Rate-Transient Analysis of Tight Oil and Gas Reservoirs

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Rate-Transient Analysis of Tight Oil and Gas Reservoirs

by

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A THESIS
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Abstract

Horizontal wells completed in multiple hydraulic fracturing stages (multi-fractured horizontal wells or MFHWs) have been critical technologies applied to low-permeability (tight) oil and gas reservoirs in recent decades, resulting in commercial production. For each stage in a MFHW, the formation is fractured by injecting water and sand at high pressure. The resulting hydraulic fracture system enhances production from tight reservoirs by increasing the effective area for flow of the reservoir fluids. Therefore, fracture conductivity and total fracture area are key parameters affecting MFHW performance. A powerful tool for characterization of MFHWs is rate transient analysis (RTA); RTA models are commonly based on analytical solutions to fluid flow equations describing flow through the rock matrix and hydraulically-induced fractures to MFHWs. In addition to MFHW characterization, RTA is used for short- and long-term production forecasting (or estimation of ultimate recovery) and estimation of fluid-in-place.

In order to obtain analytical solutions to the flow equations (for RTA purposes), simplifying assumptions have been made by practitioners such as constant formation permeability, constant properties of oil, constant hydraulic diffusivity of gas, and single-phase flow of the primary hydrocarbon phase. In this thesis, each of these assumptions are relaxed and corresponding analytical/semi-analytical solutions are developed for tight oil and gas reservoirs. Three methods are proposed for incorporation of the aforementioned nonlinearities into RTA. The methods utilize 1) the transformation of nonlinearity approach, 2) the iterative integral approach, and 3) the dynamic drainage area concept. The results of the proposed methods are compared against numerical simulation for validation, and are applied to field cases to demonstrate practical utility. Importantly, it is demonstrated that failure to incorporate corrections for the aforementioned nonlinearities into RTA can lead to significant errors in derived parameters, such as the linear flow parameter derived from transient linear flow analysis.

The RTA methods developed in this thesis are intended to provide practitioners with more robust tools for analysis of tight oil and gas reservoirs.
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Dedication

To Behjat, my beautiful wife and best friend
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Chapter 1  Introduction

1.1  Problem Statement

The economically-successful application of multi-fractured horizontal well (MFHW) technology to tight oil and gas reservoirs has changed the energy outlook in North America. Development of tight reservoirs is mainly driven by success of the MFHW technology, capital investment, and commodity prices. A key step in understanding the profitability of development of tight reservoirs is forecasting hydrocarbon production.

There are a variety of production forecasting tools available including empirical (Arps, 1945; Valko and Lee, 2010; Ilk et al., 2008a and 2008b; Duong, 2011, etc), analytical (Wattenbarger et al., 1998; Agarwal et al., 1999; Brown et al., 2011; Ozkan et al., 2011; Stalgorova and Mattar, 2013, etc), semi-analytical (Clarkson and Qanbari, 2015; Clarkson and Qanbari, 2016), and numerical models (Cipolla et al, 2010; Fan et al., 2010). A preliminary step for some of the forecasting tools is characterization of the MFHW system, which is a multi-disciplinary task. Empirical models ignore physics of fluid flow in the reservoir, whereas numerical models are flexible enough to incorporate any sort of physics into production forecasting. Analytical and semi-analytical models are designed to capture the primary flow mechanisms while being simple and fast enough to be applied to numerous wells in a short period of time.

Analytical methods, however, are not capable of accounting for some important complexities of fluid flow in tight reservoirs. In particular, the analytical models or rate transient analysis (RTA) tools are based on single-phase flow of primary hydrocarbon phase in formations with constant permeability. In addition, classic RTA tools are not accurate enough for gas wells producing with high drawdown.

The main purpose of this thesis is to develop new analytical methods, or modify the existing analytical models, for incorporation of two important nonlinearities (i.e. pressure-dependent rock and fluid properties and two-phase flow) into RTA results.

Primary challenges for data analysts hoping to extract quantitative fracture and reservoir properties from RTA of tight/shale reservoirs include data quality and the lack of available
rigorous, physics-based models. The former is essential in the current economic environment, and requires collaboration between different professionals in a company. As for the latter, all of the aforementioned characterization tools for MFHWs are indirect methods in which the collected data sets are analyzed using mathematical models with simplifying assumptions. Simplifying assumptions are inevitable primarily for two reasons: the lack of detailed data and information about the MFHW system (e.g. fracture geometry, fluid distributions, geological heterogeneity etc.); and limited capabilities of some of the existing mathematical tools, particularly analytical techniques. Standard RTA methods, in particular, are limited to the analysis of production from single-phase systems with slight pressure-dependency of rock and liquid phase properties.

For linear flow analysis of oil systems, the slope of rate-normalized pressure, \((p_i - p_{\text{wof}}) / q_o\), versus oil linear superposition time is used to calculate total \(A_c\sqrt{k_i}\), which is equal to \(4h_x\mu \sqrt{k_i}\) for a MFHW (El-Banbi 1998):

\[
\frac{p_i - p_{\text{wof}}}{q_o} = m_{cq} t_{\text{LST}}
\]

(1)

\[
A_c \sqrt{k_i} = \frac{79.71B_o}{m_{cq}} \frac{\mu_{oi}}{\phi_i c_t}
\]

(2)

In Eqs. 1 and 2, \(p_i\) (psia) and \(p_{\text{wof}}\) (psia) are initial reservoir pressure and well flowing bottomhole pressure, respectively, \(q_o\) (STB/day) is oil rate, \(t_{\text{LST}}\) (\(\sqrt{\text{day}}\)) is linear superposition time, \(m_{cq}\) (psi/STB/\(\sqrt{\text{day}}\)) is slope of linear flow plot, \(A_c\) (ft\(^2\)) is total area of hydraulic fractures, \(k_i\) (md) is permeability, \(B_o\) (rb/STB) is oil formation volume factor, and \(\mu_{oi}\) (cp), \(\phi_i\) and \(c_t\) (psi\(^{-1}\)) are oil viscosity, porosity and total compressibility at initial reservoir pressure, respectively.

Similarly, the slope of the linear flow plot for gas (plot of rate-normalized pseudopressure, \([m_g(p_i) - m_g(p_{\text{wof}})] / q_g\), versus gas linear superposition time) is used to calculate total \(A_c\sqrt{k_i}\).

\[
\frac{m_g(p_i) - m_g(p_{\text{wof}})}{q_g} = m_{cq} t_{\text{LST}}
\]

(3)
In Eqs. 3 and 4, \( m_g \) (psia\(^2\)/cp) is gas pseudopressure, \( q_g \) (Mscf/day) is gas rate, \( T \) (\(^{\circ}\)R) is formation temperature, and \( \mu_{gi} \) (cp) is gas viscosity at initial reservoir pressure.

Different methods have been introduced over the past few years for incorporation of nonlinearities associated with pressure-dependent fluid and rock properties and two-phase (gas and oil/condensate) flow in RTA of tight/shale systems. Ibrahim and Wattenbarger (2006) used an empirical correction factor (based on numerical simulation results) for linear flow analysis of single-phase gas systems. Nobakht and Clarkson (2012), Tabatabaie and Pooladi-Darvish (2016), and Behmanesh et al. (2015) employed pseudotime functions evaluated at average pressure in the investigated area during transient flow to linearize the nonlinear flow equation. Mohan et al. (2013) and Eker et al. (2014) incorporated multiphase flow into RTA of tight systems using total equivalent rate instead of the primary fluid rate.

Three methods are used in this thesis for RTA of tight reservoirs with pressure-dependent rock and fluid properties and two-phase flow, including 1) the transformation of nonlinearity method, 2) the iterative integral method, and 3) the dynamic drainage area (DDA) method. A brief review of the methods is presented in the next section. A detailed discussion of these methods is provided in Chapters 3-6.

1.2 Methods

1.2.1 Method of Nonlinearity Transformation

In the method of nonlinearity transformation, the nonlinearity of the storage term in the flow equation is used in the definition of a pseudo-pressure which, in turn, changes the flow equation into a nonlinear diffusion-type equation with hydraulic diffusivity playing the role of the concentration-dependent diffusion coefficient. Hydraulic diffusivity, as a function of the pseudopressure, is fitted with a function of desired form for which an exact analytical solution exists during transient linear flow period. A correction factor is obtained analytically for stress-sensitive tight oil reservoir, which corrects the results from linear flow plot.
\[
\frac{1}{f_c} = -\sqrt{\pi} \frac{\Delta m_2(p_w)}{\Delta m_1(p_w)} \frac{1 + M}{\beta_1 \varepsilon} \theta \bigg|_{m_{(iIni)}}
\]  

In Eq. 5, \( f_c \) is correction factor, \( m_1 \) (psi) and \( m_2 \) (psi) are oil pseudopressures based on flow and storage terms, respectively, and \( M, \beta_1, \varepsilon \) and \( \theta \) are the parameters of Fujita’s formulation. A detailed explanation of this method is provided in Chapter 2.

1.2.2 Iterative Integral Method

The iterative integral method provides an iterative scheme which uses the approximate solution to the linearized form of the flow equation (the basis of standard RTA) as the initial guess:

\[
m_D = g(\xi) = 1 - \frac{\int_0^\xi \exp\left(-2\int_0^\xi \frac{\xi}{\eta_D(m_D)} d\xi\right) d\xi}{\int_0^\xi \exp\left(-2\int_0^\xi \frac{\xi}{\eta_D(m_D)} d\xi\right) d\xi} (6)
\]

The results are then presented in the form of a correction factor, which is the ratio of the updated solution and the initial guess, for linear flow analysis. The equation for the correction factor is:

\[
\frac{1}{f_c} = \sqrt{\pi} \int_0^\xi g^{-1}(m_D) \frac{d m_D}{\eta_D} (7)
\]

In Eqs. 6 and 7, \( m_D \) is dimensionless pseudopressure, \( \xi \) is Boltzmann parameter, \( \eta_D \) is dimensionless diffusivity, and \( f_c \) is correction factor.

1.2.3 Dynamic Drainage Area Concept

The dynamic drainage area (DDA) approach, in backward mode, is used as an iterative method for analysis of transient linear flow in tight reservoirs. The method combines three reservoir engineering concepts for linear flow analysis: a time-dependent well productivity index equation for the transient flow period, material balance in the investigated area, and decoupling of saturation and pressure (which is analogous to the decoupling of geomechanics and fluid flow). In backward mode, the DDA equations for linear flow of oil and gas are as follows:
\[
\frac{m_o(\bar{p}_{\text{inv}}) - m_o(p_{\text{wf}})}{q_o} \sqrt{\frac{\mu_{oD}(\bar{p}_{\text{inv}})\phi D(\bar{p}_{\text{inv}})C_iD(\bar{p}_{\text{inv}})}{k_D(\bar{p}_{\text{inv}})}} = 57.16B_{oi} \frac{\mu_{oi}}{A_c} \sqrt{\frac{t}{\phi_Cc_i}}
\]

and

\[
\frac{m_g(\bar{p}_{\text{inv}}) - m_g(p_{\text{wf}})}{q_g} \sqrt{\frac{\mu_{gD}(\bar{p}_{\text{inv}})\phi D(\bar{p}_{\text{inv}})C_iD(\bar{p}_{\text{inv}})}{k_D(\bar{p}_{\text{inv}})}} = 576.56T \frac{\mu_{oi}}{A_c} \sqrt{\frac{t}{\phi_Cc_i}}
\]

In Eqs. 8 and 9, \(m_o\) (psi) and \(m_g\) (psi²/cp) are oil and gas pseudopressures, respectively, \(q_o\) (STB/day) and \(q_g\) (Mscf/d) are oil and gas rates, respectively, \(t\) (day) is time, \(\bar{p}_{\text{inv}}\) (psi) is average pressure in the investigated area, and dimensionless parameters are the normalized values with respect to initial values.

Eqs. 8 and 9 use pseudo-pressure drawdown with respect to average pressure in the investigated area during transient linear flow, which is obtained from material balance equations as discussed in the following subsection.

1.3 Thesis Structure

Chapter 2 presents the method of transformation of nonlinearity to account for stress-sensitivity of formation permeability in RTA of single-phase tight oil reservoirs. In this method the nonlinearity of the flow equation for stress-sensitive tight oil reservoirs (i.e. hydraulic diffusivity) is correlated with a rational function for which the flow equation has an exact analytical solution.

In Chapter 3, the iterative integral method is used to solve the nonlinear flow equation for stress-sensitive tight oil reservoirs with flowing bottom-hole pressure above or below bubble-point pressure.

The iterative integral method is applied to tight gas reservoirs in Chapter 4. A nonlinearity measure is introduced as an effective means of scaling and presenting the results of the iterative integral method for tight gas reservoirs.

The impacts of confined gas properties and adsorbed layer thickness on the results of dry gas RTA are studied in Chapter 5.
Application of dynamic drainage area (DDA) approach to RTA of tight oil and gas reservoirs is presented in Chapter 6. DDA is a semi-analytical method which combines the linear flow productivity index, material balance and decoupling of saturation and pressure.

1.4 Nomenclature

**Field Variables**

- $A_c$: area of cross-section (ft$^2$)
- $B_o$: oil formation volume factor (RB/STB)
- $c_{ti}$: total compressibility at initial reservoir pressure (psi$^{-1}$)
- $f_c$: correction factor
- $k_i$: permeability at initial reservoir pressure (md)
- $k_D$: dimensionless permeability, $k/k_i$
- $m_1$: pseudopressure based on flow term (psi)
- $m_2$: pseudopressure based on storage term (psi)
- $m_{2D}$: dimensionless pseudopressure
- $m_g$: gas pseudopressure (psi$^2$/cp)
- $m_o$: oil pseudopressure (psi)
- $M$: a parameter in Fujita’s model
- $p$: pressure (psi)
- $p_{av}$: average pressure in the investigated area (psia)
- $p_{wf}$: well flowing pressure (psia)
- $q_o$: oil production rate (STBD)
- $t$: time (day)
- $T$: reservoir temperature (°R)

**Greek Variables**
\( \beta_i \) fitting parameter in Fujita’s model

\( \epsilon \) a parameter in Fujita’s model

\( \eta_D \) dimensionless hydraulic diffusivity

\( \theta \) a parameter in Fujita’s model

\( \mu_o \) oil viscosity (cp)

\( \xi \) Boltzmann variable

\( \phi \) porosity

\( \phi_D \) normalized porosity, \( \phi/\phi_i \)

\( \phi_i \) porosity at initial pressure

### 1.5 References


Chapter 2  Analysis of Transient Linear Flow in Stress-Sensitive Formations

2.1 Abstract

Rate- and pressure-transient analysis of unconventional gas and oil reservoirs is a challenge because of the complex reservoir characteristics that control flow. Further, transient linear flow is an important flow regime for these reservoirs, which can be associated with linear flow to induced hydraulic fractures. One of the complications in the analysis of this flow regime is stress-sensitivity of porosity and permeability. This work provides a new method for analysis of transient linear flow in stress-sensitive tight oil reservoirs.

A correction factor is used to correct the results of the conventional method for analysis of transient linear flow in tight oil reservoirs. A new method is developed for calculating the correction factor using an analytical solution of the flow equation. The results indicate that the correction factor becomes more important for higher values of permeability modulus and pressure drawdown. The correction factor is used in the analysis of synthetic production generated with a numerical simulator for constant-pressure and constant-rate production. The results show that the correction factor can eliminate the considerable error of the conventional analysis method in estimating the initial reservoir permeability.

2.2 Introduction

Analytical solutions for the linearized form of the flow equation have been widely used as the basis for production data analysis of oil reservoirs. These tools give reasonable results for conventional oil reservoirs due to the moderate changes in rock and fluid properties with pressure. However, they lead to considerable error when applied to tight oil reservoirs with nonlinear flow equations due to stress-dependency of permeability and porosity.

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1 This chapter is a slightly modified version of a paper published in SPE Reservoir Evaluation and Engineering as: Qanbari, F. and Clarkson, C.R., 2014. Analysis of transient linear flow in stress-sensitive formations. SPE Reservoir Evaluation & Engineering, 17(01), 98-104. Copyright approval has been obtained from the journal (see “Copyright Permissions” section of this thesis).
Efforts have been made to develop analytical solutions for the nonlinear flow equations. Pseudopressure defined by the Kirchhoff transformation (Al-Hussainy et al., 1966) facilitates the use of the analytical solutions for conventional gas reservoirs. In pressure and production data analysis, pseudopressure in unconventional gas reservoirs plays the role of pressure in conventional oil reservoirs. For tight gas reservoirs, Ibrahim and Wattenbarger (2006) proposed a correction factor for the slope of the square-root-of-time plot during transient linear flow period. They used the results of a numerical model and obtained an empirical equation for the correction factor as a function of a dimensionless drawdown parameter. Later, Nobakht and Clarkson (2012) derived an analytical form of the correction factor for constant flowing pressure condition. This study uses a new method for solution of the nonlinear flow equation and evaluation of the correction factor for tight oil reservoirs during the transient linear flow period.

In this chapter, a new analytical method is used to solve the transient linear flow equation for single-phase oil in tight reservoirs with stress-dependent permeability. In this method, the nonlinearity is transformed into a function for which an exact solution exists. The analytical solution yields a correction factor which can be used to correct the results of the linear flow analysis in tight stress-sensitive oil reservoirs.

The structure of this chapter is as follows. First, transient linear flow of single-phase oil and the governing flow equation are discussed. Next, a method for calculation of the correction factor is presented. Following that, the results for the correction factor are given for different values of well bottom-hole pressure and permeability modulus. The correction factor is used in the analysis of synthetic production generated with a numerical simulator for constant-pressure and constant-rate production.

2.3 Theory and Methods

2.3.1 Transient Linear Flow

Transient linear flow is an important flow regime in unconventional oil and gas reservoirs. This flow regime can be associated with flow to induced hydraulic fractures. Consider for simplicity a vertical well stimulated with an infinite-conductive vertical fracture which extends all the way to the boundary of a linear undersaturated tight oil reservoir with cross-sectional area of $A_c$ (Fig.
2.1). As the well is put on production with constant bottom-hole pressure (which is greater than bubble point pressure), oil in the fracture moves toward the wellbore and pressure throughout the fracture rapidly approaches the wellbore pressure. Therefore, the fracture surface acts as a constant-pressure boundary condition for the linear system. The infinite-conductivity assumption for the hydraulic fracture implies that oil production from the well is identical to oil flow rate into the fracture, hence no mass accumulation occurs in the fracture. As production continues under the constant well bottom-hole pressure condition the pressure disturbance moves away from the fracture surface and production rate decreases. The transient flow period ends when the pressure disturbance reaches the outer boundary of the system. During this flow period, the slope of the square-root-of-time or cumulative oil production plots can be analyzed and interpreted. To do so, the flow equation in the reservoir has to be solved.

![Figure 2.1 — Schematic of a hydraulically fractured well in a linear reservoir.](image)

Appendix 2A presents the flow equation (Eq. 2A.5 in Appendix 2A) for the system considered in this chapter. The governing equation is a nonlinear PDE in which permeability (which is related to pressure by permeability modulus described by Eq. 2A.4 in Appendix 2A), porosity (Eq. 2A.3 in Appendix 2A), and formation volume factor (Eq. 2A.2 in Appendix 2A) are assumed to be exponential functions of pressure. Liquid viscosity is assumed constant for simplicity.

The procedure for rate transient analysis of these stress-sensitive formations is presented in Appendix 2B. The flow term of the main PDE (Eq. 2A.5 in Appendix 2A) is linearized by the Kirchhoff transformation. The Kirchhoff variable is called pseudopressure which is a flow potential and defined such that it ranges from 1 at wellbore pressure to 0 at initial reservoir pressure. With the Kirchhoff transformation, however, the storage term remains nonlinear with hydraulic diffusivity as the nonlinearity.
For production under constant bottom-hole pressure condition, the transient linear flow period is characterized by a straight line of slope $m_{cp}$ for the plot of $\frac{\Delta m_1(p_w)}{f_c q_o}$ versus square-root of time (i.e. square-root-of-time plot) described by:

$$\frac{\Delta m_1(p_w)}{f_c(p_w)q_o} = m_{cp} \sqrt{t}. \quad (1)$$

It is more advantageous, especially for analysis of real production data, to use cumulative oil production plot instead of the square-root-of-time plot. In this plot, time in Eq. 1 is replaced by the cumulative oil production ($Q_o$) as:

$$\left[ \frac{\Delta m_1(p_w)}{f_c(p_w)} \right]^2 \frac{1}{q_o} = \frac{m_{cp}^2}{2} Q_o. \quad (2)$$

The parameters used in Eqs. 1 and 2 are defined in Appendix 2B. The slope of the square-root-of-time plot is related to the initial permeability by:

$$m_{cp} = \frac{62.564 B_o}{A_e \sqrt{k_i}} \left[ \mu_{oi} \right] \sqrt{\phi_i c_i}. \quad (3)$$

In order to use Eq. 1 or 2 for analysis of production data, the value of the correction factor, $f_c$, needs to be specified. The analytical solution for conventional oil reservoirs (with moderate pressure-dependency of rock properties) gives a correction factor of 1. However, the conventional method leads to significant error when it is used for tight oil reservoirs due to stress-dependency of these reservoirs. In this chapter, the following general equation is obtained for the correction factor (Appendix 2B):

$$\frac{1}{f_c} = -\frac{\sqrt{\pi}}{2} \left( \frac{d m_{1D}}{d \xi} \right)_{\xi=0}. \quad (4)$$

Eq. 4 needs a solution of the flow equation for evaluation of the correction factor. In the current chapter, a new method is used for solving the flow equation for tight oil reservoirs incorporating the stress-dependency of rock properties. The method is described in the following section and Appendix 2C.
2.3.2 Calculation of the Correction Factor

In this method, the nonlinearity of the storage term in flow equation (Eq. 2A.5 in Appendix 2A) is used in the definition of Kirchhoff parameter. As shown in Appendix 2C, this transformation changes the flow equation into a nonlinear diffusion-type equation (Eq. 2C.3) with $\eta$ playing the role of concentration-dependent diffusion coefficient. This non-linear parabolic PDE arises in different fields of science and engineering including heat transfer, mass diffusion, and phase redistribution. No general analytical solution exists for this differential equation for wide ranges of nonlinearity types. However, numerous researchers have focused on special classes of nonlinearities. Among them are Crank and Henry (1949a, 1949b), Fujita (1952a, 1952b, and 1954), Philip (1955), Hill (1989), Parlang et al. (1992), and Kashchiev and Firoozabadi (2003), to name a few. More references are provided in Ames (1965) and Crank (1975). In this work, the strategy includes fitting the diffusion coefficient with a function of desired form (similar to the method introduced by Kashchiev and Firoozabadi (2003)) and use of Fujita’s method (1954) to solve the resulting diffusion-type equation. Appendix 2C provides a complete discussion of the mathematics of the method. Using this method, the equation for the correction factor becomes:

$$\frac{1}{f_c} = -\sqrt{\pi} \frac{\Delta m_z(p_w)}{\Delta m_t(p_w)} \left[ \frac{1+M}{\beta \varepsilon} \right] |_{m_{20}=1}$$

The parameters $\varepsilon$ and $\theta |_{m_{20}=1}$ in Eq. 5 are obtained by the solution presented in Appendix 2C. Appendix 2C shows that the value of the correction factor depends on permeability modulus, pressure drawdown at the wellbore, and rock and liquid compressibilities, and does not depend on rock and fluid properties at initial reservoir pressure.

2.4 Correction Factor Sensitivity

Eq. 5 is used to calculate the correction factor for a linear stress-sensitive system with initial pressure of 3000 psi, rock compressibility of $1 \times 10^{-5}$ psi$^{-1}$, oil compressibility of $1 \times 10^{-5}$ psi$^{-1}$, and constant oil viscosity. The hydraulically-fractured well at the center of this system produces under a constant flowing bottom-hole pressure condition. The correction factor is calculated for a set of permeability modulus values and different values of flowing bottom-hole pressure. The
results (Fig. 2.2) show that the correction factor decreases (becomes more important) for increased values of permeability modulus and pressure drawdown.

Figure 2.2 — Plot of correction factor versus flowing bottom-hole pressure for a set of permeability modulus for a linear system with initial pressure of 3000 psi, rock compressibility of $1 \times 10^{-5}$ psi$^{-1}$, and oil compressibility of $1 \times 10^{-5}$ psi$^{-1}$.

2.5 Applications

A one-dimensional numerical model is used to generate two sets of synthetic production data for single-phase flow of oil though 1000 ft$^2$ of a hypothetical linear system during the transient period. Consider that the rock and fluid properties for the system are: $k_i=0.05$ md, $\phi_i=0.1$, $B_{oi}=1.1564$ RB/STB, $\mu_o=0.910$ cp, $A_c=1000$ ft$^2$, $c_r=1 \times 10^{-5}$ psi$^{-1}$, $c_o=1 \times 10^{-5}$ psi$^{-1}$, and $\gamma_k=4 \times 10^{-4}$ psi$^{-1}$. The well produces under two different constraints, constant well bottom-hole pressure and constant production rate at the surface. The data from the numerical model is analyzed by the method of this chapter using the correction factor. The results are presented in the following subsections.
2.5.1 Constant Pressure Production

In this case, the simulated hydraulically-fractured well produces oil under constant flowing bottom-hole pressure of about 1000 psi. The log-log diagnostic plot presented in Fig. 2.3 confirms the transient linear flow regime. The production data and the corresponding cumulative oil production plots are demonstrated in Fig. 2.4. The slope of the cumulative oil production plot is used to calculate the initial permeability of the formation. The result is then compared with the initial permeability input to the numerical model. The analysis gives an initial permeability of 0.0516 md, which is in 3.2 percent error. However, the error of conventional analysis method (i.e. using constant correction factor of 1) is approximately 32.1 percent. This shows that the error of the conventional analysis method can be reduced significantly by using the correction factor.

