THE UNIVERSITY OF CALGARY

AN ELECTRICAL TRANSMISSION LINE MODEL OF A PIPELINE

by

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A THESIS

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The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies for acceptance, a thesis entitled, ***"An Electrical Transmission Line Model of a Pipeline"*** submitted by Zsigmond M. Pal in partial fulfillment of the requirements for the degree Master of Engineering.

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Abstract

In this thesis an analytical electrical transmission line model for an isolated buried pipeline is developed. Elements of the transmission line model are derived mathematically using both field and circuit theory methods. Propagation along the coated pipeline in a multi-layer earth is studied analytically and modeled with a computer. Performance of the transmission line is analyzed and the results are illustrated graphically. It is shown that electrical methods, such as time domain reflectometry, can provide the pipeline operators with a powerful analytical tool to monitor pipeline coating integrity.

ACKNOWLEDGEMENTS

The author is indebted to Dr.R.H. Johnston for his guidance, assistance and patience throughout the course of this work. The cooperation received from the Department of Electrical Engineering of the University of Calgary and Peace Pipe Line Ltd. as a whole is very much appreciated.

Without the cooperation of the author's wife and children, this work would not have been possible. This thesis is dedicated to them.

DEDICATION

Dedicated to my wife, Palma,
our children, Christian, and Melodie
for their love and understanding

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LIST OF FIGURES

LIST OF SYMBOLS

A, B, C, D	Scalar Coefficient
A_K, B_K, C_K, D_K	Generalized Circuit Constants, Layer No. K.
B	Magnetic Flux Density
C	Capacitance Per Unit Length
D	Distance of Overhead Wire to its Image
D_g	Equivalent Return Distance
E_x, E_y, E_z	Cartesian Component of Electric Field
E_K	Electric Field in Layer No. K.
E_m	Rotating Electric Field Vector Maxima
H_x, H_y, H_z	Cartesian Component of Magnetic Field
H_K	Magnetic Field in Layer No. K.
E_m	Rotating Magnetic Field Vector Maxima
$H(), h()$	Discrete Fourier Transforms
I, i	Current
I_L	Load Current
I_{in}	Input Current
J, J_z	Current Density
J_s	Ground Impedance Correction Term
L	Inductance Per Unit Length
L, l	Length
N	Number of Samples

Q	Lumped Charges on Electrode
R	Resistance Per Unit Length
R_{ac}	Effective a.c. Resistance
R_g	Resistance Correction Term
R_L	Load Resistance
R_Y	Shunt Resistance
T	Sample Interval in Time
V	Voltage
V_L	Load Voltage
V_S	Source Voltage
V_{in}	Input Voltage
X	Effective a.c. Inductance
X_c	Capacitive Reactance
X_g	Reactance Correction Term
$X()$	Frequency Samples of a Function
Z	Impedace
Z_c	Effective Impedance Per Unit Length
Z_L	Load Impedance
Z_S	Source Impedance
Z_s	Self Impedance of the Conductor
Z_0	Characteristic Impedance
Z_{in}	Input Impedance
Z_{oc}	Open Circuit Impedance
Z_{sc}	Short Circuit Impedance
a	Outside Radius of the Pipe

b	Outside Radius of the Pipe Coating
c	Speed of Light
d	Distance, Diameter
f	Frequency
h	Overhead Wire Distance from Ground
j	Imaginary Unit $(-1)^{1/2}$
p	Complex Ground Return Distance
q_L, q_l	Charge Per Unit Length
q	Inside Radius of the Pipe
r	Radius, Distance
s	Distance
t	Time, Wall Thickness
x, y, z	Rectangular Coordinate Variables
$x()$	Time Samples of a Function
Γ	Complex Reflection Coefficient
α	Attenuation Constant
β	Phase Constant
γ	Complex Propagation Constant
δ	Skin Depth
Δ	Infinitesimal Section
Δf	Sample Interval in Frequency
Δ_K	Width of Ground Layer No. K
ϵ_r	Relative Permittivity
ϵ_0	Free-space Permittivity
μ_r	Relative Magnetic Permeability

μ_0	Free-space Magnetic Permeability
ρ	Resistivity
σ	Conductivity
ϕ	Magnetic Flux
ξ	Distance in Ground Layer
ω	Radian Frequency

CHAPTER 1

INTRODUCTION

1.1 GENERAL

In this study an analytical electrical transmission line model for an isolated buried pipeline is developed. Oil and gas pipelines have been employed as electrical transmission lines for transmission of information *[1.1]*. The performance of this transmission channel presently is determined empirically without an overall analysis. A thorough analytical study should be performed on the pipeline as a transmission line to determine the maximum capacity of the channel before the transmission equipment is built and installed.

In addition, my goal is to determine the feasibility of using electrical methods for detection of holidays (holes) and shorts in the pipeline corrosion protection coatings. The approach is to treat the pipeline, shown in *FIG.1.1.*, the pipeline coating and the surrounding earth as an electrical transmission line. This transmission line then can be studied analytically and modeled on a computer.

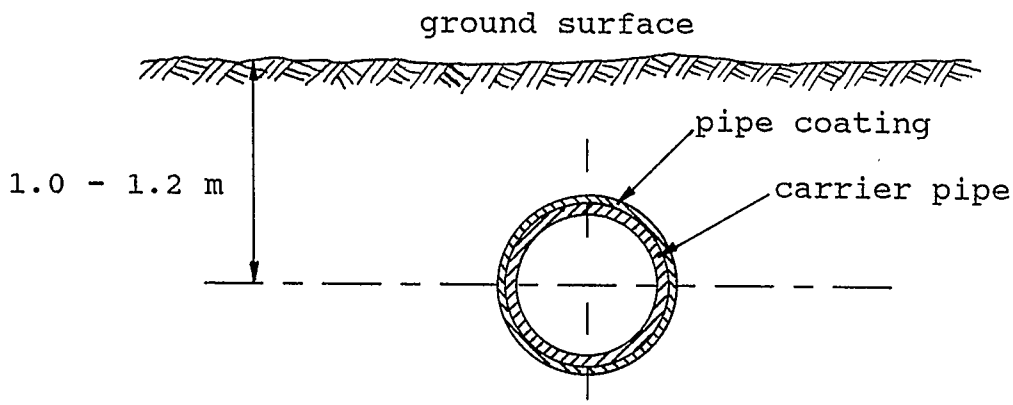


FIG.1.1 UNDERGROUND PIPELINE.

In the modeling and analysis process it will be assumed that pulses of electrical voltage are applied to the pipeline with respect to the surrounding ground. Anomalous reflections of voltage and current back on the pipeline may be indicative of coating faults. Corrosion occurs when a hole in the protective coating allows contact with the surrounding earth *[1.2]*. This electrical contact may cause anomalous electrical behaviour of the transmission line. Time domain reflectometry (TDR) may have application for the detection of these contact points. The magnitude of the reflected pulse may indicate the extent of a coating fault and the line delay may indicate the distance from the voltage injection and measurement point. Present holiday detection schemes are laborious and expensive and often require that the pipeline be taken out of service. Large holidays in the pipeline coating, if left for a long time usually lead to a development of hole in the pipeline which can lead to loss of product and often create a dangerous

or even catastrophic situation, injurious to public safety or the environment. A successful electrical method would provide a continuous method for testing the integrity of pipeline protection system, thereby providing an economical alternative for monitoring the pipeline for early signs of corrosion and leaks. This is a general industrial problem and it is a prime concern to pipeline operators. A study of electrical methods for detecting a small holiday is a worthwhile approach even if, at first, the perceived probability of success is significantly less than unity.

In this study each element of the transmission network, the pipe, the pipeline coating, the surrounding earth, the shorted pipeline casing and ground bed is studied analytically and modeled with the aid of a computer. In this study an IBM PC is used in conjunction with Microsoft Fortran, Microsoft Basic and Microsoft Chart for calculations and graphical representation of the results.

The pipeline is isolated from station to station, other lines and the ground of the existing cathodic protection system as illustrated in *FIG.1.2*. This allows the electrical measurements to be made on individual section of the pipeline.

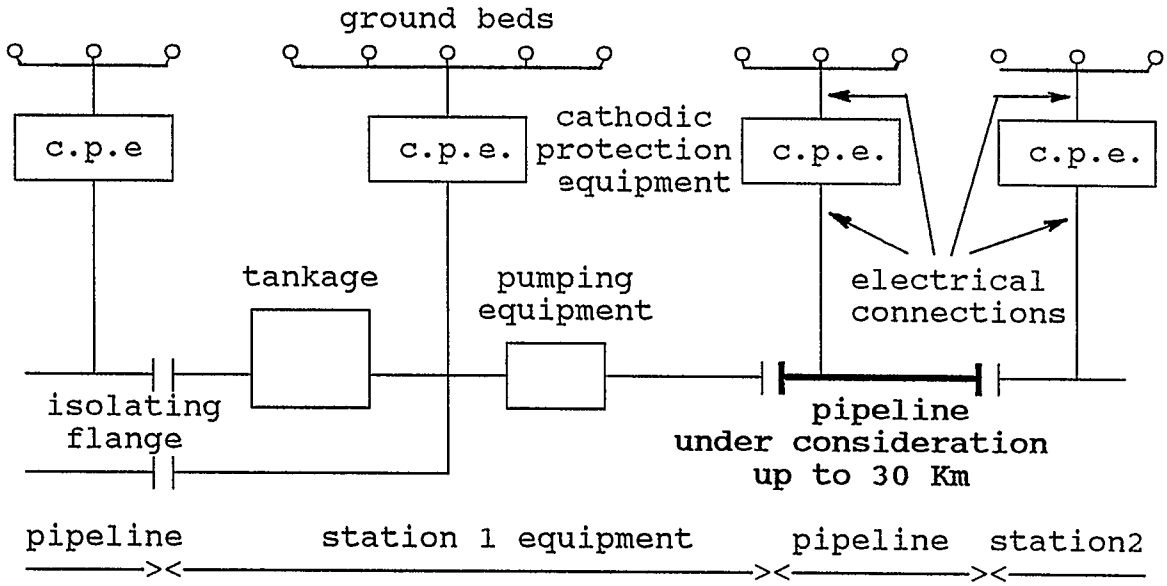


FIG.1.2 TYPICAL SEGMENT OF A PIPELINE SHOWING THE CATHODIC PROTECTION SYSTEM.

This isolation will also assist in the operation of the pipeline as an electrical transmission line. Cathodic protection is required by regulations of several governmental agencies to provide safe operation and to attempt to prevent all accidents caused by the influence of corrosion.

1.2 ANALYTICAL APPROACH

Derivation of exact closed-form analytical solutions of electromagnetic problems dealing with cylindrical current carrying conductors in the presence of earth is a formidable task, even for simple conductor geometries.

Analysis using closed-form algebraic expressions is largely intractable, owing to the large number of complicated expressions encountered. It is possible to simplify the problem considerably by making a number of judicious assumptions. Over the years there have been several papers relevant to the chosen topic in the area of power line grounding. Some of them are based on the circuit theory approach which is derived from Maxwell's equations (or field theory) using some of the following assumptions:

- The conductor is infinitely long.
- The conductor is located in the dielectric medium or is well-insulated.
- The frequency is low. If high frequencies need to be considered, then a lossless conductor is assumed or at most, the distributed lossy circuit approach is used.
- Propagation of current in an electrically long conductor is assumed to follow a decaying exponential law *[1.3, 1.4, 1.5]*.

The circuit theory approach can provide simple and accurate solutions to a number of problems which otherwise are impossible or extremely difficult to solve by field theory based methods only.

Other work that is relevant to this research are field theory based methods which have been developed for the

application of electrical prospecting methods. Electrical prospecting methods *[1.6, 1.7]* generally assume current sources of infinitesimally small spherical dimension such as point sources, electric and magnetic dipoles or perfectly insulated conductor loops located on the surface of soil. Direct current or very low frequency excitation current is often used to achieve good subsurface depth penetration.

CHAPTER 2

ELECTRICAL MODEL OF THE PIPELINE

2.1. GENERAL

The present chapter, deals with the derivation of expressions relating to the distributed circuit of an isolated buried pipeline. It involves methods that are outside the mainstream of conventional transmission line theory. Circuit analysis using the circuit element concepts of lumped resistance, inductance and capacitance, almost invariably omits any reference to ferromagnetic metals as the conducting circuit element. Any transmission line theory that is developed must account for the magnetic properties of the pipeline and the highly distributed nature of the ground return. The problem must then be treated and solved using a method that provides the engineer with a systematic analysis and design tool.

The traditional manner of treating this problem is to solve differential equations with boundary values based on Maxwell's equations. These results are used to produce values of distributed and lumped resistance and conductance. A wide variety of structures, including circular cylinders have been treated already, but the mathematical form of results is reasonably elementary only for direct currents or voltages. When the frequency is

larger than zero, even for the idealized geometries, calculation of resistance and reactance requires transcendental mathematical functions, with the consequence that approximate formulae and graphical representations are widely used. Computer analysis is needed and is applied in this study to obtain adequate solutions to the transmission line problem. The results are presented graphically.

2.2. DISTRIBUTED RESISTANCE AND INTERNAL INDUCTANCE OF THE PIPE

The most widely used transmission line conductors are solid homogeneous wires and tubular conductors of circular cross-section. The analysis that follows, shows that an approximate solution in functional form can be found for the distributed resistance and distributed internal inductance of a homogeneous tubular conductor (ie. the pipe), for all frequencies at which such a conductor can be used as transmission line.

2.2.1 SKIN EFFECT

When alternating current flows in a conductor, the alternating magnetic flux within the conductor induces eddy currents. These eddy currents, cause the main current density to decrease in the interior of the conductor and to

increase at the outer surface. The effect, known as the skin effect, becomes more pronounced the higher the frequency, or the larger the cross section of the conductor, or as in our case, when a ferromagnetic conductor material is used.

2.2.2 HISTORICAL PERSPECTIVE

The problem of determining the resistance and inductance of a round conductor was first discussed by Maxwell in 1873 *[2.1]*. He adapted an infinite series to describe the current density at any point of the cross section by which the effective resistance of the conductor can be computed at low frequencies. The importance of the results given by his formulae was not fully appreciated at that time. In 1884 Oliver Heaviside discussed the "throttling" effect in a core, that is, the increased resistance, the reduced inductance and the tendency for the current to concentrate near the surface. He used two functions M and N in his solution, which Kelvin subsequently called the *ber* and *bei* functions. Oliver Heaviside made a very important contribution by providing the approximate formula for the effective resistance of a hollow cylindrical conductor. He also gave general descriptions of how the current-density varies in the conductors. Three years later (in 1887) Lord Kelvin gave a

practical solution of the effective resistance of a solid conductor. Joseph Thomson also obtained a practical solution for high frequency current distribution. In 1909 A.Russel *[2.1]* obtained the current density variation by forming a differential equation connecting the current and the cross section dimensions of the conductors. In 1918 an alternative method was described by H.B.Dwight *[2.3]*, in which the effect of the magnetic field was presented. Successive increments of current and voltage drop were calculated, to keep the voltage uniform over the cross section. The radius was stepped in small increments which allowed the formation of a convergent series, providing the established result for a round wire and for special cases of tubes and straps. This paper also gives the asymptotic formula for the skin effect of a tube. The precise method of calculating skin effect in an isolated tube is given by Dwight *[2.4]* in 1923, then in 1929 he provides *[2.5]* tables of Bessel functions of zero and higher orders for a.c. problems. In 1934 McLachlan *[2.6]* compiled a book on Bessel functions for engineering applications, which was revised in 1955, providing substantial information about the general nature of these functions.

2.2.3 ANALYTICAL SOLUTION

The quantitative analysis of the skin effect in the pipe starts with the application of Faraday's law and Ampere's law to a solid circular conductor's longitudinal cross section which is illustrated in *FIG.2.1*. This approach is described in *[2.7]*, therefore only some intermediate results and the final formulae will be given here. The overall method presented in this chapter is original to this thesis. The analysis for the insulated buried pipe has been performed by means of a computer simulation in the frequency range of interest.

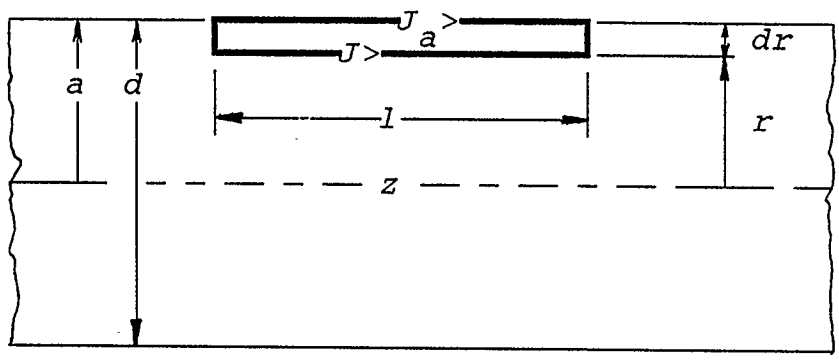


FIG.2.1 LONGITUDINAL CROSS SECTION OF SOLID CIRCULAR CONDUCTOR.

The requirement in this analysis is that an external source causes current to flow in the z direction. The resulting current density J_z at any point in the conductor's cross section is in general a function of r,

and for reasons of symmetry it is not a function of angular position around the center of the conductor. The return current path is many radii away so that the electric and magnetic fields are nearly uniform around the pipe. This is proved to be true for the case under consideration and is illustrated in the next chapter (see *FIG.3.4.a* and *b*).

The purpose of the analysis is to find the manner in which $J_z(r)$ varies with r, and from this result to find the effective resistance and internal inductance of the conductor per unit length as a function of frequency, cross section and pipe material.

At any radius r in the conductor there is an electric field $E_z(r)$ associated with the total current density $J_z(r)$ according to the time-harmonic electromagnetic relation

$$\begin{aligned} curlH &= \sigma E + \epsilon dE/dt \\ &= (\sigma + j\omega\epsilon)E \\ J_z &= \sigma E_z + j\omega\epsilon E_z \end{aligned} \tag{2.1}$$

where σ is the conductivity and ϵ is the permittivity of the conductor. For the metals used in pipelines ϵ is the same as for free space ($\epsilon_0 = 8.85 \cdot 10^{-12}$ *farads/m*). Since the conductivity of the steel pipe is about $4 \cdot 10^6$ *S/m*, it can be seen that for all practical transmission line frequencies the displacement current is negligible and the current density becomes

$$J_z(r) = \sigma E_z = J. \qquad (2.2)$$

To further simplify expressions, the $J_z(r) = J$ notation will be used, where J is time harmonic with a factor $exp(j\omega t)$.

Voltage is induced in the rectangle of *FIG.2.1.* because of the time-changing magnetic flux going through it. This flux is produced by the total conductor current contained within the radius r.

$$\oint E \cdot dl = - d\phi/dt \quad \textit{(Faraday's Law)} \qquad (2.3)$$

where ϕ is the flux linking the path and is given by

$$\phi = l \int_r^a B \, dr \, . \qquad (2.4)$$

Using $E = J/\sigma$, $J = J_m exp(j\omega t)$, $B = B_m exp(j\omega t)$ and $B_m = \mu H_m$, *(2.3)* becomes with the substitution of *(2.4)*

$$1/\sigma \; dJ_m/dr = j\omega\mu H_m \, . \qquad (2.5)$$

The second necessary equation is obtained by applying Ampere's law and writing an equation for the enclosed current around the circular path at radius r

$$2\pi rH = \int_0^r J2\pi r\ dr\ . \qquad \text{(Ampere's Law)} \tag{2.6}$$

Using $J = J_m exp(j\omega t)$, $H = H_m exp(j\omega t)$, (2.6) becomes

$$2\pi rH_m = \int_0^r J_m 2\pi r\ dr\ . \tag{2.7}$$

Taking the derivative with respect to r and rearranging, this becomes

$$dH_m/dr + 1/r\ H_m = J_m\ . \tag{2.8}$$

To eliminate H_m, we solve (2.5) for H_m and substitute the result into (2.8). This yields the differential equation for the current density

$$d^2J_m/dr^2 + 1/r\ dJ_m/dr = j\omega\mu\sigma J_m\ . \tag{2.9}$$

Equation (2.9) is a special form of Bessel's differential equation which is usually written as

$$d^2J/dr^2 + 1/r\ dJ/dr - jm^2J = 0 \tag{2.10}$$

where

$$m^2 = \omega\mu\sigma\ . \tag{2.11}$$

The solution of the Bessel's differential equation which follows is well known for the solid round conductor and the hollow tube. For a tube of inner radius q, equation *(2.10)* has two independent solutions which are known as Bessel functions of order zero of the first and second kind *[2.4, 2.6]*

$$J = (A+jB)(ber(mr)+jbei(mr)) + (C+jD)(ker(mr)+jkei(mr)) \tag{2.12}$$

where A,B,C and D are constants with values presently undetermined.

The quantity $(ber(mr)+jbei(mr))$ is the Bessel function of the first kind, order zero, and the quantity $(ker(mr)+jkei(mr))$ is the bessel function of the second kind. Their values are calculated in a computer program shown in *Appendix B*, from a series described in *Appendix A* and are illustrated in *FIG.2.2.a-d*. Note that in these figures $x = mr$.

To determine the constants $(A + jB)$ and $(C + jD)$, we require two equations. These are obtained using boundary conditions at $r = q$, and $r = a$.

At the inner surface of radius q, there is no *emf* induced by the magnetic field, so $dH/dt = 0$ and from *(2.5)* $1/\sigma \; dJ_m/dr = j\omega\mu H_m = 0$, accordingly from *(2.12)* $dJ/dr = 0$ and therefore

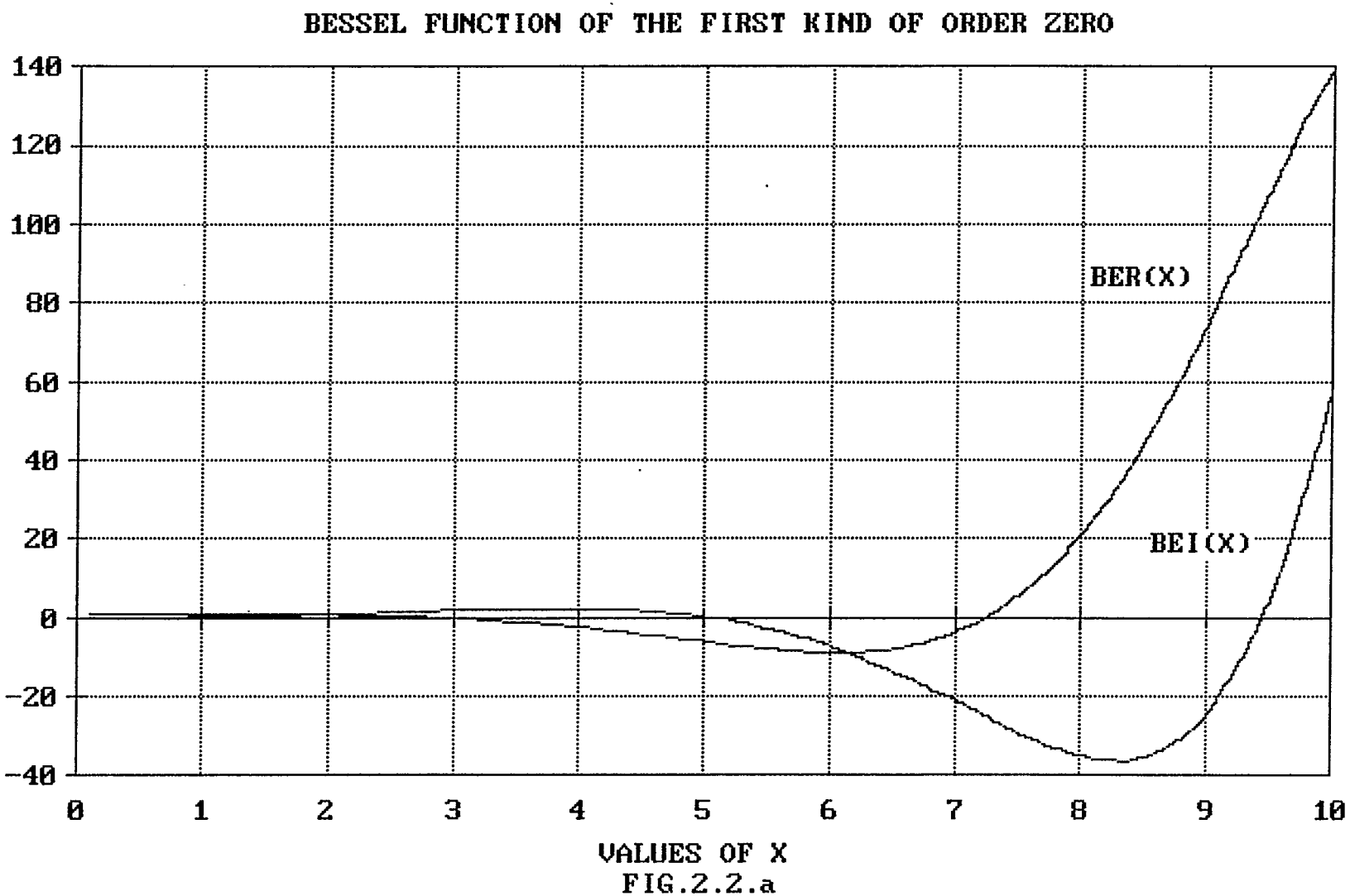

BESSEL FUNCTION OF THE FIRST KIND OF ORDER ZERO
140
120
100
80
60
40
20
0
-20
-40
0
1
2
3
4
5
6
7
8
9
10
BER(X)
BEI(X)
VALUES OF X

FIG.2.2.a

DERIVATIVE OF BESSEL FUNCTION OF THE FIRST KIND OF ORDER ZERO.

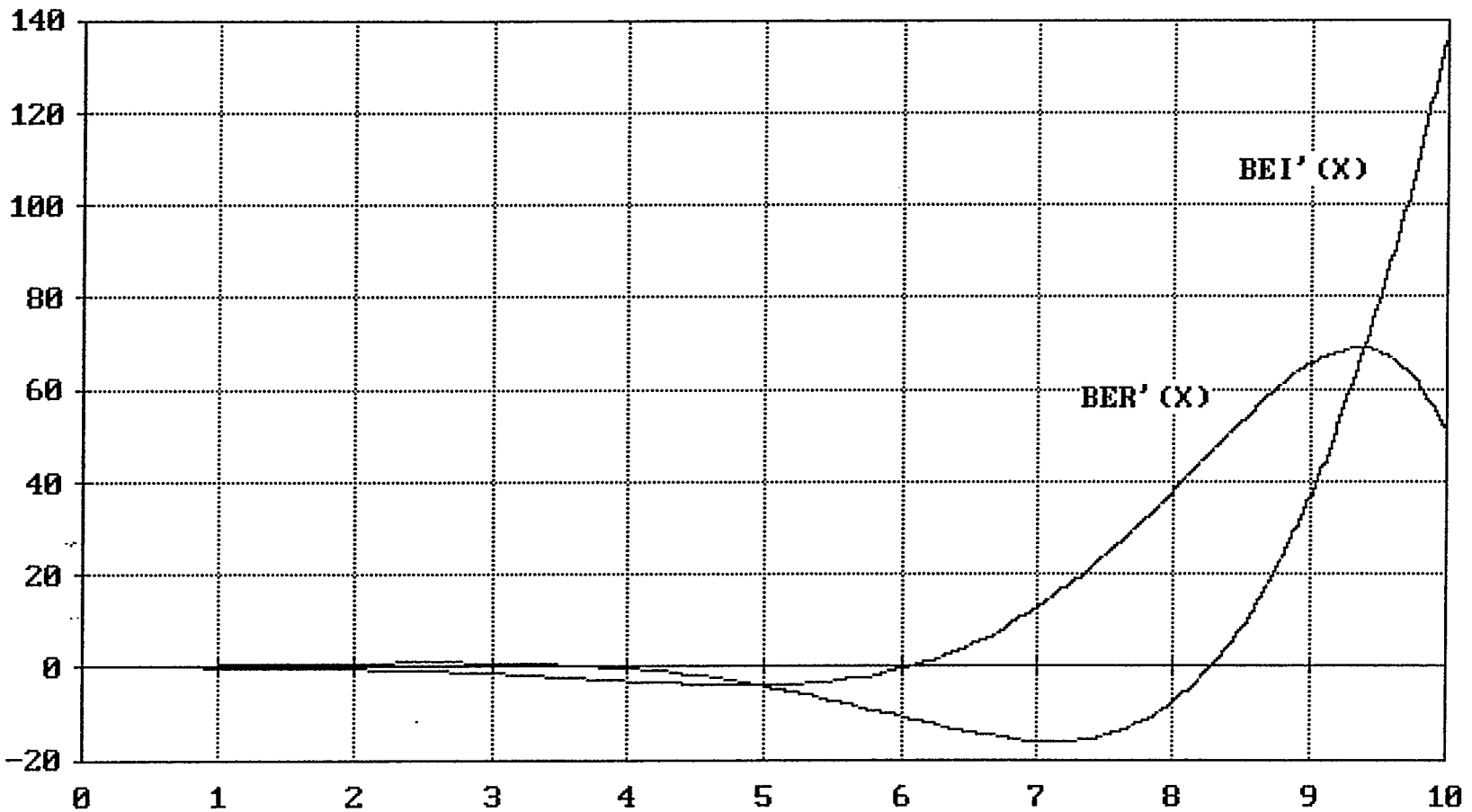


FIG.2.2.b

BESSEL FUNCTION OF THE SECOND KIND OF ORDER ZERO.

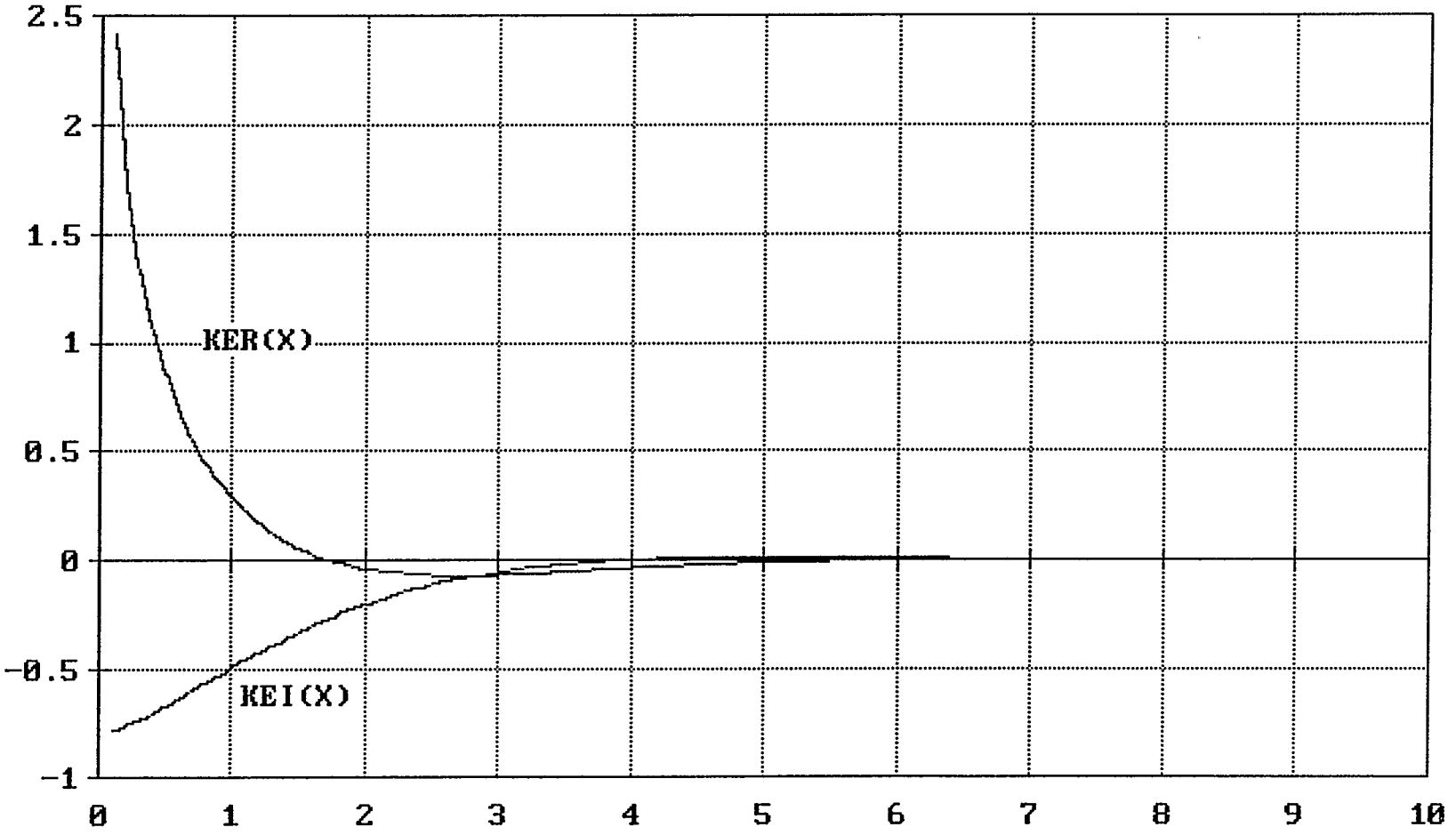
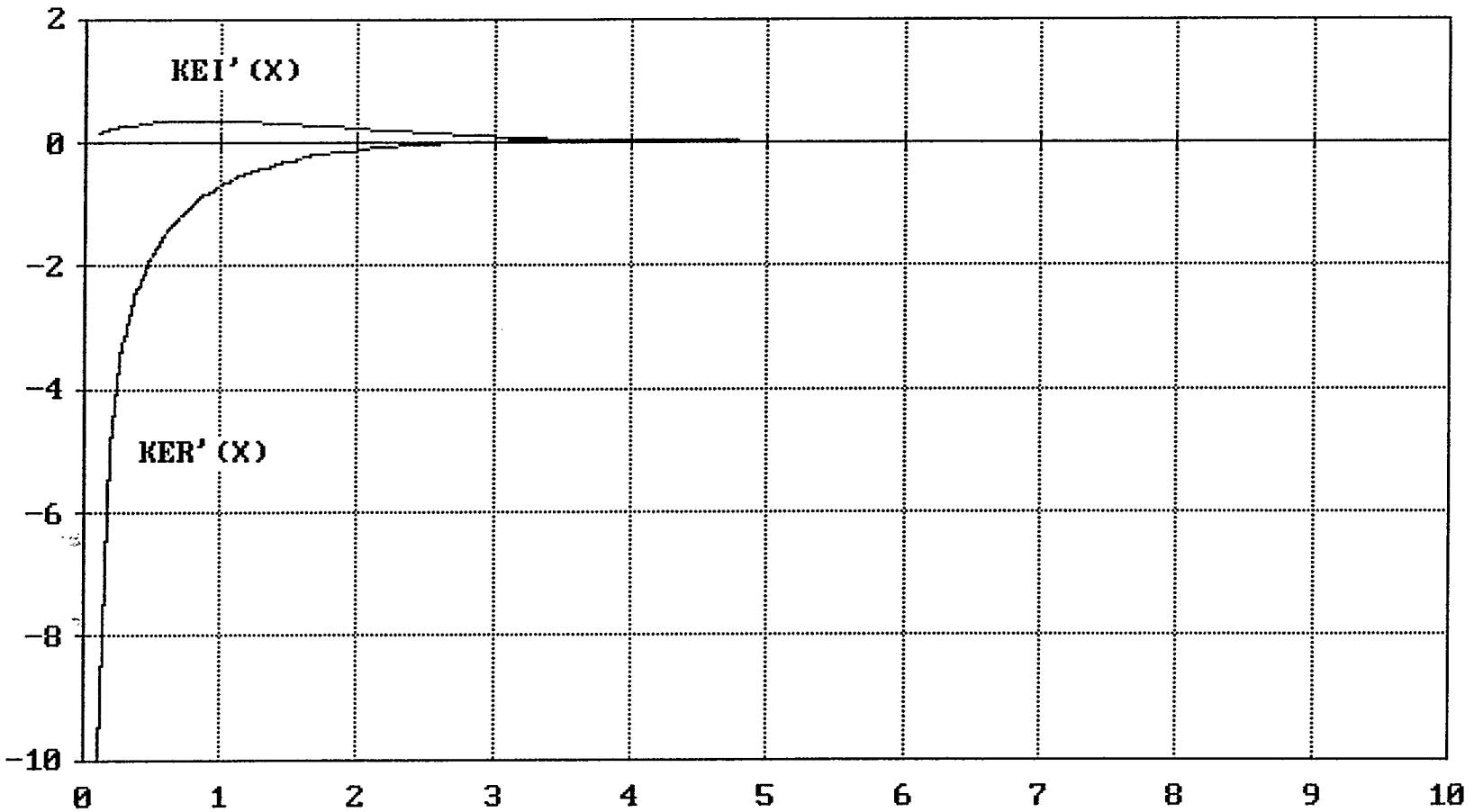


FIG.2.2.c

DERIVATIVE OF BESSEL FUNCTION OF THE SECOND KIND OF ORDER ZERO.



