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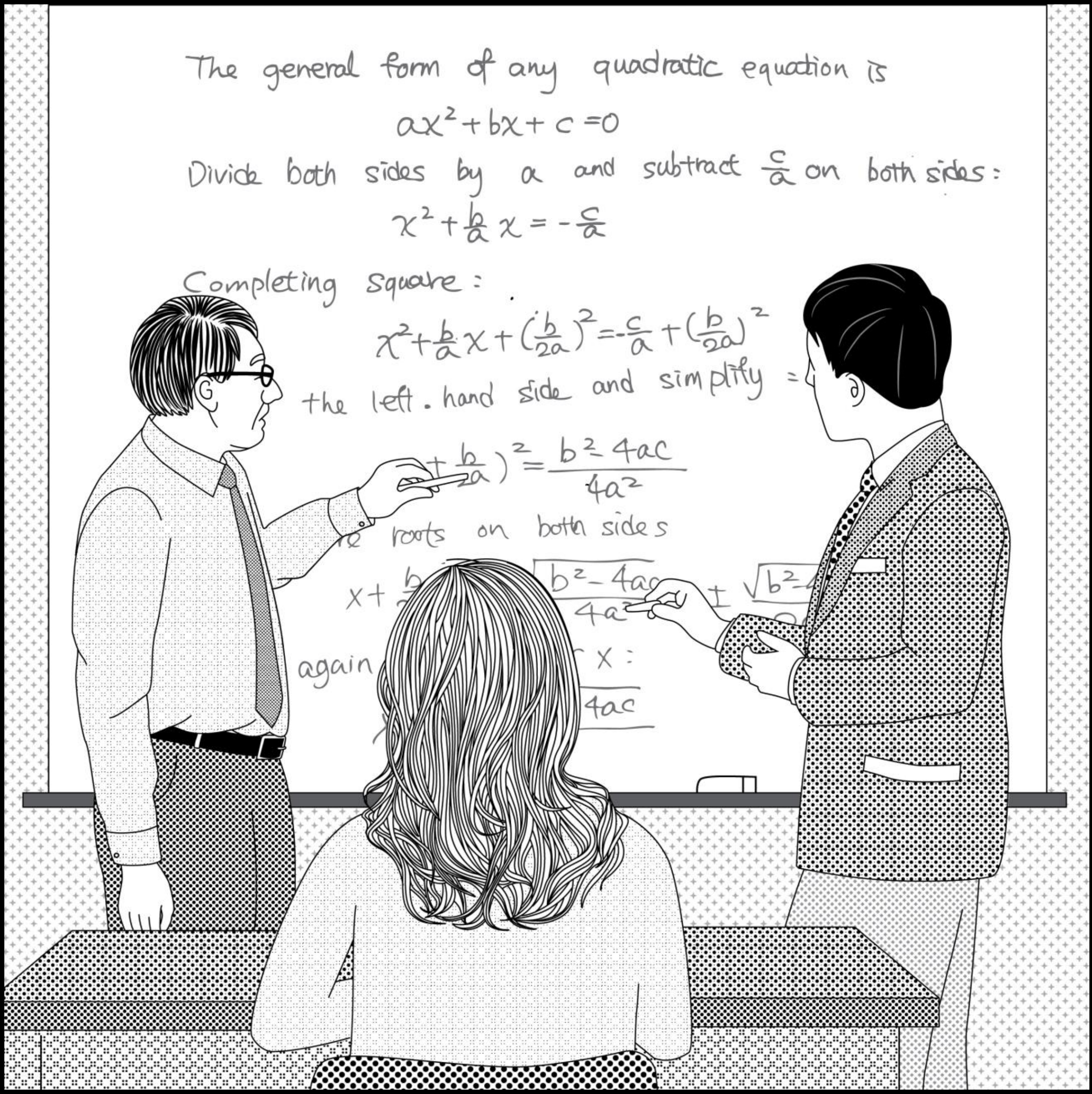

Adult Learner's Lived Experience and its Implications for Developmental Mathematics Instruction

A Doctoral Dissertation by
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Mathematics Anxiety Learning Phenomenon: Adult Learner's Lived Experience and its Implications for Developmental Mathematics Instruction

by

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A DISSERTATION

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Abstract

Previous studies of mathematics anxiety tend to focus on the objective and interobjective data of individual learner's behaviors. Using Wilber's Integral Model to identify gaps in research literature, this dissertation examined the life history of six anxious adult learners. The lived experience data from interviews and journal writing collectively disclosed the nature of Mathematics Anxiety Learning Phenomenon (MALP) at left-hand quadrants (subjective and intersubjective), responding to five research questions:

(1) What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?

(2) What are the roadblocks that prevent a learner to succeed in mathematics? What are the manifestations of these roadblocks?

(3) What are the underlying cultural beliefs in MALP, and how is the culture passed on to others and is perpetuated within and outside of the classroom?

(4) What are the social norms when learners are supporting each other? What is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?

(5) Based on the above, what could one disclose as an integral perspective about MALP and how it is cyclical and perpetual?

The data and thematic analyses disclosed MALP through all quadrants of the Integral Model, revealing that MALP is a cyclical and possibly perpetual phenomenon by festering and amplifying anxiety through episodes of learning school mathematics. Findings were consistent with Knowles' andragogy, showing mathematics anxiety to be a cumulative experience upon which learners retrieve and rely to cope with negativity toward learning. Data were compared to Givvin et al.'s (2011) hypothetical model on how college students learn developmental mathematics. The results demonstrated different paths to anxiety between two types of learners: behavioral (driven by prescribed steps and procedures) versus conceptual (driven by problem solving and logical reasoning).

Abstract (Continued)

Inspired by Dehaene's (2011) idea of overcoming nature's shortcoming in mathematical abilities through intensive nurturing, the dissertation shapes five instructional implications to foster an integral approach to mathematics education.

Acknowledgements

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Three years of doctoral study culminating to a project of this scale was impossible to complete all on my own. My tremendous thanks go to the following individuals:

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I am deeply indebted to my supervisor (more like a superhero), Dr. Veronika Bohac Clarke, who not only provided unparalleled wisdom and knowledge, but also opened my eyes to researching lived experience, allowing me to appreciate its openness, sensibility, and the integral clarity it provides to understanding mathematics anxiety.

My wholehearted gratitude goes to my supervisory committee members, Dr. Brent Davis and Dr. Ian Winchester, whose invaluable input to this project is thoroughly "lifting up" my eyes. The examination committee members, Dr. David Este and Dr. Norman Vaughan, your thoughtful comments were truly appreciated.

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Acknowledgements (Continued)

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$$\infty \times \boldsymbol{THANK\ \ YOU} \times \infty$$

All errors, shortcomings, and embarrassments remain solely my own in all aspects of this research project.

Dedications

To John, for your unending support and countless sacrifices during my pursuit of this endeavor, and for introducing me to the field of education

To Mom, Annie, who has always been there for me, through good and bad times, through thick or thin, and you are the rock whom I always count on

送給爸爸 *Charles* 的 (1946–2001)，不只為了某年對你作出的承諾，更是不曾忘記隨伴的足跡，感念不已，謝謝爸爸！

To all mathematics-anxious learners, don't give up!

YOU CAN DO IT!!

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Epigraphs

I don’t understand why they couldn’t teach something more useful than algebra.

From one of my students, 2000’s

I was never good in mathematics. I just want to pass this course. I have to. I am graduating this year!

A quote from Agnes Arvai Wieschenberg’s
Overcoming conditioned helplessness in mathematics, 1994

I dropped out of math class by grade 10 because girls are not necessarily math-science oriented.

From my mother, translated from Cantonese,
1980’s and 1990's

I chose home economics over math because my math teacher can’t stand to have me for one more year.

My mother-in-law, 2000’s

The mathematics class is not a funeral home.

Another quote from Agnes Arvai Wieschenberg’s
Overcoming conditioned helplessness in mathematics, 1994

Mathematics Anxiety Learning Phenomenon: Adult Learner's Lived Experience and its Implications for Developmental Mathematics Instruction

Chapter One: An Introduction to the Mathematics Anxiety Learning Phenomenon

This dissertation reports a study of mathematics anxiety through the lived experience of adult learners in developmental mathematics and the instructional implications of these experiences. The entire dissertation is organized into three major sections. The first section, comprised of the first three chapters, describes the nature of the research study and its methodology:

- Chapter one provides an overview of mathematics anxiety learning as a phenomenon and outlines the research questions.
- Chapter two reviews the literature on mathematics anxiety and introduces Wilber's Integral Model to show that it is a methodological void in the research landscape.
- Chapter three details the Integral Model, defends it as the underpinning framework, and discusses the methods for collecting lived experience data as well as data analysis.

The second section, comprised of the next three chapters, reports the findings and their analyses:

- Chapter four tells the lived experience of the research participants.
- Chapter five shows how the major themes are developed from the lived experience data.
- Chapter six addresses the theoretical impacts of the research study.

The third and final section of the dissertation is in chapter seven, which it concludes the research study by recommending instructional implications based on the developed major themes, outlines an emerging agenda that continues the research trajectory, and evaluates the study based on the guiding principles in mathematics education research (MER).

In this introduction chapter, I will begin with an informal and personal account of how I became familiar with mathematics anxiety in adult learners. Next, I will define mathematics anxiety as both an individual and social phenomenon, and then I will posit the problem statement and research questions, including the assumptions and delimitation of the study. Finally, the chapter will conclude with its potential significance in mathematics education research (MER).

Personal Statement

I have been teaching mathematics as a classroom instructor since 2001. In my pre-service teacher education training, I remember reading, discussing, and writing papers on many issues in mathematics education such as state policies, standards, curricula, and gender inequity, as well as mathematics phobia/anxiety. The training I received has been helpful in orienting myself to the profession, and there were plenty of remedies discussed for mathematics anxiety, such as to be reassuring with students. However, as I gain experience in the classroom, remedies that seemed to make so much sense at that time have become a lot less rosy. Namely, I find that the students are becoming more vocal and more freely express their disdain about the subject of mathematics as well as mathematics education itself. "I had one bad math teacher in 7th grade, and I have never been good with math." "I never learned this 'new math' back when I was in high school." "I am taking your course because Professor [Deleted]'s class is impossible to pass." These unsolicited confessions expressed so freely have bothered me for quite some time, mainly because to be reassuring alone is not going to resolve such negativity, which seems so indicative of a larger phenomenon at play. I find my efforts becoming more futile in combating such negativity.

Moreover, it is heartbreaking to continuously see how my students are constantly discouraged, a general frustration on the subject of mathematics. I have heard despair in their voices about whether they would succeed, sensing a covert sadness or even anger from them toward the subject matter and toward their own abilities. Seemingly, they find enjoyment in expressing those feelings to others. Such personal struggle with these feelings and the social nature of sharing them can possibly be traced to the connectedness of the anxiety, the subject matter, the learner's beliefs, their past and current learning experiences, and the self-reflection on those experiences. This is a phenomenon that is larger than myself and larger than any one individual student. Therefore, I have chosen to embark on this research study, believing that if I want to effect meaningful changes in my students, then I, as a practitioner, would have a duty to deeply and thoroughly understand the nature of mathematics anxiety and all issues that are connected to it. Given that mathematics anxiety as a phenomenon has preceded and existed long before I set foot into the classroom, what classroom strategies could I employ to optimize mathematical learning for those learners? This elusive question is what motivates me to embark on this research study.

Mathematics Anxiety Learning—A Reflection of Past Experience

To provide a descriptive account on what I will coin the Mathematics Anxiety Learning Phenomenon (MALP), I will first informally look at the following vignettes:

> **Vignette 1.** *Years ago, I advised a group of students who were enrolled in a bachelor of business administration program who were required to take a statistics course. I overheard their conversations when they were selecting from the different courses offered in a semester. Some of the comments were, "Professor* [Deleted] *is really tough, and he doesn't answer any questions that you ask," "His assignments are just impossible," and "The course goes way over what I need to know." What surprised me was that the students had started an informal support group to vent their negativity before the course had even begun. When I confronted them about what I had heard, they had no inhibitions telling me how negatively they felt toward the course and toward that professor. Fast forward to the end of the semester when the final examinations were approaching, overhearing the same group of students talking to each other: "I don't care how to do this, but as long as I can pull off a C minus, I'll be happy," "I am ready to burn the lecture notes the minute the final exam is over," and "I will never take Professor* [Deleted]*'s course ever again."*

This episode was challenging for me as an academic advisor for these students. I was conflicted. On one hand, I knew that they had a tremendous amount of apprehension, anxiety, and even anger before they entered the statistics course, and on the other hand, I found myself powerless in helping them overcome those emotions. From a greater social perspective, the fact that these students were discussing the nature of the course and the professor before the semester even began meant that they must have obtained the information from someone else, perhaps from former students who had taken the course. I wondered if the social nature of students who merely communicated their negativity toward mathematics could then induce others to sympathetically and emotionally resonate similar feelings and emotions.

> **Vignette 2.** *My mother-in-law and I had numerous dinner conversations over the past years, and whenever I became animated in discussing why studying mathematics was important, she would always bring up how proud she was to have chosen home economics over mathematics when she was in high school back in the 40's. She explained how she really had no interest in the technicality of mathematics, and she had failed the final examination by one point at the end of the school year. Her recollection was that her teacher thought that she was an arduous student, so he caved and gave her one extra point for her to pass. In the next year, she chose home economics, and she told me how much she enjoyed the sewing and cooking projects. After graduation, she married my father-in-law, who had served as a radio operator on an aircraft carrier during World War II. She raised five children while working as an occupational therapy assistant, and she could not have been more proud that the home economics skills she acquired had served well for both her family and her patients at work.*

As a mathematics teacher, I often wondered why my mother-in-law had not recognized all the mathematics involved in her cooking and sewing projects, and obviously, she was a capable person in order to be successful at home and at work. However, more deeply do I wonder how my mother-in-law interacted with her five children while raising them. Did she brag about her home economics glory to her children? To me yes, but I am not sure if that was the case with her children when they were young. Nevertheless, she had a lot to be proud of, for her children are now well into their adulthood. Each has a successful career. The three daughters are now an elementary school reading specialist, an occupational therapist, and a biochemist. Her two sons are now a technology education teacher and a director of safety and security in a major medical school. While they are all successful in their respective careers, with the exception of one daughter as a biochemist, the stereotypical gender specific career roles fit rather well with the rest. Yet the conundrum remains: how much did my mother-in-law's mathematics education experience influenced her children's subsequent learning experiences and their choices of careers?

Vignette 3. *A former classroom mathematics teacher who then became the mathematics coordinator in an urban high school is now a middle school principal. She shared with me how she has noticed incoming 7th grade students disproportionately expressing negativity toward the subject of mathematics. She observed that when students are poor in both reading and mathematics, they would be much quicker to admit that they cannot do mathematics than to admit they cannot read. In fact, incoming students had already developed their likes and dislikes among the school subjects with a large percentage of students not afraid to tell why they hate mathematics. The principal hypothesizes that the dislike of mathematics may be linked to elementary school teachers' preferences when she noticed that many multiple-subject teachers prioritize mathematics to be their least favorite subject. While she does not observe overt mathematics anxiety among the middle school mathematics teachers, she does notice that those teachers, including seasoned ones, generally have anxious feelings about teaching mathematics due to the ever changing New York state curricula and standards.*

This vignette depicts a school principal's observations on mathematics anxiety manifested twofold within a learning environment. While the incoming students exhibit the usual "I hate math" and other similar comments, the principal believes that the elementary school teachers are partially responsible, possibly due to their own mathematics anxiety. Meanwhile, the middle-school mathematics teachers, who generally have few qualms about the subject, still feel anxious about the teaching mathematics because they are pressured to meet the ever changing high-stakes state curricula and standards. In fact, when I was talking to the principal in June, 2012, she told me that they had yet to learn the new national common core learning standards that were legislated to be implemented in September, 2012. I wonder, given the current culture and politics at large, if an educator's anxiety could be directly linked to a student's anxiety toward the subject of mathematics

(McFadden, 2011). In other words, can the learners' anxiety toward mathematics be partially attributed to the classroom instructors' projection of their own anxiety?

Mathematics Anxiety Learning Phenomenon. As one can see from the above three vignettes, I am interested in the nature of mathematics anxiety for individual adult learners and the possibility of mathematics anxiety being passed from one individual to another. Each vignette portrays a different perspective: that of the learner, the parent, and the educator. But, somehow there is a connection among these perspectives. Formally, mathematics anxiety is defined as follows:

- Feelings of tension and anxiety that interfere with the manipulations of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. (Richardson & Suinn, 1972, p.551)
- Behavior which has been learned on a subconscious, automatic, reflective level by pairing previous experiences which were painful with the activity of mathematics. (Mitchell, 1984, p. 37)

Particularly interesting is Mitchell's definition that appeals to pairing previous experiences. While it is assumed the previous experiences are those of a single individual, it will be argued in this current research study that previous experiences can come from other members in the learning community, such as from a fellow learner, from a parent, or even from a mathematics instructor. This study is intended to investigate MALP, specifically situating mathematics anxiety among learners in a learning environment:

[A] Before the learning process takes place, adult learners have sets of beliefs about the subject of mathematics, about their own abilities, and about certain attitudes toward learning.

[B] The individual and social behaviors from **[A]** affect how learning takes place—i.e. learning what mathematical knowledge to memorize, learning how to survive school mathematics, and informing fellow learners of one's past experience.

[C] The individual and social behaviors from **[B]** serve as perceptions to reinforce or change the beliefs in **[A]**.

[A], **[B]**, and **[C]** altogether: How could the cycling learning phenomenon that is perpetuated in **[A]**, **[B]**, and **[C]** be sufficiently addressed (and perhaps be broken) in classroom instruction to optimize learning?

The cyclical nature of **[A]**, **[B]**, and **[C]** is defined as MALP in developmental mathematics, and it is characterized in the following figure:

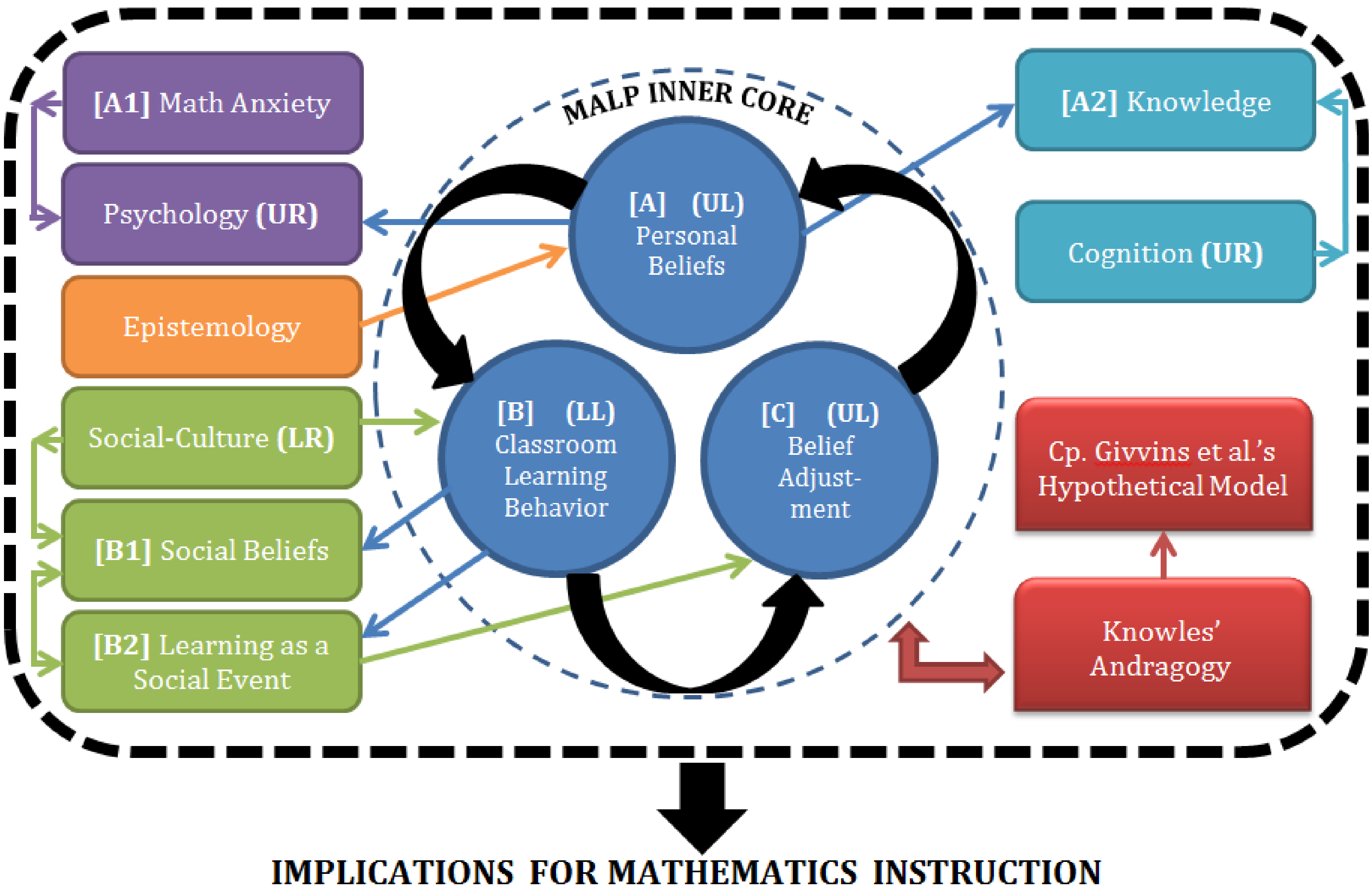


Figure 1.1. A concept map for Mathematics Anxiety Learning Phenomenon (MALP).

In the above flowchart, the circulation of **[A]**, **[B]**, and **[C]** corresponds to the description from the previous page, and this is essentially the core of MALP. In addition to the core, MALP involves several ancillary pieces. From **[A]** on personal beliefs, the effect of such beliefs forms mathematics anxiety as well as mathematical knowledge. For example, suppose a learner misbelieves that a square has three sides, then this faulty knowledge would generate anxious emotions when the textbook shows a square with four sides. For another instance, suppose a learner believes that the content of a developmental mathematics course is largely useless, then classroom behaviors such as exhibiting boredom, disinterest, and commenting to others could influence the morale of other fellow learners, and in turn influence social beliefs and group behaviors in learning. Suppose one learner says to another, "Just memorize this, and you will get through this quiz," then it is likely that the listener might stop comprehending the underlying mechanism of such mathematics. While mathematics anxiety as an emotion has been examined through psychological studies, the learner's outlook on learning mathematics can be affected by the social behaviors of other learners. Therefore, one must consider the social interaction aspect of mathematics learning and how it can play a role in mathematics anxiety. Moreover, the learning submerged in the social beliefs induced by mathematics anxiety has sociological and cultural bases that could explain such ethnographic and collective behaviors.

Theoretically, both the core and the ancillary portions of MALP tie closely to Givvin, Stigler, and Thompson's (2011) hypothetical model of college students' mathematics learning experience, and they have discovered that college students' past negative experiences result in haphazard application of rules and in being unbothered by the inconsistency in the results of procedures. These results could be argued by Knowles' andragogy based on the learner's past experience, self-directedness, and the purpose of learning mathematics. Through past learning experiences and the social nature in the current learning environment, it is not merely the subject matter that adult learners aim to engage, but rather they acquire and develop the negativity during the process as well. This enables them to strategically modify the learning process to minimize the mathematics event as well as the unpleasantness of doing mathematics. In other words, the individual and collective perspectives of mathematics anxiety are learned, and such learning directly competes with learning mathematics. Therein lies a significant problem for mathematics educators.

An Integral Approach to Study Mathematics Anxiety Learning Phenomenon. The notation of UL (Upper Left), UR (Upper Right), LL (Lower Left), and LR (Lower Right) in Figure 1.1 corresponds to, by location, the four quadrants of integral theory, which will be explained in detail in the methodology chapter. For now, it is noted that the components that correspond to the different quadrants are in different colors. Generally speaking, the four quadrants are described as different perspectives of MALP based on two dimensions: individual versus collective and interior versus exterior. An *individual* perspective focuses on a singular person while a *collective* perspective focuses on a social group of individuals. An *interior* perspective focuses on the first- and second-persons' point of view that is usually construed as subjective. Meanwhile, an *exterior* perspective focuses on the third-person's point of view that is usually construed as objective. The two dimensions altogether are realized as a quadrivium, four quadrants that co-exist and co-arise with no epistemological or ontological priority among them:

	Interior	Exterior
Individual	Methods to gain insights in the UL perspective (subjective): • Structural assessment • Phenomenological inquiry	Methods to gain insights in the UR perspective (objective): • Empirical observation • Autopoietic techniques
Collective	Methods to gain insights in the LL perspective (intersubjective): • Ethnomethodology • Hermeneutics	Methods to gain insights in the LL perspective (interobjective): • Systems analysis • Social autopoietic techniques

Figure 1.2. The research methods to gain insights to each perspective in the four quadrants (UL, UR, LL, and LR) under Wilber's Integral Model, adapted from Esbjörn-Hargens (2006b, p. 88 and 2009, p. 17).

Oftentimes, the subjective UL and intersubjective LL quadrants are referred to as the left-hand quadrants whereas the objective UR and interobjective LR quadrants are referred to as the right-hand quadrants. The inquiry into each quadrant reveals details that can be further explained through theoretical notions of levels, lines, states, and types. An integral approach to researching a phenomenon means that one comprehensively takes all perspectives into account when researching human experience. This approach, better known as Wilber's Integral Model, is often referred to All Quadrants, All Levels, All Lines, All States, and All Types or AQAL. With each perspective disclosing a unique window to reality, the composite of all perspectives forms an integral disclosure to the phenomenon, forming a comprehensive view that traditional empirical research studies usually are unable to disclose. In that sense, each perspective suggests a unique blend of research methods.

As Martin (2008) suggested, "[t]here is power in being able to see a larger picture, transcend our typical ways of approaching an inquiry, and consider where and how we can strategically use these elements" (p. 160). To attain such power and the use of different research elements, Martin argued for the use of Integral Methodological Pluralism (IMP) where a mixture of research methods is used to address the different perspectives in the AQAL model. It is worth noting that the research design for this study largely focused on the left-hand quadrants, the subjective and intersubjective perspectives in individual-interior (UL) and collective-interior (LL) quadrants. Meanwhile, the findings in this study complement past empirical psychological and sociological studies which largely focused on the right-hand quadrants, the objective and interobjective perspectives in the individual-exterior (UR) and collective-exterior (LR) quadrants. As the above Figure 1.2 suggests, the research method employed for this study adopted elements and techniques from hermeneutics (LL) and phenomenology (UL) to disclose the subjective and inter-subjective perspectives into the left-hand quadrants of MALP. The fine details of Integral Model, the different perspectives in the four quadrants, and IMP, the mixed use of methods, will be discussed in the methodology chapter.

The Integral Model serves a dual purpose for this study: (1) an examination of the current literature reveals that previous studies on mathematics anxiety retrieved objective data and drew inter-objective inferences. Broadly speaking, they explored the right-hand quadrants, resulting an incomplete disclosure under the premise of comprehensiveness of Integral Model. Based on the review of literature (in chapter two), the left-hand quadrants remain largely understudied in the research landscape. In other words, the Integral Model shows gaps in past research, and there exists a need for research to fill the void of the subjective and intersubjective nature of mathematics anxiety. (2) The Integral Model serves as an underlying framework for the design of the research such that an approach of the participants' life histories to mathematics anxiety is an appropriate mode of inquiry into the left-hand quadrants. The scope of this dissertation focuses on the perspectives in the quadrants, and the rest of AQAL, the theoretical notions of levels, lines, states, and types, is delimited and is considered to be beyond the scope of this current research study (see the Delimited section in this chapter). Together with past research and this current study, the aim is to disclose an integral view of MALP, thus providing new insights to inform mathematics instruction in an integral manner.

Research Problem. The problem investigated in this study focuses on way(s) in which classroom instruction could break the MALP cycle so that optimal learning can take place for adult learners in developmental mathematics. To probe into the problem statement, the research questions are formulated as follows:

Problem Statement: As a practitioner who teaches developmental mathematics to adult learners who often experience mathematics anxiety, what characteristics of MALP could give insights that would influence instruction to optimize learning? This problem can be dissected into five research questions:

(1) What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?

(2) What are the roadblocks (cp. Givvin et al., 2011) that prevent a learner to succeed in mathematics? And what are the manifestations of these roadblocks?

(3) What are the underlying cultural beliefs in MALP, and how is the culture passed on to others, and how is it perpetuated within and outside of the classroom?

(4) What are the social norms when learners are supporting each other? And what is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?

(5) Based on the perspectives in (1), (2), (3), and (4), what could one disclose as an integral perspective about MALP and how it is cyclical and perpetual?

Developmental Mathematics. At this point, it would be beneficial to define developmental mathematics because the major portion of the study is based on interviews with adult learners of developmental mathematics (see methodology in Chapter 3). Developmental mathematics can be loosely defined as remedial mathematics, usually below the level of pre-calculus, offered in many U.S. two-year colleges. Some common names for developmental mathematics are "Pre-Algebra" (MT 001 of Erie Community College), "Basic Math Skills" (MAT 091 of Genessee Community College), and "Introduction to Algebra" (MA 097 of Trocaire College). Oftentimes, these courses are not given college level credit that counts toward fulfilling a certificate, an associate degree, or a baccalaureate degree program. This is due to the assumption that well-prepared students entering college and university would enroll in "genuinely" college level mathematics courses such as pre-calculus and calculus. Therefore, the term "developmental" is used to distinguish the (un)preparedness of the learners entering college/university studies without the negative connotation of the term "remedial." For instance, Carnegie Foundation for the Advancement of Teaching (n.d.) claims that "60 percent of community college students who take the placement exam learn they must take at least one *remedial* course (also called *developmental* education) to build their basic academic skills" (Developmental Math, 1st paragraph, italics added). While the term "developmental mathematics" is largely understood among two-year college mathematics instructors, the American Mathematical Association for Two-Year Colleges (AMATYC) offers an explicit plea to make "no attempt to define 'college-level mathematics,' nor do its official standards address the issue of whether courses at the introductory level should be credit bearing ..." (Crossroads in Mathematics, 1995, p. 5). Hence, what constitutes developmental mathematics is open to debate within the discourse of mathematics education. For the purpose of this study, developmental mathematics is defined to be a college mathematics course that includes specific components for remediation purposes. The participants from Trocaire College whom I screened for this research study were all enrolled in a mathematics course that would meet the above definition, and I was the faculty member who wrote these courses in 2008.

Research Design: Using Life History to Uncover the Left-Hand Quadrant's Perspectives. The investigation of the MALP was through lived experience, which addresses the left-hand quadrants of the Integral Model's notion of AQAL. A highlight of this kind of research is to make a clear distinction between the individual and the social/collective aspects of the phenomenon, as well as to examine the phenomenon through the interior and exterior perspectives. Six adult learners enrolled in developmental mathematics courses in community colleges in the Western New York area participated in a series of unstructured interviews. They also wrote journals about how they supported and influenced other learners. The qualitative research data underwent thematic analysis, and the results were compared and evaluated against Givvin et al.'s hypothetical model and Knowles' andragogy.

Research Assumptions. Because the nature of inquiring life history centers one experience as primary research data, the study intends that a researcher does not enter the investigation with pre-conceived notions, there were relatively few assumptions. This research study assumes that MALP exists as an actual phenomenon in adults within social learning environments. Most notably, MALP assumes that learning is a social event in which there is a collective force. Secondly, the study employs Knowles' andragogy as a set of assumptions (Merriam & Caffarella, 1999, p. 272):

(1) As a person matures, his or her self-concept moves from that of a dependent personality toward one of a self-directing human being.

(2) An adult accumulates a growing reservoir of experience, which is a rich resource for learning.

(3) The readiness of an adult to learn is closely related to the developmental tasks of his or her social role.

(4) There is a change in time perspective as people mature—from future application of knowledge to immediacy of application. Thus an adult is more problem centered than subject centered in learning (Knowles, 1980, pp. 44–45).

(5) Adults are motivated to learn by internal factors rather than external ones (Knowles & Associates, 1984, pp. 9–12).

As such, this study assumes adult learners to be self-directed, to be making use of past experiences for learning, to relate readiness with his or her social role, to be problem-centered in learning, and to be internally motivated.

Motivation for the Research Study

Aside from my personal endeavor to become a better informed practitioner so that my students can benefit from better instruction, poor achievement among students in college developmental mathematics is an important motivator for this study. A fair amount of literature indicated that adult learners were performing alarmingly poorly in college level mathematics courses. To cite a few, the Mathematical Association of America (MAA) reported that less than one half of the students in the typical college calculus sequence were likely to complete the sequence (Kasten, Suydam, & Howe, 1988). Also, Hofacker (2006) reported that only about one half of the students enrolled in a college algebra course were likely to complete the course successfully. Furthermore, some suggested that mathematics as a subject was a "gatekeeper" such that students who performed poorly in the subject did not gain access to higher education (Jetter, 1993; Kamii, 1990; & NCTM, 1998). Therefore, many colleges/universities devoted considerable effort and resources to rectify the low completion rate and poor performance in college mathematics (Post et al., 2010, p. 275). This research study is in similar spirit as those previous reports and is motivated by low completion rates and poor performance in college level developmental mathematics courses. Through a decade of classroom teaching, I observed that students' epistemological foundations toward both the subject matter and their own abilities to succeed are key factors that play a significant role in how they perceive what learning mathematics entails. Their attitudes and perceptions may manifest in psychological, cognitive, and social behaviors, but these behaviors in turn reinforce the learner's initial perceptions, forming beliefs about the subject matter and about their own abilities.

Potential Significance of the Research Study

Because of the integral nature of this research project, the results from the each research question can be cascaded into the four quadrants to integrally disclose the nature of MALP:

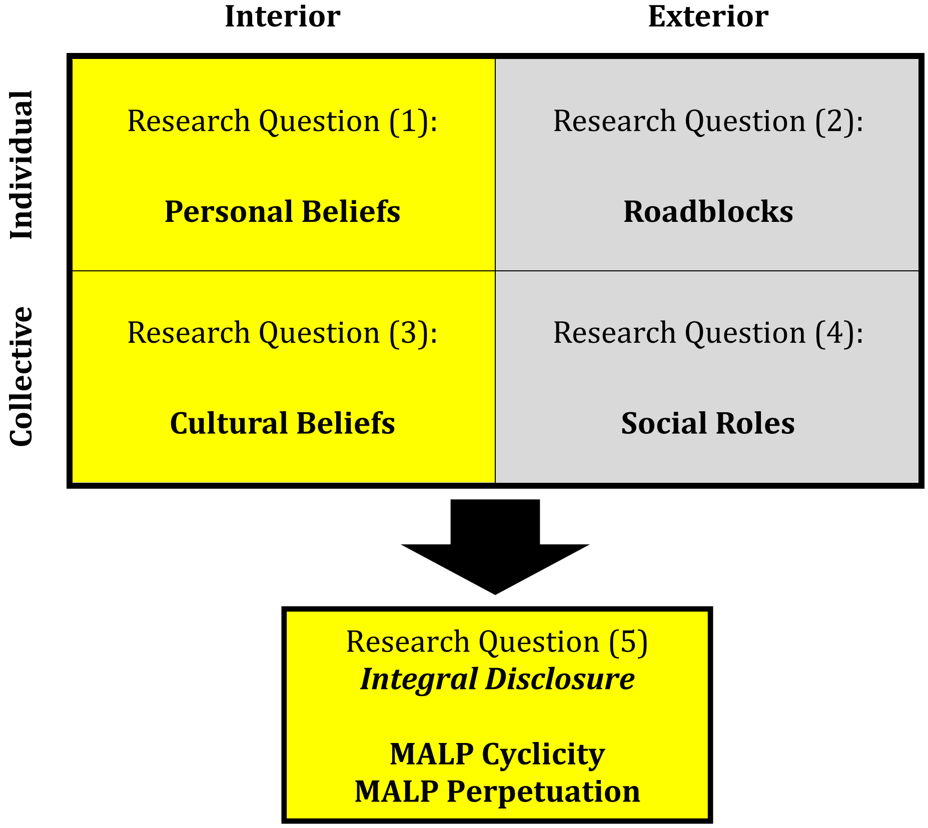


Figure 1.3. The five research questions and their relations to the four quadrants in Wilber's Integral Model, with yellow color denoting new contribution from this study and gray from past research.

The above figure shows that the research question (1) on personal beliefs discloses an individual-interior (UL) perspective, and the research question (3) on roadblocks of learning discloses a collective-interior (LL) perspective. These two questions are largely informed by the subjective and intersubjective data from this current study. Furthermore, the research questions (2) and (4) on the roadblocks of mathematics learning and on the learner's social roles are more generally informed by past empirical research studies as well as by my own third-person observations from the lived experience data. These two research questions collectively disclose the individual-exterior (UR) and collective-exterior (LR) perspectives in objective and interobjective manners. In sum, the significance of this research study (shown in yellow in the above figure) largely comes from the UL and LL quadrants. Coupled with past research findings (shown in gray in the above figure), the ultimate integral disclosure of MALP becomes a unique contribution to the research landscape as new knowledge to what is currently known about mathematics anxiety.

Moreover, Lester (2010) argued that there are three guiding principles to think about in the purpose and in the nature of mathematics education research (MER), and one would evaluate this proposed research study against each guiding principle:

(1) The goals of MER are to understand fundamental problems concerning the learning and teaching of mathematics and to utilize this understanding to investigate existing product and develop new ones that would potentially advance the quality of mathematics education.

Obviously, this study follows guiding principle (1) by studying how MALP is cyclical and perpetual. If the study indeed finds and confirms that MALP is a significant culprit in negatively impacting learning and achievement, then it could direct how to modify instruction to advance the quality of mathematics education. Although this study does not develop new products to advance the quality of mathematics education, the results of the study serve as an informed and educated opinion to raise awareness of the MALP, which could be useful in mathematics classes from pre-K to 16.

(2) To achieve these goals, MER must be theory based, which means studies in MER must be oriented within research frameworks.

As one will see in the next chapter, this study is underpinned by Givvin et al.'s (2011) hypothetical model on college student's mathematics learning experience. In turn, both MALP and Givvin et al.'s model are shown to have important implications within Knowles' andragogy. The research methodology was adapted from Wilber's (2000a; 2000b) Integral Model which presumes every perspective discloses a unique window to a phenomenon. Therefore, the investigation of MALP will be studied through a multi-perspective manner.

(3) The research framework's argued-for concepts and their interrelationships must be defined and demonstrated in context, which is entailed by Principle (1), must include mathematical context.

The current research study appeals to Renert and Davis' (2010) five mentalities of mathematical knowledge (or what entails mathematics). The investigation would reveal how the individual and collective beliefs on what constitutes mathematical knowledge would be situated in the mathematical context.

Finally, Harel (2008) stated a fourth guiding principle:

(4) The ultimate goal of instruction in mathematics is to help students develop ways of understanding and ways of thinking that are compatible with those practiced by contemporary mathematicians.

In line with the fourth principle, the intended result of this research study is to provide implications on mathematics instruction, including strategies to help adult learners overcome affective anxiety that inhibits retrieval of information from working memory. At the most basic level of learning, memory retrieval of information is one of the most fundamental abilities. Without overcoming anxiety that prevents this ability, it would be impossible for a learner to develop ways of thinking and understanding.

Lastly, one concludes this section with Stokes' (1997) "Pasteur's Quadrant" of scientific research which argues that research is inspired by both the considerations of use and the quest for understanding. A study that is considered to be a quest for understanding has goals to understand a mechanism or a phenomenon. So for instance, the study of the sub-atomic structure of an atom would be classified as a quest for understanding. Meanwhile, a study that is thought to be a consideration of use is usually applied in nature. For example, the study on how the subatomic structure of an atom changes to optimize the explosive power of an atomic bomb would be considered as a study in this category. In essence, a research study could be classified and be evaluated as in Stokes' "Pasteur's Quadrant" (as cited in Lester, 2010, p. 82, arrows added) to represent the degree/extent to which a research study successfully fulfills the quest and the considerations:

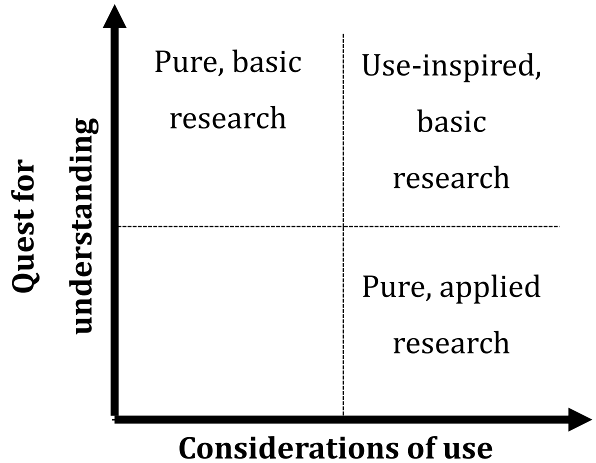


Figure 1.4. Stokes' Pasteur's quadrant.

In this research study, the quest to comprehend the nature of MALP corresponds to the quest for understanding. The quest to use such comprehension to help inform implications for classroom instruction corresponds to the consideration of use. Thus, this research study could reach an ideal place in Pasteur's Quadrant, and the results could make a significant mark within the landscape of MER.

Delimitation of the Research Study. MALP is both an individual and collective phenomenon. The collective extent can be vast—learner-to-learner, instructor-to-learner, learner-to-instructor, parent-to-child, child-to-parent, to name a few. This research study mainly focuses on learner-to-learner and instructor-to-learner experiential perspectives. The secondary focus of this study is on parent-to-child experiential perspectives. However, given that the study does not interview mathematics instructors and children, their experiential perspectives will not be included in this research study. Theoretically, the Integral Model's notion of AQAL encompasses all perspectives from all quadrants, all levels, all lines, all states, and all types to disclose a window to reality that is truly comprehensive. However, an AQAL inquiry to MALP would entail a larger scope than that of this dissertation. The current research study focuses on the inquiry into all quadrants, leaving the rest of AQAL for future research.

Conclusion

In this chapter, I have overviewed MALP and have anchored the research questions for this dissertation study. I have also begun a discussion of using the Integral Model as an underlying framework for this research, as well as how researching life history is an appropriate approach. Because there are several discussions on the model for the next few chapters to come, I will conclude this chapter with a list of key terms used in the Integral Model and their definitions:

AQAL

Stands for "All Quadrants All Levels," but it also extends to the idea of "all quadrants, all levels, all lines, all states, and all types" and it refers to the comprehensiveness and integrity of approaching to a phenomenon.

Autopoiesis

An inquiry into the 1st person "I" approach to a 3rd person "he/she" reality. The term is used in the discussion on Integral Methodological Pluralism (IMP).

Left-hand quadrants

Refers to the subjective, individual-interior (Upper Left) and intersubjective, collective-interior (Lower Left) quadrants in Integral Model.

Right-hand quadrants

Refers to the objective, individual-exterior (Upper Right) and intersubjective, collective-exterior (Lower Right) quadrants in Integral Model.

Social Autopoiesis

Refers to the study of how networks of participants and their processes to self-organize. It is a study of the 1st person "we" approach to 3rd person "They" realities, and the inside view of the exterior of a collective. The term is used in the discussion on Integral Methodological Pluralism (IMP).

In the next chapter, I will review research literature that links to MALP through learner's epistemology, psychology, cognition, and sociology. Moreover, I will discuss two particular theories: (1) Givvin et al.'s (2011) hypothetical model on community college developmental mathematics students' learning experience and (2) Knowles' andragogy. It will be argued that both theories are directly related to the study of MALP.