Figure 2.3 — The log-log diagnostic plot for the case of constant-pressure production (per 1000 ft² of fracture area).
Figure 2.4 — Production data for the constant-pressure case (a) oil production rate per 1000 ft$^2$ of the total fracture area and flowing bottom-hole pressure versus time, and (b) cumulative oil production plot.

2.5.2 Constant Rate Production

In this example, the simulated fractured well at the center of the linear system is producing under constant oil production rate of about 0.9 STBD per 1000 ft$^2$ of the total fracture area for 150 days. Similar to the constant-pressure case, the log-log diagnostic plot gives a half-slope straight line for this case, as presented in Fig. 2.5. However, the equation describing the cumulative oil production plot for constant rate production is empirically approximated by:

$$\left[ \frac{\Delta m_q(p_w)}{f_c(p_o)} \right]^2 \frac{1}{q_o} = \left( \frac{8}{\pi^2} \right) \frac{m_{Cr}^2}{2} Q_o.$$  \hspace{1cm} (6)

Fig. 2.6 show the production data and the cumulative oil production plot for the current example. Analysis of the slope of the cumulative oil production plot using Eq. 6 gives initial permeability of 0.0510 md (with about 2.0 percent error). However, if the correction factor is not used in the calculations, the error in the initial permeability will be about 25.1 percent.
Figure 2.5 — The log-log diagnostic plot for the case of constant-rate production (per 1000 ft$^2$ of fracture area).

Figure 2.6 — Production data for the constant-rate case (a) oil production rate per 1000 ft$^2$ of total fracture area and flowing bottom-hole pressure versus time, and (b) cumulative oil production plot.
2.6 Conclusions

A new method is used for calculating a correction factor for rate transient analysis of stress-sensitive oil reservoirs during transient linear flow period. The results show that the correction factor decreases (becomes more important) for increased values of permeability modulus and pressure drawdown. The correction factor is applied to the analysis of two synthetic oil production cases under constant flowing bottom-hole pressure and constant surface oil rate conditions. The results of the analysis indicate that the correction factor, combined with the pseudo-pressure function for oil, can eliminate the considerable errors associated with the conventional linear flow analysis method, which uses pressure instead of pseudo-pressure, and assumes the correction factor = 1. Based on the results of this chapter, both oil pseudo-pressure and the new correction factor are required for reliable linear flow analysis for tight, stress-sensitive oil reservoirs.

2.7 Nomenclature

Field Variables

\( A_c \quad \text{area of cross-section (ft}^2\))

\( B_o \quad \text{oil formation volume factor (RB/STB)}\)

\( c_o \quad \text{fluid compressibility (psi}^{-1}\))

\( c_r \quad \text{rock compressibility (psi}^{-1}\))

\( c_t \quad \text{total compressibility (psi}^{-1}\))

\( F \quad \text{a function a Fujita’s model}\)

\( k \quad \text{permeability (md)}\)

\( K \quad \text{a parameter in Fujita’s model}\)

\( m_1 \quad \text{pseudopressure (psi)}\)

\( m_{1D} \quad \text{dimensionless pseudopressure}\)
\( m_{2} \)  pseudopressure (psi)
\( m_{2D} \)  dimensionless pseudopressure
\( M \)  a parameter in Fujita’s model
\( p \)  pressure (psi)
\( q \)  a parameter in Fujita’s model
\( q_{o} \)  oil production rate (STBD)
\( Q_{o} \)  cumulative oil production (STB)
\( S \)  slope of the square-root-of-time plot
\( t \)  time (day)
\( x \)  distance (ft)
\( z \)  a parameter in Fujita’s model

**Greek Variables**
\( \beta_{1} \)  fitting parameter
\( \beta_{2} \)  fitting parameter
\( \gamma_{k} \)  permeability modulus (psi\(^{-1}\))
\( \varepsilon \)  a parameter in Fujita’s model
\( \eta \)  hydraulic diffusivity (ft\(^{2}\)/day)
\( \theta \)  a parameter in Fujita’s model
\( \theta_{m} \)  a parameter in Fujita’s model
\( \mu_{o} \)  oil viscosity (cp)
\( \zeta \)  Boltzmann variable
\( \phi \)  porosity
\( \phi_{i} \)  porosity at initial pressure

**Subscripts**
2.8 Acknowledgements

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2.9 References

2.10 Appendix 2A – Transient Linear Flow

The flow of a single-phase oil in a linear horizontal system in field unit system is governed by:

\[
0.00633 \frac{\partial}{\partial x} \left( \frac{k}{\mu_o B_o} \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial t} \left( \frac{\phi}{B_o} \right)
\]  (2A.1)

where \( x \) (ft) is position in the single dimension of the system, \( t \) (day) is time, \( k \) (md) is the permeability of the formation, \( \mu_o \) (cp) is oil viscosity, \( B_o \) (RB/STB) is oil formation volume
factor, \( p \) (psi) is pressure, and \( \phi \) (dimensionless) is the porosity of the rock. For a slightly compressible fluid, \( B \) is related to pressure through (Aziz and Settari, 1979):

\[
B_o = B_{oi} \exp[-c_o(p - p_i)]
\]  
(2A.2)

where \( c_o \) (psi\(^{-1}\)) is the oil compressibility factor, and \( B_{oi} \) (RB/STB) is oil formation volume factor at the initial pressure \( p_i \). Porosity of the rock is dependent on pressure due to rock compressibility \( c_r \) (psi\(^{-1}\)) and described by (Aziz and Settari, 1979):

\[
\phi = \phi_i \exp[c_r(p - p_i)]
\]  
(2A.3)

where \( \phi_i \) is porosity at \( p_i \). A similar exponential relationship is also used for the change of permeability with pressure (Pedrosa, 1986) as:

\[
k = k_i \exp[y_k(p - p_i)]
\]  
(2A.4)

where \( y_k \) (psi\(^{-1}\)) is the permeability modulus, and \( k_i \) is permeability at \( p_i \). The viscosity of oil is considered to be independent of pressure at the constant temperature of the system.

Combining Eqs. 2A.1-2A.4 yields:

\[
0.00633 \frac{\partial}{\partial x} \left( \frac{k}{\mu_o B_o} \frac{\partial p}{\partial x} \right) = \frac{\phi c_r}{B_o} \frac{\partial p}{\partial t}
\]  
(2A.5)

where the total compressibility \( c_t \) is defined by:

\[
c_t = c_r + c_o.
\]  
(2A.6)

Eq. 2A.5 is nonlinear because \( k/\mu_o B_o \) on the left side (flux differential term) and \( \phi/B_o \) on the right side (accumulation term) of the equation are functions of pressure. In order to solve this PDE, three conditions are required: the initial distribution of pressure, and the value of pressure at two different \( x \)-values (inner and outer boundary conditions).

The pressure of the semi-infinite linear system is \( p_i \) throughout, initially:

\[
p = p_i, \quad t = 0, \quad x \geq 0
\]  
(2A.7)

and remains unchanged far from the wellbore during transient period.
\[ p = p_i \quad t > 0 \quad x \rightarrow \infty. \] (2A.8)

Two kinds of boundary conditions could be assumed at \( x=0 \) (inner boundary condition) of the medium, constant pressure and constant rate, which are:

\[ p = p_w \quad t > 0 \quad x = 0 \] (2A.9)

and

\[ \frac{\partial p}{\partial x} = \frac{\mu_o B_o}{0.001127 k A_c q_o} \quad t > 0 \quad x = 0 \] (2A.10)

respectively. In Eqs. 2A.9 and 2A.10, the subscript ‘w’ is borrowed from the radial systems, where \( p_w \) (psi) is pressure at the wellbore. \( A_c \) (ft\(^2\)) is the area of the cross-section, which is constant throughout.

### 2.11 Appendix 2B – Rate Transient Analysis

A new variable (pseudopressure) is introduced in order to linearize the left side of Eq. 2A.5. The pseudopressure is essentially a potential, with its gradient proportional to the flux (Carslaw and Jaeger, 1959) and defined by:

\[ m_1(p) = \frac{\mu_i B_i}{k_i} \int_{p_0}^p \frac{k}{B \mu} d\hat{p} \] (2B.1)

where \( p_0 \) (psi) is an arbitrary reference pressure. The dimensionless form of the pseudopressure is defined as:

\[ m_{1D}(p) = \frac{\Delta m_1(p)}{\Delta m_1(p_w)} = \frac{m_1(p_i) - m_1(p)}{m_1(p_i) - m_1(p_w)} = \int_{p_i}^{p_i} \frac{k}{\mu_o B_o} d\hat{p} \] (2B.2)

Using the definition of the dimensionless pseudopressure, Eq. 2A.5 reduces to the simpler form of:

\[ \frac{\partial^2 m_{1D}}{\partial x^2} = \frac{1}{\eta(m_{1D})} \frac{\partial m_{1D}}{\partial t} \] (2B.3)
with
\[
\eta(m_{1D}) = \frac{0.00633k(m_{1D})}{\mu c_\phi(m_{1D})}
\] (2B.4)
as the hydraulic diffusivity.

The variable transformation of this kind is known as Kirchhoff transformation and dates back to Kirchhoff’s work in 1894, in which he applied this transformation for a steady state problem (Carslaw and Jaeger, 1959). In the petroleum industry, as Raghavan et al. (1972) pointed out, the Kirchhoff transformation was first used by Muskat (1949) and received marked attention when Al-Hussainy et al. (1966) introduced the concept of real gas pseudopressure. It is important to note that the Kirchhoff transformation is useful if the boundary and initial conditions of the PDE are suitably transformed (Cobble, 1967). Using the definition of the dimensionless pseudopressure (Eq. 2B.2), the conditions in Eqs. 2A.7-2A.10 change to:

\[
m_{1D} = 0 \quad t = 0 \quad x \geq 0, \\
m_{1D} = 0 \quad t > 0 \quad x \rightarrow \infty, \\
m_{1D} = 1 \quad t > 0 \quad x = 0,
\] (2B.5)

\[
\frac{\partial m_{1D}}{\partial x} = \frac{\mu}{0.001127 A_c q_o \Delta m_1(p_w)} \quad t > 0 \quad x = 0,
\] (2B.6)

respectively. Constant-pressure conditions are transformed into constant-\(m_{1D}\) counterparts, and since the gradient of \(m_{1D}\) is linearly proportional to the flux, the constant flow condition (Eq. 2A.10) (which is a pressure-dependant Neumann condition) is transferred to a Neumann-type condition of constant value (Eq. 2B.6).

If \(\eta\) on the right side of Eq. 2B.3 is assumed to be constant and evaluated at a representative pressure, say initial reservoir pressure, the PDE changes to:

\[
\frac{\partial^2 m_{1D}}{\partial x^2} = \frac{1}{\eta_i} \frac{\partial m_{1D}}{\partial t}
\] (2B.7)

where \(\eta_i\) is the value of the hydraulic diffusivity at initial pressure.
Boundary and initial conditions (Eq. 2B.5) are such that two of them can be consolidated into one using Boltzmann transformation (Ames, 1965):

$$\xi = \frac{x}{2\sqrt{\eta t}}.$$  \hspace{1cm} (2B.8)

Therefore, Eq. 2B.7 is reduced to the ordinary differential equation (ODE):

$$\frac{d^2 m_{ID}}{d\xi^2} + 2\xi \frac{dm_{ID}}{d\xi} = 0$$  \hspace{1cm} (2B.9)

with conditions:

$$m_{ID} = 0 \quad \xi \to \infty,$$
$$m_{ID} = 1 \quad \xi = 0.$$  \hspace{1cm} (2B.10)

The analytical solution to the Eq. 2B.9 (subject to 2B.10) is:

$$m_{ID}(\xi) = \text{erfc}(\xi)$$  \hspace{1cm} (2B.11)

where \(\text{erfc}\) is the complementary error function. In this paper, this approximate solution is referred to as the conventional solution of the flow equation. Replacing Eq. 2B.11 in the equation for flow rate at the wellbore:

$$q_o = -0.001127 \left( \frac{kA_c}{\mu B} \frac{\partial p}{\partial x} \right)_{\xi=0}$$  \hspace{1cm} (2B.12)

gives:

$$\frac{\Delta m_i(p_o)}{q_o} = m_{cp} \sqrt{t}.$$  \hspace{1cm} (2B.13)

where,

$$\Delta m_i(p_o) = m_i(p_o) - m_i(p)$$  \hspace{1cm} (2B.14)

and

$$m_{cp} = \frac{62.564 B_{oi}}{A_c \sqrt{k_i}} \sqrt{\frac{\mu_{oi}}{\phi_i c_i}}.$$  \hspace{1cm} (2B.15)
However, if hydraulic diffusivity in Eq. 2B.3 is not assumed constant, the equation for oil flow at the wellbore becomes:

\[
\frac{\Delta m_1(p_w)}{f_c q_o} = m_{cp} \sqrt{t}.
\] (2B.16)

where \( f_c \) is called the correction factor. The correction factor was first introduced by Ibrahim and Wattenbarger (2006). Later, Nobakht and Clarkson (2011) presented an analytical expression for the correction factor. In this chapter, the following equation is obtained:

\[
\frac{1}{f_c} = -\frac{\sqrt{\pi}}{2} \left( \frac{dm_{1D}}{d\xi} \right)_{\xi=0}
\] (2B.17)

which relates the correction factor to an exact solution of the flow equation. Eq. 2B.17 shows that a complete solution of the flow equation is required for calculating the correction factor. Appendix 2C provides a method for solution of the flow equation and calculation of the correction factor.

### 2.12 Appendix 2C – Calculation of the Correction Factor

Another transformation is defined here using the nonlinearity on the right side of Eq. 2A.5 as:

\[
m_2(p) = \frac{B_o}{\phi_i} \int_{p_i}^{p} \frac{\phi}{B_o} d\hat{p}
\] (2C.1)

with the dimensionless form as:

\[
m_{2D}(p) = \frac{\Delta m_2(p)}{\Delta m_2(p_w)} = \frac{m_2(p_i) - m_2(p_w)}{m_2(p_i) - m_2(p_w)} = \frac{\int_{p_i}^{p} \phi d\hat{p}}{\int_{p_w}^{p} \phi d\hat{p}}.
\] (2C.2)

Similar to \( m_1 \), \( m_2 \) decreases monotonically from 1 at \( p_w \) to 0 at \( p_i \). Combining Eqs. 2C.2 and 2A.5 yields:

\[
\frac{\partial}{\partial x} \left[ \eta(m_{2D}) \frac{\partial m_{2D}}{\partial x} \right] = \frac{\partial m_{2D}}{\partial t}
\] (C.3)
where
\[ \eta(m_{2D}) = \frac{0.00633k(m_{2D})}{\mu_o c_i \phi(m_{2D})}. \] (2C.4)

If the viscosity of the liquid is assumed constant and Eqs. 2A.2 to 2A.4 are used for fluid and rock properties, Eq. 2C.4 becomes:
\[ \eta(m_{2D}) = \eta_i \left[ 1 - (1 - e^{c_i(p_w - p_i)})m_{2D} \right]^{(\gamma_i - c_i)c_i}. \] (2C.5)

This transformation is suitable only for the constant pressure boundary condition at \( x=0 \), as it changes the constant rate boundary condition to a variable one. The conditions of this initial-value problem for constant pressure production read:

\[
\begin{align*}
    m_{2D} &= 0 & t &= 0 & x &\geq 0, \\
    m_{2D} &= 0 & t &= 0 & x &\to \infty, \\
    m_{2D} &= 1 & t &= 0 & x &= 0.
\end{align*}
\] (2C.6)

Applying the Boltzmann transformation to Eq. 2C.3 yields the ODE:
\[ \frac{d}{d\xi} \left[ \frac{\eta(m_{2D}) dm_{2D}}{\eta_i} \right] + 2\xi \frac{dm_{2D}}{d\xi} = 0 \] (2C.7)

with conditions:

\[
\begin{align*}
    m_{2D} &= 0 & \xi &\to \infty, \\
    m_{2D} &= 1 & \xi &= 0.
\end{align*}
\] (2C.8)

Combining Eqs. 2B.12, 2B.17 and 2C.2 gives:
\[ \frac{1}{f_c} = -\frac{\eta_w}{\eta_i} \frac{\Delta m_2(p_w)}{\Delta m_1(p_w)} \frac{\sqrt{\pi}}{2} \left( \frac{dm_{2D}}{d\xi} \right)_{\xi=0} \] (2C.9)

where \( \eta_w \) is the value of hydraulic diffusivity at the wellbore pressure. Eq. 2C.7 shows that the parameters affecting the solution of \( m_{2D} \) (as a function of \( \xi \)) are \( c_f, c_r, \gamma_k \), and pressure drawdown \( (p_w-p_i) \). On the other hand, the parameter group \( \eta_i \) has no effect on \( m_{2D}(\xi) \), and accordingly \( dm_{2D}/d\xi \).
In order to calculate \( (dm_2D/d\xi)_{\xi=0} \), it is necessary to solve Eq. 2C.7. Since the behavior of \( \eta/\eta_i \) is similar to the concentration-dependent diffusivity coefficient in Fujita’s study (1954), his solution is used for solving this equation. In order to use Fujita’s method for solving Eqs. 2C.7-2C.8, it is mathematically advantageous to approximate the nonlinearity of the equation by:

\[
\eta(m_{2D}) = \frac{1}{\beta_1 (m_{2D})^2 - 2\beta_2 m_{2D} + 1}
\]  

(2C.10)

where \( \beta_1 \) and \( \beta_2 \) are the arbitrary constants used by Fujita and are the fitting parameters in this chapter. It is important to note that the nonlinearity may also be approximated by the following exponential function:

\[
\eta(m_{2D}) = \exp \left[ -\left( 1 - e^{c(p_c-p_r)} \right) \frac{(y_k-c_r)}{c_i} m_{2D} \right]
\]  

(2C.11)

using the first two terms of the Taylor series expansion of the exponential function. With this approximation, the methods developed for diffusion problems with exponential diffusion coefficients may be used to solve Eq. 2C.7.

Fujita (1954) defines the intermediate parameter \( \theta \) (which is a complex combination of the nonlinearity, \( m_{2D} \), and \( dm_{2D}/d\xi \)) and presents \( m_{2D} \) and \( \xi \) in terms of \( \theta \) as follows:

\[
m_{2D}(\theta) = \begin{cases} 
\frac{\sqrt{\beta_1 - \beta_2^2}}{\beta_1} \tan[F(\theta;\epsilon) - \tan^{-1}(K)] + \frac{\beta_2}{\beta_1} & \text{if } 0 < \theta < 1, \quad 0 < m_{2D} < m_{2D}^* \\
\frac{\sqrt{\beta_1 - \beta_2^2}}{\beta_1} \tan[\tan^{-1}(M) - F(\theta;\epsilon) + F(\theta_m;\epsilon)] + \frac{\beta_2}{\beta_1} & \text{if } \theta_m < \theta < 1, \quad m_{2D}^* < m_{2D} < 1
\end{cases}
\]  

(2C.12)

\[
\xi(\theta) = \begin{cases} 
\frac{\beta_1}{\sqrt{\epsilon(\beta_1 - \beta_2^2)(1 + z^2)}} [z\theta + \sqrt{1 - \theta^2 - \epsilon \ln(\theta)}] & \text{if } 0 < \theta < 1, \quad 0 < m_{2D} < m_{2D}^* \\
\frac{\beta_1}{\sqrt{\epsilon(\beta_1 - \beta_2^2)(1 + z^2)}} [z\theta - \sqrt{1 - \theta^2 - \epsilon \ln(\theta)}] & \text{if } \theta_m < \theta < 1, \quad m_{2D}^* < m_{2D} < 1
\end{cases}
\]  

(2C.13)

where \( \theta_m \) and \( \epsilon \) are the roots of the system.
\[
\begin{align*}
\{ & \varepsilon \ln(\theta_m) + (1 + M^2)\theta_m^2 = 1 \\
& \tan^{-1}(K) + \tan^{-1}(M) = 2F(1; \varepsilon) - F(\theta_m; \varepsilon) 
\end{align*}
\] (2C.14)

and

\[
m^*_2D = \sqrt{\beta_1 - \beta_2^2} \tan[F(1; \varepsilon) - \tan^{-1}(K)] + \frac{\beta_2}{\beta_1} 
\] (2C.15)

with

\[
F(\theta; \varepsilon) = \int_{\theta}^{\theta_m} dq \frac{dq}{\sqrt{1 - q^2 - \varepsilon \ln(q)}}
\] (2C.16)

\[
z = \frac{\beta_1}{\sqrt{\beta_1 - \beta_2^2}} \left( m_{2D} - \frac{\beta_2}{\beta_1} \right)
\] (2C.17)

\[
K = -z_{\min} = \frac{\beta_1 \beta_2}{\sqrt{\beta_1 - \beta_2^2}}
\] (2C.18)

\[
M = z_{\max} = \frac{\beta_1 - \beta_2}{\sqrt{\beta_1 - \beta_2^2}}.
\] (2C.19)

Eqs. 2C.12 and 2C.13 give the distribution of \(m_{2D}\) over \(\zeta\). Accordingly, the value of \((dm_{2D}/d\zeta)_{\zeta=0}\) can be calculated by:

\[
\left( \frac{dm_{2D}}{d\zeta} \right)_{\zeta=0} = 2\eta_c^2 \eta_n \sqrt{\frac{1 + M}{\beta_1 \varepsilon}} \theta_{m,0=1}.
\] (2C.20)

Finally, combining Eqs. 2C.9 and 2C.20 gives the following equation for the correction factor:

\[
\frac{1}{f_c} = -\sqrt{\pi} \frac{\Delta m_c(p_w)}{\Delta m_i(p_w)} \sqrt{\frac{1 + M}{\beta_1 \varepsilon}} \theta_{m,0=1}.
\] (2C.21)
Chapter 3  Analysis of Transient Linear Flow in Tight Oil and Gas Reservoirs with Stress-Sensitive Permeability and Multi-Phase Flow²

3.1 Abstract

Horizontal wells and hydraulic fracturing are the key technologies that allow commercial production from tight oil and gas reservoirs. However, rigorous analysis of production data from these reservoirs requires incorporation of the impacts of stress-dependent permeability and multi-phase flow. Changes in the stress state of the system during production may reduce the absolute permeability. Furthermore, gas phase formation and flow in presence of supersaturated oil phase affects fluid dynamics in tight oil reservoirs. This chapter provides a rigorous methodology for incorporation of the effects of non-static permeability and multi-phase flow in rate transient analysis (RTA) of tight oil and gas reservoirs producing at variable rate/flowing pressures during transient linear flow period.

Analytical solutions for the approximate linearized form of the flow equation have been widely used as the basis for RTA tools for conventional reservoirs during transient flow period. However, they lead to considerable error when applied to tight oil and gas reservoirs. In particular, during the transient linear flow period, the slope of the square-root-of-time plot obtained from numerical solution differs from the slope calculated by analytical methods. Efforts have been made by some researchers to obtain a correction factor from the numerical solution of the flow equation to correct the slope of the square-root-of-time plot for single phase flow of gas during transient linear flow period. In this chapter, an iterative method is used for evaluation of the slope correction factor in the presence of multi-phase flow and non-static permeability for constant-pressure production during transient linear flow period. Further, the correction factor is used for analysis of production data from tight oil and gas reservoirs producing at variable rate/flowing pressures.

² This chapter is a slightly modified version of a paper presented at the SPE Unconventional Resources-Canada held in Calgary, Alberta, 5-7 November 2013 as: Qanbari, F. and Clarkson, C.R., 2013. Analysis of transient linear flow in tight oil and gas reservoirs with stress-sensitive permeability and multi-phase flow. In SPE Unconventional Resources Conference Canada. Society of Petroleum Engineers. Copyright approval has been obtained from SPE (see “Copyright Permissions” section of this thesis).
The correction factor is used in the analysis of different sets of synthetic production data for tight oil and gas reservoirs. The results show that the correction factor can reduce/eliminate the considerable errors associated with the conventional analytical methods in initial permeability estimation. For multi-phase flow cases, the producing gas-oil ratio (GOR) is used to estimate the oil saturation-pressure relationship in the reservoir, which is required for calculation pseudo-pressure and the correction factor.

The method developed in this chapter alleviates the need for using numerical simulation models to generate empirical correlations for the correction factor for the square-root-of-time plot. The easy-to-implement iterative procedure of this method only requires the pressure dependencies of the constituent elements of the hydraulic diffusivity. Therefore, this method is applicable for analysis of production profiles for variety of reservoirs with nonlinear flow equations.

3.2 Introduction

Tight oil reservoirs are stress-sensitive, even in the absence of natural fractures. The primary reason is the significant change in pore structure caused by the change in the stress state of the system during production (Lei et al., 2008). This introduces considerable error in the results of the analysis methods founded on the conventional analytical solutions of the flow equation. The error may become relatively more pronounced if free gas evolves and/or flows in the reservoir, depending on fluid and rock properties and the amount of gas saturation.

