those with high anxiety. It would be important to conduct research in the supplementary course support that this current study's model proposes to be of tremendous benefit for prospective students with weak mathematics content knowledge.

Another area that warrants further research involves the domains of MKT. Part of the complexity of MKT is distinguishing where the boundaries exist between each of the domains. "It is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions" (Ball et al., 2009, p. 403). The transition from one domain to another is not always exact and more research is required to further refine the definitions and deeply understand what each domain looks like in actual classroom practice. In addition, it would be important to identify when university course experiences represent an overlap of MKT domains. Thus, this would support preservice mathematics instructors to implement a balanced program that fosters student teachers' MKT.

Conclusion

It is critical that mathematics preservice instructors understand the differing needs of student teachers; and to expect the identical mathematical growth from all preservice teachers is unrealistic. A 'one size fits all' preservice mathematics program does not provide student teachers to develop at their own pace, nor does it fulfill their varying needs. Mathematics instructors have a responsibility to tailor their course structure based on their preservice teachers' entry points in mathematics content knowledge, efficacy, anxiety, and understandings of constructivist teaching so that course experiences are learner-centered and meaningful. Instructional decisions based on this assessment information allow for differentiated trajectories for student teachers' mathematics teaching development; thereby enabling student teachers to work toward more successful constructivist based instruction.

Preservice mathematics education programs must understand the significance of student teachers' CCK at the beginning of the academic year. This study demonstrated that CCK was correlated to anxiety and personal teaching efficacy, which further influenced pedagogical decisions. Students with higher CCK were better prepared to embrace activities that promoted specialized content knowledge (e.g., assessing students' mathematics responses and planning next steps for improvement). The model for *Preservice Elementary Mathematics Education Structure (Macro Level)* (see Figure 6.1), proposes that universities must acquire the means to identify prospective students with low CCK, in order to offer them meaningful opportunities for mathematics upgrading *before* the preservice program begins. It is also important to note that supplementary CCK support may continue into the academic year for those preservice teachers who feel they still need it. This model is intended to support university leaders of mathematics teacher education and university policy makers to employ structures that support minimal CCK admission requirements for preservice programs.

The main premise of this study was to address the problem of elementary mathematics programs dominated by transmissive oriented methods such as teacher-centeredness, rule memorization, and reliance on text books and work sheets. In an attempt to stop the vicious cycle of teachers teaching traditional mathematics much in the same way they were taught, the model *Preservice Elementary Mathematics Education Program (Micro Level)* (see Figure 6.2), was created to illustrate coherence attainment. This model is intended for mathematics instructors, mentor teachers, as well as student teachers. However, other individuals may also be interested in using this model, such as university policy makers, university curriculum leaders, practicum school principals, and researchers. Figure 6.2 demonstrates the critical importance of coherence across preservice mathematics education for instituting positive change. With this in mind, student teachers will experience transformative change in their beliefs and understanding of mathematics teaching. The outcome will be teachers who employ constructivist teaching strategies that foster children's deep understanding in mathematics; thereby putting a halt on the vicious cycle.

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APPENDICES

Appendix A.1: Reflective Journal Assignment - Description and Rubric

(Excerpt from Course Outline)

Reflective Journal Assignment – Elementary Mathematics Course

#1 Culminating Reflection Journal –term one 20% due: December, 17, 2010
#2 Culminating Reflection Journal –term two 20% due: April 11, 2011

Submit your culminating journal through the course's electronic drop box – attach a word file
Title your file name:
#1) student_nameTerm1
#2) student_nameTerm2

Approximate Pages: 3
Double Space, Times New Roman Font, 12 Point

Purposes of the Assignment

- To support the development of preservice teacher candidates by encouraging reflection on one's own practice and beliefs pertaining to mathematics instruction
- To build meta-cognitive awareness of how one thinks and learns about mathematics, and further transfer this into classroom practice
- To encourage critical thinking through the examination of assumptions underlying thoughts and actions in mathematics teaching and learning
- To explore emotional reactions to problem situations in mathematics development and understand how feelings may impact beliefs, behaviours, and actions
- To recognize accomplishments and reflect on personal development of mathematics content, teaching strategies and confidence.
- To give preservice teacher candidates a voice by describing, in their own words, the changes they are experiencing in mathematics teaching and learning

Description of the Assignment

In a mathematics journal, record your reflections, thoughts and ideas after each mathematics class (for most sessions, time will be given to you at the end of each class to work on this). You are encouraged to write about how you made sense of mathematics concepts and pedagogical practices as well as difficulties you might have encountered. At the end of term one, in December, you will compose a culminating reflective journal that includes the **significant experiences synthesized** from your regular journal entries. At the end of term two, you will again complete a culminating reflective journal that synthesizes your mathematics development of the second half of the year.

Guiding questions to facilitate your journal:

1) What specific course experiences do you believe supported your development as a mathematics teacher?
2) Please comment on each of the following areas as to how each influenced your mathematics development. Give examples of class activities, instructor behaviours, and specific incidents if at all possible.

Content Knowledge of Mathematics – refers to the breadth and depth of the mathematics knowledge possessed by individuals. Examples include: curriculum expectations, specialized understanding of the five strands in mathematics for teaching (1. number sense and numeration, 2. patterning and algebra, 3. geometry and spatial sense, 4. data management and probability, and 5. measurement), how the strands intersect, and conceptual verses procedural understanding

Pedagogical Knowledge – refers to instructional strategies and approaches to teaching. Examples include: students' learning styles, use of manipulatives, modeling, think alouds, conferencing, lesson planning, problem-based learning, technology integration, and student-focused verses teacher-focused techniques.

Mathematics Teacher efficacy –refers to two dimensions. First, it represents teachers' beliefs about their skills and abilities to be an effective teacher. Second, it represents teachers' beliefs that effective teaching can impact on student learning regardless of external factors.

Mathematics Anxiety – refers to an emotional feeling of nervousness, worry and/or apprehension that individuals may have about their ability to understand, perform mathematical functions and/or explain problems.

Instructor – Instructors' role, style, qualities, and characteristics

For your journal writing, consider the following aspects of the mathematics course. You may want to think about your mathematics development in different aspects of your life - personal, academic, as well as professional.

- Comments and reflections of mathematics class activities - their usefulness, enjoyment, effects on practice, and effects on you;
- Your professional goals as a mathematics teacher and how class activities influenced them;
- Your expectations, attitudes, values, beliefs, knowledge, skills in mathematics
- Significant events during the course and its relevance to your decisions, insights,
- Comments on relationships among professional practice, academic work, and personal life
- Your changes and/or developments in beliefs and assumptions about mathematics teaching and learning.

Your reflective journals will be assessed using the attached rubric.

