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# Three Essays on Business Taxation

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Three Essays on Business Taxation

by

Feng Wei

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
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# Abstract

This thesis contains three essays on the design of business income taxation and its effects on labor market outcomes. Given the existence of informal sector activities, especially in developing countries, and the highly regressive nature of the fixed costs of registering and complying with taxation, the first two chapters, provide theoretical frameworks for designing welfare-maximizing tax systems in countries where informal activities are pervasive. The third chapter studies the causal effects of corporate income tax on wages and hours of employment using the 2001-2004 federal tax reform in Canada as a natural quasi-experiment.

In Chapter 1, we construct a simple model where a turnover threshold separates firms paying standard corporate income tax from firms (below the threshold) who pay a tax on their sales (turnover). Thus, closed-form solutions are derived for the optimal threshold, in terms of the standard corporate income tax rate and the turnover tax rate; and closed-form solutions are derived for the optimal turnover tax rate as a function of the threshold.

Chapter 2 extends the simple model by endogenizing firms' sales levels through their input choices. We analyze a model where entrepreneurs allocate labor to the formal and informal sectors. Formal sector income is subjected either to a corporate income tax or a tax on turnover, depending on whether their turnover exceeds a threshold. Given private behavior, social welfare is optimized. We interpret the first-order conditions for welfare maximization to identify the key margins and then simulate a calibrated version of the model.

Chapter 3 studies the labor market effects of the corporate income tax by exploiting a 2001-2004 federal tax reform in Canada. This reform lowered the federal statutory corporate income tax rate by 25% (i.e., from 28% to 21%) in the sectors which were not previously under special tax treatment. Results from difference-in-differences regressions suggest that workers benefited significantly from tax reduction. Consistent with Griliches' capital-skill complementary hypothesis, this paper finds that high-educated workers benefited more from the reform compared with medium-educated or low-educated workers.

# Preface

This thesis is an original work by the author. No part of this thesis has been previously published. Chapter 1 and 2 of this thesis are co-authored with Jean-François Wen. I have obtained the permission from the author for me to include these two chapters in my thesis.

# Acknowledgements

I would like to express my first and sincerest appreciation to my supervisor, Dr. Jean-François Wen. Without his intellectual guidance and continuous support, I wouldn't become who I am today, that is, far beyond my expectation compared to the one who entered the Ph.D. program six years ago. He is not only my supervisor who is very generous with his knowledge, time and encouragement, but also an excellent researcher who I would like to emulate in my research career. He taught me how to build up a model step by step. He provided a lot of time to answer my questions. He always encouraged me when I was stuck in difficulties. I couldn't have this thesis finished without his support and advice.

Special thanks to my committee members, Dr. Daniel Gordon and Dr. Stefan Staubli. Without Dr. Gordon, I may not have pursued my academic career after my M.A.. He is the one inspiring me all the time and offered me opportunities to work with him as a teaching assistant. Dr. Staubli helped me a lot on my thesis, presentations, job market and I am really grateful to him for teaching me different empirical methods and guiding me in an empirical research. I am thankful to Dr. John Boyce, who allowed me to practice my job talk with him and provided me many valuable suggestions. Thanks to Dr. Katherine Cuff and Dr. Eugene Beaulieu for offering valuable comments for my thesis.

Also many thanks to all my colleagues and administrators from our department, who made my Ph.D. life much easier. In the end, I really want to thank all my family members, especially my parents and Yiyang Shao, I am indebted to them for their continuous and generous support.

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# Chapter 1

## Designing Presumptive Taxes in Countries with Large Informal Sectors

### 1.1 Introduction

Informal sector activities are pervasive in developing countries.<sup>1</sup> Medina and Schneider (2017) estimate that the average size of the so-called shadow economy is 36.1% of official GDP in Sub-Saharan Africa and 34.8% in Latin America and the Caribbean, compared to 18.4% in the OECD. The vast majority of informal businesses have fewer than five employees and many have none, but there are also relatively large businesses operating without licenses or tax identification numbers (see, e.g., Amin and Islam, 2015, and Benjamin and Mbaye, 2012). Bringing the larger and more profitable informal firms into the formal system would widen the tax base and probably more than justify the additional administrative costs.<sup>2</sup> There are also compelling reasons to try to tax micro-enterprises, in terms of improving horizontal equity and elevating tax morale in the formal sector (Terkper, 2003, and Torgler,

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<sup>1</sup>We adopt the common definition that a business is formal if it is registered for the relevant municipal licenses and with the tax department (Bruhn and McKenzie, 2013). However, see Kanbur and Keen (2014) for a more nuanced discussion of the meaning of informality.

<sup>2</sup>Andrade et al. (2013) suggest that inspecting informal firms earning an average of \$1,000 a month in profits in Brazil would formalize more than enough firms for the revenues to pay for the costs of such enforcement.

2005). However, widespread evasion and limited taxation capacity constrain the feasible designs of tax policy for mobilizing revenues in developing countries.

For many firms, the decision to operate in the informal economy rests on the relative costs (for example, tax and regulatory costs) and benefits (for example, access to financial sector and legal systems) of operating in the formal economy (Bird and Zolt, 2008). According to an open-ended survey of informal enterprises in the third largest city in Brazil, firm owners responded that the main disadvantages of formalizing were the initial costs of registration (62 percent of respondents stated this), having to pay taxes (58 percent), having to pay for an accountant (34 percent), and the process of registering taking too much time (32 percent) (de Andrade et al., 2016). Similarly, in a World Bank survey of mainly micro-businesses in Ethiopia (Yesegat, 2015), the single biggest disadvantage of registering for taxes was identified by respondents as being: a higher tax burden or high tax rates (34 percent), complicated tax compliance procedures (13 percent), harassment by government officials (8 percent), and frequent inspections (8 percent), with other factors accounting for the rest. The fixed costs of registering and complying with taxation are evidently highly regressive and thus can act as barriers to formalisation. Consequently, strategies for inducing formality have focused on reducing the cost of registering a business and paying taxes. Examples of initiatives to cut costs include the establishment in many countries of one-stop shops for registering a business for tax and licensing and disseminating information about how to formalize and file taxes (Joshi et al., 2014).

In a similar vein, presumptive tax regimes for small-, and medium-sized enterprises (SMEs), which use turnover or some other simple proxy for profitability as the tax base, can help deter informality by facilitating taxpayer compliance. For SMEs, complying with the standard tax regime's burdensome recordkeeping requirements not only is costly, but often exceeds the capacity and skills of the small business operator (Engelschalk, 2007). Turnover-based presumptive systems can ease the accounting requirements for filing taxes, while still obliging small businesses to keep basic books and records, thereby facilitating an

eventual transition from the presumptive to the standard tax regime. Recent work has highlighted the importance of reducing taxpayer compliance costs in encouraging entrepreneurs and small firms to voluntarily register for tax. Harju et al. (2019) show that registration for VAT in Finland increased substantially in response to simplified reporting procedures (including changing from monthly to annual VAT filing) for firms below a threshold of 25,000 euros. In Georgia, new tax regimes were introduced for micro and small businesses in 2010 to significantly lowered compliance and administrative costs of businesses and the revenue service. Specifically, firms without employees and an annual turnover below GEL 30,000 (\$18,255) were excluded from taxation (but paid a patent prior to the reform) and businesses with turnover between GEL 30,000 and GEL 100,000 (US\$ 60,850) qualified for a turnover tax regime instead of regular income tax. Bruhn and Loeprick (2016) report an 18-30% increase in the number of newly registered formal firms below the eligibility threshold of GEL 30,000 during the first year of the reform, though not in the following two years. The authors also found no evidence that the newly registered micro firms were previously formal firms producing just above the threshold. In the absence of such strategic sorting, the newly registered micro firms plausibly were drawn in from an existing stock of informal firms that decided to register when the reform was introduced. However, at the GEL 100,000 cutoff for the new small business turnover tax regime, Bruhn and Loeprick did not find a robust effect of the reform on formal firm creation in any year.

A threshold is required to limit the presumptive regime to smaller enterprises only. It is standard to define the threshold itself in terms of turnover, while putting consolidation rules into place to prevent entities from artificially splitting for tax purposes. In contrast, using the number of employees to define the threshold makes it prone to outsourcing labor.<sup>3</sup> In low-income countries, turnover taxes appear best suited for small companies, rather than for self-employed workers operating informally and without employees.<sup>4</sup> The latter group,

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<sup>3</sup>However, even if they do not serve as the presumptive tax threshold, alternative size or profit indicators, such as the number of employees, the size of the business premises, or the value of assets, can be useful information to collect for auditing a company's reported turnover.

<sup>4</sup>The case is likely different in developed countries, where the education level of self-employed individuals

which characterizes a substantial portion of the informal sector, is unlikely to formalize, even with simplified accounting and financial incentives to do so (La Porta and Shleifer, 2008, de Mel et al., 2010, de Mel et al., 2013, and Loayza, 2016). Thus, a fixed fee (patent system) may be a more realistic approach to taxing self-employed workers and microenterprises with few employees. In Uzbekistan, for instance, as of 2019, unincorporated businesses with fewer than 5 employees pay a fixed amount, rather than the 4% turnover tax applied to businesses below a turnover threshold of \$120,000. A more drastic approach is to leave untaxed microenterprises below a lower bound threshold level for the turnover tax, as described above in the case of Georgia.

Despite the simplified accounting requirements under a turnover-based presumptive tax, the experiences some countries have had with the tax are disappointing. In Tanzania, which operates a presumptive tax that is a progressively increasing proportion of turnover, the level of recordkeeping among Tanzanian SMEs has not increased much, despite a substantial tax concession for SMEs who keep (simplified) records (Engelschalk, 2007). In Kenya, a tax of 3% on declared turnover (and 3% of the threshold if the business does not keep accounts) was introduced in 2008 to curb the rapidly growing informal sector. Despite the simplification in tax compliance and tax computation, the uptake and revenue yield were weaker than expected (Wanyagathi Maina, 2017). A new presumptive tax of 15% percent of the amount payable for a business permit or trade license was introduced in 2019 in the hope of widening the tax base to include more informal firms and the small businesses into the tax ambit. Such experiences reinforce the observation that simplified procedures and presumptive tax policies by themselves are insufficient to induce enterprises to formalize.

In the case of Kenya, the need for the government to demonstrate to informal businesses that non-compliance can be detected and punished was noted in IEA (2012). In a field experiment in Brazil, de Andrade et al. (2014) estimated that receiving an inspection generated a 21 to 27 percentage point increase in the likelihood that an enterprise will of formalize. The

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is likely to be higher.

probability of detection by tax or other government authorities, resulting in penalties, raises the cost of operating a company informally. Thus, policies of simplifying taxation and taxpayer education need to be supplemented with enhanced tax enforcement and verification. Recent technological improvements may provide tax authorities with greater capabilities to observe and monitor transactions and taxpayers and hence to estimate revenue and profits (Bird and Zolt, 2008). Slemrod et al. (2019) report that innovative programs, using social and psychological factors can serve to complement standard measures for deterring tax evasion in developing countries. In Pakistan, for instance, public disclosures of tax payments boosted the rate of tax filing and the amounts of declared tax liabilities from self-employment earnings.

We construct a simple model where a turnover threshold separates firms paying standard corporate income tax from firms (below the threshold) who pay a tax on their sales (turnover). We make two extreme assumptions to allow us to study, in the simplest possible way, the behavior of firms faced by the turnover tax regime. Thus, following Kanbur and Keen (2014), we suppose that firms can costlessly adjust their output downward from an exogenous maximum potential amount and that they can escape taxation altogether by becoming ‘ghosts’ in the shadow economy.<sup>5</sup> While crude, the assumptions highlight how the option to produce informally constrains the turnover tax rate and how the turnover tax interacts with the standard corporate income tax regime to create ‘notches’ in the production levels of firms. The production inefficiencies associated with the presumptive regime and with informal sector production are also modeled very simply, as exogenous additions to the marginal cost of production.

The advantage of such a streamlined model is that it is easy to calibrate and generates practical tax policy guidelines. Thus, closed-form solutions are derived for the optimal

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<sup>5</sup>Waseem (2018) shows that the number of enterprises operating in the formal sector responds to tax pressures. When Pakistan increased the tax on unincorporated partnerships to a level comparable to the standard corporate tax, within three years of the tax increase, the number of partnerships in Pakistan had declined to 36% of the baseline level. Very few of these companies became incorporated. It is plausible that a portion of them became informal.

threshold, in terms of the standard corporate income tax rate and the turnover tax rate; and closed-form solutions are derived for the optimal turnover tax rate as a function of the threshold. The solutions depend on whether bunching below the threshold occurs or not; both types of equilibria are possible, depending on the parameter values of the model. The formulas for the optimal threshold are relatively simple and intuitive. In contrast, the formulas for the optimal tax rate are long and tedious, reflecting the complicated balancing act between two behavioral margins. That is, if the tax rate is too low, some firms will migrate away from the standard regime, but if it is too high then some firms will choose to produce in the informal sector. Nevertheless, the model optimal threshold and turnover tax rate can be computed with a spreadsheet. Numerical solutions of the model for different scenarios and comparative statics analysis are used to provide further insights.

The numerical values for the optimal threshold and turnover tax rate are both lower than the results of the more complicated model of Wei and Wen (2019), where production is endogenous through the input choices of heterogeneous firms. It is not quite evident why this is the case, but we note that the optimal threshold for VAT using Keen and Mintz’s (2004) ‘simple rule,’ which has much in common with our simple model, is also substantially lower than the optimal threshold in Keen and Mintz’s (2004) more general model, characterized by heterogeneous production technologies. Furthermore, the restraint on the optimal turnover tax rate, imposed by the option of firms disappearing into the informal sector in the present model, reduces the relative benefit of the presumptive regime over the regular regime, placing downward pressure on the optimal threshold. The main parameters of interest in our current analysis are the (constant) marginal cost of production in the informal sector and the fixed cost of taxpayer compliance in the presumptive regime. Both of these parameters can be influenced by the tax enforcement policies of the authorities. For example, the cost of operating in the shadow economy increases with the probability of inspection. The cost of compliance is diminished by disseminating information to informal enterprises on the processes of bookkeeping and filing taxes. Hence, the study contributes

to our understanding of how to design presumptive tax regimes for economies characterized by high levels of informality.

Section 2 presents the model. Section 3 describes the optimal tax policy and provides the comparative statics analysis. Section 4 calculates the optimal turnover threshold and turnover tax rate for alternative calibrations of the economy. Section 6 concludes. Proofs of lemmas and propositions are contained in an appendix.

## 1.2 A Simple Model of Firms' Behavior

We suppose that there is a population of firms endowed with differing levels of *maximum* potential sales,  $Z \in (0, Z^M)$ , where  $Z$  is exogenous and distributed according to a twice differentiable distribution function  $H(Z)$  with strictly positive density  $h(Z)$ . The upper-support of the distribution,  $Z^M$ , may be finite or infinite. To study the behavioral responses of firms in the simplest way possible, we follow Kanbur and Keen (2014) in assuming that firms can choose to adjust (costlessly) their output to any level below their maximum potential, for instance, to reduce their tax burden. The marginal cost of production is assumed to be a constant proportion of sales,  $C < 1$ , which is identical across the population. The output price is normalized to one, so output is the same as sales. Hence, a firm producing  $Z$  has a pre-tax profit of  $(1 - C)Z$ .

The tax system is as follows. Firms with actual sales equal to or exceeding a threshold  $\bar{Z}$  face a corporate income tax rate of  $t^c$  on their profits (henceforth, the ‘regular regime’), while firms selling below the threshold are taxed on their sales, with no deduction for costs, at the rate  $t$  (henceforth, the ‘presumptive regime’). The corporate income tax rate is taken to be exogenous. The turnover tax rate  $t$  and the threshold  $\bar{Z}$  are the policy choices in the model. Firms face fixed compliance costs of  $\Gamma$  and  $\Gamma'$ , in the standard and presumptive regimes, respectively. The government faces corresponding fixed administrative costs of  $A$  and  $A'$ . Since sales are easier to record and audit than profit, let  $\Gamma > \Gamma' > 0$  and  $A > A' > 0$ .



While the compliance costs are exogenous in the model, public initiatives, such as a one-stop shop for registering a business with the various licensing and tax authorities, and educational campaigns on bookkeeping and tax filing, can be regarded as efforts to reduce the size of  $\Gamma'$  for SMEs. We shall also suppose that there is an extra marginal cost  $\alpha$ , on top of  $C$ , incurred by firms in the presumptive regime, possibly representing higher borrowing costs due to a lack of verifiable information on profit (since the tax base is revenue) or, more generally, the production inefficiencies caused by the non-deductibility of costs under presumptive taxation. Finally, as in Kanbur and Keen (2014), we assume that firms can escape taxation altogether by ‘disappearing’ into the informal sector. However, in this case, their marginal cost is increased by an amount  $\lambda$ , on top of  $C$ , where  $\lambda$  is an exogenous parameter representing the inefficiencies associated with the informal sector. For example, informal enterprises face inconveniences in having to produce in ways that avoid detection by tax officials.<sup>6</sup> Indeed, advances in electronic technology has facilitated coordination between government agencies, such that a taxpayer identification number (TIN) can be required to access government services, such as obtaining passports or driver’s licenses, register cars and property, use public schools or hospitals, or to subscribe to public utility services—thus further increasing the costs of operating outside the tax system (Bird and Zolt, 2008).

We suppose that

$$(1 - t^c)(1 - C) > 1 - C - \alpha - t > 1 - C - \lambda > 0 \quad (1.1)$$

Hence, so long as the tax rate of the presumptive regime is not too high, the net profit margin is lowest in the informal sector and highest in the regular regime. The inequalities are assured for any  $t$  by the assumption that  $\lambda > \alpha > t^c(1 - C)$ . We also assume that the fixed compliance costs,  $\Gamma$  and  $\Gamma'$ , are not so large as to preclude the possibility that large producers (i.e., for  $Z$  in a neighborhood of  $Z^M$ ) can earn a strictly positive net profit in the

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<sup>6</sup>For example, informal firms have less scope for marketing or they locate in obscure locations to avoid attracting the attention of the law (Bruhn and McKenzie, 2013).

regular regime (given  $t^c$ ) and in the presumptive regime (at  $t = 0$ ), respectively.

We can summarize the structure of the economy by specifying the after-tax profit functions of four types of firms: those producing at their maximum sales level in the regular regime (*regulars*, earning  $\pi^R$ ); those who adjust their sales downward to just below the threshold (*adjusters*, earning  $\pi^A$ );<sup>7</sup> those producing at their maximum sales in the presumptive regime (*presumptive*, earning  $\pi^P$ ); and those producing at their maximum sales but escaping taxation by remaining informal (*informals*, earning  $\pi^I$ ). Each firm chooses how to behave to maximize its after-tax profits. The net profit functions are:<sup>8</sup>

$$\begin{aligned}\pi^R(Z) &= (1 - t^c)(1 - C)Z - \Gamma \\ \pi^A(\bar{Z}) &= (1 - t - C - \alpha)\bar{Z} - \Gamma' \\ \pi^P(Z) &= (1 - t - C - \alpha)Z - \Gamma' \\ \pi^I(Z) &= (1 - C - \lambda)Z\end{aligned}\tag{1.2}$$

## 1.2.1 Partitioning the distribution of firms

### 1.2.1.1 Informal and formal sectors

Recall that firms have the option of becoming informal.<sup>9</sup> There are three such pairwise comparisons to consider: informality versus presumptive taxation; informality versus adjusting; and informality versus regular taxation. Firms prefer informality to presumption if and only if

$$(1 - C - \lambda)Z > (1 - t - C - \alpha)Z - \Gamma'\tag{1.3}$$

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<sup>7</sup>Their sales are below, but arbitrarily close to, the threshold.

<sup>8</sup>In the case of adjusters, since firms must produce below the threshold to be eligible for the presumptive regime, their profit is below, but arbitrarily close to,  $\pi^A$ .

<sup>9</sup>We shall label all taxpaying firms (or, more precisely, firms registered with the tax authorities) as ‘formal,’ regardless of whether they are in the regular tax regime or the presumptive tax regime. In contrast, informal firms evade taxation altogether.

which defines a cutoff sales level,

$$Z^{IP} = \frac{\Gamma'}{\lambda - t - \alpha} \quad (1.4)$$

All firms with  $Z < Z^{IP}$  prefer informality over presumptive taxation. The expression for  $Z^{IP}$  shows how the proportion of firms in the informal sector is shaped by the taxpayer compliance cost in the presumptive regime and the relative cost disadvantage of producing informally. Similarly, firms prefer informality over adjusting if  $Z > Z^{IA}$  where,

$$Z^{IA} = \frac{(1 - t - C - \alpha)\bar{Z} - \Gamma'}{1 - C - \lambda} \quad (1.5)$$

Finally, firms prefer informality over the regular regime if  $Z < Z^{IR}$  where,

$$Z^{IR} = \frac{\Gamma}{\lambda - t^c(1 - C)} \quad (1.6)$$

### 1.2.1.2 Jumping and bunching

Firms with maximum sales of at least  $\bar{Z}$ , that choose to remain in the formal sector, must decide between producing at their maximum and being subjected to the regular regime, or reducing their output to just below the threshold and paying the presumptive tax. Given a threshold  $\bar{Z}$  and tax rates  $t^c$  and  $t$ , adjusting dominates the regular regime whenever

$$(1 - t - C - \alpha)\bar{Z} - \Gamma' > (1 - t^c)(1 - C)Z - \Gamma \quad (1.7)$$

which defines a cut-off sales level

$$\hat{Z} = \frac{(1 - t - C - \alpha)\bar{Z} - \Gamma' + \Gamma}{(1 - t^c)(1 - C)} \quad (1.8)$$

All firms with sales below  $\hat{Z}$  prefer adjusting over being regulars.

Adjusting gives rise to two distinct situations, depending on whether  $\hat{Z} \geq \bar{Z}$  or  $\hat{Z} < \bar{Z}$ .<sup>10</sup> Figure 1.1 illustrates  $\hat{Z} < \bar{Z}$ , which we shall refer to as the ‘jumping’ case. As the figure shows, firms with potential sales inferior to the threshold produce their maximum output and face the presumptive tax, earning  $\pi^P(Z)$ ; then at the point  $Z = \bar{Z}$ , profit ‘jumps’ up as firms become regulars, earning  $\pi^R(Z)$ . Figure 1.2 shows the other possibility, where  $\hat{Z} \geq \bar{Z}$ , which is the ‘bunching’ case. All the firms in the segment  $[\bar{Z}, \hat{Z})$  have  $\pi^A(\bar{Z}) > \pi^R(Z)$  and hence ‘the bunch of them’ choose to adjust their production to a level that is just below the threshold, each earning the same profit,  $\pi^A(\bar{Z})$ . When bunching occurs, there will be a notch between  $\bar{Z}$  and  $\hat{Z}$  where no production is observed.

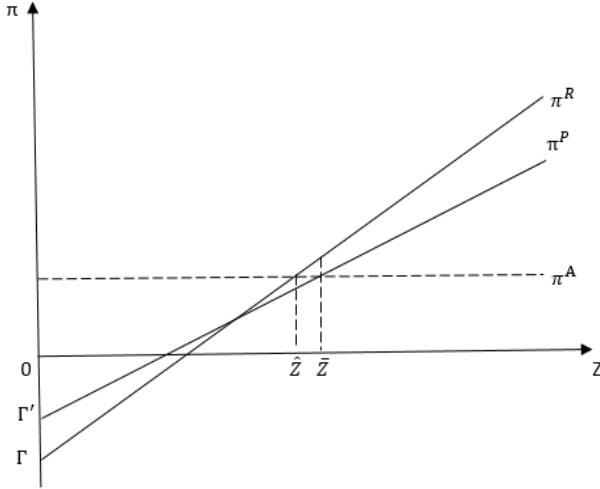


Figure 1.1: The “Jumping” Case

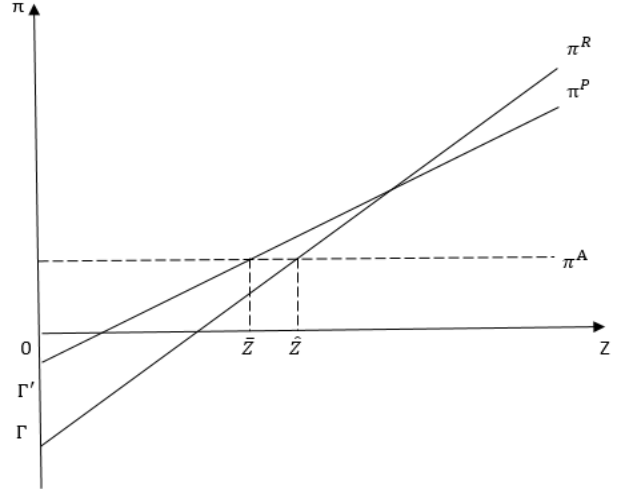


Figure 1.2: The “Bunching” Case

Using the cut-off levels defined above, we have two possible partitions of the distribution of firms in  $(0, Z^M)$ , corresponding to jumping and bunching. We will examine the jumping and bunching equilibria separately. It will turn out that either type of partition may feature as a welfare optimum, depending on parameter values and the distribution of maximum sales. Once the possible partitions have been established, we will turn to the question of welfare maximization.

<sup>10</sup>The inequalities can be expressed in terms of the tax rates: from (1.8),  $\hat{Z} \geq \bar{Z}$  when  $\bar{Z} \leq \frac{\Gamma - \Gamma'}{t + \alpha - t^c(1 - C)}$ .

### 1.2.1.2.1 Jumping

We begin the analysis of the private sector equilibrium under policy choices that generate the jumping case. The following lemma characterizes the different partitions that could arise under jumping.

**Lemma 1.1.** *If  $\hat{Z} < \bar{Z}$  (Jumping Case):*

1.  $\forall Z \in [0, \bar{Z})$ , firms choose between  $\pi^I$  and  $\pi^P$ :
  - 1.a. If  $Z^{IP} \leq \bar{Z}$ , then
    - i.  $\forall Z \in [0, Z^{IP})$ , firms locate in the informal sector
    - ii.  $\forall Z \in [Z^{IP}, \bar{Z})$ , firms locate in the presumptive regime
  - 1.b. If  $Z^{IP} > \bar{Z}$ , then
    - i.  $\forall Z \in [0, \bar{Z})$ , firms locate in the informal sector
2.  $\forall Z \in [\bar{Z}, \infty)$ , firms choose between  $\pi^I$  and  $\pi^R$ :
  - 2.a. If  $Z^{IR} \leq \bar{Z}$ , then
    - i.  $\forall Z \in [\bar{Z}, \infty)$ , firms locate in the regular regime
  - 2.b. If  $Z^{IR} > \bar{Z}$ , then
    - i.  $\forall Z \in [\bar{Z}, Z^{IR})$ , firms locate in the informal sector
    - ii.  $\forall Z \in [Z^{IR}, \infty)$ , firms locate in the regular regime

We have omitted discussing  $Z^{IA}$  in the jumping case, as any sales above the threshold  $\bar{Z}$  must be greater than  $\hat{Z}$ , implying that  $\pi^R > \pi^A$  for all  $Z > \bar{Z}$ . Therefore, firms above the threshold only need to compare  $\pi^R$  with  $\pi^I$ .

### 1.2.1.2.2 Bunching

Now we turn to the bunching case. The following partitions can arise under bunching.

**Lemma 1.2.** *If  $\hat{Z} \geq \bar{Z}$  (Bunching Case):*

1.  $\forall Z \in [0, \bar{Z})$ , firms choose between  $\pi^I$  and  $\pi^P$ :

1.a. *If  $Z^{IP} \leq \bar{Z}$ , then*

i.  $\forall Z \in [0, Z^{IP})$ , firms locate in the informal sector

ii.  $\forall Z \in [Z^{IP}, \bar{Z})$ , firms locate in the presumptive regime

1.b. *If  $Z^{IP} > \bar{Z}$ , then*

i.  $\forall Z \in [0, \bar{Z})$ , firms locate in the informal sector

2.  $\forall Z \in [\bar{Z}, \hat{Z})$ , firms choose between  $\pi^I$  and  $\pi^A$ :

2.a. *If  $Z^{IA} \leq \bar{Z}$ , then*

i.  $\forall Z \in [\bar{Z}, \hat{Z})$ , firms locate in the informal sector

2.b. *If  $Z^{IA} \in (\bar{Z}, \hat{Z})$ , then*

i.  $\forall Z \in [\bar{Z}, Z^{IA})$ , firms bunch just below the threshold

ii.  $\forall Z \in [Z^{IA}, \hat{Z})$ , firms locate in the informal sector

2.c. *If  $Z^{IA} \geq \hat{Z}$ , then*

i.  $\forall Z \in [\bar{Z}, \hat{Z})$ , firms bunch just below the threshold

3.  $\forall Z \in [\hat{Z}, \infty)$ , firms choose between  $\pi^I$  and  $\pi^R$ :

3.a. *If  $Z^{IR} \leq \hat{Z}$ , then*

i.  $\forall Z \in [\hat{Z}, \infty)$ , firms locate in the regular regime

3.b. *If  $Z^{IR} > \hat{Z}$ , then*

i.  $\forall Z \in [\hat{Z}, Z^{IR})$ , firms locate in the informal sector

ii.  $\forall Z \in [Z^{IR}, \infty)$ , firms locate in the regular regime

Note that there cannot exist situations where both  $Z^{IA}$  and  $Z^{IR}$  are greater than  $\hat{Z}$ , since this would generate a contradiction:  $\pi^R > \pi^A$ ,  $\pi^A > \pi^I$  and  $\pi^I > \pi^R$ . Thus, points 2.c and 3.b are mutually exclusive conditions. Similarly,  $Z^{IA}$  and  $Z^{IR}$  cannot both be smaller than  $\hat{Z}$  and, hence, 2.a. (2.b) invokes 3.a. We now turn to constructing and analyzing the social welfare function.

### 1.3 Social Welfare Analysis

Social welfare is the sum of aggregate private net incomes ( $\Pi$ ) and net tax revenue ( $G$ ), with the latter weighted by a factor  $\delta > 1$ , representing the marginal social value of tax revenues. The choice variables in the social welfare function are the turnover threshold  $\bar{Z}$  and the tax rate  $t$ . Hence,

$$SW(\bar{Z}, t) = \Pi(\bar{Z}, t) + \delta G(\bar{Z}, t) \quad (1.9)$$

The welfare function will consist of a series of integrals with limits of integration determined by the relevant partition of sales in  $(0, Z^M)$ , in accordance with lemma 1.1 or lemma 1.2. We simplify the problem with some preliminary observations on the optimal policy. First, in both the jumping case and the bunching case, it can never be optimal to set the threshold such that  $\bar{Z} < Z^{IP}$ , as this would cause all firms eligible for the presumptive regime, including those bunching just below the threshold, to prefer the informal sector (see part 1b of lemmas 1.1 and 1.2). Then, welfare would be at least as large (for a given  $t$ ) if the threshold were raised to the level  $Z^{IP}$ . More formally,

**Lemma 1.3.** *Any turnover threshold such that  $\bar{Z} < Z^{IP}$ , is (weakly) welfare-dominated by a threshold satisfying  $\bar{Z} \geq Z^{IP}$ .*

Lemma 1.3 allows us to drop from further consideration any policy combination  $\{t, \bar{Z}\}$  such that  $\bar{Z} < Z^{IP}$ . Second, notice that  $1 - t - C - \alpha$  must be at least slightly greater than  $1 - C - \lambda$ , as otherwise, given the fixed compliance costs, no firm would ever choose to

be subjected to the presumptive regime, since there is always the option of earning strictly positive profits in the informal sector. More formally,

**Lemma 1.4.** *Any turnover tax rate such that  $\lambda - t - \alpha \leq 0$  is (weakly) welfare-dominated by some tax rate satisfying  $\lambda - t - \alpha > 0$ , which, in turn, implies  $1 - t - C - \alpha > 0$ .*

The inequality in lemma 1.3 is used to formulate limits of integration in the welfare function, while lemma 1.4 will be useful later in the comparative statics analysis.

### 1.3.1 Welfare in the Jumping Case: $\bar{Z} > \hat{Z}$

Recall that all firms with  $Z < Z^{IR}$  would prefer to be informal over producing in the regular regime. Then, given  $Z^{IP} \leq \bar{Z}$  from lemma 1.3, the set of partitions in lemma 1.1 yield two possible cases for further consideration:  $Z^{IP} \leq \bar{Z} < Z^{IR}$  and  $Z^{IP} \leq Z^{IR} \leq \bar{Z}$ .<sup>11</sup> Then there is

**Lemma 1.5.** *The case  $Z^{IP} \leq Z^{IR} \leq \bar{Z}$  welfare-dominates the case  $Z^{IP} \leq \bar{Z} < Z^{IR}$ .*

Lemma 1.5 states that the optimal threshold should avoid firms with high-potential maximum sales finding it more profitable in the informal sector rather than staying in the regular regime.

Thus, we construct the social welfare function with  $Z^{IP} < Z^{IR} < \hat{Z} < \bar{Z}$ , which corresponds to the partition defined by 1.a and 2.a in lemma 1.1. The total profit function can be written as

$$\Pi(\bar{Z}, t) = \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ + \int_{\bar{Z}}^{Z^M} \pi^R h(Z) dZ \quad (1.10)$$

and the total tax revenue can be written as

$$G(\bar{Z}, t) = \int_{Z^{IP}}^{\bar{Z}} (tZ - A') h(Z) dZ + \int_{\bar{Z}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \quad (1.11)$$

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<sup>11</sup>We write  $Z^{IP} < Z^{IR} < \bar{Z}$  for convenience; it is also possible to have  $Z^{IR} < Z^{IP} \leq \bar{Z}$  but it will not change the argument.



Since the informal sector is an untaxed sector, there are no tax revenues collected from there. Firms with sales below the threshold (including bunchers) pay tax based on their sales, while firms with sales above the threshold pay tax based on their profit. The welfare function in a jumping configuration is then given by

$$\begin{aligned}
SW(\bar{Z}, t) = & \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ + \int_{\bar{Z}}^{Z^M} \pi^R h(Z) dZ \\
& + \delta \left\{ \int_{Z^{IP}}^{\bar{Z}} (tZ - A') h(Z) dZ + \int_{\bar{Z}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \right\}
\end{aligned} \tag{1.12}$$

The first-order condition with respect to the threshold  $\bar{Z}$  for an interior solution to the welfare maximization problem can be rearranged to give the following result.

**Proposition 1.** For given tax rates  $t$  and  $t^c$ , the optimal threshold in a jumping equilibrium (i.e., when  $\bar{Z} > \hat{Z}$ ) is given by

$$\bar{Z} = \frac{(\Gamma + \delta A) - (\Gamma' + \delta A')}{(\delta - 1)[t^c(1 - C) - t] + \alpha} \tag{1.13}$$

The expression for  $\bar{Z}$  is intuitive and independent of the distribution function  $H(Z)$ , except through its effect from  $t$ .<sup>12</sup> A small increase in the threshold causes the marginal firm to be moved from the regular regime to the presumptive regime. Since there is no bunching of firms in the jumping case, the marginal firm is unique. Thus the optimum occurs when the net welfare gain from the new arrival in the presumptive regime (equal to  $(1 - C - t - \alpha)\bar{Z} - \Gamma' + \delta(t\bar{Z} - A')$ ) balances with the net welfare loss from the firm exiting the regular regime (equal to  $(1 - t^c)(1 - C)\bar{Z} - \Gamma + \delta(t^c(1 - C)\bar{Z} - A)$ ). Proposition 1 gives a convenient formula for the optimal threshold at fixed (but not necessarily optimal) tax rates,  $t$  and  $t^c$ . It offers a guide for setting the threshold when the market equilibrium is characterized by ‘jumping,’ or, in practice, if no bunching is observed in the data and changing the tax rates themselves is not up for discussion. The formula (1.13) is akin to the ‘benchmark’ closed-

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<sup>12</sup>The proposition assumes the denominator of (1.13) is positive. If it is non-positive, then there is a corner solution, where  $\bar{Z} = 0$ .

form expression for the optimal VAT in Kanbur and Keen (2014) and Keen and Mintz (2004), which is interpreted there as an optimality condition when compliance is perfect and there are no behavioral responses (and the VAT tax rate is fixed). Along the same lines, (1.13) may serve as a benchmark for the threshold of the presumptive income tax regime. However, in the case of (1.13), behavioral responses of firms (by adjusting their sales downward) are not precluded; instead, the formula arises as an equilibrium outcome of the model, under parameterizations that result in ‘jumping’ at the optimal policy.<sup>13</sup>

From the first-order condition of welfare with respect to the tax rate  $t$  in the jumping equilibrium, we obtain the following optimality condition.