VALUES OF X

FIG.2.2.d

$$(A+jB)(ber'(mq)+jbei'(mq)) + (C+jD)(ker'(mq)+jkei'(mq))=0 \quad . \tag{2.13}$$

Since $ber'(mr) = d\ ber(mr)/d(mr)$, etc.[2.3], therefore

$$(C+jD)/(A+jB) = -(ber'(mq)+jbei'(mq))/(ker'(mq)+jkei'(mq)) \quad . \tag{2.14}$$

These complex quantities are also computed using the attached computer program.

The total current in the pipe is

$$I = \int_q^a 2\pi r J dr \tag{2.15}$$

$$=2\pi \int_q^a [(A+jB)(ber(mr)+jbei(mr))+(C+jD)(ker(mr)+jkei(mr))]rdr$$

Using the following integrals [2.3]

$$\int r\ ber(mr)\ dr = r/m\ \ bei'(mr)$$

$$\int r\ bei(mr)\ dr = -r/m\ \ ber'(mr)$$

$$\int r\ ker(mr)\ dr = r/m\ \ kei'(mr)$$

$$\int r\ kei(mr)\ dr = -r/m\ \ ker'(mr)$$

and rearranging, we get

$$I = (2\pi/m)[r(A+jB)(bei'(mr)-jber'(mr)) + r(C+jD)(kei'(mr)-jker'(mr))]_{r=q}^{r=a} .$$

The quantity in the square brackets is equal to 0 when $r = q$, as per (2.13), therefore

$$I = (2\pi a/m)[(A+jB)(bei'(ma)-jber'(ma)) + (C+jD)(kei'(ma)-jker'(ma))] . \qquad (2.16)$$

Let Z_c be the effective impedance per unit length of the pipe at a certain frequency due to its effective a.c. resistance R_{ac} and its inductance X caused by the flux inside the metal. The voltage at the surface of the pipe is found from the surface conductivity and the surface current density *(2.12)*

$$v = IZ_c = (1/\sigma)J\Big|_{r=a}$$

$$=(1/\sigma)[(A+jB)(ber(ma)+jbei(ma))+(C+jD)(ker(ma)+jkei(ma))] . \qquad (2.17)$$

The resistance of the pipe to direct current is

$$R_{dc} = 1/(\sigma\pi(a^2-q^2)) . \qquad (2.18)$$

Therefore

$$IR_{dc} = 2a/(m\sigma(a^2-q^2)) \cdot$$

$$\left\{(A+jB)(bei'(ma)-jber'(ma))+(C+jD)(kei'(ma)-jker'(ma))\right\} \tag{2.19}$$

From *(2.17) and (2.19)*,

$$Z_c/R_{dc} = [jm(a^2-q^2)/2a] \cdot$$

$$\left\{\frac{(ber(ma)+jbei(ma))+(C+jD)/(A+jB)(ker(ma)+jkei(ma))}{(ber'(ma)+jbei'(ma))+(C+jD)/(A+jB)(ker'(ma)+jkei'(ma))}\right\} \tag{2.20}$$

The above equation gives the ratio Z_c/R_{dc} as a complex quantity. The real part is equal to R_{ac}/R_{dc}, the skin effect resistance of the pipe, and the imaginary part gives the internal inductance of the pipe as a function of frequency, wall thickness and pipe material. The values of the terms in the Bessel series become very large causing numerical overflow, at about *20 Hz* for a *20"* pipe. Therefore an approximate exponential formula *[2.5]* is used instead of the precise calculation, for arguments larger than *10*. This causes no noticeable discrepency in the final values. For example *ber(x)* function gives the following two values:

calculated by series $ber(10) = 138.840$

calculated by exponential approximation $ber(10) = 138.852$,

where the difference is less than that noticeable in a graphical representation. Both the precise and the exponential approximation formulae are given in *Appendix A*

from reference *[2.5]*.

FIG.2.3.a,b are provided to illustrate the skin effect of various pipe thicknesses, as R_{ac}/R_{dc} is plotted against mt or the square root of the frequency. m is defined by equation *(2.11)* and t is the pipe wall thickness $t = a - q$. The pipe permeability is assumed to be constant, $\mu_r = 500$. The curve of R_{ac}/R_{dc} becomes approximately a straight line for high frequencies, indicating that the asymptotic solution given by Dwight *[2.3]* can be applied for values of mt greater than 4.

The asymptotic formula given below in expression *(2.21)* is given by Dwight *[2.3]* for the skin effect, which can be derived from expressions for the ber and ker functions given by Savidge *[2.2]* and is

$$R_{ac}/R_{dc} = [mt(q + a)/(2a\sqrt{2})]\ [1 + 1/(ma\sqrt{2}) + 3/(8m^2a^2)] \qquad (2.21)$$

This formula gives the curves for $mt > 4$ in *FIG.2.3.a,b*. It may be observed that the term $3/(8m^2a^2)$ in *(2.21)* is positive, therefore the curves given by *(2.20)* lie a little above their asymptotes when mt is greater than 4.

Two cases have been calculated to illustrate the skin effect on both the resistance and reactance ratios, for *20"* and *8"* pipeline, and the results are shown in *FIG.2.4.a-b*. The results for the resistance ratios only are shown in *FIG.2.4.a* separately, to provide a simpler picture.

SKIN EFFECT IN TUBES OF VARIOUS THICKNESSES
WITH ASYMPTOTES.

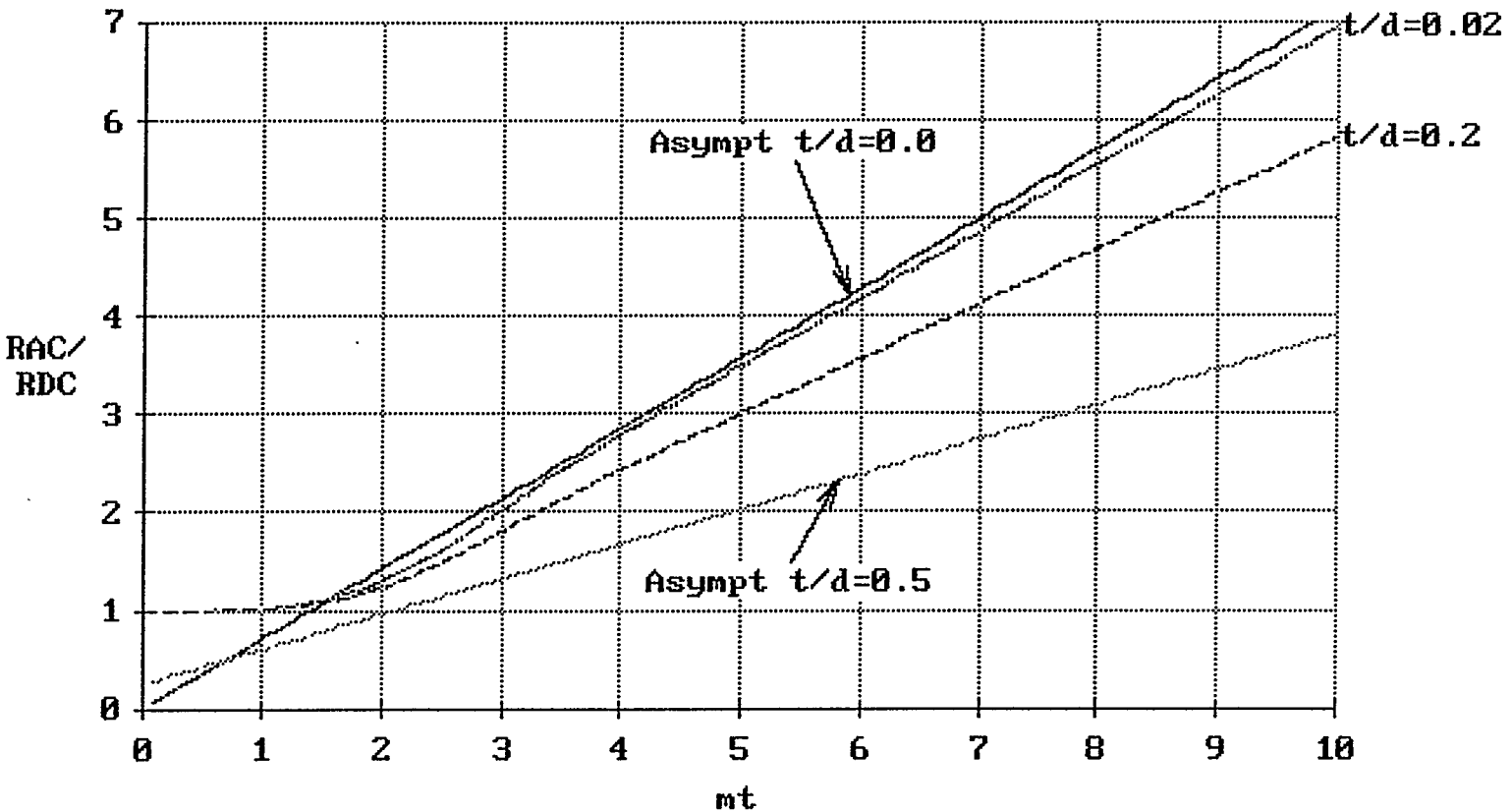


FIG.2.3.a

SKIN EFFECT IN TUBES OF VARIOUS THICKNESSES.

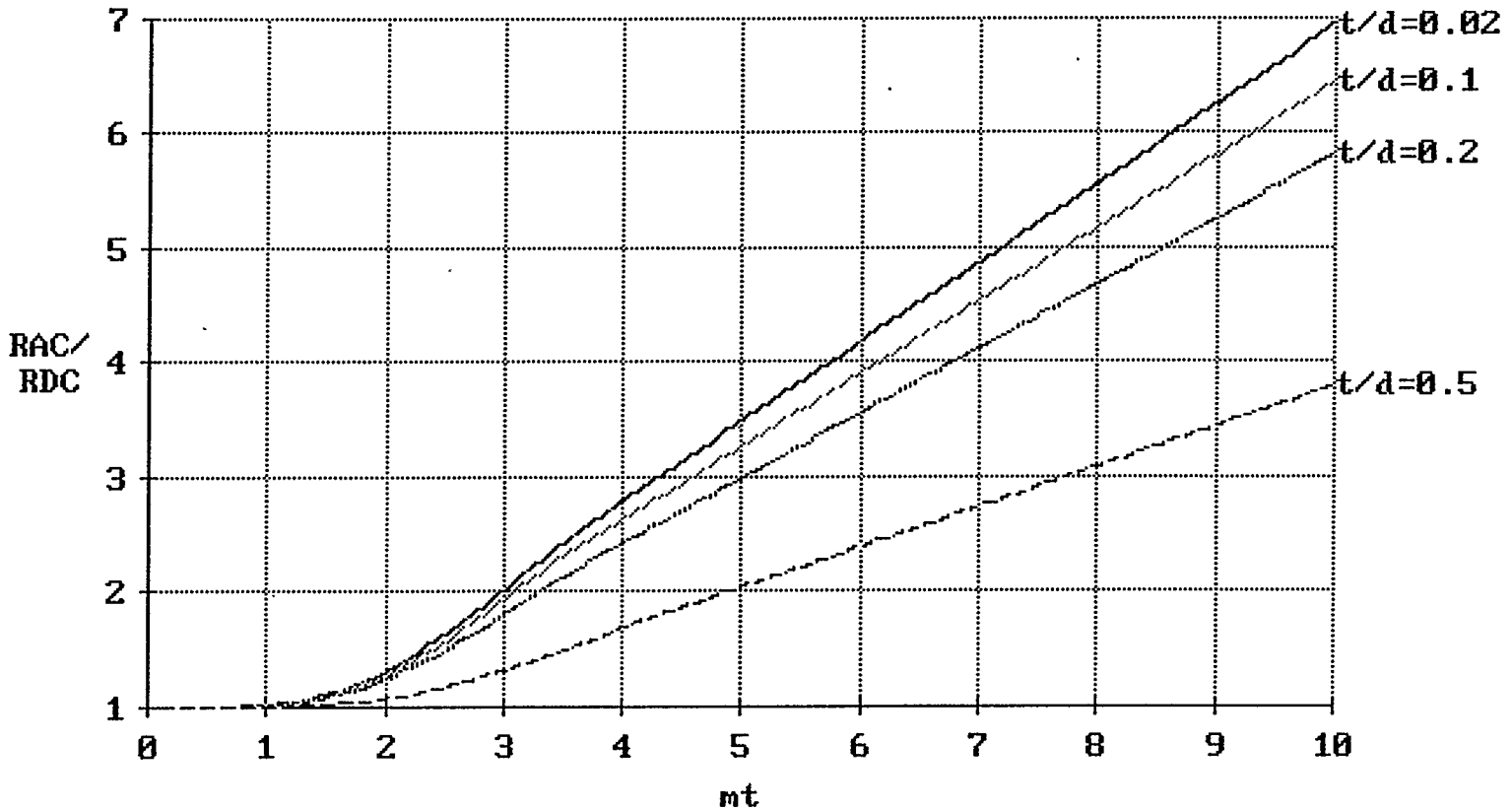


FIG.2.3.b

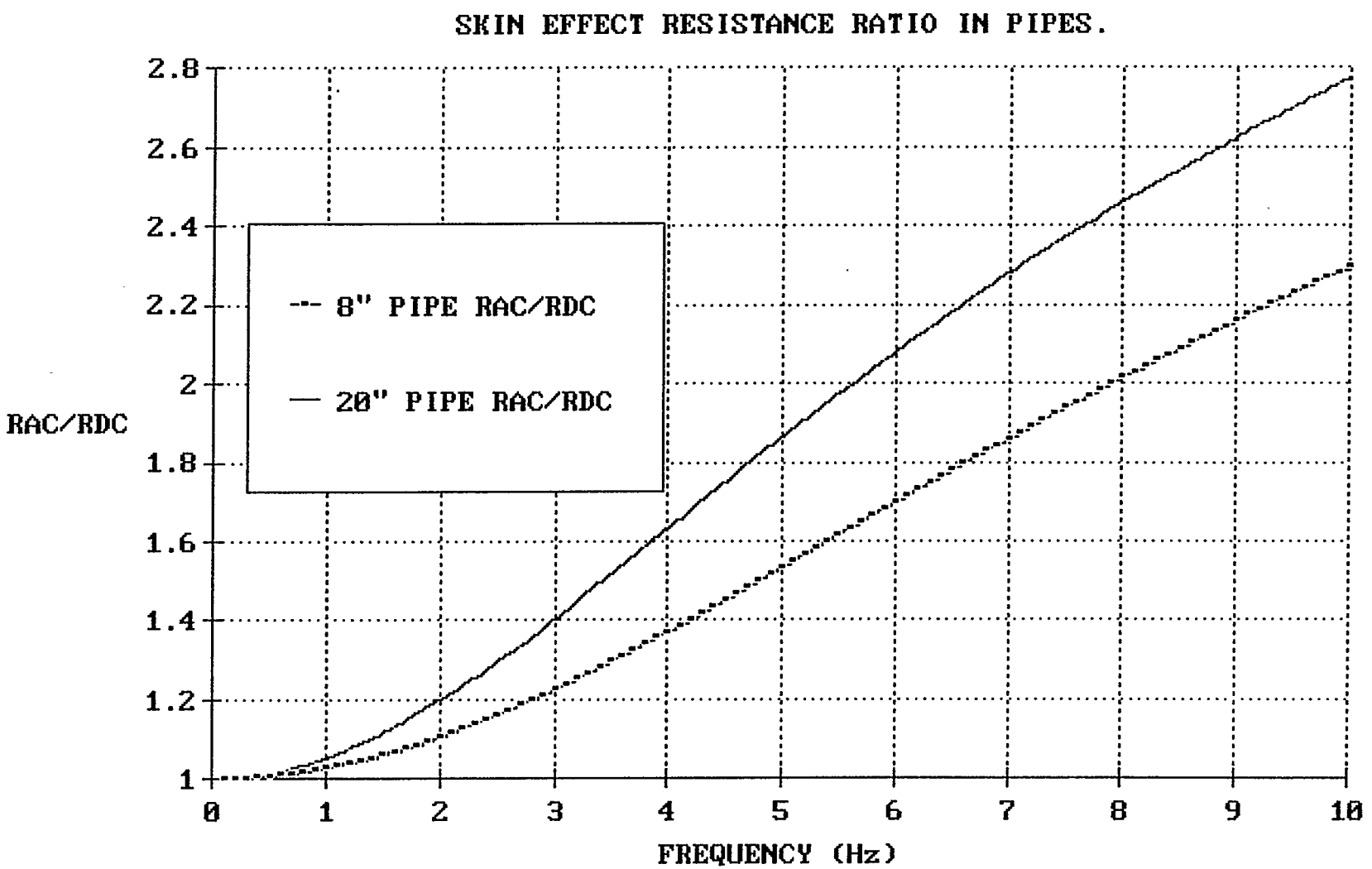

SKIN EFFECT RESISTANCE RATIO IN PIPES.
RAC/RDC
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
1
2
3
4
5
6
7
8
9
10
FREQUENCY (Hz)
8" PIPE RAC/RDC
20" PIPE RAC/RDC

FIG.2.4.a

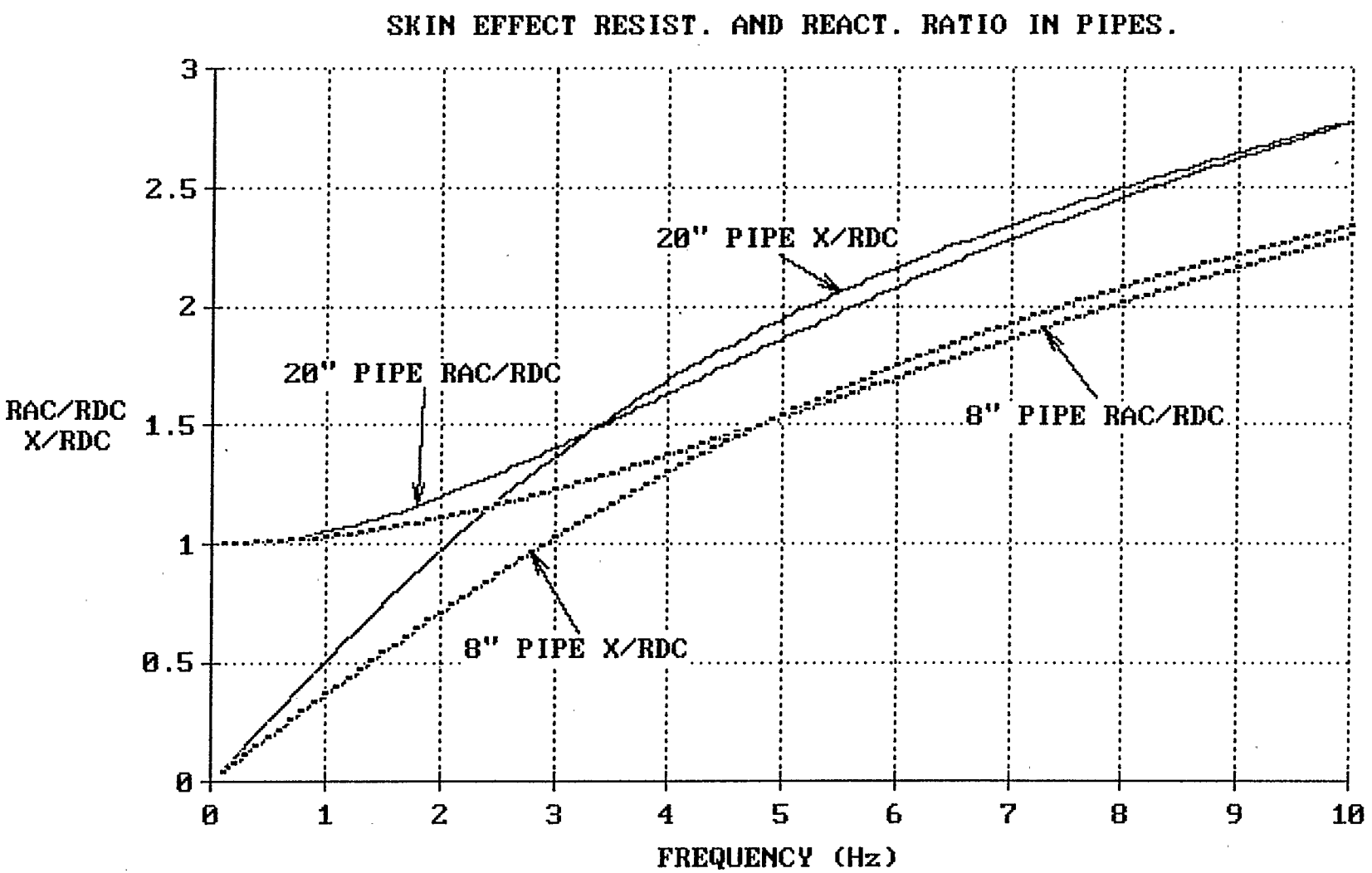

SKIN EFFECT RESIST. AND REACT. RATIO IN PIPES.
RAC/RDC
X/RDC
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
20" PIPE X/RDC
20" PIPE RAC/RDC
8" PIPE RAC/RDC
8" PIPE X/RDC
FREQUENCY (Hz)

FIG.2.4.b

2.3 DISTRIBUTED CAPACITANCE OF THE PIPE

The derivation of an expression for the distributed capacitance of an ordinary transmission line is usually a simple process, because this quantity is a function of the insulating material physical dimensions and dielectric constant only. However, in our case the physical dimensions are not necessarily fixed; distance between the outer surface of pipe coating and the hypothetical bounding surface of the ground return is a function of frequency. The skin depth will be taken to be the equivalent ground return distance D_g as shown by Wait *[2.8]*. Derivation for the equivalent return distance D_g is provided in the next chapter.

The ground return current gives rise to inductive and capacitive effects. These effects are correctly predicted if the return ground current is considered to flow at the bounding surface at radius D_g. Interpretation of this value can be best visualized with the aid of *FIG.2.5*. Capacitance distributed around the pipe is connected at D_g equivalent return distance to a hypothetical bounding surface, that is coaxial with the insulated pipe, by R_Y.

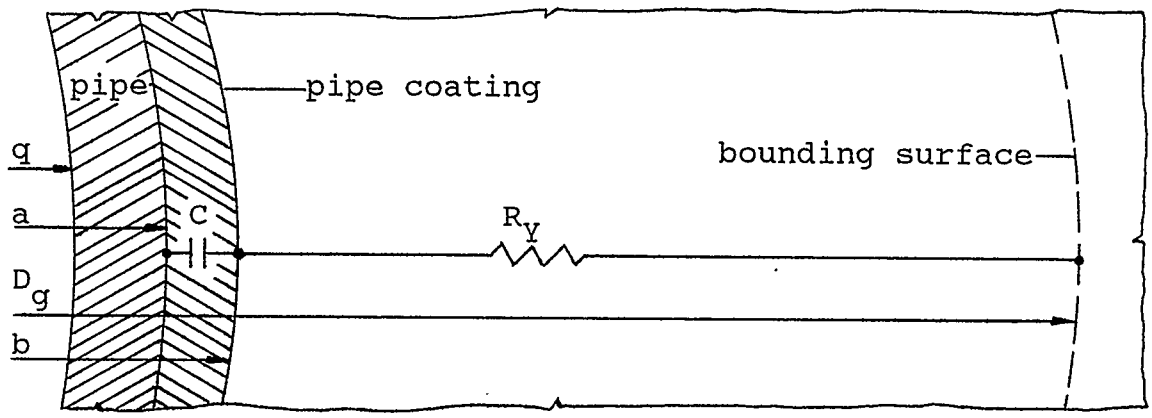


FIG.2.5 INSULATED PIPE AND GROUND RETURN.

The resistance value of R_Y is determined by the following formula

$$R_Y = V/I_Y = \int_b^{D_g} \frac{1}{2\pi r\sigma}\, dr = \frac{1}{2\pi\sigma} \ln(D_g/b) \qquad (2.25)$$

where I_Y is the shunt current per unit length

σ is the the ground conductivity .

Assuming $\sigma = 0.01\ S/m$ and $D_g \simeq 300\ m$ at $100\ Hz$, equation (2.25) gives $R_Y = 113\ Ohm$.

The capacitive reactance X_C is calculated by

$$X_C = (1/j\omega C) \qquad (2.26)$$

where the capacitance C between the two conductor elements, pipe and earth, is the ratio of the magnitude of the equal

and opposite charges on them to the potential difference produced by the charges.

$$\Delta C = \Delta Q/(V_b - V_a) \qquad (2.27)$$

and

$$V_b - V_a = \int_a^b -E_r(r)\,dr = \int_a^b \frac{q_l\,dr}{2\pi\epsilon_0\epsilon_r} = \frac{q_l}{2\pi\epsilon_0\epsilon_r}\ln(b/a)$$

where ϵ_0 is the permittivity of free space

ϵ_r is the permittivity of the coating

q_l is the longitudinal charge density, $q_l = \Delta Q/\Delta l$

The distributed capacitance of the pipe is

$$C = \Delta C/\Delta l = (\Delta Q/\Delta l)/(V_b - V_a) = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)} \qquad (2.28)$$

Using *(2.26)*, for a *20"* pipe and *100 Hz*, $X_C = 60$ *KOhm*. From here and from equation *(2.25)* we see that R_Y can be neglected for all practical earth resistivities and transmission line frequencies. Equation *(2.28)* is integrated into the final transmission model in Chapter 5 and calculated in the attached computer program in *Appendix B*.

It should be noted that identical results to *(2.28)* can be derived using a field theory approach, for insulated underground conductor as shown by Wait *[2.8]*. However, the derivation by the above method is much simpler.

CHAPTER 3

ELECTRICAL MODEL OF THE GROUND RETURN

3.1 GENERAL

The earth has been used as a conductor for electrical currents by early electrical power and communication systems. However, after a brief period of preference for sending return currents through the ground, great difficulties and hazards were found in all branches of electrical engineering *[3.1]*. The point which was overlooked was that a means had to be provided to pass current into and out of the earth, and that these means, in the form of plates, rods or pipes, have a definite resistance *[3.2]*. By the end of the 19th century, experience and analysis had indicated that long telephone circuits worked best when constructed of two copper wires, mounted as spaced open-wire pole lines, and the use of the ground-return was abandoned because of its susceptibility to crosstalk and to inductive interference from power lines. It also became obvious in power transmission that the earth should be used for fixing the neutral point of the electrical system as a return conductor, and that the earth resistance has to be included as part of the circuit *[3.3]*. Later applications for calculation of transients on power lines required further examination of the grounding.

3.2 IMPEDANCE OF THE GROUND RETURN

When alternating current flows in a pipeline, the return current in the ground does not take the shortest path between grounding points at the ends of the line, and it does not spread out to great distances from the pipe as cathodic protection current does. Instead, it follows the route of the pipeline, no matter how circuitous that may be, and it spreads out sideward and downward only to a limited extent, to avoid any large open loops. This behavior of the ground current can be explained by saying that the current density is greatest in the paths of lowest impedance and that the reactance, which is the predominant component of impedance, is lowest for those paths nearest to the pipe. This is a typical example of skin and proximity effects.

3.3 CARSON'S APPROACH

The importance of ground return parameters of transmission lines has been recognized in telecommunications theory and is explained by J.R.Carson's fundamental paper *[3.4]*, giving expressions for the line return parameters with ground return for over-head transmission lines. This is the most widely known and

accepted approach since 1926. He made the following assumptions:

- The ground resistivity is finite and uniform.
- The ground surface lies in a horizontal plane (parallel to the conductor or conductors).
- The frequency is low enough so that the capacitive displacement currents in the ground are negligible.
- The ground currents are parallel to the conductors.

The last assumption is equivalent to saying that end effects are negligible.

The calculation of line impedances according to Carson is based on an equation which contains an infinite integral with a complex argument which accounts for the effect of the finite conductivity of the ground. Carson proposed a solution for the integral (evaluated by R.M.Foster) which consisted of an infinite series. He also gave approximations for low and high frequencies.

The derivation will not be given here, but the resulting approximate formulae for uniform-earth and low frequency applications are equivalent to those given below.

Carson's original formula for self impedance with an earth return is

$$Z_s = Z_c + (j\omega\mu_0/(2\pi))\ln(4h/d) + 4\omega(P + jQ)$$

where:

$Z_c = r_c + jx_i$ = conductor internal impedance, from equation *(2.20)*

d = diameter of the conductor

h = conductor distance above ground

P and Q are correction terms, which account for the finite conductivity of the ground and are functions of the infinite integral

$$P + jQ = \int_0^\infty \frac{je^{-2h\lambda}}{\lambda + \sqrt{(\lambda^2 + j\omega\mu\sigma)}}\, d\lambda \ .$$

To evaluate P and Q, one calculates a quantity called the equivalent depth of ground return *[3.5, 3.6]*, defined as

$$D_g = 660(1/(\sigma f))^{1/2} \text{ meters}$$

where σ is the ground conductivity in S/m, and f is the frequency in *Hertz*. Comparison of the above equation with the skin depth or the equivalent depth of penetration reveals that $D_g = 1.31\delta$, where δ is the skin depth. The reactance component corresponding to D_g is

$$X_g/l = 2.91 \cdot 10^{-3} \cdot f \cdot \ln(D_g) \ \ ohms/Km \qquad (3.1)$$

where X_g/l is the ground component of reactance in *ohms/km*, and D_g is expressed in meters, the same units that are used for the other components of reactance. Since the common values of ground resistivity are between *10* and *1000 meter-ohms*, with about *100 meter-ohms* as a median value, at *f=100 Hz* D_g is commonly between *300* and *3,000* meters, with the median value of about *900 meters* , and X_g/l at *f=100 Hz* ranges from about *1.0* to *1.2 ohms/Km*.

The approximate formula of self-impedance of a single over-head wire for low frequency uniform-earth application is given by Clarke *[3.5]*

$$Z/l = R/l + R_g/l + dR_g/l + j(X + X_g + dX_g)/l \qquad (3.2)$$

where : R/l is the resistance of the conductor

$R_g/l = 10^{-3} \cdot f$ is the resistance of the ground return path in *ohms/Km*

X/l is the conductor component of reactance

dR_g/l is the earth resistance correction term

dX_g/l is the earth reactance correction term

$$dR_g/l = 10^{-3} \cdot 4.04f[-P/(3\sqrt{2}) + P^2(0.6728 + \ln(2/P))/16 + \ldots]$$
$$dX_g/l = 10^{-3} \cdot 4.04f[P/(3\sqrt{2}) - \pi P^2/64 + \ldots]$$

where P is a dimensionless parameter and defined by the following formula

$$P = D\sigma\sqrt{2} = 1.85D/D_g = 10^{-3} \cdot 8.565(\sigma f)^{1/2}$$

where D is the distance of the over-head wire to its image. In the case at hand it is the distance of the pipeline to its image.

There are some features of the low frequency approximate formula *(3.2)* which call for comment.

First, the self-impedance is essentially independent of the height of the wire or wires above ground, because the height is negligible compared with the equivalent depth of ground return. Therefore, the formula is applicable to underground conductors, in our case to a pipeline, whose height is negative.

Second, the ground resistance is independent, not counting the dR_g term, of the ground resistivity. This is explained by the fact that as the resistivity is increased the effective cross-sectional area in which the ground current flows, increases in direct proportion to the resistivity. The skin effect causes the skin depth to vary inversely as the square root of conductivity and frequency

$$\delta = (1/(\pi\sigma f\mu))^{1/2}$$

and the resistance therefore varies as

$$R_g/l = 1/(\pi\sigma\delta^2) = f$$

as given by the approximate formula of *(3.2)*. *FIG.3.1* illustrates that at *100 Hz* the approximate value of the ground resistance is *0.1 Ohm/Km* for an earth of *100 Ohm-m* resistivity. The correction term makes this value decrease with decreasing resistivity and with decreasing depth of the pipeline. The reactance increases with the same factor but to a lesser extent.

The short comings of this relatively simple asymptotic approximation are:

- A multi-layer earth is not considered.
- It is valid for a limited range of frequencies only and medium frequencies are not covered.

3.4 A CLOSED-FORM SOLUTION

Recently, a closed-form solution for the self-impedance of a conductor buried in the uniform earth has been presented by Tylavsky et al. *[3.7]*. This recent paper uses the analogy of another closed-form approximation by Deri et al. *[3.8]* intended for overhead conductors.

It should be mentioned here that many contributions in the area of developing synthetic seismograms in exploration

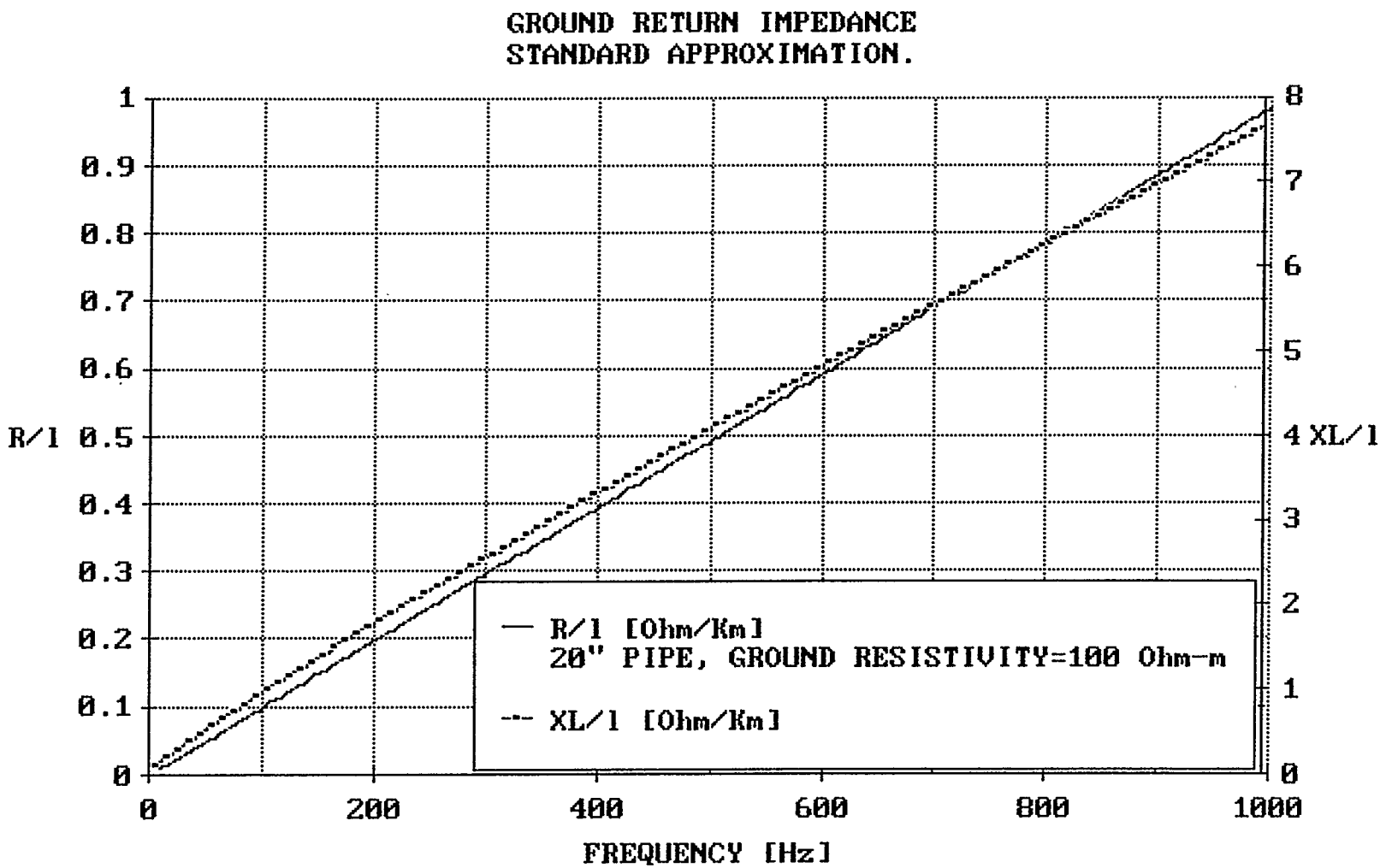

GROUND RETURN IMPEDANCE
STANDARD APPROXIMATION.
R/l
XL/l
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8
7
6
5
4
3
2
1
0
0
200
400
600
800
1000
FREQUENCY [Hz]
R/l [Ohm/Km]
20" PIPE, GROUND RESISTIVITY=100 Ohm-m
XL/l [Ohm/Km]

FIG.3.1

geophysics, like Wuenschel *[3.9]*, utilize formulations similar to those developed in *[3.8]*. Synthetic seismograms that include multiple reflections, transmission coefficients are obtained by solving the boundary value problem for the multi-layered half-space, where the reflected signal depends on the vertical distribution of acoustic impedance. The similarity is expected since acoustic wave propagation is modelled in the plane-wave case by equations analogous to that of an *EM* wave propagation into a multi-layer earth.

