THE UNIVERSITY OF CALGARY

IN SITU TENSILE STRENGTH OF SNOW IN RELATION TO SLAB AVALANCHES

by

J. Bruce Jamieson

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Abstract

Recent theories for the failure of dry-snow slab avalanches emphasize shear failure in the weak layer below the slab prior to fracturing at the slab boundaries. Appropriately, there has been considerable research into the shear failure of the weak layers which underlie slabs. However, there has been comparatively little research into the role of the slab in the release of slab avalanches.

The slip plate tensile test was used for measuring the tensile strength of snow in situ and for investigating two hypotheses which relate the tensile strength of the slab to the release of slab avalanches. During the winter of 1986-87 the technique and equipment were refined. The following winter over 550 in situ tensile tests of snow strength were made in and near the Lake Louise Ski Resort in the Rocky Mountains of Alberta, Canada.

The sources of variability associated with test conditions and with certain types of unusual fracture surfaces were identified by regression analysis. A refined set of 457 tests for studying the tensile strength of snow was obtained by rejecting 38 tests with loading times in excess of 5 seconds, 25 tests with notch radii of 1 mm, and 35 tests with fractures at the front or back of the notched tensile zone.

This refined data set consists of an average of 7 tests made on each of 66 layers. The precision of the mean for 7 tests on the same layer was approximately 15% with 90% confidence.

A photographic technique determined that, for loading times of 60 seconds or less, the tensile strain before failure is less than approximately 1%. This result, when combined with the notch sensitivity, establishes that the tests resulted in

brittle failures.

In the density range of $100 < \rho < 345$ kg/m^3, the brittle tensile strength (in kPa) of layers of new, partly settled and rounded snow is well described by:

$$\sigma = 79.7(\rho/\rho_{\text{ice}})^{2.39}$$

In the density range of 190 to 245 kg/m^3, layers of snow which show faceting can be represented by:

$$\sigma = 58.3(\rho/\rho_{\text{ice}})^{2.65}$$

Two hypotheses relating the tensile strength of the slab to the release of slab avalanches were investigated. No results were obtained to support a hypothesis relating a decrease in tensile strength of slabs to an increase in delayed-action avalanche activity.

To test a hypothesis that stronger and thicker slabs will propagate fractures to greater lengths and thus result in wider slab avalanches, an average of 4 tensile tests of the slab was made near each of 13 unconfined slab avalanches. The width of these 13 slab avalanches correlated with the thickness of the slab (correlation coefficient, r=0.84), with the tensile strength of the slab (r=0.70) and with the product of thickness and tensile strength (r=0.82). Since measurements of slab thickness and tensile strength can be made before avalanche activity and because the width of an avalanche does not appear to be affected by the character of the trigger (natural, skier-released or released by explosives), such measurements of the slab may be of use in predicting the size and destructive potential of slab avalanches.

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Notation

A	empirical constant analgous to the strength of ice
A_{cs}	area of the cross section of fracture
a	empirical constant
b	empirical constant
c	empirical constant
d_b	depth of the snow block
d_f	depth of the fracture surface
C_k	indicator variable for k^{th} test condition
c_k	regression coefficient for C_k
F_k	indicator variable for k^{th} fracture characteristic
f_k	regression coefficient for F_k
g	acceleration due to gravity
k	empirical constant
l_b	downslope length of the snow block
M_k	indicator variable for k^{th} microstucture
m_k	regression coefficient for M_k
m_{fr}	mass of the pull frames
N_{column}	column number within subplot
N_{row}	row number within subplot
n	number of tests in a sample

Notation, continued

P	pull force (gauge reading)
p	precision
R^2	coefficient of multiple determination
r	correlation coefficient
T	temperature
t	loading time to failure
$t_{\alpha;\mathrm{n}}$	critical value of t-statistic with n degrees of freedom and a probability of α associated with the tail
W	weight of the snow block and pull frames
V	indicator variable for V-shaped notches
v	coefficient of variation
w_{b}	cross-slope width of the snow block
w_{f}	width of the fracture surface
Y	indicator variable for Y-shaped notches
$\mathcal{E}$	regression residual
$\mathcal{E}_{\mathrm{i}}$	studentized residual (no effect from i^{th} observation)
α	probability of variate lying in tail of statistical distribution
β	slope angle
γ	angle of impending motion for snow on slip plate
ρ	density of snow

Notation, concluded

ρ_{ice} density of ice (917 kg/m^3)

σ tensile strength

$\dot{\sigma}$ approximate stress rate

τ shear strength

Chapter 1

Introduction

1.1 Background

The material properties of snow are important for the construction of snow roads and ski race courses, for the safety and stability of structures built on snow or threatened by the movement of snow, and for forecasting avalanche activity which threatens transportation routes and recreational areas along with the uses of them. Although this study investigates the tensile strength and other properties of snow in relation to avalanches, the findings may be of interest in the construction of snow roads, ski race courses and structures on or near snow slopes.

Avalanches are natural phenomena, which, in North America, occur mainly in thinly populated or little travelled areas. As a result, very few avalanches injure people or damage structures. However, as transportation routes and winter recreational pursuits penetrate more deeply into mountainous areas, people, vehicles and structures are increasingly exposed to avalanche hazards.

In Canada, approximately 900 people have been either injured, buried or killed by avalanches since the mid-1800s (McFarlane, 1986). In 1910, a railway work crew of 62 people was killed by a single avalanche in Rogers Pass (Schaerer, 1987, pp. 9-10). Between 1978 and 1984 in Canada, fatalities averaged 7.7 per year and property damage averaged $340,000 per year (Schaerer, 1987, p. 6). In that period all but one of those who died in avalanches were involved in recreation at the time of the accident. Slightly over half of the fatalities involved either backcountry skiers or helicopter-skiers. In Canada, avalanche accidents are approximately equally distributed between the Rocky Mountains, Columbia Mountains and Coast Mountains (Schaerer, 1987, p. 2).

In addition, closures of transportation routes because of avalanches or for avalanche control cause numerous delays for persons travelling and goods being transportated by those routes. For example, the Trans-Canada Highway through Rogers Pass has been closed an average of 70 times per year since 1965 and the duration of each closure has averaged 2 hours (Morrall, in preparation).

Some avalanche accidents involve decisions by recreationists to travel a particular route or to ski a particular slope under specific weather and snowpack conditions. However, professional avalanche forecasters with current weather and snowpack data, and assisted by computers, also continue to be surprised by the size or occurrence of a small number of avalanches.

Although avalanches are produced by failure of the snowpack, measurements of its mechanical properties have been of limited use in avalanche forecasting:

> Changes in the weather provide the best clues to when and where avalanches are likely to release. With today's technology it is not possible to observe directly subtle changes in the snowpack that signal the onset of its failure and release from the mountainside (Perla and Martinelli, 1976, p. 7).

Clearly, there is a need for measures of characteristics of the snowpack that will better assist both recreationists, professional guides and professional avalanche forecasters in making critical decisions about crossing snow slopes or opening highways and ski runs to the public. This study investigates the tensile strength of snowpack layers, and assesses the usefulness to avalanche forecasting of a particular method for measuring tensile strength. The dependence of tensile strength on more easily observed properties such as density and microstructure is also investigated.

The tensile strength of snow is a measure of the *cohesion* within the snowpack, and it is the failure of cohesive layers or *slabs* of snow that is of interest in this thesis. Such slab avalanches are more destructive and difficult to forecast than loose snow avalanches which involve the failure of less cohesive layers.

1.2 Basic Types of Avalanche Release

Avalanches release from snow slopes in two distinct ways (de Quervain, 1966). "Loose" or "point release" avalanches initiate in snow of low cohesion. A small volume of snow (< 1 m^3), typically the size of a snowball, rotates in a manner similar to the failure of a cohesionless sand (Perla, 1980). As this mass moves downslope, additional snow is set in motion (Figure 1.1). Although such avalanches may occasionally reach destructive size, most are small and harmless.

A slab consists of one or more layers of cohesive snow overlying a weaker layer. Slab avalanches occur when a plate-like portion of these cohesive layers begins to slide as a unit before breaking up (Figure 1.2). Most of the hazard to people and to structures is due to slab avalanches. Such avalanches frequently reach destructive size partly because the initial failure volume is typically orders of magnitude larger

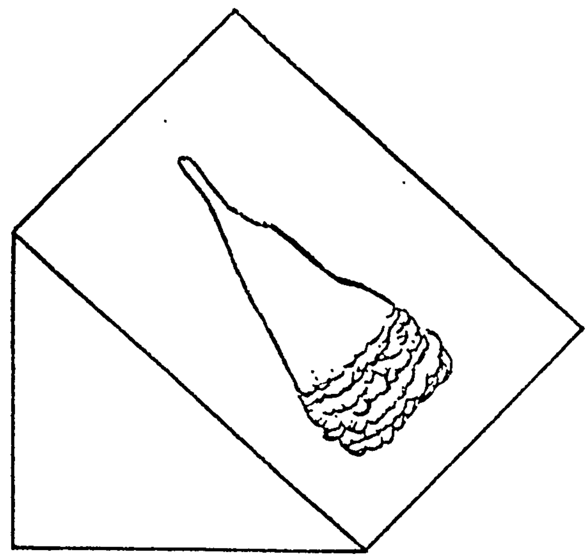

Figure 1.1: Point Release Avalanche

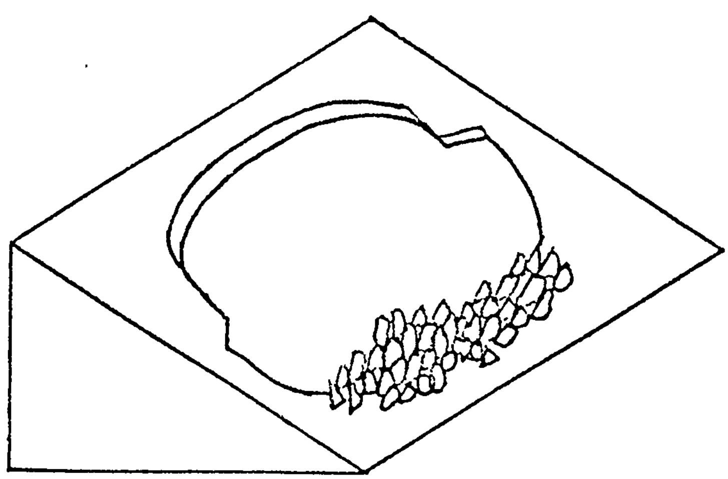

Figure 1.2: Slab Avalanche

than for the "point release" avalanches, and partly because the amount of snow set in motion on the slope below the starting zone is also greater.

Both loose and slab avalanches may occur naturally or may be triggered by stresses induced by explosives, snowmobiles, skiers, etc.

1.3 Mechanisms of Slab Failure

Dry and wet slab avalanches are released by different mechanisms. Most full depth wet slab avalanches are believed to be caused by accelerating glide on the ground-snow interface when liquid water is present (McClung, 1987). However, it is the mechanisms by which dry slabs release that are relevant to this thesis because dry-slab avalanches are a continuing phenomenon throughout the winter season and a major concern in terms of frequency and damage potential.

Perla and LaChapelle (1970), Perla (1975, 1980), McClung (1979a, 1981b, 1987) and others have argued convincingly that dry-slab failure begins in a weak layer of snow below the stronger layers of the slab (Figure 1.3). However, theories vary as to the character of this initial basal failure.

Perla and LaChapelle (1970) and Perla (1975, 1980) propose the following sequence:

1. without fracturing, *a loss of shear support* occurs in the weak layer which transfers load to the slab boundaries;

2. fracture begins at the slab boundaries either as a result of the transfer of load or from an artificial trigger such as a skier or explosion;

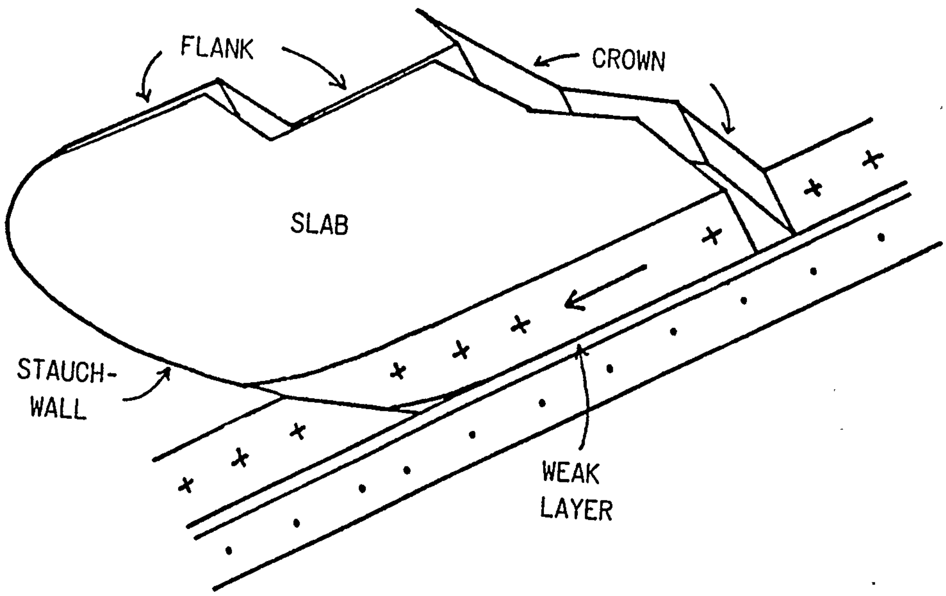

Figure 1.3: Failure of a Snow Slab

3. the initial boundary fracture releases strain energy and provides the displacement necessary to cause shear *fracture* in the weak layer; and

4. basal shear fractures and tensile crown fractures propagate in a synchronized manner.

McClung (1979a, 1981b, 1987) contends that the concentration of creep-induced deformation in the weak layer causes basal shear failure which, in turn, causes propagating basal shear *fractures*. Tensile fracture at the crown follows as a consequence of the basal shear fractures.

A third proposed mechanism involves the failure of deep slabs by the initial collapse of the basal layer (Bradley, 1966).

All of these theories are consistent with field observations and it is possible that not all dry-slab avalanches fail by the same mechanism. However, these theories agree that *basal failure precedes tensile fracture at the crown.*

1.4 Initial Basal Failure

Since shear is the predominant mode of deformation at the weak layer, much research has concentrated on shear strength (e.g. Perla et al., 1982; Sommerfeld and King, 1979) and shear deformation of snow (e.g. McClung, 1977, 1979a). There have been numerous attempts to apply a stability index based on the ratio of shear strength of the weak layer to the shear stress acting in the weak layer (Roch, 1966a; Perla, 1977; Sommerfeld and King, 1979; Perla and Beck, 1983; Conway and Abrahamson, 1984; Föhn, 1987). Methods for calculating this ratio vary as do critical values of the stability index.

The application of the stability index has been limited by uncertainty over rate effects and size effects. For example, although snow strength is known to vary with the deformation rate[1], the deformation rates within weak layers are unknown (McClung, 1981a). Also, McClung (1979a, 1981b, 1987) contends that the mechanism of basal failure is highly rate dependent.

To allow for size effects, specifically the reduced strength of larger volumes (or cross-sectional areas), the strength obtained from practically sized specimens must be statistically extrapolated to larger cross sections (Sommerfeld, 1973; Sommer-

[1] Because of this uncertainty concerning appropriate rates for measuring strength, authors such as Perla et al. (1982) and Perla and Beck (1983) refer to in situ measurements of strength as indicies. The tensile strength measurements in this thesis are also rate dependent; however the term *index* is not used.

feld and King, 1979; Sommerfeld, 1980). Such statistical extrapolation techniques may improve once the distribution of flaws within weak layers is understood. However, the size and distribution of flaws or "deficit areas" within weak layers remain an active research topic (Conway and Abrahamson, 1984, 1988; Föhn, 1988).

At present, the most promising indicator of slab stability, based on measurements of the snowpack, is the stability index, calculated as the ratio of shear strength of the weak layer to shear stress acting on the weak layer. Föhn (1987) proposes a stability index which is adjusted for normal loads, statistically extrapolated to large cross-sectional areas and adjusted for triggers such as skiers or explosions. Based on in situ shear tests, stability indices for 110 avalanche slopes are reported. Scoring a success for slopes that released when the ratio was 1 or less and a failure for slopes that released when the ratio was 1.5 or greater, Föhn obtains a success rate of 75%.

1.5 Role of the Slab in Slab Failure

Notwithstanding the primary importance of the initial failure of a weak layer in the snowpack, the overlying slab plays a role in slab release. Using fracture mechanics, McClung (1987) theorizes that increasing compliance (or decreasing stiffness) of the slab may precipitate an avalanche. Also, some forecasters suspect that stronger and thicker slabs propagate the necessary fractures to greater distances and therefore release in wider and larger slab avalanches (C. Israelson, 1987, personal communication). Although the mechanical properties of the slab are believed to play an active role both in the occurrence and size of avalanches, field data to support or contradict such ideas have been unavailable.

To study McClung's theory and the forecaster's hypothesis, measurements of slab properties in the field are required. Tensile strength appears to be the most suitable property of the slab to measure for three reasons:

1. since it is a fundamental measure of *cohesion,* it may provide a field indicator of mechanical properties such as stiffness or compliance,

2. it is likely to represent the intuitive notion of "strength" in the hypothesis concerning the width of slab avalanches, and

3. relatively large specimens of one or more contiguous layers, as is common for snow slabs, can be tested in situ using an existing test method.

Therefore, tensile strength is used as *an indicator of mechanical properties of the slab.* These applications of the tensile strength of slabs are consistent with basal failure theories, and do *not* assume or require that slab failure begins with tensile fracture at the crown.

However, little was known about the proposed test method. The literature contains only one study which reports the results of 32 tests (Conway and Abrahamson, 1984). Therefore, before the tensile strength of snow could be comprehensively studied, it was first necessary to evaluate the *slip plate* test method itself with respect to precision, limitations and sources of variability.

1.6 Objectives

The six objectives of this investigation are:

- to determine the limitations and sources of variability of the slip plate test;
- to determine the mode of failure caused by the slip plate test;
- to investigate the areal variability of snow strength and to determine the number of tests required to obtain particular levels of precision;
- to measure the tensile strength of snow in situ for the range of density and types of microstructure commonly found in snow slabs;
- to investigate whether a decrease in the tensile strength of snow slabs is associated with an increase in delayed-action avalanche activity; and
- to investigate whether stronger and thicker snow slabs tend to release in wider slab avalanches.

The microstructure of snow affects its strength. For example, Haefeli (1963) reported snow specimens of similar density but different microstructure which differed in strength by a factor of three. The microstructure of alpine snow is introduced in the next section.

1.7 Metamorphism and Microstructure

Initially, the microstructure of a snow layer depends on the shape of the ice crystals which formed in the atmosphere and on the degree to which the crystals are packed together by the wind during deposition. Typically, within a matter of days after deposition the atmospheric forms are obscured by metamorphic processes within the snowpack. The microstructural unit which is relevant to field studies is then the *grain* which may contain parts of one or more crystals. Typically, these grains range in size from 0.1 to 5 mm.

Presently, field workers use qualitative classifications of *grain shape* (Sommerfeld and LaChapelle, 1970; UNESCO, 1970). Such classification schemes are useful for studies of snow strength[2] since the character of the intergranular bonding is related to the shape of the grains.

The shape of the grains and character of the bonds change over the course of days or weeks, according to the dominant metamorphic process. Three such processes are common in the seasonal snowpack:

- low temperature gradient metamorphism[3] (dry snow)
- high temperature gradient metamorphism (dry snow)
- melt-freeze metamorphism

When snow is dry, the temperature gradient is the primary factor which determines the metamorphic process. An average temperature gradient for the snowpack may be calculated from the depth of the snowpack and the difference in temperature between the ground and the surface of the snow.

In temperate climates, the ground-snow interface is at or near 0°C throughout most of the winter. In contrast, the temperature of the snow-air interface varies widely during the winter months. When the air is cold and the snowpack is thin

[2] The UNESCO (1970) classification is used in this thesis.

[3] These names of the three processes are consistent with Gubler (1985). In the literature, low temperature gradient metamorphism is widely referred to as equi-temperature (ET) metamorphism, or sometimes as destructive metamorphism. Similarly, high temperature gradient metamorphism is referred to as temperature gradient (TG) or constructive metamorphism.

(< 1 m) as it usually is during the first half of the winter in the Canadian Rocky Mountains, the high temperature gradient process is common. In the Interior Ranges of British Columbia where the snowpack is typically thicker (2-3 m) and the air temperature is somewhat milder than in the Rocky Mountains, low temperature gradient metamorphism usually dominates.

Field workers often use 10°C/m for the critical temperature gradient that determines which type of metamorphism prevails; however, the critical value increases with decreasing temperature (Armstrong, 1980) and is believed to increase with increasing density (Colbeck, 1987).

The temperature gradient draws water vapour through porous snow layers. The flow of water vapour may vary considerably within the snowpack since it is influenced by the presence of crusts, by particularly warm or cold layers and by features of the ground cover such as rocks, bushes and "wet spots".

1.7.1 Low Temperature Gradient Metamorphism

Below a critical temperature gradient, the specific surface energy of a layer is substantially reduced by the generally complex forms of *new snow* taking on simpler, more rounded shapes. In the early stages of this process when newly fallen forms can be recognized, the grains are called *partly settled.* As the process continues *rounded* grains develop which are joined by comparatively strong neck-shaped bonds (Figure 1.4). Since the grains bond below 0°C, the process is sometimes called sintering. Large grains grow at the expense of smaller grains. Two mechanisms have been proposed for this process:

1. the vapour pressure of water over the concave surfaces of the bonds is greater than over the convex surfaces of the grains, leading to a net transfer, by sublimation, of ice to the bonds (Perla and Sommerfeld, 1987), and

2. a thin liquid-like layer on the surfaces of the ice will flow from the convex surfaces of the grain to the concave surfaces of the bond (Kuroiwa, 1975).

Both mechanisms reduce the specific surface energy of the grain structure. In temperate climates, this type of metamorphism is common, particularly in the latter part of the winter. Snow layers strengthen as a result of this process.

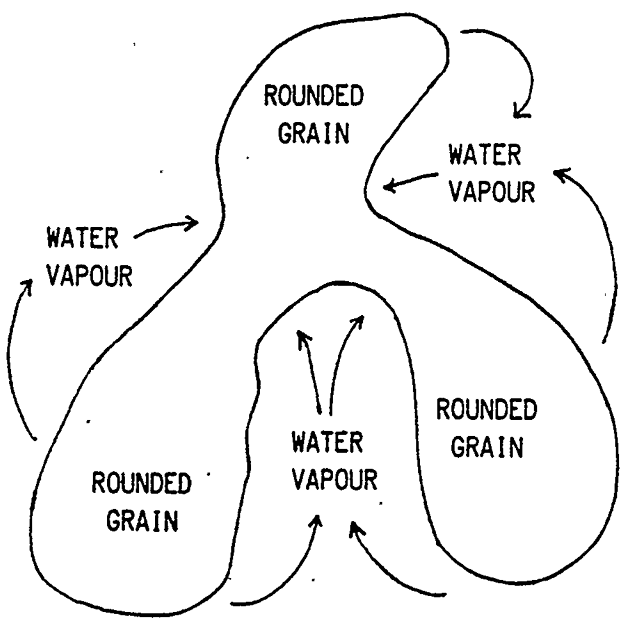

Figure 1.4: Low Temperature Gradient Metamorphism
Typical size of individual rounded grains is 0.25 to 1 mm.

1.7.2 High Temperature Gradient Metamorphism

Below freezing and above a critical temperature gradient, the flow of water vapour from warm to cold will substantially influence the grain structure. Usually, lower layers of the snowpack are warmer than upper layers. Using this common temperature profile as an example, water vapour will flow upwards causing a net transfer of ice, by sublimation, from warmer grains to the cooler, lower surfaces of grains above (Figure 1.5). These grains will grow downwards and take on angular, faceted, stepped or hollow shapes which are in marked contrast to the rounded shapes shown in Figure 1.4. Larger grains will grow at the expense of smaller grains. In the advanced stages when large, often hollow, forms are common the grains are called *depth hoar*. For densities less than approximately 260 kg/m^3, this metamorphic process will result in a progressively weaker microstructure (Akitaya, 1975). In the seasonal snowpack, layers of faceted grains typically have a density below 260 kg/m^3. Therefore, this process is generally considered to cause a decrease in cohesion and reduction in strength.

1.7.3 Melt-Freeze Metamorphism

Snow which is at 0°C may be heated by rain, by solar radiation or by warm air. Under such conditions, liquid water will form between grains. When this liquid water freezes (usually at night), the grains will be strongly bonded into clusters and the layer will become a crust.

Melt-freeze metamorphism is of little interest in this thesis. Below 0°C, layers of melt-freeze grains are comparatively strong and, unless they are quite thin, do not occur in avalanche-prone dry snow slabs. At 0°C, such layers do occur in wet

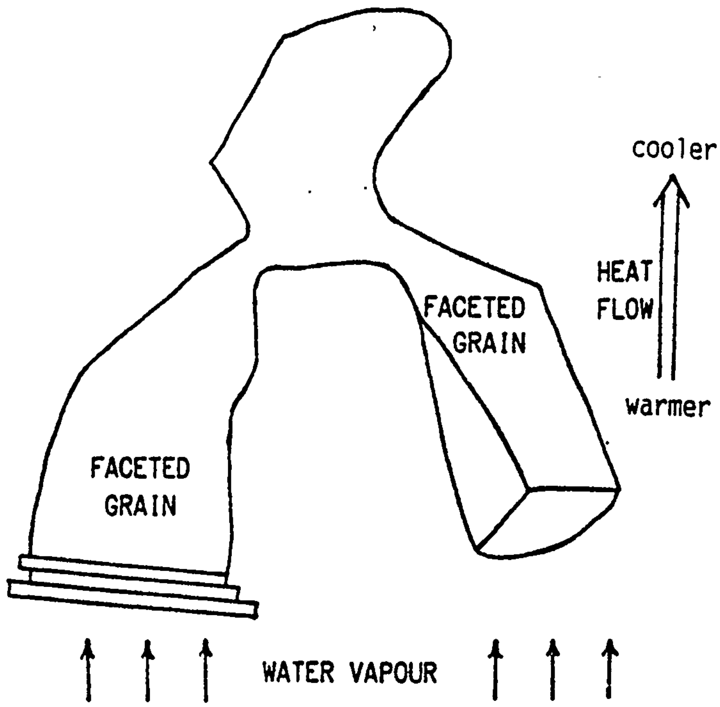

Figure 1.5: High Temperature Gradient Metamorphism
Typical size of individual faceted grains is 1 to 4 mm.

snow avalanches, but such avalanches are outside the focus of this thesis.