**Proposition 2.**

$$(\delta - 1) \int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ = \delta(tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt} \quad (1.14)$$

The left-hand side of (1.14) is the social benefit of the increased tax revenues collected from firms in the presumptive regime, as a result of raising the tax rate. On the right-hand side is the social cost of the lost tax revenues, net of administrative costs, resulting from firms now choosing to vanish from the formal sector into the informal sector ( $dZ^{IP}/dt > 0$ ). The latter amount is weighted by the density of firms at the margin of indifference between informality and operating in the presumptive regime.

An insight on the role of the effect of the informal sector on optimal tax policy can be obtained by examining the optimal turnover tax rate when the compliance cost is small.

**Proposition 3.** If  $Z$  is uniformly distributed and  $\bar{Z} \geq \hat{Z}$  (the jumping case), then the optimal presumptive tax rate approaches  $\lambda - \alpha$  as the compliance cost,  $\Gamma'$ , goes to zero.

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<sup>13</sup>If there are no behavioral responses of firms—equivalent in our model to removing the assumption that firms can strategically adjust their output—then (1.13) can serve as a benchmark even when there is bunching observed in the data, as in the interpretation of Keen and Mintz (2004), since a change in the threshold would mechanically reallocate some firms from one tax regime to the other. However, it begs the question as to why bunching may be observed in the first place.

That is,

$$\lim_{\Gamma' \rightarrow 0} t^* \rightarrow \lambda - \alpha \quad (1.15)$$

Proposition 3 is interesting because it demonstrates how the existence of an informal sector constrains the presumptive tax rate that the government can set. Since  $\lambda$  is the size of the inefficiency from producing in the informal sector, the higher is  $\lambda$  the larger the tax on turnover can be without causing firms to vanish from the view of the tax authorities. Conversely, if firms can operate relatively efficiently in the informal sector, then the tax rate in the presumptive regime must remain relatively low, as otherwise firms will prefer informality.

In the special case of  $H(Z)$  uniformly distributed on  $(0, Z^M)$ , the first-order condition for  $t$  given by (1.14) results in a cubic equation. The cubic can be written in general form as  $at^3 + bt^2 + ct + d = 0$ , where the coefficients  $a$ ,  $b$ ,  $c$ , and  $d$  are functions of the parameters of the model and the threshold  $\bar{Z}$ . The definitions of these coefficients is contained in the proof of the following proposition. Defining the discriminant as  $\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$  (Irving, 2013, Theorem 5.6),<sup>14</sup> we find that  $\Delta < 0$  in the numerical calibrations reported in Table 1 in section 1.4, when a uniform distribution is used for  $H(Z)$  and a jumping equilibrium is observed. Whenever  $\Delta < 0$  (Irving, 2013, Theorem 5.4), there is necessarily a unique real root for the cubic. This observation enables us to write a solution for the optimal turnover tax rate in terms of the threshold.

**Proposition 4.** If  $H(Z)$  is uniformly distributed and  $\Delta < 0$ , then there exist a unique real root for the first-order condition for the turnover tax rate, given by

$$t = \lambda - \alpha - \frac{2^{\frac{1}{3}}J}{(27\alpha J - 27K - 27J\lambda + \sqrt{-108J^3 + (27\alpha J - 27K - 27J\lambda)^2})^{\frac{1}{3}}} - \frac{(27\alpha J - 27K - 27J\lambda + \sqrt{-108J^3 + (27\alpha J - 27K - 27J\lambda)^2})^{\frac{1}{3}}}{3 \times 2^{\frac{1}{3}}} \quad (1.16)$$

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<sup>14</sup>In Theorem 5.6 from Irving (2013), the parameter  $a$  is set to 1.

where  $J = -\frac{(\delta+1)\Gamma'^2+2\delta\Gamma'A'}{(\delta-1)\bar{Z}^2}$  and  $K = \frac{[2\delta\Gamma'A'-(\delta-1)\Gamma'^2](\lambda-\alpha)}{(\delta-1)\bar{Z}^2}$ .

The formula for the optimal  $t$  in (1.16) is a twin of the formula for the optimal  $\bar{Z}$  in (1.13). While somewhat unwieldy-looking, (1.16) can be easily calculated with a spreadsheet to obtain the optimal tax at a given threshold and parameter values. The simultaneous solutions for (1.16) and (1.13) can also be readily computed numerically.<sup>15</sup> Although we have assumed a uniform distribution for  $H(Z)$  in deriving (1.16), our numerical analysis in section 1.4 shows that the optimal  $t$  and  $\bar{Z}$  is similar whether we assume a log-normal distribution or a uniform distribution with the same means. Furthermore, the optimal policies under jumping are not very different from the optimal policies under bunching, at the parameter values that replicate the features of developing countries. For these reasons, (1.16) and (1.13) may be regarded as benchmarks for the optimal design of a turnover-based presumptive income tax. We now turn to comparative statics analysis for further insights.

### 1.3.1.1 Comparative statics for a jumping equilibrium

We assume that the two first-order conditions necessary for an interior solution for  $t$  and  $\bar{Z}$  are satisfied and that the second-order sufficiency conditions are satisfied at the optimum.<sup>16</sup> The parameters of interest in our comparative statics analysis are  $\lambda$ ,  $\Gamma'$ ,  $t^c$ ,  $\alpha$ , and  $\delta$  on the optimal values of  $\bar{Z}$  and  $t$ . We first provide the *partial* effects of parameter changes on the optimal tax rate  $t$ , holding  $\bar{Z}$  fixed, and the optimal threshold  $\bar{Z}$ , holding  $t$  fixed. These partial effects can be useful for understanding the direction of optimal policy reform when only one policy variable is being considered for reform, such as a change in the desired threshold when the tax rate is not up for discussion. Then we present the *full* comparative statics, which can guide an overall reform in the presumptive tax regime.

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<sup>15</sup>The equations (1.16) and (1.13) can be combined to eliminate  $\bar{Z}$ , yielding another cubic equation in  $t$ .

<sup>16</sup>Conditional on the first-order conditions being satisfied, the latter conditions require  $\partial^2 SW/\partial \bar{Z}^2 < 0$ ,  $\partial^2 SW/\partial t^2 < 0$  and the Hessian to be negative definite  $((\partial^2 SW/\partial \bar{Z}^2)(\partial^2 SW/\partial t^2) - (\partial^2 SW/\partial \bar{Z}\partial t)^2 > 0$  at a locally optimal  $t$  and  $\bar{Z}$ .

**Lemma 1.6.** *In the case of a jumping equilibrium, the following are the partial effects of parameter changes on the optimal threshold, if the derivative of the density function  $h'(Z)$  is non-negative at the optimal threshold.*

$$1. \frac{d\bar{Z}}{d\lambda}|_t = -\frac{\partial^2 SW}{\partial Z \partial \lambda} \bigg/ \frac{\partial^2 SW}{\partial \bar{Z}^2} = 0$$

$$2. \frac{d\bar{Z}}{d\Gamma'}|_t = -\frac{\partial^2 SW}{\partial Z \partial \Gamma'} \bigg/ \frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$$

$$3. \frac{d\bar{Z}}{dt^c}|_t = -\frac{\partial^2 SW}{\partial Z \partial t^c} \bigg/ \frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$$

$$4. \frac{d\bar{Z}}{d\alpha}|_t = -\frac{\partial^2 SW}{\partial Z \partial \alpha} \bigg/ \frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$$

Note that the uniform distribution satisfies the requirement in the proposition, since  $h'(Z) = 0$  for all  $Z$ . It is a sufficient condition, but not a necessary one, for the comparative statics analysis.

Lemma 1.6 says that, holding the tax rate constant, the optimal threshold is unaffected by changes in the marginal cost of informality, and decreases in the cost of complying with the presumptive regime, as well as with the tax rate in the regular regime and the marginal cost of production in the presumptive regime.

**Lemma 1.7.** *In the case of a jumping equilibrium, the following are the partial effects of parameter changes on the optimal turnover tax rate, if the derivative of the density function  $h'(Z)$  is non-negative at the point of indifference between informality and the presumptive regime ( $Z^{IP}$ ).*

$$1. \frac{dt}{d\lambda}|_{\bar{Z}} = -\frac{\partial^2 SW}{\partial t \partial \lambda} \bigg/ \frac{\partial^2 SW}{\partial t^2} > 0$$

$$2. \frac{dt}{d\Gamma'}|_{\bar{Z}} = -\frac{\partial^2 SW}{\partial t \partial \Gamma'} \bigg/ \frac{\partial^2 SW}{\partial t^2} < 0$$

$$3. \frac{dt}{dt^c}|_{\bar{Z}} = -\frac{\partial^2 SW}{\partial t \partial t^c} \bigg/ \frac{\partial^2 SW}{\partial t^2} = 0$$

$$4. \frac{dt}{d\alpha}|_{\bar{Z}} = -\frac{\partial^2 SW}{\partial t \partial \alpha} \bigg/ \frac{\partial^2 SW}{\partial t^2} < 0$$

Again, a uniform distribution for  $H(Z)$  is a sufficient condition to determine the signs of the derivatives. Lemma 1.6 indicates that, holding the threshold constant, the optimal turnover tax rate increases with the marginal cost of informal sector production; this highlights how the informal sector constrains the level of the presumptive tax. The optimal threshold decreases with the cost of complying with the presumptive regime and with the marginal cost of production in the presumptive regime, but is unaffected by the regular corporate income tax rate. The latter point is interesting, because it suggests that any impact of the regular tax rate on the presumptive tax rate occurs only indirectly via changes in the threshold; for a fixed threshold, a change in the corporate tax rate has no effect on the optimal turnover tax rate. The full comparative statics analysis is given next, where  $H$  denotes the Hessian of second derivatives. It is assumed that  $H$  is negative definite at the solutions to the first-order conditions, to satisfy the second-order sufficiency conditions for a welfare maximum.<sup>17</sup>

**Proposition 5.** In the case of a jumping equilibrium, the following are the full effects of parameter changes on the optimal threshold, if the derivative of the density function  $h'(Z)$  is non-negative at the optimal threshold and at  $Z^{IP}$ .

1.  $\frac{d\bar{Z}}{d\lambda} = \frac{-(\frac{\partial^2 SW}{\partial Z \partial \lambda} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial Z \partial t} \frac{\partial^2 SW}{\partial t \partial \lambda})}{|H|} > 0$
2.  $\frac{d\bar{Z}}{d\Gamma'} = \frac{-(\frac{\partial^2 SW}{\partial Z \partial \Gamma'} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial Z \partial t} \frac{\partial^2 SW}{\partial t \partial \Gamma'})}{|H|} < 0$
3.  $\frac{d\bar{Z}}{dt^c} = \frac{-(\frac{\partial^2 SW}{\partial Z \partial t^c} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial Z \partial t} \frac{\partial^2 SW}{\partial t \partial t^c})}{|H|} < 0$
4.  $\frac{d\bar{Z}}{d\alpha} = \frac{-(\frac{\partial^2 SW}{\partial Z \partial \alpha} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial Z \partial t} \frac{\partial^2 SW}{\partial t \partial \alpha})}{|H|} < 0$

Proposition 5 reveals that a higher marginal cost of informal production results in a higher optimal threshold, while a greater compliance cost in the presumptive regime, a higher corporate income tax rate, and a higher marginal cost of production in the presumptive regime, all translate into a lower optimal threshold.

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<sup>17</sup> $H$  is negative definite when  $|H| > 0$  and  $\frac{\partial^2 SW}{\partial Z^2} < 0$  and  $\frac{\partial^2 SW}{\partial t^2} < 0$ . The latter two inequalities are satisfied automatically with the condition stated in the proposition, that  $h'(Z)$  is non-negative at the optimum. Hence, the additional assumption for an interior welfare maximum is that  $|H| > 0$ .

**Proposition 6.** In the case of a jumping equilibrium, the following are the full effects of parameter changes on the optimal turnover tax, if the derivative of the density function  $h'(Z)$  is non-negative at the optimal threshold and at  $Z^{IP}$ .

$$\begin{aligned}
1. \quad \frac{dt}{d\lambda} &= \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \lambda}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} > 0 \\
2. \quad \frac{dt}{d\Gamma'} &= \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \Gamma'}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0 \\
3. \quad \frac{dt}{dt^c} &= \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial t^c}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0 \\
4. \quad \frac{dt}{d\alpha} &= \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \alpha}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0
\end{aligned}$$

Proposition 6 shows that the optimal turnover tax rate increases with the marginal cost of informal production, but falls with the compliance cost of the presumptive regime, the corporate tax rate, and the marginal cost of production in the presumptive regime. Overall, then, countries with rampant informal sector activity—which can be interpreted in the model as economies with a low  $\lambda$  and a high  $\Gamma'$ —should set a relatively low threshold *and* a relatively low turnover tax rate, provided that the equilibrium continues to be characterized by jumping.<sup>18</sup> Countries with high corporate income tax rates should also set a lower  $t$  and a lower  $\bar{Z}$ . We consider now the optimal policy when firms bunch just below the threshold.

### 1.3.2 Welfare in the Bunching Case: $\bar{Z} \leq \hat{Z}$

Together with lemma 1.3, lemma 1.2 leaves three cases for further analysis under the possible bunching configurations:  $Z^{IP} \leq Z^{IA} \leq \bar{Z} < \hat{Z} < Z^{IR}$ ,  $Z^{IP} \leq \bar{Z} \leq Z^{IA} \leq \hat{Z} < Z^{IR}$  and  $Z^{IP} \leq \bar{Z} < Z^{IR} \leq \hat{Z} < Z^{IA}$ .<sup>19</sup> Numerical simulations, which consider all possible configurations, show that the case of  $Z^{IP} \leq \bar{Z} < Z^{IR} \leq \hat{Z} < Z^{IA}$  generates the highest social welfare. Moreover, we show with the next lemma that, in the case where  $H(Z)$  is the

<sup>18</sup>In our simulations, a significant reduction in  $\lambda$  causes the equilibrium to change from jumping to bunching, which in turn impacts the optimal threshold.

<sup>19</sup>We write  $Z^{IR} > \bar{Z}$  for convenience, but it is also possible that  $Z^{IR} < \bar{Z}$ . However, this alternative will not change the argument, i.e. if  $Z^{IR} < \bar{Z}$ , any firms with sales level between  $Z^{IP}$  and  $Z^{IR}$  ( $Z^{IR}$  and  $\bar{Z}$ ) would still choose  $\pi^P$  over  $\pi^I$ , since  $\pi^R$  is not achievable for any sales below the presumptive tax threshold.

uniform distribution, this case must be welfare-dominant. Hence, we use it to construct the social welfare function for analytical purposes below.

**Lemma 1.8.** *The case of  $Z^{IP} \leq \bar{Z} < Z^{IR} \leq \hat{Z} < Z^{IA}$  welfare-dominates the case of  $Z^{IP} \leq Z^{IA} \leq \bar{Z} < \hat{Z} < Z^{IR}$  and the case of  $Z^{IP} \leq \bar{Z} \leq Z^{IA} < \hat{Z} < Z^{IR}$ , if  $H(Z)$  is uniformly distributed.*

According to lemma 1.8, in the bunching case, the government sets the threshold to incite firms with high potential sales not to stay informal. Thus, firms with sales ranging from  $\hat{Z}$  to  $Z^{IA}$  would choose to be formal and earn  $\pi^R$ . Firms with sales at and above  $Z^{IA}$  would also choose to be regulars rather than informals, since  $Z^{IR} < Z^{IA}$ . Lemma 1.8 generates the partition defined by 1.a, 2.c, and 3.a of the lemma 1.2. Total profit is then,

$$\begin{aligned} \Pi(\bar{Z}, t) = & \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ \\ & + \int_{\bar{Z}}^{\hat{Z}} \pi^A h(Z) dZ + \int_{\hat{Z}}^{Z^M} \pi^R h(Z) dZ \end{aligned} \quad (1.17)$$

while the total tax revenue is

$$\begin{aligned} G(\bar{Z}, t) = & \int_{Z^{IP}}^{\bar{Z}} [tZ - A'] h(Z) dZ + \int_{\bar{Z}}^{\hat{Z}} [t\bar{Z} - A'] h(Z) dZ \\ & + \int_{\hat{Z}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \end{aligned} \quad (1.18)$$

Social welfare in the bunching partition is then given by

$$\begin{aligned} SW(\bar{Z}, t) = & \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ + \int_{\bar{Z}}^{\hat{Z}} \pi^A h(Z) dZ \\ & + \int_{\hat{Z}}^{\infty} \pi^R h(Z) dZ + \delta \left\{ \int_{Z^{IP}}^{\bar{Z}} (tZ - A') h(Z) dZ \right. \\ & \left. + \int_{\bar{Z}}^{\hat{Z}} (t\bar{Z} - A') h(Z) dZ + \int_{\hat{Z}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \right\} \end{aligned} \quad (1.19)$$

The first-order condition with respect to  $\bar{Z}$  can be rearranged to obtain the following



optimality condition

**Proposition 7.**

$$\begin{aligned} & [(1 - t - C - \alpha) + \delta t][H(\hat{Z}) - H(\bar{Z})] + \delta(t\bar{Z} - A')h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} \\ & = \delta[t^c(1 - C)\hat{Z} - A]h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} \end{aligned} \quad (1.20)$$

The left-hand side of (1.20) represents the marginal benefit from a small increase in the threshold, while the right-hand side is the marginal cost. As the threshold increases, adjusters would increase their sales and bunch just below the new threshold. Therefore, there are marginal gains in the profit and tax revenue in the presumptive regime for the mass  $H(\hat{Z}) - H(\bar{Z})$ , represented by the first term in (1.20). Given  $d\hat{Z}/d\bar{Z} = (1 - t - C - \alpha)/(1 - t^c)(1 - C) > 0$ , some firms that used to stay unconstrained in the regular regime now switch to bunch below the threshold: their move creates a net increase in tax revenue for the presumptive regime, which is the product of  $(t\bar{Z} - A')$  and  $h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}}$ . At the optimal sales threshold, the marginal benefit is balanced by the marginal cost, which is from the same firms switching from the regular regime to the presumptive regime, causing tax revenue loss from the regular regime, equalling  $[t^c(1 - C)\hat{Z} - A]h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}}$ .

A closed-form expression for the optimal threshold under bunching can be derived, for given tax rates  $t$  and  $t^c$ , if we again assume that  $H(Z)$  follows a uniform distribution. Then (1.20) can be simplified to give

$$\bar{Z}^* = \frac{[(1 - t - C - \alpha) + \delta t](\Gamma - \Gamma') + \delta(1 - t - C - \alpha)(A - A')}{\delta(1 - t - C - \alpha)[t^c(1 - C) - t] - [(1 - t - C - \alpha) + \delta t][t^c(1 - C) - (t + \alpha)]} \quad (1.21)$$

This optimal threshold in the case of bunching provides an analogue to the expression in (1.13) for the case of jumping (though in the latter expression for the optimal threshold for a given turnover tax rate, it was not assumed that  $H(Z)$  is uniform).

The first-order condition for optimizing welfare with respect to  $t$  can be rearranged as follows.

**Proposition 8.**

$$\begin{aligned}
& (\delta - 1) \int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ + (\delta - 1) \int_{\bar{Z}}^{\hat{Z}} \bar{Z}h(Z)dZ - \delta[t^c(1 - C)\hat{Z} - A]h(\hat{Z})\frac{d\hat{Z}}{dt} \\
& = -\delta(t\bar{Z} - A')h(\hat{Z})\frac{d\hat{Z}}{dt} + \delta(tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt}
\end{aligned} \tag{1.22}$$

In addition to the terms present in the first-order condition for  $t$  seen previously in (1.14) of the jumping case, the expression (1.22) contains two new terms appear on the left-hand side. They represent the increase in revenues from firms bunching just below the threshold and new regular income tax revenues collected from the former ‘bunchers,’ as  $d\hat{Z}/dt < 0$ . On the right-hand side, there is a new term corresponding to the revenue loss in the presumptive regime, due to fewer bunchers. The first-order condition for the turnover tax rate is clearly more complex when there is bunching behavior, than in its absence.

We are unable to establish unambiguous comparative statics results for the bunching scenario. However, in the case of a uniform distribution for  $H(Z)$ , (1.22) can be written, for a given  $\bar{Z}$ , as a quartic equation of the form  $at^4 + bt^3 + ct^2 + dt + e = 0$ . The expressions for the coefficients  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are provided in the appendix (as part of the proof of the next proposition). The discriminant of the quartic, given by  $\Delta = -\frac{\Delta_1^2 - 4\Delta_0^3}{27}$ , with  $\Delta_0 = c^2 - 3bd + 12ae$  and  $\Delta_1 = 2c^3 - 9bcd + 27b^2e + 27ad^2 - 72ace$  (Irving, 2013, Theorem 6.8),<sup>20</sup> is negative at any of the parameter values used in our simulations (see section 4). This implies that there are two distinct real roots (and two imaginary roots) (Irving, 2013, Theorem 6.5). Since the real roots are solutions to a first-order condition, one root is welfare-maximizing and the other is welfare-minimizing. There is, therefore, a unique real root for  $t$  that maximizes social welfare, for any given threshold. The following proposition provides an expression for the optimal tax rate, based on that root. The formula is long, but can be readily calculated in a spreadsheet. Together with (1.21), they provide a complete solution for the optimal policy in the case of bunching with a uniform distribution for potential

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<sup>20</sup>In Theorem 6.5,  $a$  is set to 1. For simplicity, we don’t unify  $a$  in our case.

output. We provide the formulas for the optimal policy, despite the caveats, because they may be useful in a practical setting.

**Proposition 9.** If  $H(Z)$  is uniformly distributed on  $(0, Z^M)$  and bunching occurs in the equilibrium ( $\hat{Z} \geq \bar{Z}$ ), then there is a unique turnover tax rate that maximizes social welfare, for a given threshold  $\bar{Z}$ . At the parameter values used to simulate the model (see table 1) the optimal tax rate is given by

$$t = (1/4)(\lambda - \alpha)^3 + (3/4)(\lambda - \alpha) - \frac{M}{4Q} + J - \frac{1}{2}\sqrt{-4J^2 - 2j - \frac{k}{J}} \quad (1.23)$$

where

- $M = -\frac{\delta-1}{2}\bar{Z}^2 - \frac{\delta(A-A')\bar{Z}}{(1-t^c)(1-C)} + \frac{(\delta-1)\bar{Z}[(1-C-\alpha)\bar{Z}+\Gamma-\Gamma']}{(1-t^c)(1-C)} + \frac{\delta t^c(1-C)\bar{Z}[(1-C-\alpha)\bar{Z}+\Gamma-\Gamma']}{[(1-t^c)(1-C)]^2}$
- $Q = -\frac{(2\delta-1)\bar{Z}^2}{(1-t^c)(1-C)} - \frac{\delta t^c(1-C)\bar{Z}^2}{[(1-t^c)(1-C)]^2}$
- $j = \frac{8ac-3b^2}{8a^2}; k = \frac{b^3-4abc+8a^2d}{8a^3}$
- $J = \frac{1}{2}\sqrt{-\frac{2}{3}j + \frac{1}{3a}(K + \frac{\Delta_0}{K})}; K = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}$

## 1.4 Numerical Solutions for the Optimal Threshold and Tax Rate

Since both the “jumping” and “bunching” cases are theoretically possible, this section turns to numerical simulations to explore the nature of these two cases. An important feature of the calibration is the distribution of potential sales. The pertinent distribution to use depends on the specific features of the informal sector we wish to model. In our view, a turnover tax is less appropriate for self-employed workers without employees, which constitutes a large segment of informal firms. As reported in La Porta and Shleifer (2014), in low-income countries, these individuals are typically very poor and unlikely to be choosing informality for tax purposes. Thus, the distribution of sales in our simulations should in principle reflect

only the segment of firms for which informality and formality are, arguably, both viable options. La Porta and Shleifer (2008) provide statistics on average sales from different surveys in low-income countries undertaken by the World Bank. One of these surveys (the Micro survey) targets areas of a country where there is a high concentration of businesses with fewer than five employees, but randomly selects *all* establishments in the area. In this survey, which includes firms both unregistered and registered with the central government, about 85% of the sample has two or more employees in addition to the entrepreneur. In the Micro survey, the average value of sales is about \$51,000 in 2006. At the same time, another survey by the World Bank (the Enterprise survey) drops firms with fewer than five employees and includes many large firms (more than 100 employees). The average size of firms for the same countries and year as the Micro survey is about \$1.1 million. Taking into account the number of observations in the Micro survey and the Enterprise survey, the overall average sales across the two surveys is about \$815,000. In our main simulations, we calibrate a lognormal distribution for sales to approximately this mean.

The cost of tax compliance in developing countries is subject to a wide range of estimates. Sapiei et al. (2014) estimates corporate income tax compliance cost as between 0.05% and 15% of taxable turnover in developing and transition economies. Yesegat et al. (2015) report that the total tax compliance cost (mainly bookkeeping) of businesses in Ethiopia is about 5% of turnover and that the bulk of it is attributable to business profit tax.<sup>21</sup> Survey evidence from East European transition economies in Engelschalk and Loeprick (2015) indicate that corporate income tax compliance costs are around 2% at a turnover of \$100,000, although they note that even for businesses operating at more than \$100,000 in turnover, measured compliance costs can still surpass 3%. Harju et al. (2019) estimate VAT compliance costs at 1,300 euro (about \$1,500) in Finland; however, the accounting costs of corporate income tax are typically greater than for VAT.<sup>22</sup> We set the fixed compliance cost in the regular regime

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<sup>21</sup>Although the compliance costs are for all taxes, 61% of the formal businesses reported paying profit tax and 36% reported paying turnover tax, while only 12% submit VAT and 15% pay employment related taxes.

<sup>22</sup>Yesegat et al. (2015) find that, in Ethiopia, 50% of the outsourcing component of compliance costs is attributable to business profit tax, compared to 20% for VAT.

to \$3,000, which makes compliance costs equal to about 0.75% of average turnover in the simulations. The fixed cost of tax administration is set to 20% of the compliance cost for the presumptive tax, consistent with the evidence on VAT administration costs in Cnossen (1994). However, the cost-side auditing issues relating to standard corporate income tax, such as abusive transfer pricing, suggest a relatively higher ratio of administration cost to compliance cost in the regular regime, which we set to 1/3.<sup>23</sup> Given the fact that both fixed costs are likely much lower in the presumptive regime, compared to the costs in the regular regime, the compliance cost in the presumptive regime is set to one-third of the cost in the regular regime. This is broadly consistent with Yesegat et al. (2015), which finds that, in Ethiopia, 18% of the cost of outsourcing accounting tasks stems from the turnover tax, compared to 50% for business profit tax, and 31% of in-house accounting costs relate to the turnover tax, compared to 50% for business profit tax. The additional marginal cost associated with producing informally,  $\lambda$ , is set at 0.15 to fix the relative size of the informal sector at realistic values. The base case values of all the parameters of the model are in the notes of Table 1.1. Table 1.1 shows the results for four exogenous tax rates in the standard corporate income tax regime, ranging from 8% to 24%. The simulation process covers all the possible partitions described in lemmas 1.1 and 1.2. A numerical grid search algorithm is used to find globally optimal combination of the threshold  $\bar{Z}$  and the tax rate  $t$ , given  $t^c$ . The table identifies the type of equilibrium—jumping or bunching—associated with the policy optimum for each set of parameter values considered in Table 1.

In the base case, the optimal threshold is close to \$40,000 and the turnover tax rate is close to 3%.<sup>24</sup> We observe jumping equilibria at lower values of the corporate tax rate  $t^c$  and bunching at higher values of  $t^c$ . About 30% of businesses are in the informal sector. The use of the presumptive regime reduces the amount of informality by between 6.7 and 20.8 percentage points, compared to the situation without the presumptive tax regime (equivalent

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<sup>23</sup>There appears to be almost no estimates of the administrative costs of tax systems for developing countries (Evans, 2003).

<sup>24</sup>The non-monotonic trend in the optimal threshold in the base case arises from the lognormal distribution assumed for sales.

Table 1.1: Simulation Results

Corporate income tax rate	8%	12%	16%	20%	24%
<b>Base case</b>					
Optimal turnover threshold	40700	38900	37100	35700	36300
Optimal turnover tax rate	2.54%	2.54%	2.54%	2.54%	3.02%
Type of equilibrium	Jumping	Jumping	Jumping	Bunching	Bunching
Proportion of informal firms	29.20%	29.20%	29.20%	29.20%	30.50%
Percentage point reduction in informality	6.71%	10.43%	14.12%	18.21%	20.78%
<b>Case 1. Smaller costs of compliance and administration in presumptive regime</b>					
Optimal turnover threshold	50800	48100	46100	44400	44000
Optimal turnover tax rate	3.32%	3.32%	3.32%	3.32%	3.93%
Type of equilibrium	Jumping	Jumping	Jumping	Bunching	Bunching
Proportion of informal firms	21.50%	21.50%	21.50%	21.50%	24.10%
Percentage point reduction in informality	31.31%	34.05%	36.76%	39.78%	37.40%
<b>Case 2. Less cost of Informality</b>					
Optimal turnover threshold	38900	37000	37900	43800	52900
Optimal turnover tax rate	1.36%	1.30%	1.36%	1.77%	1.87%
Type of equilibrium	Jumping	Jumping	Bunching	Bunching	Bunching
Proportion of informal firms	32.90%	32.70%	32.90%	34.30%	34.80%
Percentage point reduction in informality	3.24%	8.91%	14.99%	17.15%	23.19%
<b>Case 3. Smaller Costs of Compliance and Administration in Standard Regime</b>					
Optimal turnover threshold	22600	21600	20500	22600	24900
Optimal turnover tax rate	1.55%	1.55%	1.55%	1.72%	2.14%
Type of equilibrium	Jumping	Jumping	Bunching	Bunching	Bunching
Proportion of informal firms	26.90%	26.90%	26.90%	27.40%	28.50%
Percentage point reduction in informality	0%	5.28%	9.43%	11.90%	12.84%
<b>Case 4. Lower average sales</b>					
Optimal turnover threshold	41200	38300	36500	35500	34600
Optimal turnover tax rate	2.83%	2.25%	2.25%	2.83%	2.83%
Type of equilibrium	Jumping	Jumping	Bunching	Bunching	Bunching
Proportion of informal firms	41.00%	39.40%	39.40%	41.00%	41.00%
Percentage point reduction in informality	4.43%	11.66%	15.99%	15.98%	20.08%
<b>Case 5. Sales uniformly distributed</b>					
Optimal turnover threshold	40900	38900	36800	35600	35000
Optimal turnover tax rate	2.69%	2.50%	2.50%	2.78%	2.95%
Type of equilibrium	Jumping	Jumping	Bunching	Bunching	Bunching
Proportion of informal firms	2.60%	2.50%	2.50%	2.60%	2.70%
Percentage point reduction in informality	13.33%	24.24%	32.43%	38.10%	43.75%

*Notes:* The following parameters were used in the simulations. Base case:  $\lambda = 0.15$ ,  $\alpha = 0.05$ ,  $\Gamma = 3000$ ,  $A = 1000$ ,  $\Gamma' = 1000$ ,  $A' = 200$ ,  $C = 0.7$ ,  $\mu = 11.2$ ,  $\sigma = 2.2$ . In each alternative case, the parameters are identical to the base case, except for the parameter indicated: Case 1:  $\Gamma' = 500$ ,  $A' = 100$ ; Case 2:  $\lambda = 0.125$ ; Case 3:  $\Gamma = 2100$ ,  $A = 700$ ; Case 4:  $\mu = 10.5$ ; Case 5:  $H(Z)$  uniformly distributed on  $(0, 800,000)$ .

to forcing the threshold to be zero).

Cases 1 to 5 consider modifications of parameters values. When the costs of compliance

and administration in the presumptive regime are reduced (case 1), both the threshold and the turnover tax rate tend to increase. If the cost of informality falls (case 2), the optimal thresholds rise, while the optimal turnover tax rates fall. If the costs of compliance and administration in the regular regime decrease (case 3), then this results in very low thresholds. Case 4, where the mean of the distribution of  $H(Z)$  is lowered from around \$800,000 to around \$400,000, perhaps corresponding to a lower income country, the optimal threshold is very similar to the base case. Finally, case 5 examines the effect of assuming a uniform distribution instead of a log-normal distribution, with close to the same expected value for  $Z$ . Comparing case 5 with the base case, we observe that the results are very similar. In all cases, jumping equilibria occur at lower values of  $t^c$ , then bunching emerges.

## 1.5 Conclusions

The emphasis in this paper is on how the informal sector both motivates and constrains the design of presumptive income tax regimes. We analyze the optimal design of a presumptive income tax in the form of a tax on turnover applied to firms with sales below a threshold, when firms can make strategic choices for tax purposes, regarding their sales level and whether to produce formally or to evade taxes altogether by disappearing into the informal sector. The main purpose of the study is to generate practical insights for authorities in developing countries, seeking to reduce informal activities and to lighten the burden of taxpayer compliance and the cost of tax administration. Our recommendation for designing presumptive tax systems is to allow a fixed tax (patent) for micro enterprises with few employees, and a turnover tax for small enterprises in lieu of the standard corporate income tax and VAT. The analysis of a simple model generates formulas for the optimal threshold and the tax rate. Comparative statics and numerical simulations are provided to further guide policy choices.

Several caveats apply. The simplifying assumptions adopted for the analysis suggests that

the results should be taken as suggestive but not definitive. The calibration of the model requires some guesses on values for which reliable data is lacking. The analysis also omits two important real world issues. The first is under-reporting of income by formal firms. In our model, tax evasion only occurs by firms producing informally—that is, completely outside of the view of the tax authorities. In reality, some formal firms may under-report their sales in order to remain below the threshold separating the standard corporate income tax regime and the presumptive tax regime. Second, we abstract from a co-existing VAT threshold. On the one hand, it can be argued that economies of scope in taxpayer compliance exist when the VAT registration threshold coincides with the turnover tax threshold. On the other hand, Kanbur and Keen (2014) have argued that there are game-theoretic reasons for setting the two thresholds far apart. Specifically, setting one threshold far above the other might induce firms to profitably expand their sales until they are just below the higher threshold; hence they have now crossed the lower threshold and pay more tax on that tax instrument. If the thresholds were identical, the same firms might choose to produce just both thresholds to avoid the higher tax burdens associated with crossing the common threshold for both tax instruments. Future work could integrate these considerations into the model. Finally, as the recent literature has stressed, tax policy is not a panacea for informality in low-income countries. Reducing the cost of tax compliance through presumptive taxation may help encourage formalization, but cannot be successful without accompanying improvements in tax inspections and audits.



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# Appendix

## Proof of Lemma 1.1

The proof follows immediately from the definitions of  $Z^{IP}$ ,  $Z^{IR}$ ,  $\hat{Z}$ , and  $\bar{Z}$ .

## Proof of Lemma 1.2

The proof follows immediately from the definitions of  $Z^{IP}$ ,  $Z^{IA}$ ,  $Z^{IR}$ ,  $\hat{Z}$ , and  $\bar{Z}$ .

## Proof of Lemma 1.3

If  $\bar{Z} < Z^{IP}$ , then all firms with sales below the threshold would choose to be informal, so the presumptive regime would be empty of firms. But the same equilibrium and social welfare would be achieved by setting  $\bar{Z} = Z^{IP}$ , since, in this case too, all firms with sales below the threshold, whether they are producing at their maximum or adjusting down, would choose informality. Moreover, welfare improvements may be possible with  $\bar{Z} > Z^{IP}$ . Hence, we need only consider the thresholds  $\bar{Z} < Z^{IP}$ .

