Risk-informed inspection and maintenance planning for deteriorating structural systems

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master thesis

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Risk-informed inspection and maintenance planning for deteriorating structural systems

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

GRADUATE PROGRAM IN CIVIL ENGINEERING

CALGARY, ALBERTA

SEPTEMBER, 2021

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Abstract

Structural systems are subjected to several deteriorating processes throughout their service life. Deterioration over a long run can significantly affect the integrity of the structure and its subsequent failure can lead to severe consequences with tremendous monetary loss. Hence regular inspection and maintenance is carried out to identify and repair critical locations to keep the structure safe. The overall cost incurred during the service life of the structure can be significant, hence it is important to minimize it by adopting optimal inspection and maintenance strategies. Most of the literature works focus mainly on system level inspection and maintenance strategies whereas the strategies developed for partial inspection and maintenance is still limited. The objective of this research is to develop a comprehensive framework which will guide the engineers and decision makers to determine optimal risk-based inspection and maintenance strategies based on their requirement of faster system level analysis or a detailed component level analysis. Several options and suggestions are provided throughout the framework to choose and navigate through the different approaches. Two numerical examples are carried out to demonstrate the implementation of framework for practical applications.
Acknowledgements

First and foremost, I would like to extend my deepest gratitude to my supervisor, Dr. Markus Dann for providing me an opportunity to carry out this research under his guidance. I am thankful for his unwavering support, reassurance, patience, and enthusiasm in guiding me throughout my master’s program. Apart from sharing his immense knowledge on the subject, I am grateful for his friendly chats during meetings to check my well being. Special thanks to my exam committee members, Dr. Nigel Shrive and Dr. Ron Hugo for agreeing to be on my committee and taking the time to review my work by providing valuable feedback.

Thank you to my fellow graduate students for supporting me throughout this entire process. From sharing stories about missing people back home to making ENF 316 as our second home, from stressing about exams and deadlines to using humour as defence mechanism to alleviate stress, the journey has been truly memorable.

I am forever indebted to my parents for their unconditional support and the sacrifices they made in their life. Words fall short to extend my gratitude towards my deceased father who would have been proud to see me grow, my mother who fought against all odds and supported my decision to move abroad, my brothers who are always there to lookout for me and my husband for believing in me, supporting my goals, and always encouraging to dream big and achieve greater things in life.

I am grateful for the financial support from the Natural Sciences and Engineering Research Council (NSERC). I am also thankful to the Department of Civil Engineering for providing me with teaching opportunities.
# Table of Contents

Abstract ............................................................................................................................................. ii

Acknowledgements ........................................................................................................................... iii

Table of Contents ............................................................................................................................... iv

List of Figures ..................................................................................................................................... vi

List of Tables ....................................................................................................................................... viii

List of Symbols, Abbreviations and Nomenclature ........................................................................... ix

1. **INTRODUCTION** .................................................................................................................. 1
   1.1 Background .............................................................................................................................. 1
   1.2 Problem statement and motivation ......................................................................................... 3
   1.3 Research objectives ................................................................................................................ 4
   1.4 Thesis overview ...................................................................................................................... 5

2. **LITERATURE REVIEW** .......................................................................................................... 6
   2.1 Integrity management ............................................................................................................. 6
      2.1.1 Inspection techniques ...................................................................................................... 7
      2.1.2 Maintenance strategies .................................................................................................... 9
   2.2 Structural deterioration modelling ......................................................................................... 11
      2.2.1 Corrosion growth model ............................................................................................... 11
   2.3 Life-cycle cost optimization and decision analysis ................................................................. 13
      2.3.1 Bayesian inference ........................................................................................................ 17
      2.3.2 Bayesian inference methods .......................................................................................... 18
      2.3.3 Bayesian decision making ............................................................................................ 20
      2.3.4 Optimal inspection and maintenance approaches ........................................................ 22
         2.3.4.1 Reliability-based approach ..................................................................................... 22
         2.3.4.2 Risk based approach .............................................................................................. 23
         2.3.4.3 Risk based inspection and maintenance ................................................................ 27
         2.3.4.4 Value of information .............................................................................................. 29

3. **METHODOLOGY** ..................................................................................................................... 31
   3.1 Overview .................................................................................................................................. 31
   3.2 System modelling .................................................................................................................... 35
   3.3 Structural deterioration modelling ......................................................................................... 38
      3.3.1 Data collection and validation ....................................................................................... 39
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.2 Corrosion growth model</td>
<td>41</td>
</tr>
<tr>
<td>3.3.3 Bayesian inference</td>
<td>50</td>
</tr>
<tr>
<td>3.3.3.1 Markov chain monte carlo</td>
<td>51</td>
</tr>
<tr>
<td>3.3.3.2 Variational inference</td>
<td>52</td>
</tr>
<tr>
<td>3.4 Structural reliability analysis</td>
<td>54</td>
</tr>
<tr>
<td>3.5 Lifecycle cost modelling</td>
<td>55</td>
</tr>
<tr>
<td>3.6 Optimal inspection planning</td>
<td>60</td>
</tr>
<tr>
<td>3.7 Optimal maintenance planning</td>
<td>61</td>
</tr>
<tr>
<td>4. NUMERICAL EXAMPLES</td>
<td>66</td>
</tr>
<tr>
<td>4.1 Population-based inspection and maintenance planning</td>
<td>66</td>
</tr>
<tr>
<td>4.1.1 Structural deterioration model</td>
<td>67</td>
</tr>
<tr>
<td>4.1.2 Optimal repair criterion</td>
<td>70</td>
</tr>
<tr>
<td>4.1.3 Inspection planning</td>
<td>72</td>
</tr>
<tr>
<td>4.1.3.1 Sensitivity analysis</td>
<td>76</td>
</tr>
<tr>
<td>4.2 Defect-specific inspection and maintenance planning</td>
<td>79</td>
</tr>
<tr>
<td>4.2.1 Structural deterioration model</td>
<td>83</td>
</tr>
<tr>
<td>4.2.2 Cost analysis</td>
<td>86</td>
</tr>
<tr>
<td>4.2.3 Inspection planning</td>
<td>87</td>
</tr>
<tr>
<td>4.2.3.1 Sensitivity analysis</td>
<td>91</td>
</tr>
<tr>
<td>4.2.4 Optimal maintenance decision</td>
<td>99</td>
</tr>
<tr>
<td>4.2.4.1 Optimal repair criterion</td>
<td>100</td>
</tr>
<tr>
<td>4.2.4.2 Decision between local and global repair</td>
<td>102</td>
</tr>
<tr>
<td>5. CONCLUSION</td>
<td>105</td>
</tr>
<tr>
<td>5.1 Summary</td>
<td>105</td>
</tr>
<tr>
<td>5.2 Main contributions</td>
<td>107</td>
</tr>
<tr>
<td>5.3 Recommendations for future research</td>
<td>108</td>
</tr>
<tr>
<td>References</td>
<td>110</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1.1: Relationship of the total lifecycle costs based on degree of maintenance (modified from Nielsen & Sorenson, 2014) ..................................................3
Figure 2.1: Types of maintenance planning methods and maintenance decisions ..............10
Figure 2.2: Generic decision tree (modified from Parmigiani, G et al., 2009) ..................16
Figure 2.3: Bayesian Inference methods (modified from Birkland & Dann, 2018) ..........19
Figure 2.4: Generic decision tree for pre-posterior decision analysis (modified from Faber, M. H., 2005) .................................................................21
Figure 2.5: Generic framework for risk-based inspection & maintenance (RBIM) process (modified API, 2009) .................................................................26
Figure 2.6: Inspection plan based on RBI (modified from DNV, 2010) .....................27
Figure 2.7: Generic decision tree for risk-based inspection & maintenance (RBIM) process ....28
Figure 3.1: Framework for inspection and maintenance planning ................................28
Figure 3.2: Different approaches to model a system and their corresponding deterioration model for (a) Population-based approach (b) Discretized system approach (c) Defect-specific approach. ........................................34
Figure 3.3: Discretization of structural systems into sections or components: (a) Truss system, (b) Slab system, and (c) Pipeline segment ........................................37
Figure 3.4: Steps involved in developing a structural deterioration model ....................39
Figure 3.5: PMF of depth of defects from two inspection results ................................41
Figure 3.6: Hierarchy of uncertainties in structural deterioration modelling ................42
Figure 3.7: Schematic diagram of a typical corrosion defect in (a) a pipeline (b) a steel rod 44
Figure 3.8: Probability of detection for different values of mean detection threshold ....45
Figure 3.9: PMF of defect size of actual defect and measured defect .........................46
Figure 3.10: Probability of true call for different values of mean credible depth ..........47
Figure 3.11: Hierarchy of errors due to inspection tool considered in the corrosion model (a) population-based approach (b) discretized or defect specific approach ..........49
Figure 3.12: Schematic illustration of MCMC method .............................................51
Figure 3.13: Schematic illustration of concept of variational inference ......................52
Figure 3.14: Time value of money for different discount rates ..................................59
Figure 3.15: Schematic illustration of determination of minimal repair cost .................64
Figure 3.16: Variation of probability of failure of a defect over the service life of the structure 65
Figure 4.1: PDF of the estimated defect size for the population of defects ................68
Figure 4.2: Exceedance probability of the estimated defect size for the population of defects 5 years forward .................................................................69
Figure 4.3: Illustration of repair and failure criterion set during a maintenance event ........70
Figure 4.4: Repair cost, failure cost and total cost corresponding to different repair criterion during the service life .........................................................72
Figure 4.5: PDF of corrosion growth increment for prior and posterior analysis ............74
Figure 4.6: Convergence of VOI for different simulation size .................................................. 75
Figure 4.7: Variation of VOI for different inspection costs ....................................................... 76
Figure 4.8: Variation of VOI for different discount rates ......................................................... 77
Figure 4.9: Variation of VOI for different values of mean credible size of defect for true call of the inspection tool ................................................................. 78
Figure 4.10: Variation of VOI for different values of detection threshold of the inspection tool ... 78
Figure 4.11: Variation of VOI for different values of standard deviation of sizing error of the inspection tool ................................................................. 79
Figure 4.12: PMF of size of defects from two in-line inspections ........................................ 83
Figure 4.13: PDF of the corrosion rate per year ................................................................. 85
Figure 4.14: PDF of the size of defects for defect number 1 at every 5 years up to 20 years ...... 86
Figure 4.15: VOI as a function of the number of inspections ................................................... 90
Figure 4.16: Maximum VOI as a function of repair criterion ................................................. 91
Figure 4.17: VOI as a function of number of inspections for different repair criterion .......... 92
Figure 4.18: Optimal and maximum number of inspections as a function of repair criterion ... 93
Figure 4.19: Maximum VOI as a function of inspection cost ................................................ 94
Figure 4.20: VOI as a function of number of inspections for varying cost ratios (inspection cost : repair cost : failure cost) ................................................................. 95
Figure 4.21: VOI normalized by the VOI of cost-free inspections as a function of number of inspections for varying cost ratios (inspection cost : repair cost : failure cost) .......... 95
Figure 4.22: Normalized number of inspections versus the normalized cost of inspection ....... 96
Figure 4.23: PDF of corrosion growth rate for different groups ............................................. 98
Figure 4.24: Variation in VOI and optimal number of inspections for defects with overall corrosion growth rate versus categorised corrosion growth rate ........................................ 99
Figure 4.25: Contour plot of total lifecycle cost for different repair criterion and probability constraint ((a) Isometric view, (b) Top view) ................................................................. 101
Figure 4.26: Total lifecycle cost corresponding to maintenance events when global repair was carried out for the first time ................................................................. 104
List of Tables

Table 4.1: Number and size of defects reported during two inspections taken in the year 2008 and 2012 .................................................................81
Table 4.2: Results of VOI analysis corresponding to repair criterion of 70%wt with a probability threshold of 0.005 ..............................................................89
Table 4.3: Parameters used in corrosion growth modelling .................................................................97
List of Symbols, Abbreviations and Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Benefit</td>
</tr>
<tr>
<td>C</td>
<td>Total cost</td>
</tr>
<tr>
<td>(C_F)</td>
<td>Failure cost</td>
</tr>
<tr>
<td>(C_{\text{Ini}})</td>
<td>Initial cost</td>
</tr>
<tr>
<td>(C_{\text{Ins}})</td>
<td>Inspection cost</td>
</tr>
<tr>
<td>(C_{\text{Op}})</td>
<td>Operation cost</td>
</tr>
<tr>
<td>(C_{\text{Re}})</td>
<td>Repair cost</td>
</tr>
<tr>
<td>(C_{\text{De}})</td>
<td>Demolition cost</td>
</tr>
<tr>
<td>d</td>
<td>Index for defects</td>
</tr>
<tr>
<td>D</td>
<td>Number of defects</td>
</tr>
<tr>
<td>E[]</td>
<td>Expected value operator</td>
</tr>
<tr>
<td>g()</td>
<td>Limit state function</td>
</tr>
<tr>
<td>k</td>
<td>Failure limit in terms of metal loss</td>
</tr>
<tr>
<td>(m_R)</td>
<td>Vector of maintenance events</td>
</tr>
<tr>
<td>min</td>
<td>Minimum value operator</td>
</tr>
<tr>
<td>max</td>
<td>Maximum value operator</td>
</tr>
<tr>
<td>n</td>
<td>Section index</td>
</tr>
<tr>
<td>N</td>
<td>Number of sections in a system</td>
</tr>
<tr>
<td>p()</td>
<td>Probability density function</td>
</tr>
<tr>
<td>(P_F)</td>
<td>Probability of failure</td>
</tr>
<tr>
<td>(P_{\text{RE}})</td>
<td>Probability of repair</td>
</tr>
<tr>
<td>(P_{\text{Rt}})</td>
<td>Probability constraint for repair criterion</td>
</tr>
<tr>
<td>(P_S)</td>
<td>Probability of survival</td>
</tr>
<tr>
<td>R</td>
<td>Risk</td>
</tr>
<tr>
<td>r</td>
<td>Discount rate</td>
</tr>
<tr>
<td>t</td>
<td>Time interval</td>
</tr>
<tr>
<td>(T_{\text{Ins}})</td>
<td>Inspection time</td>
</tr>
<tr>
<td>U</td>
<td>Utility</td>
</tr>
<tr>
<td>x</td>
<td>Actual defect size</td>
</tr>
</tbody>
</table>
\( x_{De} \)  
Mean detection threshold

\( y \)  
Reported defect size

\( y_T \)  
Mean credible depth

\( z \)  
Vector of decision variables

\( * \)  
Denotes optimal decision

\( \alpha \)  
Shape parameter of gamma distribution function

\( \beta \)  
Scale parameter of gamma distribution function

\( \Delta x \)  
Corrosion growth depth

\( \varepsilon \)  
Sizing error

\( \lambda \)  
Corrosion growth rate

**Abbreviation**  
**Full form**

BMWD  
Brownian motion with drift

CGM  
Corrosion growth model

DT  
Destructive testing

GEVD  
Generalized extreme value distribution

ILI  
In-line inspection

MCMC  
Markov-chain monte carlo

NDT  
Non-destructive testing

PDF  
Probability density function

PMF  
Probability mass function

POD  
Probability of detection

POTC  
Probability of true call

RBI  
Risk based inspection

RBIM  
Risk based inspection and maintenance

RBM  
Risk based maintenance

SRA  
Structural reliability analysis

VI  
Variational inference

VOI  
Value of Information

\( wt \)  
Initial wall thickness
1. INTRODUCTION

1.1 Background

Engineering systems and structures have become indispensable to human life, economic growth, and development of any country. Gradually there is tremendous increase in construction and expansion of these facilities which usually have negligible defects when they are newly built or installed. However, many processes degrade the performance of the system over time eventually leading to its failure. For instance, reinforced concrete structures can be subject to corrosion due to presence of moisture or lack of sufficient concrete cover (Böhni, 2005). The corrosion of steel reinforcements will result in formation of rust enlarging the area around the reinforcements. This exerts pressure on the surrounding concrete resulting in generation of cracks (Alonso et al., 1998). In the due course of time, these cracks may propagate and increase in size leading to spalling of concrete cover, leaving reinforcement bars exposed to harsh environmental conditions. This may result in reduced load carrying capacity of the structure eventually leading to its failure if not taken care proactively.

In order to keep the structural system serviceable throughout its service life it is necessary to perform inspections to identify critical locations which might require corrective action plan. Based on the inspection outcomes, structural systems are repaired, and regular maintenance activities are carried out to keep the structural system operational throughout the service life. There are different methods to maintain the system, such as deterministic approach where a structural system is maintained based on the condition state of the structural system, or reliability-based approach where maintenance is carried out when the probability of failure exceeds the maximum allowable
probability of failure of the structure, etc. However, the recent focus is on lifecycle cost approach where the total cost including risk over the remaining life of the structure is minimised.