The focus of the current chapter is transient linear flow as one of the important flow regimes in tight oil and gas reservoirs associated with flow toward hydraulically-fractured wells. The new mathematical model for analysis of production data is presented in the next section based on the flow equation for undersaturated tight oil reservoirs, followed by its application to solution-gas drive and tight gas reservoir. The method is then used to analyze three sets of synthetic production data for tight oil and gas reservoirs.
3.3 Mathematical Model

3.3.1 Constant-Pressure Production

Consider a hydraulically-fractured well in a doubly-infinite linear slab reservoir producing oil at a constant well bottom-hole pressure which is above bubble point pressure. The fracture is assumed to be infinite-conductivity, implying that flow stabilizes immediately with no mass accumulation in the fracture. Under constant flowing pressure, oil flow rate declines from a high value (theoretically infinite) based on the power-law function:

\[
\frac{\Delta m(p_o)}{f_c(p_w)q_o} = m_{CP} \sqrt{t}
\]

where \( m \) (psi) is pseudo-pressure as defined in Table 3.1, \( q_o \) (STBD) is oil flow rate, \( t \) (day) is time, and \( m_{CP} \) is the slope of the square-root-of-time plot \( [\Delta m(p_o)/f_c q_o \text{ against } \sqrt{t}] \), which is defined by:

\[
m_{CP} = \frac{62.564B_{oi}}{A_c \sqrt{k_{oi}}} \sqrt{\frac{\mu_{oi}}{\phi_i c_{oi}}}
\]

for single-phase flow of oil. Equations 1 and 2 have been widely used as a tool for estimation of \( A_c \sqrt{k_i} \) or initial permeability. Another useful variation of Equation 1 is the cumulative-oil-production plot which replaces time with cumulative oil production \( Q_o \):

\[
\left[ \frac{\Delta m(p_o)}{f_c(p_w)} \right]^2 \frac{1}{q_o} = \frac{(m_{CP})^2}{2} Q_o.
\]

In this chapter, Equation 3 is preferred to Equation 1 for two reasons: first, the random noise of production data is reduced in cumulative-oil-production plot, and second, it provides a better choice for extending the constant-pressure solution to production under constant rate and the more general case of variable pressure/rate (which is dealt with at the end of this section). The correction factor in Equations 1 and 3, \( f_c \), was initially introduced by Ibrahim and Wattenbarge (2006) for tight gas reservoirs and correlated to drawdown. An analytical expression for the
correction factor was then obtained by Nobakht and Clarkson (2012a and 2012b). However, \( f_c \) was re-derived by Qanbari and Clarkson (2013) and shown to be defined by the following integral equation:

\[
\frac{1}{f_c} = \sqrt{\pi} \int_0^\xi \frac{\xi}{\eta_D} \, dm_D.
\] (4)

where the dimensionless parameters are defined in Table 3.2. Equations 1 and 4 show that the slope of the square-root-of-time plot and the correction factor are constant for a well producing under constant flowing pressure, implying that regardless of the functionality of rock and fluid properties to pressure, the log-log diagnostic plot of \( \Delta m(p_w)/f_c q_o \) (or \( 1/q_o \)) against \( t \) gives a straight half-slope line.

A solution of the flow equation is required for calculation of the correction factor using Equation 4. The simplest approach is to use conventional solution of the flow equation (erfc-solution) as an approximate solution for the original nonlinear flow equation. Therefore, the integral equation for the correction factor changes to:

\[
\frac{1}{f_c} = \sqrt{\pi} \int_0^\xi \frac{\exp(-\xi)}{\eta_D(m_D)} \, dm_D.
\] (5)

A more exact solution of the flow equation can be obtained by updating the solution using the iterative integral equation:

\[
m_D = g(\xi) = 1 - \frac{\int_0^\xi \exp\left(-2\int_0^{\xi} \frac{\xi}{\eta_D(m_D)} \, d\xi\right) d\xi}{\int_0^\xi \exp\left(-2\int_0^{\xi} \frac{\xi}{\eta_D(m_D)} \, d\xi\right) d\xi}.
\] (6)

Therefore, the equation for the correction factor becomes:

\[
\frac{1}{f_c} = \sqrt{\pi} \int_0^\xi \frac{g^{-1}(m_D)}{\eta_D} \, dm_D.
\] (7)

As Equation 6 is evaluated iteratively, the correction factor can be calculated at each step of the iteration process using Equation 7. The calculations for single-phase flow of oil show that the
method converges in the first few steps. Furthermore, the calculations show that the results of Equation 5 correlate linearly with the final results (i.e. after convergence) by:

\[
f_c = \omega_c + \frac{1 - \omega_c}{\sqrt{\pi}} \int_0^\infty \frac{erfc^{-1}(m_D)}{\eta_D(m_D)} \, dm_D
\]

(8)

with \(\omega_c\) as the correlating parameter. We have found that the correlating parameter is not sensitive to the pressure-dependent parameters and \(\omega_c = 1/3\) gives reasonably accurate results for undersaturated oil and dry gas reservoirs. Figures 3.1a and 3.1b compare the results of Equations 5-8 for an undersaturated oil reservoir and a dry gas reservoir.

Figure 3.1 — Correction factor for (a) an undersaturated oil reservoir with initial reservoir pressure of 5000 psi, permeability modulus of 5×10^{-4} psi\(^{-1}\), rock compressibility of 3×10^{-5} psi\(^{-1}\), and oil compressibility of 2×10^{-5} psi\(^{-1}\), and (b) a dry gas reservoir with the same properties as the undersaturated oil reservoir in Figure 3.1a and gas specific gravity of 0.65 (Air=1).
Table 3.1 — The governing equations, transformations, and parameter groups for oil and gas phases.

<table>
<thead>
<tr>
<th>Property</th>
<th>Oil phase</th>
<th>Gas phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow equation</td>
<td>[ \frac{\partial}{\partial x} \left( \alpha(p) \frac{\partial p}{\partial x} \right) = \beta(p) \frac{\partial p}{\partial t} ]</td>
<td>[ \frac{\partial}{\partial x} \left( a(p) \frac{\partial p}{\partial x} \right) = b(p) \frac{\partial p}{\partial t} ]</td>
</tr>
<tr>
<td>( \alpha(p) = 0.00633 \frac{k_o}{(\mu_o B_o)} )</td>
<td>( \beta(p) = \phi S_o / B_o )</td>
<td>( a(p) = 0.00633 \frac{k_g}{(\mu_g B_g)} )</td>
</tr>
<tr>
<td>Transformed flow equation</td>
<td>[ \frac{\partial^2 m_D}{\partial x_D^2} = \frac{1}{\eta_D(m_D)} \frac{\partial m_D}{\partial t_D} ]</td>
<td>[ \frac{\partial^2 m_D}{\partial x_D^2} = \frac{1}{\eta_D(m_D)} \frac{\partial m_D}{\partial t_D} ]</td>
</tr>
<tr>
<td>( \eta = \alpha / (\beta c_i) )</td>
<td>( c_i(p) = (1 / \beta)(\partial \beta / \partial p) )</td>
<td>( \eta = a / (bc_i) )</td>
</tr>
<tr>
<td>Total compressibility (c_i)</td>
<td>( m(p) = \int_{p_i}^{p} \frac{\alpha}{\alpha_i} dp )</td>
<td>( m(p) = 2 \int_{p_i}^{p} \frac{a}{a_i} dp )</td>
</tr>
<tr>
<td>Pseudo-pressure (m )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2 — Dimensionless parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionless hydraulic diffusivity (( \eta_D ))</td>
<td>( \eta_D = \eta / \eta_i )</td>
</tr>
<tr>
<td>Dimensionless time (( t_D ))</td>
<td>( t_D = t / A_e )</td>
</tr>
<tr>
<td>Dimensionless distance (( x_D ))</td>
<td>( x_D = x / \sqrt{A_e} )</td>
</tr>
<tr>
<td>Boltzmann’s variable</td>
<td>( \xi = x_D / 2 \sqrt{t_D} )</td>
</tr>
<tr>
<td>Pseudo-pressure difference (( \Delta m ))</td>
<td>( \Delta m = m(p_i) - m(p) )</td>
</tr>
<tr>
<td>Dimensionless pseudo-pressure (( m_D ))</td>
<td>( m_D = \Delta m(p) \Delta m(p_w) )</td>
</tr>
<tr>
<td>Dimensionless pressure (( p_D ))</td>
<td>( p_D = (p_i - p)/(p_i - p_w) )</td>
</tr>
</tbody>
</table>

3.3.2 Constant-Rate Production

Equations 3 and 4 have been developed for production data analysis of transient linear flow under constant flowing pressure. However, the results of this chapter show that the relationship between conventional constant-pressure and constant-rate solutions (developed based on the solution of the linearized flow equation) at the wellbore in terms of the cumulative oil production is approximately valid when we incorporate the nonlinearities of the flow equation (i.e., including the correction factor in the calculations). Therefore, as for the conventional reservoirs
(Anderson and Mattar, 2003), the slope of the cumulative-oil-production plot for constant-rate production differs from the constant-pressure counterpart by a factor of $8/\pi^2$:

$$\left[\frac{\Delta m(p_w)}{f_c(p_w)}\right]^2 \frac{1}{q_o} = \frac{4}{\pi^2} \left(\frac{m_{CP}}{Q_o}\right)^2.$$

(9)

### 3.3.3 Variable Pressure/Rate Production

For a variable pressure/rate production case, two auxiliary plots are used as the criteria to show how close the variable pressure/rate production data is to the constant-pressure or constant-rate cases. The auxiliary plots are the log-log plots of pressure function, $\Delta m(p_w)/f_c$, and the rate function, $1/q_o$, against time. For variable pressure/rate cases, both of the auxiliary plots have non-zero slopes. It is postulated that a correlation of the equations for analysis of constant-pressure and constant-rate production (Equations 3 and 9, respectively) using the slopes of the auxiliary plots can be applied to the variable pressure/rate cases. In this chapter, the results of the analysis of numerous synthetic production data (generated by numerical models) revealed that the following correlation can be used effectively for variable pressure/rate cases during transient linear flow period:

$$\left[\frac{\Delta m(p_w)}{f_c(p_w)}\right]^2 \frac{1}{q_o} = \frac{1}{m_q + m_p} \left(\frac{m_q + 8}{\pi^2} m_p\right) \left(\frac{m_{CP}}{2 Q_o}\right)^2.$$

(10)

where $m_p$ and $m_q$ are, respectively, the slopes of the auxiliary plots of pressure and rate functions. In the constant-pressure cases, $m_p$ is equal to zero and Equation 10 reduces to Equation 3. However, $m_q = 0$ for constant-rate cases, and Equation 10 changes to Equation 9. It is important to note that the pseudo-pressure difference and the correction factor in the ordinates of the cumulative-oil-production plots for constant-rate and variable pressure/rate cases (Equations 9 and 10, respectively) are functions of pressure and change with time in contrast to the constant-pressure case. At each time step, the correction factor used in Equations 9 and 10 is calculated by Equation 4, i.e. the same equation originally developed for constant-pressure case.
Figure 3.2 shows the work flow for using the correction factor for analysis of transient linear flow for the general case of variable pressure/rate production.

**Figure 3.2 — The workflow for using the correction factor for analysis of transient linear flow in stress-sensitive oil and gas reservoirs.**

### 3.4 Solution-Gas Drive Reservoirs

Figure 3.3 shows three possible zones for distribution of oil and gas during transient flow period in an initially undersaturated \( p_i > p_b \) linear oil reservoir. In Zone A far from the wellbore, pressure is higher than bubble point pressure, whereas in Zones B and C, pressure is lower than the bubble point. The front of the investigated area is located in Zone A: hence this zone is subdivided into influenced and intact regions. Saturation of free gas in Zone B is lower than the critical value for gas bulk flow initiation. Therefore, free gas in this zone remains immobile. In Zone C, saturation of the free gas is higher than the critical value and high enough for gas bulk flow to occur.

The black-oil model assigns a set of coupled partial differential equations (PDEs) for simulating the simultaneous flow of oil and gas in a solution-gas drive reservoir, which needs to be solved simultaneously for pressure and saturation distributions in the reservoir. Currently, there is no exact analytical solution for this set of PDEs. However, efforts have been made to obtain approximate analytical solutions. The preliminary step in approximate analytical methods is to
decouple the PDEs for pressure and saturation by using an approximate saturation-pressure relationship. Al-Khalifah et al. (1987a) used a linear relation between $k_o/\mu_oB_o$ and pressure, and obtained solutions for multiphase flow in terms of pressure squared. Later, Mohaghegh and Ertekin (1991) used linear pressure dependency for $k_g/\mu_gB_g$ and $k_w/\mu_wB_w$, and generated type curves for coal seam degasification wells producing under two-phase flow conditions. Analytical expressions also exist for saturation-pressure relations for two-phase flow of oil and gas (Bøe et al., 1989) and three-phase flow of oil, gas, and water (Al-Khalifah et al., 1987b).

The producing GOR has also been used to find a relation between saturation and pressure at the sandface, and was assumed applicable for the whole reservoir (Evinger and Muskat, 1942; Raghavan, 1976). The following equation is used to relate the producing GOR, sandface pressure, and oil saturation:

$$GOR = R_o + 5.615 \frac{k_{rg}}{k_{ro}} \frac{\mu_o B_o}{\mu_g B_g}. \quad (11)$$
Equation 11 is applicable when the gas saturation exceeds the critical value. However, a combination of Equation 11 and PVT data can be used to estimate the saturation-pressure relationship for reservoirs with non-zero critical gas saturation. The results of the numerical models of this chapter show that the saturation-pressure relationship obtained by the producing GOR gives a reasonable approximation of the saturation-pressure relationship in the reservoir during the transient linear flow period (Figure 3.4a shows an example). This is applicable only for a well operating under variable bottom-hole pressure (which is the most realistic case, particularly very early on for tight oil reservoirs) because multiple pressures are required to construct a complete saturation-pressure curve. The results of the numerical models show that the producing GOR stabilizes instantly at the sandface for a well producing with constant bottom-hole pressure during transient linear flow period, hence the producing GOR gives a
single point of the saturation-pressure function. However, the results for constant-pressure production show that a linear GOR-pressure relationship gives a reasonable approximation of the saturation-pressure function. The linear GOR-pressure relationship can be converted to saturation-pressure function using Equation 11. In addition to these empirical relationships, the authors are working on analytical saturation-pressure expressions for transient flow regime. Figure 3.4b compares the results of numerical simulation and Equation 11 for the oil saturation-pressure relationship of an oil reservoir with a well producing under constant-pressure condition.

It is important to note that, researchers have developed analytical (Tabatabaie and Pooladi-Darvish, 2016; Behmanesh, 2016) and empirical (Clarkson and Qanbari, 2016) methods for estimation of oil saturation as functions of pressure. The empirical equation by Clarkson and Qanbari (2016) is presented in Chapter 6 (Eq. 12).

3.5 Tight/Shale Gas Reservoirs

In tight gas reservoirs, natural gas is generated somewhere in a source rock and migrates to the tight reservoirs where it is trapped and stored in inter-particle, slot, and micro-fracture porosity (Aguilera, 2010). In shale gas reservoirs, however, natural gas is generated in the shale and remains within the shale (Spencer et al., 2011). Pore sizes steadily decrease from conventional gas to tight gas to shale gas reservoirs, with shale gas reservoirs having pore sizes in the nanoscale and permeabilities in the nanodarcy-range (Rahmanian et al., 2010). Gas flow in tight and shale gas reservoirs is affected by special transport and retention mechanisms including non-Darcy (turbulent) flow at high flow rates (e.g. in hydraulic fractures), apparent gas permeability changes due to gas slippage on pore walls (e.g. in matrix), adsorption/desorption phenomena (in organic matter), stress-sensitivity of permeability and porosity, and condensation in porous media (e.g. gas condensate reservoirs).

In this chapter, the effect of permeability change is incorporated in the RTA of tight/shale gas reservoirs during transient linear flow period by using the correction factor. Equation 10 can be used for gas reservoirs by replacing oil flow rate and cumulative production with gas flow rate and cumulative production. The slope of the square-root-of-time plot for constant-pressure production of gas, \( \Delta m(p_w)/f_c q_g \) against \( \sqrt{t} \), is as follows:
The parameters for gas reservoirs are defined in Table 3.1. Equations 10 and 12 can be used for gas condensate reservoirs with known condensate saturation-pressure relationship (Qanbari and Clarkson, 2013).

\[ m_{CP} = \frac{630TZ_i}{A_c \sqrt{k_{gi} p_i \phi_{ci}}} \sqrt{\frac{\mu_{gi}}{\phi_{ci}}}. \]  

(12)

Figure 3.4 — The results of the numerical simulation and Equation 11 for the oil saturation-pressure relationship in an oil reservoir after 50 and 100 days of production during transient linear flow period for (a) a constant-rate case and (b) a constant-pressure case.

### 3.6 Applications

Equation 10 implies that the transient linear flow period can be characterized by a straight line on the cumulative-production plot, \([\Delta m(p_w)/f_c(p_w)]^2/q_o\) versus \(Q_o\) for oil reservoirs and \([\Delta m(p_w)/f_c(p_w)]^2/q_g\) versus \(Q_g\) for gas reservoirs. Further, the slope of the cumulative-production plot is used to calculate initial permeability if fracture surface area is known (or vice versa). It is imperative to note that the straight-line analysis method for transient linear flow is analytically accurate for constant-pressure production. However, it is used as an approximate
method for analysis of production under constant flow rate and variable pressure/rate conditions. In this section, the cumulative-production plot is used to analyze three examples of synthetic variable pressure/rate production data during transient linear flow period as follows:

Example 1: an undersaturated tight oil reservoir with stress-sensitive permeability and a hydraulically-fractured well producing with variable flowing bottom-hole pressure above bubble point,

Example 2: a solution-gas drive tight oil reservoir with stress-sensitive permeability and initial reservoir pressure above bubble point and a hydraulically-fractured well producing with flowing bottom-hole pressure decreasing below bubble point, and

Example 3: a tight dry-gas reservoir with stress-sensitive permeability.

The correlations and the values of rock and fluid properties are summarized in Tables 3.3 and 3.4, respectively. In each case, a numerical model is used to generate production data based on a common well bottom-hole pressure schedule. Then the cumulative-production plot is used to estimate the value of initial permeability. In each case, the results are compared with the input to the numerical model as presented in the following sub-sections.

<table>
<thead>
<tr>
<th>Property</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity ($\phi$)</td>
<td>$\phi(p) = \phi_i \exp \left[ c_i (p - p_i) \right]$</td>
</tr>
<tr>
<td>Permeability ($k$)</td>
<td>$k(p) = k_i \exp \left[ y_i (p - p_i) \right]$</td>
</tr>
<tr>
<td>Oil relative permeability ($k_{ro}$)</td>
<td>$k_{ro} = \left( \frac{S_o}{0.2} / 0.8 \right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>Gas relative permeability ($k_{rg}$)</td>
<td>$k_{rg} = \left( \frac{S_g}{0.8} \right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>Oil formation volume factor ($B_o$)</td>
<td>$B_o(p) = B_{o_i} \exp \left[ - c_o (p - p_i) \right]$</td>
</tr>
<tr>
<td>Oil viscosity ($\mu_o$)</td>
<td></td>
</tr>
<tr>
<td>Gas viscosity ($\mu_g$)</td>
<td>Equations 4-71 and 4-76 of Danesh (1998)</td>
</tr>
<tr>
<td>Gas critical properties ($T_c, p_c$)</td>
<td>Equations 3.27 and 3.28 of Ahmed (1989)</td>
</tr>
<tr>
<td>Gas compressibility factor ($Z$)</td>
<td>Kamyab et al. (2010)</td>
</tr>
</tbody>
</table>
3.6.1 Example 1

In this case, a hydraulically-fractured well produces oil from a reservoir with initial and bubble point pressures of 5000 and 1000 psi, respectively. The production data for this example is presented in Figure 3.5a. The whole reservoir experiences single phase flow of oil as the well bottom-hole pressure is above bubble point. Oil production rate increases at very early stages of production due to the relatively sharp decrease in well-bottom-hole pressure, and then decreases after the early production peak (Figure 5a).

Table 3.4 — Rock and fluid properties for Examples 1, 2, and 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area (ft²)</td>
<td>2500</td>
</tr>
<tr>
<td>Initial reservoir pressure (psi)</td>
<td>5000</td>
</tr>
<tr>
<td>Reservoir temperature (°R)</td>
<td>580</td>
</tr>
<tr>
<td>Initial permeability (md)</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial porosity (fraction)</td>
<td>0.1</td>
</tr>
<tr>
<td>Permeability modulus (psi⁻¹)</td>
<td>5×10⁻⁴</td>
</tr>
<tr>
<td>Rock compressibility (psi⁻¹)</td>
<td>3×10⁻⁵</td>
</tr>
<tr>
<td>Undersaturated oil compressibility (psi⁻¹)</td>
<td>2×10⁻⁵</td>
</tr>
<tr>
<td>Bubble point pressure (psi) – Example 1</td>
<td>1000</td>
</tr>
<tr>
<td>Bubble point pressure (psi) – Example 2</td>
<td>4000</td>
</tr>
<tr>
<td>Oil gravity (API)</td>
<td>30</td>
</tr>
<tr>
<td>Gas specific gravity (Air=1) – Examples 1 and 2</td>
<td>0.8</td>
</tr>
<tr>
<td>Gas specific gravity (Air=1) – Example 3</td>
<td>0.65</td>
</tr>
<tr>
<td>Water saturation</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Pressure reduction at the wellbore reduces the permeability at the sandface and in the investigated area. Figure 3.5b shows the change in permeability multiplier (ratio of permeability at a specific pressure to initial permeability) at the sandface with time. In conventional RTA methods, the change in permeability is included in the calculation of the pseudo-pressure, but the correction factor is assumed unity. The advanced method used in this chapter, however, includes the correction factor in the analysis of the production data.
Figure 3.5 — Plots for Example 1 (a) well bottom-hole pressure and oil production rate versus time, and (b) permeability multiplier at the sandface versus time.

Equation 7 is used to calculate the correction factor as a function of pressure drawdown at the wellbore. The results are plotted against time in Figure 3.6a. The value of the correction factor is consequently used in the cumulative-oil-production plot as shown in Figure 3.6b, which gives the slope of 2659. The slopes of the auxiliary plots of pressure and rate functions in Equation 10, \( m_p \) and \( m_q \), are 0.371 and 0.215, respectively. Using the values of the slopes in Equation 10 and 2 gives initial permeability of 0.0994 md, which is in an error of 0.63 % with respect to the input to the numerical model. However, if the correction factor is not used in the analysis of the production data, the error in the initial permeability increases to about 53 percent. This shows that the conventional RTA method is in considerable error when applied to a stress-sensitive oil reservoir during transient linear flow period. Furthermore, the effectiveness of incorporating the correction factor in reducing the error of the conventional RTA method is illustrated.
3.6.2 Example 2

The initial and bubble point pressures of this example are 5000 and 4000 psi, respectively. The well produces with the same bottom-hole pressure schedule as Example 1. Therefore, gas phase formation occurs near the hydraulic fracture due to the pressure reduction below bubble point. Figure 3.7a presents the production data including well bottom-hole pressure, oil production rate, and producing GOR for the current example. The producing GOR remains constant and equal to solution gas ratio at the bubble point \( R_{sb} \) as long as well bottom-hole pressure is higher than the bubble point. As production continues, the GOR reduces due to extremely low gas relative permeability at low gas saturation. Later in the course of production, the GOR increases and exceeds \( R_{sb} \) due to increased value of gas relative permeability near the wellbore. The value of GOR is used to calculate oil saturation using Equation 11, and the results are presented in Figure 3.7b. This saturation-pressure relationship is assumed to be applicable for the whole reservoir and used for calculating the pseudo-pressure and the correction factor.
Figure 3.7 — Plots for Example 2 (a) well bottom-hole pressure, producing GOR, oil production rate versus time, and (b) oil saturation-pressure plot.

The correction factor is calculated at each time step using Equation 7. The results (Figure 3.8a) show that \( f_c \) decreases for increased values of pressure drawdown at the wellbore. The correction factor is used to construct the cumulative-oil-production plot as shown in Figure 3.8b. The slopes of the cumulative-oil-production plot and the auxiliary plots of pressure and rate functions are, respectively, 1253, 0.304, and 0.318. Therefore, Equations 10 and 2 give the initial permeability of 0.0984 md which is in approximately 1.6 % error. The error is far less than the error of the conventional RTA method (using the correction factor of unity) which is about 56 percent.
Figure 3.8 — Plots for Example 2 (a) the correction factor versus time, and (b) the cumulative-oil-production plot.

### 3.6.3 Example 3

This tight dry-gas reservoir (with specific gravity of 0.65) has identical reservoir properties and operational conditions as Examples 1 and 2. Figure 3.9 shows the plots of well bottom-hole pressure and gas production rate versus time. The production rate peaks at the early production stage followed by a continuous decline.
Figure 3.9 — Well bottom-hole pressure and gas production rate versus time for Example 3.

