A study of a nonwelded contact interface: exact and approximate formulas for P-SV reflection and transmission coefficients, and frequency domain raytracing

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A study of a nonwelded contact interface: exact and approximate formulas for P-SV reflection and transmission coefficients, and frequency domain raytracing

by

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ABSTRACT

The sixteen exact mathematical formulas for the P-SV particle displacement reflection and transmission coefficients for elastic plane waves incident upon a nonwelded contact interface separating two solid half-spaces, and approximations to these formulas for small amounts of nonwelded contact, small incidence angles, etc., are presented. The nonwelded contact at the interface is represented by displacement discontinuity boundary conditions. The characteristics of these conditions are that the traction is continuous across the interface but the displacement is not. The displacement discontinuity components are proportional to the corresponding stress components -- the constants of proportionality are called specific compliances. The exact formulas for the coefficients are obtained by algebraically solving the displacement discontinuity boundary conditions for each possible incident wave in the P-SV case. The formulas are expressed in the form of the coefficients for the welded contact case plus additional terms due to the presence of nonwelded contact. The formulas show that the existence of nonwelded contact makes the coefficients frequency-dependent.

For the specific case in which the media above and below the interface are the same (e.g., a fault, joint or fracture in a homogeneous medium), reflected waves exist, and there are phase changes in the transmitted wave, unlike the case for a perfect welded contact interface between two identical media, in which there are no reflected waves due to the lack of an impedance contrast. Numerical computations of the coefficients also confirm, in this case, that the nonwelded contact interface acts like a low-pass filter (Pyrak-Nolte et al., 1990b), i.e., more of the incident energy is transmitted through the interface at lower frequencies than at higher ones.
Ray-synthetic seismograms are also computed for media containing nonwelded interfaces. Due to the frequency dependence of the coefficients, the synthetic seismograms are generated by frequency domain raytracing to include the full effect of nonwelded contact for all considered frequency ranges. The generated seismograms show the change in both amplitude and phase over a nonwelded contact interface. Even for small values of the specific compliances, the phase change is significant.
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DEDICATION

To My Father and My Twin Sisters
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CHAPTER I

INTRODUCTION

1.1. Introduction

The problem of the reflection and transmission of plane elastic waves at an interface between rock layers which are not in perfect "welded contact" with each other has been addressed in the geophysical literature by several investigators. For example, Schoenberg (1980) and Pyrak-Nolte et al. (1990b) used a macroscopic approach wherein the solids are modeled as continuous media, whereas Paranjape et al. (1987) and Krebes (1987) used a microscopic approach wherein the solids are modeled as mass-spring lattices. In the macroscopic approach, displacement discontinuity boundary conditions (described below) are used to derive the reflection and transmission coefficients. In the microscopic approach, the two solids, modeled by mass-spring networks, are also connected with springs across the interface, and Newton's second law of motion \((F=ma)\) is used to describe the motion of the masses and derive the reflection and transmission coefficients. In both theories, a system of linear equations can be derived whose solutions are the reflection and transmission coefficients (like the Zoeppritz equations in the welded contact case). The coefficients obtained from both theories are identical if the respective material parameters of the two theories are related in a specific way (see Appendix I). However, the microscopic theory of the above-mentioned authors is restricted to media with specific values for the wave velocities (or Poisson's ratio).
Consequently, the study of nonwelded contact in this thesis is based on the macroscopic theory, which is more general than the microscopic theory.

For a nonwelded contact interface, Schoenberg (1980) and Pyrak-Nolte et al. (1990b) used displacement discontinuity boundary conditions (in the macroscopic approach) which state that the traction is continuous across the interface but the displacement is not (the displacement discontinuity is proportional to the traction). The displacement discontinuity for the normal component of particle displacement is equal to the normal stress component divided by the specific stiffness in the normal direction, with a similar definition holding for the tangential component of displacement. In the P SV wave case (the case of compressional and shear-vertical waves propagating in a plane), the displacement discontinuity boundary conditions result in four linear equations for the four unknown reflection and transmission coefficients, which can, of course, be solved numerically. However, even though the boundary conditions are more complicated than in the conventional welded contact case, algebraic solutions can also be derived for each possible incident wave and compared with those in the welded contact case. The algebraic solutions of the displacement discontinuity boundary conditions consist of sixteen exact formulas for the reflection and transmission coefficients for the four possible incident waves (P or SV waves incident from either above or below the interface), just as in the welded contact case (e.g., Aki and Richards, 1980, eq. 5.39). To the best of my knowledge, these exact algebraic formulas for the P-SV coefficients for the nonwelded contact case have never appeared before in the literature (although they have for the SH wave case -- see Schoenberg, 1980). They are presented in this thesis and have also been recently published (Chaisri and Krebes, 2000). These formulas generally consist of a term which is similar in form to the coefficient for the welded contact case (except for some modifications due to nonwelded contact) plus a series of imaginary terms due purely to nonwelded contact. Also, the coefficients depend on frequency, unlike those for welded contact.
The sixteen exact formulas for the coefficients in welded contact case (e.g., *Aki and Richards*, 1980, eq. 5.39) are very well known and are very useful in the study of elastic wave propagation through layered media. In the same way, although the boundary conditions in the case of nonwelded contact can be solved numerically for the reflection and transmission coefficients, closed-form analytical formulas for these coefficients are still very useful. For example, the exact formulas for the coefficients give the specific quantitative relationships between the coefficients and the parameters of the media, which cannot be obtained from the numerical solutions.

The exact formulas for the coefficients are also essential for the derivations of various approximate formulas. In this thesis, approximate formulas for small displacement discontinuities, i.e., a low degree of nonwelded contact, are derived from the exact formulas of the coefficients, as well as approximations for small incidence angles. In the welded contact case, approximate formulas for specific purposes have been developed and found to be very useful. For example, approximations of the coefficients for small differences in medium parameters across the interface (*Aki and Richards*, 1980, and *Bortfeld*, 1961) have been applied to the study of waves in vertically inhomogeneous media (*Chapman*, 1976) and elastic wave scattering (*Richards and Frasier*, 1976). Approximate formulas have also been developed, in the welded contact case, for small incidence angles and have been proven useful in AVO (amplitude versus offset) studies in exploration seismology (e.g., *Castagna and Backus*, 1993). The exact formulas for the coefficients presented in this thesis could be used to develop similar approximations in the case of nonwelded contact, which could be applied to media containing nonwelded interfaces such as cracks, joints and fractures. In computer programs for generating the ray-synthetic seismograms, the use of the closed-form reflection and transmission coefficient formulas makes the programs run faster than if the coefficients were obtained from the numerical solutions of the boundary conditions. In addition, numerical difficulties that may arise in the numerical solution of the linear equations are avoided.
The effects of nonwelded contact on elastic wave propagation have been detected in experimental work, for example, the decrease in amplitude and the phase shift for waves transmitted across fractures (Kleinberg et al., 1984), and the presence of anomalously high amplitudes for waves reflected from a single fracture (Yu and Telford, 1973). Previous studies of elastic wave propagation in fractured media, e.g., Coates and Schoenberg (1995), Frazer (1990), Hudson (1981), and Slawinski (1999), have used equivalent medium theory, in which effective elastic moduli are calculated and used to represent the fractured medium. Equivalent medium theory, however, may not perform very well in representing media in which the fractures are relatively large and sparsely spaced with spacing of the order of, or larger than, a seismic wavelength (Pyrak-Nolte et al., 1990b). If a large scale fracture can be considered to be nonwelded contact interface, then it would be more appropriate to use the exact formulas presented in this thesis in describing the effects of the fractures on elastic wave propagation. Pyrak-Nolte et al. (1990a) also used the displacement discontinuity model theory (the use of displacement discontinuity boundary conditions for wave propagation across a fracture) to analyze the anisotropy in seismic velocities and amplitudes for a purely elastic medium composed of multiple parallel fractures. They compared the calculation from both theories, the conventional equivalent medium theory and the displacement discontinuity model theory, to their physical experiments, and found that the displacement discontinuity model theory appeared to be able to represent the fractured medium better than the equivalent medium theory did.

There are a number of geological problems in which the reflection and transmission of elastic waves at a nonwelded interface could play a role. One example is the study of fractured hydrocarbon reservoirs using seismic methods, as fractures can often be modeled as nonwelded contact interfaces. A second example involves the study of seismic waves in the upper crust (the upper few kilometers): joints, fractures and faults are important features of the crust, and these features may behave more like nonwelded contact interfaces than welded contact ones in some cases, owing to the existence of relatively low effective stress levels in the upper crust (Pyrak-Nolte and Cook, 1987a).
1.2. A review of some previous work

The basic idea in the theory of elastic waves propagating across a perfectly welded interface between two solids is that the traction and displacement must be continuous across the interface. For a nonwelded interface, the modification of this basic idea, namely, that the displacement across the interface does not need to be continuous, is significant, in that it makes the study of wave propagation across an interface more general. One can say that, for a perfectly welded interface, the boundary conditions that the waves must satisfy at the interface are a special case of those for a nonwelded contact interface. Michael Schoenberg has presented a theory for nonwelded contact boundary conditions between two elastic media (Schoenberg, 1980), which were later on called the displacement discontinuity boundary conditions by Pyrak-Nolte et al. (1990b). The definition of the displacement discontinuity boundary conditions is that the traction is continuous across the interface but the displacement is not. The ratio between the traction and the discontinuity in particle displacement is called the specific stiffness of a nonwelded interface: \( \kappa_z \) and \( \kappa_t \) are the specific stiffnesses, respectively, normal and parallel to the interface. The reciprocals of the specific stiffnesses are called the specific compliances, i.e., \( c_z = 1/\kappa_z \) and \( c_x = 1/\kappa_t \).

To understand the physical mechanism in operation at a nonwelded contact interface, imagine that we have an isotropic layer situated between two homogeneous isotropic half-spaces and assume that the two interfaces are in perfect welded contact -- see Figure 1.1a. Consider the thickness \( h \) of the thin layer to be very small compared to a wavelength, and its impedance, \( Z' = \text{density } \rho' \times \text{velocity } v' \), to be very low compared to that of the uppermost medium. Considering SH waves, and applying the displacement and stress boundary conditions to the propagating waves at the two welded interfaces results in formulas for the reflection and transmission coefficients \( R' \) and \( T' \) (see Schoenberg, 1980).
As the thickness \( h \) and impedance \( Z' \) of the thin layer approach zero, \( R' \) and \( T' \) approach the reflection and transmission coefficients \( R \) and \( T \), respectively, where \( R \) and \( T \) are calculated from the displacement discontinuity boundary conditions (Figure 1.1b) with the parallel specific compliance \( c_y \) replaced by \( h/\mu' \) (Schoenberg, 1980), where \( \mu' \) is the shear modulus in the layer. In the case of normally incident P-SV waves, analogous results are obtained, i.e., \( R' \) and \( T' \) reduce to those obtained for a nonwelded interface, with the normal specific compliance \( c_z \) replaced by \( h/(\lambda' + 2\mu') \) and the parallel specific compliance \( c_x \) replaced by \( h/\mu' \), where \( \lambda' \) and \( \mu' \) are Lamé's constants (Schoenberg, 1980).

Other aspects of the effects of a nonwelded interface have been discussed in the literature. The dispersion curves for Love waves have been discussed by Schoenberg (1980). The existence of elastic interface waves along a nonwelded contact interface
between two identical media has been demonstrated by Pyrak-Nolte et al. (1987a): by applying the displacement discontinuity boundary conditions to a generalized Rayleigh wave when the seismic impedances of the media on both sides of the interface are the same, a fast and a slow weakly dispersive wave can be shown to exist along such an interface.

The displacement discontinuity theory has been confirmed by the laboratory measurements of Pyrak-Nolte et al. (1990b). In the experiment, the transmission amplitude and traveltime of a wave propagating across an intact specimen and a specimen containing a single natural fracture are measured. The specimens were compressed under an axial load normal to the fracture surface to produce an axial effective stress. The nonwelded specific stiffnesses are, in general, functions of the stress across the fracture. Therefore changing the axial effective stress across the fractured specimen gives different values of the specific stiffnesses: increasing the stress on the fracture increases the specific stiffnesses of the fracture. The effects of a nonwelded contact interface on the wave signal, observed in the experimental data, are that the transmission amplitude and the traveltime across a fracture are dependent on the frequency of the signal and on the ratio of the specific stiffness of the fracture to the seismic impedance of the rock specimen. The displacement discontinuity theory of a wave propagating across a fracture is able to match the measured data and predicts the frequency dependence of the group time delay and the amplitude as well.

1.3. Physical assumptions

The results of any study of a physical phenomenon depend on the physical assumptions made in the study. The reflection and transmission coefficients computed in this thesis are those for plane waves, and the synthetic seismograms are computed by summing plane waves of different frequencies, modified by the reflection and transmission coefficients, and geometrical spreading factors. One must remember that plane waves, being of infinite extent, do not actually exist in nature. They are a
convenient way of representing wave propagation processes mathematically, and lead to
good approximate results, but it should be kept in mind that the wavefronts propagating
in the Earth are curved in reality, and that in some situations, it is necessary to correct for
or account for this curvature to explain various features present in seismic data.

Another assumption involved in the work of this thesis is associated with the
computation of the synthetic seismograms in chapter V. Ray theory is used to compute
these seismograms, meaning that the results are valid only for high enough frequencies,
i.e., for wavelengths smaller than the distances over which the medium parameters
change substantially. If a layer is sufficiently heterogeneous so that this assumption is not
valid, then modifications to the ray computations would be necessary to produce accurate
synthetics.

Another thought that must be kept in mind is that it may be that not all fractures,
joints and cracks can be accurately modelled by the displacement discontinuity boundary
conditions used in this thesis, in spite of the experimental evidence, discussed in the
previous section, confirming these boundary conditions. In some cases, there may be
other physical effects contributing to the nonwelded nature of such interfaces. In addition,
there may be other types of nonwelded interfaces that affect seismic waves in a
substantially different way -- a completely different set of boundary conditions may be
required in these cases.

1.4. Dissertation outline

The algebraic solutions of the displacement discontinuity boundary conditions,
i.e., the reflection and transmission coefficients, are presented in Chapter II, Section 2.1,
equations 2.13 (or 2.24). The derivation involved a great deal of tedious but
straightforward mathematics which is not presented. The variables used in the formulas
were defined in the same way as in Aki and Richards (1980), and in addition, some new
variables due to the presence of nonwelded contact were introduced. These exact
formulas for the coefficients are significantly different from those for perfect welded contact obtained from the Zoeppritz equations.

*Pyrak-Nolte et al* (1990b) have shown that energy is conserved for P and SV waves normally incident upon a nonwelded interface between two identical materials (e.g., a fault or joint in a single homogeneous medium). In Section 2.2, this result is extended to the general case -- it is shown that energy is also conserved for obliquely incident waves at a nonwelded interface separating different materials.

In practice, the degree of nonweldedness of an interface in the subsurface of the earth is likely to be small in typical situations. Consequently, approximate formulas for reflection and transmission coefficients for small specific compliances, the reciprocals of the specific stiffnesses, would be useful. The approximation of the sixteen exact formulas for the reflection and transmission coefficients for this purpose is obtained in Section 2.3 by using the Taylor series approximations of the exact formulas in Section 2.1.

Chapter III discusses the specific case in which the elastic properties of the media above and below the nonwelded interface are identical, as would be the case for a crack, joint, fault or fracture in a single medium. In this Chapter, the exact formulas for the reflection and transmission coefficients in this case, and some applications, are presented. For example, the exact formulas are used to develop approximate formulas for a weakly nonwelded interface, and these results are, in turn, used to develop further approximations for small angles of incidence (which could be useful in AVO studies).

Chapter IV presents numerical examples of the behaviour of the exact and approximate formulas for the coefficients. More specifically, the amplitude and phase of the coefficients are plotted against incidence angle for different values of the specific compliances (different amounts of nonwelded contact). Also, the accuracy of the approximate formulas are examined by comparing them with the exact formulas.

The amplitudes and phases of the scattered waves at a given incidence angle are frequency-dependent even though the seismic waves are traveling in an elastic
homogeneous medium. Consequently, to trace a ray across nonwelded interfaces, and calculate its amplitude at the receiver, it is necessary to account for this frequency-dependence in the calculation. Therefore, a frequency-domain raytracing algorithm was used in this thesis for generating the synthetic seismograms. A review of frequency domain raytracing is presented in Chapter V, Section 5.2 and 5.3. The geological model used for tracing the rays is a horizontally layered medium with homogeneous layers and with some nonwelded interfaces. The reflection and transmission coefficients required in the calculation of the wave's amplitude were computed from the exact formulas presented Chapter II.

Chapter VI summarizes the conclusions of the study and discusses some possible areas of future work.

Appendix I compares the macroscopic approach to the calculation of the reflection and transmission coefficients with the microscopic approach, and shows that they give identical results when the parameters from the two theories are matched appropriately.

For the case of incident SH waves, the exact formulas for the particle displacement reflection and transmission coefficients are available in the literature (Schoenberg, 1980). A brief description of the SH case is given in Appendix II. This thesis concentrates, however, only on elastic plane P and SV incident waves.
CHAPTER II

EXACT AND APPROXIMATE FORMULAS FOR REFLECTION AND TRANSMISSION COEFFICIENTS ACROSS A NONWELDED CONTACT INTERFACE

2.1. Theory: the boundary conditions at a nonwelded contact interface

In the conventional theory for plane wave propagation across a perfect welded contact interface, the boundary conditions at the interface are the continuity of displacement and stress (or traction). These boundary conditions do not apply in the case of imperfect (nonwelded) contact. Pyrak-Nolte et al (1990b) and Schoenberg (1980) used the displacement discontinuity boundary conditions introduced by Schoenberg to treat the case of imperfect contact. In this theory, the solids on either side of a nonwelded contact interface are treated as continuous media. The boundary conditions at the interface are that the stress is continuous across the boundary but the displacement is not, the displacement discontinuity being proportional to the traction. For plane P-SV waves propagating in the x-z plane, with the interface being the z = 0 plane, the boundary conditions at the interface are

\[
\begin{align*}
 u_{x2} - u_{x1} &= c_x^1 \tau_{zx}, & u_{x2} - u_{x1} &= c_z^1 \tau_{zz} \\
 \tau_{zx1} &= \tau_{zx2}, & \tau_{zz1} &= \tau_{zz2}
\end{align*}
\]

(2.1a) (2.1b)

1 A modified portion of this chapter has been published by Siriporn Chaisri and E.S. Krebes in the Journal of Geophysical Research -- see Chaisri and Krebes (2000).
where

\[ \tau_{zx} = \mu \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \]  \hspace{1cm} (2.1c)  \\

\[ \tau_{zz} = \lambda \frac{\partial u_z}{\partial x} + (\lambda + 2\mu) \frac{\partial u_z}{\partial z} \]  \hspace{1cm} (2.1d)  

Subscripts 1 and 2 denote the upper and lower media, respectively. \( \tau_{zx} \) and \( \tau_{zz} \) are the components of the stress tensor parallel and normal to the interface, respectively, and \( u_x \) and \( u_z \) are the components of the displacement vector. \( \lambda \) and \( \mu \) are Lamé’s constants. \( c_x \) and \( c_z \) are the specific compliances parallel and normal to the interface, respectively, and each is defined as the ratio of the magnitude of the displacement discontinuity to the stress (acting at the interface) which produces it. For a perfect welded contact interface (\( c_x \to 0 \) and \( c_z \to 0 \)), the displacement discontinuity boundary conditions (2.1) reduce to the conventional boundary conditions; the right hand sides of (2.1a) are zero. The specific stiffnesses (used by Pyrak-Nolte et al., 1990b) are the reciprocals of the specific compliances, i.e., \( \kappa_x = 1/c_x \) and \( \kappa_z = 1/c_z \). In this thesis, the specific compliances are used for convenience in later applications of the solutions of the boundary conditions (2.1). In (2.1a), the stress components on the right-hand sides can be evaluated in either medium 1 or 2, as stress is continuous across the interface (see 2.1b). Either choice leads to the same solutions. Choosing medium 2 however, results in simpler equations to solve if the waves are incident from medium 1, and vice versa.

In applying the above boundary conditions to the incident and scattered plane waves, the conventions of Aki and Richards (1980, vol. I, chap 5) were used, i.e., the z-axis points downward and the x-component of each unit polarization vector is positive. Also, the same sign convention for the phase of plane waves is used. The plane waves are written as \( A \exp\left[ i\omega(s \cdot x - t) \right] \), where \( A \) is the amplitude, \( \omega \) is the angular frequency, \( s \) is the slowness vector defined as the unit vector in the direction of phase propagation divided by the phase-velocity, \( x = (x, y, z) \) is the position vector in Cartesian coordinates,
\( t \) is the time, and \( d \) is the unit vector which defines the direction of particle motion or the displacement polarization.

