Assessment of the introduction of road pricing using a Computable General Equilibrium model*

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Abstract

The introduction of road pricing has important budgetary and income distributional consequences. In countries like Denmark, due to high marginal rates of taxation, raising government revenue and redistributing income is associated with substantial distortionary and administrative costs. This paper argues that an evaluation of the introduction of road pricing needs to take into account not only the effects on congestion and on the environment, but also the effects on the government's budget and the income distributional consequences and therefore should be undertaken within a general equilibrium framework. A stylized Computable General Equilibrium (CGE) model which represents the interaction of the consumption of transport and of traffic congestion with leisure is used to illustrate this point. Model simulations show that the introduction of road pricing may be associated with a double dividend and make it desirable to reduce transport infrastructure, and furthermore, although decreasing road congestion, increase the environmental damage.

Keywords: Project evaluation, optimal taxation, externalities, separability, road pricing, CGE models, double dividend

JEL classification codes: H2, H29

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1 Introduction

The main purpose of this paper is to demonstrate the importance of conducting transport policy analysis within a second-best public economics framework and how CGE (Computable General Equilibrium) models in this context. More specifically we demonstrate how a CGE model can be used to evaluate the consequences and desirability of the introduction of road pricing and to determine its optimal level once it has been introduced.

The need for government intervention to correct for the failure of transport markets to establish an efficient resource allocation has long been recognised. Early contributions within a first-best partial equilibrium framework suggested that transport should be subjected to a so-called Pigouvian tax, corresponding to the social damage created by the associated congestion and environmental externalities. The development of optimal taxation in the 70s (as summarised in for example Atkinson and Stiglizt 1980) made it clear that the assumption of cost free lump sum taxes underlying the first-best analysis is not relevant as basis for policy recommendations. This is in particular true for transport policy analysis, as changes in transport policies often involve very substantial changes in government revenue and have important income distributional effects. Raising government revenue and redistributing income is associated with substantial costs, distortionary and as well as administrative. In evaluating changes in transport policy it is therefore not only the direct effect on traffic and the related effects on pollution, road congestion and accidents that are relevant. The effect on government revenue and the income distributional consequences may be equally important (see for example Parry and Bento 2001 who are emphatic at this point).

A number of authors have undertaken transport policy analysis within a partial equilibrium framework taking into account second-best considerations (see for example De Borger et al. 1997). But within a partial equilibrium analysis it is difficult to account for important general equilibrium effects, such as how the recycling of the revenue from transport taxes influence the cost of government funds and the effects on the income distribution.

Analysis of the optimal policies in the presence of externalities within a second-best general equilibrium framework was originally undertaken by Sandmo (1974). The theory has recently been reconsidered by Mayeres and Proost (1997, 2001) and popularised by Sandmo (2000). However, general equilibrium analysis often yields results which are difficult to interpret. To facilitate the analysis separability assumptions are often made (see for example Bovenberg 1999), which however may result in misleading conclusions. Unwarranted separability assumptions and disregard of administrative cost have for example lead to the erroneous conclusion that green tax reforms cannot be associated with a double dividend[1] (see Munk 2000).

CGE models may be used to facilitate the interpretation of analytical results (as is attempted in the present paper), but also to provide an alternative for project evaluation to replace traditional partial equilibrium cost benefit analysis. However, the use of CGE models has its own pitfalls. Restrictive functional forms may lead to erroneous conclusions. Use of additive utility functions has for example made Goulder et al. (1999) to conclude that a green policy reform cannot be associated with a double dividend. Imposing restrictive separability assumptions in a CGE model can however be avoided as illustrated by Mayeres (2000) who

[1] A green tax reform (such as the introduction of road pricing) is said to be associated with a double dividend if the increase in welfare exceeds the reduction in the environmental benefits.

explicitly models the feed-back effect of changes in transport policies on the behaviour of economic agents.

The special contribution of this paper is the derivation of optimal transport policies based on a CGE model where other contributions have only considered the effects of exogenous changes in transport policies. Furthermore, the model represents the interaction between consumption of market goods, leisure and a congested public good in a way which permits the compensated price elasticities which determine the optimal tax structure to be derived analytically. This allows the interpretation of simulation results of the introduction of road pricing to be closely related to the theory of optimal taxation. More specifically, the simulation results illustrate that a decrease in administrative costs may make possible a green tax reform associated with a double dividend, even if the tax structure prior to the decrease in administrative cost was optimal.

The structure of the paper is as follows. In Section 2 we specify the theoretical model on which the analysis is based. In Section 3 as background for the subsequent analysis we briefly review the insight provided by the theory of optimal taxation and the optimal provision of public goods of relevance for the taxation of transport and for investment in transport infrastructure. In Section 4 we specify the CGE model and in Section 5 we present the simulation results derived from the model on the introduction of road pricing. Section 6 summaries and concludes.

2 The theoretical model

The theoretical model adopted is essentially that of Sandmo (1975, 2000) who considers the optimal taxation in an economy where one commodity is associated with a public good externality which can be abated by government expenditures.

We consider an economy with many heterogeneous households who supply only one homogeneous primary factor, *labour,* labelled 0, and demand two produced commodities, *transport* and *non-transport*, labelled 1 and 2 respectively.

We represent the households' preferences by utility functions, $u^{\mathrm{h}} = u^h\left(\mathbf{x^h}, e\right)$, h$\in H$, where $\mathbf{x^h} \equiv \left(x_0^h, x_1^h, x_2^h\right)$ is the h$^{\text{th}}$ household's net demand vector, and *e*, *free road capacity*, a public good negatively related to the consumption of *transport*. The households face prices, $\mathbf{q}^h \equiv \left(q_0, q_1^h, q_2\right)$, h$\in H$, and receive lump-sum income, I^h, h$\in H$. Prior to the introduction of road pricing the price of transport to is the same for all households, i.e. $q_1^h = q_1$. After the introduction of road pricing the price of transport differs between households because how much a household pays in road pricing charge depend on how much its consumption of transport contribute to the congestion and environmental externalities.

The production side of the economy is represented by constant returns to scale production sectors, each producing one output and using only labour as input. Producer prices are $\mathbf{p} \equiv \left(p_0, p_1, p_2\right)$.

The government needs revenue for *transport infrastructure*, *G*, and for *other government consumption*, *B*. The government's expenditures are financed by commodity taxes $\mathbf{t}^h \equiv \mathbf{q}^h - \mathbf{p}$, h$\in H$ (where t_1^h, h$\in H$ are the road pricing charges), and a uniform lump-sum tax, *L*. Thus unearned income is for all households equal to the uniform lump-sum tax,[2] i.e. I^h =-*L*, h$\in H$.

Free road capacity depends negatively on the households' consumption of transport, and positively on *G*, according to $e = e\left(\left\{x_1^{\mathrm{h}}, h \in H\right\}, G\right)$. The derivative, $\frac{\partial e}{\partial x_1^{\mathrm{h}}}$, indicates to what extent the h[th] household's consumption of transport congests the roads.

