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Nonparametric and Semi-Nonparametric Approaches to the Demand for Liquid Assets

by

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Abstract

In this thesis we take a microeconomic- and aggregation-theoretic approach to the problem of the definition of money and to the demand for money in Canada. In particular we perform revealed preference tests to uncover the group of monetary assets appropriate for economic monetary aggregation. We also test some of the official monetary aggregates produced by the Bank of Canada. We conclude that broad aggregates will perform better than narrow monetary aggregates in macroeconomic models. We proceed our study in a parametric setting by estimating a demand system derived from a global flexible functional form –the Fourier. This model produces arbitrary estimates of elasticities of substitution. The elasticities of substitution between personal deposits, non-personal deposits and foreign currency deposits are found to be low and variable over the period of study. This is evidence against putting these types of assets together and against methods of aggregation that do not account for variable elasticities of substitution.

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Chapter 1

INTRODUCTION

Choosing the appropriate monetary aggregate is important in the evaluation of monetary policy in macroeconomic models. For instance, to determine the neutrality of money we must select an adequate empirical definition of money. Barnett, Fisher and Serletis (1992) notice that over the years it has been a dictum that there must exist a stable demand function for money for money to exert a predictable influence in the economy. There are two problems with previous demand for money studies. One is the linear functional form assumed and the other one is the use of official (simple-sum) monetary aggregates. The above problems raise several questions. What assets should be included in the construction of monetary aggregates? What are the best methods for aggregation? According to economic theory, how should we define money? To address these and other important questions we advocate to recent microfoundations applied to monetary economics.

In the recent literature two different but related strands of research focus on what people use as money. One approach is nonparametric and empirically evaluates revealed preference conditions to determine which assets consumers are actually using. The second strand of research estimates elasticities of substitution from a system of demand equations. If assets are found to be substitutes for each other then there is evidence that they provide similar services to people. In this study we use both approaches and explain how, instead of being competing lines of study, they actually complement each other.

We begin with the selection of assets to form valid economic aggregates. Here what is argued is that the selection of assets should be consistent with the economic theory of aggregation over assets. For an economic monetary aggregate to exist its components must at least be weakly separable from all other assets in the objective function. Weak separability implies that the marginal rate of substitution between any two assets in the subgroup is independent of the level of consumption of assets outside the subgroup. In other words, weak separability is a necessary condition for the existence of aggregates.

There are two approaches to identify which monetary aggregates are consistent with the economic theory of aggregation. These approaches are the revealed preference tests and the statistical approach based on parametric estimates. The former is nonparametric and as such has several advantages. First, it can be used with any number of observations. Secondly, it does not have to assume an underlying functional form of preferences. Finally, the approach is less computational burdensome. The main criticism to the reveal preference approach is that it is nonstochastic, that is to say, there is no way to determine the statistical degree of a rejection.

On the other hand, the statistical approach has the advantage of allowing stochastic behavior through the error term in the estimation. However, the disadvantage is that we need more observations than parameters to be estimated and even worse any separability test is a joint test of separability and the choice of functional form.[1] This

[1] For an example of statistical separability tests see Serletis (1987) who used a quasihomothetic translog functional form.

latter problem is serious when selecting a monetary aggregate. As Swofford and Whitney (1994) put it

"When any group fails a statistical test for weak separability, there is no way to know whether the failure is due to the asset not forming an economic aggregate or to the choice of an inappropriate functional form for preferences." (p.236)

The nonparametric revealed preference tests in monetary economics were initiated by Swofford and Whitney (1987). They found that a relatively narrow monetary aggregate met the conditions of weak separability and hence the conditions required for existence. This monetary aggregate was the equivalent of what Friedman and Schwartz (1963 and 1970) had already determined to be the empirical measure that best matched the theoretical construct of money.[2] In a later study Swofford (1995) tested again for weak separability of Friedman and Schwartz money. He concluded that the assets met the economic criteria to form an aggregate that we might call money for the subperiods Friedman and Schwartz studied but found evidence for a broader aggregate in post World War II data. This supported Barnett's (1980) view that a broad monetary aggregate should be used because substitution effects would be internalized between financial assets.

In our study we use Varian's (1983) procedure to test for weak separability in Canadian monetary assets. Our results are consistent with Barnett's view in that we find evidence for broad monetary aggregates. Furthermore, we group the Canadian monetary assets in three subgroups to continue our research in a parametric and semi-nonparametric framework. In the parametric stage we estimate a parameter intensive

demand system where we cannot include many monetary assets. Therefore, we use the nonparametric tests to construct three valid monetary subaggregates that we can deal with in the parametric estimation. The groups we are going to deal with are a group composed of monetary assets mainly held by households, one composed of assets held by the business sector and a third group composed of only foreign currency deposits. An interesting result from the revealed preference tests is that the household monetary aggregate is broader than the business monetary aggregate.

In our parametric exercise we estimate Gallant's (1981) Fourier demand system. The Fourier model possesses global flexibility and allows for asymptotic inferences that are, to a certain extent, free from any specification error. We test the model for consistency with the economic theory and in general we do not conclude that there is model failure. In addition, the economic content of the parameter estimates is studied through income and price elasticities and elasticities of substitution.

The focus of studies that employ this approach is mainly the substitutability/complementarity relation between monetary assets. Offenbacher (1979) estimated the elasticities of substitution among currency, demand deposits, and time deposits. His main finding is the low substitutability between assets that compose U.S. M2. Ewis and Fisher (1984 and 1985) found evidence of low and even complementarity among financial assets.[3] Serletis and Rob (1986) using a quasihomothetic translog functional form also found low elasticities of substitution. Fleissig and Serletis (2001) estimated a Fourier demand system using Canadian monetary liquid assets to find that

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[2] Friedman and Schwartz money was currency, demand deposits and savings deposits at commercial banks.
[3] Ewis and Fisher estimated a Translog and a Fourier model.

assets are substitutes for each other. The list of studies that have used this approach is endless and the functional forms used to derive the demand systems are many. However, in summary, all of them have found less than perfect substitutability among liquid monetary assets and variable elasticities of substitution over time. These two results have direct implications for the definition of money because the degree of substitutability among monetary assets has been used explicitly or implicitly to construct monetary aggregates.

In this study we estimate elasticities of substitution between personal monetary assets, non-personal monetary assets and foreign currency deposits held by the Canadian public. We find very low substitutability between monetary assets suggesting that aggregation across these types of assets is inadequate. In the sense that the elasticities of substitution are low the parametric and non-parametric results are consistent. On the other hand, foreign currency deposits (mainly U.S. dollars) are poor substitutes for Canadian assets. We also found higher variability of the estimated elasticities in the decade of the eighties (when there was more economic turmoil) than that in the nineties.

The structure of the thesis is as follows. In Chapter 2 we explore various definitions of separability and investigate how separability induces functional structure in the aggregator function. We also see how the concept of separability applies to economic problems and in particular to monetary economics. Finally, we describe the nonparametric methods used to test for weakly separable structures.

In Chapter 3 we apply the revealed preference tests to Canadian monetary data. We test for the existence of a representative consumer and for weak separability. We analyze the popular Bank of Canada definitions of money assuming that the aggregator

function is simple sum. In doing so, we uncover a different set of monetary assets than those used by the Bank of Canada and use that set of monetary assets to proceed our study in a parametric setting. We end that Chapter with a section that discusses the advantages and disadvantages of the nonparametric approach to the demand for liquid assets.

In Chapter 4 we introduce the parametric approach to the demand for money. In doing so, we use the Fourier flexible functional form to approximate the indirect utility function. We explain the reasons we select the Fourier model which is attractive in terms of its global properties. We describe in detail the Fourier model and derive the share equations for monetary assets.

In Chapter 5 we describe a stochastic specification of the Fourier model which is later estimated using nonlinear econometric techniques. We take care of some econometric considerations regarding the model's assumptions and end up estimating a dynamic version of the Fourier model. We test the model's consistency with the regularity conditions and present income and price elasticities and elasticities of substitution. Finally, in Chapter 6, we conclude with the implications of our study and we provide some suggestions for future research.

Chapter 2

SEPARABILITY AND THE NONPARAMETRIC APPROACH TO MONETARY ECONOMICS

2.1 Introduction

In this chapter and the next we will be searching for a definition of money consistent with microeconomic- and aggregation- theoretic literature. To achieve this we will conduct the analysis within a microtheoretical framework making use of two related theories, revealed preference theory and the theory of separable structures.

Revealed preference is a way to discover people's preferences from observing their behavior. We can analyze information on consumer demand to test firstly for the existence of "well behaved preferences" related to a representative consumer and secondly, for separability in the arguments of a utility function.

The set of tests we want to perform are nonparametric in the sense that we do not need to specify a functional form on the utility function. This is a clear advantage over the standard approach to demand analysis (parametric tests) where the hypothesis of separability has to be joined to the hypothesis of functional form. Nevertheless, the non-parametric tests have disadvantages on their own, mainly that they are non-stochastic in nature.

Separability of monetary assets from other goods in the utility function is an important condition for a stable relationship between broad base monetary aggregates and income. Separability has aggregation implications and the definition of money itself is an aggregation problem.

We will start our discussion by talking about assumptions economists generally make about utility functions since the definition and discussion of separability involves precisely this type of functions. Separability implies functional structure and it means that a subutility function can be defined over a sub-set of goods. Indeed, it is the properties of the utility function that we want to carry on to the separable subutility functions.

We will then look at the formal definitions of separability starting with the first notions of Leontief (1947) and Sono (1945) to eventually end up with a more general definition due to Bliss and used by Blackorby, Primont and Russell (1978). Next, we discuss the way separability implies functional structure in the utility function. Finally, the importance of separability in empirical and theoretical studies is analyzed with special emphasis on the implications for monetary economics.

2.2 Regularity Conditions (RC)

Economists usually make general assumptions to describe economic behavior. A consumer is an economic entity (e.g. a person or household) who gains satisfaction from the consumption of commodities. Denote by $X \in \Re_+^n$ the set of feasible consumption vectors. The consumer is presumed to have preferences between alternative consumption vectors in the consumption set; for example, $x^1, x^2 \in X$ are two alternative bundles and the consumer prefers x^1 over x^2 or x^2 over x^1 or she is indifferent between the two. We can give functional expression to preferences over X by assuming that the consumer has a utility function,

$$U: X \to \Re,$$

with higher values of U indicating increased satisfaction. The consumer's objective of maximizing satisfaction translates into the mathematical requirement of 'maximizing U'.

In describing preferences, in general, economists find it convenient to focus on a few general shapes of utility functions. Some call these assumptions 'Well Behaved Preferences' or 'Regularity Conditions' (RC).

Firstly, the analysis of utility maximization employs the assumption of continuity in the utility function.

$$U: \Re^n_+ \to \Re \text{ is continuous on } \Re^n_+.$$

Secondly, we will typically assume that more is better, that is that goods are always "goods" and never "bads". Let $x^1, x^2 \in X$, if x^1 is a bundle of goods and x^2 is a bundle of goods with at least as much of all n goods and more of one, then $x^2 \succ x^1$. This assumption is sometimes called monotonicity of preferences.

Given a function $U(x)$, if successively larger values x always lead to successively larger values of $U(x)$, that is if,

$$x^2 > x^1 \Rightarrow U(x^2) > U(x^1)$$

then the function $U(x)$ is said to be an increasing (or monotonically increasing) function.[1]

Next, we generally assume quasiconcavity on the utility function which is a weaker condition than concavity. Let x^1 and x^2 be any two distinct ordered n-vectors in the domain of $U(x)$ and θ a scalar with the range of values $0 \le \theta \le 1$. A function U is concave iff, for any pair of distinct points x^1 and x^2 and for θ,

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[1] A positive marginal utility $U_i(x)$ also reflects the assumption that the consumer prefers more goods to less. Monotonicity implies that indifference curves have a negative slope.

$$\theta U(x^1) + (1-\theta)U(x^2) \leq U(\theta x^1 + (1-\theta)x^2).$$

This definition is easily adaptable to strict concavity by changing the weak inequality $\leq$ to the strict inequality $<$.[2]

Strict concavity means that if the consumer is indifferent between x^1 and x^2 then the consumer will prefer any weighted average of x^1 and x^2 to either x^1 or x^2. Whilst non-strict concavity means that a weighted average of the two bundles will be at least as good as any of the two extreme bundles x^1 or x^2. Loosely, the consumer prefers 'mixtures' of goods.

Concavity of the utility function is an assumption that we would prefer to relax or generalize to quasiconcavity. It is generally desirable in mathematical economics that models or theories should rest upon as weak a set of assumptions as possible.[3]

A function U is quasiconcave iff, for any pair of distinct points x^1 and x^2 in the convex domain of U, and for $0 < \theta < 1$,

$$U(x^1) \geq U(x^2) \Rightarrow U(\theta x^2 + (1-\theta)x^1) \geq U(x^2).$$

To adopt this definition to strict quasiconcavity, the weak inequality on the right should be changed into a strict inequality. In general, a concave function must be quasiconcave, although the converse is not true.

The regularity conditions are an integral component of the functional structure

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[2] When we only have 2 goods the three assumptions together imply that contours of U (indifference curves) are convex.

[3] Another reason the replacement of concavity with quasiconcavity is desirable is to assure that a utility function or a contour-preserving transformation of it represents the same preferences. The assumption of concavity does not satisfy this requirement since, for instance, monotone increasing transformations of concave functions need not be concave. However, contour-preserving transformations of quasiconcave functions are quasiconcave.

induced by separability. There are also important conditions when estimating a (monetary) demand system in a parametric framework. The form of a demand system depends on the assumed functional form for the utility function. From a set of estimated parameters we can test if a particular functional form conforms to the regularity conditions. In this way we test the proposed functional form and/or the economic theory. This subject will be addressed in Chapters 4 and 5 when estimating monetary demand equations in a parametric framework. However, for the time being (in Chapters 2 and 3) we will concentrate on separability and nonparametric tests.

2.3 Separability

Separability is associated to valid monetary aggregates and ultimately to the definition of money since separability is related to the utility functional structure of a representative consumer. The goal is to identify the set of monetary aggregates that are actually being demanded by economic agents, Moreover, separability resolves the problem associated with few statistical degrees of freedom since it rationalizes the estimation of a small system of demand equations. In the parametric analysis to the demand for money (Chapters 4 and 5) the concept will become extremely useful. In this section we will state the formal definitions of separability to then look at the way it induces structure on the utility function.

The first notions of separability were conceived separately by Sono (1945) and Leontief (1947) when dealing with aggregation problems in consumer and production theory, respectively. According to them, if the utility (aggregator) function is twice differentiable and marginal utilities are positive,

$$U_i(x) > 0 \quad \forall x \in \Omega^n, \quad i = 1,...,n$$

then the variables i and j are separable from the kth variable if and only if[4]

$$\frac{\partial}{\partial x_k}\left(\frac{U_i(x)}{U_j(x)}\right) = 0 \quad \forall x \in \Omega^n,$$

that is if the marginal rate of substitution between i and j is independent of x_k.[5]

Leontief-Sono separability implies functional structure, as we shall discuss later in detail in Section 2.4. If a group is separable from its complement it is possible to write the utility function as

$$U(x) = \overline{U}(x^k, U^r(x^r)),$$

where $x = (x^k, x^r)$; x^r is the quantity vector corresponding to the separable set of variables and U^r is a real valued function. U^r can be thought as a sectoral utility function or a composite good. Hence, separability will allow us to form commodity aggregates.

The main disadvantage, however, of the Leontief-Sono definition is that it is local and requires differentiability. Thus, if the indifference curves over the separable variables had a kink at x_i, x_j, the Leontief-Sono definition would not be applicable.

Bliss introduced the definition of separability employed by Blackorby, Primont and Russell (1978). In order to present this definition Blackorby, Primont and Russell construct the notation necessary to identify variables and subsets of variables. Let

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[4]Separability will be defined relative to the domain of the utility function, Ω^n.

[5] Equivalently, the pair $\{i,j\}$ is separable from k if and only if $\frac{U_{ik}(x)}{U_i(x)} = \frac{U_{jk}(x)}{U_j(x)}$, which is the explicit differential statement of the previous equation and where U_{ik} are the cross partial derivatives. This identity is more useful in practice because it is easy to recognize when carrying out differential calculations.

$I=(1,2,...,n)$ be a set of integers that identify the variables over which the preferences are defined and consider a partition of the set in two subsets

$$I = \{I^k, I^r\},$$

where

$$I^k \cup I^r = I,$$

$$I^k \cap I^r = \varnothing,$$

$$I^k \neq \varnothing,$$

and

$$I^r \neq \varnothing.$$

Corresponding to the binary partition I, denote vectors in Ω^n in ways that reflect the partition. Express Ω^n as a Cartesian product of the subspaces

$$\Omega^n = \Omega^{(k)} \times \Omega^{(r)},$$

the dimensions of $\Omega^{(k)}$ and $\Omega^{(r)}$ are given by the cardinalities of I^k and I^r respectively. If the ith commodity is in the rth category, then x_i is a component of the vector $x^r \in \Omega^{(r)}$. Recall that $x = (x^k, x^r)$ and $x \in \Omega^n$.

Now partition the set of variable indices into three disjoint subsets, $\{I^k, I^q, I^r\}$, such that $x = (x^k, x^q, x^r) \in \Omega^{(k)} \times \Omega^{(q)} \times \Omega^{(r)} = \Omega^n$ and define a collection of subsets of $\Omega^{(r)}$ as

$$\Phi^r(\bar{x}^k) = \{\beta^r(\bar{x}^k, x^q, x^r) \mid x^q \in \Omega^{(q)} \wedge x^r \in \Omega^{(r)}\}$$

for some fixed vector $\bar{x}^k \in \Omega^{(k)}$ where the mapping β^r is defined by

$$\beta^r(\bar{x}^k, x^q, x^r) = \{(\hat{x}^q, \hat{x}^r) \in \Omega^q \times \Omega^r \mid U(\bar{x}^k, \hat{x}^q, \hat{x}^r) \geq U(\bar{x}^k, x^q, x^r)\}$$

Then the set of variables I^r is said to be separable in U from the set of variables I^q if $\Phi^r(\bar{x}^k)$ is nested for each fixed vector $\bar{x}^k \in \Omega^{(k)}$.[6] Any collection of subsets of $\Omega^{(r)}$, for example $B = \{B_1, B_2, ...\}$ will be nested if, for any two subsets B_1 and B_2 included in the collection, either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$.

A narrower concept that has been developed is strict separability which is equivalent to the definition used by Gorman (1968). A set of variables I^r is said to be strictly separable in U from the set of variables I^q if, for each fixed vector $\bar{x}^k$,

$$\beta^r(\bar{x}^k, x^q, x^r) = \beta^r(\bar{x}^k, \tilde{x}^q, x^r) \text{ for all}$$

$(x^q, x^r) \in \Omega^{(q)} \times \Omega^{(r)}$ and for any $\tilde{x}^q \in \Omega^{(q)}$.[7]

In general, separability does not imply strict separability but strict separability of I^r from I^q implies separability of I^r from I^q.[8]

When we have strict monotonicity we have a sufficient condition for the equivalence of separability and strict separability. And under strong regularity assumptions (differentiability and strict monotonicity) the three types of separability (Leontief-Sono, strict and non-strict) are equivalent.

2.4 Functional Structure

As already mentioned, separability induces structure. The most salient feature is

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[6] For this result, it is worth mentioning that the regularity conditions of continuity, positive monotonicity and quasiconcavity are not needed. For the definition of separability see Blackorby, Primont and Russell (1978, p.45).

[7] For Gorman (1968) a set of variables I^r is strictly separable from I^q if the conditional ordering $[\preceq^r_q \ (\bar{x}^r \succeq^r_q \bar{x}^r$ *if and only if* $U(\bar{x}^q, \bar{x}^r) \geq U(\bar{x}^q, \bar{x}^r)]$ is independent of the components of $\bar{x}^q \in \Omega^{(q)}$.

[8] See Blackorby, Primont and Russell (1978, p.47) for a proof.

that one can aggregate the separable group of variables into a single composite variable in the utility function. The following theorems state this formally and are taken from Blackorby, Primont and Russell (1978); refer to this source for proofs.

Theorem 1.1

I^r is strictly separable in U from its complement I^c if and only if there exist functions

$$U^r : \Omega^{(r)} \to \Re$$

and

$$\overline{U} : \Omega^{(c)} \times R(U^r) \to \Re$$

where $R(U^r)$ is the range of U^r such that

$$U(x) = \overline{U}(x^c, U^r(x^r)), \qquad (2.1)$$

where $\overline{U}$ is increasing in $U^r(x^r)$.

According to this theorem $U(x)$ is called the "parent" function, $\overline{U}(x^c, U^r(x^r))$ the "macro" function and U^r the "aggregator" function (for future reference).

Theorem 1.2

Assume that U is continuous and nondecreasing. Then I^r is separable in U from its complement I^c if and only if there exist functions

$$U^r : \Omega^{(r)} \to \Re$$

and

$$\overline{U} : \Omega^{(c)} \times R(U^r) \to \Re$$

such that

$$U(x) = \overline{U}(x^c, U^r(x^r)), \qquad (2.2)$$

where $\overline{U}$ is nondecreasing in $U^r(x^r)$.

Notice the differences between the two theorems. The first theorem invokes strict separability where $\overline{U}$ is increasing in $U^r(x^r)$, while the second theorem invokes separability when $\overline{U}$ is only nondecreasing in $U^r(x^r)$. When Blackorby, Primont and Russell prove theorem 1.2, they show that separability implies the existence of a complete, transitive, continuous and nondecreasing ordering $\succeq^r$ on $\Omega^{(r)}$ which may then be represented by a subutility function.

An important issue is that the macro function and the aggregator function can be picked so that they satisfy the same regularity conditions of the parent function. Therefore, the aggregator U^r can be interpreted as a sectoral utility function.

Recall the regularity conditions (RC) of U, continuity, quasiconcavity and positive monotonicity.

Theorem 1.3

Suppose that U is continuous and satisfies the strict separability condition of Theorem 1.1; then there exists $\overline{U}$ and U^r in representation (2.1) which are continuous. If, in addition U satisfies the RC, then so do $\overline{U}$ and U^r.

Theorem 1.4

Suppose that U is continuous, nondecreasing and satisfies the separability condition of Theorem 1.2; then there exist $\overline{U}$ and U^r in representation (2.2) which are continuous and nondecreasing. If, in addition U satisfies the RC, then so do $\overline{U}$ and U^r.

Theorem 1.5

Suppose U is continuous, nondecreasing and satisfies the separability assumption of Theorem 1.2; then $\overline{U}$ and U^r in representation (2.2) can be so chosen that if U has any of the following properties $\overline{U}$ and U^r possess the same property: (i) strict quasi-concavity, (ii) strict positive monotonicity, (iii) positive homotheticity, and (iv) positive linear homogeneity. If I^r is strictly separable from I^c, then (directional) differentiability of $\overline{U}$ implies (v) (partial) differentiability of $\overline{U}$ and U^r, and if U is (strictly) concave, U^r can be chosen to be (strictly) concave.

In the context of preference structures, indirect separability is a different concept from direct separability. What we have discussed up to now is separability in the direct utility function since the arguments in the function are consumption goods. The indirect utility function, on the other hand, has prices and income (or income normalized prices) as its arguments.[9] This function (to be explained in detail in Chapter 4) is an equivalent representation of the ordering of preferences represented by the direct utility function. However, the structural properties implied by these two functions is not necessarily the same. Specifically, the separability structure implied by the direct utility function over consumption goods may differ from the separability structure implied by the indirect utility function over the corresponding income normalized prices.

Consider the following functional representation equivalent to the separability condition for the indirect utility function. Assume the indirect utility function V to be continuous and nonincreasing. Let $\boldsymbol{p}/y$ be the income normalized prices. I^r is separable

in V from I^c if and only if there exist continuous functions

$$V^r : \Omega_+^{(r)} \to \Re$$

and

$$\overline{V} : \Omega_+^{(c)} \times R(V^{(r)}) \to \Re$$

such that

$$V\left(\frac{p}{y}\right) = \overline{V}\left[\frac{p^c}{y}, V^r\left(\frac{p^r}{y}\right)\right] \quad \forall \frac{p}{y} \in \Omega_+^n,$$

and $\overline{V}$ is nonincreasing in $V^r\left(\frac{p^r}{y}\right)$

The regularity conditions satisfied by the indirect utility function are continuity, negative monotonicity and quasi-convexity because $V\left(\frac{p}{y}\right)$ is the dual representation of $U(x)$. In this case $\overline{V}$ satisfies the same conditions as V while V^r reflects the properties of the reciprocal of V.[10] This stands in contrast to the structural relationship between U^r and U in the direct utility case. According to Blackorby, Primont and Russell (1978) we can think of $\overline{V}$ as an indirect utility function and of V^r as the normalized price of one unit of the rth composite commodity, in other words, as a price index.

As previously argued, in general separability of U does not imply, nor is implied by, separability of V. Therefore, a separable indirect utility function represents a different class of consumer preference orderings than does a separable direct utility function. These comments also apply when either U or V is strictly separable.

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[9] The indirect utility function indicates the indifference curve attainable at p (the corresponding price of x) and total expenditure y.

When is separability in the direct utility function going to imply separability in the indirect utility function? In the case where I^r is homothetically separable from I^c in U (where U satisfies the RC) I^r is going to be homothetically separable from I^c in V. Similarly, if the variables are homothetically separable in V they will be homothetically separable in U. [11]

In the last statement homothetic separability requires some explanation. If I^r is separable from I^c in U and if the preference ordering on $\Omega^{(r)}$ is homogeneous,[12] the representation U^r is positively homothetic. This last structure is referred as (direct) homothetic separability and it is important to notice that it does not imply $\overline{U}$ being homothetic in its arguments $(x^c, U^r(x^r))$, nor does it imply homotheticity of U.

The last argument will be of importance when deriving demand equations from an indirect utility function representation (Chapter 3). It turns out that it is easier and convenient to derive the demand system from the indirect utility function for estimation purposes.

2.5 Applying the Concept

Separability has been widely used in theoretical and empirical economics. Its importance emanates from the fact that separability is intimately related to aggregation and decentralization.

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[10] See Blackorby, Primont and Russell (1978) for reasons p. 68.
[11] Follows from theorem 3.8 in Blackorby, Primont and Russell (1978).

[12] A preference ordering on $\Omega^{(r)}$ is homogeneous if indifference between $\bar{x}^r$ and $\hat{x}^r$ implies indifference between $\lambda\bar{x}^r$ and $\lambda\hat{x}^r$ for all $\lambda > 0$.

Consider the consumer's allocation problem, which is to maximize his lifetime utility

$$E\left[\sum_{t=0}^{T}\beta^{t}U(x_t)\right] \qquad (2.3)$$

subject to his lifetime budget constraint

$$\sum_{t=0}^{T}p_t x_t(1+r)^{-t} = A_0 + \sum_{t=0}^{T}y_t(1+r)^{-t} \qquad (2.4)$$

where $\beta = \frac{1}{(1+\rho)}$; A_0 are initial financial assets, y_t is real labor income, r is the real interest rate, ρ is the rate of time preferences, x_t is a vector of goods and p_t is the corresponding vector of relative prices. x_t can be thought of being comprised of consumption goods, leisure and monetary services; then the consumer's one-period allocation problem is to maximize

$$U_t = U_t(c_t, l_t, m_t) \qquad (2.5)$$

subject to

$$p_c{}' c_t + w\, l_t + \pi' m_t = y_t \qquad (2.6)$$

where y_t is total expenditure on full income in period t; p_c is the vector of commodity prices; w is the shadow price of leisure; π is the vector of rental price of monetary assets or monetary asset user cost;[13] l is leisure and $\boldsymbol{m}$ the vector of monetary services.[14]

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[13] Following Barnett, Fisher and Serletis (1992), the ith component of π is given by $\pi_i = p^*\left(\frac{R-r_i}{1+R}\right)$, where r_i is the expected nominal interest rate on the ith asset for one period; R is the expected return on a "benchmark" asset and p^* is the true cost of living index. The expression represents the opportunity cost of holding asset i at the margin.