For the P-SV plane waves, there are four possible incident waves: the P and SV waves incident from medium one and the P and SV waves incident from medium two. Each incident wave generates four scattered waves as shown in Figure 2.1. The slowness and polarization vectors of each plane wave are shown in Table 2.1. From Table 2.1, the \( x \) and \( z \) components of displacement are:

\[
\begin{align*}
    u_{x1} &= \hat{P}_1 \sin i \exp[I \omega(px + \xi_1 z - t)] + \hat{S}_1 \cos j \exp[I \omega(px + \eta_1 z - t)] \\
            &\quad + \hat{P}_1 \sin i \exp[I \omega(px - \xi_1 z - t)] + \hat{S}_1 \cos j \exp[I \omega(px - \eta_1 z - t)] \\
    u_{x2} &= \hat{P}_2 \sin i \exp[I \omega(px - \xi_2 z - t)] + \hat{S}_2 \cos j \exp[I \omega(px - \eta_2 z - t)] \\
            &\quad + \hat{P}_2 \sin i \exp[I \omega(px + \xi_2 z - t)] + \hat{S}_2 \cos j \exp[I \omega(px + \eta_2 z - t)] \\
    u_{z1} &= \hat{P}_1 \cos i \exp[I \omega(px + \xi_1 z - t)] - \hat{S}_1 \sin j \exp[I \omega(px + \eta_1 z - t)] \\
            &\quad - \hat{P}_1 \cos i \exp[I \omega(px - \xi_1 z - t)] + \hat{S}_1 \sin j \exp[I \omega(px - \eta_1 z - t)] \\
    u_{z2} &= -\hat{P}_2 \cos i \exp[I \omega(px - \xi_2 z - t)] + \hat{S}_2 \sin j \exp[I \omega(px - \eta_2 z - t)] \\
            &\quad + \hat{P}_2 \cos i \exp[I \omega(px + \xi_2 z - t)] - \hat{S}_2 \sin j \exp[I \omega(px + \eta_2 z - t)]
\end{align*}
\]

where \( I = \sqrt{-1}, \quad \xi_n = \frac{\cos i_n}{\alpha_n}, \quad \eta_n = \frac{\cos j_n}{\beta_n}, \) where \( n = 1 \) or 2, and where the horizontal component of slowness \( p = (\sin i_n)/\alpha_n = (\sin j_n)/\beta_n \) (Snell's law).

In (2.2), \( \rho \) is the density, \( \alpha \) is the P wave velocity, \( \beta \) is the SV wave velocity, \( p \) is the ray parameter, and \( i \) and \( j \) are the angles that the P waves and SV waves, respectively, make with the \( z \)-axis (the normal to the interface).
Substitution of the displacements (2.2) into the boundary conditions (2.1), and setting $z = 0$ at the interface results in

\[
\begin{align*}
(1) \quad u_{x2} - u_{x1} &= c_x \tau_{xx} : \\
-\hat{P}_1 \sin i_1 - \hat{S}_1 \cos j_1 + \hat{P}_2 (\sin i_2 - I \omega x \chi_2 \cos i_2) + \hat{S}_2 (\cos j_2 - I \omega x \beta_2 \gamma_2) &= \hat{P}_1 \sin i_1 + \hat{S}_1 \cos j_1 - \hat{P}_2 (\sin i_2 + I \omega x \chi_2 \cos i_2) - \hat{S}_2 (\cos j_2 + I \omega x \beta_2 \gamma_2) : \\
(2) \quad u_{z2} - u_{z1} &= c_z \tau_{zz} : \\
\hat{P}_1 \cos i_1 - \hat{S}_1 \sin j_1 + \hat{P}_2 (\cos i_2 - I \omega z \alpha_2 \gamma_2) + \hat{S}_2 (-\sin j_2 + I \omega z \chi_2 \cos j_2) &= \hat{P}_1 \cos i_1 - \hat{S}_1 \sin j_1 + \hat{P}_2 (\cos i_2 + I \omega z \alpha_2 \gamma_2) - \hat{S}_2 (\sin j_2 + I \omega z \chi_2 \cos j_2) : \\
(3) \quad \tau_{xx} &= \tau_{zz} : \\
\hat{P}_1 \chi_1 \cos i_1 + \hat{S}_1 \gamma_1 + \hat{P}_2 \chi_2 \cos i_2 + \hat{S}_2 \beta_2 \gamma_2 &= \hat{P}_1 \chi_1 \cos i_1 + \hat{S}_1 \gamma_1 + \hat{P}_2 \chi_2 \cos i_2 + \hat{S}_2 \beta_2 \gamma_2 : \\
(4) \quad \tau_{zz} &= \tau_{xx} : \\
-\hat{P}_1 \alpha_1 \gamma_1 + \hat{S}_1 \chi_1 \cos j_1 + \hat{P}_2 \alpha_2 \gamma_2 - \hat{S}_2 \chi_2 \cos j_2 &= \hat{P}_1 \alpha_1 \gamma_1 - \hat{S}_1 \chi_1 \cos j_1 - \hat{P}_2 \alpha_2 \gamma_2 + \hat{S}_2 \chi_2 \cos j_2 ,
\end{align*}
\]

where \( \mu_n = \rho_n \beta_n^2 \), \( (\lambda_n + 2 \mu_n) = \rho_n \alpha_n^2 \),

and \( \chi_n = 2 \rho_n \beta_n^2 p \), \( \gamma_n = \rho_n (1 - 2 \beta_n^2 p^2) \).

These can be expressed in matrix form as

\[
\mathbf{M} [\hat{P}_1, \hat{S}_1, \hat{P}_2, \hat{S}_2]^T = \mathbf{N} [\hat{P}_1, \hat{S}_1, \hat{P}_2, \hat{S}_2]^T
\]

(2.3)

where the column vectors on the left and the right contain the amplitudes of the scattered and incident waves, respectively, the acute and grave accents represent upcoming and downgoing waves, respectively, and where
Equations (2.3) are identical to equations (5.34) of Aki and Richards (1980) for the welded contact boundary conditions, except for the additional frequency-dependent terms in the matrices, due to the presence of nonwelded contact. Equations (2.3) can be solved analytically to obtain closed-form formulas for the reflection and transmission coefficients.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Type</th>
<th>Incident waves</th>
<th>Scattered waves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$A$</td>
<td>$s$</td>
</tr>
<tr>
<td>One</td>
<td>P</td>
<td>$\hat{p}_1$</td>
<td>$(p, 0, \xi_1)$</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>$\hat{s}_1$</td>
<td>$(p, 0, \eta_1)$</td>
</tr>
<tr>
<td>Two</td>
<td>P</td>
<td>$\hat{p}_2$</td>
<td>$(p, 0, -\xi_2)$</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>$\hat{s}_2$</td>
<td>$(p, 0, -\eta_2)$</td>
</tr>
</tbody>
</table>

Table 2.1. $A$ (Amplitude), $s$ (Slowness) and $d$ (displacement polarization) for each plane wave, $A \exp[\iota \omega (s \cdot x - t)] \mathbf{d}$, at an interface.
Figure 2.1. All possible incident and scattered waves in the P-SV case. The short bold face arrows are the polarization unit vectors $d$ for each wave.
In the case of a P wave incident in medium one, the column vector of the incident waves is

\[ \begin{bmatrix} \dot{P}_1, \dot{S}_1, \dot{P}_2, \dot{S}_2 \end{bmatrix}^T = \begin{bmatrix} \dot{P}_i, 0, 0 \end{bmatrix}^T \]

Substituting this into equation 2.3, it becomes

\[
M[\dot{P}_1, \dot{S}_1, \dot{P}_2, \dot{S}_2]^T = N[\dot{P}_i, 0, 0]^T
\]

\[
M[\dot{P}_1, \dot{S}_1, \dot{P}_2, \dot{S}_2]^T = [\dot{P}_1 \alpha_1 p, \dot{P}_1 \chi_1 \cos i_1, \dot{P}_1 \alpha_1 \gamma_1]^T
\]

where \( \dot{P}_i \) denotes the ratio \( \dot{P}_i/\dot{P}_1 \), \( \dot{S}_i \) denotes the ratio \( \dot{S}_i/\dot{P}_1 \), etc.

In the same way, for an SV wave incident in medium one:

\[
M[\dot{S}_1, \dot{S}_2, \dot{S}_1, \dot{S}_2]^T = [\cos j_1, \beta_1 p, \beta_1 \gamma_1, \chi_1 \cos j_1]^T. \tag{2.6b}
\]

For a P wave incident in medium two:

\[
M[\dot{P}_i, \dot{P}_i, \dot{S}_i, \dot{S}_i]^T = \begin{bmatrix}
-\alpha_2 p - I\alpha \chi_2 \cos i_2 \\
\cos i_2 + I\alpha \chi_2 \alpha_2 \gamma_2 \\
\chi_2 \cos i_2 \\
\alpha_2 \gamma_2 
\end{bmatrix}, \tag{2.6c}
\]

and an SV wave incident in medium two:

\[
M[\dot{S}_i, \dot{S}_i, \dot{S}_i, \dot{S}_i]^T = \begin{bmatrix}
-\cos j_2 - I\alpha \chi_2 \beta_2 \gamma_2 \\
-\beta_2 p - I\alpha \chi_2 \cos j_2 \\
\beta_2 \gamma_2 \\
\chi_2 \cos j_2 
\end{bmatrix}. \tag{2.6d}
\]

From (2.6) it can be seen that the first column of \( N \) is used for the case of an incident P wave in medium 1, the second for an incident SV wave in medium 1, the third
for an incident P wave in medium 2, and the fourth for an incident SV wave in medium 2. The first, second, third and fourth equations in (2.3) and (2.6) correspond to the interface conditions for $u_x$, $u_z$, $\tau_{xx}$ and $\tau_{zz}$, respectively. Equations (2.6) are a system of linear equations, consisting of four equations in four unknowns, which can be solved analytically. The solutions of these four linear equations are the closed-form formulas for the nonwelded contact reflection and transmission coefficients.

Consider the system of linear equations,

$$MX = B,$$  \hspace{1cm} (2.7)

where $M$ is a square matrix of order $q$, $X=(x_1, x_2, x_3, ..., x_q)^T$ are the unique solutions, and $B$ is a constant column vector. From Cramer's rule, if $M$ is non-singular then the unique solution $X$ is given by

$$x_k = \frac{\det(M')}{\det(M)}$$  \hspace{1cm} (2.8)

where $M'$ is the matrix $M$ with column $k$ replaced by the column vector $B$.

det$(M)$ is the determinant of matrix $M$ which is defined as

$$\det(M) = \sum_{i=1}^{q} (-1)^{i+j} m_{ij} \det(C_{ij}(M)) \quad j = 1, 2, ..., q$$  \hspace{1cm} (2.9)

or

$$\det(M) = \sum_{j=1}^{q} (-1)^{i+j} m_{ij} \det(C_{ij}(M)) \quad i = 1, 2, ..., q$$

where $C_{ij}(M)$ is the $(q-1)\times(q-1)$ submatrix of $M$ obtained by deleting row $i$ and column $j$ of $M$, and $m_{ij}$ is the element in row $i$ and column $j$ of $M$. 
From equation (2.6), \( M \) is of order 4, and the determinant of \( M \) is

\[
\det(M) = m_{11}m_{22}m_{33}m_{44} - m_{11}m_{22}m_{43}m_{34} - m_{11}m_{32}m_{23}m_{44} + m_{11}m_{32}m_{43}m_{24} \\
+ m_{11}m_{42}m_{23}m_{34} - m_{11}m_{42}m_{33}m_{24} - m_{21}m_{12}m_{33}m_{44} + m_{21}m_{12}m_{43}m_{34} \\
+ m_{21}m_{32}m_{13}m_{44} - m_{21}m_{32}m_{43}m_{14} - m_{21}m_{42}m_{13}m_{34} + m_{21}m_{42}m_{33}m_{14} \\
+ m_{31}m_{12}m_{23}m_{44} - m_{31}m_{12}m_{43}m_{24} - m_{31}m_{22}m_{13}m_{44} + m_{31}m_{22}m_{43}m_{14} \\
+ m_{31}m_{42}m_{13}m_{24} - m_{31}m_{42}m_{23}m_{14} - m_{41}m_{12}m_{23}m_{44} + m_{41}m_{12}m_{43}m_{24} \\
+ m_{41}m_{22}m_{13}m_{44} - m_{41}m_{22}m_{33}m_{14} - m_{41}m_{32}m_{13}m_{44} + m_{41}m_{32}m_{23}m_{14}
\]

(2.10)

Using Cramer’s Rule to find the solutions of (2.6), and defining the variables in the same manner as Aki and Richards (1980), and defining some new variables, then after a lot of mathematics, one obtains the exact formulas for the sixteen possible particle displacement reflection and transmission coefficients.

Defining the variables \( a, b, c, d, E, F, G, \) and \( H \) in the same way as Aki and Richards (1980), that is,

\[
a = y_2 - y_1, \quad b = y_2 + x_1p, \quad (2.11a)
\]

\[
c = y_1 + x_2p, \quad d = 2(p_2 \beta_2^2 - p_1 \beta_1^2), \quad (2.11b)
\]

\[
\xi_n = \frac{\cos i_n}{\alpha_n}, \quad \eta_n = \frac{\cos j_n}{\beta_n}, \quad (2.11c)
\]

\[
E = b \xi_1 + c \xi_2, \quad F = b \eta_1 + c \eta_2, \quad (2.11d)
\]

\[
G = a - d \xi_1 \eta_2, \quad H = a - d \xi_2 \eta_1, \quad (2.11e)
\]

\[
K_n = \gamma_n^2 + \frac{x_n^2 \xi_n \eta_n}{\alpha_n}, \quad L_n = \gamma_n^2 - \frac{x_n^2 \xi_n \eta_n}{\beta_n}, \quad (2.11f)
\]
and defining $D$ as

$$D = EF + GHp^2 - \omega^2 c_x c_z K_1 K_2$$

$$- I\omega c_x (\rho_1 \xi_1 K_2 + \rho_2 \xi_2 K_1) - I\omega c_x (\rho_1 \eta_2 K_2 + \rho_2 \eta_2 K_1),$$ (2.12)

which is the same as the definition in *Aki and Richards* (1980) except for the last three nonwelded contact terms, one obtains the following formulas for the sixteen possible particle displacement reflection and transmission coefficients, where $\hat{P}\hat{P}$ denotes the ratio $\hat{P}_1/\hat{P}_1$, etc.

$$\hat{P}\hat{P} = [(b_\xi - c_\xi)F - (a + d_\xi \eta_2)Hp^2]D^{-1}$$

$$+ [\omega^2 c_x c_z K_2 L_1 + I\omega c_x (\rho_2 \xi_2 L_1 - \rho_1 \xi_1 K_2) + I\omega c_x (\rho_1 \eta_2 K_2 + \rho_2 \eta_2 L_1)]D^{-1},$$ (2.13a)

$$\hat{P}\hat{S} = -2\hat{\xi}_1 p(\alpha_1 / \beta_1)(ab + cd\hat{\xi}_2 \eta_2)D^{-1}$$

$$- 2\hat{\xi}_1 \gamma_1 \chi_1 (\alpha_1 / \beta_1)[\omega^2 c_x c_z K_2 + I\omega \rho_2 (c_\xi c_\xi + c_x \eta_2)]D^{-1},$$ (2.13b)

$$\hat{\beta}\hat{P} = 2\rho_1 \hat{\xi}_1 (\alpha_1 / \alpha_2)[F - I\omega(c_x \gamma_2 \chi_2 + c_x \chi_1 \chi_2 \eta_2)]D^{-1},$$ (2.13c)

$$\hat{\beta}\hat{S} = 2\rho_1 \hat{\xi}_1 (\alpha_1 / \beta_2)[Hp + I\omega(c_\xi \gamma_2 \chi_2 - c_\xi \eta_2 \chi_2)]D^{-1},$$ (2.13d)

$$\hat{S}\hat{P} = -2\hat{\eta}_1 p(\beta_1 / \alpha_1)(ab + cd\hat{\xi}_2 \eta_2)D^{-1}$$

$$- 2\hat{\eta}_1 \gamma_1 \chi_1 (\beta_1 / \alpha_1)[\omega^2 c_x c_z K_2 + I\omega \rho_2 (c_\xi c_\xi + c_x \eta_2)]D^{-1},$$ (2.13e)

$$\hat{S}\hat{S} = -[(b \eta_1 - c \eta_2)E - (a + d \xi_2 \eta_2)Gp^2]D^{-1}$$

$$- [\omega^2 c_x c_z K_2 L_1 + I\omega c_x (\rho_1 \xi_1 K_2 + \rho_2 \xi_2 L_1) + I\omega c_x (\rho_2 \eta_2 L_1 - \rho_1 \eta_2 K_2)]D^{-1},$$ (2.13f)

$$\hat{\beta}\hat{P} = -2\rho_1 \eta_1 (\beta_1 / \alpha_2)[Gp - I\omega(c_\xi \gamma_2 \chi_2 - c_\xi \eta_2 \chi_2)]D^{-1},$$ (2.13g)

$$\hat{\beta}\hat{S} = 2\rho_1 \eta_1 (\beta_1 / \beta_2)[E - I\omega(c_x \chi_1 \chi_2 \xi_2 + c_\chi \gamma_2)]D^{-1},$$ (2.13h)
\[ \dot{P} \omega = 2 \rho_2 \xi_2 (\alpha_2 / \alpha_1) [F - I \omega (c_x \gamma_1 \gamma_2 + c_z \gamma_1 \gamma_2 \eta_2)] D^{-1}, \]  
(2.13i)

\[ \dot{P} \omega = -2 \rho_2 \xi_2 (\alpha_2 / \beta_1) [G \omega (c_x \gamma_1 \gamma_2 x_1 - c_z \gamma_1 \gamma_2 x_2)] D^{-1}, \]  
(2.13j)

\[ \dot{P} \omega = -[(b \xi_1 - c \xi_2) F + (a + d \eta_2 \eta_1) G \eta^2] D^{-1} \]

\[ + \omega^2 c_x c_z K_2 L_2 + I \omega c_x (\rho_2 \xi_1 L_2 - \rho_2 \xi_2 K_1) + I \omega c_z (\rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1)] D^{-1}, \]  
(2.13k)

\[ \dot{S} \omega = 2 \rho_2 p (\alpha_2 / \beta_2) (ac + bd \xi_1 \eta_1) D^{-1} \]

\[ - 2 \xi_2 \gamma_2 x_2 (\alpha_2 / \beta_2) [\omega^2 c_x c_z K_1 + I \omega p_1 (c_x \xi_1 + c_z \eta_1)] D^{-1}, \]  
(2.13l)

\[ \dot{S} \omega = 2 \rho_2 \eta_2 (\beta_2 / \alpha_2) [H p - I \omega (c_x \gamma_1 \gamma_2 \xi_2 - c_z \eta_1 \gamma_2 x_1)] D^{-1}, \]  
(2.13m)

\[ \ddot{S} \omega = 2 \rho_2 \eta_2 (\beta_2 / \alpha_2) [E - I \omega (c_x \gamma_1 \gamma_2 \xi_2 + c_z \gamma_1 \gamma_2)] D^{-1}, \]  
(2.13n)

\[ \ddot{S} \omega = 2 \eta_2 p (\beta_2 / \alpha_2) (ac + bd \xi_1 \eta_1) D^{-1} \]

\[ - 2 \eta_2 \gamma_2 x_2 (\beta_2 / \alpha_2) [\omega^2 c_x c_z K_1 + I \omega p_1 (c_x \xi_1 + c_z \eta_1)] D^{-1}, \]  
(2.13o)

\[ \ddot{S} \omega = [(b \eta_1 - c \eta_2) E + (a + d \xi_1 \eta_2) H p^2] D^{-1} \]

\[ - \omega^2 c_x c_z K_1 L_2 + I \omega c_x (\rho_2 \xi_1 L_2 + \rho_2 \xi_2 K_1) + I \omega c_z (\rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1)] D^{-1}. \]  
(2.13p)

The formulas (2.13) have been confirmed by using Maple software. The first parts of the formulas (2.13) have the same form as the formulas in (5.39) of Aki and Richards (1980), except that \( D \) contains additional terms due to nonwelded contact. The remaining parts are due to nonwelded contact. We can rearrange the formulas (2.13) to separate out the welded contact terms so that the first parts of the formulas will be the coefficients for the case of welded contact and the rest are the nonwelded terms. The following shows the procedure for rearranging formulas (2.13).
Each formula for a coefficient in (2.13) can be written as:

\[ R = \frac{N}{D}, \quad (2.14) \]

where \( R \) is the coefficient, \( N \) is the numerator in the formula, and \( D \) is the denominator in the formula and is given by (2.12).