The total cost incurred in the inspections and maintenances of the structure along with failure costs in the event of failure due to insufficient maintenance can be considerable and add significantly to the lifecycle cost. Therefore, judicious planning is required to bring down the overall cost while maintaining the safety and serviceability of structural system. Hence, the optimum inspection and maintenance plan is selected in such a way that it minimizes the total lifecycle costs through decision making process.

Figure 1.1 shows the relationship of the total lifecycle cost based on the degree of maintenance. A very well-maintained structural system will require higher inspection and maintenance cost; however, the corresponding failure cost is minimum due to lower risk of failure. On the other hand, a poorly maintained structural system will have higher risk of failure due to lack of sufficient inspection and maintenance leading to higher failure cost. Total cost which is the sum of cost incurred in inspection and maintenance and the cost of failure will vary depending on the level of maintenance carried out resulting in a trade-off between these two costs.

Inspection and maintenance planning aims at optimizing these costs and help in maintaining the standard acceptable quality of the structural system. Frequent inspections followed by timely interventions can reduce failure costs through identification of critical locations and avoiding accidents. In some cases, consequences due to failure such as human safety and environmental impact can be more severe than the loss of functionality due to failure of the structure itself. In such cases risk-based inspection and maintenance (RBIM) planning plays an important role as it considers consequences of failure along with the likelihood of failure in evaluating the optimised economic plan for inspection and maintenance, respectively.
1.2 Problem statement and motivation

Total costs incurred in any structural system comprise of initial construction costs, operation costs and lifetime inspection and maintenance costs including failure costs. Resources spent on maintaining the integrity of the structure can be a costly affair. From time to time, efforts are being taken to formulate different inspection and maintenance strategies to bring down the overall costs and keep the structure serviceable (Abubakirov et al., 2020; Gong et al., 2019; Li et al., 2017; Sørensen & Faber, 2001; Thoft-Christensen & Sörensen, 1991). Inspection and maintenance strategies based on risk-informed decision making has been widely adopted to devise a cost optimal plan. However, most of the risk-based inspection (RBI) and risk-based maintenance
(RBM) strategies developed focus on system level rather than at component level. Strategies aimed at component level or for systems with multiple sections or components is still limited. Also, the amount of data that is available from the inspections result might be massive which can be computationally demanding and challenging. Hence there is need for efficient analysis which can provide an approximate solution with least computational time to arrive at an optimised solution.

1.3 Research objectives

The objective of this research is to develop a framework for optimal inspection and maintenance planning for deteriorating structures. The research focuses on risk-informed decision-making process namely, RBI and RBM planning to achieve minimal lifecycle costs. This framework can be applied to any deteriorating structural system whose past inspection data are available. Using this past information, structural deterioration model is developed to estimate the future deterioration state. Probability of failure and the consequences involved in case of failure are estimated to judiciously allocate resources for inspection and maintenance by prioritising the components based on their risk level.

In order to accomplish the overall research objective, following sub-objectives are achieved.

- Optimum inspection strategy will be formulated to decide if extra inspection is financially beneficial before carrying out maintenance. If further inspection is necessary, then this methodology will be useful in ascertaining which locations to be inspected first and the optimum number of inspection locations that will result in most cost-efficient plan and better knowledge about the condition state of the system.
- Optimum maintenance strategy will be formulated to determine repair criterion that will result in minimal lifecycle costs. Efforts will be taken to consider all possible decision scenarios occurring throughout the remaining service life of the structure.
• Owing to the enormous amount of data to be possibly processed from the inspection results, this framework will provide alternatives for computationally less demanding and efficient analysis resulting in more approximate solution with less computational time. Examples explaining the step-by-step methodology of this entire framework will also be included in this thesis.

1.4 Thesis overview

This thesis comprises of five chapters with the current chapter discussing the importance of inspection and maintenance planning and the need for optimal strategies. Chapter two presents some of the recent works carried out in the field of integrity management and improvements made in inspection and maintenance planning. Chapter three presents the main contribution of the research focussing on the framework for RBIM planning. Structural deterioration modelling, expected lifecycle cost modelling, and the decision-making process to accomplish optimal costs are described. Chapter four presents two numerical examples to demonstrate the implementation of the framework in practical applications. Finally, Chapter five concludes the work and recommendations for future work are provided.
This chapter provides an overview of integrity management with information about inspection techniques and maintenance strategies that are commonly adopted. It presents a brief insight on the structural deterioration modelling with detailed literature review on different types of corrosion models. This chapter also provides background information on decision analysis to make optimal decisions and preventive maintenance methods with focus on risk-informed inspection and maintenance planning as discussed in various literature.

2.1 Integrity management

As per the Canadian Infrastructure Report Card (2019), around 15% of the roads, bridges, and tunnels range between poor to very poor state of condition requiring immediate action, whereas around 25% are in fair condition, which can deteriorate to poor/very poor condition in the absence of proper action plan. In order to meet with the increased demands, in addition to construction of new facilities, it is necessary to take care of the existing facilities which might be deteriorating or reaching the end of their service life. Inspection and maintenance planning plays a crucial role in maintaining the integrity of structures as failures can be catastrophic in terms of monetary losses, environmental impact, and human life. The type of inspection and maintenance strategies employed differs based on the user requirements and their end goals. Before delving more into it, the following subsections provide an overview of some of the inspection methods and maintenance strategies that are commonly adopted in the infrastructure systems.
2.1.1 Inspection techniques

Inspection involves careful evaluation of the condition of the structural system through different testing methods. Based on the type of infrastructure system, different methods of inspection/testing are available. Testing methods can be broadly classified as destructive testing (DT) and non-destructive testing (NDT) (Shankar & Joshi, 2014). Destructive testing is a direct form of measurement where a sample of material is tested by making it fail under different load conditions. Destructive testing for an existing structure alters or damages the structure, requiring replacement of the sample section taken out for testing with a new material, and this entire process might cause interference with the functioning of the system. Also, destructive testing provides information about the mechanical properties of the material such as the compressive strength of the concrete but not the extent of deterioration such as presence of defects, crack length, or depth of corrosion (Gupta, 2018). On the other hand, non-destructive testing, which is often non-invasive in nature, is used most commonly as it does not cause any permanent damages to the system during inspection and does not affect the functioning of the system (Scott & Scala, 1982). Non-destructive methods are able to assess the degradation of the system unlike destructive testing methods. However, NDT is an indirect form of measurement and the tools that are often used for NDT evaluation, such as in-line inspection (ILI) tool used in the pipelines, are subjected to measurement errors (Dann & Dann, 2017). Measurement errors such as detection error, false call error, sizing error, and location error have to be taken into account in the structural deterioration model for accurate estimation of future deterioration while considering optimal decisions for inspection and maintenance. More details on these measurement errors are covered in the chapter 3 of the thesis. Some of the non-invasive inspection methods and their working principles are briefly explained as follows.
Visual inspection is carried out by naked eye for overall inspection of system such as to check surface defects like cracks, discontinuity in joint, etc. It is a primary method of inspection for infrastructure systems like bridges (Ryan et al., 2012) and should be supplemented with other inspection methods for more detailed results. The main shortcoming of visual inspection is accessibility as it cannot reveal the presence of internal and hidden defects (Agdas et al., 2016) and the surfaces needs to be thoroughly cleaned to avoid overlooking of any surface defects.

Non-destructive testing using ultrasonic tool involves transmission of high frequency sound waves through the component to be inspected. It employs a transmitter producing an ultrasonic pulsed wave which is propagated through the material, and the wave is reflected back when there is a change in medium such as a discontinuity in the material due to presence of an internal defect. The time taken for the signals to be reflected back are recorded to infer location, size, or orientation of the defect (Rens et al., 1997).

Another commonly adopted NDT method using magnetic flux is often used to determine defects in ferromagnetic materials such as steel pipes and structures, deterioration of steel in reinforced concrete structures, etc. In pipelines, any defect in pipe wall can be detected by the change of magnetic flux as greater the defect, the higher the leakage in magnetic flux. Based on the change of magnetic flux leakage, the size and shape of the defect can be inferred (Mandal et al., 1998).

Leak tests are carried out in transmission pipelines and pressure vessels to check any source of leakages especially where it is essential to avoid release of any pollutant in the atmosphere. The pipes are filled with water and pressure maintained in the pipe is tracked for about a day to take diurnal variations into account. Any change in parameters such as change in pressure or flow rate is monitored using sensors to identify the presence of leaks (Sadeghioon et al., 2014).
2.1.2 Maintenance strategies

Once the system is inspected, maintenance is carried out based on the inspection outcomes. Maintenance planning can be classified as corrective maintenance and preventive maintenance (Lewis, 1987). Corrective maintenance is carried out after the occurrence of failure to restore the system to its operating condition either by repair or replacement. This type of maintenance is usually adopted when the failure costs or financial consequences, threat to human safety or environmental impact due to failure is less compared to the maintenance cost of the system.

Figure 2.1 shows different types of maintenance planning methods and various actions that can be taken after the inspection. While corrective maintenance is carried out after the occurrence of failure, preventive maintenance is carried out before the occurrence of failure with an aim to reduce the probability of failure and its consequences. Some of the commonly adopted preventive maintenance methods are time-based maintenance, condition-based maintenance, reliability-based maintenance, and risk-based maintenance (Barone, 2014; Bashiri, 2011). Time-based maintenance is a scheduled periodic maintenance that is carried out regularly (Bashiri, 2011). It is similar to regular maintenance of car involving changing its oil and filter etc. at periodic intervals for proper functioning of the engine. Condition based maintenance involves monitoring governing parameters of the system at regular intervals and carrying out maintenance when the parameters reach a threshold value (Bashiri, 2011). As per ASME (2019) liquid pipelines require repair or replacement when the metal loss in the wall of pipeline is greater than 80% of its wall thickness. Reliability-based maintenance and risk-based maintenance are probabilistic based maintenance approaches (Barone, 2014) discussed in detail in Section 2.3.4.
Suitable maintenance action is carried out following the selection of type of maintenance strategy as shown in the Figure 2.1. No action is taken if the defects are under accepted limits else defects can be either repaired locally or the entire section of the system can be repaired or replaced. Decision to repair or replace one or more sections will depend upon factors such as the remaining life of the structure, amount of deterioration that has taken place and is expected in future, and the capital available that can be invested in maintenance activities. Frangopol & Liu (2007) studied the comparison among four types of maintenance actions adopted in the slab of a bridge system which are increasing the slab thickness, injecting resin, attaching steel plates and replacement. It was concluded that out of four options, repairing by injecting the resin was the cheapest option that considerably improved the condition of the slab whereas replacing the entire bridge slab section was the most expensive option, but its reliability levels will be restored to its original state.
2.2 Structural deterioration modelling

Inspection of a structural system helps in identifying and characterizing defects. Often, defects like cracks do not remain constant and tends to grow over time. While evaluating cost over the service life of the structure, it is essential to know the sizes of these defects in the future in order to estimate the total expected costs due to failure depending on its probability of failure. Modelling of structural deterioration is therefore necessary to evaluate the state of deterioration at any future instant of time based on past inspection data. Models can be developed according to different types of defects and different modes of failure. In this study focus will be on structural deterioration due to corrosion, hence detailed review on corrosion growth models is provided in this section.

2.2.1 Corrosion growth model

Corrosion growth models (CGM) can be classified into three categories: Mechanistic, semi-empirical and empirical models (Nešić, 2007). Mechanistic models are theory-based models focusing on the underlying electrochemical reactions which are responsible for corrosion. Most of the parameters considered in these models have physical meaning and they hardly require any additional data through experiments to calibrate the model. On the other hand, empirical models are data driven models which does not have any theoretical support and are purely experimental based. Hence the model will be more accurate as more data are available. Accuracy of these models are limited to the areas with identical conditions with that of the experiments, else they will lead to unreliable results. Semi-empirical models employ combination of both mechanistic and empirical models, which utilizes experiments where there is lack of theoretical knowledge about parameters.

A considerable amount of literature has been published on empirical models using deterministic and probabilistic methods (Melchers, 2003). In this study, a probabilistic empirical model will be
used; empirical model will be most suited as the main focus is on utilizing data available through inspection results. Also, due to the stochastic nature of the corrosion process, probabilistic models will be the preferred choice to account for uncertainty. Uncertainties that are present in the model are broadly classified as aleatory and epistemic uncertainties (JCSS, 2008). Epistemic uncertainties can be reduced by gathering more data whereas it is not possible to reduce the aleatory uncertainties in such a way. Example of epistemic uncertainty include measurement error which arises from imperfect inspection tool (Dann et al., 2018) but can be reduced by using better inspection tools and techniques. Example of aleatory uncertainties include temporal uncertainty in corrosion process which is stochastic in nature and not constant over time (Pandey et al., 2009).

Different CGMs have been used based on the requirement and the type of application (Dann & Maes, 2018). Vanaei et al. (2017) reviewed different CGMs that are most adopted in the pipeline industry. Authors in this paper classified the models as deterministic and probabilistic models. Under the category of deterministic models, single-value, linear and non-linear corrosion growth rate models were discussed. As the name suggests single-value models consider constant growth rate such as value of 0.4mm/year as recommended by the National Association of Corrosion Engineers (NACE). They are the most straight forward models to estimate corrosion growth rate, but it does not take into account the depth of defect and age of pipe, which is a major drawback (Caleyo et al., 2003). This limitation is overcome in linear and non-linear corrosion growth rate models which considers the growth of defects over time by using linear and non-linear growth rates, respectively. Hence, at least two sets of inspection data are required to use these models (Valor et al., 2013).

Probabilistic models such as Markov, time-dependent Generalized Extreme Value Distribution (GEVD), time-independent GEVD model, Gamma process and Brownian Motion with Drift
were also discussed by Vanaei et al. (2017). Markov models assume that the future state is influenced only by the current state and not by prior events. Markov models can be considered as a continuous time model which is discretized, and depth of defects is converted as Markov state units to determine the distribution of defect size at that state. Due to the characteristic nature of BMWD model, it can be adopted as a good approximation to replicate alternating active and passive state behavior of defects but may not be suitable for defects whose degradation is non-decreasing.

Corrosion growth rate can be used to predict the future corrosion state (Desjardins, 2001) and an accurate estimation of corrosion growth rate is essential as its underestimation may lead to unexpected failure while overestimation may lead to unnecessary resources for frequent inspection and maintenance (Kiefner et al., 2007). Gamma process models uses gamma distribution which can more accurately reflect the increasing growth of defect in a corrosion process. In literature, Gaussian models were used to a great extent due to its simplicity and mathematical ease (Younsi et al., 2013). However, the possible negative values of the Gaussian distribution cannot be justified in modelling a corrosion behavior which is monotonically increasing (van Noortwijk et al., 2007). As a result, negative values are truncated while being adopted in CGMs which seems logical but at the same time there is loss of valuable information due to this approximation (Younsi et al., 2013). Whereas gamma process models will be better suited for CGM due to the strictly positive growth increments (Pandey and van Noortwijk, 2004).

2.3 Life-cycle cost optimization and decision analysis

Lifecycle cost of a structure is the total cost incurred over its lifetime and it is an increasingly important approach in selecting the best alternative for inspection and maintenance actions in a cost optimization problem. Construction cost, operation cost and maintenance cost are the three
major costs components of the lifecycle of any structural system (Sarma & Adeli, 2002). In order to minimize the maintenance costs component of the lifecycle costs, optimal inspection planning and maintenance strategies are required. Although the overall objective is to minimise the total costs but with maintenance and inspection planning only the maintenance cost component will be affected as inspection and maintenance planning is independent of other two components. Hence the other cost components will not be considered while determining optimal inspection and maintenance strategy.

Inspection planning is often viewed as a decision problem involving decisions such as which attributes to inspect, frequency of inspection, inspection methods and the inspection locations (Faber, 2002). Similarly, maintenance planning involves decision such as whether to carry out repair, when to repair, etc. In order to minimize the lifecycle costs of the system and reach a decision point, it is essential to optimise the inspection and maintenance plan through a formal decision-making process. Hence, the prime objective of carrying out decision analysis is to make a best decision among the available options (Jordaan, 2005). The best decision is decided mostly in terms of monetary values, which provides the highest monetary benefits and lowest cost or risk (Rackwitz et al., 2005).