In most macroeconomic studies a representative agent's utility function is assumed to be intertemporally separable and then the research is focused on the one period allocation problem (most studies on separability tests are done on the one period allocation framework, see Swofford and Whitney (1988) for an example). According to Barnett, Fisher and Serletis (1992) most intertemporal decision macroeconomic models are assumed to be intertemporally strongly separable, therefore, the assumption of intertemporal 'weak' separability does not seem excessive. [15]

Now, let us address the issue of the representative agent. Demand theory provides a model of individual behavior but many interesting applications involve aggregate relationships. Unfortunately, only under very strict conditions do properties of individual demand theory carry over to aggregate relationships.

On this matter Phlips, quoted by Swofford and Whitney (1987) rationalized the use of per capita data assuming a representative consumer whose behavior reflects the average behavior of the population.[16] To support this view Fleissig, Hall and Seater (2000) as originally Phlips quote Hicks (1956 p.55, *A Revision of Demand Theory*): "To assume that the representative agent acts like an ideal consumer is a hypothesis worth testing: To assume that an actual person, the Mr. Brown or Mr. Jones who lives round the corner, does in fact act in such a way does not deserve a moment's consideration."

The basic assumption of the representative agent model is that one can treat

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[14] $\boldsymbol{m}$ is a vector of monetary assets, which provides services such as convenience, liquidity and information (Barnett, Fisher and Serletis, 1992).

[15] Weak separability is Varian's adaptation of Blackorby, Primont and Russell's terminology of separability and is explained in the next section.

[16] For this to be true there must exist individual quasi-homothetic utility functions.

aggregate data as the outcome of a single "representative" consumer's decision. In this model there is no complete microeconomic foundation, however, exact aggregation of individual preferences is not essential; "what is essential is that there exists some "representative" utility function consistent with the data, for only then can we resort to standard utility theory in conducting our analysis." (Fleissig, Hall and Seater, 2000)

When dealing with a data set one can tests for the validity of the representative agent's approach by checking for the existence of a representative agent's utility function and hence test for the possibility of aggregation across individuals.[17]

Usually, it is assumed that a representative utility function exists and that it satisfies certain separability restrictions over commodities in such a way that the analysis is performed over a convenient subset of the data. For example, it is assumed that for the representative agent $\boldsymbol{m}_t$ is separable from l and c_t in equation (2.5), hence, conveniently the analysis can ignore c_t and l_t.

Assume the utility function is separable in monetary services, then equation (2.5) may be written as

$$U = (c, l, f(\boldsymbol{m})),$$

this utility function implies a demand function

$$m_j^D = h_j(\pi, y_m), \qquad y_m = \pi' \boldsymbol{m}$$

where f defines the monetary subutility function and y_m is the total expenditure on monetary services. Under the separability assumption then the researcher can focus attention only on $\boldsymbol{m}$, reducing the data requirements and increasing the statistical degrees

of freedom.

Nevertheless, when separability of monetary services in the utility function is assumed, empirical results from aggregate data based on individual behavior are subject to misspecification of the agent's utility function. To deal with this problem, one can test for separability in the utility function.[18]

Separability implies a two-stage model for consumer behavior. In the first stage the consumer allocates expenditure among the broad categories of goods ***c***, ***l*** and ***m***. In the second stage, the consumer distributes expenditure among the goods in the ***m*** category based only on the relative prices of the goods in that category $\left(\frac{\pi_i}{\pi_j}\right)$.[19] That is the reason Swofford and Whitney (1987) note that weak separability of monetary assets is a crucial condition for the existence of a stable relationship between broad based monetary aggregates and income.

The first stage of the two-stage budgeting process is the focus of macroeconomists. They are interested, for example, in the relationship between consumption expenditure and income or money balances and income. The second stage is the starting point of microeconomists interested on how households allocate expenditure among various categories of goods.

Let us now return to issues pertaining to monetary economics; if we have

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[17] As it will be shown in the next section, this is done by checking if a data set is consistent with the Generalized Axiom of Revealed Preferences (GARP). Consistency with GARP is a necessary and sufficient condition for the existence of a well-behaved utility function capable of generating the data.

[18] Whether the utility function is weakly separable or not in monetary assets is ultimately an empirical question.

[19] Furthermore, if homothetic weak separability attains, the first stage allocation problem is based on full income and price indices for each category of goods. See Swofford and Whitney (1987).

established monetary services to be separable from leisure and consumption either by separability tests or by assumption, we can then continue in the framework of the following consumer problem,

$$\max \quad f(\boldsymbol{m}) = f(m_1, m_2, m_3, \ldots)$$

subject to

$$y_m = \pi' \boldsymbol{m}$$

where y_m is determined in the first stage of the two stage optimization problem.

Even if the utility function is weakly separable in monetary assets, now we are faced with the question of defining monetary asset subgroups. In particular, the monetary aggregates M1, M2, etc. defined by the Central Bank imply that the different components in the subaggregates are weakly separable within $\boldsymbol{m}$.[20] Separability is important because at a fundamental level the problem of the definition of money is an aggregation problem. The objective is to come out with a definition of money from the point of view of the demanders of monetary services. In practice, we can employ revealed preference theory to design a test that gives us information on definition in a very broad sense, in particular to see which grouping of the many potential monetary assets has been used by money-holders. In other words, separability is a natural criterion for defining monetary aggregates; and we can test for it using revealed preference theory.[21] Separability in the monetary utility function can be represented by:

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[20] According to Barnett, Fisher and Serletis (1992), weak separability over the component assets is both necessary and sufficient condition for the existence of stable preferences over those components of the subaggregate. This existence condition implies that if separability fails, such subaggregate has no meaning in theory.

[21] Instead of assigning assets to monetary groups in the Central Bank's ad-hoc fashion, the structure of preferences over monetary groups can be discovered by actually testing for weak separable subgroupings.

$$f(\boldsymbol{m}) \equiv f(f_1(\boldsymbol{m}^1), f_2(\boldsymbol{m}^2), f_3(\boldsymbol{m}^3))$$

where $\boldsymbol{m}^1, \boldsymbol{m}^2$ and $\boldsymbol{m}^3$ are vectors of monetary assets and the aggregator functions f_i can be treated as subaggregate measures of monetary services.[22]

Before ending this section there are two important comments. Note that there are two kinds of aggregation going on. The existence of a representative consumer implies the possibility of aggregation across individuals or households while separability considers the possibility of aggregation across goods themselves. The two concepts are, however, empirically related because (as it will be shown later), one cannot test for separability if there doesn't exist a representative consumer maximizing a utility function.[23]

Secondly, note that there are two instances where separability has to be invoked. First, to narrow the scope of the analysis, the broad category of monetary assets has to be separable from other broad categories (consumption goods, leisure). Secondly, to arrive to a definition of monetary aggregates the monetary utility function has to be separable in the different monetary assets.

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[22] For the simplified form of subaggregation, weak separability only establishes the necessary condition. The necessary and sufficient condition presumes f to be homothetically weakly separable. When homotheticity doesn't hold, the simplified form of subaggregation is not possible and the distance function can be used instead of the aggregator function.

[23] The theory of aggregation over economic agents is independent of the theory of aggregation over goods, therefore the theory of aggregation over goods remains valid for one agent regardless of the form of aggregation over agents. Further, the existing theory on aggregation over agents is much more complex than the one on aggregation over goods. See Barnett, Fisher and Serletis (1992) for a discussion.

2.6 Testing for GARP and Weak Separability

2.6.1 Testing for GARP

The standard approach to testing behavior consistency with the preference maximization problem was to postulate parametric forms to demand functions and fitting them to the observed data. The estimated demand functions were then tested for consistency with the maximization hypothesis. However, this procedure is only satisfactory when the postulated parametric forms are good approximations to the true demand functions. Since this hypothesis is not directly testable, consistency tests jointly preference maximization and functional form.

The proposed approach in this and the next chapter is nonparametric, in that it requires no ad-hoc specifications of functional form. Instead, the demand data can be used and tested with techniques of finite mathematics. This is called the revealed preference approach and originated in the work of Samuelson (1938). Samuelson derived algebraic conditions on the demand functions implied by maximizing behavior. The corollary is that all behaviors that satisfy these "revealed preferences conditions" can be viewed as maximizing behaviors.

Varian (1982) developed methods for examining if a set of observations could have been generated by a neoclassical utility maximizing consumer. He postulates the following definition:

Suppose that we are given a finite number observations on k-vectors of prices and quantities, (p^i, x^i) $i=1,...,n$.

Definition: A utility function $U(x)$ rationalizes the data $(p^i, x^i)\, i = 1,...,n$ if

$U(x^i) \geq U(x)$ for all x such that $p^i x^i \geq p^i x$, for all i.

Varian's method is based on the following theorem due to Afriat (1967):

Afriat's Theorem. The following conditions are equivalent:

(1) There exists a nonsatiated utility function that rationalizes the data.

(2) The data satisfies "cyclical consistency"; that is

$$p^r x^r \geq p^r x^s, p^s x^s \geq p^s x^t, \ldots, p^q x^q \geq p^q x^r$$
implies
$$p^r x^r = p^r x^s, p^s x^s = p^s x^t, \ldots, p^q x^q = p^q x^r$$

(3) There exist numbers $U^i, \lambda^i > 0$, $i = 1,\ldots,n$, which satisfy the Afriat inequalities.

$$U^i \leq U^j + \lambda^j p^j (x^i - x^j) \text{ for } i,j = 1,\ldots,n.$$

(4) There exists a nonsatiated, continuous, concave, monotonic utility function that rationalizes the data.

See Varian (1982) for proof.

Varian (1982) introduced an equivalent condition to Afriat's cyclical condition which is much easier to test on practice. In order to describe this formulation Varian considers the following definitions:

Definition: Given observations x^i and bundle x:

(1) x^i is directly revealed preferred to x written $x^i R^0 x$, if $p^i x^i \geq p^i x$.

(2) x^i is strictly directly revealed preferred to x written $x^i P^0 x$, if $p^i x^i > p^i x$.

(3) x^i is revealed preferred to x written

$x^i R x$, if $p^i x^i \geq p^i x^j, p^j x^j \geq p^j x^l, \ldots, p^m x^m \geq p^m x$ for some sequence of observations

$(x^1, x^2, ..., x^m)$. Here we say that the relation R is the transitive closure of the relation R^0.

(4) x^i is strictly revealed preferred to x written $x^i Px$, if there exist observations x^j and x^t such that $x^i Rx^j, x^j P^0 x^t, x^t Rx$.

Definition: A set of data satisfies the Strong Axiom of Revealed Preferences (SARP) if any of the following statements is true.

(1) $x^i Rx^j$ and $x^j Rx^i$ implies $x^i = x^j$.

(2) $x^i Rx^j$ and $x^i \neq x^j$ implies not $x^j Rx^i$.

(3) $x^i Rx^j$ and $x^i \neq x^j$ implies not $x^j R^0 x^i$.

Definition: A set of data satisfies the Generalized Axiom of Revealed Preferences (GARP) if the following statement is true.

(1) If $x^i Rx^j$ implies not $x^j P^0 x^i$.

Varian replaces condition number (2) in Afriat's theorem for the equivalent consistency with GARP.[24]

Let us return for a while to Afriat's theorem. Parts (1) and (4) tell us that if there is a utility function that rationalizes the data, it will have the properties typically assumed in consumer theory.[25] The inequality in part (3) follows directly from the concavity of the utility function and the first-order conditions of utility maximization.[26] The numbers U^i and λ^i can be used to actually construct a utility function that rationalizes the data. At the same time U^i and λ^i can be interpreted as the utility levels and marginal utilities of

[24] For proof see Varian (1982).
[25] Violations of continuity, concavity or monotonicity cannot be detected with only a finite number of demand observations.
[26] See Varian (1983).

income at the observed demands. Condition (3) simply asks whether there exists a nonnegative solution to a set of linear inequalities. The implication is that if Afriat numbers cannot be found for the data, a rationalizing utility function does not exist. Part (2), now GARP, is a mild generalization of SARP that allows the marginal rate of substitution to be constant along a section of the indifference curve. And it turns out to be the necessary and sufficient condition for the data to be consistent with utility maximization.

GARP is quite simple to check in practice. GARP can be restated as

$$\text{if } x^i R x^j \text{ then } p^j x^j \leq p^j x^i \text{ for all } i, j = 1,\ldots,n.$$

Varian's technique and specifically his computational package (NONPAR) take advantage of this reformulation. What the condition tells us is that the set of choices x^i is revealed to be preferred to x^j if the expenditures on x^i exceed or are equal to those on x^j evaluated at the original set of prices, where i and j vary over all possible combinations. In practice, we evaluate all possible pair of data (which are finite) to see if expenditures on x^i evaluated at p^j exceed expenditures on x^j evaluated at its own price (for all i,j). In a typical time series i,j will refer to dates (not necessarily consecutive).[27]

The NONPAR tests have both advantages and disadvantages. The chief advantage is that the tests are nonparametric and also the tests can handle a relatively large number of goods. One important disadvantage of the technique is that is non-stochastic, implying that a single violation leads to the rejection of the hypothesis being tested (existence of a utility function that rationalizes the data). We therefore must be especially careful about

the possibility of measurement error. With this kind of error false rejections may occur and without a distribution theory for the test we cannot judge the importance of observed rejections with statistical significant tests.[28]

One should bear in mind that there is a bias towards rejection because only one inconsistency leads towards rejection of GARP; consequently and in accordance with Fleissig, Hall and Seater (2000), passing GARP is a very strong result. According to Fleissig, Hall and Seater (2000) several approaches have extended Varian's (1982, 1983) tests to a stochastic environment by allowing for the presence of measurement error. However, we do not wish to pursue these tests or speculate whether the number of violations of GARP are too many or too large since any violation means that separability tests cannot be performed on the data set. According to Swofford and Whitney (1987), the question of how many violations of GARP can occur is a complex one and becomes even more complex when it involves sequential tests such as separability tests.

Finally, in accordance to Swofford and Whitney (1987), another important disadvantage of the technique is that its use on time series data requires the implicit assumption of strong separability of preferences over time. Barnett, Fisher and Serletis (1992) notice that nearly all intertemporal decision models currently in use in economics assume, in fact, intertemporal strong separability, hence the assumption is common in the literature.

Varian (1983) also developed the Homothetic Axiom of Revealed Preferences (HARP) to test for consistency with homothetic preferences. Homothetic functions are a

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[27] In time series, a violation occurs when a consumption bundle is revealed not preferred in one period but then is bought in another period even though the previously revealed preferred bundle is a feasible choice.

generalization of the class of homogeneous functions. A function $f:\Re^n \to \Re$ is homothetic if f is a monotone increasing transformation of some homogeneous function of degree 1.

Theorem 1.6 The following conditions are equivalent:[29]

(1) There exists a concave, monotonic, continuos, non-satiated, homothetic utility function that rationalizes the data;

(2) the data satisfies HARP: for all distinct choices of indices *(i,j,...,m)* we have

$$(p^i x^j)(p^j x^k)...(p^m x^i) \geq 1;$$

(3) There exist numbers $U^i > 0, i = 1,...,n$ such that $U^i \leq U^j p^j x^i, \quad i, j = 1,...,n;$

For a proof see Varian (1983).

For an example for (2) in theorem 1.6, when n=2 HARP implies $(p^1 x^2)(p^2 x^1) \geq 1$.

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[28] According to Drake (1996), failures of GARP can potentially occur due to slow or incomplete adjustment to relative price changes so that observed quantities are not in fact equilibrium quantities.

[29] In the following theorem prices are normalized by the level of expenditure at each observation so that $y^i = p^i x^i = 1$, for all $i = 1,...,n$.

2.6.2 Testing for Weak Separability

Varian (1983) adapts the terminology of Blackorby, Primont and Russell to define weak separability. If we partition the data into two sets of goods and prices $(p^i, x^i), (q^i, z^i) \quad i = 1,...,n$, then the utility function U is weakly separable in z goods if a subutility function $W(z)$ and a macro function $\overline{U}(x, W)$ can be found such that

$$U(x,z) \equiv \overline{U}(x, W(z))$$

where $\overline{U}$ is strictly increasing in W.

Varian provides two tests of separability; the first test is weaker than the second is since it only checks necessary conditions. The necessary conditions will be met when the data $((p^i, q^i), (x^i, z^i))$ and the data (q^i, z^i), $i=1,...,n$ both satisfy GARP. Clearly, the entire set must satisfy GARP since it comes from maximizing $U(x,z)$. The subdata z^i must satisfy GARP since it has to solve the problem

$$\begin{aligned} \max \quad & W(z) \\ \text{s.t.} \quad & q^i z \le q^i z^i \end{aligned}$$

The second test is stronger because it checks necessary and sufficient conditions for separability. Varian (1983) proves the following theorem.

Theorem 1.7 The following conditions are equivalent:

(1) There exists a weakly separable, concave, monotonic, continuos, non-satiated utility function that rationalizes the data;

(2) There exists numbers $U^i, W^i, \lambda^i > 0, \mu^i > 0$ $i=1,...n$ that satisfy

$$U^i \le U^j + \lambda^i p^j (x^i - x^j) + \left(\frac{\lambda^j}{\mu^j}\right)(W^i - W^j),$$

$$W^i \le W^j + \mu^j q^j (z^i - z^j);^{30}$$

(3) the data (q^i, z^i) and $(p^i, 1/\mu^i; x^i, W^i)$ satisfy GARP for some choices of (W^i, μ^i) that satisfy the Afriat inequalities.

One type of sufficient conditions that we are able to check is that the data satisfy GARP when the subutility functions are calculated using the Afriat inequalities. Varian's computer routine checks each subset of goods for consistency with GARP. Then it checks to see if specified subutility functions will fit together in the overall macro utility function in the appropriate way. Generating the Afriat numbers for each subutility function and seeing if these subutilities can form a consistent set does this. Once the data passes this test, the data will be consistent with the proposed structure of weak separability. However, if the data does not pass the test we cannot rule out the possibility that the separability assumption in question in fact does hold, since some other subutility representation may satisfy the Afriat inequalities. Of course, if the necessary condition fails or the sufficient condition passes, the results of the test are definite.

Finally, Varian (1983) based on Diewert's and Parkan's (1978) heuristic procedure provides the following theorem to test for homothetic separability.

Theorem 1.8 The following conditions are equivalent:

(1) There exists a homothetically separable, concave, monotonic, continuous non-satiated utility function that rationalizes the data;

(2) the data $(p^i, 1/W^i; x^i, W^i)$ satisfy GARP for some choice of W^i that satisfies the

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[30] μ^j is interpreted as the marginal utility of income at z^j.

homotheticity inequalities;

(3) there exist numbers $U^i, W^i > 0, \lambda^i > 0$ such that,

$$U^i \leq U^j + \lambda^j p^j (x^i - x^j) + \left(\frac{\lambda^j}{W^j}\right)(W^i - W^j)$$

$$W^i \leq W^j q^j z^i$$

and the proof is a straightforward application of theorems 1.6 and 1.7.

2.7 Conclusion

In this chapter we have developed the necessary theory for much of our subsequent analysis. The microeconomic and aggregation theory discussed here give basis for the two approaches of demand analysis taken in this thesis. In both, the nonparametric and parametric approaches, we are interested in modeling the representative consumer and his aggregator function that should have the properties typically assumed in consumer theory. In other words, we look for consistency with the utility maximizing hypothesis.

In Chapter 4 we will argue that the nonparametric and parametric approaches are related in the sense that data consistent with revealed preference theory is adequate in the estimation of a parametric demand system. To justify the use of a monetary aggregate or asset we can employ the revealed preference tests described by Varian (1982, 1983) and determine what groupings meet the theoretical existence conditions.

The nonparametric tools for consistency with GARP and weak separability that were described in this chapter are applied in Chapter 3 to Canadian monetary data. The narrow and broad popular monetary aggregates will be analyzed in terms of conforming

to the separability hypothesis. In doing so, a comparison with other nonparametric studies is provided.

Chapter 3

THE NONPARAMETRIC APPROACH TO THE DEFINITION OF MONEY

3.1 Introduction

In this chapter our goal is to obtain monetary aggregates consistent with the microeconomic theory discussed in the previous chapter. We will apply the revealed preference approach and specifically Varian's NONPAR tests. For that purpose we will assume that there exists an intertemporal weakly separable (in monetary services) utility function. This implies that decisions to consume monetary asset services are made independent of the decision to consume other goods. We assume that the consumer faces the following problem after having determined total expenditure on monetary services:

$$max \quad f(\boldsymbol{m}) = f(m_1, m_2, m_3, ...)$$

subject to

$$y_m = \pi' \boldsymbol{m}$$

where $\boldsymbol{m}$ is the vector of monetary asset quantities, π is the corresponding vector of user costs and y_m is determined in the first stage of the two stage optimization problem, described in Chapter 2. Because this is a strong assumption, many studies have included consumption goods, leisure and money in their analysis. Some find that all monetary components are weakly separable from consumption goods and leisure while others find that only relatively liquid monetary assets are weakly separable. This issue will be addressed later in the discussion.

The first step in our empirical research is to test if Canadian monetary data are consistent with the utility maximizing hypothesis. In doing so, we use GARP. Because the test is non-stochastic measurement errors may produce false rejections. We respond to this difficulty by faithfully reporting all violations and by adding some judgmental discussion on the validity of the results. For our monthly and quarterly data, no set is consistent with GARP for the whole sample periods. Following recent literature, the second step is to find subsamples for which GARP is not violated. We find that such subsamples exist but are relatively short rendering estimation of the utility function impractical. Our results show that narrow monetary aggregates are not consistent with weak separability but the broader the aggregate the more it conforms to the separability hypothesis. This result, somewhat at odds with the literature, suggests that narrow definitions of money do not meet the existence condition.

Monetary aggregates other than the ones defined by the Bank of Canada were uncovered by the separability tests and specific groupings were tested for the purpose of obtaining a monetary utility function defined over few monetary aggregates. This stage of our study is of crucial importance for the second part of the thesis since the parametric approach to be taken there puts a limit on the number of goods that we can deal with, since degrees of freedom become scarce.

3.2 Data

The data for this study is an updated version of the data used in Molik (1999) and Serletis and Molik (2000)[1]. Table 3.1 shows the 10 component series that the Bank of Canada uses in the construction of 5 popular monetary aggregates – M1, M1+, M1++, M2 and M3. The monetary aggregates published by the Bank of Canada are nested in the next broader definition (aggregate). For example, the three assets that form M1 are also part of M1+. The last definition, M3, is the broadest aggregate, including all 10 assets. Because revealed preference tests require data on quantities and user costs, information was not available for assets used in the construction of other monetary aggregates, such as for example, M2+ and M2++.[2]

The order of the components is from more to less liquid. According to some, the more liquid an asset the more is used for transaction purposes and less for saving purposes. [3] Also notice that the Bank of Canada makes the distinction between personal (household sector) and non-personal (business sector) deposits.

Molik (1999) approximated the user costs for each component using the following formula:

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t} p^*$$

[1] We thank Molik T.E. for making available this data set.
[2] See Molik (1999) for data availability problems.
[3] According to Aubry and Nott (1999), in the past 30 years due to "waves" of financial innovation the distinction between demand (checking) accounts and savings (notice) deposits (in Table 3.1) has become less important to banks and the public.

where p^* is the true cost of living index, R_t is the benchmark rate of return, and r_{it} is the nominal market rate of return on asset i. In theory the benchmark rate is the rate of an asset used only to transfer value from period to period and yields the highest available holding period expected rate of return in the economy.[4] According to the formula the user cost is the discounted value of the interest forgone by holding asset i for one period.

Molik (1999) set the corresponding interest rate for currency equal to zero and in calculating the user costs for demand deposits (series 2 and 3 in Table 3.1), the implicit rate of return was used, based on the following formula,

$$r_D = (1-k)r_A,$$

where r_A is the interest rate on an alternative asset and k is an estimate of the maximum required reserve ratio.[5]

The interest rates on series 4 to 10 of Table 3.1 are a composite of several interest rates published by Statistics Canada and the Bank of Canada. Table 3.2 shows the interest rates and sources used in approximating the own interest rates for different time periods.[6] Finally, interest rates on monetary assets with different terms to maturity were yield curve adjusted to make the yields comparable.[7]

Two sets of monetary and rental price data are used in this study, a quarterly set starting on January 1976 and ending on December 1999 and a monthly set running from November 1981 to December 1999. For testing purposes the monetary component data

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[4] See Molik (1999) for calculations.

[5] r_A was taken to be the interest rate on the 3-5 year government of Canada bonds and k was constructed from the primary and secondary reserve ratios against demand deposits.

[6] Because of financial innovations, the interest rate that best represents the interest rate of the monetary component changes over time for some series.

[7] Only personal fixed term deposits needed to be yield curve adjusted, for details see Molik (1999 p.91).

was further deflated using the Canadian CPI (CANSIM series P100000).[8] In addition, since the revealed preference tests invoke a representative consumer, monetary data was divided by Canadian population 15 years old and older (CANSIM series D980000) to obtain per capita data.[9]

3.3 GARP Results

Because there is no statistical test to judge the importance of GARP rejections we faithfully report all violations. For the whole quarterly data set we found 869 GARP violations while in the monthly data, for which we have more observations, we found 2063 violations. However, it is interesting to identify the periods responsible for the violations because we can then discover subsamples over which GARP is satisfied. The "revealed preference matrix" of the NONPAR program allows us to uncover the observations responsible for the inconsistencies. Once the inconsistencies are located, and re-running the GARP tests, we are able to identify the subsamples consistent with GARP. It would seem that there are too many violations in the whole sample set to uncover a period of significant length consistent with consumer maximization. However, we discovered that most violations were concentrated in the first 23 observations of the quarterly data, corresponding to the period before the introduction of foreign currency deposits (1976Q1 – 1981Q3). Because the NONPAR program performs tests over a constant number of goods, we had to include zeros for the amounts of foreign currency

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[8] CPI 1996 class, all items, 1992=100.

[9] In the literature, some studies use total population and others population 18 years old and older. In the latter case, researchers do not want to treat a six-year-old as an independent agent nor do they want to have an 18-year-old having all decisions made by his parents. Following those approaches we assume that full economic life begins at 15.

deposits for that period. Deleting the first 23 observations reduces dramatically the GARP violations from 869 to 96. The monthly data is only for the period after the introduction of foreign currency deposits so the GARP violations are more evenly distributed. The largest subsamples consistent with GARP for the monthly and quarterly data are reported in Table 3.3.

The largest subsample period consistent with GARP for the monthly data is over a 4 year period and the largest for the quarterly data is almost 6 years, which suggests that the representative consumer's utility function parameters or functional form changes quite frequently in time.[10]

3.4 HARP Results

Because many studies in the past have relied on homothetic functional forms and Varian's procedure allows us to test for homotheticity, the four largest subperiods consistent with GARP were checked for consistency with the homothetic axiom of revealed preference.[11] As can be seen in Table 3.4, no subperiod was consistent with homothetic preferences. This result is similar to Swofford and Whitney (1987) and Swofford and Whitney's (1988) result when testing US quarterly and annual data of consumption, leisure and money. They argue that studies that rely on homothetic functional forms like the Cobb-Douglas or constant-elasticity-of-substitution are therefore misspecified.

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[10] GARP tests were performed using the GDP implicit price index instead of the CPI to deflate quarterly monetary data. In that case the largest subsample consistent with GARP comprised 9 years. Further, if per capita quantities are obtained from total population (instead of population 15 and older), the maximum length consistent with GARP is near to 13 years. These results are not reported in the main text because neither monthly GDP deflator nor monthly total population series were available for comparison purposes.

Swofford and Whitney's and our results are not at all surprising since the homotheticity assumption is indeed a strong assumption, not expected to be satisfied by most economic data sets.