That is

\[ N = N_w + N_{xz} \omega^2 c_x c_z + I \omega c_x N_x + I \omega c_z N_z, \quad (2.15) \]

and

\[ D = D_w - D_{xz} \omega^2 c_x c_z - I \omega c_x D_x - I \omega c_z D_z. \quad (2.16) \]

where \( N_w \) and \( D_w \) are the welded contact parts of \( N \) and \( D \). The expressions for \( N_w, D_w, N_{xz}, N_x, \) etc., can be determined by inspection of (2.12) and (2.13).

From (2.15),

\[ R = \frac{N_w}{D} + \frac{\omega^2 c_x c_z}{D} N_x + I \frac{\omega c_x}{D} N_x + I \frac{\omega c_z}{D} N_z \quad (2.17) \]

Decomposition of the first term in (2.17) into two fractions is done as follows:

\[ \frac{N_w}{D} = \frac{N_w}{D_w \left( 1 - \frac{D_{xz}}{D_w} \omega^2 c_x c_z - I \omega c_x \frac{D_x}{D_w} - I \omega c_z \frac{D_z}{D_w} \right)} \quad (2.18) \]

\[ = \frac{N_w}{D_w} + \frac{A}{1 - \frac{D_{xz}}{D_w} \omega^2 c_x c_z - I \omega c_x \frac{D_x}{D_w} - I \omega c_z \frac{D_z}{D_w}} \quad (2.19) \]

where \( A \) is to be determined. \( R_w = \frac{N_w}{D_w} \) is the coefficient for the case of welded contact.

Further manipulation of (2.19) results in

\[ \frac{N_w}{D} = \frac{N_w - R_w D_{xz} \omega^2 c_x c_z - I \omega c_x R_w D_x - I \omega c_z R_w D_z + AD_w}{D} \quad (2.20) \]
Comparing the right hand side and left hand side results in

\[ A = \frac{R_w D_x \omega^2 c_x c_z + I \omega c_x R_w D_x + I \omega c_z R_w D_z}{D_w}. \]  

(2.21)

Substituting \( A \) from equation (2.21) back into equation (2.19), gives

\[ \frac{N_w}{D} = \frac{N_w}{D_w} + \frac{R_w D_x \omega^2 c_x c_z + I \omega c_x R_w D_x + I \omega c_z R_w D_z}{D}. \]

and

\[ \frac{N_w}{D} = R_w + \frac{\omega^2 c_x c_z}{D} (R_w D_x + N_x) + I \frac{\omega c_x}{D} (R_w D_x + N_x) + I \frac{\omega c_z}{D} (R_w D_z + N_z). \]  

(2.22)

Substituting (2.22) into equation (2.17) results in

\[ R = R_w + \frac{\omega^2 c_x c_z}{D} (R_w D_x + N_x) + I \frac{\omega c_x}{D} (R_w D_x + N_x) + I \frac{\omega c_z}{D} (R_w D_z + N_z). \]  

(2.23)

Rewriting the formulas (2.13) in the format of (2.23), the exact formulas for the P-SV 
particle displacement reflection and transmission coefficients for the nonwelded contact 
interface become

\[ \dot{\Phi} = \dot{\Phi}_w + \frac{\omega^2 c_x c_z}{D} K_2 (\dot{\Phi}_w K_1 + L_1) + I \frac{\omega c_x}{D} [\dot{\Phi}_w K_\xi + \rho_2 \xi_2 L_1 - \rho_1 \xi_1 K_1] \]

\[ + I \frac{\omega c_z}{D} [\dot{\Phi}_w K_\eta + \rho_1 \eta_1 K_1 + \rho_2 \eta_2 L_1], \]  

(2.24a)

\[ \dot{S} = \dot{S}_w + \frac{\omega^2 c_x c_z}{D} K_2 (\dot{S}_w K_1 - 2 \xi_1 \gamma_1 \chi_1 \frac{\alpha_1}{\beta_1}) + I \frac{\omega c_x}{D} [\dot{S}_w K_\xi - 2 \rho_2 \xi_1 \xi_1 \chi_1 \frac{\alpha_1}{\beta_1}] \]

\[ + I \frac{\omega c_z}{D} [\dot{S}_w K_\eta - 2 \rho_2 \xi_1 \eta_1 \chi_1 \frac{\alpha_1}{\beta_1}], \]  

(2.24b)
\[ \begin{align*}
\hat{P} \dot{P} &= \hat{P} \dot{P}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_1 K_2 \hat{P} \dot{P}_w + I \frac{\alpha_x}{D} [\hat{P} \dot{P}_w K_{\xi} - 2 \rho_{1 \xi} \gamma_{1 \gamma} \alpha_1] \\
&+ I \frac{\alpha_z}{D} [\hat{P} \dot{P}_w K_{\eta} - 2 \rho_{1 \xi} \eta_1 \eta_2 \chi_1 \chi_2 \alpha_1], \quad (2.24c) \\
\hat{P} \dot{S} &= \hat{P} \dot{S}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_1 K_2 \hat{P} \dot{S}_w + I \frac{\alpha_x}{D} [\hat{P} \dot{S}_w K_{\xi} + 2 \rho_{1 \xi} \varepsilon_{2 \gamma} \chi_2 \alpha_1] \\
&+ I \frac{\alpha_z}{D} [\hat{P} \dot{S}_w K_{\eta} - 2 \rho_{1 \xi} \eta_1 \eta_2 \chi_1 \alpha_1], \quad (2.24d) \\
\hat{S} \dot{P} &= \hat{S} \dot{P}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_2 (\hat{S} \dot{P}_w K_{1 - \xi} - 2 \eta_1 \gamma_{1 \gamma} \beta_1) + I \frac{\alpha_x}{D} [\hat{S} \dot{P}_w K_{\xi} - 2 \rho_{2 \xi} \eta_1 \gamma_{1 \gamma} \beta_1] \\
&+ I \frac{\alpha_z}{D} [\hat{S} \dot{P}_w K_{\eta} - 2 \rho_{2 \xi} \eta_1 \gamma_{1 \gamma} \beta_1] \alpha_1, \quad (2.24e) \\
\hat{S} \dot{S} &= \hat{S} \dot{S}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_2 (\hat{S} \dot{S}_w K_{1 - L_1}) + I \frac{\alpha_x}{D} [\hat{S} \dot{S}_w K_{\xi} - \rho_{1 \xi} J_{1 - \gamma} - \rho_{1 \xi} L_1] \\
&+ I \frac{\alpha_z}{D} [\hat{S} \dot{S}_w K_{\eta} - \rho_{2 \xi} L_1 + \rho_{1 \xi} K_2], \quad (2.24f) \\
\hat{S} \ddot{P} &= \hat{S} \ddot{P}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_1 K_2 \hat{S} \ddot{P}_w + I \frac{\alpha_x}{D} [\hat{S} \ddot{P}_w K_{\xi} + 2 \rho_{1 \xi} \eta_1 \gamma_{2 \gamma} \beta_1] \\
&+ I \frac{\alpha_z}{D} [\hat{S} \ddot{P}_w K_{\eta} - 2 \rho_{1 \xi} \eta_1 \gamma_{2 \gamma} \beta_1] \alpha_2, \quad (2.24g) \\
\hat{S} \ddot{S} &= \hat{S} \ddot{S}_w + \frac{\omega^2 c_x \varepsilon_z}{D} K_1 K_2 \hat{S} \ddot{S}_w + I \frac{\alpha_x}{D} [\hat{S} \ddot{S}_w K_{\xi} - 2 \rho_{1 \xi} \eta_1 \gamma_{2 \gamma} \beta_1] \\
&+ I \frac{\alpha_z}{D} [\hat{S} \ddot{S}_w K_{\eta} - 2 \rho_{1 \xi} \gamma_{2 \gamma} L_2 \beta_2], \quad (2.24h)
\end{align*} \]
\[ \dot{P} = \dot{P}_w + \frac{\omega^2 c_x c_z}{D} K_1 K_2 \dot{P} + \frac{\omega c_x}{D} \left[ \dot{P} K_\xi - 2 \rho_2 \xi_2 \gamma_2 \frac{\alpha_2}{\alpha_1} \right] + I \frac{\omega c_z}{D} \left[ \dot{P} K_\eta - 2 \rho_2 \xi_2 \eta_2 \chi_2 \frac{\alpha_2}{\alpha_1} \right], \quad (2.24i) \]

\[ \dot{S} = \dot{S}_w + \frac{\omega^2 c_x c_z}{D} K_1 K_2 \dot{S} + \frac{\omega c_x}{D} \left[ \dot{S} K_\xi - 2 \rho_2 \xi_2 \gamma_2 \frac{\alpha_2}{\beta_2} \right] + I \frac{\omega c_z}{D} \left[ \dot{S} K_\eta - 2 \rho_2 \xi_2 \eta_2 \chi_2 \frac{\alpha_2}{\beta_2} \right], \quad (2.24j) \]

\[ \dot{P} = \dot{P}_w + \frac{\omega^2 c_x c_z}{D} K_1 (\dot{P} K_2 + L_2) + \frac{\omega c_x}{D} \left[ \dot{P} K_\xi + \rho_1 \xi_1 L_2 - \rho_2 \xi_2 K_1 \right] + I \frac{\omega c_z}{D} \left[ \dot{P} K_\eta + \rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1 \right], \quad (2.24k) \]

\[ \dot{S} = \dot{S}_w + \frac{\omega^2 c_x c_z}{D} K_1 (\dot{S} K_2 - 2 \xi_2 \gamma_2 \chi_2 \frac{\alpha_2}{\beta_2} ) + \frac{\omega c_x}{D} \left[ \dot{S} K_\xi - 2 \rho_2 \xi_2 \eta_2 \chi_2 \frac{\alpha_2}{\beta_2} \right] + I \frac{\omega c_z}{D} \left[ \dot{S} K_\eta - 2 \rho_2 \xi_2 \eta_2 \chi_2 \frac{\alpha_2}{\beta_2} \right], \quad (2.24l) \]

\[ \dot{S} = \dot{S}_w + \frac{\omega^2 c_x c_z}{D} K_1 K_2 \dot{S} + \frac{\omega c_x}{D} \left[ \dot{S} K_\xi + 2 \rho_2 \xi_2 \eta_2 \chi_2 \frac{\beta_2}{\alpha_1} \right] + I \frac{\omega c_z}{D} \left[ \dot{S} K_\eta - 2 \rho_2 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_1} \right], \quad (2.24m) \]

\[ \dot{S} = \dot{S}_w + \frac{\omega^2 c_x c_z}{D} K_1 K_2 \dot{S} + \frac{\omega c_x}{D} \left[ \dot{S} K_\xi - 2 \rho_2 \eta_2 \chi_2 \xi_2 \frac{\beta_2}{\alpha_1} \right] + I \frac{\omega c_z}{D} \left[ \dot{S} K_\eta - 2 \rho_2 \eta_2 \gamma_2 \frac{\beta_2}{\alpha_1} \right], \quad (2.24n) \]
\[
\hat{SP} = \hat{SP}_w + \frac{\omega^2 c_x c_z}{D} K_1 (\hat{SP}_w K_2 - 2\eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_2}) + I \frac{\omega c_x}{D} [\hat{SP}_w K_\eta - 2\rho_1 \xi_1 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_2}]
\]

\[
+ I \frac{\omega c_x}{D} [\hat{SP}_w K_\eta - 2\rho_1 \xi_1 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_2}], \quad (2.24o)
\]

\[
\hat{SS} = \hat{SS}_w + \frac{\omega^2 c_x c_z}{D} K_1 (\hat{SS}_w K_2 - L_2) + I \frac{\omega c_x}{D} [\hat{SS}_w K_\xi - \rho_1 \xi_1 L_2 - \rho_2 \xi_2 K_1]
\]

\[
+ I \frac{\omega c_x}{D} [\hat{SS}_w K_\eta - \rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1], \quad (2.24p)
\]

where

\[
K_\xi = \rho_1 \xi_1 K_2 + \rho_2 \xi_2 K_1,
\]

\[
K_\eta = \rho_1 \eta_1 K_2 + \rho_2 \eta_2 K_1,
\]

and \(\hat{SP}_w, \hat{SS}_w\), etc., are the reflection and transmission coefficients for a welded interface, as expressed by the formulas (5.39) of Aki and Richards (1980).

Although all sixteen coefficients are given above, the last eight can actually be obtained from the first eight by switching subscripts 1 and 2 (e.g., \(\hat{SP}\) can be obtained from \(\hat{SP}\) in this way). The above formulas have been checked by evaluating them for particular numerical values of the medium parameters and verifying that the results are identical to the solutions obtained by numerical solving the matrix equations (2.3).

The above solutions can also be applied to the case of a viscous nonwelded interface (e.g., a fracture or joint containing a fluid under saturated conditions). To include the effect of viscosity, a term involving a discontinuity in the x-component of the particle velocity is included in the boundary conditions (Pyrak-Nolte et al., 1990b). Specifically, the equation involving \(u_x\) in (2.1a) is modified as follows:

\[
\kappa_x (u_{x2} - u_{x1}) + \nu_x \left( \frac{\partial u_{x2}}{\partial t} - \frac{\partial u_{x1}}{\partial t} \right) = \tau_{xx}, \quad \kappa_x = 1/c_x \quad (2.25)
\]
where $v_x$ is the specific viscosity for the interface. Since $u_x \propto \exp(-I\omega t)$ in the plane waves that are substituted into equation (2.25), the effect of viscosity can be included by simply replacing $k_x$ with $k_x - I\omega v_x$ in (2.3), (2.6) and (2.13) or (2.24), i.e., by replacing $c_x$ with $c_x/(1 - I\omega x v_x)$.

_Pyrak-Nolte et al. (1990b)_ have shown that, in the case of an SV wave normally incident upon a nonwelded viscous interface between two identical materials, the effect of viscosity, e.g., a fluid-fill crack compared to a dry crack (or one filled with a compressible gas for that matter), is to reduce the energy transmitted at low frequencies and increase the energy transmitted at high frequencies.

### 2.2. Energy conservation

It has been shown by _Pyrak-Nolte et al. (1990b)_ that energy is conserved for P and SV waves normally incident upon a nonwelded interface between two identical media. In other words, they have shown that the sum of the energy-based reflection and transmission coefficients is one. However, energy is also conserved for P and SV waves which are _obliquely_ incident upon a nonwelded interface between two _different_ materials, as is shown below.

In order to treat the question of energy conservation, we calculate the normal component of the energy flux vector, i.e., the intensity $I$ in the media. Energy is conserved if the mean (time-averaged) value of the normal component of $I$ is continuous across the interface. The formula for the $j$th component of the intensity $I$ is given by

$$I_j = -\tau_{ij} \dot{u}_i$$

(2.26)

where $i$ and $j = x, y, z$.

If the P and SV seismic waves travel in the x-z plane only, then the component of the intensity $I$ in the direction normal to the interface, i.e., the z-direction, is given by
The normal component of the intensity in medium one is

\[ I_z^{(1)} = - \sum_{n=x,z} T^{(1)}_{nz} \frac{\partial u_n}{\partial t}. \]  \hspace{1cm} (2.28)

By using the boundary conditions (2.1), equation (2.28) becomes, at the interface,

\[ I_z^{(1)} = - \sum_{n=x,z} T^{(2)}_{nz} \frac{\partial}{\partial t} \left( u_n^{(2)} - c_n r^{(2)}_{nz} \right). \]  \hspace{1cm} (2.29)

i.e.,

\[ I_z^{(1)} = I_z^{(2)} + \sum_{n=x,z} c_n r^{(2)}_{nz} \frac{\partial r^{(2)}_{nz}}{\partial t} \]  \hspace{1cm} (2.30)

with

\[ u_n^{(1)} = u_n^{(inc)} + u_n^{(RP)} + u_n^{(RS)}, \quad u_n^{(2)} = u_n^{(TP)} + u_n^{(TS)}, \]  \hspace{1cm} (2.31)

where the superscript "(j)" denotes the jth medium and inc, RP, RS, TP and TS denote the incident wave, reflected P wave, reflected SV wave, transmitted P wave and transmitted SV wave, respectively, and where we have assumed that the incident wave is in medium one.

Equation (2.30) differs from the one in the case of welded contact only in the terms containing \( c_n \) (the rightmost sum). For energy to be conserved at the nonwelded interface, the time average of these extra terms has to be zero. That it is in fact zero can be shown as follows.

The stress components in medium two have terms associated with the transmitted P and SV waves. Hence, the extra terms in equation (2.30) can be written as

\[ \sum_{n=x,z} c_n r^{(2)}_{nz} \frac{\partial r^{(2)}_{nz}}{\partial t} = \sum_{n=x,z} c_n (r_{nz}^{TP} + \dot{r}_{nz}^{TS})(\ddot{r}_{nz}^{TP} + \ddot{r}_{nz}^{TS}) \]
\[
\sum_{n=x,z} c_n \tau_{nz}^{(2)} \frac{\partial \tau_{nz}^{(2)}}{\partial t} = \sum_{n=x,z} c_n \left( r_{n_z}^{TP} \dot{t}_{nz}^{TP} + r_{n_z}^{TS} \dot{t}_{nz}^{TS} + r_{n_z}^{TP} \ddot{t}_{nz}^{TP} + r_{n_z}^{TS} \ddot{t}_{nz}^{TS} \right),
\] (2.32)

In the case of real-valued specific compliances, the time average becomes, with the real part taken for calculation,

\[
\langle \sum_{n=x,z} c_n \tau_{nz,Re}^{(2)} \frac{\partial \tau_{nz,Re}^{(2)}}{\partial t} \rangle = \sum_{n=x,z} c_n \left[ \langle r_{n_z,Re}^{TP} \dot{t}_{nz,Re}^{TP} \rangle + \langle r_{n_z,Re}^{TS} \dot{t}_{nz,Re}^{TS} \rangle + \langle r_{n_z,Re}^{TP} \ddot{t}_{nz,Re}^{TP} \rangle + \langle r_{n_z,Re}^{TS} \ddot{t}_{nz,Re}^{TS} \rangle \right],
\] (2.33)

where the subscript \( \text{Re} \) means the real part (in case the plane wave for the stress component is expressed as a complex exponential).

The plane harmonic wave for each stress component has the form \( f(x,t) = A(x)e^{-i\omega t} \). Using the rule which states that if \( f(x,t) = A(x)e^{-i\omega t} \) and \( g(x,t) = B(x)e^{-i\omega t} \) then

\[
\langle \text{Re}(f)\text{Re}(g) \rangle = \frac{1}{2} \text{Re}(f^*g^*)
\] (2.34)
results in

\[
\langle f\dot{f} \rangle = \frac{1}{2} \text{Re}(f^*\dot{f}^*) = \frac{1}{2} \text{Re}(Ae^{-i\omega t})(i\omega)A^*e^{i\omega t}) = \frac{\omega}{2} |A|^2 \text{Re}(I) = 0.
\] (2.35)

Equation (2.35) shows that the first two terms on the right-hand side of (2.33) are zero.

Let \( r_{n_z}^{TP} = f(x,t) = A(x)e^{-i\omega t} \) and \( r_{n_z}^{TS} = g(x,t) = B(x)e^{-i\omega t} \). Then, the last term in (2.33) becomes

\[
\langle r_{n_z,Re}^{TP} \ddot{t}_{nz,Re}^{TS} + r_{n_z,Re}^{TP} \ddot{t}_{nz,Re}^{TP} \rangle = \langle \dot{f}_{Re} \dot{g}_{Re} + \dot{f}_{Re} \dot{g}_{Re} \rangle
\]
\[
= \frac{1}{2} \text{Re}(f^*g^* + \dot{f} \dot{g}^*)
\]
\[
= \frac{1}{2} \text{Re}[Ae^{-i\omega t}B^*(I\omega)e^{i\omega t} + A(-I\omega)e^{i\omega t}B^*e^{i\omega t}]
\]
\[
= \frac{1}{2} \text{Re}[0] = 0.
\] (2.36)
Equation (2.36) demonstrates that the last term in (2.33) is zero.

As shown above, the time average of the right most sum in equation (2.30) is zero which leads to \( \langle I_z^{(1)} \rangle = \langle I_z^{(2)} \rangle \) for real-valued specific compliances \( c_n \). This means that the normal component of the mean intensity is continuous across the boundary. Hence, energy is conserved, i.e., the sum of the energy-reflection and transmission coefficients (the relative mean normal intensities) is one.