Using the *expenditure function approach* (see Dixit and Munk 1977), extended to represent the government's provision of a public good and external effects (see Munk 2000), the conditions for $(\mathbf{t}^h, L, G)$ to be compatible with market equilibrium may be expressed as

$$E^h(\mathbf{q}^h, e, u^h) = -L \text{, h} \in H \tag{1}$$

$$\sum_{k=0}^{2} \sum_{h \in H} t_k^h x_{\mathrm{k}}^{\mathrm{h}}(\mathbf{q}^h, e, u^h) + \mathrm{H}L - G - B = 0 \tag{2}$$

$$e = e\left(\left\{x_1^{\mathrm{h}}\left(\mathbf{q}^h, e, u^h\right), h \in H\right\}, G\right) \tag{3}$$

where $E^h(\mathbf{q}^h, e, u^h)$ is the expenditure function and $x_{\mathrm{k}}^{\mathrm{h}}(\mathbf{q}^h, e, u^h)$, k$\in$ (0,1,2) the compensated demand functions of the h[th] household.

The social preferences of the government, which is assumed to be inequality averse, are represented by a social welfare function, $W\left(u^1, u^2, ..., u^{\mathrm{H}}\right)$. The government chooses tax rates and the transport infrastructure to maximise social welfare.

3 Application of the theory of public economics to the taxation of transport and investment in transport infrastructure

The externalities associated with transport may be divided into environmental externalities and congestion externalities. *Environmental externalities*[3] (air pollution, noise and accidents) may, as an approximation, be considered separable from consumption, i.e. changes in these externalities do not impact on the pattern of consumption. However, *congestion externalities* clearly cannot be considered separable from consumption. The consumption of transport clearly depends on the level of congestion giving rise to the so-called *feedback effect*, i.e. a tax on transport reduces transport by less than the tax in isolation because the decrease in congestion increases the consumption of transport. The total effect on free road capacity of an increase in the tax on transport is therefore (see Annex)

[2] *L* is negative if it is interpreted as the fixed element in a progressive linear income tax schedule.

[3] These are not represented in the theoretical model formulated in Section 2, but have been added to the parameterised version of the model in Section 4.

$$\frac{de}{dx_1} = \frac{\partial e}{\partial x_1} \frac{1}{1 - e' E_{1e}} \tag{4}$$

$\frac{\partial e}{\partial x_1}$ the *effect on free road capacity of a marginal increase in road capacity* and $\frac{1}{1 - e' E_{1e}}$ the *feed back effect* which indicates to what extent an increase in free road capacity will increase the consumption of transport. Under first best assumptions, i.e. when the government's revenue requirement can be raised by cost-free lump sum taxes, the optimal rate of tax on transport, the so-called *Pigouvian tax*, is equal to (see Annex)

$$t_1 = -MV_e \frac{de}{dx_1} \tag{5}$$

where MV_e is *the marginal monetary evaluation of free road capacity*.

However, under the more realistic second best assumptions that the government's revenue must be raised by distortionary taxation, the optimal tax on transport must not only reflect the external effects associated with transport, but also how the taxation affects the government's two main objectives, to raise government revenue and to redistribute income. In order better to expose the interplay between these to considerations and the environmental considerations linked to the taxation of transport we will in the following identify the consumption of transport only as transport which is not linked to production[4], and make a number of specific assumptions.

Based the following stylized facts we review the conditions for optimal taxation of transport and for optimal provision of transport infrastructure[5] as background for the subsequent interpretation of the quantitative assessment of the introduction of road pricing. We assume

- that transport requires a relatively large amount of time for its consumption and, partly for that reason, may be considered complementary to the non-market use of time (leisure);
- that transport is associated with negative external effects in the form of congestion and environmental externalities (air pollution, noise, accidents);
- that the share of leisure travel in the consumption is higher for urban households than for rural households and associated with larger congestion costs;
- that urban households are better off than rural households; and
- that an increase in free road capacity will increase the supply of labour.

▪ *Taking into account that transport is complementary to leisure*

When lump sum taxation is not feasible, raising government revenue involves discouraging the supply of labour. This can be alleviated by taxing commodities which are complementary to leisure at a higher rate than commodities that are less complementary to leisure (Corlett and Hague 1953). In a one household economy when transport is not associated with externalities

[4] We thus disregard commuting and the use of transport as intermediate input.
[5] Based on the model presented in section 2, these conditions have formally been derived in Annex, however for ease of exposition only in the case of a one household economy.

the optimal ratio of the tax on transport relative to the tax on non-transport must satisfy (see Annex)

$$t_1 = \frac{\lambda - \mu}{\lambda}\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}\right) K \tag{6}$$

$$t_2 = \frac{\lambda - \mu}{\lambda}\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}\right) K \tag{7}$$

where ε_{ij} (i,j $\in$0,1,2) are *compensated demand elasticities*, μ the *net marginal social welfare of income*, λ the *marginal social value of government funds* and *K* is a positive constant. We have that $\varepsilon_{i0} = \alpha_0 \sigma_{i0}$, where $\alpha_0 = \dfrac{q_0 x_0}{q_0 \omega_0 + I}$ is the share of leisure in the household's full income, and where $\sigma_{i0} = \sigma_{0i}$ is the elasticity of substitution between commodity i and leisure. By the assumptions made transport is more complementary to leisure (non-market use of time) than other goods, i.e. $\varepsilon_{10} < \varepsilon_{20}$, and the optimal tax on transport is therefore even in the absence of externalities and distributional considerations higher than on other commodities.

It would therefore - if it were possible to tax transport at a different rate than non-transport at no supplementary administrative costs - from an efficiency point of view be desirable to tax transport at a relatively high rate.

- ***Taking into account that transport is associated with externalities***

In a one household economy the conditions for an optimal tax structure may be expressed as (see Annex)

$$t_1 = \frac{\lambda - \mu}{\lambda}\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}\right) K - \left(\frac{\mu}{\lambda} MV_e + \frac{\partial T}{\partial e}\right)\frac{de}{dx_1} \tag{8}$$

$$t_2 = \frac{\lambda - \mu}{\lambda}\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}\right) K \tag{9}$$

Taking into account that transport is associated with externalities involves that an extra tax will be levied on transport compared to when this is not the case (compare (8) with (6)). The extra tax, however, does not only depend on $MV_e \dfrac{de}{dx_1}$ as under first best assumptions, but also on the cost of government funds, $\dfrac{\lambda}{\mu}$, and on how a change in free road capacity influences the tax base, $\dfrac{\partial T}{\partial e}$. The larger the cost of government funds the smaller the importance of the externality for the optimal tax structure, and the more an increase in free road capacity expands the tax base, the higher the tax.

- ***Taking into account that transport is predominately consumed by households with a relatively high income***

Increasing the tax on transport and decreasing the tax on other commodities will redistribute income from the well-off (the urban households) to the less well off households (rural households). The optimal tax on transport will therefore be higher than if based only on efficiency considerations. The fact that the decrease in congestion is more important for the urban than for the rural households pulls in the opposite direction.

- ***Green tax reform and the possibility of a double dividend***

A *green tax reform* is a change in the tax system that reduces negative external effects. We may therefore consider the introduction of road pricing a green tax reform. A green tax reform is said to be associated with a *double dividend* if the overall benefits in terms of social welfare exceed the benefits due to the reduction in negative external effects (see Bovenberg 1999).

