Joint dynamics identification in bolted lap joints using new analytical and experimental methods

Sanati, Mehdi

doctoral thesis

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Joint Dynamics Identification in Bolted Lap Joints Using New Analytical and Experimental Methods

by

Mehdi Sanati

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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Abstract

The fierce competition in the machine tools market provokes a demand for the fast, accurate, and reliable production of machined components. This can be achieved using virtual prototyping technology which provides a computer model of the machine tool. To build an accurate virtual model of the machine tool, dynamic properties of all parts of the structure, including joints, should be identified. Mechanical joints have considerable effects on the dynamics of machine tools and ignoring them results in deviations between the virtual model and the physical structure analog.

This study proposes new analytical and experimental methods based on modal analysis and sensor development for joint dynamics identification in bolted lap joints. An analytical joint identification (AJI) technique, which employs the modal parameters of the structure, is developed using Euler-Bernoulli theory for determining the joint properties in transverse direction. The obtained results are compared with those of the inverse receptance coupling (IRC) approach. Numerical and experimental investigations verify the accuracy of the proposed methods.

A new experimental approach is proposed to determine the damping of bolted lap joints in translational and torsional directions. Because of the complex nature of the joint, this element is isolated through the addition of a mechanical resonator to the bolted structure and the frequency response function (FRF) of the combined system is used for joint damping identification. This approach overcomes difficulties associated with microslip analysis of bolted lap joints. The method is verified after comparing the results with those of the hysteresis loop approach.

In addition, a novel nanocomposite-based sensor is developed for strain and force measurements that can be used for identifying joint dynamics. Exhibiting both piezoelectric and piezoresistive properties, the developed sensors are capable of measurements over a wide
frequency range. To improve the accuracy and frequency bandwidth of the sensor, the piezoresistive and piezoelectric signals are fused. A 3D random walk model and a 2D finite element model are used to elucidate the piezoresistive and piezoelectric behaviour of the sensor, respectively. The experimental results show that the developed sensor is capable of measuring both static and high frequency loads through a fused piezoelectric/piezoresistive output, which is a unique feature of the sensor.

The proposed nanocomposite sensor is implemented in the joint interface and used for contact force measurement and joint dynamics identification through two different experimental approaches. The first approach uses the contact force and displacement data to build the hysteresis loop and subsequently extract the joint parameters. The second method employs an efficient calculation algorithm to identify the stick-slip transition in the joint interface and then obtain the energy loss and joint properties. The obtained results from both approaches are compared to each other to investigate the accuracy of these methods in determining the joint parameters.

The results of the joint dynamics identification techniques can be employed in building a database for bolted lap joints. This database can then be used in the design process in order to increase the correlation between the virtual models and actual structures.
Acknowledgements

First and foremost, I would like to extend my gratitude to my supervisor, Dr. Simon Park, who has shown me endless academic support and guidance all throughout my Ph.D. program. His ingenious feedback and continuous motivation are what made this dissertation possible.

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My deepest gratitude goes to my dear parents as well as the rest of my family, especially Hadi, Zahra, Hossein, and Diana, for their unconditional love and support.
Dedication

I dedicate this thesis to my beloved parents.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$f$</td>
<td>Force (N)</td>
</tr>
<tr>
<td>$M$</td>
<td>Moment (N.m)</td>
</tr>
<tr>
<td>$x$</td>
<td>Displacement (m)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Rotation (rad)</td>
</tr>
<tr>
<td>$H$</td>
<td>FRF of the substructures (m/N)</td>
</tr>
<tr>
<td>$K$</td>
<td>Stiffness (N/m)</td>
</tr>
<tr>
<td>$C$</td>
<td>Damping (N.s/m)</td>
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<td>$j$</td>
<td>Imaginary unit</td>
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<td>$H_J$</td>
<td>Joint FRF (m/N)</td>
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<tr>
<td>$H_{tt}$</td>
<td>Translational FRF (m/N)</td>
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<tr>
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<td>Rotational FRF (rad/N/m)</td>
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<tr>
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<td>Cross FRF (rad/N)</td>
</tr>
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<td>$Re$</td>
<td>Real part</td>
</tr>
<tr>
<td>$Im$</td>
<td>Imaginary part</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency (rad/s)</td>
</tr>
<tr>
<td>$G$</td>
<td>FRF of the assembled structure (m/N)</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity (Pa)</td>
</tr>
<tr>
<td>$I$</td>
<td>Second moment of area (m$^4$)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass per length (kg/l)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
<td>-------------</td>
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<tr>
<td>$W$</td>
<td>Transverse displacement of the beam (m)</td>
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<td>Natural frequency (rad/s)</td>
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<tr>
<td>$A_\beta$</td>
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<tr>
<td>$dq$</td>
<td>Charge density (C)</td>
</tr>
<tr>
<td>$dF$</td>
<td>Applied mechanical force (N)</td>
</tr>
<tr>
<td>$d_{33}$</td>
<td>Piezoelectric coefficient (pC/N)</td>
</tr>
<tr>
<td>$L_M$</td>
<td>Mean length of all CNTs (m)</td>
</tr>
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</table>
$L_{CNT}$ Length of each CNT inside the RVE (m)

$rand$ Random numbers between -1 and 1

$CNT_{num}$ Number of the CNTs

$Vol$ CNT volume percentage

$L_{RVE}$ Length of the RVE cube (m)

$D$ Diameter of the CNTs (m)

$\varphi$ Azimuthal angle (rad)

$\theta$ Polar angle (rad)

$R_{CNT}$ Contact resistance of CNTs (ohms)

$\sigma_{CNT}$ Electrical conductivity of CNTs (S/m)

$R_{tun}$ Tunneling resistance (ohms)

$h$ Plank’s constant

$e$ Electron charge (C)

$M$ Number of conduction channels

$\tau_p$ Transmission probability of an electron to tunnel between CNTs

$G$ Conductance matrix

$n$ Number of nodes

$P_{ij}$ Probability

$V$ Voltage (V)

$I$ Current (A)

$R$ Resistance (ohms)

$K_{uu}$ Elastic stiffness matrix

$K_{\varphi\varphi}$ Dielectric stiffness matrix
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{u\phi}$</td>
<td>Piezoelectric coupling matrix</td>
</tr>
<tr>
<td>$U$</td>
<td>Nodal displacements (m)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Nodal electrical potential</td>
</tr>
<tr>
<td>$F$</td>
<td>Nodal mechanical force (N)</td>
</tr>
<tr>
<td>$Q$</td>
<td>Nodal electrical charge (C)</td>
</tr>
<tr>
<td>$K_{\text{eff}}$</td>
<td>Effective stiffness matrix</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Stiffness matrix of the polymer</td>
</tr>
<tr>
<td>$K_{nt}$</td>
<td>Sum of all the stiffness matrices of all the nanotubes existing in the element</td>
</tr>
<tr>
<td>$S$</td>
<td>Strain</td>
</tr>
<tr>
<td>$J_a$</td>
<td>Jacobian</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Shape factor</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Fibre’s Young’s modulus (Pa)</td>
</tr>
<tr>
<td>$A$</td>
<td>Fibre’s cross-sectional area (m$^2$)</td>
</tr>
<tr>
<td>$l$</td>
<td>Fibre’s length (m)</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Young’s modulus of the polymer matrix (Pa)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of the nanotube segment</td>
</tr>
<tr>
<td>$\varepsilon_{nt}$</td>
<td>Dielectric constant of CNTs</td>
</tr>
<tr>
<td>$S$</td>
<td>Sensor signal</td>
</tr>
<tr>
<td>$P$</td>
<td>Covariance of the signal</td>
</tr>
<tr>
<td>$C$</td>
<td>Compensation coefficients</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Applied stress (Pa)</td>
</tr>
<tr>
<td>$p$</td>
<td>Contact stress (Pa)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Friction coefficient</td>
</tr>
</tbody>
</table>
\( \sigma \)  
Contact stress (Pa)

\( a \)  
Boundary radius between the contact and non-contact areas (m)

\( c \)  
Boundary radius between the sticking and sliding zones (m)

\( K_t \)  
Stiffness of the shear layer (N/m)

\( N \)  
Normal load (N)

\( r \)  
Radius (m)

\( f_{max} \)  
Maximum force (N)

\( U_{max} \)  
Maximum displacement (m)

\( K_j \)  
Joint stiffness (N/m)

\( C_j \)  
Joint damping (N.s/m)

\( \rho \)  
Density of the beam (kg/m\(^3\))

\( P \)  
Normal force (N)
Chapter One: INTRODUCTION

Machining processes are widely used in the industry for producing accurate mechanical components. Cutting operations, including milling, turning, boring, drilling, and grinding are among the most common processes employed for material removal in the manufacturing industry. High performance machining methods are needed to improve the productivity of the machining processes and stay competitive in a global market. Mechanical vibrations are consistently recognized as primary factor that hinder the accuracy and productivity of machining processes.

Despite all the advances in machining technologies, metal cutting processes still suffer from unwanted vibrations. Three different types of mechanical vibrations known as free vibrations, forced vibrations, and self-excited vibrations can be detected in metal cutting processes. The lack of dynamic stiffness in the whole system consisting of the machine tool, tool holder, cutting tool and workpiece can be considered the main source of these vibrations (Quintana and Ciurana 2011). In a cutting process, the free vibration problem can occur for example as a result of an incorrect tool path definition, which in turn causes a collision between the cutting tool and the workpiece. Unbalanced bearings or cutting tools, along with environmental vibrations can be considered as the main sources of forced vibration in the metal cutting process. In addition, forced vibration in the milling process is mainly caused when the cutting edge enters and exits the workpiece. The free and forced vibrations can be suppressed if the sources of vibrations are identified. However, self-excited vibrations, also known as regenerative chatter phenomenon, is one of the most challenging and least controllable types of vibrations in machining processes as it causes instability in the system and limits productivity. For this reason, it is of great importance to eliminate chatter
phenomenon and it has been an important topic in both academia and industry. Figure 1.1 shows the schematic of the chip thickness and wave generation when there is a chatter problem during machining operation.

![Figure 1.1. Regenerative chip thickness (Graham 2013)](image)

Chatter phenomenon causes different serious problems in the machining process such as poor surface quality, excessive noise, accelerated tool wear, machine tool damage, reduced material removal rate (MRR), and waste of energy and material (Quintana and Ciurana 2011). Figure 1.2 shows the effect of chatter on surface quality in boring operation.

![Figure 1.2. The effect of chatter on the quality of the surface in boring operation (Chen 2014)](image)
Chatter is a complex phenomenon since the dynamic of the system is a function of several parameters such as the dynamic of the tool and tool holder, the workpiece material, the cutting conditions, etc. Chatter vibrations occur near the natural frequencies of the tooltip. To eliminate the chatter problem, the optimal operating conditions for a given part and process are found through examining the physical prototype. However, this is a costly and time-consuming process; therefore, virtual prototyping technology has been recently employed to reduce the cost and time associated with physical prototype testing (Altintas et al. 2005).

Virtual prototyping is a computer simulation model of a machine tool, which accurately represents the real machine. This technique overcomes the problems associated with the traditional design and manufacturing methods (Altintas et al. 2005). A comparison between the conventional design process and the design process with virtual models is shown in Figure 1.3.

Figure 1.3. Comparing the conventional design and manufacturing processes with the design process performed by virtual prototyping technology (Altintas et al. 2005)
The models built through virtual prototyping are usually limited as, in reality, machine tools are complex structures consisting of different parts connected to each other through mechanical joints. To overcome this limitation and improve the accuracy of the created virtual models of the machine tools, the dynamics of different elements of the machine tools, especially mechanical joints, are required to be identified precisely and inserted into the virtual models.

1.1 Motivations

A fast, accurate, and reliable manufacturing process is required to produce accurate parts in the shortest time and most effective way, and therefore stay competitive in the machine tools market. This can be achieved through virtual prototyping technology and developing an accurate simulation model of the machine tool system. This should be done prior to building the actual structure to predict the dynamics of the system and obtain the optimal cutting conditions.

In order to have an accurate virtual model of the machine tool, a knowledge of the dynamic properties of all elements of the structure is required to achieve a high-fidelity model. Machine tools are complex structures consisting of several parts, including plates, bars, beams, etc. connected to each other through different types of joints. Mechanical joints play an important role as they influence the flexibility of the system and change its dynamic behaviour. They cause discontinuity in the structure and result in a high stress concentration in the connecting areas. Also, the increased contact surface between different components caused by mechanical joints results in more damping in the system. It has been demonstrated that about 60% of the total dynamic stiffness and 90% of the total damping in a machine tool structure are caused by the mechanical joints (Zhang et al. 2003). Similarly, (Beards 1982) performed a series of experiments and demonstrated that up to 90 percent of the total damping in the assembled structure is caused by the joints.
Ignoring the joints and modeling them as rigid connections results in deviations between the virtual model and the physical structure analog. In order to acquire a more accurate and reliable virtual model of the machine tool, the joint dynamic characteristics need to be accurately identified.

Different studies have been conducted to identify the joint dynamic properties through finite element (FE) model updating techniques (Friswell, Inman, and Pilkey 1998; Friswell, Mottershead, and Ahmadian 2001; Kim, Wu, and Eman 1989; Mottershead and Friswell 1993; Yuan and Wu 1985). However, these methods require accurate modal parameters, which are not easy to obtain especially in cases of heavily damped and closely coupled modes (Ibrahim and Pettit 2005). These methods, besides other sensitivity-based approaches, are highly sensitive to modal data, particularly mode shapes. Even small deviations in the measured modal data can result in high errors in the identified properties.

Difficulties associated with FE modeling and updating techniques may be overcome using response-based methods such as the inverse receptance coupling (IRC) method (Mehrpouya, Graham, and Park 2013, 2015; Mehrpouya, Sanati, and Park 2016; Park and Chae 2007; Schmitz and Donalson 2000; Schmitz and Duncan 2005). These methods require the complete frequency response function (FRF) matrices of the assembled structure and substructures, including the translational and rotational FRFs at different locations. In practice, translational FRFs can be readily measured through experimental modal analysis (EMA), but rotational FRF extraction is a physical challenge. Indirect techniques used for measuring the rotational FRFs always include errors. In addition, frequency-based methods such as IRC approach are linear techniques that eliminate the nonlinearities of the joint properties while mechanical joints have a nonlinear dynamic behaviour. When designing an active control system, it is important to understand how
the nonlinear parameters of the system change with amplitude and frequency (Ibrahim and Pettit 2005).

Considerable attempts have been made to characterize the nonlinear behaviour of the assembled structure containing mechanical joints (Cigeroglu, Lu, and Menq 2006; Gaul and Lenz 1997; Iwan 1966; Menq 1989; Ouyang, Oldfield, and Mottershead 2006; Segalman 2005; Shamoto et al. 2014). Most of the analytical and numerical approaches developed for nonlinear joint dynamic properties usually suffer from errors associated with the modeling or need experimental data for updating and calibration (Segalman 2006). Moreover, calculation time associated with these analytical methods is always considered a problem (Gregory and Martinez 2001).

This study presents new techniques for identifying the joint properties in bolted lap joints. These approaches concentrate on overcoming the limitations of the existing methods, including the difficulties in rotational and cross FRFs extraction, and FE modeling errors. In addition, the effects of nonlinearities in the contact interface are taken into consideration through developing a new type of sensor and an experimental calculation algorithm.

1.2 Objectives

The overall goal of the current study is improving the accuracy of the developed virtual prototype models of machine tools through joint dynamics identification. Hence, different methodologies are proposed, and specific aims are set to extract the joint parameters. These methodologies are applicable on structures consisting of components connected to each other through bolted lap joints. In addition, they are designed to avoid the limitations and difficulties associated with existing methods such as the need for rotational FRFs extraction and microslip boundaries identification.
Predicted joint properties can be employed in building an accurate virtual model of a machine tool. Also, the obtained results can be saved in a database and employed later in the analysis of subsequent structures that use a similar joint in their configuration. The methodologies and aims set in this study to achieve the mentioned objective, i.e. joint dynamics identification, are discussed below.

1.2.1 Identifying the joint properties in bolted lap joints

The first aim of this study is to identify the joint dynamics in bolted lap joints in transverse, tangential and torsional directions as illustrated in Figure 1.4. For the transverse direction, two different methods, i.e. the inverse receptance coupling (IRC) method and a new analytical joint identification (AJI) approach are proposed. Finite element (FE) simulation and experiments are used to investigate the effectiveness of the proposed methods. The IRC method is based on the difference between the FRF matrix of the assembled structure and the FRF matrix of each substructure. The AJI method is an analytical approach developed using the Euler-Bernoulli theory which uses the modal parameters of the whole system to extract the joint properties. This method can result in the joint properties without any need to the rotational FRFs extraction, which is always a challenge. The effect of joint conditions, such as the surface roughness and inserting an interfacial material, on its properties is investigated through both presented methods.

Figure 1.4. Schematic of an assembled structure showing the directions of the joint properties
A new experimental approach is also developed for joint damping identification in tangential and torsional directions. In this method, the lap joint is isolated through adding a mechanical resonator to the bolted structure. The FRF of this system can be easily extracted through experiments and used for joint damping identification. This method overcomes the challenges associated with analyzing the microslip motion. In addition, the hysteresis loop approach is presented to verify the accuracy of the developed approach.

1.2.2 Development of a new nanocomposite sensor with piezoelectric and piezoresistive properties

In real applications, it is more desirable to have a sensor in the joint location and identify the dynamics of the system under different conditions. Using commercial sensors for measuring the contact force and joint properties identification may affect the dynamic behaviour of the joint because of the size of these sensors. Therefore, it is necessary to have a new sensor which is thin and can be embedded in the lap joints without changing the dynamics of the structure. Hence, Aim 2 of this study is development of a novel nanocomposite sensor which can be installed in lap joints to measure the joint dynamics. The developed sensor is designed to provide high frequency bandwidth by combining the piezoelectric and piezoresistive properties. Numerical models are developed to understand the properties of the developed sensor and predict its piezoelectric and piezoresistive properties. The effectiveness of the sensor is verified after performing a set of static and dynamic experiments. Ideally, this sensor can be used for monitoring the condition of the machine tool as well as chatter detection. For example, it can be wrapped around the boring bar at the fixed end to detect chatter by monitoring the force applied to the system during machining.
1.2.3 Joint dynamics identification in bolted lap joints using the developed nanocomposite sensor

The final aim of this study is to establish experimental methods for joint dynamics identification using the developed nanocomposite sensor. The developed sensor is thin and flexible and can be mounted on any surface, and therefore, it does not have considerable effect on the dynamics of the system after inserting into the joint interface. Aim 3 addresses the determination of the stick-slip transition in the contact zone and identifying the microslip motion. A new calculation algorithm is proposed beside the hysteresis loop method to predict the energy loss and joint dynamic properties using the developed nanocomposite sensor. The established approaches use the contact force and the response of the substructures measured experimentally by the developed sensor and displacement sensors, respectively. In addition, the developed methods are employed to investigate the effect of different parameters, including normal and excitation loads, on the joint dynamic properties.

1.3 Organization of the Thesis

This study is organized as follows: Chapter 2 mainly provides an overview on the existing literatures in joint dynamics identification and nanocomposite sensor development. This chapter first covers the virtual prototyping technology with an emphasis on its applications in machining industry. Then, the importance of joint dynamics identification will be discussed, and the applications of mechanical joints in the industry will be elaborated. Different techniques used for determining the joint properties will be explained and the advantages and disadvantages of each technique will be discussed. In addition, this chapter provides a review on nanocomposite sensors, including the properties of base polymers, carbon nanotubes (CNTs) properties, different fabrication techniques, different methods used for improving the piezoelectricity of the
Polyvinylidene fluoride (PVDF) polymer, and different numerical models developed for the piezoelectric and piezoresistive properties of nanocomposites.

Chapter 3 first presents different techniques for joint dynamics identification in lap joints in transverse direction. The inverse receptance coupling (IRC) method is introduced and used for determining the joint stiffness and damping. In addition, Euler-Bernoulli theory is used to develop a new analytical joint identification (AJI) approach. The effectiveness of these methods is first investigated using the FE simulations. In addition, they are used for experimentally identifying the joint properties of a bolted lap joint in transverse direction. The joint conditions are altered through changing the surface roughness and the addition of an interfacial material to the joint and then changes on the joint properties are investigated using the developed methods. Another joint identification technique is presented in this chapter for determining the joint dynamic properties in tangential and torsional directions based on translational FRF measurement. The hysteresis loop approach is presented as a verification method. The assumptions and limitations associated with each method are discussed as well.

Chapter 4 mainly focuses on nanocomposite sensor development for joint dynamics identification. The fabrication process is presented beside the post-processing steps carried out for improving the piezoelectricity of the sensor. Piezoelectric and piezoresistive models are established for investigating the behaviour of the nanocomposite sensor. Afterwards, the accuracy of the sensor in static and dynamic measurements is verified through experiments. Finally, the sensor fusion method used for combing the piezoelectric and piezoresistive signals is explained. The limitations and assumptions of the developed sensor and presented numerical modeling are discussed at the end of the chapter.
Chapter 5 presents different methods for joint dynamics identification using the developed nanocomposite sensor. This chapter first presents an introduction on the contact model considered for the joint interface. A calculation algorithm is developed for analyzing the microslip motion using the nanocomposite sensor. The energy dissipated due to friction is then obtained and the joint properties are identified using the calculation algorithm and the hysteresis loop method. The accuracy of the developed methods is then verified through experiments. Finally, the joint properties are extracted under different conditions, i.e. the various excitation loads and different normal contact loads. The limitations and assumptions of the presented methods are discussed as well.

Chapter 6 provides a brief overview on the developed methods in this study, i.e. joint dynamics identification in transverse, tangential and torsional directions, nanocomposite sensor development, and joint properties identification using the developed nanocomposite sensor. In addition, the novel contributions and outcomes of this study are elaborated. Also, this chapter discusses the future works of this study.
Chapter Two: LITERATURE SURVEY

This study develops new analytical and experimental methods for joint dynamics identification that can be ultimately used for virtual prototyping of machine tools. A literature survey on the existing studies for determining the joint dynamics and nanocomposite sensor fabrication is presented in this chapter. In addition, the limitations and challenges associated with the existing methods are discussed. Section 2.1 elaborates general information on virtual prototyping technology with the emphasis on the machine tool centers. Section 2.2 presents different types of mechanical joints and discusses the importance of joint dynamics identification. In addition, different studies developed for identifying the joint dynamic properties are presented. Section 2.3 discusses the importance of polymeric nanocomposite (PNC) sensors along with some literature reviews on the polymer matrix, carbon nanotubes (CNTs), fabrication processes, numerical modeling methods, and different techniques for improving the piezoelectricity of the nanocomposite sensors. This chapter will be concluded with a brief summary which is provided in section 2.4.

2.1 Virtual Prototyping

Machine tools are assembled structures consisting of different substructures that are connected to each other through mechanical joints. The performance of the machine tool centers are dependent on their dynamics, machining process, and control system as shown in Figure 2.1 (Altintas et al. 2005). Every design step of the machine tool should be tested with virtual prototyping and numerical simulation to ensure the efficiency of the physical prototype.
The performance and dynamic behaviour of any structure can be analyzed by virtual prototyping technology before constructing the physical structure. A virtual prototype is a simulation model of the whole machine tool and machining processes toward producing a physical prototype. Such a prototype reduces the time and expenses of the product development. It also results in the optimal design of the physical structure by performing all optimization and design variation processes. Therefore, it avoids different steps between the design and manufacturing processes which are usually observed in the conventional machining process. Any problems associated with the machining process can be detected in the early stages if the virtual prototyping technology is employed, and the required changes are applied to the design.

Figure 2.2 shows different steps of building a finite element (FE) model of machine tools. In the first step, the computer aided design (CAD) model of the machine should be created. In the CAD modeling, the geometric details, such as chamfers or small holes that have only local effects on the dynamic behaviour of the system, are eliminated (Altintas et al. 2005). The CAD model is then meshed to build a complete FE model where the finite elements are connected by nodes and constrains.
Figure 2.2. Steps of FE analysis of a machine tool (Altintas et al. 2005)

The virtual models of different machine tool centers have been developed in different studies. Figure 2.3 shows an FE model of a milling machine which includes different parts such as the base (a), slide (b), cross rail (c), ram (d), tool (e), table (f), guideway (g), ball screw (h), and electrical motor (i).

Figure 2.3. Finite element model of a high speed milling machine (Bianchi et al. 1996)
(Hung et al. 2011) developed a finite element model of a vertical milling machine as shown in Figure 2.4 to investigate its dynamic behaviour and machining capabilities under the influence of a linear guide modulus.

![Schematic construction of a vertical column milling structure](image)

**Figure 2.4.** Schematic construction of a vertical column milling structure (Hung et al. 2011)

Different parts of the feeding mechanism, such as the ball bearing, ball screw, and linear guides, were modeled in this study. The main parts of the system were modeled as solid elements that are connected to spring elements at the rolling interface defined by Hertzian contact theory.

A modified FE model of a vertical machining center was built by (Deng et al. 2014) as shown in Figure 2.5. They developed a joint stiffness configuration method to optimize the dynamic characteristics of the whole system. The properties of the joints, including linear guides, bolts, ball screws, and bearings were identified and inserted into the modeling section to improve the accuracy of the FE model.
(Law et al. 2013) presented a computational method for improving the performance of a machine tool at the design stage.

Figure 2.5. The vertical machine center modeled by (Deng et al. 2014)

Figure 2.6. Structural synthesis steps of the machine tool (Law et al. 2013)
(Law et al. 2013) developed a position-dependent multibody dynamic model of a machine tool as shown in Figure 2.6 to investigate the dynamic behaviour of the system. The weak components of the machine tool, which affect the productivity because of the chatter problem, can be identified using the proposed modeling strategy.

(Zulaika, Campa, and Lopez de Lacalle 2011) developed a design approach for the milling machine tool to achieve high productivity with minimum mass in the mobile components of the system. The methodology is based on using a stability model of the milling process in modal coordinates to identify the design parameters that hinder the productivity of the milling process.

An FE model of a horizontal machining center, shown in Figure 2.7, was developed by (Mi et al. 2012) through modeling of linear guides, ball screws, bearings, and bolted joints. This FE model was modified and validated by experimental measurements. The effects of preloads on the ball screws and linear guides were investigated where the results indicated that the dynamic stiffness of the spindle nose is highly dependent on the applied preloads to the mechanical joints.

![Figure 2.7](image)

**Figure 2.7.** (a) Schematic a horizontal machine center; (b) FE model of each part of the horizontal machine center (Mi et al. 2012)
A new approach for modeling the milling machine behaviour was proposed by (Catania and Mancinelli 2011). They coupled the modal models of the machine frame and the spindle evaluated experimentally with the discrete modal model of the tool. However, rigid joints were used to define the connection between components. A chatter identification technique based on this model was carried out to evaluate the accuracy of the model.

