Advanced Analytical Modelling of Steam-Assisted Gravity Drainage and its Variants

Sharma, Jyotsna

doi:10.11575/PRISM/22889
http://hdl.handle.net/1880/49460
documental thesis

University of Calgary graduate students retain copyright ownership and moral rights for their thesis. You may use this material in any way that is permitted by the Copyright Act or through licensing that has been assigned to the document. For uses that are not allowable under copyright legislation or licensing, you are required to seek permission.

Downloaded from PRISM: https://prism.ucalgary.ca
UNIVERSITY OF CALGARY

Advanced Analytical Modelling of
Steam-Assisted Gravity Drainage and its Variants

by

Jyotsna Sharma

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING
CALGARY, ALBERTA
OCTOBER 2011

© Jyotsna Sharma 2011
Abstract

Advanced analytical models of Steam-Assisted Gravity Drainage (SAGD) were developed in a multiphase flow framework, accounting for convection and frontal stability. The use of solvents and non-condensable gases for improving oil recovery and enhancing thermal efficiency was also examined, based on the analytical models and also by fine-grid numerical simulation (2.5 mm grid block size).

The new SAGD models take into account multiphase flow and convective transport via modified diffusivity. Stability of the steam-bitumen interface was also investigated. Importance of these effects, missing from the existing models, is demonstrated. The analytical approach permits a comprehensive examination of the main variables, with interesting and sometimes unexpected findings. For example, contrary to the existing theories, the oil mobility reaches a maximum some distance away from the edge of the steam chamber. It was also found that convection, neglected in previous models, plays an important role in heat transport.

Co-injection of a non-condensable gas with steam was generally found to be counter-productive because of reduced condensation heat transfer by steam due to the presence of gas. Linear stability analysis showed that co-injection of solvent with steam increases interfacial instability, thus promoting increased solvent mixing at the edge of the chamber. Consequently, the oil rate of a steam-solvent process is higher than for steam-only. Fine-grid simulation results corroborate the analytical findings. SAGD field data as well as selected experimental results compared well with the predictions using the models developed.
Preface

List of original publications arising from the research documented in this thesis:

**Publications in Peer-Reviewed Journals:**


**Publications in Conference Proceedings:**


Acknowledgements

I wish to express my sincere gratitude to my supervisor, Dr. Gordon Moore, for his confidence in me and for his direction and encouragement throughout the course of this work. His invaluable guidance and fortuitous personal characteristics have inspired me to question and learn. I also extend my deepest thanks to Dr. Farouq Ali, an exceptional teacher, mentor and friend, who has provided me so much more assistance than deserved. Without his constant support through good and bad times alike, invaluable guidance, and constructive criticisms on my numerous ‘drafts of drafts’, I doubt very much that this thesis would have ever been completed.

I am grateful to Dr. Ian Gates, for giving me the opportunity to pursue my interest in analytical modelling and for helping me so well to shape and express my thoughts. Special thanks to Dr. Anthony Kovscek, for providing me with the opportunity to work at the Stanford University, and for his continued guidance and support for my research. I would also like to thank all the members of the In-Situ Combustion Research Group, particularly Dr. Raj Mehta, for their constant motivation and support. Appreciation is also extended to all the members of my supervisory committee and the examination committee. In addition, I would like to thank and acknowledge the financial support from Shell Canada, NSERC and the Department of Chemical and Petroleum Engineering.

Most of all, my love and special thanks go to my family, especially my parents, my sister, Juhi, and my partner, Daryl, for their eternal patience, love, and endless support during all these years. You all have always been and will always be the most important people in my life!
Whatever actions I may perform, 
by my body, mind, senses, intellect, or by my inner self, 
either intentionally or unintentionally, 
I offer it all to the Supreme Lord.

— Śrīmad Bhāgavatam

(Ancient Indian Scripture)
# Table of Contents

Approval Page ................................................................................................................ ii  
Abstract......................................................................................................................... iii  
Preface ........................................................................................................................... iv  
Acknowledgements ....................................................................................................... v  
Table of Contents .......................................................................................................... vii  
List of Tables .................................................................................................................. xi  
List of Figures and Illustrations ....................................................................................... xiii  
Nomenclature ................................................................................................................ xx  

## CHAPTER 1:  
INTRODUCTION .................................................................................................................. 1  
1.1 Heavy Oil and Oil Sands ........................................................................................ 1  
1.2 In-situ Extraction of Heavy Oil .............................................................................. 3  
  
1.2.1 Non-thermal Recovery Methods .................................................................... 3  
  
1.2.2 Thermal Recovery Methods ........................................................................... 4  
  
1.3 Steam-Assisted Gravity Drainage and its Variants ................................................. 6  
  
1.3.1 Original SAGD Concept ................................................................................ 6  
  
1.3.2 Non-condensable Gas Aided SAGD ............................................................... 9  
  
1.3.3 Solvent-Aided SAGD .................................................................................... 9  
  
1.4 Motivation ............................................................................................................. 12  
1.5 Specific Objectives of this Research .................................................................... 13  
1.6 Thesis Outline ...................................................................................................... 14  

## CHAPTER 2:  
LITERATURE REVIEW .................................................................................................... 18  
  
2.1 Analytical Modelling of Steam-Assisted Gravity Drainage (SAGD) ......................... 18  
  
2.2 Heat Transfer Mechanism .................................................................................... 21  
  
2.3 Non-condensable Gas Injection in SAGD ............................................................ 24  
  
2.4 Steam-Solvent Hybrid Processes ......................................................................... 27  
  
2.5 Interfacial Stability Analysis ................................................................................ 30  
  
2.5.1 Continuum Modelling Approach .................................................................. 31  
  
2.5.2 Non-continuum Modelling Approach ........................................................... 32  

## CHAPTER 3:  
MULTIPHASE ANALYTICAL MODELLING OF SAGD ..................................................... 34  
  
3.1 Original SAGD Theory and Assumptions ............................................................... 34  
  
3.2 Analytical Model .................................................................................................... 36  
  
3.2.1 Modelling Assumptions and Explanations...................................................... 36  
  
3.2.2 Oil Mobility Profile at the Edge of a Steam Chamber ................................... 37  
  
3.2.3 Oil Flow Rate at the Edge of a Steam Chamber ............................................ 41  
  
3.3 Results and Discussion. ......................................................................................... 45
CHAPTER 4: CONVECTION AT THE EDGE OF SAGD STEAM CHAMBER .......................... 63
  4.1 Introduction ......................................................................................................... 63
  4.2 Analytical Model ................................................................................................. 64
    4.2.1 Modelling Assumptions ............................................................................... 64
    4.2.2 Mathematical Formulation ........................................................................... 65
  4.3 Results and Discussion ......................................................................................... 70
    4.3.1 Relative Role of Convection vs. Conduction ................................................ 70
    4.3.2 Oil Mobility and Flow Rate ......................................................................... 74
  4.4 Validation of Theory with Field Data ................................................................... 78
  4.5 Comparison of Theory with Numerical Simulation Results ............................... 81
  4.6 Conclusions ......................................................................................................... 86

CHAPTER 5: NON-CONDENSABLE GAS AIDED SAGD PROCESS ......................... 87
  5.1 Introduction ....................................................................................................... 87
  5.2 Theory ............................................................................................................... 88
  5.3 Analytical Model ............................................................................................... 91
    5.3.1 Modelling Assumptions ............................................................................... 93
    5.3.2 Mathematical Formulation ........................................................................... 93
    5.3.3 Algorithm for Generating Fluid Flow Profiles ............................................. 105
    5.3.4 Oil Flow Rate Calculation .......................................................................... 109
  5.4 Numerical Simulation ......................................................................................... 113
  5.5 Results and Discussion ...................................................................................... 116
    5.5.1 Analytical Model ......................................................................................... 116
    5.5.2 Homogeneous Reservoir Simulation ........................................................... 121
    5.5.3 Heterogeneous Reservoir Simulation ......................................................... 128
  5.6 Validation of Theory with SAGP Experimental Data .......................................... 141
  5.7 Conclusions .................................................................................................... 145

CHAPTER 6: SOLVENT-AIDED SAGD PROCESS ............................................. 146
  6.1 Introduction .................................................................................................... 146
  6.2 Length Scales for Heat and Mass Transport at the Edge of a SAGD Chamber .... 147
  6.3 Relation with the Non-condensable Gas Aided SAGD Model ...................... 150
    6.3.1 Modified Algorithm for Solvent-Aided SAGD Process ......................... 151
  6.4 Analytical Model .............................................................................................. 153
    6.4.1 Modelling Assumptions .......................................................................... 153
6.4.2 Mathematical Formulation ................................................................. 156
6.5 Numerical Simulation ............................................................................ 159
6.6 Results and Discussion ........................................................................ 160
  6.6.1 Analytical Model Results ................................................................. 160
  6.6.2 Numerical Simulation Results ......................................................... 170
6.7 Validation of Theory with ES-SAGD Experimental Data ...................... 178
6.8 Conclusions ......................................................................................... 187

CHAPTER 7: ................................................................................................. 188

TRANSIENT COUPLED HEAT AND MASS TRANSFER MODEL ................. 188
  7.1 Introduction ......................................................................................... 188
  7.2 Coupled Heat and Mass Transfer Model .............................................. 189
    7.2.1 Theory of Heat Transfer ............................................................... 191
    7.2.2 Theory of Mass Transfer ............................................................. 192
    7.2.3 Convective Velocity Calculation .................................................. 194
    7.2.4 Coupled Heat and Mass Transfer Equations ................................... 196
  7.3 Results and Discussion ....................................................................... 197
  7.4 Difference between Current Model and Models Derived in Chapters 5 and 6 ..... 205
  7.5 Conclusions ....................................................................................... 208

CHAPTER 8: ................................................................................................. 209

INTERFACIAL STABILITY IN STEAM-SOLVENT PROCESSES ................. 209
  8.1 Introduction ......................................................................................... 209
  8.2 Theory ................................................................................................. 209
  8.3 Analytical Model .................................................................................. 210
    8.3.1 Modelling Assumptions ............................................................... 210
    8.3.2 Mathematical Formulation ............................................................ 212
  8.4 Results and Discussion ....................................................................... 219
  8.5 Validation with Experimental Data ..................................................... 227
  8.6 Conclusions ....................................................................................... 229

CHAPTER 9: ................................................................................................. 231

EXPERIMENTS AND ANALYSIS OF PORE SCALE VISCIOUS FINGERING ...... 231
  9.1 Introduction ......................................................................................... 231
  9.2 Theory ................................................................................................. 233
  9.3 Micromodel Experiments ..................................................................... 234
  9.4 Linear Stability Analysis ..................................................................... 237
  9.5 Modelling Complex Displacement Patterns ...................................... 240
    9.5.1 Capillary vs. Viscous forces ......................................................... 241
    9.5.2 Invasion-Percolation Clusters ...................................................... 243
    9.5.3 Modelling Approach ................................................................. 245
  9.6 Scaling Laws ....................................................................................... 245
  9.7 Experimental Observations and Analysis ........................................... 251
List of Tables

Table 3.1: Typical parameters for Athabasca reservoir (Ito and Hirata, 1999; Ito and Suzuki, 1996).................................................................................................................................39

Table 3.3: Comparison of SAGD models using field data. .............................................56

Table 3.4: Reservoir parameters used in the analytical model for comparison with the simulation results. ..............................................................................................................58

Table 4.1: Fluid and reservoir properties used in the analytical model.........................71

Table 4.2: Reservoir and fluid properties used in the simulation model (Ito and Suzuki, 1996; Ito and Hirata, 1999; Butler, 1997a).................................................................83

Table 5.1: List of correlations used in the analytical model. ...........................................99

Table 5.2: Reservoir and fluid properties used in the analytical and simulation models (Ito and Suzuki, 1996; Ito and Hirata, 1999; Butler, 1997a).................................106

Table 5.3: Operating conditions used in the numerical simulations. ............................115

Table 5.4a: Reservoir and fluid properties used in the SAGP experiments (Jiang et al., 1998). ..................................................................................................................142

Table 5.4b: SAGD and SAGP experimental results (Jiang et al., 1998). ........................142

Table 6.1: List of correlations used in the analytical model. .........................................158

Table 6.2: Reservoir and fluid properties used in the analytical and simulation models (Ayodele et al., 2010; Butler, 1997a).................................................................161

Table 6.3: Operating conditions used in the simulations................................................171

Table 6.4: Reservoir properties and operating conditions used in the SAGD and ES-SAGD experiments (Ayodele et al., 2010).................................................................180

Table 7.1: List of correlations and associated parameters used in this study. ..............193

Table 7.2: Reservoir properties and operating conditions used in the solvent-aided SAGD case analyzed using Matlab (Ayodele et al., 2010; Ito and Hirata, 1999)....198

Table 8.1: List of correlations and associated parameters used in this study. .................217

Table 8.2: Reservoir properties used in the analytical model. .......................................220

Table 9.1: Micromodel specifications ............................................................................262
Table 9.2: Finger width calculation

Table 9.3: Experimental parameters

Table 9.4: Results from Figures 9.4-9.8

Table A.1: Reservoir and fluid properties used in the simulation model (Ito and Hirata, 1999; Ito and Suzuki, 1996; Butler, 1997a).

Table A.2: Correlations for diffusion and dispersion coefficients and associated parameters used in this study.
List of Figures and Illustrations

Figure 1.1: Distribution of the total world oil reserves (Oilfield Review, May 2008) ........2

Figure 1.2: Viscosity of Athabasca bitumen versus temperature (Mehrotra and Svrcek, 1986) ................................................................. 5

Figure 1.3: Cross-sectional view of Steam-Assisted Gravity Drainage (SAGD) ........7

Figure 1.4: Viscosity of mixtures of Athabasca bitumen and hexane obtained by a logarithmic mixing rule ................................................................. 11

Figure 1.5: Schematic representation of the organization of this research ........17

Figure 3.1: Vertical section of SAGD steam chamber interface ........................................... 42

Figure 3.2: Temperature and bitumen viscosity profiles as a function of distance from the steam chamber edge ................................................................. 47

Figure 3.3: Oil phase mobility profile as a function of distance from the steam chamber edge ................................................................. 47

Figure 3.4: Surface plot of location of maximum oil mobility versus the viscosity parameter, \( m \), and the Corey coefficient for oil, \( a \) ................................................................. 48

Figure 3.5: Volumetric oil flux as a function of distance from the edge of the steam chamber ................................................................. 48

Figure 3.6: Surface plot of the comparison factor for Butler et al. (1981) and new model, versus viscosity parameter, \( m \), and Corey coefficient, \( a \) ................50

Figure 3.7: Comparison of oil phase mobility values from Butler’s model (1981) and current work ................................................................. 52

Figure 3.8: Comparison of volumetric oil flux values from Butler’s model (1981) and current work ................................................................. 52

Figure 3.9: Effect of thermal diffusivity, \( \alpha \), on volumetric oil flux ................53

Figure 3.10: Effect of the chamber expansion rate, \( U \), on volumetric oil flux ..........53

Figure 3.11: Homogeneous reservoir simulation results (a) Temperature distribution in the reservoir (b) Temperature profile as a function of distance from the center of the well pair (c) Oil phase mobility profile (d) Cumulative and instantaneous oil flow rates ................................................................. 59
Figure 3.12: Analytical model results (a) Temperature profile (b) Oil phase mobility profile, as a function of distance from the steam chamber edge. .................................60

Figure 4.1: Conduction and convection at the edge of a steam chamber. .......................66

Figure 4.2: Ratio of the apparent thermal diffusivity ($\alpha^*$) to the conductive thermal diffusivity ($\alpha$) versus distance from the steam chamber edge. See Table 4.1 for fluid and reservoir properties. .................................................................73

Figure 4.3: Conductive and convective heat fluxes in the conduction plus convection case. See Table 4.1 for fluid and reservoir properties. ................................................73

Figure 4.4: Comparison of temperature profiles beyond the edge of the steam chamber in conduction-only and conduction plus convection cases. ....................75

Figure 4.5: Conductive and convective heat fluxes in the conduction plus convection case at the chamber edge versus temperature. .............................................75

Figure 4.6: Comparison of oil phase mobility profiles of the conduction-only and conduction plus convection cases beyond the edge of the steam chamber. ..............77

Figure 4.7: Comparison of oil flux per metre of well length (into the page) for conduction-only and conduction plus convection cases beyond the edge of the steam chamber. ...........................................................................................77

Figure 4.8: Comparison of apparent thermal diffusivity estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot, and the predictions of the analytical theory (Equation 4.16). See Table 4.1 for fluid and reservoir properties. .................................................................................................79

Figure 4.9: Comparison of temperature distribution estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot versus predictions calculated by theory (Equation (4.15)). See Table 4.1 for fluid and reservoir properties. ........................................................................................................79

Figure 4.10: Comparison of conductive and convective heat flux components estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot versus predictions calculated by theory (Equations (4.17) and (4.18)). See Table 4.1 for fluid and reservoir properties. .................................................80

Figure 4.11: Homogeneous reservoir simulation results after 30 hours of steaming (a) Temperature distribution in the reservoir (b) Temperature profile as a function of distance from the center of the well pair. ..............................................................................84

Figure 4.12: An exponentially decaying temperature function superimposed on the temperature profile obtained from the numerical simulation. .........................................85
Figure 5.1: Variation of heat transfer coefficient with air mass fraction (Maheshwari et. al., 2004).

Figure 5.2: Cross section of the SAGD steam chamber with non-condensable gas, showing the mobile zone and the chamber edge.

Figure 5.3: Schematic of the mobile zone beyond the vapour chamber of a non-condensable gas aided SAGD process.

Figure 5.4: Comparison of the rate of methane diffusion and the rate of methane accumulation in the gas phase.

Figure 5.5: Vertical section of a SAGD chamber interface.

Figure 5.6: Analytical model results (a) Temperature & viscosity profiles (b) Phase saturation profiles (c) Relative permeability profiles (d) Methane mole fraction in the gas phase (e) Methane mole fraction in the oil phase (f) Phase mobility profiles.

Figure 5.7: Homogeneous reservoir simulation (a) Temperature distribution (b) Temperature profile versus distance from the center of the well pair.

Figure 5.8: Homogeneous reservoir simulation (a) Oil saturation distribution (b) Saturation profiles versus distance from the center of the well pair.

Figure 5.9: Homogeneous reservoir simulation (a) Three-phase relative permeability profile (b) Methane distribution in the chamber (c) Methane mole fraction in the oil and gas phases (d) Phase mobility profiles versus distance from the center of the well pair.

Figure 5.10: Homogeneous reservoir simulation: Cumulative and instantaneous oil rates for the methane co-injection case.

Figure 5.11: Heterogeneous reservoir simulation (a) Temperature distribution (Case-A) (b) Temperature distribution (Case-B) (c) Temperature distribution (Base Case) (d) Temperature profiles versus distance from the center of the well pair.

Figure 5.12: Rate of rise of temperature inside a reservoir grid-block located within the SAGD chamber at the co-ordinates [1-3-27].

Figure 5.13: Heterogeneous reservoir simulation (a) Methane mole fraction in the gas phase (Case-A) (b) Methane mole fraction in the gas phase (Case-B) (c) Effective methane concentration in the gas phase (Case-A) (d) Effective methane concentration in the gas phase (Case-B).

Figure 5.14: Heterogeneous reservoir simulation (a) Steam quality (Case-A) (b) Steam quality (Case-B) (c) Steam quality (Base Case).
Figure 5.15: Heterogeneous reservoir simulation (a) Steam quality and temperature profiles (b) Temperature and methane concentration profiles versus distance from the center of the well pair...............................................................136

Figure 5.16: Heterogeneous reservoir simulation (a) Oil phase mobility profile as a function of distance from the center of the well pair (b) Oil flow rate and well bottom hole pressures versus time. .................................................................137

Figure 5.17: Heterogeneous reservoir simulation (a) Steam quality and temperature profiles in the vertical direction across the reservoir pay (b) Temperature and methane mole fraction profiles in the vertical direction across the reservoir pay...139

Figure 5.18: Heat lost to the overburden as seen in the heterogeneous reservoir simulation. ...........................................................................................................140

Figure 5.19: Production and injection data for SAGP experiments using nitrogen as the non-condensable gas (Jiang et. al., 1998). .................................................................143

Figure 5.20: Analytical model results generated using the SAGP experimental data given in Table 5.4a (a) Temperature and oil viscosity profiles (b) Phase saturation profiles (c) Phase relative permeability profiles (d) Nitrogen mole fraction in the oil phase.................................................................144

Figure 6.1: Conductive heat transfer, and diffusive and dispersive mass transfer at the edge of a steam-solvent SAGD chamber. .................................................................154

Figure 6.2: Temperature profile beyond edge of the vapour chamber for the SAGD and solvent-aided SAGD cases. Model parameters are set to the values listed in Table 6.2.................................................................162

Figure 6.3: Comparison of rate of diffusion of hexane in the gas phase and the rate of hexane accumulation near the interface due to steam condensation. ...............164

Figure 6.4: Profile of hexane mole fraction in the gas phase beyond the edge of the chamber for the solvent-aided SAGD case. Model parameters are set to the values listed in Table 6.2.................................................................164

Figure 6.5: Profile of hexane dissolved in the oil phase beyond the chamber edge for the solvent-aided SAGD case (a) expressed as molar concentration (b) expressed as mole fraction. Model parameters are set to values listed in Table 6.2..............165

Figure 6.6: Value of effective diffusion coefficient in oil ($D_{oil,eff}$), given by Equation 6.5, beyond the edge of the chamber for the solvent-aided SAGD case. Model parameters are set to values listed in Table 6.2 .................................................................168
Figure 6.7: Profiles of oil phase viscosity beyond the chamber edge for the SAGD and solvent-aided SAGD cases. Model parameters are set to values listed in Table 6.2.................................168

Figure 6.8: Profiles of oil phase mobility beyond the chamber edge for the SAGD and solvent-aided SAGD cases. Model parameters are set to values listed in Table 6.2.................................169

Figure 6.9: Lewis number versus distance beyond the steam chamber edge for the solvent-aided SAGD case. Model parameters are set to values listed in Table 6.2.................................169

Figure 6.10: Homogeneous reservoir simulation (a) Temperature distribution in the reservoir for the SAGD case (b) Temperature distribution in the reservoir for the solvent-aided SAGD case with hexane (c) Temperature profiles for the SAGD and solvent-aided SAGD cases.................................172

Figure 6.11: Homogeneous reservoir simulations (a) Hexane mole fraction distribution in the gas phase (b) Hexane mole fraction distribution in the oil phase (c) Hexane mole fraction profile in the oil and gas phases, for the solvent-aided SAGD case.........................................................174

Figure 6.12: Homogeneous reservoir simulations (a) Oil phase viscosity for the SAGD case (b) Oil phase viscosity for the solvent-aided SAGD case with hexane (c) Temperature and oil phase viscosity profiles for the SAGD and solvent-aided SAGD cases.................................................................176

Figure 6.13: Homogeneous reservoir simulations (a) Oil phase mobility for the SAGD case (b) Oil phase mobility for the solvent-aided SAGD case with hexane (c) Temperature and oil phase mobility profiles for the SAGD and solvent-aided SAGD cases.................................................................177

Figure 6.14: Cumulative oil produced for the SAGD and solvent-aided SAGD cases as seen in the homogeneous reservoir simulations.................................................179

Figure 6.15: ES-SAGD Experiment results (Ayodele et. al., 2010) (a) Oil and solvent production rates (b) Cumulative oil produced in the ES-SAGD experiment.........181

Figure 6.16: Analytical results generated using the ES-SAGD experimental parameters listed in Table 6.4 (a) Temperature profile as a function of distance from the chamber edge (b) Comparison of the rate of hexane accumulation near the chamber walls and the rate of hexane diffusion in the gas phase (c) Hexane mole fraction profile in the oil phase (d) Hexane mole fraction profile in the gas phase (e) Phase saturation profile (f) Phase relative permeability profile..................184
Figure 6.17: Analytical results generated using the SAGD and ES-SAGD experimental parameters listed in Table 6.4 (a) Temperature profile (b) Oil phase viscosity profile (c) Oil phase mobility profile.

Figure 7.1: Conductive and convective heat transfer and convective, diffusive and dispersive mass transfer at the edge of a steam-solvent SAGD chamber.

Figure 7.2: Effect of time on the temperature profile beyond the edge of the steam-solvent chamber.

Figure 7.3: Effect of time on hexane mole fraction profile in the oil phase beyond the edge of the steam-solvent chamber.

Figure 7.4: Temperature profiles beyond the edge of steam-solvent chamber, with and without convective heat and mass transfer effects.

Figure 7.5: Convective flow velocities of the oil phase and the water phase as a function of distance from the edge of the steam-solvent chamber.

Figure 7.6: Solvent mole fraction profile in the oil phase as a function of distance from the edge of the steam-solvent chamber, with and without convective heat and mass transfer effects.

Figure 7.7: Oil phase mobility profiles beyond the edge of steam-solvent chamber, with and without convective heat and mass transfer effects.

Figure 7.8: Schematic of the mobile zone beyond the SAGD chamber in (a) Chapter 7 and (b) in Chapters 5 and 6. ($S_g$=Gas Saturation).

Figure 8.1: Schematic diagram of steam-solvent bitumen interface.

Figure 8.2: Growth rates (γ) for steam only and steam-solvent mixtures versus wavenumber (β). The pressure indicated in each plot is the total injection pressure. The mole fraction of the solvent in the vapour phase equals 0.5. The values are derived using Equation (8.17).

Figure 8.3: Growth rate of the most unstable perturbation (γ corresponding to $w_m$ in Equation (8.19)) versus mole fraction of solvent in vapour chamber at 3 MPa total pressure.

Figure 8.4: Growth rate of the most unstable perturbation (γ corresponding to $w_m$ in Equation (8.19)) versus solvent carbon number (0=steam only, 3 = propane, 4 = butane, 6 = hexane, 7 = heptane, 8 = octane) at different total pressure, with mole fraction of solvent in the vapour chamber equal to (a) 0.5 and (b) 0.6.

Figure 8.5: Oil drainage rate from Nasr and Isaacs’ (2001) experiments versus solvent carbon number (0=steam only, 1 = methane, 2 = ethane, 3 = propane, 5 = ...
pentane, 6 = hexane, 8 = octane). The experiments were conducted at an operating pressure of 2.1 MPa and injected solvent content equal to 1.5% by volume (near the interface, the concentration of solvent would have been substantially higher). ................................................................. 228

Figure 9.1: (a) Top view of a completed micromodel (b) High magnification photograph of Berea sandstone pore structure (c) Etched micromodel pattern. ..... 236

Figure 9.2: Macroscopic fingers observed during the early stage of Experiments I-V. 239

Figure 9.3: A 2-D fractal percolating cluster overlaid by a grid of squares. Mass of the cluster is related to the number of squares occupied by the cluster through the fractal dimension ................................................................. 244

Figure 9.4: Experiment I: Lost Hills, q=16x10⁻⁴ ml/min. .................................................. 252

Figure 9.5: Experiment II: South Belridge, q=16x10⁻⁴ ml/min. ........................................ 253

Figure 9.6: Experiment III: Pirawarth, q=16x10⁻⁴ ml/min. ............................................. 254

Figure 9.7: Experiment IV: Pirawarth, q=2x10⁻⁴ ml/min. ............................................... 255

Figure 9.8: Experiment V: Pirawarth, q=8x10⁻⁴ ml/min. ................................................ 256

Figure A.1: Schematic representation of the numerical simulation study done in this research .................................................................................................................. 286

Figure A.2: Permeability distribution in the reservoir for (a) Homogeneous model (b) Heterogeneous model. ................................................................................................. 288

Figure A.3: Temperature versus oil viscosity profile used in the simulation (Mehrotra and Svrcek, 1986). ...................................................................................... 292

Figure A.4: Relative permeability curves used in simulations (a) Water-oil data (b) Gas-oil data. ........................................................................................................... 293
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$, $b$</td>
<td>Corey coefficients, dimensionless</td>
</tr>
<tr>
<td>$a_R$</td>
<td>Temperature coefficient introduced by Reis (1992), dimensionless</td>
</tr>
<tr>
<td>$A$, $B$</td>
<td>Viscosity correlation coefficients, K</td>
</tr>
<tr>
<td>$C_{gas}$</td>
<td>Molar concentration of the diffusing specie (injected non-condensable gas or solvent) in the gas phase, mol/m³</td>
</tr>
<tr>
<td>$C_{oil}$</td>
<td>Molar concentration of the diffusing specie (non-condensable gas or solvent) in the oil phase, mol/m³</td>
</tr>
<tr>
<td>$c_{Po}$</td>
<td>Specific heat capacity of oil, J/(kg·°C)</td>
</tr>
<tr>
<td>$c_{PC}$</td>
<td>Specific heat capacity of the steam condensate, J/(kg·°C)</td>
</tr>
<tr>
<td>$c_{pf}$</td>
<td>Effective specific heat capacity of the flowing condensate and oil phase at the edge of the steam chamber, J/(kg·°C)</td>
</tr>
<tr>
<td>$c_{PR}$</td>
<td>Specific heat capacity of the reservoir rock, J/(kg·°C)</td>
</tr>
<tr>
<td>$c_{PW}$</td>
<td>Specific heat capacity of water, J/(kg·°C)</td>
</tr>
<tr>
<td>$D$</td>
<td>Cluster fractal dimension, dimensionless</td>
</tr>
<tr>
<td>$D_{gas,eff}$</td>
<td>Effective diffusion coefficient in the gas phase, m²/s</td>
</tr>
<tr>
<td>$D_{oil,eff}$</td>
<td>Effective diffusion coefficient in the oil phase, m²/s</td>
</tr>
<tr>
<td>$D_{g,oil}$</td>
<td>Intrinsic diffusion coefficient of the non-condensable gas (methane) at infinite dilution in the oil phase, m²/s</td>
</tr>
<tr>
<td>$D_{g,gas}$</td>
<td>Intrinsic diffusion coefficient of the non-condensable gas (methane) at infinite dilution in the gas phase, m²/s</td>
</tr>
</tbody>
</table>
$D_{s,oil}$ Intrinsic diffusion coefficient of the solvent (hexane) at infinite dilution in the oil phase, $m^2/s$

$D_{o,oil}$ Intrinsic diffusion coefficient of bitumen at infinite dilution in the oil phase, $m^2/s$

$e$ Related to the fractal dimension, dimensionless

$e_x$ Unit vector in the x direction

$e_y$ Unit vector in the y direction

$e_z$ Unit vector in the z direction

$f_1, f_2, g_1, g_2, s_1, s_2$ Scaling exponents, dimensionless

$F$ Fractional flow, dimensionless

$g$ Acceleration due to gravity, $m/s^2$

$h$ Reservoir (pay) thickness, m

$i \sqrt{-1}$

$j$ Index of discretized distance

$J$ Molar flux, mol/(m$^2$.s)

$k$ Absolute permeability of the reservoir, $m^2$

$k_o$ Effective permeability of the oil phase, $m^2$

$k_v$ Effective permeability of the steam vapour, $m^2$

$k_r$ Relative permeability, ratio

$k_{rocw}$ Relative permeability of oil at irreducible water saturation, ratio

$k_{rgcw}$ Relative permeability of gas at irreducible water saturation, ratio

$k_{rw}$ Relative permeability of the water phase, ratio
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{ro}$</td>
<td>Relative permeability of the oil phase, ratio</td>
</tr>
<tr>
<td>$k_{rg}$</td>
<td>Relative permeability of the gas phase, ratio</td>
</tr>
<tr>
<td>$k_{rww}$</td>
<td>Relative permeability of water at residual oil saturation, ratio</td>
</tr>
<tr>
<td>$k_{rwint}$</td>
<td>Relative permeability of water at the steam chamber interface, ratio</td>
</tr>
<tr>
<td>$K$</td>
<td>Equilibrium K–value, ratio</td>
</tr>
<tr>
<td>$K_{TH}$</td>
<td>Thermal conductivity of the reservoir, W/(m°C)</td>
</tr>
<tr>
<td>$k_{v1}$, $k_{v2}$</td>
<td>Raoult’s coefficients for the K-value correlation, variable dimensions</td>
</tr>
<tr>
<td>$k_{v3}$, $k_{v4}$, $k_{v5}$</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>Linear size of the network lattice expressed as the number of pores along a coordinate direction, m</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the production well, m</td>
</tr>
<tr>
<td>$m$</td>
<td>Butler’s viscosity parameter, dimensionless</td>
</tr>
<tr>
<td>mass</td>
<td>Mass of the invading fluid, kg</td>
</tr>
<tr>
<td>$M$</td>
<td>Viscosity ratio, ratio</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Mole fraction of component $i$ in a given phase, ratio</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of sites occupied by the invading fluid, dimensionless</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Capillary number, dimensionless</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure, kg/(m·s²) or Pa</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Capillary pressure, kg/(m·s²)</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Injection pressure, kg/(m·s²)</td>
</tr>
<tr>
<td>$p_o$</td>
<td>Pressure in the oil phase, kg/(m·s²)</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Partial pressure of steam, kg/(m·s²)</td>
</tr>
<tr>
<td>$p_w$</td>
<td>Pressure in the water phase, kg/(m·s²)</td>
</tr>
</tbody>
</table>
\( P_{ci} \)
Critical pressure of component \( i \), kg/(m\( \cdot \)s\(^2\))

\( P_i \)
Parachor coefficient of component \( i \), m\(^3\)(N/m\(^{1/3.6}\))/mol

\( P_L \)
Parachor coefficient of the liquid phase, m\(^3\)(N/m\(^{1/3.6}\))/mol

\( P_v \)
Parachor coefficient of the vapour phase, m\(^3\)(N/m\(^{1/3.6}\))/mol

\( q \)
Volumetric oil flow rate, m\(^3\)/s

\( Q_{\text{cond}} \)
Conductive heat flux, W/m\(^2\)

\( Q_{\text{conv}} \)
Convective heat flux, W/m\(^2\)

\( R_1, R_2 \)
Radii of curvature, m

\( \Delta S_o \)
Change in oil saturation from initial condition, ratio

\( S_{gi} \)
Gas saturation inside the steam chamber, ratio

\( S_{io} \)
Initial oil saturation, ratio

\( S_{iw} \)
Initial water saturation, ratio

\( S_o \)
Oil saturation, ratio

\( S_{or} \)
Residual oil saturation, ratio

\( S_{wD} \)
Normalized water saturation, ratio

\( S_{wr} \)
Irreducible water saturation, ratio

\( t \)
Time, s

\( t_D \)
Dimensionless time expressed as number of pore volumes, dimensionless

\( T \)
Temperature, °C

\( T^* \)
Dimensionless temperature, dimensionless

\( T_{ci} \)
Critical temperature of component \( i \), °C

\( T_r \)
Initial reservoir temperature, °C
\( T_s \)  Temperature inside the steam chamber, corresponding to the injected steam/steam-solvent/steam-non-condensable gas mixture temperature, \(^\circ\)C

\( u \)  Velocity vector, m/s

\( u_o \)  Volumetric oil flux (Darcy velocity), m/s

\( u_v \)  Volumetric flux of the steam vapour, m/s

\( U \)  Steam chamber interface velocity measured normal to the chamber edge, m/s

\( U_x \)  Steam chamber interface velocity measured in the horizontal (\( x \)) direction, m/s

\( v \)  Scaling exponent for Parachor coefficient, dimensionless

\( V \)  Fluid flow velocity, m/s

\( V_c \)  Condensate flow velocity, m/s

\( V_f \)  Effective convection velocity of the flowing oil plus water phase, m/s

\( Vol \)  Volume of bitumen, m\(^3\)

\( w \)  Wavelength, m

\( w_c \)  Critical wavelength, m

\( w_m \)  Wavelength of maximum instability, m

\( x \)  Distance measured in the horizontal direction (perpendicular to the well), m

\( x_g \)  Mole fraction of the injected non-condensable gas in the oil phase, ratio

\( x_s \)  Mole fraction of the injected solvent in the oil phase, ratio

\( y \)  Distance measured parallel to the well direction, m

\( y_g \)  Mole fraction of the injected non-condensable gas in the gas phase, ratio

\( y_i \)  Injected mole fraction of the solvent or non-condensable gas present in the gas phase inside the SAGD chamber, ratio
$y_s$  Mole fraction of the injected solvent in the gas phase, ratio

$z'$  Fundamental Fourier perturbation, m

$\bar{\epsilon}$  Initial amplitude of the fundamental Fourier perturbation mode, m

$z$  Distance measured in the vertical direction, m

Greek

$\alpha$  Thermal diffusivity of the reservoir, $m^2/s$

$\alpha^*$  Apparent thermal diffusivity, $m^2/s$

$\alpha_s$  Mechanical dispersivity, m

$\beta$  Spatial wavenumber, $1/m$

$\varepsilon$  Scaling variable, variable dimension

$\eta$  Distance measured in the direction parallel to the steam chamber interface, m

$\gamma$  Growth rate, $1/s$

$\lambda$  Phase mobility, $m^3/s/kg$

$\lambda_o$  Oil phase mobility, $m^3/s/kg$

$\lambda_w$  Water phase mobility, $m^3/s/kg$

$\varphi$  Porosity, ratio

$\varphi$  Velocity potential, $m^2/s$

$\mu_D$  Dynamic viscosity of the displaced fluid, kg/(m·s)

$\mu_I$  Dynamic viscosity of the injected fluid, kg/(m·s)

$\mu_o$  Dynamic viscosity of bitumen, kg/(m·s)

xxv
$\mu_{os}$ Dynamic viscosity of solvent diluted oil, kg/(m·s)

$\mu_{og}$ Dynamic viscosity of gas diluted oil, kg/(m·s)

$\mu_s$ Dynamic viscosity of the solvent, kg/(m·s)

$\mu_v$ Dynamic viscosity of the steam vapour, kg/(m·s)

$\mu_w$ Dynamic viscosity of the water phase, kg/(m·s)

$\mu_{wint}$ Dynamic viscosity of water at the steam chamber edge, kg/(m·s)

$\rho_c$ Density of the condensate at steam temperature, kg/m$^3$

$\rho_f$ Effective density of the flowing condensate plus oil phase at the edge of the chamber, kg/m$^3$

$\rho_L$ Density of the liquid phase, kg/m$^3$

$\rho_o$ Density of the oil phase, kg/m$^3$

$\rho_{osc}$ Density of the oil phase at standard conditions, kg/m$^3$

$\rho_s$ Density of the injected solvent, kg/m$^3$

$\rho_v$ Density of the steam vapour, kg/m$^3$

$\Delta \rho$ Difference between the oil phase density and steam density, kg/m$^3$

$\rho_{os}$ Density of the oil sands, kg/m$^3$

$\sigma$ Bulk fluid interfacial tension, N/m

$\sigma^*$ Effective interfacial tension, N/m

$\theta$ Angle between the steam chamber edge and the horizontal axis, degrees

$\tau$ Tortuosity of the reservoir, ratio

$\nu_o$ Kinematic viscosity of the oil phase, m$^2$/s

$\nu_s$ Kinematic viscosity of the oil phase at steam temperature, m$^2$/s
\( \xi \)  Distance measured from the edge of the SAGD chamber, in the direction normal to it, m

\( \xi' \)  Distance to the location of maximum solubility of the non-condensable gas or solvent in the oil phase, measured from the edge of the SAGD chamber, m

\( \xi_{\text{max}} \) Distance to the location of maximum oil mobility, measured from the edge of the SAGD chamber, m

\( \omega \)  Acentric factor, dimensionless
1.1 Heavy Oil and Oil Sands

Unconventional sources of oil such as heavy oil and oil sands (tar sands) are important hydrocarbon resources that are destined to play an increasingly important role in the oil supply of the world. Figure 1.1 shows the distribution of the world oil resources. It is evident that the amount of total unconventional oil is about three times the amount of conventional oil in place, discovered to-date, which is about 3 trillion barrels worldwide. Conventional oil is defined as oil with an API gravity\(^1\) of 25° or higher, while unconventional oil, which includes heavy oil and oil sands crude, is characterized by high viscosity and density at reservoir conditions: heavy oil (\(\mu \sim 100\text{–}10000 \text{ cp, } \rho \sim 20°\text{–}10°\text{API gravity}\)) and oil sands (\(\mu > 10000 \text{ cp, } \rho < 10°\text{API gravity}\)). Most heavy oil occurs in shallow (1000 m or less), high permeability (1 to 7 Darcy), high porosity (around 30%), poorly consolidated sand formations (Farouq Ali, 2008). The oil saturations are typically high (50–80%) and formation thicknesses are 10 to several metres.

Canada is not endowed with much conventional crude but it does have gigantic quantities of oil sands, and very substantial amounts of heavy oil. More than one-half of the oil production of Canada is from oil sands. Alberta oil sands (in Canada) contain about two trillion barrels of recoverable heavy oil and bitumen, most of it with in-situ

\[^1\text{API gravity} = (141.5/\text{SG at } 60°F) - 131.5, \text{ where SG is the specific gravity of the fluid.}\]
Figure 1.1: Distribution of the total world oil reserves (Oilfield Review, May 2008).
viscosities in the hundreds of thousands to millions of centipoise (cp) at reservoir conditions. The efficient and economic recovery of this heavy oil and bitumen is a major technical challenge.

1.2 In-situ Extraction of Heavy Oil

In view of the adverse characteristics of heavy oil and oil sands, such as high viscosity and low solution gas content, conventional recovery methods are rarely applicable. Primary recovery factors are low, averaging about 5% of the oil-in-place (Farouq Ali, 2008). Alternative recovery processes include thermal and non-thermal methods.

1.2.1 Non-thermal Recovery Methods

Non-thermal heavy oil recovery techniques can be considered for moderately viscous oils (50-200 cp), thin formations (less than 9 m), low permeability (less than 1 Darcy), and large depths (greater than 900 m). Non-thermal methods serve to reduce the viscosity of the oil, increase the viscosity of the displacing fluid, alter wettability of the rock, or reduce interfacial tension. The major non-thermal methods include polymer flooding, surfactant flooding, water flooding, caustic and emulsion flooding, light hydrocarbon flooding and carbon dioxide flooding. Overall, non-thermal methods have been only marginally successful and not cost-effective for recovering heavy oil.
1.2.2 Thermal Recovery Methods

The two requirements of any bitumen recovery technology are as follows: first, make the oil sufficiently mobile and second, provide or allow a driving force to move the mobile oil into a production wellbore. For example, the driving force can be from an imposed pressure difference or gravity drainage, water drive, or solution-gas drive or combinations thereof. If one of these requirements is not met, then the process will fail to produce oil to the surface.

Thermal techniques aim at increasing oil mobility by reducing oil viscosity through application of heat. For example, Figure 1.2 displays the viscosity of Athabasca bitumen as a function of temperature. At initial reservoir conditions (temperature typically between 7 and 15°C), the viscosity is in the millions of centipoise. Above about 200°C, the viscosity is less than 10 cp. To achieve this, many processes, including Cyclic Steam Stimulation (CSS), steam flooding and Steam-Assisted Gravity Drainage, involve steam injection into the formation. A certain fraction of the latent and sensible heat of the injected steam is transferred to the oil sands, which heats the bitumen and consequently lowers its viscosity. Other methods for heating the oil include underground combustion, such as in in-situ combustion process, hot water flooding, and electrical heating. Viscosity of immobile bitumen can also be improved by application of hybrid methods, which are a combination of the above thermal methods, or a combination of thermal and non-thermal processes, such as the hybrid steam-solvent methods.
Figure 1.2: Viscosity of Athabasca bitumen versus temperature (Mehrotra and Svrcek, 1986).
1.3 Steam-Assisted Gravity Drainage and its Variants

Steam-Assisted Gravity Drainage (SAGD) has become the process of choice to recover bitumen from shallow reservoirs where solution gas content is low, cold production is not possible, and vertical permeability is not significantly impaired. It has been piloted extensively in Athabasca and Cold Lake reservoirs in Alberta (Komery, Luhning, and Pearce, 1999; Butler, 1997a; Kisman and Yeung, 1995; Ito and Suzuki, 1996; Ito, Hirata, and Ichikawa, 2004; Edmunds and Chhina, 2001; Suggett, Gittins, and Youn, 2000; ERCB website, 2007) and is being used as a commercial technology to recover bitumen in several Athabasca reservoirs (Yee and Spargo, 2001; Farouq Ali, 1997). These pilots and commercial operations have demonstrated that this technology is technically effective and has the potential for similar success in other high viscosity, high permeability oil sands deposits around the world. SAGD offers a number of advantages in comparison to the conventional surface mining extraction techniques and alternative thermal recovery methods. For example, SAGD offers significantly greater per-well production rates, lower injection pressures, greater reservoir recovery, steam override elimination, and continuous production.