Energy is not conserved in the case of a viscous nonwelded interface (for which \( c_x \) is replaced by \( \bar{c} = c_x / (1 - I\alpha x v_x) \) in the above equations), because energy is dissipated due to the presence of a fluid in the crack. As \( \bar{c} \) is complex, the time average of the last term in equation (2.30) is no longer zero. Its value gives the amount by which the sum of the energy-based reflection and transmission coefficients differs from one.

### 2.3. Approximation of the exact formulas for small compliances

Approximations of the exact formulas for the particle displacement reflection and transmission coefficients for a small amount of nonwelded contact, i.e., for small values of the specific compliances \( c_x \) and \( c_z \), could be useful in some applications in seismology. The Taylor series expansions of the formulas are used for these approximations. Consider each formula as a function of two variables, \( c_x \) and \( c_z \), and use the definition of a Taylor series for a function of two variables \( x \) and \( y \) about \( x = a \) and \( y = b \), i.e.,

\[
f(x, y) = f(a, b) + (x - a)f_x(a, b) + (y - b)f_y(a, b)
\]

\[
+ \frac{1}{2!} \left\{ (x - a)^2 f_{xx}(a, b) + 2(x - a)(y - b)f_{xy}(a, b) + (y - b)^2 f_{yy}(a, b) \right\} + ...
\]

(2.37)

where \( f_x(a, b), f_y(a, b), ... \) denote partial derivatives with respect to \( x, y, ... \) evaluated at \( x = a, y = b \).
As we can see, the formulas \((2.13)\) or \((2.24)\) are functions of the specific compliances \(c_x\) and \(c_z\). Each of these formulas can be expanded as a Taylor series in \(c_x\) and \(c_z\) about \(c_x = 0\) and \(c_z = 0\), where the Taylor series approximation for each formula is, taking the terms up to first order in \(c_x\) and \(c_z\) only,

\[
f(c_x, c_z) = f(0,0) + c_x f_{c_x}(0,0) + c_z f_{c_z}(0,0). \tag{2.38}
\]

Applying \((2.38)\) to \((2.13)\) or \((2.24)\) yields

\[
\dot{p} \dot{p} = \dot{p} \dot{p}_w + I \frac{\alpha c_x}{D_w} [\dot{p} \dot{p}_w K_{c_x} + \rho_2 \dot{c}_z L_1 - \rho_1 \dot{c}_x K_2 ]
\]

\[
+ I \frac{\alpha c_z}{D_w} [\dot{p} \dot{p}_w K_{c_z} + \rho_1 \eta_1 K_2 + \rho_2 \eta_2 L_1 ] \tag{2.39a}
\]

\[
\dot{p} \dot{s} = \dot{p} \dot{s}_w + I \frac{\alpha c_x}{D_w} [\dot{p} \dot{s}_w K_{c_x} - 2 \rho_2 \dot{c}_z \gamma_1 \gamma_1 \dot{x}_1 \frac{\alpha_1}{\beta_1} ]
\]

\[
+ I \frac{\alpha c_z}{D_w} [\dot{p} \dot{s}_w K_{c_z} - 2 \rho_2 \dot{c}_z \gamma_2 \gamma_1 \dot{x}_1 \frac{\alpha_1}{\beta_1} ] \tag{2.39b}
\]

\[
\dot{p} \dot{p} = \dot{p} \dot{p}_w + I \frac{\alpha c_x}{D_w} [\dot{p} \dot{p}_w K_{c_x} - 2 \rho_1 \dot{c}_x \gamma_1 \gamma_2 \frac{\alpha_1}{\alpha_2} ]
\]

\[
+ I \frac{\alpha c_z}{D_w} [\dot{p} \dot{p}_w K_{c_z} - 2 \rho_1 \dot{c}_x \eta_1 \eta_1 \dot{x}_1 \frac{\alpha_1}{\alpha_2} ] \tag{2.39c}
\]

\[
\dot{p} \dot{s} = \dot{p} \dot{s}_w + I \frac{\alpha c_x}{D_w} [\dot{p} \dot{s}_w K_{c_x} - 2 \rho_1 \dot{c}_x \gamma_2 \gamma_2 \frac{\alpha_1}{\beta_2} ]
\]

\[
+ I \frac{\alpha c_z}{D_w} [\dot{p} \dot{s}_w K_{c_z} - 2 \rho_1 \dot{c}_x \eta_2 \dot{x}_1 \frac{\alpha_1}{\beta_2} ] \tag{2.39d}
\]
\[ \dot{\hat{P}} = \dot{P}_w + I \frac{\alpha_c}{D_w} [\dot{P}_w K_x - 2 \rho_2 \xi_2 \eta_2 \chi_1 \frac{\beta_1}{\alpha_1} ] + I \frac{\alpha_c}{D_w} [\dot{P}_w K_\eta - 2 \rho_2 \eta_2 \gamma_1 \chi_1 \frac{\beta_1}{\alpha_1} ] \]  

(2.39e)

\[ \dot{\hat{S}} = \dot{S}_w + I \frac{\alpha_c}{D_w} [\dot{S}_w K_x - \rho_1 \xi_1 K_2 - \rho_2 \xi_2 L_1 ] + I \frac{\alpha_c}{D_w} [\dot{S}_w K_\eta - \rho_2 \eta_2 L_1 + \rho_1 \eta_1 K_2 ] \]  

(2.39f)

\[ \dot{\hat{P}} = \dot{P}_w + I \frac{\alpha_c}{D_w} [\dot{P}_w K_x + 2 \rho_1 \xi_1 \eta_1 \gamma_1 \chi_1 \frac{\beta_1}{\alpha_2} ] + I \frac{\alpha_c}{D_w} [\dot{P}_w K_\eta - 2 \rho_1 \eta_1 \gamma_1 \chi_2 \frac{\beta_1}{\alpha_2} ] \]  

(2.39g)

\[ \dot{\hat{S}} = \dot{S}_w + I \frac{\alpha_c}{D_w} [\dot{S}_w K_x - 2 \rho_1 \xi_1 \xi_2 \eta_1 \chi_1 \chi_2 \frac{\beta_1}{\beta_2} ] + I \frac{\alpha_c}{D_w} [\dot{S}_w K_\eta - 2 \rho_1 \xi_1 \gamma_1 \gamma_2 \frac{\beta_1}{\beta_2} ] \]  

(2.39h)

\[ \dot{p}_P = \dot{p}_P + I \frac{\alpha_c}{D_w} [\dot{p}_P K_x - 2 \rho_2 \xi_2 \gamma_1 \gamma_2 \frac{\alpha_2}{\alpha_1} ] + I \frac{\alpha_c}{D_w} [\dot{p}_P K_\eta - 2 \rho_2 \xi_2 \eta_1 \eta_2 \chi_1 \chi_2 \frac{\alpha_2}{\alpha_1} ] \]  

(2.39i)

\[ \dot{p}_S = \dot{p}_S + I \frac{\alpha_c}{D_w} [\dot{p}_S K_x + 2 \rho_2 \xi_2 \xi_2 \gamma_1 \chi_1 \frac{\alpha_2}{\beta_1} ] + I \frac{\alpha_c}{D_w} [\dot{p}_S K_\eta - 2 \rho_2 \xi_2 \eta_2 \gamma_1 \chi_2 \frac{\alpha_2}{\beta_1} ] \]  

(2.39j)
\[
\hat{P} = \hat{P}_w + I \frac{\alpha x}{D_w} [\hat{P}_w K_\xi + \rho_1 \xi L_2 - \rho_2 \xi K_1]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{P}_w K_\eta + \rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1]
\]
\[\text{(2.39k)}\]

\[
\hat{P} = \hat{P}_w + I \frac{\alpha x}{D_w} [\hat{P}_w K_\xi - 2 \rho_1 \xi \xi_2 \gamma_2 x_2 \frac{\alpha_2}{\beta_2}]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{P}_w K_\eta - 2 \rho_1 \xi \eta_2 \gamma_2 \chi_2 \frac{\alpha_2}{\beta_2}]
\]
\[\text{(2.39l)}\]

\[
\hat{S} = \hat{S}_w + I \frac{\alpha x}{D_w} [\hat{S}_w K_\xi + 2 \rho_2 \xi \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_1}]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{S}_w K_\eta - 2 \rho_2 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_1}]
\]
\[\text{(2.39m)}\]

\[
\hat{S} = \hat{S}_w + I \frac{\alpha x}{D_w} [\hat{S}_w K_\xi - 2 \rho_2 \eta_2 \chi_2 \xi_2 \gamma_2 \frac{\beta_2}{\alpha_1}]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{S}_w K_\eta - 2 \rho_2 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_1}]
\]
\[\text{(2.39n)}\]

\[
\hat{P} = \hat{P}_w + I \frac{\alpha x}{D_w} [\hat{P}_w K_\xi - 2 \rho_1 \xi \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_2}]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{P}_w K_\eta - 2 \rho_1 \eta_1 \eta_2 \gamma_2 \chi_2 \frac{\beta_2}{\alpha_2}]
\]
\[\text{(2.39o)}\]

\[
\hat{S} = \hat{S}_w + I \frac{\alpha x}{D_w} [\hat{S}_w K_\xi - \rho_1 \xi L_2 - \rho_2 \xi K_1]
\]
\[
+ I \frac{\alpha x}{D_w} [\hat{S}_w K_\eta - \rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1]
\]
\[\text{(2.39p)}\]
\[
\dot{P} = \dot{P} + I \frac{\alpha z}{D_w} \left[ \dot{P} K + \rho_1 \xi_1 L_2 - \rho_2 \xi_2 K_1 \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{P} K + \rho_1 \eta_1 L_2 - \rho_2 \eta_2 K_1 \right] \\
(2.39k)
\]

\[
\dot{S} = \dot{S} + I \frac{\alpha z}{D_w} \left[ \dot{S} K - 2 \rho_1 \xi_1 \xi_2 \chi_2 \frac{\alpha_2}{\beta_2} \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{S} K - 2 \rho_1 \eta_1 \eta_2 \chi_2 \frac{\alpha_2}{\beta_2} \right] \\
(2.39l)
\]

\[
\dot{S} = \dot{S} + I \frac{\alpha z}{D_w} \left[ \dot{S} K + 2 \rho_2 \xi_2 \eta_1 \chi_2 \frac{\beta_2}{\alpha_1} \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{S} K + 2 \rho_2 \eta_1 \eta_2 \chi_2 \frac{\beta_2}{\alpha_1} \right] \\
(2.39m)
\]

\[
\dot{S} = \dot{S} + I \frac{\alpha z}{D_w} \left[ \dot{S} K - 2 \rho_2 \xi_1 \xi_2 \frac{\beta_2}{\beta_1} \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{S} K - 2 \rho_2 \eta_1 \eta_2 \chi_2 \frac{\beta_2}{\beta_1} \right] \\
(2.39n)
\]

\[
\dot{S} = \dot{S} + I \frac{\alpha z}{D_w} \left[ \dot{S} K + 2 \rho_1 \xi_1 \eta_2 \chi_2 \frac{\beta_2}{\alpha_2} \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{S} K + 2 \rho_1 \eta_1 \eta_2 \chi_2 \frac{\beta_2}{\alpha_2} \right] \\
(2.39o)
\]

\[
\dot{S} = \dot{S} + I \frac{\alpha z}{D_w} \left[ \dot{S} K - \rho_1 \xi_1 L_2 - \rho_2 \xi_2 K_1 \right] \\
+ I \frac{\alpha z}{D_w} \left[ \dot{S} K - \rho_1 \eta_1 L_2 + \rho_2 \eta_2 K_1 \right] \\
(2.39p)
\]
CHAPTER III

A NONWELDED CONTACT INTERFACE IN A SINGLE HOMOGENEOUS MEDIUM

3.1. Reflection and transmission coefficients for a nonwelded contact interface in a single homogeneous medium

A special case of interest is that of a plane interface in a single homogeneous medium (e.g., a crack, joint, or fracture). The media above and below the interface are identical. Suppose a plane P wave is incident in medium 1 upon this interface. If there were welded contact at the interface, there would be no reflected waves and no converted (P-SV) transmitted wave, only an unconverted transmitted wave with the same amplitude as the incident wave ($\hat{P}\hat{P} = 1$), because there is no impedance contrast across the interface. However, if there is nonwelded contact at the interface, both P and SV reflected and transmitted waves would be produced. Similarly, if the incident wave is an SV wave, both P and SV reflected and transmitted waves would be produced for a nonwelded interface, as opposed to the case of welded contact, in which only an unconverted transmitted wave would be produced. Substituting $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, $\rho_1 = \rho_2 = \rho$, into the formulas in (2.11), (2.12) and (2.13), and noting that Snell’s law implies $i_1 = i_2 = i$ and $j_1 = j_2 = j$, results in the following definitions,

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2 Parts of this chapter have been published in the Journal of Geophysical Research: see Chaisri and Krebes (2000).
\[ D = (2\rho \xi - I\omega c_z K)(2\rho \eta - I\omega c_x K), \]  
(3.1a)

\[ \chi = 2\rho \beta^2 \rho, \quad \gamma = \rho(1 - 2\beta^2 \rho^2), \]  
(3.1b)

\[ \xi = \frac{\cos i}{\alpha}, \quad \eta = \frac{\cos j}{\beta}, \]  
(3.1c)

\[ K = \gamma^2 + \chi^2 \xi \eta, \quad L = \gamma^2 - \chi^2 \xi \eta, \]  
(3.1d)

and in the following formulas:

\[ \dot{P}\dot{P} = [\omega^2 c_x c_z KL + 2I\omega \eta(c_x \gamma^2 - c_z \chi^2 \xi^2)]D^{-1}, \]  
(3.2a)

\[ \dot{P}\dot{S} = -2\gamma\chi\xi(\alpha / \beta)[\omega^2 c_x c_z K + I\omega \rho(c_x \xi + c_z \eta)]D^{-1}, \]  
(3.2b)

\[ \dot{P}\dot{P} = 2\rho\xi[2\rho \eta - I\omega(c_x \gamma^2 + c_z \chi^2 \eta^2)]D^{-1}, \]  
(3.2c)

\[ \dot{P}\dot{S} = 2I\omega \rho \gamma \chi \xi (\alpha / \beta)(c_z \xi - c_z \eta)D^{-1}, \]  
(3.2d)

\[ \dot{S}\dot{P} = -2\gamma \chi \eta(\beta / \alpha)[\omega^2 c_x c_z K + I\omega \rho(c_x \xi + c_z \eta)]D^{-1}, \]  
(3.2e)

\[ \dot{S}\dot{S} = -[\omega^2 c_x c_z KL + 2I\omega \rho \xi(c_x \gamma^2 - c_z \chi^2 \eta^2)]D^{-1}, \]  
(3.2f)

\[ \dot{S}\dot{P} = -2I\omega \rho \gamma \chi \eta(\beta / \alpha)(c_z \eta - c_x \xi)D^{-1}, \]  
(3.2g)

\[ \dot{S}\dot{S} = 2\rho \eta[2\rho \xi - I\omega(c_x \chi^2 \xi^2 + c_z \gamma^2)]D^{-1}. \]  
(3.2h)

In the case of welded contact, \( c_x \) and \( c_z \) are zero, leading to the results \( \dot{P}\dot{P} = 1 \) and \( \dot{S}\dot{S} = 1 \), with the other six coefficients being zero. In other words, the incident wave is transmitted through the interface without change, as expected. Note also that at zero frequency, all the coefficients are also zero except for \( \dot{P}\dot{P} = 1 \) and \( \dot{S}\dot{S} = 1 \).

It is conceivable that these results could help to explain observations that are occasionally reported of anomalous seismic reflections in zones where none are expected.
(due to the lack of significant impedance contrasts). To cite one example, Hurich et al. (1985) reported unexpectedly large seismic reflection amplitude from mylonite zones where acoustic contrasts are small. Although they suggested constructive interference due to layering as the cause, it is not unreasonable to consider the possibility that nonwelded contact at the interfaces may have enhanced the amplitudes above expectations.

The existence of reflected and transmitted waves in this case of a nonwelded contact interface separating identical media is also predicted by a simple microscopic model of a solid consisting of a lattice of masses and springs (Krebes, 1987). Also, in this case, and in the general case in which there is an impedance contrast, the theoretical and numerical results of the microscopic theory can be made to agree exactly with those of the macroscopic theory by matching appropriate microscopic parameters (involving spring and lattice constants) with the corresponding macroscopic ones (involving specific compliances). See appendix I for details.

3.2. Approximations of the reflection and transmission coefficients

3.2.1. Small specific compliances approximation

For a small amount of nonwelded contact, i.e., for small values of the specific compliances, or more generally, when the dimensionless parameter terms \( \varepsilon = \rho v \omega c \) (where \( v \) is a phase velocity, \( c \) is a specific compliance, and \( \omega \) is the angular frequency) are very small (see equation 2.42), the exact formulas (3.2) can be approximated so that to first order in \( c_x \) and \( c_z \), they become

\[
\dot{p} \dot{p} = \frac{I \omega}{2 \rho_x^2} (c_x \gamma^2 - c_z \chi^2 \xi^2),
\]  

(3.3a)
\[
\dot{P}^S = \frac{1}{2 \rho \beta \eta} \left( c_x \xi + c_z \eta \right), \quad (3.3b)
\]
\[
\dot{P}^P = 1 + \frac{1}{2 \rho \xi} \left( c_x \gamma^2 + c_x \chi^2 \xi^2 \right), \quad (3.3c)
\]
\[
\dot{S}^S = \frac{1}{2 \rho \beta \eta} \left( c_x \xi - c_z \eta \right), \quad (3.3d)
\]
\[
\dot{S}^P = \frac{1}{2 \rho \alpha \xi} \left( c_x \xi + c_z \eta \right), \quad (3.3e)
\]
\[
\dot{S}^S = \frac{1}{2 \rho \eta} \left( c_x \chi^2 \eta^2 - c_x \gamma^2 \right), \quad (3.3f)
\]
\[
\dot{S}^P = \frac{1}{2 \rho \alpha \xi} \left( c_x \xi - c_z \eta \right), \quad (3.3g)
\]
\[
\dot{S}^T = 1 + \frac{1}{2 \rho \eta} \left( c_x \gamma^2 + c_x \chi^2 \eta^2 \right). \quad (3.3h)
\]

These formulas can be obtained either by substituting the same medium parameters for both media into approximations (2.39) or by expanding the exact formulas (3.2) for small \( c_x \) and \( c_z \) (via Taylor's formula or otherwise). More generally, the formulas in (3.3) are valid for \( \rho v \alpha \xi \ll 1 \), where \( v = \alpha \) or \( \beta \) and \( c = c_z \) or \( c_x \). In other words, equations (3.3) hold even for larger values of \( c_x \) and \( c_z \), as long as the frequency values are small enough.

There are some cases, however, for which they are not valid. Inspection of equation (2.4) for the matrix \( \mathbf{M} \) shows that for very small values of \( \cos i_2 \) the nonwelded contact term containing \( c_z \) in \( M_{23} \) is no longer small compared to the welded contact term \( \cos i_2 \). Applying this to (3.3) means that for \( i \to 90^\circ \), some of the formulas in (3.3) will not be accurate. Inspection of (3.3) shows that \( \dot{P}^T \), \( \dot{P}^P \), \( \dot{S}^P \), and \( \dot{S}^P \to \infty \) as \( \xi = (\cos i)/\alpha \to 0 \), whereas they actually tend to finite values, as can be seen from the exact formulas in equations (3.2). For an incident P wave this is not very important.
because incidence angles near 90° rarely occur in practice. For an incident S wave, however, $\xi = 0$ at the critical angle $j_c = \arcsin(\beta/\alpha)$ for the scattered P waves, and is therefore of greater importance. Inspection of $M_{14}$ in (2.4) shows that a similar situation occurs for very small values of $\cos j/2$. However, applying this to (3.3), it is again not very important, as the small values of $\eta = (\cos j)/\beta$ occur only for an S wave incident near 90°.

The approximation formulas in (3.3) also show that coefficients such as $\hat{PP}$ have substantial phase values, although their amplitudes are small. Normally, such coefficients would be zero for welded contact between identical media. For example, the phase of $\hat{PP}$ is ±90°. This could be useful in data analysis as an indicator of the presence of a nonwelded contact interface.