(Kolar, Sulitka, and Janota 2010) developed a coupled model of the spindle and machine frame for the milling machine. They modeled the spindle using a reduced FE description of geometrical components and the Harris’ bearing model. They investigated the accuracy of their model through frequency response and cutting experiments.

Ignoring the mechanical joints in the FE modeling of the machine tools or inaccurate modeling of these components will cause discrepancies between the results extracted from the FE model and those of the corresponding physical structure. The next section reviews different researches which have been presented for identifying the dynamic characteristics of the mechanical joints.

### 2.2 Identification of Joint Dynamics

The main goal in the joint dynamic identification is extracting the joint parameters to improve the dynamic behaviour estimation of the entire assembled structure. An accurate understanding of the joint properties will decrease the discrepancies between the numerical and experimental responses of a structure, which usually occur because of uncertainties in the FE models.

Joint dynamics identification techniques have different applications, including machine tools and the auto industry. In machine tool centers, it is important to identify the properties of the joints,
i.e. the stiffness and damping, in order to achieve an accurate virtual model of the system (Altintas et al. 2005). There are different types of mechanical joints in the complex structures and ignoring these components in the modeling process will cause errors in structural dynamic analysis (Ibrahim and Pettit 2005).

2.2.1 Different types of joints

Dynamic characteristics of the machine tool centers are highly dependent on mechanical joints, including bolts, rivets, ball screws, and guideways. Bolted joints are the main source of damping in assembled structures because of a slip mechanism in the contact interface (Segalman 2005). Different parameters, such as material properties, surface texture, normal load, and geometry, affect the dynamic properties of the bolted joints. Rivet joints have a similar damping mechanism as bolted joints, however they provide less damping compared to the equivalent bolted joints (Walker, Aglietti, and Cunningham 2009). Linear guideways are among other important components of machine tools whose dynamic properties must be identified. Figure 2.8 shows a modeling of the linear guideways developed in different studies.

![Schematic of models for the solid structures and rolling elements of linear guideways](Majda 2012)

**Figure 2.8.** Schematic of models for the solid structures and rolling elements of linear guideways
As can be seen, a contact element is employed to model the interactions occurring in the contact area. The rolling element is modeled with a spring element of unilateral action (compression only), which connects the carriage to the guide rail at the contact points.

(Zhang et al. 2003) proposed a receptance systematic procedure for determining the dynamic behaviour of a machine tool center. They employed dynamic properties of joint interfaces identified experimentally for analyzing the dynamic characteristics of guideway joints. Ball screws in machine tools are mainly used as a feeding mechanism and the majority of the load in the feeding direction is applied to these components. The modeling of ball screws, which consist of a cylindrical shaft placed on two bearings and a movable nut along the shaft, has been investigated in different studies (Hung et al. 2011; Mi et al. 2012).

2.2.2 Different techniques for joint dynamics identification

Different methods have been presented for extracting the joint properties since an accurate modeling of the mechanical joints has an important effect on the accuracy of the virtual prototypes. Quite a few researchers have studied the linear and nonlinear behaviour of the mechanical joints. We have divided the joint dynamics identification techniques into three main categories, i.e. modeling methods, updating techniques, and pure experimental approaches, as illustrated in Figure 2.9. Each of these methods are explained with reference to some works in the literature, and the benefits and challenges associated with each method are discussed.
2.2.2.1 Modeling techniques

Different techniques presented in the literature, have tried to model the joint dynamics and identify its properties numerically. Some studies have determined the physics behind the process occurring in the joint while others have simply considered the effect of the joint without modeling the microphysics in the joint.

2.2.2.1.1 Nonlinear models

Different models have been presented for mechanical joints considering the nonlinear behaviour of these elements. (Iwan 1966) developed two types of distributed element models for modeling the hysteresis behaviour of the joint. One of which is based on an array of elastoplastic elements in parallel, while the other model considers that these elements are in series as shown in Figure 2.10. In the parallel-series model, the force is a function of displacement whereas the series-parallel formulation results in the displacement as a function of force. Iwan models can describe the joint dynamic behaviour considering both micro-slip and macro-slip movement in the contact interface. It seems that the second model presented by Iwan gives a more realistic representation.
of the deformation occurring in the vicinity of the contact interface (Menq, Bielak, and Griffin 1986). However, the parallel-series model has been used more in other studies since the equation of motion is expressed in terms of displacements as dependent variables.

![Iwan models](image)

**Figure 2.10.** Iwan models. (a) Parallel-series model; (b) series-parallel model (Menq et al. 1986)

Other studies have widely used Iwan models for investigating the hysteresis behaviour of contact interfaces. (Song et al. 2004) presented an adjusted Iwan beam element (AIBE) to analyze the dynamic response of the beams connected to each other through mechanical joints. (Quinn and Segalman 2005) developed a similar continuum model by considering a series arrangements of parallel Jenkins elements. (Miller and Dane Quinn 2009) presented an interface model for jointed structures which can be decomposed to Iwan model.

(Menq 1989; Menq et al. 1986) proposed a new continuous micro-slip model of friction in the contact interfaces to improve the ability of predicting the dynamic behaviour of frictionally damped structures.
This model was extended by (Cigeroglu et al. 2006) considering the inertia of the system as well as non-uniform normal contact force distribution. (Xiao, Shao, and Xu 2014) employed the same model to investigate the stick-slip transition along the contact interface of two connected beams. (Asadi, Ahmadian, and Jalali 2012) extended this model to a beam placed on a frictional support and subjected to transverse vibrations.

(Crawley and Aubert 1986) used a new technique named force-state mapping for identifying the joint properties. They measured the force transmission properties of the structural elements and employed them for joint dynamics identification. (Jalali, Ahmadian, and Mottershead 2007) then used this technique to identify the nonlinear lap joint parameters. (Ahmadian and Jalali 2007) represented the micro/macro slip behaviour of a bolted lap joint considering linear and nonlinear spring elements and a viscous damper. They extracted the joint parameters by minimizing the error between the predicted and measured frequency response functions (FRFs).

2.2.2.1.2 Finite element models

Different studies have modeled the joint behaviour through finite element (FE) methods. (Bograd et al. 2011) provided a detailed review on joint modeling with the FE method based on
three different approaches, i.e. node-to-node contact using the Jenkins friction model, thin layer elements, and zero thickness elements. (Chen and Deng 2005) investigated the effect of dry friction on the damping response of the frictional joint interfaces. They used two models, namely the G-K model and the M-D model, where converged finite element solutions were obtained. (Richardson, Crocombe, and Smith 1993) studied the FE modeling of adhesive joints and compared both the adhesive stress distributions and the energy release rates associated with crack growth from the two- and three- dimensional analyses. A two-dimensional (2D) thin elastoplastic layer using the MSC/Nastran QUAD4 element (MSC/Nastran) was used by (Iranzad and Ahmadian 2012) to model the nonlinear stiffness and damping effects of a bolted lap joint. They measured the nonlinear response of the structure at different force amplitudes and then found the joint parameters after comparing the results with those of the model. (Ren, Lim, and Lim 1998) predicted the response of the assembled structures using Multi Harmonic Balance (MHB) technique and obtained the properties of the nonlinear joints. (Hwang and Stallings 1994) presented a 2D axisymmetric finite element model and a 3D solid finite element model of a high pressure bolted flange joint to investigate stress behaviour. An axisymmetric finite element modeling of the bolted joints was presented by (Lehnhoff and Wistehuff 1996) to evaluate the effects of different parameters, including the magnitude and position of the external load, member thickness, and member material on the joint stiffness. (Shamoto et al. 2014) presented an analytical technique for identifying the contact stiffness and friction damping in bolted joints under torsional vibrations. Linear phenomena were extracted and solved by the FE analysis, and then a linear combination of the FEM results was calculated iteratively to obtain the boundary between the sticking and sliding zones and subsequently the joint stiffness and damping.
2.2.2.2 **Updating techniques**

Updating techniques are used for improving the accuracy of the models developed for joint dynamics identification using experimental data. The updating techniques are generally divided into two main categories, namely iterative methods and direct approaches. Iterative methods use the experimental data in several steps to update the properties or structural matrices of the FE models, and therefore minimize the deviations between the FE response and test measurements (Gang et al. 2014; Sarmadi, Karamodin, and Entezami 2016; Yuan and Liu 2011). Direct methods, on the other hand, update the properties of the developed models using experimental data in one step (Berman and Nagy 1983; Carvalho et al. 2007; Friswell et al. 2001; Wei 1990).

The iterative methods are divided into two main methods, namely the inverse eigen sensitivity method (IES) or modal based technique and the response function method (RFM) or FRF sensitivity method (Mehrpouya 2014). The IES method uses the experimental modal data, including the eigenvalues and mode shapes to update the FE models and decrease the discrepancy between the numerical and experimental data (Ehmann, Ehmann, and Wu 1991; Liu and Ewins 2002). (Shamine, Hong, and Shin 2000) identified the joint properties of a spindle system using this technique. The IES method has been also used in other studies for identifying the bolted joint properties considering the joints as lumped elements (Arruda and Santos 1993; Mottershead et al. 1996; Mottershead and Friswell 1993). One of the main problems associated with the IES method is that it is very sensitive to the eigenvectors and eigenvalues of the model (Ewins 1984). Even small variations in the modal parameters bring about substantial discrepancies in the dynamic prediction of the assembled structure. Furthermore, a sufficient number of measurement points are required in the IES method to ensure that the problem is over-determined. Employing the response function data is one of the solutions to address this problem since it provides more information
than modal data (Mottershead and Stanway 1986; Ren and Beards 1995). The response function method (RFM) is based on creating the least square problem to minimize the discrepancies between the numerical and experimental frequency response functions (FRFs). The RFM method was used by different researchers to determine joint dynamic properties (Hong and Lee 1991; Hwang 1998; Yang and Park 1993).

A high computational effort is required for the iterative approaches. The initial values of the unknown parameters should be selected carefully otherwise there would be convergence problem and it may result in different final values. These problems demonstrate the importance of introducing direct methods that can be divided into three main techniques, i.e. the error matrix method (EMM), equation error method (EEM), and direct frequency-based method (Mehrpouya 2014). The error matrix method (EMM) updates the mass and stiffness matrices of the developed model using the measured modal data of the physical structure (Baruch 1982; Berman 1979; Lee and Eun 2009a, 2009b). Different studies have used this technique for identifying joint parameters using either complete or incomplete mode shapes and eigenvalues (Baruch 2012; Berman and Nagy 1983; Yuan and Wu 1985). In the EEM approach, the equations extracted from the eigendynamic conditions along with other constraints such as the system symmetricity and orthogonality conditions are used for the updating process (Ewins 1984; Friswell et al. 2001). This technique finds the mass and stiffness matrices of the FE models that include the joint parameters using the experimental measurements (Chapman, Shaw, and Russell 1987; Fengquan and Shiyu 1996).

To overcome the challenges associated with the presented direct updating methods, the frequency-based method was proposed and has been used in different studies for joint dynamics identification. FRF-based method determines the dynamic properties of the mechanical joints
through comparing the dynamics of the assembled structure extracted experimentally with those of the rigidly attached substructures. The joints are usually modeled as stiffness and damping elements in the FRF-based methods. These elements are then identified using the FRFs of the assembled structure and substructures as their difference is related to the effect of the joints. This method is not sensitive to modal data such as eigenvalues and mode shapes, but the measurement errors and unavoidable noise still cause discrepancies in the results. Different frequency-based methods have been used by researchers to identify the joint dynamic properties. (Lee and Hwang 2007) used the FRF-based method and a gradient-based optimization technique to minimize the difference between the measured and predicted responses and subsequently identify the joint dynamics. (Movahhedy and Gerami 2006) proposed a joint model that accounts for rotational DOFs for modeling the joint between the tool and spindle and then used the frequency-based methods along with a genetic algorithm optimizing method to identify the joint parameters. (Park and Chae 2007) modified the classic receptance coupling (RC) method and identified the joint properties in machine tools through inverse receptance coupling (IRC). The IRC method was later used by Mehrpouya et al. for identifying simple joints, multiple joints, and 3D joints (Mehrpouya et al. 2013, 2015, 2016).

2.2.2.3 Experimental techniques

Another approach for measuring the joint parameters either directly or indirectly is the use of test stands. (Konowalski 2009) proposed a test stand, shown in Figure 2.12, based on the Kelvin-Voight model to extract the normal stiffness and damping of mechanical joints.
Figure 2.12. Schematic of the test stand used to estimate the normal contact stiffness and damping (Konowalski 2009)

(Shi and Polycarpou 2005) developed an experimental method based on contact resonance as shown in Figure 2.13 to obtain the contact stiffness and damping in a meso-scale interface of realistic rough surfaces under lightly loaded conditions.

Figure 2.13. Meso-scale contact stiffness and damping tester (a) schematic; (b) system model

(Shi and Polycarpou 2005)
The contact load was measured using miniature strain gauges placed on the outside surface of the lower tube spring shown in Figure 2.13(a). Based on the lumped parameter system model shown in Figure 2.13(b), the dynamic equations of the system were developed and subsequently the contact stiffness and damping were extracted using the characteristic equation.

(Eriten, Lee, and Polycarpou 2012) employed the setup proposed by Shi for measuring the tangential stiffness and damping of a bolted joint. (Eriten, Polycarpou, and Bergman 2011) presented a new setup for obtaining the tangential parameters of the lap joints using dissipated energy and fretting loops. A combination of a direct method (hysteresis loop) and an indirect method (contact resonance approach) was utilized by (Gaul and Lenz 1997). A contact resonator was suspended at each side of the joint to directly measure the force and displacement of the joint and then obtain the friction loop and joint properties.

To overcome some of the challenges associated with analytical modeling, FE updating, and even pure experimental approaches proposed for joint dynamics identification, a new nanocomposite sensor is developed in this study. This sensor can be implemented in the joint interface of the mechanical joints and used for identifying the joint parameters. Therefore, a literature review on the importance of nanocomposite sensors and their applications are provided in the next section.

2.3 The Application and Importance of Nanocomposite Sensors

Sensors are important devices for monitoring the condition of the mechanical systems and manufacturing processes. They can convert an external stimulus into a measurable signal. The demand for development of new sensors and monitoring techniques continuously increases as the industry moves towards development of the fourth industrial revolution (Industry 4.0) and higher
levels of automation for improving the quality of the manufactured goods. Polymeric nanocomposite (PNC) sensors have been widely used for different applications, especially sensing, due to their specific properties and low cost. PNCs are considered as a new class of materials, which are made of combined polymer and non-organic nano-scale fillers. Polyvinylidene fluoride (PVDF) based PNCs are great alternatives to commercial piezoelectric sensors such as piezoceramic sensors and metal foil strain gauges for sensing application under dynamic and quasi-static excitations due to their high piezoelectric properties, flexibility and high mechanical properties. Carbon nanotubes (CNTs) are considered as great candidates for nanofillers to improve the mechanical properties of the polymer matrix. They contain great mechanical properties (Gojny et al. 2004; Thostenson, Ren, and Chou 2001) and coupled electromechanical properties (Wanlin and Yufeng 2004). Therefore, adding CNTs into the polymer matrix of PNCs, provides high strength and resistance to corrosion and wear (Parmar 2015) and results in improvement in mechanical properties (Coleman et al. 2006; Gojny et al. 2005), electrothermal properties (Miaudet et al. 2007), electric conductivity (Chang et al. 2006), and ability to measure stress (Zhao, Frogley, and Wagner 2002) or strain (Chang, Su, and Chang 2008; Hierold et al. 2007).

Sensing capability of the PVDF-CNT based nanocomposite sensors highly depend on the piezoelectric properties of the PVDF polymer and the electric properties of the CNT nanoparticles. Piezoelectric properties of the PVDF polymer can be improved by developing the crystal orientations with different synthesis methods or by changing the structure of the dipoles in the polymer. In addition, the electric properties of the CNT-PVDF sensors can change by choosing different CNT parameters, such as nanotube aspect ratio, chirality and number of concentric shells (Kane et al. 1998). The literature review on PVDF as a piezoelectric polymer and CNTs as nanofillers are presented in this section. Different fabrication process used for the PNC materials
is elaborated. Also, different methods for improving the piezoelectricity of the PVDF polymer is discussed. Finally, a review on different methods developed for modeling of the piezoelectric and piezoresistive properties of PNCs are presented.

2.3.1 PVDF as a piezoelectric polymer matrix

Different polymers can be used as a piezoelectric material for sensor applications. They are divided into different categories as illustrated in Figure 2.14 (Ramadan, Sameoto, and Evoy 2014). The first group of piezoelectric polymers are the bulk polymers, which are solid films that contain the piezoelectric properties due to their molecular structure. The second group is associated with the piezoelectric composite polymers which are a combination of polymers and piezoelectric ceramics, that are the source of piezoelectricity in the composite. The third category of the piezoelectric polymer is the voided charged polymers which are completely different from the first mention group of piezoelectric polymers. In this type of polymers, gas voids are added to the polymer and the internal dipoles are created after charging the surface of the polymer. Applying stress to the polymer film changes the polarization of the dipoles and therefore results in piezoelectric response.

Semicrystalline polymers such as polyvinylidene fluoride (PVDF) (Dargaville et al. 2005), polyamides, liquid crystal polymers (Harrison and Ounaies 2002) and Parylene-C (Kim, Cheng, and Tai 2011) are in the category of bulk polymers and behave like piezoelectric inorganic materials (Ramadan et al. 2014). PVDF is the most common piezoelectric polymer used in electromechanical devices due to the largest piezoelectric coefficient (20-28 pC/N) compared to other bulk polymers (Harrison and Ounaies 2002; Ramadan et al. 2014). PVDF is synthesized by the polymerization of H2C=CF2 monomers as shown in Figure 2.15(a).
Piezoelectric properties of poled and stretched PVDF films were first observed by (Ohigashi 1976). The pyroelectric and non-linear optical behaviour of this polymer was then found by (Wada and Hayakawa 1976). PVDF piezopolymer can be used in 3 main applications, namely energy harvesting (Yoon, Arakawa, and Uchino 2015), sensing and actuation applications (Choi and Jiang 2006; Dargahi 2000; Ferreira et al. 2012) due to its flexibility, mechanical strength, low thermal

PVDF-based sensors have been used for different applications, such as machining process monitoring, cutting force measurement and online chatter detection during machining operations (Ma et al. 2012; Ma, Melkote, and Castle 2014; Nguyen et al. 2016), vibration monitoring and control (Ma, Chuang, and Pan 2011), and pressure monitoring (Kimoto and Shimada 2013; Shirinov and Schomburg 2008).

PVDF-based sensors contain higher sensitivity and frequency bandwidth compared to metal foil strain gauges when it comes to strain measurement (Ma 2013). Moreover, the voltage signals can be directly acquired from the PVDF sensor without any need to external power supply. Therefore, they are excellent candidate for remote sensing applications. PVDF provides sensors with lower Young’s modulus, higher flexibility, and smaller thickness compared to those of the Lead Zirconate Titanate (PZT). Hence, it will have less effect on the dynamics of the host structure after sensor installation. This is a very important feature when it comes to systems with complicated structure such as mechanical joints or systems with low stiffness such as plastics.

Adding CNTs into PVDF polymer provides piezoresistive properties in addition to the piezoelectricity in the sensor. In addition, (Wu and Chou 2016) showed that the addition of CNTs into PVDF improve the concentration of the $\beta$ phase which is the phase with the highest dipole moment per unit cell ($8 \times 10^{-30}$ C m) compared to the other phases (Correia and Ramos 2005; Martins et al. 2014). Next section elaborates the carbon nanotubes, their specific properties, and their sensing applications.
2.3.2 Carbon nanotubes as nanofillers

Carbon nanotubes (CNTs) are great nanofillers since they provide remarkable mechanical (Salvetat et al. 1999), electrical (Yao, Kane, and Dekker 2000) and thermal properties (Ruoff and Lorents 1995). They have remarkable physical properties, including high tensile strength, high stiffness, high Young’s modulus, high aspect ratio and good thermal conductivity (Hernández et al. 1999; Vaccarini et al. 2000). They have been widely studied in researches since their discovery in 1991 (Iijima 1991).

CNTs have one-dimensional (1D) structure and high aspect ratios (around 100-1000) which make them unique materials for developing conductive PNCs compared to other metallic particles (Sandler et al. 2003). There are two types of CNTs, namely single wall carbon nanotubes (SWCNTs) and multi wall carbon nanotubes (MWCNTs), depending on the number of concentric layers of carbon in the nanotube. SWCNTs and MWCNTs contain different properties and they can be used for different applications depending on required properties. There are different layers of graphene in each MWCNT where each layer contains different chiralities. Therefore, investigating the physical properties of MWCNTs are more complex compared to SWCNTs (Ma et al. 2010). The length of CNTs varies between some nanometers (nm) to some micrometers (\( \mu m \)), and their diameter ranges from 0.2 to 2 nm for SWCNTs while it is between 2 to 100 nm for MWCNTs. CNTs can be dispersed in molecular scale inside a polymer matrix due to their original size. Therefore, it results in composites with substantial electrochemical and electromechanical properties that makes them a unique option for sensor applications (Ajayan 1999; Thostenson et al. 2001). Electrical conductivity of PNCs can increase significantly using even low concentrations of CNTs (Bryning et al. 2005).
A CNT-PNC becomes suitable for sensing applications when the CNT concentration reaches a region called the percolation threshold as it provides a network that is fragile and susceptible to breakage due to the external forces (Miaudet et al. 2007; Zhao et al. 2002). Outside of this region there is no significant change in electrical conductivity with the PNC behaving as an insulator below the region and a conductor above it. The overall conductivity of a CNT-PNC varies by the filler volume fraction, the connectivity and topology of the network, i.e. the CNT dispersion, the CNT orientation, and the interaction between the polymer and the filler (Alig et al. 2012; Moniruzzaman and Winey 2006).

CNT-PNCs can be used for different sensing application, such as pressure (Wood and Wagner 2000), bio-molecules and flow sensors (Ghosh, Sood, and Kumar 2003), chemical (Castro et al. 2009; Kong et al. 2000), gas sensors (Li, Thostenson, and Chou 2008b), and mechanical force or strain (Georgousis et al. 2015; Park et al. 2016; Thostenson and Chou 2006). Piezoresistive properties of CNTs can be used for strain measurement. The piezoresistivity of CNT-PNCs is attributed to different contributing factors, i.e. (a) altering the conductive network due to variation in the number of contact between the CNT nanoparticles (b) change of the tunneling resistance caused by variation of the distance between the CNT nanotubes and (c) deformation of CNTs (Alamusi et al. 2011), where the first two factors contribute significantly more (Hu et al. 2012). It is shown that CNT-PNCs based strain sensors can contain higher sensitivities compared to the conventional strain gauges (Pham 2008). However, there are some challenges associate with piezoresistive CNT-PNC based strain sensors, including dispersion of CNTs inside the polymer matrix and CNT alignment. Different fabrication methods of CNT-PNCs are discussed in the next section.
2.3.3 Different fabrication methods

The aspect ratio and dispersion of the nanoparticles inside the polymer matrix have a remarkable impact on the properties of the fabricated CNT-PNC nanocomposite. Dispersion of nanoparticles inside the polymer is always challenging, especially when it comes to CNT nanoparticles mainly due to their poor interaction with the polymer matrix, strong van der Waals forces between crossed carbon nanotubes that might result in agglomeration of CNTs as can be seen in Figure 2.16 (Atif and Inam 2016; Zhbanov, Pogorelov, and Chang 2010).

![Figure 2.16](image)

**Figure 2.16.** Transmission Electron Microscopy (TEM) images of a MWCNT-PNC nanocomposite with agglomeration of CNTs (left); and without agglomeration of CNTs (right) (Pegel et al. 2008)

Different methods have been developed for mixing CNTs inside the polymer matrix to fabricate a nanocomposite with desirable dispersion and high quality. These methods are divided into two main categories, namely solution mixing and melt mixing techniques. In the solution mixing method, the polymer is dissolved in a solvent and the CNT solution is then added to it. In
order to achieve a better dispersion of CNTs, the ultrasonication method is used. Quite a few studies have used this technique for fabrication of nanocomposites (Araby et al. 2014; Kuila et al. 2011; Okamoto, Fujigaya, and Nakashima 2008; Park et al. 2016; Zhu et al. 2006). One of the challenges associated with the solution mixing method is the molecular weight degradation of the polymer. Therefore, this method is not appropriate when the polymer of a nanocomposite contains high molecular weight, such as polyethylene and polypropylene (Parmar 2015).

After preparing the nanocomposite solution, it should be deposited on a substrate using an existing method such as drop casting, inkjet printing, spray coating, spin coating, and screen printing and the desired CNT-PNC is obtained after evaporating (Kanoun et al. 2014). The schematic of each of these methods is illustrated in Figure 2.17.

![Different deposition method for fabricating the CNT-PNC nanocomposite films](image)

**Figure 2.17.** Different deposition method for fabricating the CNT-PNC nanocomposite films:

(a) Drop casting (b) Inkjet printing (c) Spray coating (d) Spin coating (e) Mayer rod coating/screen printing (Kanoun et al. 2014)

Spin coating method is used for depositing a uniform thin film and has been extensively used in different studies for depositing of CNT solutions (Choi et al. 2003; Di et al. 2009; Jo et al. 2010).
In this method, the solution is dropped in the center of a substrate which is spinning in low speed or not spinning at all and then the rotational speed of the substrate increases substantially. This process is repeated couple of times till a desired thickness of the film is fabricated.

Spray coating method is the most common technique for fabricating the CNT-PNC films due to some advantages associated with this method, including simplicity and being cost effective. In this method, the solution is poured inside a spray machine connected to an air tank and the solution is sprayed on the substrate with the air pressure. The film thickness can be controlled in this method with adjusting the number of sprays. This technique has been used in different studies for fabricating the CNT films (Kim et al. 2010; Lee et al. 2011; Park et al. 2016). It has been shown by Kim et al. that spray coating method provide films with more roughness compared to those obtained from the spin coating approach (Kim et al. 2010).

Melt mixing technique is another method that can be used for fabricating of nanocomposites. Thermoplastic polymers are melted, and nanoparticles are mixed with the polymer melt under high shear forces resulting in a better homogeneity of the composite and better dispersion of CNTs inside the polymer. This method has been used in quite a few studies in order to disperse carbon nanotubes into polymers and enhance the properties of the composite (Pötschke et al. 2003, 2005; Pötschke, Bhattacharyya, and Janke 2004; Zhang et al. 2006). Applying higher shear force during melt mixing process will result in more uniform dispersion of CNTs; however, it might cause breakage of CNTs and consequently reduce their aspect ratio. Melt mixing technique does not have the limitations of the solution mixing method, and therefore insoluble polymers and polymers with any molecular mass can be processed using this technique (Kanoun et al. 2014; Parmar 2015).