Chapter 2

Review of Relevant Literature

2.1 Background

The importance of snow strength in the avalanche problem has long been recognized, but early studies of snow strength were confined to laboratories. In the field, tests to determine hardness or resistance to penetration were used as indicators of strength because they are easier to perform. The first field study of tensile strength (Perla, 1969) was published thirty years after the first laboratory study (Haefeli, 1939).

2.2 Laboratory Tests

Twelve laboratory studies report on the tensile strength of snow for densities of 500 kg/m^3 or less. The three test methods used in the twelve studies are briefly described in this section. For each study, the number of data, the density range of tested snow and the area of maximum tension are given in Table 2.1. The results of the studies are discussed in Sections 2.4 to 2.6.

Since snow slabs can range in density from approximately 60 kg/m^3 to 460 kg/m^3 (Perla, 1977), studies involving tensile tests of snow with a density in excess of 500 kg/m^3 are excluded from this chapter.

Table 2.1: Laboratory Studies of Tensile Strength

Study	Method	Stress, Strain or Spin Rate	Area of Maximum Tension (m^2)	No. of Data	Density (kg/m^3)
Haefeli 1939	load frame	0.6 kPa/s	0.01	19	220-450
Bucher et al. 1948	spin	-	0.002	~60[a]	216-480
de Quervain 1950	spin	-	0.002	13	166-366
Bader et al. 1951	spin	-	0.002	9	384-456
Butkovich 1956	spin	-	0.002	6[a]	317-442
Butkovich 1956	ring	68 kPa/s	"fibre"[b]	1[a]	481
Roch 1966b	spin	-	0.002	82	196-480
Keeler et al. 1968	spin	10 rps[c]	0.002	~175	100-490
Keeler 1969a	spin	10 rps[c]	0.002	~150	125-440
Kovacs et al. 1969	ring	-	"fibre"[b]	32	418-500
Martinelli 1971	spin	2-3 rps[c] ($\approx$ 5 kPa/s)	0.002	53[a]	36-467
Sommerfeld 1974	spin	-	0.011	~160	60-300
Narita 1980, 1983	load frame	10^{-6}-10^{-3} s^{-1}	0.002	210	240-470

[a] number of mean strength values.

[b] test method causes bending which applies the maximum tensile stress to a "fibre" or very narrow band of negligible area.

[c] revolutions per second.

Load Frames

Haefeli (1939) and Narita (1980, 1983) froze platens to the ends of cylindrical specimens which were then placed in a load frame. Haefeli (1939) applied uniaxial tensile stress at an approximately constant rate of 0.6 kPa/s by pouring shot into a small bucket attached to the lower platen. More recently, Narita (1980, 1983) used a wide range of cross-head speeds to test specimens to failure at various constant strain rates. Failure modes and the ductile-brittle transition, as reported by Narita, are discussed in Section 2.6.

Spin Testers

Haefeli (1939) mentioned a *spin tester* that became the most common device for measuring tensile strength in laboratories (Bucher et al., 1948; de Quervain, 1950; Bader et al., 1951; Butkovich, 1956; Roch, 1966b; Keeler and Weeks, 1968; Keeler, 1969a; Martinelli, 1971; Sommerfeld, 1974). Two notches are cut, each half way along opposite "sides" of a cylindrical specimen. The specimen is rotated about the axis between the notches at an increasing rate until the notched cross section fails in uniaxial tension. Strength is calculated from the snow density and rotation rate at failure. Sommerfeld and Wolfe (1972) made numerous improvements including a larger specimen holder, a photo-electric shutoff mechanism and a massive turntable which makes the rotation rate effectively independent of the weight of the specimen. This improved spin tester was used by Sommerfeld (1974).

Ring Tests

The "ring" tensile test has been primarily used for ice. The sides of a thick-walled cylindrical specimen are compressed by a transverse load until a crack spreads from the inner surface outwards (Assur, 1960). Loading is rapid (65 kPa/s) and deformation is assumed to be elastic to failure. The inside diameter of the cylinder is kept small, typically one sixth of the outside diameter. This reduces the stress concentration which is used to calculate strength. However, only a "fibre" or band of negligible area receives the maximum tensile stress. Most seasonal alpine snow is not dense or strong enough for this technique; however, snow with a density of at least 400 kg/m^3 has been tested by Butkovich (1956) and Kovacs et al. (1969).

2.3 Field Tests

Six studies based on five techniques are reported in Table 2.2. The five test methods are briefly described in this section. The results of the six field studies are discussed, along with the results of the laboratory studies, in Sections 2.4 to 2.6.

Cantilever Beam

Perla (1969) tested newly fallen snow by undercutting cantilever beams of increasing length until they failed. With this technique only a surface "fibre" at the top of the beam receives the maximum tensile stress. The calculation of strength assumes tensile failure. However, the shapes of the failure surfaces suggest that some of the beams may have failed in shear (McClung, 1979b). Although the results show considerable scatter, the technique can be used to measure the strength of low density snow.

Table 2.2: Field Studies of Tensile Strength

Study	Method	Rate or Loading Time	Area of Maximum Tension (m^2)	No. of Data	Density (kg/m^3)
Perla 1969	beam	-	"fibre" [a]	~ 250	32-260
McCabe et al. 1978	load frame	10^{-5}-10^{-4} s^{-1}	0.011	4	186-335
McClung 1979b	tilt-table	60-495 s	0.12	38	120-350
Singh 1980	load frame	10^{-5}-10^{-4} s^{-1}	0.011	18	136-294
Conway et al. 1984	slip plate	0-15 s	0.1	14 [b]	60-250
Rosso 1987	trapezoid	-	0.1	13	140-265

[a] test method causes bending which applies the maximum tensile stress to a "fibre" or very narrow band of negligible area.

[b] number of mean strength values.

Portable Load Frame

McCabe and Smith (1978) devised a portable load frame for applying uniaxial strain at controlled rates to snow specimens in the field. As in other studies which used load frames to apply tension, cylindrical specimens were frozen to the platens. Unlike other load frame studies, strain was measured by photographing markers placed on the specimens. Singh (1980) used the same load frame but measured strain with a mechanical gauge. Singh's objective was to obtain stress-strain curves and many of his specimens exhibited visco-plastic deformation without fracture. However, in 18 of the tests, the strain rate was sufficiently rapid to cause tensile fracture. Seventeen of these tests failed with strains of 0.6% or less. The techniques appear viable but, excluding those tests which resulted in visco-plastic deformation without fracture, the two studies (McCabe and Smith, 1978; Singh, 1980) report the strength for a total of only 22 tensile tests. Clearly, many more data are required to establish precision and the influence of testing variables on strength.

Tilt-Tables

McClung (1979b) placed tilt-tables in the field. The top surface of each table consists of a fixed part and a rolling-cart. When the table is tilted, the rolling-cart is free to roll away from the fixed part. Naturally occurring snow on top of the table is tested by gently tilting the table until the snow fails under uniaxial tension through a notched zone between the fixed part and rolling-cart.

Thirty-eight tests were made on snow which ranged in temperature from -6°C to 0°C with an average of -1.7°C. Nine of the specimens included at least one layer of wet snow. The failures occurred after 60 to 495 seconds of gradual tilting. Notch sensitivity was not detected for notch shapes which varied from thin slits to arcs. This lack of notch sensitivity implies that the failures involved ductility which is likely associated with the slow loading rate and the fact that the snow specimens were at, or very near, their melting point.

Although this test method applies uniaxial tension to large cross sections (0.12 m^2), only layers of snow which are naturally deposited on the tables can be tested. Therefore, this method is not suitable to monitoring changes in the strength of snowpack layers or to making tests of arbitrarily chosen snow layers on avalanche slopes.

Slip Plate

Conway and Abrahamson (1984) developed the slip plate technique used in the present study and described in detail in Chapter 3. They measured the tensile strength of 14 slabs and variations in shear strength of the weak layers under the slabs. The tensile strength of the slab was used to calculate the critical size of a "deficit area", that is, an area of the weak layer for which the stability index was less than unity. They proposed that the minimum value of the stability index and the maximum size of a deficit area be used to assess slope stability. Fourteen mean strengths from 32 tests made on avalanche slopes are reported. This number of tests is insufficient for determining the precision of the mean or for assessing the effects of testing conditions on the results.

Trapezoidal Tests

Rosso (1987) devised the trapezoidal test and reports the results of 13 tests. The test method involves isolating a large trapezoidal portion of a snow slab by shovelling two channels in the snow which approach each other at the upslope end. Each trapezoidal block is then undercut along a weak layer with a snow saw, starting at the lower (wider) end until the block fails in tension. Attached to the following edge of the saw is a foam pad which supports the trapezoidal block and reduces the bending moment in the block.

Although the calculation of tensile strength assumes uniaxial tension, bending stresses cannot be ruled out. As well, this test cannot be performed on level terrain because the tension on the block is due to gravity. Finally, each test requires two people for approximately twenty minutes.

2.4 Laboratory Studies Versus Field Studies

Four of the test methods used in the field allow the strength of snow to be measured in situ without the specimens being disturbed by extraction from the snowpack or by transportation to a laboratory. The fifth, the portable load frame used by McCabe and Smith (1978) and by Singh (1980), allows snow to be tested in the field but requires that the specimens be extracted from the snowpack.

Laboratory studies usually store snow specimens for a period of days or weeks before testing. Changes in the microstructure and strength of the specimens are likely to occur between sampling in the field and final testing for most storage conditions.

Field studies do not allow environmental conditions to be controlled, whereas, Bucher et al. (1948) and Roch (1966b) were able to test snow specimens in the laboratory at various controlled temperatures.

Loading rates for laboratory studies of tensile strength[1] have not necessarily been better controlled than for field studies. For the most popular laboratory technique, the spin test, most authors do not state the spin rate and only Keeler (1969a) estimates the stress rate. McCabe and Smith's (1978) portable load frame and Singh's (1980) strain gauge are reported to be suitable for the full range of strain rates used by Narita (1980, 1983) in the laboratory.

In Figure 2.1, the general trends for tensile strength versus density are shown for most of the studies included in Tables 2.1 and 2.2. To facilitate comparisons, the following studies and tests are excluded from Figure 2.1 for the reasons stated:

[1] Laboratory studies of viscoelastic behaviour have used both constant strain rates (e.g. Salm, 1971) and constant stress rates (Watanabe, 1980).

- the ring tests, because only 1 test from Butkovich (1956) and 8 from Kovacs et al. (1969) are for snow within the density range (60 to 460 kg/m^3) reported for slab avalanches;
- tests on faceted grains, depth hoar, coarse grained snow and melt-freeze grains since most results from the studies listed in Tables 2.1 and 2.2 are for new, partly settled, "fine grained" or rounded snow;
- nine tests on wet snow from the data of McClung (1979b) since the presence of free water affects strength (Salm, 1981);
- the study of McCabe and Smith (1978) which includes the results of only 4 tests; and
- the studies of Narita (1980, 1983) which do not report strength as a function of density.

For the studies of Bucher et al. (1948) and Roch (1966b) which report strength of snow samples at various temperatures, only the strengths measured at or close to -10°C are used. For the spin tests of Butkovich (1956), the strengths which have been empirically adjusted to -10°C are used in Figure 2.1.

The results shown in Figure 2.1 are discussed in Sections 2.5 and 2.6.

2.5 Size Effects

Larger specimens are, on average, weaker since they may contain larger and more numerous flaws. Most commonly, size effects for tensile strength tests are discussed

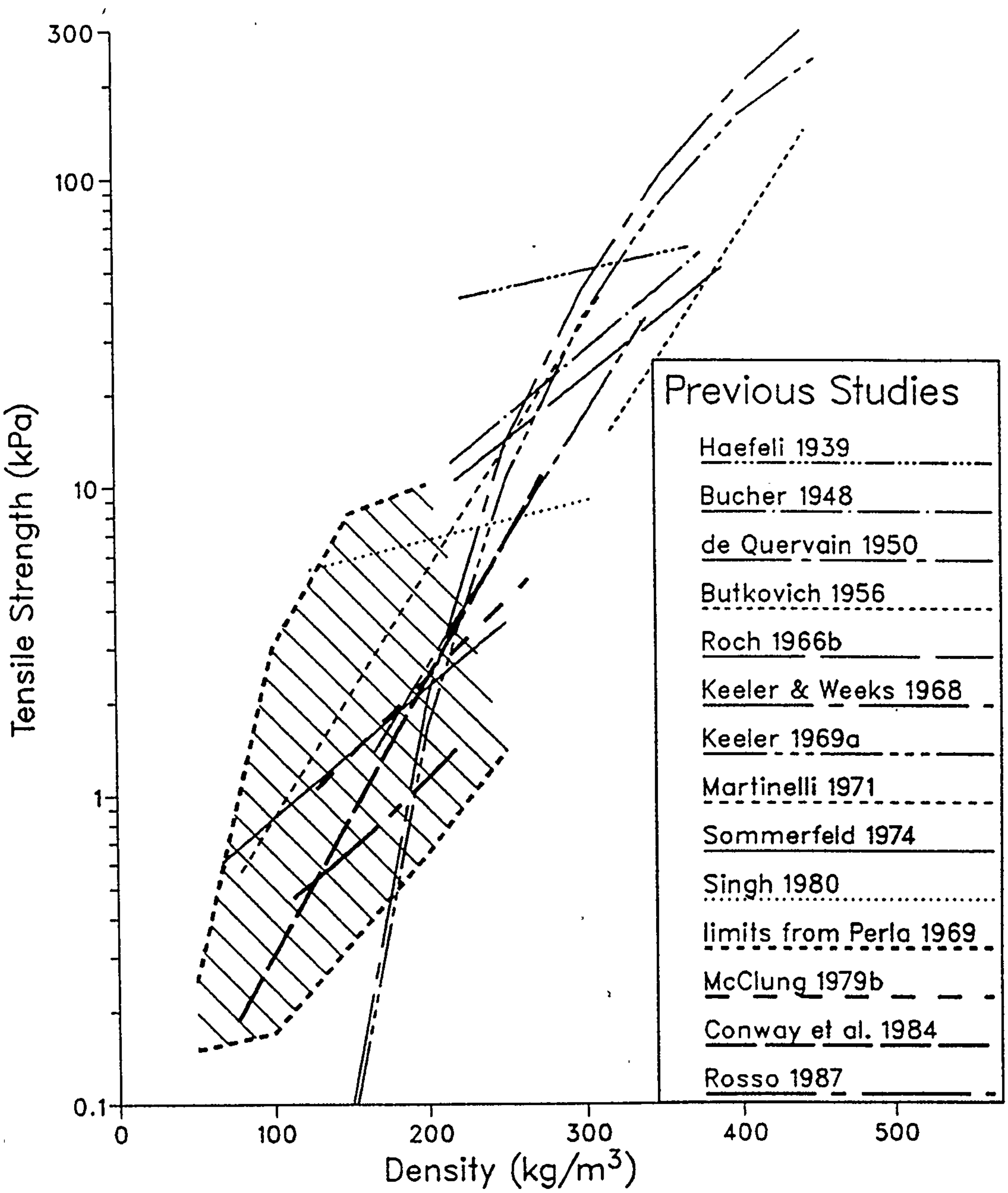

Figure 2.1: Tensile Strength from Laboratory and Field Studies
Tests on wet snow, coarse grained snow, faceted grains and depth hoar are excluded from the trends shown. For Bucher et al. (1948) and Roch (1966b), the strength of each sample determined at or closest to -10°C is used. The strengths empirically adjusted to -10°C are used from the spin tests of Butkovich (1956). In situ tests are represented by bold lines. The results of Perla's (1969) cantilever tests fall within the shaded region.

in terms of specimen *volume* (Sommerfeld, 1973, 1974, 1980) whereas shear strength is discussed in terms of area (Perla, 1977; Sommerfeld and King, 1979; Sommerfeld, 1980; Stethem and Tweedy, 1981; Föhn, 1987). However, the area of maximum tensile stress is given in Tables 2.1 and 2.2 in preference to the volume because, for most techniques including the spin test, the stress is not uniform throughout the specimen volume.

The six studies with the lowest tensile strength for a density of 150 kg/m^3 as shown in Figure 2.1 are those of Sommerfeld (1974), McClung (1979b), Conway and Abrahamson (1984), Rosso (1987) Keeler and Weeks (1968) and Keeler (1969a). Included in thses six studies are the four studies with the largest areas of maximum tension (Sommerfeld, 1974; McClung, 1979b; Conway and Abrahamson, 1984; Rosso, 1987). This is consistent with the expected trend for larger specimens to have, on average, reduced strength at any given density. The remaining two studies with generally low strengths (Keeler and Weeks, 1968; Keeler, 1969a) are discussed under Rate Effects (Section 2.6).

Although the four studies with the largest areas of maximum tension (0.01 to 0.1 m^2) each use a different test method, they show better agreement for strength as a function of density than do the eight spin test studies in which the areas of maximum tension (0.002 m^2) are smaller. This is consistent with Sommerfeld's (1980) application of Weibull and Daniels' statistics to snow, which predict a reduction in variability with an increase in specimen size.

2.6 Rate Effects and Mode of Failure

Most of the data presented in Figure 2.1 are obtained from spin tests for which the stress rate is not proportional to the acceleration rate. Also, except for tests made with the improved spin tester (Sommerfeld, 1974), the acceleration rate depends on the weight of the specimen. Keeler (1969a) discusses rate effects for the spin test and estimates the stress rate to be 5 to 7.5 kPa/s for specimens with a density of 250 kg/m^3 accelerated at 10 rps.

Narita's (1980, 1983) strain control experiments on snow specimens show that at strain rates greater than approximately 10^{-4} s^{-1}:

1. the fractures are essentially brittle[2] with no micro-cracking;
2. the critical strain is nearly constant for fixed density and microstructure;
3. strength decreases with increasing strain rate; and
4. the stress-strain curves are essentially linear to failure.

(Consistent with the third result, Hawkes and Mellor (1972) find that the tensile strength of ice decreases with increasing strain rate at rates above 10^{-3} s^{-1}.) These results indicate that, in the brittle range, increasing the strain *or stress* rate will decrease the strength.

In terms of loading times, 46 of 47 tests which failed in 50 seconds or less exhibited essentially linear stress-strain behaviour, had critical strains of 0.8% or

[2] Gubler (1978) doubts that fractures are perfectly brittle even at 10^{-3} s^{-1} and notes that viscous and plastic deformation are only negligible at strain rates greater than 1 s^{-1}.

less and resulted in brittle fractures (Narita 1980, 1983). Similarly, all 17 of Singh's (1980) tests which failed in 80 seconds or less exhibited critical strains of 0.6% or less and essentially linear stress-strain curves. With reference to Narita's (1980) loading times, Conway and Abrahamson (1984) suggest that their slip plate tests resulted in brittle fractures since their specimens were loaded to failure within 15 seconds.

Most of McClung's (1979b) loading times for the tilt-table tests were in excess of those loading times for brittle failure as reported by Narita (1980, 1983). Certainly, some of McClung's tests were ductile since the results were not sensitive to the shape of the notches.

Keeler and Weeks' (1968) and Keeler's (1969a) spin rates were 3 to 5 times faster than Martinelli's (1971), and, as shown in Figure 2.1, the strengths are reduced for specimens with a density less than 200 kg/m^3. The reduction in strength for the faster spin tests as compared to the slower tests suggests that the faster spin tests of Keeler and Weeks (1968) and Keeler (1969a) resulted in brittle fractures.

In Figure 2.1, the strengths from Haefeli (1939) show little increase with an increase in density. A possible explanation may be that the expected steeper trend is obscured by scatter resulting from jarring of the specimens as the shot was poured.

Singh's (1980) data also show little increase in strength with density. However, of all the results plotted in Figure 2.1, only Singh's are from tests made at various constant strain rates. Therefore, it is likely that the real effect of increasing density on strength is obscured by strain rate effects.

2.7 Summary

Spin tests, ring tests and load frame tests have been used to determine the tensile strength of snow in laboratories. Spin tests have been widely used, but the applied stress rates have not been accurately established, and strengths determined at different *spin* rates vary substantially. Ring tests have only been used on snow with a density greater than 400 kg/m^3. Narita (1980, 1983) used the load frame to test snow at various strain rates and investigated the ductile-brittle transition.

Five distinct test methods have been used in the field. The load frame technique (McCabe and Smith, 1978; Singh, 1980) requires that the specimens be extracted from the snowpack, but allows testing over a range of controlled strain rates. Three of the four in situ techniques apply uniaxial tension to comparatively large specimens. The four techniques with the largest areas of maximum tension give lower strengths and show less dependence on the test method than test methods with smaller areas of maximum tension.

Therefore, a case can be made supporting a test method which allows relatively large cross sections to be tested in uniaxial tension and avoids the need to extract the test specimen from the snowcover. McClung's (1979b) tilt-tables, Rosso's (1987) trapezoidal technique and Conway and Abrahamson's (1984) slip plate technique fulfill these requirements. However, the tilt-table method requires that the tables be constructed and placed days or weeks before the test is made and only snow which is naturally deposited on the tables can be tested. The trapezoidal technique is time consuming, can be done only on relatively steep slopes and bending stresses may be superimposed on the uniaxial tension. In contrast, the slip plate test method allows snow to be tested under uniaxial tension, without prior construction of tilt-tables and on level or sloping terrain. It shares a disadvantage with the other in situ techniques in that the load is applied manually and variability of the loading rate associated with manual load control seems inevitable.

Chapter 3

Experimental Methods

3.1 Field Work

Tensile tests and related snowpack measurements were conducted during the winters of 1986-87 and 1987-88 in the Rocky Mountains of Alberta. In the first winter, variations on the test method and equipment were tried during twenty-two days of field work, which was conducted primarily in Kananaskis Country of Alberta. During the second winter, forty-two days of systematic field tests were completed between December 19, 1987 and April 17, 1988 in and near the Lake Louise Ski Resort in Banff National Park. It is the results from the second winter of field work that are used to develop the conclusions for this thesis.

Conway and Abrahamson (1984) developed the test method (Figure 3.1) used in this thesis and made 32 tensile tests of snow slabs on 14 avalanche slopes in the Southern Alps of New Zealand. They also measured variations in shear strength of the weak layers under the slabs and used the *minimum* of the ratio of shear strength to shear stress for the stability index. In their model, weak layers consist of "deficit areas" and "pinned areas". Deficit areas are those regions of the weak layer for which the stability index is less than unity. Alternatively, pinned areas are those regions for which the stability index exceeds unity. They propose that the tensile strength of the slab may carry the load which is not supported by a deficit area from pinned area to pinned area. Using the tensile strength of the slab, they

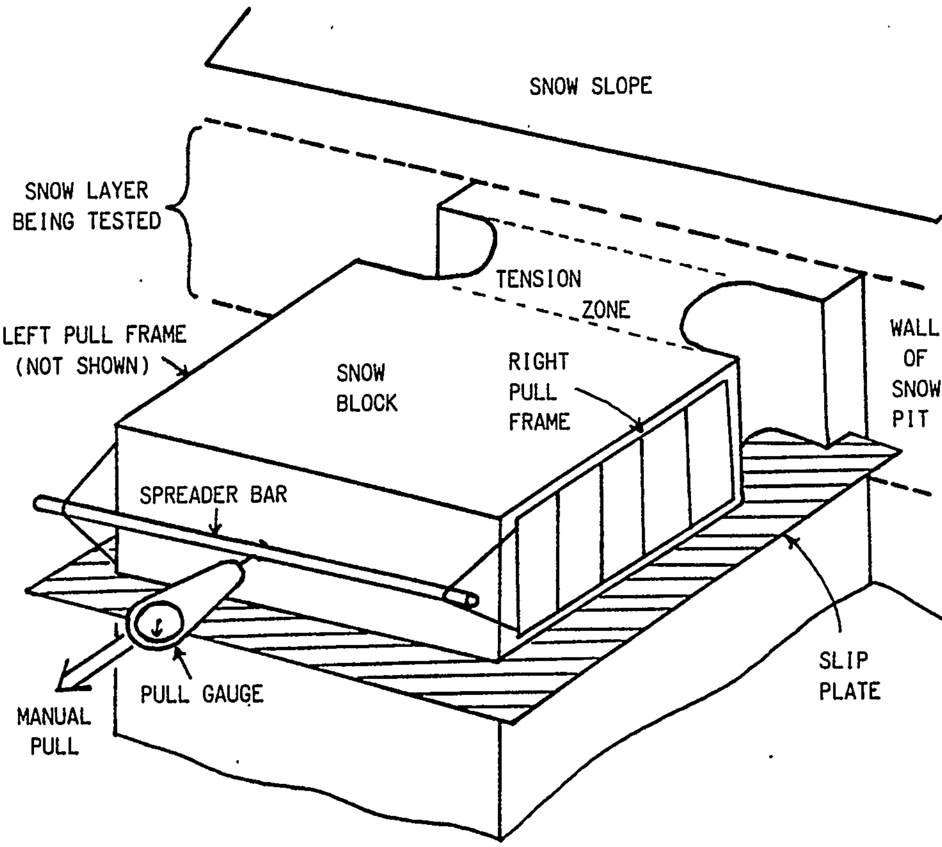

Figure 3.1: Slip Plate Tensile Test

calculate the critical size of a deficit area. They propose that the minimum value of the stability index and the size of the largest measured deficit area be used to assess slope stability. This approach requires that numerous tests be made on slopes of questionable stability. Because of the limited number of shear strength tests that were made, they were unable to confirm that slopes became unstable when the largest deficit area exceeded the critical size of a deficit area as determined from the tensile strength of the slab.

