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Essays on Child Labor and Development

Rosales Arredondo, Irving

UNIVERSITY OF CALGARY

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by

Irving Rosales Arredondo

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Abstract

This volume presents three research papers on child labor and economic development.

In Chapter 1, we argue that blanket child labor restrictions can harm the children of developing nations even in the long run. To develop our argument, we analyze the allocation of children's time among school, work and crime within a general equilibrium model with overlapping generations. Our main insight is that the social return to child labor can be higher than its private return if laws against crime and laws promoting compulsory education are not enforced. The reason is that child labor crowds out child crime, not just schooling. The perverse long-run consequences of otherwise well-intended policies against child labor are more likely in countries with poor institutions, where property rights are insecure and the returns to schooling are low.

In Chapter 2, we examine the long-run optimal policy to address the problem of child labor in developing countries. We extend the model presented in the first chapter and examine the role the imperfect enforceability of laws against crime and laws promoting compulsory schooling play in determining the long-run optimal policy. We argue that a combination of subsidies for child work and subsidies for school attendance maximizes social welfare in the long run. Such a combination of subsidies is the optimal way to reduce child labor and crime simultaneously.

In Chapter 3, I explore the effect of credit market imperfections and school quality on the joint distribution of schooling, child labor and crime. I develop an overlapping generations model of endogenous growth and human capital inequality. I argue that banning child labor can harm the children living in the poorest households if the security of property rights is sufficiently poor. Furthermore, even a temporary ban on child labor can have permanent negative effects on everyone's welfare if school quality and property rights are sufficiently poor. Finally, I show that access to credit markets may have no effect on schooling decisions if the productivity of child labor is sufficiently poor. However, an increase in school quality may eliminate child labor even in the presence of credit market imperfections.

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Overview

It is becoming increasingly recognized that blanket restrictions on child labor, as advocated in the ILO Convention No. 138, are likely to be harmful in the short run, but there remains the common presumption that they must be beneficial in the long run. The underlying logic seems compelling: child labor involves the sacrifice of a child's future human capital in exchange for current income; since future human capital determines future income, child labor is a source of future poverty. In this thesis, to the contrary, we argue that blanket child labor restrictions can harm the children of developing nations even in the long run. Our critique rests on the inextricable link between the problems of child labor and crime.

To develop our argument, in Chapter 1 we analyze the allocation of children's time among school, work, and crime within a general equilibrium model with overlapping generations. We then use the model to address the long-run consequences of restrictions on child labor. Our analysis formalizes the view that work and crime are two competing uses of a child's time and, of the two, potential crime is the greater social problem. It also illustrates how the long-run effects of alternative policies are greatly shaped by the imperfect enforceability of laws against crime and compulsory schooling laws.

In Chapter 2 we extend the model presented in Chapter 1 in order to analyze the policy implications of our theory. The optimal policy analysis presented in this chapter is designed to highlight the long-run effects of education policy and child labor policy. Our analysis suggests that the optimal complement to school subsidies is a targeted subsidy, as opposed to a tax, for child work. In practice, this could translate into conditioning transfers, whether in cash or in kind, on an optimal mix of school and work designed to crowd out alternative activities that are relatively more harmful to children.

In Chapter 3 I extend the basic model of Chapter 1 to study the effect of credit market imperfections, inequality, and school quality on the joint distribution of schooling, child labor and child crime in developing countries. I develop an overlapping generations model of endogenous growth and inequality in the distribution of human capital. The analysis addresses the short-run and long-run consequences of a ban on child labor across different households. In addition, I examine how three important aspects of institutional quality – the security of property rights, the quality of education, and the access to credit markets – can affect the activity of children. Finally, the analysis also illustrates how the activity of the children is affected by differences in inequality.

It is worth noting that the case against blanket restrictions on child labor that is made in this thesis is an economic case, not an ethical one. An argument in favor of banning the worst forms of child labor can be made on ethical grounds. In this case, the concern is about the human rights of the child. However, even though there is wide agreement that the worst forms of child labor should be banned, it is far from obvious that the typical working child would be better off if he or she were not allowed to work.

Chapter 1

Naive policies against child labor can harm the children of developing nations

with Francisco M. Gonzalez

1.1 Introduction

It is becoming increasingly recognized that blanket restrictions on child labor, as advocated in the International Labour Organization (ILO) Minimum Age Convention, 1973 (ILO Convention No. 138),[1] are likely to be harmful in the short run, but there remains the common presumption that they must be beneficial in the long run. In this paper, to the contrary, we argue that blanket child labor restrictions can harm the children of developing nations even in the long run. Our critique rests on the inextricable link between the problems of child labor and crime. According to the United Nations Human Settlements Programme (UN-HABITAT, 2011, p.23) "in Africa, 27 percent of youth are neither in school nor at work, a situation that can lead to frustration,

[1] Article 1 of the ILO Convention No. 138 demands that member countries commit "*to pursue a national policy designed to ensure the effective abolition of child labour and to raise progressively the minimum age for admission to employment*", and Article 2(4) sets the initial minimum age for admission to employment at 14 years for member countries "*whose economy and educational facilities are insufficiently developed*".

delinquency and social exclusion". This concern is specific neither to Africa nor to the twenty first century, as depicted by Charles Dickens' famous portrayal of nineteenth century London in *Oliver Twist*.

To develop our argument, we analyze the allocation of children's time among school, work, and crime within a general equilibrium model with overlapping generations. We then use the model to address the long-run consequences of restrictions on child labor. Our analysis formalizes the view that work and crime are two competing uses of a child's time and, of the two, potential crime is the greater social problem. It also illustrates how the long-run effects of alternative policies are greatly shaped by the imperfect enforceability of laws against crime and compulsory schooling laws.

Our model assumes that child labor is a source of current household income involving the sacrifice of the child's future human capital. Since human capital determines future income, child labor is a source of future poverty. Arguably, it is this logic that underlies the common view that the abolition of child labor will benefit children and promote their nation's economic development. In our model, we go a step further and consider that *child crime* also involves a similar sacrifice of a child's future human capital in exchange for current income. By crime we mean the illicit appropriation of the wealth of others. It is therefore an alternative source of current income. By child crime we mean crime committed by a child. Like child labor, it harms human capital accumulation because it also interferes with schooling. A distinguishing feature of child crime, however, is that it harms other people by taxing the returns to their work and possibly harming their human capital accumulation. Here we shall focus on child crime, taking as given the vulnerability of children to the clutches of organized crime and leaving aside crime against children more generally.

The simultaneous enforcement of laws against crime and the outright ban of child labor would be desirable in the long run, even though it would likely harm poor families in the short run, by lowering their income. Children would be forced to attend school, and the resulting increase in future human capital, together with the removal of the "crime tax", would be conducive to development. Similarly, one might expect the enforcement of compulsory full-time schooling to crowd out child labor and crime. However, neither laws against crime nor compulsory schooling laws seem enforceable in practice in the developing world.

In principle, child labor regulation is easier to enforce than either compulsory schooling or laws against crime. However, as long as the latter remain unenforced, what is the allocation of the time that is freed up by compulsory restrictions on child labor? At one extreme, if current and future time squeezed out of child labor were fully allocated to schooling, one would expect the policy to be desirable at least in the long run. At the other extreme, if all displaced child labor were instead driven to crime, one would expect the policy to be harmful even in the long run. The likely outcome lies between these two extremes, and consequently, it is necessary to understand the allocation of schooling, child labor and child crime in society jointly, rather than separately.

When we analyze this social allocation problem, our main insight is that the imperfect enforceability of laws against crime and laws in favor of compulsory schooling greatly shapes the relationship between the *social* return to child labor and its *private* return. If these laws were fully enforced, the social return to child labor would be lower than its private return, as commonly presumed. Otherwise, one must recognize that the social return to child labor can be higher than its private return. This feature has an intuitive explanation: child labor crowds out child crime, not just schooling.

This insight has remarkable policy implications. We show that restrictions on child labor are socially harmful, even in the long run, whenever the social return to child labor is larger than its private return. That restrictions on child labor have a negative short run effect is not surprising. Household income, consumption, and saving all fall in the short run, and so current households can be made worse off even if their children's human capital rises. Yet, we show that blanket restrictions on child labor can make future generations worse off as well, because restricting child labor interferes with its role in crowding out crime.

The perverse long-run consequences of otherwise well-intended policies against child labor are more likely in countries with poor institutions, where property rights are insecure and the returns to schooling are low. In this context, the short-run and the long-run effects of restrictions on child labor are linked through the intergenerational transmission of poverty. On the one hand, by lowering current and future child labor, these restrictions tend to increase human capital accumulation. On the other hand, this positive effect is counteracted by the negative effect of increased current and future crime on human capital accumulation. Moreover, current and future income fall, directly as the result of lower child labor income, and indirectly through the negative effect of lower incomes, and possibly higher crime, on saving and investment.

Previous analyses of child labor radicate the main sources of inefficiency in credit market imperfections (Baland and Robinson, 2000) and positive human capital externalities (Krueger and Donohue, 2005).[2] While our analysis takes as given the presence of credit market imperfections that prevent parents from borrowing against their children's future income, our main focus is on the negative externalities associated with crime.

We are not the first to warn against unintended consequences of standard policies to combat child labor. A central message of research on the economics of child labor during the last decade is that there is a wide variety of circumstances in which policies against child labor are likely to backfire. They may increase the incidence of child labor (Jafarey and Lahiri, 2002, Edmonds and Pavcnik, 2005a, Basu, 2005, Basu and Zarghamee, 2009)), cause households' welfare to fall (Basu and Van, 1998, Dessy and Pallage, 2005), and otherwise have perverse distributional consequences (Krueger and Donohue, 2005, Dinopoulos and Zhao, 2007, Baland and Duprez, 2009). However, while previous research has brought attention to the negative short-run effect of blanket child labor restrictions, emphasizing the loss of current income suffered by poor households, it has left unchallenged the popular view that such blanket restrictions are necessarily beneficial in the long run. By contrast, challenging precisely this view is central to our argument against enforcing the standards set in the ILO Convention No. 138.

[2] Basu (1999) and Edmonds (2008) are two excellent surveys of the literature on child labor. Doepke and Zilibotti (2005, 2010) and Dessy and Knowles (2008) consider the link between political support for child labor regulation and fertility decisions.

Section 1.2 presents some evidence on child labor and child crime as it pertains to our argument. Section 1.3 presents the basic model, and Section 1.4 characterizes the *laissez-faire* equilibrium. Section 1.5 analyzes the effect of restrictions on child labor. Section 1.6 considers two extensions. Section 1.7 concludes. Proofs are relegated to the Appendix A.

1.2 Evidence on child labor and child crime

Child labor remains pervasive around the world. Edmonds and Pavcnik (2005b) report participation rates in various activities for 124 million children between 5 and 14 years old from 36 countries in the year 2000. About 51 percent of those children combine school and work; 19 percent attend school and do not work; 18 percent work and do not attend school; the remaining 12 percent neither work nor attend school. While reliable data on actual criminal activity is lacking, it is evident that youth crime — potential as well as actual crime — is a serious concern in developing countries. According to the United Nations Office on Drugs and Crime (UNODC, 2011), about 4 million children worldwide were "brought into formal contact with the police" (i.e., arrested or cautioned) each year during the 2003-2008 period. Children working in the informal sector seem particularly vulnerable. According to e-oaxaca (June 13, 2011) more than 158,000 children working in the streets of the Mexican state of Oaxaca are believed to be vulnerable to recruitment by organized crime.

Our main insight is that the social return to child labor can be higher than its private return, because child labor crowds out child crime. This assumes that laws against crime are unenforceable, which is hardly questionable. It also assumes that compulsory schooling laws are unenforceable, and that the poor have access to only low-quality education in developing countries. All 193 members of the United Nations have signed the 1990 Convention on the Rights of the Child, whose Article 28 states that State Parties shall make primary education compulsory and available free for all.[3]

[3] All members but the United States and Somalia have also ratified the convention.

Yet, according to the United Nations Educational, Scientific and Cultural Organization (UNESCO), more than 67 million children worldwide — about 10 percent of all children of primary school age — were out of school in 2009, and furthermore, "millions of children emerge from primary school each year without having acquired basic literacy and numeracy skills" (UNESCO, 2010, p. 104). Not surprisingly, in sub-Saharan Africa alone, for instance, about 10 million children drop out of primary school each year (UNESCO, 2011, p. 47).

Evidence of the consequences of child labor regulation draws mainly on the historical record of child labor in currently developed countries, particularly the U.S. and Britain. Arguably, the overall contribution of child labor and education laws to the decline of child labor and the increase in educational attainment in the U.S. and in Britain seems to have been relatively small, with the laws being enforced only after the rise of the factory system (see Moehling (1999) and Goldin and Katz (2011) on the U.S., and Kirbi (2003) on Britain).

In England, for instance, the 1815 Report of the Committee for Investigating the Causes of the Alarming Increase of Juvenile Delinquency in the Metropolis concluded that "the improper conduct of parents", "the want of education" and "the want of suitable employment" were the main causes of juvenile delinquency (Pinchbeck and Hewitt, 1973, p. 435). Indeed, it is well documented that child labor has been encouraged historically as an effective tool to combat child crime, for instance, in nineteenth century England and the U.S. (Watson, 1896, Myers, 1933, Davidson, 1939), and in Mexico during the 1920s (Sosenski, 2008). It is also worth noting that the first efforts to combat child labor emphasized access to education, rather than the prohibition of child labor (Hindman, 2002, p. 49).

Not only rigorous evidence in support of policies against child labor is lacking, but intervention-gone-awry cases of working children seem to be the rule, rather than the exception. For instance, the threat of the Child Labor Deterrence Act in 1993, advocated by Senator Harkin in the U.S., caused garment employers in Bangladesh to dismiss an estimated 50,000 children from their factories. The 1997 UNICEF State of the World's Children noted that "*follow-up visits by UNICEF, local non-governmental organizations (NGOs) and the International Labour Organization (ILO) discovered that children went looking for new sources of income, and found them in work such as stone-crushing, street hustling and prostitution*" (UNICEF, 1997, p. 60). Similar unintended outcomes were observed in the Morocco's garment industry in 1995 after a report of the British Granada TV's *World in Action*, which investigated the labeling of garment made in Méknès (see, e.g., Bourdillon et al., 2010).

1.3 The model

Consider an economy with overlapping generations. A continuum of identical agents, with mass 1, is born every period. Each agent lives for three periods, which we refer to as childhood, adulthood and old age. Only adults face non-trivial decisions. They have preferences over current consumption (equivalently, the consumption of the child-parent pair) c_a, consumption when old c'_o, and their child's labor income next period, $w'h'$:

$$U = u(c_a) + u(c'_o) + \delta v(w'h') \equiv \ln(c_a) + \ln(c'_o) + \delta \ln(w'h'), \tag{1.1}$$

with $\delta \in (0, 1]$, where w denotes an adult's wage per effective unit of human capital, h denotes an adult's effective human capital. Primed variables denote next-period values.

Only children and adults work, and each is endowed with one unit of time. Children make no decisions. Adults allocate their children's time among three alternative activities:

$$e + x + z \leq 1, \text{ with } e, x, z \geq 0, \tag{1.2}$$

where e is the time a child spends in school, x is the time she spends at work, and z is the time she spends in criminal activities (e.g., theft).

Both child labor and crime harm human capital accumulation. We assume that

$$h' = Q(Z)(1 - ax - bz)^{\beta} h^{1-\beta}, \tag{1.3}$$

with $0 < \beta < 1$, and $0 < a \leq b < 1$, where we refer to $1 - ax - bz$ as *effective schooling*, and h is the stock of human capital children inherit from their parents. The term $Q(Z)$ in equation (1.3) reflects the fact that aggregate child crime may harm children's human capital accumulation, for a given choice of effective schooling. We assume that $Q(0) > 0$, and

$$Q(Z) = \left(\frac{Z}{\underline{Z}}\right)^{-\gamma}, \tag{1.4}$$

for all $Z > 0$, with $0 \leq \gamma < \beta$, where the term $\underline{Z} > 0$ is a normalization. It is convenient to assume $\underline{Z} \leq (1-p)\epsilon$ throughout, with $p \in [1/2, 1)$ and $\epsilon \in (0, 1/2)$, which ensures that the equilibrium level of child crime always exceeds the lower bound $\underline{Z}$. Below, $1-p$ is defined as a "crime tax". It is worth anticipating that some of our results below refer to the case with $\gamma > 0$, but we show in Section 1.6 that they also go through if, instead, the negative externality associated with crime works through the crime tax, or if the log-utility assumption is relaxed.

Human capital accumulation in the above specification is not determined by the actual time children spend in school ($e \leq 1 - x - z$), but by effective schooling ($1 - ax - bz$), where a is the opportunity cost of time allocated to work in terms of school, and b is the opportunity cost of time allocated to crime in terms to school. Thus, if a child devotes all of her time to school, effective schooling coincides with the actual time she spends in school. That is, $1 - ax - bz = 1$ if $e = 1$. At the other extreme, effective schooling remains positive even if a child does not attend school. Thus, if a child works full time, effective schooling amounts to $1 - a$ units. Our assumption that $1 - a > 0$ reflects the fact that children will retain some of their human capital even if they work full time. Similarly, effective schooling would be equal to $1 - b$ units whenever a child engages in crime full time, and our assumption that $1 - b > 0$ implies that even a full time criminal retains some human capital. Assuming that $b \geq a$ implies that crime harms human capital accumulation at least as much as work does.

There is a single final good that is produced according to the production technology

$$F\left(K, H + \phi H_c\right) = A\, K^{\alpha} \left(H + \phi H_c\right)^{1-\alpha}, \tag{1.5}$$

with $A > 0$ and $\alpha \in (0,1)$, where K is the aggregate stock of physical capital, $H = \int_0^1 h_i di$ is the aggregate stock of human capital provided by adults, $H_c = \int_0^1 x_i h_i di$ is the aggregate stock of human capital provided by children, and the productivity of children relative to that of adults is given by $\phi \leq 1$. We also maintain the assumption that $\phi > \frac{b-a}{1-b}$ throughout in order to rule out uninteresting scenarios. The aggregate production technology given by equation (1.5) reflects the fact that children and adults are perfect substitutes in production, the fact that children work x units of time rather than full time (i.e., one unit), and the fact that children are less productive than adults. We assume all markets are perfectly competitive. For simplicity, we also assume physical capital depreciates fully every period.

We model crime as the result of decentralized conflict over economic distribution,[4] and we assume, for simplicity, that crime taxes households' savings. We also assume that crime is fully unproductive, and only children engage in criminal activity. If there is some crime in the economy, a fixed proportion $1-p \in (0, 1/2]$ of all the labor income that is not consumed is subject to appropriation. A household's labor income is the sum of the adult's labor income wh and the child's labor income $w_c hx$, which is the income that a child worker with human capital h, who works x units of time gets when her wage is w_c. To formalize the aggregate consequences of decentralized crime in a simple manner, we assume each household competes against the economy's average. In particular, if a child spends z units of time in criminal activity, she will secure a proportion z/Z of the economy-wide average crime rents $(1-p)(Y_L - C_a)$, where Z is the average level of crime in the economy and Y_L denotes average labor income. Throughout the paper we use capital letters to denote economy-wide averages, which

[4] See Gonzalez (2012) for a survey of research on insecure property rights, conflict and development.

coincide with aggregates since there is a unit mass of households.

Aggregate consistency of the distribution of crime rents requires that the aggregate resources lost to child crime every period add to aggregate crime rents, that is,

$$\int_0^1 (1-p)\left((w+w_c x_i)\, h_i - c_{a,i}\right)\, di = (1-p)\int_0^1 (Y_L - C_a)\, z_i/Z\, di, \tag{1.6}$$

where the subscript i denotes an individual household. Although our focus below is on symmetric equilibria with positive levels of child crime, it remains to specify the crime rents that accrue to a criminal whenever $Z = 0$. We simply assume they are a fraction $(1-p)$ of the average labor income net of consumption $Y_L - C_a$.

Old agents at time $t+1$ simply consume their capital income,

$$c_o' = (1+r')\, s, \tag{1.7}$$

where r' is the market rate of return on savings. For simplicity, we assume there are no school fees, and so a household's savings out of labor income is given by

$$s = p\left((w+w_c x)\, h - c_a\right) + (1-p)\frac{z}{Z}\left(Y_L - C_a\right), \tag{1.8}$$

whenever $Z > 0$, where $p \in [1/2, 1)$ reflects the security of effective property rights.

We restrict attention to *symmetric* equilibria with positive consumption, which are given by a sequence of allocations $\{x_{it}, z_{it}, s_{it}\}_{t=0}^{\infty}$ for all individuals $i \in [0,1]$, a sequence of average allocations $\{X_t, Z_t, K_{t+1}\}_{t=0}^{\infty}$, with $K_0 > 0$, and a sequence of prices $\{r_t, w_t, w_{ct}\}_{t=0}^{\infty}$ such that, given prices, individuals maximize utility, their time constraint (1.2) and budget constraints (1.7) and (1.8) are satisfied, firms maximize profits, human capital for each individual evolves according to (1.3), with $h_0 = H_0 > 0$, the distribution of crime rents satisfies (1.6), every market clears, and $\{x_{it}, z_{it}, s_{it}\} = \{X_t, Z_t, K_{t+1}\}$, for all $i \in [0,1]$ and for all $t \geq 0$.

1.4 Symmetric equilibrium

In this section we characterize the unique symmetric equilibrium with positive stocks of human and physical capital, and a positive level of child labor. We begin by considering the problem of an arbitrary household. First, note that optimal saving choices of the household are interior, and they satisfy the standard Euler equation

$$\frac{\partial u\left(c_a\right)/\partial c_a}{\partial u\left(c_o'\right)/\partial c_o'} = p\left(1+r'\right),$$

which equates the marginal rate of substitution between current and future consumption of an adult and the corresponding marginal rate of transformation. The latter reflects the fact that the insecurity of property rights in the economy acts as a tax on savings.

Second, an optimal choice of child crime is always interior, equating the marginal benefits and the marginal costs of child crime:

$$\frac{\partial u\left(c_a\right)}{\partial c_a}\frac{\partial c_a}{\partial z} = -\delta\frac{\partial v\left(w'h'\right)}{\partial h'}\frac{\partial h'}{\partial z}.$$

The marginal benefits from child crime come from higher consumption, where

$$\frac{\partial c_a}{\partial z} = \left(\frac{1-p}{p}\right)\left(\frac{Y_L - C_a}{Z}\right)$$

is decreasing in the aggregate level of child crime, for given aggregate crime rents. The marginal costs of crime come from the reduction in future labor income associated with the negative impact of child crime on human capital accumulation, where

$$\frac{\partial h'}{\partial z} = \frac{-b\beta h'}{1-ax-bz}.$$

Third, the household's optimal choice of child labor satisfies

$$\frac{\partial u\left(c_a\right)}{\partial c_a}\frac{\partial c_a}{\partial x} + \delta\frac{\partial v\left(w'h'\right)}{\partial h'}\frac{\partial h'}{\partial x} \leq 0,$$

with equality whenever optimal child labor is interior. The marginal benefits from child labor come from increased current consumption associated with higher labor income, with

$$\frac{\partial c_a}{\partial x} = w_c h,$$

whereas the marginal costs come from the reduction in the child's future earnings associated with the negative impact of child labor on her human capital accumulation:

$$\frac{\partial h'}{\partial x} = \frac{-a\beta h'}{1 - ax - bz}.$$

Next, profit maximization implies that all units of human capital are paid according to their marginal product. Accordingly, the wage of an adult per unit of human capital is

$$w = (1-\alpha)\frac{F\left(K, H + \phi H_c\right)}{\left(1 + \phi X\right) H}, \tag{1.9}$$

and the wage of a child is $w_c = \phi w$. Since markets are perfectly competitive, we also have:

$$1 + r = \alpha \frac{F\left(K, H + \phi H_c\right)}{K}. \tag{1.10}$$

Recalling that population is normalized to one, in a symmetric equilibrium we have that $x = X$, $z = Z$, $c_a = C_a$, $c_o = C_o$, $s = S$. Furthermore, the labor market for adult human capital clears every period ($h = H$) and so does the market for child labor ($xh = H_c$). Finally, market clearing in the final goods market implies that aggregate income is equal to aggregate output ($Y = F\left(K, H + \phi H_c\right)$), and market clearing in the capital market every period implies that aggregate savings and aggregate investment in physical capital are equal ($S = K'$). It is easy to verify that market clearing also implies that aggregate resources lost to child crime every period add to aggregate crime rents, so equation (1.6) is satisfied.

As usual, the market clearing conditions, together with symmetry of the equilibrium, can be used to characterize equilibrium dynamics as a function of aggregate variables alone, and equations (1.9)-(1.10) can be used to eliminate prices from the resulting equilibrium conditions. To that end, note that the above Euler equation for optimal savings, together with the fact that $C_o' = (1 + r')\,S$, imply that $S = pC_a$. Thus, the aggregate resources constraint implies that aggregate consumption by adults is a fraction $\frac{1}{1+p}$ of aggregate labor income $(1 - \alpha)\,Y$. Accordingly, aggregate investment is a fraction $\frac{p}{1+p}$ of aggregate labor income

$$K' = \left(\frac{p}{1+p}\right)(1 - \alpha)\,Y. \tag{1.11}$$

Note that savings come only from labor income, because old agents are the owners of capital, do not work, and consume all of their income. With log utility, households save a constant fraction of their labor income. Intuitively, aggregate investment increases with the security of property rights, as parameterized by p. In the limit as p approaches 1, households would save exactly half of their labor income, because they do not discount future consumption.

The log-utility assumption simplifies the analysis by eliminating dynamics in child labor and crime. Noting that the resources subject to appropriation is given by $(1 - \alpha)\,Y - C_a = pC_a$, it is easy to verify that the equality of marginal costs and benefits from child crime implies the following relationship between child labor and child crime every period:

$$Z = \frac{1 - aX}{b\left(1 + \frac{\delta\beta}{1-p}\right)} \equiv g\,(X)\,. \tag{1.12}$$

The equilibrium relationship $Z = g\,(X)$ has $\partial g/\partial X < 0$ because the marginal benefit from child crime is decreasing in Z, independent of X, while the marginal cost of child crime to a household is increasing in x and z, since there are diminishing returns to effective schooling.

If the optimal choice of child labor is interior, the optimality of child labor and child crime, together, give a second equilibrium relationship between these two activities:

$$Z = \left(\frac{a(1-p)}{b\phi(1+p)} \right) (1+\phi X) \equiv f(X), \tag{1.13}$$

which has $\partial f / \partial X > 0$ because the marginal benefit from child labor is decreasing in X, independent of Z, the marginal benefit from child crime is decreasing in Z, independent of X, while the ratio of marginal costs of child labor and child crime is constant.

An equilibrium with positive levels of child labor is such that $X^* > 0$ solves $g(X) = f(X)$, with $Z^* = g(X^*)$ every period. It is then easy to verify the following (see Appendix).

Proposition 1 *There exists a symmetric equilibrium with positive child labor and schooling if and only if* $\phi/a \in \left(m^L, m^H\right)$ *and* $b \in \left(b^L, b^H\right)$*, where* $m^L \in (0,1)$*,* $m^H > m^L$*,* $b^L \in (0,1)$*, and* $b^H > b^L$ *are given in the Appendix. This equilibrium is unique, with*

$$\begin{aligned} X^* &= \frac{1+p-(1-p+\delta\beta)\left(\frac{a}{\phi}\right)}{a(2+\delta\beta)}, \\ Z^* &= \left(1+\frac{a}{\phi}\right)\left(\frac{1-p}{b(2+\delta\beta)}\right), \\ K' &= \left(\frac{p}{1+p}\right)(1-\alpha) A K^{\alpha} H^{1-\alpha} (1+\phi X^*)^{1-\alpha}, \\ H' &= \left(\frac{Z}{\underline{Z}}\right)^{-\gamma} (1-aX^*-bZ^*)^{\beta} H^{1-\beta}, \end{aligned}$$

and it converges to a steady state for all initial conditions $K_0 > 0$ *and* $H_0 > 0$*.*

Intuitively, the existence of a symmetric equilibrium with positive child labor requires the productivity of child labor, ϕ, to be sufficiently high relative to the opportunity cost of child labor in terms of schooling, a ($\phi/a > m^L$), so there is an incentive for children to allocate some time to work. It also requires that the productivity of child labor be sufficiently low relative to the opportunity cost of child labor in terms of schooling ($\phi/a < m^H$), so there remains an incentive for children to allocate some time to school. In turn, the opportunity cost of child crime in terms of schooling, given by b, needs to be sufficiently low ($b < b^H$) for schooling not to be crowed out entirely ($m^H > 0$), and also sufficiently high ($b > b^L$) for child labor and schooling to coexist ($m^L < m^H$).

