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Random Number Generation using Human Gameplay

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Random Number Generation using Human Gameplay

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Abstract

Randomness is one of the most important research areas in computer science and in particular, in cryptography. Security of almost all cryptosystems relies on random keys. Unfortunately, perfect sources of randomness are not easily accessible. However, True Random Number Generators (TRNGs) generate almost random strings, using non-perfect random sequences. A TRNG algorithm consists of an entropy source and an extractor. In this thesis, a TRNG is proposed in which a human player's input in a two-player game is used as the entropy source and the random seed required by the extractor. This means that the proposed TRNG is only dependent on user's inputs. The thesis contains the theoretical foundation of the approach, the design, and implementation of the corresponding game. To validate theories, we designed and implemented a game, and performed some user studies. The results of our experiments support the effectiveness of the proposed method in generating high-quality randomness.

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List of Symbols, Abbreviations and Nomenclature

Symbol	Definition
ACC	Anterior Cingulate Cortex
AES	Advanced Encryption Standard
CH	Cerebellar Hemispheres
CPU	Central Processing Unit
CSPRNG	Cryptographic Secure Random Number Generator
DES	Data Encryption Standard
DLPFC	Dorsolateral Prefrontal Cortex
GnuPG	Gnu Privacy Guard
IID	Independent Identically Distributed
LCG	Linear Congruential Generator
LFSR	Linear Feedback Shift Register
NIST	National Institute of Standards and Technology
NPRNG	Normal Pseudo Random Number Generator
OS	Operating System
PC	Personal Computer
PDA	Personal Digital Assistant
PGP	Pretty Good Privacy
PRNG	Pseudo Random Number Generator
RAM	Random Access Memory
RIFC	Right Inferior Frontal Cortex
RNG	Random Number Generator
SPC	Superior Parietal Cortex
SSH	Secure Shell

SSL	Secure Socket Layer
TDCS	Transcranial Direct Current Stimulation
TLS	Transport Layer Security
TRNG	True Random Number Generator
UHF	Universal Hash Function
U of C	University of Calgary

Chapter 1

Introduction

Randomness has been one of the most important research areas in computer science over the past few decades. The applications of random numbers range from randomized algorithms to routing algorithms in networks, and finally cryptography. In the world of information security, randomness is required for cryptosystems for purposes such as key generation, data padding or challenge bits in challenge-response protocols. Many security systems have been broken because they used poor sources of randomness or weak random number generators for key generation. Attacks on the Netscape implementation of the Secure Sockets Layer (SSL) protocol [GW96] and Kerberos V4 [DLS97] are well-known examples of system breakdowns due to using weak randomness sources and/or random number generation algorithms. More recent studies show that the output of the /dev/urandom interface of the Linux Random Number Generator (RNG) becomes deterministic under low entropy conditions, resulting in insecurity of the Transport Layer Security (TLS) and Secure Shell (SSH) keys [HDWH12]. Poorly generated randomness by the Linux Random Number Generator (RNG) also resulted in collisions among private and public keys, generated by individuals around the world [LHA+12].

The examples of system breakdowns due to exploiting weak randomness, on one hand, highlight the importance of randomness for security purposes, and on the other hand confirms that generating true randomness is not an easy task especially in a deterministic device, such as a computer. Any random number generator ultimately needs a physical entropy source. An entropy source uses a physical quantity such as noise in electronic circuits, or "unpredictable" software processes to output a se-

quence over an alphabet that is highly "unpredictable". However, the underlying distribution of this sequence is not necessarily uniform or even close to uniform. Thus, a postprocessing step is usually applied to the output of an entropy source to make it follow a uniform distribution. Randomness extractors are deterministic or probabilistic functions that are used for this purpose. A *True Random Number Generator* (TRNG) exploits randomness extractors to generate a random sequence from an entropy source.

In this thesis, a new user-based TRNG is proposed. This TRNG uses a human player's inputs in a two-player competitive zero-sum game as the main entropy source in its structure. The human player's input also provides the required randomness for the functionality of the randomness extractor. Therefore, the dependency of the TRNG on sources other than its user is minimized. As a proof of concept, a two-player game, and accordingly a TRNG is designed and implemented based on the proposed approach, and the output of the TRNG is evaluated using appropriate randomness tests.

1.1 Randomness

In spite of common sense on the concept of randomness, it is hard to define it mathematically. Nevertheless, for the limited purposes of creating random sequences of numbers or symbols on a computer, only some simple properties of random sequences require understanding. A sequence of symbols is considered random if it does not follow any specific order or pattern. Therefore, in a random sequence, it is not possible to predict an upcoming symbol from the previous symbols and from any given part of the sequence one cannot determine the previous symbols. Moreover, the symbols in a random sequence have no dependency on each other; that is, each symbol is generated independently from other symbols. In terms of probability theory, a sequence of

symbols is perfectly random if and only if it is sampled from a uniform distribution. Thus, all symbols in a random sequence are generated with the same probability. A random sequence of bits can be viewed as the results of repeatedly flipping an unbiased coin where one side corresponds to 0 and the other side to 1. Each outcome happens with the probability of exactly $\frac{1}{2}$. Moreover, the flips are independent of each other which means that the result of any previous coin flip will not influence future coin flips.

In this thesis, the term "randomness" is used to specify the described properties in a sequence of symbols.

1.1.1 Randomness Generation

Any resource that outputs an unpredictable value is considered as a "randomness source" in this thesis. It is unclear whether the real world's physical sources output perfectly random sequences or not. However, some sources seem to have some unpredictability, such as the low order bits of a system clock or thermal noise, nevertheless, these sources generally have biases and correlations.

Random sequences are generated by "random number generators". In a computer system, there are two main categories of random number generators: True Random Number Generators (TRNGs) and Pseudo Random Number Generators (PRNGs). TRNGs exploit randomness from a physical source such as thermal noise or from non-deterministic user based interactions such as mouse positions or times between keystrokes. PRNGs, on the other hand, take an initial random value, named the "initial seed", to run an efficient deterministic algorithm for producing a sequence that "looks random" [RSN+01]. If a PRNG does not use an external source of randomness, its output is deterministically determined by the initial seed, which means compromising the seed will result in compromise of the whole system.

1.1.2 Evaluating RNGs

It is impossible to prove definitively whether a given sequence of numbers is random simply because it is essentially impossible to prove a given sequence is sampled from a uniform distribution. The practical approach for systematic evaluation of RNGs is to take many sequences of random numbers from a given generator and subject them to a battery of statistical tests that are introduced for this purpose. If the sequences pass most of the tests, the confidence in the randomness of the numbers increases and so does the confidence in the generator.

The National Institute of Standards and Technology (NIST) has developed a package of 15 statistical tests to evaluate the randomness of a sequence. We use some tests from this package for the final evaluation of our designed TRNG.

1.2 Human as a source of randomness

Computers are fundamentally deterministic; therefore, they can not do anything unpredictable, especially randomness generation unless they use an input from a physical randomness source. Most randomness sources, such as thermal noise, are not easily available and an extra piece of hardware is required for exploiting their randomness. Therefore, in many systems, user based random number generation methods are preferable due to the availability of the user on demand. Moreover, user based RNGs add an extra level of assurance about the randomness source because users know their input will eventually be used for generating randomness in the system.

Random number generation by humans is related to specific functions of the brain such as updating and monitoring information and inhibition of automatic responses. Psychological experiments show that humans do a poor job when asked to choose random numbers [Wag72]. However, some studies demonstrate that humans become better at generating randomness by practice [Neu86, CCR+14]. This confirms that

humans, if led properly, can be considered as a relatively strong source of randomness in user based random number generators. In [RB92], Rapoport et al. used game theory to argue that the optimal strategy for rational players in some competitive zero-sum games is to select actions from a uniform distribution. They conducted a series of experiments to monitor human players' actions in a specific zero-sum game called "matching pennies". In this game, each player makes a choice between heads or tails; the first player wins if both players choose the same side or the second player wins otherwise. The game theoretic argument implies that the optimal strategy for winning the game is a uniform selection between heads and tails in each round of the game. The experimental results supported the theoretic expectation of the user's selected strategy: they almost followed uniform strategy for selecting their action in each round. This confirms that human can be considered as a good source of randomness if engaged in a strategic game and randomness generation is an indirect result of their actions.

In [HN09], Halprin et al. extended the previous experiment by enlarging players' actions into distinct positions on a two-dimensional screen. The purpose was to generate more random bits in each round so that they can use the generated randomness for practical applications such as cryptographic keys. However, their experimental results endorse that their proposed approach for enlarging the action space decreases the randomness level of generated sequence by users as they tend to avoid selecting corner points. Thus, the users' inputs are only considered as an entropy source. A seeded extractor is then applied to extract randomness from this source, and the output is fed to a robust PRNG for refreshing its states. Therefore, in this approach, the entire random number generator is very dependent on randomness sources not provided by the user. The main reason is that a relatively long, truly random sequence is required to provide a random seed for the operation of the seeded extractor that

is in charge of extracting randomness from the users' inputs. Moreover, as the game is played against a computer player, a PRNG is required for selecting the computer player's actions. Respectively, the quality of the generated randomness by the human player in the game is affected by the quality of the PRNG that determines the computer player's actions; a weak PRNG leads the human player toward weak randomness generation. This puts the efficiency of the proposed approach under serious doubt.

1.3 The Contributions

In this thesis, a TRNG that uses human gameplay against the computer as the main source of randomness is proposed. The dependency of the proposed TRNG on other randomness sources or generators is minimized. This work can be viewed as an extension of the work of Halprin et al. [HN09], but having better functionality and efficiency. The rate of randomness generation is kept high and at the same time, the human player's actions are limited to a few choices (e.g. 3 choices). It is explained how the designed game is efficiently applicable for generating a long sequence of random bits. This is a two-player game, but when only one human player is available it can be played against a computer player. Two algorithms for simulating one of the players by a computer program are discussed, and a game along with an interface for collecting data from sample human players who play the game against each of the introduced algorithms is implemented. Following this approach, the human player's input in this game corresponds to a sequence of bits which are supposed to be random. The quality of the final generated random sequence is evaluated using appropriate randomness tests. Finally, the randomness quality of the generated sequences according to each of the simulating algorithms are compared, and the results are justified. Part of this work is published in GameSec'13 [ASNS13].

When uniform selection among possible actions is the optimal strategy for winning the game in a two-player game, one can simulate the optimal computer player by choosing its actions using a PRNG. In this case, the proposed approach of using the human player's randomness will eventually generate a random sequence with better random properties than the one used on the computer player's side. An alternate approach for generating more random bits than what is used by a computer is proposed. This approach, in fact, improves the efficiency of the whole TRNG in the sense of the randomness generation rate. This is basically done by providing more actions for the human player than the computer player at each round of the game. The details of this approach are discussed in Section 4.4.

1.3.1 Game theory based TRNG

The proposed TRNG algorithm is a seeded extractor that is constructed from an expander graph. Expander graphs are well-connected d-regular graphs where each vertex is connected to d neighbors. Random walks on an expander graph are used to extract randomness from an initial distribution on a graph's vertices [AB09]. One can view the vertices as states of a process. Suppose there is an arbitrary distribution on the vertices of the graph, which means there is a probability distribution on the states. For doing a random walk on the graph, a uniform distribution on outgoing edges from each vertex is required. This distribution determines the probability of transition to corresponding neighbouring states. After one step of the random walk, each state will transit to one of the corresponding d neighboring states with the same probability. Thus, each step of the random walk results in a new distribution over the vertices (states). This distribution is closer to a uniform distribution than the initial one. By taking enough random walks, the obtained distribution becomes as close to a uniform distribution as required.

In the proposed approach, the human player's actions in the designed game are

used for providing the initial distribution as well as the uniform distribution for each step of the random walk process in the above framework. The game consists of a sequence of stages. Each stage is a simple two-player zero-sum game such as Roshambo (Rock-Paper-Scissors). At each stage the human player makes a choice among some alternatives. The game and the corresponding scores in each stage are designed so that selecting actions uniformly randomly is the best strategy for winning the game against a rational player. The first stage generates an input entropy for the TRNG, and subsequent stages provide the random seed for the extractor algorithm.

At the first stage, the user is presented with the graph and asked to randomly choose a vertex. The human player's choice at this stage is effectively a symbol of an entropy source that is generated according to some unknown distribution. The number of available actions at the first stage is relatively large and equals the number of the graph's vertices, thus, due to the experimental results mentioned in [HN09], the human player's choices are not uniformly distributed. However, the unknown distribution has some entropy. The random walks of length ℓ over the graph will be used to generate an output symbol for the TRNG with a close to uniform randomness guarantee. Subsequent intermediate stages function as the required random walk. At each intermediate stage, the player is presented with d possible actions to play a zero-sum strictly competitive game while the optimal strategy for winning the game is to select an action uniformly from the action space. Each of the d possible actions corresponds to one neighbor vertex. Since the number of possible actions at each stage is small (3, 5 or 7), the human player's input would correspond to uniform selection (based on the experiments in [RB92]) and consequently one random step on the graph.

For an estimated min-entropy of the initial vertex selection, one can determine the number of required random steps so that the output of the TRNG has the required

randomness guarantee.

Note that in the above, although the human player's actions in the intermediate game stages are effectively assumed uniformly distributed, in practice the human player's input will be close to uniform and so the proposed extraction process can be seen as approximating the random walk by a high min-entropy walk. The experimental results confirm the feasibility of this approximate approach. From a theoretical view, it is interesting to analyze the quality of the output in an expander graph extractor when the uniform random walk is replaced with a close to uniform random walk.

1.3.2 Game Design for Applicable TRNG from Human Gameplay

Two games are designed and implemented to validate the proposed TRNG. The first game is based on a 3-regular expander graph with 10 vertices. The game consists of a sequence of stages. At each stage the human and the computer make a choice from a set of vertices. If the choices coincide, the computer wins; otherwise, the human wins. In the implementation, the computer's choice is shown after the player picks his/her action, even though this choice is independent of the player's previous actions. At the first stage, the user makes a choice among 10 vertices of the whole graph. In all the subsequent stages, the player is asked to make a selection among only 3 neighbors of the previously chosen vertex.

Some experiments are performed to support the theoretical approach for designing the TRNG. The above game is implemented, and then experimented with nine human users playing the game. For the precise design of the game, the min-entropy of human choices in the initial stage is required, that is when the human player is an entropy source. For this purpose, another game is designed, which requires users to choose a vertex of the same graph and they win if their choice does not match the computer's choice. NIST [RSN+01] tests for estimating the min-entropy of the human player's

choices in this initial game are used. The number of required random walks was then determined with regards to this min-entropy. Moreover, the min-entropy of the human player's input at subsequent stages is measured and used to emulate the random walk. The estimations show that the player's choices at these stages are not exactly uniform. However, they have high min-entropy, and thus the final output of the TRNG passes the randomness tests. This confirms that the theoretical design of the TRNG is applicable in practice.

The second game is designed to generate long random sequences (e.g. 40 bits) for real world applications such as cryptographic keys. In this game, the human player initially makes a choice on a circle. The selected location is mapped to an 8-bit value. The subsequent stages are a normal Roshambo game. The expander graph is never presented to the player. Instead, random walks are tracked through the indices of neighboring vertices. The output of the TRNG is a vertex on the graph that is obtained by tracking random walks. The designed game requires two players. However, we simulate one of the players actions by a computer. Two different algorithms are used for simulating the second player's actions. In the first one, the output symbols of a very weak PRNG are mapped to the computer's actions and then the computer's actions are selected based on the PRNG's outputs and independent of the user's previous actions. The other algorithm is a predictor algorithm that keeps a history of the human player's actions over the previous rounds of the game and decides on the computer's action with respect to this history. This minimizes the computer's dependency on an outside PRNG and at the same time helps the player to modify his/her actions if they are not sufficiently random and are predictable by the computer. The details of the algorithm are explained in Section 4.3.2. The output of the proposed TRNG is compared in these two cases that verifies using a predictor algorithm for simulating one of the player's actions causes better randomness

generation by the proposed TRNG. The initial min-entropy of the player's actions is estimated similar to the previous game: a separate game is designed in which players are asked to pick a location on a circle, and they will win the game if their choice does not match the computer's. The player's min-entropy in this game is estimated using the NIST tests.

