THE UNIVERSITY OF CALGARY

THE UTILIZATION OF HORIZONTAL WELLS FOR THE PRODUCTION OF CONVENTIONAL PETROLEUM - A STUDY OF THE THEORY AND AN ANALYSIS OF THE POTENTIAL FOR APPLICATIONS IN CANADA

by

W. Selina Yeung

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN PARTIAL

FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

CALGARY, ALBERTA, CANADA

SEPTEMBER, 1987

ISBN 0-315-38095-0

THE UNIVERSITY OF CALGARY

FACULTY OF GRADUATE STUDIES

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THE UTILIZATION OF HORIZONTAL WELLS FOR THE PRODUCTION OF CONVENTIONAL PETROLEUM - A STUDY OF THE THEORY AND AN ANALYSIS OF THE POTENTIAL FOR APPLICATION IN CANADA

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ABSTRACT

Horizontal wells have been shown to increase production, reduce coning tendencies and increase recovery. This thesis reviews and summarizes the reservoir engineering aspect of horizontal well production. It includes:

1. Derivation of the steady state flow equations for horizontal wells;
2. Development and derivation of two new flow equations – pseudo-steady state flow equations for an infinite array of horizontal wells with large and small interwell spacing;
3. Evaluation of oilfield development scenarios (vertical vs horizontal wells) – includes a new approach to compare the performance of a horizontal well to different patterns of vertical wells under steady state and pseudo-steady state flow;
4. Effectiveness of horizontal wells in potential coning reservoirs - introduces a new, simplified model, which applies to heavy oil with bottom water, to predict and compare the performance of horizontal to vertical wells;
5. Applications in Canada's oilfields.

In summary, horizontal wells can be 2 to 10 times more effective than vertical wells and can be used to optimize oil production from an

oilfield. The most favourable applications are:

i) where close well spacing is required,

ii) in vertically fractured reservoirs and

iii) in reservoirs with gas cap or bottom water.

ACKNOWLEDGEMENTS

I am indebted to Dr. R.M. Butler, my dissertation director, for his guidance and encouragement during the course of this research and for his development of some of the theory described in the report - particularly that of equations 3.20, 3.22 and 4.5 and derivation in Appendix II. I am also grateful to Patricia Stuart-Bakes for typing the manuscript.

This dissertation is dedicated to my husband.

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LIST OF SYMBOLS

A, A"	area, m^2
A_S	total surface area, m^2
B, S_1	length dimension of drainage area, m
C_A	dimensionless shape factor
C_t	total compressibility, kPa^{-1}
F	ratio of oil to water viscosities
H	formation thickness, m
h′	equivalent isotropic pay thickness, m
K	permeability, μm^2
Ki	directional permeability along the principal axis, μm^2
K′	equivalent isotropic permeability, μm^2
L	horizontal well length, m
L′	ratio of horizontal well length to pay thickness
L"	total length of the multiple level horizontal drain holes, m
N	ratio of cumulative oil production to maximum water-free oil production
P	pressure, kPa
P_i	initial pressure, kPa
P_D	dimensionless pressure
$\overline{P}$	average reservoir pressure, kPa
Q	flow rate, m^3/d

Q_1	flow rate per unit reservoir thickness, m^3/d - m
Q_2	flow rate per unit well length, m^3/d - m
Q_o	cumulative oil production for horizontal well, m^3
Q_{os}	cumulative oil production for vertical well, m^3
$Q_{o,dry}$	cumulative dry oil production, m^3
Q_p	cumulative fluid production for horizontal well, m^3
Q_{ps}	cumulative fluid production for vertical well, m^3
Q_W	cumulative water production for horizontal well, m^3
Q_{WS}	cumulative water production for vertical well, m^3
$Q_{\ell C}$	critical flow rate per unit length, m^3/d - m
R	radius of drainage boundary for horizontal well at time t, m
R_S	radius of drainage boundary for vertical well at time t, m
S_o	oil saturation, fraction
V	drainage volume, m^3
W	interwell spacing, m
X_A	distance from the well to where the oil/water interface is horizontal, m
X^*	dimensionless cumulative fluid production for horizontal well
X_S^*	dimensionless cumulative fluid production for vertical well
Y	ratio of horizontal to vertical productivity index

Z	ratio of pay thickness to radius of oil/water interface at time t for vertical well
Z_S	well to cone apex distance, m
PI	productivity index, m^3/d - kPa
API	area productivity index, m^3/d - kPa - m^2
a, b	major and minor half length axes of the ellipse, m
g	acceleration due to gravity, m^2/s
h	formation thickness, m
h'	equivalent isotropic pay thickness, m
ℓ	length of horizontal drainhole, m
m	number of levels of horizontal drainholes
n	number of horizontal drainholes extended from the central shaft
q	flow rate, m^3/d
q_o	oil flow rate, m^3/d
q_ℓ^*	dimensionless flow rate
$q_{\ell c}^*$	dimensionless critical flow rate
r	radius, m
r'	equivalent radius, m
$r_{e,h}$	external drainage radius for horizontal well, m
t	time, s
t_{DA}	dimensionless time function based on drainage area
x	distance from the well, m

Greek Symbols:

ρ	density, kg/m^3
γ	water cut for horizontal well, fraction
γ_s	water cut for vertical well, fraction
μ	viscosity, mPa.s
μ_o	oil viscosity, mPa.s
μ_w	water viscosity, mPa.s
Ω	flow resistance, $kPa.s/m^3$
δ	distance from mid height of formation, m
η	dimensionless number determined in section 3.7
Φ	potential function
ψ	stream function
ϕ	porosity, fraction
θ	angle, radian
α	ratio of the radius of water/oil interface at time t to that of the radius cut, maximum dry oil production

Subscripts:

A	point A
S	point S
T	total
W	wellbore

e	external drainage boundary
h	horizontal
m	area average
o	initial
v	vertical

Superscripts:

~	dimensionless quantity

CHAPTER 1

INTRODUCTION

Horizontal holes, in the form of dykes and weeping tiles, have long been recognized as advantageous in ground water drainage applications. The potential benefits to oil and gas production provided by the increased access to the resource through horizontal holes have been investigated since the early 1920's.

The primary objective of horizontal drilling is to improve productivity and recovery. Horizontal drilling aims to change the flow conditions radically, creating a planar flow pattern instead of the conventional radial circular flow pattern. It is this change in flow pattern which is the controlling factor of the productivity and/or recovery improvement.

In conventional practice, the production well is drilled vertically to provide access to the pay zone. Typically, the wellbore will have a diameter of four inches to eight inches and perforations are made in the casing across the pay zone. In this configuration oil must flow from the pay zone to these perforations and then up through the production string. The limited access in the pay zone and the large pressure drawdown near the wellbore promotes production problems like well sanding, premature breakthrough, etc.

Horizontal wells, on the other hand, provide access to the resource much greater than that of vertical wells. It is this, effectively unlimited access, which provides the potential benefit from horizontal holes.

At present there are four methods, either available or being developed, to place horizontal holes in the pay zone. These are categorized as:

(1) Long Radius: Horizontal wells are drilled and cased using conventional directional drilling techniques. A downhole motor with a bent housing is often employed. This tends to drill along a curve and can be steered by orienting the drill pipe. If it is rotated the holes tend to be nearly straight. The downhole location is measured continuously using MWD (Measurement While Drilling).

(2) Medium Radius: Horizontal wells are directional holes turned from vertical to horizontal in a relatively short vertical section (i.e. 10-20 m) and drilled with conventional bits. Articulated drill stems are usually employed to allow relatively large curvatures.

(3) Short Radius: Horizontal drain-holes are hydraulically drilled with tubing. The turn from vertical to horizontal is made in a 3 m section and is initiated by means of a whipstock tool erected in an existing vertical wellbore.

(4) In-situ Access: Horizontal holes are drilled from a central shaft and/or tunnels.

The benefits of horizontal drilling were not realized earlier because there was no technology to place horizontal holes in the pay zone economically. Recent technology advancement coupled with declining reserves and increasing finding and lifting costs will stimulate and increase the applications of horizontal wells in future oilfield development.

CHAPTER 2

THEORY

2.1 Flow Equations for an Isolated Horizontal Well in an Infinite Reservoir

Horizontal wells have been used successfully to increase the productivity of wells in the petroleum industry. The increase of productivity due to longer perforated intervals increases as the logarithm of length. The following presents and derives two equations:

(1) for the case of an isolated horizontal well in an infinite reservoir and

(2) for the case of a long horizontal well.

The effects of anisotropy and also eccentricity are considered.

2.1.1 *Homogeneous Isotropic Reservoir*

Consider an isolated horizontal well located at the mid-height of a large homogeneous, isotropic reservoir bounded by horizontal planes. The solution to the Laplace equation for an incompressible, steady state flow can be approximated by a combination of the solutions to the following problems:

(1) potential distribution about a horizontal well in a horizontal plane.

(2) potential distribution about a horizontal well located at the centre of an infinite, homogeneous reservoir bounded by upper and lower horizontal planes.

Appendix I and II describe the solutions to the above problems. The flow equation describing an isolated horizontal well located in the centre of an infinite reservoir is approximated as follows:

Figure 2.1 shows the flow pattern to an isolated horizontal well in the horizontal plane. The horizontal well is located between the foci of the ellipse. As shown in Appendix I, both the stream function and the potential function are proportional to the well length, therefore, Figure 2.1 can be scaled to represent the flow pattern of any isolated horizontal well. The total flow resistance in the horizontal plane Ω_h, in a reservoir with thickness h is (See Appendix I).

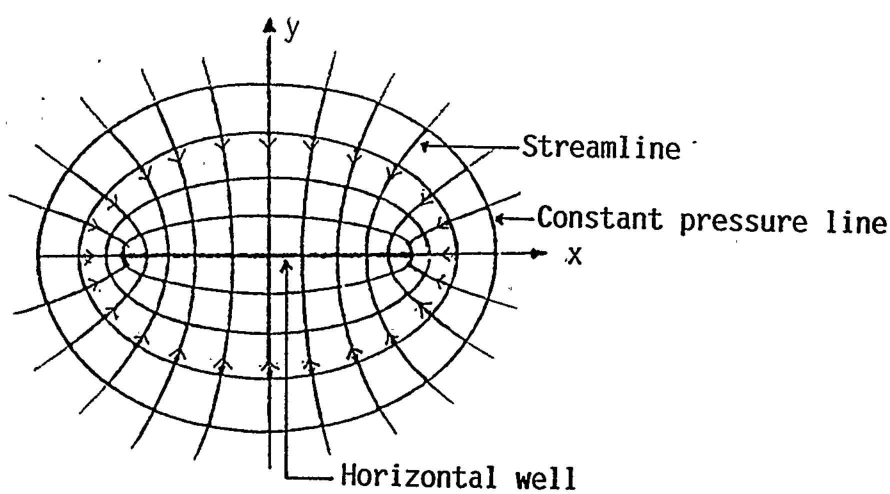


Figure 2.1 Flow Pattern to Horizontal Well in the Horizontal Plane

$$\Omega_h = \frac{\Delta P_h}{Q_1 h} \quad = \frac{\mu}{2\pi K h} \ln \left(\frac{a+b}{L/2}\right) \tag{2.1}$$

where

$a^2 = b^2 + \frac{L^2}{4}$

a, b = major and minor half lengths of an ellipse having foci located at the ends of the well

L = length of horizontal well

μ = viscosity

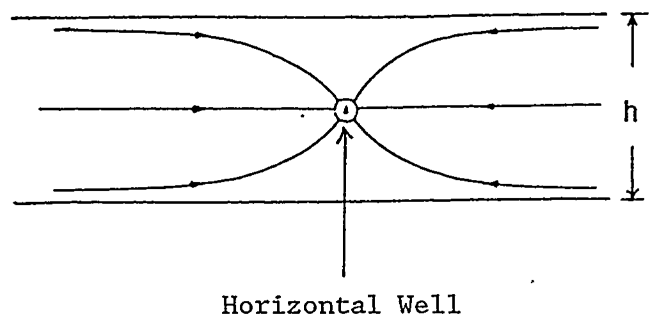
K = permeability

Q_1 = flow rate per unit reservoir thickness

ΔP_h = pressure differential between the external elliptical drainage boundary and the wellbore in the horizontal plane.

In the above equation, it is assumed that h is very small and that the resistance to vertical flow can be neglected.

Figure 2.2 shows the flow pattern to a horizontal well in the vertical plane. The flow resistance, Ω_v, due to concentration of streamlines moving towards the horizontal well in the vertical plane perpendicular to the well axis is (see Appendix II)



Horizontal Well

Figure 2.2 Flow Pattern to Horizontal Well in the Vertical Plane

$$\Omega_v = \frac{\Delta P_v}{Q_2 L}$$

$$= \frac{\mu}{2\pi K L} \ln \frac{h}{2\pi r_w} \qquad (2.2)$$

where

r_w = well radius

Q_2 = flow rate per unit length

ΔP_v = pressure differential between the external drainage boundary and the wellbore in the vertical plane.

Provided that a is large compared to h, the resistance, Ω_v, can be considered as a "skin effect" which causes additional pressure drop to the horizontal well. The sum of the flow resistances, Ω_T:

$$\Omega_T = \Omega_h + \Omega_v$$

$$= \frac{\Delta P}{Q_h}$$

where ΔP = pressure differential between the external drainage boundary and the wellbore and $\Delta P = \Delta P_v + \Delta P_h$.

and the flow rate of the horizontal well, Q_h is

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left(\frac{a+b}{L/2}\right) + \frac{h}{L} \ln \frac{h}{2\pi r_w}} \tag{2.3}$$

As shown in Appendix I, equation (2.3) can be rewritten in terms of equivalent drainage radius, $r_{e,h}$. The following tabulates the different forms of equation (2.3) which have been proposed in the literature.

1. When $\frac{L}{2a} \leq 0.5$, $r_{e,h} \simeq a$ (Giger's equation (12))

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left[\frac{r_{e,h}\left(1 + \sqrt{1-\left(\frac{L}{2r_{e,h}}\right)^2}\right)}{L/2}\right] + \frac{h}{L} \ln \left(\frac{h}{2\pi r_w}\right)} \tag{2.4}$$

2. When $\frac{L}{2b} \leq 0.5$, $r_{e,h} \simeq b$ (Borisov's equation (3))

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left[\dfrac{r_{e,h}\left(1 + \sqrt{1+\left(\dfrac{L}{2r_{e,h}}\right)^2}\right)}{L/2}\right] + \dfrac{h}{L}\ln\left(\dfrac{h}{2\pi r_w}\right)} \tag{2.5}$$

3. When $\dfrac{L}{2r_{e,h}} \ll 1$

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left[\dfrac{4r_{e,h}}{L}\right] + \dfrac{h}{L}\ln\left(\dfrac{h}{2\pi r_w}\right)} \tag{2.6}$$

Equation (2.6) can also be obtained when the equivalent radial drainage radius is defined as being equal to the average of the lengths of the two axes of the ellipse, i.e.

$$r_{e,h} = \frac{a+b}{2}$$

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left[\dfrac{4r_{e,h}}{L}\right] + \dfrac{h}{L}\ln\left(\dfrac{h}{2\pi r_w}\right)}$$

When the flow restriction term $\left(\dfrac{h}{L}\ln\dfrac{h}{2\pi r_w}\right)$ in equation (2.6) is negligible compared to $\ln\dfrac{4r_{e,h}}{L}$, the flow equation reduces to that for the production from a fully penetrating vertical fracture. Furthermore, equations (2.3) and (2.6) are equivalent, i.e. when the equivalent drainage radius is the average of the major and minor axes of the ellipse, the flow equation becomes exact, therefore, equation (2.6) is best for calculating the flow rate to the horizontal well. On the other hand, equation (2.4) will give a more optimistic flowrate for the horizontal well compared to equation (2.5). Where $r_{e,h}$ is large compared to L, equations (2.4) to (2.6) give similar values for Q_h. Under these conditions, the external elliptical drainage boundary approaches a circle; the effect of the horizontal well is then comparable to that of an extreme increase in the radius of a conventional vertical wellbore.

2.1.2 *Effect of Anisotropy*

Many reservoirs are anisotropic and this anisotropy may be represented by the permeabilities Kx, Ky and Kz.

Muskat (22) reduced the single phase flow problem in anisotropic reservoirs to flow in an equivalent "isotropic reservoir" by expanding the spatial co-ordinates x, y and z in the ratio $\left(\frac{K'}{Ki}\right)^{1/2}$ where K' is the equivalent isotropic permeability and Ki (i=x,y,z) are the directional permeabilities along the x,y and z principal axes. Therefore, for an anisotropic reservoir where K_v is different from K_h, the flow equation (2.3) will become

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left(\frac{a+b}{L/2}\right) + \frac{h'}{L} \ln\frac{h'}{2\pi r_w}} \tag{2.7}$$

where

$$h' = h\sqrt{\frac{K_h}{K_v}}$$

In reservoirs where the drainage ratios (or either a or b) are sufficiently large compared to the length of the well, L, the well productivity becomes independent of the reservoir height and/or the value of K_v.

2.1.3 *Horizontal Well Vertical Placement Eccentricity*

Equations (2.2) and (2.4) are for a horizontal well located in the vertical centre of a formation of thickness h. When the horizontal well is displaced vertically by a distance δ away from the centre of the formation, the well will experience an additional pressure drop. As shown by Muskat (22) and S.D. Joshi (18), the oil production for an off-centred well which is infinite is:

$$Q_2 = \frac{2\pi K}{\mu} \frac{\Delta P}{\ln \frac{h}{2\pi r_w} + \ln\left[1 - \left(\frac{2\delta}{h}\right)^2\right]}$$

$$= \frac{2\pi K}{\mu} \frac{\Delta P}{\ln\left[\frac{h\left(1 - \left(\frac{2\delta}{h}\right)^2\right)}{2\pi r_w}\right]} \qquad \text{for} \quad \delta < \frac{h}{2} - r_w \tag{2.8}$$

where Q_2 is the flow rate to the horizontal well in the vertical cross-section per unit length. Muskat used the Green function to derive the flow equation for an off-centred well. At $\delta = \frac{h}{2}$, the Green function vanishes and equation (2.8) becomes undefined.

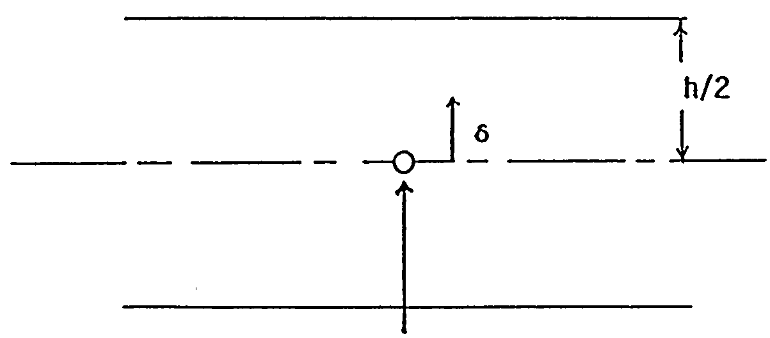


Figure 2.3 Horizontal Well Eccentricity

For the situation where $\delta < \left(\frac{h}{2} - r_w\right)$, equation (2.3) becomes (18)

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln\left(\frac{a+b}{L/2}\right) + \frac{h}{L} \ln\left[\frac{\left(\frac{h}{2}\right)^2 - \delta^2}{\frac{h\pi r_w}{2}}\right]} \tag{2.9}$$

The above equation shows that it is best to place the horizontal well in the middle of the formation when δ is small and the correction for eccentricity becomes unimportant.

2.2 Flow Equation for a Long Horizontal Well

Equation (2.6) describes the flow to a horizontal well where the length of the well is short compared to the drainage radius. In the case of a long well, the edge of the area drained becomes oval in shape. Giger (14) assumed that the potential distribution for such a system can be approximated by placing the long horizontal well in the centre of a rectangular drainage block with the flow to the well being supplied by the two vertical sides parallel to the well (Figure 2.1). He assumed that the vertical sides parallel to the well are maintained at constant pressure and the other two vertical sides are no-flow boundaries.

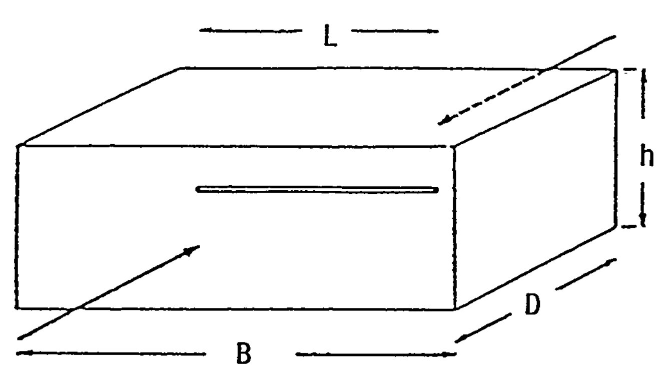


Figure 2.4 Horizontal Well Fed Laterally from Two Sides

If the vertical pressure gradients are neglected, then the potential distribution around the horizontal well in the horizontal plane is similar to the case where drainage occurs through a vertical fracture crossing the formation from top to bottom and equal in length to the horizontal well. Houpeurt (17) has shown that for this case, the potential difference, $\Delta\Phi_{ew}$, from the drainage boundary to the well is given by,

$$\Delta\Phi_{ew} = \frac{Q_1}{2\pi Kh} \operatorname{Cosh}^{-1}\left[\frac{\operatorname{Cosh} \frac{\pi}{2} \frac{D}{B}}{\operatorname{Sin} \frac{\pi}{2} \frac{L}{B}}\right] \tag{2.10}$$

where Q_1 is the flow to the horizontal well in the horizontal plane per unit pay thickness and D and B are the width and length dimensions of the rectangular drainage block as shown in Figure 2.4. In this equation, it is assumed that a constant pressure condition exists along the perimeter length, B, and a no-flow boundary condition along the side width, D, of the drainage block.

Superimposing the flow resistance at the wellbore, the flowrate of a long horizontal well (13) is

$$Q_h = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\operatorname{Cosh}^{-1}\left[\frac{\operatorname{Cosh} \frac{\pi}{2} \frac{D}{B}}{\operatorname{Sin} \frac{\pi}{2} \frac{L}{B}}\right] + \frac{h}{L} \ln \frac{h}{2\pi r_w}} \tag{2.11}$$

When the well length is the same as the length of the rectangular block, B, the flow equation (2.11) simplifies to

$$Q_h = \frac{2\pi KL}{\mu} \quad \frac{\Delta P}{\frac{\pi}{2}\frac{D}{h} + \ln\left(\frac{h}{2\pi r_w}\right)} \tag{2.12}$$

2.3 Pseudo Steady State Equation for Horizontal Well

Equations (2.6) and (2.12) developed earlier are for steady state flow and it was assumed that constant supply pressure was maintained along the "drainage boundary"

The relationship between these equations and the conditions which exist in a depleting reservoir is rather obscure, particularly when the length of the well is a large fraction of the reservoir size.

The radial flow to a conventional vertical well produces a condition in which, because of the converging flow, the area average reservoir pressure is only slightly lower than the perimeter pressure. The situation for the non-converging parallel flow to a horizontal well (neglecting the usually slight vertical component) is quite different. Here the area average reservoir pressure is considerably lower than the perimeter pressure. This phenomenon makes the comparisons given above pessimistic in predicting the improved productivity to be expected from a horizontal well. The improvement in productivity index based on average reservoir pressure is significantly greater than that predicted for indices based on perimeter pressure. This concept will be developed further in the next sections which are based on the concept of pseudo steady state flow rather than steady state flow.

The following is the development of pseudo steady state equations for parallel infinite horizontal wells with fixed large and small interwell spacing. Pseudo steady state flow is the situation which prevails in a bounded reservoir which is producing by fluid expansion and in which the pressure is falling everywhere at the same rate.

2.3.1 *Parallel Infinite Horizontal Wells With Large Interwell Spacing*

Consider a length L of a series of parallel infinite horizontal wells with interwell spacing of 2W, each of which is located in the vertical centre of a reservoir of height h (Figure 2.5).

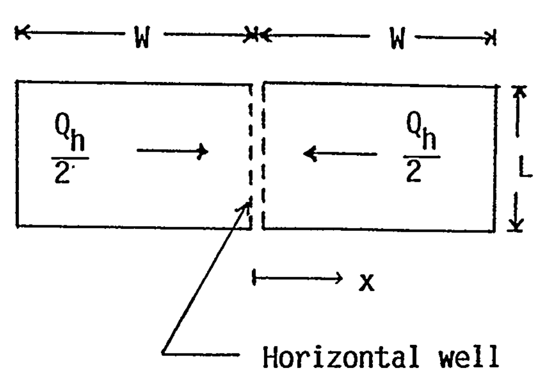


Figure 2.5 Plan View of a Section at Length L of a Single Horizontal Well in a Drainage Pattern

For the present, neglect the vertical flow convergence,

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi \mu c_t}{K} \frac{\partial P}{\partial t} \tag{2.13}$$

where c_t = total compressibility

ϕ = porosity

P = pressure

t = time

x = distance from well

In pseudo steady state, $\frac{\partial P}{\partial t}$ = constant.

$$Q_h = -\frac{\partial P}{\partial t} c_t \phi V \tag{2.14}$$

where V = drainage volume of horizontal well

= 2WLh

Q_h = oil flow rate (2 sides)

Substitute equation (2.14) into equation (2.13) and simplify, to give

$$\frac{\partial^2 P}{\partial x^2} = -\frac{Q_h \mu}{VK} \tag{2.15}$$

integrating with respect to x,

$$\frac{\partial P}{\partial x} = c_1 - \frac{Q_h \mu}{VK} x \qquad (2.16)$$

where c_1 = integration constant

Boundary condition at $x = W$, $\frac{\partial P}{\partial x} = 0$ because it is a no flow boundary.

Therefore $\quad c_1 = \frac{Q_h \mu}{VK} W$

$$\frac{\partial P}{\partial x} = \frac{Q_h \mu}{VK} (W-x) \qquad (2.17)$$

Integrate again,

$$P = c_2 + \frac{Q_h \mu}{VK} \left(Wx - \frac{x^2}{2}\right) \qquad (2.18)$$

Boundary condition at $x = 0$, $P = P_W$, the wellbore pressure.

Therefore, $\quad c_2 = P_W$

$$P - P_W = \frac{Q_h \mu}{VK} \left(Wx - \frac{x^2}{2}\right) \qquad (2.19)$$

The mean reservoir pressure is given by

$$(P_m - P_W)W = \int_0^W (P - P_W)\, dx$$

$$= \frac{Q_h \mu}{VK} \left[\frac{Wx^2}{2} - \frac{x^3}{6} \right]_o^W$$

$$(P_m - P_W)W = \frac{Q_h \mu W^3}{3VK} \tag{2.20}$$

Since $V = 2WLh$, so $P_m - P_W = \dfrac{Q_h \mu W}{6LhK}$ (2.21)

2.3.2 *Effect of Wellbore*

In addition to the pressure drop shown above, there is a pressure drop due to the convergence of the flow to the wellbore radius. This will be treated as if it is a "skin effect". The extra pressure gradient is, as before, $\Delta P = \dfrac{Q_h \mu}{2\pi KL} \ln \dfrac{h}{2\pi r_w}$.

and the overall pressure drop is

$$(P_m - P_W) = \frac{Q_h \mu W}{6LhK} + \frac{Q_h \mu}{2\pi KL} \ln \frac{h}{2\pi r_w} \tag{2.22}$$

2.3.3 *Parallel Infinite Horizontal Wells With Small Interwell Spacing*

Since equation (2.22) involves treating the near well bore flow restriction as a "skin factor", it does not apply to situations where h/W is appreciable. These situations can be accommodated by the following method.