### 3.2.2. Small $p$ approximation

The approximation formulas (3.3) can be further approximated to yield formulas for small $p$, i.e., small incidence angles. Such formulas would be useful in amplitude versus offset studies in exploration seismology. Using Taylor expansions for instance, and keeping only terms up to $O(p^2)$, and defining $r = \beta/\alpha$, we get

\begin{align*}
\hat{PP} &= \frac{1}{2} I \omega r \alpha \left\{ c_z - \frac{1}{2} \left[ 8r^4 c_x + (8r^2 - 1)c_z \right] r^2 \right\}, \quad \text{(3.4a)} \\
\hat{PS} &= -I \omega r \alpha (\beta c_x + \alpha c_z) r p, \quad \text{(3.4b)} \\
\hat{SP} &= 1 + \frac{1}{2} I \omega r \alpha \left\{ c_z + \frac{1}{2} \left[ 8r^4 c_x - (8r^2 - 1)c_z \right] r^2 p^2 \right\}, \quad \text{(3.4c)} \\
\hat{S}P &= I \omega r \alpha (\beta c_x - \alpha c_z) r p, \quad \text{(3.4d)} \\
\hat{S}\hat{P} &= -I \omega r \alpha (\beta c_x + \alpha c_z) \beta p, \quad \text{(3.4e)}
\end{align*}
\[
\delta S = -\frac{1}{2} I \omega \rho \beta \left[ c_x - \frac{1}{2} (7c_x + 8c_z) \beta^2 p^2 \right], \tag{3.4f}
\]

\[
\delta P = I \omega \rho r (\beta c_x - \alpha c_z) \beta p, \tag{3.4g}
\]

\[
\delta S = 1 + \frac{1}{2} I \omega \rho \beta \left[ c_x - \frac{1}{2} (7c_x - 8c_z) \beta^2 p^2 \right]. \tag{3.4h}
\]

In applications of equations (3.4a)-(3.4h), \( p \) should be small enough so that the singular point \( p = 1/\alpha \) is avoided. The formulas can also be written in terms of the incidence angle. For an incident P wave, in equations (3.4a)-(3.4d), replace \( \alpha p \) with \( \sin i \). For an incident SV wave, in equations (3.4e)-(3.4h), replace \( \beta p \) with \( \sin j \).

### 3.3. Note on the sensitivity of the coefficients to the specific compliances

Another interesting application of the formulas is the study of the sensitivity of the reflection and transmission coefficients to the specific compliances, \( c_x \) and \( c_z \). Chaisri and Krebes (2000) indicate that at a given angle of incidence, the amplitude of the reflected P wave, \( |\hat{P} \hat{P}| \), either decreases with \( c_x \) and increases with \( c_z \), or increases with \( c_x \) and decreases with \( c_z \). For the reflected SV wave, \( |\hat{P} \hat{S}| \) increases with both \( c_x \) and \( c_z \), and it is always more sensitive to the normal specific compliance \( c_z \) than the tangential specific compliance \( c_x \). For the transmitted P wave, \( |\hat{P} \hat{T}| \) is almost insensitive to both \( c_x \) and \( c_z \). Most of the scattered waves are more sensitive to the normal specific compliance \( c_z \) than the tangential specific compliance \( c_x \), except for the \( \delta S \) reflection for which the situation is reversed for small and large (but not medium) angles of incidence. In addition, all the sensitivities also increase with the frequency.

For the phase sensitivities of the coefficients, equation (3.3) suggests that the phases of the reflection coefficients \( \delta P \) and \( \delta S \) are always exactly 90° and that they are insensitive to the specific compliances of the nonwelded interface. However, this is not really the case. To determine the phase sensitivities of these coefficients to first order, a
second order expansion of the exact formulas is actually required. This is because the phase is computed from the arctangent of the ratio of the real and imaginary parts of the coefficients (the real and imaginary parts involve terms of even and odd order, respectively). Numerical experimentation suggests that the phase sensitivities of these coefficients tend to behave more or less like their amplitude sensitivities -- generally, they are more sensitive to $c_2$ at the smaller angles.
CHAPTER IV

NUMERICAL EXAMPLES: REFLECTION AND TRANSMISSION
COEFFICIENTS FOR NONWELDED CONTACT

4.1. Introduction

This chapter shows examples of plots of the coefficients from Chapter II and III for the discussion of the behaviour of wave propagation across a nonwelded contact interface. In Section 4.2, examples for a nonwelded contact interface in a single homogeneous medium are shown in Figure 4.1. In the case of a welded interface, in which the media above and below the interface are identical, all the coefficients are zero except for $\hat{P}\hat{P}$ and $\hat{S}\hat{S}$ (assuming the upper medium is the incidence medium), which are unity. When there is the presence of a nonwelded contact interface, even though there is no impedance contrast, we still can see energy reflected from such an interface and some changes in the transmitted waves. Figure 4.2 shows plots of the approximation formulas (3.3) and (3.4), showing how well the approximate formulas match the exact formulas.

The last section shows examples of the coefficients for a P wave incident from the first medium when medium one and two are different. The results show that the amplitudes and phases of the coefficients are changed due to the presence of a nonwelded interface compared to the case of a welded interface.

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3 Parts of this chapter have been published in the Journal of Geophysical Research -- see Chaisri and Krebes (2000).
4.2. Numerical examples: a nonwelded contact interface in a single homogeneous medium

Consider the case of a nonwelded interface between two identical materials, and consider an incident P or SV wave in the upper medium. Figure 4.1 shows examples of the eight possible coefficients: (a)-(d) are the reflection and transmission coefficients for an incident P wave and (e)-(h) are those for an incident SV wave, for three different frequencies, 1, 5, and 10 Hz. Figure 4.1 can be produced either by plotting equations (3.2a) – (3.2h) or by solving (2.3) numerically. We chose \( c_x = 2c_z \) because \( c_x \) is typically substantially larger than \( c_z \) (Pyrak-Nolte et al. 1990b). Also, by choosing \( \alpha = 2\beta \) as well, each matrix element in (2.4) and (2.5) containing a nonwelded contact term can, for normal incidence \((i, j, p = 0)\), be expressed as the conventional welded contact term times \((1 \pm I\varepsilon)\), where the plus sign is used in (2.5) and the minus sign in (2.4), and where the dimensionless variable \( \varepsilon \equiv \rho\beta\omega c_x \) (see equations 2.40 and 2.41). The specific compliance \( c_x \) was then chosen so that \( \varepsilon \approx 1 \) for the highest frequency. The specific compliances of the interface are \( c_x = 5 \times 10^{-9} \) m/Pa and \( c_z = 2.5 \times 10^{-9} \) m/Pa. These values are such that for the lowest frequency, 1 Hz, the nonwelded contact terms in (2.4) and (2.5) are small compared to the welded contact terms, whereas for the highest frequency, 10 Hz, they are comparable to the welded contact terms. The values of the medium parameters are \( \alpha = 2800 \) m/s, \( \beta = 1400 \) m/s, and \( \rho = 2300 \) kg/m\(^3\), which are typical values for the upper crust.

Even though there is no impedance contrast, figure 4.1 shows the interesting result that reflected waves are produced for \( \varepsilon \neq 0 \). For small \( \varepsilon \) \((f = 1 \) Hz\) the scattered waves are small in amplitude except for the transmitted P and SV waves in figure 4.1c and 4.1h respectively.
Figure 4.1. The amplitudes and phases of the eight scattered wave coefficients (a) $P P$, (b) $P S$, (c) $P P$, (d) $P S$, (e) $S P$, (f) $S S$, (g) $P S$, and (h) $S S$ for a nonwelded contact interface separating two identical media. The medium properties are $\rho = 2300 \text{ kg/m}^3$, $\alpha = 2800 \text{ m/s}$, and $\beta = 1400 \text{ m/s}$. $c_x = 5 \times 10^{-9} \text{ m/Pa}$ and $c_z = 2.5 \times 10^{-9} \text{ m/Pa}$. The thin solid line is the calculation for the frequency 10 Hz, the dotted line is for 5 Hz, and the thick solid line is for 1 Hz, corresponding to $\varepsilon \approx 1$, $\varepsilon \approx 0.5$, and $\varepsilon \approx 0.1$, respectively.
Figure 4.1. Continued.
Figure 4.1. Continued.
Figure 4.1. Continued
When \( \varepsilon \) increases \((f = 5, 10 \text{ Hz})\), the normally small-amplitude waves become more prominent. In the case of an incident SV wave, Figures 4.1e - 4.1g, at 30 degrees (the critical angle for the reflected and transmitted P waves), the amplitudes of scattered waves become substantially larger except for \( \hat{S}\hat{S} \), Figures 4.1h.

Figure 4.1 also shows that the coefficients have significant phase values, even for small \( \varepsilon \). For example, in Figure 4.1a, the reflected P wave \( \hat{P}\hat{P} \), although small in amplitude at small \( \varepsilon \), experiences a phase shift of about 90 degrees. In the interpretation of seismic data the observation of small \( P \rightarrow P \) reflections where none are expected, with 90° phase shifts, could be evidence of the presence of a small amount of nonwelded contact at the reflecting interface. Figure 4.1 also shows that at an incidence angle of 90 degrees (grazing incidence), all the incident P energy goes into the reflected P wave and all the incident SV energy goes into the reflected SV wave, with the other scattered waves becoming negligible, even for large amounts of nonwelded contact. This can also be seen from the exact formulas (2.13), which show that \( \hat{P}\hat{S}, \hat{S}\hat{P}, \) and \( \hat{S}\hat{P}, \hat{S}\hat{S} \approx \cos i_1 \) and \( \hat{S}\hat{P}, \hat{S}\hat{S} \approx \cos j_1 \), which approach zero for grazing incidence.

Another interesting result involves the converted wave reflection coefficients in Figures 4.1b (\( P \rightarrow SV \)) and 4.1e (\( SV \rightarrow P \)). The amplitudes of these converted waves are small for small \( \varepsilon \) but they have significant values of phase change. When interpreting data, if a small-amplitude converted reflected wave with a noticeable phase shift (around 90 degrees) is observed, when none is expected, then this could again be evidence of the presence of a nonwelded contact interface.

The curves in Figure 4.1 can also be viewed as curves for varying values of \( c_x \) at a fixed frequency, because \( \varepsilon \propto \omega c_x, \) and \( \omega \) and \( c_x \) appear in matrices \( M \) and \( N \) (equations 2.4 and 2.5) only in the combination \( \omega c_x \) (recall that \( \omega c_x = \omega c_x / 2 \) in this figure). For example, for a fixed frequency of 10 Hz, the curves labeled 1, 5, and 10 in Figure 4.1 would correspond to \( c_x = 0.5, 2.5, \) and 5 mm/MPa (or \( \kappa_x = 2.0, 0.4, \) and 0.2 MPa/mm), respectively with \( c_z = c_x / 2 \).
Pyrak-Nolte et al. (1990b) showed that, for normally incident plane P waves, the nonwelded contact interface behaves somewhat like a low-pass filter for the transmitted P wave, that is, at low frequencies, the transmitted P wave has a high amplitude whereas the reflected P wave has a low amplitude. Figure 4.1 confirms this result and, in addition, shows that the nonwelded interface behaves like a low-pass filter for the transmitted P wave at all angles of incidence except for those near 90 degrees and that the filter also attenuates the scattered converted waves at low frequencies as well.

Examples using the approximations of the coefficients are shown in Figure 4.2. Only the approximation for the reflected P wave coefficient, $\tilde{P}P$, is shown here. The exact formula (3.2a), the small compliances approximation formula (3.3a), and the formula (3.4a) for small compliances and small $p$ (i.e., the small $p$ approximation of formula 3.3a) are plotted in the same diagram. Figure 4.2 shows that when $\varepsilon$ get smaller (Figure 4.2b has the smaller value of $\varepsilon$), the amplitudes of the approximation formulas get closer to those of the exact formula. In Figure 4.2, the small compliances formula approximates the exact amplitude very well up to an incidence angle of about 80°. Figure 4.2 also shows that the approximation for small compliances and small $p$ fits the small compliances approximation very well up to an incidence angle of about 40 degrees. Regarding the phase, in the small compliance approximation (see formula 3.3a), the phase of the reflected P wave coefficient is always 90° whereas in the exact case, it is not -- it varies with $\varepsilon$ (see formula 3.2a and Figure 4.2). To get a better approximation for the phase, a second-order expansion is required. On the other hand, the exact phase does tend to get closer to 90° as $\varepsilon$ get smaller.

As mentioned above, the small $c_x$ and $c_z$ approximation formulas are valid for $\varepsilon \ll 1$, where $\varepsilon$ is the dimensionless parameter $\varepsilon = \rho \beta \omega c_x$ or $\varepsilon = \rho \alpha \omega c_z$. If the frequency is small enough, the approximation formulas hold even for larger values of the specific compliances $c_x$ and $c_z$. 
Figure 4.2. The amplitudes and phases of the reflected P wave coefficient, \( P \), for a nonwelded interface separating two identical media. The graphs compare the exact result (Figure 4.1a) with the approximation for small compliances and the approximation for small compliances and small \( p \). The medium parameters are the same as those used in Figure 4.1.

(a) The calculation at the frequency 10 Hz (\( \varepsilon \approx 1 \)).
(b) The calculation at the frequency 5 Hz (\( \varepsilon \approx 0.5 \)).
Figure 4.2 confirms this result: the accuracies of the approximations for 5 Hz are greater than those for 10 Hz, meaning that one could use larger compliance values in the 5 Hz case and still maintain an accuracy level greater than or equal to that of the 10 Hz case. This result applies to all the coefficients.

4.3. Numerical examples: a nonwelded contact interface between two different media

This section presents examples of the coefficients in the case of a nonwelded contact interface separating two different media. The phase velocities in the two media are $\alpha_1 = 2500$ m/s, $\beta_1 = 1250$ m/s, $\alpha_2 = 3200$ m/s, and $\beta_2 = 1550$ m/s. The density in both media is 2300 kg/m$^3$.

Figures 4.3 and 4.4 show the particle displacement reflection and transmission coefficients in the case of a P wave incident from medium one. For comparison purposes, the coefficients for the case of welded contact are plotted as well (the dotted lines in the figures). The exact coefficients for a nonwelded interface (equations 2.13 or 2.24) are plotted as thin solid lines for the frequencies 72 and 36 Hz in Figures 4.3 and 4.4, respectively. The thick solid lines represent the small $c_x$ and $c_z$ approximations of the coefficients (equations 2.39). In the same way as in Section 4.1, the effect of a nonwelded interface depends on the dimensionless terms $\omega c_x \rho_2 \beta_2$ and $\omega c_z \rho_2 \alpha_2$ (denoted by $\varepsilon$ in Section 4.1). Small ($<<1$) values for these terms signifies a small amount of nonwelded contact (see equation 2.42). Both of these dimensionless terms have a value of about 1 in Figure 4.3, and about 0.5 in Figure 4.4. The strong effects of a nonwelded contact interface on the amplitude and phase of the coefficients are shown in Figure 4.3 for a high frequency and high value of the dimensionless $\varepsilon$ terms.
Figure 4.3. Four coefficients for the case of a P wave incident in medium one. The phase velocities in the two media are $\alpha_1 = 2500 \text{ m/s}$, $\beta_1 = 1250 \text{ m/s}$, $\alpha_2 = 3200 \text{ m/s}$ and $\beta_2 = 1550 \text{ m/s}$. The density in both media is 2300 kg/m$^3$. The dotted line is for the case of a welded contact interface ($c_x, c_z = 0$). The thin solid line is for a nonwelded contact interface with $c_x = 6 \times 10^{-10} \text{ m/Pa}$ and $c_z = 3 \times 10^{-10} \text{ m/Pa}$, and a frequency of 72 Hz ($\varepsilon \approx 1$), and is computed using the exact formulas. The thick solid line is the small compliances approximation for nonwelded contact (with the same values of $c_x, c_z$ and frequency).
Figure 4.4. The same as Figure 4.3 except the frequency for the nonwelded contact calculations (the thin and thick solid lines) is 36 Hz ($\varepsilon \approx 0.5$).
When the frequency is decreased, the effect of a nonwelded interface is also decreased. This is indicated by the fact that the solid lines (nonwelded contact) are closer to the dotted lines (welded contact) in Figure 4.4 than in Figure 4.3. For the higher frequency, i.e., the higher $\varepsilon$ (Figure 4.3), the change in amplitude and phase, due to nonwelded contact, for the reflected P wave is large compared to the corresponding change for the transmitted P wave. In other words, a nonwelded interface has a larger effect on the reflected P wave than on the transmitted P wave. This is also true, to a lesser extent, for the lower frequency (Figure 4.4). For the reflected P wave in Figure 4.4a (the smaller frequency, i.e., the smaller $\varepsilon$), the effect of the nonwelded interface on the amplitude of the coefficient is relatively small, however, there is still a significant effect on the phase, i.e., a phase shift of about 90 degrees for incidence angles up to about 50 degrees.

In practice, the specific compliances of a nonwelded interface in the subsurface are likely to be small. Consequently, it would be easier to detect such a nonwelded interface with higher frequencies (higher frequencies mean higher $\varepsilon$ values). Detection would involve observations of reflection event amplitudes which are significantly different (because of nonwelded contact) from those expected. On the other hand, phase shifts due to nonwelded contact are quite significant, even for lower frequencies, i.e., lower $\varepsilon$ values (see Figure 4.4a, which shows a phase shift of about 90 degrees), and could be used to detect a nonwelded interface even when frequencies are lower.

For the converted P→SV wave in Figures 4.3c and 4.4c, the effect of a nonwelded interface makes the phases of the coefficients about 90° different from those in the case of a welded interface, and there is a very strong change in the amplitude, even in Figure 4.4c in which the values of $\varepsilon$ terms are small.

Consider the plots computed from the small $c_x$ and $c_z$ approximation (equations 2.39), i.e., the thick solid lines in Figures 4.3 and 4.4. The approximations are quite good in both amplitude and phase when the value of $\varepsilon$ is small (Figure 4.4). Inspection of the phase of the reflected P wave in Figures 4.3a and 4.4a shows that the approximate curves
agree better with the exact calculation curves when compared to the case of a nonwelded interface in a single homogeneous medium, as the phase values are 90° for all incidence angles (equation 3.3a and Figure 4.2).
5.1. Introduction

Synthetic seismograms are theoretically-calculated seismometer recordings showing the waveforms corresponding to the various seismic event arrivals for a given source-receiver geometry and a given geological model of the subsurface. They are produced by "synthesizing" or generating a composite of the waveforms for the various reflection events and the frequencies making up those waveforms. Synthetic seismograms are very useful for studying wave propagation in the Earth and for interpreting recorded seismic data. There are several common methods that are used for the calculation of synthetic seismograms. One such method involves raytracing, and is used in this chapter. The following definition of raytracing is given in the Encyclopedic Dictionary of Exploration Geophysics (Sheriff, 1990): “determining the arrival time at detector locations by following raypaths which obey Snell’s law through a model for which the velocity distribution is known”. However, most geophysicists involved in raytracing would also include the computation of ray amplitudes, not just arrival times, in the definition, as well as the determination of the actual raypaths. For a given source-receiver distance, an initial and important element of raytracing is to find the take-off angle or ray parameter for the ray which goes from the source to the known receiver location, while obeying Snell’s law of reflection and transmission at all the interfaces. Sections 5.2 discuss the basic raytracing algorithm used in this chapter. The geological model used for the ray tracing computations done in this thesis consists of a horizontally layered medium
in which one or more of the interfaces is nonwelded. As mentioned in the previous section, a nonwelded contact interface has significant effects on wave propagation in that both the amplitude and phase of waveforms are changed. In addition, the presence of a nonwelded contact interface makes the reflection and transmission coefficients frequency-dependent. For ideal elastic media, the calculations of the waveforms in the synthetic seismograms can normally be done in the time domain, because the reflection and transmission coefficients (involved in the waveform amplitude computations) are independent of frequency. However, for non-ideal media in which these coefficients depend on frequency, such as viscoelastic media, or as in this thesis, media containing nonwelded interfaces, the waveforms must be computed by a sum over frequencies (an inverse FFT) in order to include the effect of the frequency-dependence of the coefficients.

In this thesis, FORTRAN is used in the source code to generate ray-synthetic seismograms for a medium consisting of a sequence of flat horizontal homogeneous elastic layers with some of the layers being in nonwelded contact with each other at the interfaces. The frequency spectrum of the source signal is first obtained. Then the ray is traced for each frequency within the frequency range covered by the source pulse, the reflection and transmission coefficients and the geometrical spreading factor are applied, and finally the seismic trace in the time domain is generated by summing over the frequencies, i.e., by applying an inverse Fourier transform. The results are shown in Section 5.4.

5.2. Ray theory for homogeneous horizontally multilayered media

The particle displacement at location \( x \) and time \( t \) produced by a wave traveling in a heterogeneous medium has the form

\[
U(x,t) = U(x)f(t-T)d(x)
\]  

(5.1)
where \( t = \tau(x) \) is the wavefront equation, with \( \tau(x) \) being the traveltime, \( x = (x, y, z) \) is the position vector, \( U(x) \) is the amplitude, and \( d(x) \) is the particle motion unit vector, i.e., the polarization vector. The function \( f(t - \tau) \) is the waveform of the signal at \( x \). For a point source in a homogeneous elastic medium, the function \( f(t) \) is also the waveform of the source pulse \( s(t) \), but in general, the waveforms at the source and receiver are different. The displacement (5.1) satisfies the equation of motion and is the leading term of the ray series (Cerveny, Molotkov, and Psencik, 1977).

