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UNIVERSITY OF CALGARY

Network sensor location problem for flow observability and Origin-Destination estimation with

consideration of sensor failure

by

Mostafa Salari

A THESIS

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Abstract

The network sensor location problem (NSLP) addresses the location of traffic sensors to observe/estimate the link, route or OD flows in a traffic network. While counting sensors such as loop detectors still have an extensive application for traffic monitoring purposes, they suffer from a considerable rate of failure. In this study, I focus on two well-known problems in the NSLP known as the full link flow observability problem and the origin-destination estimation problem while considering the failure of sensors.

The full link flow observability problem is to identify the minimum set of traffic sensors to be installed in links in a road traffic network. The sensors are used to both monitor the flow of observed links and to provide flow information for the link flow inference of unobserved links. Unavoidably, the traffic sensors deployed in a traffic network are subject to failure which leads to missing the link flow observation of observed links as well as the inability to infer the link flow of unobserved links. This study aims to identify the minimum set of links in a traffic network to be instrumented with two different types of counting sensors (basic and advanced sensors) to reach full link flow observability while minimizing the effect of sensor failure on the link flow inference of unobserved links. Mathematically, I formulate two objective functions including min-max and min-sum functions. The first function attempts to minimize the maximum effect of sensor failure on the link flow inference of unobserved links while the second one minimizes the expected number of unobserved links where the flow cannot be inferred due to the failure of sensors. I select the genetic algorithm (GA) as a well-known heuristic to solve the proposed optimization model. The results recommend minimizing the number of sensors required for the link flow inference of each unobserved link as well as installing advanced sensors on links involved

in the link flow inference of multiple unobserved links. I also develop a new objective function to reflect that links in a traffic network can be either minor or major roads with different levels of importance. The results suggest installing more advanced sensors on the major roads as well as minimizing the number of major roads included in the set of unobserved links. Concerning the availability of route flow information in a network, I consider the effect of this information on evaluating the sensor deployment in a network. To maintain full link flow observability of a traffic network if any sensor fails, I study the location and type of additional sensors introduced as redundant sensors, which are more than the minimum required for full link flow observability. Finally, I discuss the applicability of the proposed model for the partial observability problem in which the full link flow observability conditions are not satisfied.

In addition to the link flow observability problem, this study also focuses on the OD estimation problem considering the failure of sensors. The OD estimation problem is to find the location of the minimum number of sensors to estimate the flow of OD pairs in a traffic network. Traffic sensors can observe the summation of OD demand flows traversing a link and through OD estimation techniques such as maximum entropy, I can estimate the OD demand flows. Contrary to the flow observability problem, the failure of a sensor, does not necessarily lead to missing the chance of estimating the OD demand of one or more OD pairs but can affect the OD demand flow information gain from OD demands.

In this study, I identify the location of counting sensors aiming to minimize the possible adverse effect of sensor failure on the OD estimation process. The input data required for the OD estimation may consist of the prior information of the OD trips that can be used to make the OD trip estimation as close as possible to the actual vehicular trips generated between each OD in the road network. However, the sensors, similar to other measurement apparatus, are subject to failure and this failure can affect the reliability of the OD trip information especially under congested traffic conditions. In this paper, I address the sensor location problem (NSLP) to identify the most reliable location set of sensors in a road traffic network with consideration of the possibility of sensors failure. I introduced two objective functions including maximization of expected OD demand flow information gain on both observed link and each OD pair. I then employed the weighted sums method (WSM) and an ε-constraint to incorporate these two objective functions. With respect to the available budget constraint, different types of sensors are considered to identify different location sets of sensors with different levels of reliability for the OD estimation. The results applied to different road traffic networks indicate the improvement in the reliability of information obtained from the selected sensor location sets.

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List of Figures and Acronyms:

:		
AVI	Automatic vehicle identification	
GA	Genetic algorithm	
HVL	Heavy vehicle load	
IR	Informative routes	
LPR	License plate recognition	
MAE	Mean absolute error	
MPRE	Maximum possible relative error	
MSE	Mean square error	
NR	Non-redundant routes	
NSLP	Network sensor location problem	
OD	Origin-Destination	
PCU	Passenger car unit	
PR	Purely redundant routes	
TAZ	Traffic analysis zone	
TDS	Total demand scale	
V2I	Vehicle to infrastructure	
V2V	Vehicle to vehicle	
WSM	Weighted sums method	

CHAPTER 1: INTRODUCTION

1.1 Background

Identifying the location of sensors in a traffic network, known as the network sensor location problem (NSLP), is a critical component of transportation network modeling. Sensors provide essential information on the trip distribution and the corresponding spatiotemporal characteristics of traffic patterns in vehicular networks. However, like all sensors these traffic sensors are subject to failure; thereby the reliability and quality of information provided by sensors is highly dependent on the failure rate of sensors (Zhu et al., 2017). According to historical records, the failure rate of counting sensors¹ exceeds 25% for all types of sensor deployments (Federal Highway Administration, 2006), yet the repair cost of these sensors may be too high for traffic agencies considering their limited budgets (Danczyk et al., 2016). Moreover, information loss is another consequence of sensor failure that imposes an implicit cost to a traffic management system.

This thesis studies NSLP with consideration of sensors failure for two sub-problems: 1) full observability problem and the sensor location problem.

1.1.1 Introduction to the sensor location problem for flow observability

For a fully observable traffic network, the flow of all links can be either directly observed (i.e., flow of observed links by sensors) or indirectly inferred (i.e., flow of

¹ For the remainder of the chapter, "counting sensors" are called "sensors". Otherwise, the type of sensors is specified

unobserved links based on information obtained from the flow of observed links). The failure of sensors not only impedes observing the flow of observed links but also results in missing link flow information associated with unobserved links since the flows of these links are inferred using the link flow information obtained from observed links. However, the sensor configuration can efficiently mitigate the adverse effect of sensor failure on information loss in a network. Therefore, there is an urgent need for a sensor configuration that minimizes the effect of sensor failure as a source of uncertainty in current traffic management applications.

The current literature in the traffic flow observability/estimation problem identifies the three most important sources of uncertainty associated with traffic flow data as follows: 1) the variability of prior Origin-Destination (OD) demand; 2) measurement errors of traffic counts; and 3) the possibility of missing the flow information due to the failure of sensors (Danczyk et al., 2016).

The variability of prior OD demand is mainly related to the flow estimation problem as it deals with the reliability of information required to be used for traffic flow estimation. For instance, Fei et al. (2007) and Fei and Mahmassani (2011) employed the Kalman filtering technique to find the sensor locations that maximize information gains through the observed data and minimize the errors of estimated OD demands. Zhou and List (2010) optimized the locations of automatic vehicle identification (AVI) sensors by maximizing the gain of observed information in an uncertain environment. Wang et al. (2012) minimized the variance in the posterior route flow estimation while considering the available information on route flows and the reliability of this information. In a similar study, Wang and Mirchandani (2013) addressed the optimum locations of sensors based on the reliability of prior route flow information.

Sensor measurement error, as another source of uncertainty, has been studied in recently published works related to the flow observability problem. For instance, Castillo et al. (2010) addressed the scanning errors while locating vehicle identification (Vehicle-ID) sensors in a traffic network. More recently, Xu et al. (2016) discussed the full link flow observability problem and proposed a robust approach that determines the links to be equipped with sensors in a traffic network regarding the flow measurement variance stemmed from sensors.

To the best of my knowledge, the failure of sensors has received less attention in flow observability/estimation research, with the exception of two studies (Li and Ouyang, 2011 and Danczyk et al., 2016). Li and Ouyang (2011) examined the location of vehicle-ID sensors in a network to maximize the information gain from OD routes considering the possibility of sensor failure, while Danczyk et al. (2016) addressed the installation of counting sensors on a freeway to minimize the overall freeway performance monitoring errors resulting from the failure of sensors. However, the consideration of sensor failure on the link flow observation/inference of links in a fully observable network is absent in current studies related to the flow observability problem.

Before proceeding, a formal definition of the terminology used in this chapter is provided below:

Observed and unobserved links: If a link is equipped with a sensor, then that link is named an observed link; otherwise it is an unobserved link.

Link flow observation and link flow inference: If the flow of a link is monitored by a sensor installed on that link then the flow of that link will be observed. This process is called *link flow observation*. On the other hand, if the flow of a link is *inferred*, using the link flow information obtained from observed links, it is then called *link flow inference*. In summary, the process of link flow observation and link flow inference occur for observed links and unobserved links, respectively.

Missing link flow inference: Link flow inference of an unobserved link will be missed if failure occurs for at least one of the sensors installed on the observed link(s) required for the link flow inference of that unobserved link.

1.1.2 Introduction to the sensor location problem for OD estimation

Origin-Destination (OD) demand reflects the OD pattern distribution in a traffic network, which is typically divided into traffic analysis zones (TAZs). OD demand cannot be observed directly, so traffic surveys have traditionally been conducted to derive the demand between OD pairs or between TAZs. As an original approach for OD data collection, traffic surveys are labor-intensive and time-consuming tasks. Technology advancements have brought new proposed traffic detection methods for OD demand estimation. Among these methods is a well-known OD demand estimate that uses traffic counts (Cascetta and Postorino, 2001).