Some parts of reference *[3.8]* will be examined in detail with corrections (numerous typographical errors occur in the equations of reference *3.8)*. The following analytical development models the current return profile in a homogeneous and multi-layer ground.

Deri et al. introduce the concept of current flow in an ideal return plane, which is placed below the ground surface at a complex distance p equal to the complex penetration depth for plane waves. They demonstrate that simple and sufficiently accurate expressions for line impedances, the Dubanton equations, are valid for the whole range of frequencies. Deri et al. *[3.8]* stated that the Dubanton equations were probably obtained by intuitive insight and that a proof of the method was still required. This proof is presented by Deri et al. *[3.8]*, and is then extended for the modelling of a multi-layer ground return.

The rewritten Carson equation for self impedance of a conductor is :

$$Z_s = (j\omega\mu_0/(2\pi))\ln(2h/r) + (\omega\mu/\pi)J_s \quad , \mu = \mu_0 \qquad (3.3)$$

where the ground correction term J_s is the infinite integral:

$$J_s = P_s + jQ_s = \int_0^\infty \frac{je^{-2h\lambda}}{\lambda + \sqrt{(\lambda^2 + j\omega\mu\sigma)}}\, d\lambda \qquad (3.4)$$

and where: h is the conductor height above ground.

r is the conductor radius.

σ is the earth conductivity .

The equivalent Dubanton equation is:

$$Z_s = (j\omega\mu_0/(2\pi))\ln(2(h+p)/r) \qquad (3.5)$$

where p is the complex penetration depth. This equation provides an accurate approximation to Carson's equation *(3.3)*, when the following concepts are introduced:

- Equivalent return distance.
- Complex depth.
- Complex ground return.

The equivalent return distance is defined with the aid of *FIG.3.2*, and the following assumptions are made:

- The conductor is assumed to be planar (and not round for this analysis) at height h above ground. The final results apply equally well for round conductor.
- The magnetic field is parallel to the ground.
- The current is time harmonic with a factor $exp(j\omega t)$ where ω is the angular frequency.

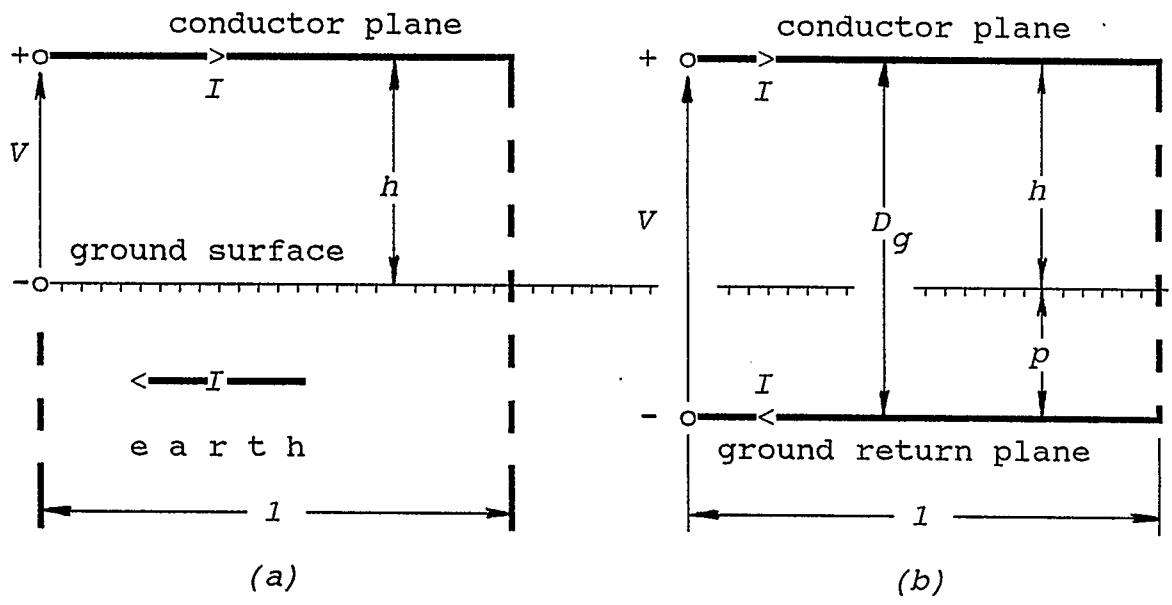


FIG.3.2 EQUIVALENT GROUND RETURN.

The impedance of the ground return loop is $Z = V/I$, as show in *FIG.3.2a*. In *FIG.3.2b* the earth is replaced by a perfectly conducting plane at distance D_g from the plane conductor. The equivalent return distance is defined by the requirement that the impedance Z of the conductor/return loop of *FIG.3.2a* be the same as of the wave impedance in *FIG.3.2b*.

Let D_g be the equivalent return distance. It turns out to be a complex length since the inductance of the loop has to account for the resistance of the earth return. Therefore $p = D_g - h$ represents the complex depth of the ground return plane below the earth surface. Because p is complex, this plane is called the complex ground return plane. The complex plane replaces the actual ground return path without loss of accuracy. Moghram et al. *[3.10]* derive this same complex depth and prove that this concept applies equally well for round conductors.

3.4.1 CALCULATION OF COMPLEX DEPTH

The following basic steps will be taken in this analytical process:

- In order to calculate the wave impedance Z, it is necessary to find the flux ϕ due to the current I.
- The current I is given, therefore we know the magnetic field H_0 at the surface.
- At infinite depth, where there is no backward wave, we calculate the magnetic field H in relationship to the surface magnetic field H_0.
- We find ϕ by integrating H for each resistivity layer and then by summation we calculate the total flux $\Sigma\phi$ from the earth surface to infinity.
- The related electric field E for each layer is found

by *ABCD* complex matrix multiplication, that is, by modeling the earth layers as transmission lines connected in cascade.

Consider in *FIG.3.2a* the closed loop formed by the source, conductor plane, descending wire, a straight horizontal plane in the earth at any depth, and an ascending wire. For this loop:

$$\oint E \cdot dl = -j\omega\phi \qquad (3.6)$$

where ϕ is the flux contained by the loop. The line integral in *(3.6)* is

$$\oint E \cdot dl = -V + Z_C I + J/\sigma \qquad (3.7)$$

where Z_C is the impedance per unit length of the conductor, and J is the current density at the depth of the return plane. From *(3.6)* and *(3.7)*

$$V = Z_C I + J/\sigma + j\omega\phi \quad . \qquad (3.8)$$

By setting the conductor impedance to zero $Z_C = 0$, thus leaving only the ground impedance, we have

$$V = J/\sigma + j\omega\phi \quad . \qquad (3.9)$$

The breakdown of V into its two components in *(3.9)* is arbitrary since the return depth was selected arbitrarily. It is therefore convenient to consider the return depth, where $J = 0$, i.e. infinite depth. Then from *(3.9)* :

$$V = j\omega\phi \qquad (3.10)$$

which is valid for a return plane at infinite depth. Equation *(3.10)* shows that the impedance of the loop is:

$$Z = V/I = j\omega\phi/I \ . \qquad (3.11)$$

The calculation of Z requires that the flux ϕ, produced by I, be determined.

By letting $h = 0$, we obtain the complex depth p as the distance between the two planes of *FIG.3.2b*, for which the flux between the planes equals the total flux in the earth, from its surface to infinity.

By denoting the current per unit width as I we have:

$$H_0 = H(0) = I \qquad in\ A/m \qquad (3.12)$$

where $H(x)$ is the magnetic field intensity at depth x. The related Maxwell's differential equations are *[3.11]*:

$$dE/dx = -j\omega\mu_0 H \qquad (3.13.a)$$

$$dH/dx = -\sigma E \qquad (3.13.b)$$

and their solution is, for the layer numbered K, referring to *FIG.3.3*

$$\begin{vmatrix} E_K \\ H_K \end{vmatrix} = \begin{vmatrix} A_K & B_K \\ C_K & D_K \end{vmatrix} \begin{vmatrix} E_{K-1} \\ H_{K-1} \end{vmatrix} \qquad (3.14)$$

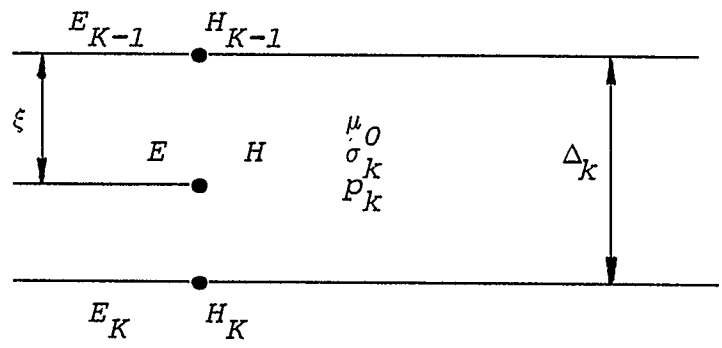


FIG.3.3 Layer no. K.

The solution of *(3.14)* is analogous to that of a number of transmission lines connected in cascade *[3.12, 3.13]*. A,B,C and D are called the Generalized Circuit Constants of the transmission line and they are defined as follows:

$$A_K = D_K = (\Theta_K + 1/\Theta_K)/2 = \cosh(\gamma\Delta_k) = \cosh(\Delta_k/p_K) \qquad (3.15)$$
$$B_K = z_k(\Theta_K - 1/\Theta_K)/2 = z_k\sinh(\gamma\Delta_k) = z_k\sinh(\Delta_k/p_K)$$

$$C_K = 1/z_k(\Theta_K - 1/\Theta_K)/2 = 1/z_k \sinh(\gamma\Delta_k) = 1/z_k \sinh(\Delta_k/p_K)$$

where

$$p_K = 1/(j\omega\mu_0\sigma_K)^{1/2} \qquad (3.16)$$
$$z_k = (j\omega\mu_0/\sigma_K)^{1/2}$$
$$\Theta_K = \exp(\Delta_K/p_k) \quad .$$

If F and B denote the forward and backward wave :

$$E = E_F + E_B \qquad (3.17)$$
$$H = H_F + H_B$$

and

$$E_F = z_K H_F \qquad (3.18)$$
$$E_B = -z_K H_B$$
$$H_F = H_{F,K-1}\exp(-\xi/p_K)$$
$$H_B = H_{B,K-1}\exp(\xi/p_K) \quad .$$

If there are $n + 1$ layers, in the last one, which extends to infinity, there will be no backward wave. Therefore:

$$E_n/H_n = z_{n+1} \qquad (3.19)$$

or in the homogeneous earth case, where $n = 0$

$$E_0/H_0 = z_1 = z \qquad (3.19a)$$

and equation *(3.18)* gives

$$H = H_0 exp(-x/p) \tag{3.20}$$

so that the total flux in the earth is

$$\phi = \mu_0 \int_0^\infty H\, dx = p\mu_0 H_0 \ . \tag{3.21}$$

The complex depth p is defined by means of *FIG.3.2b.* as the depth which gives the same flux ϕ. Therefore, for a homogeneous earth where $\mu=\mu_0$,

$$p = 1/(j\omega\mu_0\sigma)^{1/2} \ . \tag{3.22}$$

The depth p is related to the real skin depth

$$\delta = 1/(\pi f\mu_0\sigma)^{1/2} \tag{3.23}$$

by

$$1/p = (1 + j)1/\delta \ . \tag{3.24}$$

In the multi-layer case equation *(3.14)* can be applied sequentially to obtain

$$\begin{vmatrix} E_K \\ H_K \end{vmatrix} = \begin{vmatrix} A_{Ko} & B_{Ko} \\ C_{Ko} & A_{Ko} \end{vmatrix} \begin{vmatrix} E_0 \\ H_0 \end{vmatrix} \tag{3.25}$$

For $K = n$ *(3.25)* becomes

$$\begin{vmatrix} E_n \\ H_n \end{vmatrix} = \begin{vmatrix} A_{n0} & B_{n0} \\ C_{n0} & A_{n0} \end{vmatrix} \begin{vmatrix} E_0 \\ H_0 \end{vmatrix} \tag{3.26}$$

with the aid of *(3.19)*

$$\begin{vmatrix} E_0 \\ H_0 \end{vmatrix} = \begin{vmatrix} A_{n0} & -B_{n0} \\ -C_{n0} & A_{n0} \end{vmatrix} \begin{vmatrix} z_{n+1} \\ 1 \end{vmatrix} H_n \qquad (3.27)$$

or, eliminating H_n:

$$E_0 = H_0(z_{n+1}A_{n0} - B_{n0})/(A_{n0} - z_{n+1}C_{n0}) . \qquad (3.28)$$

Since $H_0 = I$, E_0 can be calculated from *(3.28)*, and all E_K, H_K will be obtained from *(3.25)*. Consequently, the flux in each layer K is obtained by integrating *(3.13a)*:

$$\phi_K = \mu_0 \int_0^{\Delta_K} H\, dx = 1/(j\omega)(E_{K-1} - E_K) \quad . \qquad (3.29)$$

This permits us to calculate the complex depth of equation *(3.21)* to give the result

$$p = \Sigma\, \phi_K/(\mu_0 H_0) \quad . \qquad (3.30)$$

The real and imaginary parts of the complex depth are shown in *FIG.3.4.a-b* for uniform and *3* layer earth. The first layer is *100 Ohm-m* and *10 m* deep, second layer *200 Ohm-m* and *20 m* deep and the third layer resistance is *300 Ohm-m* extending to infinity.

The derivation provided in this chapter indicates that the asymptotic formulation of Carson's equation is also easily evaluated, although the original equation requires

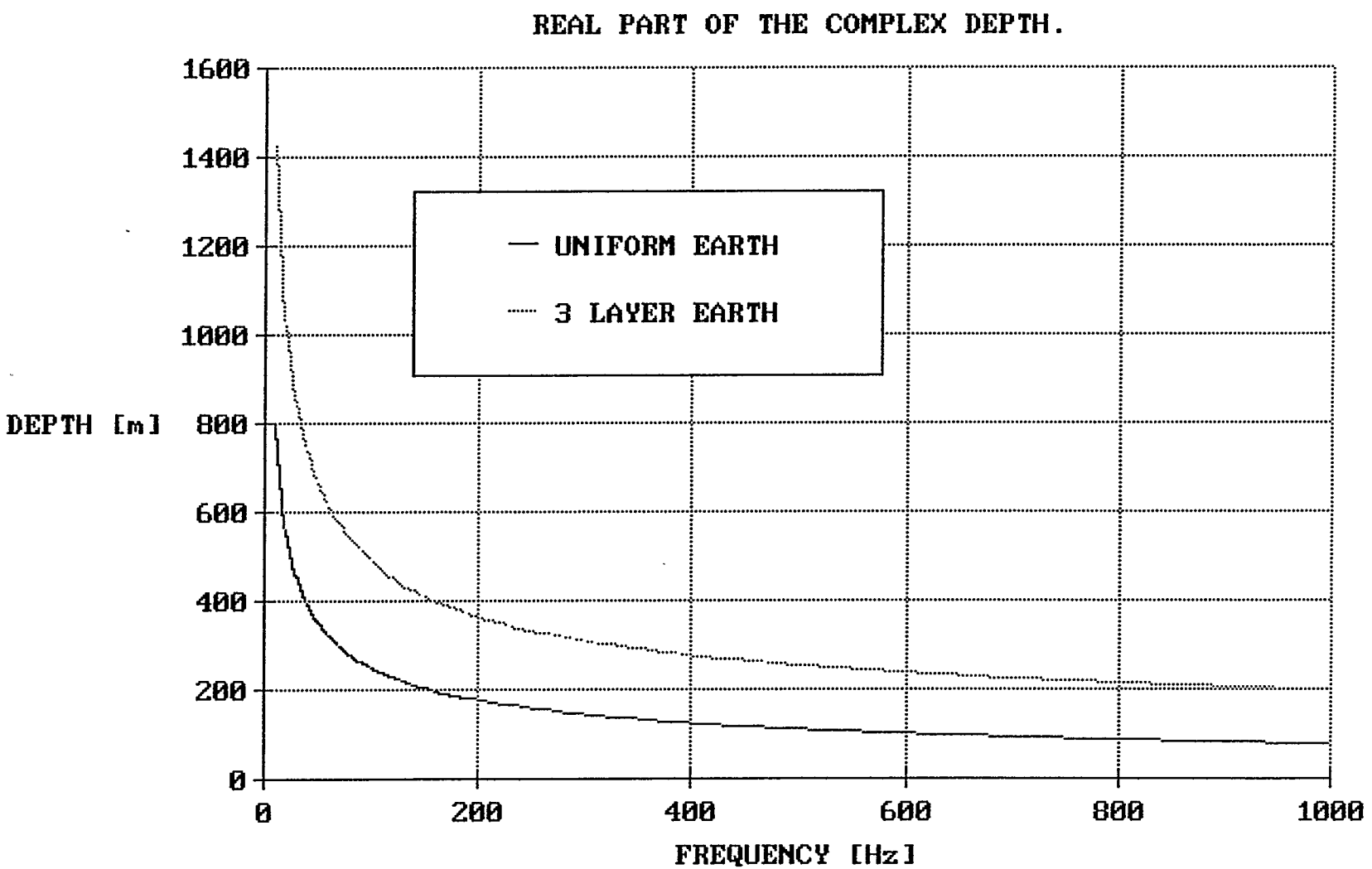

REAL PART OF THE COMPLEX DEPTH.
UNIFORM EARTH
3 LAYER EARTH
DEPTH [m]
1600
1400
1200
1000
800
600
400
200
0
0
200
400
600
800
1000
FREQUENCY [Hz]

FIG.3.4.a

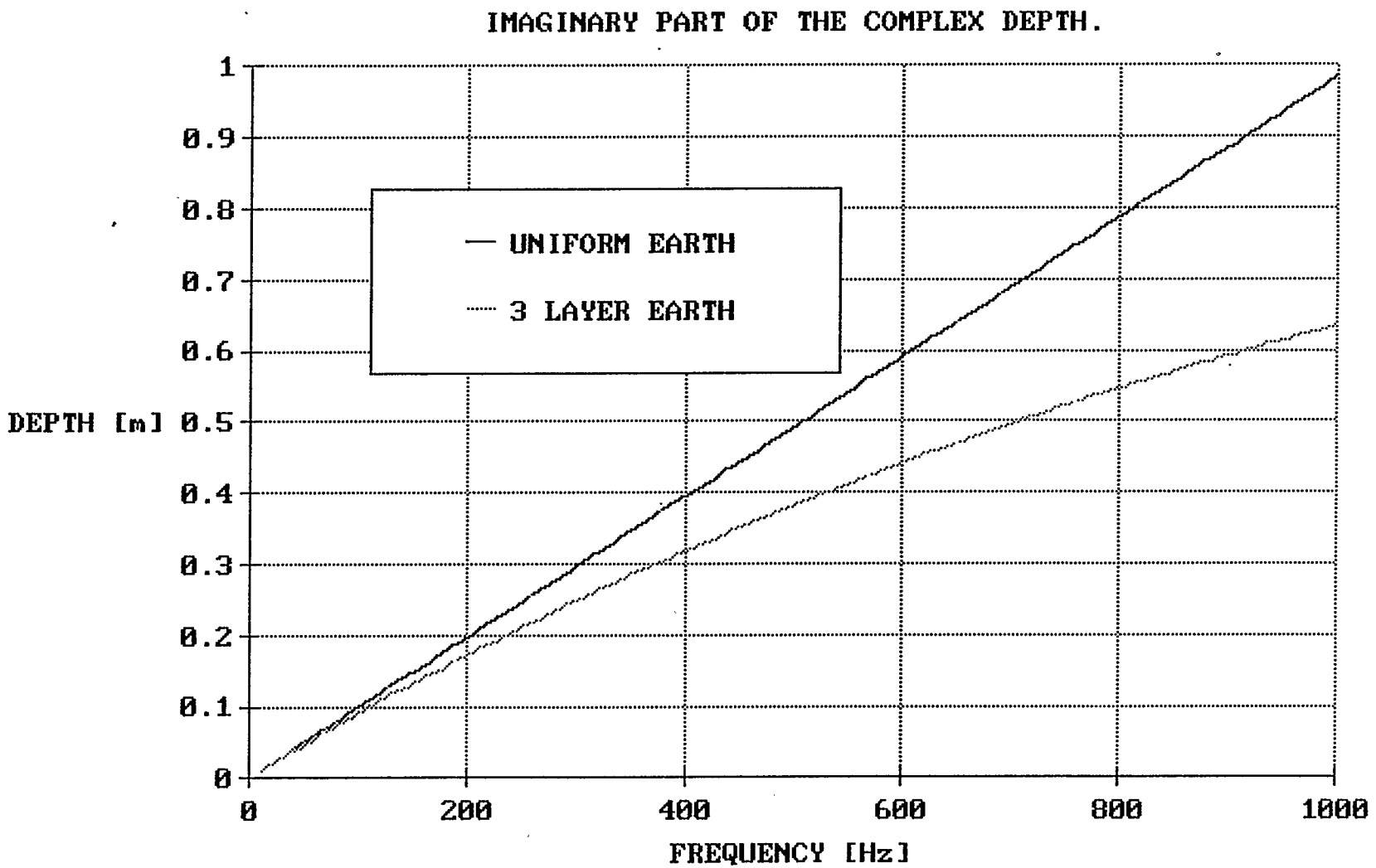

IMAGINARY PART OF THE COMPLEX DEPTH.
UNIFORM EARTH
3 LAYER EARTH
DEPTH [m]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
200
400
600
800
1000
FREQUENCY [Hz]

FIG.3.4.b

computer analysis or approximate graphical data, Carson's equations can be evaluated only for specific frequency ranges. The self-impedance, given by *(3.2)* using the equivalent depth of ground return D_g and discounting the conductor's resistance and reactance, is *9%* smaller than that given by the closed form equation *(3.5)*. The complex plane approach results in simple formulae which are valid from very low frequencies to several *MHz* and can be conveniently evaluated on a personal computer. Equations *(3.25)* to *(3.30)* are integrated into the transmission line model in *Chapter 5, Section 2*, to calculate the multi-layer ground return impedance of up to five layers. The program can be easily modified to handle more layers than five if necessary.

The final form of the series impedance of the pipeline of outside radius *a* is given by *[3.7]*

$$Z_s = j\omega\mu_0/(2\pi)\ln((a+p)/a) \quad . \qquad (3.34)$$

These values are also evaluated in the attached computer program in *Appendix B* a for multi-layer earth.

CHAPTER 4

ELECTRICAL MODEL OF THE GROUND BED AND THE SHORTED PIPELINE CASING

4.1 GENERAL

A very important aspect of the transmission line system deals with the current passage into and out of the earth. This current passage is provided for by ground electrodes or ground beds which already exist as part of the pipeline cathodic protection system *[4.1]*. The actual ground bed analysis is often quite complicated, but with practical approximations the problem can be simplified to an extent where application of classical circuit theory methods can result in closed form mathematical expressions.

Another important aspect of the transmission line model deals with the electrical model of an electrical short between the pipeline casing and the pipeline. A casing is required by codes and regulations of governmental agencies for pipelines crossing under roads and railways as illustrated in *FIG.4.1*. The shorted casing (i.e. a casing is shorted to the pipeline) forms a metallic shield around the pipe and diverts cathodic protection current from normally entering the inside of the casing, thereby leaving a large section of the pipeline unprotected. This casing short serves as a reflection point for *EM* waves on the

pipeline transmission line.

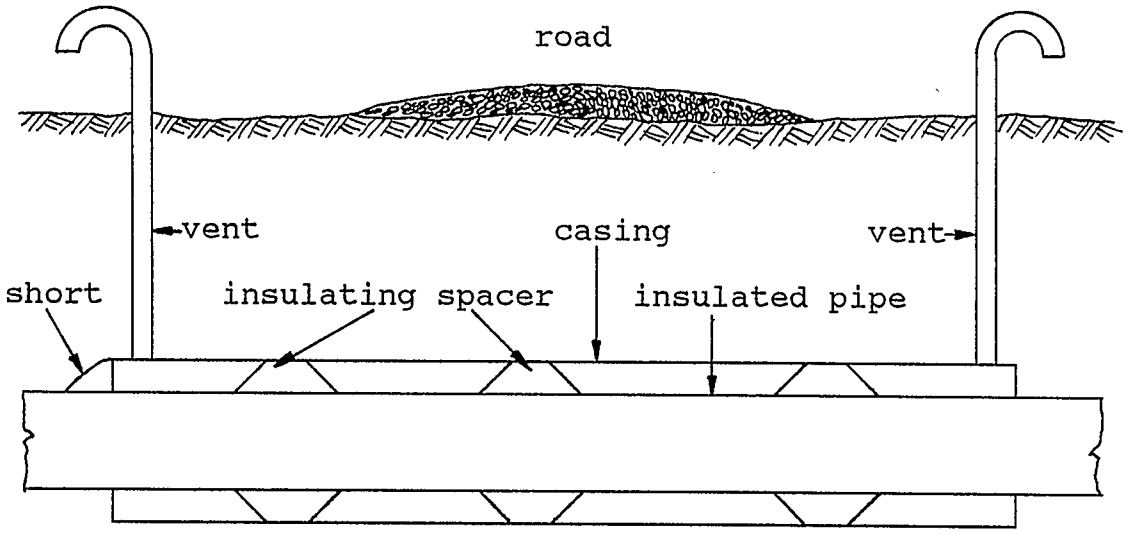


FIG.4.1 SHORTED PIPELINE CASING.

The present chapter deals with practical numerical processes for evaluating ground beds and pipeline casing located in the uniform earth, where both vertical and horizontal oriented ground beds will be considered. When the results of this chapter are combined with the result of *Chapter 2* and *3*, a complete transmission line model will be available which can be used to provide the attenuation, the phase shift and the characteristic impedance of a given buried pipeline.

4.2 ELECTRODE RESISTANCE

Although there are structural differences between electrodes used for a.c. system grounds and ground beds

used for pipeline cathodic protection, the objective is the same, and that is to provide low resistance current passage into the ground. The analysis following here is based on the following assumptions:

- The frequency is low enough that static conditions apply, therefore the resistance to ground calculation of the two grounding systems for different shapes and orientations of conductors or groups of conductors is basically the same as the calculation of capacitance and has been available for some time *[4.2, 4.3]*.

- Conductors are in direct contact with the earth. Measurements indicate that this is not strictly true, even after grounds have been installed for a long time, and as a result of contact resistance at the surface of the conductor, the resistance of grounds may be some *20%* higher than calculated *[4.4]*. At higher frequencies, this contact resistance is bridged by capacitive admittance between conductors and ground, and the resistance is thus somewhat reduced.

- The ground beds are short and the voltage drop along the bed conductor may be neglected in comparison with the potential of the ground bed structure. In general this will be the case even for ground beds with maximum dimensions of a few hundred meters.

One of the simplest forms of an electrode is the driven rod as shown in *FIG.4.2*. There is, however, no exact

formula for the capacitance of a cylinder isolated in free space. The idea is to treat the flow of electrical current from the cylinder into and through the ground in the same way as the flow of dielectric flux from an isolated cylinder. The successive approximation calculation *[4.3]* initially assumes a uniform distributions of charge and then other distributions of charge are successively added, to keep the potential of the cylinder the same throughout. This method is accurate but quite complicated to use.

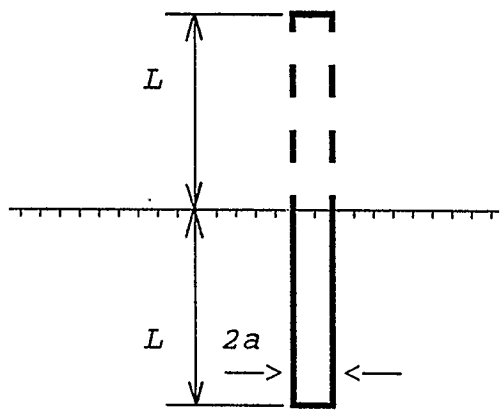


FIG.4.2 DRIVEN ROD WITH IMAGE.

There are two approximations to the above method for calculating the electrostatic capacitance of the electrode buried in the uniform earth, and they provide sufficiently good results. In both cases the electrodes are combined with their images above the surface of the ground:

1) Considering the ground electrode as half an ellipsoid of revolution in which the major axis L is very

large compared with the minor axis a. The capacitance of such an ellipsoid embedded in an infinite medium with relative permittivity ϵ_r is given by [4.5]

$$C = \frac{4\pi\epsilon_r\epsilon_0\sqrt{(L^2 - a^2)}}{\ln\left((L + \sqrt{(L^2-a^2)})/a\right)} \qquad (4.1)$$

where ϵ_0 is the permittivity of free space. It follows from the above equation that the resistance between one-half of the ellipsoid of revolution embedded in the earth of resistivity ρ and the point at infinity is

$$R = \frac{\rho}{2\pi\sqrt{(L^2 - a^2)}} \ln\left((L + \sqrt{(L^2-a^2)})/a\right) \qquad (4.2)$$

$$\simeq \frac{\rho}{2\pi L} \ln (2L/a) \ , \quad \text{for } L >> a \ .$$

2) An alternative method of calculation which becomes very useful in complicated grounding systems has been used by Dwight [4.6] and can be described simply as the uniform charge method. This consists of assuming uniform charge density over the surface of the conductor and calculating the average potential. Then the approximate capacitance is taken as equal to the total charge divided by the average potential. This method gives a resistance that is correct to within 1 percent for the cylinder, if it is very long.

Let us consider the same electrode as before as shown in *FIG.4.3.*

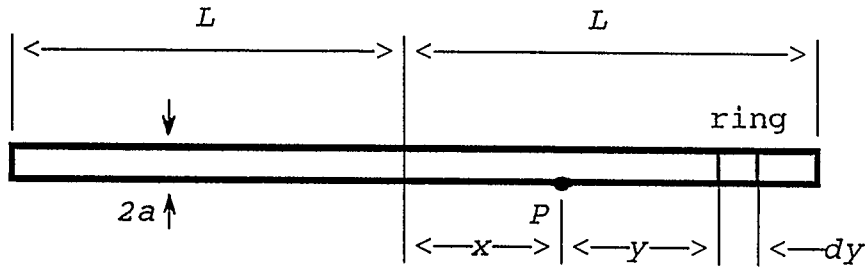


FIG.4.3 DRIVEN ROD - UNIFORM CHARGE.

Let the charge per unit length on the surface be q_L, then the charge on the incremental section of rod dy is $q_L dy$. The potential at any point P due to this charge can be shown to be *[4.6]*

$$V = dy/a\ [a/r - 1/2^2\ a^3/r^3\ (3y^2/r^2 - 1) + \ldots] \qquad (4.3)$$

where $r^2 = a^2 + y^2$ and y is the distance from P to the ring. To obtain the total potential, *(4.3)* must be integrated from $y = 0$ to $y = L - x$ and also from $y = 0$ to $y = L + x$. If this is then multiplied by dx/L and integrated from $x = 0$ to $x = L$, the average potential will be obtained. This is

$$V_{av}/(2q_L) = \ln(4L/a) - 1 + a/L(1/2 + 1/8 + 1/128 + \ldots) - a^2/L^2(3/16 - 1/32 \ldots) + a^4/L^4(1/64 - 1/1024 \ldots).$$

Since in all practical rod electrodes, the a/L ratio is very small, the terms involving powers of a/L may be neglected. The capacitance then becomes

$$1/C = V_{av}/(L2q_L) = 1/L\ [\ln(4L/a) - 1]$$

and the resistance is

$$R = \rho/(2\pi L)[\ln(4L/a) - 1) \quad . \qquad (4.4)$$

A set of curves calculated from the above two formulae *(4.2)* and *(4.4)* are given in *FIG.4.4* for earth resistance of *100 ohm-m*. In general, formula *(4.4)* gives values which are slightly lower than those given by the ellipsoid formula *(4.2)*. The actual difference is

$$\rho/(6.283L)[\ln(2L/a)-\ln(4L/a) + 1) = 0.3069\rho/(6.28L) \quad .$$

For *100 ohm-m* this becomes *4.88/L*. Therefore, for a *0.3 m* rod the difference is *16.3 ohms* or about *8* per cent, while for a *30 m* rod the difference is *.163 ohms* or about *4* per cent. Such differences may not be very important since it is doubtful whether the value of ρ is generally known with this accuracy, and other factors can also cause error. In this work the more accurate method is prefered since it is no more difficult to use.

RESISTANCE OF THE DRIVEN ROD (3" DIAMETER).

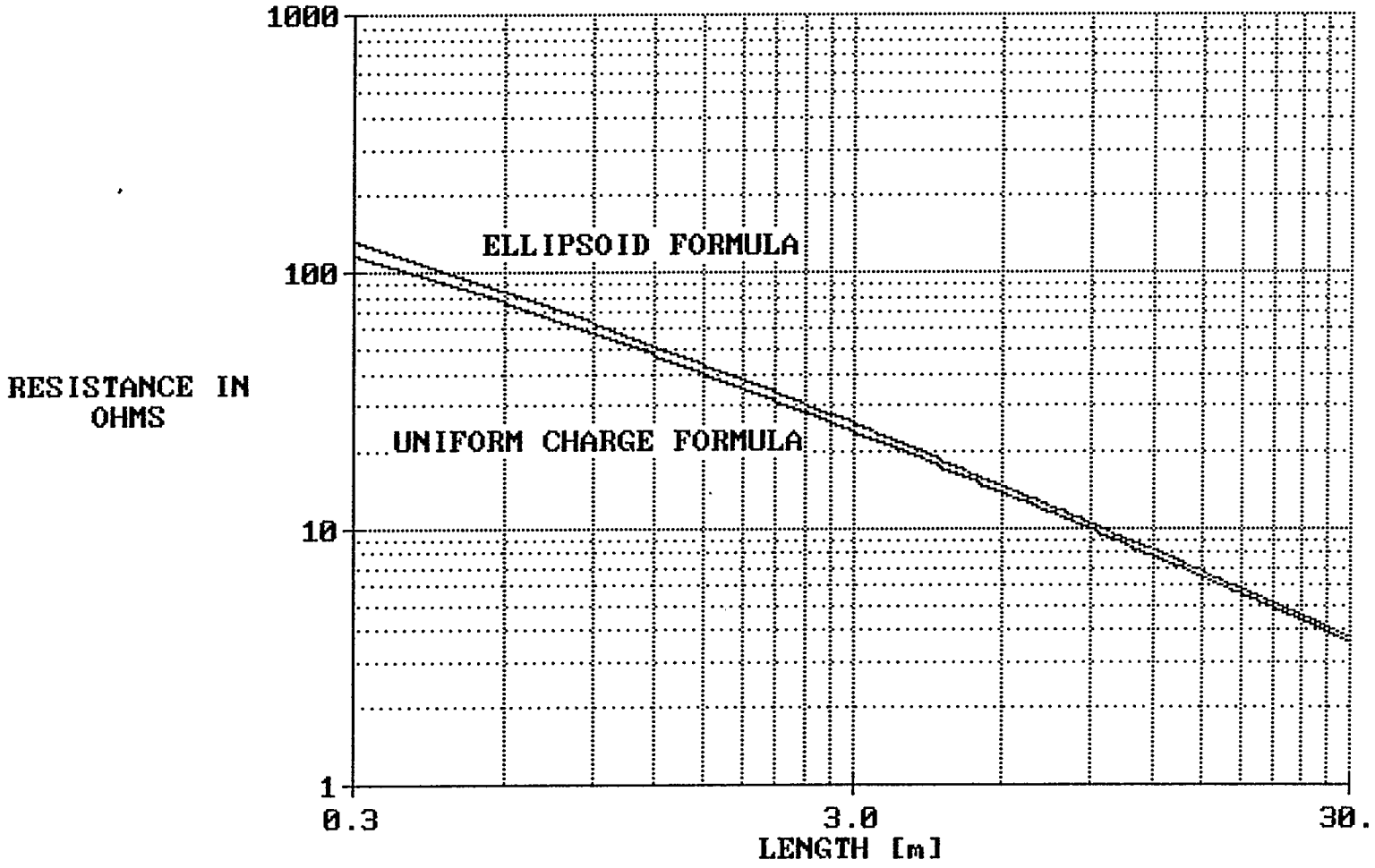


FIG.4.4

4.3 RODS CONNECTED IN PARALLEL, IN A STRAIGHT LINE

The driven rod or pipe is an economical and simple means of making connection to the earth but, except sometimes in the case of a deep-well ground *[4.1]*, the resistance of a single rod is not sufficiently low and it is necessary to use a number of them connected in parallel in a straight line. This is a typical arrangement for pipeline applications. The parallel connection does not necessarily comply with the usual law of resistance in parallel, therefore further analysis is required to determine what reduction in the total resistance can be obtained.