3.5 Separability Results

As discussed in the previous chapter, separability tests are essential to understand what monetary aggregates are in reality demanded by consumers. The purpose of the analysis is two-fold. Firstly we want to see if the monetary aggregates defined by the Bank of Canada are theoretically valid. Secondly, if these monetary aggregates fail to meet the existence condition, then we want to uncover the valid monetary component subgroupings. The second part is of crucial importance for the future objective of estimating a demand system of monetary assets. For this purpose a flexible functional form will be required. Unfortunately, a flexible functional form cannot be estimated for a large number of goods since degrees of freedom are lost exponentially as the number of goods increases. Therefore, the final objective of the separability tests is the identification of a small quantity of weakly separable monetary assets to continue our research in a parametric setting.

Separability tests are a continuation of GARP tests as discussed previously. Separability tests can only be performed over samples consistent with GARP, therefore the subsequent results refer to the largest relevant subperiods in terms of a significant length, which correspond to the periods in italics in Table 3.3.

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[11] We can only check for HARP on series that have passed GARP.

In Table 3.5 we test eight hypotheses for the Bank of Canada's definitions of money, assuming that the aggregator function is simple sum. Definition M1 fails the necessary and sufficient conditions for the 4 subsamples tested. M1+ passes the necessary condition in only one instance (quarterly data 1985:1-1990:3 in Table 3.5). M1++ does a lot better passing the necessary and sufficient conditions for both subsamples of quarterly data tested. M2 does even better passing the necessary and sufficient conditions for the two quarterly data subsamples and passing the necessary condition for one monthly subsample. M2 plus non-personal term deposits (series 9 in Table 3.1) does as well as M2 passing the two conditions for the quarterly data and the necessary condition for one monthly subperiod.

Hypotheses 9 to 36 in Table 3.6 are proposed separability –consistent subgroups. All possible combinations of groups formed with adjacent monetary components were tested.[12] We report only those in which at least one test was successful in one sample. Out of all the monetary subgroups tested only two passed the necessary and sufficient condition in the four sample periods. Personal checking accounts and current accounts form a separable subgroup from the other monetary assets in the four samples tested as well as non-personal non-chequable notice deposits with personal fixed term savings deposits. Of these two the former may be an interesting finding because this subgroup is M1 minus currency outside banks.[13] Previous studies that try to estimate a stable demand for money normally use the monetary aggregates defined by the Bank of Canada. In this regard many studies focus on narrow definitions of money like M1. What the results

[12] By adjacent we mean monetary components adjacent according to Table 3.1.

reported here suggest is a new narrow definition of money consistent with microeconomic theory. Research on a monetary aggregate composed only of personal and non-personal checking accounts might prove fruitful in future demand analysis studies.

Unlike the case just discussed, the other separable subgroup that passes all tests in the 4 samples tested does not make any appealing intuitive economic sense. Series 7 in Table 3.1 are non-chequable savings deposits held by firms whereas series 8 are fixed term savings deposits held by households. The two assets look quite different in their characteristics to be an interesting definition of money.

Several subgroups pass the necessary and sufficient conditions for the quarterly data although not for the monthly data. Of special interest are M1++ minus M1, M2 minus M1, M3 minus M1 and M3 minus M1 and foreign currency deposits.

Some studies prefer to eliminate non-personal accounts from GARP and separability tests on the basis that firms and households behave differently suggesting that the representative consumer hypothesis does better if only household data is taken into account.[14] Hypotheses 37 and 38 in Table 3.6 tests whether personal accounts are weakly separable from non-personal accounts. The answer might be yes since the necessary and sufficient conditions are met for one sample of the monthly data and one sample of the quarterly data, plus the necessary condition is met for the other quarterly data sample. On the other hand, hypotheses 39 to 41 test whether non-personal accounts are weakly separable from personal accounts including and excluding currency outside

[13] Separability tests performed over quarterly series using the GDP deflator instead of the CPI confirmed this result, m_2 and m_3 were found weakly separable from other monetary aggregates.

banks and foreign currency deposits. None of the hypotheses that included non-personal term deposits passed even the necessary condition in the four samples analyzed. However, as can be seen in hypothesis 50, current accounts, non-personal chequable notice deposits and non-personal non-chequable notice deposits (series 3,5, and 7) pass the necessary condition for one quarterly sample. Overall and in accordance to hypothesis tests 44 to 48 and 50, the household and business sectors can be thought as separable groups in one of the quarterly samples.

Many combinations of assets were tested for the final objective of coming up with a small number of component subgroupings. Recall that we are interested in uncovering a small number of aggregates for estimation purposes in a parametric framework. Separability hypothesis tests in Table 3.7 show the results of an effort to uncover a utility function over 3 or 4 separable aggregates. Hypothesis 43 which passes the necessary and sufficient condition for both quarterly data periods proposes M3 minus M1 as one weakly separable group, checking accounts as another group and currency outside banks as the third separable group. Hypothesis 42 is similar to hypothesis 43 except that foreign currency deposits form another separable group in itself. Hypotheses 44 to 48 and hypothesis 50 are over groups defined by separating the personal and non-personal sectors. In some cases currency outside banks is considered in the non-personal sector and in some others it is considered in the personal sector. Following Fisher (1989) and Fisher and Fleissig (1997) we will consider currency as being held mostly by the personal sector as exemplified in hypothesis 50. Observe that in many instances foreign currency deposits can be considered outside the personal and non-personal sector groups. Notice

[14] See Fisher (1989) and Fleissig, Hall and Seater (2000).

also that hypotheses 44 to 48 and 50 only pass the separability tests for one of the two quarterly data subperiods.

3.6 Discussion and Conclusions

In this chapter non-parametric GARP and separability tests were conducted using quarterly and monthly monetary data, assuming that a monetary weakly separable utility function exists. Regarding the existence of a weakly separable monetary utility function, Fisher (1989) does not find evidence of separability of the financial decision from the consumption decision. On the contrary, Swofford and Whitney (1988) using annual data find that monetary assets satisfy the necessary conditions for weak separability from durables, non-durables, services and leisure. However, when using quarterly data the necessary conditions for separability are only met for relatively liquid monetary assets.[15] Drake (1996) using UK quarterly data found that personal monetary assets were weakly separable from durables, non-durables, services and leisure. This may give support to our original assumption, but Drake found evidence against the two-stage budgeting assumption when performing separability tests because excluding durables, non-durables and services provided different results on the valid separable monetary aggregates.

When examining if the data is consistent with the maximization of a well-behaved monetary utility function we found that monetary data was not GARP consistent over the whole monthly or quarterly sample sets. The data revealed that a monetary utility function for a representative consumer exists only for short periods of time. This means

[15] This suggested to the authors that a relatively broad monetary aggregate is not expected to exhibit a stable relationship with income.

that there is not a constant utility function maximized by economic agents because either the parameters of the monetary utility function or the functional form change over time. On the other hand, it may mean that separability of the financial decision from the consumption decision is not valid or that the representative consumer framework is wrong altogether.

Nevertheless, we cannot ignore that passing GARP is a very strong result, since the test is biased towards rejection because a single inconsistency renders the results against the utility maximization hypothesis. In many cases we were forced to split a sample of considerable length of time because of one GARP inconsistency. As Drake (1996) argues failures of GARP may potentially occur due to slow or incomplete adjustment to relative price changes so that observed quantities are not equilibrium quantities. It may well be that GARP was rejected in many instances because of measurement errors. Remember that user costs were approximated with different techniques and different interest rates that may not be totally correct. On the other hand, the use of a different price index provided us with different periods consistent with GARP. Unfortunately, we cannot avoid measurement errors when limited by the availability of the data.

Separability tests revealed that the narrower the monetary aggregate the more it fails to meet the existence conditions, and that the broader the aggregate the more it seems to be weakly separable and meet the existence condition. This may be at odds with some literature on US data. Belongia and Chalfant (1989) tested for weak separability using only assets likely to possess the transaction characteristics of money, they considered assets in the order of more to less liquid. Subgroups were formed by

increasing one asset until separability conditions were violated. Their results showed that several narrow groups could be treated as weakly separable.[16] Fisher (1989) after several experiments argues that separability of the Federal Reserve M1, as long as it does not include NOW accounts and business deposits, may be superior to broader monetary aggregates.[17] Swofford and Whitney (1988) using annual data found that only liquid monetary assets meet the necessary and sufficient conditions for weak separability. Swofford and Whitney (1987) in another article using quarterly data found that when less liquid monetary assets are included in the monetary subutility function the necessary conditions for weak separability do not obtain. The authors concluded that a more appropriate target for monetary policy might be a relatively narrow monetary aggregate consisting of M1 plus other chequable deposits and savings deposits in institutions.[18] Fisher and Fleissig (1997) using monthly US data found that Federal Reserve M1 passed separability tests in more cases than broader monetary aggregates.[19] Furthermore, they found that M1 minus business demand deposits were weakly separable in all their subsamples tested. However, and contrary to these studies, Swofford (1995) found weak separability evidence for a broad monetary aggregate that includes mutual savings deposits and savings and loan shares.

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[16] Specifically, Belongia and Chalfant (1989), located three groupings that satisfy the necessary conditions: currency plus demand deposits, Federal Reserve's M1and M1 plus interest-bearing checking deposits.
[17] Recall, that in our case, Bank of Canada's M1 did not meet the separability conditions.
[18] There are 27 monetary assets in Swofford and Whitney's studies.
[19] Fisher and Fleissig (1997) tested for weak separability in subsamples that were consistent with GARP as done in this study. They found that M1 passed in 51.4% of the months tested while the other Federal Reserve definitions did worse.

In our case, for the Canadian data, M1 minus currency outside banks passed weak separability tests in all our subsamples analyzed.[20] Even if previous studies found different results only future research on the stability of the checking account aggregate can determine the validity of our result.

Many other subgroups passed the separability conditions for our quarterly data samples. Of special interest are subgroups of assets that exclude M1. This is consistent with Fisher and Fleissig's (1997) finding of separable groups that contain assets outside the Federal Reserve's M1. Because the traditional definitions of money all include M1 little research has been done regarding aggregates that exclude M1. Thus, this is an interesting area for future research.

Interesting enough the separability hypothesis holds a lot better for personal assets than for non-personal assets. This might mean that the representative consumer theory does better for individuals than it does for firms. Again this is consistent with previous studies that find that GARP holds a lot better when excluding the business sector.

A quick glance at Tables 3.5, 3.6, and 3.7 can show that less separability hypotheses pass the necessary and sufficient conditions for the monthly data than for the quarterly data. Recall that for the monthly data it was harder to find separable structures in long periods consistent with GARP. As discussed previously, inconsistencies with GARP may occur when economic agents have slow adjustment in their consumption bundles to relative price changes. In this sense, holdings of monetary assets may not be adjusted to optimal levels within a month. An alternative explanation given by Swofford

[20] This may be at odds with Drake's (1996) results since he could not find a monetary grouping excluding currency.

and Whitney (1988) is that agents consider expected future prices when making allocations between monetary assets, this would imply that the utility function is not intertemporally separable for monthly planning periods.[21]

Some final comments should be made regarding separability tests. First of all the likely significance of the sufficient Afriat test should be considered. Remember that if the data does not pass the sufficient condition, the hypothesized form of separability cannot be ruled out, since the Afriat numbers are not the only representation of utility that can be used for the separability tests. This fact should be taken into account specially when looking at the monthly data results.[22]

Secondly, it should be mentioned that the measurement errors discussed for the GARP tests also apply to the separability tests. Separability tests may be sensible to the price deflator or to the assumption regarding the definition of the consumption unit as Drake (1996) found in his study.[23] Because a rejection of GARP is all or nothing we cannot judge whether the violations are more likely due to measurement error or due to a more fundamental change in the model. Therefore, the nonparametric results and any model that relies in the representative consumer framework should be taken with caution until future research is undertaken. Nevertheless, although the nonparametric tests have some limitations, the alternative of not testing at all has also its shortcomings.

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[21] Observe that the same arguments can apply to the quarterly data.

[22] To overcome the sufficient test problem Swofford and Whitney (1994) performed revealed preference tests for weak separability that permit incomplete adjustment within the period under study. However, the method implies solving a very large and complex optimization problem. For that reason the authors were forced to break the sample in two.

[23] Drake's separability results differed between per capita aggregated data and per household aggregated data.

Finally, we will use the separability results obtained in this chapter for the construction of monetary aggregates consistent with economic theory to then pursue our study in a different environment, specifically in a parametric framework. This is the subject of the next chapter.

TABLE 3.1

BANK OF CANADA MONETARY AGGREGATES/COMPONENTS

Monetary Aggregate	Component	Source: Cansim series
M1	1. Currency outside banks	B2001
	2. Personal Checking Accounts	B486
	3. Current Accounts	B487
M1+	4. Personal chequable savings deposits	B452
	5. Non-personal chequable notice deposits	B472
M1++	6. Personal non-chequable savings deposits	B453
	7. Non-personal non-chequable notice deposits	B473
M2	8. Personal fixed term savings deposits	B454
M3	9. Non-personal term deposits	B475
	10. Foreign currency deposits	B482

Source: Serletis and Molik (2000). Monetary data was seasonally adjusted by Molik (1999) using the SAMA command in TSP version 4.3. The procedure uses a moving average method.

TABLE 3.2

Money Stock Data	Interest Rate Data	CANSIM Series Number
4. Personal chequable savings deposits	Selected Cda. Bond Yields, Daily Interest Checking Accounts	B14035 or DICA 0-2K (1976:1 to 1982:9), DICA 5K+ (1982:10-1999:12)
5. Non-personal chequable notice deposits	90-Day Canadian Bonds	B14043 (for the entire time series)
6. Personal non-chequable savings deposits	Selected Cda. Bond Yields, Daily Interest Savings Accounts in excess of 25K and 75K	B14019 (1976:1 to 1986:12), DI25 (1987:1 to 1988:1), Average DISA 25 & DISA 75 (1988:2 to 1999:12)
7. Non-personal non-chequable notice deposits	90-Day Canadian Bonds	B14043 (for the entire time series)
8. Personal fixed term savings deposits	5 Year Term Deposit Rate	B14045 (for the entire time series.
9. Non-personal term deposits	Prime Rate	B14020 (for the entire time series)
10. Foreign currency deposits	Euro-US Deposit Rate	B54415 (for the entire time series)

Source: Molik (1999, p.98) for a detail discussion on the interest rate data.

TABLE 3.3

LARGEST GARP-CONSISTENT SUBSAMPLE PERIODS

Quarterly Data	Monthly Data
1976Q1 – 1999Q4	**1981.11 – 1999.12**
1981Q4 – 1984Q4	*1987.01 – 1990.08*
1985Q1 – 1990Q3	*1991.04 – 1995.03*
1992Q2 – 1996Q3	
1996Q3 – 1998Q2	

TABLE 3.4

HARP TESTS

Quarterly Data	**HARP Violations**
1985Q1 – 1990Q3	23
1992Q2 – 1996Q3	18
Monthly Data	
1987.01 – 1990.08	44
1991.04 – 1995.03	48

TABLE 3.5
TEST FOR WEAK SEPARABILITY, BANK OF CANADA'S DEFINITIONS

Separability Hypothesis	Quarterly Data		Monthly Data	
	1985:1-1990:3	1992:2-1996:3	1987:1-1990:8	1991:4-1995:3
H1: $f[f_1(m_1,m_2),m_3,m_4,m_5,m_6,m_7,m_8,m_9,m_{10}]$*	X	X	X	X
H2: $f[f_1(m_1,m_2,m_3),m_4,m_5,m_6,m_7,m_8,m_9,m_{10}]$	X	X	X	X
H3: $f[f_1(m_1,m_2,m_3,m_4),m_5,m_6,m_7,m_8,m_9,m_{10}]$	X	X	X	X
H4: $f[f_1(m_1,m_2,m_3,m_4,m_5),m_6,m_7,m_8,m_9,m_{10}]$	N	X	X	X
H5: $f[f_1(m_1,m_2,m_3,m_4,m_5,m_6),m_7,m_8,m_9,m_{10}]$	X	N&S	X	X
H6: $f[f_1(m_1,m_2,m_3,m_4,m_5,m_6,m_7),m_8,m_9,m_{10}]$	N&S	N&S	X	X
H7: $f[f_1(m_1,m_2,m_3,m_4,m_5,m_6,m_7,m_8),m_9,m_{10}]$	N&S	N&S	N	X
H8: $f[f_1(m_1,m_2,m_3,m_4,m_5,m_6,m_7,m_8,m_9),m_{10}]$	N&S	N&S	N	X

Note:

m_1 to m_{10} refer to the different monetary components of Table 3.1.

N: passes the necessary condition.

N&S: passes necessary condition and a sufficient condition.

X: fails both conditions.

* Satisfies the necessary and sufficient conditions of separability for a sample of quarterly data deflated by the implicit GDP price index instead of the CPI. The period covered is 1983:2-1992:1 (9 years) and spans over the period of sample 1985:1-1990:3 on the table.

** Satisfies only the necessary condition for the data set described above.

TABLE 3.6
TEST FOR WEAK SEPARABILITY
ARBITRARY DEFINITIONS

Separability Hypothesis	Quarterly Data 1985:1-1990:3	Quarterly Data 1992:2-1996:3	Monthly Data 1987:1-1990:8	Monthly Data 1991:4-1995:3
H9: $f[m_1, f_1(m_2, m_3), m_4, m_5, m_6, m_7, m_8, m_9, m_{10}]$*	N&S	N&S	N&S	N&S
H10: $f[m_1, f_1(m_2, m_3, m_4), m_5, m_6, m_7, m_8, m_9, m_{10}]$	N	N&S	X	X
H11: $f[m_1, f_1(m_2, m_3, m_4, m_5, m_6, m_7), m_8, m_9, m_{10}]$*	N&S	X	X	X
H12: $f[m_1, f_1(m_2, m_3, m_4, m_5, m_6, m_7, m_8), m_9, m_{10}]$*	N&S	N&S	X	X
H13: $f[m_1, f_1(m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9), m_{10}]$*	X	N&S	X	X
H14: $f[m_1, f_1(m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10})]$*	N&S	N&S	X	X
H15: $f[m_1, m_2, f_1(m_3, m_4), m_5, m_6, m_7, m_8, m_9, m_{10}]$	N	N&S	X	X
H16: $f[m_1, m_2, f_1(m_3, m_4, m_5, m_6), m_7, m_8, m_9, m_{10}]$**	X	N&S	X	X
H17: $f[m_1, m_2, f_1(m_3, m_4, m_5, m_6, m_7), m_8, m_9, m_{10}]$*	X	N&S	X	X
H18: $f[m_1, m_2, f_1(m_3, m_4, m_5, m_6, m_7, m_8), m_9, m_{10}]$*	X	N&S	X	X
H19: $f[m_1, m_2, f_1(m_3, m_4, m_5, m_6, m_7, m_8, m_9), m_{10}]$*	X	N&S	X	X
H20: $f[m_1, m_2, f_1(m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10})]$	N&S	N&S	X	X
H21: $f[m_1, m_2, m_3, f_1(m_4, m_5), m_6, m_7, m_8, m_9, m_{10}]$	X	X	N	X
H22: $f[m_1, m_2, m_3, f_1(m_4, m_5, m_6), m_7, m_8, m_9, m_{10}]$*	X	N&S	X	X
H23: $f[m_1, m_2, m_3, f_1(m_4, m_5, m_6, m_7), m_8, m_9, m_{10}]$	N&S	N&S	X	X
H24: $f[m_1, m_2, m_3, f_1(m_4, m_5, m_6, m_7, m_8), m_9, m_{10}]$	N&S	N&S	N	X
H25: $f[m_1, m_2, m_3, f_1(m_4, m_5, m_6, m_7, m_8, m_9), m_{10}]$	N&S	N&S	N	X
H26: $f[m_1, m_2, m_3, f_1(m_4, m_5, m_6, m_7, m_8, m_9, m_{10})]$*	N&S	N&S	X	X
H27: $f[m_1, m_2, m_3, m_4, f_1(m_5, m_6), m_7, m_8, m_9, m_{10}]$	X	N	X	X
H28: $f[m_1, m_2, m_3, m_4, f_1(m_5, m_6, m_7), m_8, m_9, m_{10}]$	X	X	N	X
H29: $f[m_1, m_2, m_3, m_4, f_1(m_5, m_6, m_7, m_8), m_9, m_{10}]$	X	N	N	X
H30: $f[m_1, m_2, m_3, m_4, f_1(m_5, m_6, m_7, m_8, m_9), m_{10}]$	X	X	N	X
H31: $f[m_1, m_2, m_3, m_4, f_1(m_5, m_6, m_7, m_8, m_9, m_{10})]$	X	X	N	X
H32: $f[m_1, m_2, m_3, m_4, m_5, f_1(m_6, m_7), m_8, m_9, m_{10}]$*	X	N&S	N&S	N
H33: $f[m_1, m_2, m_3, m_4, m_5, f_1(m_6, m_7, m_8), m_9, m_{10}]$*	X	N&S	N&S	N
H34: $f[m_1, m_2, m_3, m_4, m_5, f_1(m_6, m_7, m_8, m_9), m_{10}]$	X	N	N&S	X
H35: $f[m_1, m_2, m_3, m_4, m_5, f_1(m_6, m_7, m_8, m_9, m_{10})]$	X	X	N	X
H36: $f[m_1, m_2, m_3, m_4, m_5, m_6, f_1(m_7, m_8), m_9, m_{10}]$*	N&S	N&S	N&S	N&S
H37: $f[m_1, m_3, f_1(m_2, m_4, m_6, m_8), m_5, m_7, m_9, m_{10}]$	N	N&S	X	N&S
H38: $f[f_1(m_1, m_2, m_4, m_6, m_8), m_3, m_5, m_7, m_9, m_{10}]$	X	N&S	X	X
H39: $f[m_1, m_2, f_1(m_3, m_5, m_7, m_9), m_4, m_6, m_8, m_{10}]$	X	X	X	X
H40: $f[m_2, f_1(m_1, m_3, m_5, m_7, m_9), m_4, m_6, m_8, m_{10}]$	X	X	X	X
H41: $f[m_2, f_1(m_1, m_3, m_5, m_7, m_9, m_{10}), m_4, m_6, m_8]$	X	X	X	X

See notes in Table 3.5

TABLE 3.7

TEST FOR WEAK SEPARABILITY

FEW GROUPINGS

Separability Hypothesis	Quarterly Data		Monthly Data	
	1985:1-1990:3	1992:2-1996:3	1987:1-1990:8	1991:4-1995:3
H42: $f[m_1, f_1(m_2, m_3), f_2(m_4, m_5, m_6, m_7, m_8, m_9), m_{10}]$	N	N&S	N	X
H43: $f[m_1, f_1(m_2, m_3), f_2(m_4, m_5, m_6, m_7, m_8, m_9, m_{10})]$ **	N&S	N&S	X	X
H44: $f[f_1(m_1, m_3), f_2(m_2, m_4, m_6, m_8), f_3(m_5, m_7, m_9), m_{10}]$	N	X	X	X
H45: $f[f_1(m_1, m_3, m_5, m_7), f_2(m_2, m_4, m_6, m_8), m_9, m_{10}]$	X	N	X	X
H46: $f[f_1(m_1, m_3, m_5, m_7), f_2(m_2, m_4, m_6, m_8, m_{10}), m_9]$	X	N	X	X
H47: $f[f_1(m_1, m_3, m_5, m_7), f_2(m_2, m_4, m_6, m_8, m_9), m_{10}]$	X	N	X	X
H48: $f[f_1(m_3, m_5, m_7), f_2(m_1, m_2, m_4, m_6, m_8, m_9), m_{10}]$	X	N&S	X	X
H49: $f[f_1(m_1, m_2), f_2(m_3, m_4, m_5, m_6, m_7, m_8, m_9), m_{10}]$ **	X	X	X	X
H50: $f[f_1(m_1, m_2, m_4, m_6, m_8), f_2(m_3, m_5, m_7), m_9, m_{10}]$	X	N	X	X
H51: $f[f_1(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_{10}), m_9]$	N	N&S	X	X

See notes in Table 3.5.

Chapter 4

THE PARAMETRIC APPROACH TO THE DEMAND FOR MONEY

4.1 Introduction

The parametric and nonparametric approaches to demand analysis are not mutually exclusive. We can use the results obtained in the nonparametric framework in the estimation of parametric demand functions. Since the parametric approach is parameter intensive we will be forced to aggregate monetary assets in a few weakly separable groups.

The parametric demand analysis involves postulating a specific functional form for the utility function and then fitting a finite amount of data to a demand system derived from maximizing the aggregator function subject to a budget constraint. However, the approach risks not being satisfactory when the postulated parametric form is not a good approximation to the true utility function. Nevertheless, as it will be argued in sections 4.3 and 4.4 there is a class of functions that provide semi-nonparametric properties that are desirable for statistical inference.

In this chapter we will learn how to derive a set of demand functions. From consumer theory we will postulate utility maximizing behavior subject to a budget constraint. The solution to that problem will provide us with the demand system. On the other hand, we will discuss the convenience of utilizing duality theory to derive the same set of demand functions from an indirect utility function.

We will also discuss, in brief, the different flexible functional forms that have been proposed in the monetary literature and the reasons we prefer to use the Fourier

model to approximate the consumer's indirect utility function. The Fourier flexible functional form is a global approximation to an unknown aggregator function that confers non-parametric properties. In addition it provides accurate approximations to partial derivatives which is a useful property when estimating elasticities. Next, we will analyze in detail the Fourier model and the implied share (demand) equations which will be used (in the next chapter) to obtain the parameter estimates of the indirect utility function. The parameter estimates themselves have no information value but once parameter estimates are in hand it is a straightforward exercise to compute price and income elasticities and elasticities of substitution. In particular we are interested on the stability and values of substitution elasticities which can provide us with insight regarding the definition of money. Specifically, if there is evidence of volatility or low substitutability the simple sum aggregates will be invalid and methods of aggregation that allow for changing elasticities of substitution will be preferred. Secondly, according to the substitutability/complementary relationship broad monetary aggregates will or will not be appropriate definitions.

The parametric analysis allows us to impose and test neoclassical restrictions on individual behavior (e.g. monotonicity and curvature restrictions). In contrast to the nonparametric framework where tests where non-stochastic in nature, the parametric analysis permits us to determine the validity of the results using probability theory.

4.2 Direct and Indirect Utility Approach

In the previous chapter we saw that the agent's problem is to choose a mix of monetary services given preferences, rental prices and income. Mathematically the problem is,

$$\max \ f(\boldsymbol{m})$$

subject to (4.1)

$$\pi' \boldsymbol{m} = y_m ,$$

where m_i (i=1,...,N) is the ith monetary asset, and π is a vector of user costs. Here we are focusing on the demand for monetary services, ignoring other goods like consumption, services and leisure.

The Lagrangean for the consumer's problem is,

$$L = f(\boldsymbol{m}) - \lambda(y_m - \pi' \boldsymbol{m})$$

and the first order conditions are,

$$f_i(\boldsymbol{m}) = \lambda \pi_i$$
$$y_m = \pi' \boldsymbol{m}$$

where $f_i(\boldsymbol{m})$ is the partial derivative with respect to m_i.

The first order conditions give us a system of simultaneous equations which when solved yield the demand system for monetary assets,

$$m_i^* = (\pi, y_m) ,$$

which, in fact, is the system we are interested in estimating to study its properties using monetary data. The expression says that asset demands depend on prices and income.

However, the form of the system depends on the specified utility functional form; this means that we will obtain different demand systems from different utility functions.