The pseudo steady state pressure for a vertical well with no flow outer boundaries is given by Earlougher et al (11) as

$$P_D = 2\pi t_{DA} + \frac{1}{2}\ln\left[\frac{A''}{r_w 2}\right] + \frac{1}{2}\ln\left[\frac{2.2458}{C_A}\right] \tag{2.23}$$

where P_D = dimensionless pressure

$$= \frac{2\pi K\ (P_i - P_W)}{Q_2\mu} \tag{2.24}$$

and

Q_2 = flow per unit length

c_A = shape factor

$$t_{DA} = \frac{Kt}{\phi\mu C_t A''} \tag{2.25}$$

t = time

A'' = drainage area

c_t = total compressibility

P_i = initial pressure

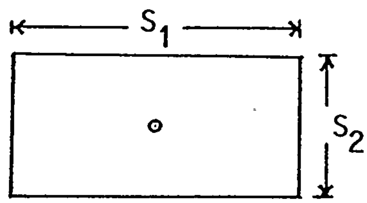
P_W = well pressure

Also, $Q_2 t$ = total production per unit length

$$= (P_i - P_m)c_t\ \phi\ A'' \tag{2.26}$$

P_m = area average pressure

The quantity $\frac{1}{2}\ln\left(\frac{2.2458}{c_A}\right)$ is a parameter which depends on the shape of the drainage pattern. The following table is taken from Earlougher's book, "Advances in Well Testing Analysis" (10).



Shape Factors For Various Closed Single-Well Drainage Areas

$\frac{S_1}{S_2}$	$\frac{1}{2}\ln\left(\frac{2.2458}{c_A}\right)$
1	− 1.3106
2	− 1.1373
4	− 0.4367
5	− 0.0249
circle	− 1.3224
hexagon	− 1.3220
equilateral triangle	− 1.2544
square	− 1.3106

Re-arrange equation (2.26), we have

$$P_i = \frac{Q_2 t}{c_t \phi A''} + P_m$$

and substitute into equation (2.23), we have

$$P_D = \frac{2\pi K}{Q_2 \mu} (P_m - P_W) + 2\pi t_{DA} \qquad (2.27)$$

Since equations (2.23) and (2.27) are equal, we have

$$\frac{2\pi K}{Q_2\mu} \ (P_m - P_W) = \frac{1}{2} \ln\left(\frac{A''}{r_w 2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{c_A}\right)$$

$$P_m - P_W = \frac{Q_2\mu}{2\pi K} \left[\frac{1}{2} \ln\left(\frac{A''}{r_w 2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{c_A}\right)\right] \qquad (2.28)$$

If the above equation is viewed as applying to an infinite horizontal well centred in a vertical, rectangular drainage area, then $S_1 = 2W$ and $S_2 = h$; $A'' = S_1 S_2$

Substitute these into equation (2.28), we have:

$$P_m - P_W = \frac{Q_2\mu}{2\pi K} \left[\frac{1}{2} \ln\left(\frac{S_1 S_2}{r_w 2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{c_A}\right)\right]$$

or

$$P_m - P_W = \frac{Q_2\mu}{2\pi K} \left[\frac{1}{2} \ln\left(\frac{2Wh}{r_w 2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{c_A}\right)\right] \qquad (2.29)$$

In the above equation, the horizontal drainage area per unit length of well is S_1 x 1 or 2W.

2.4 Reservoir Pressure Profile Under Pseudo Steady State

a) Vertical Wells

The Darcy flow equation for a vertical well under pseudo steady state is (9):

$$P - P_W = \frac{Q_v\mu}{2\pi Kh} \left(\ln \frac{r}{r_w} - \frac{r^2}{2r_e^2}\right) \tag{2.30}$$

Let $A = \pi r^2$, $A_e = \pi r_e^2$ and $A_W = \pi r_w^2$ and rewrite equation (2.30).

$$P - P_W = \frac{Q_v\mu}{2\pi Kh} \left[\frac{1}{2} \ln \frac{A}{A_W} - \frac{1}{2} \frac{A}{A_e} \right]$$

$$= \frac{Q_v\mu}{4\pi Kh} \left[\ln \frac{A}{A_W} - \frac{A}{A_e} \right] \tag{2.31}$$

When the above inflow equation is expressed in terms of the average reservoir pressure $\overline{P}$, we have

$$\overline{P} - P_W = \frac{Q_v\mu}{2\pi Kh} \left[\ln \frac{r_e}{r_w} - 0.75 \right]$$

$$= \frac{Q_v\mu}{4\pi Kh} \left[\ln \frac{A_e}{A_W} - 1.5 \right] \tag{2.32}$$

b) Parallel Horizontal Wells

For a horizontal well with length L and interwell spacing W, the pseudo steady state flow equation is:

$$P - P_W = \frac{Q_h\mu}{2WLhK} \left[Wx - \frac{x^2}{2}\right] + \frac{Q_h\mu}{2\pi KL} \ln \left(\frac{h}{2\pi r_w}\right) \tag{2.33}$$

Let $A = 2Lx$, $A_e = 2WL$ and substitute into equation (2.33), to give

$$P - P_W = \frac{Q_h\mu}{Kh}\left[\frac{W}{2L}\frac{A}{A_e} - \frac{A^2}{8A_eL^2} + \frac{1}{2\pi}\frac{h}{L}\ \ln\frac{h}{2\pi r_w}\right]$$

$$= \frac{Q_h\mu}{4\pi Kh}\left[2\pi\frac{W}{L}\frac{A}{A_e} - \pi\frac{W}{L}\left(\frac{A}{A_e}\right)^2 + \frac{2h}{L}\ \ln\frac{h}{2\pi r_w}\right] \qquad (2.34)$$

In terms of average pressure $\bar{P}$, the above equation becomes

$$\bar{P} - P_W = \frac{Q_h\mu}{2KL}\left[\frac{W}{3h} + \frac{1}{\pi}\ \ln\frac{h}{2\pi r_w}\right]$$

$$= \frac{Q_h\mu}{4\pi Kh}\left[\frac{2}{3}\pi\frac{W}{L} + \frac{2h}{L}\ln\frac{h}{2\pi r_w}\right] \qquad (2.35)$$

c) Comparison of Horizontal and Vertical Wells

Figure 2.6 shows the reservoir profiles of a vertical well and a horizontal well. It is assumed that the reservoir has a 10 metre pay zone and both wells are producing at the same rate with the same drainage area of 1 hectare. It is found that the vertical well has a larger pressure drop near the well as compared to the horizontal well and the pressure drawdown using the perimeter pressure and average pressure for the horizontal well are lower than for the corresponding vertical well. In reality, due to the layer pressure drop, more solution gas will be evolved near the vertical well and oil production decreases. However, for single phase flow, the recovery for the field using vertical wells is not drastically different from that using horizontal wells.

In this particular example, the horizontal well has 100 m of perforated length within the reservoir, whereas the vertical well has only 10 m. For the same production rate, the difference between the average reservoir pressure and the wellbore pressure is 11.3 units for the vertical well but only 1.7 for the horizontal well. The productivity index for the same reservoir area and for the same production rate is thus 11.3/1.7 = 6.6 times higher for the horizontal well. The corresponding perimeter pressures are 2.1 for the horizontal well and 11.7 for the vertical well. Based on the perimeter pressures, the improvement would be only 5.6. Under some circumstances, the difference would be greater.

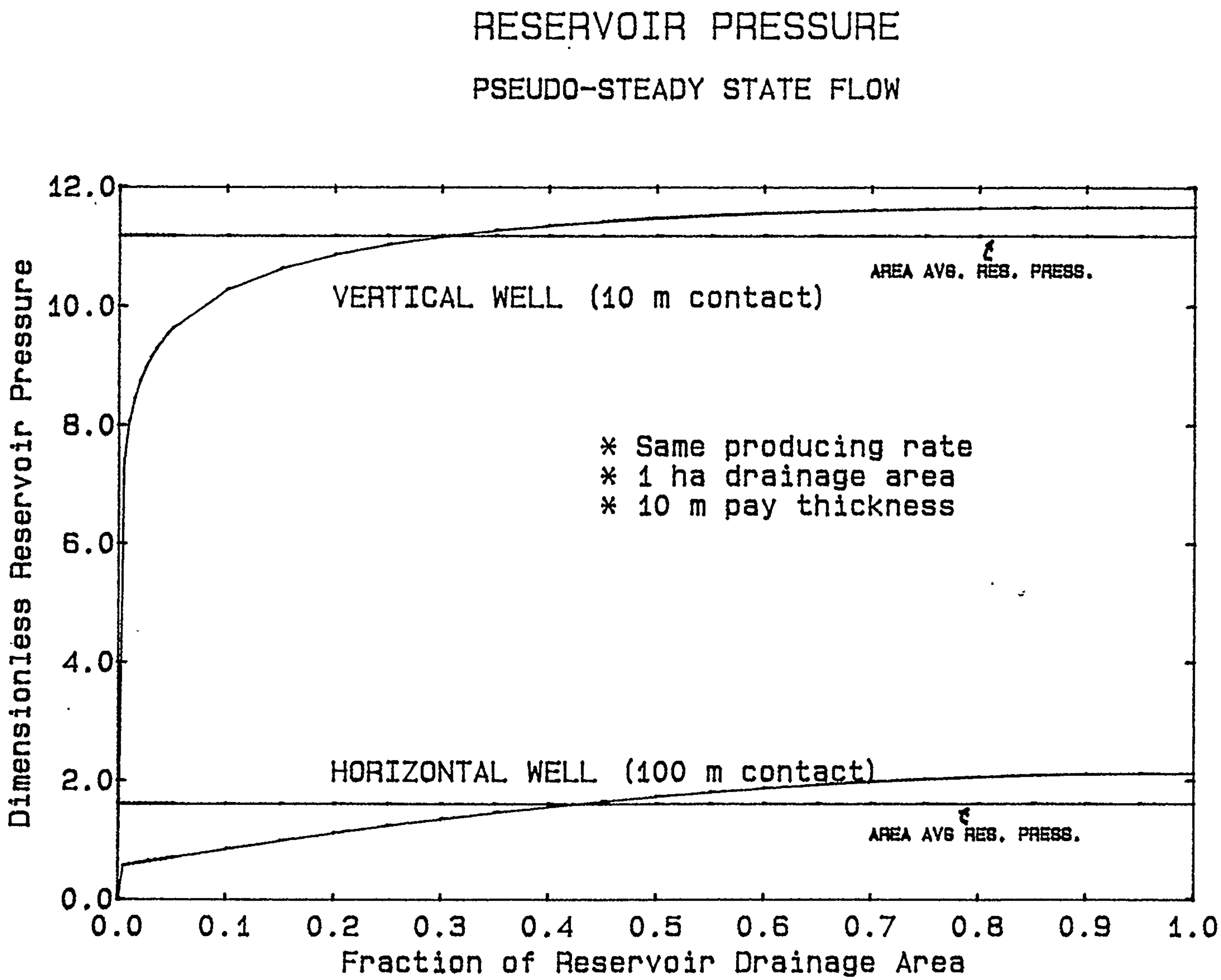

RESERVOIR PRESSURE
PSEUDO-STEADY STATE FLOW
VERTICAL WELL (10 m contact)
AREA AVG. RES. PRESS.
* Same producing rate
* 1 ha drainage area
* 10 m pay thickness
HORIZONTAL WELL (100 m contact)
AREA AVG RES. PRESS.
Dimensionless Reservoir Pressure
Fraction of Reservoir Drainage Area
12.0
10.0
8.0
6.0
4.0
2.0
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 2.6

CHAPTER 3

OILFIELD DEVELOPMENT EVALUATION

3.1 Horizontal Wells vs Vertical Wells

In an evaluation of an oilfield development where comparative studies are performed on systems of horizontal and conventional vertical wells, one of the criteria used for decision making is the additional cost vs the increase in productivity for horizontal wells.

3.1.1 *Isolated Horizontal Well*

The productivity of a vertical well, (PI_V) is expressed as follows:

$$PI_v = \frac{2\pi Kh}{\mu} \frac{1}{\ln\left(\frac{r_e}{r_w}\right)} \qquad (3.1)$$

For an isolated horizontal well with length L, the productivity index (PI_h) is:

$$PI_h = \frac{2\pi Kh}{\mu} \frac{1}{\ln\left(\frac{4r_e}{L}\right) + \frac{h}{L} \ln\left(\frac{h}{2\pi r_w}\right)} \qquad (3.2)$$

The ratio of the two productivity indices, Y, is

$$Y = \frac{PI_h}{PI_v} = \frac{\ln\left(\frac{r_e}{r_w}\right)}{\ln\left(\frac{4r_e}{L}\right) + \frac{h}{L}\ln\left(\frac{h}{2\pi r_w}\right)} \tag{3.3}$$

When $L > h$, equation (3.3) shows that for the same drainage and wellbore radius, Y is always greater than 1. Therefore, under these conditions, the productivity of a horizontal well is always greater than a vertical well and the productivity ratio increases for thinner formations and longer well lengths. The increase in the productivity ratio with horizontal well length for different formation thicknesses is shown in Figure 3.1. Alternatively, the horizontal well can be considered as equivalent to a vertical well which has the same productivity index and has an equivalent well radius r' which is determined as follows:

$$2\pi \frac{Kh}{\mu} \frac{1}{\ln \frac{r_e}{r'}} = \frac{2\pi Kh}{\mu} \frac{1}{\ln\left(\frac{4r_e}{L}\right) + \frac{h}{L}\ln\left(\frac{h}{2\pi r_w}\right)} \tag{3.4}$$

$$\ln \frac{r_e}{r'} = \ln \frac{4r_e}{L} + \frac{h}{L} \ln \frac{h}{2\pi r_w}$$

$$= \ln \left[\frac{4r_e}{L} \left(\frac{h}{2\pi r_w}\right)^{h/L}\right]$$

$$r' = \frac{r_e}{\frac{4r_e}{L}\left(\frac{h}{2\pi r_w}\right)^{h/L}} \tag{3.5}$$

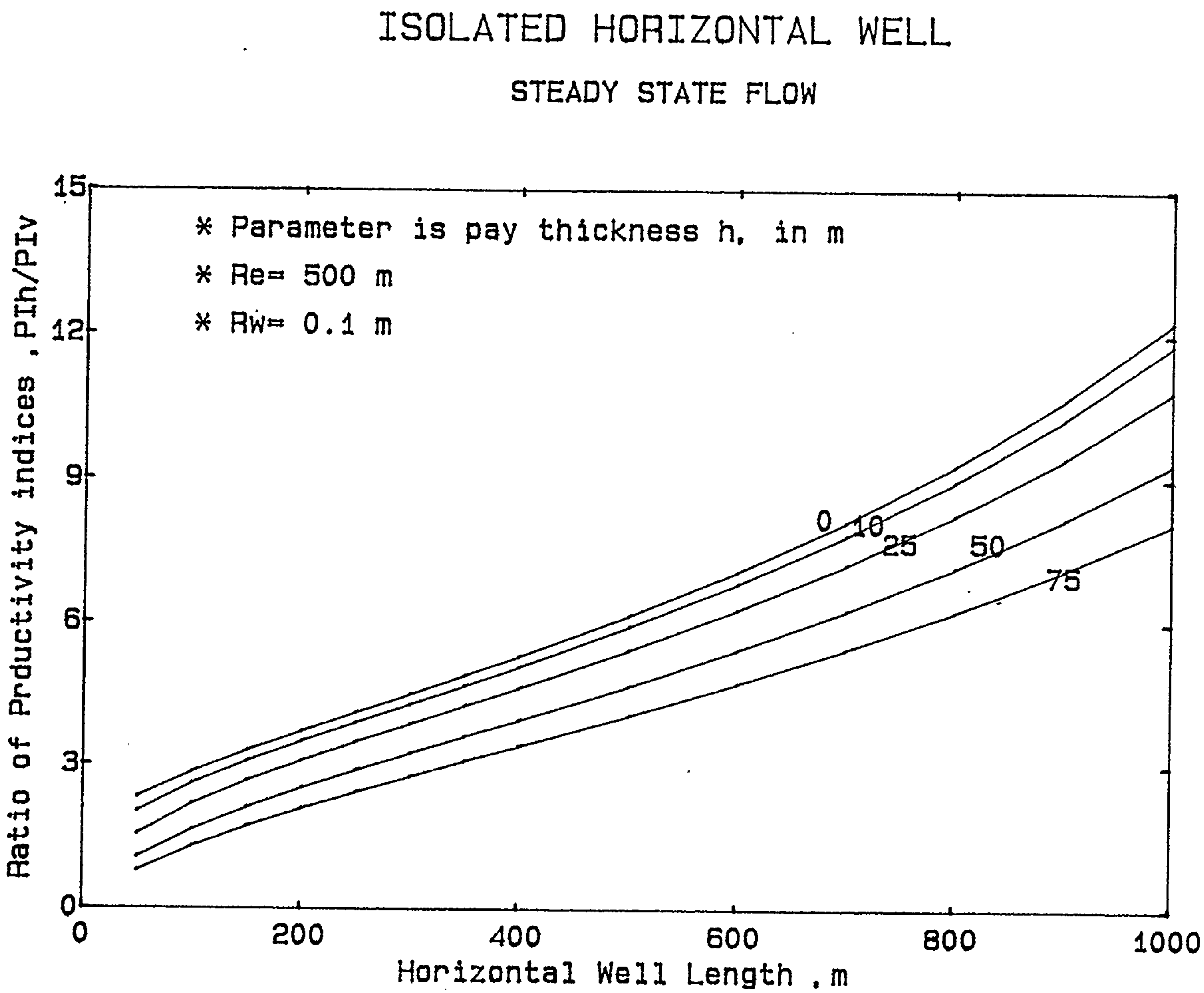

ISOLATED HORIZONTAL WELL
STEADY STATE FLOW
* Parameter is pay thickness h, in m
* Re= 500 m
* Rw= 0.1 m
Ratio of Prductivity indices ,PIh/PIv
Horizontal Well Length ,m
0
10
25
50
75
0
3
6
9
12
15
0
200
400
600
800
1000

Figure 3.1

Figure 3.2 shows the increase in equivalent well radius with well length. For a fixed well length, the equivalent well radius increases (higher productivity) as the formation becomes thinner. For small h, r' reaches an upper limit of L/4. It is noted that equation (3.2) applies to the condition when $L \ll r_e$ (refer to Appendix I).

In anisotropic reservoirs, as discussed earlier in section 2.1.2, the formation thickness, h, is replaced by the equivalent isotropic pay thickness, h', where $h' = h\sqrt{\frac{K_h}{K_v}}$. The parameter h in all of the graphs contained in this chapter can therefore be looked on as $h\sqrt{\frac{K_h}{K_v}}$ to account for the effect of anisotropy.

3.1.2 *Pattern Development*

In order to evaluate the savings between two systems of wells (horizontal vs vertical well systems) in an oilfield development, we have to assume that these two systems of wells have the same area productivity index (API). Giger (13) defined the area productivity index of a well as the ratio of the productivity index to the area drained by the well.

$$API_i = \frac{PI_i}{A_i}$$

For a system D containing n wells draining an oilfield with area

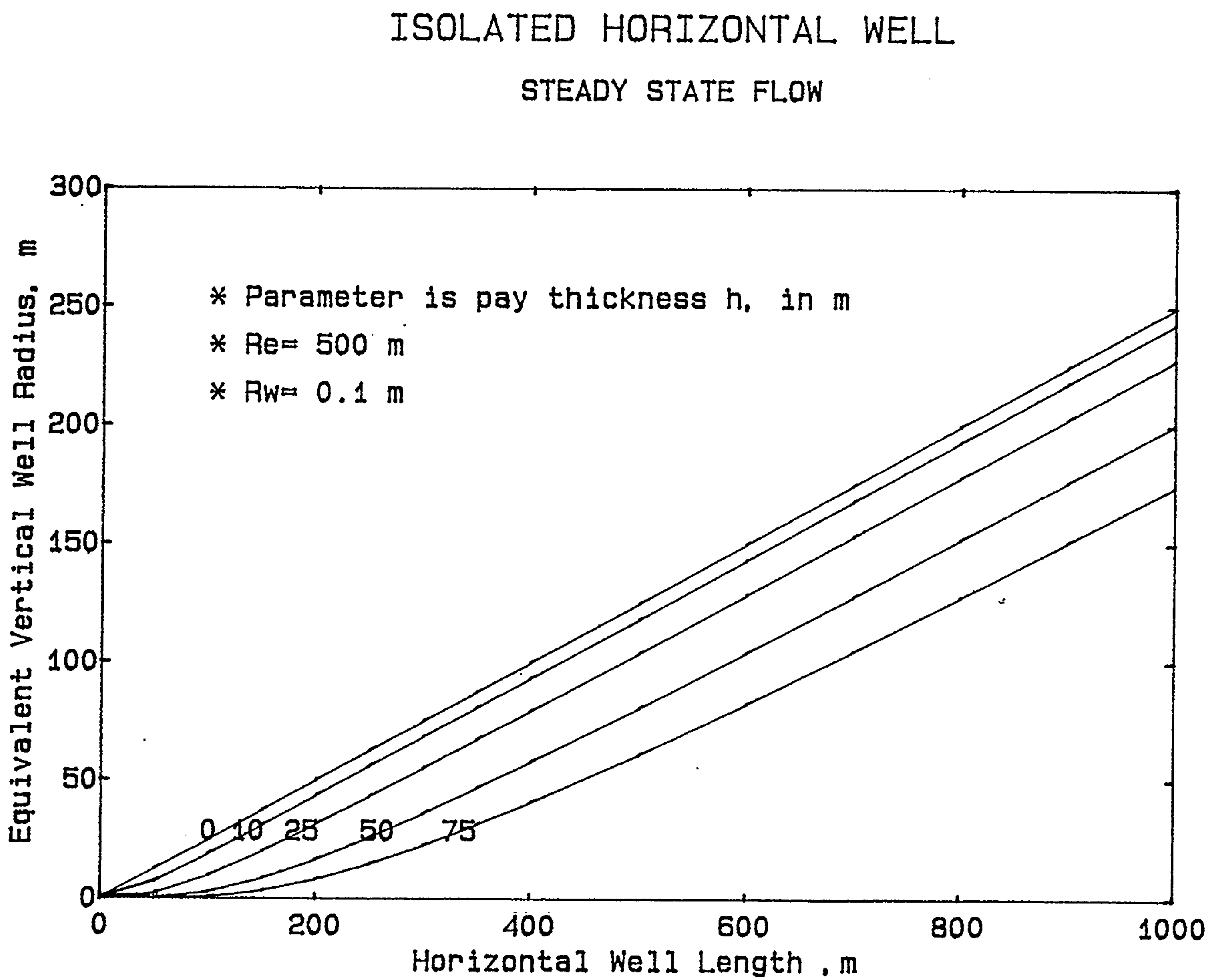

ISOLATED HORIZONTAL WELL
STEADY STATE FLOW
* Parameter is pay thickness h, in m
* Re= 500 m
* Rw= 0.1 m
0 10 25 50 75
Equivalent Vertical Well Radius, m
300
250
200
150
100
50
0
0
200
400
600
800
1000
Horizontal Well Length , m

Figure 3.2

A_D, the API of the reservoir is

$$API_D = \frac{\sum_{i=1}^{i=n} PI_i}{A_D}$$

3.2 Length of Horizontal Well Equivalent to a Vertical Well In A Pattern

Assume that the area productivities of the systems of vertical and horizontal wells are the same and wells in each system are identical.

For an array of parallel, continuous horizontal wells spaced a distance 2x apart (Figure 3.3) the area productivity index is

$$API_h = \frac{1}{2Lx} \frac{2\pi KL}{\mu} \left[\frac{1}{\frac{\pi x}{h} + \ln\left(\frac{h}{2\pi r_w}\right)} \right] \tag{3.6}$$

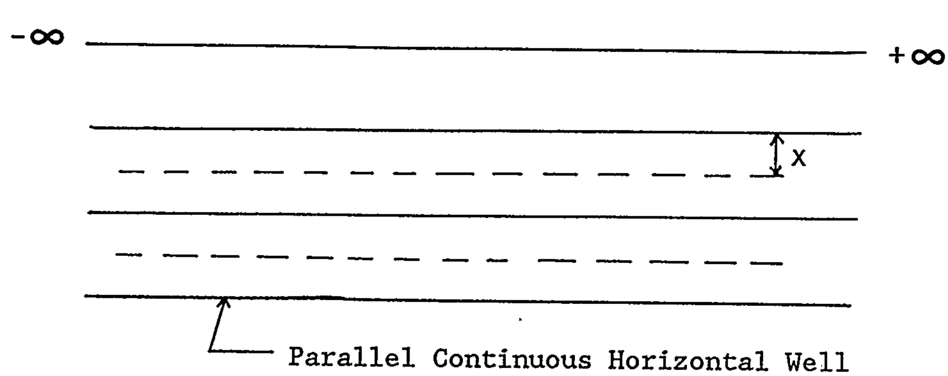


Figure 3.3 Continuous Horizontal Wells

For a vertical well, the area productivity index is

$$API_v = \frac{1}{\pi r_e^2} \frac{2\pi Kh}{\mu} \frac{1}{\ln \frac{r_e}{r_w}} \tag{3.7}$$

For equal area productivity index,

$$API_h = API_v$$

$$\frac{1}{2Lx} \frac{2\pi KL}{\mu} \frac{1}{\frac{\pi x}{h} + \ln\left(\frac{h}{2\pi r_w}\right)} = \frac{1}{\pi r_e^2} \frac{2\pi Kh}{\mu} \frac{1}{\ln \frac{r_e}{r_w}}$$

$$\frac{1}{2x} \frac{1}{\frac{\pi x}{h} + h \ln\left(\frac{h}{2\pi r_w}\right)} = \frac{1}{\pi r_e^2} \frac{1}{\ln \frac{r_e}{r_w}}$$

$$x^2 + \frac{h}{\pi} \ln\left(\frac{h}{2\pi r_w}\right)x - \frac{r_e^2}{2} \ln \frac{r_e}{r_w} = 0 \tag{3.8}$$

This is a quadratic equation and the spacing between two rows of continuous horizontal wells is two times the positive root of equation (3.8) or

$$2x = -b' + \sqrt{b'^2 + 4c'}$$

where

$$b' = \frac{h}{\pi} \ln\left(\frac{h}{2\pi r_w}\right)$$

$$c' = \frac{r_e^2}{2} \ln \frac{r_e}{r_w}$$

The length of horizontal well equivalent to one vertical well is

$$2Lx = \pi r_e^2$$

$$L = \frac{\pi r_e^2}{2x} \qquad (3.9)$$

As shown in Figures 3.4 and 3.5, the horizontal well spacing and well length increase with vertical well spacing. A thick formation will require closer spacing of horizontal wells and a longer horizontal well length to replace one vertical well.

3.3 Relative Advantage of Continuous Horizontal Wells Compared to Multiple Repeated Patterns of Vertical Wells

Consider the replacement of different arrays of equally spaced conventional vertical wells with a grid of continuous, parallel, equally spaced horizontal wells. Figures 3.6a to 3.6c show three continuing arrays of vertical wells. Figure 3.6d shows the continuous horizontal well pattern. In each vertical well array, a well drains the shaded area as shown in the diagram. For simplicity, the drainage performance in each array is assumed to be equivalent to that which would be found if the individual drainage areas were circles having the same area A.