In order to calculate the combined resistance, it is necessary first to calculate the capacitance of the analogous case as before using the uniform charge method. To reduce the complexity of the calculation the rod electrode is replaced by a hemisphere having the same resistance. The resistance of a hemisphere buried in the surface is

$$R = \rho/2\pi r$$

where r is the hemisphere radius. If this is equated to the resistance of the driven rod *(4.4)*, then

$\rho/2\pi r = \rho/2\pi L\ (ln\ (4L/a - 1)$

then

$$r = \frac{L}{ln\ (4L/a) - 1} \quad . \qquad (4.5)$$

The values of this radius for various lengths and diameters of rods or pipes are calculated and given in *FIG.4.5.*

Suppose there are two rods connected in parallel at a distance d apart. Each rod can be replaced by its equivalent hemisphere of radius r carrying a charge Q. Then the potential of either rod will be

$$V = Q/r + Q/d = Q/r(1 + r/d) = Q/r(1 + m)$$

where $m = r/d$ and r is defined by *(4.5)*.

The total charge is $2Q$, and the capacitance C is given by

$$1/C = V/2Q = 1/2r(1 + m) \quad .$$

Therefore the combined resistance of the two rods in parallel is

$$R_2 = \rho/2\pi C = \rho/4\pi r\ (1 + m) \quad . \qquad (4.6)$$

The resistance of one rod is $R_1 = \rho/2\pi r$, and so the ratio

AN EQUIVALENT HEMISPHERE.

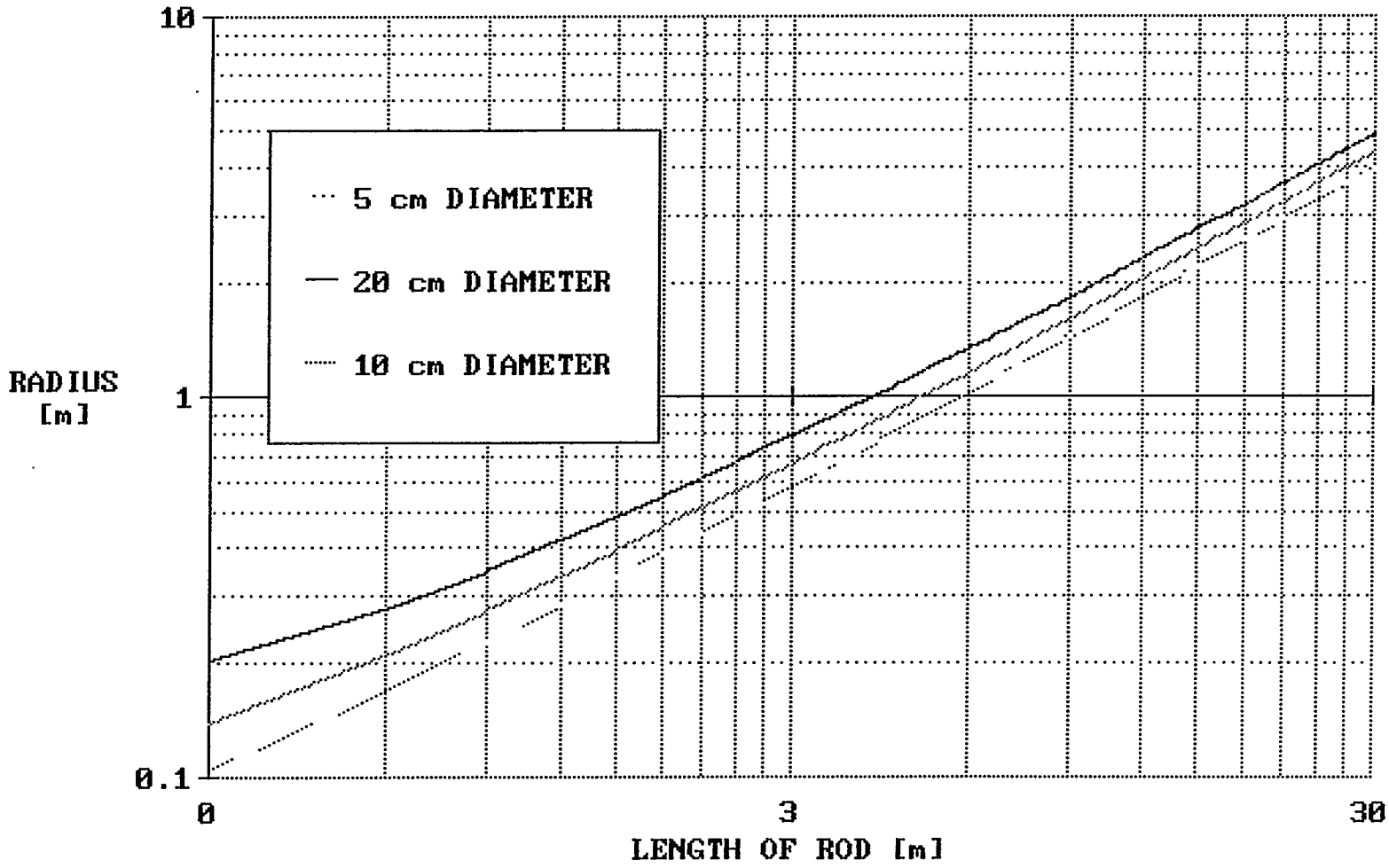


FIG.4.5

of the total resistance and the resistance of one rod is

$$R_2/R_1 = (1 + m)/2 \quad .$$

This resistance ratio has been calculated and the results are given in *FIG.4.6* for different diameters and length. The ideal value for this ratio is *0.5*, and these curves approach this value rapidly. For instance, two *3 m* rods placed *3 m* apart produce a resistance ratio of *0.615*, and the distance *d* would have to be increased very significantly to produce any substantial reduction in this ratio.

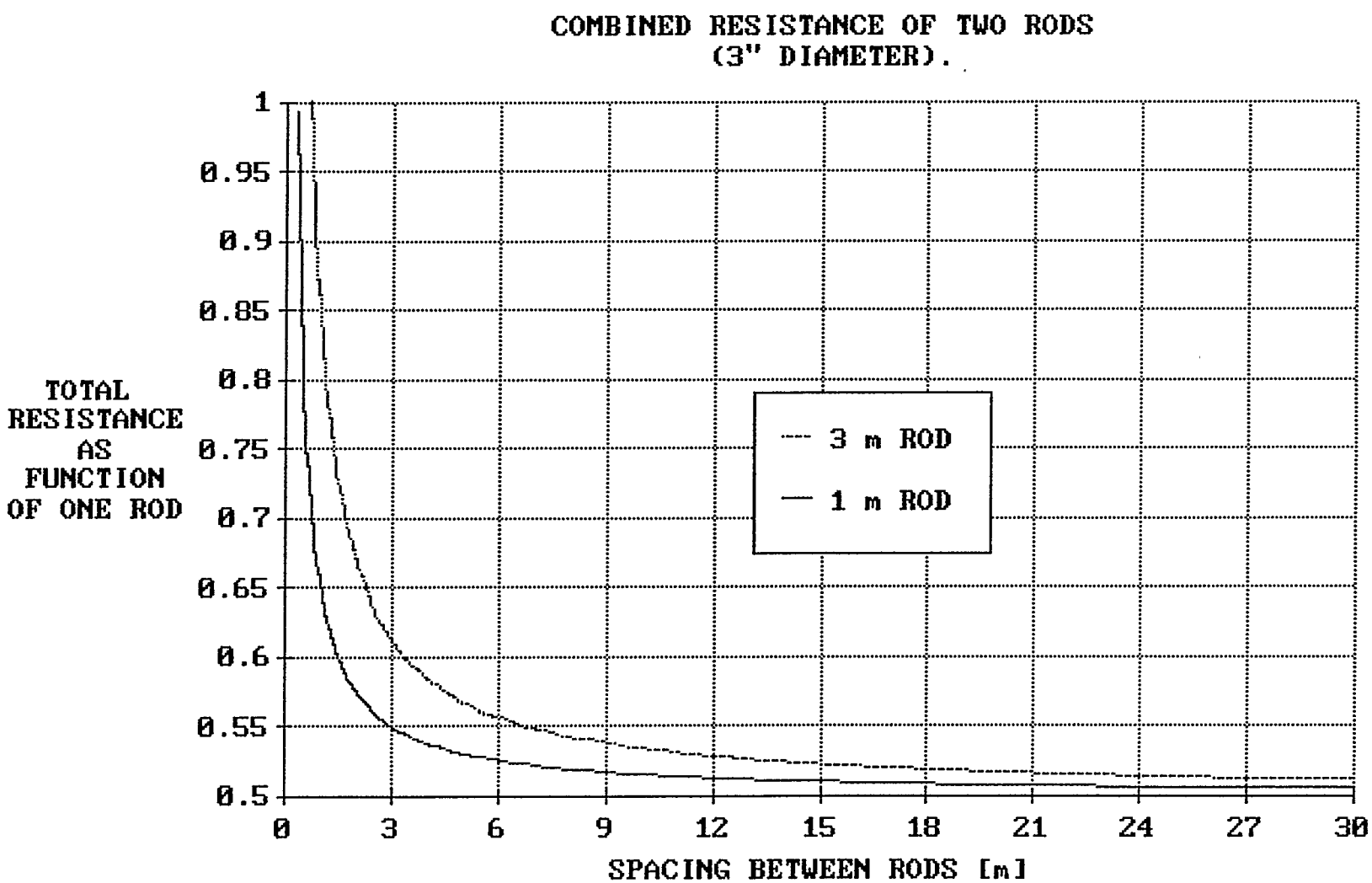
In the case of three rods in parallel the charge on the center rod will be different from those on the outer rods. Let the charge on each outer rod be Q_1 and the charge on the center rod be Q_2. Then the potential of either of the outer rods are

$$Q_1/r + Q_2/d + Q_1/2d = 1/r\ (Q_1(1 + m/2) + Q_2 m) \quad .$$

The potential of the center rod will be

$$Q_2/r + 2Q_1/d = 1/r\ (Q_1 2m + Q_2) \quad .$$

These two potentials must be equal


COMBINED RESISTANCE OF TWO RODS
(3" DIAMETER).
TOTAL RESISTANCE AS FUNCTION OF ONE ROD
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
3
6
9
12
15
18
21
24
27
30
3 m ROD
1 m ROD
SPACING BETWEEN RODS [m]
FIG.4.6

$$Q_1(1 + m/2) + Q_2 m = Q_1 2m + Q_2 \quad .$$

From here

$$Q_2 = Q_1(1 - 3m/2)/(1 - m) = kQ_1 \quad .$$

Substituting this into the potential of the center rod

$$1/r\ (Q_1 2m + kQ_1) = Q_1/r\ (2m + k) \quad .$$

The total charge $= Q_1(2 + k)$ and the capacitance is

$$1/C = 1/r\ (2m + k)/(2 + k) \quad .$$

Therefore the total resistance of three rods in parallel

$$R_3 = \rho/2\pi r\ (2m + k)/(2 + k) \quad . \qquad (4.7)$$

The resistance ratio of three rods in parallel to the resistance of one rod is

$$R_3/R_1 = (2m + k)/(2 + k) = (2 + m - 4m^2)/(6 - 7m) \quad .$$

Values of this ratio for various rods are given in *FIG.4.7.*

The resistance R_n for n number of rods in the same plane at equal spacing d can be obtained by calculating the

COMBINED RESISTANCE OF THREE RODS
(3" DIAMETER).

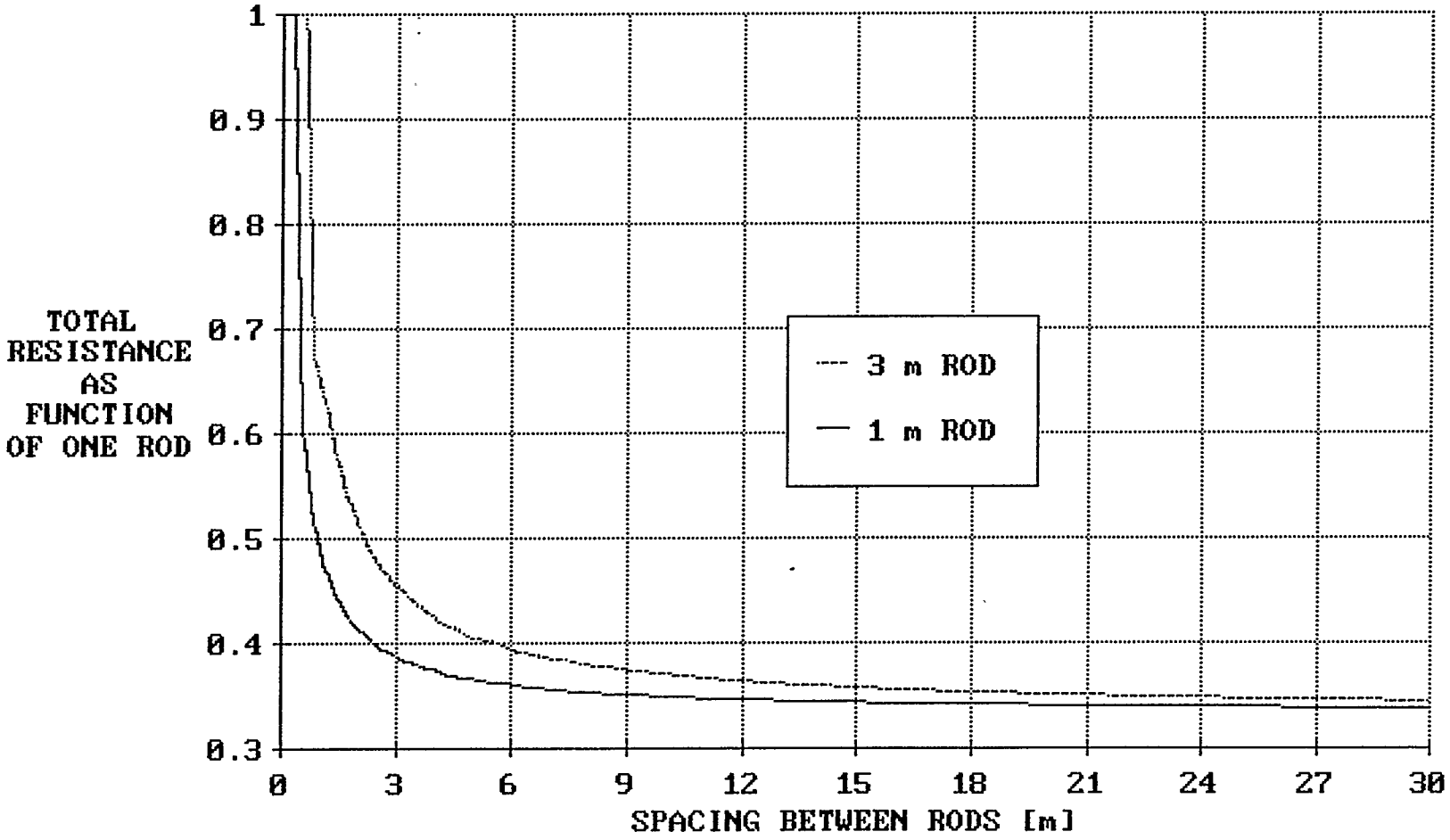


FIG.4.7

resistance with respect to each rod and taking the average value [4.6]

$$R_n = 1/n\ [R_1(a) + 2/n\ ((n-1)R_1(d) + (n-2)R_1(2d) + \ldots + R_1(nd-d))]\ .$$

When $d \geq L$, with $a = pd$, $R_1(pd) = \rho/(2\pi pd)$, the above equation becomes

$$R_n = 1/n\ [R_1(a) + \rho/(\pi d)\ (1/2 + 1/3 + 1/4 + \ldots + 1/n)\ .$$

For large number of rods $(1/2 + 1/3 + 1/4 + \ldots + 1/n)$ approaches to $ln(\gamma n/\epsilon)$, the resistance becomes

$$R_n = 1/n\ \rho/2\pi L\ [ln(4L/a) - 1 + 2L/d\ ln(\gamma n/\epsilon)] \qquad (4.8)$$

where $\gamma = 1.781\ldots$, and $\epsilon = 2.718\ldots$

4.4 THE HORIZONTAL ELECTRODE

The vertically buried rod is a convenient and economical form of electrode but there are cases where it is not practical. For instance, it is not practical if there is a layer of rock a few meters below the ground surface. In such cases, buried horizontal electrodes or ground beds are used. The ground beds are usually beds of

coke about *30-50* cm square with a metal center, buried about 1 m with lengths of up to *100 m*. The calculation for the square bed is identical to that of the circular electrode with the same circumference.

The analysis following here is also applicable to pipeline casing, when modeled as a large horizontal electrode.

As before, the resistance to ground can be determined by first calculating the electrostatic capacitance. In order to take account of the effect of the ground surface and the depth of burial, it is necessary to consider the buried electrode and its image above the surface of the ground, as illustrated in *FIG.4.8*.

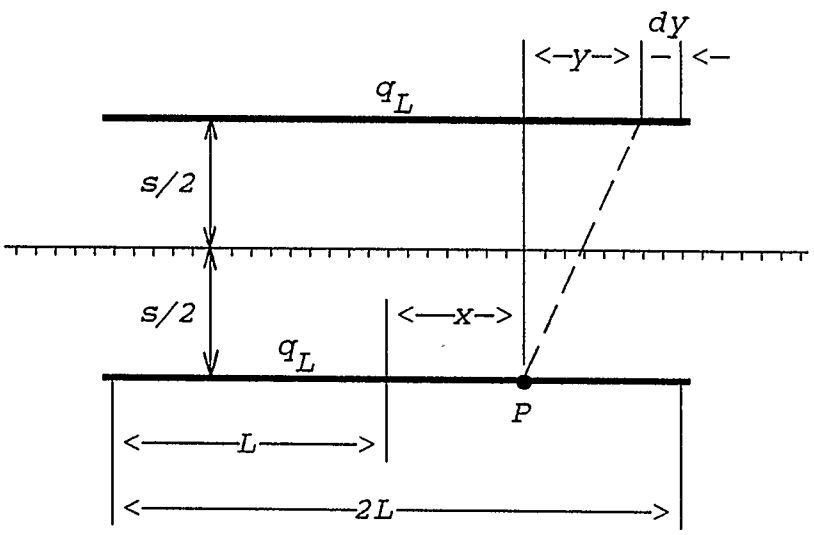


FIG.4.8 A BURIED HORIZONTAL ELECTRODE.

A uniform charge q_L per meter exists on both the electrode and its image, and it is necessary to calculate the average

potential of each electrode due to this charge *[4.3]*. The potential of the electrode due to its own charge is the same as for the driven rod *[4.7]*

$$V_{av} = 2q_L\ [ln(4L/a) - 1]\ .$$

The average potential of the electrode due to the uniform charge on the image can be calculated as follows. The potential at any point P on the electrode at distance x from the center due to an element of charge $q_L dy$ on the image is

$$V = q_L dy/\sqrt{(s^2 + y^2)}\ .$$

To calculate the potential at this point due to the whole of the image it is necessary to integrate the above expression in y from 0 to $(L + x)$ and y from 0 to $(L - x)$.

$$V = \int_0^{L+x} q_L dy/\sqrt{(s^2 + y^2)} + \int_0^{L-x} q_L dy/\sqrt{(s^2 + y^2)}$$

The average potential on an electrode due to a uniform charge on the image is obtained by multiplying the result of the above integration by dx/L and further integrating from $x = 0$ to L, and the result is

$$V = 2q_L[\ln\ (2L+\sqrt{(s^2+4L^2)})/s + s/(2L) - \sqrt{(s^2+4L^2)})/(2L)].$$

The total potential is

$$V = 2q_L[\ln(4L/a)-1 + \ln(2L+\sqrt{(s^2+4L^2)})/s + s/(2L)-\sqrt{(s^2+4L^2)})/(2L)] .$$

If this is divided by the total charge $L4q_L$ on the electrodes, this gives the values of $1/C$ for the two electrodes and this in turn leads to the following resistance

$$R = \rho/(4\pi L)\ [\ln(4L/a) - 1 + \ln(2L+\sqrt{(s^2+4L^2)})/s + s/(2L) - \sqrt{(s^2+4L^2)})/2L] . \quad (4.9)$$

Values calculated from *(4.9)* are plotted in curves of *FIG.4.8.a* for *100 ohm-m* earth resistivity. It is evident that the depth of burial and diameter has very little effect on the over all resistivity. For any other resistivity than *100 ohm-m*, the values of the curves must be multiplied by the appropriate resistivity ratio.

4.5 GROUND BEDS IN PARALLEL

By connecting ground beds in parallel that are properly

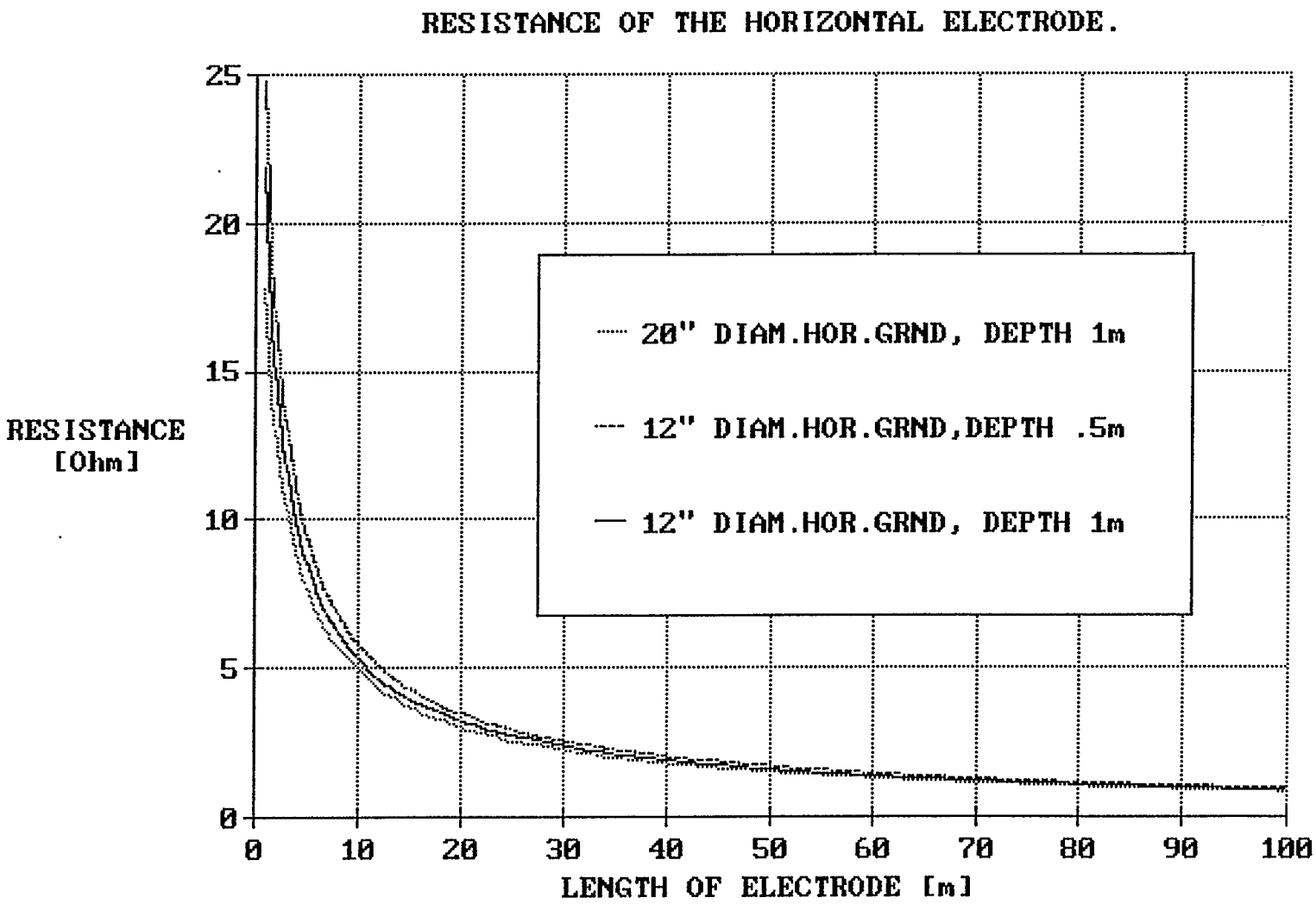

RESISTANCE OF THE HORIZONTAL ELECTRODE.
RESISTANCE [Ohm]
25
20
15
10
5
0
0
10
20
30
40
50
60
70
80
90
100
LENGTH OF ELECTRODE [m]
20" DIAM.HOR.GRND, DEPTH 1m
12" DIAM.HOR.GRND,DEPTH .5m
12" DIAM.HOR.GRND, DEPTH 1m

FIG.4.8.a

spaced, the total resistance can be reduced to less than that of a single bed *[4.7]* with twice the length and this is common practice for cathodic protection. In *FIG.4.9 AB* and *EF* are two ground beds in question. In order to carry out the calculation it is necessary to consider the images *CD* and *GH* above the surface of the ground. As before, it is assumed that there is a uniform charge of q_L per *m* length on both beds and their images.

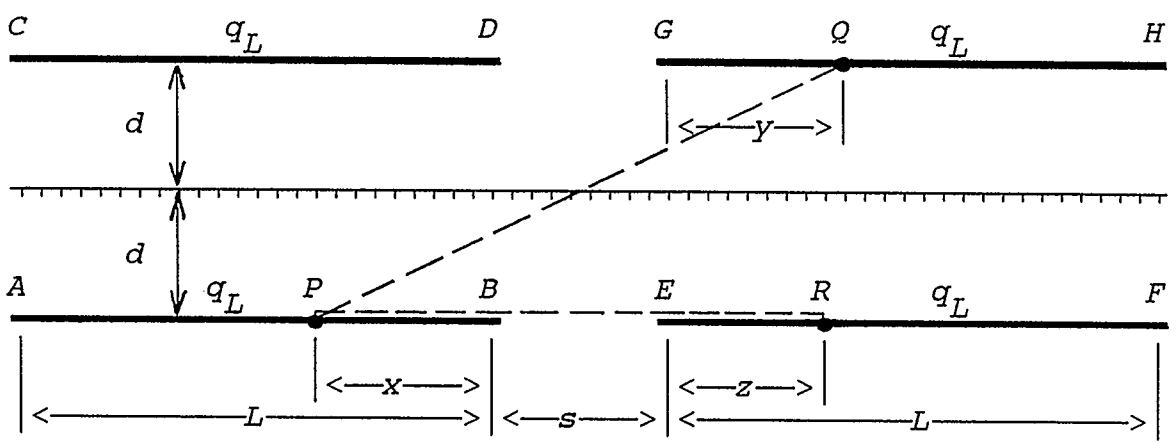


FIG.4.9 TWO GROUND BEDS CONNECTED IN PARALLEL.

The charges in the bed *AB* and its image *CD* result an average potential over *AB* equal to V_1. The total charge is $L2q_L$ and, if *AB* is the only ground bed, then the capacitance is $1/C = V/(L2q_L)$, and the resistance is $R = \rho V/(L4\pi q_L)$.

Considering the effect of the second ground bed *EF* and its image *GH*, and at first, the potential at any point *P*,

distant x from B.

1) Potential due to GH. At any point Q there is a charge $q_L dy$ and the potential at P due to this is

$$V = q_L dy/\sqrt{(4d^2 + (x + y + s)^2)} \ .$$

The total potential at P due to GH is then

$$\int_0^L q_L dy/\sqrt{(4d^2 + (x+y+s)^2)} = q_L[ln(L+x+s+\sqrt{(4d^2 + (x+y+s)^2)}$$

$$- ln((x+s)+\sqrt{(4d^2 + (x+s)^2)})] \ . \qquad (4.10)$$

2) Potential due to EF. At any point R there is a charge $q_L dz$ and the potential at P due to this is

$$q_L dz/(x + s + z) .$$

The total potential at P due to EF is given after integrating this from $z = 0$ to L

$$\int_0^L q_L dz/(x + s + z) = q_L[ln(L+x+s) - ln(x+s)] \ .$$

The total potential at P due to EF and its image GH is

$$V = q_L[\ln(L+x+s+\sqrt{(4d^2 + (x+y+s)^2)}) - \ln((x+s) + \sqrt{(4d^2 + (x+s)^2)} + [\ln(L+x+s) - \ln(x+s)] \ .$$

The average potential of AB due to EF and GH is accordingly

$$q_L B = q_L/L \int_0^L [\ln(L+x+s+\sqrt{(4d^2+(x+y+s)^2)}) - \ln((x+s)+ \sqrt{(4d^2+(x+s)^2)} + \ln(L+x+s) - \ln(x+s)]\, dx$$

$$= q_L/L \left\{ L \ln \frac{(2L+s+\sqrt{(4d^2 + (2L+s)^2)})^2 (2L + s)^2}{(L+s+\sqrt{(4d^2 + (2L+s)^2)})^2 (L + s)^2} + s\ln \frac{(2L+s+\sqrt{(4d^2 + (2L+s)^2)})^2 (2L + s)(s + \sqrt{(4d^2 + s^2)})s}{(L+s+\sqrt{(4d^2 + (2L+s)^2)})^2 (L + s)^2} - \sqrt{(4d^2 + (2L+s)^2)} + 2\sqrt{(4d^2 + (L+s)^2)} - \sqrt{(4d^2 + s^2)} \right\} \ .$$

The total potential of $AB = V_1 + q_L B$ and the total charge is $L4q_L$, and the capacitance is

$$1/C = (V_1 + q_L B)/(L4q_L) \ .$$

The resistance is

$$R = \rho(V_1 + q_L B)/(L8\pi q_L) = \rho V_1/(L8\pi q_L) + \rho B/8\pi L. \quad (4.11)$$

The first term is one half the resistance of one

isolated bed and so the quantity $\rho B/(8\pi L)$ is the amount by which the resistance of two beds in parallel exceeds half the value of one bed. The quantity $\rho B/8\pi L$ can be written as $F(\rho/L)$ and in general s/L is small, F can be written as *[4.7]*

$$F=1/(8\pi)[4\ln(2+s/L)+2s/L\ \ln((2+s/L)s/L)-4(1+s/L)\ln(1+s/L)]. \tag{4.12}$$

In *FIG.4.10* a set of curves calculated from *(4.9)* and *(4.12)* is provided to illustrate the resistance of two ground beds in parallel.

The preceding derivations can be extended to the case of two-layer and multi-layer soil. However, derivations and expressions start to become significantly more extensive and complicated, and that is outside of the scope of this work. This subject is reasonably well documented in the open literature *[4.7]*.

Equation *(4.11)* and *(4.12)* are integrated into our transmission line model to calculate the resistance at the injection point, as illustrated in *Appendix* B, in a Fortran source segment.

Formulae and curves have been given in this chapter for the change in resistance of driven rods and horizontal ground beds connected in parallel in a uniform earth. It is shown that in order to attain the full effect of

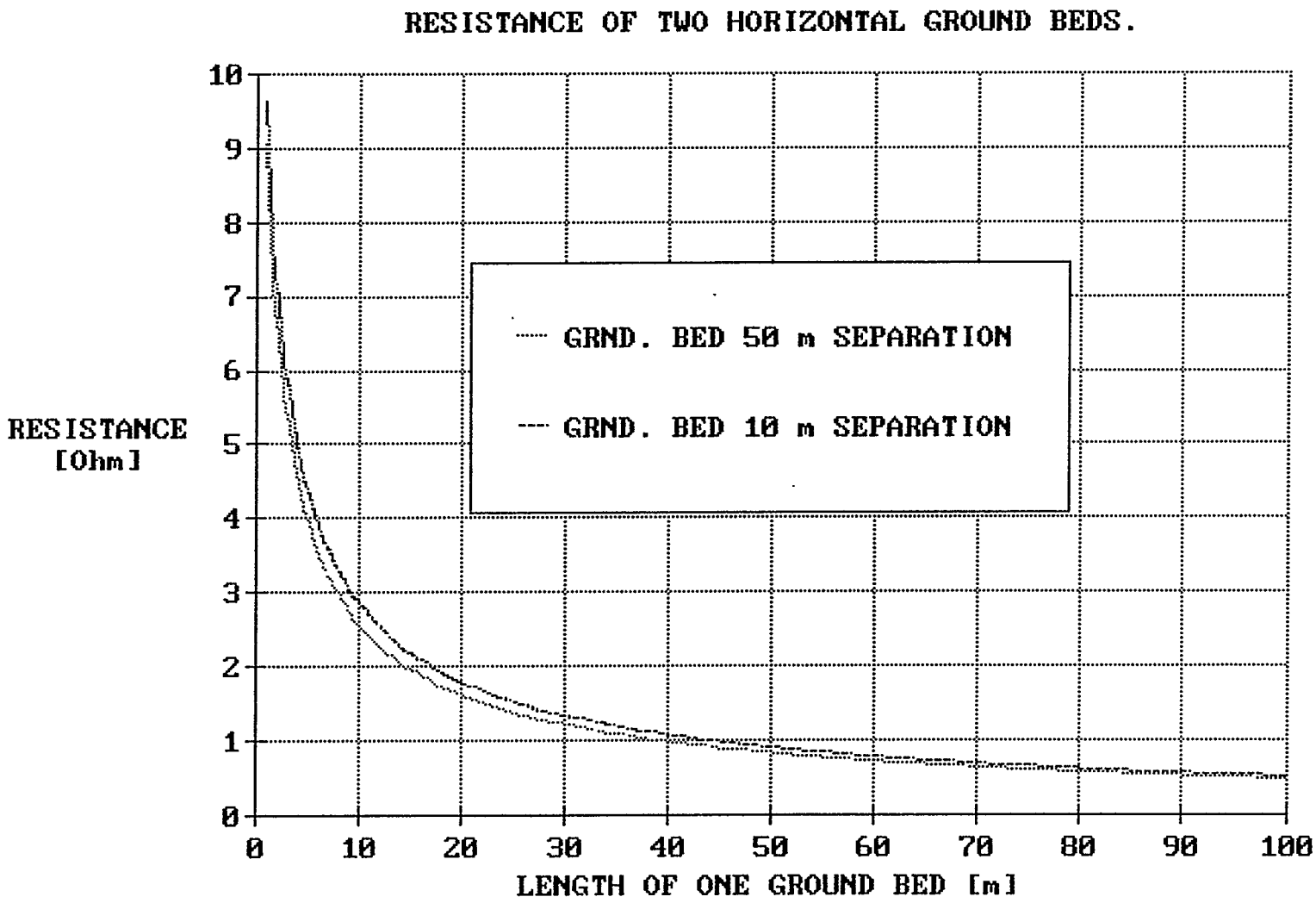

RESISTANCE OF TWO HORIZONTAL GROUND BEDS.
RESISTANCE [Ohm]
10
9
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100
GRND. BED 50 m SEPARATION
GRND. BED 10 m SEPARATION
LENGTH OF ONE GROUND BED [m]

FIG.4.10

resistance in parallel, the rods or beds must be a certain distance apart, and the rods or beds must not overlap. The calculation of the combined resistance, especially when a large number of rods is involved is difficult, and can be very lengthy and laborious. Furthermore, it is shown that two horizontal ground beds connected in parallel, properly spaced, can result in less than half of the resistance of a single bed with twice the length. More exact calculations are not necessarily advantageous over the approximate ones, as any calculation assumes ideal conditions, such as homogeneous soil, and such conditions are never met in practical cases. These practical formulae are applied directly to our transmission line model, and they complete the requirements to model the pipeline for *EM* wave propagation. Resistances derived for the single horizontal electrode, equation *(4.9)* and *FIG.4.8.a*, will also be used in our transmission line model in the next chapter for calculating load impedance, and to model a casing short as a reflection point for *EM* waves.