Traffic count information is still viable in current traffic management applications (Salari et al., 2019), but the counting sensors that provide this information are subject to considerable failure rates that adversely affect the quality and the reliability of the information the sensors can deliver (Zhu et al., 2017). Danczyk et al. (2016) indicate that the

explicit cost of repairing failed sensors might be too high for traffic management agencies' restricted budgets. Sensor failure also imposes implicit cost to these agencies due to lost information.

The location of sensors in a traffic network can determine the possibility and quality of OD demand estimates between each OD pair in a network. Their locations can also positively affect the intensity of traffic flow that can be directly observed by sensors deployed on links. A robust sensor configuration can reduce the possibility of missing OD data necessary for demand estimation and can prevent OD traffic flow information loss due to the failure of a sensor. Creating a sensor configuration that can minimize the effect of sensor failure on the OD estimation process is the main motivation behind this study.

The NSLP literature is replete with research that focuses its efforts on sensor location optimization to improve OD demand estimations. For instance, Lam and Lo (1990) prioritized OD coverage and traffic volume criteria in their sensor locations. Yang et al. (1991) developed a maximum possible relative error (MPRE) measurement to identify the maximum deviation of estimated OD demands from the true values. As an extension to the work by Yang et al. (1991), Bianco et al. (2001) attempted to reduce the value of the MPRE by developing a greedy heuristic algorithm. In another study, Yang et al. (2006) developed algorithms to extend the applicability of the MPRE criteria to the screen line-based counting sensor location problem.

Inspired by Lam and Lo (1990) and Yang et al. (1991), Yang and Zhou (1998) defined the most comprehensive set of rules to date for sensor deployment in a traffic network to estimate OD demand:

OD covering rule: Solutions to OD estimation must guarantee that all OD pairs are observed. This means that for any OD flow there must be at least one sensor to directly observe the flow of that OD.

Maximal flow fraction rule: The proportion of each OD flow directly observed by a sensor with respect to other OD demands should be maximized.

Maximal flow interception rule: Sensor locations should maximize the total flow observed by sensors.

Link independence rule: Information extracted from the sensors should be linearly independent.

Yang and Zhou (1998) employed these rules to obtain an efficient greedy algorithm that minimizes the MPRE metric. In their set of rules, the OD covering rule is the primary criterion that guarantees that the solution space is bounded and that the MPRE metric does not have infinite values. The implementations of the *OD covering* rule and the *link independence* rule do not rely on prior information such as OD flows and route fractions, but the *maximal flow fraction* and *maximal flow interception* rules cannot be implemented if no prior traffic pattern information has been determined. Larsson et al. (2010), Cipriani et al. (2006), and Yang et al. (2006) proposed complementary rules to those laid out by Yang and Zhou (1998):

Route covering rule: Solutions to OD estimation should cover all the routes connecting each OD pair.

Maximal OD demand fraction rule: For each OD demand, the fraction of demand directly observed by sensors with respect to the total demand of the same OD should be maximized.

Maximal net OD flow captured rule: The best sensor configuration is the one that maximizes the net OD flow observed by sensors. The term net in this definition excludes the double-counting of OD flows by sensors.

Maximal net route flow captured: Given a number of sensors, the best location of sensors is the one that maximizes the net route flow captured by sensors.

Gentili and Mirchandani (2012) used synthetic examples to test the eight rules outlined above to determine whether there was a noticeable dominance of any rule over the others in different contexts. They concluded that no rule could always dominate the others under distinctive scenarios. Different solution algorithms have been developed based on these rules. For instance, Chun (2001) considered the information existing in prior OD matrices and assumed specific weights for OD demand to extend the maximum flow intercepting rule. In a similar study, Ehlert et al. (2006) introduced the second-best solution for sensor configuration given a set of preinstalled sensors in a network.

There has recently been a considerable number of studies that focus on maximizing the expected information gain for OD demand estimation and link or route flow observability given a certain number of sensors. Viti et al., (2014) introduced a new metric to assess route flow observability based on different sensor configurations that lead to partial observability. Their primary goal was to maximize the route and link flow information gains in a partiallyobservable network. Rinaldi and Viti (2017) extended that research by proposing an exact and approximate route set generation model that employs graph theory and the maximum clique problem to identify a resilient counting sensor location that maximizes the information gain from a set of deployed sensors.

Other studies address information gain from an OD demand estimation perspective. For instance, Zhou and List (2010) developed an information-theoretic model that locates both automatic vehicle identification (AVI) and counting sensors to maximize the expected information gain for the OD demand estimation problem. Yang et al. (2018) and Yang and Fan (2015) attempted to improve the quality of deterministic and stochastic travel demand estimation by employing daily sensor observations. To better solve the dynamic path travel time problem, Xing et al. (2013) proposed a heterogeneous sensor assignment model and a novel measure to evaluate the quality of travel time prediction. Gomez et al. (2015) developed a fuzzy-based, bi-level optimization model to estimate real-time OD demand via counting sensors and floating car data. In another study, Yang and Fan (2015) proposed a model that considered inconsistency in the prior OD demand matrix and analyzed the error bounds of OD demand estimation. They considered both topological and operational relationships in a traffic network.

Sensor failure is a significant factor that impacts the expected information gain from a network. I identified a limited number of studies that account for sensor failure with regards to flow observability and OD demand estimation. In one of the few studies in this area, Li and Ouyang (2011) identified the optimal locations of AVI sensors in traffic to maximize the route flow information gain with consideration for sensor failure. Danczyk et al. (2016) studied the deployment of counting sensors on a freeway to minimize overall freeway performance monitoring errors with respect to sensor failure. Salari et al. (2019) addressed the full and partial link flow observability problem to minimize the adverse effect of sensor failure on the link flow inference process. Consideration for sensor failure in stochastic OD demand pertaining to the OD demand estimation problem is absent in the current literature.

1.2 Proposed Methodology and Research Contributions

This research contributes to the body of knowledge by considering the impact of sensor failure for two well-known problems in NSLP: 1) the full link flow observability problem and 2) the origin-destination estimation problem. I investigate the effects of sensor failure in identifying sensor locations for both problems. Consideration for sensor failure is often overlooked in the literature even though sensor failure can significantly affect the reliability and robustness of the NSLP. The large size and the frequently high congestion levels of actual transportation networks further exacerbate the problem.

The contribution of this study can be summarized in two distinctive categories: flow observability and flow estimation. The contributions of this research to each category are outlined below.

Contributions to the NSLP for flow observability: The model developed for this research offers several contributions. First, I develop a new mathematical optimization model for sensor configuration that considers the probability of sensor failure in reaching full link flow observability. Second, I considered the possibility of installing non-identical sensors with different failure probabilities and costs on links with different levels of importance in a network. I also consider the effect of high heavy vehicle loads (HVLs) on the failure

probability of sensors and the corresponding sensor positioning within a network. Third, I discuss the loss of this information due to the failure of sensors with respect to the availability of route flow information and I add a new consideration for sensor positioning evaluation. Fourth, I propose a new approach for locating redundant sensors in a traffic network to minimize the effect of sensor failure on the full link flow observability based on the suggested layout of sensors. Finally, I address the economic impediments to reaching full link flow observability by evaluating the effectiveness of the proposed model for partial link observability in a network.

Contributions to NSLP for OD flow estimation: In this work, I attempt to address the existing gap in the literature through three main contributions. First, I identify the location of sensors for OD demand estimation purpose by proposing a mathematical model that considers the effect of sensor failure on the OD demand information gain from links instrumented with sensors. I achieve this by developing a measure that considers the maximum possible OD demand information gain for each observed link and for each OD pair in the event of sensor failure. Second, I consider the failure rate as a function of time and study the OD demand information gain from OD pairs at useful life and wear-out phases of sensors. The consideration of time-dependent failure rate of sensors provides traffic authorities with the chance to monitor OD demand information gain through the life phases of deployed sensors. Third, with respect to sensor failure, I revise the rules for sensor deployment to maximize the OD demand information gain for installation of any extra sensors in a network.

1.3 Thesis organization

This thesis consists of four chapters that are laid out as follows:

Chapter Two describes the optimization of traffic sensor locations for complete link flow observability in a traffic network with consideration for sensor failure. This chapter begins with a comprehensive overview of the flow observability problem, followed by examples that better demonstrate the main idea behind my proposed model. This chapter also covers problem formulation, illustrative examples, discussion of the applicability of the proposed model for partial observability, and conclusions.

Chapter Three describes an analytical model that finds the optimal location of counting sensors for OD estimation purposes. This chapter includes the formulation of the proposed model and several examples that demonstrate its applicability. The chapter concludes with insightful findings from the results.

Chapter Four summarizes the findings of this research and concludes the work described in this dissertation. The contributions of this research to the greater body of literature are described. Potential applications and future research are recommended.