If the consequences of an event are well known, then the condition of certainty exist, and the best choice can be determined which yields maximum utility. Since most of the decisions lead to consequences that are uncertain owing to unknown and unpredictable future states, probabilistic analysis is required where probability distribution functions are assigned to the variables. Expected utility theory provides then a rational way of assigning utilities to each consequence and choosing the best decision which maximises the expected value of utility (Parmigiani et al., 2009). The expected utility in terms of benefit, cost and risk is then given by
E[U(z)] = E[B(z)] - E[C(z)] - E[R(z)] \quad (2.1)

Where E[B(z)] denotes the expected economic benefit achieved from the system, E[C(z)] denotes the expected cost incurred in maintenance of the system, E[R(z)] is the risk expressed in terms of expected monetary loss incurred i.e., failure cost as a consequence of failure of the system and z denotes vector of decision variables. The optimal decision is chosen as the one which maximizes the above objective function.

\[ z_{opt} = \max_z \{E[B(z)] - E[C(z)] - E[R(z)]\} \quad (2.2) \]

While carrying out decisions related to inspection and maintenance of a system, benefits associated with the system will not be altered since the objective is to restore the system to its original functionality. Hence, expected benefit can be eliminated from the above equation.

\[ z_{opt} = \min_z \{E[C(z)] + E[R(z)]\} \quad (2.3) \]

A decision-making process can be easily visualized through a decision tree, which integrates the set of decision alternatives z and possible outcomes \( \theta \) associated with each decision along with probability of the outcome \( P \) and its utility \( U \). The generic decision tree as shown in the Figure.2.2 consists of decision nodes and the event nodes. A set of available decision alternatives \( z = \{z_1, z_2, ..., z_n\} \) branches out from the decision node and each alternative leads to different outcomes \( \theta = \{\theta_1, \theta_2, ..., \theta_m\} \) which are further branched out from event nodes where \( p(\theta_j \mid d_i) \) denotes the conditional probability of occurrence of outcome \( \theta_j \) given the decision \( d_i \). Utility value \( U(z_i, \theta_j) \) is evaluated for each outcome corresponding to its respective decision. Then the best decision \( z^* \) is given by \( z^* = \max \{ E[U(\theta,z)] \} \) (Ang et al., 1975).
The concept of utility theory was first introduced by von Neumann and Morgenstern to study the human behavior influenced by economic factors (von Neumann et al., 1947). Decision analysis forms an important framework in the life-cycle cost optimization for achieving optimal results. To date, several researchers have considered principle of life-cycle cost optimization in a wide range of engineering systems and structures (Rosenblueth & Mendoza, 1971) such as offshore structures (Kubler & Faber, 2004), bridges (Kong & Frangopol, 2003), pipeline networks (Tee et al., 2014), etc. for cost optimal design and maintenance planning.

Lifecycle cost analysis considering maintenance and failure costs was carried out to estimate optimal wall thickness in designing a pipeline (Zhou & Nessim, 2011). Cost efficient, ice-resistant offshore structure was designed based on minimum expected lifecycle costs (Li et al., 2009). A framework based on lifecycle cost was developed to plan optimal inspection and maintenance actions for a corroded structure (Faber & Sorensen, 2002). Sarma & Adeli (2000) (2002)

Figure 2.2: Generic decision tree (modified from Parmigiani, G et al., 2009)

\[ z^* = \max\{E\{U(\theta, z)\}\} \]
considered multiple criteria such as weight and perimeter of the sections, economical sections that are available in market, etc. to design a steel structure with optimal life-cycle costs. Criterion of perimeter of the sections were considered as it represents the maintenance costs associated with the painting of the sections.

2.3.1 Bayesian inference

Bayesian inference is the process of drawing conclusion for the model parameters and unobserved data conditional on an observed data (Gelman et al., 2013; Congdon, 2014; Smith, 2010). For instance, the model parameters that are of interest can be number of corrosion pits or depth of corrosion or length of crack, etc. which is available from the structural deterioration model and this data represents the prior knowledge about the system. In order to make optimal decisions related to integrity management, inspection is carried out to observe the existing state of the system such as inspecting actual number of corrosion pits and these inspection results represents the evidence. Prior knowledge or belief about a system is updated once more evidence is available using Bayes theorem (Gelman et al., 2013). If the vector \( x = \{x_1, x_2, \ldots, x_n\} \) are the set of unobserved data or model parameters and the vector \( y = \{y_1, y_2, \ldots, y_n\} \) are the set of data that is being observed, then their joint density function is given as

\[
p(x, y) = p(x)p(y|x)
\]  

(2.4)

The joint probability density function (PDF) \( p(x, y) \) is obtained as a product of prior PDF \( p(x) \) based on our prior belief before taking the observable \( y \) into account and the conditional PDF \( p(y|x) \) of observing \( y \) given \( x \). Using Bayes theorem, posterior PDF can be determined as

\[
p(x|y) = \frac{p(y|x)p(x)}{p(y)}
\]  

(2.5)
The posterior PDF $p(x|y)$ represents our knowledge about the parameters $x$ after having observed the data $y$. The denominator $p(y) = \int p(y|x)p(x)dx$ is the prior predictive PDF or marginal likelihood of the data $y$. The function of the marginal likelihood is to normalize the posterior PDF and ensure that the PDF integrates to one. Most often, the above equation is expressed as $p(x|y) \propto p(y|x)p(x)$. Since the marginal likelihood may involve solving a high dimensional integral, which is difficult to compute analytically, and closed-form solutions are only available for limited family of distributions. Hence numerical approximations are required to solve for the high dimensional integral (Congdon, 2014; Gelman et al., 2013; Smith, 2010).

### 2.3.2 Bayesian inference methods

Different types of Bayesian inference methods are shown in the Figure 2.3. Apart from the exact methods, Bayesian inference can be performed by approximate methods that consist of stochastic methods and deterministic methods (Birkland & Dann, 2018). Stochastic/simulation methods involve generating random numbers from the desired posterior distribution and then estimating the parameters of any function (Gelman et al., 2013). Parameters thus obtained depends upon the generated sample and since structural reliability deals with very small probability values and tail behavior, this will require a large sample size to get accurate results. Markov Chain Monte Carlo (MCMC) method is the popularly used simulation methods. MCMC methods include algorithm such as Metropolis Hasting and Gibbs Sampling for obtaining sequence of random samples (Gelman et al., 2013)
Even though simulation methods involve simple analysis, it is suited only for small datasets owing to large computational time (Efron, 2012). When the dataset is large especially in the case of inspection results, it is challenging to run MCMC efficiently. This drawback can be overcome by using deterministic methods such as Variational Inference (VI), which tends to be much faster than the traditional MCMC method (Blei et al., 2017). Variational inference, a concept from machine learning, is widely used to solve high dimensional integrals as a numerical approximation. VI solves inference problem of intractable probability distributions $P$ through optimization from set of tractable probability distributions $Q^*$ in order to find $q$ which is similar to $P$ (Blei et al., 2017; Jordan et al., 1999). VI can be used as an alternative to simulation techniques for inferring parameters from corrosion growth models (Dann & Birkland, 2019).
2.3.3 Bayesian decision making

Bayesian decision making process involves application of Bayesian inference in decision making process where the probabilities are updated once new evidence is available (Smith, 2010). Based on the nature of information available during the decision-making process, Bayesian decision analysis can be classified as prior analysis, posterior analysis, and pre-posterior analysis. In the prior analysis, decision is made based on the already available information (Benjamin and Cornell, 1970). It is carried out when the state of nature of system is known and a decision has to be made. However, this does not indicate that the exact nature of true state is known, rather it is not possible to get more information prior to making a decision (Straub, 2004). Hence expected utilities are determined from the known probabilities and subsequently decision which yields maximum utility is selected. Examples of prior analysis can be found in most of the analysis related to decision making in environmental science for ranking different environmental project alternatives (Huang et al., 2011), in medical field for estimating health status using multi-attribute utility index (Furlong et al., 2001) among a vast variety of fields.

In order to reduce the uncertainty about the system to get better knowledge and making a particular decision, generally inspections and tests are carried out to know or get more information about the present state. Although additional cost and resources will be required in conducting the tests, such expenditure may provide valuable information about the system. Based on the test results, prior PDF is updated using Bayes theorem to get the posterior PDF and consequently expected utilities are determined to choose the best decision. Decision based on such analysis is called as posterior analysis which is similar in principle as prior analysis however it involves updated probabilities as new evidence is available. Typical examples of posterior analysis include structural health
monitoring techniques which are adopted to reduce uncertainties in structural system such as using fibre optic sensors to monitor bridges (Casas & Cruz, 2003).

In certain scenarios, extracting additional information might not be financially beneficial resulting in net negative benefit beyond an optimum level. Pre-posterior decision analysis can be used to evaluate potential benefits that can be obtained by utilizing different inspection and monitoring techniques. Figure 2.4 shows a generic pre-posterior decision tree. Pre-posterior analysis can be considered as a two-stage decision making problem which is carried out to decide if it is necessary to spend resources to get additional information about the system in the first stage and then making decision based on test results in the second stage.

**Figure 2.4: Generic decision tree for pre-posterior decision analysis (modified from Faber, M. H., 2005)**
Freeze, R. A., et al. (1992) used a pre-posterior analysis to determine the best design option with optimum site investigations. Pre-posterior analysis was used to assess the value of monitoring in buildings which are subjected to earthquake hazard (Omenzetter et al., 2016). In the literature, pre-posterior analysis is mostly implemented in optimizing inspection and maintenance plans (Vereecken et al., 2020). Goulet et al. (2015) developed a framework based on pre-posterior analysis to achieve cost optimal action plan meeting reliability constraints. Decision based on pre-posterior analysis has been widely used as an effective tool in risk-based inspection and maintenance planning (Faber, 2002).

2.3.4 Optimal inspection and maintenance approaches

One of the important applications of pre-posterior analysis is to optimize the lifecycle cost of the structure through inspection and maintenance planning. Preventive based approaches are commonly adopted to arrive at a cost optimized plan as it ensures proactive measures avoiding failure costs due to sudden failure.

2.3.4.1 Reliability-based approach

Reliability is defined as the probability that the system or component will perform its intended function without failure over a period of time and it can be determined as 1 minus the probability of failure (Elsayad, 2012). Reliability analysis is carried out to determine the reliability of the system/structure and it is classified into two categories as classical reliability analysis and structural reliability analysis. Classical reliability analysis is carried out for mechanical and engineering systems/components which are subjected to similar exposure conditions. They can be tested for failure and these results can be used as empirical data to evaluate similar other components. But same principle cannot be adopted for structures as each one is unique and testing them is not possible as they fail under extreme load conditions. Therefore, there is no prior failure
data available to evaluate in case of structures. Hence, structural reliability analysis (SRA) deals with determination of probability of failure of structures by using a limit state function (LSF). LSF (g) is determined as the resistance developed by the structure against the load acting on it. For a structure to be safe, the limit function (R - S) > 0 where R is the resistance and S is the load effect (Ditlevsen & Madsen, 2007; Thoft-Cristensen & Baker, 2012).

Decision based on reliability analysis was carried out to prioritize the selection among different ready-mix concrete supplies (Chou & Ongkowijoyo, 2015). Faber et al. (2003) proposed a framework for assessment of existing structures based on reliability theory using information obtained from inspections. Several literatures focus on inspection and maintenance planning and optimization through reliability-based decision making (Kong & Frangopol, 2003; Navarro et al., 2019).

### 2.3.4.2 Risk based approach

The risk associated with an event can be determined as the product of the probability of the occurrence of this event and the associated consequences if this event happens. Risk based approach can be considered as an extension of reliability-based approach where the risk of failure of the structure considers consequences due to failure in addition to the reliability/ probability of failure of the structure (Faber & Stewart, 2003). In risk-based approach, the risk of failure shall be kept under the maximum risk level that is acceptable. Its main goal is to maintain the integrity of the deteriorating system to keep it safe and functioning by optimally allocating resources to riskier locations. Therefore, systems that are prone to more risks are subject to frequent inspection and maintenance to keep their risk of failure under acceptable level.
In order to assess if the risk of failure is within the acceptable limit, risk can be evaluated qualitatively, quantitatively, or semi-quantitatively (DNV, 2010). Qualitative approach employs descriptive measure (such as high, medium, low risk) instead of measured quantity and gives a ranking of risk generally expressed in a matrix form. It is simple and quick form of assessment, but it is subjective in nature. Quantitative method involves assigning numerical values to quantify the outcomes and are objective in nature. Fault/event trees are utilised in estimating probability values and then the risk values are assigned accordingly. Semi-quantitative method is a combination of both the methods, for instance consequences can be assessed qualitatively and probability of failure can be evaluated quantitatively or both probability and consequence of failure can be assessed quantitatively, and risk can be expressed qualitatively. Since qualitative and quantitative approach are subjective in nature, it is not preferred since they are unreliable and subject to change from person to person (DNV, 2010).

Consequence which is the result or outcome of an event is another important component in risk-based analysis. Consequences are mostly evaluated in terms of environmental, safety and economic consideration (DNV, 2010). Environmental consequences involve impact of hazard on the environment such as release of pollutant in the atmosphere or financial costs required for cleaning and containing the pollutant. Safety consequences involve physical injury or number of fatalities and economic consequence involve monetary loss due to repair or replacement of the failed system which can be quantified directly. Indirect consequences such as societal impact is more difficult to be quantified and requires rigorous modelling (Faber & Stewart, 2003). Hence most commonly only direct consequences are considered in the risk assessment and their values are factored up to consider the indirect consequences inherently as an approximation. (JCSS, 2008).
Figure 2.5 shows a generic framework for RBIM process. The process begins with system definition or specification which involves defining the system, its components, and any dependencies among the components. In the next step, any previous inspection data are collected to gain knowledge about the deterioration state of the system. The data are then utilised to model the degradation mechanisms to predict future states of the system and to estimate its probability of failure by reliability analysis. Consequences due to possible failure events are evaluated which is then multiplied with the probability of failure to obtain the risk of failure. Risk of failure is checked against the acceptable risk criterion to ensure that it is safe and within the acceptable limit. Based on risk assessment, inspection planning is carried out through Bayesian decision analysis and maintenance actions are carried out based on the inspection outcome. New information obtained from the inspection result is utilised to update the current information about the system and this process repeats throughout the lifespan of the system (API, 2009).
Figure 2.5: Generic framework for risk-based inspection & maintenance (RBIM) process (modified API, 2009)
2.3.4.3 Risk based inspection and maintenance

Risk based inspection and maintenance involves optimization of inspection and maintenance planning using decision analysis for risk-based approach. Risk-based inspection involves optimization of inspection planning such as type of inspection methods to be adopted, timing of the inspection, locations in the system to be inspected, etc. Figure 2.6 shows decisions to be considered while formulating inspection plan based on RBI.

![Figure 2.6: Inspection plan based on RBI (modified from DNV, 2010)](image)

Similarly, risk-based maintenance involves optimization of maintenance decisions. The main objective of optimization based on risk-based approach is minimization of consequences such as inspection and maintenance costs. Figure 2.7 shows a generic decision tree for (RBIM) where RBI uses pre-posterior analysis of Bayesian decision making and RBM involves prior/posterior analysis. In the pre-posterior decision analysis of the figure, RBI aims to minimize the overall costs by deciding on number of sections to be inspected and then based on the inspection outcomes such depth of corrosion or percentage material loss, maintenance decisions are carried out whether to repair or replace the section or no action to be taken. Finally, consequences are evaluated by considering failure costs based on the system state.
Many researchers have developed framework for inspection and maintenance based on risk analysis (Balkey et al., 1998; Harnly, 1998; Kallen, 2002). A detailed overview of the risk-based inspection technique is presented by Faber (2002) and there are several literatures related to application of RBIM in engineering systems and structures. Risk-based approach is adopted in optimizing inspection and maintenance intervals in safety related systems (Vaurio, 1995) and oil refinery plants (Bertolini et al., 2009). Risk based approach was used for determining optimum maintenance plan for reducing the risk of failure in a power generating plant (Krishnasamy et al., 2005). RBI was used for determining optimum inspection intervals in pipelines subjected to corrosion (Gomes et al., 2003) and for optimal inspection planning in an offshore structure (Goyet et al., 2002) and pressurised system (Hagemeijer & Kerkveld, 1998). Optimal maintenance

Figure 2.7: Generic decision tree for risk-based inspection & maintenance (RBIM) process
planning in an offshore structure (Pui et al., 2017) and pipelines (Nessim et al., 2000; Seo et al., 2015) were carried out through risk-based decision analysis.

There are several Standards and regulations that outline various steps and recommend best practices to be adopted in RBIM. Even though the code can be used as a foundation for carrying out RBIM process, but it does not effectively provide an optimum solution (Haladuick & Dann, 2016).

2.3.4.4 Value of information

Value of information is a quantity based on pre-posterior analysis of Bayesian decision. It is analytic method of quantifying the benefit achieved with additional information and can be determined as the difference in maximum utility obtained by performing pre-posterior analysis and the maximum utility obtained by considering only prior analysis. Therefore, it is the amount a decision maker is willing to spend to make an informed decision (Straub, D. 2014).