Reflective Journal Rubric

Criterion	Level 1 D – D+	Level 2 C – C+	Level 3 B – B+	Level 4 A – A+
Reflected on how the experiences and activities from mathematics classes influenced 1) content knowledge, 2) strategies for teaching, and 3) confidence and beliefs in mathematics	-provided *limited* demonstration of criterion	-provided *some* evidence of criterion	-demonstrated criterion with *considerable effectiveness*	-demonstrated criterion with *thorough effectiveness*
Identified one's own assumptions about mathematics teaching and learning Described how these assumptions might have been challenged, questioned and/or changed	-focus of reflections were unclear and unconnected	-focus of reflections were somewhat clear with limited connections	-provided detailed reflections	-provided thoroughly detailed reflections
Made connections between personal and/or professional experiences to mathematics course materials/activities	descriptions of events and reflective thought were limited	-descriptions of events and reflective thought were somewhat effective	-focus of reflections were clear and connected -descriptions of events and reflective thought had considerable details	-focus of reflections were in-depth and well connected -descriptions of events and reflective thought had thorough details and insights

Instructor's feedback notes:

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Appendix A.2: Individual Interview Protocol and Questions – Student Teachers

PRE-INTERVIEW – September, 2010

This interview will be conducted by an external third party individual (teacher assistant from another subject area).

The following is the script and questions that the interviewer will be trained to deliver.

Hi [student teacher's name]. Thank you for participating in this interview. The purpose of this interview is to explore the influences on student teachers' mathematics content knowledge, pedagogy and teacher efficacy levels and how this may impact a teacher's effectiveness when teaching and learning mathematics. This interview will last for about one hour.

PLEASE NOTE it is important to let you know that the taped transcripts of this interview will not be analyzed by the researcher until ***after*** the submission of grades to ensure objectivity of grading and maximum protection for students.

Mathematics Anxiety Level
Mathematics anxiety refers to an emotional feeling of nervousness, worry and/or apprehension that individuals may have about their ability to understand, perform mathematical functions and/or explain problems.

1. What are your thoughts of the mathematics anxiety scale?

I want to first start off by letting you know that you were purposefully selected to be interviewed because the results of your mathematics anxiety scale was in the high/middle/low range (indicate one level), and the study wants to investigate the perspective of various mathematics anxiety levels.

2. What are your thoughts about the results of your mathematics anxiety rating scale? What do you believe are some reasons for this level?

Mathematics Content Knowledge
Content knowledge refers to the the breadth and depth of the mathematics knowledge possessed by individuals.

3. How would you describe your level of mathematics content knowledge? Explain what contributed to your content knowledge?

4. What are your mathematics goals for this academic year in terms of learning content knowledge?
   Probe*: what curriculum areas are you most interested to learn about? Do you feel you have strengths or weaknesses in a specific strand?*

Mathematics Pedagogy

In this study, pedagogy is referred to instructional strategies and/or methods for teaching mathematics.

5. What are your ideas and beliefs about good teaching of mathematics?

6. What do you hope to learn in the mathematics course in terms of instructional strategies?

7. What do you believe will be the most challenging aspects of gaining effective mathematics teaching skills?

   Probe: *Are your ideas about pedagogy shaped by the way you were taught mathematics? Were you taught through a teacher-centered approach or through a student-centered approach? How do you think this influenced your beliefs about mathematics pedagogy?*

Mathematics Efficacy

Efficacy is how capable you feel to complete a task to a high standard. This study explores mathematics efficacy of student teachers in two ways, as learners of mathematics content, and as developing mathematics teachers.

Personal Mathematics Learning Experiences—Efficacy Levels

8. How would you describe your personal mathematics learning efficacy level (high, medium, low) and explain why?

9. How would you describe your experiences as a student of mathematics?

Probe: *If describes difficulties explore when they began and why. If describes good experiences explore when they began and why*.

Mathematics Teaching Experiences—Efficacy Levels

10. How would you describe your efficacy as a developing elementary mathematics teacher? In other words how confident are you that you will make a positive impact on your students' mathematics achievement?
11. What has contributed to your perception of your efficacy?

Probe: *For personal teaching experiences that may have impacted this level.*

12. How would you compare your level of mathematics teaching efficacy with other subjects?

Probe: *Reasoning behind comparison.*

Integration of the Three Constructs

13. Of the three constructs that we discussed in mathematics development, mathematics content knowledge, pedagogy and efficacy, which one do you feel is most significant for your development as a mathematics teacher?

14. Do you see the three constructs overlapping one another? If yes, then how and why? If not, then please expand your response.

15. Do you see these three constructs impacting on your mathematics anxiety level? If yes, then how and why? If not, then please expand your response.

Thank you so much for your time to answer these questions. Is there anything that you would like to ask or further comment on?

Appendix A.3: Individual Interview Protocol and Questions – Student Teachers

POST-INTERVIEW – April, 2011

This interview will be conducted by an external third party individual (a teacher assistant from another subject area). The following is the script and questions that the interviewer will be trained to deliver.

Hi [student teacher's name]. Thank you for participating in the post-interview. Seven months have passed since our pre- interview and you've experienced a full year mathematics elementary course. Congratulations on your near completion of the B.Ed. program! This interview will last for about one hour. The purpose of this interview is to explore the specific experiences that you had during the mathematics course that influenced your mathematics content knowledge, pedagogy and teacher efficacy levels.

PLEASE NOTE it is important to let you know that the taped transcripts of this interview will not be analyzed by the researcher until ***after*** the submission of grades to ensure objectivity of grading and maximum protection for students.

Mathematics Anxiety Level
According to the mathematics anxiety scale that you completed in the spring, your level of mathematics anxiety remained unchanged/increased/decreased (select one). Mathematics anxiety refers to an emotional feeling of nervousness, worry and/or apprehension that individuals may have about their ability to understand, perform mathematical functions and/or explain problems.

1. What are your thoughts about the results of your latest mathematics anxiety rating scale?
2. Please discuss the reasons for any changes for this level over the course of the year.

Mathematics Content Knowledge
Content knowledge refers to the the breadth and depth of the mathematics knowledge possessed by individuals.

3. How would you describe your development of mathematics content knowledge after taking a year-long elementary mathematics course?

4. Do you feel that your mathematics goals in terms of learning content knowledge were achieved?

5. What did you find the most challenging in developing your mathematics content knowledge?

6. What were the most rewarding experiences both in and out of class as far as the development of your content knowledge?

7. What significant experiences in class influenced your mathematics content knowledge, for example: problem solving questions, content assessment instrumentation, use of manipulatives, journaling, lesson planning? Why and how did these support your content development?
   Probe: *Were you able to grasp the curriculum strands in mathematics in a conceptual way? Do you feel you have strengths or weaknesses in a specific strand?*

Mathematics Pedagogy

8. Did the elementary mathematics course support your development of instructional strategies for teaching mathematics? Please explain.

9. Please describe your repertoire of mathematics methods?

10. Do you feel you will be reliant on a mathematics text book? Please explain.
    Probe: *Are your ideas about pedagogy shaped by the way you were taught mathematics?*

Mathematics Efficacy

Efficacy is how capable you feel to complete a task at a high standard. This study seeks to understand your mathematics efficacy in two ways, as a mathematics learner, and as a mathematics teacher.

Personal Mathematics Learning Experiences

11. Has undertaking this math course altered your perception as a mathematics learner? Please explain.

12. Do you feel more or less confident in attempting to solve mathematics problems? Please explain.

13. Explain how the course influenced you in your personal mathematics learning experiences?

14. What were the most significant activities that you feel made a difference in your personal efficacy as a mathematics learner?