It has been claimed that the degree of dispersion of CNTs obtained from the melt mixing technique is lower compared to the solution mixing method. However, (Ke et al. 2012) investigated
the effect of different fabrication processes on dispersion of MWCNTs in a CNT-PVDF nanocomposite and found different results. They showed that the dispersion of CNTs is a function of both the concentration of nanoparticles and the fabrication method. They concluded that solution mixing method is more effective and results in better dispersion of CNTs inside the polymeric solution when the concentration of the nanoparticles is low while melt mixing provide better dispersion results when high concentration of CNTs are used. The fabricated CNT-PNC sample by melt mixing can be further processed using different method such as injection molding, compression molding, and fibre spinning to achieve a desired size and shape of the sample (Cooper et al. 2002; Parmar 2015).

In order to use the PVDF-CNT nanocomposite samples for piezoelectric measurement the piezoelectricity of the polymer needs to be improved. Different techniques for this purpose are discussed in the next subsection.

2.3.4 Improving the piezoelectricity of PVDF and polling

Polyvinylidene-fluoride (PVDF) is a semi-crystalline material consisting of four mainly conformations known as $\alpha$, $\beta$, $\gamma$, and $\delta$ phase (Fontananova et al. 2015). The C-F bonds of PVDF are polar and the highest amount of polarization can be obtained from $\beta$ phase where all dipoles of the polymer are aligned in the same direction as shown in Figure 2.18. Therefore, this phase is responsible for the piezo, pyro and ferroelectric properties of the PVDF polymer (Calvert 1975; Fukada and Takashita 1969; Furukawa et al. 1968). On the other hand, $\alpha$ phase of PVDF orients the dipole moments in random directions, and consequently zero net-polarization is observed. When the PVDF-based composite is fabricated through either melt mixing or solution mixing and solidified, $\alpha$ phase is the most dominant phase of the composite (Lovinger 1982).
The sensitivity and piezoelectricity of the PVDF-based sensors are related to the \( \beta \) phase content of the polymer (Hosseini and Makhlouf 2016). To improve the piezoelectricity of the prepared samples, the \( \beta \) crystallites of PVDF should increase and the existing \( \alpha \) phase of PVDF should be converted to \( \beta \) phase. There are different ways to do this transition and improve the piezoelectric properties of the sensor, including blending with nanofillers, mechanical stretching, electrical poling, solution casting, spin coating, and electro spinning (Davis et al. 1978; Dhakras et al. 2012; Sajkiewicz, Wasiak, and Gocłowski 1999).

Creating defects between CF\(_2\) and CH\(_2\) polymeric chains improves the polarity of the PVDF film and consequently results in a more sensitive sensor (Dargaville et al. 2005). Therefore, quite a few studies have developed different synthesis methods to improve the \( \beta \) phase of PVDF by adding different nanofillers such as trifluoroethylene (TrFE) (Sharma et al. 2012) (Figure 2.15 (b)) , silicates (Priya and Jog 2002; Shah et al. 2004), carbon black (Lallart et al. 2010), and carbon nanotubes (CNTs) (Huang et al. 2009; Manna and Nandi 2007; Ramaratnam and Jalili 2006; Yu et al. 2009). Also, the piezoelectricity of the PVDF-based sensors can be enhanced by adding different piezoelectric inorganic particles to PVDF, including BaTiO\(_3\) (Ye, Shao, and Zhen 2013), PZT (Graz et al. 2009) or ZnO (Dodds, Meyers, and Loh 2012).
Another common method for reorienting the crystallites within the polymer and enhancing the piezoelectricity is poling process and applying a high electric filed to the polymer bulk as illustrated in Figure 2.19.

![Figure 2.19. Reorienting the crystallites of the PVDF polymer and enhancing the β phase using the poling process (Dargaville et al. 2005)](image)

There are generally two methods for polling polymers, namely electrode poling (or contact poling) and corona poling (or non-contact poling). The schematic of each of these methods is shown in Figure 2.20.

![Figure 2.20. Schematic of different poling methods of polymers: (a) corona poling; (b) electrode poling (Ramadan et al. 2014)](image)
In the electrode poling, the conducting electrodes are pressed or deposited through evaporation, sputtering or painting on the surface of the polymer and a high voltage is then applied to the electrodes to create electric field across the film. Depending on the polymer, the electric field applied for poling ranges from 5 MVm\(^{-1}\) to 100 MVm\(^{-1}\) which can break down the polymer (Dargaville et al. 2005; Park et al. 2004). In order to prevent arcing problem, it is better to put the sample in a vacuum place or in an insulating fluid such as Fluorinet and silicone oil (Dargaville et al. 2005; Park et al. 2004).

If contact between the electrodes and the sample is poor, it will cause some different problems such as dielectric breakdown, discharge, arcing and non-homogenous electric field during poling. Permanent electrodes such as evaporated, sputtered, and painted electrodes bring about better contact between the electrodes and the film compared to the electrodes that are pressed on the sample. The electric field applied to the sample during electrode polling can be either constant or variable (Bauer 1983; Wegener et al. 2002). The constant electric field can be applied from 10-30 minutes till 2 hours; however, longer poling time under high voltage might cause dielectric breakdown (Harnischfeger and Jungnickel 1990; Ohigashi 1976). Therefore, applying variable electric field, which is usually sinusoidal or triangular waveforms at low frequencies (mHz), is a more appropriate way for the poling process (Dargaville et al. 2005; Li, Kagami, and Ohigashi 1992).

The final quality of reorientation of the crystallites and piezoelectric improvement depends on different contributing factors, such as the electric field and time of poling, amount of contamination between the electrodes and the film surface, and degree of temperature uniformity applied to the sample during the poling process (Ramadan et al. 2014). Mechanical stretching of the PVDF film during poling process results in better reorientation of the crystallites and improve
the piezoelectricity of the polymer after poling process (Bharti, Kaura, and Nath 1997; Harrison and Ounaies 2002). The stretching and poling procedures can be applied either simultaneously or sequentially; however, it is reported that simultaneous stretching and poling results in higher piezoelectricity in the PVDF polymer (Kaura, Nath, and Perlman 1991). It has been shown that mechanical stretching and therefore higher $\beta$ phase orientation of the PVDF film can be achieved through spin coating process (Sharma et al. 2012).

Contrary to electrode poling method, corona poling approach does not require electrode deposition and only one side of the film is required to be covered with electrodes. As can be seen in Figure 2.20, corona poling setup consists of a conductive needle which is suspended on top of the sample and a high voltage $V_c$ (8–20 kV) is applied to the needle (Dargaville et al. 2005; Park et al. 2004). In addition, there is a grid mesh at a lower voltage $V_g$ (0.2–3 kV) between the needle and the sample and the whole poling process is conducted in dry air or argon medium (Dargaville et al. 2005; Park et al. 2004). When a high voltage is applied to the conductive needle, the air around the tip of the needle get ionized and move down towards the polymer whose electrode is connected to the ground and deposit on top surface of the film. Therefore, a charge built-up is generated on top of the polymer film which in turn results in a poling electric filed between the top surface of the polymer and the ground. The grid mesh brings about an evenly distribution of ions on top of the sample’s surface and therefore a uniform charge distribution and electric filed is created (Mahadeva et al. 2013).

Although corona poling process is more complex compared to the electrode poling process, it has some advantages over the contact poling method. The most important advantage of the corona poling approach over the contact poling method is the fact that arcing and short circuit problems are minimized during the process and the electric filed is more uniform. Also, corona
poling produces an electric field which is closer to the dielectric breakdown of the polymer compared to the electrode poling and therefore the reorientation of $\beta$ phase is achieved more efficiently (Giacometti et al. 1995). In addition, the poling process will not be disrupted even if there is arcing problem in the system since only few charges will pass through the resulting damage and the rest will keep the electric field and poling process.

Different models have been developed in the literatures for predicting the properties of PNCs. Some of the modeling presented for elucidating the piezoelectric and piezoresistive properties of CNT-PNCs are discussed in the next section.

2.3.5 Modelling of electromechanical property of CNT-PNCs

Nanocomposite properties vary by changing the nanofiller’s type, concentration, alignment, aspect ratio and volume fraction as well as the polymer matrix. Therefore, studying the effect of different parameters on the nanocomposite properties through only experiments is an expensive and time-consuming procedure. Numerical and analytical modeling of nanocomposites, however, can help to decrease the cost and time associated with performing experiments and to understand and predicts the properties of the nanocomposites. Therefore, quite a few theoretical models have been developed to identify different properties of the nanocomposites, including electrical conductivity, piezoelectricity, electromechanical properties, and percolation threshold of the nanocomposites. However, the number of numerical researches in this area is fewer compared to the experimental studies.

Most of the numerical modeling for predicting the piezoresistivity of the CNT-based nanocomposites are based on the Monte Carlo simulations and the statistical percolating network theory of randomly distributed CNTs in polymers (Gong, Zhu, and Meguid 2014). The percolation
threshold was predicted by randomly distributing CNTs inside a polymer (Ounaies et al. 2003; Theodosiou and Saravanos 2010) as well as a model based on the empirical formula and the statistical percolation theory (Celzard et al. 1996). (Theodosiou and Saravanos 2010) investigated the piezoresistive properties of CNTs and then developed a numerical CNT percolating network model for studying the piezoresistivity of the CNT-polymer nanocomposites. (Behnam and Ural 2007) developed a 3D numerical model based the Monte Carlo simulation and investigated the effect of different parameters of CNTs, i.e. alignment, length, density and resistance ratio of nanotubes, on the piezoresistive properties of the nanocomposite. (Bao et al. 2011) improved this model and reported that the partially aligned CNTs will result in the maximum conductivity of the CNT network.

![Figure 2.21. Schematics of (a) uniformly distributed; and (b) aligned CNTs with different length in a representative volume element (RVE) (Bao et al. 2011)](image)

(Li, Thostenson, and Chou 2008a) employed a percolation model to compare the conductivity of wavy nanotube networks with straight nanotubes and concluded that the wavy nanotubes result in lower conductivity. (Wang and Ye 2013) investigated the mechanism and
optimization of the piezoresistive CNT-polymer composites numerically by studying the effect of different parameters including Poisson’s ratio of the polymer matrix, the diameter, orientation and concentration of CNTs inside the polymer. (Moheimani and Hasansade 2018) studied the effective thermal conductivities of the CNT-PNC nanocomposites using a closed-from micromechanical model. (Gong et al. 2014) developed a new multiscale 3D model of the CNT percolating networks considering the effect of agglomerates on the properties of the composite as shown in Figure 2.22. They concluded that the CNT agglomerates cause reduction of the electrical conductivity of the CNT networks and also bring about nonlinearity of piezoresistivity with respect to zero strain.

![Schematic of the model presented for CNT agglomeration](image)

**Figure 2.22.** Schematic of the model presented for CNT agglomeration (Gong et al. 2014)

Different studies have numerically investigated the effect of strain and external force on the electrical properties of the CNT-based nanocomposites. It has been shown in the literatures that the resistance of CNT-PNCs, which consists of the intrinsic resistance of CNTs ($R_{CNT}$) and tunneling resistance ($R_{tunnel}$), varies under external strain applied to the composite (Taya, Kim, and Ono 1998). It has been shown through analytical and numerical investigations that percolation
threshold of CNT-PNCs is a function of the tunneling length in the CNT network (Berhan and Sastry 2007; Natsuki, Endo, and Takahashi 2005). (Rahman and Servati 2012) developed a numerical modelling and investigated the effect of inter-tube distance of CNTs and alignment on the tunneling resistance. It has been reported by (Eken et al. 2011) that the increase of the tunneling reduces the percolation threshold especially in low aspect ratios. (Yasuoka, Shimamura, and Todoroki 2010) showed that the piezoresistivity changes nonlinearly under strain and it is inversely proportional to the CNT loading because of the tunneling effect. (Hu et al. 2010) developed a 3D statistical resistor network model considering the tunneling effect between CNTs inside a polymer matrix as shown in Figure 2.23 and studied the behaviour of the sensor under both tensile and compressive strains. They reported that higher sensitivity of the sensor is achieved by either increasing the tunneling resistance or the ratio of the tunneling resistance to the total resistance of the CNT network. They also concluded that lower content of CNTs close to percolation threshold, and smaller diameter of the fillers improve the sensitivity of the sensor.

![Figure 2.23. Schematic of the CNT network modeling the tunneling effect (Hu et al. 2010)](image-url)
Ray and Batra (2007) proposed a micromechanical modeling for identifying the piezoelectric and elastic moduli of a specific piezoelectric composite (NRPEC) consisting of SWCNT embedded in a lead zirconate titanate (PZT-5H) piezoceramic matrix. Ramaratnam and Jalili (2006) developed a mathematical modeling to investigate the piezoelectric properties of a PVDF-CNT nanocomposite sensor and to support their experimental results. They demonstrated that the CNT-based polymer shows a better response compared to the plain piezoelectric polymer. Pettermann and Suresh (2000) developed a unite cell model using finite element method for extracting overall elastic, dielectric, as well as piezoelectric moduli of composites whose nanotubes are unidirectional, and distributed periodically. Maxwell et al. (2010) presented a finite element simulation to study the behaviour of the three-phase piezoelectric nanocomposites comprised of a polyimide matrix, SWCNT and PZT-5A particles. Koenck (2013) developed a numerical model based on finite element analysis and Monte Carlo simulation for predicting the piezoelectric behaviour of the nanocomposites that either filler or matrix has piezoelectric properties.

2.4 Summary

A literature review on the joint dynamics identification and nanocomposite sensor development was provided in this chapter. Different studies on the virtual prototyping of machine tool centers were reviewed. Importance of identifying the mechanical joint properties in order to have an accurate virtual model of the assembled structures was discussed. In addition, different types of mechanical joints and their applications were presented. Also, different methodologies developed for identifying the joint properties were reviewed. The nonlinear models proposed for joint dynamics identification consider the nonlinearities in the joint interface and are capable of
determining the slip boundaries. However, these models are usually complicated and need the experimental data to be calibrated and verified. The FE modeling and analytical works for determining the joint properties mostly suffer from modeling errors and sometimes there are discrepancies between the numerical results and test measurements. The iterative updating methods rely on the extraction of modal parameters of the system which are sometimes inaccurate and affected by transition from the response model to the modal models. In addition, these methods are so sensitive to the accuracy of the extracted modal parameters and it might inversely affects the accuracy of the identified properties. The direct iterative methods which use the frequency response functions (FRFs) of the system, suffer from difficulties in extracting the rotational and cross FRFs.

In addition, a literature review was provided on polymeric nanocomposites (PNCs), which are a combination of polymers and nanofillers, as new class of materials. They can be used for different applications such as energy harvesting, actuation and sensing due to their specific characteristics, including flexibility, high mechanical properties and piezoelectric and piezoresistive properties. PVDF is the most important piezoelectric polymer that can be used for making PNCs. In addition, carbon nanotubes (CNTs) are among the most important nanofillers that provide remarkable mechanical, electrical, and thermal properties and can be used for creating a piezoresistive PNC applicable for sensing. Two main methods, i.e. melt mixing and solution mixing techniques, can be used for fabricating the PVDF-CNT nanocomposites. The limitations associated with each method were elaborated in this chapter. To improve the piezoelectricity of the PVDF-based samples, $\alpha$ phase of the PVDF should be converted to $\beta$ phase and then the dipoles should be reoriented in a single direction through electric poling. Finally, a literature review was
presented on several numerical and analytical modeling developed for understanding the properties of the nanocomposites.

New methods are provided in the next chapter for determining the joint dynamic properties of the bolted lap joints. In addition, this study develops new nanocomposite sensors with piezoelectric and piezoresistive properties and high frequency bandwidth. Most of the available commercial sensors have limited bandwidth and can only be used for either static or dynamic measurements. Finally, new experimental methods are developed to extract the joint properties using the nanocomposite sensor. The methods developed in this study for joint dynamics identification are designed to eliminate the limitations and problems regarding the existing approaches. Therefore, the difficulties associated with rotational FRF extraction and slip boundaries identification are addressed. The established identification techniques can be coupled with the virtual prototype of a complex structure to extract the FRFs of the system and predict its dynamic behaviour.
Chapter Three: **JOINT DYNAMICS IDENTIFICATION**

Dynamic behaviour of assembled structures is considerably dependent on joint dynamics. The exact properties of the mechanical joints are required to build accurate mathematical and numerical models of the assembled structures. In this chapter, the joint properties of bolted lap joints are identified in transverse, tangential and torsional directions. The method presented for joint dynamics identification in transverse direction is based on an analytical approach developed using Euler Bernoulli theory. For translational and torsional directions, the lap joint is isolated by adding a mechanical resonator, consisting of a mass and spring, to the bolted structure and the frequency response function (FRF) of the new system is employed to determine the joint damping. The proposed methods along with the numerical and experimental results are elaborated in this chapter.

### 3.1 Joint Dynamics Identification in Transverse Direction

For identifying the joint parameters in transverse direction, two different methods are used, i.e. the inverse receptance coupling (IRC) method and the new analytical joint identification (AJI) approach. The former approach finds the joint frequency response function (FRF) by determining the difference between the FRF of the assembled structure and those of the substructures. However, the only required data for the AJI method are modal parameters of the assembled structure, which can be measured using experimental modal analysis. The accuracies of these two approaches are investigated using the finite element (FE) method. An assembled structure consisting of two thin beams connected to each other through a bolted joint is used for experimental
investigations. In addition, the effects of varying joint conditions, including surface roughness and the addition of interfacial materials, on the properties are investigated, in order to be able to favorably change the characteristics of the joint.

3.1.1 Inverse Receptance Coupling (IRC) method

Receptance coupling (RC) method is used in this section to generally find the relationship between the FRFs of the assembled structure and the FRFs of the substructures, in order to determine those of the joint. The joint parameters can be extracted using the IRC method, which is based on the difference between the FRF matrix of the assembled structure and the FRF matrix of each substructure (Park and Chae 2007). Figure 3.1 shows the case study in this research which is an assembled structure consisting of two beams connected to each other through a bolted lap joint.

![Figure 3.1. Schematics of two beams connected through a bolted joint](image)

Figure 3.1 presents the schematics of the coupled and uncoupled states of two substructures connected to each other through a mechanical joint. Points 1 and 4 are far from the joint on each substructure and, consequently, are not involved in the joint interface; and, points 2 and 3 represent the connected points on each substructure in the joint location. $F_i$ ($i = 1,2,3,4$) represent the force and moment of the assembled structure in each location shown in Figure 3.2. Moreover, $F'_i$ ($i=2,3$) is the force and moment of the joint in each connected point in the uncoupled state.
Figure 3.2. Schematics of the coupled and uncoupled states of the connected substructures

Using the uncoupled state shown in Figure 3.2, the relationship between the displacements and forces in each substructure is defined as:

\[
\begin{bmatrix}
X_1^B \\
X_2^B
\end{bmatrix} = \begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix} \begin{bmatrix}
F_1^B \\
F_2^B
\end{bmatrix}, \quad \begin{bmatrix}
X_3^A \\
X_4^A
\end{bmatrix} = \begin{bmatrix}
H_{33} & H_{34} \\
H_{43} & H_{44}
\end{bmatrix} \begin{bmatrix}
F_3^A + F_3^J \\
F_4^A
\end{bmatrix}
\]

(3.1)

F, H and X are defined as follows:

\[F_i^S = \begin{bmatrix} f_i^S \\ M_i^S \end{bmatrix}, \quad H_{ij} = \begin{bmatrix} H_{ij,tt} & H_{ij,rr} \\ H_{ij,rt} & H_{ij,rr} \end{bmatrix}, \quad X_i^S = \begin{bmatrix} x_i^S \\ \theta_i^S \end{bmatrix} \quad (i,j = 1,2,3,4), \quad (S = A,B,J)\]

(3.2)

where \(f\) is the force; \(M\) is the moment; \(x\) is the displacement; \(\theta\) is the rotation of each substructure in location \(i\); \(H_{ij,tt} = x_i/M_j\) and \(H_{ij,rr} = \theta_i/M_j\) are translational and rotational FRFs, respectively; and, \(H_{ij,rt} = x_i/M_j\) and \(H_{ij,rt} = \theta_i/M_j\) are cross FRFs.

The joint is assumed to be an element consisting of stiffness and damping. Therefore, the equilibrium condition at the joint is given by:

\[F_2^J + F_3^J = 0\]

(3.3)

The equation of motion for the joint element can be developed as:

\[
C_x(\dot{x}_3^A - \dot{x}_3^B) + K_x(x_3^A - x_3^B) = f_2^J \\
C_\theta(\dot{\theta}_3^A - \dot{\theta}_3^B) + K_\theta(\theta_3^A - \theta_3^B) = M_2^J
\]

(3.4)
where \( K_x, K_\theta \) are joint translational and rotational stiffness, respectively; and, \( C_x, C_\theta \) are joint translational and rotational damping, respectively.

By taking the Laplace transformation from both sides of Eq. (3.4) and then substituting \( S = i\omega \), this equation can be rewritten in matrix form and the frequency domain as follows:

\[
\{ X^A_3 - X^B_2 \} = \{ H \} \{ F^J \}
\]  

(3.5)

where \( H \) is the joint FRF matrix defined by:

\[
[H] = \begin{bmatrix}
  h'' & 0 \\
  0 & h''
\end{bmatrix} = \begin{bmatrix}
  K_x + iC_x\omega & 0 \\
  0 & K_\theta + iC_\theta\omega
\end{bmatrix}^{-1}
\]  

(3.6)

Eq. (3.7) shows the relationship between the displacements and forces of the assembled structure.

\[
\begin{bmatrix}
  X^B_1 \\
  X^B_2 \\
  X^A_3 \\
  X^A_4
\end{bmatrix} = \begin{bmatrix}
  G_{11} & G_{12} & G_{13} & G_{14} \\
  G_{21} & G_{22} & G_{23} & G_{24} \\
  G_{31} & G_{32} & G_{33} & G_{34} \\
  G_{41} & G_{42} & G_{43} & G_{44}
\end{bmatrix} \begin{bmatrix}
  F^B_1 \\
  F^B_2 \\
  F^A_3 \\
  F^A_4
\end{bmatrix}
\]  

(3.7)

where \( G_{ij} (i,j=1,2,3,4) \) is the FRF of the assembled structure.

By expanding Eq. (3.1), the relationship between the displacement vector of each substructure and FRF matrixes is obtained as:

\[
X^B_2 = H_{21}F^B_1 + H_{22}F^B_2 + H_{22}F^J_2
\]

\[
X^A_3 = H_{33}F^A_3 + H_{34}F^J_3 + H_{34}F^A_4
\]

(3.8)

Substituting Eq. (3.3) and Eq. (3.8) into Eq. (3.5) results in:

\[
X^A_3 - X^B_2 = H_{33}F^A_3 - H_{33}F^J_3 + H_{34}F^A_4 - H_{21}F^B_1 - H_{22}F^B_2 - H_{22}F^J_2 = H^J F^J_2 \rightarrow F^J_2 = B^{-1}H_{33}F^A_3 + B^{-1}H_{34}F^A_4 - B^{-1}H_{21}F^B_1 - B^{-1}H_{22}F^B_2 - H_{22}F^J_2, \quad B = H^J + H_{22} + H_{33}
\]

(3.9)
Inserting Eq. (9) into Eq. (1) gives:

\[
X_1^B = H_{14}^B F_1^B + H_{12}^B F_2^B + H_{12}^F \xrightarrow{\text{Eq.(9)}}
\]

\[
X_1^B = \left( \frac{H_{11} - H_{12}^B H_{21}}{g_{11}} \right) F_1^B + \left( \frac{H_{12} - H_{12}^B H_{22}}{g_{12}} \right) F_2^B + \left( \frac{H_{12}^B H_{33}}{g_{13}} \right) F_1^B + \left( \frac{H_{12}^B H_{34}}{g_{14}} \right) F_4^B
\]

(3.10)

Similarly, all assembled structure FRF terms can be obtained as a function of the substructures’ FRFs, as given in Eq. (3.11).

\[
\begin{bmatrix}
X_1^B \\
X_2^B \\
X_3^B \\
X_4^B
\end{bmatrix} =
\begin{bmatrix}
H_{11} - H_{12} B^{-1} H_{21} & H_{12} - H_{12} B^{-1} H_{22} & H_{12} B^{-1} H_{33} & H_{12} B^{-1} H_{34} \\
H_{21} - H_{22} B^{-1} H_{21} & H_{22} - H_{22} B^{-1} H_{22} & H_{22} B^{-1} H_{33} & H_{22} B^{-1} H_{34} \\
H_{33} B^{-1} H_{21} & H_{33} B^{-1} H_{22} & H_{33} - H_{33} B^{-1} H_{33} & H_{34} - H_{33} B^{-1} H_{34} \\
H_{34} B^{-1} H_{21} & H_{34} B^{-1} H_{22} & H_{34} - H_{34} B^{-1} H_{33} & H_{44} - H_{34} B^{-1} H_{34}
\end{bmatrix}
\begin{bmatrix}
F_1^B \\
F_2^B \\
F_3^A \\
F_4^A
\end{bmatrix}
\]

(3.11)

Since the joint FRF matrix has two unknowns in Eq. (3.6), only two assembled structure FRFs are needed to find the joint stiffness and damping. Two entries of the assembled structure FRF matrix expressed in Eq. (3.11) can be rewritten as follows:

\[
\begin{bmatrix}
G_{11,r} \\
G_{12,r} \\
G_{13,r} \\
G_{14,r}
\end{bmatrix} = \begin{bmatrix}
H_{11,r} & H_{11,r} \\
H_{12,r} & H_{12,r} \\
H_{13,r} & H_{13,r} \\
H_{14,r}
\end{bmatrix} \left( \begin{bmatrix}
B_{r} & -B_{r} \\
-B_{r} & B_{r}
\end{bmatrix} \begin{bmatrix}
H_{21,r} \\
H_{21,r}
\end{bmatrix} \right)
\]

\[
\begin{bmatrix}
G_{11,s} \\
G_{12,s} \\
G_{13,s} \\
G_{14,s}
\end{bmatrix} = \begin{bmatrix}
H_{11,s} & H_{11,s} \\
H_{12,s} & H_{12,s} \\
H_{13,s} & H_{13,s} \\
H_{14,s}
\end{bmatrix} \left( \begin{bmatrix}
B_{r} & -B_{r} \\
-B_{r} & B_{r}
\end{bmatrix} \begin{bmatrix}
H_{21,s} \\
H_{21,s}
\end{bmatrix} \right)
\]

(3.12)

Points 1 and 4 are chosen for developing Eq. (3.12), since they are distant from the joint location and more accessible for measurement than points 2 and 3, which are at the joint location.