## Proof of Lemma 1.4

We have  $\pi^P(Z) - \pi^I(Z) = (\lambda - \alpha - t)Z - \Gamma'$ . Let  $t_0$  denote the tax rate such that  $\lambda - \alpha - t_0 = 0$ . Then  $\pi^P(Z) - \pi^I(Z)$  is strictly negative for all  $Z$ , since  $\Gamma' > 0$  by assumption. The same is true at any tax rate exceeding  $t_0$ . Moreover,  $\lambda - \alpha > 0$ . Therefore, there must exist a  $t_1 < t_0$  such that  $\pi^P(Z^M) - \pi^I(Z^M) = 0$ . That is, at  $t_1$ , the firm with highest potential sales is indifferent between the presumptive regime and informality, while all firms  $Z < Z^M$  prefer informality. But by continuity, there exists some  $t_2$  such that  $t_1 < t_2 < t_0$ , such that all firms, including  $Z^M$ , prefer informality. Consequently, a turnover tax of  $t_2$  generates the same equilibrium and welfare level as the tax rate  $t_0$ . Smaller values of  $t$  may increase welfare. Hence,  $t_0$  is weakly dominated by  $t_2$ . Finally, since  $\lambda - \alpha - t_0 = 0$  and  $t_2 < t_0$ , it

must be the case that  $\lambda - \alpha - t_2 > 0$ , or, equivalently,  $t_2 < \lambda - \alpha$ . Thus,  $1 - t_2 - C - \alpha > 1 - (\lambda - \alpha) - C - \alpha = 1 - \lambda - C - \alpha > 0$  by assumption.

## Proof of Lemma 1.5

We shall construct a proof by contradiction. Consider the optimization problem when  $Z^{IP} \leq \bar{Z} < Z^{IR}$ . The total profit function can be written as

$$\begin{aligned} \Pi(\bar{Z}, t) = & \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ \\ & + \int_{\bar{Z}}^{Z^{IR}} \pi^I h(Z) dZ + \int_{Z^{IR}}^{Z^M} \pi^R h(Z) dZ \end{aligned} \quad (1.24)$$

and the tax revenue is

$$G(\bar{Z}, t) = \int_{Z^{IP}}^{\bar{Z}} (tZ - A') h(Z) dZ + \int_{Z^{IR}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \quad (1.25)$$

The social welfare function is formed by is  $SW = \Pi + \delta G$ . Its first-order condition with respect to  $t$  can be written as,

$$[H(\bar{Z}) - H(Z^{IP})] = (tZ^{IP} - A') h(Z^{IP}) \frac{dZ^{IP}}{dt} \quad (1.26)$$

where we have used the fact that  $\pi^I(Z^{IP}) = \pi^P(Z^{IP})$ . Since  $\bar{Z} > Z^{IP}$ , the left-hand side of (1.26) is strictly positive, which implies the right-hand side is positive at an interior solution for  $t$ . Since  $dZ^{IP}/dt > 0$ , it must also be the case that  $tZ^{IP} - A' > 0$ . Note that this in turn implies that  $t\bar{Z} - A' > 0$ . We will use this observation in the next step of the proof.

The derivative of the social welfare function derivative with respect to  $\bar{Z}$  gives

$$\frac{dSW}{d\bar{Z}} = \pi^P(\bar{Z}) h(\bar{Z}) - \pi^I(\bar{Z}) h(\bar{Z}) + \delta(t\bar{Z} - A') h(\bar{Z}) \quad (1.27)$$

Since  $\bar{Z} \geq Z^{IP}$ , there is  $\pi^P(\bar{Z}) > \pi^I(\bar{Z})$  by the definition of  $Z^{IP}$ . Furthermore,  $t\bar{Z} - A' > 0$ .

Therefore,  $dSW/d\bar{Z} > 0$ ; welfare will keep increasing as  $\bar{Z}$  is raised until the case we have  $Z^{IP} \leq Z^{IR} \leq \bar{Z}$ , as  $Z^{IP}$  and  $Z^{IR}$  are independent of  $\bar{Z}$ . Thus the case of  $Z^{IP} \leq \bar{Z} < Z^{IR}$  is welfare-dominated.

## Proof of Lemma 1.6

The partial derivatives of (1.36) with respect to  $\bar{Z}$ ,  $t$ ,  $\lambda$ ,  $\Gamma'$  and  $t^c$  generate the following second-order partial derivatives of the welfare function in the jumping case.

- $\partial^2 SW / \partial \bar{Z}^2 = -\{(\delta - 1)[t^c(1 - C) - t] + \alpha\}h(\bar{Z}) + \{(\pi^P(\bar{Z}) - \pi^R(\bar{Z})) + \delta(t\bar{Z} - A') - \delta[t^c(1 - C)\bar{Z} - A]\}h'(\bar{Z}) < 0$  if  $h' \geq 0$
- $\partial^2 SW / \partial \bar{Z} \partial t = (\delta - 1)\bar{Z}h(\bar{Z}) > 0$
- $\partial^2 SW / \partial \bar{Z} \partial \lambda = 0$
- $\partial^2 SW / \partial \bar{Z} \partial \Gamma' = -h(\bar{Z}) < 0$
- $\partial^2 SW / \partial \bar{Z} \partial t^c = -(\delta - 1)(1 - C)\bar{Z}h(\bar{Z}) < 0$
- $\partial^2 SW / \partial \bar{Z} \partial \alpha = -\bar{Z}h(\bar{Z}) < 0$
- $\partial^2 SW / \partial \bar{Z} \partial \delta = \{[t\bar{Z} - t^c(1 - C)\bar{Z}] + (A - A')\}h(\bar{Z})$

## Proof of Lemma 1.7

From the partial derivatives of (1.37) with respect to  $\bar{Z}$ ,  $t$ ,  $\lambda$ ,  $\Gamma'$  and  $t^c$ , we obtain the following.

- $\partial^2 SW / \partial t \partial \bar{Z} = (\delta - 1)\bar{Z}h(\bar{Z}) > 0$
- $\partial^2 SW / \partial t^2 = (\delta - 1)(-Z^{IP})h(Z^{IP})\frac{dZ^{IP}}{dt} - \delta(Z^{IP} + t\frac{dZ^{IP}}{dt})h(Z^{IP})\frac{dZ^{IP}}{dt} - \delta(tZ^{IP} - A')h(Z^{IP})\frac{d^2 Z^{IP}}{dt^2} - \delta(tZ^{IP} - A')h'(Z^{IP})(\frac{dZ^{IP}}{dt})^2 < 0$  if  $h' \geq 0$



- $\partial^2 SW / \partial t \partial \lambda = (\delta - 1)(-Z^{IP})h(Z^{IP}) \frac{dZ^{IP}}{d\lambda} - \delta t \frac{dZ^{IP}}{d\lambda} h(Z^{IP}) \frac{dZ^{IP}}{dt} - \delta(tZ^{IP} - A')h(Z^{IP}) \frac{d^2 Z^{IP}}{dt d\lambda} - \delta(tZ^{IP} - A')h'(Z^{IP}) \frac{dZ^{IP}}{d\lambda} \frac{dZ^{IP}}{dt} > 0$  if  $h' \geq 0$
- $\partial^2 SW / \partial t \partial \Gamma' = [(\delta - 1)(-Z^{IP})h(Z^{IP}) \frac{dZ^{IP}}{d\Gamma'} - \delta t \frac{dZ^{IP}}{d\Gamma'} h(Z^{IP}) \frac{dZ^{IP}}{dt} - \delta(tZ^{IP} - A')h(Z^{IP}) \frac{d^2 dZ^{IP}}{dt d\Gamma'} - \delta(tZ^{IP} - A')h'(Z^{IP}) \frac{dZ^{IP}}{d\Gamma'} \frac{dZ^{IP}}{dt} < 0$  if  $h' \geq 0$
- $\partial^2 SW / \partial t \partial t^c = 0$
- $\partial^2 SW / \partial t \partial \alpha = (\delta - 1)(-Z^{IP})h(Z^{IP}) \frac{dZ^{IP}}{d\alpha} - \delta t \frac{dZ^{IP}}{d\alpha} h(Z^{IP}) \frac{dZ^{IP}}{dt} - \delta(tZ^{IP} - A')h'(Z^{IP}) \frac{dZ^{IP}}{d\alpha} \frac{dZ^{IP}}{dt} - \delta(tZ^{IP} - A')h(Z^{IP}) \frac{d^2 Z^{IP}}{dt d\alpha} < 0$  if  $h' \geq 0$
- $\partial^2 SW / \partial t \partial \delta = \int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ - (t\bar{Z} - A')h(Z^{IP}) \frac{dZ^{IP}}{dt}$

## Proof of Lemma 1.8

We again construct a proof by contradiction. First, consider the optimization problem when  $Z^{IP} \leq Z^{IA} \leq \bar{Z} < \hat{Z} < Z^{IR}$ . The total profit function can be written as

$$\begin{aligned} \Pi(t, t^c, \bar{Z}) &= \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ + \int_{\bar{Z}}^{Z^{IR}} \pi^I h(Z) dZ \\ &\quad + \int_{Z^{IR}}^{Z^M} \pi^R h(Z) dZ \end{aligned} \quad (1.28)$$

and the total tax revenue can be written as

$$G(t, t^c, \bar{Z}) = \int_{Z^{IP}}^{\bar{Z}} (tZ - A')h(Z) dZ + \int_{Z^{IR}}^{Z^M} [t^c(1 - C)Z - A]h(Z) dZ \quad (1.29)$$

Given the social welfare function  $SW(t, t^c, \bar{Z}) = \Pi(t, t^c, \bar{Z}) + \delta G(t, t^c, \bar{Z})$ , taking the derivative of  $SW(t, t^c, \bar{Z})$  with respect to  $\bar{Z}$  gives:

$$\begin{aligned} \frac{dSW}{d\bar{Z}} &= \pi^P(\bar{Z})h(\bar{Z}) - \pi^I(\bar{Z})h(\bar{Z}) + \delta(t\bar{Z} - A')h(\bar{Z}) \\ &= \delta(t\bar{Z} - A')h(\bar{Z}) \end{aligned} \quad (1.30)$$

An interior solution can exist in this case only if  $t\bar{Z} - A' = 0$  since  $h(\bar{Z}) > 0$ .

$$\frac{d^2 SW}{d\bar{Z}^2} = \delta th(\bar{Z}) + \delta(t\bar{Z} - A')h'(\bar{Z}) \quad (1.31)$$

Assume  $H(Z)$  is uniform. Then  $\frac{d^2 SW}{d\bar{Z}^2}$  can be reduced to  $\delta th(\bar{Z})$ , which is positive. Therefore, the interior solution from Equation (1.30) must be welfare minimizing. Consequently, there can only exist a corner solution if  $Z^{IP} \leq Z^{IA} \leq \bar{Z} < \hat{Z} < Z^{IR}$ : either  $\frac{dSW}{d\bar{Z}}$  is increasing for all  $\bar{Z}$  or it is decreasing for all  $\bar{Z}$ . If it is increasing as the threshold  $\bar{Z}$  keeps increasing, given  $dZ^{IA}/d\bar{Z} = (1 - t - C - \alpha)/(1 - C - \lambda) > 1$ , finally  $Z^{IA}$  will be larger than  $\bar{Z}$ . If it is decreasing, then  $\bar{Z}$  will be smaller than  $Z^{IP}$  as  $Z^{IP} > 0$ .

Second, consider the optimization problem when  $Z^{IP} \leq \bar{Z} \leq Z^{IA} < \hat{Z} < Z^{IR}$ . The total profit function can be written as

$$\begin{aligned} \Pi(t, t^c, \bar{Z}) = & \int_0^{Z^{IP}} \pi^I h(Z) dZ + \int_{Z^{IP}}^{\bar{Z}} \pi^P h(Z) dZ + \int_{\bar{Z}}^{Z^{IA}} \pi^A h(Z) dZ \\ & + \int_{Z^{IA}}^{Z^{IR}} \pi^I h(Z) dZ + \int_{Z^{IR}}^{Z^M} \pi^R h(Z) dZ \end{aligned} \quad (1.32)$$

and the total tax revenue can be written as

$$\begin{aligned} G(t, t^c, \bar{Z}) = & \int_{Z^{IP}}^{\bar{Z}} (tZ - A') h(Z) dZ + \int_{\bar{Z}}^{Z^{IA}} (t\bar{Z} - A') h(Z) dZ \\ & + \int_{Z^{IR}}^{Z^M} [t^c(1 - C)Z - A] h(Z) dZ \end{aligned} \quad (1.33)$$

Given the social welfare function  $SW(t, t^c, \bar{Z}) = \Pi(t, t^c, \bar{Z}) + \delta G(t, t^c, \bar{Z})$ , taking the derivative

of  $SW(t, t^c, \bar{Z})$  with respect to  $\bar{Z}$  gives:

$$\begin{aligned}
\frac{dSW}{d\bar{Z}} &= \pi^P(\bar{Z})h(\bar{Z}) + \int_{\bar{Z}}^{Z^{IA}} (1-t-C-\alpha)h(Z)dZ + \pi^A h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} \\
&\quad - \pi^A h(\bar{Z}) - \pi^I(Z^{IA})h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} + \delta\{(t\bar{Z} - A')h(\bar{Z}) \\
&\quad + \int_{\bar{Z}}^{Z^{IA}} th(Z)dZ + (t\bar{Z} - A')[h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} - h(\bar{Z})]\} \\
&= \int_{\bar{Z}}^{Z^{IA}} (1-t-C-\alpha)h(Z)dZ + \delta \int_{\bar{Z}}^{Z^{IA}} th(Z)dZ \\
&\quad + \delta(t\bar{Z} - A')h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}}
\end{aligned} \tag{1.34}$$

An interior solution can exist in this case only if  $t\bar{Z} - A' < 0$  since  $\frac{dZ^{IA}}{d\bar{Z}} > 0$ .

$$\begin{aligned}
\frac{d^2SW}{d^2\bar{Z}} &= (1-t-C-\alpha)[h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} - h(\bar{Z})] + \delta t[h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} - h(\bar{Z})] \\
&\quad + \delta th(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} + \delta(t\bar{Z} - A')h'(Z^{IA})(\frac{dZ^{IA}}{d\bar{Z}})^2 \\
&= [(1-C-\alpha) + (\delta-1)t][h(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} - h(\bar{Z})] + \delta th(Z^{IA}) \frac{dZ^{IA}}{d\bar{Z}} \\
&\quad + \delta(t\bar{Z} - A')h'(Z^{IA})(\frac{dZ^{IA}}{d\bar{Z}})^2
\end{aligned} \tag{1.35}$$

Assume  $H(Z)$  is uniform. Then  $d^2SW/d\bar{Z}^2$  reduces to  $[1-C-\alpha+(\delta-1)t][\frac{dZ^{IA}}{d\bar{Z}}-1]$ . Note that  $dZ^{IA}/d\bar{Z} = (1-C-t-\alpha)/(1-C-\lambda)$ . The first term of (1.35) is positive; the second term can be written as  $\frac{dZ^{IA}}{d\bar{Z}} - 1 = \frac{(\lambda-t-\alpha)}{(1-C-\alpha)}$ . The denominator of this expression is positive. In the numerator,  $t$  must be strictly less than  $\lambda - \alpha$ . Therefore, the numerator is also positive, and hence  $\frac{dZ^{IA}}{d\bar{Z}} - 1 > 0$ . This implies that if an interior solution for  $\bar{Z}$  exists, it must be welfare minimizing, rather than welfare maximizing. Consequently, there can only exist a corner solution if  $Z^{IP} \leq \bar{Z} \leq Z^{IA} < \hat{Z} < Z^{IR}$ : either  $\frac{dSW}{d\bar{Z}}$  is increasing for all  $\bar{Z}$  or it is decreasing for all  $\bar{Z}$ . If it is increasing, then as the threshold  $\bar{Z}$  keeps increasing,<sup>25</sup>

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<sup>25</sup>We also need to consider the effect of increase in  $\bar{Z}$  on  $\hat{Z}$ . Given  $d\hat{Z}/d\bar{Z} = (1-t-C-\alpha)/(1-t^c)(1-C)$  and  $dZ^{IR}/d\bar{Z} = 0$ , there is  $dZ^{IR}/d\bar{Z} < d\hat{Z}/d\bar{Z} < dZ^{IA}/d\bar{Z}$ .

the case of  $Z^{IP} \leq \bar{Z} \leq Z^{IA} < \hat{Z} < Z^{IR}$  will finally turn into  $Z^{IP} \leq \bar{Z} < Z^{IR} < \hat{Z} < Z^{IA}$  or  $Z^{IP} < Z^{IR} < \hat{Z} < \bar{Z} < Z^{IA}$ , making the latter the dominant case. If, in the contrary case, welfare is always decreasing in  $\bar{Z}$ , then eventually  $\bar{Z} = 0$ , which violates the lemma's assumption that  $Z^{IP} \leq \bar{Z}$ , since  $Z^{IP} > 0$ .

## Proof of Proposition 1

From the social welfare function (1.12) we obtain,

$$\begin{aligned}
\frac{dSW}{d\bar{Z}} &= [(1-t-C-\alpha)\bar{Z} - \Gamma']h(\bar{Z}) - [(1-t^c)(1-C)\bar{Z} - \Gamma]h(\bar{Z}) \\
&\quad + \delta\{(t\bar{Z} - A')h(\bar{Z}) - [(1-t^c)(1-C)\bar{Z} - A]h(\bar{Z})\} \\
&= \{[(1-t-C-\alpha) + \delta t]\bar{Z} - \Gamma' - \delta A'\}h(\bar{Z}) \\
&\quad - \{[(1-t^c)(1-C) + \delta t^c(1-C)]\bar{Z} - \Gamma - A\}h(\bar{Z}) \\
&= 0
\end{aligned} \tag{1.36}$$

then there is  $\bar{Z} = \{(\Gamma + \delta A) - (\Gamma' + \delta A')\} / \{(\delta - 1)[t^c(1 - C) - t] + \alpha\}$  if  $(\delta - 1)[t^c(1 - C) - t] + \alpha > 0$ .

## Proof of Proposition 2

From the social welfare function (1.12) we obtain,

$$\begin{aligned}
\frac{dSW}{dt} &= [(1 - C - \lambda)Z^{IP}]h(Z^{IP})\frac{dZ^{IP}}{dt} + \int_{Z^{IP}}^{\bar{Z}} (-Z)h(Z)dZ \\
&\quad - [(1 - t - C - \alpha)Z^{IP} - \Gamma']h(Z^{IP})\frac{dZ^{IP}}{dt} + \delta\left[\int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ\right. \\
&\quad \left. - (tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt}\right] \\
&= (\delta - 1) \int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ - \delta(tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt} \\
&= 0
\end{aligned} \tag{1.37}$$

## Proof of Proposition 3

We have

$$\frac{dSW}{dt} = (\delta - 1) \int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ - \delta(tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt} \tag{1.38}$$

As the fixed compliance cost  $\Gamma'$  in the presumptive regime goes to 0, both  $Z^{IP} \rightarrow 0$  and  $\frac{dZ^{IP}}{dt} \rightarrow 0$ , which makes  $dSW/dt > 0$ , for all  $t$ , as the first term on the right-hand of (1.38) stays positive. Therefore, raising  $t$  would increase social welfare. Since  $t$  has to be smaller than  $\lambda - \alpha$  by Lemma 1.4,  $t$  will approach  $\lambda - \alpha$  as  $\Gamma' \rightarrow 0$ .

## Proof of Proposition 4

The equation of (1.38) becomes the following if sales  $Z$  follow the uniform distribution:

$$(\delta - 1)\bar{Z}^2 = \frac{[(\delta + 1)\Gamma'^2 + 2\delta\Gamma'A']t - [2\delta\Gamma'A' - (\delta - 1)\Gamma'^2](\lambda - \alpha)}{(\lambda - t - \alpha)^3} \tag{1.39}$$

and Equation 1.39 into a function of  $t$  as  $at^3 + bt^2 + ct + d = 0$  where

- $a = -(\delta - 1)\bar{Z}^2$ ;  $b = 3(\delta - 1)\bar{Z}^2(\lambda - \alpha)$
- $c = -[3(\delta - 1)\bar{Z}^2(\lambda - \alpha)^2 + (\delta + 1)\Gamma'^2 + 2\delta\Gamma'A']$
- $d = (\delta - 1)\bar{Z}^2(\lambda - \alpha)^3 - [(\delta - 1)\Gamma'^2 - 2\delta\Gamma'A'](\lambda - \alpha)$

## Proof of Proposition 5

Write the system of totally differentiated first-order conditions in matrix form:

$$\begin{bmatrix} \frac{\partial^2 SW}{\partial \bar{Z}^2} & \frac{\partial^2 SW}{\partial \bar{Z} \partial t} \\ \frac{\partial^2 SW}{\partial t \partial \bar{Z}} & \frac{\partial^2 SW}{\partial t^2} \end{bmatrix} \begin{bmatrix} d\bar{Z} \\ dt \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} & -\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} & -\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} & -\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} & -\frac{\partial^2 SW}{\partial \bar{Z} \partial \delta} \\ -\frac{\partial^2 SW}{\partial t \partial \lambda} & -\frac{\partial^2 SW}{\partial t \partial \Gamma'} & -\frac{\partial^2 SW}{\partial t \partial t^c} & -\frac{\partial^2 SW}{\partial t \partial \alpha} & -\frac{\partial^2 SW}{\partial t \partial \delta} \end{bmatrix} \begin{bmatrix} d\lambda \\ d\Gamma' \\ dt^c \\ d\alpha \\ d\delta \end{bmatrix} \quad (1.40)$$

The determinant of the Hessian matrix  $|H|$  can be defined as follow:

$$|H| = \begin{vmatrix} \frac{\partial^2 SW}{\partial \bar{Z}^2} & \frac{\partial^2 SW}{\partial \bar{Z} \partial t} \\ \frac{\partial^2 SW}{\partial t \partial \bar{Z}} & \frac{\partial^2 SW}{\partial t^2} \end{vmatrix} = \left( \frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t^2} \right) - \left( \frac{\partial^2 SW}{\partial \bar{Z} \partial t} \frac{\partial^2 SW}{\partial t \partial \bar{Z}} \right) \quad (1.41)$$

Since  $\frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$  and  $|H|$  is assumed to be positive, the Hessian determinant is negative definite which would generate local maximum with the combination of optimal tax rate  $t$  and threshold  $\bar{Z}$ .

Applying the Cramer's rule yields the comparative statics results.

- $\frac{d\bar{Z}}{d\lambda} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t} \frac{\partial^2 SW}{\partial t \partial \lambda})}{|H|} > 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} = 0$ ,  $\frac{\partial^2 SW}{\partial t^2} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t} > 0$  and  $\frac{\partial^2 SW}{\partial t \partial \lambda} > 0$ .
- $\frac{d\bar{Z}}{d\Gamma'} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t} \frac{\partial^2 SW}{\partial t \partial \Gamma'})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} < 0$ ,  $\frac{\partial^2 SW}{\partial t^2} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t} > 0$  and  $\frac{\partial^2 SW}{\partial t \partial \Gamma'} < 0$ .
- $\frac{d\bar{Z}}{dt^c} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t} \frac{\partial^2 SW}{\partial t \partial t^c})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} < 0$ ,  $\frac{\partial^2 SW}{\partial t^2} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t} > 0$  and  $\frac{\partial^2 SW}{\partial t \partial t^c} = 0$ .
- $\frac{d\bar{Z}}{d\alpha} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} \frac{\partial^2 SW}{\partial t^2}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t} \frac{\partial^2 SW}{\partial t \partial \alpha})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} < 0$ ,  $\frac{\partial^2 SW}{\partial t^2} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t} > 0$  and  $\frac{\partial^2 SW}{\partial t \partial \alpha} < 0$ .

## Proof of Proposition 6

Applying the Cramer's rule yields the comparative statics results.

- $\frac{dt}{d\lambda} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \lambda}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} > 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$ ,  $\frac{\partial^2 SW}{\partial t \partial \lambda} > 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \lambda} = 0$  and  $\frac{\partial^2 SW}{\partial t \partial \bar{Z}} > 0$ .
- $\frac{dt}{d\Gamma'} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \Gamma'}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$ ,  $\frac{\partial^2 SW}{\partial t \partial \Gamma'} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \Gamma'} < 0$  and  $\frac{\partial^2 SW}{\partial t \partial \bar{Z}} > 0$ .
- $\frac{dt}{dt^c} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial t^c}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$ ,  $\frac{\partial^2 SW}{\partial t \partial t^c} = 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial t^c} < 0$  and  $\frac{\partial^2 SW}{\partial t \partial \bar{Z}} > 0$ .
- $\frac{dt}{d\alpha} = \frac{-(\frac{\partial^2 SW}{\partial \bar{Z}^2} \frac{\partial^2 SW}{\partial t \partial \alpha}) + (\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} \frac{\partial^2 SW}{\partial t \partial \bar{Z}})}{|H|} < 0$  since  $\frac{\partial^2 SW}{\partial \bar{Z}^2} < 0$ ,  $\frac{\partial^2 SW}{\partial t \partial \alpha} < 0$ ,  $\frac{\partial^2 SW}{\partial \bar{Z} \partial \alpha} < 0$  and  $\frac{\partial^2 SW}{\partial t \partial \bar{Z}} > 0$ .

## Proof of Proposition 7

From the social welfare function (1.19) we obtain,

$$\begin{aligned}
\frac{dSW}{d\bar{Z}} &= [(1-t-C-\alpha)\bar{Z} - \Gamma']h(\bar{Z}) + \int_{\bar{Z}}^{\hat{Z}} (1-t-C-\alpha)h(Z)dZ \\
&\quad + [(1-t-C-\alpha)\bar{Z} - \Gamma']h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} - [(1-t-C-\alpha)\bar{Z} - \Gamma']h(\bar{Z}) \\
&\quad - [(1-t^c)(1-C)\hat{Z} - \Gamma]h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} + \delta\{(t\bar{Z} - A')h(\bar{Z}) \\
&\quad + \int_{\bar{Z}}^{\hat{Z}} th(Z)dZ + (t\bar{Z} - A')h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} + (t\bar{Z} - A')h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} \\
&\quad - (t\bar{Z} - A')h(\bar{Z}) - [t^c(1-C)\hat{Z} - A]h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}}\} \\
&= \int_{\bar{Z}}^{\hat{Z}} (1-t-C-\alpha)h(Z)dZ + \delta \int_{\bar{Z}}^{\hat{Z}} th(Z)dZ \\
&\quad + \{[(1-t-C-\alpha)\bar{Z} - \Gamma' + \delta(t\bar{Z} - A')]\} \\
&\quad - [(1-t^c)(1-C)\hat{Z} - \Gamma] - \delta[t^c(1-C)\hat{Z} - A]\}h(\hat{Z})\frac{d\hat{Z}}{d\bar{Z}} \\
&= 0
\end{aligned} \tag{1.42}$$

given  $\pi^P(\bar{Z}) = \pi^A(\bar{Z})$ ,  $\pi^A(\hat{Z}) = \pi^R(\hat{Z})$  and  $d\hat{Z}/d\bar{Z} = (1-t-C-\alpha)/(1-t^c)(1-C)$ , the optimal  $\bar{Z}$  depends on the tax rate  $t$  and  $t^c$ , the multiplier of tax revenue  $\delta$ , the tax compliance cost  $\Gamma'$  and  $\Gamma$ , and the administrative cost  $A'$  and  $A$ .

## Proof of Proposition 8

From the social welfare function (1.19) we obtain,

$$\begin{aligned}
\frac{dSW}{dt} &= (1 - C - \lambda)Z^{IP}h(Z^{IP})\frac{dZ^{IP}}{dt} + \int_{Z^{IP}}^{\bar{Z}} (-Z)h(Z)dZ \\
&\quad - [(1 - t - C - \alpha)Z^{IP} - \Gamma']h(Z^{IP})\frac{dZ^{IP}}{dt} + \int_{\bar{Z}}^{\hat{Z}} (-\bar{Z})h(Z)dZ \\
&\quad + [(1 - t - C - \alpha)\bar{Z} - \Gamma']h(\hat{Z})\frac{d\hat{Z}}{dt} - [(1 - t^c)(1 - C)\hat{Z} - \Gamma]h(\hat{Z})\frac{d\hat{Z}}{dt} \\
&\quad + \delta\left\{\int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ - (tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt} + \int_{\bar{Z}}^{\hat{Z}} \bar{Z}h(Z)dZ \right. \\
&\quad \left. + (t\bar{Z} - A')h(\hat{Z})\frac{d\hat{Z}}{dt} - [t^c(1 - C)\hat{Z} - A]h(\hat{Z})\frac{d\hat{Z}}{dt}\right\} \\
&= (\delta - 1)\int_{Z^{IP}}^{\bar{Z}} Zh(Z)dZ + (\delta - 1)\int_{\bar{Z}}^{\hat{Z}} \bar{Z}h(Z)dZ \\
&\quad - \delta(tZ^{IP} - A')h(Z^{IP})\frac{dZ^{IP}}{dt} + \delta\{(t\bar{Z} - A') - [t^c(1 - C)\hat{Z} - A]\}h(\hat{Z})\frac{d\hat{Z}}{dt} \\
&= 0
\end{aligned} \tag{1.43}$$

given  $\pi^I(Z^{IP}) = \pi^P(Z^{IP})$ ,  $\pi^A(\hat{Z}) = \pi^R(\hat{Z})$ ,  $dZ^{IP}/dt > 0$  and  $d\hat{Z}/dt < 0$ , the optimal  $t$  depends on the tax rate  $t^c$  in the regular regime, the threshold  $\bar{Z}$ , the multiplier of tax revenue  $\delta$ , the tax compliance cost  $\Gamma'$  and  $\Gamma$ , and the administrative cost  $A'$  and  $A$ .

## Proof of Proposition 9

The equation (1.43) becomes

$$\begin{aligned}
&\left\{-\frac{\delta - 1}{2} + (\delta - 1)\frac{(1 - t - C - \alpha)}{(1 - t^c)(1 - C)} + \delta t^c(1 - C)\frac{(1 - t - C - \alpha)}{[(1 - t^c)(1 - C)]^2} - \delta\frac{t}{(1 - t^c)(1 - C)}\right\}\bar{Z}^2 \\
&+ \left\{(\delta - 1)\frac{\Gamma - \Gamma'}{(1 - t^c)(1 - C)} + \delta t^c(1 - C)\frac{\Gamma - \Gamma'}{[(1 - t^c)(1 - C)]^2} - \delta\frac{A - A'}{(1 - t^c)(1 - C)}\right\}\bar{Z} \\
&- \delta t\frac{\Gamma'^2}{(\lambda - t - \alpha)^3} + \delta\frac{\Gamma'A'}{(\lambda - t - \alpha)^2} - \frac{\delta - 1}{2}\frac{\Gamma'^2}{(\lambda - t - \alpha)^2} = 0
\end{aligned} \tag{1.44}$$



which can be rearranged into the standard quartic form  $at^4 + bt^3 + ct^2 + dt + e = 0$ , where

- $a = -Q$ ;  $b = (\lambda - \alpha)^3 Q + 3(\lambda - \alpha)Q - M$ ;  $c = -3(\lambda - \alpha)^2 Q + 3(\lambda - \alpha)M$
- $d = (\lambda - \alpha)^3 Q - 3(\lambda - \alpha)^2 M + S$ ;  $e = (\lambda - \alpha)^3 M + V$
- $S = -\frac{(\delta+1)}{2}\Gamma'^2 - \delta\Gamma'A'$ ;  $V = [\delta\Gamma'A' - \frac{\delta-1}{2}\Gamma'^2](\lambda - \alpha)$
- $\Delta_0 = c^2 - 3bd + 12ae$ ;  $\Delta_1 = 2c^3 - 9bcd + 27b^2e + 27ad^2 - 72ace$

# Chapter 2

## The Optimal Turnover Threshold and Tax Rate for SMEs

### 2.1 Introduction

Turnover taxes are widely used as “presumptive” or “simplified” income tax regimes for small and medium-sized enterprises (SMEs) to reduce the costs of tax compliance and administration. The rationale for this approach is that sales (turnover) are relatively easier to measure, record, and verify than profit. As the fixed costs of complying with and administering regular business income taxation makes the costs regressive,<sup>1</sup> presumptive regimes are intended for firms with sales below a threshold. However, a tax on turnover distorts the input choice of firms, unlike a well-designed tax on profits, thereby reducing productivity. Countries with presumptive regimes include France, Italy, Portugal, but especially developing and transition economies. Table 1 shows the thresholds and turnover tax rates currently in force in various countries.<sup>2</sup> There is significant variation in the observed turnover tax

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<sup>1</sup>For taxpayers, there is the time spent on bookkeeping tasks related to tax compliance and the cost of purchasing specialized accounting software. For the authorities, the cost of enforcing tax collection by visits to the premises and audits is largely independent of the amount of tax due.

<sup>2</sup>There can be additional eligibility criteria for the presumptive regime and in many countries a company below the threshold can elect to be subjected to the regular regime. For example, in Belarus firms in the simplified regime can have a maximum of 50 employees and can elect for the regular corporate income tax

rates and thresholds for SMEs. For example, in Kenya, firms with a turnover below \$49,000 are subjected to a turnover tax rate of 3% in lieu of the regular corporate income tax rate of 30%; in Seychelles the threshold and turnover tax rate are \$74,000 and 1.5%; in Mauritania they are \$84,000 and 3%; in Guinea the threshold is \$16,500 with a turnover tax rate of 5%, while in Belarus they are \$625,000 and 5%.<sup>3</sup>

Table 2.1: International practices on turnover thresholds and tax rates

Country	Threshold (USD)	Turnover tax rate (%)	Corporate income tax rate (%)
<b><i>Eastern Europe</i></b>			
Armenia	122,400	5 (trading), 3.5 (production)	20
Azerbaijan	346,000	4 (in Baku), 2 (outside Baku)	20
Belarus	625,000	5 (3 for VAT payers)	18
Latvia	45,600	15 (includes payroll tax)	20
Russia	2,250,000	6	15
Ukraine	185,000	5 (3 for VAT payers)	18
Uzbekistan	120,000	4	14
<b><i>Africa</i></b>			
Algeria	255,000	5	19
Angola	250,000	2	30
Congo (DRC)	122,000	1 (goods), 2 (services)	35
Congo (Brazzaville)	170,000	7.7	30
Cameroon	85,000	2.2	33
Guinea	16,500	5	25
Kenya	49,000	3	30
Liberia	18,600	4	25
Madagascar	56,000	5	20
Mauritania	84,000	3	25
Rwanda	22,000	3	30
Senegal	87,300	4 - 8 (progressive rates)	30
Seychelles	74,000	1.5	25
Tanzania	8,800	3 to 5.3	30
Uganda	40,500	1.5	30
Zambia	67,200	3	35
<b><i>Asia</i></b>			
Indonesia	331,200	1	25
<b><i>Western Europe</i></b>			
Austria	250,800	0.22	25
France	94,400	1.7 (industrial/commercial), 2.2 (non-commercial)	28
Italy	39,900	0.06 (food), 0.117 (professionals), etc.	27.9
Portugal	228,000	0.15	21

Sources : Miscellaneous tax guides and IBFD library. Notes : US dollar exchange rate as of December 2018.

Despite the prevalence of turnover taxes for SMEs, there is little theoretical guidance for determining the optimal threshold separating the presumptive and regular tax regimes, nor on the relationships between the threshold and the tax rates on turnover and corporate regime. In some cases, only unincorporated businesses are eligible. For example, in France the simplified regime is available only to unincorporated sole proprietors and partnerships.