Mathematics Teaching Experiences—Efficacy Levels

15. How confident are you that you will make a positive impact on students' mathematics achievement when you start teaching in the fall?

16. What course activities influenced your mathematics teaching efficacy?

Probe: *For personal teaching experiences that may have impacted these level(s).*

17. How would you compare your level of mathematics teaching efficacy with other subjects? Has this changed over the year?

Probe: *Reasoning behind comparison*

Integration of the Three Constructs

18. Of the three constructs that we discussed in mathematics development, mathematics content knowledge, pedagogy and efficacy, which one do you feel is most significant for your development as a mathematics teacher?

19. Do you think they overlap for you personally? If yes, then how and why? If not, then please expand your response.

20. Can you describe any changes you experienced in your levels of mathematics content?

21. Can you describe any changes you experienced in your levels of mathematics pedagogy?

22. Can you describe any changes you experienced in your levels of mathematics teacher efficacy?

Probe: *Were there class and field experiences that may have impacted these changes? If yes, please describe them.*
What components of the mathematics course were the most useful and how did they impact the three constructs as well as your mathematics anxiety level?
What are your suggestions to improve the course?

Thank you so much for your time in answering these questions. Is there anything that you would like to ask or further comment on?

Appendix A.4: Scoring Rubric for Mathematics Content Assessment – Open Response Items

EQAO Scoring Rubric – Grade Six Mathematics – Open Response

Retrieved from Education Quality and Accountability Office (EQAO)
http://www.eqao.com/Educators/Elementary/036/Rubrics.aspx?Lang=E&gr=036&Rubric=Math1

	Descriptor
	• blank: nothing written or drawn in response to the question
	• illegible: cannot be read; completely crossed out/erased; not written in English • irrelevant content: does not attempt assigned question (e.g., comment on the task, drawings, "?", "!", "I don't know") • off topic: no relationship of written work to the question

Level 1	• demonstration of limited understanding of concepts and/or procedures
	• application of knowledge and skills shows limited effectiveness due to
	  ○ misunderstanding of concepts
	  ○ incorrect selection or misuse of procedures
	• problem-solving process shows limited effectiveness due to
	  ○ minimal evidence of a solution process
	  ○ limited identification of important elements of the problem
	  ○ too much emphasis on unimportant elements of the problem
	  ○ no conclusions presented
	  ○ conclusion presented without supporting evidence

Level 2	• demonstration of some understanding of concepts and/or procedures
	• application of knowledge and skills shows some effectiveness due to
	  ○ partial understanding of the concepts
	  ○ errors and/or omissions in the application of the procedures
	• problem-solving process shows some effectiveness due to
	  ○ an incomplete solution process
	  ○ identification of some of the important elements of the problem
	  ○ some understanding of the relationships between important elements of the problem
	  ○ simple conclusions with little supporting evidence

Level 3	• demonstration of considerable understanding of concepts and/or procedures
	• application of knowledge and skills shows considerable effectiveness due to
	  ○ an understanding of most of the concepts
	  ○ minor errors and/or omissions in the application of the procedures
	• problem-solving process shows considerable effectiveness due to
	  ○ a solution process that is nearly complete

	- identification of most of the important elements of the problem - a considerable understanding of the relationships between important elements of the problem - appropriate conclusions with supporting evidence
Level 4	- demonstration of thorough understanding of concepts and/or procedures - application of knowledge and skills shows a high degree of effectiveness due to - a thorough understanding of the concepts - an accurate application of the procedures (any minor errors and/or omissions do not detract from the demonstration of a thorough understanding) - problem-solving process shows a high degree of effectiveness due to - a complete solution process - identification of all important elements of the problem - a thorough understanding of the relationships between all of the important elements of the problem - appropriate conclusions with thorough and insightful supporting evidence

Appendix A.5: Revised Math Anxiety Scale (RMAS)

(Please PRINT)

Name: ___________________Date: ______________ Cohort Option: __________

Gender: F M Age: _______

Highest level of mathematics education completed: ________________________________

Undergraduate Degree and Major(s): ____

Below is a series of statements. There are no correct answers for these statements. Be sure to answer every statement. Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate letters to the right of each statement.

SD **Strongly Disagree**	**D** **Disagree**	**U** **Uncertain**	**A** **Agree**	**SA** **Strongly Agree**

1	It wouldn't bother me at all to take more math courses.	SD	D	U	A	SA
2	I have usually been at ease during math tests.	SD	D	U	A	SA
3	Mathematics makes me feel uneasy and confused.	SD	D	U	A	SA
4	My mind goes blank and I am unable to think clearly when doing mathematics.	SD	D	U	A	SA
5	I have usually been at ease during math courses.	SD	D	U	A	SA
6	1 usually don't worry about my ability to solve math problems.	SD	D	U	A	SA
7	Mathematics makes me feel uncomfortable and nervous.	SD	D	U	A	SA
8	I almost never get uptight while taking math tests.	SD	D	U	A	SA
9	I get a sinking feeling when I think of trying hard math problems.	SD	D	U	A	SA
10	I get really uptight during math tests.	SD	D	U	A	SA

Appendix A.6: Revised Math Anxiety Scale (RMAS) Scoring Instructions

Step 1. Item Scoring: Items must be scored as follows: Strongly Agree = 5; Agree = 4; Uncertain = 3; Disagree = 2; and Strongly Disagree = 1.

Step 2. The following items must be reversed scored in order to produce consistent values between positively and negatively worded items.

Item 3	Item 9
Item 4	Item 10
Item 7	

Total scores could range from 10 to 50, in which 10 represents high anxiety, and 50 represents low anxiety. Therefore, higher scores indicate more positive attitudes toward mathematics, i.e., lower levels of math anxiety.

Appendix A.7: Mathematics Teaching Efficacy Beliefs Instrument (MTBEI)

Name: ____________________ Date: _________ Cohort Option: ______________

(Please PRINT)

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate letters to the right of each statement.

SD **Strongly Disagree**	**D** **Disagree**	**U** **Uncertain**	**A** **Agree**	**SA** **Strongly Agree**

1	When a student does better than usual in mathematics it is often because the teacher exerted a little extra effort.	SD	D	U	A	SA
2	I will continually find better ways to teach mathematics.	SD	D	U	A	SA
3	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	SD	D	U	A	SA
4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	SD	D	U	A	SA
5	I know how to teach mathematics concepts effectively.	SD	D	U	A	SA
6	I will not be very effective in monitoring mathematics activities.	SD	D	U	A	SA
7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	SD	D	U	A	SA
8	I will generally teach mathematics ineffectively.	SD	D	U	A	SA
9	The inadequacy of a student's mathematics background can be overcome by good teaching.	SD	D	U	A	SA

10	When a low achieving child progresses in mathematics, it is usually due to extra attention given by the teachers.	SD	D	U	A	SA
11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	SD	D	U	A	SA
12	The teacher is generally responsible for the achievement of students in mathematics.	SD	D	U	A	SA
13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	SD	D	U	A	SA
14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	SD	D	U	A	SA
15	I will find it difficult to use manipulatives to explain to students why mathematics works.	SD	D	U	A	SA
16	I will typically be able to answer students' questions.	SD	D	U	A	SA
17	I wonder if I will have the necessary skills to teach mathematics.	SD	D	U	A	SA
18	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	SD	D	U	A	SA
19	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	SD	D	U	A	SA
20	When teaching mathematics, I will usually welcome student questions.	SD	D	U	A	SA
21	I do not know what to do to turn students on to mathematics	SD	D	U	A	SA

Appendix A.8: Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)

Scoring Instructions

Step 1. Item Scoring: Items must be scored as follows: Strongly Agree = 5; Agree = 4; Uncertain = 3; Disagree = 2; and Strongly Disagree = 1.