The relationship between the translational assembled structure FRFs and substructures and joint FRFs can be extracted from Eq. (3.12).

\[
G_{11,r} = \frac{x_1}{f_1} = H_{11,r} \frac{1}{B_{r} B_{r} - B_{r} B_{r}} \left( (H_{12,r} B_{r} - H_{12,r} B_{r}) H_{21,r} + (-H_{12,r} B_{r} + H_{12,r} B_{r}) H_{21,r} \right)
\]

\[
G_{14,s} = \frac{x_4}{f_4} = \frac{1}{B_{r} B_{r} - B_{r} B_{r}} \left( (H_{12,s} B_{r} - H_{12,s} B_{r}) H_{34,s} + (-H_{12,s} B_{r} + H_{12,s} B_{r}) H_{34,s} \right)
\]

(3.13)

The translational joint FRF is obtained as a function of the translational assembled structure FRFs and substructures’ FRFs as given in Eq. (3.14).
\[ H^J_{ii} = B_{ii} - H_{12,ii} - H_{33,ii} = \]
\[ \frac{G_{14,ii}B_{ii}H_{12,ii} - H_{12,ii}^2H_{34,ii} + G_{11,ii}B_{ii}H_{34,ii} - B_{ii}H_{34,ii}H_{34,ii} + H_{12,ii}H_{12,ii}H_{34,ii}}{G_{14,ii}H_{12,ii} + G_{34,ii}H_{34,ii} - H_{11,ii}H_{34,ii}} - H_{22,ii} - H_{33,ii} \]  

(3.14)

where \( B_{ii} \) was derived using Eq. (3.13) and the MATLAB symbolic toolbox.

Substructure B shown in Figure 3.1 is a free-free beam, and its FRFs can be easily found using FE simulation. However, the FRFs of substructure A, which is a clamped-free beam, and the assembled structure translational FRFs should be measured experimentally. All of the obtained FRF terms are then substituted into Eq. (3.14) to extract the joint FRF matrix.

The joint properties can be obtained using Eq. (3.6). The joint translational FRF \( H^J_{ii} \) has two components, real and imaginary, and the joint stiffness and damping can be extracted as:

\[ H^J_{ii} = \frac{1}{K_x + C_x \omega j} \quad \rightarrow \quad K_x = \text{real} \left( \frac{1}{H^J_{ii}} \right) , \quad C_x = \frac{1}{\omega} \text{imag} \left( \frac{1}{H^J_{ii}} \right) \]  

(3.15)

The IRC method does not require rotational FRF measurements, which are not easy to obtain, and results in an explicit solution for the joint FRFs. However, some limitations still exist regarding this method: the cross FRFs between the rotational and translational FRFs are neglected in the joint FRF matrix; the measurement points are not always accessible; and, the measured FRFs are noise contaminated, which affects the results obtained for joint properties. A new approach that overcomes these limitations is presented in the next section.

3.1.2 Analytical Joint Identification (AJI) method

Another approach to identify joint parameters in transverse direction is proposed. This technique employs a combination of analytical and experimental data. The only required experimental data are the natural frequency and damping ratio of the whole structure, which can be readily measured through experimental modal analysis (EMA) (Sanati et al. 2017).
Figure 3.1(b) illustrates an equivalent model for the assembled structure shown in Figure 3.1(a). The effect of shear forces is neglected in this chapter as the joint properties are identified in transverse direction. However, these forces need to be taken into consideration if the joint dynamics are identified in tangential direction. Both connected beams have the same cross-sectional areas, but different lengths, in order to eliminate the effects of symmetry. The governing equations of motion for the model shown in Figure 3.1 are extracted using Euler-Bernoulli theory, which neglects the shear deformation and rotational inertia effects (Rao 2007; Moheimani and Ahmadian 2012; Pasharavesh et al. 2011). It is assumed that the test setup contains thin beams with high aspect ratios; consequently, shear deformation can be eliminated. The governing equations of motion for each part of the assembled structure is derived as follows:

\[
EI \frac{\partial^4 W_i(x, t)}{\partial x^4} + m \frac{\partial^2 W_i(x, t)}{\partial t^2} = 0 \quad , \quad i = 1, 2
\]

where \(E\), \(I\) and \(m\) denote the modulus of elasticity, second moment of area and the mass per unit length of the beam, respectively; and, \(W_i(x, t)\) and \(W_2(x, t)\) represent the transverse displacement of the beams A and B, respectively.

Assuming \(W_i(x, t)\) is separable in space and time, the following equation is developed as a solution for Eq. (3.16) (Guo et al. 2012; William 1996).

\[
\lambda_n^i = \omega_n \beta_j \pm \omega_n \sqrt{1 - \zeta^2}.
\]

where \(\omega_n\) and \(\zeta\) are the natural frequency and damping ratio of the assembled structure, respectively, which can be measured through experiments; and, \(j\) represents the imaginary part.

Substituting Eq. (3.17) into Eq. (3.16) results in:

\[
\frac{d^4 W_i(x)}{dx^4} - \beta_n^i \overline{W}_i(x) = 0 \quad , \quad \beta_n^i = \lambda_n \sqrt{\frac{m}{EI}} \quad , \quad i = 1, 2
\]

Eq. (3.19) represents the general solution for the above differential equation.
\[ \overline{W}_i(x) = C_1 \sin(\beta_n x) + C_2 \cos(\beta_n x) + C_3 \sinh(\beta_n x) + C_4 \cosh(\beta_n x), \quad i = 1, 2 \] (3.19)

Boundary and compatibility conditions are employed to find the unknown variables in Eq. (3.19). The left-hand endpoint of the assembled structure is clamped; therefore, the assembled structure does not experience any deflection at this location, i.e. Eq. (3.20). The beam at this point remains horizontal, so that the rotation/slope of the beam or the first derivative of the deflection function is zero, i.e. Eq. (3.21). Since the right-hand side of the assembled structure is free, there is no bending moment and shearing force at this point, resulting in Eqs. (3.22) and (3.23).

\[ x = 0: \quad y = 0 \Rightarrow W_i(0, t) = 0 \Rightarrow \overline{W}_i(0) = 0 \] (3.20)
\[ x = 0: \quad \theta = 0 \Rightarrow \frac{\partial W_i(0, t)}{\partial x} = 0 \Rightarrow \frac{d\overline{W}_i(x)}{dx} \bigg|_{x=0} = 0 \Rightarrow \overline{W}_i'(0) = 0 \] (3.21)
\[ x = L: \quad M = 0 \Rightarrow EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=L} = 0 \Rightarrow \frac{d^3 \overline{W}_i(x)}{dx^3} \bigg|_{x=L} = 0 \Rightarrow \overline{W}_i''(L) = 0 \] (3.22)
\[ x = L: \quad Q = 0 \Rightarrow EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=L} = 0 \Rightarrow \frac{d^3 \overline{W}_i(x)}{dx^3} \bigg|_{x=L} = 0 \Rightarrow \overline{W}_i'''(L) = 0 \] (3.23)

where \( M \) and \( Q \) are the bending moment and shearing force of the beam, respectively.

Four other required equations are developed using compatibility conditions. The mass effects of the joint in the assembled structure can be neglected to simplify the problem. The compatibility conditions are listed below:

\[ x = d: \quad Q_1 = Q_2 \Rightarrow EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=d} = EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=d} \Rightarrow \overline{W}_i''(d) = \overline{W}_i''(d) \] (3.24)
\[ x = d: \quad M_1 = M_2 = 0 \Rightarrow EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=d} = 0 \Rightarrow \overline{W}_i''(d) = \overline{W}_i''(d) = 0 \] (3.25)
\[ x = d: \quad Q_3 = f_k + f_c \Rightarrow -EI \frac{\partial^3 W_i(x, t)}{\partial x^3} \bigg|_{x=d} = K(W_i(d, t) - W_i(d, t)) + C(W_i(d, t) - W_i(d, t)) \] (3.26)

where \( \cdot \) denotes derivation with respect to time variable \( t \).
Applying the boundary and compatibility conditions results in the following equation.

$$[A]_{8 \times 8}[C]_{8 \times 1} = [0]_{8 \times 1}$$  \hspace{1cm} (3.27)

where $[A]$ is the coefficient matrix and $[C]$ is the unknown variables’ vector, which are shown in Eqs. (3.28) and (3.29).

\[
[A] = \begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\sin(\beta L) & -\cos(\beta L) & \sinh(\beta L) & \cosh(\beta L) \\
-\cos(\beta L) & \sin(\beta L) & \cosh(\beta L) & \sinh(\beta L) & \cos(\beta L) & -\sin(\beta L) & \sinh(\beta L) & \cosh(\beta L) \\
-\sin(\beta L) & -\cos(\beta L) & \sinh(\beta L) & \cosh(\beta L) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\sin(\beta L) & -\cos(\beta L) & \sinh(\beta L) & \cosh(\beta L) \\
A + B & P + Q & -P + Qh & -Bh & -Qh
\end{bmatrix}
\]  \hspace{1cm} (3.28)

\[
[C] = \begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8
\end{bmatrix}
\]  \hspace{1cm} (3.29)

Each cell of matrix $A$ is either a constant or a complex function of the natural frequency and damping ratio of the structure. Two separate equations may be obtained by using Cramer’s rule, i.e. $det(A) = 0$, where $det(A)$ represents the determinant of matrix $A$. Joint stiffness and damping parameters are then extracted by simultaneous solution of both equations. The natural frequency and damping ratio of the assembled structure are first measured experimentally. The captured values are then substituted into Eq. (3.28), and the joint properties are extracted using Cramer’s rule. A numerical simulation is carried out in the next section to investigate the accuracy of the proposed methods. The obtained numerical results for joint stiffness and damping using both approaches are compared.

### 3.1.3 Numerical simulations

The accuracy of the proposed methods for measuring the joint parameters is investigated numerically. In this case, beam elements are employed to model each substructure, and the joint
is modeled by linear stiffness and damping elements. Both substructures are considered to have the same material properties and cross-sectional areas, but different lengths. They are considered to have the material properties of aluminum with a modulus of elasticity \((E)\) of 72 GPa and density \((\rho)\) of 2700 kg/m3. The dimensions of each substructure and the number of elements are presented in Table 3.1.

**Table 3.1** Dimensions and number of elements used in the simulation

<table>
<thead>
<tr>
<th>Properties</th>
<th>Substructure A</th>
<th>Substructure B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width ((b))</td>
<td>25 mm</td>
<td>25 mm</td>
</tr>
<tr>
<td>Height ((h))</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>Length ((L))</td>
<td>0.45 m</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Number of elements</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

The stiffness and damping values of the joint are considered to be \(5 \times 10^5\) N/m and 150 N.s/m, respectively. The presented methods are used to find the joint parameters and then compare them with the input values to verify the accuracy of these methods. The FRF matrixes of each substructure and the assembled structure are easily extracted using the FE method. Figure 3.3 shows the translational FRF of the assembled substructure \((G_{11})\) in the frequency domain.

**Figure 3.3.** Assembled structure FRF \((G_{11})\) determined through the FE method
After inserting the FRFs of the assembled structure and each substructure into Eq. (3.14), the translational FRF of the joint is determined. The stiffness and damping of the joint are then obtained through the IRC method, i.e. Eq. (3.15). The average of the results over the frequency range are presented in Table 3.2, since the joint stiffness and damping are not perfectly constant and there are some small fluctuations over frequency domain.

In order to find the joint stiffness and damping parameters using the AJI method, the natural frequency and damping ratio of the structure need to be determined numerically. The natural frequency of the structure is obtained by the characteristic equation given in Eq. (3.30).

$$\det(K - M\omega^2) = 0$$ (3.30)

The damping ratio can be readily obtained using EMA in real applications. The peak-picking method, which is also called the half-power method, is used in this study to obtain the damping ratio (He and Fu 2001). This technique considers the FRF data in resonance areas as the data from a system with a single degree of freedom (SDOF). The frequency corresponding to the peak value of the FRF, as shown in Figure 3.4, is the natural frequency of the $r^{th}$ mode ($\omega_r = \omega_{peak}$). Damping of the $r^{th}$ mode is obtained using the half-power points at $\omega_a$ and $\omega_b$, which are located on each side of the $r^{th}$ resonance point of the FRF, with the amplitude of $\frac{H_{max}}{\sqrt{2}}\left(\zeta_r = \frac{\omega_r^2 - \omega_{peak}^2}{4\omega_r^2} \approx \frac{\omega_b - \omega_a}{2\omega_r}\right)$.

![Figure 3.4. Peak-picking method (He and Fu 2001)](image-url)
It is worth mentioning that the damping of each substructure is neglected in the calculation process, and it is assumed that all damping in the structure is caused by the joint. This is an acceptable assumption as (Gaul and Nitsche 2001) showed that material damping is much lower than the damping caused by a bolted joint. Moreover, (Beards 1982) experimentally demonstrated that up to 90 percent of the total damping in the system results from joints.

The extracted natural frequencies and the damping ratios (listed in Table 3.2) are substituted into Eq. (3.28). The stiffness and damping parameters of the joint (also listed in Table 3.2) are then found by solving the system of equations obtained from Cramer’s rule.

**Table 3.2** Dynamic characteristics of the system and identified joint parameters through the AJI and IRC methods

<table>
<thead>
<tr>
<th>Properties</th>
<th>Method</th>
<th>First Mode</th>
<th>Second Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency (Hz)</td>
<td>Peak-picking</td>
<td>13.256</td>
<td>59.899</td>
</tr>
<tr>
<td>Damping ratio (%)</td>
<td>Peak-picking</td>
<td>5.248E-4</td>
<td>1.212E-2</td>
</tr>
<tr>
<td>Joint stiffness (N/m)</td>
<td>IRC</td>
<td>4.999 E5</td>
<td>4.999 E5</td>
</tr>
<tr>
<td>Joint stiffness (N/m)</td>
<td>AJI</td>
<td>5.0282e+05</td>
<td>5.0451e+05</td>
</tr>
<tr>
<td>Error in identified joint stiffness (%)</td>
<td>IRC</td>
<td>2E-4</td>
<td>2E-4</td>
</tr>
<tr>
<td>Error in identified joint stiffness (%)</td>
<td>AJI</td>
<td>0.564</td>
<td>0.902</td>
</tr>
<tr>
<td>Joint damping (N.s/m)</td>
<td>IRC</td>
<td>150.001</td>
<td>150.001</td>
</tr>
<tr>
<td>Joint damping (N.s/m)</td>
<td>AJI</td>
<td>151.41</td>
<td>152.76</td>
</tr>
<tr>
<td>Error in identified joint damping (%)</td>
<td>IRC</td>
<td>6.6E-4</td>
<td>6.6E-4</td>
</tr>
<tr>
<td>Error in identified joint damping (%)</td>
<td>AJI</td>
<td>0.94</td>
<td>1.84</td>
</tr>
</tbody>
</table>

The numerical results show that the obtained values for joint stiffness and damping parameters using the IRC and AJI methods are close to their input values, verifying the accuracy of these approaches in joint parameters identification. Since the AJI approach only requires the
natural frequency and damping ratio of the assembled structure, which can be readily measured experimentally, it can be considered a practical alternative method in joint dynamics identification. A set of experiments is performed and the obtained results from both presented methods are compared with each other as elaborated in the next section.

3.1.4 Experimental investigation

Joint stiffness and damping parameters are experimentally obtained using the IRC and AJI methods, and the results are compared to each other. In addition, the effects of varying the joint condition, including the surface texture of the components and the addition of interfacial materials to the joint, on the joint properties are investigated.

3.1.4.1 Experimental verification

Figure 3.5 illustrates the experimental setup and test devices used in this study. The IRC and AJI approaches are utilized to obtain the stiffness and damping of the bolted lap joint. Two beams with smooth surfaces are connected to each other through a bolted joint. All pieces were made of aluminum (Al6061) with a modulus of elasticity ($E$) of 68.9 $GPa$, Poisson’s ratio ($\nu$) of 0.3 and a density ($\rho$) of 2712 kg/m$^3$. The geometric dimensions of the test setup are presented in Table 3.3.

![Figure 3.5. Experimental setup and equipment used in this study](image-url)
The FRFs terms required for the IRC method are extracted by conducting a set of experiments. The impact modal tests are performed using an instrumented force hammer (PCB 084A17) for exciting the structure by applying an impulse. In addition, an accelerometer (Kistler 8778A500) with the weight of 0.29 g is used for measuring the response as shown in Figure 3.5.

Table 3.3 Geometric dimensions of the test setup

<table>
<thead>
<tr>
<th>Properties</th>
<th>Substructure A</th>
<th>Substructure B</th>
<th>Assembled Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>25 mm</td>
<td>25 mm</td>
<td>25 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>5 mm</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>Length</td>
<td>150 mm</td>
<td>150 mm</td>
<td>271 mm</td>
</tr>
</tbody>
</table>

The required assembled structure FRFs, i.e. $G_{11}$ and $G_{14}$, are extracted by impacting at point 4 shown in Figure 3.5 and measuring the response of the system at points 1 and 4. Substructure B is a free-free beam; therefore, the accurate results for translational and rotational FRFs can be obtained through FE simulation. This substructure is modeled using the FE method, and the FRFs are numerically extracted and then updated using experimental data. Figure 3.6 shows one of the translational FRFs of substructure B ($H_{11,t}$), before and after updating using experimental results.

**Figure 3.6.** Translational FRF of substructure B ($H_{11,t}$)
Since the boundary condition of the left-hand side of the assembled structure is not perfectly clamped, the FE simulation may not give accurate results for the FRFs of substructure A. Hence, experimental tests are conducted, but the extraction of cross and rotational FRFs are challenging. In order to obtain the cross FRFs of substructure A, the finite difference method proposed by (Elliott, Moorhouse, and Pavić 2012) is used in this study. Figure 3.7 shows the impact and measurement points (points 1, 2), as well as a reference point (point 0), where the cross FRF needs to be extracted. The measurement points can be distant from the impact point.

\[ \Delta_s = \Delta_f \]

\[ \Delta_{s} - \Delta_{s} \]

\[ X_i \]

\[ X_j \]

\[ \alpha_{i} \]

\[ M_0 \]

\[ F_1 \]

\[ F_2 \]

\[ \Delta_f \]

\[ \Delta_f \]

**Figure 3.7.** Schematic of the applied force and displacement response for cross FRF measurement using the finite difference method (Elliott et al. 2012)

Considering the same values for \( \Delta_s \) and \( \Delta_f \), the cross FRF of the reference point can be developed as a function of translational FRFs in measurement points as presented in Eq. (3.31) (Elliott et al. 2012).

\[
H_{0,3} = \frac{-H_{1,x} + H_{2,x}}{4\Delta} \quad (3.31)
\]

The cross FRF of the substructure A \((H_{33,rt})\) is illustrated in Figure 3.8.
Substituting the obtained FRFs of the assembled structure and each substructure into Eq. (3.14) results in the translational FRF of the joint, as shown in Figure 3.9.

By separating the real and imaginary parts of the identified joint translational FRF \( H_{tt} \) and subsequent substitution into Eq. (3.15), the stiffness and damping of the bolted joint in transverse
direction are extracted. The obtained joint stiffness and damping parameters using the IRC method are shown in Figure 3.10 and Figure 3.11.

![Joint Stiffness](image1)

**Figure 3.10.** Identified joint stiffness using the IRC method

![Joint Damping](image2)

**Figure 3.11.** Identified joint damping using the IRC method

Experimental modal analysis (EMA) is employed to extract the modal parameters of the assembled structure. The captured FRF of the assembled structure is then curve fitted to obtain the experimental natural frequencies and damping ratios. The measured modal parameters along with
the material properties and dimensions are inserted into Eq. (3.28). The Cramer’s rule results in the joint stiffness and damping values, as presented in Table 3.4.

**Table 3.4** Identified joint properties using the AJI method, considering the first three modes

<table>
<thead>
<tr>
<th>Number of Mode</th>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
<th>Joint Stiffness (N/m)</th>
<th>Joint Damping (N.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>32.52</td>
<td>2.51</td>
<td>490.21</td>
<td>0.124</td>
</tr>
<tr>
<td>Second mode</td>
<td>184.02</td>
<td>0.686</td>
<td>1.5435E4</td>
<td>0.885</td>
</tr>
<tr>
<td>Third mode</td>
<td>590.24</td>
<td>5.19</td>
<td>9.2457E4</td>
<td>3.776</td>
</tr>
</tbody>
</table>

The comparison of the obtained results shows that there is an agreement between the two techniques, since the identified joint parameters are in the same order of magnitude. This demonstrates the accuracy of the proposed AJI method in identifying the joint properties. However, there are some discrepancies between the results obtained from the AJI and IRC methods. This can be attributed to limitations and errors in both methods. There are some uncertainties in the IRC method, especially cross FRF identification, which is extracted using the finite difference approach. This is considered one of the main challenges associated with the IRC method that may affect the final results. Moreover, the extracted FRFs used in the IRC method are noise contaminated, which may cause some discrepancies in the results. On the other hand, the AJI method is sensitive to the input values, including dimensions of the structure, material properties and modal parameters for calculating the joint properties. The effects of the joint condition on the joint parameters are investigated in the next subsection.
3.1.4.2 Effect of joint condition on joint properties

The joint condition changes in order to investigate its effects on the joint properties. The joint condition is manipulated by varying the surface texture of substructures in the joint location with sandblasting and by inserting interfacial material into the joint.

In the new experimental setup, all dimensions and material properties are the same as with the previous setup. In this case, the roughness of each substructure is changed using abrasive blasting. A rubber gasket made of chloroprene (neoprene) is also placed in between the substructures at the joint location. For each experiment, the FRFs of the entire structure and individual substructures are extracted either numerically or experimentally. The IRC method is then used to find the values of the stiffness and damping of the joint for each case. The results are shown in Figure 3.12-Figure 3.15.

The natural frequencies and damping ratios of the assembled structure are extracted using the translational FRF and curve fitting technique for each experiment. The obtained modal parameters are then utilized into the AJI method for the extraction of the stiffness and damping of the joint. Table 3.5 and Table 3.6 present the joint parameters identified through the AJI method for the abovementioned conditions.

![Figure 3.12](image-url)  
**Figure 3.12.** Identified joint stiffness using the IRC method for sandblasted surface
Figure 3.13. Identified joint damping using the IRC method for sandblasted surface

Figure 3.14. Identified joint stiffness using the IRC method for the joint with rubber interface
Figure 3.15. Identified joint damping using the IRC method for the joint with rubber interface

Table 3.5 Identified joint properties for the sandblasted surface using the AJI method, considering the first three modes

<table>
<thead>
<tr>
<th>Number of Mode</th>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
<th>Joint Stiffness (N/m)</th>
<th>Joint Damping (N.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>32.69</td>
<td>2.24</td>
<td>495.26</td>
<td>0.11</td>
</tr>
<tr>
<td>Second mode</td>
<td>185.78</td>
<td>0.596</td>
<td>1.4108E4</td>
<td>0.68</td>
</tr>
<tr>
<td>Third mode</td>
<td>594.08</td>
<td>4.71</td>
<td>9.4321E4</td>
<td>3.49</td>
</tr>
</tbody>
</table>

Table 3.6 Identified joint properties for the joint with rubber using the AJI method, considering the first three modes

<table>
<thead>
<tr>
<th>Number of Mode</th>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
<th>Joint Stiffness (N/m)</th>
<th>Joint Damping (N.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>32.29</td>
<td>3.15</td>
<td>479.59</td>
<td>0.15</td>
</tr>
<tr>
<td>Second mode</td>
<td>182.95</td>
<td>1.08</td>
<td>1.4002E4</td>
<td>1.28</td>
</tr>
<tr>
<td>Third mode</td>
<td>583.94</td>
<td>6.52</td>
<td>8.9732E4</td>
<td>4.61</td>
</tr>
</tbody>
</table>
The obtained results show that increasing the surface roughness slightly change the dynamic properties of the bolted joint with partial increase in the joint stiffness. However, the addition of interfacial material to the joint increases the joint damping, whereas the joint stiffness decreases. The effect on damping is more than that of stiffness. The interfacial material increases the contact area, resulting in more energy loss and damping. Moreover, the structure becomes more flexible, with less stiffness in the joint.

3.1.5 Limitations and assumptions

There are several assumptions and limitations associated with the proposed methods. The joint properties in lap joints in this study are modeled as linear stiffness and damping elements. However, this model is only valid while the dynamic behaviour of the structure is linear, the contact area in the joint interface remains constant, and the joint size is small. The joint preload and level of excitation applied to the structure have a significant effect on the dynamic behaviour of the system and can change the joint dynamic behaviour from linear to nonlinear (Ouyang et al. 2006). The joint structure is designed in such a way to behave in the linear range. Hence, the impacts applied to the structure during the experiments are kept at low levels and the amount of applied torque to the bolt in the joint location remains at 9 N.m to avoid the nonlinear frictional behaviour, such as micro and macro slip in the joint interface.

The AJI method is sensitive to the input data, i.e. the dimensions of the structure, which include measurement errors, and material properties that are selected from the standard material properties; however, there are always discrepancies between the material properties of standard and manufactured parts. Moreover, the AJI method is applicable where the aspect ratio of the whole structure is large enough to neglect the effects of shear deformation and rotational inertia.
In the IRC method, the joint is considered to be time-invariant and stable in the frequency range. It should be mentioned that considering the joint section as a lumped parameter system may lead to some errors in the identification. In order to minimize the noise effects and improve the consistency of the experimental results, the experimental measurements are carried out several times, and the average of the results are considered in the calculation process.

In addition to the joint properties in transverse direction, one needs to identify the joint parameters in tangential and torsional directions as well, especially for the case that the structure is under tangential and torsional excitations. Next section presents a method for determining the joint damping in other directions.

### 3.2 Joint Damping Identification in Tangential and Torsional Directions

A new experimental approach is proposed to identify the damping of bolted lap joints in tangential and torsional directions. The joint is isolated through adding a mechanical resonator to the bolted structure. The frequency response function (FRF) of the combined structure is then used for joint damping identification without any need to determine the stick-slip boundaries. The method is verified by a set of experiments.

#### 3.2.1 Proposed method for joint damping identification

An experimental method is developed in this study to eliminate the limitations of the existing approaches in joint damping identification. This method is based on translational FRF extraction that can be readily obtained using experimental modal analysis (EMA) (Sanati et al. 2018a).

Identifying the joint properties is a complex procedure due to the nonlinear nature of the joint and difficulties in extracting the stick-slip boundaries. In his study, the bolted lap joint, that
is a continuous structure with infinite modes of vibration, is converted to a component consisting of only damping and stiffness elements. It can be achieved using a mechanical resonator where a lumped mass is attached to the bolted lap joint structure using a linear spring as shown in Figure 3.16.

![Figure 3.16. The schematic of the developed approach for joint damping identification](image)

The mass and stiffness of the mechanical resonator should be significantly higher than those of the bolted lap joint structure. It is assumed that the damping of the mechanical resonator is negligible compared to that of the lap joint. The new combined structure contains only one dominant mode of vibration. The mass and spring of the mechanical resonator have a minimal effect on the obtained joint damping and they mainly change the resonance frequency of the new equivalent system.