CHAPTER 2: OPTIMIZATION OF TRAFFIC SENSOR LOCATION FOR COMPLETE LINK FLOW OBSERVABILITY IN TRAFFIC NETWORK CONSIDERING SENSOR FAILURE²

2.1 Classification of the flow observability problem

In the literature, Gentili and Mirchandani (2012) classified the network sensor location problem into two categories: (i) the sensor location flow estimation problem and (ii) the sensor location flow observability problem. The sensor location flow estimation problem concerns finding the optimum location of sensors to minimize traffic flow estimation errors, while the sensor location flow observability problem attempts to determine the location that requires the least number of sensors to make full or partial traffic flow observability possible. The solutions obtained in the flow observability problem are mainly based on network topological information, i.e., information on links connectivity and enumeration of routes. However, the flow estimation problem usually requires additional information, including the historical traffic flows in a network, since this type of problem attempts to use this historical information to estimate current traffic flow data (cf. Chootinan et al. (2005), Chen et al. (2007), Ehlert et al. (2006), Gentili and Mirchandani (2018), Hadavi and Shafahi (2016), and Zhan et al. (2018) for a further review of the literature related to the sensor location flow estimation problem).

² This content of this chapter is published in Salari et al. (2019)

The sensor location flow observability problem includes studies that attempt to maximize/minimize the objective functions related to traffic flow observations. As an example of a multi-objective problem, Viti et al. (2014) addressed the observability problem to maximize the traffic flow information gain obtained from sensors and to minimize the number of sensors to be located on links. There are also some studies which attempt to satisfy one or more predetermined objectives, known as goal-oriented problems. In one such study, Yang and Bell (1998) defined a mathematical model that determines the number of sensors to be located on fraction of the traffic flow. In a more recent work, Fu et al. (2017) used matrix arithmetic to obtain the full link observability as the goal of their model in the presence of the link-path incidence matrix.

Many of the studies which addressed the flow observability problem cannot be assigned to either category, as they usually mix multi-objective and goal-oriented problems to simultaneously pursue several aims, as well as to meet some predefined goals often defined as constraints in their formulations. As a way of illustration, He (2013), Hu et al. (2009), Ng (2012), and Xu et al. (2016) investigated the minimum number of sensors that should be installed on links with the assumption of no prior information (i.e., information on turning ratio at an intersection, link choice proportions, and the route choice behavior of users) to observe or infer the flow of all links in a traffic network. For instance, Hu et al. (2009) employed an algebraic approach to identify the location of sensors on links for full link flow observability assuming the existence of the link-path incidence matrix to represent a network. In another work, He (2013) used the topological tree characteristics of solutions

to determine the minimum set of links to be equipped with sensors for the full link observability.

The sensor location flow observability problem can also be categorized according to traffic flow observation types. Based on the classification introduced by Castillo et al. (2013), there are four types of observability problems: link flow observability, OD flow observability, route flow observability, and general case flow observability.

(1) Link flow observability: In this category, the aim is to determine which subset of links or nodes should be instrumented with sensors to enable link flow inference of unobserved links. The link flow observability problem itself can be divided into two subproblems concerning the location of sensors, which can either be on nodes (Bianco et al., 2001, 2006; Morrison and Martonosi, 2015) or on links (Bianco et al., 2014; Hu et al., 2009; Ng, 2012, 2013; Xu et al., 2016). Both of these subproblems deal with a linear system of equations and attempt to obtain the unknown values related to unobserved link flows using independent equations. (2) OD flow observability: Observing the ongoing flow between each OD pair is the purpose of OD flow observability problem. The OD flow information can be obtained counting link flows or route flows (See Castillo et al., 2008a; Mínguez et al., 2010). It is possible that the OD flow observability using link flow information has infinite solutions, known as an underspecified problem. To overcome under-specification, some researchers suggest using additional information including the prior information of OD-pair flows and network properties (Castillo et al., 2008b).

(3) *Route flow observability*: In relatively large networks, there is usually more than one path or route between each OD pair. Obtaining information about the flow of each route is the

goal of the route flow observability problem. Route flow knowledge can also provide OD flow and link flow information by using flow conservation equations. Studies addressing the route flow observability problem can be classified based on the types of sensors employed in their target network. For instance, some studies attempt to obtain route flow information using sensors (Hu et al., 2009; Rinaldi and Viti, 2017) while others use different techniques and types of sensors to acquire this information. For instance, plate scanning techniques which give information about the flow of a route or part of a route using license plate recognition (LPR) sensors is an alternative to the link counts technique (Castillo et al., 2013). (4) General case flow observability: This case happens when the flow of interest is not limited to one of the three categories described above. In fact, the goal of general flow observability could be to observe the flow of links, routes and OD flows simultaneously (Castillo et al., 2010; Fei et al., 2013). For instance, Castillo et al. (2010) developed a matrix tool for general observability in traffic networks.

Castillo et al. (2013) introduced the *link flow observability problem* as the simplest one among other observability problems. Although, recently, there have been many more studies addressing the link flow observability problem in the literature. This trend is due to the fact that this type of observability problem requires the least prior information of a traffic network while other types of problems depend on additional information which might not be available for large-scale networks. For instance, information about the number of OD pairs and the number of paths in a network must be available to solve *OD flow observability* or *route flow observability* problems, respectively. However, as the size of a traffic network increases, it becomes more challenging to obtain these pieces of information (Ng, 2012). Another reason for the current focus on the *link flow observability problem* is that most of the studies addressing the flow observability problem are designed to be employed for the strategic planning of a traffic network (Xu et al., 2016). Since it is difficult for urban planners to obtain the prior information required for other types of observability problems at the planning stage, planners prefer to rely on the least amount of prior data.

2.2 Motivating example

Consider the example of the "Fishbone network" which was first introduced by Hu et al. (2009) as a hypothetical example. This network has 18 directed links and four centroid nodes, i.e., nodes 1 and 2 as origin nodes and nodes 9 and 10 as destination nodes (See Figure 2.1). The other nodes, including nodes 3, 4, 5, 6, 7, and 8, are non-centroid nodes.



Figure 2.1 – Fishbone network

According to Ng (2012), to reach the full link flow observability in a network, I need

to equip a minimum of $\left(\frac{(|J|-|I|)}{|J|}\right) \times 100$ percent of links with sensors, where J and I

represents the set of links and non-centroid nodes in a network, respectively³. This means that at least 66.67 % of the links, i.e., 12 links, of the Fishbone network should be equipped with sensors to make this network fully observable. Note that the flow of the other six links, i.e., unobserved links, can be inferred using the link flow information of observed links, i.e., sensor-equipped links, while according to Ng (2012), different sets of links can be considered as the set of observed links in a network. The layouts introduced in Figure 2.2 are two of the many possible layouts to reach full link flow observability in the Fishbone network.



Figure 2.2 – Two possible layouts to reach full link flow observability in Fishbone network

³ To determine the minimum number of links to be equipped with sensors, Ng (2012) specifies an upper bound for the set of all possible links whose flow can be inferred to reach full link flow observability in a network.

According to Figure 2.2, there are six unobserved links as well as 12 observed links in each of layouts A and B. Figure 2.2 also introduces the linear system of equations required to infer the flow of unobserved links in each layout using the link flow information of observed links. Sensors installed in observed links similar to any other measurement apparatus are subject to failure and their failure can affect the link flow inference of unobserved links. This means that in an equation existing in either of the linear systems introduced in Figure 2.2, the flow of an unobserved link cannot be inferred if at least one of the sensors installed on the observed links, I can calculate the probability of missing/not inferring the link flow of unobserved links.

For instance, the probability of missing the link flow inference of the unobserved link 4 in layouts A and B is determined in Table 2.1.

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 $1 - (1 - p)^6$

Layout	Link flow inference equation	Possibility of missing the link flow inferen (Identical sensors)
Α	$l_4 = l_8 + l_{10} - l_6$	$1 - (1 - p)^3$

Table 2.1 – Probability of missing the link flow inference of the unobserved link 4

 $l_4 = l_{14} + l_{16} - l_6 - l_8 - l_{12} - l_{13}$

B

In Table 2.1, p is the failure probability of a sensor and it is assumed all sensors are identical having the same probability of failure and functioning independently. According to the last column of Table 2.1, the unobserved links which need a smaller number of observed links for their link flow inference tend to have a lower chance of missing their link flow

inference due to the failure of sensors. For instance, the chance of missing the link flow inference of unobserved link 4 in layout A is lower than the chance of missing the link flow inference of the same link in layout B. Therefore, a possible layout that requires fewer observed links for the link flow inference of each unobserved link is a preferred layout assuming all sensors are identical.

