A New Method For Production Data Analysis Using Superposition-Rate

Liang, Yue


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A New Method For Production Data Analysis Using Superposition-Rate

by

Peter Yue Liang

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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ABSTRACT

This research presents a new method to analyze production data – the superposition-rate. The method was developed based on the well-accepted superposition principle. It is presented in a generalized form and is applicable to data in transient flow (including radial, linear, and bilinear flow) as well as in boundary dominated flow.

The superposition-rate method is validated by synthetic data generated from reservoir modeling. Moreover, a simple yet practical workflow of implementing the superposition-rate in production data analysis is presented. Last, real field examples are utilized to demonstrate the practicality of superposition-rate.

A thorough comparison between the superposition-rate and superposition-time methods is presented. The superposition-rate shows significant advantages over the superposition-time. A key improvement of the superposition-rate in quality diagnostics and data analysis is that it does not modify time scale. Consequently the superposition-rate keeps all production data in the sequence of their occurrence.
II

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CHAPTER 1
INTRODUCTION

This thesis presents work performed for a Master of Engineering (Thesis Based) degree that was conducted at the Chemical and Petroleum Engineering Department of the University of Calgary. The M.Eng research presented herein develops a new superposition application, called superposition-rate, which is intended to aid in analyzing variable-rate/variable-pressure data in production data analysis. The superposition-rate application is new and constitutes the significant contribution of this research. The following sections present the motivation behind the study as well as the background, objectives, and organization of the thesis.

1.1 Motivation

Analysis of rate and pressure data relies on the solutions of flow equation in porous media derived using constant boundary condition. All wells can exhibit one of the two constant boundary conditions: constant production rate or constant flowing pressure. For well testing operations, the flow period is typically controlled, and constant rate solutions are chosen to analyze well testing data. On the other hand, for production operations, the flowing pressure often declines rapidly and becomes constant during a prolonged period. As a result, constant pressure solutions are considered to be more useful in analyzing production data, particularly for wells in unconventional reservoirs. However, there are numerous situations where both rate and flowing pressure continuously decline, or make step changes (discontinuously) during production
operations. These variable-rate/variable-pressure issues are typically addressed using superposition.

The superposition principle is effective in converting variable-rate/variable-pressure data to its equivalent constant boundary solution. The classical way to apply the superposition principle is to use a time function, namely superposition-time. It involves manipulation of time in accordance with the changes in rates and flow durations. Valuable as this procedure is, it suffers from many disadvantages: for instance, after manipulation, the resulting time will have been shuffled back and forth. This makes the data’s sequence difficult to be tracked and identified, and subsequently causes problems in data quality diagnostics. This is particularly evident in the presence of outliers.

Is it possible to convert variable-rate/variable-pressure data to its constant boundary equivalents while keeping data points in the sequence of their occurrence?

This thesis attempts to find an answer to the above question. It develops a new methodology which uses the same theory underlying superposition-time, but manipulates data in the opposite manner: rather than modify time (as is done classically), the new method modifies rate. Due to its nature, a name is given to this modified rate: superposition-rate. In the classical analysis approach to variable-rate/variable-pressure problems, rate and pressure are plotted against superposition-time. In the new approach developed in this thesis, superposition-rate is calculated and plotted against real time (not superposition-time). Consequently, all the data’s sequential order is preserved by the use of superposition-rate.

1.2 Background
When engineers think about the word “superposition”, most of us would say it is a well testing concept and it is used only in pressure transient analysis (PTA). Indeed, superposition is a popular technique in well testing and probably is used in every diagnostic plot in pressure transient analysis. But what most of us do not realize is that, superposition is also critical to the analysis of production data. To demonstrate the importance of superposition in production data analysis, let us first take a look at the following example.

Imagine two gas wells are producing from the same reservoir. Well A operates at constant bottom flowing pressure; while Well B is restricted at constant flow rate. The two wells’ first 3 months production profiles are shown in Figure 1.1. The goal is to compare the wells’ production and to determine which well performs better.
Since the two wells flow at different operating conditions, production rates cannot be directly compared. A typical procedure to handle this problem is to normalize the rate by dividing it by the drop in pressure \( \frac{q}{P_1-P_{wf}} \). To make the values more meaningful, we multiply \( \frac{q}{P_1-P_{wf}} \) by a fixed pressure and then compare the resulting normalized rates. According to the comparison of the two wells’ normalized rates (Figure 1.2), it appears that Well B outperforms Well A. We can even go one step further and calculate that there is 13% uplift in Well B’s first 3 months cumulative production.
Is the above comparison of the normalized rates as reliable as we think it is?

The answer is “no!” In this example, both wells’ synthetic production data were generated from an identical model with the same reservoir/well inputs. Therefore, Well B’s performance should be the same as Well A. The reason why the previous comparison was misleading is because superposition was not applied to the production data. When wells operate at different conditions, simple normalization is not enough. Superposition must be implemented in the evaluation procedure of comparing well performance. Otherwise, misleading comparison can be generated and well performance is likely to be falsely interpreted. If the production data is properly normalized with the use of superposition, then the two wells’ performance can be correctly compared (as shown in Figure 1.3).
Figure 1.3: normalized rate comparison plot (with the use of superposition)

This simplified example attempts to highlight the importance of superposition in production data analysis. It is worth reiterating that superposition is not only for well test analysis, but also a very powerful and crucial technique for analyzing production data. To avoid misinterpretation, superposition must be considered in the analysis of production data.

1.3 Objectives

The objectives of this thesis have been partially alluded to in the above sections on the motivation behind the study and the background. They are, in short:

1. Develop a new superposition application, called superposition-rate, that applies the principle of superposition while keeping all data’s consecutive order;

2. Validate that both superposition-time and superposition-rate applications are effective in converting variable rate/variable pressure data into its constant boundary condition equivalent;
3. Compare the classical superposition-time application to the newly developed superposition-rate application, to understand the difference between the two superposition applications and furthermore to identify the advantages as well as the disadvantages of each superposition application;

4. Research the effect of superposition on the data that is in a flow regime different from the chosen superposition function, and investigate whether or not one flow regime’s superposition is sufficient to analyze other flow regimes’ data; and

5. Incorporate superposition into a workflow for analyzing variable pressure/variable rate data and verify the accuracy of the proposed workflow by testing it with field production data.

1.4 Organization of the Thesis

This thesis is divided into six chapters. This chapter, Chapter 1, is the introduction and contains the motivation behind the study as well as the background, objectives, and organization of the thesis.

Chapter 2 presents a review of the literature pertinent to the development of this thesis.

Chapters 3, 4, and 5 constitute the essence of the thesis. The review of superposition principle is presented in Chapter 3, followed by the analytical development of the superposition-time and superposition-rate applications. Chapter 4 validates the effectiveness of superposition-rate, and presents a thorough comparison between superposition-time and superposition-rate within various flow systems. Chapter 5
presents the workflow of using superposition to analyze variable-rate/variable-pressure data, followed by three field examples selected from multiple fractured horizontal wells in unconventional reservoirs, and finally, a discussion on the advantages of implementing superposition-rate application in the analysis of production data.

The final chapter, Chapter 6, makes recommendations for future research to extend the work started in this thesis and provides the main conclusions resulting from the work described herein.
CHAPTER 2
LITERATURE REVIEW

The concept of superposition-rate was first published by Liang et al. in 2013. This work is the starting point for the development of this thesis. The authors developed superposition-rate function for transient linear flow without skin effect and compared the effect of superposition-rate on synthetic data to that of superposition-time. The authors also tested superposition-rate with field production data from tight gas reservoirs and demonstrated the advantages of the use of superposition-rate over superposition-time. Building on the work done in 2013, the present thesis develops the superposition-rate functions for transient linear flow, transient radial flow, transient bilinear flow as well as boundary dominated flow, both with skin and without skin situations. The thesis herein also presents a more thorough comparison of superposition-rate and superposition-time in various flow systems, namely radial flow system, linear flow system, and complex flow system.

Alberta Energy Regulator (AER) published Directive 034 in 1975 to explain well testing theory and practice. Directive 034 serves as a general guideline for the development of the analytical solutions in this work. It documented the fundamental flow equation through porous media and presented some simplifying assumptions to linearize the flow equation and solve it analytically. It showed the constant rate boundary condition analytical solutions of all possible transient flow regimes as well as boundary dominated flow regime. Moreover, it documented the principle of superposition and introduced its classical application, superposition-time, in well testing practice.
Superposition is a well-accepted mathematical application and is a standard procedure in reservoir engineering for solving multiple-rate problems. The principle of superposition, also known as Duhamel’s theorem, is usually discussed in mathematical texts: for instance, Carslaw and Jaeger (1959) and Wiley (1960).

Analytical solutions presented in this thesis are obtained from the literature. Van Everdingen and Hurst in 1949 introduced the Laplace transform solution to the diffusivity equation and derived a complete analytical solution of a cylindrical radial flow system. Based on the solution of Van Everdingen and Hurst (1949), AER (1975) presented transient radial flow and boundary dominated flow solutions to a closed radial flow system. AER (1975) also presented the boundary dominated flow solutions for various closed rectangular systems using the shape factor concept \( C_A \). Gringarten et al. (1974) presented a solution for an infinite-conductivity vertical fracture intercepting a vertical well. The authors also developed an equation to determine the time at which pseudo radial flow develops around a single infinite conductivity vertical fracture. Using the solution developed by Gringarten et al. (1974), Gringarten et al. (1975) presented type curves to analyze transient pressure behavior of infinite conductivity hydraulic fractures. Cinco-Ley et al. (1978) extended the solution presented by Gringarten et al. (1974) to investigate vertical wells with finite conductivity vertical fractures. Cinco-Ley and Samaniego (1981) utilized the finite conductivity fracture solution developed by Cinco-Ley et al. (1978) and showed the characteristics of flow regimes and pressure transient analysis procedures. The authors identified bilinear flow regime, which is observable on a log-log plot of dimensionless pressure against dimensionless time by fitting a quarter slope straight line to the data. Wattenbarger et al. (1998) presented complete solutions to
a closed linear flow system, for both constant rate and constant pressure boundary conditions. The authors also derived transient linear flow (short-term) and boundary dominated flow (long-term) approximations from the complete solutions. Bello and Wattenbarger (2009) presented transient linear flow solutions with the effect of skin for both constant rate and constant pressure boundary conditions. They illustrated that a skin effect modifies the shape of the data on type curve and specialized plot, which tends to explain the early-time curve shapes of actual wells in tight reservoirs.

Liang et al. (2012) extended the work done by Cinco-Ley and Samaniego (1981) and presented dimensionless type curves for various inter-fracture spacing to fracture half length ratios. The authors also investigated the time to the end of transient linear flow, the duration of transition, and the time to the start of compound linear flow. Thompson et al. (2012) presented dimensionless data on linear specialized plot to show the behavior of transient linear flow, transition, and compound linear flow. The authors illustrated that negative y-intercepts on specialized plots are typically created from the transition period following the transient linear flow and quantified both the magnitude and sign of the resulting y-intercept caused by the compound linear flow. These two papers form the analytical basis of the complex flow system that will be discussed in Chapter 4.

Anderson and Mattar (2003) investigated the effect of boundary dominated flow superposition-time (also known as material balance time) during transient linear flow regime and quantified a correction factor with the use of material balance time for transient radial flow and transient linear flow. They also used synthetic and field examples to show that material balance time errors significantly influence interpretation in practice when abrupt rate/pressure fluctuations occur in transient data. Liang et al.
demonstrated that superposition-time is effective in converting variable rate/variable pressure data to its constant rate equivalent. The authors investigated the effects of skin and outliers on the data after superposition-time was applied and proposed a practical workflow to analyze variable rate/variable pressure data by using material balance time.

Notwithstanding that it is a powerful tool for analyzing production data, superposition-time suffers many disadvantages. Moghadam and Mattar (2011) investigated synthetic data from various rate/pressure profiles and determined under what circumstances the choice of superposition-time function does not change the shape of the data. They revealed that an inappropriate superposition function can result in the wrong diagnosis of flow regimes and the wrong interpretation of well performance in production data analysis. Agnia et al. (2012) also showed that production data analysis is biased if inappropriate superposition-time functions are selected. Furthermore, they investigated the importance of production data diagnosis and proposed a simple yet practical technique to determine whether or not the resulting data is biased on the superposition-time specialized plots.

Moghadam et al. (2011) modified the classical P/Z gas material balance and developed an advanced gas material balance equation to account for the effects of adsorption and residual fluid and formation expansion. Rahman et al. (2006) demonstrated that conventional computation of pseudo time by direct integration can result in significant errors, notably with a large degree of depletion. They developed a rigorous method of computing pseudo time and presented an accurate definition of total compressibility. Anderson and Mattar (2005) invented a new semi-analytical method for
correcting flow equations to accommodate changing gas properties with pressure. They recommended replacing pseudo time with their newly developed corrected pseudo time when analyzing gas production data with significant transient flow. Mattar and Anderson (2005) developed a flowing material balance equation to determine gas in place without shutting in wells. It is applicable to either constant rate or variable rate/variable pressure situations. The above literature provides the theoretical principle underlying the analysis of gas production data.

Last but not least, Agnia et al. (2012) presented a workflow with proper use of superposition-time function to analyze production data for shale gas wells. This study helped the development of the workflow presented in Chapter 5. Mattar (1995 and 1997) published two papers to discuss the importance of engineering judgment and experience in pressure transient analysis. He stated that pressure transient analysis is both art (judgment/experience/practice) and science (physics/mathematics/theory). The same statement is also applicable to production data analysis and has been considered in the development of the workflow proposed in the thesis.
CHAPTER 3
SUPERPOSITION PRINCIPLE, APPLICATION AND METHOD
DEVELOPMENT

For all linear and homogeneous systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually. Such that if $y$ is the desired solution to a linear, homogeneous equation and $f(x_1), f(x_2)$ and so forth are the known particular solutions, then:

$$y = (m_1f(x_1) + b_1) + (m_2f(x_2) + b_2) + \cdots$$

$$....................................................(3.1)$$

$m_1, m_2$ as well as $b_1, b_2$, and so forth, being constants required to satisfy the boundary conditions.

Superposition is considered to be a problem-solving philosophy in which the behavior at any point and at any time is the sum of the histories of each of the effects that are considered to affect the solution.

3.1 Superposition Principle

3.1.1 Superposition in Time vs. Superposition in Space

In reservoir engineering, superposition is often carried out by two particular approaches that are important in the analysis of pressure and rate data: superposition in time and superposition in space. The difference between the two concepts is demonstrated by two simplified examples below.
When a well is producing at constant rate $q_1$ for time $t_1$ and thereafter at constant rate $q_2$ for time $t_2$, the desired solution at time $t_2$ in this multiple-rate situation can be obtained by superposing (adding) the solution due to rate $q_1$ and the solution due to rate $(q_2 - q_1)$ at time $(t_2 - t_1)$. This is called superposition in time. The same principle can be applied to any number of variable rates.

When two wells are producing from a common reservoir but at different locations, the solution at any point in the reservoir is affected by each of the two producing wells. The desired solution at Point P can be obtained by superposing (adding) the solution at Point P due to Well 1 and the solution at Point P due to Well 2. This is called superposition in space. The same principle can be applied to any number of well locations.

Despite the great analytical significance in superposition in space, this thesis only investigates the superposition in time concept because the scope of the study only focuses on a single well’s production data analysis. Future work is recommended to research the analysis method of multiple wells by applying the superposition in space concept.

3.1.2 Superposing Pressures vs. Superposing Rates

When superposing the solutions with respect to time, two approaches are available: superposing pressures and superposing rates.

Superposing pressure responses assumes that each solution’s boundary condition is constant rate. Say a well is flowing at a constant rate $q_1$ for time $t_1$, and thereafter at a constant rate $q_2$ for time $t_2$, and lastly at a constant rate $q_3$ for time $t_3$. The total pressure drop at time $t_3$ ($\Delta P_{t3}$) is obtained by adding the pressure responses due to the following three constant rate conditions:
\[ \Delta P_{t3} = \text{pressure drop due to } q_1 \text{ throughout the entire flow period (flow time } = t_3) \]
+ pressure drop due to \((q_2 - q_1)\), commencing at the time \(t_1\) (flow time \(= t_3 - t_1\))
+ pressure drop due to \((q_3 - q_2)\), commencing at the time \(t_2\) (flow time \(= t_3 - t_2\))

\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (3.2) \]

The above example of superposing pressures is illustrated in Figures 3.1 and 3.2. Note that this “multiple-rate” example is simplified for demonstration purpose. The same principle can be applied to any number of step rate changes.

![Figure 3.1: multiple-rate profile for the example of superposing pressures](image)

![Figure 3.2: pressure and total pressure profile for the example of superposing pressures](image)

Superposing rates, on the other hand, assumes that each solution’s boundary condition is constant pressure. Say a well is flowing at a constant pressure \(P_1\) for time \(t_1\), and thereafter at a constant pressure \(P_2\) for time \(t_2\), and lastly at a constant pressure \(P_3\) for
time $t_3$. The overall rate at time $t_3$ ($q_{t3}$) is obtained by adding the rate responses due to the following three constant pressure conditions:

$$q_{t3} = \text{rate due to } (P_i - P_1) \text{ throughout the entire flow period (flow time } = t_3)$$

+ rate due to $(P_1 - P_2)$, commencing at the time $t_1$ (flow time $= t_3 - t_1$)

+ rate due to $(P_2 - P_3)$, commencing at the time $t_2$ (flow time $= t_3 - t_2$) .....(3.3)

The above example of superposing rates is illustrated in Figures 3.3 and 3.4. Note that this “multiple-pressure” example is simplified and the same principle can be applied to any number of step pressure changes.