Chapter Two: Literature Review

In the previous chapter, I overviewed the Mathematics Anxiety Learning Phenomenon (MALP) by hinting at how the learner's (1) epistemology, (2) psychology, (3) cognition, (4) identity, and (5) social roles are key components to MALP. In this chapter, I will review literature in each of the above five key components to investigate how past research may provide insights for MALP. To help elucidate how MALP is situated in the research landscape, I will discuss the relationship among MALP, Givvin et al.'s (2011) hypothetical model on college students' learning experience, and Knowles' andragogy. It will be argued that both theories are directly related to the study of MALP. Then I will make a crucial distinction on how approaching the research through life history with Wilber's Integral Model as the underpinning would contrast past psychological (and some sociological) research studies' methods. Such a contrast serves as a basis for how the current study can potentially make a distinct contribution to mathematics education research.

Adult Learners' Views on Learning Mathematics

Tang (2007) reviewed learners' views toward the subject of mathematics by synthesizing existing studies of the development of their epistemic beliefs. While he makes similar psychological claims to Wieschenberg (1994) about how failure leads to helplessness, Tang appeals to the socio-cultural traditions (p. 30) that may have significant impact on how the learner might view the subject matter and how success in the subject can be achieved. He summarized that East Asian families (particularly Chinese and Japanese) have high expectations toward their children in mathematics (also in Schoenfeld, 1989, p. 345), and some of those students who struggle with the subject would cope to meet that expectation by memorizing every step required to solve a problem. Because such coping mechanisms often lead to the severe lack of understanding in mathematics, many learners develop a view of the subject that is disconnected from their real world view. Tang calls this perception "stand-alone" (p. 30). Meanwhile, U.S. parents tend to believe that the ability to succeed in school mathematics lies in "born talent," and consequently failing learners believe it is their lack of ability to succeed (p. 30; also see Schoenfeld, 1989,

p. 345). Therefore, the formation of MALP among international learning communities can be quite different, but lead to very similar perceptions and effects among learners. What is interesting in Tang's report is the absence of whether the East Asians are vocal or not with regard to their poor performance in mathematics.

Tang's study is counterpointed by Schoenfeld's (1989) study, also on students' belief systems. Schoenfeld, using NAEP (1983; pp. 27–28), implied that a group of high school geometry students from Rochester, New York, believed that learning mathematics equated to memorization (as cited in Schoenfeld, 1989, p. 344). Such memorization is described as "[s]tudents were expected to master the subject matter, by memorization, in bite-size bits and pieces" (p. 344). Furthermore, Schoenfeld cited that "there is a growing literature about misconceptions" (e.g. Helms & Novak, 1984); the literature indicates that people consistently misperceive aspects of their experiences and then act on the basis of these misperceptions" (p. 341). In sum, Schoenfeld reported on the severity of the learners' misconception toward mathematics and common poor coping mechanisms for such misconceptions. Also, he found that students would tend to view a mathematical problem as unsolvable if they could not see a glaring solution within *a couple minutes* from when they first encountered it. Moreover, Schoenfeld explained the findings through years of drill-and-kill exercise problems which students mechanically answer while spending very little time thinking deeply about them: "[Students] came to expect the problems they were asked to solve to yield to their attempts in just a few minutes, if at all" (p. 341). In a survey, Schoenfeld asked these high school students, "'[h]ow long should it take to solve a typical homework problem' averaged just under 2 minutes, and not a single response allotted more than 5 minutes" (p. 345). From the two studies, Tang and Schoenfeld together suggest that U.S. learners believe their abilities in mathematics are based on their born talent, and even if they think that they are successful, the way they define success is to mechanically solve problems without thinking deeply about how to solve these problems. This suggestion will play a significant role in the current study on MALP.

Perhaps the most directly relevant study of MALP on learners' beliefs is Wieschenberg's (1994) study. Her study on how to overcome helplessness in mathematics provides a clear perspective and gives a partial insight to MALP. She juxtaposed two ideas as grassroots, shown as follows:

- Weiner (1973, p. 11) claimed that individuals highly motivated to achieve success assume personal responsibility for success and attribute failure to lack of effort. Persons low in achievement needs do not take credit for success and ascribe failure to a lack of ability. (As cited in Wieschenberg, 1994)
- Seligman noted that we all become momentarily helpless when we fail. (As cited in Wieschenberg, 1994)

Wieschenberg used the term "helpless" to explain the learner's difficulty in making good learning progress, and she cited Weiner that those learners have accepted and believed their lack of ability for their failure (cp. Tang's and Schoenfeld's assertions on the belief of born ability). Perhaps this is the reason why adult learners who have a history of poor performance in mathematics would confess so freely of their failure as well as their negative feelings toward the subject. Wieschenberg's study is an interesting counterpoint for this research because the issues of helplessness and the learner's free admission can be seen as a manifestation of MALP in which:

Helplessness	roughly translates to	**Hindrance of Learning and Fear of Mathematics**
Free admission	roughly translates to	**Poor Perception and Passing the Fear to Others**

Figure 2.1. Analysis of Wieschenberg's (1994) key concepts on how they map to MALP.

Wieschenberg's analysis on helplessness addressed the individualistic aspect of MALP where the inner psyche of an adult learner is examined. As well, her analysis on free admission addressed the social/collective aspect on how fear is passed on from one individual to another. The contrast between the individual and social/cultural aspects will be an important distinction, as I will discuss integral theory as a research methodology later in this chapter. In addition to her work on helplessness and fear, Wieschenberg suggested several instructional strategies to combat the helplessness issue. However, her strategies are designed for after the helplessness develops, and therefore, they do not address the root cause of the helplessness (cp. Givvin's et al. (2011) hypothetical model and

Ashcraft et al. (2002; 2005; 2009)). Therefore, the issue of instruction that originally induced the poor perception of mathematics among students.

De Corte, Verschaffel, and Depaepe (2008) have confirmed that "students at all levels of education hold naïve, incorrect, and/or negative beliefs about mathematics as a domain and about mathematics learning and teaching." They, through a qualitative study, have shown that "the prevailing teaching practices and the culture in mathematics classrooms are largely responsible for the development in students of those non-availing beliefs" (p. 34). While De Corte et al. called a similar account as "abandonment of sense making," their results seem to be in agreement with Givvin et al.'s hypothetical account of the "haphazard application." Indeed, D'Amour's (2013) dissertation portrayed a perspective in accord with De Corte et al.'s notion of "the development of non-availing beliefs" and Wieschenberg's "abandonment of sense making":

> One must grant audience for sense-making in a holding environment where students explore legitimate peripheral engagement with a sense of inquiry. It is not my experience that students choose to avoid perturbations when they are at liberty to grapple with them. It is instead the experience of someone looking over one's shoulder awaiting a response that provokes anxiety. (D'Amour, 2013, p. 443)

Essentially, D'Amour hinted that the space intended for sense-making in the learning environment, such as the traditional "wait for an answer", is counterproductive to anxious learners as the space itself tends to lead to the infliction of anxiety. This social dynamic, while well intended for learning to take place, may covertly discourage anxious learners to sustain "serious play" (p. 444) in learning. Instead, anxious learners may very well opt for "pretend" to play (p. 442) in the learning environment. In other words, D'Amour argued that the learner's internal process at play has not sufficiently been respected, and mathematics teachers do not recognized the internal process because they generally are unaware of its existence or importance. Such a failure of recognition leads to a social dynamic that is not conducive to the intended authentic learning, but rather is conducive to raising anxiety among mathematics learners.

What Entails Mathematics. Davis (1996) and subsequently Renert and Davis (2010) have proposed mathematics as five stages or mentalities: *oral, pre-formalist,*

formalist, hyper-formalist, and *post-formalist* (Renert & Davis, 2010, pp. 180–181). I will briefly describe each stage:

- *The Oral Stage*: This mentality refers to mathematics knowledge by means of awareness, and the oral and verbal communication of such knowledge is often the means. For example, a young child's explanation on what the number *zero* means.
- *The Pre-Formalist Stage*: This mentality generally corresponds to the "traditional consciousness" (Renert & Davis, 2010, p. 182) which mathematics knowledge is represented through informal, written formats. For example, a drawing of a square with each side labeled as 5 cm along with a statement that says "all sides of a square have the same length." Notice that this mentality reasoning is pre-formal because a specific size of a square (i.e. 5 cm per side) cannot be a sufficient evidence to generalize that all squares share the same property.
- *The Formalist Stage*: This mentality refers to the "modernist consciousness" (Renert & Davis, 2010, p. 182) adopted by formal logic and proofs. The means of communication is reasoning and often is mathematically stylized. For example, proving the sum of two even integers is also even, $2m + 2n = 2(m + n)$. The knowledge in this mentality can be generalized with certainty, and argumentation is conducted through rationality, such as deductive logic.
- *The Hyper-Formalist Stage*: This mentality is characterized as an "extreme extension of formalist consciousness" (Renert & Davis, 2010, p. 183) in which mathematics is purely a logical and syntactic construct, and it often does not appeal to the natural world. For example, my master's thesis on the solvability of the equation $a^n + b^n = c^n$ where a, b, and c are p-adic integers, as opposed to regular integers, would be considered a form of hyper-formalist mathematics. This is mainly because p-adic integers do not correspond to the natural world: people generally do not compute with p-adic integers; engineers probably do not appeal to p-adic integers to solve problems; and scientists most likely would not use p-adic integers to

investigate natural world phenomena. This mentality requires "non-standard logics and abstract grammars" (Renert & Davis, 2010, p. 183), and it is essentially exclusively conducted as mathematical formalism. In other words, mathematicians engage themselves in the mathematics of this stage for mathematics' sake.

- *The Post-Formalist Stage*: This mentality aims to study "meta-mathematics" where different mathematical systems/logics can be compared, contrasted, reviewed, interpreted, and evaluated. For example, Godel's proof of the incompleteness of formal systems is conducted through "a socially-constructed interpretive discourse" (Renert & Davis, 2010, p. 183) would be a form of post-formalist mathematics.

While the five mentalities of mathematics provide a wide spectrum of what entails mathematics, the mathematics education practice generally sees subject content as an inert, unchanging body of knowledge, mainly as pre-formalist and formalist mathematics. This is possibly because U.S. school mathematics curricula has been highly standardized, mainly by the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards*, as it has been regulated, such as with the 2001 U.S. Act of Congress on *No Child Left Behind Act* (NCLB). Ernest (1985) has made a similar observation, "[i]n educational terms this corresponds with the view of mathematics as an inert body of knowledge which instruction transmits to the student" (p. 607). The claim of mathematics knowledge as an inert body is an important one, as one will see below on parental influence. Many parents, experience their children's mathematics materials as "new math," as opposed to the "old math" of their own early education. They find it challenging to support their children's education because they are unable to comprehend that "old math" and "new math" are different in presentation and/or argumentation (Remillard & Jackson, 2006; Ginsburg, Rashid, & English-Clarke, 2008). Such ethnographic observations are examples of seeing mathematics knowledge as an inert, unchanging body. As for the two pre-dominant mentalities—pre-formalist and formalist mathematics, those are the developmental mathematics the current study examines. While NCTM largely prescribes the principles and standards for school mathematics in the K–12 level, the American Mathematical

Association for Two-Year Colleges' (AMATYC) *Crossroads in Mathematics Standards* (1995) also achieve similar dominance in the arena of college mathematics curricula. Richardson and Suinn (1972) pioneered a widely cited definition for the phenomenon as "feelings of tension and anxiety that interfere with the manipulations of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p.551). Mitchell (1984) characterized mathematics anxiety as a "behavior which has been learned on a subconscious, automatic, reflective level by pairing previous experiences which were painful with the activity of mathematics" (p. 37).

Psychology: Mathematics Anxiety as an Individual Behavior

The Mathematics Anxiety Rating Scale as a Research Instrument. The study of the psychological nature of mathematics anxiety can be traced back to the 1970's when, as will be surveyed later in this chapter, Ashcraft suggested that mathematics anxiety is strongly linked to an individual's ability to access his/her working memory. Research effort focuses on the variables for mathematics anxiety such as ability, gender, age, ethnicity, self-efficacy, and parental influence (Hembree, 1990; Ho, Senturk, Lam, Zimmer, Hong, Okamoto, Chiu, Nakazawa, & Wang, 2000; Ma, 1999) and their effects such as motivation, achievement, test scores, course selection, and career choice. There is also research on intervention practices that seem to be effective in lowering mathematics anxiety and boosting achievement.

Psychological research studies are mainly rooted in the traditional positivism of well-controlled experimental testing (i.e. controlled-treatment comparisons and variable correlations) in which observations are documented through self-reported Likert surveys (Ashcraft, 2002; Ashcraft & Moore, 2009; Ashcraft & Ridley, 2005; Betz, 1978; Bessant, 1995; Ho et al., 2000; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998; Woodard, 2004; Ferry, Fouad, & Smith, 2000; Haynes et al., 2004; Zakaria & Nordin, 2008). Statistics, particularly multiple regression tests, ANOVA, and *t*-tests, have been traditionally used to determine the correlation among variables associated with mathematics anxiety, as well as to determine if a variable is a significant contributor. Many of these studies employ Likert surveys as research instruments, and they are largely similar to, or usually variations of, the Mathematics Anxiety Rating Scale (MARS) by Suinn (1972). Alexander and Martray (1989) have reported that MARS is used extensively in research due to its "sound psychometric attributes" and "extensive data on reliability and validity" (p. 143). Between the 70's and the 80's, researchers designed various versions of MARS, such as the abbreviated MARS or aMARS (Alexander & Martray, 1989), the revised MARS or the rMARS (Plake & Parker, 1982), the short MARS or sMARS (Richardson & Woolfolk, 1980), and the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976).

While these variations of the scales may differ in the number of items in the surveys, the designs and the contents are largely similar. They are all in the form of a Likert survey in which research participants are given a pre-set of written statements to rate from agreeable to disagreeable. While many research studies conducted through MARS or variations on MARS result in an extensive list of variables linked to mathematics anxiety with exact statistical correlations among them, research data from these studies were mainly drawn from the participants' conscious choices within the confines of the survey. A review of integral research later in this chapter (and will justify its use and its appropriateness and trustworthiness for this research study), the corpus of psychological research on mathematics anxiety has left a void in the qualitative, as well as interpretative, form of inquiry to investigate the external and internal influences of societal and cultural effects on mathematics anxiety (Ashcraft, 2002).

Variables Linked to Mathematics Anxiety. Numerous studies using the MARS as a research instrument have been conducted since the 1970's. There are too many to name and review here. A more recent study by Ho et al*.* (2000) examined gender as a specific variable in mathematics anxiety by testing it against Chinese, Taiwanese, and U.S. 6th grade students, and the study found that gender by nation student interaction is significant. The study also claims that "Asian students differ from U.S. students with respect to attitudes, beliefs, and emotions regarding mathematics" (p. 365). Such a claim is later confirmed by Tang (2007). Another well-known study by Betz (1978) on college students also links gender (p. 446) to mathematics anxiety, and Betz has identified that age, particularly in adult women, also plays a role (p. 446). Furthermore, she also found that the number of years of mathematics training in high school, as well as the level of test anxiety, are also significant variables.

Hembree's (1990) meta-analysis managed to synthesize 151 previous research studies by showing several psychological variables that correlate with mathematics anxiety. Hembree acknowledged that "the research of mathematics anxiety has prospered, spurred by increasing perceptions that the construct threatens both achievement and participation in mathematics" (p. 34). Hembree managed to show that "performance" (pp. 37–38), "attitude" (p. 38), and "gender" (pp. 39, 45) are variables that play strong roles in mathematics anxiety. In particular, Hembree concluded that "[a]cross all grades, female students report higher mathematics anxiety levels than males" (p. 45), but "the higher levels do not seem to translate into [a] more depressed performance or to greater mathematics avoidance on the part of female students [in college settings]" (p. 45). The conclusion is interesting for this current research study for two reasons: (1) Hembree's meta-analysis reveals achieving female college students can conceal their anxiety during performance, or simply put, a female student could experience mathematics anxiety without showing weakness in her performance. (2) The fact that there is not greater avoidance of mathematics in the college setting can be explained by the fact that most U.S. college degree programs require some sort of mathematics to complete. In other words, for most college students, there's no avoiding mathematics. It is yet to be seen whether college students who suffer from mathematics anxiety would avoid mathematically-oriented study if they were given a choice to do so. Regardless, Hembree's research is a summing up point for the nature of mathematics anxiety and its effect.

A follow-up meta-analysis by Ma (1999), confirmed that some of Hembree's reported variables are significant, such as grade-level groups, ethnic groups, and the instruments measuring anxiety. However, Ma did not find robust and significant interaction effects among key variables such as gender, grade, and ethnicity. Ma asserted that these different findings were different from Hembree because "mathematics anxiety can take multidimensional forms including, for example, dislike (an attitudinal element), worry (a cognitive element), and feel (an emotional element)" (p. 520)[1]. Between Hembree and Ma's meta-analyses on the conflicting results of gender and ethnicity, it becomes clear that these controversial variables will play a critical role for this research.

[1] This argument is also made by Hart (1989) and Wigfield & Meece (1998).

Individual Consequences of Mathematics Anxiety. Research reports in a much speculative manner on the individual consequences of mathematics anxiety: low motivation (Betz, 1978; Green, 1990), poor achievement (Richardson & Woolfolk, 1980; Green, 1990), low self-efficacy (Hackett, 1985), avoidance of mathematics-related courses/study (Hackett, 1985; Lopez et al., 1990), and avoidance in choosing mathematics/science-related careers (Bieschke & Lopez, 1991). In general, researchers are much more speculative because the individual consequences are directly observable. For instance, low motivation, poor achievement, and low self-efficacy can be salient features of an individual, especially if one experiences mathematics anxiety. Therefore, it seems natural that the variables of mathematics anxiety are tested much more rigorously than the consequences.

The above surveyed research studies of mathematics anxiety as an individual behavior and the identification of variables can be summarized in a diagram with a list of recurring variables and a list of individual consequences:

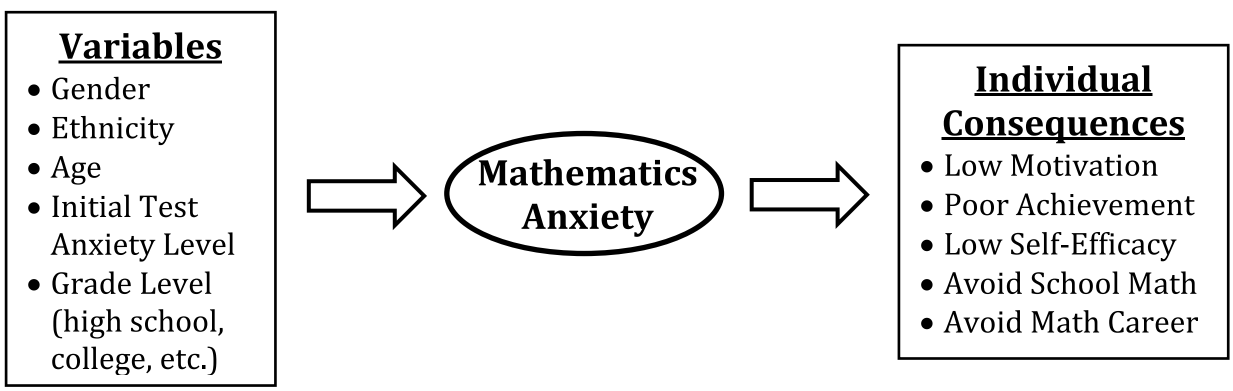


Figure 2.2. The summary of the variables and individual consequences that are linked to mathematics anxiety.

In this section, I have surveyed the research studies that investigate mathematics anxiety as an individual behavior. In the next section, I will survey research from the cognitive perspective.

Cognition: Mathematics Anxiety as Event inside the Brain and Mind

Nature, Nurture, and Plasticity. A tempting assumption to make about cognition and education is the contrast of nature versus nurture. Cognition is associated with the hardwiring of the brain and mind, and therefore, it is largely attributed to nature. While education, as a cultural and social event, molds the mind, and therefore, it is largely attributed to nurture. Dehaene's (2011) work aimed to bridge this dichotomy. He addressed the hardwiring of mathematics by comparing the human species with other intelligent animal species. "Our brain seems to be equipped from birth with a number sense. Elementary arithmetic appears to be a basic [*sic*] biologically determined ability in our species (and not just our own—since we share it with many animals)" (pp. 169–170). To strengthen his position, Dehaene asserted that "[p]reverbal human infants have elementary numerical abilities" (p. 173). He reported research observations of "surprises" from five-month-old infants when they witnessed unexpected sums and differences. For example, two objects being placed behind a screen drop, but the removal of the screen magically showed the unexpected sums of one object $1 + 1 = 1$ and three objects $1 + 1 = 3$, suggesting that preverbal infants seem to have some innate basic numerical abilities.

The observation that development of mathematical ability through nurture is obvious, as seen in educational training both inside and outside of the school environment. Dehaene reported some culture may have a linguistic advantage than other, such as Chinese children whose native language[2] has a much simpler word morphology for representing numbers comparing to those who speak French or English (p.175). Other observations that explain why some learners are more talented in numeracy and mathematics include better classroom organization and other qualitative comparison in mathematics education. The idea of different people with varying abilities in mathematics is a strong argument that education plays a nurturing role despite the nature's hardwiring is relatively the same for all. As Dehaene asserted, "innumeracy may be our normal human condition, and it takes us considerable effort to become numerate" (p. 176), and "[w]e all start out in life with very similar brains, all endowed with an elementary number sense that has some innate

[2] Earlier reports from Dehaene showed that numbers are represented in memory twofold as if the person is bilingual. (For instance, Dehaene & Akhavein, 1995, p. 314; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999, p. 970). For example, the number 351 is stored in both the Arabic format as well as the English format "three hundred and fifty-one." Also, Dehaene (1992) further argued that single-digit operations, such as $7 + 8 = 15$ or $2 \times 4 = 8$, are also stored in memory as part of a "mathematics" lexicon for efficient calculation of more involved operation, as in $7 + 2 \times 4 = 7 + 8 = 15$ (p. 6). These earlier works provide an interdisciplinary perspective on how language and mathematics are related. Also among these studies, memory and its retrieval seem to be a crucial assumption in order for a person to use numbers and operations successfully in a larger mathematical problem solving context.

structure, but also a degree of plasticity that allows it to be shaped by culture" (p. 178). By juxtaposting those two ideas, one can reasonably conclude that the goal of mathematics education is to unleash the Dehaene's notion of plasticity to impart numeracy in learners. Hence, the discussion bridges nature and nurture, or cognition and education.

Emotional Stress and Physical Pain that Affect Memory and Learning. While stress is different from the notion of anxiety, it is nevertheless an aspect that could influence learning, and anxiety could reasonably lead to stress. LePine, LePine, and Jackson (2004) performed a study for stress and adult learning, and they found that even though stress does not significantly affect one's cognitive abilities (p. 887), it nonetheless can negatively impact these adults in their learning abilities (p.889). On the other hand, Christianson (1992) reviewed how emotional stress could impact eyewitnesses in recalling important events from memory. Yet, they did better when prompted by visual and audio cues. In other words, the study found that emotional stress can significantly impact recall, but not necessarily recognition (p. 293). One notes that Christianson's subjects were not learners and the emotional stress from the anxiety induced through learning mathematics can be qualitatively different from Christianson's notion of emotional stress. Still, it is of this current study's interest to explore how stress could impact memory and learning.

LeDoux and Muller (1997) also suggested an intricate relationship between (1) conditioned fear and trauma and (2) unconsious and conscious memories. Conditioned fear refers to a learned signal that precedes a fearful or anxious event. For example, a fire alarm in a building begins to siren, and people in the building may begin to experience anxiety and fear without actually witnessing an open fire. The research suggested that "[d]uring an aversion experience, associations are formed between painful stimuli and other information being processed at that time" (p. 1724), and "trauma is understood as an intense aversion experience" (p. 1724). The learning as a result of an aversion experience involve the processing of external signals by the amygdala (p. 1724), and "the unconscious emotional memories formed by the amygdala and related brain areas can never be converted into conscious memories" (p. 1725). Perhaps LeDoux and Muller's findings are relevant to the study of mathematics anxiety because the initial aversion experience in learning mathematics could be interpreted as a basis that unconsciously form fearing and anxious memory that affect later episodes of learning.

It is interesting to note that LeDoux discussed the issue of fear and pain in his study. In a more recent study by Lyons and Beilock (2012), their report on highly anxious individuals compared to individuals who were not: “high levels of math anxiety predict increased pain-related activity during anticipation of doing math, but not during math performance itself” (p. 5). In essence, when highly anxious individuals were anticipating a mathematical task, the researchers found increased activity in brain regions (seen through fMRI) associated with visceral threat detection, similar to experiencing physical pain. Using neuroscience, Lyons and Beilock managed to show some direct correlates between mathematics anxiety and physical responses, and in fact, they claimed that this is one of the first findings that physical pain is inflicted by a cultural event like mathematics as opposed to by a physical event, such as pain inflicted by getting tattoo on one’s arm or having one’s chest hair waxed. While how physical pain associates with cultural events is not a focus of this research study, it is reasonable to believe, if Lyon and Beilock’s interpretation was correct, that physical pain inflicted by mathematics anxiety may likely cause emotional stress. In turn, this affects memory recall, and the effect becomes a major handicap for developing numeracy and ultimately learning mathematics.

To synthesize the above research literature, one observes that highly mathematics anxious adults may likely experience physical pain when they anticipate a learning event, set by the conditioned fear from previous negative experience. The fear and pain may translate into stress, aversion experience, or even trauma. Compounded by the fact that stress and anxiety can very well compromise the reliability of conscious memory, all the above seem to lead to a recipe of unnecessary challenges for learning mathematics. Therefore, the nurturing component in education is of particular important in order for a learner to succeed. The paragraphs above provide a general context on how physical pain and emotional stress associate with mathematics anxiety, as well as how cognition plays role in memory and learning. In the second half of this section of cognition, I will review studies that directly focused on mathematics anxiety and its effects working memory.

Working Memory. Ashcraft took a clinical research approach to mathematics anxiety by conducting a series of psychological research studies on its nature. His first study (2002) investigated mathematics-anxious adults’ error rates of adding whole numbers, and the research noted differences in lower accuracy of addition with carry-over, such as 46 + 27 compared to that of addition without, such as 46 + 21, illustrated below:

Without Carry-Over	**With Carry-Over**
$\begin{array}{r} \\ 46 \\ \underline{+\ \ 21} \\ 67 \end{array}$	$\begin{array}{r} {\scriptstyle 1}\ \ \\ 46 \\ \underline{+\ \ 27} \\ 73 \end{array}$

Figure 2.3. An illustration of "carry-over" of addition in real numbers.

He hypothesized that anxious individuals tend to answer quickly and sacrifice accuracy, and "[b]y speeding through problems, highly anxious individuals minimized their time and involvement in the lab task, much as they probably did in math class" (p. 183). Ashcraft also speculated about the results of his research findings, that general anxiety may disrupt working memory processes, "because anxious individuals devote attention to their intrusive thoughts and worries, rather than the task at hand" (p. 183). Confirmed by Perry (2006) who examined the interface between adult learning and neuro-processing, he found that:

> The adult learner in a persistent low-level state of fear retrieves information from the word differently than do adults who feel calm. … Even if an adult has successfully stored information in cortical areas, this information is inaccessible while the learner feels so fearful. (Perry, 2006, p. 26)

Indeed, both Ashcraft's and Perry's findings seem to be in agreement that anxious/fearful learners may find it challenging to access working memories—a common and frequent task often required in standardized assessment that could hinder anxious learners to perform to their true abilities.

Ashcraft and Ridley (2005) performed a second study by investigating whether mathematics anxiety affects mental processes during problem solving (p. 321). Similar to the first (2002) study, Ashcraft and Ridley concluded that "high-anxious participants were using a speed-accuracy tradeoff rather strategically," namely "sacrificing accuracy so as to hurry the experimental session along" (p. 322). What is interesting from this study is the acknowledgement that mathematics anxiety is a result of both "external and internal influences that have an impact on the individual's autonomous learning behaviors, for example, spending time on homework, asking questions in math class, deciding to take additional math courses" (p. 318). As seen earlier, Schoenfeld (1989) detailed a similar account of how little time students are accustomed to devote in solving mathematics problems. Apparently, a typical clinical research methodology in psychology (i.e. in this case, controlled-treatment comparison) will find that "… it is exceedingly difficult to separate possible social and cultural effects …" (p. 319). This is an acknowledgement that there is much more to mathematics anxiety than investigating error rates of adding whole numbers. Therefore, the series of Ashcraft et al.'s studies provide a strong argument for an alternative methodology in order to investigate the external and internal influences of mathematics anxiety through the societal and cultural perspectives. In turn, I argue that integral theory can be such an alternative.

The third study of the series is Ashcraft and Moore's (2009) in which the researchers attested that the working memory issue is caused by mathematics anxiety instead of by test anxiety. The researchers asked individuals to read controlled passages of ordinary narratives and treatment passages where content words are replaced by mathematical words. The study found that the error rates in reading the treatment passages were significantly higher than those in the controlled passages. These findings aim to isolate test anxiety from mathematics anxiety, and to isolate memory recall as a significant attributing factor for lower performance:

> It was as if the high math-anxious participants were participating in a three-way competition for their limited working memory resources: difficult math, letter retention and recall, and their own math anxiety. The load on working memory became so pronounced that their performance deteriorated markedly—affective drop. (p. 202)

And:

> It was working memory that was compromised in our study of college students' performance and math anxiety; working memory suffered the brunt of the math anxiety effect because of the inner-worries and self-doubts that are reported by math-anxious individuals. (p. 203)

Not only did Ashcraft and Moore (2009) find that mathematics anxiety is more likely responsible for inhibiting an individual from access his/her working memory, but they were also successful at countering Hopko, McNeil, Lejuez, Ashcraft, Eifert, and Riel's (2003) criticism by clarifying the distinction between mathematics and test anxieties. Ashcraft and Moore's results also suggest that mathematics anxiety, while inhibiting working memory, can decrease an individual's performance in arithmetic as well as his/her reading ability, and such ability is crucial for an individual to succeed in school mathematics (e.g. reading mathematics test directions and reading word problems).

Amidst these findings, Ashcraft et al. (2002; 2005; 2009) admitted that the series of research studies do not provide many insights on what the root causes of mathematics anxiety might be. While the current research study does not actively pursue the origins, the integral research methodology on social and cultural perspectives[3] may reveal certain clues. The review of the series of studies by Ashcraft et al. (2002; 2005; 2009) provides a detailed account on the issue of working memory and mathematics anxiety. The findings provide a detailed explanation of how an individual's ability to add whole numbers may operate on mathematics tests and examinations. However, it is yet unclear how the compromised working memory may affect classroom learning. Recalling the five mentalities of mathematical knowledge, "doing mathematics" can mean a lot more than simply adding whole numbers (oral and pre-formalist stages), and so, further study of mathematics anxiety on a broader scope of the knowledge (such as the formalist stage) is necessary to account for the phenomenon. Furthermore, the challenge of investigating the external and internal influences of societal and cultural effects on mathematics anxiety reveals a gap that this study may be able to fill.

Affective and Cognitive Anxieties. Another aspect of how mathematics anxiety manifests in an individual is the distinction of affective (emotionality) versus cognitive (worry). Originally proposed by Liebert and Morris (1967), the distinction can be seen as follows (Ho et al., 2000, p. 363):

- *Affective anxiety* refers to the emotional component of anxiety, feelings of nervousness, tension, dread, fear, and unpleasant physiological reactions to testing situation.

[3] See Ashcraft (2002, p. 184) for a discussion on a research gap in social and cultural perspectives.

- *Cognitive anxiety* refers to the worry component of anxiety, which is often displayed through negative expectation, preoccupation with and self-deprecatory thoughts about the anxiety-causing situation.

The distinction, in which the affective anxiety is likely responsible for poor mathematics performance while the cognitive anxiety is most likely responsible for general test anxiety, is interesting (Bandalo et al., 1995, p. 620; Ho et al., 2000, p. 375). While Ashcraft and Moore (2009) referred to performance deterioration as "affective drop" (p. 202), they alluded to how an anxious individual fails to reason and be rational so that the retrieval of working memory would not be possible to perform the computation task at hand. In other words, Ashcraft et al. seem to agree with Ho et al., pinpointing the affective anxiety as a delibilitator to poor mathematics performance. Furthermore, this is a distinction that can be investigated in this current research study. Namely, how willingly an individual would share the feelings of affective anxiety versus the feelings of cognitive anxiety is yet to be discovered. Above all, this investigation has the potential to provide new insights by disclosing such emotions from an individual's perspective.

Socialization: Mathematics Anxiety as Cultural Phenomenon

The Parental Role. Some of the sociological studies on mathematics anxiety were conducted in a similar manner to the quantitative psychological studies cited earlier. A salient feature of sociological studies is the link between parental influence and the learner's outlook in mathematics education experience. Geist and King (2008) reviewed Campbell and Clewell (1999), Campbell, Storo, and Educational Resources Information (1996), Laster (2004), and Levi (2000), who all suggested that parental attitudes and expectations "have a direct correlation to their children's achievement in mathematics" (p. 44). Ferry et al. (2000) also studied the role of family contexts, and they linked parental encouragement as a significant factor in mathematics learning experiences and outcome expectancies (p. 360). They argued that self-efficacy, personal attitudes, and outcome expectations play a significant and influential role in college mathematics students (p. 359). Perhaps the most robust finding came from Dahmer's (2001) dissertation, in which she studied sixty-six families and found that parents' own mathematics anxiety is a significant contributing factor to their children's mathematics achievement, such as with test scores.

Even though none of these studies claimed a cause and effect relationship between parental anxiety and the learner's mathematics anxiety nor claimed a correlation between the two, MALP does in fact seem to be perpetuated among the participants in mathematics education—educators, learners, and parents.

While the quantitative studies determined parental influence as a significant factor linked to their children's mathematics outlook, there are several diverging qualitative studies that are worth mentioning here. Peressini (1996; 1998) examined the parental role of mathematics education and concluded that "parents, because of their incomplete knowledge of this true discourse, are unable to engage in various activities that are part of mathematics education (e.g. assisting their children with mathematics homework)" (1998, p. 575), and "… parents were involved in only a limited and passive fashion" (1996, p. 3). Remillard and Jackson (2006) found some evidence and provided a reason to explain Peressini's claim. "…[P]arents did not see the connection that are readily apparent to most mathematics educators and teachers" (p. 255). On the other hand, Ginsburg et al. (2008) acknowledged parents and children struggle to make sense of school mathematics, and consequently, they perceived that "they are learning mathematics within the home as they work with their children" (p. 24), and Pritchard (2004) found that parental support among the New Zealander subjects of the study was "generally positive" (p. 483).

The review of parental influence remains unclear on how such influence plays a role in MALP. Even though some literature portrayed a positive image of parent-child co-learning mathematics at home, a careful perusal of some of the parental quotes in these studies supports the perpetuation of MALP, such as:

> When he [the child] does his [math] homework, I learn with him, 'cause a lot of the stuff they're doing now, it's new stuff … I *never heard* of that when I was going to school, *never*. (Ginsberg et al., 2008, pp. 23 – 24, italics added)
>
> I remember like back in the day it was kind of easy, but [my daughter's class was] doing this, divide by this and you add this … and I'm like "No way! I don't get it!" I know the old way, that's it. So I told [the teacher], "I'm sticking to the old way," and she was like, "Well … I think the new way is kind of easy." (Remillard & Jackson, 2006, p. 252).

These quotes are particularly interesting, especially the second one where the parent attempted to exert power in the discourse. Parents are supportive of their children's mathematics education, albeit the relative ease of confessing and emphasizing "old stuff" versus "new stuff" shows that parents are potentially harboring discomfort with mathematics. They may in turn pass on such negative views to their children. The overall picture on parental support within the context of school mathematics is that parents are seemingly supportive of their children's mathematics education, to the extent that they are willing to learn "new math" with their children at home. However, the social-educational environment that encompasses parents, children, mathematics teachers, and other educators as active participants, has not been conducive to parental involvement. It has insulated them from the core discussion on pedagogy, on the curricular content of school mathematics, and on the reform efforts (Peressini, 1996). The result may be that parental support has not been optimized in their children's learning because negativity toward school mathematics is harbored among parents, children as learners, and the educational system. In other words, one possible argument is that negative attitudes towards mathematics, while they could be easily explained by the technical and challenging nature of the subject, could also be a manifestation of parents who play a relatively passive and powerless (Peressini 1996; 1998) role in educational discourse.

Even though this current study aims to research adult learners and not children, the findings on parental influence are also important to adult learners of developmental mathematics in that they all must have been influenced by their parents or caretakers during their childhood. In turn, these adult learners may enter the mathematics classroom with pre-conceived lack of self-efficacy, personal attitudes, and outcome expectations, with these beliefs have been established long beforehand. A related study on attitude toward science and technology by Roberts, Reid, Schrouder, and Norris (2011) about how public "buy-in" in specific technology without the attitude of trusting science in general. They found an asymmetry in trust that "trust in generalized science and technology affects trust in specific technologies, but not vice versa" (p. 638). While their study was not directly on the public attitude of toward mathematics, a possible idea to contemplate is whether it holds true on the learners' "buy-in" of specific mathematics learning experience without the attitude of trusting mathematics and mathematics education in general. Apply this "buy-in" idea to MALP, the potential argument for parental influence as a sociological factor in mathematics anxiety may provide initial insights to MALP about how it could be perpetuated. Secondly, power in mathematics education seems to be a suspect in playing an important role for MALP, and many adult learners in college developmental mathematics courses often play a dual role both as learners and as parents. Such a dynamic would be of interest in this current research study.

School as a Learning Environment. Perhaps the most relevant study that is closely related to MALP is Givvin et al.'s (2011) hypothetical model on how the school environment plays a role in the behavior of adult learners. Before Givvin et al. (2011), Wieschenberg (1994) documented how anxious mathematics learners suffer from the notion of helplessness, a condition that De Corte et al. (2008) referred to as "sense making abandonment":

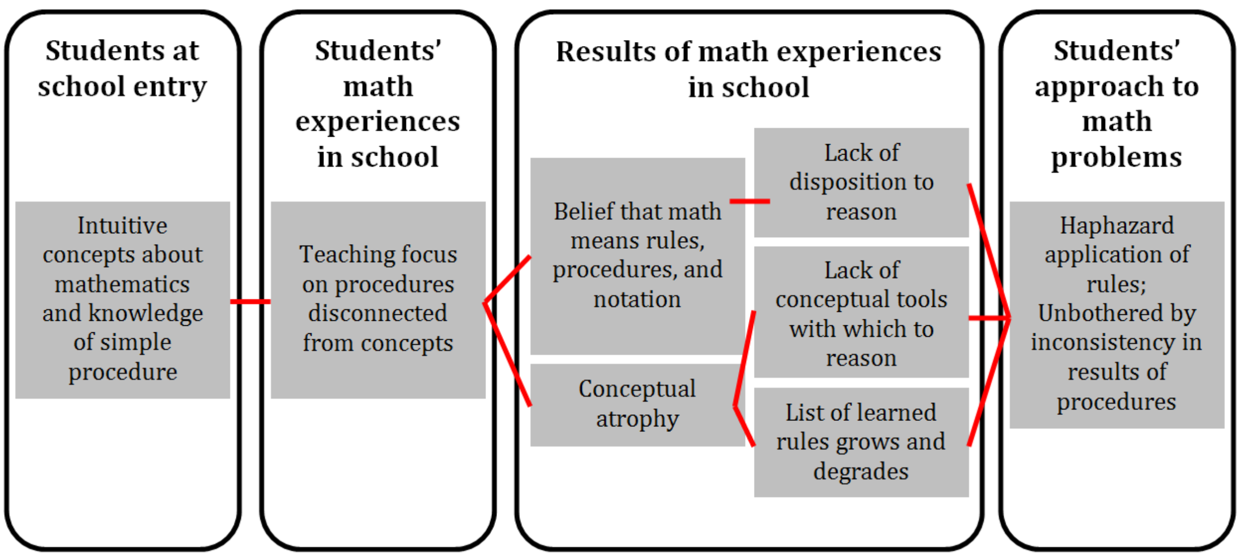


Figure 2.4. Givvin et al.'s (reproduced from p. 6, 2011) model on the making of a community college developmental math student: A hypothetical account [colors added].

Givvin et al.'s hypothetical account of mathematics instruction includes, "teachers with narrow views of what it means to know and do mathematics," instruction that "never made the underlying concepts explicit," and instruction that "emphasized procedures and paid relatively little attention to conceptual connections" (p. 5). They also claimed that “students who were curious, who tried to understand why algorithms worked, were often discouraged by the teacher…” (p. 5). Furthermore, such MALP related instruction produces three kinds of adult learners: (1) “Some students learned on their own … the value of connecting rules and procedures to concepts,” (2) “Still others … were able to rely on a *strong memory*” [italics added], and (3) “… College students … [who were w]ithout conceptual supports and without a strong rote memory, the rules, procedures, and notations they had been taught started to degrade and get buggy over time” (p. 5). One common thread among these three kinds of learners is how they all rely on memory to play a critical role in doing mathematics. As reviewed earlier, Ashcraft et al. (2002; 2005; 2009) revealed that working memory would become problematic for a learner experiencing mathematics anxiety.

Indeed, Schoenfeld (1989) asserted that many students were exposed for years to a “drill and kill” style of instruction and assignments. Bessant’s (1995) research on college students reveals similarities to Schoenfeld’s (1989) assertion that “… math anxious students who prefer to learn mathematics through detailed step-by-step procedures are more apt to experience math test anxiety” (p. 338). Juxtaposing Givvin et al.’s model—conceptual atrophy, haphazard applications of rules, and inconsistency in procedures—along with Schoenfeld’s “drill and kill” and Bessant’s link between step-by-step procedural learning, one could summarize how an anxious learner might experience the following:

(1) Learners’ who prefer detailed mathematics procedures as instruction often rely on memories to execute mathematics problems in a step-by-step manner.

(2) When memories begin to fail a learner, affective anxiety begins to take over the learner’s cognitive process, further inhibiting the retrieval of information in working memory.

(3) The result is a conceptual atrophy with “buggy” memory and isolated concepts in the learner’s mind, and thus, such an anxiety event would reinforce itself, ready to be set-off in the next event when the slightest mathematics challenge is posed.

The totality of Givvin et al. (2011), Wieschenberg (1994), De Corte et al. (2008), Schoenfeld (1989), and Bessant (1995), shows a strong evidence that school experience plays an important role in how learners may operate during learning mathematics, and in turn how the social aspect in school may play a role in MALP.

Tying MALP to Knowles' Andragogy

In this section, I will first review Knowles' andragogy, and then I will tie adult learners' psychology, cognition, and sociology in mathematics anxiety to Knowles' andragogy.

Knowles' Andragogy. Knowles (1968, 1973, 1975) proposed the concept of andragogy as a set of assumptions for adult learning to set it apart from pedagogy. The development of the concept was based on Lindeman's (1961) insights by combining "aspects of humanist, constructivist, and cognitivist orientations toward learning" (Taylor, Marienau, & Fiddler, 2000, p. 359). While pedagogy is described as "the art and science of helping *children* learn" (Knowles, 1980, p. 43, italics added), the term *andragogy* refers to "the art and science of helping *adults* learn" (Knowles, Holton, & Swanson, 1998; Merriam & Caffarella, 1999, p. 272). The motivation to distinguish the two lies in the learning psychology of adults in which self-concept, self-directedness, and the role of learners' experience are radically present (Knowles et al., 1998, pp. 64–65).