Step 2. The following items must be reversed scored in order to produce consistent values between positively and negatively worded items. Reversing these items will produce high scores for those high and low scores for those low in efficacy and outcome expectancy beliefs.

Item 3	Item 17
Item 6	Item 18
Item 8	Item 20
Item 15	Item 21

In SPSSx, this reverse scoring can be accomplished by using the recode command. For example, recode ITEM3 with the following command:

RECODE ITEMS (5=1) (4=2) (2=4) (1=5)

Step 3. Items for the two scales are scattered randomly throughout the MTEBI. The items designed to measure <u>Personal Mathematics Teaching Efficacy Belief</u> (SE) are as follows:

Item 2	Item 11	Items 18
Item 3	Item 15	Item 19
Item 5	Item 16	Item 20
Item 6	Item 17	Item 21
Item 8		

Items designed to measure <u>Outcome Expectancy</u> (OE) are as follows:

Item 1	Item 9	Item 13
Item 4	Item 10	Item 14
Item 7	Item 12	

Note: In the computer program, DO NOT sum scale scores before the RECODE procedures have been completed. In SPSSx, this summation may be accomplished by the following COMPUTE command:

COMPUTE SESCALE = ITEM2 + ITEM3 + ITEM5 + ITEM6 + ITEM8 + ITEM11 + ITEM15 + ITEM16 + ITEM17 + ITEM18 + ITEM19 + ITEM20 + ITEM21

COMPUTE OESCALE = ITEM1 + ITEM4 + ITEM7 + ITEM9 + ITEM10 + ITEM12 + ITEM13 + ITEM14

Appendix A.9: Mathematics Content Assessment, Pre-Test

Mathematics Content Assessment

Based on Released Assessment Questions from EQAO*

Grade Six Mathematics Test

FALL

(Please Print)

Name:__________________________

Date:___________________________

Cohort Option:_________________________

Gender: F M

*Education Quality and Accountability Office – questions downloaded from www.eqao.com

1) Number Sense and Numeration

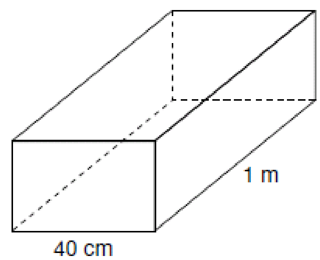
When Chang and Dillon visit another country, they find two types of coins are used there; one with a Q on it and one with an E on it. Chang has 13 Q coins and Dillon has 5 Q coins and 7 E coins. If Chang's coins have a total value of $0.65 and Dillon's coins have a total value of $3.75, what is the value of each type of coin?

Show your work:

Your thoughts, reactions and ideas about solving this problem:

2) Measurement

Jude’s fish tank, shown below, holds 100 000 cm3 of water when full. Jude decides to pour in water to a height of 5 cm below the top of the tank.



How much water, in cm3, will Jude need to pour into the tank so that the water is 5 cm below the top?

Show your work.

Your thoughts, reactions and ideas about solving this problem:

3) ***Patterning and Algebra***

Reena and Neetu go fishing.

The results of their activity are shown in the chart below.

Number of Fish Caught	**Time (Minutes)**
1	2
2	5
4	17
6	37

If the pattern continues, how many fish will they have caught after 90 minutes?

Explain your answer.

Your thoughts, reactions and ideas about solving this problem:

4) Data Management and Probability

The faces of a number cube are labelled 1, 2, 2, 3, 4, and 5. The number cube is rolled 114 times.

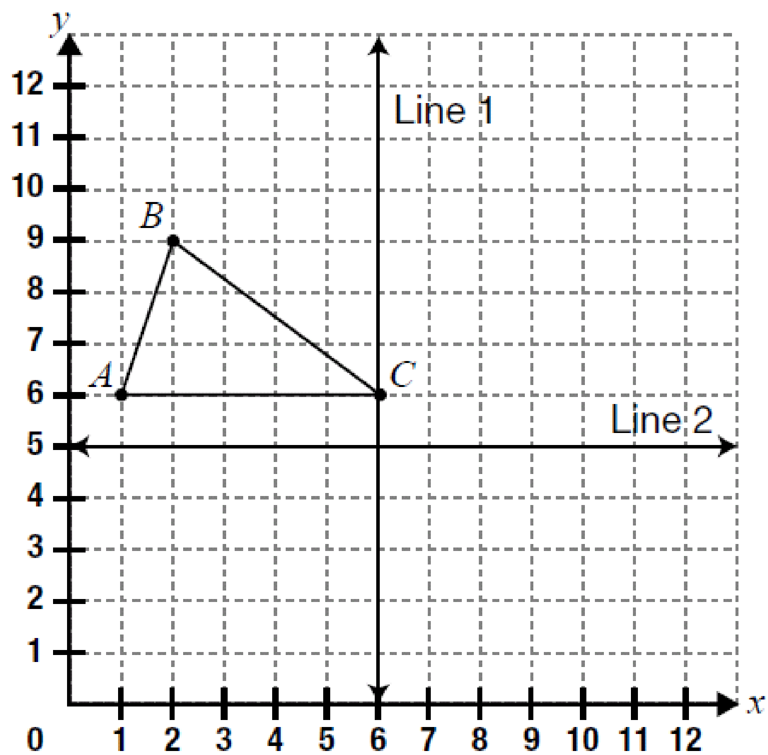
How many times would you expect the number 2 to appear?

Justify your answer.

Your thoughts, reactions and ideas about solving this problem:

5) Geometry and Spatial Sense

The drawing below shows a grid with ΔABC, Line 1 and Line 2. On the grid, reflect ΔABC across Line 1 and then reflect the new triangle across Line 2.



Your thoughts, reactions and ideas about solving this problem:

Multiple Choice Items

1) Joseph has a measuring wheel that clicks once for every metre he walks. How many times will the wheel click when Joseph walks 2.6 km?

a 2

b 26

c 260

d 2600

2) Chandra, Brittany, Ben, and Daniel buy different sandwiches and salads for lunch. Their choices are shown below.

Prices for Lunch

	Salad	Sandwich
Chandra	4.48	3.99
Brittany	4.48	4.99
Ben	3.49	4.99
Daniel	3.49	3.99

Which person should receive about $2.50 change from $10.00?

a Chandra

b Brittany

c Ben

d Daniel

3) Ravi makes 2.80 L of pudding. He wants to completely fill 350 mL cups with pudding.

Which of the following expressions can be used to find how many 350 ml cups Ravi can fill?