Even if the direct utility approach just described above has a lot of intuitive appeal there is an easier way to obtain the demand system. Substitution of m_i * into the direct utility function gives us the indirect utility function,

$$V(\pi, y_m) = V[m_1(\pi, y_m), m_2(\pi, y_m), ..., m_N(\pi, y_m)]$$

$$= \max \ f(\boldsymbol{m}) \ \text{ subject to } \ \pi'\boldsymbol{m} = y_m$$

The indirect utility function, $V(\pi, y_m)$, is an equivalent representation of the structure of preferences represented by the direct utility function, $f(\boldsymbol{m})$, and states that utility depends indirectly on prices and income. It turns out that $V(\pi, y_m)$ is the dual representation of $f(\boldsymbol{m})$ because maximization of $f(\boldsymbol{m})$ with respect to $\boldsymbol{m}$ given π is equivalent to minimization of $V(\pi, y_m)$ with respect to π given $\boldsymbol{m}$.[1]

The indirect utility function turns out to be more satisfactory in econometric work. The reason is that the price variables enter into the function as exogenous variables. Since individual consumers cannot affect market prices, they are supposed to take rental prices as given and choose monetary asset quantities. In this way the rental prices may not be correlated with error terms in the statistical preference relationship. This property is desirable from a statistical point of view. Another advantage is that we

---

[1] The following are properties of the indirect utility function:

(1) $V(\pi, y_m)$ is nonincreasing in π and nondecreasing in y_m.

(2) $V(\pi, y_m)$ is homogeneous of degree zero in (π, y_m).

(3) $V(\pi, y_m)$ is quasiconvex in π.

(4) $V(\pi, y_m)$ is continuous at all $\pi_i > 0$, $y_m > 0$.

can obtain the demand system from the indirect utility function by applying Roy's identity, without having to solve a system of simultaneous equations,

$$m_i(\pi, y_m) = -\frac{\partial V(\pi, y_m)}{\partial \pi_i} \Big/ \frac{\partial V(\pi, y_m)}{\partial y_m}$$

Let $v_i = \pi_i / y_m$ be the expenditure normalized user cost so that $V = V(v)$, then the budget share equations will be given by

$$s_i = \frac{\pi_i m_i}{y_m} = v_i \frac{\partial V(v)}{\partial v_i} \Big/ \sum_{i=1}^{N} v_i \frac{\partial V(v)}{\partial v_i} \quad i=1,2,...,N$$

4.3 Functional Forms

The final objective of the parametric approach is to accurately estimate income, price and substitution elasticities. This is done by means of estimating a monetary demand system derived from an indirect utility function. The first step is to choose a functional form capable of approximating the true indirect utility function and its partial derivatives. Precise estimates of partial derivatives are important because elasticities of substitution are functions of the partial derivatives of the utility function.

Unfortunately, parametric functions like the Cobb-Douglas do not approximate well the true data generating function and often restrict the substitutability/ complementary relationship between assets. However, to overcome these limitations we can use a flexible functional form that can provide an order approximation to an arbitrary function and give unrestricted estimates of elasticities of substitution. In the macroeconomics literature there are many flexible functional forms in use to approximate

an unknown indirect utility function. The purpose of this section is to survey among the alternatives and choose the most appropriate for our monetary demand system estimation.

Flexible functional forms differ in their specific parameterization and approximation properties. In particular we will analyze three general types of flexible functional forms that are frequently used in the literature. These are the locally-flexible functional forms, the effectively global regular forms and the globally flexible functional forms.

The locally-flexible forms only provide a local approximation to the true data generating function at a single point in a delta neighborhood. The effectively global regular forms are locally-flexible functional forms that have larger regularity regions or are higher rank models that can better approximate non-linear Engel curves. Finally, the globally flexible forms or semi-nonparametric (SNP) functions provide an asymptotically global approximation to the true data generating process and their partial derivatives.

Fisher, Fleissig and Serletis (2001), argue that the locally-flexible functional forms have shown several shortcomings in previous studies. Specifically, these forms can fail to provide satisfactory approximations to the true data generating process for the elasticities of substitution. More importantly, the local approximation to the true data generating function is at a single point in a delta neighborhood of an unknown and often small size. Furthermore, the regularity region can be relatively small.[2] The effectively global regular functional forms give a partial solution to the latter problem by providing larger theoretical regularity regions that are capable of approximating more general Engel

[2] Another limitation of the locally-flexible forms pointed out by Fleissig and Serletis (2001) is that they fail to measure how accurately partial derivatives of the true utility function are approximated.

curves. However, even if they are better approximations than the initial flexible forms, in the limit they may not be regular and therefore not provide an effective approximation to the derivatives –and hence the curvature of the true data generating function. The semi-nonparametric functions provide a more general approach to approximate the true data generating function to tackle the problems just discussed. The SNP functions are referred to as global approximations because asymptotically they are capable of approximating the unknown underlying general function at all points and hence of generating arbitrarily accurate elasticities at all points. The SNP functions converge asymptotically to the true data generating process (and therefore the true elasticities of substitution) by expanding the order of the expansion series as sample size increases.

Fisher, Fleissig and Serletis (2001) analyzed three locally-flexible functional forms (the Generalized Leontief (GL) model, the Basic Translog (BTL) model, and the Almost Ideal Demand System (AIDS)); three effectively global regular functional forms (the Full Laurent Model, the Quadratic AIDS (QUAIDS), and the General Exponential Form (GEF)); and two globally flexible functional forms (the Fourier Model and the Asymptotically Ideal Model (AIM)). The authors empirically assessed the performance of these flexible forms according to the relative satisfaction of the regularity conditions for the aggregate consumer, expenditure responses, forecasting exercises, estimates of elasticities of substitution and information criteria.

Using U.S. consumption data Fisher, Fleissig and Serletis (2001) found that the QUAIDS, GEF, AIM, and Fourier were the best models in terms of conforming to regularity conditions. In terms of consistency of the elasticities of substitution to economic expectations, the results from the Fourier and AIM (the SNP functions) were

the most plausible. Overall, the globally flexible functional forms or SNP were more adept in picking up the interactions between business cycle turning points (characterized with large changes in user costs) and the highly nonlinear utility function of the aggregate consumer. In terms of forecasting results, the AIM and Fourier models dominated in comparison to the others. Finally, in terms of information criteria (the Akaike Information Criterion and the Bayesian Schwarz information Criterion) there were no large differences found among all the models tested. In the end of their analysis Fisher, Fleissig and Serletis (2001), conclude that the QUAIDS, the AIM, and the Fourier models stand out across all tests. But the global models (AIM and Fourier) possess greater advantages when utility surfaces are highly nonlinear, when user costs fluctuate considerably over time, and consequently when estimated elasticities of substitution are quite volatile over time.

According to the approximation theoretical properties and empirical results just discussed, it is apparent that the semi-nonparametric functions will perform better in the estimation of the monetary demand system and the corresponding elasticities of substitution. Here, we will use the Fourier flexible functional form from the two globally flexible forms discussed by Fisher, Fleissig and Serletis (2001) because the Fourier model has been widely used in the monetary economics literature. Applications of the model to monetary data can be found in Ewis and Fisher (1985), Fisher (1992), Fisher and Fleissig (1994), Fisher and Fleissig (1997), and Fleissig and Serletis (2001).[3]

[3] Other globally flexible functional forms used with monetary data are the Minflex Laurent Generalized Leontief and the Minflex Laurent Translog. See Barnett (1985); Barnett and Lee (1985); Barnett, Lee and Wolfe (1985); Barnett, Lee and Wolfe (1987).

4.4 The Fourier Flexible Form

According to Gallant (1981) a multivariate Fourier series expansion of order K is denoted as

$$\sum_{|k|^* \le K} a_k e^{ik'v} \qquad (4.2)$$

where k denotes a multi-index,[4]

$v \in V, \quad a_k = u_k + iw_k,$

where i is the imaginary unit, $u_k, w_k \in \Re$, and $|k|^* = \sum_{l=1}^{N} k_l$ is the length of the multi-index.

We can re-express (4.2) as,

$$\sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha} e^{ijk_\alpha' v} \qquad (4.3)$$

where the sequence of multi-indices $\{k_\alpha$ are constructed from the set

$$\{k : |k|^* \le K\} \subset \{jk_\alpha : \alpha = 1,..., A; j = 0, \pm 1, ..., \pm J$$

The sum in (4.3) is real valued if $a_{0\alpha}$ is real valued and $a_{j\alpha} = \bar{a}_{-j\alpha}$ (where the overbar denotes complex conjugation). Equivalently if,

$$w_{0\alpha} = 0, \quad u_{j\alpha} = u_{-j\alpha}, \quad w_{j\alpha} = -w_{-j\alpha}$$

are imposed, where $a_{j\alpha} = u_{j\alpha} + iw_{j\alpha}$, $\alpha = 1,2,..., A$ and $j = 0, \pm 1, 2, ..., \pm J$, expression (4.3) will be real valued.

[4] A multi-index is an N vector of integers denoting partial differentiation of the utility function.

Because the multi-indices k with $|k|^* \leq K$ will occur in pairs of opposite sign except for the multi-index 0=(0,0,...,0) then, $a_{0\alpha} e^{i0'v} = u_{0\alpha}$ real valued, and

$$a_{j\alpha} e^{ijk_\alpha' v} = 2u_{j\alpha} \cos(jk_\alpha' v) - 2w_{j\alpha} \sin(jk_\alpha' v)$$

which is also real valued.[5] Therefore, we can express the following equivalence,

$$\sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha} e^{ijk_\alpha' v} = \sum_{\alpha=1}^{A} \left\{ u_{0\alpha} + 2\sum_{j=1}^{J} \left[u_{j\alpha} \cos(jk_\alpha' v) - w_{j\alpha} \sin(jk_\alpha' v) \right] \right\}$$

In practice it is more convenient to work with the sine/cosine representation.

A Fourier series is a periodic function in each of its arguments but an indirect utility function is not. According to Gallant (1981), experience acquired in other contexts suggests that the number on sine/cosine terms in a Fourier approximation of a non-periodic function can be considerably reduced if a linear term $b'v$ is included. In addition, if a quadratic term $v'Cv$ is included curvature restrictions may be imposed.

If x denotes the N-vector of commodities, p is the vector of prices, and y_m is income, let $v = p/y_m$ and s is an N-1 vector of budget shares $s_i = p_i x_i / y_m, i = 1,..., N-1$, then the Fourier flexible form approximation to the indirect utility function is,

$$V(v) = u_0 + b'v + \tfrac{1}{2} v'Cv + \sum_{\alpha=1}^{A} \left\{ u_{0\alpha} + 2\sum_{j=1}^{J} u_{j\alpha} \cos(jk_\alpha' v) - w_{j\alpha} \sin(jk_\alpha' v) \right\} \qquad (4.4)$$

where $C = -\sum_{\alpha=1}^{A} u_{0\alpha} k_\alpha k_\alpha'$

[5] See Gallant (1981 p.214).

According to this equation, the elements of the multi-index $\{k_\alpha\}$ can be considered like weights (when multiplied by v) of the normalized price index.

Following the economic theory described in Section 4.2, the consumer's utility is maximized when expenditure is allocated according to the expenditure system obtained from applying Roy's identity,

$$s_i = \frac{p_i x_i}{y_m} = v_i \frac{\partial V^*(v)}{\partial v_i} \Big/ \sum_{i=1}^{N} v_i \frac{\partial V^*(v)}{\partial v_i}, \qquad i=1,\ldots,N$$

where $V^*(v)$ is the true indirect utility function and it is assumed that $V^*(v)$ has continuous partial derivatives and that $\frac{\partial V^*(v)}{\partial v_i} \leq 0$ for all v in the region of approximation. In this way expenditure shares will always be between zero and one.

A careful inspection of Roy's identity reveals that it is actually the partial derivatives of the indirect utility function which need to be accurately approximated by the global flexible form and not just the indirect utility function. The advantage of using the Fourier series is that it can approximate the true indirect utility function as closely as desired in Sobolev norm (in other words it is dense in a Sobolev norm). The Sobolev norm is a measure of the distance between $V^*(v)$ and $V(v)$ which takes derivatives into account and at the same time it is a global measure over the region of approximation, (V). The purpose of the approximation is to make this distance arbitrarily small.[6]

Gallant (1982) uses the term Diewert-flexible when referring to flexible forms that can provide a second order approximation to an arbitrary twice differentiable function at a given point but uses the term Sobolev-flexible to denote the ability to

achieve close approximation in Sobolev norm. This ability is what sets the Fourier flexible form apart from the Translog, generalized Leontief and other Diewert-flexible forms. The important distinction is that Sobolev flexibility confers non-parametric properties on a functional form and Diewert-flexibility does not. The non-parametric properties imply that, even though the flexible form is treated as if it were the true model in an econometric study, in fact the inferences are valid against all reasonable true states of nature at least in large samples.

Now we will apply Roy's identity on (4.4) to obtain the share equations. First we will obtain $\frac{\partial V(v)}{\partial v_i}$ by breaking $V(v)$ in its additive elements.

(1) Linear term,

$$\frac{\partial b'v}{\partial v_i} = b_i \qquad (4.5)$$

(2) Quadratic term,

$$\frac{\partial \frac{1}{2}v'Cv}{\partial v_i} = -\sum_{\alpha=1}^{A}(u_{0\alpha}v'k_\alpha)k_{i\alpha} \qquad (4.6)$$

Proof:

let A=3 and N=2 then, $k_\alpha' = [k_{1\alpha}, k_{2\alpha}]$ and

$$C = -u_{01}\begin{bmatrix} k_{11}^2 & k_{11}k_{21} \\ k_{21}k_{11} & k_{21}^2 \end{bmatrix} - u_{02}\begin{bmatrix} k_{12}^2 & k_{12}k_{22} \\ k_{22}k_{12} & k_{22}^2 \end{bmatrix} - u_{03}\begin{bmatrix} k_{13}^2 & k_{13}k_{23} \\ k_{23}k_{13} & k_{23}^2 \end{bmatrix}, \text{ so,}$$

[6] See Gallant (1981, p.218) for more details.

$$
\begin{aligned}
v'Cv = {} & v_1^2(-u_{01}k_{11}^2 - u_{02}k_{12}^2 - u_{03}k_{13}^2) \\
& + v_1v_2(-u_{01}k_{21}k_{11} - u_{02}k_{22}k_{12} - u_{03}k_{23}k_{13}) \\
& + v_2v_1(-u_{01}k_{11}k_{21} - u_{02}k_{12}k_{22} - u_{03}k_{13}k_{23}) \\
& + v_2^2(-u_{01}k_{21}^2 - u_{02}k_{22}^2 - u_{03}k_{23}^2)
\end{aligned}
$$

taking the partial derivative with respect to v_1, we have

$$
\frac{\partial v'Cv}{\partial v_1} = 2v_1(-u_{01}k_{11}^2 - u_{02}k_{12}^2 - u_{03}k_{13}^2) + 2v_2(-u_{01}k_{11}k_{21} - u_{02}k_{12}k_{22} - u_{03}k_{13}k_{23})
$$

$$
= -2\left[u_{01}(v_1k_{11} + v_2k_{21})k_{11} + u_{02}(v_1k_{12} + v_2k_{22})k_{12} + u_{03}(v_1k_{13} + v_2k_{23})k_{13}\right]
$$

From the last expression it is easy to see that the terms $u_{0\alpha}(v_1k_{1\alpha} + v_2k_{2\alpha})$ are equal to $u_{0\alpha}v'k_\alpha$, therefore,

$$
\frac{\partial \frac{1}{2}v'Cv}{\partial v_1} = -\sum_{\alpha=1}^{3}(u_{0\alpha}v'k_\alpha)k_{1\alpha}
$$

from here we can generalize for any v_i and any A,

$$
\frac{\partial \frac{1}{2}v'Cv}{\partial v_i} = -\sum_{\alpha=1}^{A}(u_{0\alpha}v'k_\alpha)k_{i\alpha}
$$

(3) Sine-cosine term

$$
\frac{\partial}{\partial v_i}\left(\sum_{\alpha=1}^{A}u_{0\alpha} + 2\sum_{j=1}^{J}[u_{j\alpha}\cos(jk_\alpha'v) - w_{j\alpha}\sin(jk_\alpha'v)]\right)
$$

$$
= \sum_{\alpha=1}^{A}\sum_{j=1}^{J}2[-u_{j\alpha}jk_{i\alpha}\sin(jk_\alpha'v) - w_{j\alpha}jk_{i\alpha}\cos(jk_\alpha'v)]
$$

$$
= \sum_{\alpha=1}^{A}\sum_{j=1}^{J}2j[-u_{j\alpha}\sin(jk_\alpha'v) - w_{j\alpha}\cos(jk_\alpha'v)]k_{i\alpha} \qquad (4.7)
$$

Putting the terms (4.5), (4.6) and (4.7) together and multiplying by v_i yields,

$$v_i \frac{\partial V(v)}{\partial v_i} = v_i b_i - \sum_{\alpha=1}^{A}\left[u_{0\alpha} v' k_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha' v) + w_{j\alpha}\cos(jk_\alpha' v)]\right] k_{i\alpha} v_i \qquad (4.8)$$

The denominator in Roy's identity is the summation of (4.8) over the number of assets *i*, hence the expenditure system is,

$$y_i(v,\theta) = \frac{v_i b_i - \sum_{\alpha=1}^{A}\left[u_{0\alpha} v' k_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha' v) + w_{j\alpha}\cos(jk_\alpha' v)]\right] k_{i\alpha} v_i}{b' v - \sum_{\alpha=1}^{A}\left[u_{0\alpha} v' k_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha' v) + w_{j\alpha}\cos(jk_\alpha' v)]\right] k_\alpha' v} \qquad (4.9)$$

for i=1,...,N, $\{b\}$, $\{u\}$ and $\{w\}$ are the parameters to be estimated and $b_N = -1$. System (13) can be written compactly as,

$$y = y(v,\theta)\ , \qquad (4.10)$$

The parameters A and J are the number of terms and the degree of approximation respectively. The selection of these is an empirical problem that can be solved (according to Fleissig and Serletis (2001)) by empirical testing.[7] $b_N = -1$ is a convenient normalization rule given that the system is homogeneous of degree zero in its parameters and not identified if not normalized.[8] $y(v)$ and s are N-1 dimensional hence the *Nth* expenditure and predicted expenditure are obtained by subtraction,

$$y_N(v) = 1 - \sum_{i=1}^{N-1} y_i(v), \qquad s_N = 1 - \sum_{i=1}^{N-1} s_i$$

As a final remark we should mention that the scaling of the data is important since the Fourier approximation is a periodic function in its arguments but the indirect utility function is not. To avoid the approximation from diverging from the true indirect utility

[7] Fleissig and Serletis (2001) used a standard F-test.
[8] See Gallant (1981, p.224).

function the data should be re-scaled so that the income normalized prices are between 0 and 2Π.[9] Let (Y_L, Y_U) with $Y_L > 0$ be the interval if incomes over the desired approximation and (p_{Li}, p_{Ui}) with $p_{Li} > 0$ be the price intervals. Units and prices should be re-scaled so that,

$$0 < v_{Li} \leq v_i \leq v_{Ui} < 2\Pi, \qquad \text{for } i = 1,...,N.$$

4.5 Conclusions

In this chapter we have introduced the parametric framework by stating first the representative agent's problem. According to Serletis (2001), the use in recent years of the simple representative consumer paradigm in monetary economics is opening the path to the progressive introduction of the entire microfundations, aggregation theory, and micro-econometrics literature into the field of monetary economics.

It was argued that preferences could be represented by either the direct or indirect utility function. The latter representation is preferred because of the exogenous price variables. Nevertheless, as discussed in Chapter 2, the structural properties of the direct utility function do not imply the same structure of preferences in the indirect utility function. Serletis (2001) argues that in order to implement a demand system based on the indirect utility function which satisfies properties of the direct utility function, a correspondence has to exist between the direct and indirect properties. In this regard, if the direct utility function is weakly separable with homothetic aggregator functions, the indirect utility function will be separable with the same structure across expenditure

[9] See Gallant (1981, p.217).

normalized prices. Unfortunately and despite all the advantages of the Fourier model we can loose the correspondence between structures of preferences because the Fourier form is not homothetic.

In order to approximate the unknown indirect utility function, several flexible functional forms were discussed. We chose the Fourier flexible form because as Fisher (1992, p.145) puts it "this method produces an expenditure system (and estimates of the elasticities of substitution) with the asymptotic property that the average prediction bias may be made arbitrarily small by increasing the number of terms in the expansion". The model produces unrestricted estimates of elasticities of substitution that can come arbitrarily close to the elasticities implied by the true (but unknown) aggregate indirect utility function. In the next chapter the Fourier model will be evaluated in terms of satisfying the regularity conditions (consumer maximizing behavior) and the economic content and validity of the parameter estimates will be studied through the estimated price elasticities, income elasticities and elasticities of substitution.

Chapter 5

ESTIMATION AND EMPIRICAL RESULTS

5.1 Introduction

In this Chapter we derive a stochastic specification for our Fourier monetary demand system for the purpose of estimation. As discussed in the previous chapter, the Fourier model is parameter intensive and a large system cannot be estimated, therefore we will discuss the reduction of its dimension to only three monetary assets. In doing so we want to use a method of aggregation that preserves the economic characteristics. The Divisia index is a method consistent with the utility maximization hypothesis that has been widely employed by monetary economists. We will explain in detail the Divisia index.

Before estimation, many specification details have to be taken care of. We will carefully explain the assumptions regarding our stochastic model. When working with monetary time series it is common to find evidence of autocorrelation of the residual terms. Like in the past, we solve this problem by re-specifying the stochastic system assuming that the error terms follow a specific kind of process. The results of the parameter estimates are reported with statistics to judge their validity along with measures of goodness of fit. We will see that the Fourier model fits well the data even when the utility surfaces seem to be highly nonlinear and when there are economic turning points characterized with volatility in user costs, this is more evident in the decade of the eighties than the nineties.

After we test if the parameter estimates are consistent with the utility maximizing hypothesis underlying the Fourier model, we explain how to derive price and income elasticities and elasticities of substitution (and their standard errors). The analysis of these has bearing on the definition of money itself and on why traditional log-linear money-demand functions have performed particularly bad in the past. As we have already mentioned in previous chapters, the official monetary aggregates are calculated as simple unweighted sums of their components, this procedure effectively implies that the elasticities of substitution between monetary assets cannot be less than infinite. A more reasonable approach to the definition of money, which has already been proposed in the literature, is to regard monetary assets as joint products with different degrees of moneyness. According to our results the variations of the elasticities of substitution over time, but more importantly the apparent low substitution/complementarity, suggest that simple sum monetary aggregation of household assets with business assets is part of the problem of poor fits of the monetary aggregates, in various applications.

The results also show low substitutability between Canadian monetary assets and foreign currency deposits (most of which are composed of American dollars). This may say something about the way the Canadian consumer regards American monetary holdings compared to Canadian holdings, and although still very premature, ultimately this can have a bearing on the discussion of a monetary union.

5.2 Stochastic Specification

In this section we will specify the expenditure system to be estimated. From the previous chapter we saw that the share equations were derived from applying Roy's

identity to the Fourier flexible functional form. Basically we are interested in estimating expression (4.9) with a vector of error terms appended,

$$s_{it}(v,\theta) = \frac{v_{it}b_i - \sum_{\alpha=1}^{A}\left[u_{0\alpha}v'k_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha'v) + w_{j\alpha}\cos(jk_\alpha'v)]\right]k_{i\alpha}v_{it}}{b'v - \sum_{\alpha=1}^{A}\left[u_{0\alpha}v'k_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha'v) + w_{j\alpha}\cos(jk_\alpha'v)]\right]k_\alpha'v} + u_{it}$$

which can be compactly written as

$$s_t = y(v_t,\theta) + u_t \qquad (5.1).$$

Observe that we have attached a time subscript since we are dealing with time series data. Remember that i=1,...,N and $\{b\}$, $\{u\}$ and $\{w\}$ are the parameters to be estimated. A is the number of terms and J the degree of approximation to be determined *a priori* or by empirical testing. In our study we will set, as most authors do, J=1 to preserve a significant number of degrees of freedom. In addition, J=1 has the easy intuition that we can capture the influence of changes in relative prices on the budgetary shares. A in turn, according to Fisher (1992), has to be increased by the number of assets each time, which is expensive in terms of degrees of freedom. In our study we choose A=6 in order to get a sufficient number of terms. Ewis and Fisher (1985) determined that the next available A would be 9 which is expensive in degrees of freedom.[1] According to Gallant (1982) the number of terms when fitting a Fourier demand system is no different than the problem of determining the correct order of a distributed lag, the correct order of (p,q) when fitting an ARMA process to time series data. The conventional approach is to

[1] Ewis and Fisher (1985) and Fisher (1992) were the only authors to mention that the number of terms A has to be increased by the number of assets each time. To not contradict the literature we chose A equal to a multiple of our number of assets but in practice we did not find any reason why A could not be increased by one each time.

use upward or downward selection by means of t-tests or F-tests that one or more terms can be deleted from the model. Gallant (1982) says that the downward selection involves deleting first the terms that correspond to the largest multi-index length $|jk_\alpha|$. This is the reason why in our study we use the simplest multi-indices with the smallest length which were also used by Fisher (1989) when estimating the Fourier model with American monetary data. The multi-indices employed are:

$$k_\alpha = \begin{pmatrix}1\\0\\0\end{pmatrix} \bullet \begin{pmatrix}0\\1\\0\end{pmatrix} \bullet \begin{pmatrix}0\\0\\1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} \bullet \begin{pmatrix}0\\1\\1\end{pmatrix} \bullet \begin{pmatrix}1\\0\\1\end{pmatrix}$$

A demand system like $f(\boldsymbol{m})$ (described in Chapter 3 with 10 monetary assets) has too many assets and is almost impossible to estimate because the Fourier model is parameter intensive. $f(\boldsymbol{m})$ is too large to provide precise parameter estimates and reliable results. To reduce the dimension of our monetary demand system we impose the following restrictions:

(1) x_1 = monetary group composed of $m_3, m_5,$ and m_7

(2) x_2 = monetary group composed of $m_1, m_2, m_4, m_6,$ and m_8

(3) x_3 = monetary group composed of m_{10}

These subsets of assets can legitimately be combined to form meaningful monetary groupings when the elements of x_i are weakly separable in the monetary utility function $f(\boldsymbol{m})$. In Chapter 3 the tests for weak separability revealed that these groupings satisfy the necessary condition for weak separability in the quarterly data sample 1992:2-1996:3. Ideally the weak separability tests would satisfy the necessary and

sufficient conditions across the entire data sample. However, since the entire quarterly sample does not pass the GARP test and because we were looking for groupings that made economic sense we chose the above groups. The monetary utility function for our parametric study is then defined as,

$$f(x_1, x_2, x_3)$$

where the x_i are subaggregates over elements of $\boldsymbol{m}$. Observe that x_1 corresponds to most non-personal assets, x_2 corresponds mostly to personal assets and x_3 corresponds to foreign currency deposits held by both sectors.