Merriam and Caffarella (1999, p. 272) have summarized andragogy as five assumptions:

(1) *Self-Directedness:* As a person matures, his or her self-concept moves from that of a dependent personality toward one of a self-directing human being.

(2) *Resevoir of Experience:* An adult accumulates a growing reservoir of experience, which is a rich resource for learning.

(3) *Readiness to Learn:* The readiness of an adult to learn is closely related to the developmental tasks of his or her social role.

(4) *Problem Centeredness:* There is a change in time perspective as people mature—from future application of knowledge to immediacy of application. Thus an adult is more problem centered than subject centered in learning (Knowles, 1980, pp. 44–45).

(5) *Internal Motivation:* Adults are motivated to learn by internal factors rather than external ones (Knowles & Associates, 1984, pp. 9–12).

Assumption (1) is described as “the need to know”, the awareness of an adult capable of evaluating the benefits one would gain from learning and the consequences of not learning. Also, the assumption deals with an adult’s self-concept of being responsible for his/her decisions and for his/her life, and in essence, being responsible for directing themselves to learn (Knowles et al. 1998, pp. 64–65). Assumption (2) appeals to the role of learners’ experiences, with the act of adult learning relying on both the quantity and quality of past experiences (Knowles et al. 1998, pp. 68). This assumption is particularly important for this current research study, because the adult learner’s behaviors that contribute to MALP are motivated by one’s past negative mathematics learning experiences. Assumptions (3) and (4) deal with adults’ self-concept and self-direction, but with a caveat that learning is tied to applications, functions, and pragmatics as adults play individual and social roles. In other words, an adult is ready to learn whatever he or she believes is useful and applicable both in the immediate and the future uses. As pointed out earlier in the research on parental support, (Remillard & Jackson, 2006; Ginsburg et al., 2008), parents seem to be motivated to learn school mathematics with their children as a social role, to support their education. This assumption is also important to the study of MALP because there is evidence, such as Peressini (1996; 1998), revealing a disconnect between mathematics knowledge and using the knowledge to engage in the educational discourse. Potentially, in the current study, one could argue that it is this disconnect that contributes to negativity in learning mathematics, and such negativity becomes a driving force for MALP. Assumption (5) makes a distinction between internal and external motivations. Potent and more authentic learning takes place by means of internal motivation, such as job satisfaction, self-esteem, and quality of life, etc. (Knowles et al. 1998, p. 68). Conversely, external motivation alone does not provide as an effective driving force as internal motivation for an adult to engage in a learning activity.

MALP and Knowles’ Andragogy. Based on Knowles’ andragogy of the five assumptions on adult learning, one can assert a relationship between MALP and Knowles’ andragogy. In order to elucidate such a relationship, one should first recall that this research study focuses on adult learners for developmental mathematics. Givvin et al. (2011) hypothesized that learners apply mathematical rules haphazardly and De Corte et al. (2008) and Wieschenberg (1994) respectively called the similar notion as “sense making abandonment" and "helplessness." Certainly one could assume that adult learners are not taught to achieve such atrocious results. This begs the question that these results are actually learned through self-direction (assumption (1)) through an accumulation of a growing reservoir of negative and anxious experiences (assumption (2)). Because many adults find it challenging to connect mathematical knowledge to immediate and future applications (assumptions (3) and (4)), there is little internal motivation (assumption (5)) for one to prevent conceptual atrophy. From the learners' perspective, learning mathematics through a prescribed step-by-step recipe without processing the underlying mathematical reasoning seems to be a relatively successful strategy to get through a semester long college mathematics course. Furthermore, self-directness, the relative success as past experience, negative learning experience as an internal motivator, and the social identity as a fellow learner collectively formed a desire to inform other learners (Remillard & Jackson, 2006; Ginsberg et al., 2008) to help them through the arduous mathematics learning journey. In sum, there are many threads that intertwine between MALP and Knowles' andragogy, and the current study has substantial theoretical value to strike the research landscape.

Integral Model as a Basis for the Research Methodology

Recalling the discussion of Wilber's Integral Model in the introductory chapter, the model serves a dual purpose for this study. First, the above literature review shows that past research studies focused largely on the right-hand quadrants, leaving gaps in the left-hand quadrants in the research landscape. Second, the mode of inquiry into the left-hand quadrants requires a method that accesses the subjective and intersubjective lived experience of research participants; thus, approaching the study through angle of life history is an appropriate approach for this current study. Up to this point, I have only discussed in general terms about Integral Model as the research approach. While the fine details of the model will be discussed in the methodology chapter, I will briefly review the mechanics of Integral Model and show that the previous related research on MALP can be classified into the four quadrants of integral theory: inquiry on life history, behavioral analyses, cultural investigation, and social assessments. In the methodology chapter, it will be explained in detail as to why interviews and journaling of life history are appropriate methods for the underlying model.

Wilber's (2000a; 2000b) Integral Model, as he called it, was the "theory of everything" in which the way of knowing relies on the idea that every perspective discloses a unique window to the phenomenon. For example, the reviewed psychological studies on mathematics anxiety have disclosed an individualistic perspective, while the sociological studies have disclosed a collective perspective, but what about a personal experiential perspective? Integral theory would argue that such a perspective discloses yet another unique insight about MALP, and it is exactly the epistemology of integral theory that allows this current research study to fill a void in the research landscape.

The underlying model encompasses two major contrasts: (1) individual versus collective and (2) interior versus exterior. The contrast of individual versus collective discloses the phenomenon through psychology versus sociology. Meanwhile, the contrast of interior versus exterior discloses the phenomenon through the first-person's perspective versus the third-person's perspective, as illustrated in the four quadrants below:

	Interior	**Exterior**
Individual	**Upper Left (UL) Quadrant** **Interior-Individual** • The "I" 1st person perspectives • Experiential • Inquiries into emotions, beliefs, and attitude	**Upper Right (UR) Quadrant** **Exterior-Individual** • The "It" 3rd person perspectives • Behavioral • Inquiries into cognition and psychological behaviors
Collective	**Lower Left (LL) Quadrant** **Interior-Collective** • The "We" 1st person perspectives • Cultural • Inquiries into philosophy, education and interaction	**Lower Right (LR) Quadrant** **Exterior-Collective** • The "They" 3rd person perspectives • Social • Inquiries into environments and sociological behaviors

Figure 2.5. An illustration of the characteristics of the quadrants, adapted from Esbjörn-Hargens (2006b).

Each quadrant is referred to as UL, UR, LL, and LR for the different perspectives each could draw respectively: experiential, behavioral, cultural, and social. So far in this literature review chapter, past research shows varying extents to the behavioral, cultural, and social perspectives. The experiential aspect of the individual-interior perspective is largely lacking in past research. This is because of the predominantly positivististic approach in methodology for both psychology and sociology in which the Likert surveys are used as data collection instruments. Such a mode of inquiry requires the researchers to pre-set the surveys with statements that likely draw relevant information from individuals, based on (dis)agreement or (un)likelihood. This entails a potential that the design of the instrument is infused with pre-conceived notions of the phenomenon. In other words, much of surveyed research may potentially suffer from the "placebo effect." A placebo effect occurs when a research participant experiences the authentic medication's effect but has been given a sugar pill. Similarly, as a research instrument used to elicit the degree of mathematics anxiety, the MARS survey encompasses both statements that describe scenarios that might cause authentic anxiety and statements that act more like a placebo. For example, a learner "walking into a math class" (Alexander & Martray, 1989, p. 145) may experience more authentic anxiety than, say "buying a math textbook" (Alexander & Martray, 1989, p. 145). However, by mixing all of these Likert statements together in a survey, there is a potential that research participants may have reported "false-positives" of mathematics anxiety in the past research studies through placebo statements. Therefore, I used life history, that is, interviewing adult learners and asking them to write journals about their mathematics learning experiences. Such a method would allow, through an unfiltered means, first-hand experience to become authentic research data. The collection of data will be relatively less controlled, compared to those research studies that employed the Likert surveys, and the analysis of these data will be less cut-and-dry. Nevertheless, one anticipates that the results of the current study would disclose new information on MALP, particularly through an experiential perspective, that the past studies were unable to do so.

Research Problem. In the introductory chapter, I overviewed MALP and presented the research questions, but in this case, each research question is matched against the four quadrants under the integral research methodology:

> *Problem Statement*: As a practitioner who teaches developmental mathematics to adult learners who often experience mathematics anxiety, what characteristics of

MALP could give insights that would influence instruction to optimize learning? This problem can be dissected into five research questions:

(1) *What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?*

This sub-question corresponds to the UL, or the individual-interior quadrant, aiming to gain an experiential understanding of MALP.

(2) *What are the roadblocks (cp. Givvin et al., 2011) that prevent a learner to succeed in mathematics? And what are the manifestations of these roadblocks?*

This sub-question examines the UR, or the individual-exterior quadrant, targeting to gain psychological/cognitive insights to MALP.

(3) *What are the underlying cultural beliefs in MALP, and how is the culture passed on to others, and how is it perpetuated within and outside of the classroom?*

This sub-question relates to the LL, or the collective-interior quadrant, focusing the research to gain cultural insights to MALP.

(4) *What are the social norms when learners are supporting each other? And what is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?*

The sub-question signifies the LR, or the collective exterior quadrant, concentrating the research to gain sociological insights to MALP.

(5) *Based on the perspectives in* (1)*,* (2)*,* (3)*, and* (4)*, what could one disclose as an integral perspective about MALP and how it is cyclical and perpetual?*

This is a summation-question that draws from the previous sub-questions to find new meanings into MALP.

As one can see from the above sets of research questions, the investigation of MALP appeals to the learner's belief system, the learner's psychology, cognition, memory, the learner's cultural role within the learning community, and the social learner's identity, power, and passiveness within the educational discourse. Furthermore, integral research as a methodology seems to be an appropriate means to provide unique insights to the phenomenon.

Conclusion

In this literature review chapter, I reviewed past research that has major links to MALP. A summary argument can be made that past research studies characterized negativity towards mathematics learning as “helplessness” (Wieschenberg, 1994) and “abandonment of sense making” (De Corte et al. 2008). These claims seem to be in line with Givvin et al.’s (2011) “conceptual atrophy” in their hypothetical model. Meanwhile, Renert and Davis’ (2010) investigation revealed that the general perception of mathematical knowledge lies in the pre-formalist and formalist mentalities, narrowing the popular view on what constitutes mathematics. Of the psychological studies reviewed in this chapter, all of them collected data on mathematics anxiety through the MARS surveys in which experiential information is filtered. The results of these studies provide a basis for the current study on what variables are relevant to MALP. Cognitive studies have shown that mathematics anxiety may cause difficulty in retrieval of working memory (Ashcraft et al. 2002; 2005; 2009; Hopko et al., 2003), and other studies showed that mathematics anxiety is linked to the affective domain of emotion (Ho et al., 2000) which negatively affected learners’ mathematics performance and, in turn, their achievement. The consequence can be immediate (such as not succeeding in a mathematics test) to long term (such as choosing a mathematics/science related career). On the other hand, reports show that mathematics achievement and learning attitudes are linked to parental influence (Ferry et al., 2000; Dahmer, 2001). Furthermore, Peressini (1996; 1998) claimed that parents, albeit their effort to take part in their children’s mathematics education, are often marginalized and insulated from the educational discourse, resulting in them playing a rather powerless role in the educational community. Such a summation argument paints a picture that is much richer and contextual than Givvin et al.’s (2011) model on college students’ mathematics experience, and I will argue through Knowles’ andragogy that the experience of mathematics anxiety, instead of authentic mathematics knowledge, is the driving force of learning, thus resulting the social behavior of free admission of mathematics anxiety and the passing the anxiety from one learner to another.

While I have briefly introduced integral research as a fourfold investigation to MALP, I will discuss integral theory as the research methodology in detail in the next chapter.

Chapter Three: Research Methodology

In the previous two chapters, I introduced and overviewed the Mathematics Anxiety Learning Phenomenon (MALP) and reviewed previous research that is related to MALP. In this chapter, I will detail how integral theory as a research framework addresses the research questions laid forth. This chapter is divided into two parts: (1) It begins with a brief introduction of Wilber's Integral Model and All Quadrants All Levels (AQAL) as a underlying framework that addressses the research questions at hand. Next, I will overview the research inquiry landscape in traditional educational research and examine how integral theory is situated in the landscape. At the end of the first section, I will defend the choice of using integral theory as a research framework among the traditional qualitative, traditional quantitative, and mixed-method research methodologies. (2) Following the section on research inquiry, I will detail the design of the research based on integral theory (including the limitations), and I will discuss how data will be collected and analyzed.

The adopted theoretical framework for the research study as a method of inquiry is based on Wilber's Integral Model (Wilber, 2000a; Esbjörn-Hargens, 2006b, 2009; Martin, 2008; Renert, 2011), in which an AQAL approach to investigate a phenomenon. As previously discussed, it is through this model that this study revealed gaps in past research. Also, this underlying framework is an ideal choice because of its comprehensive nature to "unveil" the psychosocial phenomenon through its participants, through the MALP in which it is situated, and through 3rd person (objective and interobjective) perspectives. Several perspectives are contrasted within the framework that makes it an ideal choice for this research study. For example, interior versus exterior, 1st, 2nd, versus 3rd person perspectives, internal versus external, subjective versus objective, and intersubjective versus interobjective are included.

The Epistemology of Wilber's Integral Model

In the foreword of Frank Visser's (2003) book entitled *Ken Wilber: Thought as Passion*, Wilber ambitiously stated that "[i]ntegral theory weaves together the significant insights from all the major human disciplines of knowledge, including the natural and social sciences as well as the arts and humanities." It was Wilber's belief that a radically different framework such as integral theory can offer a mode of inquiry to study human experiences in a post-modern research context. Unlike other traditional frameworks, such as positivism and constructivism, Wilber's framework does not bracket the subjectivity in the inquiry process, and he believed that subjectivity provides a unique blend of values. The underlying epistemology of Wilber's Integral Model subsumes the following:

(1) Every perspective discloses a unique window to reality.

(2) All perspectives form an integral conference to reality.

(3) An investigation of only a partial collection of the perspectives would compromise the integrality of the phenomenon.

(4) The mode of inquiry is to seek and to embrace divergent paths toward the truth.

Esbjörn-Hargens and Wilber (2006) laid the epistemological foundation by stating that "all … approaches have at least some partial truths to offer an integral conference" (p. 529). Esbjörn-Hargens (2006b) also suggested an intuition that "everyone is right" and that "each practice or injunction enacts and therefore discloses a different reality" (p. 86). In other words, every perspective one holds is a window to reality, and if one leaves out "one or more perspectives, a fundamental aspect of the integral whole would be lost and our ability to understand it and address it would be compromised" (Esbjörn-Hargens 2009, p. 6). Indeed, losing the ability to understand the integral "wholeness" when leaving out one or more perspectives is a criticism of the bracketing of subjectivity, a notion deeply ingrained in positivism and constructivism, in which those inquirers believe that maintaining a pristine objectivity in their research would produce superior results. Such criticism was subtly posited by Martin (2008):

> There is power in being able to see a larger picture, [to] transcend our typical ways of approaching an inquiry, and [to] consider where and how we can strategically use these elements … Increasing the depth of inquiry by using IR [integral research] both intra-study and also inter-study is a powerful concept, one not readily available to the vast majority of today's scholars (p. 160).

As a result, the epistemology of integral theory arguably offers a unique edge to research inquiry that traditional inquiry seldom yields. Specifically relevant to this research study, MALP of developmental mathematics among adult learners has much to gain through the integral approach, as it is indeed a psychosocial phenomenon whose stakeholders are the learners and the educators. Consequently, it is the interaction of the stakeholders with the subject matter by which MALP is being induced. While the epistemological debate is likely to continue, it is important to point out that the investigation of a learning phenomenon is exclusively induced by human experiences from the learners, educators, and their interactions with the subject matter. Without these human experiences, MALP would not exist, and conversely, MALP only exists through the human experiences and the subject matter. In other words, one could argue that the human experiences, the subject matter, and MALP all co-exist and co-arise. In this sense, the epistemology of integral theory seems to be an appropriate choice to an inquiry of the research topic at hand, through all the perspectives that co-arise in MALP.

A Research Methodology Based on Integral Theory

The basic five recurring elements of integral theory, often known as AQAL, are: quadrants, levels, lines, states, and types.

All Quadrants. Esbjörn-Hargens (2006a) described the quadrants as the "basic perspectives an individual can take on reality" (p. 5). This includes the interior and exterior of the individuals and the collectives, and each of the permutation, called "quadrant," is irreducible, with its own valid claim, and with its mode of investigation. The following figure illustrates the characteristics of the four quadrants as a fourfold, co-arising lens:

<table>
<tr>
<td>Upper Left (UL): Interior-Individual

“I” – Intentional (Subjective)
1st person perspectives
Experiential phenomena
Phenomenological Inquiry:
• Emotions
• Beliefs
• Attitude</td>
<td>Upper Right (UR): Exterior-Individual

“IT” – Behavior (Objective)
3rd person (singular) perspectives
Behavioral phenomena
Behavior Analyses:
• Cognitive
• Behavioral</td>
</tr>
<tr>
<td>Lower Left (LL): Interior-Collective

“WE” – Cultural (Intersubjective)
1st and 2nd person perspectives
Cultural phenomena
Cultural & Worldview Investigation:
• Philosophical
• Educational
• Interactive</td>
<td>Lower Right (LR): Exterior-Collective

“ITS” – Behavior (Interobjective)
3rd person (plural) perspectives
Social phenomena
Social Assessments:
• Environmental (i.e. the learning environment)</td>
</tr>
</table>

Figure 3.1. An illustration of the characteristics of the quadrants, adapted from Esbjörn-Hargens (2006b).

The *All Quadrants* approach inquires a psychosocial phenomenon through four different perspectives: (1) interior-individual’s (UL) inquiry on life history, (2) exterior-individual’s (UR) behavioral analysis, (3) interior-collective’s (LR) cultural and worldview investigations, and (4) exterior-collective’s (LL) social assessments. The inquiry into MALP could reveal its nature in great detail through the four perspectives (A.K.A. the quadrivium) because “[t]he quadrants provide a particularly helpful lens for researchers in that the left-hand and right-hand quadrants are associated with qualitative and quantitative methods respectively” (Esbjörn-Hargens, 2006b, p. 84). Essentially, this means that the *All Quadrants* approach is a mixed-method approach.

As a means of understanding how mathematics anxiety is cyclical and perpetuated in an adult learners’ psyche, the four quadrant perspective in the investigation of MALP is comprised of the following correspondences:

- The UL quadrant corresponds to the individual learner’s learning experiences from the first-person perspective.

- The UR quadrant corresponds to reported observations (including from past research) on the individual learner's cognitive process when learning mathematics, and his/her belief systems towards the subject matter, towards his/her own ability to learn the subject matter, and towards what constitutes learning mathematics.
- The LL quadrant corresponds to (including from past research) how the culture of the mathematics learning community, such as the socially shared opinions among the learners in the mathematics learning community, and how the culture relates to MALP.
- The LR quadrant corresponds to the social identity of learners in the mathematics learning community and the social role they play in the mathematics education discourse, and how the social dynamics are at play in MALP.

In sum, the research study assumes that the quadrivium of the four co-arising perspectives would provide rich and detailed data that disclose the nature of MALP.

Levels, Lines, States, and Types. Levels refer to the complexity of the psychosocial phenomenon in each of the basic quadrants. Esbjörn-Hargens (2009) made a distinction of the level of *depth* for the left-hand quadrants and the level of *complexity* for the right-hand quadrants, and therefore, "[e]ach quadrant serves as a map of different terrains of reality" (p. 7). Renert (2011) described the notion as the "development through which phenomena in each quadrant have evolved and complexified …" (p. 17). Furthermore, he elaborated on "development" that it is "complex and nonlinear, with moments of progress and regress, stagnation and transcendence" (p. 17). Finally, Esbjörn-Hargens (2009) advocated the importance of levels because they represent many potential layers of development in each quadrant, and "practitioners [could] gain valuable traction by aiming their efforts at the appropriate scale [levels] and thereby finding the key leverage point" (p. 9). Meanwhile, lines refer to the different paths of development within each quadrant. Esbjörn-Hargens (2006b; subsequently 2009) described the lines of development as the various distinct capacities that develop through levels in each aspect of reality as presented by the quadrants. As for states and types, his description was that states are "temporary occurrences of aspects of reality," while types refer to the "variety of styles that aspects of reality assume in various domains" (p. 84). Further asserted, Esbjörn-Hargens (2006b) explained that integral theory "assigns no ontological and epistemological priority to any of these [five] elements" (p. 84) as they co-arise simultaneously, and "each of the five elements is understood to be part of each and every moment" (p.84).

Integral Methodological Pluralism (IMP). One of the most important aspect of integral theory that drives an eclectic mix of methodology in the quadrivium is Wilber's (2000a; 2000b) proposal of Integral Methodological Pluralism (IMP) that calls on researchers to use a variety of methodologies that are suitable and appropriate to each of the quadrants. Specifically, each quadrant as a perspective could be divided into two zones, which could be studied through the "inside" and through the "outside." Inside refers to first-person perspectives, and outside refers to the third-person perspectives. This distinction results in eight distinctive *zones*, that are often referred to as eight primodial perspectives (8PP) in which there are methodologies that are operationalized (Esbjörn-Hargens, 2006b; Martin, 2008) and that are aligned to each zone:

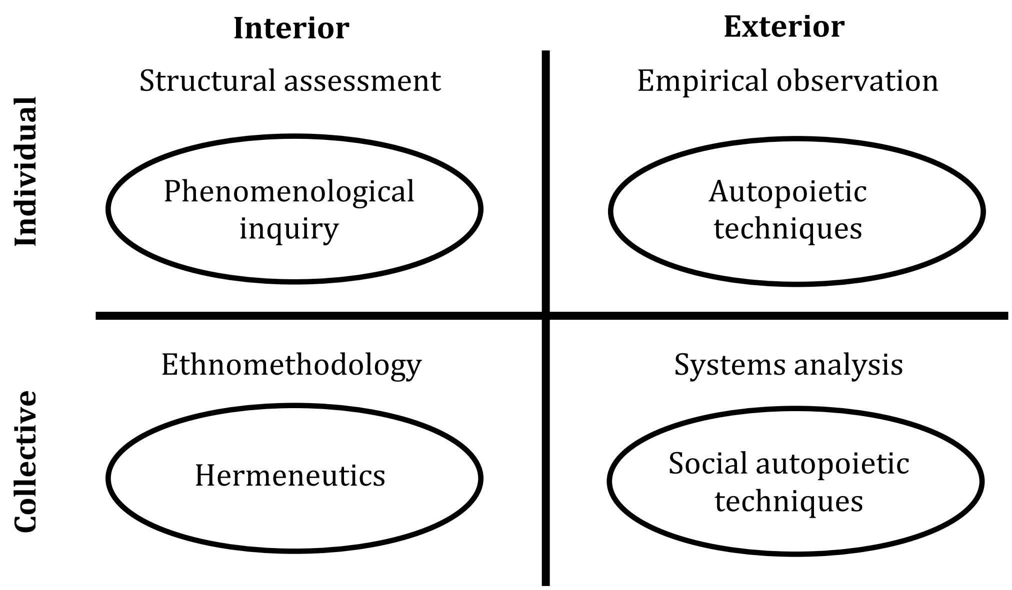


Figure 3.2. The eight methodological zones, adapted from Esbjörn-Hargens (2006b, p. 88 and 2009, p. 17).

The above figure shows the four quadrants that are divided into eight zones. Each quadrant has an "inside" first-person perspective and an "outside" third-person perspective. The operationalized methodologies for the first-person perspective are labeled inside of the oval while those of the third-person perspective are labeled outside of the oval. As can be seen, each zone is aligned to a specific methodology that is also aligned within positivism, constructivism, feminism, and other forms of epistemology. Brown (2005) cited Wilber (2004) and argues for the multi-methodological approach, that uses an eclectic mix of the above methodology in research which would produce superior research results:

> The whole point about any truly [i]ntegral approach is that it touches bases with as many important areas of research as possible before returning very quickly to the specific issues and applications of a given practice … [T]his inescapably means that all of those approaches have at least some partial truths to offer an integral conference, and … can be very rigorous in standards of evidence and efficacy …" (as cited in Brown, 2005, pp. 8–9).

Recall the underlying epistemology that:

- Every perspective discloses a unique window to reality,
- All perspectives form an integral conference to reality, and
- The mode is to seek and embrace divergent paths toward the truth.

Wilber's plea for the multi-methodological approach and the significance of the epistemology, together, can infer to a pragmatic argument that "[e]ach methodology discloses an aspect of reality that other methods cannot" (Esbjörn-Hargens, 2006b). Therefore, IMP becomes a paramount staple in research inquiry based on integral theory. Furthermore, the development of the research methodology to study MALP as a psychosocial phenomenon is a reasonable choice. Indeed, based on the methods aligned in each quadrant under the notion of IMP, the study employed a research method with elements that resonate with an phenomenological inquiry (UL) with hermeneutic analyses (LL).

Defending Integral Theory as a Mixed-Method Approach

Similar to other theoretical frameworks, integral theory as a form of pragmatism assumes the following epistemological basis:

- Integral theory assigns no ontological or epistemological priority to any of these elements because they co-arise and "tetra-mesh" simultaneously (Esbjörn-Hargens, 2006b, p. 84).
- All perspectives are "right", and each holds a piece of the reality. The tetra-meshed perspectives together form a reasonably close description of the "Truth" in reality. (Wilber, 2006; Esbjörn-Hargens, 2006b; Martin, 2008)

The historical discussion in research methodology often divides and debates between qualitative and quantitative approaches to research. This can be traced back to the Chicago Schools in the 1920's (Esbjörn-Hargen 2006b, p. 79). Positivists, who largely employ quantitative methods, believed that "social observations should be treated as entities in much the same way that physical scientists treat physical phenomena," and that the "observer is separated from the entities that are subject to observation" (Johnson & Onwuegbuzie, 2004, p. 14). This leads to a general research culture that values "time- and context-free generalizations" (Nagel, 1986), and that causes outcomes to be thought reliable and valid (Johnson & Onwuegbuzie, 2004, p. 14). Meanwhile, constructivists and interpretivists, who largely employ qualitative methods, reject positivism's time- and context-free generalizations as impossibilities, and they are more content with the idea of "multiple-constructed realities" (Johnson & Onwuegbuzie, 2004, p. 14). Thus, qualitative methods do not strive to fully differentiate causes and effects nor separate the knower and the known, since the subjective knower is a part of and a source of the reality (Guba, 1990).

Pragmatism, on the other hand, takes on an epistemological maxim that "the current meaning or instrumental or provisional truth value … of an expression … is to be determined by the experiences or practical consequences of belief in or use of the expression in the world" (Johnson & Onwuegbuzie, 2004, p. 16). The methodology based on pragmatism, as argued previously in Johnson and Turner (2003) as the fundamental principle of mixed research, that "… researchers should collect multiple data using different strategies, approaches, and methods in such a way that the resulting mixture or combination is likely to result in complementary strengths and nonoverlapping weaknesses …" (Johnson & Onwuegbuzie, 2004, p. 18). Furthermore, they argued that "effective use of this principle is a major source of justification for mixed methods research because the product will be superior to monomethod studies" (p. 18). Among these details, research inquiry based on integral theory clearly follows a similar stance to pragmatism. Furthermore, the use of IMP provides a unique integral conference to MALP, and it is a gap in the past research study (albeit at the epistemological "space") that could provide unprecedented results that the objectivist's perspectives may not disclose.

While there are plenty of differences in the epistemologies of traditional quantitative and qualitative methods, there are striking similarities. Researchers who employ these methods rely on "empirical observations to address research questions," and they "describe their data, construct explanatory arguments from their data, and speculate about why the outcomes they observed happened as they did." Finally, to "minimize confirmation bias and other sources of invalidity (or lack of trustworthiness)," These researchers "incorporate safeguards into their inquiries" (Johnson & Onwuegbuzie, 2004, p. 15). While traditional qualitative and quantitative researchers revere validity to maximize objectivity, such a notion does not reconcile with integral theory. Rather, the notion of trustworthiness (see "research design" in the next section) would be more applicable. Collected data on lived experience would be verified by the participants that they are genuine and are spoken exclusively through their first-person "voice." Furthermore, the studying of lived experience aims to gain a deeper understanding of the nature of the phenomenon, and it does not aim to explain and/or control reality (van Manen, 1990, p. 9). In this sense, inquiry based on integral theory—with trustworthiness in place—would offer one more direct contact with reality (van Manen, 1990, p. 9) and simultaneously would bring nearness to "that which tends to be obscure" and "that which tends to evade the intelligibility" (van Manen, 1990, p. 32). In similar spirit, Morgan (2011) agreed that the study of human experience is "intended to complement rather than [to] replace projects derived from other ways of knowing, primarily by conveying a sense of direct engagement with the phenomenon of interest" (p. xiii).

To summarize the above section, integral theory has an epistemology of embracing a divergence of perspectives, in which each perspective discloses a unique window of reality. By investigating these perspectives through AQAL, using IMP, the integral conference provides a rich and deep description of human experience. While IMP calls for an eclectic mix of research methods to elucidate multiple co-arising perspectives, one argues that such a mode of research conforms with the ideals of pragmatism with trustworthiness as a safeguard and as an analogous parallel to "validity" in traditional qualitative and quantitative research approaches.

The Design of the Research Methods

Recall that MALP is cyclical and that this study intends to understand how mathematics anxiety is perpetuated through an investigation of the lived experiences of adult learners. The current research study pursued the following inquiry:

- *Problem Statement*: As a practitioner who teaches developmental mathematics to adult learners who often experience mathematics anxiety, what characteristics of MALP could give insights that would influence instruction to optimize learning? This problem can be dissected into five research questions:

(1) (UL; zone 1) What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?

(2) (UR; zone 6) What are the roadblocks (cp. Givvin et al., 2011) that prevent a learner to succeed in mathematics? And what are the manifestations of these roadblocks?

(3) (LL; zone 3) What are the underlying cultural beliefs in MALP, and how is the culture passed on to others, and how is it perpetuated within and outside of the classroom?

(4) (LR; zone 7) What are the social norms when learners are supporting each other? And what is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?

(5) (LR; zone 8) Based on the perspectives in (1), (2), (3), and (4), what could one disclose as an integral perspective about MALP and how it is cyclical and perpetual?

Participants. Participants of the study were six adult learners, each in a developmental mathematics course. Criteria for participation were as follows:

- Non-traditional adult learners who have had a hiatus of school of at least three years between high school graduation/GED and the current college study.
- They are engaged in learning some form of developmental mathematics, such as enrolling in a community colleges' developmental mathematics course.
- As a suggestion to the participants, they are willing to write journal entries.

Recruitment procedures are outlined in Appendix B.

Lived Experience Data from Interviews and Journals. Laverty (2003) indicated that studying human experience as a methodology does not govern a set of rules to guide the research process (p. 28). Therefore, I have chosen a blend of suggestions from van Manen (1990) and Morgan (2011) in devising procedures for this research study. As suggested by the bolder designs (Smith, Flowers, & Larkin 2009, p. 52) to investigate beyond mere one interview per participant, six participants were asked to participate in a series of two individual interviews and were asked to write three journal entries. The multiple points of interaction in two different formats ensure the trustworthiness of the data, a triangulation suggested by Elliott, Fischer, and Rennie (1999; as cited in Smith, Flowers, & Larkin (2009)). Each participant addressed the following five core topics:

(1) Past and current personal learning experiences in mathematics

(2) Study habits for the current mathematics course

(3) Experiences in helping with his/her child's (or another learner's) mathematics study outside of the class. Also, his/her understanding of his/her child's (or another learner's) mathematics ability and academic expectations

(4) Comments on their beliefs/emotions/attitude of what constitutes mathematics, of his/her abilities in the subject. Artifact materials, such as homework assignments and tests, from the current course are encouraged to be used to show how the participants came to those beliefs

(5) The participant's role and identity as a mathematics learner in the contexts of a learning community and of the mathematics education discourse.

Each interview was audiotaped and transcribed. At the beginning of the first interview, an abbreviated MARS (or aMARS) and a short survey (see Appendix D) was administered to ensure that each participant suffered from mathematics anxiety. Interviews and journal entries were conducted using the guidelines adapted from van Manen (1990, pp. 64–65, 66–68) and Morgan (2011, pp. 17 – 24). The interaction between the participants and myself, as the researcher, was one that treated participants as experts and the interviewer as an appreciative perceiver (Morgan, 2011, pp. 17–18). In other words, each participant was treated as the only one who has the immediate encounter to his/her experience. Meanwhile, as the researcher, I "attended continuously to the participant's meaning instead of thinking ahead to the next question or some other future task" (Morgan, 2011, p. 18). A pilot interview was conducted to ensure that the procedures for questioning and guiding techniques were sound and proper. The data from the pilot interview are not used for analysis in this study. See Appendix F for further specifics.

To cover all the aspects of the research questions and to allow for iterative self-reflection, two (45 minutes to one hour) interviews were necessary. The first interview aimed to have each participant share his/her experiences of the above data items (1) to (4). The second interview explored the participant's experiences on how he/she interacted with others regarding the subject of mathematics (item (5)). This interview also served as a supplemental interview to the data collected from the first. The format of the second interview was similar to the first one in order to provide an opportunity for the participant to add any additional information on the above data items (1) to (5) before the series of interviews were concluded. Transcripts from both interviews were provided for each participant to assure the collected data were accurate and trustworthy.

In addition, the participants were asked to write three journal entries (one per week), documenting and reflecting on his/her experiences with helping his/her child or another individual with studying mathematics. The journal data provided an internal LL perspective as opposed to the second interview (see above) in which the aim was to disclose an external LL perspective. The materials generated from the participants' mathematics course of study were also a part of the research. These materials could include, but are not limited to, course notes, homework assignments, quizzes, tests, and examination papers. The recruitment, data collection, and data management/storage of the research study adhered to the regulations and guidelines of the Conjoint Faculties Ethics Research Board (CFREB) and the study (File No. 7494), was approved by CFREB in October 2012. The study was also approved by the Institutional Research Board (IRB) of University of Phoenix[4]. Per policy of the Faculty of Graduate Studies, the approval letters are submitted as separate documents from this dissertation.

Data Analysis for Developing Major Themes. The entire corpus of data includes: (1) the interview transcripts, (2) journal entries, and (3) artifact materials from the participants' mathematics work. In particular to the artifact materials, they were discussed with the participants during interviews so that contexts of the materials could be made clear for thematic analysis. All of these data underwent an interpretative analysis for major themes, and then they were sorted into the four quadrants. Morgan (2011) defined a theme as experiential similarities with mutually related aspects of a more comprehensive pattern (p. 33); these similarities run twofold—within a participant and across participants. Recurring themes from different quadrants would become significant elements used to identify implications for instruction. When considering what a theme would be, I adopted the following criteria from van Manen (1990, pp. 87–88):

- Theme is the experience of focus, of meaning, of point.
- Theme is a form of capturing the phenomenon one tries to understand.
- Theme is the needfulness to make sense, is the sense we are able to make of something, is the openness to something, and ultimately is the process of insightful disclosure.

The thematic analysis is conducted to disclose an andragogic understanding of MALP. Themes will be developed according to Morgan's (2011) reading procedure, and van Manen's (1990) sentential analysis. These two authors' paradigms work in tandem:

- *Reading Procedure*: Both pre-readings and re-readings took place multiple times. Pre-readings were used to determine preliminary themes, and they

[4] University of phoenix did not provide a reference number for their IRB clearance.

were galvanized through external supports from other places in the data, identified through re-readings. (Morgan, 2011, p. 35).

- *Sentential Analysis*: I first filtered through the corpus of data by identifying "statement(s) and phrase(s) that seem particularly essential about the phenomenon or experience being described" (van Manen, 1990, p. 93). These statements would then be utilized to make deeper sense of the phenomenon.
- *Developed Themes*: After readings and sentential analysis, themes were developed, and each theme would be presented as a written paragraph and/or as a graphical representation. By cascading the developed themes into each of the four quadrants, I would then be able to underscore the lived experience of these six adult learners in mathematics anxiety, to find deep and sensible meanings to MALP, and to identify ideas for improved instruction.

Once the thematic analysis was completed, a further round of analysis on the major themes would be performed to obtain theoretical significance of these themes, and how MALP linked to Givvin et al.'s (2011) hypothetical model on community college developmental mathematics students' learning experience. Furthermore, the theoretical analysis should also reveal how MALP links to Knowles' andragogy in terms of affective versus cognitive anxiety, contingent versus authentic knowledge, and personal versus social beliefs. Essentially, it was my hope that this research study would contribute to the academic community fourfold, as integral theory's AQAL has prescribed.

Limitation of the Research Methodology. The research methods for lived experience can be a relatively flexible research methodology to investigate a phenomenon. The strength of the unstructured interviews and journal writings is the collection of uninhibited experiential data that past research may possibly have overlooked. However, the limitation of the method lies in how to effectively analyze these data, and how to draw meaningful conclusions from them. Moreover, the traditional positivist approach strives for validity, reliability, and objectivity, while this research study strives for trustworthiness and the embrace of the divergent truth. While the mode of knowledge sought in this study is simply different from those of the traditional positivist approach, one could argue that the distinction of trustworthiness versus validity is a limitation, as educational research has been rooted and grounded in the former. Although integral research has yet to reach wide-spread popularity, the results of this research could make a unique contribution in academics.

Conclusion

The cleverness of Wilber's Integral Model lies in its epistemological foundations that each perspective discloses a unique window to reality, and therefore, all perspectives, four quadrants, eight primordial (zones) perspectives, levels, lines, states, and types are all essential components as an examination in the research inquiry process. Furthermore, the Integral Methodological Pluralism (IMP) can be argued successfully through pragmatism that the divergent paths to the psychosocial reality, justify IMP as an eclectic mix of traditional qualitative and quantitative methods for inquiry. While the quantitative methods—that largely align with the Right-Hand quadrants—do not seem to pose any major contradiction to the positivist's epistemological stances, the qualitative methods—that largely align with the Left-Hand quadrants—are controversial portions of the IMP in regard to bracketing subjectivity. In constructivism, qualitative methods strive to bracket as much subjectivity in the data to achieve a pristine research result. Yet Wilber's theory unapologetically embraces subjectivity and intersubjectivity by arguing that the Right-Hand quadrants are perspectives that could disclose windows to reality.

Due to the divide deeply rooted in epistemological foundations, it is challenging to argue which is better as there is no one single perfect epistemology, nor is there one perfect methodology. To achieve this impossibility is like making a judgment value for either Euclidean or non-Euclidean geometry. Using mathematics as an analogy, every mathematical system has irreducible postulates that a system must assume, such as "the shortest distance between two points is a line segment." In this case, arguing that a specific epistemology is superior to another would not be as productive as asking whether IMP would produce fruitful results in this research study. Davis (2008) states that the quality includes "1) that the participants are not fooling the researcher, [*sic*] 2) that the participants and the researcher are not fooling themselves; and 3) that the researcher is not fooling the reader of the research. And therefore, making sure that “aligning the questions and methodologies is crucial for ensuring valid contributions of the inquiry to understanding of the phenomena studied” (Davis, 2008, p. 8). Furthermore, the goal of using integral theory to investigate a phenomenon is to gain understanding of “a myriad of dimensions of reality as it reveals itself” (Esbjörn-Hargens, 2006a, p. 22). Therefore, integral theory is a well-chosen research framework for studying the lived experience of MALP of adult learners in developmental mathematics.

Chapter Four: Research Findings
An Account of Each Participant's Lived Experience

In the previous chapters, this dissertation has discussed the context of Mathematics Anxiety Learning Phenomenon (MALP) and past research studies on mathematics anxiety as well as their methodologies. The research methodology chapter contrasted this dissertation study with past studies and showed how it set itself apart from them. This chapter documents the findings from stories of the six participants who contributed to this study. This chapter serves as the first of the three chapters of data analysis, which addresses the findings from an individual-interior's (UL) perspective in this chapter to a collective-interior's (LL) perspective in the next chapter, in which major themes will be developed and discussed through the exterior (UR and LR) perspectives. The third of the analysis chapters will serve as a synthetic chapter in which I use both the findings in this current chapter and major themes as means to compare these analyses to the theoretical aspects of the study, namely to Knowles' andragogy and to Givvin et al.'s (2011) hypothetical model.

The organization of this chapter is straightforward. I first will present a general descriptive summary of the six participants. Then I will describe each participant's contributions from a synopsis format, and also I will follow each synopsis with the interpretive individual themes. Finally, I will conclude with participants' contribution as raw materials for further interpretive analysis for the next chapters to come.

Overview of the Participants

Six adult native English speakers from the Western New York area were recruited to participate in the study. The study group comprised two male and four female college students, who had all graduated from their respective high schools. Also, they each self-identified that they have suffered from mathematics anxiety, and they were generally interested in how their anxiety has affected their learning of mathematics. The interviews were conducted over a course of two months, from December 2012 to January 2013, all in the Western New York area. The interviews were all conducted in a pleasant atmosphere at quiet areas in their respective colleges' libraries. None of the participants had difficulty or showed reluctance in the journal portion of the study, and the participants were all interested in the topics discussed. It was noted that the participants took a liberal approach in writing their journal entries, and therefore, the topics and foci varied widely in the contents of the entries. To determine the intensity of mathematics anxiety for each participant, I used the same benchmark from Ashcraft and Moore (2009) for aMARS (abbreviated) by rating the 25 Likert scales from 0 (no anxiety) to 4 (extreme anxiety) so that the range of the score is 0 to 100. Ashcraft and Moore (2009) reported a mean of 36 with a standard deviation of 16 (p. 199). Their method specified that one standard deviation above the mean, or a score of 52, was statistically defined to be highly anxious. All participants but one scored greater than 52 in the survey, confirming that they are highly anxious. One participant received a score of 51, which is statistically borderline highly anxious. Below is a summary of the six participants with their information tabulated.

Table 4.1

Summary Information of the Six Participants

	High School Graduation	**College Major**	**Math Courses Taken OR Taking**	**aMARS Score**
M1 “Carl”	1986	Communications and Business	Statistics	96
M2 “Jon”	2000	Hospitality Management	Statistics	87
F1 “Gerri”	1976	Education	Algebra and Calculus	54
F2 “Anne”	2007	Business Management and Education	Accounting and Statistics	58
F3 “Ellen”	1990	Education	Statistics and GRE Preparation	51
F4 “Sue”	*Did not list*	Nursing	Dosage Math and Statistics	57

All six participants chose non-mathematics/science majors as their fields of study, and it is interesting to note that all but one either took or were taking statistics at the time of their participation. The original design of the research proposed a three males and three females composition for the six participants, and after recruitment, two males and four females participated. This was not intended, but rather reflected the pool of participants that came forward to the study. The slight deviation from the gender composition did not pose any substantial difference in the research findings.

Each Participant's Story—The Lived Experience

This section provides a synopsis for each participant on his/her account of lived experiences on mathematics anxiety. While the narratives are given in third person, it is intended for the readers to have a brief view of the individual-interior (UL) and collective-interior (LL) perceptions on how they have become aware of their own mathematics anxiety. Following each synopsis is the individual themes for each participant. The synopses are written through pre-reading and re-reading so that each participant's overall contributing story would be presented in a coherent manner. Individual themes are developed through the reflection of sentential analysis, as outlined in the previous methodology chapter. Starting from this point forward, all references to the research data are indexed in the format of [Participant's ID].I/J[Number].L[Number] which participants' identifications are M1, M2, F1, F2, F3, and F4. The letter "I" or "J" denotes the source is respectively from an interview or a journal entry. For example "F1.J2" denotes the F2's second journal entry. The last code beginning with the letter "L" denotes the line number on the transcription of an interview or of a journal entry.