Statistical tests in a battery of tests called Rabbit [LS07] are used to evaluate the output of the proposed TRNG. The details of the experiments are given in Chapter 5.

1.4 Thesis Structure

This thesis is divided into six chapters. Chapter 1 gives an overview of the studied problem, which is randomness generation, and addresses the contributions for generating randomness from human gameplay. Chapter 2 is the definitions and the related background knowledge. In Chapter 3 randomness measures and methods for generating randomness are elaborated and the psychological studies on randomness generation by human beings are briefly reviewed. Chapter 4 contains the main contributions and methodology for constructing a TRNG from human gameplay. Chapter 5 explains the experimental set-up for evaluating the proposed TRNG and contains the results and analysis of the experiments. Finally, the conclusion and future work are in Chapter 6.

1.4.1 Theorems and proofs

In this thesis, whenever a theorem or lemma is used from other works, the original reference proving it is cited. If a theorem or lemma is not cited, the proof is from this work.

Chapter 2

Preliminaries and Background

In this chapter, some of the concepts and backgrounds which are extensively used in this work are recalled. For definitions in probability theory section [S+02] is used, and for the graph theory part [Gra95] is used. Information theoretic concepts are mainly from [CT12]. For describing entropy estimation methods [BK12] is used. The game theory concepts and definitions are from [Osb04] and [OR94] and the repeated games part is mainly from [BS08].

2.1 Linear Algebra

For two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, the inner product is defined as $\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{i=1}^{n} \mathbf{u}_i, \mathbf{v}_i$. These two vectors are orthogonal when $\langle \mathbf{u}, \mathbf{v} \rangle = 0$.

L_2-norm for a vector $\mathbf{u} \in \mathbb{R}^n$ is denoted by $\|\mathbf{u}\|_2$ and is defined as $\sqrt{\langle \mathbf{u}, \mathbf{u} \rangle} = \sqrt{\sum_{i=1}^{n} \mathbf{u}_i^2}$. For *$L_2$-norm* of the sum of two vectors we have $\|\mathbf{u} + \mathbf{v}\|_2 \leq \|\mathbf{u}\|_2 + \|\mathbf{v}\|_2$, where equality happens only when two vectors are orthogonal. The *L_1-norm* of $\mathbf{u}$ is denoted by $|\mathbf{u}|_1$, and is defined as $\sum_{i=1}^{n} |\mathbf{u}_i|$. The following lemma shows the relationship between L_1 and L_2 norms:

Lemma 2.1.1 *[AB09] For every vector $\mathbf{u} \in \mathbb{R}^n$, we have:*

$$\frac{|\mathbf{u}|_1}{\sqrt{n}} \leq \|\mathbf{u}\|_2 \leq |\mathbf{u}|_1$$

In this thesis, bold lower-case letters such as $\mathbf{u}$ are used to denote vectors, and bold upper-case letters such as $\mathbf{U}$ are used to denote matrices. For row representation of a column vector $\mathbf{u}^t$ is used and the transpose of a matrix $\mathbf{U}$ is denoted by $\mathbf{U}^t$.

2.2 Probability Theory

A probability space is a triple $(\Omega, \mathcal{F}, P)$ where Ω is a set of "*outcomes*", $\mathcal{F}$ is a set of "*events*", and $P : \mathcal{F} \to [0, 1]$ is a function that assigns probabilities to events so that the sum of probabilities over the whole sample space becomes 1 where the sample space is the collection of all possible outcomes. A real-valued function defined on the outcome of a probability experiment is called a "*random variable*". A random variable whose set of possible values is either finite or countably infinite is called discrete.

Upper-case letters are used to denote discrete random variables, and lower-case letters are used to denote a random variable's realizations. By $Pr(X = x)$ we mean the assigned probability to the realization of random variable X when it equals x. The calligraphic letters $\mathcal{X}$ are used to show sets and subsets of elements. In this thesis, $|\mathcal{X}|$ is used to denote the number of elements in a set and $X \in \mathcal{X}$ means the random variable's distribution is over $\mathcal{X}$, and $x \in \mathcal{X}$ means the realization of the random variable is one of the elements of $\mathcal{X}$.

P_X denotes the distribution of the random variable X. Since any random variable corresponds to a distribution, the terms "random variable" and "distribution" are used interchangeably, unless for emphasising their difference. For two random variables $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$, P_{XY} denotes their joint distribution, and $P_{X|Y}$ denotes their conditional distribution. The conditional probability of a random variable X given that Y takes a value $y \in \mathcal{Y}$ with $P_Y(y) > 0$, is denoted by $P_{X|Y}(x|y)$ and is given by:

$$P_{X|Y}(x|y) = \frac{P_{XY}(x, y)}{p_Y(y)} \quad \text{for } x \in \mathcal{X}.$$

The expected value of a random variable X with probability distribution P_X is de-

noted by $\mathbb{E}[X]$ and is given by:

$$\mathbb{E}(X) = \sum_{x \in X} xPr(X = x).$$

Theorem 2.2.1 ***(A Chernoff Bound)****[Hoe63] Suppose $X_1, X_2, ..., X_n$ are independent random variables taking values in the interval $[0, 1]$. Then for $X = \frac{\sum_{i=1}^{n} X_i}{t}$, we have:*

$$Pr[|X - \mathbb{E}(X)| \geq \epsilon] \leq 2exp(\frac{-t\epsilon^2}{4}).$$

Definition 2.2.1 ***Markov Chain****[S+02] Consider a sequence of random variables $X_0, X_1, ...$ and suppose that the set of possible values for these random variables is $\{0, 1, ..., M\}$ for an integer M. The sequence of random variables form a Markov chain if and only if*

$$P\{X_{n+1} = j | X_n = i_n, X_{n-1} = i_{n-1}, ..., X_0 = i_0\} = P_{ij}.$$

X_n can be interpreted as the state of the system at time n. With this interpretation, forming a Markov chain means the system changes from state i to j with some constant probability P_{ij}. The values of P_{ij} are called *transition probabilities* of the Markov chain and they satisfy:

$$P_{ij} \geq 0 \quad and \quad \sum_{j}^{M} P_{ij} = 1.$$

The Markov chain of order κ for the sequence as $X_0, X_1, ...$ is defined if and only if

$$P\{X_{n+1} = j | X_n = i_n, ..., X_{n-\kappa+1} = i_{n-\kappa+1}\} = P_{ij}.$$

It is convenient to show transition probabilities p_{ij} in a square matrix called a *tran-*

sition probability matrix as follows:

$$\mathbf{P} = \begin{pmatrix} P_{0,1} & P_{0,1} & \cdots & P_{0,M} \\ P_{1,0} & P_{1,1} & \cdots & P_{1,M} \\ \vdots & \vdots & \ddots & \vdots \\ P_{M,0} & P_{M,1} & \cdots & P_{M,M} \end{pmatrix}.$$

In theory, by knowing the transition probability matrix of a Markov chain, one can compute all probabilities of interest.

2.3 Measures of Randomness

A random variable is perfectly random when its corresponding distribution is uniform. However, we would like to quantify *imperfect* randomness for other distributions so that they become comparable. This is done by defining the notion of the *entropy* of a given distribution. There are multiple types of entropy that one can define [Rrn61]. In this work, only two notions are considered: *Shannon entropy* [Sha01] which is the most common notion of entropy in cryptography, and *min-entropy* which is the most suitable notion for our application, generating randomness.

2.3.1 Information Theoretic Measures of Randomness

The most common information theoretic measure of randomness is Shannon's definition of entropy. Shannon entropy measures the average amount of uncertainty in a random variable and is defined as follows.

Definition 2.3.1 ***Shannon entropy:*** *For a random variable $X \in \mathcal{X}$ with distribution $P_X(x)$, the Shannon entropy is denoted by $H(X)$ and is equal to:*

$$H(X) = -\sum_{x \in \mathcal{X}} P_X(x) \log P_X(x).$$

For $X \in \mathcal{X}$, Shannon entropy is maximized when we have uniform distribution over $\mathcal{X}$. In this case $H(X) = -\log|\mathcal{X}|$ and any other distribution on this set will have smaller Shannon entropy. One can view Shannon entropy as a measure of existing randomness in a random variable. The bigger Shannon entropy represents more randomness. Nevertheless, this claim is not precise since it is possible to define pathological distributions that have high Shannon entropy but are useless for randomness extraction. To see why this is a problem, consider $X \in \{0,1\}^n$ with the following distribution:

$$Pr[X = 0^n] = 1 - \frac{1}{2^{100}}.$$

and for $a \in \{0,1\}^n \setminus 0^n$:

$$Pr[X = a] = \frac{1}{2^{100}}(\frac{1}{2^n - 1}) \sim \frac{1}{2^{n+100}}.$$

Consequently the Shannon entropy for a random variable X is:

$$H(X) = (1 - \frac{1}{2^{100}}) \log \frac{1}{1 - \frac{1}{2^{100}}} + \sum_{a \neq 0^n} \frac{1}{2^{100}}(\frac{1}{2^n - 1}) \log \frac{1}{\frac{1}{2^{100}}(\frac{1}{2^n - 1})} \sim \frac{n + 100}{2^{100}}.$$

Although the amount of Shannon entropy in the above example is linear in n, we cannot extract bits that are close to uniform by sampling X since it is a constant value (0^n) most of the time.

Min-entropy is an alternative entropy notion which is more useful for randomness extraction applications. The min-entropy measures the minimum information contained in a random variable.

Definition 2.3.2 ***Min-entropy:*** *For a random variable $X \in \mathcal{X}$ with distribution $P_X(x)$, min-entropy is denoted by $H_\infty(X)$ and is :*

$$H_\infty(X) = -\log(\max_{x \in \mathcal{X}}(P_X(x))).$$

In the mentioned example, $H_\infty(X) = -log(1 - \frac{1}{2^{100}})$, which is quite a small number and matches our intuition about the amount of randomness in the described distribution.

An alternative approach to evaluate the randomness of a distribution is to compare it with the uniform distribution. The *statistical distance* of two distributions is a measure of their difference and is defined as follows.

Definition 2.3.3 ***Statistical distance:*** *For two distributions X and Y defined over the set $\mathcal{U}$, the statistical distance between X and Y is defined as:*

$$SD(X;Y) \triangleq \frac{1}{2}\sum_{u\in\mathcal{U}} |Pr(X=u) - Pr(Y=u)| = \frac{1}{2}|X-Y|.$$

When $SD(X;Y) \leq \epsilon$ we say the distribution X is ϵ-close to distribution Y.

2.3.2 Entropy Estimation

Finding the entropy of a source requires knowledge of the exact source distribution. However, when the source distribution is unknown, one can still estimate the source entropy by sampling (as much as required) from the source. Apparently the existence of certain structures in the source simplifies entropy estimation. In particular, estimating the min-entropy of an independently and identically distributed (IID) source is straightforward. Since the samples are not correlated in such a source, the most common value among the samples is more likely to be the most probable one. The upper and lower bounds for the min-entropy is calculated based on this observation. Details of finding the upper bound and lower bound for the min-entropy of an IID source are explained in [BK12].

The testing method introduced in the NIST draft [BK12] initially, checks whether the source can be considered an IID. NIST suggests the following set of tests for checking if a source is IID: shuffling tests and statistical tests. The source output is

considered IID if the dataset passes all the tests. Then, based on the number of the observations of the most common output value, the min-entropy is estimated.

In the shuffling tests the following scores are calculated for the shuffled datasets: Compression score, Runs score, Excursion score, Directional Runs score, Covariance score and Collision score. For the samples following an IID distribution, the shuffled datasets would follow the same distribution as the original dataset and therefore, the derived scores for the original dataset and shuffled datasets are expected to be similar. But, if the samples do not follow an IID distribution, then some test scores may be very different for the original and shuffled datasets.

NIST only introduces one statistical test to check if a source is IID. The test is called Chi-square test and consists of two different types of test: a test for independence, and a test for goodness of fit. The independence test is aimed to find dependencies in the existing samples of the dataset. The goodness of fit test checks whether ten data subsets produced from the main dataset follow the same distribution.

A failure of any of the shuffling or statistical tests stops IID testing, and a number of tests are used to estimate the min-entropy, assuming the source is non-IID. For a non-IID source, the dependencies in time or state may result in entropy overestimation. However, in [BK12], five tests are suggested to minimize the possibility of overestimating the source entropy. Each of these tests is designed to measure specific statistics of the samples to capture the structure of the distribution. These five tests are collision, collection, compression, Markov, and frequency tests. Each test outputs a value as the estimation of the min-entropy. The final min-entropy will be the minimum over all of these estimated values.

2.3.2.1 Collision Test

The collision test is designed to estimate the probability of the most-likely state in a dataset. For this purpose it measures the mean time to the first collision in the dataset. To avoid entropy overestimation in non-IID sources, this test selects the minimum entropy estimation of all the tests as the expected entropy of the source [BK12, p. 62].

2.3.2.2 Partial Collection Test

The partial collection test computes the entropy of a dataset with regards to the number of distinct values in the output space. If the output dataset contains a small number of distinct values, the test estimates low entropy and if the dataset has high variation, high entropy is estimated [BK12, p. 65].

2.3.2.3 Markov Test

A Markov model in Definition 2.2.1 is used as a template for describing sources with dependencies. The Markov tests estimate min-entropy by measuring the dependency between successive outputs of the source. In the Markov tests, instead of single samples, chains of samples are considered. By the careful estimation of the transition probability matrix, the Markov model is used for entropy estimation. An accurate estimation of this matrix requires a large dataset. Nevertheless, in practice, a large data requirement is avoided by overestimating the low transition probabilities [BK12, p. 67].

2.3.2.4 Compression Test

The comparison test is designed based on the Maurer Universal Statistic [Mau92]. The test does not have any assumption of independence. The entropy rate of the dataset is calculated based on the compression rate of the dataset. The test initially generates a dictionary of values, and then computes the average number of samples

required to write an output with respect to the dictionary [BK12, p. 69].

2.3.2.5 Frequency Test

In the frequency test, min-entropy is estimated based on the rate of occurrence of the most probable sample value. Similar to the Markov test, an accurate estimation of min-entropy requires a large dataset. When only a small dataset is available, this test ignores unlikely sample values and therefore underestimates the min-entropy [BK12, p. 71].

2.4 Graph Theory

A graph G consists of a non-empty set $V(G)$ of elements, called *vertices* or *nodes*, and a set $E(G)$ of elements called *edges*, together with a relation of incidence that associates each edge with two vertices, called its ends. An edge with identical ends is a *loop* and one with distinct ends is a *link*. A *multigraph* is one with no loops. A *simple* graph is one with neither loops nor multiple edges. A *directed graph (digraph)* D is a graph in which a direction is assigned to each edge.

The degree of a vertex v in a graph G is the number of edges of G incident with v. A graph is *d-regular* if each vertex has degree d.

The adjacency matrix $\mathbf{A}(G)$ is the matrix whose rows and columns are indexed by the vertices of G and the element a_{uv} is the number of edges of G joining vertices u and v.