If (5.1) is substituted into the equation of motion, \( \sigma_{ij} = \rho(\partial^2/dt^2)u_i \), and if the high frequency approximation, or weak heterogeneity approximation, is made, then one obtains the eikonal equation, i.e.,

\[
\nabla \tau^2 = v^{-2}
\]

(5.2)

\( v \) is the velocity (P or S) and \( \nabla \tau \) is the slowness vector, which is normal to the wavefront.

In order to trace the ray, and calculate its amplitude, quantitative expressions for \( \tau(x) \) and \( U(x) \) are needed. These are given in the following sections.

5.2.1. Ray geometry and traveltime

The raypath is the trajectory followed by the traveling wavefront as it propagates through the medium. The ray is normal to the wavefront, and a given point on the raypath has the coordinates \( x_i(s) \) where \( x = (x_1, x_2, x_3) = (x, y, z) \), and where \( s \) is the arc length along the raypath. Referring to figure 5.1, the wavefront takes a time \( dt \) to travel the distance \( ds \) between points A and B. Points A and B have the position vectors \( x \) and \( x + dx \), respectively. Consequently, one can see that the distance \( ds \) has the same magnitude as \(|dx|\), hence,

\[
ds = |dx| \quad \text{i.e.,} \quad ds^2 = dx^2 + dy^2 + dz^2
\]
implying

$$\left( \frac{dx}{ds} \right)^2 + \left( \frac{dy}{ds} \right)^2 + \left( \frac{dz}{ds} \right)^2 = 1. \quad (5.3)$$

From equation 5.3, we have

$$\frac{dx}{ds} \cdot \frac{dx}{ds} = 1, \quad (5.4)$$

where \( \frac{dx}{ds} \) is the unit vector normal to the wavefront and parallel to the raypath at A.

Figure 5.1. The ray geometry, as the wavefront advances from A to B, over a distance \( ds \) or \( dx \) in a time \( d\tau \).
Consider the eikonal equation, (5.2):

\[ \nabla \tau^2 = \frac{1}{v^2} \]

Rearranging yields

\[ v \nabla \tau \cdot v \nabla \tau = 1. \] \hfill (5.5)

This means that \( v \nabla \tau \) is also a unit vector normal to the wavefront. Hence, equations (5.4) and (5.5) give

\[ \frac{dx}{ds} = v \nabla \tau. \] \hfill (5.6)

These are the "normal" equations, or the equation for the ray geometry.

As the wavefront advances from point A to B with a velocity \( v \) in a time \( d\tau \), the distance traveled by the wave is \( ds = vd\tau \). Substituting this distance \( ds \) into the normal equations (5.6) yields

\[ \frac{dx}{d\tau} = v^2 \nabla \tau \] \hfill (5.7)

or, in terms of components,

\[ \frac{dx_i}{d\tau} = v^2 \frac{\partial \tau}{\partial x_i}, \quad i = 1, 2, \text{ and } 3. \]

These equations can be used to trace the ray. For a vertically heterogeneous medium, it can be assumed that the wave travels in the \( x-z \) plane only. In this case, equation (5.7) becomes

\[ \frac{dx}{d\tau} = v^2 S_x, \] \hfill (5.8a)

and

\[ \frac{dz}{d\tau} = v^2 S_z, \] \hfill (5.8b)
where \( \mathbf{x} = (x_1, x_2, x_3) = (x, y, z) \), \( S_x \) is the horizontal component of the slowness vector, i.e., the ray parameter \( p \) which is constant along the ray trajectory, and \( S_z \) is the vertical component of the slowness. In terms of ray angles, the definitions for \( S_x \) and \( S_z \) are

\[
S_x = p = \frac{\sin \theta}{v},
\]

and

\[
S_z = \frac{\cos \theta}{v} = \frac{\sqrt{1 - p^2 v^2}}{v},
\]

where \( \theta \) is the angle that the raypath makes with the z-axis (the direction normal to the interface).

Substituting equations (5.9) and (5.10) into (5.8), we get

from (5.8b),

\[
d\tau = \frac{dz}{v \sqrt{1 - p^2 v^2}},
\]

and from (5.8a),

\[
dx = v^2 p d\tau.
\]

Upon substituting \( d\tau \) from (5.11) into this last equation, it becomes

\[
dx = \frac{v pdz}{\sqrt{1 - p^2 v^2}}.
\]

For a flat-layered medium with homogeneous layers, the raypath within each layer (a ray segment) is a straight line because the velocity is constant in each layer. In this case, the ray equations (5.11) and (5.12) can be written as

\[
\tau_i = \frac{h_i}{v_i \sqrt{1 - p^2 v_i^2}};
\]

\[
x_i = \frac{v_i ph_i}{\sqrt{1 - p^2 v_i^2}}
\]
where the subscript $i$ is the number of $i$th ray segment (with the first segment leaving the source and the last entering the receiver), $d\tau \rightarrow \tau_i$ is the traveltime along the $i$th ray segment, $dz \rightarrow h_i$ is the thickness of the layer containing the $i$th ray segment, and $dx \rightarrow x_i$ is the horizontal distance spanned by the $i$th ray segment (see Figure 5.2). Each ray segment could be either a P or an SV wave.

If a source and receiver are on the surface of the horizontally layered medium, and if the ray is a primary ray (a ray consisting of only one reflection point at the base of the bottom layer), and if the down-going and up-coming raypaths are symmetric about a vertical axis through the reflection point, then they have the same traveltimes and span the same horizontal distance. Consequently, the total traveltime between source and receiver (the "two-way" time) can be written as

$$\tau = 2 \sum_{j=1}^{n} \tau_j = 2 \sum_{j=1}^{n} \frac{h_j}{v_j \sqrt{1 - \frac{v_j^2}{v_i^2}}},$$

(5.15)

and the distance $x$ between source and receiver is

$$x = 2 \sum_{j=1}^{n} x_j = 2 \sum_{j=1}^{n} \frac{p h_j}{v_j \sqrt{1 - \frac{v_j^2}{v_i^2}}},$$

(5.16)

where the subscript $j$ is the layer number, $n$ is the number of layers (or the number of the bottom or base layer), and $v_j$ is the wave speed (P or SV) associated with the two ray segments (down- and up-going) in the $j$th layer (see figure 5.2).

When the ray encounters an interface, as it propagates through the medium, it must obey Snell’s law. Referring to the angles shown in Figure 5.2, Snell's law states that

$$p = \frac{\sin \theta_{i-1}}{v_{i-1}} = \frac{\sin \theta_i}{v_i} = \frac{\sin \theta_{i+1}}{v_{i+1}},$$

(5.17)
where $\theta_i$ is the angle that the ray segment makes with the $z$-axis (the direction normal to the interface).

\[ \theta_{i-1} = \theta_i \quad \text{from Snell's law} \]

Figure 5.2. A diagram showing the ray trajectory as it travels through layer $n$ (the bottom or base layer). $i$ is the ray segment number.

### 5.2.2. Amplitude calculation

The particle displacement vector of the ray traveling in such a medium is given by equation (5.1), i.e.,

\[ U(x,t) = U(x)f(t - \tau)d(x). \]

For a layered medium with homogeneous layers, the final formula for the amplitude $U(x)$ is given by Cervený, Molotkov, and Psencik (1977) and Cervený and Ravindra (1971) as
where \( g_o \) is a quantity representing the directional characteristics of the source (for simplicity, we assume a spherically symmetric point source, for which \( g_o = 1 \)), \( R \) is the product of the reflection and transmission coefficients at the interfaces, and \( L \) is the geometrical spreading function. The phase \( \psi \) of \( R \) (which is generally a complex number, i.e., \( R = |R|e^{i\psi} \)) is implicitly incorporated in (5.1), resulting in a waveform \( f(t) \) at \( x \) which is, in general, different from the source waveform \( s(t) \). It can be shown (Aki and Richards, 1980; Cerveny, Molotkov, and Psencik, 1977; Cerveny and Ravindra, 1971) that when \( R, L \) and \( d \) are independent of frequency (as they are in elastic media with welded interfaces), \( f(t) \) can be written as

\[
f(t) = s(t) \cos \psi - H\{s(t)\} \sin \psi \tag{5.18}
\]

where \( H\{s(t)\} \) is the Hilbert transform of \( s(t) \). As can be seen, if \( \psi \) is other than 0 or \( \pi \) (i.e., if \( R \) is not real), then \( f(t) \) is different from \( s(t) \) in form.

In the conventional welded contact case, the signal at \( x \) can be computed in the time domain from the above formula and equation (5.1). However, for a medium containing nonwelded interfaces, \( R \) is explicitly dependent on frequency. Consequently, the computations of the ray amplitudes at \( x \) must account for the effects of this frequency dependence. To do this, it is necessary to determine the amplitude first in the frequency domain. To change from a time domain calculation into a frequency domain calculation, consider the Fourier time transform of the particle displacement (5.1), that is

\[
\tilde{U}(x, \omega) = U(x) \delta(x) \int_{-\infty}^{\infty} f(t - \tau)e^{i\omega t} \, dt
\]

\[
= U(x) \delta(x) \int_{-\infty}^{\infty} f(t - \tau)e^{i\omega(t-\tau)}e^{i\omega \tau} \, d(t - \tau)
\]
\[ \tilde{U}(x, \omega) = U(x) F(\omega) e^{i\omega t} d(x) \]  

(5.19a)

where \( \tilde{U}(x, \omega) \) is the frequency spectrum of the signal at \( x \), and \( F(\omega) \) is the frequency spectrum of the waveform at \( x \). For our layered medium, the difference between the waveforms at the source and receiver is caused by the phase shift \( \psi \) (due to \( R \)) experienced by the wave. In mathematical terms, \( F(\omega) = S(\omega) e^{i\omega} \). Inserting this into equation (5.19) yields

\[ \tilde{U}(x, \omega) = U_c(x) S(\omega) e^{i\omega t} d(x), \quad \text{where} \quad U_c(x) = g \frac{R}{L} \]  

(5.19b)

and where \( R \) is a function of frequency \( \omega \) if nonwelded interfaces are present. The inverse Fourier transform (over \( \omega \)) of equation (5.19b) gives the displacement \( U(x,t) \) at the receiver. As \( R \) must now be included in the integral for the inverse transform, equation (5.18) no longer applies. As a side remark, if the medium was anelastic (with welded interfaces, say), then \( R, L \) and \( d \) would all be complex and frequency-dependent, and would all have to be included in the integral over frequency.

As can be seen from equation (5.19b), the computation of the ray amplitude involves the calculation of the product \( R \) of the reflection and transmission coefficients, the geometrical spreading factor \( L \), the polarization vector \( d \), and the frequency spectrum \( S(\omega) \) of the source pulse \( s(t) \). In addition, if the receiver is on the surface, then it responds to both the incoming wave and the waves reflected by the surface, meaning that \( d \) must be replaced by a vector accounting for this "free surface effect" (see below).

**The reflection and transmission losses**

The product \( R \) of the reflection and transmission coefficients is defined as

\[ R = \prod_{i=1}^{m-1} R_i \]  

(5.20)
where \( m \) is the total number of ray segments in the ray, and \( R_i \) is the reflection or transmission coefficient for the incident wave represented by the \( i \)-th ray segment, i.e., the coefficient connecting ray segments \( i \) and \( i+1 \).

At the welded interfaces, the solutions of the conventional Zoeppritz equations can be used to compute the coefficients \( R_i \). However, at the nonwelded interfaces, the coefficients \( R_i \) are computed from the formulas in equation (2.13). These formulas reduce to the solutions of the Zoeppritz equations when the specific compliances are zero.

**The geometrical spreading losses**

The geometrical spreading function \( L \) can be calculated for a medium consisting of a sequence of horizontal homogeneous layers. It is given by (Cerveny and Ravindra, 1971)

\[
L = \frac{\cos \theta_1}{v_1} \left[ \left( \sum_{i=1}^{m} \frac{v_i h_i}{\cos \theta_i} \right) \left( \sum_{i=1}^{m} \frac{v_i h_i}{\cos^2 \theta_i} \right) \right]^{1/2} \tag{5.21}
\]

where \( \theta_1 \) is the angle that the \( i \)-th ray segment makes with the vertical, and \( v_i \) and \( h_i \) are the velocity and the thickness of the layer in which the \( i \)-th ray segment is found, respectively. The velocity \( v_i \) is either the P or SV wave speed, depending on whether the \( i \)-th ray segment is a P or SV wave.

**The surface conversion coefficients**

If the receiver is on the surface, it will be affected by both the incoming wave (associated with the last ray segment) and the reflected waves (see Figure 5.3). Consequently, the particle displacement at the receiver will be different from that given by the \( \mathbf{d} \) vector for the incoming wave (Cerveny and Ravindra, 1971, p.66-68).
From equation (5.19b), the displacement $\vec{U}$ at the receiver for a given frequency can be written as $\vec{U} = \vec{U}d$, where $\vec{U} = U_c S(\omega)e^{i\omega t}$ and $U_c = g_0(R/L)$. The displacement for each reflected wave will differ from that for the incoming wave only by the surface reflection coefficient and the direction of particle motion. Hence, the total displacement $\vec{U}$ at the receiver in Figure 5.3 is

$$\vec{U} = \vec{U}_I + \vec{U}_P + \vec{U}_S = \vec{U}_I d_I + R_P \vec{U}_I d_P + R_S \vec{U}_I d_S = \vec{U}_I g \quad (5.22a)$$

where

$$g = d_I + R_P d_P + R_S d_S \quad (5.22b)$$

and where the subscripts $I$, $P$ and $S$ denote the incoming, reflected P and reflected SV waves at the receiver, respectively. The incoming wave is associated with the last or $m$th ray segment. $R_P$ and $R_S$ are the free surface reflection coefficients for the reflected P and SV waves (they satisfy the boundary conditions $\sigma_x=\sigma_z=0$ at the surface), and $d_I$, $d_P$, and $d_S$ are the polarization vectors for the incoming wave and the reflected P and SV waves, respectively. $R_P$ and $R_S$ can be obtained from equations (2.13k) and (2.13l) for an incoming P wave and from equations (2.13o) and (2.13p) for an incoming SV wave by setting $c_x=c_z=\rho_1=\alpha_1=\beta_1=0$ in these equations. Equation (5.22) shows that the displacement at the receiver can be written in terms of $\vec{U}_I$ as long as $d_I$ is replaced by $g$, the surface conversion vector. Note that $g$ is not a unit vector. Note also that $g$ can be a complex vector -- for an SV wave incident at an angle greater than the critical angle for the reflected P wave, $R_P$, $R_S$, and $d_P$ are complex (because $\cos \theta$ is imaginary), making $g$ complex as well.

For example, for an incoming P wave, from Table 2.1, the expressions for the particle motion unit vectors are

$$d_I = (\alpha p, 0, -\cos \theta) \quad d_P = (\alpha p, 0, \cos \theta) \quad d_S = (\cos \phi, 0, -\beta p). \quad (5.23)$$

where $p = (\sin \theta)/\alpha = (\sin \phi)/\beta$. 

The cosines can also be written as $\cos \theta = \sqrt{1-\alpha^2 p^2}$ and $\cos \phi = \sqrt{1-\beta^2 p^2}$. For an evanescent P wave ($p>1/\alpha$, in the case of an incident SV wave), $\cos \theta = \pm i \sqrt{\alpha^2 p^2 - 1}$ where the positive (negative) sign is chosen if the frequency is positive (negative). Hence, the components of $g$ for an incoming P wave are given by

$$g_x = \alpha p + R_p \alpha p + R_s \cos \phi$$

$$g_z = -\cos \theta + R_p \cos \theta - R_s \beta p$$

(5.24a)  

(5.24b)

These are called conversion coefficients by Cerveny and Ravindra (1971). As mentioned above, $R_P$ and $R_S$ can be obtained from equations (2.13k) and (2.13l), or from the standard expressions found in various texts (e.g., Aki and Richards, 1980, eq. 5.26 and 5.27). They are given by

$$R_p = \frac{(4\beta^2 p^2 r \cos \theta \cos \phi - (1 - 2\beta^2 p^2)^2) / D}{D},$$

(5.25a)

and

$$R_s = \frac{4(1 - 2\beta^2 p^2)p \beta \cos \theta / D}{D},$$

(5.25b)

where $D = (1 - 2\beta^2 p^2)^2 + 4\beta^2 p^2 r \cos \theta \cos \phi$, and $\beta = \beta / \alpha$.

---

**Figure 5.3.** A receiver on the surface records the displacement $\vec{U} = \vec{U}_I + \vec{U}_P + \vec{U}_S = \vec{U}_I + \mathbf{g}$, where $\mathbf{g}$ is called the surface conversion vector.
Substituting (5.25) into (5.24), we obtain

\[ g_x = 4 \beta p \cos \theta \cos \phi / D \quad (5.26a) \]

and

\[ g_z = -2 \left( 1 - 2 \beta^2 p^2 \right) \cos \theta / D. \quad (5.26b) \]

These are the same as those given by Cerveny and Ravindra (1971, eq. 2.89) except for the minus sign on the right side of (5.26b) -- their \( z \) axis points upwards.

Incorporating the free surface effect into the amplitude at the receiver amounts to replacing, in equation (5.19b), the polarization vector \( d \) associated with the last ray segment with the surface conversion vector \( g \). One obtains

\[ \bar{U}(x, \omega) = U_c(x) S(\omega) e^{i \omega r} g(x). \]  

\textbf{The waveform of the source pulse}

The waveform \( f(t) \) at the receiver can be computed from the source pulse \( s(t) \) using equation (5.18) if \( R \) is independent of frequency (i.e., if all the interfaces are welded). In this case, it is not necessary to know the source pulse's frequency spectrum \( S(\omega) \) -- in principle, only \( s(t) \) is required (although in practice, the Hilbert transform can sometimes be more easily obtained if the spectrum \( S(\omega) \) is known). However, if \( R \) is dependent on frequency, then the waveform at the receiver can be computed from the inverse Fourier transform of equation (5.27), meaning that \( S(\omega) \) is required.

Figure 5.4 shows one possible waveform (out of many) for the source pulse. The frequency spectra are shown in the top row and their corresponding time-waveforms (the wavelets obtained from an inverse Fourier transform) are shown in the bottom row. The source waveform is the causal pulse used by Emmerich and Korn (1987), given by

\[ s(t) = \sin(\omega_o t) - 0.5 \sin(2\omega_o t), \quad 0 \leq t \leq (2 \pi / \omega_o) \]  

\textbf{(5.28)}

where \( \omega_o = 2\pi f_o \) and \( f_o \) is the dominant frequency.
Figure 5.4. The causal pulse used by Emmerich and Korn (1987) with (a) $f_o = 20$ Hz, and (b) $f_o = 10$ Hz. The upper figures show the amplitude and phase of the spectrum $S(\omega)$ and the lower ones show the pulse $s(t)$ in the time domain.
Its Fourier transform is

\[
S(\omega) = \begin{cases} 
\frac{1\pi}{\omega_o} & \text{when } \omega = \omega_o \\
-\frac{1\pi}{2\omega_o} & \text{when } \omega = 2\omega_o \\
-6i\omega_o^3 \exp\left(\frac{1\pi \omega/\omega_o}{\omega^2 - 4\omega_o^2}\right) \sin\left(\frac{\pi \omega/\omega_o}{\omega^2 - \omega_o^2}\right) & \text{when } \omega \neq 2\omega_o \text{ and } \omega \neq \omega_o.
\end{cases}
\]

(5.29)

In Figure 5.4, the top diagram shows the amplitude spectrum of (5.29) for a dominant frequency of 20 Hz and 10 Hz in Figure 5.4a and 5.4b, respectively, with the phase spectrum shown as an insert, and the bottom diagram shows the corresponding wavelet (5.28).

The synthetic raytracing seismogram in the next section will use the source spectrum in Figure 5.4a for choosing the specific compliances at a given dominant frequency. However, any input source function could be used in the frequency domain raytracing program as long as the frequency spectrum of the source function is known.

### 5.3. Procedure for the calculation in the frequency domain

The frequency domain raytracing is performed as follows.

1) The initial conditions of the model, the medium properties, the thicknesses of the layers, and the source-receiver distances are chosen.

2) The source signal is generated in the frequency domain, i.e., \(S(\omega)\), the Fourier transform of \(s(t)\), the source pulse, is computed.