"Carl" Male #1

Carl, a father of three daughters, was a "straight A" student who studied communications and business administration until he had to face a statistics course, which he described to be a "horrible" experience. It is worth noting that Carl scored an extreme 96 out of 100 in the aMARS survey, and it was not a surprise when he shared how he struggled all his life with mathematics. Handicapped by two different learning disabilities, Carl shared a rather surprising piece of his history that he enjoyed his high school geometry course when the teacher took the students out to a tree grove to measure tree trunks. The hands-on learning experience related well to the concepts of diameter and circumference, and he thought that was probably one of the most positive episodes he ever had when learning mathematics. For the most part, as Carl explained, because of his terrible fear of doing calculations by hand, he had adapted in life to solve mathematical problems by using calculators and computer spreadsheet software. These tools took away the unpleasantness of the complicated hand calculations, and Carl would focus on "designing" a solution to whatever the problem at hand.

This was the strategy he employed for the statistics course; he was completely articulate in the concepts, such as the bell curve and hypothesis testing, but he relied heavily on the computer to crunch numbers to obtain results from statistical tests/procedures. In a metaphor, Carl pointed out how using the computer to take away the hand calculation was like taking a snake off him as he recounted how much he was petrified of snakes just as much as mathematics. Back in his teenage years, he experienced an episode when someone threw a copperhead snake on to him, and he knew that they were poisonous. Getting that snake off him somehow had a similar feeling to taking hand-calculation away from the overall problem solving process. Carl insisted that having the "computers tak[ing] care of this [fear of mathematics] issue" (M1.I1.L364) for him was his saving grace for the statistics course, which he received a not-so-deserving a grade of "B-."

Because Carl's three daughters were in school, I asked if he would help his daughters with their school work and document his experience in his journal entries. As Carl put it, his wife, who was an educator, did most of the home support for them. However, Carl thought it would be an interesting experiment in helping out on the mathematics homework for a change. The result was part amusing and part unproductive. All of his children thought that their dad's newfound interest was funny. "Dad, why are you trying to help? <u>You hate math</u>" (M1.J2.L31–32)! In another episode, one daughter thought Dad was not really helping, and she requested dad to leave her alone so that her sister and her mother would help instead. Despite the humor and discouragement, Carl managed to look at puzzles like "flipping numbers" and "lines connecting numbers," which he dubbed as "new math":

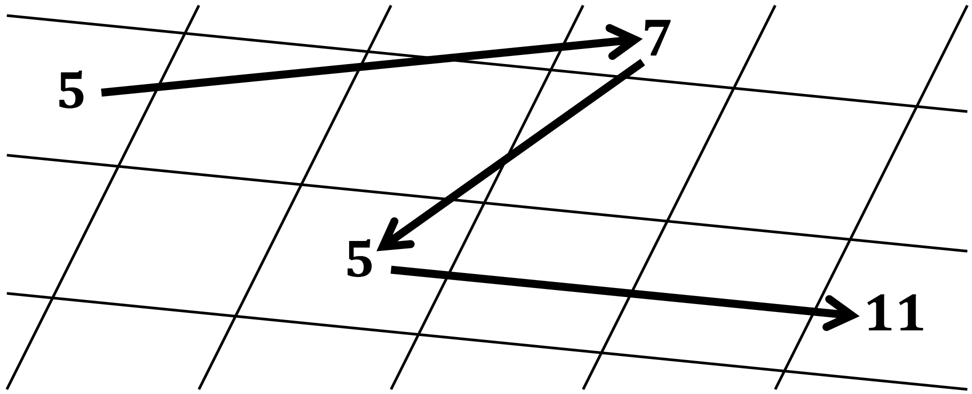


Figure 4.2. A representation of Carl's drawing of what he believed as "New Math." Carl explained while drawing that this was a mathematical puzzle in his daughter's homework assignment, and there were numbers connected by lines in a web grid. Exactly how the connected lines represented as a relationship to these numbers were both a mystery to Carl and me, and he was unable to elaborate how this puzzle was supposed to be solved. He contrasted "new math" like the above against what he had learned in the past "normal math," and he concluded that helping his daughters to write out solutions was challenging for them. He also added that without any parental resources from school, Carl found his help in general to be rather futile because he could not figure out how to write out the solution to any of the daughter's liking. He felt horrible for his daughters to go through these seemingly senseless puzzles, and the daughters did not take his new interest in helping their mathematics homework seriously. In fact, in our second meeting, I asked about the school support system, and Carl thought that the school districts did a marvelous job on parental involvement such as parent-child school field trips for history classes, and for English classes' school plays for parents to appreciate the fruits their children's labor. When I asked if there were similar parent-child activities for mathematics classes, Carl was astounded by his own response, and he told me that parent-teacher conference was the extent of his participation. Furthermore, it was noted that there were neither resources nor guides for students to bring home with their mathematics work, making him suspect most parents would find themselves clueless in helping their children to do "new math." If there were one wish, Carl would have wanted a better ability to understand "new math" so that he and his daughters would have an easier time with school mathematics.

M1's Individual Theme (A)—A quest to adapt in mathematics. A prominent recurring theme for Carl is how he described himself as learning disabled, and perhaps that was the reason why he perceived his mathematics learning experiences as "horrible":

> Okay well that brings horrible memories back to me too, just just [*sic*] for the record, and he [Carl's mathematics teacher] would say "write out all the answers" and do this and do this and do this and the … in the final [pause] what I was able to show was that, I tried. (M1.I1.L127–129)

Despite the negative experience, Carl asserted that he had tried learning mathematics. It is noteworthy to point out that not only did Carl consider the memories horrible, but also was the word "too" suggested that the "horrible memories" of these experiences were repeated events. A further interpretation of the repeated "and do this" is that Carl's mathematics teacher could be showing prescribed steps, and Carl found these steps unbearable to perform. On the other hand, Carl contrasted how he had positive experiences with his geometry teacher:

> He [Carl's geometry teacher] was a great great great [*sic*] teacher. … He, um, the fact that he took it out of the classroom, made it real, used real examples when … he would try to explain something like a pond, okay, or something that doesn't have normal shapes to it, and figure out the circumference or figure out how much space it's taking, um for me and actually for most of the class actually, the fact that we were outside tying strings to the ground and doing mathematical formulas without even knowing we were doing it. (M1.I1.L207–214)

In the above quote, Carl described how his geometry teacher took the students outdoor to work on measurements and other geometric concepts. While Carl explained that the learning experience in geometry was a lot more hands-on and contextualized than those of his algebra course, it seemed that he was given tasks in the field to be solved, and Carl was able to devise solutions to these tasks. The act of carrying out these devised solutions made learning mathematics pleasant and meaningful. In other words, Carl could be productive in learning mathematics when in context. On the other hand, Carl found that carrying out prescribed steps to solutions without appealing to reasons or context for these steps to be a meaningless experience. As an accomplished adult, Carl has now relied on technology to help him through the mundane steps:

I: And, do you actually believe that you have the ability to learn math at …

M1: At my age now? No.

I: Or or [*sic*] at any age…

M1: I believe that the school system is completely different when I started [the number of years has been deleted] years ago. There was a possibility, would I be a math genius, no. I also think though when I was born there weren't computers, there wasn't Excel, there wasn't, um, there possibilities way to learn math. Because, I can do Excel and all "if" statements, fancy calculations, but to me that's not math. (M1.I1.L240–247)

Not only did Carl seemed to have compensated by using technology to help him carry out mathematical steps that he found difficult to perform, but also is it interesting he thought using technology to "engineer" solutions to problem such as "if" statements and other logical Booleans and conditionals were not considered to be mathematics. This showed that Carl might have compartmentalized his horrible experiences as mathematics, but the mathematics coping skills he had adapted using technology as an aid were considered to be non-mathematics by him.

M1's Individual Theme (B)—"New math" versus "normal math". Carl made a quest to help his three teenage daughters with their mathematics homework, and they thought the whole idea was hilarious. Carl made a distinction between the kind of mathematics that he used to learn as "normal math" and the "new math" that his daughters were learning:

> Nope, I could not do it [new math] … I could think it out, yet I could not diagram it out the way the teacher wanted. My youngest was not teaching me how to do her homework, which of course made me feel sort of like an idiot. … New math is frustrating for those of us that learned "normal" math. … Plus as a father I should be able to help here, which I could not, so there is a feeling of uselessness. (M1.J1.L13–23)
>
> Normal math to me is you put it on a piece of paper, and it will say two plus two equals four not to be over simplistic or or [*sic*] you're dividing and you have a line and and [*sic*] the way I learned math. New math, I just see these diagonal and

somehow they put together. I could probably draw it, … , and it goes from one place to another (see the above figure on the web of numbers). (M1.I2.L67–71)

While Carl could not explain what he perceived "new math" was to him, it was clear that there exists a distinction (or perhaps a barrier) in his mind that "new math" was something that he had not seen in his past learning experiences. This "new math" was difficult for him to verbalize. It took him more effort to process than he already knew. As a father who wanted to support his daughters' mathematics work, this "new math" became a barrier, and possibly became the reason why his daughters laughed at Carl. When I draw commonality among individual themes, the distinction between "new math" and "normal math" will become important.

M1's Individual Theme (C)—Lack of resources for parental support. Even though the whole idea of helping his daughters was perceived as hilarious, Carl noticed how little parental resources and support for mathematics came from school:

> She [Carl's wife] actually asked the school district [on "new math"], and it was one of those standard answers "well every school district is doing it nowadays and this is how we teach it." (M1.I2.L258–260)
>
> *While we were discussing what school events parents were invited to, and it seemed like there were events for every subject but mathematics:*
>
> I: I see, but nothing in math that would invite parents to …
>
> M1: Absolutely nothing. I can't even think of one even close thing, except for an e-mail or phone call I got from a teacher telling me one of my kids wasn't handing in their homework on time.
>
> I: That was rather disappointing … So, how do you think the other parents are, um, coping with this "new math" with their kids' homework?
>
> M1: I think either they don't or, I'd say there are three things. Either they don't, the kids are picking it up on their own, or the parents are picking it up on their own … Either the kids are picking it up on their own or they're just not paying attention. (M1.I2.L343–357)

From the discussion, Carl painted a picture that the mathematics teachers in school generally played a rather authoritative role, making sure that the students were doing what they were supposed to, turning in homework on time and such. However, it seemed that the parents really did not play a contributing or a collaborative role in their children's mathematics education. When a child brought home a "new math" homework assignment and needed help, it seemed that many parents would be left to their own devices. In Carl's case, he was fortunate that his wife had been savvy in homework support. As he claimed, he would be no good in playing the supportive role.

Thematic Cascade of Carl's Individual Themes. Carl's three individual themes encompassed three of the four quadrants in integral theory. Individual Theme M1(A) on adaptation was on Carl's view on his own mathematical ability, and therefore it falls under the individual-interior (UL) perspective. Individual Theme M1(B) on the distinction between "new math" and "normal math" was his own reflection after helping his daughters to do their homework, and therefore, the theme falls under individual-exterior. Finally, Individual Theme M1(C) on parental support was an experience Carl had with the school system on the social identities that he, his wife, and the school mathematics teachers respectively played. Therefore, it is classified as collective-exterior:

	Interior	**Exterior**
Individual	**M1(A)** A quest to adapt in mathematics	**M1(B)** "New math" versus "normal math"
Collective		**M1(C)** Lack of resources for parental support

Figure 4.3. A cascade of M1 Carl's individual themes into the quadrivium.

"Jon" Male #2

Jon was a young adult who grew up in a small town in Upstate New York. He studied business and hospitality in college because he was interested in hotel and restaurant operations and management. He was raised by a single mother who owned a family bar and grill. Jon described that in his high school days, he did his homework assignments all by himself, since his mother did not provide much help. While Jon did not think twice that his experience was all that unique, a vivid episode during high school was his algebra teacher who, in Jon's eyes, was more of a mathematician than of a mathematics teacher:

> I think … the analogy of the mathematician and then the math teacher knew the material, was great at the material. When teaching students that one, had an interest, or two were already good at math, they were phenomenal. But if you had a student at fourteen, fifteen, sixteen that had no interest in math or was just simply not good at math, the teachers would get extremely frustrated. (M2.I1.L80–85)

Jon identified himself as the "fourteen, fifteen, sixteen years old" who had no interest in mathematics, and his learning experience was largely lacking engagement, enjoyment, and entertainment. The "mathematician" teacher often sped through the materials because the better half of the class could catch on, while Jon identified himself belonging to the other not-so-good half. When he went home to do homework, his mother took the independent approach, and he was left to his own devices. The algebra textbook, in Jon's words, was written in "a mathematician's language" (M2.I1.L144–145), which did not aim to be instructive. With no help availed to him, Jon attempted to find assistance from the "mathematician" teacher, but he would only make his teacher extremely frustrated because Jon could not find a "right" way to ask his questions. In fact, "wrong questions made to this teacher so mad that Jon once saw him "flip a desk" to the wall. Jon felt embarrassed and intimidated, and he could not learn algebra in this kind of environment, especially when he recalled witnessing the same teacher screaming and yelling at an inattentive student by implying that he was stupid. Despite the terrorizing bedside manner and the traumatizing experience, Jon still described the teacher as a "nice guy" (M2.I1.L217–273) who had lecture notes that were faded to yellow and brown, ready for his retirement.

When Jon entered college, he chose to study a business-oriented field mainly because he found working with money quite motivating. Business mathematics, to Jon, was largely arithmetic, and he could handle addition and subtraction. Algebra, on the other hand, was in a different echelon, and he would at all costs avoid revisiting the subject. After hearing from other business students that the "Business Statistics" course was a horror in its own class, Jon was nervous when it came time for him to take the course. To his surprise, it was taught by a young, attractive female teacher who was easy on his eyes. Her engaging appeal and charisma, along with an unending supply of patience for him, made a huge difference for his learning. Jon thought that the individualized interaction with her engaged him well, and he was at ease enough to joke that the teacher was a “statistical model.” Nevertheless, even though Jon felt that he had success in that course, he would “lose sleep” (M2.I1.L535) if he were to take it again. When asked what traits would make a student successful in mathematics, Jon suggested that while some students were better at the mathematics than he was, for him the engagement and interaction made all the difference. He further asserted that entertaining instruction, while not a requirement, was yet an important element to ease his learning.

Jon explained to me how he felt about engagement and interaction with a memorable experience. One day he walked by a classroom at his college where a class was in session. He saw a mathematics professor through the glass pane on the door teaching, yet facing the chalkboard the whole time without once turning himself to face his students. Meanwhile, his students were seemingly lost, many of them looking at each other, making fun of the professor’s socially inept behaviors, and texting and perusing social media on their mobile devices. Jon told me that he would not want (anyone) to learn in such a lackluster environment that obviously did not engage the students, nor was there any teacher-student interaction. In fact, Jon took his ideas of interaction, engagement, and entertainment to heart as a volunteered mentor who tutored young teens in an inner city school district. While Jon was good at tutoring the English and history subjects, he oftentimes needed to “investigate” the mathematics and science subjects along with his protégés, and he admitted that he could not be tutoring, investigating, and engaging all at the same time to make the experience an entertaining one. Moreover, he felt that some of the mathematics materials taught in school nowadays looked quite different from what he had back when he was a high school student. Of all the subjects he worked with his teenage protégés, Jon admitted that mathematics was the least successful. Reflecting on all the above experience as a whole, Jon noted that he had used to blame the school system on his fear of mathematics when he was younger, but he was more mature at the time of the interview, and he wished that he had taken more responsibility for himself.

M2's Individual Theme (A)—Jon's perception on the subject of mathematics. Jon directly mentioned that arithmetic is addition, subtraction, and multiplication. Nevertheless, several passages in Jon's interviews and his journal entries suggest that Jon established a belief that mathematics is prescribed steps, and the mimicking of these steps was construed as the act of learning. To Jon, a successful episode of mathematical learning is all about repetition, as he described how he achieved comfort in arithmetic:

> Arithmetic is something you use every day so with repetition you become good at it. You know, I compare it to if I haven't, haven't [*sic*] been skiing in ten years, if I try going skiing tomorrow I'm probably not going to be great at it. But if I went skiing every day for ten years, I'd probably be pretty decent. So I think the arithmetic is just something you do every day. (M2.I2.L144–149)

In this instance, the use of the skiing metaphor was used as a similarity to learning mathematics. It was as if Jon could do mathematics on a daily basis, then he would be able to strengthen his ability to carry out a set of prescribed steps, and he would excel in the subject. Part of the reason why Jon insisted describing mathematics as a series of steps is that he experienced the chain of events after an erroneous step that led him to have a wrong solution:

> We're doing something [be]cause we, we're told we need to do it, and here's the steps that we have to do. And I think it would always be frustrating too. Math, it feels like, if you're if you're [*sic*] wrong, one little area, it messes up the whole answer; if you're wrong here, the, at the end you're wrong. So, there was no, there was no, uh, I guess, no error, there couldn't be any error. I feel like in other subjects you might be, you're close to right. (M2.I1.L287–292)

Even though some criticized a professor that teaching steps was not teaching mathematics, Jon defended the teacher because, in Jon's view, getting through all the steps was essentially the predominant act in doing mathematics:

> [An individual] says, you know, well he [referred to a specific professor] doesn't. You know, it's not real math, he's teaching it in steps, and that's not, whatever, the students understood it. They learned it and did well in his class and they found math fun to an extent. (M2.I2.L465–468)

M2's Individual Theme (B)—Roadblocks to learning mathematics. There were multiple reasons why Jon did not thrive in the mathematics learning environment. In his eyes, Jon blamed the support system that he had practically no help at home or at school (M2.I1.L140–141). Even though Jon admitted that he should have taken more responsibility since he did not ask for extra help (M2.I1.L409–412), he certainly had asserted that the non-interactive learning environment was a cause:

> Teachers would write on the board, quickly explain and my fellow classmates and myself would simply copy down what was being written on the board. I truly never felt that the material that was being supposedly "taught" to us was actually being taught. (M2.J1.L6–9)

In fact, some of the environments that Jon described was difficult to listen to:

> … [A]nd it was literally … *Goodwill Hunting*, when he's [the professor] just scribbling, scribbling, oh I made a mistake, erase it, fix it, not explain, so you're writing a problem, you made a mistake, you just erase it and fix it? You don't say "oh folks, sorry, let's go back." I mean it was, eh… And there was not once, no turn around to look at the students, it was his back the whole time, with scribbling every five seconds would make a mistake, erase it, write again, and students just, for an hour and fifteen minutes, [would] just copy down what he did. (M2.I2.L450–458)

As I listened attentively to Jon's account on these learning environments, it seemed that Jon did not want to accept the above learning condition, but it was a tacit assumption that copying down as many as details as possible was a learner's default behavior in the classroom. During Jon's interview, he revealed a parade of episodes of these very similar learning environments, resulting in his conception of how poor mathematics learning environments generally were.

Specifically, Jon described his identity among learners belonging to one of the two halves of students. One half of these learners could understand the materials, and the other half in which he belong generally did not catch on:

> … [T]he fifty-percent that didn't know the material, um, I almost felt like she [the teacher] didn't even, didn't even try to teach to those individuals. The fifty percent, so if the left side of the classroom understands it, she's over there talking while we're over here doodling on the desk or, just, there's no attention, there's no, there

> was no [very long pause] there was no passion from that teacher to actually want to teach or want to engage a learner that, really and and [*sic*] that's the point of education. … So there was no no [*sic*] engagement at all. There was nothing fun about math. (M2.I1.L246–251 & L260–262)

This was not the only place when Jon identified himself in the group that did not do well. Indeed, how Jon viewed his own identity in the learning environment, which was a passive, low-functioning individual with a collective who also suffered a similar fate, was revealed throughout the interviews.

M2's Individual Theme (C)—Expectations in engagement. Because Jon believed that copying complete solutions to mathematical problems was of utmost importance to learning:

> … [Y]ou went through the motions. You followed, you wrote down everything in your notebooks, you went home, you tried to study it, math to me er, it's it's [*sic*] numbers, it's here's the problem, there needs to be an answer. Um, and how you get there… I don't know if I ever got there. (M2.I1.L475–479)

Notice the phrase "through the motions" Jon implies that a typical way of learning mathematics was through an unexciting and tedious routine. The importance of writing down everything in a notebook suggested how Jon perceived prescribed steps as mathematics. Also, Jon found it difficult to articulate the path from the problem to the answer such that the path was largely a vague unknown. This indicates the absence of the underlying reasoning to connect the sequential steps into a coherent solution. In fact, the idea of Jon's expression that he never got "there" could mean that he had difficulty getting *all* of the details of the steps down, and if the steps were incomplete, it could be the reason why Jon failed to develop the underlying reasoning.

Perhaps this is one of the most prominent themes when Jon described his experience. He couldn't seem to emphasize enough about positive interactions through engagement of learners:

> I remember she was a young, energetic, compassionate professor. She truly took time to explain the material and did not make students feel as if they were "stupid" or incapable of understanding the material being taught. (M2.J2.L33–35)

> …[I]f you're breaking it up into pieces, saying, making sure, "does everyone understand?" That's, you're stopping in the middle and you're looking at everyone, going around the room to make sure people, okay, that's engaging [everyone] because you're not going at full-throttle through something, you're taking your time. I think the pace easily engages someone. (M2.I2.L509–519)

In Jon's own words, the expectation of engagement is to be positive, as he implied that it is commonplace for mathematics teachers to make students feel incapable. Also, the use of the word "understand" implies being able to follow through with all the pieces. Therefore, the interpretation of Jon's perception is that he expected that learning mathematics is about prescribed steps, and being able to carry out the steps in a procedure is learning mathematics. Furthermore, Jon drew positive experience from subjects other than mathematics to make his point:

> …[Y]ou can remember [in] English, you can remember the exciting teacher, that even if you didn't like Shakespeare you didn't like, still try really passionately to make it interesting. With math it was just … eh. (M2.I1.L310–313)

Ultimately, while Jon thought that he bore some individual responsibility to his unremarkable experience, he gave an account that he expected mathematics teachers to be marvelously engaging. A good teacher in Jon's opinion should be moving at a pace that he as a learner would be comfortable at, speak in a language that transcend from the prescribed steps with repetitious practice for him to succeed.

Thematic Cascade of Jon's Individual Themes. To classify the above themes, Individual Theme M2(A) is Jon's own perception on the subject, and therefore it falls into the individual interior (UL) quadrant. Individual Theme M2(B) on roadblocks discusses the cultural traditions of Jon's mathematics learning, and hence, it would be classified as collective-interior (LL). Finally, Individual Theme M2(C) on the expectations in engagement reveals a wishful role that Jon would like to play in the mathematics education discourse. Thus, it is classified as a collective-exterior (LR) perspective.

	Interior	Exterior
Individual	**M2(A)** Jon's perception on the subject of mathematics	
Collective	**M2 (B)** Roadblocks of learning mathematics	**M2(C)** Expectations in engagement

Figure 4.4. A cascade of M2 Jon's individual themes into the quadrivium.

"Gerri" Female #1

Gerri had been a master teacher for years, working in a high school, and just recently received a promotion to become an instructional coach who would oversee the instructional delivery of other teachers. Because of the demands of coaching other teachers, especially in mathematics, and the occasional tutoring of students in the same subject area, she decided to go back to college to "brush up" her mathematical skills. She went to two different local colleges because she did not find a professor whose instruction was conducive to her learning at the first. The professor at the second college, as well as the college's resources for their students, were more helpful.

The first professor, as Gerri described, would spend time on a chalkboard to illustrate sample algebra problems, and then he would allow students to solve similar problems on their own in class before they went home with a "plethora" of homework problems (F2.I1.L26–27). When the students came back for the next class, the professor would poll them for any questions that he could go over from the homework, yet his expectation was that there would be no questions from any student. Despite his unwelcoming demeanor, Gerri took the invitation as an opportunity anyway, which only ended up in his chastising her way of asking questions. She could not receive much clarification from the professor, which left her frustrated as to why she must memorize so much about the different types of equations for each algebraic topic. Gerri admitted that she was not a good “memorizer,” and this became apparent when she did not learn well with this professor (F1.I2.L95–96). In fact, Gerri described that the professor really preferred no questions from the students so that he could carry on and continue lecturing. Because of this experience, Gerri went to a different college and enrolled herself in another course. The next professor was helpful in taking questions and provided individual interaction. Also, she sought help from the “Math Lab.” While the materials had been a struggle for her, she solicited so much help from the lab that she “knew the Math Lab people very well” (F1.I1.L101–102). In both colleges, Gerri asserted that the textbooks for both courses were a “struggle” to read and comprehend, and it was challenging for her to follow many of the step-by-step examples from those texts. Gerri mentioned that she had to memorize plenty of steps, and it was perplexing to her why both the in-class instruction and textbook explanation largely focused on procedural steps instead of the underlying reasoning/logic and problem solving techniques.

In fact, Gerri herself identified that she learned best when a problem was presented to her with multiple ways to reach a solution. This was the same attitude she took when she tutored other students. In her journal entries, Gerri described how frustrating it was when she helped her granddaughter on her homework. They were doing division, but the granddaughter insisted using her school teacher’s “Big 7” strategy instead of using traditional long division to find a quotient (F1.J1). Gerri described that showing the long division to her granddaughter did not produce a mutually satisfying learning experience since much of the discussion was focused on what the school teacher would and would not consider acceptable work. In another entry, Gerri helped a high school student on ratio and proportion problems, and the student surprised her with how she memorized the steps to solve proportion equations by using cross multiplication. So, she helped the student to explore the underlying reasoning, which in her mind was an aspect which any educator should be aiming for any student’s learning to achieve. To her amazement, working with the student on the underlying reasoning of what a ratio was and how it related to fractions and percentages ended up frustrating the student more so than before the tutoring session. The student did not see the point of probing deeper by arguing that she had the steps already memorized, and she was able to complete the mathematics assignment merely using these steps. As Gerri described, the probing for deeper understanding was a waste of time to the student. In the last journal entry, Gerri documented a much more positive tutoring session with a high school student, who was graphing linear equations on paper in a superficial way. So, Gerri asked him what linearity meant to him, and he drew a blank stare. She worked hard with him on exploring the x and y variables, changing them unit-by-unit and investigating how these changes of one variable could affect the other. After 45 minutes of intense reasoning, the student exhibited both satisfaction and happiness for a deeper understanding of the linear equations, which he told her that he could not have achieved this in his class alone. However, this episode led Gerri to wonder how K–12 teachers could possibly develop such deepness when they had limited amounts of instructional time. Also, they are "under the gun" for standardized testing on a "mile wide and inch deep" curriculum.

When we met the second time, Gerri expressed her concern that the current infrastructure of the K–12 educational system could undercut educators' efforts. The poorly written standardized tests that largely rewarded rote memorization, the flawed teacher evaluation based on students' test scores, and the school evaluation by the state based on overall passing rates—they all had played a role in how instruction in the classroom was delivered. In particular, Gerri described a commonality many students shared, including her own granddaughter, who preferred a "conformity" style of working on homework problems, leading them to follow step-by-step procedures without any deviation and without questioning if there were better ways to solve the same problems. This worried her, as she put it, because this kind of education would produce a generation of students who would be unable to solve problems creatively, nor were they adequately prepared to solve novel problems that were not seen previously. Gerri hoped that the new common core curricula that had recently been gaining popularity among U.S. schools, which would focus instruction in a contextualized manner, would give new meanings and reasons to students why they learn mathematics.

F1's Individual Theme (A)—The reliance of memorization causes anxiety. In fact, Gerri often emphasized how the reliance of memory had caused her problems in learning mathematics. This followed how she observed her granddaughter learned in conformity:

> I spent a lot of time in the math lab, trying to get extra help, And, um, most of them, you just have to memorize exactly what to do when you saw it. (F1.I1.L36–39)

Gerri further elaborated:

> For me personally, pure memorizing is difficult, so that would make anxiety as well. (F1.I1.L92–93)
>
> Yes, I think sometimes it would take me a little longer to learn things. And maybe I had to really understand them, not just memorize. There is a difference between doing it and understanding it. Sometimes some math teachers want to move fast, and they don't wait for you… (F1.I1.L126–130)

These quotes showed how Gerri perceived the norm of learning mathematics, and the heavy reliance of memorization was off-putting for her, succinctly told:

> I get frustrated with kids that don't [attempt to understand], I guess I'm not a memorizer, I'm an "understander." (F1.I2.L95–96)

Plainly put, Gerri perceived a subtle dynamic in the mathematics learning environment that memorization was covertly encouraged as part of the learning process. Meanwhile, the quest to understand the inner working of mathematical problems was of little emphasis. This had become a source of frustration for Gerri.

F1's Individual Theme (B)—A Perception of conformity in learning mathematics. Gerri told a grim story about how mathematics educators are stuck in a tough spot in teaching young students mathematics while balancing a satisfactory pass rate on standardized testing. She referred to both her granddaughter and one of her students who were doing mathematics homework in a "programmatic", "algorithmic", and "plug and chuck" manner. If they deviate from the prescribed steps, then they became frustrated:

> It was common for her [granddaughter] to tell me that I was doing it incorrectly, regardless of the answer I was able to derive. My methods were often different from the methods that her school math programs were teaching, and she was very upset that I did not know the "correct" method. (F1.J1.L5–8)
>
> *Gerri was helping a high school student on ratio and proportion:*
>
> It was obvious from the start that the female student had memorized the mathematical process of cross multiplying… After that sunk in we went back to the

> original problem I had put before her. I had hoped that she could see that there was a relationship between them. It didn't click with her and I could sense her frustration, she just wanted me to show her how to solve the problems. She had no desire to truly understand why, she just wanted to solve it. I find this so disturbing. (F1.J2.L3–4 & L16–20)

In both instances, Gerri's journal documented that both students were reluctant to engage in mathematical thinking that is beyond the set of prescribed steps that their respective teachers had demonstrated in class. Gerri's granddaughter developed the ideas that not conforming to the prescribed steps would be considered incorrect with undesirable results. While this theme is not totally unexpected, it is a concern because Gerri described what a typical student would consider what doing mathematics equates to a behavioral exercise through a single path to a correct solution. The idea of multiple correct solutions to a problem is not cultivated in the learning environment:

> I don't know if that's normal that kids think they can only do things one way in math. They don't always see that there is more than one way to get to the same answer. (F1.I2.L49–51)
>
> I seem to feel a resistance to understand math. They just want to know how to do the process and get the answer; they don't really want to understand it. They don't really have that questioning … [of] whys and how it fits together, where somebody who can do math can follow a preprogrammed, you know, almost like a computer. (F1.I2.L87–92)

Furthermore, in Gerri's experience, many students did not seem to be motivated to comprehend the underlying concepts because that kind of learning was not reflected in better school grades and higher standardized test scores. In Gerri's belief, this is the reason why many students resist comprehension and prefer conforming to prescribed procedures. In other words, conforming to prescribed steps would be rewarded and reflected through the grading system and in standardized testing, but ingenuity in mathematical problem solving would not.

F1's Individual Theme (C)—A keen observer in mathematics instruction. I felt fortunate to have Gerri as a participant in the study because she is a seasoned educator. She provided a detailed account of how mathematics instruction was delivered:

> It doesn't matter what you're writing about, but you're writing essays. You're working on that one skill set all year, where a math teacher has all these little piecemeal. They're not working on, you know, this year you're going to be comfortable with numbers. You know there, it's all these tiny skill sets that they're trying to push together, and that's where I hope common core is really looking, you know, more holistically at bigger skill sets and saying here's the pieces that will help you to get to do, you know, statistical analysis, like we're really going to focus on that topic and look at the skill sets. I'm hoping it will be more meaningful for students because I think, right now, it's very fragmented in all these little things. I know I was observing a classroom and they were, uh, if they were doing equations, but I, let's say they were doing equations. She had done the testing and the kids didn't do that great and I said what do they need to go onto the next unit to be successful, cause if you don't… And she said, "oh nothing, the next unit's on [blank blank blank…]" Oh, so, and the good news was that gave her some time to remediate after school and work with kids on the equations, but the fact that there was a topic that was totally disconnected, to me, was a, was really kind of funny. That's like, okay, we're going to teach Biology for three weeks then we're going to teach Chemistry for two weeks, you know like disconnecting like that, shouldn't math flow? (F1.I2.L159–176)

Gerri pinpointed a major flaw in mathematics instruction that its delivery was fragmented and disconnected. This disconnected manner may have conditioned students to miss the overall big picture and underlying reasoning in mathematics. She contrasted the subject of mathematics to the subject of English, in which a teacher could continually help students to develop essay skills in a holistic manner. Therefore, Gerri was really hopeful that the up-and-coming (as of 2013) national common core learning standards which focus on contextualized learning would improve student's attitudes and mode of learning so that they may become better "understanders" as opposed to memorizers:

> I'd like to think common core is going to make it because the shifts in common core, one of them is contextualizing and I think the idea of contextualizing math gives

reason for learning it. Gives it a visual to go along with something that can be very abstract. (F1.I2.L381–384)

This was how Gerri ended her story, advocating a goal in mathematics education to strive for a holistic approach to problem solving where creativity and ingenuity should be valued as opposed to conformity of following steps.

Thematic Cascade of Gerri's Individual Themes. Gerri's individual themes can be classified as follows: Individual Theme F1(A) on memorization addresses Gerri's own perception on how to study mathematics, and therefore, it is considered as an individual-interior (UL) perspective. The second Individual Theme F2(B) on conformity is a cultural tradition that Gerri observed about what was considered appropriate in the learning environment. Henceforth, it is classified as a collective-interior (LL) perspective. Finally, Gerri's observation on mathematics instruction in Individual Theme F2(C) was third-person's account of a collective experience. Thus it is a collective-exterior (LR) perspective.

	Interior	**Exterior**
Individual	**F1(A)** The reliance of memorization causes anxiety	
Collective	**F1(B)** A perception of conformity in learning mathematics"	**F1(C)** A keen observer in mathematics instruction

Figure 4.5. A cascade of F1 Gerri's individual themes into the quadrivium.

"Anne" Female #2

Anne was an articulate student, and I will begin her synopsis with a quote from her:

I never used to [like] math I think may be in between third and fourth grade is when I started really struggling with it to the point where I was very scared to like take the tests and take the quizzes. So, my parents were concerned because I was doing fine

> in everything else, but I was having issues with math. So I actually went and got tested at like a, something like a [private learning center; deleted], ... And they did all these tests on me, and pretty much came to the conclusion that nothing was wrong with me. I was just… nervous about mathematics. (F2.I1.L33–39)

Anne had just graduated from college with double majors in business management and education at the time of writing. She identified herself as suffering from mathematics anxiety since third grade. During high school, she had to participate in team/group work in mathematics class, and she had always taken the "back seat approach" (F2.J3) because she worried that her skills would not match the other teammates in the group. In fact, Anne described that her contributing to the team/group learning experience as "feeding her to the sharks." Also, Anne had been discouraged from asking how "force-fed" school mathematics could be useful to her. Because of her lackluster participation, as well as unfulfilled curiosity, she struggled with mathematics, particularly with word problems, throughout her childhood. Since her parents figured out Anne had not been doing well, they arranged a few tutors, including her mother for several summers, to remediate her skills. The calming décor in one of the tutors' homes had positive effects on her learning, and some of her successes included memorization tricks and skills. Particularly salient was her memory of her own mother, an exceptionally accomplished educator, who tutored her during summer vacations to help her retain mathematical skills. Anne dubbed the tutoring as "Mommy School" (F2.J2), where they did a lot of test prep exercises and recipe fractions when they baked together, and she found the experience to be engaging, enjoyable, and edible.

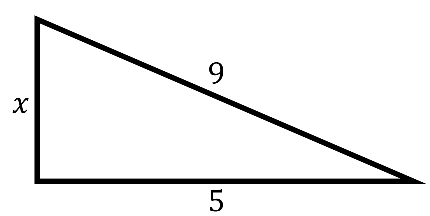
In her college days, Anne reported, she struggled in algebra and accounting. In fact, she dropped accounting on her first attempt and retook the same course in a local business college during a subsequent summer when she returned home (F2.I1.L118–126). She noted that the lack of individual time with the professors and the lack of peer help were the main reasons she was not successful. She also commented that the accounting problems required a mastery of myriad formulas, and she found that to be more challenging than she could handle. Meanwhile, when asked how she was successful in the second attempt in the same course, Anne thought that lots of patience, practice, and hard work were her keys to success because she would not trust her own reasoning. Later in her college career, she took statistics for her second major in education. While she did not find the struggles in statistics as intense as her previous mathematics learning experiences, she maintained that it was not enjoyable.

When asked whether she was anxious in other life situations, Anne equated mathematics anxiety with the anxiety she experienced during job finding:

> … [W]hen I was like trying to find a full-time job, I was very anxious about that, and that was kind of the same intensity that I felt with math anxiety. I got to get a job, I got to get out of the house, I got to get my own place, like how can I do this, I have student loans to pay so it's more financial, um, anxiety that I have that is anything that I can, is almost more intense than mathematics. (F2.I1.L333–339).

Anne immediately pointed to her anxiety during a job search when she was worried about her finances. The uncertainty of whether she could pay bills had given her an overwhelming emotion. When she finally secured a job, that overwhelming emotion went away—unlike in learning, when her mathematics anxiety did not. Through this analogy, she identified the never ceasing anxiety in learning mathematics as the lack of control she felt about the subject. When she landed a job, she knew that her financial situation would improve, and it would be under her control, but the same feeling of being in control never surfaced throughout any of her experiences in learning mathematics.

One aspect of this study was to have Anne come up with a mathematics problem that would make her anxious, and multiple times she expressed she could not do so, nor could she remember much from her previous experiences. After some encouragement and very reluctantly, Anne came up with a geometric problem (reproduced below):



> …[T]hey draw the shapes like where they draw like a triangle like this, and they say like, this is nine and this is five, and what is this? (F2.I1.L277–278)

Figure 4.6. Anne's own example on a geometric problem that inflicted anxiety.

While Anne was able to produce this problem, she did not recall that it relied on the figure to be a right triangle, and it would be solved with the use of the Pythagorean Theorem. Anne provided relatively little discussion on it, and this is an indication that she indeed remembered little content from her past mathematical learning.

On the topic of what changes could have made a difference in her past experiences, Anne pointed out that her questioning techniques were poor. Furthermore, her pride led her to mask her insecurities during group, work which had made it difficult for her to learn mathematics in a productive manner. To learn mathematics better, she thought that a more individualized one-on-one interaction and collaboration would be instrumental, and a healthy dose of connections between mathematics and real-life applications should be made clear to the learners. This leads to my remark on Anne's concept of what mathematics is, which she had difficulties describing. She pointed out that mathematics was "lots of x's and y's" (F2.I1.L89), and it was something not so applicable in life, leading her to conclude that statistics (which she did well with relative ease) was a separate "category" from mathematics.

F2's Individual Theme (A)—Patience, practice, and memorization. Anne's idea on how to study mathematics was interesting:

> I: What kind of skillsets do you believe that you need to have to be successful [in mathematics]?
>
> F2: Patience, you have to be patient, um, you have to be hard working, you have to ask for help when you need it, um, you need to practice. (F2.I1.L154–157)

When I wanted to find out why practice and patience were important to her learning, Anne suggested:

> In math I feel like there's something logical, I know there is lots of logic in math, obviously, is very logical, but when I'm doing math I don't think "well you know, yeah that one makes sense, like I had to know, that's why, I don't trust my reasoning, um, a piece of my brain to go with math I guess. (F2.I1.L182–185)

The idea of not trusting her own reasoning made me wonder if Anne relied on memorization to get her through mathematics:

> … Like you said here A squared plus B squared equals C squared, I just feel like, that's basic, I can get this, but when you start piling things upon it, that's when you

start losing me. And … I wasn't able to just memorize all the equations and when they were supposed to be used and how they were meant to be used, and I just [pause] would freak out. (F2.I1.L305–309)

… [W]hen you're younger and just learning that concept, it's easier to memorize those little things. But then when you, um, get into the more advanced math where more [there are] equations you need to know, memorization is harder because I have to try and juggle all of these things, so yeah. I I [*sic*] think that, I think that I have to memorize. (F2.I1.L190–194)

There are other discussions in the journal entries and interviews that show how Anne relied heavily on memorization, but the above really described how Anne felt when she was overwhelmed with mathematical discourse. Because of that, she could not rely on her own reasoning to make sense of the materials and the subject. Hence, my interpretation is that Anne used patience and practice to "survive" learning mathematics through memorization.

F2's Individual Theme (B)—Lack of transparent application. One of Anne's difficulties with which I could sympathize was:

I think that math should be taught with some form of application to it 'cause I I [*sic*] remember being in school and being like "how is this going to help me in life," and the teacher would be like "Anne, just be quiet!" (F2.I1.L207–209)

I remember my one teacher saying I think with geometry that all this is what you need if you want to be an engineer, or something. I honestly asked the question in a lot of classes, why am I here? Why am I taking this? (F2.I1.L265–268)

Not only did Anne not understand how learning mathematics could be beneficial for her life, but then her teacher chastised her questioning its mathematical purpose. Perhaps this is the reason, as one will see in the next theme, why Anne became quite passive in learning mathematics.

F2's Individual Theme (C)—Lack of pacing and control, so pretend to learn mathematics. For Anne to learn mathematics well, she considered pacing in instruction a key component:

> My managerial accounting course was an hour and half long. And for the first hour, she would … it felt like she was talking about nothing but her personal life and her family. And the last half hour would be like jam-packed of everything. … She was going too fast … (F2.I1.L92–96)

And Anne further elaborated on the pacing of "going too fast" that many mathematics teachers were "down to business":

> I: What do you mean by down to business?
>
> F2: They well, teachers, um, you know, [and] they have a certain syllabus they want to stick to throughout the whole year. And, you know, they want to get this stuff done by then … (F2.I2.L58–61)

These two above quotes are similar to how Ellen (Participant F3, see below) described she could not keep up with the professor. In some way, Anne also found the pacing of the instructional delivery an element that could overwhelm her. This leads to a discussion on the important of learner's control in the instructional environment:

> There was no, I couldn't, I mean the only way was up [higher mathematics] [laughs]. So like I would take the courses and the only way that you could control it is if you wanted, if you weren't, if you felt like you weren't being challenged enough then you went, and took higher level courses. That that's [*sic*] the only way you can control it. There's no control for "slowdown." (F2.I2.L307–311)

Anne compared the lack of control in her past mathematics courses to those experiences of another subject:

> In English, if you're interested more in Shakespearian, you know, literature. You can take a class in that, or you can take a class in romanticism, like, but you don't have a whole lot of movement at all with math. (F2.I2.L338–341)

As I interviewed Anne, it became clear that the lack of learner's control could be a detriment to how Anne might perceive her own learning. In fact, she admitted that she took a passive role:

> If my study group had come to consensus that we needed to ask the teacher a question, I was all for that. But, if I had a question, if they were going too fast, and I didn't know how they did that step, I wouldn't ask my peers. (F2.I2.L92–95)

I would sit back [laughs] I I [*sic*] wouldn't, you know, I'd be the one who kinda sits back, but, doing enough like head nodding, like, "oh yeah. I agree with that," but secretly thinking, "hmm, I have no idea." So, the teacher wouldn't know. (F2.I2.L97–99)

Ultimately, this theme illuminates how Anne, through years of playing a passive learning role in mathematics, did not possess much control as a learner. Therefore, she developed a masquerade for her peers and teachers, as if she knew what she were doing.

Thematic Cascade of Anne's Individual Themes. The classification of Anne's individual themes is as follows: Individual Theme F2(A) on the qualities that helped her study mathematics was Anne's own individual reflection of her learning journey, and so it is an individual-interior (UL) perspective. Through the culture of interaction between Anne and her teachers, she discovered the lack of application in mathematical topics. Therefore, the Individual Theme F2(B) is a collective-interior (LL) perspective. The third Individual Theme F2(C) is a third-person's reflection, as well as Anne's social role to pretend to be learning, addressing her own learning experience in the collective exterior (LR) perspective.