2.5 Game Theory

Game theory is the study of strategic decision making. Decision makers are considered the *players* of the game. There is a set of available *actions* for each player and a specification of payoffs for each combination of actions which models the interaction

between the players. An action profile is a list of all the players' actions. An assumption in game theory is that players are rational, which means they aim to maximize their utility, given some belief about the other players' actions. Thus, each player has *preferences* about the action profile. For describing the players' preferences a *utility function* is used. In general, a strategic game is defined as follows:

Definition 2.5.1 *A strategic game with ordinal preferences consists of*

- *A set of* ***players****,* $\mathcal{N} = 1, ..., N$ *is a finite set,*
- *A set of* ***actions*** *for player* i *denoted by* A_i*,*

  (If for every player i*, the set* A_i *of actions is finite then the game is "finite".)*

  Action profile*:* $\mathbf{a} = (a_1, ..., a_n) \in A = A_1 \times ... \times A_n$.
- ***Preferences*** *over the set of action profiles based on the* utility function *that captures payoffs.*
- ***Utility function*** *for player* i*:* $u_i : A \to R$ *, which* R *is the set of real numbers. We say that player* i *prefers* a *to* b *iff* $u_i(a) > u_i(b)$.

A finite two-player strategic game can be represented by a table. In such a representation, the *row* player is player 1 and the *column* player is player 2; i.e. rows correspond to actions $a_1 \in A_1$ and columns correspond to actions $a_2 \in A_2$. The two numbers in each cell list payoffs of each player: the row player first and the column player second.

2.5.1 Strategic Games

In game theory, two kinds of games are considered: competitive games and cooperative games. In competitive games players have exactly opposed interests (conse-

quently the number of players is always 2), while in games of cooperation players have the same interest.

An example of a strategic game is the "*Prisoner's Dilemma*". In this game, two criminals are arrested for their individual minor crimes, while they also have associated in a major crime but there is not enough evidence for proving the major crime. The prisoners are put into separate cells. If they both confess, each will be sentenced to three years in prison. If only one of them confesses, he/she will be freed and used as a witness against the other, who will receive a sentence of four years. If neither confesses, they will both spend one year in prison for the minor crime. Table 2.1 shows the payoff table of this game. Note that negative numbers are used for denoting the payoffs of this game since each prisoner tends to minimize prison sentences for himself/herself:

-	Stay silent	Confess
Stay silent	(-1,-1)	(0,-4)
Confess	(-4,0)	(-3,-3)

Table 2.1: Prisoner's Dilemma payoffs

Definition 2.5.2 ***Nash Equilibrium:*** *When the game is played by the players, each player forms a belief about other players' actions. This belief is under the assumption that each player plays rationally. Playing based on these beliefs would conclude in achieving a* Nash Equilibrium. *If an equilibrium profile is played, nobody has an incentive to deviate from his/her action and an action profile is not equilibrium if someone has incentive to deviate from the action profile.*

Definition 2.5.3 ***Best Response****: Let the list of all the players' actions except i be denoted by* $\mathbf{a}_{-i}$*, then the set of player i's best actions is denoted by* $B_i(\mathbf{a}_{-i})$ *and is defined as follows:*

$$B_i(\mathbf{a}_{-i}) = a_i \in A_i : u_i(a_i, \mathbf{a}_{-i}) \geq u_i(a'_i, a_{-i}) \ \ for \ \ all \ \ a'_i \ \ in \ \ A_i$$

We say that $a_i^* \in BR(a_i)$ *iff* $\forall a_i \in A_i : u_i(a_i^*, \mathbf{a}_{-i}) \geq u_i(a_i, a_{-i})$*, where* $\mathbf{a}_{-i} = (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n)$ *and* n *is the number of the players.*

The *minmax value* for player i is defined as $v_i = \min_{\mathbf{a}_{-i} \in A_{-i}} \max_{a_i \in A_i} u_i(\mathbf{a}_{-i}, a_i)$. We say that players of the game are playing with *pure strategy* when they play only one action with positive probability. Now we can define the *pure strategy Nash equilibrium*.

Definition 2.5.4 ***Pure strategy Nash equilibrium:*** $\mathbf{a} = (a_1, ..., a_n)$ *is a pure strategy Nash equilibrium iff* $\forall i, a_i \in BR(a_{-i})$

In the *prisoner's dilemma* example the best response for each of the players regardless of the other player's action is to "confess". Thus, the pure strategy Nash equilibrium of this game is *(Confess, Confess)*.

2.5.2 Mixed Strategy Equilibrium

There are many games that do not have a pure strategy Nash equilibrium. Another kind of equilibrium, called a *mixed strategy Nash Equilibrium*, is defined for these games. A mixed strategy of a player in a strategic game means that more than one action is played with positive probability by the player. The set of actions with non-zero probability form the support of a mixed strategy.

Suppose the set of all strategies for i is S_i, and the set of all strategy profiles is $S = S_1 \times S_2 \times ... \times S_n$. If all players follow a mixed strategy $\mathbf{s} \in \mathbf{S}$, then their payoffs are:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

And

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

Definition 2.5.5 ***Mixed strategy Nash equilibrium:*** *Suppose* $\mathbf{s}$ *is the action profile of* n *players playing mixed strategies in a strategic game.* $\mathbf{s} = (s_1, s_2, ..., s_n)$ *is a Nash Equilibrium if* $\forall i, s_i \in BR(\mathbf{s_{-i}})$

One of the most well-known games in game theory is *"matching pennies"*. This game is an example of a strictly competitive game. In the matching pennies game, one player wants to match the other player's choice and the other player wants to mismatch. Table 2.2 shows the player's payoffs in this game:

-	Head	Tail
Head	(1,-1)	(-1,1)
Tail	(-1,1)	(1,-1)

Table 2.2: Matching Pennies payoffs

The matching pennies game is called a zero-sum game because for any action profile $\mathbf{a}$ we have $u_1(\mathbf{a}) + u_2(\mathbf{a}) = 0$. From the table one can see that there is no pure strategy Nash equilibrium for matching pennies.

Theorem 2.5.1 *[Nas51] Every strategic finite game, in which players have finitely many actions has a mixed strategy Nash equilibrium.*

Example 2.5.1 ***(Matching pennies Nash equilibrium)****: Suppose the first player chooses Head or Tail with corresponding probabilities p and $1-p$. This player must randomize to make player 2 indifferent in his/her choices, so that he/she will not have incentive to change his/her strategy. Thus:*

$$u_2(Head) = u_2(Tail),$$

$$-p + 1(1-p) = p - (1-p) \Rightarrow p = \frac{1}{2}.$$

With a similar argument, we conclude the mixed strategy Nash equilibrium for the second player is also choosing *Head* or *Tail* with probability $\frac{1}{2}$. This means the best strategy for both players in the matching pennies game is to play completely randomly.

2.5.3 Repeated Games

In game theory, when the same set of the players play a given game multiple times, the game is called a *repeated game*, and the game that is repeated is called the *stage game*.

When a game is repeated for finitely many times it can be represented by a *game tree* (in the sense of graph theory) in which each node represents the choice of one of the players, each edge represents a possible action, and the leaves represent final outcomes over which each player has a utility value. This form of representing payoffs of a game is known as the *extensive form*. Suppose the prisoner's dilemma game in Table 2.1 is played twice and the payoff function of each player is additive.

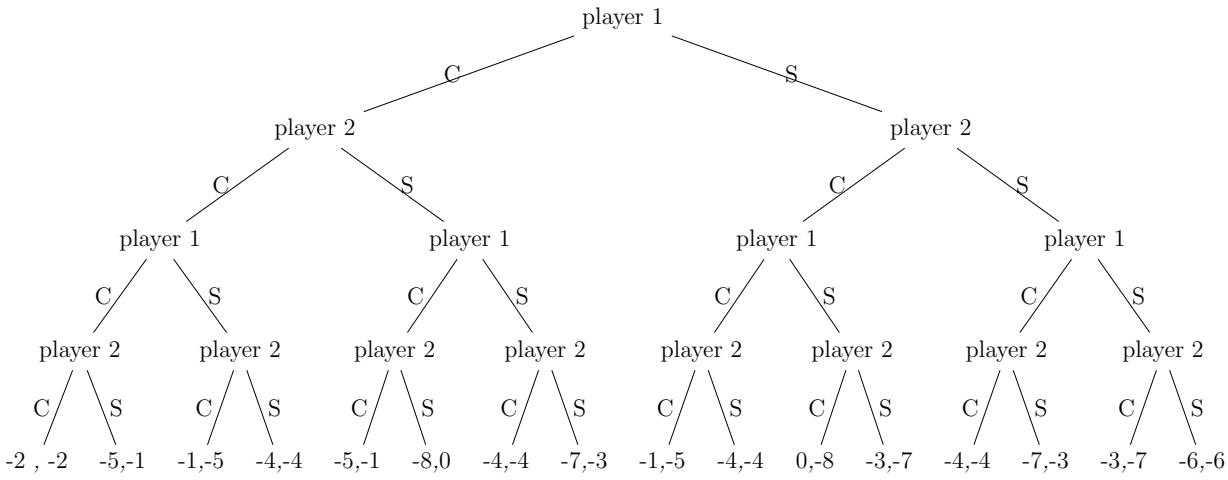


Figure 2.1: Two stage prisoner's dilemma extensive form ([BS08])

When a game is infinitely repeated, the extensive form representation of the game is an infinite tree, and the payoffs would be infinite if we define them as the sum of the payoffs in each stage-game. There are two meaningful ways to define a finite payoff for an infinitely repeated game: the *average reward*, which is the average of the infinite sequence of payoffs for player i, and the *discounted reward*, which is the sum of player i's payoff in the immediate stage-game, plus the sum of future rewards discounted by a constant factor.

A payoff profile $\mathbf{r} = (r_1, r_2, ..., r_n)$ in an n-player game is *enforceable* when each player's payoff is bigger or equal to its corresponding *minmax* value, and is *feasible* if it is a convex, rational combination of the outcomes in the game.

Folk theorems are a family of theorems about possible Nash equilibrium payoff profiles in a repeated game. An instance of a folk theorem family for an infinitely repeated game with an average reward is as follows:

Theorem 2.5.2 ***Folk Theorem****[BS08] Consider an n-player infinitely repeated game with average reward payoff profile $\mathbf{r} = (r_1, r_2, ..., r_n)$.*

1. *If $\mathbf{r}$ is the payoff profile for any Nash equilibriums of the game, then for each player i, r_i is enforceable.*
2. *If $\mathbf{r}$ is both feasible and enforceable, then $\mathbf{r}$ is the payoff profile for some Nash equilibrium of the game.*

2.6 Related Works

User inputs are widely used to provide background entropy for random number generators in computer systems. For example, in Linux based systems the operating system continuously runs a background process to collect entropy from users' inputs [GPR06]. However, due to repetitive patterns of mouse movements or keystrokes, these entropy sources, in general, provide lower entropy level when used for on-demand collection of entropy.

Psychological studies show that humans are neither capable of generating randomness nor recognizing it [Bru97, Wag72]. Nevertheless, some psychological experiments confirm that humans' random behaviour will improve by trial and error. In other words, a human asked to continuously generate random sequences will do better by receiving feedback on the randomness of previously generated sequences [Neu86].

Some competitive two-player games are capable of leading a human toward more random behaviours. In such games, the opponent would exploit non-random behaviour of the human player to win the game. Thus, each of the players tries to play as randomly as possible to win. Fortunately, such games are very well studied in the "game theory" context. In particular, it has been shown that in some competitive zero-sum games the best strategy for winning is to play uniformly randomly. The idea of using such games for leading a human toward random behaviour and consequently generate randomness was initially proposed by Rapoport and Budescu [RB92]. Their experimental results showed that it is possible to generate a sequence with relatively good randomness properties from a series of human actions in a two-player competitive zero-sum game with uniform choices as the best strategy of players. They used the*matching pennies* game as an example of the desired game in their study. In the matching pennies game, each of the players makes a choice between "head" and "tail". One of the players wins the game in case of matching choices while the other one wins if the choices mismatch. Due to the game theory, the optimal strategy for players in this game is to choose uniformly randomly (with probability $\frac{1}{2}$) between head and tail. The experimental results in [RB92] showed that players almost followed a uniform random strategy for playing the game. This result confirms that humans engaged in a strategic game can be a good source of entropy and entropy generation could be an indirect result of their actions. Indeed, one can view the two-player competitive game as an approach to provide feedback for players on their previous actions. In this sense, the later experimental results confirm the psychological experiments that emphasise the role of feedback in training human for randomness generation.

Although the approach for generating randomness from human gameplay in [RB92] is important from a theoretical point of view, it can not be considered as a practical framework since in each stage of the game the players have two possible choices

which means in each stage they can at most generate one random bit. Considering that modern cryptographic keys are relatively long, e.g. at least 128 bits for AES key, by using the above approach one needs to play the matching pennies game at least 128 times in order to generate an appropriate key for a cryptographic purpose. Playing the game so many times on one hand is boring for the player and on the other hand is not efficient since the randomness generation process is very slow.

Halprin et al. by using the above studies, proposed a framework for extracting randomness from human gameplay against computer [HN09]. They designed an extension of the matching pennies game named "hide and seek", in which the player is provided with an $n \times m$ matrix displayed on the computer screen and is asked to choose a matrix location. The player wins if his/her choice is the same as the computer player's choice. It turns out that this game itself is a zero-sum game with a uniform optimal strategy for the players. Theoretically, by taking the optimal strategy, a human player would be able to generate $\log n + \log m$ random bits in each stage of the game. Nevertheless, visual implementation of the game showed that players' choices are biased as they tend to avoid choosing ending points. To overcome the problem of non-uniformity of choices, an explicit t-resilient extractor from [BST03] is used. Players' choices in the designed game are given to the seeded extractor as an entropy source to generate a sequence that is ϵ-close to a uniform distribution. The output of the extractor is then used as an input for the refresh() function in Barak-Halevi's robust PRNG [BH05]. Figure 2.2 shows the construction of a robust PRNG based on human gameplay.

Barak and Halevi's robust PRNG is constructed from any cryptographically secure PRNG $G : \{0,1\}^m \rightarrow \{0,1\}^{2m}$. The system has an internal state $State_i \in \{0,1\}^m$. On request for next(), the system runs G on the internal state $State_i$. One half of the result is returned as the output of the PRNG, and the other half becomes the new

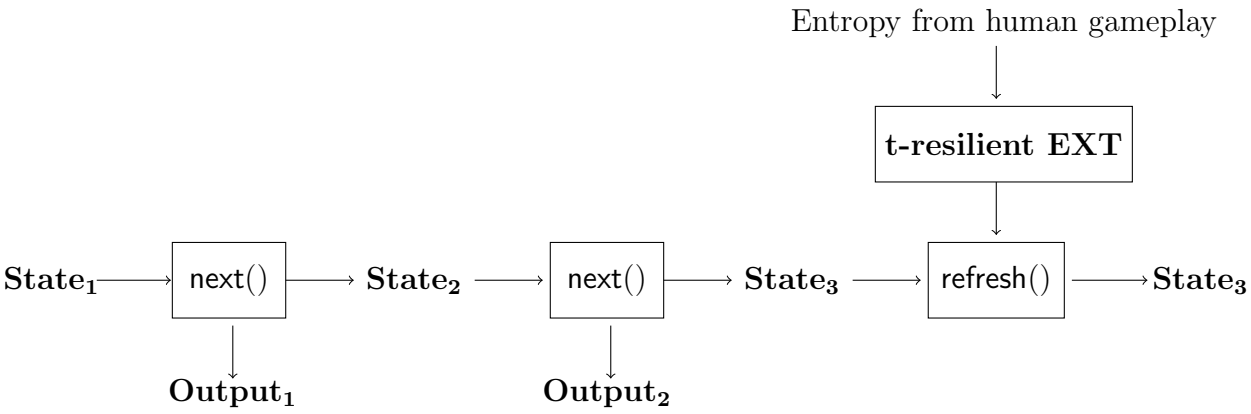


Figure 2.2: Robust PRNG with human provided entropy

state of the system $State_{i+1}$. When refresh() is requested, the system takes m bits of the extracted random sequence from the entropy source and applies the XOR on the current state with it to obtain the new state.

Indirect approaches for generating randomness from human gameplay are considered in [Dey14] and [ASN14]. In particular Dey [Dey14] uses the randomness produced in the video game by the interaction of the player (e.g. Super Mario) with game elements for the purpose of obfuscation. In [ASN14] a video game is designed to indirectly collect entropy from human errors in the gameplay of a user against a computer.