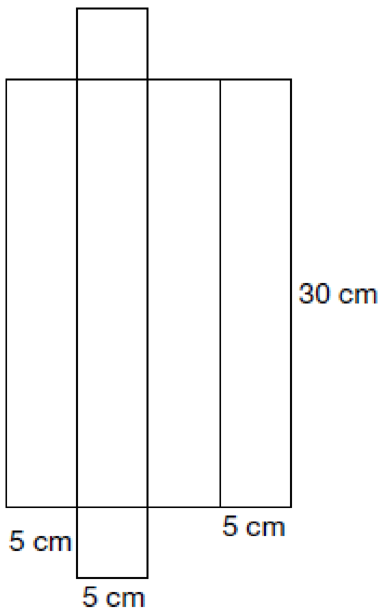
a 2.80 X 1000 ÷ 350

b 2.80 X 1000 + 350

c 2.80 X 350 X 1000

d 2.80 X 350 X 1000

4) Rebecca creates a net of a rectangular prism, as shown below. What is the total surface area of the rectangular prism?



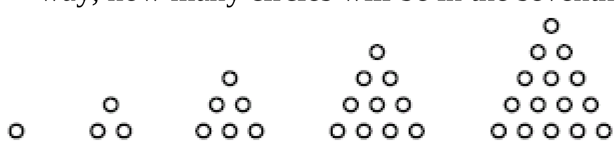
a 450 cm2

b 600 cm2

c 650 cm2

d 750 cm2

5) Consider the five terms in the following pattern. If the pattern continues in the same way, how many circles will be in the seventh term?



a 21

b 25

c 28

d 36

6) A regular polygon is created with angles of 60° and sides of 4 cm in length. Which statement below describes this polygon?

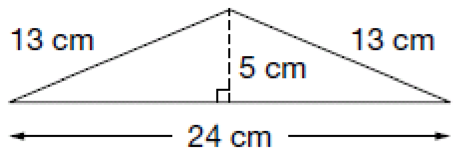
a triangle with perimeter of 12 cm

b triangle with perimeter of 16 cm

c rhombus with perimeter of 12 cm

d rhombus with perimeter of 16 cm

7) What is the area of the triangle shown below?



a 60 cm2

b 65 cm2

c 120 cm2

d 156 cm2

8) It takes Nadeem 22 minutes to walk 1 kilometre. At this rate, approximately how long will it take Nadeem to walk 5 kilometres?

a 1 hour

b 2 hours

c 100 hours

d 110 hours

9) Which expression is equivalent to 128 ÷ 2?

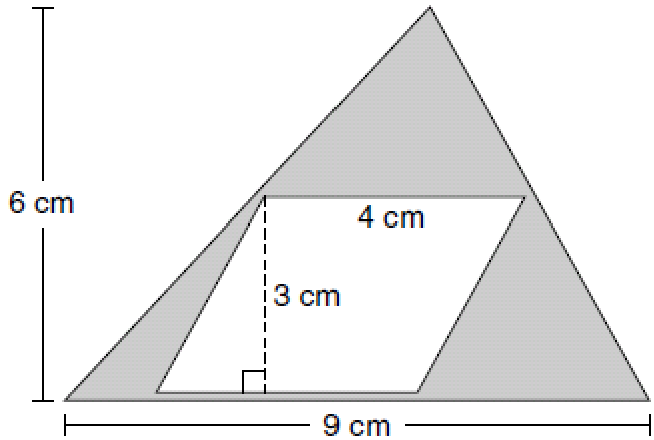
a (120 ÷ 2) + (8 ÷ 2)

b (120 ÷ 2) ÷ (8 ÷ 2)

c (120 + 2) + (8 + 2)

d (120 + 2) ÷ (8 + 2)

10) Which expression can be used to find the area of the shaded region?



a 54 ÷2 - 12

b 54 - 4 x 12 ÷ 2

c 12 ÷ 2 - 54

d 1 2 - 54 ÷2

Appendix A.10: Mathematics Content Assessment, Post-Test

Mathematics Content Assessment

Based on Released Assessment Questions from EQAO*

Grade Six Mathematics Test

SPRING

(Please Print)

Name:___________________________

Date:____________________________

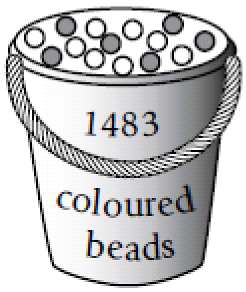
Cohort Option:__________________________

Gender: F   M

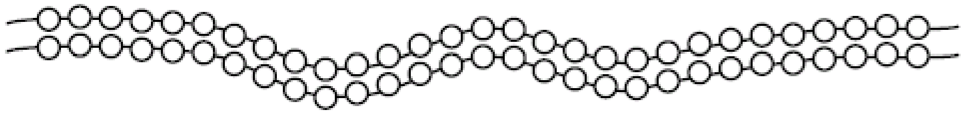
*Education Quality and Accountability Office – questions downloaded from www.eqao.com

1) Number Sense and Numeration

There are 1483 beads in the bucket.



Tina wants to make bracelets with 2 rows. A 2-row bracelet uses 58 beads.



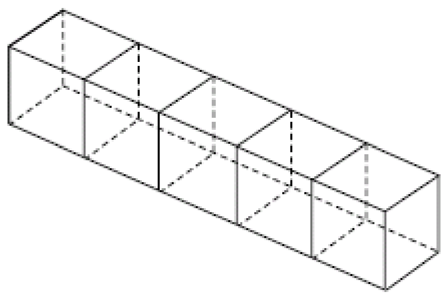
Tina estimates that she can make 30 bracelets. Is her estimate reasonable?

Explain your answer.

Your thoughts, reactions and ideas about solving this problem:

2) Measurement

Daneen builds a model train with 5 cubes as shown below. The dimensions of each cube are 2 cm X 2 cm X 2 cm.



Daneen wants to paint the outside of the model train with red paint. The cost to paint 1 cm2 of the train is $0.75. How much will it cost to paint the outside of the model train?

Show your work.

Your thoughts, reactions and ideas about solving this problem:

3) Patterning and Algebra

A researcher observes the distance travelled by a pod of grey whales. She records her observations on the chart below.

Hours Spent Travelling	**Distance Travelled in Total (km)**
3	5
6	11
12	23
24	47
48	95

Assuming the pattern continues, how many hours will it take for the pod to travel 377 km?
Explain your thinking.

Your thoughts, reactions and ideas about solving this problem:

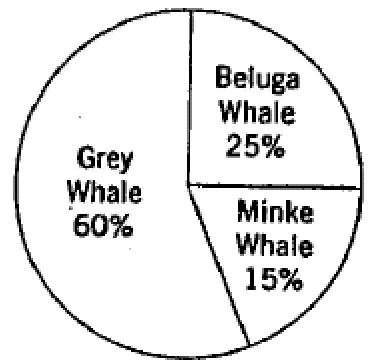
4) Data Management and Probability

A scientist records the number of species of whales along a section of the coast. In total, he sees 50 whales. His observations are recorded in the chart below. He compares these numbers with last year's data recorded on the circle graph below.

Whales Observed This Year

Species of Whale	Number of Whales Observed (out of 50)
Grey Whale	28
Beluga Whale	13
Minke Whale	9

Whales Observed Last Year



He concludes that the two sets of data are almost the same.

Is this a reasonable conclusion?