1.5 Long-run consequences of child labor restrictions

In this section we use the above model to argue that the presence of child crime can greatly shape the long-run implications of child labor restrictions. Formally, we consider an enforceable upper bound to the time a child can devote to work, which we denote by $\overline{x}$, with $0 \leq \overline{x} < X^*$. Larger values of $\overline{x}$ correspond to weaker child labor restrictions, which are binding as long as they constrain child labor to be below the equilibrium level X^*. The case where $\overline{x} = 0$ corresponds to an outright child labor ban.

That restrictions on child labor have a negative short run effect is not surprising. Even though the resulting fall in child labor comes with a rise in child crime, household income, saving, and investment all fall in the short run. Consequently, current households can be made worse off, even if their children's future human capital rises. More importantly, the following proposition shows that restrictions on child labor can make future generations worse off as well.

Proposition 2 ***(i)*** *A permanent cap $\overline{x}$ on child labor, with $0 \leq \overline{x} < X^*$, reduces long-run utility if and only if $\overline{x} < x_U$;* ***(ii)*** *$x_U > 0$ if and only if $\phi/a > n_U$;* ***(iii)*** *x_U rises with γ, and $x_U \geq X^*$ if and only if $\gamma \geq \beta\left(1 - \frac{1-p+\delta\beta}{\left(1+\frac{\delta}{2}\right)(1+p)}\right)$, where x_U and n_U are given in the Appendix.*

Part *(i)* of the proposition says that child labor restrictions harm long-run utility whenever they lower child labor sufficiently. Part *(ii)* provides the conditions for a full ban to be harmful. More generally, it says that there is a non-empty interval of child labor restrictions $[0, x_U]$ that harms long-run utility if and only if the productivity of child labor, relative to the opportunity cost of child labor in terms of schooling, is sufficiently high. Part *(iii)* implies that a given child labor restriction is more likely to be harmful when the effect of the human capital externality associated with child crime is stronger. It also says that even a marginal restriction on child labor may harm long-run utility.

The short-run and the long-run consequences of restrictions on child labor are linked through the intergenerational transmission of poverty. Following the enforcement of a permanent cap on child labor, increased current and future crime counteracts the positive effect on human capital accumulation of decreased current and future child labor. Moreover, current and future income fall, directly as the result of lower child labor, and indirectly through the negative effect of lower incomes on saving and investment.

It is somewhat remarkable that Proposition 2 holds even in the log-utility case, and even though the crime tax $1-p$ is independent of aggregate crime. These two assumptions will be relaxed in Section 1.6. However, under these assumptions, not only child labor and crime are unaffected by the dynamics of capital accumulation, but the aggregate investment *rate* is independent of the levels of child labor and crime. Accordingly, the negative effect of lower income on investment is particularly weak, since the aggregate *investment rate* is unaffected by the enforcement of a cap on child labor.

The above two assumptions weaken the negative effect of crime on human capital accumulations as well. Thus, it is easy to verify that human capital increases, both in the short run and in the long run, following the enforcement of a permanent cap on child labor. However, the increase in human capital cannot compensate for the fall in capital accumulation if the human capital externality associated with crime is sufficiently strong.

The intuition behind the results given in Proposition 2 is better understood by contrasting the equilibrium of the model with two alternative benchmark planning problems. It is worth mentioning in advance that these benchmark planning problems will also play a useful role in our long-run optimal policy analysis in Chapter 2. The first one is the problem of a planner that allocates all resources in the economy in order to maximize the representative household's long-run utility over all feasible allocations. Formally, let the utility of the representative household in period t be

$$V_t = \ln(\theta_t C_t) + \ln((1-\theta_{t+1}) C_{t+1}) + \delta \ln\left(\frac{(1-\alpha) F(K_{t+1}, (1+\phi X_{t+1}) H_{t+1})}{(1+\phi X_{t+1})}\right),$$

where $\theta_t \equiv C_{at}/C_t$, and where the above objective function assumes that adult labor is rewarded according to its social marginal product every period. The relevant planning problem consists of choosing an allocation $\{X_t, Z_t, K_{t+1}, \theta_t\}_{t\geq 0}$ in order to solve:

$$\begin{aligned} \max \lim_{t\to\infty} V_t \text{ subject to } \quad C_t &= F(K_t, (1+\phi X_t) H_t) - K_{t+1}, \\ H_{t+1} &= \left(\frac{Z_t}{\underline{Z}}\right)^{-\gamma} (1 - aX_t - bZ_t)^{\beta} H_t^{1-\beta}, \qquad (1.14) \\ X_t &\geq 0,\ Z_t \geq \underline{Z},\ X_t + Z_t \leq 1,\ \theta_t \in [0,1],\ \text{for all } t \geq 0. \end{aligned}$$

It is easy to verify that any solution exhibits constant values of X and Z. As indefinitely maintainable values of C, K, and H satisfy $C = F(K, (1+\phi X) H) - K$ and $H = \left(\frac{Z}{\underline{Z}}\right)^{-\gamma/\beta} (1 - aX - bZ)$, Problem (1.14) reduces to the following two-step problem. First, suppose that the planner allocates aggregate consumption period by period between the old and the adults, with $C_a = \theta C$, and $C_o = (1-\theta) C$. It is easy to see that $\theta = 1/2$ solves:

$$\max_{\theta} \ln(\theta C) + \ln((1-\theta) C) \text{ for any } C > 0. \qquad (1.15)$$

Now, Problem (1.14) reduces to:

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha} H^{1-\alpha} (1+\phi X)^{1-\alpha} - K\right) + \delta \ln\left(\frac{(1-\alpha) AK^{\alpha} H^{1-\alpha} (1+\phi X)^{1-\alpha}}{(1+\phi X)}\right)$$

$$\text{subject to } H = (Z/\underline{Z})^{-\gamma/\beta} (1 - aX - bZ),\ X \geq 0,\ Z \geq \underline{Z},\ X + Z \leq 1. \qquad (1.16)$$

The second planning problem is just like Problem (1.14), except that the planner cannot control child crime, and so she faces the additional constraint $Z_t = g(X_t)$

every period, where $g(\cdot)$ is given by equation (1.12). The only difference between the two planning problems is that the second one must take into account that child crime decisions are made optimally by the households. The above argument now implies that the new planning problem also solves Problem (1.15). However, instead of solving Problem (1.16), it solves:

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha}-K\right)+\delta\ln\left(\frac{(1-\alpha)AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha}}{(1+\phi X)}\right)$$

$$\text{subject to } Z=g(X),\ H=(Z/\underline{Z})^{-\gamma/\beta}(1-aX-bZ),\ X\geq 0,\ Z\geq \underline{Z},\ X+Z\leq 1. \tag{1.17}$$

Let $\{X_1, Z_1, K_1\}$ and $\{X_2, Z_2, K_2\}$ solve Problem (1.16) and Problem (1.17), respectively. Comparing these allocations and the steady-state equilibrium allocation $\{X^*, Z^*, K^*\}$ implied by Proposition 1, one can show the following result (see Appendix).

Proposition 3 ***(i)*** *$X_1 < X^*$ if and only if $X^* > 0$, and $Z_1 = \underline{Z} < Z^*$.* ***(ii)*** *$X_2 > 0$ is a necessary condition for a permanent cap $\overline{x}$ on child labor, with $0 \leq \overline{x} < X^*$, to reduce long-run utility.* ***(iii)*** *The following three statements are equivalent: (a) $X_2 \geq X^*$, (b) $Z_2 \leq Z^*$, and (c) $x_U \geq X^*$, where x_U is given in Proposition 2.*

Part *(i)* says that equilibrium child labor and crime are both inefficiently large relative to the long-run optimal allocation $\{X_1, Z_1, K_1\}$. Thus, it is socially desirable to eliminate both child labor and crime. Part *(ii)* implies that child labor restrictions necessarily increase long-run utility if the constrained long-run optimal allocation $\{X_2, Z_2, K_2\}$ requires the elimination of all child labor. Part *(iii)* states that the condition that equilibrium child labor be inefficiently low relative to the *constrained* long-run optimal allocation $\{X_2, Z_2, K_2\}$ is equivalent to the condition given in Proposition 2 to have that even a marginal restriction on child labor will harm long-run utility. It also implies that equilibrium child labor is too low if and only if equilibrium child crime is too high, relative to $\{X_2, Z_2, K_2\}$.

To appreciate the full implications of Proposition 3, it is useful to consider the solutions to Problem (1.16) and Problem (1.17) in some detail. In both cases, optimal investments require equating the marginal product of capital to its cost:

$$\frac{2}{C}\left(\frac{\partial F}{\partial K} - 1\right) = -\delta \frac{1}{Y}\frac{\partial F}{\partial K}.$$

Using the fact that $\frac{\partial F}{\partial K} = \alpha \frac{Y}{K}$ and $\frac{C}{K} = \frac{Y}{K} - 1$, the long-run optimal investment rate is:

$$\frac{K_1}{Y_1} = \frac{K_2}{Y_2} = \alpha\left(\frac{2+\delta}{2+\alpha\delta}\right), \tag{1.18}$$

which is greater than the capital share in the production of output, α, because $\delta > 0$, and individuals do not care about their children's consumption, but rather about their labor income. It is easy to verify that the steady-state equilibrium investment rate is lower than the long-run optimal investment rate, that is, $K^*/Y^* < K_1/Y_1$, if and only if $\frac{p}{1+p} < \frac{\alpha}{1-\alpha}\left(\frac{2+\delta}{2+\alpha\delta}\right)$. For instance, $\alpha \geq 1/3$ ensures that this is the case for all $p < 1$.

Now, consider Problem (1.16). Since crime harms human capital accumulation, the long-run optimal choice of crime is simply $Z_1 = \underline{Z}$. Furthermore, if the long-run optimal level of child labor is interior, the *social* return to child labor in the long run must be zero:

$$2\phi \frac{\partial F}{\partial (1+\phi X)} - (2+\delta)\frac{\partial F}{\partial H}\left[-\frac{\partial H}{\partial X}\right] = 0. \tag{1.19}$$

The first term in the left side of the equation is the *social* marginal benefit of child labor. The second term is its *social* marginal cost. For a comparison, in equilibrium, child crime satisfies $Z^* = g(X^*)$, and it is the *private* return to child labor that is zero:

$$(1+p)\phi \frac{\partial F}{\partial (1+\phi X)} - \delta \frac{\partial F}{\partial H}\left[-\frac{\partial H}{\partial X}\right] = 0. \tag{1.20}$$

The first term in the left-hand-side is the *private* marginal benefit of child labor evaluated in utility terms. It accounts for the fact that individuals consume only a fraction $\frac{1}{1+p}$ of additional income from child labor. The second term is the *private* marginal cost of child labor, which reflects the marginal cost from the forgone future income, discounted by the factor δ, due to the fact that child labor harms the child's human capital.

Evaluated at the equilibrium allocation, the *social* marginal benefit of child labor is greater than the *private* marginal benefit. This comes from the fact that child labor increases output, which increases the consumption of both the adult and the old. Evaluated at the equilibrium allocation, the social marginal cost of child labor is also greater than its private marginal cost. This comes from the fact that child labor harms a child's human capital, which depresses consumption of both the adult and the old. It also depresses future labor income, and this cost is discounted by the factor δ. It is easy to see that the social return (that is, the social benefit net of cost) of child labor is always negative, when evaluated at the equilibrium allocation, since $\frac{2}{2+\delta} < \frac{1+p}{\delta}$. Accordingly, $X_1 < X^*$.

Now, consider Problem (1.17). Child crime satisfies $Z_2 = g\left(X_2\right)$ and X_2 satisfies

$$2\phi\frac{\partial F}{\partial\left(1+\phi X\right)} - (2+\delta)\frac{\partial F}{\partial H}\left[-\frac{\partial H}{\partial X} - \frac{\partial H}{\partial Z}\frac{\partial Z}{\partial X}\right] = 0. \tag{1.21}$$

As in Problem (1.16), it is the social return to child labor that is being maximized. The only difference is that Problem (1.17) takes into account the fact that child labor crowds out child crime, which in turn affects the calculation of the marginal cost of child labor in terms of human capital. Since $\frac{\partial g(X)}{\partial X} < 0$, this additional effect reduces the social marginal cost of child labor, relative to the unconstrained optimum, whenever the human capital externality associated with crime is present, which implies that $X_2 > X_1$ whenever $\gamma > 0$. Moreover, Proposition 2 and Proposition 3 together imply that the social (marginal) returns to child labor are positive (when evaluated at the equilibrium allocation) if and only if $X_2 > X^*$, which is the case if and only if the human capital externality associated with crime is sufficiently strong that even a marginal child labor restriction will harm long-run utility. Note that the harmful effects of child labor restrictions come from the *potential*, as opposed to the actual, participation of children in criminal activities.

It is easy to show that

$$X_1 = \begin{cases} \frac{(1-b\underline{Z})\frac{1}{a}-\left(1+\frac{\delta}{2}\right)\frac{1}{\phi}}{2+\frac{\delta}{2}} & \text{if} \quad \phi/a \geq \frac{1+\frac{\delta}{2}}{1-b\underline{Z}} \\ 0 & \text{if} \quad \phi/a \leq \frac{1+\frac{\delta}{2}}{1-b\underline{Z}}. \end{cases} \tag{1.22}$$

Recall that we have assumed that $\underline{Z} \leq (1-p)\,\epsilon$, with $p \in [1/2, 1)$ and $\epsilon \in (0, 1/3)$, in order to ensure that the equilibrium level of child crime exceeds the lower bound $\underline{Z}$. In addition, note that Z_1 converges to 0 as ϵ approaches 0, and so

$$\lim_{\epsilon \to 0} X_1 = \begin{cases} \frac{\frac{1}{a}-\left(1+\frac{\delta}{2}\right)\frac{1}{\phi}}{2+\frac{\delta}{2}} & \text{if} \quad \phi/a \geq 1+\frac{\delta}{2} \\ 0 & \text{if} \quad \phi/a \leq 1+\frac{\delta}{2}. \end{cases}$$

Similarly, it is easy to verify that

$$X_2 = \begin{cases} \frac{\frac{1}{a}-\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)\frac{1}{\phi}}{1+\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)} & \text{if} \quad \phi/a \geq \left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right) \\ 0 & \text{if} \quad \phi/a \leq \left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right), \end{cases} \tag{1.23}$$

where it is evident that $X_2 \geq X_1$ whenever $\gamma \geq 0$, with $X_2 = X_1$ if and only if $\gamma = 0$.

It should be noted that $X_2 > 0$ if and only if a/ϕ is sufficiently small, and also that this situation is compatible with $X_1 = 0$, which requires that a/ϕ be sufficiently large. In this sense, our main results apply to an economy where child labor is unambiguously harmful to children, but not too harmful. As a mater of interpretation, our argument against child labor restrictions therefore excludes the worst cases of *hazardous work* as well as all *unconditionally worst forms of child labor*.[5] Importantly, however, we think that our analysis does apply to many forms of child labor that harm children's human capital accumulation. An implication is that the fact that child labor is harmful to children does not justify the imposition of child labor restrictions. This is the case only if child labor is sufficiently harmful.

[5]Worldwide, about 115 million children (under 18 years old) are estimated to do *hazardous work* (ILO, 2011) — "work which, by its nature or the circumstances in which it is carried out, is likely to harm the health, safety or morals of children" (ILO Convention 182 (1999), Article 3(d)). At least another 8.4 million children are involved in *unconditional worst forms of child labor*, including all forms of slavery, prostitution and pornography, and drug production and trafficking (ILO Convention 182 (1999), Article 3(a,b,c)).

Consider the case in which property rights are perfectly secure. That is, suppose that $p = 1$, and $\gamma = 0$. In this extreme case, it is easy to verify that child labor restrictions unambiguously increase long-run utility. However, it should be noted that they also make current generations worse off. In particular, the current old suffers from decreased capital rents, and the current households forgo child labor income, which is not compensated by the increase in adult labor income. This scenario formalizes the common perception that child labor restrictions are desirable in the long run because they allow children to accumulate human capital, even though current generations may be worse off.

Arguably, the simultaneous enforcement of laws against crime and the outright ban of child labor would be desirable in the long run, and so would be the enforcement of compulsory full-time schooling. However, neither laws against crime nor compulsory schooling laws seem enforceable in developing countries. The following policy analysis recognizes these facts.

1.6 Extensions

The above formulation of the negative externalities associated with child crime, working through human capital accumulation, allowed for a simple analysis of the long-run effects of restrictions on child labor. In this section, we briefly discuss two extensions of our basic model that would lead to the same qualitative results, even in the absence of human capital externalities associated with child crime. The first one allows aggregate child crime to affect the magnitude of the "crime tax". The second one relaxes the assumption of log-utility.

There are various ways to think about how child criminality works. For example, it may teach children to be opportunistic. One implication is that it reduces the effectiveness of schooling, as in our basic model. The relevance of this channel is documented in studies of school violence.[6] Another is that it may lower institutional

[6] For instance, a study of school violence in ten developing countries concludes that *"violence at school is costly not only in financial terms, but also in terms of the long-term damage it inflicts on*

quality.[7] A way to formalize the latter is to allow aggregate child crime to affect the crime tax $1 - p$ in our basic model. We have the following analog of Proposition 2.

Proposition 4 *Suppose that* $1 - p = P(Z)$, *with* $P(0) = 0$, $\partial P / \partial Z > 0$, *and* $\partial^2 P / \partial Z^2 < 0$, *and consider an equilibrium with positive crime.* ***(i)*** *A binding permanent cap* $\overline{x}$ *on child labor reduces long-run utility if and only if* $\overline{x} < \widetilde{x}_U$; ***(ii)*** $\widetilde{x}_U > x_U$, *where* $\widetilde{x}_U$ *is characterized in the Appendix, and* x_U *is given in Proposition 2.*

This proposition considers the case where the "crime tax" $(1 - p)$ increases, at a decreasing rate, with the aggregate level of child crime, focusing on equilibria with positive crime. Part *(i)* of the proposition says that child labor restrictions harm long-run utility whenever they lower child labor sufficiently. Part *(ii)* says that the effect of crime on the crime tax makes any given child labor restriction relatively less desirable in the long run. The problem is that any restriction on child labor increases the aggregate level of child crime in the economy, which increases the crime tax, which in turn promotes crime. The resulting equilibrium level of crime is higher, while long-run investment, income, and utility all fall relative to the case where the crime tax is exogenous.

It is easy to construct numerical examples to ensure that any restriction on child labor will reduce long-run utility even in the absence of human capital externalities (i.e., even if $\gamma = 0$). One such example is the following: let $P(Z) = 1 - \exp\{-\eta Z\}$, with $\delta = 1$, $\alpha = 0.7$, $\beta = 0.8$, $\phi = 0.85$, $a = b = 0.95$, and let $\gamma = 0$ and $\eta = 1.28$, which gives $X^* = 0.38$, $Z^* = 0.03$, and $p = 1 - P(Z^*) = 0.96$, in the unique equilibrium with positive crime. In this case, a cap on child labor still leads to higher human capital in the long-run as well as in the short run. Relative to the case where the crime tax is exogenous, the counteracting effect of increased crime on human capital is larger; moreover, increased crime raises the crime tax, which in turn depresses the economy's

the individual's healthy personality growth and development, the loss of his and her quality of life, its interference with the individual's learning of pro-social behaviours, and, above all, its impact on the vital task of developing human resources for national development." (UNESCO, 1997, p. 7).

[7] One instance of this is documented in Al Jazeera's (02/09/2012) broadcast on the river traders of Brazil, in particular, on the participation of children in both trade and piracy along the Tajaparu River (http://www.aljazeera.com/programmes/2011/05/201153142852595854.html).

investment rate; consequently, the negative impact that a cap on child labor has on investment, and thus on future income, is exacerbated.

Finally, it should be noted that the assumption of log utility in our basic model facilitates the analysis by eliminating dynamics in the allocation of children's time. Alternatively, suppose that an adult's utility is given by

$$U = \frac{c_a^{1-\sigma} - 1}{1-\sigma} + \frac{(c_o')^{1-\sigma} - 1}{1-\sigma} + \delta\left(\frac{(w'h')^{1-\sigma} - 1}{1-\sigma}\right),$$

where $\sigma \geq 0$ is the inverse of the elasticity of intertemporal substitution, and where $\sigma = 1$ corresponds to log utility. One can verify that there is a continuum of equilibrium paths whenever $\sigma \neq 1$, which are parameterized by arbitrary initial conditions (X_0, Z_0). However, one can also show that this feature does not translate into a continuum of steady-state equilibrium allocations. It is not difficult to ensure that there exists a unique steady-state equilibrium, although it has no analytical solution.

In this case, it can be shown that a permanent cap $\overline{x}$ on child labor, with $0 \leq \overline{x} < X^*$, reduces long-run utility if and only if $\overline{x} < \widehat{x}_U$, with $\widehat{x}_U > x_U$ if and only if $\sigma > 1$, where x_U is given in Proposition 2. That is, the fact that the elasticity of intertemporal substitution $(1/\sigma)$ is lower than one makes any given child labor restriction relatively less desirable in the long run.

Furthermore, it can also be shown that there exists a non-trivial level $\sigma_L > 1$ such that a permanent cap on child labor reduces long-run utility if and only if $\sigma > \sigma_L$, for any $\gamma \geq 0$. The reason is that larger values of σ tend to exacerbate the negative consequences of restrictions on child labor, so much so that any restriction will lead to lower utility in the long run if the elasticity of intertemporal substitution is sufficiently low $(\sigma > \sigma_L)$, even in the absence of human capital externalities associated with child crime (i.e., even if $\gamma = 0$), and even if the crime tax $(1-p)$ is exogenous. For example, to show σ_L is a non-trivial level, simulations of the model with $\delta = \phi = 1$, $\alpha = 0.7$, $\beta = 0.8$, $a = 0.45$, $b = 0.5$, $p = 0.95$, and $\gamma = 0$, indicate that even a marginal restriction on child labor causes utility to fall in the long run whenever $\sigma \geq 4$, with

$X^* = 0.31$ and $Z^* = 0.004$, for $\sigma = 4$, and where X^* and Z^* decrease with σ.

Intuitively, the greater the value of σ, the less willing individuals are to sacrifice current consumption, either in exchange for future consumption, or for the sake of their children's future human capital. Accordingly, for greater values of σ, the loss of current income arising from child labor restrictions induces households to sacrifice relatively more of their child's future human capital, by increasing child crime.

Indeed, it is possible to show that there exists a non-trivial level $\sigma_H > \sigma_L$ such that $\frac{\partial(1-a\overline{x}-bZ)}{\partial\overline{x}} > 0$ if and only if $\sigma > \sigma_H$, for any $\gamma \geq 0$. That is, if the elasticity of intertemporal substitution is sufficiently low such that $\sigma > \sigma_H$, then any restriction on child labor would unambiguously decrease not only utility but also human capital in the long run, even in the absence of human capital externalities associated with child crime. In this case, it is easy to find numerical examples where the outright ban of all child labor would lower not only utility but also human capital in the long run for non-trivial values of σ. For example, simulations of the model, with $\delta = \phi = 1$, $\alpha = 0.5$, $\beta = 0.8$, $a = 0.3$, $b = 0.5$, $p = 0.93$, and $\gamma = 0$, indicate that banning all child labor would cause human capital to fall in the long-run if $\sigma \geq 15$ (with $X^* = 0.25$ and $Z^* = 0.007$ if $\sigma = 15$). This effect would reinforce, rather than offsetting, the negative welfare effect of reduced investment, and so it is sufficient to ensure that long-run utility would fall as well.

1.6.1 Alternative policies

There are a number of other implications of our analysis that deserve emphasis. Here we consider the effect of alternative policies that are commonly advocated. The first class of policies refers to the use of international sanctions as a tool to pressure developing countries to combat child labor. Examples include import bans and consumer boycotts. The second refers to the abolition of school fees, and more generally, reductions in school fees.

Although our basic model above does not involve international trade, we think that a primary effect of international sanctions against firms, industries, or countries that use child labor, can be formalized in terms of a reduction in the productivity of child labor in the context of our basic model. Our reading of the literature is that economists agree by now that international sanctions can have undesirable consequences in the short run, as they may induce reductions in labor income that may increase the supply of child work, and otherwise be welfare reducing. In contrast, we argue that international sanctions can be undesirable even in the long run. In particular, we have the following result.

Proposition 5 *International sanctions, formalized as a reduction in the productivity of child labor, ϕ, reduce long-run utility if and only if $X_2 > 0$, where X_2 is given by equation (1.23).*

This proposition says that the necessary and sufficient condition for international sanctions to reduce long-run utility is that the level of child labor that maximizes long-run utility, constrained by the fact that child crime is determined by the equilibrium response of the households, be positive.

Intuitively, international sanctions have two different effects. On the one hand, they directly reduce household's income through the reduction in the effective wage earned by child workers, inducing households to increase crime. On the other hand, these sanctions decrease the opportunity cost of child crime relative to child labor (see equation (1.13)). That is, for a given level of child labor, households are willing to increase the proportion of children's time allocated to crime. These effects combined increase child crime.

It should be noted that the unconstrained optimal that maximize long-run utility would also respond to reductions in the productivity of child labor by lowering long-run utility, whenever the corresponding optimal allocation exhibit positive levels of child labor. However, the exact conditions refer to whether or not the constrained optimal allocation would demand child labor to be positive. This highlights once again the key social role of child labor in crowding out child crime.

Next, consider the possibility that schooling is costly. For simplicity, suppose that households must pay a school fee that is equal to a proportion $f > 0$ of the economy's average labor income Y_L if their child is to attend school. It is a simple matter to extend our analysis for $f = 0$ to the case where school fees are positive, and show the following result.

Proposition 6 *Abolishing school fees increases long-run income and long-run utility. It also reduces child crime, but it increases child labor if and only if* $p < \frac{1}{1+\delta\beta}$.

The proposition states that the abolition of school fees is welfare improving, and that it also results in increased long-run income. While this corresponds to the conventional view, it is so for a different reason. Abolishing school fees is a desirable policy against child crime. Indeed, child labor may even increase permanently in response to the policy. However, it would be a mistake to interpret the resulting increase in child labor as a policy failure, because it simply reflects the fact that child labor helps crowding out child crime, which is really the more fundamental social problem.

1.7 Conclusion

The dominant view within developed countries is that international labor standards aimed at the eradication of child labor must be immediately enforced. This view underlies significant international activism aimed at compelling developing countries to enforce the standards set in the ILO Convention No. 138. Thus, on August 29, 2012, the Union cabinet of India approved an amendment to existing child labor laws that, if adopted by parliament, would impose significant penalties to parents and employers of children younger than 14 in any work at all. Not surprisingly, it has been noted that "*[i]mage is very important now since India is promoting itself as the fastest-emerging economic power in the world, ...[t]hey can't afford legislation which goes against that image*" (Wall Street Journal, November 23, 2012).

Those welcoming India's proposed law rejoice that *"[t]hey've really recognised that the long-term benefits of education are far more consequential than the short-term gains of child labor"* (Financial Times, August 29, 2012). Dissenting voices regret that "*strategies have been designed for all children based on generalised examples of children in hazardous and intolerable forms of labour ... that account for a very small percentage of the child work force*", warning that "*[t]his new amendment will be even more difficult to enforce and will further push children into more invisible, unmonitored and therefore hazardous situations*" (Deccan Herald, September 4, 2012).