Until now, I have only considered the *probability* of missing link flow inference for a given set of unobserved links. In addition, I am also interested in exploring the *effect* of a sensor's failure on the link flow inference of unobserved links. For instance, I compare the failure effect of the sensor installed on observed link 9 in layouts A and B (See Table 2.2). According to Table 2.2, the observed link 9 appears in four different equations of the linear system related to layout A while this observed link exists in three different equations of the linear system related to layout B. This means that the flow of 4 unobserved links (links 1, 13, 16 and 17) in layout A cannot be inferred if the sensor installed on link 9 breaks down. However, the flow of only three unobserved links (links 1, 15, and 17) cannot be inferred if the sensor installed on link 9 in layout B stops functioning. Considering the failure probability of the sensor installed on link 9, I can also find the expected number of unobserved links for which flow cannot be inferred due to the failure of the sensor located on link 9 in each layout. The last column of Table 2.2 represents this expected value assuming that the sensors installed on link 9 in both layouts A and B are identical. According to this column, the lower the number of appearances of an observed link in different equations required for the link flow inference of unobserved links, the lower the expected number of unobserved links for which flow cannot be inferred due to the failure of the sensor installed on that observed link.

Layout	Equations including link 9	Set of unobserved links requiring link 9	Expected number of unobserved links (Identical sensors)
A	$\begin{cases} l_1 = l_5 + l_9 - l_7 \\ l_{13} = l_9 + l_{11} + l_{14} - l_{15} \\ l_{16} = l_9 + l_{10} + l_{11} + l_{12} - l_{15} \\ l_{17} = l_9 + l_{10} + l_{11} + l_{12} - l_{18} \end{cases}$	{1,13,16,17}	4 <i>p</i>
В	$\begin{cases} l_1 = l_5 + l_9 - l_7 \\ l_{15} = l_9 + l_{11} + l_{14} - l_{13} \\ l_{17} = l_9 + l_{11} + l_{14} + l_{16} - l_{13} - l_{18} \end{cases}$	{1,15,17}	3 <i>p</i>

Table 2.2 – Failure effect of the sensor installed on link 9 in layouts A and B

In Tables 2.1 and 2.2, I assumed that the sensors are identical, each having similar failure probability. In this study, I am also interested in examining the possibility of installing different types of sensors, i.e., non-identical sensors, with dissimilar failure probability. The probability of missing the link flow inference of unobserved links due to the failure of identical or non-identical sensors, as well as the effect of each sensor failure on the link flow inference of unobserved links, motivated me to determine the location of sensors in a network to minimize the adverse effect of sensor failure.

2.3 Link flow inference: Locating sensors on links

Inferring link flows based on the traffic flow information of links instrumented with sensors is referred to as the link flow inference. Node-based and route-based methods are two main techniques to infer link flows. Node-based approaches use the flow conservation rule for non-centroid nodes to infer link flows, while route-based methods are based on linkpath incidence equations. Ng (2012) suggested the use of a node-based approach to avoid the path enumeration problem. Therefore, in this work, I also employ this approach to infer link flows. The flow conservation rule for a set of links connected to a node can be shown as follows:

$$\sum_{j \in In(i)} l_j - \sum_{j \in Out(i)} l_j + BF_i = 0$$
(2.1)

Where l_i represents the ongoing flow on the link j. In(i), Out(i) and BF_i are the set of links with a head at node i, a tail at node i, and the balancing flow at node i, respectively. For a non-centroid node, the *BF* should be equal to zero. Therefore, for each non-centroid node, the above equation changes to Equation (2.2).

$$\sum_{j \in In(i)} l_j - \sum_{j \in Out(i)} l_j = 0 \qquad \forall i \in I$$
(2.2)

For a traffic network consisting of |I| non-centroid nodes and |J| links, Equation (2.2) can be written as the following system of linear equations:

$$T_{|I| \times |J|} L_{|J| \times 1} = 0 \rightarrow \begin{cases} t_{11} l_1 + t_{12} l_2 + \dots + t_{1|J|} l_{|J|} = 0 \\ t_{21} l_1 + t_{22} l_2 + \dots + t_{2|J|} l_{|J|} = 0 \\ \vdots \\ \vdots \\ t_{|I|} l_1 + t_{|I|2} l_2 + \dots + t_{|I||J|} l_{|J|} = 0 \end{cases}$$

$$(2.3)$$
where τ is the node-link incidence matrix, which is defined for non-centroid nodes. For this linear system, t_{ij} is equal to +1 if *j* is an incoming link to node*i*. Otherwise, if *j* is an outgoing link from node*i*, then t_{ij} equals -1, and it equals 0 if *j* is neither an incoming nor an outgoing link to/from node *i*. The number of non-centroid nodes, |I|, and the number of links, |J|, respectively comprise the number of equations and variables of this system. A linear system has a unique solution when the number of equations equals the number of unknown variables and all equations are linearly independent. Thus, in the linear system of Equation (2.3), I can determine the value of a maximum number of |I| unknown variables when the remainder variables, |J|-|I|, are known, and all |I| equations of this system are independent. The flow conservation rule using a node-based approach for a traffic network can also be written as follows (Ng, 2012):

$$\underbrace{\left(T_{u} \quad T_{o}\right)}_{T} \begin{pmatrix} l_{u} \\ l_{o} \end{pmatrix} = 0$$

$$(2.4)$$

where T_o and T_u are the sub-matrices of T relating to observed and unobserved links, respectively. The linear system of Equation (2.4) can also be shown as follows:

$$T_{u}l_{u} + T_{o}l_{o} = 0 \implies l_{u} = -T_{u}^{-1}T_{o}l_{o}$$
(2.5)

According to Equation (2.5), the node-link matrix corresponding to unobserved links, T_u , should be an invertible matrix with the rank of |I| to make it possible to infer the flow of unobserved links, l_u , while according to Ng (2012), the matrix T_u is not necessarily unique.

In what follows, I introduce one of the methods for constructing the matrix of unobserved links.

2.4 The concept of new links

Initially introduced by Castillo al. (2014), the *new link* method is further developed by Xu et al. (2016) as a means to build the matrix of unobserved links. According to this method, each link assigned to a set of new links related to a non-centroid node should be a link connected to that node and shouldn't already be assigned to other sets of new links. In other words, the set of new links associated with a non-centroid node includes *all links* connected to that node excluding the links which are already allocated to other sets of new links. To create the sets of new links, I usually start from the first non-centroid node and continue constructing these sets for all other non-centroid nodes. It is possible that some sets of new links become empty sets in a network, as the links connected to those nodes are already assigned to the sets of new links associated with other non-centroid nodes. The following equations show the rules to be followed in constructing the set of new links:

$$J = \bigcup_{i \in I} H_i \tag{2.6}$$

$$H_i \cap H_i = \emptyset \qquad \forall i, i' \in I \tag{2.1}$$

 $(\mathbf{n} \mathbf{n})$

where *J* and *I* are the set of links and non-centroid nodes in a network, respectively. H_i and H_i represent the set of new links assigned to the non-centroid nodes *i* and *i*. According to Equation (2.6), the union of all sets of new links should be equal to the set of links. Moreover, Equation (2.7) guarantees that there is no intersection between any set of new links in a network. As a way of illustration, Table 2.3 shows the set of new links for the network depicted in Figure 2.1

Table 2.3 – Set of links for the Fishbone network

Non-centroid node	Connected links	Set of new links to nodes
3	1,5,7,9	$\{1, 5, 7, 9\}$
4	2,3,5,6,7,8,11,12	{2, 3, 6, 8, 11, 12}
5	4,6,8,10	{4,10}
6	9,11,13,14,15	{13,14,15}
7	10,12,13,14,16	{16}
8	15,16,17,18	{17,18}

The second column of Table 2.3 shows the links connected to each non-centroid node in the node-link incidence matrix belonging to the network in Figure 2.1 The last column of this table shows the sets of *new links* assigned to each non-centroid node. As can be seen for each set, the link(s) allocated to a non-centroid node are not assigned to any other noncentroid node. Xu et al. (2016) proposed that "*the maximum set of unobserved links to be inferred from observed links can be found by selecting any single new link assigned to each non-centroid node*".⁴ Note that the term "*maximum*" indicates that depending on the topology of a network, the number of sets of new links can be less than the number of non-centroid nodes (i.e., some sets of new links associated with non-centroid nodes become empty sets). The links selected from each non-empty set of new links are then used to create the matrix

⁴ For the proof, please refer to Castillo et al. (2014) or Xu et al. (2016).

of unobserved links, i.e., T_{u} , while the structure of sets of new links prevents the selected links from inducing a cyclic graph and becoming a non-invertible matrix.

2.5 The effect of sensor failure on the link flow inference of unobserved links

In this section, I investigate the importance of sensor failure in determining the set of observed links in a network to reach full link flow observability. To do so, I explore the objective function(s) required to minimize the effect of sensor failure on the link flow inference of unobserved links. According to the motivating example, two contributing factors in determining the set of observed links, considering the failure of sensors, include the probability of missing the link flow inference of unobserved links as well as the effect of sensor failure on the link flow inference of unobserved links. I further discuss these two factors with respect to situations in which the sensors are identical or not. The probability of failure of identical sensors is assumed to be the same regardless of their cost and their employed technology. This assumption can be beneficial when there are not enough historical records about the failure of different type of sensors. However, if this information is available, I should be able to develop a more specific model considering non-identical sensors having different probabilities of failure. Note that in both scenarios, I assume that the sensors are installed independently and there is no correlation between the failure of any pair of sensors.