Explain your thinking?

Your thoughts, reactions and ideas about solving this problem:

5) Geometry and Spatial Sense

Construct a pentagon on the grid below that meets the following conditions.

- exactly 1 line of symmetry
- 2 obtuse angles
- 2 right angles
- 1 acute angle
- at least 1 side with a length of 3 units

Draw the line of symmetry on your pentagon.

Your thoughts, reactions and ideas about solving this problem:

Multiple Choice Items

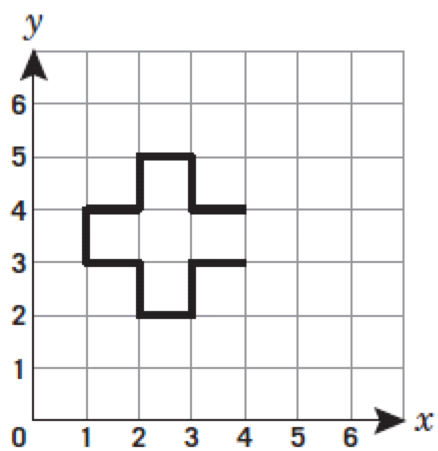
1) Which set is in order from least to greatest?

a 1.153, 1.062, 0.13, 0.054
b 0.13, 0.054, 1.162, 1.153

c 0.054, 0.13, 1.153, 1.062

d 0.054, 0.13, 1.062, 1.153

2) Jacob draws almost of an addition symbol on the Cartesian plane below.



Which two ordered pairs represent the location on the grid of the two points that should be connected to complete the addition symbol?

a (3, 4) and (4, 4)

b (4, 3) and (3, 3)

c (3, 4) and (4, 3)

d (4, 4) and (4, 3)

3) What value, when placed in the box, would make the following equation true?
6 X □ – 4 = 56 + 6

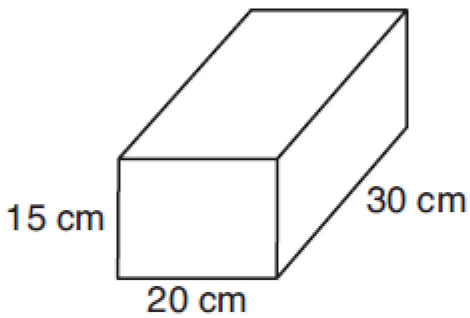
a 10

b 11

c 31

d 62

4） Four students calculate the volume of the shoe box shown below.



The following number sentences show the students' calculations. Which calculation is correct?

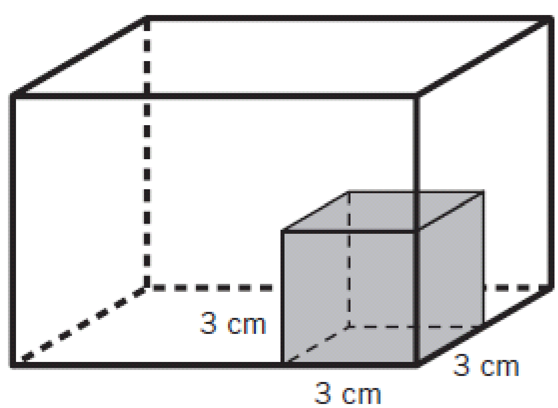
a 15 cm X 20 cm = 300 cm2

b 20 cm X 30 cm = 600 cm2

c 20 cm + 30 cm + 15 cm = 65 cm3

d 15 cm X 20 cm X 30 cm = 9000 cm3

5） Twelve cubes measuring 3 cm by 3 cm by 3 cm fit perfectly into the rectangular prism shown below. What is the volume of the rectangular prism in cm3?



a 36 cm3

b 162 cm3

c 288 cm3

d 324 cm3

6) A class records the colour of the cars that drive past the school in a short period. These data are shown in the table below.

Car Colour

Colour	**Number of Cars**
Black	2
Blue	3
Grey	1
Red	3
White	1

Based on these data, if 40 cars drive past the school, how many cars could be expected to be blue?

a 3

b 10

c 12

d 30

7) Chloe's parents are buying a car. They want to pick 1 colour at random from 4 possible car colours. Which of the following methods should they use?

a Flip a coin.

b Toss a 6-sided number cube with 1 through 6 on the faces.

c Use a spinner with 4 equal-sized sections labelled with the 4 possible colours.

d Pick one card from 10 cards with 1 of the 4 colours written on each face.

8) In a hockey arena, the first row has 276 seats, the second row has 288 seats and the third row has 300 seats. Each row after this continues to increase by the same number. If the arena has a total of 6 rows, how many seats are in the arena?

a 1836

b 1176

c 972

d 312

9) The same number is added to each term in a pattern to get the value of the next term. Below are the fourth, fifth and sixth terms in the pattern.

… 95, 98, 101, …

What are the first, second and third terms in the pattern?

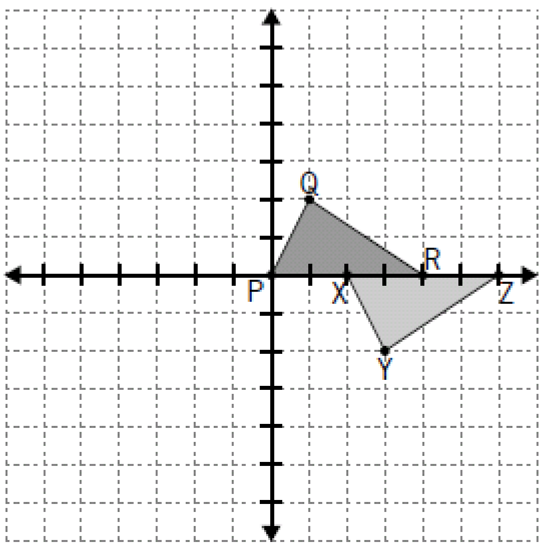
a 83, 85, 87

b 83, 86, 89

c 86, 88, 92

d 86, 89, 92

10) The triangle PQR has been transformed to the position XYZ.



What is a correct description of the transformation?

a translation to the right by 2 units and down by 2 units

b translation to the right by 4 units and reflection about the horizontal axis

c reflection about the horizontal axis followed by reflection about the vertical axis

d translation to the right by 2 units and reflection about the horizontal axis

Appendix A.11: Script for Communication About and Invitation to Participate in Study

- To be presented by external third party individual (teacher education program assistant)
- To be presented after the completion of the mathematics surveys, content test, and the description of the journal assignments
- Researcher-instructor will not be present during this presentation

Hello Student Teachers,

Good day. I am here on behalf of Mary Reid to discuss her research project. Mary Reid is a doctoral candidate from the University of Calgary, in the Graduate Division of Educational Research (GDER). The purpose of her study is to examine how specific teaching and learning experiences in a preservice education mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy and 3) pedagogy, and how do these constructs relate and/or influence one another?

You all completed three mathematics instruments that will help your instructor program for your specific mathematics development. These three instruments were:

1) Revised Math Anxiety Survey (RMAS);
2) Mathematics Teaching Efficacy Beliefs Instrument (MTEBI); and
3) Mathematics content assessment.