Evidently, arguments against blanket child labor restrictions have not found much support in the policy arena, even though it is increasingly recognized that they are likely to be harmful in the short run. We think this is because of the persistent belief that such blanket restrictions are likely to be beneficial in the long run. In this paper, to the contrary, we have argued that enforcing the standards set in the ILO Convention No. 138, as India is proposing to do, can harm the children of developing nations even in the long run. That some forms of child labor are abhorrent is not in dispute. Neither is the spirit of Convention No. 138. However, our analysis does call on policy makers to avoid blanket restrictions on child labor, lest they violate the well-known Hippocratic injunction to *do no harm*.

Not surprisingly, working children organizations across the world point out not only that child work does not necessarily interfere with schooling, but also that it may itself be an important source of human capital. Our analysis strengthens their case by formalizing the common observation that "*there are worse things that can happen to children than having to work*" (Basu, 1999, p. 1115) and analyzing the long-term implications. Formally, we have focused on criminal activity, recognizing both the vulnerability of children to the clutches of organized crime and the potential negative externalities associated with child crime.

Our main insight is that the imperfect enforceability of laws against crime and laws in favor of compulsory schooling greatly shapes the relationship between the *social* return to child labor and its *private* return. If these laws were fully enforced, the social return to child labor would be lower than its private return, as commonly presumed. Otherwise, one must recognize that the social return to child labor can be higher than its private return. This feature has an intuitive explanation: child labor crowds out child crime, not just schooling.

The perverse long-run effects of otherwise well-intended policies against child labor are more likely in countries with poor institutions, where property rights are insecure and the returns to schooling are low. It is in this context that the blanket restrictions advocated by the ILO, national ministries, trade unions and “ethical” consumers from developed countries are representative of the naive policies to which the title of this paper alludes. Our analysis illustrates that their case for the abolition of child labor is fallacious, not only because it presumes that high-quality education is the relevant alternative to child labor, but also because it fails to recognize how the short-term and the long-term effects of restrictions on child labor are linked through the intergenerational transmission of poverty.

Chapter 2

Optimal policy to combat child labor and child crime

with Francisco M. Gonzalez

2.1 Introduction

In Chapter 1 we showed why blanket restrictions on child labor, as advocated in the ILO Convention No. 138, are undesirable since they can harm the children of developing nations even in the long run. Thus, an important question is to determine how to address the problem of child labor in developing countries. In this Chapter, we argue that a combination of subsidies for child work and subsidies for school attendance maximizes social welfare in the long run.

To develop our argument, we extend the model presented in Chapter 1 and examine the role the imperfect enforceability of laws against crime and laws promoting compulsory schooling play in determining the long-run optimal policy. The policy analysis presented here is designed to highlight the long-run effects of education policy and child labor policy. We first introduce a class of policies rich enough to target the multiple distortions present in the *laissez-faire* environment. We then characterize the long-run consequences of alternative subsets of policies in turn. Our goal is to characterize the policy that maximizes social welfare in the long run.

In order to analyze the policy implications of our argument against the ILO Convention No. 138, we first consider the effect of child labor tax policy in the absence of education policies. We show that the effect of compulsory child labor restrictions can be always replicated by a positive tax on child labor. Furthermore, we show that the long-run optimal child labor policy, in the absence of education policy, requires a subsidy, as opposed to a tax, whenever the social return to child labor exceeds its private return.

Then, we analyze the effect of education policies in the absence of child labor policy. In this case, we argue that a subsidy for school attendance is a useful instrument because it taxes child labor and child crime simultaneously. In particular, we find that long-run optimal subsidies for school attendance always dominate positive taxes on child labor, and in general tend to dominate child labor subsidies as well. The long-run advantage of targeted subsidies for school attendance over child labor subsidies reflects the fact that the latter crowd out schooling as well as crime, whereas the former crowd out child labor and child crime, both of which are detrimental to long-run social welfare.

Finally, we present the analysis of the long-run optimal policy. In the context of our model, we argue that a combination of subsidies for child work and subsidies for school attendance maximizes social welfare in the long run. Thus, the optimal complement to school subsidies is a targeted subsidy, as opposed to a tax, for child work. There are two main aspects to this.

First, the key point is that combining both subsidies is the optimal way to *reduce* child labor and crime simultaneously, not that the optimal level of child labor is necessarily high. This result is due to the fact that school subsidies depress child labor and so they interfere with the social value of child labor arising from its role in crowding out crime. Indeed, even if the long-run optimal allocation requires eliminating child labor, we show that it is optimal to subsidize, as opposed to prohibit, child labor as long as compulsory education is not enforceable.

The second aspect concerns the implementation of our proposed optimal policy in developing countries. In the formal model, we focus on conditional income transfers (positive or negative) in order to illustrate the relevant incentive problems. In practice, this could translate into conditioning transfers, whether in cash or in kind, on an optimal mix of school and work designed to crowd out alternative activities that are relatively more harmful to children.

The evidence suggests that subsidies for school attendance have been successful in increasing school enrollment and reducing child labor incidence (Ravallion and Wodon, 2000, and Bourguignon, Ferreira and Leite, 2003). Programs such as Progresa in Mexico, Bolsa Escola in Brazil, and the Food-For-Education (FFE) program in Bangladesh are well known cases. Yet, the resulting increases in school enrollment tend to be significantly larger than the declines in child labor. With respect to this, our theory suggests that, where the potential for child crime is an important consideration, a relatively low response of child labor to targeted subsidies for school attendance is the counterpart of a relatively high response of child crime.[1]

Section 2.2 presents the model, and Section 2.3 characterizes the equilibrium under the tax policy and presents the long-run optimal policy problem. In Section 2.4, we study the effects of alternative policies. We consider the effect of child labor tax policy in the absence of education policies. We then analyze the effect of education policies in the absence of child labor policy. Finally, we show that the unconstrained long-run optimal policy always consists of a combination of subsidies for school attendance and child work. Section 2.5 concludes. Proofs are relegated to the Appendix B.

[1] The evidence for the U.S. strongly suggests that education is an effective tool to reduce juvenile crime. Lochner (2011) offers an excellent survey of the empirical evidence on education and crime.

2.2 The model

Consider an economy with overlapping generations. A continuum of identical agents, with mass 1, is born every period. Each agent lives for three periods: childhood, adulthood and old age. Only adults face non-trivial decisions and they have preferences over current consumption c_a, consumption when old c_o', and their child's labor income next period, $w'h'$:

$$U = u(c_a) + u(c_o') + \delta v(w'h') \equiv \ln(c_a) + \ln(c_o') + \delta \ln(w'h'), \tag{2.1}$$

with $\delta \in (0,1]$, where w denotes an adult's wage per effective unit of human capital, h denotes an adult's effective human capital. Primed variables denote next-period values.

Only children and adults work, and each is endowed with one unit of time. Children make no decisions. Adults allocate their children's time among three alternative activities: $e + x + z \leq 1$, with $e, x, z \geq 0$, where e is the time a child spends in school, x is the time she spends at work, and z is the time she spends in criminal activities.

Both child labor and crime harm human capital accumulation. We assume that

$$h' = Q(Z)(1 - ax - bz)^{\beta} h^{1-\beta}, \tag{2.2}$$

with $0 < \beta < 1$, and $0 < a \leq b < 1$, where h is the stock of human capital children inherit from their parents. We refer to $1 - ax - bz$ as *effective schooling*, where a is the opportunity cost of time allocated to work in terms of school, and b is the opportunity cost of time allocated to crime in terms to school. Our assumption that $1 - a > 0$ reflects the fact that children will retain some of their human capital even if they work full time. Similarly, our assumption that $1 - b > 0$ implies that even a full time criminal retains some human capital. Assuming that $b \geq a$ implies that crime harms human capital accumulation at least as much as work does.

The term $Q(Z)$ in equation (2.2) reflects the fact that aggregate child crime may harm children's human capital accumulation, for a given choice of effective schooling. We assume $Q(0) > 0$, and $Q(Z) = (Z/\underline{Z})^{-\gamma}$, for all $Z > 0$, with $0 \leq \gamma < \beta$, where the term $\underline{Z} > 0$ is a normalization. We also assume $\underline{Z} \leq (1-p)\,\epsilon$ throughout, with $p \in [1/2, 1)$ and $\epsilon \in (0, 1/2)$, where $1-p$ is defined as a crime tax.

There is a single final good that is produced according to the production technology

$$F\left(K, H + \phi H_c\right) = A\, K^{\alpha} \left(H + \phi H_c\right)^{1-\alpha}, \tag{2.3}$$

with $A > 0$ and $\alpha \in (0,1)$, where K is the aggregate stock of physical capital, $H = \int_0^1 h_i di$ is the aggregate stock of human capital provided by adults, $H_c = \int_0^1 x_i h_i di$ is the aggregate stock of human capital provided by children, and the productivity of children relative to that of adults is given by $\phi \leq 1$. As in Chapter 1, we assume $\phi > \frac{b-a}{1-b}$. The aggregate production technology given by equation (2.3) reflects the fact that children and adults are perfect substitutes in production, the fact that children work x units of time, and the fact that children are less productive than adults. We assume all markets are perfectly competitive. For simplicity, we also assume physical capital depreciates fully every period.

Given our ultimate focus on steady-state equilibria, it is sufficient to consider linear tax instruments. We restrict attention to the class of policies $\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\}$, where, for simplicity, we assume that every period t each adult faces a saving tax equal to $\tau_s s$, a tax on child labor equal to $\tau_x x w_c h$, and a tax on education equal to $\tau_e e w h$. In addition, each old individual in period $t+1$ faces a tax on capital gains equal to $\tau_o \left(1 + r'\right) s$. Finally, firms face a payroll tax $\tau_w \left(wH + w_c H_c\right)$.

We model crime as the result of decentralized conflict over economic distribution. We assume that crime is fully unproductive and only children engage in criminal activity. If there is some crime in the economy, a fixed proportion $1-p \in (0, 1/2]$ of all the after-tax labor income that is not consumed is subject to appropriation. A household's labor income is the sum of the adult's labor income wh and the child's labor income $w_c hx$. To formalize the aggregate consequences of decentralized crime, we assume each household competes against the economy's average. In particular, if a child spends z units of time in criminal activity, she will secure a proportion z/Z of the economy-wide average crime rents $(1-p)\left(Y_L - C_a - T_x - T_e\right)$, where Z is the average level of crime in the economy, Y_L denotes average labor income, and where T_x and T_e denote the aggregate tax revenue associated with the tax rates τ_x and τ_e, respectively, in period t. In what follows, we use capital letters to denote economy-wide averages, which coincide with aggregates since there is a unit mass of households.

Aggregate consistency of the distribution of crime rents requires that the aggregate resources lost to child crime every period add to aggregate crime rents, that is,

$$\int_0^1 (1-p)\left(\left(w+(1-\tau_x)\,w_c x_i\right)h_i - c_{a,i} - \tau_e e w h\right)\,di = (1-p)\int_0^1 \left(Y_L - C_a - T_x - T_e\right)\frac{z_i}{Z}\,di, \tag{2.4}$$

where the subscript i denotes an individual household. In order to specify the crime rents that accrue to a criminal whenever $Z = 0$, we simply assume they are a fraction $(1-p)$ of the average labor income net of consumption and taxes $Y_L - C_a - T_x - T_e$.

For simplicity, we assume there are no school fees. Then, individuals face the budget constraint

$$\begin{aligned}(1+\tau_s)\,s \;&=\; p\left(\left(w+(1-\tau_x)\,w_c x\right)h - c_a - \tau_e e w h\right) \qquad (2.5)\\ &\quad+\frac{(1-p)z}{Z}\left[\left(w+w_c X\right)H - C_a - T_x - T_e\right],\end{aligned}$$

when adults, whenever $Z > 0$, where $p \in [1/2, 1)$ reflects the security of effective property rights. Individuals also face the budget constraint

$$c_o' = (1-\tau_o)\,(1+r')\,s, \tag{2.6}$$

when old, where r' is the market rate of return on savings. Moreover, since firms face a payroll tax $\tau_w\,(wH + w_c H_c)$, after-tax profits are:

$$F\left(K, H+\phi H_c\right) - (1+\tau_w)\left(wH + w_c H_c\right) - (1+r)\,K. \tag{2.7}$$

Each of the above taxes can, in principle, be positive (i.e., actual taxes) or negative (i.e., subsidies). We restrict attention to the class of policies that balance the government budget period by period, so they satisfy

$$T_s + T_o + T_w + T_x + T_e = 0,$$

every period, where T_j is the aggregate tax revenue associated with the tax rate τ_j, in period t. We will continue to avoid time subscripts whenever possible.

For a given policy $T = \{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\}$, a *symmetric equilibrium* consists of a sequence of allocations $\{x_{it}(T), z_{it}(T), s_{it}(T)\}_{t=0}^{\infty}$ for all individuals $i \in [0,1]$, a sequence of average allocations $\{X_t(T), Z_t(T), K_{t+1}(T)\}_{t=0}^{\infty}$, with $K_0 > 0$, and a sequence of prices $\{r_t(T), w_t(T), w_{ct}(T)\}_{t=0}^{\infty}$ such that, given prices, individuals maximize utility, their time constraint and budget constraints are satisfied, firms maximize after-tax profits, human capital for each individual evolves according to (2.2), with $h_0 = H_0 > 0$, aggregate consistency of the distribution of crime rents is satisfied, every market clears, and $\{x_{it}(T), z_{it}(T), s_{it}(T)\} = \{X_t(T), Z_t(T), K_{t+1}(T)\}$, for all $i \in [0,1]$ and for all $t \geq 0$.

2.3 The policy problem

It is useful to anticipate that we shall adopt the standard primal approach to optimal taxation, by solving a series of primal planning problems and then show how to implement their solutions via policy instruments, instead of attacking directly the dual optimal taxation problems. Let $\left\{X_t^*(T), Z_t^*(T), K_{t+1}^*(T)\right\}_{t=0}^{\infty}$ denote a symmetric equilibrium allocation under the policy T. We say that a policy T implements allocation $\{X_t, Z_t, K_{t+1}\}_{t=0}^{\infty}$ if there is a symmetric equilibrium with $\left\{X_t^*(T), Z_t^*(T), K_{t+1}^*(T)\right\} = \{X_t, Z_t, K_{t+1}\}$, for all $t \geq 0$.

Our objective is to characterize the policy that maximizes the steady-state equilibrium level of utility within the class of feasible policies. Formally, letting $U_t^*(T)$ denote the adults' equilibrium utility at date t, and letting Ψ be the class of feasible policies, we aim to solve

$$\max_{T} \lim_{t\to\infty} U_t^*(T), \text{ subject to } T \in \Psi. \tag{2.8}$$

Recall that the class of feasible policies is restricted to those that balance the government budget every period. Of course, the solution to the policy problem also depends on the precise tax instruments available. To fix terminology, we say that a solution to Problem (2.8) is a long-run optimal policy within the class of feasible policies Ψ.

In order to understand the solution to Problem (2.8), it is useful to characterize first the steady-state equilibrium allocation that a policy $\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\}$ implements, subject to the balanced-budget condition. This is easily done by proceeding as we did in the case of the *laissez-faire* equilibrium in Chapter 1.

Given that population is normalized to one, in a symmetric equilibrium we have $x = X$, $z = Z$, $c_a = C_a$, $c_o = C_o$, $s = S$. In addition, the labor market for adult human capital clears every period ($h = H$) and so does the market for child labor ($xh = H_c$). Market clearing in the final goods market implies that aggregate income is equal to aggregate output ($Y = F(K, H + \phi H_c)$), and market clearing in the capital market every period implies that aggregate savings and aggregate investment in physical capital are equal ($S = K'$). Finally, it is easy to verify that market clearing also implies that aggregate resources lost to child crime every period add to aggregate crime rents, so equation (2.4) is satisfied.

Next, profit maximization implies that all units of human capital are paid according to their after-tax marginal product. Then, given the payroll tax, the wage of an adult per unit of human capital is given by

$$w = \left(\frac{1-\alpha}{1+\tau_w}\right)\frac{F(K, H + \phi H_c)}{(1+\phi X)H},$$

and the wage of a child is a fraction ϕ of the wage of an adult, $w_c = \phi w$.

Since markets are perfectly competitive, the rental price of physical capital is equal to its marginal product

$$1 + r = \alpha\frac{F(K, H + \phi H_c)}{K}.$$

Now consider the problem of an arbitrary household. First, note that the optimal saving choices of households are interior, and they satisfy the Euler equation

$$\frac{\partial u\left(c_a\right)/\partial c_a}{\partial u\left(c_o'\right)/\partial c_o'}=\frac{p\left(1-\tau_o\right)\left(1+r'\right)}{1+\tau_s},$$

where in this case, the tax rates on savings and capital gains affect the marginal rate of transformation due to their effect on adult's and old's consumption, respectively.

Second, an optimal choice of child crime is always interior, equating the marginal benefits and the marginal costs of child crime:

$$\frac{\partial u\left(c_a\right)}{\partial c_a}\frac{\partial c_a}{\partial z}+\delta\frac{\partial v\left(w'h'\right)}{\partial h'}\frac{\partial h'}{\partial z}=0.$$

One can verify that the optimality of child crime can be written as

$$\frac{1-p}{Z}+\tau_e\left(\frac{1+p}{1+\phi\left(1-\tau_x\right)X-\tau_e\left(1-X-Z\right)}\right)-\frac{\delta\beta b}{1-aX-bZ}=0. \tag{2.9}$$

The first term in the left side is the marginal benefit from crime rents, evaluated in utility terms. It accounts for the fact that the resources subject to appropriation are simply pC_a. The second term is the marginal effect on utility that child crime has on the education's tax burden through its effect on the level of education. The term in parentheses is simply the ratio wH/C_a, which depends on both τ_x and τ_e. The last term is the marginal cost of child crime, which reflects the utility cost associated with the foregone future income that comes from the fact that crime harms children's human capital.

Next, the household's optimal choice of child labor satisfies

$$\frac{\partial u\left(c_a\right)}{\partial c_a}\frac{\partial c_a}{\partial x}+\delta\frac{\partial v\left(w'h'\right)}{\partial h'}\frac{\partial h'}{\partial x}\leq 0,$$

with equality whenever optimal child labor is interior. The optimality of child labor implies that

$$\left(\phi\left(1-\tau_x\right)+\tau_e\right)\left(\frac{1+p}{1+\phi\left(1-\tau_x\right)X-\tau_e\left(1-X-Z\right)}\right)-\frac{\delta\beta a}{1-aX-bZ}\leq 0, \tag{2.10}$$

with strict inequality only if $X = 0$ is optimal. The term in the second parentheses is the ratio wH/C_a, as before. The term $\phi(1-\tau_x)(wH/C_a)$ is the marginal benefit from after-tax child-labor income, evaluated in utility terms. The term $\tau_e(wH/C_a)$ is the marginal effect on utility that child labor has on the education's tax burden through its effect on the level of education (i.e., the ratio wH/C_a). The last term on the left-hand-side of equation (2.10) is the marginal utility cost of child labor associated with lower human capital accumulation.

One can verify that the government budget each period can be written as

$$\frac{1+\tau_w}{1-\alpha}\left[p\tau_s\left(\frac{1-\alpha(1-\tau_o)}{1+p+\tau_s}\right)+\alpha\tau_o\right]+\tau_w=-\left(\frac{\tau_x\phi X+\tau_e(1-X-Z)}{1+\phi X}\right), \quad (2.11)$$

the Euler equation for optimal saving, evaluated in steady-state equilibrium, gives

$$\frac{C_o}{C_a}=\left(1+\frac{p}{1+\tau_s}\right)\frac{\alpha(1-\tau_o)}{1-\alpha(1-\tau_o)}, \quad (2.12)$$

and the steady-state equilibrium investment rate is given by

$$\frac{K}{Y}=p\left(\frac{1-\alpha(1-\tau_o)}{1+p+\tau_s}\right). \quad (2.13)$$

It will become clear that equations (2.9)–(2.13) convey the relevant information needed to understand the long-run optimal policy problem.

2.4 Policy analysis

In this section, we examine how alternative policies address the different distortions that are present in the steady-state equilibrium of the *laissez-faire* environment. We introduce, in turn, saving policy, child labor policy, and education policy, and then show that a combination of these policies is the long-run optimal policy. It will be convenient to define the class of balanced-budget policies

$$\Psi_0=\left\{\{\tau_s,\tau_o,\tau_w,\tau_x,\tau_e\}:T_s+T_o+T_w+T_x+T_e=0,\ \text{for all } t\geq 0\right\}.$$

2.4.1 Saving policy

Consider the case where saving policy is feasible, but neither child labor policy nor education policy are. That is, consider the class of feasible policies

$$\widehat{\Psi} = \left\{ \{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\} \in \Psi_0 : \tau_x = \tau_e = 0 \right\}.$$

Clearly, these class of policies can solve the inefficiency of saving behavior and balance the government budget. It is also clear that the instruments $\{\tau_s, \tau_o, \tau_w\}$ alone cannot affect the allocation of children's time. Note that the two equilibrium conditions (2.9) and (2.10), when $\tau_x = \tau_e = 0$, coincide with the optimality conditions for child crime and child labor in the *laissez-faire* equilibrium. The arguments presented in Section 1.5 of Chapter 1 continue to apply here, and so it is easy to see that the long-run optimal policy within the class of policies $\widehat{\Psi}$ solves the problem

$$\max_{\theta} \ln(\theta C) + \ln((1-\theta) C) \text{ for any } C > 0, \tag{2.14}$$

which gives the optimal intergenerational allocation of aggregate consumption, and it also implements the allocation that solves the constrained planning problem:

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha} - K\right) + \delta \ln\left(\frac{(1-\alpha)AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha}}{(1+\phi X)}\right)$$

$$\text{subject to } H = (Z/\underline{Z})^{-\gamma/\beta}(1 - aX - bZ), \; X \geq 0, \; Z \geq \underline{Z}, \; X + Z \leq 1, \tag{2.15}$$

and to equations (2.9)–(2.10), evaluated at $\tau_x = \tau_e = 0$.

Let $\left\{\widehat{X}, \widehat{Z}, \widehat{K}\right\}$ denote the allocation that solves this problem. Clearly, the allocation of children's time is exactly the *laissez-faire* equilibrium allocation, that is, $\widehat{X} = X^*$, and $\widehat{Z} = Z^*$, where X^* and Z^* are given by

$$X^* = \frac{1 + p - (1 - p + \delta\beta)\left(\frac{a}{\phi}\right)}{a(2+\delta\beta)},$$

$$Z^* = \left(1 + \frac{a}{\phi}\right)\left(\frac{1-p}{b\,(2+\delta\beta)}\right),$$

respectively.

Moreover, the investment rate $\widehat{K}/\widehat{Y}$ is identical to the unconstrained optimum K_1/Y_1 given by

$$\frac{K_1}{Y_1} = \alpha\left(\frac{2+\delta}{2+\alpha\delta}\right).$$

It is easy to find the taxes $\{\widehat{\tau}_s, \widehat{\tau}_o\}$ that ensure that the equilibrium conditions (2.12) and (2.13) hold with $C_o/C_a = 1$ and $K/Y = \alpha\left(\frac{2+\delta}{2+\alpha\delta}\right)$, which give the efficient intergenerational allocation of aggregate consumption and also the efficient investment rate. One can then calculate the value of the payroll tax $\widehat{\tau}_w$ that balances the government budget every period.

Proposition 7 *The policy $\{\widehat{\tau}_s, \widehat{\tau}_o, \widehat{\tau}_w, 0, 0\}$, with*

$$\widehat{\tau}_s = \frac{p\,(1-\alpha)}{\alpha\,(2+\delta)} - 1,\ \ \widehat{\tau}_o = 1 - \frac{1-\alpha}{\alpha\,(2+\alpha\delta)},\ \textit{and}\ \widehat{\tau}_w = \frac{1-p+\alpha\delta}{1+p},$$

is the unique long-run optimal policy within the class of feasible policies $\widehat{\Psi}$. It implements the solution to Problem (2.14), and also the allocation $\left\{\widehat{X}, \widehat{Z}, \widehat{K}\right\}$ that solves Problem (2.15).

The tax rates $\{\widehat{\tau}_s, \widehat{\tau}_o, \widehat{\tau}_w\}$ ensure that the investment rate and the intergenerational allocation of consumption are long-run optimal, while balancing the government budget every period. Clearly, $\widehat{\tau}_w > 0$ always. Recalling from the *laissez-faire* equilibrium analysis in Chapter 1 that $K^*/Y^* < K_1/Y_1$, if and only if $\frac{p}{1+p} < \frac{\alpha}{1-\alpha}\left(\frac{2+\delta}{2+\alpha\delta}\right)$, then it is easy to verify that $\widehat{\tau}_s < 0$ and $\widehat{\tau}_o > 0$ whenever $K^*/Y^* < K_1/Y_1$.

2.4.2 Child labor policy in the absence of education policy

Now, let us consider the case where saving policy and child labor policy are feasible, but education policy is not. That is, consider the class of feasible policies

$$\Psi_2 = \left\{ \{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\} \in \Psi_0 : \tau_e = 0 \right\}.$$

Clearly, the tax instruments $\{\tau_s, \tau_o, \tau_w, \tau_x\}$ are sufficient to ensure that the investment rate and the intergenerational allocation of consumption are long-run optimal, while balancing the government budget every period. However, access to child labor taxes (positive or negative) is insufficient to fully solve the problem of the inefficient allocation of children's time. Formally, in the absence of education policy (i.e., if $\tau_e = 0$), the tax τ_x influences the first-order condition for an optimal child labor choice (equation (2.10)), but it does not enter the first-order condition for an optimal choice of child crime (equation (2.9)). The latter equation, evaluated at $\tau_e = 0$ becomes exactly the optimality condition for child crime in the *laissez-faire* equilibrium, given by

$$Z = \frac{1 - aX}{b\left(1 + \frac{\delta\beta}{1-p}\right)} \equiv g\left(X\right), \tag{2.16}$$

which is a constraint that any policy within the class of feasible policies Ψ_2 must respect. Then, it is easy to see that the long-run optimal policy within the class of policies Ψ_2 implements the allocation that solves the following constrained planning problem:

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha}H^{1-\alpha}\left(1+\phi X\right)^{1-\alpha} - K\right) + \delta \ln\left(\frac{\left(1-\alpha\right)AK^{\alpha}H^{1-\alpha}\left(1+\phi X\right)^{1-\alpha}}{\left(1+\phi X\right)}\right)$$

$$\text{subject to } Z = g\left(X\right),\ H = \left(Z/\underline{Z}\right)^{-\gamma/\beta}\left(1 - aX - bZ\right),\ X \geq 0,\ Z \geq \underline{Z},\ X + Z \leq 1. \tag{2.17}$$

Let $\{X_2, Z_2, K_2\}$ solve Problem (2.17). We have the following result.

Proposition 8 ***(i)*** *There is a unique long-run optimal policy* $\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\}$ *within the class of feasible policies* Ψ_2*. It implements the solution to Problem (2.14), and also the allocation* $\{X_2, Z_2, K_2\}$ *that solves Problem (2.17).* ***(ii)*** $\{\tau_{s2}, \tau_{o2}\} = \{\widehat{\tau}_s, \widehat{\tau}_o\}$*, as given in Proposition 7, and* $\tau_{x2} < 0$ *if and only if* $\gamma > \beta\left(1 - \frac{1-p+\delta\beta}{\left(1+\frac{\delta}{2}\right)(1+p)}\right)$.

Proposition 8 shows that the unique long-run optimal policy within the class of policies Ψ_2 implements the constrained optimal allocation discussed in Section 1.5, $\{X_2, Z_2, K_2\}$, and so it maps exactly into our analysis of child labor restrictions from Chapter 1. In particular, it implies that the long-run optimal child labor policy, in the absence of education policy, requires a subsidy whenever the social return to child labor exceeds its private return.