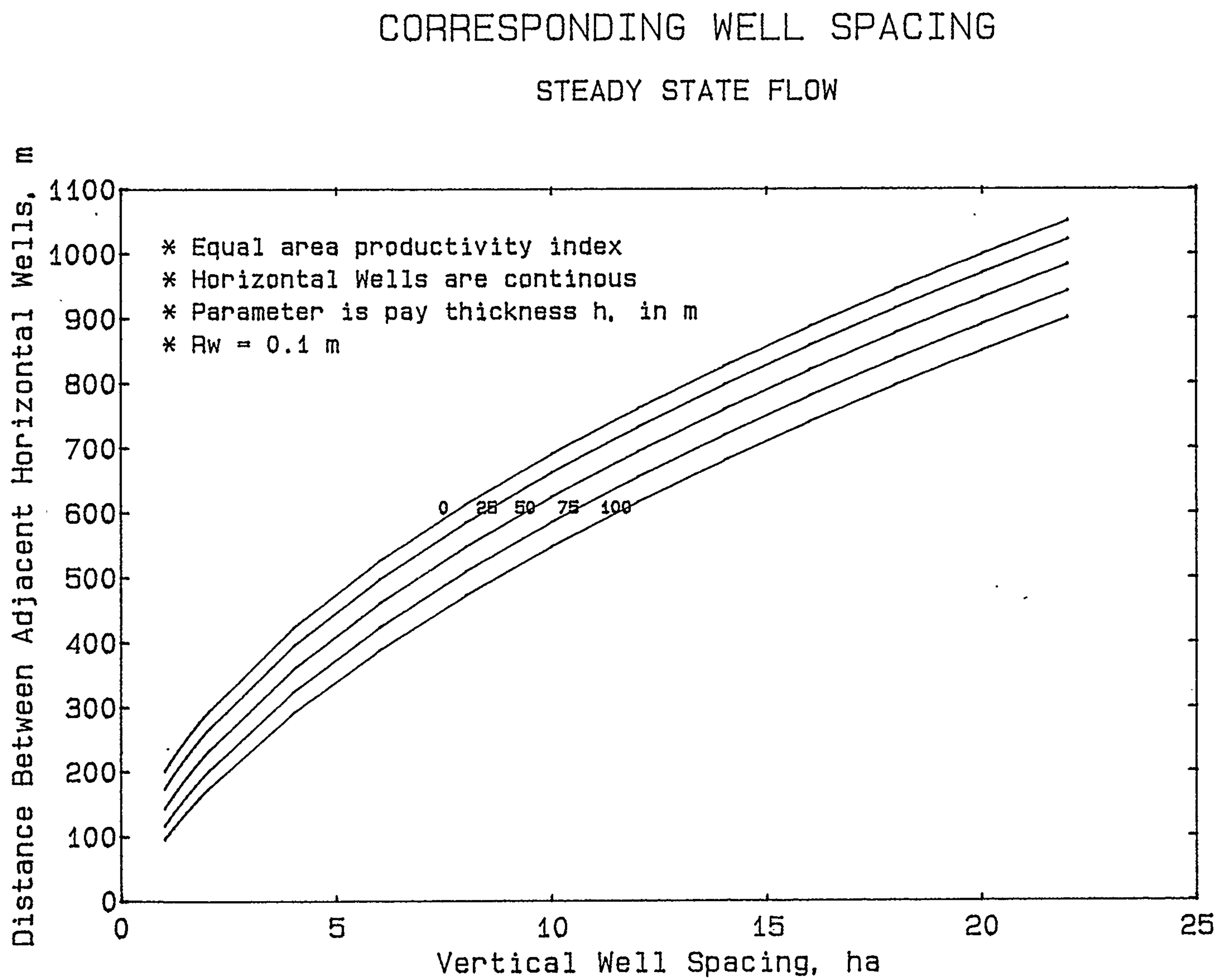

CORRESPONDING WELL SPACING
STEADY STATE FLOW
* Equal area productivity index
* Horizontal Wells are continous
* Parameter is pay thickness h, in m
* Rw = 0.1 m
0
25
50
75
100
Distance Between Adjacent Horizontal Wells, m
Vertical Well Spacing, ha
0
100
200
300
400
500
600
700
800
900
1000
1100
0
5
10
15
20
25

Figure 3.4

EQUIVALENT LENGTH OF HORIZONTAL WELL

STEADY STATE FLOW

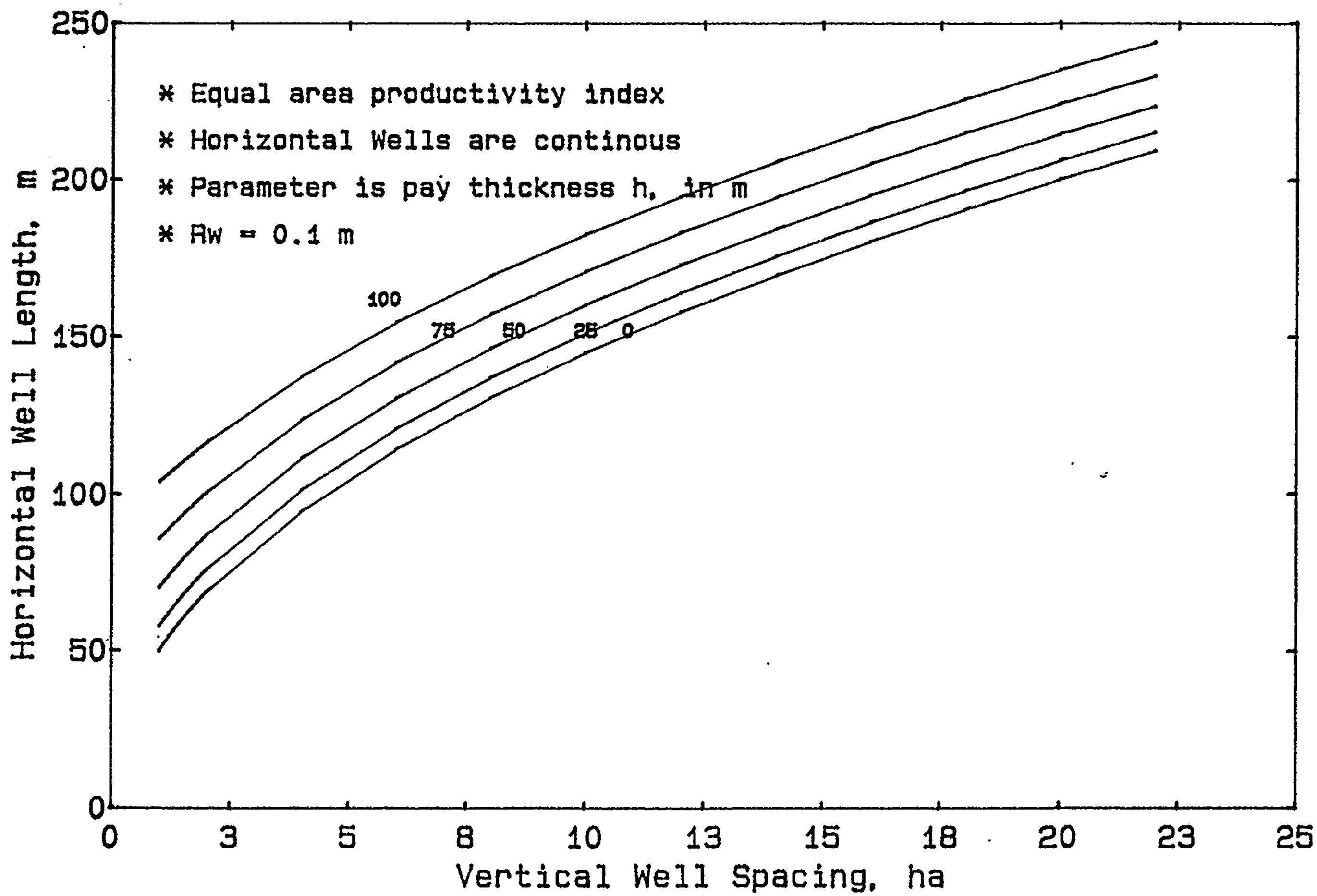
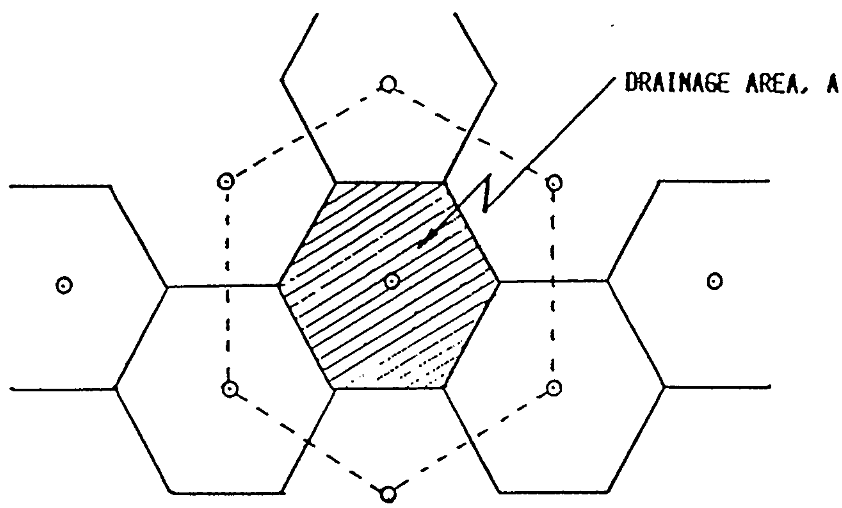


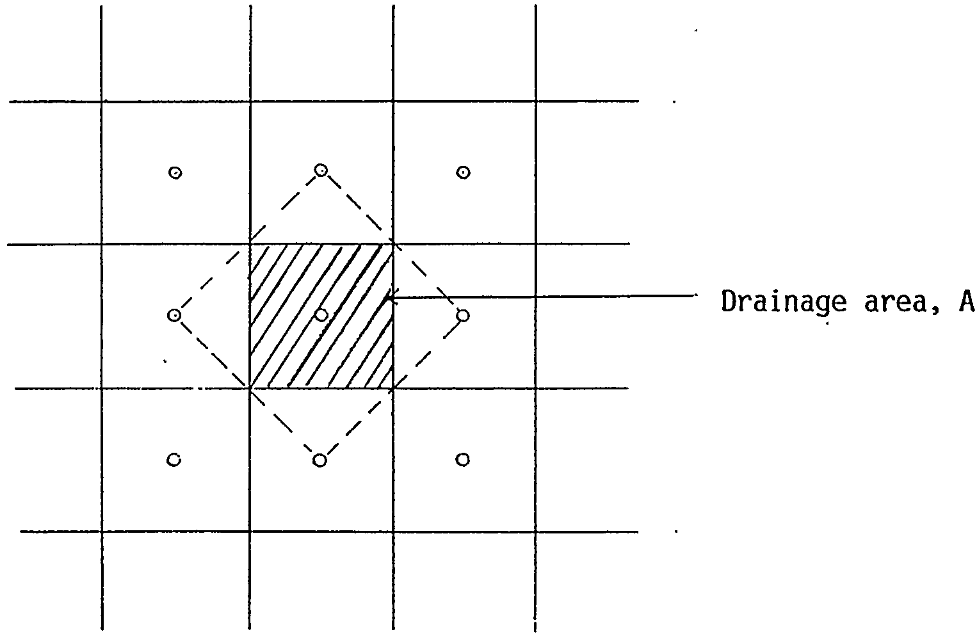
Figure 3.5

Figures 3.7 to 3.9 show the corresponding spacing between the horizontal wells and the vertical wells for different vertical well patterns (square, seven spot, five spot and hexagon shaped patterns). In general, the spacing between the horizontal wells is larger than that between adjacent vertical wells and the hexagonal arrangement of the vertical wells gives the largest well spacing ratio between the horizontal and vertical wells.



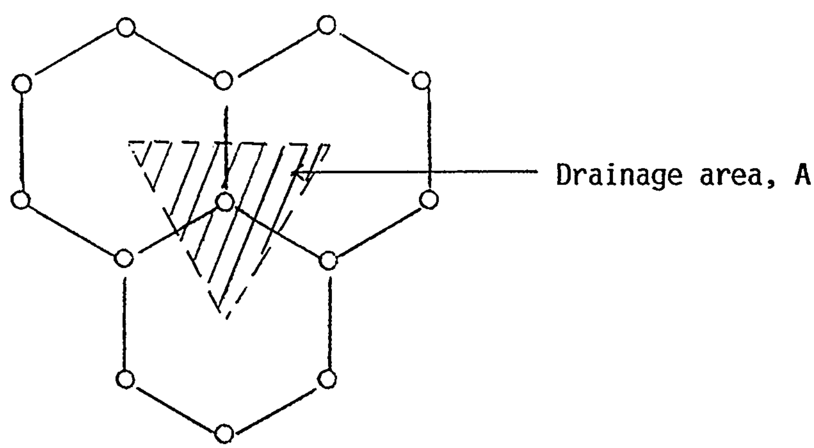
Well Spacing $= 1.0746 \sqrt{A}$

Figure 3.6a **7 Spot Well Pattern**



Well Spacing $= \sqrt{A}$

Figure 3.6b 5 - Spot Pattern



Well Spacing $= 0.8774 \sqrt{A}$

Figure 3.6c Hexagon Pattern

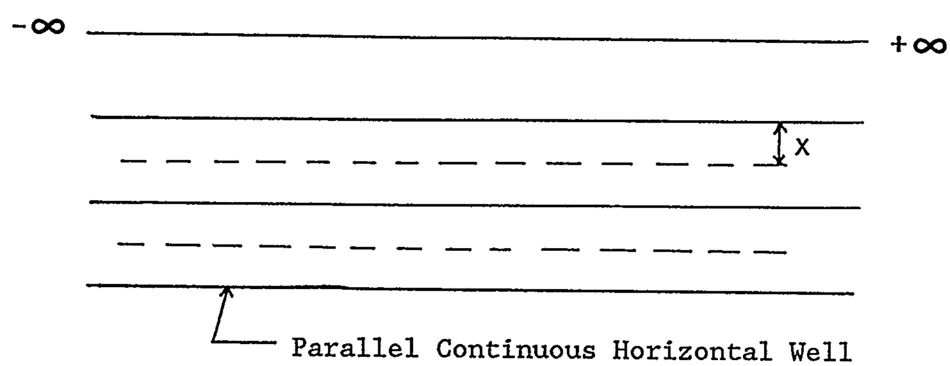


Figure 3.6d Continuous Horizontal Well Pattern

CORRESPONDING WELL SPACING

STEADY STATE FLOW

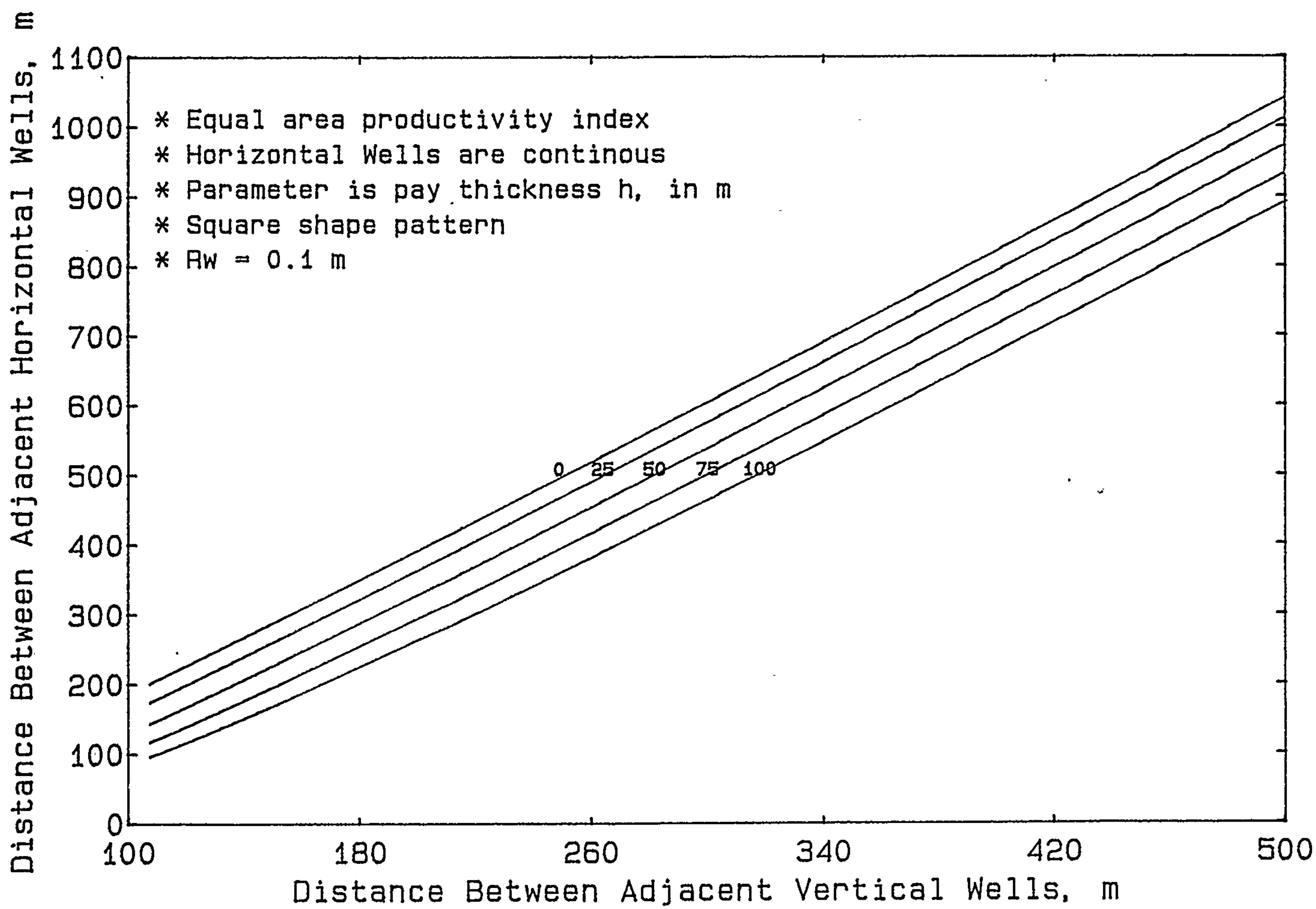


Figure 3.7

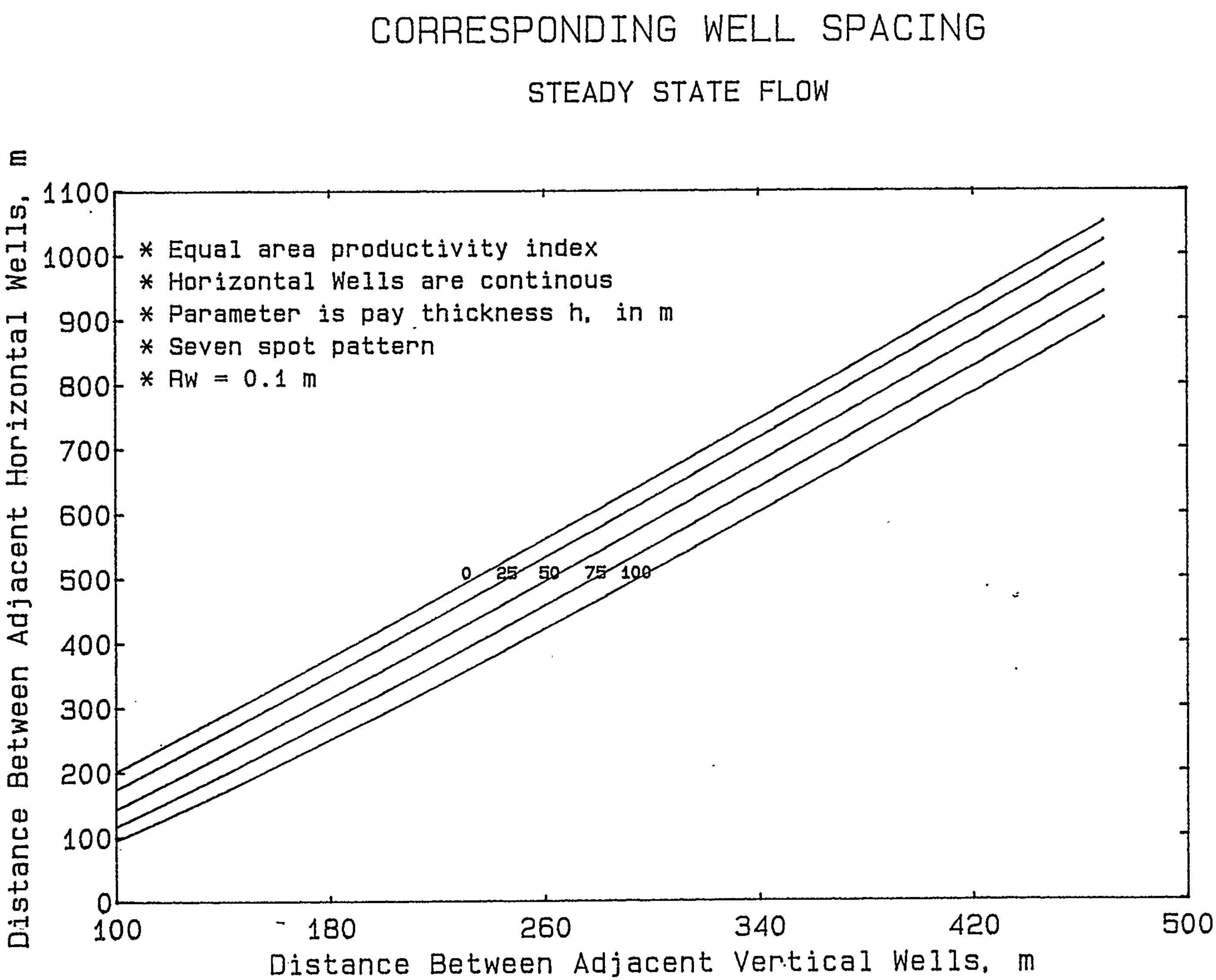

CORRESPONDING WELL SPACING
STEADY STATE FLOW
* Equal area productivity index
* Horizontal Wells are continous
* Parameter is pay thickness h, in m
* Seven spot pattern
* Rw = 0.1 m
0
25
50
75
100
Distance Between Adjacent Horizontal Wells, m
0
100
200
300
400
500
600
700
800
900
1000
1100
100
180
260
340
420
500
Distance Between Adjacent Vertical Wells, m

Figure 3.8

CORRESPONDING WELL SPACING

STEADY STATE FLOW

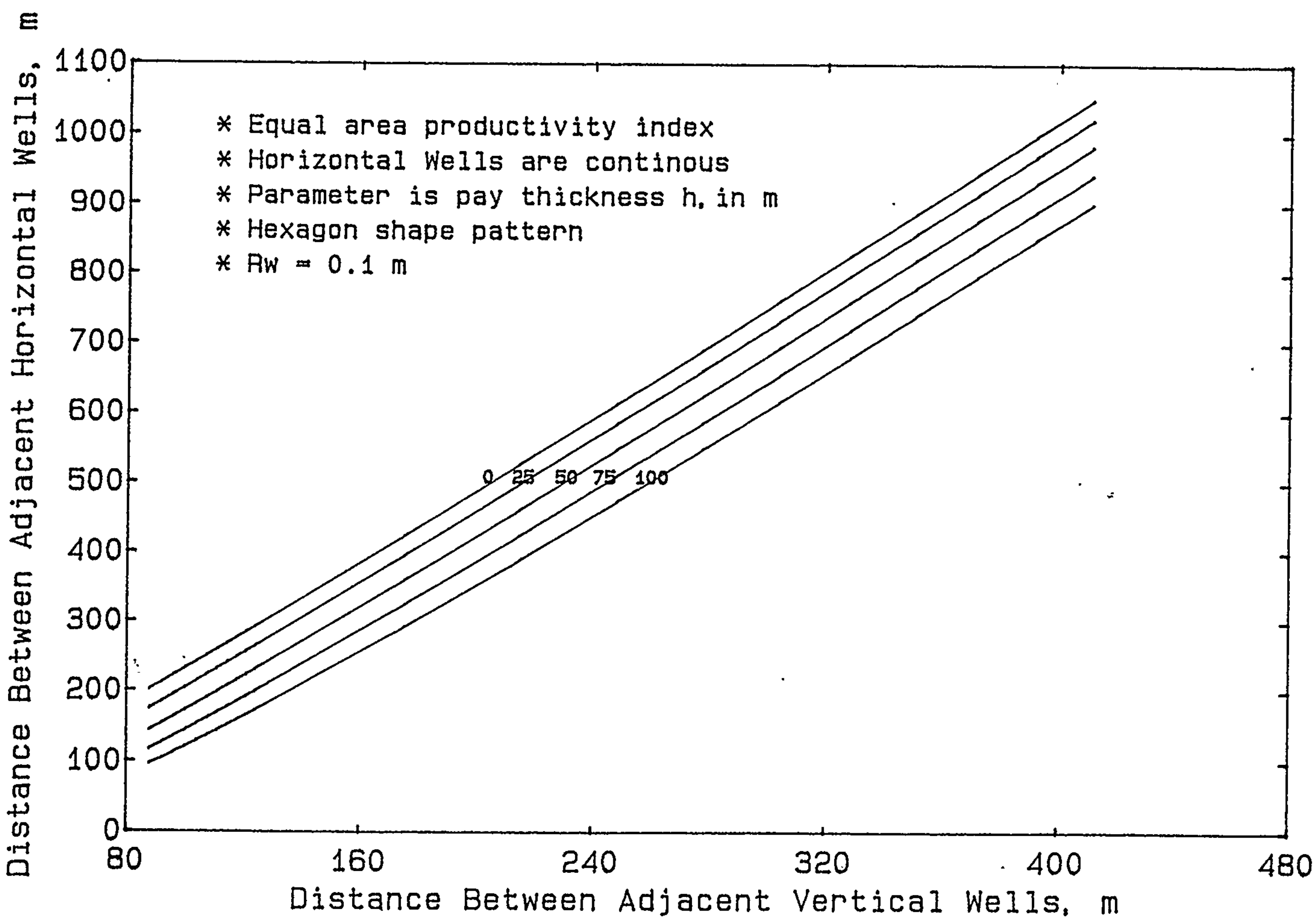
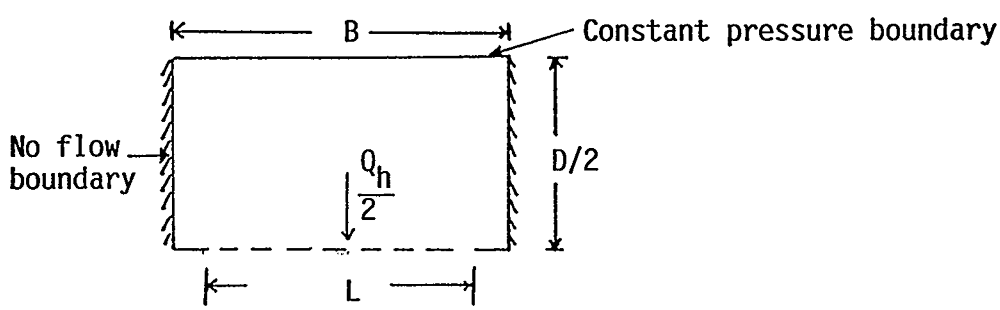


Figure 3.9

3.4 Relative Advantage of Long Horizontal Wells in an Array to a Repeated Pattern of Vertical Wells

Giger et al (14) showed that the flow to a long horizontal well which is in an array of a repeated pattern approximates that of a well located in a rectangular box of length B (in the same direction as the well of length L), width S and height h. Referring to Section 2.2, equation (2.11), the flow equation is:

$$Q_h = \frac{2\pi Kh\Delta P}{\mu} \; \frac{1}{\text{Cosh}^{-1}\left[\dfrac{\text{Cosh}\left(\frac{\pi}{2}\frac{D}{B}\right)}{\text{Sin}\left(\frac{\pi}{2}\frac{L}{B}\right)}\right] + \frac{h}{L}\ln\left[\dfrac{h}{2\pi r_w}\right]} \qquad (3.10)$$



Houpert (17) derived the above equation for a constant pressure boundary along the length B and a no-flow boundary along the side D of the rectangular drainage box.