You will also complete the same instruments in the spring in order to measure any changes over the academic year. Another requirement of this course is reflective journals, which was explained to you earlier this morning as part of the course outline. Mary Reid is requesting your permission to analyze your instrumentation results and reflective journals as part of her data.

I will distribute the consent form. Let's go over the consent form now:

- discuss how participants can withdraw at any time and to communicate withdraw to external third party individual
- discuss confidentiality and anonymity
- discuss consent can be given to all the instrumentation and journal data, or just some
- encourage student teachers to speak to third party individual, or email her if they have any questions concerning the data collection
- inform potential participants to hand in signed consent forms to third party individual, either by person, or in her mailbox

Thank you very much for your time. Have a great day.

Appendix A.12: Consent Form for use of Instrumentation Data and Journal Data

UNIVERSITY OF CALGARY

Name of Research, Faculty, Department, Telephone & Email:
Mary Reid, Faculty of Education, Ed.D. Candidate, Graduate Division of Educational Research (GDER), [contact information]
Supervisor:
Dr. Shelleyann Scott, Ph. D., Associate Professor, Faculty of Education
Title of Project:
Mathematics Content Knowledge, Mathematics Teacher Efficacy, and Pedagogy: An Examination of the Three Constructs in Mathematics Preservice Elementary Education.
***Sponsor*:**
N/A

Dear Elementary B. Ed. Student Teacher:

My name is Mary Reid and I am a doctoral candidate from the University of Calgary, in the Graduate Division of Educational Research (GDER).

This consent form, a copy of which has been given to you, is only part of the process of informed consent. If you want more details about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

The University of Calgary Conjoint Faculties Research Ethics Board has approved this research study.

Purpose of This Study:
The purpose of my study is to examine how specific teaching and learning experiences in a preservice education mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy and 3) pedagogy, and how do these constructs relate and/or influence one another?

What Will I Be Asked To Do?
As an elementary B.Ed. preservice teacher candidate, the following are some required assignments for this mathematics course:
Instrumentation

At the beginning of the academic year (September, 2010) and at the end of the academic year (April, 2011), three instruments will be given to student teachers for completion. These are: 1) the Revised Math Anxiety Survey (RMAS) 2) the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) and 3) a mathematics content assessment. The use of these instruments are usual components of the mathematics course and the results help the instructor to determine students` levels of mathematics anxiety, mathematics teacher efficacy, and content knowledge, and whether or not any changes in these levels occurred over the course of the year. These various instruments are important diagnostic tools as the results inform the differentiated instruction. As a regular component of this process, the instructor provides more support to those preservice teachers who are in need. This is a usual aspect of the course. Examples of additional support structures include, small group tutoring to further develop mathematics content knowledge, mixed ability groupings for collaborative projects, and providing more time and assistance during problem solving activities to alleviate anxiety and promote learning.

Reflective Journal

Another required assignment of the course is for all student teachers to complete a reflective journal. Culminating reflective journals are due at the end of each term, December 2010, and April 2011. Reflective journal writing is a valuable process that can support preservice teachers to observe, reflect, and question practice and beliefs pertaining to mathematics instruction. Specifically, for student teachers who are weak in mathematics content knowledge, have low mathematics teacher efficacy, or feel high mathematics anxiety, the reflective process of journal writing can reduce mathematics anxiety and build confidence as mathematics learners.

The purpose of this letter is to request your permission to analyze your instrumentation results and reflective journals as part of my research data.

The instruments and the reflective journals are required portions of this course. If you are interested in participating by having your results included in the study, please indicate so below, and return the signed consent form to the third party individual charged with recruitment. No adverse actions will be taken against you or your grades if you choose this option. You will still participate in all the same assignments and other classroom activities as the rest of the class. The researcher will not know who participates until after course grades are posted.

What Types of Personal Information Will Be Collected?

Participation is absolutely voluntary and that the decision regarding an individual's participation, non-participation or withdrawal will not be known to the researcher until after final course grades are posted. Participation, non-participation or withdrawal will have no effect on the individual's continuing relationship with OISE/UT. Should you give permission to use your instrumentation and journal data in the study, your gender age may be noted by the researcher. However, all study data will be reported in an anonymous format and no personally identifying information will be included in the final report. The final research report will identify the university setting in a large urban city located in southern Ontario, and briefly describe the university as a research institute.

Are There Risks or Benefits If I Participate?

There are no reasonably foreseeable risks, harms, or inconveniences to the participant.

The potential benefits of the research involve a better understanding of one's own mathematics development. The instrumentation results act as important diagnostic tools that inform differentiated instruction. As a regular component of this process, the instructor provides more support to those preservice teachers who are in need. This is a usual aspect of the course. Examples of additional support structures include, small group tutoring to further develop mathematics content knowledge, mixed ability groupings for collaborative projects, and providing more time and assistance during problem solving activities to alleviate anxiety and promote learning.

The process of the journal writing will support your reflection on mathematics teaching and learning experiences. Reflective journal writing is a valuable process that can support preservice teachers to observe, reflect, and question practice and beliefs pertaining to mathematics instruction. Specifically, for student teachers who are weak in mathematics content knowledge, have low mathematics teacher efficacy, or feel high mathematics anxiety, the reflective process of journal writing can reduce mathematics anxiety and build confidence as mathematics learners.

What Happens to the Information I Provide?

As part of the course, the instructor will examine the results of your survey instruments (i.e., RMAS, MTEBI, and the Mathematics Content Assessment) and share this with you. The results will then be securely stored in a locked cabinet, only accessible by the researcher and her supervisor. After your instructor has assessed your journals, these files will be secured in a password-protected computer, only accessible by the researcher and her supervisor. Stored data will be destroyed after the two year storage period. The primary use of data will be to inform this Ed.D. research project The results of this study will also be used to help inform teacher education and professional development programs in mathematics.

There are no potential risks to the release of your results. The research will maintain the confidentiality of all participants, through aggregation of data. Most data will be reported in aggregate form, and individual quotes will be used only if there is no risk of identifying the participant or the course involved Also, the anonymity of individuals involved in the research will be protected, in that, no real names will be published in the final or subsequent documents. Participants will have the opportunity to withdraw their results at any time during the year, without penalty. Should a participant choose to withdraw their results; only the data prior to the withdrawal will be included in the project. Participants can affect their withdrawal by communicating to the third party individual (the teaching assistant who presented this study to your class), without the knowledge of the researcher. No remuneration or compensation will be awarded for release of data results.

There are several options for you to consider if you decide to take part in this research. You can choose all, some or none of them. Please put a check mark on the corresponding line(s) that grants me your permission to:

I grant permission for you to use my RMAS results: Yes: ___ No: ___

I grant permission for you to use my MTEBI results: Yes: ___ No: ___

I grant permission for you to use my Mathematics Content Assessment results: Yes: ___ No: ___

I grant permission for you to use my reflective journals: Yes: ___ No: ___

Signatures (written consent)
Your signature on this form indicates that you 1) understand to your satisfaction the information provided to you about your participation in this research project, and 2) agree to participate as a research subject.
In no way does this waive your legal rights nor release the investigators, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from this research project at any time. You should feel free to ask for clarification or new information throughout your participation.