2.5.1 Probability of missing the link flow inference of unobserved links

2.5.1.1 Identical sensors

The probability of missing the link flow observability of any observed link is equal to the probability of failure of the sensor installed on that link. However, for an unobserved link, the failure of at least one of the sensors installed in the observed links required for the link flow inference of that unobserved link can prevent the link flow inference of that link. According to Equation (2.5), the flow of unobserved links can be inferred by multiplying the matrix $-T_u^{-1}T_o$ by the matrix of observed links, i.e., I_o . The elements of the matrix $-T_u^{-1}T_o$ can help me to identify the observed links that should be used for inferring the flow of an unobserved link. For instance, I can consider that the observed link j^{-1} should be used to infer the flow of the unobserved link j^{-1} if the element a_{jj} of row j^{-1} and column j^{-1} of the matrix $-T_u^{-1}T_o$ is a non-zero value. The maximum probability of missing the link flow inference of an unobserved link can be obtained using Equation (2.8):

$$Y_{1} = \max_{j \in J} \left(1 - \prod_{j \in J} (1 - a_{jj}^{2} p) \right)$$
(2.8)

where *p* represents the failure probability of identical sensors. Moreover, I squared the element of the matrix $-T_u^{-1}T_o$ in Equation (2.8) to cancel the effect of negative signs for counting the number of observed links in each row of the matrix $-T_u^{-1}T_o$. Equation (2.8) is helpful in identifying the unobserved links that have a higher chance of missing their link flow inference. When all sensors are identical, the probability of missing the link flow

inference of an unobserved link depends on the number of observed links which need to be used to infer the flow of that unobserved link. In other words, the higher the number of observed links required for the link flow inference of an unobserved link, the higher the probability of missing the link flow inference of that link. I can identify the unobserved link(s) that need the maximum number of observed links for the link flow inference by counting the number of non-zero elements in each row of matrix $-T_u^{-1}T_o$.

In addition to determining the maximum possibility of missing the link flow inference of an unobserved link, I can also calculate the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors installed on the observed links.

$$Y_{2} = \sum_{j \in J} \left(1 - \prod_{j \in J} (1 - a_{jj}^{2} p) \right)$$
(2.9)

By minimizing Y_2 , I can reduce the expected number of unobserved links for which link flow cannot be inferred as a result of sensor failure. To obtain the average number⁵ of observed links required for the link flow inference of each unobserved link, I can use the following equation:

$$Y_{3} = \frac{\sum_{j,j \in J} a_{jj}^{2}}{|j'|}$$
(2.10)

⁵ In this work, the term "average number" refers to the arithmetic mean of a variable.

where |j'| is the number of unobserved links in a network. If a particular layout of sensors leads to a lower value of γ_3 compared to other layouts, then I expect this layout also has a lower value of γ_2 which means that this layout also has a lower expected number of unobserved links for which flow cannot be inferred due to the failure of sensors. The reason for this is the fact that γ_3 calculates the average number of observed links required to infer the flow of an unobserved link, and when sensors are assumed to be identical, the probability of missing the link flow inference of an unobserved link depends on the number of observed links needed to infer the flow of that link. Therefore, a lower value of γ_3 indicates that, on average, a smaller number of observed links are required for the link flow inference of unobserved links, and correspondingly, the lower the probability of missing the link flow inference of an unobserved link. However, if two layouts have the same value of γ_3 , they could still have a dissimilar value of γ_2 . I provide an example in Appendix I to discuss this situation in more detail.

2.5.1.2 Non-identical sensors

Unlike identical sensors, non-identical sensors can have different probabilities of failure depending mainly on their employed technology. The maximum probability of missing the link flow inference of an unobserved link in the presence of non-identical sensors can be calculated as follows:

$$Y_{1}^{'} = \max_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^{2} y_{fj}^{*} p_{f}) \right)$$
(2.11)

where y_{g} is a binary variable to determine if the sensor type f is installed on the observed link j^{*} or not, and p_{f} represents the failure probability of the sensor type f. Equation (2.11) is of importance in detecting the unobserved link that has the highest chance of missing the link flow inference. In addition to the calculation of the maximum probability of missing the link flow inference, I can calculate the expected number of unobserved links whose flow will be missed due to the failure of non-identical sensors:

$$Y_{2}^{*} = \sum_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^{2}, y_{fj}, p_{f}) \right)$$
(2.12)

where in Equation (2.12), the higher value of Y_2^{-} indicates that the flow of a higher number of unobserved links cannot be inferred if one or more sensors installed on observed links break down.

2.5.2 Effect of a sensor failure on the link flow inference of unobserved links

2.5.2.1 Identical sensors

To discuss the effect of a sensor's failure on the link flow inference of unobserved links, I need to identify the appearance of observed links in different equations used to infer the flow of unobserved links. To determine the number of appearances of an observed link in these equations, I need to count the non-zero values of the column vector associated with that observed link in the matrix $-T_u^{-1}T_o$. Equation (2.13) calculates the maximum expected number of unobserved links for which their flow will be missed due to the failure of a sensor installed on an observed link:

$$Y_4 = \max_{j \in J} \left(\left(\sum_{j \in J} \alpha_{jj}^2 \right) p \right)$$
(2.13)

In Equation (2.13), $\left(\sum_{j \in J} \alpha_{jj}^2\right)$ counts the number of non-zero elements in the column

vector associated with the observed link j^{*} and it can show the number of appearances of the observed link j^{*} in different equations used for the link flow inference of unobserved links. I also can count the number of non-zero elements in each column vector of the matrix $-T_{u}^{-1}T_{o}$ to obtain the maximum number of observed links that is used in different equations required for the link flow inference. This way I can determine the maximum number of unobserved links for which their flow cannot be inferred due to the failure of a sensor installed on an observed link. The average number of unobserved links whose flow cannot be inferred due to the failure of a sensor can be calculated as:

$$Y_{5} = \frac{\left(\sum_{j:j^{*} \in J} \alpha_{j^{*} j^{*}}^{2}\right)}{\left|j^{*}\right|}$$
(2.14)

where in Equation (2.14), |j'| is the number of observed links in a network. Comparing Equations. (2.14) and (2.10), I can conclude that by minimizing the average number of observed links used to infer the flow of an unobserved link, I can also minimize the average number of unobserved links where link flow cannot be inferred due to the failure of a sensor.

2.5.3 Non-identical sensors

For non-identical sensors, the expected number of unobserved links for which their flow cannot be inferred due to the failure of a sensor depends not only on the number of appearances of an observed link in different equations used for the link flow inference of unobserved links but also on the failure probability of the sensor installed on that observed link. The maximum expected number of unobserved links where their flow cannot be inferred due to the failure of a sensor is calculated as:

$$Y_4^{"} = \max_{j \in J} \left(\sum_{f \in F} y_{fj} p_f \left(\sum_{j \in J} \alpha_{jj}^2 \right) \right)$$
(2.15)

where in Equation (2.15), the observed link with the highest impact on the link flow inference of unobserved links can be identified.

2.6 Mathematical formulation

In this section, I present the constraints and objective functions required to determine the type and the location of sensors on links in a traffic network. To deal with the objective functions introduced in the previous section, I suggest using min-max and min-sum methods. my mathematical formulation is introduced as follows:

Subscript:

- i = non-centroid node of a traffic network
- j =links of a traffic network
- j = unobserved links of a traffic network
- r = routes of a traffic network

- j = observed links of a traffic network
- f = type of the sensor to be located on a link

Sets:

- $I = \text{set of non-centroid nodes: } \{1, ..., |I|\}$
- $J = \text{set of links in a traffic network: } \{1, ..., |J|\}$
- R = set of routes in a traffic network: $\{1, ..., |R|\}$
- $M = \text{set of major roads}, M \subset J$
- $F = \text{set of the sensor type to be located on links: } \{1, ..., |F|\}$
- H_i = set of new links assigned to node *i*
- $H = \text{set includes all sets of new links } \{1, ..., |H|\}^{6}$

Parameters:

- T =node-link incidence matrix $(T_{|I| \times |J|})$
- p_f = probability of failure of sensor type f
- $c_f = \text{cost of sensor type } f$ that can be located on a link
- w_j = relative importance of link j

⁶ Based on the definition of the set of new links, the inequality $|H| \leq |I|$ is always valid.

 h_{ij} = a binary parameter that shows if link j is assigned to the set of new links associated with the node i, i.e., $h_{ij} = 1$ if $j \in H_i$

 q_{jr} = a binary parameter that shows if route *r* traverses link *j* or not. q_{jr} = 1 if route *r* traverses link *j* and q_{jr} = 0 otherwise

 ϑ = Budget constraint

Decision variables:

 x_{ij} = binary variable indicating whether link *j* which is assigned to the node *i* as a new link is an unobserved link (x_{ij} = 1) or not (x_{ij} = 0).

 o_j = binary variable determining if link *j* is an observed link or not.

 y_{ij} = binary variable indicating whether the sensor type f is installed on link j ($y_{ij} = 1$) and ($y_{ij} = 0$) otherwise.