	Interior	**Exterior**
Individual	**F2(A)** Patience, practice, and memorization	
Collective	**F1(B)** Lack of transparent application	**F1(C)** Lack of pacing and control, so pretend to learn

Figure 4.7. A cascade of F2 Anne's individual themes into the quadrivium.

"Ellen" Female #3

Ellen, a successful U.S.-born college student who studied English education, grew up in a traditional Asian-Indian household. At the time of her participation, she was preparing for her Graduate Record Examination (GRE) and applying to graduate schools to study speech therapy. One experience that Ellen shared was that she took the same mathematics course twice, once in high school, and then again in college because she wanted to attend medical school down the road. The logic behind taking the same course twice was to double the chances to succeed in this challenging subject for her. While taking the course in high school for the first time, Ellen described that she had a cohort of classmates since the sixth grade, and they had been studying together for years. The group had positive dynamics, collegiality, and healthy competitiveness (and sometimes rivalry, which is a very common cultural expectation among Asian-Indian students) to strive to succeed. The teacher welcomed questions, and Ellen got the one-on-one attention when needed. Ellen thought that her learning experience, while challenging and not always stress-free, was a decent one. Nevertheless, I sensed a hint of disappointment in Ellen's voice when she disclosed that she earned a either "B" or "B-" in the course. When asked what her shortcoming was for not "acing" the course, Ellen explained that she was better at learning the concept or "the big picture", but when it came to attention toward details, she had a difficult time getting all the procedural minutiae executed correctly:

> …[I]f I'm trying to figure out the concept, and then there's a calculation that needs to be done, and I'm still thinking about the concept and he's moved on to the calculation. I need a minute, I need him to stop and wait for me to calculate. I have a tendency to make mathematical errors, I have a tendency to skip over things, be jumping really fast. That's one of the things in math I have to work at, is to slow down and do things methodically and precisely, it doesn't come naturally to me. (F3.I1.L242–248)

Ellen's plan to take the same mathematics course for the second time was nothing like how she had envisioned. With the familiar cohort of study group no longer in place, and the professor who went over the materials so fast in a lecture hall that sat hundreds, she just could not keep up. She began falling behind in the lecture when she made careless mistakes and skipped essential steps when writing her own notes. In her mind, she had this overwhelming spinning thought of "I can't do this; I cannot do this" (F3.I1.L306). In the end, her inability to keep pace with instruction was one of the reasons why Ellen gave up her dream to go to medical school, and she ended up studying English education instead.

It was not until Ellen decided to study for her GRE examination that she came to the realization of the significance of her lack of attention to details. One day, Ellen was reviewing the counting theory unit in which there was a sample question asking for the number of permutations in a situation of which a calculation of a factorial "11!" was required. Ellen had thought that the concept of factorial was within her reach:

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

Figure 4.8. The concept of factorial where n is any positive integer.

So, Ellen did the multiplication as prescribed, and then checked her solution against the answer key at the back of the preparation guidebook. Perplexing to her was a different answer from her own work, she tried the calculation again, and this time her solution matched the guidebook's. This episode made her wonder what had happened, and she went back to the original wrong solution to troubleshoot. After some perusal of her own work, she came to find out she carelessly omitted the "7" in her multiplication:

Guidebook's answer key:

$$11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 39{,}916{,}800$$

Ellen's solution:

$$11! = 11 \times 10 \times 9 \times 8 \quad \underbrace{(omitted)} \quad \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}702{,}400$$

Figure 4.9. Ellen's careless mistake on a problem of factorial and her troubleshooting the flawed solution.

The troubleshooting was like an epiphany to Ellen as she began to make a distinction between conceptual and procedural errors. In this case, the careless omission of a number was obviously a procedural error, but indeed she understood the use of the concept of factorial that multiplies descending positive integers. As we spoke, she came to realize a stark contrast: that she had total control of the pacing when she was studying for the GRE examination, which was not so in a classroom with a professor lecturing. Ellen concluded that this contrast of having control must have made a difference in her learning mathematics, as she could comfortably work with the GRE mathematics materials, but she had not been able to do so in her past college experience.

In our second meeting, Ellen shared a great deal about her seven year-old son Victor (the name was changed for the purpose of reporting). She described that his first grade teacher would adhere a star-shaped sticker for every perfect homework assignment he did. Victor got star stickers for almost all of his homework assignments in various subjects, but usually not in mathematics. In Victor's eyes, getting a star on his homework assignment was a must, and he began disliking mathematics for this reason. Ellen, who felt uneasy about how her son began to suffer from mathematics anxiety so early in his schooling, had been helping him with his homework on a daily basis. In one episode, they were working together on the "Math Mountains", an exercise to help students develop the abstract notion of addition and subtraction as a single algebraic operation, as well as numeracy, illustrated in the two examples below:

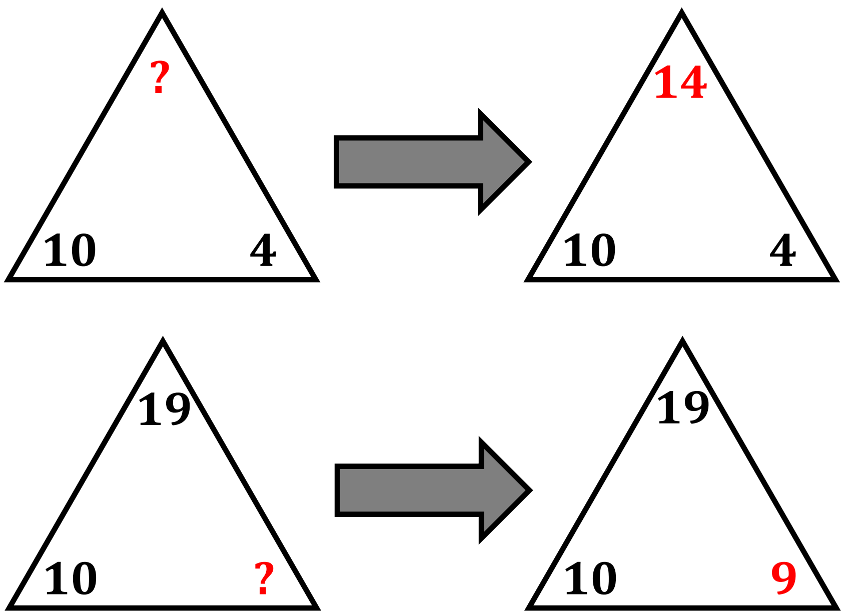


Figure 4.10. Sample "Math Mountains" that Victor was working on in his first grade mathematics homework assignment.

On the day that Victor came home, he was upset that he did not get a star in his "Math Mountain" assignment because he erroneously wrote 41 instead of the correct 14 in the first sample question above. Ellen thought it was important to help him count up from 10 and down from the sum (the number on the top) to obtain the correct answers. Victor, after some encouragement, counted up and down quickly and accurately, showing off how he knew the "Math Mountains" on all the problems. This convinced Ellen that Victor indeed knew how the "Math Mountains" worked, leading her to believe that there was a different reason why Victor got the one problem wrong. Ellen then explained to me in detail that Victor had recently developed a habit of transposing letters in spelling and digits in mathematics. Thus, she began to realize that it could have been the same transposing error that cost Victor his star. So, Ellen attempted to lift Victor's spirit, "I told him he was not bad at math at all to which he replied, 'Yes I am, [but] I did not get a star'" (F3.J1.L21–22). Through Ellen's observations, Victor's self-concept of success was binary: perfect and not perfect. His view of evaluating his own performance troubled Ellen because she worried that this could be the beginning of long-term mathematics anxiety in Victor. When I asked Ellen how she knew what to do with Victor's mathematics school work, Ellen pointed out that the school teacher provided parents with resource guides, so that the parents could understanding what to do with some of the not-so-traditional mathematics. She further elaborated that she and her husband moved to the suburb because of the excellent school district, knowing that other schools may not necessarily be this supportive to parents. However, due to budget cuts, Ellen heard through the grapevine that her son's school would be eliminating their accelerated academic program in the upper grades, which worried her a great deal. Our interview ended on the topic of do-over: What would Ellen do differently if she had a chance to do it all over? She brought up an interesting point that how she compared herself with other Asian-Indian students who showed off how studious they were by showing that they had study notes filling up all of the margins on every page of their textbooks. Ellen, on the other hand, was a student who had much more average work ethics, and she had none of those "impressive" evidences to show off. She wished that she was more of the show-off type.

F3's Individual Theme (A)—Difficulty with college mathematics. Ellen devoted an elaborated discourse on her experience in college mathematics. She attributed her lack of success due to her lack of detail orientation.

> This is one of the things I struggled with I think, as a student in high school and especially in college is if I'm figuring it out, if I'm trying to figure out the concept, and then there's a calculation that needs to be done, and I'm still thinking about the

concept and he's moved on to the calculation. I need a minute, I need him [the mathematics professor] to stop and wait for me to calculate. I have a tendency to make mathematical errors, I have a tendency to skip over things, be jumping really fast. That's one of the things in math I have to work at, is to slow down and do things methodically and precisely, it doesn't come naturally to me. So as I'm doing that, if he's moving on to the next part of the problem, then by the time I get back to him, there's a disjoint. And that grows, the space between me and that lecture grows over time until, by the end of the class, I'm still making sure I've computed things correctly, and then I'm moving onto the next step. I need to make sure that, step-by-step, I'm doing everything the way it needs to be done and that I understand at each step. If I can't stop it, that's really stressful [laughs] for me. (F3.I1.L240–255)

The more anxious I am the less I'm able to actually see what's on the paper in front of me. Sometime if I get really worked, sometimes if I'm really worked up already when I sit down to do a problem, I could make the dumbest mistake at the very beginning and just not be able to see it. I mean, like this omit, omitting a number or, um, you know. (F3.I1.L389–393)

Ellen's assumption here is that mathematics is sequential where one mistake early in the solution could have an adverse effect toward the final numerical solution. While I determine this theme is related to detail orientation in a learner, this could be argued as a pre-theme for the next theme on control and pacing. Ellen's experience does support the notion that the lack of control and pacing in learning mathematics would lead to unsatisfactory results in the learner's perception.

F3's Individual Theme (B)—The control and pacing in learning mathematics. Ellen expressed more about how the pacing of instruction may have affected her learning. Two contrasting pieces in the interviews showed a dramatic difference in Ellen's perception of success:

When discussion her learning in an accelerated program:

I really thought that my math anxiety was very much related to my early experiences in math, um, but, and a certain amount of pressure that I felt for a couple of reasons. First of all because it was a very accelerated pace at the school I went to, but I had nothing to compare it to so it wasn't accelerated it was just normal for me and, um, but my parents, we did go to India when I was in kindergarten and I remember, um, coming back from India without having done any of my work and I remember, I don't remember being behind or having difficulty. I remember being able to do the work they were asking me to do, but I remember standing with my workbook, my math workbook, like this… and like all the other kids had theirs completely filled out and mine was totally empty. (F3.I2.L478–488)

When asked about Ellen's GRE learning experience:

I guess it is, but there's something different about it … because right now I'm studying for the GRE and I decided to do the [The publisher's name of test preparation material has been removed] course, and I'm doing just the advanced math tutorial, and uh, it's very interesting because, even though, you know, I can't ask questions it's going pretty well. And maybe it's, maybe it's more about pacing, because I can stop the video if I need to, I can rewind it. (F3.I1.L231–238)

The contrast was clear between how the accelerated program set the pace for Ellen's learning versus Ellen herself set the pacing in the GRE course. It was not surprising that when Ellen had control over of her own learning, she was satisfied with the results. This theme resonates with Anne's experience in accounting when she could not learn from the professor who rushed through the materials toward the end of the lectures. In sum, this theme captures two important ideas: how the pacing of instructional delivery can significantly affect learner's outlook toward their own learning and how learner's control in his/her learning can affect the feeling of the learner's ownership in the content materials.

F3's Individual Theme (C)—Conceptual versus procedural. The previous two themes highlight Ellen's perception of mathematics, leading to her distinction between concept and procedure in her general learning of the subject. After years of experience, she finally decided that there exists a duality of mathematics:

I would consider math, [pause] I think it a combination of concepts and procedural knowledge, but, and then to go along with that it's a, it's a conscious combination of concepts and procedural knowledge so, in other words you have to use, you have to

> know how to use procedural knowledge to address concepts, and concepts to address procedural knowledge. (F3.I1.L339–344)

Because of this duality of mathematics, Ellen discovered that her mistakes can be classified by those categories. This was a breakthrough for Ellen because she then developed a self-regulating mechanism on how well she executed solutions to mathematical problems:

> That's part of the problem for me is that sometimes I'm making a procedural error, and sometimes there is a conceptual error, and sometimes I can't tell the difference. (F3.I1.L291–293)
>
> It was only after I was able to sort of, with maturity I didn't have then, able to settle down and say "wait a minute [pause] this is not a conceptual error," there has to be, I mean, I went into the video it said, you know, twelve things taken seven at a time, okay, I'm using the right combinatoric formula, you know, I've got it memorized, I've got it right. So like you know what, the only thing left is an arithmetical error. There is something wrong with my numbers. So then I was able to go back and figure that out. (F3.I1.L306–312)

Because of Ellen's perception of mathematics, this leads to her worry for her son who did not share the same perception.

F3's Individual Theme (D)—The worry of not achieving perfection. As Ellen described her son's expectation on mathematics work, his view was binary that it was either perfectly done or not:

> He has less trouble with the actual math work than he does with overcoming his anxiety about sitting down to do it. I also noticed how much like me he is in that he is very anxious until he is engaged in the problem solving. Once he is working, he is actually quite happy. He answered all the questions correctly without help, so I don't think he lacks skills. I wondered as I watched him what being good at math means to him. I also noticed that how he feels about math is more problematic than how he actually does. I don't think he has any sense of the degrees of success. There is only perfect and not perfect. (F3.J2.L40–47)

> It has more to do with, judged by his own standard which we can't really figure out where he gets this idea. It's sort of like if he even makes one mistake that's awful. There's no degree for him. He either, it was perfect or it wasn't perfect, and not perfect is not acceptable. (F3.I2.L25–28)

> I: In one interaction you had with him you actually, after some coaching, you actually told him he wasn't bad at math, and then he said "yes I am [but] I didn't get a star".
>
> F3: Right. So perfect equals good, and everything else equals bad. (F3.I2.L105–107)

It is believable that Victor's goal was to obtain a star for his mathematics homework. Because he set such a high bar of perfection to achieve, he was often disappointed, and Ellen believed that this could be a potential source for Victor's mathematics anxiety. Ellen expressed how the binary distinction of success was not adequate:

> But there's no sense that like, a paper with all the computation done correctly could still be good work even if a seven is written backwards, like one seven written backwards is equivalent to him, at this point he reacts the same way to that as he would to all the answers being wrong, like everyone. He just has no sense that, like, this could still be good work even if you missed one or two [and] that could still be good. That's not good for him, that's terrible, and I think that's where the anxiety comes from. (F3.I2.L115–121)

Indeed, Victor's view was not all that different from how Gerri's granddaughter's behavior on conformity. If a problem was not solved in a prescribed procedure, then the solution would not be considered correct. This theme is quite revealing also in explaining how many students developed the notion of and be conditioned to achieve perfection in learning mathematics.

Thematic Cascade of Ellen's Individual Themes. In terms of the perspectives for each of the Ellen's individual themes, Individual Theme F3(A) about Ellen's difficulty is an individual first person's reflection, and thus it is considered to be an individual-interior (UL) perspective. The second Individual Theme F3(B) on control and pacing is a reflection of the cultural reflection, and therefore it is a collective-interior (LL) perspective. On Ellen's “after the fact” personal perception of the distinction between conceptual and procedural mathematics, this third Individual Theme F3(C) is classified as an individual-exterior (LR) perspective. Finally, when Ellen described her observation on her son’s learning in Individual Theme F3(D), this is a third-person observation outside of her son’s collective learning environment between him and his teacher. Therefore, it is classified as a collective-exterior (LL) perspective.

	Interior	**Exterior**
Individual	**F3(A)** Difficulty with college mathematics	**F3(C)** Conceptual versus procedural
Collective	**F3(B)** The control and pacing in learning mathematics	**F3(D)** The worry of not achieving perfection

Figure 4.11. A cascade of F3 Ellen’s individual themes into the quadrivium.

“Sue” Female #4

Sue was a nurse who had been out of work because of an injury, and she decided to return to school to further her education. In the degree program she was in, she had to take dosage mathematics as well as statistics. She was very happy and proud to receive “A+” grades on both courses. While it was a positive experience for Sue to receive training as a nurse, that had not been the case in her past. Sue recounted her high school days when she failed algebra: “I feel in algebra I was just completely in the dark” (F4.I1.L148). Sue described the teacher as “cut and dry” and not helpful. That was not how she could learn algebra. Since then, the school had moved her into business mathematics, a course in which she did well. When asked what the differences were between algebra and the rest of her successful learning experiences, Sue claimed that both business and dosage mathematics required plenty of memorization on addition and subtraction. Apparently, Sue excelled doing mathematics through memorization, and the same trick, Sue admitted, was not so applicable in algebra. She attributed the success to the fact that the course was based on problems that could be solved through procedures that could be prescribed in a step-by-step manner. Meanwhile, the previous algebra course's problems were more equation-like, and that overwhelmed her with a great deal of anxiety. In fact, Sue lamented that she was unable to receive a New York State Regents High School Diploma, a more prestigious graduation diploma for public schools than the regular "local" diploma she ended up receiving. To the day of Sue's participation in this study, the failure of receiving a prestigious Regents diploma was a disappointment to her. If Sue had the chance to go back to high school now, she would have had a much more mature attitude to tackle algebra, and she always wished to have obtained the more prestigious diploma.

We discussed how she felt when she was anxious. Sue compared her mathematics anxiety to when she first started working as a nurse on a job:

> …[W]ell, when I started working as a nurse, because, you know, you're responsible for people's lives, so, you know, it was… just being new, you have to, you had to learn a lot of different things, and being in the real world is a little bit different than just reading out of a book, so it took a little while to to [*sic*] relax into it a little bit because you really can never fully relax being a nurse because you're responsible for people's lives. So, you know, that I guess something like that… (F4.I1.L280–285)

On asking how long it took her to begin to relax, Sue was me that there was no *total* relaxation when human lives were involved, but I was told that it took her over a year to be comfortable in her nursing role. As she recalled it, "you learned your job," so, I followed through by asking what mathematics she had to use on the job. Sue thought that those were all easy mathematics, such as checking and double checking the dosage of medications for her patients.

In one of Sue's journal entries, she elaborated about how she and her teenage son interacted. Sue asserted that her 15-year-old son was smart, naturally gifted, and excelled in mathematics. In fact, Sue felt embarrassed when she sought help from him when she was working on the statistics coursework. I pointed out that it was unlikely that her son would have had the statistics materials memorized, and I wondered how he knew what to do and how to help. Sue thought her son was born that way, that he was talented with numbers, and she did not have the same blessing. As a matter of fact, Sue shared some of her son’s mathematics homework problems as examples of the mathematics that would have made her anxious:

Example (1):

$$\frac{1}{2} \cdot 2^2 + 3 \cdot 2 + 6$$

Example (2):

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

Figure 4.12. Sue’s 15 year-old son’s mathematics homework problem that made her anxious.

Despite the wounded pride, Sue enjoyed the success she had in statistics, and in fact, she even offered help to her classmates, which she described as a pleasant experience to have felt for once “not so lost.” Overall, Sue was still anxious toward mathematics despite her positive experiences, which could never “negate” her past negative experiences. In essence, Sue claimed that she would continue to be anxious when she was faced with new, unfamiliar mathematics materials.

F4’s Individual Theme (A)—Memorization is the key to understanding mathematics. Perhaps the most prominent and recurring theme from Sue is how she described that learning mathematics requires a significant portion of memorization:

I: Oh, there is algebra in dosage math?

F4: Yeah, [pause] but you know you [pause] if [pause] you have a teacher, they teach you a certain way, and you basically memorize the way to do it. And then, it it’s [*sic*] [pause] it’s confusing, but it’s doable. (F4.I1.L119–122)

The interpretation here for Sue is that she perceived learning mathematics as a form of prescribed procedures, expecting a teacher to show step-by-step processes, and she followed these steps. The way she describes how she understood mathematics is consistent to this behavioral manner of working on mathematics problems, and to aim for correct numerical solutions:

I: What kind of ability in you that you felt that, um, that you had that made you successful in some of the math you have taken, especially the one that you got an "A plus" in it?

F4: Well memorization, I I [*sic*] have very good memorization skills, and um, understanding basic concepts um, you know I can read something and understanding it very well. (F4.I1.L258–263)

> With the help of my son, classmates and teachers I have realized that math problems, if solved step-by-step, are very simplified. (F4.J3.L47–48)

Sue believed the memorization strategy was the key to understanding mathematics as she juxtaposed the two ideas in one single sentence. Sue also described in detail her inability to "understand" mathematics when:

> … [I]f you put a whole bunch of symbols and formulas in front of me, I just don't get it, I just don't understand it. (F4.I1.L52–54)

> The one thing I have to admit, if it got complicated (math equations) my anxiety would be sky high… I wish I could understand certain problems better. … I feel very unsure about any algebraic coursework. Formulas seem to be my problem. (F4.J2.L26–27 & L32–34)

I: And if you were to follow through with that [the strategy of memorization], did you still feel that you were in the dark?

F4: No, because I was getting the right answers. (F4.I1.L164–167)

In other words, Sue's main goal in doing mathematics is to obtain right answers, and the bunch of symbols and/or formulas were the barrier to achieve her goal because they were not conducive to her strategy of memorization. Sue showed those kind of "bunches" in the previously illustrated examples (1) and (2). The metaphor of "in the dark" will be interpreted in the next individual theme.

F4's Individual Theme (B)—"In the dark" versus "Black and white". A very interesting metaphor that Sue used for several times was when she described her feeling of confusion as being "in the dark." When she did understand mathematics, she felt that the comprehension was "black and white":

> I think the business teacher give me more one-on-one time than the math [algebra] teacher did because he was very cut and dry. He just gave you the problems, he may have given you a simple explanation of something that kind of left you still in the dark, [laughs] and that was it, that was all the help you were getting from him. (F4.I1.L387–392)

And later, Sue added:

> Her [the business math teacher] teaching, the way she taught, it was just very easy for me to follow along; everything was just in black and white and she just had it, you know, in a certain way and you learned it. She asked if you had any questions, you asked, you were corrected if you were wrong, and you know it just kept flowing, it flowed very well. (F4.I1.L422–427)

The metaphoric distinction further explains how Sue had a strategy to follow prescribed steps which these steps can "flow" in a linear manner, and she was able to follow through to achieve her goal of obtaining right answers. However, if confronted by a bunch of symbols like an equation or a formula, then Sue found it challenging to work with, and she would feel that she was in the dark. This contrasts with Gerri who preferred the opposite. Gerri preferred to play a role of an "understander" for solving mathematics problems as opposed to a role of a memorizer for prescribed steps. Sue somehow could not put herself into the same role as Gerri in learning mathematics. In her eyes, prescribed steps were the key to help her perceive the mathematical materials as "black and white."

F4's Individual Theme (C)—The mystery of having talent in mathematics. Perhaps one of the most interesting topics of discussion in Sue's story is how her teenage son managed to help with her college mathematics homework. Sue's son, as she described, exceled in mathematics in high school, and the skills came naturally to him. By the time we were on this topic, Sue already mentioned multiple times how she relied on memorization of prescribed steps as a way to understand mathematics. So I asked if her son relies on the same strategy:

> I: But it's so interesting that you pointed out that he [the teenage son] never had a college level statistics course, and let me try to put the two and two

together. So, you think memorization has to be a big part in math, but he had not memorized anything [from your] statistics [course], so where do you think this automatic ability comes from?

F4: I really don't know. [laughs] I, you know I I [*sic*] do, it seems the way I partake certain people when they're doing math, it just seems like they just are gifted in some way. That's the way I interpreted it, anyway, you know. And some people when they do their, um, you know they do working out formulas and everything like that, they just know what to do. [laughs] I don't know … I know my father was was [*sic*] very gifted in math, you know, so I, I [*sic*] don't know if it follows along the line, you know, genetically, but I mean, it wasn't from me that my son got this math talent, that's for sure. (F4.I2.L119–127 & L223–225)

For Sue to bring up her genetic line reveals her belief that having talent in mathematics could be a born ability; perhaps that her son did well in mathematics can be traced back to his grandfather. This is consistent to Schoenfeld's (1989) and Tang's (2007) assertions that many Americans felt that their abilities to learn mathematics were induced through nature, not nurture. This also begs the question that Sue might feel that understanding mathematics through memorization was a strategy for her to succeed in school mathematics, but she would not consider *understanding* mathematics to be the same as *knowing* mathematics. This is to say that Sue conceptualized that there were different levels of learning mathematics: (1) Not understanding mathematics when there are a bunch of symbols in equations and formulas, (2) understanding mathematics when Sue can follow prescribed steps, and (3) knowing mathematics when her son could make sense of the bunch of symbols in equations and formulas and transform them into workable steps.

Thematic Cascade of Sue's Individual Themes. As for the perspectives for each of the Sue's individual themes, Individual Theme F4(A) on memorization is an individual first person's reflection, and thus it is considered to be an individual-interior (UL) perspective. The second Individual Theme F4(B) on distinguishing a duality in mathematics is an "after the fact" reflection, and therefore, it is an individual-exterior (UR) perspective. Finally, when Sue described her observation of her son's mysterious talent in mathematics in

Individual Theme F4(C), this is a third-person observation outside of her son's learning. Therefore, it is classified as a collective-exterior (LL) perspective.

	Interior	Exterior
Individual	**F4(A)** Memorization is the key to understanding mathematics	**F4(B)** "In the dark" versus "black and white"
Collective		**F4(C)** The mystery of having talent in mathematics

Figure 4.13. A cascade of F4 Sue's individual themes into the quadrivium.

Conclusion

The goal of this chapter was to provide synopses of the lived experience of the six participants and their contribution to this research study. The individual themes were developed from each participant's own words and utterances. As seen in the individual cascades, the 19 individual themes span across all four quadrants of Wilber's Integral Model. This is significant because the findings achieved the intended outcomes of the theoretical framework chosen at the proposal stage of the dissertation. The integral cascade among all six participants are illustrated below both in visual and in table formats:

	Interior	Exterior
Individual	M1(A)—Adapt M2(A)—Jon's Perception F1(A)—Memorization F2(A)—Memorization F3(A)—Difficulty in Math F4(A)—Memorization	M1(B)—Duality F3(C)—Duality F4(B)—Duality
Collective	M2(B)—Roadblocks F1(B)—Conformity F2(B)—Application F3(B)—Control/Pacing	M1(C)—Support M2(C)—Engagement F1(C)—Keen Observer F2(C)—Pacing/Control F3(D)—Worry/Perfection F4(C)—Talent

Figure 4.14. A cascade of all 19 individual themes into the quadrivium.

Table 4.15

A Perspectival Classification of the Individual Themes

Participant	Individual Theme	Perspectival Classification		
M1 "Carl"	M1(A)—A quest to adapt in mathematics	Individual	Interior	UL
	M1(B)—"New math" versus "normal math"	Individual	Exterior	UR
	M1(C)—Lack of resources for parental support	Collective	Exterior	LR
M2 "Jon"	M2(A)—Jon's perception on the subject of mathematics	Individual	Interior	UL
	M2(B)—Roadblocks of learning mathematics	Collective	Interior	LL
	M2(C)—Expectations in engagement	Collective	Exterior	LR

Table 4.15 (Continued)

A Perspectival Classification of the Individual Themes

Participant	Individual Theme	Perspectival Classification		
F1 "Gerri"	F1(A)—The reliance of memorization causes anxiety	Individual	Interior	UL
	F1(B)—A perception of conformity in learning mathematics	Collective	Interior	LL
	F1(C)—A keen observer in mathematics instruction	Collective	Exterior	LR
F2 "Anne"	F2(A)—Patience, practice, and memorization	Individual	Interior	UL
	F2(B)—Lack of transparent application	Collective	Interior	LL
	F2(C)—Lack of pacing and control, so pretend to learn	Collective	Exterior	LR
F3 "Ellen"	F3(A)—Difficulty with college mathematics	Individual	Interior	UL
	F3(B)—The control and pacing in learning mathematics	Collective	Interior	LL
	F3(C)—Conceptual versus procedural	Individual	Exterior	UR
	F3(D)—The worry of not achieving perfection	Collective	Exterior	LR
F4 "Sue"	F4(A)—Memorization is the key to understanding mathematics	Individual	Interior	UL
	F4(B)—"In the dark" versus "black and white"	Individual	Exterior	UR
	F4(C)—The mystery of having talent in mathematics	Collective	Exterior	LR

As seen in both illustrations above, the lived experience data, with their essence shown in the individual themes, are scattered in all four quadrants, making the research study possible for an integral disclosure. Two premises of Wilber's Integral Model are important in the discussion here: (1) every perspective discloses a unique window to a phenomenon, and (2) a partial collection of perspectives would compromise the integrality. Henceforth, the lived experience data, as well as their essence in the 19 individual themes, all need to be verified for integrality. In this case, I argued that the study achieved the underlying epistemology of Wilber's Integral Model.

In the next chapter, I will draw some similarities and differences among these individual themes, and I will also highlight disclosures from the lived experience data that have not been discussed in the surveyed literatures from Chapter 2. These disclosures, along with the 19 individual themes, will be used for the development of the major themes for MALP in the next chapter. Also, after major themes are developed, I will revisit the definition of MALP.

Chapter Five: Major Themes for MALP and their Relations to its Definition

In the previous chapter, 19 individual themes for all six participants were developed and documented through stories that captured the essence and richness of their lived experience. The goal of this second of the three analysis chapters is to develop overall major themes of the Mathematics Anxiety Learning Phenomenon (MALP). This chapter is organized in as follows: first, I will draw some similarities and uniqueness from the six participants' lived experience, and second, I will summarize the individual themes from the previous chapter and classify them by how they reflect the overall major themes for MALP. Third, I will present the major themes for MALP, revealing a coherent and integral disclosure of the phenomenon. In particular, I will revisit the three components of MALP, recalled in the following:

[A] Before the learning process takes place, adult learners have sets of beliefs about the subject of mathematics, about their own abilities, and about certain attitudes toward learning.

[B] The individual and social behaviors from **[A]** affect how learning takes place—i.e. learn what mathematical knowledge to memorize, learning how to survive school mathematics, and informing fellow learners of one's past experience.

[C] The individual and social behaviors from **[B]** serve as perceptions to reinforce or change the beliefs in **[A]**.

[A], **[B]**, and **[C]** altogether: How could the cycling learning phenomenon that is perpetuated in **[A]**, **[B]**, and **[C]** be sufficiently addressed (and perhaps be broken) in classroom instruction to optimize learning?

The cyclical nature of **[A]**, **[B]**, and **[C]** is defined as MALP in developmental mathematics.

The goal of this chapter is to use the collected lived experience data and relate them to the definition of MALP, and provide informed details to each of the components. Because the major themes were developed through my interpretation of the lived experience data, these major themes essentially present a 3rd person's perspective on MALP, making the results as a disclosure in the individual-exterior (UR) and collective-exterior (LR) within the integral theory's quadrants.

Some Similarities and Uniqueness of the Lived Experience

In this section, I will outline and describe some similarities and differences among the six participants' lived experience. The purpose of this section is to anchor an overall observation to their experiences as an exploration of possible materials for the development of the major themes. Furthermore, it serves as an intermediate phase, transitioning from each participant's synopsis to the eventual major themes for MALP. It is obvious that all six participants experienced mathematics anxiety, which can be confirmed by both their aMARS scores (UR) and their own stories (UL). By the research design and the intended sampling, all participants' college studies were not related to science or mathematics. They all described their firsthand experience through specific events. In their own unique ways, they all experienced traumatic events either directly from their learning experiences or through metaphors to their mathematics learning:

Table 5.1

Traumatic Events Experienced by the Six Participants and One Child

Participant	Traumatic Event	Type
M1 "Carl"	Experiencing the fear of snake	Metaphoric
M2 "Jon"	Teacher got angry and belittling	Direct
F1 "Gerri"	Berated by a professor by asking questions	Direct
F2 "Anne"	Study group as "being fed to sharks"	Metaphoric
F3 "Ellen"	Could not keep up with a professor	Direct
"Victor"	Did not get a star on his homework assignment	Direct
F4 "Sue"	Anxious like saving patients in a life or death situation	Metaphoric

All participants expressed some kind of regret or "I should have done…" This is unexpected as none of the surveyed literatures from Chapter 2 of the dissertation reported the feeling of guilt or regret. In the next chapter, this original finding could be used to determine implications that fosters mathematics anxious-friendly learning environment. Briefly described in the following table is each participant's regret:

Table 5.2

Regret or Disappointment Experienced by the Six Participants

Participant	Regret or Disappointment
M1 "Carl"	Wished to have the ability to understand "new math" so that his daughters would have an easier time.
M2 "Jon"	Wished to have taken more responsibility for himself, instead of blaming the school system for his shortcoming in the subject.
F1 "Gerri"	Wished the textbooks were better written.
F2 "Anne"	Wished that she could have asked better questions in class, as well as have spoken up more in group work, not letting her pride got in the way of learning.
F3 "Ellen"	Wished to have behaved more like other Asian-Indians in high school who were visibly studious.
F4 "Sue"	Wished to be more mature back in high school, and have a different attitude in studying mathematics so that she could pass the algebra course and received the Regents high school diploma.

In addition to the feeling of regret that came with mathematics anxiety, another important element that stood out from the lived experience among all six participants (plus one child) was how they perceive mathematics in a dual manner. While the justification of this area will come later in the chapter, the following is a tabulation of their perception of duality in mathematics:

Table 5.3

A Perception of Duality in Mathematical Learning

Participant	Duality	Comments
M1 "Carl"	Normal Math vs. New Math	Carl could make sense of "normal math," but he found "new math" meaningless to him.
M2 "Jon"	Understandable vs. Impossible to Understand	Jon claimed that he would have done well if the mathematics presented was understandable, delivered in an engaging and entertaining manner.
F1 "Gerri"	Reasonable vs. Memorizing	Gerri admitted that she was not a good memorizer. She needed to learn mathematics through reasoning, and not through rote memorization.
F2 "Anne"	Applicable vs. Not Applicable	Much of Anne's experience toward mathematics to be not applicable, and she considered statistics that was applicable to business and other fields as a "different" category.
F3 "Ellen"	Conceptual vs. Procedural	Ellen found that the key to solve problems was to find reasoning in the concept. The procedure was largely the technical work that could be tricky and error-prone.
"Victor"	Perfect vs. Not Perfect	Victor believed that doing well in mathematics meant that he had to do the work perfectly. Otherwise, all effort was futile.
F4 "Sue"	Memorizable vs. Not Memorizable	Sue believed that mathematics could be manageable for her if she could memorize how to do them. She did not understand how other individuals (like her son) could solve problems without memorization.

Excluding Victor's perception, it seems that there are two main types of expectations about mathematics learning for the participants. One type is the expectation that the act of learning mathematics should be focused on memorization and prescribed steps (Carl, Jon, and Sue). The second type of expectation is that reasoning and generalization should be applied to the problem solving that Anne, Gerri, and Ellen seemed to prefer. Because the expectations of these two types of learners varied in a fairly dichotomizing manner, they will have profoundly different implications for classroom instruction as well as theoretical consequences. All the above are brief perspectives that disclose the individual-interior (UL) views of MALP.

As for the collective-interior's (LL) perspective, all participants described that teacher-student engagement/interaction as well as patience and good explanations/skills and tricks were essential for success in learning mathematics. However, their expectations toward engagement/interaction were not all the same. One participant, Jon, insisted that entertainment was an important element in instruction, but none of the rest of the participants expected to be entertained during learning. During interactions, three participants (Jon, Gerri, and Ellen) found a worrisome feeling when working with another individual on mathematical work. In particular, Jon worried about not being entertaining and engaging enough; Gerri worried about the development of richness and depth in reasoning and thinking in the confinement of limited time and resources; and Ellen worried that the grading system could be a culprit that caused mathematics anxiety for her son. Two participants (Anne and Sue) described their collective experiences as learners. Sue reported that it was embarrassing to have her son help her with her mathematics homework, and Anne discussed how she often pretended to understand but wished she had asked more questions. One participant (Carl) attempted to help his daughters with their homework, but he could not make much sense of it, and his daughters could not take him seriously.

Another interesting contrast to be pointed out is the perception of parental support/resources from school from two of the participants. Both Carl and Ellen thought that their children were going to excellent school districts, but Carl's district did not share the same mathematics resources as Ellen's for parents to play a role in their children's school mathematics work. This was apparent when Ellen could articulate and help Victor on the “Math Mountains” while Carl drew a web of senseless numbers connected by arrows. While the difference in grade levels (Victor in grade 1 and Carl’s daughters in middle and high school grades) could explain Carl’s senseless rendition of mathematics work, their contrasting accounts both had supporting details in their interviews. Hence, this contrast seemed to be credible and trustworthy. In the same category as resources is the learners’ opinions on how mathematics was communicated to them. Gerri noted a salient idea that mathematics texts should focus on problem solving skills by targeting reasoning instead of prescribed steps. On the other hand, Jon thought that mathematics texts should not be written in mathematician’s language, and Sue thought that big lumps of algebraic symbols, formulas, and equations, presented as single objects, looked particularly intimidating to her because she could not “break them down” into manageable steps.

In every episode of the social situations among the participants, there existed a necessity of purpose as an expectation on the interaction. In each case, the interaction was met with an unexpected element. The following table outlines interactions of the six participants, their purposes, and surprises:

Table 5.4

An Inventory of Purposes, Unexpected Elements, and Corresponding Results among the Six Participants' Learning Interactions

Participant	Learner(s) OR Audience	Purpose	Unexpected Element	Result
M1 "Carl"	Carl's three daughters	To make sense of "new math"	"New math" seemed impossible to make sense of	Negative to Carl
M2 "Jon"	Inner city school's teenagers	Be engaging and entertaining	Failed to do so	Negative to all
	Jon	To ask for clarification	Teacher got upset and flipped a desk	Negative to Jon
F1 "Gerri"	High school students	To develop deep conceptual understanding	Students resisted, claimed that memorization was good enough	Negative to all
	Gerri	To ask for clarification	Resulting in being chastised	Negative to Gerri
F2 "Anne"	Anne	To sit quietly, to go with the flow, to not be embarrassed	Felt like "being fed like sharks"	Negative to Anne
	Anne	*Reluctant to be in "Mommy School"*	Engaging, enjoyable, and edible	Positive to Anne

Table 5.4 (Continued)

An Inventory of Purposes, Unexpected Elements, and Corresponding Results among the Six Participants' Learning Interactions

Participant	Learner(s) OR Audience	Purpose	Unexpected Element	Result
F3 "Ellen"	Ellen	To study mathematics with the cohort of Asian-Indian classmates	While Ellen studied math (B or B- in the course), the group was very competitive, making Ellen felt like an underdog all the time.	Mixed to Ellen
	Victor	To obtain correct answers (for Ellen)	Victor already knew how to obtain correct answers, but he was too focused to get a star instead	Mixed to Ellen; Negative to Victor
F4 "Sue"	Sue	To correctly solve the statistics homework problems	Felt embarrassed afterward because she was asking for her son's help	Mixed to Sue
	A classmate	A classmate wanted help on statistics	Sue felt great despite of limited math ability	Positive to Sue

In Jon's case, he aimed to be engaged and entertained by his instructor, but that did not happen. Gerri aimed to develop a deep understanding of the mathematical concepts, but she met with resistance from some of her students. Anne aimed to be quiet in a group situation so that she would not be embarrassed, saving her pride in a short run, but hurting her learning in a long run. Ellen wanted to help her son to understand his own capability to do mathematics and to take ownership of his ability, but she found that Victor was only categorizing his ability in a binary "perfect" and "not perfect" manner. Moreover, Sue got what she needed in statistics, but she felt embarrassed to ask her son for the help it took. Aside from Anne's enjoyable and edible experience at "Mommy School", these expectations were generally met with negative experiences; positive experiences were far and few in between. This particular collective-interior (LL) finding is interesting because it provides a basis for how the social interaction component of MALP (recalling component **[B]** in the definition) could fuel negativity among participants, making MALP a cyclical phenomenon. As one will see by the end of this chapter, the development of the major themes for MALP will play an important role in addressing component **[B]** in the definition.

The Development of the Major Themes for MALP

As outlined in the previous chapter, individual themes were developed for each participant. Collectively, there are plenty of overlaps among these themes. These overlaps are paramount to connecting the themes to several reflective elements that will provide essence for the development of the major themes.

Table 5.5

A Summary for Individual Themes and Their Essence to Various Reflective Elements

Participant	Individual Theme	Reflective Element
M1 "Carl"	M1(A)—A quest to adapt in mathematics	Control
	M1(B)—"New math" versus "normal math"	Duality
	M1(C)—Lack of resources for parental support	Roadblock

Table 5.5 (Continued)

A Summary for Individual Themes and Their Essence to Various Reflective Elements

Participant	Individual Theme	Reflective Element
M2 "Jon"	M2(A)—Jon's perception on the subject of mathematics	Duality
	M2(B)—Roadblocks to learning mathematics	Roadblock
	M2(C)—Expectations in engagement	Belief
F1 "Gerri"	F1(A)—The reliance of memorization causes anxiety	Memory
	F1(B)—A perception of conformity in learning mathematics	Belief
	F1(C)—A keen observer in mathematics instruction	Belief
F2 "Anne"	F2(A)—Patience, Practice, and memorization	Memory
	F2(B)—Lack of transparent application	Belief
	F2(C)—Lack of pacing and control, so pretend to learn	Control
F3 "Ellen"	F3(A)—Difficulty with college mathematics	Roadblock
	F3(B)—The control and pacing in learning mathematics	Control
	F3(C)—Conceptual versus procedural	Duality
	F3(D)—The worry of not achieving perfection	Belief
F4 "Sue"	F4(A)—Memorization is the key to understanding mathematics	Memory
	F4(B)—"In the dark" versus "black and white"	Duality
	F4(C) —The mystery of having talent in mathematics	Belief

The essence of each of the 19 individual themes is reflected in five different elements among the participants. With their analysis, the major themes for MALP are presented below.

Major Theme (1A)—The Learner's Beliefs

F1(B)—A perception of conformity in learning mathematics
F3(D)—The worry of not achieving perfection
F4(C)—The mystery of having talent in mathematics

A mathematics-anxious learner often plays a behavioral role in learning school mathematics by relying on a set of prescribed steps to be mimicked. Conditioned to conform to the steps as opposed to deviate from the prescription for an alternative or better problem solving experience, the learner comes to believe that this is how mathematics should be done. When confronted with deviating from the prescribed steps, anxiousness may brew in the learner because the prescribed steps would no longer be of use to him/her, and the trigger may originate from past disappointments and traumas. Meanwhile, requiring a learner to navigate and reflect his/her own problem solving process without prescribed steps is an anxiety-inflicting event, and there are two dynamics involved. First, the learner lacks security to devise his/her own steps in a problem solving strategy, and second, instructors often do not encourage or cultivate ingenuity to solve mathematical problems in a creative manner. When an anxious learner observes another talented learner deviating from prescribed steps and managing to solve problems, the success is often attributed to born talent or nature's gift that no amount of nurturing could make up for.