2.4.3 Education policy in the absence of child labor policy

We now study the long-run effect of education policy when child labor policy is not feasible. A school subsidy is a useful instrument because it taxes child labor and child crime simultaneously. However, we shall be careful to restrict attention to policies that do not surreptitiously rely on taxes on child labor and criminal activity that are in fact unavailable. In particular, we wish to avoid the unrealistic case where sufficiently large school subsidies are in effect equivalent to a pair of distinct taxes on child labor and child crime. It will become clear below that this is not a concern whenever the unconstrained optimum has $X_1 > 0$. However, if the unconstrained optimum has $X_1 = 0$, a large enough school subsidy can (sometimes) implement the unconstrained long-run optimal allocation, by eliminating child labor entirely, then raising the school subsidy further, thereby taxing crime directly. In order to exclude this extreme case, we consider the following class of feasible policies:

$$\Psi_3 = \left\{\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\} \in \Psi_0 : \tau_x = 0, \text{ equation (2.10) holds with equality}\right\}.$$

The requirement that equation (2.10) holds with equality is consistent with $X = 0$ as well as $X > 0$, but it does restrict the class of feasible policies by requiring that equilibrium child labor choices satisfy the corresponding first-order condition for an *interior* optimal choice.

Once again, the available tax instruments are sufficient to ensure that the investment rate and the intergenerational allocation of consumption are long-run optimal, while balancing the government budget every period. However, access to (feasible) education policy (taxes or subsidies) is insufficient to fully solve the problem of the inefficient allocation of children's time. Formally, in the absence of child labor policy (i.e., if $\tau_x = 0$), the tax τ_e enters both equation (2.9) and equation (2.10). Combining both equations to eliminate the tax, one can derive a constraint that any policy within the class of policies Ψ_3 must respect (the last constraint in Problem (2.18)), and verify that the long-run optimal policy within the class of policies Ψ_3 implements the allocation that solves the constrained planning problem

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha} - K\right) + \delta \ln\left(\frac{(1-\alpha)\,AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha}}{(1+\phi X)}\right)$$

$$\text{subject to } H = (Z/\underline{Z})^{-\gamma/\beta}(1-aX-bZ)\,,\ X \geq 0,\ Z \geq \underline{Z},\ X+Z \leq 1, \text{ and} \quad (2.18)$$

$$\phi(1+p) = (1+\phi-\phi Z)\left[\frac{1-p}{Z} - \frac{\delta\beta b}{1-aX-bZ}\right] + (1+\phi X)\frac{\delta\beta a}{1-aX-bZ}.$$

Let $\{X_3, Z_3, K_3\}$ solve Problem (2.18). We have the following result.

Proposition 9 ***(i)*** *There is a unique long-run optimal policy* $\{\tau_{s3}, \tau_{o3}, \tau_{w3}, 0, \tau_{e3}\}$ *within the class* Ψ_3*. It implements the solution to Problem (2.14), and also the allocation* $\{X_3, Z_3, K_3\}$ *that solves Problem (2.18).* ***(ii)*** $\underline{Z} < Z_3 < Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$*, and* $X_3 \leq X_1$*, with* $X_3 = X_1$ *if and only if* $X_1 = 0$*.* ***(iii)*** $\{\tau_{s2}, \tau_{o2}\} = \{\widehat{\tau}_s, \widehat{\tau}_o\}$*, as given in Proposition 7, and* $\tau_{e3} < 0$*.* ***(iv)*** $U^*(\{\tau_{s3}, \tau_{o3}, \tau_{w3}, 0, \tau_{e3}\}) > U^*(\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\})$ *if and only if* $\gamma < \overline{\gamma}$*, for some* $\overline{\gamma} > \beta\left(1 - \frac{1-p+\delta\beta}{(1+\frac{\delta}{2})(1+p)}\right)$*, where* $\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\}$ *is characterized in Proposition 8.*

Part *(i)* of the proposition states the existence and uniqueness of the optimal policy, and characterizes the corresponding optimal allocation, following our discussion above. Part (ii) implies that the *laissez-faire* levels of both, child labor and child crime, are too high, relative to $\{X_3, Z_3, K_3\}$, and not just relative to the unconstrained optimal allocation $\{X_1, Z_1, K_1\}$. It also implies that, whereas $Z_3 > \underline{Z}$, it is always the case that $X_3 \leq X_1$, with $X_3 < X_1$ whenever $X_1 > 0$. Part *(iii)* says that the presence of education policy does not distort the optimal saving policy, and it is optimal to subsidize school attendance, as one would expect.

Part *(iv)* says the "education policy" $\{\tau_{s3}, \tau_{o3}, \tau_{w3}, 0, \tau_{e3}\}$ characterized in Proposition 9 dominates the "child labor policy" $\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\}$ characterized in Proposition 8 if and only if the externality associated with crime is not too strong. In particular, recalling that $\beta\left(1 - \frac{1-p+\delta\beta}{\left(1+\frac{\delta}{2}\right)(1+p)}\right)$ is the cutoff value of γ at which the long-run optimal child labor policy has exactly $\tau_{x2} = 0$, education policy dominates child labor policy whenever the optimal child labor policy is a positive tax on child labor.

To understand the nature of the optimal allocation in this case, it is useful to note that X_3 maximizes the social return to child labor. The corresponding first-order condition can be written as

$$2\phi \frac{\partial F}{\partial (1+\phi X)} - (2+\delta)\frac{\partial F}{\partial H}\left[-\frac{\partial H}{\partial X} - \frac{\partial H}{\partial Z}\frac{\partial Z}{\partial X}\right] = 0.$$

This is exactly equation (1.21) from Section 1.5 in Chapter 1, which also maximizes the social return to child labor in Problem (2.17). The difference is that the constraint in Problem (2.18) implies that $\frac{dZ}{dX} > 0$, whereas the relevant constraint in Problem (2.17) implies that $\frac{dZ}{dX} = \frac{\partial g(X)}{\partial X} < 0$. Our arguments presented in Section 1.5 continue to apply here, implying that $X_3 \leq X_1 \leq X_2$.

2.4.4 Long-run optimal policy

Finally, let us examine the interaction of child labor policy and education policy. To that end, consider the class of feasible policies

$$\Psi_1 = \left\{\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_e\} \in \Psi_0 : \text{equation (2.10) holds with equality}\right\},$$

where we maintain the requirement that child labor choices satisfy the relevant first-order condition for an *interior* optimal choice in order to avoid the unrealistic case where education subsidies alone can mimic perfectly the combined effect of taxes on child labor and crime.

Let $\{X_1, Z_1, K_1\}$ solve the problem

$$\max_{X,Z,K} 2\ln\left(AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha} - K\right) + \delta \ln\left(\frac{(1-\alpha)AK^{\alpha}H^{1-\alpha}(1+\phi X)^{1-\alpha}}{(1+\phi X)}\right)$$

$$\text{subject to } H = (Z/\underline{Z})^{-\gamma/\beta}(1-aX-bZ),\ X \geq 0,\ Z \geq \underline{Z},\ X+Z \leq 1. \quad (2.19)$$

We have the following result.

Proposition 10 ***(i)*** *There exists a unique long-run optimal policy* $\{\tau_{s1}, \tau_{o1}, \tau_{w1}, \tau_{x1}, \tau_{e1}\}$ *within the class* Ψ_1*. It implements the solution to Problem (2.14), and also the allocation* $\{X_1, Z_1, K_1\}$ *that solves Problem (2.19), for* $\underline{Z} > \widetilde{Z}$*, where* $\widetilde{Z} \in (0, Z_3)$ *is given in the Appendix.* ***(ii)*** $\{\tau_{s1}, \tau_{o1}\} = \{\widehat{\tau}_s, \widehat{\tau}_o\}$*, as given in Proposition 7,* $\tau_{x1} < 0$*,* $\tau_{e1} < 0$*, and* $\tau_{w1} > 0$*.*

Part *(i)* implies that, although the optimal policy $\{\tau_{s1}, \tau_{o1}, \tau_{w1}, \tau_{x1}, \tau_{e1}\}$ implements the unconstrained optimal allocation $\{X_1, Z_1, K_1\}$, it does so only if the lower bound $\underline{Z}$ on criminal activity exceeds a certain level $\widetilde{Z} > 0$. This is due to the fact that child labor and school subsidies raise crime rents, and thus they encourage crime indirectly, effectively constraining their ability to crowd out crime. The fact that $\widetilde{Z} < Z_3$ implies that the optimal policy $\{\tau_{s1}, \tau_{o1}, \tau_{w1}, \tau_{x1}, \tau_{e1}\}$ does implement the unconstrained optimum in a large set of non-trivial cases, where $Z_1 < Z_3$. Part *(ii)* of the proposition states that the unconstrained optimal policy involves a combination of subsidies to child labor as well as school attendance. The reason is that subsidies for school attendance depress child labor and so they interfere with the social value of child labor associated with its beneficial role in crowding out crime. Consequently, combining child labor subsidies and school subsidies is the optimal way to reduce child labor and crime simultaneously.

One can also verify that if equation (2.10) were allowed to hold with strict inequality, the optimal school subsidy τ_{e1} would remain unchanged. The only difference is that the optimal child labor tax rate τ_x would become indeterminate, with any rate $\tau_x \geq \tau_{x1}$ being optimal.

It is worth noting that labor income taxes and child labor subsidies are in effect equivalent from the viewpoint of the maximization of long-run social welfare in the present environment. In particular, it can be shown that a positive labor income tax makes households poorer, which encourages child labor, which in turn crowds out crime. In this sense, labor income taxation has a useful role in the present environment only to the extent that it replicates the effect of child labor subsidies. Accordingly, the optimal combination of labor income taxation and education policy would involve a positive school subsidy and a positive income tax. In the context of actual developing countries, however, one would expect child labor subsidies to be politically feasible when labor income taxation would not be.

Finally, suppose that taxes on child crime as well as child labor are feasible. For instance, suppose that households face a tax on criminal activity equal to $\tau_z zwh$. Given a feasible tax policy $\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_z\}$ individuals face the budget constraint

$$\begin{aligned}(1+\tau_s)\,s &= p\left(\left(w+(1-\tau_x)\,w_c x\right)h - c_a - \tau_z zwh\right) \\ &\quad + \frac{(1-p)z}{Z}\left[\left(w+w_c X\right)H - C_a - T_x - T_z\right],\end{aligned}$$

when adults, whenever $Z > 0$. Otherwise, the model economy is as before.

One can verify that in this case the optimal choice of child crime satisfies

$$\frac{1-p}{Z} - \tau_z\left(\frac{1+p}{1+\phi\,(1-\tau_x)\,X - \tau_z Z}\right) - \frac{\delta\beta b}{1-aX-bZ} = 0,$$

and the optimal choice of child labor satisfies

$$\phi\,(1-\tau_x)\left(\frac{1+p}{1+\phi\,(1-\tau_x)\,X - \tau_z Z}\right) - \frac{\delta\beta a}{1-aX-bZ} \leq 0.$$

with equality whenever optimal child labor is interior.

Then, it is easy to verify that there is a feasible policy $\{\tau_s, \tau_o, \tau_w, \tau_x, \tau_z\}$ that implements the unconstrained long-run optimal allocation $\{X_1, Z_1, K_1\}$, for any $\underline{Z} > 0$. Moreover, such a policy necessarily involves a positive tax on child labor as well as a positive tax on criminal activity. The reason is that child labor in the *laissez-faire* equilibrium is always inefficiently high in first-best terms, and criminal activity can be taxed away. However, this is akin to assuming that laws against crime are perfectly enforceable, which is clearly at odds with the actual evidence on developing countries. Instead, the tax rate τ_z can be viewed as parameterizing the degree of enforcement of laws against crime. Cross-country differences may then be thought of in terms of variation in τ_z. One can easily extend our analysis to verify that higher tax rates τ_z are associated with long-run optimal policies featuring lower school subsidies and lower child labor subsidies. Only when the "tax" on criminal activity is sufficiently large, taxation of child labor becomes optimal from the viewpoint of maximizing long-run social welfare.

2.5 Conclusion

In this Chapter we have examined the effects of alternative fiscal policies. In the analysis, we introduced a class of policies rich enough to target the multiple distortions present in the laissez-faire environment. We then characterized the long-run consequences of alternative subsets of policies in turn.

In order to analyze the policy implications of our argument against the ILO Convention No. 138, we began by considering the effect of targeted child labor policies in the absence of education policies. We showed that the effect of compulsory child labor restrictions can be always replicated by a positive tax on child labor. However, if the social return to child labor is larger than its private return, subsidizing child labor, as opposed to taxing it, is a desirable policy because it promotes child labor at the expense of child crime.

We then analyzed the effect of education policies in the absence of child labor policy. We found that long-run optimal subsidies for school attendance always dominate positive taxes on child labor, and in general tend to dominate child labor subsidies as well. The long-run advantage of targeted subsidies for school attendance over child labor subsidies reflects the fact that the latter crowd out schooling as well as crime, whereas the former crowd out child labor and child crime, both of which are detrimental to long-run social welfare.

In the context of our model we have argued that a combination of subsidies for child work and subsidies for school attendance maximizes social welfare in the long run. Thus, the optimal complement to school subsidies is a targeted subsidy, as opposed to a tax, for child work. There are two main aspects to this.

First, the key point is that combining both subsidies is the optimal way to *reduce* child labor and crime simultaneously, not that the optimal level of child labor is necessarily high. Indeed, targeted subsidies to child labor would continue to be optimal even if the maximization of social welfare required eradicating child labor. This is because school subsidies depress child labor and thereby interfere with the social value of child labor arising from its role in crowding out crime.

The second aspect concerns the implementation of our proposed optimal policy in developing countries. In the formal model, we focused on conditional income transfers (positive or negative) in order to illustrate the relevant incentive problems. However, there are various options in terms of how to implement these policies. In practice, our policy proposal amounts to modifying conditional transfer programs such as the Mexican program Oportunidades to accommodate working children. Within existing programs, children's access to food and health care is ensured in exchange for the children's attendance to school. Instead, our analysis implies that an optimal mix of school and work ought to be designed in order to crowd out alternative activities that are relatively more harmful to children. Such programs ought to take into account the circumstances and the best interest of children, as advocated by an increasing number of grass-roots working children movements worldwide.

Finally, our analysis also implies that greater emphasis should be given to complementary interventions aiming at improving the quality of education, the children's work environment, and the enforcement of laws against crime. The problem of child labor is related not only to poverty, but also to poor institutions and poor school quality. While these issues remain unresolved in developing economies, it is the eradication of child abuse and neglect, rather than the eradication of child labor, that is imperative.

Chapter 3

Learn, sweat or steal: a theory of development and the activity of children

3.1 Introduction

The goal of deterring child labor is enshrined in countless agreements and human rights documents signed in most of countries around the world. Nonetheless, the latest estimates of the ILO indicate that at least 215 million children are involved in child labor (14% of the global child population). Conventional views call for the elimination of child labor practices to ensure the welfare of the children, who are affected both cognitively and physically by the often repetitive and demanding nature of the work. However, the issues involved in deterring child labor are not as clear-cut as they may initially appear. When implementing policies to deter child labor, countries and organizations rarely take into account the fact that children often work to obtain much-needed resources for their families and when these well-intentioned policies render hazardous jobs out of bounds for children, the children themselves are driven into the underground economy where they are not protected at all, resulting in highly counterproductive outcomes. For children in the poorest conditions, school is not necessarily an option. Under certain circumstances, such as extreme poverty or lack of additional opportunities, households often face a tradeoff not between work and school, but between work and other activities to obtain resources such as child crime and prostitution. An example of this occurred in the garment industry in Bangladesh in 1993, where about 50,000 children were removed from the industry. Visits conducted by UNICEF found that children subsequently began engaging in activities such as prostitution, hustling, and stone crushing (UNICEF, 1997, p. 60).

In this paper I argue that in economies characterized by low institutional quality and human capital inequality, banning permanently child labor can harm the development of the children living in poor households and their entire dynasties. Furthermore, if the quality of education is sufficiently low, even a temporary ban on child labor can have permanent negative effects not only for the current generation of the poor children, but also for all individuals in the economy in the long run.

To this end, I develop and analyze an overlapping generations model of endogenous growth and inequality in the distribution of human capital. The model formalizes the fact that in developing countries where the institutional quality is low, it is easier for children to access illegal activities as a means to obtain resources needed for consumption. This study sheds light on the short run and long run consequences, for both the children and the economy in aggregate, of poor institutional quality and inequality in the economy.

The analysis addresses the short-run and long-run consequences of a permanent and a temporary ban on child labor across different households. In addition, I examine how three important aspects of institutional quality – the security of property rights, the quality of education, and the access to credit markets – affect the tradeoff different households face when choosing whether to have their children attending school, working, or involved in crime. I argue that even without credit market imperfections, child labor can still be present in the economy. Furthermore, I show that having access to credit markets may have no effect on schooling decisions if the productivity of child labor is sufficiently poor. However, a sufficient increase in school quality may eliminate child labor even in the presence of credit market imperfections. Finally, the analysis also illustrates how the activity of the children is affected by differences in inequality.

As in the model presented in Chapter 1, I consider that child labor and child crime are two competing uses of a child's time and both are a source of current household income. In addition, I also consider that crime is a non-productive activity that harms the human capital of the child. However, the model presented here differs from that in Chapter 1 in two important ways, leading to new results. First, I depart from the symmetric equilibrium analysis and consider that individuals are different. The focus is on the incentives different households have when allocating their children's time, and their implications for policy. In particular, I assume the economy is characterized by inequality on the distribution of human capital. I incorporate this assumption into an endogenous growth model with human capital accumulation as the engine of growth. The specification of the human capital accumulation structure used here allows examining the role played by the quality of education in the economy on the activity of the children. Second, I assume that households have access to credit markets in the sense that, if necessary, households can borrow resources. This assumption allows identifying the effects of credit market imperfections.

I show that a child's activity is determined by the income of the household she lives in. Children living in the poorest households are involved in crime, while children living in households with the highest levels of income attend school full time. Within the intermediate levels, the children of those households with relatively low levels of income work full time, and the children of those households with relatively large levels of income combine their time between school and work. In the long run, the distribution of human capital is an endogenous outcome that is a function of the quality of education and the security of property rights. This sorting is consistent with the evidence found in different studies. For instance, studies conducted by the ILO shows that school enrolment is determined by poverty, and the accessibility, quality and cost of education. These studies also show that poverty leads children to alternative mechanisms of subsistence, such as crime, mainly to help their families.[1]

In order to examine the effects of a permanent ban and a temporary ban on child

[1] See, for instance, ILO (2002a), ILO (2002b), and Dowdney (2003).

labor I consider two cases: *i*) a small open economy and *ii*) a closed economy.

In the first case, I show that a permanent ban on child labor can have permanent negative effects for the children living in the poorest households if the level of property rights in the economy is sufficiently low. The ban leads some of the poor households to involve their children in crime, when otherwise they would have preferred to send their children to work. Consequently, the ban on child labor directly affects the development of a child living in poor households, by reducing her accumulation of human capital. In turn, this leads the affected children to involve their own children in crime in the future. Moreover, I show that even a temporary ban on child labor can have permanent negative effects for the children living in the poorest households if the quality of education and the level of property rights in the economy are sufficiently low. Intuitively, economies that prematurely implement policies to ban child labor exacerbate the problem of child crime whenever property rights and quality of education are sufficiently poor, reducing the accumulation of human capital and physical capital in the long run.

In the second case I show that, in the short run, a ban on child labor harms the children living in the poor households and benefits the children living in the rich households. Furthermore, I show an interesting result that has not been previously identified: if the quality of education and the level of property rights in the economy are sufficiently low, even a temporary ban on child labor can have permanent negative effects for all individuals in the economy. This result is due to the fact that, in the closed economy case, the temporary ban not only affects the households in the poorest conditions by driving children into crime, but it also decreases the physical capital-human capital ratio of the economy in the long run, reducing in turn the labor income and consumption of all individuals.

Next, in the context of the model, I examine the relationship between economic institutions and the activity of children. First, I show that an increase in either the quality of education or the security of property rights increases the proportion of children attending school full time, and it decreases the proportion of children out from school. Following this result, I examine the effects of a policy designed to increase the institutional quality in the economy financed through taxes. The results suggest that it is possible that investing on institutional quality might result in lower proportions of children working full time and children involved in crime. However, for this to occur, certain conditions must hold. Specifically, either the institutional quality should be very sensitive to the investment, or the costs associated with the investment on institutional quality should be covered by those households with the largest levels of income.

Second, I consider the case of credit market imperfections where the resources households can borrow from the capital markets are restricted to a certain level. I show that credit market imperfections might decrease the proportion of children attending school full time, increasing in turn the proportion of children combining school and work. In particular, this occurs only if the productivity of child labor is sufficiently large.

Finally, the implications of differences in inequality are discussed. In order to measure inequality, the approach used here is based on the concept of second-order stochastic dominance. In this case, I argue that the long-run level of child crime is greater in economies with larger inequality. In turn, this implies that the growth rate of economies with larger inequality will be lower during the transition to the steady state.

There is a growing literature examining the causes and consequences of child labor.[2] In the theoretical work it is well established that child labor is a persistent phenomenon associated primarily with poverty and credit market imperfections (Basu & Van, 1998; Baland & Robinson, 2000; Ranjan, 2001). It has been recognized that policies to control child labor may cause child labor to rise (Basu, 2005; Basu & Zarghamee, 2009; Baland & Duprez, 2009), or a household's welfare to fall if the loss in child labor earnings is not compensated by an equivalent increase in adult labor income

[2] Basu (1999), Basu & Tzannatos (2003), and Edmonds (2008) are excellent surveys of this literature.

(Basu & Van, 1998; Dessy & Pallage, 2005). Different studies have also analyzed the effects of child labor restrictions through international labor standards (Basu et. al., 2003), and the endogenous adoption of child labor laws from a political economy perspective (Doepke & Zilibotti, 2005; Dessy & Knowles, 2008; Doepke & Zilibotti, 2010). Few studies have focused on the hazardous forms of child labor, in which policy interventions in the hazardous sector might be undesirable (Dessy & Pallage, 2005), or welfare improving (Rogers & Swinnerton, 2008).[3]

As has been presented in previous literature, I show that economies exhibit child labor in the short run because of poverty. However, distinct from the literature, I bring in an explanatory factor for this reality and argue that the persistence of child labor over time is due to low school quality. In economies with high levels of quality of education, the development path of the economy leads to an endogenous elimination of child labor relatively sooner without the need for policy intervention. An implication of this result is that a policy that increases the quality of education in developing economies is superior to policies that deter child labor.

Historically, the successful stories of eliminating child labor come from the experience of developed countries, like Great Britain and the U.S. However, the evidence suggests that these results were not driven by legislative measures against child labor. For instance, Moehling (1999) and Kirby (2003) show that the U.S. and Britain's measures against child labor including compulsory schooling, respectively, were enforced once the level of child labor was negligible, finding no statistically significant effects.[4]

Interestingly, the quality of education in the U.S. a century ago was greater than the quality of education currently available in countries with child labor. Evidence of this

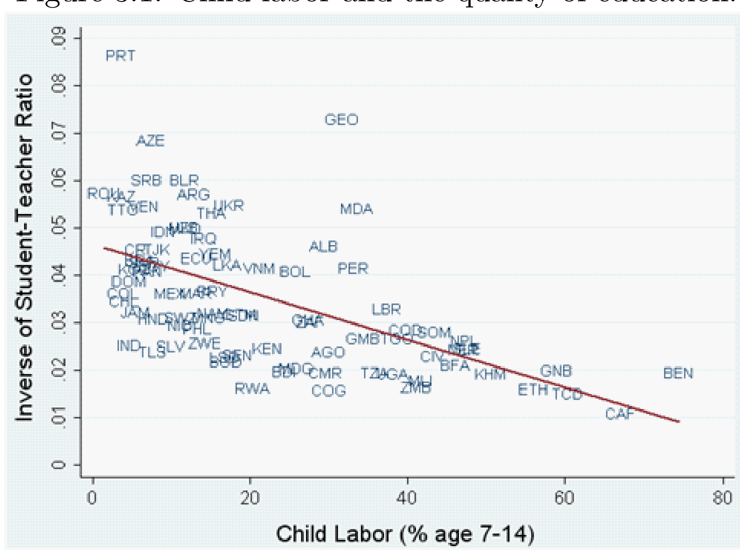
[3]Dessy & Pallage (2005) argue that banning child labor in the hazardous sector may be undesirable since this sector maintains child labor wages in the good sector high enough to allow for human capital accumulation. In a model of exploitative labor, Rogers & Swinnerton (2008) argue that exploited child laborers are paid less than the value of their marginal product of labor, such that there is room for intervention in the exploitative sector that is welfare improving.

[4]Moehling (1999) finds that the minimum age limits legislation and compulsory education had no effects on the likelihood of manufacturing child labor in the US: the decrease in child labor in US during 1880 − 1930 was not driven by the legislative success of the child labor movement. Kirby (2003) argues that the state legislation and education laws in Great Britain had no effect upon child labor. Moreover, by the time the child labor legislation and compulsory schooling were introduced, child employment had declined to statistically insignificant levels.

can be found in the pupil-teacher ratio, a measure commonly used as a proxy for the quality of education. During the period when child labor fell in the U.S. (1880-1930), this country had an average ratio of 34.3 points while low income countries today have on average 50.2 points. As a reference, the ratio for the OECD countries today is 14.4 points.[5] It is worth noticing that during this period the U.S. observed a relatively constant ratio.[6]

Recent evidence is given in Figure 1. This Figure plots the correlation between child labor and quality of education across 82 countries. The horizontal axis denotes child labor, defined as the proportion of children between the ages 7-14 that work. The vertical axis denotes the quality of education, defined as the inverse of the student-teacher ratio. As the figure shows, across countries there is a negative correlation between child labor and the quality of education.

Figure 3.1: Child labor and the quality of education.



Empirical evidence of this relationship is presented in Handa (2002) who studies the importance of school quality on school attendance. The author argues that school

[5] Sources: US data (for 1880-90) Statistical Abstract of the United States, 1892 Fifteenth Number, Bureau of Statistics; (for 1900-30) Historical Statistics of the United States, Colonial Times to 1970; Bureau of the Census, Department of Commerce. Present data: The World Bank.

[6] The ratios for the US are: 34.2 (1880), 35.0 (1890), 35.4 (1900), 34.4 (1910), 33.6 (1920), and 33.2 (1930). Actually, the pupil-teacher ratio remained roughly at the same levels until the 1960s.

enrolment in Mozambique is sensitive to the number of trained teachers. Moreover, building more schools or raising adult literacy has a larger impact on primary school enrolment rates than interventions that raise household income.

The next section presents the basic model of a small open economy, and section 3.3 characterizes the equilibrium. The analysis of the effects of a ban in child labor is presented in section 3.4. Section 3.5 considers the relationship between different economic institutions and the activity of the children. In particular, section 3.5.1 examines the effect of the quality of education and property rights on the activity of the children; section 3.5.2 presents the case of imperfect credit markets; and section 3.5.3 discusses the effect of inequality. Section 3.6 presents the closed economy case and section 3.7 concludes. Proofs are relegated to the Appendix C.

3.2 A model of child labor, crime and schooling

Consider an endogenous growth model of a small open overlapping-generations economy with no population growth. Every period, a continuum of agents with mass one is born. Each agent lives for three periods: childhood, adulthood and old age. All individuals can work and they are endowed with one unit of time each period. A household is defined as the parent-child pair. Following the unitary model of the household, parents are the decision makers of the family, and they exhibit altruistic behavior towards their children in the form of human capital bequest (investment in education). Each individual i seeks to maximize

$$U = u(c_{ai}) + u(c'_{oi}) + u(l'_{oi}) + \delta v(h'_i) \equiv \ln(c_{ai}) + \ln(c'_{oi}) + \ln(l'_{oi}) + \delta \ln(h'_i), \quad (3.1)$$

where c_{ai} is the household i's current consumption, c'_{oi} is individual i's consumption when old, $l'_{oi} \in [0,1]$ is individual i's leisure when old, and h'_i is the level of human capital of individual i's offspring once she is an adult. This last term represents the intergenerational altruism from the parent to the child. Primed variables denote next-period values.