![Figure 3.3: multiple-pressure profile for the example of superposing rates](image)

In theory, if a well is operated with rate step changes (as shown in Figure 3.1), then superposing pressures is straightforward to implement and it will convert the
“multiple-rate” system to its single constant rate equivalent; if a well is operated with pressure step changes (as shown in Figure 3.3), then superposing rates is straightforward to implement and it will convert the “multiple-pressure” system to its single constant pressure equivalent; moreover, in a variable-rate/variable-pressure situation, both superposing techniques can be used and should be equally effective in converting data to their respective constant boundary condition solution. However, to the best of the author’s knowledge, none of the pressure and rate transient analysis selects the option of superposing rates, even when analyzing data from constant pressure condition. This is because the diffusivity equation is hard to be solved with the constant pressure boundary condition so that the constant pressure solutions are often expressed in terms of complicated mathematical functions. This makes the resulting formulation after superposing rates intricate and difficult to be understood. Furthermore, the pressure drop due to skin damage is not constant and diminishes over time at constant pressure boundary condition. Therefore, in the presence of skin, the constant pressure solution becomes somewhat a surprise and difficult to be analyzed (Bello and Wattenbarger, 2009). This adds significant complexity to the implementation of constant pressure superposition.

Because of the challenging nature of constant pressure solutions, the approach of superposing rates will not be further investigated. The remainder of the thesis only applies the approach of superposing pressures (as is done in conventional pressure and rate transient analysis).

In summary, after the comparison of different superposition concepts and techniques, superposition in time and superposing pressures are chosen as the most
suitable options to fulfill the requirement of this thesis. The thesis will focus on single-well analysis and implement the approach of superposing pressures from the known solution derived from the constant rate boundary condition.

### 3.2 Superposition Application – Superposition-Time vs. Superposition-Rate

The superposition procedure is a simple yet powerful tool. Simply stated, an anticipated pressure response from a well in a variable-rate/variable-pressure situation can be modeled by assuming constant rate during the time interval between different rates and thereafter combining pressure response of each time interval for which a simple and straightforward solution is available.

Superposition (sometime is also called convolution) can be mathematically expressed as follows:

\[ P(t) = P_i - \int_0^t q(t - \tau)P_u'(\tau)d\tau \]  

**Equation 3.4** deals with continuously changing rates. If the rates change discretely (i.e. step rate changes), the superposition equation becomes:

\[ P(t) = P_i - \sum_{i=1}^{n}(q_i - q_{i-1})P_u(t - t_{i-1}) \]  

where \( P_u \) is called unit rate function. Within the scope of this study, \( P_u \) can be simplified to the following equation:

\[ P_u = mf(t) + b \]  

To demonstrate the superposition principle, this section adopts the simplified multiple-rate example previously presented in Figures 3.1 and 3.2. For each single rate, \( q \), the pressure drop can be written as follows:

\[ \frac{\Delta P}{q} = mf(t) + b \]
Subsequently the total pressure drop of multiple-rate system at $t_3$ (Equation 3.2) can be re-written as:

$$\Delta P_{t3} = q_1 \times [mf(t_3) + b] + (q_2 - q_1) \times [mf(t_3 - t_1) + b] + (q_3 - q_2) \times [mf(t_3 - t_2) + b] \tag{3.8}$$

$$\Delta P_{t3} = m \times [q_1 f(t_3) + (q_2 - q_1)f(t_3 - t_1) + (q_3 - q_2)f(t_3 - t_2)] + q_3 b \tag{3.9}$$

It is instructive to transform Equation 3.9 into an equivalent single rate solution as follows:

$$\Delta P_{t3} = m \times [q_1 f(t_3) + (q_2 - q_1)f(t_3 - t_1) + (q_3 - q_2)f(t_3 - t_2)] + q_3 b = m \times q_x f(t_x) + q_3 b \tag{3.10}$$

where $q_x$ and $t_x$ are unknowns.

To continue the calculation, we must select one of the following two options:

1. make $q_x$ equal $q_3$ and thereafter calculate $t_x$; or
2. make $t_x$ equal $t_3$ and thereafter calculate $q_x$

The first option is the classical application of the superposition principle and the calculated $t_x$ has a well-known name: Superposition-Time ($t_{sp}$). The second option is the application being developed in this thesis, and the calculated $q_x$ is given the name of Superposition-Rate ($q_{sp}$). First, the implementation of superposition-time is demonstrated below. This is followed by a demonstration of superposition-rate.

After replacing $q_x$ by $q_3$ in Equation 3.10, superposition-time is calculated as follows:

$$f(t_{sp}) = \frac{q_1 f(t_3) + (q_2 - q_1)f(t_3 - t_1) + (q_3 - q_2)f(t_3 - t_2)}{q_3} \tag{3.11}$$
The same superposition principle can be applied to any number of multiple-rate conditions. For any number of step rate changes, n, the generalized formula of superposition-time is written as follows:

\[
f(t_{sp}) = \sum_{i=1}^{n} \frac{(q_i - q_{i-1})f(t_n - t_{i-1})}{q_n}
\]

(3.12)

**Figures 3.5** and **3.6** demonstrate how the superposition-time works. An equivalent single rate system is created for the same well, in which the well operates at constant rate \( q_3 \) until time \( t_{sp} \). The pressure drop due to \( q_3 \) equals the overall pressure drop, \( \Delta P_{t3} \), caused by the entire multiple-rate system.

**Figure 3.5**: superposition-time demonstration - well flows at constant rate \( q_3 \) for the duration of \( t_{sp} \)

**Figure 3.6**: superposition-time demonstration - pressure drop caused by \( q_3 \) at \( t_{sp} \) equals multiple-rate’s pressure drop at \( t_3 \)
Alternatively, to develop the formulation for superposition-rate, $t_x$ in Equation 3.10 is replaced by $t_3$:

$$
q_{sp} = \frac{q_1 f(t_3) + (q_2 - q_1)f(t_2 - t_1) + (q_3 - q_2)f(t_3 - t_2)}{f(t_3)}
$$

(3.13)

For any number of step rate changes, $n$, the generalized formula of the superposition-rate is written as follows:

$$
q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1})f(t_n - t_{i-1})}{f(t_n)}
$$

(3.14)

Figures 3.7 and 3.8 demonstrate how the superposition-rate works. An equivalent single rate system is created, in which the well operates at constant rate, $q_{sp}$, for the whole duration $(t_3)$. The pressure drop due to $q_{sp}$ equals the overall pressure drop at $t_3$ caused by the entire multiple-rate system.

![Figure 3.7: superposition-rate demonstration - well flows at equivalent constant rate $q_{sp}$ for the entire period of $t_3$](image)

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Figure 3.7: Superposition-rate demonstration - well flows at equivalent constant rate $q_{sp}$ for the entire period of $t_3$. 

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3.3 Development of Superposition Function for Individual Flow Regime

3.3.1 Development of Analytical Solutions

The fundamental equation for liquid (or fluid of small compressibility) flow through porous media is given by:

\[ \nabla \left( \rho \frac{1}{\mu} k \nabla P \right) = \frac{\partial (\Phi p)}{\partial t} \]

In the forthcoming work of this thesis, the above flow equation will be solved for various reservoir geometries and the following boundary conditions:

1. inner boundary condition: the flow rate at the wellbore \( r_w \) is constant, or
   \[ \frac{\partial P}{\partial r} |_{r_w} = \text{constant}, \text{ for } t > 0 \]

2. outer boundary condition: no fluid flows across the reservoir boundary \( r_e \), or
   \[ \frac{\partial P}{\partial r} |_{r_e} = 0, \text{ for } t > 0 \]

The flow equation through porous media given above is in generalized form. To further linearize it and solve it analytically, some simplifying assumptions must be applied. The assumptions of interest are summarized below (AER Directive034, 1975):

1. isothermal condition prevails
2. gravitational effects are negligible
3. flowing fluid is single phase
4. Flow is laminar
5. the porous medium is homogeneous, isotropic and incompressible, and porosity is constant
6. permeability is constant and independent of pressure
7. flowing fluid viscosity is constant and independent of pressure
8. flowing fluid compressibility is small and constant
9. pressure gradient is small

In addition to the assumptions given above, any heterogeneous characteristics in the reservoir are out of the scope of this study and consequently are not considered throughout the theoretical development. These heterogeneities include, but are not limited to, dual porosity, dual permeability, multiple-layer, composite geometry (i.e. stimulated reservoir volume), and so on.

Because of their inherent simplicity, dimensionless terms will be used whenever expedient. Expression in dimensionless terms reduces the number of variables and makes analytical solutions be able to be presented in form of Equation 3.7.

When assumptions 1 - 9 are applied to Equation 3.15, the generalized flow equation for a slightly compressible fluid can be written in dimensionless terms as follows:

$$\nabla^2 (P_D) = \frac{\partial (P_D)}{\partial (z_D)}$$ .................................................................(3.18)

For radial flow geometry (i.e. cylindrical shape reservoir with a vertical well at the center), Equation 3.18 becomes:
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P_D}{\partial r} \right) = \frac{\partial P_D}{\partial t_D} \tag{3.19}
\]
\[
\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \tag{3.20}
\]

where

\[
P_D = \frac{kh(P_i - P)}{141.2qB_{\mu}} \tag{3.21}
\]
\[
t_D = \frac{0.00633kt}{\phi \mu c r_w^2} \tag{3.22}
\]
\[
r_D = \frac{r}{r_w} \tag{3.23}
\]
\[
r_{ed} = \frac{r_e}{r_w} \tag{3.24}
\]

The complete radial flow solution at the wellbore \((r_w)\) is given by van Everdingen and Hurst (1949):

\[
P_{wd} = \frac{2}{r_{ed} - 1} \left( \frac{1}{4} + t_D \right) - \frac{3r_{ed}^4 - 4r_{ed}^4 \ln r_{ed} - 2r_{ed}^2 - 1}{4(r_{ed}^2 - 1)^2} + 2 \sum_{n=1}^{\infty} \frac{e^{-\beta_n^2 t_D} J_1^2(\beta_n r_{ed})}{\beta_n^2 [J_1^2(\beta_n r_{ed}) - J_1(\beta_n)]} \tag{3.25}
\]

where \(\beta_n\) are the roots of

\[
J_1(\beta_n r_{ed})Y_1(\beta_n) - J_1(\beta_n)Y_1(\beta_n r_{ed}) = 0 \tag{3.26}
\]

\(J_1\) and \(Y_1\) being the Bessel functions of the first and second kind of order 1.

\[
P_{wd} = \frac{kh(P_i - P_{wd})}{141.2qB_{\mu}} \tag{3.27}
\]

The early-time (i.e. \(\frac{t_D}{r_{ed}^2} < 0.25\)) transient approximation of Equation 3.25 is given by:

\[
P_{wd} = \frac{1}{2} \ln t_D + 0.4045 \tag{3.28}
\]

The late-time (i.e. \(\frac{t_D}{r_{ed}^2} > 0.25\)) boundary dominated approximation of Equation 3.25 is given by:
\[ P_{WD} = \frac{2t_D}{r_{eD}} + \ln r_{eD} - \frac{3}{4} \] .................................................................(3.29)

Alternatively, Equation 3.29 can be expressed in terms of \( t_{DA} \) as follows:

\[ P_{WD} = 2\pi t_{DA} + \ln r_{eD} - \frac{3}{4} \] .................................................................(3.30)

where

\[ t_{DA} = \frac{0.00633kt}{\phi \mu c A} = \frac{0.00633kt}{\phi \mu c \pi r_{eD}^2} \] .................................................................(3.31)

For linear flow geometry (i.e. a rectangular shape reservoir and a vertical well located at the center with an infinite-conductivity, full-penetration, full-extension vertical fracture), Equation 3.18 becomes:

\[ \frac{\partial^2 P_D}{\partial x_D^2} = \frac{\partial P_D}{\partial t_{Dxf}} \] .................................................................(3.32)

\[ P_D = \frac{k h (P_i - P_{
 完整的线性流动解在井口由 Gringarten et al. (1974) 和 Wattenbarger et al. (1998) 给出。
\[ P_{WD} = \frac{\pi}{2} \left( \frac{x_f}{y_e} \right) t_{Dxf} + \frac{\pi}{6} \left( \frac{y_e}{x_f} \right) \] .................................................................(3.38)

Alternatively, \textbf{Equation 3.38} can be expressed in terms of \( t_{DA} \) as follows:

\[ P_{WD} = 2\pi t_{DA} + \frac{\pi}{6} \left( \frac{y_e}{x_f} \right) \] .................................................................(3.39)

where

\[ t_{DA} = \frac{0.00633 k t}{\phi \mu c_l A} = \frac{0.00633 k t}{4 \phi \mu c_l x_f y_e} \] .................................................................(3.40)

Note that the analytical solutions developed above are derived for the linear flow geometry with a vertical infinite-conductivity fracture. For the linear flow geometry with a finite-conductivity fracture, transient bilinear flow regime exists and occurs prior to the transient linear flow. The analytical solution of the bilinear flow regime (Cinco-Ley et al. 1981) is presented as follows:

\[ P_{WD} = \frac{2.45}{\sqrt{FCD}} \sqrt{t_{Dxf}} \] .................................................................(3.41)

where

\[ FCD = \frac{k_r w_f}{k x_f} \] .................................................................(3.42)

As a review, the analytical solution to transient radial flow is presented in \textbf{Equation 3.28}; the analytical solution to transient linear flow is presented in \textbf{Equation 3.37}; the analytical solution to transient bilinear flow is presented in \textbf{Equation 3.41}; and the solutions to boundary dominated flow are presented in \textbf{Equations 3.30} and \textbf{3.39}.

\textit{3.3.2 Effect of Skin}

All the analytical solutions in the previous section are derived for the ideal fluid flow system without skin effect. But in real life, there are many additional pressure drops (\( \Delta P_s \)) that can occur but that have not been accounted for in the ideal system. These can be caused by near wellbore effects such as flow convergence, turbulence in the fracture,
phase trapping, and non-reservoir effects such as liquid loading in the wellbore (Thompson et al. 2012). Each individual and/or the combination of these effects all manifest themselves as a positive skin.

Non-Darcy flow introduces an additional pressure drop that is flow rate dependent. Therefore it is not constant over the period of production. The Non-Darcy flow effect is not considered within the scope of this thesis, but worth further investigation in future research.

Skin is defined as an additional pressure dimensionless term given by:

\[ P_{D_{\text{actual}}} = P_{D_{\text{ideal}}} + S \] .................................................................(3.43)

Note that the formulation of the skin term must be somewhat consistent with the definition of the dimensionless pressure. In this thesis, the skin definition of a vertical well without fracture is:

\[ S = \frac{kh\Delta P_s}{141.2qB\mu} \] .................................................................(3.44)

It is convenient to express the skin of a well with fracture (for both infinite-conductivity and finite-conductivity) in dimensionless terms as:

\[ S_f = \frac{k(2\pi\epsilon)\Delta P_s}{141.2qB\mu} \] .................................................................(3.45)

Consequently

\[ S = S_f \frac{h}{2\pi r_f} \] .................................................................(3.46)

In the presence of skin, the analytical solution to transient radial flow (Equation 3.28) becomes:

\[ P_{WD} = \frac{1}{2} \ln t_D + 0.4045 + S \] .................................................................(3.47)

The analytical solution to transient linear flow (Equation 3.37) becomes:
P_{WD} = \sqrt{\pi t D_{xf}} + S f \frac{h}{2x_f} \hspace{3cm} (3.48)

The analytical solution to transient bilinear flow (Equation 3.41) becomes:

\[ P_{WD} = \frac{2.45}{\sqrt{FCD}} \sqrt{t D_{xf}} + S f \frac{h}{2x_f} \hspace{3cm} (3.49) \]

The following two equations represent the analytical solutions to boundary dominated flow (Equations 3.30 and 3.39) with skin effect in the radial flow geometry and linear flow geometry, respectively:

\[ P_{WD} = 2\pi t_{DA} + \ln r eD - \frac{3}{4} + S \hspace{3cm} (3.50) \]
\[ P_{WD} = 2\pi t_{DA} + \frac{\pi}{6} \left( \frac{v e}{x f} \right) + S f \frac{h}{2x_f} \hspace{3cm} (3.51) \]

3.3.3 Superposition for Transient Radial Flow

As alluded in Equation 3.47, in transient radial flow, the time function, f(t), becomes ln(t) (or log(t) which is more commonly used). Subsequently, the generalized formula of superposition-time (Equation 3.12) can be re-written as below:

\[ \log(t_{sp}) = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \log(t_n - t_{i-1})}{q_n} \hspace{3cm} (3.52) \]

Substitute log(t_{sp}) for f(t_x) in Equation 3.10 and make q_x equal to q_n, thereafter Equation 3.10 becomes:

\[ \Delta P_t = q_n m \log(t_{sp}) + q_n b \hspace{3cm} (3.53) \]

After re-arrangement of the above equation, the flow equation of transient radial flow regime can be expressed in terms of superposition-time as follows:

\[ \frac{\Delta P}{q_n} = m \log(t_{sp}) + b \hspace{3cm} (3.54) \]

where m and b are constant. They can be obtained by combining Equation 3.22, 3.27, 3.44 and 3.47, and the resulting equations are presented as follows:
Similar to superposition-time, the generalized formula of superposition-rate (Equation 3.14) can be re-written by substituting \( \log(t) \) for \( f(t) \):

\[
q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \log(t_n - t_{i-1})}{\log(t_n)}
\]

Substitute \( q_{sp} \) for \( q_x \) in Equation 3.10 and make \( t_x \) equal to \( t_n \), thereafter Equation 3.10 becomes:

\[
\Delta P_t = q_{sp} m \log(t_n) + q_n b
\]

After re-arrangement of the above equation, the flow equation can be expressed in terms of superposition-rate as follows:

\[
\frac{\Delta P - q_n b}{q_{sp}} + b = m \log(t) + b
\]

where \( m \) and \( b \) are constant and have the same definitions as stated in Equations 3.55 and 3.56.