Based on these stylised facts, there are thus four reasons why a policy-maker may want to tax transport at a higher rate than other commodities:

- it is more complementary to leisure (non-market use of time) than other commodities,
- it is consumed by high income households relatively more than by the low income households,
- its consumption is associated with environmental and congestion externalities, and
- a decrease in the congestion externality may stimulate the labour supply and thus expand the tax base.

Starting from a proportional tax structure $\overline{\mathbf{t}} = \left(t_0, \overline{t}, \overline{t}\right)$ with the same rate of tax on the transport good and the non-transport good, a green tax reform changing the tax system into the optimal tax system, $\mathbf{t}^* \equiv \left(t_0^*, t_1^*, t_2^*\right)$, would therefore be associated with a double dividend as the increase in the tax on transport relative that on other goods would not only decrease the externalities associated with transport but also lead to a more efficient tax system and a desirable redistribution of income. However, if the tax system is already optimal if follows from the envelope theorem that a change in the tax on transport although it will further reduce congestion and other externalities will not increase welfare and that a green tax reform in this situation cannot be associated with a double dividend.

- ***The optimal size of the transport infrastructure***

Under first best assumptions the condition for the optimal provision of the transport infrastructure is (see Annex)

$$MV_G = MC_G \tag{10}$$

where MV_G is the *marginal social evaluation of transport infrastructure,* and MC_G the marginal costs of its production. However, under second best assumptions the condition is

$$MV_G = \frac{\lambda}{\mu}\left(MC_G - \frac{dT}{dG} \right) \tag{11}$$

where $\frac{\lambda}{\mu}$ the *marginal costs of government funds*, and $\frac{dT}{dG}$ is the *effect of an increase in transport infrastructure on the tax base*.

If an increase in transport infrastructure expands the tax base (via its effect on the free road capacity), then the optimal level of transport infrastructure will be larger than if this effect is not taken into account.

- ***The effect of the introduction of road pricing on the optimal taxation of transport and the optimal provision of transport infrastructure***

The administrative costs of monitoring the individual consumption of transport are considerable, but with the advent of new technologies, as those tested in the AKTA project (see Nielsen and Vuk 2003), they are not any longer prohibitive. Whereas it has not previously been possible to differentiate the price of transport between household according to the external damage associated with their consumption of transport, this is now possible. Within the framework of our model we interpret this technological change as a reduction in administrative cost. We indicate the optimal tax system before the change, $\mathbf{t}^* \equiv \left(t_0^*, t_1^*, t_2^*\right)$, and after the change, $\mathbf{t}^{**h} \equiv \left(t_0^{**}, t_1^{**h}, t_2^{**}\right)$, h$\in H$ and in order to simplify the exposition we assume that administrative costs associated with $\mathbf{t}^*$ before the change in technology are equal to those associated with $\mathbf{t}^{**h}$, h$\in H$ after the change. The introduction of road pricing will thus increase social welfare as it provides the government with extra instruments.

The introduction of road pricing the optimal tax on the consumption of transport for urban households will increase whereas the optimal tax for rural household will decrease. As the taxation of transport has become a more efficient instrument, the introduction of road pricing will justify an increase in the average level of taxation of transport. The introduction of road pricing will thus result in a reduction in the level of congestion. The change from $\mathbf{t}^*$ to $\mathbf{t}^{**h}$, h$\in H$ therefore represent a green tax reform.

The optimal tax structure with road pricing will essentially be determined by the same considerations as without road pricing. With tax rates chosen optimally both before and after the change in technology, the introduction of road pricing will increase social welfare through three different channels: *first*, by the reduction in congestion; *second*, by the redistribution of income from the relatively rich urban households to the rural households; and *third* by the stimulation of the labour supply due to the higher taxation of transport and the decreased congestion. This latter effect is the result of two opposite effects: on the one hand the decrease in congestion will encourage the consumption of transport and thus discourage the supply of labour; on the other hand it will diminish the amount of time required to achieve a given amount of transportation.

It is not possible a priori to establish how the introduction of road pricing will influence the total demand of transport. As the effect on environmental externalities is related to the total consumption of transport, road pricing may increase the environmental externalities associated with road pricing although it will decrease congestion.

As it reduces the urban households' demand for transport, the introduction of road pricing decreases the marginal social evaluation of transport infrastructure, MV_G. It also decreases the marginal costs of government funds, $\frac{\lambda}{\mu}$, as road pricing increases tax efficiency, which justify an increase in transport infrastructure; however on balance the optimal level of transport infrastructure is likely to be smaller after the introduction of road pricing.

- ***The introduction of road pricing will be associated with a double dividend***

The introduction of road pricing will therefore, as argued above, be associated not only with a *first dividend* in the form of a reduction the external effects, but also a *second dividend* due to income distributional and tax efficiency gains, even if the tax system was optimal before the change in administrative costs. If the tax system was not optimal, the scope for the introduction of road pricing to yield a substantial double dividend is naturally far greater.

This contradicts with the established wisdom that a green tax reform cannot be associated with a double dividend (see Bovenberg 1999[6]). The contradiction may be explained by the fact that we take into account administrative costs and distributional considerations and that we do not impose separability between the externality and leisure (see Munk 2000).

4 The parameterised model

Specification of functional forms for free road capacity and environmental externalities

Free road capacity, *e*, is negatively linked to the *congestion externalities* and positively linked to government investments in *transport infrastructure* according to

$$e = \omega_e - \sum_{h \in H} a_1^h C_1^h \left(x_1^h, c_0^h \right) + G \tag{12}$$

where ω_e is the endowment of road capacity, $a_1^h C_1^h \left(x_1^h, c_0^h \right)$ the congestion externality generated by the hth household (as a function of its consumption of the transport good, x_1^h, and the time use for transportation, c_0^h, where a_1^h is the transport congestion coefficient associated with the hth household), and G the government's provision of transport infrastructure.

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[6] Bovenberg (1999) concludes, *"The overall message of this paper is disappointing for those who expect substantial non-environmental benefits from green tax reforms. The analysis shows that stringent conditions need to be met in order for an environmental tax reform to yield a double dividend".*

In addition, in order to represent environmental damage (air pollution, noise, accidents) associated with the consumption of transport, we extend this framework to include *environmental externalities* specified as

$$\tilde{e} = \sum_{h \in H} x_1^h \tag{13}$$

Specification of household preferences

Many CGE models that have been used to evaluate changes in environmental policies impose separability between consumption and leisure and between consumption and the externality (see for example Goulder et al 1999). For the analysis of transport policy issues to impose such assumptions is highly unrealistic and may result in misleading conclusions. We have therefore chosen to specify the households' preferences using a utility function with explicit representation of the use of time, (the CES-UT) (see Munk 2002) extended with a public good to represent how the provision of free road capacity influences household behaviour, and with an additive term $E_{\tilde{e}}\tilde{e}$ to represent the effect on welfare of the environmental externalities, $\tilde{e}$, associated with the consumption of transport (see Figure 1)

$$u = U\left(C\left(C_T\left(C_1\left(x_1, c_0^1; \sigma^{C1}\right), e; \sigma^T\right), C_2\left(x_2, c_0^2; \sigma^{C2}\right); \sigma^D\right), c_0^0; \sigma^L\right) + E_{\tilde{e}}\tilde{e} \tag{14}$$