CHAPTER 5

COMPUTER SIMULATION OF DATA TRANSMISSION ON THE PIPELINE

5.1. GENERAL

In this chapter the performance of a given arrangement of pipeline with ground return and ground bed, for data transmission will be considered using a computer simulation method. The purpose of the analysis is to find the maximum channel capacity of the transmission line for digital signal transmission and to provide specifications related to actual field conditions. The methodologies applied here are similar to those developed for telephone cable twisted pair transmission lines *[5.1]*. First, transmission line equations are obtained, using classical transmission line theory to suit the pipeline application, then the transfer function of the transmission network is found, to assist in the final evaluation. Different patterns of eight bit words are applied to the transmission line by the Fast Fourier Transfer *(FFT)* process, the resulting spectrum is multiplied by the pipeline transfer function and then it is transferred back to the time domain by the inverse Fast Fourier Transfer (FFT^{-1}). The advantage of using digital signal processing, which deals with transformations of signals that are discrete in both amplitude and time, can

be realized using the flexibility of general-purpose digital computers.

5.2 TRANSMISSION LINE MODEL

The pipe, the coating and the ground return are treated as a uniform electrical transmission line as shown in *FIG.5.1*.

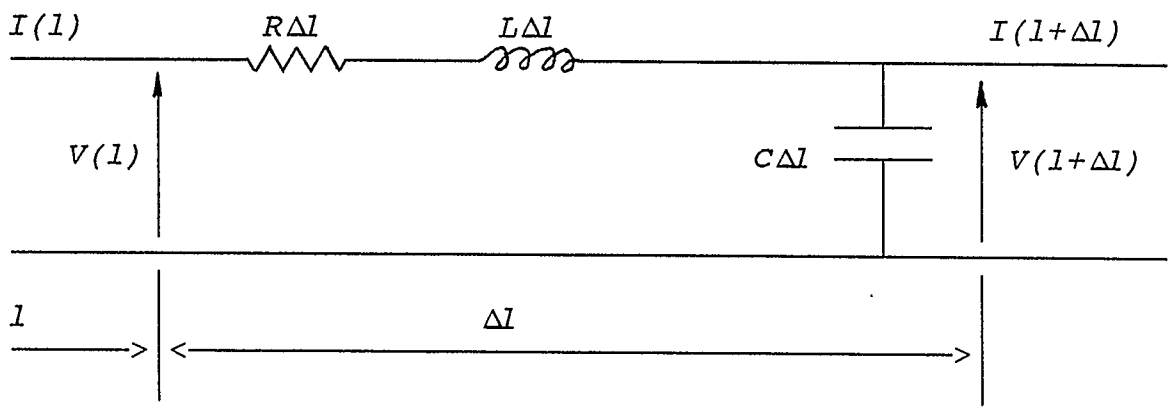


FIG.5.1. EQUIVALENT CIRCUIT OF AN INFINITESIMAL PORTION OF THE UNIFORM TRANSMISSION LINE.

The differential equations for a uniform transmission line are found by focusing on an infinitesimal section of length Δl, located at coordinate l on the line, remote from the line termination. This line section has the following values :

$R\Delta l$ series resistance, where $R = R_{pipe} + R_{ground}$

$L\Delta l$ series inductance, where $L = L_{pipe} + L_{ground}$

$C\Delta l$ shunt capacitance, where $C = C_{pipe}$

The pipe resistance and inductance values are derived from equation *(2.20)* and *(2.21)*. When the pipe parameters (diameter, thickness, conductivity and permeability) are known, R_{pipe} and L_{pipe} are found by using the computer program in *Appendix B*, entitled: Skin Effect Resistance and Reactance Calculation. The ground resistance R_{ground} and inductance L_{ground} are found using equations *(3.30)* and *(3.34)*. These values are computed using the above mentioned program listed in *Appendix B*. The capacitance of the transmission line model is calculated using equation *(2.28)*. The inside and the outside radii and the permittivity of the pipe coating must be known, in these analysis the pipe permittivity is taken to be $\epsilon_r = 3.7$.

The shunt conductance, usually considered in a transmission line model, of the pipeline plastic material coating is very low over the frequency range of interest and is equated to zero.

First we write two phasor equations from *FIG.5.1.*,

$$V(l+\Delta l) - V(l) = \Delta V(l) = - R\Delta l \; I(l) - j\omega L\Delta l \; I(l)$$

$$I(l+\Delta l) - I(l) = \Delta I(l) = - j\omega C\Delta l \; V(l)$$

and then we form the differential equations by letting Δl approach to zero

$$dV/dl = -(R + j\omega L)I \tag{5.1}$$

$$dI/dl = -j\omega CV \ . \tag{5.2}$$

Solving these simultaneous linear first order equations with constant coefficients for separate equations in V and I, produces two second order equations

$$d^2V/dl^2 - (R + j\omega L)j\omega CV = 0 \tag{5.3}$$

$$d^2I/dl^2 - (R + j\omega L)j\omega CI = 0 \tag{5.4}$$

The solution of *(5.3)* and *(5.4)* gives the transmission line equations for the pipeline *[5.2]*

$$V_{in} = V_L \cosh\gamma l + I_L Z_0 \sinh\gamma l \tag{5.5}$$

$$I_{in} = I_L \cosh\gamma l + V_L/Z_0 \sinh\gamma l \tag{5.6}$$

Where the complex propagation costant γ and the complex characteristic impedance Z_0 are defined by the expressions:

$$Z_0 = [(R + j\omega L)/j\omega C]^{1/2} \tag{5.7}$$

$$\gamma = [(R + j\omega L)j\omega C]^{1/2} \tag{5.8}$$

When the pipeline is driven by a Thevenin equivalent source, the resulting electrical transmission network, as illustrated in *FIG.5.2.*, is suitable for computer simulation. The source impedance is equal to the ground

bed resistance. The load impedance is either equal to the characteristic impedance for a transmission system, or it is equal to the casing short resistance connected in parallel with the impedance of the remaining pipeline as for a Time-Domain Reflectometry system.

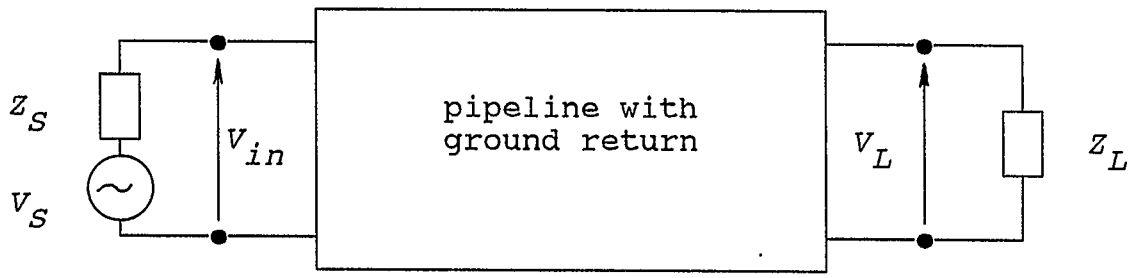


FIG.5.2 THE TRANSMISSION NETWORK.

To find the transfer function of the transmission line from *FIG.5.2* we write the following equations:

$$V_{in} = V_S - I_{in}Z_S \tag{5.9}$$

$$I_L = V_L/Z_L \tag{5.10}$$

where

$$Z_L = \frac{R_L \cdot Z_0}{R_L + Z_0}$$

and R_L is the load resistance. Substituting *(5.5)* and *(5.10)* into *(5.9)*

$$V_L[\cosh\gamma l + Z_0/Z_L \sinh\gamma l] = V_S - V_L[Z_S/Z_L \cosh\gamma l + Z_S/Z_0 \sinh\gamma l]$$

The transfer function is then found to be:

$$V_L = \frac{V_S}{(1 + Z_S/Z_L)\cosh\gamma l + (Z_0/Z_L + Z_S/Z_0)\sinh\gamma l} \quad . \qquad (5.11)$$

5.3 PULSE RESPONSE OF THE PIPELINE

To determine the performance of the pipeline for data transmission, we assume that a repetitive eight bit word, representing one mark and seven spaces, is applied to the transmission line input. The output pulse of the transmission line is then examined for distortion and data recognizability. This analysis is performed with the application of the Discrete Fourier Transform.

The first step in applying the discrete transform is to choose the number of samples N and the sample interval T. One of the properties of the discrete transform is that it approximates rather poorly the continuous transform for the higher frequencies. To approximate the continuous transform it is necessary to make the number of samples

large, until the computational time is intolerable. The graphic program, Microsoft Chart, which is used for graphical presentation of the data limits the size of N. The number of data points that the chart program can import from another program is limited. Therefore the chosen values are: $N = 32$ and $T = 0.01$ *sec*. The samples, 0 to 31, are illustrated in *FIG.5.3*. The eight bit word, one mark and 7 spaces is sampled $N = 32$ times or 4 times per bit, and transfered into the frequency domain with the aid of a FFT program *[5.3]*.

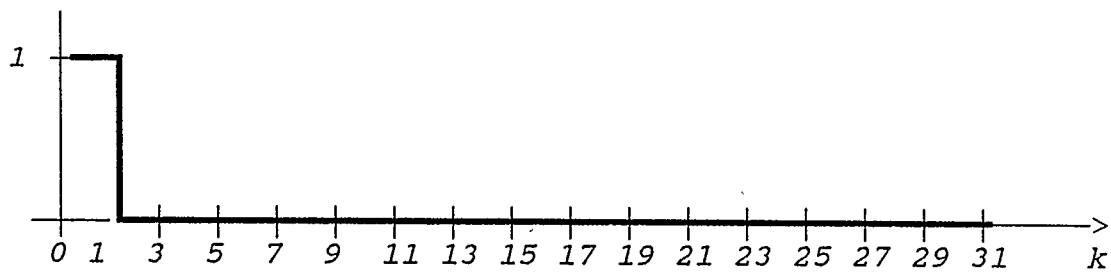


FIG.5.3 THE PULSE APPLIED TO THE TRANSMISSION LINE.

The discrete Fourier transform is computed by the following formula *[5.4]*:

$$H(n/NT) = T \sum_{k=0}^{N-1} x(k)\, e^{-j2\pi nk/N} \quad ; \quad n = 0,1,\ldots,N-1 \qquad (5.12)$$

The results are shown in *FIG.5.4* up to $n = N/2$. We note that the real part of the discrete transform is symmetrical

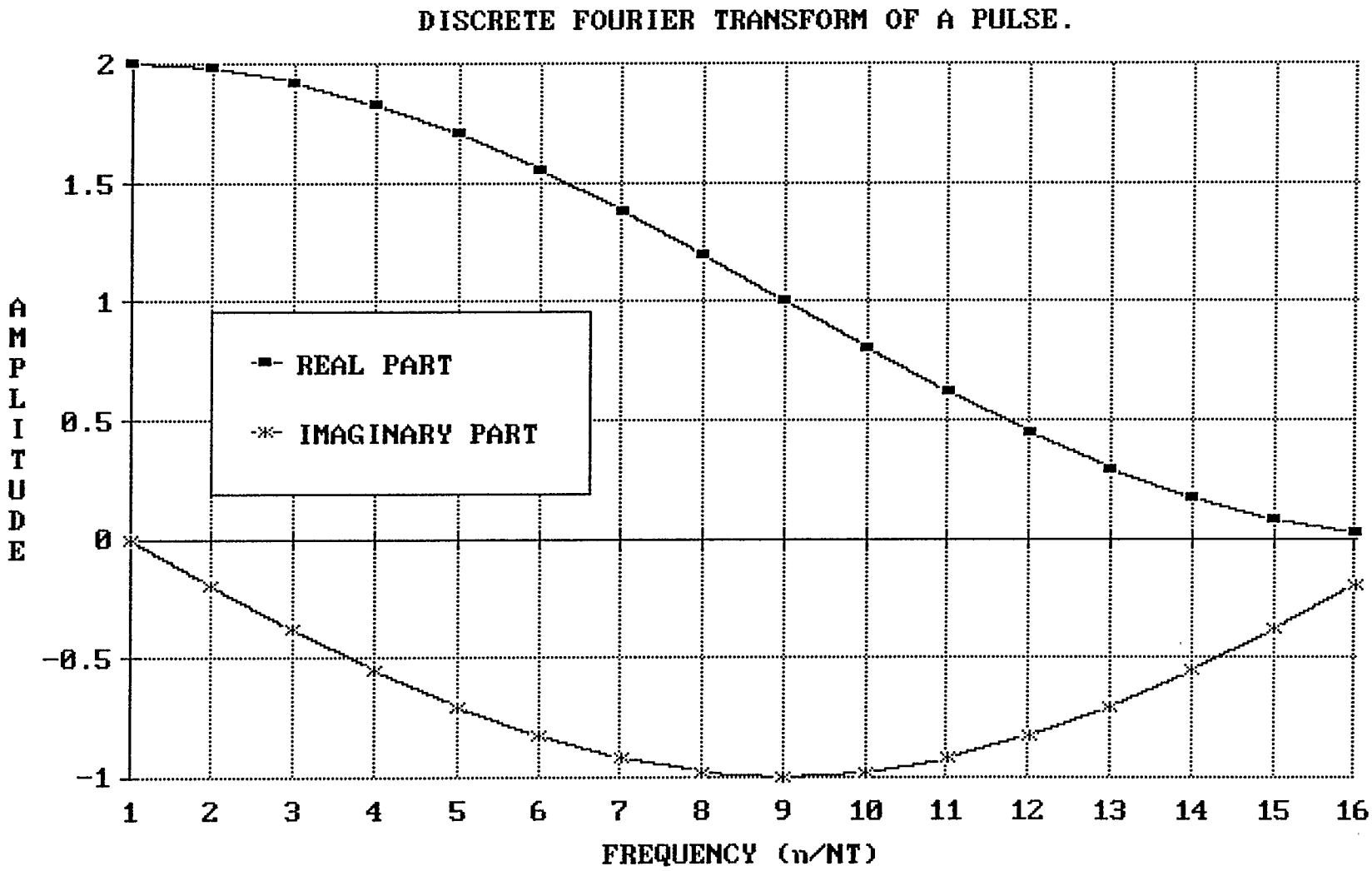

DISCRETE FOURIER TRANSFORM OF A PULSE.
AMPLITUDE
2
1.5
1
0.5
0
-0.5
-1
REAL PART
IMAGINARY PART
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
FREQUENCY (n/NT)

FIG.5.4

about $n = N/2$. This follows from the fact that the real part of the transform is an even function and that results for $n > N/2$ are simply negative frequency results. The imaginary part is an odd function with respect to $n = N/2$, and those results for $n > N/2$ are also to be interpreted as negative frequency results.

In the frequency domain the spectrum of the pulse is multiplied by the pipeline transfer function and then transfered back to the time domain by means of an inverse discrete Fourier transform [5.4].

$$h(kT) = \Delta f \sum_{n=0}^{N-1} X(n) e^{j2\pi nk/N} \quad ; \qquad k = 0,1,\ldots,N-1 \qquad (5.13)$$

where

$$X(n) = [R(n\Delta f) + jI(n\Delta f)]$$

and Δf is the sample interval in frequency. Assuming $N = 32$ and $\Delta f = 6.25$.

Since we know that $R(f)$, the real part of the complex frequency function, must be an even function, we can then fold $R(f)$ about the sample point $n = N/2$. We simply sample the frequency function up to the point $n = N/2$ and then fold these values about $n = N/2$ to obtain the remaining

samples. To simplify the computation process, an alternate inversion formula *[5.4]* is used instead of *(5.13)*, which provides equivalent results.

$$h(kT) = \Delta f \left[\sum_{n=0}^{N-1} X^*(n) e^{j2\pi nk/N} \right]^* \quad ; \quad k = 0,1,\ldots,N-1 \qquad (5.14)$$

To use this formula, we first conjugate the complex frequency function. Since the resulting time function is real, the final conjugation operation illustrated in equation *(5.14)* can be omitted, and we simply compute

$$h(kT) = \Delta f \sum_{n=0}^{N-1} [R(n\Delta f)+j(-1)I(n\Delta f)]e^{j2\pi nk/N} \quad ; \; k=0,1,\ldots,N-1 \qquad (5.15)$$

Computation of *(5.15)* yields a complex function whose imaginary part is approximately zero and whose real part is the transmission network time domain response for different field conditions, as illustrated in *FIG.5.5*. From an examination of this figure it appears that the transmission system would function for distances of up to *30* and possibly *40 Km*. The output pulse of the *50 Km* line has broadened substantially and probably would not be distinguishable amongst multiple ones and zeros.

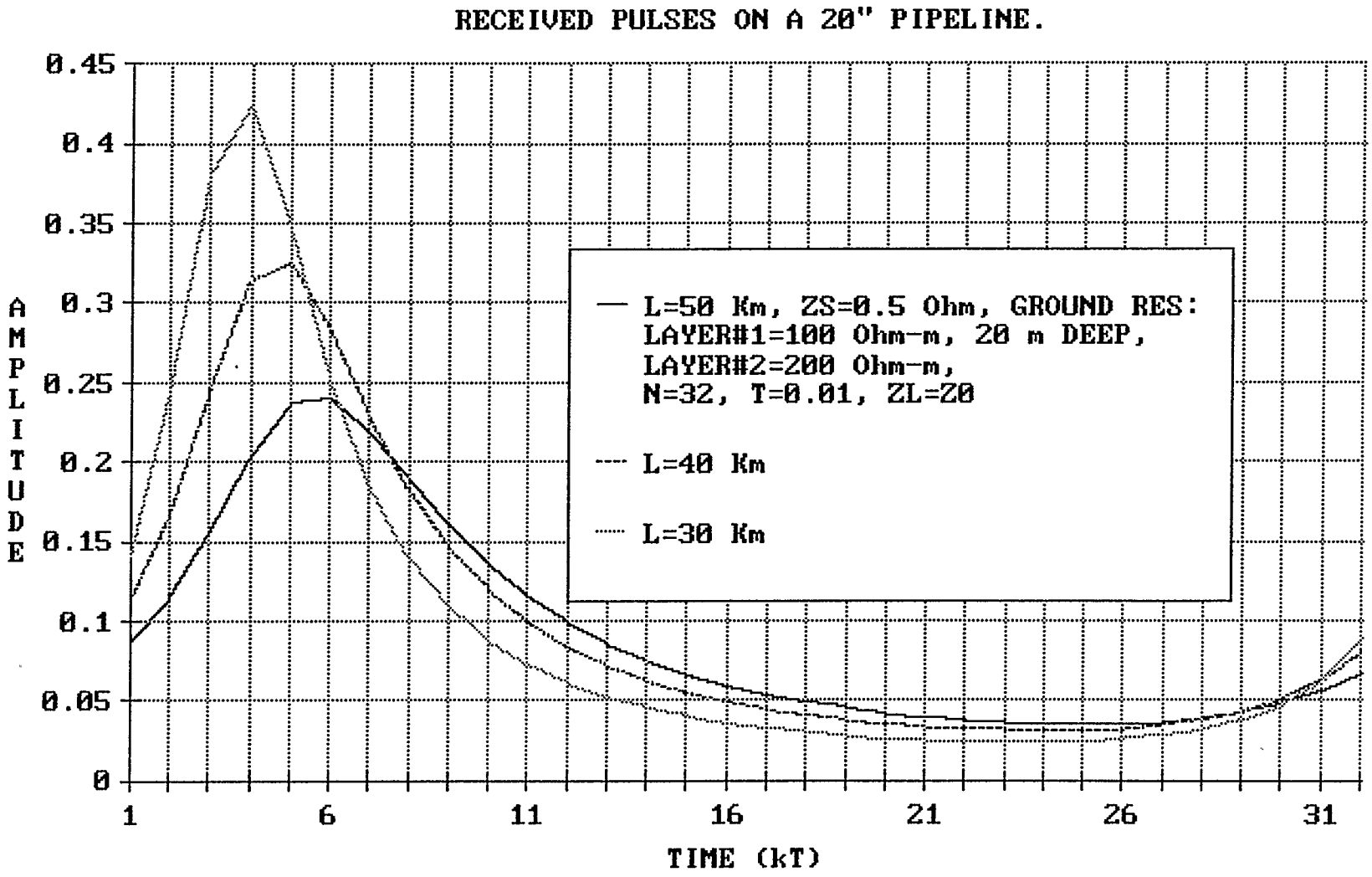

RECEIVED PULSES ON A 20" PIPELINE.
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
AMPLITUDE
1
6
11
16
21
26
31
TIME (kT)
L=50 Km, ZS=0.5 Ohm, GROUND RES:
LAYER#1=100 Ohm-m, 20 m DEEP,
LAYER#2=200 Ohm-m,
N=32, T=0.01, ZL=Z0
L=40 Km
L=30 Km

FIG.5.5

5.4. PULSE CODE MODULATION SIMULATION

A more clear indication of the effects of pulse spreading and pulse amplitude changes on data recognizability can be gained by creating a simulated eye diagram. This diagram is used for evaluation of degradation of data while transmitted on a channel.

This simulation treats Return to Zero Alternate Mark Inversion *(RZAMI)* pulses, one word is one mark and 7 spaces, the other word is alternating mark zero mark, as illustrated in *FIG.5.3* and *FIG.5.6* respectively. These words are treated separately, they are Fourier transformed *(FFT)* and the spectrum is multiplied by the pipeline transfer function and then the results are inverse FFTed. Each of the time domain output waveforms is plotted with one bit time shift and voltage inversions to obtain a comprehensive eye diagram.

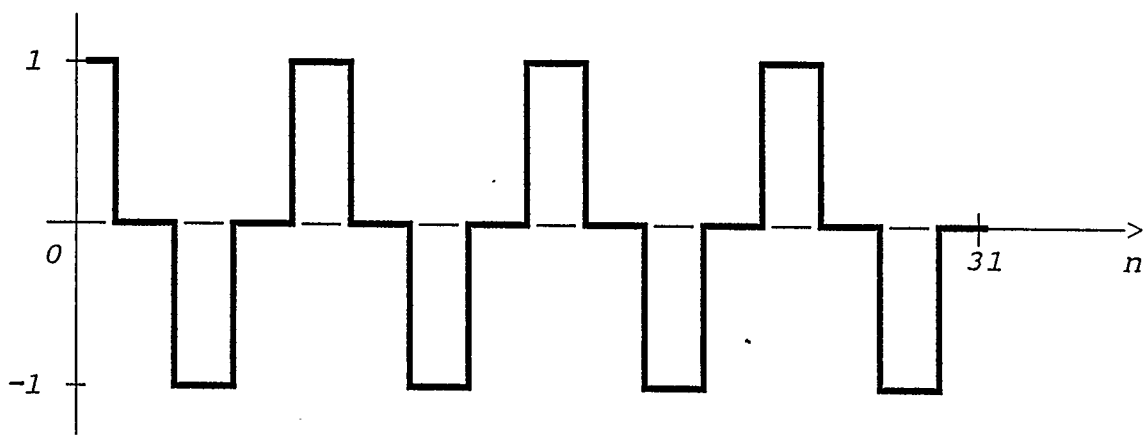


FIG.5.6 RETURN TO ZERO ALTERNATE MARK PULSES.

The final results are illustrated in *FIG.5.7.a-c*. These eye diagrams provide engineers with a powerful analysis tool. The eye diagrams as derived by the computer simulation of a digital signal on a pipeline provide a guide for the evaluation of the pipeline as an electrical transmission medium. The size of the openings in the eye diagram compared with noise levels at the receiver end of the pipeline are critical for good communication. The simulations produce the required eye diagrams for PCM signals over a very large range of variables, such as:

- pipeline length
- line loading such as casing short
- bit rate
- pipeline inside and outside diameters
- pipe permeability
- pipe insulation
- pipe resistivity
- ground bed resistance
- ground bed burial depth
- ground bed length
- ground bed separation
- ground return resistance for multi-layered earth
- layers separation.

The sample eye diagrams indicate that data transmission

EYE DIAGRAM OF AN RZAMI PCM SIGNAL TRANSMITTED
ON A 20", 30 Km PIPELINE, RL=20 Ohm.

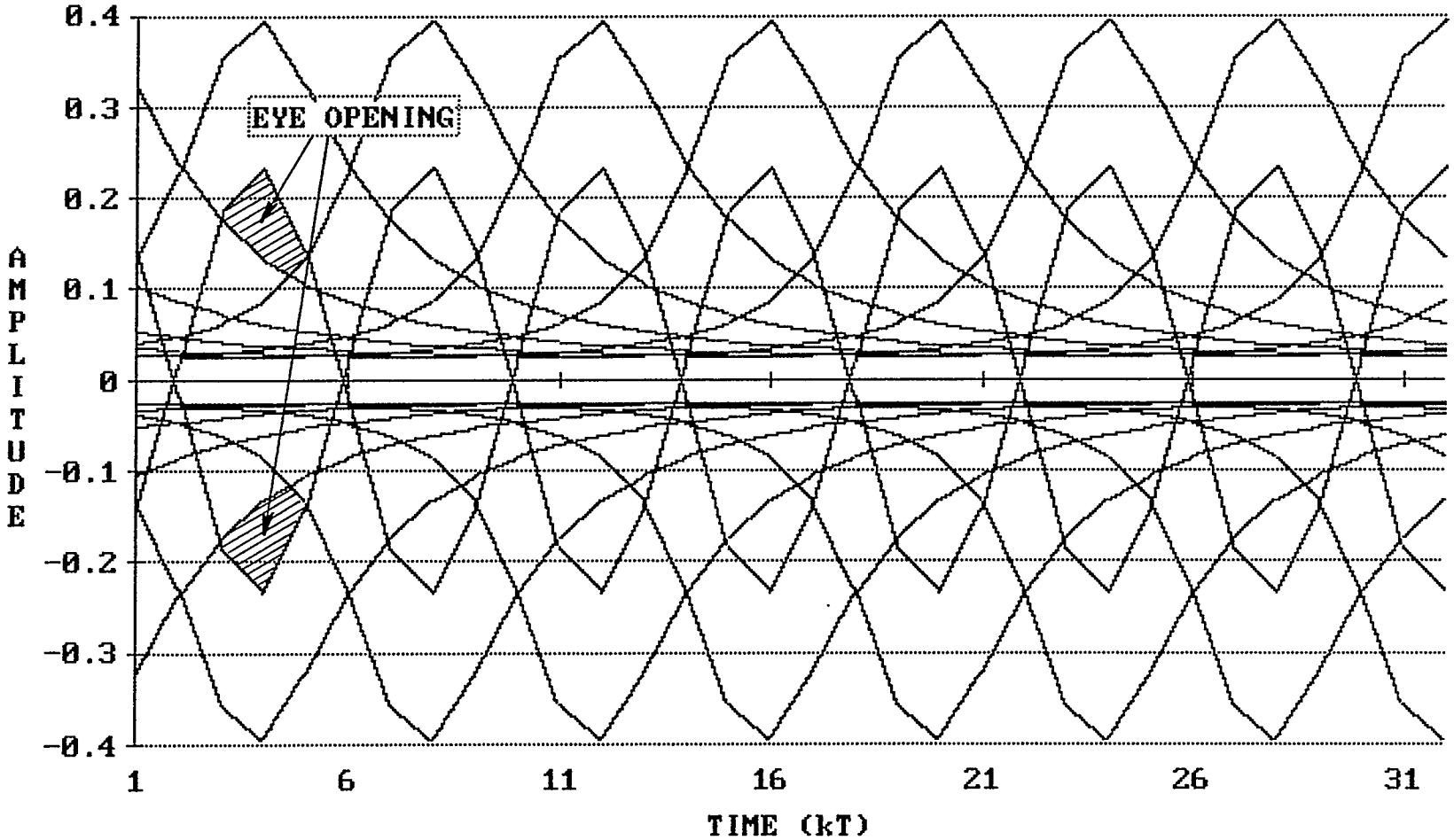


FIG.5.7.a

EYE DIAGRAM OF AN RZAMI PCM SIGNAL TRANSMITTED
ON A 20", 30 Km PIPELINE, RL=6 Ohm.

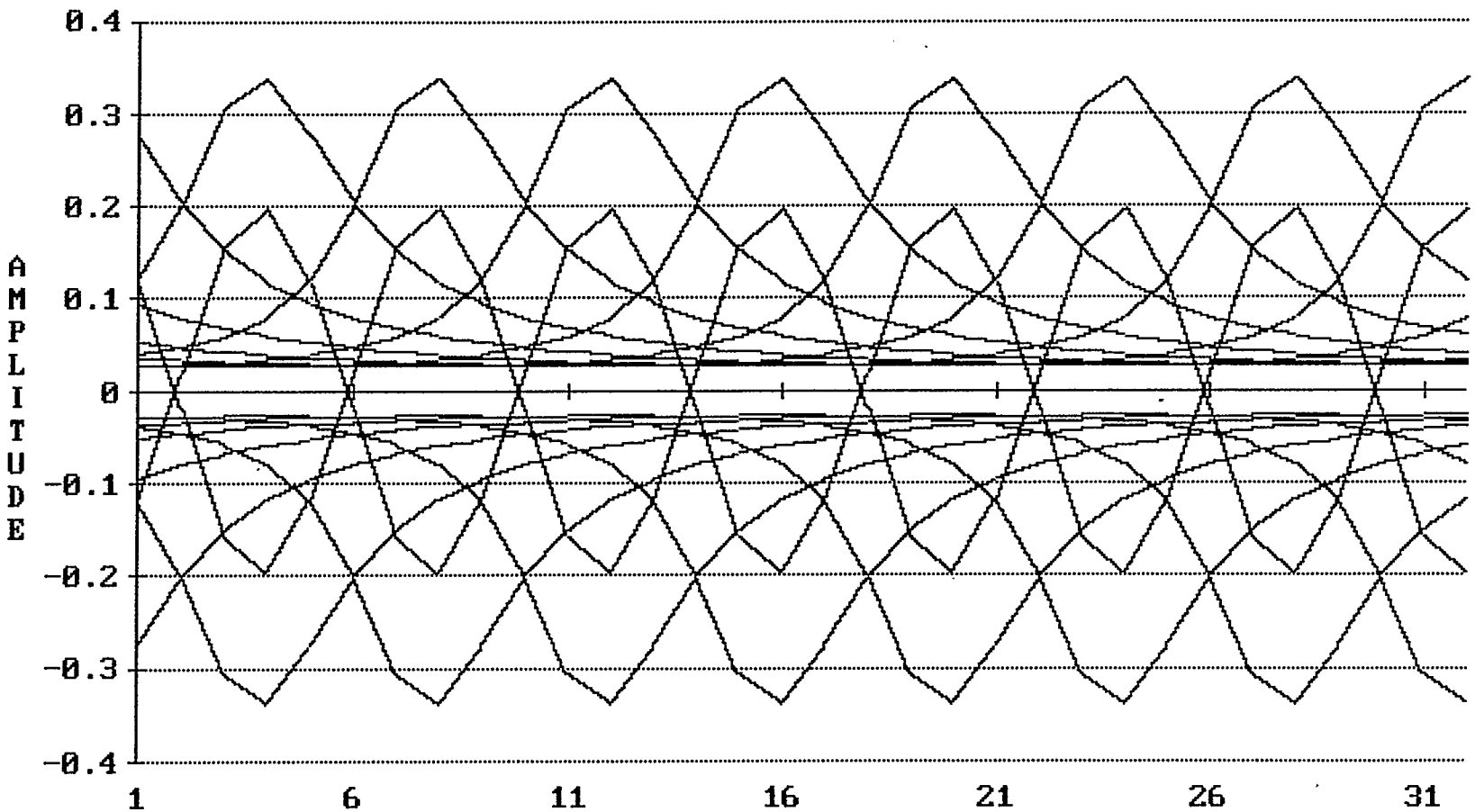


FIG.5.7.b

EYE DIAGRAM OF AN RZAMI PCM SIGNAL TRANSMITTED
ON A 20", 50 Km PIPELINE, NO-LOAD CONDITION.

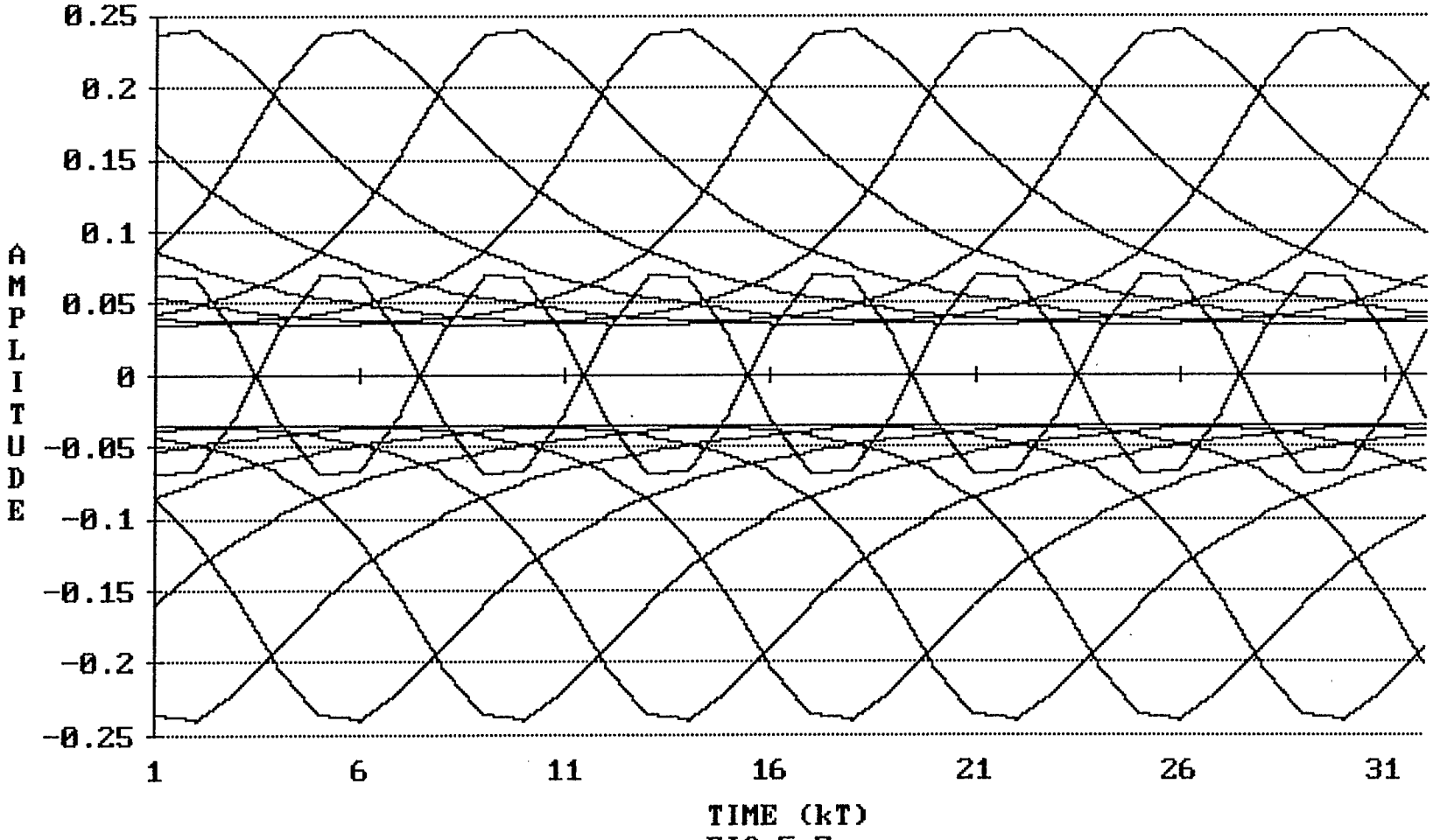


FIG.5.7.c

rate of *12.5 Hz* can be achieved on a *20" 30Km* pipeline.