 T_u = variable associated with a set of column vectors structured by selecting a link from each set of new links and putting the column vector related to that link from node-link incidence matrix in T_u . The binary variable o_j associated with each link selected to be in T_u should be equal to zero.

2.6.1 Objective functions and constraints regarding identical roads

To install sensors in a network, I can assume all links are equally significant and find the preferred location of sensors so as to minimize the effect of sensor failure on the link flow inference. The reason for introducing the min-max functions is to minimize the maximum effect of sensor failure on the link flow inference of unobserved links. Depending on whether sensors are identical or not, I can define different min-max objective functions to identify the location of sensors. Equation (2.16) introduces two possible objective functions assuming that sensors are not identical.

$$\begin{cases} Z_1 = \min\left(\max_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^2, y_{fj}, p_f)\right)\right) & I \\ Z_2 = \min\left(\max_{j \in J} \left(\sum_{f \in F} y_{fj}, p_f\left(\sum_{j \in J} \alpha_{jj}^2\right)\right)\right) & II \end{cases}$$
(2.16)

Equation (2.16-I) minimizes the maximum probability of not inferring the flow of an unobserved link due to the failure of sensors, while Equation (2.16-II) minimizes the maximum effect of a sensor's failure on the link flow inference of unobserved links. Equation (2.16) can be used for identical sensors by considering that $p_f = p$, $\forall f$ and excluding the binary variable from the equation that determines the type of sensor installed on an observed link. The min-sum objective function is introduced here to minimize the average number of unobserved links whose link flow inference will be affected by a sensor's failure. To be more specific, I introduced Equation (2.17) to minimize the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors.

$$Z_{3} = \min\left(\sum_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^{2} y_{fj}^{*} p_{f})\right)\right)$$
(2.17)

Similar to Equation (2.16), Equation (2.17) can be used for identical sensors, assuming all sensors have the same probability of failure and by removing the binary variable y_{fj} from the equation. The following constraints relate to the above mentioned objective functions:

s. <i>t</i> .,		
$\sum_{j\in H_i} h_{ij} x_{ij} = 1$	$\forall i \in I$	(2.18)
$1 - \sum_{i \in I} x_{ij} = o_j$	$\forall \ j \in J$	(2.19)
$\sum_{f \in F} y_{fj} = o_j$	$\forall j \in J$	(2.20)
$\sum_{j \in J} \sum_{f \in f} c_f y_{fj} \leq \mathcal{G}$		(2.21)

Constraint (2.18) allows a link to be an unobserved link in the set of links assigned to the node i, if that link is already available in the set of new links associated with node i. Constraint (2.19) identifies the set of observed links in a network that should be equipped with sensors. According to the constraint (2.20), if a link is considered as an observed link in any set of new links, then the flow of that link should be observed by a sensor installed on that link. Constraint (2.21) imposes the budget constraint required for installing sensors in a network, while the budget should be sufficient to allow installation of the minimum number of sensors required for full link flow observability.

2.6.2 Objective functions and constraints considering major roads

In real networks, all links are not of the same importance -some are major roads, such as highways or major arterials, and others are minor roads such as collectors. Regarding the failure of sensors, I am interested in incorporating the relative importance of links in finding the optimal location of sensors. On one hand, if the number of links required to be instrumented with sensors exceeds the number of major links in a network, then traffic sensors can be manually assigned to all major roads, i.e., major roads become observed links, and the rest of the observed links can be selected from the remaining links in the set of links. This assignment can be justified by the fact that the chance of missing the link flow inference of an unobserved link is usually higher than the chance of missing the link flow observability of an observed link. The rest of the links required to complete the set of observed links can be selected from the minor roads in a network while using the min-max or min-sum objective functions introduced in Equation (2.16) and Equation (2.17), respectively. On the other hand, if the number of links required to be equipped with sensors is less than the number of links in the set of major roads, then either of Equation (2.16) or (2.17) can still be used as the objective function of the model while enforcing that all members in the set of observed links should be from the set of major roads. The following are the objective function and constraints that consider major roads in instrumenting links with sensors for the two situations described above:

(2.22)

$$Z_{4} = \min\left(\sum_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^{2}, y_{fj}, p_{f})\right)\right) \text{ OR } \begin{cases} Z_{1} = \min\left(\max_{j \in J} \left(1 - \prod_{f \in F} \prod_{j \in J} (1 - a_{jj}^{2}, y_{fj}, p_{f})\right)\right) \\ Z_{2} = \min\left(\max_{j \in J} \left(\sum_{f \in F} y_{fj}, p_{f}\left(\sum_{j \in J} \alpha_{jj}^{2}\right)\right)\right) \end{bmatrix} \text{ II } \end{cases}$$

s.t.

Equation (2.20) & Equation (2.21)

$$\begin{cases} \sum_{j \in F} y_{jj} = 1 & \forall j \in M & \text{if } |M| \le |J| - |H| & \text{I} \\ \\ \text{OR} \\ \sum_{j \in F} y_{jj} = 0 & \forall j \in J \setminus M & \text{if } |M| > |J| - |H| & \text{II} \end{cases}$$
(2.23)

$$\sum_{j \in J} o_j = |J| - |H| \qquad \text{if } |T_u| \neq 0 \tag{2.24}$$

As I explained above, the objective function selected to be used in both situations can be either Equation (2.16) or Equation (2.17). Constraint (2.23-I) is associated with the case where the number of major links is less than the number of observed links required to reach full link flow observability, while, constraint (2.23-II) is formulated for the case where the number of major roads is greater than the number of observed links required to guarantee full link flow observability. Constraint (2.24) ensures that the number of links is equal to the number of links minus the total number of sets of new links. According to this constraint, the observed links should be selected in a way that the node-link incidence matrix related to unobserved links has a non-zero determinant. This non-zero determinant condition needs to be satisfied, because by enforcing that the set of observed links be from the set of major roads, it is possible that more than one link in a set of new links is selected as the unobserved link which means the concept of new links explained in Section 2.4 cannot be applied.

Moreover, equipping all major roads with sensors could also lead to a situation where it is not possible to construct the matrix of unobserved links to be invertible. I provide an example in the small Fishbone network in Appendix II to elaborate on this situation in more detail. To avoid the occurrence of the above-mentioned situation, I could update the sets of new links by removing the major links to be instrumented with sensors from the sets of new links in such a way as to avoid a set of new links becoming an empty set. Employing this approach, I can still use the concept of new links to generate initial feasible solutions. However, as Equation (2.23) is no longer applicable to this approach, I need to relax this constraint as well as update the objective function to cause more major links to become observed links in the optimum solution. A possible way to achieve this objective is to assign weights to links to signify their relative importance considering different factors including the capacity of links and so on. The updated objective function is provided below:

$$Z_{5} = \min\left(\sum_{j' \in J} w_{j'} \left(1 - \prod_{f \in F} \prod_{j'' \in J} \left(1 - \left(a_{j'j''}^{2} y_{fj''} p_{f}\right)\right)^{\frac{1}{w_{j'}}}\right)\right)$$
(2.25)

where in Equation (2.25), w_j and w_j , $(0 < w_j, w_j \le 1, \forall j', j' \in J)$ are the weights assigned to a link in a traffic network to emphasize the relative importance of the link, with higher values of the weight for a link indicating greater significance of that link. In Equation (2.25), if a link is unobserved, then its relative weight is equal to w_i . In this way, the objective function

assigns the links with lower weights as unobserved links to minimize the value of Z_3 . Moreover, if the link becomes an observed link, then its weight is represented as $\frac{1}{w_c}$ as the

power of the term $(1-(a_{j'j'}^2y_{jj''}p_f))$, $0 \le (1-(a_{jj'}^2y_{jj'}p_f)) \le 1 \forall j', j \in J, f \in F$, to maximize this term and to prompt the model to assign the links with higher weights as observed links. Considering Equation (2.25) as the objective function, then I should set Equations. (2.18-2.21) as the constraints of the model. Equation (2.25) is also useful for the situation where it is easier to incorporate the relative importance of links compared to each other, instead of separating the links into two distinctive sets of major roads and minor roads. Employing Equation (2.25) as the objective function and Equations. (2.18-2.21) as the constraints, I also can directly construct the set of unobserved links from the original sets of new links, not the updated ones.

2.7 Full link flow observability considering route flow information

Although access to route flow information in the strategic planning phase of a network is not always feasible, this type of information can provide urban planners with useful knowledge to reduce the number of sensors required to be installed in a network (Fu et al., 2016). Note that my aim in this section is not to separately investigate the full/partial route flow observability problem considering the failure of sensors, but to discuss how the existence of route flow information will lead to a better evaluation of the solutions to reach full link flow observability, which is the main focus of this work. Depending on the

uniqueness of information that can be obtained from link flow observation, routes can be divided into three distinctive sets:

 R^1 : All links traversed by this type of route are not traversed by any other route.