Chapter 3

Randomness Generation

In this chapter, the different methods for randomness generation are introduced. Random number generators (RNGs) are classified and an overview of practical RNGs as well as theoretical functions and their explicit constructions that can extract randomness from an entropy source is conducted. To answer the question, "Is a human able to generate randomness?", a brief survey on human psychology is provided, and some hypotheses and experiments that aim to explain the random behaviour of a human are mentioned.

3.1 Randomness Importance

Randomness plays an important role in computer science, in particular in cryptography and information security. The security of any cryptographic algorithm and protocol directly depends on the randomness of the key. Indeed, it is impossible to have a deterministic fixed secret. Randomness can be exploited for generating the required cryptographic keys in encryption or challenge-response protocols (e.g. the key in SSH or SSL/TLS connections), or for randomizing cryptographic algorithms to achieve a desirable level of security. The unpredictability of a random sequence is its pivotal property that makes it suitable for the mentioned applications. Using weak randomness in security systems may result in a security breakdown of the whole system.

A weak random number generator exposes the system to a variety of attacks. Several attacks on Netscape browser Version 1.1 are addressed in [GW96]. These attacks exploit weakness in the system's RNG to guess the secret key, and break the

security of the system. Kerberos 4 session keys, generated for a DES block cipher, are shown to have much smaller entropy than what is required for their security [DLS97]. This means that the brute force attack can break the security of the system with much fewer trials than what is expected (in particular when time and the host-id of Kerberos server are known, the number of possible keys is only 2^{20}, while DES offers 2^{56} possible keys for providing the security of the encrypted block). More recent studies observed weak-key-generation vulnerability in the Debian Linux version of OpenSSL [YRS+09] and a level of predictability in RSA public keys [LHA+12] due to a bug in the random number generation algorithm.

The importance of having a good random number generator is well recognized from the above examples. Indeed, a huge part of a system's security depends on the quality of randomness, and thus improving the random number generators as well as introducing appropriate applicable entropy sources is of crucial importance for enhancing the security level of cryptographic systems.

3.1.1 Random Number Generators

An RNG is an algorithm that could be implemented by a computer program, and is intended to have an output with desirable randomness properties. There are mainly two different types of RNGs: True Random Number Generators (TRNGs) and Pseudo Random Number Generators (PRNGs).

3.1.1.1 True Random Number Generators

A TRNG (also referred to as an *entropy harvester* in the literature [VM03]) applies a processing function to an entropy source to generate (extract) randomness. The entropy source is obtained from a non-deterministic physical quantity such as noise in an electrical circuit or mouse positions during a user's specific interactions with a computer program. Although TRNGs are presumably secure under most circum-

stances, their speed is limited by the speed of the physical phenomenon, and thus they are usually slow. For increasing the throughput of random number generation, the output of a TRNG might be fed into a PRNG.

Quantum TRNGs, on the other hand, are very fast, but using them requires special hardware and resources. This limits their application to very restricted and critical security purposes.

3.1.1.2 Pseudo Random Number Generators

A PRNG is an efficient deterministic algorithm that expands a short perfectly random (or almost perfectly random) seed to a much longer sequence that "looks random" in a meaningful and well-defined way [Gol10, RSN+01]. This usually means that no efficient algorithm can distinguish the output of the CSPRNG (Cryptographic Secure Pseudo Random Number Generator) from a perfectly random sequence.

There are two kinds of PRNGs: Normal Pseudo Random Number Generators (NPRNGs) and Cryptographic Secure Pseudo Random Number Generators (CSPRNGs).

An NPRNG deterministically produces a sequence of values depending only on the initial seed (also known as initial internal state). Since there are only a limited number of states, the output of an NPRNG eventually repeats itself and becomes periodic. If the initial seed is an n-bit vector, the maximum period for the NPRNG is 2^n [VGLS12], however some NPRNGs such as a Linear Feedback Shift Register (LFSR) might have an even smaller period. Nevertheless, with a long enough period the output random sequence could be appropriate for practical applications.

A simple example of an NPRNG is the Linear Congruential Generator (LCG) [PM88]. The generator function of a LCG is:

$$X_{n+1} = a.X_n + b \quad mod \;\; m$$

Where a, b, and m are chosen constant integers and $a, b < m$. X_0 is the initial seed and the sequence of $(X_n)_{n\geq 1}$s is the generated pseudo random sequence.

An LFSR (Figure 3.1) is another important example of an NPRNG, which is a shift register whose input bit is a linear function of its previous state. The initial value of the LFSR is its seed and its input is a linear function of some bits of the shift register value. Since the operation of the register is deterministic, the stream of values produced by the register is completely determined by its current state, and because the register has a finite number of possible states, it ultimately enters a repeating cycle. However, with an appropriate feedback function, one can obtain a sequence of bits which appears random with a long period.

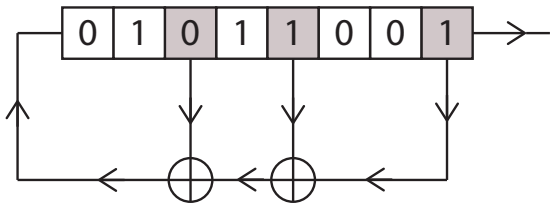


Figure 3.1: An 8-bit LFSR
([Kle13])

The most commonly used linear function of single bits for an LFSR is exclusive-or (XOR). Thus, an LFSR is most often a shift register whose input bit is derived from the XOR of some bits of the overall shift register. The bits in the LFSR state that influence the input are called taps. A maximum-length LFSR with n bits output, cycles through all possible $2^n - 1$ states within the shift register except the state where all bits are zero [Kle13].

A CSPRNG is a PRNG that satisfies cryptographic conditions besides statistical conditions [VGLS12]. Although, CSPRNGs are still predictable from their internal state, they do not quickly reveal significant information about their internal state when seeded with sufficient entropy. An example of a CSPRNG is the CryptMT3 random number generator [MSNH08].

3.2 Statistical Tests

Generally, the quality of produced random numbers is evaluated by statistical tests; this means that for making any statement about the quality of an RNG, we should specify the tests this statement is based on. NIST has developed a package of 15 statistical tests to evaluate the randomness of (arbitrarily long) binary sequences. Different types of predictability in a random sequence are considered in the NIST statistical package [RSN+01]. The 15 tests included in the NIST statistical test suite are: The Frequency Test, Frequency Test within a Block, The Runs Test, Tests for the Longest-Run-of-Ones in a Block, The Binary Matrix Rank Test, The Discrete Fourier Transform (Spectral) Test, The Non-overlapping Template Matching Test, The Overlapping Template Matching Test, Maurer's "Universal Statistical" Test, The Linear Complexity Test, The Serial Test, The Approximate Entropy Test, The Cumulative Sums (Cusums) Test, The Random Excursions Test, and The Random Excursions Variant Test.

Note that some desirable properties for random numbers can only be checked by non-polynomial-time tests, such as the *spectral test*. Thus, it is not sufficient to limit the definition of PRNGs to those RNGs whose output could not be distinguished from uniform sequences [Röc05] using polynomial-time algorithms.

3.3 Human Psychology for Generating Randomness

Random number generation by human subjects is a procedurally-simple task which is related to specific executive functions in the brain such as updating and monitoring of information and inhibition of automatic responses. There is a relatively large body of literature on human random number generation in psychology. The failure of human subjects to behave randomly is a robust finding (for reviews see [Wag72] and [Bru97]). The experiments for evaluating human random behaviour vary from calling

out digits, letters of the alphabet, or nonsense syllables, to writing these same symbols on paper, pressing push-buttons, touching metal disks with a stylus, or drawing lines on a paper. The effect of different experimental conditions such as required speed of responses, age, mathematical sophistication of subjects, and competing attentional demands have systematically been extensively studied and various statistical tests have been used for evaluating the generated randomness [TN98].

There are two different explanations for human randomness generation ability in the psychology community: explanation by *trait* and explanation by *skill*.

3.3.1 Explanation by Trait for Humans' Random Behaviour

In a random sequence, each symbol is approximately equally repeated over a long run. An explanation by *trait* claims that humans are incapable of generating random sequences due to their inherent limitations. According to one hypothesis, humans fail in generating randomness because their memory is not able to keep track of all the generated symbols [Bad66]. According to another hypothesis, attentional processes do not permit subjects to completely ignore their previous responses while it is essential for random behaviour [Wei64]. The third hypothesis highlights humans' difficulty in distinguishing randomness (when presented with two series of numbers, subjects usually cannot discriminate random from non-random series) as the trait limitation that prevents randomness generation by a human [Wag70].

3.3.2 Explanation by Skill for Humans' Random Behaviour

Due to this explanation, "Randomlike behaviours are learned and controlled by environmental feedback, as are other highly skilled activities" [Neu86]. If so, random behaviour skill should be trained and practiced to be improved. Indeed, humans can learn how to generate randomness during a process. The role of feedback in improving generated randomness by humans is considered in [Neu86], and the result confirms

that humans generates better random sequences when feedback on their previous actions is provided.

More recent studies showed that random number generation involves several mental processes. In fact, it requires adopting the correct strategy, based on instructions and the subject's concept of randomness, monitoring the output, and eventually modifying the strategy of randomness generation. Generating random numbers requires the activation of different brain regions consisting of the Left Dorsolateral Prefrontal Cortex (DLPFC), the Anterior Cingulate Cortex (ACC), the Superior Parietal Cortex (SPC), the Right Inferior Frontal Cortex (RIFC) and the Cerebellar Hemispheres (CH) [CCR$^+$14]. The functionality of each part is briefly explained below [MC07].

-*The Left Dorsolateral Prefrontal Cortex (DLPFC)* is an area in the prefrontal cortex of the human brain that is "known to be involved in classical executive functions, includng working memory, set shifting, sequencing, planing, inhibition and abstract reasoning" [MC07, P. 355].

-*The Anterior Cingulate Cortex (ACC)* plays an important role as an integrative center in lots of the body's autonomic functions as well as controlling emotions. It is also involved in cognitive behavioural tasks, such as reward anticipation, decision-making and attention motivation.

-*The Superior Parietal Cortex (SPC)* is involved with divided attention and receives a great deal of visual input as well as sensory input from one's hands.

-*The Right Inferior Frontal Cortex (RIFC)* mediates response inhibition. It is typically implicated in go/no-go tasks.

-*The Cerebellar Hemispheres (CH)* play a significant role in some of the motor and perceptual tasks like spatial attention, verbal working memory, and language processing and in regulating fear and pleasure responses.

Transcranial Direct Current Stimulation (TDCS) is a safe and reliable technique

that can produce transient behavioural changes and influence cognitive functions. In this method, a weak current is applied constantly over time to increase (anodal stimulation) or decrease (cathodal stimulation) the excitability of the neuronal populations underlying the active electrode.

TDCS can produce transient behavioural changes and influence cognitive functions. When TDCS is combined with cognitive training protocols, seemingly it can enhance the protocol effects [LMF12].

The experiments in [CCR+14] demonstrate that practice consistently induces training plasticity and influences specific features of humans' performance in randomness generation. The experiments also suggest that TDCS could transiently modify the random behaviour of a human.

3.4 Randomness Extraction

Any function that extracts almost uniform bits from an entropy source is called a *randomness extractor*. Randomness extractors require a guarantee on the randomness property such as min-entropy of their input entropy source in order to guarantee randomness level of the output. A TRNG thus can be built from a randomness extractor together with an entropy source. There are two different types of extractors: deterministic extractors and seeded extractors. Deterministic extractors, like TRNGs, use a source of imperfect randomness for generating randomness, but do not require a random seed. On the other hand, seeded extractors and PRNGs are similar due to using a random seed for randomness generation. Nevertheless, extractors and PRNGs are totally different objects. The output of the extractor is statistically close to uniform distribution, while the output of a PRNG is computationally indistinguishable from a uniform distribution. Moreover, a source of imperfect randomness is not a required input for a PRNG, while it is necessary for randomness extractors. It

turns out that PRNG constructions of a certain kind are extractors. The connection between extractors and PRNGs is demonstrated in [Tre01].

3.4.1 Deterministic extractors

Deterministic extractors are ideal extractors that extract randomness from an entropy source without investing any extra randomness. However, it turns out that having a deterministic extractor as a general imperfect randomness source is impossible. Respectively, the challenge in constructing an optimal extractor is to minimize the amount of required extra randomness known as the seed.

Definition 3.4.1 ***(Deterministic Extractors)*** *For a class of distributions $\mathcal{C}$ over $\{0,1\}^n$, the function $Ext : \{0,1\}^n \rightarrow \{0,1\}^m$ is (n,ϵ)-extractor if for any $X \in \mathcal{C}$, $Ext(X)$ is ϵ-close to U_m, where U_m is the uniform distribution over $\{0,1\}^m$.*

A well-known example of a deterministic extractors are the von Neumann extractor [von63] that extracts perfectly uniform bits from a source generated by a finite-state Markov chain. Suppose $X_1, X_2, ..., X_n$ is an identical independent distributed binary sequence with bias δ so that $Pr[X_i = 1] = \delta$. A Von Neumann extractor breaks all the variables in pairs and for each non-identical pair (01 or 10) outputs the first bit and skips otherwise. With this extractor, after processing $\frac{1}{2\delta(1-\delta)}$ pairs, on average, we will have an unbiased random bit sequence.

The Definition 3.4.1 follows the observation that an extractor is a *single* function that works for all sources in a class. For determining the class of a source, we do not need the exact distribution of each source since only having special properties is sufficient for our purpose. Some examples of specific class of distributions are as follows:

- *Bit fixing source:* A block of n bits consisting of k random and independent bits together with $n - k$ fixed bits.

- *Adaptive bit-fixing sources:* A block of n bits consisting of k independent and random bits and $n - k$ bits that are dependent on the k independent bits.

- *Flat k-source:* Uniform distribution on a subset $S \in \{0,1\}^n$ of size 2^k.

The min-entropy of sources is a general measure that can be used to classify sources.

Definition 3.4.2 ***(k-source)*** *A random variable X is a k-source if $H_\infty(X) \geq k$, i.e. if $Pr[X = x] \leq 2^{-k}$.*

All the above examples of sources are considered k-sources. An appropriate deterministic extractor should work for a wide class of random sources. But it turns out that it is impossible to construct a deterministic extractor that extracts even one random bit from a general k-source. Further studies on deterministic extractors can be found in [SV86].

3.4.2 Seeded Extractors

Although deterministic extractors do not exist for general k-sources, any randomly chosen function is a good extractor for all flat k-sources (and therefore for all k-sources because any k-source is a convex combination of flat k-sources [CG88]) with high probability. One can show this by allowing the extractor to depend on the source and then apply the Chernoff bound (Theorem 2.2.1) to bound the failure probability of the extractor. By allowing extractors to use an additional random input called a *seed*, extraction from general k-sources becomes possible. The additional random input indeed randomizes over the choice of extracting function from a family of extractors. As a result, the seeded extractor is able to extract almost all entropy from all k-sources with high probability.

Definition 3.4.3 ***seeded extractors [NZ96]:*** *A function* $Ext : \{0,1\}^n \times \{0,1\}^d \rightarrow \{0,1\}^m$ *is a* (n, k, ϵ)*-extractor if for every* k*-source* X *on* $\{0,1\}^n$*,* $Ext(X, U_d)$ *is* ϵ*-close to* U_m*. An extractor is explicit if it is polynomial time computable.*

Note that in seeded extractors, since output length is bigger than the seed length, the overall generated randomness is more than the amount of randomness we invest as the seed. The goal of seeded extractors is to produce as many output bits as possible while minimizing the number of random bits used for the seed. The existence of an optimal (n, k, ϵ)-extractor with seed length $d = \log(n-k) + 2\log(\frac{1}{\epsilon}) + O(1)$ and output length $k + d - 2\log(\frac{1}{\epsilon}) - O(1)$ has been shown by using probabilistic methods [RTS00].