Similar to Examples 1 and 2, the correction factor is calculated using Equation 7 and plotted in Figure 3.10a. The results indicate that the correction factor becomes more important for higher values of pressure drawdown. Moreover, the correction factor for this example is relatively lower (more important) than the values for Examples 1 and 2. The slopes of the cumulative-gas-production plot (Figure 3.10b) and the auxiliary plots of pressure and rate function become 7.803, 0.394, and 207, respectively. This gives initial permeability of 0.0999 md (0.1 percent error). On the other hand, the error of the conventional RTA method (using $f_c = 1$) in estimating the initial permeability of the current example is about 95 percent. This shows that the new RTA method using the correction factor can eliminate the considerable error of the widely used conventional method. Furthermore, comparing the results of the analysis of these three examples shows that the conventional method is less effective when the correction factor deviates more from unity. This can be explained by the following equation for the error of the conventional RTA method in estimating the initial permeability, $E_{Conv}$ (Qanbari and Clarkson, 2013):


\[
E_{\text{Conv}} = \frac{1-(f_c)^2}{(f_c)^2} \times 100.
\]  

(13)

Figure 3.10 — Plots for Example 3 (a) the correction factor versus time, and (b) the cumulative-gas-production plot.

3.7 Conclusions

A new method is used to incorporate stress-sensitivity of permeability in rate transient analysis of tight oil and gas reservoirs and correct for multi-phase flow in tight oil cases. A correction factor is used to correct the slopes of the square-root-of-time and cumulative-production plots. The method is applied to RTA of different examples of tight oil and gas wells producing under variable pressure/rate conditions. For multi-phase flow cases, we provide an approximate method for generating a saturation-pressure relationship, which is required for pseudo-pressure calculations, that has proven effective. The results using simulated cases show that the correction factor can reduce/eliminate the considerable error of the conventional RTA technique in estimating the initial reservoir permeability of stress-sensitive reservoirs experiencing single or multiphase flow.
3.8 Nomenclature

Field Variables

\( a \) function of pressure in gas flow equation

\( A_c \) area of cross-section (ft\(^2\))

\( b \) function of pressure in gas flow equation

\( B_o \) oil formation volume factor (RB/STB)

\( B_{oi} \) oil formation volume factor at initial pressure (RB/STB)

\( c_o \) oil compressibility (psi\(^{-1}\))

\( c_r \) rock compressibility (psi\(^{-1}\))

\( c_t \) total compressibility (psi\(^{-1}\))

\( E_{Conve} \) error of the conventional RTA method in permeability estimation (\%) 

\( f_c \) correction factor

\( k \) permeability (md)

\( k_i \) permeability at initial pressure (md)

\( m \) pseudo-pressure (psi)

\( m_{CP} \) slope of the square-root-of-time plot for constant-pressure production

\( m_D \) dimensionless pseudo-pressure

\( m_p \) slope of the log-log plot of \( p_w \) function vs. time

\( m_q \) slope of the log-log plot of rate function vs. time

\( p \) pressure (psi)

\( p_0 \) reference pressure (psi)

\( p_b \) bubble point pressure (psi)

\( p_i \) initial pressure (psi)
\( p_w \) — well bottom-hole pressure (psi)

\( q_g \) — gas flow rate (MSCFD: 1000 standard cubic feet/day)

\( q_o \) — oil flow rate (STBD)

\( Q_g \) — cumulative gas production (MSCF: 1000 standard cubic feet)

\( Q_o \) — cumulative oil production (STB)

\( R_s \) — solution gas-oil ratio (SCF/STB)

\( R_s \) — solution gas-oil ratio at bubble point (SCF/STB)

\( t \) — time (day)

\( t_D \) — dimensionless time

\( x \) — distance (ft)

\( x_D \) — dimensionless distance

**Greek Variables**

\( \alpha \) — function of pressure in oil flow equation

\( \beta \) — function of pressure in oil flow equation

\( \gamma_k \) — permeability modulus (psi\(^{-1}\))

\( \Delta \) — difference

\( \eta \) — hydraulic diffusivity (ft\(^2\)/day)

\( \eta_i \) — hydraulic diffusivity at initial pressure (ft\(^2\)/day)

\( \eta_D \) — dimensionless hydraulic diffusivity

\( \mu_o \) — oil viscosity (cp)

\( \mu_{oi} \) — oil viscosity at initial pressure (cp)

\( \zeta \) — Boltzmann variable

\( \phi \) — porosity

\( \phi_i \) — porosity at initial pressure
Subscripts

\( c \) cross-section, correction, capillary
\( CP \) constant pressure
\( D \) dimensionless
\( i \) initial
\( k \) permeability
\( o \) oil
\( r \) rock
\( t \) total
\( w \) well
\( \theta \) reference

Abbreviations

GOR gas-oil ratio
PDE partial differential equation
PVT pressure-volume-temperature
RTA rate transient analysis

3.9 Acknowledgments

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3.10 References

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Chapter 4  A New Method for Production Data Analysis of Tight and Shale Gas Reservoirs during Transient Linear Flow Period³

4.1 Abstract

Pseudo-pressure has historically been used in analytical solutions of the diffusivity equation for analysis of real gas flow in conventional gas reservoirs. The accuracy of analytical solutions during the transient flow period is contingent on the validity of the assumption of constant hydraulic diffusivity which is implicit in the background formulations. However, the assumption of pseudo-pressure-independent hydraulic diffusivity during transient flow is not valid for the cases of high pressure drawdown at the wellbore. For tight and shale gas reservoirs, this dependency is more pronounced due to the complexities associated with non-Darcy, adsorption/desorption phenomena, stress-sensitivity of permeability and porosity, and condensation in porous media.

The current chapter focuses on rate transient analysis of tight and shale gas reservoirs during transient linear flow period for a single fractured well producing under constant well bottom-hole pressure. The results of the analytical solution of real gas flow in porous media are corrected for the effects of high pressure drawdown, non-static permeability, and condensate formation. The method proposed in this chapter includes three key elements: introducing a measure of nonlinearity (departure of dimensionless hydraulic diffusivity from linearity); differential and integral formulation of the correction factor (used to correct the slope of the square-root-of-time plot); implementing the iterative integral method for solution of flow equation; and evaluating the correction factor for constant-pressure production during transient linear flow period. The results show that the correction factor becomes more important for higher values of drawdown, permeability modulus, and condensate saturation.

³ This chapter is a slightly modified version of a paper published in the Journal of Natural Gas Science and Engineering as: Qanbari, F. and Clarkson, C.R., 2013. A new method for production data analysis of tight and shale gas reservoirs during transient linear flow period. Journal of Natural Gas Science and Engineering, 14, 55-65. Copyright approval has been obtained from the journal (see “Copyright Permissions” section of this thesis).
4.2 Introduction

Development of production data analysis tools requires solutions of the flow equations in space and time domains. Flow equations are nonlinear partial differential equations (PDE) with pressure and saturation (multi-phase flow cases) as the unknown functions of space and time. The generally accepted solutions for the diffusivity equation (Van Everdingen and Hurst, 1949), flow of real gases (Aronofsky and Jenkins, 1954; Al-Hussainy et al., 1966), and multiphase flow in solution gas drive reservoirs (Raghavan, 1976; Bøe et al., 1989) are founded on the assumption of constant hydraulic diffusivity. If hydraulic diffusivity is a moderate function of pressure (in diffusivity equation) and/or pseudopressure (for real gases, stress-sensitive formations, and solution gas drive reservoirs) and/or pressure drawdown is not high, the assumption is acceptable. Otherwise, the analysis leads to considerable errors. In particular, for tight and shale gas reservoirs the assumption is violated due to the special flow and storage mechanisms. This calls for more elaborate solution methods and/or correction of the conventional solutions.

In this chapter, an iterative integral method is used to calculate a correction factor (which corrects the slope of the square-root-of-time plot obtained by the analytical solution) for constant pressure production during transient linear flow in dry gas and gas condensate reservoirs incorporating the effects of pressure drawdown at the wellbore, stress-sensitive permeability, and multiphase flow.

This paper is organized as follows. The mechanisms affecting flow and storage of gas in tight and shale gas reservoirs and their impacts on well performance are discussed in Section 4.3. Transient linear flow is discussed in Section 4.4. Section 4.5 introduces the nonlinearity measure of dimensionless hydraulic diffusivity. The production data analysis tools for transient linear flow of gas are discussed in Section 4.6. Section 4.6 also explains the differential and integral formulation of the correction factor and discusses implementation of iterative integral method for evaluation of the correction factor. The results of the calculations for dry gas and gas condensate reservoirs are presented and discussed in Section 4.7. Finally, we end with Section 4.8, conclusions.
4.3 Tight and Shale Gas Reservoirs

A tight gas reservoir is a reservoir that cannot be produced at economic flow rates nor recover economic volumes of natural gas unless the well is stimulated by a large hydraulic fracture treatment or produced by use of a horizontal wellbore or multilateral wellbores (Holditch, 2006). In tight gas reservoirs, natural gas is generated somewhere else (usually is a shale formation) and migrates to the tight reservoirs where it is trapped and stored in inter-particle, slot, and micro-fracture porosity (Aguilera, 2010). In shale gas reservoirs, however, natural gas is generated in the shale and remains within the shale (Spencer et al., 2011). Moving from conventional gas to tight gas to shale gas reservoirs, pore sizes steadily decrease, with shale gas reservoirs having pore sizes in the nano-scale and permeabilities measured in nanodarcies (Rahmanian et al., 2010).

Gas flow in tight and shale gas reservoirs is affected by special transport and retention mechanisms including non-Darcy flow at high flow rates (e.g. in hydraulic fractures), apparent gas permeability due to gas slippage on pore walls (e.g. in matrix), adsorption/desorption phenomena, stress-sensitivity of permeability and porosity, and condensation in porous media (in gas condensate reservoirs). In the following subsections, these mechanisms and their impacts on well performance in tight and shale gas reservoirs are reviewed.

4.3.1 Non-Darcy Flow Due to Inertial Effects

At high flow rates in porous media, pressure drop is not proportional to fluid velocity (non-Darcy flow). Firoozabadi and Katz (1979) summarize different authors’ views on flow mechanisms at high flow rates. Some researchers (Fancher et al., 1933; Katz et al., 1959) connect non-Darcy flow to the turbulent flow by comparing flow of fluids through porous media and fluid flow in pipes. The Forchheimer number (Ma and Ruth, 1993; Zeng and Grigg, 2006) has also been used for identifying the beginning of non-Darcy flow in porous media. However, Geertsma (1974) believes that within the flow range normally experienced in oil and gas reservoirs, energy losses caused by actual turbulence can be ignored, and the observed departure from Darcy’s law is the result of convective accelerations and decelerations of the fluid particles on their way through the pore space. Hassanizadeh and Gray (1987) used the general continuum approach to develop a
nonlinear relationship between the pressure gradient and the flow velocity. They concluded that the nonlinear dependence of interfacial drag forces on the flow velocity can give rise to the nonlinear behavior of flow at high velocities. Regardless of the true mechanisms for non-Darcy flow, Forchheimer’s nonlinear relationship (1901) or its variations are commonly used for non-Darcy flow at high flow rates. In terms of well performance analysis, non-Darcy flow increases pressure drop in the high-velocity region near wellbore, and within hydraulic fractures for fractured wells. This phenomenon may lead to underestimation of formation permeability if Darcy’s law is used. Several researchers (Holditch and Morse, 1976; Umnuayponwiwat et al., 2000; Gil et al., 2003, to name a few) have studied the effects of non-Darcy flow on the performance of gas wells.

4.3.2 Non-Darcy Flow Due to Slip-Flow, Transitional Flow and Diffusion

Apparent permeability of formations with comparable pore diameter to gas molecular size is different from intrinsic permeability, which is a property of the porous media. This effect can be explained by taking into account the phenomena of slip, which are related to mean free paths of the gas molecules and approximated by a linear function of the reciprocal mean pressure (Klinkenberg, 1941). The apparent gas permeability of a tight gas reservoir is related to the Knudsen number as the ratio of mean-free-path of molecules to the hydraulic (pore) radius (Civan, 2010; Civan et al., 2011). Swami et al. (2012) used different approaches for prediction of apparent gas permeability of tight gas reservoirs. They found that the apparent permeability becomes more important for reservoirs with nano-pores, that is mainly shale gas and some tight gas reservoirs. Wu et al. (1998) obtained analytical solutions for gas flow in porous media with Klinkenberg effects. Clarkson et al. (2012) incorporated the dynamic slippage concept of Ertekin et al. (1986) in production data analysis of tight and shale gas reservoirs.

4.3.3 Adsorption/Desorption Phenomena

Gas molecules under pressure have tendencies to adsorb physically on organic solid surfaces in coal and shale gas reservoirs due to the molecular interaction forces. Pressure disturbance during production changes the thermodynamic equilibrium conditions and leads to desorption of the gas molecules. There are two different groups of methods proposed for modeling the adsorption and
desorption phenomena; equilibrium and non-equilibrium methods. The equilibrium methods consider no intermediate stages with adsorption or desorption, i.e. the adsorbed and the free phase systems are in immediate equilibrium. Equilibrium methods include Langmuir method, Dubinin Radushkevich/Astakhov equations, ideal adsorption model (Myers and Prausnitz 1965), modified vacancy solution model (Clarkson, 2003), simplified local density model (Rangarajan et al., 1995), two-dimensional equations of state (Zhou et al., 1994). The non-equilibrium methods are used to model the time-dependency of the adsorption and desorption phenomena. The proposed non-equilibrium methods are absolute rate theory (Elliott and Ward, 1997; Rudzinski and Panczyk, 2002), sticking probability approach (Becker and Hartman, 1953; Ehrlich, 1956; Kisliuk, 1957), statistical rate theory (Ward, 1983; Rudzinski and Panczyk, 2002), and statistical rate theory of interfacial transport (Rudzinski and Panczyk, 2001).


4.3.4 Non-Static Permeability

In all reservoirs, the changes in pressure and temperature induced by recovery operations are accompanied by changes of stress state (Settari et al., 2005). In stress-sensitive reservoirs, including tight oil and gas reservoirs, changes in the stress state of the system during production may reduce the absolute permeability, which in turn results in productivity loss. According to the theory of poro- and thermo-elasticity, supported by laboratory evidence, porosity and absolute permeability are functions of effective stress (Settari et al., 2005). Reservoir rocks may exhibit an exponential or linear relationship between absolute permeability and mean effective stress or a power-law relationship between effective permeability and porosity (Raghavan and Chin, 2004). The exponential relationship between absolute permeability and mean effective stress is:
\[ k = k_0 e^{-b_k (\sigma'_m - \sigma'_{m0})} \] 

(1)

where \(k\) (md) is permeability at mean effective stress \(\sigma'_m\) (psi), \(b_k\) (psi\(^{-1}\)) is characteristic parameter of the rock, and \(k_0\) (md) is the permeability at reference mean effective stress \(\sigma'_{m0}\).

Eq. 1 is commonly used to correlate the experimental data for permeability changes with mean effective stress. However, a relationship between permeability and pore pressure is required for uncoupled models commonly used for fluid flow in reservoirs. Yilmaz et al. (1994) defined the permeability modulus (compliance) \(\gamma_k\) as the sensitivity of the permeability to the pore pressure \(p\) at constant confining pressure \(p_{con}\):

\[
\gamma_k = \frac{1}{k} \left( \frac{\partial k}{\partial p} \right)_{p_{con}}.
\]

(2)

Therefore, for invariant permeability modulus, permeability is related to pore pressure by:

\[ k = k_0 e^{\gamma_k (p - p_0)} \]

(3)

where \(k_0\) (md) is permeability at reference pressure \((p_0)\). Settari et al. (2005) introduced a systematic approach to translate the rock mechanics lab data (including permeability measured as a function of effective stress) to functions of pore pressure. Clarkson et al. (2012) obtained a relationship between permeability modulus and the rock characteristic parameter \(b_k\) for uniaxial stress state as:

\[
\gamma_k = b_k \alpha_B \left[ 1 - \frac{2}{3} \left( \frac{1 - 2\nu}{1 - \nu} \right) \right]
\]

(4)

where \(\alpha_B\) is Biot’s constant and \(\nu\) is Poisson’s ratio. For the less realistic case of free reservoir deformation (in which deformations are free in all directions) the relationship between the permeability modulus and \(b_k\) changes to:

\[ \gamma_k = b_k \alpha_B \]

(5)

Non-static permeability has been accounted for shale gas and CBM reservoirs (Thompson et al., 2010; Clarkson et al., 2009). Qanbari and Clarkson (2012) developed a method for production
data analysis of undersaturated tight oil reservoirs during transient linear flow period. They studied the effect of non-static permeability on the slope of the square-root-of-time plot.

4.3.5 Gas Condensation in Porous Media

Pressure reduction below dew point pressure in gas condensate reservoirs leads to condensation of the intermediate and heavy molecules from the gas phase. The results of experimental studies on the effect of porous media on equilibrium behavior of gas condensates are somewhat contradictory with respect to whether or not equilibrium is attained in these systems (Saeidi and Handy, 1974).

The condensate phase remains immobile until its saturation reaches a critical value. It is expected that capillary forces, which are the major factors in governing multiphase flow in oil reservoirs, play a less important role relative to gravity and viscous forces in gas condensate reservoirs (Danesh et al., 1988). The key factors that control gas and condensate flow in porous media are critical condensate saturation and relative permeability relationships. The measured values of critical condensate saturation are in the range of 10 to 50 percent (Li and Firoozabadi, 2000).

Fang et al. (1996) used a network of circular capillary tubes for the representation of porous media and modeled the effects of surface tension and contact angle hysteresis on critical condensate saturation. They found that critical condensate saturation increases with increase in surface tension and contact angle hysteresis. Wang and Mohanty (1999) used a pore network model to study the effects of phase trapping and pore connectivity on critical condensate saturation. Their study showed that the critical condensate saturation is a function of pore geometry, water saturation, and interfacial tension.

Experimental (Gravier et al., 1986) and modeling (Li and Firoozabadi, 2000) studies show abrupt changes of relative permeability to gas near critical condensate saturation. Field data (Engineer, 1985; Barnum et al., 1995) shows that condensate formation can result in severe loss in flow capacity of individual wells and therefore gas recovery. Numerous researchers (Fussel et al., 1973; Jones and Raghavan, 1988; Vo et al., 1989) have used analytical models for well performance analysis of gas condensate reservoirs.
4.4 Transient Linear Flow

Linear flow in tight gas reservoirs can be associated with hydraulic fractures or special geometrical effects (Arévalo-Villagrán et al., 2006). Other examples of reservoir geometries for which linear flow can be developed are channel sands, wells between parallel faults, and stratified reservoirs in which low-permeability layers drain into high-permeability ones (Kohlhaas et al., 1982). El-Banbi (1998) explains various linear models for fractured wells and wells producing in reservoirs with high-permeability streaks. Figure 4.1 shows the schematic for an ideal linear slab reservoir with a hydraulically-fractured well. As the well starts producing with constant bottom-hole pressure, the pressure disturbance propagates progressively throughout the reservoir with the front (which has different practical definitions) moving at the rate of:

\[ v_f = \alpha_c \sqrt{\frac{\eta_{max}}{t}} \]  

(6)

where \( \alpha_c \) is a numerical factor, and \( \eta_{max} \) is the maximum value of hydraulic diffusivity in the pressure range of well bottom-hole pressure and initial reservoir pressure. For this ideal linear system, the transient flow period spans from the start of production to the time that the front of pressure disturbance reaches the boundary of the reservoir. The transient flow period lasts a few weeks to half a year for conventional gas reservoirs (Walsh and Lake, 2003). However, based on Eq. 6, tight gas reservoirs have prolonged transient flow period due to low permeability (and low hydraulic diffusivity). Field data shows that this flow regime may continue for years for tight gas reservoirs (Stright and Gordon, 1983; Hale, 1986; Wattenbarger et al, 1998; El-Banbi and Wattenbarger, 1998).

![Figure 4.1 — The schematic for an ideal linear flow in a linear slab reservoir with a hydraulically-fractured well.](image)
Flow of real gases in a linear system is governed by the nonlinear partial differential equation (PDE) presented in Appendix 4A. For dry gas reservoirs, the parameters are all functions of pressure. The commonly used approximate analytical method is the one introduced by Aronofsky and Jenkins (1954) and Al-Hussainy et al. (1966). They use the Kirchhoff transformation and linearize the flow term of the PDE (Appendix 4B). However, the transformed flow equation remains nonlinear with dimensionless hydraulic diffusivity as the nonlinearity on the right hand side:

\[
\frac{\partial^2 \psi_D}{\partial x_D^2} = \frac{1}{\eta_D(\psi_D)} \frac{\partial \psi_D}{\partial \tau_D}.
\]

For constant pressure production, boundary and initial conditions for transient linear flow becomes:

\[
\begin{align*}
\psi_D = 0 & \quad t_D = 0 \quad x_D \geq 0, \\
\psi_D = 0 & \quad t_D > 0 \quad x_D \to \infty, \\
\psi_D = 1 & \quad t_D > 0 \quad x_D = 0.
\end{align*}
\]

Analytical solutions to the transformed flow equation are required for production data analysis. This chapter attempts to find solutions under the conditions of Eq. 8.

### 4.5 Nonlinearity Measure

If dimensionless hydraulic diffusivity is assumed constant and evaluated at a representative pressure \( (p_R) \), Eq. 7 can be solved analytically. During transient flow period, initial reservoir pressure (Aronofsky and Jenkins, 1954; Al-Hussainy et al., 1966) and average pressure in the region of influence (Anderson and Mattar, 2007; and Nobakht and Clarkson, 2012a, 2012b) have been used as the representative pressure. The validity of these assumptions depends on the degree of nonlinearity of dimensionless hydraulic diffusivity. A quantitative measure of the nonlinearity \( (M_n) \) of dimensionless hydraulic diffusivity is defined as the root mean square (RMS) of its deviation from the specified constant value and formulated as (Emancipator and Kroll, 1993):
\[
M_n = \sqrt{\int_0^1 (\eta_D - 1)^2 \, d\psi_D}
\] (9)

Dimensionless hydraulic diffusivity \( \eta_D \) is the only nonlinearity of the transformed flow equation (Eq. 7). The constituent elements of \( \eta_D \) are effective permeability, porosity, gas saturation, gas viscosity and formation volume factor, as described by:

\[
\eta_D = \frac{\eta}{\eta_i}
\] (10)

where \( \eta_i \) is hydraulic diffusivity at initial pressure and:

\[
\eta = \frac{a}{b'_p}
\] (11)

\[
a(p) = 0.00633 \frac{k(p)k_{rg}(S_g)}{\mu_g(p)B_g(p)}
\] (12)

\[
b(p) = \frac{\phi(p)S_g}{B_g(p)}.
\] (13)

\( b'_p \) in Eq. 11 is derivative of \( b \) with respect to pressure. These parameters, except for gas saturation, are explicit functions of pressure. Table 4.1 lists these parameters and the corresponding relationships used in this chapter. Gas saturation and pressure distributions are obtained simultaneously by numerical solution of flow equations for gas and liquid phases. For analytical purposes, however, the relationship between gas saturation and pressure during transient linear flow period is assumed to be known (decoupling of saturation and pressure). Therefore, the gas flow equation is the only equation to be solved for pressure distribution.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability ( k )</td>
<td>( k = k_i \exp \left[ \psi_i(p - p_i) \right] )</td>
</tr>
<tr>
<td>Gas relative permeability ( k_{rg} )</td>
<td>( k_{rg} = S_g^2 )</td>
</tr>
<tr>
<td>Porosity ( \phi )</td>
<td>( \phi = \phi_i \exp \left[ \psi_i(p - p_i) \right] )</td>
</tr>
</tbody>
</table>

Table 4.1 — The correlations for rock and fluid properties.
Gas viscosity ($\mu_g$)  
Eqns. 2-71 and 2-76 of Danesh (1998)

Gas formation volume factor ($B_g$)  
$$B_g(p) = \frac{p_{sc}}{T_{sc}} \frac{Z(p)T}{p} = 0.02827 \frac{Z(p)T}{p}$$

Gas critical properties  
Eqns. 3.27 and 3.28 of Ahmed (1989)

Gas compressibility factor ($Z$)  
Kamyab et al. (2010)

---

We postulate that the accuracy of the analytical solution obtained under constant $\eta_D$ assumption depends on the value of the nonlinearity measure, $M_n$. This is explained in the following sections in the context of the development of a tool for analysis of production data for gas reservoirs during transient linear flow period.

### 4.6 Production Data Analysis Tools

Transient linear flow period is characterized by a straight line for plots of $\Delta \psi(p_w)/q_g$ versus $\sqrt{t}$ or $1/q_g$ versus $\sqrt{t}$ with slopes of $m_{CP}$ and $m_{CP}'$, respectively. These slopes can be used to estimate properties of the reservoir or hydraulic fracture. However, suitable models are required to relate the slopes to the reservoir and well completion properties. These relationships are obtained by solution of the flow equation. The generally accepted solution is the pseudo-pressure analogy (Aronofsky and Jenkins, 1954; Al-Hussainy et al., 1966) of the solutions initially identified by Carslaw (1921) for heat conduction in solids, Theis (1935) for groundwater hydraulics, and Van Everdingen and Hurst (1949) for oil reservoirs. In this method, dimensionless hydraulic diffusivity in the transformed flow equation is assumed constant and evaluated at initial reservoir pressure (i.e. $\eta_D=1$). Therefore, the slopes of the square-root-of-time plots become:

$$m_{CP,\eta_D=1} = \frac{630TZ}{\frac{1}{\phi_c c_i}} \frac{\mu_g}{A_c \sqrt{k_{gi} p_i}} \sqrt{\frac{\mu_g}{\phi_c c_i}}$$  \hspace{1cm} (14)

and
Eq. 15 is used to estimate one of the reservoir parameters, initial permeability or permeability modulus, provided that fracture half-length is known. Initial permeability is calculated directly from Eqs. 14 and 15. However, iterative procedures are used to calculate permeability modulus for known values, or probability distribution function, of initial permeability.

