An Approach to Stellar Photometry Using Simulated Annealing

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Abstract

Simulated annealing can be quite effectively used to fit functions of many parameters to a set of data. This paper discusses the application of simulated annealing to the fitting of Moffat functions to crowded field star images. Overlapping star profiles can be fit simultaneously and missed stars can be located based on the χ^2 value of the fit, and the resulting functional model yields excellent photometric results. This method has been integrated into a photometry system which allows the user to select the accuracy of the results at the expense of time needed to perform the calculations, and is but one example of such an application.

1. Introduction

Astronomical researchers are often interested in the brightness and color of stars, especially when studying star clusters in general and globular clusters in particular. A problem is that stars are often too close together to obtain an accurate measurement of brightness. Moreover, there may be many thousands of stars in an image, a fact that makes computer assisted analysis very important. Computers have been applied for some time to crowded field photometry, and computer programs for this purpose are in common use [2]. Many different approaches may be taken, but measuring the brightness of a star is equivalent to fitting a function to the data and integrating that function over the area of the star. This can be difficult when the frames contain so many stars that the their images overlap; many stars have to be fitted simultaneously.

Optimizing this fit requires of a method able to explore the parameter space involved and not get caught in local minima, since the best result is that corresponding to the global minimum. It is proposed here to apply the technique of simulated annealing to the optimization of this fit. Section 2 of this paper describes the use of a annealing in the context of crowded field stellar photometry. Section 3 describes the problems encountered in computer assisted photometry of crowded star fields and shows why this is interesting. Section 4 summarizes some of the work and gives some hints about future efforts.

2. Simulated Annealing

Many methods have been devised to optimize the value of a function in one of more parameters. These methods usually employ a figure of merit that determines how good the optimization is. They then optimize this value by changing the parameters repeatedly. The most straight forward approach is to choose new values of the parameters by changing them in the direction that reduces the value of the figure of merit. Though this would work fine for functions with a single minimum, it has the unpleasant problem of trapping the optimization process in local minima.

Simulated annealing has been used to minimize continuous real functions of many parameters since it tends not to get trapped in local minima. In order to fit a function to a sampled surface a measure of goodness of fit is minimized. The figure of merit employed here to determine the goodness of the fit is the χ^2 value, which gets smaller as the fit improves. An example, introduced by Bohachevsky [1], can be used to illustrate these how to fit a function using a simulated annealing process. The function to be minimized is:

$$f(x, y) = x^2 + 2y^2 - 0.3\cos(3\pi x) - 0.4\cos(4\pi y) + 0.7$$

This function is an effort to model the sort of convoluted surface that a fitness function might achieve as a worst case. As can be seen in Figure 1a this function has many local minima and one well defined global minimum at (0,0). Any downhill method could get caught in one of these local valleys and never reach (0,0). To test the algorithm we start at (1,1) and let the algorithm proceed. For this specific purpose we use a very simple implementation. Figure 1b shows the path followed by the algorithm, where the dark lines indicate the steps that it followed. This plot illustrates that annealing can be considered a biased random walk.

For every pixel in the region being fitted it is necessary to locate all those objects near enough to contribute some overlap. This is called *grouping*, and is done from a list of all located objects and their positions. One object is selected and then those which lie close enough are located and removed from the list. The process is recursively repeated for all these objects until no other overlapping object can be found. At this point a new object is selected from the list and the whole process repeated until no more objects remain in the list.

3. Stellar Photometry

If the image of a star as seen through a telescope were just a dot of light, one would be tempted to just measure one pixel in our image and consider that to be the brightness of the star. Unfortunately, the light has been spread over a region of the image according to the transfer function of the telescope. Instead of just measuring one pixel what should be done is to integrate all this light back into one value. This is equivalent to undoing the spreading of the transfer function. If no other stars are present a simple way of doing this integration is simply to select a region of the image and add up the light in all those pixels. This is known as *synthetic aperture* photometry [2] because the region simulates how the aperture of a single detector would have measured the signal.

A second approach to photometry uses the transfer function of the device. Here we use a function to account for the fact that the star was a point source that was spread into the resulting image. Because of this it is common to refer to this function as the point spread function (PSF). If we can come up with an analytic expression for the PSF then given the image it should be possible to adjust this expression until the function best fits the data. We can now compute the brightness of the star by integrating this function. For clear isolated stars synthetic aperture offers the advantage of being very fast. Astronomers are however more interested in clusters of stars, and these result in what are called crowded fields or images. If two stars are close enough their PSFs may overlap. It is then impossible to use synthetic aperture reliably. This is because it is not possible to know how much of the light in the overlap region corresponds to each star.