Despite an existence proof of extractors, using extractors for randomness generation requires an explicit construction of the extractor. These constructions are mentioned in [Sha04]. One example of the explicit construction of extractors is obtained by using pairwise independent hash functions [HILL99]. The other explicit construction uses *expander graphs* for obtaining extractors. This type of construction will be elaborated in the next section.

Definition 3.4.4 *[BST03] A seeded extractor is called a* t*-resilient extractor if its output is* ε*-close to uniform for* 2^t *pre-determined source distributions with the probability of at least* $1 - \varepsilon$*, when the seed is chosen uniformly random.*

Explicit constructions of t-resilient extractors from ℓ-wise independent Universal Hash Function (UHF) are given in [BST03].

3.5 Expander Graphs for Randomness Extraction

Expander graphs are a family of well-connected graphs with a broad range of applications in theoretical computer science as well as computer networks. They have been

used for designing algorithms, error correcting codes, pseudo random generators and robust computer networks. In particular, they can be used as seeded extractors for randomness extraction from an entropy source.

3.5.1 Measures of expansion

For a formal definition of an *"expander graph"*, we require a quantitative invariant for measuring connectedness of the graph. Minimum number of neighbor vertices for all sub-graphs or minimum number of edges leaving any sub-graphs are two classic measures for well-connectedness [HLW06].

3.5.1.1 Vertex Expansion

This measure requires that every "not-too-large" set of vertices has "many" neighbors:

Definition 3.5.1 ***(Vertex Expander)*** *A graph G is a (K, A)* vertex expander *if for all sets S of at most K vertices, the* neighborhood $N(S) \stackrel{def}{=} \{u|\ \exists v \in S|\ (u, v) \in E\}$ *is of size at least $A \times |S|$.*

For an ideal d-regular expander graph, A is as close to d as possible. $d = O(1)$ and $K = \Omega(n)$, where n is the number of vertices. In the above definition, if instead of $N(S)$ we use the number of edges leaving S, we will have *edge expansion*.

3.5.1.2 Random Walks and Spectral Expansion

Suppose $X = i$ and $i = 1, 2, ..., n$ is an outcome of a random experience, and vector $\mathbf{p}^t = (p_1, p_2, ..., p_n)$ indicates the corresponding probabilities for each of the outcomes such that $Pr(X = i) = p_i$. An undirected d-regular graph G with n vertices labelled from 1 to n, depicts the outcomes of the random experience. Consequently, the probability of picking vertex i is p_i. The edge between vertices i and j is denoted by e_{ij}. Suppose there is probability distribution on d connected edges to each vertex. The

corresponding probability for e_{ij} is depicted by p_{ij}. The corresponding probability for e_{ij} is regardless of starting and ending vertices order that is $p_{ij} = p_{ji}$. An edge is selected due to the outcome of another independent random experience. Starting from vertex i and picking the edge e_{ij} will conclude in reaching to vertex j. This is called *one random walk* on the graph G.

Let $\mathbf{A}$ be the normalized adjacency matrix of graph G. Obviously $\mathbf{A}$ is a symmetric matrix and the sum of the entries on each row or column is exactly one. The product of $\mathbf{A}$ and $\mathbf{p}$, i.e. $\mathbf{q} = \mathbf{Ap}$, gives a vector $\mathbf{q}^t = [q_1, q_2, ..., q_n]$, which is actually the new probability distribution on the vertices of graph G after one random walk. Elements of $\mathbf{q}$ are obtained from the inner product of each row of $\mathbf{A}$ with vector $\mathbf{p}$. Thus we have:

$$q_i = \sum_{j=1}^{n} a_{ij} p_j$$

Since each element of $\mathbf{q}$ has a non-negative value and $\sum_{i=1}^{n} q_i = 1$ (because $\sum_{j=1}^{n} a_{ij} = 1$ and $\sum_{j=1}^{n} p_j = 1$), we conclude that the vector $\mathbf{q}^t = [q_1, q_2, ..., q_n]$ is a probability distribution vector.

Intuitively, well-connectedness of the graph implies that random walks on graph would converge quickly to uniform distribution. The convergence rate of random walks is captured by the second largest eigenvalue of the graph's adjacency matrix. This value is denoted by λ from now on. *Spectral expansion* is the random walk based measure of well-connectedness for the expander graph.

Definition 3.5.2 *For $\gamma \in [0, 1]$, a regular graph G has spectral expansion γ if $\lambda \leq 1 - \gamma$.*

The following lemma shows the exact relation between the convergence rate of random walks and the second largest eigenvalue of the normalized adjacency matrix in an expander graph:

Lemma 3.5.1 *[AB09, Lemma 21.3] Let G be an n-vertex regular graph and $\mathbf{p}$ the initial distribution on the nodes of the graph and $\mathbf{u}$ the vector of uniform distribution. Then we have:*

$$||\mathbf{A}^l\mathbf{p} - \mathbf{u}|| \leq \lambda^l$$

Note that since $\mathbf{A}$ is a stationary matrix, all of its eigenvalues would be less than or equal to 1 [AB09, p. 424]. Lemma 3.5.1 shows that good expander graphs have smaller λ, since with smaller λ, one needs a smaller number of random walks to get close to a uniform distribution.

The relation between spectral expansion, vertex expansion, and edge expansion has been studied in [Tan84], [AM84] and [Che70]. These studies show that any of the expansion measures implies others.

3.5.1.3 Explicit Expander Graphs

Due to the enormous number of applications of expander graphs in computer science, having an explicit construction for them is essential. One simple construction for an expander graph is d-regular graphs. The following theorem shows that d-regular graphs have small λ and consequently are good expanders.

Theorem 3.5.1 *[AB09, Section 21.2.1] For every constant $d \in \mathbb{N}$ any d-regular n-vertex graph G satisfies $\lambda \geq \frac{2\sqrt{d-1}}{d - o(1)}$ where the $o(1)$ term vanishes as $n \to \infty$ and d is constant.*

Ramanujan graphs are the representation of an explicit construction for expander graphs such that $\lambda < \frac{2\sqrt{d-1}}{d}$, which means for *Ramanujan graphs* as $n \to \infty$ we would have $\lambda = \frac{2\sqrt{d-1}}{d}$.

For a fixed d and large n , the d-regular n-vertex Ramanujan graph minimizes λ. Thus Ramanujan graphs are the best expanders. The explicit construction for Ramanujan graphs is given in [LPS86] that uses deep mathematical results to prove

that the construction is a Ramanujan graph. There are other simple explicit constructions of good expander graphs that can be efficiently generated. That is, there exists a polynomial-time algorithm that for vertices indexed by $i \in I$, generates the indexes of each vertex's neighbors. In Chapter 4, a game is designed based on expander graphs. The explicit construction of expander graphs is very critical for realizing the designed game. Here we bring two examples of simple explicit constructions for 5-regular and 3-regular expander graphs. The designed game is realized based on these two graphs in Chapter 4.

Construction 3.5.1 ***(discrete torus expanders)*** *[GG81] Let G be a graph with vertex set $V = \mathbb{Z}_M \times \mathbb{Z}_M$ and edges from each node (x, y) to the nodes (x, y), $(x+1, y)$, $(x, y+1)$, $(x, x+y)$ and $(-y, x)$ where all arithmetic is modulo M. This graph is a good 5-regular graph.*

Construction 3.5.2 ***(p-cycle with inverse chords)*** *[Lub10] For a prime number p, let $V = \mathbb{Z}_p$ be the set of vertices in graph G. The vertices are labelled by $x \in \{0, p-1\}$. Neighbor vertices have indexes $x-1$, $x+1$ and x^{-1} where all arithmetic is mod p and 0^{-1} is defined to be 0. This is a construction of a good 3-regular expander graph.*

Other explicit constructions of expander graphs use graph product techniques such as the Zig-Zag product and replacement product [RVW00].

3.5.2 Expander Graphs as Extractors

Lemma 3.5.1 yields that a random walk on an expander graph can be interpreted as an extractor. In particular we have the following extractor construction from random walks on an expander graph.

Lemma 3.5.2 *[AB09, lemma 21.27] For $0 < \epsilon \leq 1$ and every n and $k \leq n$, there exists an explicit (k, ϵ)-extractor $Ext : \{0,1\}^n \times \{0,1\}^t \to \{0,1\}^n$ where $t = O(n -$*

$k + log\frac{1}{\epsilon})$.

The above lemma assumes an expander graph with $\lambda = \frac{1}{2}$. However, in general for an arbitrary λ and min-entropy k, we have:

Theorem 3.5.2 *[ASNS13] Let G be a 2^n-vertex d-regular expander graph with normalized adjacency matrix $\mathbf{A}$ and X be a k-source with probability distribution vector $\mathbf{p}$ over $\{0,1\}^n$. For a random walk of length ℓ over the graph starting from a vertex selected according to distribution $\mathbf{p}$, we have:*

$$SD(\mathbf{A}^\ell \mathbf{p}, U_n) \leq \frac{1}{2}\lambda^\ell \sqrt{n}(2^{-k/2} + 2^{-n/2})$$

Proof:

$$\begin{aligned} SD(\mathbf{A}^\ell \mathbf{p}, U_n) &= \frac{1}{2} \sum_{a \in \{0,1\}^n} |Pr[\mathbf{A}^\ell \mathbf{p} = a] - Pr[U_n = a]| && (3.1) \\ &\leq \frac{1}{2}\sqrt{n}\|\mathbf{A}^\ell \mathbf{p} - U_n\|_2 && (3.2) \\ &\leq \frac{1}{2}\sqrt{n}\lambda^\ell \|\mathbf{p} - U_n\|_2 && (3.3) \\ &\leq \frac{1}{2}\sqrt{n}\lambda^\ell (2^{-k/2} + 2^{-n/2}) && (3.4) \end{aligned}$$

Equation 3.1 is the definition of statistical distance. Inequality 3.2 follows from Lemma 2.1.1. Inequality 3.3 is indicated in the proof of Lemma 3.5.2 in [AB09] and Inequality 3.4 is because $\|\mathbf{p} - U_n\|_2 \leq \sqrt{\|\mathbf{p}\|_2^2 + \|U_n\|_2^2}$ and $\|\mathbf{p}\|_2^2 \leq 2^{-k}$ (since X with probability distribution $\mathbf{p}$ is a k-source).

Note that since there are explicit constructions for expander graphs, Theorem 3.5.2 introduces a family of explicit constructions for randomness extractors.

Chapter 4

True Random Number Generator from Human Gameplay

In this chapter, an integrated approach for true randomness generation from human gameplay is proposed. Game theory is used for designing the game. We show that the game design is adjustable to several popular existing games, such as Roshambo. The proposed TRNG is compared with previous existing RNGs that use human gameplay, and the advantages of the proposed approach are described.

4.1 Introduction

Every RNG (even PRNG) needs at least one source of randomness to produce random numbers. The challenge is where to find randomness in a deterministic device, such as a computer. The commonly applicable entropy sources in a computing device include noise in electrical circuits, the current system time or user input (such as mouse movements or time intervals between keystrokes). Any of these sources has their advantages and drawbacks. System time is probably the worst source of randomness among the above mentioned sources. This is because, if the execution time of the RNG is almost known, the time as a source of randomness will only contain a few bits of entropy [RSN+01]. This was the biggest problem in the implementation of the Netscape browser v1.1 [GW96] and Kerberos V4 [DLS97]. Gathering entropy from system-based noise sources often requires extra hardware. Moreover, such noise sources are not always available. User based entropy sources, on the other hand, are an attractive alternative entropy source, which is easily available while requiring no

extra hardware for exploiting their entropy.

Using the input of a computer game is a tempting idea for providing a user based entropy source. The user willingly plays the game as much as required, while producing entropy for a random number generator. Note that not just any game is appropriate for this purpose. The desired game should lead the human player to generate as much randomness as possible while staying interesting for the user.

4.2 The Contribution: True Random Number Generator from Human Gameplay

A TRNG that uses human gameplay against a computer as the only source of randomness is proposed. This TRNG is a randomness extractor that uses human input as the initial entropy source and the random seed. The user can improve the randomness of the output at any time and to any level of closeness to a uniform distribution, simply by playing the game for more rounds. Thus, the final results have a guaranteed and adjustable level of randomness.

In comparison, in the construction of Halprin et al. [HN09], (i) although the entropy source is based on human input, a second external source of perfect randomness is required to provide a seed for the extractor, and (ii) the size and quality of the final output depends on the extractor that is used after obtaining the output of the entropy source. Therefore, changing the quality of the final output requires replacing the extractor and performing the calculations from the beginning.

4.2.1 Applications

As mentioned before, RNGs are essential components for security systems. User-based TRNGs add an extra level of assurance about the randomness source since the TRNGs exploit randomness from the user's input. An example application of

this approach is generating random cryptographic keys for software such as PGP, OpenSSL, and GnuPG. These applications rely on the random number generation of the OS, which collects its required entropy from a physical entropy source. However, enough entropy for randomness extraction may not be available at the time of the request. In this case, the proposed approach can be used to generate the required randomness from users by asking them to play a simple game. According to the conducted experiments in this thesis, for generating a 64 bit key that is 2^{-18}-close to uniform distribution, the user should choose 8 locations on a presented circle (with 256 distinct points) and then play Roshambo for 15 times against the computer.

Another application of the proposed approach can be in virtual environments in which multiple users share some hardware such as CPU and RAM. In such environments using the shared hardware as an entropy source for the system's RNG is risky due to the dependence between entropy pools of different users, especially if the RNG is used for generating a cryptographic key of a security protocol. In these scenarios using a human player's gameplay for generating randomness is much more reliable, since the generated randomness would be independent of the underlying shared hardware and the other users' randomness.

4.2.2 The TRNG's Structure

The proposed TRNG, similar to any other TRNGs, consists of two main modules: (i) An *entropy source* that uses a physical process to generate a sequence of symbols with a lower bound on its min-entropy and (ii) a *seeded randomness extractor* that transforms the entropy source into an almost perfect random sequence using the extra randomness provided by the seed.

In the proposed approach, the gameplay between a human player and a computer player is used to realize the two modules of a TRNG. The required entropy for the proposed construction is provided by the user's initial choices among the vertices of

an expander graph, while the expander graph is used as a randomness extractor and the user's choices in the subsequent stage-games provide the seed for the randomness extractor.

4.2.2.1 Fitting a Two-player Game to a TRNG

There is a d-regular expander graph at the heart of the proposed construction. The game consists of a sequence of stage-games. At the first stage-game, the whole expander graph is presented to the user who will be asked to choose a vertex. The number of vertices of the graph is determined by the length of the required random sequence, such that if n random bits are required, the number of vertices is 2^n. The human's choice is effectively a symbol of an entropy source with some unknown distribution that is not uniform but has some min-entropy.

A subsequent random walk of length ℓ over the graph will be used to obtain an output of the TRNG, which is desirably close to uniform randomness. On each vertex of the graph, the user is provided with a simple game which effectively requires him/her to choose among the set of neighboring vertices. The presented game at this stage is a zero-sum game with uniform optimal strategy. The required randomness to generate random walks is provided from human choices in this game.

One of the players is simulated with the computer but note that it is possible to consider two human players playing the game as well. The computer player's choices would not affect the order of the rounds, but is effective in gaining or losing scores by the other player. In fact, it is the computer player's strategy, besides the appropriate design of the payoffs in stage-games, that leads the human player to play randomly at each stage. The number of required random walks to achieve a specific level of randomness, i.e. ϵ-closeness to a uniform distribution, is determined by a desired ϵ and the min-entropy of the initial vertex selection. Consequently the designed game consists of two parts:

1. *Initial stage-game:*

   The initial stage-game is an extension of the matching pennies game. The user chooses a vertex from the set of 2^n possible vertices in the graph, and wins if the computer cannot "guess" his/her choice.