Because of these assumptions, equation (3.10) is not strictly applicable in an oilfield drilled with an array of long horizontal producers. In primary depletion, each horizontal well is surrounded by no-flow boundaries and therefore, the pseudo-steady state equation (refer to section 3.6 of this chapter) better approximates the reservoir production performance. For completeness, the following summarizes the development of the equation to calculate the number of vertical wells equivalent to one horizontal well in an array.

Referring to equation (3.10), the area productivity index, API_h, of the horizontal well is

$$API_h = \frac{2\pi Kh}{DB\mu} \frac{1}{\operatorname{Cosh}^{-1}\left[\frac{\operatorname{Cosh}\left(\frac{\pi}{2}\frac{D}{B}\right)}{\operatorname{Sin}\left(\frac{\pi}{2}\frac{L}{B}\right)}\right] + \frac{h}{L}\ln\left[\frac{h}{2\pi r_w}\right]} \tag{3.11}$$

This is equal to the area productivity index, API_v, for the vertical wells.

$$API_v = \frac{2\pi Kh}{\mu} \frac{1}{\pi r_e^{\,2}} \frac{1}{\ln\left(\frac{r_e}{r_w}\right)} \tag{3.12}$$

therefore

$$DB\left\{\operatorname{Cosh}^{-1}\left[\frac{\operatorname{Cosh}\left(\frac{\pi}{2}\frac{D}{B}\right)}{\operatorname{Sin}\left(\frac{\pi}{2}\frac{L}{B}\right)}\right] + \frac{h}{L}\ln\left[\frac{h}{2\pi r_w}\right]\right\} = \pi r_e^{\,2}\ln\left(\frac{r_e}{r_w}\right) \tag{3.13}$$

r_e can be found from the above equation using the Newton-Raphson technique. That is,

if

$$G = DB\left\{\mathrm{Cosh}^{-1}\left[\frac{\mathrm{Cosh}\left(\frac{\pi}{2}\frac{D}{B}\right)}{\mathrm{Sin}\left(\frac{\pi}{2}\frac{L}{B}\right)}\right] + \frac{h}{L}\ln\left[\frac{h}{2\pi r_w}\right]\right\} \tag{3.14}$$

and $r_{e,i}$ is the i^{th} approximation for r_e,

then the next approximation is

$$r_{e,i+1} = r_{e,i} + \frac{G - \pi r_{e,i}^{\ 2}\ln\left(\frac{r_{e,i}}{r_w}\right)}{\pi r_{e,i}\left[2\ln\left(\frac{r_{e,i}}{r_w}\right) + 1\right]} . \tag{3.15}$$

The number of vertical wells equivalent to the horizontal well is equal to the ratio of the two areas, i.e.

number of vertical wells equivalent to one horizontal well

$$= \frac{DB}{\pi r_e^{\ 2}}$$

3.5 Relative Advantage of Horizontal Wells in Repeated Elliptical Pattern

Section 3.3 considers the relative advantage of continuous horizontal wells. In this section, we will consider the case where the horizontal wells are arranged in a staggered pattern and will approximate this as a pattern of slightly overlapping ellipses.

Figure 3.10 shows a staggered array of horizontal wells with the assumed elliptical drainage boundaries.

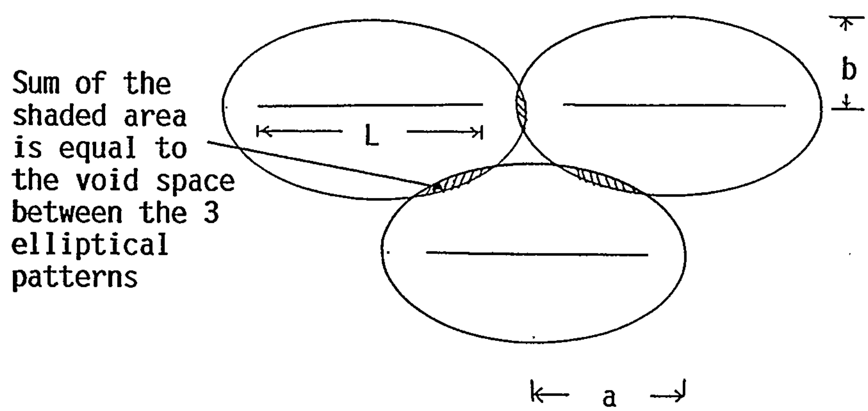


Figure 3.10 Staggered Elliptical Drainage Pattern

Similar to the above sections, using equation (2.3) in 2.1.1, and with equal area productivity indices of the horizontal and the equivalent vertical wells, we have,

$$\frac{1}{\pi ab} \frac{1}{\ln\left(\frac{2(a+b)}{L}\right) + \frac{h}{L} \ln \frac{h}{2\pi r_w}} = \frac{1}{\pi r_e^2} \frac{1}{\ln \frac{r_e}{r_w}} \qquad (3.16)$$

where $$b = \sqrt{a^2 - (\tfrac{L}{2})^2}$$

The number of vertical wells equivalent to the horizontal well is given by $\frac{ab}{r_e^2}$.

Figure 3.11 shows the number of vertical wells equivalent to a 500 metre long horizontal well for different formation thicknesses. This figure shows the advantage of horizontal wells decreases as the formation thickness increases.

3.6 Length of Horizontal Well Equivalent to a Vertical Well in a Regular Array Based on Pseudo-Steady State Concept

The pseudo-steady state equation for a vertical well with closed drainage boundary is described in 2.33, equation (2.28).

$$P_m - P_W = \frac{Q_2 \mu}{2\pi K} \left[\frac{1}{2} \ln\left(\frac{A''}{r_w{}^2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{c_A}\right) \right]$$

The area productivity index for a reservoir with height h is given by,

$$\frac{Q_2 h}{A''(P_m - P_W)} = \frac{4\pi K h}{\mu A''} \left[\frac{1}{\ln\left(\frac{A''}{r_w{}^2}\right) + \ln\left(\frac{2.2458}{c_A}\right)} \right] \tag{3.17}$$

for a square pattern with the vertical well located at the centre, the

NO. OF VERTICAL WELLS EQUIV. TO A HORIZONTAL WELL

STEADY STATE FLOW

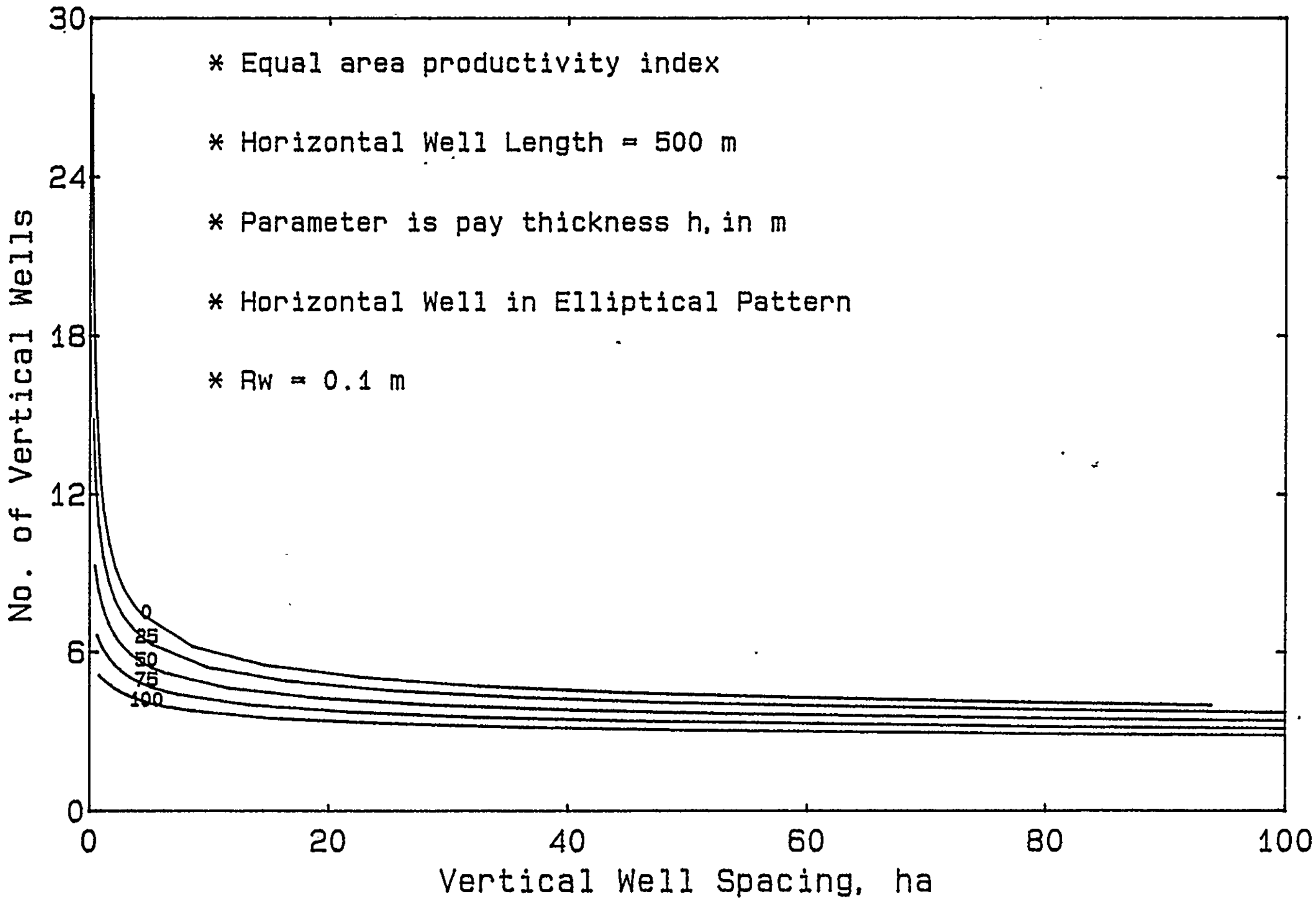


Figure 3.11

area productivity index, PI_v, of the vertical well is:

$$API_v = \frac{4\pi Kh}{\mu A''} \left[\frac{1}{\ln\left(\frac{A''}{r_w{}^2}\right) - 2.6212} \right]$$

Let S_1 be the side of the square pattern, then $A'' = S_1^2$; substitute into the above equation, we have,

$$PI_v = \frac{4\pi Kh}{\mu S_1{}^2} \left[\frac{1}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212} \right] \qquad (3.18)$$

For an infinite horizontal well with large interwell spacing W, the pseudo-steady state equation is described in 2.3.1, equation (2.22):

$$P_m - P_W = \frac{Q_h W}{6LhK} + \frac{Q_h \mu}{2\pi KL} \ln \frac{h}{2\pi r_w}$$

with the drainage area of the horizontal well being 2WL. The area productivity of such a horizontal well is,

$$API_h = \frac{Q_h}{(P_m - P_W) 2WL}$$

$$= \frac{Kh}{\mu W^2} \frac{1}{\left(\frac{1}{3} + \frac{1}{\pi} \frac{h}{W} \ln \frac{h}{2\pi r_w}\right)} \qquad (3.19)$$

For equal area productivity index, $API_v = API_h$.

$$\frac{4\pi Kh}{\mu S_1^{\,2}} \; \frac{1}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212} = \frac{Kh}{\mu W^2} \left[\frac{1}{\frac{1}{3} + \frac{1}{\pi}\frac{h}{W}\ln\left(\frac{h}{2\pi r_w}\right)}\right]$$

$$S_1 = 2W \sqrt{\frac{\frac{\pi}{3} + \frac{h}{W}\ln\left(\frac{h}{2\pi r_w}\right)}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212}} \qquad (3.20)$$

This equation can be solved rapidly by the successive substitution method.

Similarly, for an infinite horizontal well with small interwell spacing, the pseudo-steady state equation (refer to 2.3.3, equation (2.29)) is:

$$P_m - P_W = \frac{Q\mu}{2\pi K} \left[\frac{1}{2}\ln\left(\frac{2Wh}{r_w 2}\right) + \frac{1}{2}\ln\left(\frac{2.2458}{c_A}\right) \right]$$

and the horizontal drainage area is 2W x 1. The area productivity index is

$$API_h = \frac{2\pi K}{W\mu} \; \frac{1}{\ln\left(\frac{2Wh}{r_w 2}\right) + 2\left[\frac{1}{2}\ln\left(\frac{2.2458}{c_A}\right)\right]}$$

The constant term, $\frac{1}{2}\ \ln\left(\frac{2.2458}{c_A}\right)$, depends on the vertical drainage area of the horizontal well.

Let $N = \frac{2W}{h} = \frac{S_1}{S_2}$, values of $\frac{1}{2}\ \ln\left(\frac{2.2458}{c_A}\right)$ are available for $N = 5,4,2,1$ from Earlougher et al (11). See page 21.

For equal area productivity index, $API_v = API_h$

$$\frac{4\pi Kh}{\mu S_1^2}\ \frac{1}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212} = \frac{2\pi K}{W\mu}\ \frac{1}{\ln\left(\frac{2Wh}{r_w 2}\right) + 2(\frac{1}{2})\ \ln\frac{2.2458}{c_A}} \tag{3.21}$$

Substitute $W = \frac{Nh}{2}$ into equation (20) and re-arrange,

$$S_1^2 = Nh^2 \left[\frac{\ln\left(\frac{Nh^2}{r_w 2}\right) + 2(\frac{1}{2})\ \ln\frac{2.2458}{c_A}}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212}\right]$$

$$S_1 = \sqrt{\frac{Nh^2\left[\ln\left(\frac{Nh^2}{r_w 2}\right) + 2(\frac{1}{2})\ \ln\frac{2.2458}{c_A}\right]}{2\ln\left(\frac{S_1}{r_w}\right) - 2.6212}} \tag{3.22}$$

The above equation can be solved using the successive substitution method.

The length of horizontal well, L, equivalent to a vertical well is:

$$L = \frac{S_1^{\,2}}{2W} \qquad (3.23)$$

Figures 3.12 to 3.14 are generated using equations (3.20), (3.22) and (3.23), and assuming a 0.1 m wellbore radius.

For the case of an infinite horizontal well with large interwell spacing (Figures 3.12 amd 3.13), equation (3.20) is used to calculate the vertical well spacing. For a reservoir with 15 metres of pay, Figure 3.12 shows that a 46 m vertical well spacing corresponds to a 100 m horizontal well spacing, and Figure 3.13 shows that for a 15 m pay zone, each 21 m of a long horizontal well is equivalent to a vertical well having a drainage area of 0.21 hectares.

In Figure 3.14, both equations (3.20) and (3.22) are used to plot the corresponding interwell spacing between horizontal and vertical wells. Since the results of the two equations fall on smooth curves, then equations (3.20) and (3.22) can be used to predict the equivalent length of horizontal well for large and small vertical well drainage areas respectively.

CORRESPONDING WELL SPACINGS

PSEUDO-STEADY STATE FLOW

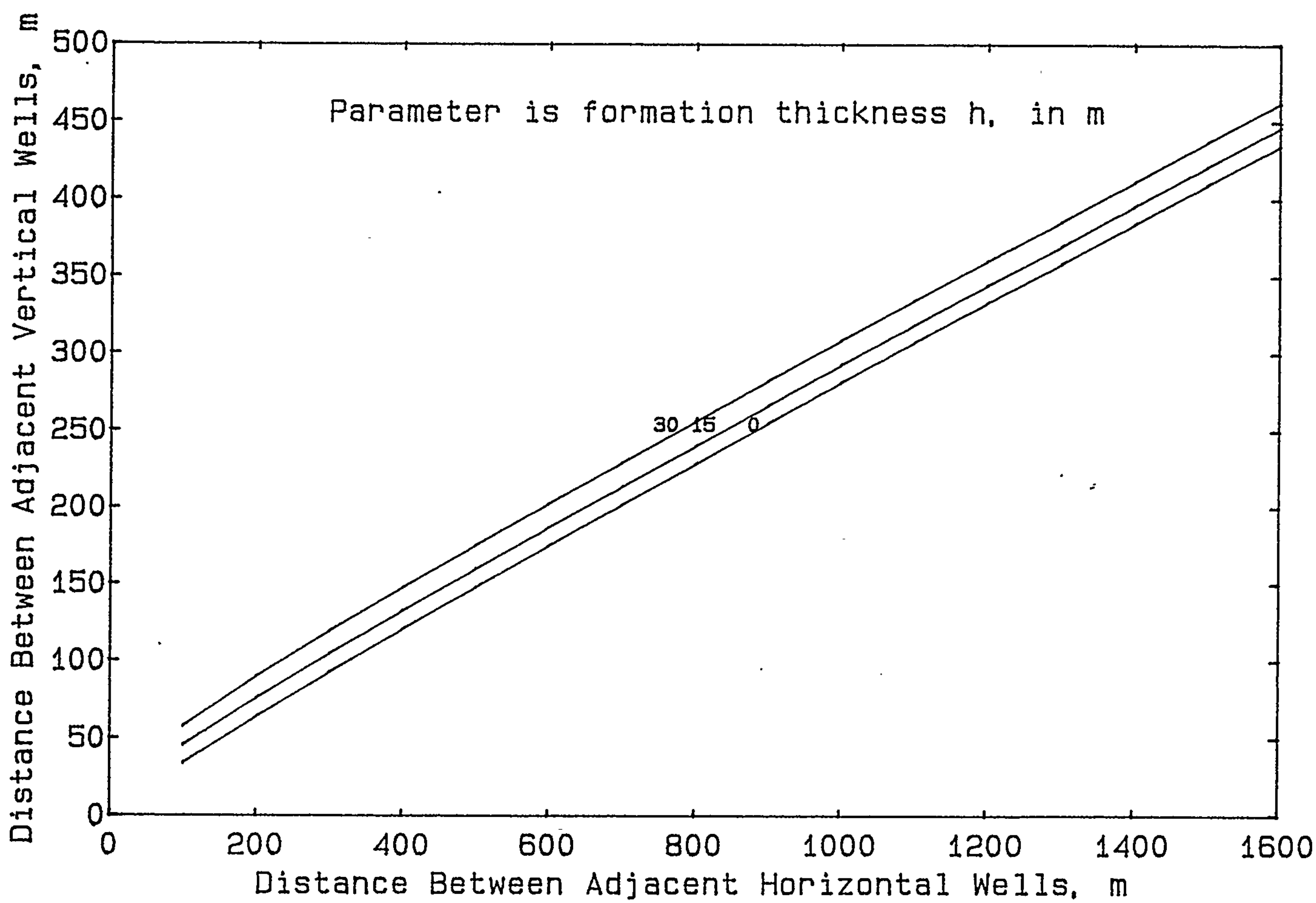


Figure 3.12

LENGTH OF HORI. WELL EQUIV. TO A VERT. WELL

PSEUDO-STEADY STATE FLOW

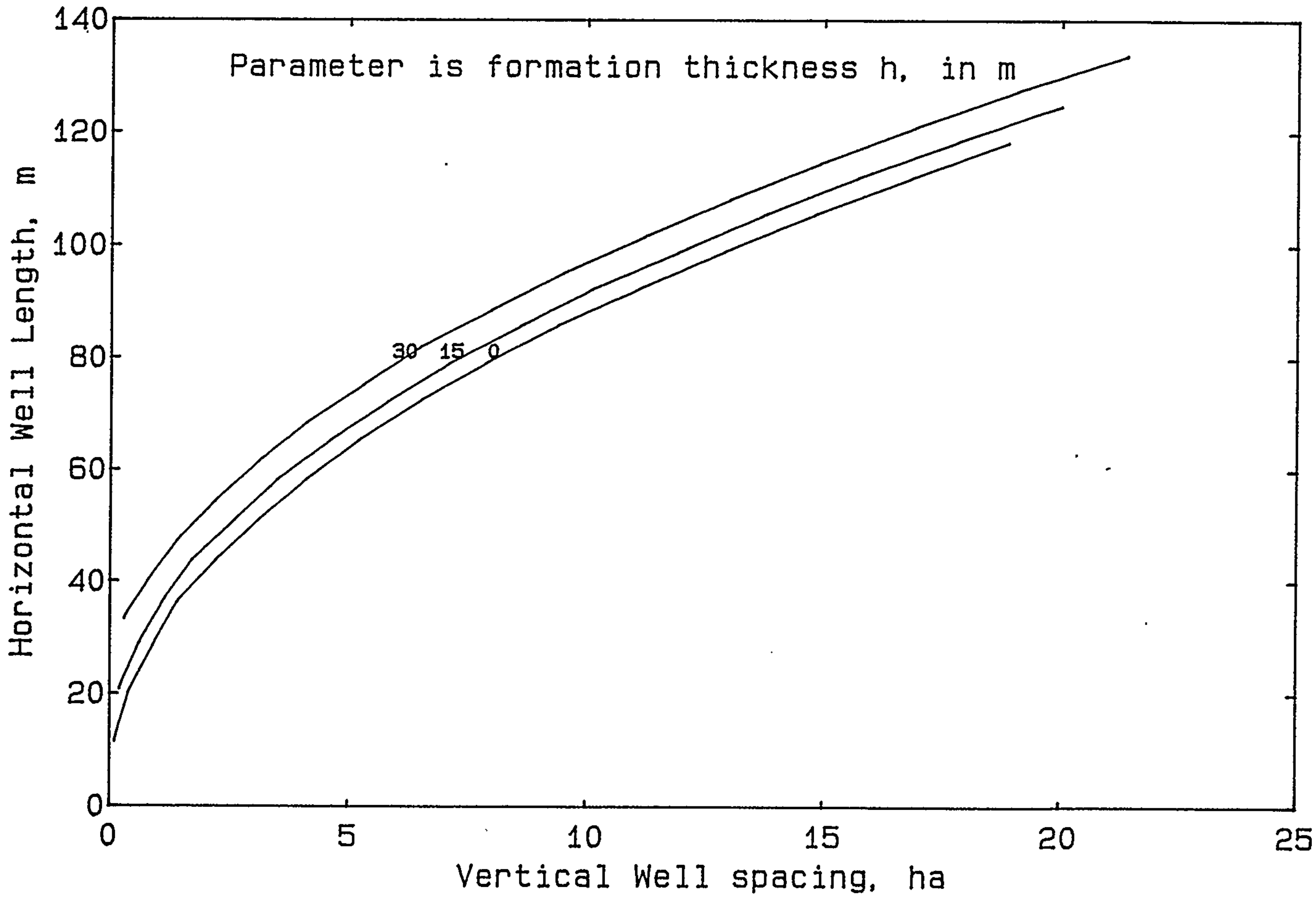


Figure 3.13

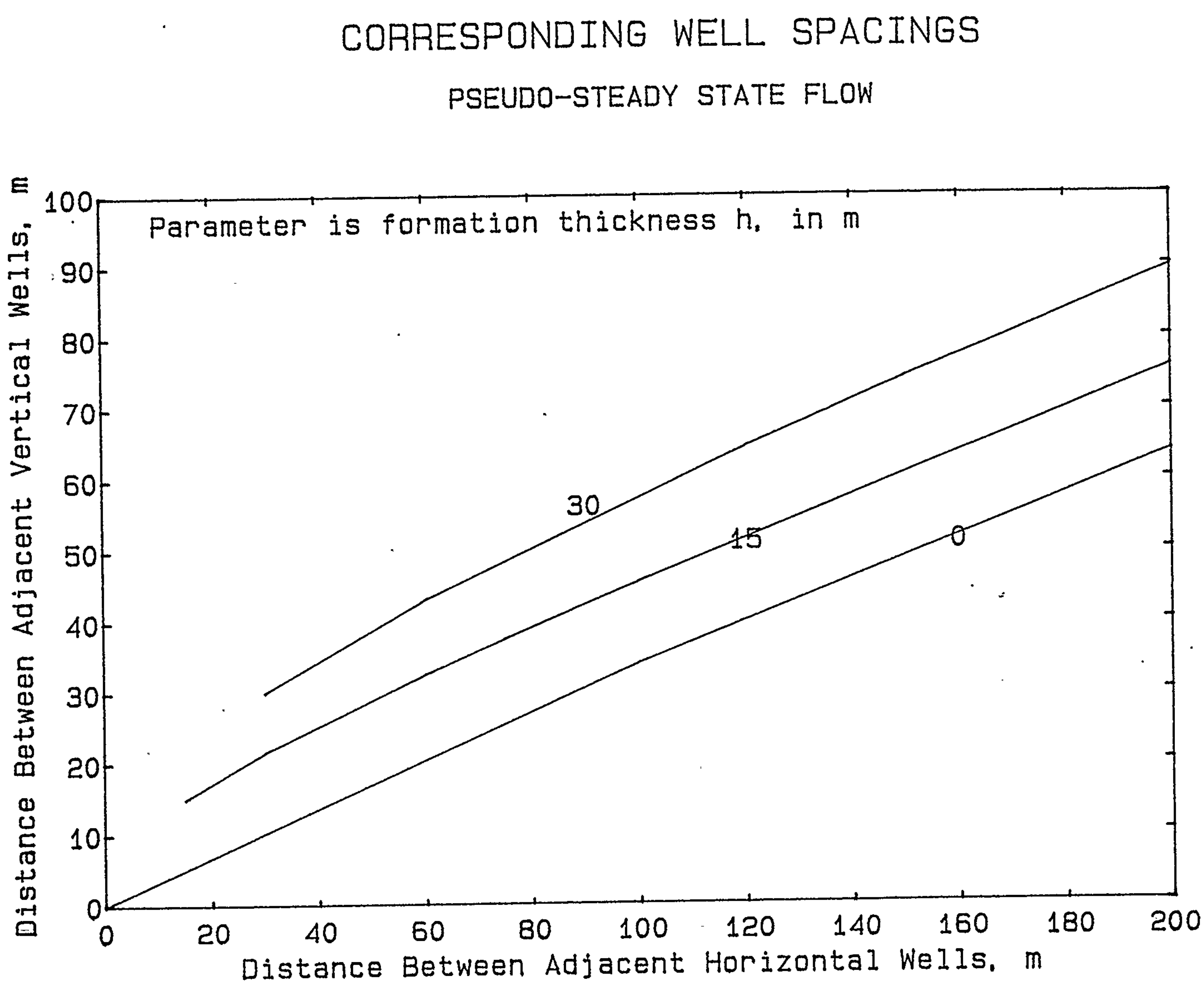

CORRESPONDING WELL SPACINGS
PSEUDO-STEADY STATE FLOW
Parameter is formation thickness h, in m
30
15
0
Distance Between Adjacent Vertical Wells, m
Distance Between Adjacent Horizontal Wells, m
0
10
20
30
40
50
60
70
80
90
100
120
140
160
180
200

Figure 3.14

3.7 Star Shaped Drainage Pattern

Drilling horizontal holes from a central shaft system can reduce the cost of drilling several isolated horizontal holes. Borisov (3) has shown that the flow rate for such a star shaped drainage pattern is:

$$Q = \frac{2\pi Kh\Delta P}{\mu} \frac{1}{\ln \frac{\eta r_e}{L} + \frac{h}{Ln} \ln \frac{h}{2\pi r_w}} \tag{3.22}$$

where

n = number of horizontal holes extended from the central shaft

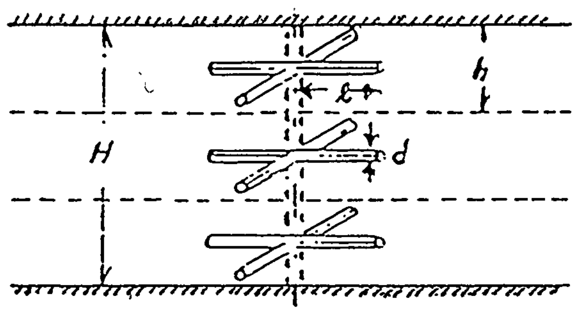
η = a dimensionless number and is determined from the following table.

n	1	2	3	4
η	4	2	1.86	1.78

For $n = 1$, $\eta = 4$, equation (3.22) becomes, as before (equation 2.6),

$$Q = \frac{2\pi Kh}{\mu} \frac{\Delta P}{\ln \frac{4r_e}{L} + \frac{h}{L} \ln \frac{h}{2\pi r_w}} \tag{3.23}$$

In a very thick formation, one of the production methods which might be considered is to drill horizontal holes in different plane surfaces (or levels) as shown in the following.