<sup>3</sup>See Engelschalk and Loeprick (2015) and International Tax Dialogue (2007) for more examples of presumptive income taxes. See Logue and Vettori (2001) for a discussion of presumptive regimes in the context of the tax compliance of SMEs in the United States.

income. A very commonly advocated rule of thumb for the threshold of the presumptive income tax regime is to use the VAT threshold. The logic is that firms large enough to comply with the bookkeeping requirements of the VAT ought also to be able to comply with business income taxation. However, the optimal threshold for income taxation is driven by various margins that are not the same as for the optimal VAT. One such consideration is the gap between the regular corporate income tax rate and the presumptive tax rate on sales. A “good practice” recommendation in reports by international organizations is that the effective tax rate on income, implied by the turnover tax, should be more onerous at the threshold than the burden under the regular tax, in order to encourage firms to “graduate” to the regular system. But this advice neglects the cost of tax compliance, the avoidance of which is the very purpose of the presumptive regime. Another common policy recommendation is that the presumptive tax should be “neutral,” in the sense of equalizing the after-tax profit margins across tax regimes. However, in any interior equilibrium there will, by definition, be some firms that are indifferent between the two tax regimes, so that the recommendation for setting the optimal policy is vacuous without a detailed model, unless it is assumed that all firms have identical pre-tax profit margins. At the same time, moreover, care must be taken in setting the turnover tax rate, so as not to push firms in the presumptive system down into the untaxed but low-productivity informal sector, or, in some cases, into a fixed tax regime (“patent system”) intended for subsistence self-employment activities (Coolidge and Yilmaz, 2016). Thus, the optimal design of a presumptive tax regime is a complex issue.

This paper is the first to study the optimal sales threshold separating the presumptive and regular corporate income tax regimes and the corresponding optimal tax rates. We identify the key margins determining the welfare optimum and show how the optimal policies vary with the marginal cost of public funds, with administrative costs, and with productivity shifts. Additionally, we show how the optimal threshold and turnover tax rate are affected by changes in the corporate income tax rate. Our study is related to several strands of the

literature. First, our analysis of an optimal turnover threshold separating two tax regimes is complementary to Keen and Mintz (2004) on the VAT threshold and Dharmapala et al. (2011) on the threshold between a tax on sales and a fixed fee regime, while contributing to our understanding of the behavior of firms confronted by “notches” in tax schedules (Kleven and Waseem, 2013, Kanbur and Keen, 2014). Important differences with Kanbur and Keen (2014) are our inclusion of an intensive margin and our characterization of not only the optimal threshold, but also the optimal tax rates. While solutions for the optimal threshold can be derived in the absence of behavioral responses, akin to the “benchmark” in Kanbur and Keen (2014), marginal adjustments are crucial for interior solutions of optimal tax rates.<sup>4</sup>

The structure of our model resembles the model of Keen and Mintz (2004), except for an important distinction. While the heterogeneity of firms in Keen and Mintz (2004) stems from differences in productivity, in our model the heterogeneity is in terms of marginal costs of production. This is crucial for studying turnover taxes, because it is precisely the non-deductibility of costs that generates the inefficiencies associated with turnover taxation. Thus, an interesting finding arising from adjustments along the intensive margin in our model is that, depending on the tax rates and the size of compliance costs, *both* the higher-cost firms and the lower-cost firms may locate in the presumptive regime, leaving only middle-cost firms in the regular regime. Best et al. (2015) analyze a related tax system, whereby firms are taxed on profits, provided the tax liability is greater than an alternative minimum tax levied on turnover. The turnover tax in this case does not economize on compliance costs, since every firm must calculate and report its liabilities under the regular regime.<sup>5</sup> The heterogeneity of firm’s marginal costs in our model also makes it suitable for considering the effects of another approach used to simplify taxation for small businesses, in which a turnover threshold is used to separate larger businesses subject to VAT and smaller businesses subject to an alternative turnover tax system (Zu, 2018).

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<sup>4</sup>However, unlike Kanbur and Keen (2014), we do not extend the analysis to consider multiple thresholds and income concealment.

<sup>5</sup>The purpose of the minimum tax is to reduce the opportunity for evading the corporate income tax.

Our main findings are that the optimal threshold is generally between about \$100,000 and \$150,000, depending on the value added per firm of a country, and the optimal turnover tax rate is close to 3% in our benchmark calculation, if a single tax rate is being applied to all sectors of the economy. However, according to our estimates, the optimal turnover tax rate is higher and the optimal threshold is lower for Sub-Saharan Africa. Comparing our results with actual the practices described in Table 1, we find that, while many countries have appropriate policies, others deviate substantially from our prescriptions for welfare maximization.

The paper is structured as follows. Section 2 provides the basic description of the model. Section 3 contrasts the theoretical properties of the corporate income tax and the turnover tax by supposing that only one regime is used. Section 4 analyzes the choices of firms when the presumptive and regular tax regimes coexist. Section 5 examines the first-order conditions of the social welfare function with respect to the threshold and tax rates, given the private sector equilibrium responses. Section 6 provides numerical simulations of the optimal policies for a benchmark case and for countries at differing levels of economic development. Section 7 concludes. Proofs are in Appendix 1 unless they follow directly from the discussion.

## 2.2 The general setup

We assume that every individual allocates one unit of labor time between an amount  $L$  for production in the formal sector and  $1 - L$  for production in the informal sector. Both sectors produce final goods, but the production technologies differ. An individual's output in the formal sector is  $f(L)$ , where  $f$  is increasing and strictly concave (with  $f(0) = 0$  and derivatives indicated by  $f' > 0$  and  $f'' < 0$ ). In contrast, there is a constant rate of productivity  $w$  in the informal sector. The informal-sector good serves as the numeraire and, by definition of informality, the earnings  $w(1 - L)$  are untaxable. Production in the formal sector requires, in addition to labor supply, some amount  $\lambda > 0$  of an imported intermediate

good, per unit of output. The country is small in world markets, with the price of the formal-sector final good and the imported intermediate good fixed at  $p$  and  $p_I$ , respectively. The value of  $\lambda$  is individual-specific. Thus, for a given value of  $\lambda$  the cost of the intermediate good per unit of output produced is  $c = p_I \lambda$  and we can differentiate between the heterogeneous abilities of individuals, or “firms” by assuming directly that  $c$  is distributed according to a twice differentiable cumulative distribution function  $H(c)$ , with density  $h(c)$  and support  $c \in [0, 1]$ .<sup>6</sup>

Two linear tax regimes are considered for the income earned in the formal sector. In the regular regime, the cost of the intermediate input is tax deductible, with formal-sector profits taxed at the rate  $t^c < 1$ , while in the presumptive regime the tax rate is  $t < 1$  and costs are not deductible. Thus, the regular regime represents a corporate income tax and the presumptive regime corresponds to a tax on turnover. Firms with sales inferior to a fixed threshold  $\bar{Z}$  are placed in the presumptive regime. Finally, it is assumed that an individual subjected to the regular regime faces a fixed compliance cost  $\Gamma \geq 0$  and imposes a fixed administrative cost  $A > 0$  on the tax authority. For simplicity, we assume that there are no compliance or administration costs associated with the presumptive regime. We can represent net profits by

$$\pi(L) \equiv \rho f(L(\rho)) + w(1 - L(\rho)) - I^R \Gamma \quad (2.1)$$

where the “net price” is

$$\rho = \begin{cases} p^P \equiv (1 - t)p - c & \text{if in the presumptive regime} \\ p^R \equiv (1 - t^c)(p - c) & \text{if in the regular regime} \end{cases} \quad (2.2)$$

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<sup>6</sup>Similarly to Keen and Mintz (2004), an alternative interpretation of the model makes the reference to “firms” more natural. The variable  $L$  can be thought of as the amount of capital a firm invests in the taxed sector, subject to a fixed required rate of return, represented by  $w$ . Aside from an irrelevant constant stemming from the fixed time endowment, this interpretation gives the same formal structure as the self-employed labor model that we have described. In this case, however, no inferences can be drawn about the prevalence of informal activities, since  $1 - L$  is not constrained to be positive.

and  $I^R$  is an indicator function, which equals 1 when the firm is subjected to the regular regime and 0 if it is in the presumptive regime.

Ignore the sales threshold for the moment. The first-order condition for unconstrained profit-maximization by a type- $c$  firm in a given tax regime is

$$\rho f'(L^*) - w = 0 \quad (2.3)$$

with the net price  $\rho$  being a function of  $c$  via (2). The second-order condition,  $\rho f''(L^*) < 0$ , requires  $\rho$  to be positive. Let  $L(\rho) \equiv L^*$  and define the optimized profit function using (2.3) and (2.1) as

$$\pi^*(\rho) = \pi(L(\rho)) \quad (2.4)$$

and let

$$\pi^R(c) \equiv \pi^*(p^R(c)) \quad (2.5)$$

$$\pi^P(c) \equiv \pi^*(p^P(c)) \quad (2.6)$$

Since informal sector activities are untaxed, the minimum net profit of a firm in the regular regime is  $\pi^R = w - \Gamma$  (obtained by setting  $L = 0$ , which becomes optimal as  $t^c$  approaches unity).<sup>7</sup> Similarly, a firm in the presumptive regime may encounter a tax rate  $t$  such that its net price,  $p^P$ , is negative. In this case, the solution for  $L^*$  given by (2.3) would violate the second-order condition. Such a firm would choose the corner solution of retreating entirely to the informal sector with  $L^* = 0$  and earn  $w$  (recall there is no compliance cost for the presumptive regime).

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<sup>7</sup>An alternative assumption would be that the compliance cost is escaped when the firm chooses  $L = 0$ . This would introduce a kink in the profit function of firms in the regular regime. However, the alternative assumption does not affect our findings on the optimal policies.



Observe that, at interior solutions for  $L^*$ ,

$$\frac{d\pi^*}{d\rho} = f(L^*) > 0 \quad (2.7)$$

$$\frac{d^2\pi^*}{d\rho^2} = f'(L^*)\frac{dL^*}{d\rho} > 0 \quad (2.8)$$

where (2.7) uses the envelope theorem and the inequality in (2.8) is implied by the differential of (2.3) and the strict concavity of the production function. Hence, the profit function is an increasing and strictly convex function of the net price. However, the effect on  $\rho$  arising from changes in  $c$  will generally differ between the two regimes, due to the lack of deductibility of costs under presumptive taxation. To consider this relationship, rewrite the optimized profit function directly as a function of  $c$ ,  $\pi^*(c)$ , and calculate its derivatives, as follows:

$$\frac{d\pi^*}{dc} = \frac{d\pi^*}{d\rho} \frac{d\rho}{dc} = f(L^*) \frac{d\rho}{dc} < 0 \quad (2.9)$$

$$\frac{d^2\pi^*}{dc^2} = \frac{d^2\pi^*}{d\rho^2} \left(\frac{d\rho}{dc}\right)^2 > 0 \quad (2.10)$$

where the linearity of the tax systems ( $d^2\rho/dc^2 = 0$ ) is used in deriving (2.10) and the relationship between cost and the net price ( $d\rho/dc$ ) is given by

$$\frac{dp^P}{dc} = -1 \quad \text{in the presumptive regime} \quad (2.11)$$

$$\frac{dp^R}{dc} = -(1 - t^c) \quad \text{in the regular regime} \quad (2.12)$$

The signs of the derivatives (2.9) and (2.10) follow from (2.7)–(2.8) and (2.11)–(2.12). Now consider the effect of the sales threshold, whereby firms face the presumptive tax only if their sales are below the threshold. It may cause some firms to achieve a constrained profit maximum by producing just below the threshold (but arbitrarily close to  $\bar{Z}$ ) while facing the presumptive tax, or producing exactly at the threshold while facing the regular tax. We examine these situations later.

On the demand side, for simplicity, all individuals are assumed to have identical quasi-linear preferences, defined over the two final goods, with all income effects attached to the informal-sector good. Since  $p$  is fixed on world markets, individual demand for the formal-sector good  $x(p)$  is independent of tax policy. Tax revenues, net of administrative costs in the case of the regular regime, are used to pay for public expenditure  $G$ , which is assumed to generate a constant marginal utility,  $\delta > 1$ .<sup>8</sup> An individual's indirect utility is then of the form  $v(p) + \pi + \delta G$ , where  $v(p)$  is a constant that is independent of tax policy. If the tax payment of a type- $c$  individual net of any administration cost is denoted by  $g(c)$ , then the objective function of a utilitarian government can be represented by an expectation on the unit continuum for  $c$ :<sup>9</sup>

$$SW = E[v(p) + \pi(c) + \delta(g(c))] \quad (2.13)$$

The model delivers optimal values for  $t$ ,  $t^c$  and  $\bar{Z}$  simultaneously. We shall also consider how  $t$  and  $\bar{Z}$  should vary with the regular regime tax rate  $t^c$ , taking the latter variable as exogenous. This will allow us to comment on optimal reforms to a presumptive regime when the corporate income tax rate is taken as a given, but not necessarily at its optimal value; such partial reforms appear common in practice. Before proceeding to an analysis of the optimal threshold, it is interesting to contrast the presumptive and regular tax regimes, if all firms were placed in a single tax regime.

## 2.3 Comparing the presumptive and regular regimes

We first consider the simple case in which all firms are placed in a single fiscal regime. The comparison is useful for identifying the benefits and costs of each type of tax. Observe that, if there were no variation in the unit cost  $c$  across firms (i.e., if  $H(c)$  were a degenerate distribution), then, for any value of  $t^c$ , there would be an equivalent value of  $t$ , such that

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<sup>8</sup>In equilibrium, the marginal cost of public funds will be identical to the marginal utility of public spending.

<sup>9</sup>The expectation will consist of sets of integrals, corresponding to segments of firms in the regular regime and segments of firms in the presumptive regime.

$p^P = p^R$  and  $L^{P*} = L^{R*}$ . In that case, clearly the presumptive regime would dominate the regular regime, because of the compliance and administrative costs associated with the regular regime. However, when there is dispersion in the unit costs, there can be no value of  $t$  that is equivalent to  $t^c$  for every firm. Thus, as a result of the nondeductibility of costs in the presumptive regime, the output of many firms in the presumptive regime will be distorted, which results in a loss of social welfare.<sup>10</sup> These observations are illustrated with the square root production function  $f(L) = L^{1/2}$ , which allows us to solve explicitly for the optimal tax rate in each regime and to show how welfare in the presumptive regime is a decreasing function of the variance of unit costs.

### 2.3.1 All firms are in the regular regime

With a square root production function and all firms placed in the regular regime, individual firms' profits and net tax payments, along with social welfare, are given by the expressions:

$$\pi^R(c) = \left(\frac{1}{4w}\right)(p^R)^2 + w - \Gamma \quad (2.14)$$

$$g^R(c) = t^c(p - c)\left(\frac{1}{2w}\right)(p^R)^2 - A \quad (2.15)$$

$$\begin{aligned} SW^R = & v(p) + \frac{1}{4w}E([(1 - t^c)(p - c)]^2) \\ & + \delta \frac{1}{2w}t^c E[(p - c)(1 - t^c)(p - c)] + w - (\Gamma + \delta A) \end{aligned} \quad (2.16)$$

From the first- and second-order conditions for welfare maximization, we obtain the optimal tax rate in the regular regime,  $t^{c*}$ .

**Lemma 2.1.** *The optimal tax rate in the regular regime is an increasing function of the marginal value of public funds and is independent of the distribution of unit costs. It is given by*

$$t^{c*} = \frac{\delta - 1}{2\delta - 1}$$

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<sup>10</sup>This distortion compounds the distortion already present in either regime, due to the nondeductibility of the opportunity cost  $w$  of  $L$  supplied to the formal sector.

### 2.3.2 All firms in the presumptive regime

When all firms are placed in the presumptive regime, the solutions for the same variables as above are:

$$\pi^P(c) = \left(\frac{1}{4w}\right)(p^P)^2 + w \quad (2.17)$$

$$g^P(c) = \left(\frac{tp}{2w}\right)((1-t)p - c) \quad (2.18)$$

$$SW^P = v(p) + \frac{1}{4w}E([(1-t)p - c]^2) + \delta \frac{tp}{2w}E[(1-t)p - c] + w \quad (2.19)$$

Maximizing  $SW^P$  with respect to  $t$  yields the optimal presumptive tax rate,  $t^*$ .

**Lemma 2.2.** *The optimal tax rate in the presumptive regime is an increasing function of the average profit margin of the sector and is smaller than the optimal tax rate in the regular regime. It is given by*

$$\begin{aligned} t^* &= \left(\frac{\delta - 1}{2\delta - 1}\right) \left(\frac{p - E(c)}{p}\right) \\ &= t^{c*} \left(1 - \frac{E(c)}{p}\right) \end{aligned}$$

The average profit margin is  $(p - E(c))/p$ . The last equation implies that  $0 < t^* < t^{c*}$ , since  $p > E(c)$ .<sup>11</sup>

### 2.3.3 Comparison of welfare between the assigned regimes

Substituting the optimal tax rates  $t^{c*}$  and  $t^*$  into (2.16) and (2.19), respectively, yields the maximized social welfare functions. The difference between the maximized values of  $SW^R$  and  $SW^P$  is given by following proposition.

**Proposition 10.** With a square root production function, the difference between social

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<sup>11</sup>We are assuming here interior solutions for  $L$  for all firms, which is readily satisfied if  $p$  is large enough, so that no firm faces a negative net price at  $t^*$ .

welfare in the regular regime and the presumptive regime is increasing in the variance of the unit costs  $\sigma^2$ , but decreasing in both the fixed compliance and administrative costs. Depending on these two forces, either regime could be optimal, if only one regime is possible and tax rates are linear. The welfare difference is given by

$$SW^R - SW^P = -(\Gamma + \delta A) + \frac{1}{4w} \frac{(\delta - 1)^2}{(2\delta - 1)} \sigma^2 \quad (2.20)$$

Hence, the presumptive regime works best when there is little dispersion in the unit costs of firms in the presumptive regime, or, more generally speaking, in their profit margins. This observation can justify the common practice of categorizing firms in the presumptive regime by their types of economic activities and applying different tax rates to each category. Doing so reduces the variance of costs in each category, but it also makes the tax system more complex and may increase the cost of administering it.<sup>12</sup>

## 2.4 Presumptive regime with a sales threshold

We now analyze the tax structure in which only firms with sales below some threshold  $\bar{Z}$  are subjected to the presumptive regime, while firms with sales at or above  $\bar{Z}$  are obliged to be in the regular regime. We proceed by characterizing the private sector equilibrium for an arbitrary policy triplet  $\{t, t^c, \bar{Z}\}$ , including which regime each firm faces, subject to the constraint imposed by the threshold. This is accomplished in several steps. We calculate the desired sales levels of firms at each tax rate,  $t^c$  and  $t$ , and compare these outcomes with the turnover threshold,  $\bar{Z}$ , to characterize the choices effectively available to each firm. Comparisons of profit under the alternatives then determine each firm's optimal production decision. Thus, given the tax policy and the resulting market equilibrium, social welfare can

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<sup>12</sup>It is worth noting that the Keen and Mintz (2004) model of the optimal VAT, where all firms have the same input cost but differ in terms of productivity, is unsuitable for analyzing the questions addressed in our paper. Since every firm in their model has an identical unit cost,  $p^P$  will be the same for every firm. Given  $t^c$ , a unique value of  $t$  exists that makes  $p^P = p^R$  for every firm. Thus the presumptive regime must dominate the regular regime, due to the fixed costs associated with the regular regime.

be computed and the government, proceeding in this manner, searches for a global optimum. Of particular interest in the characterization of the behavior of firms is the possibility that some will “bunch” at, or just below, the sales threshold.

### 2.4.1 Partitioning the distribution of firms

To analyze the effect of the threshold on the behavior of firms, according to their unit costs, it is convenient to transform the profit function to express it in terms of sales. That is, maximizing (2.1) by choosing  $L$  is equivalent to choosing  $Z$  to maximize

$$\pi(Z, \rho) \equiv \rho \frac{Z}{p} + w[1 - f^{-1}(\frac{Z}{p})] - I^R \Gamma \quad (2.21)$$

in which  $Z = pf(L)$  and  $f^{-1}(Z/p) = L$  is the inverse of the production function. The desired sales level  $Z(\rho) \equiv Z^*$  solves the first-order condition

$$\frac{d\pi(Z, \rho)}{dZ} = \frac{\rho}{p} - \frac{w}{pf'(f^{-1}(Z^*/p))} = 0 \quad (2.22)$$

Differentiating (2.22) and using the properties of inverse functions, the desired sales function is decreasing in the unit cost  $c$ :

$$\frac{dZ(\rho)}{dc} = \frac{dZ}{d\rho} \frac{d\rho}{dc} = -\frac{p}{w} \frac{(f')^3}{f''} \frac{d\rho}{dc} < 0 \quad (2.23)$$

since  $\frac{d\rho}{dc} < 0$  and  $f'' < 0$ . A simple illustration of the desired sales curves and profit functions is provided by the square root production function  $f(L) = L^{1/2}$ :

$$Z(\rho) = \frac{p}{2w} \rho \quad (2.24)$$

and

$$\pi(\rho) = \frac{\rho^2}{4w} + w - I^R \Gamma \quad (2.25)$$

Since  $\rho$  is a linear function of  $c$  and from (2.24)  $Z(\rho)$  is itself linear in  $\rho$ , desired sales  $Z(c)$  is linear in  $c$ :

$$Z^R(c) = \frac{p}{2w}[(1 - t^c)(p - c)] \quad (2.26)$$

$$Z^P(c) = \frac{p}{2w}[(1 - t)p - c] \quad (2.27)$$

The desired sales functions characterized by (2.22) and illustrated with (2.26)–(2.27) can be used to construct four mutually exclusive sets of unit costs that exhaust the domain of  $H(c)$ . These sets are determined by whether the desired sales level of a given firm with unit cost  $c$  is below or above the threshold  $\bar{Z}$ . We will say that a firm is “constrained” by the threshold, if its desired turnover, when facing the presumptive tax, exceeds the threshold permitted for firms in the presumptive regime; or, if its desired turnover, when facing the regular tax regime, is below the threshold. Given any policy  $\{t, t^c, \bar{Z}\}$ , the four sets are

1.  $Z^P(c) \geq \bar{Z}$  and  $Z^R(c) \geq \bar{Z}$

(Only firms in the presumptive regime are constrained)

Define the set  $S_1(c) = \{c | Z^P(c) \geq \bar{Z} \text{ and } Z^R(c) \geq \bar{Z}\}$

2.  $Z^P(c) \geq \bar{Z}$  and  $Z^R(c) < \bar{Z}$

(Firms are constrained in both regimes)

Define the set  $S_2(c) = \{c | Z^P(c) \geq \bar{Z} \text{ and } Z^R(c) < \bar{Z}\}$

3.  $Z^P(c) < \bar{Z}$  and  $Z^R(c) \geq \bar{Z}$

(Firms are unconstrained in both regimes)

Define the set  $S_3(c) = \{c | Z^P(c) < \bar{Z} \text{ and } Z^R(c) \geq \bar{Z}\}$

4.  $Z^P(c) < \bar{Z}$  and  $Z^R(c) < \bar{Z}$

(Only firms in the regular regime are constrained)

Define the set  $S_4(c) = \{c | Z^P(c) < \bar{Z} \text{ and } Z^R(c) < \bar{Z}\}$

### 2.4.2 Constructing the sets using sales diagrams

The two curves described by (2.22) for  $Z^R$  and  $Z^P$  can be depicted on the same diagram, together with an arbitrary sales threshold  $\bar{Z}$ , with  $c$  on the horizontal axis, in order to determine which firms are constrained by the threshold. The slopes and intercepts of  $Z^R$  and  $Z^P$  will depend on the (arbitrary) values of  $t^c$  and  $t$ . The sales threshold is a horizontal line. We illustrate the procedure in Figure 2.1 using the linear case of (2.26) and (2.27), which cross where  $p^R(c) = p^P(c)$ , i.e., at  $\hat{c} = \frac{(t^c - t)p}{t^c}$ . Let  $\hat{Z} = Z^R(\hat{c}) = Z^P(\hat{c})$ . Figure 2.1 depicts a situation where  $Z^R$  has a lower intercept than  $Z^P$  (corresponding to  $t^c > t$ ) and the lines cross in the interior at  $\hat{c} > 0$  with the threshold  $\bar{Z} > \hat{Z}$  (Case A).  $Z^R$  is necessarily shallower than  $Z^P$  in the linear case since  $t^c < 1$ . In contrast, Figure 2.2 shows  $\bar{Z} < \hat{Z}$  (Case B). In both figures, define  $c'$  by  $Z^P(c') = \bar{Z}$  and  $c''$  by  $Z^R(c'') = \bar{Z}$ . Similarly, the values of  $c'$  and  $c''$  can be constructed for the cases where  $Z^R$  has a higher intercept than  $Z^P$  (corresponding to  $t^c < t$ ) or the lines do not intersect in the interior of the space. We omit these analyses for the sake of brevity.

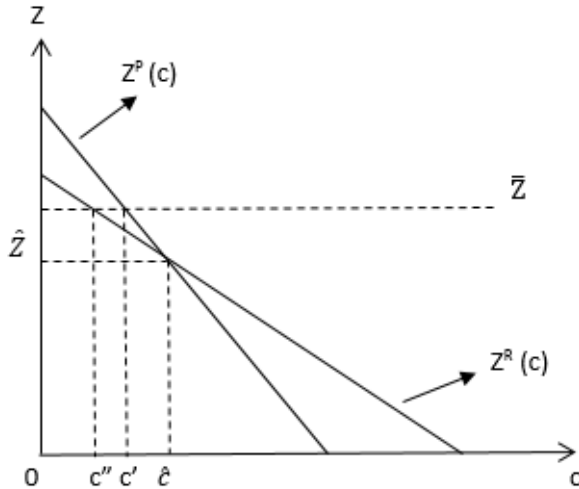


Figure 2.1: Case A

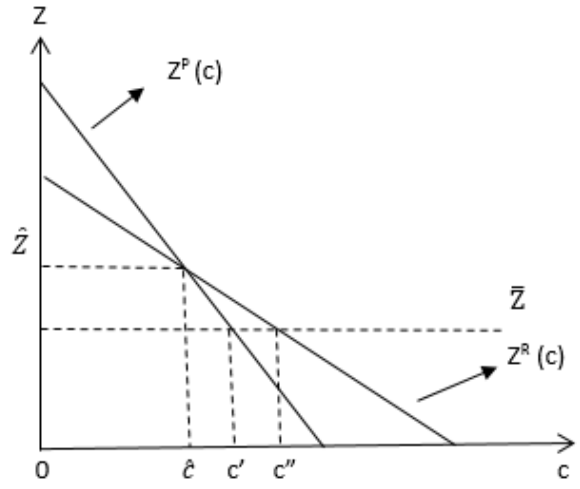


Figure 2.2: Case B

In Figure 2.1, it can be seen that firms in the segment  $[0, c'')$  would, if they are in the regular regime with the given tax rate  $t^c$ , desire a level of sales exceeding the threshold  $\bar{Z}$ .



Hence, their optimal sales in the regular regime is unconstrained by the threshold. However, firms  $[c'', 1]$  are constrained in the regular regime, in that their preferred sales level is below the threshold. In the presumptive regime, the segment  $[0, c']$  is constrained by the threshold, because the threshold level of sales is inferior to their preferred sales level when facing the given presumptive tax rate  $t$ ; firms in  $[c', 1]$  are unconstrained in the presumptive regime, because their preferred level of sales is below the threshold. In Figure 2.2, the situation is slightly different, because  $c''$  lies to the right of  $c'$ . We have

- Case A: shown in Figure 2.1  $\Rightarrow$  if  $\bar{Z} \geq \hat{Z}$ , there is  $0 < c'' \leq c' \leq \hat{c}$
- Case B: shown in Figure 2.2  $\Rightarrow$  if  $\bar{Z} < \hat{Z}$ , there is  $0 < \hat{c} < c' < c''$

From the discussion of Figure 2.1, the sets  $S_1$  to  $S_4$  corresponding to Case A are given by

1.  $S_1(c) = \{c : c \in [0, c'']\}$
2.  $S_2(c) = \{c : c \in [c'', c']\}$
3.  $S_3(c) = \emptyset$
4.  $S_4(c) = \{c : c \in [c', 1]\}$

Similarly, from Figure 2.2, the sets in Case B are given by

1.  $S_1(c) = \{c : c \in [0, c']\}$
2.  $S_2(c) = \emptyset$
3.  $S_3(c) = \{c : c \in [c', c'']\}$
4.  $S_4(c) = \{c : c \in (c'', 1]\}$

Note that, in each case, the union of the four sets equals  $[0, 1]$ . With a given sales threshold, there are four types of profit functions associated with a firm's choice of tax regime:

1. Unconstrained in presumptive regime:  $\pi^P(p^P) = p^P \frac{Z(p^P)}{p} + w[1 - f^{-1}(\frac{Z(p^P)}{p})]$

2. Constrained in presumptive regime:  $\pi^B(p^P) = p^P \frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))$
3. Unconstrained in regular regime:  $\pi^R(p^R) = p^R \frac{Z(p^R)}{p} + w[1 - f^{-1}(\frac{Z(p^R)}{p})] - \Gamma$
4. Constrained in regular regime:  $\pi^A(p^R) = p^R \frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z})) - \Gamma$

where  $\mu = f^{-1}(\frac{\bar{Z}}{p})$  is the labor supply that makes sales just equal to the threshold. The profit  $\pi^B(p^P)$  corresponds to firms that “bunch” just below the threshold. If their sales were any higher, they would have to switch to the regular regime. Similarly,  $\pi^A(p^R)$  is the profit of firms in the regular regime, who bunch at the threshold. If their sales were any lower, they would be obliged to face the presumptive tax regime.<sup>13</sup> Each firm compares the profits it can earn from alternative choices of regime and sales level. This is done with profit-difference functions, as follows.

1. For firms in  $S_1$ :  $D_1(c) = \pi^R(c) - \pi^B(c)$

Assume that firms choose the regular regime if and only if  $D_1(c) > 0$ .

2. For firms in  $S_2$ :  $D_2(c) = \pi^A(c) - \pi^B(c)$

Assume that firms bunch at the threshold if and only if  $D_2(c) > 0$ .

3. For firms in  $S_3$ :  $D_3(c) = \pi^R(c) - \pi^P(c)$

Assume that firms choose the regular regime if and only if  $D_3(c) > 0$ .

4. For firms in  $S_4$ :  $D_4(c) = \pi^A(c) - \pi^P(c)$

Assume that firms bunch at the threshold if and only if  $D_4(c) > 0$ .

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<sup>13</sup>All firms that are restricted from achieving their desired sales level by the threshold would choose the threshold level of sales as their constrained optimum—hence the notion of bunching.

The roots of the profit-difference curves partition the cost space into firms that are better off under one tax regime or the other. Thus the roots identify every firm's optimal choice of sales and hence the fiscal regime each is subjected to, given the tax policies  $\{t, t^c, \bar{Z}\}$  and the compliance cost  $\Gamma$ .

**Proposition 11.** The profit-difference curves have the following characteristics.

1. The graph of  $D_1(c)$  is strictly convex and has at most two real roots. If the roots are imaginary, then all firms in  $S_1$  choose the regular regime.
2. The graph of  $D_2(c)$  is linear and is positively sloped for any  $t^c > 0$ . Let  $c^*$  denote its single root. Firms in  $S_2$  with  $c^* \leq c \leq 1$  choose to bunch at the threshold. If  $c^* > 1$ , then all firms in  $S_2$  choose to bunch just below the threshold.
3. The graph of  $D_3(c)$  is strictly quasi-concave and has at most two real roots, if  $3(f'')^2 - f'f'''$  is positive for all  $L > 0$ . If the roots are imaginary, then all firms in  $S_3$  choose the presumptive regime.<sup>14</sup>
4. The graph of  $D_4(c)$  is strictly quasi-concave and has at most two real roots. If the roots are imaginary, then all firms in  $S_4$  choose the presumptive regime.

The quasi-concavity or convexity of the profit-difference curves (except for  $D_2$ ) can give rise to situations where the curves have a hump-shape or an inverted hump-shape, respectively. Take, for example, the case of  $D_3(c) = \pi^R(c) - \pi^P(c)$ . It is an increasing function at low values of  $c$  if  $dD_3(0)/dc > 0$ .<sup>15</sup> However, it eventually starts decreasing, because  $\pi^P$  flattens as  $p^P$  approaches 0; meanwhile,  $\pi^R$  continues to decline with  $c$ , since  $p^R > 0$  and  $dp^R/dc < 0$  for all  $c$ . Thus,  $D_3$  would initially rise, reach a peak, and then fall, yielding two possible real roots for  $D_3(c) = 0$ .

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<sup>14</sup>The sufficiency condition is weaker than requiring  $f(L(\rho))$  to be strictly concave in  $\rho$  (i.e.,  $2(f'')^2 - f'f''' > 0$ ), which is satisfied by standard production functions, such as the exponential, the logarithmic, and the quadratic forms of  $f(L)$ .

<sup>15</sup> When  $f(L) = L^\alpha$ , the parametric condition for  $dD_3(0)/dc > 0$  is  $(1 - t^c) < (1 - t)^\alpha$ .

Each of the profit-difference curves can be superimposed on the sets  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ , with the roots of the functions used to characterize the equilibrium choice of each firm  $c \in [0, 1]$ . To illustrate the approach, consider again the case of  $D_3 = \pi^R - \pi^P$  and assume it has a hump-shape. Define  $c_l$  and  $c_h$  as the lower and upper roots of  $D(c)$ , respectively.

**Proposition 12.** If the profit-difference curve  $\pi^R(c) - \pi^P(c)$  is hump-shaped with real roots  $c_l$  and  $c_h$ , then

1.  $\forall c \in [0, c_l), \pi^P > \pi^R \Rightarrow$  firms are better off in the presumptive regime
2.  $\forall c \in [c_l, c_h], \pi^R \geq \pi^P \Rightarrow$  firms are better off in the regular regime
3.  $\forall c \in (c_h, 1], \pi^P > \pi^R \Rightarrow$  firms are better off in the presumptive regime

To ensure that the roots are confined to the unit interval, in the statements above, for  $i = \{l, h\}$ , if  $c_i < 0$ , replace  $c_i$  with  $c_i = 0$ , while if  $c_i > 1$ , replace it with  $c_i = 1$ .

The following lemma shows how to apply these observations.

**Lemma 2.3.** Define  $c'$  by  $Z^P(c') = \bar{Z}$  and  $c''$  by  $Z^R(c'') = \bar{Z}$  and suppose that tax policy  $\{t, t^c, \bar{Z}\}$  generates Case B in Figure 2.2. Then the behavior of the firms in the interval  $S_3 = [c', c'']$  is characterized by the intersection of  $S_3$  and each of the three intervals described in Proposition 12.

The proposition records an interesting and novel observation: *both* the relatively low-cost and the relatively high-cost firms can prefer the presumptive regime, with only the middle-cost firms preferring the regular regime. Intuitively, non-deductibility is unimportant for low-cost firms and hence they will favor the presumptive regime, either to avoid the compliance cost of the regular regime or because  $t^c > t$ . At the other end of the cost spectrum, high-cost firms have relatively small operating profits and so they will tend to prefer the presumptive regime to avoid the compliance cost. It is the middle cost firms that are confronted with an important tradeoff between the nondeductibility of costs in the presumptive regime and

the cost of compliance in the regular regime. If, instead of having a hump-shape, the profit-difference curve is monotonically decreasing, then there will be a single root, which will again partition the firms in an obvious way. In general, it will be clear from the graphs of the various profit-difference curves how firms in each of the different segments of  $[0, 1]$  behave (in terms of labor supply and hence of sales) for any given tax policy.