We now know that N=3, J=1, and A=6. Given that the system is homogeneous of degree zero in its parameters we set $b_3 = -1$, and using the multi-indices described above, let:

$$\begin{aligned} ss_1 = \{&b_1 - u_{01}v_1 - 2[u_{11}\sin(v_1) + w_{11}\cos(v_1)] \\ &- u_{04}(v_1 + v_2) - 2[u_{14}\sin(v_1 + v_2) + w_{14}\cos(v_1 + v_2)] \\ &- u_{06}(v_1 + v_3) - 2[u_{16}\sin(v_1 + v_3) + w_{16}\cos(v_1 + v_3)]\}v_1; \end{aligned}$$

$$\begin{aligned} ss_2 = \{&b_2 - u_{02}v_2 - 2[u_{12}\sin(v_2) + w_{12}\cos(v_2)] \\ &- u_{04}(v_1 + v_2) - 2[u_{14}\sin(v_1 + v_2) + w_{14}\cos(v_1 + v_2)] \\ &- u_{05}(v_2 + v_3) - 2[u_{15}\sin(v_2 + v_3) + w_{15}\cos(v_2 + v_3)]\}v_2; \end{aligned}$$

$$\begin{aligned} ss_3 = \{&-1 - u_{03}v_3 - 2[u_{13}\sin(v_3) + w_{13}\cos(v_3)] \\ &- u_{05}(v_2 + v_3) - 2[u_{15}\sin(v_2 + v_3) + w_{15}\cos(v_2 + v_3)] \\ &- u_{06}(v_1 + v_3) - 2[u_{16}\sin(v_1 + v_3) + w_{16}\cos(v_1 + v_3)]\}v_3 \end{aligned}$$

and let $ss = ss_1 + ss_2 + ss_3$, then our specific stochastic share equations of the Fourier model look like:

$$s_1 = ss_1 / ss + u_1$$
$$s_2 = ss_2 / ss + u_2$$
$$s_3 = ss_3 / ss + u_3$$

where the s_i are the positive expenditure shares and the u_i are the error terms. This is what we estimate using the seemingly unrelated regression procedure. The errors are assumed to have the "classical" properties,

$$E(u_t) = 0, \qquad E(u_t, u_s') = \begin{matrix} \Sigma & \text{for } s = t \\ 0 & \text{for } s \neq t \end{matrix} \qquad \text{for all } s,t$$

where Σ is a symmetric and positive semidefinite covariance matrix, and 0 is a matrix of zeros. These assumptions permit correlation among the disturbances at time t but forbids the possibility of autocorrelated disturbances. Because of these assumptions and because the expenditure shares s_i sum to unity at each observation, it follows that $\sum_i u_{it} = 0$ at each observation so the system is singular and an equation must be deleted if it is to be estimated. Our method of estimation guarantees parameter estimates invariant to the deleted equation and to check invariance we estimate the models more than once, deleting a different equation each time.

5.3 The Divisia Aggregator

Now that we have a stochastic specification it seem that we can go straight on to the estimation stage. However, as previously discussed and before any further step we have to reduce the dimension of the system to obtain reliable results. We already discussed in depth the weak separability issue to form valid monetary subaggregates and arrived to the following monetary utility function specification

$$f(x_1, x_2, x_3).$$

Here x_i has two mutually consistent interpretations. It can be thought as a specific category utility function or as a subaggregate measure of monetary services. Now, the question of how we should aggregate or group the elements in each monetary asset x_i remains. In the construction of these subcategories we want to preserve the economic characteristics and constructing Divisia index numbers from individual quantities and user costs can do this. The supporting theory behind Divisia index numbers is also that of the constrained maximization of the aggregate consumer's intertemporal utility function. It turns out that the Divisia index is the transformed first-order conditions for constrained optimization.

We could use simple-sum aggregation in the construction of the monetary assets x_i in fashion to the Central Bank's methodology but simple-sum aggregation methods imply dollar-for-dollar perfect substitutability. This would imply infinite elasticities of substitution between each pair of component assets and empirical evidence suggests that the observed elasticities of substitution between pairs of assets are actually quite low.[2] Because we do not expect perfect substitutability and because user costs fluctuate significantly over time, the simple-sum method will not account well for substitution effects. On the contrary, the Divisia quantity index internalizes the substitution effects and measures only the income effect of a relative price change. Moreover, the Divisia index has been widely applied to monetary aggregation.

[2] See for example Offenbacher (1979).

In the past, statistical index number theory and aggregation theory developed independently. It was Erwin Diwert (1976, 1978) who provided the link between the two. Diwert attached economic properties to statistical indices. These properties depend upon how well the indices track a particular functional form of the unknown aggregator function. And the reader should remember that the subutility function itself is the appropriate aggregator function.

Using a particular index is equivalent to using a particular functional form that describes the agent's aggregator function. In the literature these indices are called exact. However, we now encounter the problem of not knowing the particular form of the aggregator function. This conflict is solved by choosing an index linked to a functional form which provides a second order approximation to an arbitrary and unknown aggregator function. Such statistical index numbers are called superlative. It turns out that the Divisia index is exact for the linearly homogeneous and flexible translog thereby approximating an arbitrary and unknown exact aggregator function up to a third-order remainder term.[3] What we gain using this methodology is the ability to do precise work even without knowing the exact form of the aggregator function.

In addition to tracking the unknown utility function the Divisia index satisfies a large number of statistical properties. Irving Fisher (1922) who assessed the quality of particular statistical indices by means of tests described these properties.[4] According to Fisher (1922) an important property of an index is self-duality assessed by the factor reversal test. An index will be self-dual if the price and quantity indices comprise a dual

[3] According to Barnett, Fisher and Serletis (1992) even if the aggregator function is not homogeneous the Divisia index remains exact for the distance function, which in that case, is the true aggregator function.

pair in the sense that the product of the price and quantity indices equals total expenditure. In the case of the Divisia index the price and quantity indices are not dual. This is a common failure among the most popular indices. However, even if the Divisia price index is used to measure the price of the Divisia quantity index, the size of the error due to failure on the factor reversal test would be very small (third order in changes).

The discrete time (Törnqvist) Divisia index for period t is,

$$M_t = M_{t-1}\Pi\left(\frac{m_{j,t}}{m_{j,t-1}}\right)^{\frac{1}{2}(w_{j,t}+w_{j,t-1})} \qquad (5.2)$$

where

$$w_{j,t} = \frac{\pi_{j,t}m_{j,t}}{\sum_{j=1}^{n}\pi_{j,t}m_{j,t}}$$

Taking the term M_{t-1} to the left-hand side of (5.2) and applying logs yields,

$$\log M_t - \log M_{t-1} = \sum_{j=1}^{n} w_{j,t}^{*}(\log m_{j,t} - \log m_{j,t-1}) \qquad (5.3)$$

where $w_{j,t}^{*} = (\frac{1}{2})(w_{j,t} + w_{j,t-1})$.

Expression (5.3) says that the growth rate of the Divisia aggregate is the share-weighted average of the growth rates of the component quantities.[5]

Following Serletis (1987), Divisia user cost indices for the monetary subaggregates can be constructed using Fisher's weak factor reversal test.

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[4] Fisher (1922) tested the indices for proportionality, circularity, determinateness, commensurability and factor reversal.

[5] For a source on the construction of the Divisia Index see Molik (1999) or Anderson, Jones and Nesmith (1997).

$$p_t = p_{t-1}\left[\frac{\sum_{j=1}^{n}\pi_{j,t}m_{j,t}}{\sum_{j=1}^{n}\pi_{j,t-1}m_{j,t-1}}\right]\Bigg/\left[\frac{M_t}{M_{t-1}}\right]$$

A final comment, so far the Divisia index as used in the monetary literature is only capable of measuring the flow of monetary services provided by the individual monetary assets, thus it should not be seen as a measure of the consumer's stock of monetary wealth.

At this stage we should take a break and put together the pieces we have been analyzing from the very first chapters. We have monetary data and user costs that partly satisfy empirical revealed preference tests. We have an aggregation method that preserves the economic characteristics in the event of changes in relative prices. And finally, we propose to estimate a model and in the end elasticities with a global flexible form that can come arbitrarily close to the elasticities implied by the true (but unknown) aggregate indirect utility function.

5.4 Data and Econometric Considerations

We use quarterly data covering the period 1982Q1-1999Q4. The data is the same that was explained in Chapter 3, section 3.2. Remember that the per capita monetary data was obtained by dividing total quantities by population 15 years old and older and deflated with the CPI (all items). Using these monetary data and their respective user costs we constructed the Divisia monetary quantity indices according to the methodology described in the previous section and with the help of the DIVIND command in the TSP International program (Version 4.5). With the same command and program we

constructed the Divisia user cost indices according to Fisher's weak factor reversal tests. According to the restrictions we imposed in Section 5.2 we obtained the following:

$x_1 = \text{Divisia}(m_3, m_5, m_7)$,
$x_2 = \text{Divisia}(m_1, m_2, m_4, m_6, m_8)$,
$x_3 = \text{Divisia}(m_{10})$,

and

$p_1 = \text{Divisia price index of } x_1$,
$p_2 = \text{Divisia price index of } x_2$,
$p_3 = \text{Divisia price index of } x_3$.

Recall that $v_i = p_i / y_m$, then the Fourier demand system (5.1) is estimated with the across equation restrictions using the seemingly unrelated procedure in TSP International (Version 4.5) with convergence set at 0.00001. After many lesser approximations characterized by failure to obtain convergence or very few significant coefficients we obtained the results presented in Table 5.1. Each share equation provides an accurate approximation to the data in terms of the R-square. Of the 20 parameters estimated 18 turn out to be significant at a 95% significance level. The Durbin-Watson statistic, however, provides evidence of serial correlation in the error terms. This may be caused by institutional restrictions that prevent agents from adjusting monetary asset holdings to relative price changes within one period. In these cases the usual practice is to assume that the disturbance terms follow a first-order autoregressive process of the kind

$$u_t = \rho u_{t-1} + e_t$$

where $\rho = [\rho_{ij}]$ is a matrix of unknown parameters and e_t is a non-autocorrelated vector of disturbance terms with constant covariance matrix. Autocorrelation in demand systems

was first analyzed in detail by Berndt and Savin (1975) who showed that if one assumes no autocorrelation across equations (ρ is diagonal) then the autocorrelation coefficients for each equation will be the same ($\rho_{11} = \rho_{22} = \rho_{33}$).[6] By lagging equation (5.1) one period, multiplying it by ρ, and subtracting it from (5.1) yields the following stochastic budget share equations to be estimated,

$$s_t = y(v_t, \theta) + \rho s_{t-1} - \rho y(v_{t-1}, \theta) + e_t .$$

The estimation results of this system are displayed in Table 5.2. As expected the value of the Durbin-Watson statistics increased in both equations but not enough to completely disregard the autocorrelation problem. The R-squares indicate a better fit of this model compared to the previous and 15 out of twenty parameters turn out to be significant. In particular this is a very good ratio if we compare our model to many in the literature. In most cases only half or less parameters are statistically significant, however, this poses no problem because as with vector autoregressions and other time series methods, there can be many parameters to estimate and not all need to be significant. There is no drawback, as long as the fit is good and many of the estimated parameters are actually significant.

[6] If we were not to restrict the autocorrelation parameter to be the same for each of the equations in the system the results would differ depending on which equation is dropped during estimation. Verification of such an argument can be found in Tibibian (1980).

5.5 Results

5.5.1 Regularity Tests Results

We are now interested in checking whether the parameter estimates are consistent with the utility maximizing hypothesis underlying the model. By imposing the restriction $b_3 = -1$ we maintained the hypothesis of homogeneity of degree zero in the parameters. In this model adding up and symmetry are guaranteed and no additional restrictions are imposed.[7]

We do test, however, at each observation, the theoretical restrictions that are not part of the maintained hypothesis. The fact that the direct utility function has several properties implies that the indirect utility function possess properties of its own. We are interested in verifying the restrictions on the form of the indirect utility function from the Fourier parameter estimates. The regularity conditions are nonnegativity, monotonicity and curvature. The positivity restriction is checked by direct computation of the values of the fitted shares. The monotonicity restriction requires that the indirect utility function be monotonically decreasing, $\frac{\partial V(v)}{\partial v_i} < 0$ for all $v \in \overline{V}$ and can be checked by computing the gradient vector of the estimated indirect utility function. Finally, quasiconvexity (curvature condition) can be tested by examining the Allen elasticity matrix (σ_{ij}^a) provided the monotonicity conditions hold.[8] It is required this matrix to be negative semidefinite. The results of these tests are presented in Table 5.3. The numbers in the

[7] The general restrictions that a demand system must satisfy are: homogeneity of degree zero in prices and income, adding up (increases in the total expenditure are completely allocated to all monetary assets), symmetry (symmetric Allen elasticities of substitution), and negativity of the own substitution effect.
[8] Non-positive own-elasticities of substitution are a necessary but not sufficient requirement of the curvature condition.

table represent the number of times in which the regularity tests were satisfied. Nonnegativity is satisfied in 100% of the observations, monotonicity in 79% (which corresponds to all observations after 1985) but the curvature condition is only satisfied in 15.4% of the observations. Nevertheless, to determine whether model failure was a strong result, the own-elasticities of substitution were checked for cases in which the value of the parameter is significantly positive. There were only two such cases indicating model failure. In Table 5.8 the reader can verify that in most cases where the own-elasticities of substitution are positive they are however, insignificant.

5.5.2 Price and Income Elasticities and Elasticities of Substitution

Finally, to go into the economic questions we are interested on the information content of the estimated parameters. We will therefore, derive price and income elasticities and the elasticities of substitution. The stability of these will give us insight on the appropriate monetary aggregate definitions of money. Specifically, if there is evidence of volatility in the elasticities of substitution the simple sum aggregates will be invalid and methods of aggregation that allow for changing elasticities of substitution will be preferred. Secondly, according to the substitutability/complementary relationship broad monetary aggregates will or will not be appropriate definitions.

Following Gallant (1981) we can obtain the ordinary demands (Marshaling) from the parameter estimates and from the derivatives of the Fourier indirect utility function,

$$q(p, y_m) = [v'(\partial / \partial v)V(v)]^{-1}(\partial / \partial v)V(v),$$

where $(\partial/\partial v)V(v)$ is the first derivative with respect to income normalized prices of the Fourier indirect utility function defined by

$$(\partial/\partial v)V(v) = b + Cv - 2\sum_{\alpha=1}^{A}\sum_{j=1}^{J} j[u_{j\alpha}\sin(jk_\alpha' v) + w_{j\alpha}\cos(jk_\alpha' v)]k_\alpha,$$

where $C = -\sum_{\alpha=1}^{A} u_{0\alpha}k_\alpha k_\alpha'$.

Writing q for $q(p, y_m)$ and evaluating at the point $v = p/y_m$ we have

$$(\partial/\partial p')q(p, y_m) = [p'(\partial/\partial v)V(v)]^{-1}[(\partial^2/\partial v\partial v')V(v) - qv'(\partial^2/\partial v\partial v')V(v) - (v'(\partial/\partial v)V(v))qq'],$$

where $(\partial^2/\partial v\partial v')V(v)$ is the second derivative of the indirect utility function

$$(\partial^2/\partial v\partial v')V(v) = -\sum_{\alpha=1}^{A}\left\{u_{0\alpha} + 2\sum_{j=1}^{J} j^2[u_{j\alpha}\cos(jk_\alpha' v) - w_{j\alpha}\sin(jk_\alpha v)]\right\}k_\alpha k_\alpha'.$$

Let $P = diag(p_1, p_2, p_3)$ and $Q = diag(q_1, q_2, q_3)$, then the uncompensated price elasticities are given by

$$\eta_M = Q^{-1}[(\partial/\partial p')q(p, y_m)]P,$$

and the income elasticities are

$$\eta_T = -Q^{-1}[(\partial/\partial p')q(p, y_m)]p$$

which are the quantities typically of interest in demand studies.

The price elasticities are presented in Table 5.4; most of them are negative indicating that assets are gross complements, although the effects are generally small. Although we do not capture the variability of the elasticities presented in Table 5.4 in Table 5.6 we summarize the results by looking at the price elasticities evaluated at the mean. All of them are below unity negative and significant. We computed the standard

errors by linearizing the elasticity formulas around the estimated parameter values and then by using the standard formulas for the variance of linear functions of random variables. This was carried out using the ANALYZ command in TSP.

Expenditure elasticities and standard errors are presented in Table 5.5.[9] The elasticities that are statistically significant are indicated in the table with an asterix. Volatility is present for the three assets in the eighties (some are even negative) but the magnitudes stabilize in the nineties. The income elasticity for the business asset (x_1) stabilizes below unity but for the household asset (x_2) and foreign currency deposits (x_3) it stabilizes above unity. The income elasticities at the mean for the whole period are reported in Table 5.7. Foreign currency deposits have a significant elasticity well above unity. The also significant elasticity for households is greater than one but very close to unity. Finally, the income elasticity at the mean for the business sector asset is below unity and significant.

To continue our assessment of the validity of the parameter estimates we compute Allen and Morishima elasticities of substitution. Diewert (1974) demonstrates that the Allen partial Elasticity of Substitution (AES) between two liquid assets i and j, σ_{ij}^a can be derived directly from the estimates of the indirect utility function,

$$\sigma_{ij}^a = \frac{\sum_k (v_k V_k) V_{ij}}{V_i V_{ij}} - \frac{\sum_k v_k V_{jk}}{V_j} - \frac{\sum_k v_k V_{ik}}{V_i} + \frac{\sum_m \sum_k v_k V_{km} v_m}{\sum_n v_n V_n}$$

where V_i and V_{ij} denotes elements of $(\partial / \partial v)V(v)$ and $(\partial^2 / \partial v \partial v')V(v)$ respectively.[10]

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[9] Standard errors were calculated using the ANALYZ command in TSP. The same procedure is used in the calculation of standard errors of subsequent elasticities.

[10] The Allen elasticity of substitution can also be computed from the income and price elasticities by

Gallant (1981) argues that global approximation to within arbitrary accuracy of the elasticities of substitution appears to be a far more appealing property than equality a single point. That is if σ_{ij}^{*} corresponds to the true indirect utility function and σ_{ij}^{a} corresponds to a flexible form which satisfies the proposed definition, then by choosing ε sufficiently small there is a k with

$$\left|\sigma_{ij}^{*}(v)-\sigma_{ij}^{a}(v)\right|<\varepsilon, \qquad \text{for all } v \in \overline{V}$$

where $\varepsilon > 0$ and $\overline{V}$ is the region of approximation (the overbar denotes closure of a set). If an increase in the price of j produces a decrease in the demand for i $(\sigma_{ij}^{a} < 0)$ assets are said to be Allen complements. If, on the contrary $\sigma_{ij}^{a} > 0$, assets are Allen substitutes.

Traditionally, the AES has been used to study substitution behavior and structural instability. However, when there are more than 2 goods the AES may be uninformative. For two assets the relationship is unambiguous –the assets must be substitutes. When there are more than 2 assets the substitution relationship becomes complex and depends on things like the direction taken towards the point of approximation. Fleissig and Serletis (2001) and Serletis (2001) argue that in that case the Morishima Elasticity of Substitution (MES) is the correct measure of substitution elasticity. The MES is defined as,

$$\sigma_{ij}^{m} = s_i(\sigma_{ji}^{a} - \sigma_{ii}^{a})$$

where s_i is the share of asset i in the consumer's budget. The MES looks at the impact on the ratio of two goods (x_i/x_j). Assets will be Morishima complements (substitutes) if an

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using the Slusky equation $\sigma_{ij}^{a} = \eta_i + \eta_{ij}/s_j$. In this thesis the elasticities were calculated by both methods.

increase in the price of j causes x_i/x_j to decrease (increase). Finally, Morishima elasticities may be nonsymmetric ($\sigma_{ij}^m \neq \sigma_{ji}^m$) because the ratio x_i/x_j may be affected differently by a change in p_i or p_j. Only in the case where the aggregator function is a member of the CES family will the MES be symmetric.

We present estimates of Allen own-elasticities of substitution and standard errors in Table 5.8. For the business and household assets, the own elasticities are relatively low, negative and significant in the eighties and early nineties. However, they turn positive, low in magnitude and insignificant in the late nineties. In contrast, the own-price elasticity for foreign currency deposits appears to be large in magnitude, negative and significant through the entire sample. These results are confirmed when analyzing the AES at the mean (Table 5.13). The business and household own-price elasticities at the mean are negative, very low and insignificant; however, the foreign currency deposits elasticity at the mean is greater than 2 negative and significant. Cross Allen elasticities of substitution are displayed in Table 5.9. Very few AES between the business asset and the household asset are significant. The ones that are significant occur in the eighties are low and positive. Overall it appears that there is no relationship between these two assets, specially in the nineties. The relationship between the business asset and foreign currency deposits appears to be weak too. However, there seems to be changes in the type of relationship since for some periods the substitution elasticity is positive and significant (e.g. 1983 and 1990) and for some others is negative and significant (e.g. 1988 and 1992). Regarding the substitution elasticity between the household asset and foreign currency deposits there is no apparent change in the kind of relationship. The significant

elasticities are all positive and low in magnitude. The Allen elasticity at the mean between these two assets is 0.3 and significant.

Morishima elasticities of substitution and standard errors are presented in Tables 5.10 to 5.12. The elasticities that relate the business sector asset and the households sector asset σ_{12}^{m} and σ_{21}^{m} are low specially in the nineties and the few that look significant occur in the eighties and are positive. The Morishima elasticities of substitution involving the business sector asset and foreign currency deposits are also low as shown in Table 5.11, nevertheless, in the case where the business asset user cost varies σ_{31}^{m} the elasticities are in many cases significant. At the mean (Table 5.14) this elasticity amounts to approximately 0.1 and it is statistically significant. Table 5.12 shows the MES that involve the household sector and foreign currency deposits, again the magnitudes are low but in this case a large ratio of elasticities are significant. Evaluated at the means both elasticities are statistically significant, less than one and positive.

5.6 Discussion

Because the estimation of a semi-nonparametric functional form is parameter intensive, we were forced to group monetary liquid components in 3 broad assets, one comprising only business sector assets (excluding non-personal term deposits), another one mostly including household assets and the third one including only foreign currency deposits. In this regard, we have monetary data that only partially satisfy nonparametric empirical tests. Although the discrepancies between the sample periods for the non-parametric and parametric demand analysis provides a possible caveat in respect to

assessing the latter, the use, as far as possible, of legitimate asset groupings should, at the very least, ensure that our money demand results will be superior to similar studies conducted without any weak separability evidence.[11]

In order to produce the three monetary asset groups and later estimate a demand system of three asset shares, we advocated consistency with the underlying preference of asset holders. To do so we used the Divisia monetary index which provides an aggregation-theoretic measure widely applied to monetary economics.

At the estimation stage we found evidence of autocorrelation in the error terms and corrected the problem to a certain extent using Berndt and Savin's (1975) procedure. However and according to the new Durbin-Watson statistic some degree of autocorrelation persists. Agents may face various constraints that limit full adjustment of asset holdings each period, in other words, the pattern of demand may not adjust to a change in exogenous variables instantaneously. In fact, Swofford and Whitney (1994) find incomplete adjustment on US monetary data. To solve this problem Fisher and Fleissig (1994) propose to estimate a dynamic Fourier model. The idea behind Fisher and Fleissig's (1994) dynamic model (following economic theories) is to use past variables – in the utility function, in the constraints, or by time series methods to model adjustment costs, habit persistence and/or expectations. We suggest the application of Fisher and Fleissig's dynamic model on Canadian monetary assets as an alternative for further research in this area.

[11] Remember that the weak separability tests are valid only for certain subsample periods. Because of scarcity in degrees of freedom parametric econometric estimation is impractical in small sample sizes.

Related to the issue of the nonparametric tests are the regularity test results performed over the Fourier model. In principle we know that our data set does not pass the revealed preference tests. The fact that quasiconvexity of the indirect utility function holds in few observations should not be very surprising since the data sample had many GARP violations. Recall that x_3 (foreign currency deposits) appeared in the last quarter of 1981, accordingly, we would expect some adjustment period after its implementation. This may well affect the equilibrium quantities and the regularity conditions in the first years of our sample (where the monotonicity condition does not hold). Nevertheless, the fact that only two own elasticities of substitution are significantly greater than zero indicates that in most instances we cannot advocate model failure. In addition, the monotonicity condition does a lot better and the nonnegativity conditions holds at every data point. [12]

If the researcher is interested in imposing *a priori* the curvature condition, Gallant (1981) provides computer code for imposing sufficient (but not necessary) curvature conditions globally on the Fourier model. However, the span of the model is reduced to a degree that is rejected by the data according to Barnett, Geweke and Wolfe (1991). A better method to impose the restriction is provided by Gallant and Golub (1984) in which consistency of the estimators is not affected. However, the restrictions are not exclusive on the parameters but jointly on the parameters and on the data. A further disadvantage is the method's degree of difficulty.

[12] In Ewis and Fisher's (1985) Fourier monetary demand model the necessary and sufficient curvature conditions are also the less likely to hold.

If we turn our attention to the expenditure elasticities, it is clear that the numbers are quite variable in the eighties but relatively more stable and significant in the nineties for all three assets. The expenditure elasticity at the mean for the business sector at 0.69 is expenditure inelastic, accordingly monetary assets in the business sector are inferior goods. One might be tempted to conclude erroneously that there are economies of scale for the business assets in money holdings. However, in this analysis total expenditure is defined as the sum of expenditures on individual monetary services and not as total spending. Consequently, no inferences can be made on whether real money holdings increase less than proportionately when total spending is raised. The story is different for the household asset which behaves more like a normal good. For every increase in total money spending, households raise monetary holdings in a proportionate manner. Finally, foreign currency deposits are a luxury good, according to our results.

The Allen own-elasticities of substitution at the mean are negative, as required by the theory, but only statistically significant for foreign currency deposits. For the business and household assets they are very low and insignificant, which suggests that the net demand for these kind of assets is very inelastic. Contrary to these, the demand for foreign currency deposits looks quite elastic, in the net being very sensitive to a change in its own price.

Turning to the substitutability/complementarity relationship among monetary assets and according to the cross Allen elasticity of substitution financial assets are net substitutes or there is no relationship at all. However, as already mentioned above, the substitutability/complementarity relationship is better studied by the Morishima elasticities of substitution. Table 5.10 and Figure 1 show the Morishima elasticities

between household and business assets. Figure 1 shows how the Morishima elasticities behave very similar independently of which of the two user costs is varied leading us to conclude that the elasticity of substitution between these two kind of assets might be constant. Spikes occur in the third quarter of 1985 and first of 1986, indeed this is a result that repeats itself in the income elasticities, Allen and other Morishima elasticities of substitution. The elasticities are much more stable after 1989 and the behavior may be revealed in a more interesting way by looking at several recent years under the microscope. Figure 5 shows that after 1989 the Morishima elasticities between households and business assets actually do not follow the same pattern leading to the conclusion that assets do not necessarily have constant elasticity of substitution.

By inspection of Table 5.10, we can say that there is no apparent relation between household assets and business assets, a result that does not surprise us at all. However, in the cases where there is a significant relation the assets behave like substitutes (mostly this happens in the eighties) but the effect is low. Our conclusion regarding an acceptable definition of money, although much work remains to be done, is that the aggregation of business assets with household assets using a simple sum method requiring infinite elasticities could be unwise more because of the low substitutability than for the variability. Moreover, we are inclined to pose a question, should the central bank consider the impact on two monetary aggregates or have two monetary targets, one for households and another one for the business sector when making its decisions?

The subject matter does not have to be necessarily if household assets and business assets are complements or substitutes but how similar they are in comparison with the third asset, foreign currency deposits. We do this by looking at the Morishima

elasticities that involve foreign currency deposits. Figure 2 shows the elasticities between the business asset and foreign currency deposits, (and Figure 4 for the period of relative stability). It is clear that the elasticity is more sensitive when the user cost of foreign deposits changes but still both elasticities seem to be closely correlated. At the mean unambiguously we can say that when the price of business assets increases the ratio x_3/x_1 also increases but by a small amount. On the other hand, Table 5.12 and Figure 3 display the Morishima elasticities for the household assets and foreign currency deposits, again the pattern of high volatility in the eighties and low in the nineties is repeated. At the mean (Table 5.14) both elasticities are well below unity and significant. In this case household assets and foreign currency deposits behave like substitutes irrespective of which price induces the changes. Thus, while there is little evidence that household and business assets are substitutes, they appear alike so far as comparisons with a third, benchmark, asset are concerned. The only difference is that household assets have a higher degree of substitutability than business assets have with foreign currency deposits. Figure 6 displays the Morishima elasticities of substitution for households and business when varying the price of foreign currency deposits. In many instances they seem to behave in opposite direction, in fact the correlation coefficient confirms this behavior (-0.6175). That is to say, when business assets and foreign currency deposits become more substitutable between each other, household assets and foreign currency deposits become less substitutable and the opposite also occurs.