A variation of the solution above is to evaluate dimensionless hydraulic diffusivity at a representative pressure chosen between well bottom-hole pressure and initial reservoir pressure. Therefore, the relationships for the slopes of the square-root-of-time plots for this method change to:

\[ m'_{\eta_{DR}} = \sqrt{\eta_{DR}} m_{\eta_{DR}=1} \]  

(16) and

\[ m'_{\eta_{DR}} = \sqrt{\eta_{DR}} m'_{\eta_{DR}=1} \]  

(17)

where \( \eta_{DR} \) is dimensionless hydraulic diffusivity at the representative pressure.

Ibrahim and Wattenbarger (2006) defined a correction factor to correct the slope of square root of time plot obtained by the liquid solution for dry gas during transient linear flow period. It is equivalent to \( \sqrt{\eta_{DR}} \) in Eqs. 16 and 17. The correction factor \( f_c \) is defined as:

\[ f_c = \frac{m}{m_{\eta_{DR}=1}} = \frac{m'}{m'_{\eta_{DR}=1}}. \]  

(18)

They used the results of a numerical model and obtained an empirical equation for the correction factor as a function of a dimensionless drawdown parameter. They concluded that the analytical solution deviates from the exact gas solution for higher levels of drawdown. Nobakht and Clarkson (2012a, and 2012b) analytically derived the correction factor for shale gas reservoirs.

As explained in Appendix 4C, the correction factor is equal to:
The correction factor is unity if dimensionless hydraulic diffusivity is assumed constant and equal to 1 (corresponding to the initial reservoir pressure). However, in most cases, the change of dimensionless hydraulic diffusivity with dimensionless pseudopressure has to be included in calculation of the correction factor. If an exact solution of the flow equation is available in the form of $\psi_D(\zeta)$, Eq. 19 can be used to calculate the correction factor. However, no general analytical solution exists for wide ranges of nonlinearity ($\eta_D$) types. On the other hand, the problem with approximate methods is that they do not necessarily give a good approximation of the derivative of the dimensionless pseudopressure with respect to $\zeta$ (and the correction factor). However, they may give good approximations of the integral of the dimensionless pseudopressure and its functions over the domain. Therefore, a new method is used as follows for calculation of the correction factor from an approximate solution of the flow equation. Eq. 4B.11 is rearranged to the form:

$$\frac{d}{d\psi_D} \left( \frac{d\psi_D}{d\zeta} \right) = -\frac{2}{\eta_D} \frac{d\zeta}{d\psi_D}.$$  \hspace{1cm} (20)

Integrating both sides of Eq. 20 gives:

$$\left( \frac{d\psi_D}{d\zeta} \right)_{\psi_D=1} = -2 \int_{\eta_D}^{1} \frac{d\zeta}{d\psi_D} d\psi_D$$  \hspace{1cm} (21)

assuming that $\left( \frac{d\psi_D}{d\zeta} \right)_{\psi_D=0} = 0$ during transient flow period. Therefore, the correction factor becomes:

$$\frac{1}{f_c} = \sqrt{\pi} \int_{\eta_D}^{1} \frac{d\zeta}{d\psi_D} d\psi_D.$$  \hspace{1cm} (22)

Eq. 22 relates the correction factor to an integral (with the solution of the flow equation in the integrand) rather than the derivative (Eq. 19). Therefore, this equation is more advantageous than Eq. 19 for calculation of the correction factor.
An approximate solution of the flow equation is required in order to evaluate the integral in Eq. 22. In this study, the iterative integral method is used as follows. Integrating Eq. 4B.11 twice and applying the initial conditions in Eq. 4B.12 gives an approximate solution of the PDE as follows:

$$
\psi_D(\xi) = 1 - \frac{\int_0^\xi \exp\left(-2\int_0^\xi \frac{\xi}{\eta_D} d\xi\right) d\xi}{\int_0^\infty \exp\left(-2\int_0^\xi \frac{\xi}{\eta_D} d\xi\right) d\xi}
$$

(23)

The distribution of the dimensionless pseudopressure is iterated in Eq. 23 and a new value for the correction factor is evaluated. The iteration process continues until the correction factor converges. The initial guess for the distribution of the dimensionless pseudopressure is assumed to be the distribution corresponding to $\eta_D=1$, which is essentially the conventional analytical (error function) solution of the problem.

Comparing Eq. 18 with 16 or 17 gives the relationship between the suitable representative pressure and the correction factor as:

$$
\eta_{DR} = f_c^2.
$$

(24)

Equivalently, if real time ($t$) is changed to the pseudo-time ($t_a$):

$$
t_a = (f_c^2)t,
$$

(25)

in nonlinear flow equations, the conventional linearized solution will give accurate results for transient linear flow period.

### 4.7 Results and Discussion

The correction factor is introduced to correct the slope of the square-root-of-time plot obtained by analytical solution of the flow equation under the condition $\eta_D=1$. Results are presented for constant-pressure production during transient linear flow period for stress-sensitive dry gas and gas condensate reservoirs. The correction factor is calculated for different values of well bottom-hole pressure, permeability modulus, and condensate saturation-pressure relationships for a dry gas reservoir and a gas condensate reservoir. The idea is to present the correction factor in the
form of graphs for practical usage in production data analysis. Therefore, the correction factor is plotted against the nonlinearity measure (as a combination of the key parameters affecting the correction factor).

### 4.7.1 Example 1 – Simulated Dry Gas Reservoirs

A dry gas reservoir with initial pressure of 4000 psi, gas specific gravity of 0.65 and rock compressibility of $1 \times 10^{-5}$ psi$^{-1}$ is assumed. Reservoir rock and fluid properties are calculated by the equations provided in Table 4.1. Dimensionless hydraulic diffusivity is obtained as a function of dimensionless pseudo-pressure for different values of well bottom-hole pressure for permeability modulus of $5 \times 10^{-4}$ psi$^{-1}$ (Figure 4.2a) and different values of permeability modulus for well bottom-hole of 2000 psi (Figure 4.2b).

![Figure 4.2 — Dimensionless hydraulic diffusivity versus dimensionless pseudo pressure for a Example 1for (a) different values of well bottom-hole pressure and permeability modulus of $5 \times 10^{-4}$ psi$^{-1}$, and (b) different values of permeability modulus and well bottom-hole pressure of 2000 psi.](image)

Figures 4.2a and 4.2b show that $\eta_D$ deviates from unity at higher values of drawdown and permeability modulus. Deviation of $\eta_D$ from unity can be presented using the nonlinearity measure described above. Figure 4.3 plots the nonlinearity measure versus well bottom-hole pressure for different values of permeability modulus. The figure shows that the nonlinearity
measure increases with increased value of pressure drawdown at the wellbore and permeability modulus.

Figure 4.3 — Nonlinearity measure versus well bottom-hole pressure for Example 1 for different values of permeability modulus.

The iterative integral method is used to evaluate the correction factor for the dry gas reservoir of Example 1. The results show that the correction factor becomes more important for higher values of drawdown and permeability modulus (Figure 4.4a). If the correction factor is plotted against the nonlinearity measure, the plots of Figure 4.4a collapse onto a single line as shown in Figure 4.4b. Therefore, the nonlinearity measure is introduced as a parameter that reflects the combined effects of pressure drawdown and the permeability modulus on the correction factor of dry gas reservoirs. The values of the correction factor for this example are less than unity, which implies that the analytical method (without correction) underestimates the initial permeability if it is applied to production data analysis of dry gas reservoirs. The relative error in initial permeability estimated by the analytical method ($E_{\text{analytical}}$) is related to the correction factor by:

$$E_{\text{analytical}} (%) = 100 [1 - (f_c)^2].$$

(26)
which is higher for higher values of drawdown and permeability modulus (lower values of the correction factor). The results show that $E_{\text{analytical}}$ for this example can be as high as 60 percent. Therefore, the correction factor is necessary for correction of the results of the analytical methods.

![Figure 4.4 — Correction factor versus (a) well bottom-hole pressure, and (b) nonlinearity measure, for dry gas reservoir of Example 1 for different values of the permeability modulus.](image)

### 4.7.2 Example 2 – Simulated Gas Condensate Reservoirs

Consider three gas condensate reservoirs A, B, and C with identical gas specific gravity of 0.85, permeability modulus of $5 \times 10^{-4}$ psi$^{-1}$, initial pressure of 4000 psi, and upper dew point pressure ($p_{ud}$) of 3000 psi. The relationship between condensate saturation and pressure for the three reservoirs are plotted in Figure 4.5. The maximum condensate saturation for reservoirs A, B, and C (which occurs at the pressure of 2000 psi for the all the reservoirs) during transient linear flow period are 0.03, 0.06, and 0.09, respectively. The critical condensate saturation for all the reservoirs is 0.15. It is assumed that the condensate-pressure relationships of Figure 4.5 do not change for constant pressure production during the transient flow period. As noted in Chapter 3, researchers have developed analytical (Tabatabaie and Pooladi-Darvish, 2016; Behmanesh, 2016) and empirical (Clarkson and Qanbari, 2016) methods for estimation of oil saturation as
functions of pressure. The empirical equation by Clarkson and Qanbari (2016) is presented in Chapter 6 (Eq. 12). The saturation-pressure correlation of Clarkson and Qanbari (2016) for a gas condensate system is:

\[ S_o(p) = \frac{S_{o,\text{base}}(p)}{1 + (k_{ro})_{S_{o,\text{base}}}(p)} \cdot \frac{p_i Z_i (p_d)}{p_d} \cdot \frac{1}{\text{erfc}^{-1} \left( \frac{p_i - p_d}{p_i} \right)} \]  

(27)

where the base saturation values are calculated as:

\[ S_{o,\text{base}}(p) = 5.61 \left( \frac{R_{v,dew} - R_s(p)}{10^6 - R_s(p)R_v(p)} \right) \cdot \frac{B_o(p)}{B_{gd,dew}(p)} \]  

(28)

All the parameters of Eqs. 27 and 28 are presented in the Nomenclature section.

![Figure 4.5](image.png)

Figure 4.5 Condensate saturation versus pressure for gas condensate reservoirs A, B, and C.

The results of the calculations show that presence of condensate phase in the reservoir increases the nonlinearity measure of the hydraulic diffusivity, which in turn affects the value of the correction factor. Figure 4.6a shows the correction factor corresponding to gas condensates A, B,
and C plotted against well bottom-hole pressure for constant pressure production during transient linear flow period. Plotting the correction factor against the nonlinearity measure collapses the curves of condensate reservoirs A, B, and C onto the one for the dry gas reservoir, as shown in Figure 4.6b. Figure 4.6b is used for transient linear flow analysis of wells producing with constant pressure in stress-sensitive dry gas and gas reservoirs. The nonlinearity measure is calculated by Eq. (9) and the correction factor is read from Figure 4.6b. The correction factor is then used in Eq. (18) to correct the analytical expression for the slope of the square-root-of-time plot.

This study focused on the effects of the key parameters, well bottom-hole pressure, permeability modulus, and condensate saturation, on the correction factor using the correlations given in Table 1 and condensate saturation-pressure relationships of Figure 4.5. More studies are required to investigate the effects of other pressure-dependent rock and fluid properties and correlations.

![Figure 4.6](image)

**Figure 4.6** Correction factor versus (a) well bottom-hole pressure, and (b) nonlinearity measure, for gas condensate reservoirs A, B, and C.

### 4.7.3 Example 3 – Gas Condensate Field Case

Production data from a shale gas condensate well in North America is analyzed in this section. Initial reservoir pressure and reservoir temperature are 5100 psia and 120°F, respectively.
Production data (including gas rate, condensate-gas ratio, and bottom-hole pressure) is shown in Figure 4.7. Production data is smooth in the first four months; however, the well experiences operational problems (probably due to liquid loading) later in the life of the well. Similar to the previous simulated cases, the correction factor is calculated using the PVT data (Figure 4.8). Condensate saturation in Figure 4.8 is estimated using Eqs. 27 and 28. The correction factor at each well flowing pressure is calculated using the iterative integral method developed in this study and plotted in Figure 4.9. From Figure 4.9, it is observed the correction factor for low flowing bottomhole pressures is very large, meaning that conventional linear flow analysis assuming single-phase flow in the reservoir would be in significant error.

Figure 4.7 Production data for the gas condensate field case.
Figure 4.8 PVT properties of the gas condensate field case.

Figure 4.9 Correction factor versus well flowing pressure for the gas condensate field case.
4.8 Conclusions

In this work we have provided, for the first time, a practical method for correcting the square-root of time plot for the effects of drawdown, stress sensitivity and condensate dropout in low permeability gas condensate reservoirs. This correction can be significant and of importance for reservoir characterization, where transient linear flow is observed and one wishes to obtain an estimate of hydraulic fracture half-length or permeability (they cannot be determined independently from transient linear flow alone).

A nonlinearity measure is introduced as a measure of the departure of dimensionless hydraulic diffusivity from unity. The differential and integral representations of the correction factor are obtained. Furthermore, the iterative integral method is used to solve the flow equation and evaluate the correction factor for constant-pressure production during transient linear flow period. The method is used for dry gas and gas condensate reservoirs. The results show that the correction factor becomes more important for higher values of drawdown, permeability modulus, and condensate saturation. More studies are required to investigate the effects of other pressure-dependent rock and fluid properties and correlations.

4.9 Nomenclature

Field Variables

\( a \) function of pressure in flow equation

\( A_c \) area of cross-section (ft\(^2\))

\( b \) function of pressure in flow equation

\( b_k \) a characteristic parameter of the rock (psi\(^{-1}\))

\( B_g \) gas formation volume factor (RB/STB)

\( B_{gd} \) gas formation volume factor based on dry gas only (rcf/scf)

\( B_{gd,dew} \) gas formation volume factor based on dry gas only at dew-point pressure (rcf/scf)

\( B_o \) oil formation volume factor (RB/STB)
$c_r$  rock compressibility (psi$^{-1}$)

$c_t$  total compressibility (psi$^{-1}$)

$c_{ti}$  total compressibility at initial reservoir pressure (psi$^{-1}$)

$f_c$  correction factor

$E_{analytical}$  error of the analytical method (%)

$k$  permeability (md)

$k_0$  permeability at $p_0$ or $\sigma'_w$ (md)

$k_g$  gas effective permeability (md)

$k_{gi}$  gas effective permeability at initial reservoir pressure (md)

$k_i$  permeability at initial pressure (md)

$k_{rg}$  gas relative permeability

$k_{ro}$  oil relative permeability

$m_{CP}$  slope of the square root of time plot

$m_{CP}'$  slope of the square root of time plot

$M_n$  nonlinearity measure

$p$  pressure (psi)

$p_0$  reference pressure (psi)

$p_{con}$  confining pressure (psi)

$p_d$  dew point pressure (psi)

$p_i$  initial reservoir pressure (psi)

$p_{sc}$  pressure at standard conditions (psi)

$p_w$  well bottom-hole pressure (psi)

$q_g$  gas flow rate (Mscf/day)

$q_{gD}$  dimensionless gas flow rate
$R_s$ solution gas-oil ratio (scf/STB)

$R_v$ solution condensate-gas ratio (STB/MMscf)

$R_{vi}$ solution condensate-gas ratio at initial reservoir pressure (STB/MMscf)

$S_c$ condensate saturation

$S_g$ gas phase saturation

$S_{g,\text{base}}$ gas base saturation (in Eqs. 27)

$S_o$ oil saturation

$S_{o,\text{base}}$ oil base saturation (in Eqs. 28)

$t$ time (day)

$t_a$ pseudotime (hr)

$t_D$ dimensionless time

$T$ temperature (°R)

$T_{sc}$ temperature at standard conditions (°R)

$x$ distance (ft)

$x_D$ dimensionless distance

$Z$ gas compressibility factor

$Z_i$ gas compressibility factor at initial reservoir pressure

Greek Variables

$\alpha_B$ Biot’s constant

$\alpha_c$ a numerical factor

$\gamma_k$ permeability modulus (psi$^{-1}$)

$\Delta$ difference

$\eta$ hydraulic diffusivity (ft$^2$/day)

$\eta_D$ dimensionless hydraulic diffusivity
\( \eta_{DR} \) dimensionless hydraulic diffusivity at representative pressure

\( \eta_i \) hydraulic diffusivity at initial reservoir pressure

\( \eta_{max} \) maximum hydraulic diffusivity (ft\(^2\)/day)

\( \mu_g \) gas viscosity (cp)

\( \mu_{gi} \) gas viscosity at initial reservoir pressure (cp)

\( \nu \) Poisson’s ratio

\( \xi \) Boltzmann variable

\( \sigma'_m \) mean effective stress (psi)

\( \sigma'_{w0} \) reference mean effective stress (psi)

\( \phi \) porosity

\( \phi_i \) porosity at initial pressure

\( \psi \) pseudopressure (psi)

\( \psi_D \) dimensionless pseudopressure

**Subscripts**

\( a \) apparent or pseudo

\( B \) Biot

\( c \) cross-section, correction, capillary

\( CP \) constant pressure

\( D \) dimensionless

\( g \) gas phase

\( i \) initial

\( k \) permeability

\( p \) pressure
4.10 Acknowledgments

The authors would like to thank sponsors of Tight Oil Consortium (TOC) for their support of this research.

4.11 References


4.12 Appendix 4A – Flow Equation

Flow of gas phase in a linear horizontal system is governed by:

\[
\frac{\partial}{\partial x} \left[ a(p) \frac{\partial p}{\partial x} \right] = \frac{\partial b(p)}{\partial t}
\]  

(4A.1)

where

\[
a(p) = 0.00633 \frac{k(p)}{\mu_g(p)B_g(p)}
\]  

(4A.2)

and

\[
b(p) = \frac{\phi(p)}{B_g(p)}
\]  

(4A.3)

for dry gas reservoirs, and:

\[
a(p) = 0.00633 \frac{k(p)k_{rg}(S_g)}{\mu_g(p)B_g(p)}
\]  

(4A.4)

and

\[
b(p) = \frac{\phi(p)S_g}{B_g(p)}
\]  

(4A.5)

for gas condensate reservoirs. Eqs. 4A.1-4A.5 are in field unit system where \(x\) (ft) is position in the single dimension of the system, \(t\) (day) is time, \(p\) (psi) is pressure, \(k\) (md) is permeability, \(\phi\) (dimensionless) is the porosity of the rock, \(k_{rg}\) is relative permeability to gas, \(S_g\) is gas saturation,
μg (cp) is gas viscosity, and \(B_g\) (RB/STB) is gas formation volume factor. Boundary and initial conditions for production under constant wellbore pressure \(p_w\) during transient period read:

\[
\begin{align*}
  p &= p_i & t = 0 & x \geq 0 \\
  p &= p_i & t > 0 & x \rightarrow \infty \\
  p &= p_w & t > 0 & x = 0.
\end{align*}
\]  

(4A.6)

4.13 Appendix 4B – Variable Transformations

Similar to the pseudopressure defined by Aronofsky and Jenkins (1954) and Al-Hussainy et al. (1966), a new variable (called pseudopressure) is defined based on the nonlinearities on the left side of equation above as:

\[
\psi(p) = \frac{2}{a_i} \int_{p_i}^{p} a(p) d\hat{p}
\]  

(4B.1)

where \(p_0\) (psi) is the reference pressure and subscript ‘i’ denotes values at initial pressure. The values of the parameters at the initial pressure are used to give the pseudopressure the same dimension as the real pressure. The pseudopressure is essentially a potential, with its gradient proportional to the flux (Carslaw and Jaeger, 1959). The variable transformation of this kind is known as Kirchhoff transformation and dates back to Kirchhoff’s work in 1894, in which he applied this transformation for a steady state problem (Carslaw and Jaeger, 1959). The dimensionless form of the pseudopressure is:

\[
\psi_D(p) = \frac{\psi(p_i) - \psi(p)}{\psi(p_i) - \psi(p_w)} = \frac{\int_{p_i}^{p} a(p) d\hat{p}}{\int_{p_w}^{p_i} a(p) d\hat{p}}
\]  

(4B.2)

which varies from 0 at initial pressure to 1 at wellbore pressure. Combining Eqs. 4A.1 and 4B.2 yields:

\[
\frac{\partial^2 \psi_D}{\partial x^2} = \frac{1}{\eta(\psi_D)} \frac{\partial \psi_D}{\partial t}
\]  

(4B.3)

where the hydraulic diffusivity \((\eta_D)\) is:
\[ \eta(\psi_D) = \frac{a(\psi_D)}{b_p(\psi_D)}. \] \hspace{1cm} (4B.4)

Dimensionless distance, time, and hydraulic diffusivity are defined by:

\[ x_D = \frac{x}{\sqrt{A_c}} \] \hspace{1cm} (4B.5)

and

\[ t_D = \frac{\eta_i - t}{A_c} \] \hspace{1cm} (4B.6)

and

\[ \eta_D = \frac{\eta}{\eta_i} \] \hspace{1cm} (4B.7)

respectively, where \( \eta_i \) is hydraulic diffusivity at initial pressure. Using the dimensionless parameters, Eq. 4B.3 changes to:

\[ \frac{\partial^2 \psi_D}{\partial x_D^2} = \frac{1}{\eta_D(\psi_D)} \frac{\partial \psi_D}{\partial t_D}. \] \hspace{1cm} (4B.8)

Boundary and initial conditions change to:

\[
\begin{align*}
\psi_D &= 0 \quad t_D = 0 \quad x_D \geq 0, \\
\psi_D &= 0 \quad t_D > 0 \quad x_D \to \infty, \\
\psi_D &= 1 \quad t_D > 0 \quad x_D = 0.
\end{align*}
\] \hspace{1cm} (4B.9)

Boundary and initial conditions are such that two of them can be consolidated into one using the Boltzmann transformation (Ames, 1965):

\[ \xi = \frac{x_D}{2\sqrt{t_D}}. \] \hspace{1cm} (4B.10)

Therefore, Eq. 4B.8 is reduced to the nonlinear ordinary differential equation (ODE):

---

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\[
\frac{d^2 \psi_D}{d\xi^2} + \frac{2}{\eta_D} \xi \frac{d\psi_D}{d\xi} = 0 \tag{4B.11}
\]

with conditions:

\[
\psi_D = 0 \quad \xi \to \infty, \\
\psi_D = 1 \quad \xi = 0. \tag{4B.12}
\]

### 4.14 Appendix 4C – Production Data Analysis Tools

Gas flow rate at standard conditions \(q_g\) from a hydraulically fractured well located at the center of a linear system is given by:

\[
q_g = -6.33 \times 10^{-6} \times 2 \left( \frac{k_A c \partial p}{\mu_B g \partial x} \right)_{x=0} \tag{4C.1}
\]

where \(q_g\) is in Mscf/day. Using the definition of \(\psi_D\) and the Chain Rule, Eq. 4C.1 becomes:

\[
\frac{1}{q_{gD}} = 2\pi f_c \sqrt{\pi \xi_D} \tag{4C.2}
\]

where \(q_{gD}\) is dimensionless flow rate:

\[
q_{gD} = \frac{711 q_T \mu_g Z_i}{k_g p_t \sqrt{A_c \left[ \psi(p_t) - \psi(p_w) \right]}} \tag{4C.3}
\]

and \(f_c\) is a correction factor:

\[
\frac{1}{f_c} = \frac{\sqrt{\pi} \left( \frac{d\psi_D}{d\xi} \right)_{\xi=0}}{2}. \tag{4C.4}
\]

The value of \(f_c\) depends on the variation of dimensionless hydraulic diffusivity with dimensionless pseudopressure, and does not depend on rock and fluid state properties at initial reservoir pressure. Eq. 4C.2 implies that the log-log plot of reciprocal of dimensionless rate versus dimensionless time is a straight line of slope 0.5 during transient linear flow period. Another useful form of Eq. 4C.2 is:
where \( m_{CP} \) is the slope of the plot of \( \frac{\psi(p_i) - \psi(p_w)}{q_g} \) versus \( \sqrt{t} \):

\[
m_{CP} = \frac{630Tf_c Z_i}{A_c \sqrt{k_{gi} p_i \phi_i c_{ii}}} \sqrt{\frac{\mu_{gi}}{\phi_i c_{ii}}}.
\]

(4C.6)

The value of slope \( m_{CP} \) is used to evaluate \( A_c \sqrt{k_{gi}} \) or permeability modulus. If the permeability modulus is known, \( A_c \sqrt{k_{gi}} \) is calculated from:

\[
A_c \sqrt{k_{gi}} = \frac{630Tf_c Z_i}{m_{CP} p_i} \sqrt{\frac{\mu_{gi}}{\phi_i c_{ii}}}.
\]

(4C.7)

However, an iterative procedure is required to evaluate the permeability modulus for known value of \( A_c \sqrt{k_i} \) because both pseudopressure and \( f_c \) depend on the permeability modulus. Two methods are used to evaluate the permeability modulus from the production data. The first method is to use the slope of the plot of the reciprocal of gas flow rate versus square root (\( m'_{CP} \)) to evaluate the permeability modulus:

\[
m'_{CP} = \frac{630Tf_c Z_i}{A_c \sqrt{k_{gi} p_i [\psi(p_i) - \psi(p_w)]} \sqrt{\phi_i c_{ii}}} \sqrt{\frac{\mu_{gi}}{\phi_i c_{ii}}}.
\]

(4C.8)

The second method is to use an initial guess for the permeability modulus, then find the slope \( m_{CP} \), and reevaluate the permeability modulus from the slope. This process continues until the convergence is achieved. In this method, calculation of slope is a part of the iteration process.
Chapter 5   Effects of Pore Confinement on Rate-Transient Analysis of Shale Gas Reservoirs

5.1 Abstract

It is common practice to use PVT data measured in laboratories (i.e. bulk fluid properties) for reservoir modeling and production data analysis purposes. However, theoretical studies have shown that fluid properties (including critical properties, phase behavior, viscosity, density, etc.) change under nano-scale confinement in shale reservoirs due to interactions between the fluid molecules and the pore walls (pore proximity effects). In addition to these effects, in nanopores, an adsorbed layer can form which could cause changes in the apparent permeability to the free gas phase. The purpose of this chapter is to incorporate the effects of pore proximity and adsorbed layer thickness changes into rate-transient analysis. The transient linear flow period in particular is studied for this purpose as it is often the dominant flow period in shale gas wells.