3) Trace calculation

   - The value of the ray parameter \(p\) for a given offset \(x\) is computed for each ray by solving equation (5.16). A simple Newton-Raphson root-finding
algorithm is used. If desired, the takeoff angle can be computed from this $p$ value by using Snell's law, equation (5.17). Equation (5.15) is then used to compute the traveltime $\tau$ for the ray.

- For each frequency within the range of the frequencies under consideration (the frequencies contained in $S(\omega)$), the product of the reflection and transmission coefficients, equation (5.20), is computed. This product is frequency dependent if nonwelded interfaces are present. Also, the geometrical spreading factor, equation (5.21), is computed, which is frequency independent for elastic media.

- The phase shift term $e^{i\omega r}$ is computed.

- The $z$ component of the particle displacement unit vector $d_z$ is computed at the receiver, or, if it is needed (if the receiver is on the surface), the surface conversion coefficient $g_z$ at the receiver is computed.

4) Computation of the wave form frequency spectrum $\tilde{U}(x,\omega)$
   - Equation (5.19b) is used when the receiver is not on a free surface (e.g., if it is inside a homogeneous half-space).
   - Equation (5.27) is used when the receiver is on the free surface.

5) The particle displacement, $U(x,t)$, in time domain is constructed by application of the inverse Fourier transform of the frequency spectrum $\tilde{U}(x,\omega)$.

6) Scaling is applied, if it is needed.
5.4. Raytracing synthetic seismogram examples

The subsurface models that are used for investigating the effects of nonwelded contact interfaces are shown in Figure 5.5. Model A, shown in Figure 5.5a, is used to study the case of a nonwelded contact interface in a single homogeneous medium. Model B, shown in Figure 5.5b, is used to study the effects of a sequence of nonwelded interfaces in a single homogeneous medium. Model C, shown in Figure 5.5c, is used to study the effects of the presence of nonwelded contact interfaces between two different media. The source pulse spectrum of Emmerich and Korn (1987), given in equation (5.29), was used with a dominant frequency of 20 Hz -- see Figure 5.4a. The density for every layer in all the models is 2300 kg/m$^3$. For models A and C, the synthetic seismograms have a time sample rate of 2 ms and a trace interval of 50 m. For model B, the time sample rate is 2ms and the trace interval is 75 m. The maximum offsets for models A, B, and C are 2500, 3600 and 1250 m, respectively.

The selection of the numerical values for the specific compliances $c_x$ and $c_z$ in models A and B is based on the dimensionless parameter $\varepsilon = \rho \beta \omega c_x = \rho \alpha \omega c_z$, with $c_z = c_x / 2$ (see Section 4.1), where $\omega = 2\pi f$ and $f$ is the frequency of the signal. The upper homogenous layer in both models has a velocity of 2800 m/s for P waves and 1400 m/s for S waves. A frequency of 60 Hz is used to evaluate the specific compliances (60 Hz is about the maximum frequency in the source pulse spectrum of Figure 5.4a, if we ignore the small side lobe between 60 and 80 Hz). The dimensionless parameter $\varepsilon$ is set to 1, 0.5, and 0.1, and with a frequency of 60 Hz we then have $c_x = 8.24 \times 10^{-10}$, $c_x = 4.12 \times 10^{-10}$, and $c_x = 8.24 \times 10^{-11}$ m/Pa, respectively. The reason for using a frequency of 60 Hz instead of the dominant frequency (20 Hz) is to make sure that the parameter $\varepsilon$ stays $\leq$ 1, 0.5, or 0.1, for all frequencies in the range of consideration in each case. If the dominant frequency was used, then from Figure 5.4a, the right side of the amplitude spectrum (in the frequency range 21-60 Hz) would make the parameter $\varepsilon$ (for frequencies in that range) have higher values than the original values of 1, 0.5 and 0.1, in
each case. In other words, the values of 1, 0.5 and 0.1 are chosen to be the maximum values of $\varepsilon$.

Figure 5.6a shows the synthetic seismograms for model A1, in which there is no nonwelded interface in the upper homogeneous layer -- this is used as a reference. Figures 5.6b, 5.6c and 5.6d are the synthetic seismograms for model A2, with the specific compliances for the nonwelded interface corresponding to $\varepsilon \approx 1$, $\varepsilon \approx 0.5$, and $\varepsilon \approx 0.1$, respectively. Figures 5.7a, 5.7b, 5.7c and 5.7d show magnified views of the trace at 600m offset from Figures 5.6a, 5.6b, 5.6c and 5.6d, respectively, for closer inspection. The synthetic seismograms for model A2 show that there is reflected energy coming from the nonwelded contact interface, with the first event (P-P) arriving at a time of 357 ms, even though there is no impedance contrast. A phase change in the P-P waveform of about 90 degrees (see Figure 5.7), compared to the reference reflection, is also present. This phase shift is predicted by the theoretical formulas of the previous chapters.

The amplitude of the reflection from the nonwelded contact interface is dependent on the dimensionless parameter $\varepsilon$. The effect of the nonwelded interface is diminutive when $\varepsilon$ is about 0.1, i.e., when the specific compliance of the nonwelded interface is $c_x = 8.24 \times 10^{-11}$ m/Pa, in model A2. In order to detect a nonwelded interface, or fracture etc., with a specific compliance of about $10^{-11}$ m/Pa or, equivalently, a specific stiffness $\kappa$ of about $10^{10}$ Pa/m, higher frequencies, of the order of 100 Hz, are required (Pyrak-Nolte, 1988). Figure 5.6 confirms this. The effect of the nonwelded interface on amplitude and phase is very strong when the parameter $\varepsilon$ is of the order of 1 or more. The chosen values of the specific compliances of the interface are for the highest frequency in the amplitude spectrum of the waveform. That means that for the rest of the frequencies in the waveform spectrum, the actual value of $\varepsilon$ is smaller. In particular, at the dominant frequency, 20 Hz, of the source pulse, the value of $\varepsilon$ is of the order of 0.1.
Figure 5.5. Subsurface models and their configurations. The density in all layers is 2300 kg/m$^3$. 
Figure 5.6. (a) The generated seismograms for model A1, and (b)-(d) the corresponding seismograms for model A2, a single nonwelded interface in a homogeneous medium, with the specific compliances of the nonwelded contact interface being for (b), $c_x = 8.24 \times 10^{-10} \text{ m/Pa (} \varepsilon \approx 1\text{)}$, for (c), $c_x = 4.12 \times 10^{-10} \text{ m/Pa (} \varepsilon \approx 0.5\text{)}$, and for (d), $c_x = 8.24 \times 10^{-11} \text{ m/Pa (} \varepsilon \approx 0.1\text{)}$. 
Figure 5.7. Zoom-in views of the trace at 600m offset from the synthetic seismograms in Figure 5.6. The traces in (a), (b), (c) and (d) are close-up views from Figures 5.6a, 5.6b, 5.6c and 5.6d, respectively.
The plot of the P wave reflection coefficient, Figure 4.1a, shows that the amplitude is small and the phase is about 90° for $\varepsilon \approx 0.1$ and Figure 4.1c shows that the transmission coefficient is very close to that for the case of a welded contact interface, i.e., an amplitude of one and zero phase. This is consistent with the observation that the reference reflection for model A2 shown in Figure 5.6 shows no effect from the transmitted wave coming from the nonwelded interface (there is no obvious change in the second event at the time 714 ms).

In Figures 5.6 and 5.7, the converted P-SV wave from the nonwelded reflector is also present as expected with a smaller amplitude which is dependent on the parameter $\varepsilon$, and a phase shift about 90° compared to the reference reflection.

The synthetic seismograms for model B are shown in Figure 5.8. Figure 5.8a shows the response for model B1, in which there are no nonwelded interfaces, and Figures 5.8b-d show the response for model B2, in which there is a sequence of five nonwelded interfaces within the upper homogeneous layer, each with the same value of the normal and tangential specific compliances. These values are $c_x = 8.24 \times 10^{-10}$ m/Pa ($\varepsilon \approx 1$), $c_x = 4.12 \times 10^{-10}$ m/Pa ($\varepsilon \approx 0.5$), and $c_x = 8.24 \times 10^{-11}$ m/Pa ($\varepsilon \approx 0.1$) for Figures 5.8b, 5.8c, and 5.8d, respectively (with $c_z = c_x/2$, as before).

In Figure 5.8b, where the parameter $\varepsilon$ is large ($\approx 1$), it can be seen that the amplitudes of the reflections from the nonwelded interfaces are relatively high compared to those from the lowermost welded interface (the lowest reflection), and also that the phases of the waveforms are substantially different. These high amplitudes and phase changes are due to the effect of all the nonwelded reflection and transmission coefficients taken together. In Figure 5.8c where $\varepsilon \approx 0.5$, the phase shift of about 90° in each waveform coming from a nonwelded reflector is mainly due to the reflection coefficient. In Figure 5.8d, where $\varepsilon \approx 0.1$, the effects of the nonwelded interfaces are very small. For small specific compliances, higher frequencies are required to detect nonwelded interfaces, as mentioned above.
Figure 5.8. (a) The synthetic seismograms for model B1 in which there is one welded interface separating two homogeneous layers, and (b)-(d) the corresponding synthetic seismograms for model B2, in which there is a sequence of nonwelded contact interfaces within the upper homogeneous layer, above the welded interface. The specific compliance values are, for (b), $c_x = 8.24 \times 10^{-10} \text{ m/Pa} \ (\varepsilon \approx 1)$, for (c), $c_x = 4.12 \times 10^{-10} \text{ m/Pa} \ (\varepsilon \approx 0.5)$, and for (d), $c_x = 8.24 \times 10^{-11} \text{ m/Pa} \ (\varepsilon \approx 0.1)$. 
Figure 5.9. The corresponding synthetic seismograms for model B2, in which there is a sequence of nonwelded contact interfaces within the upper homogeneous layer, above the welded interface. The specific compliance values are $c_x = 8.24 \times 10^{-10}$ m/Pa and $c_z = 4.12 \times 10^{-10}$ m/Pa. The dominant frequencies for the source pulse are (a) 40 Hz ($\varepsilon \approx 2.0$), (b) 20 Hz ($\varepsilon \approx 1.0$), (c) 10 Hz ($\varepsilon \approx 0.5$) and (d) 5 Hz ($\varepsilon \approx 0.25$).
Figure 5.10. The zero offset traces from the synthetic seismograms in Figure 5.8. The traces in (a), (b), (c) and (d) are close-up views from Figures 5.8a, 5.8b, 5.8c and 5.8d, respectively.
Figure 5.11. The zero offset traces from the synthetic seismograms in Figure 5.9. The traces in (a), (b), (c) and (d) are close-up views from Figures 5.9a, 5.9b, 5.9c and 5.9d, respectively.
Figure 5.9 shows synthetic seismograms generated for model B2 for a variety of dominant frequencies of the source pulse. In this figure, $c_x = 8.24 \times 10^{-10}$ m/Pa and $c_z = c_x / 2 = 4.12 \times 10^{-10}$ m/Pa for all the nonwelded interfaces, and dominant frequencies of 40 Hz, 20 Hz, 10 Hz and 5 Hz are used for the source pulse in Figure 5.9a ($\varepsilon \approx 2.0$), 5.9b ($\varepsilon \approx 1.0$), 5.9c ($\varepsilon \approx 0.5$) and 5.9d ($\varepsilon \approx 0.25$), respectively. Figure 5.9a shows an attenuation of the amplitude along the traces. The amplitude becomes very small for the last reflector (it is hard to see the reflection event at 1.1 sec.). When the parameter $\varepsilon$ is large, the effect of a nonwelded interface on an incident wave becomes more manifest; more of the energy of the wave is reflected and less of it is transmitted (see Figures 4.1a and 4.1c). If the parameter $\varepsilon$ is made even larger, almost all of the energy of the wave is reflected by the first nonwelded interface, and there is little or no energy transmitted through the interface.

Figures 5.10 and 5.11 show the zero offset trace from synthetic seismograms in Figures 5.8 and 5.9, respectively. These 2 figures show the "group time delay" effect discussed by Pyrak-Nolte et al. (1990a). In this effect, a wave pulse propagating across a nonwelded interface, such as a fracture, is delayed in time. In other words, the transmitted wave at a nonwelded interface experiences a traveltime delay. This effect can be seen by comparing the traveltime, at zero offset say, of the reference reflection from the welded reflector in Figure 5.10a with its counterpart in Figure 5.10b. Close inspection reveals a traveltime delay in Figure 5.10b of about 22 ms, due to the transmission of the wave through several nonwelded interfaces. In Figures 5.10c and 5.10d, the group traveltime delays are about 13 and 3 ms, respectively. They are smaller because there is less nonwelded contact in these cases (the value of $c_x$ is smaller).

The group traveltime delay produced by a nonwelded interface depends on the products of the specific compliances of the interface with the impedances of the medium (as in the parameter $\varepsilon$) and on the frequency as well. The group time delays are largest at low frequencies, and they decrease with increasing frequency (Pyrak-Nolte et al., 1987b). Figures 5.11c and 5.11d confirm these results. Figures 5.11c and 5.11d are synthetic
seismograms from model B2 with \( c_x = 8.24 \times 10^{-10} \) m/Pa and dominant frequencies of 10 Hz (see Figure 5.4b) and 5 Hz for the source pulse, respectively. Comparing Figure 5.11c and Figure 5.11d with Figure 5.11b (where the dominant frequency of the source pulse was 20 Hz), shows that the first peak of the waveform for the deepest reflection (from the welded interface) arrives about 42 ms later in the 10 Hz case and about 76 ms in the 5 Hz case (Figures 5.11c and 5.11d). A summary of these group traveltime delays is shown in Table 5.1.

<table>
<thead>
<tr>
<th>Figure ID</th>
<th>Frequency (Hz)</th>
<th>( c_x ) m/Pa</th>
<th>( \varepsilon )</th>
<th>Peak (s)</th>
<th>Time delay (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.10a</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>1.088</td>
<td>-</td>
</tr>
<tr>
<td>5.10b/5.11b</td>
<td>20</td>
<td>8.24 \times 10^{-10}</td>
<td>1.0</td>
<td>1.110</td>
<td>22</td>
</tr>
<tr>
<td>5.10c</td>
<td>20</td>
<td>4.12 \times 10^{-10}</td>
<td>0.5</td>
<td>1.101</td>
<td>13</td>
</tr>
<tr>
<td>5.10d</td>
<td>20</td>
<td>8.24 \times 10^{-11}</td>
<td>0.1</td>
<td>1.091</td>
<td>3</td>
</tr>
<tr>
<td>5.11a</td>
<td>40</td>
<td>8.24 \times 10^{-10}</td>
<td>2.0</td>
<td>1.096</td>
<td>8</td>
</tr>
<tr>
<td>5.11c</td>
<td>10</td>
<td>8.24 \times 10^{-10}</td>
<td>0.5</td>
<td>1.130</td>
<td>42</td>
</tr>
<tr>
<td>5.11d</td>
<td>5</td>
<td>8.24 \times 10^{-10}</td>
<td>0.25</td>
<td>1.164</td>
<td>76</td>
</tr>
</tbody>
</table>

Table 5.1. This table shows the group time delay of the bottom reflector in model B2, where “Peak” is the traveltime of the peak of the waveform.
Figure 5.12. The synthetic seismograms in (a), (b), (c), and (d) are those for models C1, C2, C3, and C4, respectively. The specific compliances for the nonwelded contact interfaces are $c_x = 6.0 \times 10^{-10}$ m/Pa and $c_z = 3.0 \times 10^{-10}$ m/Pa.
Figure 5.13. Zoom-in views of the trace at offset 450m from the synthetic seismograms in Figure 5.12. (a), (b), (c) and (d) are close-up views from Figures 5.12a, 5.12b, 5.12c and 5.12d, respectively.
Figure 5.12 shows the synthetic seismograms for model C, which consists of three different layers joined by two interfaces, which can be welded, nonwelded, or both -- see Figure 5.5c. The specific compliances are \( c_x = 6.0 \times 10^{-10} \text{ m/Pa} \) and \( c_z = 3.0 \times 10^{-10} \text{ m/Pa} \) for all the nonwelded interfaces, and they are, of course, zero for the welded interfaces. The specific compliance values for the nonwelded interfaces were selected to have an \( \varepsilon \) parameter value of about 0.1-1 (to keep it on the order of 1 in value). Figure 5.12a is the seismic response for model C1 in which both interfaces are in welded contact. Figures 5.12b, 5.12c, and 5.12d are the synthetic seismograms for models C2, C3, and C4, respectively. In Figure 5.12b, the second interface is nonwelded. In Figure 5.12c, it is the first interface that is nonwelded, and in Figure 5.12d, both interfaces are nonwelded. Figure 5.13 shows close-up views of the traces at 450m offset in Figure 5.12.

Inspection of Figures 5.12 and 5.13 shows that replacing a welded interface with a nonwelded one produces substantial amplitude and phase changes in the waveforms associated with the reflection from the interface, with the phase shifts being about 45°. For example, let us compare the first reflections in Figures 5.13a and 5.13c. These reflections come from the same interface except that in Figure 5.13a the interface is welded and in Figure 5.13c it is non-welded (see models C1 and C3, respectively, in Figure 5.5). Comparing these reflections, we can clearly see a phase shift of about 45°, and a noticeable amplitude increase in the nonwelded reflection. The same effect can also be seen by comparing the lowermost reflections in Figures 5.13a and 5.13b, with both the amplitude increase and phase change for the nonwelded reflection being more substantial (see models C1 and C2, respectively, in Figure 5.5). All the changes in the waveform are mostly due to the reflection coefficient -- see Figures 4.3a and 4.3b (\( \varepsilon \approx .3 \) at the frequency 20 Hz). In this case, the nonwelded interfaces have a small effect on the transmitted P wave. In Figures 5.12b and 5.12d, the group travel time delays are too small to be detected.

The converted P-SV reflected wave from the first interface can also be seen in Figure 5.12. It is the second reflection, i.e., the low amplitude reflection around 0.8 s
lying between the two major reflections. In comparing the converted wave reflection in Figure 5.12a or 5.12b, in which the first interface is welded, with the converted wave reflection in Figure 5.12c or 5.12d, in which the first interface is nonwelded, shows that substantial amplitude and phase changes are produced by the nonwelded contact. The converted wave amplitudes are larger for the nonwelded interface and the phase shifts are about 90°. This can be seen more clearly in Figure 5.13 -- comparing Figures 5.13a or 5.13b with Figures 5.13c or 5.13d shows that the nonwelded reflections have higher amplitudes and are shifted in phase by about 90° (they are approximately zero-phase wavelets).

All of the effects of a nonwelded interface discussed above, i.e., the unexpected and substantial changes in the amplitudes and phases of the reflection waveforms, and the group traveltime delays, could be used as evidence of the presence of nonwelded contact interfaces when interpreting real seismic data. For example, these results could conceivably be applied to time lapse seismic monitoring: if some reflection event would be recorded at different times (i.e., in different seismic surveys), and if it exhibited waveform amplitude and phase changes of the type presented above over time (e.g., see Figure 5.12), it could indicate that a welded interface was becoming nonwelded over time, perhaps because of tectonic stresses, or because of the infiltration of a fluid between the layers, etc. Such amplitude and phase changes could, of course, be caused by something other than nonwelded contact, or by a combination of nonwelded contact and other effects. It would require the judgment of the seismic data analyst to determine whether or not nonwelded contact should be considered in the seismic interpretation of a data set exhibiting such amplitude and phase changes.
6.1. The effect of nonwelded contact on seismic waves

When an elastic wave travels across a nonwelded contact interface, the effects of such an interface on the elastic wave are changes in the amplitude and phase which are different from those for a welded interface. For the case of a nonwelded interface in a single homogeneous medium, such as a fracture, crack or joint, the exact formulas show that reflected waves (both unconverted and converted) are generated by the interface, and that a transmitted converted wave is generated as well, even though there is no impedance contrast across the interface (for a welded interface, no reflected waves would be generated, only a unit-amplitude unconverted transmitted wave). For small $\varepsilon$, where $\varepsilon = \omega c/\rho v$, i.e., a small amount of nonwelded contact, the scattered waves are small in amplitude except for the unconverted transmitted wave (whose amplitude is near unity). As $\varepsilon$ increases, the small amplitude scattered waves become more outstanding. From the graphs of the coefficients vs. incidence angle for a small amount on nonwelded contact, even though the degree of the nonweldedness is small, $\varepsilon << 1$, the phase shifts of the reflected waves have significant values (about $90^\circ$). Also, since the coefficients for the nonwelded case are frequency dependent, a nonwelded interface can act like a frequency filter on the incident wave. This thesis has confirmed the result obtained by other investigators that a nonwelded contact interface behaves somewhat like a low-pass filter, in that the incident wave passes through the interface to the other side with a higher
amplitude if the frequency is lower. Or, in other words, the amplitude of the unconverted transmitted wave increases with decreasing frequency.