Since children make no decisions, parents allocate their child's unit of time between school, e_i, work, x_i, or illegal activities (crime), z_i, with

$$e_i + x_i + z_i \leq 1. \tag{3.2}$$

For simplicity, crime requires the full unit of the child's time ($z_i = \{0, 1\}$), while schooling and child labor can be combined ($e_i, x_i \geq 0$).

All individuals i born at any time t inherit the level of human capital of their parents. The child's accumulation of human capital requires time and resources devoted to education, and it is governed by a dynamic function of the form

$$h_i' = \left(1 + \theta e_i^{\beta} b_i^{1-\beta} - \varepsilon z_i\right) h_i, \tag{3.3}$$

with $\beta \in \left(\frac{1}{2}, 1\right)$ and $\varepsilon \in (0, \overline{\varepsilon})$, where $\overline{\varepsilon} < 1$ is given in the Appendix, h_i is the stock of human capital of the household i; $e_i \in [0, 1]$ is the time the child of household i spends in education; $z_i = \{0, 1\}$ is an indicator equal to one if the child is involved in crime, zero otherwise; $b_i \geq 0$ is the household i's investment on the education of its child; and $\theta > 0$ is a parameter that reflects the quality of education in the economy.

The properties of the human capital accumulation function reflect some of the key elements in the related literature:[7] (i) the individual's level of human capital is an increasing function of the parental level of human capital, the time spent in education, and the resources invested in education; (ii) there are diminishing returns to the time and resources invested in education; and (iii) for a given level of parental human capital, time spent in education, and resources invested in education, the returns to education are higher the larger the quality of education.

From the above specification of human capital accumulation, note that a child devoting all of her time to school ($e_i = 1$) will have a level of human capital in the next period equal to $(1 + \theta b_i^{1-\beta})h_i$. In turn, a child spending all of her time working ($x_i = 1$) will keep constant her level of human capital. At the other extreme, if a

[7]See, for instance, Glomm and Ravikumar (1992) and Galor and Tsiddon (1997).

child engages in crime ($z_i = 1$) her level of human capital in the next period is equal to $(1-\varepsilon)\,h_i$. The assumption $1-\varepsilon > 0$ implies that even a full time criminal retains some human capital, and the assumption $\varepsilon > 0$ implies that crime harms human capital accumulation more than work.

The technology of human capital accumulation is identical for all individuals (i.e. all individuals have the same ability). However, they may differ in their level of human capital inherited from their parents. In particular, I assume the economy starts at time 0 with a distribution of human capital of the initial parent generation given by the density function $g_0(h_{i,0})$ with a continuous support defined over the nonnegative real line. It follows that

$$G_t(m) = \int_0^m g_t(h_{i,t})\,dh_{i,t} \quad \forall \;\; m \in \mathbb{R}_+ \quad \text{and} \quad \int_0^\infty g_t(h_{i,t})\,dh_{i,t} = 1 \quad \forall \;\; t. \tag{3.4}$$

Every period the economy produces a single final good using physical capital and the human capital of the working individuals. Given the small open economy assumption, the supply of physical capital every period is determined by the aggregate savings in the economy, in addition to *net* international borrowing. Factor and product markets are competitive. The final good is produced according to the production technology

$$F(K,H) = AK^{\alpha}H^{1-\alpha}, \tag{3.5}$$

with $A > 0$, $\alpha \in (0,1)$, where K is the aggregate stock of physical capital that fully depreciates every period, and H is the aggregate stock of *effective* human capital, with

$$H \equiv H_a + \phi H_c + H_o = \int_0^1 h_i di + \phi \int_0^1 x_i h_i di + \int_0^1 (1-l_{oi})\,h_{oi} di, \tag{3.6}$$

where H_a is the aggregate stock of human capital provided by adults, H_c is the aggregate stock of human capital provided by children, H_o is the aggregate stock of human capital provided by old individuals, and $\phi \in (0,1]$ is the productivity of children relative to that of adults. Notice from equation (3.6) that I assume all worker individuals are perfect substitutes in production, reflecting the facts that a child works $x_i \in [0,1]$

units of time and they are not more productive than adults, and the fact that an old individual works $1 - l_{oi} \in (0, 1)$ units of time.

During childhood, individuals may receive schooling and/or go to work and receive a wage rate per effective unit of human capital w_c, or they may be involved in crime appropriating crime rents. I assume that crime is a fully unproductive activity and only children can engage on it. If at least one household has its child involved in crime ($z_i = 1$ for some i), a fixed proportion $(1 - p)$ of *total* labor earnings in the economy is subject to appropriation, i.e. parents with children involved in crime are not exempt from appropriation. The parameter $p \in (0, 1)$ reflects the security of effective property rights in the economy, with $p = 1$ denoting perfectly secure property rights. Hence, a child involved in crime can claim a proportion $(1 - p)/n$ of the labor earnings Y_L, where n is the total number of children involved in crime.

When adults, individuals supply their unit of labor inelastically in the labor market receiving a wage rate per effective unit of human capital w, and decide the allocation of time of their child $(e_{i,t}, x_{i,t}, z_{i,t})$. In addition, from the household's income, parents decide how much to invest in the education of their child b_i, how much to save (or borrow) s_i, and how much to consume c_{ai}. In order to ensure boundedness on the long-run growth rate of the economy, I assume $b_i \leq b_H$, for some $b_H > 0$ to be defined below. Formally, household i's consumption is given by

$$c_{ai} = p\,(w + w_c x_i)\,h_i + \frac{(1 - p)\,Y_L}{n} z_i - s_i - b_i. \tag{3.7}$$

When old, individuals decide the proportion of time they allocate into leisure and work receiving a wage rate w_o, spending all the resources generated from *net* capital income and labor in their consumption. That is, whenever an adult borrow resources, she pays the debt when old. Thus, consumption for an old individual is

$$c'_{oi} = R s_i + p\,(1 - l'_{oi})\,w'_o h_{oi}, \tag{3.8}$$

where the gross interest rate, $R = 1 + r$, is given by international capital markets and assumed to be constant over time.[8]

Every period, the aggregate resources lost to child crime add to aggregate crime rents. That is, the distribution of crime rents satisfy the aggregate consistency condition

$$\int_0^1 (1-p)\left(w + w_c x_i\right) h_i \, di + \int_0^1 (1-p)\left(1 - l_{oi}\right) w_o h_{oi} \, di = (1-p) \int_0^1 Y_L \frac{z_i}{n} \, di. \quad (3.9)$$

In this economy, a competitive equilibrium is a sequence of individual allocations $\{e_i(t), x_i(t), z_i(t), b_i(t), l_{oi}(t+1), s_i(t)\}_{t=0}^{\infty}$ for all agents $i \in [0,1]$, with $s_i(-1) > 0$ given, a sequence of distributions of human capital $\{G_t\left(h_i(t)\right)\}_{t=0}^{\infty}$, with $g_0\left(h_i(0)\right)$ given, a sequence of allocations $\{K(t)\}_{t=0}^{\infty}$, and a sequence of prices $\{R(t), w(t), w_c(t), w_o(t)\}_{t=0}^{\infty}$ such that, given prices, each individual maximizes her utility, firms maximize profits, human capital for each individual evolves according to (3.3), with $h_i(0) = h_{i0} > 0$ for all $i \in [0,1]$, the distribution of crime rents satisfies (3.9), and every market clears for all $t \geq 0$.

3.3 Equilibrium analysis

Consider first the firms. Competitive international capital markets imply that the stock of physical capital in the economy at any time t is determined by

$$\alpha \frac{F\left(K, H\right)}{K} = R, \quad (3.10)$$

where R is constant and given in the international market.

Next, profit maximization implies that all units of human capital are paid according to their marginal product. Then, the wage per unit of human capital for adults, child labor, and old labor are, respectively, given by:

$$w = (1-\alpha)\frac{F\left(K, H\right)}{H}, \quad (3.11)$$

$$w_c = \phi w, \quad (3.12)$$

[8] The small open economy assumption is relaxed in Section 3.6.

$$w_o = w. \tag{3.13}$$

The small open economy assumption, by which the rental rate of physical capital is determined by the international market, simplifies the analysis by eliminating dynamics in the physical capital-effective human capital ratio of the economy. From equation (3.10), this ratio is stationary and given by

$$K = \left(\frac{\alpha A}{R}\right)^{\frac{1}{1-\alpha}} H. \tag{3.14}$$

In turn, this implies from equations (3.11)-(3.13) that the wage rates of the economy are constant over time, with

$$w = (1-\alpha)\, A^{1/(1-\alpha)} \left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3.15}$$

Now consider the problem of the households. First, optimal saving satisfy the Euler equation

$$\frac{\partial u\left(c_{ai}\right)}{\partial c_{ai}} = R\frac{\partial u\left(c'_{oi}\right)}{\partial c'_{oi}}, \tag{3.16}$$

for all i, which equates the marginal rate of substitution between current and future consumption of an adult with the (constant) marginal rate of transformation.

Second, the optimal choice of leisure is always interior and it satisfies

$$\frac{\partial u\left(c'_{oi}\right)}{\partial c'_{oi}}\frac{\partial c'_{oi}}{\partial l'_{oi}} + \frac{\partial u\left(l'_{oi}\right)}{\partial l'_{oi}} = 0, \tag{3.17}$$

for all i, where the marginal benefits come from the utility leisure gives to an individual, and the marginal costs come from the reduction in future consumption associated with lower labor income, with

$$\frac{\partial c'_{oi}}{\partial l'_{oi}} = -pw'_o h_i.$$

Using the fact that $w_c = \phi w$, one can easily verify that the system given by equations (3.16) and (3.17) results in the following general expressions for savings and leisure for all individuals i:

$$s_i = \frac{2}{3}pwh_i\left[1+\phi\left(1-e_i\right)+\frac{\left(1-p\right)Y_L}{npwh_i}z_i-\frac{b_i}{pwh_i}-\frac{1}{2R}\right], \tag{3.18}$$

$$l'_{oi} = \frac{1}{2}+\frac{Rs_i}{2pwh_i}. \tag{3.19}$$

Consequently, from equations (3.7) and (3.18), consumption when adult is given by

$$c_{ai} = \frac{1}{3}pwh_i\left[1+\phi\left(1-e_i\right)+\frac{\left(1-p\right)Y_L}{npwh_i}z_i-\frac{b_i}{pwh_i}+\frac{1}{R}\right], \tag{3.20}$$

and from the Euler equation in (3.16), consumption when old satisfies $c'_{oi} = Rc_{ai}$.

Third, the optimal allocation of schooling satisfies.

$$\frac{\partial u\left(c_{ai}\right)}{\partial c_{ai}}\frac{\partial c_{ai}}{\partial x_i}\frac{\partial x_i}{\partial e_i}+\delta\frac{\partial v\left(h'_i\right)}{\partial h'_i}\frac{\partial h'_i}{\partial e_i}\leq 0, \tag{3.21}$$

with equality whenever optimal schooling is interior. The marginal benefits come from the increase in the child's future human capital associated with schooling

$$\frac{\partial h'_i}{\partial e_i} = \beta\theta\left(\frac{b_i}{e_i}\right)^{1-\beta}h_i,$$

which are increasing on the quality of education, θ. The marginal costs come from the reduction in the household's current consumption associated with lower child labor income

$$\frac{\partial c_{ai}}{\partial x_i}\frac{\partial x_i}{\partial e_i} = -pw_c h_i.$$

Next, the optimal choice of investment in education b_i satisfies

$$\frac{\partial u\left(c_{ai}\right)}{\partial c_{ai}}\frac{\partial c_{ai}}{\partial b_i}+\delta\frac{\partial v\left(h'_i\right)}{\partial h'_i}\frac{\partial h'_i}{\partial b_i}\leq 0, \tag{3.22}$$

with equality whenever the optimal investment in education is interior. The marginal benefits come from the increase in the child's future human capital

$$\frac{\partial h_i'}{\partial b_i} = (1-\beta)\,\theta \left(\frac{e_i}{b_i}\right)^{\beta} h_i,$$

while the marginal costs come from the reduction in the household's current consumption associated with the investment on education

$$\frac{\partial c_{ai}}{\partial b_i} = -1.$$

The equality of the returns to schooling and the returns for the investment on education given in equations (3.21) and (3.22), respectively, give the following relationship between schooling and investment on education

$$b_i = \frac{1-\beta}{\beta}\phi p w h_i e_i. \tag{3.23}$$

Thus, the equality of returns to both activities implies that the investment on education is increasing in both, the optimal allocation of schooling and the level of human capital of the household.

The condition for the optimality of schooling in equation (3.21) together with the equality of the returns to schooling and the investment on education in equation (3.23) determine the optimal level of schooling at time t, which is given by

$$e_i = \frac{3}{3+\delta}\left[\frac{\delta\beta}{3\phi}\left(1+\phi+\frac{1}{R}\right) - \frac{1}{\theta}\left(\frac{\beta}{(1-\beta)\,\phi p w h_i}\right)^{1-\beta}\right], \tag{3.24}$$

with $\partial e_i/\partial h_i > 0$ and $\partial e_i/\partial \theta > 0$. Together, equations (3.23) and (3.24) imply that both, the optimal choice of schooling and investment on education are increasing in the level of human capital of the household and the quality of education.

Notice from equation (3.24) that we have $e_i = 0$ if and only if $h_i \leq h_L$, with

$$h_L = \frac{\beta}{(1-\beta)\,\phi p w}\left[\frac{3\phi}{\theta\delta\beta\left(1+\phi+\frac{1}{R}\right)}\right]^{\frac{1}{1-\beta}}, \tag{3.25}$$

where w is given by equation (3.15). Similarly, we have that $e_i = 1$ if and only if

$h_i \geq h_H$, with

$$h_H = \frac{\beta}{(1-\beta)\,\phi p w}\left[\frac{3\phi}{\theta\left[\delta\beta\left(1+\phi+\frac{1}{R}\right)-(3+\delta)\,\phi\right]}\right]^{\frac{1}{1-\beta}}, \tag{3.26}$$

where the right-hand-side of this expression is defined if and only if $\delta > \delta_L$, with

$$\delta_L \equiv \frac{3\phi}{\beta\left(1+\phi+\frac{1}{R}\right)-\phi},$$

where $\delta_L > 0$ since $\beta > 1/2$. Thus, whenever h_H exists, i.e. $\delta > \delta_L$, we have $h_H > h_L$.

It is worth noticing that, since $e_i \leq 1$, the left-hand-side of equation (3.21) is positive whenever $h_i > h_H$. In this case, equation (3.23) does not hold. Instead, from equation (3.22) one can verify that the optimal investment on education is given by the unique solution to the following expression:

$$\frac{3}{\theta}b_i^{\beta} + (3+\delta\,(1-\beta))\,b_i = \delta\,(1-\beta)\left(1-\frac{1}{2R}\right)pwh_i, \tag{3.27}$$

with $db_i/dh_i > 0$ and $db_i/d\theta > 0$.

Finally, in order to analyze the relationship between child labor, child crime and human capital, recall that the crime rents for a household with its child involved in crime are given by $(1-p)\,Y_L/n$, where $Y_L = (1-\alpha)\,Y$. Given that crime involves the whole unit of time of the child, for child crime to be optimal a necessary condition is that the crime rents must be greater than the full-time child labor earnings. However, it is not a sufficient condition since crime harms the accumulation of human capital of the child more than full time child labor. Therefore, child crime is optimal if and only if the household's utility from child crime is greater or equal than the utility from full time child labor. Using equations (3.7), (3.8), (3.18), and (3.19), one can verify that $z_i = 1$ if and only if

$$(1-\varepsilon)^{\delta/3}\left(1+\frac{(1-p)\,(1-\alpha)\,Y}{npwh_i}+\frac{1}{R}\right) \geq 1+\phi+\frac{1}{R}. \tag{3.28}$$

Substituting equation (3.11) into this expression, and rearranging terms, we have $z_i = 1$

if and only if $h_i \leq \underline{h}_c$, where $\underline{h}_c$ is given by the unique solution to the following expression

$$\underline{h}_c G\left(\underline{h}_c\right) = \frac{(1-\varepsilon)^{\delta/3}(1-p)\,H}{p\left[\phi + \left(1+\frac{1}{R}\right)\left(1-(1-\varepsilon)^{\delta/3}\right)\right]}, \tag{3.29}$$

where I used the fact that $n = G\left(\underline{h}_c\right) = \int_0^{h_c} g\left(h_i\right) dh_i$, and $G\left(\underline{h}_c\right) \in (0,1)$ is the proportion of the households with a level of human capital up to $\underline{h}_c$. Thus, $G\left(\underline{h}_c\right)$ is the proportion of children involved in crime. Notice from this equation that the threshold $\underline{h}_c$ increases over time whenever the aggregate stock of effective human capital, H, increases over time.

We have then the following

Proposition 11 *(i) There exists a threshold level of human capital* $\underline{h}_c > 0$*, such that* $z_i = 1$ *if and only if* $h_i \leq \underline{h}_c$*. (ii) There exists a threshold level of human capital* $h_L > 0$ *with* $h_L \geq \underline{h}_c$ *if and only if* $p \geq p_L$*, with* $p_L < 1$*, such that* $x_i = 1$ *if and only if* $h_i \in \left(\underline{h}_c, h_L\right)$*. (iii) There exists a threshold level of human capital* $h_H > 0$ *if and only if* $\delta > \delta_L$*, with* $h_H > h_L$*, such that* $e_i = 1$ *if and only if* $h_i \geq h_H$*. (iv) If* $h_i \in (h_L, h_H)$*, then* $x_i > 0$*,* $e_i > 0$ *with* $x_i + e_i = 1$ *and* $\partial e_i / \partial h_i > 0$*, where* p_L *is given in the Appendix.*

Part *(i)* of the Proposition says that the children of those households with the lowest levels of human capital in the economy ($h_i \leq \underline{h}_c$) are the children involved in crime. Part *(ii)* provides the condition for full time child labor to exist. It says that there exists a non-empty interval of human capital $\left(\underline{h}_c, h_L\right)$ if and only if the security of effective property rights is sufficiently large, such that the children of those households with levels of human capital within this interval work full time. Otherwise, the rents from child crime are sufficiently large such that households have no incentives for full-time child labor, even though child crime harms the accumulation of human capital of the child more than full-time child labor. Part *(iii)* provides the condition for full-time schooling to exist. It says that the children of those households with the largest levels of human capital $h_i \geq h_H$ attend school full-time, where h_H exists if and only if the degree of altruism of the parent to her child, δ, is sufficiently large. Part *(iv)* says that the children of those households with intermediate levels of human capital, $h_i \in (h_L, h_H)$, combine both child labor and schooling, and that the level of schooling is increasing in the level of human capital of the household.

Intuitively, households with relatively large levels of human capital ($h_i \geq h_H$) receive a sufficiently high income from adult labor such that they do not need additional income from child labor, sending their children to school full time. On the opposite side of the distribution, for those households with relatively low levels of human capital ($h_i \leq h_L$), the labor income parents receive is sufficiently low such that their children do not attend school. Instead, the children participate in either full-time child labor or child crime in order to obtain the resources needed for the consumption of the household.

In what follows I assume $p > p_L$ and $\delta > \delta_L$, such that both full-time child labor and full-time schooling exist in equilibrium.

Consequently, from equation (3.18) we have four possible cases for optimal savings depending on the level of human capital of the household. The first case is given by those households with their children participating in crime. That is, for all $h_i \leq \underline{h}_c$

$$s_i = \frac{2}{3} pwh_i \left[1 + \frac{(1-p)\,H}{G\left(\underline{h}_c\right) ph_i} - \frac{1}{2R} \right], \tag{3.30}$$

where $s_i > 0$ for all $h_i < \underline{h}_c$, with $\partial s_i / \partial h_i > 0$, and w is constant and given by equation (3.15). In the second case we have those households with their children working full time, where for all $h_i \in (\underline{h}_c, h_L]$

$$s_i = \frac{2}{3} pwh_i \left[1 + \phi - \frac{1}{2R} \right], \tag{3.31}$$

where $s_i > 0$ for all $h_i \in (\underline{h}_c, h_L]$, with $\partial s_i / \partial h_i > 0$. Next, we have those households with their children combining school and work. That is, for all $h_i \in (h_L, h_H)$

$$s_i = \frac{2}{3} pwh_i \left[1 + \phi \left(1 - \frac{e_i}{\beta} \right) - \frac{1}{2R} \right]. \tag{3.32}$$

By substituting equation (3.24) into equation (3.32), one can verify that there exists a level $\widehat{h} \in (h_L, h_H)$ such that $\partial s_i/\partial h_i \leq 0$ if and only if $h_i \geq \widehat{h}$. Furthermore, $s_i < 0$ at $e_i = 1$ if and only if $\phi > \frac{\beta}{1-\beta}\left(1-\frac{1}{2R}\right) \equiv \widehat{\phi}$, where $\widehat{\phi} < 1$ if and only if $\beta < 1/\left(2-\frac{1}{2R}\right) \equiv \widehat{\beta} \in (1/2, 1)$.

Finally, the last case is given by those households with their children attending school full time, where for all $h_i \geq h_H$ we have

$$s_i = \begin{cases} \frac{2}{3}pwh_i\left[1-\frac{1}{2R}-\frac{b_i}{pwh_i}\right] & \text{if } \; b_i < b_H \\ \frac{2}{3}pwh_i\left[1-\frac{1}{2R}-\frac{b_H}{pwh_i}\right] & \text{if } \; b_i \geq b_H \end{cases}, \tag{3.33}$$

where b_i in this last equation is given by the unique solution to equation (3.27). To simplify, it is assumed that b_H satisfies

$$b_H = \frac{1-\beta}{\beta}\phi p\left(1-\alpha\right)A^{1/(1-\alpha)}\left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}} h_H, \tag{3.34}$$

where h_H is given by equation (3.26). This assumption implies $b_i = b_H$ for all $h_i \geq h_H$, and $b_i < b_H$ for all $h_i < h_H$ (see equation (3.23)). That is, once the child of a household attends school full time, the investment on education is constant. Thus, equation (3.33) becomes

$$s_i = \frac{2}{3}pwh_i\left[1-\frac{1}{2R}-\frac{\phi\left(1-\beta\right)}{\beta}\frac{h_H}{h_i}\right]. \tag{3.35}$$

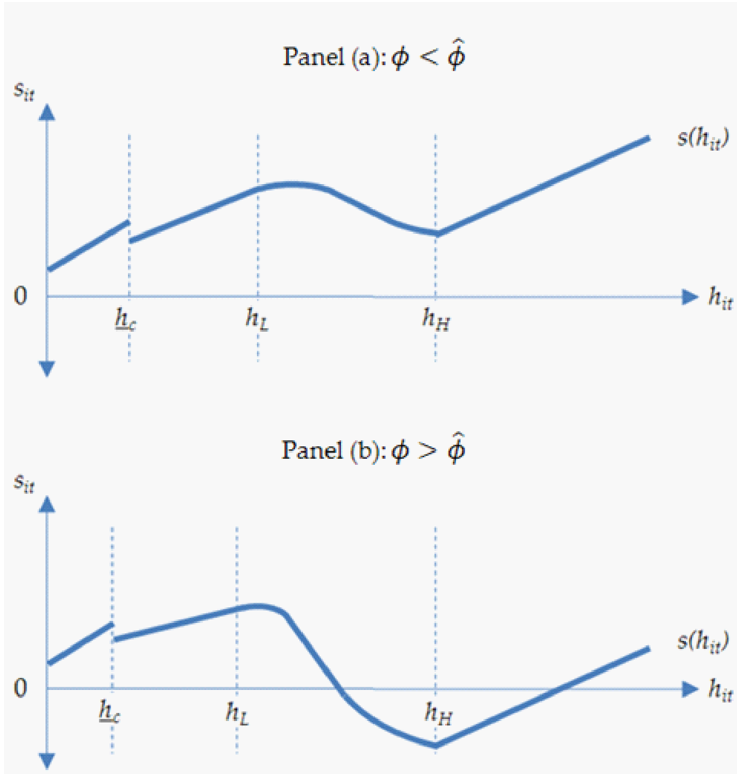
Graphically, the optimal household's saving as a function of the human capital of the household is given in Figure 3.2.

Let us now analyze the evolution over time of the distribution of human capital in the economy. In order to do so, let $\overline{h}_{t,[a,b]}$ denote the average level of human capital in the interval $[a, b]$ at time t, and $\widehat{h}_{t,[a,b]}$ denote the $g_t\left(h\right)$-weighted average of $h_{i,t}$ in the interval $[a, b]$. Thus,

$$\overline{h}_{t,[a,b]} = \int_a^b h_{i,t}g_t\left(h_{i,t}\right)dh_{i,t} = \widehat{h}_{t,[a,b]}\int_a^b g_t\left(h_{i,t}\right)dh_{i,t},$$

where the last equality follows from the *Mean Value Theorem* since $g_t\left(h_{i,t}\right) \geq 0$ for any

Figure 3.2: Household's saving



$h_{i,t} \in \mathbb{R}_+$.

Let $\Delta B \equiv B_{t+1} - B_t$ denote the change over time of the variable B. Recalling that the level of human capital of a household with a child working full time remains constant, the aggregate level of human capital in the economy, $h = \int_0^1 h_i di$, evolves according to

$$\Delta h_{t,[0,\infty]} = \Delta\widehat{h}_{t,[0,\underline{h}_c]} G\left(\underline{h}_c\right) + \Delta\widehat{h}_{t,[h_L,h_H]} \left[G\left(h_H\right) - G\left(h_L\right)\right] + \Delta\widehat{h}_{t,[h_H,\infty]} \left[1 - G\left(h_H\right)\right], \tag{3.36}$$

where $\Delta\widehat{h}_{t,[0,\underline{h}_c]} < 0$, $\Delta\widehat{h}_{t,[h_L,h_H]} > 0$, and $\Delta\widehat{h}_{t,[h_H,\infty]} > 0$. One can easily verify that $\partial \Delta h_{t,[0,\infty]}/\partial\theta > 0$, that is, the evolution of the level of human capital in the economy is increasing in the quality of education.

Notice from equation (3.3) that, at any time t, for all the children living in households with a level of human capital $h_i \leq \underline{h}_c$, since $z_i = 1$, we have $h_i'/h_i < 1$. In turn, this implies from equation (3.29) that once in adulthood, children living in households with human capital levels within this interval will have their own children in involved crime as well.

For the children working full time, i.e. $h_i \in (\underline{h}_c, h_L]$, since $x_i = 1$ and $z_i = e_i = 0$, their human capital next period is constant ($h_i' = h_i$). However, note from equations (3.25) and (3.29) that h_L is constant and $\Delta \underline{h}_c > 0$. Therefore, there exists some period $T < \infty$ with $\partial T / \partial \theta < 0$ in which $\underline{h}_c = h_L$, such that the dynasties of those households that in the beginning had their children working full time, will have their children engaged in child crime. This implies that, in the long-run, $\lim_{t \to \infty} \Delta \widehat{h}_{t,[0,h_L]} = 0$.

Next, for all $h_i \in (h_L, h_H)$ we have $h_i' > h_i$ since $e_i \in (0,1)$ and $b_i > 0$. Moreover, given that both e_i and b_i are increasing in h_i, every generation accumulates more human capital and will increase their adult labor earnings compared to the previous generation, choosing in turn to send their own child more time to school and investing additional resources on her education. Following this process, there exists some period $T' < \infty$ with $\partial T' / \partial \theta < 0$ in which the dynasties of the households within this segment of the distribution will choose full-time schooling.

Thus, the development path of this economy leads to an endogenous elimination of child labor in the long-run. More importantly, the facts that $\partial T / \partial \theta < 0$ and $\partial T' / \partial \theta < 0$ imply that the persistence of child labor in an economy is lower the larger the quality of education.

In the steady state equilibrium, the distribution of human capital is characterized by a unique sequence of optimal choices for the child's time allocation: a constant proportion $G\left(\underline{h}_c^*\right)$ of dynasties have their children involved in crime, and a constant proportion $1 - G\left(\underline{h}_c^*\right)$ have their children attending school full-time, where $\underline{h}_c^* = h_L$. Thus, in the long run, the growth rate of the aggregate level of human capital in the economy converges asymptotically to $\lim_{t \to \infty} \frac{\Delta h_{t,[0,h_L]}}{h_{t,[0,h_L]}} = \theta b_H^{1-\beta}$.