The FRF amplitude at the resonance frequency of the obtained single dominant mode can be used for extracting the joint damping, since the main source of damping and energy dissipation in the system is through the bolted joint; and, the damping from other sources, such as structural damping, is not considerable compared to the joint damping. The developed method eliminates the difficulties associated with the identification of the microslip boundaries in the contact zone and indirectly results in the joint damping parameter.
The experimental setup used in this study for the joint damping identification in translational direction is shown in Figure 3.17. The excitation was applied to the mechanical resonator and the bolted structure using a shaker (Bruel & Kjaer 4808). A flexible rod was used for transferring the excitation from the shaker to the combined structure. The input force to the system was measured using a 3-component force sensor (Kistler 9018B). The displacement of the mechanical resonator and bolted beam were measured using a fiber optic displacement sensor (Philtec RC20-Q) and a wide frequency bandwidth capacitive sensor (Lion Precision DMT20), respectively.

![Experimental setup used for joint damping identification in translational direction](image)

**Figure 3.17.** Experimental setup used for joint damping identification in translational direction

Some considerations should be taken into account during the experimental process. It is preferable to not use fast, continuous sine sweep excitations to extract the FRF of the combined structure since the joint dynamics are nonlinear. Also, this type of fast, continuous sweeping excitation causes discrepancies between the measured and actual FRFs. Therefore, the sine sweep
excitation is only used to first estimate the resonance frequency of the system (Ahmadian and Jalali 2007). Then, sinusoidal excitations in a frequency band around the estimated resonance frequency are applied to the structure. The measured signals of the force input to the system and the lumped mass displacement are used to extract the FRF of the system around the resonance frequency and subsequently identify the joint damping through the developed approach. In addition, the force and displacement signals at the resonance frequency are employed to find the hysteresis loop of the system and then extract the joint damping.

The conditions used in the experimental investigations are shown in Table 3.7.

**Table 3.7** Experimental conditions used in this study

<table>
<thead>
<tr>
<th>Properties</th>
<th>Experimental Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of the bolted beam</td>
<td>100 × 40 × 10 mm</td>
</tr>
<tr>
<td>Material properties of the bolted beam</td>
<td>Steel 1018</td>
</tr>
<tr>
<td>Applied bolt torque – translational excitation</td>
<td>2 N.m</td>
</tr>
<tr>
<td>Applied bolt torque - torsional excitation</td>
<td>8 N.m</td>
</tr>
<tr>
<td>Surface roughness in the contact interface</td>
<td>Ra=2.31 µm</td>
</tr>
<tr>
<td>Moment arm length</td>
<td>45 mm</td>
</tr>
<tr>
<td>Mass of the mechanical resonator</td>
<td>1.4 kg</td>
</tr>
<tr>
<td>Stiffness of the mechanical resonator</td>
<td>2.693e7 N/m</td>
</tr>
</tbody>
</table>

As presented in Table 3.7, the bolt torque applied to the structure for translational and torsional are different to assure that the joint interface stays in the microslip regime during the experiment and no macroslip motion occurs. Also, it is worth mentioning that the stiffness of the mechanical resonator presented in Table 3.7 is obtained using the FE simulation.
In order to employ the presented method for measuring the torsional damping of the bolted lap joints, the experimental setup needed to be modified as shown in Figure 3.18. As can be seen, the mechanical resonator is attached in perpendicular to the assembled structure. Therefore, the translational excitation of the mechanical resonator resulted in torsional movement of the bolted beam, with the energy loss occurring from the torsional friction.

![Experimental setup used for torsional joint damping measurement](image)

**Figure 3.18.** Experimental setup used for torsional joint damping measurement

The frequency response function (FRF) of a system with one dominant mode of vibration is derived as:

$$H(\omega) = \frac{1}{k} \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{\omega}{\omega_n}\right)j}$$

(3.32)

where $\omega$ is the excitation frequency, $\omega_n$ is the natural frequency, $k$ is the static stiffness of the structure, $\zeta$ is the damping ratio, and $j$ represents the imaginary unit.
The real and imaginary parts of the FRF can be obtained by multiplying the numerator and
denominator of Eq. (3.32) by the complex conjugate of the denominator. Considering \( r = \omega / \omega_n \),
the real and imaginary parts of the FRF are given by:

\[
\begin{align*}
\text{Re}(H(\omega)) &= \frac{1}{k} \frac{1-r^2}{(1-r^2)^2 + (2\zeta r)^2} \\
\text{Im}(H(\omega)) &= \frac{1}{k} \frac{-2\zeta r}{(1-r^2)^2 + (2\zeta r)^2}
\end{align*}
\] (3.33)

Therefore, the amplitude of the FRF can be obtained from the real and imaginary parts as
shown in Eq. (3.34).

\[
\left| H(\omega) \right| = \sqrt{\text{Re}(H(\omega))^2 + \text{Im}(H(\omega))^2} = \frac{1}{k} \sqrt{\frac{1}{(1-r^2)^2 + (2\zeta r)^2}}
\] (3.34)

The maximum of the FRF occurs at the resonance frequency where the excitation frequency
is equal to the natural frequency of the system, i.e. \( r = 1 \). Therefore,

\[
\left| H(\omega) \right|_{\text{max}} (i) = \frac{1}{2k\zeta(i)}
\]

\[
C(i) = 2\zeta(i)\sqrt{mk} = \frac{1}{\left| H(\omega) \right|_{\text{max}} \times \omega_n}
\] (3.35)

where \( \zeta \) is the damping ratio of the structure, \( H_{\text{max}} \) is the FRF amplitude at the resonance
frequency \( (\omega_r) \), \( \omega_n \) represents the step associated with each excitation amplitude. Also, \( C \) represents
the damping of the structure where the natural frequency \( (\omega_n) \) and maximum FRF amplitude
\( (H_{\text{max}}) \) are extracted from the FRF. It is assumed that damping ratio of the structure is not
considerable and therefore \( \omega_n \approx \omega_r \).
Another method of joint damping identification is presented in the next section and subsequently the results of both approaches are compared with one another.

3.2.2 Hysteresis loop approach

To verify the obtained joint damping data, the results need to be compared with another reliable method such as the hysteresis loop approach. Figure 3.19 shows a typical hysteresis loop that could be obtained in the experiments by measuring the force input to the lumped mass and its displacement.

![Schematic of a typical hysteresis loop](image)

**Figure 3.19.** Schematic of a typical hysteresis loop

The area inside the hysteresis loop represents the dissipated energy in the joint interface through friction. Eq. (3.36) represents the energy loss of the developed system since the joint interface is considered as the main source of energy dissipation in the structure.

\[
W_d = \oint F_d \, du = \oint C \dot{u} \, du = \oint C \dot{u}^2 \, dt
\]

(3.36)

where \( W_d \) is energy loss, \( u \) is displacement of the lumped mass, and \( C \) is damping of the structure. The damping is mainly caused due to friction at the joint interface.
Considering the harmonic excitation applied to the system, the response of the system will be harmonic with the same excitation frequency. Therefore,

\[ u(t) = U \sin(\omega t - \varphi) \quad , \quad \dot{u}(t) = U \omega \cos(\omega t - \varphi) \quad (3.37) \]

where \( U \) is displacement amplitude, \( \omega \) is the excitation frequency, and \( \varphi \) represents the phase shift between the excitation and the response.

Substituting Eq. (3.37) into Eq. (3.36) results in:

\[ W_d = C \omega^2 U^2 \int_0^{2\pi/\omega} \cos^2 (\omega t - \varphi) dt = \pi C \omega U^2 \quad (3.38) \]

Therefore, the equivalent damping of the system, i.e. the joint damping, is obtained by (Shamoto et al. 2014):

\[ C(i) = \frac{W_d(i)}{\pi \omega U^2(i)} \quad (3.39) \]

where the energy loss (\( W_d \)) and \( U \) can be obtained from calculating the area inside the hysteresis loop and measuring the displacement amplitude of the lumped mass, respectively.

The damping results of a bolted lap joint under different levels of excitation are obtained using both developed approaches and presented in the next section. The discussion on the results along with the future works of this study are discussed as well.

3.2.3 Results and discussion

The accuracy of the developed approach in joint damping identification is investigated through a set of experiments for both translational and torsional directions.

In order to find the joint damping using the proposed method, the FRF of the combined structure is needed to be determined. Figure 3.20 depicts that the prepared system only contains
one dominant mode of vibration, while the bolted structure itself has several modes in the same frequency range. Different levels of excitation are applied to the system in the translational direction to cause different levels of microslip motion in the joint interface; and, the changes in the damping are tracked as a function of the excitation amplitude.

Figure 3.20. FRF of the bolted structure with and without the mechanical resonator

It is expected to observe consistent FRFs even if different levels of excitation are applied to a structure that has no mechanical joints. However, the FRFs change slightly under varying loads when it comes to the assembled structures with bolted lap joints. This is due to the excitation of different microslip and macroslip regimes corresponding to different force amplitudes that affect the resulting FRF of the system. Figure 3.21 shows the FRFs of the system for different excitation levels in the translational direction. As can be seen in this figure, increasing the excitation level decreases the amplitude of the FRF at the resonance frequency which indicates the increase of damping in the system. The resonance frequency also decreases when increasing the excitation load, since the slip area expands, and the stiffness of joint interface reduces.
Figure 3.21. FRFs of the combined structure for different excitations in translational direction

Figure 3.22 shows the hysteresis loop for each excitation level plotted using the input force to the structure and the displacement of the lumped mass at the resonance frequency. Vpp shown in Figure 3.21 and Figure 3.22 represents the peak to peak voltage of the sinusoidal signal input to the shaker.

Figure 3.22. Hysteresis loops in translational direction for different excitation levels
The area inside the hysteresis loops \((W_d)\) expanded by increasing the excitation load and consequently the energy loss due to friction increased. By calculating the area inside each loop \((W_d)\), the energy loss is calculated. Figure 3.23 shows the changes of the energy loss by increasing the displacement amplitude of the bolted beam.

![Energy Loss Graph](image)

**Figure 3.23.** Energy loss in translational excitation vs lumped mass displacement for different excitation levels

The energy loss in Figure 3.23 shows an increasing slope with increase of the displacement amplitude which indicates the fact that only microslip motion occurs in the contact interface since the macroslip results in a constant slope for the change of the energy loss versus displacement amplitudes (Gaul and Lenz 1997).

Figure 3.24 shows the translational joint damping for each excitation level for both the newly developed and the hysteresis loop methods. The joint damping is identified using Eq. (3.35), based on the developed approach, and Eq. (3.39) is employed to obtain the joint damping using the hysteresis loop method.
The comparison of the translational joint damping obtained from both approaches demonstrates that there is a good agreement between the results. It verifies the accuracy of the developed method in joint damping identification.

![Graph showing joint damping comparison](image)

**Figure 3.24.** Joint damping in translational direction for different displacements of the bolted beam

The accuracy of this method in determining the joint damping in torsional direction is investigated as well. The experimental setup shown in Figure 3.18 is used for this purpose. The process for torsional damping identification of a bolted lap joint is the same as that for the translational direction: the external force is applied to the mechanical resonator; and, the joint damping is obtained from the FRF.

Figure 3.25 shows the FRFs of the system when the bolted beam is under torsional excitation with different levels. Same as the translational direction, force and displacement signals at the resonance frequency are used to extract the hysteresis loop at each excitation level as depicted in Figure 3.26. These results similarly prove that higher excitation levels provide wider hysteresis loops and more energy dissipation as shown in Figure 3.27.
Figure 3.25. FRFs of the combined system for different excitation levels in torsional direction

Figure 3.26. Hysteresis loops in torsional direction for different excitation levels

Eq. (3.35) is employed to obtain the joint damping in torsional direction using the developed method. Also, this parameter is identified using the hysteresis loop method through Eq. (3.39). The results extracted from both approaches are compared to one another in Figure 3.28 where a good agreement is achieved.
Figure 3.27. Energy loss in torsional excitation vs lumped mass displacement for different excitation levels

Figure 3.28. Joint damping in the torsional direction for different rotational displacements of the bolted beam

As with the translational direction, increases in the excitation level decreased the resonance frequency, due to development of the sliding area. By comparing the results shown in Figure 3.24 and Figure 3.28, it can be concluded that the torsional joint damping of a bolted structure under
the same level of excitation is higher than the translational joint damping due to the moment that is applied to the bolted structure.

There are some assumptions in this study that might cause discrepancies in the results. The assumptions and limitations are discussed in the next section.

3.2.4 Limitations and assumptions

Some assumptions were made in this study that might affect the accuracy of the obtained results. The identified damping using the developed method involves all sources of damping, including the joint damping and structural damping. However, the resulting damping effect is assumed to be largely dominated by the joints. The assumption is justified since the structural damping of the mechanical resonator without being attached to the bolted lap joint was experimentally found negligible. In addition, Eqs. (3.35) and (3.39) involve some simplifications and using them for joint damping identification might cause discrepancies. Also, the natural frequency is assumed to be approximately the same as the resonance frequency due to small damping ratio in the system. In the developed setup, the mechanical resonator is attached to the bolted beam through some different connections including mechanical bolts and it affects on the accuracy of the results as they add damping to the system. In order to have more accurate results the mechanical resonator can be attached to the bolted beam using more rigid connections such as welding; however, aligning two systems would be challenging in this case and should be done with care.
3.3 Summary

Different techniques for identifying the joint properties of bolted lap joints in transverse, tangential and torsional directions were proposed in this chapter. Joint dynamic properties were identified in transverse direction using two different methods, i.e. (a) the inverse receptance coupling (IRC) technique, which is an FRF-based method; and, (b) the analytical joint identification (AJI) method, which only requires the natural frequency and damping ratio of the structure. The accuracies of these methods were verified numerically using the finite element (FE) method. These approaches were also used to experimentally identify the stiffness and damping of a lap joint, and the results show that there is a good agreement between the two methods. The effects of the joint condition on the joint properties were experimentally investigated through both presented methods. While increasing the surface roughness of substructures in the joint location has a small effect on the joint dynamics, the addition of interfacial materials into the joint decreases the joint stiffness but increases the damping. The AJI method has some advantages over the IRC method; however, it is sensitive to the input variables. The AJI approach needs only one measurement, whereas the IRC method requires a set of experiments to extract the FRFs of the whole structure and substructures. In addition, the rotational and cross FRF measurements, which are necessary in the IRC method, are challenging; and, the measurement points are sometimes inaccessible. However, the modal parameters of the structure can be readily obtained using experimental modal analysis (EMA) and then used in the AJI method.

In addition, a new experimental method was developed for identifying the translational and torsional damping of bolted lap joints. This technique allows for more practical prediction of the damping of assembled structures. In this approach, the joint of a bolted structure was isolated using a mechanical resonator consisting of mass and spring elements. The joint damping was then
obtained using the FRF of the new combined system. This technique eliminates the difficulties associated with slip boundaries identification. The accuracy of this method was verified after comparing its results with those of the hysteresis loop approach. The developed method can be extended into other types of mechanical joints.

Notwithstanding the effectiveness of the presented methods for joint dynamics identification, it has been always required to develop a flexible and thin sensor that can be implemented in the joint interface and predict the joint properties directly. Next chapter presents the fabrication process of the nanocomposite sensor along with the experimental results.
Chapter Four: DEVELOPMENT OF NANOCOMPOSITE SENSOR WITH PIEZOELECTRIC AND PIEZORESISTIVE PROPERTIES

Sensors provide an interface between mechanical systems and the physical world. With the move towards Industry 4.0 and cyber-physical systems, demands for cost-effective sensors are rapidly increasing. Conventional sensors used for monitoring manufacturing processes are often bulky and need complex processes. It is desirable to develop a new type of sensors that can be inserted into the joint interface and used for joint dynamics identification and other monitoring applications while the dynamic of the original system is almost unchanged. A novel nanocomposite sensor is developed in this chapter. The fabrication process of the sensor is discussed and the procedure for improving the piezoelectricity of the sensor is elaborated. The sensor can measure both static and dynamic loads over a wide range of frequencies since it exhibits both piezoelectric and piezoresistive properties. The acquired signals are fused to improve the accuracy and frequency bandwidth of the sensor. The performance of the sensor is evaluated through both static and dynamic tests and the results are compared with those of the commercial sensors. A 3D random walk model and a 2D finite element (FE) model are presented for modeling the piezoresistive and piezoelectric characteristics of the sensor, respectively. The accuracy of the models is verified after comparing the numerical results with the experiment measurements.

4.1 Fabrication Process and Experimental Setup

The fabrication process of the nanocomposite sensor, including the solution preparation and spray coating is presented (Sanati et al. 2018b). In addition, the process required for improving the
piezoelectric $\beta$ phase of the polymer matrix and subsequently increasing the piezoelectric coefficient of the sensor is discussed. Also, the experimental setups used for validation of the sensor are presented.

4.1.1 Fabrication of the Nanocomposite Sensor

Polyvinylidene fluoride (PVDF) is used as the polymer matrix of the developed sensor due to its high piezoelectricity. In addition, PVDF can form semi-crystalline structure (Ferroelectric) after poling. PVDF powder with the size of 100 nm was prepared from Sigma Aldrich. A solution of PVDF and N-N dimethylformamide (DMF) with 0.1 g/mL is stirred on the hot plate at 80 °C for 3 h until the polymer is fully dissolved. CNTs with 0.0004 g/mL concentration are separately mixed in DMF and sonicated at room temperature for 30 min to achieve a homogeneous dispersion of nanoparticles. The mixture is then added to the PVDF-DMF solution and stirred for 1 additional hour at room temperature. The schematic of the fabrication process is illustrated in Figure 4.1.

![Figure 4.1. The schematic of the fabrication process](image-url)
Once the mixture is prepared, a thin layer with 70 μm thickness is deposited using spray coating. The spray system used in this study is an air brush system connected to a compressed air supply. The nanocomposite mixture is poured inside the air brush system and fine droplets are deposited onto a glass substrate. The sprayed layer of the nanocomposite is heated at 80 °C till the solvent is fully evaporated and then the film is peeled off of the substrate.

Figure 4.2 shows the different steps associated with the fabrication process of the nanocomposite sensor.

![Figure 4.2](image)

*Figure 4.2. The fabrication process steps of the nanocomposite sensor*

The CNT network inside the polymer matrix results in a conductive network and, therefore, provides piezoresistive properties needed for measurement of static and low frequencies loads and strains. However, it is still required to activate the piezoelectric properties of the PVDF polymer to measure higher frequencies with increased sensitivity. There are many ways of accomplishing this, but for this study two sequential processes of mechanical stretching and high-voltage poling
are used to change the structure of the PVDF polymer and increase its piezoelectric properties. These processes are elaborated in the next subsection.

4.1.2 Sequential Stretching and Poling

To improve the piezoelectricity of the prepared samples, the $\beta$ crystallites of PVDF should increase and the existing $\alpha$ phase of PVDF should be converted to the $\beta$ phase. Mechanical stretching, and electrical poling of the samples are utilized sequentially in this study for improving the $\beta$ phase and subsequently the piezoelectricity of the sensor. It is presented in the literature that the sequential stretching and poling can result in improved piezoelectric properties in PVDF-based samples (Mahadeva et al. 2013).

The prepared samples were first stretched mechanically at the temperature of 80 °C. It is shown that the transition from $\alpha$ phase to $\beta$ phase is less effective at temperatures above 80 °C (Sencadas, Gregorio, and Lanceros-Méndez 2009). A custom device was built for stretching the nanocomposite samples as shown in Figure 4.3. The sample is held between a fixed and moving clamp. A stepper motor driver and linear stage is used to provide the movement. The speed of the moving clamp is adjusted using an electronic speed controller and the sample is stretched to 500% elongation. It has been shown by (Li et al. 2014) that there is a direct relationship between the stretching ratio and transformation from $\alpha$ phase to $\beta$ phase of PVDF. They showed that the amount of $\beta$ phase rapidly increases by increasing the stretching ratio up to about 300% elongation. However, after this value, the $\beta$ phase transition reaches a constant value and does not change considerably. Applying more stretching ratio, such as 500% in this study, results in a thinner film and subsequently allows a higher electric field during the poling process with the same voltage
potential applied to the system. Therefore, alignment of the dipoles of the polymer occurs more efficiently and a higher piezoelectric coefficient is achieved.

Figure 4.3. (a) Custom mechanical stretching device; (b) stretching of nanocomposite; (c) sample of the stretched nanocomposite film

Fourier transform infrared spectroscopy (FTIR) spectra of the sample before and after stretching provides insight about the structure of the PVDF polymer. FTIR allows the identification of different crystalline forms and to monitor the transition between different phases. Among different phases of the PVDF polymer, $\gamma$ and $\beta$ phases are similar in their polymer structure and can be identified by the absorbance peak at certain wavenumbers. For example, the band at 840 cm$^{-1}$ is attributed to both $\gamma$ and $\beta$ phases, and this peak carries a dual signature of both phases (Martins et al. 2014). However, the band at 1279 cm$^{-1}$ is exclusively attributed to the $\beta$ phase and consequently monitoring the absorbance peak in this wavenumber can provide information about the $\beta$ phase transition in the sample after stretching and poling (Martins et al. 2014).
The FTIR spectra of the prepared samples, i.e., pure PVDF, unstretched PVDF–CNT, stretched PVDF–CNT and polled PVDF–CNT, are presented in Figure 4.4.

**Figure 4.4.** Fourier transform infrared spectroscopy (FTIR) of the PVDF and PVDF-carbon nanotube (CNT) samples before and after stretching.

The results illustrated in Figure 4.4 show that the unstretched PVDF sample contains mostly α phase due to the high absorbance peaks at 763, 795 and 974 cm\(^{-1}\), and almost no peak corresponding to β phases is observed which demonstrates that the polymer mainly consists of the non-polar α phase.

(Gregorio, Jr. and Cestari 1994) calculated the absorption coefficients, \(K_\alpha\) and \(K_\beta\), at the receptive wavenumbers 766 and 840 cm\(^{-1}\) after assuming that the FTIR absorption follows the Lambert–Beer law and presented Eq. (4.1) for calculating the β phase of PVDF considering that the polymer is comprised of only α and β phases:

\[
\beta = \frac{A_\beta}{\left(\frac{K_\beta}{K_\alpha}\right) A_\alpha + A_\beta} \tag{4.1}
\]
where $\beta$ represents the beta phase content, $A_\alpha$ and $A_\beta$ are the absorbance in wavenumbers 766 and 840 cm$^{-1}$, respectively, and $K_\alpha$ and $K_\beta$ equal to $6.1 \times 10^4$ and $7.7 \times 10^4$ cm$^2$mol$^{-1}$, respectively.

Adding CNTs into PVDF show an increase in the absorbance at wavenumber 840 cm$^{-1}$ corresponding to the $\gamma$ and $\beta$ phases. However, there is no considerable absorption peak at 888 and 1234 cm$^{-1}$ corresponding to $\gamma$ phase which implies that the polymer is mainly made of $\alpha$ and $\beta$ phases. Based on Eq. (4.1) adding CNTs into PVDF increases the $\beta$ phase content from 32% to 78%. After stretching the $\beta$ phase increases to 83% and the associated peaks at 840 and 1280 cm$^{-1}$ became more pronounced, indicating an improvement of the polar $\beta$ phase in the samples. In addition, the obtained results prove that reheating and poling of the samples after stretching do not have any remarkable effect on the $\beta$ phase of the samples, but it is necessary to align the newly formed $\beta$ phase crystallites in a single direction to improve the piezoelectricity of the sample.

To see the effect of stretching on the piezoelectricity of the prepared samples, the piezoelectric charge coefficient of the sample along the thickness direction is measured. This parameter is defined as the ratio of electric charges per unit area generated in response to an applied force. Considering that the thickness of the nanocomposite is negligible, and the poling occurs in the thickness direction, i.e., in the $z$ direction, the only charge coefficient component which needs to be measured is $d_{33}$. This component is defined as the ratio of the charge density and the applied mechanical force to the sample as shown in Eq. (4.2).

$$d_{33} = \frac{dq}{dF}$$  \hspace{1cm} (4.2)

where $dq$ is the charge density and $dF$ is the applied mechanical force.
The above equation shows that there is a linear relationship between the applied force to the piezoelectric materials and the electric charge generated in the sample. A schematic detailing the process of measuring the piezoelectric coefficient \( d_{33} \) is shown in Figure 4.5a.

A wide range \( d_{33} \) tester (APC YE2730A \( d_{33} \) meter) was used to measure the piezoelectric coefficient of the sample. The measured value for the stretched sample shows that there is no improvement in the piezoelectric coefficient of the sample after stretching although the \( \beta \) phase content improved significantly compared to the pure unstretched PVDF. The reason behind this is the fact that the dipoles associated with the \( \beta \) phase are randomly arranged inside the polymer and consequently the net piezoelectric coefficient is almost zero.

![Diagram and image](image)

**Figure 4.5.** (a) Schematic of the piezoelectric coefficient \( d_{33} \) measurement; (b) measured piezoelectric coefficient \( d_{33} = -31.2 \text{ pC/N} \) of the stretched sample after poling

The problem regarding random orientation of dipoles and aligning them in a single direction can be solved by including a high voltage poling process. There are usually two main methods for poling the samples, namely contact poling and corona poling. Issues associated with contact poling approach include arcing problems at high voltages, damaging the sample, and disrupting the poling process. Therefore, the corona poling method is chosen, although this approach is more complex.
A schematic of the corona poling setup used in this study along with a figure of an actual poling process are shown in Figure 4.6. Parameter $d$ in this figure shows the distance between the needle and the copper electrode connected to the ground. As can be seen, a copper electrode connected to the ground is placed on a hot plate and the nanocomposite sample is mounted on top of it. Also, a corona needle connected to a high-voltage power supply (15 kV) is placed above the sample and the grid mesh. A metallic grid mesh is used to generate a uniform electric field during poling. A voltage divider made of high power resistors are built to create a voltage of 3 kV applied to the grid mesh. During the process, the air inside the enclosure becomes ionized when the corona needle is connected to the high voltage power supply. The ionized particles move through the grid mesh and towards the ground plane and deposit on the surface of the nanocomposite sample. The resulting electric field provides alignment of the $\beta$ phase crystallites in a single direction, which is the thickness direction in this study.

Figure 4.6. (a) Schematic of the corona poling; (b) Actual image of the corona poling process

The piezoelectric coefficient of the nanocomposite sample after poling was measured using the $d_{33}$ tester where an improvement of $-1.6 \text{ pC/N}$ to $-31.2 \text{ pC/N}$ was observed, as shown in Figure 4.5b.
With a completed sensor, different static and dynamic tests are performed to verify the performance and accuracy of the nanocomposite sensor in strain measurement. The experimental setups used in this study are discussed in the next subsection.

4.1.3 Experimental Setup

The performance of the developed sensor is verified through different experiments. The experimental setup used in this study is shown in Figure 4.7. As depicted in the figure, the developed sensor is mounted on an aluminum cantilever. Both sides of the fabricated nanocomposite film are connected to copper electrodes so the generated charge and resistivity changes of the sensor under strain are captured.