1.3.1 Original SAGD Concept

SAGD was first conceptualized by Dr. Roger Butler and his coworkers at Imperial Oil in the late 1970’s (Butler, McNab, and Lo, 1981; Butler and Stephen, 1981; Butler, 1985). Figure 1.3 shows the cross-section of a typical SAGD chamber. In the reservoir, steam flows from the top horizontal injection well into a chamber that is largely depleted of oil
Figure 1.3: Cross-sectional view of Steam-Assisted Gravity Drainage (SAGD).
(and hence also known as the depletion chamber). The production well is typically located a few metres above the base of the oil column whereas the injection well is located between 5 and 10 metres above the production well. A liquid saturation at the base of the chamber surrounds the production well and acts as a steam trap, preventing injected steam from being directly produced from the reservoir. Typically, the lengths of the injection and production wells are between 500 and 1000 m. Given that the dominant drive mechanism of SAGD is gravity, relatively shallow reservoirs (that cannot be taken to high pressure and produced by formation re-compaction) or ones with low solution gas, such as Athabasca reservoirs, can be produced by SAGD.

The steam flows to the edge of the steam chamber and releases its remaining latent heat to the cool oil sands at the edges. In this work, the ‘edge of the chamber’ is defined mathematically as the interface beyond which the temperature declines below the saturated steam temperature inside the SAGD chamber. Here steam refers to the steam-vapour phase and the steam condensate is considered as water phase. Due to the heating effect of steam, the viscosity of the bitumen is lowered and this mobilized oil then flows under gravity to the lower horizontal production well. In a SAGD process, the oil layer adjacent to the expanding steam chamber is mobile, and this region will therefore be referred to as the ‘mobile zone’ (within the steam chamber, the oil phase is at residual oil saturation and beyond the heated zone the cold oil phase is practically immobile because of its high viscosity).
1.3.2 Non-condensable Gas Aided SAGD

In one variation of SAGD, a small amount of non-condensable gas is added to the injected steam for reasons such as to maintain pressure in the chamber, utilize energy in place, reduce water consumption, and provide thermal insulation to lower the overburden heat losses. When non-condensable gases are present in the SAGD chamber, they move with the steam vapour towards the condensing surface, where the steam condenses and the gases that are left behind the interface concentrate. The presence of gas reduces the steam partial pressure and consequently steam saturation temperature within the chamber. Most experiments and numerical simulations seem to indicate that steam-oil ratios and overburden heat losses are reduced by the injection of non-condensable gases with steam during SAGD. The recovery and production performance, however, indicates mixed observations, both positive and negative, depending on the reservoir and the operating conditions.

1.3.3 Solvent-Aided SAGD

In SAGD, steam delivers energy in the form of latent and sensible heat, but it requires excessive amount of energy for its generation, which is often by combustion of natural gas. This in turn implies large amounts of emitted greenhouse gases and water usage. Typically, 2 to 10 m$^3$ of steam (expressed as cold-water equivalent, CWE) are required per m$^3$ of produced bitumen. This means that more than 300 m$^3$ of carbon dioxide are emitted to the atmosphere per m$^3$ of produced bitumen. Given the large energy and emission-intensities of steam based processes, and the limitations of heat transfer rates,
there is a strong motivation to develop enhanced recovery processes with reduced steam requirements and improved thermal efficiency. One option is the combined use of heat and solvent, that is, the use of mass transfer.

In solvent-aided SAGD processes, such as the Expanding Solvent Steam-Assisted Gravity Drainage (ES-SAGD) recovery technology (Nasr and Isaacs, 2001), steam and solvent are co-injected into the formation, and both heating and solvent dilution act simultaneously to lower the oil phase viscosity. For example, Figure 1.4 displays the oil phase (mixture of bitumen and solvent) viscosity as a function of the mole fraction of solvent (hexane) for different temperatures. Similar to the viscosity-temperature relationship (displayed in Figure 1.2), providing sufficient solvent is added to the bitumen, the oil phase viscosity can fall below 10 cp.

The key benefit realized from steam-solvent bitumen recovery processes is that it lowers the net energy injected into the formation per unit volume of oil produced. This translates to reduced steam requirement, which in turn implies lower amounts of natural gas being combusted for steam generation, as a result of which, the greenhouse gas emissions drop below that of a steam-only recovery process. Solvent injection helps in further reduction of the oil phase viscosity, below that achieved by steam temperature alone, which leads to higher production rates. Also, potentially lower injection pressure is required since less steam is injected. This implies that the saturation temperature is lowered which in turn means that heat losses from the chamber are lower and the overall thermal efficiency of the process is improved. Furthermore, if the solvent can be separated
Figure 1.4: Viscosity of mixtures of Athabasca bitumen and hexane obtained by a logarithmic mixing rule.
from the produced bitumen, it can be re-injected and re-used throughout the life of the operation. One other benefit that results from solvent-based processes is solvent upgrading e.g. propane de-asphalting which leads to a value-added product at the wellhead.

1.4 Motivation
The purpose of the work described in this thesis is to extend our fundamental understanding of the SAGD process and its variants. The original analytical SAGD model and a few of its variants, were derived under certain idealistic assumptions and do not include important physical phenomenon. For example, most previous models do not take multiphase flow of oil and condensate at the edge of the SAGD chamber into account. In addition, in most analysis of heat transfer at the SAGD chamber edge, the major mode of heat transfer is taken to be heat conduction, while convective heat transfer by the warm condensate is not considered. These missing physical phenomena, as well as the idealistic assumptions, limit the scope of physical understanding and predictive capability that can be derived from these simple theories.

For solvent-aided SAGD process, the dynamics at the edge of the chamber, especially with respect to the interactions of heat transfer and solvent transport, remains unclear. Also, the overall effect of non-condensable gas injection in SAGD, in terms of its effects on chamber development, bitumen flow rate and heat losses, is not fully understood. There are no closed-form analytical models currently available that fully
analyze the gas or injected solvent behaviour in a SAGD setting. This severely limits the
ability to efficiently design and optimize solvent or gas-aided SAGD processes.

The motivation of this research is to improve our understanding of the underlying
physics of thermal and thermal-solvent gravity drainage processes, filling a critical gap in
this area.

1.5 Specific Objectives of this Research

The specific objectives of this research are as follows:

1. Develop an analytical model of SAGD that includes multiphase flow and relative
   permeability effects.

2. Examine the relative roles of conductive and convective heat transfer beyond the
   steam chamber edge, and conditions for the predominance of each.

3. Examine the role of non-condensable gas injection with steam in chamber
   development, oil flow rate and heat losses in SAGD.

4. Examine the coupled heat and mass transfer mechanism in steam-solvent
   co-injection techniques, within the framework of SAGD analytics.

5. Investigate the impact of interfacial stability on the performance of oil recovery
   processes.

6. Test the results of the proposed models through detailed comparison with field
   data, fine-grid numerical simulations, and selected laboratory experimental data.

7. Compare the results yielded by the new models with those given by previous
   analytical SAGD models.
1.6 Thesis Outline

Chapter 2 presents a literature review of the application and analysis of the SAGD process and its variants. In doing so, it highlights the strengths and major shortcomings of the previous analytical models.

In Chapter 3, the flow conditions at the edge of the chamber are re-examined and a new model for gravity drainage of mobilized bitumen at the edge of a SAGD steam chamber is derived, that includes relative permeability and multiphase flow effects.

In Chapter 4, the heat transfer mechanism at the edge of a SAGD chamber is analyzed and a new theory to account for convection of warm condensate into the oil sands beyond the edge of the chamber is developed. The relative roles of conduction and convection in the overall oil mobilization are examined.

The objective of Chapter 5 is to analyze the role of non-condensable gas injection in SAGD, in terms of its impact on chamber development, bitumen flow rate and overburden heat losses. A new analytical model for non-condensable gas aided SAGD process is derived, to predict the fluid flow behaviour and phase saturation profiles in the mobile zone.

For solvent-aided SAGD processes, the dynamics at the edge of the chamber, especially with respect to the interaction of heat transfer and solvent transport, plays a key role. The analysis of the coupled heat and mass transfer behaviour at the edge of a steam-solvent chamber is presented in Chapter 6.
The analytical theories presented in Chapters 3–6 assume that the SAGD chamber is expanding under a pseudo steady-state condition, which assumes that the time-scale of reservoir heating is similar to the time-scale of the chamber edge motion. Also, in Chapters 5 and 6, convective mass transport beyond the chamber edge is neglected. Chapter 7 presents an extension of the models derived in Chapters 3–6, by relaxing the above assumptions. Specifically, a system of coupled heat and mass transfer equations is derived, that accounts for the conductive-convective heat transfer, coupled with the convective-diffusive mixing of solvent into the oil phase, under unsteady state condition.

One of the factors that influence the efficiency and rate at which oil is mobilized in steam based recovery processes is the stability of the steam-oil interface. In Chapter 8, the stability of the interface at the edge of the SAGD chamber is examined to determine how solvent co-injection with steam affects the vapour-bitumen interface, and how this influences the overall oil mobilization and production rate in solvent-aided SAGD processes.

The material presented in Chapter 9, aims to supplement the understanding gained from the analysis done in the main body of the thesis. The analytical theories derived in Chapters 3–8, are based on the continuum approach for modelling flow in porous media, incorporating the multiphase flow extension of Darcy’s law. However, for pore-scale displacements, the conventional Darcy-type modelling of fluid flux is not predictive under unstable, immiscible, imbibition conditions. Chapter 9 outlines a framework for interpretation of pore-scale displacement patterns using statistical theories and concepts.
of fractal analysis. The work described in Chapter 9 was done by the author at Stanford University, as part of a research project.

The analytical models derived in this research were tested via comparison with fine-grid numerical simulation results. Appendix A gives the details of the numerical simulation study conducted in this research.

Figure 1.5 illustrates the scheme of this research, which may be helpful in locating specific sections.
Objectives

1. Improving analytical modelling of SAGD
2. Comparing analytical results with numerical simulation
3. Testing the analytical theory via comparison with field and experimental data

Improved analytical models

- Include multiphase flow (Chapter 3, Sec. 3.2)
- Include convective heat transport (Chapter 4, Sec. 4.2)
- Include gas injection with steam (Chapter 5, Sec. 5.3)
- Include solvent injection with steam (Chapter 6, Sec. 6.4)

Numerical simulation results

- Include multiphase flow (Chapter 3, Sec. 3.2)
- Include convective heat transport (Chapter 4, Sec. 4.2)
- Include gas injection with steam (Chapter 5, Sec. 5.3)
- Include solvent injection with steam (Chapter 6, Sec. 6.4)

Comparison with selected field data and experimental results

- Transient, coupled heat and mass transfer model (Chapter 7, Sec. 7.2)
- Interfacial stability analysis (Chapter 8, Sec. 8.3)
- Experiments and analysis of pore scale viscous fingering (Chapter 9, Sec. 9.6)

Figure 1.5: Schematic representation of the organization of this research.
2.1 Analytical Modelling of Steam-Assisted Gravity Drainage (SAGD)

The first paper on SAGD was presented by Butler, McNab and Lo, at the Oil sands Symposium in Jasper, Canada, in 1979. The key physics of SAGD are relatively well established and simple theories to describe oil drainage rate and steam usage have been derived from first principles (Butler, McNab, and Lo, 1981; Butler and Stephens, 1981; Butler, 1985; Ferguson and Butler, 1988; Reis, 1992, 1993; Butler, 1997a; Akin, 2005). The original analysis done by Butler and his associates (1981, 1985), underlies all SAGD theories that have been published (Ferguson and Butler, 1988; Reis, 1992, 1993; Akin, 2005). Despite its simplicity, Butler’s theory identifies the relationship between oil production rate and reservoir and fluid physical parameters such as permeability, mobile oil saturation range, thermal diffusivity, oil viscosity dependence on temperature, height of the reservoir, and density of the oil phase, and is given by:

\[ q = 2L \sqrt{\frac{2kg\alpha\phi\Delta S_o h}{m \nu_s}} \]  

(2.1)

where \( q \) is the volumetric bitumen production rate, \( L \) is the length of the production well, \( k, \alpha, \phi, \Delta S_o \), are the permeability, thermal diffusivity, porosity, and mobile oil saturation range of the reservoir, respectively, \( m \) is a constant used to reflect the dependence of viscosity on temperature, and \( \nu_s \) is the kinematic viscosity of bitumen, defined as \( \mu/\rho \), at the steam temperature.
The original theory was revised by Butler and Stephen (1981) to model the chamber shape such that it remained attached to the production well. The oil production rate predicted by this revised model, referred to as the “Tandrain” model, is given by

\[ q = 2L \sqrt{\frac{1.5kg \alpha \phi \Delta S_o h}{m \nu_s}} \]  

(2.2)

Ferguson and Butler (1988) reported a calculation procedure to predict the impact of variable steam injection rates and pressures on SAGD. Their theory provides an improvement over the original SAGD theory because it could handle varying steam pressures. However, the resulting theory was developed in a numerical framework and required the solution of a complex differential equation that approximated heat transfer at the boundary of the steam chamber as a constant temperature boundary layer at the edge of the chamber.

Reis (1992) derived a steam-based gravity drainage theory similar to Butler’s (1985) theory for horizontal wells. The steam chamber shape was assumed as an inverted triangle. This shape has been observed in Hele-Shaw and sandpack laboratory models (Chung and Butler, 1988) but thermocouple data (Encana report, 2007; CNRL report, 2007; ConocoPhillips report, 2008) from field operations and detailed simulation studies (Ito, Hirata, and Ichikawa, 2001b; Gates, Kenny, Hernandez-Hdez, and Bunio, 2005) suggest that the steam chamber is more elliptical in cross-sectional shape. Reis (1992) derived a similar theory for steam-based gravity drainage in radial geometry around

\[ \text{In this work, the ‘edge of the chamber’ is defined mathematically as the interface beyond which the temperature declines below the temperature inside the SAGD chamber } (T_s > T) \]
vertical wells. The steam chamber was assumed to be an inverted cone and material and 
energy balance equations were used to determine a steam to oil ratio. Reis used his theory 
to demonstrate that vertical well SAGD may have potential as a bitumen recovery 
process. The oil drainage rate predicted by Reis for a linear geometry is given by:

\[ q = 2L \sqrt{\frac{kg \alpha \phi \Delta S \cdot h}{2a_R \nu_s m}}, \quad (2.3) \]

where, \( a_R \) is an empirical constant equal to 0.4. Reis (1992, 1993) showed that Butler’s 
model overpredicts the oil production rates compared to the experimental results. 
However, his model did not predict production during the rise of the chamber.

Akin (2005) derived a steam-based gravity drainage theory that accounted for 
steam distillation and asphaltene deposition effects, where the steam chamber shape is 
approximated as an inverted triangle. His analysis showed that at late times, steam-
distillation and asphaltene deposition are the controlling features of the process rather 
than steam-chamber size and lateral heat transfer.

The SAGD theories discussed above are similar and treat the ideal homogeneous 
reservoir case, where latent heat lost by steam occurs uniformly along a two-dimensional, 
symmetric steam chamber-bitumen interface. In reality, however, the interface can be 
wavv across any cross-section of the chamber perpendicular to the well pair. Also, the 
chamber could vary in shape and size along the length of the wells due to reservoir 
heterogeneity and non-uniform steam distribution along the injector and non-uniform 
production of reservoir fluids along the production well. Other reasons that cause this 
departure from ideal behaviour are non-parallel wells, shale layers within the reservoir,
and non-uniform oil column thickness. The previous analytical SAGD models do not encompass all of the physics of the flow at the interface. For example, they do not take relative permeability or geomechanics into account at the chamber edge. These missing physical phenomena, as well as the idealistic assumptions listed above, limit the scope of physical understanding that can be derived from these simple theories.

2.2 Heat Transfer Mechanism

Heat transfer at the edge of the chamber is central to bitumen mobilization and drainage in SAGD. In most analytical models of SAGD, the major mode of heat transfer at the steam chamber edge is taken to be heat conduction (Butler et al., 1981; Butler and Stephen, 1981; Butler, 1985; Ferguson and Butler, 1988; Reis, 1992, 1993; Butler, 1997; Akin, 2005; Gotawala and Gates, 2008; Sharma and Gates, 2010a). However, steam condensate, with a volume two to five times as large as the volume of oil, flows between the steam chamber and the adjoining oil rich part of the reservoir (Farouq Ali, 1997). With so much condensate flowing, convective heat transfer by the flow of warm condensate would be expected to play an important role in the process.

In SAGD, several studies have examined convective steam flow and fingering at the edge of the steam chamber (Butler, 1987, 1994; Ito and Ipek, 2005; Gotawala and Gates, 2008; Sharma and Gates, 2011a). Butler (1987, 1994), and Ito and Ipek (2005), both suggested that steam vapour fingers would penetrate the oil sands at the edge of the chamber. The potential benefit is that if heat transfer could be enhanced at the SAGD chamber edge, oil production could be improved.
Ito and Suzuki (1996) performed a simulation study of a SAGD process applied to the Hangingstone oil sands reservoir to gain a visual understanding of the flow behaviour of steam, oil, and condensate. They presented the temperature and oil saturation profiles ahead of a steam chamber edge after 2.5 years of steaming. The simulation results clearly indicated that there is a significant contribution of convection in the total heat transfer at higher temperatures. A convex temperature profile for temperature beyond the edge of the SAGD chamber was obtained, which also indicated that the main transport of energy is from fluid convection. They verified their claim by test runs by assuming one-hundredth and one-thousandth thermal conductivities and showed that indeed water flow ahead of steam chamber and convective heat transfer are playing main roles in the oil recovery mechanism.

Birrell (2001) analyzed temperature data from vertical observation wells in the Dover Phase B SAGD pilot to calculate the apparent thermal diffusivity and relative roles of conductive and convective heat transfer. From the analysis of the field data, he obtained a high value of thermal diffusivity and significant convective heat flux at high temperature which clearly indicated the important role of convection in the heat transfer ahead of the SAGD steam chamber.

Farouq Ali (1997) criticized the assumption of purely conductive heat transfer beyond the SAGD steam chamber. In response to that critic, Edmonds (1999) stated that based on the associated change in enthalpy, the liquid water could carry and deposit at most about 18% of the heat of condensation of the same water. Convection due to oil is around 1/5th of this; conduction is the only available mechanism to carry the remaining
78%. From his analysis, Edmunds concluded that convective heat transfer is probably less than 5% of that due to conduction. However, the analytical and simulation results reported in this research demonstrate that convection can provide a relatively large contribution to the overall heat transfer at the edges of a steam chamber, in the presence of a mobile water phase.

There is field evidence for mobile water in oil sands reservoirs. A cold-water injectivity test was done at the Underground Test Facility (UTF) Phase A SAGD pilot in November, 1987 (Aherne and Maini, 2006). The three SAGD well pairs of the Phase A SAGD pilot were each 60 m long, and drilled from the UTF tunnel. Prior to operating in SAGD mode, water was injected into the central well pair. The original pressure of the reservoir was 510 kPa. During water injection, the injection pressure ranged from 1100 to 1550 kPa, and in the span of 17 days, the water injection rate started at 12 m$^3$/day and declined to 8 m$^3$/day. Aherne and Maini (2006) examined the pressure data from piezometers located in the bitumen zone in three nearby observation wells and determined that water did move horizontally through the bitumen zone. The injection test demonstrated that water is mobile in the oil sands reservoir, which implies that the initial water saturation is higher than the initial irreducible water saturation. Water mobility is commonly many times that of bitumen because of the large difference in the viscosities of water and oil at native reservoir conditions. If the water saturation is greater than the irreducible water saturation, and if the water is at elevated temperature in these reservoirs, there is potential that it could provide additional heat transfer at the chamber edge in the case where chamber pressure exceeds the original reservoir pressure.
The above discussion indicates that heat transfer beyond the edge of the steam chamber can be enhanced beyond that from conduction alone by convection. However, it remains unclear how large convection is relative to conduction. Currently there are no closed form analytical models that examine the relative roles of convective and conductive heat transfer at the SAGD steam chamber edge.

2.3 Non-condensable Gas Injection in SAGD

The injection of non-condensable gas with steam in SAGD is a concept that has been extensively discussed in the literature and tested via numerous simulation studies, laboratory experiments and a small number of field pilots. Butler (1997b) proposed the injection of a non-condensable gas with steam during the SAGD process and called it “SAGP (Steam and Gas Push)”. He contended that the gas accumulation near the edge of the chamber and at the top of the reservoir would result in a lower average chamber temperature and much lower heat loss to the overburden, resulting in a low steam-oil ratio. Butler, Jiang, and Yee, (1997b, 1998, 1999, 2000a, 2000b, and 2001) conducted physical model experiments, which confirmed that the addition of a small amount of non-condensable gas has a beneficial effect on the SAGD process primarily due to the gas insulation effect. The non-condensable gas accumulated at the leading lateral edge of the vapour chamber is believed to be able to re-saturate to the middle of the steam chamber. This, they believed, was accomplished through the lateral counter-current flow to the stream of injecting gas by molecular diffusion and results in a low temperature zone even directly above the horizontal wells. The lower temperature in the entire upper part of the
chamber observed in the laboratory experiments was believed to scale under the field conditions. During the laboratory experiment, most of the injected gas was produced (80%) back from the production well. From the experimental observation, they concluded that although the oil production rate could be somewhat lower for the SAGP process, the steam-oil ratio was much better than in SAGD.

Ito et al. (2001a) studied the effect of hydrocarbon gas injection on oil production during SAGD projects, using numerical simulation of the UTF Phase B pilot. Their results supported Butler et al.’s observations that gas would accumulate on the top of the reservoir as well as at the steam-oil interface. The results further indicated that the oil production rate is significantly reduced when gas is injected with steam from an early period of a SAGD operation. However, the results suggested that if the gas injection is initiated during later periods of the process, an improved steam-oil ratio is obtained without significant reduction in oil production rate. In this case, the injected non-condensable gas migrated to the upper part of the reservoir and did not prevent the growth of the steam chamber, since the chamber had already grown to the desired size. This mode of growth of steam chamber also resulted in an improvement in steam-oil ratio. From these finding, they concluded that the understanding of the recovery mechanism involved in gas injection and monitoring of the expansion of steam chamber is the key for determining the optimal timing to initiate gas injection.

Kisman and Yeung (1995) predicted that a small amount of non-condensable gas in SAGD would improve oil production rate while a large amount would prove detrimental. Yuan, Nasr, and Law (2006) conducted large-scale laboratory experiments,
which confirmed the observations of Ito (2001a) and Butler et al. (2000, 2001), and showed that the injected gas would accumulate at the interface and at the top of the reservoir.

The effect of solution gas on the performance of the SAGD process was studied by Edmunds, Kovalsky, Gittins, and Pennacchio (1991). Their simulation study predicted that the small volumes of non-condensable gases released from bitumen move to the top and towards the leading lateral edges of the steam chamber, leading to reduced frontal advance rate. However, they believed that most of gases released into the steam chamber were rapidly removed with the produced bitumen and condensate. Edmunds (2007) presented evidence of gas effects on the chamber rise rate in several Athabasca SAGD projects. He concluded that the rate of rise of chamber depends on the thickness of the gas blanket above the steam.

Good (1997) also investigated the effect of non-condensable gas injection on the SAGD process using numerical simulation. His results showed that injection of a non-condensable gas with steam during the late stage of a SAGD project results in slowing down of the lateral growth of the steam chamber, and improves reservoir conformance. He called this phenomenon, “blunting” and suggested that higher injected non-condensable gas content (25% to 100% non-condensable gas) should be used when the purpose is to wind-down the SAGD process.

All the experiments and numerical simulation studies seem to indicate that the injected non-condensable gas tends to accumulate at the steam condensation front and this ‘gas buffer’ leads to reduced overburden heat losses. The recovery and production
performance, however, indicates mixed observations, both positive and negative, depending on the reservoir and operating conditions. There are currently no simple analytical models available in the literature that can predict the fluid flow rate or phase saturation distribution for SAGD, in the presence of a non-condensable gas. Thus, an overall assessment of the influence of non-condensable gas on the SAGD performance remains poorly understood and inconclusive.

2.4 Steam-Solvent Hybrid Processes

Farouq Ali and Abad (1976) introduced the idea of hybrid steam-solvent processes, where solvent is co-injected or alternatively injected with steam and both heating and solvent dilution act simultaneously to lower the oil phase viscosity. The key benefits realized from steam-solvent bitumen recovery processes, as shown by several authors (Nasr and Isaacs, 2001, 2003; Gates and Chakrabarty, 2008; Nasr and Ayodele, 2005) are that it lowers the net energy injected into the formation per unit volume of oil produced. Mokrys and Butler (1993) compared SAGD to solvent-aided SAGD and observed a 30% reduction in steam requirement, and 99% recovery of solvent (propane). There are several papers that have examined the oil phase viscosity behaviour in the solvent-aided SAGD process (Gates, 2007; Gates and Chakrabarty, 2008). As described by Gates (2007), several benefits can be realized from steam-solvent co-injection:

1. reduced steam requirement which results in lower water use, reduced greenhouse gas emissions and lower operational costs,

2. produced solvent can be separated from the produced oil and re-injected,
3. further reduction of the oil phase viscosity below that achieved by steam temperature alone which leads to higher production rate, and

4. potentially lower injection pressure since less steam injected which also implies that the saturation temperature is lowered, which in turn means that heat losses from the chamber are lower and the overall thermal efficiency of the process is improved.

Butler and Mokrys (1989) published their first paper on solvent analog of SAGD in 1989. The original concept of the VAPEX (Vapour Extraction) process is like SAGD, with a solvent vapour replacing the steam. However, due to the limited oil mobilization achieved from solvent-dilution alone, the process met with limited success on the field. Zhao (2007) proposed the Steam Alternating Solvent (SAS) process in which steam and solvent are injected alternately using well configuration similar to the SAGD process. This process combines the advantages of SAGD and VAPEX (Vapour Extraction) to increase oil recovery and reduce energy consumption. Through field scale simulations and preliminary laboratory experiments, Zhao (2007) showed that SAS process lowers the energy requirement by almost 47% for recovering the same amount of oil.

Nasr and Isaacs (2001) developed the Expanding-Solvent Steam-Assisted Gravity Drainage (ES-SAGD) process that combines the benefits of steam and solvent for recovering heavy oil. In this process, the hydrocarbon additive is selected in such a way that it would evaporate and condense at the same conditions as the water phase. ES-SAGD process has been successfully field-tested, and has resulted in improved oil rates
and steam-oil ratios, and lower energy and water requirements as compared to conventional SAGD.

Steam-solvent co-injection has been tested in field operations for bitumen recovery. In Cold Lake, Alberta, a small amount of diluent (gas condensate) was co-injected with steam in late-cycle CSS wells in Imperial Oil’s Liquid Addition to Steam for Enhanced Recovery (LASER) process pilot (Leaute and Carey, 2005). The bitumen production rate of the LASER wells were reported to be about 100% higher than that of a neighbouring CSS wells (without solvent injection). EnCana (then PanCanadian), in 2002, co-injected butane with steam in the Solvent-Aided Process (SAP) pilot in Senlac, Saskatchewan, in one of the Phase C well pairs (Gupta, Gittins, and Picherack, 2005; Gupta and Gittins, 2005). In this pilot, SAGD was conducted for the first half year and thereafter, butane was added to the injected steam for several months. After butane injection started, the oil production rate increased by more than 50%. After butane injection ceased, the oil rate dropped to a value similar to that of the projected SAGD oil rate. EnCana is now employing SAP in their Christina Lake project (Gupta and Gittins, 2005). So far, the uplift of oil rate is over 50%. An ES-SAGD test was performed on Pair-3 of the Nexen’s Long Lake pilot from February to April 2006. Jet B was selected as the hydrocarbon solvent, which is a mixture of heavier petroleum fraction (C_7 to C_{12}). The oil rate increased slightly, by about 8%, and the SOR decreased by about 7%, during the two-month test period. The performance forecast from numerical simulation, however, showed more promising results in the long term.
For solvent-aided thermal recovery processes, the overall interactions of heat transfer and solvent transport, remains unclear. There are currently no analytical models available, which fully describe the coupling between the steam heating and solvent mixing effects to predict the oil rate in steam-solvent processes.

2.5 Interfacial Stability Analysis

Fingering at the interface between phases is a common phenomenon encountered in petroleum recovery (Homsy, 1987). The immiscible displacement of one fluid by another in porous media leads to a wide range of dynamic flow behaviours, depending on the relative magnitude of viscous, capillary, and gravity forces. Feder (1988) indicated that the dynamics of flow instability in porous media have two main components: (i) the global pressure distribution controlled by a Darcy-type law for fluid flux and (ii) the local fluctuations in the pore geometry. Conventional modelling is based on a continuum approach that describes the porous medium with macroscopic properties defined as averages of the corresponding microscopic quantities. Sahimi and Yortsos (1990) suggested that the continuum treatment is valid only when there is a scale separation between the process parameters, and the physical properties of the medium. When continuum conditions are not met, such as when the pore-scale fluctuations are not negligible, a non-continuum system definition must be taken.

Thus, two distinct approaches to stability analysis emerge; the continuum approach based on Darcy’s law and its multiphase extension, and the non-continuum approach, such as pore network modelling and fractal analysis.
2.5.1 *Continuum Modelling Approach*

Problems related to the stability of the interface when a fluid of lower viscosity (higher mobility) displaces a fluid of greater viscosity (lower mobility) have been studied widely (e.g., Hill, 1952; van Meurs, 1957; Saffman and Taylor, 1958; Perkins and Johnston, 1963; Gupta and Greenkorn, 1974; Tang and Kovscek, 2011). It has been pointed out (Hill, 1952; Saffman and Taylor, 1958; Taylor, 1950) and verified experimentally (Lewis, 1950) that when a viscous fluid contained in porous media is driven by a lower viscosity fluid, the interface is unstable resulting in wavy perturbations on the boundary surface between both fluids which grow with time. Pritchard (2004), by using linear stability analysis, examined the stability of viscous fingering of an injected fluid into a porous medium that contains a fluid of different composition and temperature. His results showed that the rate of growth of the interfacial instability depends on the coupling between the thermal and compositional effects.

The first mathematical linear stability analysis of an interface under the influence of viscous, gravity, and capillary forces was done by Chouke, Van Meurs, and Vander Poel (1959) for the displacement of two immiscible fluids. By considering capillary forces at the interface, they found that there is a threshold for the displacement rate, above which the flow is unstable. The analysis of viscous fingering in thermal and thermal-solvent processes using the linear stability theory is discussed in Chapter 8.
2.5.2 Non-continuum Modelling Approach

From experimental studies and network model simulations, Aker (1996) concluded that when capillary effects are small, viscous instabilities manifest themselves as macroscopically growing viscous fingers. However, when capillary forces become more important compared to viscous and gravity effects, the observed structures consist of rough and wide fronts, with trapped clusters of displaced fluid, ranging from the pore size to the length of the system. In the capillary dominated regime, the finger widths are found to be of the same order as the pore size and the overall shape of the pattern depends strongly on the pore structure and geometry.

For modelling pore-scale fluid transport in disordered porous media, Lenormand, Touboul, and Zarcone (1988) suggested a non-continuum approach based on a discrete system definition that requires microscopic description of the pore geometry and the physical laws of flow within the pores. Lenormand et al. (1988, 1983) demonstrated the existence of three limiting cases for the basic displacements in a porous medium: capillary fingering, when capillary forces are strong in comparison to viscous forces, viscous fingering, when a less viscous fluid displaces a more viscous fluid, and stable displacement in the opposite case. He described these flow regimes using statistical theories: invasion percolation, Diffusion-Limited Aggregation (DLA) and anti-DLA, depending on the relative magnitudes of capillary and viscous forces. Further, Lenormand developed a phase diagram with the mobility ratio ($M$) and capillary number ($N_c$) as the axes, and calculated the boundaries of the above limiting flow conditions and
the critical values of $N_c$ and $M$ for crossover among the different regimes. His theory was based on network model simulations and physical model experiments.

Several studies have shown that for pore-scale fluctuations, the morphology of the growing phase is often describable in terms of fractal geometry (Ferer, Sams, Geisbrecht, and Smith, 1993, 2007; Vicsek, 1989; Meakin, Feder, Frette, and Jossang, 1992; Chen and Wilkinson, 1985; Måløy, Feder, and Jøssang, 1985). Mandelbrot (1982) described a fractal as "a rough or fragmented geometric shape that can be split into parts, where each part is (at least approximately) a reduced-size copy of the whole". Theoretical and experimental studies by Riaz, Tang, Tchelpi, and Kovscek (2007) and Tang and Kovscek (2011) have shown that for micro (or pore) scale and meso (or core) scale displacements, the conventional Darcy-type modelling of fluid flux are not predictive under unstable, immiscible, imbibition conditions. It has been recognized by many authors that capillary fingering is a manifestation of a complex geometry amenable to fractal treatment (Ferer et al., 1993; Peters and Flock, 1981; Måløy et al., 1985). Recently, much work has been done in studying pore-scale fluid transport in disordered porous media using statistical models that utilize this approach (Ferer et al., 1993; Peters and Flock, 1981; Måløy et al., 1985; Lenormand et al., 1988; Dullien, 1992). The analysis of pore-scale fingering using statistical theories and concepts of fractal analysis is discussed in Chapter 9.
3.1 Original SAGD Theory and Assumptions

SAGD process and the associated theory have been described in a series of papers by Butler and his associates (Butler, McNab, and Lo, 1981; Butler and Stephen, 1981; Butler, 1985). In the original theory, it is assumed that the reservoir is homogeneous, and that the steam chamber is symmetric, two-dimensional (2-D) and at constant temperature and pressure. A single-phase flow of mobilized oil is assumed at the edge of the steam chamber (where the ‘edge of the chamber’ is defined mathematically as the distance from the well pair beyond which the temperature starts declining below the temperature inside the SAGD chamber). The original model is based on a pseudo steady-state approximation, which assumes that the time-scale of reservoir heating is similar to the time-scale of the steam chamber edge motion. The heat transfer ahead of the edge is assumed to be by conduction only, for which the temperature profile ahead of the interface is given by (Butler et al., 1981):

\[
\frac{T - T_r}{T_s - T_r} = e^{U\xi/\alpha}
\]  

(3.1)

where \( T_s \) and \( T_r \) are the steam chamber temperature and initial reservoir temperature, respectively, \( \xi \) is the distance measured from the edge of the steam chamber, \( \alpha \) is the thermal diffusivity of the reservoir, and \( U \), is the rate of chamber expansion measured in the direction normal to the chamber edge. If we define a dimensionless temperature \( T^* \) as:
\[ T^* = \frac{T - T_r}{T_s - T_r} \]  

Equation (3.1) becomes:

\[ T^* = e^{\frac{U_s}{a}} \]  

(3.2b)

In the original model, the dependence of temperature on kinematic viscosity is assumed to be given by the simple relation:

\[ \frac{\nu_v}{\nu_s} = \left( \frac{T - T_s}{T_s - T_r} \right)^m = (T^*)^m \]  

(3.3)

For heavy crude oils and bitumen, the value of \( m \) is typically between 3 and 4 (Butler, 1985).

The original theory does not encompass all of the physics of the flow at the interface. For example, it does not take the flow of steam condensate and relative permeability effects at the chamber edge into account. In addition, convective heat transfer by the warm condensate beyond the chamber edge is not considered. These missing physical phenomena limit the scope of physical understanding that can be derived from the original SAGD theory and other analytical SAGD models based on a similar approach.

In the following section, the flow at the edge of a SAGD chamber is re-examined and a new theory is derived which includes multiphase flow and relative permeability effects. This new theory is compared with Butler’s model and other analytical SAGD theories, and with the production data from several existing SAGD operations, as well as experimental data. The predictions of the analytical model are also tested via comparison
with the results from fine-grid homogeneous reservoir simulations, for qualitative and quantitative similarity.

### 3.2 Analytical Model

#### 3.2.1 Modelling Assumptions and Explanations

The following assumptions were made in the development of the analytical model:

1. Porous medium is homogeneous (constant porosity and permeability).
2. Analysis is done for a mature SAGD steam chamber that has reached the maximum height, \( h \), at time, \( t=0 \), and is expanding at a constant velocity \( U \), in the direction perpendicular to the chamber walls.
3. Heat transfer beyond the chamber edge is only by conduction.
4. Overburden and underburden heat losses are absent.
5. Reservoir oil does not initially have any solution gas.
6. Heat transfer and mass transfer is directed normal to the edge of the chamber.
7. Boussinesq approximation (that is, variation in density is neglected everywhere except in the buoyancy term).
8. Density and viscosity are dependent on temperature.

Homogeneity is assumed in order to be able to derive a closed form analytical solution for the oil flow rate. The condition of pseudo steady-state expansion is assumed following the original theory (Butler et al., 1981; Butler and Stephen, 1981). However, in Chapter 7 this assumption is relaxed and equations are derived under unsteady state condition. There, it is also shown that the steady-state assumption is valid for a mature steam chamber that has reached the top of the formation.

Heavy oil and bitumen often contain only a small amount of solution gas (Farouq Ali, 1997) and therefore in the analytical model, the reservoir oil is assumed to be devoid
of any solution gas content for the ease of modelling. This implies that in the ‘mobile zone’, which is defined as the narrow region beyond the chamber edge where oil and water phases are mobile, a multiphase flow of condensate and oil occurs.

Heat transfer beyond the chamber edge is assumed to be purely conductive; this assumption is made following Butler’s original analysis (Butler et al., 1981). In the next chapter, however, the heat transfer model is extended to include convective heating effects. There, it is also shown that the assumption of purely conductive heat transfer beyond the chamber edge is reasonable when the initial water phase saturation in the reservoir is equal to (or less than) the irreducible water saturation ($S_{wr}$). Accordingly, the water saturation was maintained close to $S_{wr}$ in the analytical and simulation models, used in the analysis presented in this chapter.

3.2.2 Oil Mobility Profile at the Edge of a Steam Chamber

In steam-based gravity drainage processes, the controlling factor for the oil phase movement in the reservoir is the oil phase mobility, $\lambda_o$, which depends on its relative permeability and viscosity as:

$$\lambda_o = \frac{k_{ro} k}{\rho_o \nu_o}$$  \hspace{1cm} (3.4)

The relative permeability, in turn, depends on the oil saturation, which varies with the relative distance from the steam chamber. The relative permeability of oil and water with respect to each other can be determined from lab-based relative permeability
measurements, which are often correlated in the form of the Corey’s (Brook’s and Corey, 1964; Corey, 1954) equations as:

\[ k_{ro} = k_{recw} (1 - S_{wD})^a \]  

(3.5a)

and

\[ k_{rw} = k_{rwoo} (S_{wD})^b \]  

(3.5b)

where \( k_{ro} \) is the relative permeability of the oil phase, \( k_{rw} \) is the relative permeability of the water phase, and \( k_{rwoo} \) is the relative permeability of the water phase at residual oil saturation, and \( k_{recw} \) is the relative permeability of the oil phase at irreducible water saturation. \( S_{wD} \) is the normalized water saturation defined as:

\[ S_{wD} = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} \]  

(3.6)

The exponents \( a \) and \( b \) are the Corey coefficients, which set the curvatures of the oil and water relative permeability curves, respectively. Typical values of Corey coefficients for Athabasca reservoirs are listed in Table 3.1.

In the steam chamber, the oil saturation is low and approaches the residual oil saturation, \( S_{or} \). In the flowing oil zone (or the mobile zone), it ranges from the residual oil saturation up to the original oil saturation of the reservoir. To first approximation, the oil saturation can be assumed to be linear with respect to temperature ahead of the steam chamber ranging from the residual oil saturation at the edge of the chamber \( (S_o=S_{or}, T=T_s) \) to the initial reservoir oil saturation within the cold reservoir \( (S_o=S_{io}, T=T_r) \). Thus, the oil saturation profile ahead of the interface can be approximated as:

\[ S_o = S_{or} + (S_{io} - S_{or})(1 - T^*) \]  

(3.7)
Table 3.1: Typical parameters for Athabasca reservoir (Ito and Hirata, 1999; Ito and Suzuki, 1996).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial reservoir temperature, $T_r$, °C</td>
<td>10</td>
</tr>
<tr>
<td>Injected steam temperature, $T_s$, °C</td>
<td>260</td>
</tr>
<tr>
<td>Oil density, $\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>Permeability, $k$, m$^2$</td>
<td>4$x$10^{-12}</td>
</tr>
<tr>
<td>Thermal diffusivity, $\alpha$, m$^2$/s</td>
<td>7$x$10^{-7}</td>
</tr>
<tr>
<td>Initial oil saturation, $S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Irreducible water saturation, $S_{wr}$</td>
<td>0.19</td>
</tr>
<tr>
<td>Viscosity of oil at steam temperature, $\mu_o(T_s)$, cp</td>
<td>4.2</td>
</tr>
<tr>
<td>Chamber expansion rate, $U$, cm/day</td>
<td>20</td>
</tr>
<tr>
<td>Initial reservoir pressure, kPa</td>
<td>2000</td>
</tr>
<tr>
<td>Pay height, $h$, m</td>
<td>20</td>
</tr>
<tr>
<td>Well length, $L$, m</td>
<td>500</td>
</tr>
<tr>
<td>Butler’s viscosity parameter, $m$</td>
<td>3</td>
</tr>
<tr>
<td>Corey Coefficients</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>2</td>
</tr>
<tr>
<td>$b$</td>
<td>4</td>
</tr>
</tbody>
</table>
The above approximation is supported by observations from the fine-grid numerical simulation done in this research as well as simulation studies done by Ito and Suzuki (1996) which show that in the mobile zone (temperatures ranging from $T_s \sim 0.5 T_s$), the oil saturation increases linearly with decreasing temperature.

By using Butler’s temperature-distance relationship ahead of the steam chamber (Equation 3.2a), and assuming that conductive heat transfer dominates beyond the chamber edge, the saturation profile can be written as:

$$S_o = S_{or} + (S_{io} - S_{or})(1 - e^{-\frac{U}{\alpha}})$$

(3.8)

Once the oil saturation is known, the relative permeability of the oil phase with respect to distance can be determined by substituting Equation (3.8) in Corey’s equations:

$$k_{ro} = k_{recw} \left( \frac{S_o - S_{or}}{1 - S_{wr} - S_{or}} \right)^{\alpha} (1 - e^{-\frac{U}{\alpha}})^{\alpha}$$

(3.9)

The oil mobility as a function of distance is obtained by combining Equations (3.3), (3.4), and (3.9) to give:

$$\lambda_o = \frac{kk_{recw}}{\rho_o v} \left( \frac{S_{io} - S_{or}}{1 - S_{wr} - S_{or}} \right) (1 - e^{-\frac{U}{\alpha}})^{\alpha} \left( e^{-\frac{U}{\alpha}} \right)^{m}$$

(3.10)

The ratio of saturation differences in Equation (3.10) is equal to unity and can be removed from the equation (this will be done in the derivations below). The location of maximum oil mobility, $\xi_{max}$, can be found by differentiating Equation (3.10) with respect to distance and setting the result equal to zero. The result is:

$$\xi_{max} = \frac{\alpha}{U} \ln(1 + \frac{a}{m})$$

(3.11)
which shows that the location of the highest oil phase mobility depends on the curvature of the oil phase relative permeability curve, thermal diffusivity of the reservoir, viscosity coefficient $m$, and the lateral spreading speed of the steam chamber. Equation (3.11) implies that the faster the chamber grows, the shorter is the distance beyond the chamber edge at which the maximum oil mobility is found.