From the synthetic seismograms presented in the previous chapter, at a nonwelded contact interface, there is also a group travel time delay for the transmitted waves in addition to the changes in the amplitude and phase of the signal (beyond those produced by welded contact). The changes in travel time, due to the group delay, are higher at the lower frequencies and decrease when the frequency increases. These synthetics also confirm the work of Pyrak-Nolte et al. (1988).

As stated at the end of Chapter V, the above-mentioned effects of nonwelded contact (i.e., amplitude and phase changes, and group traveltime delays) could be used to identify nonwelded interfaces in real data. Applications could include the analysis of seismic data from time-lapse monitoring projects, and from regions containing joints, cracks and fractures.

6.2. Exact formulas for the coefficients and their applications

In Chapter II, the exact formulas for the particle displacement reflection and transmission coefficients for P and SV elastic waves incident upon a nonwelded contact interface have been derived. A nonwelded interface is represented by the displacement discontinuity boundary conditions, which state that the stress is continuous across the interface but the displacement is not, and that the displacement discontinuity for each component of displacement is proportional to the corresponding stress component. The formulas are obtained by algebraically solving these boundary conditions. The expression for each formula can be written as a sum of two parts: the first part is the coefficient for the case of welded contact and the second part is due to nonwelded contact and contains the nonwelded specific compliances. The exact formulas show that the coefficients in the case of nonwelded contact are frequency-dependent. It is also shown that the energy is conserved for all oblique incidence angles at a nonwelded interface as long as the specific
compliances, $c_x$ and $c_z$, are real. The exact formulas for the coefficients can also be applied to the case of a viscous nonwelded interface, e.g., a fracture containing a fluid in a saturated condition, by replacing $c_x$ with $c_x/(1 - I\alpha_x v_x)$.

Using the Taylor series approximation, I derived approximate formulas for the reflection and transmission coefficients for a weakly nonwelded contact interface. The approximation is only made up to the first order in the specific compliances. These approximate formulas might be useful in AVO studies. Since the frequency $\omega$ always appears in the formulas in combination with a specific compliance in the form $\alpha_x$ or $\alpha_z$, the approximate formulas are valid even for larger values of $c_x$ and $c_z$ as long as the frequency values are small enough.

The exact and approximate formulas for the coefficients have also been applied to the specific case in which the media above and below a nonwelded interface are identical. Approximate formulas have been obtained for a small degree of nonweldedness and also for small incidence angles. These approximate formulas have been used in the study of the sensitivity of the coefficients to the specific compliances (Chaisri and Krebes, 2000).

In computer programs for generating synthetic seismograms via raytracing, it is generally more convenient to evaluate the required reflection and transmission coefficients from analytical formulas rather than from numerical solutions of the boundary conditions. In this way, one can avoid potential numerical problems that may arise in the numerical solution of the boundary conditions. Also, the programs tend to run a little faster when analytical formulas are used. In the case of a nonwelded contact interface, the coefficients are frequency-dependent, and so the computer program must perform the calculations in the frequency domain to include the effect of nonwelded contact in the range of frequencies of interest.
6.3. Possible future work

Another interesting topic for investigation on the effect of a nonwelded contact interface on seismic wave propagation, which has not yet been studied, is the topic of seismic wave diffraction. As we know that the presence of a nonwelded contact interface affects both the amplitude and phase of the source signal, it might also play a role in the calculation of diffraction waveforms during the computation of synthetic seismograms, as diffractions appear where the medium parameters of the earth model change substantially over a distance which is small compared to the seismic wavelength.

The seismic wave traveltime delay from a nonwelded interface might also affect the location of a nonwelded reflector on the CMP stack section produced by conventional seismic data processing schemes. The stack section is an uncorrected, i.e., unmigrated, image of the subsurface. The process of seismic migration, which attempts to move the reflectors seen on the stack section to their true spatial positions, gives a more correct image of the subsurface. Consequently, it might be necessary to take nonwelded contact into account in the migration process, if the reflectors of interest are nonwelded.

The analytic function for the group traveltime delay, i.e., the mathematical formula for it, has been presented by Pyrak-Nolte (1988), but only for a wave normally incident upon a nonwelded contact interface. Again, future work in this topic would involve the derivation of a formula for the group delay for a wave incident upon a nonwelded interface at an arbitrary angle.

If we are confident of the existence of a nonwelded interface on a seismic data section, how do we obtain the nonwelded specific compliances for the interface from the seismic section? A study of how to invert seismic reflection data for the specific compliances associated with a nonwelded interface has not yet been done, and would be of interest in the analysis and interpretation of seismic data for zones containing cracks, fractures, and other nonwelded interfaces.
REFERENCES


APPENDIX I

MACROSCOPIC VERSUS MICROSCOPIC THEORY

A1.1. Introduction

As mentioned in Chapter I, there are two theories found in the literature for computing P-SV plane wave reflection and transmission coefficients for elastic waves propagating across a plane nonwelded contact interface separating two homogeneous isotropic elastic materials. One of the theories is based on a microscopic approach in which each material is represented by a simple mass-spring lattice, with springs also connecting the two lattices at the interface (Paranjape et al., 1987; Krebes, 1987). The other is based on a macroscopic approach in which the materials are treated as continuous media, with displacement discontinuity boundary conditions being used at the interface (e.g., Pyrak-Nolte et al., 1990b; Schoenberg, 1980). This chapter will show that the two theories produce identical results for the coefficients if the respective parameters of the two theories are related in a specific way.

A1.2. The macroscopic theory

The details for this theory are shown in Chapter II. The solids on either side of the nonwelded contact interface are treated as continuous media. The boundary conditions at the interface are such that the stress is continuous across the boundary but the displacement is not -- see equation (2.1) for the boundary conditions at the interface. In
equations (2.1a), the stress components on the right-hand sides can be evaluated in either medium 1 or 2, as stress is continuous across the interface (equation 2.1b). Either choice leads to the same solutions. Choosing medium 2, the transmission medium (Pyrak-Nolte et al., 1990b; Schoenberg, 1980), results in slightly simpler equations to solve for an incident wave in medium 1, and vice versa. Here, however, medium 1 is chosen to facilitate comparison with the microscopic theory. Substitution of the usual mathematical expressions for the displacement components of the incident and scattered plane waves in media 1 and 2 into equations (2.1) results in the following equations for a wave incident from medium one:

\[
\begin{bmatrix}
-\alpha_1 p + I\omega c_x \chi_1 \cos i_1 & -\cos j_1 + I\omega c_x \beta_1 \gamma_1 & \alpha_2 p & \cos j_2 \\
\cos i_1 - I\omega c_x \alpha_1 \gamma_1 & -\beta_1 p + I\omega c_x \chi_1 \cos j_1 & \cos i_2 & -\beta_2 p \\
\chi_1 \cos i_1 & \beta_1 \gamma_1 & \chi_2 \cos i_2 & \beta_2 \gamma_2 \\
-\alpha_1 \gamma_1 & \chi_1 \cos j_1 & \alpha_2 \gamma_2 & -\chi_2 \cos j_2
\end{bmatrix}
\begin{bmatrix}
RP \\
RS \\
TP \\
TS
\end{bmatrix}
\]

\[
= \begin{bmatrix}
P(\alpha_1 p + I\omega c_x \chi_1 \cos i_1) + S(\cos j_1 + I\omega c_x \beta_1 \gamma_1) \\
P(\cos i_1 - I\omega c_x \alpha_1 \gamma_1) + S(-\beta_1 p + I\omega c_x \chi_1 \cos j_1) \\
P(\chi_1 \cos i_1) + S(\beta_1 \gamma_1) \\
P(\alpha_1 \gamma_1) + S(-\chi_1 \cos j_1)
\end{bmatrix}
\]

(A1.1)

where \(P=1\) and \(S=0\) for an incident P-wave, and \(P=0\) and \(S=1\) for an incident SV-wave. \(RP, RS, TP\) and \(TS\) are the P-wave reflection, SV-wave reflection, P-wave transmission and SV-wave transmission coefficients, respectively. The first, second, third and fourth equations in (A1.1) correspond to the interface conditions for \(u_z, u_x, \tau_{zz}\) and \(\tau_{zx}\), respectively.

The imaginary terms are proportional to the specific compliances \(c_z\) or \(c_x\). For perfect welded contact \((c_z \to 0\) and \(c_x \to 0\)), the equations reduce to the well-known Zoeppritz equations for the reflection and transmission coefficients.
A1.3. The microscopic theory

In this theory, the medium on each side of the interface is modeled as a cubic crystal lattice, with $a$ being the distance between a particle and its nearest neighbor, as well as the distance across the interface (see Figure A1.1).

![Lattice model of two solids in contact. The figure shows the x-z plane.](image)

The forces between nearest and next-nearest neighbors are supplied by springs, characterized by force constants $A_i$ and $B_i$, respectively, where $i=1$ for the medium of incidence and $i=2$ for the medium of transmission. $A$ and $B$ are the force constants across the interface. $M_1$ and $M_2$ are the masses of each particle at the lattice nodes in medium 1 and 2, respectively. Only purely longitudinal and purely transverse waves are allowed to propagate in the medium, which leads to the restriction that the ratio of the P-wave velocity to the SV-wave velocity must equal $\sqrt{3}$ (corresponding to a Poisson ratio of $\frac{1}{3}$).
This limits the general applicability of the theory, but that is not relevant for the purposes of this appendix.

Assuming plane harmonic waves propagating in the x-z plane (the plane of Figure A1.1), and applying Newton's second law of motion to the particles on each side of the interface results in four equations of motion in the horizontal and vertical components of displacement, which correspond to the four boundary conditions in the macroscopic case. Substituting the expressions for the harmonic plane waves into these equations, taking appropriate linear combinations of the equations to facilitate comparison with the macroscopic equations, and taking the long wavelength approximation, leads to the equations

\[
\begin{bmatrix}
-a_1 p + I\eta \sin 2\gamma_i \\
\cos \gamma_i - I\eta \left(3 \cos^2 \gamma_i + \sin^2 \gamma_i\right) / 3 \\
\chi_1 \cos \gamma_i \\
-a_0 \gamma_1
\end{bmatrix}
= \begin{bmatrix}
\alpha_1 p \\
\beta_1 \gamma_1 \\
\chi_1 \cos \gamma_i \\
-a_0 \gamma_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
-cos \gamma_1 + I\eta (k/k) \cos 2\gamma_j \\
-\beta_1 p + I\eta (k/k) \cos 2\gamma_j \\
\chi_2 \cos \gamma_2 \\
-\chi_2 \cos \gamma_2
\end{bmatrix}
\begin{bmatrix}
\alpha_2 p \\
\cos \gamma_2 \\
-\beta_2 p \\
-\beta_2 \gamma_2
\end{bmatrix}
= \begin{bmatrix}
P(a_x p + I\eta \sin 2\gamma_i) \\
P(\cos \gamma_i - I\eta \left(3 \cos^2 \gamma_i + \sin^2 \gamma_i\right) / 3) \\
P(\chi_1 \cos \gamma_i) \\
P(a_x \gamma_1)
\end{bmatrix}
+ \begin{bmatrix}
P(\alpha_1 p + I\eta \sin 2\gamma_j) \\
P(-\beta_1 p + I\eta (k/k) \cos 2\gamma_j) \\
P(\beta_1 \gamma_1) \\
P(-\chi_1 \cos \gamma_j)
\end{bmatrix}
\]

where \(k\) and \(\bar{k}\) are the wavenumbers of the P- and SV-waves, respectively, in the incidence medium, and the parameter \(\eta = A_1 k A/k A\) is a measure of the nonwelded contact (see Paranjape et al., 1987, for details). The above-mentioned restriction means that \(\bar{k}/k = \sqrt{3}\). In the case of perfect welded contact, the bonds between particles across the boundary and between particles in the same medium are all of the same order of magnitude, i.e., \(A_1, A_2 \sim A\), and in the long wavelength limit \((ka << 1)\), the equations reduce to the Zoeppritz equations \((\eta \to 0)\). In the case of imperfect contact, the bonds between particles across the interface are much weaker than the bonds between particles...
in the same medium, i.e., $A << A_1, A_2$. For $(A_i/A_j) \sim ka$, the parameter $\eta \sim 1$, and the imaginary terms cannot be neglected.

**A1.4. Equivalency of the two theories**

To demonstrate that the two theories give identical results, we equate (A1.1) and (A1.2), and then use Snell's law, along with the restriction $\bar{k}/k = Z_{p1}/Z_{s1} = \sqrt{3}$, where $Z$ is the density times the velocity, to obtain

$$\eta = \omega c_x Z_{p1} = \omega c_x \frac{Z_{s1}^2}{Z_{p1}}$$  \hspace{1cm} (A1.3)

If the parameter $\eta$ of the microscopic theory is related to the specific compliances $c_x$ and $c_z$ of the macroscopic theory according to equation (A1.3), then the two theories will give identical numerical results for the reflection and transmission coefficients. Equation (A1.3) also implies that $c_x/c_z = 3$, in accord with the restriction in the microscopic theory (although, in the macroscopic theory, $c_x$ and $c_z$ are independent parameters).

By substituting $\eta = A_1 ka/A$ into equation (A1.3), it can be easily shown that $c_z = (A_1/A) \left[ a/(\rho_1 \alpha_1^2) \right]$ and $c_x = (A_1/A) \left[ a/(\rho_1 \beta_1^2) \right]$, in basic agreement with the results of Schoenberg (1980), who showed that if the nonwelded contact interface is replaced by a thin layer of thickness $h$ situated between the two sides, and in perfect welded contact with them, then in the long wavelength limit, the reflection coefficient at the top of the layer and the transmission coefficient at the bottom of it are the same as those for the nonwelded contact interface with the specific compliances $c_x$ and $c_z$ replaced by $h/(\rho \alpha^2)$ and $h/(\rho \beta^2)$, respectively, where the densities and velocities are those of the thin layer.
A1.5. Conclusions

In this appendix, it has been shown that the reflection and transmission coefficients for a nonwelded contact interface computed using the macroscopic approach, i.e., by application of the displacement discontinuity boundary conditions (2.1) in a continuous medium, are identical to those computed using a microscopic approach, i.e., by modeling the media as mass-spring lattices and solving the equations of motion, if the parameters in the two approaches are related by equation (A1.3). This suggests that phenomena associated with nonwelded contact may be possibly studied by a simple microscopic approach as well. The microscopic theory is limited to the case \( \alpha/\beta = \sqrt{3} \), but it should be possible to generalize it. Preliminary work by E.S. Krebes (unpublished materials) though suggests that this cannot be done by simply including more two-body forces (e.g., 3\textsuperscript{rd} nearest neighbors), but would entail accounting for more complicated angular-type forces in the equations of motion, such as three-mass bond-angle forces.

A1.6. Summary

The P-SV plane wave reflection and transmission coefficients for a nonwelded contact interface computed by a microscopic approach (wherein the solids are modeled as mass-spring lattices) are identical to those computed by a macroscopic approach (wherein the solids are modeled as continuous media), if the respective parameters of the two approaches are related appropriately.
A2.1. Displacement discontinuity boundary conditions for SH waves

The propagation of SH waves is not coupled with that of P or SV waves, because the displacement polarization of SH waves is in the y-direction, whereas that of P and SV waves is in the x-z plane. Therefore the case of an incident SH wave on a nonwelded contact interface is relatively simple. If the SH waves are propagating in the x-z plane, with \( z = 0 \) at the interface, then the boundary conditions at the interface are

\[
\begin{align*}
\frac{\partial u_y}{\partial z} |_{z=0} &= c_y \tau_{zy} \\
\tau_{zy1} &= \tau_{zy2}
\end{align*}
\]

where

\[
\tau_{zy} = \mu \left( \frac{\partial u_y}{\partial z} \right)
\]

\( \tau_{zy} \) is the component of the stress tensor parallel to the surface, \( c_y \) is the specific compliance (\( c_y \to 0 \) for a welded interface), and \( u_y \) is the SH wave displacement.
A2.2. Reflection and transmission coefficient formulas for SH waves

As for the P-SV case, we assume that the positive z direction is downwards. There are two possible incident SH waves and two possible scattered waves. The displacement vectors for these SH waves are as follows:

Incident SH wave in medium one, \( u_{i1} = (0, \hat{S}_1, 0) \exp[I\omega(px + \eta_1 z - t)] \), \hspace{1cm} (A2.2a)

Incident SH wave in medium two, \( u_{i2} = (0, \hat{S}_2, 0) \exp[I\omega(px - \eta_2 z - t)] \), \hspace{1cm} (A2.2b)

Scattered SH wave in medium one, \( u_{s1} = (0, \hat{S}_1, 0) \exp[I\omega(px - \eta_1 z - t)] \), \hspace{1cm} (A2.2c)

Scattered SH wave in medium two, \( u_{s2} = (0, \hat{S}_2, 0) \exp[I\omega(px + \eta_2 z - t)] \) \hspace{1cm} (A2.2d)

where \( \eta_n = \frac{\cos j_n}{\beta_n} \), \( n = 1, 2 \), is the vertical component of slowness.

To solve for the reflection and transmission coefficients for an incident SH wave, the displacements (A2.2), which satisfy the equation of motion, are applied to the boundary conditions (A2.1). This results in the following equations:

\[-\hat{S}_1 + \hat{S}_2(1 + I\omega c_y Z_2) = \hat{S}_1 - \hat{S}_2(1 - I\omega c_y Z_2) \] \hspace{1cm} (A2.3a)

\[Z_1 \hat{S}_1 + Z_2 \hat{S}_2 = Z_1 \hat{S}_1 + Z_2 \hat{S}_2, \] \hspace{1cm} (A2.3b)

where \( Z \) is the seismic impedance, i.e., density times velocity.

In matrix form, equations A2.3 become

\[\mathbf{M}[\hat{S}_1, \hat{S}_2]^T = \mathbf{N}[\hat{S}_1, \hat{S}_2]^T, \] \hspace{1cm} (A2.4)

where

\[\mathbf{M} = \begin{bmatrix} 1 & -(1 - I\omega c_y Z_2) \\ Z_1 & Z_2 \end{bmatrix}, \hspace{1cm} \text{and} \hspace{1cm} \mathbf{N} = \begin{bmatrix} -1 & (1 + I\omega c_y Z_2) \\ Z_1 & Z_2 \end{bmatrix}. \] \hspace{1cm} (A2.5)

In the case of an incident SH wave from medium one, \( \hat{S}_2 = 0 \). Then (A2.4) becomes
\[ M[\hat{S}S', \hat{S}S']^T = [-1 \ Z_1]^T, \quad (A2.6) \]

and in the case of an incident SH wave from medium two, \( \hat{S}_1 = 0 \), and we have

\[ M[\hat{S}S', \hat{S}S']^T = [(1 + i\omega c_y Z_2) \ Z_2]^T, \quad (A2.7) \]

Equations (A2.6) and (A2.7) are each a set of linear equations, with two equations and two unknowns. By using the Cramer’s rule, the analytical solutions for the reflection and transmission coefficients are easily obtained (Schoenberg, 1980). The results are

\[ \hat{S}S = (Z_1 - Z_2 - i\omega c_y Z_1 Z_2)/D, \quad (A2.8a) \]
\[ \hat{S}S = 2Z_1/D, \quad (A2.8b) \]
\[ \hat{S}S = 2Z_2/D, \quad (A2.8c) \]
\[ \hat{S}S = (Z_2 - Z_1 - i\omega c_y Z_1 Z_2)/D, \quad (A2.8d) \]

where
\[ D = Z_1 + Z_2 - i\omega c_y Z_1 Z_2. \]

The formulas (A2.8) can also be written as

\[ \hat{S}S = \hat{S}S_w + i\omega c_y Z_1 Z_2 (\hat{S}S_w - 1)/D, \quad (A2.9a) \]
\[ \hat{S}S = \hat{S}S_w + i\omega c_y Z_1 Z_2 \hat{S}S_w/D, \quad (A2.9b) \]
\[ \hat{S}S = \hat{S}S_w + i\omega c_y Z_1 Z_2 \hat{S}S_w/D, \quad (A2.9c) \]
\[ \hat{S}S = \hat{S}S_w + i\omega c_y Z_1 Z_2 (\hat{S}S_w - 1)/D, \quad (A2.9d) \]

where the subscript \( w \) indicates the familiar reflection or transmission coefficient for the welded contact case, which can obtained by substituting \( c_y = 0 \) into equations (A2.8). These equations (A2.9) are useful in that they show that each coefficient for the nonwelded contact case can be obtained from the corresponding one for the welded contact case by the addition of a term due to nonwelded contact.