One last comment regarding the variability of elasticities of substitution. In general we see less variability than the one reported by Fleissig and Serletis (2001) who estimated a Fourier model over Canadian monetary assets. Fleissig and Serletis calculated

Morishima elasticities of substitution between cash, chequable savings deposits and non-chequable savings. They find these monetary assets to be substitutes for each other. In general their Morishima elasticities are nonsymmetrical and variable over time. In contrast to our findings their elasticities vary through out the eighties and nineties and in some cases the variability increases in the nineties. They attribute this to the recent volatility in the financial environment. In our case the variability decreases in the nineties possibly reflecting an environment of less economic turning points and smaller changes in user costs.

5.7 Conclusions

In this chapter we have argued that by using the Fourier flexible functional form we are able to obtain consistent estimates of price and income elasticities and elasticities of substitution for the demand for money. Our main result is that the various financial assets are substitutes in use for each other with the values not being very large or that there is no relationship at all. This is particularly useful as it enables us to provide empirical evidence that bear indirectly on the demand for money definition of money. The fact is that the degree of substitutability among monetary assets has been used to provide –explicitly or implicitly– a rational for the choice of assets to be included in a monetary aggregate.

The income elasticities, Allen and Morishima elasticities of substitution between household and business assets are certainly not constant especially in the decade of the eighties when Canada experienced recession and higher rates of inflation. As Ewis and Fisher (1985) argue the reason the elasticities of substitution may vary is that they

respond to changes in the relative prices among monetary assets and not necessarily that technology or tastes have changed. These changes occur more frequently around business cycle turning points. There is no guarantee that the climate of stability will continue forever. If the elasticities of substitution are not constant then simple sum aggregations that require infinite and constant elasticities of substitution will not produce a satisfactory result. The problem is that substitution effects are being incorrectly accounted for resulting in money definitions that do not accurately measure the actual quantities of monetary services that economic agents select in the aggregate. As a counter argument we should mention that the Morishima elasticities of substitution between household assets and business assets appear to be quite correlated indicating that the elasticity of substitution may, in fact, be constant over the period studied. Even so, the elasticities are clearly not infinite. By implication a simple sum aggregate of M2 containing household and business monetary assets together would be unwise. This confirms a problem that has already been identified in the literature, and it strongly suggests the use of other than simple sum methods of aggregation like the Divisia index. We should acknowledge, however, that once a proper definition of money is obtained our results do not provide strong evidence against using conventional money-demand functions (as the estimated Morishima elasticities of substitution exhibit correlation, Figure 1).

The patterns of the elasticities of substitution between foreign currency deposits and all the other forms of assets are non-constant and low over the sample period analyzed. Here more than discrediting a simple sum aggregate M3 we feel we can say something about the substitution between Canadian monetary assets and monetary holdings of American dollars by Canadian consumers (as most of foreign currency

deposits are hold in this form). The fact that the elasticities are quite low indicates that Canadian and American assets are serving quite different purposes in both the business sector and the household sector. Although a considerable amount of work remains to be done regarding a monetary union between Canada and The United States, so far we cannot conclude that the American dollar works as a substitute for Canadian monetary assets. Moreover, the negative relation among elasticities of substitution between business and household deposits with foreign currency deposits displayed in Figure 6 indicates that the business and household sectors regard in a different way the American currency.

TABLE 5.1

FOURIER FLEXIBLE FUNCTIONAL FORM

	R-Square	Durbin-Watson
Share 1	0.942730	0.586837
Share 2	0.962696	0.720149

Parameter	Estimate	Standard Error	t-statistic
b_1	-3.05623	.395407	-7.72934
b_2	-12.6514	1.04234	-12.1375
u_{01}	.226661	.046666	4.85708
u_{11}	.059129	.01003	5.89502
w_{11}	0.18006	.030998	.580869
u_{02}	-1.50367	.18689	-8.04574
u_{12}	.256705	.051138	5.0198
w_{12}	.606796	.096787	6.26937
u_{03}	.161918	.00879744	18.4051
u_{13}	.031790	.00498778	6.37349
w_{13}	-.0096265	.00333249	-2.88869
u_{04}	-.403359	.055185	-7.30917
u_{14}	.067402	.033625	2.00449
w_{14}	-.223363	.033111	-6.74589
u_{05}	-.145737	.00385269	-37.8273
u_{15}	-.015349	.00421171	-3.64436
w_{15}	-.043909	.00497153	-8.83206
u_{06}	-.041646	.00272603	-15.2772
u_{16}	.00334775	.00171742	1.94929
w_{16}	-.0011194	.00122317	-.915161

TABLE 5.2

FOURIER FLEXIBLE FUNCTIONAL FORM WITH AUTOREGRESSIVE DISTURBANCES

	R-Square	Durbin-Watson
Share 1	0.992555	0.927539
Share 2	0.993094	0.859696

Parameter	Estimate	Standard Error	t-statistic
b_1	-2.53746	.333937	-7.59861
b_2	-6.46171	.725963	-8.90089
u_{01}	.249039	.021911	11.3658
u_{11}	.00581309	.00436977	1.3303
w_{11}	.016434	.00833501	1.97173
u_{02}	-.392966	.137172	-2.86467
u_{12}	.012008	.027288	.440049
w_{12}	.00064397	.056623	.011373
u_{03}	.162142	.00305658	53.047
u_{13}	.00767256	.00127705	6.00801
w_{13}	-.00556939	.000938139	-5.93664
u_{04}	-.331917	.031187	-10.6429
u_{14}	.045581	.015169	3.00496
w_{14}	-.014572	.010164	-1.43371
u_{05}	-.133548	.00412597	-32.3677
u_{15}	-.0065182	.0011326	-5.75506
w_{15}	-.011448	.00173119	-6.61275
u_{06}	-.051541	.00366422	-14.0659
u_{16}	-.00144863	.000533068	-2.71753
w_{16}	.000229886	.000434141	.52952
ρ	.988099	.010798	91.5113

TABLE 5.3

REGULARITY-TESTS RESULTS

Test	Number of Times
Nonnegativity	72
Monotonicity	56
Quasiconvexity	11

TABLE 5.4

PRICE ELASTICITIES

Obs	η_{11}	η_{22}	η_{33}	η_{12}	η_{21}	η_{13}	η_{31}	η_{23}	η_{32}
1982.Q2	-0.2117	-0.5793	-0.7898	-0.7726	-0.236	0.0057	-0.9331	-0.0119	-3.1113
Q3	-0.3642	-0.5975	0.766	-1.0649	-0.2178	-0.167	-0.266	-0.076	-0.0348
Q4	-0.7354	-0.5278	0.0161	-1.0807	-0.1378	-0.053	0.312	-0.0486	0.0096
1983.Q1	-0.6858	-0.5089	-0.5273	-1.0691	-0.1513	0.0078	0.3059	-0.0316	-0.6457
Q2	-0.6593	-0.5146	-0.6379	-0.9238	-0.1298	0.0014	-0.2714	-0.0208	-1.6556
Q3	-0.6124	-0.5243	-0.6077	-0.9129	-0.1379	-0.0056	-0.4347	-0.0181	-1.8699
Q4	-0.6627	-0.556	-0.5426	-0.8672	-0.1334	-0.0022	-0.1876	-0.0237	-1.3941
1984.Q1	-0.75	-0.5655	-0.4893	-0.7817	-0.1115	-0.0108	-0.112	-0.0265	-1.0354
Q2	-0.7878	-0.4481	-0.5155	-0.9641	-0.0565	-0.0429	-0.7511	-0.0073	-1.8235
Q3	-1.3801	0.3991	-0.4783	-2.3679	0.3561	-0.2227	-2.7854	0.0842	-3.1726
Q4	-0.2886	-1.0122	-0.3571	0.1903	-0.3877	0.0827	0.089	-0.099	-1.0513
1985.Q1	-0.3845	-1.0087	-0.3873	0.1766	-0.3354	0.0789	0.1226	-0.093	-1.0111
Q2	-0.3972	-1.1804	-0.3238	0.475	-0.3531	0.1165	0.411	-0.1198	-0.8087
Q3	0.4238	-2.5665	-2.0627	5.0108	-0.3223	0.4754	-4.9347	-0.1546	-11.9829
Q4	-0.5444	-0.9707	-0.6459	0.0027	-0.3735	0.145	1.2401	-0.1155	-0.3675
1986.Q1	5.1264	-5.439	3.8592	0.5369	-5.3736	-0.2918	15.075	-0.4093	34.589
Q2	-2.1938	1.9186	-0.6694	-6.4753	0.4936	-0.4715	0.6667	0.1887	1.0172
Q3	-0.4161	-0.1148	-0.5815	-1.4495	-0.255	-0.0267	-0.3566	-0.0281	-1.6674
Q4	-0.3097	-0.5014	-0.1373	-0.8601	-0.3404	-0.0578	-0.174	-0.0565	-0.5262
1987.Q1	-0.6891	-0.4941	-0.4104	-1.4288	-0.1029	-0.1542	-0.16	0.0081	0.3449
Q2	-0.5121	-0.7936	0.0885	-0.7038	-0.162	-0.1331	-0.1814	-0.0315	0.636
Q3	-0.5296	-0.9931	-0.1162	-0.326	-0.1588	-0.1216	-0.0007	-0.0298	1.2859
Q4	-0.3972	-1.3077	0.1667	0.499	-0.2071	-0.0881	0.1288	-0.0603	1.8903
1988.Q1	-0.4316	-1.084	-0.9448	0.3633	-0.1541	0.0062	-0.3505	-0.0067	-0.3931
Q2	-0.3747	-1.0062	-0.3157	-0.0187	-0.1759	-0.078	-0.2997	-0.0321	0.1465
Q3	-0.3102	-1.0401	0.0807	0.0865	-0.1941	-0.0853	-0.3032	-0.053	0.1754
Q4	-0.2784	-0.9577	0.001	-0.0843	-0.1945	-0.0885	-0.3976	-0.0486	-0.2114
1989.Q1	-0.2631	-0.8716	-0.0635	-0.285	-0.2029	-0.0926	-0.4641	-0.0432	-0.4577
Q2	-0.1955	-0.6711	-0.456	-0.407	-0.2374	-0.0236	-0.6142	-0.0491	-1.6468
Q3	-0.1324	-0.6371	-0.996	0.035	-0.209	0.1285	-0.8703	-0.0454	-3.4604
Q4	-0.0772	-0.6477	-1.0603	0.2634	-0.201	0.1759	-0.9006	-0.051	-3.8571
1990.Q1	-0.0239	-0.6189	-0.9256	0.3621	-0.2213	0.2242	-0.865	-0.0853	-3.9902
Q2	0.0661	-0.5533	-1.1707	0.7318	-0.2497	0.3997	-1.034	-0.123	-5.2687
Q3	0.0425	-0.5563	-1.1732	0.6436	-0.2485	0.3716	-1.0341	-0.1141	-5.2166
Q4	-0.0769	-0.6451	-1.0292	0.2444	-0.2044	0.1732	-0.8898	-0.0541	-3.8046
1991.Q1	-0.1187	-0.6661	-0.9287	0.0709	-0.2044	0.1131	-0.8325	-0.0459	-3.2317
Q2	-0.1874	-0.7449	-0.377	-0.3665	-0.2199	-0.0663	-0.6049	-0.0414	-1.2333
Q3	-0.1655	-0.7072	-0.2801	-0.4219	-0.2483	-0.0648	-0.5872	-0.0545	-1.2322
Q4	-0.1364	-0.6922	-0.1009	-0.4669	-0.2895	-0.0848	-0.5598	-0.0699	-0.9968
1992.Q1	-0.1226	-0.7082	-0.1081	-0.4395	-0.2799	-0.091	-0.5796	-0.0685	-1.0035
Q2	-0.1404	-0.7033	-0.1283	-0.4917	-0.2998	-0.0931	-0.5883	-0.0549	-0.8215
Q3	-0.1398	-0.6897	-0.1582	-0.4893	-0.3265	-0.0821	-0.5966	-0.0534	-0.817
Q4	-0.0741	-0.7183	-0.0501	-0.3511	-0.2725	-0.095	-0.5859	-0.089	-1.1352
1993.Q1	-0.0876	-0.7582	-0.1651	-0.3892	-0.265	-0.1063	-0.6262	-0.0534	-0.8971
Q2	-0.0993	-0.7449	-0.2649	-0.3895	-0.2935	-0.0697	-0.6261	-0.0466	-1.0007
Q3	-0.1	-0.7436	-0.3091	-0.3712	-0.3047	-0.0482	-0.6048	-0.0472	-1.1048
Q4	-0.0922	-0.7531	-0.3008	-0.3513	-0.3017	-0.0367	-0.5654	-0.0508	-1.1779

TABLE 5.4

PRICE ELASTICITIES

(...CONT)

Obs	η_{11}	η_{22}	η_{33}	η_{12}	η_{21}	η_{13}	η_{31}	η_{23}	η_{32}
1994.Q1	-0.0929	-0.743	-0.3079	-0.3587	-0.3168	-0.0322	-0.5639	-0.0521	-1.1756
Q2	-0.0738	-0.7571	-0.3068	-0.3482	-0.2717	-0.0534	-0.5413	-0.0548	-1.1198
Q3	-0.0855	-0.7702	-0.2562	-0.346	-0.2712	-0.0372	-0.4925	-0.056	-1.1877
Q4	-0.0942	-0.7828	-0.1893	-0.3544	-0.2746	-0.0267	-0.455	-0.054	-1.2321
1995.Q1	-0.085	-0.7875	-0.2435	-0.3411	-0.2255	-0.0527	-0.4395	-0.0593	-1.1354
Q2	-0.0947	-0.7897	-0.2105	-0.3571	-0.2294	-0.0387	-0.4032	-0.0593	-1.1801
Q3	-0.1003	-0.792	-0.1712	-0.3739	-0.239	-0.0311	-0.3891	-0.0569	-1.1974
Q4	-0.1049	-0.7926	-0.1383	-0.3936	-0.2477	-0.0282	-0.3821	-0.054	-1.1849
1996.Q1	-0.103	-0.7799	-0.1438	-0.3971	-0.2718	-0.0288	-0.4154	-0.0538	-1.1598
Q2	-0.1023	-0.7775	-0.1322	-0.3985	-0.2875	-0.0267	-0.4318	-0.0515	-1.1586
Q3	-0.1081	-0.7715	-0.1049	-0.4112	-0.3163	-0.0228	-0.451	-0.0461	-1.1394
Q4	-0.1295	-0.7687	-0.0405	-0.4591	-0.3422	-0.0163	-0.4347	-0.0311	-1.0593
1997.Q1	-0.1339	-0.7632	-0.0506	-0.4547	-0.3462	-0.0164	-0.4367	-0.0333	-1.0681
Q2	-0.1331	-0.7819	-0.0354	-0.4589	-0.3272	-0.0126	-0.4096	-0.0276	-1.1143
Q3	-0.1386	-0.787	-0.0177	-0.4812	-0.3255	-0.0095	-0.3944	-0.0197	-1.0867
Q4	-0.1322	-0.8002	-0.0083	-0.4903	-0.3147	-0.0075	-0.3904	-0.014	-1.0777
1998.Q1	-0.1192	-0.822	-0.0179	-0.4546	-0.283	-0.0091	-0.3767	-0.0196	-1.192
Q2	-0.1181	-0.8256	-0.0236	-0.4403	-0.2769	-0.0091	-0.3731	-0.0219	-1.2372
Q3	-0.1108	-0.8039	-0.0635	-0.4098	-0.2879	-0.0152	-0.4036	-0.0368	-1.2269
Q4	-0.1109	-0.7535	-0.1524	-0.4001	-0.3348	-0.0234	-0.485	-0.0501	-1.14
1999.Q1	-0.1041	-0.7573	-0.1618	-0.3937	-0.3233	-0.0255	-0.4823	-0.0519	-1.1462
Q2	-0.1089	-0.7656	-0.1305	-0.4006	-0.3218	-0.0224	-0.4656	-0.0483	-1.1572
Q3	-0.1147	-0.7674	-0.1078	-0.4111	-0.3266	-0.0204	-0.4568	-0.0449	-1.1498
Q4	-0.1142	-0.7749	-0.0825	-0.4213	-0.3214	-0.0197	-0.4429	-0.0414	-1.1388

TABLE 5.5

INCOME ELASTICITIES

Obs	η_1		η_2		η_3	
1982.Q2	0.9786	(0.7806)	0.8272*	(0.2767)	4.8342*	(1.4938)
Q3	1.5960*	(0.4908)	0.8913*	(0.1684)	-0.4651	(0.6138)
Q4	1.8691*	(0.6348)	0.7141*	(0.2350)	-0.3377	(0.9884)
1983.Q1	1.7471*	(0.7909)	0.6918*	(0.2912)	0.8671	(1.2940)
Q2	1.5816*	(0.5956)	0.6652*	(0.2334)	2.5649*	(0.7988)
Q3	1.5309*	(0.5321)	0.6803*	(0.2081)	2.9123*	(0.7602)
Q4	1.5321*	(0.4179)	0.7131*	(0.1577)	2.1242*	(0.5819)
1984.Q1	1.5425*	(0.3531)	0.7035*	(0.1311)	1.6367*	(0.5122)
Q2	1.7949*	(0.5209)	0.5119*	(0.2191)	3.0902*	(0.8066)
Q3	3.9707*	(2.2182)	-0.8394	(1.2197)	6.4363	(4.6135)
Q4	0.0156	(1.1693)	1.4988*	(0.5490)	1.3194	(1.3410)
1985.Q1	0.1290	(0.9018)	1.4371*	(0.4517)	1.2757	(1.1045)
Q2	-0.1943	(1.1486)	1.6533*	(0.6290)	0.7216	(2.4024)
Q3	-5.9100	(17.1937)	3.0434	(5.8940)	18.9803	(39.2689)
Q4	0.3967	(1.7499)	1.4597	(1.0163)	-0.2267	(3.6774)
1986.Q1	-5.3716	(26.5780)	11.2219	(36.9802)	-53.5231	(92.902)
Q2	9.1406	(7.7651)	-2.6010	(3.4145)	-1.0145	(7.4781)
Q3	1.8922*	(1.0885)	0.3979	(0.4544)	2.6054	(2.2145)
Q4	1.2277*	(0.3836)	0.8983*	(0.1989)	0.8374*	(0.4975)
1987.Q1	2.2721*	(1.0776)	0.5890*	(0.3327)	0.2254	(0.9649)
Q2	1.3490*	(0.7333)	0.9871*	(0.2415)	-0.5432	(0.8332)
Q3	0.9772	(1.1317)	1.1817*	(0.3678)	-1.1691	(1.6027)
Q4	-0.0137	(1.4576)	1.5751*	(0.5254)	-2.1858	(2.3173)
1988.Q1	0.0622	(1.4528)	1.2448*	(0.4443)	1.6884	(1.6552)
Q2	0.4715	(0.7243)	1.2142*	(0.2229)	0.4689	(0.8882)
Q3	0.3089	(0.5641)	1.2872*	(0.1740)	0.0470	(0.7007)
Q4	0.4512	(0.3793)	1.2007*	(0.1093)	0.6080	(0.4414)
1989.Q1	0.6407*	(0.2599)	1.1178*	(0.0732)	0.9853*	(0.2934)
Q2	0.6261*	(0.2419)	0.9576*	(0.0704)	2.7170*	(0.3535)
Q3	-0.0311	(0.3538)	0.8915*	(0.0914)	5.3266*	(0.8901)
Q4	-0.3620	(0.3646)	0.8997*	(0.0877)	5.8180*	(0.9442)
1990.Q1	-0.5624	(0.3691)	0.9255*	(0.0794)	5.7809*	(0.9094)
Q2	-1.1975*	(0.4946)	0.9260*	(0.1093)	7.4734*	(1.4360)
Q3	-1.0578*	(0.4798)	0.9188*	(0.1058)	7.4239*	(1.4082)
Q4	-0.3407	(0.3596)	0.9037*	(0.0856)	5.7236*	(0.9188)
1991.Q1	-0.0653	(0.3321)	0.9165*	(0.0823)	4.9929*	(0.7687)
Q2	0.6203*	(0.2229)	1.0062*	(0.0595)	2.2152*	(0.2645)
Q3	0.6523*	(0.2020)	1.0099*	(0.0576)	2.0996*	(0.2351)
Q4	0.6881*	(0.1781)	1.0515*	(0.0571)	1.6575*	(0.1871)

TABLE 5.5

INCOME ELASTICITIES

(...CONT)

Obs	η_1	η_2	η_3
1992.Q1	0.6531* (0.1849)	1.0565* (0.0557)	1.6912* (0.1914)
Q2	0.7252* (0.1665)	1.0580* (0.0568)	1.5381* (0.1660)
Q3	0.7113* (0.1585)	1.0696* (0.0595)	1.5718* (0.1631)
Q4	0.5202* (0.2080)	1.0798* (0.0547)	1.7712* (0.2151)
1993.Q1	0.5831* (0.2046)	1.0766* (0.0543)	1.6884* (0.2042)
Q2	0.5585* (0.1953)	1.0849* (0.0585)	1.8918* (0.2149)
Q3	0.5195* (0.1980)	1.0956* (0.0606)	2.0187* (0.2311)
Q4	0.4803* (0.2101)	1.1056* (0.0613)	2.0441* (0.2430)
1994.Q1	0.4838* (0.2087)	1.1118* (0.0644)	2.0474* (0.2437)
Q2	0.4753* (0.2284)	1.0836* (0.0592)	1.9678* (0.2360)
Q3	0.4687* (0.2244)	1.0974* (0.0568)	1.9364* (0.2363)
Q4	0.4753* (0.2194)	1.1113* (0.0554)	1.8764* (0.2335)
1995.Q1	0.4788* (0.2239)	1.0723* (0.0465)	1.8184* (0.2211)
Q2	0.4905* (0.2178)	1.0784* (0.0463)	1.7938* (0.2156)
Q3	0.5052* (0.2149)	1.0880* (0.0477)	1.7577* (0.2123)
Q4	0.5267* (0.2122)	1.0944* (0.0493)	1.7053* (0.2085)
1996.Q1	0.5290* (0.2147)	1.1056* (0.0553)	1.7189* (0.2158)
Q2	0.5275* (0.2142)	1.1166* (0.0582)	1.7226* (0.2206)
Q3	0.5422* (0.2070)	1.1339* (0.0625)	1.6953* (0.2253)
Q4	0.6049* (0.1899)	1.1420* (0.0646)	1.5345* (0.2182)
1997.Q1	0.6051* (0.1867)	1.1428* (0.0648)	1.5554* (0.2197)
Q2	0.6045* (0.1873)	1.1367* (0.0609)	1.5593* (0.2180)
Q3	0.6293* (0.1831)	1.1323* (0.0601)	1.4989* (0.2132)
Q4	0.6300* (0.1878)	1.1289* (0.0590)	1.4764* (0.2095)
1998.Q1	0.5829* (0.1990)	1.1246* (0.0537)	1.5866* (0.2101)
Q2	0.5675* (0.2000)	1.1243* (0.0522)	1.6338* (0.2121)
Q3	0.5359* (0.2059)	1.1286* (0.0552)	1.6941* (0.2211)
Q4	0.5344* (0.2023)	1.1384* (0.0657)	1.7773* (0.2322)
1999.Q1	0.5233* (0.2084)	1.1325* (0.0646)	1.7903* (0.2316)
Q2	0.5319* (0.2053)	1.1356* (0.0633)	1.7533* (0.2303)
Q3	0.5462* (0.2008)	1.1389* (0.0633)	1.7144* (0.2286)
Q4	0.5552* (0.201)	1.1378* (0.0624)	1.6642* (0.2247)

*Elasticities that are statistically significant.

TABLE 5.6

PRICE ELASTICITIES AT THE MEAN

Asset	η_{i1}	η_{i2}	η_{i3}
x_1	-0.19284* (0.10465)	-0.45201* (0.096112)	-0.049117* (0.0096264)
x_2	-0.28267* (0.022486)	-0.74048* (0.041165)	-0.039655* (0.0042661)
x_3	-0.54725* (0.10691)	-0.98468* (0.12806)	-0.22076* (0.043678)

*Elasticities that are statistically significant.

TABLE 5.7

INCOME ELASTICITIES AT THE MEAN

η_1	η_2	η_3
0.69397* (0.15546)	1.06281* (0.0499)	1.75269* (0.19496)

*Elasticities that are statistically significant.