Similarly, from equations (3.6) and (3.36), we have that the evolution over time of the aggregate stock of effective human capital, H, is given by

$$\Delta H_t = \Delta h_{t,[0,\infty]} + \phi \Delta \widehat{x_t h}_{t,[h_L,h_H]} \left[G\left(h_H\right) - G\left(h_L\right) \right] + \Delta \widehat{(1-l_{ot})}\, h_{t,[0,\infty]}. \qquad (3.37)$$

Noticing that leisure is constant in the long run, then the growth rate of the economy converges asymptotically to

$$\lim_{t \to \infty} \frac{\Delta Y_t}{Y_t} = \theta b_H^{1-\beta}, \qquad (3.38)$$

where $\frac{\Delta Y_t}{Y_t} = \frac{\Delta H_t}{H_t} = \frac{\Delta K_t}{K_t}$ for all $t \geq 0$.

To provide further intuition regarding the evolution of the children's activities, Figure 3.3 illustrates in a simple manner the transitional dynamics of the allocations of children's time. The vertical axis denotes the mass of children, i.e., a point in the vertical axis represents the child in household $i \in [0, 1]$. For expositional reasons, households are conveniently indexed so that children are ranked in order of increasing human capital. That is, the family at the bottom of the graph has the lowest level of human capital in the economy at time 0, and the family at the top of the graph has the highest level of human capital. The horizontal axis denotes time. Thus, along any horizontal line we are following the dynasty of the same household throughout time.

Figure 3.3: Allocations of children's time across dynasties over time

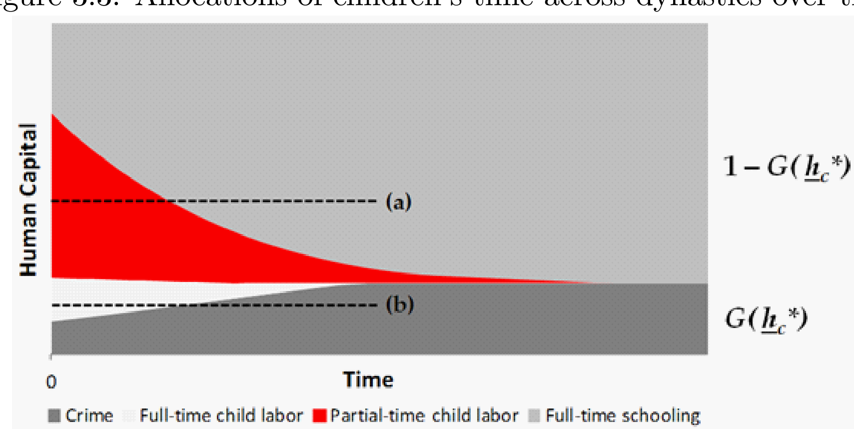


Figure 3.3 shows two examples. Example (a) follows the dynasty of a household that at time 0 has its children working part-time, and after some generations, the child

of the corresponding dynasty attends school full time. The reason is that the level of human capital of this dynasty increases over time. Then, the household's adult labor income increases over time such that at some point it is optimal for this dynasty to send its child full-time to school. Example (b) follows a dynasty with a lower level of human capital that at time 0 has its child working full-time, and after some generations, the child is engaged in crime. The reason is that, as the economy develops, the rents from child crime increase over time, increasing in turn the relative earnings between child crime and child labor. Thus, this increase over time in the relative earnings induces this dynasty to reallocate its child time from child labor to child crime in the future.

3.4 Analysis of a child labor ban

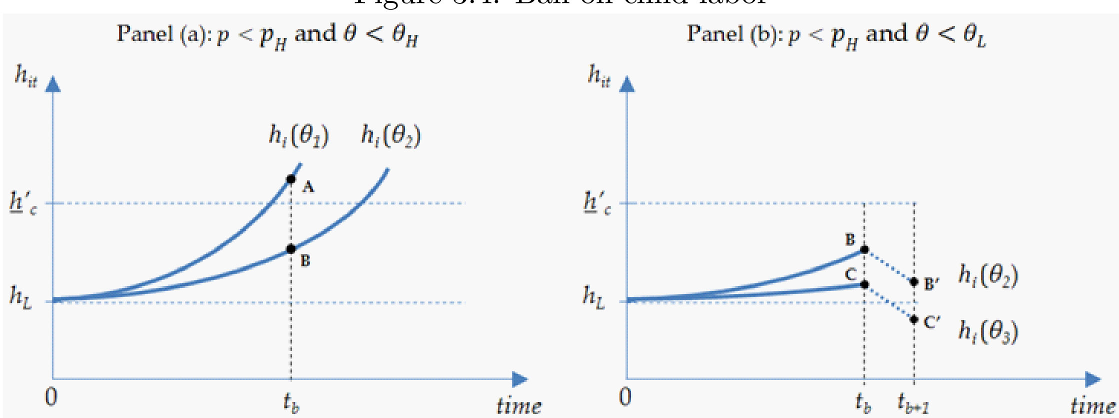
Formally, consider an enforceable full-ban on child labor occurring at some period $t_b > 0$, such that $x_i = 0$ for all i. After the ban is imposed, it is optimal for a household to have its child in school full time if and only if the utility from schooling is greater than the utility from child crime. In the Appendix, I show that there exists a threshold level $\underline{h}_c' \in (\underline{h}_c, h_H)$ such that, after the ban, a household will have its child attending school if and only if $h_i > \underline{h}_c'$. Furthermore, $\underline{h}_c' > h_L$ if and only if $p < p_H$ where $p_H > p_L$. That is, the new threshold level of human capital that determines whether a household prefers schooling over child crime after the ban, $\underline{h}_c'$, is greater than the threshold level of human capital that determines whether a household starts sending its child to school part time before the ban, h_L, if and only if the level of property rights is sufficiently low. We have then the following

Proposition 12 *(i) A permanent ban on child labor increases the proportion of children involved in crime in the long run if and only if $p < p_H$ and $\theta < \theta_H$. (ii) A temporary ban on child labor increases the proportion of children involved in crime in the long run if and only if $p < p_H$ and $\theta < \theta_L$, with $\theta_L > 0$ and $\theta_L < \theta_H$. (iii) Both θ_L and θ_H are decreasing in t_b. (iv) If $p < p_H$ and $\theta < \theta_L$, even a temporary ban has permanent negative effects for the children living in the poorest households.*

Parts *(i)* and *(ii)* of the Proposition state that both a permanent ban and a temporary ban on child labor will permanently increase the proportion of children engaged in crime if and only if the security of effective property rights and the quality of education in the economy are sufficiently low. Part *(iii)* of the Proposition implies that, the earlier a ban on child labor is implemented, the more likely is that the ban will increase the proportion of children in crime.

Figure 3.4 illustrates in a simple manner the intuition behind this result. The figure follows the same household i, and its dynasty, that at time $t = 0$ has a level of human capital just above the level h_L, and at the time when the ban is implemented, t_b, it has a level of human capital $h_i(\theta)$. Figure 3.4 shows the case $p < p_H$ such that $\underline{h}_c' > h_L$. Panels (a) and (b) depict the conditions for a permanent ban and a temporary ban to increase child crime in the long run, respectively.

Figure 3.4: Ban on child labor



Recall from the analysis of the previous section that the proportion of households with their children involved in crime in the long run is given by $G(h_L)$. Thus, a necessary condition for a permanent child labor ban to increase the proportion of children involved in crime in the long run is $p < p_H$, such that $\underline{h}_c' > h_L$. However, this is not a sufficient condition since a child combining school and work before the ban, i.e. $h_i \in (h_L, h_H)$, will be involved in crime once the ban is implemented if and only if $h_i \leq \underline{h}_c'$. Together, $p < p_H$ and $\theta < \theta_H$ are necessary and sufficient conditions for a ban to increase child crime in the long run.

To see this, consider panel (a) in the figure. Everything else equal, this panel presents two cases for $p < p_H$: case 1, in which the quality of education is relatively large such that $\theta_1 > \theta_H$; and case 2, in which the quality of education is relatively low such that $\theta_2 < \theta_H$. In case 1, the quality of education is sufficiently large such that at the time the policy is implemented, there are no households with a level of human capital between h_L and $\underline{h}_c'$ (point A in panel (a)). In this case, the permanent ban has no long run effects. However, if the quality of education is sufficiently low as in case 2 ($\theta_2 < \theta_H$), the accumulation of human capital of the children that attend school part time is not sufficiently large, such that, at the time the ban is implemented, there exists a non-empty set of dynasties with $h_i \in (h_L, \underline{h}_c']$ (point B in panel (a)). In this case, the permanent ban on child labor increases child crime in the long run since $G\left(\underline{h}_c^P\right) > G\left(\underline{h}_c^*\right)$. Finally, notice from the figure that, for a given θ, the earlier a ban is implemented (lower t_b), the more likely the policy will increase child crime in the long run.

Consider now the case of a temporary ban on child labor (panel (b) in Figure 3.4). For simplicity, suppose a ban on child labor is implemented at time t_b and it is abrogated at time t_{b+1}. Notice that once the policy is abrogated, the threshold level h_L applies again. In this case, since crime harms the human capital accumulation of the child, if the quality of education in the economy is sufficiently low such that $\theta < \theta_L$ (case 3 in panel (b)), there exists a non-empty set of dynasties negatively affected by the policy that before the policy implementation would have had their children in school in the long-run, but after the temporary implementation will have their children involved in crime in the long-run. Therefore, if the quality of education is sufficiently low such that $\theta < \theta_L$, the long-run proportion of children in crime after the temporary ban on child labor is greater relative to the same proportion without the policy intervention, i.e. $G\left(\underline{h}_c^T\right) > G\left(\underline{h}_c^*\right)$.

Finally, let us analyze the effects of the child labor ban on output and welfare in the long run. Notice that, at the period the ban is implemented (t_b), the aggregate stock of effective human capital in the economy (H) decreases since there are no working children anymore. Next, given the small open economy assumption, notice from equation (3.14) that the aggregate stock of physical capital decreases to such a level that the ratio K/H remains constant. From equation (3.11), this implies that the wage per unit of human capital for adults is not affected by the policy. In turn, this implies that in the long run, the dynasties of all those households with a level of human capital $h_i > \underline{h}_c'$ at the time of the policy implementation, are not affected by the child labor ban.

Either in the case of a permanent ban or a temporary ban on child labor, if the quality of education is sufficiently low such that any policy increases the level of crime in the long run (i.e. $G\left(\underline{h}_c^P\right) > G\left(\underline{h}_c^*\right)$), the aggregate stock of effective human capital after the policy (H_t^P) is lower relative to the no-ban case (H_t) every period (i.e. $H_t^P < H_t$ for all $t \geq t_b$), which implies that both physical capital and output are also lower every period because of the ban ($K_t^P < K_t$ and $Y_t^P < Y_t$ for all $t \geq t_b$). Furthermore, the fact that lower output induces lower aggregate labor income, together with the fact that $G\left(\underline{h}_c^P\right) > G\left(\underline{h}_c^*\right)$, imply that crime rents per household decrease. Thus, the dynasties of all those households with $h_i \leq \underline{h}_c^P$ are worse off because of the ban.

3.5 Institutions and the activity of children

In this section, I use the above model to examine how the children's activities are affected by the institutional quality on the economy, by the presence of credit market imperfections, and finally I briefly discuss the effects of inequality.

3.5.1 Property rights and the quality of education

In this section, we examine the relationship between the institutional quality in the economy and the incidence of child labor and child crime. In particular, the focus is on the quality of education (θ) and the security of effective property rights (p). Historically, the successful stories of eliminating child labor come from the experience of developed countries, like Great Britain and the U.S. Interestingly, the quality of education in the U.S. a century ago was greater than the quality of education currently observed in developing countries with child labor. Accordingly, from equations (3.25), (3.26), and (3.29), we have the following result:

Proposition 13 *An exogenous increase in*

(i) the quality of education: increases the proportion of children attending school full time and decreases the proportion of children working full time. Furthermore, it reduces both the time it takes to eliminate child labor from the economy and the level of child crime in the long run;

(ii) the security of effective property rights: increases the proportion of children attending school full time, decreases the proportion of children out from school, and decreases the level of child crime in the short and the long run.

This result suggests that countries with higher institutional quality will observe relatively larger levels of children attending school, and relatively lower levels of children outside from school. The reason is simple, the greater the quality of institutions, the greater the effective returns to education, whether they come from increasing future human capital (θ), or from increasing future net labor income (p).

Investment on institutional quality

We now explore the case where the institutional quality of an economy can be improved by investing resources on it. The purpose of this section is to present a simple analysis of the effects that the investment on either school quality or the security of property rights can have on the children's activities. An analysis of optimal investment on institutional quality is beyond the scope of the paper.

Suppose that every period t the levels of quality of education and effective property rights are determined by

$$\begin{aligned} \theta &= \Psi_\theta\left(I_\theta\right), \\ p &= \Psi_p\left(I_p\right), \end{aligned} \tag{3.39}$$

where I_j denotes the aggregate resources invested on institution j, with $\partial\Psi_j/\partial I_j > 0$ for $j = \{\theta, p\}$, $\Psi_\theta(0) = \theta_o > 0$, $\Psi_p(0) = p_o \in (0,1)$, $\lim_{I_\theta\to\infty} \Psi_\theta(I_\theta) = \overline{\theta}$, and $\lim_{I_p\to\infty} \Psi_p(I_p) = 1$.

In addition, suppose the investment on institutions is financed through taxes. In particular, assume that every period t each adult i faces a tax $T_{\theta i}$ to finance the investment on the quality of education, and a tax T_{pi} to finance the investment on the security of property rights. Accordingly, adult consumption is now given by

$$c_{ai} = pwh_i + px_i w_c h_i + \frac{(1-p)\,Y_L}{n} z_i - s_i - b_i - T_{\theta i} - T_{pi}. \tag{3.40}$$

Every period, aggregate investment on an institution is given by the aggregate tax revenue associated with it,

$$\begin{aligned} I_\theta &= \textstyle\int_0^1 T_{\theta i} di, \\ I_p &= \textstyle\int_0^1 T_{pi} di. \end{aligned} \tag{3.41}$$

To maintain the analysis as simple as possible, assume T_{ji} is proportional to the adult labor income of the household, such that $T_{\theta i} = \tau_\theta pwh_i$ and $T_{pi} = \tau_p pwh_i$, where τ_j is the tax rate. Thus, $I_j = \tau_j pwh$ for $j = \{\theta, p\}$, where $h = \int_0^1 h_i di$.

It is straightforward to verify that the optimal level of schooling at time t is now

$$e_i = \frac{3}{3+\delta}\left[\frac{\delta\beta}{3\phi}\left(1+\phi+\frac{1}{R}-\tau_\theta-\tau_p\right) - \frac{1}{\theta}\left(\frac{\beta}{(1-\beta)\,\phi pwh_i}\right)^{1-\beta}\right]. \tag{3.42}$$

Following the same steps as before, we have that $e_i = 0$ if and only if $h_i \leq h_L$, with

$$h_L = \frac{\beta}{(1-\beta)\,\phi pw}\left[\frac{3\phi}{\theta\delta\beta\left(1+\phi+\frac{1}{R}-\tau_\theta-\tau_p\right)}\right]^{\frac{1}{1-\beta}}, \tag{3.43}$$

where w, θ and p are given by equations (3.11) and (3.39), respectively. Next, we have that $e_i = 1$ if and only if $h_i \geq h_H$, with

$$h_H = \frac{\beta}{(1-\beta)\,\phi pw}\left[\frac{3\phi}{\theta\left[\delta\beta\left(1+\phi+\frac{1}{R}-\tau_\theta-\tau_p\right)-(3+\delta)\,\phi\right]}\right]^{\frac{1}{1-\beta}}. \tag{3.44}$$

Similarly, one can easily verify that $z_i = 1$ if and only if $h_i \leq \underline{h}_c$, where $\underline{h}_c$ is given by the unique solution to the following expression

$$\underline{h}_c G\left(\underline{h}_c\right) = \frac{\left(1-\varepsilon\right)^{\delta/3}\left(1-p\right)H}{p\left[\phi + \left(1+\frac{1}{R}-\tau_\theta-\tau_p\right)\left(1-\left(1-\varepsilon\right)^{\delta/3}\right)\right]}, \tag{3.45}$$

where $G\left(\underline{h}_c\right) \in (0,1)$ is the proportion of child crime in the economy.

Next, we examine how the two alternative policies affect the activity of children. As indicated previously, a child's activity depends on the level of human capital of the household relative to these thresholds' levels. Thus, whether a child switch between activities or not depends on the effect the investment on institutional quality has on the different thresholds.

Consider first the case of the investment on the quality of education, θ. Given equations (3.39) and (3.41), total differentiation of equation (3.43) with respect to h_L, θ, and τ_θ yields

$$dh_L < 0 \iff \frac{\partial\Psi_\theta\left(I_\theta\right)/\partial I_\theta}{\Psi_\theta\left(I_\theta\right)} > \frac{1}{pwh\left(1+\phi+\frac{1}{R}-\tau_\theta\right)}. \tag{3.46}$$

That is, the threshold h_L decreases with the investment on the quality of education if and only if the proportional increase on the school quality is sufficiently large. This implies that, if this proportional increase in θ is sufficiently large, there exists a non-empty interval of the level of human capital such that, due to the investment, the children of the households with a human capital level within this interval will attend school part time when otherwise they would have worked full time. However, it is the case that some children would now work full time if the increase in θ is not sufficiently large.

Similarly, from equation (3.44) we have that the threshold h_H decreases with the investment on the quality of education if and only if the proportional increase on θ is sufficiently large. That is,

$$dh_H < 0 \iff \frac{\partial\Psi_\theta\left(I_\theta\right)/\partial I_\theta}{\Psi_\theta\left(I_\theta\right)} > \frac{1}{pwh\left(1+\phi+\frac{1}{R}-\tau_\theta-\left(\frac{3+\delta}{\delta\beta}\right)\phi\right)}. \tag{3.47}$$

Notice from equations (3.46) and (3.47), the increase in θ needed to decrease h_H is relatively larger than the increase needed to decrease h_L.

Thus, if the increase in θ is sufficiently low, such that the inequality in equation (3.46) is reversed, the policy designed to increase the quality of education in the economy ends up driving children out from school. The reason is that, on the one hand, an increase in the quality of education gives incentives to increase the allocation of the child's time in school. On the other hand, the fact that all households have to pay for the investment on the school quality gives incentives to households to allocate their child's time away from school in order to obtain resources for the household. Overall, if the increase in θ does not compensate for the resources lost with the investment, households will prefer to decrease the allocation of time in school.

Next, notice from equation (3.45) that the threshold $\underline{h}_c$ necessarily increases with the investment on school quality. Then, because of the policy, the children of some households will participate in child crime when otherwise they would have worked full time. The reason is that the tradeoff the poorest households face when deciding whether the child will work full time or will be engaged in child crime is independent of θ, while at the same time, these households spend resources on the tax.

Consider now the case of the investment on the security of effective property rights, p. Following the same steps as before, from equations (3.43) to (3.45) we have

$$dh_L < 0 \iff \frac{\partial \Psi_p\left(I_p\right)/\partial I_p}{\Psi_p\left(I_p\right)} > \frac{1}{(1-\beta)\, pwh\left(1+\phi+\frac{1}{R}-\tau_p\right)}, \tag{3.48}$$

$$dh_H < 0 \iff \frac{\partial \Psi_p\left(I_p\right)/\partial I_p}{\Psi_p\left(I_p\right)} > \frac{1}{(1-\beta)\, pwh\left(1+\phi+\frac{1}{R}-\tau_p-\left(\frac{3+\delta}{\delta\beta}\right)\phi\right)}, \tag{3.49}$$

$$d\underline{h}_c < 0 \iff \frac{\partial \Psi_p\left(I_p\right)/\partial I_p}{\Psi_p\left(I_p\right)} > \frac{1-p}{pwh\left(1+\frac{\phi}{1-(1-\varepsilon)^{\delta/3}}+\frac{1}{R}-\tau_p\right)}. \tag{3.50}$$

Similarly, we have from equations (3.48) and (3.49) that both thresholds h_L and h_H decrease with the policy if and only if the proportional increase in the level of effective property rights is sufficiently large. In this case, however, the threshold for the crime-work tradeoff, $\underline{h}_c$, can indeed decrease with the policy if and only if the proportional increase in p is sufficiently large, such that the inequality in equation (3.50) holds. Otherwise, even an investment policy that increases the security of property rights in the economy can drive children into crime.

Finally, it is worth noticing that, if we have access to differentiated tax rates across households, i.e. $\tau_{\theta i}$ and τ_{pi}, we can implement a tax policy that necessarily decreases the different threshold levels, i.e. that increases schooling and decreases crime. For instance, suppose $\tau_{\theta i} = \tau_{pi} = 0$ for all households with $h_i < \widetilde{h}$, for some $\widetilde{h} > h_H$, and $\tau_{\theta i} > 0$, $\tau_{pi} > 0$ for all households with $h_i \geq \widetilde{h}$. In this case, the relatively poor households would not incur in any cost associated with the investment in institutional quality, and they would be favored by the improvement on institutions. However, this would be at the expense of the relatively wealthy households.

3.5.2 Credit market imperfections

In this section we examine how credit market imperfections may affect the optimal decisions over the children's activities. In particular, I focus on the case where credit market imperfections prevent households from borrowing resources. Formally, consider that household's saving is restricted by a non-positive lower bound. For simplicity, assume that households cannot borrow more than a proportion $\pi \geq 0$ of the household's adult labor income, such that $s_i \geq -\pi p w h_i$. Larger values of π correspond to weaker credit market imperfections, and if $\pi = 0$, households simply cannot borrow. Otherwise, the model is as before.

Let $\lambda_i \geq 0$ denote the shadow value associated with the borrowing constraint $s_i \geq -\pi p w h_i$. Given equations (3.3), (3.7) and (3.8), the Lagrangian associated with household i's maximization problem is

$$\mathcal{L}\left(s_i, l'_{oi}, e_i, b_i, z_i\right) = \ln\left(c_{ai}\right) + \ln\left(c'_{oi}\right) + \ln\left(l'_{oi}\right) + \delta \ln\left(h'_i\right) + \lambda_i\left(s_i - \pi p w h_i\right), \quad (3.51)$$

where $\lambda_i > 0$ whenever the restriction binds. Clearly, whenever $\lambda_i = 0$, the optimal allocations are the same as before. It is worth noticing that $\phi > \widehat{\phi}$ is a necessary condition for $\lambda_i > 0$ for some i, since otherwise optimal saving is positive for all households (see Figure 3.2). In addition, notice that $\lambda_i = 0$ for all those households with a positive level of optimal saving.

Suppose $\phi > \widehat{\phi}$. One can easily verify that, if and only if $\pi < \overline{\pi}$, there exists a non-empty interval of human capital level $\left[\underline{h}, \overline{h}\right]$ such that $\lambda_i > 0$ if and only if $h_i \in \left(\underline{h}, \overline{h}\right)$, where $\overline{\pi} = \frac{2}{3}\left[\phi\left(\frac{1-\beta}{\beta}\right) + \frac{1}{2R} - 1\right]$, $\underline{h} \in (h_L, h_H)$ and $\overline{h} > h_H$. That is, if the credit market imperfections are sufficiently weak, i.e. $\pi \geq \overline{\pi}$, there are no borrowing constraints in the economy.

Whenever $\pi < \overline{\pi}$ and $\lambda_i > 0$, the optimal level of schooling at time t is given by

$$e_i = \frac{1}{1+\delta}\left[\frac{\delta\beta}{\phi}(1+\phi+\pi) - \frac{1}{\theta}\left(\frac{\beta}{(1-\beta)\,\phi p w h_i}\right)^{1-\beta}\right], \tag{3.52}$$

where $\partial e_i/\partial\pi > 0$. That is, for those households restricted by the borrowing constraint, the optimal choice of schooling decreases with the strength of the credit market imperfections.

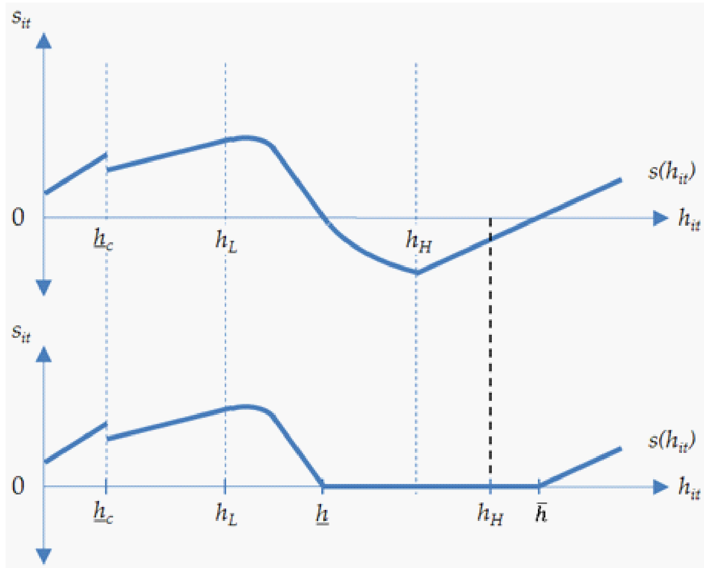
Next, since π affects the optimal allocation of time in school, it also affects the threshold level h_H that determines whether a household has its child attending school full time. From equation (3.52) we have that $e_i = 1$ if and only if $h_i \geq h'_H$, with

$$h'_H = \frac{\beta}{(1-\beta)\,\phi p w}\left[\frac{\phi}{\theta\left[\delta\beta\,(1+\phi+\pi) - (1+\delta)\,\phi\right]}\right]^{\frac{1}{1-\beta}}, \tag{3.53}$$

where $h'_H > h_H$ if and only if $\phi > \widehat{\phi}$, and with $\partial h'_H/\partial\pi < 0$. That is, the threshold h'_H increases with the strength of the credit market imperfections.

Figure 3.5 presents the optimal household's saving for two different cases: $\pi > \overline{\pi}$ and $\pi = 0$. Notice from the figure that in the short run, borrowing constraints decrease the proportion of children attending school full time, increasing in turn, the proportion of children working part-time. That is, credit market imperfections tend to decrease the allocation of time in school in the short run whenever the productivity of child labor is sufficiently large. In the long run, however, since π does not affect the threshold h_L the economy converges to the same optimal allocations as before.

Figure 3.5: Credit market imperfections: $\pi > \overline{\pi}$ (above) and $\pi = 0$ (below)



3.5.3 Inequality

In this Section we briefly discuss some implications of differences in inequality on the distribution of human capital. To maintain the analysis as simple as possible, the concept of inequality used here is based on the concept of generalized Lorenz dominance.[9] The convenience of this approach relies on the fact that the concept of generalized Lorenz dominance is equivalent to the concept of second-order stochastic dominance. In particular, a distribution A (generalized) Lorenz dominates another distribution B if and only if the distribution A dominates B stochastically at a second order (i.e. inequality in B is larger). Formally, distribution A dominates B stochastically at a second order if $\int_{-\infty}^{m} [B(x) - A(x)]\, dx \geq 0$ for all m, and $\int_{-\infty}^{m} [B(x) - A(x)]\, dx > 0$ for some m.

With the concept of second-order dominance in place, it is straightforward to examine the implications of inequality in the context of this model. Suppose we start at time 0 with two different distributions of human capital in the economy, $G_1(h)$ and $G_2(h)$, continuously defined over the same support and with the same initial mean, and where

[9] This concept is based on the generalized Lorenz curve introduced by Shorrocks (1983), where the ordinates of the generalized Lorenz curve are the standard Lorenz curve ordinates multiplied by the average level of the distribution.

the distribution $G_2(h)$ strongly dominates the distribution $G_1(h)$ stochastically at a second order, i.e. inequality in $G_1(h)$ is greater. Notice that this assumption implies that the (standard) Lorenz curve for distribution $G_1(h)$ lies below the Lorenz curve for distribution $G_2(h)$.