5.5 TIME-DOMAIN REFLECTOMETRY (TDR)

TDR is a special form of impulse or step-function test in which the signal viewed is the series of reflections produced by imperfections in the transmission line. In earlier frequency-domain reflectometers in which either the standing wave ratio or the reflection coefficient is measured as a function of frequency, the interpretation of the results for what might be causing the reflections is often quite difficult, especially when many reflections are present. In *TDR* the various echos are spread out in time just as in a radar display, the distance along the time axis corresponds to distance down the transmission line, and the relative amplitude of the reflected signal correlates with the magnitude of the impedance discontinuity.

Results of the simulated reflections on a *20"* pipeline are shown in *FIG.5.8*. The pulse is viewed at the load with different values of the load resistance or casing short. The value of the casing short is calculated in the previous chapter, equation *(4.9)*, and it represents realistic field conditions. The transfer function of the network is calculated by equation *(5.11)* and with the aid of *FIG.5.9*. The FFT process, described earlier, is applied

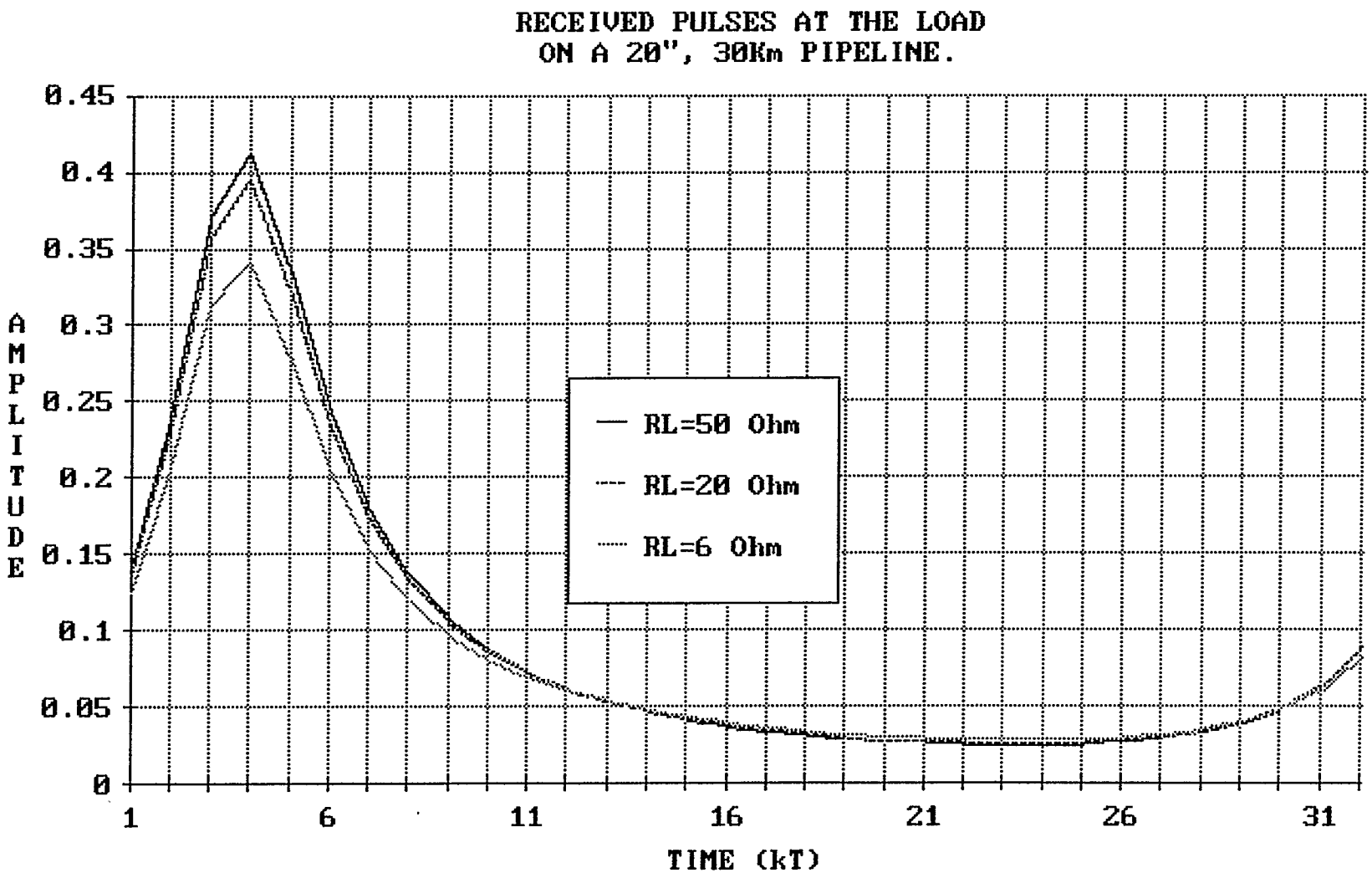

RECEIVED PULSES AT THE LOAD
ON A 20", 30Km PIPELINE.
AMPLITUDE
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
6
11
16
21
26
31
TIME (kT)
RL=50 Ohm
RL=20 Ohm
RL=6 Ohm

FIG.5.8

to the one mark seven zero word which is illustrated in *FIG.5.3*.

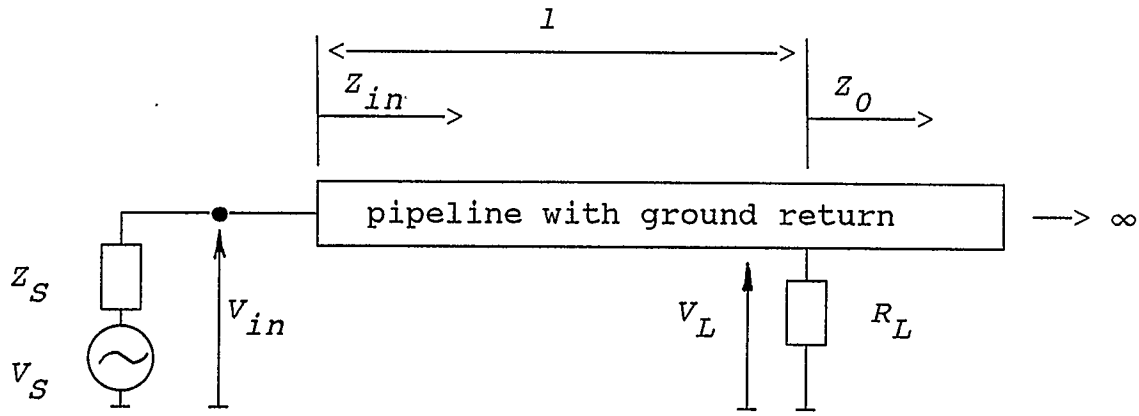


FIG.5.9 TRANSMISSION NETWORK WITH DISCONTINUITY.

Again, equation *(5.9)* is

$$V_{in} = V_S - Z_S I_{in} = V_S - (Z_S/Z_{in})\ V_{in}$$

and the transfer function is given by

$$V_{in} = \frac{V_S \cdot Z_{in}}{Z_{in} + Z_S} \qquad (5.16)$$

where Z_{in} is defined with the aid of *(5.5)* and *(5.6)*

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{\cosh\gamma l + (Z_0/Z_L)\sinh\gamma l}{(1/Z_L)\cosh\gamma l + (1/Z_0)\sinh\gamma l} \qquad (5.17)$$

and

$$Z_L = \frac{R_L \cdot Z_0}{R_L + Z_0} \quad .$$

The reflected pulses are calculated and plotted without

the incident pulse in *FIG.5.9.a*. The transfer function in this case is given by

$$V_{in} = V_S Z_0 / (Z_0 + Z_S) e^{-2\gamma l} \cdot \Gamma \qquad (5.18)$$

where Γ is the complex reflection coefficient defined as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} . \qquad (5.19)$$

Analyzing the time delay, magnitude, and shape of these reflected waveforms under field conditions, would permit us to determine the approximate location and nature of the impedance variation in the pipeline. In this simulation they clearly show that casing short of *6 Ohm* or less can be detected and measured on a *30Km* pipeline. This is illustrated in *FIG.5.9.b* The distance of the short from the signal input can be determined by the the time-domain pulse separation between incident and reflected pulse.

Simulations of three shorts *20 Km* apart are shown in *FIG.5.10.a* and the effect of load variation is further emphasized in *FIG.5.10.b*. The transfer function in this case is calculated with the aid of *FIG.5.11*.

TDR OF ONE SHORT
ON A 20", 30Km PIPELINE.

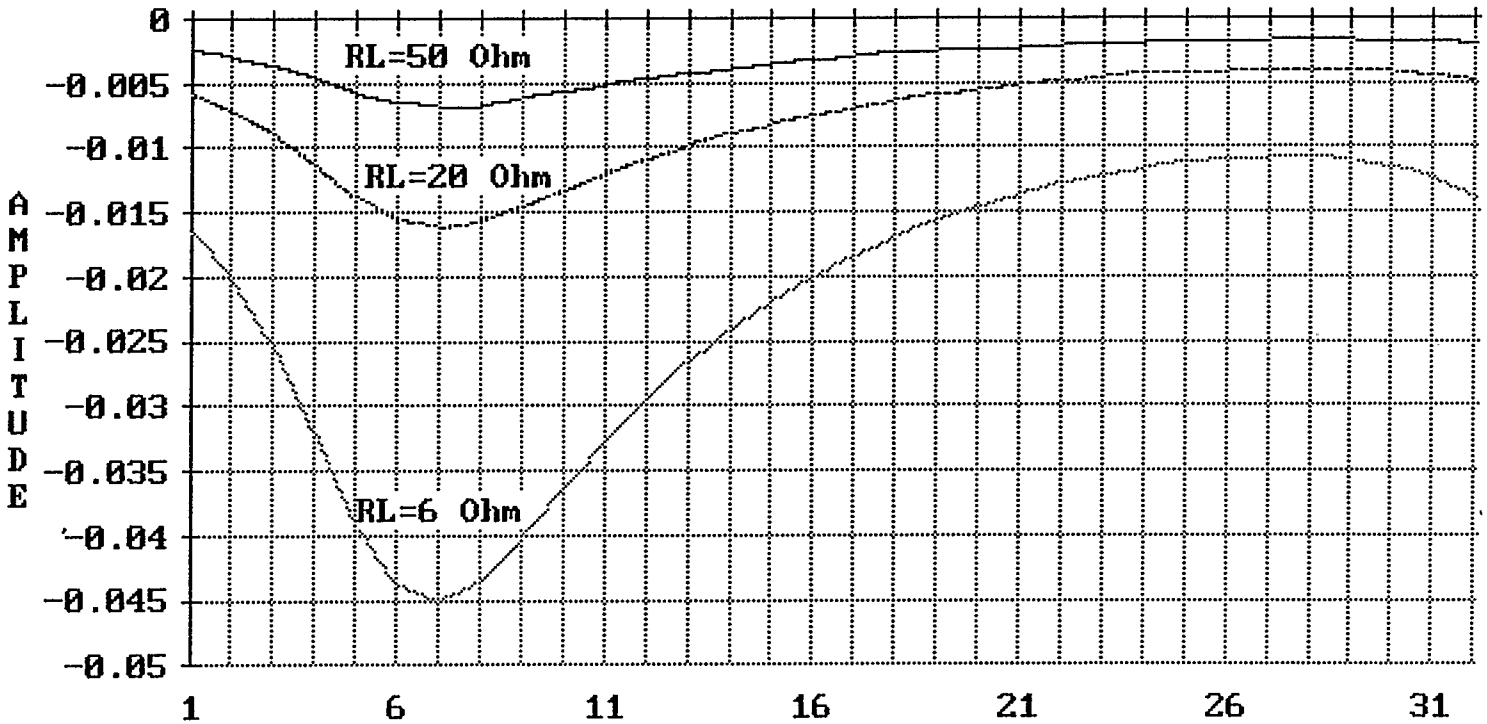


FIG.5.9.a

TDR OF ONE SHORT
ON A 20" PIPELINE, RL=6 Ohm.

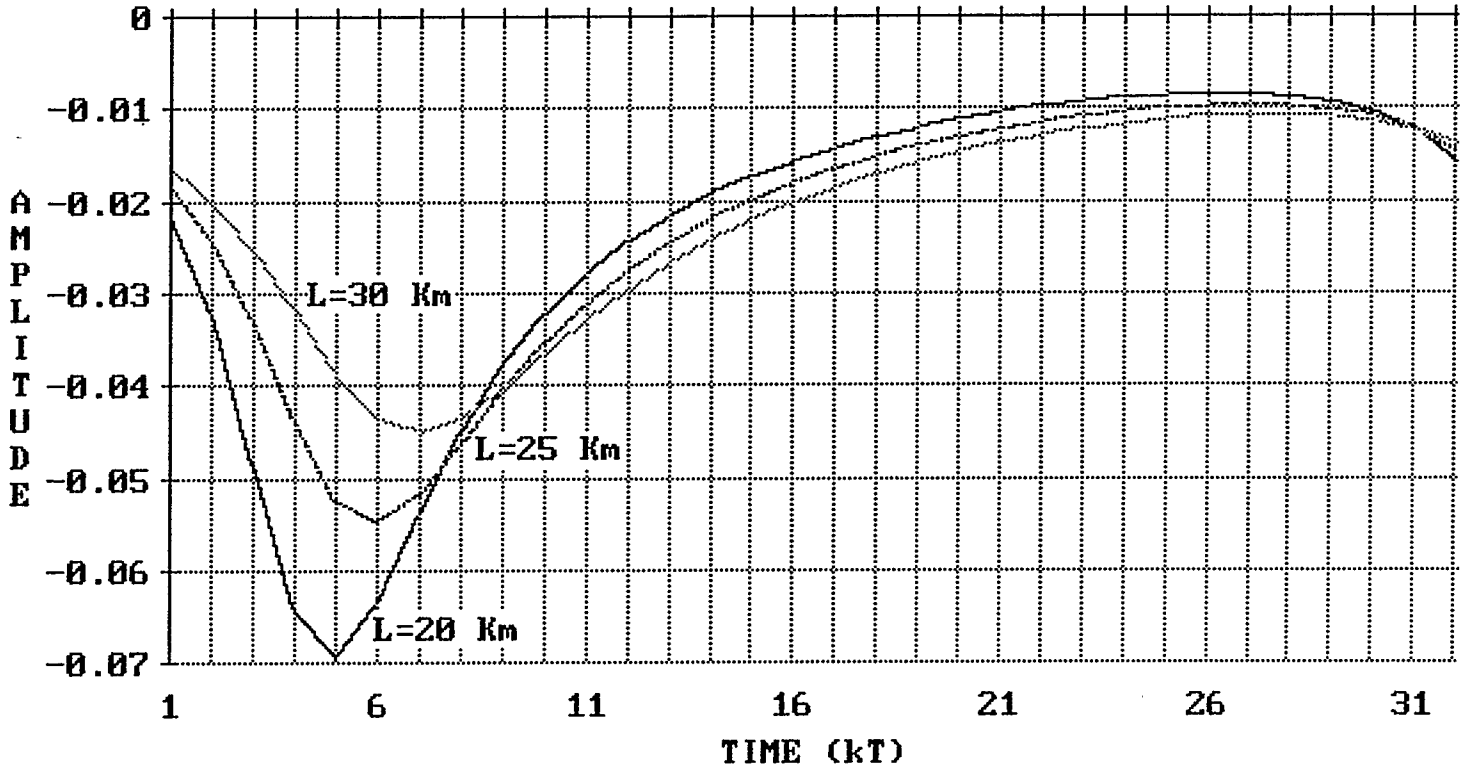


FIG.5.9.b

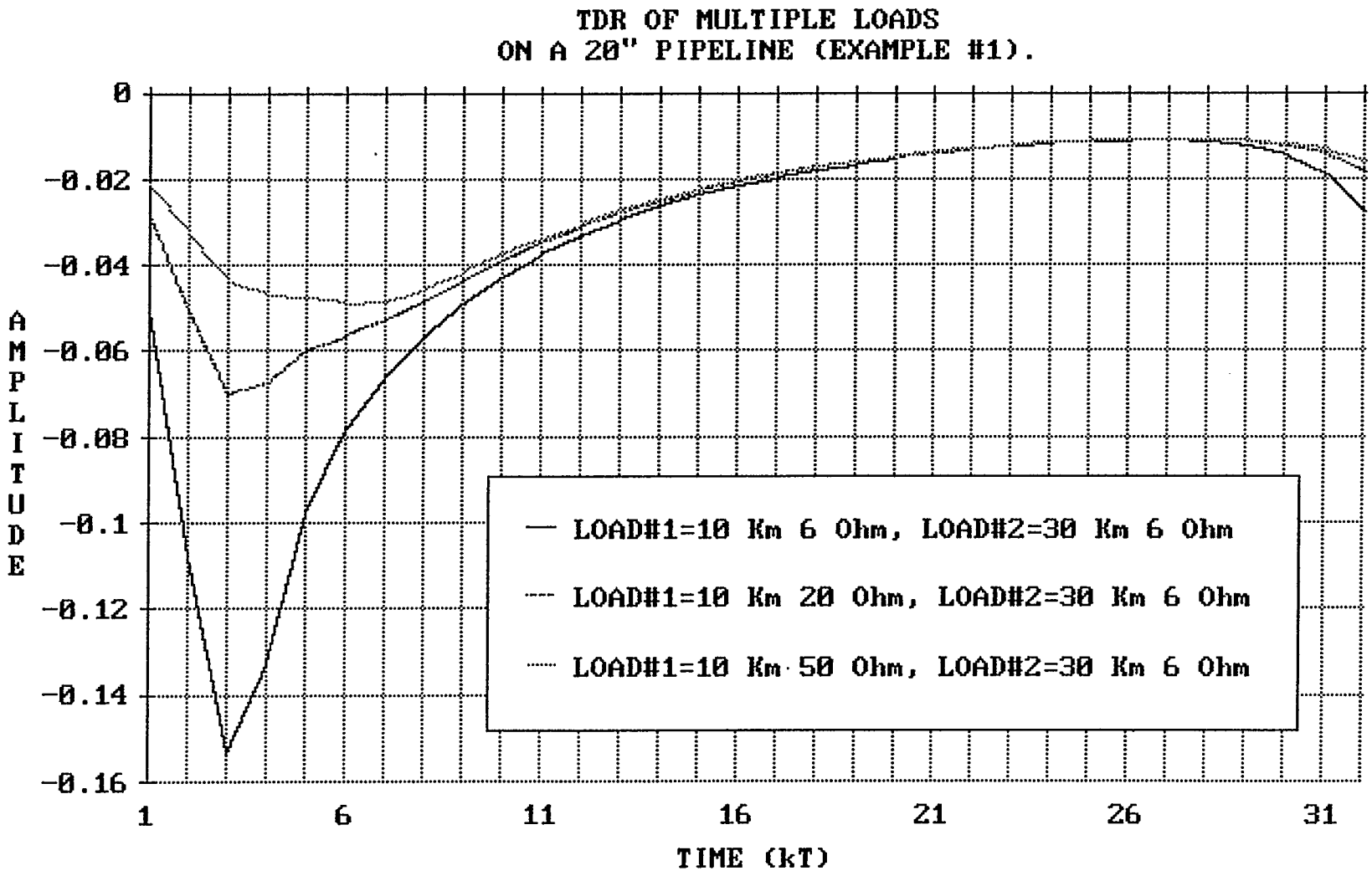

TDR OF MULTIPLE LOADS
ON A 20" PIPELINE (EXAMPLE #1).
AMPLITUDE
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
1
6
11
16
21
26
31
TIME (kT)
LOAD#1=10 Km 6 Ohm, LOAD#2=30 Km 6 Ohm
LOAD#1=10 Km 20 Ohm, LOAD#2=30 Km 6 Ohm
LOAD#1=10 Km 50 Ohm, LOAD#2=30 Km 6 Ohm

FIG.5.10.a

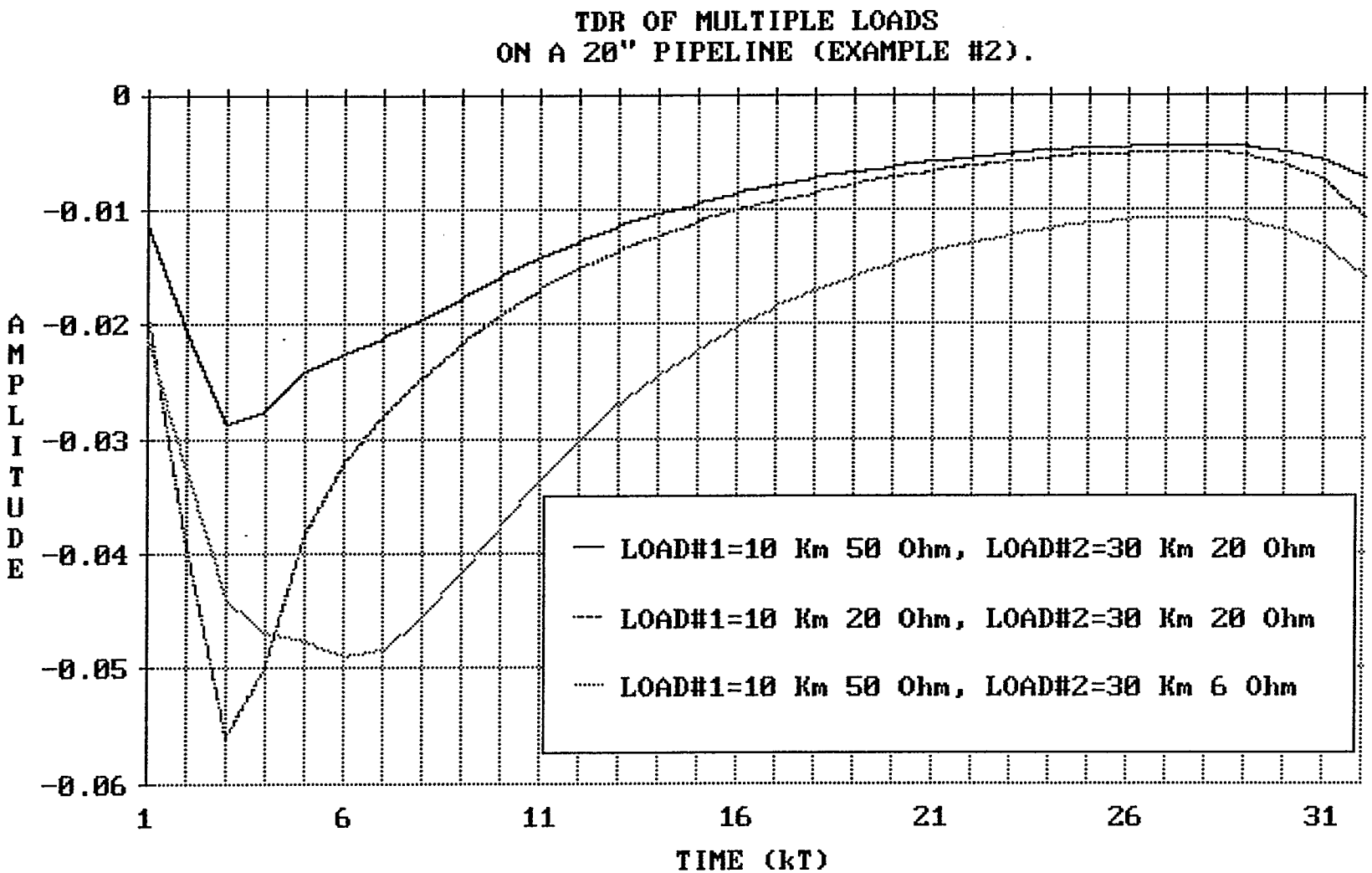

TDR OF MULTIPLE LOADS
ON A 20" PIPELINE (EXAMPLE #2).
AMPLITUDE
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
1
6
11
16
21
26
31
TIME (kT)
LOAD#1=10 Km 50 Ohm, LOAD#2=30 Km 20 Ohm
LOAD#1=10 Km 20 Ohm, LOAD#2=30 Km 20 Ohm
LOAD#1=10 Km 50 Ohm, LOAD#2=30 Km 6 Ohm

FIG.5.10.b

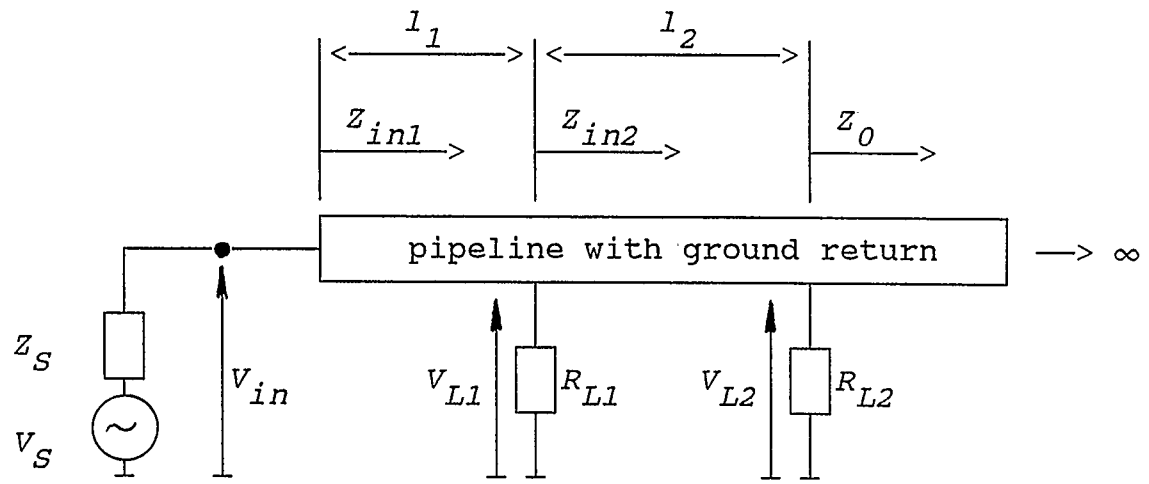


FIG.5.11 TRANSMISSION NETWORK WITH TWO LOADS.

Reflected pulses are again calculated and separated from the incident pulse and the transfer function can be calculated as follows. First, we find the input impedance of the above network

$$Z_{in1} = \frac{V_{in}}{I_{in}} = \frac{\cosh\gamma l_1 + (Z_0/Z_{L1})\sinh\gamma l_1}{(1/Z_{L1})\cosh\gamma l_1 + (1/Z_0)\sinh\gamma l_1} \qquad (5.19)$$

where

$$Z_{L1} = \frac{R_{L1} \cdot Z_{in2}}{R_{L1} + Z_{in2}}$$

and

$$Z_{in2} = \frac{\cosh\gamma l_2 + (Z_0/Z_{L2})\sinh\gamma l_2}{(1/Z_{L2})\cosh\gamma l_2 + (1/Z_0)\sinh\gamma l_2}$$

where

$$Z_{L2} = \frac{R_{L2} \cdot Z_0}{R_{L2} + Z_0} \quad .$$

The transfer function of this multiple reflection case is then given by

$$V_{in} = \frac{V_S \cdot Z_0}{Z_0 + Z_S} \cdot \frac{Z_{in1} - Z_0}{Z_{in1} + Z_0} \qquad (5.20)$$

where Z_{in1} is defined by *(5.19)*. The above transfer function shows that the initial voltage pulse at the input terminals of the line is

$$V_{in} = \frac{V_S \cdot Z_0}{Z_0 + Z_S} \, ,$$

since the input impedance of the line is equal to its characteristic impedance when there is pulse traveling in only one direction on the line. This initial pulse then will undergo multiple reflections, caused by the loads, and this effect is taken care of by the second term of equation *(5.20)*.

Comparison of the simulated data and form of *FIG.5.10.a* and *FIG.5.10.b* shows that multiple casing shorts of 6 to 50

Ohm, 20 Km apart can be detected, if the order and location of the shorts are favorable to the simulation, that is, they must be in the range of the applied *FFT* process accuracy. Simulation accuracy may be improved by increasing the word length of the applied *FFT*.

CHAPTER 6

DETERMINATION OF TRANSMISSION LINE CHARACTERISTICS FROM IMPEDANCE MEASUREMENTS

6.1 GENERAL

During the course of this study a series of measurements were conducted at the University of Calgary to prove the correctness of the methodologies adapted in the previous chapters for impedance and capacitance calculation. The experiment was conducted on a #18 AWG insulated wire with three different arrangements:

1.) Wire was laid on the ground on a straight line.

2.) Lifted *0.1 m* above ground.

3.) Lifted *0.2 m* above ground and looped back to itself.

Our objective was to obtain characteristic values from the measurement of the wire, from which we can find the capacitance as a function of distance above ground. Furthermore, our expectation was that the results obtained could be tabulated and applied for the case of a pipe buried underground. The measurements were made in the frequency domain and we used high frequencies so that the wire would simulate a pipeline with low frequency excitation. Setup of the measurement is shown in *FIG.6.1*.

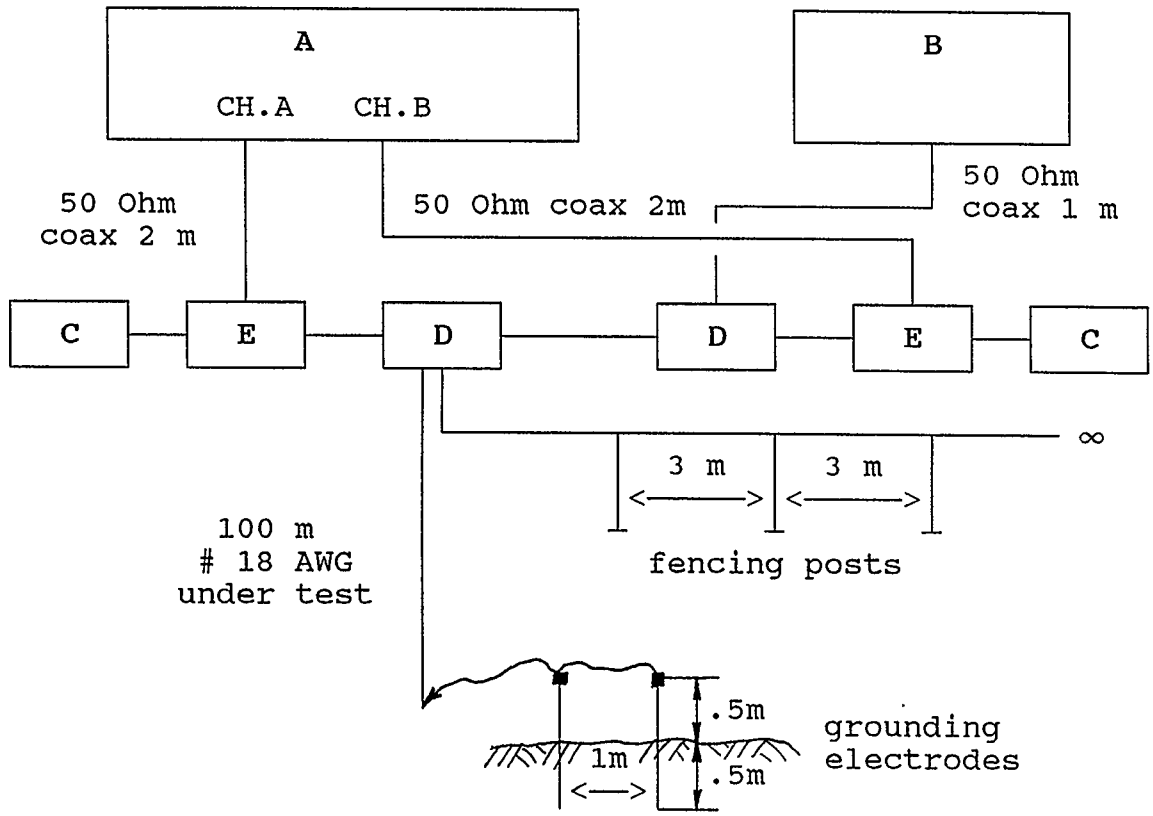


FIG.6.1 IMPEDANCE MEASUREMENT ON A #18 AWG WIRE.