To illustrate further, consider the case of  $f(L) = L^{1/2}$ . The profit-difference  $D_3(c)$  is then a quadratic function, over the interval of costs for which  $p^P(c) \geq 0$ :

$$\pi^R - \pi^P = \frac{1}{4w}[(1-t^c)^2 - 1]c^2 + \frac{p}{2w}[(1-t) - (1-t^c)^2]c + \left\{ \frac{p^2}{4w}[(1-t^c)^2 - (1-t)^2] - \Gamma \right\} \quad (2.28)$$

For the interval of costs for which  $p^P(c) < 0$  and hence  $\pi^P = w$ , the profit-difference curve in that case is simply

$$\pi^R - \pi^P = \frac{1}{4w}[(1-t^c)(p-c)]^2 - \Gamma \quad (2.29)$$

The roots of (2.28) are

$$c = \frac{p[(1-t) - (1-t^c)^2] \pm p\sqrt{t^2(1-t^c)^2 - \frac{4w}{p^2}[1 - (1-t^c)^2]\Gamma}}{1 - (1-t^c)^2} \quad (2.30)$$

Real roots exist if and only if  $\Gamma \leq \Gamma'$ , where

$$\Gamma' = \frac{p^2}{4w} \left( \frac{t^2(1-t^c)^2}{1 - (1-t^c)^2} \right) \quad (2.31)$$

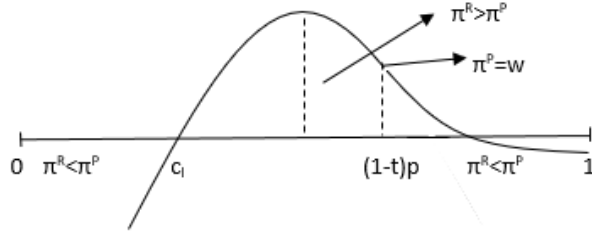
Suppose the compliance cost is  $\Gamma = 0$ . The two roots in (2.30) become:

$$c_l = \frac{(t^c - t)p}{t^c}; \quad c_h = \frac{(2 - t^c - t)p}{2 - t^c} \quad (2.32)$$

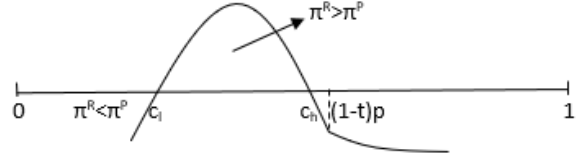
The upper root  $c_h$  is inadmissible because  $p^P(c_h) < 0$ .<sup>16</sup> If  $t < t^c$ , then  $c_l > 0$  and firms with  $c \in [0, c_l)$  prefer the presumptive regime, while firms with  $c \in [c_l, 1]$  prefer the regular regime

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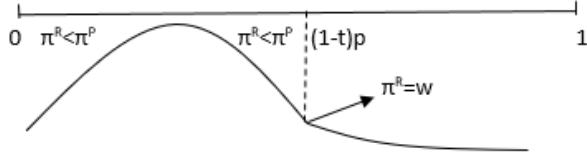
<sup>16</sup>In that case, the admissible upper root is obtained from (2.29), which is  $c_h = p - 2(w\Gamma)^{1/2}/(1-t^c)$ .



(a) Case with  $\Gamma \in [0, \Gamma'')$



(b) Case with  $\Gamma \in [\Gamma'', \Gamma']$



(c) Case with  $\Gamma \in (\Gamma', \infty)$

Figure 2.3:  $D_3(c) = \pi^R(c) - \pi^P(c)$  with different scales of  $\Gamma$

(firm- $c_l$  is indifferent). If  $t < t^c$  but  $c_l > 1$ , then all firms prefer the presumptive regime. Finally, if  $t \geq t^c$ , then  $c_l \leq 0$  and all firms (weakly) prefer the regular regime. For larger values of  $\Gamma$ , both  $c_l$  and  $c_h$  can be admissible roots of the profit-difference curve. Figure 2.3 (panels *a* to *c*) provides graphs of the profit-difference curve  $D_3(c) = \pi^R(c) - \pi^P(c)$  for different values of  $\Gamma$ . The inflexion point in each graph corresponds to the point at which  $p^P(c)$  becomes negative, so the profit-difference curve equation switches from (2.28) to (2.29). In panel *a*,  $\Gamma$  is small, so the only root for (2.28) is  $c_l$ ; firms with unit costs exceeding  $(1-t)p$  will switch to a corner solution, in which  $L^* = 0$  and  $\pi^P$  becomes a constant equal to  $w$ . For larger values of  $\Gamma$ , both roots of (2.28) become admissible, as illustrated in panel *b*. Finally, in panel *c*,  $\Gamma$  is so large that there are no real roots for  $D_3(c)$ —every firm prefers to be subjected to the presumptive regime instead of the regular regime.

Figures 2.4 and 2.5 illustrate how the four  $S$  sets and the profit-difference curves are combined to determine the market equilibrium. The sets  $S_1$  to  $S_4$  partition the cost space  $[0, 1]$  and the corresponding profit-difference curves  $D_1$  to  $D_4$  determine the behaviors of

the firms in each segment, depending on the values of the roots. Figure 2.4 shows the Case A configuration of  $Z^P(c)$ ,  $Z^R(c)$  and  $\bar{Z}$  such that  $\bar{Z} \geq \hat{Z}$  and  $0 < c'' \leq c' \leq \hat{c}$ , as previously illustrated in Figure 2.1. Figure 2.5 shows the Case B configuration, where  $\bar{Z} < \hat{Z}$  and  $0 < \hat{c} < c' < c''$ , as previously illustrated in Figure 2.2. The bottom portions of Figure 2.4 and Figure 2.5 provide graphs of the profit-difference curves for the sets  $S_1(c)$ ,  $S_2(c)$ ,  $S_3(c)$ , and  $S_4(c)$ , associated with Case A and Case B, respectively. Observe that the profit-difference curves corresponding to each segment can be “stitched” together to form a continuous curve. To see this, recall that  $S_3$  is empty in Case A, so  $D_3$  becomes irrelevant. The curves  $D_1$  and  $D_2$  are therefore joined, using the fact that  $\pi^R = \pi^A$  at the unique cost at which the desired sales curve equals the threshold,  $Z^R(c'') = \bar{Z}$ . Similarly,  $D_2$  is joined to  $D_4$  at the cost level at which  $\pi^P = \pi^B(c')$ . In Case B,  $S_2$  is empty and  $D_2$  becomes irrelevant. Then,  $D_1$  is joined to  $D_3$  where  $\pi^P = \pi^B(c')$  and  $D_3$  is joined to  $D_4$  where  $\pi^R = \pi^A(c'')$ . Thus, we can speak of an overall profit-difference curve as the outcome of joining the specific profit-difference curves over the  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$  segments of the unit interval.

Figure 2.4 exhibits a unique root for  $D_1(c)$  within the set of costs  $S_1 = [0, c'')$ , indicated as  $c_1$ . In the segment  $S_2 = [c'', c')$ , there are no real roots. In the segment  $[c', 1]$ , there are two admissible roots, labelled  $c_4$  and  $c_5$ ; hence, only the firms with unit costs  $c_4 < c < c_5$  prefer a sales level that puts them at the threshold for the regular regime, rather than in the presumptive regime. In Figure 5, there are two admissible roots in the segment  $S_1 = [0, c')$ , shown as  $c_1$  and  $c_2$ ; thus, among these firms, the ones in the subintervals  $[0, c_1)$  and  $[c_2, c')$  prefer the regular regime, while the “intermediate” firms prefer to bunch just below the threshold. Figure 2.5 also shows that all firms in the set  $S_3 = [c', c'']$  choose the regular regime over the presumptive regime, while firms in the set  $S_4 = (c'', 1]$  bunch at the threshold. Thus, Figures 2.4 and 2.5 illustrate how, for a given tax policy, we can characterize the behavior of all of the firms to determine which tax regime each firm is subjected to in equilibrium. It is then straightforward to compute the social welfare value corresponding to the given tax policy. A global welfare optimum can be determined by performing a grid

search across all combinations of  $\{t^c, t, \bar{Z}\}$ .

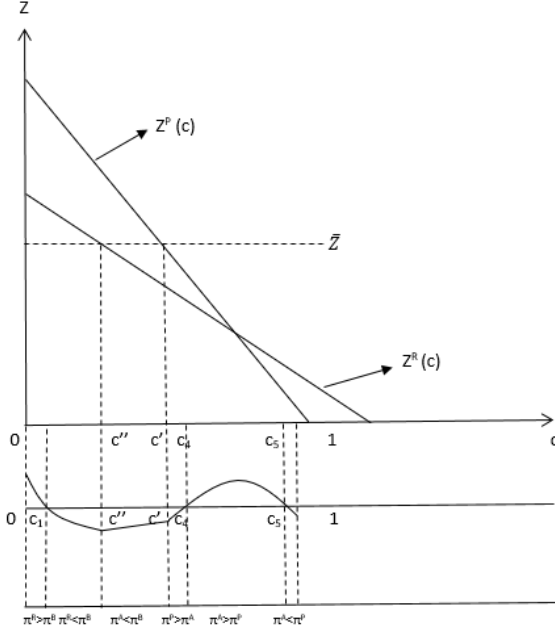


Figure 2.4: Case A

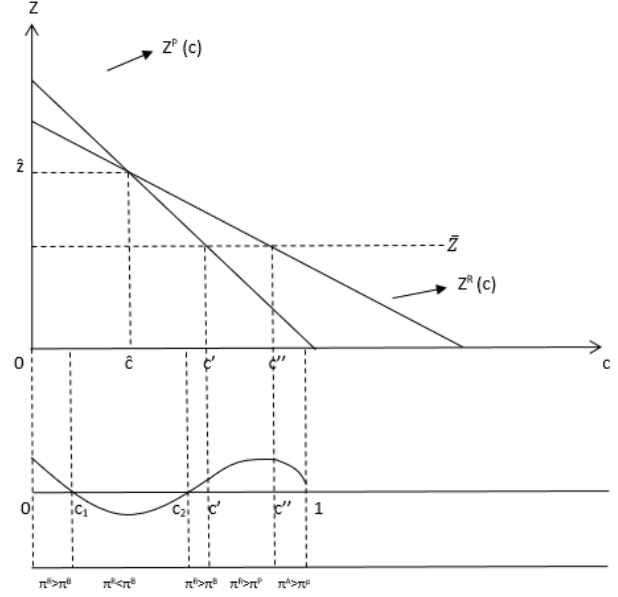


Figure 2.5: Case B

## 2.5 Social welfare optimization

Due to the discontinuous nature of the private sector equilibria—i.e., the number of admissible roots can change as tax rates or the sales threshold change—using calculus to optimize the global value of social welfare is infeasible. However, it is possible to characterize the first-order conditions for welfare maximization at an interior local optimum. From our numerical simulations, it turns out that the globally optimal tax policy generates the configuration shown in Figure 2.6, which is analogous to Figure 2.5 in the previous section, except that there is now a single root, denoted by  $c_1$ , for the overall profit-difference curve. The welfare function in the neighborhood of this optimal policy can then be constructed using the variables  $c'$  and  $c_1$  from Figure 2.6 as limits of integration.<sup>17</sup> The aggregate net profit function

<sup>17</sup>Note that  $c_1$  is the lower root of  $\pi^R - \pi^B = 0$ , which determines which firms in the set  $S_1$  for Case B will choose the regular regime versus bunching below the threshold. In Figure 2.6,  $S_1 \equiv [0, c'] = [0, c_1) \cup [c_1, c']$



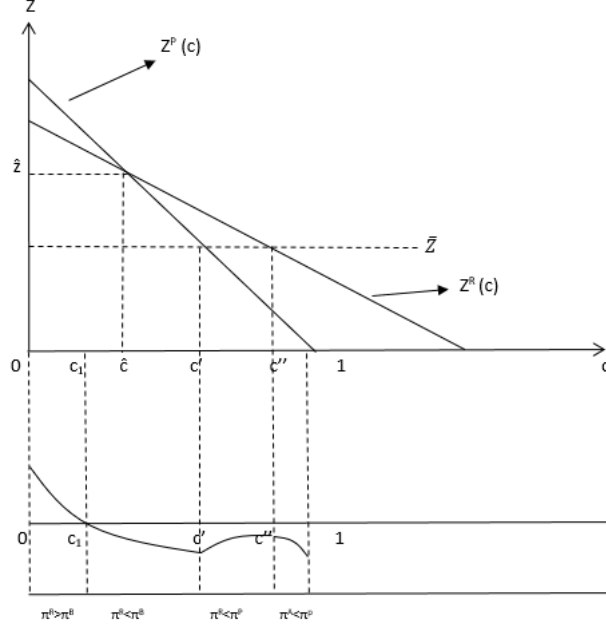


Figure 2.6: Figure from Simulation Results

is given by

$$\begin{aligned}
\Pi(\bar{Z}, t, t^c) = & \int_0^{c_1} \{p^R f(L(p^R)) + w[1 - L(p^R)] - \Gamma\} h(c) dc \\
& + \int_{c_1}^{c'} \{p^P \frac{\bar{Z}}{p} + w[1 - \mu(\bar{Z})]\} h(c) dc \\
& + \int_{c'}^1 \{p^P f(L(p^P)) + w[1 - L(p^P)]\} h(c) dc
\end{aligned} \tag{2.33}$$

where  $\mu(\bar{Z})$  is the labor supply at which a firm's sales equals the threshold. The first term on the right-hand side of (2.33) is the net profit of firms in the regular regime; the second term is the net profit of firms just below the threshold; the third term is the net profit of

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and firms in  $[0, c_1)$  are unconstrained in the regular regime, while those in  $[c_1, c']$  choose to be constrained just below the threshold; the set  $S_2$  is empty, while in  $S_3 \equiv [c', c'']$  and  $S_4 \equiv (c'', 1]$  all the firms prefer to be unconstrained by the threshold in the presumptive regime over any alternative behavior. The equilibrium depicted in Figure 2.6 can be referred to as R-B-P, meaning that relatively low-cost firms ( $c \in [0, c_1)$ ) are unconstrained in the regular regime (R), mid-cost firms ( $c \in [c_1, c']$ ) bunch just below the threshold (B), while higher-cost firms ( $c \in (c', 1]$ ) are unconstrained in the presumptive regime (P).

firms unconstrained in the presumptive regime. Government spending is given by

$$\begin{aligned}
G(\bar{Z}, t, t^c) &= \int_0^{c_1} [t^c(p - c)f(L(p^R)) - A]h(c)dc \\
&+ \int_{c_1}^{c'} t\bar{Z}h(c)dc \\
&+ \int_{c'}^1 tpf(L(p^P))h(c)dc
\end{aligned} \tag{2.34}$$

The first term on the right-hand side of (2.34) is the tax revenues in the regular regime, net of administrative costs; the second term is the tax revenues collected from the firms just below the threshold; the third term is the revenue generated by firms in the presumptive regime that are unconstrained by the threshold. Social welfare can be written in terms of these aggregates as,

$$SW = \Pi(\bar{Z}, t, t^c) + \delta G(\bar{Z}, t, t^c) + V(p) \tag{2.35}$$

where  $V(p)$  is the aggregate consumer surplus from sales in the formal sector, which is a constant and hence of no consequence for the welfare optimization. The first-order condition for welfare-maximization with respect to the threshold  $\bar{Z}$  can be rearranged to obtain the following result.

**Proposition 13.** The optimal threshold is characterized by:

$$\begin{aligned}
&\delta[(t^c(p - c_1)f(L(p^R(c_1))) - A - t\bar{Z})]h(c_1)\frac{dc_1}{d\bar{Z}} \times (-1) \\
&= \int_{c_1}^{c'} [\frac{p^P}{p} - w\mu_{\bar{Z}}]h(c)dc + \delta t[H(c') - H(c_1)]
\end{aligned} \tag{2.36}$$

where  $\mu_{\bar{Z}} \equiv d\mu/d\bar{Z}$ . The left-hand side of (2.36) (in absolute value) is the net effect along the *extensive margin* (EM) that results from raising the threshold by \$1, while the right-hand side is the net effect along the *intensive margin* (IM) of raising the threshold by \$1. At the optimum, the extensive and intensive margins are balanced.<sup>18</sup> The extensive margin refers to the change in welfare arising from the relocation of some firms from the regular

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<sup>18</sup>Letting  $dSW/d\bar{Z} = EM + IM = 0$ , the optimum requires  $-EM = IM$ .

regime toward bunching just below the threshold in the presumptive regime. In contrast, the intensive margin refers to the change in welfare from the increased threshold, holding the mass of firms in each regime constant. Beginning with the extensive margin, on the left side of (2.36), the mass of firms moving from the regular regime to now bunching below the threshold is  $h(c_1)(dc_1/d\bar{Z})$ , where  $dc_1/d\bar{Z}$  is the leftward shift of  $c_1$  in Figure 2.6 and  $h(c_1)$  is the density of firms at  $c_1$ .<sup>19</sup> This shift in mass is multiplied by the total tax revenue change per affected firm:  $t^c(p - c_1)f(L(p^R(c_1))) - A$  is the tax revenue loss in the regular regime, net of savings of administrative costs, while  $t\bar{Z}$  is the gain from the presumptive tax on each new “buncher.” Turning to the intensive margin, the first term on the right side of (2.36) is positive and represents the gain in production efficiency: when the threshold is increased, firms that used to bunch below the threshold would expand their output.<sup>20</sup> The second term is the extra tax revenue in the presumptive regime directly due to the higher threshold: every firm previously bunching in the presumptive regime (i.e., the mass of firms with unit costs between  $c_1$  and  $c'$ ) pay an additional  $t \times \$1$ , with the increased tax revenues weighted by the marginal utility of public spending.<sup>21</sup>

The first-order condition for welfare with respect to the presumptive tax rate  $t$  gives the following result.<sup>22</sup>

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<sup>19</sup>Since firms constrained below the threshold desire to expand sales, their profits rise as  $\bar{Z}$  is raised. At the same time, the profits of firms unconstrained in the regular regime are unaffected by the threshold. Consequently,  $\pi^R(c) - \pi^B(c)$ , which has an inverted hump-shape (i.e., a parabola that opens upward in the square root production function case), must sink at every  $c$ , as  $\bar{Z}$  increases.  $c_1$  is the lower root of the profit-difference curve  $D_1$ ; hence,  $dc_1/d\bar{Z} < 0$ .

<sup>20</sup>Recall that  $\bar{Z} = pf(\mu(\bar{Z}))$  and hence  $\mu_{\bar{Z}} = \frac{1}{pf'}$ . In equation (2.36), all firms with cost between  $c_1$  and  $c'$  are constrained by the threshold and would choose, in its absence, to produce more. So, for them,

$$p^P f' - w > 0$$

and it follows, after substituting in the expression for  $\mu_{\bar{Z}}$ , that  $\frac{p^P}{p} - w\mu_{\bar{Z}} > 0$ .

<sup>21</sup>Note that the compliance cost,  $\Gamma$ , does not explicitly enter the social welfare first-order conditions, because the marginal firm,  $c_1$ , already balances the discontinuous gain in net profit in moving to the regular regime with the compliance cost incurred.

<sup>22</sup>For brevity, we omit from the discussion the first-order condition with respect to the regular tax rate  $t^c$ . Thus, the results can be interpreted as optimizing welfare by choosing  $\bar{Z}$  and  $t$  for a given value of  $t^c$ . The first-order condition with respect to  $t^c$  is given in the appendix.

**Proposition 14.** The optimal  $t$  is characterized as:

$$\begin{aligned}
& \delta[t^c(p - c_1)f(L(p^R(c_1))) - A - t\bar{Z}]h(c_1)\frac{dc_1}{dt} \times (-1) \\
& = (\delta - 1)\bar{Z}[H(c') - H(c_1)] + (\delta - 1) \int_{c'}^1 pf(L(p^P))h(c)dc \\
& \quad - \delta \int_{c'}^1 tp^2 f' \frac{dL}{dp^P} h(c)dc
\end{aligned} \tag{2.37}$$

The left side of (2.37) (in absolute value) is the effect along the extensive margin of increasing the presumptive tax rate  $t$  and the right side is the effect along the intensive margin. When  $t$  increases by 1%, the mass  $h(c_1)\frac{dc_1}{dt}$  of firms with cost  $c_1$ , who previously stayed just below the threshold, would now move to the regular regime,<sup>23</sup> resulting in a welfare change equal to  $\delta$  times the net revenue gain in the regular regime,  $t^c(p - c_1)f(L(p^R(c_1))) - A$ , minus the revenue loss from the former “bunchers,”  $t\bar{Z}$ . As for intensive margin, the first pair of terms on the right-hand side of (2.37) gives the welfare change due to the increased revenues from both the bunchers and unconstrained firms in the presumptive regime, respectively, while the last term is the welfare loss from the lower tax revenues caused by the reduction in the output of the firms in the presumptive regime,<sup>24</sup> as a result of the increased tax rate. In summary, Propositions 13 and 14 can provide guidance on tax reforms. Consider the impacts of slightly raising  $t$  from a given level. The following effects must be considered.

1. The net revenue change from the firms switching to the regular regime.
2. The greater administrative costs generated by the firms switching to the regular regime.
3. The tax revenue gain from firms that remain in the presumptive regime.
4. The reduction in the output of firms in the presumptive regime.

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<sup>23</sup> $\pi^R(c) - \pi^B(c)$  has an inverted hump shape and  $\pi^R$  is independent of  $t$ , while  $\pi^B$  is decreasing in  $t$ . Consequently, an increase in  $t$  lifts the graph of the profit-difference curve, making  $c_1$  shift to the right. Hence,  $dc_1/dt > 0$ .

<sup>24</sup>The term,  $-tp^2 f' \frac{dL}{dp^P}$ , stems from  $tp(f' \frac{dL}{dp^P} \frac{dp^P}{dt})$  with  $\frac{dp^P}{dt} = -p$ . It can also be written as the product of sales and the elasticity of output with respect to the presumptive tax rate:  $Z^P(\frac{df}{dt})(\frac{t}{f})$  where  $Z^P = pf(L(p^P))$ .

The weight  $\delta$  is applied to the net change in tax revenues arising from shifts in the equilibrium allocation of firms or from changes in their output, while the weight  $(\delta - 1)$  is applied when firms pay more of their current profits as tax revenues. The proportion of firms switching from bunching just below the threshold to entering the regular regime (point 1) and the efficiency loss from raising the presumptive tax rate (point 4) are empirical matters.

## 2.6 Numerical simulations

For quantitative insights, we calibrate a numerical simulation model to replicate the broad characteristics of an economy with both regular and presumptive regimes. The production function is  $f(L) = \beta L^\alpha$  and parameter values are selected such that: the average value added of a firm is about \$120,000;<sup>25</sup> there is some bunching of firms just below the optimal threshold;<sup>26</sup> and the compliance cost is about 0.4% of the average turnover of firms in the regular regime.<sup>27</sup> The formal sector output price is 1 USD and the informal sector good's price is normalized to 1. The distribution of firms' unit costs is  $H(c) = 0.2c^2 + 0.8c^3$ , with  $c \in [0, 1]$ , capturing the preponderance of relatively high cost firms in most economies and generating a ratio of average value-added to sales equal to 26%. The marginal value of public funds is set to 1.3. A summary of the baseline parameterizations is in lower-left cell of Table 2.2.

Table 2.2 reports the optimal turnover tax rate and the optimal threshold at corporate tax rates ranging (exogenously) from 11% to 29%. The row entitled "Placement of firms" shows the tax regimes firms choose (implicitly through their sales level) starting from the

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<sup>25</sup>The figure corresponds, e.g., to the value added per firm in Latvia, of 106,369 euro in 2017 (see European Commission, 2018).

<sup>26</sup>Bruhn and Loeprick (2016) provide evidence of bunching below the thresholds of the turnover tax regimes in Georgia.

<sup>27</sup>Corporate income tax compliance cost is between 0.05% and 15% of taxable turnover in developing and transition economies (Sapiei et al., 2014). Surveys of companies in Armenia and Ukraine suggest compliance costs comparable to our calibration for turnovers in the range of \$150,000 to \$1 million (Engelschalk and Loeprick, 2015). To fix  $A$ , we used the ratio of administrative cost to taxpayer compliance cost, based on estimates for VAT reported in Keen and Mintz (2004).

lowest cost segment of firms to the highest cost segment.<sup>28</sup> When the corporate tax rate is low ( $t^c = 11\%$ ), the regular regime is not worth the compliance/administrative costs and the optimal threshold is very high, with all firms allocated to the presumptive regime. If  $t^c$  is raised to 14%, there is bunching *at* the threshold in the regular regime, whereas at higher corporate tax rates there is bunching just below the threshold. Increasing the corporate tax rate in steps from 17% to 26%, the optimal threshold falls and the optimal turnover tax rate rises.<sup>29</sup> Eventually, when  $t^c$  is high enough ( $t^c = 29\%$ ), the distortion induced by the corporate income tax makes it optimal to again have all firms in the presumptive regime by setting an elevated threshold.

The last set of rows in Table 2.2 provide the global optimum at the baseline parameter values and the comparative statics analysis. The overall optimal policy occurs when the corporate income tax rate is 20.9% and the optimal turnover tax rate is 2.7%, while the optimal threshold is \$113,000 and about 7% of firms bunch just below the threshold.<sup>30</sup> A slight increase in the cost of compliance and administration (Case 1:  $\Gamma = \$1300$  and  $A = \$260$ ) leads to a decrease in the optimal threshold, as well as an increase in both the corporate tax rate and the presumptive tax rate. A slight increase in the marginal value of public funds (Case 2:  $\delta = 1.35$ ) leads to a decrease in the threshold, accompanied by an increase in both the corporate tax rate and the turnover tax rate. A slight increase in the productivity parameter  $\beta$  (Case 3:  $\beta = 810$ ) leads to an increase in the threshold and in the tax rates.

Going beyond comparative statics, in Table 2.3 we fix the corporate income tax rate at 20% and consider how the optimal threshold and turnover tax rates vary at significantly different levels of economic development. However, such an exercise can only be taken

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<sup>28</sup>At the calibration used for Table 2.2, there is no case of multiple equilibria at the optimal policy.

<sup>29</sup>Raising the corporate tax rate makes bunching more attractive, resulting in lost revenue. Lowering the threshold and increasing the turnover tax rate both serve to counter the incentive to bunch below the threshold.

<sup>30</sup>Though not shown in Table 2.2, the gross profit margins of firms bunching just below the threshold varies from 9% to 12% at the equilibrium. This shows the difficulty of applying the conventional wisdom, that the tax rates should be chosen to equalize the net returns of the marginal firm across tax regimes. When firms are heterogeneous, there is no unique profit margin on which to base the calculation of net returns.

as suggestive, rather than definitive, as we do not recalibrate the whole model for each country. Instead, we simply adjust the production function's shift parameter,  $\beta$ , to control the level of productivity in the economy. In reality, richer or poorer countries, relative to the baseline, likely have quite different distributions of firms in terms of marginal costs and their governments will have different marginal values of public funds. But there is insufficient data on most developing countries to calibrate individually and, moreover, attempting to do so could obscure the impacts of the different parameter configurations on the country-specific optimal tax policies. We take the following approach instead. The parameter  $\beta$  is adjusted to achieve a target level of average value added per firm. As we do not have systematic data on the value added per firm for countries outside of the EU, we use data on GDP per person employed from the ILOSTAT database of the International Labour Organization to shift the value added per firm proportionately, relative to our benchmark simulation. We do this for three categories of countries, based on their group average, as reported in the ILOSTAT data: Sub-Saharan Africa, Latin America, and the Euro area, in ascending order of GDP per person employed. As Table 2.3 shows, the lower the average value added per firm, the lower is the optimal threshold and the higher is the turnover tax rate. The result for Sub-Saharan Africa stands out, as the optimal turnover tax rate is high, at 6.8%, and a large proportion of firms are in the presumptive regime (including those bunching below the threshold), despite a relatively low threshold. These results are not unrealistic. For example, in Madagascar, firms with sales below \$56,000 face a turnover tax rate of 5%, while the corporate income tax rate is 20%.

Table 2.2: Simulation Results

Tax rate in the regular regime		11.0%	14.0%	17.0%	20.0%	23.0%	26.0%	29.0%	
Optimal tax rate in the presumptive regime		6.00%	4.40%	2.40%	2.60%	2.80%	3.10%	6.00%	
Optimal sales threshold (\$ thousand)		924	339	141	120	97	86	924	
Placement of firms		P	R - A - P	R - B - P	R - B - P	R - B - P	R - B - P	P	
Proportion of firms		100%	42% - 14% - 44%	65% - 9% - 26%	68% - 8% - 24%	73% - 6% - 21%	75% - 4% - 21%	100%	
Average value-added per firm		1.26E+05	1.27E+05	1.23E+05	1.23E+05	1.16E+05	1.12E+05	1.26E+05	
Average compliance cost / turnover		n.a.	0.39%	0.39%	0.41%	0.42%	0.43%	n.a.	
Compliance cost / turnover at the threshold		n.a.	0.35%	0.85%	1.00%	1.24%	1.40%	n.a.	
Social Welfare		82185.22	82185.68	82476.49	82626.64	82611.02	82419.20	82185.22	
Comparative statistics		$t^{c*}$	$t^*$	$\bar{Z}^*$ (\$ thousand)	Place of firms	Proportion of firms	Average value-added per firm	Average compliance cost / turnover	Compliance cost / turnover at the threshold
Baseline: $\Gamma = 1200$ , $A = 240$ , $\delta = 1.3$ , $\alpha = 0.45$ and $\beta = 800$		20.9%	2.7%	113	R - B - P	70% - 7% - 23%	1.18E+05	0.41%	1.06%
Case 1. $\Gamma = 1300$ , $A = 260$		21.1%	2.8%	112	R - B - P	70% - 7% - 23%	1.18E+05	0.45%	1.16%
Case 2. $\delta = 1.35$		23.1%	2.8%	99	R - B - P	72% - 7% - 21%	1.16E+05	0.42%	1.21%
Case 3. $\beta = 810$		21.3%	2.7%	114	R - B - P	70% - 8% - 22%	1.20E+05	0.40%	1.05%



Table 2.3: Selected Regimes Comparison

Selected economy	$t^*$	$Z^*$ (\$ thousand)	Place of firms	Proportion of firms	Average compliance cost / turnover	Compliance cost / turnover at the threshold
Sub-Saharan Africa	6.8%	46	R - B - P	34% - 14% - 52%	2.55%	2.61%
Latin America	3.6%	92	R - B - P	57% - 11% - 32%	0.72%	1.30%
Euro Area	2.0%	150	R - B - P	76% - 6% - 18%	0.25%	0.80%

## 2.7 Conclusions

Turnover-based presumptive business income tax systems are very common in developing and transition economies, where the compliance and administrative costs associated with corporate income taxation are highly regressive. In several OECD countries, including France, Italy, and Portugal, turnover taxes are applied to sole proprietorships meeting the sales threshold. This paper is the first to provide a theoretical analysis of the optimal turnover threshold and tax rate. The analysis provides insights on the key margins for setting the turnover threshold and tax rate, in relation to the corporate income rate, the importance of revenues for the government, and the size of compliance and administrative costs. A calibrated model suggests an optimal turnover tax rate of between 2 and 3 percent, with a threshold of around \$115,000, with some variation for countries at different levels of economic development. While the threshold value resembles the rule-of-thumb often used to recommend the VAT threshold, the margins of behavior between the VAT and a turnover tax are very different. A potentially important omission from our model is the economies of scope for taxpayer compliance and tax administration that may arise from using the same threshold for the presumptive tax on turnover as the VAT threshold. The joint determination of the thresholds for VAT and presumptive income taxation is an area for future research.<sup>31</sup> The model assumes full compliance but allows for firms to restrict their output to remain below the threshold (bunching). Concerns frequently expressed, that presumptive tax regimes discourage small firms from growing, because they prefer not to be subjected to the regular corporate income tax, attest to the relevance of this form of adjustment of production. Future work can consider the additional possibility that firms remain below the threshold by concealing their actual sales (Waseem, 2018).

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<sup>31</sup>Kanbur and Keen (2014) show some of the complexities arising from the interplay of the thresholds of different instruments.

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# Appendix

## Proof of Proposition 11

1. We have

$$\begin{aligned} D_1(c) &= \pi^R - \pi^B \\ &= [p^R f(L(p^R(c))) + w(1 - L(p^R(c))) - \Gamma] - [p^P(\frac{\bar{Z}}{p}) + w(1 - \mu(\bar{Z}))] \end{aligned} \quad (2.38)$$

$$\frac{d(\pi^R - \pi^B)}{dc} = -(1 - t^c)f(L(p^R)) + \frac{\bar{Z}}{p} \quad (2.39)$$

$$\frac{d^2(\pi^R - \pi^B)}{dc^2} = (1 - t^c)^2 f' \frac{dL^*}{dp^R} > 0 \quad (2.40)$$

where  $\mu(\bar{Z})$  is the labor supply needed to generate the sales level  $\bar{Z}$ . Since  $d^2(\pi^R - \pi^B)/dc^2 > 0$ , the function is strictly convex. It can therefore have at most two real roots.

2. We have

$$\begin{aligned} D_2(c) &= \pi^A - \pi^B \\ &= p^R \frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z})) - \Gamma - [p^P \frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))] \\ &= [(t - t^c)p + t^c c] \frac{\bar{Z}}{p} - \Gamma \end{aligned} \quad (2.41)$$

$$\frac{d(\pi^A - \pi^B)}{dc} = t^c \frac{\bar{Z}}{p} > 0 \quad (\forall t^c > 0) \quad (2.42)$$

Hence,  $D_2$  is linear and its unique real root is  $c^* = \frac{(p\Gamma/\bar{Z}) + (t^c - t)p}{t^c}$ . Consequently,  $\pi^A > \pi^B$  for all  $c \in [c^*, 1]$ .

3. For strict quasi-concavity, we will establish that  $D_3(c)$  is increasing over an interval  $[0, c^*)$  and then decreasing over the remaining interval  $[c^*, 1]$ , or it is either decreasing

or increasing throughout. Suppose, first, that  $p^P(c) \geq 0$  for all firms. We have

$$\begin{aligned} D_3(c) &= \pi^R - \pi^P \\ &= [p^R f(L(p^R)) + w(1 - L(p^R)) - \Gamma] - [p^P f(L(p^P)) + w(1 - L(p^P))] \end{aligned} \quad (2.43)$$

$$\frac{d(\pi^R - \pi^P)}{dc} = -(1 - t^c)f(L(p^R)) + f(L(p^P)) \quad (2.44)$$

using (2.9), (2.11) and (2.12). If  $D_3$  attains an interior local maximum at some  $c^*$ , then it must be a solution to

$$(1 - t^c)f(L(p^R(c))) = f(L(p^P(c))) \quad (2.45)$$

Now, ignore for the moment the term  $(1 - t^c)$  in (2.45) and consider graphing the curves  $J \equiv f(L(p^R(c)))$  and  $K \equiv f(L(p^P(c)))$ .  $J(c)$  and  $K(c)$  are both continuously decreasing in  $c$ :

$$\begin{aligned} \frac{df(L(\rho(c)))}{dc} &= f' \frac{dL}{d\rho} \frac{d\rho}{dc} \\ &= -\frac{(f')^2}{\rho f''} \left( \frac{d\rho}{dc} \right) < 0 \end{aligned} \quad (2.46)$$

using  $dL/d\rho = -f'/(\rho f'')$  from the differentiation of (2.3). We also know that  $J(c) < K(c)$  if and only if  $p^R < p^P$ ; that is,  $c < \frac{(t^c - t)p}{t^c}$ , using the definitions of  $p^R$  and  $p^P$ . Similarly,  $J(c) > K(c)$  if and only if  $p^R > p^P$ . Thus,  $J(c)$  intersects  $K(c)$  from below at a unique value  $\hat{c}^*$ . (We do not require the curves to be concave or convex.)