![Figure 4.7. The experimental setup used in this study](image)

The nanocomposite sensor is attached to the cantilever beam using double-sided tape. The sensor is mounted near the clamped end of the cantilever where the predicted strain is highest. A charge amplifier system and a voltage divider are used for the piezoelectric and piezoresistive sensors, respectively. The schematic of the charge amplifier and voltage divider are shown in Figure 4.8.
Figure 4.8. The schematic of (a) voltage divider; and (b) charge amplifier used for piezoresistive and piezoelectric measurements, respectively.

A vibration exciter (B&K Type 4808) is used to apply excitations to the beam for forced vibration tests; but it is removed for free vibration and static tests. A commercial metal foil strain gauge (Omega SGD-2/350-LY11) and an accelerometer (Kistler type 8778A500) are used for verifying the performance of the nanocomposite sensor in both static and dynamic tests. The applied force to the cantilever causes elastic strain in the beam which in turn results in generating electric charge and changes in the resistivity of the sensor that can be captured using a data acquisition system. For the impulse excitation test, the nanocomposite sensor is placed on a beam and the frequency response function (FRF) of the structure is measured experimentally. A piezoelectric hammer (PCB 084A17) is employed for applying an impulse excitation to the beam structure.

In the next section, two numerical modeling approaches are introduced for studying the piezoelectric and piezoresistive properties of the CNT-PVDF nanocomposite sensor. The established finite element (FE) simulations are verified after comparing the numerical and experimental results.
4.2 Numerical Modeling

To further understand the behaviour of the sensing mechanism and the effect of strain on the electro-mechanical properties, numerical models are developed. As the sensor exhibits both piezoresistive and piezoelectric properties, two separate studies are performed to analyze the contributing effects of each phenomenon.

4.2.1 Piezoresistive Modeling

To predict the piezoresistive behaviour of the CNT-based nanocomposites, a 3D model is developed based on random walks on a finite network of CNTs (Li 2011; TabkhPaz et al. 2016). In this model, the random walkers are simulated by electrical particles which are inserted into the CNT network starting at the source which has the highest electrical potential. They then move through the CNT network towards the nodes with the lowest potential called the drain. The electrical particles choose the pathways in the circuit depending on probability. Each walker is considered in the simulation and then the current and voltage of each node are calculated based on the paths taken by the particles. The total electrical current passing through the CNT network can be obtained and, when combined with the known source-drain potential, results in a value for the electrical resistivity of the network. The schematic of Random Walk model simulation for calculating the resistivity of a CNT network is shown in Figure 4.9.

Figure 4.9. Schematic of random walk model for calculating the resistivity of a CNT network
In CNT-PNC nanocomposites, CNTs are in contact with each other through either direct contact or electron tunneling. Therefore, the resistance of the CNT nanocomposites mainly depends on the intrinsic resistance of the CNTs ($R_{CNT}$) and the tunneling resistance ($R_{tun}$) which is higher than the direct CNT resistance (Li, Thostenson, and Chou 2007). The tunneling resistance is usually observed when the insulating layer of the polymer is thin.

Different steps of the modelling process used in this study are presented in Figure 4.10. The FE simulation includes generating the representative volume element (RVE) using randomly distributing CNTs with variable lengths, determining the conductive pathways in the CNT network through identifying CNT–CNT contacts and tunneling distances, and finally forming the resistive network and calculating the resistivity of the CNT network.

**Figure 4.10.** Process modeling of a piezoresistive CNT network using random walks method

The first step for modelling the CNT network is creating the RVE which represents the random distribution of CNTs inside a polymer matrix. A random distribution of CNTs with
different lengths are generated using different parameters of the CNTs and the RVE as given in Eqs. (4.3) and (4.4):

\[ L_{CNT} = L_M + (0.3 \times L_M) \times \text{rand} \]  
\[ CNT_{num} = 4 \times Vol \times \left( \frac{L_{RVE}^3}{L_M \times \pi \times D^2} \right) \]

where \( L_M, L_{CNT} \) and \( \text{rand} \) are the mean length of all CNTs, the length of each CNT inside the RVE, and random numbers between -1 and 1, respectively. Also, \( CNT_{num} \) is the number of the CNTs inside the RVE, \( Vol \) represents the CNT volume percentage, \( L_{RVE} \) is the length of the RVE cube, and \( D \) is the diameter of the CNTs. The random distribution of CNTs provides an anisotropic and disordered network of CNTs. The ratio of the length of the RVE to the mean length of the CNTs ranges between 1 and 30 (Wichmann 2011) and a ratio equal to 3.3 was chosen in this study since it is suggested as a safe ratio in literatures (Grujicic, Cao, and Roy 2004; Wichmann 2011) and dramatically reduces the computational time. The length of the RVE cube was selected 1 \( \mu m \times 1 \mu m \times 1 \mu m \) which is large enough to represent the nanocomposite’s morphology with reasonable accuracy. Also, the mean length and diameter of CNTs were considered 0.3 \( \mu m \) and 20 \( nm \), respectively.

After picking random values for the lengths of CNTs, different random points are selected as starting points for generating the CNT network inside the RVE. The end point of each CNT is defined using the length of each CNT and random selection of angles as given in Eq. (4.5):

\[ x_{end} = x_{start} + L_{CNT} \cos(\theta) \]
\[ y_{end} = y_{start} + L_{CNT} \sin(\theta)\cos(\varphi) \]
\[ z_{end} = z_{start} + L_{CNT} \sin(\theta)\sin(\varphi) \]

where \( \varphi \) is the azimuthal angle, and \( \theta \) represents the polar angle as shown in Figure 4.11.
In the next step, the generated CNTs that pass the boundaries of the RVE are identified and the out-of-boundary portion of the CNTs are cut to make sure all generated CNTs stay inside the RVE. In addition, the developed model eliminates the CNTs that are not connected to other CNTs or the electrodes. The CNTs that have a distance less than tunneling distance (1.8 nm (Li et al. 2007; Wichmann 2011)) with their surrounding CNTs are not eliminated since the electrons and random walkers can travel between them and create a conductive path.

Figure 4.12 shows an example of the CNT network before and after eliminating the non-contact and non-tunneling CNTs.
Figure 4.12. The generated CNT network (a) before eliminating the isolated CNTs; (b) After eliminating the non-connected and non-tunneling CNTs

The updated CNT network is now used for calculating the resistivity of the nanocomposite and investigating the effect of strain on the piezoresistive effect. The conductance values of each contact and electron tunneling inside the CNTs network, which are reciprocals of resistance values, are calculated and stored in a conductance matrix which is an $N \times N$ matrix where $N$ is the number of CNTs after updating. The CNT contact resistance is a function of the geometry and the electrical conductivity of the CNTs and is calculated using Eq. (4.6) (Bao et al. 2011):
\[ R_{CNT} = \frac{4L_{CNT}}{\pi \sigma_{CNT} D^2} \]  \hspace{1cm} (4.6)

where \( R_{CNT} \), \( L_{CNT} \) and \( D \) are the contact resistance, length of each CNT, and diameter of CNTs, respectively, and \( \sigma_{CNT} \) is the electrical conductivity of CNTs (5000 S/m).

The tunneling resistances can be obtained using Eq. (4.7) (Bao et al. 2011):

\[ R_{tun} = \frac{h}{2e^2} \frac{1}{M \tau_p} \]  \hspace{1cm} (4.7)

where \( R_{tun} \) is the tunneling resistance, \( h \) is the Plank’s constant (\( 2\pi\hbar \)), \( e \) is the electron charge (\( \frac{h}{2e^2} \approx 12.9054 \text{ k}\Omega \)), \( M \) is the number of conduction channels, and \( \tau_p \) is the transmission probability of an electron to tunnel between CNTs that can be obtained using the Schrodinger equation developed previously (Bao et al. 2011). Contact resistance (Eq. (4.6)) is considered for those CNTs which overlap or are in contact with each other while the tunneling resistance (Eq. (4.7)) is considered for those CNTs that have a gap distance less than tunneling distance.

Random walk approach is used for calculating the overall resistivity of the network under different strain conditions. Figure 4.13 shows a simple circuit representing the CNTs network. The conductance matrix of this circuit can be developed as given in Eq. (4.8) where nodes 1 and 4 represent the source and drain electrodes, respectively and nodes 2 and 3 are two contact points.

**Figure 4.13.** Schematic of a circuit representing a simple CNT network
\[ G = \begin{bmatrix}
0 & G_{12} & G_{13} & 0 \\
G_{21} & 0 & G_{23} & G_{24} \\
G_{31} & G_{32} & 0 & G_{34} \\
0 & G_{42} & G_{43} & 0
\end{bmatrix} \] (4.8)

where \( G \) is the conductance matrix, \( G_{ij} = \frac{1}{R_{ij}} \) and \( G_{ij} = G_{ji} \) are the conductance values.

After extracting the conductance matrix from the obtained resistance values, the nodes connected to the source and drain are identified. Based on the method developed by (Doyle and Snell 2000), the electrical particles and random walkers are imported into the CNT network from the source nodes and move from one node to another through the conductive path till they finally reach the drain and leave the circuit. The random walkers choose the path to move from source to drain based on the probability of each path inside the RVE. The probability of choosing node \( j \) when the electrical particle is on node \( i \) \( (P_{ij}) \) can be calculated using the conductance of the path between \( i \) and \( j \) and the overall conductance of the whole network as presented in Eq. (4.9):

\[ P_{ij} = \frac{G_{ij}}{\sum_{l=1}^{n} G_{lj}} \] (4.9)

where \( G_{ij} \) is the conductance value of the CNT path between node \( i \) and \( j \), and \( n \) is the number of nodes connected to node \( i \) in the CNT network through either direct contact or tunneling.

An electrical potential of 1 V is applied across the source and drain nodes and the voltage at any node \( i \) of the network \( (V_i) \) can be obtained using Eq. (4.10). Voltage \( V_i \) is a function of voltage \( j \) which is in contact with point \( i \) and the probability that a walker might move from point \( i \) to point \( j \), i.e.,

\[ V_i = \sum_j P_{ij}V_j \] (4.10)
To calculate the current $I_{ij}$ along a path from $i$ to $j$, it is assumed that a walker might pass several times along the path from $i$ to $j$, and in the opposite direction from $j$ to $i$. Therefore, it is assumed that current $I_{ij}$ is proportional to the net number of movement along the path from $i$ to $j$, where the movement from $j$ to $i$ is considered as negative (Doyle and Snell 2000). Then,

$$I_{ij} = \frac{(V_i - V_j)}{R_{ij}} \quad (4.11)$$

A random walker starts from a node on the source and pass a conductive path to reach the drain but if it returns to the source before reaching the drain, it keeps going. Finally, the total voltage between the source and the drain along with the total current given in Eq. (4.12) are used to calculate the equivalent resistance of the CNT network as presented in Eq. (4.13):

$$I_{total} = \sum_{j=2}^{n} I_{1j} \quad (4.12)$$

$$R_{eq} = \frac{V_{total}}{I_{total}} \quad (4.13)$$

where $n$ is the number of CNTs connected to the source electrode.

Changes in geometry of nanocomposites due to applied strains may result in contraction or expansion of the CNT network. In case of contraction of the network, number of connections between CNTs may increase, as schematically shown in Figure 4.14, which in turn results in less resistivity in the nanocomposite. Conversely, positive strain will decrease the number of CNT connections and, therefore, higher resistivity is achieved.
Figure 4.14. Schematic of changes in CNT interaction due to strain

To investigate the piezoresistive effect of the CNT network, strains are applied to the RVE in a particular direction and the changes in the resistivity of the system are calculated. Figure 4.15 shows the numerical and experimental results of the piezoresistive effect of the nanocomposite under different strains applied harmonically to the sensor.

Figure 4.15. Piezoresistive results of the nanocomposite obtained from simulation and experiments

As can be seen, the numerical results for the resistivity of the nanocomposite under different strains follow a similar trend with the experimental results. Also, it is observed that the developed
modeling approach results in higher resistance of nanocomposite when positive strain (tension) is applied to the system compared to when negative strain (compression) is applied to the nanocomposite as was expected. It can be seen in the obtained results that using more random walkers result in more accurate results; however, it increases the processing time significantly. The standard deviation value calculated for the simulation with 10 random walkers \(σ^2 = 0.0877\) is much lower than that of the simulation with 1 random walker \(σ^2 = 0.1302\).

The piezoelectric properties of the nanocomposite are studied numerically in the next section and the discussed approach is verified with the experimental results.

4.2.2 Piezoelectric modeling

The piezoelectric coefficient of the nanocomposites is predicted using a 2D Monte Carlo simulation. The model used in this study is based on a study conducted by (Koenck 2013), in which randomness in nanotube size, location and orientation were incorporated in the determination of polymer nanocomposite piezoelectric parameters. The piezoelectric properties of nanocomposites can be predicted using a finite element approach as given in Eq. (4.14) (Koenck 2013):

\[
\begin{bmatrix}
K_{uu} & K_{u\phi} \\
K_{u\phi}^T & K_{\phi\phi}
\end{bmatrix}
\begin{bmatrix}
U \\
\Phi
\end{bmatrix}
=
\begin{bmatrix}
F \\
Q
\end{bmatrix}
\]  

(4.14)

where \(K_{uu}\) and \(K_{\phi\phi}\) are the elastic stiffness matrix and dielectric stiffness matrix, respectively, \(K_{u\phi}\) is the piezoelectric coupling matrix, \(U\) and \(\Phi\) are nodal displacements and nodal electrical potential, respectively, \(F\) is the nodal mechanical force, and \(Q\) is the nodal electrical charge. The model follows a three-step process to incorporate both the properties of the carbon nanotubes and surrounding polymer. In steps one and two, the elastic stiffness, piezoelectric coupling and dielectric stiffness matrices are calculated for the carbon nanotubes and polymer separately and in the third step, the effective piezoelectric parameters are computed using Eq. (4.15):
\[ K_{\text{eff}}^{\text{ele}} = K_{m}^{\text{ele}} + \sum_{i=1}^{n} K_{nt}^{i} \]  

(4.15)

where \( K_{\text{eff}} \) represents the effective stiffness matrix for a single element, \( K_{m} \) is the stiffness matrix of the polymer in the element, and \( K_{nt} \) is the sum of all the stiffness matrices of all the nanotubes existing in the element. The following shows how to find the matrices in Eqs. (4.14) and (4.15) associated with both polymer element and embedded CNTs.

4.2.2.1 Calculation of polymer piezoelectric parameters

The effective matrices in Eq. (4.14) can be obtained by considering the properties of the polymer and the CNTs as given in Eq. (4.15). The stiffness, piezoelectric coupling and dielectric stiffness matrices can be obtained using the formulations given in Eq. (4.16):

\[ K_{uu} = \int B_{u}^{T} C B_{u} dV \]
\[ K_{u\phi} = \int B_{u}^{T} C B_{\phi} dV \]
\[ K_{\phi\phi} = -\int B_{\phi}^{T} C B_{\phi} dV \]  

(4.16)

where \( B_{u} \) is the strain and displacement and \( B_{\phi} \) is associated with the electrical field and potential. There is a direct relationship between the strain of an element and the displacement as given in Eq. (4.17):

\[ S = B u \]  

(4.17)

where \( S \) is the strain and \( u \) is the displacement. Eq. (4.18) gives a method for approximating the displacement using a series of shape functions \( N \) within each element:

\[ u = N_{u} U \]  

(4.18)
Combining Eqs. (4.17) and (4.18) yields:

\[ S = B N_u U = B_u U \]  \hfill (4.19)

In the analysis, a four-node quadrilateral element with three degrees of freedom is chosen. In this case four shape factors, \( N_1 \ldots N_4 \), per element are used and their partial derivatives are taken with respect to local coordinates \( r \) and \( s \). \( N_u \) and \( B_u \) are then assembled as:

\[
N_u = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \end{bmatrix} \hfill (4.20)
\]

\[
B_u = \begin{bmatrix}
\frac{\partial N_1}{\partial r} & 0 & \frac{\partial N_2}{\partial r} & 0 & \frac{\partial N_3}{\partial r} & 0 & \frac{\partial N_4}{\partial r} & 0 \\
0 & \frac{\partial N_1}{\partial s} & 0 & \frac{\partial N_2}{\partial s} & 0 & \frac{\partial N_3}{\partial s} & 0 & \frac{\partial N_4}{\partial s} \\
\frac{\partial N_1}{\partial s} & \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial s} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial s} & \frac{\partial N_3}{\partial r} & \frac{\partial N_4}{\partial s} & \frac{\partial N_4}{\partial r}
\end{bmatrix} \hfill (4.21)
\]

where shape factors are given by:

\[
N_1 = \frac{1}{4} (1 - r)(1 - s) \quad , \quad N_2 = \frac{1}{4} (1 + r)(1 - s) \hfill (4.22)
\]

\[
N_3 = \frac{1}{4} (1 + r)(1 + s) \quad , \quad N_4 = \frac{1}{4} (1 - r)(1 + s)
\]

The presented model is a two-dimensional (2D) square element, therefore,

\[ r = s = \pm \frac{1}{\sqrt{3}} \hfill (4.23) \]

To map the local coordinates of each node to the global coordinates \( x \) and \( y \), the Jacobian matrix presented in Eq. (4.24) is used:

\[
J_a = \begin{bmatrix} dx & dy \\ dr & dr \\ ds & ds \end{bmatrix} \hfill (4.24)
\]
A similar approach can be used for determining $B_{\phi}$. To solve the volume integrals and determine the matrices in Eq. (4.16), Gauss quadrature is employed. Therefore, Eq. (4.16) is reformulated as:

$$K_{m}^{ele} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{u}^{T} C B_{u} \| J_{a}(r,s) \|$$  \hspace{1cm} (4.25)

This equation represents the elastic stiffness matrix of the polymer element. The piezoelectric coupling and dielectric stiffness matrixes associated with the polymer element can be obtained in a similar approach. Similarly, the corresponding matrixes need to be calculated for the embedded CNTs in the element as presented in the next subsection.

4.2.2.2 Calculation of carbon nanotube (CNT) piezoelectric parameters

In the modelling of the carbon nanotubes, the embedded fibre finite element (FE) method, first proposed by (Esteva and Spanos 2009) is used. In this method, an RVE populated with carbon nanotubes, in the desired concentration and orientation, is discretized into multiple elements. For the finite element model to function properly, each carbon nanotube crossing between elements must be partitioned. This is accomplished by use of the Liang-Barsky algorithm wherein each CNT is described parametrically:

$$x = x_{1} + (x_{2} - x_{1})t$$
$$y = y_{1} + (y_{2} - y_{1})t$$  \hspace{1cm} (4.26)

where $(x_{1},y_{1})$ and $(x_{2},y_{2})$ are the start and end points of the carbon nanotube, respectively, and $t$ is any value between 0 and 1. The overall number of CNT lines in each element is determined after applying the Liang-Barsky algorithm to each CNT line with respect to each FE element. CNTs cross multiple elements when a denser mesh is used in the simulation; therefore, they are divided.
to multiple sections without affecting their properties as previously proved (Esteva and Spanos 2009). The resulting grid mesh of carbon nanotubes is shown in Figure 4.16.

![Figure 4.16](image)

**Figure 4.16.** Embedded fibre method grid mesh with element partitioning

To determine the piezoelectric parameters of the carbon nanotubes, each nanotube is approximated as a line element which results in efficient computation and sufficient accuracy (Konrad and Graovac 1996). Natural coordinates of the CNTs are used to determine the corresponding shape functions. To do so, the transformation matrix is defined as:

\[
T = \begin{bmatrix}
N_u(\xi_i, \eta_i) \\
N_u(\xi_j, \eta_j)
\end{bmatrix}
\] (4.27)

where \( i \) and \( j \) represent the start and end points of the CNTs and each \( N_u \) is a shape factor similar in form to Eq. (4.21), and \( r \) and \( s \) are the coordinates of each carbon nanotube. The stiffness of each CNT within the element can be obtained using the following transformation matrix (Koenck 2013).
\[ k = \frac{(E_f - E_m)A}{l} \begin{bmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha & -\cos^2 \alpha & -\cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha & -\cos \alpha \sin \alpha & -\sin^2 \alpha \\ -\cos^2 \alpha & -\cos \alpha \sin \alpha & \cos^2 \alpha & \cos \alpha \sin \alpha \\ -\cos \alpha \sin \alpha & -\sin^2 \alpha & \cos \alpha \sin \alpha & \sin^2 \alpha \end{bmatrix} \quad (4.28) \]

where \( E_f, A \) and \( l \) are the fibre’s Young’s modulus, cross-sectional area and length, respectively; and \( E_m \) is the Young’s modulus of the polymer matrix material. The angle of the nanotube segment is given by \( \alpha \). The stiffness of all CNTs embedded inside the element can be found by a similar procedure as above. Then, the effective stiffness matrix in Eq. (4.15) may be calculated.

As carbon nanotubes are not piezoelectric materials, the piezoelectric coupling matrix is 0. The dielectric stiffness matrix is formulated as follows (Koenck 2013):

\[ K_{\psi \phi}^{nt} = \frac{-\varepsilon_{nt}A}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (4.29) \]

where \( \varepsilon_{nt} \) is the dielectric constant of CNTs.

After computing the matrices for each embedded CNT in the element, the effective matrices of a single element can be obtained. Finally, the boundary conditions are applied to the RVE and the effective piezoelectric coefficient of the nanocomposite can be then calculated.

### 4.2.2.3 Finite Element (FE) model solution

The material properties of MWCNT/PVDF nanocomposites were used for computing the effective piezoelectric charge coefficient, \( d_{33} \), of the composite material using the described FE simulation. The simulation boundary conditions include a fixed relation along the bottom edge with a force of 10 N applied on the top edge. In addition, no electrical potential was applied to both edges and, therefore, an electrical field was induced in the nanocomposite material due to the
piezoelectric nature of the PVDF polymer. The model parameters used for the simulation are shown in Table 4.1 (Parmar 2015).

Table 4.1. Parameters for CNT/PVDF nanocomposites cited

<table>
<thead>
<tr>
<th>PVDF/CNT</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVDF</td>
<td>Elastic Modulus</td>
<td>GPa</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Piezoelectric coupling coefficient ($e_{13}$)</td>
<td>cm$^{-2}$</td>
<td>4.63</td>
</tr>
<tr>
<td></td>
<td>Piezoelectric coupling coefficient ($e_{33}$)</td>
<td>cm$^{-2}$</td>
<td>−3.03</td>
</tr>
<tr>
<td></td>
<td>Dielectric permittivity constant ($\varepsilon_{11}$)</td>
<td>Fm$^{-2}$</td>
<td>$110 \times 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>Dielectric permittivity constant ($\varepsilon_{33}$)</td>
<td>Fm$^{-1}$</td>
<td>$110 \times 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>Film thickness ($t$)</td>
<td>µm</td>
<td>15</td>
</tr>
<tr>
<td>CNT</td>
<td>Elastic modulus</td>
<td>GPa</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>Dielectric permittivity constant ($\varepsilon_{33}$)</td>
<td>Fm$^{-1}$</td>
<td>$120 \times 10^{-12}$</td>
</tr>
</tbody>
</table>

To verify the accuracy of the developed Monte Carlo model, an FE simulation was performed on about 100 different microstructures with the RVE geometry of $1 \times 1 \mu$m$^2$. The piezoelectric charge coefficient ($d_{33}$) is extracted by averaging the results over 100 randomly generated RVEs and the obtained result is compared with that of the experiments as shown in Table 4.2.

Table 4.2. Piezoelectric charge constant results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulated</th>
<th>Experimental</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric charge coefficient, $d_{33}$ (pC/N)</td>
<td>24.2</td>
<td>31.2</td>
<td>22</td>
</tr>
</tbody>
</table>

Figure 4.17 shows the numerical and experimental results of the piezoelectric properties of the nanocomposite under different strains applied to the sensor. In this case, reference values from
a strain gauge were used and applied as boundary conditions to the model. The resulting charge was determined from the finite element simulation and compared to the experimental results. This simulation was performed under quasi-static conditions and dynamic effects such as charge leakage and parasitic capacitance were neglected.

![Graph showing piezoelectric results](image)

**Figure 4.17.** Piezoelectric results of the nanocomposite obtained from simulation and experiments

The result obtained for piezoelectric coefficient in Table 4.2 presents a difference of 22% with respect to the experimental results. In addition, the results extracted from Figure 4.17 show a good agreement between the FE simulation and experimental results. Therefore, it can be concluded that the described model can predict the piezoelectric properties of the PVDF–CNT nanocomposites.

The performance of the nanocomposite sensor is verified through experiments and comparing the results with those of the commercial sensors. Next section presents the results obtained from the experimental tests and then discusses on the results.
4.3 Results and Discussion

A set of experiments was performed to validate the accuracy of the sensor developed. The FRF of a structure was obtained using the nanocomposite sensor through impulse excitation test. The response of the sensor under both static and dynamic loads were then extracted and compared with the strain gauge results as an existing commercial sensor. Finally, the piezoelectric and piezoresistive signals extracted from the nanocomposite sensor were fused to achieve an integrated sensor with higher sensitivity and accuracy compared to each signal separately.

4.3.1 Impulse excitation and frequency response function (FRF) measurement

An impulse excitation was used for investigating the performance of the nanocomposite sensor in FRF measurement. The impulse excitation test was conducted by exciting the beam structure using a piezoelectric hammer. The resulting charge (piezoelectric response) of the nanocomposite sensor was collected along with the impact hammer signal. The FRF of the structure can be obtained by capturing the data of the applied force and the sensor response. It is important to assure that impact hammer has sufficient energy throughout the desired frequency range, and that the input energy is sufficiently transferred to the output.

The time domain responses of the developed sensor are compared with the results obtained from the Kistler accelerometer as a reference sensor in Figure 4.18. The ratio of the response of the sensor to the applied force to the system in frequency domain results in the FRF of the structure as shown in Figure 4.19. The results of the nanocomposite sensor are compared with those of the accelerometer as a commercial sensor.
Figure 4.18. Time-domain response of the piezoelectric hammer (top), and nanocomposite sensor and accelerometer responses under impulse excitation (bottom)

Figure 4.19. Frequency response function (FRF) of the beam structure obtained from the nanocomposite sensor measurements and Kistler accelerometer

The FRF of the structure shows that there is a good agreement between the FRF results obtained from the nanocomposite sensor and the commercial piezoelectric sensor. Contrary to
existing commercial piezoelectric sensors, the developed sensor can measure the static and low-frequency excitations as presented in next section.

4.3.2 Static and dynamic measurement results

The piezoresistive characteristic of the sensor is only used for static measurement due to the charge leakage problem associated with the piezoelectric sensor. A static force was applied to the free end of the cantilever shown in Figure 4.7 and the strain of the cantilever at the sensor location was measured using the piezoresistive output of the sensor. After applying a bending load to the system, the free end of the cantilever was held for about 4 seconds at maximum displacement and returned to its original state. The results obtained from both nanocomposite sensor and the commercial strain gauge sensor are shown in Figure 4.20. A Savitzky–Golay filter was applied to the recorded signals in this study to minimize the effect of noise.