3.2.3 *Oil Flow Rate at the Edge of a Steam Chamber*

The flow of oil is determined by examining the material balance about a differential element located at the chamber interface, as shown in Figure 3.1 (here, $x$ is in the horizontal direction perpendicular to the well, $y$ is parallel to the well direction, and $z$ is in the vertical direction). The drainage rate of oil for a section of thickness $d\xi$ and length $L$ (where $L$ is the well length, going into the paper) is given by Darcy’s law for gravity flow as:

$$
 dq = \frac{(d\xi \times L) k_o \Delta \rho g \sin \theta}{\mu_o} 
$$

(3.12)

where $g$ is the acceleration due to gravity, and $\theta$ is the local inclination of the chamber interface. Since the density of steam is much lower than that of oil, that is, $\rho_o \ll \rho$, $\Delta \rho \sim \rho_o$. Using the oil mobility relationship derived in Equation (3.10), the oil phase velocity (given by the flow rate per unit area) can be written as:

$$
 u_o = \frac{dq}{d\xi \times L} = \frac{kk_{roch} s g \sin \theta}{\rho_o \nu_s} \left(1 - \frac{U_o}{a} \frac{U_o}{a} \right) \left(\frac{U_o}{e^a} \right)^m
$$

(3.13)

From a material balance around the differential element displayed in Figure 3.1, the
Figure 3.1: Vertical section of SAGD steam chamber interface.
amount of oil flowing into the element is equal to the sum of oil flowing out of the element, and the accumulation of oil volume within the element in time \(dt\). If the oil density does not vary significantly, the mass balance around a thin vertical element becomes:

\[
q_{\eta + d\eta} dt = q_{\eta} dt + (Vol\big|_{r+dt} - Vol\big|_{r})
\]

where \(\eta\) corresponds to the direction tangential to the flow direction along the chamber interface. The volume of bitumen, \(Vol\), in the differential element is given by:

\[
Vol = \phi \Delta S_o L d\xi d\eta
\]

Substituting Equation (3.15) into Equation (3.14) and re-arranging gives:

\[
\frac{\partial q}{\partial \eta} = \phi \Delta S_o L \frac{\partial \xi}{\partial t}
\]

After substituting the values of effective permeability and viscosity into Equation (3.12), the result is:

\[
\frac{dq}{d\xi} = \frac{Lk\kappa_{recw} g \sin \theta}{\nu_s} \left( \frac{-\nu}{\alpha \xi} \right)^a \left( e^{\frac{-\nu}{\alpha \xi}} \right)^m
\]

Integrating Equation (3.17) with respect to \(\xi\) yields the flow rate across a two-dimensional cross-section in the \(\xi\) direction:

\[
q = \frac{Lk\kappa_{recw} \alpha g \sin \theta \Gamma(m)\Gamma(a+1)}{\nu_s U \Gamma(m + a + 1)}
\]
where $\Gamma(\cdot)$ is the gamma function\(^3\). From geometry considerations, $U = \partial \xi/\partial t = \cos \theta \partial z/\partial t = \sin \theta \partial x/\partial t$ and $d\eta = dx / \cos \theta$. After substituting these relations into Equation (3.16), the result is:

$$U = \frac{\cos \theta}{\phi \Delta S_o L} \partial q / \partial x$$

(3.19)

which can be re-arranged to give:

$$\frac{U}{\sin \theta} = \frac{\cot \theta}{\phi \Delta S_o L} \partial q / \partial x = \frac{\partial q / \partial z}{\phi \Delta S_o L}$$

(3.20)

Substituting Equation (3.20) in Equation (3.18) gives:

$$q = \frac{L^2 \alpha k_{\text{recw}} g \phi \Delta S_o}{v_s} \frac{\Gamma(m)\Gamma(a+1)}{\Gamma(m+a+1)} \frac{dz}{dq}$$

$$\Rightarrow \int_{0}^{h} q dq = \int_{z=0}^{h} \frac{L^2 \alpha k_{\text{recw}} g \phi \Delta S_o}{v_s} \frac{\Gamma(m)\Gamma(a+1)}{\Gamma(m+a+1)} dz$$

(3.21)

which can be integrated over the height of the chamber, $h$, to give the total oil rate from both sides of the chamber:

$$q = 2L \sqrt{\frac{2\alpha k_{\text{recw}} g \phi \Delta S_o h \Gamma(m)\Gamma(a+1)}{v_s \Gamma(m+a+1)}}$$

(3.22)

Equation (3.22) reveals that the volumetric oil flow rate as predicted by this new equation is different from Butler et al.’s (1981) equation by a factor equal to:

$$\sqrt{\frac{\Gamma(m+1)\Gamma(a+1)}{\Gamma(m+a+1)}}$$

(3.23)

---

\(^3\) $\Gamma(z) = \int_{0}^{\infty} t^{z-1} e^{-t} dt$
From Equations (3.20) and (3.22), the horizontal steam chamber speed is given by:

\[
U_x = \frac{\partial x}{\partial t} \bigg|_{z} = L \sqrt{\frac{\Gamma(m+1)\Gamma(a+1)}{\Gamma(m+a+1)}} \sqrt{\frac{\phi \Delta k \rho c_{p} \rho_{o}}{2 \phi \delta S_{o} \nu_{m} m_h}}
\]  

(3.24)

which is the same as that calculated by Butler et al. (1981) except that it is discounted by the factor displayed in Equation (3.23). Thus, the lateral spreading rate of the steam chamber predicted by the theory derived here is significantly lower than that given by the original SAGD theory.

The multiphase model, derived above, reduces to the original SAGD model (Butler et al., 1981) when the relative permeability of the oil phase in the mobile zone, is assumed to be unity (implying a single phase oil flow) or a constant value (implying saturation independent relative permeability).

### 3.3 Results and Discussion

For processes where heat transfer controls the viscosity of the oil phase, the key to understanding the process dynamics results from an examination of the viscosity profile that propagates beyond the chamber edge due to the temperature profile. Figure 3.2 displays plots of the dimensionless temperature and dynamic viscosity profiles calculated from Equations (3.2) and (3.3), respectively, for parameters typical of an Athabasca reservoir listed in Table 3.1. The results show that the heat transfer length scale over which the reservoir is significantly affected by heat conduction from the steam chamber is about 10 m. This is similar to the SAGD field data and reservoir simulation data which suggests that the thermal boundary layer at the edge of a steam chamber is between 5 and
10 m thick (Ito and Suzuki, 1996; Birrell et al., 2001). At the edge of the chamber, the oil viscosity is low, less than 10 cp, whereas it rises to about 10,000 cp at 10 m distance from the chamber edge.

This viscosity profile would suggest that the most mobile oil is that which is at the chamber edge. However, this is not the case due to relative permeability effects. Figure 3.3 displays the oil mobility as a function of distance from the edge of the chamber, for an Athabasca reservoir with properties listed in Table 3.1. The plot reveals that the highest mobility does not occur at the steam chamber edge ($\xi=0$) but rather at a location beyond the edge of the steam chamber ($\xi=\xi_{\text{max}}$).

Figure 3.4 displays a surface plot demonstrating the effect of the viscosity parameter, $m$, and the Corey coefficient for oil, $a$, on the distance at which the oil mobility is maximum ($\xi_{\text{max}}$). The result shows that the larger the viscosity coefficient and smaller the curvature of the oil phase relative permeability curve, the closer is the maximum to the steam chamber edge. As shown by Butler (1985), the viscosity parameter, $m$, is a measure of the change of the oil viscosity with respect to temperature. The higher the value of $m$, the higher is the rate of change of the viscosity with respect to temperature at steam temperature. Thus, Equation (3.11) reveals that the slower the ability of the oil viscosity to decline with heating, the farther the location of the maximum oil mobility is, with respect to the edge of the chamber.

Figure 3.5 plots the volumetric oil flux as a function of distance, $\xi$, which reveals that the flow behaviour at the edge of a steam chamber is complex and is not simply a monotonic function with its maximum at the edge of the steam chamber. It is clear that
Figure 3.2: Temperature and bitumen viscosity profiles as a function of distance from the steam chamber edge.

Figure 3.3: Oil phase mobility profile as a function of distance from the steam chamber edge.
Figure 3.4: Surface plot of location of maximum oil mobility versus the viscosity parameter, $m$, and the Corey coefficient for oil, $a$.

Figure 3.5: Volumetric oil flux as a function of distance from the edge of the steam chamber.
the oil flux profile resulting from Equation (3.13) exhibits zero oil-phase velocity at the
chamber edge, where the oil saturation is at the residual saturation, rises to a maximum at
the location of maximum oil mobility ($\xi_{\text{max}}$), and then declines further into the oil sands
because the oil viscosity is rising due to cooler temperature.

The factor that differentiates the new model with the previous models is displayed
in Equation (3.23) and plotted in Figure 3.6 as a function of $a$, and $m$, for reservoir
parameters listed in Table 3.1. The results show that the factor can be significantly less
than unity for a wide range of the viscosity parameter ($m$) and Corey coefficient ($a$)
values. This implies that the previous models provide an optimistic estimate of the oil
the oil flux profile resulting from Equation (3.13) exhibits zero oil-phase velocity at the
chamber edge, where the oil saturation is at the residual saturation, rises to a maximum at
the location of maximum oil mobility ($\xi_{\text{max}}$), and then declines further into the oil sands
because the oil viscosity is rising due to cooler temperature.

The factor that differentiates the new model with the previous models is displayed
in Equation (3.23) and plotted in Figure 3.6 as a function of $a$, and $m$, for reservoir
parameters listed in Table 3.1. The results show that the factor can be significantly less
than unity for a wide range of the viscosity parameter ($m$) and Corey coefficient ($a$)
values. This implies that the previous models provide an optimistic estimate of the oil
production rate from SAGD. For typical Athabasca reservoirs, the viscosity parameter
has value equal to about 4 and the Corey coefficient for oil is typically between 3 and 4.
The results in Figure 3.6 demonstrate that the oil production rate can be as low as 20% of
that predicted from Butler’s formula.

49
Figure 3.6: Surface plot of the comparison factor for Butler et al. (1981) and new model, versus viscosity parameter, m, and Corey coefficient, a.
Figures 3.7 and 3.8 compare the oil phase mobility and oil flux profiles as a function of distance for the new model and Butler’s formula, respectively. The results show that the oil mobility and oil flux profiles obtained from the model derived here are significantly lower than that calculated from Butler’s model.

Figure 3.9 displays the oil volumetric flow rate profile as a function of distance beyond the edge of the steam chamber at several thermal diffusivities. The results show that the larger the thermal diffusivity, the farther the peak value of the oil flow rate is from the chamber edge. Figure 3.10 shows the effect of the lateral spreading rate of the steam chamber on the oil volumetric flow rate profile as a function of distance. The results reveal that the larger the spreading rate, the closer the maximum oil flow rate is to the chamber edge. This is because of the competition between the steam chamber-spreading rate and conductive heat transfer length scale. When heat conduction length scale is large relative to the spreading rate, the maximum oil flow rate is moved further away from the edge of the steam chamber.

3.4 Validation of Theory with Experimental Data
Table 3.2 compares the previous analytical SAGD models and Equation (3.22) derived here, for two physical model experiments (Sasaki et al., 2001; Chung and Butler, 1988). For completeness, the properties of the systems are also listed in Table 3.2. The Corey
Figure 3.7: Comparison of oil phase mobility values from Butler’s model (1981) and current work.

Figure 3.8: Comparison of volumetric oil flux values from Butler’s model (1981) and current work.
Figure 3.9: Effect of thermal diffusivity, $\alpha$, on volumetric oil flux.

Figure 3.10: Effect of the chamber expansion rate, $U$, on volumetric oil flux.
Table 3.2: Comparison of SAGD models for two physical model experiments.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$, °C</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_s$, °C</td>
<td>106</td>
<td>109</td>
</tr>
<tr>
<td>$\rho_o(T_r)$, kg/m$^3$</td>
<td>998</td>
<td>998</td>
</tr>
<tr>
<td>$h$, m</td>
<td>0.3</td>
<td>0.21</td>
</tr>
<tr>
<td>$L$, m</td>
<td>0.0045</td>
<td>0.03</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>$\alpha$, m$^2$/s</td>
<td>2.6 x10$^{-7}$</td>
<td>5.86 x10$^{-7}$</td>
</tr>
<tr>
<td>$k$, m$^2$</td>
<td>1.15 x10$^{-10}$</td>
<td>2.39 x10$^{-9}$</td>
</tr>
<tr>
<td>$k_{ro}$</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>$S_{io}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\nu_s$, m$^2$/s</td>
<td>1.2 x10$^{-4}$</td>
<td>1.04 x10$^{-4}$</td>
</tr>
<tr>
<td>$m$</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Temperature coefficient (Reis), $a_R$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Corey coefficient, $a$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Oil flow rate $q$, cm$^3$/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Butler, McNab and Lo (1981)</td>
<td>8</td>
<td>326</td>
</tr>
<tr>
<td>Butler and Stephen (1981)</td>
<td>7</td>
<td>283</td>
</tr>
<tr>
<td>Reis (1992)</td>
<td>6</td>
<td>258</td>
</tr>
<tr>
<td>Current work</td>
<td>6</td>
<td>251</td>
</tr>
<tr>
<td><strong>Observed oil flow rate</strong></td>
<td><strong>6</strong></td>
<td><strong>275</strong></td>
</tr>
</tbody>
</table>
coefficient for the oil-water relative permeability curves were taken to be equal to one since the relatively large size of the grains used in the experiments causes capillary pressure influences to be small. This implies that the relative permeability curves are straight lines. A comparison of oil flow rate obtained from the analytical models and the observed experimental rates demonstrates that the new model derived here provides an improved estimate of the oil rate compared to the other analytical SAGD models.

3.5 Validation of Theory with Field Data
Table 3.3 compares the previous analytical SAGD models and Equation (3.22) derived here, for seven SAGD field operations. The results reveal that the new model provides an improved estimate of the oil production rate, again demonstrating that relative permeability effects must be included in the analysis of oil flow at the edge of a steam chamber.

3.6 Comparison of Theory with Numerical Simulation Results
Numerical simulation of a SAGD operation was performed on a 2-D, finely gridded, reservoir model. In order to compare the results from the analytical theory with the numerical simulation, a homogeneous reservoir model, with constant porosity and permeability and no overburden/underburden heat loss effect, was used. The reservoir model is described in detail in Appendix A, Section A.3.1, and its properties are listed in Table A.1. By using the same reservoir and fluid properties, as used in the simulation model, oil phase mobility and oil flow rate were predicted using the analytical theory
Table 3.3: Comparison of SAGD models using field data.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Christina Lake</th>
<th>Cold lake Hilda Lake, Shell</th>
<th>Conoco Phillips Surmont</th>
<th>Dover UTF Phase B, Athabasca Reservoir</th>
<th>Hangingstone Athabasca Reservoir</th>
<th>Wolf lake, Cold lake Bitumen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellpair A3</td>
<td>I3/P3</td>
<td>A</td>
<td></td>
<td></td>
<td>B10 Grand Rapids, SD9 Pad, Average of 6 Well pairs</td>
<td></td>
</tr>
<tr>
<td>$T_r$, °C</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$T_s$, °C</td>
<td>220</td>
<td>245</td>
<td>225</td>
<td>220</td>
<td>260</td>
<td>250</td>
</tr>
<tr>
<td>$\rho_o$, kg/m$^3$</td>
<td>880</td>
<td>880</td>
<td>880</td>
<td>880</td>
<td>980</td>
<td>980</td>
</tr>
<tr>
<td>$h$, m</td>
<td>26</td>
<td>24</td>
<td>30</td>
<td>21</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td>$L$, m</td>
<td>690</td>
<td>1000</td>
<td>850</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.33</td>
<td>0.35</td>
<td>0.33</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>$\alpha$, m$^2$/s</td>
<td>$7 \times 10^{-7}$</td>
<td>$7 \times 10^{-7}$</td>
<td>$7 \times 10^{-7}$</td>
<td>$7 \times 10^{-7}$</td>
<td>$7 \times 10^{-7}$</td>
<td>$7 \times 10^{-7}$</td>
</tr>
<tr>
<td>$k$, m$^2$</td>
<td>$6 \times 10^{-12}$</td>
<td>$5 \times 10^{-12}$</td>
<td>$5 \times 10^{-12}$</td>
<td>$7 \times 10^{-12}$</td>
<td>$5 \times 10^{-12}$</td>
<td>$3 \times 10^{-12}$</td>
</tr>
<tr>
<td>$\overline{k_{ro}}$, m$^2$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$S_{so}$</td>
<td>0.8</td>
<td>0.63</td>
<td>0.79</td>
<td>0.85</td>
<td>0.77</td>
<td>0.75</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.1</td>
<td>0.12</td>
<td>0.12</td>
<td>0.15</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>0.19</td>
<td>0.1</td>
<td>0.19</td>
<td>0.15</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>$\nu$, m$^2$/s</td>
<td>$6.81 \times 10^{-6}$</td>
<td>$3.41 \times 10^{-6}$</td>
<td>$3.41 \times 10^{-6}$</td>
<td>$7.95 \times 10^{-6}$</td>
<td>$4.28 \times 10^{-6}$</td>
<td>$4.08 \times 10^{-6}$</td>
</tr>
<tr>
<td>$m$</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Temperature coefficient (Reis), $a_R$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Corey coefficient, $a$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Oil Flow rate $q$, m$^3$/day(per well pair)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>262</td>
<td>359</td>
<td>379</td>
<td>148</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>227</td>
<td>311</td>
<td>329</td>
<td>128</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>207</td>
<td>284</td>
<td>300</td>
<td>117</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>173</td>
<td>72</td>
<td>75</td>
<td>80</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td><strong>Field Data</strong></td>
<td>170</td>
<td>65</td>
<td>50</td>
<td>77</td>
</tr>
</tbody>
</table>
derived in this chapter. The parameters used in the analytical model for comparison with the simulation results are listed in Table 3.4.

The results from the numerical simulation were analyzed after 30 hours (1.25 days) of steaming, which on scaling up using Butler’s (1985) scaling parameter translates to 8 years of SAGD operation. Figures 3.11a and 3.11b display the 2-D temperature distribution in the reservoir and temperature profile as a function of distance from the well pair, respectively. The results reveal that the edge of the chamber is located at a distance of about 17 cm from the well pair. The length scale for heat transfer, for the set of operating conditions used in the simulation (Table 3.4), is roughly 1 m. Figure 3.11c displays the oil phase mobility profile as a function of distance from the well pair. The oil phase mobility is zero at the chamber edge where the oil saturation is at the residual saturation condition, and peaks to about 5 mD/cp at a distance of 10 cm (≈ξ_{max}) beyond the chamber edge. Figure 3.11c also reveals that the mobile zone extends from the edge of the chamber to about 25 cm beyond it. Figure 3.11d displays the cumulative and instantaneous production rates, which shows that the average oil production rate observed in the numerical simulations is about 2.56 x10^{-5} m^3/day.

The results generated using the analytical model, using the parameters of the simulation model, are displayed in Figure 3.12. Figure 3.12a displays the temperature profile calculated from Equation (3.1), as a function of distance from the edge of the steam chamber (which is located at ξ =0). The result reveals that beyond the chamber edge, the temperature monotonically declines below the SAGD chamber temperature, Ts (=220 °C). The observed heat transfer length scale is about 1 m beyond the chamber edge.
Table 3.4: Reservoir parameters used in the analytical model for comparison with the simulation results.

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$, °C</td>
<td>10</td>
</tr>
<tr>
<td>$T_s$, °C</td>
<td>220</td>
</tr>
<tr>
<td>$\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>$k$, m$^2$</td>
<td>5x10$^{-13}$</td>
</tr>
<tr>
<td>$a$, m$^3$/s</td>
<td>3x10$^{-6}$</td>
</tr>
<tr>
<td>$S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$S_{ow}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>0.19</td>
</tr>
<tr>
<td>$\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
<tr>
<td>$U$, cm/day</td>
<td>20</td>
</tr>
<tr>
<td>Initial reservoir pressure, kPa</td>
<td>2000</td>
</tr>
<tr>
<td>Injection pressure, $p_i$, kPa</td>
<td>2363</td>
</tr>
<tr>
<td>$h$, m</td>
<td>0.1125</td>
</tr>
<tr>
<td>$L$, m</td>
<td>0.0025</td>
</tr>
<tr>
<td>Butler’s viscosity parameter, $m$</td>
<td>3</td>
</tr>
<tr>
<td>Corey Coefficients</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>3</td>
</tr>
<tr>
<td>$b$</td>
<td>3</td>
</tr>
</tbody>
</table>
Figure 3.11: Homogeneous reservoir simulation results (a) Temperature distribution in the reservoir (b) Temperature profile as a function of distance from the center of the well pair (c) Oil phase mobility profile (d) Cumulative and instantaneous oil flow rates.
Figure 3.12: Analytical model results (a) Temperature profile (b) Oil phase mobility profile, as a function of distance from the steam chamber edge.
(for the parameters listed in Table 3.4). The heat transfer length scale as well as temperature profile obtained from the analytical model is similar to the simulation results (Figure 3.11b).

Figure 3.12b displays the oil phase mobility profile as a function of distance from the chamber edge, calculated from Equation (3.10). The plot reveals that the oil phase mobility is zero at the chamber edge and has a maximum value of about 5.5 mD/cp at a distance of 10 cm beyond the chamber edge. This implies that $\xi_{\text{max}}$, from Equation (3.11), equals 10 cm. The oil mobility profile is qualitatively and quantitatively similar to the simulation results shown in Figure 3.11c.

Equation (3.22) predicts an oil production rate of $1.88 \times 10^{-5}$ m$^3$/day for the reservoir properties used in the numerical simulation, listed in Table 3.4. For the same reservoir parameters, Butler and Stephen’s (1981) equation predicts a value of $8 \times 10^{-5}$ m$^3$/day. These results show that the analytical theory derived here agrees favourably with the results obtained from the fine-grid numerical simulations and predicts more accurate oil production rate compared to the previous SAGD theories.

### 3.7 Conclusions

A new model for gravity drainage of mobilized bitumen at the edge of a SAGD chamber is derived, that includes relative permeability and multiphase flow effects. These effects are important, but missing from the previous analytical SAGD models. The new model reveals that the highest oil flow velocity is not at the steam chamber edge but rather at some distance away from the chamber edge, due to relative permeability effects. The
theory reveals that the location of the maximum mobility is set by the balance between the spreading rate and the conductive heat transfer into the oil sands at the edge of the chamber. A comparison of the oil flow rate obtained from the new analytical model with field, experimental, and fine-grid simulation data, demonstrates that the model derived here provides an improved estimate of the oil rate compared to the previous SAGD theories.
Chapter 4: CONVECTION AT THE EDGE OF SAGD STEAM CHAMBER

4.1 Introduction

A key control on the productivity of SAGD is the extent of heating of oil sands at the edge\(^4\) of the steam chamber. The larger the heated layer of oil sands beyond the chamber walls, the greater is the thickness of the mobile oil layer and lower is the viscosity of the bitumen in the mobile layer. Enhancement of heat transfer at the chamber edge can be done by several means: 1. raising the temperature difference between the steam chamber and the initial reservoir temperature, 2. increasing the thermal conductivity of the oil sands, and 3. increasing the heat transfer surface area. Given the range of steam injection pressures (recall steam chamber at saturation conditions, so temperature and pressure are coupled) possible in the process, a rise of temperature will be limited. For example, it will not be practical to double the temperature difference with a steam-based process. The thermal conductivity of oil sands is nearly constant in the reservoir provided the fluid saturations and porosity are not varying substantially. Also, the surface area is not a controllable quantity in these systems. Thus, it is not possible to enhance conductive heat transfer within the reservoir to raise oil production rate from the reservoir.

However, heat transfer can be enhanced by other means: 1. convective heat transfer, 2. exothermic reactions that occur at the edge of the chamber (may be possible

\(^4\) In this work, the ‘edge of the chamber’ is defined mathematically as the interface beyond which the temperature declines below the temperature inside the SAGD chamber \((T<T_s)\).
through in-situ combustion), and 3. electromagnetic energy transfer through induction (requires ferromagnetic materials seeded in the reservoir) or microwave heating. Reactions and electromagnetic heating are both complex, whereas if water is mobile in these reservoirs, there is potential that steam condensate could provide additional heat transfer at the chamber edge in the case where chamber pressure exceeds original reservoir pressure. However, it remains unclear how large convection is relative to conduction.

In this chapter, the heat transfer mechanism at the edge of a SAGD chamber is re-examined. Specifically, a new theory is derived to account for convection of warm condensate into the oil sands, and to compare the relative roles of convective and conductive heat transfer at the edge of a SAGD steam chamber. The theory has been tested via comparison with field data taken from the Dover Phase B SAGD pilot, as well as with fine-grid numerical simulation results.

4.2 Analytical Model

4.2.1 Modelling Assumptions

The following assumptions were made in the derivation of the analytical model:

1. Porous medium is homogeneous (constant porosity and permeability).
2. Analysis is done for a mature SAGD steam chamber that has reached the maximum height, \( h \), at time, \( t=0 \), and is expanding at a constant velocity \( U \), in the direction perpendicular to the chamber walls.
3. Overburden and underburden heat losses are neglected.
4. Heat transfer and mass transfer is directed normal to the edge of the chamber.

5. No solution gas is present in the oil and therefore in the mobile zone, multiphase flow of only condensate and oil occurs.

6. Boussinesq approximation (that is, variation in density is neglected everywhere except in the buoyancy term).

7. Density and viscosity are dependent on temperature.

4.2.2 Mathematical Formulation

Figure 4.1 displays a schematic of the domain at the edge of the SAGD chamber. Heat transfer ahead of the steam chamber occurs by convection and conduction according to the following equation:

\[
K_{TH} \left\{ \left( \frac{\partial^2 T}{\partial x^2} \right) + \left( \frac{\partial^2 T}{\partial y^2} \right) + \left( \frac{\partial^2 T}{\partial z^2} \right) \right\} - V_c \rho_o c_p \left\{ \left( \frac{\partial T}{\partial x} \right) + \left( \frac{\partial T}{\partial y} \right) + \left( \frac{\partial T}{\partial z} \right) \right\} = \rho_{os} c_p \left( \frac{\partial T}{\partial t} \right)
\]

(4.1)

where \( V_c \) is the convective velocity of the condensate, normal to the chamber edge, \( \rho_{os} \), \( c_p \), and \( K_{TH} \) are the density, specific heat capacity, and thermal conductivity of the reservoir, respectively. Subscript \( c \) denotes properties of the flowing condensate. The temperature gradient in the direction parallel to the chamber interface and along the well direction is considered much smaller than that perpendicular to the interface. Equation (4.1) in the horizontal direction then reduces to:

\[
K_{TH} \left( \frac{\partial^2 T}{\partial x^2} \right) - V_c \rho_o c_p \left( \frac{\partial T}{\partial x} \right) = \rho_{os} c_p \left( \frac{\partial T}{\partial t} \right)
\]

(4.2)
Figure 4.1: Conduction and convection at the edge of a steam chamber.
If the steam chamber interface is moving with velocity $U$, normal to the chamber edge, we can transform the reference axis to the moving chamber by the following substitution of variables:

$$\xi \rightarrow x - Ut$$

resulting in:

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial T}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial T}{\partial \xi} \frac{\partial \xi}{\partial z} = \frac{\partial T}{\partial \xi}$$

(4.4a)

and

$$\frac{dT(\xi, t)}{dt} = \frac{\partial T}{\partial \xi} \frac{d\xi}{dt} + \frac{\partial T}{\partial t} \frac{dt}{dt}$$

(4.4b)

Thus, substituting Equations (4.4a) and (4.4b) into Equation (4.2) yields:

$$K_{TH} \left( \frac{\partial^2 T}{\partial \xi^2} \right) - V_c \rho_c c_{pc} \left( \frac{\partial T}{\partial \xi} \right) = \rho_w c_p \left( -U \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t} \right)$$

(4.5)

Similar to the derivation in Chapter 3, it was assumed that the chamber is undergoing pseudo steady-state expansion, for which Equation (4.5) simplifies to:

$$K_{TH} \left( \frac{\partial^2 T}{\partial \xi^2} \right) = (V_c \rho_c c_{pc} - U \rho_w c_p) \left( \frac{\partial T}{\partial \xi} \right)$$

(4.6)

The condensate velocity profile normal to the chamber interface depends on the relative water permeability and viscosity variations in that direction. The condensate velocity at any point can be determined from the following relation:

$$V_c = U \frac{k_{rw}}{k_{rwint}} \frac{\mu_w}{\mu_{win}}$$

(4.7)
where \( k_{rw} \) is the relative permeability of the water phase (condensate) and \( \mu_w \) is the viscosity of water at temperature \( T \). At 200°C, \( \mu_w \) declines to less than 0.2 cp. Subscript int denotes the respective property at the chamber interface. Substituting Equation (4.7) in Equation (4.6) yields:

\[
K_{TH} \left( \frac{\partial^2 T}{\partial \xi^2} \right) = -U \left( \rho_w c_p - \rho_c c_{pc} \frac{k_{rw}}{k_{rwint}} / \mu_w \right) \left( \frac{\partial T}{\partial \xi} \right) \quad (4.8)
\]

On rearranging Equation (4.8) we obtain:

\[
\alpha^* \left( \frac{\partial^2 T}{\partial \xi^2} \right) = -U \left( \frac{\partial T}{\partial \xi} \right) \quad (4.9)
\]

where

\[
\alpha^* = \frac{K_{TH}}{\left( \rho_w c_p - \rho_c c_{pc} \frac{k_{rw}}{k_{rwint}} / \mu_w \right)} \quad (4.10)
\]

The oil and water relative permeability can be determined from laboratory based relative permeability data which are often correlated in the form of Corey’s equations (Corey, 1954; Brooks and Corey, 1964) as described in Chapter 3 (Section 3.2):

\[
k_{ro} = k_{rcw} (1 - S_{wD})^a, \text{ for oil and } k_{rw} = k_{rcw} (S_{wD})^b, \text{ for water} \quad (4.11)
\]

Substituting Corey’s relations into Equation (4.10) gives:

\[
\alpha^* = \frac{K_{TH}}{\left( \rho_w c_p - \rho_c c_{pc} \frac{k_{rw}}{k_{rwint}} / \mu_w \right) \left( \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} \right)^b} \quad (4.12)
\]

where \( \alpha^* \) is an “apparent” thermal diffusivity due to convective as well as conductive heat transfer. From Equation (4.12), it is evident that at \( S_w = S_{wr} \), \( \alpha^* = \alpha \) and if \( S_w > S_{wr} \), then
\( \alpha^* > \alpha \). When \( S_w > S_{wr} \), there is mobile water in the reservoir and so convection also contributes to heat transfer thereby increasing the apparent thermal diffusivity. At irreducible water saturation, the water is immobile and heat transfer occurs through conduction alone. Equation (4.12) can be rearranged to yield:

\[
\alpha^* = \frac{\alpha}{1 - \frac{\rho c_p}{\rho_m c_p} \frac{k_{rw}}{k_{rwu}} \left( \frac{S_w - S_{wr}}{1 - S_{wr}} \right)}
\]

(4.13)

To calculate the apparent thermal diffusivity with respect to temperature, the variation of water saturation with temperature is needed. As a first approximation for modelling multiphase flow effects, the oil saturation beyond the steam chamber edge is assumed to be a linear function of the dimensionless temperature \( T^* \), as described in Chapter 3 (Section 3.2):

\[
S_o = S_{or} + (S_{io} - S_{or})(1 - T^*)
\]

(4.14)

where \( S_o, S_{io}, \) and \( S_{or} \) are oil saturation, initial oil saturation, and residual oil saturation, respectively, at the dimensionless temperature \( T^* \). The temperature distribution ahead of the steam chamber is given by:

\[
T^* = \frac{T - T_r}{T_s - T_r} = e^{-\frac{U}{\alpha^*T}}
\]

(4.15)

Therefore, substituting Equations (4.14) and (4.15) into Equation (4.13) yields the following nonlinear equation for the apparent conductive and convective thermal diffusivity, \( \alpha^* \):
Equation (4.16) can be solved by Newton’s method for $\alpha^*$ as a function of $\zeta$, given values of the relative permeability endpoints, water viscosity, and the density and specific heat capacity of the steam condensate and oil sands, as a function of temperature.

If the steam chamber pressure is larger than the initial reservoir pressure, the analytical relationship between the apparent thermal diffusivity and conductive thermal diffusivity, given by Equation (4.16), can be used to determine the temperature profile, computed from Equation (4.15). Once the temperature distribution is found, the conductive and convective heat fluxes can be calculated, respectively, from:

$$Q_{\text{conv}} = V_c c_{pc} \rho_c (T - T_r)$$ \hspace{1cm} (4.17)

and

$$Q_{\text{cond}} = -K_{TH} \frac{\partial T}{\partial x}$$ \hspace{1cm} (4.18)

### 4.3 Results and Discussion

#### 4.3.1 Relative Role of Convection vs. Conduction

Table 4.1 lists the reservoir and fluid properties used to analyze the theory derived above. These values are typical of an Athabasca oil sands reservoir. The conductive thermal diffusivity of the oil sands, $\alpha$, is equal to $7 \times 10^{-7}$ m$^2$/s. Figure 4.2 plots the ratio of the apparent thermal diffusivity, $\alpha^*$, to the conductive thermal diffusivity, $\alpha$, as a function of the distance from the edge of the steam chamber. The results show that under the
Table 4.1: Fluid and reservoir properties used in the analytical model.

<table>
<thead>
<tr>
<th>References</th>
<th>Analytical Model (Figures 4.2 to 4.7)</th>
<th>Comparison with Field Data from Dover UTF Phase B, Athabasca Reservoir (Figures 4.8 to 4.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_r, ^\circ C$</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>$T_s, ^\circ C$</td>
<td>260</td>
<td>205</td>
</tr>
<tr>
<td>$h$, m</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>$L$, m</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\alpha$, m$^3$/s</td>
<td>7x10$^{-7}$</td>
<td>7x10$^{-7}$</td>
</tr>
<tr>
<td>$k$, m$^3$</td>
<td>5x10$^{-12}$</td>
<td>6x10$^{-12}$</td>
</tr>
<tr>
<td>Average $k_{rw}$ at the interface</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$k_{roco}$</td>
<td>0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>$k_{roco}$</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.14</td>
<td>0.1</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$S_{io}$</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho_o$, kg/m$^3$</td>
<td>980</td>
<td>880</td>
</tr>
<tr>
<td>$\mu_o(T_s)$, cp</td>
<td>5.2</td>
<td>7</td>
</tr>
<tr>
<td>$m$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$a$</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>$b$</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>$U$, cm/day</td>
<td>1</td>
<td>1.7</td>
</tr>
<tr>
<td>$\mu_w$, cp (Reid et al., 1977)</td>
<td>log($\mu_w$) = $A \left( \frac{1}{T} - \frac{1}{B} \right)$, where $A=658.25$ and $B=283.16$;</td>
<td>$\mu_w$ in cp, $T$ in K</td>
</tr>
</tbody>
</table>

71
reservoir conditions listed in Table 4.1, the apparent thermal diffusivity is several times larger than the conductive thermal diffusivity at the edge of the chamber (at distance, $\xi = 0$ m). The apparent thermal diffusivity drops rapidly moving beyond the edge of the chamber and reaches within 5% of the conductive thermal diffusivity at a location about 3.5 m beyond the edge of the steam chamber. Given that the conduction length scale is between 10 and 15 m beyond the SAGD chamber, the results reveal that the convection-active zone is narrower than the conduction zone. Figure 4.3 plots the conductive and convective components of the total heat flux for the conduction plus convection case. The results reveal that the convective heat flux dominates over the first 1.5 m, whereas the conductive heat flux is larger beyond. The convective heat transfer component becomes minor beyond about 3.5 m from the edge of the chamber.

Figure 4.4 displays a comparison of temperature profiles beyond the edge of the steam chamber in the conduction-only and conduction plus convection cases. The results show that in the zone where the convective heat transfer is significant, the temperature profile of the conduction plus convection case is slightly larger than that of the conduction case. The enhancement of temperature beyond the edge of the steam chamber is up to about 13°C within the convection-active zone. This temperature enhancement may seem small but this yields a bitumen viscosity drop of up to 16%, at about 2 m into the convection-active zone beyond the chamber edge. In addition, the shape of the temperature profile for the conduction plus convection case no longer has the exponential decay, typical of pure conduction, but is concave downwards within the convection-active zone.
Figure 4.2: Ratio of the apparent thermal diffusivity ($\alpha^*$) to the conductive thermal diffusivity ($\alpha$) versus distance from the steam chamber edge. See Table 4.1 for fluid and reservoir properties.

Figure 4.3: Conductive and convective heat fluxes in the conduction plus convection case. See Table 4.1 for fluid and reservoir properties.
Figure 4.5 compares the conductive and convective heat fluxes in the conduction plus convection case at the chamber edge as a function of temperature, where the steam temperature in the chamber equals 260°C. The results reveal that convection at the chamber edge dominates beyond about 225°C whereas conduction is the largest heat flux below 225°C. Convective heat transfer becomes a minor heat transfer mechanism below 125°C. This can be explained from the multiphase flow behaviour of condensate and oil near the chamber edge which results in significant relative permeability effects. The amount of condensate present near the edge of the steam chamber can be three to five times in volume in comparison to the amount of flowing oil (Farouq Ali, 1997). Thus the presence of high water saturation near the chamber edge leads to an increase in the relative water permeability which results in an enhanced convective flow of the hot condensate into the oil sands. Consequently, a higher convective heat flux is observed at high temperatures near the steam chamber edge, as seen in Figure 4.5.

4.3.2 Oil Mobility and Flow Rate

Figure 4.6 shows a comparison of oil phase mobility (defined as the oil phase effective permeability divided by the oil phase viscosity) profiles for the conduction-only and conduction plus convection cases beyond the edge of the SAGD chamber. The results reveal that, similar to the predictions of the multiphase theory derived in Chapter 3, Section 3.2, the most mobile oil is not at the edge of the steam chamber but rather some distance beyond the chamber edge. This is due to the relative permeability of the oil phase. At the chamber edge, it is low and thus the mobility is low despite the low oil
Figure 4.4: Comparison of temperature profiles beyond the edge of the steam chamber in conduction-only and conduction plus convection cases.

Figure 4.5: Conductive and convective heat fluxes in the conduction plus convection case at the chamber edge versus temperature.
viscosity there. Deeper within the reservoir, the oil relative permeability increases as the oil saturation increases and a peak in the oil mobility is found. Further into the colder reservoir region, the temperature drops and consequently, the oil viscosity approaches the original oil viscosity of the reservoir and as a result, the oil mobility declines. Within the convection-dominated zone, the oil mobility profile of the conduction-only case is higher than that of the conduction plus convection case. This, at first, seems surprising because it would be expected that enhanced heat transfer due to the combined effects of conduction and convection would yield more mobile oil than that resulting from conduction alone. The reason for the lowering of the oil mobility in the conduction plus convection case arises from relative permeability effects. Since the mobile hot condensate enters the edge of the oil sands, the oil saturation drops and as a consequence, so too does the oil relative permeability. This means that the oil effective permeability drops and despite the reduction of the oil viscosity due to slightly higher temperature arising from convection, the overall oil mobility declines as a result. In other words, the impact of the convective flow of water into the oil sand at the chamber edge on the oil relative permeability is larger than the impact it has on the temperature distribution with respect to the oil mobility. The impact of the oil mobility profile on the oil flux (per metre of well length) is displayed in Figure 4.7. The results show that condensate convection into the oil sands lowers the oil flux. The consequence is reduced oil volumes being delivered to the production well at the base of the steam chamber.
Figure 4.6: Comparison of oil phase mobility profiles of the conduction-only and conduction plus convection cases beyond the edge of the steam chamber.

Figure 4.7: Comparison of oil flux per metre of well length (into the page) for conduction-only and conduction plus convection cases beyond the edge of the steam chamber.
4.4 Validation of Theory with Field Data

Birrell (2001) analyzed temperature data from vertical observation wells in the Dover Phase B SAGD pilot to calculate the apparent thermal diffusivity and relative roles of conductive and convective heat transfer. From his analysis of the temperature distribution, he determined periods of time where the steam chamber was growing and other times where it had stopped moving due to a barrier. During the periods of time that the chamber growth has stopped, Birrell analyzed the temperature profiles to estimate the apparent thermal diffusivity. Table 4.1 lists the average reservoir properties of the Dover Phase B site. Figure 4.8 compares the apparent thermal diffusivity at the edge of the chamber determined by Birrell’s analysis versus values calculated from Equation (4.16) as a function of temperature. The comparison reveals that the theory provides a reasonable representation of the field data. If convection was not taken into account, the apparent thermal diffusivity would be equal to about $7 \times 10^{-7} \text{m}^2/\text{s}$ at all temperatures, which is not the case beyond about 160°C.

Figure 4.9 displays a sample temperature profile from Birrell’s analysis as a function of vertical distance beyond the top of the steam chamber (collected from vertical observation well). To compare, the temperature profiles of the conduction-only and conduction plus convection cases are also plotted. In the region near the edge of the chamber, the results show that the conduction plus convection case provides an improved fit with the field data. The field data, similar to the conduction plus convection case, exhibits the concave downwards profile for the first 1.5 m beyond the top of the steam chamber. Figure 4.10 compares the conductive and convective heat flux components
Figure 4.8: Comparison of apparent thermal diffusivity estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot, and the predictions of the analytical theory (Equation 4.16). See Table 4.1 for fluid and reservoir properties.

Figure 4.9: Comparison of temperature distribution estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot versus predictions calculated by theory (Equation (4.15)). See Table 4.1 for fluid and reservoir properties.
Figure 4.10: Comparison of conductive and convective heat flux components estimated from field data obtained from vertical observation wells in the Dover Phase B SAGD pilot versus predictions calculated by theory (Equations (4.17) and (4.18)). See Table 4.1 for fluid and reservoir properties.
estimated from Birrell’s analysis of the field data versus predictions calculated by Equations (4.17) and (4.18). The results show that the theory generates similar trends as evident in the field data. This suggests that the theory captures the underlying behaviour of convection at the edge of a steam chamber.

### 4.5 Comparison of Theory with Numerical Simulation Results

The analytical theory derived above establishes that in the overall heat transfer mechanism beyond the edge of a SAGD chamber, the convective heat component is dominant at high temperatures close to the edge of the chamber, if the water phase is mobile. In the SAGD simulations discussed in the previous chapter (Chapter 3, Section 3.6), the initial water saturation in the reservoir model was close to the irreducible water saturation ($S_{wir} = 0.19$, $S_{iw} = 0.2$). The simulation results revealed that near the edge of the chamber, the water saturation was close to the irreducible water saturation ($S_{wi} \approx S_{wr}$), and thus the water phase mobility at the chamber edge was low. Consequently, the effect of convective heat transfer near the chamber edge was present, but not significant. This was also evident in the simulation results which showed an exponentially decaying, concave upwards, temperature profile beyond the edge of the chamber (Figure 3.11b), typical of conductive heat transfer. For this reason, the temperature profile from the simulation could be compared with the conductive heat transfer equation, Equation (3.1) (Chapter 3).

In order to clearly establish the presence of convective heat transfer effect near the steam chamber edge, the homogeneous reservoir simulation model used in Chapter 3
was modified by specifying a lower irreducible water saturation in the reservoir ($S_{ir}=0.05$), to make the initial water phase in the reservoir more mobile ($S_{iw}>S_{ir}$). Table 4.2 summarizes the reservoir and fluid properties used in the simulation model.