As the title of this major theme suggests, the learner's belief is considered to be an individual-interior (UR) element because it is situated and often developed within an individual. However, the actual beliefs are negotiated through the learner's experience, such as pretending to conform in learning mathematics, or realizing that achieving perfection is the learning expectation. Therefore, despite the UR nature of this major theme, there are aspects of it that are related to the collective (LL and LR) perspectives.

Major Theme (1B)—Cultural Beliefs

F1(C)—A keen observer in mathematics instruction
M2(C)—Expectations in engagement
F2(B)—Lack of transparent application

This theme is further developed when one examines how mathematics instruction is delivered. Many mathematics instructors may deliver instruction through the manner of prescribed steps because they could have the same beliefs as the learners in Major Theme (1A). While some prefer this behavioral learning strategy of mimicking steps, others may find the strategy frustrating. Oftentimes, the instruction that relies on such a strategy presumes one single way to do mathematics, therefore frustrating the group of learners who wants to develop skills beyond following prescribed steps. If a learner begins to question a particular method and/or its application, many instructors would find it challenging to provide a satisfactory response for the learner, leaving him/her a sense of mystery about doing mathematics in a particular fashion. Moreover, this mystery could become yet another disappointment when the expectation is not fulfilled. Because of the sense of insecurity among individual learners, many of them expect some sort of individualized engagement. They believe that the remediation and re-assurance of the execution of prescribed steps are appropriate during such engagements. On the contrary, engagements targeting beyond the prescribed mathematical steps could be construed by some learners as deliberately obtuse for no meaningful reason. This kind of remediation would push them out of their "comfort zone" and further reinforce their insecure and anxious feeling further. Perhaps this troubling emotion they feel leads them back to their former traumas and regrets with learning mathematics.

Major Theme (2)—Roadblocks to Learning Mathematics

M1(C)—Lack of resources for parental support
M2(B)—Roadblocks to learning mathematics

The participants in the study pointed out several roadblocks that prevent learners from thriving through a litany of issues in the school learning environment:

- Lack of individual engagement and one-on-one tutoring
- Incomprehensible mathematics texts
- Lack of resources to support doing homework
- Frustrated and angry mathematics instructors
- Excessively fast pacing in instructional delivery
- The belittling of asking not-so-clever questions
- Lack of reasoning in word problems, equations, and symbolisms
- Lack of opportunity for one to take control over his/her own learning (this will be addressed as a separate major theme)
- Lack of transparency on the application value of mathematical concepts

While the above may be seen as a "dirty laundry" list of blaming the educational system for one's failure in learning mathematics, the collective of these roadblocks paint a despairing reality about how anxious learners perceive the social learning environment. Many of them felt that asking questions in class would be a futile exercise because they may not receive answers that could be meaningful to their own learning experience. Other learners may feel a disconnection between the classroom experience and the homework experience. When they attempt to reconcile the two, either there are little resources available, or the resources are not written in a way that is accessible. This leaves these learners very few alternatives to play an active role (see Major Theme (5)) in learning mathematics. Also, a frustrated or angry instructor may become the learners' roadblock because the instructor could become unapproachable, or be seen by adult learners as uncooperative. All of these became a horrendous concoction of social norms that are not conducive to learning. Simply put, the learners enter the school environment with their expectations of themselves, the learning experience, and how to interact with other individuals within the subject discourse. However, often the expectations are not met, and without sufficient control in one's learning, an anxious adult is unable to find ways to meet those expectations independently. Henceforth, this becomes the roadblock for learning mathematics. The results of these roadblocks, found in the cultural contexts of social learning, may yet root themselves in the learner's psyche as another disappointment and trauma.

Major Theme (3A)—Duality as a Way to Perceive Mathematics

M1(B)—"New math" versus "normal math"
M2(A)—Jon's perception on the subject of mathematics
F3(C)—Conceptual versus procedural
F4(B)—"In the dark" versus "black and white"

When an anxious learner arrives at the mathematics classroom with their beliefs and then experiences numerous roadblocks, how can they find a way to make sense of their learned mathematics? Certainly, if a learner continues to leave the anxious and incomprehensible portion of mathematics unreconciled, then it is unlikely they will be able to survive school mathematics for a prolonged period of time. In other words, a learner must find some resolution to the portion of mathematics that is inaccessible to them, and that could be the reason many learners develop a perception of mathematics as two independent entities, or as a duality. Perhaps the most prominent view is Sue's perception of mathematics that puts her "in the dark" (not comprehensible) and "black and white" (comprehensible and easy to memorize). However, the same notion can be subtle to the learning such as Ellen's perception of conceptual and procedural mathematics, a distinction she needed in order to gauge the errors in her own mathematical work. While duality is one way to perceive mathematics in the learner's eyes, the lived experience data also showed that one could engage in a meta-learning event by attempting to perceive one's own performance. This is the case of Victor who perceived his performance as either "perfect" or "not perfect." Even though he was just seven years old, Victor already developed a cognitive system savvy enough to perceive that the star sticker on his school work had an external motivation for him. While the purpose of feedback is to facilitate learning, the act actually had a negative impact on him in that he viewed the sticker to be as important, if not more important than, the learning event itself. In other words, the learning habits for school mathematics can be conditioned at a young age, and in turn become for the learner the foundation of how to perceive the subject matter and how to perceive doing school mathematics appropriately. Overall among the participants, it seems that their mathematics anxiety tends to set in when the mathematics discourse in a learning event is inaccessible to them. As one will see in the next chapter on the theoretical implications, this major theme on the perception of duality in mathematics will have significant implications to both Knowles' andragogy and Givvin et al.'s hypothetical model of learning mathematics.

Major Theme (3B)—Discrepancy of Expectations in Social Interaction

F1(B)—A perception of conformity in learning mathematics
F4(C)—The mystery of having talent in mathematics
F1(C)—A keen observer in mathematics instruction
M2(C)—Expectations in engagement

While I did not specifically peg down individual themes that exclusively contribute to this major theme in the previous table, essentially, it is above four themes that associate the belief component that forms this major theme. As a continuation from Major Theme (3A), this theme captures the essence that there are different expectations in teacher-learner and learner-learner interactions. In teacher-learner interaction, every participant described some form of dissatisfaction. For example, Carl was doodling in class because he was disengaged; Jon's teacher expected him to ask better questions; and Anne attributed her inability to learn due to the fact that her professor's pacing was too quick for her. In each case, there was some kind of discrepancy in the teacher's and learners' expectations. Simply put, the teachers instructed students with particular kinds of outcomes in mind, but adult learners expected instruction to be different because they had certain beliefs about how they could best learn mathematics.

Similarly, a discrepancy of expectations was also seen in the learner-learner interaction. Carl's embarrassing experience in helping his daughters and Gerri's frustrating moments with her granddaughter and with one of her students were examples showing that different expectations about the social interaction in learning mathematics led to frustrating experiences for both participants. This theme is of particular importance because I believe this discrepancy is a potential trigger for the anxious emotion among adult learners. One possible explanation for this major theme is that adult learners may view mathematics dualistically (see Major Theme (3A)), and this difference in perception leads to an ineffective communication among parties.

Major Theme (4)—The Necessity of Strong Rote Memorization Skills in Learning Mathematics

F1(A)—The reliance of memorization causes anxiety
F2(A)—Patience, practice, and memorization
F4(A)—Memorization is the key to understanding mathematics

Among the six participants, three of them explicitly described that strong rote memorization skills are necessary to learn mathematics. As described in Major Themes (1A) and (1B), the beliefs and expectations are to follow prescribed steps and to employ a behavioral learning strategy. Furthermore, because of the roadblocks outlined in Major Theme (2), the successful learning of mathematics (A.K.A. passing school mathematics courses) necessitates strong rote memorization in order to do well on tests and examinations. For those whose rote memorization comes naturally, they generally could pass school mathematics, course after course, semester after semester, year after year, as long as learning and instruction do not target beyond the behavioral strategy. For those whose rote memorization skills are not strong enough, they may adapt by having an unending amount of patience and repetitious practice to "survive" school mathematics.

There are two reasons why rote memorization is a necessity for the behavioral learning strategy. The first has to do with the nature of prescribed steps and procedures that the learners attempt to mimic without sufficient processing to explain how the steps are connected in a coherent manner. This makes memorizing each step in a procedure an independent process. The second has also to do with the reasoning that could have made connections among different procedures and concepts. As Gerri described, the instructional delivery for a mathematics course is characterized as "piecemeal," making each procedure and concept an independent entity for the learners. Without developing relational connections among mathematical concepts, the learners rely on rote memorization to survive the grading process of their mathematics courses. The lack of connection among prescribed steps, procedures, and concepts could also explain how the learners could forget them so easily once they have been assessed. Receiving a good grade can be a strong external motivation for many learners, and once that has been attained, there are relatively few reasons to continue to retain those memories of piecemeal steps and procedures. Certainly many anxious learners are not compelled by the application value of school mathematics.

The above described two groups of learners: one possesses strong memorization skills, and the other does not. However, there exists a third group of learners, whether they have strong rote memorization skills or not, who prefer the mathematics learning experience to be less rote oriented but more logic-and-reasoning oriented. A teaching style that targets the behavioral learning strategy of memorizing steps could be a source of anxiety for this particular group of learners. Furthermore, this could be the source of regret for some of these learners, such as regretting not asking better questions in class so that the teacher's response could have furthered reasoning and understanding in the learner's conceptual development while learning mathematics.

Major Theme (5)—Learner's Perceptions about Control of Their Learning Process

M1(A)—A quest to adapt in mathematics
F2(C)—Lack of pacing and control, so pretend to learn
F3(B)—The control and pacing in learning mathematics

This major theme can be viewed as hopeful. Obviously, all participants were successful in school mathematics, each in his/her own unique ways. Initially, however, they arrived at the classroom with their own beliefs, struggled with roadblocks, suffered from past and current disappointments, created their own perception of duality in mathematics, and arduously memorized prescribed steps and concepts. Ultimately, this theme reveals the significance of taking control of one's own learning, where each participant took control in some fashion. For example, Carl found technology such as a computer spreadsheet software that could address his own shortcomings and weaknesses in mathematics, and Ellen took control over the pacing of instructional delivery by replaying and rewinding the video lectures that came with her mathematics text. On the other hand, some of the participants had means of control that were much more drastic. Gerri and Anne changed schools until they found the learning environments conducive to their needs. Sue failed her high school algebra course, but she found refuge in business mathematics. Each participant had a unique story, but everyone eventually found ways to take control over their own learning. Perhaps this is the reason why they found eventual success, but even more importantly, it seems that taking control in one's own learning can overcome past traumas, regrets, and disappointments. Thus, this major theme provides key implications for improvements in mathematics instruction. One caveat is that the actual action of gaining learner's control can be a covert and subconscious act. Most participants discussed what they did, but the realization only came during the interviews and the journal writing which elucidated the ways they had compensated for their less than perfect learning environments, which they spoke with negative feeling.

Thematic Relation to the Definition of MALP

In this section, I will use the major themes of MALP to inform its original hypothetical definition, proposed in Chapter 1.

Initial Beliefs [A]. The initial belief component speaks to the adult learners who have sets of beliefs about the subject of mathematics, about their own abilities, and about certain attitudes toward learning. Major Themes (1A) and (1B), about learner's beliefs and on cultural beliefs respectively, clearly relate to this component as the learner uses his/her beliefs to envision how learning mathematics would take place. Many learners also established beliefs through past learning experience, disappointments, and traumas. As an unspoken hope, these learners assume those strategies could, should, and would work again as they enter the learning environment. Oftentimes, they already have had a clear idea of their own ability (Major Theme (4)), and they have formed opinions on how their learning can be optimized for the school mathematics assessment. Many adult learners also arrived at a mathematics classroom with guardedness. While they may not be able to overtly articulate what may trigger their anxiety, they certain are able to voice those emotions when a trigger is present. Being disappointed before, these learners therefore maintain a guarded stance when they are in the learning environment.

Individual and Social Behaviors [B]. This is the component when a learner behaves based on the belief system from **[A]**, such as how to face the roadblocks for learning (Major Theme (2)), how to survive school mathematics through rote memorization (Major Theme (4)), and how to gain learner's control (Major Theme (5)). Furthermore, the conflicting expectations (Major Theme (3B)) are of particular importance in this component. In a social environment, individuals do tend to caution each other in hopes that all learners are taking control. For example, Gerri's granddaughter insisted that using the traditional long division would not be acceptable in her teacher's eyes, and so she negotiated with Gerri. Even though this was done with the best intention, the result often inflicts frustration on both parties and further engraves the feeling of mathematics anxiety on both. Interesting in this social behavior component **[B]**, often the roadblocks are culturally inflicted during social learning activities, and therefore, the social behaviors can be more harmful than helpful in mathematics learning. One thing that is worth noting is that there are two kinds of intentions: one kind of intention is for Individual A to help Individual B so that B would learn how to do mathematics in A's eyes. The other kind of intention is for A to help B understand what strategy would be best to survive/pass the mathematics course they are in. Both intentions can converge on their purposes to avoid roadblocks.

Belief Adjustment [C]. Based on both initial belief and the social interaction during learning, adult learners eventually would reflect on and make sense of the mathematics learning process. They reflect on why their questioning techniques may not be successful (Major Theme (5)), and they construct their own version of what constitutes mathematics (Major Themes (3A) and (3B)). The reflection process in turn informs the adjustment of the learner's belief system. This becomes a cycle until next time the learner reenters him/herself into the mathematics learning environment, as shown in the following figure:

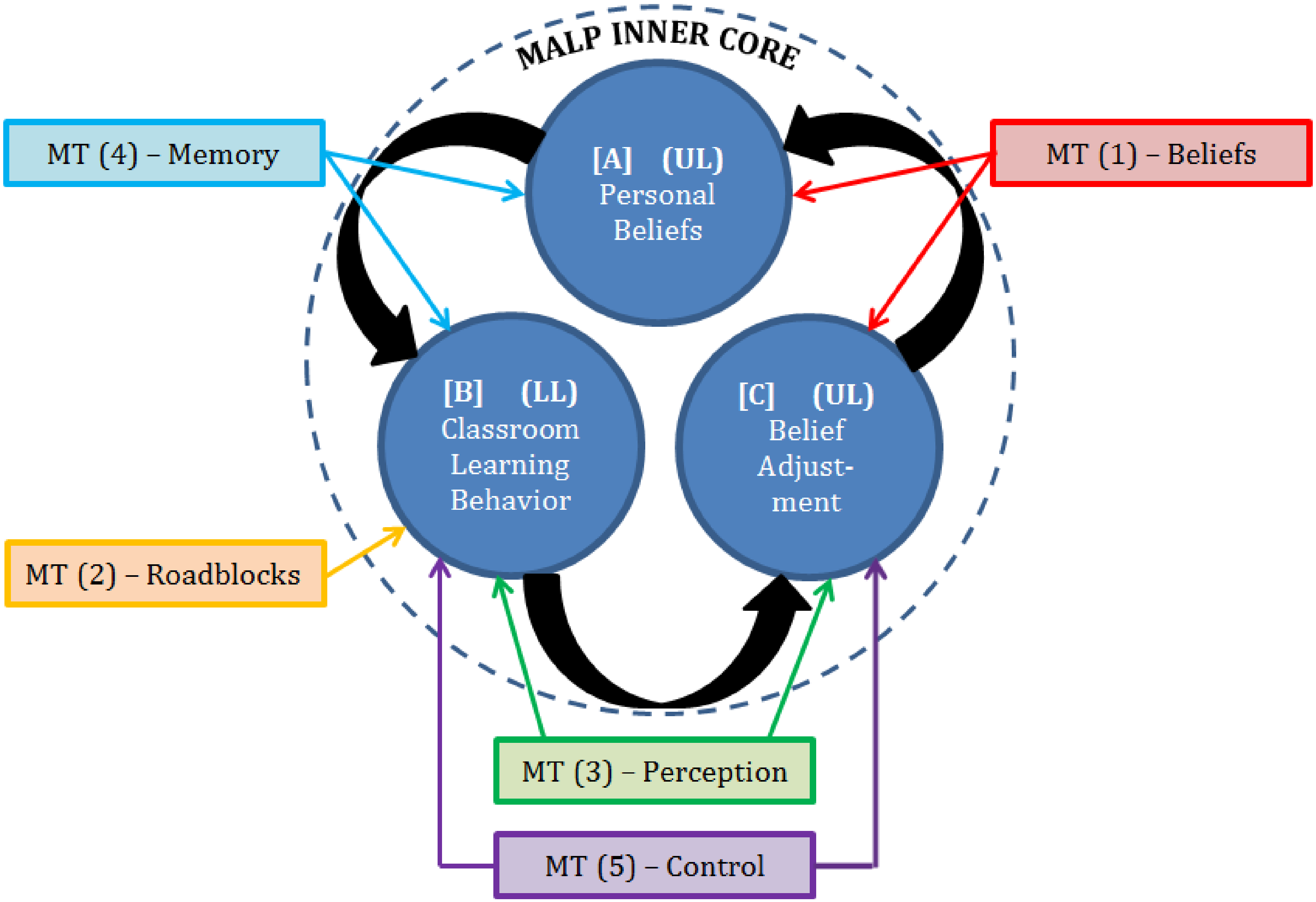


Figure 5.6. Thematic relation to the definition of MALP.

As the learner manages to adjust his/her belief system in **[C]** based on the learning experience, the system would eventually reach equilibrium or a constant state. This means that the learner has established a belief system that reflect his/her learning experience. In turn, it becomes a part of the overall belief in **[A]**, and the cycle repeats itself next time the learner enters the mathematics learning environment, such as taking another mathematics course in the next semester.

Conclusion

In this chapter, I have developed the major themes for MALP and related them to the definition of MALP. The lived experience of the participants played a crucial role in providing details on how the MALP components could be informed through the major themes. Also important are the discoveries through sieving and reflecting on the participants' lived experience, and these discoveries are quite different from the results of the past traditional quantitative studies. One concluding remark for the major themes is that every major theme can be seen as a trigger for anxiety in mathematics learning. While Ashcraft et al. (2002; 2005; 2009) alluded that the cause for mathematics anxiety is not clearly known, these triggers are probable suspects, and therefore, following Ashcraft et al.'s comment, the causes for these triggers have potential for future research.

As I reflected on my own past teaching experience and the participants' lived experience, I found the inquiry of life history reached full circle in which the participants' lived experience are the elements for developing the major themes for MALP. Based on the richness and essence of these experiences, major themes are formed, and they are related to the definition of MALP, which was my original idea before conducting the research study. The exercise of major theme development in itself helped me develop the depth, richness, and inter-relatedness of the collective experiences explored in this study. Also, because of the interpretative nature of thematic development, the overall reality that the five themes have concocted and disclosed would be considered as external (i.e. UR and LR) in integral theory. In the next chapter, I will use the lived experience and major themes to revisit the theoretical frameworks that I surveyed in the past chapters.

Chapter Six: Theoretical Implications of MALP

In the past chapter, I developed the major themes of the Mathematics Anxiety Learning Phenomenon (MALP) through the lived experience of six participants, and I found connections between the themes and the three components of MALP. In this chapter, I will examine and discuss the theoretical implications of the findings in Knowles' andragogy and in Givvin et al's (2011) hypothetical model. The result of the discussion will be used to respond to the research questions set forth at the beginning of the research study, cascaded in the fourfold perspective of Wilber's Integral Model, recalled here:

> *Problem Statement*: As a practitioner who teaches developmental mathematics to adult learners who often experience mathematics anxiety, what characteristics of MALP could give insights that would influence instruction to optimize learning? This problem can be dissected into five research questions:
>
> (1) *Personal Beliefs:* What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?
>
> (2) *Roadblocks:* What are the roadblocks (cp. Givvin et al., 2011) that prevent a learner to succeed in mathematics? And what are the manifestations of these roadblocks?
>
> (3) *Cultural Beliefs:* What are the underlying cultural beliefs in MALP, and how is the culture passed on to others, and how is it perpetuated within and outside of the classroom?
>
> (4) *Social Roles:* What are the social norms when learners are supporting each other? And what is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?
>
> (5) *Integral Disclosure:* Based on the perspectives in (1), (2), (3), and (4), what could one disclose as an integral perspective about MALP and how it is cyclical and perpetual?

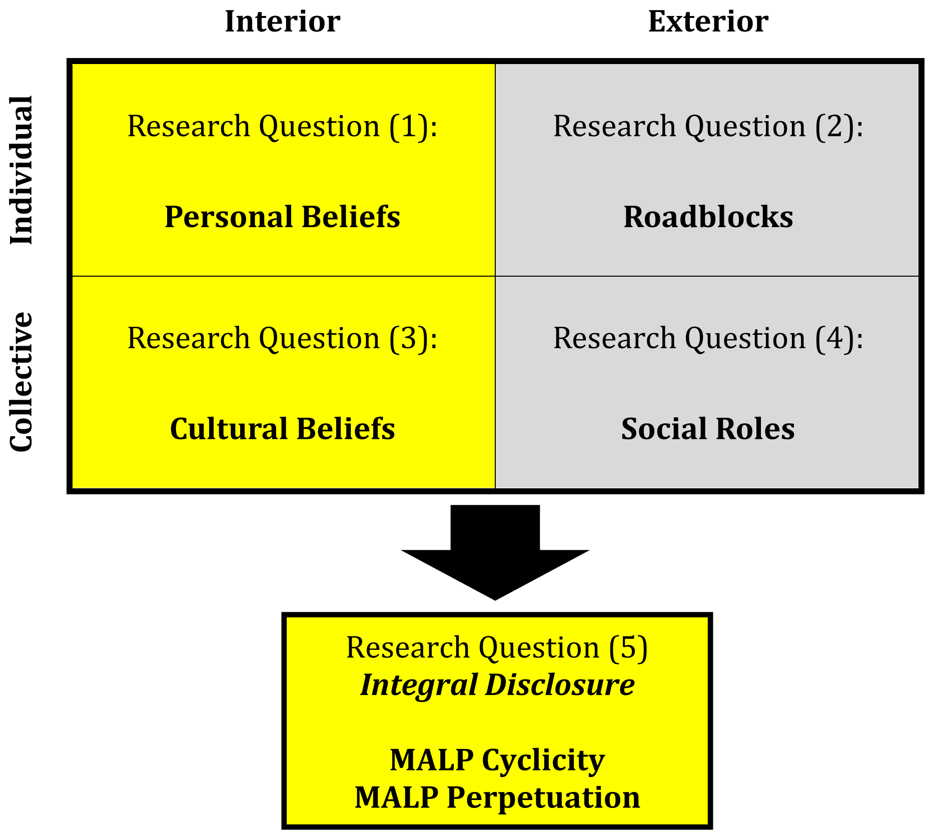


Figure 6.1 (Idem 1.3). The five research questions and their relations to the four quadrants in Wilber's Integral Model, reproduced from Chapter One. Yellow color denotes the distinct contribution of this study to the research landscape.

The goal of this chapter is to connect the research study to its theoretical underpinning. Through discussion of the research data, I will be able to fold the findings into each of the four quadrants of integral theory. I will provide clear and concise responses to the research questions. In the spirit of Stokes' Pasteur's quadrant (Figure 1.2), the aim of this chapter is the quest to understand. I will discuss the implications for classroom instruction in the following concluding chapter.

The Lived Experience and Knowles' Andragogy

This section on examining the lived experience and Knowles' andragogy aims to provide a basis to respond to Research Questions (1) on personal beliefs and (4) on social roles. To perform an analysis of MALP and Knowles' Andragogy, I will first recall the five assumptions of Knowles' andragogy:

(1) *Self-Directedness:* As a person matures, his or her self-concept moves from that of a dependent personality toward one of a self-directing human being.

(2) *Reservoir of Experience:* An adult accumulates a growing reservoir of experience, which is a rich resource for learning.

(3) *Readiness to Learn:* The readiness of an adult to learn is closely related to the developmental tasks of his or her social role.

(4) *Problem Centeredness:* There is a change in time perspective as people mature—from future application of knowledge to immediacy of application. Thus an adult is more problem centered than subject centered in learning (Knowles, 1980, pp. 44–45).

(5) *Internal Motivation:* Adults are motivated to learn by internal factors rather than external ones (Knowles & Associates, 1984, pp. 9–12).

Self-Directedness. The first assumption is about the self-directedness of the adult learner. This assumption originates from the idea that adult learners are motivated to learn, and therefore they insert themselves into the college education environment. However, many U.S. college degree programs require a significant portion of general education prior to embarking on the major studies. The general education component usually requires writing, mathematics, sciences, social sciences, humanity, foreign language, and/or some form of artistic study (either visual or performing art). Many adults, particularly non-traditional learners who return to school after working for a while, enter college with fairly clear ideas on what they would like to embark upon studying. However, they may soon find the general education component burdensome, as they may not connect the immediate applications of these requirements to their lives. Financially, many of them realize that a significant portion of tuition and fees would go toward these general education courses, and emotionally, often they feel that those courses are barriers for them to get through before they can "really" study the major courses, which they are motivated to do. Even if one is not pursuing a mathematics or science major study, inevitably, mathematics courses must be taken to fulfill the general education requirement. Furthermore, if students receive low scores on their entrance placement tests, they are generally required to take developmental mathematics in addition to the regular college level mathematics course, and then the burdensome feeling becomes even greater.

The arguable point here is that the adult learners generally are not, as described in Knowles' andragogy, self-directed in *taking* developmental mathematics courses. Nevertheless, they are self-directed in working toward a passing grade and toward exiting the course. This means the followings:

- Adult learners' use pre-conceived beliefs and expectations for various aspects of education to self-direct. Major Themes (1A) and (2) address the individual and personal beliefs formed through past experience and roadblocks, and they reflect on the psyche and attitudes of many adult learners as they enter the classroom.
- Therefore, they are self-directed in strategizing how to pass and exit a developmental mathematics course through Major Theme (4) on rote memorization. The motivation to pass mathematics would be that they can move forward to their major courses which they originally decided to embark upon. For a learner, this may take precedence over meaningful learning, however meaningful learning may mean to him/her.
- The results from the above two bullets affect the adult learner's belief system and his/her perception toward mathematics, as seen in Major Theme (3) on the duality of mathematics. With a categorization of what mathematics is accessible, and what mathematics is perceived to be inaccessible, a learner is likely to have self-directed himself/herself to an anxious feeling when engaging in the inaccessible mathematics discourse. Sue is a good example: when she saw that her son's mathematics work that resembled a lump of equations and symbols to her, she directed herself to stop attempting to make sense of that work.
- A successful result of self-directedness in MALP is Major Theme (5) on gaining learner's control. Obviously, if an adult learner aims to succeed in developmental mathematics, he/she would be motivated to gain control over his/her own learning. In the lived experience data, some of the actions to

achieve this can be quite drastic, including Gerri's changing schools to find instructors and resources that were acceptable to her.

Reservoir of Experience. The second assumption of Knowles' andragogy has to do with one's past experience and how it could facilitate learning. As I reflect on my own past teaching, I see that this assumption should have worked well for mathematics learners. Their past successful problem solving experiences ought to have been a positive resource for the learners when faced with future problems, similar or otherwise. However, the lived experience data showed a very different reality. It is likely that the adult learners in the study had a reservoir of experiences which triggered mathematics anxiety. These experiences were amplified, and perhaps manifested into physical pain, when they even as much as anticipated mathematics learning (Lyon & Beilock, 2012, p. 5). Often, a learner's positive attitude could be marred by the anticipation of painful unpleasantness. As seen in the lived experience data, it is a culture that mathematics instructors have often taken advantage of the learner's belief in prescribed steps, believing that learning would be at least facilitative if they mirrored those steps in instruction. This practice results in reinforcing the trigger of anxiety for a learner when the act of doing mathematics does not conform to the prescribed steps. In particular to the major themes:

- The growing reservoir of past experiences (including past disappointments and traumas) became the basis for both the learner's beliefs and cultural beliefs (Major Themes (1A) and (1B)). Past experiences with negativity attached to them established what a learner should believe about what the act of learning mathematics should be like. As described above, past experiences also mold how some instructors perceive what the teaching of mathematics should be like. Furthermore, negative experiences become a barrier to learning mathematics because highly anxious learners tend to rely on these experiences as their resource. They may already have defense mechanisms set up whenever the trigger of anxiety is present in the learning environment and mathematical discourse.
- On a positive note, some learners established studying methods such as the behavioral learning strategy and rote memorization (Major Theme (4)) from

their past school mathematics experiences. While rote memorization without any underlying conceptual understanding should not be condoned in mathematics education, this is one way that the learners could gain control over their own learning processes and play active roles (Major Theme (5)).

- For those learners who strive to understand the underlying concepts over prescribed steps, they bring forth past learning experiences that have had a learning strategy, different from that of behavioral style. Therefore, their mathematics anxiety when triggered by non-sensible rote memorization can also be explained through their reservoir of past learning experiences.

Readiness to Learn. It is generally thought that an adult learner is ready to learn a task that is related to his/her social role. The traditional assumption in mathematics education is that the mathematical concepts that are taught in developmental courses should be related to daily life, so that learners can find applicable value to the concepts. In turn, the learners are ready for the materials. However, the lived experience data show that the traditional assumption is not true. Most prominently in Anne's story, she was told to "just be quiet" when she asked about the practical of the mathematics she was learning. Certainly it is believable that Anne's story is not an isolated case. In a sense, previous reports already indicated that low performance and completion rate were due to poor preparation (Post et al., 2010, Jetter 1993, Kamii, 1990; NCTM, 1998, & Prichard, 1995). These reports show that many adult learners arrived at the college mathematics courses unprepared, lacking readiness for instruction.

Nevertheless, the six participants were ready in their own ways. Based on the Major Theme (2) on roadblocks, adult learners face many difficulties and challenges in the developmental mathematics learning environment. While they may not be overtly aware of all of these roadblocks, they have at least a certain level of awareness of some of them. Thus, they are motivated and ready to take control over their education, so that they can remove these roadblocks. In the learners' eyes, they view themselves being inserted into the school system, such as the structure of their degree program requirements, and they perform a social role to pass and exit the barrier mathematics courses. Therefore, they are ready to take control in their learning to achieve these goals. This is the explanation for why rote memorization (Major Theme (4)) has a tendency to become the anxious learner's' choice of strategy. Memorizing prescribed steps and being able to recite them on homework, tests, and examinations can most likely get them enough points to pass and exit a developmental mathematics course. In other words, the learners' social role—in their lives, in their career, in their family, and in all non-school occasions where mathematics is involved—is no longer relevant for them in the developmental mathematics courses. The course and the school environment become a context for them to play a social role in succeeding within the system.

Problem Centeredness. Adult learners are more problem centered than subject centered. This is obvious as one examines the goals of many college adult learners. If the goal is then to exit developmental mathematics and then transition to their major study courses, this goal becomes a problem in itself, and adult learners definitely orient themselves and are motivated to exit in an efficient manner. Major Themes (4) and (5) on rote memorization and learner's control, as described previously in this section, are both related to this assumption of the andragogy. This is not how one may have envisioned mathematics education, but the flip side is that the conceptual learners who dislike rote memorization are centered in how the mathematical problems related to their livelihood while learning. As Gerri put it, she was motivated to be the "understander" so that she could fulfill her personal goal to brush up mathematics in order to become a better instructional coach.

One interesting, but subtle nuance for this assumption, is how Jon and Anne made comparisons with the applicable value to the subject of English. Jon admitted that learning Shakespeare in English literature did not seem to have an immediate impact to his social life, but that did not stop him from excelling in that topic. As he claimed, the English instructor had brought Shakespeare to life, while the algebra instructor did not even come close to doing the same. In comparison, Anne's teacher explained to class that geometry could be used by engineers and architects. This did not fulfill the immediacy for application in Anne's eyes because she did not envision herself in one of those careers. That being said, one could interpret that the problem centeredness for adult learners has to do with how the learning can *parallel* real life application, as opposed to quoting exactly how the learning *is* applicable. In other words, since many adult learners play a social role in the context of the learning environment, one could improve instruction by situating the problem in the environment itself for immediate application.

As for the social aspect of mathematics learning, it is also seen from the lived experience that learners were problem centered. The episode of Carl helping his daughters was an excellent illustration that they actually preferred their father to leave them alone. Given that Carl visualized the "new math" as inaccessible (see Figure 4.2 on Carl's drawing on what he believed as "New Math"), he was most likely not helping in a productive manner, and his daughters realized that their father's help was more distracting than helping. As problem centered learners, the quickest way to finish their homework assignments was to have their father stop helping. In short, they were indeed driven by this assumption of Knowles andragogy.

Internal Motivation. Internal motivation, as assumed in the current discussion, is the drive for a learner to act and interact within the educational discourse without an explicit reward system. Perhaps this assumption is the most challenging to link to MALP. When Anne's algebra instructor told her to be quiet when she asked for the applicable value of a mathematical concept, her internal motivation for learning such a concept was not fulfilled. Similarly, many learners who employ the behavioral learning strategy for prescribed steps rely on external motivation of passing and exiting their mathematics courses to get through. Perhaps this is the reason why the participants, particularly Anne and Sue, were unable to come up with their own examples of mathematics problems during their interviews. Seemingly, rote memorization is merely stored in short-term memory for the purpose of homework, tests, and examinations. Once those assessments were completed, the motivation dissipated because the general education requirement for mathematics was no longer a problem. In other words, the external motivation alone, such as passing grades and course credits, is not enough to compel the anxious adult to learn in an authentic manner. Also relevant here are the different perceptions of mathematics as well as learning expectations among teachers and learners. These differences, often not reconciled, become a source of lowering internal motivation for learning.

However, a deeper observation through the lived experience data is that there seems to be an internal motivation to perceive mathematics as dualistic. Recall that an adult learner has a need to find a way to reconcile with the inaccessible portion of mathematics

(Major Theme (4)) to his/her own frame of reference, so he/she is internally motivated to transition the portion in and out of his/her path of thinking as minimally as one could. Some teachers might refer to this as "in in one ear, and out in the other." Anne's experience in her accounting course when the professor rushed through the concepts and procedures in a short amount of time is a good example. She mostly tuned out during the instructional delivery. Conversely, having the reconciliation and forming two mathematical categories, albeit one of them being inaccessible, seems to be more desirable for the learner to continue to participate in the learning without triggering overwhelming anxiousness. In this sense, the internal motivation for the learner is to keep oneself level-headed so that one could finish the social learning activity. However, the result of such a haphazard processing of mathematics has a side effect. Once the learner leaves the classroom and studies at home, the inaccessible portion of mathematics can still overwhelm him/herself when reviewing course notes, textbooks, and other resources. Therefore, the lived experience data cataloged a list of roadblocks (Major Theme (2)) to the participants, who thought that the materials and resources provided were not helpful.

In summary, adult learners in developmental mathematics are indeed operating their own learning, in the context of MALP, through Knowles andragogy. Every major theme finds its way to relate to at least one of the assumptions, and the andragogy could be used to explain how some of the behaviors seen in the lived experience. Furthermore, the set of assumptions is useful in providing implications for instruction, which I will address in the next concluding chapter.

The Lived Experience and Givvin et al.'s (2011) Hypothetical Model

In order to respond to Research Question (2) on roadblocks, which requires a comparison to Givvin et al.'s (2011) hypothetical model, one recalls the model here, along with its interface to the MALP inner core:

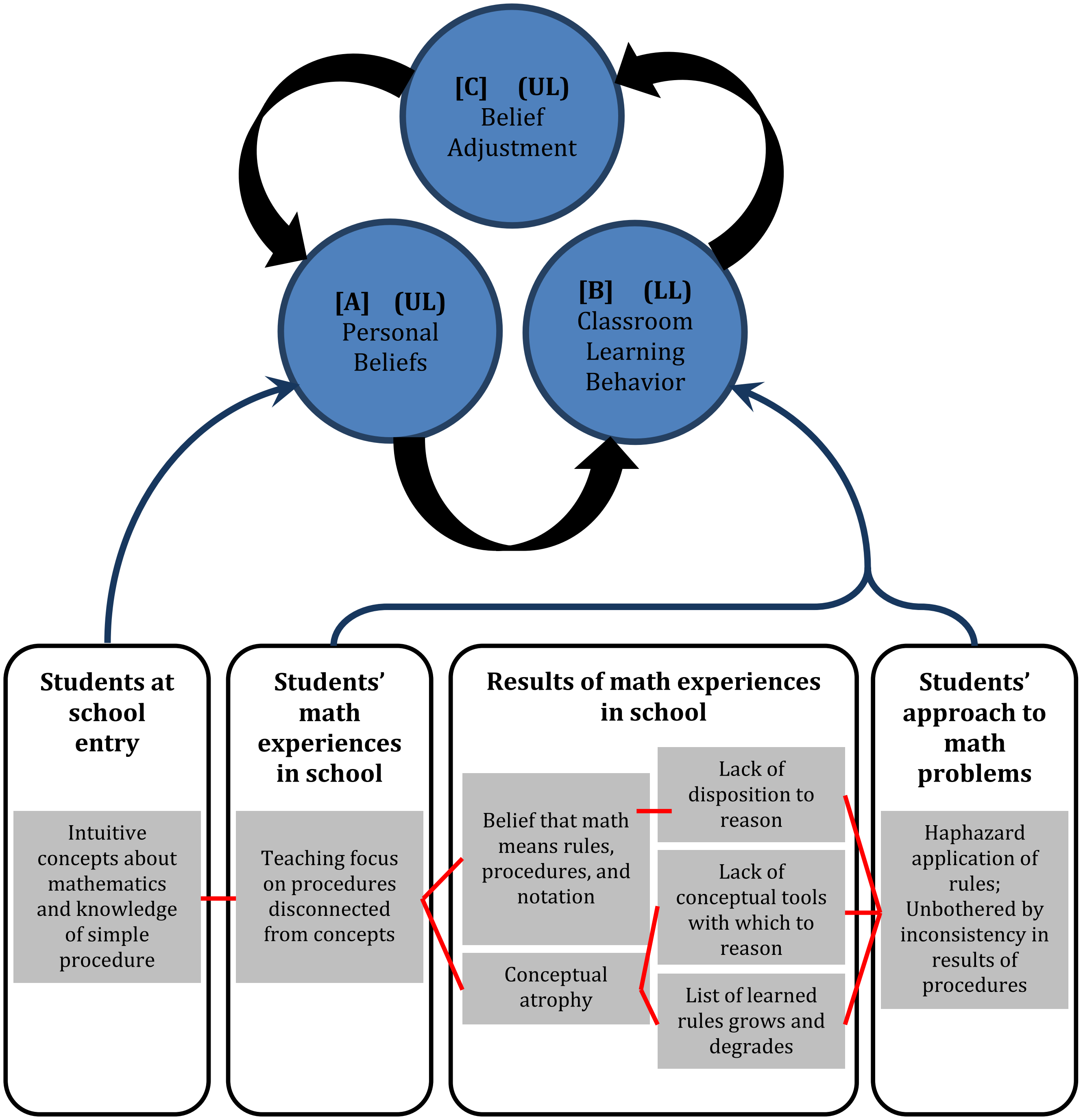


Figure 6.2. Givvin et al.'s (reproduced from p. 6, 2011) model on the making of a community college developmental math student: A hypothetical account [colors added] and its interface to the MALP inner core [rotated for presentation].

The model clearly addresses components **[A]** on initial belief and **[B]** on the social classroom behaviors of MALP. Clearly, the five major themes suggest that both of those components span across the individual-collective as well as the interior-exterior perspectives. Givvin et al.'s account is a counterpoint between the instructor's narrow view of what constitutes school mathematics and the learner's reaction to consequent psychological behaviors. The essential premises of the model include:

- [Instruction that] never made the underlying concepts explicit, and emphasized procedures and paid relatively little attention to conceptual connections.
- [S]tudents who were curious, who tried to understand why algorithms worked, were often discouraged by the teacher.
- Some students learned on their own … the value of connecting rules and procedures to concepts.
- College students … [who were w]ithout conceptual supports and without a strong rote memory, the rules, procedures, and notations they had been taught started to degrade and get buggy over time.
- Learners … all rely on memory to play a critical role in doing mathematics.

The lived experience data practically confirmed that Givvin et al.'s model is quite accurate to the following extent.

On Instructional Explicitness. Whether instruction has made underlying concepts explicit would be difficult to judge in this study alone. This is mainly because of the design of the research, wherein I confined the data through the participants' accounts when they were outside of the classroom. However, through my interaction with Ellen working on the GRE examination question, there is a limited indication that she did not connect $\frac{x+y+z}{3}$ to the concept of the arithmetic mean of $\{x, y, z\}$. Albeit her raving comments that she felt like she was learning mathematics by controlling the video lectures that came with the examination preparation book, it seemed that her learning produced a limited understanding, and the underlying concepts remained relatively unexplored. Also, Gerri's encounter when tutoring a student on ratio and proportion seemed to be a fitting example illustrating that the tutee's teacher probably did not emphasize the inner working of the concept and focused more on procedures instead. This led the tutee to be frustrated and upset when Gerri changed the constants and variables around in a problem to help her understand ratio and proportion. In that sense, it is believable that this aspect of the hypothetical model is realistic, and the Major Theme (3B) on discrepancy of expectations could explain how conceptual learners tend to be frustrated when instructors tend to emphasize the significance of procedural steps.

On Discouragement to Understand Underlying Concepts. This can be seen through Anne's lived experience when she asked her teacher how certain mathematics could help her, and the teacher responded "Anne, just be quiet!" Jon also had discouraging moments when his teacher thought he was not asking the right questions and intimidated him by flipping a desk. In fact, the most prominent example was Gerri's experience that the professor did not really answer her questions because he did not welcome questions in the first place. In addition to the obvious discouragement from the instructors toward learners, there is also a sense that some learners choose to self-discourage. Perhaps this is most obviously seen in Anne's story: when she could not keep up with the learning, she would simply sit back and nod, pretending she understood whatever topic was at hand. While this behavior may be seen as an attempt to fit in, one interpretation is that Anne was motivated to covertly play the acting because she knew that there would be a finite ending to school mathematics. After the course ended, whatever she pretended to know would no longer matter. Hence, the underlying concept is really not a priority during the learning process. Rather, as one will see in the next section, memorization is the priority instead. Major Themes (1A) and (1B) both show how adults behave and interact given the learning environment in *status quo*. Major Theme (2) that describes roadblocks for learning seems to be in consonance with this premise of Givvin et al.'s model. Major Theme (5) on learner's control seems to be a consequence of how a learner may act in order to be perceived as fully learning in the social environment.

On Memorization and Memory Degradation. This premise of the hypothetical model is robustly seen in the lived experience data. All six participants discussed at length the necessity of rote memorization in order to get through school mathematics. As for memory degradation, the lived experience data appeal to the idea because of the fact that almost all participants found it uncomfortable to produce any previously learned mathematical problems during their interviews. If produced, the problems were often ill-formed or incomplete. For example, Carl's attempt at reproducing "new math" ended in a non-sensible web of numbers. Anne produced a geometric problem that involved Pythagorean Theorem, yet her illustration was incomplete without stating that the triangle must have a right angle in order for the theorem to be applied correctly. Sue produced a bunch of "lumps of numbers" examples, but she could not articulate what they represented. All of these examples reveal that what the students could remember about mathematics had been degraded. Major Theme (4) on strong rote memorization along with this premise in the Givvin et al.'s model are two ideas that complement each other in describing how rote memorization is ineffective for adults to learn mathematics.

I suspect that part of the cyclical nature of MALP can be exhibited through this idea of memory degradation. When a learner ends up with buggy ideas from a previous mathematics course, naturally he/she enters the next course that relies on the previous course as a prerequisite with tremendous anxiety. The instructor assumes that the previous concepts remained intact, but the learner knows how degraded his/her memory is. However, expressing his/her own degradation into the social learning environment is seen as a personal shortcoming. Henceforth, one often bottles those feelings and lets anxiety and frustration brew. When the latter class begins to develop concepts, whether the instructor targets procedure and/or concept, the path of least resistance in order to get through the course is to use rote-memorized steps without being bothered to develop the inter-relatedness among steps and procedures. When the learner exits the course with a barely passing grade, he/she moves onto the next mathematics course, and the cycle begins yet again. In this sense, the discovery of MALP in this study has added details to Givvin et al.'s model.