They did not study the dependence of tensile strength on density or microstructure although the test method is well suited to such purposes since it allows the uniaxial tensile strength of relatively large snow specimens to be measured in situ. The specimens may comprise, in cross section, one or more contiguous layers with distinct microstructures, so either a single layer or composite of several layers may be tested.

3.2 Procedure

From a pit wall facing down-slope, a column of snow approximately 40 cm by 40 cm was isolated as shown in Figure 3.1. One or more contiguous layers of snow with a total thickness greater than 10 cm was selected for testing. Snow over the selected layers was carefully removed, exposing the top surface of the snow to be tested.

One pull frame was gently pushed into each side of the column parallel to, and close to, the exposed surface as shown in Figures 3.1 and 3.2. Slightly below and parallel to the snow layer to be tested, a slip plate was inserted. (The slip plate was carefully waxed beforehand to minimize friction and adhesion with the snow.) Two notches, each with a 50 mm radius, were cut with downward strokes of a notching tool between the frames and the undisturbed snowpack. After the notches were cut, the resulting width of the tensile zone was approximately 20 cm. Thus, a block of snow was isolated on top of the slip plate, between the two pull frames and remained attached to the snowpack only by the notched tensile zone (Figures 3.1 and 3.2).

Load was applied to each pull frame by means of a cord attached to one end of a spreader bar. Slope-parallel pull was applied manually to the mid-point of the

Figure 3.2: Tensile Test on an 18° Slope

spreader bar with a pull gauge (Figures 3.1 and 3.2). The load was applied at an approximately uniform rate until fracture occurred at which time the maximum load was automatically recorded. Loading times ranged from 0.5 to 5 seconds with a mean of 2.2 seconds. (The results of 38 tests with loading times in excess of 5 seconds were not used to develop the conclusions but are used to discuss rate effects in Section 5.7.)

After failure the fracture surfaces were inspected. If the fracture was not confined between the extremes of the notch diameter (radius usually 50 mm), the test was rejected and the procedure was repeated. Fracture surfaces which were not planar and fractures which did not occur through the smallest cross section of the notched zone were noted. These unusual fractures and their effect on the results are analyzed in Chapter 4.

Friction between the snow and the slip plate was measured by tilting the plate and noting the angle with a carpenter's slope gauge at which a snow block on the plate began to slide. Friction angles for the snow blocks on the waxed slip plate ranged from 2° to 10° with a mean of 3.6°. Static friction coefficients from the literature for snow on waxed surfaces are somewhat higher than the mean values obtained in the present study by inclining the plate. Mellor (1975) suggests that the best estimates of static friction are kinetic friction coefficients obtained from snow sliding slowly. For the temperature range 0°C to -15°C, slowly sliding kinetic friction coefficients range from 0.1 to 0.2 (Ericksson, 1949; Bowden and Tabor, 1964, pp. 150-151). This is up to 3.5 times greater than the average value of 0.06 obtained in this study by inclining the plate. Since measurements of friction angles were repeatable in the field and are believed to indicate actual changes in plate friction due to air and snow temperatures, these values are used to calculate strength. (In the only previous slip plate tests, Conway and Abrahamson (1984) used zero for the friction coefficient.)

For one person to dig a pit and make a tensile test with density and temperature measurements required between 15 and 30 minutes depending largely on the depth of the snow to be tested from the surface. In and near the Lake Louise Ski Resort during the winter of 1987-88, the depth of the snowpack was typically in the range of 0.8 to 1.2 m and the pits were usually dug to the ground. Additional tests in the same pit required approximately 10 minutes per test.

3.3 Strength Calculation

The following measurements were taken: maximum pull force P, the loading time t, length of the block (parallel to the pull direction) l_{b}, width of the block w_{b}, depth of the block d_{b}, the width and depth of the fracture surface w_{f}, d_{f}, slope angle β, friction angle γ for the snow block on the plate, mean snow density ρ and the mass of the frames m_{fr}.

The weight of the block and frames is simply:

$$W = (l_{\mathrm{b}} w_{\mathrm{b}} d_{\mathrm{b}} \rho + m_{\mathrm{fr}}) g \qquad (3.1)$$

where g is the acceleration due to gravity. From statics, the average tensile stress at failure in the cross-sectional area of the fracture is:

$$\sigma = (W \sin\beta + P - W \tan\gamma \cos\beta)/(w_{\mathrm{f}} d_{\mathrm{f}}) \qquad (3.2)$$

3.4 Equipment

The pull gauge, slip plate, pull frames, notching tools and other implements are described in this section. The pull gauge, slip plate, pull frames, notching tools, spreader bar and slope gauge are shown in Figure 3.3.

3.4.1 Pull Gauge

Chatillon-brand spring gauges were used. The needle on the load dial of these gauges displays the present load unless the locking button has been set in which case the needle displays the maximum load. For these tests the needle was set to zero and the locking button was set prior to the load being applied. Thus, the

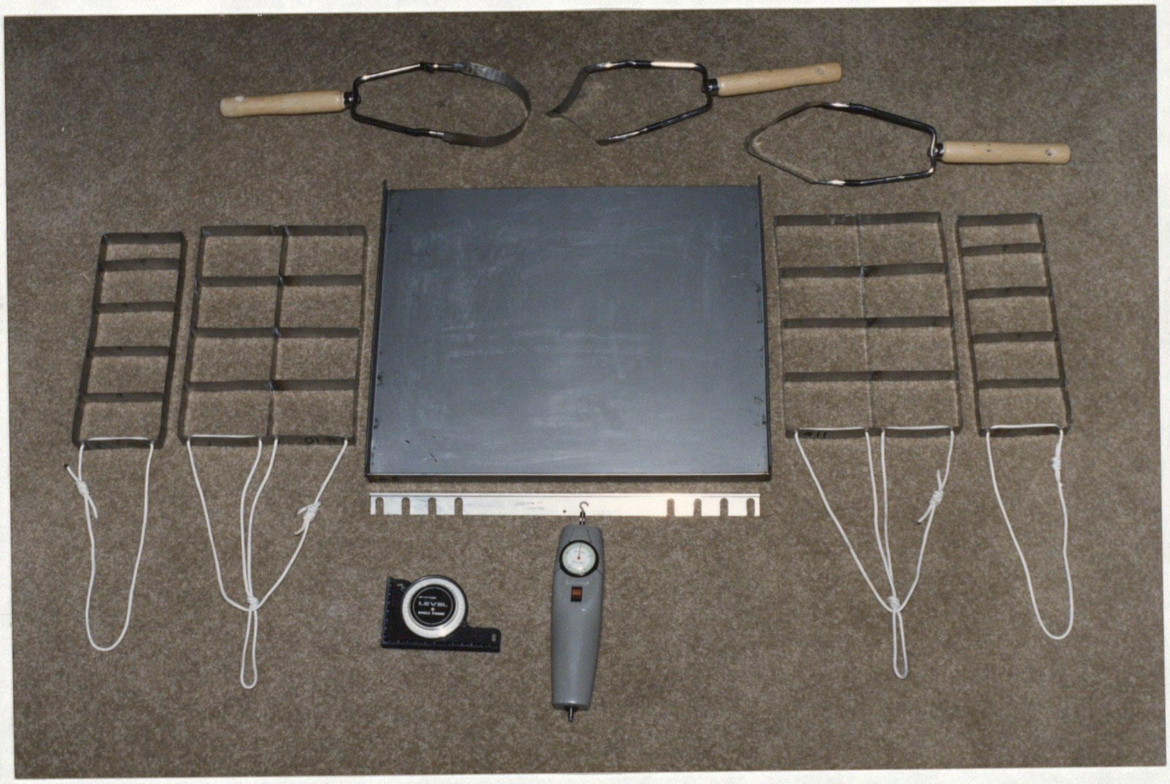

Figure 3.3: Equipment for Slip Plate Tensile Tests

maximum load at failure was recorded on the gauge. Most tests were made with a gauge with a maximum capacity of 250 N and scale divisions of 2.5 N. Typical loads at failure were 20 to 100 N.

Pull gauges with a capacity of 20, 50 and 100 N were also used during the first winter, but a 250 N capacity proved necessary for the denser snow layers.

3.4.2 Slip Plate

To minimize disturbance or prestressing of the tensile zone, a thin straight cutting edge and a low-friction flat surface were considered essential for the slip plate.

Several slip plates constructed of sheet metal and two made from plastic "sliding carpets" were initially used. The plastic carpets were abandoned during the first winter because they dented more easily and were difficult to flatten in cold weather. Sheet metal plates of 0.6 mm stainless steel were finally selected because they were sufficiently stiff to resist denting and bending under normal handling. The selected plate was 45 cm across, 40 cm in the down-slope direction and had 2.5 cm stiffening flanges on three sides at right angles to the surface of the plate. The cutting edge without the flange was sharpened to facilitate gentle insertion under the snow block.

Snow may adhere (bond) to contact surfaces within seconds (Mellor, 1975) and on occasion, the snow blocks tended to adhere to the stainless steel plate. Surface preparations of ski wax, silicon lubricant for ski bindings and a "no stick" spray for cooking pans were tried. The best results were obtained from several coats of paste ski wax which restricted the adhesion problem to days when the air temperature (which affects the plate temperature) was much colder or much warmer than the temperature of the snow in contact with the plate. Also, the plate was buried in the wall of the shady side of the snow pit between tests to further reduce the occurrence of snow blocks adhering to the plate.

3.4.3 Pull Frames

Eight combinations of designs and sizes of frames were tried. Frames soldered from 25 mm wide strips of 0.45 mm stainless steel sheet metal proved adequately sturdy for testing snow up to a density of 360 kg/m^3. Given the dimensions of the plate and tensile zone, 28 cm was the maximum and the preferred length of the frames.

Four equally spaced cross pieces were soldered in place. Since the width of the frames should approximately span the thickness of the snow being tested, frames with widths of 10, 20 and 30 cm were constructed and used sucessfully. The cutting edges of the frames were sharpened to permit gentle penetration of the sides of the snow block.

3.4.4 Notching Tools

During the first winter, the notches were cut with the "toothless" edge of a snow saw to avoid stress concentrations that might have been associated with the grooves cut by the saw teeth. Because of concern that cutting notches with such a thick and blunt edge might be disturbing the snow specimens, specialized notching tools were made for the second winter of field work. These tools allowed the notches to be cut with a single downward stroke. The "blade" of these tools was made from 0.7 mm stainless steel sheet metal. The cutting edge of each blade was sharpened. These notching tools are shown in Figure 3.3 and the effect of notch shape on strength is discussed in Section 5.8.

The strengths from the first winter were systematically lower compared to the strengths from the second winter during which the notching tools were used. It is believed that cutting notches with a snow saw (first winter) weakened the tensile specimens. For this reason, only the results from the second winter are used in this thesis.

3.4.5 Other Implements

The carpenters slope gauge shown in Figure 3.3 was used to measure slope angles and the angle of impending motion for the snow block on the plate.

A folding ruler was used to measure the dimensions of the fracture surface and the block.

The density of cylindrical specimens in a stainless steel tube of volume 100 cm^3 were measured with a balance type gauge (manufactured by Strong Stitch) shown in Figure 3.4.

3.5 Repeated Tests

Tests on a particular layer or group of contiguous layers were repeated either to obtain a mean value of strength, to study variability of the mean, or to compare the effect of various pull rates or of different notching tools on strength.

The number of tests on a particular layer, or group of contiguous layers, on a given day ranged from 1 to 42 with an average of 6.9. The surface area disturbed by each test was approximately 1 m^2. The tests were repeated in adjacent but undisturbed portions of the study site.

Each set of repeated tests on a particular layer disturbed a rectangular portion of a test site called a *subplot* (Figure 3.4). The location of each test within a subplot, that is, with respect to the other tests on the same layer(s) on the same day, was identified by a row and column number. The location of the first test in any subplot was row one and column one. In all but two of the study sites the slope was perceptible; each row was approximately 0.6 m upslope from the previous row and each column was approximately 1.4 m across the slope from the previous column.

Figure 3.4: A Subplot Used for a Number of Tensile Tests

In the two level study sites, the direction of increasing row numbers was chosen parallel to the direction of pull for consistency with the sloping study sites. The row and column numbers of a test within a subplot are used to study the areal variability of strength (Section 5.5.2) and to "pair up" tests on the same layer for which the notch shapes were different (Section 5.8).

Chapter 4

Sources of Variability

4.1 Introduction

The results of a strength test depend not only on the properties of the material being measured, but also on the test methods and conditions under which measurements are made. This chapter considers certain test conditions and unusual fracture characteristics as possible sources of variability. Factors which are found to increase variability are identified. The results associated with such factors are either accepted along with the consequent increase in the variability of the strength data or are rejected for statistical or physical reasons, thereby reducing the variability of the data. By this screening process, a refined data set of 457 tests is obtained from a raw data set of 555 tests as described in Section 4.9.

Eight test conditions and fourteen fracture characteristics were investigated for their effect on the results. Examples of test conditions include loading time, notch shape, slope angle and notch shape. Fracture characteristics considered include the "crown" angle between the plate and fracture surface, fractures which did not occur through the smallest cross section of the notched zone and several types of non-planar fracture surfaces. The complete list of sources of variability considered is presented.in Table 4.1.

Table 4.1: Sources of Variability

Measurement Scale[1]	Factor
	Snow Properties
ratio	density
categorical	microstructure
interval	temperature
categorical	moisture level
	Test Conditions
categorical	site
interval	days since December 19, 1987
interval	replicate number for fixed site, day and snow layer
ratio	load time to failure
interval	static friction angle for snow on plate
interval	slope angle
ratio	width of fracture surface
ratio	depth of fracture surface
categorical	notch shape
categorical	plate moved continuously to prevent bonding
	Fracture Characteristics
interval	"crown" angle between plate and fracture surface
categorical	across back of notched zone
categorical	slightly back of centre of notched zone
categorical	slightly forward of centre of notched zone
categorical	across front of notched zone
categorical	fracture angles towards back of one notch
categorical	fracture angles toward front of one notch
categorical	protruberance on fracture surface
categorical	two fracture planes meet at slight angle ($< 20°$)
categorical	two fracture planes meet at distinct angle ($> 20°$)
categorical	fracture produced a chunk
categorical	at back of plate
categorical	cup-shaped fracture
categorical	fracture misses a V- or Y-shaped notch

[1] Terms defined in Appendix A.

4.2 Statistical Model

The independent or predictor variables resulting from the field study are the material properties, the test conditions and the fracture characteristics. However, a conventional analysis of variance is inappropriate for detecting which of these independent variables are associated with variability in the results since the number of tests with particular combinations of test conditions and fracture characteristics is not constant. For analyzing the sources of variability in such *unbalanced* data a multivariate regression is appropriate (Neter and Wasserman, 1974, p. 633). The regression analysis in this chapter follows the approach of Weisberg[1] (1980, pp. 152-162).

The following considerations are used to develop an appropriate regression model in Sections 4.2.1 to 4.2.4:

1. snow strength is a strong non-linear function of density (Keeler and Weeks, 1968; Martinelli, 1971; and others);
2. as snow strength increases, so does its variance (Keeler and Weeks, 1968);
3. some of the predictor variables represent categorical data (Table 4.1); and
4. strong correlations between the predictor variables can affect the results of the regression analysis.

[1] Weisberg analyzed the effect of a number of factors, such as cloud seeding, on rainfall.

4.2.1 Non-Linear Dependence of Strength on Density

A non-linear model is required to allow for the dependence of tensile strength on snow density. Ballard and Feldt (1965) propose an exponential relationship between snow strength and density which is based on bond growth in sintering snow. Since many of the tests in the present study were made on faceted snow which usually shows progressive weakening rather than sintering, the exponential model of Ballard and Feldt is judged inappropriate.

The empirical model of Perla et al. (1982):

$$\sigma = A\,(\rho/\rho_{\text{ice}})^{\,a} \tag{4.1}$$

is used as the basis for the regression equation since it is simple and because it fits the data well, as shown in Figure 5.1.

4.2.2 Non-Constant Variance

When tensile strengths and densities from the 555 tests are fitted to Equation 4.1 by the IMSL least squares procedure ZXSSQ, the variance is not constant, as shown in Figure 4.1. Keeler and Weeks (1968) also reported a systematic increase in variance with strength, which gives the higher strengths greater influence than the lower values over the regression parameters. This non-constant variance (heteroscedasticity) is stabilized by a log-log transformation (Weisberg, 1980, pp. 120-128):

$$\ln\sigma = \ln A + a\ln(\rho/\rho_{\text{ice}}) \tag{4.2}$$

and results in an improved distribution of variance as shown in Figure 4.1. For comparison, the variances for each interval are normalized by the variance for the entire strength range.

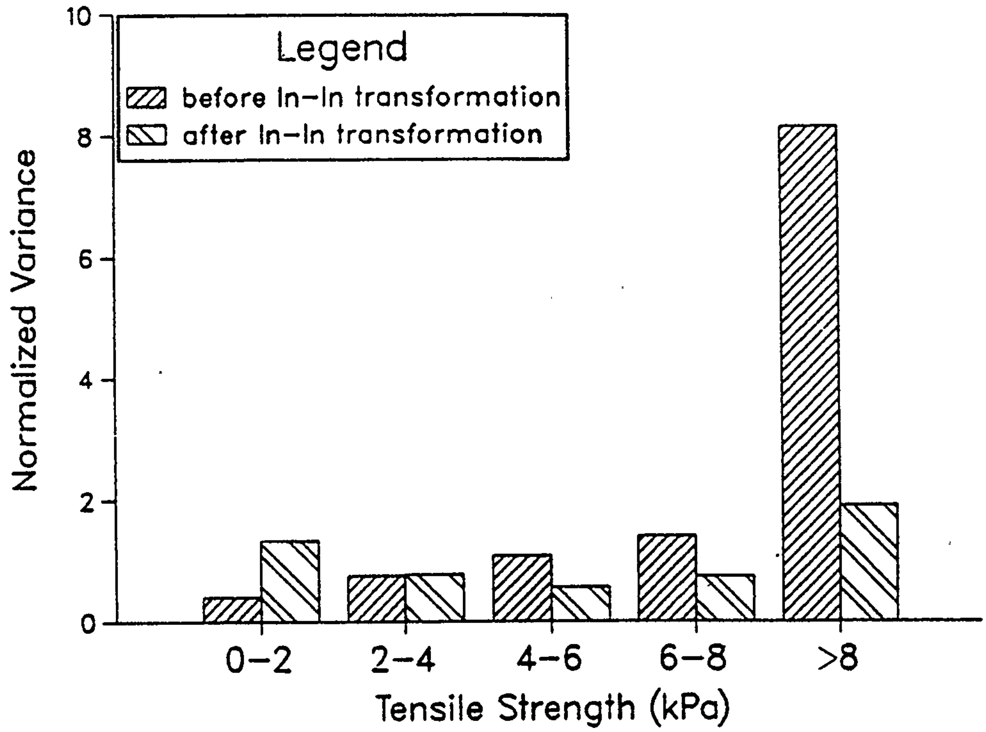

Figure 4.1: Effect of ln-ln Transformation on Normalized Variance

4.2.3 Additional Factors

To investigate the dependence of strength on the factors other than density, Equation 4.2 is expanded to include the variables for temperature, microstructure, moisture, test conditions and fracture characteristics.

Grain size is not considered for two reasons. First, a field study of shear strength (Perla et al., 1982) found that grain size did not correlate well with strength. Second, in 22 of the 555 tests in this study, the predominant grain size could not be recorded since the specimen consisted of more than one layer, each with a distinct grain size. The use of grain size as a factor would exclude these 22 tests from the regression analysis, thereby reducing the influence of material properties such as density.

4.2.4 Categorical Variables

Using categorical variables as predictor variables in a regression is somewhat controversial. Stevens (1968) argues that regression techniques should only be used on "measurements", i.e., ratio and interval data. Authors such as Harris (1975, p. 226), Weisberg (1980, pp. 152-162) and Draper and Smith (1981, pp. 241-246) allow categorical data into regressions provided the categorical data are represented by indicator variables. These authors emphasize the importance of plotting the residuals against the predicted values to assess the *aptness* of the model. The latter approach and recommendations are followed in this analysis.

All but three of the categorical variables are dichotomous, that is, each has two levels. For example, either the fracture surface crossed the back of the notched tensile zone or it did not.

The remaining three categorical variables, microstructure, site and notch shape, each have several levels. Since, in general, categorical levels are not ordered, each of the multi-level variables is represented by several dichotomous indicator variables (Weisberg, 1980, pp. 160-162). For example, the three notch shapes are represented by two indicator variables: V and Y. Tests with a V-shaped notch are represented by $V = 1$, $Y = 0$. Tests with a Y-shaped notch are represented by $V = 0$, $Y = 1$ and tests with a 50 mm radius rounded notch are represented by $V = 0$, $Y = 0$.

Similarly, the six types of microstructure are represented by five indicator variables and the seven sites are represented by six indicator variables.

Terms representing interaction between the indicator variables are not included for two reasons. First, to keep the statistical model simple, only "additive" factors, those which consistently increase (or consistently decrease) the predicted values, are considered. (Similarly, interactions are not considered in the "ruggedness" program (ASTM C-1067) for detecting significant interlaboratory factors.) Second, the complete set of interaction terms would exceed the degrees of freedom present in the data.

4.2.5 Interdependence of Predictor Variables

The dependence of any predictor variable on a combination of the other predictor variables may affect the regression coefficients and attempts to assess the effects of individual predictor variables (Neter and Wasserman, 1974, pp. 339-341). Those pairs of predictor variables with correlation coefficients greater than 0.55 or less than -0.55 are listed in Table 4.2 and are discussed in this section.

The slope angle is strongly correlated (r=0.91) with the indicator variable for "slide paths in or near the Lake Louise Resort" because the slope angles in the avalanche starting zones were much steeper (30° to 50°) than the slope angles in the remaining study sites ($\leq$ 17°). The snow temperature increased with "days since December 19, 1987" (r=0.81) indicating progressive warming of the snowpack during the winter. The indicator variable for rounded grains correlated with $\ln(\rho/\rho_{ice})$ (r=0.64) because rounded grains are often denser than layers with other types of microstructure. The indicator variable for the Richardson Study Plot is

Table 4.2: Strong Correlations Between Possible Predictor Variables

Correlated Variables		r-value
slope angle	slide paths in or near Lake Louise Resort[1]	0.91
temperature	days since December 19, 1987	0.81
$\ln(\rho/\rho_{ice})$	rounded grains[1]	0.64
Wolverine Study Plot[1]	Richardson Study Plot[1]	-0.60
fracture depth	rounded grains[1]	-0.59
moist snow[1]	static friction angle for snow on waxed plate	0.58
Lipalian Study Plot[1]	replicate number within sample	0.57
new snow[1]	slope angle	0.56

[1] indicator variable

negatively correlated (r=-0.60) with the indicator variable for the Wolverine Study Plot because most of the tests were made at one of these two study plots. The negative correlation (r=-0.59) between the depth of the fracture surface and the indicator variable for rounded grains suggests that many of the thinner layers that were tested consisted of rounded grains. The indicator variable for moist snow correlated with the static friction angle for snow on the plate (r=0.58) indicating that specimens of moist snow had greater friction with the waxed plate than did specimens of dry snow. The positive correlation between the replicate number and the indicator variable for the Lipalian Study Plot (r=0.57) results from the large sample of 42 tests which were the only tests made at this study plot. Finally, the correlation between the slope angle and the indicator variable for new snow (r=0.56) results from most of the tests on new snow being made in avalanche starting zones which are consistently steep.

To reduce the interdependence of those predictor variables to be included in the regression, the indicator variables for study sites were excluded as was the variable for "days since December 19, 1987". As a result of these exclusions, the strongest correlation between any two predictor variables included in the regression is 0.64.

4.3 Multivariate Regression

Consistent with the previous discussion, a logical statistical model for analyzing the sources of variance is:

$$\ln \sigma = \ln A + a \ln(\rho/\rho_{\text{ice}}) + bT + \sum_{i} m_i M_i + \sum_{i} f_i F_i + \sum_{i} c_i C_i + \mathcal{E} \qquad (4.3)$$

where T is the temperature, M_i, F_i and C_i are the indicator variables for the microstructure, fracture characteristics and test conditions, respectively, and $\mathcal{E}$ is the "unexplained" error or residual. In Equation 4.3, the terms a, b, m_i, f_i and c_i are the regression coefficients.

4.4 Aptness of Statistical Model

To demonstrate the aptness of a statistical model, Weisberg (1980, pp. 120-122) recommends that a graph of the studentized residuals against the predicted values meet the following criteria:

1. the residuals should lie in a horizontal band about 0,
2. 95% of the studentized residuals should lie between +2 and -2, and
3. only about 1 in 1000 studentized residuals should lie outside the ±3 band.

The studentized residuals from the multiple regression are plotted against the predicted values of $\ln \sigma$ in Figure 4.2. Inspection of this graph shows that, although 1 of the 555 studentized residuals falls below -3, the statistical model essentially meets the above criteria.