An iterative integral method is used to solve the nonlinear partial differential equation (PDE) for fluid flow in the presence of pore proximity effects and changing adsorbed layer thickness. Fluid properties (i.e. viscosity and density) under confinement are simulated using critical property adjustments and conventional fluid property correlations. The simplified local density model is used to estimate the thickness of the adsorbed layer, and this is coupled with an apparent permeability equation, which accounts for diffusion and slippage phenomena, to quantify permeability alteration in the presence of an adsorbed layer. Stress-sensitivity of permeability is also accounted for.

The results of our analysis using simulated data show that neglecting proximity effects in linear flow analysis leads to overestimation of the linear flow parameter (i.e. $x_f \sqrt{k_i}$) in nanoporous shale reservoirs.

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4 This chapter is a slightly modified version of a paper presented at SPE/CSUR Unconventional Resources Conference–Canada held in Calgary, Alberta, 30 September-02 October 2014 as: Qanbari, F., Haghshenas, B. and Clarkson, C.R., 2014, September. Effects of Pore Confinement on Rate-Transient Analysis of Shale Gas Reservoirs. In SPE/CSUR Unconventional Resources Conference–Canada. Society of Petroleum Engineers. Copyright approval has been obtained from SPE (see “Copyright Permissions” section of this thesis).
reservoirs. This in turn could cause errors in the derivation of fracture half-length, if permeability is known, or vice-versa.

The new modified analytical rate-transient analysis tools and procedures provided in this work will lead to improved linear flow analysis, should pore proximity and confinement effects be important. In general, this method can be used for inclusion of different pressure-dependent fluid and rock properties and processes in the analysis of shale gas reservoirs.

5.2 Introduction

Transient linear flow is often the dominant flow regime in multi-fractured horizontal wells, and is characterized by a straight line on a square-root-time plot, which is a plot of rate-normalized pseudopressure (for gas reservoirs) or rate-normalized pressure (for oil reservoirs) against square-root of time (Wattenbarger et al. 1998). Square-root-time plots are used to estimate fracture half-length provided that an estimation of the initial formation permeability is given. The slope of the pseudopressure-based square-root-time plot (used for gas reservoirs) overestimates fracture half-length, whereas the pressure-based version (used for oil reservoirs) underestimates fracture half-length. Therefore, the slope of the square-root-time plot needs to be corrected for high drawdown (Ibrahim and Wattenbarger 2006; Nobakht and Clarkson 2012a,b), stress-sensitivity of permeability (Qanbari and Clarkson 2013a,b), and multi-phase flow (Qanbari and Clarkson 2013c) using a correction factor; pseudopressure should also be used for oil reservoirs before calculating the correction factor. The combination of pseudopressure and correction factor accounts for the impact of pressure-dependent rock and fluid properties and multi-phase flow on the slope of square-root-time plot and consequently improves the accuracy of the calculated fracture half-length.

With the exception of non-Darcy flow, the effects of pore confinement have never been considered in rate-transient analysis. It has recently been suggested that fluid properties in nanopores differ from bulk fluid properties, and methods for correcting gas viscosity and compressibility factor have been provided (Zarragoicoechea and Kuz 2002, 2004; Singh et al. 2009; Ma et al. 2013). Because these properties affect flow of gas through the nanoporous
matrix of shales, failure to correct for alteration of the properties due to pore confinement may lead to errors in rate-transient analysis.

Another effect that has not been considered in rate-transient analysis is the possible change in hydraulic radius of nanopores due to change in adsorbed layer thickness (Haghshenas et al. 2014). A gradient in fluid density exists within nanopores with an increase in fluid density in proximity to the pore walls. The existence of this higher density adsorbed layer, whose thickness changes with pressure, alters the flow of free gas through the nanoporosity, causing an additional change in gas permeability.

As with other pressure-dependent properties, we can account for the effects of pore confinement and adsorbed layer thickness change in linear flow analysis by including them in pseudopressure calculations and the correction factor for the square-root-time plot.

In the following Theory section, we first explain how we account for alteration of fluid properties due to pore confinement, and then summarize how non-Darcy flow, adsorbed layer thickness changes, and stress-dependent permeability are calculated. Next we describe how these pressure dependencies are used in the calculation of pseudopressure and the correction factor. Finally, in the Application section, a step-by-step procedure for rate-transient analysis is provided using a simulated example of a dry shale gas reservoir. The impact of neglecting the effects of pore confinement and pore radius changes are demonstrated with this example.

5.3 Theory

5.3.1 Effect of Pore Confinement on Fluid Critical Properties

Gas adsorption-desorption hysteresis at low temperatures is a characteristic of mesoporous adsorbents characterized by distinct step changes in the adsorbed amount with pressure (Burgess et al. 1989; Machin 1994). It is associated with capillary condensation under pore confinement (Evans et al. 1986). The hysteresis loops, however, shrink with increasing temperature and eventually disappear at some capillary critical temperature that depends on the average pore size but lies substantially below the critical temperature of the bulk fluid (Ball and Evans 1989). Morishige et al. (1997) showed experimentally that capillary critical temperatures for argon,
nitrogen, oxygen, ethylene, and carbon dioxide decrease substantially with decreased pore size. They further demonstrated that plots of normalized capillary critical temperature against the ratio of molecular diameter to pore radius for all the fluids except CO$_2$ form a single line passing through the origin. Zarragoicoechea and Kuz (2002, 2004) used the van der Waals equation of state and Lennard-Jones potential equations to predict vapor-liquid equilibria and capillary critical properties of the confined fluids in nanopores and found good agreement with the Morishige et al.’s (1997) experimental results (the critical pressure shift was not measured experimentally). They presented the following quadratic equation for critical temperature and pressure adjustment:

$$\frac{T_c - T_{cp}}{T_c} = \frac{p_c - p_{cp}}{p_c} = 0.9409 \frac{\sigma}{r_p} - 0.2415 \left( \frac{\sigma}{r_p} \right)^2$$

(1)

where $T_c$ (°R) is the critical temperature of bulk gas, $T_{cp}$ (°R) is the critical temperature of confined gas, $p_c$ (psi) is the critical pressure of bulk gas, $p_{cp}$ (psi) is the critical pressure of confined gas, $\sigma$ (nm) is the size parameter of Lennard-Jones potential, and $r_p$ (nm) is pore radius. Figure 5.1 illustrates the normalized critical temperature shift using the Lennard-Jones parameter of $\sigma = 0.244 \sqrt{T_c / p_c}$ (Teklu et al. 2014). The modification of gas compressibility and viscosity as a result of pore confinement is illustrated in Figure 5.2a and 5.2b, respectively; gas properties are calculated with conventional correlations (Danesh 1998) with the use of the modified critical properties. Teklu et al. (2014) used Eq. 1 and liquid-gas capillary pressure to investigate the effect of pore confinement on the phase envelope of a Bakken oil sample. Singh et al. (2009) used configurational-bias grand-canonical transition-matrix Monte Carlo simulations to model critical property alterations for alkanes. Ma et al. (2013) presented correlations for the critical property alterations based on Singh et al.’s results. The results of Singh et al.’s simulation studies, however, are not verified experimentally.
Figure 5.1 — Normalized critical temperature shift for methane as a function of pore radius.

Figure 5.2 — Methane compressibility factor (a) and viscosity (b) versus pressure for different pore sizes.
5.3.2 Gas Slippage/Diffusion (Non-Darcy Flow)

Theoretical and experimental studies have shown that low-pressure gas flow in nanoporous shales is influenced by the interaction between gas molecules and the pore walls. Gas flow at low pressures exceeds values calculated using bulk (viscous) flow and Knudsen diffusion models. The reason is that the assumption of zero gas velocity at the pore wall in continuum theory is violated in nanopores at low pressure values; that is, gas molecules slide (or “slip”) along the pore walls. To describe this behavior, a slippage factor ($F>1$) is used to correct the viscous flow assumption for the positive impact of gas slippage on flow rate (Javadpour 2009; Swami et al. 2012; Swami and Settari 2012) and an apparent permeability is defined which accounts for slippage and diffusion. Figure 5.3a illustrates the ratio of apparent to absolute permeability versus pressure for methane (calculated using Javadpour’s model); apparent permeability increases dramatically with decreased pressure.

Although the current work is concerned with gas flow in nanopores, we note that liquid flow in nanopores may also be greater than that explained by continuum theory. In Appendix 5A we have provided a brief discussion of this point, which will be explored in future work and applied to experimental measurement and modeling of oil and condensate flow in nanoporous shales.

5.3.3 Adsorbed Layer Changes

Apparent permeability models that account for non-Darcy flow (slippage/diffusion) assume constant pore radius and bulk gas properties, neglecting the effect of the adsorbed layer thickness and its change with pressure, and gas property change due to pore proximity. Haghshenas et al. (2014) implemented the simplified local density model to calculate the thickness of the adsorbed layer and used the effective radius for gas flow (pore radius minus the adsorbed layer thickness) in the gas slippage/diffusion models. Figure 5.3b illustrates the ratio of apparent to absolute permeability versus pressure for methane after including the adsorbed layer thickness. We observe that the effect of adsorbed layer thickness change is most pronounced for pore radii less than 4 nm, causing a reduction of apparent permeability (relative to the constant pore radius case shown in Figure 5.3a) particularly at high pressures.
Gas adsorbed on the pore walls acts as a source during depressurization and a sink during pressurization. As a source/sink term, desorption/adsorption is expressed in the form of a compressibility term added to the total compressibility of the system (Bumb and McKee 1988; Clarkson et al. 2007). Total compressibilities of a methane-saturated system with and without adsorption term are shown in Figure 5.4; the adsorption term makes a difference, particularly at higher pressures.
5.3.4 Stress-Sensitive Permeability

Laboratory experiments have also demonstrated that permeability and porosity of shales change significantly with net-confining stress (Brezovski and Cui 2013; Cui et al. 2010) or pore pressure (for constant overburden pressure). In this work, permeability modulus is used to relate permeability to pore pressure (Yilmaz et al., 1994) which is related to Biot’s constant and Poisson’s ratio (Clarkson et al. 2013).

5.3.5 Correction to Linear Flow Analysis

An analytical method for calculation of the correction factor for the square-root-time plot as a function of pressure drawdown and pressure-dependent fluid and rock properties was presented by Qanbari and Clarkson (2013a,c) and is summarized in the Appendix 5B. The corrected square-root-time plot is represented by:
\[
\frac{\psi(p_i) - \psi(p_w)}{f_c q_g} = m_{CP} \sqrt{t}
\] .......................................................... (2)

In this work, we also account for pore proximity effects, and pore radius changes due to adsorption. The effect of pore confinement on gas compressibility factor (Figure 5.2a) and viscosity (Figure 5.2b), change of apparent permeability with slippage, diffusion and adsorbed layer thickness (Figure 5.3b), pressure dependency of total system compressibility (Figure 5.4) and stress-dependency of permeability and porosity are all incorporated into the calculation of pseudopressure (Eq. 5B.7 in Appendix 5B) and correction factor (Eq. 5B.28 in Appendix 5B) which in turn are used in the corrected square-root-time plot. The slope of the corrected square-root-time plot (Eq. 5B.25 in Appendix 5B) is then used to calculate the fracture half-length. The relationship between estimated fracture half-length (from square-root-time plot with correction factor) using confined and bulk gas properties is as follows:

\[
\frac{(x_f)_{\text{Confined}}}{(x_f)_{\text{Bulk}}} = \frac{\left( \frac{Z_i}{k_{ai}} \sqrt{\frac{\mu_{gi}}{c_{ai}} \frac{f_c}{\psi(p_i) - \psi(p_w)}} \right)_{\text{Confined}}}{\left( \frac{Z_i}{k_{ai}} \sqrt{\frac{\mu_{gi}}{c_{ai}} \frac{f_c}{\psi(p_i) - \psi(p_w)}} \right)_{\text{Bulk}}}
\] .......................................................... (3)

### 5.4 Application

A dry shale gas reservoir (saturated with 100 percent methane) initially at 5000 psi is simulated for demonstration of the effects of pore proximity and confinement on rate-transient analysis. The reservoir is assumed to have a stimulated reservoir volume with no flow beyond the fracture tips (similar to the geometry assumed by Nobakht and Clarkson 2012a); simulation model inputs are given in Table 5.1. As an element of symmetry, one side of a fracture is considered in the simulation model. A logarithmic gridding scheme is used near the hydraulic fracture to minimize the numerical simulation error.
Table 5.1 — Input parameters for the simulated case.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial reservoir pressure (psi)</td>
<td>5000</td>
</tr>
<tr>
<td>Reservoir temperature (°R)</td>
<td>630</td>
</tr>
<tr>
<td>Well flowing pressure (psi)</td>
<td>1000</td>
</tr>
<tr>
<td>Permeability modulus (psi⁻¹)</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>Rock compressibility (psi⁻¹)</td>
<td>$3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Pore radius (nm)</td>
<td>2</td>
</tr>
<tr>
<td>Fracture half-length (ft)</td>
<td>300</td>
</tr>
<tr>
<td>Fracture height (ft)</td>
<td>200</td>
</tr>
<tr>
<td>Fracture spacing (ft)</td>
<td>500</td>
</tr>
<tr>
<td>Formation permeability (md)</td>
<td>0.0002</td>
</tr>
<tr>
<td>Langmuir pressure (psi)</td>
<td>2000</td>
</tr>
<tr>
<td>Langmuir volume (scf/ton)</td>
<td>334</td>
</tr>
<tr>
<td>Fluid composition</td>
<td>100% CH₄</td>
</tr>
</tbody>
</table>

The following steps are taken to calculate the correction factor for the square-root-time plot, incorporating the effects of pore confinement, adsorption/desorption, and stress-sensitivity:

**Step 1.** Calculate critical temperature and pressure adjustment in nanopores. The critical properties of methane under confinement are calculated using Eq. 1 and the Lennard-Jones parameter ($\frac{3}{244.0} c c p T = \sigma$) (Teklu et al. 2014). Bulk critical temperature and pressure of methane are, respectively, 343 R and 667 psia. The Lennard-Jones parameter becomes 0.3936 nm which results in adjusted critical temperature and pressure values of 283 R and 550 psi, respectively.

**Step 2.** Calculate gas compressibility factor and viscosity using available correlations (Danesh, 1998). The adjusted critical properties from Step 1 are used in the correlations, assuming that the available gas property correlations are valid for confined gases. Figure 5.5 illustrates the corrected gas compressibility factor and viscosity for this example.
Step 3. Evaluate apparent permeability as a function of pressure, accounting for the effects of slippage, diffusion, adsorbed layer thickness, and stress-sensitivity. Permeability change with pressure for the current example is shown in Figure 5.6. The adsorption compressibility term and total compressibility as a function of pressure are also calculated (see Figure 5.4).
Figure 5.6 — Apparent permeability for methane in 2 nm radius pores accounting for diffusing, slippage, adsorption layer, and stress-sensitivity ($\gamma_k=1\times10^{-4}$ psi$^{-1}$).

Step 4. Calculate the correction factor using the procedure given by Qanbari and Clarkson (2013a) and summarized in the Appendix 5B. The correction factor for this example equals 0.5701.

Step 5. Calculate the correction factor using the bulk properties of methane and calculate the ratio of the two values of fracture length using Eq. 3. For this example, the ratio equals 0.6695; therefore, ignoring the confinement effects results in overestimation of fracture half-length by approximately 50 percent.

5.5 Discussion

In the previous section, we have illustrated our procedure for accounting for pore confinement, adsorbed layer thickness and stress-sensitive permeability in the analysis of transient linear flow of nanoporous shales for a specific set of conditions of pore size and permeability modulus. However, the correction factor applied to the square-root-time plot is very sensitive to pore size, permeability modulus and flowing pressure as illustrated in Figure 5.7. The results show that the correction factor is more important for smaller pore sizes, higher pressure drawdown values, and
higher permeability moduli. As a rough estimation, permeability (in nano Darcy) is related to pore size (in nm) by \( k = 125r_p^2 \) implying that the effect of confinement on gas flow is more pronounced in lower permeability shales.

It is important to note that, in order to account for the effect of adsorbed layer thickness on gas flow in nanopores, we assumed that there is a definite boundary between the adsorbed layer and the bulk phase and hence adsorption layer reduces the effective radius for gas. We further assumed that there is no flow within the adsorbed layer. A gradual phase transition from dense adsorbed layer near the pore wall to the bulk gas phase at the center of the pore seems to be more realistic. In addition, molecular flow in the adsorbed phase might happen. Experimental studies are required to clarify these important dynamic processes. Further, experimental studies are needed to gauge the validity of the results of the models describing changes in fluid properties, especially hydrocarbon fluids in nanoporous systems.

![Correction factor versus flowing pressure for dry gas reservoirs with different pore radii and permeability moduli.](image)

**Figure 5.7** — Correction factor versus flowing pressure for dry gas reservoirs with different pore radii and permeability moduli.

### 5.6 Conclusions

Changes in gas properties due to pore confinement, apparent permeability changes (due to non-Darcy flow, stress-sensitivity and adsorbed layer thickness), and total system compressibility
(gas+rock+adsorption term) are analytically incorporated into transient linear flow analysis of nanoporous shale gas reservoirs. While the impact of permeability changes caused by stress-sensitivity and non-Darcy flow on rate-transient analysis have previously been investigated, this study is the first to evaluate the impact of gas property changes caused by pore confinement and hydraulic pore radius changes caused by the dependence of adsorption layer thickness on pressure. A correction factor for the square-root-time plot was developed to account for these affects and other pressure-dependent properties of the fluid and the reservoir. This study indicates that neglecting these complexities results in overestimation of fracture half-length.

We have demonstrated, using a simulated dry shale gas example, that ignoring the pore confinement effect results in a 50 percent overestimation in the fracture half-length. A sensitivity analysis was performed to demonstrate the impact of pore size and permeability modulus (which impacts the degree of stress-sensitivity) on correction factor magnitude. The effects of pore confinement and adsorption layer are clearly greater for shales with pore radii smaller than 4 nm for the conditions studied, meaning that shale plays with dominant pore throat sizes in this range will be most affected.

5.7 Nomenclature

Field variables

\[ a \] function of pressure in flow equation
\[ b \] function of pressure in flow equation
\[ c_r \] rock compressibility (psi\(^{-1}\))
\[ c_t \] total compressibility (psi\(^{-1}\))
\[ f_c \] correction factor (dimensionless)
\[ h \] fracture height
\[ k \] permeability (md)
\[ k_a \] apparent permeability (md)
\[ l_s \] slip length (nm)
\[ m \] pseudopressure (psi)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{CP}$</td>
<td>slope of the square root of time plot</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure (psi)</td>
</tr>
<tr>
<td>$p_i$</td>
<td>initial reservoir pressure (psi)</td>
</tr>
<tr>
<td>$r_p$</td>
<td>pore radius (nm)</td>
</tr>
<tr>
<td>$R$</td>
<td>radius (nm)</td>
</tr>
<tr>
<td>$t$</td>
<td>time (day)</td>
</tr>
<tr>
<td>$t_a$</td>
<td>pseudotime (day)</td>
</tr>
<tr>
<td>$t_D$</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (°R)</td>
</tr>
<tr>
<td>$x_f$</td>
<td>fracture half-length (ft)</td>
</tr>
<tr>
<td>$x$</td>
<td>distance (ft)</td>
</tr>
<tr>
<td>$x_D$</td>
<td>dimensionless distance</td>
</tr>
<tr>
<td>$Z$</td>
<td>gas compressibility factor (dimensionless)</td>
</tr>
<tr>
<td>$\gamma_k$</td>
<td>permeability modulus (psi$^{-1}$)</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>difference</td>
</tr>
<tr>
<td>$\eta$</td>
<td>hydraulic diffusivity (ft$^2$/day)</td>
</tr>
<tr>
<td>$\eta_D$</td>
<td>dimensionless hydraulic diffusivity</td>
</tr>
<tr>
<td>$\eta_{DR}$</td>
<td>dimensionless hydraulic diffusivity at representative pressure</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>hydraulic diffusivity at initial reservoir pressure</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity (cp)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Boltzmann variable (dimensionless)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Lennard-Jones parameter (nm)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>porosity</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>porosity at initial pressure</td>
</tr>
<tr>
<td>$\psi$</td>
<td>pseudopressure (psi)</td>
</tr>
<tr>
<td>$\psi_D$</td>
<td>dimensionless pseudopressure</td>
</tr>
</tbody>
</table>
Subscripts

\( 0 \) reference
\( Bulk \) property under bulk conditions
\( c \) critical value in bulk
\( cp \) critical value in pore
\( Confined \) property under confined conditions
\( CP \) constant pressure
\( d \) desorption
\( D \) Dimensionless
\( g \) gas
\( i \) initial
\( k \) permeability
\( r \) rock
\( R \) representative
\( s \) slip
\( sc \) standard conditions
\( t \) total
\( w \) wellbore

5.8 Acknowledgements

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5.9 References


5.10 Appendix 5A: Liquid Flow in Nanopores

Experimental (Majumder et al. 2005; Holt et al. 2006; Whitby et al. 2008; Chen et al. 2008) and simulation (Sokhan et al. 2002; Joseph and Aluru 2008) studies have shown that pressure-induced liquid flow rate in synthetic nanotubes exceeds values calculated from the conventional continuum theory (i.e. Hagen-Poiseuille equation which is based on the assumption of zero liquid velocity at the tube wall). Enhanced liquid flow in nanotubes has been explained using two alternative concepts: liquid slippage and viscosity reduction. In slippage theory (Majumder et al. 2011; Whitby et al. 2008), a non-zero velocity is used in the derivation of the Hagen-Poiseuille equation. As an alternative, higher liquid velocity in nanotubes is explained by liquid viscosity reduction due to interaction between nanotubes and liquid molecules (Chen et al. 2008). Both of the explanations can be combined in an enhanced liquid mobility equation as follows:

\[
\frac{k}{\mu} = \left(1 + \frac{4l_s}{R}\right) \frac{k}{\mu}
\]

where \(l_s\) is slip length defined as the difference between a hypothetical radius at which liquid velocity is zero and the tube radius (\(R\)).

5.11 Appendix 5B: Calculation of Correction Factor

Flow of the gas phase in a linear horizontal system is governed by:

\[
\frac{\partial}{\partial x} \left[ a(p) \frac{\partial p}{\partial x} \right] = \frac{\partial b(p)}{\partial t}
\]

where
\[ a(p) = 0.00633 \frac{k(p)}{\mu_g(p) B_g(p)} \] .................................................................................(B.2)

and

\[ b(p) = \frac{\phi(p)}{B_g(p)} \] .........................................................................................(B.3)

for dry gas reservoirs, and:

\[ a(p) = 0.00633 \frac{k(p) k_{rg}(S_g)}{\mu_g(p) B_g(p)} \] .................................................................................(B.4)

and

\[ b(p) = \frac{\phi(p) S_g}{B_g(p)} \] .........................................................................................(B.5)

for gas condensate reservoirs.