It can be expressed as,

\[
\text{Value of Information} = \text{Cost without inspection} – \text{Cost with inspection}
\]  

(2.6)

It means that decision to carry inspection is acceptable when the value of obtaining the information exceeds the cost that is spent on the inspection. The seminal work on VOI was carried out by Schlaifer and Raiffa (1961) and subsequently VOI analysis has found its application in variety of fields including medical field (Claxton et al., 2001; Tuffaha et al., 2014), management of environmental health risk (Yokota & Thompson, 2004), etc. Even though Bayesian pre-posterior analysis is adopted in various fields of engineering and infrastructure systems for several years, quantifying the benefits explicitly through VOI has gained popularity only in the most recent years. In structures, majority of the research focusses on optimal placing of sensors for efficient structural
health monitoring (Malings & Pozzi, 2016a; Malings & Pozzi, 2016b; Krause, 2008; Pozzi & Der Kiureghian, 2011; Zonta et al., 2014). VOI is applied to the structures subjected to natural disasters by employing it as one of the metrics for quantification of benefits achieved through optimal sensing in bridges post earthquake (Malings & Pozzi, 2016a). Also, in case of offshore structures subjected to extreme wave loads, the potential benefit of using an improved climate model was quantified using VOI analysis (Garré, & Friis-Hansen, 2013).

In terms of choosing best inspection technique in the upcoming inspection, heuristic framework based on VOI was developed (Haladuick & Dann, 2018). In this framework, decision is based only on the next inspection instead of the entire lifecycle, hence approximation in analysis is done by considering independent analysis of the upcoming inspection and separating it from the lifecycle decision analysis. Due to this approximation, VOI is estimated as a range with an upper and lower bound instead of single value estimate. As a result, inspection decisions are acceptable if the cost of inspection lies outside the bounds of VOI then it can be used as an alternative to life-cycle analysis else as an initial estimate for the LCC analysis.

In this study, VOI analysis will be carried out to decide whether inspection is required and the extent of inspection. The detailed methodology for evaluating value of information will be provided in Chapter 3 of this thesis. Numerical examples based on this framework will provided in Chapter 4 of this thesis.
3. METHODOLOGY

The main contribution of this research, which is developing a framework to achieve minimal lifecycle costs through risk-based inspection and maintenance planning, is presented in this chapter. Section 3.1 provides an overview of the steps involved in the proposed framework and the subsequent sections explain these steps in more detail along with the assumptions and considerations in each step. Section 3.2 presents different approaches to model the system that can be adopted in this framework. Section 3.3 presents structural deterioration modelling to predict the corrosion state in future based on past inspection data. Section 3.4 provides an insight on structural reliability analysis to determine the probability of failure. Section 3.5 covers cost modelling to formulate the objective function for lifecycle cost analysis and determination of total lifecycle cost. Section 3.6 focuses on determining optimal inspection strategies while Section 3.7 is centered around optimal maintenance actions.

3.1 Overview

Inspection and maintenance are carried out throughout the lifetime of the structure to retain its integrity, where the related strategies can significantly affect the overall lifecycle cost of the structure. Although different strategies and optimization criteria can be considered to lower the total costs, minimizing lifecycle cost is the most comprehensive approach (Rackwitz, 2005). The objective of the proposed framework is to minimize the total lifecycle cost through risk-based lifecycle cost optimization. It will help the engineers and decision makers to realistically assess the cost that would be incurred in future and thereby reducing the lifecycle costs by adopting optimal strategy over the remaining lifespan of the structure. To achieve this objective, the proposed framework can be broadly classified into following sections:
a) system modelling to define and model the structural system (Section 3.2)
b) structural deterioration modelling to model the deterioration in the structure (Section 3.3),
c) structural reliability analysis to determine probability of failure (Section 3.4),
d) lifecycle cost analysis to determine costs due to inspection, repair, and failure (Section 3.5),
e) inspection decision making to determine optimal decisions related to inspection of the structural system (Section 3.6), and
f) maintenance decision making to determine optimal maintenance plan conditional on the inspection outcome (Section 3.7).

Based on the abovementioned broader sections in the thesis, various steps involved in the framework are illustrated in Figure 3.1. The framework begins with modelling the structural system using an appropriate approach depending on the user requirement. The user has an option to select a condensed approach for a faster system level analysis or a detailed approach focusing on the component level. The next step in the framework involves collecting previous inspection data to know the presence of any defects and to predict their growth in future. Before utilising this data, it is essential to carry out preliminary data analysis to check if the data is consistent and error-free. If the data seems to be inconsistent, then check the reason for such inconsistencies and eliminate any noise present in the data. Once the data are validated, then it will be used for developing the corrosion growth model in order to predict the future growth of these defects until the end of system’s lifetime.

A probabilistic model will be utilised for modelling the corrosion growth using the information obtained from previous inspections. Over the period of time, as more information about the system becomes available through inspections, the unknown parameters of the corrosion model can be updated through Bayesian inference methods. Based on the complexity of the data available from
earlier inspection results, the user can choose whether to analyze the corrosion growth model using Markov-Chain Monte Carlo (MCMC) or Variational Inference (VI). In case of a small inspection dataset, MCMC can be employed to get accurate results. In the case of larger dataset, an approximate method of Bayesian Inference namely Variational Inference (VI) can be employed to ease the complexity and reduce the computational time. The corrosion growth model is then updated, and future states of corrosion is estimated.

In the next step, structural reliability analysis is carried out to determine the probability of failure of the structural system based on the limit state functions. Depending on the number of failure modes, limit state function for each case is evaluated to determine the probability of failure. This step is followed by cost modelling to determine the total costs incurred in the lifecycle of the structural system. Apart from inspection costs and maintenance costs, it also includes risk in terms of failure cost of the structural system. Following this step, optimal inspection and maintenance strategies are determined resulting in minimal lifecycle cost.

Appropriate inspection and maintenance decisions, such as whether or not to inspect and/ or repair, resulting in minimal lifecycle cost of the system, will be considered. The optimal number of inspections can be determined by quantifying the benefits achieved from additional inspection based on the value of information. Decisions related to maintenance actions will depend on the information obtained from inspection. Optimal repair criterion and optimal probability of failure for the repair criterion can be determined to achieve maintenance decision at the lowest cost level to carry out the maintenance action. After the last step, the loop continues from the beginning as the maintenance event will update the future inspection data and this cycle continues till the end of service life.
Figure 3.1: Framework for inspection and maintenance planning
3.2 System modelling

In the first step of the framework, system modelling for the corrosion growth analysis and the subsequent decision making for optimal inspection and maintenance strategies can be carried out through any of the following approaches as shown in Figure 3.2.

a) Population-based approach where all the defects are considered as one population (no discretization, no defect-specific approach)

b) Discretized-system approach where system is split into sections/components and the most critical section or the most severe deterioration per section may be considered

c) Defect-specific approach where all the defects are considered individually

![Diagram showing different approaches to model a system and their corresponding deterioration model.](image)

**Figure 3.2**: Different approaches to model a system and their corresponding deterioration model for (a) Population-based approach (b) Discretized system approach (c) Defect-specific approach.
The user or decision-maker needs to choose a suitable approach to model the system depending on the requirement. A population-based approach can be carried out for system level inspection and maintenance. The deterioration model in such a case is developed by considering all the defects as one population and the subsequent decisions are made at system level i.e., decision to inspect and/or maintain the system. In case of a defect-specific approach, each defect in the system is considered individually and the deterioration model is developed for individual defects. Decision related to inspection and maintenance is also carried out specific to defects.

In defect-specific approach, the individual defects from different inspections are first matched and the growth of defect for each of the matched pair is considered for analysis. Whereas in case of population-based approach, no such matching is carried out and the defect growth is estimated for the entire set of defects. Therefore, population-based approach is advantageous if the number of defects is large, and a faster analysis is required as matching these defects from different inspection outcomes can be cumbersome process.

The third approach mentioned in the above list is discretizing the system into their components or smaller sections. In such case either the critical sections can be considered for the analysis or only the most critical defect in each section can be considered where the decision made for critical defect in a section is applied to all the defects in that particular section.

To illustrate the system specification, a structural frame can be discretized into beam and columns as its components or the beam itself can be discretized into smaller sections if the length of the beam or the number of defects is large. The size of sections for discretization and the basis to choose the corrosion analysis approach and subsequent decision making will depend on a case-by-case basis. The objective of discretization is to achieve approximately homogenous deterioration within a section. Figure 3.3 shows few examples of different structural systems where this
framework can be employed and schematic illustration of its discretization. A truss system can be discretized by its members, whereas slabs and pipelines can be discretized into smaller strips.

Figure 3.3: Discretization of structural systems into sections or components: (a) Truss system, (b) Slab system, and (c) Pipeline segment.
Depending on the size of the structural system, the number of defects present can be fewer or vast in nature. More defects lead to more datasets which in turn increases the complexity and computational time of the analysis. Hence, the framework also gives consideration to the complexity of the data available as a trade off between faster or accurate results depending on the priority of the user.

3.3 Structural deterioration modelling

The second step of the framework is to develop a structural deterioration model to predict the future physical state of the system. Although the strength and serviceability of structural systems can be affected by numerous factors requiring consideration of various deterioration mechanisms and their corresponding models, the emphasis of the framework is on optimising inspection and maintenance decisions rather than focussing on the deterioration model or mechanism to be employed. Hence, for simplicity, only corrosion is considered in this framework as the leading deterioration mechanism causing local corrosion in the structure. Local corrosion is a type of corrosion where the attack is targeted at localised spots as opposed to global corrosion that occurs uniformly across the structure or a section of it. The nature of corrosion and the resulting defects can be external or internal depending on the type of system and the corrosive medium. For instance, in case of a buried transmission pipeline, external corrosion causes metal loss on the outer surface of the pipeline due to environmental factors, whereas the internal corrosion causes metal loss on the inner surface of the pipeline due to corrosive nature of the materials being transported. The proposed framework can be applied to any structural system subjected to local or pitting corrosion due to internal or external corrosion effects. That being said, this framework can be also adopted for structures subjected to other deterioration mechanisms with appropriate assumptions. Figure 3.4 outlines the steps involved in developing a structural deterioration model.
The framework for developing the model begins with collecting the data from previous inspections and matching the data from multiple inspections when a defect-specific corrosion growth analysis is carried out. No such matching is required in case of a population-based analysis. Although this section is not the focus of the research, it is briefly explained here for the sake of presenting a complete framework. After gathering the data, a suitable deterioration model is developed which can best represent the deterioration process considered in the system. Since the model is developed over the remaining life of the system, which can be considerably a long time, temporal uncertainties need to be considered in addition to spatial uncertainties, model uncertainties and uncertainties related to errors from the inspection tool. The final step in this process will involve updating the model through Bayesian inferencing to properly assess the uncertainties with regards to state of condition of the system. These above-mentioned steps are discussed in detail in the following sub sections.

### 3.3.1 Data collection and validation

As mentioned earlier, the framework can be adopted to any type of deterioration model. To introduce complete framework, a generic corrosion growth model will be presented. The first and foremost step involved in this process is collection of existing data about the system to develop the mathematical model. In order to model the individual defects or population of defects,
inspection results will be required to see the trend of past growth of corrosion defects and to further predict future growth.

While carrying out defect specific corrosion growth analysis, it is necessary to match the defects reported from multiple inspections for an accurate analysis. The defects can be matched either manually or through automated algorithms or by incorporating both methods (Dann & Dann, 2017). The objective of matching the defects is to determine corrosion growth of individual defects and check the consistency of the data and remove any outliers if present. One simple technique to validate the matched data from different inspections is by plotting a histogram or PMF of the defects size to check the similarity between the distributions. With the passage of time, there will be shift of PMF values from lower defect sizes to the higher defect sizes if there is active corrosion present. The plot may also reveal if there is any inconsistency present between data from different inspections. Some reasons for inconsistencies in the data may be due to the errors resulting from the inspection tool which are covered in more detail in Section 3.3.2

Figure 3.5 shows an example of PMFs of size of defects expressed as a percentage of its initial wall thickness (%wt) recorded during two different inspections taken a few years apart. It is evident from the figure that the PMF values of the defect sizes observed during the second inspection have shifted right towards higher metal loss indicating an increase in size of the corrosion defect and that the corrosion is active. If the shift was on the opposite side, then it means that defect size has decreased in size which is impractical and incorrect.
3.3.2 Corrosion growth model

Corrosion is a stochastic process involving spatial and temporal uncertainties, meaning there is variability in the corrosion growth process in space and time. In addition, there is uncertainty associated with the inspection tool measuring the corrosion defect giving rise to several measurement errors. An overview of the different types of corrosion growth models is explained in the chapter 2 of this thesis. User has variety of corrosion models to choose from and can adopt any corrosion model as long as it effectively considers all the uncertainties involved in a corrosion growth process. As the focus of the research is on decision making process with an aim to provide an overall framework in achieving optimal inspection and maintenance strategies, generic corrosion growth model will be considered which is capable of considering all the uncertainties. Maes et al. (2008) proposed a hierarchical Bayes framework to account for uncertainties involved.
in a structural deterioration model. Figure 3.6 shows hierarchy of uncertainties considered in the deterioration model where the variables in each level are conditional upon the variables in the higher level. The bottom-most-level shows uncertainty due to inspection tool which is epistemic in nature while the next levels show spatial uncertainty and temporal uncertainty which are aleatory in nature. Finally, there is model uncertainty which is again epistemic in nature. Unlike aleatory errors, epistemic errors can be reduced with better inspection tools and better knowledge about the models.

Figure 3.6: Hierarchy of uncertainties in structural deterioration modelling

Spatial uncertainty refers to the difference in corrosion at different locations at a given time. Spatial uncertainty can be addressed by appropriately modelling the condition state considering the spatial variability among defects at different locations. For instance, this can be achieved by discretizing the system into smaller sections leading to homogeneity in the population of defects within the section. This is due to the fact that, defects closer to each other might be exposed to similar
conditions compared to the ones faraway ensuring homogeneity within smaller sections. If the defects are faraway then it is suitable to consider individually to account for the heterogenous nature of the corrosion process. To demonstrate that the framework developed in this research can be applied to any deterioration model with spatial variability, population of defects within a section is considered in the first example whereas individual defects are considered in the second example of Chapter 4.

Temporal uncertainty refers to the difference in corrosion at a given location during different time. Developing the corrosion growth model can be implemented in two main steps i.e., determining the past corrosion growth and predicting the future corrosion growth. At least two inspection results are required to estimate the growth of defect in the past. If only one inspection result is available, it will be difficult to estimate the corrosion growth rate in the past as the slope of growth will be unknown. Else an assumption has to be made that the corrosion defect is initiated immediately after construction, which is a very strong assumption as the defect might not have generated immediately after the construction. While predicting the size of defect in the future state, temporal uncertainty can be addressed by considering independent corrosion growth increments. Since the growth increments needs to be non-negative, strictly positive they can be modelled using a gamma process. If defect specific corrosion growth analysis is carried out, then the past and future corrosion growth is calculated for individual defects. In case of a population-based approach, the past and future corrosion growth is determined for the population of defects.

Finally, there is uncertainty due to errors generated in the inspection tool while reporting number and size of the defects. Figure 3.7 shows an illustration of a typical corrosion defect in a pipeline and a steel rod, where the maximum depth of corrosion reported by the inspection tool will be used to develop the model.
Figure 3.7: Schematic diagram of a typical corrosion defect in (a) a pipeline (b) a steel rod

The non-destructive techniques adopted to inspect the system are subjected to various errors since the techniques involve indirect form of measurement. This can lead to different types of measurement errors as follows:

a) Detection error: Inspection tool fails to detect defects.

b) Sizing error: Inspection tool reports defect size different from the actual defect size.

c) False call error: Inspection tool reports a non-existent defect.

d) Reporting error: Inspection tool does not report defects when their size is smaller than reporting threshold
Detection error depends on the detection capability of the inspection tool which in turn depends on the actual size of the defect i.e., higher the defect size, greater chance for a defect to get detected. Therefore, probability of detection (POD) can be defined by an exponential function with a mean detection threshold $x_{De}$ (Maes & Salama, 2008).

$$\text{POD}(x_d) = 1 - \exp\left(\frac{-x_d}{x_{De}}\right) \quad \forall \ d = 1,2, \ ...D$$ \hspace{1cm} (3.1)

Figure 3.8 shows POD as a function of actual defect size $x$ for different values of $x_{De}$. Higher values of mean detection threshold has higher probability of detection as seen in the figure.