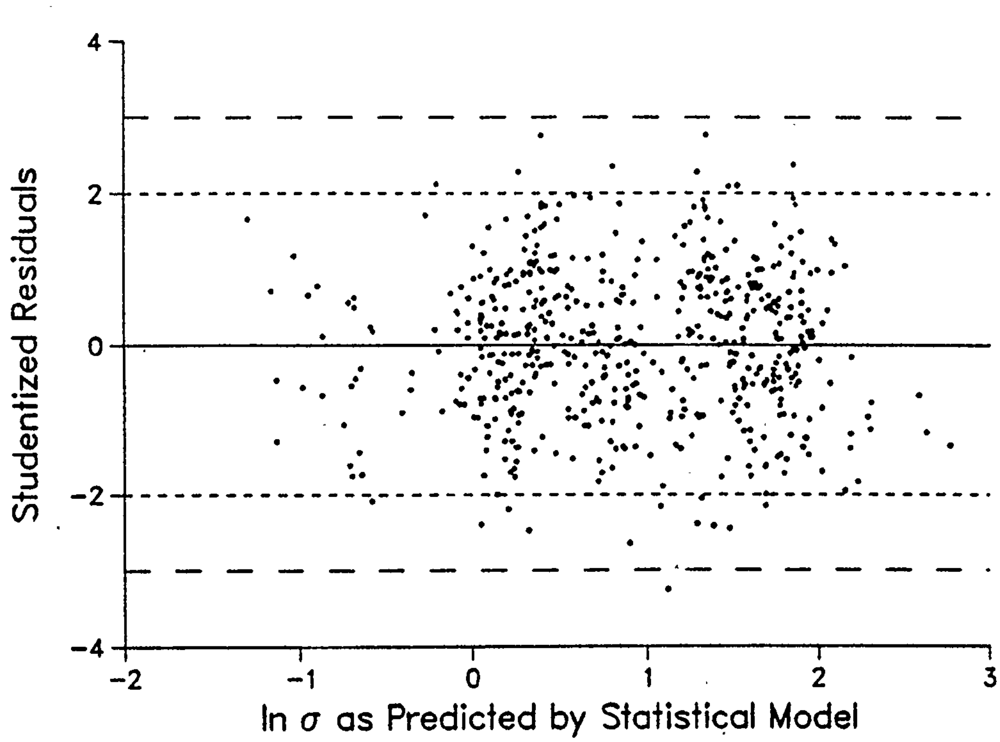

Figure 4.2: Distribution of Studentized Residuals from Statistical Model

As mentioned in Section 4.2, the results of the field study constitute unbalanced data; that is, the numbers of tests in the various cells are not equal. However, a graph of studentized residuals provides a discriminating test of the aptness of a regression model applied to unbalanced data. Consider the hypothetical data set in which there is only one *cup-shaped* fracture. The indicator variable for this fracture characteristic will be zero except for the single test for which it will be 1. The regression coefficient for the *cup-shaped* indicator variable will therefore be free to fit the particular result to the regression equation with a residual of zero. However, the studentized residual is calculated *without* any effect from this test, Therefore, if the result does not fit with the other results in the regression, its studentized residual will be large and will be obvious on any graph or histogram of studentized residuals[2].

4.5 Results of Regression Analysis

The regression was performed using the Regress routine of the Minitab statistical package. Results are presented in Tables 4.3, 4.4 and 4.5. Variables for which the t-value is greater than 2.59 or less than -2.59 are significant at the 99% level. The list of factors with this significance level are presented in Table 4.6 and discussed in Section 4.6 to 4.8.

4.6 Dependence of Strength on Material Properties

4.6.1 Density

In agreement with previous snow strength studies (for example, Keeler and Weeks, 1968), the most influential factor on strength is density. For subsequent graphs and comparisons, tensile strength is presented as a function of density.

[2] Statistical terms are defined in Appendix A

Table 4.3: Regression Results for Material Properties

Regression Coefficient	t-value	Predictor Variable
2.061	25.59	$\ln(\rho/\rho_{ice})$
-0.002	-0.36	snow temperature (°C)
-0.096	-1.23	indicator variable for moist snow
		Indicator Variables for Microstructure
-0.050	-0.58	new snow
0.193	2.30	partly settled grains
0.448	4.48	rounded grains
-0.371	-4.07	faceted grains
-0.481	-5.04	faceted and rounded grains

Table 4.4: Regression Results for Test Conditions

Regression Coefficient	t-value	Predictor Variable
-0.003	-1.65	replicate number within sample
0.015	9.32	loading time to failure (s)
0.031	2.52	static friction angle for snow on waxed plate (°)
0.014	5.62	slope angle (°)
-0.182	-3.96	width of fracture surface (mm)
0.064	2.74	depth of fracture surface (mm)
		Indicator Variables for Controlled Factors
-0.041	-0.51	V-shaped notch
-0.288	-4.58	Y-shaped notch
-0.039	-0.21	plate moved continuously to prevent adhesion

Table 4.5: Regression Results for Fracture Characteristics

Regression Coefficient	t-value	Predictor Variable
0.001	0.11	"crown" angle between plate and fracture surface
		Indicator Variables for Unusual Fractures
-0.193	-3.26	fracture at back of notched zone
0.214	2.34	fracture slightly back of centre of notched zone
-0.202	-1.06	fracture slightly forward of centre of notched zone
-0.278	-2.87	fracture at front of notched zone
-0.257	-1.42	fracture angles towards back of one notch
-0.036	-0.95	fracture angles toward front of one notch
0.141	1.76	protruberance on fracture surface
-0.003	-0.02	two fracture planes meet at slight angle (< 20°)
0.231	1.54	two fracture planes meet at distinct angle (> 20°)
0.032	0.23	fracture branched and released a chunk
0.060	0.76	fracture at back of plate
0.026	0.47	cup-shaped fracture
-0.049	-0.50	fracture missed a V- or Y-shaped notch

Table 4.6: Significant Factors Affecting the Results of Tensile Tests

Material Properties	Density Microstructure
Test Conditions	Loading Time Notch Shape Slope Angle Width of Fracture Surface Depth of Fracture Surface
Fracture Characteristics	Fracture at Back of Notched Zone Fracture at Front of Notched Zone

4.6.2 Microstructure

Most of the tests were made on specimens consisting of a single layer of snow. The predominant microstructure of each single layer specimen is classified into one of four categories: newly fallen snow (also called new snow), partly settled grains, rounded grains, and faceted grains (UNESCO, 1970). The grains in certain specimens showed strong evidence of both rounding and faceting. These are placed in a fifth category called faceted and rounded grains. In addition, 26 tests were made on multi-layer specimens, the layers of which exhibited distinct microstructures. These 26 tests are placed in a sixth category.

Inspection of the regression coefficients in Table 4.3 shows that two classes of microstructure, faceted grains and rounded and faceted grains were significantly weaker than the other classes of microstructure and the multi-layer specimens. Thus, for subsequent regressions on the refined data set (for example, as shown in Figure 5.1), test results are partitioned into the following two groups by microstructure:

Group I	new snow
	partly settled grains
	rounded grains
	multi-layer specimens
Group II	faceted grains
	faceted and rounded grains

4.6.3 Snow Temperature

The effect of snow temperature is not significant at the 95% or 99% levels. This factor is considered further for the refined data set in Section 5.2.1.

4.6.4 Moisture Content

The snow was dry for all but 22 of the tests. For these 22 tests, the snow was classified as "moist" since free water was not visible under low magnification, yet the snow tended to stick together when squeezed lightly with a gloved hand (NRCC and CAA, 1986). This level of moisture turned out not to be a significant source of variability in this particular set of data, so the 22 tests on moist snow are included in subsequent analyses.

No tests were conducted on "wet" snow with visible free water.

4.7 Significant Test Conditions

4.7.1 Loading Time

The loading time to failure is the most statistically significant test condition. A graph of the studentized residuals from regression on Equation 4.2 against loading time (Figure 4.3) shows that these residuals do not average zero for loading times greater than 5 seconds. Instead, strength increases markedly for loading times in excess of 5 seconds. The 38 tests with loading times greater than 5 seconds are rejected from the refined data set but are discussed further under Rate Effects (Section 5.7).

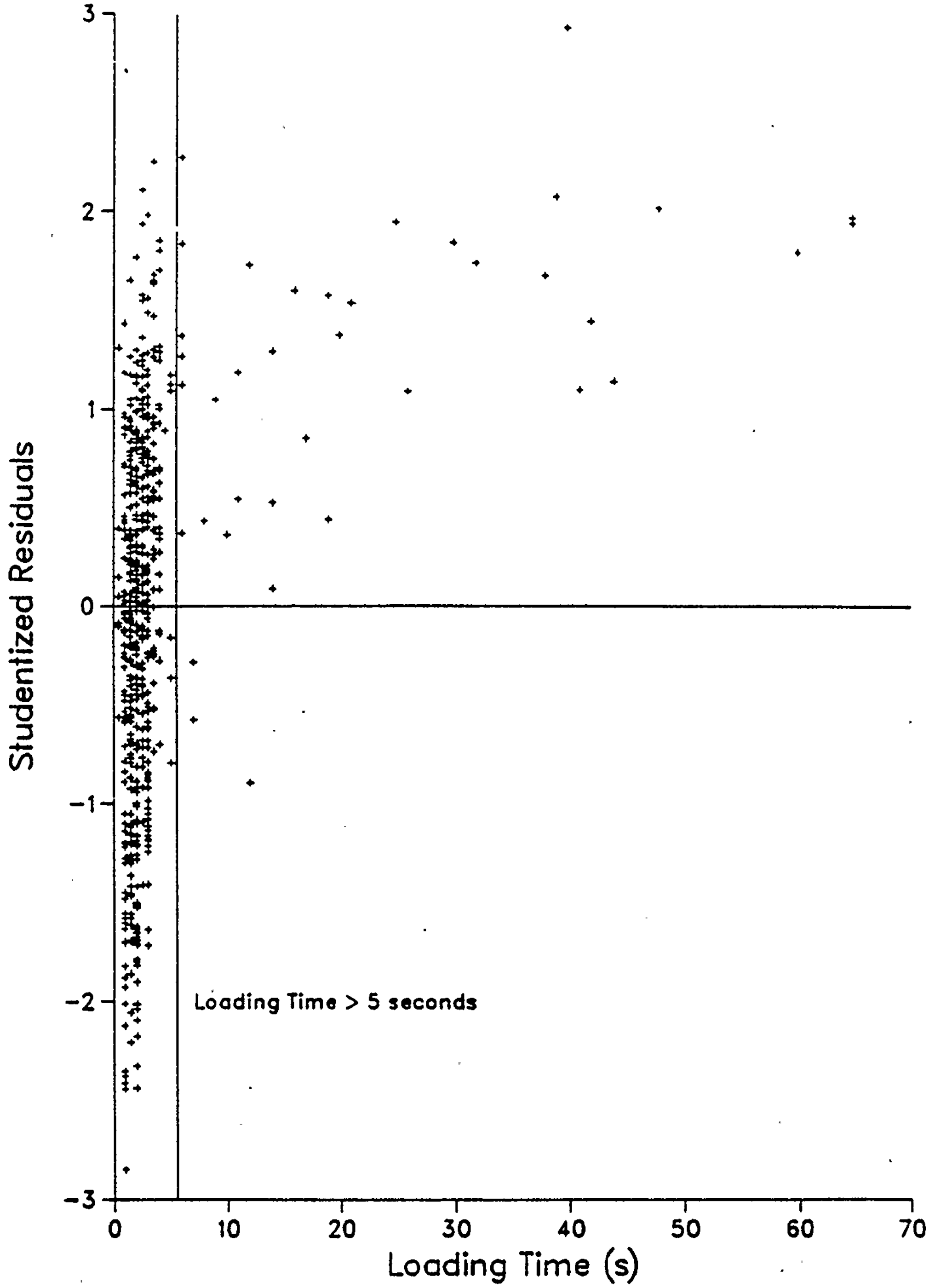

Figure 4.3: Effect of Extended Loading Time
on Studentized Residuals from $\ln\sigma = \ln A + a\ln(\rho/\rho_{\text{ice}})$

4.7.2 Slope Angle

The analysis associates an increase in slope angle with a substantial increase in measured strength. Most of the tests were made on low angle terrain in clearings below tree line. However, 50 of the 555 tests were made at avalanche starting zones on slopes ranging from 30° to 50°. Wind packing of grains at these exposed locations may have produced a stronger microstructure which would not have been detected by the visual classification of grain structure. Sommerfeld (1973) tested specimens of windpacked snow and found them stronger than loosely deposited snow of similar microstructure and density.

A second possible explanation involves creep on the steeper slopes. Laboratory tests by Brown (1977) show that snow specimens first subjected to viscous deformation exhibit greater strength when rapidly strained to failure than snow specimens that are initially unloaded. Therefore, snow on steep slopes where creep is more prevalent can be expected to be stronger than on shallower slopes where it is less prevalent.

The combined effect of windpacking and creep may explain the influence of slope angle on the strength results. The tests made on steeper slopes are included in the refined data set and slope effects are accepted as a source of variability.

4.7.3 Notch Shape

Rounded notches of 50 mm radius were used for 514 of the 555 tests. For the remaining 41 tests, V- and Y-shaped notches were used to investigate whether the fractures were notch-sensitive. The regression coefficients given in Table 4.4 show that the 25 tests with the Y-shaped notches give values of strength that are significantly reduced. Except for the discussion of notch effects (Section 5.8), these 25 tests are rejected and excluded from the refined data set. Because the 16 tests with V-shaped notches did not give reduced strengths there was no need to reject these tests.

4.7.4 Width of Fracture Surface

The analysis associates wider fracture surfaces with reduced values of strength. Studies of many different materials dating back to Griffith (1920) support the idea that larger specimens may contain larger flaws and thus, on average, be weaker. Similarily, for snow, decreasing strength for larger specimens has been reported by Sommerfeld (1971, 1974).

In this work, many of the specimens with wide fracture surfaces and low strength values are those in which the fracture crossed the front or back of the notched zone. These unusual fractures are rejected for reasons discussed in Section 4.8. Size effects are discussed for the remainder of the data in Section 5.6.

4.7.5 Depth of Fracture Surface

The regression analysis associates a increase in strength with a increase in the depth of the fracture surface. This trend may be due to a limited number of tests made on wind-deposited slabs which were comparatively strong and quite thick.

4.8 Significant Fracture Characteristics

The 28 fractures which crossed the back of the notched zone and the 7 fractures which crossed the front of the notched zone yield strengths substantially lower than those for normal fractures when the strength calculation is based on the actual fracture area. Since fractures may spread from a weakness or flaw in the snow, they may not always occur through the smallest cross section. However, the reduction in strength for these two types of unusual fracture is also statistically significant when the calculations are repeated using the narrowest cross section of the notched zone. Therefore, it is more likely that these unusual fractures were due to disturbance of the snow block prior to loading than to fractures which propagated from flaws at the front or back of the notched zone. Accordingly, these 35 tests are excluded from the refined data set.

4.9 Screening of Test Results

As described above, some of the variability of the data was attributed to specific tests conditions and unusual fracture characteristics. Thus, a refined set of data for studying the tensile strength of snow is obtained by rejecting:

- 25 tests made with Y-shaped notches,
- 38 tests with loading times greater than 5 seconds,
- 28 tests in which the fracture is at the back of the notched zone, and
- 7 tests in which the fracture is at the front of the notched zone.

Except for the discussion of notch-sensitivity and rate effects, the refined data set representing 457 tests is used to present the results and develop the conclusions.

Chapter 5

Results

5.1 Introduction

In situ uniaxial tensile tests allow large specimens to be tested without extraction from the snowpack or transportation to the laboratory, as discussed in Chapter 2. The three previous studies which used in situ uniaxial test methods (McClung, 1979b; Conway and Abrahamson, 1984; Rosso, 1987) report a total of 83 tests. For these test methods the precision was not determined and the effect of loading rates and specimen size was not studied. Further, although these test methods are suited to studying the dependence of tensile strength on microstructure, the strength of faceted grains which plays an important role in many slab avalanches has not been studied. Finally, except for McClung's (1979b) tilt-table tests which were not sensitive to notch shape, limited information has been provided regarding the failure mode.

In this chapter the results of 457 in situ uniaxial tensile tests are used to investigate the tensile strength of snow with respect to temperature, density and microstructure (Section 5.2), specimen size (Section 5.6), areal variability (Section 5.5) and statistical distribution and precision (Sections 5.3 and 5.4). Investigations into rate effects (Section 5.7), notch sensitivity (Section 5.8) and critical strain (Section 5.9) are used to show that the fractures are essentially brittle (Section 5.10). The present results are compared to previous in situ uniaxial tensile studies (Section 5.11) and to a previous shear strength study (Section 5.12).

5.2 Dependence of Strength on Material Properties

This section discusses the effects of density, microstructure and temperature on tensile strength based on the refined set of 457 tests. The results for the refined set of data are tabulated in Appendix D.

5.2.1 Temperature

If density and microstructure are held constant, colder snow is expected to be stronger than warmer snow (Mellor and Smith, 1966). Perla et al. (1982) investigated the effect of temperature on in situ shear strength. For each of the investigated microstructures, the correlation between the residuals $\tau - \tau_{\text{ice}}(\rho/\rho_{\text{ice}})^{a}$ is reported. The equivalent expression for tensile strength residuals is $\sigma - A(\rho/\rho_{\text{ice}})^{a}$. However, because of the non-constant variance of the present data, the tensile strength residuals $\mathcal{E}$ were calculated after a log-log transformation:

$$\mathcal{E} = \ln \sigma - [\ln A + a \ln(\rho/\rho_{\text{ice}})] \tag{5.1}$$

The resulting correlation coefficients and confidence levels (based on a two-tailed t-test), for the shear strength study and the present tensile strength study, are shown in Table 5.1.

Both studies show that if density is held constant, the strength of faceted snow increases with decreasing temperature. However, for Group I microstructures the correlation coefficients are less significant and show a trend toward positive values. Perla et al. (1982) explain this trend by noting that temperature, density and

Table 5.1: Temperature Effects for In Situ Strength Tests

Microstructure	Shear Strength $\tau - \tau_{ice}(\rho/\rho_{ice})^a$ vs Temperature (Perla et al., 1982)			Tensile Strength $\ln\sigma - [\ln A + a\ln(\rho/\rho_{ice})]$ vs Temperature (Present Study)		
Group I	No. of Tests	Correlation Coefficient	Confid. Level	No. of Tests	Correlation Coefficient	Confid. Level
new snow	27	-0.141	52%	24	-0.123	43%
partly settled	66	0.272	97%	34	-0.082	36%
rounded grains	118	0.112	77%	209	0.038	42%
multilayer specimens	-	-	-	24	0.426	96%
Group II						
faceted grains	341	-0.189	99%	51	-0.202	85%
faceted and rounded grains	-	-	-	96	-0.185	93%

strength all tend to increase with burial depth and age. Specifically, a low temperature gradient tends to strengthen snow layers through the growth of intergranular bonds (Keeler, 1969b). Thus, it is likely that, for Group I microstructures, the expected negative correlation for strength versus temperature is obscured by the age-sintering process that tends to accelerate with time as the winter progresses and both snowpack depth and air temperatures increase.

5.2.2 Density and Microstructure

The microstructures of the 457 test specimens are partitioned in Table 5.2.

There are 66 samples which contain, on average, 7 tests. Each test in a sample was obtained by the same test method applied consecutively on the same day to

Table 5.2: Partition of Tests by Microstructure

	Microstructure	No. of Tests	No. of Samples
Group I	newly fallen snow	24	6
	partly settled grains	34	5
	rounded grains	230	26
	multilayer specimens	24	6
Group II	faceted grains	94	15
	faceted and rounded grains	51	8

adjacent or nearby specimens from the same layer(s) of snow. The number of tests per sample ranged from 1 to 42.

In Figure 5.1, mean strength is plotted against mean density for each of the 66 samples. The non-linear dependence of strength on density is apparent, as is the relative weakness of Group II microstructures compared to Group I microstructures.

The strength of the multilayer specimens is similar to that of the other Group I microstructures. This is expected since these specimens are portions of snow slabs, and these snow slabs typically consist of layers of new snow, partly settled grains and rounded grains. (In the Canadian Rockies, the weak layers which underlie snow slabs typically consist of Group II microstructures.)

Using Equation 4.2 and the Regress routine of the Minitab statistical package, the 312 test results in Group I and the 145 test results in Group II are regressed separately. The resulting regression parameters and coefficients of determination are summarized in Table 5.3.

Although only the means of the 66 samples are plotted in Figure 5.1, all 457

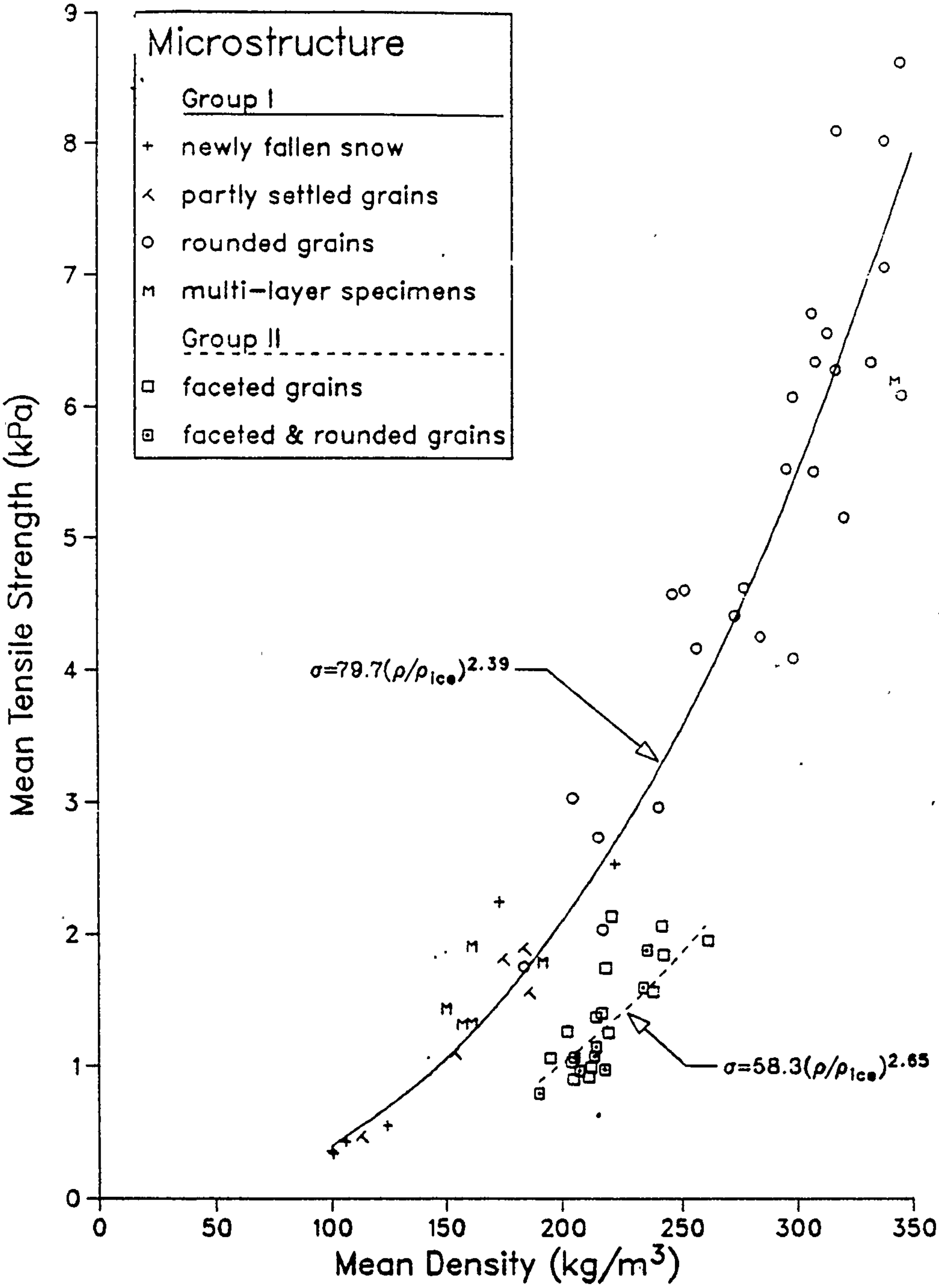

Figure 5.1: Dependence of Tensile Strength on Density and Microstructure. The 66 mean strengths and mean densities are based on a total of 457 tests made in the Alberta Rockies during the winter of 1987-88.

Table 5.3: Estimates of Regression Parameters for $\ln \sigma = \ln A + a \ln(\rho/\rho_{\text{ice}})$

Microstructure	No. of Tests	A (kPa)	a	Coefficient of Determination $100\%R^2$
Group I	312	79.7	2.39	88.4%
Group II	145	58.3	2.65	39.0%

tests are used to calculate the two regressions which are presented in exponential rather than logarithmic form.

5.2.3 Discussion

Group II microstructures were tested in the density range 190 to 260 kg/m^3. Throughout this range, Group II microstructures were approximately half as strong as Group I microstructures. However, because blocks of faceted snow occasionally broke when either the frames or plate were inserted, mean strengths and the regression for Group II microstructures may be biased towards higher values.

The coefficient of determination for the Group II regression is substantially lower than for the Group I regression (Table 5.3). This suggests that the strength of Group II microstructures may vary more widely than that of Group I microstructures. Sommerfeld (1973) also reports that the strength of faceted snow is more variable than new, partly settled or rounded snow.

5.3 Statistical Distribution of Strength

Many statistical techniques are based on an assumption of normality of the data, although most such techniques are "robust" under a relaxation of this assumption

(Harris, 1975, p. 231). In support of the multivariate regression and of the t-tests which follow in this chapter, this section investigates the statistical distribution of repeated tensile strength measurements.

Two study plots were judged to be uniform by inspection and by probing. In the first study plot, one layer of rounded grains was tested 42 times. In the second plot, 30 tests were made on a layer of rounded grains. Test conditions such as load time and cross-sectional area were kept as constant as possible. The results are summarized in Table 5.4.

(It should be noted that the tested snow layers in the two study plots were deposited by the same snow storm, by the time they were tested their material properties were quite different due to the combined effects of wind, elevation, orientation to sun, temperature gradient, etc.)

Table 5.4: Results from the Two Large Samples of Replicates

Study Plot		Lipalian Shoulder	Richardson Bench
Date		88/02/06	88/02/07
Mean Temperature (°C)		-9.4	-7.3
Density	Mean (kg/m^3)	216	251
	St. Dev.	8	6
	Coef. of Var. (%)	4	2
Tensile Strength	No. of Tests	42	30
	Mean (kPa)	2.03	4.60
	St. Dev. (kPa)	0.41	0.93
	Coef. of Var. (%)	20	20

The normality of a set of replicates can be assessed by inspection of a normal probability plot, or by a statistical test of normality. Normal probability graphs (Figures 5.2 and 5.3) of the two sets of replicates were obtained by plotting the strengths against the normal scores for a population of the same size, mean and standard deviation (Minitab Reference Manual, 1985, p. 46). The relative "straightness" of the plotted points suggests the data are from a normal distribution.