The results from the modified simulation model are shown in Figure 4.11. The temperature distribution in the reservoir after 30 hours of steaming (equivalent to 8 years of SAGD operation on scaling-up) is shown in Figure 4.11a, and the temperature profile as a function of distance from the well pair is shown in Figure 4.11b. The edge of the steam chamber can be located on the temperature profiles at about 17 cm from the well pair. It is evident from the results, that the shape of the temperature profile no longer has the typical exponential decay but is concave downwards within the convection-active zone, close to the chamber edge. This trend becomes clear when the temperature profile is superimposed with an exponentially decaying temperature function which will result for purely conductive heat transfer (that is, Equation (4.15) with a constant thermal diffusivity). The result is displayed in Figure 4.12, which shows that convective heat transfer component dominates over the first 15-20 cm beyond the edge of the chamber. The enhancement of temperature within the convection-active zone beyond the chamber edge is up to about 16°C. These results are consistent with the predictions of the analytical theory and establish the presence of convective heat transfer effect near the chamber edge. The reason for the difference in the heat transfer length scale between the simulation results (Figure 4.11) and the analytical results discussed in Section 4.3 (Figure 4.4), is the faster chamber expansion rate observed in the numerical simulation.
Table 4.2: Reservoir and fluid properties used in the simulation model (Ito and Suzuki, 1996; Ito and Hirata, 1999; Butler, 1997a).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir temperature, $T_r$, °C</td>
<td>5</td>
</tr>
<tr>
<td>Initial reservoir pressure, kPa</td>
<td>2000</td>
</tr>
<tr>
<td>Oil density, $\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>Rock/Sand compressibility, 1/kPa</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>Permeability, $k$, m$^2$</td>
<td>$5 \times 10^{-13}$</td>
</tr>
<tr>
<td>Initial oil saturation, $S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Initial water saturation, $S_{iw}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Irreducible water saturation, $S_{wr}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Viscosity of oil at steam temperature, $\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thermal conductivity, J/(m-day-°C)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>$6.048 \times 10^5$</td>
</tr>
<tr>
<td>Oil</td>
<td>$1.339 \times 10^4$</td>
</tr>
<tr>
<td>Gas</td>
<td>4320</td>
</tr>
<tr>
<td>Water</td>
<td>$5.8147 \times 10^4$</td>
</tr>
</tbody>
</table>

**Operating Conditions**

| Injection pressure, $p_i$, kPa                          | 2363                   |
| Steam chamber temperature, $T_s$, °C                    | 215.5                  |
| Steam quality                                           | 1                      |

83
Figure 4.11: Homogeneous reservoir simulation results after 30 hours of steaming (a) Temperature distribution in the reservoir (b) Temperature profile as a function of distance from the center of the well pair.
Figure 4.12: An exponentially decaying temperature function superimposed on the temperature profile obtained from the numerical simulation.
(\(U\approx 20 \text{ cm/day} \) in the simulation as opposed to 1-2 cm/day for the results discussed in Section 4.3)

\textbf{4.6 Conclusions}

A mathematical model was derived to account for the convection of warm condensate into the oil sands at the edge of a SAGD chamber. An apparent thermal diffusivity term, which includes both the conductive and convective heat components, was derived to examine the relative roles of convection and conduction in the overall heat transfer mechanism beyond the steam chamber edge. The results demonstrate that convection can provide a relatively large contribution to heat transfer close to the edge of the steam chamber. However, enhanced heat transfer does not necessarily mean enhanced oil mobility and oil rates, due to the relative permeability effects at the chamber edge. As condensate invades the oil sands, the oil saturation drops and consequently the oil relative permeability declines. This in turn results in the reduction of the oil mobility, despite the lowered oil viscosity due to higher temperature arising from convective heat transfer. The analytical theory was tested via comparison with field data taken from the Dover Phase B SAGD pilot, as well as with fine-grid numerical simulation results.
Chapter 5:

NON-CONDENSABLE GAS AIDED SAGD PROCESS

5.1 Introduction

In one variation of SAGD, a small amount of a non-condensable gas is added to the injected steam to maintain pressure in the chamber, while utilizing the energy in place, reducing steam consumption, and providing thermal insulation from overburden heat losses. Non-condensable gases are gases, which can be present in the SAGD steam chamber but do not condense into the liquid phase to any large degree at steam temperatures, such as methane, nitrogen, air and others (Aherne and Birrell, 2002). The presence of non-condensable gas (which will be interchangeably referred to as ‘gas’) can significantly affect the displacement of oil and make the SAGD process more complicated.

The role of gas during a steam-gas co-injection processes, in terms of its effects on chamber development, bitumen flow rate, and heat losses is not fully understood and is, therefore, the focus of this chapter. There are currently no simple analytical models available in the literature, which can predict the fluid flow rate for SAGD in the presence of non-condensable gas. The object of the present study is to understand the above effects using analytical modelling and fine-grid numerical simulation.

Methane was used as the non-condensable gas for the purpose of this study as it is the main solution gas in most reservoirs. However, it is believed that the findings of this study are applicable to other non-condensable gases such as nitrogen, air, or others. From the results, calculations are performed to make qualitative and quantitative assessments of
the role of gas in terms of its impact on chamber development, fluid flow rate and overburden heat losses.

5.2 Theory
When non-condensable gases are present in the SAGD chamber, as a result of in-situ generation or co-injection with steam, they move with the vapour as one phase within the chamber towards the condensing surface. The temperature inside the chamber corresponds to the partial pressure of the steam and thus is reduced proportionally to the amount of non-condensable gas injected. As the steam condenses, the volume shrinkage creates a low-pressure region ($\Delta p$), which leads to an increased non-condensable gas concentration at the condensing interface. The concentration gradient of the non-condensable gas thus created, leads to counter-current diffusion of gas towards the SAGD chamber, which opposes its further accumulation near the condensing interface. Simultaneously, the non-condensable gas also dissolves and diffuses into the adjacent oil phase. Thus, the equilibrium concentration of gas near the chamber interface is dictated by the complex interaction between diffusion, dispersion, and dissolution mechanisms.

Another effect resulting from the presence of non-condensable gas layer near the chamber wall will likely be the modification of the rate of heat transfer from steam to the adjacent oil. Inhibitive effect of non-condensable gases on steam condensation and heat transfer is a well-known phenomenon and has been studied extensively in the context of industrial applications in nuclear reactors and steam boilers (Jensen, 1988; Vierow, 1990; Siddique, Golay, and Kazimi, 1993; Maheshwari, Patel, Vijayan, Saha, and Aritomi,
These studies indicate that the presence of even a small amount of non-condensable gas (e.g. air, N₂, He, etc.) in the condensing vapour leads to a significant reduction in heat transfer during condensation. The buildup of non-condensable gases near the condensate film inhibits the diffusion of vapour steam from the bulk mixture to the liquid film. The net effect is to reduce the effective driving force for heat and mass transfer. Othmer (1929) showed experimentally that the heat transfer rate is reduced by over fifty percent due to small amount of air or other gases. The drastic reduction in the condensation heat transfer coefficient in the presence of non-condensable gas can be explained as follows: when the vapour mixed with a non-condensable gas condenses, the non-condensable gas remains in the vicinity of the condensate layer. This gas acts as a barrier between the vapour and the condensate layer (the ‘gas buffer’) and makes it difficult for the vapour to reach the surface. The vapour now must diffuse through the non-condensable gas first before reaching the surface and this reduces the effectiveness of the heat transfer process. Variation of heat transfer coefficient with the mass fraction of non-condensable gas is shown in Figure 5.1 (Maheshwari et al., 2004). Thus, it is expected that the presence of high concentration of non-condensable gas at the SAGD chamber edge would reduce the condensation heat transfer by the steam vapour to the oil phase. In some cases, this would be a desirable effect, as it would limit the overburden heat losses.

The above discussion indicates that the presence of non-condensable gas in the SAGD chamber leads to a number of effects which occur simultaneously, making the analytical modelling a challenging task. During the SAGD process, the oil layer adjacent
Figure 5.1: Variation of heat transfer coefficient with air mass fraction (Maheshwari et al., 2004).
to the expanding chamber is mobile while the oil phase within the chamber (which is at residual oil saturation) and farther in the cold reservoir is practically immobile. Therefore, the theoretical calculations discussed in the next section focus on predicting the temperature, saturation, relative-permeability, and phase mobility profiles in this ‘mobile zone’. Numerical simulations were conducted to visually and quantitatively understand the gas distribution in the oil depleted chamber and the effect of gas on in-situ steam quality, overburden heat losses, phase saturations, and fluid flow rate. The theoretical calculations were also compared with the results from physical model experiments of the Steam and Gas Push (SAGP) process, which is a thermal oil recovery process that is similar to SAGD, but involves the addition of a small concentration of a non-condensable gas to the injected steam.

5.3 Analytical Model

Figure 5.2 displays a schematic of a section of the interface of a SAGD steam chamber with non-condensable gas, and the mobile zone adjacent to it. In this work, the ‘mobile zone’ is defined as the narrow zone beyond the steam condensation front where multiphase flow of oil, water, and gas takes place. The ‘edge of the chamber’ is mathematically defined as the vapour-oil interface beyond which the temperature drops below the injected steam/non-condensable gas mixture temperature ($T<T_s$). An analytical model was developed to examine the effect of non-condensable gas injection on the overall heat and mass transfer in the mobile zone.
Figure 5.2: Cross section of the SAGD steam chamber with non-condensable gas, showing the mobile zone and the chamber edge.
5.3.1 Modelling Assumptions

The following assumptions were made in the development of the analytical model:

1. Porous medium is homogeneous (constant porosity and permeability).
2. Overburden and underburden heat losses are neglected.
3. Heat transfer and mass transfer are directed normal to the edge of the chamber.
4. Convective heat and mass transfer beyond the chamber edge are neglected.
5. Analysis is done for a mature SAGD chamber that has reached the maximum height, $h$, at time, $t=0$, and is expanding at a constant velocity $U$, in the direction perpendicular to the chamber walls.
6. Negligible solubility of methane in the water phase (which is a reasonable assumption at typical SAGD operating conditions).
7. Reservoir oil does not initially have any solution gas.
8. Boussinesq approximation (that is, variation in density is neglected everywhere except in the buoyancy term).

5.3.2 Mathematical Formulation

The heat transfer ahead of a moving SAGD chamber is governed by the one-dimensional Fourier’s equation, which can be written in the reference coordinates of the moving SAGD chamber as:

$$\frac{\partial^2 T}{\partial \xi^2} - U \frac{\partial T}{\partial \xi} = \frac{\partial T}{\partial t}$$

(5.1)
As discussed in Chapter 3, the temperature profile ahead of the interface can be derived from the above equation for the case of pseudo steady-state expansion, and is given by:

\[
\frac{T - T_r}{T_s - T_r} = e^{-\frac{\xi U}{a}}
\]

(5.2)

where \(T_s\) and \(T_r\) are the SAGD chamber temperature and the initial reservoir temperature, respectively, \(U\) is the interface velocity measured normal to the chamber edge, \(\alpha\) is the thermal diffusivity of the reservoir (which is defined as the ratio of the thermal conductivity and specific heat capacity of the reservoir), and \(\xi\), is the distance measured from the chamber edge in the direction normal to it. The above relation is obtained from the following boundary conditions: at the edge of the SAGD chamber, \(\xi = 0\), \(T = T_s\) and as \(\xi \to \infty\), then \(T \to T_r\). Inside the SAGD chamber, steam vapour and non-condensable gas coexist in the gas phase and thus the chamber temperature is dictated by the steam partial pressure, which is a function of the mole fraction of the injected non-condensable gas according to the Dalton’s Law:

\[
p_s = (1 - y_g) p_i
\]

(5.3)

where, \(p_s\) is the steam partial pressure, \(p_i\) is the total injection pressure and \(y_g\) is the mole fraction of the injected non-condensable gas. Inside the SAGD chamber the mole fraction of the non-condensable gas is assumed to correspond to the injected gas composition. However, in the mobile zone beyond the chamber edge, as the temperature decreases below the chamber temperature, the partial pressure of the saturated steam also begins to drop. Accordingly, from Equation (5.3) this implies that the mole fraction of steam in the gas phase, \((1 - y_g)\), will also decrease. As a result, the mole fraction of the non-
condensable gas in the gas phase will continuously increase, which will create a concentration gradient near the condensing surface. This concentration gradient would in turn lead to counter-current diffusion of the non-condensable gas back into the SAGD chamber, which will in-effect limit its further accumulation near the edge due to steam condensation. Thus, the movement of the non-condensable gas in the mobile zone depends on its rate of counter-current diffusion in the gas phase versus the rate of steam condensation. As long as the rate of counter-current diffusion remains less than the rate of steam condensation, non-condensable gas concentration in the gas phase will continue to increase, while it is being continuously injected. However, once diffusion becomes the dominating effect, non-condensable gas concentration in gas phase will not increase any further and the gas phase saturation beyond that point would drop to zero. This mechanism is schematically explained in Figure 5.3 and further mathematically described below.

In the reference frame of the chamber edge, which is translating at a constant velocity $U$, the rate of change of a physical quantity with time will be directly proportional to its rate of change with respect to distance from the chamber edge and the chamber velocity. For example, the rate of change of non-condensable gas mole fraction in the gas phase ($y_g$) with respect to distance from the SAGD chamber, can be written as (assuming temperature inside the chamber is constant and hence $p_s$ is constant):

$$\frac{dy_g(\xi, t)}{dt} = \frac{\partial y_g}{\partial \xi} \frac{d\xi}{dt} + \frac{\partial y_g}{\partial t} \frac{dt}{dt}$$

(5.4)

Since it was assumed that the SAGD chamber is undergoing pseudo steady-state
Figure 5.3: Schematic of the mobile zone beyond the vapour chamber of a non-condensable gas aided SAGD process.
expansion, Equation (5.4) becomes:

\[
\frac{dy_g}{dt} = -\frac{\partial y_g}{\partial \xi} U
\]

\[
\Rightarrow \frac{dy_g}{dt} \propto \frac{\partial y_g}{\partial \xi}
\]

The rate of steam condensation can be obtained by evaluating the rate of change of steam partial pressure in the mobile zone. The temperature profile derived in Equation (5.2) was used to evaluate the steam partial pressure profile using the following pressure-temperature correlation for saturated steam (Butler, 1997a):

\[
\log_{10} p_s = 9.8809 - \left[ 2.42223 + \frac{326.57}{T + 273.15} \right]^2
\]

(5.6)

In gravity drainage processes such as SAGD, gravity, rather than pressure gradient, is the major driving mechanism for fluid production and thus the operating pressures are often kept close to the reservoir pressures. Consequently pressure gradients across the edge of the chamber are small, typically less than 5 kPa/m. This was also seen in numerical simulations. Thus within the mobile zone, for a fully developed SAGD chamber, pressure can be assumed to be constant. Using Equations (5.3) and (5.6), the rate of non-condensable gas accumulation near the chamber walls, due to steam condensation, can be evaluated as:

\[
\frac{dy_g}{d\xi} = -\frac{\partial}{\partial \xi} \left( \frac{p_s}{p_i} \right)
\]

(5.7)
The non-condensable gas concentration gradient created due to steam condensation leads to its counter-current diffusion towards the chamber. This process can be quantitatively described by the Fick’s first law, which in the absence of convection, can be written as:

\[
\frac{\partial C_{\text{gas}}}{\partial t} = D_{\text{gas,eff}} \frac{\partial^2 C_{\text{gas}}}{\partial x^2}
\]  \hspace{1cm} (5.8)

where, \( D_{\text{gas,eff}} \) is the effective diffusion coefficient in the gas phase that includes the contribution from molecular diffusion as well as hydrodynamic dispersion (Perkins and Johnston, 1963), and \( C_{\text{gas}} \) is the molar concentration (mol/m\(^3\)) of the non-condensable gas in the gas phase. By transforming the frame of reference to the moving chamber interface and by assuming that the system is undergoing pseudo steady-state expansion, diffusion equations can be derived in a similar fashion as the temperature equations (Chapter 5, Equation (4.9)) to give:

\[
U \frac{\partial C_{\text{gas}}}{\partial \xi} = -D_{\text{gas,eff}} \frac{\partial^2 C_{\text{gas}}}{\partial \xi^2}
\]  \hspace{1cm} (5.9)

Since the non-condensable gas concentration increases beyond the chamber edge, its diffusion would be directed towards the SAGD chamber and will oppose its further accumulation at the interface due to steam condensation. The diffusion coefficient in the gas phase is a function of temperature, reservoir porosity, and turtuosity, as shown in Table 5.1, and hence varies with the distance from the chamber edge. Equation (5.9) was discretized using finite difference approximation and solved for small incremental distance steps \((0, \Delta \xi \ldots j\Delta \xi, (j+1)\Delta \xi \ldots)\) to give:

\[
C_{\text{gas}}^{j+1} = C_{\text{gas}}^j e^{-\frac{U\Delta \xi}{D_{\text{gas,eff}}}}
\]  \hspace{1cm} (5.10)
Table 5.1: List of correlations used in the analytical model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlation</th>
<th>Constants</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of Athabasa bitumen</td>
<td>$\frac{\nu_o}{\nu_s} = \left( \frac{T - T_r}{T_s - T_r} \right)^m$</td>
<td>$m=3.5$</td>
<td>Butler(1985)</td>
</tr>
<tr>
<td></td>
<td>$\nu_o$ in m$^2$/s, $T$ in °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>$\log \mu_g = A \left( \frac{1}{T} - \frac{1}{B} \right)$</td>
<td>$A=57.6$</td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>$\mu_g$ in cp, $T$ in K</td>
<td>$B=114.14$</td>
<td></td>
</tr>
<tr>
<td>Viscosity of saturated water</td>
<td>$\mu_w = \left( \frac{969 - T}{25.6T + 405.7} \right)$</td>
<td></td>
<td>Gottfried (1965)</td>
</tr>
<tr>
<td></td>
<td>$\mu$ in cp, $T$ in °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity of gas-oil mixture</td>
<td>$\ln(\mu_{\text{og}}) = (1-x_g) \ln(\mu_g) + x_g \ln(\mu_g)$</td>
<td></td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>$\mu$ in cp, $x_g$ in mole fraction</td>
<td></td>
<td>CMG (2009)</td>
</tr>
<tr>
<td>Solvent solubility</td>
<td>$K$-value = $\left( k_{v1} + k_{v2} P_i + k_{v3} \right) e^{\frac{k_{v4}}{T+k_{v5}}}$</td>
<td></td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>$T$ in °C, $P_i$ in kPa</td>
<td></td>
<td>CMG (2009)</td>
</tr>
<tr>
<td>Diffusion coefficients</td>
<td>$D_{g,oil}(T^\circ C) = D_{g,oil}(20^\circ C) \left[ \frac{T + 273.15}{20 + 273.15} \right]^{0.545}$</td>
<td></td>
<td>Jamialahmadi et al. (2006)</td>
</tr>
<tr>
<td></td>
<td>$D_{g,oil}(20^\circ C) = 3 \times 10^{-6}$</td>
<td></td>
<td>Das and Butler (1996)</td>
</tr>
<tr>
<td></td>
<td>$D_{g,oil}(T^\circ C) = D_{g,oil}(20^\circ C) \left[ \frac{T + 273.15}{20 + 273.15} \right]^{0.545}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{g,oil}(20^\circ C) = 1 \times 10^{-8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{\text{gas,eff}} = \frac{D_{g,oil} \phi}{\tau} + \alpha_t U$</td>
<td>$\tau=\sqrt{2}$</td>
<td>Butler and Yee (1986)</td>
</tr>
<tr>
<td></td>
<td>$D_{\text{oil,eff}} = \frac{D_{g,oil} \phi}{\tau} + \alpha_t U$</td>
<td>$\alpha_t=1 \times 10^{-4}$</td>
<td>Perkins and Johnston (1963)</td>
</tr>
</tbody>
</table>
The rate of change of non-condensable gas concentration in the gas phase due to diffusion can then be obtained by differentiating the above relation:

\[
\frac{dC^j_{\text{gas}}}{d\xi} \bigg|_{\Delta\xi \to 0} = -\frac{C^j_{\text{gas}} U}{D^j_{\text{gas,eff}}} \tag{5.11}
\]

In order to compare the rate of change of gas concentration derived in Equation (5.11) with the rate of gas accumulation derived in Equation (5.7), the molar concentration units \((C_{\text{gas}})\) were converted to mole-fraction units using the ideal gas law. When the rate of counter-current diffusion becomes greater than the rate of steam condensation, the non-condensable gas accumulation near the chamber edge stops and the gas phase saturation beyond that distance becomes zero. The distance measured from the chamber edge at which the non-condensable gas concentration in the gas phase peaks, and beyond which the gas phase saturation becomes zero, would be referred to as \(\xi^*\) (see Figure 5.3).

The partitioning of the non-condensable gas between the oil and gas phases is obtained by assuming that instantaneous phase equilibrium is realized. At the condensing interface, the mole fraction of non-condensable gas that dissolves into the oil phase is determined by the equilibrium \(K\)-values defined as:

\[
K = \frac{y_g}{x_g} \tag{5.12}
\]

where \(y_g\) and \(x_g\) are the mole fractions of the non-condensable gas in the gaseous and oleic phases, respectively. These equilibrium \(K\)-values are often correlated empirically or theoretically as a function of pressure and temperature (Reid, Prausnitz, and Sherwood, 1977):
\[ K = \left( \frac{k_{v1}}{p_i} + k_{v2}p_i + k_{v3} \right) e^{\frac{k_{v4}}{T}} \]  (5.13)

where \( T \) is the temperature, \( p_i \) is the pressure and \( k_{v1} \) through \( k_{v5} \) are the constant coefficients. For example, for methane dissolution in oil, the values of these constants are \( k_{v1} = 5.4 \times 10^5, \ k_{v2} = k_{v3} = 0, \ k_{v4} = -879.84, \ k_{v5} = -265.99, \) (CMG, 2009) when the temperature is expressed in °C and pressure is in kPa.

In the mobile zone, the total pressure remains almost constant and the major impact on the solubility, therefore, results from the temperature profile derived from Equation (5.2). In the mobile zone as the temperatures declines, the solubility of non-condensable gas in oil as well as its mole fraction in the gas phase continues to increase up to the distance \( \xi^* \), beyond which the gas phase saturation becomes zero (Figure 5.3). Beyond \( \xi^* \), i.e., for \( \xi > \xi^* \), the non-condensable gas dissolved in oil further mixes with the oil phase by dispersive and diffusive transport. This process can be quantitatively described in a similar fashion as the diffusion in the gas phase (Equation (5.8)) as:

\[ U \frac{\partial C_{oil}}{\partial \xi} = -D_{oil,eff} \frac{\partial^2 C_{oil}}{\partial \xi^2}, \text{ for } \xi > \xi^* \]  (5.14)

where, \( C_{oil} \) is the molar concentration of the injected non-condensable gas in the oil phase, and \( D_{oil,eff} \) is the effective diffusion coefficient in the oil phase, that includes the contribution from molecular diffusion as well as hydrodynamic dispersion (Perkins and Johnston, 1963). The molar concentration of the non-condensable gas in oil is related to the mole fraction of the non-condensable gas in the oil phase, through the density of the gas-oil solution.
The molecular diffusion coefficient is a function of temperature, viscosity, molecular weight, and gas concentration. However, in the model developed in this chapter, the variation of $D_{oil,eff}$ with the non-condensable gas concentration was assumed to be negligible. This assumption seems justified for the following reasons: 1) molecular diffusion coefficient of a light gas (methane in this case) is much higher than that of bitumen, 2) the dependence of $D_{oil,eff}$ on temperature is much stronger than on dissolved gas concentration, and 3) for the ease of modelling, as gas concentration itself depends nonlinearly on temperature and its inclusion will increase the complexity of the analytical model, without significant benefit. The effect of concentration on diffusion coefficient is studied in Chapter 6, for the case of steam-solvent co-injection processes. The equations used to calculate the overall diffusion coefficients are summarized in Table 5.1.

In the mobile zone, the oil saturation ranges from the residual oil saturation at the edge of the chamber and increases up to the original oil saturation deeper into the cold reservoir. As a first approximation, the oil saturation was assumed to vary linearly with the normalized temperature, as done in Chapters 3 and 4 (Sharma and Gates, 2011a and 2010a):

$$S_o = (1 - T^*) (1 - S_g - S_{wr}) + S_{or}$$  \hspace{1cm} (5.15a)

The dimensionless temperature $T^*$ used in Chapters 3 and 4 was defined for the entire temperature range beyond the edge of the SAGD chamber ($T_e \leq T \leq T_s$). However, experiments and field observations have shown that below 90 °C, heavy oil and bitumen remain practically immobile. To account for this effect, the normalized temperature in the
above formulation was modified to model temperature range existing within the mobile region, which would roughly translate to temperature from $T = T_s$ to $T = T_s/2$, that is:

$$T^* = \left( \frac{T_s - T / 2}{T_s - T_s / 2} \right)$$  \hspace{1cm} (5.15b)

The above approximation is supported by the observations from fine-grid numerical simulation study done in this work, as well as simulation studies done by Ito and Suzuki (1996), which show that in the mobile zone (temperatures ranging from $T_s$ to $\sim 0.5T_s$), the oil saturation increases linearly with decreasing temperature.

Inside the chamber, the oil saturation will be close to the residual oil saturation, $S_{or}$, and the water saturation can be approximated by the irreducible water saturation, $S_{wr}$. With the above assumptions, the gas saturation within the chamber, $S_{gi}$, can be written as:

$$S_{gi} = 1 - S_{or} - S_{wr} \hspace{1cm} (5.16)$$

Beyond the chamber edge, as the temperature drops and steam condensation begins, the gas saturation continuously decreases below $S_{gi}$, until it drops to zero at $\xi^*$. Thus from $\xi=0$ to $\xi=\xi^*$, the gas saturation can be assumed to vary linearly from $S_g=S_{gi}$ to $S_g=0$ (see Figure 5.3) and written as:

$$S_g = S_{gi} \left( 1 - \frac{\xi}{\xi^*} \right) \hspace{1cm} (5.17)$$

Water saturation beyond the edge of the chamber can then be evaluated using the oil and gas saturations derived above:

$$S_w = 1 - S_g - S_o \hspace{1cm} (5.18)$$
The movement of oil, water and gas phases in the reservoir is controlled by their respective phase mobility, $\lambda$, defined as the ratio of the effective permeability and viscosity, $\mu$, of that phase:

$$\lambda = \frac{k \cdot k}{\mu}$$

(5.19)

The relative permeability, $k_r$, in turn, depends strongly on the phase saturations. The multiphase flow of gas, oil, and condensate in the mobile zone necessitates the use of three-phase relative permeability. In this work, the relative permeability was modeled using the Naar, Wygal, and Henderson (1962) equations that were derived for the case of three phase imbibitions. Their equations were based on the concept of flow in straight capillaries with the added complication of random interconnections between pores, and of storage capacity, which allows possible phase trapping. The final equations are integrals of $dS_w/p_c^2$, where $p_c$, is the capillary pressure. By assuming $1/p_c^2$ to be linear with $S_w$, the final relative permeability equations were derived (Naar and Wygal, 1961; Naar, Wygal, and Henderson, 1962). Their original equations were modified in this work to account for the endpoint relative permeability effects, resulting in the following set of equations:

$$k_{ro} = k_{ro,0} \frac{S_o^3(1 - S_g + S_w - 2S_{wr})}{(1 - S_{wr})^4}$$

(5.20a)

$$k_{rw} = k_{rwo} \frac{(S_w - S_{wr})^4}{(1 - S_{wr})^4}$$

(5.20b)

$$k_{rg} = k_{rgw} \frac{S_g^3(2 - S_g - 2S_{wr})}{(1 - S_{wr})^4}$$

(5.20c)
The viscosities for oil, water, gas and their mixtures were modeled using the correlations listed in Table 5.1. For example, following Butler (1985) the dependence of kinematic viscosity of oil on temperature was modeled using the following relation:

\[
\frac{\nu_o}{\nu_s} = \left( \frac{T - T_r}{T_s - T_r} \right)^m
\]  

(5.21)

Thus by knowing the viscosity and relative permeability values, the mobility of each phase can be evaluated using Equation (5.19). From the mobility profiles, oil flow rate in the presence of non-condensable gas can be evaluated; this is discussed in Section 5.3.4.

5.3.3 Algorithm for Generating Fluid Flow Profiles

Methane was selected as the non-condensable gas for co-injection with steam in SAGD. The fluid flow profiles are generated using the equations discussed in Section 5.3.2 and the reservoir and fluid properties listed in Table 5.2. The calculation steps outlined below can be performed by formulating discretized steps (0, Δξ, 2Δξ, .. jΔξ,..) in a numerical computing software such as Matlab or Microsoft Excel. The calculation procedure is summarized in the flowing steps (also see Figure 5.3 for reference):

1) Generate the temperature profile, \( T \), as a function of distance, \( \xi \), using Equation (5.2) and the corresponding steam partial pressure profile, \( p_s \), vs. \( \xi \), using Equation (5.6).

2) Generate methane mole fraction profile in the gas phase, \( y_g \), vs. \( \xi \), using Equation (5.3) and the pressure profile \( (p_s) \) derived in Step–1.
Table 5.2: Reservoir and fluid properties used in the analytical and simulation models (Ito and Suzuki, 1996; Ito and Hirata, 1999; Butler, 1997a).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$, °C</td>
<td>5</td>
</tr>
<tr>
<td>$T_s$, °C</td>
<td>220</td>
</tr>
<tr>
<td>$\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>$k$, m$^2$</td>
<td>5x10$^{-15}$</td>
</tr>
<tr>
<td><strong>Heterogeneous reservoir</strong></td>
<td></td>
</tr>
<tr>
<td>$\phi$ (average)</td>
<td>0.33</td>
</tr>
<tr>
<td>$k$ (average), m$^2$</td>
<td>5x10$^{-15}$</td>
</tr>
<tr>
<td>$\alpha$, m$^2$/s</td>
<td>7x10$^{-1}$</td>
</tr>
<tr>
<td>$S_{iw}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>0.19</td>
</tr>
<tr>
<td>$\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
<tr>
<td>$U$, cm/day</td>
<td>20</td>
</tr>
<tr>
<td>Injected methane mole fraction, $y_i$</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial reservoir pressure, kPa</td>
<td>2000</td>
</tr>
<tr>
<td>Injection pressure, $p_i$, kPa</td>
<td>2363</td>
</tr>
</tbody>
</table>
[2b] Using the methane mole fraction profile evaluated in Step−2a, calculate the rate of methane accumulation due to steam condensation:

\[
\left. \frac{dy_g}{d\xi} \right|_j = \frac{y_g|_{i+1} - y_g|_j}{\Delta \xi}
\]

(5.22)

[3a] Evaluate the effective diffusion coefficient of methane in the gas phase, \(D_{\text{gas,eff}}\), using the temperature profile from Step−1 and the correlation given in Table 5.1.

[3b] Calculate the rate of diffusion of methane in the gas phase using Equations (5.10) and (5.11). Convert the molar concentration of methane in gas phase, to methane mole fraction units, using the ideal gas law.

[4] Compare the rate of methane diffusion and methane accumulation in gas phase, to determine the distance \(\xi^*\), where the rate of diffusion becomes greater than the rate of methane accumulation. The diffusion and accumulation rates for the analytical model developed in this study are compared in Figure 5.4. The plot reveals that \(\xi^*\sim 17\) cm from the SAGD chamber edge.

[5] Assuming a constant pressure (equal to the injection pressure, \(p_i\)) in the mobile zone and temperature profile from Step−1, evaluate the \(K\)-values using Equation (5.13). Calculate the mole fraction of methane dissolved in the oil phase, \(x_g\), using the \(K\)-values, Equation (5.12), and \(y_g\) from Step−2.

[6a] Evaluate the effective diffusion coefficient of methane in the oil phase, \(D_{\text{oil,eff}}\), using the temperature profile from Step−1 and the correlations in Table 5.1.
Figure 5.4: Comparison of the rate of methane diffusion and the rate of methane accumulation in the gas phase.
[6b] Generate the methane concentration profile in the oil phase beyond $\xi^*$ using Equation (5.14) and $D_{oil,eff}$ from Step–6a:

$$C_{oil}^{j+1} = C_{oil}^{j} e^{\frac{U_\xi}{D_{oil,eff}}}, \xi > \xi^*$$  \hspace{1cm} (5.23)

[7a] Calculate the gas saturation inside the chamber, $S_{g'}$, using Equation (5.16) and gas saturation in the mobile zone, $S_g$ vs. $\xi$, using Equation (5.17), where $\xi^*$ is from Step–4.

[7b] Evaluate the oil saturation profile in the mobile zone using Equation (5.15), and the water saturation profile using Equation (5.18).

[8] Calculate the oil, water, and gas phase relative permeability values from Equations (5.20a), (5.20b) and (5.20c), respectively.

[9] Evaluate the water, methane, and oil phase viscosity in the mobile zone, using the correlations in Table 5.1, temperature profile from Step–1 and methane concentration in oil phase evaluated in Step–5 and Step–6. Note that the oil phase viscosity will depend on both the temperature and the dissolved methane concentration.

[10] Calculate the oil, water, and gas phase mobility profiles using Equation (5.19).

5.3.4 Oil Flow Rate Calculation

The flow of oil can be determined by examining the material balance about a differential element located at the SAGD chamber interface as shown in Figure 5.5. The drainage rate of oil for a section of thickness $d\xi$ and length $L$ (where $L$ is the well length, going into the paper) is given by Darcy’s law for gravity flow as:

\[ Q = -\frac{k}{ho_g \mu_g} \frac{dP}{dx} \]
Figure 5.5: Vertical section of a SAGD chamber interface.
\[
dq = \frac{(d\xi \times L)k_o \Delta \rho g \sin \theta}{\mu_o}
\]

(5.24)

where \(g\) is the acceleration due to gravity, \(\theta\) is the local inclination of the interface.

Because the density of steam vapour is much less than that of oil, that is, \(\rho_v \ll \rho_o\), we can substitute \(\Delta \rho\) by \(\rho_o\) in the above equation to give:

\[
dq = \frac{Lk_o \rho_o g \sin \theta}{\mu_o} d\xi
\]

(5.25)

Following Butler et al. (1981), the dependence of kinematic viscosity on temperature was modeled using Equation (5.21), which can be combined with Equation (5.2) to give the following relationship between kinematic viscosity and distance beyond the edge of the chamber:

\[
\nu_o = \nu_s e^{\left(\frac{mU_s}{\alpha}\right)}
\]

(5.26)

The oil flow rate can be re-written using the above relations as:

\[
dq = \frac{Lk_o \rho_o g \sin \theta}{\nu_s} e^{\left(\frac{mU_s}{\alpha}\right)} d\xi
\]

(5.27)

Effective permeability of oil, \(k_o\), is a function of oil, water and gas phase saturations (Equations (5.20a), (5.20b) and (5.20c)) which in turn depend on the distance from the edge of the SAGD chamber, \(\xi\) (from Equations (5.15), (5.17) and (5.18)) making \(k_o\) a function of \(\xi\). Thus, Equation (5.27) can be integrated to give:

\[
q = \frac{Lk_o g \sin \theta}{\nu_s} \int_{\xi=0}^{\xi=\infty} k_o e^{\left(\frac{mU_s}{\alpha}\right)} d\xi
\]

(5.28)
where \( k \) is the absolute permeability of the reservoir and \( k_{ro} \) is the relative permeability of the oil phase. From a material balance around the differential element displayed in Figure 5.5, the amount of oil flowing into the element is equal to the sum of oil flowing out of the element and the accumulation of oil volume within the element in time \( dt \). If the oil phase density does not vary significantly, the mass balance around the differential slice becomes:

\[
q_{\eta+d\eta} \, dt = q_{\eta} \, dt + (\text{Vol}|_{t+dt} - \text{Vol}|_{t})
\]  

(5.29)

where \( \eta \) corresponds to the direction tangential to the flow direction along the chamber interface. The volume of bitumen in the differential element is given by:

\[
\text{Vol} = \phi \Delta S_\eta L d\xi d\eta
\]  

(5.30)

Substituting Equation (5.30) into (5.29) and re-arranging gives:

\[
\frac{\partial q}{\partial \eta} = \frac{\phi \Delta S_\eta L}{\partial t} \frac{\partial \xi}{\partial t}
\]  

(5.31)

From geometry considerations, \( U = \partial \xi/\partial t = \cos\theta \partial z/\partial t = \sin\theta \partial x/\partial t \) and \( d\eta = dx/\cos\theta \). After substituting these relations into Equation (5.31) and rearranging, the result is:

\[
\frac{U}{\sin\theta} = \cot\theta \frac{\partial q/\partial x = \partial q/\partial z}{\phi \Delta S_\eta L}
\]  

(5.32)

Substituting Equation (5.32) in Equation (5.28) gives:

\[
qdq = \frac{UL^2 k \phi \Delta S_\eta}{v_s} \left[ \int_{\xi=0}^{\xi=\pi} k_{ro} e^{-\frac{mU\xi}{\alpha}} d\xi \right] d\xi
\]  

(5.33)

which can be integrated in the \( z \) direction, over the height of the chamber above the injector, \( h \), to give the oil flow rate down the side of the chamber:
\[
\frac{q^2}{2} = \frac{UL^2 h_{kg} \phi \Delta S_o}{v_s} \left[ \int_{\xi=0}^{\xi=\xi_0} k_m e^{-\frac{m U \xi}{\alpha}} d\xi \right]
\]

(5.34)

The above relation can be simplified to give the total oil flow rate from both sides of the chamber as:

\[
q = 2L \sqrt{\frac{2 \alpha h_{kg} \phi \Delta S_o}{m v_s} \left[ \int_{\xi=0}^{\xi=\xi_0} k_m e^{-\xi \frac{m U \xi}{\alpha}} d\xi \right]}
\]

(5.35a)

where,

\[
\tilde{\xi} = \left( \frac{m U \xi}{\alpha} \right)
\]

(5.35b)

The non-condensable gas aided SAGD model derived in this chapter, reduces to the multiphase flow model (derived in Chapter 3), when the gas phase saturation in the mobile zone and injected non-condensable gas mole fraction \(y_i\) are assumed to be zero.

### 5.4 Numerical Simulation

In order to test the results from the analytical model, the case of steam-methane co-injection in SAGD was simulated on a homogeneous reservoir model, where, 0.1 mole fraction of methane was continuously co-injected with 0.9 mole fraction steam of steam quality 1 (or 100%). The mole fraction of the injected methane is the same as that used by Butler et al. (2000), in their initial SAGP experiments with methane as the non-condensable gas. The fluid and reservoir model properties for the homogeneous simulation model are discussed in detail in Appendix A, Section A.3.1.
In order to get a visual understanding of the effect of non-condensable gas on SAGD performance, in terms of its impact on fluid saturations, in-situ steam quality, overburden heat losses, and fluid flow rate, steam-methane co-injection was also simulated on a heterogeneous reservoir model which is described in Appendix A, Section A.3.2 (also see Figure A.1).

Three cases were studied on the heterogeneous model; the Base Case was a SAGD operation with steam-alone injection. This was compared with two scenarios of steam-methane co-injection; in Case-A, the temperature in the steam-methane chamber was maintained the same as the steam chamber in the Base Case by increasing the injection pressure, and in Case-B the injection pressure was kept the same as the SAGD Base Case. This was done in order to differentiate the effect of non-condensable gas on temperature reduction (due to steam partial pressure reduction) from other effects caused by the presence and movement of non-condensable gas in the SAGD chamber. In both, Case-A and Case-B, 0.1 mole fraction of methane was continuously co-injected with 0.9 mole fraction steam of quality 1 (or 100%) from the horizontal injector. The operating conditions for all the different cases are summarized in Table 5.3.

The numerical simulator was programmed to include the effects of molecular diffusion in the oleic and the gaseous phases, hydrodynamic dispersion, and overburden heat losses (in the heterogeneous model). The dependence of molecular diffusion on temperature and viscosity was modelled using the same relationship which was used in the analytical model (as summarized in Table 5.1). In all the cases, reservoir oil did not have any initial solution gas content. The rock and fluid properties were similar to a
Table 5.3: Operating conditions used in the numerical simulations.

<table>
<thead>
<tr>
<th></th>
<th>Homogeneous Model</th>
<th>Heterogeneous Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Operation conditions</strong></td>
<td><strong>Steam-Methane co-injection</strong></td>
<td><strong>SAGD</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Injection pressure, (p_i), kPa</strong></td>
<td>2363</td>
</tr>
<tr>
<td></td>
<td><strong>Injected methane mole fraction, (y_i)</strong></td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td><strong>SAGD chamber temperature, (T_s), °C</strong></td>
<td>215.5</td>
</tr>
</tbody>
</table>
typical heavy oil reservoir as summarized in Table 5.2. The details of the simulation models are given in Appendix A. The fluid distribution and flow rates were analyzed after 30 hours (or 1.25 days) of steaming, which on scaling up using Butler’s (1985) scaling parameter translates to 8 years of SAGD operation. The results from simulations and analytical model were compared for qualitative and quantitative similarity.

5.5 Results and Discussion

5.5.1 Analytical Model

The results from the analytical model are presented in Figure 5.6 (a through e). These were obtained from the equations developed in Section 5.3, using the reservoir and fluid properties summarized in Table 5.2. Figure 5.6a displays the temperature and dynamic viscosity profiles calculated from Equations (5.2) and (5.21), respectively, for parameters listed in Table 5.2. Distance is measured from the edge of the chamber which is defined at \( \xi = 0 \). The results show that the conductive heat transfer length scale is about 1 m. The rate of chamber expansion, \( U \), was the average chamber expansion rate observed in the numerical simulation of the homogeneous model (\(~20\) cm/day) and is of the order of typical SAGD chamber expansion rates observed in the field (Butler, 1987; Gotawala and Gates, 2008). At the edge of the chamber, the oil viscosity is low, less than 10 cp, whereas it rises to about 100,000 cp at 1 m distance from the chamber edge. This viscosity profile would suggest that the most mobile oil is that which is at the chamber edge. However, this is not the case due to relative permeability effects.
Figure 5.6: Analytical model results (a) Temperature & viscosity profiles (b) Phase saturation profiles (c) Relative permeability profiles (d) Methane mole fraction in the gas phase (e) Methane mole fraction in the oil phase (f) Phase mobility profiles.
Figure 5.6b plots the saturation profiles for the oil, gas, and water phases, derived from Equations (5.15), (5.17), and (5.18), respectively. Oil saturation increases from residual oil saturation within the chamber to the initial oil saturation, beyond the mobile zone. Gas saturation declines linearly beyond the chamber edge until the rate of countercurrent diffusion of methane becomes greater than its rate of accumulation in the gas phase (at $\xi^* \approx 17$ cm, Figure 5.4), as described in Section 5.3.3. Water saturation is close to the connate water saturation within the chamber and farther in the colder oil sands. However, in the mobile zone, as the temperature drops below the SAGD chamber temperature, water saturation increases due to the condensation of steam. Because the gas phase saturation is decreasing while the oil phase saturation is increasing with distance, water saturation has a maximum within the mobile zone, at a distance of about 15 cm from the edge of the chamber.