On Students Learning on Their Own. The lived experience data showed some examples of this. In particular, Carl's analogy that he acted as an engineer to solve problems and used aids to help him through the tedious and arduous calculations, was an excellent example that he most likely developed conceptual understanding of the problem presented to him. Also, the fact that he had the ability to analyze the problem as a whole so that he could devise a plan on how to execute a solution to it was an indication that he developed concepts that transcend from the mere prescribed steps. Another example is Ellen's troubleshooting strategy, in which she attempted to detect whether her conceptual development was flawed. These two examples both have shown that some learners do behave as described in the hypothetical model, confirming the premise that they learn on their own and make connections from rules and procedures to concepts. However, it was not readily seen in the data that many learners consider such reflection and self-learning as authentic learning of mathematics. This could be explained through the Major Themes (3A) and (3B), which entail both the perception of duality and social expectation—a cultural belief that many learners perceive school mathematics as largely procedures, steps, and "lumps of numbers and symbols." The underlying conceptual understanding and self-discovered applications are perceived as "non-mathematics."

MALP Contributions to Givvin et al.'s (2011) Model. Overall, Givvin et al.'s model clearly depicts how a typical behavioral learner experiences school mathematics, and this current study discloses practically all facets of the model. Nevertheless, the lived experience data also show how conceptual learners tended to become became dissatisfied with their mathematics learning experience. Using Givvin et al.'s model as a framework, it is speculated that:

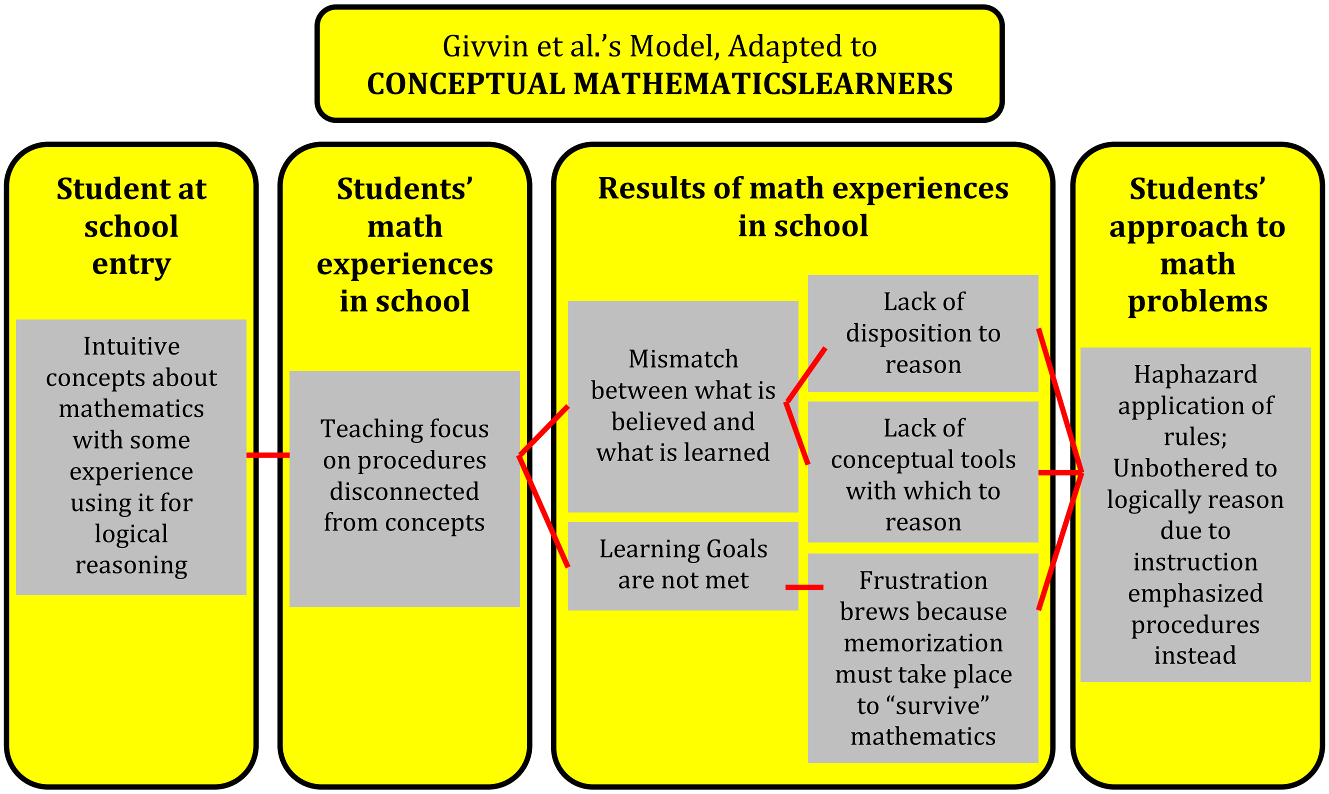


Figure 6.3. A modification of the Givvin et al.'s model - adapted for conceptual learners' experience in developmental mathematics.

In particular, the adapted model shows that there is a mismatch between the teaching and learning goals, and the desire to develop conceptual understanding is not met. Because of the lack of learner's control, there is not much a conceptual learner can do when the instructor merely develops prescribed steps on procedures in a disconnected manner. The result is that the conceptual learners must still rely on memorization to "survive" the mathematics course, leading to a very similar result of haphazard application of rules and prescribed steps.

Thematic Cascade in Integral Model and Responses to the Research Questions

Recall that a paramount goal of this project is to use integral theory as a theoretical underpinning to inform a qualitative study of MALP. Now that I have made a detailed analysis on the major themes and related them to Knowles' andragogy, the present lived experience, its richness, depth, essence, and details can be cascaded into the four quadrants of Wilber's Integral Model to respond to the research questions.

Individual-Interior (UL). The first question deals with the learner's personal belief system on what one believes to be mathematics, one's ability on learning mathematics, as well as how learning mathematics should be. This line of questioning directly addresses the individual-interior (UL) quadrant of MALP:

(1) What are the learner's personal beliefs on learning mathematics, on their own abilities in learning mathematics, and on what constitutes mathematics in their eyes? When grouped with other individuals, how do they interact in the learning environment?

Major Theme (1A) on individual beliefs essentially responds to the first part of this question. Often adult learners overtly believe that the act of doing mathematics is to carry out prescribed steps. Both behavioral and conceptual learners seem to share this overt belief. Few participants believe, however, that the conceptual aspect also constitutes mathematics. The covert belief, on the other hand, relies on Major Theme (3A) on duality of mathematics, that an adult learner often compartmentalizes mathematics that he/she could process into a category and another category for what he/she could not. This duality falls into the individual-interior (UL) quadrant, and it is radically different the five mentalities of mathematics from Renert and Davis (2010), which would fall in the collective-exterior (LR) quadrant. Also different from the five mentalities is the perception that duality speaks to how a learner's consciousness may perform such compartmentalization based on his/her own perceived ability. For example, Sue's duality of memorizable versus not memorizable mathematics, foretold her ability in mathematics when it came to memorizing prescribed steps. Meanwhile, Anne's duality of applicable versus not applicable, showed that her self-directedness in learning mathematics is based on the immediacy of application for her. Ellen, furthermore, compartmentalized her perception of mathematics as "memorizer" and "understander," and she identified with the mathematics that she could understand, as a conceptual learner.

When confronted with another learner's mathematics work, many learners default into their identified compartment of mathematics which I will refer to as a "singularity." In fact, these singularities are usually quite different among the learners engaging in a social mathematics discourse. Such a discrepancy of perception and expectation could be a significant source of frustration for all individuals during a social learning event. To confine the response to this research question in the UL quadrant, a learner being instructed often expects the discourse to reflect the mathematics that he/she perceives as identifiable (cf. Gerri tutoring a student). Because different people have different identifiable mathematics, a social discussion often leads to myriad non-commutable ideas. As discussed previously on Knowles' self-directedness in adult learners, they frequently do not move beyond the identified learning space to reach consensual communication. In other words, the learning spaces of different adults do not generally intersect. The result of student interaction is all too often not a learning gain, rather it is the gain of more frustration and disappointment as part of a learner's reservoir of experience, strengthening the trigger for anxiety. In sum, the findings for this research question complement De Corte et al.'s (2008) claims that "students at all levels of education hold naïve, incorrect, and/or negative beliefs about mathematics as a domain and about mathematics learning and teaching."

Individual-Exterior (UR). The second research question deals with roadblocks and their manifestations addressing the individual-exterior (UR) quadrant of MALP:

(2) What are the roadblocks (cp. Givvin et al., 2011) that prevent a learner to succeed in mathematics? And what are the manifestations of these roadblocks?

Obviously, Major Theme (2) on roadblocks largely provides myriad concrete examples of roadblocks for learning mathematics. However, I will respond to this research question in twofold because the two types of learning preferences face different types of roadblocks. For those who preferred prescribed steps and the behavioral learning strategy, an instructor who caters to such a preference may at face value be beneficial. However, because the relation among steps and among other prescribed procedures are not well-developed, they become isolated entities in the learner's conceptual space. Consequently, this isolation becomes the central root for the roadblocks that would be manifested. For example, a learner would not be able to comprehend how certain mathematics could be applicable in real life when he/she maintains an isolated list of prescribed steps without their inter-relationship being developed as well. Also, the isolation is a likely source of mathematics anxiety when there are excessively many (in the learner's mind) prescribed procedures to be memorized (Major Theme (4) and Givvin et al.'s conceptual atrophy). Therefore, in day-to-day teaching, when an instructor continues to deliver more procedures, the demand of rote memorization can become overwhelming to these learners. In the case of an instructor developing conceptual understanding and attempting to discourage memorization, this type of learner would still be overwhelmed because (1) this kind of instructional delivery is not within their preference, (2) they do not have the inter-relatedness developed in their conceptual space to fully appreciate conceptual understanding, and (3) through previous experience, they have developed defense mechanisms against learning in this fashion. In other words, learners of this type have conditioned themselves to a "no-win" situation for learning school mathematics.

The other type of learner, namely those who prefer the conceptual learning strategy, can be frustrated as well. When prescribed steps are delivered, the instructor's intention may be to cater to many learners. But this type of learner does not prefer rote memorization, and their conceptual learning space craves deeper understanding to fill the void for the inter-relatedness among steps and procedures. When the void is not given due attention, the learner becomes frustrated because he/she may perceive learning to be incomplete (Major Theme (3B)). To survive school mathematics by rote memorization alone would be a source of frustration, as the learner does not have an outlet for reasoning to be developed. On the other hand, if an instructor targets learner's conceptual understanding, then this would help them. Unfortunately, the former type of behavioral learner who vocalizes dissatisfaction with conceptual development would negatively affect conceptual learners by discouraging the instructor to deliver in this fashion. This results in a social environment that still might not yet suit the learning style of the conceptual learner.

To address the comparison to the Givvin et al's hypothetical model in terms of individual learner's roadblocks, the discussion in this chapter provided a strong basis that conceptual atrophy can be a significant roadblock. In addition, it is through MALP cyclicity that such atrophy exemplifies from past to present school mathematics experience that this roadblock gradually strengthens and prevents a learner from transcending to his/her conceptual development from prescribed steps and procedures. Overall, one recognizes that Givvin et al.'s model provides a fairly accurate individual-exterior's account of mathematics anxiety, whereas this study of MALP adds more details to the model in the same quadrant, as well as in other quadrants.

Collective-Interior (LL). The third research question deals with the underlying cultural beliefs, and how MALP as a culture passes anxiety from one learner to another:

(3) What are the underlying cultural beliefs in MALP, and how is the culture passed on to others, and how is it perpetuated within and outside of the classroom?

Most notably, Schoenfeld's (1989) finding on students' expectation to master the subject through memorization in "bite-sized bits and pieces" is confirmed and seen through Gerri's observations of "piecemeal" instructional delivery. This confirmation shows that the nature of instruction could be a cultural norm because it largely models a behavioral learning strategy. Furthermore, such a strategy may lead learners to mechanically "drill-and-kill" on mathematical problems, as Schoenfeld claimed. For those adult learners who prefer the behavioral learning strategy, they end up spending insufficient amounts of time thinking deeply about mathematical problems. On the other hand, those who prefer conceptual learning strategy would feel the prescribed steps to be meaningless exercises. This synthetic response shares a large similarity to De Corte et al.'s (2008) claims that (1) "students at all levels of education hold naïve, incorrect, and/or negative beliefs about mathematics as a domain and about mathematics learning and teaching," and (2) "the prevailing teaching practices and the culture in mathematics classrooms are largely responsible for the development in students of those non-availing beliefs" (p. 34).

Among the different learners helping each other, there is a sense of self-centeredness in the learner's expectation (Major Themes (1B) and (3B)). When a learner has an expectation of how to learn the materials, one expects the person who acts in the helper's role to meet that expectation. Gerri's encounters with her granddaughter and her students were good examples. When she attempted to help the student on the proportion problem and probed her conceptual understanding, the student not only did not comprehend Gerri's underlying intention, but she also resisted being helped in this fashion. Similarly, when Carl helped his daughters with their mathematics homework, they belittled his attempt, and made fun of his own negative mathematics view. Also seen in above examples is how learners consistently attempt to gain control of the learning experience

(Major Theme (5)), resulting in both of these incidents which illustrate mismatches of expectations.

The successful counterpart of the interaction comes from the examples that a learner goes beyond his/her learning space and style to help others. Sue's son, who attempted to explain how to perform calculations using those visually lumped together symbols and numbers in order for Sue to manage computing numerical solutions to her homework assignment, was an exceptional example. This is particularly revealing because Sue described her son to be talented in mathematics, and he had abilities to do mathematics that were beyond her grasp. From her observation, one could safely assume that he probably has excellent conceptual understanding of Sue's homework problems, but he chose to show her how to perform computations in a way that Sue felt comfortable with. What is unclear in this example is whether the familiarity between mother and son contributed to the success of this social interaction. Therefore, one must merely note that these successful incidents were beyond the cultural norm.

Collective-Exterior (LR). The fourth research question deals with the social norms, identity and roles in learning mathematics in the social environment:

(4) (LR; zone 7) What are the social norms when learners are supporting each other? And what is the identity of an adult learner in the mathematics education community? What role does the learner play in the mathematics education discourse?

Recall in the literature review chapter, several studies (Geist & King, 2008; Campbell & Clewell, 1999; Laster, 2004; Levi, 2000; & Campbell, Storo, & Educational Resources Information, 1996) that discussed how parental attitude may correlate with their young children's attitudes as mathematics learners. Peressini's (1996) finding showed that parents played a limited and passive role in mathematics education, and this has been seen in the lived research data. It is also seen that many adult learners also played a similar role. The most prominent was the vivid story that Jon told about how he learned mathematics in just the same way as how he saw the professor teaching from outside of a classroom. Sue's story exemplified how she played a passive role throughout the school system. Albeit her wish to obtain a prestigious high school diploma, she was tracked into the business mathematics course that would lead her to a lower diploma.

Every mathematics teacher knows that a learning environment with diverse learners can be quite challenging for instructional delivery. Obviously, as previously discussed, the two types of learners, behavioral and conceptual, are quite dichotomizing. Instructional delivery driven by one of them is often the source of frustration for the other. Even when I attempt to do both in my classroom, many learners will disengage themselves from instruction when they feel that a certain aspect of the delivery does not "speak" to them. As one can see in the lived experience data, Carl often practiced doodling in class, and Anne preferred to pretend to understand, to go with the flow, and to not ask questions. These are just some examples of disengagement when a learner does not feel the instructional delivery is important to him/her. The end result is a mediocre classroom environment with adult learners observing others putting half-hearted energy into learning, creating a stale aura that is not a conducive or optimized educational setting. In fact, learners seem to become vocal in this stale aura, with questions such as: "How is this useful to me?" or "Why am I learning this?" (cp. Anne's comments). They do this to gain communal support from other learners, as well as to gain some form of learner's control. This gives a sense of identity for the learner that he/she is fitting within the social learning environment, and that he/she is not alone in feeling this way.

Fortunately, the adult learners are not entirely passive. There are some elements of active roles seen in the lived experience data. Carl decided to make use of computers to compensate for his shortcomings with computation and calculation. Gerri decided to enroll in a different college for a more satisfying learning experience. Sue made sure that she kept asking for help, and received resources that aligned with her learning needs. All of these are elements of playing an active role, and they could be explained through self-directedness of Knowles' andragogy. What remains to be seen is how adult learners' own perceptions can be meaningfully captured in the mathematics educational discourse. Anne's story exhibited this point clearly when she asked how she could apply the mathematical concepts into her daily life, and the response from the instructor did not address her own perception. The response did not give her any immediacy to continue. Finally, there is a tacit assumption among the six learners that the *status quo* on the extent of their involvement in the mathematics educational discourse is accepted. This means that no matter how little they contribute to the discourse, either for their own education or for their children's education, they do not strive to engage more for the sake of education. Overall, the lived experience data showed that the adult learners do engage in a limited active role in community, and they were focused on their immediate learning. Yet, they often assumed the school system was the *status quo*, and they were not informed that they have opportunities to engage themselves in the educational discourse to make effective changes in improving the overall learning experience.

Integral Disclosure. The fifth and final research question, although technically is a collective-exterior perspective, is also a fourfold integral disclosure of MALP. Specifically, the question seeks how the above responses collectively form an integral conference to show how MALP can be cyclical and perpetual:

(5) (LR; zone 8) Based on the former four perspectives, what could one disclose as an integral perspective about MALP, and how it is cyclical and perpetual?

The lived experience data suggest that MALP cyclicity can be seen in the individual learners. The individual belief system, particularly the duality of mathematics, played a crucial role in dictating how an individual behaves in the social mathematics learning environment. In fact, an adult learner has already established these beliefs before coming into a classroom. Also, because of past experience, many learners developed defense mechanisms when they were confronted with a territory in which they were not comfortable. It is seen that the learner's perception of mathematics is radically different from how mathematicians and educators view the subject. Thus, in the social learning environment, oftentimes the different views clash, and this leads to a not-so-productive learning gain and conceptual development. During interactions among learners, the self-centeredness of their own expectations and how they learn often undercuts the productivity of discussing mathematics. Many adults have expectations about how to learn mathematics, and they assume others would expect the same. This can explain the interesting finding: a learner often has a desire to fill a void in his/her own learning space, and that desire is not always met, resulting frustration, negativity, and anxiety.

The manner of instructional delivery often caters to a single learning style of prescribed steps, and this leaves most learners to focus on rote memorization rather than developing underlying concepts. This sociology leaves little room for a learner to play an active role. Consequently, learners seek outlets to exercise their power, such as controlling their behaviors to conceal their lack of understand in class, expressing overtly their negativity toward how the subject is taught, or changing course/instructor/school in order to find something that they are looking for. It is seen that the learners would go to great lengths to play an active role, but they do not necessarily prioritize conceptual understanding as their ultimate learning goal.

As for MALP cyclicity, the current research findings that complement Givvin et al.'s hypothetical model together essentially provided details on how MALP cyclicity takes place: an adult learner comes into a college mathematics classroom knowing what mathematics he/she has forgotten (memory degradation). Influenced by other learners misunderstanding, misdirection, and misguidance, the learner finds that the interactions among other learners generally lead to more frustration and negativity. Meanwhile, the instructor tends to assume what has been forgotten as a prerequisite. With the practice of delivering prescribed steps and procedures and the culture of discouraging learners to deviate from the instructor's intents, most adults ended up choosing to learn by rote memorization because it is the path of least resistance within the confines of the school setting. Furthermore, Givvin et al.'s ideas on conceptual atrophy both during the school term when the mathematics course takes place and after the course has ended are worth mentioning. Because many college adults would take multiple mathematics courses, the beliefs and degraded memory would feed into the next course. The degradation also amplifies and reinforces the need for the learner to choose rote memorization, where the cycle begins again. MALP cyclicity can be illustrated in the following manner:

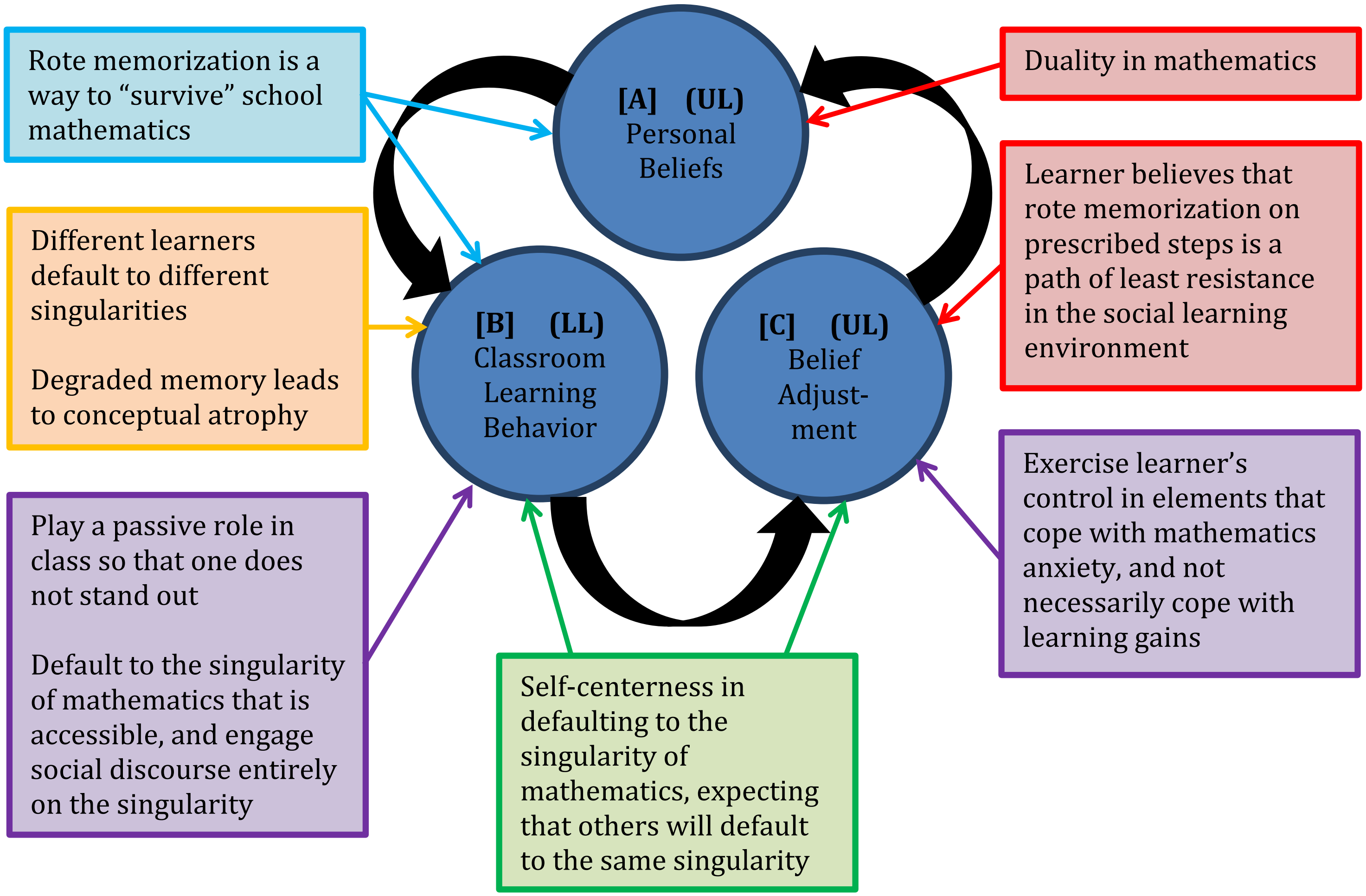


Figure 6.4. An overall of integral disclosure of MALP through the current lived experience data with colored boxes correspond to their respective major themes.

How MALP is perpetuated is not clearly seen in the research data. Specifically, I am unable to ascertain that mathematics anxiety could be passed on from one learner to another. However, the data did suggest that the mismatched learning expectations between two learners can lead to frustrating social learning experiences. Also, because rote memorization is a key source in MALP leading to conceptual voids in one's learning space, learners' interactions could encourage the use of behavioral learning strategy. When these voids in a learner fester into negativity and anxiety, one suspects that MALP would be perpetuated. In sum, the perpetuation of MALP remains an elusive possibility, yet a direct link of mathematics anxiety from one learner to another remains to be seen.

In this section, I have responded to the research questions based on the lived experience data and the major themes. The responses weave through the major themes of MALP, Knowles' andragogy, and Givvin et al.'s hypothetical model. These theoretical elements provided a co-arising integral conference that discloses MALP in an enriching manner, allowing me to examine the details and uniqueness in close proximity. I cannot measure the physical distance between me and MALP as compared to other past studies, because the proximity is a mere metaphor. However I felt that my experience conducting this piece of research did allow me to submerge my researcher self in MALP, whereas my previous mere self as a practitioner did not. This fulfills the main goals of the research study.

Conclusion

In this chapter, I have attempted to contribute the lived experience data from six participants to the theoretical realm, specifically to Knowles' andragogy and to Givvin et al's (2011) hypothetical model. Furthermore, based on the research data, I responded to the five research questions that cascaded into the fourfold quadrants of integral theory. I will conclude this theoretical chapter by writing a post-hoc reflection on the research methodology. This chapter indeed uncovered multitudes of results that reveal a unique disclosure to the cyclical and perpetual nature of MALP, and these results complemented the research landscape in a way that many past quantitative research studies could not. However, perhaps the most significant finding of MALP is the description of how it is cyclical, and this finding is unlikely achievable through a quantitative study. The second important finding is the two different types of adult learners, namely behavioral and conceptual. The chosen methodology yielded richly detailed, divergent descriptions of each kind of learners' mathematics anxiety and how the formation of each is different from the other. Again, it is unlikely that using a traditional modernistic research approach would lead to this finding. Theoretically, past research studies that utilized any variant forms of the MARS surveys assumed linearity on the severity of mathematics anxiety, and the current research study has uncovered that mathematics anxiety, while similar among behavioral and conceptual learners, can be cycled differently in a qualitative manner. Consequently, this study has widened the future trajectory of how mathematics anxiety can be researched.

One lurking question was from Ashcraft et al.'s (2002; 2005; 2009) charge to find the root cause of mathematics anxiety. This current research substantiated how mathematics anxiety could be reinforced and magnified within the MALP cycle, and the research data helped me see the driving force within the cycle. However, Ashcraft et al.'s charge can be interpreted deeper yet; in fact, one could continue to research how the cycle is originated. As I posited earlier in the dissertation, the goal of this study is not intended to directly respond to the charge. However, the examination of MALP provided a rich and sensible description to the nature of the phenomenon, and it brought a nearness of the topic to me. Ultimately, the charge leaves opportunities for future research, and it is one charge that has weight for me, both as a researcher and as an educational practitioner.

The next chapter will be the final and summation chapter, and I will address instructional implications based on the research findings.

Chapter Seven: Instructional Implications of MALP and Conclusion

In the past three chapters, I have analyzed the lived experience data from six participants, and through cascading the data into the different quadrants of integral theory, I have uncovered both interior and exterior perspectives of MALP. The goal of this conclusion chapter is to provide informed implications of developmental mathematics instruction for adult learners based on the findings of MALP. One caveat that should be made clear: the target audience of this research study is developmental mathematics instructors who are interested in how to foster a mathematics anxiety friendly instructional delivery. Therefore, the implications below are not merely for my own instruction. In this sense, they are written so as to reach a broad audience that could find them useful. In this chapter, I first will briefly describe the notion of integral education. Then, using it as a basis, I will propose five instructional implications. Also included in this chapter is an emerging research agenda to show how one might continue the research trajectory of the current research. The chapter will conclude with some personal reflections on the importance of integrally understanding the lived experience of mathematics-anxious learners.

Integral Education as an Extension of Wilber's Integral Model

While it is clear that the instructional implications in this chapter will be directly motivated by the lived experience data, the major themes, and the analyses with Knowles' andragogy and Givvin et al.'s hypothetical model, these implications are also a manifest of the AQAL model, or better known as "integral education." Thus, it is important to provide a brief background of integral education as an extension of Wilber's Integral Model. Intuitively, integral education follows a similar set of assumptions as its meta-theoretical Integral Model:

(1) Every learning perspective discloses a unique window to knowledge.

(2) All learning perspectives form an integral conference to knowledge.

(3) An investigation of only a partial collection of the learning perspectives would compromise the integrality of knowledge.

(4) The mode of inquiry in learning is to seek and to embrace divergent paths toward knowledge.

Because the instructional implications are integral, I will present two versions of integral education below.

Esbjörn-Hargens' Integral Education. Obviously, there are differing ways to interpret how the Integral Model is extended to education. Esbjörn-Hargens (2007) realized twelve commitments to education through the Integral Model:

	Interior	**Exterior**
Individual	**Upper Left (UL) Quadrant** **Interior-Individual** *Educational Experience* that is: • Contemplative • Critical • Somatic	**Upper Right (UR) Quadrant** **Exterior-Individual** *Educational Behavior* that is: • Skillful • Practical • Active
Collective	**Lower Left (LL) Quadrant** **Interior-Collective** *Educational Culture* that is: • Connective • Perspectival • Ethical	**Lower Right (LR) Quadrant** **Exterior-Collective** *Educational System* that is: • Ecological • Social • Global

Figure 7.1. Twelve commitments of integral education, adapted from Esbjörn-Hargens (2007; p. 9).

The fourfold consideration of education (and specifically in this case, instruction) is to attend to the individualistic experience and behavior as well as to the collectivistic instructional culture and system. Esbjörn-Hargens suggested a simple and practical way to apply these commitments by filling in the blank, "In what way does my course fulfill [Insert Commitment]?" (p. 10). He also emphasized that "certain situations or courses might require an emphasis on some of these over others," and "any educator or student can quickly assess the [i]ntegral value of their course by scanning it to see what is being left out or underemphasized" (p. 10). Therefore, when determining the instructional implications for developmental mathematics, I attempt to encompass all 12 commitments. However, particular commitments are emphasized: individualistic experience to be contemplative and critical, individualistic behaviors to be active, collectivistic culture to be connective and perspectival, and collectivistic system to be ecological and social. Esbjörn-Hargens' notion of integral education can be viewed as a macroscopic application of how integral theory pertains to every aspect of education, from learning to instructing, from curriculum to policy, and from administration to leadership. The scope of integral education is much larger than the central concern of this dissertation about how learning takes place under the duress of mathematics anxiety. Edwards' (2005) notion of integral learning, on the other hand, can be viewed as a microscopic application, specific to instruction and learning.

Edwards' Integral Learning. Similar to integral education, integral learning is also a manifestation of the AQAL model where it highlights the inter-dynamic of the four quadrants. The difference between agentic and communal learning depicts the individual-collective distinction while the difference between abstract and concrete learning depicts the interior-exterior distinction:

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The figure contained a diagram entitled, "Integral Cycle of Learning and Knowledge Development"

In

Edwards, M. (2005). The integral holon: A holonomic approach to organizational change and transformation. *Journal of Organizational Change Management*, *18*(3), 269–288.

Figure 7.2. The inter-dynamics of each quadrant in integral learning, from Edwards (2005; p. 284).

A key feature in Edwards' interpretation of integral learning is that it exemplifies that learning is a cyclical process[5]. Edwards advocates a dynamic learning process that cycles through four learning strands:

- *Illuminative Strand*: What is happening?
- *Interpretative Strand*: What does it mean?
- *Validative Strand*: What have we learned?
- *Injunctive Strand*: What do we do?

Through these four basic inquiry strands, integral learning strives to achieve both agentic and communal learning, as well as concrete and abstract learning. While both Esbjörn-Hargens' (2007) and Edwards' (2009) interpretations on integral education and integral learning were realized quite differently, the commonality remains central to Wilber's Integral Model that it will take multiple perspectives in order to integrally disclose knowledge to learners. In essence, both interpretations advocate the wholeness approach to knowledge, and advocate for divergent paths to seek knowledge to its depth, breadth, richness, and proximity to the learner. In this sense, it is important to devise instructional implications based on a combination of these interpretations.

Toward an Integral Approach of Mathematics Education: Instructional Implications of MALP

Recall that Dehaene's (2011) discussion about nature and nurture in human's abilities of numeracy and learning mathematics, he strongly advocated for improvements in education to better help learners to develop those abilities. In this section, I will propose five instructional implications based on the major themes of MALP. In the spirit of integral theory, they are ordered from individualistic to collectivistic, or more specific to integral learning, from agentic to communal.

Facilitating Active Control in the Learning Process. As discussed in the previous chapters, affecting active learner's control remains a prominent feature that repeatedly appears in the lived experience data, and a truly learner-centered instruction would take control into account. In particular, Ellen pointed out that her control over the pace for

[5] As in many other established learning models, such as the Biggs and Collis' (1995) and Pegg and Tall's (2005) Structure of the Observed Learning Outcome or the SOLO taxonomy, for example.

instructional delivery had made her feel more confident in her own learning than passively following a live lecture. In fact, she identified the element of control as the reason that her preparation for the GRE examination was more successful than her past college mathematics experiences. Therefore, when an instructor designs learning activities for adults, one way to foster some positivity into the social learning environment is to consider elements that allow the learners to take the lead.

For example, a popular instructional model that takes advantage of the learner's control is the "Flipped Classroom" instructional model that calls for students to engage in instructional delivery outside of the classroom and develop problem solving skills in class with the guidance of an instructor. The outside classroom component of the flipped model usually includes video lectures, internet resources, and other hands-on learning activities/manipulatives for learners to develop the underlying understanding of a topic/concept. The advantage of this component is that a learner controls the pacing and delivery of instruction, and therefore, an anxious individual could perceive the experience to be less frustrating, perhaps even resulting in a more positive experience than before. Also, often learners can sense an unwelcoming attitude from some live instructors (e.g. Jon's hallway observation of a mathematics class, Gerri's first college instructor who asked the class to pose questions but expected none), and the outside use of class resources can prevent this effect from happening. For example, video lectures can be done as PowerPoint presentations, and interactive manipulatives can be demonstrated entirely through a Whiteboard or through the Geometer's Sketchpad. The elimination of seeing a live person's facial expressions while materials are presented can minimize unintentional transference of negative attitudes to the learners. In addition, the inside classroom component is intended as a workshop to facilitate the development of problem solving techniques with live guidance from an instructor. This means that the activities of this component remain interactive between the instructor and the learner and/or among learners, allowing the learner to maintain more control during the problem solving instead of being a passive listener during a traditional lecture. Because of the outside classroom component, this would mean that there is more meaningful time spent on facilitation of problem solving in the classroom. The "Flipped Classroom" instructional approach is one specific example of allowing learners to gain control and play an active role within a designed learning element. More generally, any design element for instruction that aims to gain individual access to learner's control is a worthwhile response to the implications of Major Theme (5). This implication is designed in line with the integral education commitments to be "active," "contemplative," "connective," and "ecological" and in line with injunctive and validative strands of integral learning.

Helping Learners to Reconceive Mathematics—A Dual Instructional Approach. Another prominent theme from this current study is how learners perceive mathematics as a duality (Major Theme (3A)). Traditional instruction, however, often treats mathematical topics as a single process—for example, applying the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find the measures of the sides in a right triangle. Or, traditional instruction comes in the form of multiple representations—for example, solving quadratic equations $ax^2 + bx + c = 0$ (with $a \neq 0$) both algebraically and graphically. Because mathematics is traditionally conceived through a formal, logical system, the inter-relationship among steps and procedures is often assumed and implicit. To improve instruction, one could help learners to reconceive mathematics in a dual manner—both conceptual and procedural—respectively as the rational and functional approaches:

- *Rational Approach*: Instruction that targets the concept and the inner workings of a mathematical idea. For example, the illustration of completing square to derive the quadratic formula is a rational approach to solving quadratic equation.
- *Functional Approach*: Instruction that targets procedural, step-wise nuances and the presentation of solutions. The illustration of the step-wise use of the quadratic formula to solve equations and the presentation of its mathematical work to make the solution readable is an example of instruction that places emphasis in the functional approach.

In my past teaching, I have attempted to counsel my students that every topic could be learned twice. The rational approach to understanding a concept demands a very different skill set than the functional approach to mastering the procedural mechanism and nuances for presenting mathematical solutions in a readable manner. The distinction between the two approaches is analogous to teaching language arts and writing. Understanding a mathematical concept is a process like brainstorming and collecting logical and reasonable ideas for writing, and this is what the rational approach should target. Mastering the steps, mechanisms, and nuances for a procedure is like putting the ideas down on paper coherently with conventional grammar and mechanics, and that is what the functional approach should target. While the teaching of writing generally differentiates the two as separate learning skills (beginners working on grammar and mechanics and advanced writers working on refining ideas), traditional mathematics instruction does not distinguish the two. Here, I argue that using the dual instructional approach is more learner-oriented, and one could achieve better learning gains in an anxious-friendly environment. This implication addresses the commitments of integral education such as "skillful" for the functional approach and "critical" for the rational approach, thus providing a perspectival culture, making the system sustainable or "ecological." Furthermore, this implication targets the illuminative and interpretative strands of integral learning; that is to say, "What is happening?" as a functional approach and "What does it mean?" as a rational approach.

Fostering and Sustaining Positivity in the Learning Environment. This implication responds to Major Themes (1B), cultural beliefs and (3B), discrepancy of expectations in social interaction. Recall from the previous chapter on MALP cyclicity that frustration and negativity fester within the cycle and consequently engender anxiety in a learner, and this instructional implication addresses the learner's psyche by fostering and sustaining positivity in instruction. This means that the instructor must be vigilant in actively listening to the learners' interaction and counsel them when the interaction becomes a fellowship ground that develops negativity. Consider the interaction between Gerri and her granddaughter on using the "Big 7" strategy for division. Both were frustrated because Gerri wanted to show her granddaughter how to divide through long division. Her granddaughter, on the other hand, knew that her teacher may not accept the quotient solutions unless they were obtained through the "Big 7" strategy. While it is beyond the scope of this study to find out why Gerri's granddaughter did not perceive long division as a suitable procedure, I speculate that this was purely a preference of instructional design made by the granddaughter's mathematics teacher. A further conjecture is that the teacher did not make it clear to the students that long division is generally an acceptable mathematical procedure, for it seems the instructional goal on that particular assignment was to have the students practice the alternative “Big 7” method. As a result, the interaction did not produce meaningful learning gain for the granddaughter, and it also affected Gerri—who is a conceptual learner—in a negative way. Suppose this incident were to take place in a classroom where an instructor was present. The negativity and frustration could be completely averted through redirection in social learning. Recall that adult learners are self-directed. They can be motivated when the appropriate reason is made known to them, giving them a purpose and internal motivation to practice whatever topic at hand. Hence, the implication for mathematics instructors is a reminder to carefully observe any social learning interaction and to preemptively redirect and/or intervene among the learners when signs of negativity appear.

The idea of fostering and sustaining positivity in the classroom environment and instructional discourse also helps revitalize the stale aura that was discussed in the past chapter. Recall that the staleness was caused mostly by unmet needs of various learning styles, resulting in intimidation among the learners, who stayed tacit and passive during learning. This is also a sign of frustration developing during the individuals’ perception of their own (inadequate) learning. Therefore, this qualifies as an opportunity for an instructor to intervene to break the stale aura and redirect the learners’ attention into a more positive attitude toward the learning activity.

When it came to the question of how to change the perception of mathematics in a learner, Perry (2006), motivated by neuro-processing, suggested learning interactions as follows:

> A creative and respectful educator can create safety by making the learning environment more familiar, structure, and predictable. Predictability, in turn, is created by consistent behavior. This implies not rigidity but rather consistency of interaction. The invisible yet powerful web of relationships that effective educators cerate between themselves and learners, and between and among learners, is crucial to an optimal learning environment. (p. 27)

Perry’s suggestion clearly advocates a consistency in the mathematics learning environment when recognizing and addressing the learners’ dual perception in mathematics. Agreeing with Daloz (1999), Perry concluded that “consistent, nurturing, and sensitive attention to the learner's state of mind" can develop a sense of safety in adult learners (as cited in Perry, 2006; p. 27). In turn, one could possibly be able to effectively and adequately address the learners' perceptions in a non-threatening/fearful/anxious manner. This implication deals with the following the commitments of integral education: "active," "somatic," "ethical," and "global." It also corresponds to the interpretative and validative strands of integral learning.

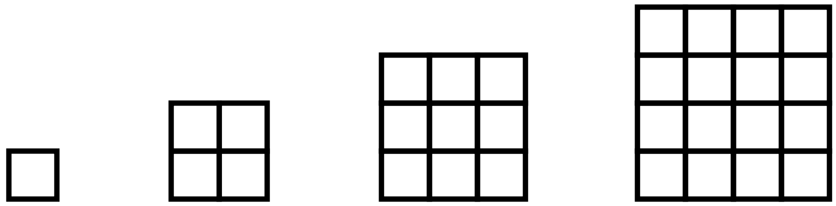
Creating a Parallel Context for Application. This instructional implication is inspired by Jon's story about how he was engaged to learn about Shakespeare, and more generally it is inspired by the Major Theme (2) on the roadblocks of learning mathematics. The teacher in the story made Shakespeare's literature applicable to students by showing them that it paralleled their own sensibility in a way to which they could relate. Similarly in mathematics instruction, one could also strive to create a parallel context to help anxious learners engage themselves so that they could relate to the mathematical skills at hand. Instead of using the passé canned response of telling the learners that geometry is useful for mechanical engineering, it might be more motivating for learners if the classroom environment is contextualized as a problem that the learners must solve by using the developed skills. This way, one could strive for instruction to have a sense of immediacy in application, and adult learners could readily attempt applying mathematics skills to problem solve. In a hands-on manner, adult learners could find meaning and interrelations of mathematical skills and procedures while problem solving, thus allowing them to gain control in their own learning.

I will cite an example from my own teaching. Instead of organizing the developmental mathematics curriculum by skills and procedures such as the "Logarithm" unit, I organize it by contextualized topics, such as "The 2011 Japanese Sendai earthquake and tsunami" a unit of study on logarithm. In this example, I first engaged the learners through the videos and visuals of the devastation of the earthquake and tsunami, allowing them to "feel" the severity of the natural disaster. Then we discuss how intensity of energy is measured, through a logarithmic scale, and segue into a more formal discussion of logarithm. This 2011 natural disaster is particularly fitting because both earthquake energy, measured in Richter's scale, and radiation energy, measured in micro-Sievert (μSv), are logarithmic. Learners who experienced the entire topic were surrounded by how the measurement works. Furthermore, when I began asking students to solve problems on logarithm, I made sure that the problems were relevant and related to calculation of energy. Finally, when the learners were comfortable working with those particular scales, I introduced other more traditional logarithmic scales such as how to solve problems with half-life and depreciation. The end result was that while the learners may not have immediately related to an earthquake and/or tsunami, they still felt a sense of sympathy toward what happened in Japan, and that emotion was a link to help them connect the mathematics and science involve in situation like this. Compared to the traditional instruction by which one usually introduces the application of logarithm in half life and depreciation, learners could more immediately engage themselves to thoroughly appreciate the mathematical topic through a contextualized theme (Casper & Yuen, 2011). This particular implication can be generalized however an instructor sees fit to parallel a context in the classroom for learners to make a concept immediately applicable. This implication realizes the commitments of integral education such as "practical," "contemplative," "perspectival," and "social." Meanwhile, it also corresponds to the illuminative, interpretative, and validative strands of integral learning.