The "straightness" of the data plotted in Figures 5.2 and 5.3 and thus the normality of the data can be tested by calculating the correlation coefficients for the data plotted in each of the two graphs and comparing these r-values with tabulated values required at the 99% confidence level (Minitab Reference Manual, 1985, p. 46). Coefficients of the correlation between the strengths and their normal scores are given in Table 5.5 with the tabulated values required at the 99% confidence level.

Table 5.5: Test of Normality for Largest Samples of Replicates

Study Site	No. of Tests	Correlation Coefficient	Value Required at 99% Level
Lipalian Shoulder	42	0.989	0.960
Richardson Bench	30	0.987	0.949

In statistical terms, Table 5.5 indicates that the hypothesis of normality cannot be rejected at the 99% level. This provides strong evidence that the replicates come from normal distributions.

5.4 Precision

The precision of mean strength values depends on the variability of the results and on the number of tests. From the coefficient of variation, the number of tests

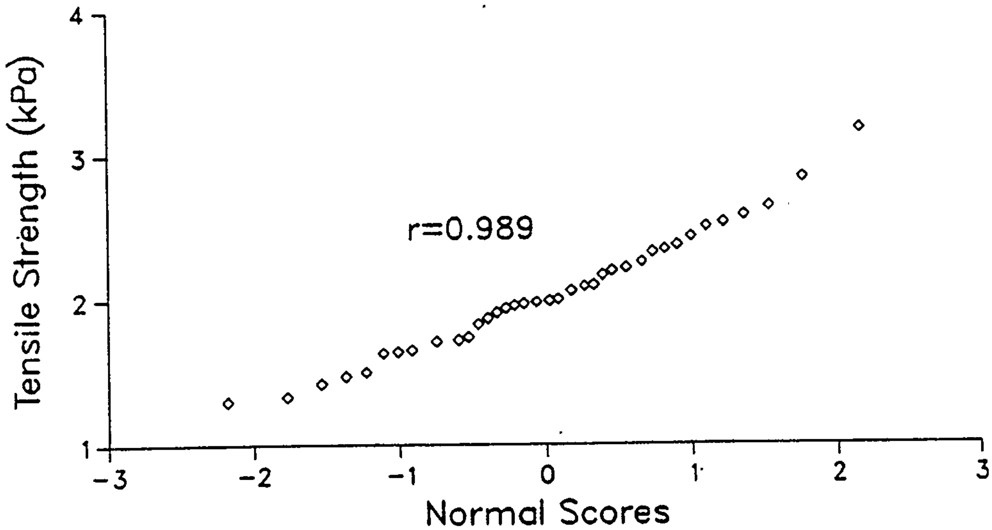

Figure 5.2: Normal Probability Graph of Sample of 42 Replicates. Tensile strength tests made on a layer of rounded snow with a density of 216 kg/m^3 at Lipalian Study Plot on February 6, 1988.

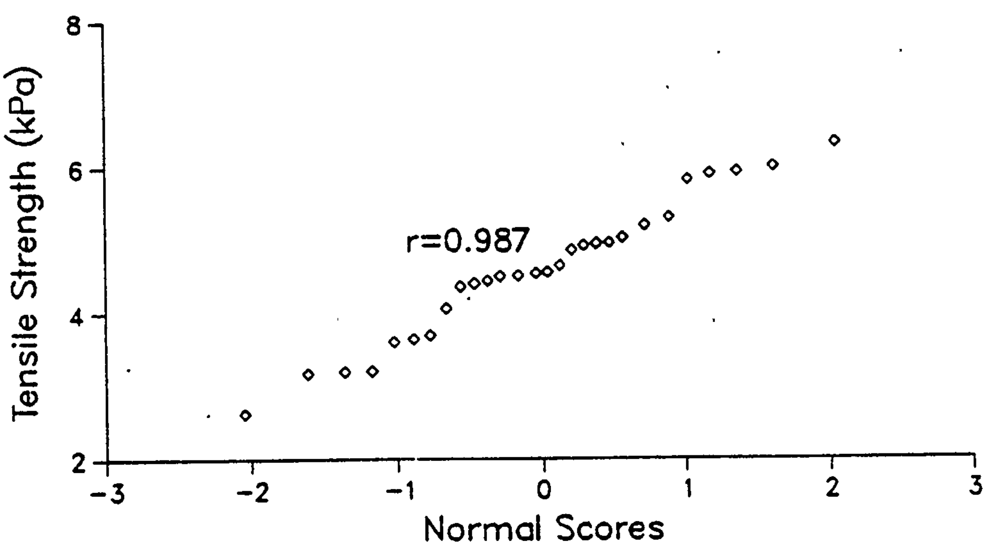

Figure 5.3: Normal Probability Graph of Sample of 30 Replicates. Tensile strength tests made on a layer of rounded grains with a density of 251 kg/m^3 at Richardson Study Plot on February 7, 1988.

required to obtain specified levels of precision can be estimated.

The coefficient of variation (v) of tensile strength for each of the two large samples considered in the previous section is 0.202. Considering all the 61 samples of more than 1 test, the coefficients of variation ranged from 0.05 to 0.45 with a mean of 0.192 (and a weighted mean of 0.196).

Therefore, using 0.20 as a representative value for v, the number of tests which must be averaged to obtain precision p at the $1 - 2\alpha$ confidence level is given by:

$$n = (t_{\alpha;\text{n-1}} v/p)^2 \tag{5.2}$$

in which $t_{\alpha;\text{n-1}}$ is the tabulated value from a Students t-distribution with $n - 1$ degrees of freedom which has a probability of α associated with each tail. Equation 5.2 is solved by trial and error and the results are given in Table 5.6.

Table 5.6: Number of Tests for Required Precision

Required Precision of Mean (p)	Confidence Level $100\%(1-2\alpha)$	Estimated Number of Tests (n)
10	90	13
10	95	18
15	90	7
15	95	10

On average, there were 7 tests per sample in this study. With 90% confidence, the mean strength determined from 7 replicates should have a precision of 15%.

The precision of previous in situ *tensile* tests has not been determined. However, Perla (1977) reports that depending on the uniformity of the snow in a study site, 4 to 10 *shear* frame tests are required for 15% precision at 90% confidence.

5.5 Spatial Distribution of Strength

5.5.1 Between Study Sites

Tensile tests were made at many different sites (Table 5.7), all but one of which were in or near the Lake Louise Ski Resort. The snow strength at the various sites differed, either because different layers were tested, or in those cases for which the same layer was tested, because the strength of the layer was affected by environmental and terrain factors associated with the location.

Table 5.7: Partition of Tests by Site

No. of Tests	No. of Samples	Study Site
162	23	Richardson Bench[1] (elev. 2335m) at Lake Louise Resort
16	3	Top of Larch Chairlift[1] (elev. 2400m) at Lake Louise Resort
171	24	Wolverine[1] (elev. 2240m) near Lake Louise Resort
42	1	Lipalian Shoulder[1] (elev. 2215m) near Lake Louise Resort
6	1	Pika Corner[1] (elev. 2235m) at Lake Louise Resort
50	13	Avalanche starting zones in or near Lake Louise Resort
11	1	Flats below Mt. Blackprince (elev. 1890m) in Kananaskis Country

[1] Location of study plot is shown in Appendix B.

Factors which determine the initial cohesion of a newly fallen layer include windpacking and the shape of the new snow forms. For example, *graupel* or soft hail is deposited as uncohesive spheres, whereas *stellar* crystals have branches which tend to interlock. Subsequently, the combined effects of overburden pressure, density, temperature and temperature gradient may cause the strength to either increase or decrease (de Quervain, 1963; Perla and Martinelli, 1976, p. 42-52). Therefore, the strength of newly fallen and *old* snow, depends on factors such as the elevation and orientation to wind and sun which vary with the location. An example of this is given in Section 5.3. Several weeks after a layer of snow was deposited by a particular storm, the layer was more than twice as strong at one study plot as at another site 3.5 km away.

Shear strength tests of weak layers and penetration tests of the snowpack made in study plots are two of many measurements used by professional avalanche forecasters to forecast the avalanche hazard in slide paths often several kilometres from the study plot. Usually study plots are easily accessible and sheltered from the wind, while the starting zones of the slide paths are difficult to access and are exposed to strong winds. In most cases the properties of the snowpack in the starting zones are not measured, but are estimated by extrapolation from snowpack measurements at one or more study plots and by considering readings made at several meteorological stations. In addition, observations of avalanche activity and explosive tests are often used to form and confirm the forecast.

5.5.2 Areal Variability Within Study Plots

In all but a few rare situations, the snowpack consists of more than one distinct layer. For this reason, the heterogeneity of the snowpack is discussed in terms of the *areal variability* of individual layers (Keeler and Weeks, 1968; Conway and Abrahamson, 1988; Föhn, 1988). This section considers the areal variability of layers within study plots and assumes each layer to be vertically homogeneous.

The guidelines for selecting study plots (NRCC and CAA, 1986, p. 34) suggest that sites be as close as possible to avalanche starting zones and be either sheltered from the wind or subject to minimal drifting. Clearly, substantial drifting will deposit layers of uneven thickness and varying material properties.

Wind is not the only factor which affects the areal variability of individual snow layers. Features of the ground cover such as rocks, small bushes and poorly drained soil may influence the temperature gradient and vapour flow in the overlying snow and consequently may affect the material properties of the snow layers.

Areal variability is investigated using the data from repeated tests of the same layer on the same day within a rectangular *subplot* of a larger study plot. The position of any test within a subplot is identified by row and column numbers as described in Section 3.5. Strength measurements are not available for every position within a subplot, either because the snow at a particular position was disturbed or because the test made at that position was rejected for reasons discussed in Section 4.9. Since some tests are "missing" from the rectangular subplots, a regression is used in place of a conventional two-way analysis of variance. (Minitab Reference Manual, 1985, p. 106). For simplicity, a linear additive model is used to regress the strength on the row and column numbers:

$$\ln \sigma = a + b(N_{\text{row}}) + c(N_{\text{column}}) + \mathcal{E} \tag{5.3}$$

in which a, b and c are regression coefficients, $\mathcal{E}$ is the residual and N_{row} and N_{column} are the row and column numbers. (Since the spacing between rows and between columns is fairly constant, the row and column numbers represent interval data.) For the regression, the variance of the dependent variable (σ) is stabilized by a logarithmic transformation (Section 4.2.2).

The IMSL routine RLSEP is used to calculate the regression and the following quantities: the t-ratios for the row and column terms, R^2 and the $1-\alpha$ probability level. The coefficient of determination (R^2) is the percentage of the total variance associated with independent linear trends along rows and columns of a subplot. The probability that the set of replicates does not come from a uniform layer with only random variability is $1-\alpha$.

Using the 24 tests made at Wolverine Study Plot on March 2, 1988 as an example, the regression equation is:

$$\ln \sigma = 1.34 - 0.013(N_{\text{row}}) + 0.098(N_{\text{column}}) + \mathcal{E}$$

The t-ratios for the row and column coefficients are -0.65 and 2.30 respectively, which indicate a substantial linear change in strength with column number but not with row number. This is shown graphically in Figure 5.4.

For the regression on this set of 24 replicates, $R^2 = 0.20$ and $1-\alpha = 0.91$. Therefore, 20% of the variance is associated with linear trends along the rows and columns, and there is a 91% probability that the data did not come from a uniform layer.

The regression procedure and inferences shown in the above example were applied to 24 sets of 6 or more replicates for which the row and column numbers were recorded. For each of these data sets, the coefficient of variation v, R^2 and $1-\alpha$ are given in Table 5.8.

As detected by the linear model, most of the subplots do not show consistent changes in strength along the rows or columns. However, for three of the 24 regressions, R^2 is significant at the 90% level. Therefore, within some of the subplots, substantial *areal* trends in tensile strength are apparent. Such trends

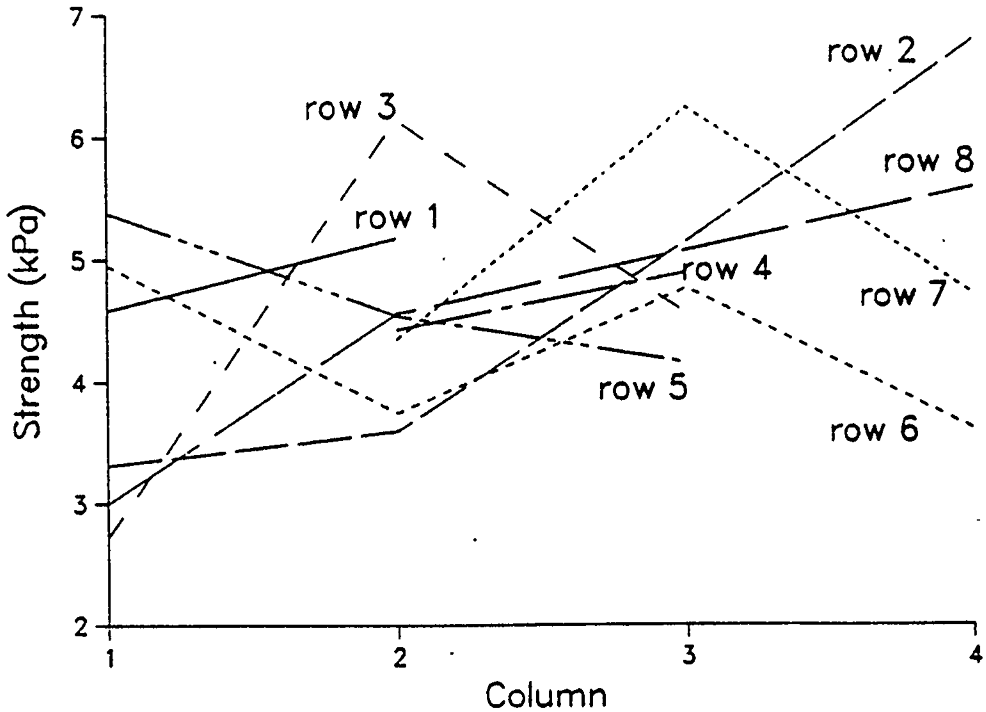

Figure 5.4: Strength Variation Within a Rectangular Subplot
Twenty-four tensile tests of rounded snow with a density of 277 kg/m^3 made in Wolverine Study Plot on March 2, 1988.

Table 5.8: Variability Associated with Position Within Subplots

Study Plot	Date	Micro-structure	Density (kg/m^3)	No. of Tests	100v (%)	100R^2 (%)	100(1-α) (%)
Richardson	88/01/01	faceted	211	8	15	43	76
"	88/01/04	faceted	214	6	16	3	1
"	88/01/07	"	218	11	10	15	48
"	88/01/10	"	220	8	16	62	91
"	88/01/16	"	219	6	22	43	57
"	88/02/07	rounded	251	30	20	12	82
"	88/03/14	"	313	11	32	36	83
Wolverine	87/12/28	faceted	205	6	21	58	73
"	88/01/01	"	204	8	20	29	58
"	88/01/04	"	216	7	9	18	32
"	88/03/02	rounded	277	24	22	20	91
"	88/03/09	"	307	13	18	23	73
"	88/03/09	faceted	261	8	27	35	65
"	88/03/20	rounded	284	7	32	39	63
"	88/03/22	partly settled	113	10	23	9	28
"	88/03/24	"	154	7	18	12	55
"	88/03/26	"	184	10	16	16	46
"	88/03/26	rounded	298	10	22	44	87
"	88/03/30	"	215	6	34	78	90
"	88/04/02	"	317	6	19	35	48
"	88/04/17	rounded	338	9	17	27	84
"	88/04/17	"	332.	10	19	8	57
Lipalian	88/02/06	"	217	42	20	1	20
Bl. Prince	88/03/16	"	204	11	16	37	84

are probably indications of wind affecting the deposition of certain layers and thus reflect the compromise in study site location between proximity to starting zones and shelter from wind.

In a study of shear strength throughout the depth of the snowpack, Keeler and Weeks (1968) separated the effects of position within a plot from random error at two of the ten test levels within the snowpack. This shear strength result and the present result for tensile strength show that areal strength trends within study plots are occasionally substantial.

5.6 Size Effects

Since larger specimens may contain larger flaws, the mean strength in a tensile test is expected to decrease with increasing volume. Based on tensile spin tests of two different sizes of specimens, Sommerfeld (1971, 1974) reports that the specimens with larger volumes had, on average, lower strengths. Applying this size effect to a uniaxial tensile test, the mean strength is expected to decrease with increasing cross-sectional area provided the specimen length remains constant.

In this section, two approaches are used to study such size effects. The first approach is based on 24 tensile tests made on the same layer of rounded grains on the same day (Wolverine Study Plot, March 2, 1988). The cross section was deliberately varied from 1.4×10^{-2} m^2 to 4.1×10^{-2} m^2 while the loading rate and other conditions were kept as constant as possible. For the correlation of strength with cross-sectional area, A_{cs}, the r-value is -0.129 (Table 5.9).

In the second approach, all 457 tests are used. Because of the distinct microstructures and the range of density in this data set, *residuals* $\ln\sigma - [\ln A + a\ln(\rho/\rho_{ice})]$ are correlated with cross-sectional area for each of the two microstructure groups. The correlation coefficients are 0.187 for the 312 tests in Group I and -0.153 for the 145 tests in Group II (Table 5.9).

Table 5.9: Correlations with Cross-Sectional Area

Correlation	Micro-structure	No. of Tests	r-value
σ *vs* A_{cs}	rounded	24	-0.129
$\ln\sigma - [\ln A + a\ln(\rho/\rho_{ice})]$ vs A_{cs}	Group I	312	0.187
$\ln\sigma - [\ln A + a\ln(\rho/\rho_{ice})]$ vs A_{cs}	Group II	145	-0.153

Although two of the three correlations show the expected negative trend indicating a decrease in strength with an increase in cross-sectional area, the low r-values and the inconsistent sign suggest that the effect of size on the results is minor or is obscured by factors inherent to the slip plate test such as the variability of the manual pull rate.

5.7 Rate Effects

In the laboratory, Narita (1980, 1983) used various constant strain rates to test snow ranging in density from 250 to 450 kg/m^3. At strain rates greater than 2x10^{-4} s^{-1}, 40 tests were made, each of which exhibited essentially linear stress-strain curves with failure strains of 0.8% or less. Therefore, all 40 tests in which failure occurred in less than 40 seconds exhibited linear stress-strain curves. For these tests, a decrease in strength is associated with an increase in strain rate.

Typically, the *laboratory* strength of snow with a density of 300 kg/m^3 was reduced from 50 kPa to 25 kPa by increasing the strain rate from $3x10^{-4}$ s^{-1} to $10x10^{-4}$ s^{-1}. (Similarly, Hawkes and Mellor (1972) found the tensile strength of ice was reduced by increasing the strain rate above 10^{-3} s^{-1}.)

Singh (1980) removed snow samples which ranged in density from 136 to 294 kg/m^3 from the snowpack and applied tension and compression at constant strain rates in the field. Eighteen tension tests resulted in fractures; seventeen of these exhibited essentially linear stress-strain curves and failed in 80 seconds or less.

The proportionality between stress and strain suggests that, for constant stress rate tests with loading times of 40 seconds or less, and possibly as high as 80 seconds, strength is expected to decrease with increasing stress (or strain) rate.

In the present study, the load was manually increased as uniformly as possible. Although control over the loading rate was poor, the applied stress rate during any one test is assumed to be approximately constant. Thus:

$$\dot{\sigma} \approx \sigma / t \tag{5.4}$$

in which σ and t are the stress and loading time at failure, respectively.

There were 457 tests with loading times of 5 seconds or less and an additional 38 tests with loading times of up to 65 seconds. For all these tests, failure times are approximately within the range studied by Narita (1983) and Singh (1980) which corresponded to linear stress-strain curves. Therefore, it is expected that increases (or decreases) in the loading rate in the present study should affect the strength in the same manner as did increases (or decreases) in the strain rate as measured by Narita and Singh.

The tensile strengths from the present study generally decrease with increasing stress rate, but show wide scatter. Eleven tests at the Wolverine Study Plot on March 24, 1988 were made by deliberately varying the loading rate. Figure 5.5 shows the decrease in strength with an increase in loading rate for these eleven tests. Two other similar loading rate experiments show the same trend but with more scatter.

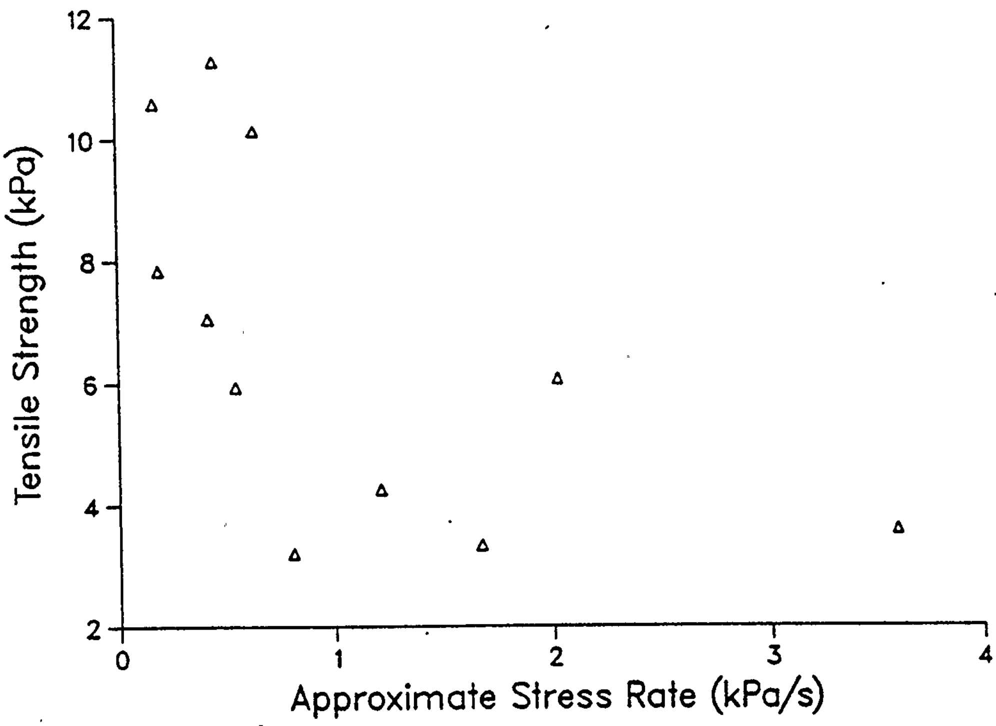

Figure 5.5: Effect of Stress Rate on Strength
Eleven tensile tests made on a layer of rounded grains with a density of 298 kg/m^3 at the Wolverine Study Plot on March 24, 1988.

For the range of snow density and loading times, stress can be assumed to be proportional to strain. Therefore, the general trend for strength to decrease with increasing stress rate is consistent with the strain control experiments of Narita (1980, 1983).

5.8 Notch Sensitivity

For studying notch sensitivity, three distinct notch shapes were used. Each notch was cut with a single downward stroke with one of the three notching tools shown in Figure 3.3.

All but 41 of the tests employed U-shaped notches with a 50 mm radius. V-shaped notches with a 10 mm radius were used for 16 tests. There were 25 tests with a Y-shaped notch. The tip of each Y-shaped notch had a radius of approximately 1 mm.

Using the U-shaped notch as a standard, the effects of V- and Y-shaped notches are analyzed by pairing each V- or Y-shaped notch test with the closest test on the same layer made with a U-shaped notch. The analysis is presented in Appendix C and summarized in Table 5.10.

Table 5.10: Summary of Notch Sensitivity

Notch Shapes Compared	No. of Pairs	Average Reduction in Strength	Confidence Level for Detecting Reduction in Strength
V vs U	14	1%	50%
Y vs U	19	22%	99%

At the 99% level, Y-shaped notches are associated with a reduction in strength as compared to U-shaped notches. This decrease averages 22%. A reduction in strength associated with V-shaped notches (10 mm radius) is not apparent. Therefore, the fractures are sensitive to small radius notches. This result is used in Section 5.10 to show that the fractures are brittle.

McClung (1979b) used notched tensile specimens for tilt-table tests. No sensitivity to notch shape was detected for shapes ranging from thin slits to arc-shaped notches. The difference in notch sensitivity between the two studies may be due to differences in loading rates and in snow temperatures. The temperatures of the snow specimens on the tilt-tables averaged -1.7°C compared with an average of -6.7°C for the present study. As well, loading times for the tilt-table tests averaged 103 seconds compared with an average of 2.2 seconds for the present study.

5.9 Critical Strain

Strain has not been measured in previous in situ studies of the tensile strength of snow. In the present study, measurements of strain just before failure (called the critical strain) were made for a limited number of slip plate tests to permit comparison with critical strains determined in the laboratory by Narita (1980, 1983).

Narita reports the critical strain and strain rate for 117 constant strain rate tests on snow which ranged in density from 250 to 450 kg/m^3. Brittle fractures are characterized by essentially linear stress-strain curves and critical strains of 0.8% or less. Of the 76 ductile failures reported, only five occurred at strains of less than 1%.

A simple photographic technique was used with the present test method to measure approximate values of critical strain. A ruler was inserted across one of the two notches, as shown in Figure 5.6. While the load was being applied, a motor-driven camera, shown in Figure 3.4, photographed the ruler and notch boundary up to 5 times per second. The displacement of the snow block and fracture surfaces *after* fracture is shown in Figure 5.7.

The deformation at failure was determined by subtracting the exposed length of the ruler in the last photograph before fracture or cracking from the exposed length of the ruler in the first photograph taken before the load was applied. If, for example, an additional 1 mm of ruler was exposed in the last photograph, then the 100 mm notched tensile zone must have strained 1%.

The divisions marked on the ruler restrict the accuracy of the measurements to about ± 0.25 mm which is approximately equal to the deformation being measured. In spite of this low degree of accuracy, the technique was useful for establishing a limit on the critical strain.

The notches were photographed for fifteen tests, each of which involved approximately constant loading rates. Critical strains for these 15 tests are shown in Table 5.11. The three tests presented in the bottom three rows of this table are for loading times between 7 and 14 seconds. Even if the three tests with extended loading times are included, the critical strains do not exceed 1%. Therefore, the critical strains are comparable to those reported by Narita (1980, 1983) for brittle failure.