Figure 5.6c displays the relative permeability profiles that result from Equations (5.20a), (5.20b), and (5.20c). The gas phase has the highest relative permeability near the edge of the chamber as the oil saturation is low and water saturation is close to the irreducible water saturation near the chamber edge. As the oil phase saturation increases its relative permeability also increases, until the endpoint relative permeability value, $k_{roCW}$ is reached. The water phase relative permeability is considerably lower than that of the oil and gas phases, since the reservoir is water wet and water saturation is only slightly higher than the irreducible water saturation in the mobile zone. Water phase relative permeability follows the water saturation profile and also has a maximum within the mobile zone.
Figure 5.6d displays the methane mole fraction profile in the gas phase as a result of steam condensation near chamber walls. The highest methane mole fraction in the gas phase is about 0.8, which occurs at a distance of about 17 cm (=\(\xi^*\)) from the chamber edge. Beyond \(\xi^*\), the gas phase saturation and hence methane mole fraction in gas phase drops to zero (as explained in Section 5.3 and Figures 5.3 and 5.4). Up to the distance \(\xi^*\), the amount of methane that dissolves in oil is given by Equation (5.12). Beyond \(\xi^*\), methane further mixes with the adjacent oil phase by diffusion and dispersion and the resulting concentration profile is given by Equation (5.23). Figure 5.6e displays the overall methane mole fraction profile in the oil phase. The result reveals that the active mass transfer length scale is about 22 cm in extent, with the majority of the methane existing within the first 20 cm of the edge of the chamber. This is about an order of magnitude smaller than the heat transfer length scale. Figure 5.6e shows that the maximum solubility of \(~0.03\) mole fraction of methane in oil phase occurs at a distance of 17 cm (=\(\xi^*\)) from the chamber edge.

Mobility profiles for the three phases are displayed in Figure 5.6f. It is evident that the mobile zone, which was defined in Section 5.3 as the region where multiphase flow of oil, water, and gas takes place, extends from the edge to the chamber to approximately 0.6 m beyond it. Mobility depends on the phase viscosity and relative permeability by Equation (5.19). Figures 5.6a and 5.6c show that the viscosity and relative permeability of the oil phase increase with distance beyond the chamber edge. Oil mobility, being the ratio of these two effects, therefore shows a maximum some distance away from the edge of the chamber (at a distance of \(~14\) cm), and not at the
chamber edge, where oil has minimum viscosity. Similarly, water phase mobility also has a maximum within the mobile zone. It is clear that the resulting water mobility profile exhibits zero phase mobility at the chamber edge and deeper in the cold reservoir because at either places, water phase is at irreducible water saturation condition. Figure 5.6f also shows that the gas phase mobility is much higher than the water and oil phase mobilities.

5.5.2 Homogeneous Reservoir Simulation

The results from the analytical theory were compared with the numerical simulation results of steam-methane co-injection in SAGD, using a homogeneous reservoir model with no overburden heat losses. Fluid and rock properties used in the simulation were similar to the analytical model (as summarized in Table 5.2). It must be noted that the analytical theory does not account for the effect of chamber wall inclination for generating the fluid flow profiles, while the simulation results depend on the inclination of the growing chamber. Therefore, to compare the two data, the simulation results for the flow profiles generated midway across the chamber were compared with the analytical results, to average out the inclination effect. This roughly translated to profile across grid block [1-1-27] to grid block [1-400-27] expressed in their respective $x$, $y$, $z$, coordinates. This may, however, limit the ability of the analytical theory to exactly match the values in the simulation results.

The simulation results are displayed in Figures 5.7 through 5.10. Figure 5.7a displays the two-dimensional distribution of temperature in the reservoir and Figure 5.7b
Figure 5.7: Homogeneous reservoir simulation (a) Temperature distribution (b) Temperature profile versus distance from the center of the well pair.
plots the lateral temperature profile after 30 hours of steaming (which is equivalent to 8 years of SAGD operation, on scaling up). The edge of the chamber can be located on the temperature profile as roughly at a distance of 0.17 m from the well pair (where the temperature starts declining below the SAGD chamber temperature). Temperature profile in Figure 5.7b shows a monotonically decreasing trend beyond the chamber edge, with a length scale of 0.8-1 m. Maximum temperature is ~215.5 °C, corresponding to a steam partial pressure of 2127 kPa inside the chamber. The results are similar to the analytical temperature profile displayed in Figure 5.6a.

Oil saturation distribution in the reservoir in displayed in Figure 5.8a while oil, water, and gas phase saturation profiles, as a function of distance from the well pair, are displayed in Figure 5.8b. On comparing the simulation results with the corresponding results from the analytical model (Figure 5.6b) we see that the profiles are qualitatively and quantitatively similar. In the mobile zone, gas saturation decreases linearly with distance beyond the chamber edge (located at ~0.17 m from the well pair). Oil saturation increases with distance from the chamber edge until it reaches the original reservoir oil saturation at a distance of about 0.3 m from the edge, where the temperature is ~100 °C or close to $T_s/2$. This justifies the assumption made in the analytical model (in Equation 5.15).

Relative permeability data generated by the simulator, displayed in Figure 5.9a, is compared with the corresponding results from the analytical model (Figure 5.6c). Similar to the analytical model, the simulator predicts much lower water phase relative permeability compared to the gas and oil phases. Also near the chamber edge, gas phase
Figure 5.8: Homogeneous reservoir simulation (a) Oil saturation distribution (b) Saturation profiles versus distance from the center of the well pair.
Figure 5.9: Homogeneous reservoir simulation (a) Three-phase relative permeability profile (b) Methane distribution in the chamber (c) Methane mole fraction in the oil and gas phases (d) Phase mobility profiles versus distance from the center of the well pair.
relative permeability is much higher than oil and water phases. The results reveal that the theory provides reasonable estimates of three phase relative permeability, in terms of both trends and values.

At the edge of the chamber, methane mixes with the oil phase as a result of dispersion, diffusion and dissolution. The resulting methane distribution in the chamber is shown in Figure 5.9b. Methane mole fraction profile in the oil and gas phases is displayed in Figure 5.9c. The results reveal that the maximum methane mole fraction in the oil phase is about 0.03, and in the gas phase is about 0.85, which occur at a distance of about 0.28 m from the edge of the chamber. These values are similar to the analytical results (Figures 5.6d and 5.6e). The results reveal that the active mass transfer length scale is about 25 cm in extent beyond the chamber edge, which is similar to the prediction of the analytical theory (Figure 5.6e).

Figure 5.9d displays the phase mobility profiles as a function of distance from the well pair. The mobile zone extends from the edge to the chamber to about 0.5 m beyond it, which is comparable to the length scale of the mobile region seen in the analytical model (Figure 5.6f). It is also evident that similar to the predictions of the multiphase theory derived in Chapter 3, Section 3.2 and the analytical theory derived here, the most mobile oil is not at the edge of the chamber but rather some distance beyond the edge.

Figure 5.10 displays the cumulative and instantaneous oil rates obtained from the simulation of the homogeneous reservoir model. The average oil production rate is about $1 \times 10^{-5}$ m$^3$/day. To calculate the analytical oil flow rate, Equation (5.35) was integrated using the relative permeability profile evaluated in Step–8 of the algorithm (Section
Figure 5.10: Homogeneous reservoir simulation: Cumulative and instantaneous oil rates for the methane co-injection case.
5.3.3) which is displayed in Figure 5.6c, and the reservoir properties and operating conditions from Table 5.2. The resulting value for oil production rate is, \( q = 1.7 \times 10^{-5} \) m\(^3\)/day. The comparison of the oil flow rate obtained from the analytical model and numerical simulation demonstrates that the analytical theory derived here reasonably estimates the oil rate when methane is co-injected with steam during SAGD.

5.5.3 **Heterogeneous Reservoir Simulation**

In this section, results from the three cases examined using the heterogeneous reservoir model are discussed. Simulation parameters for the three cases are summarized in Tables 5.2 and 5.3. Figure 5.11 shows the temperature distribution in the reservoir for the three cases after 30 hours of steaming. The highest temperature inside the SAGD chamber for the **Base Case** is 220 °C, corresponding to steam saturation pressure of 2363 kPa. For **Case-A**, the injection pressure was such that the temperature in the SAGD chamber is same as the **Base Case**. To do this steam-methane mixture was injected at a higher pressure than the **Base Case**. From Dalton’s law it would be expected that in order to have the same temperature in the chamber with 0.1 mole fraction methane, the injection pressure should be \( 2363 / 0.9 = 2626 \) kPa

However, in the simulation the injection pressure in **Case-A** was slightly higher than the above value to account for heat losses. With an injection pressure of 2790 kPa, the chamber temperature close to 220 °C was achieved.

For **Case-B**, the steam partial pressure in the injected stream is reduced to 2126 kPa when 0.1 mole fraction of methane is co-injected, according to the Dalton’s law
Figure 5.11: Heterogeneous reservoir simulation (a) Temperature distribution (Case-A) (b) Temperature distribution (Case-B) (c) Temperature distribution (Base Case) (d) Temperature profiles versus distance from the center of the well pair.
of partial pressure. Consequently, the temperature inside the chamber drops from 220 °C to 215.5 °C. Figure 5.11 shows that the size of the chamber is biggest for the Base Case and smallest for Case-B, where steam partial pressure is the lowest. The edge of the SAGD chamber is roughly located at 0.18 m from the well pair for the Base Case, 0.122 m for Case-A, and 0.102 m for Case-B, after 30 hours (1.25 days) of steaming. This means that the chamber is growing at the fastest rate for the Base Case and slowest for Case-B. For Case-A, although the chamber temperature is the same as the Base Case, the chamber is smaller. This can be explained from the presence of a “methane buffer” or high methane concentration near the edge of the chamber, which reduces the heat transfer rate from the steam vapour to the adjacent oil. The rate of temperature rise in a block located inside the SAGD chamber (grid block [1-3-27]) is shown in Figure 5.12. It is evident that the rate of temperature increase in the grid block is highest for the Base Case and lower for Case-A and Case-B. This indicates lower rate of condensation heat transfer from the injected steam due to the presence of non-condensable gas inside the chamber. This also confirms the experimental findings by Maheshwari et al. (2004, see Figure 5.1) and Othmer (1929). The lower rate of heat transfer results in lower oil mobilization and lower oil drainage rate, which retards the growth of the chamber.

As steam condenses, the volume shrinkage creates a low pressure region (Δp) that leads to an increased gas concentration at the condensing interface. Figure 5.13a and 5.13b show the distribution of methane in the reservoir in terms of methane mole fraction in the gas phase, after 30 hours of steaming, for Case-A and Case-B, respectively. Most
Figure 5.12: Rate of rise of temperature inside a reservoir grid-block located within the SAGD chamber at the co-ordinates [1-3-27].
Figure 5.13: Heterogeneous reservoir simulation (a) Methane mole fraction in the gas phase (Case-A) (b) Methane mole fraction in the gas phase (Case-B) (c) Effective methane concentration in the gas phase (Case-A) (d) Effective methane concentration in the gas phase (Case-B).
numerical simulators (including STARS), are programmed to output mole fraction of a component even for an absent phase (i.e; zero phase saturation) from the $K$-value correlations (CMG, 2009). Therefore, to get a true estimate of methane distribution, the product of gas saturation and methane mole fraction was used ($S_g y_g$), which gives the effective methane concentration ($m^3$ methane per $m^3$ core) in the gas phase. This is shown in Figures 5.13c and 5.13d. The results reveal that the highest concentration of methane exists near the edge of the SAGD chamber due to the steam condensation effect. A higher concentration is observed near the top edge of the chamber, as compared to the bottom, which is a consequence of: (1) higher heat transfer flux near the extreme corners of the chamber where steam is exposed to the overburden on one side and the cold reservoir on the other, and (2) due to the of the buoyancy effect of gas.

Many numerical simulation studies that analyze the effect of non-condensable gas in SAGD chamber show that the mole fraction of the injected non-condensable gas inside the chamber is negligible (~0) while all the gas is accumulated at the chamber edges. From a thermodynamic point of view, this is not possible, as inside the chamber, steam and gas will exist as a well-mixed mixture in a single gas phase. Thus, the mole fraction of the non-condensable gas inside the chamber would be close to the injected gas mole fraction (or higher if there are higher heat losses), and not zero. A well-mixed single gas phase inside the SAGD chamber is also observed in experiments that studied the effect of non-condensable gas in SAGD (Yuan et al., 2006). The simulations done in this work capture this effect accurately (Figure 5.13).
High steam quality is one of the most essential requirements for a successful SAGD operation (Farouq Ali, 2008; Butler, 1985). The presence of high methane concentration affects the steam quality near the condensing interface, resulting in a steam quality gradient. Figure 5.14 displays the in-situ steam quality distribution in the reservoir after 30 hours of steaming for the three cases on a log scale. The results show that when methane is injected with steam, the steam quality declines more sharply near the steam-oil interface as compared to the Base Case. This is the result of increased methane accumulation near the chamber edge, which lowers the heat transfer rate from steam to oil. The temperature, steam quality and methane concentration profiles plotted in Figures 5.15a and 5.15b clearly demonstrate the effect of methane accumulation on lowering the steam quality near the chamber edges in Case-A and Case-B.

For optimal results, SAGD requires high steam quality in the vicinity of the chamber walls to achieve maximum heat transfer and oil mobilization. Thus, the high methane concentration near the edge leads to lower heat transfer from steam to oil, resulting in lower oil mobility, as seen in Figure 5.16a. Oil phase mobility in the Base Case is almost 33% higher than Case-A and Case-B. Figure 5.16b compares the resulting oil flow rates and well bottom-hole pressures for the three cases. The results reveal that the highest oil rate is seen for the Base Case followed by Case-A and Case-B. The results reveal that the oil flow rate is reduced by almost 50% when methane is co-injected at the same injection temperature (Case-A) and by 62% when methane is co-injected at the same total injection pressure (Case-B). This is a consequence of the above-mentioned
Figure 5.14: Heterogeneous reservoir simulation (a) Steam quality (Case-A) (b) Steam quality (Case-B) (c) Steam quality (Base Case).
Figure 5.15: Heterogeneous reservoir simulation (a) Steam quality and temperature profiles (b) Temperature and methane concentration profiles versus distance from the center of the well pair.
Figure 5.16: Heterogeneous reservoir simulation (a) Oil phase mobility profile as a function of distance from the center of the well pair (b) Oil flow rate and well bottom hole pressures versus time.
disadvantages of using methane, i.e. lower heat transfer rate, reduced oil phase mobility, and lower steam quality because of the gas accumulation near the chamber edge.

A similar effect of methane accumulation on temperature and steam quality is seen near the top and bottom of the reservoir, where heat is lost to the overburden and the underburden, respectively. This is shown in Figures 5.17a and 5.17b where the temperature, steam quality and methane mole fraction profiles are displayed as a function of vertical distance from the top of the reservoir. The profile is plotted across grid blocks [1-1-1] to [1-1-50]. As steam condenses near the top of the reservoir, methane concentration increases, resulting in poor heat transfer and lower steam quality. Methane accumulation and steam quality gradient is higher for Case-B because the chamber temperature is lowest and steam starts condensing earlier.

Figure 5.18 plots the overburden and underburden heat loss for the three cases. It is evident that heat losses are highest for the Base Case and lowest for Case-B. This is a result of methane accumulation at the top of the reservoir and the resulting lower temperature, as seen in Figure 5.15b. The smaller size of the chamber in Case-A and Case-B also reduces the area exposed to the overburden. The above results explain the often-cited advantage of using non-condensable gases in SAGD for lowering overburden heat losses.

From the results of the heterogeneous reservoir simulations, it can be concluded that the accumulation of methane near the top and at the edges of the SAGD chamber has a significant impact on reducing heat transfer rate from steam to oil, resulting in significantly lower oil flow rate. Impaired oil production and slower chamber expansion
Figure 5.17: Heterogeneous reservoir simulation (a) Steam quality and temperature profiles in the vertical direction across the reservoir pay (b) Temperature and methane mole fraction profiles in the vertical direction across the reservoir pay.
Figure 5.18: Heat lost to the overburden as seen in the heterogeneous reservoir simulation.
rate results even when the chamber temperature is maintained equal to the *Base Case*, due to the adverse effect of gas accumulation at the chamber walls.

5.6 Validation of Theory with SAGP Experimental Data

Butler (1997b) proposed injection of a non-condensable gas during the SAGD process and called it “SAGP (Steam and Gas Push)”. Jiang, Butler, and Yee (1998) conducted scaled physical model experiments, using nitrogen as the non-condensable gas, and compared the results with SAGD experiments, to evaluate the effect of non-condensable gas on the overall performance. The properties of their experimental set up and the results from their experiments are summarized in Tables 5.4a and 5.4b, respectively (Jiang et al., 1998). Figure 5.19 displays the cumulative production and injection data for their SAGP experiments. From these results, they concluded that although the oil production rate could be somewhat lower for the SAGP process, the steam-oil ratio is much smaller.

The analytical theory derived in this chapter was used to predict the oil flow rate for Jiang et al.’s(1998) SAGP experiments, by using identical set of reservoir and fluid properties as used in their experiments (Tables 5.4a). The results from the analytical model are displayed in Figure 5.20. To calculate the oil production rate, Equation (5.35) was integrated using the relative permeability profile displayed in Figure 5.20c and reservoir properties and operating conditions from Table 5.4a, to give oil rate of \( q = 4 \times 10^{-5} \text{ m}^3/\text{hr} \) or roughly 40 g/hr. The analytical result is close to the average oil rate observed in the SAGP experiments (Table 5.4b). These results confirm that the theory
Table 5.4a: Reservoir and fluid properties used in the SAGP experiments (Jiang et al., 1998).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model dimensions, cm</td>
<td>35.5 x 21.6 x 3.2</td>
</tr>
<tr>
<td>( \mu_o(20^\circ C) ), cp</td>
<td>28000</td>
</tr>
<tr>
<td>( \mu_o(T_s) ), cp</td>
<td>117</td>
</tr>
<tr>
<td>( T_r, ^\circ C )</td>
<td>5</td>
</tr>
<tr>
<td>Injected steam temperature, ( ^\circ C )</td>
<td>115</td>
</tr>
<tr>
<td>SAGP chamber temperature, ( T_s, ^\circ C )</td>
<td>80</td>
</tr>
<tr>
<td>( \rho_o(T_r) ), kg/m(^3)</td>
<td>980</td>
</tr>
<tr>
<td>( p_i ), kPa</td>
<td>170.5</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.34</td>
</tr>
<tr>
<td>( k, \text{ m}^2 )</td>
<td>2.2x10(^{-10})</td>
</tr>
<tr>
<td>( \alpha , \text{ m}^2/\text{s})</td>
<td>5x10(^{-7})</td>
</tr>
<tr>
<td>( S_{io} )</td>
<td>0.9</td>
</tr>
<tr>
<td>( S_{or} )</td>
<td>0.24</td>
</tr>
<tr>
<td>Injected nitrogen mole fraction, ( y_i )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 5.4b: SAGD and SAGP experimental results (Jiang et al., 1998).

<table>
<thead>
<tr>
<th>Process</th>
<th>SAGD</th>
<th>SAGP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time, hours</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Cumulative oil produced, g</td>
<td>430</td>
<td>351</td>
</tr>
<tr>
<td>Cumulative water produced, g</td>
<td>2784</td>
<td>1590</td>
</tr>
<tr>
<td>Cumulative steam injected, g</td>
<td>3065</td>
<td>1659</td>
</tr>
<tr>
<td>Cumulative gas injected, g</td>
<td>n/a</td>
<td>3.5</td>
</tr>
<tr>
<td>Cumulative SOR, g/g</td>
<td>7.1</td>
<td>4.8</td>
</tr>
<tr>
<td>Average oil rate, g/hr</td>
<td>71.8</td>
<td>50.1</td>
</tr>
</tbody>
</table>
Figure 5.19: Production and injection data for SAGP experiments using nitrogen as the non-condensable gas (Jiang et. al., 1998).
Figure 5.20: Analytical model results generated using the SAGP experimental data given in Table 5.4a (a) Temperature and oil viscosity profiles (b) Phase saturation profiles (c) Phase relative permeability profiles (d) Nitrogen mole fraction in the oil phase.
developed in this chapter can be used for estimating oil flow rate when nitrogen is co-injected with steam in SAGD.

5.7 Conclusions
Analytical theory was derived to predict phase distribution profiles in the mobile zone and oil production rate when a non-condensable gas is co-injected with steam during a SAGD operation. The equilibrium concentration of gas accumulating at the interface as a result of steam condensation is dictated by the complex interaction between diffusion, dispersion and dissolution effects at the chamber edge. The results from the fine-grid numerical simulations support the predictions of the mathematical theory, which suggests that the theory captures the underlying mechanism of non-condensable gas behaviour in a SAGD chamber. The analytical theory was also tested via detailed comparison with SAGP experimental data, using nitrogen as the injected non-condensable gas with steam.

The results from this analytical and numerical study demonstrate that non-condensable gas co-injection with steam in SAGD is in general unfavorable. The injected non-condensable gas tends to accumulate at the steam condensation front, which lowers the rate of condensation heat transfer of steam to the adjacent oil. The result is reduced steam quality, and consequently lower oil production rate and slower growth of the chamber. Another effect of methane accumulation at the top of the chamber is the reduction in overburden heat loss. However, the reduction in oil flow rate and the final recovery would likely negate the benefit of such heat loss reduction.
Chapter 6: 

SOLVENT–AIRED SAGD PROCESS 

6.1 Introduction

In solvent-aided SAGD processes, such as Expanding Solvent Steam-Assisted Gravity Drainage (ES-SAGD), Steam Alternating Solvent (SAS), Solvent-Aided Process (SAP), and others, both steam and solvent are injected into the SAGD chamber and heating and solvent dilution mechanisms act together to lower the oil phase viscosity. The controlling feature of these processes is the mobilization and drainage of oil at the chamber edge. Furthermore, both oil phase viscosity – mobility – and density – drainage – depend strongly on the temperature and solvent concentration. This means that the solvent content, pressure, and temperature at the edge of the SAGD chamber are important because they set the amount of solvent that mixes in the oil phase, which in turn sets its viscosity. Here the ‘edge of the chamber’ is defined as the vapour-oil interface beyond which the temperature drops below the injected steam/steam-solvent mixture temperature ($T < T_s$).

For solvent-aided SAGD processes, the dynamics at the edge of the chamber, especially with respect to the interactions of heat transfer and solvent transport, remains unclear. The interactions are complex: oil mobility is controlled by the oil phase viscosity and its effective permeability, which in turn depend on temperature and solvent content. Solvent content is controlled by the diffusion and dispersion coefficients, which in turn are functions of temperature and viscosity. Since the heat and solvent content in the
mobile bitumen are the primary driving forces in steam-solvent processes, it becomes imperative to fully understand the heat and mass transport at the interface.

The object of this study is to investigate the extent to which the heat transfer and solvent mass transfer length scales affect oil mobility and oil viscosity at the edge of the vapour chamber in a solvent-aided SAGD process. The analytical model for the non-condensable gas aided SAGD process (developed in Chapter 5) is extended and modified to predict the performance of solvent-aided SAGD processes. In this study, hexane is used as the hydrocarbon solvent for co-injection with steam in SAGD. Fine-grid numerical simulations were performed to gain a visual understanding of fluid distribution and flow behaviour in the reservoir when solvent is co-injected with steam. The results from the analytical model were tested via comparison with the numerical simulation results, as well as with ES-SAGD experimental data.

6.2 Length Scales for Heat and Mass Transport at the Edge of a SAGD Chamber

For a typical Athabasca oil sands reservoir, the thermal diffusivity, $\alpha$, ranges from $2 \times 10^{-7}$ to $3 \times 10^{-6}$ m$^2$/s (Butler, 1985; Farouq Ali, 1997). For a mature SAGD operation, the edge propagation speed lies in the range of about $U = 1$ to 25 cm/day, for a typical Athabasca reservoir (Butler, 1987; Gotawala and Gates, 2008). If the temperature profile at the edge of the chamber is invariant as the chamber expands into the oil sands, then the length scale of heat conduction is roughly equal to $\alpha U$, which is typically between 0.1 and 15 m. In other words, at the edge of a SAGD or mature CSS steam chamber, the penetration of heat into the oil sands beyond the chamber edge would range from 0.1 to
15 m. Temperature profiles observed in vertical observation wells confirm this length scale for heat conduction beyond the edge of the steam chamber (CNRL report, 2007; JACOS report, 2007; ConocoPhillips report, 2007; Devon report, 2005; EnCana report, 2007; Shell report, 2007). Since the thermal diffusivity does not change significantly for most oil sands reservoir, the edge propagation speed, $U$, largely controls the extent of the conduction length scale. The edge propagation speed in turn is controlled by the formation permeability. The higher the permeability, the larger the edge propagation speed and the smaller the conduction length scale. On the other hand, the lower the permeability, the smaller the edge propagation speed and the larger is the conduction length scale. The conduction length scale sets the extent of oil sands beyond the chamber edge, which has sufficient mobility to move under gravity.

Inside the SAGD chamber, the solvent concentration in the gas phase is (assumed) as the injected solvent concentration. The solvent mass transfer into the oil sands beyond the chamber is largely controlled by the solubility of the solvent and its molecular diffusion and dispersion in the oil phase. For a typical solvent-bitumen system, the molecular diffusion coefficient in the oil phase ranges from $1 \times 10^{-11}$ to $5 \times 10^{-10}$ m$^2$/s (Das and Butler, 1996; Etminan, Maini, Hassanzadeh, and Chen, 2009). At moderate flow rates, the porous media would create asymmetrical mixing zones because of stream splitting and unequal mass transfer resulting in mechanical dispersion with the longitudinal dispersion coefficient approximately proportional to the first power of average fluid velocity. If this interstitial fluid velocity is large enough, it can lead to formation of a region dominated by longitudinal dispersion. The mechanical dispersivity
in the direction normal to the edge of the chamber, $\alpha_x$, can be estimated to be in the order of 5 to 200 $\mu$m (Perkins and Johnston, 1963). Given that the edge propagation speed, $U$, in the order of 1 to 25 cm/day, this implies that the mechanical dispersion coefficient, $\alpha_x U$, is of order $10^{-13}$ to $10^{-10}$ m²/s. The total effective diffusion coefficient is then the sum of the molecular diffusion and mechanical dispersion coefficients and thus, the length scale of mass transfer is roughly equal to $(D_{oil,eff} + \alpha_x U) / U$ which is typically between 0.001 and 0.3 m. In other words, at the edge of a steam-solvent SAGD chamber, the penetration of solvent into the oil sands beyond the chamber edge would range up to 0.3 m.

A comparison of the length scales of heat and mass transfer reveals that the mass transfer length scale is more than an order of magnitude smaller than the heat transfer length scale. This implies that beyond the edge of a steam-solvent SAGD chamber, two zones emerge, first, a zone near the chamber edge where both temperature and solvent content influence the oil phase viscosity (of the order of tens of centimetres), and second, a zone where temperature alone influences the oil phase viscosity (of the order of metres).

From an oil production point of view, the larger the mobile zone (defined as the narrow zone beyond the edge of a SAGD chamber where multiphase flow of oil, water and gas takes place), the greater the amount of oil that drains under gravity to the base of the SAGD chamber. Thus, to enhance production, the process conditions and solvent choice should be designed to increase the heat and mass transport length scales. Given that the thermal conductivity of oil sands cannot be substantially changed, it is not
expected that the heat conduction length scale can be enlarged significantly. However, the solvent content at the chamber edge and mass transfer length scale can be altered by changing the solubility of the solvent in the bitumen (by raising pressure or temperature or by changing the solvent itself) or the mass transport properties i.e. diffusion and dispersion coefficients of the solvent in the fluid phases at the chamber edge.

6.3 Relation with the Non-condensable Gas Aided SAGD Model

In Chapter 5, an analytical theory was derived to predict fluid flow profiles in the mobile zone when a non-condensable gas, such as methane or nitrogen, is co-injected with steam during SAGD. The results revealed that near the edge of the chamber, methane concentration in the gas phase gradually increases as the steam vapour condenses. This creates a methane concentration gradient near the condensing interface, which results in the counter-current diffusion of methane back towards the vapour chamber, and limits its further accumulation at the chamber wall. The competition between these two opposing forces (diffusion versus accumulation) determines the equilibrium methane concentration near the chamber walls. Further, the amount of methane that dissolves in the adjacent oil phase beyond the edge of the chamber is dictated by the equilibrium solubility value ($K$-value factor) of the injected gas in oil, and its effective diffusivity in the oil phase (see Figure 5.3). The analytical and simulation results obtained in Chapter 5 revealed that the concentration profile for methane in oil and gas phases has a maxima at some distance from the edge of the SAGD chamber (Figure 5.6). A similar mechanism of heat and mass transfer would occur when a solvent, which exists in the vapour phase inside the
chamber, is co-injected with steam in SAGD. There will be, however, some modifications to the modelling approach, as outlined below.

In the analytical model developed in Chapter 5, the dependence of the effective diffusivity coefficient of the non-condensable gas in the oil phase was assumed to be independent of gas concentration (Section 5.3.2). This assumption is acceptable when the diffusing specie is a light gas such as methane, whose intrinsic molecular diffusion coefficient is much higher than that of bitumen. However, for heavier hydrocarbon solvents such as butane, pentane, hexane, octane, and others, solvent diffusivity coefficients in oil are lower than for most non-condensable gases, and therefore the effective diffusivity in the solvent-oil mixture is a strong function of the solvent concentration in the oil phase (Oballa and Butler, 1989). In this chapter, the analytical model derived in Chapter 5 is modified to include a concentration and temperature dependent effective diffusivity coefficient for solvent transport in the oil phase. Since solvent concentration depends nonlinearly on distance, the result is a complex non-linear mass transfer equation, which must be solved numerically.

6.3.1 Modified Algorithm for Solvent-Aided SAGD Process

In Chapter 5 (Section 5.3.3), a 10-Step algorithm was developed for generating the fluid flow profiles in the mobile zone, when a non-condensable gas is co-injected with steam in SAGD (also see Figure 5.3). When a solvent (hexane in this case) is co-injected with steam in SAGD, similar steps for calculation can be followed, with an exception of Step-6 where the solvent concentration in the oil phase is evaluated.
The temperature, steam partial pressure, and hexane concentration profiles in the gas phase can be evaluated by following Step−1 to Step−5 of the algorithm, using the reservoir and fluid properties, and the corresponding correlations summarized in Tables 6.1 and 6.2. Following Step−4 of the algorithm, the rates of hexane diffusion and hexane accumulation in the gas phase near the chamber walls are compared to determine the distance $\xi^*$, where the rate of countercurrent diffusion of hexane becomes greater than the rate of hexane accumulation due to steam condensation.

Up to the distance $\xi^*$ beyond the chamber edge, hexane mole fraction in the gas phase is calculated from hexane partial pressure in the gas phase at the corresponding temperature (from Step−2 of algorithm) and the amount of hexane dissolved in the oil phase is calculated from the corresponding $K$-value (following Step−5 of the algorithm):

$$y_s = 1 - \frac{P_s}{P_i}, \text{ for } 0 < \xi < \xi^*$$  \hspace{1cm} (6.1a)

and

$$x_s = \frac{y_s}{K}, \text{ for } 0 < \xi < \xi^*$$  \hspace{1cm} (6.1b)

where $x_s$ and $y_s$ are the mole fractions of the injected solvent in the oil and gas phases respectively, $p_i$ is the injection pressure, and $p_s$ is the steam partial pressure. At $\xi^*$, the solvent concentration in the gas phase peaks to a maximum value and beyond $\xi^*$, the gas phase saturation within the mobile oil zone drops to zero (see Figures 5.3 and 5.6d for the methane injection case).

If the effective diffusivity coefficient in the oil phase is assumed to be independent of the solute concentration, as was done for the case for non-condensable gas
injection in Chapter 5, an exponentially decaying concentration profile is obtained in Step–6 for small incremental distance steps $(0, \Delta \xi \ldots j \Delta \xi, (j+1) \Delta \xi \ldots)$ to give:

$$C_{oil}^{j+1} = C_{oil}^j e^{-\frac{U \Delta \xi}{D_{oil,eff}}}, \text{ for } \xi > \xi^*$$

(6.2)

However, for the case of solvent co-injection with steam, the above equation cannot be used, as the diffusion coefficient will vary with solvent concentration in the oil phase.

Solvent concentration profile for $\xi > \xi^*$ is derived in the next section. Once the solvent concentrations in the oil and gas phases are known, the fluid flow profiles in the mobile zone are obtained by following Step–7 to Step–10 of the algorithm in Chapter 5 (Section 5.3.3).

### 6.4 Analytical Model

Solvent-aided SAGD processes are characterized by coupled heat and mass transfer beyond the edge of the chamber whose interaction controls the dynamics of the process. Figure 6.1 displays the edge of the chamber, as would be the case in a solvent-aided SAGD process. The ‘mobile zone’ is defined as the narrow zone beyond the steam condensation front where multiphase flow of oil, water and gas takes place. The ‘edge of the chamber’ is mathematically defined as the vapour-oil interface beyond which the temperature drops below the injected steam/steam-solvent mixture temperature ($T < T_s$).

#### 6.4.1 Modelling Assumptions

The following assumptions were made in the development of the analytical model:
Figure 6.1: Conductive heat transfer, and diffusive and dispersive mass transfer at the edge of a steam-solvent SAGD chamber.
1. Porous medium is homogeneous (constant porosity and permeability).

2. Analysis is done for a mature SAGD chamber that has reached the maximum height, $h$, at time, $t=0$, and is expanding at a constant velocity $U$, in the direction perpendicular to the chamber walls.

3. Heat transfer and mass transfer are directed normal to the edge of the chamber.

4. Overburden and underburden heat losses are neglected.

5. Heat transfer ahead of the SAGD chamber is by conduction.

6. Convective mixing of solvent in oil is neglected.

7. Boussinesq approximation (that is, variation in density is neglected everywhere except in the buoyancy term).

8. Density and viscosity are dependent on temperature and solvent concentration.

Convective heat and mass transport beyond the edge of the chamber were neglected in order to keep the heat and mass transfer equations decoupled. This condition as well as the condition of pseudo steady-state expansion are, however, relaxed in an extension of this model which is discussed in Chapter 7, where both convective heating and convective mixing of solvent in oil are examined under transient conditions. The resulting set of coupled partial differential equations are more complex and are solved using Matlab.
6.4.2 Mathematical Formulation

For conductive heat transfer ahead of a steam chamber whose edge is translating through the oil sands at a constant speed $U$, the temperature profile was derived in Chapter 3 (Section 3.1) as:

$$\frac{T - T_r}{T_s - T_r} = e^{-\frac{U\xi}{\alpha r}}$$

(6.3)

where $\xi$ is the distance measured from the edge of the SAGD chamber, $\alpha$ is the thermal diffusivity of the oil sands, and $T_r$ and $T_s$ are the original reservoir temperature and saturated steam temperature, respectively.

In the absence of convection, change in the solvent concentration field with time is predicted from a mass balance of the solvent that accounts for mass accumulation and diffusive transport as given by the Fick’s law. By considering the case that the system is undergoing pseudo steady-state expansion, the solvent concentration profile in the oil phase is given by (Chapter 5, Section 5.3.2):

$$U \frac{\partial C_{oil}}{\partial \xi} = -D_{oil,eff} \frac{\partial^2 C_{oil}}{\partial \xi^2} \text{ for } \xi > \xi^*$$

(6.4)

where, $D_{oil,eff}$, is the effective diffusion coefficient in the oil phase that includes the contribution from molecular diffusion as well as hydrodynamic dispersion effects (Perkins and Johnston, 1963), and $C_{oil}$ is the molar concentration of the injected solvent in the oil phase expressed as moles of solvent per unit volume of oil.

The effective diffusion coefficient in the oil phase ($D_{oil,eff}$) depends on the porosity and tortuosity of the reservoir, as well as on the intrinsic diffusion coefficients of
oil and the injected solvent (Oballa and Butler, 1989; Reid et al., 1977). In this analysis, the dependence of the diffusion coefficient on solvent concentration was assumed to follow the intrinsic diffusion coefficients weighted by the solvent mole fraction in a power law fashion (Reid et al., 1977). Diffusion coefficient is generally inversely proportional to the phase viscosity or viscosity to some power, and proportional to the temperature or temperature to some power. In this work, the correlation developed by Das and Butler (1989) were used for calculation of the effective diffusion coefficient, as summarized in Table 6.1. The effective diffusion coefficient will also include the effect of mass transport due to mechanical dispersion, which can be approximated as the product of the hydrodynamic dispersivity, \( \alpha_s \), and the flow velocity in the direction perpendicular to the edge of the chamber, \( U \) (Perkins and Johnston, 1963), as:

\[
D_{oil,eff} = \left( D_{o,oil} \right)^{x_s} \left( D_{s,oil} \right)^{1-x_s} \left( \frac{\phi}{\tau} \right) + \alpha_s U
\]  

(6.5)

where \( D_{o,oil} \) and \( D_{s,oil} \) are the intrinsic molecular diffusion coefficients of the oil and the solvent, respectively, \( \alpha_s U \) is an effective mechanical dispersion coefficient, \( \tau \) is the reservoir tortuosity and \( x_s \) is the solvent mole fraction in the oil phase. A value of \( \sqrt{2} \) was used to allow for the effect of tortuosity, as suggested by Butler and Yee (1986). Since the intrinsic diffusion coefficients depend on temperature and viscosity (Reid et al., 1977) and the viscosity in turn depends on temperature and solvent concentration, and the solvent diffusion coefficient varies with solvent concentration (Oballa and Butler, 1989), the effective diffusion coefficient varies spatially in the mobile oil zone beyond the edge of the SAGD chamber. After combining Equation (6.4) with Equation (6.5), the result is:
Table 6.1: List of correlations used in the analytical model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlation</th>
<th>Constants</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of Athabasca bitumen</td>
<td>$\ln(\mu_o) = A\ln(T + 273.15) + B$</td>
<td>$A = -3.5738$</td>
<td>Mehrotra and Svrcek (1986)</td>
</tr>
<tr>
<td></td>
<td>$\mu_o$ in cp, $T$ in °C</td>
<td>$B = 22.8379$</td>
<td></td>
</tr>
<tr>
<td>Viscosity of solvent</td>
<td>$\log \mu_s = A\left(\frac{1}{T} - \frac{1}{B}\right)$</td>
<td>Hexane:</td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>$\mu_s$ in cp, $T$ in K</td>
<td>$A = 362.79$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B = 207.09$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diffusion coefficients</td>
<td>$D_{s,\text{gas}}(T^\circ\text{C}) = D_{s,\text{gas}}(20^\circ\text{C})\left[\frac{T + 273.15}{20 + 273.15}\right]^{1.8}$</td>
<td>Das and Butler (1996)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{s,\text{gas}}(20^\circ\text{C}) = 4\times10^{-8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{s,\text{oil}}(T^\circ\text{C}) = D_{s,\text{oil}}(20^\circ\text{C})\left[\frac{T + 273.15}{20 + 273.15}\right]\frac{\mu_o @ 20^\circ\text{C}}{\mu_s @ T^\circ\text{C}}^{0.545}$</td>
<td>Butler and Yee (1986)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{s,\text{oil}}(20^\circ\text{C}) = 4\times10^{-10}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{o,\text{oil}}(T^\circ\text{C}) = D_{o,\text{oil}}(20^\circ\text{C})\left[\frac{T + 273.15}{20 + 273.15}\right]\frac{\mu_o @ 20^\circ\text{C}}{\mu_s @ T^\circ\text{C}}^{0.545}$</td>
<td>Ayodele et al. (2010)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D_{o,\text{oil}}(20^\circ\text{C}) = 5\times10^{-11}$</td>
<td></td>
<td>Perkins and Johnston (1963)</td>
</tr>
<tr>
<td>Concentration dependence:</td>
<td>$D_{\text{oil},\text{eff}} = \left(D_{o,\text{oil}}\right)^{-\gamma}\left(D_{s,\text{oil}}\right)^{-\alpha_s}\frac{\phi}{\tau} + \alpha_s U$</td>
<td>$\tau = \sqrt{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D$ in m$^2$/s, $T$ in °C, $\mu$ in cp, $\alpha_s$ in m, $U$ in m/s, $x_s$ is the mole fraction of injected solvent</td>
<td>$\alpha_s = 1\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Solvent solubility</td>
<td>$K$-value = $\left(\frac{k_{v1} + k_{v2}p_i + k_{v3}}{p_i}\right)e^{\frac{k_{v4}}{T - k_{v5}}}$</td>
<td>Hexane:</td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>$T$ in °C, $p$ in kPa</td>
<td>$k_{v1} = 1.0606\times10^6$</td>
<td>CMG (2009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{v2} = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{v4} = -2697.55$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_{v5} = -224.37$</td>
<td></td>
</tr>
<tr>
<td>Viscosity of solvent-oil mixtures</td>
<td>$\ln(\mu_{so}) = (1-x_s)\ln(\mu_o) + x_s\ln(\mu_s)$</td>
<td></td>
<td>CMG (2009)</td>
</tr>
<tr>
<td></td>
<td>$\mu$ in cp, $x_s$ in mole fraction of injected solvent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the solution of which, given its nonlinearity, is difficult to obtain in a closed analytic form. Here, Equation (6.6) was discretized by using a finite difference approximation for the derivatives. This yields a tridiagonal matrix system. The molar concentration of solvent in oil is related to the mole fraction of solvent in the oil phase through the density of the solvent-oil solution. Equation (6.6) holds for the region: \( \xi^* \leq \xi \leq \infty \). Where, \( \xi^* \) is the distance measured from the SAGD chamber edge beyond which the gas phase saturation in the mobile oil zone drops to zero (see Figure 5.3). The boundary conditions were as follows. First, at \( \xi = \xi^* \), temperature is calculated from Equation (6.3) and the solvent mole fraction in the oil phase, \( x_s \), is set by the solubility of the solvent at the solvent partial pressure and the temperature at \( \xi = \xi^* \), which is calculated from Equations (6.1a) and (6.1b). Second, deeper into the cold reservoir, that is, as \( \xi \to \infty \), then \( T \to T_r \) and \( x_s \to 0 \). By using these boundary conditions, the set of finite difference approximation equations representing Equation (6.6) were solved by Newton’s method.

6.5 Numerical Simulation

The objective of the numerical simulation study was to verify the results from the analytical model and to get a visual understanding of the effect of solvent on SAGD performance in terms of its impact on chamber development, fluid phase distribution and fluid flow rates. In order to compare the simulations with the analytical results, a homogeneous reservoir model, with uniform porosity and permeability, and no
overburden heat losses was used, to match the assumptions made in the analytical theory. The reservoir and fluid properties for the homogeneous reservoir model are summarized in Table 6.2 and discussed in detail in Appendix A (Section A.3.1).

6.6 Results and Discussion

6.6.1 Analytical Model Results

In this study, hexane was used as the solvent for co-injection with steam in SAGD. The reservoir and fluid properties listed in Table 6.2, and correlations summarized in Table 6.1 were used in the analytical calculations. Figure 6.2 displays the temperature profiles beyond the edge of the SAGD chamber, with and without solvent injection. For steam-alone injection case the highest temperature inside the SAGD chamber is 220 °C, corresponding to the steam saturation pressure of 2363 kPa. When 0.1 mole fraction of hexane is co-injected with steam, the steam partial pressure in the chamber is reduced to 2126 kPa in accordance with the Dalton’s law of partial pressure. Consequently, the chamber temperature drops from 220 °C to 215.5 °C. The temperature profiles reveal that the length scale for heat transfer is of the order of 1 m for the set of operating conditions used in this study (Table 6.2). This is comparable to the range of conductive heat transfer length scale observed from thermocouple data from active SAGD field operations (CNRL report, 2007; JACOS report, 2007; ConocoPhillips report, 2007; Devon report, 2005; EnCana report, 2007; Shell report, 2007).