 R^2 : There is at least one link in the set of links traversed by route type 2 which is not traversed by any other route.

 R^3 : All links traversed by route type 3 are also traversed by other routes.

Note that the definition of route types, i.e., sets R^1 , R^2 and R^3 , is analogous to that provided by Rinaldi and Viti (2017). They also categorized routes into three categories; namely, non-redundant routes (NR), redundant while informative routes (RI), and purely redundant (PR) routes. The definition of categories NR, RI and PR routes are very similar to sets R^1 , R^2 and R^3 , respectively. One important difference is that Rinaldi and Viti (2017) defined a route as belonging to NR category if the links traversed by it were not *previously* crossed by any other route, whereas in my definition, a route is classified as type 1 if the links traversed by that route are not crossed by *any other* route. I proposed this definition for R^1 as it makes it easier to interpret the failure effect of sensors installed on links. Equation (2.26) represents the characteristics of sets R^1 , R^2 and R^3 :

$\int R^1 \bigcap R^2 = \emptyset$	Ι	
$\int R^1 \bigcap R^3 = \emptyset$	П	(2,26)
$\int R^2 \cap R^3 = \emptyset$	III	(2.20)
$\left(R^1 \bigcup R^2 \bigcup R^3 = R\right)$	IV	

According to Equation (2.26), there is no intersection between any pair of sets R^1 , R^2 and R^3 . Moreover, Equation (2.26-IV) declares that the union of sets R^1 , R^2 and R^3 equals R which is the set of all routes in a network.

The flow of a link in a network can consist of the summation of flow of routes from sets R^1 , R^2 or R^3 . The following equation demonstrates the relationship between flow of routes which traverse link *j*:

$$v_j = \sum_{r \in \mathbb{R}^1 \bigcup \mathbb{R}^2 \bigcup \mathbb{R}^3} e_r \tag{2.27}$$

Where e_r is the ongoing flow of route r. In Equation (2.27), if r, where $r \in R^1 \cup R^2$, is the only route traversing link j, then by instrumenting link j with a sensor, I can obtain the flow of this route. In a different situation, if $r \in R^2 \cup R^3$ and there is more than one route crossing link j, then I may still find the exact flow of all or some routes traversing link j by equipping this link with a sensor while also using the route information obtained from other routes in R^1 and R^2 . This latter situation is discussed by Castillo et al. (2014) and Rinaldi and Viti (2017) when they addressed the fact that a route in the set of R^3 may still contribute to increasing the gain in route information by using sensor-equipped link information. In other words, these authors pointed out that considering link-route incidence matrix, the column vector related to the route r' in R^3 may be independent from the union of column vector of routes in $R^1 \cup R^2$. Rinaldi and Viti (2017) identify those routes in R^3 in a network to ensure there is no other route that can contribute to partial/full route flow observability. In my work,

I suggest installing a sensor on at least one link from the set of links traversed by each route in R^1 :

$$\sum_{j \in J} q_{jr} o_j \ge 1 \qquad \forall r \in R^1$$
(2.28)

In Equation (2.28), q_{jr} is a binary parameter that shows whether route *r* traverses link *j* or not. The value of this parameter can be obtained from the link-route incidence matrix. Moreover, for the set of links traversed by a route in R^2 , I recommend equipping the link which is not traversed by other routes with a sensor:

$$q_{jr} - \left(\sum_{r' \in R \setminus \{r\}} q_{jr'}\right) \le o_j \qquad \forall j \in J, r \in \mathbb{R}^2$$
(2.29)

According to Equation (2.29), if a link is traversed by a route in R^2 , and is not traversed by any other route in R, then that link should be equipped with a sensor. Finally, by using the results of the model developed by Rinaldi and Viti (2017), I can recognize which routes in R^3 are contributing to route flow observability and install a sensor on at least one of the links traversed by these routes:

$$\sum_{j \in J} q_{jr} o_j \ge 1 \qquad \forall r \in \mathbb{R}^3$$
(2.30)

Where R^{3} is the set of routes in R^{3} which are independent from all routes in R^{1} and R^{2} . According to Equation (2.30), at least one of the links available in the set of links traversed by a route in R^{3} should be instrumented with a sensor.

I should note that I may not be able to reach full link flow observability employing Equations. (2.28), (2.29) and (2.30) as hard constraints. This situation could arise when the set of links which create a cyclic graph are selected to be instrumented with sensors according to these three constraints⁷. To avoid this possibility, I suggest finding the location of sensors to reach full link flow observability in the first level optimization and then maximizing the information gain of routes, i.e., minimizing the probability of missing the route flow observability in the second level by using the pool of optimum solutions obtained in the first level as feasible solutions for the second level optimization. I also can combine both objective functions, including minimizing the effect of sensor failure on link flow inference of unobserved links, as well as minimizing the effect of this failure on route flow observability of routes in a single level optimization, using weighted sums method (WSM) or ε-constraint. However, I prefer two-level optimization as the feasible solution used in the second level guarantees to position sensors in a way to have minimum failure effect on link flow inference of unobserved links. The objective function and the constraint of the second level can be defined as:

$$Z_6 = \sum_{r \in \mathbb{R}^1 \cup \mathbb{R}^2 \cup \mathbb{R}^3} \left(\prod_{j \in J} \prod_{f \in F} \left(p_f \right)^{y_{j_i} q_{j_r}} \right)$$
(2.31)

s.t.

Equations. (2.18-2.21)

⁷ Please refer to Appendix II where I discuss a similar situation in which instrumenting all major links with sensors leads to the creation of a cyclic graph.

$$\begin{cases} Z_4 \leq Z_4^{opt} & \text{I} \\ OR & \\ Z_5 \leq Z_5^{opt} & \text{II} \end{cases}$$

$$(2.32)$$

Equation (2.31) attempts to minimize the expected number of routes in R^1 , R^2 , and R^3 for which route flow observability will be missed due to the failure of sensors. According to this equation, the flow of a route will be missed if all sensors installed on links traversed by this route break down. In other words, the higher the number of sensor-equipped links traversed by a route, the lower the probability of missing that route flow observability. In the second level optimization, Equations. (2.18-2.21) are employed to ensure full link flow observability. Moreover, Equation (2.32) ensures that, depending on the objective function used in the first level optimization, i.e., Z_4 or Z_5 , the optimum solution of the second level is also the optimum solution of the first-level optimization model.

2.8 Redundant sensors in a traffic network

After identifying the location of sensors, I can also investigate the preferred location of *redundant sensors* – the *minimum* number of extra sensors needed to maintain the full link flow observability of a network if one or more of the sensors installed on observed links fails to observe the link flows. These redundant sensors provide us with two main benefits for traffic monitoring purposes: 1) most importantly, they can maintain full link flow observability when sensor failure occurs in the system; thus, adding to the robustness of the network observability and 2) to a lesser degree, they can participate in reducing the link flow inference error when there is no failure among sensors in a network. I developed the idea of redundant sensors in this work to account for the possibility of sensor failure when the initial

location of sensors is determined using the optimization model introduced in Section 2.6. The following steps outline the procedure required for finding the type and location of redundant sensors:

Step 1. Determine the initial location of sensors

According to this step, I need to determine the optimum location of sensors to reach full link flow observability. The objective function and the constraint introduced in Section 2.6 can be employed to find the location of sensors.

Step 2. Consider all combinations of failure among sensors

This step mainly deals with the possible failure of sensors already installed on links according to Step 1. For instance, if I assume layout A introduced in the motivating example as the optimum location of sensors determined in Step 1, then in Step 2, I need to find each possible combination of failure between these installed sensors. According to layout A, in which there are 12 sensors installed in the Fishbone network, I need to consider 2¹²-1 combinations of failure among sensors. However, considering all failure combinations presents its own combinatorial complexity and might not be feasible for relatively large networks. To deal with this situation, I suggest considering the failure combinations among sensors that are most susceptible to failure while sensors are assumed to be Non-identical. Moreover, as already discussed, I suggest examining the failure of only those sensors installed on major roads as these roads are more important for traffic monitoring purposes.

Step 3. Find the location of redundant sensors for each combination of failures

Step 3 attempts to find the location of redundant sensors for each combination of sensor failure determined in Step 2. To find the location of these extra sensors, I use the objective

functions and the constraints introduced in Section 2.6. However, I need to add the following constraints as well:

i The redundant sensors cannot be installed on a link which already has failed sensors.

ii The location of sensors, excluding the redundant sensors, should follow what I already determined in Step 1.