![Figure 3.8: Probability of detection for different values of mean detection threshold](image)

Sizing error $\varepsilon$ is the difference between the reported or the measured defect size $y$ and the actual defect size $x$. Therefore, the measured defect size is given by,

$$y_d = x_d + \varepsilon_d \quad \forall \ d = 1,2, \ ...D$$ \hspace{1cm} (3.2)
where $D$ is the total number of defects. Unlike detection error, sizing error is assumed to be independent of the actual size of the defect. In literature, it is assumed that the sizing error is usually normally distributed with a mean value of zero, and a given standard deviation, $\sigma_s$ depending on the inspection tool (Dann & Huyse, 2018).

$$\varepsilon_d | \sigma_s \sim Normal (0, \sigma_s)$$ (3.3)

Figure 3.9 shows a comparison between the probability mass function (PMF) of actual defect size and the measured defect size. The distribution of the reported defect size is much wider than the actual size due to the sizing uncertainty involved with inspection tool.

**Figure 3.9: PMF of defect size of actual defect and measured defect**
False call error is produced when the inspection tool reports defects that do not exist maybe due to some noise produced in the inspection tool. Similar to probability of detection, probability of true call is given by

\[ POTC(y_d) = 1 - \exp \left( \frac{-y_d}{y_T} \right) \quad \forall \ d = 1, 2, \ldots D \quad (3.4) \]

where \( y_T \) is the mean credible depth. Figure 3.10 shows values of POTC for different values of mean credible depth.

Figure 3.10: Probability of true call for different values of mean credible depth
Finally, reporting threshold is used to remove very small defects from the reported data as the likelihood is very high that many of these small defects are noise. Defects having size smaller than the reporting threshold are truncated and not included in the inspection data.

While determining optimal strategies, it is essential to make sure that the inspection data are perfect else the uncertainties related to inspection tool has to be taken into consideration (Maes & Salama, 2008). These uncertainties are considered explicitly or sometimes they are included in the corrosion growth model. Figure 3.11 shows hierarchical level of measurement errors considered within the inspection tool uncertainty for population of defects (Dann & Maes, 2015) and individual defects. For defect-specific approach only the sizing error is considered whereas in case of population-based approach, the number of defects or the measurement errors at each level is conditional upon the number of defects at higher levels as shown in the figure. The top-most level represents the actual number of defects whereas the bottom most level represents the number of defects reported by the inspection tool after being subjected different levels of errors. This hierarchy of errors is demonstrated in the first example of Chapter 4 while carrying out pre-posterior analysis for a population of defects.
Figure 3.11: Hierarchy of errors due to inspection tool considered in the corrosion model (a) population-based approach (b) discretized or defect specific approach
3.3.3 Bayesian inference

The next step in this framework is to update the unknown variables using the information obtained from the inspection. This updating is carried out through Bayesian inference where the knowledge about the unknown variable is updated by combining it with the new information gathered from the inspection outcome. The corrosion growth model which depicts the metal loss in future can be uncertain due to long time horizon. Hence, by applying additional information through Bayesian inference will lead to better knowledge about the state of corrosion in future. For instance, Bayesian inference will be used to make inference about the future size of the defect \( x_d \) conditional upon the size of defects \( y_d \) reported by the inspection tool using Equation (2.5) presented in Chapter 2 of this thesis. The posterior PDF is given by

\[
p(x_d | y_d) = \frac{p(y_d | x_d)p(x_d)}{\int p(y_d | x_d)p(x_d)dx_d} \quad \forall \ d = 1, 2, \ldots D \quad (3.5)
\]

The denominator term \( \int p(y_d | x_d)p(x_d)dx_d \) in the above equation represents the marginal likelihood of \( y_d \) which normalises the posterior PDF to unity. Solving this integral is quite complex due to non-availability of a closed form solution. Hence approximate Bayesian inference methods which are either stochastic or deterministic in nature are adopted. MCMC based on sampling techniques is the most adopted approximate Bayesian method. However, MCMC analysis can be time consuming even though it yields accurate results if the sample size is large. To overcome this drawback, variational inference method can be utilised for a less accurate solution, however it provides much faster result compared to MCMC when the dataset is large. Either of these methods can be implemented which is briefly explained in the following sections and the user can decide which method to include in the framework depending on the number of defects that needs to be analysed and the level of accuracy expected.
3.3.3.1 Markov chain monte carlo

MCMC is a type of stochastic sampling method which combines Markov chain with the Monte Carlo technique to evaluate the posterior distribution of the unknown model variables (Gelman, 2004). Monte Carlo is a sampling method that rely on generation of random numbers from a target distribution and the numbers generated in each iteration are independent of the numbers generated in the previous iteration. Whereas Markov chain is a sequence of numbers where each number is dependent on the previous number in the sequence and the MCMC method involves sampling from the Markov chains resulting in convergence to a target distribution. In MCMC analysis, arbitrary values are assigned to the unknown model parameters for initialisation and then the simulations are carried out. Figure 3.12 shows schematic illustration of MCMC method. After the initial sequence of iterations, commonly referred to as burn-in or warm-up period as shown in figure, the sequence starts to converge to the posterior PDF of the unknown variables. As the number of unknown variables increases, the time to carry out simulation will be higher.

Figure 3.12: Schematic illustration of MCMC method
3.3.3.2 Variational inference

Variational inference method is one of the approximate Bayesian inference methods based on optimization (Blei et al., 2017). Instead of direct inference, VI uses series of iterations to find the most optimal approximate distribution \( q^*(x) \) from a family distributions \( q(x) \) to determine the posterior pdf \( p(x|y) \). Hence, the main principle of variational inference is to transform the inference problem into an optimization problem. Figure 3.13 shows schematic illustration of variational inference method where the optimal parameter \( \theta^* \) is determined which will reduce the discrepancy between the approximate distribution \( q^*(x) \) and the posterior posterior pdf \( p(x|y) \).

![Figure 3.13: Schematic illustration of concept of variational inference](image)

Kullback-Leibler divergence is used to minimise the variation between the approximate distribution \( q^*(x) \) and the posterior pdf \( p(x|y) \). Kullback-Leibler divergence, also known as relative entropy, measures the similarity of a probability distribution with respect to another reference probability distribution. The value of KL divergence is zero when one distribution is exactly same as the other distribution, hence minimizing the value of KL divergence will lead to achieving most optimal approximate distribution. The KL divergence between \( q(x) \) and \( p(x|y) \) is given by (Blei et al., 2017),
\[ KL (q(x) || p(x|y)) = E[\log q(x)] - E[\log p(x|y)] \]  

(3.6)

Since \( p(x|y) = p(x,y)/p(y) \), then the last term in the above equation can be re-written as follows,

\[ E[\log p(x|y)] = E[\log p(x,y)] - E[\log p(y)] \]

(3.7)

Where \( p(x,y) \) is the joint PDF of \( x \) and \( y \). The last term \( E[\log p(y)] \) is independent of \( x \), it can be eliminated from the equation since it is a constant term.

Now Equation (3.6) is written as

\[ KL (q(x) || p(x|y)) = -(E[\log p(x,y)] - E[\log q(x)]) \]

(3.8)

Where the term on the right-hand side of the above equation is called as the Evidence lower bound (ELBO) (Blei et al., 2017). As per Equation (3.8), maximising ELBO is equivalent to minimising the KL divergence. Therefore, the optimal distribution \( q^*(x) \) is obtained by maximizing ELBO with respect to \( q(x) \),

\[ q^*(x) = \arg\max\{E[\log p(x,y)] - E[\log q(x)]\} \]

(3.9)

Mean field assumption is then applied in the above equation to get \( q(x) \) as product of individual PDFs \( q_i(x_i) \) and co-ordinate ascent variational inference (CAVI) algorithm is employed for optimization to give optimal PDF \( q^*(x) \).

\[ q^*_i(x_i) \propto \exp\{E_{-i}[\log p(x_i,X_{-i},y)]\} \]

(3.10)

Where index \(-i\) refers to all components of vector \( y \) excluding \( x_i \). This optimization is carried out until all the parameters have been determined. Either of the above-mentioned Bayesian inference methods can be utilised depending on the user requirement to update the model with the inspection results.
3.4 Structural reliability analysis

The next step in the framework is to evaluate probability of failure as a function of time, depending on the corrosion state in the future obtained from the structural deterioration model. The main objective of determining probability of failure of the structural system is to later quantify the costs associated with the risk of its failure. In structural reliability analysis, the probability of failure is estimated using a limit state function which is generally expressed as resistance offered by the structural system minus the load acting on it (Melchers, 2018). A structural system is said to be safe when the resistance is higher than the load acting on the structure and a case of failure occurs when the limit state is violated.

Structural system can be subject to different failure modes; for example, in case of corroding pipelines, burst failure is one of the main failure modes apart from leak failure (CSA, 2012). A burst failure occurs when the wall of the pipeline is weakened to resist the internal pressure. Limit state function in this case can be determined as the difference between operating pressure and the failure pressure. It is essential to determine limit state function for each of these failure modes and consider any dependency between them (Melchers, 2018). In order to determine the probability of failure of the pipeline subjected to both leak and burst failure, a system level reliability analysis will be required where probability of failure corresponding to each of the limit state function will be considered.

Leak failure of a corroded pipeline occurs when the defect depth exceeds the wall thickness of the pipeline (CSA, 2012). In this case, depth of corrosion is the load which increases with time whereas resistance is offered by the thickness of the wall. Limit state function in this case can be expressed as follows

\[ g(k, x_d) = k - x_d \quad \forall d = 1, 2, \ldots, D \quad (3.11) \]
where \( g(k, x_d) \) is the limit state function, \( k \) is the critical thickness of the wall or the failure limit and \( x_d \) is the size of corrosion defect. Probability of failure is then given by

\[
P_F = p(x_d > k) \quad \forall \; d = 1, 2, \ldots, D
\]  

(3.12)

Reliability analysis can be either time dependent or independent (Melchers, 2018). In the above scenario while the resistance is time independent and remains constant, the corrosion defect is time dependent and increases monotonically. Closed form solution may not be available to determine the probability of failure. In such cases, decision maker can use either transformation methods such as first-order reliability method, second-order reliability method or simulation methods such as Monte Carlo simulation for solving. For simplicity, only one failure mode will be considered for both the examples in Chapter 4.

3.5 Lifecycle cost modelling

The next step in the framework is to determine the total lifecycle cost. The objective function considering the benefits and the total lifecycle costs associated with a structure is given by (Rackwitz, 2005)

\[
U(z) = B(z) - C(z)
\]

(3.13)

where \( B(z) \) is the benefit, \( C(z) \) is the lifecycle cost and \( z \) is the vector of decision variables that describe the inspection and maintenance strategies. Typical lifecycle cost includes initial cost related to planning, design, and construction of the system, then inspection and maintenance cost to inspect the condition of the system and further maintenance action based on the inspection outcome, operating cost for the functioning of the system, and finally probable failure cost in the event of system failure (Sarma & Adeli, 2002). Sometimes demolition cost or clean up cost involved at the end of service life of the structure is also accounted in the total lifecycle cost.
(Plebankiewicz et al., 2016). Since the benefits and cost are uncertain, decisions are made based on expected utility according to decision theory (Rackwitz, 2005). The objective function, in terms of expected utility, is expressed as

\[ E[U(z)] = E[B(z)] - E[C(z)] \] (3.14)

The lifecycle cost of a structure in the above equation considers all the incurred cost over the lifetime of the structure including failure cost as a consequence to risk of failure. The lifecycle cost is given by

\[ C(z) = C_{Ini}(z) + C_{Ins}(z) + C_{Re}(z) + C_{F}(z) + C_{Op}(z) + C_{De}(z) \] (3.15)

where \( C(z) \), \( C_{Ini}(z) \), \( C_{Ins}(z) \), \( C_{Re}(z) \), \( C_{F}(z) \), \( C_{Op}(z) \), \( C_{De}(z) \) correspond to the lifecycle cost, initial cost for planning, design and construction, inspection cost, repair cost, failure cost, operating cost, and demolition cost respectively. Failure cost can be categorised as direct failure cost and indirect failure cost (JCSS, 2008). Direct failure cost considers the cost directly related to failure of the system, such as cost incurred in repairing or replacing the system, economic cost for causalities, environmental cost. Indirect failure cost considers the cost associated with the loss of functionality of the system, damage to the reputation of the company, reduced usage due to loss of public trust, etc. (Saad et al., 2016). Although the consequences such as environmental impact and human safety use different units of measurement, but they have to be converted into monetary units to carry out this analysis. Expected benefit is considered to be independent of the inspection and maintenance decisions, therefore, it can be removed from the objective function. Now, Equation (3.14) can be written as,

\[ E[U(z)] = -E[C_{Ini}(z)] - E[C_{Ins}(z)] - E[C_{Re}(z)] - E[C_{F}(z)] - E[C_{Op}(z)] - E[C_{De}(z)] \] (3.16)
where $E[U(z)]$ is the expected utility considering the decision variables $z$ related to inspection and maintenance strategies, such as frequency of inspection and maintenance, type of inspection and maintenance. To achieve minimal lifecycle costs, the expected utility is maximized over the lifetime of the system. The above objective function is generic and can be tailor-suited as per desired needs and requirements. While the lifecycle costs in Equation (3.16) include all costs associated with the considered system, not all costs terms are relevant for the decision-making problem at hand. It is possible to remove the cost terms that are independent of the decision variables and develop a reduced lifecycle cost equation. Considering maintenance and inspection planning for an existing system, initial construction costs and demolition costs at the end of the lifecycle are not relevant and can be removed from the objective function. Now, the expected lifecycle costs can be expressed as

$$E[C(z)] = E[C_{Ins}(z)] + E[C_{Re}(z)] + E[C_F(z)]$$  \hspace{1cm} (3.17)

The cost of inspection and repair will depend on whether inspection and repair are carried out for the entire system or specific locations (Faber et al., 1996). Considering a structural system which is discretized into $N$ sections, then the expected inspection cost for the entire system can be obtained as summation of inspection cost of individual sections. As per Faber et al., (1996), the expected costs for inspection, repair and failure are calculated as follows,

$$C_{Ins}(z) = \sum_{n=1}^{N} [C_{Ins,n}(z)][1 - P_F(T_{Ins,n}, z)] \frac{1}{(1+r)^{T_{Ins,n}}}$$  \hspace{1cm} (3.18)

where $C_{Ins,n}(z)$ is the inspection cost for the $n^{th}$ section considering that failure has not occurred before inspection. If the failure has already occurred, then the section would not be available for inspection. $T_{Ins,n}$ is the time interval between past inspection i.e., $t=0$ and the time at which inspection is carried out. $P_F$ is the probability of failure of the section considered in the time
interval from $t=0$ until $T_{Ins,n}$, hence the term $1 - P_f(T_{Ins,n})$ indicates the probability of survival of the section that is available for the inspection. $r$ is the discount rate to account for the time value of money and it is briefly explained at the end of this section.

The expected repair cost for the system based on the assumption that the section has not failed yet and available for repair is given by (Faber et al., 1996),

$$C_{Re}(z) = \sum_{n=1}^{N} C_{Re,n}(z) P_{Re,n}(T_{Re,n}, z) \frac{1}{(1+r)^{T_{Re,n}}}$$  \hspace{1cm} (3.19)

where $C_{Re,n}$ is the repair cost for an individual section, $T_{Re,n}$ is the time at which repair is carried out and $P_{Re,n}$ is the probability of repair of the section.

The expected failure cost of the system is given by (Faber et al., 1996),

$$C_F(z) = \sum_{n=1}^{N} P_{S,n} \int_{0}^{T} C_{F,n}(t, z) P_{F,n}(t, z) \frac{1}{(1+r)^{t}} dt$$  \hspace{1cm} (3.20)

where $P_{S,n}$ denotes the probability of survival of the component, $C_{F,n}$ is the cost of failure of the section and $P_{F,n}$ is the probability of failure of the section $n$.

The above equations can be used to calculate the total cost over the remaining lifetime of the structure, $T$. The cost equations given above are provided in terms of sections i.e., cost of inspecting a section, cost of repairing section, etc. Depending on the type of approach to be adopted, these costs can be replaced by cost of inspection and maintenance of a system in a population-based approach and by cost of inspection and maintenance of a defect in a defect-specific approach.
The costs considered in the objective function corresponding to different times need to be transformed to a single present value. The term $\frac{1}{(1 + r)^t}$ in the above equations is used to convert future costs into its present value by considering the discount rate $r$ to account for the time value of money (Sarma & Adeli, 2002). Figure 3.14 compares the discounting effect over a period of 50 years based on different discount rates of 0, 1%, 3%, and 5% respectively. The figure indicates that there is no effect of future cost on the present value of money and the cost remains constant throughout the period when there is no discounting ($r=0$) whereas a discount rate of 3% accounts to only 23% of the cost at $t= 50$ years. As the discount rate increases, the value of money decreases with increase in time. This indicates that its better to delay the inspection and repair of a defect as its cost would be comparatively lower in the future provided that the probability of failure is low.