![Graph showing strain over time](image)

**Figure 4.20.** The piezoresistive response of the system under static loading obtained from the nanocomposite sensor and the metal foil strain gauge
The results obtained prove that there is a good agreement between the results of piezoresistive sensor and the metal foil strain gauge as reference sensor in static measurements. However, there is a small deviation between the results that can be attributed to the electric connections, i.e., the copper tape used in this study. Also, the nanocomposite sensor is attached to the beam using double-sided tape, while the strain gauge sensor is attached using super glue. Therefore, when a static force is applied to the system, the reference sensors react to this change immediately, but the difference in the bond properties of the double-sided tape does not allow the nanocomposite sensor to follow the behaviour of the reference sensor accurately enough. The above results also show that the developed sensor can detect the sign of the strain. When the force is applied upward at the end of the cantilever resulting in negative strain, the resistance of the sensor decreases due to decrease of the distance between the CNT networks. Similarly, the length of the strain gauge decreases when a force is applied upward resulting in lower resistance in the sensor. The calibration factor for the piezoresistive sensor was picked $1.5 \times 10^{-3} \text{ V}^{-1}$ and this calibration constant was used for the free and forced vibration tests.

Dynamic excitations including the free and forced vibrations were applied to the free end of the cantilever and the strain of the beam at the sensor location was measured using the nanocomposite sensor. For the free vibration test, the beam was first pushed down 10 mm at the free end of the cantilever and then released. The results of the free vibrations for both developed sensor and the commercial strain gauge are shown in Figure 4.21.
Figure 4.21. Free vibration response of the piezoelectric nanocomposite sensor and the metal foil strain gauge as reference sensor

The free vibration results show that there is a good agreement between the piezoelectric results and strain gauge results. For the piezoelectric results, a calibrating factor of $6.1 \times 10^{-6} \ pC^{-1}$ was chosen to convert the charge output from the nanocomposite sensor to strain after matching the results obtained from both sensors. This calibration factor was used for all subsequent piezoelectric testing.

The calibration factors obtained for the piezoelectric and piezoresistive properties vary for different sensors as they are dependent on the fabrication process, polling process, and electrode connection of the sensor. Therefore, they need to be found separately for each sensor prior to measurements to have more accurate results.

To investigate the linearity of the developed sensor, harmonic excitation was applied to the system with different amplitudes and the strain of the beam for each excitation level was measured using the nanocomposite sensor and the strain gauge as the reference sensor. The strain results
were then plotted versus applied force to the system measured using a 3-component piezoelectric force sensor (Kistler 9018B), as shown in Figure 4.22.

![Graph showing strain vs. force](image)

**Figure 4.22.** The strain of the beam under different excitation loads measured with both developed nanocomposite and metal foil strain gauge as reference sensor

The results obtained prove that there is almost a linear relationship between the applied force to the system and the resultant strain measured by the developed and reference sensors, verifying the linearity of the developed sensor. It also shows that the calibration factor is almost constant for a wide range of strains since the nanocomposite sensor and the reference sensor are closely matched with each other.

With both the piezoresistive and piezoelectric properties calibrated, the sensor underwent additional forced vibrations under different excitation frequencies to evaluate the performance of the nanocomposite sensor. The results of the forced vibrations with different excitation frequencies for both piezoresistive and piezoelectric responses are compared with the strain gauge as a reference sensor as shown in Figure 4.23.
The strain of the cantilever under forced vibration at different excitation frequencies measured with piezoresistive sensor, piezoelectric sensor, and metal foil strain gauge as the reference sensor: (a) 0.1 Hz (b) 1 Hz (c) 100 Hz (d) 1000 Hz.

The above results show that there is a good agreement between the forced vibration results of the piezoresistive nanocomposite sensor in low frequencies and the piezoelectric nanocomposite sensor in frequencies higher than 5 Hz, and those of the strain gauge. However, the piezoelectric sensor is not accurate in low frequencies and it can be seen in the above results that there is very small and mostly noisy response from the piezoelectric sensor at the low excitation frequency equal to 0.1 Hz. Also, the piezoresistive sensor is not capable of measuring excitations above 160 Hz and it can be seen in the above results that the piezoresistive response at 1000 Hz is predominantly noise.
To improve the accuracy of the measured strains and increase the frequency bandwidth of the developed sensor, the piezoelectric and piezoresistive signals are combined and the fused signals are compared with those of the reference sensor in the next section.

4.3.3 Fusion of piezoresistive and piezoelectric measurements

Sensor fusion techniques combine signals from different sensors to achieve more accurate results with less uncertainty compared to each signal individually. To obtain improved information of the piezoelectric and piezoresistive nanocomposite sensors, the output signals are fused, and a combined signal is achieved.

Quite a few sensor fusion techniques have been developed in different studies using different algorithms. A method that we intend to use in this study for fusion of the piezoelectric and piezoresistive nanocomposite sensors is similar to an optimum linear smoother developed by (Fraser and Potter 1969) for the combining two independent estimates. This method is based on the central limit theorem asserting that the mean of some independent random variables which are distributed identically can be approximated by normally distributed variables (Einicke 2012). Based on the Fraser–Potter smoother, the fused signal is a linear combination of the two measured signals weighted by their according covariances (Einicke 2012; Fraser and Potter 1969). Accordingly, Eq. (4.30) was developed after modifying the Fraser–Potter smoother for fusing the piezoelectric and piezoresistive signals:

\[
S_3 = ((C_1P_1)^{-1} + (C_2P_2)^{-1})^{-1} \times ((C_1P_1)^{-1}S_1 + (C_2P_2)^{-1}S_2)
\]  

(4.30)

where \(S_3\) is the fused signal, \(S_1\) and \(S_2\) are the piezoelectric and piezoresistive signals, \(P_1\) and \(P_2\) are the covariances associated with \(S_1\) and \(S_2\) signals, respectively, and \(C_1\) and \(C_2\) represent the compensation coefficients for each signal. For frequencies lower than 5 Hz, \(C_1\) and \(C_2\) are picked...
as 1 and 200, respectively, while for frequencies higher than 5 Hz these coefficients are modified to 200 and 1, respectively.

The results of the dynamic tests after fusing the piezoelectric and piezoresistive properties for different excitation frequencies are shown in Figure 4.24.

![Graphs showing strain vs. time for different excitation frequencies](image)

**Figure 4.24.** The fused and strain gauge signals for the strain of the cantilever under forced vibration at different excitation frequencies: (a) 0.1 Hz (b) 10 Hz (c) 100 Hz (d) 1 kHz (e) 4 kHz (f) 10 kHz.
The reason behind picking the presented values for coefficients $C_1$ and $C_2$ is the fact that the response of the piezoresistive sensor at high frequency and piezoelectric at low frequency are very small and mostly noise, resulting in very low covariances. Therefore, the covariance of each signal needs to be weighted using the compensation coefficients, otherwise, the signal with lower variance will be the dominant based on Eq. (4.30) and the fused signal will be more matched to the weaker signals which causes discrepancies in the fused results.

The obtained results in Figure 4.24 show that sensor fusion technique improves the performance and accuracy of the nanocomposite sensor; however, the very low frequencies fused signals are almost same as the piezoresistive signals as it is dominant in the sensor fusion and the high frequency signals (above 100 Hz) are almost the same as the piezoelectric signals since they are dominant. This also explains the need for the compensation coefficients to be frequency dependent. Below 5 Hz, the piezoresistive signal is weighted much higher than the piezoelectric signal and above 5 Hz, the piezoelectric signal is weighted higher. Also, it can be seen in the obtained results that at high excitation frequencies of 10 kHz, the reference sensor is no longer able to track the input while the developed sensor continues to accurately capture the signal. It shows a novel contribution of this study where the developed nanocomposite sensor is capable of measuring both low and high frequency signals while existing commercial sensors only measure either high or low frequencies.

### 4.4 Limitations and Assumptions

There are some limitations and assumptions associated with the fabrication process of the developed nanocomposite sensor and numerical models. It is assumed that the sensor exhibits elastic behaviour and there is no effect due to the Poisson’s ratio in the developed random walks
model. In addition, it is considered that the $\beta$ phase is the main structure of PVDF (100% beta phase), however the polymer consists of 4 different phases and this assumption might affect the numerical results. It is assumed that the used method for mixing the solution of nanoparticles and polymer provides uniform dispersion of the nanoparticles. Also, we assumed that the random dispersion of CNTs within the polymer matrix creates isotropic behaviour. Finally, the simulations were performed under quasi-static conditions and it was assumed that transient effects were negligible.

One of the main limitations observed in this study is that the effect of poling on improving the piezoelectricity decreases with increasing CNT concentration. Although it has been shown that increasing CNT concentration reduces the electrical potential required for poling to occur, it causes a reduction in the potential piezoelectric performance and reduces overall sensor sensitivity. This observation resulted in the use of different CNT concentrations to optimize the piezoelectric and piezoresistive performance, respectively. For optimal piezoelectric performance, 0.1 wt.% of CNT was used, and 2 wt.% of CNT was chosen to reach electrical percolation and optimize piezoresistive performance. The solution-mixing method used for fabricating the nanocomposite sensors contains its own challenges including imperfect dispersion of CNTs inside the polymer matrix. Other mixing techniques including melt-mixing technique might result in better performance of the nanocomposite since they provide better distribution of CNTs when the concentration of CNTs is higher. Also, adding other nanoparticles to the nanocomposite sensor might result in higher piezoelectricity and a more sensitive sensor. Finally, copper tape was used as electrodes for the sensor. For improved performance, employing other methods of depositing electrodes, such as sputtered silver, might reduce electrical contact resistance and improve the sensitivity of the developed sensor.
The fusion method chosen may also be improved. As the sensor exhibits good piezoresistive response at low frequency with almost no piezoelectric response and the opposite occurring at high frequencies, a simple Fraser–Potter smoother may not be the most appropriate choice. The fusion algorithm is only applicable to frequency bands in which both piezoresistive and piezoelectric signals are accurate. In future work, a frequency-based fusion algorithm may result in further improvements.

4.5 Summary

A new nanocomposite-based sensor was developed in this study for strain measurements and joint dynamics identification. This sensor can be also used for measuring the strain of the tool and then indirect measurement of cutting forces in machining operations. The developed sensor combines piezoelectric and piezoresistive properties to achieve improved sensitivity and higher frequency bandwidth. This sensor is cost effective compared to the existing commercial sensors and is flexible and can be mounted on any surface. Two different numerical models were presented for investigating the piezoelectric and piezoresistive properties of the sensor. The accuracy of these models was verified using the experimental results.

The performance of the developed sensor in strain measurement was investigated through static and dynamic experiments. The accuracy of the sensor was verified after comparing the experimental results with those of commercial sensors. The static results showed that the developed sensor can measure static forces and can detect the sign of the strain applied to the system. Also, the dynamic results showed that this sensor can be used for free and forced vibration measurements after calibration with a reference sensor. To further improve the performance of the sensor, a sensor fusion algorithm was implemented. Both piezoelectric and piezoresistive signals
were combined using a modified Fraser–Potter smoother approach resulting in an integrated sensor with improved performance. The developed sensor can measure the strains as low as 8 μstrain and in the frequency range of 0 to 10 kHz; however, the sensitivity and measurement range of the sensor can improve if the fabrication process is modified and the packaging process is improved.

The developed sensor is used for joint dynamics identification after inserting into the joint interface. Next chapter presents two different methods that employ the nanocomposite sensor for determining the joint dynamic properties.
Chapter Five: **JOINT DYNAMICS IDENTIFICATION USING NANOCOMPOSITE SENSOR**

Different techniques for indirectly determining the joint properties in bolted structures were previously presented in Chapter 3. However, it is always desirable to use a sensor in the joint interface for directly identifying the joint dynamics. Therefore, the developed nanocomposite sensor is implemented in the joint interface to measure the contact force and then predict the joint parameters using two new experimental methods. The first method employs the contact force and displacement data to find the hysteresis loops and identify the joint properties. The second approach is based on an efficient calculation algorithm that uses the force and displacement data to predict the stick-slip boundary and then determine the energy loss and joint dynamic properties. Experiments are conducted on a structure consisting of two beams attached to one another using a bolted lap joint to verify the accuracy of the developed sensor-based methods.

The contact model considered for the joint interface of the bolted structure is first presented in this chapter. The process of identifying the joint properties using the new sensor-based methods are then elaborated. The effect of different conditions, i.e. contact normal load and excitation load, on the joint properties are investigated as well. This chapter is concluded with discussions on the limitations and assumptions associated with each method and also a summary of the presented methods and obtained results.
5.1 Development of the Contact Model

A contact model is presented in this section for microslip analysis using the nanocomposite sensor. The stick-slip boundaries are then identified and employed for extracting the friction force, energy loss, and joint dynamic properties.

Two types of motion are considered for bolted lap joints during vibration in tangential direction, i.e. microslip and macroslip (Ames et al. 2009; Cigeroglu, Lu, and Menq 2006). As an external force is applied to the joint, part of the interface area begins slipping; these localized motions are known as microslip. Increasing the force applied to the jointed structure causes a larger part of the interface to slip; finally, the whole contact area will be slipping, which is known as macroslip. The actual parts connected to each other through a lap joint do not have relative movement on the macro-level during microslip, but small portions of the interfacial area experience the motion. Most joints under reasonable force levels just experience the microslip without occurring the macroslip. It means that a small localized motion in microslip results in energy dissipation in the joint and consequently causes damping in the assembled structure. Thus, modelling the frictional slip in the joint as macroslip causes some deviations from the real physical model.

The contact interface in the microslip phenomenon is divided into three different regions, namely the sticking zone, sliding zone, and non-contact zone (Shamoto et al. 2014). When the dimensions of substructures attached to each other through bolted lap joints are large enough, there would be no contact in the outer area of the connection since the normal contact stress is zero in this area. In the inner region of the contact area, i.e. sticking zone, the substructures stick and no relative movement occurs in the connection. The outer region of the contact area is sliding zone where microslip occurs in the contact interface due to external excitations applied to the structure.
Figure 5.1(a) shows an assembled structure consisting of two substructures connected to each other through a bolted lap joint. The nanocomposite sensor is implemented in the joint interface to measure the shear force applied to the structure in the joint location.

![Diagram of assembled structure and nanocomposite sensor](image)

Figure 5.1. (a) The schematic of the assembled structure (Ames et al. 2009); (b) The schematic of different boundaries at the joint interface

The contact model developed for the given system in this study is shown in Figure 5.1. During each cycle, the width of the slip annulus (a-c) grows from zero to maximum, corresponding to the maximum displacement applied to the structure, and then shrinks back to zero (Ames et al.)
2009) when there is no external excitation applied to the system. The radius \( r=a \) illustrates the boundary between the contact zone and non-contact zone, where the contact stress is equal to zero. Also, the radius \( r=c \) shows the boundary between the sticking and sliding zones.

One new method is developed based on using the nanocomposite sensor in this study to extract the joint properties. This method is based on identifying the stick-slip boundary, i.e. \( r=a \) and \( r=c \) in Figure 5.1(a). An effective calculation algorithm is developed to first find this boundary and then extract the friction force and joint dynamic parameters. In addition, another sensor-based technique, i.e. the hysteresis loop method, is presented in this study for joint dynamics identification. Next section presents the experimental setups designed for verifying the accuracy of these methods.

### 5.2 Experimental Setups

The experimental setup shown in Figure 5.2 is used for joint dynamics identification using the sensor-based approaches. The setup consists of two beams which are connected to each other through a bolted lap joint and the whole system is excited by two shakers that are placed on each side of the bolted joint. Both beams of the assembled structure are made of stainless steel 303 with a modulus of elasticity (E) of 190 GPa, Poisson’s ratio (\( \nu \)) of 0.25 and density (\( \rho \)) of 8000 kg/m. The dimensions of the connected beams are given in Table 5.1.

**Table 5.1** Dimensions of substructures used for building the assembled structure

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>70 mm<em>38 mm</em>10 mm</td>
</tr>
</tbody>
</table>
Harmonic excitations are exerted into the assembled structure using two shakers (Bruel & Kjaer 4808) and the response of the system is measured using two wide frequency bandwidth capacitive sensors (Lion DMT20) with the sensitivity of 80 V/mm.

In order to find the contact force, the developed nanocomposite sensor is inserted into the joint interface as shown in Figure 5.2. The output of the function generator is fed into both shakers shown in Figure 5.2 to ensure that the same signal is applied to both sides of the beams. Also, the gains of the power supplies connected to the shakers are picked in a way to ensure that the same excitation amplitude is applied to each side of the beams. A 3-component piezoelectric force sensor (Kistler 9018B) is placed between the shaker and the bolted structure to calibrate the nanocomposite sensor. The calibration factor of the sensor at the frequency of $\omega = 400 \text{ Hz}$ was experimentally measured 301 N/V. This calibration factor is almost constant for a certain frequency at all excitation amplitudes; however, the calibration factor varies for different
frequencies mostly due to the fact that there is a pressure on the nanocomposite sensor and it affects the performance of the sensor. The piezoelectric characteristic of the sensor is used for the measurements in this chapter as the piezoresistive sensor does not provide proper signals at the high frequency, i.e. $\omega = 400 \text{ Hz}$. The experimental conditions employed in this study for joint dynamics identification using the sensor-based methods are presented in Table 5.2.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt torque</td>
<td>5 N.m</td>
</tr>
<tr>
<td>Excitation frequency</td>
<td>400 Hz</td>
</tr>
<tr>
<td>Excitation level</td>
<td>20 Vpp</td>
</tr>
</tbody>
</table>

Friction coefficient in the joint interface can be obtained if there is a macroslip movement in the system. Therefore, a bolt torque of 2 N.m and an excitation load with the frequency of 100 Hz are applied to the system. Therefore, a high amplitude was observed in the displacement response which ensures that there is macroslip in the joint interface and consequently the force measured by the nanocomposite sensor is attributed to the dynamic friction force. Hence, the friction coefficient is found using Eq. (5.1) where the normal force is measured experimentally by the commercial piezoelectric force sensor (Kistler 9018B).

$$ Friction \ coefficient : F = \mu N \Rightarrow \mu = \frac{F}{N} = \frac{161}{956} = 0.168 $$

(5.1)

The assembled structure’s FRF is extracted before and after inserting the nanocomposite sensor to the joint interface to ensure that the dynamics of the system do not change considerably. The impulse excitation tests and experimental modal analysis are performed for this purpose using an instrumented piezoelectric force hammer (PCB 2222). In addition, an accelerometer (Kistler
8778A500) with a weight of 0.29 g and the sensitivity of 10.47 mV/g is used for the response measurement as illustrated in Figure 5.3.

![Test setup used for FRF extraction](image)

**Figure 5.3.** Test setup used for FRF extraction

The FRF results of the assembled structure are shown in Figure 5.4. The obtained results show that adding the nanocomposite sensor to the joint interface does not change the dynamics of the structure considerably. Therefore, it is assumed that the obtained results for the joint properties with and without the sensor in the joint interface are similar.

![Assembled structure’s FRF before and after inserting the nanocomposite sensor into the joint interface](image)

**Figure 5.4.** Assembled structure’s FRF before and after inserting the nanocomposite sensor into the joint interface obtained using the impulse excitation tests
The developed methodologies for joint dynamics identification are presented in the next section. Also, the results of the joint properties are presented and discussed. The effect of different conditions, including different bolted torques and excitation loads on the joint dynamics are also investigated after verifying the accuracy of the sensor-based methods.

5.3 Methodologies and Results

Dynamic properties of the bolted lap joint are identified in this section using the hysteresis loop approach and the calculation algorithm method as two developed nanocomposite sensor-based techniques. The joint identification methods are presented in this section and the obtained results from both sensor-based methods are compared with each other to investigate the accuracy of the nanocomposite sensor in joint dynamics identification.

5.3.1 Joint dynamics identification using the hysteresis loop approach

Hysteresis loop method is used as the first sensor-based method for joint dynamics identification in this section. Plotting the measured force in the bolted connection \( F_{\text{shear}} \) versus the relative displacement response of the beams \( x_{\text{rel}} \) results in a hysteresis loop similar to the typical one shown in Figure 3.19. As presented in Chapter 3, the energy loss in a system can be obtained by computing the area inside the hysteresis loop. For a structure containing a bolted lap joint, the energy loss can be mostly attributed to the energy dissipated through friction in the joint interface.

Similar to Chapter 3, the damping of the joint can be extracted after calculating the energy loss as below.
\[ C_j = \frac{W_d}{\pi \omega U^2} \]  \hspace{1cm} (5.2)

where \( W_d \) is the energy loss, \( \omega \) is the excitation frequency, and \( U \) represents the displacement amplitude of the structure under harmonic excitation.

The slope of the hysteresis loop is employed for finding the stiffness of the joint. It is shown that the joint stiffness in tangential direction equals to the slope of the hysteresis loop, i.e. \( K_j = \text{Slope of the hysteresis loop} \) (Bograd, Schmidt, and Gaul 2008).

Figure 5.5 shows the hysteresis loop of the system extracted experimentally. This loop is obtained by plotting the contact force, measured by the nanocomposite sensor placed in the joint interface, versus the relative tangential displacement of the beams, which was measured using the capacitive sensors.

![Hysteresis loop](image)

**Figure 5.5.** Hysteresis loop extracted experimentally for the bolt torque of 5 N.m and excitation level of 20 Vpp

The area inside the hysteresis loop is computed for obtaining the energy loss. The joint damping is then identified using Eq. (5.2). Also, the joint stiffness is obtained after extracting the
slope of the hysteresis loop. The values of the joint properties and energy loss extracted using the nanocomposite sensor and hysteresis loop approach are presented in Table 5.3.

Table 5.3 Energy loss and joint dynamic properties identified using the nanocomposite sensor and hysteresis loop approach

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values from hysteresis loop method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy loss ((W_d))</td>
<td>0.809 mN.m</td>
</tr>
<tr>
<td>Joint damping ((C_j))</td>
<td>1867 N.s/m</td>
</tr>
<tr>
<td>Joint stiffness ((K_j))</td>
<td>1.18 e7 N/m</td>
</tr>
</tbody>
</table>

Another sensor-based method is developed and presented in this study that identifies the joint parameters through microslip analysis. The process of the stick-slip boundaries identification and joint properties extraction along with the obtained results are elaborated in the next section.

5.3.2 Joint dynamics identification using the calculation algorithm

In order to find the joint dynamic properties, the stick-slip boundary should be first identified. Therefore, the unknown values \(r=a\) and \(r=c\) shown in Figure 5.1(a), namely the contact-noncontact and sticking-sliding boundaries, respectively, need to be determined. The process of finding these unknown parameters is presented in this section. Then, joint dynamic properties of the bolted lap joint can be obtained after finding the friction force and total energy loss in the joint interface using an efficient calculation algorithm.

The boundary between the contact and noncontact zones is obtained using the contact stress distribution. Since there is no contact stress in the noncontact zone, the contact-noncontact boundary, i.e. \(r=a\) in Figure 5.1(a), is where the contact stress distribution equals to zero. Eq. (5.3)
shows a non-dimensional form of the contact stress distribution in the contact interface of layered structures jointed with connecting bolts as shown in Figure 5.6 (Nanda and Behera 1999).

\[ \frac{p}{\sigma_s} = A_1 + A_2 \left( \frac{R}{R_B} \right)^2 + A_3 \left( \frac{R}{R_B} \right)^4 + A_4 \left( \frac{R}{R_B} \right)^6 + A_5 \left( \frac{R}{R_B} \right)^8 + A_6 \left( \frac{R}{R_B} \right)^{10} \] (5.3)

where \( A_1, A_2, A_3, A_4, A_5 \) and \( A_6 \) are constants given in Table 5.4, and \( \sigma_s \) is the applied stress under the washer and bolt. Applied stress, \( \sigma_s \), can be found by measuring the normal force exerted to the structure and the area of the washer used in the experiments.

**Table 5.4** Constants of the polynomial pressure distribution in the contact interface (Nanda and Behera 1999)

<table>
<thead>
<tr>
<th>Constant</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
<th>( A_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.68517</td>
<td>0.10122</td>
<td>0.94205E-2</td>
<td>0.23895E-2</td>
<td>0.29487E-3</td>
<td>0.11262E-4</td>
</tr>
</tbody>
</table>

From Eq. (5.3), the border between contact and noncontact zones is independent of the tightening load applied to the joint. Therefore, there exists a contact zone in the form of a circle whose radius is 3.5 times of the connecting bolt radius, i.e. \( a=3.5 \ R_B \) (Nanda and Behera 1999).
The only remaining unknown is the boundary between sticking and sliding zones. This parameter can be obtained using a constraint implying that the tangential forces in the sticking zone and sliding zone at the boundary are equal, i.e. \( f_{\text{sliding}} = f_{\text{sticking}} \).

The frictional force in the sliding zone is readily obtained using Eq. (5.4) and the experimental data.

\[
f_{\text{fractional}} : f_f = \mu N = \mu \sigma A = \int_{a}^{c} \mu \sigma (2\pi r) dr
\]

where \( \mu \) is the friction coefficient that was measured through experiments as shown in Eq. (5.1), \( \sigma \) is the contact stress distribution given in Eq. (5.3), \( a \) is the boundary radius between the contact and non-contact areas and \( c \) is the boundary radius between the sticking and sliding zones.

In order to find the force in the stick zone, an elastoplastic shear layer of negligible thickness, which permits elastic deformation of the beam in the contact area before slipping, is considered in the joint interface (Menq et al. 1986). This elastic layer acts such as a shear spring connected to the beams in the sticking zone and allows every contact point to have an elastic displacement relative to the support before slipping (Cigeroglu et al. 2006). This linear deformation of the contact points in the sticking zone occurs in the reality likely due to asperities or bonding contact stress (Menq et al. 1986). Thus, the force in the sticking zone is obtained using Eq. (5.5) (Menq et al. 1986).

\[
f_{\text{sticking}} : f_{st} = \begin{cases} K_t u & |K_t u| \leq f_{f, \text{max}} \\ f_{f, \text{max}} & \text{otherwise} \end{cases}
\]

where \( K_t \) is the stiffness of the shear layer in the tangential direction found experimentally as will be discussed, \( f_{f, \text{max}} \) is the yield level (usually \( f_{f, \text{max}} = \mu N \)), and \( u \) represents the beam displacement in axial direction, measured experimentally using capacitive sensors.
Eqs. (5.4) and (5.5) are used in an efficient calculation algorithm developed in this study for finding the stick-slip boundary. Subsequently, friction force, energy loss, and joint dynamic properties are identified as will be presented.

The assembled structure shown in Figure 5.1 is excited by harmonic excitations in axial direction using two shakers. It is shown that the amount of energy dissipated due to friction in one vibration cycle is four times the energy dissipated within the first quarter cycle (Shamoto et al. 2014). Thus, the amount of energy dissipated in the first quarter is calculated in this study and the result is multiplied by four to find the energy loss in the whole cycle. The following steps are designed in the calculation algorithm process to identify the stick-slip boundary and joint properties.