We have then $G_1(h_L) > G_2(h_L)$ and $1 - G_1(h_H) < 1 - G_2(h_H)$. That is, the proportion of households with their children out from school is greater in the economy with larger inequality, while the proportion of households with children attending school full time is greater in the economy with lower inequality.

Since $G_1(h_L) > G_2(h_L)$, in the long run, the level of child crime is larger in the economy with greater inequality. Furthermore, given that $G_1(h_H) > G_2(h_H)$, notice from equations (3.37) and (3.38) that, even though both economies converge to the same growth rate in the limit as $t \to \infty$, the growth rate in the economy with larger inequality is lower during the transition.

3.6 The closed economy case

As mentioned above, the small open economy assumption greatly simplifies the analysis by eliminating dynamics in the determination of the physical capital-effective human capital ratio. In this section, I depart from this assumption and consider the case of a closed economy.

Nevertheless, to simplify the analysis and in order to obtain an analytical solution to the model, it is assumed that old individuals do not work. Thus, only adults face non-trivial decisions having preferences of the form

$$U = u(c_{ai}) + u(c'_{oi}) + \delta v(h'_i) \equiv \ln(c_{ai}) + \ln(c'_{oi}) + \delta \ln(h'_i), \tag{3.54}$$

where, c_{ai} is the household i's current consumption, c'_{oi} is individual i's consumption when old, and h'_i is the level of human capital of individual i's offspring once she is an adult.

When old, individuals simply consume their capital income

$$c'_{oi} = R' s_i, \tag{3.55}$$

where in this case, the rental rate of capital is not determined in the international capital market, but by the equilibrium condition given in equation (3.10). Otherwise, the economy model is as before.

From the production technology in equation (3.5), the aggregate stock of effective human capital, H, is now given by

$$H \equiv H_a + \phi H_c = \int_0^1 h_i di + \phi \int_0^1 x_i h_i di, \tag{3.56}$$

where H_a is the aggregate stock of human capital provided by adults, H_c is the aggregate stock of human capital provided by children.

A competitive equilibrium is a sequence of allocations $\{e_i(t), x_i(t), z_i(t), b_i(t), s_i(t)\}_{t=0}^{\infty}$ for all individuals $i \in [0,1]$, a sequence of distributions of human capital $\{G_t(h_i(t))\}_{t=0}^{\infty}$, with $g_0(h_i(0))$ given, a sequence of allocations $\{K(t)\}_{t=0}^{\infty}$, with $K(0) = K_0 > 0$, and a sequence of prices $\{R(t), w(t), w_c(t)\}_{t=0}^{\infty}$ such that, given prices, each individual maximizes her utility, firms maximize profits, human capital for each individual evolves according to (3.3), with $h_i(0) = h_{i0} > 0$ for all $i \in [0,1]$, the distribution of crime rents satisfies (3.9), and every market clears for all $t \geq 0$.

In order to focus our attention in the more interesting case, I assume that the initial stock of physical capital satisfies $K_0 < h_0 \left[(1-\alpha) A/2\right]^{1/(1-\alpha)}$. This assumption ensures that the K/H ratio increases over time.

The equilibrium analysis of this Section parallels that of Section 3.3. In this case, however, the market clearing condition in the physical capital market implies that every period aggregate savings and aggregate investment in physical capital are equal, i.e. $K' = \int_0^1 s_i di$. In addition, the wage rate is not constant over time and it is not determined by equation (3.15). Instead, from equation (3.11), the wage rate for adult labor every period is given by

$$w = (1-\alpha) A \left(\frac{K}{H}\right)^{\alpha}, \tag{3.57}$$

with $w_c = \phi w$. Similarly, the gross interest rate every period is given by

$$R = \alpha A \left(\frac{H}{K}\right)^{1-\alpha}. \tag{3.58}$$

Following the same steps as before, the Euler equation together with $c'_{oi} = R' s_i$ imply that $s_i = c_i$ for all i and for all t. Then, optimal saving for all i is given by the general expression

$$s_i = \frac{1}{2} pwh_i \left[1 + \phi (1 - e_i) + \frac{(1-p) Y_L}{npwh_i} z_i - \frac{b_i}{pwh_i}\right], \tag{3.59}$$

where $s_i > 0$ for all i. In addition, one can easily verify that the optimal level of schooling at time t is given by

$$e_i = \frac{2}{2+\delta} \left[\frac{\delta\beta}{2\phi}(1+\phi) - \frac{1}{\theta}\left(\frac{\beta}{(1-\beta)\phi pwh_i}\right)^{1-\beta}\right], \tag{3.60}$$

where w is given by equation (3.57). Thus, we have that $e_i = 0$ if and only if $h_i \leq \widehat{h}_L$, with

$$\widehat{h}_L = \frac{\beta}{(1-\beta)\phi pw} \left[\frac{2\phi}{\theta\delta\beta(1+\phi)}\right]^{\frac{1}{1-\beta}}. \tag{3.61}$$

Similarly, $e_i = 1$ if and only if $h_i \geq \widehat{h}_H$, with

$$\widehat{h}_H = \frac{\beta}{(1-\beta)\phi pw} \left[\frac{2\phi}{\theta\left[\delta\beta(1+\phi) - (2+\delta)\phi\right]}\right]^{\frac{1}{1-\beta}}, \tag{3.62}$$

where the right-hand-side of this expression is defined if and only if $\delta > \widehat{\delta}_L \equiv \frac{2\phi}{\beta(1+\phi)-\phi}$, with $\widehat{\delta}_L > 0$ since $\beta > 1/2$. Thus, whenever $\widehat{h}_H$ exists, we have $\widehat{h}_H > \widehat{h}_L$. Notice from equations (3.57), (3.61) and (3.62) that both $\widehat{h}_H$ and $\widehat{h}_L$ decrease over time if and only if the ratio K/H increases over time.

From equation (3.22), the optimal investment on education for all $h_i \in \left[\widehat{h}_L, \widehat{h}_H\right]$ continues to be given by equation (3.23), and for all $h_i > \widehat{h}_H$ it is now given by the unique solution to the following expression:

$$\frac{2}{\theta} b_i^{\beta} + (2 + \delta (1 - \beta)) b_i = \delta (1 - \beta) p w h_i, \tag{3.63}$$

with $db_i/dh_i > 0$ and $db_i/d\theta > 0$. As before, it is convenient to assume that b_i is bounded by some level b_H.

Next, from the optimality of child crime we have that $z_i = 1$ if and only if $h_i \leq \underline{\widehat{h}}_c$, where $\underline{\widehat{h}}_c$ is given by the unique solution to the following expression

$$\underline{\widehat{h}}_c G\left(\underline{\widehat{h}}_c\right) = \frac{(1 - \varepsilon)^{\delta/2} (1 - p) H}{p \left[1 + \phi - (1 - \varepsilon)^{\delta/2}\right]}, \tag{3.64}$$

where $G(\underline{\widehat{h}}_c) \in (0, 1)$ is the proportion of child crime in the economy. As before, one can verify that $\underline{\widehat{h}}_c < h_L$ if and only if $p > \widehat{p}_L$, with $\widehat{p}_L < 1$,. In addition, the threshold $\underline{\widehat{h}}_c$ increases over time whenever the aggregate stock of effective human capital, H, increases over time.

Finally, from equation (3.59), aggregate investment every period is determined by

$$K' = \frac{1}{2} \left[(1 - \alpha) A K^{\alpha} H^{1-\alpha} - \int\limits_0^1 b_i di \right]. \tag{3.65}$$

In the long run, the economy converges asymptotically to a unique steady state equilibrium where the ratios Y/H, K/H, and the growth rate of the economy are all constant over time. In particular, one can verify that $\lim_{t\to\infty} H_t = \int_0^1 h_{i,t} di = h_t$, and then we have

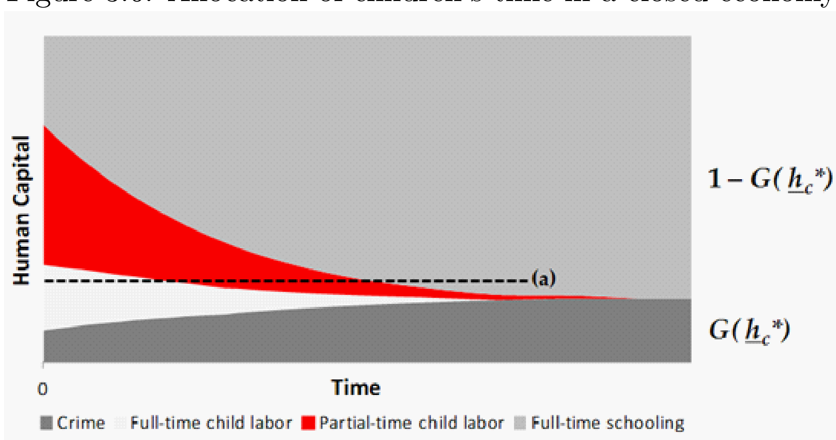
$$\lim_{t\to\infty} \frac{K_t}{H_t} = \left[\frac{(1 - \alpha) A}{2\left(1 + \theta b_H^{1-\beta}\right)} \right]^{\frac{1}{1-\alpha}}, \tag{3.66}$$

$$\lim_{t\to\infty} \frac{Y_t}{H_t} = A^{1/(1-\alpha)} \left[\frac{1-\alpha}{2\left(1+\theta b_H^{1-\beta}\right)} \right]^{\frac{\alpha}{1-\alpha}}, \tag{3.67}$$

$$\lim_{t\to\infty} \frac{\Delta Y_t}{Y_t} = \lim_{t\to\infty} \frac{\Delta K_t}{K_t} = \lim_{t\to\infty} \frac{\Delta H_t}{H_t} = \theta b_H^{1-\beta}. \tag{3.68}$$

Figure 3.6 illustrates the transitional dynamics of the allocations of the children's time in the closed economy case. As in Figure 3.3, the vertical axis denotes the mass of children conveniently indexed and the horizontal axis denotes time.

Figure 3.6: Allocation of children's time in a closed economy



Different from the open economy case, in the closed economy case there exists a non-empty set of dynasties in which the children start by working full time, and in the long run, the corresponding children of the same dynasty attend school full time (see example (a) in Figure 3.6). In this case, as the economy develops, the K/H ratio in the economy increases over time increasing in turn the wage rate. The progressive rise in the household's adult labor earnings together with the increase on the child labor wage rate, allow the child of the corresponding dynasty to attend school part time at some point in the future. Following this process, the proportion of the child's time devoted to school increases in the following generations, up to the point where the child will attend school full time.

Using the above model of a closed economy, we have the following result.

Proposition 14 *If $p < \widehat{p}_H$ and $\theta < \widehat{\theta}_L$, a temporary ban on child labor has permanent negative effects in the long run for all individuals in the economy.*

That is, different from the open economy (see Proposition 12), in this case a temporary ban can have permanent negative effects not only for the children living in the poorest households, but for all individuals in the economy.

To see this, suppose a temporary ban is implemented at some time t and it is abrogated at $t+1$. Given that the analysis from this Proposition parallels that of Proposition 12, I shall discuss only the new results.

Consider first the contemporaneous effects. At the time of the implementation, the aggregate stock of effective human capital H falls with the ban since there are no working children. This effect induces an increase on the K/H ratio because the existing stock of physical capital K is already determined, which in turn increases the wage rate for adult labor w and the household's adult labor earnings. Therefore, contemporaneously, those households with the largest levels of human capital, $h_i \geq \widehat{h}_H$ (i.e. with children attending school full time), are unambiguously better off with the ban because the increase on adult labor earnings increase current consumption.

Next, even though the wage rate for adult labor increases, aggregate labor income, $Y_L = (1-\alpha)Y$, falls with the ban. In turn, the decrease in Y_L induces a fall on aggregate investment relative to the no-ban case. Then, the aggregate stock of physical capital at $t+1$ is relatively lower due to the ban.

Now, once the ban is abrogated at time $t+1$, the aggregate stock of effective human capital H is lower relative to the no-ban case. This comes from the fact that, if the level of effective property rights and the quality of education are sufficiently low, the temporary ban increases permanently the level of child crime in the economy. Furthermore, H is lower relative to the no-ban case every period. This, in turn, generates also physical capital and output to be lower relative to the no-ban case.

Relative to the no-ban case, if $p < \widehat{p}_H$ and $\theta < \widehat{\theta}_L$ the fall in the aggregate stock of effective human capital induces a more than proportional decrease on aggregate investment in the future. Together, these two effects induce a fall on the K'/H' ratio.

Then, even though the economy after the temporary ban on child labor converges in the limit as $t \to \infty$ to the same ratios K/H and Y/H given in equations (3.66) and (3.67), respectively, and the same growth rate given in equation (3.38), it does so from below. That is, every period these ratios and the growth rate of the economy are lower due to the temporary ban.

Finally, the decrease on the K/H ratio implies a reduction on the wage rate for adult labor w. Therefore, in the long run not only the level of child crime is greater because of the temporary ban, but in addition, consumption decreases for *all* households in the economy because of the reduction on adult labor earnings.

3.7 Conclusion

This Chapter analyzes the role that institutional quality – security of property rights, quality of education, and credit market imperfections – plays on the relationship between child labor, child crime, and schooling, in an economy characterized by inequality in the distribution of human capital. The study illustrates the important role of the institutional quality and the distribution of human capital in the activity of children. It shows that the interplay between economic institutions and the incentives households face when allocating their children's time greatly shapes the consequences of a child labor ban.

The study suggests that, in economies where the quality of education and the security of property rights are relatively low, a ban on child labor tends to harm the very same children it is intending to help: the working children living in the poorest households. The reason is that a policy designed to ban child labor exacerbates the problem of child crime by leading these working children into criminal activities. Furthermore, even a temporary ban on child labor can have permanent negative effects for all individuals in the economy if the security of property rights and the quality of education are sufficiently low.

A further implication of the model is that child labor is not eliminated from the economy by having perfect capital markets, although it might be reduced. On the other hand, a policy that increases the quality of education may completely eliminate child labor, even in the presence of credit market imperfections. The results also suggest that an economy may find it beneficial to invest on institutional quality if this investment is financed by households with the largest levels of human capital, or if the efficacy of the investment to increase the quality of institutions in the economy is sufficiently large.

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Appendix A

Proof of Proposition 1

The analysis leading to Proposition 1 shows that an equilibrium with $X^* > 0$ and $Z^* > 0$ is such that (X^*, Z^*) solves equations (1.12) and (1.13), every period. Since both equations are linear, and they have different slopes, there is at most one solution. It is straightforward to verify that the solution (X^*, Z^*) is the one given in the proposition.

Next, one must ensure that $X^* > 0$, $Z^* > \underline{Z}$, and $X^* + Z^* < 1$. These three conditions are necessary and sufficient to ensure $X^* \in (0,1)$, $Z^* \in (\underline{Z}, 1)$, and $1 - X^* - Z^* \in (0,1)$. It is straightforward to verify that $X^* > 0$ if and only if $\phi/a > m^L$, with $m^L \equiv \frac{1-p+\delta\beta}{1+p}$, and also that $X^* + Z^* < 1$ if and only if $\phi < m^H a$, with

$$m^H \equiv \frac{\left(1 + \frac{\delta\beta}{1-p}\right)\frac{b}{a} - 1}{\left(\frac{1+p}{1-p}\right)\frac{b}{a} + 1 - b\left(1 + \frac{\delta\beta}{1-p} + \frac{1+p}{1-p}\right)}.$$

Note that $m^L \in (0,1)$, for all $p \in [1/2, 1]$. It remains to show that $m^H > m^L$, with $m^H > 0$. One can easily verify that $m^H > 0$ if and only if

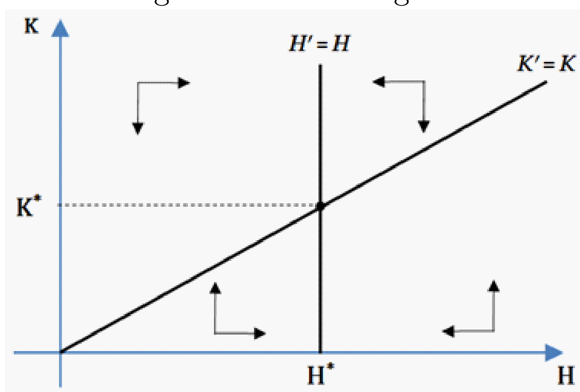
$$b\left(1 + \frac{\delta\beta}{1-p}\right) < 1 + \left(\frac{1}{a} - 1\right)\left(\frac{1+p}{1-p}\right)b,$$

and also that $m^H > m^L$ if and only if $b\left(1 + \frac{\delta\beta}{1-p}\right) > 1$. The above conditions define the interval $\left(b^L, b^H\right)$ given in the proposition in the obvious way.

Next, recalling that $\underline{Z} \leq (1-p)\,\epsilon$, it is easy to verify that $Z^* > \underline{Z}$ if $\epsilon < \frac{1+\frac{a}{\phi}}{b(2+\delta\beta)}$, which is the case for all $\epsilon \in (0, 1/2]$ since $\phi < m^H a$. It is easy to verify that the above existence conditions are consistent with the assumption that $\phi > \frac{b-a}{1-b}$.

The difference equation for K follows from (1.5) and (1.11), and the difference equation for human capital accumulation follows from evaluating (1.3) in equilibrium. Equilibrium dynamics are characterized by these two difference equations, together with (K_0, H_0). Moreover, interior schooling allocations ensure that the equilibrium exhibits positive human and physical capital for all positive initial capital stocks. The equilibrium is obviously unique.

Figure 7: Phase diagram



Furthermore, it converges to a unique steady state for all $K_0 > 0$ and $H_0 > 0$, as seen in the phase diagram depicted in Figure 7. The locus of points with $H' = H$ is given by $H = (Z^*/\underline{Z})^{-\gamma/\beta}(1 - aX^* - bZ^*)$, and the locus of points where $K' = K$ is given by $K = BH(1 + \phi X^*)$, where $B = \left(\left(\frac{p}{1+p}\right)(1-\alpha)A\right)^{\frac{1}{1-\alpha}}$. The sign of K'/K and that of H'/H in the different regions of the phase diagram can be easily inferred from the dynamic equations for K and H that are given in the proposition. This concludes the proof. QED

Proof of Proposition 2

One can verify that steady-state equilibrium utility, for any given $\overline{x}$ with $X = \overline{x} \leq X^*$, is equal to $\ln[B_1\Psi(\overline{x})]$, where $B_1 > 0$ is some constant, with $\Psi(\overline{x}) = (1 - a\overline{x})^{\left(1-\frac{\gamma}{\beta}\right)(2+\delta)}(1 + \phi\overline{x})^2$. The function Ψ is strictly concave, with its maximum at $\overline{x} = X_2$, where X_2 is given by (1.23), and where $X_2 > 0$ if and only if $\phi/a \geq (1 - \gamma/\beta)(1 + \delta/2)$. It is easy to see that there exists a number $x_U \in [0, X^*]$ such that $\Psi(\overline{x}) < \Psi(X^*)$ if and only if

$\overline{x} < x_U$. Let $x_U = X^*$ if and only if $X_2 \geq X^*$. Else, if $X_2 < X^*$, let x_U be the unique solution to $\Psi(x_U) = \Psi(X^*)$ with $x_U \in (0, X^*)$. Note that $x_U > 0$ if and only if $X_2 > 0$ and $\Psi(0) < \Psi(X^*)$, where $\Psi(0) = 1$. Accordingly, let $x_U = 0$ if and only if $\Psi(0) \geq \Psi(X^*)$. This proves Part (i).

Consider Part (ii). It is easy to verify that the necessary and sufficient condition for $\Psi(X^*) > 1$ can be written as

$$\left(1+\frac{a}{\phi}\right)^{\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)}\left(1+\frac{\phi}{a}\right) > \left(1+\frac{1+p}{1-p+\delta\beta}\right)^{\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)}\left(1+\frac{1-p+\delta\beta}{1+p}\right).$$

The left side of the inequality falls with a/ϕ if and only if $\frac{\phi}{a} > \left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)$, for $0 \leq \gamma < \beta$. Above, we have noted that this is the necessary and sufficient condition for $X_2 > 0$. Let n_U be the unique value of $\phi/a \geq (1-\gamma/\beta)(1+\delta/2)$ that equates both sides of the above inequality. A value of $n_U \geq (1-\gamma/\beta)(1+\delta/2)$ can always be found since the right side of the inequality is minimized at $\frac{1+p}{1-p+\delta\beta} = [(1-\gamma/\beta)(1+\delta/2)]^{-1}$. This proves Part (ii).

Now consider Part (iii). It is easy to verify that x_U, as defined above, is increasing in γ, for all $x_U \in (0, X^*)$. Recall that $x_U < X^*$ if and only if $X_2 < X^*$, and note that X_2 is an increasing function of γ, whereas X^* is independent of γ. Finally, one can verify that $x_U < X^*$ if and only if $\gamma < \beta\left(1-\frac{1-p+\delta\beta}{(1+p)(1+\delta/2)}\right)$. This concludes the proof. QED

Proof of Proposition 3

Consider Part (i). First, as noted in the proof of Proposition 1, $\epsilon \in (0, 1/2]$ is a sufficient condition for $Z^* > \underline{Z}$. Next, to compare X^* and X_1, recall that $X^* > 0$ if and only if $\phi/a > m^L$, where $m^L \in (0,1)$ is given in the proof of Proposition 1. From equation (1.22), note that $X_1 > 0$ if and only if $\phi/a \geq (1+\delta/2)/(1-b\underline{Z})$, where $(1+\delta/2)/(1-b\underline{Z}) \geq 1$, for all $\underline{Z} \geq 0$. Next, note that $X_1 > 0$ solves the equilibrium condition (1.19) whenever $\phi/a \geq (1+\delta/2)/(1-b\underline{Z})$. Substituting equation (1.18) into (1.19) one can show that an interior solution $X_1 > 0$ must solve

$$\frac{\phi}{1+\phi X} = \left(1+\frac{\delta}{2}\right)\frac{a}{1-aX-b\underline{Z}}. \tag{A.1}$$

Moreover, taking into account that $Z^* = g\left(X^*\right)$, equation (1.20) can be written as

$$\frac{\phi}{1+\phi X} = \left(\frac{1-p+\delta\beta}{1+p}\right)\frac{a}{1-aX}. \tag{A.2}$$

Comparing equations (A.1) and (A.2), one can see that $X^* > X_1$ if and only if

$$\left(1+\frac{\delta}{2}\right)\frac{1-aX_1}{1-aX_1-b\underline{Z}} > \frac{1-p+\delta\beta}{1+p}, \tag{A.3}$$

which is always the case, since the left side is greater than 1 and the right side is lower than 1, for all $p \in [1/2, 1)$. It is easy to verify that existence of the allocation (X^*, Z^*, K^*) with $X^* > 0$ implies existence of (X_1, Z_1, K_1). This concludes the proof of Part *(i)*.

Part (ii) was shown in the proof of Proposition 2.

Consider Part (iii). Noting that $Z^* = g\left(X^*\right)$, $Z_2 = g\left(X_2\right)$, and $\partial g/\partial X_2 < 0$, it follows that $Z^* \geq Z_2$ if and only if $X^* \leq X_2$. Next, note that $X_2 > 0$ if and only if $\phi/a \geq (1-\gamma/\beta)(1+\delta/2)$, and also that $X_2 > 0$ solves the equilibrium condition (1.19). In turn, this condition implies that an interior solution $X_2 > 0$ must solve

$$\frac{\phi}{1+\phi X} = \left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)\frac{a}{1-aX}. \tag{A.4}$$

Noting that $X^* > 0$ if and only if $\phi/a > \frac{1-p+\delta\beta}{1+p}$, and $X_2 > 0$ if and only if $\phi/a > (1-\gamma/\beta)(1+\delta/2)$, and comparing equations (A.4) and (A.2), one can see that, whenever $X^* > 0$, one has that $X^* \leq X_2$ if and only if $\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right) \leq \frac{1-p+\delta\beta}{1+p}$. It is easy to verify that this condition holds if and only $x_U \geq X^*$, as shown in the proof of Proposition 1. It is now easy to verify that existence of the equilibrium allocation (X^*, Z^*, K^*) with $X^* > 0$ implies existence of the constrained optimal allocation $\{X_2, Z_2, K_2\}$, whenever $X^* \geq X_2$. Instead, if $X^* < X_2$, note that the existence of the allocation $\{X_2, Z_2, K_2\}$ requires that $X_2 + Z_2 < 1$, where $Z_2 = g\left(X_2\right)$. It is easy to see that $X + g(X)$ is an increasing function of X. Moreover, since X_2 is increasing in γ whenever $X_2 > 0$, it is easy to verify that there exists a number $\gamma_{\max} \in (\gamma_U, \beta)$, where $\gamma_U \equiv \beta\left(1 - \frac{1-p+\delta\beta}{(1+p)(1+\delta/2)}\right)$, such that the allocation $\{X_2, Z_2, K_2\}$ exists if and only if $\gamma \in [0, \gamma_{\max})$. Finally, to prove that $Z_2 > \underline{Z}$, note the minimum level of Z_2, denoted by $Z_2^{\min}$, is given by $X_2 + Z_2 = 1$ since $X + g(X)$ is increasing on X and $g'(X) < 0$. Note that $Z_2^{\min} = \frac{(1-p)(1-a)}{b(1-p+\delta\beta)-a(1-p)} > \underline{Z}$ for all $\epsilon \in (0, 1/2]$ since $\phi/a > \frac{1-p+\delta\beta}{1+p}$ and $b < \frac{a+\phi}{1+\phi}$. This concludes the proof. QED

Proof of Proposition 4

One can verify that the optimality of crime choices implies that $z = G(x, P(Z))$, where $G(x, 1-p) = g(x)$, and g is given by equation (1.12). Suppose there is an equilibrium with positive crime. Paralleling the arguments in the proof of Proposition 2, equilibrium long-run utility can be written as $\ln[B_2(1 - P(Z))\Psi(X)]$, where $B_2(p) = B_1$, and where B_1 and Ψ are the same as in the proof of Proposition 2. Moreover, note that the facts that $P(0) \geq 0$, $\partial P/\partial Z > 0$, and $\partial^2 P/\partial Z^2 < 0$, imply that $(\partial P(Z)/\partial Z)\, Z \leq P(Z)$, for $Z \in [0, 1)$, which in turn implies that $dZ/dX < 0$, with $dZ/dX = \frac{\partial G(X,P(Z))/\partial X}{1-\partial G(X,P(Z))/\partial Z}$. The rest of the proof parallels the proof of Proposition 2, noting that, if $dZ/dX < 0$, then $\ln[B_2(1 - P(Z))\Psi(X)]$ is a strictly concave function of X with its maximum at $X = \widetilde{X}_2$, where $\widetilde{X}_2 > X_2$, since $\frac{dB_2(1-P(Z))}{dZ}\frac{dZ}{dX} > 0$. QED

Proof of Proposition 5

As noted in the proof of Proposition 2, U^* is given by $U^* = \ln[B_1\Psi(X^*)]$, where X^* is given in Proposition 1 and $B_1 > 0$ is some constant independent of ϕ. By substituting the expression for X^*, it is easy to verify that equilibrium long-run utility is given by

$$U^* = \ln\left[\widehat{B}_1\left(1 + \frac{a}{\phi}\right)^{\left(1-\frac{\gamma}{\beta}\right)(2+\delta)}\left(1 + \frac{\phi}{a}\right)^2\right],$$

where $\widehat{B}_1 > 0$ is some constant independent of ϕ. It is now straightforward to verify that $\frac{\partial U^*}{\partial \phi} > 0$ if and only if $\phi/a > (1 - \gamma/\beta)(1 + \delta/2)$, which is the same condition for $X_2 > 0$. This concludes the proof. QED

Proof of Proposition 6

Considering that schooling is costly modify only the expression for household's savings, leaving the rest of the original model unchanged. Given that the school fee is equal to a proportion $f \in (0,1)$ of the economy's average labor income Y_L, a household's savings out of labor income is now given by

$$s = p\left(\left(w + w_c x\right)h - c_a\right) + (1-p)\frac{z}{Z}\left(Y_L - C_a\right) - fY_L. \tag{A.5}$$

It is then a simple matter to characterize equilibrium dynamics. The Euler equation for optimal savings together with the fact that $C_o' = (1+r')\,S$, imply that $S = pC_a$. Thus, consumption by adults is a fraction $\frac{1-f}{1+p}$ of labor income $(1-\alpha)\,Y$. Accordingly, aggregate investment is given by

$$K' = \left(\frac{p}{1+p}\right)(1-f)\,(1-\alpha)\,Y. \tag{A.6}$$

Next, the total resources subject to appropriation are $(1-\alpha)\,Y - C_a = C_a\left(\frac{p+f}{1-f}\right)$. Then, it is easy to verify that the equality of marginal costs and benefits from child crime implies

$$Z = \frac{1 - aX}{b\left(1 + \frac{p\delta\beta(1-f)}{(1-p)(p+f)}\right)} \equiv g\left(X\right), \tag{A.7}$$

with $\partial g/\partial X < 0$. Now, the equality of marginal returns to child labor and child crime gives the second equilibrium relationship between child work and child crime:

$$Z = \left(\frac{a\,(1-p)\,(p+f)}{b\phi p\,(1+p)}\right)(1+\phi X) \equiv f\left(X\right), \tag{A.8}$$

where $\partial f/\partial X > 0$. Following the same steps as those in the proof of Proposition 1, one can verify that there exists a unique stable steady state equilibrium with

$$X^* = \frac{1 + p - \frac{a}{\phi}\left(1 - p + \delta\beta - \frac{f[p(1+\delta\beta)-1]}{p}\right)}{a\left(2 + \delta\beta - \frac{f[p(1+\delta\beta)-1]}{p}\right)} \text{ and } Z^* = \frac{(1-p)\left(1+\frac{a}{\phi}\right)\left(1+\frac{f}{p}\right)}{b\left(2 + \delta\beta - \frac{f[p(1+\delta\beta)-1]}{p}\right)}, \tag{A.9}$$

where $Z^* > \underline{Z}$. Note from (A.9) that $\frac{\partial Z^*}{\partial f} > 0$ for all $f \in (0,1)$, and $\frac{\partial X^*}{\partial f} < 0$ if and only if $p < \frac{1}{1+\delta\beta}$. Furthermore, $\frac{\partial(1-aX^*-bZ^*)}{\partial f} < 0$ for all $f \in [0,1)$. This proves the second part of the proposition.