2. *Walk stage-games:*

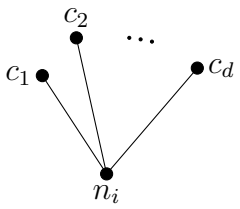
   Each subsequent stage-game corresponds to a single step of the random walk. At vertex V, the player is presented with a zero-sum game with d possible actions. The game payoffs are designed in such a way that a uniform choice between the presented actions is the optimal strategy, and so the user's input would correspond to uniform selection among the possible actions. Each of the d actions corresponds to one of the neighbouring vertices of V. Thus, the user's random choice in the zero-sum game is used to generate a random walk on the expander graph. Using a d-regular graph ensures that the number of choices at every vertex is the same.

The main difference between the initial and the subsequent stage-games is the number of choices available to the user and the way these choices are used.

Suppose n_i represents the corresponding vertex of a player's choice at the i^{th} walk stage-game and c_1 to c_d represent the neighbouring vertices of n_i. Figure 4.1 shows how player choices in walk stage-games are mapped to a random walk.

4.2.2.2 Choosing the expander graph

The initial step toward constructing the proposed TRNG is to choose an appropriate d-regular expander graph. The number of vertices is determined by the length of the required random string. An expander graph with 2^n vertices outputs n bits that are



Summary of the walk stage-games:

- n_i: The choice of the player in the previous round that we interpret as the player's current state.
- $c_1 - c_d$: d available actions for the human player in round i.
- c_i: The choice of the player in round i.
- $n_{i+1} = c_i$.

Figure 4.1: The graph representation of i^{th} stage-game

ϵ-close to uniform distribution. After selecting the graph, each vertex is labeled with a binary string of length n. The two parameters n and d are directly related to the computational efficiency of the system in generating random bits: larger n means a longer output string and more random bits, and larger d means faster convergence to the uniform distribution and thus a shorter walk to reach the same level of closeness to the uniform distribution (see Theorem 3.5.1). However, there are some considerations in choosing these parameters: first of all, because the graph is the basis of the game's visual presentation to the user, one needs to consider the usability of the system and so the choice of n and d should take this factor into account. In Section 4.2.3.1, a solution for overcoming the visualization restrictions is suggested. Nevertheless, storage limitations of the computer may cause other constraints on choosing these parameters as storing a graph construction with a large number of vertices on a computer requires a relatively large amount of available memory on the computer.

The other essential requirement for the proposed TRNG is that the steps of the

random walk must correspond to an independent and uniformly distributed random variable. Experimental results in [HN09, Wag72] show that bias will increase in the human player's choices for larger sets of possible choices as human beings naturally tend to avoid picking end points. This implies a restriction on choosing relatively large values for d.

Example 4.2.1 *Suppose a sequence of* 64 *random bits that is* 2^{-18}*-close to a uniform sequence is required for a real world application (e.g. to construct a cryptographic key). Using the proposed TRNG requires constructing a d-regular expander graph with* 2^{64} *vertices. According to Theorem 3.5.1, there exists an explicit construction for expander graphs named Ramanujan graphs that, for large amounts of N (number of vertices), have a second largest eigenvalue, that is approximately equal to* $\lambda \approx \frac{2\sqrt{d-1}}{d}$*. To find the required number of random walks for obtaining the desirable random sequence, we need an estimate for the min-entropy of the user's initial choices. Assume the amount of existing min-entropy is 0.6 per bit (38 bits in 64 bits) that is the estimated min-entropy of human players in [HN09]. For the choice of* $d = 3$, $\lambda \approx 0.92$*. In this case, for obtaining 64 bits that are* 2^{-18}*-close to uniform distribution:*

$$2^{-19} \leq \frac{1}{2} 0.92^{\ell} \sqrt{64} (2^{-19.2} + 2^{-32})$$

$$\Rightarrow \ell \geq 14.3 \Rightarrow \ell = 15$$

Table 4.1 lists the number of required random walks for different amounts of d to get 64 random bits that are 2^{-18} close to uniform distribution. As the table shows, the number of required rounds significantly decreases as d grows.

4.2.3 Game Design

A two-player game based on the described approach is designed. For the game design, a Petersen graph is used, which is a 3-regular graph with n vertices. The second

d	λ	ℓ
3	0.92	15
5	0.8	7
7	0.7	4
10	0.6	3

Table 4.1: Number of required random walks (ℓ) for different choices of d

largest eigenvalue of the adjacency matrix of this graph is 0.94. In the designed game, initially, the graph is presented to the human player and the player chooses a vertex out of 10 vertices. In the corresponding stage-game, the player is presented with neighboring vertices and is asked to choose a vertex from neighboring vertices. Both the initial stage-game and walk stage-games are an extension of the matching pennies game. The human player plays against the computer player, and he/she wins the game if his/her choice does not match the computer's choice. Behavioural strategy for simulating the computer's player actions is used, that is, at each stage of the game the computer player plays its optimal strategy. The optimal strategy is choosing an action uniformly. Thus, a PRNG is used for determining the computer player's actions.

4.2.3.1 Visualizing large graphs

The number of required random bits for real world applications is relatively large, e.g. 64 bits. However, the proposed game will generate at most $\log 10 \approx 3.3$ random bits. One way to generate the appropriate sequence is to iterate the game (consisting of initial stage-game and walk stage-games) until 64 bits are generated. This method, however, is not practical as in this example one needs to play the game roughly 21 times, which is neither efficient nor interesting. Instead, for generating long random sequences of length n, it is preferred to start with a large graph having 2^n vertices and ask the human player to choose a vertex. In this case, the player only needs to play a few walk stage-games for generating the appropriate random sequence. Nevertheless,

with the proposed approach for game design, visualizing the whole graph on the screen would be a challenge.

For solving the visualization problem for generating sequences of length n, Construction 3.5.2 (*p-cycle with inverse chords*) is used for the expander graph where p is the smallest prime number smaller than 2^n. All the vertices of the initial graph are presented as 2-dimensional points on the screen labeled from 0 to $p-1$. In the initial stage-game, the player is just asked to pick a location on the screen. This choice is mapped to one of the vertices of the original graph. Subsequent walk stage-games are just a simple zero-sum game, such as Roshambo with 3 possible actions for players. Since the neighbour vertices of vertex x in (*p-cycle with inverse chords*) expander graphs are $x-1$, $x+1$ and x^{-1}, each of the player's choices is mapped to one of these vertices. For example:

$$\begin{array}{cc} \text{Rock} & x-1 \\ \text{Paper} & x+1 \\ \text{Scissors} & x^{-1} \end{array}$$

The final output is then calculated by following this mapping.

Example: For $p = 17$, suppose a player's initial choice corresponds to a vertex with the label 10 and the subsequent choices in Roshambo are: *Paper*, *Paper* and *Rock*. Using the above mapping, the sequence of random walks is as follows:

$$10 \xrightarrow{Paper} 11 \xrightarrow{Paper} 12 \xrightarrow{Rock} 11$$

Thus 11 is the output of this random walk process.

Remark 4.2.1 *Enlarging human choices in the initial game can significantly decrease the initial entropy as the player will avoid choosing corner points due to experiments in [HN09, Wag72]. Our suggestion is using a continuous circle instead of the whole screen in the initial stage-game so that each point on the circle represents a graph*

vertex. The user chooses a random point on the circle, which corresponds to a random point in the initial stage-game and continues to walk stage-games as described above. The advantage of a circular representation of the graph is that all the vertices have almost equal chance of being selected by the human player as there is no corner to be avoided. However, it is likely that players have bias in their choices of points . To avoid this, we randomly change the corresponding number to each point at each round of the game. Therefore, the player's initial min-entropy would become larger by using this game.

4.3 Simulating one of the Players by Computer

Although the designed game can be played by two players, in some systems, such as a Personal Digital Assistant (PDA), private randomness is preferred as a second player may not always be available. Furthermore, any communication for generating randomness might be eavesdropped, which is not desirable if the random value is supposed to generate a cryptographic key. Thus, it is natural to simulate one of the players in the game by a computer.

Two different algorithms are used for simulating one of the players. One of the algorithms simply uses a pseudo random sequence for determining the player's actions while the second algorithm adaptively picks an action based on the opponent's game-play. Game theory is used to analyze the human player's expected actions against each of these algorithms and to find the equilibrium of the game. In Chapter 5, the theoretical predictions are compared with experimental results.

4.3.1 Algorithm "**A**": PRNG on the Computer Side

In this algorithm, the output of a PRNG determines the computer player's choices. These choices will look uniformly random to the human player. The human player is

not able to remember long sequences. Thus, there is no need to use a cryptographic PRNG for this application, as the output of even weak PRNGs looks uniformly random to human players. In this case, the game is analyzed as an *imperfect information game with behavioural strategy*.

In an extensive form game, which is obtained from a finitely repeated game, sometimes players do not know all the actions of the other player or cannot recall all their past actions. In this case, some nodes in the game tree (Section 2.5.3) are indistinguishable to the player simply because (i) the player does not have the sequence of previous actions and (ii) the available actions in the indistinguishable nodes are identical. These types of games are called *imperfect information* games. Each group of indistinguishable nodes forms an *information set*. *Behavioural* strategies in the extensive form game are the strategies in which each agent's (potentially probabilistic) choice at each node is made independently of his/her choices at other nodes.

This algorithm does not use any history of previous actions. Thus, the game is an *imperfect information* game and the computer player's *information set* consists of all the nodes that require its action. Since the action at each node is made independent of other nodes, the computer's strategy is considered a *behavioural strategy*. Repeating a Nash strategy in each stage-game will be an equilibrium in behavioural strategies simply because a Nash strategy is the optimal strategy at each node. The human player may remember the previous 1 or 2 actions nevertheless, because of the symmetry of the game and the computer player's random strategy, this will not change his/her optimal strategy which is playing uniformly randomly.

Note that one can view the game with a PRNG on the computer side as a non-repetitive game: at each stage the computer's choices are independent of previous actions, and the human player does not need to remember previous actions (because this will not change his/her optimal strategy).

4.3.2 Algorithm "**B**": Predictor Algorithm on the Computer Side

For two-player zero-sum repeated games, the folk theorem (Theorem 2.5.2) is still true [OR94, Proposition 156.1], but it becomes vacuous. Suppose we iterate a two-player zero-sum game G with payoffs U and $-U$. If each of the players uses a minimax strategy against the opponent, the maximum payoff is U. Thus in the iterated game, the only Nash-equilibrium payoff profile is $(U, -U)$ and the only way to get this is if each player always plays his/her minimax strategy, which is randomizing over the possible actions with uniform distribution. Thus, a Nash equilibrium strategy is optimal if the other players also use the Nash equilibrium strategies. In reality, however, players will not use Nash equilibrium strategies simply because they may be trying to predict the opponent's actions. In this case, the player who can predict better and plays more unpredictably wins the game.

The attempt to predict the opponent's action naturally appears when a human is involved in playing the game. This makes the game more competitive and thus more interesting.

To have a more realistic simulation of the human player, a predictor algorithm named "Iocaine Powder" [Egn00] by Dan Egnor is used. This algorithm took the first place in the "First International Roshambo Programming Competition". This algorithm uses a heuristically designed compilation of strategies and attempts to predict what the opponent will do. Thus, the optimal strategy or metastrategy is chosen based on past performance. The main strategies that are employed in this algorithm are random guessing, frequency analysis, and history matching:

- *Random guess*

  The algorithm may choose a move by random selection. This algorithm can be called a hedge. If somebody predicts and defeats this algorithm, it means that he/she is able to predict the output of the PRNG, which

is assumed to be impossible. At that point, the meta-strategy will make sure that the program "eliminates its losses" then starts playing randomly to avoid a destructive loss.

- *Frequency analysis*

  The frequency analyzer looks at the players' past actions to find the move they have made most frequently to predict their future moves. However, it is not very useful against complex opponents. It can be used to defeat other competitors who use it as a predictive method.

- *History matching*

  This is the strongest predictor in the Iocaine Powder algorithm, and several forms of this technique are used in other strong entries. This algorithm looks for an order in the history similar to the last few moves. For instance, if in the last four moves, the player played paper against rock, scissors against scissors, rock against paper, and scissors against rock, the algorithm will look for sequences in the history when the same four moves happened. In fact, a sequence of the last 30 moves by the opponent is considered superior to the last four moves. When such a sequence is found, the history matcher assesses the opponent's move after that sequence and assumes they will make the same move in the future.

  Once the history is created, this predictor can easily defeat a broad range of weak contestants. The use of meta-strategy allows this algorithm to defeat other strong opponents.

This algorithm is used in the second experiment in Chapter 5, where the human player plays Roshambo against the computer repeatedly and then the final output of

the TRNG is calculated based on the user's choices.

4.4 Enlarging Choices of the Human Player

So far, we have considered the game in which the number of actions for both players is equal and the optimal strategy for both players is to play uniformly randomly. When the game is played against a computer, this effectively means that the computer should make a random choice for any random move of the human player. The computer's random choice is provided by a PRNG. When the number of actions for both players is equal, any generated randomness by the human requires an equivalent amount of randomness from the computer. This puts the efficiency of such a human based RNG into serious doubt.

Note that in the proposed TRNG, the equality of the actions for both players will not disturb the efficiency of the TRNG, as truly random bits are obtained from human choices even if we spend some pseudo random bits for the computer player's choices. In other words, the quality of generated randomness is improved by the proposed game design. Moreover, by using algorithm "B" (Section 4.3.2) the amount of required pseudo random bits is much less than the generated bits.

Halprin et al. [HN09] use the two dimensional extension of the matching pennies game (named Hide and Seek) to enlarge the choices of the players and subsequently increase the number of generated random bits in one round of the game. Since in this approach, the number of available actions for both of the players is equal, the game is not an efficient way of generating randomness from human gameplay. In other words, this approach cannot produce more random bits than what is spent on the computer player's side. For example, assume there are 2^n locations on a two dimensional screen and each player is asked to choose one of the locations. The computer player wins if the choices match and the human player wins otherwise.

The computer player's actions are determined by a PRNG. Suppose the PRNG on the computer side generates 2^{n-1} instead of 2^n outputs. As a result, the computer player only chooses half of the locations on the screen. For example, the computer player chooses locations on the right half of the screen. The human player after a while figures this out and chooses locations on the left half of the screen to avoid matching the choices. Therefore, the generated random bits by the human player, corresponding to his/her choices in the game, would at most be $n-1$ bits.

In a two-player computer game, if we provide the human player with more choices than the computer player, he/she can generate more random bits than a computer player. For this purpose, we suggest using a non-cooperative game in which the number of available actions for one of the players (the human player) is more than the other player (the computer player). The important issue in designing such a game is setting the payoffs so that randomly choosing among possible actions becomes the human player's best strategy.

For setting the payoffs in the desired game, let's assume the computer's strategy is pre-determined (i.e. we know the probability distribution over the computer player's available actions). The payoffs are set such that they lead the human player to play uniformly randomly over the available actions.

Suppose the size of action profile (Definition 2.5.1) for the human player is n and for the computer player is m, with $m < n$. We assume a_{ij} is the payoff for the first player resulting from picking the i^{th} action by the first player and j^{th} action by the second player, and b_{ij} is the corresponding payoff for the second player. Table 4.2 shows the payoff table of a general two-player game between the human player and the computer player.

Assume that the computer player picks action X_i with probability of p_i, thus we have $\sum_{i=1}^{n} p_i = 1$.

Human ╲ Computer	X_1	X_2	...	X_n
Y_1	a_{11}, b_{11}	a_{12}, b_{12}	...	a_{1n}, b_{1n}
Y_2	a_{21}, b_{21}	a_{22}, b_{22}	...	a_{2n}, b_{2n}
...				
Y_m	a_{m1}, b_{m1}	a_{m1}, b_{m2}	...	a_{mn}, b_{mn}

Table 4.2: Payoff table of a general two-player game

In order to gain the mixed strategy, the computer player has to randomize its moves to make the human player indifferent between his/her choices, i.e.

$$\forall i : \sum_{j=1}^{n} p_j a_{ij} = K$$

where K is an arbitrary constant.