Step 1: \(N\) samples of the displacement of the beams corresponding to the first quarter of the cycle are selected as below. \(u\) shows the overall displacement of the beams which is the relative displacements measured with the capacitive sensors.

\[
u_i = u(i) \quad , \quad i = 1, 2, 3, \ldots, n
\]
\[
u(1) = 0 \quad , \quad u(n) = u_{\text{max}}
\]

Step 2: For each \(u_i\), the stick-slip boundary is found using the following equation.

\[
\int_{c_i}^{u_i} \mu \sigma(2\pi r) \, dr = K_i u_i
\]

where \(\sigma\) is the contact stress distribution obtained from Eq. (5.3) and \(K_i\) is the stiffness of the elastoplastic shear layer considered for the sticking zone and identified experimentally. Eq. (5.7) results in the sticking-slip boundary, i.e. \(r = c_i\), for each step. This parameter is then used for calculating the friction force and energy loss in the joint interface.
Step 3: The energy loss is first calculated using the obtained results through microslip analysis. The total amount of energy loss in each step is equal to:

\[(W_{d})_i = 4 \int_{c_i}^{a} (2\pi\mu\sigma r \times (u_i - u_{i-1}))dr\]  \hspace{1cm} (5.8)

Then, the equivalent damping for the joint is obtained by Eq. (5.9).

\[\begin{align*}
(C_j)_i &= \sum_{i=2}^{n} \frac{(W_{d})_i}{\pi\omega U_i^2} = \frac{4}{\pi\omega U_i^2} \sum_{i=2}^{n} \int_{c_i}^{a} (2\pi\mu\sigma r \times (u_i - u_{i-1}))dr
\end{align*}\]  \hspace{1cm} (5.9)

In addition, the joint stiffness in each step is equals to:

\[\begin{align*}
(K_j)_i &= \frac{(F_j)_i}{u_i} = \frac{K_iu_i + F_j}{u_i} = \frac{2K_iu_i}{u_i} = 2K_i
\end{align*}\]  \hspace{1cm} (5.10)

In order to complete the calculation steps presented above, the stiffness of the shear layer should be first extracted. This parameter can be determined using Eq. (5.10) and the joint stiffness which is the slope of the hysteresis loop as given in Table 5.3. Therefore, the stiffness of the elastic shear layer is found equal to  \(K_i = \frac{1.18 \times 10^7}{2} = 5.9 \times 10^6 \text{ N/m}\). Subsequently, this parameter is used to obtain the stick-slip transition, friction force in the sliding zone, energy loss, and joint damping using Eqs. (5.7) - (5.9). The obtained results are then compared with those of the hysteresis loop method.

The normal force measured experimentally is used in Eq. (5.3) to find the contact stress distribution as shown in Figure 5.7.
Figure 5.7. Contact stress distribution in the contact interface

$N$ samples of the experimental displacement data of the first quarter are picked for joint damping identification using the calculation algorithm. For each sample the stick-slip boundary, i.e. $c_i$, is obtained using Eq. (5.7). Figure 5.8 shows the width of the sliding zone, i.e. $(a - c_i)$.

Figure 5.8. Identified stick-slip boundary $(a-c_i)$ using the calculation algorithm
After identifying the stick-slip boundary and calculating the joint friction force, Eq. (5.8) is employed to obtain the energy loss as shown in Figure 5.9. Comparing the value for energy loss from the calculation algorithm and the hysteresis loop approaches demonstrates that there is a small difference between the results and it verifies the accuracy of both nanocomposite sensor-based methods.

![Energy Loss Graph](image)

**Figure 5.9.** Dissipation energy due to friction in the joint \( (W_d) \) obtained using the calculation algorithm method

After extracting the energy loss, the joint damping is calculated using Eq. (5.9) as presented in Figure 5.10. The results show that there is no joint damping in the system at the beginning of the excitation as there is no sliding and energy loss. However, the joint damping increases due to the expansion of the sliding zone and the increase of the energy loss. Table 5.5 compares the percentage difference in extracted joint parameters using two different described sensor-based methods.
Figure 5.10. Nonlinear joint damping \((C_j)\) extracted using the calculation algorithm method.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values from calculation algorithm method</th>
<th>Values from the Hysteresis loop method</th>
<th>Percentage difference from the hysteresis loop method (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy loss ((W_d))</td>
<td>0.672 mN.m</td>
<td>0.809 mN.m</td>
<td>17.05</td>
</tr>
<tr>
<td>Joint damping ((C_j))</td>
<td>1542 N.s/m</td>
<td>1867 N.s/m</td>
<td>17.4</td>
</tr>
</tbody>
</table>

The above results verify the accuracy of the developed methods as there is a small deviation between the joint parameters obtained from the hysteresis loop approach and the calculation algorithm method when the nanocomposite sensor is implemented in the joint interface for measuring the contact force. The discrepancies between the results can be attributed to the limitations and assumptions in each method as will be explained in the discussion section.

After verifying the accuracy of the sensor-based methods for joint dynamics identification, these approaches are used for determining the joint properties in tangential direction under different conditions. Next section presents the effect of changing the joint conditions on its properties.
5.4 Discussions

The nanocomposite sensor-based approaches are used to identify the joint properties after changing the conditions of the experiments. These changes include varying the bolt torque and excitation loads applied to the system. The results for each case are presented in the next sections. Also, the limitations and assumptions associated with each method are discussed.

5.4.1 Investigating the effect of excitation loads

The excitation loads applied to the assembled structure change and the effect on the identified joint properties using the hysteresis loop and calculation algorithm methods are investigated in this section. The hysteresis loop for each excitation load is extracted using the force and displacement signals measured by the nanocomposite and capacitive sensors, respectively. The obtained hysteresis loops are illustrated in Figure 5.11.

![Hysteresis loops under different excitation loads applied to the assembled structure under the bolt torque of 5 N.m](image)

Figure 5.11. Hysteresis loops under different excitation loads applied to the assembled structure under the bolt torque of 5 N.m

As can be seen, increasing the excitation loads results in wider hysteresis loops indicating that more energy is dissipated in the system through friction in the joint interface. Also, it can be
observed that the stiffness of the joint is almost constant for all excitation levels as the slope of all hysteresis loops shown in Figure 5.11 are approximately unchanged. Therefore, the same stiffness is considered for the elastic shear layer and the energy loss and joint parameters are extracted using the calculation algorithm method as presented in Figure 5.12 and Figure 5.13 and Table 5.6.

![Energy loss](image1)

**Figure 5.12.** Energy loss ($W_d$) for different excitation levels at the bolt torque of 5 N.m identified by the calculation algorithm method

![Joint damping](image2)

**Figure 5.13.** Joint damping ($C_j$) for different excitation levels at the bolt torque of 5 N.m identified by the calculation algorithm method
Table 5.6 Joint parameters using both sensor-based approaches at 5 N.m bolt torque and under different excitation loads

<table>
<thead>
<tr>
<th>Excitation level</th>
<th>Properties</th>
<th>Values using calculation algorithm method</th>
<th>Values using hysteresis loop method</th>
<th>Difference between the results (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>2.052e-3 mN.m</td>
<td>2.289e-3 mN.m</td>
<td>10.35</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1543 N.s/m</td>
<td>1727 N.s/m</td>
<td>10.65</td>
</tr>
<tr>
<td>4 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>2.979e-2 mN.m</td>
<td>3.518e-2 mN.m</td>
<td>15.32</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1538 N.s/m</td>
<td>1827 N.s/m</td>
<td>15.81</td>
</tr>
<tr>
<td>8 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>0.114 mN.m</td>
<td>0.129 mN.m</td>
<td>11.62</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1540 N.s/m</td>
<td>1766 N.s/m</td>
<td>12.79</td>
</tr>
<tr>
<td>12 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>0.268 mN.m</td>
<td>0.298 mN.m</td>
<td>10.06</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1541 N.s/m</td>
<td>1726 N.s/m</td>
<td>10.71</td>
</tr>
<tr>
<td>16 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>0.439 mN.m</td>
<td>0.517 mN.m</td>
<td>15.08</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1543 N.s/m</td>
<td>1826 N.s/m</td>
<td>15.5</td>
</tr>
<tr>
<td>20 Vpp</td>
<td>Energy loss ($W_d$)</td>
<td>0.672 mN.m</td>
<td>0.809 mN.m</td>
<td>17.05</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1542 N.s/m</td>
<td>1867 N.s/m</td>
<td>17.4</td>
</tr>
</tbody>
</table>

The presented results show that there are small deviations between the calculation algorithm method and hysteresis loop approach in all excitation amplitudes with the maximum difference of 17.4%. In addition, it can be seen that the amount of joint damping is approximately constant in all excitation levels while the energy loss increases by increasing the excitation load applied to the assembled structure. A similar conclusion was driven by (Bograd et al. 2008) where they did not observe a significant change in the joint stiffness and damping by varying the excitation load when the joint pressure and bolt preload were high enough.
5.4.2 Investigating the effect of contact normal load

In order to investigate the effect of normal load on the joint properties using the presented sensor-based methods, the bolt torque changes. Therefore, two different torque loads, i.e. 3 N.m and 8 N.m, are applied to bolt joint and a certain excitation load, i.e. 20 Vpp, is applied to the structure and the joint properties are extracted using the calculation algorithm and the hysteresis loop methods.

Figure 5.14 shows the hysteresis loop of the structure under 20 Vpp when there are different normal forces applied to the bolted joint. It can be seen that the joint stiffness drops slightly from 1.18e7 N/m to 1.08 e7 N/m when the bolt torque decreases from 5 N.m to 3 N.m as the slope of the hysteresis loop decreases marginally. On the other hand, the slope of the joint stiffness and subsequently the stiffness of the shear layer remains almost constant with very small increase when the bolt torque increases from 5 N.m to 8 N.m. The stiffness of the shear layer is updated based on the new slope of the hysteresis loop under bolt torque of 3 N.m and 8 N.m and the new values are used for extracting the energy loss and joint dynamics using the calculation algorithm method. The results of the energy loss are presented in Figure 5.15 and Table 5.7.

![Hysteresis loops for excitation level of 20 Vpp and different bolt torques](image)

**Figure 5.14.** Hysteresis loops for excitation level of 20 Vpp and different bolt torques
The obtained energy loss values are used to find the joint damping using Eqs. (5.2) and (5.9) as presented in Figure 5.16 and Table 5.7.

**Figure 5.15.** Energy loss ($W_d$) for the excitation level of 20 Vpp and different normal loads identified by the calculation algorithm method

**Figure 5.16.** Joint damping ($C_j$) for the excitation level of 20 Vpp and different normal loads identified by the calculation algorithm method
Table 5.7 Joint parameters using both sensor-based approaches under 20 Vpp and for different normal loads

<table>
<thead>
<tr>
<th>Bolt torque</th>
<th>Properties</th>
<th>Values using calculation algorithm method</th>
<th>Values using hysteresis loop method</th>
<th>Difference between the results (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 N.m</td>
<td>Energy loss ($W_d$)</td>
<td>0.762 mN.m</td>
<td>0.871 mN.m</td>
<td>12.51</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1410 N.s/m</td>
<td>1624 N.s/m</td>
<td>13.1</td>
</tr>
<tr>
<td>5 N.m</td>
<td>Energy loss ($W_d$)</td>
<td>0.672 mN.m</td>
<td>0.809 mN.m</td>
<td>17.05</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1542 N.s/m</td>
<td>1867 N.s/m</td>
<td>17.4</td>
</tr>
<tr>
<td>8 N.m</td>
<td>Energy loss ($W_d$)</td>
<td>0.665 mN.m</td>
<td>0.728 mN.m</td>
<td>8.65</td>
</tr>
<tr>
<td></td>
<td>Joint damping ($C_j$)</td>
<td>1569 N.s/m</td>
<td>1729 N.s/m</td>
<td>9.25</td>
</tr>
</tbody>
</table>

Similar to the previous results, there is a small difference between the calculation algorithm method and the hysteresis loop approach. The obtained joint dynamic properties show that there is more energy loss in the system when less bolt torque is applied to the joint, i.e. 3N.m; however, the joint damping slightly changes compared to two other cases which is in line with the results found by (Bograd et al. 2008). It can be attributed to the fact that the normal load applied to the joint is high enough and its changes does not have considerable effect on the joint dynamics (Bograd et al. 2008).

5.4.3 Limitations and Assumptions

Some assumptions were made in this study which may affect the obtained results. A circular area was considered for the sticking and sliding zones of the contact interface when equal forces are applied to each end of the structure. However, applying exactly the same amount of forces to each side of the bolted structure is challenging using the available power supplies and shaker.
systems. In addition, the clamping pressure is assumed to be evenly distributed and an analytical approach was used for extracting the contact stress distribution in the joint interface. The considered contact stress distribution may include some errors and it may cause some discrepancies in the obtained results. Inserting the nanocomposite sensor to the joint interface does not change the dynamics of the system considerably but it changes the nature of the joint and might affect the accuracy of the developed model and calculation algorithm. Moreover, the displacement of the beam at the stick-slip boundary was assumed to be the same as the displacement at the end of the beam measured experimentally. It can only be acceptable if the joint interface is small. In order to have more precise results, the contact interface should be modelled analytically, and the accurate displacement of the beam should be predicted and employed in the calculation algorithm. In addition, it is assumed that the whole damping in the structure is caused by the friction in the joint and the other sources of damping, such as the connection between washers and beams, are eliminated in this study. Finally, the sticking zone is assumed to behave as an elastic spring with a constant stiffness in the whole area which is not necessarily true in reality.

Using the developed sensor for joint dynamics identification include some limitations. The developed sensors are polymer-based nanocomposite sensors and applying high normal load might result in plastic deformation and creep behaviour and negatively affect their performance. Also, the nanocomposite sensor needs to be calibrated each time that it is implemented in the joint interface using commercial sensors.

5.5 Summary

Nanocomposite sensors were inserted into the joint and employed for the contact force measurement and joint dynamics identification using two difference methods. Experimental data,
including the contact force and beams displacements, were used to first find the hysteresis loop and then extract the joint properties. Similarly, the stick-slip transition in the contact interface was identified using the experimental data along with a developed calculation algorithm. It was then used to obtain the energy dissipation and damping of the joint. The developed techniques consider the nonlinearities in the joint which have been usually eliminated for simplicity. The extracted joint dynamics results were compared to each other where small deviations were observed that can be attributed to the assumptions made for each method. The maximum difference between the identified joint parameters from the hysteresis loop and the calculation algorithm methods was found to be approximately 17.4%. The effect of varying the joint conditions, i.e. normal force and excitation load, was investigated experimentally through both sensor-based methods where similar conclusions were obtained. It was observed that changing the conditions does not have considerable effect on the joint dynamics if the normal pressure applied to the system is high enough.
Improving the accuracy of machined parts is of great importance in the manufacturing industry. Chatter vibration is a challenging phenomenon that occurs during the machining process and hinders the safety and productivity. This problem can be addressed effectively if optimal cutting conditions are appropriately chosen through virtual prototyping technology that provides an FE model of the machine tool. It decreases the cost and time associated with testing and iterative changing of physical prototypes. Dynamic properties of all components of the machine tool, especially mechanical joints that add flexibility and damping to structures, need to be identified to build an accurate virtual prototype of the system and subsequently prevent chatter by selecting optimal cutting conditions. This chapter provides a summary of this study along with the novel contributions. Finally, the future works of this study are elaborated.

6.1 Summary

A summary of this study is discussed in this section. In Chapter 2, a brief introduction and literature survey was provided. The importance of virtual prototyping technology was discussed, and also different virtual models of the machine tools were reviewed. Different techniques presented for identifying the joint dynamic behavior, including modeling techniques, direct and iterative updating techniques, and experimental approaches were reviewed. The advantages and disadvantages of each method were then discussed and the importance of developing new techniques which eliminate the limitations of the existing methods was emphasized. In addition, the importance and applications of PNC sensors were reviewed. Different polymer base selections,
especially PVDF as a piezoelectric polymer, were discussed and the importance of CNTs as nanofillers with unique properties were presented. Different fabrication methods, including solution mixing and melt mixing techniques were reviewed and the challenges associated with each method were elaborated. Finally, different simulation models presented for predicting the properties of the CNT-PNC nanocomposites were reviewed.

In Chapter 3, different approaches were presented for predicting the joint dynamics behaviour in tangential, torsional and transverse directions. The method proposed for joint dynamics identification in transverse direction is an analytical technique which employs the modal parameters of the whole structure that can be readily measured through experimental modal analysis. The accuracy of this method was verified through FE simulation and experiments after comparing the results with those of an existing joint dynamics identification method, i.e. inverse receptance coupling (IRC) method. In addition, the proposed method for the torsional and tangential directions is based on isolating the bolted lap joint through adding a mechanical resonator to the system and then using the FRF of the combined structure. The accuracy of this method was verified after comparing the experimental results with those of the hysteresis loop method.

Chapter 4 presented the fabrication process of the CNT-PVDF nanocomposite sensor that exhibits both piezoelectric and piezoresistive properties. In addition, the process of improving and reorienting the $\beta$ phase of PVDF which contains the highest dipole moment and is in charge of piezoelectricity of the polymer was elaborated. The accuracy of the developed sensor was verified with experimental tests including static and dynamic experiments. Finally, the piezoelectric and piezoresistive signals were combined using a sensor fusion technique to improve the accuracy and bandwidth of the sensor.
In Chapter 5, two different methods were developed to identify the joint dynamics using the nanocomposite sensor inserted into the joint interface. The established methods use the contact force and the displacement of the bolted beams measured by the nanocomposite sensor and displacement sensors, respectively. A model was proposed for different zones in the contact interface and the stick-slip boundaries were then predicted using the experimental data and a calculation algorithm. Therefore, the challenges associated with analytical and numerical microslip analysis were eliminated. The proposed methods were verified through experimental tests. In addition, the effect of different parameters, including contact normal load and excitation load on the identified joint properties were investigated using the sensor-based techniques.

6.2 Novel Scientific Contributions

The novel contributions of this study fall into four main categories, including new techniques for joint dynamics identification, and development of a new nanocomposite sensor. The main contributions of this study are discussed below.

6.2.1 Joint dynamics identification in lap joints in transverse direction

The first contribution of this study is development of an identification technique that can be applied to lap joints. This technique predicts the transverse joint properties of bolted lap joints. The presented technique avoids the limitations associated with the existing methods, including modeling errors and rotational FRF extraction. Also, this method requires no mode shape information, which is challenging and requires quite a few measurements. The only parameters needed in this approach are the natural frequencies and damping ratios of the whole structure. These parameters can be extracted through experimental modal analysis (EMA) with only one
measurement. Investigation of the effects of different factors, such as surface texture and inserting interfacial materials to the joint interface are also other contribution of this study. This information can be saved and used as a comprehensive database for the properties of bolted lap joints under various conditions. This database can be offered to the manufacturers for updating the virtual models of their machine tools and therefore improving the accuracy of the FE modeling.

6.2.2 Joint damping identification in lap joints in tangential and torsional directions

Another main contribution of this research is identifying the joint damping of bolted lap joints in tangential and torsional directions. Mechanical joints are the main source of nonlinearity in the system, depending on the applied bolt torque and excitation force to the system. The proposed method only uses the FRF of the structure after adding a mechanical resonator, consisting of a lumped mass and spring, to the bolted structure. Adding the mechanical resonator to the system results in a new combined structure containing only one dominant mode of vibration. Therefore, this method identifies the joint damping indirectly without any need for determining the stick-slip boundaries in the contact interface. This method can be employed for both translational and torsional joint damping extractions using only translational FRFs that can be readily obtained by measuring the input force to the system and the displacement of the mechanical resonator.

6.2.3 Development of a new nanocomposite sensor with piezoelectric and piezoresistive properties

Development of a new nanocomposite sensor for strain and force measurement over a wide frequency range is another novel contribution of this study. The piezoelectric and piezoresistive properties are combined in a single sensor to achieve high frequency bandwidth compared to commercially available sensors that they are only suited for measurement of static and low frequency signals, or dynamic and high frequency signals. Combining and fusing both the
piezoelectric and piezoresistive signals of the sensor is a novel work developed in this study to achieve a wider frequency bandwidth suitable for low and high frequency measurements in a single sensing package along with a higher accuracy. The nanocomposite sensors are thin and flexible and therefore they can be implemented in the joint interface to measure the contact force. In addition, the developed sensors can be used for other applications such as condition health monitoring beside the machining process applications. These sensors can be attached to the boring bars or the machine tools to monitor the condition of the system and detect chatter.

6.2.4 Development of new methods for joint dynamics identification using the nanocomposite sensor

Using the commercial piezoelectric sensors to measure the force and displacement is sometimes challenging because of their size. Also, the measurement areas might not be easily accessible. The developed nanocomposite sensors can be inserted into the joint interface of the assembled structures without considerably changing the dynamics of the system as proved experimentally. It is a novel contribution of this study to use the sensor signals for joint dynamics identification using new established techniques. These methods can result in the energy loss and predicting the boundary between the sticking and sliding zones using an efficient calculation algorithm and without any need to go through all the complicated microslip modeling processes.

6.3 Future Works

The present study develops different effective methods for joint dynamics identification. However, the accuracy of the predicted joint properties can be improved if some of the limitations and assumptions associated with each method are addressed. Future of this study can be divided into three main categories, i.e. (a) improving the models and techniques presented for joint
dynamics identification (b) improving the sensitivity of the developed nanocomposite sensor (c) employing the developed nanocomposite sensors for other applications.

6.3.1 Improving the models and techniques presented for joint dynamics identification

Euler-Bernoulli theory was used in this study for determining the joint properties in transverse direction and the effects of shear deformation and rotational inertia were neglected in the model. However, Timoshenko theory overcomes these limitations and therefore it might provide more accurate results for practical applications.

The experimental setup proposed for the joint properties prediction in tangential and torsional directions can be modified if the mechanical resonator is attached to the bolted structure using rigid connections and therefore more accurate results might be achieved. Also, the change of joint damping properties due to varying joint conditions can be studied. Parameters of study would include joint preload, material properties, inclusion of interfacial layers, and interface roughness. Also, the developed method can be used for finding the optimized joint damping by changing the number and location of the bolted lap joints.

In addition, an analytical modeling can be developed for finding the displacement of the beam in the stick-slip boundary instead of using the experimental measurements. It can be accomplished by developing an analytical model as below.

Figure 6.1. Schematic of the analytical model for calculating the response of the bolted beam
After considering a shear layer with the stiffness of \( K \) for the sticking zone of the joint interface, the equations of motion can be derived as given in Eq. (6.1).

\[
\begin{align*}
EA \frac{\partial^2 u}{\partial x^2} &= \rho A \omega^2 \frac{\partial^2 u}{\partial \theta^2} \quad 0 \leq x \leq L_1 \\
EA \frac{\partial^2 u}{\partial x^2} - \mu P(x) &= \rho A \omega^2 \frac{\partial^2 u}{\partial \theta^2} \quad L_1 \leq x \leq L_2 \\
EA \frac{\partial^2 u}{\partial x^2} - 2ku &= \rho A \omega^2 \frac{\partial^2 u}{\partial \theta^2} \quad L_2 \leq x \leq L_3 \\
EA \frac{\partial^2 u}{\partial x^2} - \mu P(x) &= \rho A \omega^2 \frac{\partial^2 u}{\partial \theta^2} \quad L_3 \leq x \leq L_4 \\
EA \frac{\partial^2 u}{\partial x^2} &= \rho A \omega^2 \frac{\partial^2 u}{\partial \theta^2} \quad L_4 \leq x \leq L
\end{align*}
\]  

(6.1)

where \( E, A \) and \( \rho \) are the module of elasticity, cross-sectional area, and density of the beam, respectively, \( u \) is the displacement of the beam, \( P \) represents the normal force in the contact interface, \( \omega \) is the excitation frequency and \( \theta = \omega t \), \( 0 \leq \theta \leq \frac{\pi}{2} \).

The boundary and compatibility conditions are given in Eqs. (6.2) and (6.3), respectively.

\[
EA \left. \frac{\partial u}{\partial x} \right|_{x=0} = u'(0) = 0 \quad , \quad EA \left. \frac{\partial u}{\partial x} \right|_{x=L} = EAu'(L) = F_0 \sin \theta \quad (6.2)
\]

\[
\begin{align*}
u(L_1, \theta) &= u(L_1^+, \theta) \quad , \quad u'(L_1^+, \theta) = u'(L_1^+, \theta) \\
u(L_2, \theta) &= u(L_2^+, \theta) \quad , \quad u'(L_2^+, \theta) = u'(L_2^+, \theta) \\
u(L_3, \theta) &= u(L_3^+, \theta) \quad , \quad u'(L_3^+, \theta) = u'(L_3^+, \theta) \\
u(L_4, \theta) &= u(L_4^+, \theta) \quad , \quad u'(L_4^+, \theta) = u'(L_4^+, \theta)
\end{align*}
\]

(6.3)

where \( F_0 \) is the amplitude of the force applied to the system.

Solving the equations of motion given in Eq. (6.1) and applying the boundary and compatibility conditions can result in an accurate estimation of the response of the system.
6.3.2 Improving the sensitivity of the developed nanocomposite sensor

To improve the sensitivity of the nanocomposite sensor, different fabrication methods such as melt mixing technique can be explored. Also, different nanoparticles, especially inorganic piezoelectric particles such as BaTiO3 (Ye et al. 2013), PZT (Graz et al. 2009) or ZnO (Dodds et al. 2012) can be added to the nanocomposite sensor in order to improve its piezoelectricity. In addition, adjusting the poling conditions can be investigated in future studies in order to achieve the highest amount of β phase reorientation during polling (Mahadeva et al. 2013). The effect of CNT concentration on the performance of the sensor can be investigated through both numerical simulations and experimental tests. A hand spray system was used for fabricating the nanocomposite sensor and it might cause inconsistency in the developed sensors. This problem can be partially eliminated if an automatic spray system is employed. Efficient electrode deposition methods such as sputtered silver and appropriate electrical connections could also improve the performance and consistency of the sensor. Improvements in performance can also be achieved with improved fusion algorithms.

6.3.3 Employing the developed nanocomposite sensor for other applications

The nanocomposite sensor was inserted into the joint interface and used for predicting the joint properties. In addition, this sensor can be employed for other applications such as cutting force measurement in both boring and milling operations after developing a wireless communication module. Figure 6.2 shows that the nanocomposite sensor can be mounted on the boring bars and spindle to measure the cutting forces in boring and milling operations, respectively.
In addition, we have already used the nanocomposite sensors for building a table dynamometer for cutting force measurement in milling operations (Park et al. 2016). A schematic and the constructed image of the developed CNT-PNC based force sensing system is illustrated in Figure 6.3.

Similarly, the developed nanocomposite sensor can be used in a smart jaw system to monitor the gripping forces during boring and turning operations and prevent unwanted incidents.


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