Let m be the number of levels of horizontal drainage holes. As shown earlier, the flow rate in one level of formation is

$$q = \frac{2\pi Kh}{\mu} \quad \frac{\Delta P}{h \dfrac{\eta r_e}{\ell} + \dfrac{h}{1n} \ln \dfrac{h}{2\pi r_w}} \tag{3.24}$$

Since H = mh, the total flow rate of m multiple level wells is:

$$Q = qm = \frac{2\pi HK}{\mu} \quad \frac{\Delta P}{\ln \dfrac{\eta r_e}{\ell} + \dfrac{H}{\ell mn} \ln \dfrac{h}{2\pi m r_w}} \tag{3.25}$$

The total length of the multiple level horizontal drain holes must be greater than the formation thickness, L" (i.e. $L'' = nm\ell > H$) or drilling horizontal holes would be senseless.

Equation 3.25 shows that for given m and L", Q will be a maximum when $n = 1$ and $\eta = 4$, i.e.

$$Q = 2\pi \frac{KH}{\mu} \frac{\Delta P}{\ln \frac{4r_e}{L''} + \frac{H}{L''} \ln \frac{H}{2\pi r_w} + (1 - \frac{H}{L}) \ln m} \tag{3.26}$$

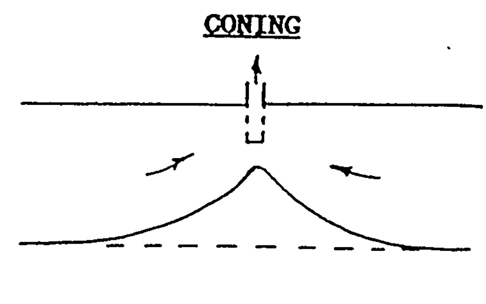
The above equation shows that when L" > H, the flow rate will be highest for m = 1.

Thus it is preferable, in an isotropic reservoir, to drill a single long horizontal hole in the centre of the formation rather than several levels of multiple horizontal drain holes. However, for heterogeneous beds, the multiple level star shaped drainage pattern may be more practical, especially when the producing formation is divided into separate horizons by tight or impermeable barriers and/or the drilling drainage area available is restricted. In this case, drilling multiple level wells with a level for each horizon can ensure the most rational drilling through the formation – particularly if the sidetracks can be controlled independently. However, this facility is only available at present at considerable cost.

CHAPTER 4

WATER CONING

In a reservoir which contains a layer of bottom water, the conventional production practice is to drill a partially penetrating vertical well completed over a short section near the top of the reservoir.



Conventional theoretical analysis considers the pressure gradients required within the oil layer to cause flow to the well. These pressure gradients are reflected in the equilibrium level of the water layer. The interface rises beneath the well and, if the velocity is not too great, reaches an equilibrium position. In this equilibrium position, the differential weight of the water column is supported by the pressure differential and the corresponding critical oil production velocity is proportional to the difference in density of oil and water and inversely proportional to the oil viscosity. For heavy oils, the viscosity is high and the density difference term is small. As a result, the maximum critical rate is found to be far below economic production rates. Numerical simulations of such cases (1) show that the producing water/oil ratio is then essentially independent of production rates. Under such a condition, the economic production rate will cause the water cone to rise rapidly as stabilization by gravity becomes insignificant when oil density and viscosity increase.

The following sections compare the performance of horizontal to vertical wells and assume the density of oil is less than or equal to that of water. This condition is found in most reservoirs.

4.1. Oil Production from a Vertical Well and a Horizontal Well

Figure 4.1 shows a well completed at the top of the reservoir immediately below a layer of impervious overburden. There is a layer of bottom water lying beneath the reservoir. Assume that the water and oil have equal mobility and density. The permeability of the reservoir is the same in all directions. Under these conditions, the flow lines are straight and radial. The drainage volume is equal to half of a spherical volume, i.e.

$$V_v = \frac{1}{2}\left(\frac{4\pi}{3} h^3\right)$$

$$= \frac{2}{3} \pi h^3 \qquad (4.1)$$

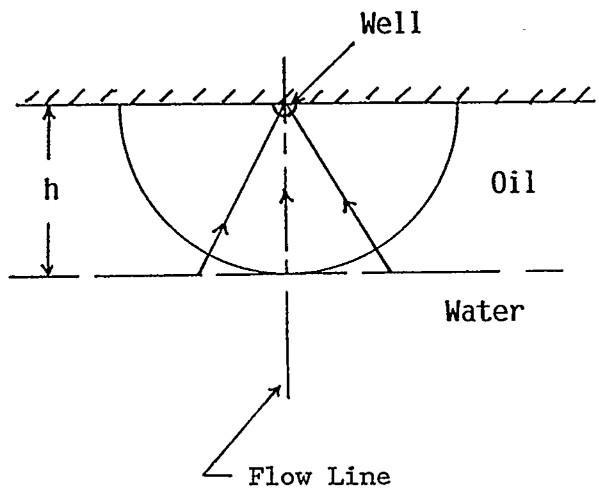


Figure 4.1 **Drainage Volume for Water Free Oil**

If the vertical well is replaced by a horizontal well of length L, the drainage volume will increase. The drainage volume for such a horizontal well is equal to the sum of the volume of a hemisphere plus the hemicylinder, or

$$V_h = \frac{2}{3}\pi h^3 + \frac{1}{2}\pi h^2 L \tag{4.2}$$

In most cases, the horizontal permeability is greater than the vertical and the drainage boundary will change from a circle to an ellipse as shown in Figure 4.2.

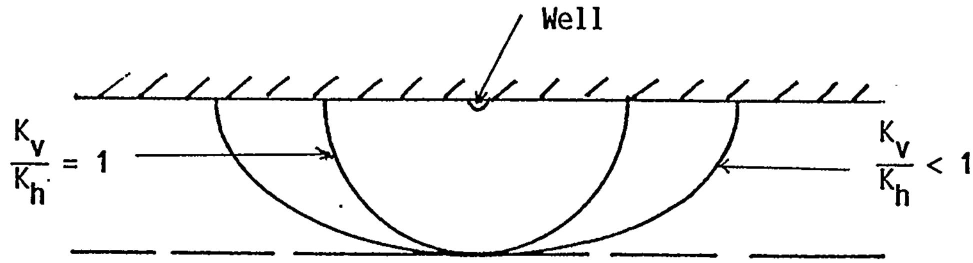


Figure 4.2 Drainage Volume in Anisotropic Reservoir

The drainage distance in the horizontal direction has been increased by the square root of the ratio of the horizontal to the vertical permeability and the volume of water-free oil produced from a vertical well compared to that of a horizontal well is:

$$Q_v'' = \frac{2}{3} \pi h^3 \frac{K_h}{K_v} \phi \Delta So \qquad (4.3)$$

$$Q_h'' = \left[\frac{2}{3} \pi h^3 \frac{K_h}{K_v} + \frac{1}{2} \pi h^2 L \sqrt{\frac{K_h}{K_v}} \right] \phi \Delta So \qquad (4.4)$$

$$\frac{Q_h''}{Q_v''} = 1 + \frac{3}{4} \frac{L}{h} \sqrt{\frac{K_v}{K_h}} \qquad (4.5)$$

Drainage from both vertical and horizontal wells is improved when K_h is greater than K_v, but the effect is larger for the vertical well. Conversely, the relative performance of a horizontal well improves when K_v is larger than K_h (eg. in reservoirs with vertical fractures).

Equation 4.5 shows that the horizontal well will always produce more water-free oil than the vertical well and the ratio of the volumes will increase with the length of the horizontal well. Furthermore, the incremental length of horizontal well, ΔL, required to replace an additional vertical well is

$$\Delta L = \frac{4}{3} h \sqrt{\frac{K_v}{K_h}} \qquad (4.6)$$

Figure 4.3 shows the effect of anisotropy on the ratio of oil production (horizontal well vs vertical well) vs horizontal well length in a 10 m thick reservoir. As expected, the advantage of the horizontal well decreases when the vertical permeability of the reservoir is smaller than the horizontal permeability.

4.2 Comparison of the Performance of Horizontal vs Vertical Wells in a Reservoir with Bottom Water

In a reservoir with bottom water, the horizontal well is drilled near the top of the producing formation (Figure 4.4). Assuming that the fluids move radially towards the horizontal well, let x be defined as the dimensionless fluid production, with respect to the cumulative water-free oil production, i.e.

$$X^* = \frac{\text{cum. fluid production, } Q_p}{\text{cum. water-free oil production, } Q_{o,dry}}$$

$$= \frac{Q_p}{\frac{\pi}{2} h^2 L} \tag{4.7}$$

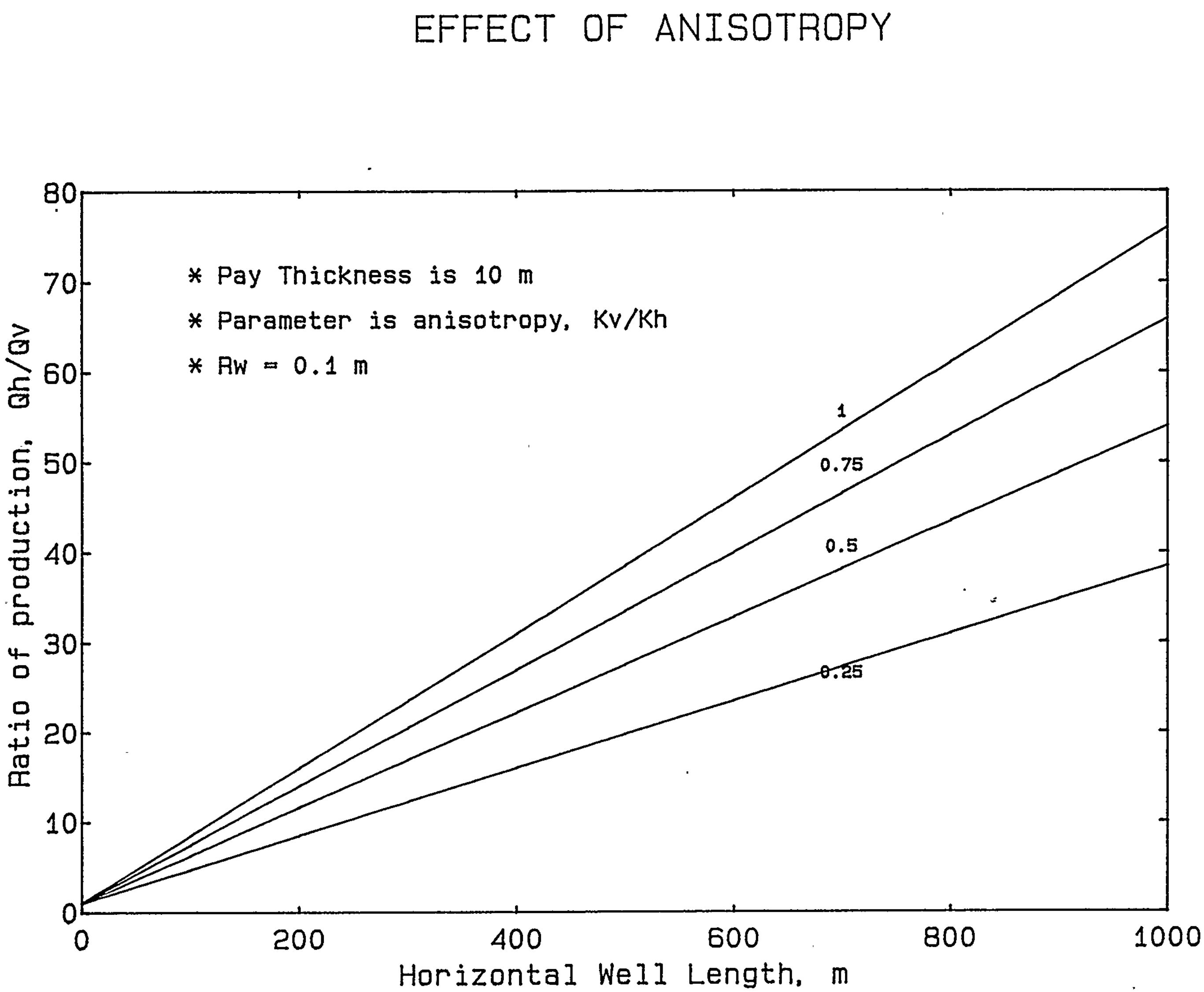

EFFECT OF ANISOTROPY
* Pay Thickness is 10 m
* Parameter is anisotropy, Kv/Kh
* Rw = 0.1 m
1
0.75
0.5
0.25
Ratio of production, Qh/Qv
Horizontal Well Length, m
0
10
20
30
40
50
60
70
80
0
200
400
600
800
1000

Figure 4.3

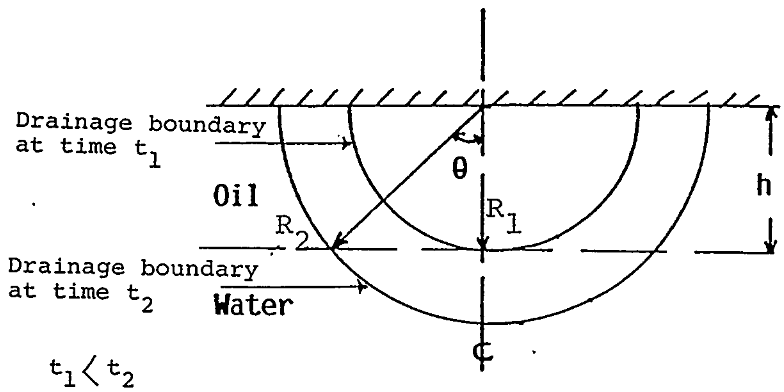


Figure 4.4 Position of the Drainage Boundary at Different Time

Let L' be the ratio of the length of horizontal well, L to the formation thickness h and α be the ratio of the radius of the drainage boundary at time t, R, to the radius of the drainage boundary at maximum water-free oil production, i.e.

$$L' = \frac{L}{h} \tag{4.8}$$

$$\alpha = \frac{R}{h} \tag{4.9}$$

at time t, the cumulative fluid production is

$$Q_p = \frac{\pi}{2} R^2 L + \frac{2\pi}{3} R^3 \tag{4.10}$$

The maximum water-free oil production is

$$Q_{o,dry} = \frac{\pi}{2} h^2 L + \frac{2}{3} \pi h^3$$

$$= \frac{\pi}{2} h^2 (L'h) + \frac{2}{3} \pi h^3$$

$$= \frac{\pi}{2} h^3 L' + \frac{2}{3} \pi h^3 \qquad (4.11)$$

For dimensionless production, we divide equations (4.10) and (4.11) by h^3 , and substituting equation (4.9), we have

$$\tilde{Q}_p = \frac{\pi}{2} L'\alpha^2 + \frac{2}{3} \pi\alpha^3 \qquad (4.12)$$

and

$$\tilde{Q}_{o,dry} = \frac{\pi}{2} L' + \frac{2}{3} \pi \qquad (4.13)$$

Where $\tilde{Q}_p$ and $\tilde{Q}_{o,dry}$ are the dimensionless fluid production at time t and dimensionless water-free oil production respectively.

Since from equation (4.7), $X^* = \dfrac{Q_p}{Q_{o,dry}}$,

and X^* is also equal to the ratio of the dimensionless production, i.e.

$$X^* = \frac{\tilde{Q}_p}{\tilde{Q}_{o,dry}} \qquad (4.14)$$

or

$$\tilde{Q}_p = X^* \tilde{Q}_{o,dry} \quad . \tag{4.15}$$

substituting equation (4.13) in (4.15) and equating it to equation (4.12), we have,

$$\tilde{Q}_p = \frac{\pi}{2} L'\alpha^2 + \frac{2}{3} \pi\alpha^3$$

$$= X^* \left(\frac{\pi}{2} L' + \frac{2}{3} \pi\right) \tag{4.16}$$

Therefore, for given values of x and L', the corresponding value of α can be found by solving equation (4.16) using the Newton-Rapson method.

Figure 4.4 shows that the angle θ is related to the parameter α as follows.

$$\cos\theta = \frac{h}{R} = \frac{1}{\alpha}$$

and at time, t, when the drainage boundary is at R distance away from the well, the volume of water contained in the crest bounded by the drainage boundary can be calculated as follows.

$$Q_w = \frac{\pi}{2} R^2 L' \left(\frac{2\theta}{\pi}\right) - h \sqrt{R^2 - h^2}\, L' + 1/3\pi (R-h)^2 \ (2R+h)$$

$$= L'h^3 \left(\alpha^2\theta - \sqrt{\alpha^2 - 1}\right) + 1/3\pi\ h^3\ (\alpha - 1)^2 (2\alpha + 1) \tag{4.17}$$

and the corresponding dimensionless water volume is

$$\tilde{Q}_w = L' \left[\alpha^2\theta - \sqrt{\alpha^2 - 1}\right] + 1/3\pi\ (\alpha - 1)^2(2\alpha + 1) \tag{4.18}$$

The water cut, γ, is:

$$\gamma = \frac{Q_w}{Q_p} = \frac{\tilde{Q}_w}{\tilde{Q}_p} \tag{4.19}$$

$$= \frac{L'\left(\alpha^2\theta - \sqrt{\alpha^2 - 1}\right) + 1/3\ \pi(\alpha-1)^2(2\alpha + 1)}{\frac{\pi}{2} L'\alpha^2 + 2/3\ \pi\alpha^3} \tag{4.20}$$

Figure 4.5 shows the effect on the horizontal well production for increasing well length/pay thickness ratio. For a given cumulative production, as the length to pay thickness ratio increases, more oil will be pushed into the wellbore resulting in a lower water cut.

For a vertical well completed at the top of the formation, the fluid volume produced under the same operating conditions is less than the horizontal well. Let Q_{ps} be the volume of fluid produced by the vertical well when the drainage boundary at a distance R_s from the well reaches the vertical wellbore,

$$Q_{ps} = \frac{2}{3}\pi R_s^3 \tag{4.21}$$

Let Q_{ws} be the volume of water contained in Q_{ps}, such that

$$Q_{ws} = \frac{1}{3} \pi (R_s - h)^2 (2R_s + h) \tag{4.22}$$

and assume $Z = \frac{h}{R_s}$, the fraction of water cuts, γ_s, contained in the fluid volume Q_{ps} is

$$\gamma_s = \frac{Q_{ws}}{Q_{ps}}$$

$$= \frac{1/3 \pi (R_s - h)^2 (2R_s + h)}{2/3 \pi R_s^3} \tag{4.23}$$

Substituting $Z = \frac{h}{R_s}$ into equation (4.23), we have

$$\gamma_s = \frac{1}{2} (1 - Z)^2 (2 + Z) \tag{4.24}$$

Comparison of equations (4.20) and (4.23) show that for the same total fluid production, the horizontal well will yield a lower water cut since more oil is displaced by a water crest than by a cone (Figure 4.6).

When the horizontal well and vertical well water cuts are equal, i.e.

$$\gamma = \gamma_s = \frac{1}{2} (1 - Z)^2 (2 + Z)$$

EFFECT ON CYLINDRICAL DISPLACEMENT

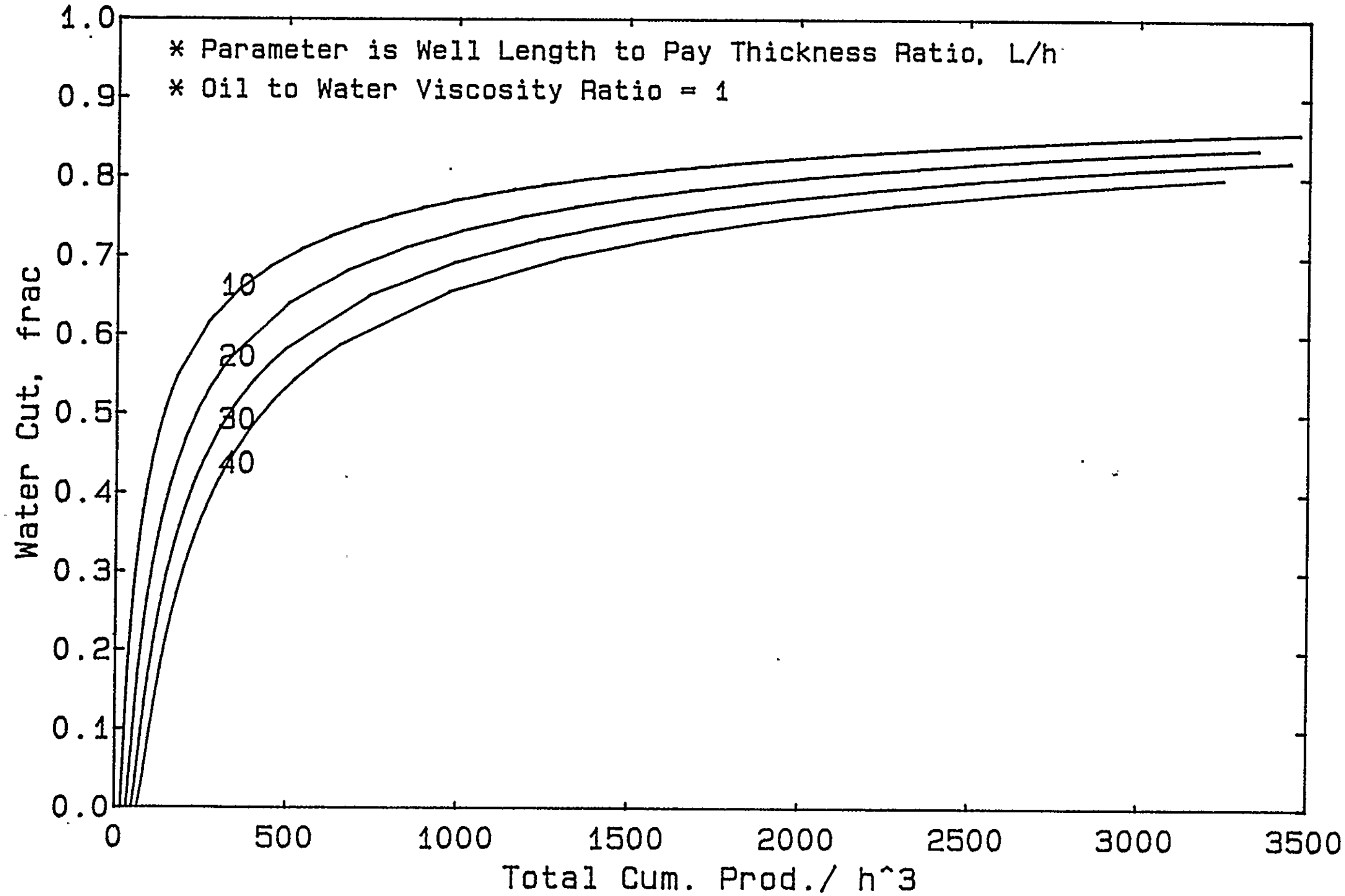


Figure 4.5

Using the Newton-Rapson method, the corresponding Z value can be found. Similar to the horizontal well, let X_s^* be the dimensionless fluid production with respect to the cumulative water-free oil production for the vertical well, i.e.

$$X_s^* = \frac{Q_{ps}}{2/3 \; \pi \; h^3} \qquad (4.25)$$

Substitute equation (4.21) into (4.25)

$$X_s^* = \frac{R^3}{h^3} \qquad (4.26)$$

or, X_s^* can be written as $X_s^* = \left(\frac{1}{Z}\right)^3$ (4.27)

The dimensionless fluid production expressed as a function of X_s is:

$$\tilde{Q}_{ps} = \frac{2/3 \; \pi R_s^{\;3}}{h^3} = 2/3 \; \pi X_s^* \qquad (4.28)$$

For a given water cut and cumulative fluid production, the volume of oil produced, Q_o is

$$Q_o = (1 - \gamma) \; Q_p \qquad (4.29)$$

for a horizontal well and,

$$Q_{os} = (1 - \gamma) \; Q_{ps} \qquad (4.30)$$

for a vertical well.