6.1.1 LIST OF INSTRUMENTS:

A ; Vector Voltmeter
Model No.: HP 8405A
Scale on meter A : -70 to +10 db
Scale on meter B : 0 to 180 degree

B ; Signal Generator
Model No.: Marconi Inst. #2022
Output Impedance = 50 Ohm

C ; 50 Ohm Termination

D ; Directional Coupler MCL
Model No.: ZFDC-20-5
20 db coupling

E ; HP 50 Ohm Tee

6.2 MEASUREMENTS AND CALCULATIONS

Transmitted and reflected signal magnitude and phase angle values along with the calibration data are recorded and tabulated in the attached computer program data segments. From this data we calculate the open and short circuit impedance of the wire. When the wire or transmission line is terminated in an open circuit or a short circuit, its input impedance is determined by the propagation factors α and β, the characteristic impedance Z_0 and the line length l. The input impedance is defined with the aid of *(5.5)* and *(5.6)*

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{\cosh\gamma l + (Z_0/Z_L)\sinh\gamma l}{(1/Z_L)\cosh l + (1/Z_0)\sinh\gamma l} \tag{6.1}$$

and the normalized input impedance is

$$\frac{Z_{in}}{Z_0} = \frac{Z_L/Z_0 + \tanh\gamma l}{1 + (Z_L/Z_0)\tanh\gamma l} \ . \tag{6.2}$$

It is easily seen that in the simplest case of $Z_L = Z_0$, this above equation reduces to $Z_{in}/Z_0 = 1$, consistent with transmission line terminated by a matched load. When $Z_L = \infty$, the input impedance Z_{oc} of a line of length l with an open circuit termination is

$$Z_{oc} = Z_0 coth\gamma l \quad . \tag{6.3}$$

The input impedance Z_{sc} of the same line with short circuit termination is

$$Z_{sc} = Z_0 tanh\gamma l \quad . \tag{6.4}$$

If Z_{sc} and Z_{oc} are measured at the same frequency, then Z_0, α,β and l will have the same values in both of the equation *(6.3)* and *(6.4)*. Multiplying together the corresponding sides of these equations gives for a general case

$$Z_0 = (Z_{sc}Z_{oc})^{1/2} \quad . \tag{6.5}$$

In our case $Z_L = 0$ cannot be physically achieved, any grounding electrode has a definite resistance, as shown in *Chapter 4*. The measured value of the rods in parallel were *110 Ohm*, which is in good agreement with the theoretically obtainable values shown in *FIG.4.4* and *FIG.4.6*. This measured resistance has to be accounted for, and the general solution for equation *(6.5)* is not applicable. Therefore, Z_0 is derived with the aid of *(6.2)* and *(6.3)*, and the result is

$$Z_0 = [Z_{in}(Z_{oc} + Z_L) - Z_L Z_{oc}]^{1/2} \quad . \tag{6.6}$$

The propagation constant γ can also be calculated from the measured impedances Z_{oc} and Z_{in}. Using equation *(6.4)*

$$\tanh\gamma l = \frac{Z_0}{Z_{oc}} = \frac{1 - e^{-2\gamma l}}{1 + e^{-2\gamma l}} \tag{6.7}$$

which gives

$$e^{2\gamma l} = \frac{1 + Z_0/Z_{oc}}{1 - Z_0/Z_{oc}} \quad . \tag{6.8}$$

Taking logarithms of both sides gives

$$\gamma = \frac{1}{2l} \ln \left\{ \frac{1 + Z_0/Z_{oc}}{1 - Z_0/Z_{oc}} \right\} \quad . \tag{6.9}$$

From *(5.7)*, *(5.8)* and *(6.9)* we can determine the capacitance per unit length

$$C = \gamma/(j\omega Z_0) \, . \tag{6.10}$$

FIG.6.2.1 to *FIG.6.2.3* illustrate the results which are calculated in the attached computer program for the #18 AWG wire. These results show a good deal of scatter as a function of frequency. Also shown on these charts is the theoretical calculation of Z_0 based on the theory of the

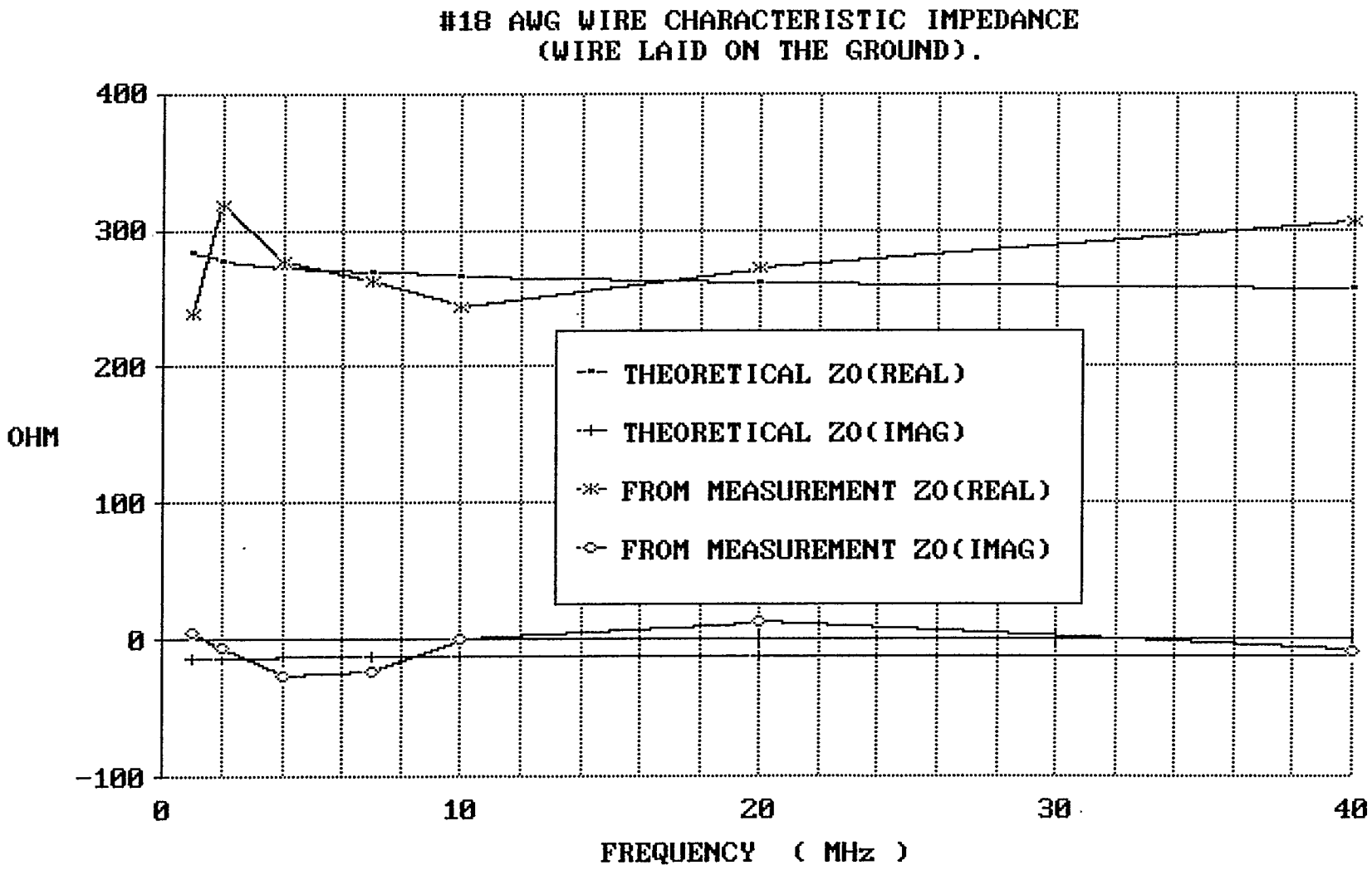

#18 AWG WIRE CHARACTERISTIC IMPEDANCE
(WIRE LAID ON THE GROUND).
400
300
200
100
0
-100
OHM
0
10
20
30
40
FREQUENCY (MHz)
THEORETICAL ZO(REAL)
THEORETICAL ZO(IMAG)
FROM MEASUREMENT ZO(REAL)
FROM MEASUREMENT ZO(IMAG)

FIG.6.2.1

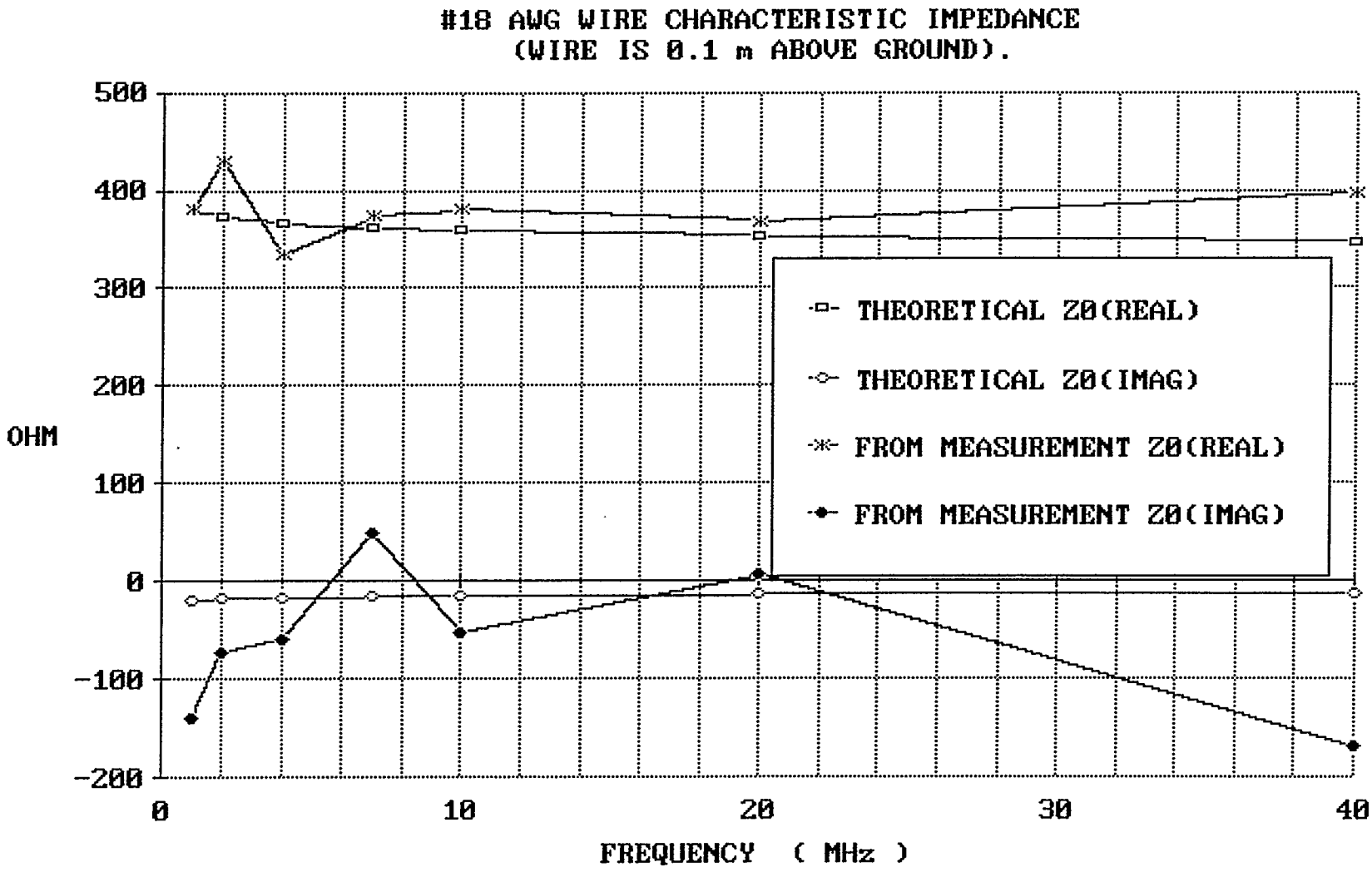

#18 AWG WIRE CHARACTERISTIC IMPEDANCE
(WIRE IS 0.1 m ABOVE GROUND).
500
400
300
200
100
0
-100
-200
OHM
0
10
20
30
40
FREQUENCY (MHz)
THEORETICAL Z0(REAL)
THEORETICAL Z0(IMAG)
FROM MEASUREMENT Z0(REAL)
FROM MEASUREMENT Z0(IMAG)

FIG.6.2.2

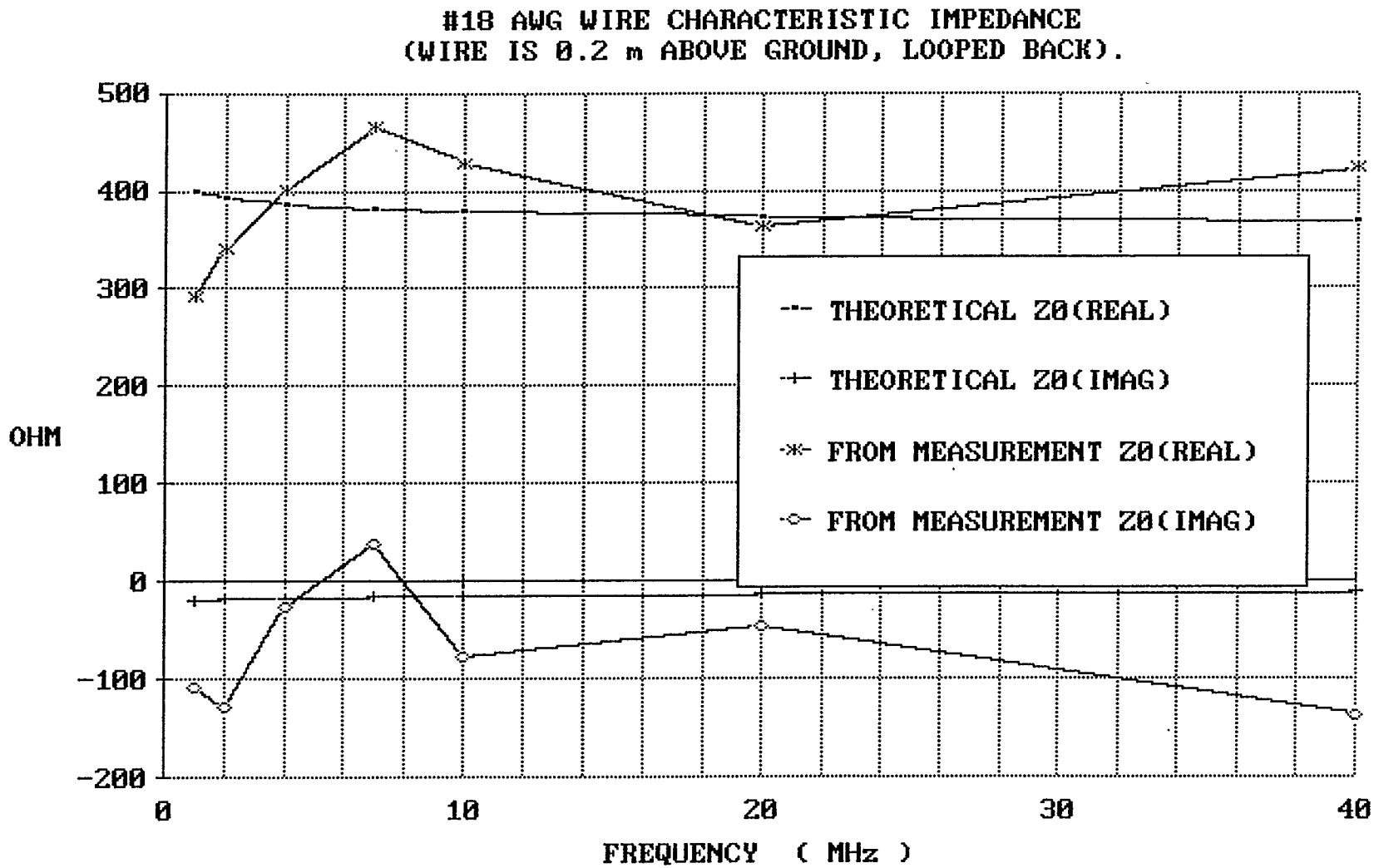

#18 AWG WIRE CHARACTERISTIC IMPEDANCE
(WIRE IS 0.2 m ABOVE GROUND, LOOPED BACK).
500
400
300
200
100
0
-100
-200
OHM
0
10
20
30
40
FREQUENCY (MHz)
THEORETICAL Z0(REAL)
THEORETICAL Z0(IMAG)
FROM MEASUREMENT Z0(REAL)
FROM MEASUREMENT Z0(IMAG)

FIG.6.2.3

foregoing chapters. The probable cause of the scatter is the measurement frequencies and the near surface layering of the earth. The frequency was high and therefore the skin-depth of the ground return was small, in the range of *0.5 - 4 m*. Dipole-dipole apparent resistivity measurements on the earth surface, with change of dipole separation also revealed a highly variable ground resitivity with depth. At such a small skin-depth the ground layer variation will dominate the impedance variation as the frequency varies. In order to obtain reliable results we would have to make the measurement over a homogeneous ground that is homogeneous over the full range of skin-depths.

Furthermore simple analysis indicated that the pipeline insulation completely dominates the capacitive susceptance term in the transmission line equations. Capacitive radial currents and radial resistances in the earth have negligible effect compared with pipeline insulation capacitance.

Our conclusions, resulting from the test, is that for definitive results further measurements are needed under more favorable conditions. These conditions would be high frequency measurement over previously quantified uniform earth. Values derived from our test should only be used as a general indication of the correctness of the foregoing theory.

CHAPTER 7

SUMMARY AND CONCLUSIONS

This work has used existing theory from a broad range of fields to investigate an original topic of research. These theories had to be adapted to fit the research topic and the computer simulation applied to the problem.

The mathematical development has been given for modeling an isolated buried pipeline as an electrical transmission line. The derivations also include mathematical approximations which are required for evaluating complex trancendental functions. Both field theory and circuit theory have been employed to provide a simple and accurate computer model. In one case we see that the exact calculation of skin effect and multi-layer earth return must be performed by a computer to be practical. In another case we see that earth layered effects for ground rod resistance calculation are neglected to keep the scope of work manageable. A number of useful formulae which have been developed over the years for calculation of series impedances are derived and presented. When multiple solutions or approximations were encountered during the derivation process, this work presented them in increasing accuracy.

Computer simulations performed on sample pipelines show that the pipeline may be used as a communication channel.

The simulations provide a method for designing and evaluating this communication channel under a wide variety of conditions and determining its feasibility before installation.

Time domain reflectometry methods appear to have applicability to holiday detection in pipelines. Time domain reflectometry methods require a significant impedance discontinuity at the holiday. Thus the holiday itself must have a low resistance to ground because the pipeline characteristic impedance is low ($Z_0 \approx 3$ *Ohm*). An initial coating resistance profile along a new pipeline could serve as a reference to which similar data taken in later years may be compared. Such comparison could reveal information on the long term performance of the coating.

A substantial amount, about *50%*, of all pipeline leakages occur at road and river crossings. In a road crossing failure the pipeline first contacts the pipeline casing. This has a low impedance to ground and should give a good impedance discontinuity to give a good reflected pulse. In a typical scenario of a failing river crossing, the pipeline "floats" out of the river bed into the river at spring break-up. Rocks and debris being carried down the river remove the coating, giving a large surface connection to the ground via the water. This again should give a good return signal for *TDR* measurement.

It is not anticipated that small holidays will be

detectable with *TDR*, and due to pulse dispersion within the pipeline (as a transmission channel) the time delay of the returned pulse will give only an approximate location of the large holiday.

It would appear that *TDR* holiday detection methods merit further investigation with possibly a field trial.

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APPENDIX A

1. BESSEL FUNCTIONS OF ORDER ZERO, WITH SMALL VALUES OF x (x < 10)

1.1 BESSEL FUNCTION OF THE FIRST KIND OF ORDER ZERO

$$ber\ x = 1 - \frac{(1/2\ x)^4}{(2!)^2} + \frac{(1/2\ x)^8}{(4!)^2} - \ldots$$

$$+ (-1)^{n-1} \frac{(1/2\ x)^{4(n-1)}}{(2n-2)!(2n-2)!} + \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1/2\ x)^{4(n-1)}}{(2n-2)!(2n-2)!}$$

$$bei\ x = \frac{(1/2\ x)^2}{(1!)^2} - \frac{(1/2\ x)^6}{(3!)^2} + \frac{(1/2\ x)^{10}}{(5!)^2} - \ldots$$

$$+ (-1)^{n-1} \frac{(1/2\ x)^{4n-2}}{(2n-1)!(2n-1)!} + \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1/2\ x)^{4n-2}}{(2n-1)!(2n-1)!}$$

$$ber'x = -\frac{2(1/2\ x)^3}{(2!)^2} + \frac{4(1/2\ x)^7}{(4!)^2} - \frac{6(1/2\ x)^{11}}{(6!)^2} - \ldots$$

$$+ (-1)^n \frac{2n(1/2\ x)^{4n-1}}{(2n)!(2n)!} + \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{2n(1/2\ x)^{4n-1}}{(2n)!(2n)!}$$

$$bei'x = \frac{1/2\ x}{(1!)^2} - \frac{3(1/2\ x)^5}{(3!)^2} + \frac{5(1/2\ x)^9}{(5!)^2} - \ldots$$

$$+ (-1)^{n-1} \frac{(2n-1)(1/2\ x)^{4n-3}}{(2n-1)!(2n-1)!} + \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)(1/2\ x)^{4n-3}}{(2n-1)!(2n-1)!}$$

1.2 BESSEL FUNCTION OF THE SECOND KIND OF ORDER ZERO

$$ker\ x = ber\ x(ln2 - \gamma - lnx) + \frac{\pi}{4} bei\ x - (1+1/2)\frac{(1/2\ x)^4}{(2!)^2}$$

$$+ (1 + 1/2 + 1/3 + 1/4) \frac{(1/2\ x)^8}{(4!)^2} - \ldots$$

$$+ (-1)^n (1 + 1/2 + 1/3 \ldots + 1/2n) \frac{(1/2\ x)^{4n}}{(2n)!(2n)!} + \ldots$$

$$= \text{ber}\ x(\ln 2 - \gamma - \ln x) + \frac{\pi}{4} \text{bei}\ x +$$

$$+ \sum_{n=1}^{\infty} (-1)^n (1 + \frac{1}{2n}) \frac{(1/2\ x)^{4n}}{(2n)!(2n)!}$$

$$\text{kei}\ x = \text{bei}\ x(\ln 2 - \gamma - \ln x) - \frac{\pi}{4} \text{ber}\ x + \frac{(1/2\ x)^2}{(1!)^2}$$

$$- (1 + 1/2 + 1/3) \frac{(1/2\ x)^6}{(3!)^2} - \ldots$$

$$+ (-1)^{n-1} (1 + 1/2 + 1/3 \ldots + \frac{1}{2n-1}) \frac{(1/2\ x)^{4n-2}}{(2n-1)!(2n-1)!}$$

$$= \text{ber}\ x(\ln 2 - \gamma - \ln x) + \frac{\pi}{4} \text{bei}\ x +$$

$$+ \sum_{n=1}^{\infty} (-1)^{n-1}(1 + \frac{1}{2n-1}) \frac{(1/2\ x)^{4n-2}}{(2n-1)!(2n-1)!}$$

$$ker'x = ber'x(ln2 - \gamma - lnx) - \frac{1}{x} ber\ x + \frac{\pi}{4} bei'x$$

$$-(1+1/2)\frac{(1/2\ x)^3}{2!1!} + (1 + 1/2 + 1/3 + 1/4)\frac{(1/2\ x)^7}{4!3!} -$$

$$\ldots + (-1)^n(1+ 1/2 + 1/3 \ldots + \frac{1}{2n}) \frac{(1/2\ x)^{4n-1}}{(2n)!(2n-1)!}$$

$$= ber'x(ln2 - \gamma - lnx) - \frac{1}{x} ber\ x + \frac{\pi}{4} bei'x$$

$$+ \sum_{n=1}^{\infty} (-1)^n(1 + \frac{1}{2n}) \frac{(1/2\ x)^{4n-1}}{(2n)!(2n-1)!}$$

$$kei'x = bei'x(ln2 - \gamma - lnx) - \frac{1}{x} bei\ x - \frac{\pi}{4} ber'x$$

$$+ \frac{1/2\ x}{(1!)^2} - (1 + 1/2 + 1/3\ ) \frac{(1/2\ x)^5}{3!2!} + \ldots$$

$$+ (-1)^{n-1}(1 + 1/2 + 1/3 \ldots + \frac{1}{2n-1}) \frac{(1/2\ x)^{4n-3}}{(2n-1)!(2n-2)!}$$

$$= bei'x(ln2 - \gamma - lnx) - \frac{1}{x} bei\ x - \frac{\pi}{4} ber'x$$

$$+ \sum_{n=1}^{\infty} (-1)^{n-1}(1 + \frac{1}{2n-1}) \frac{(1/2\ x)^{4n-3}}{(2n-1)!(2n-2)!}$$

where γ is the Euler's constant ; $\gamma = 0.5772157$

2. BESSEL FUNCTIONS OF ORDER ZERO, WITH LARGE VALUES OF x (x > 10)

$$ber\ x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left\{ L_0(x)\ cos(\frac{x}{\sqrt{2}} - \frac{\pi}{8}) - M_0(x)\ sin(\frac{x}{\sqrt{2}} - \frac{\pi}{8}) \right\}$$

$$bei\ x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left\{ M_0(x)\ cos(\frac{x}{\sqrt{2}} - \frac{\pi}{8}) + L_0(x)\ sin(\frac{x}{\sqrt{2}} - \frac{\pi}{8}) \right\}$$

where

$$L_0(x) = 1 + \frac{1^2}{1!8x} cos\frac{\pi}{4} + \frac{1^2 \cdot 3^2}{2!(8x)^2} cos\frac{2\pi}{4} + \ldots$$

$$+ \frac{1^2 \cdot 3^2 \ \ldots (2p-1)^2}{p!\,(8x)^p} \cos \frac{p\pi}{4} + \ldots$$

$$M_0(x) = - \frac{1^2}{1!8x} \sin \frac{\pi}{4} - \frac{1^2 \cdot 3^2}{2!\,(8x)^2} \sin \frac{2\pi}{4} - \ldots$$

$$- \frac{1^2 \cdot 3^2 \ \ldots (2p-1)^2}{p!\,(8x)^p} \sin \frac{p\pi}{4} - \ldots$$

$$ber'x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left\{ S_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) - T_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right\}$$

$$bei'x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left\{ T_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) + M_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right\}$$

where

$$S_0(x) = 1 - \frac{1 \cdot 3}{1!8x} \cos \frac{\pi}{4} - \frac{1^2 \cdot 3 \cdot 5}{2!\,(8x)^2} \cos \frac{2\pi}{4} + \ldots$$

$$+ \frac{1^2 \cdot 3^2 \ \ldots (2p-3)^2 (2p-1)(2p+1)}{p!\,(8x)^p} \cos \frac{p\pi}{4} + \ldots$$

$$T_0(x) = \frac{1\cdot 3}{1!8x} \sin\frac{\pi}{4} - \frac{1^2\cdot 3\cdot 5}{2!(8x)^2} \sin\frac{2\pi}{4} + \ldots$$

$$+ \frac{1^2\cdot 3^2 \ldots (2p-3)^2(2p-1)(2p+1)}{p!(8x)^p} \sin\frac{p\pi}{4} + \ldots$$

$$\ker x = \frac{\sqrt{\pi}}{\sqrt{(2x)}} e^{-x/\sqrt{2}} \left\{ L_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) \right.$$

$$\left. + M_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) \right\}$$

$$\mathrm{kei}\ x = \frac{\sqrt{\pi}}{\sqrt{(2x)}} e^{-x/\sqrt{2}} \left\{ M_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) \right.$$

$$\left. - L_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) \right\}$$

where $L_0(-x)$ and $M_0(-x)$ are obtained by changing x to $-x$ in equation $L_0(x)$ and $M_0(x)$.

$$\ker' x = \frac{\sqrt{\pi}}{\sqrt{(2x)}} e^{-x/\sqrt{2}} \left\{ S_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) \right.$$

$$+ T_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \Bigg\}$$

$$kei'x = \frac{\sqrt{\pi}}{\sqrt{(2x)}} e^{-x/\sqrt{2}} \Bigg\{ T_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right)$$

$$- S_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \Bigg\}$$

where $S_0(-x)$ and $T_0(-x)$ are obtained by changing x to $-x$ in equation $S_0(x)$ and $T_0(x)$.