Eq. 5B.1-5B.5 are in field units where \( x \) (ft) is position in the single dimension of the system, \( t \) (day) is time, \( p \) (psi) is pressure, \( k \) (md) is permeability, \( \phi \) (dimensionless) is the porosity of the rock, \( k_{rg} \) is relative permeability to gas, \( S_g \) is gas saturation, \( \mu_g \) (cp) is gas viscosity, and \( B_g \) (RCF/SCF) is gas formation volume factor. Boundary and initial conditions for production under constant wellbore pressure \( p_w \) during transient period read:

\[
\begin{align*}
  p &= p_i \quad t = 0 \quad x \geq 0 \\
  p &= p_i \quad t > 0 \quad x \to \infty \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots 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\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
\]

Similar to the pseudopressure defined by Aronofsky and Jenkins (1954) and Al-Hussainy et al. (1966), pseudopressure is defined based on the nonlinearities on the left side of Eq. 5B.1 as:

\[
\psi(p) = \frac{2}{a_i} \int_{p_0}^{p} a(p)d\hat{p} \] .................................................................................(B.7)

where \( p_0 \) (psi) is the reference pressure and subscript ‘i’ denotes values at initial pressure. The values of the parameters at the initial pressure are used to give the pseudopressure the same dimension as the real pressure. The dimensionless form of the pseudopressure is:
\[ \psi_D(p) = \frac{\psi(p_i) - \psi(p)}{\psi(p_i) - \psi(p_w)} = \frac{\int_{p_i}^{p} a(p)dp}{\int_{p_w}^{p} a(p)dp} \]  \hspace{1cm} \text{(B.8)}

which varies from 0 at initial pressure to 1 at wellbore pressure. Combining Eq. 5B.1 and 5B.8 yields:

\[ \frac{\partial^2 \psi_D}{\partial x^2} = \frac{1}{\eta(\psi_D)} \frac{\partial \psi_D}{\partial t} \]  \hspace{1cm} \text{(B.9)}

where the hydraulic diffusivity \((\eta_D)\) is:

\[ \eta(\psi_D) = \frac{a(\psi_D)}{b'_p(\psi_D)} \]  \hspace{1cm} \text{(B.10)}

Dimensionless distance, time, and hydraulic diffusivity are defined by:

\[ x_D = \frac{x}{\sqrt{A_c}} \]  \hspace{1cm} \text{(B.11)}

and

\[ t_D = \frac{\eta_D}{A_c} \]  \hspace{1cm} \text{(B.12)}

and

\[ \eta_D = \frac{\eta}{\eta_i} \]  \hspace{1cm} \text{(B.13)}

respectively, where \(\eta_i\) is hydraulic diffusivity at initial pressure. Using the dimensionless parameters, Eq. 5B.9 changes to:

\[ \frac{\partial^2 \psi_D}{\partial x_D^2} = \frac{1}{\eta_D(\psi_D)} \frac{\partial \psi_D}{\partial t_D}. \]  \hspace{1cm} \text{(B.14)}

Boundary and initial conditions change to:

\[ \psi_D = 0 \quad t = 0 \quad x_D \geq 0, \]

\[ \psi_D = 0 \quad t_D > 0 \quad x_D \to \infty, \]  \hspace{1cm} \text{(B.15)}

\[ \psi_D = 1 \quad t_D > 0 \quad x_D = 0. \]
Boundary and initial conditions are such that two of them can be consolidated into one using the Boltzmann transformation (Ames, 1965):

\[ \xi = \frac{x_D}{2 \sqrt{t_D}} \]  

(B.16)

Therefore, Eq. 5B.14 is reduced to the nonlinear ordinary differential equation (ODE):

\[ \frac{d^2 \psi_D}{d\xi^2} + \frac{2 \xi}{\eta_D} \frac{d\psi_D}{d\xi} = 0 \]  

(B.17)

with conditions:

\[ \psi_D = 0 \quad \xi \to \infty, \quad \psi_D = 1 \quad \xi = 0. \]  

(B.18)

Gas flow rate at standard conditions \( q_g \) from a hydraulically fractured well located at the center of a linear system is given by:

\[ q_g = -6.33 \times 10^{-6} \times 2 \left( \frac{k_g A_e \partial p}{\mu_g B_g \partial x} \right)_{x=0} \]  

(B.19)

where \( q_g \) is in Mscf/day. Using the definition of \( \psi_D \) and the Chain Rule, Eq. 5B.19 becomes:

\[ \frac{1}{q_{gD}} = 2\pi f_c \sqrt{\pi \tau_D} \]  

(B.20)

where \( q_{gD} \) is dimensionless flow rate:

\[ q_{gD} = \frac{711 q_g T \mu_g Z_i}{k_g P_i \sqrt{A_e \left[ \psi(p_i) - \psi(p_w) \right]}} \]  

(B.21)

and \( f_c \) is a correction factor:

\[ \frac{1}{f_c} = -\sqrt{\pi} \left( \frac{d\psi_D}{d\xi} \right)_{\xi=0} \]  

(B.22)

Another useful form of Eq. 5B.20 is:

\[ \frac{\left[ \psi(p_i) - \psi(p_w) \right]}{f_c q_g} = m_{CP} \sqrt{t} \]  

(B.23)

where \( m_{CP} \) is the slope of the plot of \( \left[ \psi(p_i) - \psi(p_w) \right]/(f_c q_g) \) versus \( \sqrt{t} \):
The value of slope $m_{CP}$ is used to evaluate $x_f \sqrt{k_{gi}}$:

$$x_f \sqrt{k_{gi}} = \frac{630TZ_i}{m_{CP} h \sqrt{\phi \mu_c}}.$$  \hfill (B.25)

The value of $f_c$ depends on the variation of dimensionless hydraulic diffusivity with dimensionless pseudopressure, and does not depend on rock and fluid state properties at initial reservoir pressure. A more useful equation for calculation of the correction factor can be derived as follows. Eq. 5B.17 is rearranged to the form:

$$\frac{d}{d\psi_D} \left( \frac{d\psi_D}{d\xi} \right) = -\frac{2}{\eta_D} \xi.$$  \hfill (B.26)

Integrating both sides of Eq. 5B.26 gives:

$$\left( \frac{d\psi_D}{d\xi} \right)_{\psi_D=1} = -2 \int_0^{\psi_D} \frac{\xi}{\eta_D} d\psi_D.$$  \hfill (B.27)

assuming that $\left( \frac{d\psi_D}{d\xi} \right)_{\psi_D=0} = 0$ during transient flow period. Therefore, the correction factor becomes:

$$\frac{1}{f_c} = \sqrt{\pi} \int_0^1 \frac{\xi}{\eta_D} d\psi_D.$$  \hfill (B.28)

Eq. 5B.28 relates the correction factor to an integral (with the solution of the flow equation in the integrand) rather than the derivative (Eq. 5B.22). Therefore, this equation is more advantageous than Eq. 5B.22 for calculation of the correction factor.

An approximate solution of the flow equation is required in order to evaluate the integral in Eq. 5B28. In this study, the iterative integral method is used as follows. Integrating Eq. 5B.17 twice and applying the initial conditions in Eq. 5B.18 gives an approximate solution of the PDE as follows:
\[\psi_d(\xi) = 1 - \frac{\int_{0}^{\xi} \exp\left(-2\int_{0}^{\xi} \frac{\xi}{\eta_D} d\xi\right) d\xi}{\int_{0}^{\infty} \exp\left(-2\int_{0}^{\infty} \frac{\xi}{\eta_D} d\xi\right) d\xi}\] (B.29)

The distribution of the dimensionless pseudopressure is iterated in Eq. 5B.29 and a new value for the correction factor is evaluated using Eq. 5B.28. The iteration process continues until the correction factor converges. The initial guess for the distribution of the dimensionless pseudopressure is assumed to be the distribution corresponding to \(\eta_D=1\), which is essentially the conventional analytical (error function) solution of the problem.

Eq. 5B.14 can also be solved analytically assuming a representative constant value for dimensionless hydraulic diffusivity, \(\eta_{DR}\). With this assumption, the relationship between the suitable representative pressure and the correction factor would be:

\[\eta_{DR} = f_c^{2} \] (B.30)

Equivalently, if \(\eta_{DR}=1\) is used in Eq. 5B.14, real time (t) should be changed to the pseudo-time (\(t_a\)):

\[t_a = (f_c^{2})t \] (B.31)
Chapter 6  Rate-Transient Analysis of Liquid-Rich Tight/Shale Reservoirs Using the Dynamic Drainage Area Concept: Examples from North American Reservoirs

6.1 Abstract

The early-time performance of multi-fractured horizontal wells is mainly controlled by fracture geometry, total effective area of the fractures, and conductivity of the primary fracture system. Inverse modeling using rate-transient analysis (RTA) methods has historically been used to characterize MFHWs at different stages of well life, including the early-time performance. In particular, linear flow analysis is used to estimate the total effective fracture area from online production data, provided that reservoir and fluid properties are known. However, a primary complication in analytical linear flow analysis is the incorporation of nonlinearities such as multi-phase flow and pressure-dependent rock/fluid properties into the calculations.

A new linear flow analysis technique is presented in the current study, which can be applied to tight/shale systems with multi-phase flow and pressure-dependent rock/fluid properties. The method combines three important reservoir engineering concepts for linear flow analysis: dynamic drainage area (DDA), material balance, and decoupling of saturation and pressure (which is analogous to the decoupling of geomechanics and fluid flow). The DDA approach has been used previously by the authors for history-matching and forecasting using a semi-analytical model, but not for inverse modeling (RTA). The DDA concept, which uses a time-dependent well productivity index equation for the transient flow period, facilitates the incorporation of any sort of nonlinearity (including decoupled saturation functions) and operational constraints in modeling and RTA of linear flow in MFHWs.

The method is validated against numerical simulation and applied to various sets of field production data from tight/shale gas and oil wells with different levels of condensate- (oil-) gas

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5 This chapter is a slightly modified version of a paper published in Journal of Natural Gas Science and Engineering as: Qanbari, F. and Clarkson, C.R., 2016. Rate-Transient Analysis of Liquid-Rich Tight/Shale Reservoirs Using the Dynamic Drainage Area Concept: Examples from North American Reservoirs. Journal of Natural Gas Science and Engineering 35, 224-236. Copyright approval has been obtained from the journal (see “Copyright Permissions” section of this thesis).
ratio. For all the field cases, total effective fracture area obtained from the new analytical RTA method is in reasonable agreement with numerical modeling results.

Regarding accuracy and practicality, the new method represents an improvement in RTA of liquid-rich tight/shale reservoirs, particularly for cases with multi-phase flow and pressure-dependent rock/fluid properties. Further, the concepts used in the new model development are easy to understand and implement.

6.2 Introduction

Application of the game-changing multi-fractured horizontal well (MFHW) technology in tight/shale reservoirs results in a volume of the reservoir accessed through a complex fracture system created by high pressure injection of fracturing fluid. Characterization of such a complex system using physics-based flow models is a goal in reservoir engineering of tight/shale reservoirs. As summarized by Clarkson et al. (2016), there are multiple independent methods for characterization of MFHWs including fracture (geomechanical) modeling, fracture monitoring (microseismic), early-time fracturing fluid flowback analysis, and long-term (online) production data analysis (PDA) or RTA. Each method utilizes specific data collected during completion and production: integration of the results of the whole suite of analyses is a reasonable practice to better characterize the system and reduce the uncertainty. In a study by Clarkson et al. (2014), fracture half-lengths obtained from RTA were shown to be smaller than those derived from fracture modeling/monitoring or flowback analysis: the authors attributed this to various mechanisms reducing the effective contributing half-length of the fractures during long-term production.

Primary challenges for data analysts hoping to extract quantitative fracture and reservoir properties from RTA of tight/shale reservoirs include data quality and the lack of available rigorous, physics-based models. The former is essential in the current economic environment, and requires collaboration between different professionals in a company. As for the latter, all of the aforementioned characterization tools for MFHWs are indirect methods in which the collected data sets are analyzed using mathematical models with simplifying assumptions. Simplifying assumptions are inevitable primarily for two reasons: the lack of detailed data and information about the MFHW system (e.g. fracture geometry, fluid distributions, geological
heterogeneity etc.); and limited capabilities of some of the existing mathematical tools, particularly analytical techniques. Standard RTA methods, in particular, are limited to the analysis of production from single-phase systems with slight pressure-dependency of rock and liquid phase properties. For linear flow analysis of oil systems, the slope of rate-normalized pressure, \( \frac{p_i - p_{wof}}{q_o} \), versus oil linear superposition time is used to calculate total \( A \sqrt{k_i} \), which is equal to \( 4h \sqrt{\frac{k_i}{k}} \) for a MFHW (El-Banbi 1998):

\[
\frac{p_i - p_{wof}}{q_o} = m_{eq} t_{LST} \tag{1}
\]

\[
A \sqrt{k_i} = \frac{79.71 B_o}{m_{eq}} \frac{\mu_{oil}}{\phi c_i} \tag{2}
\]

Similarly, the slope of the linear flow plot for gas (plot of rate-normalized pseudopressure, \( \frac{m_g (p_i) - m_g (p_{wof})}{q_g} \), versus gas linear superposition time) is used to calculate total \( A \sqrt{k_i} \).

\[
\frac{m_g (p_i) - m_g (p_{wof})}{q_g} = m_{eq} t_{LST} \tag{3}
\]

\[
A \sqrt{k_i} = \frac{8034T}{m_{eq}} \frac{1}{\phi \mu_{go} c_i} \tag{4}
\]

Different methods have been introduced over the past few years for incorporation of nonlinearities associated with pressure-dependent fluid and rock properties and two-phase (gas and oil/condensate) flow in RTA of tight/shale systems. Ibrahim and Wattenbarger (2006) used an empirical correction factor (based on numerical simulation results) for linear flow analysis of single-phase gas systems. Qanbari and Clarkson (2013a, 2013b) developed an analytical correction factor for linear flow analysis of gas and oil systems with stress-dependent permeability and two-phase flow of gas and oil using an iterative integral method. Nobakht and Clarkson (2012), Tabatabaie and Pooladi-Darvish (2016), and Behmanesh et al. (2015b) employed pseudotime functions evaluated at average pressure in the investigated area during transient flow to linearize the nonlinear flow equation. Qanbari and Clarkson (2014) correlated pressure-dependent diffusivity with a pressure function for which the corresponding flow equation has an exact analytical solution. Mohan et al. (2013) and Eker et al. (2014) incorporated
multiphase flow into RTA of tight systems using total equivalent rate instead of the primary fluid rate.

In this chapter, the concept of dynamic drainage area (DDA), combined with material balance and decoupling of saturation and pressure, is used to construct a modified linear flow plot (plot of rate normalized pseudopressure vs. square-root of time), referred to herein as the DDA-corrected linear flow plot. In the previous work by the authors (Clarkson and Qanbari 2016a,b), the DDA concept was used for forecasting, while in the current work, it is used for inverse modeling. This is an alternative method for incorporation of nonlinearities into the RTA of tight/shale reservoirs. In the following sections, the method is introduced, validated against synthetic cases (generated by numerical models), and applied to field cases.

6.3 Theory and Method Development

6.3.1 Dynamic Drainage Area

Recently, Clarkson and Qanbari (2016a) used the dynamic drainage area (DDA) concept as an approximate semi-analytical method for history matching and forecasting MFHWs in tight/shale reservoirs; the method was later extended to history matching and forecasting horizontal wells completed in low-permeability, undersaturated coalbed methane reservoirs Clarkson and Qanbari (2016b). In this method, the distance of investigation is calculated at each time step during the transient flow period and a time-dependent linear productivity index equation with pseudo-pressure is used for rate calculation. As noted by Clarkson and Qanbari (2016a), various forms of the time-dependent productivity index equation have been used by Muskat (1937), Lee et al. (1998), and Shahamat et al. (2014). Figure 6.1 illustrates the distance of investigation concept and the parameters that are calculated at two time steps (including distance of investigation, average pressure and saturation, gas and oil rates).

In the current work, the DDA method is applied in backward mode as an RTA tool in which gas and oil rates are known. In backward mode, the DDA equations for linear flow of oil and gas are as follows (derivations are provided in the Appendix 6A for completeness):
\[
\frac{m_o(p_{\text{inv}}) - m_o(p_{\text{wf}})}{q_o} \sqrt{\frac{\mu_o D(p_{\text{inv}}) \phi D(p_{\text{inv}}) c_i D(p_{\text{inv}})}{k_D(p_{\text{inv}})}} = 57.16 B_{oi} \sqrt{\frac{\mu_{oi}}{A_e k_i}} \sqrt{\phi_i c_{ii}}
\]  
\(5\)

and

\[
\frac{m_g(p_{\text{inv}}) - m_g(p_{\text{wf}})}{q_g} \sqrt{\frac{\mu_g D(p_{\text{inv}}) \phi D(p_{\text{inv}}) c_i D(p_{\text{inv}})}{k_D(p_{\text{inv}})}} = 576.56 T \sqrt{\frac{\mu_{gi} c_{ii}}{\phi_i k_i}}
\]  
\(6\)

Eqs. 5 and 6 use pseudo-pressure drawdown with respect to average pressure in the investigated area during transient linear flow, which is obtained from material balance equations as discussed in the following subsection. Oil and gas pseudopressesures in Eqs. 5 and 6 are defined as:

\[
m_o(p) = \int_{p_o}^{p} \left\{ \frac{k_D(\hat{p}) k_{rg}(S_o) \hat{p}}{\mu_o D(\hat{p}) B_o D(\hat{p})} \right\} d\hat{p}
\]  
\(7\)

\[
m_g(p) = \int_{p_o}^{p} \left\{ \frac{2 k_D(\hat{p}) k_{rg}(S_o) \hat{p} B_g(\hat{p})}{k_i \mu_g(\hat{p}) Z_g(\hat{p}) B_{go}(\hat{p})} \right\} d\hat{p}
\]  
\(8\)

It should be noted Eqs. 5 and 6 are semi-analytical with empirical elements and inevitable limitations as discussed by Clarkson and Qanbari (2016a). Oil and gas pseudopressures (i.e. Eqs. 7 and 8) incorporate the pressure-dependency of fluid properties, permeability, and relative permeability. In Eq. 8, the classic definition of gas pseudopressure, for example, is modified to account for pressure-dependency of permeability, relative permeability, and dry-gas-based formation volume factor.

Figure 6.1 — Illustration of the dynamic drainage area concept. Modified from Clarkson and Qanbari (2016a).
6.3.2 Material Balance

The average pressure in the investigated area at each time step during the transient linear flow period (Figure 6.1), which is required for evaluation of Eqs. 5 and 6, is calculated at each time step using the following material balance equations for oil and gas (Clarkson and Qanbari, 2016a):

\[
N_p = 4x_{ft}y_{inv} \left[ \frac{\phi_i S_{oi} + R_{si} \frac{\phi_i S_{gi} - \phi_i(\bar{p}_{inv})S_{o,inv}}{5.615B_{oi}} - R_i(\bar{p}_{inv}) \frac{\phi_i(\bar{p}_{inv})S_{g,inv}}{10^6 B_{oi}(\bar{p}_{inv})}}{5.615B_{oi}} \right]
\]

(9)

\[
G_p = 4x_{ft}y_{inv} \left[ \frac{\phi_i S_{gi} + R_{si} \frac{\phi_i S_{oi} - \phi_i(\bar{p}_{inv})S_{g,inv}}{5.615B_{oi}} - R_i(\bar{p}_{inv}) \frac{\phi_i(\bar{p}_{inv})S_{o,inv}}{10^6 B_{oi}(\bar{p}_{inv})}}{5.615B_{oi}} \right]
\]

(10)

The distance of investigation \(y_{inv}\) in Eqs. 9 and 10 is calculated using the following time-dependent power function (Wattenbarger et al., 1998):

\[
y_{inv} = \text{Min} \left\{ \alpha \sqrt[\phi_i \mu_i c_{ii}] {y_e} \right\}
\]

(11)

where \(y_e\) is fracture half-spacing (see Figure 6.2). Coefficient \(\alpha\) depends on the operational conditions (rate and flowing bottomhole pressure). In this study, \(\alpha = 0.159\) (which corresponds to production under constant bottomhole pressure conditions) is used for all the cases, including those with variable flowing bottomhole pressure, and demonstrated to be reasonably accurate. This is an empirical element of the current DDA method. However, the reader is advised to test the sensitivity of analysis results to the choice of \(\alpha\). Behmanesh et al. (2015a) recently derived \(\alpha\) using analytical methods.

Cumulative production at each time step from well production history is used in Eqs. 9 and 10 to calculate average pressure and saturation. Average pressure, in turn, is used in Eq. 5 or 6, depending on the system of interest. The complication, however, is that \(x_{ft} \sqrt{k_i}\) (or \(A \sqrt{k_i}\)) is required for calculation of average pressure and saturation from Eqs. 9 and 10. Considering \(A \sqrt{k_i}\) as the main unknown of transient linear flow analysis, the new linear flow plot generated
using the DDA approach is, therefore, iterative. The following steps are recommended to evaluate \( x_{k_i} \) (or \( A_{k_i} \)):

1) Initialize \( x_{k_i} \) (or \( A_{k_i} \)).
2) Calculate average pressure and saturation from Eq. 9 and 10.
3) Use the average pressure from Step 2 and construct the linear flow plot using Eq. 5 or 6.
4) Update \( x_{k_i} \) (or \( A_{k_i} \)) from of the slope of the plot from Step 3.
5) Go to Step 2 if \( x_{k_i} \) (or \( A_{k_i} \)) does not converge.

This iterative procedure can be handled through different methods, such as following the steps above, using graphical methods (similar to the flowing material balance method – see Mattar et al., 2006), or employing robust optimization tools.

### 6.3.3 Decoupling of Pressure and Saturation

A difficulty arising from application of Eqs. 7 and 8 is that a relationship between pressure and saturation is required for evaluation of gas and oil pseudo-pressures. However, fortunately, the relationship remains practically unchanged with time and flowing bottom-hole pressure during transient linear flow (Qanbari and Clarkson, 2013a; Tabatabaie and Pooladi-Darvish, 2016; Behmanesh et al., 2015b; Clarkson and Qanbari, 2016a). This uniqueness has encouraged the authors to predict the relationship so that saturation and relative permeability in the pseudo-pressure equations can be represented by pressure functions and treated similarly to other pressure functions (such as viscosity, FVF, permeability etc.). This is a very important simplification which facilitates application of the solutions for transient flow in single-phase systems (with pressure-dependent properties) to highly nonlinear transient two-phase flow problems by replacing saturation with a unique pressure function (which can be obtained independently). The process of replacing saturation with a pressure function during transient flow is referred to as “decoupling of pressure and saturation” in this study, which is analogous to decoupling of geomechanics and fluid flow in reservoir engineering (where simple pressure-dependent porosity and permeability functions are used in flow models to avoid coupled geomechanical-flow calculations). In the current work, the saturation-pressure correlations of Clarkson and Qanbari (2016a) are used, which are reproduced below for completeness:
where the base saturation values are calculated as:

\[
S_{o,\text{base}}(p) = \frac{5.615 B_o(p)}{5.615 B_o(p) + B_g(p) [R_{sh} - R_s(p)]} \quad \text{black oil}
\]

\[
S_{o,\text{base}}(p) = \frac{5.615 \left( \frac{R_{v,\text{dew}} - R_v(p)}{10^6 - R_v(p) R_s(p)} \right) B_s(p)}{B_g(p) \left( \frac{R_{v,\text{dew}} - R_v(p)}{10^6 - R_v(p) R_s(p)} \right) B_g(p)} \quad \text{gas condensate}
\]

6.4 Method Validation

Six different synthetic cases (generated using CMG IMEX) are used to check the accuracy of the DDA-corrected linear flow analysis method: two cases (one with constant permeability and one with stress-sensitive permeability) for each of the hydrocarbon systems (dry gas, gas condensate and oil systems) are considered. The numerical model input parameters are listed in Table 6.1. The first case for each hydrocarbon system has constant permeability whereas the second case includes stress-sensitive permeability. In each case, a multi-fractured horizontal well is modeled and the resulting production data is analyzed using a linear flow plot to back calculate the linear flow parameter, i.e. total \( A \sqrt{k_i} \) (which is compared to the input value of 15000 ft²md⁰.⁵ used in all of the synthetic cases).

In this study, the numerical model setup of Clarkson and Qanbari (2015) is used. The hydraulic fractures are assumed identical and the element of symmetry (Figure 6.2) is discretized perpendicular to fracture direction. The size of \( n \)th grid block is calculated from:

\[
(\Delta y)_n = (\Delta y)_0 e^{au}, \quad n = 0,1,2,\ldots, N_{\text{grids}}
\]

where \( (\Delta y)_0 \) is the size of fracture grid, \( N_{\text{grids}} \) is total number of matrix grid blocks, and \( a \) is a factor controlling the size of the grids, which can be calculated fracture half-distance equation:

\[ \text{Method Validation} \]
\[ y_c = \sum_{n=0}^{N_{\text{grids}}} (\Delta y)_n \]  

(15)

For all of the synthetic cases of the current study, the element of symmetry is discretized by 51 grid blocks, with a fracture grid size of 0.25 ft and fracture half distance of 250 ft (which gives \( a=0.00381 \)). For all of the cases, the model is run with variable bottomhole pressure starting from an initial bottomhole pressure \((p_{wf,i})\) of 4000 psia which exponentially decreases to a terminal flowing pressure \((p_{wf,f})\) of 2000 psia over 400 days using the following expression:

\[ p_{wf}(t) = p_{wf,f} + \left[ p_{wf,i} - p_{wf,f} \right] \exp(-a_p t) \]  

(16)

where \( a_p = 0.01 \). Eq. 16 (which has three input parameters \( p_{wf,i}, p_{wf,f}, \) and \( a_p \)) is based on the authors’ observation of change in flowing bottomhole pressure in a large population of liquid-rich shale/tight wells. However, it should be noted that Eq. 16 is not a universal equation for flowing bottomhole pressure in shale/tight systems.