It is worth mentioning that there is a reason to keep the \( b \) term on the both sides of Equation 3.59. It may seem to be redundant. However, it was decided to express the flow equation in this way deliberately, because it makes the solution look like its constant rate equivalent.

### 3.3.4 Superposition for Transient Linear Flow

In transient linear flow, the time function, \( f(t) \), becomes \( \sqrt{t} \). Subsequently the generalized formula of superposition-time (Equation 3.12) can be re-written as below:

\[
\sqrt{t_{sp}} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{q_n}
\]
Substitute \( \sqrt{t_{sp}} \) for \( f(t_x) \) in Equation 3.10 and make \( q_x \) equal to \( q_n \), thereafter Equation 3.10 becomes:

\[
\Delta P_t = q_n m \sqrt{t_{sp}} + q_n b ..........................................................(3.61)
\]

After re-arrangement of the above equation, the flow equation of transient linear flow regime can be expressed in terms of superposition-time as follows:

\[
\frac{\Delta P}{q_n} = m \sqrt{t_{sp}} + b ..........................................................(3.62)
\]

where \( m \) and \( b \) are constant. They can be obtained by combining Equation 3.27, 3.34, 3.45 and 3.48, and the resulting equations are presented as follows:

\[
m = \frac{141.2B}{x_{hf}} \sqrt{\frac{0.00633\mu}{k\phi \epsilon}} ..........................................................(3.63)
\]

\[
b = \frac{141.2B\mu}{k(2x_f)} S_f ..........................................................(3.64)
\]

Similar to superposition-time, the generalized formula of superposition-rate (Equation 3.14) can be re-written by substituting \( \sqrt{t} \) for \( f(t) \):

\[
q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{\sqrt{t_n}} ..........................................................(3.65)
\]

Substitute \( q_{sp} \) for \( q_x \) in Equation 3.10 and make \( t_x \) equal to \( t_n \), thereafter Equation 3.10 becomes:

\[
\Delta P_t = q_{sp} m \sqrt{t_n} + q_n b ..........................................................(3.66)
\]

After re-arrangement of the above equation, the flow equation can be expressed in terms of superposition-rate as follows:

\[
\frac{\Delta P - q_n b}{q_{sp}} + b = m \sqrt{t} + b ..........................................................(3.67)
\]

where \( m \) and \( b \) are constant and have the same definitions as stated in Equations 3.63 and 3.64.
3.3.5 Superposition for Transient Bilinear Flow

In transient bilinear flow, the time function, $f(t)$, becomes $\sqrt[4]{t}$. Subsequently the generalized formula of superposition-time (Equation 3.12) can be re-written as below:

$$\frac{4}{\sqrt{t_{sp}}} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1})\sqrt[n]{(t_n - t_{i-1})}}{q_n}$$

The flow equation of transient bilinear flow regime can be expressed in terms of superposition-time as follows:

$$\frac{\Delta P}{q_n} = m\frac{4}{\sqrt{t_{sp}}} + b$$

where $m$ and $b$ are constant. They can be obtained by combining Equation 3.27, 3.34, 3.45 and 3.49, and the resulting equations are presented as follows:

$$m = \frac{97.58B^4}{\sqrt{k_f w_f h}} \frac{1}{\sqrt{\kappa \mu c_t}}$$

$$b = \frac{1412B^4}{k(2x_f)} S_f$$

Similar to superposition-time, the generalized formula of superposition-rate (Equation 3.14) can be re-written by substituting $\sqrt[4]{t}$ for $f(t)$:

$$q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1})\sqrt[n]{(t_n - t_{i-1})}}{4\sqrt{t_n}}$$

The flow equation is expressed in terms of superposition-rate as follows:

$$\frac{\Delta P - q_n b}{q_{sp}} + b = m\sqrt[4]{t} + b$$

where $m$ and $b$ are constant and have the same definitions as stated in Equations 3.70 and 3.71.

3.3.6 Superposition for Boundary Dominated Flow
In boundary dominated flow, the time function, \( f(t) \), becomes \( t \). Evaluation of the generalized formula of superposition-time \((\text{Equation } 3.12)\) shows that the superposition-time during boundary dominated flow is equivalent to:

\[
t_{sp} = \frac{Q_n}{q_n} \tag{3.74}
\]

Note the superposition-time defined in \((\text{Equation } 3.74)\) has another name, Material Balance Time. This name is well-known and comes from the fact that the boundary dominated flow regime is greatly under the influence of depletion and material balance considerations. Material balance time is a popular time function that has been widely applied in modern production data analysis.

The flow equation of boundary dominated flow regime can be expressed in terms of material balance time as follows:

\[
\frac{\Delta P}{q_n} = mt_{sp} + b \tag{3.75}
\]

\( m \) is a function of reservoir pore volume (material balance considerations), and does not depend on reservoir shape (flow geometry). Thus, for both radial and linear flow geometries, \( m \) is given by:

\[
m = \frac{141.2B}{h} \left( \frac{2\pi \times 0.00633}{\phi \Omega A} \right) \tag{3.76}
\]

\( b \), on the other hand, has different definitions, depending on the reservoir shape.

For a well centered in a circular reservoir (radial flow geometry), \( b \) is defined as:

\[
b = \frac{141.2B}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + S \right] \tag{3.77}
\]

For a well with a fully penetrating hydraulic fracture centered in a rectangular reservoir (linear flow geometry), \( b \) is defined as:

\[
b = \frac{141.2B}{kh} \left[ \frac{\pi}{6} \left( \frac{v_e}{x_f} \right) + \frac{h}{2x_f} S_f \right] \tag{3.78}
\]
Similar to superposition-time, the generalized formula of superposition-rate (Equation 3.14) can be re-written by substituting \( t \) for \( f(t) \):

\[
q_{sp} = \frac{Q_n}{t_n} \tag{3.79}
\]

The flow equation is expressed in terms of superposition-rate as follows:

\[
\frac{\Delta P - q_n b}{q_{sp}} + b = mt + b \tag{3.80}
\]

where \( m \) and \( b \) are constant and have the same definitions as stated in Equations 3.76, 3.77, and 3.78.

As a review, all of the definitions of superposition-time and superposition-rate as well as the flow equations derived for different flow regimes are summarized in Tables A1 and A2 in the Appendix.

3.3.7 Gas Consideration

All equations developed in the previous sections strictly apply to slightly compressible fluid, but they can be used to analyze gas flow if pseudo pressure is substituted for pressure and pseudo time is substituted for time.

The pseudo pressure is rigorous and straightforward and can be calculated as follows:

\[
p_p = 2 \int_{p_0}^{p} \frac{p}{\mu z} dp \tag{3.81}
\]

The pseudo time definition can change and is dependent on its application. In well test analysis, during shut in period, the pseudo time is calculated based on the bottom hole pressure at the wellbore. The pseudo time of build up or fall off is expressed as follows:

\[
t_a = \mu_1 c_{tl} \int_0^{t} \frac{dt}{\mu_w c_{tw}} \tag{3.82}
\]
On the other hand, in production analysis, pseudo time calculation becomes iterative and requires the knowledge of the average pressure within the investigated drainage area (Anderson and Mattar, 2005). The resulting time function has a new name and is called corrected pseudo time. The definition of corrected pseudo-time is shown below:

\[ t_{ca} = \mu_l c_t i \int_0^t \frac{dt}{(\mu_i c_t)_{inv}} \] (3.83)

It was concluded that pseudo pressure and corrected pseudo time are effective in accounting for changing gas properties in production data and making the gas dimensionless type curve look the same as its liquid equivalent (Liang et al. 2011).

In the analysis of production data, calculating pseudo pressure is straightforward but calculating corrected pseudo time not only becomes iterative but the calculation changes at each data point. The diagnostic improvement in corrected pseudo time can be easily masked by the presence of data scatter and other sources of errors, and the overall effect of corrected pseudo time is insignificant. In other words, although it is recommended to use corrected pseudo time, the complexity of its use in the analysis is not justified in practice. Therefore, in the remainder of this thesis, a more simplified time function, pseudo time (as opposed to corrected pseudo time), is used to analyze gas production. While pseudo time calculation is still iterative, it is much more convenient because it only requires calculating the average pressure of the reservoir. In other words, pseudo time is corresponding to the total reservoir volume and not to the changing volume of investigation. The definition of pseudo-time is given by:

\[ t_a = \mu_l c_t i \int_0^t \frac{dt}{\mu_i c_t} \] (3.84)
It should be emphasized that the implementation of pseudo pressure and pseudo time only changes the constants (m and b) in the flow equations of each flow regime, but it does not change the generalized form of the flow equations as well as the definitions of superposition-time and superposition-rate. As a review, all equations and definitions\textsuperscript{1} for gas system are summarized in Tables A3 and A4 in the Appendix.

\textsuperscript{1} All equations and definitions for gas system were developed based on the standard conditions in Alberta, Canada (14.65 psia and 60 °F).
CHAPTER 4

COMPARISON BETWEEN SUPERPOSITION-TIME AND SUPERPOSITION-RATE

As mentioned in Chapter 3, both superposition-time and superposition-rate are derived from the same superposition principle. Furthermore, these two superposition applications attempt to achieve the same purpose in production data analysis, which is to convert variable rate/variable pressure data into its equivalent constant rate solution. But during the analysis, superposition-rate and superposition-time manipulate the data in completely different ways, which leads to a significant difference in their resulting diagnostic plots.

To illustrate the difference, a simple synthetic data set was created from an infinite-conductivity vertical fractured well model in which only transient linear flow exist and skin is zero. The well is modeled for a simplified rate step change situation as shown in Figure 4.1. From inspection of the analytical solution to transient linear flow for a single constant rate (Equation 3.48), it is evident that plotting the data on Cartesian coordinate with normalized pressure \( \frac{P_i - P_{wf}}{q_n} \) on the y-axis and square-root of time on the x-axis will yield a straight line. This “straight line” analysis plot (Figure 4.2) is called Linear Flow Specialized Plot. In this example, when \( \frac{P_i - P_{wf}}{q_n} \) is plotted against square-root of real time, only the first flow period’s constant rate data yields a meaningful straight line (the red line). Since the constant rate data from the second and third flow periods do not form a single straight line, analysis cannot be done for the entire data set.
Figure 4.1: synthetic gas rates in a three-step rate change situation

![Graph showing synthetic gas rates](image)

Figure 4.2: normalized pressure vs. square-root of real time on linear flow specialized plot

![Graph showing normalized pressure vs. square-root of real time](image)

Figure 4.3 demonstrates how superposition-time manipulates the data in this rate step change example. With the use of superposition-time, $\frac{P_i - P_{wf}}{q_n}$ is plotted against square-root of superposition-time. Superposition-time modifies the time (x-axis) and moves the data back and forth in the horizontal direction. The data that occurred at early time (the dots in blue) are moved to late time; whereas the data that occurred at late time (the dots in green) are moved to early time. Superposition-time helps to unify the data of all three
flow periods into a common analysis straight line (the red line). Note that the blue dots have been moved to times that are beyond the actual flow duration.

![Diagram showing superposition-time unification](image)

**Figure 4.3:** how superposition-time unifies the data into a straight line

In contrast to the “time shuffle” caused by superposition-time, the effect of superposition-rate is illustrated in **Figure 4.4.** \( \frac{P_1-P_{wr}-qnb}{q_{sp}} + b \) is plotted against square root of real time (not superposition-time). Superposition-rate modifies the normalized pressure (y-axis) and moves the data up and down in the vertical direction. Superposition-rate unifies all data into a common straight line with the same slope and y-intercept as those from **Figure 4.3.** Note that as superposition-rate does not change time scale, all three flow periods stay in their consecutive time order. The blue dots have been moved down and the green dots up during the actual flow duration.
Figure 4.4: how superposition-rate unifies the data into a straight line

The example given above is a simplified case and only has three step changes. The same superposition principle can be applied to any number of step rate changes.

It must be emphasized that although calculating superposition-rate is simple and straightforward, implementing superposition-rate on the specialized plot is tricky and becomes an iterative process. As shown in Figure 4.4, the y-intercept (b) is used in the calculation of the normalized pressure on y-axis. But b is unknown prior to plotting the analysis straight line. Therefore iteration must be performed on b until data forms a straight line and the straight line’s y-intercept is consistent with the b value used in the calculation of normalized pressure. Fortunately this iteration process typically converges fast so it does not make specialized plots too cumbersome to analyze.

4.1 Introduction to Diagnostic Plots and Comparison Method

4.1.1 Diagnostic Plots
Before comparing the two superposition applications, let us first review two types of diagnostic plots that frequently appear in the analysis of production data. These diagnostic plots will be extensively utilized in the forthcoming sections of this study.

The first commonly used diagnostic plot is called type curve. Type curve is a graphic representation of the complete analytical solutions to the flow equation and is plotted on logarithmic coordinate (also known as log-log scale). It is a powerful method for identifying flow regimes, especially in a changing flow regime situation when multiple flow regimes exist. A type curve diagnostic plot typically consists of two curves: normalized pressure and derivative. Normalized pressure was introduced previously and is sometimes called the unit rate function. Derivative is a well-known concept in well test analysis for identifying flow regimes from pressure transient data. By definition, the derivative is the slope of the pressure difference versus time when is plotted using a semi-log coordinate. It can be expressed in a mathematical form as follows:

\[
\text{DER} = \frac{d(\Delta P)}{d(\ln(t))} = t \frac{d(\Delta P)}{d(t)} \tag{4.1}
\]

In production data analysis, when using superposition-time \(t_{sp}\) and normalized pressure \(\frac{P_{1-P_{wf}}}{q_{in}}\), the derivative is defined as:

\[
\text{DER}_{t_{sp}} = \frac{d\left(\frac{P_{1-P_{wf}}}{q_{in}}\right)}{d(\ln(t_{sp}))} = t_{sp} \frac{d(\frac{P_{1-P_{wf}}}{q_{in}})}{d(t_{sp})} \tag{4.1a}
\]

When using superposition-rate \(q_{sp}\) and normalized pressure \(\frac{P_{1-P_{wf}}-q_{n}b}{q_{sp}} + b\), the derivative is defined as:

\[
\text{DER}_{q_{sp}} = \frac{d\left(\frac{P_{1-P_{wf}}-q_{n}b}{q_{sp}} + b\right)}{d(\ln(t))} = t \frac{d\left(\frac{P_{1-P_{wf}}-q_{n}b}{q_{sp}} + b\right)}{d(t)} \tag{4.1b}
\]
From the inspection of the constant rate solutions developed in Chapter 3, it is evident that the derivative will appear as straight line on a log-log plot and the slope of each flow regime’s straight line is unique. In fact, the slope of the straight line is characteristic of flow regimes: transient radial flow exhibits a zero slope straight line, transient linear flow exhibits a half (\(\frac{1}{2}\)) slope straight line, transient bilinear flow exhibits a quarter (\(\frac{1}{4}\)) slope straight line, and boundary dominated flow exhibits a unit (1) slope straight line. The characteristic slopes of the flow regimes are much more clearly evident in the derivative than in the normalized pressure.

Another significant advantage of the derivative is that it is not affected by skin. In the presence of skin, the normalized pressures of early-time transient linear flow and transient bilinear flow are no longer straight lines on log-log plot. Instead, they become curves and consequently lose their diagnostic value. Derivative, on the other hand, always retains the flow regimes’ straight line signature regardless of the presence of skin, which makes derivative a very powerful tool to identify flow regimes.

It is worth mentioning that derivative should not be confused with superposition. Despite the fact that derivative and superposition are often used simultaneously in type curve analysis (e.g. derivative is calculated based on superposition-time), the intentions of their use are completely different. Superposition serves to convert variable rate/variable pressure solutions to their constant rate equivalents; while derivative serves to identify the flow regimes. They both have significant diagnostic value but cannot be replaced by each other.

Albeit powerful in identifying flow regimes, derivative suffers a serious problem which limits its use in production data analysis. It is a fact that derivative significantly
amplifies the noise in production data. In well test analysis, the quality of the pressure data collected from downhole recorders is good, thus the noise during build up or fall off is insignificant. However, in production data analysis, flow rates and pressures are measured at wellhead and their quality is usually poor, which often makes the derivative too noisy to be analyzed. There are several smoothing techniques available to reduce the noise in the derivative. However, smoothing should be used with caution because over smoothing the derivative can completely change the shape of the derivative and disguise flow regimes which may exist (or falsely display flow regimes which do not exist). Because this chapter uses synthetic data, derivative can be safely used and smoothing the derivative is not necessary. Data filtering and smoothing techniques will be discussed in the next chapter in which real field production is investigated.