![Figure 3.14: Time value of money for different discount rates](image-url)
Considering discounted rate is important, especially for decisions related to inspection and maintenance planning which are carried out for a long period of service life of the structure. It tends to reduce the effect of any long-term assumptions that are considered in the analysis. Generally, a discount rate of around 2-3% above the inflation rate is considered to be reasonable in a lifecycle cost analysis (Sarma & Adeli, 2002).

3.6 Optimal inspection planning

The next step in this framework is to determine an inspection strategy that would result in minimum lifecycle costs. The optimal inspection strategy can be achieved in several ways such as by optimising the interval time between the inspections, optimizing the type of inspection tool to be employed, etc. Ideally, they should all be done simultaneously to find the true optimum, but it is simplified to avoid large optimization problem. Since inspection tools and intervals are often pre-selected, optimization for inspection strategy will be achieved in the proposed framework by determining the need for inspection in case of a population-based analysis determining the optimal number of inspection locations and prioritizing their locations in the next inspection in case of defect-specific analysis. This optimization is achieved by maximizing the Value of Information (VOI).

Value of information is a quantity which determines the financial benefit of performing inspection and the subsequent decisions such compared to not performing inspections (Straub, D. 2014). i.e., the difference of cost incurred to carry out maintenance actions without performing planned inspection and the cost incurred to carry out maintenance actions by performing the planned inspection. If \( E[C'] \) is the expected total cost in case of no inspection and \( E[C(z)] \) is the total expected cost with inspection, then VOI for a particular inspection scheme \( z \) is given by

\[
VOI(z) = E[C'] - E[C(z)]
\] (3.21)
The total expected cost for the case without inspection case is given by

\[ E[C'] = E[C_{Re}] + E[C_F] \]  

(3.22)

where \( E[C_{Re}] \) and \( E[C_F] \) is the expected repair cost and expected failure cost. Similarly, the total expected cost for the case of inspection is given by

\[ E[C(z)] = E[C_{Ins}(z)] + E[C_{Re}(z)] + E[C_F(z)] \]  

(3.23)

where \( E[C_{Ins}(z)] \) is the expected inspection cost and expected failure cost for a given inspection scheme \( z \). The optimal inspection strategy \( z^* \) is obtained by maximising VOI which is given by

\[ z^* = \arg \max_z VOI(z) \]  

(3.24)

In this framework, the above maximization is carried out by an exhaustive search which may not be feasible for larger optimization problems. In such cases, advanced optimization algorithms has to be adopted. In chapter 4, system level VOI analysis would be carried out in first example whereas in the second example VOI analysis would be performed for defect specific analysis.

3.7 Optimal maintenance planning

The final step in this framework is to develop a maintenance strategy that would result in an optimum lifecycle cost. When a defect is detected, the operator needs to decide whether to repair the defect immediately or not. This decision will affect the overall costs since if the defect is critical then by carrying out repair immediately, the chance of its failure in future is reduced and thereby reducing the lifecycle cost in the event of failure. On the other hand, if the size of the defect is not critical and the repair is carried out, then resources are spent unnecessarily on a defect which could have survived for a longer time without any intervention.
In most cases, this decision is taken based on the condition of the system when the defect size reaches its repair criterion i.e., section is repaired if the size of the defect is greater than the allowable critical size. For instance, as per ASME (2019) repair or replacement of a pipeline section is carried out whenever the size of the metal loss exceeds 80% of its initial wall thickness. The decision carried out based on this criterion will be purely based on the condition of the system without uncertainty rather than considering risks or consequences associated with the failure of the system. Hence in this framework, optimal repair criterion will be calculated for the given set of defects based on its repair probability or failure probability and the corresponding economic consequences related to each decision. Additionally, a constraint on probability of repair will be applied for a given repair criterion to check if the repair is required or not. The use of this constraint is essential to judiciously decide whether repair is required or not.

Therefore, the decision to repair or not will be made depending on the optimum repair criterion for a given constraint or based on the optimum probability constraint for a given repair criterion. There is also an option to decide both optimum repair criterion and optimum probability constraint for repair criterion based on this optimization analysis which will result in minimum lifecycle costs.

\[
z^* = \arg\min\{E[C_{Re}(z)] + E[C_F(z)]\} \quad (3.25)
\]

In case of a leak failure of a pipeline section, if \( z \) denotes the repair criterion i.e., the size of defect when repair is to be carried out and \( k \) denotes the size of the defect when failure occurs which is usually considered to be complete metal loss, then the repair probability for a single defect \( x_d \) between a particular time interval \( t \) is given by,

\[
P_{RE}(z < x_d < k) = \int_z^k p(x_d)dx_d \quad \forall \ d = 1, 2, \ldots D \quad (3.26)
\]
Where \( x_d \) is the size of the defect considered. Similarly, the probability of failure for a single defect using structural reliability analysis is given by

\[
P_F(x_d \geq k) = \int_k^\infty p(x_d) \, dx_d \quad \forall \, d = 1, 2, \ldots D \tag{3.27}
\]

Once the repair probability and failure probability are determined then the total repair costs and total failure costs for all the defects over the remaining lifespan of the structure are determined based on the Equation (3.19) and (3.20), respectively.

Figure 3.15 shows a decision tree of possible maintenance actions available and the cost resulting from each action. For each defect, there exists two maintenance actions i.e., to repair or not to repair. This decision is carried out by comparing the repair probability of the defect with the probability constraint for repair criterion \( P_{Rt} \). If the repair probability exceeds this value, then a decision to carry repair is carried out. The cost associated with this decision would be equal to the repair cost associated with this defect. On the other hand, if the repair probability does not exceed the value of the optimum probability constraint, then the defect is not repaired at that moment. Then the failure cost associated with this defect is considered as there exists a possibility of failure in future before it is decided to repair the defect. Such decision-making process is carried out for all the defects \( d = \{1,2,\ldots D\} \) present in the system. In order to estimate the cost over the lifetime of the structure, then these costs are considered for all maintenance events, \( m_R = \{m_{R1}, m_{R2},\ldots m_{Rj}\} \) and summed up together to get the total repair costs over the lifetime of the structure. In order to minimise the cost, optimal repair criterion \( z^* \) and an optimal probability constraint of repair \( P_{Rt}^* \) is determined by finding the values based on the Equation (3.25).
Figure 3.15: Schematic illustration of determination of minimal repair cost

Figure 3.16 illustrates the variation of probability of failure associated with a defect over the lifetime of the structure. It can be seen that the probability of failure gradually increases over time until its repaired. Whenever a repair is carried out, it is assumed that the defects are not completely removed as the repairs might not be completely perfect and a minimum probability of failure $P_{F(min)}$ due to the presence of that respective defect still exists as shown in Figure 3.16. In such a case, the corrosion rate which was used earlier for the defect will be adopted again assuming that the defect is subjected to same exposure condition before and after the repair. The growth of defect is again
predicted in the future and the corresponding repair and failure probabilities will be evaluated in the further lifecycle analysis.

Figure 3.16: Variation of probability of failure of a defect over the service life of the structure
4. NUMERICAL EXAMPLES

The framework presented in Chapter 3 is illustrated in this chapter using numerical examples. As discussed in Chapter 3, the framework begins with selecting a suitable approach for corrosion growth analysis and the subsequent decision making to achieve minimal lifecycle costs. In order to demonstrate that the framework can be adopted for either of these approaches, it will be illustrated with two sets of examples using population-based approach in the first example and defect specific approach in the second example.

Section 4.1 presents the first example where a VOI analysis is carried out using the population-based approach to make optimal inspection decisions regarding whether inspection is required. In this case, decision is made whether to inspect the entire system irrespective of the condition or size of individual defects. While VOI analysis can be carried out assuming any repair criterion, however, optimal repair criterion is determined first for the population of defects and then used in the VOI analysis.

Section 4.2 presents the second example where a VOI analysis is carried out for a specific set of defects in order to find whether local inspections are necessary and subsequently to determine the extent and prioritize the locations for the local inspections. Thereafter, decision making related to optimal maintenance is carried out.

4.1 Population-based inspection and maintenance planning

A VOI analysis is carried out for population of defects in a pipeline to determine if inspecting the pipeline is financially beneficial before the next maintenance event. Additionally, an optimum repair criterion is determined for the population of defects to achieve minimum lifecycle cost. In total, there are 1753 corrosion defects that were reported in the pipeline during past inspection. It
is assumed that the remaining life of the pipeline is 30 years with a total of 6 maintenance events carried out at every 5 years. Leak failure is assumed to be the only mode of failure in the pipeline. The cost of repairing a defect is assumed as 100 monetary units (mu), the cost of failure of a defect is assumed as 200 times the cost of repair of the defect, and the cost of inspecting the pipeline is assumed as 1 mu in the analysis. A discount rate of 3% is considered to account for the time value of money. Measurement uncertainty is considered in the pre-posterior analysis.

The analysis in this example is divided into three parts covering most of the steps involved in the framework.

a) Gamma process model for population-based corrosion growth analysis is applied to predict the future size and the probability of failure is determined for leak failure mode (Section 4.1.1).

b) Optimal repair criterion using lifecycle cost optimisation for the population of defects is determined (Section 4.1.2).

c) Optimal inspection strategy using VOI analysis is carried out to determine the financial benefit of additional inspection (Section 4.1.3).

4.1.1 Structural deterioration model

The first step of the framework begins with collecting the past inspection data to develop a deterioration model. Based on past inspection data, the initial size of the population of defects at the time of previous inspection \((t = 0)\) is determined to be gamma distributed with a shape parameter \(\alpha\) and scale parameter \(\beta\) of 3 and 5, respectively. i.e,

\[
x|\alpha, \beta \sim gamma(\alpha = 3, \beta = 5)
\]  

\[\text{(4.1)}\]
The corrosion growth $\Delta x$ for 5 years is determined to be gamma distributed with a shape parameter of 1 and scale parameter of 5 based on the inspection data. i.e.,

$$\Delta x|\alpha, \beta \sim \text{gamma}(\alpha = 1, \beta = 5)$$

(4.2)

The PDF of the future corrosion defect size during the first maintenance time is obtained by adding growth increment to the initial defect size. Figure 4.1 shows the PDF of the defect size expressed in terms of its initial wall thickness (%wt) at $t = 5$ years obtained by convoluting the PDF of the initial defect size with the PDF of the corrosion growth increment (Benjamin & Cornell, 1970). It is evident from the figure that the PDF of the future size of the defect has shifted right towards higher size.

Figure 4.1: PDF of the estimated defect size for the population of defects
In the next step, probability of failure of the pipeline is determined as per Equation (3.27) where the failure limit for leak failure is considered as 100%wt i.e., failure occurs when the size of the defect reaches 100% of the initial wall thickness. Figure 4.2 shows the exceedance probability of the estimated population depth at $t = 5$ years. Probability that the size of a defect is greater than the critical size can be shown using an exceedance probability plot. The probability that the size of population of defects have reached the failure criterion, i.e., probability of failure of 100%wt after 5 years can be obtained directly from the Figure 4.2 as $3.2 \times 10^{-6}$. This probability of failure value is then multiplied with the failure cost of the defect to get the total failure cost.

![Figure 4.2: Exceedance probability of the estimated defect size for the population of defects 5 years forward](image)

Similarly, total repair cost is obtained as a product of probability of repair and the repair cost of the defect. In order to find the probability of repair, the value for repair criterion needs to be set. When the size of defect reaches repair criterion, it is decided to repair the defect to maintain the integrity and prevent failure in future. For example, if repair criterion is set at 50%wt then the
corresponding probability of repair after 5 years as per Equation (3.26) can be obtained from Figure 4.2 as $(0.01 - 3.2 \times 10^{-6})$ where the value 0.01 corresponds to the exceedance probability of 50% wt.

It is to be noted that generation and growth of new defects is not considered in the analysis.

4.1.2 Optimal repair criterion

In order to estimate the optimal repair criterion, total lifecycle cost is estimated for different values of repair criterion $z = \{30, 31, \ldots, 96\} \%\text{wt}$. Reason for selecting this discretized range of repair criterion is to speed up the analysis. Continuous range of values from 0 to 100 can be selected for more accuracy but the computation time will be extremely high. The optimal repair criterion is selected in such a way that it results in minimal total lifecycle cost as per Equation (3.25). The repair criterion corresponding to minimum total lifecycle cost is selected as optimum repair criterion $z^*$. This means that every 5 years when the maintenance is performed, all the defects that exceeds this level needs to be repaired as illustrated in the Figure 4.3.

![Figure 4.3: Illustration of repair and failure criterion set during a maintenance event](image-url)
It also means that few defects will exceed this level before the maintenance event and may fail as shown in the figure. While evaluating lifecycle costs, the failed and repaired defects are removed from further analysis after every maintenance event.

Figure 4.4 shows the variation of repair cost, failure cost and total cost for different values of repair criterion considered in this analysis. It is evident from the figure that the repair cost is maximum when the repair criterion is set to 30%wt. This is because setting a lower repair criterion means that even defects with smaller sizes are repaired much earlier than their failure resulting in higher repair cost. However, this will lead to lower failure cost as the defects are already repaired and there is not any risk of failure associated with the repaired defects anymore. As the value of repair criterion increases, there is decline in the repair cost due to a lower number of required repairs, but the failure cost keeps increasing. The increasing failure cost is associated with the increased risk of failure of the defects which are not repaired yet.

The curve for total cost is obtained by summing repair and failure cost throughout the service life as per Equation (3.22) and the lowest value of this curve will correspond to the minimum total lifecycle cost as shown in the Figure 4.4. From the figure, this point corresponds to the repair criterion of 74%wt resulting in the minimum total lifecycle cost of $1.03 \times 10^4$ monetary units. Therefore, as per prior analysis, all the defects in this pipeline, unless they have already failed, are repaired when the size is greater than 74%wt during a maintenance event.
4.1.3 Inspection planning

In Section 4.1.2, future corrosion growth is estimated based on the past information available about the defects to determine lifecycle cost. This corresponds to the case of prior analysis where total lifecycle cost is determined for the maintenance decision without carrying out an extra inspection. In this section, the VOI is determined after carrying out pre-posterior analysis to assess if extra inspection is financially beneficial. The future corrosion growth is updated considering all the possible inspection outcomes, which in turn affects the subsequent maintenance decisions. The corresponding expected lifecycle cost is then determined for this case to evaluate VOI. As the quantified VOI depends on several factors, a sensitivity analysis is carried out at the end of this example to study the impact of the parameters on the VOI.

Figure 4.4: Repair cost, failure cost and total cost corresponding to different repair criterion during the service life
In Section 4.1.2 total cost without inspection based on prior analysis is determined as $1.03 \times 10^4$ mu. However, determining total cost for posterior analysis is not as straightforward as the prior analysis. Posterior decision analysis involves additional uncertainty as the future inspection outcomes are unknown beforehand. To address this issue, simulations for large number of samples is carried out to generate different inspection outcome i.e., samples for different reported size of the defects are generated to account for the different case scenarios. Additionally, measurement errors generated by the inspection tool needs to be considered. For the posterior analysis, it is assumed that the next inspection is carried out ten years since the last inspection.

The posterior analysis begins with predicting the future corrosion growth by developing a corrosion growth model and updating the parameters based on the possible inspection outcomes. The corrosion growth model used is a hierarchical population-based corrosion growth model proposed by Dann & Maes (2015) which uses gamma process to model deterioration. Reason for using this model is to account for the uncertainties associated with the inspection tool, as this model inherently considers multiple level of errors to estimate the actual number and size of the corrosion defects as discussed earlier and shown in the Figure 3.11. The actual number and size of defects are inferred from the reported inspection data by assuming the values of reporting threshold as 1%wt, $y_f$ as 20%wt, standard deviation of sizing error as 5%wt and $x_{De}$ as 10%wt in this example.

Once the actual number and size of defects are inferred then based on the PDF of actual size of the defects, the PDF of corrosion growth increment is obtained using updated shape and scale parameter for the corrosion growth and used for further analysis. Figure 4.5 shows the PDF of corrosion growth increment considered for prior analysis and PMF of corrosion growth increment obtained after generating first inspection outcome sample for posterior analysis.
Figure 4.5: PDF of corrosion growth increment for prior and posterior analysis

Based on this updated corrosion model, corresponding probability of failure with a failure criterion of $100\%$wt of metal loss and repair probability for different value of repair criterion $z = \{30, 31, \ldots, 96\} \%$wt is calculated once again in a similar fashion as done earlier in the prior analysis case. While calculating the total lifecycle cost, in addition to failure and repair cost considered during the service life, inspection cost incurred in inspecting the pipeline is also included to determine total lifecycle cost for posterior case as per Equation (3.23).