COMPARISON OF CYLINDRICAL & VERTICAL DISPLACEMENT

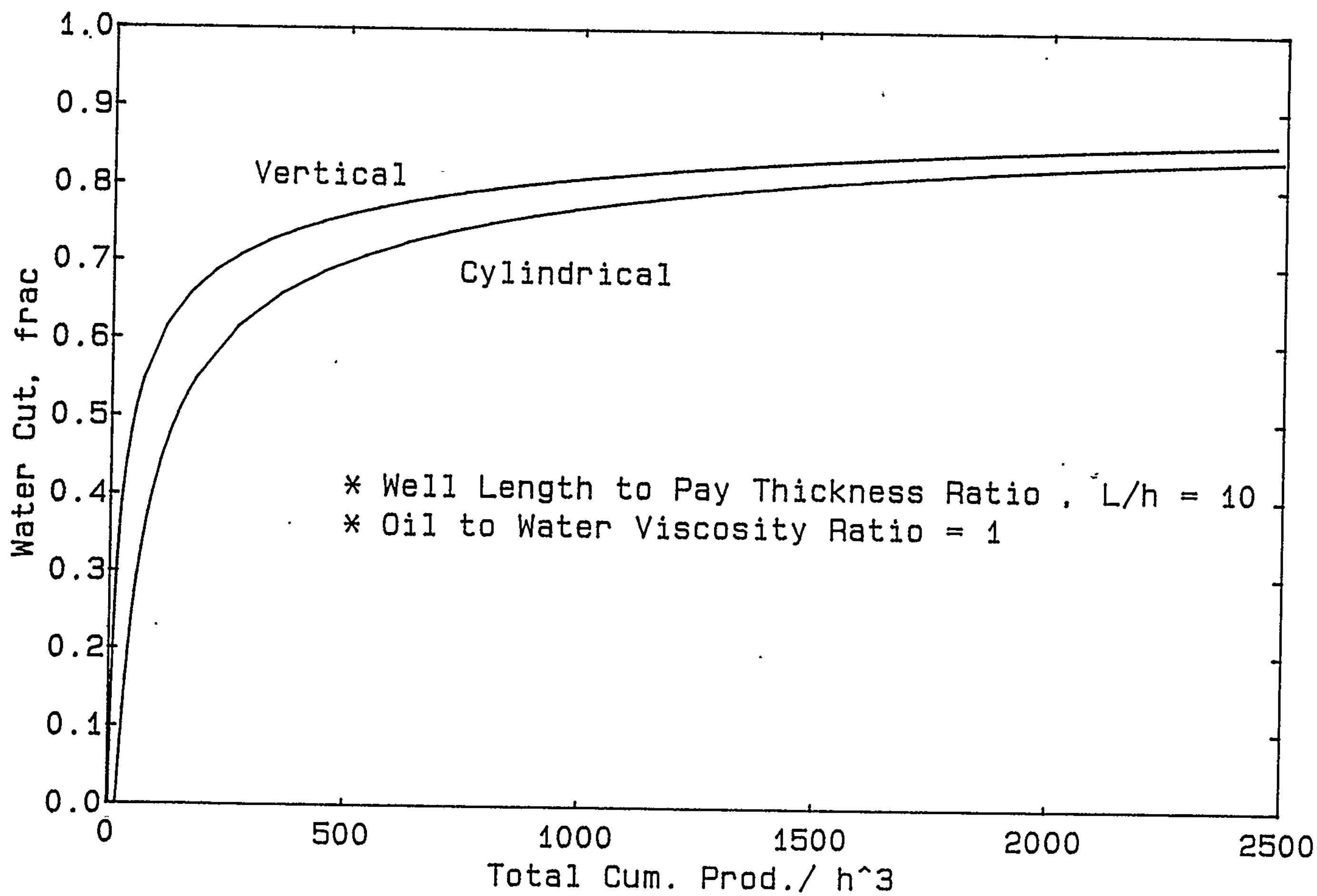


Figure 4.6

So far, we have assumed that the viscosities of oil and water are the same. If oil viscosity is larger than the water viscosity by a factor, F, for the same oil production, the water production will be increased proportional to the ratio of the viscosities and the resulting water cut will be increased accordingly, thus

$$\frac{\mu_o}{\mu_w} = F \tag{4.31}$$

$$\gamma_F = \frac{F\,\gamma}{1 + (F - 1)\,\gamma} \tag{4.32}$$

where γ_F is the water cut at that viscosity ratio.

Figures 4.7 and 4.8 show the effect of horizontal well length and viscosity ratio on horizontal well production. As the viscosity ratio increases, water cut increases rapidly and then levels off to approximately 95%.

4.3 Advance of Waterfront for Horizontal and Vertical Well

In a reservoir with bottom water and equal viscosities of oil and water, the advancement of water/oil contact is a function of the configuration of the drainage volume.

Let N be defined as the ratio of oil produced to the cumulative water-free oil production. For a vertical well, the drainage volume is spherical, and $N = \dfrac{\text{produced oil volume}}{2/3\ \pi h^3 \phi \Delta S_o}$.

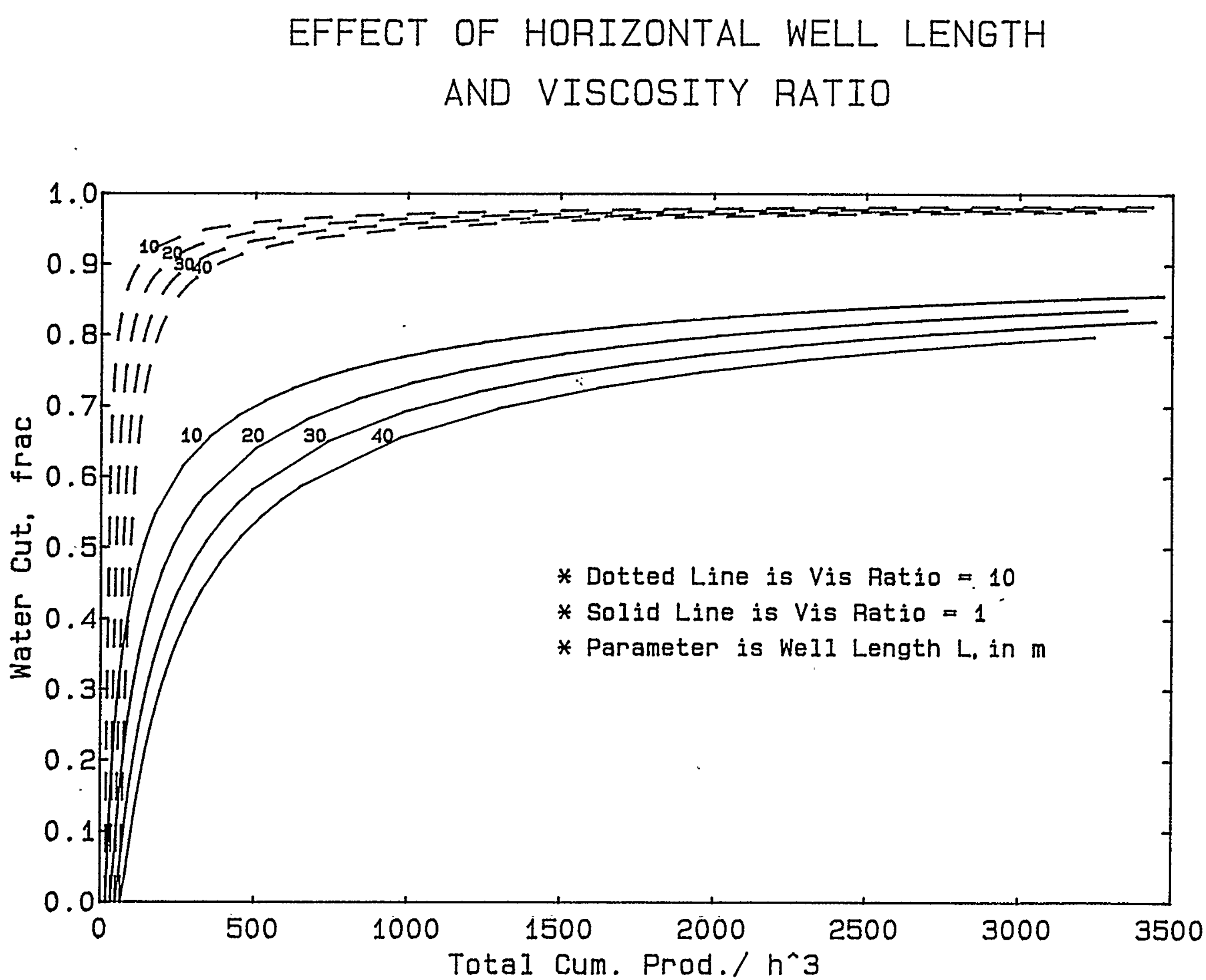

EFFECT OF HORIZONTAL WELL LENGTH
AND VISCOSITY RATIO
Water Cut, frac
Total Cum. Prod./ h^3
* Dotted Line is Vis Ratio = 10
* Solid Line is Vis Ratio = 1
* Parameter is Well Length L, in m
10
20
30
40
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
500
1000
1500
2000
2500
3000
3500

Figure 4.7

EFFECT OF HORIZONTAL WELL LENGTH
AND VISCOSITY RATIO

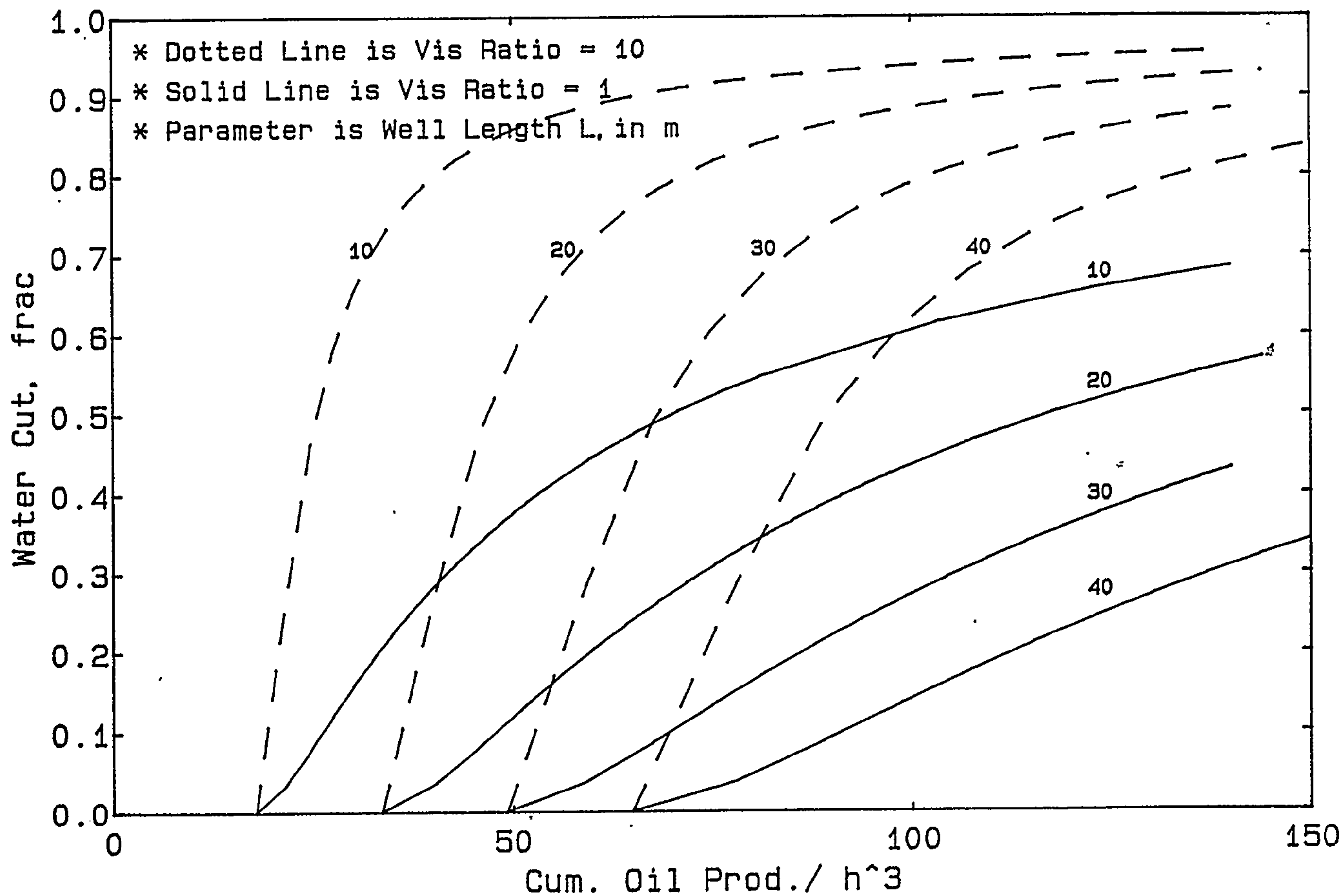


Figure 4.8

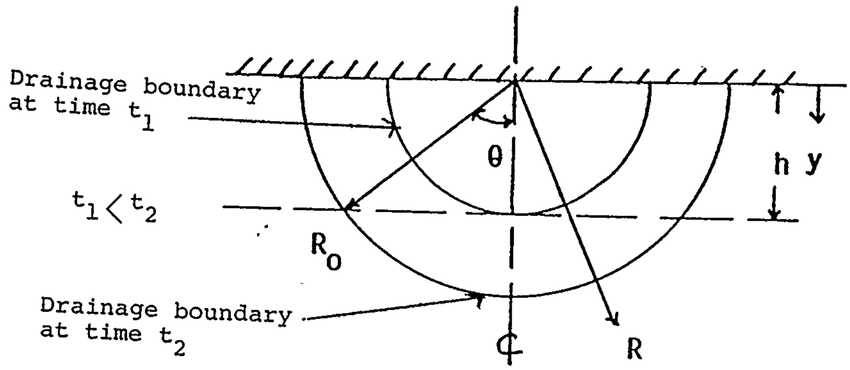


Figure 4.9. Positions of Drainage Boundary at Different Times

For a vertical well penetrating the top of the formation, the oil flow rate in the bottom half of the sphere is

$$q_o = \frac{-K}{2\mu} 4\pi y^2 \frac{\partial P}{\partial y} \tag{4.33}$$

$$\text{velocity} = \frac{q_o}{A} = \frac{q}{2\pi R^2} = -\frac{dR}{dt} \phi \Delta S_o \tag{4.34}$$

$$\text{therefore} \quad \frac{dR}{dt} = \frac{-q_o}{2\pi R^2 \phi \Delta S_o} \tag{4.35}$$

The total volume of oil produced when the drainage boundary moves from R_o to R can be obtained by integrating equation (4.35), i.e.

$$-q_o t = 2\pi\phi\Delta S_o \int_{R_o}^{R} R^2 dR$$

$$= \frac{2}{3}\pi\phi\Delta S_o R^3 \Big|_{R_o}^{R} \tag{4.36}$$

Since N is defined as $\dfrac{q_o t}{2/3\ \pi h^3 \phi\Delta S_o}$, equation (4.36) becomes

$$R^3 - R_o{}^3 = NH^3$$

or

$$\frac{R}{h} = \left\{\left(\frac{R_o}{h}\right)^3 - N\right\}^{1/3} \tag{4.37}$$

Since $\dfrac{R_o}{h} = \dfrac{1}{\text{Cos}\theta}$

Therefore

$$\frac{R}{h} = \left\{\left(\frac{1}{\text{Cos}\theta}\right)^3 - N\right\}^{1/3} \tag{4.38}$$

The position of the water/oil contact in the case of a horizontal well is derived similar to that of the vertical well and shown as the following:

$$N = \frac{q_o t}{\frac{\pi}{2} h^2 L\Delta S_o \phi + 2/3\ \pi\ h^3 \phi\Delta S_o} \tag{4.39}$$

Total surface of the produced fluid volume

$$A_s = \pi RL + 2\pi R^2$$

Since $$\frac{q_o}{\pi RL + 2\pi R^2} = -\frac{dR}{dt} \Delta S_o \phi \tag{4.40}$$

$$q_o t = -\Delta S_o \phi \int_{R_o}^{R} (\pi RL + 2\pi R^2)\, dR \tag{4.41}$$

Substituting equation (4.39) into equation (4.41), we have

$$N \left(\frac{\pi}{2} h^2 L\right) - \frac{\pi}{2} L (R_o^2 - R^2) = \frac{2\pi}{3} (R_o^3 - R^3) - \frac{2\pi}{3} h^3 N \tag{4.42}$$

Since $\text{Cos}\phi = \frac{h}{R_o}$, equation (4.42) becomes

$$\frac{3}{4} \frac{L}{h} \left[\left(N - \frac{1}{\text{Cos}^2\phi}\right) - \left(\frac{R}{h}\right)^2\right] = \left(\frac{1}{\text{Cos}^3\phi} - N\right) - \left(\frac{R}{h}\right)^3 \tag{4.43}$$

For given values of N, ϕ and $\frac{L}{h}$, we can use the Newton-Rapson method to find $\frac{R}{H}$, i.e.

$$f\left(\frac{R}{h}\right) = \frac{3}{4} \frac{L}{h} \left[\left(N - \frac{1}{\text{Cos}^2\phi}\right) - \left(\frac{R}{h}\right)^2\right] = \left(\frac{1}{\text{Cos}^3\phi} - N\right) + \left(\frac{R}{h}\right)^3 = 0 \tag{4.44}$$

$$f'(\frac{R}{h}) = 3(\frac{R}{h})^2 - \frac{3}{2}\frac{L}{h}(\frac{R}{h}) \qquad (4.45)$$

and

$$(\frac{R}{h})_i = (\frac{R}{h})_{i-1} - \frac{f(\frac{R}{h})_{i-1}}{f'(\frac{R}{h})_{i-1}}$$

Figures (4.10) and (4.11) show the advancing waterfront as a function of dimensionless oil production for a vertical well and a horizontal well respectively. For the same dimensionless oil production, the waterfront of a horizontal well is always lower than that of a vertical well, therefore more oil could be recovered before water breakthrough to the producer. Figure (4.11) shows the advancing waterfront to a horizontal well when L/h is unity. For larger ratios of L/h, the waterfront will rise more slowly (refer to equation (4.43)).

4.4 Displacement of Oil Vertically by Water

Consider an infinite array of horizontal wells, each of which is located at the top of the reservoir and separated from each other by a distance W (Figure 4.12). The recovery mechanism is that of a bottom water drive. The resultant pressure distribution due to the individual wells of the array is given by Muskat and shown as follows:

$$\Phi = \frac{Q_2\mu}{2\pi K}\left\{\ln\left[\cosh\frac{2\pi y}{W} - \cos\frac{2\pi x}{W}\right] + \frac{1}{2}\ln^2\right\} \qquad (4.46)$$

POSITION OF WATER FRONT ADVANCING UPWARDS TO A POINT WELL

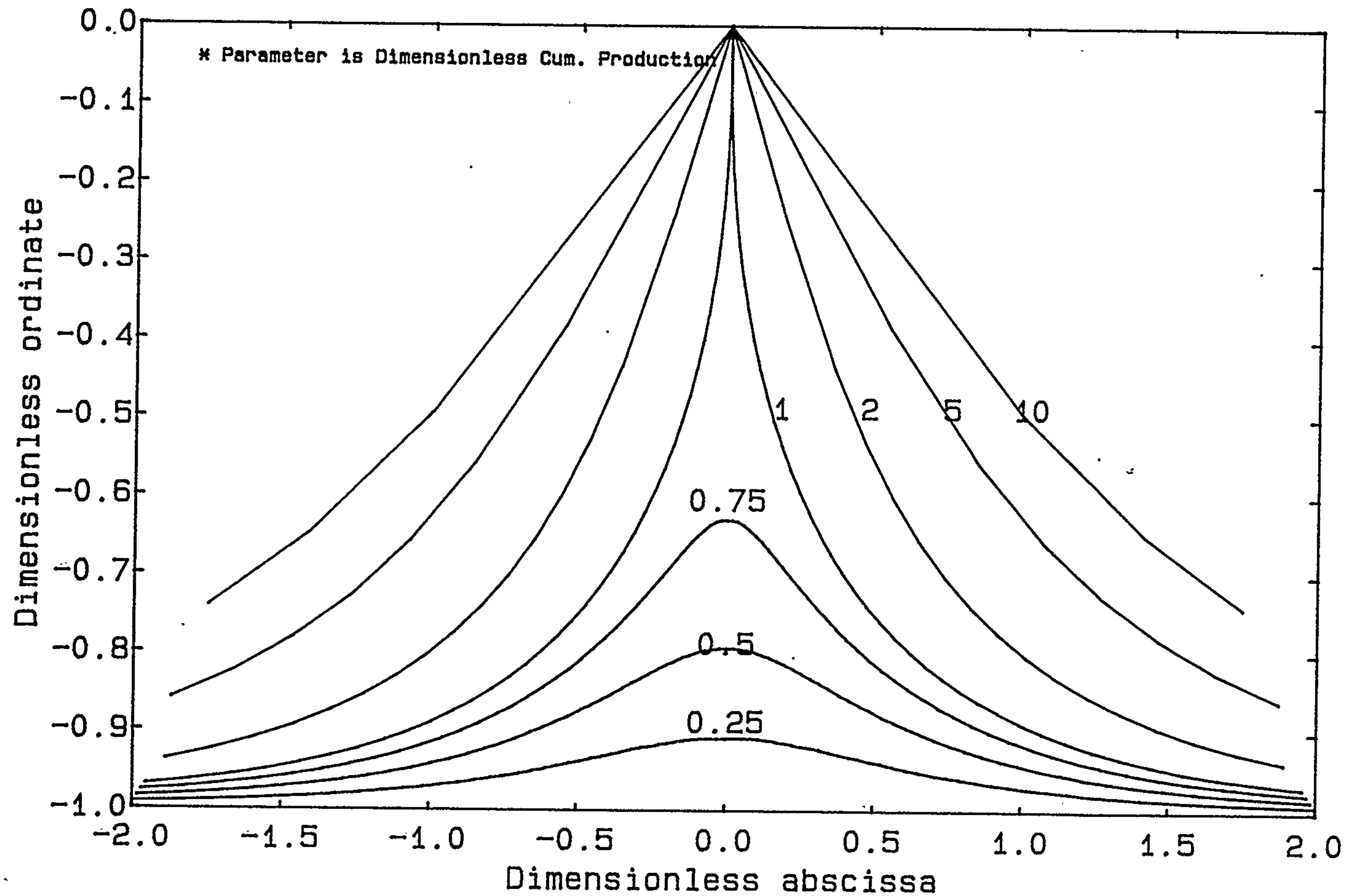


Figure 4.10

POSITION OF WATER FRONT ADVANCING UPWARDS
TO A HORIZONTAL WELL

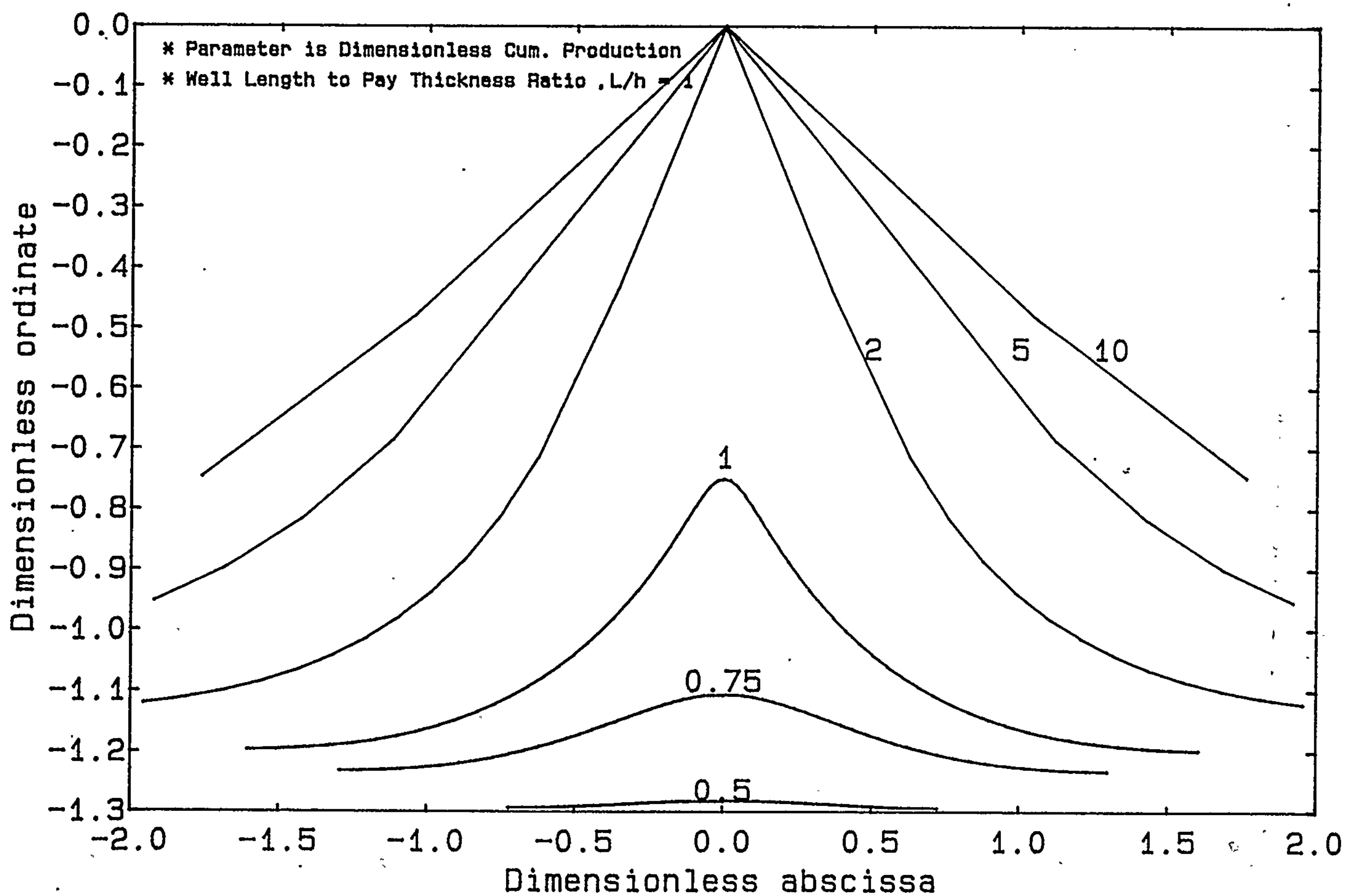


Figure 4.11

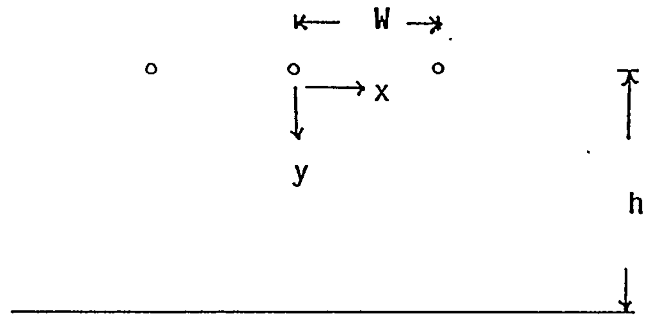


Figure 4.12. Infinite Array of Horizontal Wells

Along the centre line, x = 0.

$$\Phi = \frac{Q_2\mu}{2\pi K}\left\{\ln\left[\operatorname{Cosh}\frac{2\pi y}{W} - 1\right] + \frac{1}{2}\ln^2\right\} \tag{4.47}$$

$$\frac{d\Phi}{dy} = \frac{Q_2\mu}{2\pi K}\left\{\frac{1}{\operatorname{Cosh}\frac{2\pi y}{W} - 1}\ \frac{2\pi}{W}\ \operatorname{Sinh}\frac{2\pi y}{W}\right\} \tag{4.48}$$

Since $V = -\frac{K}{\mu}\frac{d\Phi}{dy}$

$$= -\frac{K}{\mu}\ \frac{Q_2\mu}{2\pi K}\ \frac{2\pi}{W}\left[\frac{\operatorname{Sinh}\frac{2\pi y}{W}}{\operatorname{Cosh}\frac{2\pi y}{W} - 1}\right] \tag{4.49}$$

or, $$\frac{dy}{dt} = -\frac{Q_2}{W}\ \frac{\operatorname{Sinh}\frac{2\pi y}{W}}{\operatorname{Cosh}\frac{2\pi y}{W} - 1} \tag{4.50}$$

Integrate t from time t_o.

$$t_o = -\frac{W}{Q_2}\int_h^o \left[\frac{\text{Cosh}\,\frac{2\pi y}{W} - 1}{\text{Sinh}\,\frac{2\pi y}{W}}\right] dy \tag{4.51}$$

$$= \frac{W}{Q_2}\int_o^h \left[\text{Coth}\,(\frac{2\pi y}{W}) - \text{Csch}\,\frac{2\pi y}{W}\right] dy \tag{4.52}$$

$$= \frac{W^2}{2Q_2\pi}\,\ln\left[\frac{\text{Sinh}\,(\frac{2\pi h}{W})}{2\tanh\,(\frac{\pi h}{W})}\right] \tag{4.53}$$

Recovery assuming piston-like displacement is

$$\text{Recovery} = \frac{Qt_o}{Wh}$$

$$= \frac{W}{2\pi h}\;\ln\left[\frac{\text{Sinh}\,(\frac{2\pi h}{W})}{2\tanh\,(\frac{\pi h}{W})}\right] \tag{4.54}$$

$$\text{dimensionless production} = \frac{\text{Recovery x W x h}}{\frac{\pi}{2}\,h^2} \tag{4.55}$$

Figure 4.13 shows the effect of well spacing on recovery and production. As the well spacing/pay thickness ratio increases, the dimensionless oil production increases and the recovery factor falls. As the interwell spacing increases, the efficiency of oil recovery will decrease whereas more oil will be recovered per individual well due to larger area per well spacing.