Also note that the classic definition of type curve refers to a set of pre-plotted solutions to a particular reservoir system and is plotted with dimensionless variables. This thesis does not adopt this conventional definition. Henceforth every log-log scale diagnostic plot will be called type curve, regardless of the dimension of the variables.

The other commonly used diagnostic plot is the specialized plot. As previously shown in Figures 4.3 and 4.4, a plot on Cartesian coordinate with square root of time on the x-axis is called linear flow specialized plot. Transient radial flow, transient bilinear flow, and boundary dominated flow also have their own unique specialized plots: a Cartesian plot with logarithm of time on the x-axis is called radial flow specialized plot, a Cartesian plot with fourth root of time on the x-axis is called bilinear flow specialized plot, and a Cartesian plot with elapsed time (without any mathematical functions applied) on the x-axis is called boundary dominated flow specialized plot. When data’s flow
regime and the type of specialized plot are consistent, the constant rate solution appears as a straight line.

In contrast to the type curve which is able to identify flow regimes from a complete solution, a specialized plot is only useful in “segmental” analysis. This means the complete solution must be dissected into segments based on data’s flow regimes. Then each segment is plotted and analyzed individually on its corresponding specialized plot. The segmental analysis relies on the existence of the straight lines so sometimes it is also called “straight line” analysis. The straight lines exist and exhibit the correct slope and y-intercept only when data’s flow regime matches the type of the specialized plot. Therefore the identification on data’s flow regime is crucial to segmental analysis. A falsely determined flow regime leads to the wrong type of specialized plot selection, and subsequently results in an incorrectly positioned analysis line. In other words, a completely wrong interpretation can be made if the flow regimes are mistakenly identified.

It is also noticed that the specialized plot maintains its diagnostic value when skin is present. In the existence of skin, the normalized pressure after superposition moves parallel and always retains its straight line signature on specialized plots.

4.1.2 Comparison Method

Two objectives are expected to be achieved by comparing superposition-time with superposition-rate. First the comparison serves to accentuate the difference between superposition-time and superposition-rate and furthermore to identify the advantages as well as the pitfalls of each of these two superposition applications. Second the comparison attempts to validate the hypothesis that superposition-time and superposition-
rate have the same analytical functionality, which is to convert variable rate/variable pressure data into its constant rate equivalent.

To meet the above objectives, two synthetic data sets were created. The first set represents a well operating at constant rate; while the second set represents the same well but operating at constant pressure. After superposition-time and superposition-rate were applied separately to the constant pressure data, the results were compared to the constant rate data to see if they would overlay on top. The reason why constant pressure data was chosen is because if superposition could successfully convert the constant pressure data into its corresponding constant rate data, then it is likely able to convert all possible variable rate/variable pressure data into their constant rate equivalents. It is expected that all of variable rate/variable pressure solutions are intermediate between the constant rate and the constant pressure solutions (Liang et al. 2011), because constant rate and constant pressure are the two extreme operating conditions. The effects of superposition-time and superposition-rate on the constant pressure data were studied on both type curves and specialized plots.

In addition to the two objectives previously mentioned, this thesis also attempts to investigate the effect of superposition on the data that is in the flow regime different from the chosen superposition. When a superposition function is applied to the data that is purely in the superposition’s corresponding flow regime, then the data follows the constant rate solution and any resulting analysis is rigorous. However, in a changing flow regime situation, the data may span several different flow regimes. Superposition is no longer a single function, and will not necessarily force all the data to follow the constant rate solution faithfully. The choice of superposition function depends on the existing flow
regime, which is unknown prior to analysis. This means that, in practice, only one flow regime’s superposition can be used at a time, even though multiple flow regimes may exist. The question is: whether or not one flow regime’s superposition is sufficient to analyze other flow regimes’ data? If the answer is yes, then which of the flow regimes’ superposition function is able to generate meaningful results for other flow regimes? If the answer is no, then which (if any) of the flow regimes’ superposition function is useful for flow regime identification, so that rigorous “segmental” analysis can be done after?

To answer these questions, the effect of each flow regime’s superposition on the complete constant pressure data set was investigated. The investigation was conducted in three flow systems that are commonly encountered in real life. These three flow systems (or reservoir models) will be presented in the forthcoming sections. These include two simple flow systems (for modeling pure radial flow and pure linear flow) as well as one complex flow system (for modeling the flow of a multiple-stage fractured horizontal well).

### 4.2 Radial Flow System

The first system is radial flow system. The schematic of this flow system is shown in Figure 4.5. It represents a cylindrical shape reservoir with a vertical well drilled at the center. The reservoir is bounded by outer no-flow boundaries.
Figure 4.5: schematic of radial flow system with no-flow outer boundary

A constant rate synthetic data set and a constant pressure synthetic data set were generated using an identical single-phase (water\(^1\)) analytical model\(^2\). When normalized pressures and their derivatives were plotted against time (without any superposition) on the type curve shown in Figure 4.6, two flow regimes were clearly observed: transient radial flow followed by boundary dominated flow. The constant rate derivative shows that the transient radial flow is a zero-slope straight line and the boundary dominated flow is a unit-slope straight line. The constant pressure derivative exhibits the same zero-slope straight line as that of constant rate derivative during transient radial flow. However, it diverges from the constant rate unit-slope straight line once boundary dominated flow begins, and becomes concave upward during boundary dominated flow, corresponding to an exponential curve.

The constant rate analytical solutions of the early-time transient radial flow and late-time boundary dominated flow were developed in Chapter 3. Since plotting superposition-rate requires the y-intercept (b) of each flow regime, the definitions of b are reviewed below:

---

\(^1\) To avoid any complexity caused by variations in fluid properties, all of the synthetic data presented in this chapter were generated by choosing water as the flowing fluid.

\(^2\) All of the analytical models presented in this thesis were created using IHS Harmony software suite.
For transient radial flow:

\[ b = \frac{162.6B\mu}{kh} \left[ \log\left( \frac{k}{\phi \mu c r_w^2} \right) - 1.847 + 0.8686S \right] \] ..............................................(4.2)

For boundary dominated flow:

\[ b = \frac{141.2B\mu}{kh} \left[ \ln \left( \frac{r_a}{r_w} \right) - \frac{3}{4} + S \right] \] ......................................................(4.3)

The radial flow system’s model parameters as well as the calculated b values are presented in Table 4.1.

<table>
<thead>
<tr>
<th>s=</th>
<th>0</th>
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</thead>
<tbody>
<tr>
<td>h=</td>
<td>100 ft</td>
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<tr>
<td>\Phi=</td>
<td>0.05</td>
</tr>
<tr>
<td>\mu=</td>
<td>0.1 cP</td>
</tr>
<tr>
<td>Ct=</td>
<td>2.0E-6 1/psi</td>
</tr>
<tr>
<td>Bw=</td>
<td>1.0 bbl/stb</td>
</tr>
<tr>
<td>k=</td>
<td>0.001 md</td>
</tr>
<tr>
<td>rw=</td>
<td>1 ft</td>
</tr>
<tr>
<td>re=</td>
<td>500 ft</td>
</tr>
</tbody>
</table>

| b-Radial | 512.7 psi/bbl |
| b-BDF    | 771.6 psi/bbl |

To investigate the effect of superposition, transient radial flow and boundary dominated flow’s superposition functions were applied to the constant pressure data. Transient linear flow and transient bilinear flow’s superposition functions were not applied as they do not exist within the radial flow system.

The effect of radial superposition is shown in Figure 4.7. Since constant rate and constant pressure solutions are visually indistinguishable during transient radial flow, it is not surprising that radial superposition makes the superposed data behave like a constant rate. However, during boundary dominated flow, the superposed data does not follow the constant rate data. This is because superposition works rigorously only when the data’s flow regime matches the choice of the superposition function. In addition, from the
comparison between radial superposition-time (left plot of Figure 4.7) and radial superposition-rate (right plot of Figure 4.7), it is noticed that the boundary dominated flow data is affected by the two superposition applications in different ways. Superposition-time distorts the boundary dominated flow derivative unduly, and makes the late-time portion of boundary dominated flow look like transient radial flow. Superposition-rate, on the other hand, distorts the original data to a much lesser degree, and qualitatively, at least, aids in preserving the derivative’s effectiveness in flow regimes identification.

The effect of boundary dominated flow superposition is shown in Figure 4.8. During transient radial flow, it looks like BDF superposition-time (left plot of Figure 4.8) does not move the constant pressure data at all, and consequently retains the zero-slope signature of its derivative. On the other hand, BDF superposition-rate (right plot of Figure 4.8) causes the transient data to separate from the constant rate at the beginning but the two eventually converge at the end of transient radial flow. During boundary dominated flow, BDF superposition works properly and both superposition-time and superposition-rate make the constant pressure data look exactly the same as the constant rate. Furthermore, by comparing BDF superposition-time to BDF superposition-rate, the use of BDF superposition-time shows an advantage: BDF superposition-time makes the derivative appear to be zero-slope during transient radial flow as well as unit-slope during boundary dominated flow. In other words, BDF superposition-time retains the characteristics of both flow regimes, which makes it a better choice to identify flow regimes in radial flow system.
It is worth reiterating that in the plots on the left hand side (with the use of superposition-time), the normalized pressure is defined as \( \frac{P_l-P_{wf}}{q_n} \) and the derivatives were calculated from Equation 4.1a. But in the plots on the right hand side (with the use of superposition-rate), the superposition normalized pressure is defined as \( \left( \frac{P_l-P_{wf}-q_n b}{q_{sp}} + b \right) \) and the derivatives were calculated from Equation 4.1b. The same definition is applicable to the type curves in the remainder of this thesis.

In addition, for all of the comparison plots in the remainder of this chapter, the unit of the y-axis is in psi/(bbl/day) and the unit of the x-axis is in days.

**Figure 4.6:** type curve of radial flow system (y-axis is in psi/bbl; x-axis is in days)

**Figure 4.7:** type curve after radial superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
In addition to type curves, specialized plots of the radial flow system are also presented in this section. Figures 4.9 and 4.10 demonstrate radial flow specialized plots. Figure 4.9 shows the original constant rate data and constant pressure data; while Figure 4.10 shows the effect of radial superposition-time (left) and radial superposition-rate (right) on the constant pressure data. Figures 4.11 and 4.12 demonstrate boundary dominated flow specialized plots. Figure 4.11 shows the constant rate and constant pressure data; while Figure 4.12 shows the effect of BDF superposition-time (left) and BDF superposition-rate (right) on the constant pressure data.

For our comparison purpose, the specialized plots do not contribute additional diagnostic value than type curves. Specialized plots often are prepared for the use of segmental analysis. Their primary value is in estimating reservoir characteristics and well performance from the straight lines. More details about how to use specialized plots to perform segmental analysis will be discussed in Chapter 5.
Figure 4.9: radial flow specialized plot of radial flow system (y-axis is in psi/bbl; x-axis is in days)

Figure 4.10: radial flow specialized plot after radial superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.11: BDF specialized plot of radial flow system (y-axis is in psi/bbl; x-axis is in days)
All of the plots presented above were made based on synthetic data from the reservoir model with zero skin. A new reservoir model with positive skin was also created and its type curves are presented below. The superposition-time and superposition-rate type curves with positive skin show a behavior consistent with the type curves without skin. Therefore it can be concluded that the superposition operations are independent of the presence of skin and they manipulate the data the same way, no matter whether skin exists or not.

Figure 4.13: type curve of radial flow system with positive skin (y-axis is in psi/bbl; x-axis is in days)
4.3 Linear Flow System

The second system is linear flow system (as shown in Figure 4.16). The linear flow system represents a rectangular shape reservoir with a vertical well drilled at the center. This vertical well is completed by a bi-wing hydraulic fracture that has infinite fracture conductivity. Moreover the tips of the hydraulic fracture are located on the reservoir’s outer no-flow boundaries.
A constant rate synthetic data set and a constant pressure synthetic data set were generated for the linear flow system using a single-phase (water) analytical model. When normalized pressures and their derivatives were plotted against time (without any superposition) on the type curve shown in Figure 4.17, two flow regimes were observed: transient linear flow followed by boundary dominated flow. The constant rate data shows that both the normalized pressure and derivative exhibit half-slope straight lines during transient linear flow and unit-slope straight lines during boundary dominated flow. The constant pressure data’s normalized pressure and derivative are also half-slope straight lines during transient linear flow, but they are separated from their corresponding constant rate straight lines by a factor of $\frac{\pi}{2}$. Once boundary dominated flow begins, both the constant pressure’s normalized pressure and derivative quickly depart from their half-slope straight lines and curve upward.

The constant rate analytical solutions of the early-time transient linear flow and late-time boundary dominated flow were developed in Chapter 3. Since plotting superposition-rate requires the y-intercept (b) of each flow regime, the definitions of b are reviewed below:

For transient linear flow:
\[ b = \frac{141.2B\mu}{k(2x_f)}S_f \] ..............................(4.4)

For boundary dominated flow:

\[ b = \frac{141.2B\mu}{kh} \left[ \frac{\pi}{6} \left( \frac{y_e}{x_f} \right) + \frac{h}{2x_f}S_f \right] \] ..............................(4.5)

The linear flow system’s model parameters as well as the calculated b values are presented in Table 4.2.

<table>
<thead>
<tr>
<th>sf=</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=</td>
<td>100 ft</td>
</tr>
<tr>
<td>( \Phi = )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \mu = )</td>
<td>0.1 cP</td>
</tr>
<tr>
<td>( Ct = )</td>
<td>2.0E-06 1/psi</td>
</tr>
<tr>
<td>( Bw = )</td>
<td>1.0 bbl/stb</td>
</tr>
<tr>
<td>k=</td>
<td>0.001 md</td>
</tr>
<tr>
<td>( xf = )</td>
<td>100 ft</td>
</tr>
<tr>
<td>( Ye = )</td>
<td>500 ft</td>
</tr>
<tr>
<td>b-Linear</td>
<td>0.0 psi/bbl</td>
</tr>
<tr>
<td>b-BDF</td>
<td>369.7 psi/bbl</td>
</tr>
</tbody>
</table>

To investigate the effect of superposition, transient linear flow and boundary dominated flow’s superposition functions were applied to the constant pressure data. Transient radial flow and transient bilinear flow’s superposition functions were not applied as they do not exist within the linear flow system.

The effect of linear superposition is shown in Figure 4.18. During transient linear flow, both linear superposition-time and linear superposition-rate make the constant pressure data look identical to the constant rate data. But during boundary dominated flow, linear superposition no longer forces the constant pressure data to follow the constant rate data. From the comparison between linear superposition-time (left plot of Figure 4.18) and linear superposition-rate (right plot of Figure 4.18), it is noticed that the boundary dominated flow data is affected by the two superposition applications in
different ways. Superposition-time distorts the boundary dominated flow data unduly and makes boundary dominated flow look very similar to transient linear flow. Superposition-rate, on the other hand, distorts the data to a much lesser degree, and in fact makes the data look like a unit slope line (although is not the correct one, it nevertheless helps to identify the flow regime).

The effect of boundary dominated flow superposition is shown in Figure 4.19. During transient linear flow, BDF superposition-time (left plot of Figure 4.19) shifts the constant pressure data very close to the constant rate, while still maintaining the data’s half slope signature. On the other hand, BDF superposition-rate (right plot of Figure 4.19) shows half slope characteristic in the derivative (although offset a little), but unfortunately the normalized pressure is not a straight line anymore. During boundary dominated flow, both BDF superposition-time and BDF superposition-rate make the constant pressure data look exactly the same as the constant rate. The advantage of using BDF superposition-time is obvious in this case: BDF superposition-time makes the constant pressure data (including both normalized pressure and derivative) appear to be half-slope during transient linear flow as well as unit-slope during boundary dominated flow. In other words, BDF superposition-time retains the characteristics of both flow regimes, which makes it a better choice to identify flow regimes in linear flow system.
In addition to type curves, specialized plots of the linear flow system are also presented in this section. Figures 4.20 and 4.21 demonstrate linear flow specialized plots.
Figure 4.20 shows the original constant rate data and constant pressure data; while Figure 4.21 shows the effect of linear superposition-time (left) and linear superposition-rate (right) on the constant pressure data. Figures 4.22 and 4.23 demonstrate boundary dominated flow specialized plots. Figure 4.22 shows the original constant rate data and constant pressure data; while Figure 4.23 shows the effect of BDF superposition-time (left) and BDF superposition-rate (right) on the constant pressure data.

Once again, these specialized plots do not contribute additional diagnostic value than type curves for our comparison purpose. Specialized plots are often generated to conduct segmental analysis. More details about the use of specialized plots will be discussed in Chapter 5.
Figure 4.21: linear flow specialized plot after linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.22: BDF specialized plot of linear flow system (y-axis is in psi/bbl; x-axis is in days)

Figure 4.23: BDF specialized plot after BDF superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

All of the plots presented above were made based on synthetic data from the reservoir model with zero skin. A new reservoir model with positive skin was also created and its type curves are presented below. The type curves with positive skin are consistent with the type curves without skin. This means that superposition manipulates the data the same way, no matter whether skin exists or not.
Figure 4.24: type curve of linear flow system with positive skin

Figure 4.25: type curve with positive skin after linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.26: type curve with positive skin after BDF superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

4.4 Complex Flow System
Subsequent to the two simple flow systems already presented, a more complicated flow system is presented in Figure 4.27. This complex flow system looks very similar to the linear flow system investigated previously, and it also represents a vertical well at the center of the reservoir with infinite conductivity vertical fracture. The only difference between this complex flow system and the linear flow system is that in the linear flow system, the tips of the hydraulic fracture extend fully to the reservoir’s no-flow boundaries; whereas in the complex flow system, the tips of the fracture do not.