Now, consider the effect of the term  $(1 - t^c)$  on the left-hand side of (2.45). Its effect is to rotate  $J(c)$  downward from a fixed base at  $c = p$ ; each point on the curve moves down by a fixed proportion  $1 - t^c$ . Figure 2.7 illustrates the rotation in the curve. The curves  $(1 - t^c)J(c)$  and  $K(c)$  intersect at  $c^*$ . The slope of  $(1 - t^c)J(c)$  is always greater than the slope of  $J(c)$  (i.e., “less negative”). Furthermore, we will show that, at  $c^*$ , the slope of  $J(c)$  is greater than the slope of  $K(c)$  (“less negative”), which implies that

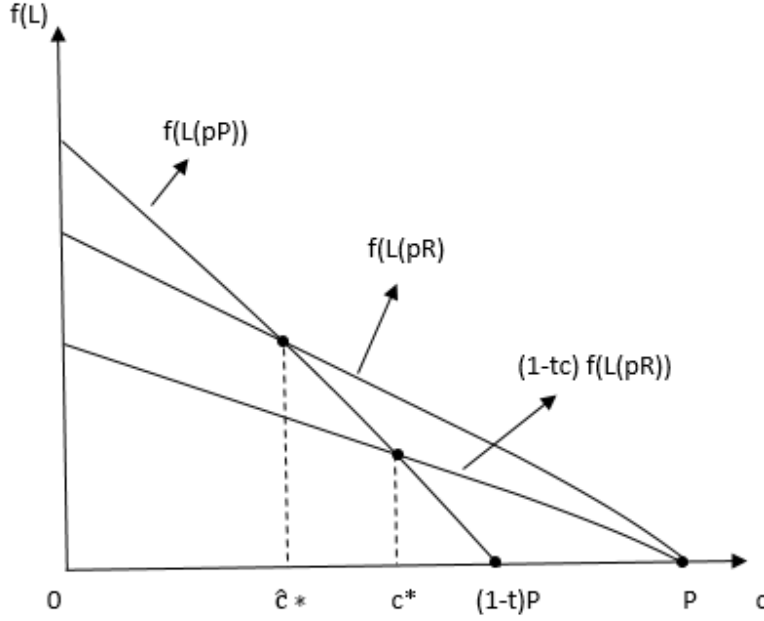


Figure 2.7: Unique maximum of  $\pi^R(c) - \pi^P(c)$

the curve  $(1 - t^c)J(c)$  must always cut the curve  $K(c)$  from below. We have  $p^R > p^P$  at every point to the right of  $\hat{c}^*$  and  $c^* > \hat{c}^*$ ; it follows that the slope of  $J(c)$  is greater (“less steep”) than the slope of  $K(c)$  at  $c^*$ , since

$$\frac{d}{d\rho} \left( \frac{df(L(\rho))}{\rho} \right) = \frac{-(f')^2}{\rho^2(f'')^3} (3(f'')^2 - f'f''') > 0 \quad (2.47)$$

using (2.46) and the assumption that  $3(f'')^2 - f'f''' > 0$ .

Now suppose that the intersection between  $(1 - t^c)J(c)$  and  $K(c)$  at  $c^*$  is not unique. Then there is another point, to the right of  $c^*$ , where the curve  $(1 - t^c)J(c)$  intersects  $K(c)$  from above. But we have just shown that this is impossible. Hence, there can only be one interior local maximum point of  $D_3(c)$ .

So far, we have ignored the possibility that there are values of  $c \in [0, 1]$  such that  $p^P \leq 0$ . At all  $c$  such that  $c \geq (1 - t)p$ , the firms in the presumptive regime would choose a corner solution with  $L^* = 0$  and  $\pi^P = w$ . Meanwhile,  $d\pi^R/dc < 0$  for all  $c < 1$ . Hence,  $D_3(c)$  is decreasing for all  $c \geq (1 - t)p$ . Therefore,  $D_3$  is decreasing over

the whole interval  $[c^*, 1]$  even when there exists a  $\tilde{c} < 1$  such that  $p^P < 0$  for all  $c > \tilde{c}$ . Finally, it is possible that  $c^*$  (defined above) occurs outside of  $c \in [0, 1]$ , in which case  $D_3$  is decreasing throughout (when  $c^* \leq 0$ ) or is increasing throughout (when  $c^* \geq 1$ ). Both cases satisfy the definition of strict quasi-concavity.

4. For strict quasi-concavity, we will establish that  $D_4(c)$  is increasing over an interval  $[0, c^{**}]$  and then decreasing over the remaining interval  $[c^{**}, 1]$ , or it is decreasing throughout. Consider, first, firms with  $p^P > 0$  (i.e.,  $c < (1 - t)p$ ). We have

$$\begin{aligned} D_4(c) &= \pi^A - \pi^P \\ &= [p^R \frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z})) - \Gamma] - [p^P f(L(p^P)) + w(1 - L(p^P))] \end{aligned} \quad (2.48)$$

$$\frac{d(\pi^A - \pi^P)}{dc} = -(1 - t^c) \frac{\bar{Z}}{p} + f(L(p^P)) \quad (2.49)$$

$$\frac{d^2(\pi^A - \pi^P)}{dc^2} = -f' \frac{dL}{dp^P} < 0 \quad (2.50)$$

Hence, for all  $p^P > 0$ ,  $D_4(c)$  is strictly concave. So far, we have ignored the possibility that there are values of  $c \in [0, 1]$  such that  $p^P \leq 0$ . At all  $c$  such that  $c \geq (1 - t)p$ , the firms in the presumptive regime would choose a corner solution with  $L^* = 0$  and  $\pi^P = w$ . Meanwhile  $d\pi^A/dc < 0$  for all  $c < 1$ . Hence,  $D_4(c)$  is decreasing for all  $c \geq (1 - t)p$ . Thus,  $D_4$  is either a decreasing function throughout  $c \in [0, 1]$  or it is increasing over some range  $[0, c^{**})$  and then decreasing over the range  $[c^{**}, 1]$ . In either case,  $D_4$  is strictly quasi-concave.



## Proof of Proposition 13

$$\begin{aligned}
\frac{dSW}{d\bar{Z}} &= [p^R(c_1)f(L(p^R(c_1))) + w(1 - L(p^R(c_1))) - \Gamma]h(c_1)\frac{dc_1}{d\bar{Z}} \\
&+ [p^P(c')\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c')\frac{dc'}{d\bar{Z}} - [p^P(c_1)\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c_1)\frac{dc_1}{d\bar{Z}} \\
&+ \int_{c_1}^{c'} [p^P\frac{1}{p} - w\mu_{\bar{Z}}]h(c)dc - [p^P(c')f(L(p^P(c'))) + w(1 - L(p^P(c')))]h(c')\frac{dc'}{d\bar{Z}} \\
&+ \delta\{[t^c(p - c_1)f(L(p^R(c_1))) - A]h(c_1)\frac{dc_1}{d\bar{Z}} + \int_{c_1}^{c'} tp\frac{1}{p}h(c)dc \\
&+ tp\frac{\bar{Z}}{p}[h(c')\frac{dc'}{d\bar{Z}} - h(c_1)\frac{dc_1}{d\bar{Z}}] - tpf(L(p^P(c'))h(c')\frac{dc'}{d\bar{Z}}\} \\
&= 0
\end{aligned} \tag{2.51}$$

Using the definitions of  $c_1$  and  $c'$  to cancel terms, the expression simplifies to

$$\begin{aligned}
\frac{dSW}{d\bar{Z}} &= \int_{c_1}^{c'} [p^P\frac{1}{p} - w\mu_{\bar{Z}}]h(c)dc + \delta \int_{c_1}^{c'} tp\frac{1}{p}h(c)dc \\
&+ \delta[t^c(p - c_1)f(L(p^R(c_1))) - A - tp\frac{\bar{Z}}{p}]h(c_1)\frac{dc_1}{d\bar{Z}} \\
&= 0
\end{aligned} \tag{2.52}$$

Rearranging the equation generates the proposition.

## Proof of Proposition 14

$$\begin{aligned}
\frac{dSW}{dt} &= [p^R(c_1)f(L(p^R(c_1))) + w(1 - L(p^R(c_1))) - \Gamma]h(c_1)\frac{dc_1}{dt} \\
&+ [p^P(c')\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c')\frac{dc'}{dt} - [p^P(c_1)\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c_1)\frac{dc_1}{dt} \\
&+ \int_{c_1}^{c'} (-p\frac{\bar{Z}}{p})h(c)dc - [p^P(c')f(L(p^P(c'))) + w(1 - L(p^P(c')))]h(c')\frac{dc'}{dt} \\
&+ \int_{c'}^1 [-pf(L(p^P))]h(c)dc \tag{2.53} \\
&+ \delta\{[t^c(p - c_1)f(L(p^R(c_1))) - A]h(c_1)\frac{dc_1}{dt} + \int_{c_1}^{c'} (p\frac{\bar{Z}}{p})h(c)dc \\
&+ tp\frac{Z}{p}[h(c')\frac{dc'}{dt} - h(c_1)\frac{dc_1}{dt}] + \int_{c'}^1 [pf(L(p^P))]h(c)dc \\
&- tpf(L(p^P(c')))]h(c')\frac{dc'}{dt} + \int_{c'}^1 [-tp^2f'\frac{dL}{dp^P}]h(c)dc\} = 0
\end{aligned}$$

Using the definitions of  $c_1$  and  $c'$  to cancel terms, the expression simplifies to

$$\begin{aligned}
\frac{dSW}{dt} &= - \int_{c_1}^{c'} p\frac{\bar{Z}}{p}h(c)dc - \int_{c'}^1 pf(L(p^P))h(c)dc \\
&+ \delta \int_{c_1}^{c'} p\frac{\bar{Z}}{p}h(c)dc + \delta \int_{c'}^1 pf(L(p^P))h(c)dc - \delta \int_{c'}^1 tp^2f'\frac{dL}{dp^P}h(c)dc \\
&+ \delta[t^c(p - c_1)f(L(p^R(c_1))) - A - tp\frac{\bar{Z}}{p}]h(c_1)\frac{dc_1}{dt} \tag{2.54} \\
&= 0
\end{aligned}$$

Rearranging the equation generates the proposition.

## First-Order Condition of Welfare for $t^c$

$$\begin{aligned}
\frac{dSW}{dt^c} &= \int_0^{c_1} [-(p-c)f(L(p^R))]h(c)dc \\
&\quad + [p^R(c_1)f(L(p^R(c_1))) + w(1 - L(p^R(c_1))) - \Gamma]h(c_1)\frac{dc_1}{dt^c} \\
&\quad + [p^P(c')\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c')\frac{dc'}{dt^c} - [p^P(c_1)\frac{\bar{Z}}{p} + w(1 - \mu(\bar{Z}))]h(c_1)\frac{dc_1}{dt^c} \\
&\quad - [p^P(c')f(L(p^P(c')))) + w(1 - L(p^P(c')))]h(c')\frac{dc'}{dt^c} \\
&\quad + \delta\left\{\int_0^{c_1} (p-c)f(L(p^R))h(c)dc + \int_0^{c_1} [-t^c(p-c)^2 f' \frac{dL}{dp^R}]h(c)dc \right. \\
&\quad \left. - tp\frac{\bar{Z}}{p}h(c_1)\frac{dc_1}{dt^c}\right\} \\
&= 0
\end{aligned} \tag{2.55}$$

Using the definitions of  $c_1$  and  $c'$  to cancel terms, the expression simplifies to

$$\begin{aligned}
\frac{dSW}{dt^c} &= (\delta - 1) \int_0^{c_1} (p-c)f(L(p^R))h(c)dc \\
&\quad - \delta \int_0^{c_1} [t^c(p-c)f' \frac{dL}{dp^R}(p-c)]h(c)dc \\
&\quad + [t^c(p-c_1)f(L(p^R(c_1))) - A - tp\frac{\bar{Z}}{p}]h(c_1)\frac{dc_1}{dt^c} \\
&= 0
\end{aligned} \tag{2.56}$$

## Chapter 3

# Corporate Tax Reform and the Labor Market in Canada

### 3.1 Introduction

The corporate income tax (CIT) is an important source of revenue for many governments. However, its distributional impacts—the incidence of the tax—is widely debated. Most of the empirical studies on the labor market effects of corporate taxation rely on variations in tax policy across jurisdictions for identification. However, potentially omitted variables relating to non-tax differences and trends between countries or states could confound the measured effects of tax policies on wages (Fuest, Peichl, and Siegloch, 2018). In contrast, this paper takes advantage of a unique tax reform in Canada, which excluded manufacturing industries from a seven-point general CIT rate reduction over the 2001 to 2004 period. Prior to 2001, manufacturing and processing industries benefited from a federal tax credit that effectively reduced the federal tax rate from the generally applicable rate of 28 percent to 21 percent. The tax reform extended the same treatment to services-producing industries and by 2004 the effective statutory rate was identical, at 21 percent, in services and manufacturing. We exploit the selective nature of the tax reform and the large size of the tax rate change to

estimate the effects of lower taxes on wages and employment using a difference-in-differences (DiD) approach.

Previous studies have elaborated on two channels for corporate tax incidence. The so-called direct effect assumes quasi-rents earned by firms in imperfectly competitive industries are bargained over between owners and workers. Under this hypothesis, a change in the corporate tax burden can affect wages directly by altering the after-tax quasi-rents available for distribution. The direct effect was introduced by Arulampalam, Devereux, and Maffini, 2012. Related papers estimating a direct effect include Felix and Hines Jr, 2009; Dwenger, Rattenhuber, and Steiner, 2011, Liu and Altshuler, 2011. The distinguishing feature of this approach is the inclusion of an explanatory variable that controls for tax-induced capital investments, such as the capital-labor ratio or the value added per worker. In contrast, the indirect effect of CIT on wages arises from tax-induced changes in capital investments in competitive industries. In a small open economy, but with limited international labor mobility, the marginal product of labor would respond to the investments and national changes in the CIT rate would thereby pass through to workers' wages in competitive labor markets. The indirect effect is associated with the seminal theoretical work of Harberger (1962), which was extended to small open economies by Harberger (1995, 2008), Mutti and Grubert (1985), Gravelle and Smetters (2006). In open economy general equilibrium models, when there is perfect capital mobility and perfect product substitution, the share of the corporate tax burden falling on workers is inversely related to the size of the country. The relative capital intensity of manufacturing industries and the internationally tradeable nature of manufactured goods would also contribute to domestic labor bearing the CIT burden in small economies. Gravelle (2014) provides a survey of studies on the direct effect of the literature on the indirect effect of CIT on tax incidence.

A study by the federal Department of Finance of Canada (Parsons, 2008) used a DiD estimation strategy to examine changes in the real capital stock among 22 service industries that saw large reductions in their tax burden in the 2001-2004 tax cut period against 21

manufacturing industries that saw little or no change. His finding, that a 10 per cent reduction in the user cost of capital arising from changes in the tax parameters resulted in a 7 per cent increase in the capital stock, provides support for using the tax reform episode to identify the indirect effect of CIT on wages. We use the individual-level monthly Canadian Labor Force Survey (LFS) to explore labor market effects of the tax reform. This microdata not only provides the key variables of interest—the hourly wage rate and weekly working hours—but also the characteristics of individuals and the industry in which they work.

We test the capital-skill complementarity hypothesis (Griliches, 1969) by examining the wage effects of the CIT reforms across educational categories. Under this hypothesis, capital is more complementary with skilled labor than with unskilled labor. Consequently, the wage rate of skilled labor should fall (rise) more compared to unskilled labor, when there is an increase (decrease) in the CIT rate. Given the existence of a gender wage gap (Blau and Kahn, 2017), we also conduct an analysis for males and female earners separately. In addition, we disaggregate the labor market effects of CIT by size of firm. As noted by McKenzie (2017), small open economy models of tax incidence presume that corporations are large enough to access international capital markets, which is less likely in the case of small corporations. Furthermore, large firms may pay differently than smaller firms to deter shirking and raise productivity (Oi and Idson, 1999).

Most of the existing empirical estimates of the wage effects of CIT are based on cross-country tax variations. These include Hassett, Mathur, et al., 2006 and 2015; Felix, 2007; Clausing, 2012. See Fuest, Peichl, and Siegloch (2018) for a broad critique of this cross-jurisdiction approach.<sup>1</sup> McKenzie and Ferede (2017) used aggregate provincial data for Canada to examine, simultaneously, the effects of provincial variations of CIT rates on capital-labor ratios, and the effects of capital-labor ratios on wages. In this way, they established the link between provincial CIT and wages via the indirect effect. However, the CIT rates are weighted averages across industries and hence abstract from tax variations

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<sup>1</sup>See also Gravelle and Hungerford (2008) and Clausing (2012) for specific criticisms of the empirical specifications of some of the papers in this stream of the literature.

between industries. Clausing (2012) had applied a similar strategy to OECD countries from 1981 to 2009. In a first-stage regressions relating the capital-to-labor ratio to the CIT, she found no impact, casting doubt on previous estimates of the indirect channel for tax incidence in small open economies. Recently, Fuest, Peichl, and Siegloch (2018) exploited the substantial tax variation in the German local business tax across municipalities to estimate DiD models of wages paid by firms in different locations, using administrative employer-employee micro data. They found that on average workers bear approximately 51 percent of the total tax burden.

Our main results are that the seven percentage point in the CIT rate in service industries increased the real hourly wage rate by 1.4% and the number of weekly working hours by 0.7%. Since the combined federal-provincial CIT rate in the service sector fell by about 20% between 2000 and 2004 (the combined CIT rate being 43.4% in 2000), the elasticity of the wage rate with respect to the CIT rate is estimated from the DiD at about  $-0.7$ . The value is very similar to the result obtained from a continuous DiD regression in which the event dummy variable is replaced by the actual CIT rate. A back-of-the-envelope calculation based on the elasticity estimates suggests that annual wages increase by 118 cents per dollar of reduced corporate tax revenue collection in Canada. Furthermore, highly-educated workers benefited the most from the reform, compared with medium-educated or low-educated workers. The wage increases resulting from the reform are 2.9%, 1.6%, and 1.1% for high-educated, medium-educated, and low-educated male workers, respectively. For females, only the high-educated group saw wage increases (5.3%) during the reform period. The only significant change in the number of working hours was for medium-educated female workers (an increase of 1.1%). Medium-sized firms are found to increase the wage rate by 1.1%, while large-sized firms increased both the wage rate and the working hours by 1.2% and 0.7%, respectively. No wage effects of CIT were found for small firms, defined as having fewer than 20 employees. The findings are robust by including a comprehensive set of individual and aggregate-level variables.

A caveat on the DiD estimates is the comment by Fuest, Peichl, and Siegloch (2018), that service sector products are generally less tradeable than manufactured goods, making services relatively more susceptible to forward-shifting of the tax burden onto their customers. As a result, the wage and employment effects of changes in the CIT rate applied to services may be expected to be smaller than in a case where the CIT rate changes for manufacturing, making our DiD estimates a lower bound for the tax incidence effects in manufacturing industries.

The rest of the paper is organized as follows. Section 2 describes the structure of the Canadian corporate tax system and the 2001-2004 federal tax reform and the labor force data. Section 3 describes the identification methods. Section 4 gives the empirical results. Section 5 provides a series of checks on the validity of the identification assumptions required for the DiD estimation strategy. Section 6 concludes.

## **3.2 Canadian CIT and description of the data**

### **3.2.1 Corporate income tax in Canada**

The CIT system in Canada consists of both federal and provincial taxes. At the federal level, there are three main elements: the basic federal CIT rate, the federal corporate surtax rate, and the federal abatement rate (Cahill et al., 2007). In addition, there is a manufacturing and processing (M&P) profit tax credit to encourage investment in these activities. The February 2000 federal budget announced a five-year, seven percentage point CIT rate reduction plan, beginning with a one-percentage point reduction in the CIT rate, effective January 1, 2001; the October 2000 federal budget update confirmed the additional six percentage point CIT reductions to be phased in by 2 percentage points per year from January 1, 2002 to January 1, 2004. These reductions were implemented as a “general tax reduction,” deducted in the same manner as used for manufacturing and processing (Cahill et al., 2007). Importantly, the general tax reduction only applied to income not already benefiting from preferential corporate tax treatment. Hence, the manufacturing, processing, and resource sectors, which



were already beneficiaries of special tax reductions, were excluded from the reform. Each province also imposes its own CIT. Provincial corporate taxes are about one-third of the income taxes on Canadian corporations. The provincial CIT rates were unchanged or changed by little during the reform period in all but two provinces.<sup>2</sup> The total effective CIT rate is the sum of the federal and provincial rates, where the latter were aggregated using capital weights from the Department of Finance.<sup>3</sup> In 2000, the federal CIT rate inclusive of the surtax was 29.12% for services and 22.12% for manufacturing, while the weighted-average provincial CIT rate was 14.2% for services and 12.8% for manufacturing. In 2004 the federal rate was 22.12% in both services and manufacturing, while the weighted-average provincial rates were 12.8% and 11.7%, respectively.

### 3.2.2 Data description

The main data source is the public-use Canadian Labor Force Survey (LFS) (January 1997 to December 2007.<sup>4</sup>) The Canadian LFS is a monthly survey including approximately 56,000 households with 100,000 individuals in total. The LFS is used by Statistics Canada to evaluate the current state of the Canadian labor market and the related results are used by governments to make important economic decisions.

The data set contains rich information at the individual level including demographic information (such as age, gender, the highest level of educational attainment, marital status, etc.) and social-economic status (such as employment status, earnings and working hours).

The employment status includes several key features, i.e., labor force status, industry of main

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<sup>2</sup>British Columbia reduced the CIT rate by 3 percentage points in 2002; New Brunswick decreased its rate by 4 percentage points between 2000 and 2003.

<sup>3</sup>For example, the combined federal-provincial CIT tax rate = (basic federal CIT rate - federal abatement rate)  $\times$  (1 + federal surtax rate) - M&P credit rate + provincial CIT. Note that corporations that pay provincial/territorial corporate income tax receive a 10-percentage-point federal abatement and the federal surtax rate is constant at 1.12% across the period covered in the study. The federal CIT statutory rate that we describe in the paper is the “basic federal rate” minus the federal abatement rate, which exists for political historical reasons. Thus, in 2000, the federal CIT rate of 28 percent is given by the basic federal rate of 38 percent, net of the 10 percent federal abatement.

<sup>4</sup>Due to the global financial crisis and a new tax reduction policy implemented in 2008, the years after 2008 are excluded.

job, class of worker and firm size. Labor force status tells whether an individual is employed, unemployed or not in the labor force. Industry of main job links each individual's main job with the North American Industry Classification System (NAICS) code, showing which sector an individual belongs to. Class of worker divides all workers into three categories: wage and salary worker, self-employed worker and unpaid family workers. For wage and salary workers, class of worker also provides information about whether a worker is in the public or private sector and firm size indicates how many employees there are in the same firm. Earnings are defined as usual hourly wages and the working hours are the usual working hours at main job.

We restrict the sample to workers aged 25-54 because it is standard in the labor economics literature to study the age group corresponding to workers' prime years of work. Only workers in the private sector are studied because wage and salary for employees in public sector are partially affected by governments' budget as Mueller (2000) discussed.<sup>5</sup> The range of years for the main results is from 1997 to 2004 because the wage data becomes available in the data set starting from 1997 and the manufacturing sector starts to experience a general downturn after 2004;<sup>6</sup> moreover, the federal government introduced a tax policy of significant accelerated depreciation for manufacturing in 2007. These various factors are likely to confound the DiD analysis. However, the years 2005 to 2007 are added to the DiD as a robustness check in subsection 5.5. The results show that restricting the post-reform period to 2001-2004 understates the incidence of the corporate tax cut on wages, making the main estimates a lower bound.

Table 3.1 reports summary statistics of wage rates, number of working hours, and other

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<sup>5</sup>The services-producing sector includes wholesale and retail trade, transportation and warehousing, professional, scientific and technical services, business, building and other support services, information, culture and recreation and other services. Accommodation and food service is excluded because it is most likely to be affected by the Law of Minimum Wage. Public or quasi-public industries, such as public administration, education services, healthcare services and utilities are excluded. Finance, insurance, real estate, rental and leasing which qualify for other special tax provisions are also excluded.

<sup>6</sup>Although Bernard (2009) argues that more than one in seven manufacturing jobs (322,000) disappeared in Canada between 2004 and 2008, Baldwin and Macdonald (2009) finds that the growth rate of productivity in manufacturing sector stayed close to constant at 1.1% since the 1960s until 2009.

Table 3.1: Summary Statistics

	All		Service		Manufacturing		Difference-in-Differences
	Pre	Post	Pre	Post	Pre	Post	
% of Female Workers	41.09%	42.18%	46.94%	47.53%	25.72%	27.19%	-0.88%
% of Married Workers	86.45%	72.77%	86.04%	71.64%	87.55%	75.96%	-2.81%
Education Group:							
% of Low-educated	41.36%	38.60%	39.47%	36.93%	46.30%	43.29%	0.47%
% of Medium-educated	45.35%	46.59%	45.91%	46.95%	43.87%	45.59%	-0.68%
% of High-educated	13.30%	14.80%	14.92%	16.12%	9.83%	11.12%	-0.09%
Age Group:							
25-34	35.41%	31.74%	37.05%	33.32%	31.09%	27.31%	0.05%
35-44	38.61%	38.16%	37.92%	37.66%	40.42%	39.55%	0.61%
45-54	25.98%	30.10%	25.03%	29.02%	28.49%	33.14%	-0.66%
Wage Rate	16.77	17.05	16.03	16.46	18.72	18.73	0.42
Working Hours	38.12	38.15	37.39	37.48	40.06	40.01	0.14
# of Observations	727,776	728,406	527,284	536,741	200,492	191,665	18284

Notes: This table reports summary statistics using data from the Canadian LFS 1997 - 2004. Samples are separated by sectors and time periods. Service sector is the treated sector and manufacturing sector is the control sector. The pre-policy period is from 1997 to 2000 and the post-policy period is from 2001 to 2004.

individual-level observable variables, separately for the service and manufacturing sectors, using the manufacturing industries which pass the placebo test for parallel trends with the service sector (see section 3).<sup>7</sup> The statistics show that workers in the manufacturing sector have higher wage rates on average than those in the service sector; the average wage rate increases by 43 cents in service sector but only by 1 cent in the manufacturing sector after the tax reform. On the other hand, the number of working hours rose by 0.07 hours in the service sector but declined by 0.05 hours in the manufacturing sector. The proportion of female workers is higher in the services than in manufacturing, while the proportion of married workers is lower. The workers have lower level of education, higher percentages of the mid-age and older-age groups in the manufacturing sector compared with those in the service sector.

<sup>7</sup>Five industries are excluded: wood product, computer & electronics, transportation equipment, textile mills & product and clothing & leather, as these industries do not satisfy the common trend assumption.

## 3.3 Identification methods

### 3.3.1 Identification methods

A multivariate regression analysis is used to isolate the effects of demographic factors. In particular, a difference-in-differences (DiD) approach is used to evaluate the causal effect of the five-year tax reduction plan. Since the tax rate reduction is applied to the service sector, the treatment group is defined as workers in private service sector. The definition of the comparison group is crucial, as it should not be affected by the tax policy and should be able to capture the counterfactual economic trends in the absence of the tax reform. Specific industries within the manufacturing sector are tested for a parallel trend with the services sector, in order to determine the best comparison group for the DiD analysis. More details regarding the comparison group will be discussed later. The DiD analysis is implemented by estimating the following regression function:

$$\ln(Y_{ijt}) = \alpha + \beta(\text{Service}_j \times \text{Post}_t) + \gamma_j + \lambda_t + X_{ijt}^T \delta + \epsilon_{ijt} \quad (3.1)$$

where  $i$  denotes individual (worker),  $j$  industry,  $t$  time, and  $Y_{ijt}$  is the outcome variable of interest (wage rate, number of working hours). The variable  $\text{Service}_j$  is a dummy variable for the industries in service sector (1 if in service sector, 0 otherwise).  $\text{Post}_t$  is a time dummy which takes on the value of 0 from 1997 to 2000, and 1 in 2001 and in the following years.<sup>8</sup>  $\gamma_j$  captures the industry fixed effect and  $\lambda_t$  controls the Year $\times$ Month time fixed effect, i.e., the monthly trend for each year will be captured.  $X_{ijt}^T$  is a matrix of individual-specific characteristics to control for any observable differences that might confound the analysis, including indicator variables for age, gender, highest educational attachment, marital status, and province that a worker lives in. The coefficient  $\beta$  is the primary interest which captures

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<sup>8</sup>The tax reduction is effective on Jan. 1st, 2001. Since there is a gap between the date of announcement and the date of implementation, the anticipatory effects of treatment will be checked in Section 5.4.

the average treatment effect of the tax cut on services. In an alternative specification, in order to estimate the average effect of a change in the CIT rate on the dependent variable, the term  $Service_j \times Post_t$  in Equation 3.1 is replaced by  $\ln(Tax_{jt})$ , where  $Tax_{jt}$  stands for the tax rate in either service or manufacturing sector in a given year. Recall that the weighted average tax rate in the service sector across all provinces is reduced from 43.63% in 1997 to 34.90% in 2004, while there is only 1.19% reduction in the manufacturing sector from 1997 to 2004. Hence, the variation over time in  $Tax_{jt}$  is primarily driven by changes in the CIT rate applicable to services. Then the regression equation is given by

$$\ln(Y_{ijt}) = \alpha + \beta \ln(Tax_{jt}) + \gamma_j + \lambda_t + X_{ijt}^T \delta + \epsilon_{ijt} \quad (3.2)$$

in which  $\beta$  captures the average effect of a 1% change in the CIT rate on the dependent variable.

In accordance with Bertrand, Duflo, and Mullainathan (2004) and Angrist and Pischke (2008), all standard errors are clustered at the level of 20 industries, 10 provinces, and 6 age groups, totalling 1200 clusters. The standard errors use the wild cluster bootstrap method proposed by Cameron, Gelbach, and Miller (2008) and are calculated to deal with the issue of a small number of clusters. As the standard errors using the two estimation methods introduced above do not alter the main findings of the paper, only the standard errors of the former method are reported in the paper.

### 3.3.2 Identification assumptions

The identification assumptions of DiD are: (1) the difference in the outcome variables between the service and manufacturing sectors is constant over time in the absence of the tax reduction policy; (2) the stable unit treatment value assumption (SUTVA) holds;<sup>9</sup> (3) there are parallel trends in the outcome variables of interest among industries in the manufacturing

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<sup>9</sup>There is no spillover effect, indicating that the tax reduction policy has no effect on the control group, i.e., the manufacturing sector.

sector;<sup>10</sup> and (4) there is no anticipatory effect.

Assumption (1) is the common trend assumption, which requires that the service and manufacturing sectors have parallel trends in the outcome variables in the absence of the tax reduction policy. According to Bernard (2009), some industries within the manufacturing sector, including textile mills & its related products, clothing, woods products, and motor vehicle products were hit heavily by extraneous factors, rather than the tax reform. LaLonde (1986) argues that the first step in a non-experimental evaluation is to find a comparison group which is truly comparable with the treatment group. To be more precise, the treatment and comparison groups should share the same trend in the absence of the policy. In order to test the common trend, each industry within the manufacturing sector is checked by a placebo test, which uses each year before the year 2001 as a “fake” treatment year in Equation (3.1) to identify a closely comparable group for the service sector. Using the information about each individual workers’ industry and wage rate, Table 3.2 shows the results of the placebo test with the null hypothesis of  $\beta = 0$ . For any  $\beta$  significantly different from zero, the common trend assumption is violated. There are 13 out of 18 industries within the manufacturing sector that proved to be in parallel with the service sector, which are non-metallic mineral, primary metal mineral, fabricated metal, machinery manufacture, electronic equipment & appliance, furniture & related, food & beverage & tobacco product, paper manufacturing, printing & related, petro & coal products, chemical manufacturing, plastics & rubber, and miscellaneous manufacturing. The results from Table 3.2 match the fact that some manufacturing industries went into downturn before the tax reform, as indicated by Bernard (2009).

Assumption (2) states that the policy should not directly affect the labor market outcome outside of the service sector. For example, if the wage rate in the service sector increases due to the tax reduction, some workers from the manufacturing sector may try to move into the service sector, which may lead to an increase in the manufacturing wage rate. This

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<sup>10</sup>There is no change in outcome variables among industries within the manufacturing sector across years.

Table 3.2: Placebo Tests between the Service Sector and Manufacturing Sector

Treatment group	Control group	Fake treatment year		
		1998	1999	2000
Service Sector	<b>Manufacturing Sector</b>			
	A. Wood Product	0.491***	0.418***	0.263
	B. Non-Metallic Mineral	-0.384	-0.0445	-0.327
	C. Primary Metal Manufact.	-0.0778	0.0799	-0.130
	D. Fabricated Metal	0.303*	0.302*	-0.0684
	E. Machinery Manufacture	-0.122	0.0720	-0.138
	F. Computer & Electronics	-0.552	-0.667**	-0.333
	G. Elec. Equip & Appliance	-0.380	0.545	0.513
	H. Transport Equipment	0.173	0.369***	0.335***
	I. Furniture and Related	-0.0587	-0.00267	-0.241
	J. Food & Bev. & Tobacco Prod.	0.274*	0.230*	0.0712
	K. Textile Mills & Product	0.668**	0.741***	0.709**
	L. Clothing & Leather	0.457**	0.394**	0.393*
	M. Paper Manufacturing	0.218	0.236	0.0547
	N. Printing and Related	0.275	0.213	-0.0711
	O. Petro. & Coal Products	0.0224	0.175	-0.286
	P. Chemical Manufacturing	0.0196	-0.383	-0.0194
	Q. Plastics and Rubber	0.0207	-0.188	0.414*
	R. Misc Manufacturing	0.424	0.212	-0.263

Notes: A placebo test is conducted to examine whether the common trend assumption is satisfied by comparing each industry within manufacturing sector with the whole service sector. The null hypothesis of the placebo test is  $\beta = 0$ , as described in the content. If the null hypothesis is not rejected, the common trend assumption before the policy changes is valid. The industries covered by green are the ones having parallel trends with the service sector before the policy. The outcome variable used is the wage rate per hour. Significance levels: \*\*\* = 1%, \*\* = 5%, \* = 10%. The null hypothesis is rejected if p-value < 5%.

assumption will be discussed in section 3.5.1. Assumption (3) is tested by another type of placebo test among the above 13 potential control groups which have the common trend with the service sector. The test is conducted by using each industry within the manufacturing sector as a pseudo-treatment group to compare with the remaining manufacturing industries in the pool across all sample years. The detailed discussion of this test is in section 3.5.3. Assumption (4) rules out the anticipatory effect. Since the tax reduction policy is first announced on Feb. 28th, 2000, there may exist a case that some firms in the service sector decide to expand production and increase the wage rate for hiring more labor during the period of announcement, then the estimates of the policy becomes the lower bound of the

actual effect. The tests of the anticipatory effect is discussed in section 3.5.4.

## 3.4 Results

### 3.4.1 The aggregate effects of the tax reduction

Table 3.3 presents the regression results for the aggregate effects of the tax-reduction policy using equations (3.1) and (3.2). The outcome variables are the hourly wage rate and weekly working hours (both in logarithm). Before the tax policy is implemented, the average wage rate is \$16.03 and the average working hours is 37.39 in the service sector. The variable *Tax* in Table 3.3 refers to the log value of the tax rate. The estimation is a simple fixed-effects estimation with only two periods and two groups. Columns (1) and (3) include only industry and month $\times$ year fixed effects, and Columns (2) and (4) also include the dummies for gender, age, education level, marital status, and provinces.

Panel A gives the results of the wage regressions. The estimates in Column (1) and (2) suggest that the tax cut increased the wage rate in the service sector by 0.9%-1.4%. The elasticity of the wage rate with respect to the CIT rate in (4), which has the full set of controls, is negative with a magnitude of 0.077, which means that a 1 percent decrease in the CIT rate for the service sector is associated with a 0.077 percent increase in the wage rate in that sector. These results are statistically significant at the one percent level. As the weighted average tax rate in service sector was reduced from 43.63% to 34.90%, accounting for a 20-percent decrease, using the elasticity of the CIT rate on wage rate of  $-0.077$ , the wage rate should increase by 1.54%, which is close to the estimated coefficient of Service $\times$ Post in Column (2).