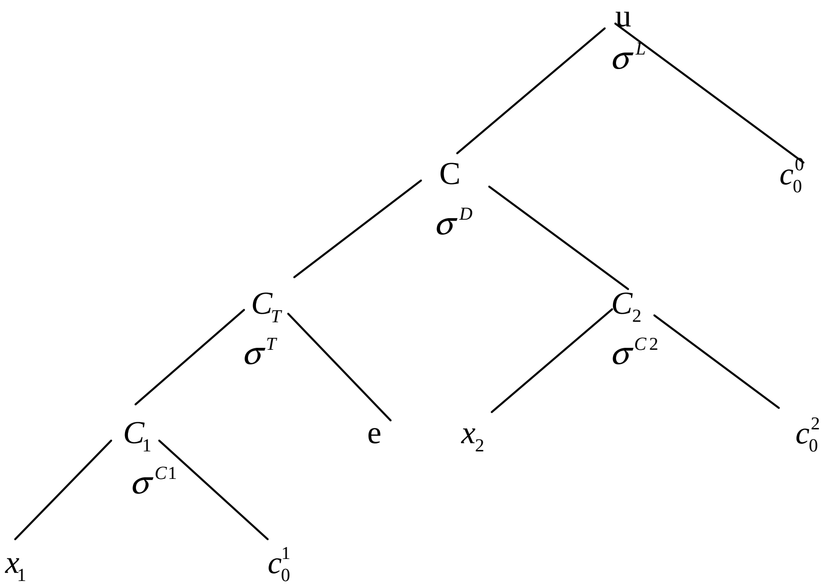
$C_i = C_i\left(x_i, c_0^i; \sigma^{Ci}\right)$ expresses for the transport input composite (i=1) and for the other good composite (i=2) the preference for the amount purchased of *commodity i*, x_i, relative to the *time used for its consumption*, c_0^i. The elasticity of substitution σ^{Ci} indicates the degree of substitutability between the two.

$C_T\left(e, C_1; \sigma^T\right)$ expresses for the transportation composite the preferences of *free road capacity* **,** e **,** relative to *the transport input composite* **,** C_1 **.** The elasticity of substitution σ^T indicates the degree of substitutability between the two.

$C = C\left(C_T, C_2; \sigma^D\right)$ expresses the preference for the *transportation composite***,** C_T, and the *other good composite,* C_2**.** The elasticity of substitution, σ^D, indicates the degree of substitutability between the two.

$U\left(C, c_0^0; \sigma^L\right)$ expresses the preference for *aggregate consumption*, C, relative to *pure leisure,* $c_0^0 = \omega_0 - \sum_{i=1,2} c_0^i + x_0$ where ω_0 is the household's time endowment. The elasticity of substitution, σ^L, indicates the degree of substitutability between the two.

Fig 1 Nested structure of utility function with explicit representation of the use of time



By specifying the share of time used in the consumption of transport to be larger than for non transport, and the rate of substitutions, $\sigma^{Ci}, i = 1, 2$, within the composites, $C_i\left(x_i, c_0^i; \sigma^{Ci}\right), i = 1, 2$, as smaller than the rate of substitution, σ^D, between the composites, $\varepsilon_{10} < \varepsilon_{20}$ (see Munk 2002). The CES-UT representation of household preferences thus allows transport to be more complementary to leisure than non-transport. It also makes it possible to represent that a decrease in congestion stimulates the labour supply (See Table 1).

Real income and social welfare

We assume an additively separable, symmetric social welfare function defined on the households' real income

$$W = \sum_{h \in H} w\left(R^h\right) \tag{15}$$

where $w(R)$ is a concave function and R^h the real income of the h$^{\text{th}}$ household defined as

$$R^h \equiv Y^{h.0} + E^h\left(\mathbf{q}^0, v^h(\mathbf{q}, I^h; e, \tilde{e});\ e^0,\ \tilde{e}^0\right) - I^{h.0} \tag{16}$$

where $E^h\left(\mathbf{q}^0, u^h;\ e^0,\ \tilde{e}^0\right)$ is the expenditure function evaluated at the benchmark vector of household prices, free road capacity and environmental externality, $\left(\mathbf{q}^0,\ e^0,\ \tilde{e}^0\right)$, $u^h = v^h(\mathbf{q}, I^h; e, \tilde{e})$ the utility function evaluated at the vector of household prices, lump sum income, free road capacity and environmental externality $(\mathbf{q}, I^h; e, \tilde{e})$ and $Y^{h.0}$ the h$^{\text{th}}$ household's nominal income in the benchmark situation. In the benchmark situation $E^h\left(\mathbf{q}^0, u^h;\ e^0,\ \tilde{e}^0\right) = I^h$. The real income in the benchmark situation is thus by design equal to the nominal income in the benchmark situation, $Y^{h.0}+I^{h}$. The changes in real income for different households due to the implementation of a project are not affected by how the real income in the benchmark situation is defined. The real income changes only depend on the behaviour characteristics of the household, which in principle can be established objectively. Given the choice of the social welfare function, the definition of the benchmark level of the real income function has normative significance, however. It has in other words it has importance for the change in social welfare associated with a project. The h[th] household's marginal evaluation in monetary terms of the congestion externality is $E_{e}^{h}=\frac{\partial E^{h}}{\partial e}$, and its marginal evaluation of the environmental externality is $E_{\tilde{e}}^{h}\equiv\frac{\partial E^{h}}{\partial \tilde{e}}$.

We assume that there in the economy are two types of households, urban households, indexed U, who use the congested roads and rural households, indexed R, which do not.

The parameter values required for the model are share values derived from the benchmark dataset, substitution elasticities and the benchmark marginal evaluation of the environmental and congestion externalities. The benchmark dataset in the form of a *Social Accounting Matrix* (SAM) as well as the other parameter values are provided in Munk (2003).

From the parameters the share parameters derived from the benchmark dataset and substitution elasticities and other values of the extended CES-UT utility function of the model the matrix of compensated price elasticities, ε_{ij}, i,j=0,1,2, contingent on the level of the free road capacity, and the and expansion elasticities with respect to free road capacity, ε_{ie}, i=0,1,2, has been calculated as they are important for understanding the behaviour of the model (see Table 1). Notice that transport is more complementary to leisure than the non-transport good.

Table 1: Compensated price elasticities and expansion elasticities for the reference set of parameters in the benchmark situation

Urban household	ε_{ij}			ε_{ie}
	Price of			Quantity of
Quantity of	Labour	Non-transport	Transport	Free road capacity
Labour	0.28	-0.30	0.02	0.01
Transport	-0.10	0.40	-0.31	0.56
Non-transport	0.19	-0.25	0.06	-0.30

Rural household	ε_{ij}			ε_{ie}
	Price of			Quantity of
Quantity of	Labour	Non-transport	Transport	Free road capacity
Labour	0.24	-0.25	0.00	0.00
Transport	0.00	0.27	-0.27	0.00
Non-transport	0.15	-0.20	0.05	0.00

The marginal social welfare of income is defined as $\beta^h \equiv \frac{\partial w}{\partial R^h}$. As rural households are assumed to have lower income than urban households, $\beta^R > \beta^U$ (see Table 2).