Beyond the chamber edge, as steam condenses the concentration of hexane in the gas phase continuously increases until the rate of counter-current diffusion of hexane into
Table 6.2: Reservoir and fluid properties used in the analytical and simulation models (Ayodele et al., 2010; Butler, 1997a).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial temperature, $T_r$, °C</td>
<td>10</td>
</tr>
<tr>
<td>Steam temperature, $T_s$, °C</td>
<td>220</td>
</tr>
<tr>
<td>Oil density, $\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>Solvent (hexane) density, $\rho_s$, kg/m$^3$</td>
<td>664</td>
</tr>
<tr>
<td>Injection pressure, $p_i$, kPa</td>
<td>2363</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>Thermal diffusivity, $\alpha$, m$^2$/s</td>
<td>$1.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Permeability, $k$, m$^2$</td>
<td>$5.0 \times 10^{-13}$</td>
</tr>
<tr>
<td>End point relative permeability, $k_{zocw}$</td>
<td>0.9</td>
</tr>
<tr>
<td>Mechanical dispersivity, $\alpha_x$, m</td>
<td>0.0001</td>
</tr>
<tr>
<td>Initial oil saturation, $S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Irreducible water saturation, $S_{wr}$</td>
<td>0.19</td>
</tr>
<tr>
<td>Oil viscosity, $\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
<tr>
<td>Thermal conductivity of the reservoir, $K_{TH}$, W/(m-K)</td>
<td>1.54</td>
</tr>
<tr>
<td>Chamber expansion rate, $U$, cm/day</td>
<td>20</td>
</tr>
<tr>
<td>Injected mole fraction of hexane, $y_i$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Figure 6.2: Temperature profile beyond edge of the vapour chamber for the SAGD and solvent-aided SAGD cases. Model parameters are set to the values listed in Table 6.2.
the SAGD chamber becomes greater than the rate of hexane accumulation due to steam condensation (also see Figure 5.3). Figure 6.3 compares the rate of hexane diffusion and hexane accumulation in the gas phase as a function of distance from the chamber edge. The result reveals that $\xi^*$, the distance at which the rate of hexane accumulation drops below the rate of hexane diffusion, is about 5 cm beyond the edge of the chamber. This means that up to this distance hexane will continue to accumulate in the gas phase near the chamber walls. The mole fraction of hexane in the gas phase is shown in Figure 6.4. This was obtained from Equation (6.1a). The result reveals that the highest hexane concentration in the gas phase is about 0.45 mole fraction, which occurs at the distance $\xi^* \approx 5 \text{ cm from the chamber edge}$.

The other important factor that determines the performance of a steam-solvent process is the solvent dilution of the oil phase action ahead of the SAGD chamber. The more diluted the oil phase, the lower its viscosity. The amount of solvent that dissolves and disperses in the bitumen in the mobile zone is controlled by the solvent concentration profile in the oil phase ahead of the edge of the chamber. Up to the distance $\xi^*$, the amount of hexane that dissolves in oil is given by Equation (6.1b). Beyond $\xi^*$, hexane further mixes with the oil phase by diffusion and dispersion and the resulting concentration profile is given by the solution of Equation (6.6). Figures 6.5a and 6.5b display the overall solvent molar concentration and the equivalent solvent mole fraction profiles in the oil phase, respectively, as a function of distance from the chamber edge. The results reveal that the active mass transfer length scale is about 0.25 m in extent, with
Figure 6.3: Comparison of rate of diffusion of hexane in the gas phase and the rate of hexane accumulation near the interface due to steam condensation.

Figure 6.4: Profile of hexane mole fraction in the gas phase beyond the edge of the chamber for the solvent-aided SAGD case. Model parameters are set to the values listed in Table 6.2.
Figure 6.5: Profile of hexane dissolved in the oil phase beyond the chamber edge for the solvent-aided SAGD case (a) expressed as molar concentration (b) expressed as mole fraction. Model parameters are set to values listed in Table 6.2.
the majority of the solvent within 20 cm of the edge of the chamber. This is about an
order of magnitude smaller than the heat transfer length scale seen in Figure 6.2.

Figure 6.6 plots the variation of the effective diffusion coefficient in the oil phase,
$D_{oil,eff}$, beyond the edge of the chamber, for the steam-solvent system. The results show
that there is a maximum of the overall diffusion coefficient at about 10 cm from the edge
of the chamber. This results from the combination of the temperature, viscosity, and
solvent content profiles. An overall effect of steam heating and solvent dilution on oil
viscosity reduction results in enhanced mobilization of bitumen in a solvent-aided SAGD
process. Figure 6.7 compares the oil viscosity profiles as a function of distance from the
edge of the chamber, for the steam-only and steam-solvent cases for the parameters listed
in Table 6.2. The oil viscosity profile for the SAGD case arises from the corresponding
temperature profile shown in Figure 6.2, whereas in the solvent-aided SAGD case it
results from both the temperature and solvent content profiles displayed in Figures 6.2
and 6.5, respectively. A comparison of the steam-only and steam-solvent cases reveals
that the oil phase viscosity for the steam-solvent case is less than that of the steam-only
case near the edge of the chamber. The reduced oil viscosity for the steam-solvent case
persists for about the first 20 cm of the region beyond the edge of the chamber. However
beyond this distance the oil phase viscosity for the steam-solvent system becomes slightly
greater than the steam-only case. The difference in the length scales for heat and mass
transfers, as discussed above, leads to the formation of the two distinct zones in the
viscosity profile ahead of the SAGD chamber, dominated respectively by solvent dilution
effect in the first 20 cm beyond the chamber edge, and steam heating effect beyond that.
Since the temperature in the SAGD chamber is lowered when hexane is co-injected with steam, the viscosity beyond the solvent diluted region ($\xi > 20 \text{ cm}$) is higher for steam-solvent case as compared to the steam-alone case. The oil viscosity is lowest in the region where both heat and mass transfer are present and rises to the original reservoir value far into the cold oil sands.

Figure 6.8 displays the oil phase mobility profiles for the SAGD and solvent-aided SAGD cases. The oil mobility also takes oil effective permeability into account, which ultimately depends on the oil saturation. Here, the oil saturation, and relative permeability were evaluated in a similar manner as done for the case of non-condensable gas injection, using the parameters listed in Table 6.2. This was outlined in Step−7 and Step−8 of the calculation algorithm developed in Chapter 5 (Section 5.3.3). The results reveal that the oil mobility for the solvent-aided SAGD case is several orders of magnitude greater than that for the SAGD case, for the first few centimetres beyond the edge of the chamber (~20 cm) due to the solvent dilution effect. Based on the gravity-drainage theory derived in Chapter 3, this would mean that the bitumen production rate will be enhanced by a factor of between 2 and 8. Beyond the solvent diluted region, the oil phase mobility values for the steam-solvent and steam-only cases are practically the same.

Figure 6.9 displays the Lewis number, $\text{Le} \equiv \alpha / D_{\text{oil, eff}}$, a measure of the ratio of thermal diffusivity to mass diffusivity. The results show that the thermal diffusivity is about 400 times the solvent diffusivity. However, there is a slight depression of the Lewis number near the edge of the chamber, which corresponds to the peak in the effective
Figure 6.6: Value of effective diffusion coefficient in oil ($D_{oil,eff}$), given by Equation 6.5, beyond the edge of the chamber for the solvent-aided SAGD case. Model parameters are set to values listed in Table 6.2.

Figure 6.7: Profiles of oil phase viscosity beyond the chamber edge for the SAGD and solvent-aided SAGD cases. Model parameters are set to values listed in Table 6.2.
Figure 6.8: Profiles of oil phase mobility beyond the chamber edge for the SAGD and solvent-aided SAGD cases. Model parameters are set to values listed in Table 6.2.

Figure 6.9: Lewis number versus distance beyond the steam chamber edge for the solvent-aided SAGD case. Model parameters are set to values listed in Table 6.2.
diffusion coefficient displayed in Figure 6.6. Although the depression is relatively small and exists over a thin region just beyond the edge of the chamber, the enhancement of the oil mobility at the edge of the chamber is sufficient to increase the oil rate by potentially several times.

6.6.2 Numerical Simulation Results

The case of solvent-aided SAGD process, with hexane as the injected solvent, was simulated on a homogeneous reservoir model, with constant porosity and permeability and no overburden heat losses. Fluid and rock properties for the homogeneous reservoir model are summarized in Appendix A, Section A.3.1. The properties are similar to those used in the analytical model (summarized in Table 6.2). The operating conditions for the SAGD and solvent-aided SAGD processes are summarized in Table 6.3. It must be noted that the analytical theory does not account for the effect of chamber wall inclination for generating the fluid flow profiles, while the simulation results do depend on the inclination of the growing chamber. Therefore to compare the two results, the simulation results for the flow profiles generated midway across the chamber were compared with the analytical results, to average out the inclination effect. This roughly translated to profile across grid block [1-1-27] to grid block [1-400-27], expressed in their respective $x$, $y$, $z$, coordinates. This may, however, limit the ability of analytical theory to exactly match the values in the simulation results.

The simulation results are shown in Figures 6.10 through 6.14. Figures 6.10a and 6.10b display the two-dimensional distribution of temperature in the reservoir for the
Table 6.3: Operating conditions used in the simulations.

<table>
<thead>
<tr>
<th>Operation conditions</th>
<th>SAGD</th>
<th>Solvent-Aided SAGD (with hexane)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection pressure, $p_i$, kPa</td>
<td>2363</td>
<td>2363</td>
</tr>
<tr>
<td>Injected hexane mole fraction, $y_i$</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>SAGD chamber temperature, $T_s$, °C</td>
<td>220</td>
<td>215.5</td>
</tr>
<tr>
<td>Steam quality</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 6.10: Homogeneous reservoir simulation (a) Temperature distribution in the reservoir for the SAGD case (b) Temperature distribution in the reservoir for the solvent-aided SAGD case with hexane (c) Temperature profiles for the SAGD and solvent-aided SAGD cases.
SAGD and solvent-aided SAGD cases, respectively. Figure 6.10c compares the lateral temperature profiles for the two cases after 30 hours (1.25 days) of steaming which on scaling up using Butler’s (1985) scaling parameter translates to 8 years of SAGD operation. The edge of the chamber can be located on the temperature profile as roughly at a distance of 0.27 m for the SAGD case, and at 0.2 m for the solvent-aided SAGD case, from the well pair. Temperature profiles in Figure 6.10c show a monotonically decreasing trend beyond the chamber edge with a length scale of 0.8-1 m, which is similar to the analytical model results (Figure 6.2). The maximum temperature for the SAGD case is about 220 °C corresponding to steam saturation pressure of 2363 kPa, while that for solvent-aided SAGD case it is about 216 °C, corresponding to a steam partial pressure of 2127 kPa. The simulation results are similar to the analytical temperature profiles displayed in Figure 6.2. An early drop in temperature near the chamber edge is observed for the case of solvent-aided SAGD. This is a result of increasing hexane accumulation near the interface which lowers the steam partial pressure and saturation temperature.

At the chamber edge, as steam condenses, the volume shrinkage creates a low-pressure region (Δp), which leads to an increased hexane concentration in the gas phase at the condensing interface. The resulting hexane mole fraction distribution in the gas phase is displayed in Figures 6.11a and 6.11c. The maximum concentration of hexane in the gas phase is roughly 0.45 mole fraction, which occurs at a distance of about 6 cm from the edge of the chamber (which implies, \( \xi^* = 6 \) cm). These results compare well with the distance \( \xi^* \), and the maximum hexane concentration derived in the analytical model.
Figure 6.11: Homogeneous reservoir simulations (a) Hexane mole fraction distribution in the gas phase (b) Hexane mole fraction distribution in the oil phase (c) Hexane mole fraction profile in the oil and gas phases, for the solvent-aided SAGD case.
(Figures 6.3 and 6.4, respectively). Beyond the chamber wall, hexane mixes with the oil phase as a result of dispersion, diffusion and dissolution. The resulting hexane mole fraction distribution in the oil phase is displayed in Figures 6.11b and 6.11c. The length scale of hexane mass transfer is about 0.3 m beyond the chamber edge, which is similar to the prediction of the analytical theory (see Figure 6.5).

The oil phase viscosity resulting from solvent and/or temperature dilution effect, for the two cases, is shown in Figure 6.12. In the mobile zone (which roughly extends from the edge of the chamber to about 0.4 m beyond it) the oil phase viscosity is several orders of magnitude lower (< 50 cp) than the original oil viscosity (> 10^6 cp) for both the cases. However, due to the solvent dilution effect close to the chamber edge in the solvent-aided SAGD case, the oil phase viscosity is about an order of magnitude lower than the SAGD case. Beyond the solvent diluted zone, the oil phase viscosity for the solvent-aided SAGD case becomes slightly greater than the SAGD case. This is due to the lower chamber temperature, corresponding to the lower steam partial pressure on solvent co-injection. The simulation results are qualitatively and quantitatively similar to the predictions of the analytical theory (Figure 6.7).

Figure 6.13 displays the phase mobility profiles as a function of distance. The mobile zone extends from the edge to the chamber to approximately 0.4 m beyond it, which is comparable to the thickness of the mobile zone seen in the analytical model (Figure 6.8). Oil phase mobility for the solvent-aided SAGD case is several times higher than the SAGD case for the first 20 cm beyond the chamber edge. This corresponds to the solvent mass transfer length scale which is about 20 cm as seen in Figure 6.11. Beyond
Figure 6.12: Homogeneous reservoir simulations (a) Oil phase viscosity for the SAGD case (b) Oil phase viscosity for the solvent-aided SAGD case with hexane (c) Temperature and oil phase viscosity profiles for the SAGD and solvent-aided SAGD cases.
Figure 6.13: Homogeneous reservoir simulations (a) Oil phase mobility for the SAGD case (b) Oil phase mobility for the solvent-aided SAGD case with hexane (c) Temperature and oil phase mobility profiles for the SAGD and solvent-aided SAGD cases.
the solvent diluted zone, the oil phase mobility in the solvent-aided SAGD case is dominated by only the steam heating effect, like the SAGD case. It is also evident that similar to the predictions of the analytical theory derived in Chapter 3, the most mobile oil is not at the edge of the chamber but rather some distance beyond the edge.

Figure 6.14 compares the cumulative oil rates for the two cases. The average oil production rate is \( \sim 1 \times 10^{-4} \text{ m}^3/\text{day} \) for the SAGD case and \( \sim 7 \times 10^{-3} \text{ m}^3/\text{day} \) for the solvent-aided SAGD case. This significant enhancement in the oil rate is consistent with the enhanced productivity observed in field applications of steam-solvent processes (Gupta and Gittins, 2005; Leaute and Carey, 2005).

The fine-grid numerical simulation results support the predictions of the analytical theory, which suggests that the theory captures the underlying flow mechanisms in solvent-aided SAGD process.

6.7 Validation of Theory with ES-SAGD Experimental Data
Scaled laboratory test of ES-SAGD, with hexane as the co-injected solvent, was conducted in a 2-D high pressure, high temperature test facility at the Alberta Research Council (Ayodele et al., 2010). The ES-SAGD test results showed higher oil production rate as compared to equivalent SAGD experimental tests, for the same amount of injected steam. A summary of the operating conditions, steam-solvent ratios, and reservoir and fluid properties for the SAGD and ES-SAGD experiments is given in Table 6.4. The oil and solvent production rates for both the experiments are shown in Figure 6.15a and the cumulative oil production for the ES-SAGD test is shown in Figure 6.15b. The average
Figure 6.14: Cumulative oil produced for the SAGD and solvent-aided SAGD cases as seen in the homogeneous reservoir simulations.
Table 6.4: Reservoir properties and operating conditions used in the SAGD and ES-SAGD experiments (Ayodele et al., 2010).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>SAGD</th>
<th>ES-SAGD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment cell dimensions, cm</td>
<td>24 x 80 x 10</td>
<td>24 x 80 x 10</td>
</tr>
<tr>
<td>Initial temperature, $T_r$, °C</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Injected steam temperature, $T_s$, °C</td>
<td>220</td>
<td>220</td>
</tr>
<tr>
<td>Injected steam pressure $p_i$, kPa</td>
<td>2100</td>
<td>2100</td>
</tr>
<tr>
<td>Initial cell pressure, kPa</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>Steam quality</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Oil type</td>
<td>Athabasca bitumen (UTF)</td>
<td>Athabasca bitumen (UTF)</td>
</tr>
<tr>
<td>$\rho_o(T_r)$, kg/m$^3$</td>
<td>1.0007</td>
<td>1.0007</td>
</tr>
<tr>
<td>$\rho_s$, kg/m$^3$</td>
<td>N/A</td>
<td>664</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.316</td>
<td>0.317</td>
</tr>
<tr>
<td>$\alpha$, m$^2$/s</td>
<td>1.83x10$^{-6}$</td>
<td>1.83x10$^{-6}$</td>
</tr>
<tr>
<td>$k$, m$^2$</td>
<td>120x10$^{-12}$</td>
<td>120x10$^{-12}$</td>
</tr>
<tr>
<td>$k_{ocw}$</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha_s$, m</td>
<td>2.4x10$^{-4}$</td>
<td>2.4x10$^{-4}$</td>
</tr>
<tr>
<td>$S_{io}$</td>
<td>0.937</td>
<td>0.969</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\mu_o(T_s)$, cp</td>
<td>7.26</td>
<td>7.26</td>
</tr>
<tr>
<td>Average solvent injection rate, g/hr</td>
<td>N/A</td>
<td>198.2</td>
</tr>
<tr>
<td>Average steam injection rate, g/hr</td>
<td>2.03x10$^3$</td>
<td>1.96x10$^3$</td>
</tr>
<tr>
<td>Molecular diffusion coefficient of hexane in the vapour phase, m$^2$/s</td>
<td>N/A</td>
<td>4.2x10$^{-8}$</td>
</tr>
<tr>
<td>Molecular diffusion coefficient of hexane in the oleic phase, m$^2$/s</td>
<td>N/A</td>
<td>4.2x10$^{-10}$</td>
</tr>
</tbody>
</table>
Figure 6.15: ES-SAGD Experiment results (Ayodele et. al., 2010) (a) Oil and solvent production rates (b) Cumulative oil produced in the ES-SAGD experiment.
ES-SAGD incremental recovery over SAGD was approximately 11% in the first 500 minutes.

The fluid and reservoir properties listed in Table 6.4 were used to predict the fluid flow profiles and oil flow rates for the ES-SAGD case using the analytical theory derived in this chapter. The results from the analytical theory are displayed in Figure 6.16 and 6.17. The temperature profile beyond the edge of the chamber (which is located at $\xi=0$) for the ES-SAGD case is shown in Figure 6.16a. These values were calculated from Equation (6.3). The rates of hexane diffusion and accumulation were compared to get the distance $\xi^*$, measured from the edge of the chamber, beyond which the gas phase saturation in the mobile zone becomes zero. The result is displayed in Figure 6.16b, which shows that $\xi^*$ is approximately located at a distance of 2 cm beyond the chamber edge. The resulting hexane mole fraction profile in the oil phase, obtained by solving Equation (6.6), is displayed in Figures 6.16c and the mole fraction of hexane in the gas phase obtained by solving Equation (6.1a) is displayed in Figure 6.16d. The phase saturation and relative permeability profiles are displayed in Figures 6.16e and 6.16f, respectively.

The temperature, viscosity, and mobility profiles for the SAGD and ES-SAGD cases are compared in Figure 6.17. The temperature profiles beyond the edge of the chamber for the two cases are compared in Figure 6.17a. The results show that the profiles for both the cases are similar. This is because the amount of co-injected solvent (hexane) in the ES-SAGD case is small ($\approx0.02$ mole fraction) and therefore the reduction
Figure 6.16: Analytical results generated using the ES-SAGD experimental parameters listed in Table 6.4 (a) Temperature profile as a function of distance from the chamber edge (b) Comparison of the rate of hexane accumulation near the chamber walls and the rate of hexane diffusion in the gas phase (c) Hexane mole fraction profile in the oil phase (d) Hexane mole fraction profile in the gas phase (e) Phase saturation profile (f) Phase relative permeability profile.
Figure 6.17: Analytical results generated using the SAGD and ES-SAGD experimental parameters listed in Table 6.4
(a) Temperature profile (b) Oil phase viscosity profile (c) Oil phase mobility profile.
in steam partial pressure is negligible. The oil phase viscosity profiles for the two cases are compared in Figure 6.17b. The results clearly show that due to the combined effect of steam heating and solvent dilution in ES-SAGD, the oil phase viscosity near the edge of the chamber is slightly lower than the SAGD case. The lower viscosity persists for the first few centimetres beyond the edge of the chamber in the region where solvent has penetrated. The oil phase mobility profiles for the two cases are compared in Figure 6.17c. The results reveal that the oil mobility for the ES-SAGD case is greater than that for the SAGD case for the first few centimetres (~2 cm) beyond the edge of the chamber due to the solvent dilution effect. Beyond the solvent diluted zone, the oil phase mobility for the SAGD case is higher than that for the ES-SAGD case because of slightly higher steam temperature.

In Chapter 5 (Section 5.3.4), the following analytical equation was derived for prediction of oil production rate when a non-condensable gas is co-injected with steam in SAGD:

\[
q = 2L \sqrt{\frac{2\alpha h k g \Delta S_o}{m \nu_s} \sum_{\xi=0}^{\xi=\infty} \int_{\xi=0}^{\xi=\xi} k_{ro} e^{-\xi} d\xi}
\]

The oil production rate for the ES-SAGD case was calculated by integrating Equation (6.7) using the relative permeability profile displayed in Figure 6.16f and the reservoir properties and operating conditions listed in Table 6.4, to predict an oil rate of \( q = 4 \times 10^{-7} \, \text{m}^3/\text{s} \) (about 1000 g/hr). The average oil production rate observed in the ES-SAGD experiment in the first 400 minutes (see Figure 6.15) is about \( 2.1 \times 10^{-7} \, \text{m}^3/\text{s} \) (=}
The results show that the analytical theory reasonably predicts the oil flow rate, however, the experimental oil production rate is lower than the analytical prediction. This could be due to the heat losses through the walls of the experimental vessel. The presence of significant amount of heat losses during the ES-SAGD experiments was also reported by Ayodele et al. (2010).

6.8 Conclusions

In this chapter, a simple analytical model is derived to describe the dynamics of heat transfer and solvent mass transfer effects in the mobile zone at the edge of a steam-solvent SAGD chamber. A double front system evolves: compositional and thermal fronts result with different length scales. The extent to which these fronts affect oil mobility at the edge of the chamber is investigated. The results reveal that solvent provides an additional means to raise oil phase mobility beyond that achieved with heat alone. Due to the enhanced oil mobility, steam-solvent processes yield higher oil production rates than the steam-only ones. The results from the fine-grid numerical simulation corroborate the analytical findings, which suggests that the theory captures the underlying mechanism of solvent-aided SAGD process. The predictions of the analytical theory also compare well with the ES-SAGD experimental data.
Chapter 7:

TRANSIENT COUPLED HEAT AND MASS TRANSFER MODEL

7.1 Introduction

In Chapters 3–6, heat, mass, and momentum transport equations were applied in various forms to derive analytical models for the SAGD process and its variants. In Chapter 3, conductive heat transfer equation was used to predict oil flow rate in SAGD. The model developed in Chapter 3 was extended to included convective heat transfer effects beyond the SAGD steam chamber in Chapter 4. In Chapters 5 and 6, coupled heat and mass transfer equations were employed to predict oil flow rates in non-condensable gas-aided SAGD and solvent-aided SAGD operations, respectively. However, as noted in Sections (3.2), (4.2), (5.3.1) and (6.4.1), these analytical models were derived under some simplifying assumptions; such as, the SAGD chamber is expanding under pseudo steady-state condition, and convective heat transfer and convective solvent transport are absent. This chapter presents an extension of the models derived in the previous chapters. Specifically, a set of coupled heat and mass transfer equations are derived, that account for the conductive and convective heat transfer, coupled with the diffusive and convective mixing of solvent in the oil phase, under unsteady-state( or transient) conditions, in the mobile zone adjacent to the SAGD chamber.

Model development as well as discussion of selected results and their significance is presented. These effects can be important even though they were assumed to be absent in the models derived in the previous chapters, as the primary objective was to develop robust yet simple analytical models that do not require advanced computing capability,
without compromising the underlying physics. This chapter presents a framework for solving the coupled heat and mass transfer equations under unsteady-state conditions and aims to supplement the understanding gained from the analytical models discussed earlier.

7.2 Coupled Heat and Mass Transfer Model

Figure 7.1 displays the edge of the SAGD chamber, as would be the case in a solvent–aided SAGD operation. The ‘mobile zone’ is defined as the narrow zone beyond the steam condensation front where multiphase flow of oil, water, and gas takes place. The ‘edge of the chamber’ is mathematically defined as the vapor-oil interface beyond which the temperature declines below the injected steam/non-condensable gas or steam/solvent mixture temperature ($T < T_s$).

The results in Chapter 4 established that if there is mobile water phase in the reservoirs and the steam chamber pressure exceeds the original reservoir pressure, there is potential that convection of steam condensate could provide an additional means for heat transfer at the steam chamber edge beyond conduction. The mobilized warm oil will also carry some amount of heat beyond the edge of the chamber, which would be an additional contribution to the overall convective heat flux. Similarly, it can be argued that there will potentially be convective mixing of the injected solvent or non-condensable gas with the oil phase, beyond the diffusive-dispersive mass transport, in the presence of a mobile oil phase. The next subsections examine these effects mathematically.
Figure 7.1: Conductive and convective heat transfer and convective, diffusive and dispersive mass transfer at the edge of a steam-solvent SAGD chamber.
7.2.1 Theory of Heat Transfer

A key control on the productivity for thermal recovery processes, such as SAGD, is the extent of heating of oil sands at the edge of the steam chamber. The larger the heated layer of oil sands at the chamber edge, the greater is the thickness of the mobile zone and the lower is the viscosity of bitumen in the mobile layer. Heat transfer ahead of the SAGD chamber occurs by convection and conduction according to the following equation:

\[
K_{TH} \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right\} = V_f \rho_f c_{pf} \left\{ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right\} - \rho_b c_{pb} \left( \frac{\partial T}{\partial t} \right) \]  

(7.1)

Subscript \( f \) denotes the flowing liquid phase in the mobile zone, comprising of the water and oil phases, \( V_f \) is the effective convective flow velocity and \( c_{pf} \) is the effective heat capacity for the flowing phase, \( \rho_b c_{pb} \) is the bulk volumetric heat capacity of the reservoir, and \( K_{TH} \) is the thermal conductivity of the reservoir. The effective convective velocity of the flowing liquid phase can be calculated from a simple heat balance:

\[
V_f \rho_f c_{pf} = S_a V_a \rho_a c_{po} + S_w V_w \rho_w c_{pw} \]  

(7.2)

where \( \rho_w c_{pw} \) and \( \rho_w c_{pw} \) are the bulk volumetric heat capacity of the mobile water phase and the mobile oil phase, respectively. The temperature gradient in the direction parallel to the chamber interface and along the well direction is considered to be much smaller.
than that perpendicular to the interface. Thus, Equation (7.1) in the horizontal direction reduces to:

\[ K_{Th} \left( \frac{\partial^2 T}{\partial x^2} \right) - V_f \rho_f c_{pf} \left( \frac{\partial T}{\partial x} \right) = \rho_b c_{pb} \left( \frac{\partial T}{\partial t} \right) \]  \hspace{1cm} (7.3)

The above equation can be transformed to a reference axis aligned with the edge of the moving chamber by the substitution of variables, \( \xi \rightarrow x - Ut \), to get the following result:

\[ \frac{K_{Th}}{\rho_b c_{pb}} \left( \frac{\partial^2 T}{\partial \xi^2} \right) + \left( U - V_f \frac{\rho_f c_{pf}}{\rho_b c_{pb}} \right) \left( \frac{\partial T}{\partial \xi} \right) = \frac{\partial T}{\partial t} \]  \hspace{1cm} (7.4)

In the above equation the densities, heat capacities and convective phase velocities are all non-linear functions of temperature and solvent concentration, as evident from the correlations given in Table 7.1.

### 7.2.2 Theory of Mass Transfer

Mass transfer of non-condensable gas and solvent, by molecular diffusion and longitudinal dispersion mechanisms, was discussed in Chapters 5 and 6, respectively. This can be described mathematically by the Fick’s first law (in one dimension) as:

\[ J = -D_{eff, oil} \frac{\partial C_{oil}}{\partial x} \]  \hspace{1cm} (7.5)

where \( J \) is the molar diffusive flux, \( D_{eff, oil} \) is the effective diffusion coefficient in the oil phase that includes the contribution from molecular diffusion as well as hydrodynamic dispersion (Perkins and Johnston, 1963), and \( C_{oil} \) is the molar concentration of the diffusing specie (solvent or non-condensable gas) in the oil phase. In addition to diffusion, convective mixing in oil takes place due to the bulk velocity of the mobile oil
Table 7.1: List of correlations and associated parameters used in this study.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlation</th>
<th>Constants</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of Athabasa bitumen</td>
<td>( \ln(\mu_o) = A \ln(T + 273.15) + B )</td>
<td>( A = -3.5738 ) ( B = 22.8379 )</td>
<td>Mehrotra and Svreck (1984)</td>
</tr>
<tr>
<td></td>
<td>( \mu_o ) in cp, ( T ) in °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity of diluent-like solvents</td>
<td>( \log \mu_s = A \left( \frac{1}{T} - \frac{1}{B} \right) )</td>
<td>Hexane: ( A = 362.79 ) ( B = 207.09 )</td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td></td>
<td>( \mu_s ) in cp, ( T ) in K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diffusion coefficients for hexane and oil</td>
<td>( D_{s,oil}(T^\circ C) = D_{s,oil}(20^\circ C) \left[ \frac{T + 273.15}{20 + 273.15} \right]^{0.545} \left( \frac{\mu_o @ 20^\circ C}{\mu_s @ T^\circ C} \right) )</td>
<td>( D_{s,oil}(20^\circ C) = 3 \times 10^{-11} )</td>
<td>Das and Butler (1989)</td>
</tr>
<tr>
<td></td>
<td>( D_{o,oil}(20^\circ C) = 3 \times 10^{-11} )</td>
<td></td>
<td>Butler and Yee (1986)</td>
</tr>
<tr>
<td></td>
<td>( D_{s,oil}(T^\circ C) = D_{s,oil}(20^\circ C) \left[ \frac{T + 273.15}{20 + 273.15} \right]^{0.545} \left( \frac{\mu_s @ 20^\circ C}{\mu_s @ T^\circ C} \right) )</td>
<td>( D_{s,oil}(20^\circ C) = 3 \times 10^{-10} )</td>
<td>Ayodele et al. (2010)</td>
</tr>
<tr>
<td></td>
<td>( D_{o,oil}(T^\circ C) = D_{o,oil}(20^\circ C) )</td>
<td></td>
<td>Perkins and Johnston (1963)</td>
</tr>
<tr>
<td>Concentration dependence:</td>
<td>( D_{oil,eff} = (D_{o,oil})^{x_o} (D_{s,oil})^{x_s} \left( \frac{\phi}{T} \right) + \alpha_s U )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( D_o, D_s ) in m²/s, ( T ) in °C, ( \mu ) in cp, ( x_s ) in mole fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solvent solubility</td>
<td>( K)-value = \left( \frac{k_{v1} + k_{v2}p + k_{v3}}{p_i} \right)^{\frac{k_{v4}}{k_{v5}}} )</td>
<td>( k_{v1} = 1 \times 10^6 ) ( k_{v2} = k_{v3} = 0 ) ( k_{v4} = -2697.5 ) ( k_{v5} = -224.37 )</td>
<td>Reid et al. (1977) CMG (2009)</td>
</tr>
<tr>
<td></td>
<td>( T ) in °C, ( p ) in kPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity of solvent-oil mixtures</td>
<td>( \ln(\mu_{oil}) = (1 - x_s) \ln(\mu_o) + x_s \ln(\mu_s) )</td>
<td></td>
<td>Reid et al. (1977) CMG (2009)</td>
</tr>
<tr>
<td></td>
<td>( \mu ) in cp, ( x_s ) is solvent mole fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat capacity as a function of temperature</td>
<td>( c_{Po} = 1.605 + 4.361 \times 10^3 T - 4.04 \times 10^7 T^2 )</td>
<td></td>
<td>Butler (1997a) tortike and Farouq Ali (1989)</td>
</tr>
<tr>
<td></td>
<td>( c_{Pw} = 4.182 - 1.5 \times 10^{-4} T + 3.44 \times 10^7 T^2 + 4.2 \times 10^{-8} T^3 )</td>
<td>( c_{Pv} = 0.715 + 1.71 \times 10^{-3} T - 1.908 \times 10^{-6} T^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \rho_b c_{Ph} = (1 - \phi) \rho_o c_{Pr} + \phi \rho_s \rho_o c_{Po} + \phi \rho_w \rho_o c_{Pw} )</td>
<td>( \rho_b c_{Ph} = (1 - \phi) \rho_v c_{Pv} + \phi \rho_s \rho_v c_{Ps} + \phi \rho_w \rho_v c_{Pw} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T ) in °C, ( cp ) in kJ/(kg·°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density as a function of temperature</td>
<td>( \rho_o = 1009 - 0.634 T )</td>
<td></td>
<td>Tortike and Farouq Ali (1989)</td>
</tr>
<tr>
<td></td>
<td>( \rho_w = 1000.7 - 0.1616 T - 0.00262 T^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T ) in °C, ( \rho ) in kg/m³</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

193
phase. Thus, the overall change in the concentration field with time is predicted from a mass balance of the solute that accounts for mass accumulation and convective, as well as diffusive-dispersive transport as given below (Bird et al., 2001):

$$\frac{1}{\rho_o} \frac{\partial}{\partial x} \left( \rho_o D_{\text{eff,oil}} \frac{\partial C_{\text{oil}}}{\partial x} \right) - V_o \frac{\partial C_{\text{oil}}}{\partial x} + \frac{\partial C_{\text{oil}}}{\partial t} = 0$$  \hspace{1cm} (7.6)

where $V_o$ is the convective velocity of the mobilized oil phase, in the direction normal to the chamber edge. The above equation can be transformed to a reference axis aligned with the edge of the moving chamber by the substitution of variables, $\xi \rightarrow x - Ut$, to get the following result:

$$\left(U - V_o\right) \frac{\partial C_{\text{oil}}}{\partial \xi} + \frac{1}{\rho_o} \frac{\partial}{\partial \xi} \left( \rho_o D_{\text{eff,oil}} \frac{\partial C_{\text{oil}}}{\partial \xi} \right) = \frac{\partial C_{\text{oil}}}{\partial t}$$  \hspace{1cm} (7.7)

In the above equation the density, effective diffusivity, and convective phase velocity are all non-linear functions of temperature and dissolved solvent or non-condensable gas concentration, as evident from the correlations listed in Table 7.1.

### 7.2.3 Convective Velocity Calculation

The convective velocities of the water and oil phases, in the direction normal to the chamber edge, depend on their relative permeability and viscosity variations in that direction. Following the derivation in Chapter 4, the water and oil phase velocities at any point beyond the chamber interface can be obtained from Darcy’s law as:
\[ V_w = U \frac{k_{rw} / \mu_w}{k_{rw\text{int}} / \mu_{w\text{int}}} \]
\[ V_o = U \frac{k_{ro} / \mu_o}{k_{ro\text{int}} / \mu_{o\text{int}}} \]

where \( k_{rw} \) is the relative permeability and \( \mu_w \) is the viscosity of the water phase, and \( k_{ro} \) and \( \mu_o \) are the relative permeability and viscosity, respectively, for the oil phase, at temperature \( T \). Subscript \( \text{int} \) denotes the respective property at the chamber interface. In the above analysis, the pressure difference required for convective flow is implied through the translational velocity \( U \).

Similar to the derivation in Chapters 3 and 4, the oil and water phase relative permeabilities are correlated in the form of Corey’s equations as:

\[ k_{ro} = k_{rcw} (1 - S_{wD})^n, \text{ for oil phase, and } k_{rw} = k_{rco} (S_{wD})^n, \text{ for water phase} \quad (7.9) \]

In the analytical and numerical results obtained in Chapters 5 and 6, it was observed that in the mobile zone, the penetration of the gas phase is limited to only few centimeters (5-10 cm) beyond the chamber edge (see Figures 5.6d, 5.9c, 6.4 and 6.16d). Therefore, in the analysis presented in this chapter, the gas phase saturation in the mobile zone was neglected. The oil saturation profile beyond the SAGD chamber edge is assumed to be a linear function of the dimensionless temperature, \( T^* \), as was done in the derivations in the previous chapters:

\[ S_o = S_{or} + (S_{io} - S_{or})(1 - T^*) \quad (7.10a) \]
where,

\[ T^* = \frac{T - T_r}{T_s - T_r} \]  (7.10b)

and, \( S_o \) is the oil saturation at the dimensionless temperature \( T^* \), \( S_{io} \) is the initial oil saturation, and \( S_{or} \) is the residual oil saturation. The results from Equations (7.9) and (7.10), along with the viscosity correlations listed in Table 7.1, were substituted in Equation (7.8) to obtain the convective velocities for the oil and water phases.

### 7.2.4 Coupled Heat and Mass Transfer Equations

Solvent and non-condensable gas assisted steam based processes are characterized by coupled heat and mass transfer beyond the edge of the chamber, whose interaction controls the dynamics of the process. In the mobile zone, the oil phase viscosity strongly depends on the temperature and dissolved solvent or gas concentration profiles, which are obtained by solving Equations (7.4) and (7.7), respectively. As seen from the correlations listed in Table 7.1, the two equations are coupled and interdependent on each other through common fields. Therefore, to obtain the overall oil mobilization in solvent or non-condensable gas assisted steam based processes, the heat and mass transfer equations must be solved simultaneously with specified boundary conditions.

The boundary conditions are as follows: first, at the interface, \( \xi = 0 \), \( T = T_s \), the injected steam-solvent or steam-gas mixture temperature, and \( C_{oil} = C_i \), the dissolved

---

\[ \text{In this analysis, } T^* \text{ is defined for the temperature range } T_s \text{ to } T_r, \text{ however in order to model the temperatures occurring in a typical ‘mobile zone’, } T^* \text{ can be modified to model the temperature range } T_s \text{ to } T_s/2, \text{ as done in Chapters 5 and 6. This would alter the thickness of the mobile zone; however, the general results would remain unaffected.}\]
solvent or non-condensable gas concentration that will exist in the oil phase at the edge of the chamber, which is set by the solubility at the pressure, and the temperature at the chamber edge. Second, deeper into the cold reservoir, that is, as \( \xi \to \infty \), then \( T \to T_r \), the original reservoir temperature, and \( C_{oil} \to 0 \). The overall heat and mass transfer equations can be summarized as:

\[
\frac{\partial C_{oil}}{\partial t} = (U - V_o) \frac{\partial C_{oil}}{\partial \xi} + \frac{1}{\rho_o} \frac{\partial}{\partial \xi} \left( \rho_o D_{oil,eff} \frac{\partial C_{oil}}{\partial \xi} \right)
\]

\[
\frac{\partial T}{\partial t} = \left( U - V_f \frac{\rho_f c_p_r}{\rho_b c_p_b} \right) \frac{\partial T}{\partial \xi} + \frac{K_{th}}{\rho_d c_p_b} \frac{\partial^2 T}{\partial \xi^2}
\]

Given the non-linearity and time variance inherent in the above equations, it is difficult to obtain an analytical closed-form solution for the heat and mass transfer problem. The above set of coupled, partial differential equations, were therefore programmed in the technical computing software, Matlab (Matlab, 2008) and solved numerically.

7.3 Results and Discussion

Table 7.2 lists the properties of the oil sands reservoir and the operating conditions used in this study. The effect of convective heat and mass transport in the mobile zone of solvent-aided SAGD process was studied with hexane as the injected solvent. In this work, the ‘mobile zone’ is defined as the narrow zone beyond the steam condensation front where multiphase flow of oil, water and gas takes place and the ‘edge of the chamber’ is mathematically defined as the vapor-oil interface beyond which the
Table 7.2: Reservoir properties and operating conditions used in the solvent-aided SAGD case analyzed using Matlab (Ayodele et al., 2010; Ito and Hirata, 1999).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial reservoir temperature, $T_r$, °C</td>
<td>5</td>
</tr>
<tr>
<td>Oil density, $\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>Solvent (hexane) density, $\rho_s$, kg/m$^3$</td>
<td>664</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>Thermal diffusivity, $\alpha$, m$^2$/s</td>
<td>$7 \times 10^{-7}$</td>
</tr>
<tr>
<td>Permeability, $k$, m$^2$</td>
<td>$2 \times 10^{-12}$</td>
</tr>
<tr>
<td>End point oil relative permeability, $k_{roCW}$</td>
<td>0.9</td>
</tr>
<tr>
<td>Mechanical dispersivity, $\alpha_x$, m</td>
<td>0.0001</td>
</tr>
<tr>
<td>Initial oil saturation, $S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Residual water saturation, $S_{wr}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Oil viscosity, $\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
<tr>
<td>Reservoir thermal conductivity, $K_{TH}$, W/(m-K)</td>
<td>1.54</td>
</tr>
<tr>
<td>Chamber expansion rate, $U$, cm/day</td>
<td>30</td>
</tr>
<tr>
<td>Corey coefficients</td>
<td>$\frac{a}{b}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection pressure, $p_i$, kPa</td>
<td>2363</td>
</tr>
<tr>
<td>Injected steam temperature, °C</td>
<td>220</td>
</tr>
<tr>
<td>Injected hexane mole fraction, $y_i$</td>
<td>0.1</td>
</tr>
<tr>
<td>SAGD chamber temperature, $T_s$, °C</td>
<td>215.5</td>
</tr>
<tr>
<td>Steam quality</td>
<td>1</td>
</tr>
</tbody>
</table>
temperature declines below the injected steam and non-condensable gas or steam and solvent mixture temperature ($T<T_s$). The results from Equation (7.11) were compared with the case without convective heat and mass transfer effects.

In the analytical theories presented in Chapters 3–6, pseudo steady-state expansion of the chamber was assumed, for which the heat and mass transfer equations were time invariant. Such an assumption is acceptable for the case of a mature SAGD chamber at late times, when the transient effects are subdued. On the basis of this reasoning, the analytical equations in Chapters 3–6 were derived for a mature SAGD chamber which has reached the top of the primary pay zone, and is only expanding sideways. It can be shown mathematically, that the solutions to the transient heat transfer equation, Equation (7.4), approaches the pseudo steady-state solution roughly for time, $t > \alpha/U^2$. Similarly, the solution to the transient mass transfer equation, Equation (7.7), will asymptotically approximate the pseudo steady-state solution, roughly for time, $t > D/U^2$. This implies that for the set of parameters listed in Table 7.2, for time, $t > 16$ hrs, the solution to the transient equations will approximately approach the pseudo steady-state solution.

The effect of time on the temperature and hexane concentration profiles (which was converted to hexane mole fraction in the oil phase) obtained from Equation (7.11), are displayed in Figures 7.2 and 7.3, respectively. Distance, $\xi$, is measured from the edge of the translating steam-solvent chamber. The results reveal that after about 12 hours the profiles show little variation with time. This is consistent with the mathematical reasoning discussed in the previous paragraph. The time invariance of the results at late
Figure 7.2: Effect of time on the temperature profile beyond the edge of the steam-solvent chamber.

Figure 7.3: Effect of time on hexane mole fraction profile in the oil phase beyond the edge of the steam-solvent chamber.
times also confirms the assumption of pseudo steady-state expansion and time invariance made in Chapters 3–6 for mature SAGD operations.

Following the observations in Figures 7.2 and 7.3, the temperature, hexane mole fraction, and oil phase mobility profiles were obtained at t=18 hrs (for the pseudo steady-state period), and compared with the case without convective heat and mass transfer effects. Figure 7.4 shows a comparison of the temperature profiles beyond the SAGD chamber edge, with and without convective heat and mass transfer effects. The highest temperature for both cases is about 215.5 °C, corresponding to the reduced steam partial pressure (= 2127 kPa) due to solvent injection. The length scale for heat transfer is about 1 m corresponding to the operating conditions listed in Table 7.2. This is similar to the results obtained in the previous chapters (Figures 3.12a, 4.11, 5.6a, 6.2). The results reveal that the temperature is enhanced (by up to 14 °C) when convective effects are included. However, the enhancement is limited to only first few centimeters beyond the chamber edge. The reason for this is that close to the chamber walls, water saturation is high and consequently water phase has high relative permeability, which results in higher convective velocity. Also, the shape of the temperature profile for the convective transport case no longer has the exponential decay, typical of pure conduction, but is concave downwards within the convection-dominated zone. This result is similar to the observations in Chapter 4 (Figures 4.4 and 4.12).