Panel B reveals the positive effect of the policy on the intensive margin of the employment. The estimates in Columns (1) and (2) indicate that the average working hours in service sector is lifted up by 0.7%. The estimates in Columns (3) and (4) imply that the elasticity of the CIT rate on the number of working hours is between  $-0.044$  and  $-0.055$  and are



Table 3.3: The Aggregate Effect: Wage Rate and Working Hours

<b>Difference-in-Differences Analysis</b>				
	(1)	(2)	(3)	(4)
<b>A. Dependent Variable: ln(Real Wage Rate)</b>				
Serv $\times$ Post	0.009** (0.004)	0.014*** (0.004)		
Serv $\times$ Tax			-0.038 (0.031)	-0.077*** (0.027)
Observations	1,456,182	1,456,182	1,456,182	1,456,182
Adjusted $R^2$	0.168	0.354	0.168	0.354
<b>B. Dependent Variable: ln(Weekly Working Hours)</b>				
Serv $\times$ Post	0.007*** (0.002)	0.007*** (0.002)		
Serv $\times$ Tax			-0.055*** (0.014)	-0.044*** (0.014)
Observations	1,455,198	1,455,198	1,455,198	1,455,198
Adjusted $R^2$	0.048	0.122	0.048	0.122

Notes: All specifications includes the industry and the month $\times$ year fixed effects. In column (2) and (4), the dummies for gender, age, education level, marital status and provinces are also included in the specifications. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

statistically significant at the one percent level.

In summary, the tax-reduction policy aiming at the service sector increased both the wage rate and the number of working hours, relative to the control group of manufacturing industries, suggesting that the burden of the corporate tax does fall on labor in Canada, which is eased as the the CIT rates are reduced. The findings on the elasticity of CIT rate on wage rate are supported by the existing literature which studied the indirect transmission mechanism with the estimated range of the elasticity from 0 to 0.5 percent.<sup>11</sup> On the other

<sup>11</sup>Clausing (2012) only finds a significant positive relationship between capital labor ratio and wage rate, but insignificant relationship between CIT rates and capital labor ratio. However, Hassett and Mathur (2015) expand their study by incorporating the spatial effects and find the elasticity of CIT rate on wage rate is 0.5. Fuest et al. (2018) finds that labor bears about half of the corporate tax incidence.

side, there are few studies which have looked at the effect of CIT on working hours, among which Mertens and Ravn (2013) finds no impact of a CIT cut on working hours. One possible source of the increase in working hours is that some part-time jobs switch to full-time jobs, which is supported by our results shown later.

To evaluate the labor share of the corporate tax burden, we use year 2002 as an example calculation. In 2002, the total federal and provincial tax revenue from the private service sector was \$11.07 billion.<sup>12</sup> Thus, a 1 percent reduced in the CIT rate would be expected to have a \$110.7 million less tax collection from the private service sector (in the absence of behavioral changes). The estimated elasticities imply that such a decline in corporate tax collections would have lifted the average hourly wage in services by 0.077 percent. Using the average wage rate in services, which was \$16.48 in the same year, the annual wage would have risen by about \$24.73 per worker.<sup>13</sup> Aggregating over the 5.26 millions workers in the private services sector,<sup>14</sup> this adds up to an increase of about \$130.08 millions in wages. That implies that the annual wage increases by 118 cents for every dollar less collected from the corporate tax.

### 3.4.2 The heterogeneous effects of the tax reduction

The heterogeneous effects of the policy are investigated across gender and education levels in this subsection. This enables us to uncover how the corporate tax burden is distributed among different types of labor. Table 3.4 presents the empirical results using equation (3.1). The models are estimated by including industries and month×year fixed effect and dummies for age, marital status, and province. Genders and education levels are separated by columns and rows, respectively. High-educated workers are those who achieve bachelor’s degree or above, medium educated are those who have post some secondary degree or diploma and

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<sup>12</sup>This figure is the aggregated corporate income taxes paid by the service industries and is obtained using the Financial and Taxation Statistics for Enterprises, 2002.

<sup>13</sup>Given the average wage rate of \$16.48 and the average working hours of 37.48 in 2002, the amount of the increase in the annual income per worker equals to  $16.48 \times 0.00077 \times 37.48 \times 52$ , which is \$27.43.

<sup>14</sup>The figure is the sum across private service industries. The source of the data is Survey of Employment, Payrolls and Hours, 2002.

low educated are those with 13-year education or below. Panel A uses the interaction term of  $Serv \times Post_t$  to look at the overall effect of the tax policy and Panel B investigates the elasticity of CIT rate on the outcome variables with the term  $\ln(Tax)$ .

While estimates from Table 3.4 are a bit noisier on gender/education level due to the smaller size of observations, some important points emerge from each sub-sample estimates. First, the coefficients for working hours in Panels A and B become insignificant or marginally significant, implying that the effect of the policy has a limited impact on the number of working hours across gender or education level. Second, the treatment effect on the wage rate is statistically significant across education levels for males in Panel A but only significant for high-educated female.

Third, the results in Table 3.4 support the capital-skill complementarity hypothesis. In fact, the tax reduction is estimated to increase the wage rate of high-educated males by 2.9% and medium-educated males by 1.6%, which in dollar values are 69.2 cents and 29.4 cents, respectively.<sup>15</sup> For females, only the high-educated workers benefit from the policy significantly, as there is a larger increase in the wage rate which is 5.3 percents and accounts for 97.5 cents, compared to the high-educated males. The finding that the CIT incidence is relatively largest for workers with high levels of education is opposite to the results in Fuest et al. (2018).

In summary, the increase in the wage rate is mainly from the males across all education levels and the high-educated females. Within the group of the medium-educated and low-educated workers, the gender premium between males and females would increase as a result of a reduction of the CIT rate.

### 3.4.3 The annual effects of the tax reduction

This subsection examines the annual trend between the service and manufacturing sector before and after the tax reduction policy. Since the tax reform was implemented in four

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<sup>15</sup> Average wage rates in the service sector for high-educated, medium-educated and low-educated males are \$23.85, \$18.36 and \$16.06.

Table 3.4: The Heterogeneous Effects: Wage Rate and Working Hours

<b>Difference-in-Differences Analysis</b>				
	(1)	(2)	(3)	(4)
Dependent Variable	<b>ln(Real Wage Rate)</b>		<b>ln(Weekly Working Hours)</b>	
Gender	<b>M</b>	<b>F</b>	<b>M</b>	<b>F</b>
<b>Panel A</b>				
	<b>High - Educated</b>			
Serv $\times$ Post	0.029** (0.012)	0.053*** (0.018)	-0.004 (0.004)	0.010 (0.011)
Adjusted $R^2$	0.198	0.139	0.027	0.054
	<b>Medium - Educated</b>			
Serv $\times$ Post	0.016*** (0.006)	-0.001 (0.008)	0.003 (0.002)	0.011* (0.006)
Adjusted $R^2$	0.234	0.221	0.029	0.045
	<b>Low - Educated</b>			
Serv $\times$ Post	0.011* (0.006)	-0.007 (0.008)	0.001 (0.003)	0.006 (0.006)
Adjusted $R^2$	0.208	0.203	0.045	0.053
<b>Panel B</b>				
	<b>High - Educated</b>			
Serv $\times$ Tax	-0.143 (0.088)	-0.341*** (0.123)	0.042 (0.027)	-0.068 (0.080)
Adjusted $R^2$	0.198	0.138	0.027	0.054
	<b>Medium - Educated</b>			
Serv $\times$ Tax	-0.081** (0.039)	0.027 (0.061)	-0.013 (0.016)	-0.070* (0.039)
Adjusted $R^2$	0.234	0.221	0.029	0.045
	<b>Low - Educated</b>			
Serv $\times$ Tax	-0.065 (0.043)	0.038 (0.057)	-0.006 (0.020)	-0.072 (0.044)
Adjusted $R^2$	0.208	0.203	0.045	0.053

Notes: All specifications includes the industry and the month $\times$ year fixed effects and the dummies for gender, age, education level, marital status and provinces. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

years starting from 2001 and the level of tax rate reduction is different across years, the study of the annual effect can provide more information going beyond the study of the aggregate effect, including whether there was a lead or lagged effect of the policy. It also

serves as an internal validity check for the common trend assumption: before the policy is implemented, the outcome variables in the service sector should have a similar trend as in the manufacturing sector. By replacing  $Service_j \times Post_t$  from Equation (3.1) with a full set of treatment  $\times$  year interaction terms, the following equation is used to explore the dynamic impact of the reform:

$$Y_{ijt} = \alpha + \sum_{t=1997}^{2004} \beta_t (Service_j \times d_t) + \gamma_j + \lambda_t + X_{ijt}^T \delta + \epsilon_{ijt}, \quad t \neq 2000, \quad (3.3)$$

where  $d_t$  is a year dummy, equal to 1 if a respondent is observed in year  $t$  and 0 otherwise. Using Equation (3.3), the coefficient  $\beta_t$  of the interaction term  $Service_j \times d_t$  captures the annual effect, which is the difference of the outcome variable between the service and manufacturing sector in year  $t$ , relative to the reference year 2000. The industries and  $year \times month$  fixed effects and the dummies for gender, age and provinces are controlled for in all regressions and the dummy of education levels are controlled for in the general regression (all workers).

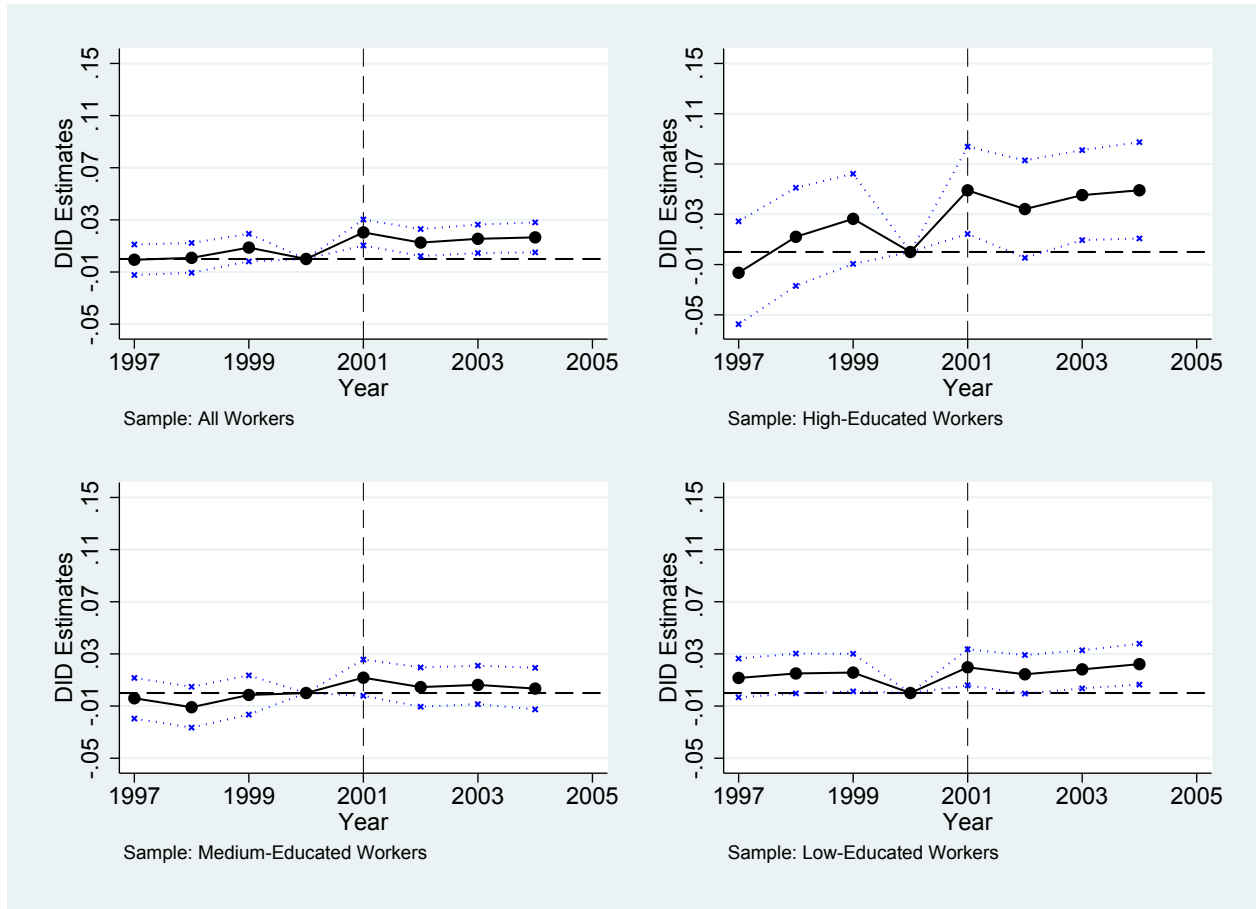
The leads of Equation (3.3) are the estimated  $\beta_t$ s with year  $t$  prior to the reference year 2000. If the common trend assumption is satisfied, one should expect that these  $\beta_t$ s are not significantly different from zero. The  $\beta_t$ s with year  $t$  greater than or equal to 2000 are the lags of Equation (3.3). As the results shown in the previous subsections indicate that the CIT reduction increases the wage rate, the annual effect is expected to rise in each additional post-reform year, due to two reasons: first, CIT affects the wage rate through capital investment, which could take time to be respond to the tax rate changes; second, the rate of the reduction becomes 2 percentage points following the 1 percentage point in 2001, and the larger is the tax reduction, the larger expected impact.

Figures 3.1 and 3.2 plot the estimated coefficients of the interaction term  $Service \times d_t$  over the period from 1997 to 2004 for the wage rate and the number of working hours, respectively. Each dot represents the coefficient of the interaction term and the blue dashed lines give the 95 percent confidence interval. The  $\beta$  in the year 2000, which is one year before the first year of reform, is normalized to 0 as the base year used to compare with the other

years, and the vertical black dashed line represents the first year that the tax reduction is implemented.

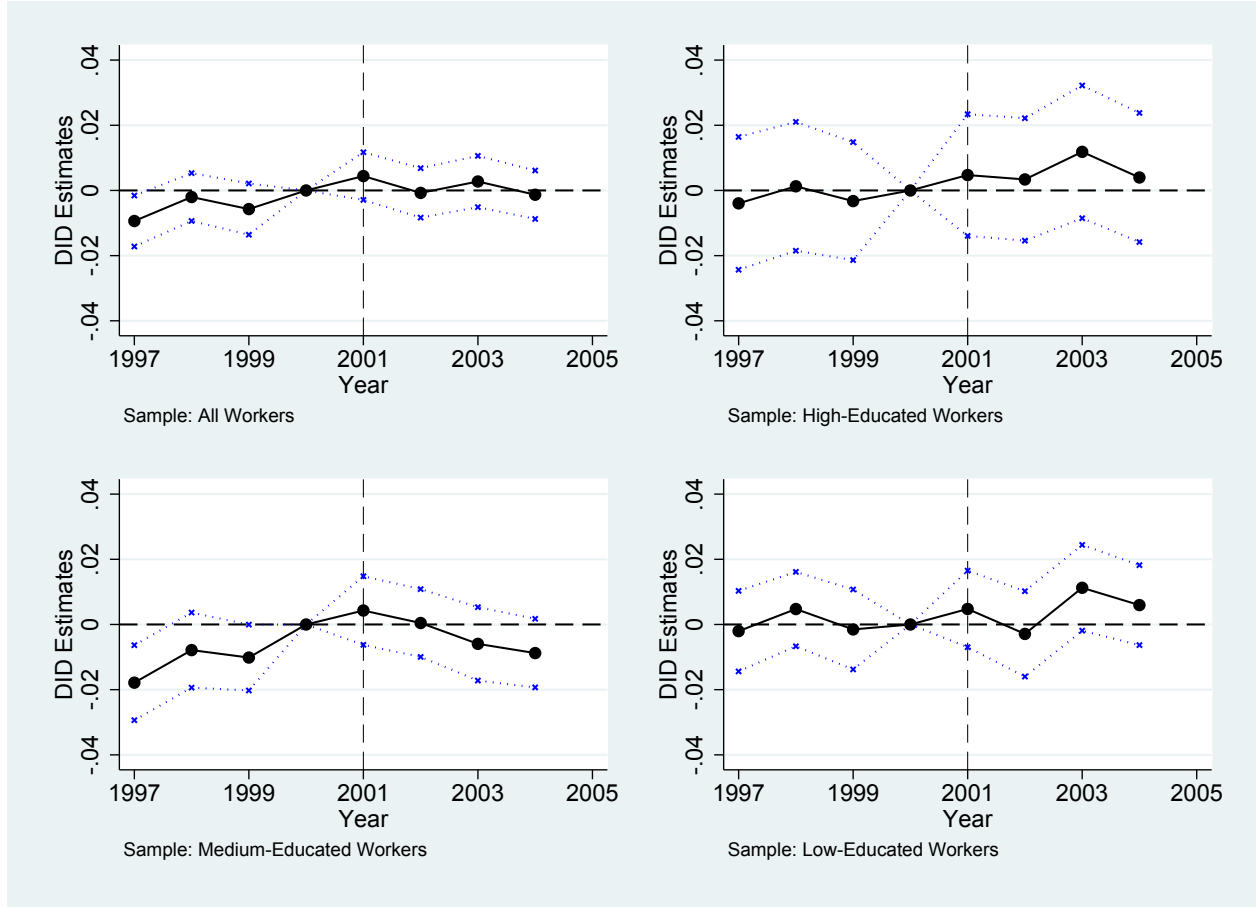
Looking at the dots before the year 2000 from Figures 3.1 and 3.2 using the whole sample, all of them are not significantly different from zero. These sub-figures on the upper left of each figure indicate that the outcome variables are parallel between the service and the manufacturing sector before the policy is implemented, which provides strong support to the satisfaction of the common trend assumption required for the validity of the difference-in-differences method.

Figure 3.1: The Annual Effects on the Wage Rate



Notes: This table presents estimated coefficients from a linear model for wage rate. The dependent variable is log transformed and CPI adjusted. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines. Service sector is the treatment group and selected industries within manufacturing sector is the control group.

Figure 3.2: The Annual Effects on the Number of Working Hours



Notes: This table presents estimated coefficients from a linear model for number of working hours. The dependent variable is log transformed. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines. Service sector is the treatment group and selected industries within manufacturing sector is the control group.

The impact of the tax reduction on the wage rate increases in dollar values gradually, as shown in Figure 3.1. Although the growth rate of the wage rate across all samples retreats a bit between the year 2001 and 2002, it is still positive. There is a significant and consecutive rise in the wage rate for the high-educated from the year 2003 to 2004, implying that the wage rate of the high-educated workers responded most strongly to the tax cut toward the end of the reform period. The medium-educated workers experience non-significant increase in the wage rate across most of the sample years. However, the low-educated workers experienced

a modest but consistent gain in each year of the reform period.

The number of working hours does not change significantly following the implementation of the tax reduction, as shown in Figure 3.2. The results indicate that the tax reduction has limited impact on the intensive margin.

In summary, looking at the impact on either the wage rate or the number of working hours, the results suggest that the common trend assumption is satisfied. Medium-educated workers experience no increase in either the wage rate or number of working hours, high-educated and low-educated workers only benefit from an increase in the wage rate, but not in the working hours.

### **3.4.4 Who offers more: small-, medium- or large-sized firms?**

A CIT reduction in the tax burdens of firms may have different effects across firm sizes, perhaps due to financial constraints on smaller firms and different exposures to the labor market. This subsection investigates the wage and working hours effects of tax cuts on different sizes of firms.

Table 3.5 presents both the wage and working-hour effect on workers from firms of different sizes using equation (3.1). Samples are restricted to workers in small-, medium-, and large-sized firms. Small-sized firms are defined as those with less than 20 employees, medium-sized firms are defined as those with 21-500 workers and large-sized firms are defined as those with more than 500 employees.

The results suggest that the wage and working-hour effects on workers for small-sized firms are negligible with insignificant estimates. One possible reason for this result is that Canadian controlled private corporations with active incomes below \$200,000 already benefited from a tax deduction prior to the 2001 tax reform and hence were precluded from a further reduction on this portion of their income. Another possible explanation is that smaller firms are less capital intensive than larger firms and hence are less affected by the CIT rate. The estimates on medium-sized firms indicate that the average wage in the service



sector increases by 1.1% but the average weekly working hours are not significantly changed. The estimates on large-sized firms suggest that the wage and working-hour effects are statistically significant, as there are 1.2% and 0.6% increases in the average wage rate and working hours, respectively. The last column combines the medium- and large-sized firms. It shows that the wage rate increases by 1%, while working hours is not significantly affected.

Table 3.5: Different Size of Firms

Difference - in - Difference Analysis				
	Small Firms	Medium Firms	Large Firms	Medium & Large Firms
<b>Dependent Variable: ln(Real Wage Rate)</b>				
Serv $\times$ Post	-0.002 (0.008)	0.011** (0.005)	0.012** (0.005)	0.010** (0.004)
Observations	303,651	433,260	540,121	973,381
Adjusted $R^2$	0.292	0.325	0.407	0.363
<b>Dependent Variable: ln(Weekly Working Hours)</b>				
Serv $\times$ Post	0.003 (0.007)	0.000 (0.003)	0.006** (0.003)	0.002 (0.002)
Observations	303,373	432,978	539,856	972,834
Adjusted $R^2$	0.12	0.099	0.15	0.121

Notes: All specifications includes the industry, the year and the months fixed effect, the dummies for gender, age, marital status, provinces. Samples are restricted to the workers who are employed in the private sector. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

## 3.5 Discussions

### 3.5.1 Assessing the stable unit treatment value assumption

To make sure that the stable unit treatment value assumption is satisfied, i.e., there is no significant change in the manufacturing sector as the control group after the tax policy is implemented on the service sector as the treated group, the policy effects on other indicators are checked as follows:

First, the tax reduction policy does not cause important changes in the labor market structure for either the service or the manufacturing sector. The employment ratio and the average number of education levels are the two indicators which are used as proxies to check whether the CIT reform induced a labor movement from the service sector to the

manufacturing sector.<sup>16</sup> The employment ratio for the service sector is always between 34.1% and 35.1%, and for the selected manufacturing sector it is around 8.1% - 8.8%. The stable employment share in both sectors suggest that there is no significant change in the extensive margin due to the tax reduction. On the other hand, the average number of education levels, which captures the level of workers' skill, increases from 4.08 to 4.26 in the service sector and from 3.7 to 3.97 in the manufacturing sector. Furthermore, the summary results for the percentage of workers in the different education groups in Table 3.1 show that changes in the proportion in each educational category changes in the same direction for manufacturing and services between the pre- and post-reform periods. These structural parallels between the two sectors suggest that there is no obvious transformation in the skill distribution in the economy in 2001-2004 as a result of the tax reform.

Second, a more precise method is used to check whether the policy reform changes the demographic variables by running a difference-in-differences regression on the demographic variable as the outcome variable. Table 3.6 suggests that the tax cut had no effects on the composition of gender, age and education level, the results provide extra evidence that there is no change in the characteristics of the workers which are relevant to the labor market outcomes.<sup>17</sup>

In summary, little evidence is found that the tax policy changes the structure of the labor market. Both the employment ratio and the education level are stable between the service and manufacturing sector. The results from using the DiD approach on the demographic variables such as gender, age and education level provide extra evidence that the structure of the labor market has not changed as a result of the tax reduction in the service sector. However, if it is the case that manufacturing wages are impacted by the tax cut in services, then the main results should be interpreted as a lower bound of the CIT incidence on wages.

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<sup>16</sup>The education levels are divided into 7 categories: 1. 0-8 years; 2. some secondary; 3. Grade 11-13, graduate; 4. some post secondary; 5. post secondary certificate or diploma; 6. bachelor's degree; 7. postgraduates.

<sup>17</sup>Although the policy effect on marital status is significant, it is less important for the validity of the DiD, compared to gender, age or education level.

Table 3.6: Selection on observations

**Difference-in-Differences Analysis**

Dependent Variable	Female	Marriage	Age	Education
Serv $\times$ Post	-0.003 (0.004)	-0.029*** (0.008)	-0.013 (0.011)	-0.008 (0.007)
Observations	1,456,182	1,456,182	1,456,182	1,456,182
Adjusted $R^2$	0.096	0.056	0.016	0.089

Notes: All specifications includes the industry and the month $\times$ year fixed effects. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

**3.5.2 Assessing the exclusion restriction assumption**

One concern in the DiD approach is that the change in the wage rate or the number of working hours may occur not only because of the tax reduction, but also due to other factors. For example, it could be that the service sector became more unionized after 2001. For the indirect transmission mechanism to be valid, the tax reduction must affect the wage rate or the number of working hours only through their effects on capital investment. The validity of the exclusion restriction assumption is addressed in the following two ways.

First, a direct way to rule out the effect of greater unionization in services (or less unionization in manufacturing) over the sample period is to include a variable in the regression that captures the rate of unionization. The LFS data provides the information on whether a worker is a union member or covered by some collective agreement; in each case we treat the worker as a union member. Unionization may be a key factor in determining the bargaining power of workers (Felix and Hines Jr, 2009). Thus, controlling for unionization is also a device for controlling for the direct effect of the corporate tax burden on wages. Columns (1) and (4) in Table 3.7 show that the estimated results of the tax reduction on the wage rate and the number of working hours are significant and robust after controlling for unionization, respectively. The magnitudes and signs of the coefficients of Serv  $\times$  Post

and Serv x Tax are very similar to the values reported in Columns (1) and (4) in Table 3, suggesting that the indirect effect is the principal channel for the tax incidence in Canada, and perhaps for small open economies more generally.

Second, whether the tax reduction decreases the ratio of the number of part-time jobs over the number of employment is another potential policy concern. As a business in the service sector receives more investment as its tax burden is reduced, some of the part-time positions in this business are likely to become the full-time positions. As most of the part-time jobs are low-paid and vulnerable, the wage rate and the number of working hours are likely affected by the ratio of part-time employment in the overall employment (Aaronson and French, 2004). The results of the CIT cut on the wage rate in Columns (2) and (5) of Table 3.7 show that the dummy variable indicating a part-time job has a limited impact on changing the parameter of interest and its significance level. However, the results for the number of working hours become insignificant after controlling for the dummy variable of the part-time job, which indicates that the change in the number of working hours appears to be mainly through the channel of switching from the part-time into full-time positions.

In summary, the exclusion restriction assumption is satisfied for both the wage rate and the number of working hours. The results are robust and significant when controlling for the effect of unionization. The impact on the wage rate is also robust and significant when including a dummy variable for part-time jobs. The insignificance of the tax reform on hours of work after controlling for part-time jobs suggests that the channel through which the tax reduction increases the number of working hours is through part-time work.

### **3.5.3 Assessing the common trend assumption among the comparison industries**

Identification of causal effects requires common trends between the treatment and control groups in the pre-treatment period, in order to render credible the assumption that the outcome variables in the treatment group would have been parallel with the same outcome

Table 3.7: Checks on the Exclusion Restriction Assumption

<b>Difference-in-Difference Analysis</b>						
	(1)	(2)	(3)	(4)	(5)	(6)
<b>A. Dependent Variable: ln(Real Wage Rate)</b>						
Serv $\times$ Post	0.013*** (0.004)	0.012*** (0.004)	0.012*** (0.004)			
Serv $\times$ Tax				-0.073*** (0.026)	-0.066** (0.026)	-0.062** (0.026)
Observations	1,456,182	1,456,182	1,456,182	1,456,182	1,456,182	1,456,182
Adjusted $R^2$	0.363	0.373	0.382	0.363	0.373	0.382
<b>B. Dependent Variable: ln(Weekly Working Hours)</b>						
Serv $\times$ Post	0.007*** (0.002)	0.000 (0.001)	0.000 (0.001)			
Serv $\times$ Tax				-0.045*** (0.014)	-0.002 (0.008)	-0.003 (0.008)
Observations	1,455,198	1,455,198	1,455,198	1,455,198	1,455,198	1,455,198
Adjusted $R^2$	0.122	0.641	0.641	0.122	0.641	0.641

Notes: All specifications includes the industry, the month $\times$ year fixed effects, the dummies for gender, age, education level, marital status and provinces. In column (1) and (4), the dummy for unionization is included in the specifications. In column (2) and (5), the dummy for part-time jobs is included. Column (3) and (6) include all the variables. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

variables in the control group during the post-treatment period in the counter-factual case that the tax reduction had not been implemented. However, it is also necessary to check that the common trend assumption is satisfied using each industry within the manufacturing sector as a pseudo-treatment industry, compared with the rest within the manufacturing sector. This is needed to support the assumption that there are no significant changes in the outcome variables among the control industries. The wage and working-hour effect are examined by using Equation (3.3) separately and the analysis is repeated for each of the 13 industries used in the control group in the empirical analysis.

Figures A1-A3 plot the estimated results of wage rates from Equation (3.3). These figures suggest that most of the industries within the manufacturing sectors share the same

trends with the rest of the manufacturing industries, except for machinery manufacture and electronic equipment & appliance, which experienced some shocks after and before the year 2001, respectively. To carefully address this issue, Equation (3.1) is re-estimated by excluding the industries of machinery manufacture and electronic equipment & appliance. Table A.1 shows that the results are robust after restricting the sample.

Figures A4-A6 illustrate the estimated results of working hours from Equation (3.3). The trends of the number of working hours are parallel for most of the industries within the manufacturing sector, except furniture & related and petro & coal products which have some slight changes after the year 2001. Similarly, Equation (3.1) is estimated by excluding these two industries. Compared with the results in Table 3.3, the results shown in Table A.2 are also robust.

#### **3.5.4 Assessing the anticipatory effect**

The existence of the anticipatory effect can make the results biased. Since there is a 10-month gap between the date of the announcement and the implementation of the tax reduction, whether companies in the service sector responded to the policy before it took effect should be checked, as the previous analysis based on the assumption of non-anticipatory effect.

The policy was initially announced on February 28, 2000 and the first tax cut was implemented on January 1, 2001. Therefore, the pre-announcement period can be defined as January and February, and the post-announcement period is defined as the months between March and December in 2000. Therefore, the first difference captures the difference in the outcome variables between the pre- and post-announcement periods. The second difference takes seasonality into account by using the observations in 1998 and 1999 to control for it. The last difference is used to control for the permanent heterogeneity across sectors using

dummy variables for the sector. The estimation function can be written as follows:

$$\begin{aligned}
Y_{ijt} = & \alpha + \beta_1 Mar-Dec_t + \beta_2 2000_t + \beta_3 Serv_j \\
& + \beta_4 Serv_j \times Mar-Dec_t + \beta_5 2000_t \times Mar-Dec_t + \beta_6 Serv_j \times 2000_t \\
& + \beta_7 Serv_j \times 2000_t \times Mar-Dec_t + X_{ijt}^T \delta + \epsilon_{ijt}
\end{aligned} \tag{3.4}$$

in which the dependent variable is wage rate or working hours.  $Mar-Dec_t$  equals 1 if the observations are between March and December, and 0 otherwise.  $2000_t$  equals 1 if the observations are in 2000, and 0 in the other sample years.  $Serv_j$  equals 1 if the observations are in the service sector, and 0 if the observations are in the selected industries in the manufacturing sector. Age, gender, marital status, province, and the month-year fixed effects are controlled for.

Table 3.8 and 3.9 show the estimation results of Equation (3.4) for the wage rate and working hours, respectively. The columns with odd (even) numbers at the top of the table report the results using the sample of male (females). Also, education is divided into three levels: high-educated, medium-educated and low-educated, each of which is reported in the column of (1) and (2), (3) and (4), and (5) and (6), respectively.

$\beta_7$  is the main parameter which captures the announcement effect on the outcome variables of the service sector in 2000. As shown,  $\beta_7$  is statistically insignificantly different from zero across all sample groups, indicating that the wage rate or the working hours in the service sector did not respond to the announcement of the tax reduction policy.  $\beta_5$  is another parameter with  $Mar-Dec$  in the interaction term, which captures the overall announcement effect including the manufacturing sector in 2000. Again all  $\beta_5$ s regardless of gender or educational level are not significantly different from zero, which provides extra evidence of non-anticipatory effects, such that neither the service nor the manufacturing sectors were affected by the announcement of the tax reduction policy. Moreover, Appendix B repeats the same procedure of obtaining the main results but excluding the announcement year 2000 to show that the main results are robust. The results without year 2000 follow exactly the

Table 3.8: Checks on the Anticipatory Effect of the Wage Rate

	(1)	(2)	(3)	(4)	(5)	(6)
Education	<b>High-Educated</b>		<b>Medium-Educated</b>		<b>Low-Educated</b>	
Gender	<b>M</b>	<b>F</b>	<b>M</b>	<b>F</b>	<b>M</b>	<b>F</b>
Serv $\times$ 2000 $\times$ Mar-Dec	0.037 (0.033)	-0.074 (0.052)	-0.006 (0.013)	0.003 (0.024)	-0.023 (0.014)	-0.014 (0.018)
Serv $\times$ 2000	-0.037 (0.031)	-0.001 (0.051)	0.024* (0.013)	-0.014 (0.022)	0.017 (0.014)	-0.013 (0.017)
Serv $\times$ Mar-Dec	0.022 (0.018)	0.008 (0.028)	0.001 (0.007)	-0.011 (0.012)	0.004 (0.008)	0.017 (0.011)
2000 $\times$ Mar-Dec	-0.010 (0.023)	0.071* (0.038)	0.001 (0.010)	0.026* (0.016)	0.010 (0.010)	0.025* (0.013)
Observations	44,837	29,261	148,382	100,817	129,721	95,608
Adjusted $R^2$	0.197	0.164	0.246	0.225	0.225	0.216

Notes: Dependent variable is  $\ln(\text{wage rate})$ . All specifications includes the industry, the year and the months fixed effects, the dummies for gender, age, marital status, provinces, the dummies of unionization and part-time jobs. Samples are restricted to the workers who are employed in the private sector and divided by three education levels and two genders. The service sector between March and December in 2000 is the treatment group. The selected industries within manufacturing sector between March and December in 2000 and the service sector between March and December for 1998-1999 are the two control groups. Data come from the Canadian LFS 1998-2000. The post-treatment period is from March to December in 2000. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

same trends as shown in the main results, which imply the anticipatory effect is absent.

### 3.5.5 The years after the tax implementation period

As the manufacturing sector experienced a significant downtown after 2004, this paper uses 2004 as a cut-off point for the main result. Between 2005 and 2007, the CIT rate at the federal level for the service and manufacturing sector remains the same after the tax reduction policy, although the federal government did introduce accelerated depreciation for machinery and equipment, as well as buildings, used in manufacturing and processing. These factors risk confounding the analysis. Nevertheless, Table 3.10 presents the effects of the tax reduction on the estimation of equation 3.1 by extending the post-treatment year to 2007. Similar to Table 3.4, male (female) samples are examined in columns of odd (even) numbers and samples of high-, medium-, and low-educated workers are examined in different rows. All



Table 3.9: Checks on the Anticipatory Effect of the Number of Working Hours

	(1)	(2)	(3)	(4)	(5)	(6)
Education	<b>High-Educated</b>		<b>Medium-Educated</b>		<b>Low-Educated</b>	
Gender	<b>M</b>	<b>F</b>	<b>M</b>	<b>F</b>	<b>M</b>	<b>F</b>
Serv $\times$ 2000 $\times$ Mar-Dec	-0.013 (0.013)	-0.029 (0.034)	0.007 (0.007)	0.024 (0.018)	0.001 (0.008)	0.030 (0.019)
Serv $\times$ 2000	0.010 (0.012)	0.040 (0.030)	-0.004 (0.007)	-0.013 (0.017)	-0.003 (0.008)	-0.023 (0.018)
Serv $\times$ Mar-Dec	0.010 (0.007)	0.035** (0.018)	0.010** (0.004)	-0.005 (0.011)	0.009* (0.005)	-0.007 (0.011)
2000 $\times$ Mar-Dec	-0.002 (0.010)	0.018 (0.023)	-0.004 (0.005)	-0.004 (0.012)	0.001 (0.006)	0.012 (0.013)
Observations	44,791	29,253	148,260	100,790	129,556	95,589
Adjusted $R^2$	0.023	0.052	0.023	0.043	0.040	0.054

Notes: Dependent variable is  $\ln(\text{working hours})$ . All specifications includes the industry, the year and the months fixed effects, the dummies for gender, age, marital status, provinces, the dummies of unionization and part-time jobs. Samples are restricted to the workers who are employed in the private sector and divided by three education levels and two genders. The service sector between March and December in 2000 is the treatment group. The selected industries within manufacturing sector between March and December in 2000 and the service sector between March and December for 1998-1999 are the two control groups. Data come from the Canadian LFS 1998-2000. The post-treatment period is from March to December in 2000. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

specification include the industry and the Year  $\times$  Month fixed effects and the dummies for age, marital status and provinces.