Table 2: Social welfare weights

	Household types	
	Urban	Rural
Social welfare weights, β^h	1.10	1.00

5 Presentation and interpretation of simulation results

The change from taxation of transport to road pricing involves a differentiation of the tax on transport to reflect differences in the associated externalities. To capture this we have represented the introduction of road pricing as the move from a situation where the consumption of transport by all household types is taxed at the same rate, to a situation where the tax on transport can be differentiated between urban households and rural households, the consumption of the first being associated with congestion externalities and that of the latter with none. We present three types of analysis. We first calculate the consequences of two different projects based on different assumptions about the use of the revenue generated from road pricing (*Consequence analysis*). We then evaluate these two projects based on a set of supplementary value judgements (*Project evaluation*), and finally calculate the optimal policies under three different sets of assumptions with respect to which policy instruments are available to the government (*Optimality analysis*).

Consequence analysis

The political debate about the introducing of road pricing suggests that the way the revenue generated from road pricing is used will affect the popular support for such a reform. This is, related to the fact that reforms that provide an approximate Pareto improvement are politically easier to implement than reforms that imply substantial redistributions between social groups. We therefore consider different assumptions with respect to how the tax revenue is used. We calculate the consequences of the introduction of road pricing in the form of two alternative projects, each with a closure corresponding to alternative assumptions about how the revenue generated by road pricing is used. In the *first case*, we assume that the revenue from road pricing is used to reduce other transport taxes such as taxes on car ownership and on petrol. This corresponds to the view that for the introduction of road pricing to be politically feasible, the revenue needs to be used to reduce other taxes on transport such as taxes on petrol and car ownership. Since the tax on transport is the sum of these taxes and the road pricing tax, the introduction of road pricing will in this case be represented as the tax on the transport for urban households being increased and that for rural households being decreased. In the *second case* the revenue from road pricing is used to reduce the rate of income tax. This corresponds to the view generally held by economists that such revenue should enter into the government's budget with no strings attached.

For both projects, the tax on the urban households' consumption of transport is increased due to the introduction of road pricing from the initial level of 80% to 139%. Technically in the case of *Project 1*, the tax on the rural households' consumption of transport is endogenous and all other tax instruments, other than the tax on the urban households' consumption of transport, are kept fixed. Conversely, in the case of *Project 2*, the income tax rate is endogenous and all other tax instruments, including the tax on the rural households' consumption of transport, are kept fixed.

Table 3a shows the tax changes associated with *Project 1* and *Project 2*. *Project 1* involves a decrease in other transport-related taxes by 17 percentage points such that the government revenue generated by taxes on transport remains more or less the same. Combined with the increase in the tax on transport due to the introduction of road pricing, the tax on transport thus increases from 80% to 139%-17%=122% for urban households and decreases from 80% to 63% for rural households.

Project 2 involves the same increase in the tax on transport as Project 1, i.e. the rate of tax on transport for urban households increases from 80% to 139%, and for rural households it is kept at 80%, but is not combined with reductions in other transport related taxes. Instead, there is a decrease in the rate of tax on labour from 60% to 59%. *Project 2* thus implies that the introduction of road pricing increases the rate of tax on the consumption of transport.

Table 3a: Consequences of introduction of road pricing: Tax rates

	Benchmark	***Project 1: Revenue from road pricing used to reduce other taxes on private road transport***	***Project 2: Revenue from road pricing used to reduce income tax***
Household taxes			
Labour	60%	60%	59%
Transport in urban areas	80%	122%*	139%
Transport in rural areas	80%	63%	80%
Other goods	25%	25%	25%

* 139%-17%

Table 3b shows the consequences of the two projects in terms of changes in the consumption of transport, free road capacity and the evaluation of the externalities. It also indicates the effects on the supply of labour, on the real incomes of the rural and urban households and on the social welfare of society. *Project 1* reduces the urban households' consumption of transport and increases that of the rural households. As a result the congestion externality is reduced (free road capacity increased). The decrease in consumption of transport for the urban households is relatively small compared to the increase for the rural household, because of the feedback effect, i.e. because the decrease in road congestion stimulates the consumption of transport. In aggregate, *Project 1* results in an increase of the consumption of transport and thus increases the environmental externalities. The supply of labour increases for urban households, but it decreases for the rural households. This reflects the assumption built into the model that transport is complementary to non-market use of time. The net effect on the supply of labour is positive, but rather small.

Table 3b: Consequences of introduction of road pricing: Changes in transport, externalities and real income

	Benchmark	***Project 1: Revenue used to reduce other taxes on private road transport***	***Project 2: Revenue used to reduce income tax***
		Changes compared to benchmark	
Consumption of transport			
Consumption of transport by urban households	70	-2.0	-2.6
Consumption of transport by rural household	30	2.9	0.9
Total	100	0.9	-1.7
Transport infrastructure			
Free road capacity	80	1.9	2.7
Marginal evaluation	1.01	-0.14	-0.17
Externalities			
Evaluation of congestion externalities		2.0	2.2
Evaluation of environmental externalities		-0.2	0.4
Total		1.8	2.5
Labour			
Supply of labour by urban household	1084	10.1	13.1
Supply of labour by rural household	566	-9.8	-3.4
Total	1650	0.3	9.7
Real income and social welfare			
Real income of urban households	813	-10.4	-6.2
Real income of rural households	417	12.2	15.4
Social welfare	1311	3.0	10.8
Double Dividend		1.1	8.3

Source: Own calculations based on stylised CGE model
Note: The *double dividend* is calculated as the change in social welfare minus the evaluation in the change in the externalities.

This contrasts with the results obtained for *Project 2*, where the revenue from road pricing is used, rather than to reduce other transport related taxes, to reduce the income tax rate. In this case the project results in a significant increase in the supply of labour and a significant increase in social welfare. The consumption of transport declines not only as a result of the increase in the price of the transport good, but also as a consequence of the decrease in the income tax rate, which increases the cost of the time needed for transportation.

Project 1 expands the tax base much less than *Project 2* due to the much smaller expansion of the supply of labour. *Project 2* is thus associated with a considerable greater gain in efficiency (as measured by the increase in aggregate real income).

The marginal evaluation of road capacity is in the benchmark situation 1.01. This means that one kr investment in transport infrastructure increases social welfare by 1.01 units (disregarding the costs of financing the investment). The introduction of road pricing reduces the marginal evaluation of transport infrastructure (from 1.01 to 0.86 in *Project 1* and to 0.83 in *Project 2*). The projects thus decrease the value of investment in transport infrastructure, which reflects that the projects decrease the consumption of transport and thus increase the amount of free road capacity, limiting the need for extra road capacity.

Project evaluation

Project evaluation may involve comparing each of the two projects with the status quo, or comparing one project with the other. *Project 2* dominates *Project 1* according to the Pareto criterion, since the real income of both households is higher under *Project 2* than under Project 1, but based on the Pareto criterion neither project is comparable with the status quo as the urban households lose while the rural households gain in both cases.

However, introducing supplementary value judgements also makes the projects comparable with the status quo. In this context it should be emphasized that to assume that the government attaches the same social welfare weight to the income of all household types, i.e. that the government is inequality neutral, as done in traditional cost-benefit analysis, naturally also implies the adoption of supplementary value judgements. Although such value judgements are convenient from a computational point of view, they are often not very relevant, i.e. they do not correspond to the value judgements of the political decision-maker for whom the analysis is prepared. On the basis of the social welfare weights in Table 2, the introduction of road pricing by either project is desirable, but the increase in social welfare is far greater when the revenue generated from road pricing is used to reduce the tax on labour income than to reduce other transport related taxes.