EFFECT OF WELL SPACING ON RECOVERY & PRODUCTION

FOR ARRAY OF HORIZONTAL WELLS

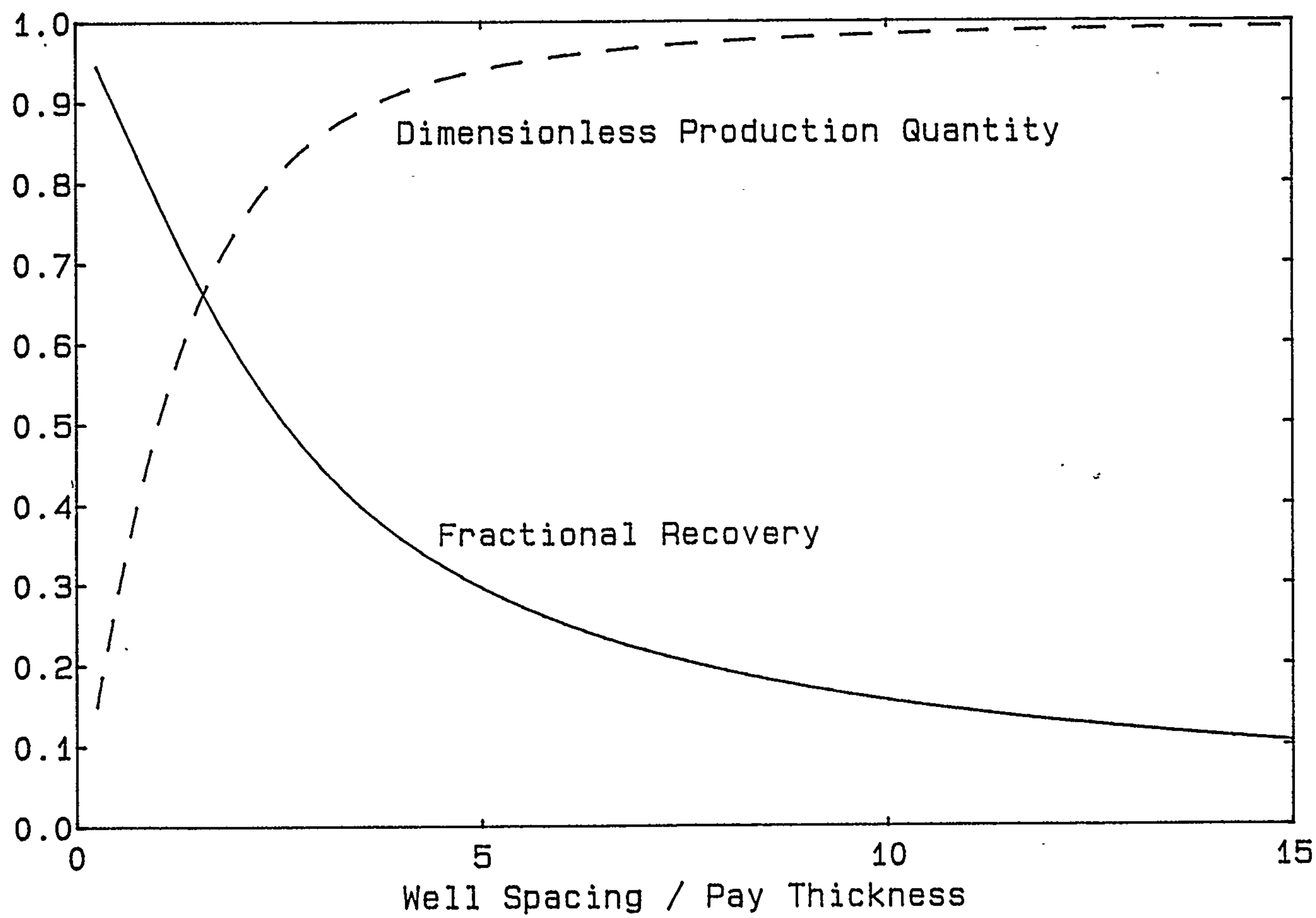
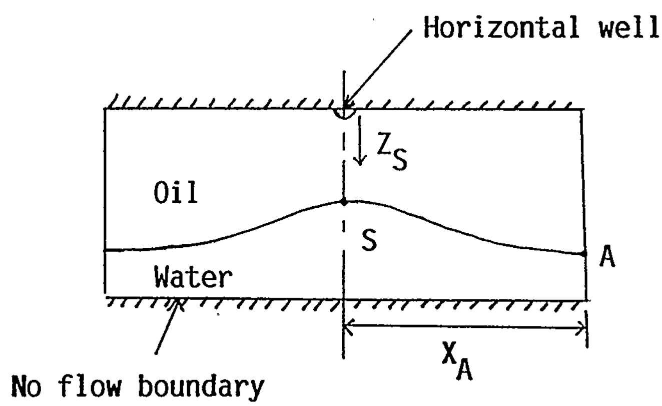


Figure 4.13

4.5 Other Approaches to Coning

I. Chaperon (8) and Giger et al (16) have discussed and published analytical solutions to predict gas/water coning towards horizontal well and oil recovery before breakthrough. Giger's paper (16) is not discussed here because we have not been able to reproduce the results shown in his paper. The following summarises the work of I. Chaperon.



In an isotropic reservoir with the horizontal well placed close to the top of the stratum, the flow potential Φ_A, as derived by Houpeurt, (17) gives,

$$\Phi_A = \frac{Q_h \mu}{2\pi LK} \ln \left[\text{Cosh} \left(\frac{\pi X_A}{h} \right) + 1 \right] \tag{4.56}$$

where X_A = distance from the well where the oil/water interface is horizontal

point A = location of the water/oil interface at the drainage boundary of the well point S = location of cone apex

The potential at the apex of the cone is Φ_S,

$$\Phi_S = \frac{Q_h\mu}{2\pi LK} \ln \left[1 - \text{Cos}\frac{\pi Z_S}{h}\right] \tag{4.57}$$

where Z_S = well to cone apex distance.

The viscous potential is $\Phi_A - \Phi_S$.

$$\Phi_A - \Phi_S = \frac{Q_h\mu}{2\pi LK} \ln \left[\frac{\left(1 + \text{Cosh}\frac{\pi X_A}{h}\right)}{\left(1 - \text{Cos}\frac{\pi Z_S}{h}\right)}\right] \tag{4.58}$$

The gravity potential is

$$\Phi_A - \Phi_S = \Delta\rho g\ (h - Z_S) \tag{4.59}$$

where $\Delta\rho$ = density difference between the oil phase and the water phase

g = gravity constant

For static equilibrium, viscous flow potential = gravity potential, and

$$\frac{Q_h \mu}{2\pi LK} \quad \ln \left[\frac{\left(1 + \mathrm{Cosh} \frac{\pi X_A}{h}\right)}{\left(1 - \mathrm{Cos} \quad \frac{\pi Z_S}{h}\right)} \right] \quad = \quad \Delta\rho g \ (h - Z_S)$$

$$\frac{Q_h}{L} \quad = \quad \Delta\rho g \ \frac{hK}{\mu} \left\{ \frac{(1 - \frac{Z_S}{h}) \quad 2\pi}{\ln \left[\frac{1 + \mathrm{Cosh} \frac{\pi X_A}{h}}{1 - \mathrm{Cos} \quad \frac{\pi Z_S}{h}} \right]} \right\} \tag{4.60}$$

At the limit of stability,

$$\frac{\partial \Phi_S}{\partial Z} \quad = \quad \Delta\rho g \quad = \quad \frac{Q_h \mu}{2LKh} \ \frac{\mathrm{Sin} \ \frac{\pi Z_S}{h}}{1 \ - \ \mathrm{Cos} \ \frac{\pi Z_S}{h}} \tag{4.61}$$

If Z_{SC} is the critical well to cone apex distance, then for critical stability,

$$\pi \left(1 - \frac{Z_{SC}}{h}\right) \ \frac{\mathrm{Sin}\pi \ \frac{\pi Z_{SC}}{h}}{1 \ - \ \mathrm{Cos} \ \frac{\pi Z_{SC}}{h}} \quad = \quad \ln \left[\frac{1 + \mathrm{Cosh} \frac{\pi X_A}{h}}{1 - \mathrm{Cos} \quad \frac{\pi Z_S}{h}} \right] \tag{4.62}$$

Let the dimensionless flow rate be defined as q_ℓ^* such that from equation (4.60),

$$q_\ell^* = \frac{2\pi\left(1 - \frac{Z_S}{h}\right)}{\ln\left[\frac{1 + \mathrm{Cosh}\,\frac{\pi X_A}{h}}{1 - \mathrm{Cos}\,\frac{\pi Z_S}{h}}\right]} \tag{4.63}$$

Substitute $Z_S = Z_{SC}$ at critical condition; the critical dimensionless flow rate is therefore $q_{\ell C}^*$.

$$q_{\ell C}^* = 2\left(\frac{1 - \mathrm{Cos}\,\frac{\pi Z_{SC}}{h}}{\mathrm{Sin}\,\frac{\pi Z_{SC}}{h}}\right) \tag{4.64}$$

Therefore, the critical flow rate per unit length $Q_{\ell C}$ given by I. Chaperon is

$$Q_{\ell C} = \Delta\rho g h \frac{K}{\mu}\; q_{\ell C}^* \tag{4.65}$$

Let $\alpha = \frac{X_A}{h}$. For an anisotropic reservoir, α becomes

$\alpha = \frac{X_A}{h}\left(\frac{K_v}{K_h}\right)^{1/2}$ and equation (4.65) will be modified as:

$$Q_{\ell C} = \Delta\rho g h \frac{K_h}{\mu}\left(\frac{K_v}{K_h}\right)^{1/2} q_{\ell C}^* \tag{4.66}$$

Figures 4.14 and 4.15 show the effect of the critical well to cone distance and the dimensionless critical rate on α. Equation (4.62) is

HORIZONTAL WELL

EFFECT OF CONE APEX LOCATION

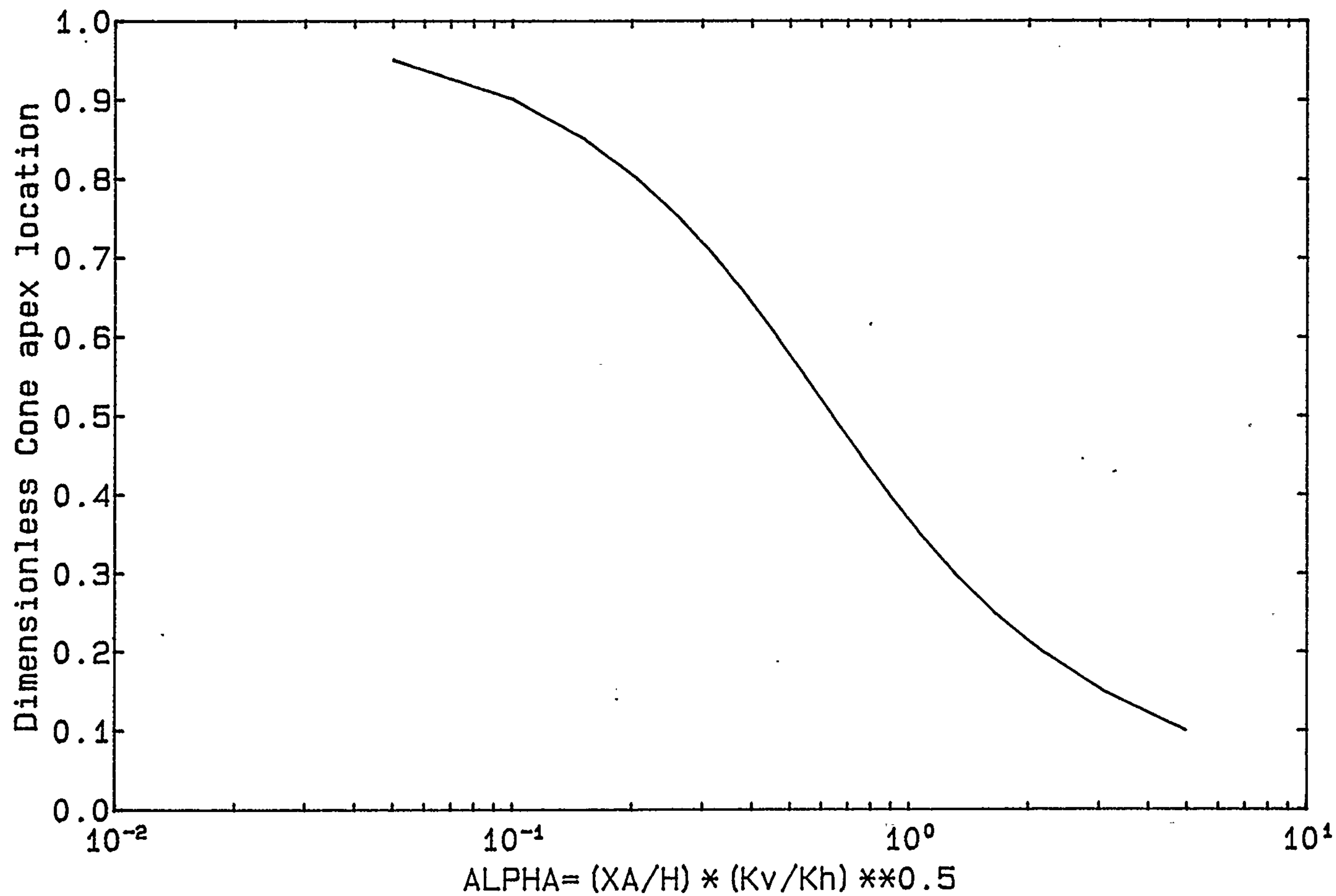


Figure 4.14

HORIZONTAL WELL

EFFECT OF DIMENSIONLESS CRITICAL RATE

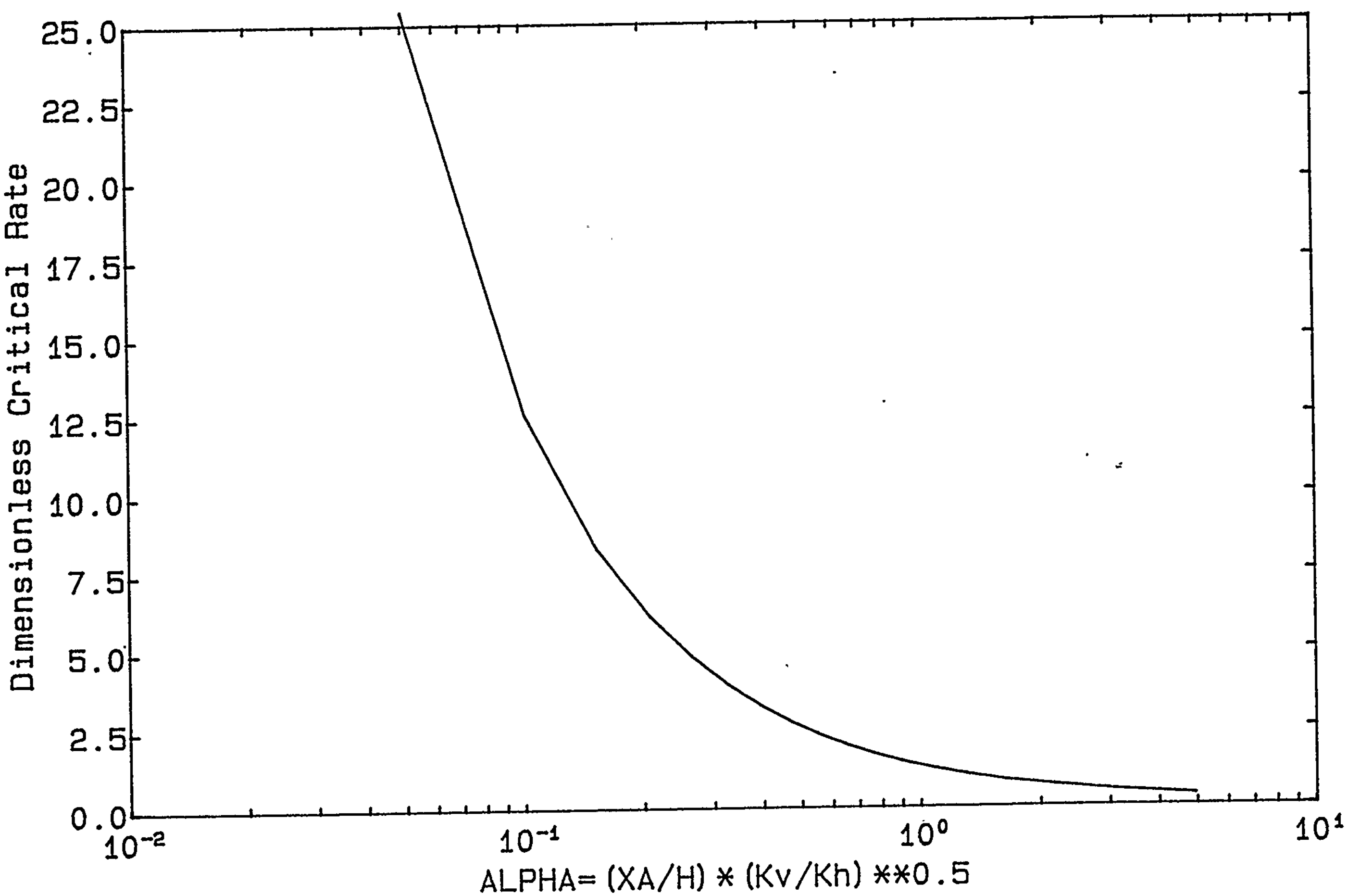


Figure 4.15

solved for $\frac{Z_{SC}}{h}$ by assuming values of α and using the Newton-Rapson method. The procedure is given as follows:

$$\text{Let} \quad x = \frac{Z_{SC}}{h}, \qquad \alpha = \frac{X_A}{h} \quad \text{for isotropic reservoir}$$

$$\alpha = \frac{X_A}{h}\left(\frac{K_v}{K_h}\right)^{1/2} \quad \text{for anisotropic reservoir}$$

Equation (4.62) can be rewritten as

$$\pi\,(1 - x)\,\frac{\text{Sin}\ \pi x}{1 - \text{Cos}\ \pi x} = \ln\left[\frac{1 + \text{Cosh}\ \pi x}{1 - \text{Cos}\ \pi x}\right] \tag{4.67}$$

$$\text{let } f(x) = \frac{1 - \text{Cos}\ \pi x}{\text{Sin}\ \pi x}\ \ln\left[\frac{1 + \text{Cosh}\ \pi x}{1 - \text{Cos}\ \pi x}\right] - \pi\,(1-x) = 0$$

$$f'(x) = \ln\left[\frac{1 + \text{Cosh}(\pi x)}{1 - \text{Cos}\ \pi x}\right]\left[\frac{\pi\ \text{Cos}\ \pi x(\text{Cos}\ \pi x - 1)}{\text{Sin}^2\ \pi x} + \pi\right] - 2\pi$$

$$\text{and} \quad x_i = x_{i-1} - \frac{f(x_i)}{f'(x_{i-1})}$$

One observes that when the vertical permeability decreases with horizontal permeability being constant, α becomes smaller and the critical cone stays further away from the well. However, the resulting critical rate becomes smaller. The same effect can be observed in an isotropic reservoir by increasing the interwell spacing of horizontal wells. The decrease in the critical rate results in a decrease in fractional recovery.

In the above mathematical development, I. Chaperon used the Muskat Model (22) to derive equation (4.66). The Muskat Model assumes no flow boundaries at the top and bottom of the reservoir. The model also assumes that the water/oil interface at point A remains horizontal at all times without considering water inflow in the region. This is physically impossible since without external oil and water influx, the water/oil interface at the drainage boundary will fall below the original level after some time of production. Due to the limitation of the Muskat Model, equation (4.66) should be used with caution.

CHAPTER 5

ADVANTAGES OF HORIZONTAL DRILLING

The advantages of horizontal drilling depend upon the cost ratio of horizontal to vertical wells. This cost ratio used to be 4 to 1 (27). Now, due to the recent advances in technology, this ratio can be reduced to as low as 1.3 to 1 (27).

Horizontal wells have shown an increase in production rate of 2-5 times and, in some cases, even 10 times that of the vertical wells. In reservoirs overlying an aquifer or located under a gas cap, the horizontal wells can produce even more. Since horizontal wells can improve production rates and recoveries by a variety of mechanisms, they are being used, on a commercial scale, for infill drilling in Prudhoe Bay, for the development of the new karstic, Rospo Mare Field in the Adriatic Sea, by UNOCAL in the Helder and Helm fields in the North Sea and by ARCO for the development of the Bima field offshore Indonesia.

There are numerous advantages of drilling horizontal wells. The following highlights some of the important factors.

(1) *Thickness*

The use of horizontal wells may reduce the cost of developing the field as compared to the drilling of vertical wells. This is because the productive length of a horizontal well is greater than that of a vertical well. The increase in productivity as shown earlier in equation (3.3) is proportional to the logarithmic function of the well length. Figure (3.1) shows that for a fixed horizontal well length, the increase in productivity is higher in thin pay reservoirs.

(2) *Multiple Fracture Reservoirs*

In fields having parallel vertical fractures such as the Rospo Mare Field (24), horizontal well drilling has a higher chance of cutting across these fractures and will have a greater increase in productivity. The effect of the vertical fractures is also similar to making K_v/K_h large, and, as shown in section 2.1.2, the equivalent isotropic pay thickness will be smaller. Under this condition, horizontal wells can reduce the cost of developing the field because conventional vertical wells are usually closely spaced in this type of oilfield.

(3) *Heterogeneous Reservoirs*

In oilfields with very heterogeneous permeability, a horizontal well drilled will also have a greater chance of encountering highly productive zones. Heterogeneity has the same effect as reducing the effective formation thickness (refer to section 2.1.2) and thereby increasing the productivity ratio of horizontal to vertical wells (refer to figure (3.1)).

(4) *Reservoirs with Gas Cap or Bottom Water*

A horizontal well is particularly useful in a reservoir with a gas cap (eg. Unocal, North Sea and Standard, Prudhoe Bay) or bottom water (eg. Rospo Mare Field) since it can be drilled near the bottom or the top of the reservoir to obtain as much clearance as possible. Also, with still greater total production, production per unit length is less, thus locally causing less stress on the oil/gas or oil/water interface. This effect is small in heavy oil reservoirs with bottom water because the density difference between the oil and water phase is small.

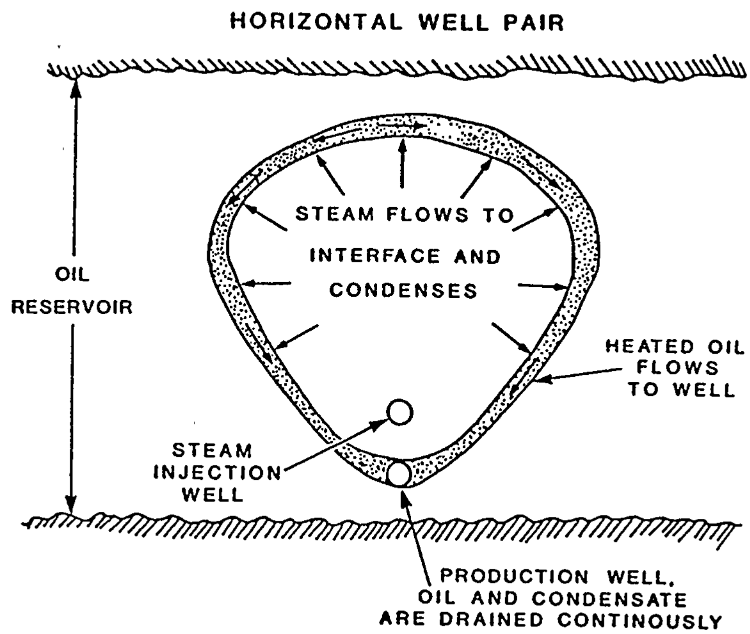
Figures (4.10) and (4.11) show the position of the water front advancing upward towards the vertical and horizontal well respectively. For the same dimensionless cumulative production, the position of the water/oil interface is higher in the case of the vertical well. A more important factor for viscous oil production is that a larger area is drained by a horizontal well since the interface forms a crest (rather than a cone as in the case of a vertical well) thereby increasing the volume of oil recovered prior to breakthrough.

(5) *Heavy Oil Reservoirs*

The advantage of horizontal well drilling is more significant in multiphase, adverse mobility ratio reservoirs. Horizontal wells will help to slow down the breakthrough of the water front since the water breakthrough will occur more rapidly under unstable flow. The steadiness of drainage in horizontal wells will provide more homogeneous progression to fronts which is especially advantageous at the start of production.

(6) *Thermal Recovery of Heavy Oil*

Butler et al (7) and S.D. Joshi (19) have shown significant advantages of thermal application in horizontal wells. Field applications such as the UTF project, Esso's Cold Lake project, etc. have been built to demonstrate horizontal well technology in improving thermal oil recovery. Injecting steam into a horizontal well will cause the steam zone to expand steadily while decreasing the heat loss into the over/underburden.



Schematic Diagram for Steam-Assisted Gravity Drainage Process

(Reference: S.D. Joshi (19))

The above figure shows a schematic diagram of thermal assisted gravity drainage. Two horizontal wells are closely spaced one above the other. The injected steam travels vertically upward and eventually forms a steam chamber. The steam condenses at the chamber boundaries and transfers heat to the heavy oil. A thin, heated layer of oil drains along the chamber boundaries to the production well. As time progresses, the steam chamber spreads upward and sideways, eventually contacting the top of the reservoir; then it starts descending, giving a high sweep efficiency. The process is similar to that of a gas cap overlying the oil where the gas/oil interface slowly moves down toward the production well as production continues. Horizontal production wells allow production at an economic rate without the coning (cresting) of steam into the production well.

(7) *Geological Information*

Logging and coring in horizontal wells will provide lateral evolution of facies and of fluid distribution in the reservoir. In the Rospo Mare Field, Elf Aquitaine (24) has been successful in obtaining eight cores and recorded several logs in the horizontal well. The logs recorded included Cement Bond Log (CBL), Gamma Ray (GR), Casing Collar Locator (CCL), Borehole Compensated Induction Sonic (BHL-ISF), Variable Density Log (VDL), Borehole Geometry Tool (BHT) and Dual Laterolog (DLL). It was reported that the production geology data was invaluable in determining future development of the oilfield.