The convective oil and water phase velocities are displayed in Figure 7.5. The plot reveals that the highest water phase velocity (which primarily contributes to the convective heat transfer) occurs at the chamber edge, while the highest oil phase velocity
Figure 7.4: Temperature profiles beyond the edge of steam-solvent chamber, with and without convective heat and mass transfer effects.

Figure 7.5: Convective flow velocities of the oil phase and the water phase as a function of distance from the edge of the steam-solvent chamber.
(which primarily contributes to the convective solvent mixing), occurs some distance beyond the SAGD chamber edge. The result also shows that the convective velocity for the water phase is several times higher than that for the oil phase. This is because water has significantly lower viscosity as compared to bitumen.

Figure 7.6 shows a comparison of the solvent (hexane) mole fraction profiles in the oil phase beyond the edge of the SAGD chamber, with and without convective heat and mass transfer effects. At the chamber edge, the mole fraction of hexane dissolved in the oil phase corresponds to the hexane solubility in oil at the temperature and pressure at the edge of the chamber, which can be calculated from the $K$-value correlation listed in Table 7.1. The results reveal that the solvent mass transfer length scale as well as solvent mole fraction in the oil phase is slightly enhanced when convective transport effects are included. The enhancement is up to about 0.05 mole fraction hexane within the convection-dominated zone. The enhancement is highest where the oil phase velocity is highest, as seen from Figure 7.5.

Figure 7.7 shows a comparison of the oil phase mobility (defined as the ratio of the oil phase effective permeability and the oil phase viscosity) profiles with and without the convective transport effects, beyond the edge of the SAGD chamber. The results reveal that similar to the results obtained in the previous chapters (Figures 3.12b, 4.6, 5.6f, and 6.8), the most mobile oil is not at the edge of the SAGD chamber but rather some distance beyond it. The maximum oil mobility value for both the cases are almost identical (about 22 mD/cp). Close to the chamber edge, the oil phase mobility for the convective transport case is lower than that for the case without convection; however,
Figure 7.6: Solvent mole fraction profile in the oil phase as a function of distance from the edge of the steam-solvent chamber, with and without convective heat and mass transfer effects.

Figure 7.7: Oil phase mobility profiles beyond the edge of steam-solvent chamber, with and without convective heat and mass transfer effects.
beyond the first few centimeters, the mobility profile for the convective transport case becomes slightly higher than the non-convective case. This result corresponds directly with the regions where enhanced temperature and enhanced solvent mole fraction due to convection occur, as seen in Figures 7.4 and 7.6, respectively.

For the convective transport case, the lower oil mobility in the region close to the chamber edge is due to the relative permeability effect. As the condensate invades the oil sands, the oil saturation drops, and consequently the oil relative permeability declines; this in turn results in the reduction of the oil mobility despite the lowered oil viscosity due to higher temperature arising from convective heat transfer. Beyond the first few centimeters within the mobile zone, the enhanced solvent penetration (as seen in Figure 7.6) due to the convective solvent transport is the reason for higher oil phase mobility for the convective transport case. Overall, these results show that as a result of convective transport, the heat and solvent transfer beyond the chamber edge are enhanced individually; however, the overall impact on the oil phase mobility is slight to negligible.

7.4 Difference between Current Model and Models Derived in Chapters 5 and 6
The coupled heat and mass transfer model derived in this chapter, will reduce to the analytical models derived in Chapters 5 and 6, when the convective solvent mixing and coupling of the heat and mass transfer equations (that is, the dependence of temperature on solvent concentration via the convection term) and the effect of time, are removed. However, apart from the above distinctions, there is another key difference between the
heat and mass transfer model derived here, and the derivations presented in Chapters 5 and 6. This is schematically explained in Figure 7.8, and described below.

In the model described in this chapter, the gas saturation in the mobile zone was assumed to be zero. This resulted in a monotonically decreasing solvent concentration profile in the oil phase beyond the chamber edge (Figures 7.3 and 7.6). In the heat and mass transfer models derived in Chapters 5 and 6, the gas phase saturation in the mobile zone was not neglected, which resulted in a concentration profile which had a maxima some distance beyond the chamber edge (in Figures 5.6 and 6.5, gas/solvent concentration peaks at $\xi^*$) because of the effect of increasing solvent solubility with decreasing temperature. The assumption of no gas saturation in the mobile zone, in the model described in this chapter, was made for the following reasons: (1) the analytical and numerical results obtained in Chapters 5 and 6, showed that the penetration of the gas phase in the mobile zone, is limited to only few centimeters (5-10 cm) beyond the chamber edge (see Figures 5.6b, 5.8b, 6.4 and 6.11), and (2) as postulated in Chapters 5 and 6, the diffusive mixing of solvent or non-condensable gas in the oil phase will not begin until the gas phase saturation in the mobile zone becomes zero (at $\xi^*$). Since the objective of this chapter was to present a framework for solving the diffusive-convective heat and mass transfer equations, only the diffusion-active region in the mobile zone was considered, where the gas saturation is zero.
Figure 7.8: Schematic of the mobile zone beyond the SAGD chamber in (a) Chapter 7 and (b) in Chapters 5 and 6. ($S_g=$Gas Saturation)
7.5 Conclusions

A set of coupled transient heat and mass transfer equations were derived to examine the role of convective transport in solvent and non-condensable gas assisted steam based gravity drainage processes. The results demonstrate that the solution to the transient heat and mass transfer equations will asymptotically approach the pseudo steady-state solution at late times. The length of time to reach pseudo steady-state depends mainly on the chamber expansion rate and the thermal diffusivity of the reservoir \((t > \alpha/U^2)\). By including the convective transport effect, the heat transfer and solvent mass transfer beyond the SAGD chamber edge are enhanced, individually. However, the enhancement in the overall oil mobility (and hence oil production rate) by inclusion of convective transport effects is insignificant. The main reasons for this are: (1) the enhancement in temperature and solvent concentration due to convective transport occur in different regions within the mobile zone, with dissimilar length scales, and (2) the relative permeability effect competes against the convective heat transfer effect resulting in lower oil mobility in spite of enhanced temperature. The results from this analysis indicate that, given the advanced computation requirement and insignificant impact on flow rate, the effect of convective transport may be ignored for the derivation of closed form analytical models, as done in Chapters 3–6.
Chapter 8:

INTERFACIAL STABILITY IN STEAM-SOLVENT PROCESSES

8.1 Introduction

There are many factors that influence the efficiency and rate of oil mobilization in steam based recovery processes such as SAGD. One of them is the stability of the steam-oil interface, which is controlled by the relative magnitudes of viscous, gravity (buoyancy), and capillary forces acting at the interface. In this chapter, theoretical evidence for occurrence of such instability in steam and steam-solvent processes is presented. The stability of the interface at the edge\(^6\) of a SAGD chamber is examined to determine how solvent co-injection with steam in SAGD, affects the performance of the process.

8.2 Theory

Within the SAGD chamber, oil drains under the action of gravity until it reaches the residual oil saturation, \(S_{or}\). The temperature of the vapour phase is equal to the saturation temperature of the steam at its partial pressure. Thus, the temperature of the oil phase at the edge of the chamber is equal to the steam temperature, \(T_s\), corresponding to its partial pressure. The solubility of the solvent in the oil phase at the edge of the chamber is set by the solubility of the solvent at the partial pressure of the solvent in the vapour phase and the temperature at the edge of the chamber.

\(^6\) In this work, the ‘edge of the chamber’ is defined mathematically as the steam-oil interface beyond which the temperature declines below the temperature inside the SAGD chamber \((T<T_s)\).
Here, following Chouke et al. (1959), one-dimensional displacement of mobilized oil by steam and steam-solvent mixtures has been considered beyond the edge of the chamber. Figure 8.1 displays a schematic of a section of the interface of the vapour chamber as would be seen for a solvent-aided SAGD operation. The system under study consists of a two-dimensional homogeneous porous medium. Steam, or solvent, or both are below the interface whereas bitumen is above. The solvent remains in the vapour phase within the chamber. At the interface, the solvent condenses and mixes with the oil phase according to its solubility at the solvent partial pressure in the vapour phase in the chamber and the temperature at the edge of the chamber.

8.3 Analytical Model

8.3.1 Modelling Assumptions

To study the stability of the interface, the following assumptions were made:

1. The SAGD chamber is rising in the vertical direction (z direction).
2. Porous medium is homogeneous (constant porosity and permeability).
3. The fluids are initially separated by a sharp interface with vapour phase below the interface and mobilized bitumen above. Since bitumen and steam are immiscible, the assumption of a sharp interface is reasonable for the purpose of mathematical analysis (Saffman and Taylor, 1958; Chouke et al., 1959).
4. Heat transfer and mass transfer are directed normal to the edge of the chamber.
5. Boussinesq approximation (that is, variation in density is neglected everywhere except in the buoyancy term).
Figure 8.1: Schematic diagram of steam-solvent bitumen interface.
6. Density and viscosity depend on temperature and solvent concentration.

### 8.3.2 Mathematical Formulation

The conservation of momentum for the steam and oil phases can be expressed by using Darcy’s law as:

\[
\begin{align*}
    u_o &= -\frac{k_o}{\mu_o} (\nabla p + \rho_o g e_z) \\
    u_v &= -\frac{k_v}{\mu_v} (\nabla p + \rho_v g e_z)
\end{align*}
\]

where the subscripts \(o\) and \(v\) denote the oil phase and steam (or steam-solvent) vapour phase, respectively; \(u_i\) is the Darcy phase velocity, \(\mu_i\) is the viscosity, \(\rho_i\) is the density, and \(k_i\) is the effective permeability, for the corresponding phase \(i\); \(p\) is the pressure, and \(e_z\) is the unit vector in the vertical (\(z\)) direction. Here, Cartesian coordinates are chosen such that the positive \(z\) axis is directed from the vapour phase to the oil phase, as shown in Figure 8.1. Thus, the velocity of the interface in the direction opposite to gravity is considered positive. Equations (8.1a) and (8.1b) can be re-written in terms of the velocity potential for each phase, \(\phi\), as:

\[
    u = -\nabla \phi
\]

where,

\[
    \phi = \frac{k}{\mu} p + \frac{k}{\mu} \rho g z
\]

which can be rearranged as,
\[ p = \frac{\mu}{k} \varphi - \rho g z \]  
(8.4)

where \( z \) denotes the vertical position of the interface. To study the stability of the interface, we examine the growth of perturbations of wavelength \( 2\pi/\beta \), which can be described as (Chouke et al., 1959):

\[ z' = \overline{z} \exp\left[ \gamma t + i(\beta_x x + \beta_y y) \right] \]  
(8.5)

where, \( z' \) denotes the fundamental Fourier perturbation of initial amplitude \( \overline{z} \), \( \gamma \) is the growth rate of the perturbation, and \( \beta_x e_x + \beta_y e_y \) is the spatial propagation vector of the normal modes of the perturbation (where \( e_x \) and \( e_y \) are the unit vectors in the \( x \) and \( y \) directions, respectively) of magnitude:

\[ \beta = \sqrt{\beta_x^2 + \beta_y^2} \]  
(8.6)

In Equation (8.5), \( \gamma \) is a measure of the time scale of the rise of the steam chamber interface shown in Figure 8.1, while \( \beta \) gives the wave number (=\( 2\pi \)/wavelength) associated with the disturbance \( z' \). The wavelength of the perturbation physically represents the peak-to-peak separation between the fingers at the interface in Figure 8.1.

By assuming that the fluids are incompressible and the oil sands matrix has uniform porosity, the equation of continuity is given by:

\[ \nabla \cdot u = 0 \]  
(8.7)

On combining Equations (8.2) and (8.7) we get:

\[ \nabla^2 \varphi = 0 \]  
(8.8)
At the interface, for continuity of velocity, the following condition on the velocity potentials holds:

\[
\frac{\partial z'}{\partial t} = -\frac{\partial \varphi_o}{\partial z}
\bigg|_{z=z'} = -\frac{\partial \varphi_v}{\partial z}
\bigg|_{z=z'}
\] (8.9)

Appropriate solutions for the velocity potential for the oil phase for which, the disturbance vanishes at infinity, under the assumption of first order perturbation, are:

\[
\varphi_o = \frac{i\alpha}{\beta} \exp\left[-\beta \cdot z + \gamma \cdot t + i\left(\beta_x x + \beta_y y\right)\right]
\] (8.10)

which vanishes as \(z \to +\infty\), and for steam,

\[
\varphi_v = -\frac{\gamma}{\beta} \exp\left[\beta \cdot z + \gamma \cdot t + i\left(\beta_x x + \beta_y y\right)\right]
\] (8.11)

which vanishes as \(z \to -\infty\). From Equation (8.4), the pressure drop across the oil and steam phases at the interface is given by:

\[
(p_v - p_o)_{z=z'} = \left(\frac{\mu_v}{k_v} \varphi_v - \frac{\mu_o}{k_o} \varphi_o\right) - (\rho_v - \rho_o)gz'
\] (8.12)

The interfacial tension acts to shorten the macroscopic perturbation at the interface and thus stabilizes the interface. The capillary pressure gradient is inversely proportional to the radius of curvature of the perturbed interface and therefore is large for the small fingers. The pressure discontinuity arising from capillary forces across the interface at \(z = z'\) is given by:

\[
(p_v - p_o)_{z=z'} = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)
\] (8.13)
Here $\sigma$ is the interfacial tension at the interface, and $R_1$ and $R_2$ are the principle radii of curvatures at the interface. Thus, from Equations (8.12) and (8.13), the pressure gradient required to make the interface deform must be such that:

$$
\left(\frac{\mu_v}{k_v} \varphi_v - \frac{\mu_o}{k_o} \varphi_o\right) - (\rho_v - \rho_o)gz > \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)
$$

(8.14)

For infinitesimal disturbances, the slope of the interface is small compared to unity, which allows the curvature of the interface to be approximated as (Muskat, 1937):

$$
\frac{1}{R_1} = -\left(\frac{\partial^2 z'}{\partial x^2}\right)
$$

(8.15a)

$$
\frac{1}{R_2} = -\left(\frac{\partial^2 z'}{\partial y^2}\right)
$$

(8.15b)

Equations (8.15a) and (8.15b) are substituted into Equation (8.14) by using the values of $z'$, $\varphi_o$ and $\varphi_s$, which yields the following inequality for first order perturbations:

$$
-\frac{\gamma \cdot \vec{z}}{\beta} \left(\frac{\mu_v}{k_v} + \frac{\mu_o}{k_o}\right) - (\rho_v - \rho_o)gz > (\sigma \cdot \vec{z})\beta^2
$$

(8.16)

Thus, the characteristic equation becomes:

$$
\sigma \beta^3 + \gamma \left(\frac{\mu_v}{k_v} + \frac{\mu_o}{k_o}\right) - \beta (\rho_o - \rho_v)g = 0
$$

(8.17)

When the growth rate of perturbation, $\gamma$, is positive, any small disturbance at the interface grows exponentially, in other words, the system is unstable. From Equation (8.17), to obtain a positive $\gamma$, the Fourier decomposition of the perturbation must contain a
wavelength — greater than a minimum critical wavelength, \( w_c \), which can be derived as follows:

\[
\gamma \left( \frac{\mu_v}{k_v} + \frac{\mu_o}{k_o} \right) = \beta (\rho_o - \rho_v)g - \sigma \beta^3 > 0
\]

\[
\Rightarrow \frac{(\rho_o - \rho_v)g}{\sigma} > \beta^2 \Rightarrow \beta_c = \sqrt{\frac{(\rho_o - \rho_v)g}{\sigma}}
\]

Thus, the critical wavelength is given by:

\[
w_c = \frac{2\pi}{\beta_c} = 2\pi \sqrt{\frac{\sigma}{(\rho_o - \rho_v)g}}
\]

(8.18b)

The perturbation wavelength with the largest growth rate, \( w_m \), is found by differentiating Equation (8.17) with respect to \( \beta \) and equating the result to zero. This can be shown to be related to the critical wavelength, \( w_c \), as:

\[
w_m = \sqrt{3}w_c = 2\sqrt{3}\pi \sqrt{\frac{\sigma}{(\rho_o - \rho_v)g}}
\]

(8.19)

In Equation (8.17), the densities and viscosities of the oil and steam-solvent phases depend on the temperature and pressure at the interface. For the oil phase, the solvent content depends on the solvent partial pressure and temperature in the vapour chamber. Also, the interfacial tension depends on the temperature. Table 8.1 lists the correlations used here for phase properties and interfacial tension for Athabasca bitumen and solvents under consideration. In this study, the stability of oil displacement in steam-only system is compared to that of steam and propane, steam and butane, steam and hexane, steam and heptane, and steam and octane systems. The solvent content in the vapour chamber is
Table 8.1: List of correlations and associated parameters used in this study.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlations</th>
<th>Constants (if needed)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of Athabasca bitumen ( \mu_o ) in cp, ( T ) in °C</td>
<td>( \ln \ln(\mu_o) = A \ln(T + 273.15) + B )</td>
<td>( A = -3.5738 ), ( B = 22.8379 )</td>
<td>Mehrotra and Svrcek (1986)</td>
</tr>
</tbody>
</table>
| Viscosity of diluent like solvents \( \mu_s \) in cp, \( T \) in K | \( \log \mu_s = A \left( \frac{1}{T} - \frac{1}{B} \right) \) | Propane: \( A = 222.67 \), \( B = 133.41 \)  
Butane: \( A = 265.84 \), \( B = 160.2 \)  
Hexane: \( A = 362.79 \), \( B = 207.09 \)  
Heptane: \( A = 436.73 \), \( B = 232.53 \)  
Octane: \( A = 473.7 \), \( B = 251.7 \) | Reid et al. (1977) |
| Viscosity of saturated water \( \mu_w \) in cp, \( T \) in °C | \( \mu_w = \left( \frac{969 - T}{25.6T + 405.7} \right) \) | | Gottfried (1965) |
| Viscosity of steam vapour \( \mu_v \) in cp, \( T \) in °C | \( \mu_v = \left[ 0.2 \left( \frac{9}{5}T + 32 \right) + 81.97 \right] \times 10^{-4} \) | | Gottfried (1965) |
| Density of Athabasca bitumen \( \rho_o \) and \( \rho_{osc} \) in kg/m³, \( T \) in °C | \( \rho_o = \frac{\rho_{osc}}{1 + \frac{T - 20}{1047}} \) | | Butler (1997a) |
| Density of saturated water \( \rho_w \) in kg/m³, \( T \) in °C | \( \rho_w = \left[ \frac{0.001 + 1.436 \times 10^{-6}}{0.00212868 T^2 + 0.134186 T - 4.8872} \right]^{-1} \) | | Butler (1997a) |
| Interfacial tension \( \sigma \) in mN/m, \( T_{ci} \) in K, \( P_{ci} \) in bar, \( \rho_L \) and \( \rho_v \) in kg/m³, \( P_L \) and \( P_v \) in \( \sum_i n_i P_i \) | \( \sigma = (P_L \rho_L - P_v \rho_v)^{\frac{3}{6}} \)  
\( P_i = (8.21307 + 1.97473 \omega) T^{1.03406_i} P_{ci}^{0.82636} \)  
\( P_L = \sum_i n_i P_i \), \( P_v = \sum_i n_i P_i \) | | Hugill and Welsenses (1986)  
Reid et al. (1977) |
<table>
<thead>
<tr>
<th>Property</th>
<th>Correlations</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent Solubility</td>
<td>[ K-value = \left( \frac{k_{v1}}{p_i} + k_{v2} p_i + k_{v3} \right) e^{\frac{k_{v4}}{k_{v5}} T - k_{v6}} ]</td>
<td>Reid et al. (1977)</td>
</tr>
<tr>
<td>( T ) in °C, ( p ) in kPa</td>
<td>Propane: ( k_{v1} = 900850, k_{v4} = -1872.46, k_{v5} = -247.99 )</td>
<td>CMG (2009)</td>
</tr>
<tr>
<td></td>
<td>Butane: ( k_{v1} = 859000, k_{v4} = -2154.9, k_{v5} = -238.73 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hexane: ( k_{v1} = 1006200, k_{v4} = -2697.55, k_{v5} = -224.37 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Heptane: ( k_{v1} = 1.04 \times 10^6, k_{v4} = -2911.32, k_{v5} = -216.6 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Octane: ( k_{v1} = 1.12 \times 10^6, k_{v4} = -3120.29, k_{v5} = -209.52, )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k_{v2} = k_{v3} = 0 ) (all solvents)</td>
<td></td>
</tr>
<tr>
<td>Viscosity of solvent-bitumen</td>
<td>[ \ln \mu_s = (1 - x_s) \ln \mu_o + x_s \ln \mu_s ]</td>
<td>CMG (2009)</td>
</tr>
<tr>
<td>mixtures</td>
<td>( \mu ) in cp, ( x_s ) is the solvent mole fraction</td>
<td></td>
</tr>
</tbody>
</table>
taken to be equal to 0.5 (in mole fraction) which is within the range of maximum solvent mole fraction in oil phase observed at the edge of the SAGD chamber in solvent-aided SAGD simulations reported in Chapter 6 (Section 6.6) as well as in other works (Gates and Chakrabarty, 2008). The solubility of the solvent in the oil phase is set by its partial pressure and temperature in the vapour phase at the chamber interface. The temperature is equal to the saturation temperature corresponding to the partial pressure of the steam.

8.4 Results and Discussion

Table 8.2 lists the properties of the oil sands reservoir used in this study. Figure 8.2 displays the growth rate (γ) of perturbation (z') for the steam-only and steam-solvent cases at 1, 2, 3, and 4 MPa total pressures. The growth rate was calculated from Equation (8.17) by substituting different values of β, or wavenumbers (0 ≤ β ≤ 800 in Figure 8.2), and using respective phase properties. The growth rate (γ) measures the time scale of the growth of disturbances at the interface. For a SAGD chamber, the growth rate of the disturbance is the rise rate of the chamber. The greater the growth rate, the higher the rise rate, which in turn implies the larger the oil production rate. Physically, the growth rate can be considered as an index of the mixing of the solvent in the vapour chamber and oil in the region near the interface. In addition, the area of the interface grows as the disturbance grows. The greater the area of the interface, the greater the solvent mixing and heat transfer at the interface, which implies higher mobilization of bitumen and higher oil rate.
Table 8.2: Reservoir properties used in the analytical model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability, ( k ), ( m^2 )</td>
<td>( 5 \times 10^{-13} )</td>
</tr>
<tr>
<td>Average relative permeability at the steam-oil interface</td>
<td></td>
</tr>
<tr>
<td>Gas phase, ( k_{rg} )</td>
<td>0.3</td>
</tr>
<tr>
<td>Oil phase, ( k_{ro} )</td>
<td>0.228</td>
</tr>
<tr>
<td>Initial reservoir temperature, ( T_r, ^\circ C )</td>
<td>10</td>
</tr>
<tr>
<td>Porosity, ( \phi )</td>
<td>0.39</td>
</tr>
<tr>
<td>Saturated steam pressure (in MPa) and corresponding temperature (in °C)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>234</td>
</tr>
<tr>
<td></td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>264</td>
</tr>
</tbody>
</table>
Figure 8.2: Growth rates ($\gamma$) for steam only and steam-solvent mixtures versus wavenumber ($\beta$). The pressure indicated in each plot is the total injection pressure. The mole fraction of the solvent in the vapour phase equals 0.5. The values are derived using Equation (8.17).
The results displayed in Figure 8.2 show that in all cases, the growth rates ($\gamma$) are positive over a range of wavenumbers. In other words, the systems are all unstable. At large wavenumber (small wavelength or narrow fingers), the growth rates become negative exhibiting stabilization due to capillary action. For the steam-only case, although the results reveal that the growth rates are relatively insensitive to pressure, the higher the pressure, the more unstable the system. At 4 MPa total pressure, the maximum growth rate ($\gamma$ in Equation (8.17)), corresponding to $w_m$ in Equation (8.19)) is roughly equal to $2.34 \times 10^{-5}$ s$^{-1}$. This implies that the time scale for the growth of the instability is of order of 12 hours. The perturbation wavelength with the largest growth rate is equal to about 2.1 cm (= twice the finger-width of the perturbation). Thus, the length scale of the fastest growing perturbation is small relative to the size the SAGD chamber, which is of order of tens of metres wide and high, and hundreds of metres long. The growth rates for the propane-steam system are larger than that of the steam-only system. Similar to the steam-only case, the higher the total pressure, the greater was the growth rate. At 4 MPa total pressure, the timescale for the growth of the instability with the maximum growth rate ($\gamma$ corresponding to $w_m$) is about 6.2 hours. For propane-steam, the wavelength of the fastest growing perturbation ($w_m$) is equal to that of the steam-only case. The results show that the addition of propane does not appreciably impact the likelihood of having fingers but that the fingers will grow more rapidly. Similarly, butane and hexane further increase the growth rate of steam fingers. At 4 MPa, the time scales for the growth of the instability for butane and hexane
are equal to about 3.2 and 0.73 hours, respectively. The wavelength of the fastest growing disturbance for the butane and hexane cases is roughly equal to that of the propane case. For heptane, the maximum growth rate is lower than that of hexane, and for octane, it is even lower. These results reveal that hexane appears to yield the most rapidly growing unstable system at 4 MPa total pressure with 0.5 mole fraction solvent mole fraction.

Figure 8.3 shows the impact of the solvent mole fraction in the vapour chamber on the maximum growth rate ($\gamma$ corresponding to $w_m$ in Equation (8.19)), at 3 MPa total pressure. The solvent content in the oil phase at the edge of the chamber is given by the $K$-value relationship for the solvent at the temperature and pressure of the chamber. For propane, the results show that the maximum growth rate is relatively insensitive to the concentration of solvent. For butane, over the range of solvent mole fraction considered, the larger the solvent mole fraction, the higher is the maximum growth rate, that is, the more unstable is the interface. For hexane, the maximum growth rate exhibits a maximum at 0.5 mole fraction solvent. For heptane, the value of the maximum growth rate is similar to that with hexane and occurs at 0.3 mole fraction solvent. Finally, for octane, the maximum occurs below 0.2 mole fraction solvent.

Figures 8.4a and 8.4b display the maximum growth rate ($\gamma$ corresponding to $w_m$ in Equation (8.19)) versus type of solvent at different total pressures, for 0.5 and 0.6 mole fraction solvent in the chamber, respectively. In the case where the solvent mole fraction at the edge of the chamber is equal to 0.5, the results show that at pressures above about 1 MPa, hexane yields the least stable system with the largest growth rates, for the expected
Figure 8.3: Growth rate of the most unstable perturbation (\( \gamma \) corresponding to \( w_m \) in Equation (8.19)) versus mole fraction of solvent in vapour chamber at 3 MPa total pressure.
Figure 8.4: Growth rate of the most unstable perturbation ($\gamma$ corresponding to $w_m$ in Equation (8.19)) versus solvent carbon number (0=steam only, 3 = propane, 4 = butane, 6 = hexane, 7 = heptane, 8 = octane) at different total pressure, with mole fraction of solvent in the vapour chamber equal to (a) 0.5 and (b) 0.6.
operating steam pressures in SAGD and solvent-aided SAGD operations. At total pressure equal to 1 MPa, heptane yields the least stable system. In the case where the solvent mole fraction at the edge of the chamber is equal to 0.6, the results show that hexane yields the largest growth rates between 1 and 5 MPa total pressure. The results show that there is an intermediate solvent (between the lightest and heaviest ones considered here) that gives the most unstable system.

The results of this study demonstrate that instability and higher growth rate of steam-solvent fingers could be the reason why steam-solvent recovery processes of Athabasca bitumen have better performance, both in terms of oil rate and thermal efficiency, than steam-only processes (Gupta and Gittins, 2005; Leaute and Carey, 2005). Therefore, there are three fundamental contributions to enhanced production by solvent-steam processes: first, the oil phase viscosity is lowered due to solvent dilution of oil, second, the oil phase saturation is slightly enhanced at the edge of the chamber due the mixing of solvent in oil, and third, the vapour-oil interface becomes more unstable on solvent injection with steam, which promotes more mixing at the chamber edge.

8.5 Validation with Experimental Data

The results reported in the previous section confirm the findings of the gravity drainage experiments conducted by Nasr and Isaacs (2001), displayed in Figure 8.5, which revealed that an intermediate solvent yielded the highest oil rates, among propane through nonane solvents, for their solvent-aided SAGD experiments. The experiments were conducted at an operating pressure of 2.1 MPa and injected solvent content was
Figure 8.5: Oil drainage rate from Nasr and Isaacs’ (2001) experiments versus solvent carbon number (0=steam only, 1 = methane, 2 = ethane, 3 = propane, 5 = pentane, 6 = hexane, 8 = octane). The experiments were conducted at an operating pressure of 2.1 MPa and injected solvent content equal to 1.5% by volume (near the interface, the concentration of solvent would have been substantially higher).
equal to 1.5% by volume (near the interface, the concentration of solvent would have been substantially higher). Nasr and Isaac’s (2001) experiments with Athabasca bitumen revealed that this intermediate solvent was hexane and they ascribed this to the saturation curve of hexane being similar to that of steam at their set of experimental operating conditions, that is, the solvent condenses under similar conditions as steam.

The results of this study also suggest that there is an intermediate solvent where the instability is maximized. However, the results of this research indicate that this is due to the competition between the solubility of the solvent versus the capability of the solvent to reduce the viscosity of the oil phase. That is, at constant pressure and solvent content in the vapour phase, of the solvents examined here, the larger its carbon number, the higher is its solubility in the oil phase. However, the higher the carbon number, the more viscous is the solvent phase, and its capability to dilute the bitumen and reduce its viscosity is eroded. Thus, there is an intermediate solvent that yields good solubility together with viscosity-breaking action.

8.6 Conclusions

A linear stability theory has been developed for the interface at the edge of a vapour chamber as would be found in SAGD and solvent-aided SAGD processes. The results of the stability theory reveal that the addition of solvent increases the growth rate of interfacial instability. Greater instability implies that there will be more mixing, and potentially more heat transfer, at the interface, which in turn means that, a larger amount of oil will be mobilized resulting in higher oil production rates. The results reveal that there is an intermediate solvent where the instability is maximized. Physically, this is due
to the balance between the solubility of the solvent versus the capability of the solvent to lower the viscosity of the oil phase. Under typical operating pressures of current SAGD operations, the results of this study reveal that hexane or heptane yield the most unstable conditions.
Chapter 9:

EXPERIMENTS AND ANALYSIS OF PORE SCALE VISCOUS FINGERING

9.1 Introduction

The problem of viscous fingering in porous media is of central importance to most oil recovery processes. Immiscible multiphase displacement exhibits a wide range of behaviours depending on the relative magnitude of viscous, capillary, and gravity forces. A greater understanding of the flow patterns that evolve during such displacements yields insights to improve predictive capability and increase oil recovery.

Viscous fingering in thermal and thermal-solvent recovery processes was discussed in Chapter 8, and analyzed using the continuum model for flow in porous media incorporating the multiphase flow extension of Darcy’s law. However, theoretical and experimental studies (Riaz et al., 2007; Tang and Kovscek, 2011) have shown that for micro (or pore) scale and meso (or core) scale displacements, the conventional Darcy-type modelling of fluid flux are not predictive under unstable, immiscible, imbibition conditions. When a less viscous fluid displaces a more viscous fluid at low capillary numbers, the displacement patterns show fractal, self-similar characteristics arising from viscous instabilities as well as local capillary control of interface movement. Under such

7 The material presented in this appendix was done as part of a research project at the Stanford University in collaboration with Dr. Anthony Kovscek (professor at the Energy Resources Engineering Department, Stanford University).
conditions, a detailed consideration of the progression of imbibing phase through the pores network of a porous medium is required.

The purpose of this study is to understand analytically, the flow patterns observed during forced imbibition experiments in silicon micromodels, carried out under unstable conditions and low flow rates. For the specific range of flow rates \( (N_c \sim 10^{-7}) \) and oil to water viscosity ratios \( (M \sim 8-400) \) considered in these experiments, both capillary and viscous forces are important and the displacement shows features similar to the invasion-percolation regime in the presence of a viscous gradient. Using statistical theories and concepts of fractal analysis, scaling power laws are derived to describe quantitatively, the functional forms for saturation, fractional flow, and capillary dispersion profiles, using the self-similarity characteristics inherent in the displacement patterns. Results show that functional relations of the scaling model are in considerable agreement with the experimental data.

The next section reviews the dynamics of fingering in capillary dominated flow regimes. The following section lays out the details of the etched-silicon micromodel experiments. A linear stability analysis is then applied to these experimental results and the need for statistical modelling to analyze complex flow patterns is discussed. The concepts of the percolation theory for capillary dominated displacements, and fractal properties are reviewed and applied to analyze the invading fluid pattern. Based on these fractal properties, scaling relations are derived and verified using the experimental results.
9.2 Theory

The immiscible displacement of one fluid by another in a porous media leads to a wide range of dynamic flow behaviours, depending on the relative magnitude of viscous, capillary, and gravity forces. Fingering instabilities are observed in two-phase flows either due to viscous effects, when the mobility of the displacing phase is greater than the mobility of the displaced phase, and/or due to the lack of local capillary equilibrium. When capillary effects are small, viscous instabilities manifest themselves as macroscopically growing viscous fingers. When capillary forces become more important compared to viscous and gravity effects, the finger widths are found to be of the same order as the pore size, and the overall shape of the pattern depends strongly on the pore structure and geometry.

Both continuum and non-continuum, discrete model approaches are used for study of transport properties through porous media. Conventional modelling is based on a continuum approach that describes the porous medium with macroscopic properties defined as averages of the corresponding microscopic quantities. In several situations, however, the conditions for the validity of averaging are not quite satisfied because of difficulty in describing macroscopic transport coefficients for the complex flow geometry.

When continuum conditions are not met, such as when the pore-scale fluctuations are not negligible, a non-continuum system definition must be taken. The non-continuum method often employs a network representation of the pore space in which any disordered porous medium is mapped onto an equivalent random network of
interconnected pore bodies and pore throats. Once this mapping is complete, one then studies transport and reaction processes in porous media and the complex displacement pattern in a more realistic manner. In this discrete representation, one employs the ideas and concepts of the statistical physics of disordered media such as percolation (invasion-percolation or ordinary percolation), scaling, and diffusion limited aggregation (Lenormand et al., 1988). Pore-scale fluctuations and the driving force influence the geometry of the resulting interface. Depending on the values of these parameters, a great variety of patterns is found experimentally.

9.3 Micromodel Experiments
Micromodels allow direct pore-scale observation of multiphase flow through porous media at length scales that vary from pore to ensembles of pores to the entire micromodel. They contain an etched-flow pattern that is viewed with a microscope. The flow pattern is virtually anything that can be etched onto glass or silicon. One limitation that is inherent to all micromodels is that they are only two-dimensional. Extrapolation of results to three-dimensional flow must be done carefully. Micromodels, however, provide the best means to visualize fluid movement at the pore scale while honoring both geometric and topologic properties of real rocks. A detailed literature review is provided by Buckley (1991).

Etched-silicon-wafer micromodels of the type described by Rangel-German and Kovscek (2006), initially developed by Hornbrook et al. (1992), were used. They contain a repeated pattern obtained from a scanning electron microscope image of a thin section
of Berea sandstone. These micromodels offer 1:1 size scaling of typical sandstone pores. The depth of flow channels is 25 µm and grains range in size from 30 to 200 µm. The porosity is roughly 0.2 and the permeability is approximately 1000 mD (as summarized in Table 9.1, given at the end of the chapter). The total network dimension is 5 cm square, Figure 9.1. Such micromodels have been used to study multi-phase flow, solution gas drive, contaminant hydrogeology, and foam generation (e.g., Buckley, 1991; George et al., 2005; Keller, Blunt, and Roberts, 1997; Kovscek, Patzek, and Radke, 1997).

Micromodels were fabricated at the Stanford Nanofabrication Facility (SNF). There are two main steps in their construction: fabrication and anodic bonding. Fabrication begins with the design and construction of a mask. The mask is placed on a silicon wafer previously coated with a photosensitive chemical. The pattern of the mask is impressed on the wafer by means of passing ultraviolet light through the mask. The silicon wafer is developed and later etched with hydrofluoric acid (HF) to a depth of around 25-30 µm. In this way, pore cavities are created.

The silicon wafer with the etched pattern is cleaned in a sulfuric acid (H$_2$SO$_4$) cleaning solution (NOCHROMIX, Godax Laboratories), air dried, and then bonded to a glass plate to create a two-dimensional porous medium. Anodic bonding is accomplished at 500 °F and 1000 V for 50 minutes (Inwood, 2008; Buchgraber, Clemens, Castanier, and Kovscek, 2011). The electricity is turned off and the new micromodel is allowed to cool to room temperature.

Displacements were carried out using three different types of crude oils: South Belridge (µ=8 cp), Lost Hills (µ=90 cp) and Pirawarth (µ=400 cp). Water was used as the
Figure 9.1: (a) Top view of a completed micromodel (b) High magnification photograph of Berea sandstone pore structure (c) Etched micromodel pattern.
displacing phase in all cases under constant injection rate conditions. Lost Hills and South Belridge are significantly less viscous than Pirawarth, and since the focus of this study was to observe fingering instabilities, Pirawarth oil was more extensively studied because of its greatest mobility contrast with water, for three different injection rates. High-resolution, microvisual data, in the form of photographs and video footage, of the fingering process were captured from the microscope using a Nikon Coolpix P5100 camera. Pictures were taken at a number of time steps until breakthrough of injectant at the production end, and digitized using image analysis software. The corresponding saturations were calculated using the normalized pixel density in the binary image obtained after digitization. These values were used to plot the saturation profiles as a function of distance at different times, and were used for validating the scaling hypothesis derived in this study.

9.4 Linear Stability Analysis

When a viscous fluid contained in a porous media is driven by a lower viscosity fluid, the interface is unstable resulting in wavy perturbations on the boundary surface between both fluids which grow with time. To describe the normal modes of small disturbances from a plane interface between immiscible fluids and their rate of growth, an assumption is often made that the fluids remain completely separated along a definite interface. This leads to formulae representing the instability of accelerated interfaces between fluids of different densities and viscosities.
The first mathematical linear stability analysis of an interface under the influence of viscous, gravity, and capillary forces was conducted by Chouke et al. (1959) for one dimensional displacement of two immiscible fluids (which also discussed in Chapter 8, Section 8.3). By considering capillary forces at the interface, they found that there is a threshold for the displacement rate \(U_c\) above which the flow is unstable. The wavelength of maximum instability in immiscible displacements, \(w_m\), (or the peak-to-peak separation between the perturbation fingers at the interface) was shown to be:

\[
w_m = 2\sqrt{3\pi} \sqrt{\frac{\sigma^* \times k}{(M - 1)(V - U_c)\mu_w}}
\]

where \(\sigma^*\) is the effective (macroscopic) interfacial tension, \(M\) is the viscosity ratio, \(k\) is the absolute permeability of the medium, \(\mu_w\) is the viscosity of water, and \(V\) is the flow velocity of the displacing phase, that is assumed to be constant. The physical interpretation of the above relation is that surface tension dampens short waves and thus stabilizes the interface. Microscopic interface length is different from the length of the macroscopic fluid-fluid interface. Consequently, the interfacial tension at the two scales is also different in a heterogeneous system. Chouke et al. (1959) used effective surface tension as the value of the average surface tension seen at macroscopic scale and experimentally found this value to be approximately eight times the bulk fluid interfacial tension.

Figure 9.2 shows the initial fingering pattern observed in the five micromodel experiments. In all experiments distinctive macroscopic fingering patterns are observed. Additionally, in all cases, the interface shows “roughness” at multiple scales. Chouke’s
Figure 9.2: Macroscopic fingers observed during the early stage of Experiments I-V.
analysis (Equation (9.1)) is used to calculate the finger widths \((w_m/2)\) observed during the initial stage of the experiments, when the linear growth theory and Chouke’s analysis are presumably valid. It was found that the effective surface tension must be 25 times the bulk interfacial tension to match the observed finger widths. The values of calculated and observed finger widths are tabulated in Table 9.2 (given at the end of the chapter).

Although Chouke’s theory gives a representative scale for the finger widths, it does not sufficiently explain the multi-scale fingering pattern observed during the later stage of the experiments. Chouke’s theory does capture the inverse relationship between the capillary number and finger width which is seen in the experimental results. In most of the experiments conducted in this study a single dominating finger does not emerge, rather, fingers seem to grow nonlinearly at all scales. The results show that Chouke’s analysis, although insightful, is insufficient in analyzing later-time formation and growth of these complex fingering patterns.

9.5 Modelling Complex Displacement Patterns

An approach to porous media hydrodynamics that has been successful in capturing the pore-scale dynamics is based on treating the flow in a porous medium as a stochastic, rather than as a deterministic process. For example, the flow of a fluid particle may be represented as random walk. There are two fundamental random walk models that are possible: one puts the randomness on the particle and the other considers the randomness that is inherent in the porous medium. Lenormand et al. (1988) associated the various fluid flow patterns for the basic flow mechanisms (capillary fingering, viscous fingering
and stable displacement) with well-defined statistical models: diffusion limited aggregation (DLA), when randomness is ascribed to the fluid and percolation (or invasion percolation), when the randomness is due to the medium itself. Invasion percolation is used to describe capillary fingering that is affected by the randomness of the pore sizes. It is a ‘local model’ in which fluid invades one pore at a time, depending on the capillary forces acting at the throat. Viscous displacements, either stable or unstable, are governed by the continuum pressure field as well as discrete displacements of the interface.

9.5.1 Capillary vs. Viscous forces

Lenormand et al. (1988, 1983) demonstrated the existence of three limiting cases for the basic displacements in a porous medium: capillary fingering, when capillary forces are strong in comparison to viscous forces, viscous fingering when a less viscous fluid displaces a more viscous fluid, and stable displacement in the opposite case. He described these flow regimes using statistical theories: invasion percolation, DLA and anti-DLA depending on the relative magnitudes of capillary and viscous forces. The local competition between capillary and viscous forces is characterized by the capillary number, \( N_c \), and the viscosity ratio, \( M \), defined as:

\[
N_c = \left( \frac{u \mu_i}{\sigma} \right)
\]  \hspace{1cm} (9.2)

\[
M = \left( \frac{\mu_D}{\mu_i} \right)
\]  \hspace{1cm} (9.3)
where \( u \) is the injection velocity and, \( \mu_I \) and \( \mu_D \) are the viscosities of the injected and displaced fluids, respectively and \( \sigma \) is the bulk fluid interfacial tension. Further, Lenormand developed a phase diagram with \( M \) and \( N_c \) as the axes, and calculated the boundaries of the above limiting flow conditions and the critical value of \( N_c \) and \( M \) for crossover among regimes. His theory was based on simulations using network models and experiments. For an immiscible drainage displacement in square network models he established the limiting regimes of percolation and DLA domains to small capillary numbers \( N_c \sim O(10^{-9}) \) and large viscosity ratios \( M \gg 1 \), and large capillary numbers \( N_c \sim O(10^{-5}) \) and small viscosity ratios \( M \ll 1 \), respectively.

In capillary dominated flows of immiscible fluids, pore structure and geometry play a significant role in determining the displacement behaviour. For such cases Darcy-type continuum models do not sufficiently represent the physics of the process at the pore scale. Due to the underlying randomness of the geometry of the porous media, the spread of the fluid in such displacements is likely described more accurately by percolation models. Percolation effects arise due to the instabilities in capillary equilibrium. Wilkinson (1986) noted that percolation effects in immiscible displacement in porous media occur when (1) flow rate and density differences are small so that capillary effects dominate at the pore level (2) the medium is random, and (3) the pore space is multiply connected.