Compared with the main results in Table 3.4, the male workers experience a further increase in the wage rate across different education levels but their working hours have not increased. The high-educated males still show the largest increase in the wage rate at 3.5%, compared with the medium or low-educated workers. The hourly wage rate increases by 1.8% among the low-educated males, which is more than 1.7% among the medium-educated workers. For the female workers, the size of increase in the wage rate among the high-educated workers shrinks to 4.7% and the wage rate does not rise for the medium- and low-educated female workers. For the number of working hours, the medium- and low-educated female workers show increases of 1.8% and 1.4%, respectively.

Because of the presumably exogenous negative shock to manufacturing, the resulting fall

Table 3.10: The Extended Heterogeneous Effects: Wage Rates and Working Hours

<b>Difference-in-Differences Analysis</b>				
	(1)	(2)	(3)	(4)
Dependent Variables	<b>ln(Real Wage Rate)</b>		<b>ln(Weekly Working Hours)</b>	
Gender	M	F	M	F
<b>High-Educated</b>				
Serv $\times$ Post	0.035*** (0.011)	0.047*** (0.017)	-0.004 (0.004)	0.015 (0.010)
Observations	175,253	123,033	175,127	123,011
Adjusted $R^2$	0.194	0.132	0.028	0.048
<b>Medium-Educated</b>				
Serv $\times$ Post	0.017*** (0.005)	0.002 (0.008)	0.003 (0.002)	0.018*** (0.006)
Observations	543,094	384,106	542,636	384,017
Adjusted $R^2$	0.224	0.213	0.032	0.045
<b>Low-Educated</b>				
Serv $\times$ Post	0.018*** (0.006)	-0.002 (0.008)	0.002 (0.003)	0.014** (0.006)
Observations	448,488	334,482	447,963	334,414
Adjusted $R^2$	0.2	0.197	0.048	0.053

Notes: All specifications includes the industry and the month $\times$ year fixed effects and the dummies for gender, age, education level, marital status and provinces. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2007. The post-policy period is from 2001 to 2007. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

in the labor demand in manufacturing could cause a downward trend of the wage rate and the working hours across different educational levels in the manufacturing sector. Therefore, the estimated results when including the years from 2005 to 2007 are likely to be overestimated, making our tax incidence results for the restricted period ending in 2004 likely lower bound estimates. Combined with the main results, the “true” estimates might lie between the estimates from Table 3.4 and Table 3.10; i.e., the gain in the wage rate for the high-educated male workers as a result of the CIT rate cut is bounded between 2.9% and 3.5%.

### 3.6 Conclusion

This paper exploits the selective nature and the large size of the 2001-2004 CIT reform in Canada to identify the impact of corporate tax rate on the outcomes of the labor market.

Using the individual-level data from 1997 to 2004, and then extended to 2007, this study is able to estimate the causal effect of corporate taxation using the difference-in-differences method under the satisfaction of the required assumptions, such as the common trend and non-anticipatory assumptions. Through studying the tax reduction, the results support the notion that workers benefited from the 2001 tax reduction. The estimates are a 1.4% and 0.7% increase in the overall wage rate and working hours in the service sector over the period of the tax implementation.

This paper also investigates the heterogeneous effects across different groups. The results strongly confirm the capital-skill hypothesis among the male workers, as the high-educated male workers top the percentage gain in the wage rate compared with the medium- and low-educated male workers. For the female workers, the high-educated workers experience an increase in the wage rate leaving the medium and low-educated workers unchanged. The increase in working hours only appears in the group of the medium-educated females workers, as more part-time positions become full-time positions. Furthermore, examining the labor outcome impacts across different firm sizes, we find that the larger a firm is, the more it increases the wage rate and working hours for its employees in response to the tax cut.

One challenge of the paper is the length of the period covered in the study, as the manufacturing sector starts to face a downturn after 2004. Since the potential downward trend of the wage rate and working hours would widen the gain in service sector wages relative to manufacturing wages for reasons not having to do with the tax cut in services, including the years 2005-2007 in the post reform period risks overestimating the impact of the reform on wages and hours of work. Indeed, when the study extends the sample period to 2007, we find mainly some modest increases in the impacts. The results for the sample period ending in the last year of the reform, 2004, may be interpreted as a lower bound of the estimated effects. Based on the estimates, we find that the incidence of the CIT falls on labor and that CIT rate cuts increase wages significantly. A back-of-the-envelope calculation based on our main DiD estimate suggests that workers gain 118 cents per dollar

of CIT revenue forgone through a tax rate cut. CIT tax incidence estimates can be used to inform income distribution analysis. For example, if wage earnings are relatively more important for below-average income households than is capital income, then the finding that the incidence of the CIT burden falling on workers would suggest the CIT may be a regressive tax.

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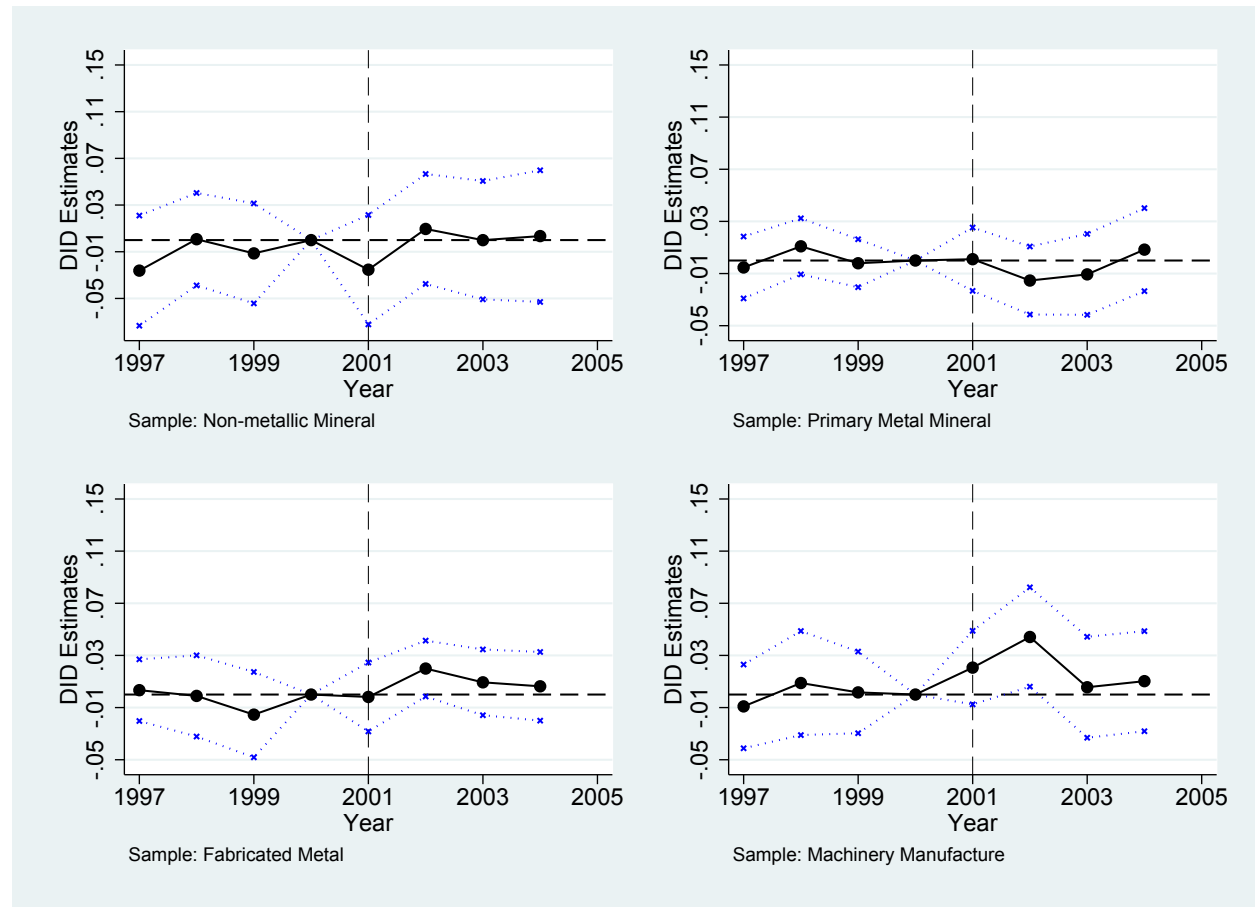
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# Appendix

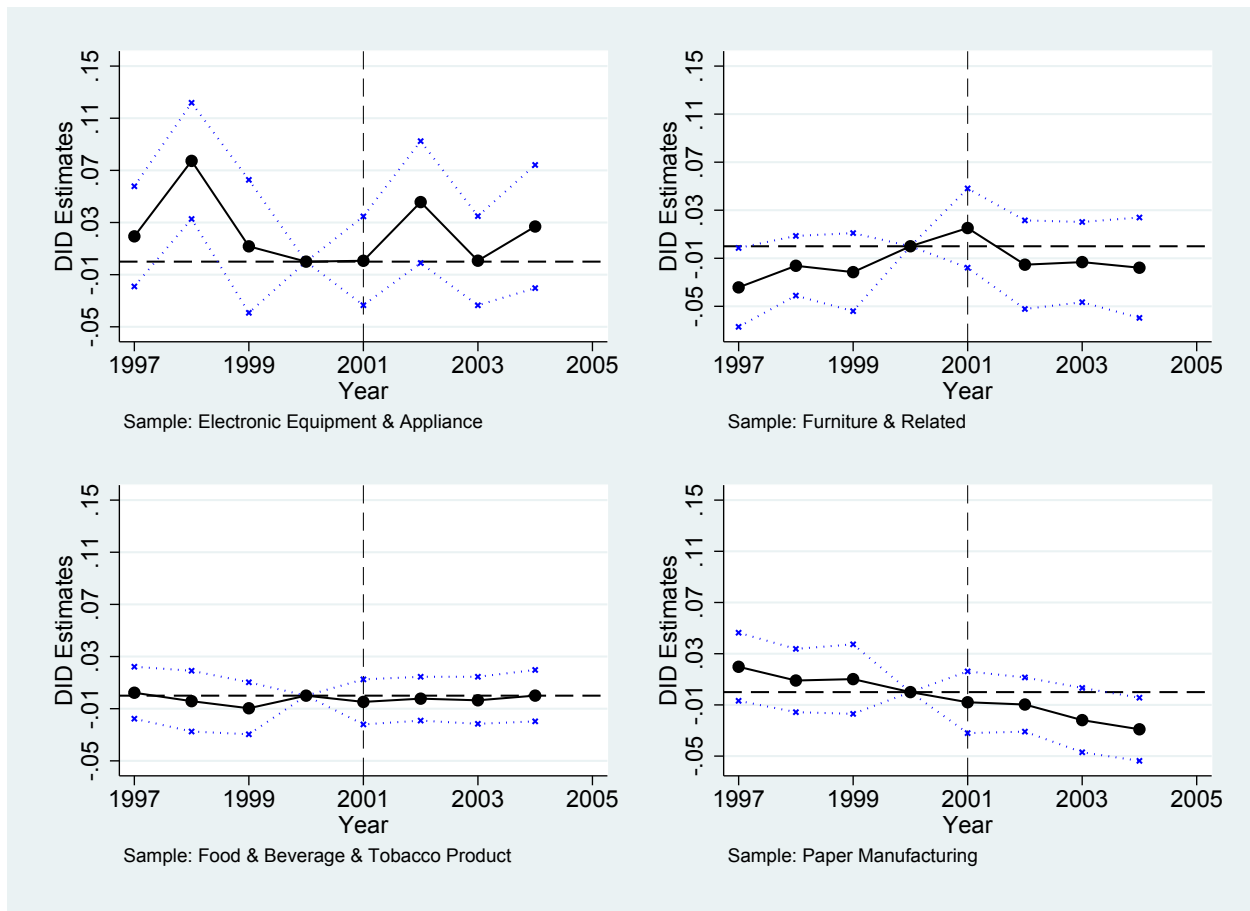
## Appendix A: Trends among the Comparison Industries

Figure A.1: Placebo Test: Trends in Wage Rate among the Comparison Industries



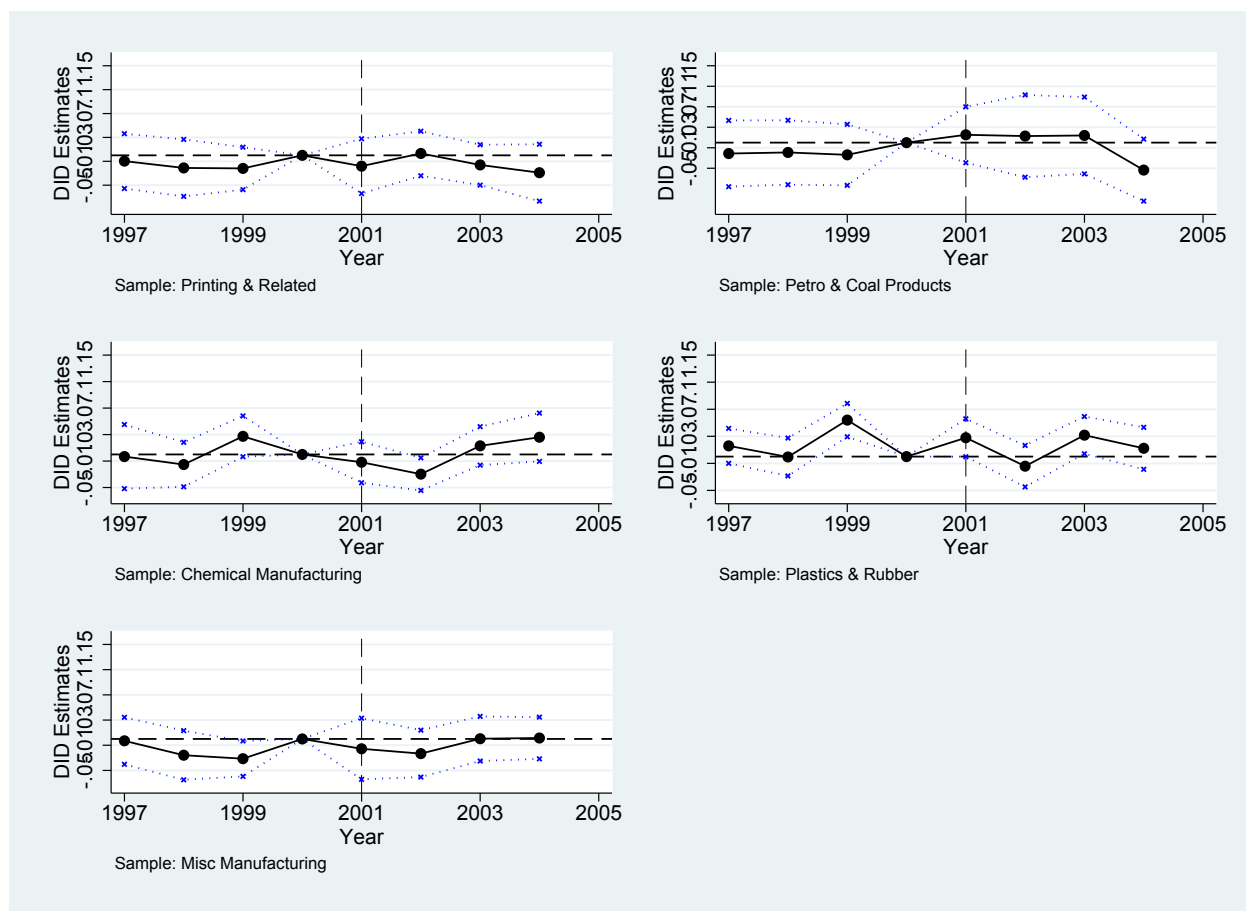
Notes: This table presents estimated coefficients from a linear model for wage rate. The dependent variable is log transformed and CPI adjusted. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Figure A.2: Placebo Test: Trends in Wage Rate among the Comparison Industries



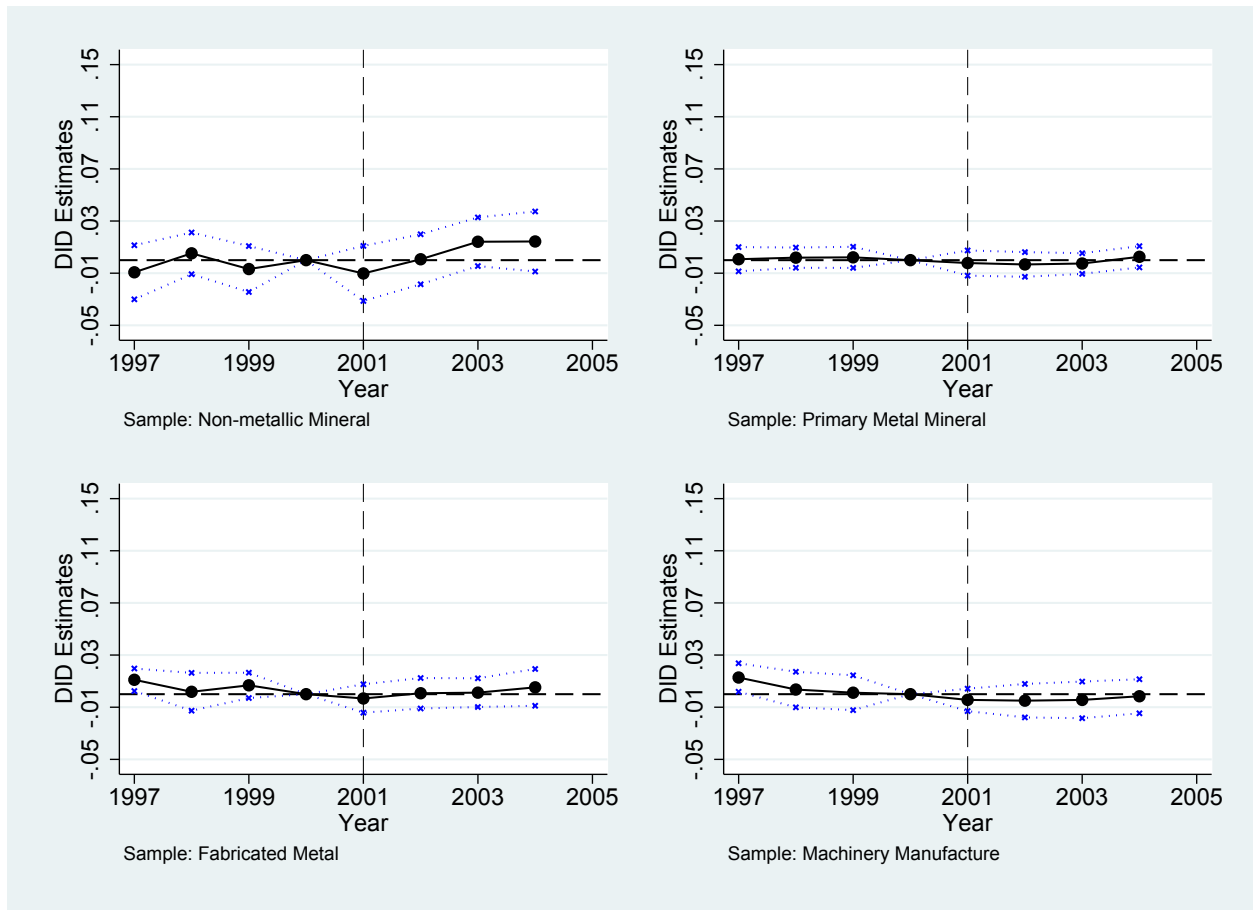
Notes: This table presents estimated coefficients from a linear model for wage rate. The dependent variable is log transformed and CPI adjusted. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Figure A.3: Placebo Test: Trends in Wage Rate among the Comparison Industries



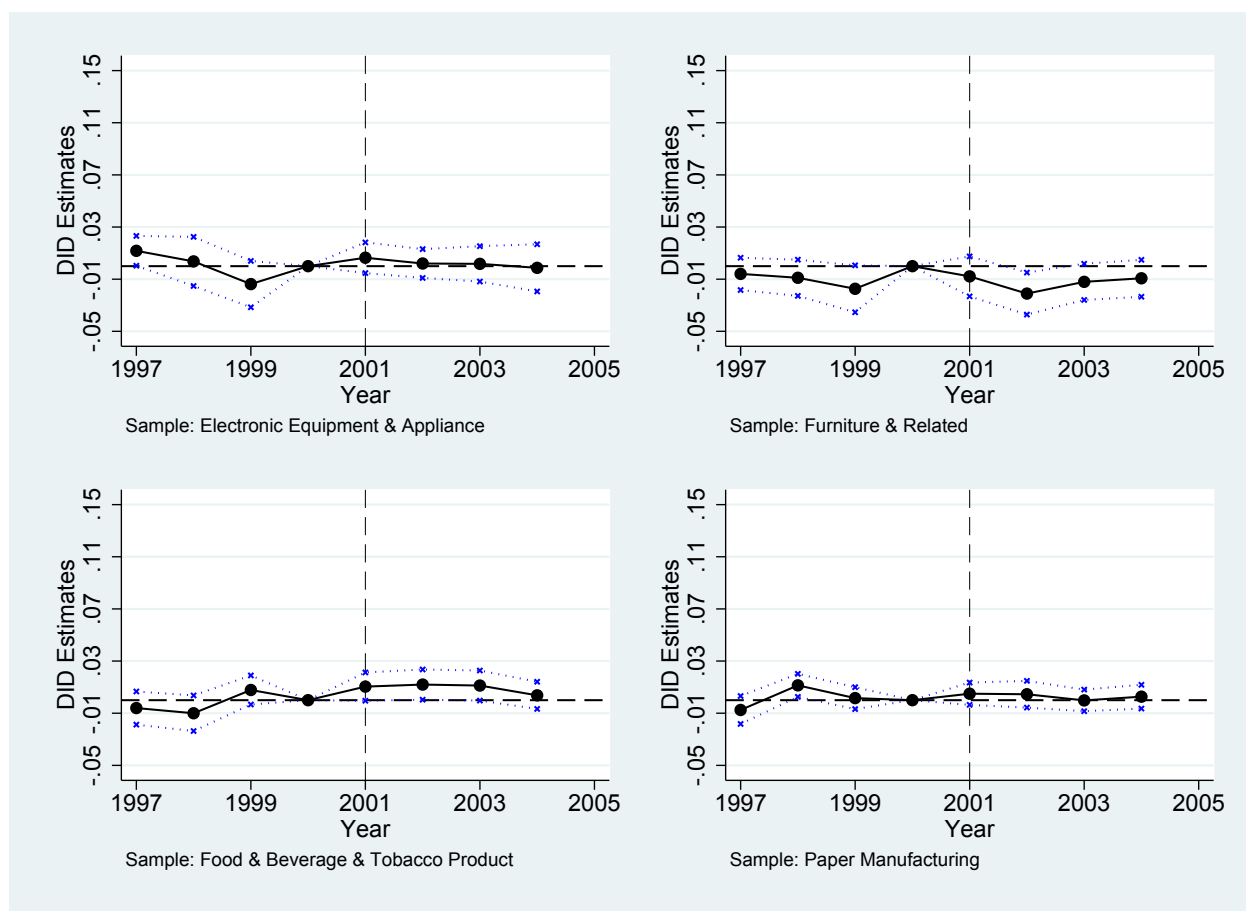
Notes: this table presents estimated coefficients from a linear model for wage rate. The dependent variable is log transformed and CPI adjusted. The time periods are 1997-2006. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Figure A.4: Placebo Test: Trends in Number of Working Hours among the Comparison Industries



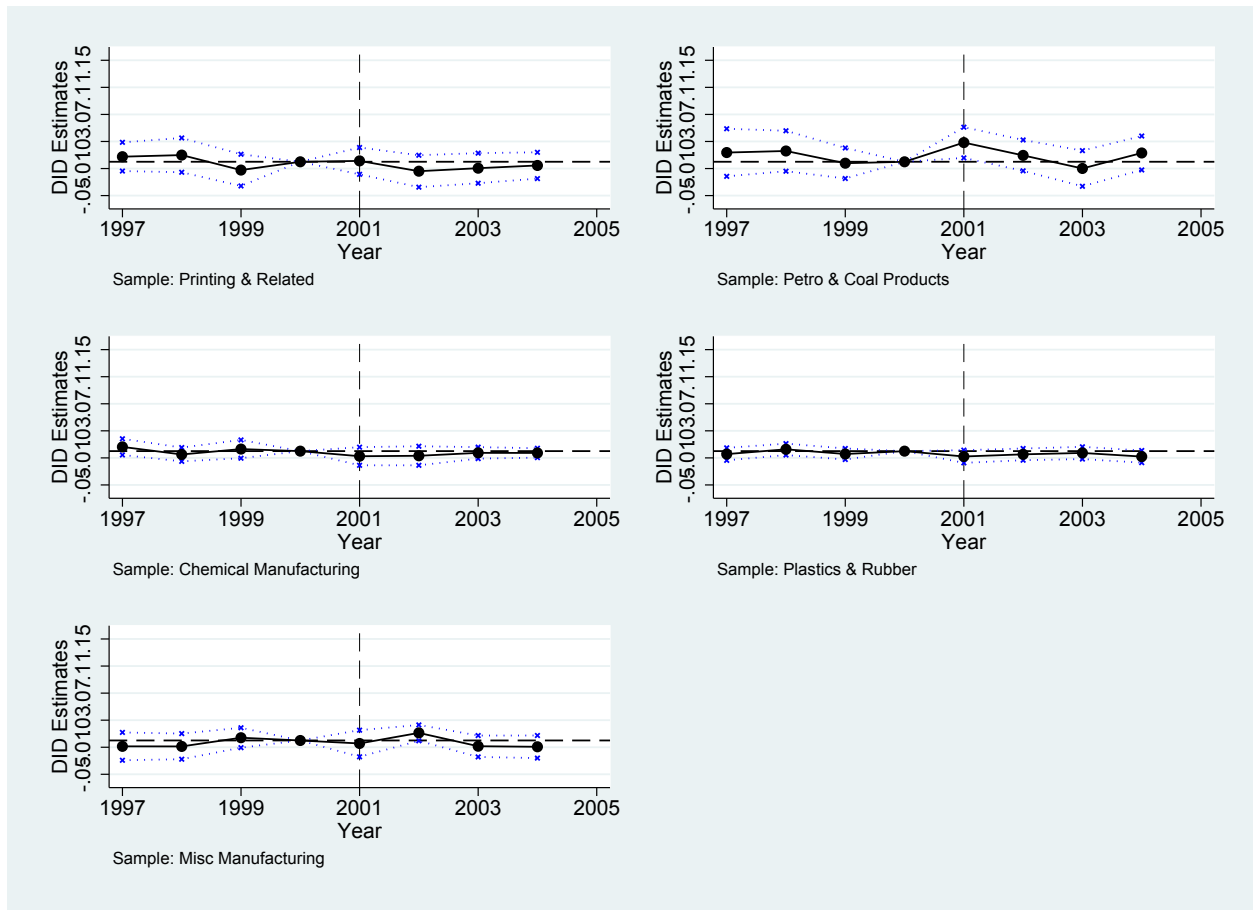
Notes: This table presents estimated coefficients from a linear model for number of working hours. The dependent variable is log transformed. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Figure A.5: Placebo Test: Trends in Number of Working Hours among the Comparison Industries



Notes: This table presents estimated coefficients from a linear model for number of working hours. The dependent variable is log transformed. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Figure A.6: Placebo Test: Trends in Number of Working Hours among the Comparison Industries



Notes: This table presents estimated coefficients from a linear model for number of working hours. The dependent variable is log transformed. The time periods are 1997-2004. Each dot is the estimated coefficients of the interaction terms in Equation (3.3). A 95-percent confidence interval is shown by the blue dashed lines.

Table A.1: Difference-in-Difference Analysis

<b>Difference-in-Differences Analysis</b>						
Serv $\times$ Post	0.016*** (0.004)	0.014*** (0.004)	0.015*** (0.004)			
Serv $\times$ Tax				-0.083*** (0.027)	-0.076*** (0.027)	-0.081*** -0.028
Sample	E	G	E & G	E	G	E & G
Restriction	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded

Notes: E is machinery manufacture industry and G is the electronic equipment & appliance industry. All specifications includes the industry and the month $\times$ year fixed effects. The dummies for gender, age, education level, marital status and provinces are also included in the specifications. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

Table A.2: Difference-in-Difference Analysis

<b>Difference-in-Differences Analysis</b>						
Serv $\times$ Post	0.007*** (0.002)	0.007*** (0.002)	0.007*** (0.002)			
Serv $\times$ Tax				-0.043*** (0.014)	-0.045*** -0.014	-0.044*** (0.014)
Sample	I	O	I & O	I	O	I & O
Restriction	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded

Notes: I is the furniture and related industry and O is the petro & coal products industry. All specifications includes the industry and the month $\times$ year fixed effects. The dummies for gender, age, education level, marital status and provinces are also included in the specifications. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

## Appendix B: Robustness Check for Anticipatory Effect

Table B.1: The Anticipatory Effect across Education Levels

	All Samples	High-Educated	Medium-Educated	Low-Educated
<b>A. Dependent Variable: ln(Real Wage Rate)</b>				
$Serv \times 2000 \times Mar - Dec$	-0.011 (0.008)	-0.001 (0.026)	-0.005 (0.011)	-0.017 (0.011)
$Serv \times 2000$	0.004 (0.008)	-0.018 (0.026)	0.010 (0.010)	-0.001 (0.011)
Observations	548,626	74,098	249,199	225,329
Adjusted $R^2$	0.366	0.230	0.337	0.325
<b>B. Dependent Variable: ln(Weekly Working Hours)</b>				
$Serv * 2000 * Mar-Dec$	0.009* (0.005)	-0.013 (0.014)	0.013* (0.007)	0.013 (0.009)
$Serv \times 2000$	-0.003 (0.005)	0.019 (0.013)	-0.004 (0.007)	-0.010 (0.008)
Observations	548,239	74,044	249,050	225,145
Adjusted $R^2$	0.126	0.090	0.126	0.138

Notes: All specifications includes the industry, the year and the months fixed effects, the dummies for gender, age, marital status, provinces, the dummies of unionization and part-time jobs. Samples are restricted to the workers who are employed in the private sector and divided by three education levels and two genders. The service sector between March and December in 2000 is the treatment group. The selected industries within manufacturing sector between March and December in 2000 and the service sector between March and December for 1998-1999 are the two control groups. Data come from the Canadian LFS 1998-2000. The post-treatment period is from March to December in 2000. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.



Table B.2: The Aggregate Effect excluding 2000: Wage Rate and Working Hours

	(1)	(2)	(3)	(4)
A. Dependent Variable: $\ln(\text{Wage Rate})$				
Serv $\times$ Post	0.006 (0.005)	0.013*** (0.004)		
Serv * Tax			-0.022 (0.017)	-0.068** (0.015)
Observations	1,271,685	1,271,685	1,271,685	1,271,685
Adjusted $R^2$	0.167	0.352	0.167	0.352
B. Dependent Variable: $\ln(\text{Working Hours})$				
Serv $\times$ Post	0.008*** (0.002)	0.008*** (0.002)		
Serv * Tax			-0.060*** (0.008)	-0.051*** (0.008)
Observations	1,180,444	1,180,444	1,180,444	1,180,444
Adjusted $R^2$	0.047	0.121	0.048	0.121

Notes: All specifications includes the industry and the month $\times$ year fixed effects. In column (2) and (4), the dummies for gender, age, education level, marital status and provinces are also included in the specifications. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The pre-policy period is from 1997 to 1999. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

Table B.3: The Heterogeneous Effects excluding 2000: Wage Rate and Working Hours

<b>Difference-in-Differences Analysis</b>				
	(1)	(2)	(3)	(4)
Dependent Variable	<b>ln(Wage Rate)</b>		<b>ln(Working Hours)</b>	
Gender	M	F	M	F
<b>Panel A</b>				
	<b>High - Educated</b>			
Serv $\times$ Post	0.030** (0.014)	0.038* (0.020)	-0.003 (0.005)	0.019 (0.012)
Adjusted $R^2$	0.218	0.180	0.028	0.054
	<b>Medium - Educated</b>			
Serv $\times$ Post	0.020*** (0.006)	-0.006 (0.009)	0.004* (0.003)	0.013* (0.007)
Adjusted $R^2$	0.256	0.269	0.028	0.045
	<b>Low - Educated</b>			
Serv $\times$ Post	0.012** (0.006)	-0.004 (0.008)	0.000 (0.003)	0.006 (0.007)
Adjusted $R^2$	0.263	0.293	0.044	0.051
<b>Panel B</b>				
	<b>High - Educated</b>			
Serv $\times$ Tax	-0.139 (0.091)	-0.269** (0.128)	0.040 (0.029)	-0.102 (0.082)
Adjusted $R^2$	0.198	0.138	0.028	0.054
	<b>Medium - Educated</b>			
Serv $\times$ Tax	-0.100** (0.042)	0.044 (0.064)	-0.016 (0.017)	-0.079* (0.042)
Adjusted $R^2$	0.232	0.219	0.029	0.045
	<b>Low - Educated</b>			
Serv $\times$ Tax	-0.059 (0.046)	0.066 (0.060)	-0.004 (0.021)	-0.075 (0.047)
Adjusted $R^2$	0.205	0.203	0.045	0.053

Notes: All specifications includes the industry and the month $\times$ year fixed effects and the dummies for gender, age, education level, marital status and provinces. All samples are restricted to the employed in private sectors. Service sector is the treatment group and selected industries within manufacturing sector are in the control group. Data come from the Canadian LFS 1997-2004. The pre-policy period is from 1997 to 1999 The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

## Appendix C: Including all Industries in the Manufacturing Sector

Although there is strong evidence showing that some industries in the manufacturing sector have experienced downturn before the tax reduction policy, the following regression results by including all industries in the manufacturing sector using equation 3.1 provide additional supports for the robustness of the main results.

Table C.1: the Aggregate Effect including all Industries in the Manufacturing Sector: Wage Rate and Working Hours

<b>Difference-in-Differences Analysis</b>				
	(1)	(2)	(3)	(4)
<b>A. Dependent Variable: ln(Real Wage Rate)</b>				
Serv * Post	0.006* (0.004)	0.011*** (0.003)		
Serv $\times$ Tax			-0.031 (0.028)	-0.064*** (0.024)
Observations	1,655,858	1,655,858	1,655,858	1,655,858
Adjusted $R^2$	0.1823	0.3672	0.182	0.367
<b>B. Dependent Variable: ln(Weekly Working Hours)</b>				
Serv * Post	0.009*** (0.002)	0.008*** (0.002)		
Serv $\times$ Tax			-0.061*** (0.013)	-0.050*** (0.013)
Observations	1,654,867	1,654,867	1,654,867	1,654,867
Adjusted $R^2$	0.0543	0.1222	0.054	0.122

Notes: All specifications includes the industry and the month $\times$ year fixed effects. In column (2) and (4), the dummies for gender, age, education level, marital status and provinces are also included in the specifications. All samples are restricted to the employed in private sectors. Service sector is the treatment group and the manufacturing sector is the control group. Data come from the Canadian LFS 1997-2004. The post-policy period is from 2001 to 2004. \* indicates significance at the 10 percent level, \*\* significance at the 5 percent level, and \*\*\* significance at the 1 percent level.

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To Whom It May Concern,

I allow Feng Wei to use the materials from working papers “Designing Presumptive Taxes in Countries with Large Informal Sector” and “The Optimal Turnover Threshold and Tax Rate for SMEs” for his dissertation.

Sincerely,

Dr. Jean-Francois Wen  
Department of Economics  
University of Calgary