CHAPTER 6

APPLICATIONS IN CANADA'S OILFIELDS

Any horizontally and vertically permeable reservoir can be exploited by horizontal wells. However, the economics for drilling horizontal wells depends on the production ratio of the horizontal well being greater than its cost ratio as compared to that of the conventional vertical well. Therefore, potential applications of horizontal wells involve estimating the production ratios relative to cost.

In a confined, single phase reservoir, overall recovery is not drastically improved by horizontal wells when compared to the recovery of vertical wells. The gain lies in the productivity of horizontal wells as a result of increased effective drainage area. The comparison of the productivity indices between horizontal wells and vertical wells will be highest when long horizontal wells are drilled in reservoirs where the vertical wells have small drainage radii. In multiphase flow, the recovery factor can be improved drastically by horizontal wells especially before gas or water breakthrough.

In general, most reservoirs containing natural or artificial fractures exhibit vertical or subvertical fractures. In this type of reservoir, fractures behave like permeability heterogeneities and are generally detrimental to vertical wells. Horizontal wells, on the other hand, are favourable under these circumstances.

The best technical applications of horizontal drilling are in the following conditions.

1. Sparsely distributed vertically fractured reservoirs where a vertical well has little chance of intersecting with a fracture.

2. Reservoirs with thief zones (bottom water or gas cap) and where a horizontal well can be positioned away from the fluid contact.

3. Thin-pay, low recovery reservoirs which produce poor economics for conventional vertical well projects.

4. Edge water or gas drive reservoirs where a horizontal well combines with the effect of a more efficient gravity drainage and a wider drainage area.

5. Secondary or tertiary recovery processes where horizontal wells can improve the injectivity and the areal sweep efficiency. Heavy oil reservoirs for both thermal and non thermal recovery techniques.

6. As evaluation wells in order to study the lateral evolution of the facies and provide valuable information for development decisions.

In Canada, applications of horizontal drilling are mainly in light oil reservoirs and tar sands. Govier (1983), compared Canadian energy resources and concluded that heavy oil reserves in Canada are approximately four times larger than the conventional light oil reserves. In particular, the "Lloydminster" type reservoirs represent a sizeable percentage of the heavy oil reserves.

In a typical Lloydminster heavy oil reservoir, the formation is too thin for application of thermal recovery processes. The relatively high oil density and viscosity make heavy oil production low and marginally economical. Secondary recovery processes like waterflooding are not effective in most cases due to adverse mobility ratios. In cases where the heavy oil is associated with thief zones, the chance of recovering the petroleum reserves is further decreased. This type of reservoir is a potential candidate for horizontal well application.

The benefits of horizontal drilling and its application were not readily realized due to the fact that there were not practical methods or technology available to economically drill horizontal holes. Recent technical advancement, coupled with declining reserves and increasing finding and lifting costs should stimulate the use of horizontal drilling technology.

CHAPTER 7

CONCLUSIONS

Usually, enhanced oil recovery is achieved either by providing additional energy to the reservoir or by changing the physical characteristics of the in-situ fluids; horizontal wells provide a different process by changing the flow geometry.

This thesis summarises and reviews the steady state flow equations for horizontal wells which were proposed in the literature (8), (12), (18). Since steady state flow conditions are not really applicable to depleting reservoirs, the flow equations for an array of infinite horizontal wells (with large and small interwell spacing) producing under pseudo-steady state have been developed here. It is found that under pseudo-steady state the reservoir pressure drawdown (using either perimeter or area average reservoir pressure) is considerably larger for vertical wells as compared to horizontal wells. For single phase flow, the ultimate recovery for a vertical well may not be drastically lower than that of the horizontal well. However, for multiphase flow, due to the larger pressure drawdown near the vertical well, more solution gas may be evolved and oil production may decrease.

The advantage of horizontal wells over vertical wells may be utilized by achieving higher production rates with approximately the same overall recovery or by operating at approximately the same rate but with a high overall recovery.

For thin pay reservoirs, the productive length of a horizontal well is considerably greater than that of a vertical well. The productivity of a horizontal well increases with its length, although more slowly because of the logarithmic function. Analytical derivations of the flow equation for horizontal wells show that the increase in productivity may regularly be three to five times than for vertical wells. Recently, the increase in productivity of horizontal wells has been demonstrated in North America (26) and Western Europe (20) and improvements in performance of the same order as those predicted in this thesis have been found (eg. 2-5 times the productivity).

Under equivalent conditions, analytical calculations show that the number of horizontal wells required to obtain the same production is less than that for a system of vertical wells. However, this production method will be economically successful only if the site has been properly selected and the system of wells carefully designed. A thorough study of the reservoir and overall economics must be conducted before any decision is taken to drill horizontal wells.

In reservoirs with gas cap or bottom water, horizontal drilling provides increased standoff from the fluid contacts, thereby improving the production rate. Additionally, the longer wellbore length serves to reduce the pressure drawdown and further reduce coning tendencies. An important factor in heavy oil reservoirs with bottom water layers is that the water cone which is drawn upwards to the well is much larger for horizontal wells than for the vertical wells and, as a result, a much larger volume of oil is displaced. This steadiness of drainage in horizontal wells provides a more efficient sweep and hence, higher recovery.

The benefits of horizontal drilling were not realized earlier due to the lack of technology to place horizontal holes in the formation economically. Although recent technology has made some advancement, horizontal production as an emerging technology still has much room for improvement.

Stimulated by the declining reserves and increasing costs of exploration and transportation, industrial development with horizontal wells is already under way. Undoubtedly, horizontal wells are considered to be a new enhanced recovery tool and there is a bright future ahead.

Recommendations

This thesis uses simplified analytical models to study the flow to a horizontal well under primary depletion. The mathematical models described earlier include many assumptions and, therefore, results obtained using these equations should be compared with the field data and that from the numerical simulation using a commercially available black-oil model. In addition, laboratory experiments should be performed to investigate the flow mechanism and recovery of a horizontal well and compared to the analytical solutions.

Throughout this research work, single phase flow is assumed and future work should extend the flow theory to multiphase flow including the effects of anisotropy. Future research is proposed to include the study of injectivity and area sweep efficiency on the miscible and immiscible flooding applications.

APPENDIX I

STEADY STATE POTENTIAL DISTRIBUTION ABOUT A HORIZONTAL WELL IN A HORIZONTAL PLANE

Below, it is shown that the potential distribution of a horizontal well of length L can be described by a series of confocal ellipses and hyperbolas intercepting orthogonally to each other (Figure I.1). The ellipses represent constant pressure lines and the hyperbolae represent the stream lines. At the outer drainage boundary, the constant pressure is P_e.

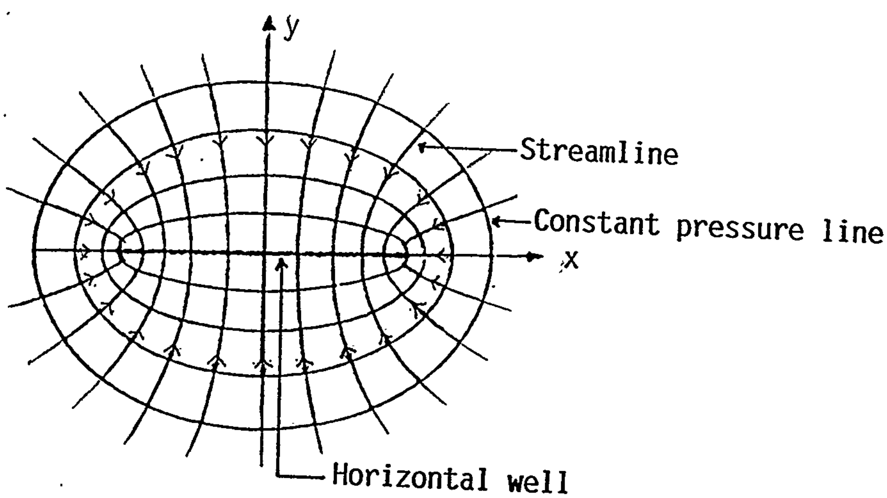


Figure I.1 Schematic Diagram of Potential Flow to a Horizontal Well in the Horizontal Plane

The complex potential F is obtained using the analytic function $w = \cosh^{-1}(z/c)$ (23). The mapping function can be re-written as

$$z = c \cosh w \tag{I.1}$$

where $z = x + iy$

and $w = u + iv$

Expanding the hyperbolic function and equating the real and imaginary parts, we have

$$x = c \cosh u \cos v \tag{I.2}$$

$$y = c \sinh u \sin v \tag{I.3}$$

Eliminating u and v, the above two equations become

$$\frac{x^2}{c^2 \cosh^2 u} + \frac{y^2}{c^2 \sinh^2 u} = 1 \tag{I.4}$$

$$\frac{x^2}{c^2 \cos^2 v} - \frac{y^2}{c^2 \sin^2 v} = 1 \tag{I.5}$$

Equations (I.4) and (I.5) represent a series of confocal ellipses and hyperbolae respectively. Since the complex potential F is

$$F = \frac{Q_1 \mu}{2\pi K} w \tag{I.6}$$

where Q_1 is the flow rate per unit reservoir thickness. It is assumed a positive sign for flow into the well.

The real (Φ) and imaginary (ψ) potentials can be reformulated as

$$\Phi = \frac{Q_1 \mu}{2\pi K} \operatorname{Cosh}^{-1} H^*_\Phi \tag{I.7}$$

$$\psi = \frac{Q_1 \mu}{2\pi K} \operatorname{Cos}^{-1} H^*_\psi \tag{I.8}$$

where

$$H^*_\Phi = \left[\frac{x^2 + y^2 + c^2 + \sqrt{(x^2 + y^2 + c^2)^2 - 4c^2x^2}}{2c^2}\right]^{1/2}$$

$$H^*_\psi = \left[\frac{x^2 + y^2 + c^2 - \sqrt{(x^2 + y^2 + c^2)^2 - 4c^2x^2}}{2c^2}\right]^{1/2}$$

Potential Function at the x-axis (i.e. y=0)

Along the x-axis, the potential function Φ becomes

$$\Phi = \frac{Q_1 \mu}{2\pi K} \operatorname{Cosh}^{-1} \frac{x}{c}$$

The inverse hyperbolic function becomes undefined for $\frac{x}{c} < 1$. Therefore, along the horizontal well length, the potential function is zero. ($\Phi_w = 0$). For distances larger than c, the potential function is:

$$\Phi = \frac{Q_1\mu}{2\pi K} \quad \text{Cosh}^{-1} \left(\frac{x}{c}\right)$$

The above expression shows that the potential function Φ is proportional to the ratio of $\frac{x}{c}$. Therefore, different values of the potential function can be obtained by adjusting the ratio of $\frac{x}{c}$.

In particular, at the outer elliptical drainage boundary a,

$$\Phi_e = \frac{Q_1\mu}{2\pi K} \quad \text{Cosh}^{-1} \left(\frac{a}{c}\right)$$

For

$$\frac{a}{c} \geq 1,$$

$$\Phi_e = \frac{Q_1\mu}{2\pi K} \left[\ln \left(\frac{a}{c} + \sqrt{\frac{a^2}{c^2} - 1}\right)\right]$$

$$= \frac{Q_1\mu}{2\pi K} \ln \left(\frac{a + \sqrt{a^2 - c^2}}{c}\right) \qquad (I.9)$$

The potential difference between the outer drainage boundary and the well is therefore

$$\Delta\Phi_{ew} = \Phi_e - \Phi_w = \frac{Q_1\mu}{2\pi K} \ln \left(\frac{a + \sqrt{a^2 - c^2}}{c}\right) \qquad (I.10)$$

The geometry of an ellipse with c being the foci gives $a^2 = b^2 + c^2$.

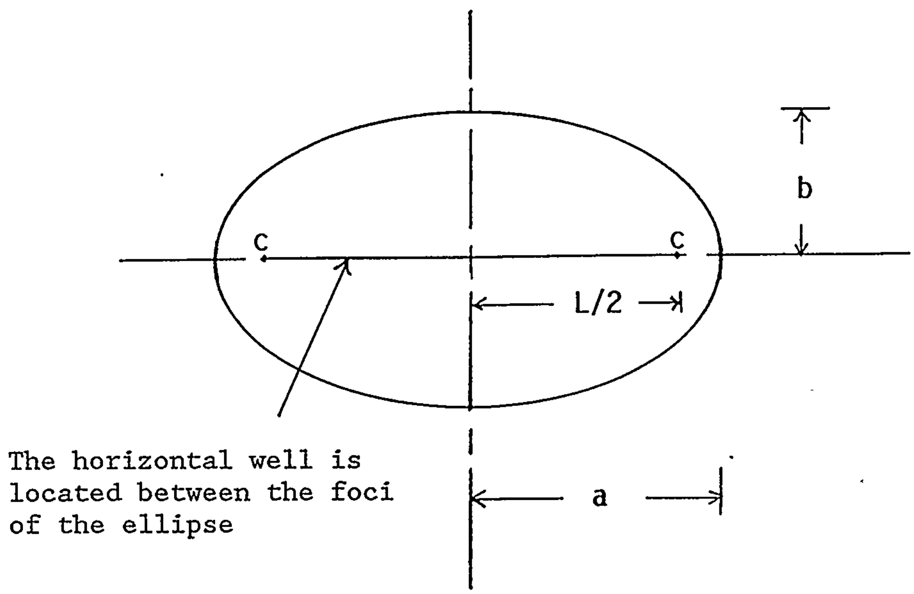


Figure I.2 Schematic Diagram of an Ellipse

Furthermore, let c = L/2,

$$\Delta\Phi_{ew} = \frac{Q_1 \mu}{2\pi K} \ln\left(\frac{a+b}{L/2}\right) \qquad \text{(I.11)}$$

The flow rate per unit thickness of formation, Q_1, is

$$Q_1 = \frac{2\pi K}{\mu} \Delta\Phi_{ew} \frac{1}{\ln\left(\frac{a+b}{L/2}\right)} \qquad \text{(I.12)}$$

Equivalent Drainage Radius

Assuming the area of an ellipse equals that of a circle, the equivalent radial drainage of an ellipse is $r_{e,h} = \sqrt{ab}$. The following shows the different forms of the equation for the flow rate Q_1 as a result of the different approximations in evaluating $r_{e,h}$.

(1) If $r_{e,h} \simeq \frac{a+b}{2}$ for $a \simeq b$,

$$Q_1 = \frac{2\pi K}{\mu} \Delta\Phi_{ew} \frac{1}{\ln\left(\frac{r_{e,h}}{L/4}\right)} \tag{I.13}$$

(2) If $r_{e,h} \simeq a$ for $\frac{L}{2a} \leq 0.5$, (Giger (12))

$$Q_1 = \frac{2\pi K}{\mu} \Delta\Phi_{ew} \frac{1}{\ln\left\{\frac{r_{e,h}\left[1 + \sqrt{1 - \left(\frac{L}{2r_{e,h}}\right)^2}\right]}{L/2}\right\}} \tag{I.14}$$

(3) If $r_{e,h} \simeq b$ for $\frac{L}{2b} \leq 0.5$, (Borisov (3))

$$Q_1 = \frac{2\pi K}{\mu} \Delta\Phi_{ew} \frac{1}{\ln\left\{\frac{r_{e,h}\left[1 + \sqrt{1 + \left(\frac{L}{2r_{e,h}}\right)^2}\right]}{L/2}\right\}} \tag{I.15}$$

When $r_{e,h}$ is very large compared to L, equations (I.13) to (I.15) give similar values for Q_1. On the other hand, equation (I.14) give the most optimistic value for Q_1 compared to equations (I.13) and (I.15). Equation (I.13) is exact since the equivalent drainage radius is the average of the major and minor axes of the ellipse. Therefore, this is the best equation for calculating Q_1.

APPENDIX II

POTENTIAL DISTRIBUTION ABOUT A HORIZONTAL WELL LOCATED AT THE CENTRE OF AN INFINITE HOMOGENEOUS RESERVOIR

Consider an infinitely long horizontal well located at the centre of an infinite, homogeneous reservoir. The reservoir thickness is h and is bounded above and below by impervious boundaries. The well is producing at a steady constant rate per unit length Q_2 (Figure II.1).

The above problem is analogous to an infinite series of horizontal wells located above each other with spacing h (Figure II.2). The steady production rate per unit length Q_2 of each well is contributed by two corresponding source wells, each of strength $-Q_2/2$, located at $x = -\alpha$ and $x = +\alpha$.

The complex potential function F is obtained by the conformal mapping of the $z = x + iy$ plane (Figure II.1) to the plane $w = u + iv$ (Figure II.2) using the analytic function $w = e^{\pi z/h}$.

As shown in Figure II.3, all the source wells on the left side of the z-plane maps on to the origin of the w-plane and all the source wells on the right of the z-plane lie alternately at $+\alpha$ and $-\alpha$ in the w-plane. The vertical axis in the z-plane becomes a circle of unit radius with its centre at the origin of the w-plane. The sink wells in the z-plane lie alternately at (1,0) and (−1,0) in the w-plane.

For each set of wells the flow potential F is given by

$$F = \frac{Q_2\mu}{2\pi K} \left[\ln(w+1) - 2(\tfrac{1}{2})\ln(w) + \ln(w-1)\right]$$

$$= \frac{Q_2\mu}{2\pi K} \ln(w - \frac{1}{w})$$

$$= \frac{Q_2\mu}{2\pi K} \ln\left(2 \text{ Sinh } \frac{\pi z}{h}\right) \qquad \text{(II.1)}$$

The potential Φ is the real part of the complex flow F. The real part of $\ln\left(2 \text{ Sinh } \frac{\pi z}{h}\right)$ is the logarithm of the modulus of 2 $\text{Sinh}\left(\frac{\pi z}{h}\right)$. So, the potential Φ is represented as

$$\Phi = \frac{Q_2\mu}{4\pi K} \left\{\ln \left(\text{Cosh } \frac{2\pi x}{h} - \text{Cos } \frac{2\pi y}{h}\right) + \frac{1}{2} \ln 2\right\} \qquad \text{(II.2)}$$

Along the centre plane of the reservoir (i.e. y=0), and dropping the constant term $\frac{1}{2}$ ln 2, the potential Φ becomes

$$\Phi = \frac{Q_2\mu}{4\pi K} \ln \left(\text{Cosh } \frac{2\pi x}{h} - 1\right) \qquad \text{(II.3)}$$

Potential at Wellbore Where $x = r_w$

$$\Phi_w = \frac{Q_2\mu}{4\pi K} \ln \left\{ \frac{1}{2}\left(e^{\frac{2\pi r_w}{h}} + e^{-\frac{2\pi r_w}{h}} \right) - 1 \right\}$$

Expressing the exponentials as power series and assuming $h \gg r_w$

$$\Phi_w = \frac{Q_2\mu}{4\pi K} \ln \left\{ \frac{1}{2}\left(\frac{2\pi r_w}{h}\right)^2 \right\} \tag{II.4}$$

The potential difference between a point at a considerable distance from the well and the wellbore is:

$$\Phi - \Phi_w = \frac{Q_2\mu}{4\pi K} \left\{ \ln \left(\cosh \frac{2\pi x}{h} - \cos \frac{2\pi y}{h} \right) - \ln\left(\frac{1}{2} \left(\frac{2\pi r_w}{h}\right)^2 \right) \right\}$$

If x is large, the cosine term becomes negligible in comparison to the hyperbolic cosine and the hyperbolic cosine becomes $\frac{1}{2} e^{\frac{2\pi x}{h}}$ and $\Phi - \Phi_w$ reduces to

$$\Phi - \Phi_w = \frac{Q_2\mu}{4\pi K} \left[\frac{2\pi x}{h} + 2 \ln \frac{h}{2\pi r_w} \right] \tag{II.5}$$

The first term corresponds to the linear potential gradient and the second term to the additional gradient which is required to produce convergance to the horizontal well of radius r_w.

This extra potential gradient is

$$\Delta\Phi = \frac{Q_2\mu}{2\pi K} \ln \frac{h}{2\pi r_w} \tag{II.6}$$

Potential at Stagnation Point (0, h/2)

The point B (0, h/2) and the corresponding point B' (0, -h/2) are of interest because the fluid velocity is zero at these points. The potential difference between this point and the well is obtained by substituting x=0 and y=h/2 in equation (II.4).

Therefore

$$\Phi_B - \Phi_w = \frac{Q_2\mu}{2\pi K} \left[\ln \left(\frac{2h}{\pi r_w} \right) \right] \tag{II.7}$$

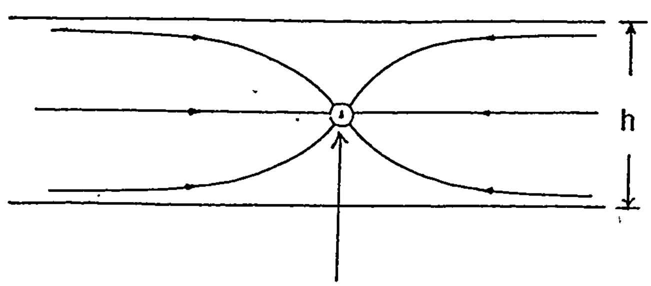


Figure II.1 Schematic Diagram of Potential Flow to a Horizontal Well in a Vertical Plane

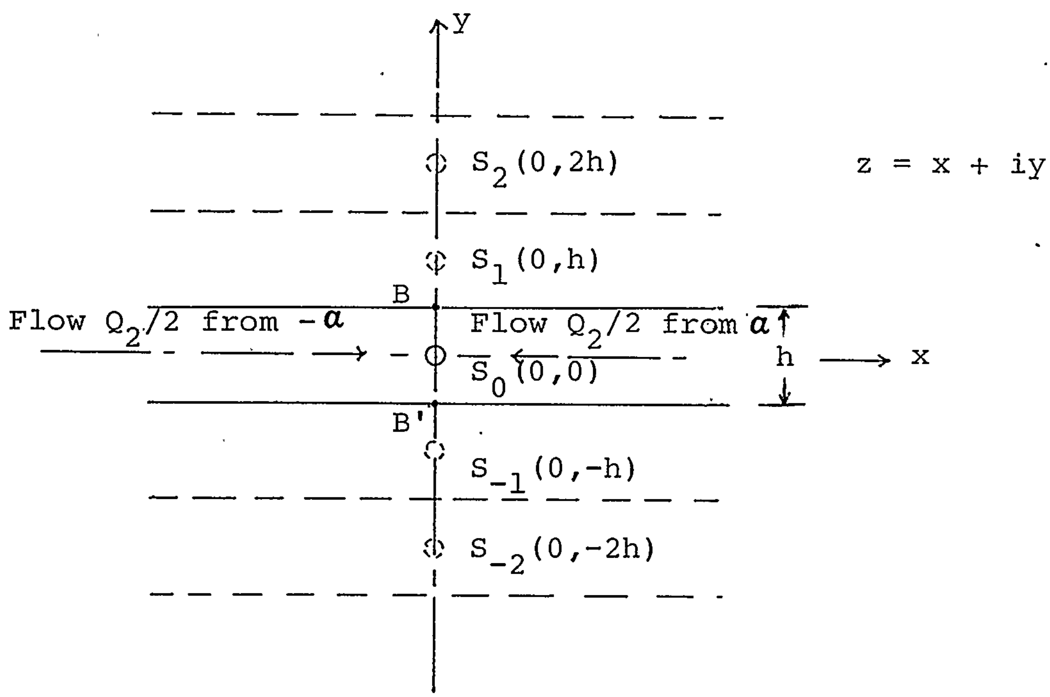


Figure II.2 Horizontal Well Potential Distribution - Z-Plane

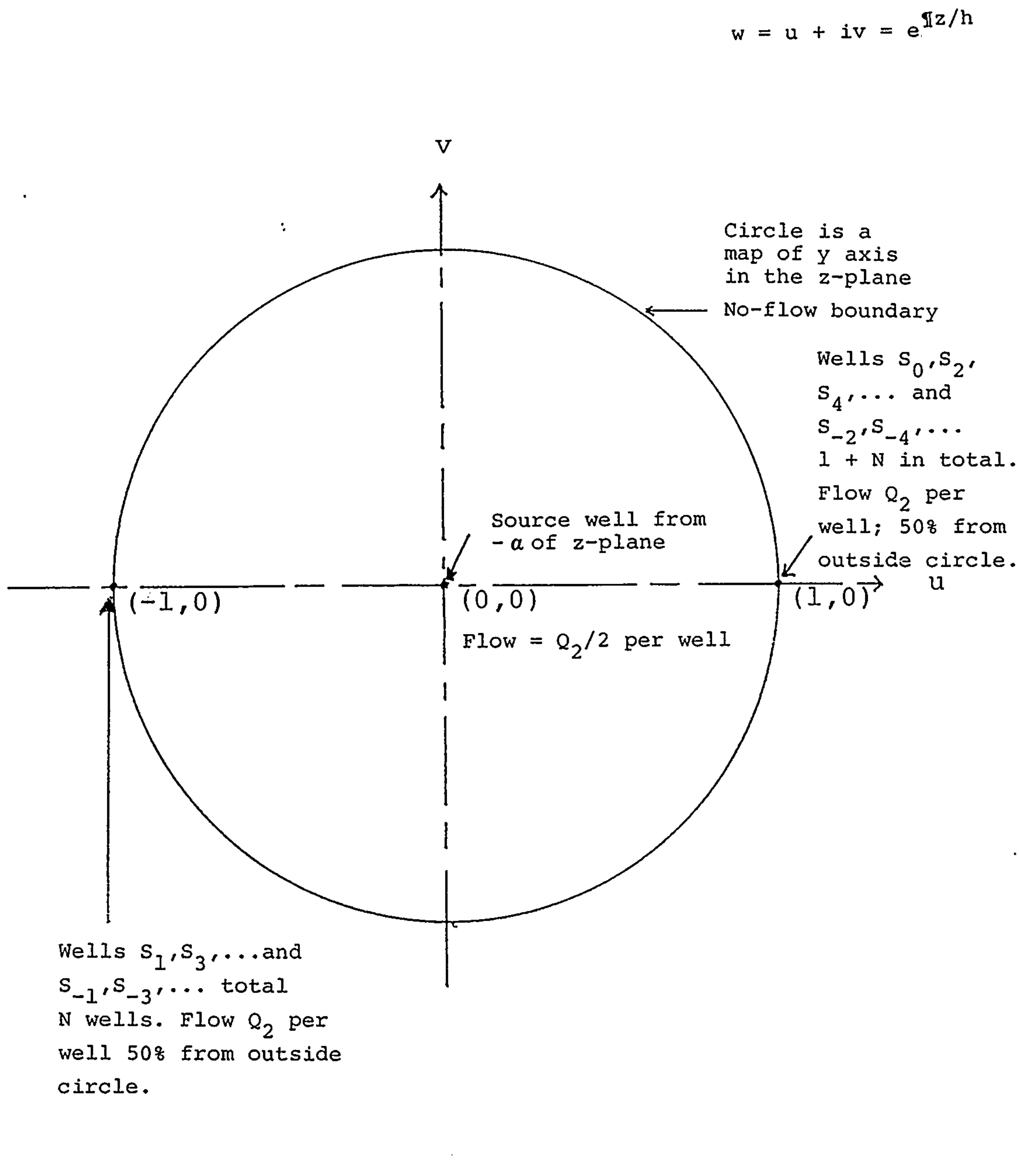


Figure II.3 Horizontal Well Potential Distribution - W-Plane

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