APPENDIX B

```
*=====================================================================
*
*                    EYE - DIAGRAMS ON PIPELINE
*
*=====================================================================
*
*                      UNIVERSITY OF CALGARY
*
*              DEPARTMENT OF ELECTRICAL ENGINEERING
*
*                            THESIS :
*
*              AN ELECTRICAL TRANSMISSION LINE MODEL
*                         OF A PIPELINE
*
*                       BY : ZSIGMOND PAL
*                       ID : 809797
*
*=====================================================================
* THIS FORTRAN SOURCE PROGRAM ALLOWS THE USER TO DETERMINE THE
* FREQUENCY RESPONSE OF A PIPELINE BY PLOTTING THE EYE DIAGRAM OF THE
* OUTPUT OF THE TRANSMISSION LINE.
* THE PROGRAM CALCULATES THE PARAMETERS FOR A PIPELINE
* WHICH IS MODELED AS AN ELECTRICAL TRANSMISSION LINE FOR A GIVEN
* ARRANGEMENT. EACH ELEMENT OF THE TRANSMISSION LINE, THE PIPE, THE
* COATING, THE SURROUNDING EARTH AND THE GROUND BED IS CALCULATED BASED
* ON THE USER'S ENTRY IN SUBROUTINES, WHICH ARE CALLED FROM THIS MAIN
* PROGRAM.
* FORTRAN SOURCE SEGMENT NAMED PBESS COMPUTES THE SKIN EFFECT RESISTANCE
* AND REACTANCE RATIO OF THE PIPE IN THE FORM OF SERIES .
* THE PROGRAM USES RETURN TO ZERO ALTERNATE MARK INVERSION PULSES .
*=====================================================================
      IMPLICIT COMPLEX*16 (Z)
      INTEGER IN
      REAL DAT1(32),DAT2(32),LENGTH,TAU,RL1
      COMPLEX GAMMA(17),ZO(17), DAT(32),TRANS(32)
      COMPLEX SPR(2,17),CSHYP,SNHYP,TEMP(2,64)
      DATA DAT1/1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     C          0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/
      DATA DAT2/1,1,0,0,-1,-1,0,0,1,1,0,0,-1,-1,0,0,
     C          1,1,0,0,-1,-1,0,0,1,1,0,0,-1,-1,0,0/

      OPEN(1,FILE='EYE.PRN')
*
* READ INPUT PULSE TRAIN  DAT1 AND TAKE THE FFT
*
      DO 20 IN=1,32
```

```
          DAT(IN)=CMPLX(DAT1(IN),0.0)
20      CONTINUE
        N=32
        ISI=-1
        CALL FOUREA (DAT,N,ISI)

        DO 23 IN=1,16
          SPR(1,IN)=DAT(IN)
23      CONTINUE

*
* READ INPUT PULSE TRAIN  DAT2 AND TAKE THE FFT
*
        DO 25 IN=1,32
          DAT(IN)=CMPLX(DAT2(IN),0.0)
25      CONTINUE
        N=32
        ISI=-1
        CALL FOUREA (DAT,N,ISI)

        DO 27 IN=1,16
          SPR(2,IN)=DAT(IN)
27      CONTINUE

*
* TRANSMISSION LINE TRANSFER FUNCTION
*
        WRITE(*,*)'ENTER LENGTH OF THE PIPELINE IN METER'
        READ(*,*)LENGTH
        WRITE(*,*)'ENTER THE LOAD RESISTANCE'
        READ(*,*)RL1
        ZL1=CMPLX(RL1,0.0)
        PI2=8.0*ATAN(1.0)
        TAU=0.01
        ZIMAG=CMPLX(0.0,1.0)

        CALL PBESS(TAU,RDC,ZS,ZO,GAMMA)

        WRITE(*,*)'RDC [Ohm/m] =',RDC
        WRITE(*,*)
        WRITE(*,*)'  FREQ              Zin                    ZO
     C        Zload'
        WRITE(*,*)'   Hz        REAL        IMAG      REAL       IMAG
     C    REAL       IMAG'
        WRITE(*,*)

        DO 10 IN=1,17
           FREQ=(IN-1)/(TAU*8.0)
           IF (IN.EQ.1) THEN
               ZO(IN)=CMPLX(SQRT(RDC/1.0E-06),0.0)
               GAMMA(IN)=CMPLX(SQRT(RDC*1.0E-06),0.0)
```

```
          ENDIF
          GAMMA(IN)=LENGTH*GAMMA(IN)
          CSHYP=(CEXP(GAMMA(IN)) + CEXP(-GAMMA(IN)))/2.0
          SNHYP=(CEXP(GAMMA(IN)) - CEXP(-GAMMA(IN)))/2.0
          ZL=ZO(IN)*ZL1/(ZO(IN)+ZL1)

*
* ASSUMING THE SOURCE VOLTAGE (Vs) TO BE 2.0V
* EYE DIAGRAM AT THE LOAD
*

         TRANS(IN)=2.0/((1+ZS/ZL)*CSHYP+(ZO(IN)/ZL+ZS/ZO(IN))*
     C                SNHYP)

          WRITE(*,200)FREQ,ZIN,ZO(IN),ZL
200       FORMAT(E9.3,2x,6(E9.3,2x))
10      CONTINUE

*
* CALCULATE OUTPUT SPECTRUM
*
        DO 80 IK=1,2
        IN=1
        DAT(IN)=SPR(IK,IN)*REAL(TRANS(IN))
        DO 50 IN=2,17
          I=34-IN
          DAT(IN)=SPR(IK,IN)*TRANS(IN)
*
* MAKE DATA SYMMETRICAL AT THE 17 TH. DATA POINT
*
          DAT(I)=CMPLX(REAL(DAT(IN)),-AIMAG(DAT(IN)))
50      CONTINUE
        N=32
        ISI=1

        CALL FOUREA (DAT,N,ISI)

        DO 90 IN=1,32
        TEMP(IK,IN)=DAT(IN)
        TEMP(IK,IN+32)=DAT(IN)
90      CONTINUE
80      CONTINUE

*
* SAVE DATA
*
        DO 100 IN=1,32
          WRITE(1,40)(REAL(TEMP(1,IK*4+IN)),REAL(-TEMP(1,IK*4+IN)),
     C    IK=0,7),REAL(TEMP(2,IN)),REAL(-TEMP(2,IN))
40      FORMAT(16(E12.6,','),E12.6,',',E12.6)
100     CONTINUE
```

```
WRITE(1,*)
CLOSE(1,STATUS='KEEP')
STOP
END
```

```
*=====================================================================
*
*              TIME-DOMAIN REFLECTOMETRY ON PIPELINE
*
*=====================================================================
*
*                     UNIVERSITY OF CALGARY
*
*             DEPARTMENT OF ELECTRICAL ENGINEERING
*
*                           THESIS :
*
*             AN ELECTRICAL TRANSMISSION LINE MODEL
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*
*=====================================================================
* THIS FORTRAN SOURCE PROGRAM ALLOWS THE USER TO DETERMINE THE
* TDR OF A PIPELINE BY PLOTTING THE REFLECTED PULSE AT THE INPUT OF
* THE TRANSMISSION LINE.
* THE PROGRAM CALCULATES THE PARAMETERS FOR A PIPELINE
* WHICH IS MODELED AS AN ELECTRICAL TRANSMISSION LINE FOR A GIVEN
* ARRANGEMENT. EACH ELEMENT OF THE TRANSMISSION LINE, THE PIPE, THE
* COATING, THE SURROUNDING EARTH AND THE GROUND BED IS CALCULATED BASED
* ON THE USER'S ENTRY IN SUBROUTINES, WHICH ARE CALLED FROM THIS MAIN
* PROGRAM.
* FORTRAN SOURCE SEGMENT NAMED PBESS COMPUTES THE SKIN EFFECT RESISTANCE
* AND REACTANCE RATIO OF THE PIPE IN THE FORM OF SERIES .
*=====================================================================
      IMPLICIT COMPLEX*16 (Z)
      INTEGER IN,ANSWER
      REAL DAT1(32),DAT2(32),LEN1,LEN2,TAU,RL1,RL2
      COMPLEX GAMMA(17),GAMMA1(17),GAMMA2(17),ZO(17),DAT(32),TRANS(32)
      COMPLEX SPR(2,17),CSHYP,SNHYP,TEMP(2,64)
      DATA DAT1/1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     C          0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/

      OPEN(1,FILE='TDR.PRN')
*
* READ INPUT PULSE TRAIN  DAT1 AND TAKE THE FFT
*
      DO 20 IN=1,32
        DAT(IN)=CMPLX(DAT1(IN),0.0)
20    CONTINUE
      N=32
      ISI=-1
      CALL FOUREA (DAT,N,ISI)

      DO 23 IN=1,16
```

```
          SPR(1,IN)=DAT(IN)
23      CONTINUE

*
* TRANSMISSION LINE TRANSFER FUNCTION
*
        WRITE(*,*)'TIME DOMAIN REFLECTOMETRY ON PIPELINES'
        WRITE(*,*)'THE FOLLOWING OPTIONS ARE AVAILABLE'
        WRITE(*,*)'1 = MONITOR SENDING AND RETURNING PULSE AT THE INPUT'
        WRITE(*,*)'2 = MONITOR ONLY RETURNING PULSE AT THE INPUT'
        WRITE(*,*)'3 = MONITOR ONLY SENDING PULSE AT THE LOAD'
        WRITE(*,*)'4 = MONITOR MULTIPLE REFLECTION AT THE INPUT'
        WRITE(*,*)'ENTER YOUR CHOICE'
        READ(*,*)ANSWER
        WRITE(*,*)'PIPELINE SECTION IS MEASURED FROM SOURCE TO LOAD'

        IF (ANSWER.EQ.4) THEN
          WRITE(*,*)'ENTER LENGTH OF THE FIRST SECTION OF PIPELINE'
          WRITE(*,*)'UPTO THE LOAD, IN METER'
          READ(*,*)LEN1
          WRITE(*,*)'ENTER THE FIRST LOAD RESISTANCE'
          READ(*,*)RL1
          WRITE(*,*)'NOW ENTER THE SECOND SECTION PARAMETERS'
        ENDIF

        WRITE(*,*)'LENGTH OF THE PIPELINE UPTO THE LOAD, IN METER ?'
        READ(*,*)LEN2
        WRITE(*,*)'LOAD RESISTANCE ?'
        READ(*,*)RL2

        IF (ANSWER.EQ.4) THEN
          LEN2=LEN2-LEN1
        ENDIF

        PI2=8.0*ATAN(1.0)
        TAU=0.01
        ZIMAG=CMPLX(0.0,1.0)

        CALL PBESS(TAU,RDC,ZS,ZO,GAMMA)

        WRITE(*,*)
        WRITE(*,*)'  FREQ              Zin                    ZO
     C        Zload'
        WRITE(*,*)'   Hz        REAL        IMAG      REAL       IMAG
     C   REAL       IMAG'
        WRITE(*,*)

        DO 10 IN=1,17
           FREQ=(IN-1)/(TAU*8.0)
           IF (IN.EQ.1) THEN
               ZO(IN)=CMPLX(SQRT(RDC/1.0E-06),0.0)
```

```
            GAMMA(IN)=CMPLX(SQRT(RDC*1.0E-06),0.0)
          ENDIF
          GAMMA2(IN)=LEN2*GAMMA(IN)
          CSHYP=(CEXP(GAMMA2(IN)) + CEXP(-GAMMA2(IN)))/2.0
          SNHYP=(CEXP(GAMMA2(IN)) - CEXP(-GAMMA2(IN)))/2.0
          ZL2=ZO(IN)*RL2/(ZO(IN)+RL2)
          ZIN2=(CSHYP+ZO(IN)/ZL2*SNHYP)/
     C         (1.0/ZL2*CSHYP+(1.0/ZO(IN))*SNHYP)
*
* ASSUMING THE SOURCE VOLTAGE (Vs) TO BE 2.0V
*
*
* TDR DIAGRAM AT THE INPUT WITH BOTH SENDING AND RETURNING
* PULSES
*
          IF (ANSWER.EQ.1) THEN
          TRANS(IN)=2.0*ZIN2/(ZIN2+ZS)
*
* TDR DIAGRAM AT THE INPUT WITH RETURNING PULSES ONLY
*
          ELSEIF (ANSWER.EQ.2) THEN
          TRANS(IN)=2.0*CEXP(-2.0*GAMMA2(IN))*(ZL2-ZO(IN))/
     C    (ZL2+ZO(IN))*ZO(IN)/(ZO(IN)+ZS)
*
* TDR DIAGRAM AT THE LOAD WITH SENDING PULSES ONLY
*
          ELSEIF (ANSWER.EQ.3) THEN
          TRANS(IN)=2.0/((1+ZS/ZL2)*CSHYP+(ZO(IN)/ZL2+ZS/ZO(IN))*
     C                SNHYP)
*
* MULTIPLE TDR DIAGRAM AT THE INPUT
*
          ELSEIF (ANSWER.EQ.4) THEN
          GAMMA1(IN)=LEN1*GAMMA(IN)
          CSHYP=(CEXP(GAMMA1(IN)) + CEXP(-GAMMA1(IN)))/2.0
          SNHYP=(CEXP(GAMMA1(IN)) - CEXP(-GAMMA1(IN)))/2.0
          ZL1=ZIN2*RL1/(ZIN2+RL1)
          ZIN1=(CSHYP+ZO(IN)/ZL1*SNHYP)/
     C         (1.0/ZL1*CSHYP+(1.0/ZO(IN))*SNHYP)
          TRANS(IN)=2.0*ZO(IN)/(ZO(IN)+ZS)*(ZIN1-ZO(IN))/(ZIN1+ZO(IN))
          ENDIF

300       CONTINUE
          WRITE(*,200)FREQ,ZIN2,ZO(IN),ZL2
200       FORMAT(E9.3,2x,6(E9.3,2x))
10      CONTINUE

*
* CALCULATE OUTPUT SPECTRUM
*
        IK=1
```

```
      IN=1
      DAT(IN)=SPR(IK,IN)*REAL(TRANS(IN))
      DO 50 IN=2,17
        I=34-IN
        DAT(IN)=SPR(IK,IN)*TRANS(IN)
*
* MAKE DATA SYMMETRICAL AT THE 17 TH. DATA POINT
*
        DAT(I)=CMPLX(REAL(DAT(IN)),-AIMAG(DAT(IN)))
50    CONTINUE
      N=32
      ISI=1

      CALL FOUREA (DAT,N,ISI)

      DO 90 IN=1,32
      TEMP(IK,IN)=DAT(IN)
      TEMP(IK,IN+32)=DAT(IN)
90    CONTINUE

*
* SAVE DATA
*
      DO 100 IN=1,32
         WRITE(1,40)REAL(TEMP(1,IN))
40    FORMAT(E12.6)
100   CONTINUE
      WRITE(1,*)
      CLOSE(1,STATUS='KEEP')
      STOP
      END
```

```
*=====================================================================
*
*       SKIN EFFECT RESISTANCE AND REACTANCE CALCULATION
*
*=====================================================================
*
*                     UNIVERSITY OF CALGARY
*
*              DEPARTMENT OF ELECTRICAL ENGINEERING
*
*                           THESIS :
*
*              AN ELECTRICAL TRANSMISSION LINE MODEL
*                        OF A PIPELINE
*
*                       BY : ZSIGMOND PAL
*                       ID : 809797
*
*=====================================================================
* THIS FORTRAN SOURCE PROGRAM CALCULATES THE PARAMETERS FOR A PIPELINE
* WHICH IS MODELED AS AN ELECTRICAL TRANSMISSION LINE FOR A GIVEN
* ARRANGEMENT. EACH ELEMENT OF THE TRANSMISSION LINE, THE PIPE, THE
* COATING, THE SURROUNDING EARTH AND THE GROUND BED IS CALCULATED BASED
* ON THE USER'S ENTRY.
* THE SKIN EFFECT RESISTANCE  AND REACTANCE RATIO OF THE PIPE
* IN THE FORM OF SERIES ARE CALCULATED.
* BESSEL FUNCTIONS OF THE FIRST KIND AND SECOND KIND OF ORDER ZERO
* ARE COMPUTED BY A SUBROUTINE FOR LARGE AND FOR SMALL VALUES OF
* THE ARGUMENT .
* THE OUTPUT IS STORED IN THE OUTPUT DATA FILE FOR FURTHER PROCESSING
*
*==========================CONSTANTS==================================
*
*
*       EULER = .5772157                EULER'S CONSTANT
*       PI = 3.14159265                 CONSTANT
*       MUO = 1.256E-06                 PERMEABILITY OF FREE SPACE [H/m]
*       MUR = 500                       PIPE RELATIVE PERMEABILITY
*       EPS = 4*PI*8.854E-12            PERMITTIVITY OF FREE SPACE [F/m]
*       EPSR = 3.7                      PIPE INSOL. REL. PERMITTIVITY
*       RHOP = 2.5E-07                  PIPE RESISTIVITY [Ohm*m]
*       INS = 0.003                     INSOLATION THICKNESS [m]
*
*==========================VARIABLES==================================
*
*       R=RR/2                          OUTSIDE RADIUS OF THE PIPE
*       Q=QQ/2                          INSIDE RADIUS OF THE PIPE
*       RHOE                            GROUND RESISTIVITY [Ohm*m]
*       GAMMA                           PROPAGATION CONSTANT
*       ZO                              CHARACTERISTIC IMPEDANCE
*       TAU                             PULSE WIDTH
```

```
*       ZS                              GROUND BED RESISTANCE
*
* ZO AND GAMMA ARE ONE DIMENSIONAL COMPLEX ARRAYS
*
*=======================================================================
        SUBROUTINE PBESS(TAU,RDC,ZS,ZO,GAMMA)

        IMPLICIT REAL*8 (A-H,K,M,X),COMPLEX*16 (Z),INTEGER (I-J,L)
        COMPLEX GAMMA(17),ZO(17),RADC(17)
        REAL*4 INS,TAU,SKIN,MAGNITU
        INTEGER*4 P,ANS
        DIMENSION BES(8,2),BESS(8,100),Z12(1,100),SKIN(100)

        WRITE(*,*)'OUTSIDE DIAMETER OF THE PIPE [m] ='
        READ(*,*)RR
        WRITE(*,*)'INSIDE DIAMETER OF THE PIPE [m] ='
        READ(*,*)QQ

        EULER = .5772157
        PI = 3.14159265
        MUO = 1.256E-06
        MUR = 500.0
        EPS = 4.0*PI*8.854E-12
        EPSR = 3.7
        RHOP = 2.5E-07
        INS = .003

*=======================================================================
*
*       GROUND BED RESISTANCE CALCULATION
*
*=======================================================================

        WRITE(*,*)'GROUND RESISTIVITY FOR GROUND BED [Ohm-m] ='
        READ(*,*)RHO
        WRITE(*,*)'GROUND BED DIAMETER [m] ='
        READ(*,*)D
        WRITE(*,*)'GROUND BED LENGTH [m] ='
        READ(*,*)L
        WRITE(*,*)'DEPTH OF GROUND BED [m] ='
        READ(*,*)S
        WRITE(*,*)'SEPARATION OF THE GROUND BEDS [m]'
        READ(*,*)T
        S=S*2
        A=D/2
        ZS=RHO/(8*PI*L)*((LOG(4*L/A)-1+LOG((2*L+SQRT(S**2+4*L**2))/S)+
     c  S/(2*L)-(SQRT(S**2+4*L**2))/(2*L))+4*LOG(2+T/L)+2*T/L*LOG(T/L*
     c  (2+T/L))-4*(1+T/L)*LOG(1+T/L))
        WRITE(*,*)
        WRITE(*,*)' GROUND BED RESISTANCE IN Ohm =',REAL(ZS)
        WRITE(*,*)
```

```
      RT2=SQRT(2.0)
      R=RR/2.0
      Q=QQ/2.0
      T=R-Q
*
* PIPE DC RESISTANCE /m
*
      RDC=RHOP/(PI*(R**2-Q**2))

*=====================================================================
*
* CALCULATE MULTI-LAYER GROUND RESISTANCE
*
*=====================================================================

      CALL GROUND(RR,TAU,Z12,SKIN)

*=====================================================================
*
*CALCULATE SKIN EFFECT RESISTANCE AND REACTANCE RATIO OF THE PIPE
*
*=====================================================================

      WRITE(*,*)'  FREQ       MT        RAC/RDC         GAMMA/m '
      WRITE(*,*)'   Hz                                REAL       IMAG'
      WRITE(*,*)

      DO 100 IN=2,17
      FREQ=(IN-1)/(TAU*8.)
      M=SQRT(2.0*PI*FREQ*MUR*MUO/RHOP)
      MR=M*R
      MQ=M*Q
      MT=M*T
      Z7=CMPLX(0.0,1.0)

*
* FOR  MT  GREATER  THAN 4 ,THE SKIN EFFECT IN A TUBE IS  SOLVED
* BY THE ASYMPTOTIC FORMULA
*

      IF (MT.GT.4) THEN
        RADC(IN)=M*(R**2-Q**2)/(2.0*R*RT2)*(1.0+1.0/(MR*RT2)+3.0/
     c  (8.0*MR**2))
        GOTO 10
      ENDIF

*
* FOR  MT  LESS THAN 4 ,THE ASYMTOTIC FORMULA CAN NOT BE USED TO
* OBTAIN PRACTICAL VALUES OF SKIN EFFECT RATIOS . IN A CASE LIKE THIS
* THE PRECISE METHODE OF CALCULATION IS REQUIRED
```

```
*

      DO 30 I=1 ,2
        IF (I.EQ.1) THEN
           X=MR
        ELSEIF (I.EQ.2) THEN
           X=MQ
        ENDIF

      CALL BESSEL(X,BER,BEI,BERR,BEII,KER,KEI,KERR,KEII)

      BES(1,I)=BER
      BES(2,I)=BEI
      BES(3,I)=BERR
      BES(4,I)=BEII
      BES(5,I)=KER
      BES(6,I)=KEI
      BES(7,I)=KERR
      BES(8,I)=KEII
30    CONTINUE

*
*  USING CALCULATED BESSEL FUNCTIONS SOLVE REST OF THE EQUATION
*

      Z1=CMPLX(BES(1,1),BES(2,1))
      Z2=CMPLX(BES(5,1),BES(6,1))
      Z3=CMPLX(BES(3,1),BES(4,1))
      Z4=CMPLX(BES(7,1),BES(8,1))
      Z5=CMPLX(BES(3,2),BES(4,2))
      Z6=CMPLX(BES(7,2),BES(8,2))
      Z=Z7*M*(R**2.0-Q**2.0)/(2.0*R)*(Z1+(-Z5/Z6)*Z2)/(Z3+(-Z5/Z6)*Z4)
      RADC(IN)=Z
10    CONTINUE
*
* PIPE CAPACITANCE TO GROUND /m
*
      CAP = 2.0*PI*EPS*EPSR/LOG((R+.004)/R)

*
* PROPAGATION CONSTANT Z10 AND CHARACTERISTIC IMPEDANCE CALCULATION
*

      Z10=CDSQRT(Z7*2*PI*FREQ*CAP*(RADC(IN)*RDC+REAL(Z12(1,IN))))
      GAMMA(IN)=Z10
      Z0(IN)=CDSQRT((RADC(IN)*RDC+Z12(1,IN))/(Z7*2*PI*FREQ*CAP))
      WRITE(*,700)FREQ,MT,REAL(RADC(IN)),GAMMA(IN)
700   FORMAT(E9.3,2x,4(E9.3,2x))
100   CONTINUE
      WRITE(*,*)
      WRITE(*,*)' CAPACITANCE =',CAP
```

```
      WRITE(*,*)
      RETURN
      END

*=====================================================================
*                    SUBROUTINE
*
*             BESSEL FUNCTION CALCULATION
*
* THIS SUBROUTINE CALCULATES BESSEL FUNCTIONS OF ORDER ZERO IN THE
* FORM OF SERIES FOR LARGE VALUES ( X > 10.0 ) AND SMALL VALUES
* ( X < 10.0 ) OF THE ARGUMENT
*
* BER,BEI,BERR AND BEII ARE THE BESSEL FUNCTION OF THE FIRST KIND,
* ORDER ZERO
* KER,KEI,KERR AND KEII ARE THE BESSEL FUNCTION OF THE SECOND KIND,
* ORDER ZERO
*=====================================================================

      SUBROUTINE BESSEL(X,BER,BEI,BERR,BEII,KER,KEI,KERR,KEII)
      IMPLICIT REAL*8 (A-H,K,M,X),INTEGER (I-J,L)

      BER=0.0
      BEI=0.0
      BERR=0.0
      BEII=0.0
      KER=0.0
      KEI=0.0
      KERR=0.0
      KEII=0.0

      EULER = .5772157
      PI = 3.14159265
      RT2=SQRT(2.0)
      D=0.0

*
*BER FUNCTION CALCULATOION
*

      IF (X.LT.10.0) GOTO 110
      A=.3989425*EXP(X/RT2)/SQRT(X)
      BER=A*((1.0+.7071/(8.0*X))*COS(X/RT2-.3927)+.7071/(8.0*X)*SIN(X/
     c RT2-.3927))
      GOTO 120
110   DO 50 I=1 ,10
         A=(-1.0)**(I-1)
         B=(.5*X)**(4.0*(I-1))
         L=2*I-2
         CALL FACTOR(L,D)
```

```
         BER=BER+A*B/D**2.0
50     CONTINUE
120    CONTINUE

*
*BEI FUNCTION CALCULATION
*

       IF (X.LT.10.0) GOTO 130
       A=.3989425*EXP(X/RT2)/SQRT(X)
       BEI=A*((1.0+.7071/(8.0*X))*SIN(X/RT2-.3927)-.7071/(8.0*X)*COS(X/
     c RT2-.3927))
       GOTO 140
130    DO 51 I=1 ,10
         A=(-1.0)**(I-1)
         B=(.5*X)**(4.0*I-2.0)
         L=2*I-1
         CALL FACTOR(L,D)
         BEI=BEI+A*B/D**2.0
51     CONTINUE
140    CONTINUE

*
*BER' FUNCTION CALCULATION
*

       IF (X.LT.10.0) GOTO 150
       A=.3989425*EXP(X/RT2)/SQRT(X)
       BERR=A*((1.0-2.1213/(8.0*X))*COS(X/RT2+.3927)-2.1213/(8.0*X)*SIN
     c (X/RT2+.3927))
       GOTO 160
150    DO 52 I=1 ,10
         A=(-1.0)**I
         B=2.0*I*(.5*X)**(4.0*I-1)
         L=2*I
         CALL FACTOR(L,D)
         BERR=BERR+A*B/D**2.0
52     CONTINUE
160    CONTINUE

*
*BEI' FUNCTION CALCULATION
*

       IF (X.LT.10.0) GOTO 170
       A=.3989425*EXP(X/RT2)/SQRT(X)
       BEII=A*((1.0-2.1213/(8.0*X))*SIN(X/RT2+.3927)+2.1213/(8.0*X)*COS
     c (X/RT2+.3927))
       GOTO 180
170    DO 53 I=1 ,10
         A=(-1.0)**(I-1)
```

```
          B=(2.0*I-1)*(.5*X)**(4.0*I-3.0)
          L=2*I-1
          CALL FACTOR(L,D)
          BEII=BEII+A*B/D**2.0
53      CONTINUE
180     CONTINUE

*
*KER FUNCTION CALCULATION
*

        H=0.0
        IF (X.LT.10.0) GOTO 190
        A=1.2533141*EXP(-X/RT2)/SQRT(X)
        KER=A*((1.0-1.0/(8.0*X))*COS(X/RT2+.3927)+.7071/(8.0*X)*SIN
     c  (X/RT2+.3927))
        GOTO 200
190     DO 54 I=1 ,10
          A=(-1.0)**I
          B=(.5*X)**(4.0*I)
          E=0.0
          DO 55 J=1 ,(2*I)
            EE=1.0/J
            E=E+EE
55        CONTINUE
          L=2*I
          CALL FACTOR(L,D)
          H=H + A*E*B/D**2.0
54      CONTINUE
        KER=(LOG(2.0)-EULER-LOG(X))*BER + BEI*PI/4.0+ H
200     CONTINUE

*
*KEI FUNCTION CALCULATION
*

        G=0.0
        IF (X.LT.10.0) GOTO  210
        A=1.2533141*EXP(-X/RT2)/SQRT(X)
        KEI=A*(-(1.0-.7071/(8.0*X))*SIN(X/RT2+.3927)+.7071/(8.0*X)*COS
     c  (X/RT2+.3927))
        GOTO 220
210     DO 56 I=1 ,10
          A=(-1.0)**(I-1)
          B=(.5*X)**(4.0*I-2.0)
          E=0.0
          DO 57 J=1 ,(2*I-1)
            F=1.0/J
            E=E+F
57        CONTINUE
          L=2*I-1
```

```
          CALL FACTOR(L,D)
          G=G + A*E*B/D**2.0
56      CONTINUE
        KEI=(LOG(2.0)-EULER-LOG(X))*BEI - BER*PI/4.0+ G
220     CONTINUE

*
*KER' FUNCTION CALCULATION
*

        G=0.0
        H=0.0
        IF (X.LT.10.0) GOTO 230
        A=-1.2533141*EXP(-X/RT2)/SQRT(X)
        KERR=A*((1.0+2.1213/(8.0*X))*COS(X/RT2-.3927)-2.1213/(8.0*X)*
     c  SIN(X/RT2-.3927))
        GOTO 240
230     DO 59 I=1 ,10
          A=(-1.0)**I
          B=(.5*X)**(4.0*I-1)
          E=0.0
          DO 58 J=1 ,(2*I)
            F=1.0/J
            E=E+F
58        CONTINUE
          L=2*I-1
          CALL FACTOR(L,D)
          DD=D
          L=2*I
          CALL FACTOR(L,D)
          G=G + A*E*B/(DD*D)
59      CONTINUE
        KERR=(LOG(2.0)-EULER-LOG(X))*BERR - BER/X+BEII*PI/4.0+ G
240     CONTINUE

*
*KEI' FUNCTION CALCULATION
*

        G=0.0
        H=0.0
        IF (X.LT.10.0) GOTO 250
        A=-1.2533141*EXP(-X/RT2)/SQRT(X)
        KEII=A*(-(1.0+2.1213/(8.0*X))*SIN(X/RT2-.3927)-2.1213/(8.0*X)*
     c  COS(X/RT2-.3927))
        GOTO 260
250     DO 60 I=1 ,10
        A=(-1.0)**(I-1.)
        B=(.5*X)**(4.0*I-3.0)
        E=0.0
        DO 61 J=1 ,(2*I-1)
```

```
          F=1.0/J
          E=E+F
61      CONTINUE
        L=2*I-2
        CALL FACTOR(L,D)
        DD=D
        L=2*I-1
        CALL FACTOR(L,D)
        G=G + A*E*B/(DD*D)
60      CONTINUE
        KEII=(LOG(2.0)-EULER-LOG(X))*BEII-BEI/X-BERR*PI/4.0+G
260     CONTINUE
        RETURN
        END

*
* SUBROUTINE FOR FACTORIAL CALCULATION
*

        SUBROUTINE FACTOR(L,D)
        REAL*8 D
        INTEGER*4 J,L

        IF (L.EQ.0) THEN
        L=1
        ENDIF
        D=1.0
        DO 20 J=1 ,L
           D=D*J
20      CONTINUE
        RETURN
        END


*=====================================================================
*                        SUBROUTINE
*
*              PIPELINE GROUND RETURN SELF IMPEDANCE
*                        CALCULATION
*
*=====================================================================
        SUBROUTINE GROUND(RR,TAU,Z12,SKIN)

        IMPLICIT REAL*4 (M,R),COMPLEX*16 (A-H,P,T,Z),INTEGER (I-L)
        REAL*4 PI,SIGMA,FREQ,DELTA,TAU,SKIN
        DIMENSION RHOE(5),SIGMA(5),A(2,2,5),C(2,2),DELTA(4)
        DIMENSION Z12(1,100),Z10(1,100),SKIN(100)

        WRITE(*,*)'ENTER THE NUMBER OF GROUND LAYERS 2,3,4 OR 5 '
        READ(*,*)KLAYER
        WRITE(*,*)'GROUND RESISTIVITY OF EACH LAYER'
```

```
      WRITE(*,*)'STARTING FROM THE TOP [Ohm*m] ='

      DO 300 K=1,KLAYER
         READ(*,*)RHOE(K)
300   SIGMA(K)=1.0/RHOE(K)

      WRITE(*,*)'ENTER THE VALUE OF EACH LAYER THICKNESS '
      WRITE(*,*)'STARTING FROM THE TOP [m] '

      DO 301 K=1,KLAYER-1
         READ(*,*)DELTA(K)
301   CONTINUE

      PI=3.14159265
      MUO=4*PI*1.0E-07
      R=RR/2
      ZIMAG=CMPLX(0.0,1.0)
      WRITE(*,*)'TAU=',TAU
      WRITE(*,*)
      WRITE(*,*)'  FREQ    COMP.DEPTH MLAYER [m]     GROUND IMP.'
      WRITE(*,*)'   Hz        REAL        IMAG      REAL       IMAG'
      DO 100 L=2,17
         FREQ=(L-1)/(TAU*8.)

*=====================================================================
* COMPLEX PENETRATION DEPTH FOR HOMOGENEOUS EARTH , MUO=MUO0
*=====================================================================

         PUNIF=CDSQRT(1.0/(ZIMAG*2*PI*FREQ*MUO*SIGMA(1)))
*        WRITE(*,*)'P uniform earth= ',PUNIF
*
* CLOSED FORM SOLUTION FOR UNDERGROUND IMPEDANCE [/m] CALCULATION
*

         Z10(1,L)=ZIMAG*FREQ*MUO*CDLOG((R+PUNIF)/R)
*        WRITE(*,*)'Z (series) uniform earth=',Z10(1,L)

*======================================================================
*          COMPLEX DEPTH CALCULATION FOR MULTI-LAYER EARTH
*======================================================================

         DO 500 K=1 ,KLAYER-1

            P=1.0/CDSQRT(ZIMAG*2.0*PI*FREQ*MUO*SIGMA(K))
            ZETA=CDSQRT(ZIMAG*2.0*PI*FREQ*MUO/SIGMA(K))
            THETA=CDEXP(DELTA(K)/P)
*
* CALCULATION OF GENERALIZED CIRCUIT CONSTANTS  A,B,C AND D ,A=D
* IN THE MATRIX A(ROW,COL,K). NOTE THE WAY A(1,1,K) AND A(1,1,K+1)
* IS PASSED TO THE SUBROUTINE MATMUL
*
```

```
            A(1,1,K)=(THETA+1.0/THETA)/2.0
            A(1,2,K)=ZETA*(THETA-1.0/THETA)/2.0
            A(2,1,K)=(THETA-1.0/THETA)/(2.0*ZETA)
            A(2,2,K)=A(1,1,K)
500       CONTINUE

*
*LOOK OUT FOR TWO LAYERED EARTH
*
          IF (KLAYER .EQ. 2) THEN
            DO 602 I=1,2
              DO 602 J=1,2
                C(I,J)=A(I,J,1)
602         CONTINUE
            GOTO 601
          ENDIF


          DO 600 K=1 ,KLAYER-1
            IF (K .EQ. KLAYER-1) GOTO 603
              CALL MATMUL (A(1,1,K+1),A(1,1,K),C)
603       CONTINUE

            DO 600 I=1,2
              DO 600 J=1,2
                A(I,J,K+1)=C(I,J)
600       CONTINUE
601       CONTINUE

*
* CALCULATE ZETA FOR THE LAST LAYER
*
          ZETA=CDSQRT(ZIMAG*2.0*PI*FREQ*MUO/SIGMA(KLAYER))
*
* EO CAN BE CALCULATED , AT THE TOP OF THE FIRST LAYER
* HO=CURRENT  AND THEREFORE
*
          EO=(ZETA*C(1,1)-C(1,2))/(C(1,1)-ZETA*C(2,1))
          P=EO/(ZIMAG*2.0*PI*FREQ*MUO)
          Z12(1,L)=ZIMAG*FREQ*MUO*CDLOG((R+P)/R)
          SKIN(L)=REAL(P)
          WRITE(*,700)FREQ,P,Z12(1,L)
700       FORMAT(E9.3,2x,4(E9.3,2x))

100     CONTINUE
      WRITE(*,*)
      RETURN
      END

*=====================================================================
*                        SUBROUTINE
```

```
*                  MATRIX MULTIPLICATION
*
*               A(L,M) IS MULTIPLIED BY B(M,N)
*               RESULT IS STORED IN C(L,N)
*
*=====================================================================

      SUBROUTINE MATMUL (A,B,C)
      COMPLEX*16 A,B,C,SUM
      DIMENSION A(2,2),B(2,2),C(2,2)

      DO 10 I=1,2
         DO 10 K=1,2
            SUM=(0.0,0.0)
            DO 20 J=1,2
20          SUM=SUM+A(I,J)*B(J,K)
10    C(I,K)=SUM
      RETURN
      END
```

```
*=====================================================================
*
*                    SUBROUTINE :FOUREA
*             COOLEY-TUKEY FAST FOURIER TRANSFORM
*
*=====================================================================
*
*                    UNIVERSITY OF CALGARY
*
*             DEPARTMENT OF ELECTRICAL ENGINEERING
*
*                          THESIS :
*
*             AN ELECTRICAL TRANSMISSION LINE MODEL
*                       OF A PIPELINE
*
*                      BY : ZSIGMOND PAL
*                      ID : 809797
*
*=====================================================================
* THIS FORTRAN SOURCE PROGRAM ALLOWS THE USER TO CALCULATE THE
* FAST FOURIER TRANSFORM OF A DISCRETE FUNCTION.
* DATA IS A ONE-DIMENSIONAL COMPLEX ARRAY WHOSE LENGTH, N, IS A
* POWER OF TWO.  ISI IS +1 FOR AN INVERSE TRANSFORM AND -1 FOR A
* FORWARD TRANSFORM.  TRANSFORM VALUES ARE RETURNED IN THE INPUT
* ARRAY, REPLACING THE INPUT.
* TRANSFORM(J)=SUM(DATA(I)*W**(J-1)), WHERE I AND J RUN
* FROM 1 TO N AND W = EXP (ISI*2*PI*SQRT(-1)/N).  PROGRAM ALSO
* COMPUTES INVERSE TRANSFORM, FOR WHICH THE DEFINING EXPRESSION
* IS INVERT (J)=(1/N)*SUM(DATA(I)*W**((I-1)*(J-1))).
* RUNNING TIME IS PROPORTIONAL TO N*LOG2(N), RATHER THAN TO THE
* CLASSICAL N**2.
* THIS IS A VERY SHORT VERSION OF THE FFT AND IS INTENDED
* MAINLY FOR DEMONSTRATION. PROGRAMS  ARE AVAILABLE IN
* IEEE COLLECTION WHICH RUN FASTER AND ARE NOT
* RESTRICTED TO POWERS OF 2 OR TO ONE-DIMENSIONAL ARRAYS.
* SEE -- IEEE TRANS AUDIO (JUNE 1967), SPECIAL ISSUE ON FFT.
*=====================================================================
      SUBROUTINE FOUREA (DATA, N, ISI)
      COMPLEX DATA(1)
      COMPLEX TEMP, W
*
      PI = 4. *ATAN(1.)
      FN = N
*
* THIS SECTION PUTS DATA IN BIT-REVERSED ORDER
*
      J = 1
      DO 80 I=1,N
*
* AT THIS POINT, I AND J ARE A BIT REVERSED PAIR (EXCEPT FOR THE
```

```
* DISPLACEMENT OF +1)
*
          IF (I-J) 30, 40, 40
*
* EXCHANGE DATA(I) WITH DATA(J) IF I.LT.J.
*
30        TEMP = DATA(J)
          DATA(J) = DATA(I)
          DATA(I) = TEMP
*
* IMPLEMENT J=J+1, BIT-REVERSED COUNTER
*
40        M = N/2
50        IF (J-M) 70, 70, 60
60        J = J - M
          M = (M+1)/2
          GO TO 50
70        J = J + M
80      CONTINUE
*
* NOW COMPUTE THE BUTTERFLIES
*
        MMAX = 1
90      IF (MMAX-N) 100, 130, 130
100     ISTEP = 2*MMAX
        DO 120 M=1,MMAX
          THETA = PI*FLOAT(ISI*(M-1))/FLOAT(MMAX)
          W = CMPLX(COS(THETA),SIN(THETA))
          DO 110 I=M,N,ISTEP
            J = I + MMAX
            TEMP = W*DATA(J)
            DATA(J) = DATA(I) - TEMP
            DATA(I) = DATA(I) + TEMP
110     CONTINUE
120     CONTINUE
        MMAX = ISTEP
        GO TO 90
130     IF (ISI) 160, 140, 140
*
* FOR INV TRANS -- ISI=1 -- MULTIPLY OUTPUT BY 1/N
*
140     DO 150 I=1,N
          DATA(I) = DATA(I)/FN
150     CONTINUE
160     RETURN
        END
```

```
*=====================================================================
*
*       DETERMINATION OF TRANSMISSION LINE CHARACTERISTICS FROM
*                       IMPEDANCE MEASUREMENT
*
*=====================================================================
*
*                      UNIVERSITY OF CALGARY
*
*              DEPARTMENT OF ELECTRICAL ENGINEERING
*
*                             THESIS :
*
*              AN ELECTRICAL TRANSMISSION LINE MODEL
*                        OF A PIPELINE
*
*                       BY : ZSIGMOND PAL
*                       ID : 809797
*
*=====================================================================
*
* THIS PROGRAM CALCULATES THE CHARACTERISTIC IMPEDANCE
* OF A WIRE, BASED ON IMPEDANCE MEASUREMENT. THE WIRE IS 100 m LONG
* AND IT IS LAID ON THE GROUND IN THE FIRST CASE, LIFTED 0.1 m
* ABOVE GROUND IN THE SECOND CASE, AND LOOPED BACK TO ITSELF,
* 0.2 m ABOVE THE GROUND, IN THE THIRD CASE.
*
* PCALIB IS THE RECEIVED POWER AT THE TIME  OF CALIBRATION
* FROM THE OPEN  COAX LINE NOT CONNECTED TO THE WIRE
* ZOPEN(I) AND ZTERM(I) IMPEDANCE VALUES FOR OPEN AND
* TERMINATED CIRCUITS
* Z(0) CHARACTERISTIC IMPEDANCE
*=====================================================================


      IMPLICIT COMPLEX (Z)
      COMPLEX PROCOM,PRTCOM,GAMMA,Y
      REAL PCALIB,PHASE1,PHASEO,PHASET,PRMAGO,PRMAGT,FREQ,CAP
      DIMENSION PCALIB(7),PHASE1(7),PROCOM(7),PRTCOM(7)
      DIMENSION FREQ(7),CAP(7),ZOPEN(7),ZTERM(7),Z0(21),GAMMA(21)
      DIMENSION PRMAGO(21),PHASEO(21),PRMAGT(21),PHASET(21)

      DATA PRMAGO/-21.3,-19.5,-18.5,-18.6,-18.9,-18.9,-19.0,
     C            -17.0,-17.3,-18.1,-17.3,-17.2,-17.9,-17.8,
     C            -16.9,-18.2,-18.0,-17.0,-17.1,-17.9,-17.8/

      DATA PHASEO/-31.0,-15.0,-42.0,-75.0,-103., 145.,-60.0,
     C            -11.0,-20.0,-41.0,-71.0,-105., 143.,-62.0,
     C            - 8.5,-18.0,-41.0,-72.0,-106., 142.,-62.0/

      DATA PRMAGT/-19.2,-18.3,-18.7,-18.5,-18.8,-18.8,-18.8,
```

```
C              -19.4,-18.3,-18.0,-17.6,-17.6,-18.0,-17.9,
C              -21.1,-18.0,-17.5,-17.1,-17.2,-18.0,-17.9/

      DATA PHASET/ -3.0,-22.0,-42.0,-75.0,-103., 145.,-58.0,
C              -10.5,-24.0,-44.0,-71.0,-105., 145.,-64.0,
C              -15.0,-30.0,-41.0,-72.0,-105., 142.,-62.0/

      DATA PCALIB/-16.2,-15.9,-15.5,-15.2,-15.2,-15.6,-16.0/

      DATA PHASE1/-7.5,-20.0,-40.0,-73.0,-103.0,144.0,-58.0/

      DATA FREQ/1.0,2.0,4.0,7.0,10.0,20.0,40.0/

      OPEN(1,FILE='WIRETEST.PRN')
      RAD=57.2957
      RTERM=120.0
*
* CALCULATE REFLECTED POWER OF THE OPEN CIRCUITED LINE
*
      DO 40 K=0,2
         WRITE(*,*)
         WRITE(*,*)
         WRITE(*,*)'VALUES FOR OPEN CIRCUIT'
      DO 10 I=1,7

         WRITE(*,*)
         WRITE(*,*)'FREQUENCY IN MHz =',FREQ(I)
         PRO = PRMAGO(K*7+I) - PCALIB(I)
         WRITE(*,*)'POWER MAGNITUDE IN db =',PRO
         PRO = 10.0**(PRO/20.0)
         PHASE=PHASEO(K*7+I) - PHASE1(I)
         WRITE(*,*)'PHASE ANGLE =',PHASE
         PHASE=PHASE/RAD
         PROCOM(I)=CMPLX(COS(PHASE)*PRO,SIN(PHASE)*PRO)
         WRITE(*,*)'REFLECTION COEFFICIENT =',PRO

10    CONTINUE
      WRITE(*,*)
      WRITE(*,*)
*
* CALCULATE REFLECTED POWER OF THE TERMINATED WIRE
*
      WRITE(*,*)'VALUES FOR TERMINATED CIRCUIT'
      WRITE(*,*)

      DO 20 I=1,7

         WRITE(*,*)'FREQUENCY IN MHz =',FREQ(I)
         PRT = PRMAGT(K*7+I) - PCALIB(I)
         WRITE(*,*)'POWER MAGNITUDE IN db =',PRT
         PRT = 10.0**(PRT/20)
```

```
         PHASE=PHASET(K*7+I) - PHASE1(I)
         WRITE(*,*)'PHASE ANGLE =',PHASE
         PHASE=PHASE/RAD
         PRTCOM(I)=CMPLX(COS(PHASE)*PRT,SIN(PHASE)*PRT)
         WRITE(*,*)'REFLECTION COEFFICIENT =',PRT
         WRITE(*,*)
20     CONTINUE

       WRITE(*,*)
*
* CALCULATE INPUT IMPEDANCE OF THE OPEN AND TERMINATED  WIRE
*
       DO 30 I=1,7
         ZOPEN(I)=(1.0+PROCOM(I))/(1-PROCOM(I))*50.0
         WRITE(*,*)' OPEN CIRCUITED WIRE INPUT IMPEDANCE =',ZOPEN(I)

         ZTERM(I)=(1.0+PRTCOM(I))/(1.0-PRTCOM(I))*50.0
         WRITE(*,*)'TERMINATED WIRE INPUT IMPEDANCE =',ZTERM(I)
*
* CALCULATE CHARACTERISTIC IMPEDANCE
*
         ZO(K*7+I)=CSQRT(ZTERM(I)*(ZOPEN(I)+RTERM)-ZOPEN(I)*RTERM)
         GAMMA(K*7+I)=1.0/(2*100.0)*LOG((1.0+ZO(K*7+I)/ZOPEN(I))/
     C          (1.0-ZO(K*7+I)/ZOPEN(I)))
         WRITE(*,*)'GAMMA(K*7+I) =',GAMMA(K*7+I)
         Y=GAMMA(K*7+I)/ZO(K*7+I)
         CAP(I)=AIMAG(Y)/(6.28*FREQ(I)*1.0E+06)
         WRITE(*,*)'CHARACTERISTIC IMPEDANCE IN OHMS=',ZO(K*7+I)
         WRITE(*,*)'CONDUCTANCE IN MHOS=',REAL(Y)
         WRITE(*,*)'CAPACITANCE IN FARADS=',CAP(I)
         WRITE(*,*)
30     CONTINUE
40     CONTINUE

       DO 50 I=1,7
         WRITE(1,1000)(ZO(K*7+I),GAMMA(K*7+I),K=0,2)
1000   FORMAT (11(E12.6,',',E12.6),',',E12.6,',',E12.6)
50     CONTINUE

       CLOSE(1,STATUS='KEEP')
       STOP
       END
```