On the other hand, we want the human player to make the computer player indifferent between its choices by playing each action with probability of $\frac{1}{m}$. Consequently:

$$\forall j : \frac{1}{m} \sum_{i=1}^{m} = K'$$

where K' is another arbitrary constant.

The above criteria do not specify unique payoffs for the game, but any set of payoffs satisfying those criteria will result in the desired Nash Equilibrium.

Example 4.4.1 *Consider the following extension of matching pennies: A human player has 4 colors to choose from. Two of these colors are warm colors (Red and Yellow), and the other two are cold (Blue and Green). The computer player has only 2 choices: warm or cold. If the computer player guesses the kind of the color picked by the human player correctly, then it wins, otherwise the human player will win. Satisfying the mentioned criteria, we have adjusted the payoffs of the game so that the best strategy for the human player is to pick each color with a probability of* $\frac{1}{4}$. *The payoff table is shown in Table 4.3.*

The Nash Equilibrium of this matrix game can be calculated using a "graphical solution" or alternatively linear programming [BOC+95]. The Nash equilibrium is

	Cold	Warm
Red	(1,2)	(-1,-2)
Blue	(-3,1)	(3,-1)
Yellow	(4,-3)	(-4,3)
Green	(-2,4)	(2,-4)

Table 4.3: Payoff table for color picking game

equal to $\frac{1}{4}$, which means the human player will play uniformly randomly. We interpret each color as 2 bits assigned in Table 4.4.

Color	Output bits
Red	00
Blue	01
Yellow	10
Green	11

Table 4.4: Mapping human choices to binary strings

By playing this game in each round, we will get 2 random bits while the computer has used just one random bit. After several rounds, we can generate twice as many random bits than the computer.

4.5 Comparison to Halprin et al. approach

Initial psychological experiments on human random gameplay confirmed that human actions in competitive zero-sum games correspond to almost uniform randomness. However, available actions in these games were limited to a small number (2 or 3 choices) [RB92]. Halprin et al. [HN09] used an extension of the matching pennies game in which available actions are from a two-dimensional action space. Players were asked to choose a location on the screen, one of them wins when the choices match, and the other wins otherwise. This extension is done to increase the entropy rate as more available actions will collect more entropy and eventually generate more random bits.

Despite the theoretic expectations, experiments in [HN09] showed that human players avoid choosing corner locations in their designed game. To compensate for the bias of human choices, a seeded randomness extractor was used to extract randomness from the human player's choices in the game. They used a t-resilient extractor to ensure the extraction procedure is resilient against limited adversarial influence on the entropy source after the public seed is chosen. To provide sufficient entropy for randomness extraction, the human player has to play the game for a relatively large number of time. This requirement decreases the speed and efficiency of the randomness generation process. For better efficiency, the output of the extractor is given to a robust PRNG that uses the extracted randomness for state refreshment and generates s longer random sequences. Table 4.5 shows a setting of parameters in Halprin et al.'s game design.

Parameter	**Value**
Output(bits)	128
Required clicks	28
t resiliency	46
Random seed (bits)	15232
Initial state of PRNG (bits)	128
ϵ closeness to uniform	2^{-64}

Table 4.5: Parameters setting in Halprin et al.'s design

The length of the required seed for Halprin et al.'s extractor is relatively long and providing this random seed itself needs a TRNG. This creates a loop in their construction. Nevertheless, the output is a pseudo random sequence which means that the quality of the generated randomness is decreased from true randomness to pseudo randomness. Moreover, when one of the players is simulated by a computer, a PRNG should generate a sequence of actions for the computer player.

In contrast, in the proposed game-theoretic based game design, the human player has a few choices in each round of the game while the entropy rate is still high and

long sequences of random bits are generated. The randomness extraction is done as part of the game design with no need for an extra seed, and furthermore, the number of rounds that are required to be played is significantly less than what is required in the simple form of the matching pennies game.

In the proposed two-player game, we can simulate one of the players with a computer. In this case, however, the amount and quality of the generated sequence is improved. Note that the number of random bits used by the computer player is determined by the number of required random walks, which is significantly smaller than the final generated random sequence.

Table 4.6 demonstrates a parameter setting in the proposed game design for comparison with Table 4.5.

Parameter	**Value**
Output(bits)	128
Required stage-games	37
d-regularity	5
Random seed (bits)	not required
ϵ closeness to uniform	2^{-64}

Table 4.6: Parameter setting in our design

Another advantage of the proposed game is that it can adjust to some of the popular and respectively interesting two-player games (such as Roshambo) while Halprin et al.'s approach needs a new design of a two-player game with the supporting game-theoretic analysis to show the optimal strategy is uniform. The new game is supposed to be interesting, even though ultimately the users decide if the game is interesting.

Chapter 5

Experiments and Results

5.1 Introduction

As mentioned in Section 4.2.2, a TRNG consists of two modules: (i) an entropy source that generates a sequence of symbols and (ii) a randomness extractor module that transforms the input from an entropy source into an almost uniformly distributed sequence. The parameters of the extractor module directly depend on the minimum available min-entropy in the entropy source. By conservatively estimating the min-entropy, one can guarantee a reliable design of the extractor module. The output of the TRNG is finally evaluated using statistical tests to ensure appropriate functionality of the designed TRNG.

In this chapter, the output of the proposed TRNG is assessed. Two different games with the same structures are designed for this purpose. One is a graphical game, named *Wolf and Sheep*, in which the human player is initially asked to choose a vertex on a Petersen (10 vertex, 3-regular) graph. In the subsequent walk stage-games, the player chooses a vertex from 3 neighbors of the previously selected vertex. In each stage-game, the player wins the game in case of a mismatch. The other game is called *C-Roshambo*, which is choosing a location on a circle followed by the regular Roshambo game for a number of rounds [1]. The theoretical approach, proposed in this thesis, enables us to generate random sequences from human gameplay in these two games. The details of the game design and the final evaluation of the output are explained in this chapter.

[1]The University of Calgary Conjoint Faculties Research Ethics Board has approved this research study

5.2 Game Design and Experiment Set-up

In this chapter, two different simple games are designed to examine the proposed TRNG: *Wolf and Sheep* and *C-Roshambo*. Each game is played between a human and a computer player. For estimating the min-entropy of human choices in the initial stage-game, a separate sub-game is designed for each of the main games. The potential players were contacted, and the willing players participated in the experiment. The set of players who participate in the first game (Wolf and Sheep) is different from those who participate in the second game (C-Roshambo). Nine players played the first and seven players played the second game. All of them were graduate computer science students at the University of Calgary. The instructions of the game were explained to the players. Each player was asked to play each game for at least 1000 rounds, trying his/her best to win the game. This is sufficient to run the required min-entropy and statistical tests. The objectives of the experiments are as follows:

1. Estimate the min-entropy of human choices in the initial stage-game, which is required for an appropriate design of the extraction module for the TRNG. With regards to Theorem 3.5.2, the number of required random walks on the expander graph, and consequently the number of walk stage-games is only determined if the min-entropy of the input is lower-bounded.

2. Examine the statistical properties (e.g. min-entropy) of the user's random walks to verify whether they are a good approximation of uniform random walks.

3. Examine the statistical properties of the final output.

4. Compare the output's statistical properties with that of a pseudo random sequence generated by the computer to show the generated random

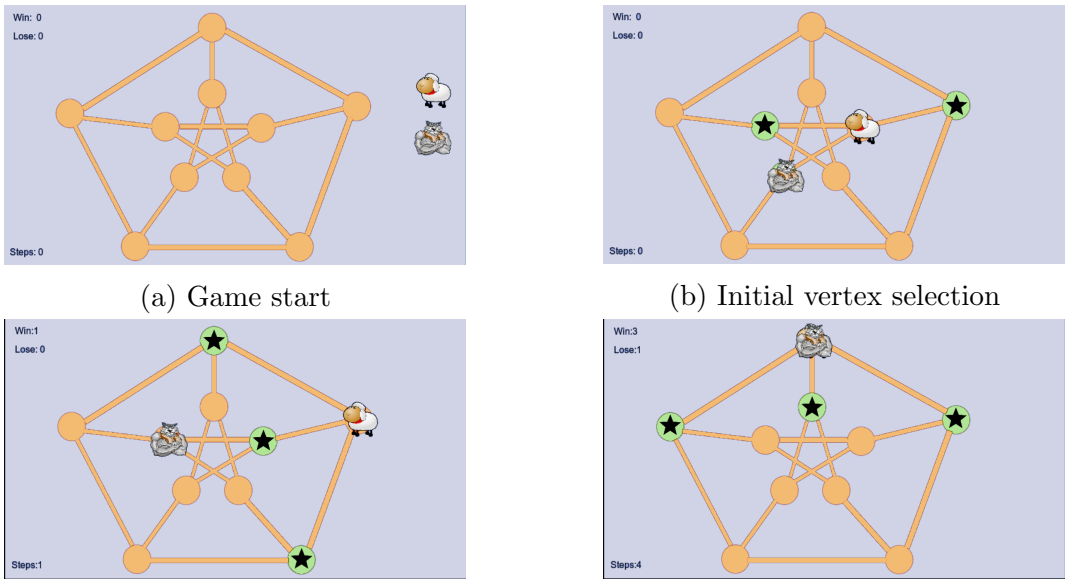
sequence is more random than what is used for simulating the computer player.

Remark 5.2.1 *The only requirement for the players is to be able to play the game rationally to maximize their overall score in the game. Students are participants who can play rationally and thus are appropriate participants for the experiment.*

5.2.1 First Game: "Wolf and Sheep"

This game is a version of matching pennies in which a human player tries to hide his/her sheep from the computer's wolf. A Petersen graph, which is a 3-regular graph with 10 vertices is used as the expander graph, for the game design. In the initial stage-game, the human player chooses one of the 10 vertices of the presented graph. This will place a sheep on the chosen vertex (Figure 5.1a). The computer responds by placing a wolf on a random vertex. The player loses if the wolf and sheep are on the same vertex. Otherwise, he/she wins the game. If the player wins, then the game will highlight vertices that are adjacent to the selected vertex (marked by the stars in Figure 5.1b). Then the same game is played again, but this time, the available actions are only the highlighted vertices, instead of all the vertices. The same as the initial game, the user wins (Figure 5.1c) if the choices are different and loses otherwise (Figure 5.1d). The final winner of the game is the player with a higher final score. Note that in the initial stage-game the number of possible actions is equal to the total number of vertices (10), and in the walk stage-games the number of possible actions is equal to the number of neighboring vertices (3).

The game is implemented using HTML 5 technology so that it can be run and played on any system with an Internet browser and even on touchscreen devices, such as tablets or smartphones. From now on we refer to the experiment related to this game as *Experiment I*.



(a) Game start

(b) Initial vertex selection

(c) First step of random walks

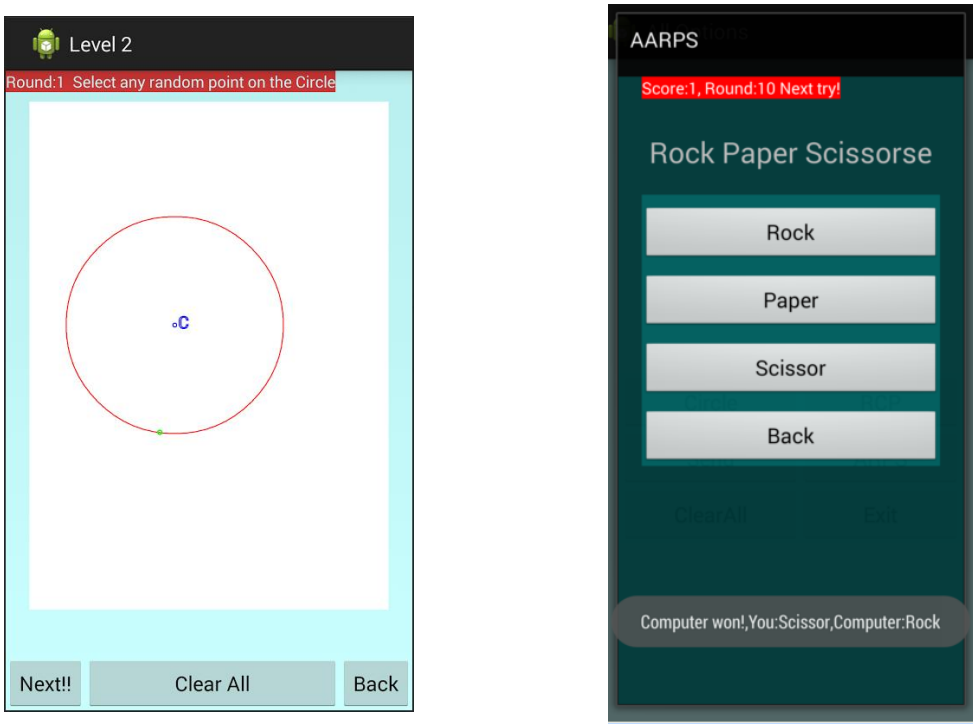
(d) Computer wins

Figure 5.1: Wolf and Sheep game
[ASNS13, Figure 1]

5.2.2 Second Game: "C-Roshambo"

This game is for generating long random sequences. Since the corresponding expander graph for this purpose is too big (i.e. has too many vertices). The whole graph is never presented to the player. Instead, the player is asked to choose a random location on a circle to start the game (Figure 5.2a). This initial selection corresponds to some unknown distribution that provides initial min-entropy for the TRNG. In the subsequent stage-games, the player is asked to play the basic *Roshambo* game (Figure 5.2b). The optimal strategy for winning Roshambo is to play uniformly randomly. Thus, the actions of a rational player in these stage-games determine a random walk on the expander graph. For generating random 8-bit sequences, 251 points (which is the largest prime number less than $2^8 = 256$) are specified on the circle and Construction 3.5.2 is used for the expander graph. Each of the points are mapped to an integer from 0 to 250. For increasing the min-entropy of the player's choices, the corresponding numbers to each point are changed for each round of the

game. As explained in Section 4.2.3.1, the final output of the TRNG is determined by tracking the sequence of choices in the Roshambo game.



(a) Initial Game

(b) Roshambo

Figure 5.2: C-Roshambo Game

This game is implemented in Java and one can run and play it on any Android device. We refer to experiment related to this game as *Experiment II*.

5.3 Entropy Estimation

To measure the min-entropy (or Shannon entropy) of a source, initially, one needs to figure out whether the source is an IID source. For an IID source, having enough samples from the source allows estimating the probability distribution of the source with high confidence by estimating the occurrence probability of the most repeated sample. Shuffling tests and Chi-square tests check whether the source follows an IID distribution. If any one of these tests fails, then the source is considered a non-

IID source, and the min-entropy is estimated through the corresponding tests for a non-IID source (Section 2.3.2).

Any anomaly in the source should be detected before applying IID and non-IID tests. Otherwise, the min-entropy would be over- or underestimated. NIST defines a set of sanity checks to prevent this. The sanity checks contain two tests: a compression test and a collision test. No estimation will be given if the sanity checks fail.

For the experiments in this thesis, an unofficial version of the NIST tests for entropy estimation are obtained (the code is not released yet) and used to estimate the min-entropy of the user's inputs. It is admitted in [BK12, Section 9.3.1] that the approximation from the NIST tests are very much dependent on the number of samples given to the tests (which is quite intuitive and acceptable). In our case, the subset of users and consequently the sample size is relatively small which implies a very conservative estimate of min-entropy for our experiment.

5.3.1 Entropy Estimation in the Initial Stage-Game

To measure the min-entropy of the initial stage-game, an independent sub-game is designed for both of the experiments. The players are asked to play this initial game for at least 1000 times that is sufficient for estimating the min-entropy of player's choices in the initial stag-game.