This small difference in geometry leads to significant complexity when solving the flow equation of the complex flow system. Gringarten et al. (1974) utilized source functions and Green’s function to obtain the complete analytical solution for a vertical well with infinite conductivity fracture. This analytical solution is complex and presented in terms of complicated mathematical functions and infinite series. In order to simplify the math and provide a better understanding of the progression of flow regimes, a conceptual flow sequence of the complex flow system is presented below.
The first flow regime in the complex flow system is transient linear flow as shown in Figure 4.28. This is identical to the transient linear flow that appears in the linear flow system.

The second flow regime starts to become difficult to envision. An additional transient flow regime may follow the first transient linear flow, but its existence relies on reservoir size. As reservoir size gets small, the effect of reservoir boundaries is observed sooner, which prevents the full establishment of the second transient flow regime. In an ideal case of large reservoir size, the second transient flow regime is able to be fully developed, but there are still three possibilities. The second flow regime could be compound linear flow (as depicted in Figure 4.29) or pseudo radial flow (as depicted in Figure 4.30) or the combination of the two. Compound linear flow is defined as linear flow towards the region depleted by the fracture and its flow direction is perpendicular to that of the first transient linear flow. Pseudo radial flow is defined as radial flow towards the vertical wellbore after the investigated drainage area has expanded far beyond the tips of the fracture.

Boundary dominated flow is the last flow regime in the complex flow system. If the reservoir size is too small, boundary dominated flow would replace the compound linear flow or pseudo radial flow and become the second flow regime following the transient linear flow.
The above conceptual sequence of flow regimes was validated by analytical modeling, with an added condition that $x_e$ is infinite to ensure the second transient flow regime is fully developed. (Consequently boundary dominated flow no longer exists.) Constant rate synthetic data sets were generated to evaluate various reservoir geometries. The constant rate data’s signatures are presented on dimensionless type curves to reveal
the behavior of the transient linear, compound linear, or pseudo radial flow regimes. The dimensionless normalized pressure type curve is shown in Figure 4.31 and the dimensionless derivative type curve is shown in Figure 4.32. Note that the type curves were generated for a range of aspect ratios, namely $ye/xf$ equals 0.05, 0.1, 0.25, 0.5, 1 and 2.5. Ratios less than 0.05 and greater than 2.5 are not considered in this work as these geometries are unlikely to be observed in real life (Liang et al. 2012). Infinite $ye/xf$ was also added to the type curves to ensure explicit presence of pseudo radial flow.

Figure 4.31: dimensionless normalized pressure type curve of complex flow system
From the inspection of the derivative type curve in Figure 4.32, it is evident that the derivative signature is consistent with the conceptual sequence of flow regimes previously discussed. Moreover, as illustrated in the example shown in Figure 4.33, the complex flow system is the fundamental “unit” model for analyzing the production data of multiple-stage fractured horizontal wells (MFHW). A MFHW can be treated as equivalent to the complex flow system, when the following assumptions are taken into consideration:

1. Ignore any production contribution from the areas beyond the lateral length of horizontal wellbore
2. All fractures are equally spaced along horizontal well
3. Properties of each individual fracture are identical
4. Horizontal wellbore do not contribute to total production
It is worth pointing out that the boundaries’ definition is changed from complex flow system to MFHW. In a complex flow system, \( x_e \) and \( y_e \) are the distance from the well to the reservoir outer boundaries and these boundaries are physically existing. However, in a MFHW, the boundaries become “imaginary” and \( x_e \) and \( y_e \) are altered to represent \( \frac{1}{2} \) inter-well spacing and \( \frac{1}{2} \) inter-fracture spacing, respectively.

\[
\begin{array}{c}
\text{Figure 4.33: the total production of a multiple-stage fractured horizontal well is equivalent to the production of a complex flow system multiplied by the number of fractures}
\end{array}
\]

Returning to the subject of superposition, in order to thoroughly understand the behavior of superposition, the complex flow system is tested by two cases, namely compound linear flow system and pseudo radial flow system. Both systems were designed with “extreme” \( \frac{y_e}{x_f} \) aspect ratios. They will be presented separately in the following sections.

4.4.1 Compound Linear Flow System

The compound linear flow system represents a complex flow system with a small \( \frac{y_e}{x_f} \) aspect ratio. A constant rate synthetic data set and a constant pressure synthetic data set were generated using a single-phase (water) analytical model. When normalized pressures and their derivatives were plotted against time (without any superposition) on the type curve shown in Figure 4.35, three flow regimes were observed: transient linear flow, compound linear flow, and boundary dominated flow. The constant rate data shows
that the normalized pressure and derivative exhibit half-slope during both transient linear flow and compound linear flow, and unit-slope during boundary dominated flow. As expected, the half-slope signature is more evident in the derivative. The constant pressure data’s normalized pressure and derivative, on the other hand, exhibit half-slope during transient linear flow and quickly depart from the half-slope after the end of transient linear flow. The behavior of the derivative is found to be unusual during compound linear flow: notwithstanding the significant departure, the derivative somewhat recovers the half-slope signature immediately after the compound linear flow starts. After the end of compound linear flow, boundary dominated flow emerges and both the normalized pressure and derivative become concave upward.

The constant rate analytical solutions to transient linear flow and boundary dominated flow were developed in Chapter 3. The constant rate solution of compound linear flow can be obtained from the solution of transient linear flow, by replacing $x_f$ with $\frac{y_e}{2}$. In addition, there is always a negative apparent skin term associated with the compound linear flow. Thompson et al. (2012) studied the apparent skin of compound linear flow ($S_{CL}$) in depth and quantified the magnitude of $S_{CL}$ based on various fracture half length and spacing combinations. The authors presented a plot of $S_{CL}$ as a function of $\frac{y_e}{x_f}$ as follows:
Figure 4.34: apparent skin of compound linear flow ($S_{CL}$) against aspect ratio

Since the implementation of superposition-rate requires the y-intercept ($b$) of each flow regime, the definitions of $b$ are reviewed below.

For transient linear flow:

$$b = \frac{141.2B\mu}{k(2x_f)} S_f$$

(4.6)

For compound linear flow:

$$b = \frac{141.2B\mu}{kh} \left( \frac{h}{2x_f} S_f + S_{CL} \right)$$

(4.7)

For boundary dominated flow:

$$b = \frac{141.2B\mu}{kh} \left[ \frac{\pi}{6} \left( \frac{y_e}{x_f} \right) + \frac{h}{2x_f} S_f + S_{CL} \right]$$

(4.8)

The compound linear flow system’s model parameters as well as the calculated $b$ values are presented in Table 4.3.

Table 4.3: model parameters and $b$ values for the compound linear flow system
To investigate the effect of superposition, transient linear flow, compound linear flow, and boundary dominated flow’s superposition functions were applied to the constant pressure data. It must be emphasized that although transient linear flow and compound linear flow use the same formula to calculate superposition, the procedures of implementing superposition-time and linear superposition-rate are different. There is no difference between transient linear flow superposition-time and compound linear flow superposition-time. However when superposition-rate is chosen and is applied to data, the resulting diagnostic plots show significant differences between transient linear flow and compound linear flow, because the definitions of b are different for these two linear flow regimes.

The effect of transient linear flow superposition is shown in Figure 4.36. In general, the superposition-time and superposition-rate yield similar results during transient flow regimes and both applications convert the constant pressure data to the equivalent constant rate data during transient linear flow as well as during compound linear flow. But differences appear once boundary dominated flow begins. Transient linear superposition does not make the constant pressure data follow the constant rate
data anymore. Moreover, superposition-time distorts the boundary dominated flow data unduly, and makes boundary dominated flow look like the extension of compound linear flow. Superposition-rate, on the other hand, distorts the data to a much lesser degree, which aids in identifying the start of boundary dominated flow.

The effect of compound linear flow superposition is shown in Figure 4.37. Unlike transient linear flow, compound linear flow’s superposition-time and superposition-rate have completely different behaviors during transient flow regimes. The superposition-time successfully transforms the constant pressure data to its constant rate equivalent during transient linear flow as well as during compound linear flow; whereas the superposition-rate only performs the proper transformation during compound linear flow. Differences also occur during boundary dominated flow. The superposition-time makes boundary dominated flow look like the extension of compound linear flow. Superposition-rate, on the other hand, distorts the data to a much lesser degree, making boundary dominated flow easier to detect.

The effect of boundary dominated flow superposition is shown in Figure 4.38. BDF superposition-time is exceptionally successful and shifts the entire constant pressure data close to the constant rate. On the other hand, BDF superposition-rate makes the constant pressure data behave strangely prior to boundary dominated flow, despite the fact that it effectively converts the constant pressure data to the constant rate equivalent during boundary dominated flow. By comparing BDF superposition-time to BDF superposition-rate, the advantage of using BDF superposition-time is easily noticed. BDF superposition-time preserves the signature shape of every flow regime, which makes it a better choice to identify flow regimes for compound linear flow system.
Figure 4.35: type curve of compound linear flow system (y-axis is in psi/bbld; x-axis is in days)

Figure 4.36: type curve after transient linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.37: type curve after compound linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
In addition to type curves, specialized plots generated from the compound linear flow system are also presented in this section, from Figure 4.39 to 4.44. As before, the different superposition functions do not present any distinct advantage and specialized plots do not appear to enhance the interpretation methodology.
Figure 4.40: linear flow specialized plot after transient linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.41: linear flow specialized plot to illustrate compound linear flow (y-axis is in psi/bbl/day; x-axis is in days)

Figure 4.42: linear flow specialized plot after compound linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
Figure 4.43: BDF specialized plot to illustrate boundary dominated flow (y-axis is in psi/bbl; x-axis is in days)

Figure 4.44: BDF specialized plot after BDF superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

All of the plots presented above were based on the synthetic data from the reservoir model with zero skin. A new reservoir model with positive skin was also created and its type curves are presented below. The type curves with positive skin are consistent with the type curves without skin, which indicates that superposition manipulates the data the same way, no matter whether skin exists or not.
Figure 4.45: Type curve of compound linear flow system with positive skin (y-axis is in psi/bbl; x-axis is in days)

Figure 4.46: Type curve with positive skin after transient linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.47: Type curve with positive skin after compound linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
4.4.2 Pseudo Radial Flow System

The pseudo radial flow system represents a complex flow system with a large $\frac{y_e}{x_r}$ aspect ratio. A constant rate synthetic data set and a constant pressure synthetic data set were generated using a single-phase (water) analytical model. When normalized pressures and their derivatives were plotted against time (without any superposition) on the type curve shown in Figure 4.49, three flow regimes were observed: transient linear flow, pseudo radial flow, and boundary dominated flow. The constant rate data shows that the derivative exhibits half-slope during transient linear flow, zero-slope during pseudo radial flow, and unit-slope during boundary dominated flow. The constant pressure data follows the overall trend of the constant rate and its derivative also exhibits half-slope and zero-slope during transient linear flow and pseudo radial flow, respectively. But the constant pressure data starts to deviate once boundary dominated flow begins. Constant pressure’s normalized pressure and derivative become concave upward during boundary dominated flow.

The constant rate analytical solution to transient linear flow was developed in Chapter 3. The constant rate solution to pseudo radial flow is identical to the solution of
transient radial flow developed in Chapter 3 except that there is an apparent skin term associated with the pseudo radial flow. This apparent skin ($S_{PR}$) is due to the additional pressure drop caused by the previous transient linear flow, and is related to $x_f$ as follows:

$$\frac{x_f}{2} = r_w e^{-SPR} \text{.................................................................(4.9)}$$

Furthermore, the constant rate analytical solution to the boundary dominated flow in pseudo radial flow system is slightly different from the analytical solutions developed previously. The BDF solution herein utilizes a dimensionless shape factor ($C_A$) and is defined as follows:

$$P_{WD} = 2\pi t_{DA} + \frac{1}{2} \ln\left(\frac{4A}{1.781r_w^2C_A}\right) + \frac{h}{2x_f} S_f + S_{PR} \text{.................................................................(4.10)}$$

where the value of $C_A$ can be found in Table 2-7 of AER Directive034 (1975).

Since the implementation of superposition-rate requires the y-intercept (b) of each flow regime, the definitions of b are reviewed below.

For transient linear flow:

$$b = \frac{141.2B_{\mu}}{k(2x_f)} S_f \text{.................................................................(4.11)}$$

For pseudo radial flow:

$$b = \frac{162.6B_{\mu}}{kh} \left[\log\left(\frac{k}{\phi_{\mu}c_r r_w^2}\right) - 1.847\right] + \frac{141.2B_{\mu}}{kh} \left(\frac{h}{2x_f} S_f + S_{PR}\right) \text{.................................................................(4.12)}$$

For boundary dominated flow:

$$b = \frac{141.2B_{\mu}}{kh} \left[\frac{1}{2} \ln\left(\frac{4A}{1.781r_w^2C_A}\right) + \frac{h}{2x_f} S_f + S_{PR}\right] \text{.................................................................(4.13)}$$

The pseudo radial flow system’s model parameters as well as the calculated b values are presented in Table 4.4.

Table 4.4: model parameters and b values for the pseudo radial flow system
To investigate the effect of superposition, transient linear flow, pseudo radial flow, and boundary dominated flow’s superposition functions were applied to the constant pressure data.

The effect of linear flow superposition is shown in Figure 4.50. The linear superposition-time and linear superposition-rate yield similar results during transient flow regimes. Both shift the constant pressure data close to the constant rate data during transient linear flow as well as during pseudo radial flow. But a difference occurs during boundary dominated flow. Superposition-time distorts the boundary dominated flow data excessively; while superposition-rate distorts the data to a much lesser degree, which makes boundary dominated flow a bit easier to detect.

The effect of radial flow superposition is shown in Figure 4.51. Unlike transient linear flow superposition, radial flow superposition-time and superposition-rate show very different signatures. The superposition-time successfully converts the constant pressure data to its constant rate equivalent during transient linear flow as well as during pseudo radial flow; whereas the superposition-rate only performs the proper transformation during pseudo radial flow. A difference also occurs during boundary flow data.
dominated flow. The superposition-time distorts boundary dominated flow excessively. The superposition-rate, on the other hand, distorts the data mildly.

The effect of boundary dominated flow superposition is shown in Figure 4.52. BDF superposition-time causes almost the entire constant pressure data to overlay the constant rate, with only negligible difference at early time prior to pseudo radial flow. On the other hand, BDF superposition-rate makes the constant pressure data behave strangely prior to boundary dominated flow, despite the fact that it effectively converts the constant pressure data to the constant rate equivalent during boundary dominated flow. Once again, BDF superposition-time preserves the signature shape of every flow regime, which makes it a better choice to identify flow regimes for pseudo radial flow system.

Figure 4.49: type curve of pseudo radial flow system (y-axis is in psi/bbl/d; x-axis is in days)

Figure 4.50: type curve after linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
In addition to type curves, specialized plots generated from the pseudo radial flow system are also presented in this section, from Figure 4.53 to 4.58. As before, the different superposition functions do not present any distinct advantage and specialized plots do not appear to enhance the interpretation methodology.
Figure 4.53: linear flow specialized plot to illustrate transient linear flow (y-axis is in psi/bbl; x-axis is in days)

Figure 4.54: linear flow specialized plot after linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.55: radial flow specialized plot to illustrate pseudo radial flow (y-axis is in psi/bbl; x-axis is in days)
Figure 4.56: radial flow specialized plot after radial superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

Figure 4.57: BDF specialized plot to illustrate boundary dominated flow (y-axis is in psi/bbld; x-axis is in days)

Figure 4.58: BDF specialized plot after BDF superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)

All of the plots presented above were based on the synthetic data from the reservoir model with zero skin. A new reservoir model with positive skin was also
created and its type curves are presented below. The type curves with positive skin are consistent with the type curves without skin, which indicates that superposition manipulates the data the same way, no matter whether skin exists or not.

Figure 4.59: type curve of pseudo radial flow system with positive skin (y-axis is in psi/bbl/d; x-axis is in days)

Figure 4.60: type curve with positive skin after transient linear superposition is applied to constant pressure data (left: superposition-time; right: superposition-rate)
4.5 Comparison Summary

Based on the investigation of the effect of superposition on the constant pressure data, the differences between superposition-time and superposition-rate were illustrated in two simple flow systems and one complex flow system. The primary findings are summarized below.

1. Both superposition-time and superposition-rate are effective in converting constant pressure data (consequently any variable rate/variable pressure data) to its constant rate equivalent, but only for the data within the chosen superposition function’s flow regime.

2. Superposition-time sometimes (but not always) forces the data to look like the chosen superposition function’s flow regime, regardless of the intrinsic flow regime of the data itself. This problem is more pronounced when early-time transient flow’s superposition-time is applied to late-time boundary dominated flow data. And this problem becomes less significant when late-
time boundary dominated flow’s superposition-time is applied to early-time transient flow data. In other words, the transient superposition-time functions make boundary dominated flow look like transient flow; while boundary dominated flow superposition-time does not make transient flow look like boundary dominated flow.

3. Among the different flow regimes, boundary dominated flow is the flow regime that is most prone to being affected by the choice of superposition-time.

4. In contrast to superposition-time, superposition-rate does not tend to force all the data to look like the chosen superposition function’s flow regime. Superposition-rate does the rigorous conversion only during the chosen superposition function’s flow regime. When data’s flow regime is inconsistent with the choice of the superposition function, superposition-rate often (but not always) makes the data look significantly different than its equivalent constant rate solution.