Now the cost obtained for posterior case is carried out for several simulations to account for uncertainty and get a good estimate for expected total lifecycle cost. In total 250,000 simulations were carried out to determine the expected value of 9,526.1 mu as the total expected lifecycle cost.
and the corresponding VOI is 773.8 μu. The positive VOI indicates that carrying out inspection of the pipeline is financially beneficial.

Figure 4.6 shows the convergence of VOI. It is evident that for smaller number of simulations, the VOI changes drastically as not enough possible inspection outcomes have been generated. Beyond 200,000 simulations, the value of VOI seems to stabilize and converge. Considering 250,000 simulations for the analysis is therefore reasonable.

Figure 4.6: Convergence of VOI for different simulation size
4.1.3.1 Sensitivity analysis

In this section, a sensitivity analysis is carried out by varying the values of different parameters and check its effect on the VOI.

Figure 4.7 shows the VOI for different inspection costs. The VOI is always a maximum when the cost of inspection is zero which means that it is most financially beneficial when free information is available i.e., better decisions are made based on the extra information that is available for no extra cost. The VOI reaches the theoretical value of 774.53 mu when the inspection cost is zero. As the cost of inspection increases, the value of VOI gradually decreases and becomes zero when the cost inspection cost is 1,040 mu. Beyond this value of inspection cost, the VOI is negative. A negative VOI indicates that the amount spent on inspection is more than the amount saved by making better decisions afterwards. It is financially not beneficial to carry out inspection, and maintenance action can be carried out based on prior analysis. Therefore, inspection can be carried out for inspection costs up to 1,040 mu when the value of VOI is just positive.

![Figure 4.7: Variation of VOI for different inspection costs](image)

- Break-even point at 1,040 mu
- VOI values: 774.53, 402.52, 30.52, -341.48, -713.49, -1,457.50
- Cost of inspection (mu) range: 0 to 3000
- VOI (mu) range: -2000 to 1000
Figure 4.8 shows variation of VOI for different values of discount rate. A maximum VOI of 1,866 mu is achieved when there is no discounting for time value of money. As the value of discount rate increases, the value of VOI gradually decreases.

**Figure 4.8: Variation of VOI for different discount rates**

As the VOI depends on parameters affecting the inspection outcome, variation of VOI with respect to accuracy of inspection tool is analysed. Figures 4.9, 4.10 and 4.11 show the variation of VOI for different values of mean credible size of defect for true call, threshold for detection and standard deviation considered for the sizing error. For all the three cases, as these values of considerations increase, the VOI decreases. The value of VOI is maximum when the inspection is perfect and free of errors, which is significant especially in the case of zero sizing error.
Figure 4.9: Variation of VOI for different values of mean credible size of defect for true call of the inspection tool

Figure 4.10: Variation of VOI for different values of detection threshold of the inspection tool
4.2 Defect-specific inspection and maintenance planning

In this example, two previous ILI data obtained from inspection carried out in 2008 and 2012 respectively, for a section of an upstream pipeline are considered. The ILI results reported several defects during the previous inspection, however, there are 69 critical corrosion defects that may require high resolution local inspection for further assessment. This high-resolution inspection will be taken from outside the pipeline which will require additional resources in excavation and accessing the defect locations as they are buried underground. Due to the large number of defects, it is not feasible to excavate all the locations for local inspection. Therefore, the proposed framework will be used to determine the optimal number and locations for local inspections that will result in overall optimal cost. Additionally, repair criterion for these defects are also determined using the proposed framework. Sizes of these 69 defects recorded during the previous
two inspections are shown in Table 4.1 (Dann & Huyse, 2014). The remaining service life is assumed as 20 years and the maintenance events are carried out every 5 years till the end of service life. Leak failure is assumed to be the only mode of failure in the pipeline. The cost of repairing a single defect and cost of failure of single defect is considered as 100 μ and 10,000 μ, respectively while the cost of inspecting the defect is considered as 1 μ. A discount rate of 3% is considered to account for the time value of money. Since different types of inspection errors and the method to consider them is demonstrated in the previous example, in this example it is assumed that the defects are perfectly matched and are not subjected to any errors for simplicity.

The objective of using this example is to present different steps involved in the framework to find optimal inspection and maintenance criterion resulting in minimal costs using a defect-specific approach when a detailed analysis at defect level is required. The analysis in this example is divided into four parts covering most of the steps involved in the framework.

a) Corrosion rate model for defect specific corrosion growth analysis is developed to predict the future size of the individual defects and thereby probability of failure for leak failure is determined (Section 4.2.1).

b) Costs involved in the analysis is briefly discussed (Section 4.2.2).

c) VOI analysis is carried out to find the optimal number of inspections and their specific locations (Section 4.2.3).

d) Decision analysis related to maintenance is carried out to determine optimal repair criterion and to determine if global repair is beneficial (Section 4.2.4).
Table 4.1: Number and size of defects reported during two inspections taken in the year 2008 and 2012

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<tr>
<th>Defect ID, d</th>
<th>Reported size $x_{d,1}$ at first inspection (% wt)</th>
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<tr>
<td>69</td>
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<td>18</td>
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</tr>
</tbody>
</table>

Mean = 15.90  Mean = 21.84  Mean = 5.94
4.2.1 Structural deterioration model

The first step of the process begins with checking for any inconsistency in the data by plotting a frequency graph or PMF values of the size of defects. Figure 4.12 shows PMFs of size of defects recorded during two different inspections taken 4 years apart. It is evident from the figure that the PMF values of the defect sizes observed during the second inspection have shifted right towards higher metal loss indicating an increase in size of the corrosion defect and that the corrosion process seems to be active. The mean value of defect size during the first inspection and second inspection is 15.90%wt and 21.84%wt, respectively, with an average growth of 5.94%wt.

Figure 4.12: PMF of size of defects from two in-line inspections
The next step in the framework is to use the inspection data in developing a structural deterioration model. To predict future corrosion growth using a corrosion rate model, the corrosion rate for all the defects in the pipeline section is modelled using a gamma distribution where the parameters used in the distribution are calculated based on the past inspection data (Younsi et al., 2013).

If \( x_{d,1} \) is the actual size of the defect taken during the first inspection at the year 2008 and \( x_{d,2} \) is the actual size of the defect taken during the second inspection at the year 2012, then the growth of defect is given by,

\[
\Delta x_d = x_{d,2} - x_{d,1} \quad \forall \ d = 1, 2, \ldots, 69
\]

(4.3)

Where \( \Delta x_d \) is the actual corrosion growth for defect \( d \). The corrosion growths for all the defects are shown in the last column of Table 4.1. Since the time elapsed between these two inspections are known, the rate of corrosion growth for defect \( d \) between two inspection results is obtained as

\[
\lambda_d = \frac{x_{d,2} - x_{d,1}}{t_2 - t_1} \quad \forall \ d = 1, 2, \ldots, 69
\]

(4.4)

Where \( \lambda_d \) is the rate of corrosion growth for defect \( d \) between year 2008 and 2012. Once the corrosion rate for individual defects is calculated then using the mean and standard deviation of the corrosion growth rate, the shape parameter \( \alpha \) and scale parameter \( \beta \) for the gamma distribution is obtained as 1.294%wt and 0.871%wt, respectively.

Figure 4.13 shows the PDF of the corrosion growth rate per year plotted using the above-mentioned values of shape and scale parameter.
In the next step, PDF of the defect size in future is estimated individually for all the defects based on the following equation

\[
p_{x_d(t)}(x_d) = \frac{1}{t} \frac{\beta^\alpha}{\Gamma(\alpha)} \left( \frac{x_d - x_{d,2}}{t} \right)^{(\alpha-1)} \exp\left( -\beta \frac{(x_d - x_{d,2})}{t} \right) \quad \forall \ d = 1, 2, \ldots, 69 \tag{4.5}
\]

Figure 4.14 shows the PDF of the estimated size of defect number 1 in future which had a size of 10%wt during the last inspection. Figure shows the plots for 5, 10, 15 and 20 years forward respectively since the last inspection. As the time in future increases, the curves become more and more flatter representing more uncertainty in the future. In the next step, probability of repair and probability of failure for all the defects are determined as per Equation (3.26) and (3.27), respectively.
4.2.2 Cost analysis

Once probability of failure and repair of all the defects are determined, the next stage of the framework involves determining costs as shown in the Figure 3.1. Cost analysis is carried out to evaluate risks and decide inspection and maintenance strategies which will minimise the overall costs. While the strategies developed for repair criterion depend on the lifecycle cost optimization with costs considered during the entire service life for analysis, in case of analysis for optimal inspection strategy only the costs involved in the next inspection is considered for optimization to simplify the analysis. Reason for this simplification is, unlike population-based approach where decision is carried out for entire system, defect-specific analysis involves several defects to be considered during each inspection event and determining the priority of locations to be inspected. When considered over the entire lifecycle it will lead to several possible event combinations to be

86
considered, that it would not be feasible to carry out analysis. Therefore, the costs involved in each of these strategies are discussed and considered separately in their respective sections.

4.2.3 Inspection planning

In this section VOI analysis is carried out to develop an optimal inspection strategy to decide whether to inspect a particular defect and to determine the optimal number of defect locations to be inspected. For the prior analysis i.e., analysis without further inspection, it is assumed that the defects are repaired when the size of defect is expected to reach 70% wt with a probability of $5 \times 10^{-3}$ and failure occurs when the size of defect reaches 100% wt. Based on this criterion considered in the prior analysis, 9 out of 69 defects would be repaired in the next maintenance event which would be carried out 5 years since the last inspection had taken place. Total cost as per Equation (3.22) which includes repair cost of repairing the 9 defects and failure cost considered for the remaining defects sums to around 902.73 μ. Failure cost for the non-repaired defects are considered based on the probability that these defects grow from below the repair criterion to the failure limit between the maintenance events. Since the value of probability of failure is very negligible in this case it didn’t add much to the total cost. Reason for this very small probability of failure lies in the fact that most of the defects available in the data are small having an average defect size of 21.84% wt in the second inspection with an average growth of 5.94% wt in 4 years as seen in Table 4.1. Due to this only 9 defects which have larger defect size with larger corrosion growth are beyond critical limit for repair whereas remaining defects are comparatively very small leading to lower probability values beyond failure criterion of 100% wt.

For the posterior analysis i.e., analysis with inspection, it is assumed that the next inspection would be carried out after 4 years since the last inspection. To determine posterior costs, total cost as a result of inspection and its corresponding outcome is to be determined as per Equation (3.23).
Since the inspection outcome is not known beforehand, all the possible outcomes of the defect size since the last inspection are considered and weighted with their prior probability to obtain the total expected cost for each of the 69 defects. With this, total cost without inspection for the prior case and the total cost with inspection for the posterior case is known for determining VOI as per Equation (3.21) for all the defects.

Next, as per Equation (3.24), VOI is maximised in order to determine optimal number of inspection locations. To find the optimal inspection location, the analysis begins by assuming that only the first defect is inspected with other defects uninspected and then VOI is evaluated in this scenario. If this VOI is higher than the initialized value of VOI then this is set as maximum value of VOI, and the corresponding location is marked as the optimal location. Next, VOI is determined by assuming that only the second defect is inspected and the corresponding VOI is determined and checked against VOI value from the previous case. If it is higher than the previous value, then it is set as new value of maximum VOI, and the corresponding location is now marked as the optimal location. These steps are carried out for all the defects to determine the first optimal location in terms of VOI. Then this location is removed from further analysis to determine the second optimal location in similar way and this process is repeated until the order of inspection for all the defects are determined.

Table 4.2 lists the results of the VOI analysis in terms of inspection location, and it is corresponding VOI. The maximum value of VOI obtained is 835.03 μ after inspecting 9 locations which are defect number 37, 26, 66, 27, 59, 67, 12, 58 and 63.
Table 4.2: Results of VOI analysis corresponding to repair criterion of 70%wt with a probability threshold of 0.005.

<table>
<thead>
<tr>
<th>No. of inspection</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>6</th>
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<tr>
<td>VOI</td>
<td>97.61</td>
<td>194.33</td>
<td>291.06</td>
<td>387.38</td>
<td>481.27</td>
<td>573.22</td>
<td>663.96</td>
<td>751.59</td>
<td>835.03</td>
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<td>17</td>
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<td>23</td>
<td>34</td>
</tr>
<tr>
<td>VOI</td>
<td>834.01</td>
<td>832.99</td>
<td>831.97</td>
<td>830.96</td>
<td>829.94</td>
<td>828.92</td>
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<td>40</td>
<td>60</td>
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<tr>
<td>VOI</td>
<td>824.86</td>
<td>823.84</td>
<td>822.81</td>
<td>821.78</td>
<td>820.76</td>
<td>819.73</td>
<td>818.70</td>
<td>817.67</td>
<td>816.64</td>
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<td>61</td>
<td>3</td>
<td>7</td>
<td>14</td>
<td>18</td>
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<tr>
<td>VOI</td>
<td>815.62</td>
<td>814.58</td>
<td>813.55</td>
<td>812.51</td>
<td>811.47</td>
<td>810.43</td>
<td>809.38</td>
<td>808.33</td>
<td>807.29</td>
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<th>40</th>
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<td>69</td>
<td>5</td>
<td>52</td>
<td>56</td>
<td>10</td>
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<tr>
<td>VOI</td>
<td>806.24</td>
<td>805.18</td>
<td>804.12</td>
<td>803.05</td>
<td>801.99</td>
<td>800.91</td>
<td>799.84</td>
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<td>797.67</td>
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<td>33</td>
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<tr>
<td>VOI</td>
<td>796.58</td>
<td>795.49</td>
<td>794.40</td>
<td>793.28</td>
<td>792.16</td>
<td>791.01</td>
<td>789.86</td>
<td>788.72</td>
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<td>Defect ID</td>
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<td>36</td>
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<td>35</td>
<td>38</td>
<td>47</td>
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<tr>
<td>VOI</td>
<td>786.34</td>
<td>785.14</td>
<td>783.90</td>
<td>782.66</td>
<td>781.38</td>
<td>780.10</td>
<td>778.76</td>
<td>777.43</td>
<td>776.10</td>
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<td>46</td>
<td>54</td>
<td>45</td>
<td>15</td>
<td>43</td>
</tr>
<tr>
<td>VOI</td>
<td>774.71</td>
<td>773.31</td>
<td>771.77</td>
<td>770.13</td>
<td>768.38</td>
<td>766.62</td>
</tr>
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These results can also be represented graphically to show the optimal number of inspections and maximum number of inspection locations as shown in the Figure 4.15. The figure shows the variation of value of VOI as a function of number of inspection locations. The value of VOI steeply increases for the first nine inspections beyond which it gradually decreases. Although it decreases beyond nine inspections, it is still beneficial to inspect all the defects since the VOI value remains positive. Hence the maximum number of defects/locations that can be inspected are 69 whereas the optimal number is 9. Interestingly, these 9 defects resulting in maximum VOI had largest defect sizes out of all defects in the last inspection and corresponds to the 9 defects that required repair as per the prior analysis.

![Figure 4.15: VOI as a function of the number of inspections](image-url)
4.2.3.1 Sensitivity analysis

The value of repair criterion has an impact on the VOI and the maximum VOI that can be achieved. If the size of the defect allowed or the repair criterion considered is small, then it means that a greater number of defects have higher chance of exceeding the repair criterion. This leads to higher repair cost in the prior analysis scenario for lower repair criterion. In such cases maximum VOI achieved would be more due to lower posterior costs when compared with the prior costs. This is reflected in the Figure 4.16 which shows the maximum values of VOI corresponding to different repair criterion. Maximum VOI with 3,786.38 μ corresponds to 50%wt repair criterion is the highest. As the allowable defect size or the repair criterion increases, maximum VOI gradually decreases indicating decreased requirement for inspection, reaching almost to zero at 90%wt repair criterion.

![Figure 4.16: Maximum VOI as a function of repair criterion](image)

**Figure 4.16: Maximum VOI as a function of repair criterion**
Figure 4.17 shows value of VOI as a function of number of inspection locations for different repair criterion. VOI for different repair criterion are increasing up to an optimal number beyond which its decreasing slightly.

**Figure 4.17: VOI as a function of number of inspections for different repair criterion**
Figure 4.18 shows the optimal and maximum number of inspection locations corresponding to different repair criterion. In conjunction with the previous figure, the optimal number of inspections is decreasing from 43 to 5 corresponding to increased repair criterion from 50\%wt to 90\%wt. The maximum number of inspections remains constant as 69 until 80\%wt repair criterion which means that the VOI is still positive. The maximum number of inspections drops to 13 for 90\%wt repair criterion indicating that the VOI is negative beyond 13 number of inspections.