The implementation of both projects reduces, as already mentioned, the social evaluation free road capacity. This suggests that after the introduction of road pricing, a project to reduce the government's expenditures on road infrastructure would be desirable even if it had not been so prior to the reform.

Comparing the value of the change in the externalities (1.8 in the case of *Project 1* and 2.5 in the case of *Project 2*) with the change in social welfare (3.0 and 10.8) shows that the introduction of road pricing is associated with a significant *double dividend*[7], i.e. that the total benefits of the introduction of road pricing exceed the benefits of the reduction of the externalities (by 1.1 in the case of *Project 1* and by 8.3 in the case of *Project 2*). There are three reasons for this. The first reason is that the in the benchmark situation the tax on transport is smaller than its complementarity to leisure would justify, i.e. that a further increase in the tax on transport would increase social welfare even disregarding the effect on

[7] The first dividend is calculated as the change in welfare at the initial prices due to the change in the externalities. The double dividend is thus the change in welfare minus the first dividend.

the externality. The second reason is that the decrease in congestion reduces the use of non-market use of time for transportation and thus stimulates the labour supply (although only marginally, see Table 3). Both effects produce an efficiency gain. The third reason is that the introduction of road pricing redistributes income from the urban to the rural households, who have a higher marginal social welfare of income than the urban households.

Optimality analysis

In order to calculate the optimal tax structure (in terms of the income tax rate and in terms of taxes on transport and on other goods) and the optimal level of the public good (transport infrastructure), supplementary value judgements have to be specified as in the case of project evaluation. We assume that the government attaches a higher social welfare weight to the real income of the rural household than to the real income of the urban household (see Table 2), and on this basis we calculate the optimal policy for the government under three different sets of assumptions with respect to which policy instruments the government is effectively able to use. In the first case we consider the situation before a dramatic reduction of the administrative costs associated with road pricing has made road pricing feasible. We represent prohibitive administrative costs by the assumption that the government is not able to differentiate the tax on transport between the urban and the rural households. Taking a long-term perspective, we assume that the government is able to adapt the transport infrastructure to the optimal tax policy. In the second and third cases road pricing has become feasible. The government therefore in these cases is assumed (for simplicity at no costs) to be able to differentiate the tax on the consumption of transport between the rural and the urban households. In the second case we adopt a short-term perspective assuming that it is not possible to adapt the road infrastructure after the reduction in the administrative costs, whereas in the third case we adopt a long-term perspective as in the first case and assume that such adjustment is possible.

In all three cases we assume that the value of the lump-sum transfers remains unchanged in terms of the price of labour and therefore that the tax on labour can be fixed as a matter of normalisation.

The optimal solutions for the three sets of assumptions are provided in Table 4. In Part a of the table, the values of the instrument variables are indicated, whereas Part b contains values of a number of goal variables and other endogenous variables.

The optimal solution in all three cases involves taxes on transport that are much higher than in the benchmark situation and a level of transport infrastructure much lower than the benchmark level.

Compared with the benchmark situation, the optimal solution in the first case, *OPT1*, involves a considerable reduction in the consumption of transport.. The policy changes result in a reduction in congestion externalities valued at 6.4. This reduction stimulates the consumption of transport of the urban households, but does not affect the rural households. Both types of households benefit from the reduction in environmental externalities, but in the model this has no effect on behaviour. The increase in the taxation of transport increases the tax base directly by increasing the supply of labour, and indirectly by decreasing congestion, which in turn also increases the supply of labour. The policy change increases the real income of the rural household but decrease the real income of urban households which has to carry the burden of the increased tax on transport, but have to share the benefit of the reduction in other taxes with the rural households. The policy change results in a considerable increase in social welfare, in part explained by the benefits due to redistribution of income.

The optimal solution in the second situation, *OPT2,* shows that the introduction of road pricing (the possibility of differentiating the tax on the consumption of transport for the rural and for the urban households) makes it possible to increase social welfare, not only compared with the benchmark situation, but also compared with the optimal situation prior to the introduction of road pricing. The introduction of road pricing leads to an increase in the consumption of transport by rural households that is greater than the reduction for urban households. This results in an increase in the environmental externalities, but a reduction in the congestion externalities which only depend on consumption of transport by the urban households. Compared with the optimal solution without road pricing, *OPT1*, the change in the social value of the reduction of the externalities is 5.0. The policy change leads to a reduction in the supply of labour. The increase in urban households' labour supply as a consequence of the increase in the tax on transport is, due to the feed back effect, smaller for urban households than the corresponding decrease in the supply of labour for rural households. The use of an extra policy instrument results, as one would expect, in an increase in social welfare. Part of this increase is due to the reduction in the externality, part is due to the redistribution from urban to rural households. There is, however, compared to OPT1, no benefit due to an increase in the supply of labour - on the contrary.

In the third case, the optimal solution, OPT3, the introduction of road pricing by reducing the demand for transport infrastructure in urban areas, where it is associated with congestion, reduces the optimal amount of transport infrastructure. The possibility of adjusting the transport infrastructure justifies a further increase in the tax on transport in urban areas compared with the optimal solution in the previous case, *OPT2*. This is because the reduction of the government provision of road infrastructure increases in the marginal evaluation of the externality.

Table 4a: Optimal solutions: Transport infrastructure and tax rates

	Benchmark	***OPT1***	***OPT2***	***OPT3***
Transport infrastructure				
Government provision of transport infrastructure	50	31	31	21
Household taxes				
Labour	60%	60%	60%	60%
Transport in urban areas	80%	202%	446%	495%
Transport in rural areas	80%	202%	118%	112%
Other goods	25%	8%	9%	8%

Table 4b: Optimal solutions: Changes in transport, externalities and real income

	Benchmark	OPT1	OPT2	OPT3
		Changes compared to benchmark	Changes compared to OPT 1	
Consumption of transport				
Consumption of transport by urban households	70	-11.5	-4.2	-5.9
Consumption of transport by rural household	30	-12.0	8.8	9.7
Total	100	-23.5	4.6	3.8
Transport infrastructure				
Free road capacity	80	-2.7	4.4	-3.7
Marginal evaluation	1.01	0.2	-0.6	-0.4
Externalities				
Evaluation of congestion externalities		6.4	5.9	7.7
Evaluation of environmental externalities		4.9	-1.0	-0.8
Total		11.3	5.0	7.0
Labour				
Supply of labour by urban households	1084	26.7	31.5	37.9
Supply of labour by rural households	566	10.3	-33.4	-38.8
Total	1650	37.0	-1.9	-0.9
Real income				
Real income of urban households	813	-1.0	-41.4	-50.8
Real income of rural households	417	23.1	44.5	53.4
Social welfare	1311	24.4	7.5	7.9
Double dividend		13.1	2.5	1.0

Source: Own calculations based on stylised CGE model
Note: The double dividend is calculated as the change in social welfare minus the evaluation in the change in the externalities.