5. Early-time flow regime’s superposition-rate always has minimal effects on changing late-time flow regime. But on the other hand, late-time flow regime’s superposition-rate is likely to excessively distort early-time flow regime.

6. Boundary dominated flow superposition-time is the most benign of all of the different superposition-time and superposition-rate functions, for our purpose of identifying flow regimes. BDF superposition-time is the function that most
often retains the characteristics of each individual flow regime, which makes it the best superposition function to use in identifying flow regimes.
Chapter 5 proposes a practical workflow to analyze variable rate/variable pressure data with the implementation of superposition. To validate the proposed workflow, real field data are analyzed and the results are compared against modeling.

Previous chapters show the theoretical principles of superposition and use synthetic data to illustrate the applications of superposition-time and superposition-rate. In practice, many problems associated with the use of superposition are likely to be encountered when analyzing field data. The forthcoming sections investigate some major practical issues, and subsequently discuss how to handle these issues and to minimize the errors in analysis. Based on our investigation, the newly invented superposition-rate demonstrates many diagnostic advantages over superposition-time. These advantages are presented at the end of this chapter.

5.1 Workflow

The diagram (Figure 5.1) is a generalized workflow chart for analyzing production data with the implementation of superposition functions. Every step presented in this workflow will be thoroughly discussed in this section.
Figure 5.1: workflow for analyzing production data

The first step of the proposed workflow is to decide on the type of flow system: i.e. the well and reservoir model as well as the possible flow regimes that are likely to be encountered prior to analyzing the data. This decision relies on an understanding of the subject wells and reservoirs that derives from previous experience and practice. This step is the foundation of production data analysis. Once a flow system is decided, the decision should not be changed unless significant inconsistency is discovered in the analysis. Moreover, it is recommended to always start with the simplest flow system. It is not
worth further complicating the analysis or the model if a simple system suffices to do the work.

The second step is data diagnostics. Whether or not production data analysis extracts true reservoir signals relies on how much effort engineers put on diagnosing raw data quality and understanding its behavior. Unfortunately the importance of this step often has been underestimated or somewhat forgotten by many engineers. In an ideal case when production data is consistent and of good quality, meaningful results can be obtained without data diagnostics. However, in practice, the quality of production data is likely to be questionable. Therefore engineers must be critical of all data. Outliers must be filtered out and inconsistent data must be removed prior to analysis. Blind application of production data analysis without careful data diagnostics can result in misinterpretation of the reservoir characteristics. If outliers and inconsistent data are not properly identified, production data analysis may lead to an interpretation that appears to be mathematically correct yet be completely meaningless. The impact of outliers will be demonstrated in the second field example.

The third step is to identify flow regimes using a type curve. As stated in Chapter 4, boundary dominated flow superposition-time is the most benign of all of the superposition functions, for the purpose of identifying flow regimes from a complete data set. Therefore, normalized pressure as well as derivative are plotted against boundary dominated flow superposition-time on type curve (log-log scale) to identify data’s flow regimes. Note that in the presence of a skin term, the normalized pressure of early-time transient flow is no longer characteristic of straight line signature on type curve. Instead, it appears to be a curve on log-log scale and consequently loses its diagnostic value.
Derivative, on the other hand, always retains the flow regimes’ straight line signature regardless of the presence of skin. This promotes derivative to be a better diagnostic tool to identify flow regimes. However, derivative suffers from its own pitfall: derivative is often noisy in production data analysis and is unusable unless it is smoothed. Data smoothing can be misleading because over smoothing can completely mask flow regimes that may exist. Therefore, a minimum amount of smoothing is always recommended to be applied. But in real life, because of the typically poor quality of production data, the noisiness of the derivative is difficult to reduce and the shape is hard to recognize in analysis unless significant data smoothing is applied. Generally speaking, derivative should always be used with caution in production data analysis. Often it results in either mistaken identification of flow regimes (i.e. due to over smoothing) or unrecognizable flow characteristics (i.e. due to insufficient smoothing). The pitfall of derivative will be illustrated in the second field example.

The flow regimes identified need to be consistent with the conceptual flow system previously selected in the first step. If flow regimes are consistent, then move to the next step; if they are not consistent, then go back to data diagnostics (the second step) and have another look at the raw production data. The flow system should be the last thing to change in the entire workflow. When inconsistencies are observed, most of the times they are likely to be associated with errors in the data (because of the challenging nature of production data quality), but are less likely to be caused by the selected flow system being wrong.

The fourth step is to utilize the boundary dominated flow specialized plot to estimate the current drainage volume (if the system is still in transient flow) or reservoir
volume (if already in boundary dominated flow). For liquid systems, this step is straightforward. First calculate boundary dominated flow superposition-rate (alternatively superposition-time). Second, plot superposition-rate normalized pressure \( \left( \frac{P_i - P_{wf}}{q_{bp}} \right) \) versus time, or alternatively normalized pressure \( \left( \frac{P_i - P_{wf}}{q_{bp}} \right) \) versus superposition-time, on Cartesian plot and draw a straight line to fit through the late-time portion of the data. From the slope of the straight line \((m)\), drainage volume can be calculated as follows:

\[
V = \frac{5.615}{mC_t} \text{..................................................} (5.1)
\]

For gas systems, the objective of this step is twofold: 1) to evaluate drainage volume; and 2) to calculate pseudo time based on the drainage volume estimated from 1. To fulfill these two objectives at the same time, the calculation becomes iterative. Each step in this iteration process is listed as follows:

1. Assume drainage volume \((V)\)

2. Use Gas Material Balance equation to calculate \(\frac{P}{Z}\) and subsequently \(\bar{P}\) for every data point

\[
\frac{P}{Z} = \frac{P_i}{Z_l} \left( 1 - \frac{V_p}{V} \right) \text{..................................................} (5.2)
\]

3. Estimate \(\bar{\mu}\) and \(\bar{c}_g\) at \(\bar{P}\) for every data point

4. Estimate \(\bar{c}_t\) at \(\bar{P}\) for every data point using Equation 5.3 derived by Rahman et al. (2006)

\[
\bar{c}_t = c_{ti} \left( 1 - \bar{c}_g (P_i - \bar{P}) \right) + S_g \left( \bar{c}_g - c_{gi} + \bar{c}_g c_{gi} (P_i - \bar{P}) \right) \text{.............} (5.3)
\]

5. Calculate pseudo time \((t_a)\) based on Equation 5.4

\[
t_a = (\mu_t c_{ti}) \int_0^t \frac{dt}{\rho c_t} \text{..................................................} (5.4)
\]
6. Calculate boundary dominated flow superposition pseudo time \( (t_{spa}) \)

\[
t_{spa} = \frac{\int_0^{t_a} q_n dt_a}{q_n}
\]

(5.5)

7. Plot normalized pseudo pressure \( \left( \frac{\Delta P_p}{q_n} \right) \) against \( t_{spa} \) on Cartesian plot and draw a straight line to fit the late-time portion of the data.

8. Based on the definitions of the boundary dominated flow in **Tables A3** and **A4** of the Appendix, calculate drainage volume from the slope of the straight line \( (\text{m}) \) as follows:

\[
V = \frac{5.636 \times 10^3 \times T}{m \mu c \xi B_{gi}}
\]

(5.6)

9. Return to 1 and replace initial guess with the newly calculated drainage volume.

10. Repeat Steps 1 - 9 until drainage volume converges.

Note if the system still exhibits transient flow, the drainage volume calculated above becomes current (or minimum) drainage volume at the end of production. Then pseudo time is calculated based on the average pressure within the current drainage volume. It is worth mentioning that this is not equivalent to calculating corrected pseudo time. Corrected pseudo time calculates each data point’s current drainage volume at every single time step. Consequently, corrected pseudo time not only makes the calculation iterative but forces the iteration to change at each data point, resulting in a large amount of iterative steps.

The above iteration uses boundary dominated flow superposition-time. It becomes more complicated if boundary dominated flow superposition-rate is used, instead. If
boundary dominated flow superposition-rate replaces superposition-time, then Steps 6 and 7 are changed as follows:

6. Calculate boundary dominated flow superposition-rate ($q_{sp}$) based on pseudo time

$$q_{sp} = \int_{t_0}^{t_a} q_n dt_a$$

\[ (5.7) \]

7. 1. Plot superposition normalized pseudo pressure \( \left( \frac{\Delta P - q_n b}{q_{sp}} + b \right) \) against $t_a$ on Cartesian plot and draw a straight line to fit the late-time portion of the data (b equals to 0 for initial guess)

7. 2. Estimate y-intercept of the straight line (b)

7. 3. Return to 7.1 and replace b with the newly estimated b value

7. 4. Repeat 7.1 – 7.3 until b converges

Despite the fact that boundary dominated flow superposition-rate adds one more iteration to the drainage volume calculation, the whole process converges rapidly if the initial guess is reasonable. Both the boundary dominated flow superposition-rate and superposition-time will be implemented in the field examples to estimate drainage volume, for comparison purpose.

It should be noticed that the above drainage volume calculation is equivalent to the flowing material balance analysis (Mattar and Anderson, 2005). The theory underlying both methods is identical. Both methods are iterative for gas systems and they yield the same results.

After the analysis in the boundary dominated flow specialized plot, the next (fifth) step is to estimate transient flow parameters from the transient flow specialized plots. According to the flow regimes identified in the previous step (the third step), the
complete data set is segmented into discrete subsets and then each subset is analyzed separately with its own simple analytical solution. If transient linear flow is detected, then the linear flow portion of the data is analyzed using a linear specialized plot (square root time function in x-axis); whereas if transient radial flow is detected, then the radial flow portion of the data is analyzed using a radial specialized plot (logarithmic time function in x-axis). Superposition-rate (alternatively superposition-time) must be calculated and consistent with the identified flow regime. When the data is plotted on its corresponding specialized plot, it exhibits the proper straight line. A straight line is drawn through the appropriate data, and flow parameters can be evaluated from the slope (m) and y-intercept (b) of the straight line using the following constant rate solutions.

For transient linear flow in liquid systems:

\[ x_f h \sqrt{k} = \frac{141.2B}{m} \sqrt{\frac{0.00633\pi\mu}{\phi c_t}} \] \hspace{1cm} (5.8)

\[ S_f = \frac{bk(2x_f)}{141.2B\mu} \] \hspace{1cm} (5.9)

For transient linear flow in gas systems (i.e. after pressure is replaced by pseudo pressure and time is replaced by pseudo time):

\[ x_f h \sqrt{k} = \frac{199.8 \times 10^3 \times T}{m} \sqrt{\frac{1}{\phi\mu c_t \varepsilon_{ti}}} \] \hspace{1cm} (5.10)

\[ S_f = \frac{bk(2x_f)}{1.417 \times 10^6 \times T} \] \hspace{1cm} (5.11)

For transient radial flow in liquid systems:

\[ kh = \frac{162.6B\mu}{m} \] \hspace{1cm} (5.12)

\[ S = 1.151 \left[ \frac{bkh}{162.6B\mu} - \log \left( \frac{k}{\phi\mu c_t \varepsilon_{ti}^2} \right) + 1.847 \right] \] \hspace{1cm} (5.13)
For transient radial flow in gas systems (i.e. after pressure is replaced by pseudo pressure and time is replaced by pseudo time):

\[ \text{kh} = \frac{1.631 \times 10^6 \times T}{m} \] ...........

..............(5.14)

\[ S = 1.151 \left[ \frac{bkh}{1.631 \times 10^6 \times T} - \log \left( \frac{k}{\phi \mu_\ell \nu^2} \right) + 1.847 \right] \] ...........

..............(5.15)

The sixth step is to compile the results from all of the diagnostic and analysis plots previously investigated and to interpret individual reservoir parameters. As seen in **Equations 5.8 - 5.15**, the specialized plots only result in combination of multiple reservoir parameters. Take transient linear flow for instance. The linear specialized plot results in the flow parameter, \( x_f h \sqrt{k} \). It is a combination of three independent parameters, which cannot be separated unless external information is provided. The external information may be obtained from the analysis of other flow regimes. But in the case when only transient linear flow regime exists, the solution becomes non-unique. Then it is important to incorporate the external information from other sources, including but not limited to log analysis, core analysis, micro-seismic, chemical tracer, offset well interference, and so on. The information obtained from other sources always helps to lend credence to the interpretation.

The final step of the proposed workflow is verification. Build a model based on the results of the analyses and then run the model, modifying the parameters as necessary, until an acceptable history match is achieved. The analyses and the modeling must be consistent with each other and with the pre-selected flow system. It is important to keep in mind that our role is to understand the data and not simply to history match.

### 5.2 Field Examples
5.2.1 Production Data Analysis - Example 1

The production data of the first field example comes from a multiple-stage fractured horizontal well in the Montney tight gas reservoir. The horizontal well was completed by hydraulically fracturing 11 stages along its lateral wellbore with an average inter-fracture spacing of 500 ft. The reservoir’s initial properties are displayed in Table 5.1. The well was on commercial production for approximately 13 months. The production was restricted at constant rate (approximately 7 MMscf/day). After 8 months of production, when the wellhead pressure had reached line pressure, the production rate began to decline and the flowing pressure became constant. It is also noticed that after 3 months of production, the well was shut in for approximately 1.5 months due to plant maintenance. The well’s production rate and bottom hole flowing pressure are shown in Figure 5.2.

Table 5.1: initial reservoir parameters for example 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>4481.7 psia(a)</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>0.968</td>
</tr>
<tr>
<td>$T$</td>
<td>194°F</td>
</tr>
<tr>
<td>$h$</td>
<td>131.2 ft</td>
</tr>
<tr>
<td>porosity</td>
<td>0.04</td>
</tr>
<tr>
<td>$C_f$</td>
<td>7.11E-06 1/psi</td>
</tr>
<tr>
<td>$C_{gi}$</td>
<td>1.56E-04 1/psi</td>
</tr>
<tr>
<td>$S_g$</td>
<td>0.85</td>
</tr>
<tr>
<td>$C_{ti}$</td>
<td>1.40E-04 1/psi</td>
</tr>
<tr>
<td>$\mu_{gi}$</td>
<td>0.0233 cP</td>
</tr>
<tr>
<td>$B_{gi}$</td>
<td>0.0040 ft³/scf</td>
</tr>
</tbody>
</table>
Linear flow was considered to be the conceptual flow system for this well. Based on the author’s previous experience, during the first 13 months of production transient linear flow is likely to be the dominant flow regime. It is possible that, in the late-time data, the end of transient linear flow may be evident. Note that the data quality of this data set is exceptionally good. In spite of the small amount of outliers, the production rate and pressure are consistent for the entire data set. Further data filtering is unnecessary in this example.

The normalized pressure and derivative were plotted against boundary dominated flow superposition-time on type curve (as shown in Figure 5.3). The normalized pressure shows concave upward signature from beginning to approximately 100 days and then exhibits half slope straight line for the rest of the data, indicating transient linear flow with the existence of skin damage. In spite of smoothing having been applied, the derivative is still noisy. Fortunately its shape is recognizable in this example. Since the derivative is not affected by the skin effect, it exhibits a characteristic of half slope straight line (i.e. linear flow) for the entire data set. From the normalized pressure and derivative, it is evident that the well was produced under transient linear flow for the
entire 13 months of production, and the end of transient linear flow was not observed from the type curve.

Figure 5.3: type curve plot with BDF superposition-time for example 1

Boundary dominated flow specialized plot was made to evaluate the current drainage volume at the end of production as well as to calculate pseudo time. Both boundary dominated flow superposition-rate (left in Figure 5.4) and superposition-time (right in Figure 5.4) were implemented in the calculation. Fast convergence occurred in both iterative methods. Although the two plots look different, the late-time data forms the same analysis straight line (red line). From the slope of the straight line, the current drainage volume was estimated to be 6.4 Bcf. Pseudo time was calculated correspondingly and was used for analyzing transient flow in the next step.
Since the well is in transient linear flow during its entire production period, only transient linear flow specialized plot was made for analysis. Prior to applying superposition, the normalized pressure versus square root of time (without any superposition) was plotted in Cartesian coordinates as shown in Figure 5.5, in order to observe the behavior of the data independently without the effect of superposition. Following this preliminary plot, the superposition functions will be applied to demonstrate the value of using superposition in overcoming this plot’s disadvantages. Although no superposition has been applied to the data, this plot still showed some diagnostic value in flow regimes identification. However, when discontinuities (i.e. shut ins or step changes) occur, its value diminishes. As shown in Figure 5.5, instead of unifying one single straight line, the data was shown as three segments with their own shapes. The data exhibits a straight line and behaves properly prior to the shut in (this is to be expected as the rate is constant during that period). But shortly after the shut in, the data no longer retains its straight line signature and takes considerable amount of time to go back to the previous trend. Lastly, once operating conditions change (e.g. from constant rate to constant pressure in this example), the data exhibits a new straight line with a different slope. Moreover, this late-time straight line yields a negative y-intercept, which could be mistakenly interpreted as a negative skin.
Subsequently, both linear flow superposition-rate and superposition-time were applied to the data to illustrate the effect of superposition. As shown in Figure 5.6, the advantage of superposition is immediately obvious: superposition helps to unify the entire data into a single straight line with the correct y-intercept regardless of the discontinuities and changes in production data.