![Base geometry for the synthetic cases with an element of symmetry. From Clarkson and Qanbari (2015).](image)

Figure 6.2 — Base geometry for the synthetic cases with an element of symmetry. From Clarkson and Qanbari (2015).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>240</td>
<td>240</td>
<td>220</td>
<td>220</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 6.1 — Model input for synthetic cases.
<table>
<thead>
<tr>
<th>Initial pressure (psi)</th>
<th>10000</th>
<th>10000</th>
<th>9000</th>
<th>9000</th>
<th>8000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid properties[^2]</td>
<td>Figure 5.3</td>
<td>Figure 5.3</td>
<td>Figure 5.8</td>
<td>Figure 5.8</td>
<td>Figure 5.13</td>
<td>Figure 5.13</td>
</tr>
<tr>
<td>Porosity (%)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Permeability modulus</td>
<td>0</td>
<td>5×10^{-4}</td>
<td>0</td>
<td>5×10^{-4}</td>
<td>0</td>
<td>5×10^{-4}</td>
</tr>
<tr>
<td>(psi[^1])[^3]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial gas saturation (%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Initial oil saturation (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Gas relative permeability</td>
<td>$S_g^3$</td>
<td>$S_g^3$</td>
<td>$S_g^3$</td>
<td>$S_g^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil/condensate relative permeability</td>
<td>$S_c^3$</td>
<td>$S_c^3$</td>
<td>$S_c^3$</td>
<td>$S_c^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input total $A \sqrt{k_i}$</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
</tr>
</tbody>
</table>

[^1] Permeability modulus, $\gamma_k$, is used in the exponential function $k(p)=k_i \exp[\gamma_k (p-p_i)]$ (Yilmaz et al., 1994).

[^2] SC, DG, GC, and BO stand for synthetic case, dry gas, gas condensate and black oil, respectively.

[^3] Default black-oil and extended black-oil properties of IHS Harmony are used for all cases.

**6.4.1 Dry Gas Cases**

Synthetic dry gas cases, SC-DG1 (constant permeability) and SC-DG2 (with stress-sensitive permeability), have an initial reservoir pressure of 10000 psia and reservoir temperature of 240 °F. The gas compressibility factor and viscosity for these cases are plotted against pressure in Figure 6.3 and production data are presented in Figures 6.4 and 6.5.

In Figure 6.6b and 6.7b, both a standard linear flow plot using Eq. 3 and the DDA-corrected linear flow plot using Eq. 6 are constructed and the linear flow parameter is evaluated from the slopes of these plots. The corresponding linear flow diagnostic plots are presented in Figure 6.6a.
and 6.7a, respectively. The results are listed in Table 6.2. Application of the DDA-corrected linear flow analysis method results in total $A_i \sqrt{k_i}$ estimates of 15300 and 15900 ft$^2$md$^{0.5}$ (compared to 18200 and 6200 ft$^2$md$^{0.5}$ using the standard approach for gas systems, i.e. Eqs. 3-4) for SC-DG1 and SC-DG2, respectively. For both cases, the DDA-modified approach gives more accurate results compared to numerical simulation. In SC-DG1, the error associated with the standard method is due primarily to strong dependence of gas compressibility factor on pressure and high pressure drawdown and the error in SC-DG2 is due to gas property change with pressure and stress-sensitivity of permeability.

Figure 6.3 — Gas properties for the synthetic dry gas cases SC-DG1 and SC-DG2.
Figure 6.4 — Production data for synthetic case SC-DG1.

Figure 6.5 — Production data for synthetic case SC-DG2.
Figure 6.6 — Synthetic case SC-DG1: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.

Figure 6.7 — Synthetic case SC-DG2: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.
6.4.2 Gas Condensate Cases

The initial condensate-gas ratio of synthetic gas condensate cases SC-GC1 and SC-GC2 is 200 STB/MMscf, and the dew point pressure is 4000 psia. The condensate-gas ratio (CGR) decreases as pressure drops below dew point pressure (Figure 6.8) leaving some liquid condensate in and near hydraulic fractures where the highest pressure drawdown occurs. Production data for SC-GC1 and SC-GC2 are plotted in Figures 6.9 and 6.10 and the linear flow plots are shown in Figures 6.11b and 6.12b, respectively. The corresponding linear flow diagnostic plots are presented in Figure 6.11a and 12a, respectively. Application of the DDA-corrected linear flow analysis method results in total \( A_i \sqrt{k_i} \) estimates of 14500 and 13600 ft\(^2\)md\(^{0.5}\) (compared to 15500 and 5900 ft\(^2\)md\(^{0.5}\) for the standard approach for gas systems, i.e. Eqs. 3-4) for SC-GC1 and SC-GC2, respectively (see Table 6.2). The DDA-corrected approach therefore provides more accurate results for SC-GC2 compared to the standard method.

![Graph showing gas properties](image)

Figure 6.8 — Gas properties for the synthetic cases SC-GC1 and SC-GC2.
Figure 6.9 — Production data for synthetic case SC-GC1.

Figure 6.10 — Production data for synthetic case SC-GC2.
Figure 6.11 — Synthetic case SC-GC1: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.

Figure 6.12 — Synthetic case SC-GC2: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.
6.4.3 Black Oil Cases

PVT properties for SC-BO1 and SC-BO2 are presented in Figure 6.13. Production data for SC-BO1 and SC-BO2 are plotted in Figures 6.14 and 6.15, respectively. Linear flow plots for SC-BO1 and SC-BO2 are shown in Figures 6.16 and 6.17. Application of the DDA-corrected linear flow analysis method results in total $A_k \sqrt{k_i}$ estimates of 14600 and 15200 ft$^2$md$^{0.5}$ for SC-BO1 and SC-BO2, respectively, and is therefore more accurate than the standard method for oil systems, i.e. Eqs. 1-2, which resulted in $A_k \sqrt{k_i}$ estimates of 17200 and 8000 ft$^2$md$^{0.5}$ (see Table 6.2).

![Graphs showing oil and gas properties for SC-BO1 and SC-BO2](image)

Figure 6.13 — (a) oil properties and (b) gas properties for synthetic oil cases SC-BO1 and SC-BO2.
Figure 6.14 — Production data for synthetic case SC-BO1.

Figure 6.15 — Production data for synthetic case SC-BO2.
Figure 6.16 — Synthetic case SC-BO1: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.

Figure 6.17 — Synthetic case SC-BO2: (a) log-log diagnostic plot, (b) standard and DDA-corrected linear flow plots.
Table 6.2 — Results of linear flow analysis for the synthetic cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SC-DG1</th>
<th>SC-DG2</th>
<th>SC-GC1</th>
<th>SC-GC2</th>
<th>SC-BO1</th>
<th>SC-BO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input total $A_c \sqrt{k_i}$ (ft²md⁰.⁵)</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
</tr>
<tr>
<td>Total $A_c \sqrt{k_i}$ (ft²md⁰.⁵) from Eqs. 2 and 4</td>
<td>18200</td>
<td>6200</td>
<td>15500</td>
<td>5900</td>
<td>17200</td>
<td>8000</td>
</tr>
<tr>
<td>Total $A_c \sqrt{k_i}$ (ft²md⁰.⁵) from DDA-created linear flow plot</td>
<td>15300</td>
<td>15900</td>
<td>14500</td>
<td>13600</td>
<td>14600</td>
<td>15200</td>
</tr>
</tbody>
</table>

6.5 Application to Field Cases

The new DDA-corrected method is applied to two real MFHW cases (RC1 and RC2) from North American shale reservoirs. The formation, location of the wells, completion and stimulation details are withheld to preserve operators confidentiality. The input parameters used for modeling both cases are provided in Table 6.3. The DDA-corrected linear flow analysis and numerical history matching are used to obtain linear flow parameters for the real cases. The same numerical setup presented in the Method Validation section (Figure 6.2) is used for numerical history matching of the real cases.
Table 6.3 — Properties of real cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RC1*</th>
<th>RC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure (psi)</td>
<td>7600</td>
<td>4700</td>
</tr>
<tr>
<td>Temperature (°F)</td>
<td>240</td>
<td>205</td>
</tr>
<tr>
<td>Fluid properties</td>
<td>Figure 6.18</td>
<td>Figure 6.21</td>
</tr>
<tr>
<td>Porosity (%)</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Permeability modulus (psi⁻¹)**</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Initial gas saturation (%)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Initial oil saturation (%)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gas relative permeability</td>
<td>$s_g^4$</td>
<td></td>
</tr>
<tr>
<td>Oil/condensate relative permeability</td>
<td>$s_c^3$</td>
<td></td>
</tr>
</tbody>
</table>

* RC stands for real case.
** Permeability modulus, $\gamma_k$, is used in the exponential function $k(p)=k_i \exp[\gamma_k (p-p_i)]$ (Yilmaz et al., 1994).

6.5.1 Wet Gas Case

RC1 is a wet gas case with initial reservoir pressure of 7600 psia (one of the field cases used by Clarkson and Qanbari 2016a). PVT properties for RC1 are plotted against pressure in Figure 6.18. Production data and history match results using a numerical simulator are shown in Figure 6.19. The gas linear flow plot generated with the DDA approach is illustrated in Figure 6.20. The linear flow parameter $A \sqrt{k_i}$ input into the numerical model is 8200, compared to 7400 ft²md⁰.⁵ derived from the DDA-corrected linear flow plot (see Table 6.4). The error from DDA-corrected linear flow analysis is therefore approximately 10 percent, or within acceptable engineering error.
Figure 6.18 — Gas properties for field gas case RC1.

Figure 6.19 — History match of field case RC1 data using numerical simulation (a) gas rate, (b) cumulative gas production.
6.5.2 Rich Gas Condensate Case

RC2 is a rich gas condensate case with initial reservoir pressure $p_f$ 4700 psia and initial CGR of 220 STB/MMscf. The PVT properties of real case RC2 are shown in Figure 6.21. Gas and oil production data, and the corresponding history matches using numerical simulation, are provided in Figures 6.22 and 6.23, respectively. The DDA-corrected linear flow plot (shown in Figure 6.24) gives a linear flow parameter within 12\% of error of the numerical simulation input (26200 vs. 23500 ft$^2$md$^{0.5}$, respectively).

Figure 6.20 — (a) Log-log diagnostic plot, and (b) DDA-corrected linear flow plot for field case RC1.
Figure 6.21 — Gas properties for field case RC2.

Figure 6.22 — History match of field case RC2 data using numerical simulation (a) gas rate, (b) cumulative gas production.
Figure 6.23 — History match of field case RC2 data using numerical simulation (a) condensate rate, (b) cumulative condensate production.

Figure 6.24 — (a) Log-log diagnostic plot, and (b) DDA-corrected linear flow plot for field case RC2.
Table 6.4 — Results of linear flow analysis for real cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RC1</th>
<th>RC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total $A_c \sqrt{k_i}$ (ft$^2$ md$^{0.5}$) from numerical simulation</td>
<td>8200</td>
<td>23500</td>
</tr>
<tr>
<td>Total $A_c \sqrt{k_i}$ (ft$^2$ md$^{0.5}$) from DDA-corrected linear flow plot</td>
<td>7400</td>
<td>26200</td>
</tr>
</tbody>
</table>

6.6 Discussion

In this study, a semi-analytical method, based on the dynamic drainage area concept, was used in backward mode for linear flow analysis. The resulting modified linear flow plot incorporates pressure-dependent fluid and rock properties and two-phase flow into linear flow analysis. The method was validated against different synthetic (numerically-generated) cases and applied to two real cases. For the synthetic and the real cases studied, the DDA-corrected linear flow plot gives reasonably accurate results compared to numerical simulation. This method, however, has limitations that need to be considered in future studies. One of the limitations is that the oil saturation-pressure correlations (Eqs. 12 and 13) are used for simplicity. These correlations have been tested for a limited number of cases; more rigorous and general methods for evaluation of the relationship between oil saturation and pressure can also be used (see Tabatabaie and Pooladi-Darvish, 2016; Behmanesh et al., 2015). Further, as noted by Clarkson and Qanbari (2016a), the DDA method is unable to accurately reproduce transient rate periods immediately following major flowing pressure changes or shut-ins; this can be handled by reinitialization of the model after major pressure changes and shut-ins. Finally, the iterative nature of the method makes it computationally more expensive compared to standard linear flow analysis tools. However, the method gives more accurate results which is important for reliable characterization and production forecasting of MFHWs producing from reservoirs exhibiting multi-phase flow and/or stress-sensitive reservoir properties.

6.7 Conclusions

A new RTA method based on the dynamic drainage area concept is presented in this paper for the first time. The DDA approach facilitates the inclusion of nonlinearities (such as pressure-
dependent fluid and rock properties and two-phase flow of gas and oil/condensate) into RTA of the transient linear flow period exhibited by multi-fractured horizontal wells in tight/shale systems. The accuracy of the new approach is evaluated against multiple synthetic cases (generated by numerical models) and found to be acceptable. Real field examples from North American tight/shale reservoirs are also evaluated using the new approach to demonstrate the practical applicability. The DDA-corrected method, however, will require additional testing and validation due to the inherent empirical elements and simplifying assumptions.

6.8 Acknowledgements

The sponsors of Tight Oil Consortium (TOC), hosted at the University of Calgary, are acknowledged for their support. Chris Clarkson would like to acknowledge Shell, Encana and Alberta Innovates Technology Futures (AITF) for support of his Chair position in Unconventional Gas and Light Oil Research at the University of Calgary, Department of Geoscience.

6.9 Nomenclature

Field Variables

\( a \) a constant parameter in grid size equation (Eq. 14)

\( a_p \) a constant parameter in flowing bottom-hole pressure equation (Eq. 16)

\( A_c \) total fracture area (ft\(^2\))

\( B_g \) gas formation volume factor based on dry gas and condensate (rcf/scf)

\( B_{gd} \) gas formation volume factor based on dry gas only (rcf/scf)

\( B_{gd,dew} \) gas formation volume factor based on dry gas only at dew-point pressure (rcf/scf)

\( B_{gd,i} \) gas formation volume factor based on dry gas only at initial reservoir pressure (rcf/scf)

\( B_o \) oil formation volume factor (RB/STB)

\( \bar{B}_o \) oil formation volume factor at average reservoir pressure (RB/STB)

\( B_{oD} \) dimensionless oil formation volume factor, \( B_o/B_{ol} \)
$c_{tD}$  dimensionless total compressibility, $c_t/c_{ti}$

$c_{ti}$  total compressibility at initial reservoir pressure (1/psi)

$G_p$  cumulative gas production (Mscf)

$h$  reservoir thickness (ft)

$k$  absolute permeability (md)

$k_{D}$  dimensionless permeability, $k/k_i$

$k_i$  absolute permeability at initial reservoir pressure (md)

$k_{rg}$  gas relative permeability (dimensionless)

$k_{ro}$  oil relative permeability (dimensionless)

$m_{cq}$  slope of square-root time plot (psi/STB/$\sqrt{\text{day}}$, psi$^2$/cp/scf/$\sqrt{\text{day}}$)

$m_g$  gas pseudopressure (psi$^2$/cp)

$m_o$  oil pseudopressure (psi)

$N_p$  cumulative oil production (STB)

$N_{grids}$  number of grid blocks

$p$  pressure (psia)

$p_d$  dew point pressure (psia)

$p_i$  initial reservoir pressure (psia)

$\bar{p}$  average reservoir pressure (psia)

$\bar{p}_{av}$  average pressure in the investigated area (psia)

$p_{wf}$  well flowing pressure (psia)

$p_{wf,i}$  initial well flowing pressure (psia)

$p_{wf,f}$  terminal well flowing pressure (psia)

$q_g$  gas rate (Mscf/d)
\( q_o \)  oil rate (STB/d)

\( R_s \)  solution gas-oil ratio (scf/STB)

\( R_{sb} \)  solution gas-oil ratio at bubble point pressure (scf/STB)

\( R_{si} \)  solution gas-oil ratio at initial reservoir pressure (scf/STB)

\( R_v \)  solution condensate-gas ratio (STB/MMscf)

\( R_{vi} \)  solution condensate-gas ratio at initial reservoir pressure (STB/MMscf)

\( R_{v,dew} \)  solution condensate-gas ratio at dew-point pressure (STB/MMscf)

\( RNP \)  rate-normalized pressure (psi/STBD, psi²/cp/scfD)

\( S_c \)  condensate saturation (dimensionless)

\( S_g \)  gas saturation (dimensionless)

\( S_{g,base} \)  base gas saturation from Eq. 13 (dimensionless)

\( S_{gi} \)  initial gas saturation (dimensionless)

\( \bar{S}_{g,inv} \)  average gas saturation in investigated area (dimensionless)

\( S_o \)  oil saturation (dimensionless)

\( \bar{S}_o \)  average oil saturation (dimensionless)

\( S_{o,base} \)  base oil saturation from Eq. 13 (dimensionless)

\( S_{oil} \)  initial oil saturation (dimensionless)

\( \bar{S}_{o,inv} \)  average oil saturation in investigated area (dimensionless)

\( t \)  time (day)

\( T \)  reservoir temperature (°R)

\( t_{LST} \)  linear superposition time (\( \sqrt{\text{day}} \))

\( x_e \)  well half-spacing (ft)

\( x_{ft} \)  total fracture half-length (ft)
Greek Variables

\( y_e \) fracture half-spacing (ft)

\( y_{inv} \) distance of investigation (ft)

\( Z_g \) gas compressibility factor (1/psi)

\( \alpha \) constant of distance of investigation equation (Eq. 11)

\( \gamma_{API} \) oil gravity (API)

\( \gamma_k \) permeability modulus (1/psi)

\( \Delta t \) time step (day)

\( (\Delta y)_n \) size of nth grid block

\( (\Delta y)_0 \) size of fracture grid block

\( \mu_g \) gas viscosity (cp)

\( \mu_{gD} \) dimensionless gas viscosity, \( \mu_g / \mu_{gi} \)

\( \mu_{gi} \) gas viscosity at initial reservoir pressure (cp)

\( \mu_o \) oil viscosity (cp)

\( \mu_{oD} \) dimensionless oil viscosity, \( \mu_o / \mu_{oi} \)

\( \mu_{oi} \) oil viscosity at initial reservoir pressure (cp)

\( \phi \) porosity (dimensionless)

\( \phi_D \) normalized porosity, \( \phi/\hat{\phi} \)

\( \hat{\phi} \) porosity at initial reservoir pressure (dimensionless)
References


6.11 Appendix 6A – Dynamic Drainage Area

Using the DDA concept (Clarkson and Qanbari, 2016a), oil and gas rates during transient linear flow period are, respectively, approximated by:
\[
q_o = k_i h \left[ m_o(\bar{p}_{inv}) - m_o(p_{wf}) \right] + \frac{R_s(p_{wf})}{1000} k_i h \left[ m_g(\bar{p}_{inv}) - m_g(p_{wf}) \right] \]
\[
141.2 \mu_w B_o \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] 1424T \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] \]

\[
q_g = k_i h \left[ m_g(\bar{p}_{inv}) - m_g(p_{wf}) \right] + \frac{R_s(p_{wf})}{1000} k_i h \left[ m_o(\bar{p}_{inv}) - m_o(p_{wf}) \right] \]
\[
1424T \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] 141.2 \mu_o B_o \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] \]

For simplicity, the second terms on the right-hand sides of Eqs. 6A.1 and 6A.2 are ignored in this study; that is, oil flow in gas phase and gas flow in condensate phase are neglected, which results in:

\[
q_o = k_i h \left[ m_o(\bar{p}_{inv}) - m_o(p_{wf}) \right] \]
\[
141.2 \mu_w B_o \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] \]

\[
q_g = k_i h \left[ m_g(\bar{p}_{inv}) - m_g(p_{wf}) \right] \]
\[
1424T \left[ \frac{2}{\pi} \left( \frac{y_{inv}}{x_p} \right) \right] \]

Therefore, the DDA-corrected rate normalized pressure (RNP) equations for oil and gas systems are obtained by substituting \( y_{inv} \) from Eq. 11 rearranging Eqs. 6A.3 and 6A.4 as follows:

\[
\frac{m_o(\bar{p}_{inv}) - m_o(p_{wf})}{q_o} = 57.16 B_o \sqrt{\frac{H_o}{A_i k_i \phi c_i \sqrt{t}}} \]

\[
\frac{m_g(\bar{p}_{inv}) - m_g(p_{wf})}{q_g} = 576.56 T \sqrt{\frac{1}{A_i k_i \phi g c_i u}} \]

There are four differences between DDA-corrected RNP equations (Eqs. 6A.5 and 6A.6) and standard RNP equations (Eqs. 1 and 3): for the DDA-corrected RNP equations, 1) pseudopressure is used for oil systems; 2) average pseudopressure is used instead of initial pseudopressure; 3) the coefficients are different (due to average pseudopressure being used in DDA-corrected RNP), and 4) superposition time is replaced with square-root of time. A plot of DDA-corrected RNP versus square-root of time gives a straight line from which the linear flow
parameter, \( A_i \sqrt{k_i} \), may be calculated using Eqs. 6A5 and 6A6. In this study, Eqs. 6A5 and 6A6 were applied to different dry gas, gas condensate and oil systems. It is concluded that using average rock and fluid properties (i.e. evaluated at average pressure in the investigated area, \( \bar{p}_{inv} \)) in the definition of distance of investigation provides more accurate results. Therefore, Eqs. 6A3 and 6A4 change to:

\[
\frac{m_o(p_{inv}) - m_o(p_{wf})}{q_o} \sqrt{\frac{\mu_{oD}(\bar{p}_{inv})\phi_D(\bar{p}_{inv})c_{iD}(\bar{p}_{inv})}{k_D(\bar{p}_{inv})}} = \frac{57.16B_{iD}}{A_i \sqrt{k_i} \sqrt{\phi c_{iD}}} (6A7)
\]

\[
\frac{m_g(p_{inv}) - m_g(p_{wf})}{q_g} \sqrt{\frac{\mu_{gD}(\bar{p}_{inv})\phi_D(\bar{p}_{inv})c_{gD}(\bar{p}_{inv})}{k_D(\bar{p}_{inv})}} = \frac{576.56T}{A_i \sqrt{k_i} \sqrt{\phi c_{gD} c_{u}}} (6A8)
\]

The dimensionless parameters of Eqs. 6A7 and 6A8 are defined as:

\[
\mu_{oD} = \frac{\mu_o}{\mu_{oi}}
\]

\[
\mu_{gD} = \frac{\mu_g}{\mu_{gi}}
\]

\[
\phi_D = \frac{\phi}{\phi_i}
\]

\[
k_D = \frac{k}{k_i}
\]

\[
c_{iD} = \frac{c_i}{c_{ii}}
\]
Chapter 7 Conclusions and Future Work

7.1 Conclusions

Three methods are proposed for analysis of transient linear flow in tight oil and gas reservoirs with pressure-dependent rock and fluid properties and two-phase flow. The methods include the transformation of nonlinearity method, iterative integral method, and dynamic drainage area (DDA) method. The methods are validated against numerical models, and their practical applicability tested with field cases.

The results show that the new methods can effectively correct the results of standard RTA methods for tight oil and gas reservoirs. The main takeaways from this thesis are as follows:

1. In Chapter 2, the method of transformation of nonlinearity is successfully applied to RTA of tight oil reservoirs with stress-dependent permeability. The results show that oil pseudo-pressure and the analytical correction factor are necessary for reliable linear flow analysis of tight stress-sensitive oil reservoirs.

2. In Chapter 3, the correction factors calculated from iterative integral method are shown to effectively reduce the error of the results from classic linear flow plots for tight oil wells operating above and below bubble-point pressure.

3. The application of the iterative integral method to tight gas reservoirs is shown in Chapter 4. The results demonstrate that, in addition to gas pseudo-pressure, a correction factor is required to obtain reliable results from linear flow analysis of tight gas reservoirs.

4. The impacts of pore proximity and adsorbed layer thickness are incorporated in RTA of dry gas systems in Chapter 5. The results demonstrate that ignoring pore confinement effects and adsorption layer thickness in the analysis of linear flow may introduce considerable error in the results of tight gas/shale gas RTA.

5. Finally, the DDA-corrected RTA method is successfully applied to tight oil and gas reservoirs in Chapter 6. The DDA-corrected RTA provides a new perspective for analysis of tight reservoirs by using average pressure in the investigated area instead of initial reservoir pressure, and time instead of superposition time.
7.2 Future Work

This thesis represents an effort to improve RTA of tight oil and gas reservoirs exhibiting complexities such as stress-dependent permeability and two-phase flow. Each chapter of the thesis provides the bases for a set of studies that the author intends to do in future including:

1. Chapter 2: the method of transformation of nonlinearity can be expanded to more complex cases including two-phase flow of oil+gas, gas+condensate and hydrocarbon+water.

2. Chapters 3 & 4: the methods developed in these chapters will be applied to a range of oil and gas condensate cases with different oil-gas ratio values. The results will then be compared to those using other advanced RTA methods provided in the literature, and against the DDA method.

3. Chapter 5: the study of the impacts of pore confinement, adsorption layer, and condensation on RTA results will continue to be evaluated; further, the impact of pore confinement effects on the performance of rich gas condensate wells will be evaluated.

4. Chapter 6: application of DDA for analysis of boundary-dominated flow and communicating tight oil and gas wells can help optimize well and fracture spacing. The results of the application of the DDA method for transient linear flow will be compared with the methods developed in Chapters 3 & 4.
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