Now, from (1.3), (A.6), and (A.9), it is easy to verify that equilibrium long-run income is given by

$$Y^* = B_2 \frac{\eta\varphi\,(1-f)^{\alpha/(1-\alpha)}}{(1+\eta+\varphi)^{2-\gamma/\beta}}, \tag{A.10}$$

where $B_2 > 0$ is some constant independent of f, $\eta \equiv \frac{(1-f)\delta\beta p}{(1-p)(p+f)}$, and $\varphi \equiv \frac{(1+p)p}{(1-p)(p+f)}$. It is tedious but straightforward to verify that a sufficient condition for $\frac{\partial Y^*}{\partial f} < 0$ is given by

$$\frac{(1-p)(p+f)+(1+p)\,p}{(1-f)\,p} + \frac{(1-p)(p+f)+(1-f)\,\delta\beta p}{(1+p)\,p} > \left(1-\frac{\gamma}{\beta}\right)(1+\delta\beta)\,,$$

which is always the case for all $p \in [1/2,1]$ and for all $f \in [0,1)$. Hence $\frac{\partial Y^*}{\partial f} < 0$ for all $f \in [0,1)$. Finally, one can verify that equilibrium long-run utility can be expressed as

$$U^* = \ln\left[\frac{\alpha\,(1-\alpha)^{1+\delta}}{1+p}\,(Y^*)^{2+\delta}\,\frac{(1-f)}{(1+\phi X^*)^{\delta}}\right].$$

Given that $\frac{\partial Y^*}{\partial f} < 0$, a sufficient condition for $\frac{\partial U^*}{\partial f} < 0$ is $\partial\left(\frac{(1-f)}{(1+\phi X^*)^{\delta}}\right)/\partial f < 0$. Note from (A.9) that $1+\phi X^* = \left(1+\frac{\phi}{a}\right)\left(1+\frac{1+\eta}{\varphi}\right)^{-1}$. where η and φ are as defined above. It is now easy to verify that $\partial\left(\frac{(1-f)}{(1+\phi X^*)^{\delta}}\right)/\partial f < 0$ if and only if

$$\frac{1-p\,(1+\delta\beta)}{(1+p)\,p} < \frac{1}{\delta\,(1-f)}\left(1+\frac{(1-p)(p+f)+(1-f)\,\delta\beta p}{(1+p)\,p}\right),$$

which is always the case since the left side is lower than 1 while the right side is greater than 1, for all $f \in [0,1)$. Hence $\frac{\partial U^*}{\partial f} < 0$ for all $f \in [0,1)$. This concludes the proof. QED

Appendix B

Proof of Proposition 7

The discussion leading to Proposition 7 implies that the taxes $\{\tau_s, \tau_o\}$ ensuring that equations (2.12) and (2.13) hold with $C_o/C_a = 1$ and $K/Y = \alpha\left(\frac{2+\delta}{2+\alpha\delta}\right)$ solve Problem (2.14), and they also implement the allocation $\left\{\widehat{X}, \widehat{Z}, \widehat{K}\right\}$ that solves Problem (2.15). It is easy to verify that there are only two such values of the tax rates $\widehat{\tau}_s$ and $\widehat{\tau}_o$, which are given in the proposition. One can then verify that the payroll tax rate $\widehat{\tau}_w$ that balances the government budget is the unique solution to equation (2.11) evaluated at $\tau_s = \widehat{\tau}_s$, $\tau_o = \widehat{\tau}_o$ and $\tau_x = \tau_e = 0$. QED

Proof of Proposition 8

To prove Part *(i)*, note that equation (2.10), evaluated at $Z = g(X)$ and $\tau_e = 0$, gives $X^*(\{\tau_s, \tau_o, \tau_w, \tau_x, 0\})$ as a decreasing function of τ_x alone:

$$X^*(\{\tau_s, \tau_o, \tau_w, \tau_x, 0\}) = \frac{1 + p - (1 - p + \delta\beta)\,\frac{a}{\phi}\left(\frac{1}{1-\tau_x}\right)}{a(2 + \delta\beta)}.$$

Using equation (1.23), the unique tax rate τ_{x2} that solves $X^*(\{\tau_s, \tau_o, \tau_w, \tau_{x2}, 0\}) = X_2$ is:

$$\tau_{x2} = \begin{cases} 1 - \frac{\frac{a}{\phi}(1-p+\delta\beta)\left[1+\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)\right]}{\left(1+p-\frac{a}{\phi}(2+\delta\beta)\right)\left(1-\frac{\gamma}{\beta}\right)\left(1+\frac{\delta}{2}\right)-(1-p+\delta\beta)} & \text{if} \quad \phi/a \geq \left(1 - \frac{\gamma}{\beta}\right)\left(1 + \frac{\delta}{2}\right) \\ 1 - \frac{a}{\phi}\left(\frac{1-p+\delta\beta}{1+p}\right) & \text{if} \quad \phi/a \leq \left(1 - \frac{\gamma}{\beta}\right)\left(1 + \frac{\delta}{2}\right). \end{cases}$$

From equation (2.9), $Z^*(\{\tau_s, \tau_o, \tau_w, \tau_{x2}, 0\}) = g(X_2) = Z_2$. These facts, together with the discussion leading to Proposition 8, imply that τ_{x2} and the tax rates $\{\widehat{\tau}_s, \widehat{\tau}_o\}$

given in Proposition 7, implement the unique allocation $\{X_2, Z_2, K_2\}$ that solves Problem (2.14) and Problem (2.17). Obviously, no policy within the class Ψ_2 can do any better. Finally, one can verify that the payroll tax rate τ_{w2} that balances the government budget is the unique solution to equation (2.11) evaluated at $\tau_s = \widehat{\tau}_s$, $\tau_o = \widehat{\tau}_o$, $\tau_x = \tau_{x2}$, $\tau_e = 0$ and $X = X_2$. Hence, the optimal policy within the class Ψ_2 exists and is unique. This proves Part *(i)*.

Consider Part *(ii)*. From Proposition 2, $X^*(\{\tau_s, \tau_o, \tau_w, 0, 0\}) < X_2$ if and only if $\gamma > \beta\left(1 - \frac{1-p+\delta\beta}{(1+p)(1+\delta/2)}\right)$. The above arguments imply that $X^*(\{\tau_s, \tau_o, \tau_w, \tau_x, 0\}) < X_2$ if and only if $\tau_x > \tau_{x2}$. Therefore, $\tau_{x2} < 0$ if and only if $\gamma > \beta\left(1 - \frac{1-p+\delta\beta}{(1+p)(1+\delta/2)}\right)$, since $X^*(\{\tau_s, \tau_o, \tau_w, \tau_x, 0\})$ decreases with τ_x. The rest of Part *(ii)* is proven in Part *(i)*. QED

Proof of Proposition 9

Consider Part (ii) first. One can verify that there is an allocation $\{X_3, Z_3, K_3\}$ that solves Problem (2.18), with $X_3 \geq 0$ and $Z_3 \geq \underline{Z}$. Writing the last constraint in Problem (2.18) as $m(X, Z) = 0$, we know that X_3 satisfies equation (1.21), with $dZ/dX = \frac{-\partial m/\partial X}{\partial m/\partial Z}$. One can verify that the problem is nicely behaved, so the solution X_3 is unique. One can also verify that our assumption that $b < \frac{a+\phi}{1+\phi}$ implies that $dZ/dX > 0$, for all $Z \in (0,1)$. Since $dZ/dX > 0$, the solution Z_3 is unique. Comparing the above equation with equation (1.19), it follows that $X_3 \leq X_1$, and moreover, $X_3 < X_1$ if and only if $X_1 > 0$, since $dZ/dX > 0$, for all $Z \in (0,1)$.

Since $m(X^*(\{\tau_s, \tau_o, \tau_w, 0, 0\}), Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})) = 0$, and $X^*(\{\tau_s, \tau_o, \tau_w, 0, 0\}) > X_1 \geq X_3$, then $Z_3 < Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$, where $X^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$ and $Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$ are the equilibrium levels given in Proposition 1. It remains to prove that $Z_3 > \underline{Z}$. Since $m(X, Z) = 0$ implies that $dZ/dX > 0$, for all $Z \in (0,1)$, we know that $Z_3 > Z'$, where Z' is the unique value that solves $m(0, Z') = 0$ in the interval $(0,1)$. Next, taking into account that $b < \frac{a+\phi}{1+\phi}$, and noting that total differentiation of $m(0, Z) = 0$ with respect to Z and b gives $\frac{dZ}{db} < 0$, for $Z \in (0,1)$, one can verify that $Z' > \frac{(1-p)(1+1/\phi)}{2+\delta\beta+(1-p)(1+a/\phi)}$. One can also verify that $\frac{(1-p)(1+1/\phi)}{2+\delta\beta+(1-p)(1+a/\phi)} > \underline{Z}$ for all $\epsilon \in (0, 1/2]$ and for all $p \in [1/2, 1)$.

This proves that $Z_3 > \underline{Z}$. This concludes the proof of Part *(ii)*.

Now consider Part *(i)*. From Part (ii), we know that there exists a unique allocation $\{X_3, Z_3, K_3\}$ that solves Problem (2.18), with $X_3 \geq 0$ and $Z_3 > \underline{Z}$. Obviously, an optimal policy within the class Ψ_3 cannot do better than to implement $\{X_3, Z_3, K_3\}$. Replicating our previous arguments, the only way to implement $\{X_3, Z_3, K_3\}$ is to choose the tax rates $\{\widehat{\tau}_s, \widehat{\tau}_o\}$ given in Proposition 7, together with

$$\tau_{e3} = \frac{-\left(\frac{1-p}{Z_3} - \frac{\delta\beta b}{1-aX_3-bZ_3}\right)}{1+p-X_3\left(\frac{1-p}{Z_3} - \frac{\delta\beta(b-a)}{1-aX_3-bZ_3}\right) - (1-X_3-Z_3)\left(\frac{1-p}{Z_3} - \frac{\delta\beta b}{1-aX_3-bZ_3}\right)}.$$

One can then verify that the payroll tax rate τ_{w3} that balances the government budget is the unique solution to equation (2.11) evaluated at $\tau_s = \widehat{\tau}_s$, $\tau_o = \widehat{\tau}_o$, $\tau_x = 0$, $\tau_e = \tau_{e3}$, $X = X_3$ and $Z = Z_3$. This concludes the proof of Part *(i)*.

Consider Part *(iii)*. The constraint $m(X, Z) = 0$ implies that the denominator in the above equation for τ_{e3} is positive. The numerator is negative because $X_3 < X^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$ and $Z_3 < Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$, and

$$\frac{1-p}{Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})} - \frac{\delta\beta b}{1 - aX^*(\{\tau_s, \tau_o, \tau_w, 0, 0\}) - bZ^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})} = 0.$$

The rest of Part *(iii)* is shown in Part *(i)*.

To prove Part *(iv)*, one can verify that $U^*(\{\tau_{s3}, \tau_{o3}, \tau_{w3}, 0, \tau_{e3}\}) > U^*(\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\})$ when $\gamma = 0$. Next, using the envelope theorem one can write

$$\frac{\partial}{\partial\gamma}\left(U^*(\{\tau_{s3}, \tau_{o3}, \tau_{w3}, 0, \tau_{e3}\}) - U^*(\{\tau_{s2}, \tau_{o2}, \tau_{w2}, \tau_{x2}, 0\})\right) = -\left(\frac{2+\delta}{2}\right)\left(\ln\left(\frac{Z_3}{Z_2}\right)\right),$$

where recall that Z_3 is independent of γ, whereas Z_2 is a decreasing function of γ. Part *(iv)* now follows from the fact that $Z_3 < Z_2$ for all $\gamma \leq \beta\left(1 - \frac{1-p+\delta\beta}{\left(1+\frac{\delta}{2}\right)(1+p)}\right)$. In turn, this follows from the facts that Z_3 is independent of γ, $Z_3 < Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$, Z_2 is a decreasing function of γ, and $Z_2 = Z^*(\{\tau_s, \tau_o, \tau_w, 0, 0\})$ when $\gamma = \beta\left(1 - \frac{1-p+\delta\beta}{\left(1+\frac{\delta}{2}\right)(1+p)}\right)$.
QED

Proof of Proposition 10

Consider Part *(i)*. Equations (2.9) and (2.10) can be solved for a unique pair of tax rates

$$\tau_e = \frac{-\left(\frac{1-p}{Z} - \frac{\delta\beta b}{1-aX-bZ}\right)}{1+p-X\left(\frac{1-p}{Z} - \frac{\delta\beta(b-a)}{1-aX-bZ}\right) - (1-X-Z)\left(\frac{1-p}{Z} - \frac{\delta\beta b}{1-aX-bZ}\right)}, \tag{B.1}$$

$$1-\tau_x = \frac{\frac{1}{\phi}\left(\frac{1-p}{Z} - \frac{\delta\beta(b-a)}{1-aX-bZ}\right)}{1+p-X\left(\frac{1-p}{Z} - \frac{\delta\beta(b-a)}{1-aX-bZ}\right) - (1-X-Z)\left(\frac{1-p}{Z} - \frac{\delta\beta b}{1-aX-bZ}\right)}, \tag{B.2}$$

as a function of X and Z alone, assuming that equation (2.10) holds with equality. Let τ_{e1} and τ_{x1} be given by the above two equations, evaluated at $X = X_1$ and $Z = Z_1$. Clearly, an optimal policy must give $wH/C_a > 0$, and it can be verified that $wH/C_a > 0$ implies that the denominator in the above equations must be positive. This, together with our previous arguments, implies that $\{\tau_{s1}, \tau_{o1}, \tau_{x1}, \tau_{e1}\}$ with $\{\tau_{s1}, \tau_{o1}\} = \{\widehat{\tau}_s, \widehat{\tau}_o\}$, implements the allocation $\{X_1, Z_1, K_1\}$ that solves Problem (2.14) and Problem (2.19), provided that

$$1+p-X_1\left(\frac{1-p}{\underline{Z}} - \frac{\delta\beta\left(b-a\right)}{1-aX_1-b\underline{Z}}\right) - \left(1-X_1-\underline{Z}\right)\left(\frac{1-p}{\underline{Z}} - \frac{\delta\beta b}{1-aX_1-b\underline{Z}}\right) > 0.$$

The left side of the inequality approaches $-\infty$ as $\underline{Z}$ approaches zero, and it is increasing in $\underline{Z}$. Hence, the above inequality is satisfied if and only if $Z > \widetilde{Z}$, where $\widetilde{Z}$ is the unique value that equates the left side of the above inequality to zero, and where X_1 is given by equation (1.22). To prove that $\widetilde{Z} \in (0, Z_3)$, recall that $\underline{Z} \leq (1-p)\,\epsilon$, with $p \in [1/2, 1)$ and $\epsilon \in (0, 1/2)$. Replacing $\underline{Z}$ with $(1-p)\,\epsilon$ in the above inequality, one can show that $Z > \widetilde{Z}$ if and only if $\epsilon > 1/\left(2+n\right)$, where n is a positive number, and $\widetilde{Z} = (1-p)\,/\left(2+n\right) > 0$. That $\widetilde{Z} < Z_3$ follows from the fact that $Z_3 > (1-p)\,\epsilon$, for $\epsilon \in (0, 1/2)$.

Finally, the optimal payroll tax rate τ_{w1} is the unique solution to equation (2.11) evaluated at $\tau_s = \widehat{\tau}_s$, $\tau_o = \widehat{\tau}_o$, $\tau_x = \tau_{x1}$, $\tau_e = \tau_{x1}$, $X = X_1$, and $Z = Z_1$. This proves Part *(i)*.

Consider Part *(ii)*. To prove that $\tau_{e1} < 0$ note that the numerator of the right side of equation (B.1) is negative when $(X, Z) = (X_1, Z_1)$, because the numerator is equal to zero when $(X, Z) = (X^* (\{\tau_s, \tau_o, \tau_w, 0, 0\}), Z^* (\{\tau_s, \tau_o, \tau_w, 0, 0\}))$, and $X_1 < X^* (\{\tau_s, \tau_o, \tau_w, 0, 0\})$ and $Z_1 < Z^* (\{\tau_s, \tau_o, \tau_w, 0, 0\})$. That $\tau_{e1} < 0$ now follows from the fact that the denominator of the right side of equation (B.1) is positive, as shown above. To prove that $\tau_{x_1} < 0$, totally differentiate equation (B.2) to get $d\tau_x = A_0 dZ + A_1 dX$, where $A_0 > 0$, and $A_1 < 0$ since $b < \frac{a+\phi}{1+\phi}$. The fact that $\tau_{x_1} < 0$ follows from noting that the right side of equation (B.2) is equal to one, when evaluated at $(X, Z) = (X_3, Z_3)$, and noting that $X_3 \leq X_1$, and $Z_3 > Z_1$. The rest of Part *(ii)* is proven in Part *(i)*. QED

Appendix C

Proof of Proposition 11

Consider Part (i). One can verify that, every time t, utility for individual i is equal to $\ln\left[B_1 c_{ai}^3 \left(h_i'\right)^\delta / h_i\right]$, where $B_1 > 0$ is some constant. Then, a household will prefer child crime over full-time child labor if and only if $\left.\frac{c_{ai}^3\left(h_i'\right)^\delta}{h_i}\right|_{z_i=1} \geq \left.\frac{c_{ai}^3\left(h_i'\right)^\delta}{h_i}\right|_{x_i=1}$. Evaluating equations (3.3) and (3.20) at $z_i = 1$ and $x_i = 1$, respectively, it is easy to verify that $z_i = 1$ if and only if equation (3.28) holds, which implies $z_i = 1$ if and only if $h_i \leq \underline{h}_c$ where $\underline{h}_c$ is the solution to equation (3.29). Since $\underline{h}_c G\left(\underline{h}_c\right)$ is strictly increasing in $\underline{h}_c$ for all $\underline{h}_c \geq 0$, with $\underline{h}_c G\left(\underline{h}_c\right) = 0$ when $\underline{h}_c = 0$ and $\underline{h}_c G\left(\underline{h}_c\right) \to 0$ when $\underline{h}_c \to 0$, there exists a unique solution to equation (3.29).

Consider Part (ii). From the analysis leading to Proposition 11, we have that $x_i = 1$ if and only if $\underline{h}_c < h_i \leq h_L$ where h_L is given by equation (3.25). Next, one must ensure that $\underline{h}_c < h_L$. From equations (3.25) and (3.29) one can verify that $\underline{h}_c \leq h_L$ if and only if $p \geq 1 - \Psi_1 \equiv p_L$, where Ψ_1 is some positive number.

Parts (iii) and (iv) follow from the analysis leading to Proposition 11. This concludes the proof. QED

Proof of Proposition 12

Consider Part (i). After the imposition of the full ban on child labor, as noted in the proof of Proposition 11, a household will prefer child crime over full-time schooling if and only if $\left.\frac{c_{ai}^3\left(h_i'\right)^\delta}{h_i}\right|_{z_i=1} \geq \left.\frac{c_{ai}^3\left(h_i'\right)^\delta}{h_i}\right|_{e_i=1}$. Evaluating equations (3.3) and (3.20) at $z_i = 1$ and $e_i = 1$, one can verify that $z_i = 1$ if and only if $h_i \leq \underline{h}_c'$ where $\underline{h}_c'$ is the unique solution to the following expression

$$\underline{h}_c' G\left(\underline{h}_c'\right) = \frac{\left(1-\varepsilon\right)^{\delta/3}\left(1-p\right)H\left(1+\frac{b_H G\left(\underline{h}_c'\right)}{(1-p)wH}\left(\frac{1+\theta b_H^{1-\beta}}{1-\varepsilon}\right)^{\delta/3}\right)}{p\left(1+\frac{1}{R}\right)\left[\left(1+\theta b_H^{1-\beta}\right)^{\delta/3}-\left(1-\varepsilon\right)^{\delta/3}\right]}, \tag{C.1}$$

where one can easily verify that $\underline{h}_c' \in (\underline{h}_c, h_H)$. Next, a permanent ban increases the proportion of child crime in the long run only if $\underline{h}_c' > h_L$. Comparing equations (3.29) and (C.1), one can verify that $\underline{h}_c' > h_L$ if and only if $p < p_H \equiv 1 - \Psi_2 + \varphi > 0$, with $p_H > p_L$ because $\Psi_2 < \Psi_1$ whenever $\varepsilon \in [0, \overline{\varepsilon}]$, and φ is some positive number, with $\overline{\varepsilon} \equiv 1 - \frac{\delta\beta(1+\phi+1/R)-\delta}{(2+\phi/(1+1/R))^{3/\delta}[\delta\beta(1+\phi+1/R)-(3+\delta)\phi]} \in (0,1)$. Noticing that the function h_i' is continuous and increasing in θ for all $\theta > 0$, with $h_i'/h_i = 1$ when $\theta = 0$ and $h_i'/h_i \to \infty$ as $\theta \to \infty$, the rest of Part (i) follows from Panel (a) of Figure 3.4.

Parts (ii) to (iv) follow from the fact that $\underline{h}_c' > h_L$ if and only if $p < p_H$, and from the analysis of the Panel (b) of Figure 3.4 in the text. This concludes the proof. QED

Proof of Proposition 13

Consider Part (i). From equation (3.26), the proportion of children attending school full time increases since $\partial h_H/\partial\theta < 0$. From equations (3.25) and (3.29), the proportion of children working full time decreases since $\partial h_L/\partial\theta < 0$ and $d\underline{h}_c/d\theta = 0$. In the long run, both the time it takes to eliminate child labor from the economy and the level of child crime decreases since $\partial h_L/\partial\theta < 0$.

Consider Part (ii). The proportion of children attending school full time increases since $\partial h_H/\partial p < 0$. The proportion of children out from school decreases since $\partial h_L/\partial p < 0$. The level of child crime in the short run decreases since $d\underline{h}_c/dp < 0$, and also in the long run since $\partial h_L/\partial p < 0$. This concludes the proof. QED

Proof of Proposition 14

At the time of the temporary ban, one can verify that $z_i = 1$ if and only if $h_i \leq \widehat{\underline{h}}_c'$ where $\widehat{\underline{h}}_c'$ is given by the unique solution to the following expression

$$\widehat{\underline{h}}_c' G\left(\widehat{\underline{h}}_c'\right) = \frac{(1-\varepsilon)^{\delta/2}(1-p)H\left(1+\frac{b_H G(\widehat{\underline{h}}_c')}{(1-p)wH}\left(\frac{1+\theta b_H^{1-\beta}}{1-\varepsilon}\right)^{\delta/2}\right)}{p\left[\left(1+\theta b_H^{1-\beta}\right)^{\delta/2}-(1-\varepsilon)^{\delta/2}\right]}, \tag{C.2}$$

with $\widehat{\underline{h}}_c' \in (\widehat{\underline{h}}_c, \widehat{h}_H)$. From equations (3.61) and (C.2) one can verify that there exists a level of effective property rights $\widehat{p}_H > 0$ such that $\widehat{\underline{h}}_c' > \widehat{h}_L$ if and only if $p < \widehat{p}_H$, with $\widehat{p}_H > \widehat{p}_L$. Now, following the same arguments presented in the proof of Proposition 12, there exists a level $\widehat{\theta}_L$ such that a temporary ban increases the level of child crime in the long run if and only if $p < \widehat{p}_H$ and $\theta < \widehat{\theta}_L$. This concludes the first part of the proof.

Next, we prove that $\frac{K'}{H'} > \frac{\widehat{K}'}{\widehat{H}'}$, where $\frac{K'}{H'}$ is the physical capital-human capital ratio next period if no ban were implemented. Recalling that $h = \int_0^1 h_i d_i$, from equation (3.65) one can verify that $\frac{K'}{H'} > \frac{\widehat{K}'}{\widehat{H}'}$ if and only if

$$\frac{(1-\alpha)AK^{\alpha}H^{1-\alpha} - \int_{h_L}^{\infty} b_i di}{(1-\alpha)AK^{\alpha}h^{1-\alpha} - \int_{\widehat{\underline{h}}_c'}^{\infty} b_i di} > \frac{h' + \phi\left[G(h_L) - G(\underline{h}_c')\right] + \phi\int_{h_L}^{h_H} x_i' di}{\widehat{h}' + \phi\left[G(h_L) - G(\widehat{\underline{h}}_c')\right] + \phi\int_{h_L}^{h_H} \widehat{x}_i' di}, \tag{C.3}$$

where $h' = h + \theta\left[\int_{h_L}^{h_H} e_i^{\beta} b_i^{1-\beta} h_i di + \int_{h_H}^{\infty} b_i^{1-\beta} h_i di\right] - \varepsilon G(\underline{h}_c)$ and $\widehat{h}' = h + \theta\int_{\widehat{\underline{h}}_c'}^{\infty} b_i^{1-\beta} h_i di - \varepsilon G(\widehat{\underline{h}}_c)$ with $\widehat{h}' > h'$. By substituting these two last expressions into equation (C.3) one can verify that the inequality always hold whenever $\varepsilon < \overline{\varepsilon}$, $p < \widehat{p}_H$, and $\theta < \widehat{\theta}_L$, where $\overline{\varepsilon}$ is the constant given in the proof of Proposition 12. Finally, note that if $\frac{K'}{H'} > \frac{\widehat{K}'}{\widehat{H}'}$ at some time t, then $\frac{K'}{H'} > \frac{\widehat{K}'}{\widehat{H}'}$ for all $t + a$ for any $a \geq 1$, with $\lim_{t\to\infty} \frac{K_t}{H_t} = \lim_{t\to\infty} \frac{\widehat{K}_t}{\widehat{H}_t}$. That is, the difference decreases over time. This concludes the proof. QED