Participant's Name: (please print) ____

Participant's Signature ____Date: ________

Researcher's Name: (please print) Mary Reid

Researcher's Signature: ____Date: _________

Questions/Concerns
If you have any further questions or want clarification regarding this research and/or participation, please contact:

Teacher Education Program Assistant (External Third Party Individual for Mary Reid's Ed.D. Research) [contact information]

OR

Supervisor: Dr. Shelleyann Scott, Associate Professor [contact information]

If you have any concerns about the way you've been treated as a participant, please contact the Senior Ethics Resource Officer, Research services Office, University of Calgary at (403) 220-3782; email rburrows@ucalgary.ca.

A copy of this consent form has been given to you to keep for your records and reference. The investigator has kept a copy of this consent form.

Appendix A.13: Email Invitation to Student Teachers to Participate in Interviews

Dear [student teacher's name to be added],

I am emailing you on behalf of Mary Reid.

Mary is an Ed. D. candidate at the University of Calgary. Her research is seeking to understand how specific teaching and learning experiences in a preservice education mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy and 3) pedagogy, and how do these constructs relate and/or influence one another?

You are invited to participate in two interviews, one pre-interview and one post-course interview. The purpose of the interviews will be to discuss your goals and beliefs about mathematics teaching from a student teachers' perspective; and further talk about any mathematics course activities you feel support the mathematics development of student teachers. The interview will take place at a time that is convenient for you, in the main campus building. The interview will take approximately one hour to complete, this is beyond your regular school work.

Please see the attached consent form for more information (Appendix A.14).

If you are interested, please reply back to this email, and we can set up a pre-interview time.

Thank you for your consideration.

Sincerely,

External Third Party Interviewer for Mary Reid's Ed. D. research interviews

[contact information]

Appendix A.14: Consent Form for Student Teacher Individual Interviews

UNIVERSITY OF CALGARY

Name of Research, Faculty, Department, Telephone & Email:
Mary Reid, Faculty of Education, Ed.D. Candidate, Graduate Division of Educational Research (GDER), [contact information]
Supervisor:
Dr. Shelleyann Scott, Ph. D., Associate Professor, Faculty of Education
Title of Project:
Mathematics Content Knowledge, Mathematics Teacher Efficacy, and Pedagogy: An Examination of the Three Constructs in Mathematics Preservice Elementary Education.
***Sponsor*:**
N/A

Dear Elementary B. Ed. Student Teachers,

My name is Mary Reid and I am an Ed.D. candidate from the University of Calgary, in the Graduate Division of Educational Research (GDER).

This consent form, a copy of which has been given to you, is only part of the process of informed consent. If you want more details about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

The University of Calgary Conjoint Faculties Research Ethics Board has approved this research study.

Purpose of This Study
The purpose of my study is to examine how specific teaching and learning experiences in a preservice education mathematics course contribute to student teachers' 1) mathematics content knowledge, 2) mathematics teacher efficacy and 3) pedagogy, and how do these constructs relate and/or influence one another?

What Will I Be Asked To Do?
As a voluntary participant in the research, you will have the opportunity to participate in two individual interviews, one pre-interview during September, and one post-interview during April. The interview will take place at a time that is convenient for you, in the

OISE/UT building. The interview will take approximately one hour to complete, this is beyond your regular school work. During the interview, you will be asked questions about goals and beliefs about mathematics teaching and learning; and how any mathematics course activities influenced your development as a mathematics teacher. The interview will be recorded and the audio recordings stored as digital files. Some comments that you make may be recorded in written point form. All written and digital recordings will be used as study data. In order to reduce potential bias by the participant or researcher, a third party external interviewer (a teacher assistant from another subject area) will be conducting the interview. **The taped transcripts and any written notes from the interviews will NOT be analyzed until *after* the submission of final grades.**

You may refuse to participate altogether in the study or withdraw from the study at any time without penalty or loss. Participants can withdraw after completion of either interview without the knowledge of the researcher by communicating to the third party external interviewer. If you initially agree to participate in the research study but subsequently withdraw, then any data gathered to that point will be retained and used in the study.

What Types of Personal Information Will Be Collected?
Participation is absolutely voluntary and that the decision regarding an individual's participation, non-participation, or withdrawal will not be known to the researcher until after final course grades are posted. Participation, non-participation, or withdrawal will have no effect on the individual's continuing relationship with the university. Should you agree to participate in the study your gender and mathematics learning and teaching experiences may be noted by the researcher. However, all study data will be reported in an anonymous format and no personally identifying information will be included in the final document or any subsequent reports. Most data will be reported in aggregate form, and individual quotes will be used only if there is no risk of identifying the participant or the course involved. The final research report will identify the university setting in a large urban city in southern Ontario, and briefly describe the university as a research institute. There will be no remuneration or compensation for participating in this study.

Are There Risks or Benefits If I Participate?
There are no reasonably foreseeable risks, harms, or inconveniences to the participant.

The potential benefits of the research involve a better understanding of one's own mathematics development. The process of the interview will support your reflection on mathematics teaching and learning experiences.

What Happens to the Information I Provide?
Participation is completely voluntary and confidential. You are free to discontinue participation at any time during the study. No one except the researcher and her supervisor will be allowed to examine the written or digital recordings of your interviews. Any written notes taken during the interviews will be securely stored in a locked cabinet, only accessible by the researcher. The digital recordings of the interviews will be secured

in a password-protected computer, only accessible by the researcher. Stored data will be destroyed after the two year storage period. The primary use of data will be to inform this Ed.D. research project. The results of this study will be used to help inform teacher education and professional development programs in mathematics.

Signatures (written consent)

Your signature on this form indicates that you 1) understand to your satisfaction the information provided to you about your participation in this research project, and 2) agree to participate as a research subject.

In no way does this waive your legal rights nor release the investigators, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from this research project at any time. You should feel free to ask for clarification or new information throughout your participation.

Participant's Name: (please print)

____

Participant's Signature __________________________________Date: _______________

Researcher's Name: (please print) Mary Reid

Researcher's Signature: __________________________________Date: ______________

Questions/Concerns

If you have any further questions or want clarification regarding this research and/or participation, please contact:

Teacher Education Program Assistant (External Third Party Individual for Mary Reid's Ed.D. Research)
[contact information]

OR

Supervisor: Dr. Shelleyann Scott, Associate Professor
[contact information]

If you have any concerns about the way you've been treated as a participant, please contact the Senior Ethics Resource Officer, Research services Office, University of Calgary at (403) 220-3782; email rburrows@ucalgary.ca.

A copy of this consent form has been given to you to keep for your records and reference. The investigator has kept a copy of this consent form.

Appendix A.15: Confidentiality Agreement for External Third Party Interviewer

UNIVERSITY OF CALGARY

Name of Researcher: ***Mary Reid***

Title of Project: ***Mathematics Content Knowledge, Mathematics Teacher Efficacy, and Pedagogy: An Examination of the Three Constructs in Mathematics Preservice Elementary Education***

Before we can select you to conduct research interviews, we must obtain your explicit consent not to reveal any of the contents of the interviews, nor to reveal the identities of the participants (i.e. the students interviewed and their place of education). If you agree to these conditions, please sign below.

____

Print Name

____

Signature

A teacher assistant from another subject area fulfilled this role.