Table 5.11: Critical Strains for Fifteen Slip Plate Tests

Date	Density (kg/m^3)	No. of Photos. Before Fracture	Loading Time (s)	Strain (%)
88/03/30	215	10	3.5	0.5
"	"	3	1	0.5
"	"	6	3	0.25
"	"	3	1	0.5
"	"	4	1.3	0.5
88/04/15	320	4	1.5	0.25
"	300	7	3.5	0.25[a]
"	340	7	1.5	0.0[a]
"	"	5	1.5	0.0[a]
"	"	17	5	0.25[a]
"	345	7	2	0.5[a]
"	345	10	2.5	0.5[a]
88/03/30	215	8	14[b]	0.5
"	"	9	12[b]	0.75
88/04/15	340	5	7.5[b]	0.5

[a] Test rejected from the refined data set because the fracture crossed the back of the notched zone.

[b] Test rejected from the refined data set because the loading time exceeded 5 seconds.

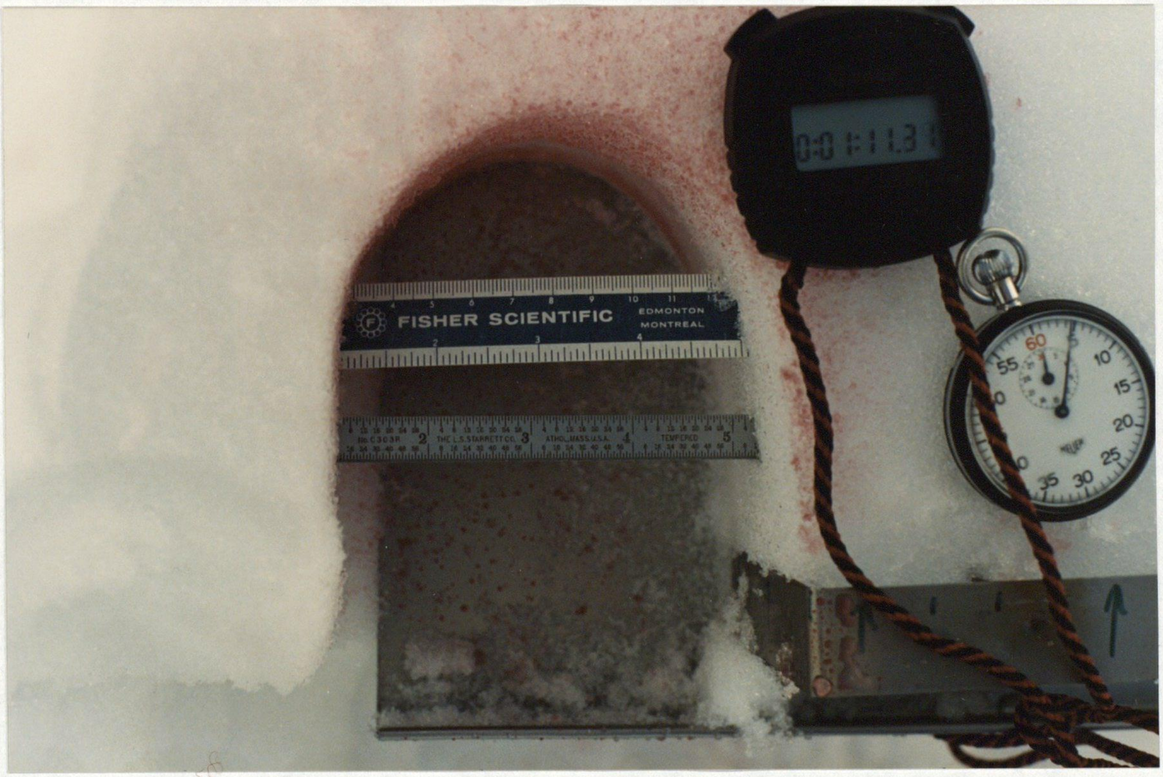

Figure 5.6: Measurement of Notch Deformation During Loading

5.10 Failure Mode

Brittle fractures are usually defined to be those which fail suddenly with little or no *plastic* deformation.

In a thorough laboratory study of tensile fractures, Narita (1980, 1983) identifies the following three characteristics of brittle fractures:

1. sharp fractures perpendicular to the axial load,

2. sudden failure with no microcracking at strains of 0.8% or less, and

3. essentially linear stress-strain behaviour to failure.

Figure 5.7: Displacement of Snow Block and Fracture Surfaces

As a consequence of these results and the observed decrease in strength for an increase in the strain rate, the strength is also expected to decrease with increasing stress rate.

In the present study:

1. fractures were observed to be approximately perpendicular to the axial load;
2. the critical strain was $\leq 1\%$ (Section 5.9); and
3. the strength decreased with loading rate (Section 5.7).

As well, the results are sensitive to the shape of notches (Section 5.8). Therefore, the mode of failure for the present tests is essentially brittle.

5.11 Comparison with Other In Situ Tensile Tests

Previous studies of tensile strength are reviewed in Chapter 2. There have been four previous in situ studies of tensile strength and in three of these studies (McClung, 1979b; Conway and Abrahamson, 1984; Rosso, 1987) uniaxial loads were applied to specimens with cross-sectional areas of 0.1 m^2 or larger.

The results of these three studies are compared with the present results in Figure 5.8. However, tests on faceted grains and tests on moist or wet snow are excluded to reduce the sources of variability in the compared data. This excludes 3 tests on "early TG" (probably faceted grains) from the 13 trapezoidal tests (Rosso, 1987) and 145 tests on Group II (faceted) microstructures from the present study. Also excluded are 9 tests on wet snow from the 38 tilt-table tests (McClung, 1979b) and 21 tests on moist snow from the present study.

Below a density of 200 kg/m^3 the present tensile strengths are consistent with the three previous in situ studies which used uniaxial loading. Above 200 kg/m^3, it appears that the strengths from the present study may be lower than the 18 strengths reported by McClung (1979b) and Conway and Abrahamson (1984) in this density range. The greater strengths from these two latter studies may be due to more gradual loading; Conway and Abrahamson loaded their specimens over 0 to 15 seconds and McClung tilted the tables over 60 to 495 second time spans.

However, only 3 of 11 results from McClung (1979b) and 3 of 7 results from Conway and Abrahamson (1984) are distinctly higher than the present results, so it is not possible to conclude that the present results are systematically reduced compared to these previous studies.

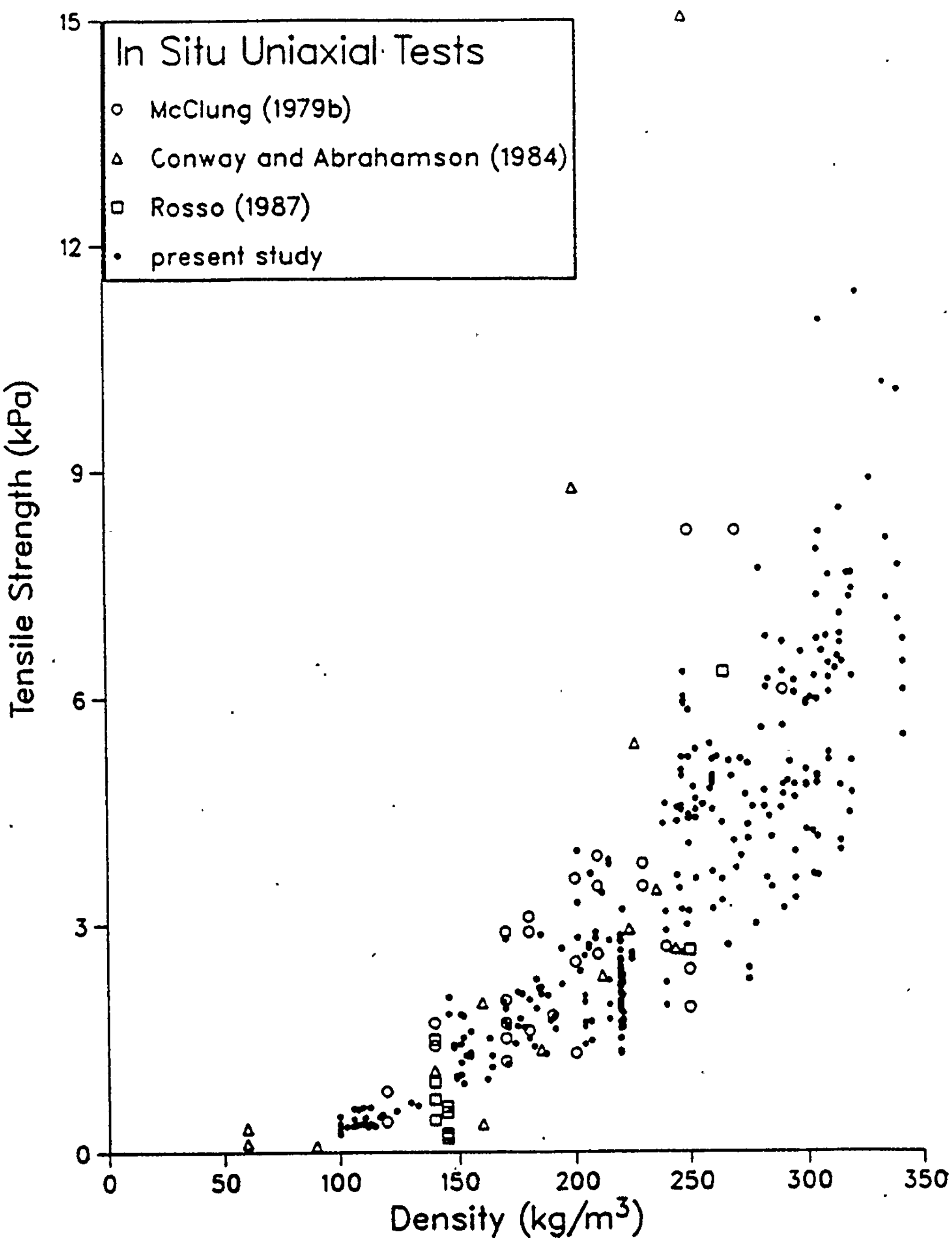

Figure 5.8: Comparison of Results from In Situ Uniaxial Tensile Strength Studies. Only results for dry snow classified as new, partly settled or rounded are included.

Each of the three in situ uniaxial test methods shows a similar amount of variability of strength as a function of density. This suggests that much of the variability of strength is independent of the test method and may be associated with a natural dispersion of material properties.

5.12 Ratio of Tensile Strength to Shear Strength

The ratio of tensile strength to shear strength has been used to compare the results of tensile strength studies with shear strength studies. However, a wide range of values for the ratio of tensile strength to shear strength (hereafter denoted by σ/τ) are reported for snow. Roch (1966b) gives values ranging from 1 to 5 for which the lower values apply to snow with an uncohesive (probably faceted) microstructure. Keeler and Weeks (1968) and Keeler (1969a) report ratios of 10 and 6.5 respectively but suggest that the high ratio may be due to the stress rate which was 4 times greater for the shear tests than for the tension tests. McClung (1979b, pp. 327-328) hypothesizes that σ/τ may approach 1:

> It is not possible to (theoretically) relate the shear strength to the tensile strength of snow at present, because the failure surface is unknown. Common failure theories for metals predict that the shear strength is one-half to two-thirds of the tensile strength. Granular materials such as snow, being much stronger in compression than in tension, might, however, be expected to have a shear strength which approaches the tensile strength due to possible widening of the failure envelope. ... If failure strengths could be compared for homogeneous samples, presum-

ably the tensile strength would be approximately equal to the shear strength.

The tensile strength data from the present study are well suited for comparison by the σ/τ ratio with the shear strength data of Perla et al. (1982) for the following reasons:

- both studies are based on in situ tests in which the specimen is manually pulled to failure within 5 seconds;
- the failure area in the present study averages 0.028 m^2 which compares very well with a constant value of 0.025 m^2 for the shear strength study;
- both studies collected field data from the Alberta Rocky Mountains;
- the same two-parameter model is used to regress strength on density; and
- the two studies include results for the same four types of microstructure.

For each of the common microstructures, regressions based on the two-parameter model are presented in Table 5.12. However, the variance in the present data is stabilized by the log-log transformation (Section 4.2.2) before the regression is computed by the Minitab routine Regress.

Using the regressions from Table 5.12, the ratio σ/τ is computed for the four microstructures. These ratios are plotted in Figure 5.9 for the range of density for which the particular microstructure is commonly found in slab avalanches. For newly fallen snow, σ/τ ranges from 1 to 4. For the other three microstructures the ratio is close to 1, which is consistent with McClung's hypothesis. Clearly, such values are more reasonable than many of the previously reported large values.

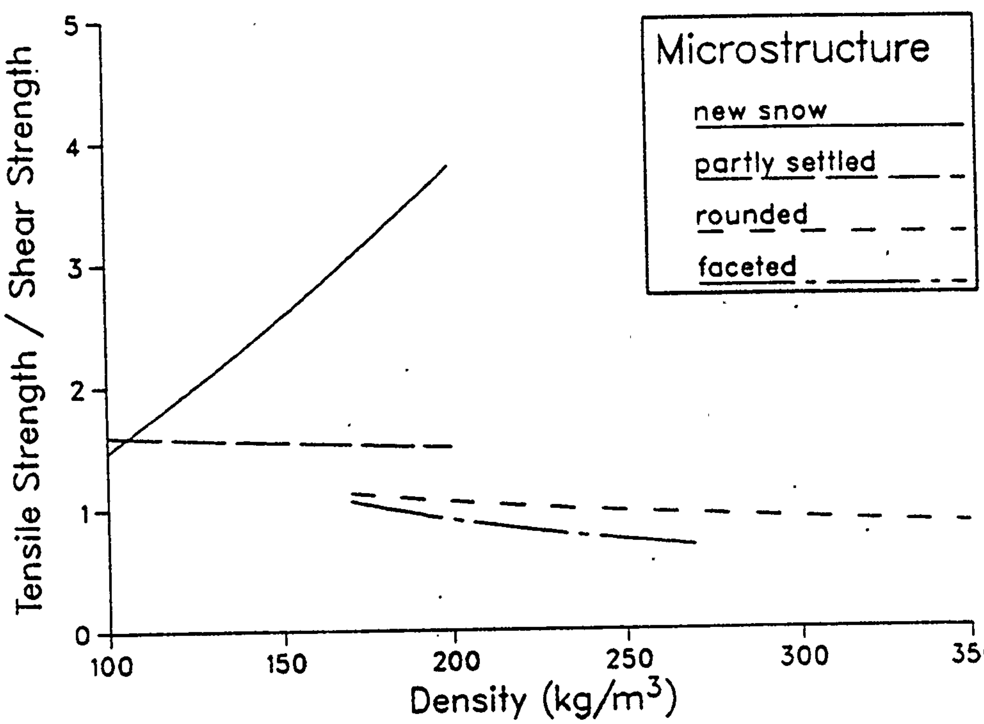

Figure 5.9: Ratio of Tensile Strength Regression to Shear Strength Regression for Four Types of Microstructure.
Shear strength regressions from Perla et al. (1982) and tensile strength regressions from the present study.

Table 5.12: Shear and Tensile Strength Regressions

	Shear Strength Perla et al. 1982 $\tau = \tau_{ice}(\rho/\rho_{ice})^k$				Tensile Strength Present Study $\ln\sigma = \ln A + a\ln(\rho/\rho_{ice})$			
Microstructure	No. of Tests	τ_{ice} (kPa)	k	$100R^2$ (%)	No. of Tests	σ_{ice} (kPa)	a	$100R^2$ (%)
New Snow	27	5	1.38	36	24	153	2.75	92
Partly Settled	66	134	2.92	88	34	175	2.83	90
Rounded	118	143	2.82	70	230	85	2.44	69
Faceted	341	238	3.47	68	94	51	2.52	39

5.13 Summary

Sixty layers and six groups of contiguous layers were each tested an average of 7 times for a total of 457 tests. On a generally uniform layer, 7 tests are required to obtain a precision of 15% with 90% confidence. Strengths from repeated tests are shown to be normally distributed.

The tensile strength for new snow, partly settled snow and rounded snow is presented for the density range of 100 to 360 kg/m^3. The tensile strength of faceted snow and of snow which is both rounded and faceted is presented for the density range of 190 to 260 kg/m^3. In this range, the latter types of microstructure were approximately half as strong as the snow with a partly settled or rounded microstructure.

The failures are shown to be essentially brittle by determining approximate values for critical strain, by demonstrating a decrease in strength for an increase in the loading rate and by showing that the strength is sensitive to notch shape.

By excluding the tests on moist, wet and faceted snow, the strengths from the present study are compared with the strengths from the three previous in situ axial studies and are in good agreement for densities between 100 and 200 kg/m^3. Between 200 and 300 kg/m^3, two of the three previous studies show some higher strengths than the present study.

Using the present tensile strength regressions and the shear strength regressions of Perla et al. (1982), the ratio of tensile to shear strength approximates unity as hypothesized by McClung (1979b).

Chapter 6

Applications

Two questions are fundamental to avalanche forecasting:

1. When are avalanches likely to occur?

2. How large are potential avalanches likely to be?

Applications of the slip plate tensile test to the two fundamental questions of avalanche forecasting were attempted during the winter of 1987-88. In this chapter, these questions are discussed in regard to slab avalanches which are reported in the literature to be generally larger and more difficult to forecast than loose snow slides.

Early in the project, a decrease in the tensile strength of important slab layers was considered as a possible indicator of an increase in avalanche activity. Four studies of change in the tensile strength of particular layers over time were completed. However, the results of these time studies presented in Section 6.1 do not support a relationship between changes in the tensile strength of snow slabs, as measured by the slip plate test, and avalanche occurrences. Further, slip plate tests of snow slabs are not a promising approach to the first fundamental question regarding avalanche occurrences (Section 6.1.5).

In Section 6.2, results are presented to support a relationship between the width of slab avalanches and both the tensile strength and thickness of snow slabs. Since measurements of the tensile strength and thickness of snow slabs can be made prior to avalanche activity, such measurements may provide a method of predicting the width and thus estimating the size of potential avalanches.

6.1 Time Studies of Strength and Delayed Avalanches

Most natural slab avalanches occur during or soon after the starting zones are loaded by snowfall or by the redistribution of snow by wind. However, it is the "delayed action avalanches" that are not related to an increase in the snow load which most challenge avalanche forecasters.

The failure of a snow slab under constant load has been investigated by McClung (1981b, 1987) by applying the principles of progressive growth of slip surfaces in over-consolidated clays (Palmer and Rice, 1973). This theory deals with the growth by shearing, of a flaw called a slip band in a weak layer below a stronger layer. Using Griffith's (1920) energy criterion, a critical length is derived beyond which the slip band is unstable. The critical length depends, in part, on the stiffness of the overlying slab. For a linear elastic model of a snow slab, this stiffness is represented by Young's modulus; for a viscous slab, the stiffness is represented by the reciprocal of the creep compliance. If the slab becomes less stiff (or more compliant), then a particular length of slip band may become critical. Such changes in stiffness may be due to warming or to metamorphic changes in the microstructure of the snow slab (McClung, 1987).

Lacking methods of directly measuring the creep compliance or Young's modulus in situ, the tensile strength was examined as a possible indicator of changes in the mechanical properties of the slab that might lead to instability.

Delayed action avalanches are believed to be more common in the Canadian Rockies than in the Interior Ranges of British Columbia or in the Coast Ranges. In fact, the avalanche forecaster for Banff National Park estimated that up to 12 such avalanches might occur during the winter in the mountains in and near the Lake Louise Ski Resort. Therefore, this area was well suited to investigating the hypothesis that decreases in the tensile strength of snow slabs might contribute to the occurrence of delayed action avalanches.

A series of tensile tests and other measurements of a particular slab made over a period of days or weeks is referred to as a "time profile".

Four time profiles were completed. In each case, the slab being measured consisted of a single layer at the start of the time profile. Additional layers deposited by snowfall during a time profile were carefully scooped off the top of the "indicator layer" before each test was made. The tensile strength measurements on a particular indicator layer were discontinued after the underlying weak layer strengthened sufficiently to stabilize the slab.

The mechanical properties of snow slabs vary with aspect and exposure to wind as discussed in Section 5.5. Two study plots were used in which only minimal wind effects were observed: one at the 2335 m level on the south side of Mt. Richardson; and one with a northeast aspect at the 2240 m level in Wolverine Basin. Initially, a third site, the Larch Study Plot at the 2400 m level on Mount Lipalian, was also used but measurements at this site were discontinued after five days because of drifting and associated areal variability of strength.

At the Richardson Study Plot, time profiles of 45- and 49-days were completed. In addition, two time profiles of 11-days were made at the Wolverine Study Plot.

The changes in strength for each of these time profiles are discussed in Sections 6.1.1 to 6.1.4.

6.1.1 First Richardson Time Profile

In the first profile, a layer of faceted grains was tested on 14 occasions in the 45-day period between December 19, 1987 and February 1, 1988. During this interval, the density of the layer increased from 190 to 242 kg/m^3. On each observation day, the tensile strength, density and temperature of the indicator slab were measured an average of 6 times. Initially, the temperature was measured in the middle of the indicator slab. However, starting on January 1, 1988 (Day 13), temperatures were taken at the top and bottom of the indicator slab to permit both the average temperature and the average temperature gradient within the slab to be calculated.

In Figure 6.1, the tensile strength, density, temperature and temperature gradient are shown over the 45-day period. For each value of mean strength, the 95% confidence interval is also shown. In addition, the mean values of the approximate stress rate (Equation 5.4) are plotted because changes in the stress rate may have systematically affected the measured strengths.

On the night of the third day of this time profile, an avalanche occurred in the West Bowl of Mount Redoubt 2 km from the Richardson Study Plot. This was considered to be a delayed action avalanche since the snowfall totalled 1 cm in the preceeding three days and winds had been light. However, at this time the tensile strength of the indicator layer was increasing slowly.

Between the twenty-second and twenty-seventh days, the average of the tensile strength measurements decreased from 2.1 to 1.3 kPa while changes in the density,

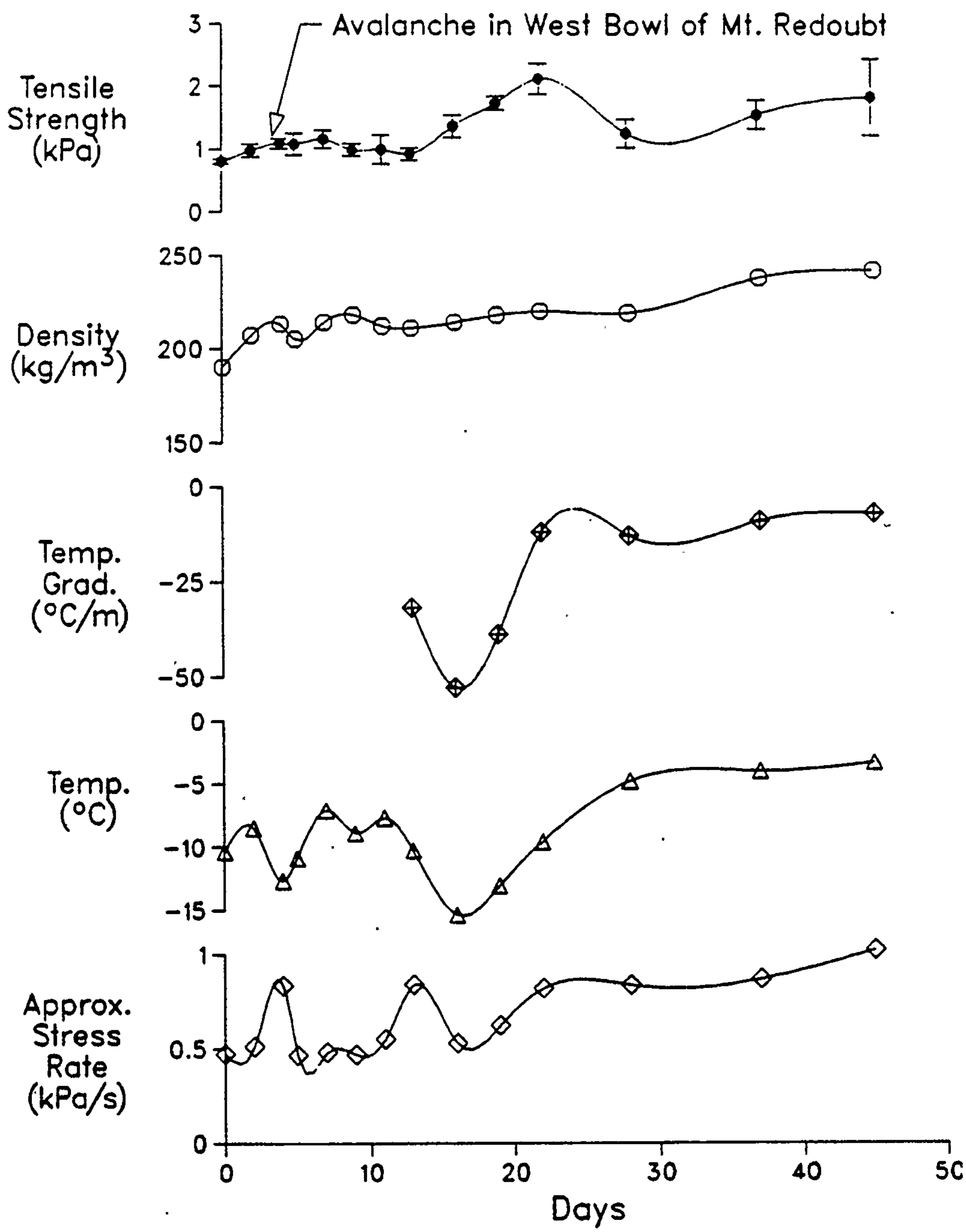

Figure 6.1: Tests of a Layer of Faceted Grains at the Richardson Plot Between Dec. 19, 1987 and Feb. 1, 1988.

temperature gradient and stress rate were negligible. This apparent decrease in tensile strength may have been due to the 5°C increase in snow temperature, or possibly due to areal variability of the snow layer within the study plot. No delayed action avalanches occurred during this apparent decrease in strength.