From the comparison of the two superposition applications shown in Figure 5.6, it is evident that both applications are effective in linearizing the transient linear flow data on the linear flow specialized plot. Moreover, the two superposition applications show consistent results and the same analysis line (red line) was generated. Based on the slope of the straight line, $x_f h \sqrt{K}$ was calculated to be 29689 ft$^2 \sqrt{\text{md}}$. External information was taken into account in order to estimate individual parameters. Core analysis suggests the permeability of the Montney formation is in the micro-darcy range in the subject area. Micro-seismic, chemical tracer, and offset well interference tests show that the vertical growth of the hydraulic fractures is approximately 130 ft for similar completion designs. Therefore, by assuming 0.001 md permeability and 130 ft fracture height for all of the 11 hydraulic fractures along the lateral wellbore, the fracture half length ($x_f$) was evaluated.
to be 650 ft. This number is consistent with observations from the offset well interference tests. Skin damage was also estimated from the y-intercept of the analysis line, which results in a positive $S_f$ of 0.028.

![Figure 5.6: linear flow specialized plot for example 1](image)

To validate the interpretation of the reservoir parameters obtained from the analysis, an analytical gas reservoir model was created for this 11-stage horizontal well in order to history match its production. The parameters estimated from the analysis provide an excellent starting point for history matching. In this example, satisfactory match had been achieved once the analysis results were directly input to the model. After fine-tuning of the model to optimize the history match, the analytical model results in 0.001 md reservoir permeability, 131 ft fracture height, 673 ft fracture half length, and 0.022 skin factor. These numbers are consistent with that of the analysis, which confirms the validity of the workflow proposed in this thesis.

5.2.2 Production Data Analysis - Example 2

The production data of the second field example also comes from a multiple-stage fractured horizontal well in the Montney tight gas formation. The horizontal well was completed with a more intense fracturing design, by hydraulically fracturing 36 stages
along its lateral wellbore with an average inter-fracture spacing of 167 ft. The reservoir’s initial properties are displayed in Table 5.2. The well has a lengthy commercial production history for approximately two and half years. The production rate was initially restricted at approximately 7 MMscf/day. After 6 months of production, the rate began to decline. The flowing pressure monotonously declined for the entire period. The well’s production rate and bottom hole flowing pressure are shown in Figure 5.7.

Table 5.2: initial reservoir parameters for example 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>4264.1 psia(a)</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>0.942</td>
</tr>
<tr>
<td>$T$</td>
<td>194 °F</td>
</tr>
<tr>
<td>$h$</td>
<td>164 ft</td>
</tr>
<tr>
<td>porosity</td>
<td>0.04</td>
</tr>
<tr>
<td>$C_f$</td>
<td>7.11E-06 1/psi</td>
</tr>
<tr>
<td>$C_{gi}$</td>
<td>1.64E-04 1/psi</td>
</tr>
<tr>
<td>$S_g$</td>
<td>0.85</td>
</tr>
<tr>
<td>$C_{ti}$</td>
<td>1.46E-04 1/psi</td>
</tr>
<tr>
<td>$\mu_{gi}$</td>
<td>0.0232 cP</td>
</tr>
<tr>
<td>$B_{gi}$</td>
<td>0.0041 ft³/scf</td>
</tr>
</tbody>
</table>

Figure 5.7: production rate and pressure profile for example 2

Linear flow system was considered as the conceptual flow system for this well. Due to the well’s intense inter-fracture spacing and its extended production history, the end of transient linear flow is likely to be observed and boundary dominated flow behavior is expected to be seen in the late-time data. The data quality in this example is
somewhat poor, which is not surprising in production data analysis. Although the overall trends of rate and pressure are consistent for the entire period, a significant amount of outliers exist. Outliers must be identified and removed prior to analysis. In this example, data filtering was done by calculating the moving average of every 10 data points. Then each data was compared to its corresponding moving average. If the difference is more than 1%, then the data was identified as outlier and was removed from the data set.

Normalized pressure and derivative were plotted against boundary dominated flow superposition-time on type curve (as shown in Figure 5.8). The plot on the left hand side shows the raw data without filtering data. The normalized pressure is concave upward at early-time and transitions to a unit slope straight line. As mentioned previously, in the presence of skin, the normalized pressure during transient linear flow is no longer a straight line at early-time. It usually takes a long time to reestablish its half slope straight line signature. However, in this example, boundary dominated flow effect occurred prior to the full reestablishment of the transient linear flow signature. Thus the half slope straight line was not clearly observed from the normalized pressure. On the other hand, although the derivative is not affected by skin, the derivative in this example is very noisy. Despite significant data smoothing, the shape is still unrecognizable. Therefore, without filtering the data, flow regimes cannot be properly identified in this example.

The plot on the right hand side shows the data after filtering. It is very interesting to observe that the raw data’s normalized pressure exhibits a unit slope straight line at late-time but this straight line disappears after filtering the data. As mentioned in Chapter 4, the superposition-time function tends to make any data look like that particular
superposition’s flow regime. This problem is more pronounced in the presence of outliers. In addition, as it shuffles time back and forth, anomalous data (outliers) become difficult to be tracked and identified. In this example, boundary dominated flow superposition-time shuffles the outliers to such a distance, that it makes the outliers look (falsely) like boundary dominated data. If this is not recognized, it is easy to arrive at the wrong interpretation. Without filtering the data, it would appear that the well had definitely reached boundary dominated flow, as the “later” data clearly manifests itself as a unit slope straight line. However, from the right hand side plot of Figure 5.8, it is obvious that the unit slope straight line was caused only by the outliers that got shuffled by the superposition-time operation. Therefore, when a superposition-time function is used to analyze the data from the flow regimes that are inconsistent with the chosen superposition-time, it becomes particularly susceptible to the existence of outliers, as it tends to make meaningless data (outliers) appear as if they were meaningful.

Due to the large amount of outliers, a significant smoothing had been applied to the derivative. Although the derivative is still noisy after the smoothing, the flow regime’s characteristics can still be identified: a half slope straight line until circa 100 - 500 days (precise value cannot be identified) followed by a unit slope straight line. This means proper data filtering with careful data smoothing may bring back some diagnostic value to derivative. It is worth reiterating that it is not recommended to over filter and smooth the data, as it is likely to change the derivative’s original shape and cause problematic interpretation. In practice, the derivative should always be used with caution, notably in the presence of outliers.
The boundary dominated flow specialized plot was made to evaluate the drainage volume as well as to calculate pseudo time. Both boundary dominated flow superposition-rate (left plot in Figure 5.9) and superposition-time (right plot in Figure 5.9) were implemented in the calculation. Fast convergence occurred in both iterative methods. The late-time data forms similar analysis straight lines (red lines). From the slope of the straight line, the drainage volume was estimated to be 10.6 Bcf. Pseudo time was calculated correspondingly and was used for analyzing transient flow in the next step.
The normalized pressure versus square root of time (without any superposition) was plotted in Cartesian coordinates as shown in Figure 5.10. In this example, since most of the data is continuously decreasing during the entire production period, straightforward normalization reveals its diagnostic value in identifying flow regimes. Early-time straight line with positive y-intercept indicates transient linear flow with skin damage. Late-time concave upward signature suggests the effect of boundary dominated flow. The end of linear flow happened at approximately 225 days (or $15\sqrt{\text{day}}$).

It should be emphasized that although Figure 5.10 is able to identify the time to the end of transient linear flow, it cannot be used to estimate the transient flow parameter from its early time straight line. Without superposition, variable rate/variable pressure data may appear to exhibit a straight line, but this straight line’s slope and y-intercept are incorrect.

Subsequent to the linear flow specialized plot without superposition, rigorous linear flow superposition-rate and superposition-time were applied to the data to illustrate the effect of superposition. From the comparison of the two superposition methods shown in Figure 5.11, during transient linear flow, superposition-rate and superposition-time are
in good agreement. Similar linear flow analysis lines (red lines) were obtained. However, during boundary dominated flow, superposition-time makes boundary dominated flow data appear to be another straight line, which could be mistakenly interpreted as a new transient linear flow regime; whereas superposition-rate, on the other hand, distorts the original data to a much lesser degree, helping to prevent false interpretation. Therefore, when both linear and boundary dominated flow regimes exist, linear flow superposition-rate illustrates its advantage over superposition-time. It gives the correct slope of the linear flow straight line and meanwhile accentuates the transition from transient linear flow to boundary dominated flow, thus aiding in the identification of the end of linear flow.

Moreover, there is one more problem that is associated with the use of superposition-time. Once the time to the end of linear flow is found from the linear flow specialized plot made by superposition-time, it is difficult to convert this superposition-time to its equivalent real time because the superposition-time operation shuffles data back and forth and makes data’s real time hard to track. This significantly impairs the practical use of the superposition-time. On the contrary, superposition-rate shifts the data in the vertical direction and keeps all of the data in the time sequence of their occurrence. Therefore, once the time to the end of linear flow is found from the linear flow specialized plot made by superposition-rate, this value can be directly used in other analyses, such as radius of investigation calculation.

Based on the slope of the straight line, \( x_t h \sqrt{K} \) was calculated to be 59987 ft^2/\( \sqrt{\text{md}} \). Combining the results from drainage volume estimation, core analysis, and offset well interference tests, the permeability, the vertical growth of the hydraulic fractures as well
as the fracture half length were estimated as 0.00035 md, 164 ft, and 543 ft respectively. These values are assumed the same for all of the 36 hydraulic fractures along the lateral wellbore. Skin damage was also estimated from the y-intercept of the analysis line, which results in a positive $S_f$ of 0.023.

![Figure 5.11: linear flow specialized plot for example 2 (left: superposition-rate; right: superposition-time)](image)

To validate the interpretation of the reservoir parameters obtained from the analysis, an analytical gas reservoir model was created for this 36 stages horizontal well to history match its production. After fine-tuning of the model to optimize the history match, the analytical model results in 10.0 Bcf reservoir volume, 0.00035 md reservoir permeability, 164 ft fracture height, 607 ft fracture half length, and 0.067 skin factor. It is noticed that the difference between analysis and modeling is larger in this example. But given the complexity in the data, the results from analysis are quite acceptable. They certainly are excellent starting points for history matching in modeling. This example also confirms the validity of the workflow proposed in this thesis.

5.2.3 Well Test Analysis - Example 3

Notwithstanding that the primary focus of this work is production data analysis, one well test example was investigated to illustrate the significance of superposition in
pressure transient analysis. As the concept underlying well test analysis is essentially the same as production data analysis, the workflow of well test analysis is similar to Figure 5.1, but only with minor modifications. The difference will be introduced in this section.

The third field example is an oil well example that comes from a multiple-stage fractured horizontal well in Duvernay. The horizontal well was completed by plug and perf technique. Overall 66 effective fractures were placed along its lateral wellbore with an average inter-fracture spacing of 109 ft. The reservoir’s initial properties are displayed in Table 5.3. Tandem pressure recorders were landed downhole and shortly thereafter the well was opened to flow for 215.8 hours at a constant oil rate of 723 bbl/day. Subsequent to the flow, the well was shut in to conduct a pressure build up test. After 613.1 hours of shut in, the downhole recorders were retrieved. The well’s production rate and bottom hole pressure are shown in Figure 5.12.

**Table 5.3: initial reservoir parameters for example 3**

<table>
<thead>
<tr>
<th>T</th>
<th>201.2 °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>147.6 ft</td>
</tr>
<tr>
<td>porosity</td>
<td>0.04</td>
</tr>
<tr>
<td>Cf</td>
<td>6.48E-06 1/psi</td>
</tr>
<tr>
<td>Coi</td>
<td>6.82E-06 1/psi</td>
</tr>
<tr>
<td>So</td>
<td>0.9</td>
</tr>
<tr>
<td>Cti</td>
<td>1.28E-05 1/psi</td>
</tr>
<tr>
<td>μoi</td>
<td>0.6750 cP</td>
</tr>
<tr>
<td>Boi</td>
<td>1.3509 bbl/stb</td>
</tr>
</tbody>
</table>
Similar to production data analysis, the first step in well test analysis is to determine the flow system as well. Linear flow system was selected as the conceptual flow system. Transient linear flow is expected to be the dominant flow regime during build up. Transient bilinear flow signature (characteristic of quarter slope straight line) is also possible to be observed in the data. Often data quality is good in well test analysis. In this example, the rate and pressure are smooth and consistent for the entire data set. Downhole electrical gauges were deployed to take pressure measurements. The gauges recorded a pressure data every second, creating a significant amount of data to be analyzed. To accelerate the speed of calculation, the number of data points was reduced by performing a logarithmic filter on the build up data and an arithmetic filter on the flow data.

Note that the boundary dominated flow superposition is not meaningful during build up and fall off. Instead, in this example, the linear flow superposition was chosen to use in type curve analysis in order to identify flow regimes. Moreover, a new concept will be used to analyze the shut in pressure ($P_s$): superposition equivalent function. Because the initial reservoir pressure is unknown in well test analysis, $\Delta P$ is no longer
(P_1 - P_{wf}) as shown in the previous production data analysis, and it becomes (P_s - P_{wf}). As a consequence of this, superposition equivalent function is used to replace superposition function. As is done classically, the shut in period’s normalized pressure \( \left( \frac{P_s - P_{wf}}{q} \right) \) and derivative was plotted against linear flow superposition equivalent time on type curve (as shown in Figure 5.13). The derivative exhibits wellbore storage (early unit slope straight line) followed by transient linear flow (half slope straight line). In addition to the type curve, shut in pressure was plotted against square root of linear flow superposition-time on Cartesian coordinates (Figure 5.14), in order to evaluate reservoir’s initial pressure from the y-intercept. (This is the only diagnostic plot in which superposition function was applied.) This well’s extrapolated initial pressure (P*) was estimated to be 7526 psi. In this two-rate example (one constant rate flow and one shut in), the definitions of linear flow superposition equivalent time (t_{sp, equiv}) and linear flow superposition-time (t_{sp}) can be simplified as follows:

\[
\sqrt{t_{sp, equiv}} = \sqrt{t_s} + \sqrt{t - t_s} - \sqrt{t} \quad \text{.................................}(5.16)
\]

\[
\sqrt{t_{sp}} = \sqrt{t} - \sqrt{t - t_s} \quad \text{.................................}(5.17)
\]

where t_s is the flow duration (or the time at the beginning of shut in).

Figure 5.13: type curve plot with linear flow superposition equivalent time for example 3
A problem is likely to happen with the classical use of the linear flow superposition equivalent time. As seen in its definition, superposition equivalent time is limited by flow duration ($t_s$). In other words, for a case in which a well is open to flow for only a short period, the calculated superposition equivalent time for build up is always small even if the well is shut in for an extended period. Therefore late-time build up data would be so compressed toward $t_s$ that its shape is difficult to recognize. An easy fix to this problem has been commonly implemented in well test analysis: first calculate the derivative by using superposition equivalent time (as is done in Figure 5.13) and then plot the resulting data against real time on type curve. This is effective for radial flow regime, because the derivative of radial flow is characteristic of a “flat” straight line so shifting data along the x-axis does not change its shape. However, it causes problems to data in other flow regimes. In Figure 5.15, the same normalized pressure and derivative data as shown in Figure 5.13 was plotted against real time. The shape of the derivative is altered. The late-time derivative shows departure from the half slope straight line and curves downward, which can lead to a wrong interpretation: i.e. false transition to pseudo radial flow between 10 and 100 hours.
Alternatively, linear flow superposition equivalent rate can be implemented to fix the problem mentioned above. As shown in Figure 5.16, the normalized pressure calculated using linear flow superposition equivalent rate \( \left( \frac{P_s-P_{wfp}-q_n b}{q_{sp,\equiv}} + b \right) \) and derivative were plotted against real time on type curve. The derivative exhibits the same signature as that shown in Figure 5.13: wellbore storage followed transient linear flow until the end of the build up. Meanwhile, as the data is plotted on real time, late-time data will honor the actual shut in time and will no longer be compressed. In other words, superposition equivalent rate retains the characteristics of the flow regime while eliminating the problem of late-time data compression caused by superposition equivalent time. It is certainly a more reliable method than superposition-time. In this two-rate example (one flow plus one build up), linear flow superposition equivalent rate \( q_{sp,\equiv} \) can be simplified as follows:

\[
q_{sp,\equiv} = \frac{q(\sqrt{t_s}+\sqrt{t-t_s}+\sqrt{t})}{\sqrt{t-t_s}} \tag{5.18}
\]

![Figure 5.15: type curve plot with real time for example 3](image-url)
Figure 5.16: type curve plot with linear flow superposition equivalent rate for example 3

Linear flow specialized plots were made and are shown in Figure 5.17. Both linear flow superposition equivalent rate and superposition equivalent time were applied to the data. Consistent linear flow analysis lines (red lines) were obtained from the specialized plots. Based on the slope of the straight line, \( x_f h \sqrt{K} \) was calculated to be 18243 ft\( ^2 \sqrt{\text{md}} \). By assuming 0.00025 md permeability and 147.6 ft fracture height for all of the 66 hydraulic fractures along the lateral wellbore, the fracture half length (\( x_f \)) was evaluated as 118 ft.

Figure 5.17: linear flow specialized plot for example 3 (left: superposition equivalent rate; right: superposition equivalent time)

To validate the interpretation of the reservoir parameters obtained from the analysis, an analytical oil reservoir model was created to history match the well’s
production. After fine-tuning of the model to minimize the errors in history match, the analytical model resulted in 7455 psi initial reservoir pressure, 0.00025 md reservoir permeability, 147.6 ft fracture height, 108 ft fracture half length. These numbers are consistent with that of the analysis, which confirms the validity of the workflow proposed in this thesis.