Supporting Learners to Redefine Mathematics as a Subject for Logical Reasoning. Last, but not least, the fifth instructional implication is to help behavioral learners redefine the subject of mathematics. This implication is inspired by Major Themes (1A), personal beliefs; (3A), duality; and (4), the necessity of strong rote memorization. Throughout this study, the lived experience data showed that the learners who prefer prescribed steps are indeed similar to how Givvin et al.'s (2011) model described them—a belief that mathematics means rules, procedures, and notation. Together, this belief and the vocalization of preferring learning in this fashion create a stale aura not conducive to conceptual learning. To rectify this particular social learning issue, it is of utmost importance for instructors to help learners perceive mathematics as a tool in logical reasoning. In other words, a learner's view should be developed and facilitated through the dual rational and functional approach so that one believes that doing mathematics is just problem solving. Obviously, altering one's personal belief is not an overnight event. Rather, this should be an ongoing endeavor, perhaps throughout the entire developmental mathematics course, to help adult learners gain experience in using mathematics for problem solving. This may result in greater meaning for the learner that mathematics becomes a logical reasoning tool that guides and assists them in navigating the problem solving process through the rational approach. When behavioral learners begin to perceive mathematics as a logical reasoning entity, then they might begin to attempt to learn mathematics as conceptual learners. In line with the previous implication, the stale aura of the mathematics classroom may then begin to dissipate. In sum, this instructional implication calls for utilizing the rational and functional approach strategically, during direct instruction and during social learning among learners, to nurture them into operating in a more conceptual manner. Through consistency (Perry 2006) and MALP Component **[C]** of belief adjustment, learners will accumulate successful and positive problem solving experience through logical reasoning. Gradually, these learners will recreate a different vision of mathematics and come to see the importance of logical reasoning and its relationship to and usefulness in learning and doing mathematics.

An example that reflects this implication is to help learners define the notion of a "perfect square." Of course, the formal definition is often used: an integer n is a perfect square if $n = m^2$ for $m \in \mathbb{Z}$. However, the reliance of the definition alone (and such reliance is common place in a purely functional instructional approach) reinforces the procedural habits of behavioral learners. However, a problem solving activity for learners to arrange a number of square tiles into a large square could be an activity that helps learners to "visualize" the notion of perfect square:

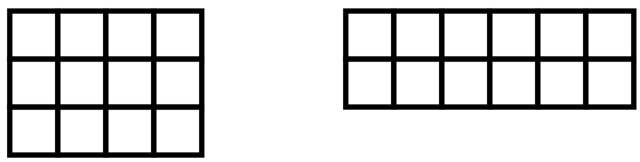
- One, four, nine, and sixteen are perfect square because each respective number of square tiles can be arranged into a larger square:



$\mathbf{1 \times 1 = 1}$ $\mathbf{2 \times 2 = 4}$ $\mathbf{3 \times 3 = 9}$ $\mathbf{4 \times 4 = 16}$

Figure 7.3A. Four sample visual representations of perfect square integers.

- Twelve is not a perfect square because twelve square tiles cannot be arranged into a larger square. Rather they could be arranged into either a 3×4 or a 2×6 rectangle:

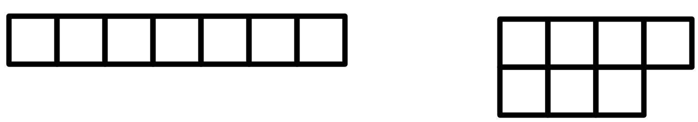


$\mathbf{3 \times 4 = 12}$ $\mathbf{2 \times 6 = 12}$

Figure 7.3B. Two sample visual representations of non-perfect square integers.

To help a learner visualize the inter-connectedness of integers, one can also introduce the notion of prime number as the number of tiles that could only be formed into a rectangle as a single row:

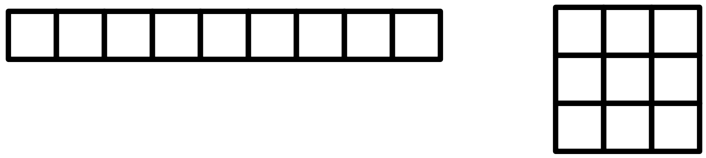
- Seven is a prime number because a single row of seven tiles is the only arrangement that forms a 1×7 rectangle:



Reasoning Achieved **Needs More Development**

Figure 7.4A. A sample visual representation of the conceptual development of prime numbers.

- Nine is a composite number because there are multiple ways to arrange nine tiles into larger rectangles with an understanding that a square is a rectangle, such as 1×9 and 3×3:



$\mathbf{1 \times 9 = 9}$ $\mathbf{3 \times 3 = 9}$

Figure 7.4B. A sample visual representation of the conceptual development of composite numbers.

The idea of using investigative activities such as the above is to help procedurally-oriented learners to break-free from heavy reliance on procedures, rules, and memorization. By continually reinforcing such breaking-free, one may be able to alter the deeply ingrained learner's belief, resulting toward a more conceptually-oriented discussion and environment that is conducive for learning gains. . This implication advocates for the following commitments of integral education: "skillful," "critical," "ethical," and "ecological", and it encompasses all four strands of integral learning.

Paralleled to Wilber's Integral Model, the above five implications are meant to be presented from individualistic to collectivistic perspectives. They aim to help learners to gain active control by taking advantage of the dual perception in mathematics, and ultimately, they aim to foster positivity in the learner's mind so that one can reconceived mathematics as problem solving and logical reasoning. In that sense, the implications respond to D'Amour's (2013) idea of recognizing learner's internal learning process. In addition, each implication is motivated by the major themes developed in the previous chapter as well as by the nature of MALP. Each also responds to the past research literatures such as Wieschenberg's (1994) idea of helplessness and Schoenfeld's description on students' ritualistic way of problem solving (i.e. in five minutes or less and then give up). More importantly, the implications also aim to deemphasize reliance on rote memorization, which Ashcraft et al. (2002; 2005; 2009) documented to be problematic for mathematics anxious individuals to retrieve. In short, the implications are informed by both current and past research.

Concluding Remarks for the Instructional Implications. Although not usually discussed in the context of the Integral Model, some of the implications are already being advocated in literature. For example, the National Center for Academic Transformation (Twigg, 2011) suggests the Emporium Model in which learners are engaged in highly interactive (and in small group) lab sessions, similar to the "Flipped Classroom" model described earlier. The goal is to engage learners to constructive sense making in their own learning experience, as opposed to default to helplessness or haphazard applications that lead to conceptual atrophy. This is certainly in line with helping learners to reconceive mathematics and with supporting learners to redefine mathematics for logical reasoning. The Emporium Model also advocates for learners to choose and engage in their own materials, helping them to develop a sense of active control in their own learning. While there are similarities between the proposed implications to various instructional models, one argues that reconsidering any of the models through Integral Model can produce a more robust analysis for practitioners. In turn, learners would gain a richer, deeper, and integral learning experience to shape their outlook toward what and how they learn.

To sum up the five instructional implications toward an integral approach of mathematics education: their qualities span all 12 commitments in integral education and all four strands in integral learning. Altogether as an integral approach, implementing these instructional implications could improve developmental mathematics instruction to foster a mathematics anxious-friendly learning environment for adult learners.

Self–Evaluation on the Guiding Principles for Mathematics Education Research

One final analysis before concluding the dissertation is to evaluate this research study against the Lester (2010) and Harel's (2008) guiding principles for Mathematics Education Research (MER), recalled as below:

(1) The goals of MER are to understand fundamental problems concerning the learning and teaching of mathematics and to utilize this understanding to investigate existing product and develop new ones that would potentially advance the quality of mathematics education.

(2) To achieve these goals, MER must be theory based, which means studies in MER must be oriented within research frameworks.

(3) The research framework's argued-for concepts and their interrelationships must be defined and demonstrated in context, which is entailed by Principle (1), must include mathematical context.

(4) The ultimate goal of instruction in mathematics is to help students develop ways of understanding and ways of thinking that are compatible with those practiced by contemporary mathematicians.

The current research on MALP fulfills Principle (1) successfully. From the beginning, I have aimed to study and strived to understand the nature of anxiety as a fundamental problem concerning the learning and teaching of mathematics. Through the fourfold perspectival disclosure of MALP, I discovered implications that could improve instruction to advance the quality of mathematics education. As for Principle (2), the research was underpinned by Givvin et al.'s (2011) hypothetical model and Knowles' andragogy. Furthermore, the mode of inquiry was framed through Wilber's Integral Model. These theoretical relations help this study to anchor its findings in the research landscape in an inter-related manner, both responding to past research and adding new insights to mathematics anxiety as a research topic. As for Principle (3), the five instructional implications in this conclusion chapter were argued based on the lived experience data. While they are not "proven" in context, my own expertise as a seasoned practitioner provides insights that they would be well suited for adult learners in developmental mathematics. Furthermore, one of the most important findings in this study showed that the six mathematics-anxious adult learners each perceived through instruction and conceived in their own conceptual development that mathematics is a construct of duality. This is unlike Renert and Davis' (2010) claim of the five mentalities of mathematical knowledge. This finding speaks directly to the Principle (3) on mathematical context. Finally, Principle (4) suggests that MER should contribute to help learners develop ways of understanding and thinking, and the instructional implications fulfill this principle.

Obviously, Givvin et al.'s account that learners see mathematics as rules and procedures showed that learners of developmental mathematics do not view mathematics as a manner of inquiry for problem solving. Advocating learner's control, changing a learner's perception by fostering positivity, facilitating application, and redefining mathematics are all aimed to help learners re-conceptualize how to better engage in mathematical problem solving. At the level of developmental mathematics, this addresses Renert and Davis' (2010) oral, pre-formalist, and formalist stages, and therefore, I argue that guiding principle (4) is fulfilled. The above evaluation sums up the potential impact of this study on the current research landscape.

Emerging Research Agenda

This current research study has begun to address one of my most mystifying observations as a practitioner—mathematics learners often openly express their own negative experiences. In turn, this observation begs several questions:

- How can these expressions inform educators and researchers?
- What are the value systems of the learners?
- What are their assumptions, cultural norms, and sociological consequences?
- And what are the implications of these expressions among the triad of learners, educators, and the subject matter?

The findings of this dissertation show further opportunities for research in this topic. First and foremost, the small number of six participants in this study is a mere beginning to achieve an integral view of MALP. Thus, the most obvious continuation of this research would be to expand the lived experience data to include in-service and pre-service mathematics teachers, school administrators, parents, as well as other individuals who might have a stake in mathematics education. All of these could be further examined through the Integral Model's notion of AQAL, an inquiry which this dissertation has started. The notion of AQAL, recalled from the discussions in the previous chapters, represents a comprehensive disclosure of rich details of a phenomenon in all quadrants, all levels, all lines, all states, and all types. The results of the current research study have begun to understand mathematics anxiety in an integral manner by concentrating on the details in the quadrants. Nevertheless, the lived experience data already showed varying levels of complexity of the psychosocial interplays in and among learners and their instructors.

For example, in the individual upper quadrants and among the behavioral learners, some of them might be more sophisticated than merely memorizing steps to solve mathematical problems, attempting to conceptually understand the underlying reasoning of some, but not all, of the steps. These "mixed" style learners would have different levels of development and perhaps different lines to underscore the varying paths showing their shifts from behavioral to conceptual learning. When the developmental levels and cognitive lines are studied carefully, one perhaps can document the different learning styles are temporary states, and there may be different types of patterns of how they learn mathematics or how their dual views of mathematics have evolved. Similarly, a continuation of integral disclosure of MALP can be advanced by examining the levels and lines in the collective lower quadrants to illustrate the cultural and social interactions and tensions between the behavioral and conceptual learners in classroom environments. By expanding the lived experience data as well as the deepened integral analyses, one could operationalize the essence of MALP to all levels, all lines, all states, and all types. In sum, one would disclose a more integral, unobscured view of MALP so that one could reach a richness and nearness to the phenomenon and simultaneously make use of the newly found knowledge and insights to further inform instruction and mathematics education.

Furthering the research trajectory with the continuation to adopt Wilber's Integral Model and its notion of AQAL, mathematics anxiety has been primarily studied in the fields of psychology and cognitive science. However, the integral research trajectory can study mathematics anxiety distinctly within the educational perspective. If this were the case, one would ultimately broaden and deepen the understanding of how mathematical concepts develop under the anxious state and how transforming learning environments can positively impact those learners. By continuing to employ the Integral Model as the underlying framework, one may continue to obtain new findings, unique to the educational perspective, that traditional and empirical research may be unable to disclose. To further engage in the integral research, one can extend Wilber's Model to developmental learning theories. For example, Biggs and Collis' (1995) and Pegg and Tall's (2005) Structure of the Observed Learning Outcome (or the SOLO taxonomy) as a developmental model appeals to the notions of uni-structural, multi-structural, and relational conceptual objects. These elements could then be investigated through respective lines of development in each quadrant. This area of research, when pursued, can possibly bridge the gap of differing views of mathematics among mathematicians, mathematics educators, mathematics learners, and the general public. Nevertheless, in addition to using the expected frameworks for andragogy, such as Knowles and Associates (1984) and Knowles et al. (1998), it would be fruitful to pursue the investigation of the bridge between andragogy and pedagogy using Robert Kegan's (1994) developmental model, which is highly compatible with Wilber's Integral Model. Wilber's own analysis of Kegan's model leads to this enthusiastic endorsement:

> Kegan's approach is especially important, in my view, because he so clearly elucidates the nature of embedding (identifying) and de-embedding (transcending), which marks each major wave of self-development. His books *The Evolving Self* and *In Over Our Heads* show why a developmental approach is so important… (Wilber, 2000c, pp 42-43)

In sum, the emerging research exploration within theoretical frameworks is tremendous.

From a practitioner's point of view, I believe it is my duty to continue to refine my own craft of teaching through the implications found in this study. Particularly, if I were in a position to educate pre-service teachers, then I could act as a bridge between the researcher and their potential approach to mathematics instruction. It would be prudent to show them how Wilber's Integral Model can be useful to disclose educational phenomenon as well as fostering integral education so that they can begin to understand the significance of the MALP implications and to apply them as they develop their own crafts of teaching. Similarly, it is of equal importance to work with in-service mathematics teachers and school administrators. While mathematics anxiety is well-known to students and educational professionals, its nature is rather illusive among educational professionals. Consequently, working with these professionals as well as with parents will benefit K–12 students by fostering mathematics anxious-friendly learning environments. Moreover, it is my hope, after developing a good working relationship with school districts, to initiate an after school program to support positive mathematics learning by involving both parents and children. This program would aim to empower both parents and learners to take control over their own education instead of maintaining passive roles. It is my belief that empowerment could be the key to abate mathematics anxiety in the social learning setting, and subsequently. Such empowerment would likely generate a more positive social attitude toward the subject of mathematics.

This research study has profound potential impact on my future, both as a post-modernistic researcher and an "integralist" practitioner. The emerging agenda is for my own continual professional growth, for working with pre-service and in-service educational professionals, and for initiating an after school program. The agenda strikes a well-balanced equilibrium between the quest for understanding and the considerations of use, an ideal in the Stokes' Pasteur's quadrant discussed in Chapter 1.

Final Thoughts

At the beginning of this dissertation, I started with my own personal statement and an informal capture of my past experience as an educational practitioner with three different vignettes. I will end the dissertation with one last vignette that happened in my classroom not too long ago (I had just completed one of the dissertation drafts when this happened, and the instructional implications were clearly fresh in my mind.)

Vignette 4. *I was finishing up a group investigative activity with a class, and most students finished on time. However, one student was struggling with the activity, and her group mates completed the task and left. She sat at her desk thinking intently without realizing that her group and the rest of the class were gone. As the students from the next class began walking into the room, one of them saw her and decided to help. They were talking and discussing how to answer some of the mathematics problems in the activity sheet, and she clarified her thinking in just a mere few minutes. She thanked him for his help, and she left the classroom. While this particular interaction was quite common, what struck me as intriguing was the following. When the next class began and I explained the (same) investigative activity to the class, I thought that the male student would be all set, and he would need very little of my attention compared to the other students. Contrary to my thinking, he sat there as if he saw a ghost. He couldn't do it. When I asked him what happened, he told me that he froze, and he couldn't. "But didn't you just help the girl in the previous class?" He responded that he could do the mathematics then, but he did not know what happened and he just couldn't do the same mathematics when he was at his desk. As a result, I asked him what was the difference between the mere ten minutes time when he was helping the girl and now he was working in class. He couldn't quite tell, and so*

I suggested that working in the hallway might make a difference. Sure enough, he finished the activity in the hallway without needing any help.

Before I embarked on this research study, I never thought that mathematics anxiety could be as paralyzing for the learner as in the above vignette. Clearly, the learner had good mathematical skills and was ready to participate in the learning activity, but the formal classroom setting induced the anxiety to the point that he could not function properly. By giving him some choices in a positive manner so that he could play an active learner's role, he managed to overcome the paralyzing effect and to reengage himself back into the learning activity. This story is one of the many specific reasons why submerging myself in the lived experience of mathematics-anxious learners is ever so important. This study is intended to allow the research a nearness and openness to the experiences of the participants and to search for new meanings from them. Ultimately, such a study aims to reach a sensibility about MALP. Through the stories told by my own students and other mathematics instructors, I became interested in the possibility of finding a way to research one's experience to inform education. Also, through the culmination of years of hope and doctoral training, I am finally able to achieve my goal of researching the nature of mathematics anxiety. In the end, it is through engaging myself as an appreciative perceiver that has made me more conscious and more sensitive to the needs of anxious adult learners. As I continue to reflect on the trajectory of my own path as a researcher, I envision that the collection of lived experience from different groups of individuals will continually provide a refined integral perspective of MALP, making it a life-long quest to improve and refine my teaching to strive for a mathematics anxiety-friendly environment for my students.

When I started my doctoral training, I was not sure how to conduct research based on the learners' lived experiences, which I, as a mathematics teacher, witnessed on a daily basis. Now, I hold these stories and experiences personally and professionally as they are dear my heart because they were the eyes, the windows to the soul of this dissertation. After conducting research, I now see my students' experiences in an integral light. Looking back at my educational experience at University of Calgary, the training I received has been true to its motto *Mo Shùile Togam Suas*; it has indeed lifted up my eyes.

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Appendix A: Informed Consent Form as Approved by the CFREB (Page 1 of 3)

UNIVERSITY OF CALGARY

Name of Researcher, Faculty, Department, Telephone & Email:

Chris L. Yuen, Ed.D. student, Faculty of Education, 716-803-2635, CLKYUEN@ucalgary.ca

Supervisor:

Dr. Veronika Elizabeth Bohac-Clarke, Associate Professor,
Faculty of Education, 403-220-3363, bohac@ucalgary.ca

Title of Project:

Mathematics Anxiety Learning Phenomenon: Adult Learner's Lived Experience and its Implications for Developmental Mathematics Instruction

***Sponsor*:**

General Research Grant Award, University of Phoenix

This consent form, a copy of which has been given to you, is only part of the process of informed consent. If you want more details about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

The University of Calgary Conjoint Faculties Research Ethics Board has approved this research study.

Purpose of the Study:

The purpose of this study is to seek understanding of your personal experience of mathematics anxiety and how your past experience may have affected how you learn school mathematics. It is expected that the results of the project will help improve future instruction to reduce mathematics anxiety in learning experiences.

What Will I Be Asked To Do?

You, the participant, will complete a short survey and will have up to two hours within two individual interview sessions to describe your experience. Each interview is expected to last between 45 minutes to one hour; interview length varies depending on how much time each participant needs to describe his or her experience(s). You also are asked to write three journal entries, a free writing exercise, describing and reflecting your experiences of how you support others in studying mathematics.

Participation of this study is voluntary. You may decline to participate without penalty. If you decide to participate, you may choose to end the interview at any time without fear of any kind of reprisal. If you withdraw from the study before data collection is complete, any collected data to this point will be retained/used.

Appendix A: Informed Consent Form as Approved by the CFREB (Page 2 of 3)

What Type of Personal Information Will Be Collected?

Should you agree to participate, you will be asked to provide your gender, age, educational level, field of study, and educational history.

Are there Risks or Benefits if I Participate?

Minimal risks are foreseen deriving from your participation in this study. While most adults have no qualms in admitting that they are fearful or anxious about mathematics, the admission of having such feeling may potentially be embarrassing or upsetting to some individuals. In most cases, you will be assured that it is OK to have such fear or anxiousness, and that many people do feel the same way. You may take time to recollect your thoughts and composure before continuing, and you have the right to terminate, without penalty, the participation at any time.

There is no predicted direct benefit to you, individually, for participating in this research. One potential benefit of the research is that it may add to the body of knowledge on this topic.

As a token of appreciation, you will receive a gift card of a value of US$25.00 upon completion of your participation in the research study. You may choose a vendor for the gift card, such as Amazon.com or Target.

What Happens to the Information I Provide?

Anonymity will be guaranteed from the point when you are invited to participate in the study. The researcher or a professional transcriber will transcribe the interview using a word processor. Your name will not be included in the interview transcript or in the final report. The pseudonyms of M1, M2, M3, F1, F2, and F3 will be assigned to the participants by gender.

The interview will be digitally recorded on two audio recording devices. The interviewer or a professional transcriber will transcribe the interview using a word processor. Your name will not be included in the interview transcript or in the final report. A pseudonym will be used instead. The names of cities or people you may mention and other identifying information will also be changed.

Transcribed interviews will be reviewed by you, and you have the opportunity up to two weeks to modify any portion of the transcriptions. Nevertheless, it is up to the researcher included any portion of the transcriptions and/or journal entries in the final report. Printed transcripts of the interviews and journal entries will be analyzed solely by the researcher. No one else will have access to the transcripts to maintain your anonymity.

Audio recordings, transcribed interviews, and journal entries will be stored on a password-protected area of the researcher's computer. They may also be preserved on flash drives and/or compact discs stored in a locked file. Printed transcripts of the interview and printed journal entries may also be kept in a locked file. Audio recordings, transcribed interviews, and journal entries, printed and digital storage will be destroyed by December 31, 2016.

Appendix A: Informed Consent Form as Approved by the CFREB (Page 3 of 3)

Findings, which may include quotes from your interview and journal entries, will be prepared for a dissertation, publication and/or presentation at academic conferences.

Signatures (written consent)

Your signature on this form indicates that you 1) understand to your satisfaction the information provided to you about your participation in this research project, and 2) agree to participate as a research subject.

In no way does this waive your legal rights nor release the investigators, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from this research project at any time. You should feel free to ask for clarification or new information throughout your participation.

Participant's Name: (please print) ____

Participant's Signature ____Date: _______________

Researcher's Name: (please print) ____

Researcher's Signature: ____Date: ________________

Questions/Concerns

If you have any further questions or want clarification regarding this research and/or your participation, please contact:

Dr. Veronika Bohac-Clarke
Associate Professor
Faculty of Education
403-220-3363, bohac@ucalgary.ca

If you have any concerns about the way you've been treated as a participant, please contact the Senior Ethics Resource Officer, Research Services Office, University of Calgary at (403) 220-3782; email rburrows@ucalgary.ca.

A copy of this consent form has been given to you to keep for your records and reference. The investigator has kept a copy of the consent form.

Appendix B: Recruitment Procedures for Research Participants

"Call for participants" was advertised through two channels with a reward system:

(1) "Call for participants" posters were placed in the student lounges of Erie Community College, Trocaire College, and Bryant & Stratton College.

(2) With the permissions of instructors, I spent about 10 minutes in front of developmental mathematics courses to personally invite participants for the study.

(3) Participants who completed the study will receive gift certificate from Barnes and Noble Bookstore. This compensatory practice is common among the research participation conducted at the University at Buffalo, State University of New York.

Participation Procedures:

- Consent form was given to each participant initially to request permission to use his/her lived experiences as research data for the project. The researcher discussed the extent of the participation (this was also printed on the consent form) –recorded interviews and journal entries.
- Recorded interviews were conducted in a small group study room in the participant's college library. The room was a quiet enclosed area that was conducive to recorded interviews, and the college library was a familiar location for the participant. All recordings were transcribed. Both the audio recording and the transcriptions were stored in a password protected USB memory drive.
- The journal entries by each participant were written in the comfort of his/her own home. Submissions of the entries were through a secured email address. The collected data were stored in a password protected USB memory drive.

Appendix B: Recruitment Procedures for Research Participants (Continued)

- All research data were stored in a password protected USB memory drive and in a disc-format as a back-up. All materials were stored in a locked filing cabinet for the duration of the research project. At the end of the project, the data will be stored in the same secured manner for no less than three years. All data will be destroyed afterwards.
- Each participant received a gift card as a token of appreciation for participating in all the recorded interviews and for writing the journal entries. This practice was similar to other research performed through other major research universities in the Western New York area.
- Reporting of the lived experiences of the participants was be anonymous. Participants were coded as M1, M2, M3, F1, F2, and F3 where M and F denoted the gender of the participants, and they were referred to in this report with agreed-upon pseudo-names.

Appendix C: Call for Participants in a Research Study

- Do you want to know whether the way you are learning mathematics is efficient?
- Are there better ways to learn mathematics?
- Do you want your first-hand experience on learning mathematics be heard?

A research study is calling for participants who can share their first-hand experience on learning mathematics, and the study investigates how such experience impact learning. Participants will:

- Be attending two recorded interviews (approximately one hour each).
- Write three weekly journal entries documenting first-hand learning experience.

You may be eligible to participate in the study if:

- You are out of school for at least three years between last you attended school and your current college study.
- You are enrolled in a developmental mathematics courses such as "Introduction to Algebra" or "Survey of Mathematics."
- You enjoy sharing your experience and enjoy writing about them.

Participants who complete the study will receive a gift certificate for Barnes and Nobles Book Store.

If you are interested in the study, please contact:

Chris L. Yuen, Researcher
716-218-8828
CLKYUEN@ucalgary.ca

Appendix D: MALP Research Participant's Short Survey and aMAR Rating Scale

Identifier:__________________________ Date:______________________________________

(1) Have you attended high school? ______________________

(2) Have you ever received a high school diploma? ______________________

(3) If so, what year did you receive your high school diploma? ______________________

(4) If you did not receive a high school diploma, how many years did you attend high school? ______________________

(5) Did you receive a General Equivalency Diploma (GED)? ______________________

(6) If so, what year did you receive your GED? ______________________

(7) What year did you enter college? ______________________

(8) What is your current field of study (i.e. major) in college? ______________________

(9) What math course are you (will you be) taking? ______________________

(10) Do you have any children? ______________________

(11) If so, are they currently in elementary, middle, or high school? ______________________

(12) Do you help them with their school work on a regular basis? ______________________

(13) Do you help them with their **<u>math</u>** school work on a regular basis?

____

(14) If you are asked to write journals about the experience of helping another individual's math school work, would you be comfortable with that? ______________________

(15) Is English your native tongue? ______________________

(16) What kind of math is most difficult for you? If you want, create (and attempt to solve) a problem in the space below to show the kind of math that is difficult to you.

Appendix D: MALP Research Participant's Short Survey and aMAR Rating Scale (Continued)

aMARS Rating Scale

Identifier:________________________ Date:_________________________________

Direction: Rate your anxiety level by checking a box for each of the following statements.

Anxiety Level	*Not At All* ←——————→				*Extreme Anxiety*
1. Opening a math or stat book and seeing a page full of problems	☐	☐	☐	☐	☐
2. Reading a cash register receipt after your purchase	☐	☐	☐	☐	☐
3. Watching a teacher work on an algebraic equation on the blackboard	☐	☐	☐	☐	☐
4. Buying a math textbook	☐	☐	☐	☐	☐
5. Taking an exam (final) in a math course	☐	☐	☐	☐	☐
6. Picking up math textbook to begin a difficult reading assignment	☐	☐	☐	☐	☐
7. Being given a set of subtraction problems to solve on paper	☐	☐	☐	☐	☐
8. Thinking about an upcoming math test one week before	☐	☐	☐	☐	☐
9. Signing up for a math course	☐	☐	☐	☐	☐
10. Picking up math textbook to begin working on a homework assignment	☐	☐	☐	☐	☐
11. Listening to another student explain a math formula	☐	☐	☐	☐	☐
12. Thinking about an upcoming math test one hour before	☐	☐	☐	☐	☐
13. Studying for a math test	☐	☐	☐	☐	☐
14. Being given a set of division problems to solve on paper	☐	☐	☐	☐	☐

Appendix D: MALP Research Participant's Short Survey and aMAR Rating Scale (Continued)

aMARS Rating Scale

Direction: Rate your anxiety level by checking a box for each of the following statements.

Anxiety Level	*Not At All* ←				→ *Extreme Anxiety*
15. Being given a set of multiplication problems to solve on paper	☐	☐	☐	☐	☐
16. Thinking about an upcoming math test one day before	☐	☐	☐	☐	☐
17. Taking an exam (quiz) in a math course	☐	☐	☐	☐	☐
18. Realizing you have to take a certain number of math classes to fulfill requirements	☐	☐	☐	☐	☐
19. Being given a "pop" quiz in a math class	☐	☐	☐	☐	☐
20. Getting ready to study for a math test	☐	☐	☐	☐	☐
21. Taking math section of college entrance exam	☐	☐	☐	☐	☐
22. Receiving your final math grade	☐	☐	☐	☐	☐
23. Walking into a math class	☐	☐	☐	☐	☐
24. Being given homework assignments of many difficult problems that are due the next class meeting	☐	☐	☐	☐	☐
25. Being given a set of numerical problems involving addition to solve on paper	☐	☐	☐	☐	☐

Credit: The abbreviated MARS is reproduced with the permissions of Dr. Livingston and Dr. Martray from their 1989's article "The development of an abbreviated version of the MARS."

Appendix E: Guidelines for Writing Weekly Journals

Record, describe, and reflect your experiences of how you support others in studying mathematics.

- Describe the experience as you live through it. Avoid explanations, generalizations, and interpretations. For example, it does not help to state what caused you to be happy about a test score, why you are bored with certain tasks, and why you feel your child tends to do homework in front of the television.
- Describe the experience from the inside, as it were; almost like a state of mind: the feelings, the mood, the emotions, etc.
- Focus on a particular example or incident of the object of experience: describe specific events, an adventure, a happening, a particular experience.
- Try to focus on an example of the experience which stands out for its vividness, or as it was the first time.
- Attend to how the body feels, how things smell(ed), how they sound(ed), etc.
- Avoid trying to beautify your account with fancy phrases or flowery terminology.

Appendix F: Guidelines for Interviewing Participants

Five core topics:

1. Past and current personal learning experiences in mathematics
2. Study habits for the current mathematics course
3. Experiences in helping with another learner's mathematics study outside of the class. Also, his/her understanding another learner's mathematics ability and academic expectations
4. Comments on their beliefs/emotions/attitudes of what constitutes mathematics, of his/her abilities in the subject. Artifact materials, such as homework assignments and tests, from the current course are encouraged to be used to show how the participants came to those beliefs
5. The participant's role and identity as a mathematics learner in the contexts of a learning community and of the mathematics education discourse

Interviews are conducted by making it clear to participants that there are no right or wrong answers, with the following characteristics:

- Largely unstructured interviews, but centered to the core topics of discussions
- Concrete: each participant will be asked to describe his/her experiences through specific instance, situation, person, or event, and these specifics would be used to explore the whole experience to the fullest.
- The flow of the interviews:
    - Initial Question: This is the question to initiate the participant to share his/her experience. This question is generally asked in "what" and "when" formats.
    - Leading Questions: These are subsequent comments and questions that are based on the content that the participant has brought forward: for example: "Can you think of other similar instances?"

Appendix F: Guidelines for Interviewing Participants (Continued)

- Follow-up Questions: These are questions that assist the participant to unpack details of situations he or she broaches, such as:
    - In what way?
    - Can you give an example?
    - What was it like to discover … ?
    - How did you become aware of it?
    - What did it feel like?
    - What was it like to tell others?
- Final questions: These are questions that close the interview, for instance: "Is there anything more you would like to add?" and the summation question, "Here's what I heard you say … Am I correct?"
- Redirections: If a participant strays into a 3*rd* person experience, then it may warrant redirection such as, "Can you think of an instance that you have experienced a similar event?" Also, silences among interchanges would be respected to provide the participant time to think, recall, and reflect. If a prolonged silence arises, then a redirect may occur, such as "let me see if I heard you correctly, [then paraphrase]."
- Avoidance of "why" questions: these questions may lead the participant to rationalize and to form abstract ideas that are not necessary. Also, these questions may potentially be construed as confrontational. Therefore, it is best to avoid the "why" questions during the interviews.

• Transcripts will be provided in the second interview for the participant to determine the trustworthiness of the data.

Appendix G: Letters of Permission

Subject: Re: Requesting Permission to Use Your 1989's Abbreviated MARS
From: "Alexander, Livingston" <lalexand@pitt.edu>
Date: Sun, November 18, 2012 8:04 pm
To: "clkyuen@ucalgary.ca" <clkyuen@ucalgary.ca>
Options: View Full Header | View Printable Version | Download this as a file | Add to Address Book

Dear Mr. Yuen,

I'm traveling and cannot print and return the form you sent as an attachment, however, I hereby grant my permission with all conditions stated in your email.

Signature Removed Per University Policy

On Nov 18, 2012, at 4:59 PM, "clkyuen@ucalgary.ca" <clkyuen@ucalgary.ca> wrote:

November 13, 2012

Dear Dr. Alexander:

My name is Chris Yuen, and I am currently conducting a dissertation study on mathematics anxiety through the University of Calgary. The funding agency, University of Phoenix's IRB, has requested me to secure a written permission in order for me to use the abbreviated MARS from your 1989's article "The development of an abbreviated version of the Mathematics Anxiety Rating Scale" as part of the research study. I ask if it is possible to obtain the permission with the following conditions:

•I will use this survey only for my research study and will not sell or use it with any compensated management or curriculum development activities.

•I will include the copyright statement and a statement of credit to you on all copies of the instrument.

•I will send your research study and one copy of reports, articles, and related publications that make use of this survey data promptly to your attention.

•I am able to obtain permission to use the instrument at no charge.

•The expected date of completion of the research is December 31, 2013.

I have included a form for your response. Should you have any questions and/or comments, please do not hesitate to contact me by email at CLKYUEN@ucalgary.ca or by phone at 716-803-2635. Thank you for your help in this matter, and I look forward to hearing from you soon.

Sincerely,

Chris L. Yuen
<Survey Permission Letter - Alexander.pdf>
<Survey Permission Letter - Reply Portion.docx>

Appendix G: Letters of Permission (Continued)

Permission to Use an Existing Survey

I, __Carl R. Martray________________________________, give permission for Chris Yuen to use the abbreviated MARS from my 1989's article "The development of an abbreviated version of the Mathematics Anxiety Rating Scale" with the following conditions:

- The survey will be used solely for Chris Yuen's dissertation study and will not sell or use it with any compensated management or curriculum development activities.
- Chris Yuen will include the copyright statement and a statement of credit on all copies of the instrument.
- Chris Yuen will send your research study and one copy of reports, articles, and related publications that make use of this survey data promptly to your attention.
- Chris Yuen will be able to obtain permission to use the instrument at no charge.
- The expected date of completion of the research is December 31, 2013.

Print Name: Carl R. Martray

Signature: *Signature Removed Per University Policy*

Date:

Current version 032012

Appendix G: Letters of Permission (Continued)

Subject: FW: Edwards, M.G. 2005, vol. 18, no. 3, pp. 269-288
From: "Chris Tutill" <CTutill@emeraldinsight.com>
Date: Thu, August 1, 2013 4:09 am
To: "'clkyuen@ucalgary.ca'" <clkyuen@ucalgary.ca>
Cc: "Laura Jenkins" <LJenkins@emeraldinsight.com>
Options: View Full Header | View Printable Version | Download this as a file | Add to Address Book

Dear Chris

Many thanks for your email.

Please allow me to introduce myself, my name is Chris Tutill and I am the Editorial
Support Assistant here at Emerald.

With regards to your request, Emerald is happy for you to reuse the Figure in your
dissertation subject to full referencing and credit of the original publication. I
should also stress that this is on the understanding that the content does not
feature on Proquest or any other commercial aggregator site.

I hope the above has answered your query but should you require any further
assistance, please do not hesitate to contact me.

Signature Removed Per University Policy

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CTutill@emeraldinsight.com| www.emeraldinsight.com

Π Please consider the environment before printing this email

-----Original Message-----
From: Chris L. Yuen [mailto:clkyuen@ucalgary.ca]
Sent: 26 July 2013 23:26
To: Permissions
Cc: mark.edwards@uwa.edu.au
Subject: Edwards, M.G. 2005, vol. 18, no. 3, pp. 269-288

To Whom It May Concern:

Hi, my name is Chris Yuen, and I am a doctoral student at the University of Calgary.
The reason I am writing is to seek permission to use Figure
10 on p. 284 in the following article:

Edwards, M.G. 2005, 'The integral holon: A holonomic approach to organisational
change and transformation', Journal of Organizational Change Management,

vol. 18,
no. 3, pp. 269-288.

The figure will be used in writing my dissertation. Please let me know if you
require any specifics. Thank you for your help in this matter.

Yours truly,

--
Chris L. Yuen

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Bingley, BD16 1WA United Kingdom. Registered in England No. 3080506, VAT No. GB 665
3593 06

Appendix G: Letters of Permission (Continued)

Subject: RE: Your Article: Integral Cycle of Knowledge
From: "Mark Edwards" <mark.edwards@uwa.edu.au>
Date: Mon, July 22, 2013 8:14 pm
To: "clkyuen@ucalgary.ca" <clkyuen@ucalgary.ca>
Options: View Full Header | View Printable Version | Download this as a file | Add to Address Book

Attachments:

Edwards, M.G. 2009, 'An integrative metatheory for organisational learning and sustainability in turbulent environments.pdf	177.56 kB	[application/pdf]	**Edwards, M.G. 2009, 'An integrative metatheory for organisational learning and sustainability in turbulent environments.pdf**	Download
Edwards, M. G. (2005) The Integral Holon.pdf	3.99 MB	[application/pdf]	**Edwards, M. G. (2005) The Integral Holon.pdf**	Download

Hi Chris,

Happy to see my work is of some use to you in your research. I've attached
the
original paper and another paper which might be of interest to you.

(1) Are you the author of the figure?
Yes I am

(2) From which of your articles did the figure originate?

Edwards, M.G. 2005, 'The integral holon: A holonomic approach to
organisational
change and transformation', Journal of Organizational Change Management,
vol. 18,
no. 3, pp. 269-288. (see attached)

(3) If I would like to use it for my thesis, what might be a proper way to
credit
your work?

Seek permission from the publishers - Emerald Group Publishing Limited.
Then use
the appropriate citation and referencing depending on what referencing
system you
are using.

Best wishes

Signature Removed Per University Policy

Dr Mark Edwards

Appendix G: Letters of Permission (Continued)

Assistant Professor

Business School

The University of Western Australia

M261, 35 Stirling Highway

Crawley, WA 6009, Australia

Email: mark.edwards@uwa.edu.au

Telephone: +61 8 6488 5869

www.business.uwa.edu.au

CRICOS Provider Code: 00126G

-----Original Message-----
From: Chris L. Yuen [mailto:clkyuen@ucalgary.ca]
Sent: Tuesday, 23 July 2013 5:11 AM
To: Mark Edwards
Subject: Your Article: Integral Cycle of Knowledge

Dear Dr. Edwards:

Hi, my name is Chris Yuen, and I am a graduate student at the University of Calgary.

The reason I am writing to you is to ascertain a source for a figure (see attached) on integral learning:

http://www.integralworld.net/edwards2.html
http://indistinctunion.wordpress.com/2009/05/23/the-integral-learning-cycle-and-the-map/

Apparently, the figure was blogged by Chris Dierkes', and he referred the originality to your article on integral cycle. My questions are:

(1) Are you the author of the figure?
(2) From which of your articles did the figure originate?
(3) If I would like to use it for my thesis, what might be a proper way to credit
your work?

Thank you for your help in this matter, and I look forward to hearing from you.

Yours truly,

--

Appendix G: Letters of Permission (Continued)

Subject: RE: Edwards, M.G. 2005, vol. 18, no. 3, pp. 269-288
From: "Rowena Wake" <rwake@ucalgary.ca>
Date: Wed, August 7, 2013 1:08 pm
To: "'clkyuen@ucalgary.ca'" <clkyuen@ucalgary.ca>
Options: View Full Header | View Printable Version | Download this as a file | Add to Address Book

Hello Chris,

Thank you for your email.

There is no commercial component to the Vault nor is it a commercial aggregator. The Vault is a collection of University of Calgary theses that are openly accessible to anyone with Internet access at no cost.

It is possible for other sites and aggregators to harvest material from the Vault. It is currently harvested by Google, Google Scholar, Yahoo and Bidu. Once harvested we do not have direct control over the harvested material however as you are the copyright holder for your thesis no one should be profiting from selling copies of it or providing access to it unless you grant them permission.

Please let me know if you have any further questions.

Signature Removed Per University Policy

Rowena Wake
Copyright Specialist / IR Coordinator
Taylor Family Digital Library
University of Calgary
rwake@ucalgary.ca
403-210-6753
http://library.ucalgary.ca/copyright

June 4, 2013

Chris Yuen
EOC Assistant Professor of Mathematics
Educational Opportunity Center
University at Buffalo, SUNY
465 Washington Street
Buffalo, NY 14203

Dear Dr. Chris Yuen:

With approval from the authors of the article *What Community College Developmental Mathematics Students Understand About Mathematics, Part 2: The Interviews*, published in the *Math*AMATYC *Educator*, Vol. 2, No. 3. May, 2011, you are hereby granted permission, without any royalties, to republish or display with proper attribution any content of the article. Please find below a copy of the author's approval letter.

Sincerely,

Signature Removed Per University Policy

David Tannor
Editor, *Math*AMATYC *Educator*
Department of Mathematics
Muskegon Community College
221 S. Quarterline Road
Muskegon, MI 49442

Appendix G: Letters of Permission (Continued)

UNIVERSITY OF CALIFORNIA, LOS ANGELES **UCLA**

BERKELEY • DAVIS • IRVINE • LOS ANGELES • RIVERSIDE • SAN DIEGO • SAN FRANCISCO SANTA BARBARA • SANTA CRUZ

DEPARTMENT OF PSYCHOLOGY
LOS ANGELES, CA 90095–1563

June 3, 2013

David Tannor,
Editor, *MathAMATYC Educator*

Dear Mr. Tannor,

We're pleased to hear of Dr. Chris Yuen's interest in our article "What Community College Developmental Mathematics Students Understand About Mathematics, Part 2: The Interviews (*MathAMATYC Educator*, 2(3), 4-18). We would be happy for you to allow him to reproduce any portion of it, with attribution.

Sincerely,

Signature Removed Per University Policy

Karen Givvin, Ph.D.
Karen.givvin@ucla.edu

Phone: (310) 664-2301 email: stigler@psych.ucla.edu

Appendix G: Letters of Permission (Continued)

5/24/13

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By Jan Travers at 10:48 am, Aug 15, 2013

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Appendix G: Letters of Permission (Continued)

Trocaire College

Thursday, October 25, 2012

To Whom It May Concern:

Chris L. Yuen has been granted permission to conduct research with students at Trocaire College for his dissertation study entitled *Mathematics Anxiety Learning Phenomenon: Adult Learner's Lived Experience and its Implications for Developmental Mathematics Instruction*. Mr. Yuen is permitted to recruit students, use facilities at Trocaire, and use the name of the college on any and all of the research documentation related to this project. This permission is in conjunction with the approval of the Conjoint Faculties Research Ethics Board of the University of Calgary.

Sincerely,

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Ryan Hartnett, Ph.D.

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