5.3.1.1 Entropy Estimation of Initial Stage-Game in the "Wolf and Sheep" Game

In *Experiment I*, players are asked to initially play the designed separate sub-game. The min-entropy of player's choices in this game is indeed, the min-entropy of the player's initial selection of a vertex on the graph in the main game. At each round of this game, the player is asked to choose a vertex out of 10 existing vertices and wins if his/her choice does not match the computer's. Table 5.1 shows the estimated min-entropy of 9 users who played the described game for some rounds. NIST tests showed

that users' inputs to this game are not following an IID distribution. Therefore, the min-entropy is estimated using tests for a non-IID source (Section 2.3.2). The first row is the number of total shots played by each user and the second row is the min-entropy of the player's choices per bit.

User	1	2	3	4	5	6	7	8	9
Total shots	1770	2141	3980	3439	2021	652	1685	905	983
Min-entropy (per bit)	0.45	0.49	0.61	0.65	0.52	0.49	0.47	0.55	0.51

Table 5.1: Min-entropy of users' input in the first stage-game of Wolf and Sheep game [ASNS13, Table 1]

In conclusion, for the first stage-game the lowest min-entropy is 0.45 per bit, and on average a min-entropy of 0.52 per bit is expected. This amount of min-entropy is sufficient to: 1) consider user provided inputs to the first stage-game of *Experiment I* as an appropriate entropy source and 2) design the rest of the TRNG based on the estimated min-entropy.

5.3.1.2 Entropy Estimation of Initial Stage-Game in the "C-Roshambo" Game

The designed sub-game for *Experiment II* is selecting a location on a circle for collecting random scores. This sub-game is designed for estimating the min-entropy of player's choices in the initial stage-game of the main game. Seven players played this game. The min-entropy of player's choices on the circle is estimated by NIST tests. The tests showed that player's choices do not follow an IID distribution. Thus, the min-entropy is estimated using tests for non-IID sources. The results are showed in Table 5.2.

User	1	2	3	4	5	6	7
Total shots	1093	731	987	1000	993	1115	2163
Min-entropy (per bit)	0.58	0.61	0.42	0.56	0.47	0.44	0.54

Table 5.2: Min-entropy of users' inputs in first stage-game of C-Roshambo game

The lowest min-entropy is 0.42 per bit and on average we expect 0.52 per bit entropy. This amount of min-entropy is sufficient to: 1) consider user provided inputs to the first stage-game of *Experiment II* as an appropriate entropy source and 2) design the rest of the TRNG based on the estimated min-entropy.

5.3.2 Entropy Estimation of Random Walks

To use an expander graph as an extractor for randomness extraction, random walks should follow a uniform distribution (Section 3.5.1). In both of the designed games, the human player's random choices in walk stage-games are used for doing a random walk. In spite of theoretical expectation, the human player's choices in these stage-games are not uniformly random. However, because of the particular design of the games (in which playing uniformly random is the optimal strategy), the sequence of the human player's choices is very close to uniform randomness. The result of experiments in this section verifies that the sequence of the human player's random walks in both designed games has high min-entropy and thus is appropriate for using as random walks to extract entropy from the initial game.

The players are asked to play the walk stage-game for at least 1000 rounds. Data from game play of participants is collected, and the corresponding min-entropy for both games is estimated.

5.3.2.1 Entropy Estimation of Walk Stage-games in Wolf and Sheep Game

In this experiment, NIST tests for IID sources are passed for player's inputs in walk stage-games. Therefore, the tests for IID sources are used for estimating the min-entropy of the data. Table 5.3 shows the estimated min-entropy for each player during the walk stage-game in *Experiment I*.

From the above table, the average expected min-entropy in walk stage-games for *Experiment I* is 0.68 per bit. One can compare the average expected min-entropy

User	1	2	3	4	5	6	7	8	9
Total shots	370	369	1009	1786	560	1071	4821	1190	1065
Min-entropy (per bit)	0.49	0.51	0.64	0.75	0.78	0.72	0.83	0.61	0.79

Table 5.3: Min-entropy of users' inputs in walk stage-games in "Wolf and Sheep" [ASNS13, Table 3]

of two tables or the amount of min-entropy for each player in Table 5.1 and Table 5.3 to confirm that the min-entropy of the human player's choices is more when the alternatives are less. Relatively high min-entropy of players' choices in these stage-games validates our assumption regarding uniformity of choices in walk stage-games.

5.3.2.2 Entropy Estimation of Walk Stage-games in C-Roshambo Game

The human players were asked to play two types of C-Roshambo game, each for at least 1000 times. The computer player is once simulated with a PRNG (Algorithm "A") and once with the "Iocain Powder" algorithm (Algorithm "B") (Section 4.3). A very weak PRNG for simulating the computer's player is used in the first case. In particular, a 5-bit maximum length LFSR with a period of 31 states was utilized for this purpose. This LFSR takes taps from 3rd and 5th bits, and the initial seed value is 10110.

NIST tests for IID distribution were passed for user's inputs in these walk stage-games, and so the samples are considered as an IID samples for min-entropy estimation. Table 5.4, shows the min-entropy of human players' choices in walk stage-games of *Experiment II*, once playing against Algorithm "A" and once against Algorithm "B".

User	1	2	3	4	5	6	7
Total shots playing against "A"	1034	948	1171	1000	1016	939	1592
Min-entropy (per bit)	0.64	0.55	0.61	0.32	0.57	0.49	0.74
Total shots playing against "B"	1098	1112	976	1000	1240	1458	832
Min-entropy (per bit)	0.73	0.65	0.58	0.44	0.79	0.62	0.67

Table 5.4: Min-entropy of users in walk stage-games in "C-Roshambo"

The expected average min-entropy for player's choices against Algorithm "A" is 0.56 per bit and for playing against Algorithm "B" is 0.64 per bit. This shows that the min-entropy of the human player's actions is more when playing against Algorithm "B", that is the human player plays more randomly against Algorithm "B". This observation is intuitive as Algorithm "B" indeed gives feedback to the player on his/her previous actions. Any predictability in the player's actions is exploited by the computer and concludes in the player's loss of the game. Thus, the human player modifies his/her actions to become unpredictable and therefore wins the game.

The other point to note is that, because of the short period of the LFSR, none of the random tests are passed for its output and its estimated min-entropy is very small (0.08 per bit). However, the results of experiment in this chapter show that the human player plays randomly even against this weak PRNG.

5.4 Measuring Statistical Property of the Output

To examine statistical properties of a sequence, the statistical tests in a battery of tests called Rabbit [LS07] are used. The Rabbit set of tests includes tests from NIST [RSN+01] and DIEHARD [Mar98] with modified parameters tuned for smaller sequences and hardware generators.

5.4.1 Evaluating the Output of the "Wolf and Sheep" Game

Using the results in Theorem 3.5.2, the distribution of the final output of the Wolf and Sheep game will be ϵ-close to uniform distribution if the walk is uniformly random. The previous experiments (Section 5.3.2) showed that the human player's actions in walk stage-games are almost uniformly distributed, and there is a min-entropy guarantee on the actions of the players in the initial stage-game. Consequently, we expect to have a random sequence as the output of the game (which in fact is the

output of the designed TRNG). The set of statistical tests on the final output is run, and the results are summarized in Table 5.5. The data in the table is from a sample player.

Statistical Test	p-value
LinearComplexity	0.51
LempelZiv	0.73
SpectralFourier	0.34
Kolmogorov-Smirnov	0.23
PeriodsInString	0.13
HammingWeight	0.55
HammingCorrelation	0.69
HammingIndependence	0.29
RunTest	0.60
RandomWalk	0.67

Table 5.5: Statistical tests for the "Wolf and Sheep" game [ASNS13, Table 2]

The numbers in Table 5.5 are the p-values of each test. The details of p-value calculation are given in [LS07]. In statistical studies with a limited dataset, it is possible to fix an acceptance margin for p-values (for example 0.05 or 0.001) and accept the randomness of the sequence only if the p-value is not in the margin area. However, it is often recommended to report p-values as well, because this provides more information than reporting an "accept" or " reject" based on the fixed margin.

For our case, we set the margin of p-value to be 0.001. Thus, the test is passed if these values are more than 0.001 or less than 0.999. Therefore, the tests in Table 5.5 are passed for our dataset.

The data from other players is also examined with all statistical tests, and all the tests were passed for other player's data. Overall the experiments show the viability of the proposed approach in practice.

5.4.2 Evaluating the Output of the "C-Roshambo" Game

Besides statistical tests, the output min-entropy of an arbitrarily chosen sample user (user 1) is estimated to compare it with the initial min-entropy in the first stage-game:

Game	Estimated min-entropy
Initial stage-game	0.58 per bit
Playing against "A"	0.63 per bit
Playing against "B"	0.71 per bit

Table 5.6: Min-entropy comparison of user 1's input and output

Table 5.6 shows that the min-entropy of the TRNG's output is more than the min-entropy of the initial input. This means that the output of the TRNG is more uniformly distributed than its initial input. This is because of using random-walks in the walk stage-games. As expected, when the computer player is simulated by Algorithm "B", the output's min-entropy is larger than the min-entropy of the output when Algorithm "A" is used. This shows that the TRNG's output is more random when Algorithm "B" determines the computer player's choices. This is because the human player plays more randomly against Algorithm "B" that provides feedback on his/her actions. Similar to the previous game, the set of statistical tests are run on the final output of C-Roshambo when the computer player is simulated with Algorithm "B" (*Iocain Powder* algorithm). The results are summarized in Table 5.7, which is generated from an arbitrarily chosen sample user's inputs (user 1).

The amount of the p-values for the above tests shows that the randomness tests are passed for the output of the designed TRNG. The statistical tests were run for the other user's data as well. The *PeriodsInString* test failed for user 4. It is probable that this user has followed a specific pattern during his/her gameplay. However, the overall results of the experiments are close to our expectation from human player's gameplay.

Statistical Test	p-value
LinearComplexity	0.33
LempelZiv	0.18
SpectralFourier	0.49
Kolmogorov-Smirnov	0.81
PeriodsInString	0.21
HammingWeight	0.55
HammingCorrelation	0.76
HammingIndependence	0.08
RunTest	0.67
RandomWalk	0.42

Table 5.7: Statistical tests for "C-Roshambo" game

5.5 Conclusion

Two games were designed and implemented for doing experiments on the proposed TRNG. The min-entropy of player's actions in the initial stage-games and corresponding walk stage-games were estimated. The results show that human actions in walk stage-games are an appropriate choice for doing a random walk on an expander graph. The statistical properties of the final output were also tested. In spite of limited sample data, modified statistical tests passed for player's provided data which confirms the TRNG output is sufficiently random.

The results of experiments with the second designed game, C-Roshambo, showed that a human player plays more randomly against a predictor algorithm that tries to predict his/her actions based on the history of previous actions. Subsequently, the output of the TRNG is more random when a predictor algorithm is simulating the computer player's actions. However, the output of the TRNG is still considered random when a PRNG simulates the computer's actions. Even using a weak PRNG does not change this conclusion. The justification is that since a human is not able to detect patterns in a weak PRNG, he/she plays as if the computer's actions are uniformly random. A critical point here is that the output of the designed TRNG

(generated by human gameplay) is much more random than the initial PRNG.

Chapter 6

Conclusion and Future Works

The main contribution in this thesis was proposing a new TRNG that is capable of using the human player's input as the only source of randomness. This TRNG is mainly a two-player game that takes human gameplay as the input and generates a sequence of random bits. We designed and implemented two games as a proof of concept for the proposed TRNG. The results of statistical tests on the output of the TRNG confirm its efficiency and applicability.

6.1 Concluding Remarks

Random number generators are an important part of any security system. Good randomness guarantees the security of cryptographic protocols. User based randomness generation is preferable to other ways of randomness generation especially in shared environments like cloud services. In such environments other traditional sources of entropy (e.g. hardware consisting of CPU and RAM) are shared among users and consequently user based random generation is the only way that ensures generation of non-correlated randomness. Moreover, randomness generation from users doesn't need any extra hardware for entropy collection which eases its application for various systems with limited resources, especially hardware.

This work opens a new direction for independent randomness generation in shared environments without any requirement for extra computational capability or randomness generation subsystems.

6.1.1 Advantages of the Proposed TRNG

The proposed TRNG uses human gameplay, from interesting games, for generating randomness which is interesting to users. The structure of this TRNG is very flexible and can be fit to several popular games such as Roshambo. The only condition for the game is that the optimal strategy requires the players to play uniformly randomly for winning the game. The basic game of TRNG effectively asks players to choose an action among a group of alternatives. Although the proposed TRNG can generate long random sequences, it always provides players with a few alternatives, except in the initial stage-game. This feature causes human players to play more randomly since they behave more predictably when choosing among many alternatives.

The game is capable of being played by two users. In this case, the generated random sequence is completely independent from the computer. However, the game can be played against a computer player as well. For this purpose, we require a PRNG to simulate the computer player's actions. The experimental results showed that even very weak PRNGs are suitable for being used on the computer player's side because the human mind is unable to memorize relatively long patterns and thus cannot distinguish between truly random sequences and pseudo random ones. In other words, the experiments show that human players follow their optimal strategy, which is playing uniformly randomly against a computer even if the computer uses a weak PRNG for selecting its actions. One can also use a predictor algorithm on the computer's side for predicting the human player's actions. In this case, the human would play more randomly since the predictor algorithm indeed gives feedback to the players on their previous actions. To summarize, we can say that dependency of the proposed TRNG on computer randomness is minimized through an appropriate game design. This feature is ideal for shared environments in which users require an independent source of randomness for critical security applications.

6.2 Future Works

There are a number of extensions and directions for future work. On a theoretical side, characterizing a randomness extractor with guaranteed min-entropy on its seed is an interesting open problem. In particular it is challenging to analyze non-uniform random walks on an expander graph based extractor.

On the implementation and experimental side, for generating large strings, for example 64 or 80 bit strings, we need to collect as much entropy as possible in the initial round. Here one needs to find ways for enabling the user to make the initial selection of the random value with "high" initial min-entropy. In this work, Roshambo is used as a zero-sum game with 3 choices in walk stage-games. Other interesting zero-sum games with a greater number of available actions will conclude in faster randomness generation of the whole game. Creating an interesting game and interface to encourage random selection will improve the efficiency of the approach. The strings generated by sample users were analyzed. Wider user studies are required to measure min-entropy and statistical properties of strings at different stages (entropy source, random walk and final output) as well as usability of the system.

A predictor algorithm such as Iocain Powder used in this experiment improved human random behaviour by giving indirect feedback of the player's actions. Better predictor algorithms for simulating human players in zero-sum games will lead the player to generate better unpredictable sequences.

Hardware faults or malicious tampering with entropy sources may result in biases in the randomness source that are not easily detectable. Using human input protects a system against such faults or malicious tampering. Providing a secure interface for playing the game by two users allows randomness generation independent of the computer and improves trustworthiness of the generated randomness.

This work is the first construction of a full TRNG that uses human gameplay as

the entropy source and the seed for the extractor. Using human users to construct TRNGs with a higher rate of randomness generation is an interesting direction for future work.

Chapter 7

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Appendix A

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January 4, 2016

To: Faculty of Graduate Studies

I am writing this letter to clarify the contribution of the authors for the paper "*A true random generator using human gameplay*". The paper is based on a course project of Setareh Sharifian in the course CPCS 630, taught by Dr. Rei Safavi-Naini. The project report described the idea and a very basic implementation.
Dr. Safavi-Naini found the idea novel and worth further development.
At the time, I was a PhD student working on a different topic which gave me a good background for the project. I also had experience with software developments for games which was necessary for a larger experiment that was required for the project.

Dr. Rei Safavi-Naini asked me to join the project and help with the implementation and experiments. I joined the project and developed a game that was played by users. The data was collected and together with Setaeh Sharifian, we did the data cleaning, statistical analysis and graphing. The paper is based on the course project report, with the added experiment and analysis, and is written by all three coauthors.

Sincerely,

Alimomeni, Mohsen

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