6.1.2 Second Richardson Time Profile

The second time profile at the Richardson Study plot represents the period between January 25, 1988 and March 14, 1988. During this 49-day period, a layer of rounded grains was tested an average of 8 times on each of 9 observation days.

No delayed action avalanches occurred and there was no substantial decrease in tensile strength. The gradual increase in tensile strength shown in Figure 6.2 is consistent with the densification and generally moderate temperatures and temperature gradient. The increase in the stress rate is probably due to the increase in strength and the operator's attempts to keep the loading time constant.

6.1.3 First Wolverine Time Profile

The first time profile at the Wolverine Study Plot consists of the period between December 24 and January 4, 1988. During this 11-day period, a layer of faceted grains was tested an average of 6 times on each of 5 observation days.

There were no delayed action avalanches and the apparent reduction in mean tensile strength around Day 4 may not be substantial in view of the 95% confidence intervals shown in Figure 6.3.

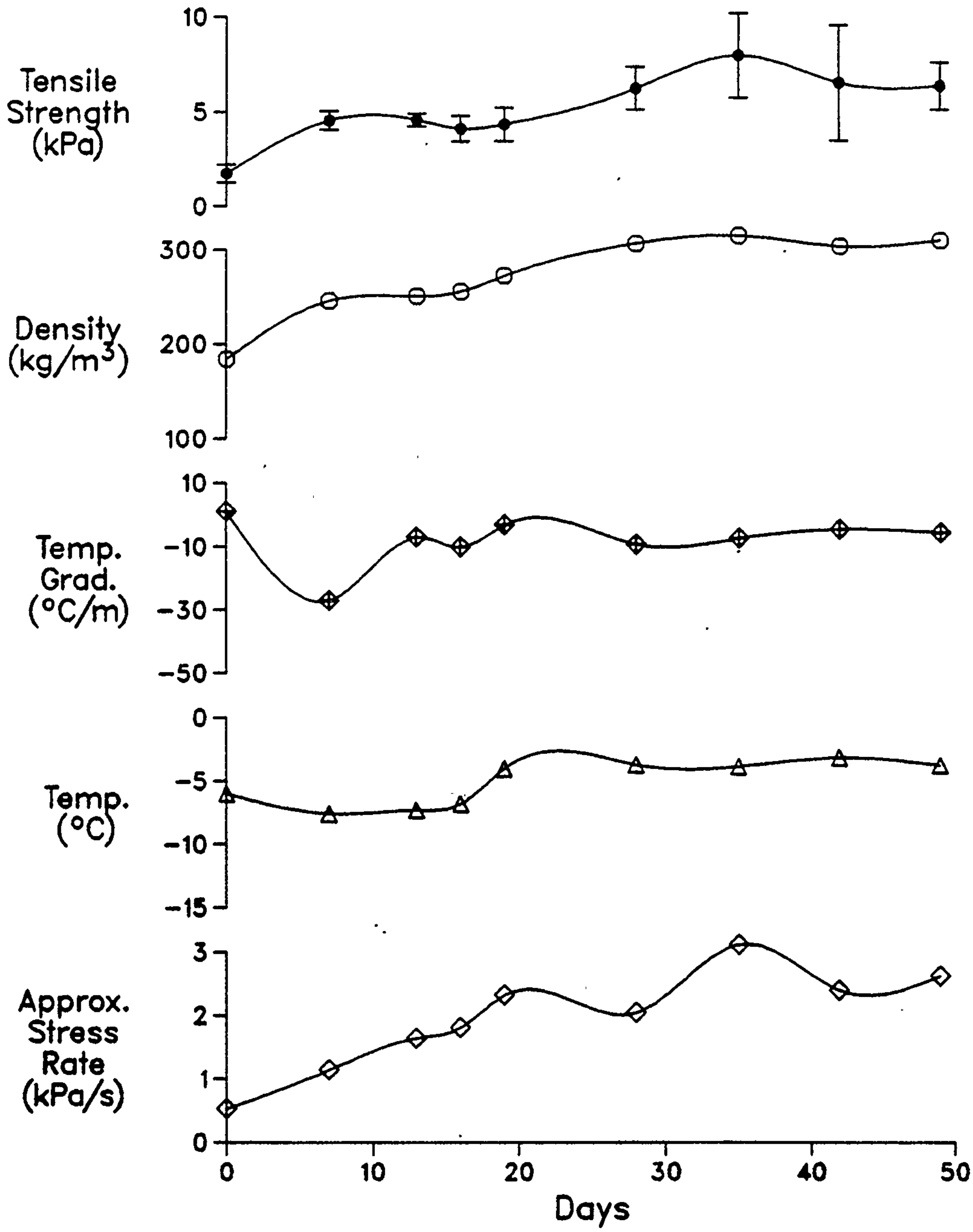

Figure 6.2: Tests of a Layer of Rounded Grains at the Richardson Plot Between Jan. 25 and Feb. 1, 1988.

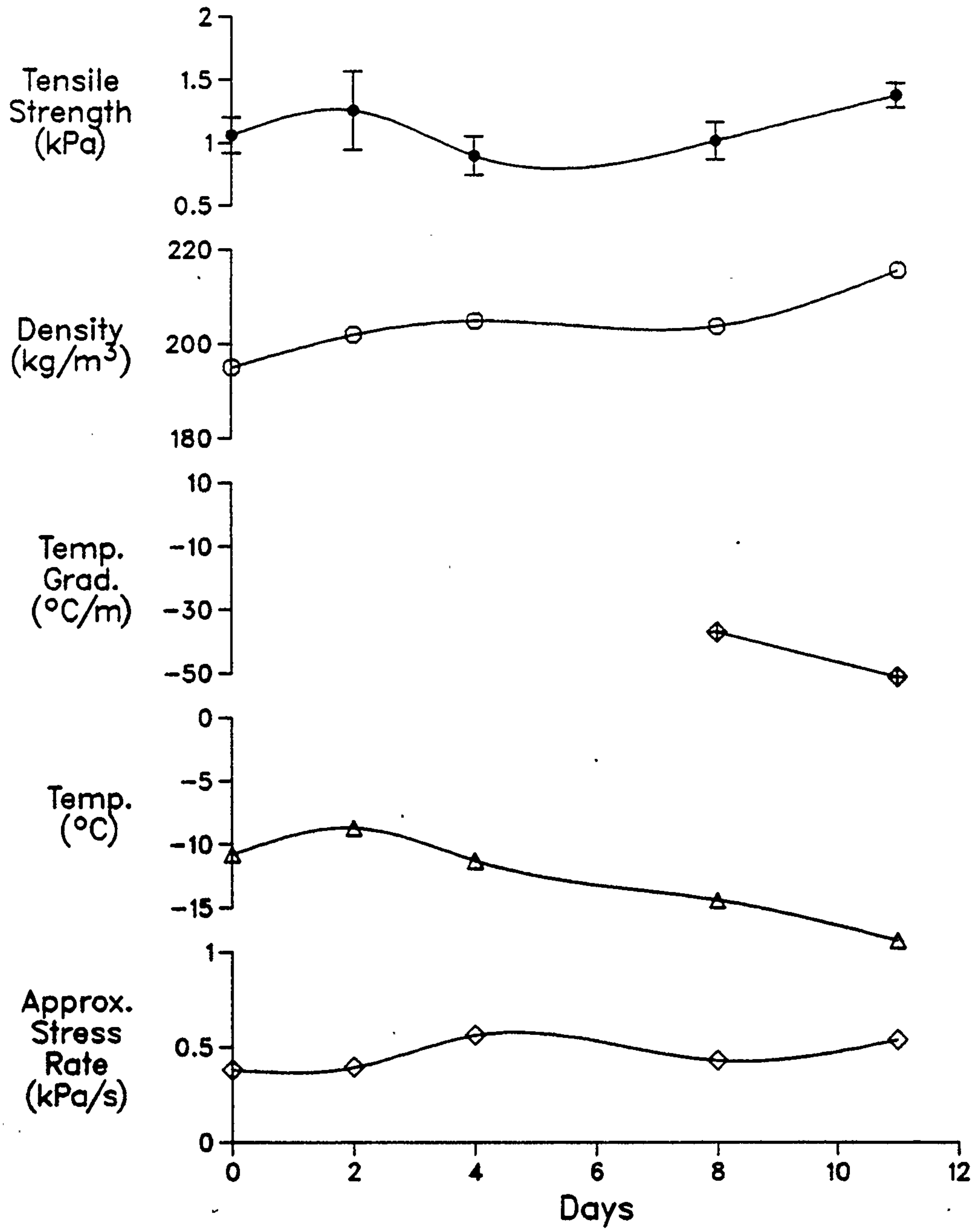

Figure 6.3: Tests of a Layer of Faceted Grains at the Wolverine Plot Between Dec. 24, 1987 and Jan. 4, 1988.

6.1.4 Second Wolverine Time Profile

On five occasions in the 11-day period between March 22 and April 2, 1988, an average of 8 tests were made on a layer of rounded grains. No delayed action avalanches occurred and the density and tensile strength increased steadily as shown in Figure 6.4.

6.1.5 Discussion of Time Profiles

There were no important decreases in tensile strength of the indicator layers and only one delayed action avalanche. Since this avalanche occurred during a period of increasing strength, there is no evidence to support a relationship between a decrease in tensile strength and the occurrence of delayed action avalanches.

Even if there had been more delayed action avalanches, this approach may not have provided a useful indicator of delayed action avalanche activity for the following reasons:

- It is difficult to choose study plots in which the changes in mechanical properties of the snow layers *accurately* reflect the trends occurring in starting zones exposed to the wind. Study plots in which the snowcover is quite similar to the snowcover in exposed starting zones are likely to exhibit wind drifting and therefore areal variability of mechanical properties of the snow layers.

- To measure changes in mean tensile strength with the precision necessary to detect subtle changes in slab properties requires many tests in a study plot with minimal areal variability.

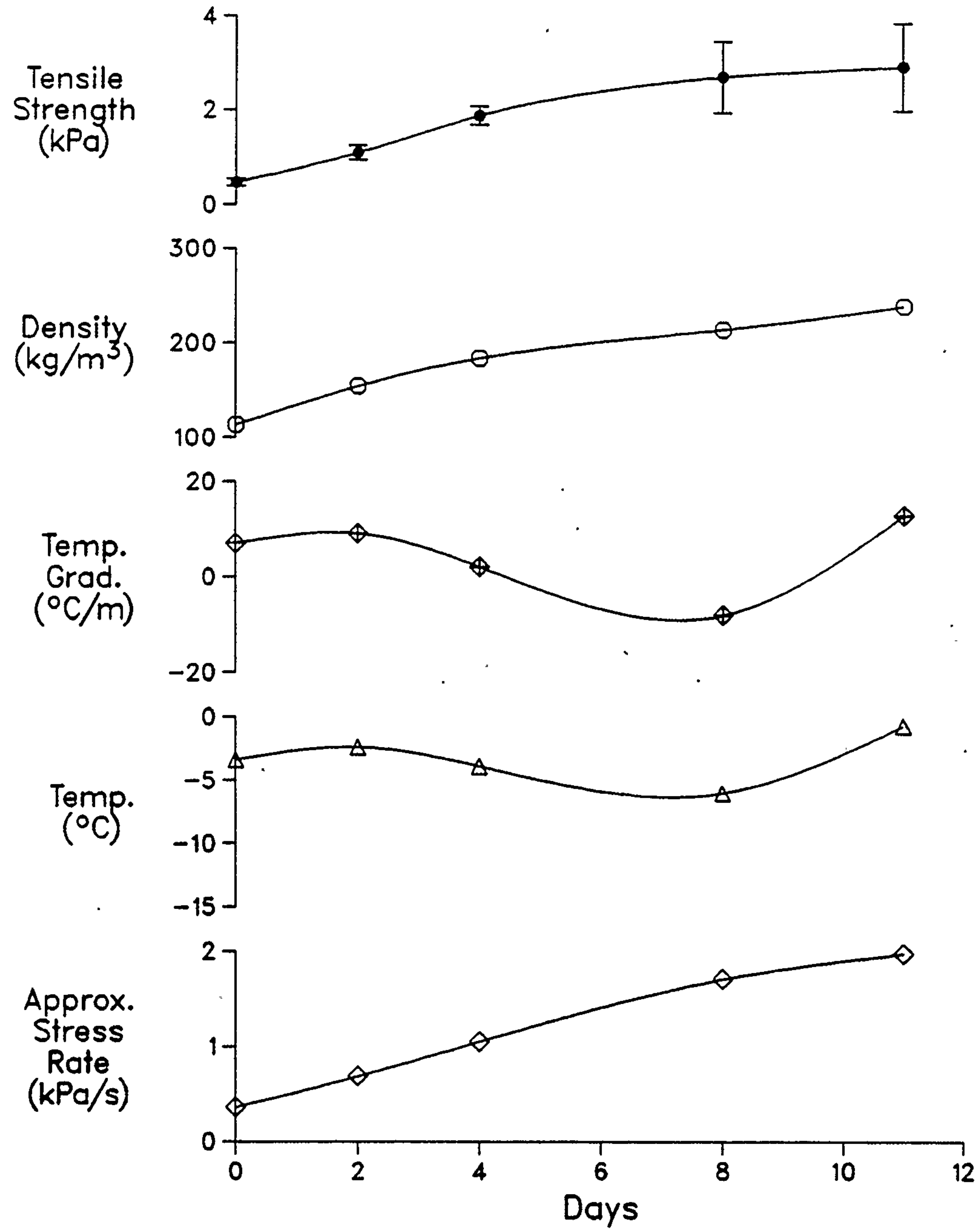

Figure 6.4: Tests of a Layer of Rounded Grains at the Wolverine Plot Between March 19 and April 2, 1988.

- Visco-elastic effects need to be considered in the stable growth of slip bands (McClung, 1987) and although the brittle tensile strength might be expected to exhibit the same trends as Young's modulus, the result of such a brittle strength test is probably a poor indicator of creep compliance primarily because the loading is too rapid to allow for viscous effects.

6.2 Crown Width of Slab Avalanches

Avalanche workers have often reported that stronger, thicker slabs propagate wider crown fractures which result in larger slab avalanches.

To study this hypothesis, a total of 50 measurements of tensile strength and slab thickness were made at the crown fractures of 13 *unconfined* slab avalanches. For eleven of these avalanches, the measurements were made within two days of the occurrence. Of these eleven, four avalanches were triggered by explosives and seven were released by skiers. The remaining two avalanches occurred naturally and the measurements were made 6 days after the occurrences during which the temperature ranged from -15°C to -31°C.

For each of these unconfined avalanches, measurements were made at a site along the crown fracture at which the slab was average in thickness and where there were no unusual features in the crown fracture such as rock outcrops or trees. At these crown fractures, thickness measurements (perpendicular to the slope) and an average of four tensile tests were made. The mean values of these measurements are summarized in Table 6.1. In each case the slab consisted of one or more layers of new snow, partly settled grains or rounded grains. Faceted grains

Table 6.1: Summary of Measurements at Crown Fractures

Date of Slide (1988)	Starting Zone	Trig-ger[1]	Crown Width (m)	Mean Thick-ness (m)	Mean Tensile Strength (kPa)	No. of Tests	Time Before Measure-ments[2]
01/25	Swedes	E	30	0.31	1.8	1	6 h
01/29	Wolv. Bench	N	30	0.27	1.56	2	6 d
01/29	Wolv. S.W.	N	70	0.31	1.81	5	6 d
02/13	Splitrock	S	15	0.22	0.55	4	2 h
02/20	Richardson	S	90	0.47	6.21	4	2 d
03/21	Eagle Ridge 3	E	12	0.10	0.36	1	2 h
03/27	Lake Pitch	S	10	0.11	2.53	5	1 h
03/27	Splitrock	S	8	0.11	2.24	5	2 h
04/03	Lipalian 3	S	40	0.25	1.33	5	1 d
04/03	Purple Bowl	S	40	0.27	1.44	5	1 d
04/06	Lipalian 1	S	8	0.19	0.43	4	4 h
04/07	Lipalian 2	E	30	0.26	1.32	5	1 h
04/08	Out of Bounds	E	45	0.44	1.89	4	1 d

[1] N - natural, S - triggered by skier, E - triggered by explosive.
[2] h - hour, d - day.

were not observed in any slab layer.

Correlations of crown width with mean strength, mean thickness and with the product of mean strength and mean thickness are reported in Table 6.2. Although strong conclusions are not possible based on 13 data points, the r-values are consistent with the hypothesis that stronger and thicker slabs result in wider slab avalanches. The confidence levels in the rightmost column show that the probability of obtaining such r-values from a set of 13 points taken from a large set of *uncorrelated* data is less than 1%.

The correlation of crown width, B, with the product of tensile strength, σ, and thickness, D, is of particular interest since it best represents the reports of field

Table 6.2: Correlations With Crown Width

Slab Property	Min.	Max.	r-value	Confidence Level (%)
Thickness, D (m)	0.10	0.47	0.84	99.9
Tensile Strength, σ (kPa)	0.36	6.2	0.70	99.2
Thickness x Tensile Strength, $D\sigma$ (kN/m)	36.	2.92	0.82	99.9

workers.

The crown width is plotted against the product of thickness and tensile strength in Figure 6.5. By regressing crown width, in metres, on this product, in kN/m, the following ratio is obtained:

$$B/D\sigma \approx 40 \tag{6.1}$$

Excluding the point for the 90 metre wide fracture results in a better fit of the remaining points to the ratio:

$$B/D\sigma \approx 50 \tag{6.2}$$

Although the number of data points is very limited, there is no apparent effect of different triggers on the results. .

Brown et al. (1972) report that there is no known theoretical upper limit to the width of slab avalanches. However, the trend apparent in Figure 6.5 suggests that the width of a crown fracture may be a property of the slab and independent of the triggering mechanism. Further research is necessary to investigate mechanisms which may limit the width of slab avalanches.

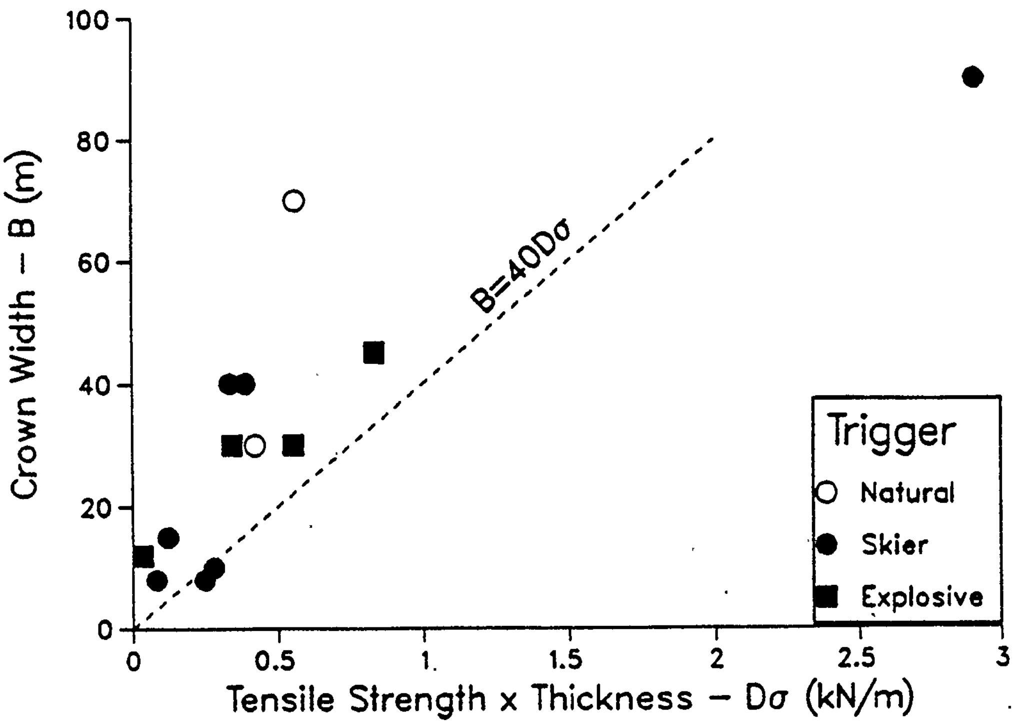

Figure 6.5: Crown Width vs Slab Thickness x Strength
For 13 unconfined slab avalanches which occurred between January 25, and April 8, 1988 in and near the Lake Louise Ski Resort.

Chapter 7

Conclusions

7.1 Overview

The failure of cohesive snow layers on slopes results in slab avalanches which are typically larger and more hazardous than the loose snow avalanches which result from the failure of uncohesive snow layers (Mellor, 1968, pp. 17-19).

Slab failure begins with the shear failure or collapse of a weak layer of snow below the slab. Much research has focussed on shear strength (e.g. Perla et al., 1982) and on the mechanical behaviour of the weak layer (e.g. McClung, 1979a). Although the emphasis on the initial failure of the weak layer is appropriate, the role of the overlying slab in the release of slab avalanches has received limited attention. For example, McClung's (1987) theory that increasing slab compliance may precipitate slab release and the hypothesis put forward by field workers that stronger and thicker slabs release as wider, larger and more destructive slab avalanches have not been verified in the field.

To study these ideas, systematic measurements of the tensile strength of snow, in the ranges of density and microstructure which commonly occur in snow slabs, were considered appropriate. The Slip Plate Test as developed by Conway and Abrahamson (1984) was chosen as the test method since it allows the uniaxial tensile strength of relatively large snow specimens to be measured in situ.

Investigations of the test method itself were also required since the limitations, precision and sources of variability were unknown.

The six objectives of this project were:

- to determine the limitations and sources of variability of the slip plate test;
- to determine the mode of failure caused by the slip plate test;
- to investigate the areal variability of snow strength and to determine the number of tests required to obtain particular levels of precision;
- to measure the tensile strength of snow in situ for the range of density and types of microstructure commonly found in snow slabs;
- to investigate whether a decrease in the tensile strength of snow slabs is associated with an increase in delayed action avalanche activity; and
- to investigate whether stronger and thicker snow slabs tend to release in wider slab avalanches.

During the winter of 1986-87, variations on the test method and equipment were investigated during 22 days of field work. Using the techniques and equipment selected during the first winter, 42 days of systematic measurement of tensile strength and other snowpack properties were conducted in and near the Skiing Louise Resort during the winter of 1987-88. From the complete set of 555 tensile strength tests, a refined set of 457 tests was obtained by rejecting 38 tests with loading times in excess of 5 seconds, 25 tests made with sharp-tipped notches and 35 tests in which the fracture occurred across the front or back of the notched tensile zone.

This set of 457 tests adds to the 83 uniaxial tensile in situ tests reported in the literature (McClung, 1979b; Conway and Abrahamson, 1984; Rosso, 1987).

Except for the conclusions regarding notch sensitivity and rate effects, the refined set of 457 tests is used to develop the following conclusions.

7.2 Limitations and Sources of Variability of the Slip Plate Test

7.2.1 Limitations

Layers of rounded snow grains with a strength as low as 0.25 kPa can be tested with this technique. For faceted grains, the lower strength limit is approximately 0.5 kPa. Snow strength ranging up to 10 kPa was measured with the method and equipment described in Chapter 3. However, 10 kPa does not constitute an upper limit for the equipment or the test method.

7.2.2 Rate Effects

For loading times ranging from 0 to 60 seconds, strength decreases with an increase in the loading rate.

7.2.3 Notch Sensitivity

Strength is not substantially reduced by changing the notches in the specimens from U-shaped notches with 50 mm tip radii to V-shaped notches with 10 mm tip radii. However, the strength is significantly reduced by changing the notches in the specimens from U-shaped notches with 50 mm tip radii to Y-shaped notches with 1 mm tip radii.

7.2.4 Size Effects

The cross sections of the tensile zone were varied from 1.4×10^{-2} m^2 to 4.1×10^{-2} m^2 without any prominent effect on strength. This suggests that, within the range of cross sections that are practicable, specimen size is not a substantial source of variability.

7.3 Failure Mode

Loading times to failure of 5 or less seconds, and possibly times as high as 60 seconds, result in essentially brittle failure.

A related conclusion is that for loading times of 60 seconds or less, the deformation before failure is less than approximately 1 mm for specimens with a length of 100 mm This corresponds to a strain at failure of less than 1%.

7.4 Areal Variability and Precision

Substantial trends in the strength of a snow layer may occur within generally sheltered study plots. Using the procedure described in Chapter 3 to test snow layers in such plots, approximately 13 tests are required to obtain a precision of 10% with 90% confidence. Similarly, to obtain a precision of 15% with 90% confidence, approximately 7 tests are required. Areal variability within study plots is expected to increase with exposure to wind.

7.5 Tensile Strength of Snow

A total of 312 tests was made on layers of new snow, partly settled grains and rounded grains, which ranged in mean density from 100 to 345 kg/m^3. The brittle tensile strength of these layers is well represented by:

$$\sigma = 79.7(\rho/\rho_{\text{ice}})^{2.39} \tag{7.1}$$

in which ρ is the density of the snow and σ is the tensile strength in kPa. Since snow slabs commonly consist of these types of microstructure and have a mean density of 206 kg/m^3 and a standard deviation of 77 kg/m^3 (Perla, 1977), Equation 7.1 is applicable to most snow slabs.

Layers of grains which show faceting are sometimes found in snow slabs and are more commonly found in the weak layers below snow slabs. A total of 145 tests was made on snow which exhibited faceting (and faceting in conjunction with rounding) which ranged in mean density from 190 to 260 kg/m^3. The brittle tensile strength of these layers can be represented by:

$$\sigma = 58.3(\rho/\rho_{\text{ice}})^{2.65} \tag{7.2}$$

This empirical equation may be biased upwards because a number of the weaker specimens of faceted snow broke before the load was applied.

In the density range of 190 to 260 kg/m^3, the tested layers of faceted snow were approximately half as strong as the group of layers of new snow, partly settled grains and rounded grains.