When deciding on a function to approximate the PSF of a telescope the best choice is fairly obvious. Moffat [3] studied the way in which an image forms in a telescope, and he showed that a simple function with 2 free parameters was enough to account for most of the distortions. The function suggested has come to be known as the Moffat function and is of the form

$$I(r) = I_0\left(1+\left(\frac{r}{\rho}\right)^2\right)^{-\beta}$$

where I_0 is the intensity of the star, I(r) is the intensity observed at a distance r from the center of the image and ρ and β are shape parameters. If the PSF is space invariant then so are the shape parameters. For example, given a value for peak intensity and full width at half maximum, only one possible shape exists for both the Gaussian and Lorentzian curves. For the Moffat curve there is an infinite combination of values of ρ and β that still give the same full width at half maximum. It is this flexibility that makes this function a more suitable choice.

The photometry system was tested in two ways: first on an extensive suite of simulated images, and then on real images for which good photometry has been published. Testing on synthetic images was done to explore the ability of the algorithm to solve the different problems crowded-field photometry imposes in a controlled environment. Several sets of test data were created and different types of noise added to evaluate performance under conditions that went from ideal to realistic.

In a test image a star is represented by a two dimensional peak. A pair of x and y coordinates determine the center of the star, and a value of I in its brightness. The PSF of the image is controlled through the shape of the star function. The algorithm attempts to minimize the χ^2 value as measured

FIGURE 1 (a) Bohachevsky's function; it is hard to find a global minimum because of all of the local ones. (b) The path that simulated annealing takes to find the minimum.

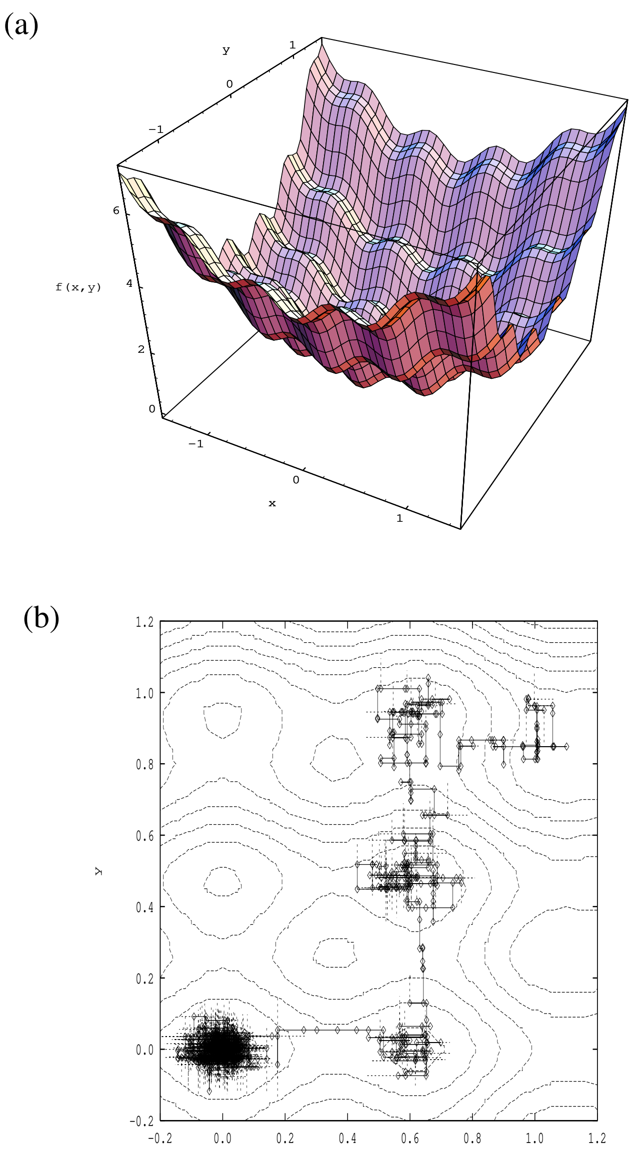
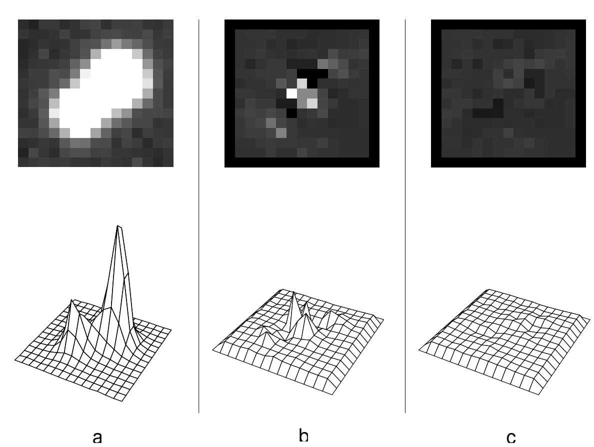