TABLE 5.8

ALLEN OWN-ELASTICITIES OF SUBSTITUTION

Obs	σ_{11}^a		σ_{22}^a		σ_{33}^a	
1982.Q2	0.1427	(0.5759)	0.0150	(0.0805)	-18.7025*	(4.2224)
Q3	0.1856	(0.3774)	0.0226	(0.0616)	13.7176*	(4.5364)
Q4	-0.6657*	(0.3084)	-0.0818	(0.0564)	0.0063	(2.3586)
1983.Q1	-0.6566*	(0.3444)	-0.0629	(0.0602)	-12.1765*	(2.4399)
Q2	-0.7138*	(0.3814)	-0.0970	(0.0621)	-14.3665*	(2.6879)
Q3	-0.6370*	(0.3817)	-0.0885	(0.0610)	-14.1753*	(2.5692)
Q4	-0.7720*	(0.3820)	-0.1094*	(0.0572)	-12.7856*	(2.1091)
1984.Q1	-0.8880*	(0.5038)	-0.1650*	(0.0701)	-10.5118*	(1.5169)
Q2	-0.7796	(0.5415)	-0.1702*	(0.0853)	-10.8258*	(2.0142)
Q3	-0.2525	(0.9164)	-0.2123	(0.2109)	-6.5728*	(3.1172)
Q4	-0.8592	(1.0425)	-0.1404	(0.1643)	-5.4713*	(2.3447)
1985.Q1	-1.0441	(1.0789)	-0.1889	(0.1620)	-6.1863*	(2.2528)
Q2	-1.4104	(1.4797)	-0.2510	(0.2129	-5.3245*	93.0023)
Q3	-4.5789	(11.7878)	-1.0238	(2.3988)	-21.7576	(32.0338)
Q4	-1.1460	(1.0721)	-0.1663	(0.2584)	-13.1016*	(5.3797)
1986.Q1	9.5865	(33.5340)	1.9599	(6.7313)	1.5736	(27.6889)
Q2	1.7949	(1.4267)	0.3883	(0.3965)	-12.2618	(8.3706)
Q3	0.5564	(0.4877)	0.2147	(0.1374)	-6.7877*	(2.7708)
Q4	0.2606	(0.2890)	0.0867	(0.0730)	-1.3783	(91.4157)
1987.Q1	-0.4126	(0.4359)	-0.1318*	(0.0565)	-6.8804*	(1.5593)
Q2	-0.6928	(0.4753)	-0.1493*	(0.0617)	1.1963	(2.3140)
Q3	-1.2470*	(0.7466)	-0.2263*	(0.0829)	-3.2227*	(1.6134)
Q4	-1.7170*	(0.9877)	-0.2605*	(0.1118)	0.8776	(2.4860)
1988.Q1	-1.8158*	(0.9729)	-0.2835*	(0.0994)	-13.8241*	(3.8749)
Q2	-1.1666	(0.7250)	-0.1994*	(0.0732)	-4.8413*	(1.2945)
Q3	-1.0625	(0.6831)	-0.1561*	(0.0700)	1.5637	(1.7204)
Q4	-0.7892	(0.5949)	-0.1280*	(0.0594)	0.6256	(1.2778)
1989.Q1	-0.4960	(0.4767)	-0.1050*	(0.0506)	-0.1524	(1.0481)
Q2	-0.1441	(0.3612)	-0.0377	(0.0411)	-3.6238*	(0.9498)
Q3	-0.5832	(0.4990)	-0.0375	(0.0487)	-8.0706*	(1.8428)
Q4	-0.7008	(0.5989)	-0.0347	(0.0505)	-7.6242*	(1.9065)
1990.Q1	-0.6659	(0.5942)	0.0189	(0.0501)	-4.9598*	(1.6903)
Q2	-0.9263	(0.5743)	0.0953*	(0.0576)	-5.4903*	(2.7174)
Q3	-0.8846*	(0.5464)	0.0854	(0.0557)	-6.0519*	(2.6543)
Q4	-0.6758	(0.5867)	-0.0295	(0.0501)	-7.2669*	(1.8377)

TABLE 5.8

ALLEN OWN-ELASTICITIES OF SUBSTITUTION

(...CONT)

Obs	σ_{11}^a		σ_{22}^a		σ_{33}^a	
1991.Q1	-0.5756	(0.5458)	-0.0475	(0.0491)	-7.1475*	(1.6115)
Q2	-0.1620	(0.4311)	-0.0748*	(0.0444)	-3.0710*	(0.8511)
Q3	-0.0036	(0.3599)	-0.0395	(0.0407)	-1.7001*	(0.8437)
Q4	0.1808	(0.3002)	-0.0055	(0.0411)	0.3344	(0.8660)
1992.Q1	0.1849	(0.3299)	-0.0154	(0.0415)	0.2948	(0.8539)
Q2	0.2168	(0.2718)	-0.0187	(0.0433)	-0.2773	(0.9163)
Q3	0.2306	(0.2479)	-0.0077	(0.0494)	-0.7224	(0.9120)
Q4	0.2240	(0.4109)	-0.0041	(0.0432)	1.1957	(0.7755)
1993.Q1	0.2300	(0.3876)	-0.0432	(0.0418)	-0.5157	(0.7856)
Q2	0.1829	(0.3194)	-0.0300	(0.0420)	-2.0399*	(0.6787)
Q3	0.1476	(0.3078)	-0.0198	(0.0418)	-2.7868*	(0.6026)
Q4	0.1299	(0.3273)	-0.0117	(0.0395)	-2.7437*	(0.5425)
1994.Q1	0.1418	(0.3140)	-0.0041	(0.0422)	-2.8614*	(0.5335)
Q2	0.1720	(0.4019)	-0.0232	(0.0376)	-2.2486*	(0.5089)
Q3	0.1136	(0.3862)	-0.0109	(0.0340)	-2.0475*	(0.4856)
Q4	0.0843	(0.3679)	-0.0003	(0.0320)	-1.5768*	(0.4623)
1995.Q1	0.0760	(0.4645)	-0.0245	(0.0284)	-1.6111*	(0.4605)
Q2	0.0442	(0.4382)	-0.0135	(0.0271)	-1.4599*	(0.4487)
Q3	0.0442	(0.4126)	-0.0052	(0.0274)	-1.1975*	(0.4327)
Q4	0.0563	(0.3926)	0.0001	(0.0280)	-0.9181*	(0.4117)
1996.Q1	0.0981	(0.3657)	0.0046	(0.0313)	-1.0151*	(0.4127)
Q2	0.1160	(0.3465)	0.0091	(0.0330)	-0.9573*	(0.4090)
Q3	0.1364	(0.3057)	0.0175	(0.0356)	-0.7799*	(0.3953)
Q4	0.1494	(0.2565)	0.0257	(0.0361)	0.0452	(0.3493)
1997.Q1	0.1397	(0.2492)	0.0258	(0.0368)	-0.1919	(0.3598)
Q2	0.1218	(0.2574)	0.0209	(0.0330)	0.0611	(0.3582)
Q3	0.1257	(0.2499)	0.0205	(0.0319)	0.4469	(0.3413)
Q4	0.1347	(0.2655)	0.0189	(0.0311)	0.7958*	(0.3260)
1998.Q1	0.0955	(0.3144)	0.0115	(0.0294)	0.5342	(0.3417)
Q2	0.0766	(0.3226)	0.0097	(0.0291)	0.3832	(0.3543)
Q3	0.0885	(0.3237)	0.0119	(0.0306)	-0.2728	(0.3878)
Q4	0.1374	(0.2862)	0.0190	(0.0396)	-1.4302*	(0.4263)
1999.Q1	0.1403	(0.3052)	0.0156	(0.0385)	-1.4298*	(0.4290)
Q2	0.1296	(0.2982)	0.0170	(0.0367)	-1.1492*	(0.4162)
Q3	0.1273	(0.2857)	0.0192	(0.0364)	-0.9332*	(0.4040)
Q4	0.1322	(0.2907)	0.0195	(0.0351)	-0.5606	(0.3850)

*Elasticities that are statistically significant.

TABLE 5.9

ALLEN ELASTICITIES OF SUBSTITUTION

Obs	σ_{12}^a	σ_{13}^a	σ_{23}^a
1982.Q2	-0.1047 (0.2161)	1.1494* (0.6405)	0.4718* (0.2058)
Q3	0.0478 (0.1375)	-1.4955* (0.6585)	-0.5158* (0.2424)
Q4	0.2392* (0.1256)	0.7377 (0.5044)	-0.3233 (0.2620)
1983.Q1	0.1616 (0.1404)	1.9393* (0.6964)	-0.0906 (0.2673)
Q2	0.2133 (0.1492)	1.6199* (0.6092)	0.1126 (0.2218)
Q3	0.1922 (0.1483)	1.3737* (0.5535)	0.1702 (0.2028)
Q4	0.2492* (0.1419)	1.4720* (0.5441)	0.0620 (0.2033)
1984.Q1	0.3420* (0.1829)	1.2738* (0.6609)	0.0466 (0.2593)
Q2	0.3273 (0.2104)	0.6355 (0.6391)	0.3145 (0.2674)
Q3	0.2502 (0.4395)	-2.0874* (1.2246)	1.4514* (0.5949)
Q4	0.3238 (0.4112)	1.5892 (1.5410)	-0.3832 (0.8291)
1985.Q1	0.4136 (0.4201)	1.6498 (1.5441)	-0.3541 (0.8291)
Q2	0.5720 (0.5773)	1.9799 (2.0707)	-0.5831 (1.1608)
Q3	2.0309 (5.1443)	3.4796 (9.3998)	-0.0096 (2.4228)
Q4	0.4011 (0.5211)	3.2876 (2.2238)	-0.8423 (1.1085)
1986.Q1	-4.4573 (15.4452)	-9.5370 (32.4709)	5.3783 (18.7965)
Q2	-0.9481 (0.7825)	1.2179 (1.8785)	0.5703 (0.6323)
Q3	-0.4209* (0.2350)	1.4607 (0.9568)	-0.0555 (0.3936)
Q4	-0.1646 (0.1371)	0.2942 (0.3585)	-0.0143 (0.1920)
1987.Q1	0.1880 (0.1490)	-0.3977 (0.5027)	0.7286* (0.2156)
Q2	0.3411* (0.1590)	-1.2663* (0.6149)	0.3676* (0.2051)
Q3	0.5150* (0.2391)	-1.1720 (0.7417)	0.6542* (0.2484)
Q4	0.6867* (0.3234)	-1.6335* (1.0022)	0.4676 (0.2882)
1988.Q1	0.5744* (0.3030)	0.1633 (0.7493)	1.1342* (0.3947)
Q2	0.4452* (0.2227)	-0.8412 (0.6300)	0.6747* (0.2097)
Q3	0.4290* (0.2072)	-1.2936* (0.6575)	0.2905* (0.1754)
Q4	0.3343* (0.1766)	-1.1634* (0.5607)	0.3147* (0.1457)
1989.Q1	0.2409* (0.1444)	-1.0201* (0.4651)	0.3431* (0.1248)
Q2	0.0225 (0.1162)	0.2979 (0.3588)	0.2744* (0.1303)
Q3	0.0199 (0.1649)	1.6972* (0.5658)	0.2815 (0.1912)
Q4	0.0180 (0.1859)	1.8678* (0.6444)	0.2533 (0.2066)
1990.Q1	-0.0319 (0.1831)	2.0387* (0.6756)	-0.0642 (0.2207)
Q2	-0.0989 (0.1951)	3.2284* (0.9296)	-0.4363 (0.3302)
Q3	-0.0935 (0.1881)	3.2110* (0.8967)	-0.3915 (0.3126)
Q4	0.0128 (0.1828)	1.8457* (0.6344)	0.2202 (0.2038)

TABLE 5.9

ALLEN ELASTICITIES OF SUBSTITUTION

(...CONT)

Obs	σ_{12}^a		σ_{13}^a		σ_{23}^a	
1991.Q1	0.0373	(0.1707)	1.4128*	(0.5520)	0.3158*	(0.1828)
Q2	0.0884	(0.1280)	-0.3099	(0.3886)	0.4256*	(0.1193)
Q3	0.0262	(0.1097)	-0.2270	(0.3471)	0.2710*	(0.1161)
Q4	-0.0249	(0.0964)	-0.4241	(0.3127)	0.1352	(0.1154)
1992.Q1	-0.0121	(0.1009)	-0.5218	(0.3313)	0.1723	(0.1139)
Q2	-0.0276	(0.0933)	-0.5923*	(0.3053)	0.2804*	(0.1099)
Q3	-0.0531	(0.0972)	-0.4796*	(0.2947)	0.2957*	(0.1140)
Q4	-0.0096	(0.1138)	-0.5708	(0.3769)	0.0583	(0.1201)
1993.Q1	0.0082	(0.1088)	-0.8362*	(0.3718)	0.3634*	(0.1023)
Q2	-0.0245	(0.1040)	-0.4753	(0.3231)	0.3939*	(0.0996)
Q3	-0.0374	(0.1054)	-0.2300	(0.3071)	0.3616*	(0.0992)
Q4	-0.0410	(0.1083)	-0.1047	(0.3089)	0.2966*	(0.0984)
1994.Q1	-0.0550	(0.1100)	-0.0297	(0.3014)	0.2817*	(0.1025)
Q2	-0.0337	(0.1145)	-0.2580	(0.3496)	0.3309*	(0.0968)
Q3	-0.0292	(0.1106)	-0.1093	(0.3300)	0.2273*	(0.0933)
Q4	-0.0279	(0.1071)	-0.0117	(0.3118)	0.1267	(0.0922)
1995.Q1	0.0038	(0.1093)	-0.2640	(0.3633)	0.2369*	(0.0821)
Q2	-0.0034	(0.1063)	-0.1072	(0.3395)	0.1621*	(0.0820)
Q3	-0.0108	(0.1049)	-0.0312	(0.3229)	0.1051	(0.0834)
Q4	-0.0168	(0.1037)	-0.0081	(0.3114)	0.0693	(0.0844)
1996.Q1	-0.0317	(0.1054)	-0.0189	(0.3049)	0.0817	(0.0902)
Q2	-0.0401	(0.1055)	-0.0145	(0.2971)	0.0724	(0.0932)
Q3	-0.0528	(0.1032)	0.0033	(0.2763)	0.0466	(0.0969)
Q4	-0.0618	(0.0957)	0.0051	(0.2430)	-0.0039	(0.0956)
1997.Q1	-0.0604	(0.0952)	0.0378	(0.2386)	-0.0078	(0.0962)
Q2	-0.0504	(0.0922)	0.0733	(0.2398)	-0.0309	(0.0924)
Q3	-0.0505	(0.0894)	0.0660	(0.2332)	-0.0363	(0.0901)
Q4	-0.0501	(0.0909)	0.0137	(0.2427)	-0.0185	(0.0885)
1998.Q1	-0.0327	(0.0964)	0.0460	(0.2677)	-0.0275	(0.0871)
Q2	-0.0270	(0.0971)	0.0825	(0.2709)	-0.0365	(0.0872)
Q3	-0.0334	(0.0996)	0.0651	(0.2773)	-0.0101	(0.0909)
Q4	-0.0599	(0.1047)	0.0412	(0.2704)	0.0838	(0.1011)
1999.Q1	-0.0574	(0.1064)	0.0155	(0.2818)	0.0998	(0.1002)
Q2	-0.0534	(0.1034)	0.0329	(0.2741)	0.0625	(0.0982)
Q3	-0.0536	(0.1011)	0.0463	(0.2648)	0.0370	(0.0976)
Q4	-0.0528	(0.1003)	0.0239	(0.2655)	0.0207	(0.0961)

*Elasticities that are statistically significant.

TABLE 5.10

MORISHIMA ELASTICITIES OF SUBSTITUTION x_1 / x_2

Obs	σ_{12}^{m}		σ_{21}^{m}	
1982.Q2	-0.0627	(0.2001)	-0.085	(0.2114)
Q3	-0.0356	(0.1316)	0.0173	(0.1359)
Q4	0.2625*	(0.1259)	0.2129*	(0.1177)
1983.Q1	0.2334*	(0.1395)	0.1514	(0.1336)
Q2	0.2663*	(0.1508)	0.2095	(0.1416)
Q3	0.2343	(0.1476)	0.1915	(0.1420)
Q4	0.2937*	(0.1449)	0.2424*	(0.1343)
1984.Q1	0.3795*	(0.1960)	0.3301*	(0.1684)
Q2	0.3387	(0.2188)	0.3268*	(0.1968)
Q3	0.1643	(0.4350)	0.2944	(0.4183)
Q4	0.3903	(0.4591)	0.2866	(0.3575)
1985.Q1	0.4778	(0.4688)	0.3738	(0.3652)
Q2	0.6474	(0.6488)	0.5101	(0.4936)
Q3	2.1043	(5.3404)	1.9276	(4.7804)
Q4	0.5459	(0.5749)	0.3388	(0.4572)
1986.Q1	-4.8131	(16.576)	-3.7684	(13.113)
Q2	-0.8192	(0.6639)	-0.8577	(0.7450)
Q3	-0.3044	(0.2220)	-0.3983*	(0.2236)
Q4	-0.1362	(0.1367)	-0.1553	(0.1270)
1987.Q1	0.1542	(0.1539)	0.2192	(0.1364)
Q2	0.2593	(0.1631)	0.3424*	(0.1493)
Q3	0.4196*	(0.2394)	0.5229*	(0.2230)
Q4	0.5605*	(0.3108)	0.6748*	(0.3063)
1988.Q1	0.5494*	(0.3023)	0.6085*	(0.2804)
Q2	0.3687	(0.2271)	0.4588*	(0.2055)
Q3	0.3373	(0.2115)	0.4216*	(0.1947)
Q4	0.2522	(0.1831)	0.3332*	(0.1651)
1989.Q1	0.1705	(0.1512)	0.2466*	(0.1346)
Q2	0.0423	(0.1231)	0.0406	(0.1041)
Q3	0.1446	(0.1668)	0.0394	(0.1443)
Q4	0.1639	(0.1889)	0.0365	(0.1614)
1990.Q1	0.1466	(0.1886)	-0.0347	(0.1569)
Q2	0.2015	(0.1977)	-0.1294	(0.1626)
Q3	0.1942	(0.1898)	-0.1194	(0.1581)
Q4	0.1580	(0.1861)	0.0292	(0.1587)
1991.Q1	0.1425	(0.1749)	0.0586	(0.1494)
Q2	0.0600	(0.1371)	0.1124	(0.1156)
Q3	0.0075	(0.1185)	0.0442	(0.0994)
Q4	-0.0553	(0.1027)	-0.0127	(0.0893)

TABLE 5.10

MORISHIMA ELASTICITIES OF SUBSTITUTION x_1 / x_2

(...CONT)

Obs	σ_{12}^m		σ_{21}^m	
1992.Q1	-0.0516	(0.1089)	0.0021	(0.0929)
Q2	-0.0675	(0.0964)	-0.0058	(0.0886)
Q3	-0.0825	(0.0959)	-0.0290	(0.0937)
Q4	-0.0584	(0.1264)	-0.0036	(0.1027)
1993.Q1	-0.0550	(0.1186)	0.0348	(0.1003)
Q2	-0.0549	(0.1079)	0.0037	(0.0969)
Q3	-0.0498	(0.1074)	-0.0117	(0.0978)
Q4	-0.0450	(0.1110)	-0.0198	(0.0996)
1994.Q1	-0.0534	(0.1111)	-0.0339	(0.1015)
Q2	-0.0500	(0.1209)	-0.0072	(0.1035)
Q3	-0.0344	(0.1164)	-0.0127	(0.1004)
Q4	-0.0270	(0.1120)	-0.0195	(0.0981)
1995.Q1	-0.0153	(0.1192)	0.0203	(0.0979)
Q2	-0.0101	(0.1142)	0.0074	(0.0960)
Q3	-0.0120	(0.1112)	-0.0041	(0.0957)
Q4	-0.0163	(0.1089)	-0.0122	(0.0954)
1996.Q1	-0.0310	(0.1098)	-0.0257	(0.0972)
Q2	-0.0388	(0.1090)	-0.0345	(0.0977)
Q3	-0.0504	(0.1053)	-0.0486	(0.0968)
Q4	-0.0600	(0.0964)	-0.0603	(0.0919)
1997.Q1	-0.0576	(0.0957)	-0.0589	(0.0913)
Q2	-0.0475	(0.0933)	-0.0499	(0.0888)
Q3	-0.0485	(0.0903)	-0.0502	(0.0870)
Q4	-0.0493	(0.0917)	-0.0497	(0.0892)
1998.Q1	-0.0313	(0.0980)	-0.0326	(0.0937)
Q2	-0.0249	(0.0989)	-0.0272	(0.0941)
Q3	-0.0302	(0.1025)	-0.0326	(0.0945)
Q4	-0.0551	(0.1055)	-0.0531	(0.0980)
1999.Q1	-0.0537	(0.1079)	-0.0495	(0.0989)
Q2	-0.0495	(0.1051)	-0.0482	(0.0967)
Q3	-0.0496	(0.1025)	-0.0499	(0.0951)
Q4	-0.0499	(0.1020)	-0.0501	(0.0948)

*Elasticities that are statistically significant.

TABLE 5.11

MORISHIMA ELASTICITIES OF SUBSTITUTION x_1 / x_3

Obs	σ_{13}^m		σ_{31}^m	
1982.Q2	0.2550	(0.1770)	0.6662*	(0.1486)
Q3	-0.4340*	(0.2363)	-0.8216*	(0.2532)
Q4	0.4072*	(0.2017)	0.0342	(0.1175)
1983.Q1	0.7406*	(0.2652)	0.5706*	(0.1026)
Q2	0.6703*	(0.2473)	0.6023*	(0.1026)
Q3	0.5680*	(0.2184)	0.5530*	(0.0932)
Q4	0.6454*	(0.2262)	0.5188*	(0.0786)
1984.Q1	0.6671*	(0.3181)	0.4747*	(0.0764)
Q2	0.4330	(0.3240)	0.4246*	(0.0866)
Q3	-0.5996	(0.6065)	0.1649	(0.1501)
Q4	0.8078	(0.7899)	0.3713*	(0.1360)
1985.Q1	0.8829	(0.8017)	0.4067*	(0.1339)
Q2	1.1072	(1.1068)	0.3912*	(0.1632)
Q3	2.5654	(6.6312)	1.2779	(2.0670)
Q4	1.5645	(1.1644)	0.8223*	(0.3576)
1986.Q1	-6.5540	(22.2419)	-0.7782	(3.0202)
Q2	-0.1723	(0.5028)	0.8022	(0.5722)
Q3	0.2817	(0.3861)	0.5106*	(0.2075)
Q4	0.0108	(0.1802)	0.1036	(0.0991)
1987.Q1	0.0038	(0.2133)	0.3744*	(0.0999)
Q2	-0.1438	(0.2281)	-0.1253	(0.1317)
Q3	0.0179	(0.2867)	0.1160	(0.1059)
Q4	0.0195	(0.3371)	-0.1366	(0.1627)
1988.Q1	0.4549	(0.3463)	0.8519*	(0.2318)
Q2	0.0744	(0.2660)	0.2378*	(0.0847)
Q3	-0.0523	(0.2496)	-0.1520	(0.1089)
Q4	-0.0840	(0.2202)	-0.0981	(0.0857)
1989.Q1	-0.1213	(0.1884)	-0.0484	(0.0722)
Q2	0.1122	(0.1749)	0.2821*	(0.0848)
Q3	0.5468*	(0.2551)	0.7262*	(0.1592)
Q4	0.5856*	(0.2881)	0.7487*	(0.1769)
1990.Q1	0.6252*	(0.3053)	0.6031*	(0.1796)
Q2	1.0120*	(0.3886)	0.7873*	(0.2877)
Q3	1.0053*	(0.3733)	0.8064*	(0.2706)
Q4	0.5786*	(0.2851)	0.7220*	(0.1725)
1991.Q1	0.4624*	(0.2563)	0.6548*	(0.1463)
Q2	-0.0354	(0.1811)	0.1969*	(0.0785)
Q3	-0.0564	(0.1639)	0.1086	(0.0783)
Q4	-0.1627	(0.1436)	-0.0578	(0.0790)

TABLE 5.11

MORISHIMA ELASTICITIES OF SUBSTITUTION x_1 / x_3

(...CONT)

Obs	σ_{13}^{m}		σ_{31}^{m}	
1992.Q1	-0.1851	(0.1510)	-0.0632	(0.0806)
Q2	-0.2235*	(0.1329)	-0.0223	(0.0773)
Q3	-0.2065	(0.1270)	0.0167	(0.0753)
Q4	-0.1988	(0.1734)	-0.1539*	(0.0873)
1993.Q1	-0.2645*	(0.1595)	-0.0240	(0.0768)
Q2	-0.1741	(0.1435)	0.1054*	(0.0605)
Q3	-0.1016	(0.1409)	0.1645*	(0.0519)
Q4	-0.0617	(0.1460)	0.1658*	(0.0460)
1994.Q1	-0.0466	(0.1443)	0.1776*	(0.0447)
Q2	-0.1046	(0.1597)	0.1449*	(0.0523)
Q3	-0.0537	(0.1546)	0.1247*	(0.0438)
Q4	-0.0231	(0.1495)	0.0858*	(0.0356)
1995.Q1	-0.0718	(0.1573)	0.0956*	(0.0485)
Q2	-0.0321	(0.1512)	0.0875*	(0.0420)
Q3	-0.0164	(0.1476)	0.0676*	(0.0363)
Q4	-0.0144	(0.1445)	0.0480	(0.0318)
1996.Q1	-0.0280	(0.1451)	0.0524*	(0.0316)
Q2	-0.0324	(0.1439)	0.0465	(0.0293)
Q3	-0.0355	(0.1374)	0.0332	(0.0242)
Q4	-0.0410	(0.1229)	-0.0011	(0.0136)
1997.Q1	-0.0293	(0.1211)	0.0067	(0.0147)
Q2	-0.0134	(0.1224)	0.0003	(0.0120)
Q3	-0.0164	(0.1190)	-0.0064	(0.0082)
Q4	-0.0323	(0.1218)	-0.0095	(0.0059)
1998.Q1	-0.0121	(0.1317)	-0.0083	(0.0089)
Q2	0.0014	(0.1335)	-0.0057	(0.0100)
Q3	-0.0058	(0.1376)	0.0109	(0.0183)
Q4	-0.0269	(0.1350)	0.0699*	(0.0281)
1999.Q1	-0.0339	(0.1398)	0.0726*	(0.0302)
Q2	-0.0262	(0.1368)	0.0532*	(0.0263)
Q3	-0.0222	(0.1331)	0.0399*	(0.0232)
Q4	-0.0292	(0.1332)	0.0217	(0.0205)

*Elasticities that are statistically significant.

TABLE 5.12

MORISHIMA ELASTICITIES OF SUBSTITUTION x_2 / x_3

Obs	σ_{23}^m		σ_{32}^m	
1982.Q2	0.3258*	(0.1730)	0.6434*	(0.1392)
Q3	-0.3700*	(0.1960)	-0.7680*	(0.2373)
Q4	-0.1601	(0.1803)	-0.0154	(0.1170)
1983.Q1	-0.0186	(0.1880)	0.4886*	(0.0836)
Q2	0.1415	(0.1521)	0.5455*	(0.0884)
Q3	0.1764	(0.1442)	0.5102*	(0.0830)
Q4	0.1158	(0.1363)	0.4675*	(0.0676)
1984.Q1	0.1377	(0.1479)	0.4253*	(0.0552)
Q2	0.3184*	(0.1557)	0.4127*	(0.0740)
Q3	1.0589*	(0.3798)	0.2950*	(0.1268)
Q4	-0.1500	(0.4829)	0.2676*	(0.1485)
1985.Q1	-0.1025	(0.4666)	0.3027*	(0.1405)
Q2	-0.2058	(0.6376)	0.2540	(0.1985)
Q3	0.6400	(0.8879)	1.1012	(1.5147)
Q4	-0.4035	(0.5863)	0.6151*	(0.2365)
1986.Q1	2.0074	(7.5262)	0.2665	(2.4903)
Q2	0.1168	(0.5625)	0.7637	(0.4964)
Q3	-0.1693	(0.2915)	0.4167*	(0.1641)
Q4	-0.0624	(0.1310)	0.0845	(0.0941)
1987.Q1	0.5898*	(0.1535)	0.4395*	(0.1023)
Q2	0.3610*	(0.1514)	-0.0422	(0.1212)
Q3	0.6210*	(0.1872)	0.2193*	(0.1017)
Q4	0.5187*	(0.2315)	-0.0223	(0.1405)
1988.Q1	1.0055*	(0.3075)	0.9110*	(0.2502)
Q2	0.6222*	(0.1551)	0.3279*	(0.0864)
Q3	0.3218*	(0.1343)	-0.0677	(0.0941)
Q4	0.3191*	(0.1098)	-0.0171	(0.0727)
1989.Q1	0.3195*	(0.0925)	0.0276	(0.0613)
Q2	0.2105*	(0.0901)	0.2804*	(0.0741)
Q3	0.2188*	(0.1285)	0.6209*	(0.1346)
Q4	0.1997	(0.1401)	0.6214*	(0.1461)
1990.Q1	-0.0567	(0.1483)	0.4219*	(0.1422)
Q2	-0.3541	(0.2214)	0.4564*	(0.2404)
Q3	-0.3183	(0.2090)	0.4928*	(0.2242)
Q4	0.1726	(0.1378)	0.5932*	(0.1417)
1991.Q1	0.2510*	(0.1236)	0.5709*	(0.1225)
Q2	0.3448*	(0.0841)	0.2494*	(0.0676)
Q3	0.2092*	(0.0835)	0.1453*	(0.0677)
Q4	0.0922	(0.0876)	-0.0152	(0.0698)

TABLE 5.12

MORISHIMA ELASTICITIES OF SUBSTITUTION x_2 / x_3

(...CONT)

Obs	σ_{23}^m		σ_{32}^m	
1992.Q1	0.1240	(0.0865)	-0.0095	(0.0699)
Q2	0.1954*	(0.0875)	0.0394	(0.0687)
Q3	0.1942*	(0.0944)	0.0702	(0.0671)
Q4	0.0413	(0.0901)	-0.0991	(0.0711)
1993.Q1	0.2753*	(0.0788)	0.0658	(0.0623)
Q2	0.2832*	(0.0766)	0.1640*	(0.0492)
Q3	0.2543*	(0.0747)	0.2025*	(0.0408)
Q4	0.2078*	(0.0706)	0.1910*	(0.0340)
1994.Q1	0.1903*	(0.0750)	0.1971*	(0.0330)
Q2	0.2423*	(0.0711)	0.1877*	(0.0360)
Q3	0.1656*	(0.0641)	0.1463*	(0.0295)
Q4	0.0895	(0.0600)	0.0934*	(0.0239)
1995.Q1	0.1877*	(0.0569)	0.1312*	(0.0306)
Q2	0.1270*	(0.0555)	0.1050*	(0.0270)
Q3	0.0799	(0.0553)	0.0755*	(0.0234)
Q4	0.0501	(0.0555)	0.0521*	(0.0204)
1996.Q1	0.0546	(0.0599)	0.0577*	(0.0203)
Q2	0.0444	(0.0613)	0.0508*	(0.0189)
Q3	0.0201	(0.0632)	0.0350*	(0.0159)
Q4	-0.0204	(0.0614)	-0.0013	(0.0091)
1997.Q1	-0.0229	(0.0624)	0.0053	(0.0100)
Q2	-0.0363	(0.0569)	-0.0022	(0.0081)
Q3	-0.0402	(0.0546)	-0.0082	(0.0055)
Q4	-0.0270	(0.0532)	-0.0099*	(0.0037)
1998.Q1	-0.0288	(0.0520)	-0.0096*	(0.0055)
Q2	-0.0342	(0.0517)	-0.0079	(0.0064)
Q3	-0.0159	(0.0552)	0.0085	(0.0120)
Q4	0.0437	(0.0688)	0.0719*	(0.0193)
1999.Q1	0.0571	(0.0681)	0.0769*	(0.0204)
Q2	0.0311	(0.0648)	0.0545*	(0.0178)
Q3	0.0122	(0.0636)	0.0395*	(0.0157)
Q4	0.0009	(0.0616)	0.0216	(0.0136)

*Elasticities that are statistically significant.