![Graph showing optimal and maximum number of inspections as a function of repair criterion](image)

**Figure 4.18: Optimal and maximum number of inspections as a function of repair criterion**

Figure 4.19 shows the maximum values of VOI for different values of inspection cost. The value of maximum VOI is highest for cost-free inspection with 844.02 mu. As the cost of inspection increases, the maximum VOI achieved gradually decreases.
Figure 4.19: Maximum VOI as a function of inspection cost

Figure 4.20 shows VOI as a function of number of inspections for different cost ratios. VOI is maximum when the cost of inspection is zero where the corresponding VOI curve continuously increases from no inspection to inspection of all the locations. As the cost of inspection increases, the VOI reaches to different maximum values beyond which it gradually decreases but still remaining positive throughout for the different cost ratios considered in the figure.

Figure 4.21 shows VOI for different cost ratios normalised by the VOI of cost-free inspection. The normalized VOI curves begins at different values for different cost ratios and remain constant for initial few inspections beyond which they start to decline as the inspection cost increases.
Figure 4.20: VOI as a function of number of inspections for varying cost ratios (inspection cost : repair cost : failure cost)

Figure 4.21: VOI normalized by the VOI of cost-free inspections as a function of number of inspections for varying cost ratios (inspection cost : repair cost : failure cost)
Figure 4.22 shows the variation of extent of inspection with the cost of inspection normalised by the cost of repair. The maximum number of inspections remains constant initially at the maximum value up to around 12% of normalised inspection cost. This means that full system, i.e., all defects can be inspected when the cost of inspection is up to 12% of the repair cost. Beyond this point, as the cost of inspection with respect to repair cost increases, the percentage of inspected locations decreases. It is not recommended to inspect the defects when the cost of inspection reaches close to 100% of the repair costs.

Figure 4.22: Normalized number of inspections versus the normalized cost of inspection
While considering the growth of defect for the analysis, individual size of defects were considered instead of a population approach, and future defect sizes of the respective defects were determined depending on their size in the last inspection value. However only one corrosion growth rate was determined overall for all the defects instead of corrosion growth for individual defects for the simplification of the analysis. This simplification might not result in accurate results as the average corrosion growth rate will underestimate the actual higher corrosion growth values and overestimate the lower values. This simplification was carried out to focus more on the inspection and maintenance strategies rather achieving an accurate corrosion growth model. However, since the final results depend on the accuracy of corrosion model, the following sensitivity analysis was carried out to observe the variation in VOI due to this simplification. Since considering individual corrosion growth will lead to a tedious process in the sensitivity analysis, hence as a middle ground, all the defects are categorised into 4 groups depending on their actual corrosion growth rate. Mean and standard deviation of the corrosion growth rates are determined for these 4 groups using which the shape and scale parameter for the gamma distributed corrosion growth rate of the pipeline section is determined as shown in Table 4.3. These parameters will be used for corrosion growth analysis depending to which group of corrosion growth rate the respective defect belongs.

**Table 4.3: Parameters used in corrosion growth modelling**

<table>
<thead>
<tr>
<th>Group No.</th>
<th>No. of defects</th>
<th>Corrosion growth rate per year (%wt)</th>
<th>Shape parameter $\alpha$ of corrosion growth rate</th>
<th>Scale parameter $\beta$ of corrosion growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>42</td>
<td>0.75-1.25</td>
<td>3.11</td>
<td>0.19</td>
</tr>
<tr>
<td>II</td>
<td>11</td>
<td>1.25-2.5</td>
<td>32.77</td>
<td>0.06</td>
</tr>
<tr>
<td>III</td>
<td>13</td>
<td>2.5-3.75</td>
<td>70.94</td>
<td>0.05</td>
</tr>
<tr>
<td>IV</td>
<td>3</td>
<td>3.75-5.5</td>
<td>59.02</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Figure 4.23 shows the PDF of the corrosion growth rate per year plotted using the above-mentioned values of shape and scale parameter for the four groups. The future size of the defects are determined using these different corrosion growth rates.

![PDF of corrosion growth rate for different groups](image)

**Figure 4.23: PDF of corrosion growth rate for different groups**

Analysis was carried out similarly by only changing these values of corrosion growth rate to determine prior and posterior cost in evaluating VOI. Figure 4.24 shows the variation of VOI with number of inspection locations for single corrosion growth considered earlier vs the four different corrosion growth rates considered now. All other values of analysis remains the same. From the figure, it is evident that even though VOI remains positive if all the 69 defects are inspected, however there is decline in the number of optimal inspection location and the maximum VOI achieved. The reason for this difference comes from the lowered prior cost of around 400 mu while considering different corrosion growth rates as it is more accurate and doesn’t overestimate the growth of higher defect sizes.
This sensitivity study shows the importance of considering accurate spatial model depending on the location of the defects to get accurate results as the defects closer to each other may follow the same corrosion growth but the defects which are far apart may grow differently independent of each other even under the same exposure condition.

![Variation in VOI and optimal number of inspections for defects with overall corrosion growth rate versus categorised corrosion growth rate](image)

**Figure 4.24:** Variation in VOI and optimal number of inspections for defects with overall corrosion growth rate versus categorised corrosion growth rate

### 4.2.4 Optimal maintenance decision

The final step in this framework is formulating maintenance strategies resulting in minimum lifecycle cost of this pipeline section. Some of the maintenance decisions such as threshold for the size of defect to carry out repair, threshold for probability restraint corresponding to the size of defect and the decision whether to repair only the critical defects or all the defects during a
particular maintenance event is analysed in this section. The optimal strategies are adopted by minimising the total lifecycle costs.

4.2.4.1 Optimal repair criterion

One of the ways to achieve optimal maintenance strategy is by deciding the optimal repair criterion beyond which the defects are repaired. Based on Equation (3.25), user has an option to decide the repair limit for a given probability threshold or to decide the probability threshold for a given repair limit which will result in minimal lifecycle costs, or the overall minimum lifecycle cost can be determined for a specific repair limit and probability threshold. Defect by defect analysis is carried out to evaluate individual defect’s repair and failure probabilities as per Equation (3.26) and (3.27) respectively based on the results of the corrosion growth model. When the defects are repaired during a particular maintenance activity, they are assumed to be regenerated again following the same corrosion growth pattern prior to their failure and considered in the subsequent analysis throughout the lifecycle.

Figure 4.25 shows the results of the analysis in the form of 3D contour plot of the total lifecycle costs plotted for various values of repair limit and probability constraints. The surface plot is U-shaped with higher lifecycle costs when the repair criterion is 40% wt. The surface plot gradually decreases and reaches minimum values of the contour plot between 55% wt to 90% wt for the repair criterion with probability constraint ranging between 0.001 to 0.04. For the higher repair criterion, probability constraint values are low and vice versa which is justifiable since higher constraint values are required to carryout repair for smaller size and low probability constraints are sufficient to repair larger defect sizes.
Figure 4.25: Contour plot of total lifecycle cost for different repair criterion and probability constraint ((a) Isometric view, (b) Top view)
The lowest lifecycle cost of 5,232.65 mu corresponds to repair limit of 85% wt with a probability constraint of $6 \times 10^{-3}$. However, the variation between the range of 55% wt-90% wt for the lowest lifecycle cost is very small, hence any of those points for repair criterion with corresponding probability constraint can be considered for achieving optimal values. This gives flexibility to the field applications where maintenance can be performed at different repair criterion within this range and still achieve more or less the same minimal costs.

**4.2.4.2 Decision between local and global repair**

In Section 4.2.4.1, the minimum lifecycle costs were determined while repairing critical defects during a maintenance event. If a structure has a number of defects, not all the defects would be critical during a given maintenance event. While only the critical ones are repaired, the others are left untouched until the next maintenance event when they become critical. This process of repairing the defects as and when they become critical continues over several maintenance events planned during remaining service life of structure. This plan will lead to minimum lifecycle cost when optimal repair criterion is adopted. However, in certain cases, it might be financially beneficial to repair all the defects (global repair) during a maintenance event irrespective of their criticality, rather than repairing defects individually (local repair) over several maintenance events. This is a possibility when the cost of global repair is cheaper and cost effective than the local repairs of several individual defects. There can be several reasons when a global repair cost will be cheaper than the cost of repairing defects individually. For example, when a group of defects are closer to each other, it might be economical to treat the defects altogether especially when the mobilisation cost is higher. To consider this scenario in this example, cost of repair of all the defects is assumed as 50% of the total cost of repair of the all the defects considered individually.
Apart from the decision to consider global repair instead of local repair during a maintenance event, there is an additional layer of decision making to decide during which maintenance event such consideration should be made. For instance, if there are 4 number of maintenance events planned during the service life as assumed in this example, then one of the possibilities is considering global repair during the first maintenance event and local repair during the last 3 events. Next possibility is to consider global repair during the second maintenance event and local repair in the first and last 2 events and so on. All the possible decision scenarios related to the time of replacing local repair with global repair shall be considered to determine minimum lifecycle cost as the corresponding repair and failure costs will vary. Since same corrosion growth rate is assumed for all the defects in this example, it is to be noted that in this scenario, once a global repair is carried out then the further growth of all the regenerated defects will be same and reach critical size in future at the same time. This also means that once a global repair is carried out during one of the maintenance events, then carrying out global repair will be the only option for the rest of the following maintenance events due to this assumption. Based on this condition, an algorithm was developed and following scenarios were considered.

Scenario 1: Global repair during all the maintenance events.

Scenario 2: Local repair during first maintenance event and global repair during rest of the maintenance events.

Scenario 3: Local repair during first two maintenance event and global repair during last two maintenance events.

Scenario 4: Local repair during first three maintenance events and global repair in the last maintenance event.
Lifecycle cost for each of these scenarios were determined and compared to the minimum lifecycle costs of 5,232.65 μu obtained in the Section 4.2.4.1 where only local repairs were carried out during all the maintenance events. Figure 4.26 shows the variation of total lifecycle costs corresponding to the maintenance event when the first global repair was carried out. The total lifecycle cost is 3,459.90 μu when the global repair is carried out for the first time during the first maintenance event corresponding to Scenario 1 whereas the total lifecycle cost increases to 6,285.80 μu when the global repair carried out for the first time during the last maintenance event corresponding to Scenario 4. Compared to the minimum total lifecycle cost achieved by the local repairs, carrying out the first global repair either during the first or second maintenance event seems to be beneficial.

![Graph](image-url)

**Figure 4.26**: Total lifecycle cost corresponding to maintenance events when global repair was carried out for the first time
5. CONCLUSION

The purpose of this research is to develop a comprehensive framework to achieve minimal lifecycle cost through risk-based inspection and maintenance planning for structural systems. This chapter briefly summarises the performed work, lists the main contributions, and concludes with a discussion on the limitations and suggested directions for future work.

5.1 Summary

Chapter one discusses the importance of the integrity management through inspection and maintenance of the structures, introduces problem statement and motivation to carry out the research, and sets out objectives to address the problem.

Chapter two contains the literature review for the various topics involved in the framework. It begins with an overview of the inspection tools and maintenance strategies commonly adopted for infrastructures, provides background information on the cost analysis with focus on risk-based lifecycle cost optimization, presents a detailed review of corrosion growth models, and outlines risk-based framework commonly adopted for inspection and maintenance planning.

Chapter three presents the main contribution of the research with the development of the framework and a step-by-step guideline to achieve optimal inspection and maintenance strategies. In addition to determining the strategies to achieve minimum lifecycle costs, the main idea is to provide user with a streamlined process. The framework comprises of six main sections: system modelling, structural deterioration modelling, structural reliability analysis, consequences modelling, inspection decision-making and maintenance decision-making. Under each section detailed explanation for the analysis and the methods adopted is covered, along with any assumption made for the simplification of the analysis. Further, recommendations for alternative
approaches are presented which will give flexibility to the user to navigate through the process depending on their requirements and end goals.

Chapter four presents two numerical examples, following most of the steps adopted in the framework to demonstrate its applicability to the real-world problems. To show the versatility of the framework and to demonstrate that the framework works well with different corrosion models, different approaches for corrosion analysis and subsequent decision making were considered in both examples.

In the first example, the framework was applied to a population-based approach of defects subjected to imperfect inspection data. A population-based corrosion growth model with hierarchical framework to account all the uncertainties was adopted. Model employs gamma process to determine the corrosion growth and probability of failure in the future is estimated using structural reliability analysis. In the next step, cost analysis is carried out based on the probability of failure of the population of defects. An optimum repair criterion is then determined based on the lifecycle cost optimisation. In the next step VOI analysis is carried out to determine if performing system level inspection would be financially beneficial. Since the inspection outcomes are unknown prior to the analysis, 250,000 simulations were carried out to generate different inspection outcomes. Finally in the last step, sensitivity analysis is carried out to study the effect of certain parameters on the analysis.

In the second example, framework was applied for a defect-specific approach considering inspection data to be perfect and free from measurement errors. Corrosion growth rate model is developed based on the past inspection data to determine future growth size of individual defects. Next, cost analysis for the cases with inspection and without inspection were determined to evaluate VOI. Optimal and maximum number of inspections along with their locations are then
determined. Corresponding to final section of the framework, lifecycle cost analysis is carried out to determine the optimal repair criterion which will give user the flexibility to use their resources and carry out repair. In the final step, decision to whether repair only the critical defects or all the defects during a maintenance event is analysed.

5.2 Main contributions

The main contribution of this research lies in the development of a comprehensive framework to achieve minimum cost using RBIM planning, streamlining the process and providing alternatives throughout the framework. The framework can be implemented in any infrastructure industry, to optimally plan the course of action and minimise the expense while retaining the integrity of the structure. For instance, in pipeline industry, this framework can be adopted to arrive at optimal repair criterion for carrying out inspection and maintenance decision based on risk of failure of a particular location. Codal provisions such as ASME provide repair criterion as 80 percent metal loss purely based on the condition of the system which is not a cost-effective method. Whereas applying this framework based on risk-based approach will result in higher cost savings by suitably planning and allocating the resources to locations with higher risk of failure. Overall, the main contributions can be listed as follows:

a) Contribution to the field of integrity management, where the framework developed in this research can be adopted in any type of infrastructure systems with application to pipeline, bridges, tunnels, etc. to minimise their lifecycle costs. Also, it can be applied for any deterioration mechanism by considering suitable deterioration model accordingly.

b) Contribution to the field of risk-based inspection, where the extent of inspection is determined through optimal number of inspections and maximum number of inspections in case of partial
inspection. Framework also considered VOI at system level, i.e., for population of defects in a section for faster decision support related to inspection planning.

c) Contribution to the field of risk-based maintenance, where optimal repair criterion is determined and decision to carry out repair of critical defects or all the defects is analysed, providing flexibility to the users to allocate their limited resources accordingly. Efforts were made to consider all the possible decision scenarios throughout the remaining service life of the structure.

d) The entire framework can be applied to a population of defects for a faster analysis or/and individual defects for in-depth analysis, providing alternative options to user. The framework is completely automated with minimal manual intervention.

5.3 Recommendations for future research

Although efforts were taken to account all the scenarios and considerations, however, certain assumptions were made either to simplify the analysis, or due to generalisation of the process, or due to lack of sufficient exposure. Following are the list of limitations and a recommended course of actions to overcome these limitations.

a) Only single deterioration mechanism is considered for the structural system in the numerical example, whereas in reality, the structure can be subject to multiple deterioration and failure mechanisms. Therefore, all the failure modes must be carefully evaluated and any dependency between them must be considered for accurate results.

b) When defects are repaired, they are either removed from the further analysis or assumed to be regenerating after repair following same corrosion growth rate as earlier assuming same exposure condition before and after repair. However, in reality, it is uncertain to know to what extent the regenerated defects will follow the earlier corrosion growth rate. To overcome this
limitation, consider a range of possible corrosion growth values with mean as the old corrosion growth and suitable standard deviation. Evolution of new defects shall also be considered which is ignored in this analysis.

c) In cost analysis, only direct costs related to maintenance and failure were considered as consequence of failure whereas other costs related to long-term economic and environmental impact are neglected. These costs can significantly contribute to the lifecycle costs. Hence, they should be included for a more comprehensive cost analysis.

d) The remaining service life and the various costs considered for the lifecycle analysis is usually uncertain. Hence it should be accounted in the analysis for precise results.

e) While carrying out VOI analysis for defect-specific approach, the analysis is simplified by considering only the cost in the next inspection instead of lifecycle cost due to complexity of the analysis. Considering all the possible scenarios throughout the lifecycle while determining VOI for defect-specific approach would be a good avenue for future study.
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