In recent years, the so-called *double dividend* issue has attracted considerable attention both among policy-makers and economists. The issue is whether replacing existing taxes with taxes on commodities causing environmental damage will increase social welfare, not only by internalising the negative external effects, but also by reducing the distortionary costs of the tax system as a whole. Based on the idea that the tax revenue obtained from environmental taxes could be used to reduce pre-existing distortionary taxes, the initial contributions to the analysis of the issue suggested that a *green tax reform* in general would be associated with a double dividend. However, it has subsequently become clear that the intuition behind the initial suggestion was flawed by not taking into account the distortionary effects of the environmental taxes. Now, it seems that the established wisdom is that a green tax reform is unlikely to generate a significant double dividend, and, if previous policies have been economically rational, that a green tax reform cannot generate a *double dividend* at all. The present analysis indicates that, in a situation where the tax on transport is too low (according to the relevant social welfare function), a higher tax on transport increase the social welfare beyond the social value of the decrease in the externalities associated with transport, i.e. is associated with a *double dividend*.

When the initial situation is not optimal (i.e. in the benchmark situation), adopting an optimal tax on transport (*OPT1*) or introducing road pricing at the optimal level justified by the reduction of the administrative costs (*OPT2* or *OPT2*) associated with a substantial *double dividend* (by 13.1, 15,6, and 14.1, respectively)

When the initial situation has been optimal given the administrative costs initially associated with introducing road pricing (*OPT1*), the *double dividend* associated with the introduction of road pricing is still positive, but far smaller; by 2.5 if the tax reform is not associated with an adjustment in the road infrastructure (*OPT2*), and by 1 when it is (*OPT3*).

The theoretical model allows a double dividend to arise from three sources: first that the change in the tax rates increases the tax base (in general by increasing the labour supply), second that the increase in the public good (free road capacity) has the same effect, and third that the tax changes have desirable income distributional effects (see Munk 2000).

The main source of the double dividend associated with the increase of the tax on transport from the benchmark to the optimal level (*OPT1*) is the increase in the supply of labour.

In the case of the introduction of road pricing leading from *OPT1* to *OPT2,* the *double dividend* is explained by the increase of the free road capacity resulting in a greater supply of labour and the redistribution of income from the urban to the rural households.

Finally, in the case of the introduction of road pricing leading from OPT1 to OPT3 the double dividend is explained only by the redistribution of income from the urban to the rural households.

Sandmo 2000 has suggested that when a green tax reform would be combined with an adjustment of the level of abatement, this would be associated with a third dividend. Comparing OPT3 with OPT2 shows that reducing the amount of road infrastructure after the introduction of road pricing naturally has resulted in increased of social welfare. In this sense Sandmo's observation is naturally correct, but notice that the larger increase in social welfare associated with OPT3 compared with OPT2 (7.9 compared with 7.4), the *double dividend* contribute less (1.0 compared to 2.5).

6 Summary and concluding remarks

We have demonstrated that analysing transport policy within a second-best framework leads to significant different conclusions than if conducted within a first best partial equilibrium framework. We have shown that when the initial situation is non-optimal, how the revenue generated by road pricing is recycled has important implications for the evaluation of transport projects, whereas at the optimum the choice of tax instrument to balance the government's budget is not important. Furthermore, we have shown that a tax on leisure travel following a reduction in the administrative costs of taxing transport is likely, contrary to the established wisdom, to be associated with a double dividend even if policies prior to the change have been optimal. The analysis has also highlighted that the introduction of road pricing will have important income distributional effects among regions and may affect the environment adversely, and that these are sensitive to how the government's budget is balanced. Finally, we have demonstrated that the introduction of road pricing justify a reduction in the size of transport infrastructure.

We have also demonstrated the potential of CGE models to provide an alternative to traditional cost benefit analysis as a framework for project evaluation. However, for CGE models to become a sound instrument for transport policy analysis substantial theoretical and empirical work still need to be undertaken.

First, the theoretical framework for transport policy analysis needs to represent the use of transport not only for leisure travel, as in the present model, but also for commuting and for use as intermediate input in production. The optimal road pricing charges for these uses are different than those for leisure travel. Furthermore also investments in public transport and the pricing policies for the use of public transport need to be taken into account in devising transport policy recommendations. This may be achieved by expanding on work already undertaken by Peary and Bento (2001), De Borge and Dender (2003), Calthorp, De Borge and Proost (2003) and others.

Second, but not of less importance, the empirical basis for CGE based transport policy analysis need to be improved. Important work has been undertaken under the EU financed UNITE project in establishing transport accounts, but further work need be undertaken to provide data in the form required for CGE modelling.

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Annex: Derivation of conditions for optimal tax and optimal abatement in a one household economy with public goods and external effects

Formulation of the government's maximisation problem for a one household economy

Using the expenditure function approach the government's maximisation problem may be expressed as

$$\underset{\{q_i,\ i=0,..,N\},L,e,u}{Max} \ u \ \text{s.t.}$$

$$L - E(\mathbf{q}, e, u) = 0\,, \qquad (\mu)$$

$$\sum_{k=0}^{2} t_i x_k(\mathbf{q}, e, u) + \mathrm{H}L - G - B = 0, \qquad (\lambda)$$

$$e\left(x_1\left(\mathbf{q},e,u\right)\right) - e = 0\,, \qquad (\rho) \tag{17}$$

The Lagrangian corresponding to the maximisation problem may be written as[8]

$$\begin{aligned} \mathrm{L} &= u \\ &+ \mu\left(-L - E(\mathbf{q}, e, u), G\right) + \lambda\left(\sum_{i=0}^{2} t_i E_i\left(\mathbf{q}, e, u\right) + L - G - B\right) \\ &+ \rho\left(e\left(E_1\left(\mathbf{q},e,u\right), G\right) - e\right) \end{aligned} \tag{18}$$

Using the expenditure function approach the first order conditions for an optimal solution with respect to u; L; $\mathbf{q} \equiv \left(q_0, q_1, q_2\right)$; and e, respectively, are

$$1 - \mu E_u + \lambda \sum_{i=0}^{2} t_i E_{\mathrm{iu}} + \rho e' E_{1\mathrm{u}} \qquad = 0 \tag{19}$$

$$-\mu + \lambda \qquad = 0 \tag{20}$$

---

8 We utilize the derivative notation writing $E_i \equiv \dfrac{\partial E}{\partial q_i}$, $i = 0,1,2$, $E_e \equiv \dfrac{\partial E}{\partial e}$, $E_{ij} \equiv \dfrac{\partial^2 E}{\partial q_i\, \partial q_j}$, $i, j = 0,1,2$ and $E_{ie} \equiv \dfrac{\partial^2 E}{\partial q_i\, \partial e}$, $i = 0,1,2$

$$-\mu x_k + \lambda x_k + \lambda \sum_{i=0}^{2} t_i E_{ik} + \rho e' E_{1k} \qquad = 0 \qquad\qquad k \in (0,1,2) \quad (21)$$

$$-\mu E_e + \lambda \sum_{i=0}^{2} t_i E_{ie} + \rho e' E_{1e} - \rho \qquad = 0 \qquad\qquad (22)$$

where $e' \equiv \frac{\partial e}{\partial x_1}$ is marginal change in free road capacity due to an increase in the consumption of transport.