For modelling fluid displacement in capillary dominated flows, the invasion percolation model is typically used. It is the dynamic variant of the classical percolation model. Invasion percolation is a kinetic growth phenomenon as opposed to an
equilibrium system approach used in ordinary percolation. The difference between them is that in invasion percolation the displacing fluid grows only in a single cluster while in ordinary percolation the displacing fluid consists of many randomly placed clusters (Wilkinson, 1986).

9.5.2 Invasion-Percolation Clusters

Invasion percolation was introduced to simulate the displacement of one fluid by another in porous media when capillary forces are dominant (Lenormand and Bories, 1980; Chandler et al., 1982). For a growing percolation cluster, the number of sites occupied by the invading fluid, \( N \), at the moment the invader fluid first percolates (i.e., percolation threshold) is expressed as (Mandlebrot, 1982):

\[
N \sim l^D
\]

(9.4)

where \( l \) is the linear size of the network lattice and \( D \) is the cluster fractal dimension. This is illustrated in Figure 9.3. One often interprets \( N \) as the mass \((mass \sim l^D)\) and hence the cluster dimension is often called the mass dimension. A fractal cluster has a property that the density decreases as its size increases as described by the exponent of the fractal relation in the above equation. The cluster fractal dimension is a quantitative measure of how the cluster fills the space it occupies (Feder, 1988). A similar fractal relationship for the mass and the first moment of mass, \( <x> \), gives:

\[
\text{mass} \sim <x>^{D-1}
\]

(9.5)

From network simulations, Wilkinson and Willemsen (1983), found \( D \approx 1.89 \) in two dimensions and \( D \approx 2.52 \) in three dimensions for the case of invasion percolation without
Figure 9.3: A 2-D fractal percolating cluster overlaid by a grid of squares. Mass of the cluster is related to the number of squares occupied by the cluster through the fractal dimension.

Number of sites occupied, $N(l) \propto l^D$

$D = \text{Fractal dimension}$

$1 < D < 2$
trapping. When the “trapping” of the defender fluid by the invading fluid is also considered, the fractal dimension was found to be \( D \approx 1.82 \) for a two dimensional cell (Chandler et al., 1982).

### 9.5.3 Modelling Approach

From the above discussion and review, we find that a continuum approach based on Darcy’s law does not fully describe the ramified aspect of both capillary and viscous fingers such as those illustrated in Figure 9.2. The behaviour of systems at intermediate capillary numbers and mobility ratios, such as those used in the experiments reported here \( N_c \sim O(10^{-6}-10^{-7}) \) and \( M \sim O(10) \), are not fully understood. Under these conditions both capillary forces as well as viscous forces play significant roles although the relatively low capillary numbers are indicative of a capillary dominant regime. Consequently, alternative models, based on a microscopic description of the medium and stochastic displacement of the interface between the fluids, are used to explain the flow patterns observed during the imbibition experiments reported here. Such stochastic models are capable of useful predictions of the geometry and transport properties of a given network, and of how these properties depend on the network size. The objective here is not to describe these theories in detail, but rather to apply their main features in the analysis.

### 9.6 Scaling Laws

The structural self-similarity and scale invariance of fractals render them feasible for treatment with scaling laws. The scaling theory follows closely the critical point scaling
pioneered by Fisher and others (Fisher, 1967; Ferer et al., 1991). The conventional description of two-phase flow in porous media begins with Darcy’s law:

\[ \nabla p_i = -\left( \frac{\mu_i}{k_{ri}} \right) u_i \]  

(9.6)

where, \( u_i \) is the velocity, \( \mu_i \) is the viscosity, and \( k_{ri} \) the relative permeability of phase \( i \); \( k \) is the absolute permeability of the medium, and \( p_i \) is the pressure of phase \( i \). In order to compute the saturation profiles for incompressible, one dimensional displacement in the horizontal direction, the flow equations for oil and water are solved in conjunction with the capillary pressure (Wilkinson, 1986):

\[ p_o - p_w = p_c \]  

(9.7)

and continuity equations to yield:

\[ \phi \frac{\partial S_i}{\partial t} + \frac{\partial u_i}{\partial x} = 0 \]  

(9.8)

The pressure fields are eliminated to obtain a single equation for water (injected phase) as:

\[ \phi \frac{\partial S}{\partial t} + \frac{\partial}{\partial x} \left( VF - G \frac{\partial S}{\partial x} \right) = 0 \]  

(9.9)

where,

\[ F = \frac{\lambda_w}{\lambda_w + \lambda_o} \]  

(9.10)

\[ G = \frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o} \frac{\partial P_c}{\partial S} \]  

(9.11)
The symbol $F$ is the fractional flow, $V$ is the total fluid velocity, and $\lambda$ is the mobility of the phase. Here the notation is consistent with that used by Wilkinson (1986).

In the case of stable, equilibrium (compact) flow, where capillary phenomena are comparatively less important, relative permeability and the fractional flow ($F$) of the phase are typically described as a function of saturation of the phase. In this case, the saturation scales as the wavespeed ($x/t$), that is, the saturation front moves linearly with time (Buckley and Leverett, 1942). In the limits of unfavorable mobility ratios and when capillary forces (or pore-scale fluctuations) become more important compared to viscous and gravity effects, we observe fractal patterns during forced displacement. Formation of these fractals is an unstable non-equilibrium process, and for such flows we expect variables with non-classical indices (Ferer et al., 1993).

Ferer et al. (1991, 1993) showed using general arguments that for fractal-like displacements, the relative permeability and fractional flows are no longer solely a function of saturation but also of time. This implies that the relative permeability is history dependent. They derived these scaling functions for two-dimensional miscible floods at high viscosity ratios ($M=10,000$). In the following section, the functional relations of saturations, relative permeability and fractional flow are derived starting from the basic flow equations to arrive at similar functional relationships. Here the analysis is based on the fractal nature of invading fluid pattern and not particularly an invasion-percolation or DLA type regime, due to the intermediate $N_c$ and $M$ values. In addition to fractional flow and saturation, the scaling of capillary dispersion has also been derived as capillary pressure plays an important role in stabilizing the fractal nature of flow.
Following are the scaling forms that were assumed for the above functions:

\[ F(S,t) = t^{f_1} f \left( \frac{S}{t^{f_2}} \right) \]  
(9.12a)

\[ G(S,t) = t^{g_1} g \left( \frac{S}{t^{g_2}} \right) \]  
(9.12b)

\[ S(x,t) = t^{s_1} \left( \frac{x}{t^{s_2}} \right) = t^{s_1} s(\varepsilon) \]  
(9.12c)

Inserting the above relations into Equation (9.9) yields:

\[ \frac{\partial S}{\partial t} = \phi \left[ s_1 t^{s_1-1} s(\varepsilon) - s_2 t^{s_2-1} \frac{\partial s(\varepsilon)}{\partial \varepsilon} \right] \]  
(9.13)

and

\[ \frac{\partial}{\partial x} \left( V F - G \frac{\partial S}{\partial x} \right) = \frac{1}{t^{s_1}} \frac{\partial}{\partial \varepsilon} \left[ V f^1 f \left( t^{s_1-f_2} s(\varepsilon) \right) - \frac{\partial s(\varepsilon)}{\partial \varepsilon} \right] \]  
(9.14)

Adding Equations (9.13) and (9.14) produces:

\[ \phi \left[ s_1 t^{s_1-1} s(\varepsilon) - s_2 t^{s_2-1} \frac{\partial s(\varepsilon)}{\partial \varepsilon} \right] + \frac{1}{t^{s_1-f_1}} \frac{\partial}{\partial \varepsilon} \left[ V f^1 f \left( t^{s_1-f_2} s(\varepsilon) \right) - \frac{\partial s(\varepsilon)}{\partial \varepsilon} \right] = 0 \]  
(9.15)

Equation (9.15) is only a function of \( \varepsilon \), the scaling variable. Accordingly, the following relations for the exponents emerge:

\[ s_1 + s_2 - f_1 = 1 \]  
(9.16a)

\[ s_1 = f_2 \]  
(9.16b)
\[ s_1 = g_2 \]  \hspace{1cm} (9.16c)

\[ s_1 + g_1 = f_1 + s_2 \]  \hspace{1cm} (9.16d)

For a fractal object, such as a percolating cluster, the fractal mass, \( mass \), is related to the first moment of mass, \( \langle x \rangle \), by Equation (9.5). Because the flow rate is constant, the relationship becomes:

\[ mass \approx t = C \langle x \rangle^{D-1} \]  \hspace{1cm} (9.17)

where \( C \) is a constant and \( D \) is the fractal dimension. Thus for fractal patterns, the first moment of saturation is a function of time. To be consistent with the terminology used by Ferer et al. (1993), we rewrite Equation (9.17) as:

\[ \langle x \rangle^{(D-1)} \sim t \text{ or } \langle x \rangle \sim t^{1+e} \]  \hspace{1cm} (9.18a)

where \( e \) is a factor introduced for mathematical convenience, which is related to the fractal dimension, \( D \), as:

\[ \text{——} \]  \hspace{1cm} (9.18b)

The first moment of saturation is given by:

\[ \frac{\int xS(x,t)dx}{\int S(x,t)dx} \approx \langle x \rangle \]  \hspace{1cm} (9.19)

which is rewritten using the scaling functions in Equation (9.12) and fractal relationship as:

\[ \frac{\int t^{\eta+2s} s(\varepsilon) d\varepsilon}{\int t^{\eta+s} s(\varepsilon) d\varepsilon} \approx t^{1+e} \]  \hspace{1cm} (9.20)

On equating the exponents of \( t \) on both sides the result is:
From fluid conservation for incompressible flow in two dimensions, the following holds:

$$\int S(x,t)dx \approx t$$  \hspace{1cm} (9.22)

Rewriting the above equation using the scaling functions and equating exponents, we get:

$$\int t^{s_1+s_2} S(\varepsilon)d\varepsilon \approx t$$  \hspace{1cm} (9.23)

which gives the following relations:

$$s_1+s_2 = 1$$ \hspace{1cm} (9.24)

$$s_1 = -e$$

Using the relations obtained from Equations (9.16), (9.23), and (9.24), the values of other exponents are:

$$f_1 = 0$$ \hspace{1cm} (9.25)

$$f_2 = -e$$ \hspace{1cm} (9.26)

$$g_2 = -e$$ \hspace{1cm} (9.27)

$$g_1 = 1+2e$$ \hspace{1cm} (9.28)

Thus the scaling relations become:

$$F(S,t) = f(t^e S)$$ \hspace{1cm} (9.29)

$$G(S,t) = t^{1+2e} g(t^e S)$$ \hspace{1cm} (9.30)

$$S(x,t) = t^{-e} \left( \frac{x}{t^{1+e}} \right)$$ \hspace{1cm} (9.31)

From the above equations, for fractal like displacements, fractional flow and hence relative permeability are indicated to be functions of both local saturation and time. For compact flow in two dimensions, \(D=2\) and \(e=0\) and the above functional relationships
reduce to the conventional Buckley-Leverett relationships of linear flow where saturation scales as $x/t$ (position/time). Thus, the above scaling relations, based on fractal flow theory, give a more generic displacement relationship.

The validity of the above scaling relationships viz. Equations (9.29), (9.30) and (9.31), was tested on imbibition displacement experiments in micromodels.

### 9.7 Experimental Observations and Analysis

The parameters for the five set of experiments are listed in Table 9.3 (given at the end of the chapter). Figures 9.4 through 9.8 correspond to Experiment sets I-V. In each figure from Figure 9.4 through 9.8: (a) displays $\log\langle x \rangle$ versus $\log(t)$, where $\langle x \rangle$ is the maximum distance moved by the interface at time $t$ and represent the moment of lowest water saturation, (b) displays the time evolution of the (normalized) saturation profiles, (c) displays the saturations as a function of wavespeed ($x/t_D$), where $t_D$ is the dimensionless time in terms of number of pore volumes injected, and (d) displays the scaled saturation profiles, $(t_D^{-e}S)$ vs. $(x/t_D^{1+e})$.

The fractal dimension ($D$) and the exponent $e$ for the invading fluid clusters is found from the slope of the graph of $\log\langle x \rangle$ versus $\log(t)$, using Equation (9.18). The values of $e$ and $D$ for all the experiments are summarized in Table 9.4 (given at the end of the chapter). The values for $e$ and $D$ for experiments (I, II, III and IV) fall in the range of, $e \sim [0.07-0.21]$ and $D \sim [1.83-1.93]$. These results are in good agreement with fractal dimensions typically seen in Invasion-Percolation regimes ($\sim 1.82$) for nonwetting phase.
Figure 9.4: Experiment I: Lost Hills, q=16x10^{-4} ml/min.
Figure 9.5: Experiment II: South Belridge, $q=16 \times 10^{-4}$ ml/min.
Figure 9.6: Experiment III: Pirawarth, q=16x10^{-4} ml/min.
Figure 9.7: Experiment IV: Pirawarth, q=2x10^-4 ml/min.
Figure 9.8: Experiment V: Pirawarth, q=8x10^{-4} \text{ ml/min}.
saturating a medium initially filled with water (Meakin et al., 1992). This establishes the
dominance of local capillary forces on the advancing front. The difference in observed
fractal dimensions compared to a typical invasion-percolation case arises from the
presence of viscous gradients, that are not negligible in the experiments. For experiment
V, the value of $e=0.64$, that implies $D=1.57$, which is much lower than the expected value
for an invasion percolation regime. This could be caused by the edge effects during
experiment V, which can be seen in Figure 9.2. In this context, an edge effect is relatively
rapid movement of the injected fluid along the boundaries of the model. The origin is
likely a somewhat greater depth of etching of the micromodel relative to the etch depth in
the centre. Due to these effects, the relation between invading cluster mass and its linear
size may not truly have the fractal characteristics.

Figures 9.4b through 9.8b display (normalized) saturation profiles at different
times. Notice that these saturations are moving forward with time, as is expected. In
Figures 9.4c to 9.8c we see that for experiments I, III, IV and V the saturation curves do
not collapse when saturation is scaled with wavespeed ($x/t_D$), and thus they diverge from
a linear profile as predicted by conventional linear frontal advance theory (Buckley and
Leverett, 1942). However, in Figure 9.5c for experiment II, we see that the saturation
profiles do collapse to within an acceptable error margin. This is a consequence of the
low value of $e\sim0$ ($D\sim1$) that indicates an almost compact flow. The oil is comparatively
lighter ($\mu=8$ cp) and thus a comparatively stable displacement regime is expected.

In Figure 9.4c through 9.8c, the accuracy of the scaling relationships derived in
the previous section is tested for each experiment. For experiments I, III, IV and V the
data collapses within acceptable error margins when the respective values of \( e \) are used for the scaling. However, for experiment II, the scaling coefficient is low (\( e=0.07 \)) and thus the linear scaling (Buckley and Leverett, 1942) seems to be sufficient for explaining displacement.

The above results demonstrate that the saturation profiles do not scale linearly with \( x/t_D \) (position/time) as predicted by the conventional linear flow models, and rather show self-similarity when \( (t_D^e S_w) \) is plotted versus \( (x/t_D)^{1+e} \). Significant non-equilibrium effects are found at the micromodel scale during unstable, capillary-dominated forced displacements. Complementary experiments examining spontaneous imbibition in three-dimensional porous media also find evidence of non-equilibrium effects (Leguen and Kovscek, 2006; Schembre and Kovscek, 2006). Leguen and Kovscek (2006) performed simulations and core scale experiments that illustrate that capillary behaviour is an inherently non-equilibrium process that cannot be modeled using the traditional multiphase reservoir simulation equation. Several studies (Barenblatt and Vinnichenko, 1980; Barenblatt and Gilman, 1987; Barenblatt, Silin, and Patzek, 2003; Silin and Patzek, 2004) have suggested that equilibrium is not reached in porous media during spontaneous equilibrium and that non-equilibrium effects should be taken into account. It follows that for transient non-equilibrium processes, capillary pressure and relative permeability are no longer functions of water saturation alone, but also depend on saturation history and time. The scaling laws developed in this study quantitatively model the time dependence of capillary pressure, saturation and fractional flow profiles, thus confirming the presence of non-equilibrium effects within an unstable forced-imbibition regime.
9.8 Discussion

Two-phase fluid displacements in porous media have been studied extensively in fields such as hydrology, soil science, and petroleum engineering. A great variety of structures are observed by changing the physical parameters of the fluids including viscosity contrast, wettability, interfacial tension, and displacement rate; yet, for the most part, the characterization and modelling of such displacement patterns remains a challenging and complex problem.

The purpose of this study was to understand analytically, the flow patterns observed during forced imbibition experiments carried out under unstable conditions and low flow rates. For the specific range of flow rates ($N_c \sim 10^{-7}$) and mobility ratios ($M \sim 8-400$) that are encountered in the experiments, neither viscous nor capillary forces are negligible. It was found that the multiphase flow extension of Darcy’s law is not sufficient to describe the displacement of the two fluids. Evidence of non-equilibrium effects is seen, under which the capillary pressure and relative permeability are no longer functions of water saturation alone, but also depend on saturation history and time. The analysis presented here is based on a limited set of experiments in a particular porous medium. The limitations of the experimental work that could potentially affect the calculations need to be pointed out:

- There were problems related with capturing the photographs such as: (1) pictures were taken at an angle slightly less than normal to the micromodel, (2) pictures have less detail as a consequence of a shorter exposure time, (3) pictures may be
affected by glare. These factors were kept in mind while analyzing the scanned pictures and corrective measures were used as necessary.

- It is difficult to measure the time elapsed as a function of fluid injected because of the small volume of the micromodel relative to the length of tubing and dead volume within the micromodel holder.
- Some of the experiments suffered from edge effects in which the water entered through the corner of the inlet and traveled preferentially along one of the two parallel edges of the etched pore space. In experiments that experienced edge effects, sweep efficiency declined dramatically because the finger along the edge broke through while the rest of the water front remained nearly static.

Experiments in which the above problems were severe were not considered for this analysis. This reduced the number of sample sets. A complete set of experimental results is presented by Inwood (2008). The analysis presented here, in spite of its limitations, demonstrates the complexity of forced unstable imbibition, and the limitations of a continuum approach to explain its behaviour. Additional experiments examining flow behaviour for the same range of flow parameters \( (N_c, M) \) can be used for further validation of the scaling model.

9.9 Conclusions

The work presented here outlines a framework for interpretation of high-resolution images from multiphase flow visualization experiments. A condition of forced imbibition was maintained in all cases. The results displayed a range of behaviour that varied
according to the magnitude of viscous and capillary forces as characterized by the viscosity ratio and capillary number, respectively.

For unstable, immiscible, incompressible, multiphase flow, it was found that the multiphase flow extension of Darcy’s law is inadequate to describe flow through the micromodel ensemble of pores. For the specific range of flow rates ($N_c \sim 10^7$) and mobility ratios ($M \sim 8–400$) considered in the present experiments, evidence of non-equilibrium effects is seen. The results reveal that the flux of a phase under two-phase flow conditions is not solely a function of pressure gradient and phase saturation, but also depends on saturation history and time. Experimental results show self-similarity when $(t_D e S_w)$ is plotted versus $(x/t_D^{1+e})$ (where $e < 1$), rather than $S_w$ versus $(x/t_D)$ as predicted by the Buckley-Leverett linear flow theory.
### Table 9.1: Micromodel specifications

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions, cm</td>
<td>5 x 5</td>
</tr>
<tr>
<td>Unit cells per 5 cm</td>
<td>104</td>
</tr>
<tr>
<td>Unit cell dimensions, µm</td>
<td>480 x 400</td>
</tr>
<tr>
<td>Etched depth, µm</td>
<td>25</td>
</tr>
<tr>
<td>Etched pore volume, mL</td>
<td>0.029</td>
</tr>
<tr>
<td>Porosity, φ</td>
<td>0.47</td>
</tr>
<tr>
<td>Absolute permeability, k, Darcy</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 9.2: Finger width calculation

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Oil</th>
<th>Finger width, mm (Chouke et al., 1959)</th>
<th>Observed finger width (average), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Lost Hills</td>
<td>2.417</td>
<td>2-4</td>
</tr>
<tr>
<td>II</td>
<td>South Belridge</td>
<td>8.618</td>
<td>9-10</td>
</tr>
<tr>
<td>III</td>
<td>Pirawarth</td>
<td>1.142</td>
<td>~1</td>
</tr>
<tr>
<td>IV</td>
<td>Pirawarth</td>
<td>3.229</td>
<td>2-3</td>
</tr>
<tr>
<td>V</td>
<td>Pirawarth</td>
<td>1.614</td>
<td>2-3</td>
</tr>
</tbody>
</table>

### Table 9.3: Experimental parameters

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Oil</th>
<th>μI, cp</th>
<th>μD, cp</th>
<th>q x 10^-4 ml/min</th>
<th>Nc x 10^-6</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Lost Hills</td>
<td>1</td>
<td>90</td>
<td>16</td>
<td>1.42</td>
<td>90</td>
</tr>
<tr>
<td>II</td>
<td>South Belridge</td>
<td>1</td>
<td>8</td>
<td>16</td>
<td>1.42</td>
<td>8</td>
</tr>
<tr>
<td>III</td>
<td>Pirawarth</td>
<td>1</td>
<td>400</td>
<td>16</td>
<td>1.42</td>
<td>400</td>
</tr>
<tr>
<td>IV</td>
<td>Pirawarth</td>
<td>1</td>
<td>400</td>
<td>2</td>
<td>0.18</td>
<td>400</td>
</tr>
<tr>
<td>V</td>
<td>Pirawarth</td>
<td>1</td>
<td>400</td>
<td>8</td>
<td>0.71</td>
<td>400</td>
</tr>
</tbody>
</table>

### Table 9.4: Results from Figures 9.4-9.8

<table>
<thead>
<tr>
<th>Exp. no.</th>
<th>Oil</th>
<th>e</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Lost Hills</td>
<td>0.21</td>
<td>1.83</td>
</tr>
<tr>
<td>II</td>
<td>South Belridge</td>
<td>0.07</td>
<td>1.93</td>
</tr>
<tr>
<td>III</td>
<td>Pirawarth</td>
<td>0.11</td>
<td>1.90</td>
</tr>
<tr>
<td>IV</td>
<td>Pirawarth</td>
<td>0.18</td>
<td>1.85</td>
</tr>
<tr>
<td>V</td>
<td>Pirawarth</td>
<td>0.74</td>
<td>1.58</td>
</tr>
</tbody>
</table>

262
Chapter 10: CONCLUSIONS AND RECOMMENDATIONS

This concluding chapter brings together the analytical and numerical findings presented in the previous chapters, by revisiting the main research objectives posed in Chapter 1 (Section 1.5). In doing so, it highlights the original contributions of this thesis, as well as suggests directions for further research in the field of thermal gravity drainage processes.

10.1 Research Objectives Revisited

10.1.1 Objective 1

*Develop an analytical model of SAGD that includes multiphase flow and relative permeability effects.*

**Multiphase Flow Model for SAGD** (Chapter 3):

A new model for gravity drainage of mobilized bitumen at the edge\(^8\) of a SAGD chamber was derived, that takes into account the impact of multiphase flow and relative permeability on the oil mobility profile.

- The new model reveals that the highest oil velocity is not at the edge of the steam chamber but rather at some distance away from the chamber walls, which depends on the curvature of the oil relative permeability curve with respect to water, and the rate of change of oil viscosity with temperature.

---

\(^8\) The ‘edge of the chamber’ is mathematically defined as the vapour-oil interface beyond which the temperature declines below the temperature inside the SAGD chamber \((T<T_s)\).
• The theory shows that the larger the lateral spreading rate of the steam chamber, the closer is the location of the maximum oil mobility and flow rate to the edge of the chamber. In addition, the larger the thermal diffusivity, the farther away the location of the maximum oil mobility and flow rate is from the steam chamber edge.

• The theory reveals that the location of the maximum oil mobility is set by the balance between the chamber spreading rate and the conductive heat transfer into the oil sands at the edge of the chamber.

• The results from the new analytical model were validated through comparison with field data, fine-grid numerical simulation results, as well as with selected experimental data.

10.1.2 **Objective 2**

Examines the relative roles of conductive and convective heat transfer beyond the steam chamber edge, and conditions for the predominance of each.

**Convective Heat Transfer Model** (Chapter 4):

An analytical theory was developed to account for convection of warm condensate into the oil sands at the edge of a SAGD chamber, in a multiphase flow framework. An apparent thermal diffusivity term, which includes both the conductive and convective heat components, was derived to examine the relative roles of convective and conductive heat transfer beyond the chamber edge.
• The results reveal that the convected condensate carries energy into the oil sands, which promotes a concave downwards temperature profile beyond the edge of the steam chamber.

• The results demonstrate that convection can provide a relatively large contribution of heat transfer at the edges of a steam chamber. However, enhanced heat transfer does not necessarily mean enhanced oil mobility and oil rate, due to the relative permeability effects.

• The theory was tested via comparison with field data taken from the Dover Phase B SAGD pilot as well as with fine-grid numerical simulation results.

10.1.3 Objective 3

Examine the role of non-condensable gas injection with steam in chamber development, oil flow rate and heat losses in SAGD.

Non-condensable Gas Aided SAGD Model (Chapter 5):

A mathematical theory was derived to predict saturation distribution and fluid flow profiles in the mobile zone, when a non-condensable gas is co-injected with steam during a SAGD operation.

• The results from the analytical and numerical study demonstrate that non-condensable co-injection with steam in SAGD is in general unfavourable.

---

9 The ‘mobile zone’ is defined as the narrow zone beyond the steam condensation front where multiphase flow of oil, water, and gas takes place.
• The injected non-condensable gas tends to accumulate at the steam condensation front, which lowers the rate of condensation heat transfer from steam to the adjacent oil.

• The accumulation of gas at the chamber edge results in reduced steam quality, and consequently lower oil production rate, and slower growth of the chamber.

• Another effect of methane accumulation at the top of the chamber is the reduction in overburden heat loss (as observed in the numerical simulation results). However, the reduction in oil flow rate and the final recovery would likely negate the benefit of such heat loss reduction.

• The theory was validated via comparison with fine-grid numerical simulation results as well as with Steam-Assisted Gas Push (SAGP) experimental data.

10.1.4 **Objective 4**

*Examine the coupled heat and mass transfer mechanism in steam-solvent co-injection techniques, within the framework of SAGD analytics.*

**Solvent-Aided SAGD Model** (Chapter 6):

A simple model was derived to describe the dynamics of heat transfer and solvent mass transfer effects in the mobile zone at the edge of a steam-solvent chamber. The model was used to investigate the extent to which the heat and mass transport length scales affect oil mobility and oil viscosity profiles at the edge of the steam-solvent SAGD chamber.
• From an oil mobility point of view, two distinct regions are observed beyond the edge of the chamber. First, a zone near the chamber edge where both temperature and solvent content influence the oil phase viscosity (of the order of tens of centimetres), and second, a zone where temperature alone influences the oil phase viscosity (of the order of metres).

• Due to the enhanced oil mobility of steam-solvent processes, steam-solvent processes yield higher oil production rates as compared to steam-alone ones. For the same oil volume produced, a steam-solvent process will generate less greenhouse gas emissions than a steam-only one.

• The theory was validated via comparison with fine-grid numerical simulation results as well as Expanding-Solvent SAGD (ES-SAGD) experiments.

**Coupled Transient Heat and Mass Transfer Model (Chapter 7):**

A set of transient, coupled heat and mass transfer equations were derived to account for the conductive-convective heat transfer, coupled with the convective-diffusive mixing of solvent in the oil phase, in the mobile zone adjacent to a SAGD chamber.

• The results demonstrate that the solution to the transient heat and mass transfer equations will asymptotically approach the pseudo steady-state solution at late times. The length of the time to reach pseudo steady-state depends mainly on the chamber expansion rate and the thermal diffusivity of the reservoir \( t > \alpha / U^2 \).

• By including the convective transport effect, the heat transfer and solvent mass transfer beyond the steam chamber edge are enhanced, individually.
• However, the enhancement in the overall oil mobility (and hence oil production rate) by inclusion of convective transport effects is insignificant. The main reasons for this are: (1) the enhancement in temperature and solvent concentration due to convective transport occur in different regions within the mobile zone, with dissimilar length scales, and (2) the relative permeability effect competes against the convective heat transfer effect, resulting in lower oil mobility in spite of enhanced temperature.

10.1.5 **Objective 5**

*Investigate the impact of interfacial stability on the performance of oil recovery processes.*

**Linear Stability Model for SAGD and Solvent-aided SAGD** (Chapter 8):

A frontal stability theory was developed for the interface at the edge of a vapour chamber as would be found in SAGD and solvent-aided SAGD processes. Theoretical evidence for occurrence of interfacial instability, and its impact on the performance of the process, was investigated.

• The results of the stability theory reveal that the addition of solvent increases the growth rate of interfacial instability. Greater instability implies that there will be more solvent mixing, and potentially more heat transfer, at the interface, thus promoting higher oil mobilization and enhanced oil production rate.

• The higher the injection pressure, the more unstable is the interface. This is because higher pressure implies greater solubility of the solvent in the oil phase.
• There is an intermediate solvent where the instability is maximized. Physically, this is due to the balance between the solubility of the solvent versus the capability of the solvent to lower the viscosity of the oil phase. This confirms the results of experiments conducted by Nasr and Isaacs (2001) which demonstrated that an intermediate solvent (hexane) resulted in the highest oil drainage rate.

• Under typical operating pressures of current SAGD operations, the results of this study reveal that hexane or heptane yield the most unstable conditions.

• The theory was validated through comparison with ES-SAGD experimental results.

Analysis of Pore-scale Viscous Fingering (Chapter 9):

Using statistical theories and concepts of fractal analysis, scaling power laws were derived to describe quantitatively the pore-scale imbibition displacement patterns, observed under unstable conditions.

• The results reveal that for unstable, immiscible, incompressible, pore-scale displacements, the usual continuum model for flow in porous media incorporating the multiphase flow extension of Darcy’s law is not predictive.

• For describing complex displacement patterns arising from instabilities in local capillary equilibrium, statistical theories and fractal analysis is more convenient and appropriate.

• The displacement patterns, for the specific range of flow rates (capillary number, $N_c \sim 10^{-7}$) and mobility ratios ($M \sim 8$–400) reported in the study, show evidence of
non-equilibrium effects. Under these conditions, the relative permeability and fractional flow are no longer solely a function of saturation, but also of time.

- Results show that the functional relations of the scaling model derived in this study are in considerable agreement with the experimental data.

10.2 Directions for Future Work

1) The following effects were neglected in deriving the analytical theories presented in this research, and therefore should be considered for future analytical study of the SAGD process and its variants:
   - effect of ashphaltene precipitation
   - geomechanical effects on the SAGD process
   - formation of water-oil emulsions
   - wettability alteration in thermal and thermal-solvent processes
   - economic evaluation of the process
   - the impact of wellbore dynamics
   - the problem of formation of non-uniform and discrete steam chambers
   - overburden and underburden heat loss effects
   - effect of gas saturation in the native reservoir outside of the steam chamber

2) Large scale, high pressure, physical model experiments are needed to enable better understanding of the flow behaviour, and the dynamics of heat and mass transfer at the edge of the vapour chamber in SAGD and solvent or gas aided SAGD processes.
3) Non-linear stability analysis of the steam-oil interface can shed more light on the dynamic behaviour of the SAGD process.
References


49. Encana Christina Lake Thermal Project. 2007. In-situ Update Available at ERCB website: www.ercb.ca.


presented at the SPE International Thermal Operations and Heavy Oil Symposium, Calgary, Canada, 1-3 November.


CIM International Conference on Horizontal Well Technology, Calgary, Canada, 6–8 November.


A.1. Objectives of Numerical Simulation Study

The objectives of the numerical simulation study were to:

1) Verify and compare the results from the analytical models developed in this research.
2) Gain visual understanding of the fluid flow behaviour and phase distribution in the reservoir, during a Steam-Assisted Gravity Drainage (SAGD) process.
3) Understand the effect of non-condensable gas and hydrocarbon solvent injection on SAGD performance, in terms of its impact on fluid phase distribution, chamber development, in-situ steam quality, overburden heat losses, and fluid flow rates.

A.2. Simulation Model

A two-dimensional (2-D) reservoir model was used for simulating SAGD, non-condensable gas aided SAGD and solvent-aided SAGD operations. CMG’s (CMG, 2009) thermal simulator STARS was used for the purpose of this study. Given that the length scales for heat and mass transfer are of the order of a few centimetres to few metres (Sharma and Gates, 2011b, 2010b), the selection of a proper grid block size is critical in resolving the flow behaviour near the edge\(^{10}\) of the chamber. The model used in this work

\(^{10}\) In this work, the ‘edge of the chamber’ is defined mathematically as the interface beyond which the temperature declines below the temperature inside the SAGD chamber \((T<T_s)\)
was, therefore, finely gridded with a grid block size of 2.5 mm in all directions. A 2-D rectangular grid system was used with 500 x 1 x 50 grid blocks in the x, y, and z directions respectively (where x is in the horizontal direction perpendicular to the well, y is parallel to the well direction and z is in the vertical direction). The physical dimensions of the simulation model are 125 cm x 0.25 cm x 12.5 cm, which is of the order of typical SAGD experimental models (Chung and Butler, 1988; Sasaki, Akibayashi, Yazawa, Doan, and Farouq Ali, 2001). The producer well was placed close to the bottom of the reservoir and the injector was positioned five grid blocks above the producer. The centre of the well pair was located in a line of symmetry, therefore only half of the reservoir was modelled and used for the purpose of the simulation study. The results from the simulation model were analyzed after 30 hours (or 1.25 days) of steaming, which on scaling up using Butler’s (1985) scaling parameter translates to 8 years of SAGD operation. Figure A.1 illustrates the overall scheme for the numerical simulation study conducted in this research.

A.3. Reservoir Models Used

A.3.1. Homogeneous Reservoir Model

In order to verify the results of the analytical theories, a homogeneous reservoir model with uniform porosity and permeability, and no overburden heat loss effects was used, to match the assumptions made in the analytical theory. The homogeneous reservoir model was used for simulating SAGD, non-condensable gas aided SAGD, and solvent aided SAGD processes.
Objectives
1. Verify results from the analytical models
2. Visualize the fluid flow behaviour in SAGD
3. Understand the effect of non-condensable gas, and solvent injection on SAGD performance

Homogeneous Reservoir Model
- Constant porosity & permeability
- No overburden and underburden heat losses

Heterogeneous reservoir model
- Randomly distributed porosity & permeability
- Includes overburden and underburden heat losses

Compare solvent-aided SAGD model
(Chapter 6, Sec. 6.6.2)

Compare SAGD multiphase flow model
(Chapter 3, Sec. 3.6)

Compare convective heat transport model
(Chapter 4, Sec. 4.5)

Compare non-condensable gas aided SAGD model
(Chapter 5, Sec. 5.5.2)

Compare solvent-aided SAGD model
(Chapter 6, Sec. 6.6.2)

Analyze overburden heat losses in SAGD
(Chapter 5, Sec. 5.5.3)

Analyze overburden heat losses in non-condensable gas aided SAGD
(Chapter 5, Sec. 5.5.3)

Modified $S_{oi}$

Figure A.1: Schematic representation of the numerical simulation study done in this research.
A.3.2. Heterogeneous Reservoir Model

The heterogeneous reservoir model included overburden and underburden heat loss effects, and had randomly distributed porosity and permeability values. The main purpose of using this model was to understand the effect of the presence of non-condensable gas in a live SAGD chamber on overburden heat losses, chamber development, and in-situ steam quality. The average rock and fluid properties for the heterogeneous model were similar to the homogeneous model and are summarized in Table A.1. The two reservoir models are shown in Figures A.2a and A.2b.

A.4. Simulation Cases Analyzed

A.4.1. SAGD

The base case for comparison for all the numerical simulations was a SAGD operation with steam-alone injection. This case was simulated on both the homogeneous as well as the heterogeneous reservoir models.

The results from the homogeneous reservoir simulation, that is, SAGD (homogeneous), were used for testing the predictions of the analytical SAGD model developed in Chapter 3. The comparison of the simulation and analytical model results is presented in Section 3.6 of Chapter 3.

In order to clearly see the convective heat transfer effect in SAGD, and to compare the analytical results of the convective heat flow model developed in Chapter 4, the homogeneous reservoir model was slightly modified to make the initial water phase more mobile, by specifying a lower irreducible water saturation ($S_{wr}=0.05$). The
Figure A.2: Permeability distribution in the reservoir for (a) Homogeneous model (b) Heterogeneous model.
Table A.1: Reservoir and fluid properties used in the simulation model (Ito and Hirata, 1999; Ito and Suzuki, 1996; Butler, 1997a).

<table>
<thead>
<tr>
<th>Physical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir temperature, $T_r$, °C</td>
<td>10</td>
</tr>
<tr>
<td>Initial reservoir pressure, kPa</td>
<td>2000</td>
</tr>
<tr>
<td>Oil density, $\rho_o(T_r)$, kg/m$^3$</td>
<td>980</td>
</tr>
<tr>
<td>Rock compressibility, 1/kPa</td>
<td>$3\times10^{-6}$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.33</td>
</tr>
<tr>
<td>Permeability, $k$, m$^2$</td>
<td>$5\times10^{-13}$</td>
</tr>
<tr>
<td>$\phi$ (average), $k$ (average), m$^2$</td>
<td></td>
</tr>
<tr>
<td>Initial oil saturation, $S_{io}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Initial water saturation, $S_{iw}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Irreducible water saturation, $S_{wr}$</td>
<td>0.19</td>
</tr>
<tr>
<td>Viscosity of oil at steam temperature, $\mu_o(T_s)$, cp</td>
<td>7.5</td>
</tr>
<tr>
<td>Thermal conductivity, J/(m-day-°C)</td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>$6.048\times10^5$</td>
</tr>
<tr>
<td>Oil</td>
<td>$1.339\times10^4$</td>
</tr>
<tr>
<td>Gas</td>
<td>4320</td>
</tr>
<tr>
<td>Water</td>
<td>$5.8147\times10^9$</td>
</tr>
<tr>
<td>Hexane $K$-value coefficients (CMG, 2009)</td>
<td>$k_{v_1}=1006200, k_{v_2}=k_{v_3}=0$,$k_{v_4}=-2697, k_{v_5}=-224.3$</td>
</tr>
<tr>
<td>Methane $K$-value coefficients (CMG, 2009)</td>
<td>$k_{v_1}=545470, k_{v_2}=k_{v_3}=0$, $k_{v_4}=-879.84, k_{v_5}=-265.99$</td>
</tr>
</tbody>
</table>
comparison between the analytical convection model results and the simulation results is presented in Section 4.5 of Chapter 4.

The results from the heterogeneous reservoir simulation, that is, SAGD (heterogeneous), were used for comparing the overburden heat loss rates in SAGD with the non-condensable gas aided SAGD process. This is presented in Section 5.5.3 of Chapter 5.

A.4.2. Non-Condensable Gas Aided SAGD

Methane was used in this study as the non-condensable gas for co-injection with steam in SAGD, since it is the main solution gas in heavy oil reservoirs. However, it is believed that the findings of this work are applicable to other non-condensable gases such as nitrogen, air, and helium, which do not have significant solubility in bitumen at steam temperatures.

The case of steam-methane co-injection was simulated on both the homogeneous as well as heterogeneous reservoir models. The results from the homogeneous reservoir simulation were compared with the predictions of the analytical non-condensable gas aided SAGD model developed in Chapter 5. The comparison of the analytical and simulation results is presented in Section 5.5.2 of Chapter 5.

The results from the heterogeneous reservoir simulation were compared with the results of SAGD (heterogeneous) simulations, to understand the impact of non-condensable gas (methane) co-injection with steam on overburden heat loss rate, phase
distribution, in-situ steam quality and fluid flow rate. This is presented in Section 5.5.3 of Chapter 5.

**A.4.3. Solvent-Aided SAGD**

Hexane was used in this study as the solvent for co-injection with steam for the solvent-aided SAGD process. The case of steam-hexane co-injection was simulated on the homogeneous reservoir model. The results were compared with SAGD (homogeneous) simulation to understand the role of solvent (hexane) co-injection on the performance of SAGD. The results from the simulation were also compared with the predictions of the analytical solvent-aided SAGD model developed in Chapter 6, for qualitative and quantitative similarity. The comparison of the simulation and analytical model results is presented in Section 6.6.2 of Chapter 6.

**A.5. Reservoir and Fluid Properties**

The rock and fluid properties for the homogeneous and heterogeneous reservoir models used in this simulation study are summarized in Table A.1. These values are based on typical Athabasca oil sands reservoir properties. The $K$-value constants ($k_v1$, $k_v2$, $k_v3$, $k_v4$ and $k_v5$) for the different components were taken from the CMG (2009) manual, and are also summarized in Table A.1. In all the cases, reservoir oil did not have any initial solution gas content. The temperature-dependent viscosity data used in the simulation was based on the experimentally determined viscosity values for Athabasca bitumen by Mehrotra and Svrcek (1986). The viscosity profile is displayed in Figure A.3. The water-
oil and gas-oil relative permeability curves used in the simulation study are displayed in Figure A.4.

The numerical simulator was programmed to include the effects of molecular diffusion in the oleic and the gaseous phases, as well as hydrodynamic dispersion. The dependence of the molecular diffusion coefficient on temperature and viscosity was modelled using the same relationship as used in the analytical models. The correlations used to model mechanical dispersion and molecular diffusion coefficients in the simulation are summarized in Table A.2.

![Figure A.3: Temperature versus oil viscosity profile used in the simulation (Mehrotra and Svrcek, 1986).](image-url)
Figure A.4: Relative permeability curves used in simulations (a) Water-oil data (b) Gas-oil data.
Table A.2: Correlations for diffusion and dispersion coefficients and associated parameters used in this study.

<table>
<thead>
<tr>
<th>Property</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion coefficient of non-condensable gas (methane) in the gas phase, (m^2/s)</td>
<td>(D_{g,gas}(T^\circ C) = D_{g,gas}(20^\circ C) \left[\frac{T + 273.15}{20 + 273.15}\right]^{1.8}) (D_{g,gas}(20^\circ C) = 3 \times 10^{-6})</td>
<td>Butler and Yee (1986)</td>
</tr>
<tr>
<td></td>
<td>(D_{g,oil}(T^\circ C) = D_{g,oil}(20^\circ C) \left[\frac{T + 273.15}{20 + 273.15}\right]^{0.54}) (D_{g,oil}(20^\circ C) = 1 \times 10^{-8})</td>
<td>Jamialahmadi, Emadi, and Müller-Steinhagen (2006)</td>
</tr>
<tr>
<td></td>
<td>(D_{s,gas}(T^\circ C) = D_{s,gas}(20^\circ C) \left[\frac{T + 273.15}{20 + 273.15}\right]^{1.8}) (D_{s,gas}(20^\circ C) = 4 \times 10^{-7})</td>
<td>Butler and Yee (1986)</td>
</tr>
<tr>
<td></td>
<td>(D_{s,oil}(T^\circ C) = D_{s,oil}(20^\circ C) \left[\frac{T + 273.15}{20 + 273.15}\right]^{0.54}) (D_{s,oil}(20^\circ C) = 4 \times 10^{-10})</td>
<td>Jamialahmadi et al. (2006)</td>
</tr>
<tr>
<td></td>
<td>(D_{o,oil}(T^\circ C) = D_{o,oil}(20^\circ C) \left[\frac{T + 273.15}{20 + 273.15}\right]^{0.54}) (D_{o,oil}(20^\circ C) = 5 \times 10^{-11})</td>
<td>Butler and Yee (1986)</td>
</tr>
<tr>
<td>Dispersion coefficient, (m)</td>
<td>(\alpha_x = 0.0001)</td>
<td>Perkins and Johnston (1963)</td>
</tr>
</tbody>
</table>