FIGURE 2 Determining the number of stars by using the residuals of multiple attempts.



against the star image, which is discrete. This means that the objective function is quite time consuming, requiring the comparison of the integral of the Moffat function over each pixel with the pixel value at the same point. For example, the fit of a Moffat star obtained through a genetic algorithm is compared with a fit found through simulated annealing as follows:

Method	Beta	Rho	X	Y	I	χ^2	Evals
Genetic	5.006	9.824	8.000	8.000	101.563	0.0040	14000
Annealing	4.658	9.431	8.003	7.999	101.332	0.0039	3366

After extensive testing on images containing a single simulated star, a series of tests were run using a set of test images having two test stars each. At first the test images corresponded to pairs of stars of equal intensity. One star is kept centered on a pixel while the other one is placed at different pixel offsets. Then experiments were performed using stars of variable brightness. Another test uses a sequence of two star images where the two stars are moved progressively closer to each other. They correspond to a separation of 3, 2.8, 2.5, 2.1, 1.8 and 1.4 pixels between the centers. Even at a distance of only 1.4 pixels the genetic algorithm does a good job of estimating both the center coordinates and intensities of the stars:

Tests were also performed to verify the ability to fit stars by iterating when a star is missed in a first pass, and later added to the fitting list. For example, an image was created with three stars in it. Two were large and separated enough to make sure they would be detected in the first pass, and a third smaller star was placed between these two so it would not be possible to see it in a first pass. The parameters for this image can be seen in Figure 3, which also shows both the image and a three dimensional representation of the data. The original image clearly seems to contain only two stars. When only two stars are fitted the median filtered residuals look like those in Figure 3b. Based on this a third star is added and all three fitted again. The resulting median filtered residuals can be seen in Figure 3c.

4. Conclusions and Further Work

While synthetic test images provide control situations were the performance of the algorithm can be tested, the final test is the application to real data. For this purpose several CCD images of the globular cluster NGC6397 have been obtained and reduced. In astronomical work the independent values obtained for each star in the frame are rarely considered individually. Usually two frames are available for reduction, one taken with a V filter (meaning visual, a yellowish color) and another one with a B (blue) filter. Once the stars are measured in both frames it is possible to establish their color from the two sets of results. A normal way of describing the color is by expressing the difference between these values, for example B-V.

The usefulness of annealing can be established by comparing the B-V diagram it generates with one created by standard techniques. The quality of the result was ascertained by finding how well the diagram compares with one obtained by standard methods. This was done for NGC6397[4], and the comparison of the diagrams can be seen in Figure 4. There is a linear relationship between the two reduced sets of data de-