7.6 Delayed Action Avalanches

No relationship was determined between decreases in tensile strength of important slab layers and delayed action avalanche activity partly because only one delayed action avalanche occurred during any of the four periods in which the tensile strength of a slab layer was being monitored. For reasons discussed in Section 6.1, the particular approach to the question of delayed action avalanches used in this project is not promising.

However, it was possible to detect changes in layer strength with time and weather conditions. Therefore, this approach may be useful for studying the effect of metamorphic processes on in situ strength and cohesion.

7.7 Width of Slab Avalanches

Measurements of the thickness and the strength of a slab may be of use in predicting the width and the size of potential avalanches for three reasons:

1. Based on an average of 4 measurements made at each of 13 unconfined slab avalanches, the width of a slab avalanche increases with the thickness of the slab (correlation coefficient, r=0.84), with the tensile strength of the slab (r=0.70) and with the product of thickness and tensile strength (r=0.82).

2. Measurements of slab thickness and tensile strength can be made prior to avalanche activity.

3. The relationship linking slab thickness and strength to the width of slab avalanches appears to be independent of whether the avalanches occur naturally,

or are triggered by a skier or by an explosive.

Chapter 8

Recommendations

The present study tested snow over the density range of 100 to 345 kg/m^3, somewhat smaller than the density range of 60 to 460 kg/m^3 reported for slab avalanches (Perla, 1977). The in situ tests of Perla (1969) and Conway and Abrahamson (1984) provide limited strength data in the density range of 60 to 100 kg/m^3. However, in situ tests are required in the range of 350 to 460 kg/m^3. The slip plate tensile test appears suited to measuring snow with a density greater than 350 kg/m^3 for which data are lacking.

In this thesis and in other field studies, the microstructure of snow is classified in terms of grain shape and grain size based on inspection under low magnification. While convenient in the field, such classifications are qualitative and, to a degree, subjective. Quantitative measures of microstructure, suitable for field use, are required.

The present study includes only 21 tests on snow *qualitatively classified* as *moist* by the glove test (NRCC and CAA, 1986) and no tests were made on snow with visible free water. In situ tensile tests in conjunction with *measurements* of free water content are required to study the effect of water content on tensile strength.

A method of measuring the creep compliance in situ is also needed, so that the role of the slab in the occurrence of delayed action avalanches can be investigated.

Before the apparent correlation of the thickness and strength of slabs with the width of resultant slab avalanches can be used by avalanche forecasters, either the empirical relationship must be verified by additional crown fracture measurements, or the relationship between the mechanical properties of a slab and the width of slab avalanches must be established analytically.

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Appendix A

Glossary

A.1 Statistical Terms

Interval Data Interval data are numbers which represent a property; equal differences between the numbers represent equal differences in the level of the property. For example, temperatures in degrees Celsius are interval data. The classification of avalanches by destructive potential is *not* interval data since the difference in destructive potential between a class 4 and a class 3 avalanche is not the same as the difference between a class 2 and a class 1.

Nominal Data The numbers assigned to the elements of a set which serve as *labels* but which do not imply an order for the elements are nominal data. For example, the numbers on the uniforms of members of a football team are nominal.

Ordinal Data The numbers assigned to *rank* elements of a set according to a property or attribute are ordinal data. For example, the classification of avalanches by destructive potential is ordinal since a class 2 avalanche is more destructive than a class 1.

Ratio Data Ratio data are interval data in which "zero" represents an absolute minimum for the property. Since 0°C is not an absolute minimum, temperatures in degrees Celsius are *not* ratio data. Temperatures in degrees Kelvin are ratio data since 0°K is the absolute minimum, and as a consequence, 20°K is twice as warm as 10°K.

Residual A residual is the deviation of a particular value of the dependent variable from its value predicted by the regression. Using strength as function of density as an example, if a specimen has a density of 200 kg/m^3 and a strength of 2.5 kPa and the regression predicted a strength of 2.0 kPa for a density of 200 kg/m^3 then the residual would be 0.5 kPa.

Standardized Residual The standardized residual is the dimensionless ratio of the residual to the standard deviation of the residuals. Using the example above, if the standard deviation of the residuals is 0.25 kPa, then the standardized residual is 2.0.

Studentized Residual The i-th studentized residual is the standardized residual for the regression calculated *without* the i-th data point. Therefore, if the ninth point greatly influenced a regression which was based on ten points, then the ninth studentized residual would be much greater than the ninth standardized residual. The studentized residual is a measure of the deviation of the i-th point from the trend (as modelled by the regression) of the *other* points.

A.2 Terms for Snow Microstructure

Faceted Grains Faceted grains have characteristic flat faces.

Depth Hoar The grains have ledges or steps and re-entrant angles and some grains are cup-shaped or are fragments of cup-shaped grains (UNESCO, 1970).

Ice Grain In snow, a grain or ice grain is a particle of ice discernible under low magnification.

Ice Crystal In an ice crystal, the ice molecules are regularly arranged in a lattice.

Melt-Freeze Grains Melt-freeze grains are amorphous or clusters of grains caused by several melt-freeze cycles (NRCC and CAA, 1986).

Newly Fallen Snow Newly fallen snow or new snow consists of crystals which are unchanged or are close to the forms of solid precipitation (UNESCO, 1970).

Partly Settled Grains Partly settled grains (also called *initially metamorphosed),* have been metamorphosed but elements of the original new snow crystals are still recognizable (UNESCO, 1970).

Rounded Grains Rounded grains are rounded but not amorphous or clustered as a result of melt-freeze cycles.

Appendix B

Field Study Area

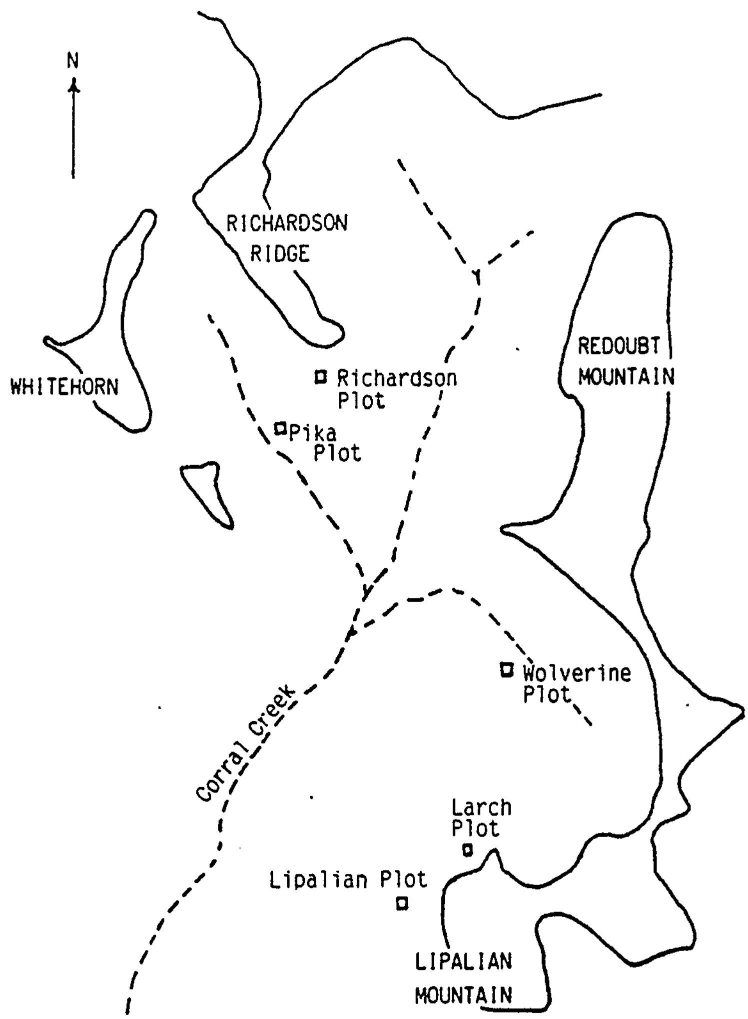

Figure B.1: Map of Field Study Area
Five study plots and 2500 m contour line are shown.
Facilities of Lake Louise Ski Resort are not shown.

Appendix C

Analysis of Notch Effects

Notation

D	variate for difference between matched pairs
n	number of data in a set
s	standard deviation of a sample
$t_{\alpha;n-1}$	critical value of t-statistic with a probability of α and $n-1$ degrees of freedom
σ_U	strength for test made with U-shaped notches
σ_V	strength for test made with V-shaped notches
σ_Y	strength for test made with Y-shaped notches

C.1 Pairing of Tests

A matched pairs design (Koch and Link, 1970, pp. 154-157) is used to study the effects of notch shape on strength.

There were 25 tests made with Y-shaped notches (called Y-tests) and 19 tests made with V-shaped notches (V-tests). The remainder of the tests had U-shaped notches (U-tests). V-tests are paired with nearby U-tests, and Y-tests are paired with nearby U-tests, based on the following criteria:

- Only tests from the refined data set may be used.

- Both tests of a pair must be made at the same site on the same layer of snow on the same day.
- Within the study site, paired tests should be as close together as possible.
- A U-test may be used at most once in each set of pairs.

The matched pairs are shown in Tables C.1 and C.2. However, based on the above criteria, 2 V-tests and 6 Y-tests could not be paired with a U-test.

Table C.1: V-Tests Paired with U-Tests

Date 1988	Micro-structure	Mean Density (kg/m^3)	V-Tests Row	V-Tests Col.	V-Tests Strength (kPa)	U-Tests Row	U-Tests Col.	U-Tests Strength (kPa)
03/26	partly settled	184	3	1	2.27	2	1	1.73
			5	2	2.06	4	2	1.29
03/26	rounded	298	1	1	4.10	2	1	4.85
			2	2	5.62	1	2	11.76
			3	1	7.94	4	1	6.34
			4	2	5.95	3	2	4.67
			5	1	7.61	6	1	6.06
04/02	rounded	240	1	1	3.16	1	2	2.91
			3	1	2.23	3	2	4.59
			3	1	2.23	3	2	4.59
			2	3	1.93	-	-	-
04/02	rounded	317	1	1	5.17	1	2	6.45
			2	3	4.74	2	2	7.43
			3	1	7.64	3	2	6.28
04/02	rounded	338	1	1	7.03	1	2	8.10
			3	1	7.74	3	2	10.07
			2	3	7.31	-	-	-

Table C.2: Y-Tests Paired with U-Tests

Date 1988	Micro-structure	Mean Density (kg/m^3)	Y-Tests Row	Y-Tests Col.	Y-Tests Strength (kPa)	U-Tests Row	U-Tests Col.	U-Tests Strength (kPa)
03/20	rounded	284	1	1	2.48	2	1	3.49
			1	3	3.06	3	4	4.84
			2	2	2.31	1	2	3.96
			3	2	2.73	3	1	4.13
			4	1	2.38	5	1	2.27
			4	3	2.75	4	4	4.31
			5	2	2.65	5	3	6.73
			6	2	2.37	-	-	-
03/22	partly settled	113	6	1	0.45	5	1	0.56
03/24	partly settled	154	5	2	1.03	6	2	1.46
			6	1	1.34	5	1	1.31
04/02	rounded	240	2	1	3.68	1	2	2.91
			3	3	4.33	3	2	4.59
			1	3	1.67	-	-	-
			4	1	3.34	-	-	-
04/02	rounded	317	1	3	4.99	1	2	6.45
			2	1	5.35	2	2	7.43
			3	3	7.08	3	2	6.28
			4	1	4.84	-	-	-
04/02	rounded	338	1	3	5.25	1	2	8.10
			3	3	5.50	3	2	10.07
			2	1	6.10	-	-	-
			4	1	5.72	-	-	-
			6	2	4.18	4	2	4.48
			6	1	3.86	5	1	6.22

C.2 Calculations for Y-Tests Paired with U-Tests

To test for the effect on strength of Y-shaped notches compared to U-shaped notches confidence intervals are required for the *difference* between strengths from matched pairs: D_{UY}. However, since the variability of the results increases with strength (Section 4.2.2), the variance is stabilized by a logarithmic transformation. Thus:

$$D_{\mathrm{UY}} = \ln \sigma_{\mathrm{U}} - \ln \sigma_{\mathrm{Y}} \tag{C.1}$$

If the confidence interval for the population mean of D_{UY} includes zero, then at that confidence level, the effect of the Y-shaped notch is insignificant.

For a sample with mean $\bar{D}_{\mathrm{UY}}$ and standard deviation s_{D}, the $1-2\alpha$ confidence interval, based on n pairs, is bound by:

$$\bar{D}_{\mathrm{UY}} \pm t_{\alpha;\mathrm{n\text{-}1}} s_{\mathrm{D}}/\sqrt{n} \tag{C.2}$$

Using the smaple mean and standard deviation for the set of 19 Y-tests paired with U-tests, the 90% confidence interval is bound by:

$$\begin{aligned} \bar{D}_{\mathrm{UY}} \pm (t_{0.05;18}) s_{\mathrm{D}}/\sqrt{19} &= 0.289 \pm (1.73)0.286/\sqrt{19} \\ &= 0.289 \pm 0.114 \end{aligned}$$

and the average reduction in strength in per cent due to the Y-shaped notches is:

$$100\% \left(\sum(\sigma_{\mathrm{U}} - \sigma_{\mathrm{Y}})/\sigma_{\mathrm{U}}\right)/19 = 22\%$$

C.3 Effect of Notch Shape

The 90, 95 and 99% confidence intervals for D_{UV} and D_{UY} are presented in Table C.3. Since the 99% confidence interval for the population mean of D_{UY} excludes zero, strength is substantially reduced by Y-shaped notches. Alternatively, the 90% confidence interval for the population mean of D_{UV} includes zero, so there is no evidence to indicate that V-shaped notches affect the strength.

Table C.3: Confidence Intervals for D_{UV} and D_{UY}

Notch Shapes Compared:	V vs U	Y vs U
No. of Pairs	14	19
Average Strength Reduction	1%	22%
Mean Difference $\bar{D}$	0.071	0.289
Standard Deviation s_{D}	0.379	0.286
90% Confidence Interval	-0.11 to 0.25	0.18 to 0.40
95% Confidence Interval	-0.15 to 0.29	0.15 to 0.43
99% Confidence Interval	-0.23 to 0.38	0.10 to 0.48
Critical Confidence Level Which Excludes a Mean Difference of 0	50%	99.9%

Appendix D
Summary of Data

Table D.1: Results for New Snow, Partly Settled Snow and Multilayer Specimens

		Microstructure		No.			Mean		Mean	Fracture	
Date 1988	Site	Grain Shape[1]	Grain Size (mm)	of Tests	Tensile Strength[2] (kPa)	Density[2] (kg/m^3)	Temperature (°C)	Loading Time[2] (s)	Slope Angle (°)	Mean Width (10^{-2}m)	Mean Depth (10^{-2}m)
01/25	Swedes	m	-	1	1.79	191	-9.0	4.0	38	26	31
02/04	Wolv. Bench	ps	0.8	2	1.56 ± 0.09	186 ± 8	-9.5	2.0 ± 0.0	26	18	27
02/04	Wolv. S.W.	ps	0.8	5	1.81 ± 0.29	175 ± 2	-10.7	2.2 ± 0.8	28	18	31
02/13	Splitrock	n	1.0	4	0.55 ± 0.09	124 ± 8	-5.3	0.8 ± 0.3	33	28	22
02/22	Mt. Richard.	m	-	4	6.21 ± 0.55	342	-7.5	3.1 ± 1.5	34	11	47
03/21	Eagle 3	n	1.0	1	0.36	100	-3.5	0.5	45	29	10
03/21	Pika Corner	n	1.5	5	0.34 ± 0.09	101 ± 2	-3.0	1.0 ± 0.4	8	17	20
03/22	Wolverine	ps	1.0	10	0.47 ± 0.11	113 ± 4	-3.4	1.3 ± 0.5	10	17	16
03/24	Wolverine	ps	0.8	7	1.10 ± 0.20	154 ± 6	-2.4	1.6 ± 0.8	8	15	14
03/26	Wolverine	ps	1.0	10	1.89 ± 0.31	184 ± 4	-3.9	1.8 ± 0.4	9	16	15
03/27	Lake Pitch	n	0.8	5	2.53 ± 0.18	222 ± 3	-8.0	1.6 ± 0.4	41	21	11
03/27	Splitrock	n	0.8	5	2.24 ± 0.57	173 ± 15	-7.8	1.4 ± 0.7	48	22	11
04/04	Lipalian 3	m	-	5	1.33 ± 0.18	161 ± 11	-8.5	2.2 ± 0.4	37	25	25
04/04	Purple Bowl	m	-	5	1.44 ± 0.23	150 ± 2	-9.0	2.2 ± 0.6	32	20	27
04/06	Lipalian 1	n	1.0	4	0.43 ± 0.11	106 ± 1	-1.5	1.6 ± 1.1	41	32	19
04/07	Lipalian 2	m	-	5	1.32 ± 0.35	157 ± 7	-7.0	2.9 ± 1.1	30	19	26
04/08	Out-Bounds	m	0.5	4	1.89 ± 0.16	161 ± 12	-3.9	3.6 ± 0.5	36	20	44

[1] n - new snow, ps - partly settled, r - rounded (UNESCO, 1970), m - multilayer specimen.

[2] mean ± standard deviation.

Table D.2: Results for Rounded Snow at Richardson, Lipalian and Blackprince Study Plots

		Microstructure								Fracture	
Date 1988	Study Plot	Grain Shape[1]	Grain Size (mm)	No. of Tests	Tensile Strength[2] (kPa)	Density[2] (kg/m^3)	Mean Temp-erature (°C)	Loading Time[2] (s)	Mean Slope Angle (°)	Mean Width (10^{-2}m)	Mean Depth (10^{-2}m)
01/25	Richardson	r	0.5	3	1.75 ± 0.42	184 ± 10	-6.0	3.3 ± 0.6	9	17	25
02/01	Richardson	r	0.5	2	4.57 ± 0.34	246 ± 9	-7.6	4.0 ± 1.4	7	14	21
02/07	Richardson	r	0.8	30	4.60 ± 0.93	251 ± 6	-7.3	2.8 ± 0.8	7	16	20
02/10	Richardson	r	0.8	6	4.16 ± 0.82	256 ± 9	-6.8	2.3 ± 0.6	9	15	16
02/13	Richardson	r	0.8	6	4.41 ± 1.09	273 ± 3	-4.0	1.9 ± 0.7	8	15	17
02/22	Richardson	r	1.0	6	6.35 ± 1.39	308 ± 6	-3.7	3.1 ± 1.1	8	15	16
02/29	Richardson	r	1.0	4	8.12 ± 2.23	317 ± 4	-3.8	2.6 ± 0.9	9	15	15
03/07	Richardson	r	1.0	4	6.72 ± 3.06	306 ± 2	-3.1	2.8 ± 0.9	10	15	17
03/14	Richardson	r	1.0	11	6.57 ± 2.07	313 ± 11	-3.7	2.5 ± 1.1	8	17	17
02/06	Lipalian	r	1.0	42	2.03 ± 0.41	216 ± 8	-9.4	1.7 ± 0.3	2	16	12
03/16	Blackprince	r	0.5	11	3.03 ± 0.49	204 ± 5	-4.9	1.9 ± 0.7	3	16	18

[1] r - rounded (UNESCO, 1970).

[2] mean ± standard deviation.

Table D.3: Results for Rounded Snow at Wolverine Study Plot

Date 1988	Study Plot	Microstructure: Grain Shape[1]	Microstructure: Grain Size (mm)	No. of Tests	Tensile Strength[2] (kPa)	Density[2] (kg/m^3)	Mean Temperature (°C)	Loading Time[2] (s)	Mean Slope Angle (°)	Fracture: Mean Width (10^{-2}m)	Fracture: Mean Depth (10^{-2}m)
03/02	Wolverine	r	1.0	24	4.62 ± 1.00	277 ± 11	-4.0	2.1 ± 0.7	11	14	18
03/09	Wolverine	r	1.0	13	5.51 ± 0.99	307 ± 6	-2.8	2.2 ± 0.8	13	14	11
03/20	Wolverine	r	1.0	7	4.25 ± 1.36	284 ± 9	-3.0	2.2 ± 0.8	8	14	9
03/22	Wolverine	r	0.5	2	5.53 ± 0.97	295 ± 0	-3.5	3.8 ± 0.4	9	17	10
03/24	Wolverine	r	1.0	5	4.09 ± 1.18	298 ± 8	-2.9	2.7 ± 1.2	8	13	9
03/26	Wolverine	r	1.0	10	6.08 ± 1.34	298 ± 10	-3.2	3.0 ± 0.9	9	15	10
03/30	Wolverine	r	1.0	6	2.73 ± 0.92	215 ± 0	-6.0	1.6 ± 1.0	9	12	12
04/02	Wolverine	r	0.5	5	2.96 ± 1.04	240 ± 0	-0.7	1.5 ± 0.5	8	15	11
04/02	Wolverine	r	1.0	6	6.29 ± 1.17	317 ± 7	-1.5	2.9 ± 1.2	8	14	11
04/02	Wolverine	r	1.5	5	8.05 ± 1.20	338 ± 3	-1.7	2.9 ± 0.8	9	13	11
04/15	Wolverine	r	1.5	1	5.16	320	-0.8	2.5	9	18	7
04/17	Wolverine	r	1.0	9	7.08 ± 1.23	338 ± 10	0.0	3.2 ± 0.6	8	17	12
04/17	Wolverine	r	1.0	10	6.35 ± 1.21	332 ± 10	0.0	2.6 ± 0.4	7	17	12
04/17	Wolverine	r	1.0	1	8.65	345	0.0[3]	3.0	8	18	12
04/17	Wolverine	r	1.0	1	6.10	345	0.0[3]	3.5	9	22	12

[1] r - rounded (UNESCO, 1970).
[2] mean ± standard deviation.
[3] m - moist by glove test (NRCC and CAA, 1986).

Table D.4: Results for Faceted Snow at Richardson, Wolverine and Larch Study Plots

Date	Study Plot	Microstructure Grain Shape[1]	Microstructure Grain Size (mm)	No. of Tests	Tensile Strength[2] (kPa)	Density[2] (kg/m^3)	Mean Temperature (°C)	Loading Time[2] (s)	Mean Slope Angle (°)	Fracture Mean Width (10^{-2}m)	Fracture Mean Depth (10^{-2}m)
87/12/30	Rich.	f	1.0	3	0.99 ± 0.20	212 ± 3	-7.7	1.8 ± 0.3	6	14	23
88/01/01	Rich.	f	1.0	8	0.92 ± 0.14	211 ± 4	-10.3	1.1 ± 0.4	7	13	22
88/01/04	Rich.	f	1.5	6	1.37 ± 0.22	214 ± 8	-15.4	2.6 ± 0.7	7	14	22
88/01/07	Rich.	f	1.5	11	1.74 ± 0.18	218 ± 3	-13.1	2.8 ± 0.6	8	11	23
88/01/10	Rich.	f	1.5	8	2.13 ± 0.35	220 ± 7	-9.6	2.6 ± 0.9	9	10	24
88/01/16	Rich.	f	1.5	6	1.25 ± 0.28	219 ± 8	-4.8	1.5 ± 0.4	9	12	22
88/01/25	Rich.	f	2.0	3	1.56 ± 0.20	238 ± 8	-4.0	1.8 ± 0.3	7	13	24
88/02/01	Rich.	f	2.0	3	1.84 ± 0.53	242 ± 2	-3.3	1.8 ± 0.3	5	12	22
87/12/24	Wolv.	f	1.0	5	1.06 ± 0.16	195 ± 4	-10.8	2.8 ± 0.4	10	16	32
87/12/26	Wolv.	f	1.0	5	1.26 ± 0.35	202 ± 10	-8.7	3.2 ± 1.3	13	16	32
87/12/28	Wolv.	f	1.0	6	0.90 ± 0.19	205 ± 17	-11.3	1.6 ± 0.5	10	15	26
88/01/01	Wolv.	f	1.0	8	1.03 ± 0.21	204 ± 11	-14.4	2.4 ± 0.5	10	14	28
88/01/04	Wolv.	f	1.0	7	1.40 ± 0.13	216 ± 9	-17.6	2.6 ± 0.6	10	12	25
88/03/09	Wolv.	f	2.0	8	1.95 ± 0.53	261 ± 5	-2.3	1.4 ± 0.3	12	16	13
87/12/23	Larch	f	1.0	7	2.06 ± 0.16	242 ± 7	-14.8	2.3 ± 0.5	17	18	16

[1] f - faceted (UNESCO, 1970).

[2] mean ± standard deviation.

Table D.5: Results for Rounded-Faceted Snow at Richardson and Larch Study Plots

Date	Study Plot	Microstructure		No. of Tests	Tensile Strength[2] (kPa)	Density[2] (kg/m^3)	Mean Temperature (°C)	Loading Time[2] (s)	Mean Slope Angle (°)	Fracture	
		Grain Shape[1]	Grain Size (mm)							Mean Width (10^{-2}m)	Mean Depth (10^{-2}m)
87/12/19	Rich.	rf	1.0	5	0.80 ± 0.04	190 ± 7	-10.4	1.7 ± 0.6	5	14	27
87/12/21	Rich.	rf	1.0	6	0.97 ± 0.13	207 ± 8	-8.5	1.9 ± 0.7	7	16	24
87/12/23	Rich.	rf	1.0	6	1.08 ± 0.10	213 ± 3	-12.7	1.3 ± 0.4	8	15	21
87/12/24	Rich.	rf	1.0	8	1.07 ± 0.25	205 ± 12	-10.9	2.3 ± 0.7	6	15	22
87/12/26	Rich.	rf	0.8	9	1.15 ± 0.22	214 ± 4	-7.1	2.4 ± 0.6	6	16	23
87/12/28	Rich.	rf	0.8	8	0.98 ± 0.14	218 ± 9	-8.9	2.1 ± 0.7	7	16	22
87/12/19	Larch	rf	0.8	3	1.88 ± 0.19	235 ± 4	-12.2	2.7 ± 0.6	16	17	19
87/12/21	Larch	rf	0.8	6	1.60 ± 0.22	234 ± 5	-10.6	1.7 ± 0.8	16	16	16

[1] rf - grains are rounded and faceted (UNESCO, 1970).

[2] mean ± standard deviation.