5.3 Advantages of Superposition-Rate

Although implementing superposition-rate is more complex and iterative, the newly invented superposition-rate demonstrates many diagnostic advantages over superposition-time. Based on the examples studied:

1. Superposition-rate does not modify the time scale in any way (whereas superposition-time modifies time in accordance with the rate changes). This keeps all data in the sequence of their occurrence, resulting in a significant improvement in quality diagnostics and data analysis.

2. The improvement to data diagnostics caused by superposition-rate becomes more pronounced in the presence of outliers. Superposition-rate does not make outliers look like that particular superposition’s flow regime (whereas superposition-time often does). In addition, since superposition-rate does not shuffle time back and forth, outliers become easier to be tracked and identified. Although outliers must be removed prior to analysis, the use of superposition-rate makes analysis less susceptible to outlier problems.

3. Linear flow superposition-rate accentuates the transition from transient linear flow to boundary dominated flow, thus aiding in the identification of the end
of linear flow. In addition, once the time to the end of linear flow is found from the linear flow specialized plot made by using superposition-rate, this value can be directly used in other analyses.

4. Superposition equivalent rate retains the characteristics of all of the flow regimes and meanwhile eliminates the problem of late-time data caused by superposition equivalent time. Superposition equivalent rate should be the preferred application in well test analysis. It is reliable for the identification of flow regimes.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

This thesis has demonstrated the importance of superposition in production data analysis. Superposition must be considered in the analysis of production data. Furthermore, a new superposition application, superposition-rate, has been developed and presented that acknowledges the same superposition principle underlying superposition-time, but manipulates data in the opposite manner. The validity and practicality of the new superposition-rate has been demonstrated by the use of synthetic data and field examples.

This chapter presents major properties, advantages, and disadvantages of the new superposition-rate function as well as the classical superposition-time function as discovered throughout the development of this work. It also presents areas that may warrant future study.

6.1 Conclusions

The main conclusions of this thesis are summarized below.

Both superposition-time and superposition-rate are effective in converting constant pressure data (consequently any variable rate/variable pressure data) to its constant rate equivalent, but only for the data within the chosen superposition function’s flow regime.

Superposition-time sometimes (but not always) forces the data to look like the chosen superposition function’s flow regime, regardless of the intrinsic flow regime of
the data itself. This problem is more pronounced when early-time transient flow’s superposition-time is applied to late-time boundary dominated flow data. And this problem becomes less significant when late-time boundary dominated flow’s superposition-time is applied to early-time transient flow data.

In contrast to superposition-time, superposition-rate does not tend to force the data to look like the chosen superposition function’s flow regime. When data’s flow regime is inconsistent with the choice of the superposition-rate function, superposition-rate often (but not always) makes the data look significantly different from its equivalent constant rate solution. This effect is more pronounced when late-time boundary dominated flow’s superposition-rate is applied to early-time transient flow data. And it becomes less significant when early-time transient flow’s superposition-rate is applied to late-time boundary dominated flow data.

Boundary dominated flow superposition-time is the most benign of all of the different superposition-time and superposition-rate functions, for the purpose of identifying flow regimes. BDF superposition-time is always effective in retaining the characteristics of every flow regime.

From the field examples studied in this work, superposition-rate demonstrates many diagnostic advantages over superposition-time. These advantages are summarized as follows:

1. Superposition-rate does not modify the time scale in any way (whereas superposition-time shuffles time as the rate changes). This keeps all data in the sequence of their occurrence, resulting in a significant improvement in quality diagnostics and data analysis.
2. The advantage of using superposition-rate in data diagnostics becomes more evident in the presence of outliers. Superposition-rate does not make outliers look like that particular superposition’s flow regime (whereas superposition-time often does). In addition, since superposition-rate does not shuffle time back and forth, outliers become easier to be tracked and identified. Although outliers should be removed prior to analysis, the use of superposition-rate makes analysis less prone to outlier problems.

3. Linear flow superposition-rate accentuates the transition from transient linear flow to boundary dominated flow, thus aiding in the identification of the end of linear flow. In addition, once the time to the end of linear flow is found from the linear flow specialized plot made by superposition-rate, this value can be directly used in other analyses.

4. Superposition equivalent rate retains the characteristics of all of the flow regimes and meanwhile eliminates the problem of late-time data caused by superposition equivalent time. Superposition equivalent rate is reliable for the identification of flow regimes in well test analysis.

6.2 Recommendations

1. One area that may warrant future study is to investigate how to combine the superposition-time and superposition-rate in order to yield better results. For instance, as alluded before, implementation of the superposition-rate is iterative but the superposition-time is not. As a consequence of this, there is
value in utilizing the superposition-time to estimate an initial guess for the
superposition-rate’s iteration.

2. The effect of superposition-time and superposition-rate needs to be
investigated within some more complicated flow systems, such as dual
porosity and a system with stimulated rock volume.

3. Further research needs to be done by incorporating corrected pseudo time into
the workflow of production data analysis for gas systems.

4. The newly developed superposition-rate functions should be more
extensively tested by field data, notably pressure transient data for well test
analysis.

5. The superposition-rate concept should be expanded to analyze multiple wells’
production data. To do this, the superposition in space principle needs to be
investigated.

6. A user-friendly software package should be developed to utilize the proposed
workflow for analyzing production and well test data.
NOMENCLATURE

A  Drainage area (ft$^2$)
B  Formation volume factor (bbl/stb in liquid system; ft$^3$/scf in gas system)
b  y-intercept of analysis line on specialized plot (psi/stb/d in liquid system; psi$^2$/cp/MMscf/d in gas system)

C_A  Shape factor
c  Compressibility (psi$^{-1}$)
c_f  Formation compressibility (psi$^{-1}$)
c_t  Total compressibility (psi$^{-1}$)
\bar{c_t}  Total compressibility based on average reservoir pressure (psi$^{-1}$)

DER  Well testing derivative (psi)

FCD  Dimensionless fracture conductivity

h  Formation thickness (ft)
h_f  Fracture thickness (ft)
k  Permeability (md)
k_f  Fracture permeability (md)
m  Slope of analysis line on specialized plot (unit depends on the type of specialized plot selected, with reference to Appendix)
n_f  number of hydraulic fractures

P  Pressure (psia)
P_p  Pseudo pressure (psi$^2$/cp)

P_s  Well’s build up pressure during shut in period
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{wff}$</td>
<td>Well’s final flowing pressure prior to shut in</td>
</tr>
<tr>
<td>$\bar{P}$</td>
<td>Average pressure (psia)</td>
</tr>
<tr>
<td>$q$</td>
<td>Flow rate (stb/d in liquid system; MMscf/d in gas system)</td>
</tr>
<tr>
<td>$q_{sp}$</td>
<td>Superposition-rate (stb/d in liquid system; MMscf/d in gas system)</td>
</tr>
<tr>
<td>$q_{sp_equiv}$</td>
<td>Superposition equivalent rate (stb/d in liquid system; MMscf/d in gas system)</td>
</tr>
<tr>
<td>$r_e$</td>
<td>External radius of cylindrical shape reservoir (ft)</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Wellbore radius (ft)</td>
</tr>
<tr>
<td>$S$</td>
<td>Skin</td>
</tr>
<tr>
<td>$S_g$</td>
<td>Gas saturation</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature (°R)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (day)</td>
</tr>
<tr>
<td>$t_a$</td>
<td>Pseudo time (day)</td>
</tr>
<tr>
<td>$t_{ca}$</td>
<td>Corrected pseudo time (day)</td>
</tr>
<tr>
<td>$t_{sp}$</td>
<td>Superposition-time (day)</td>
</tr>
<tr>
<td>$t_{spa}$</td>
<td>Superposition pseudo time (day)</td>
</tr>
<tr>
<td>$t_{sp_equiv}$</td>
<td>Superposition equivalent time (day)</td>
</tr>
<tr>
<td>$V$</td>
<td>Reservoir volume (stb in liquid system; scf in gas system)</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Cumulative gas production (scf)</td>
</tr>
<tr>
<td>$w_f$</td>
<td>Fracture width (ft)</td>
</tr>
<tr>
<td>$x_f$</td>
<td>Fracture half length (ft)</td>
</tr>
<tr>
<td>$x_e$</td>
<td>$\frac{1}{2}$ of reservoir size in x-direction, rectangular shape reservoir (ft)</td>
</tr>
<tr>
<td>$y_e$</td>
<td>$\frac{1}{2}$ of reservoir size in y-direction, rectangular shape reservoir (ft)</td>
</tr>
<tr>
<td>$Z$</td>
<td>Compressibility factor</td>
</tr>
</tbody>
</table>
Δ Difference operator
∇ Derivative operator

GREEK
μ Viscosity (cp)
μ̅ Viscosity based on average reservoir pressure (cp)
ρ Density (lbm/ft^3)
φ Porosity
π 3.1416

SUBSCRIPTS
D Dimensionless
e External boundary
f Fracture
g Gas phase
i Initial
n Data’s consecutive number, starting from 1
o Oil phase
w Internal boundary (wellbore) or water phase
wf wellbore flowing
REFERENCES


Canadian Unconventional Resources Conference, Calgary, Alberta, Canada, 5-7 November.


Mattar, L. 1997. Computer - Black Box or Tool Box. JCPT 36 (03). PETSOC 97-03-GE.


### APPENDIX

#### Table A1: analytical solutions for liquid systems with the use of superposition-time

<table>
<thead>
<tr>
<th></th>
<th>Superposition-Time Definition</th>
<th>Liquid Flow Equation</th>
<th>m</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Radial Flow</td>
<td>( \log(t_{sp}) = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \log(t_n - t_{i-1})}{q_n} )</td>
<td>( \frac{\Delta P}{q_n} = m \log(t_{sp}) + b )</td>
<td>( \frac{162.6B\mu}{kh} )</td>
<td>( \frac{162.6B\mu}{kh} \left[ \log \left( \frac{k}{\phi \mu c_pr_w^2} \right) - 1.847 + 0.8686s \right] )</td>
</tr>
<tr>
<td>Transient Linear Flow</td>
<td>( \sqrt{t_{sp}} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{q_n} )</td>
<td>( \frac{\Delta P}{q_n} = m \sqrt{t_{sp}} + b )</td>
<td>( \frac{19.91B\mu}{\mu \sqrt{x_l h}} )</td>
<td>( \frac{141.2B\mu}{k(2x_l) Sf} )</td>
</tr>
<tr>
<td>Transient Bilinear Flow</td>
<td>( \sqrt{t_{sp}} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{q_n} )</td>
<td>( \frac{\Delta P}{q_n} = m \sqrt{t_{sp}} + b )</td>
<td>( \frac{97.58B\mu}{\sqrt{k_f w_l h}} \left( \frac{1}{k\phi m c_t} \right) )</td>
<td>( \frac{141.2B\mu}{k(2x_l) Sf} )</td>
</tr>
<tr>
<td>Boundary Dominated Flow</td>
<td>( t_{sp} = Q_n \frac{Q_n}{q_n} )</td>
<td>( \frac{\Delta P}{q_n} = m t_{sp} + b )</td>
<td>( \frac{5.615B}{A\phi c_t h} )</td>
<td>( \frac{141.2B\mu}{kh} \left[ \ln \left( \frac{r_h}{r_w} \right) - \frac{3}{4} + S \right] ) (radial flow geometry)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \frac{141.2B\mu}{kh} \left( \frac{r_h}{r_w} \right) + \frac{k}{2x_l} S_f ) (linear flow geometry)</td>
</tr>
</tbody>
</table>

#### Table A2: analytical solutions for liquid systems with the use of superposition-rate

<table>
<thead>
<tr>
<th></th>
<th>Superposition-Rate Definition</th>
<th>Liquid Flow Equation</th>
<th>m</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Radial Flow</td>
<td>( q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \log(t_n - t_{i-1})}{\log(t_n)} )</td>
<td>( \Delta P - q_{n}b \frac{t_{sp}}{q_{sp}} + b = m \log(t) + b )</td>
<td>( \frac{162.6B\mu}{kh} )</td>
<td>( \frac{162.6B\mu}{kh} \left[ \log \left( \frac{k}{\phi \mu c_pr_w^2} \right) - 1.847 + 0.8686s \right] )</td>
</tr>
<tr>
<td>Transient Linear Flow</td>
<td>( q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{\sqrt{t_n}} )</td>
<td>( \Delta P - q_{n}b \frac{\sqrt{t_{sp}}}{q_{sp}} + b = m \sqrt{t} + b )</td>
<td>( \frac{19.91B\mu}{\mu \sqrt{x_l h}} )</td>
<td>( \frac{141.2B\mu}{k(2x_l) Sf} )</td>
</tr>
<tr>
<td>Transient Bilinear Flow</td>
<td>( q_{sp} = \sum_{i=1}^{n} \frac{(q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})}}{\sqrt{t_n}} )</td>
<td>( \Delta P - q_{n}b \frac{\sqrt{t_{sp}}}{q_{sp}} + b = m \sqrt{t} + b )</td>
<td>( \frac{97.58B\mu}{\sqrt{k_f w_l h}} \left( \frac{1}{k\phi m c_t} \right) )</td>
<td>( \frac{141.2B\mu}{k(2x_l) Sf} )</td>
</tr>
<tr>
<td>Boundary Dominated Flow</td>
<td>( q_{sp} = \frac{Q_n}{t_{n}} )</td>
<td>( \Delta P - q_{n}b \frac{t_{sp}}{q_{sp}} + b = m t + b )</td>
<td>( \frac{5.615B}{A\phi c_t h} )</td>
<td>( \frac{141.2B\mu}{kh} \left[ \ln \left( \frac{r_h}{r_w} \right) - \frac{3}{4} + S \right] ) (radial flow geometry)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \frac{141.2B\mu}{kh} \left( \frac{r_h}{r_w} \right) + \frac{k}{2x_l} S_f ) (linear flow geometry)</td>
</tr>
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</table>
Table A3: analytical solutions for gas systems with the use of superposition-time

<table>
<thead>
<tr>
<th>Superposition-Time Definition</th>
<th>Liquid Flow Equation</th>
<th>m</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Radial Flow</td>
<td>[ \log(t_{sp}) = \sum_{i=1}^{n} (q_i - q_{i-1}) \log(t_n - t_{i-1}) ]</td>
<td>[ \frac{\Delta P_p}{q_n} = m \log(t_{sp}) + b ]</td>
<td>[ 1.631 \times 10^6 T ]</td>
</tr>
<tr>
<td>Transient Linear Flow</td>
<td>[ \sqrt{t_{sp}} = \sum_{i=1}^{n} (q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})} ]</td>
<td>[ \frac{\Delta P_p}{q_n} = m \sqrt{t_{sp}} + b ]</td>
<td>[ 199.8 \times 10^3 T ]</td>
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<tr>
<td>Transient Bilinear Flow</td>
<td>[ \sqrt[4]{t_{sp}} = \sum_{i=1}^{n} (q_i - q_{i-1}) ^{1/4} (t_n - t_{i-1}) ]</td>
<td>[ \frac{\Delta P_p}{q_n} = m \sqrt[4]{t_{sp}} + b ]</td>
<td>[ 979.2 \times 10^3 T ]</td>
</tr>
<tr>
<td>Boundary Dominated Flow</td>
<td>[ t_{sp} = \frac{Q_n}{q_n} ]</td>
<td>[ \frac{\Delta P_p}{q_n} = m t_{sp} + b ]</td>
<td>[ 56.36 \times 10^3 T ]</td>
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</table>

Table A4: analytical solutions for gas systems with the use of superposition-rate

<table>
<thead>
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<th>Superposition-Rate Definition</th>
<th>Liquid Flow Equation</th>
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<th>b</th>
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<tr>
<td>Transient Radial Flow</td>
<td>[ q_{sp} = \sum_{i=1}^{n} (q_i - q_{i-1}) \log(t_n - t_{i-1}) ]</td>
<td>[ \frac{\Delta P_p - q_a b}{q_{sp}} + b = m \log(t) + b ]</td>
<td>[ 1.631 \times 10^6 T ]</td>
</tr>
<tr>
<td>Transient Linear Flow</td>
<td>[ q_{sp} = \sum_{i=1}^{n} (q_i - q_{i-1}) \sqrt{(t_n - t_{i-1})} ]</td>
<td>[ \frac{\Delta P_p - q_a b}{q_{sp}} + b = m \sqrt{t} + b ]</td>
<td>[ 199.8 \times 10^3 T ]</td>
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<tr>
<td>Transient Bilinear Flow</td>
<td>[ q_{sp} = \sum_{i=1}^{n} (q_i - q_{i-1}) ^{1/4} (t_n - t_{i-1}) ]</td>
<td>[ \frac{\Delta P_p - q_a b}{q_{sp}} + b = m \sqrt[4]{t} + b ]</td>
<td>[ 979.2 \times 10^3 T ]</td>
</tr>
<tr>
<td>Boundary Dominated Flow</td>
<td>[ q_{sp} = \frac{Q_n}{t_n} ]</td>
<td>[ \frac{\Delta P_p - q_a b}{q_{sp}} + b = m t + b ]</td>
<td>[ 56.36 \times 10^3 T ]</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( x_i )</th>
<th>( h )</th>
<th>( S_f )</th>
<th>( h )</th>
<th>( S_f )</th>
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<tr>
<td>( x_i )</td>
<td>( y_i )</td>
<td>( L )</td>
<td>( S_f )</td>
<td>( L )</td>
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