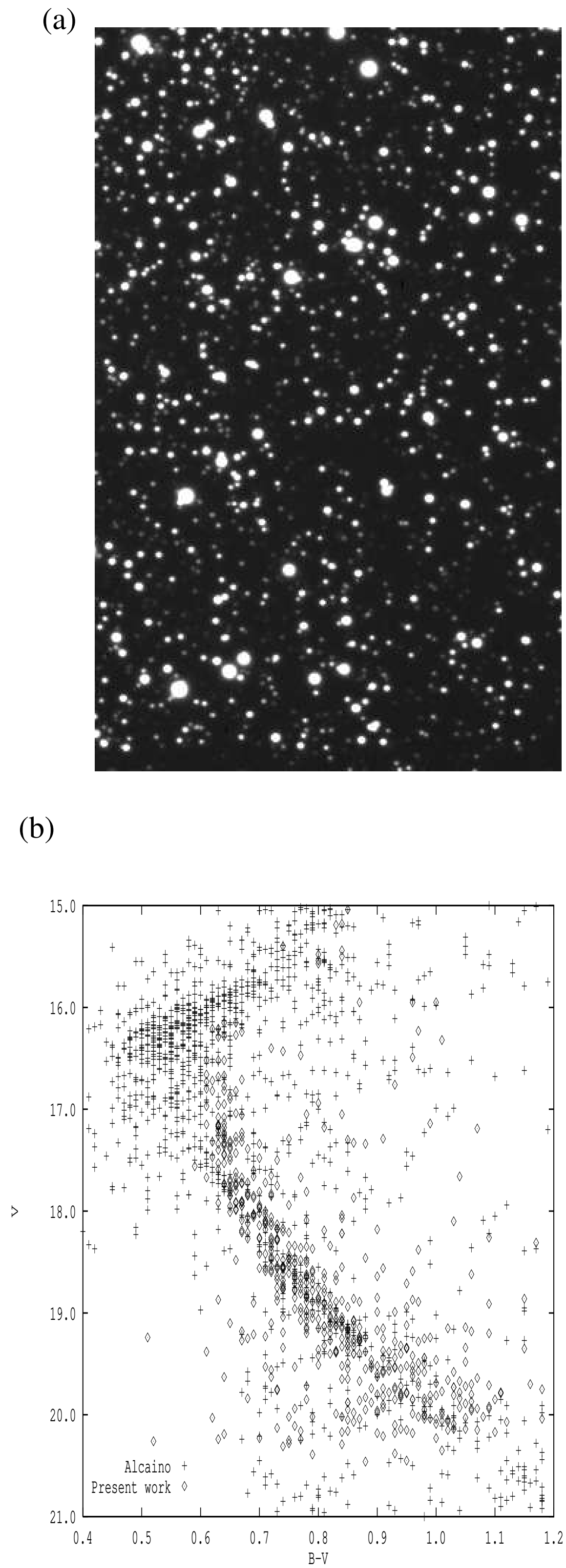


FIGURE 3 (a) Portion of the cluster NGC6397 (b) Color-magnitude diagram showing both annealing data and that obtained by traditional methods.

fined by:

$$B_{pub} = 25.21 + 0.953 B_{anneal}$$
$$V_{pub} = 24.91 + 0.987 V_{anneal}$$

where B_{anneal} was obtained by annealing and B_{pub} by standard methods.

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5. References

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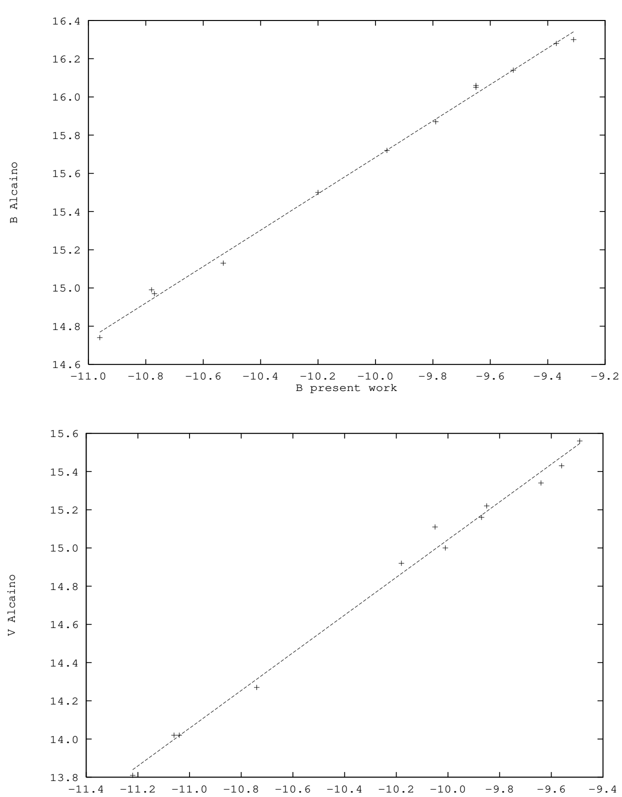

FIGURE 4 Relationship between the data reduced using annealing and data reduced using other techniques. A linear relationship is apparent.