Characterisation of the optimal tax system

The social value of a marginal increase in the externality

Defining the *value in monetary terms of a marginal increase in the externality* as $MV_e \equiv -E_e$ from (22) we then have that the social value of a marginal increase in the externality is

$$\rho = \left(\mu MV_e + \lambda \sum_{i=0}^{2} t_i E_{ie}\right) \frac{1}{1 - e' E_{1e}} \qquad (23)$$

It is thus determined by three different elements

1) The value in terms of social welfare of the increase in the externality

μMV_e ,

2) The opportunity cost value of the change in tax revenue due to the income and the substitution effect of the increase in the externality

$\lambda \sum_{i=0}^{2} t_i E_{ie}$, and

3) The feed back factor

$\frac{1}{1 - e' \mathrm{E}_{1\mathrm{e}}}$.

In the case of separability between the externality and household net trade, $\mathbf{x}$, i.e. where the household utility functions may be written as $u(U(\mathbf{x}), e)$, the feed back factor is 1 and the effect on tax revenue zero. The expression for ρ therefore becomes

$$\rho = \mu MV_e \qquad (24)$$

The net marginal social value of income

From (19), using that $E_{\mathrm{u}} = 1/\frac{\partial \mathrm{V}}{\partial I}(\mathbf{q}, e, I)$, $E_{\mathrm{iu}} = \frac{\partial \mathrm{x}_i}{\partial I^{\mathrm{h}}}(\mathbf{q}, e, I) / \frac{\partial V}{\partial I}(\mathbf{q}, e, I)$, we have

$$\mu = \beta + \lambda \sum_{i=0}^{2} t_i \frac{\partial x_i}{\partial I} + \rho e' \frac{\partial x_1}{\partial I} \tag{25}$$

where $\beta \equiv \frac{\partial V}{\partial I}$ is the *marginal value of income*. We may thus interpret μ, in analogy with Diamond (1975) 's *net marginal social value of income,* as the increase in social welfare if the income of the household were increased by one unit from outside the economy.

The first order conditions for an optimal solution with respect to changes lump sum tax

From (20) we have when a lump sum tax (linear income tax) is feasible, then at the optimum

$$\lambda = \mu \tag{26}$$

i.e. that the *opportunity cost price of government funds,* λ, is equal to the average net marginal social value of income, μ.

Tax formulae

The *marginal excess burden relative to the costs of government funds* is $\theta \equiv \frac{\lambda\text{-}\mu}{\lambda}$. If lump-sum taxation is feasible $\lambda\text{-}\mu = 0$ and thus $\theta = 0.$ If lump sum taxation is not feasible we have from (21)

$$\sum_{i=0}^{2} t_i E_{ik} = -\frac{\lambda - \mu}{\lambda} x_k - \frac{\rho}{\lambda} e' E_{1k} \qquad \text{k} \in (0,1,2) \ (27)$$

Multiplying on both sides by t_k, summing over k and reordering, we have

$$\sum_{k=0}^{2} \sum_{i=0}^{2} t_i t_k E_{ik} + \frac{\rho}{\lambda} e' \sum_{k=0}^{2} t_k E_{1k} = -\frac{\lambda - \mu}{\lambda} \left(G + B \right) \tag{28}$$

where $\left(G + B \right) = \sum_{k=0}^{2} t_k x_k > 0$.

The first term is negative because $\left\{ E_{ik}, i, k \in (0,1,2) \right\}$ negative semi-definite, the second term is in general positive because $\rho < 0$ and since the optimal tax system will in general discourage the compensated demand of the dirty good, i.e. $\sum_{k=0}^{2} t_k E_{1k} > 0$. It therefore possible the marginal costs of government funds, λ, may be smaller than the net social value of income, μ. However for relatively high levels of taxation $\lambda - \mu > 0.$

Setting $t_0 = 0$ as a matter of normalisation, we then obtain from (27) the following conditions to characterise an optimal tax structure

$$\sum_{i=C} t_i E_{i1} = -\theta x_1 - \frac{\rho}{\lambda} e' E_{11}$$

$$\sum_{i=C} t_i E_{i2} = -\theta x_2 - \frac{\rho}{\lambda} e' E_{12} \tag{29}$$

We therefore obtain the following expression for the tax rates on the produced commodities

$$t_1 = \theta \frac{\left(-E_{22} X_1 + E_{12} X_2\right)}{D} - \frac{\rho}{\lambda} e'$$

$$t_2 = \theta \frac{\left(-E_{11} X_2 + E_{21} X_1\right)}{D} \tag{30}$$

where D= $E_{11}E_{22} - E_{21}E_{12} = \left|\mathbf{E}^0\right| > 0$

Rewriting we get

$$t_1 = \theta\left(\varepsilon_{12} - \varepsilon_{22}\right)\frac{q_1 q_2}{D} - \frac{\rho}{\lambda} e' \tag{31}$$

$$t_2 = \theta\left(\varepsilon_{21} - \varepsilon_{11}\right)\frac{q_1 q_2}{D} \tag{32}$$

where $\varepsilon_{ij} \equiv E_{ij} \Big/ \frac{x_i}{q_j}$.

By the homogeneity of degree zero of compensated demand, $E_j(\mathbf{q},u)$, we have that $\sum_{j \in FC} \varepsilon_{ij} = 0$, $i \in FC$, and therefore that $\varepsilon_{12} = -\varepsilon_{11} - \varepsilon_{10}$ and $\varepsilon_{21} = -\varepsilon_{22} - \varepsilon_{20}$, the optimal tax structure may therefore also be expressed as

$$t_1 = \theta\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}\right)\frac{q_1 q_2}{D} - \left(\frac{\mu}{\lambda} MV_e + \sum_{i=0}^{2} t_i E_{ie}\right)\frac{e'}{1 - e' E_{1e}} \tag{33}$$

$$t_2 = \theta\left(-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}\right)\frac{q_1 q_2}{D} \tag{34}$$

The relative marginal excess burden, θ, may therefore be interpreted as the relative weight to be given to reducing the distortionary costs of financing the government's requirement and to internalising the environmental effects.

If a lump sum tax can be used such that $\lambda = \mu$, then θ=0 and

$$t_1 = -\frac{\rho}{\lambda} e' \tag{35}$$

$$t_2 = 0 \tag{36}$$

i.e. the tax on the dirty good is in that case equal to the Pigouvian tax as known from first best analysis.

The criteria for the optimal provision of transport infrastructure

Differentiating (18) with respect to G we have

$$MV_G = \frac{\lambda}{\mu}\left(MC_G - \frac{dT}{de}\frac{\partial e}{\partial G} \right) = 0 \tag{37}$$

where $MV_G \equiv E_e \frac{\partial e}{\partial G}$ is the marginal social evaluation of the transport infrastructure, $\frac{\lambda}{\mu}$, the marginal costs of government funds, $MC_G = 1$ the marginal cost of transport infrastructure and $\frac{dT}{dG} \equiv \sum_{i=0}^{2} t_i E_{ie} \frac{\partial e}{\partial G} \Big/ \left(1 - e' E_{1e}\right)$ the effect on the government's tax revenue of an increase in transport infrastructure.

Under first best assumptions $\frac{\lambda}{\mu} = 1$ and the condition for the optimal provision of the transport infrastructure become

$$MV_G = MC_G \tag{38}$$