TABLE 5.13

ALLEN ELASTICITIES AT THE MEAN

Asset	σ_{i1}^{a}	σ_{i2}^{a}	σ_{i3}^{a}
x_1	-0.012777 (0.23322)	0.025227 (0.084889)	-0.24858 (0.2792)
x_2		-0.034454 (0.036102)	0.30061* (0.093578)
x_3			-2.48036* (0.77808)

*Elasticities that are statistically significant.

TABLE 5.14

MORISHIMA ELASTICITIES OF SUBSTITUTION AT THE MEAN

Asset	σ_{i1}^{m}	σ_{i2}^{m}	σ_{i3}^{m}
x_1		0.010337 (0.086681)	-0.064139 (0.12046)
x_2	0.040202 (0.080169)		0.2257* (0.069265)
x_3	0.12136* (0.053507)	0.15123* (0.045804)	

*Elasticities that are statistically significant.

FIGURE 1

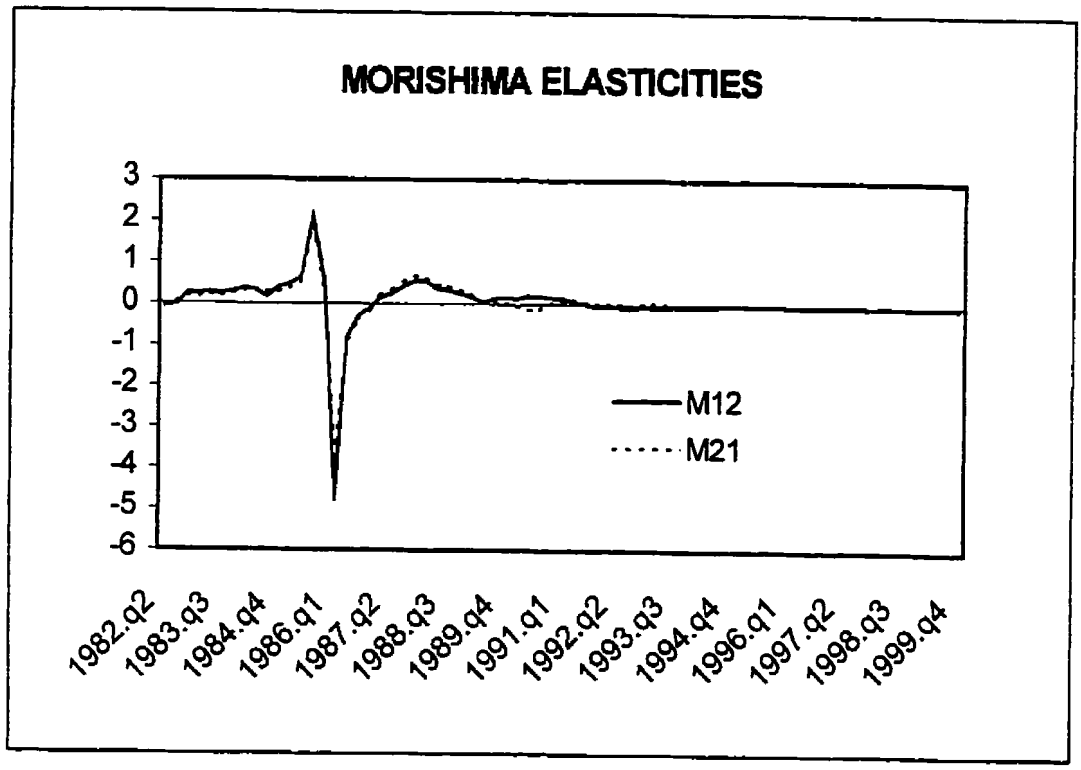

MORISHIMA ELASTICITIES
3
2
1
0
-1
-2
-3
-4
-5
-6
M12
M21
1982.q2
1983.q3
1984.q4
1986.q1
1987.q2
1988.q3
1989.q4
1991.q1
1992.q2
1993.q3
1994.q4
1996.q1
1997.q2
1998.q3
1999.q4

FIGURE 2

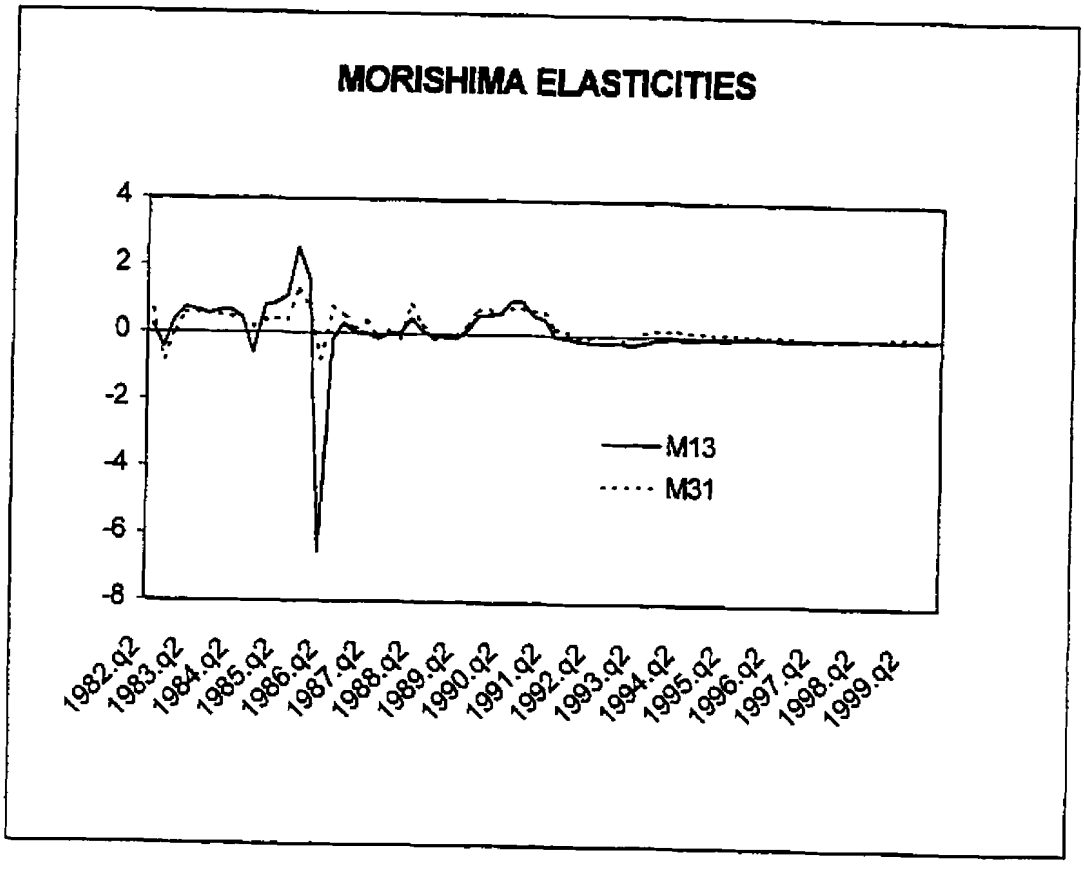

MORISHIMA ELASTICITIES
4
2
0
-2
-4
-6
-8
M13
M31
1982.q2
1983.q2
1984.q2
1985.q2
1986.q2
1987.q2
1988.q2
1989.q2
1990.q2
1991.q2
1992.q2
1993.q2
1994.q2
1995.q2
1996.q2
1997.q2
1998.q2
1999.q2

FIGURE 3

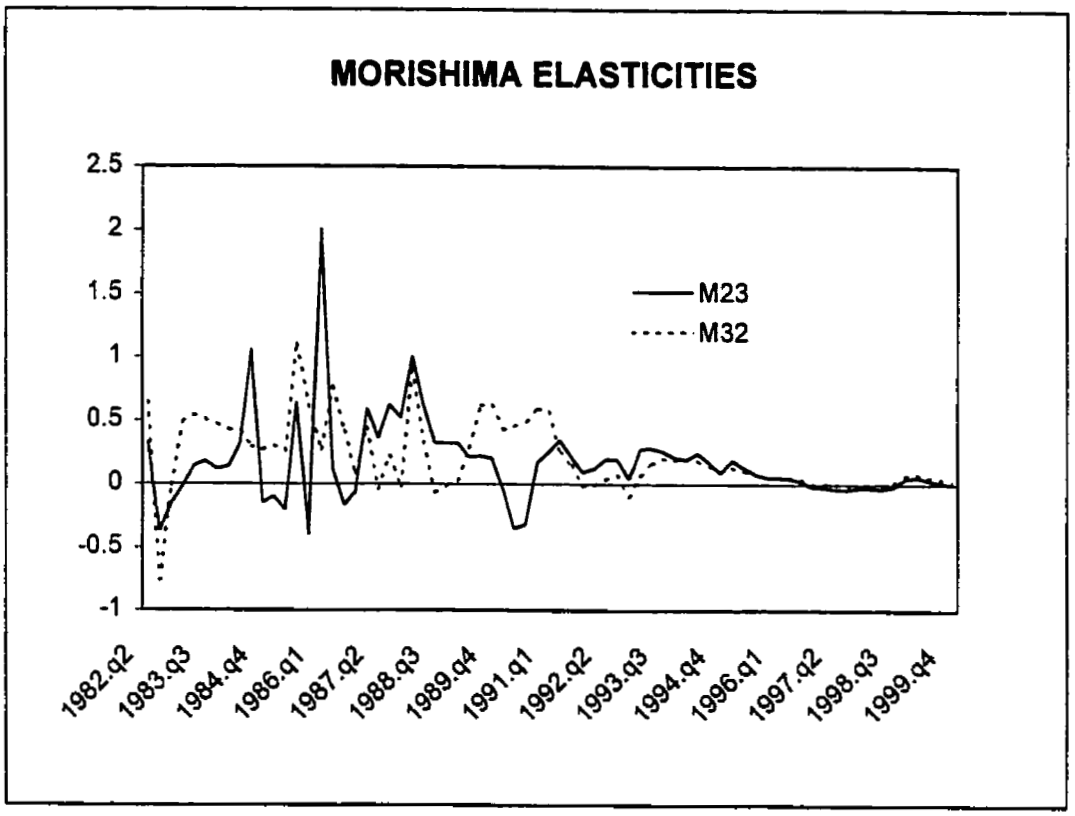

MORISHIMA ELASTICITIES
2.5
2
1.5
1
0.5
0
-0.5
-1
M23
M32
1982.q2
1983.q3
1984.q4
1986.q1
1987.q2
1988.q3
1989.q4
1991.q1
1992.q2
1993.q3
1994.q4
1996.q1
1997.q2
1998.q3
1999.q4

FIGURE 4

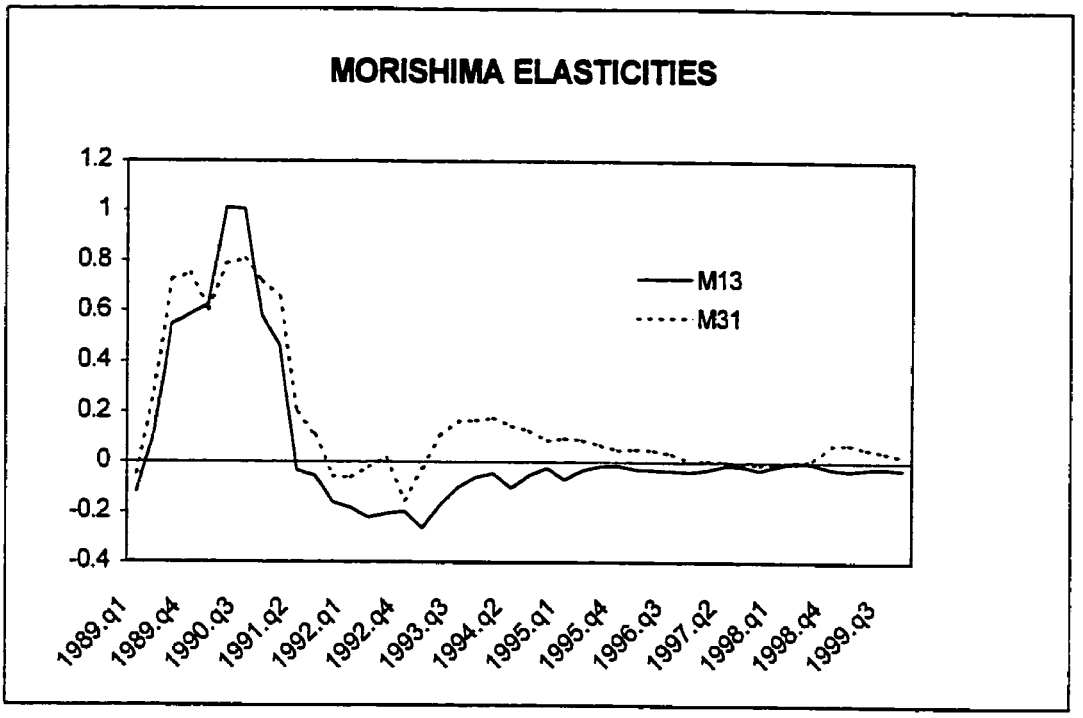
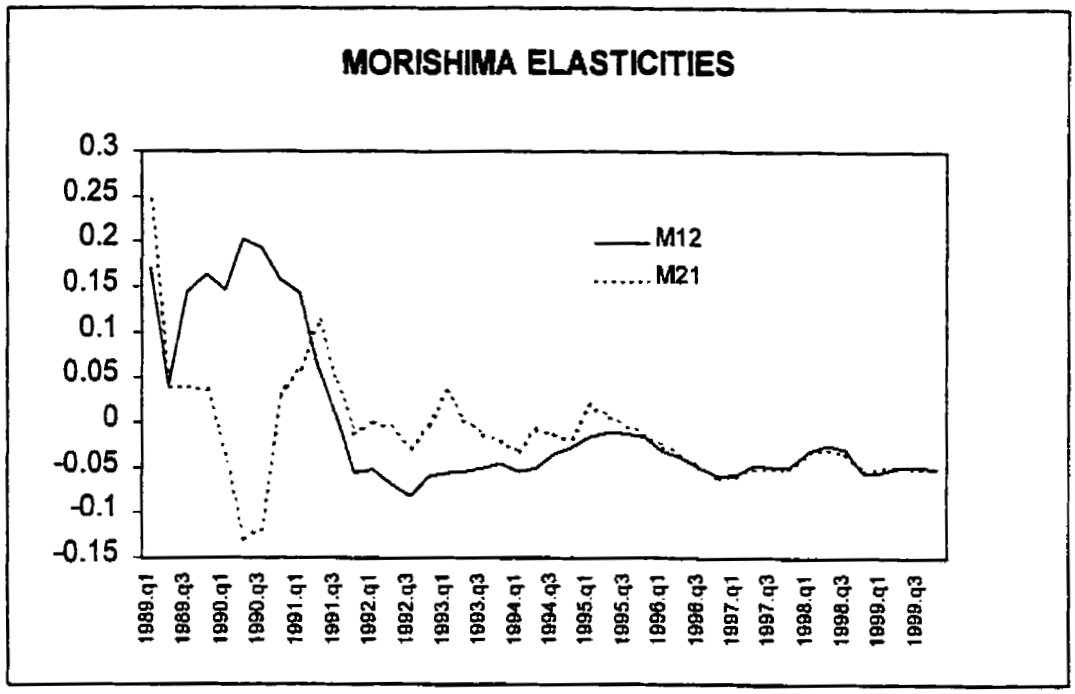

MORISHIMA ELASTICITIES
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
M13
M31
1989.q1
1989.q4
1990.q3
1991.q2
1992.q1
1992.q4
1993.q3
1994.q2
1995.q1
1995.q4
1996.q3
1997.q2
1998.q1
1998.q4
1999.q3

FIGURE 5


MORISHIMA ELASTICITIES
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
M12
M21
1989.q1
1989.q3
1990.q1
1990.q3
1991.q1
1991.q3
1992.q1
1992.q3
1993.q1
1993.q3
1994.q1
1994.q3
1995.q1
1995.q3
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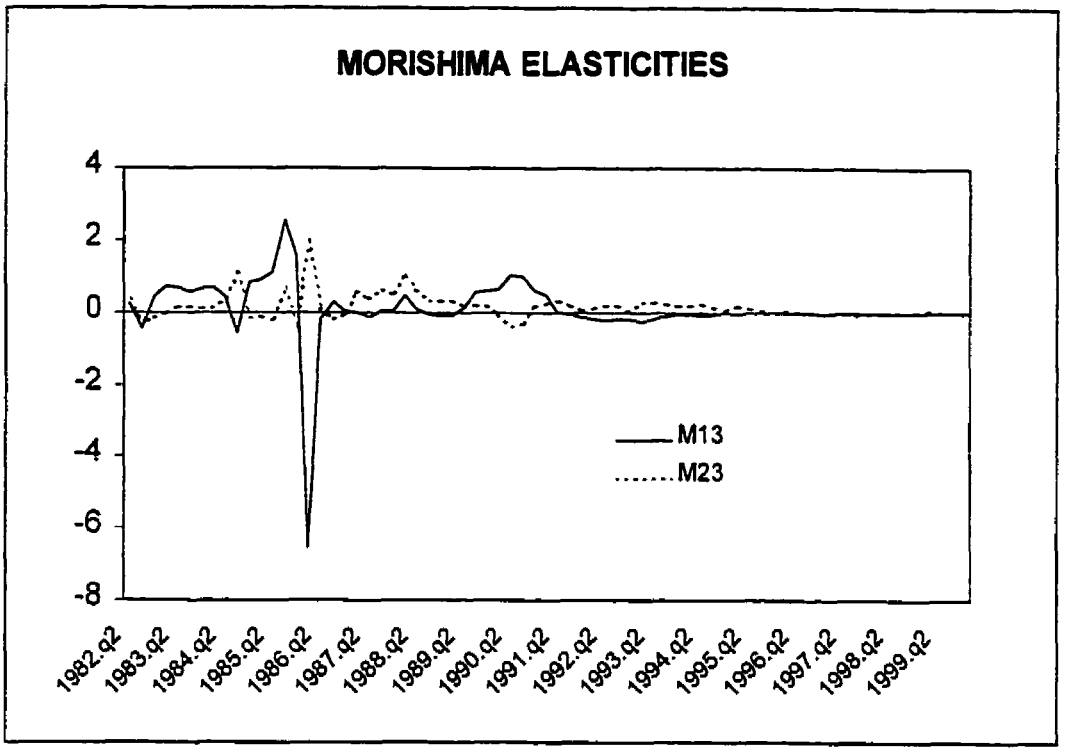
1998.q3
1999.q1
1999.q3

FIGURE 6


MORISHIMA ELASTICITIES
4
2
0
-2
-4
-6
-8
M13
M23
1982.q2
1983.q2
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1985.q2
1986.q2
1987.q2
1988.q2
1989.q2
1990.q2
1991.q2
1992.q2
1993.q2
1994.q2
1995.q2
1996.q2
1997.q2
1998.q2
1999.q2

Chapter 6

CONCLUSIONS

We have analyzed Canadian monetary data with the use of recent developments in microeconometrics. In doing so, we took advantage of important microfoundations like the theory of constrained utility maximization. Through out the analysis we used many other paradigms like the theory of separable structures, aggregation theory and index number theory just to mention some. Important is the fact that we put together two approaches to the demand for money that were at some point different strands of research. In fact, in the literature, few studies perform nonparametric separability tests to then estimate elasticities among financial assets. In most cases demand systems are estimated under the assumption of an ad hoc weakly separable structure.

In this study we performed Varian's (1983) revealed preference weak separability tests to uncover the components of optimal monetary aggregates. We found a collection of assets that does not necessarily match the set employed by the Bank of Canada. On the other hand, we argued that a broad monetary aggregate like M2 will perform better in macroeconomic models than narrower definitions. We also found that personal monetary assets are weakly separable from non-personal monetary assets and vice-versa. In this regard, we saw that it is possible to have a broader monetary aggregate for the personal sector (that includes term deposits) than for the non-personal sector.

We found other potentially interesting narrow definitions of money like an aggregate composed of personal and non-personal demand deposits. Offenbacher (1979) and Ewis and Fisher (1985) have already given evidence of lower substitutability

between currency and demand deposits than between currency and time or savings deposits. All this is evidence against using narrow definitions of money like the Bank of Canada's M1. We think that only further research on the relation between demand deposits and other assets will give a better understanding about the proper place that these components have in monetary aggregation. The final result of our nonparametric exercise was the grouping of monetary components in a personal subaggregate, a non-personal subaggregate and a foreign currency deposit asset. These three assets were subsequently analyzed in a parametric setting to study the relationships among them.

Like any empirical test in economics, the nonparametric tests were not without their problems. The most important one was not being able to find a weakly separable structure across our full sample period. One reason was that the monetary sample had many GARP violations so that we had to break our sample in subperiods. There are many possible explanations for GARP violations starting with the problem that we did not include other important goods in the objective function. Goods like consumption and leisure were not included in the GARP and separability tests. Another explanation might be that the representative consumer's utility functional form or parameters are changing over time or that the representative consumer paradigm does not hold.

In our parametric investigation we considered the estimation of a globally flexible functional form capable of producing arbitrary estimates of elasticities of substitution at any point. An intermediate step was the construction of three monetary subaggregates capable of keeping economic properties. We employed an "ideal" index number, the Divisia, to perform the aggregation over the components identified in the nonparametric stage.

A static Fourier demand system was initially estimated but a model that accommodated some dynamics was later adopted because of its better performance. Unfortunately, the Fourier model does not, in this case, satisfy the regularity conditions at every data point. However, there were very few statistical significant violations of the necessary curvature condition which led us to believe that model failure was not a big problem.

We then computed price and income elasticities. We concluded that on average non-personal assets are inferior goods, personal assets are normal goods and foreign currency deposits are luxury goods. The cross-price elasticities indicate that all assets are gross complements. To study the net effects we computed Allen and Morishima elasticities of substitution. We found that personal and non-personal assets hardly respond to changes in their own user costs. In contrast, foreign currency deposits are quite price elastic.

The analysis of Morishima elasticities of substitution demonstrates that there is hardly any relationship between personal and non-personal assets, in other words, they are not substitutes but they are not complements either. This is evidence against any attempt of putting these two kinds of assets together. It implies that there are two different demands for money in the economy, one for each sector. It might be that the personal monetary assets perform differently compared to the non-personal asset in macroeconomic models. But what are the implications of this for monetary policy? The Bank of Canada should probably expect different responses in the demand for money from each sector when it considers a policy change.

Personal and non-personal assets are substitutes for foreign currency deposits but the degree of substitutability is low. The implications of this (that we can think of) have to do with the demand for foreign currency and possibly even for a monetary union. Are foreign currency deposits (U.S. dollars) a substitute for Canadian dollars according to the average consumer? The answer is yes, but they are far from being perfect substitutes. In general, we can say that "moneyness" of foreign currency is not high. People and firms are using the U.S. dollar as a means of storing value or as a hedge to cover future transactions. On the other hand, we believe the information provided by the elasticities of substitution that involves foreign currency deposits is useful to the Bank of Canada when designing monetary policy. When changes occur in relative prices people and firms adjust their optimum levels of national and foreign currencies. The elasticities of substitution can give decision-makers an idea of the extent of the adjustments that ultimately have a bearing on the exchange rate.

In general, all of our estimated elasticities show greater variability in the eighties than in the nineties. We expect to have greater instability in both user costs and elasticities when the country experiences periods of recession and when inflation is higher. The result of variable elasticities over time is consistent with other studies that use flexible functional forms to approximate the aggregator function.

Our conclusions regarding an appropriate economic monetary aggregate do not significantly differ from conclusions reached by authors that have used the elasticity of substitution approach. Variable elasticities, low substitutability among financial assets (and sometimes complementarity) are some common outcomes. Examples in the monetary literature are found in Offenbacher (1979), Ewis and Fisher (1984 and 1985),

Serletis (1988), Fisher (1989), Fisher (1992), Fisher and Fleissig (1994), Fleissig and Swofford (1996), Fleissig and Serletis (2001) and many others.[1] As Fisher (1992) puts it regarding the implications of the evidence:

"One direct implication is that monetary policy may well be improved significantly if money demand is modeled in a way that permits internalization of this variation, as with a system of demand equations featuring variable elasticities of substitution over time." (p.150)

As striking as results are in the recent monetary literature, striking is too that they have provided very few links to monetary policy implementation. So far no central bank follows the proposed approach. One reason is that these studies are difficult to reproduce, particularly in the case of global flexible functional forms where there is no well established procedure to determine the number of terms or order of the approximation.

The implications of our results concerning monetary policy are several. In the past, the Bank of Canada has used the official monetary aggregates as policy targets. Unfortunately the monetary target will not be stable over a long period of time. According to our findings the popular monetary aggregates have the wrong components and they do not account for variable elasticities of substitution. At the very best the official monetary aggregates are temporary.

On the other hand, the elasticities of substitution are indicative of the effects and extent of monetary policy changes. When influencing changes in the relative usercosts,

[1] The most important difference between these studies and ours is the elasticities of substitution being calculated among different monetary sets. A common approach in the literature is to calculate elasticities of substitution between currency, demand deposits, savings deposits and/or small time deposits held by both personal and non-personal sectors.

the Bank of Canada could assess the redistribution among financial assets that will take place. The elasticities of substitution indicate the degree of "moneyness" of the assets under study. By studying the elasticities of substitution the Bank of Canada may resolve the "missing money" puzzle. We should keep in mind that the demand for money is something the consumer determines not the monetary authority or the researcher. Once we are able to understand better the demand for money we will be able to better predict and understand the effect that monetary policy has on the economy.

A final comment concerns suggestions for future research. The issues discussed in this study were many that it may not be difficult for the reader to uncover numerous opportunities. We suggest further separability tests with Canadian monetary data. The studies in the literature are mainly with U.S. data. The fact that the composition and definition of monetary assets is institutionally different calls for further analysis in the Canadian context. In particular a study including consumption goods, leisure and money would be interesting in determining weak separability of a group of financial assets from the rest of the goods. Results could be contrasted with this study.

In our case we estimated a Fourier demand system. It would be interesting to use other functional form specifications with the same data as the one used here. For example, results from a Translog model or an Almost Ideal Model could be compared to the ones provided here. Models could be compared in terms of conforming to the regularity conditions to see which one performs better. We consider this particular line of study of extreme importance because only models that satisfy the economic theory are useful for policy purposes.

A way of extending our analysis is to estimate a dynamic Fourier demand system in the spirit of Fisher and Fleissig's (1994) dynamic specifications. Dynamics modeled in that fashion may produce a better model in terms of the assumptions regarding the error terms. Again, the estimated elasticities of substitution could be compared to ours.

Another potentially productive area of future research would be the investigation of the substitutability/complementarity relationship in the context of consumption goods and personal monetary assets, without including any business asset. This would ensure that we are exclusively in the representative consumer's context. The first step would be to test for weak separability to find the groups that satisfy the condition. Having these in hand, we would estimate a demand system using a flexible functional form to calculate the elasticities of substitution. Certainly it would be informative to know the relationship between monetary assets and other goods. Further, the investigation of the relationship between personal financial assets particularly among demand deposits, chequable savings deposits, non-chequable savings deposits and term deposits in the Canadian context would be productive.

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