For instance, considering layout A introduced in the motivating example (i.e., Fishbone network), I know that one of the possible combinations of failure is the situation in which sensors installed on links 3 and 5 stop functioning. Considering this scenario, according to Step 3, I need to find the location of extra sensors in the Fishbone network to reach full link flow observability using the optimization model introduced in Section 2.6. The new constraints that should be considered include the fact that redundant sensors cannot be installed on links 3 and 5 anymore (constraint i) and that the rest of the sensors, excluding the redundant sensors, should follow the sensor positioning introduced in layout A (constraint ii). Note that depending on the network topology, I can disregard some of the combinations of failures among sensors determined in Step 2 as the link flow observability is impossible in those situations.

Step 4. Sensor assignment

In this step, I use the results obtained from Step 3 which specify the location of redundant sensors for each combination of sensor failure. In Step 4, I assigned sensors to links based on the frequency of being selected in Step 3. This means that the links that have been selected more often to be instrumented with redundant sensors will be equipped with more advanced sensors. The sensor assignment, however, is subject to budget constraints which limit the

type of sensors to be installed on links as well as the number of links to be equipped with sensors.

2.9 Solution algorithm

The optimization problem presented in Section 2.6 is a nonlinear problem. The nonlinearity arises in the objective functions introduced in Equations. (2.16) and (2.17) [presented altogether in Equation (2.22)], Equation (2.25) and (2.31). Moreover, due to the existence of binary decision variables y_{g} and o_{j} , it becomes an integer optimization problem. The combinatorial complexity introduced in the problem is due to the fact that there is no general and explicit function to express the relationship between the sensor location scheme and the number of observed links required for link flow inference of unobserved links (Castillo et al., 2014; Xu et al., 2016). Moreover, with respect to redundant sensors, there is a scalability concern introduced in Step 2 that motivates us to employ an efficient solution algorithm to solve the proposed problem.

To solve the proposed optimization problem, I employed the progressive genetic algorithm (GA) initially developed by Guan and Aral (1999) which is designed for optimization problems with nonlinear equality and inequality constraints. The output of the GA is the location of observed links in a traffic network to reach full link flow observability, as well as the type of sensors to be installed at these locations in the case of non-identical sensors. Note that I implemented the proposed GA using MATLAB R2014 software on a personal computer with 3.4 GHz Intel CoreTM i7-6700 processor and 16 Gb of memory.

Figure 2.3 provides the flow chart of the employed algorithm. According to the figure, the algorithm initiates when the required inputs, chromosome representation approach, and population sizes, are defined. Note that all parameters introduced in Section 2.6 will be the input of the GA in the initialization procedure. Moreover, in the initialization procedure, I built three different initial populations introduced as P^1 , $P^2 \& P^3$ with distinctive sizes depending on the size of a network to find the best number of iterations and also to assure I reached the best possible solution. This step is followed by chromosome evaluation, i.e., sorting, using fitness functions. The fitness function introduced as Z_{trimes} is used in a GA to guide the simulation procedure toward optimal solutions. Crossover and mutation procedures are then employed to reproduce more premium chromosomes and to keep the diversity of feasible solutions, respectively. In the next steps, the number of iterations introduced as "MaxItr" is determined by evaluating the points for each initial population in which the fitness function reaches a plateau.



Figure 2.3 – Flowchart of algorithm

Table 2.4, below describes the parameter settings related to the proposed GA. The lower and upper bounds of the possible range of crossover rate are higher than for mutation rate as I will give the chromosomes with higher fitness value a higher chance of reproduction. Note that I provide explanations related to the number of iterations in Section 2.9.4.

Table 2.4 – Settings related to the proposed GA

Parameters	Description
Initial nonulation size	Three different population sizes. Population size varies depending on the
Initial population size	size of network
Mutation rate	Discrete uniform distribution. Range: [0.2, 0.7]
Crossover rate	Discrete uniform distribution. Range: [0.4,0.9]

In the following section, I provide in-depth details related to GA structure, as well as the approach used to deal with nonlinearity of objective functions.

2.9.1 Chromosome generation and representation

In the designed GA, a chromosome length equals the number of links in a network. The cells associated with the randomly selected links from each set of new links, which are considered as the unobserved links, should be set to zero. For the remaining cells of the chromosome, a type of sensor is randomly selected and the number given in each cell represents the sensor type that should be installed on that link. For instance, if the j^{th} cell of a chromosome is equal to f, it means that the sensor type f should be installed on link j. For the initial population, I used the concept of new links to avoid an exhaustive search in generating feasible solutions, while allowing more than one link to be selected from each set of new links in subsequent populations to keep the diversity of possible solutions. The feasibility of solutions is then evaluated considering the budget constraint introduced in Equation (2.21) as well as determining if the matrix of unobserved links, i.e., T_s , is an invertible matrix.

2.9.2 Fitness function

In the proposed GA, Equations. (2.16), (2.17), (2.25) or (2.31) can be used as the fitness function. However, the nonlinearity in these objective functions can contribute to running time of the proposed algorithm. So, I attempted to linearize these functions in order to decrease the running time as much as possible. This process of linearization is explained for Equation (2.17), while a similar approach can be chosen to linearize other objective functions. One of the variables that contributes to the nonlinearity of Equation (2.17) is a_{jj} , which could be -1, 0 or 1. In Equation (2.17), a_{jj} is taken to the power of 2 which reduces the possible values for this variable to be either 0 or 1. I introduced a new binary variable, β_{jj} , that takes the value of 1 if $|a_{jj}|=1$ and 0 otherwise. Equation (2.17) can be rewritten using this new binary variable:

$$Z_{3} = \min\left(\sum_{j \in J} \left(1 - \prod_{f \in F} \prod_{j' \in J} (1 - \beta_{j'j'} y_{fj'} p_{f})\right)\right)$$
(2.33)

Where in Equation (2.33), the term $\prod_{f \in F} \prod_{j' \in J} (1 - \beta_{jj'} y_{jj'} p_f)$ includes the multiplication of two binary variables, i.e., $\beta_{jj'}$ and $y_{jj'}$, that can be replaced with a new binary variable η_{jjf} . By including the variable η_{jjf} , I also need to consider the following constraints:

$$\eta_{jjf} \le \beta_{jj} \qquad \forall j, j \in J, f \in F$$
(2.34)

$$\eta_{jjf} \le y_{fj} \qquad \forall j, j \in J, f \in F$$
(2.35)

$$y_{jj} + \beta_{jj} - 1 \le \eta_{jjf} \qquad \qquad \forall j, j' \in J, f \in F$$

$$(2.36)$$

Equations. (2.34) and (2.35) restrict the value of η_{jff} to be zero when either of y_{ff} or β_{ff} is zero. Moreover, according to Equation (2.36), η_{ff} is equal to 1 when both y_{ff} and β_{ff} are 1. The new form of Equation (2.33) will be as follows:

$$Z_3 = \min\left(\sum_{j \in J} \left(1 - \prod_{f \in F} \prod_{j^* \in J} (1 - \eta_{jjf} p_f)\right)\right)$$
(2.37)

The next step is to linearize the multiplications terms in $\prod_{f \in F} \prod_{j \in J} (1 - \eta_{jjf} p_f)$. I employed the log function to linearize this term and replaced the linearized form in Equation (2.37):

$$\log\left(\prod_{f \in F} \prod_{j \in J} (1 - \eta_{jjf} p_f)\right) = \sum_{f \in F} \sum_{j \in J} \log\left(1 - \eta_{jjf} p_f\right) \quad \forall j' \in J \quad \Rightarrow \\
Z_3' = \min\left(\sum_{j \in J} \left(1 - \sum_{f \in F} \sum_{j \in J} \log\left(1 - \eta_{jjf} p_f\right)\right)\right)$$
(2.38)

Equation (2.38) represents the linear form of Equation (2.17) which means that the minimum value of Z_3 also guarantees the minimum value of Z_3 . Note that I employed the linear form of objective functions to solve the illustrative examples.

2.9.3 Mutation and crossover procedures

The single-point crossover procedure is applied to the proposed GA to increase the chance of reproduction of the chromosomes that stand in a higher rank considering the fitness function. Moreover, the mutation procedure is employed to keep the genetic diversity in the generation of a population. In both the mutation and crossover procedures, the chromosomes are allowed to select links as unobserved links which do not necessarily originate from the set of new links.

2.9.4 Stopping criteria

The generational process in GA is repeated until a termination condition is reached. The common terminating conditions are satisfying predefined criteria, reaching a fixed number of repetitions, reaching the budget cap, and reaching no improvement in the fitness function through successive iterations. In this work, among all the above mentioned conditions, I consider the number of repetitions to terminate the GA. Based on this condition, the GA stops when it reaches the number of repetitions determined as an input. Note that I repeated the GA with different initial populations and defined the number of repetitions as four times the maximum repetition among different populations in which the highest-ranking solution reaches a plateau such that successive iterations no longer produce better results. This approach is a useful method to avoid exhaustive iterations when there is a low probability of reaching a better solution.

2.10 Illustrative examples

This section includes three illustrative cases to examine the applicability of the proposed model for various networks with different topologies and sizes. The first example is the Fishbone network which is already introduced in the motivating example in Section 2.2. The second case, the Sioux Falls network, belongs to the city of Sioux Falls in the state of South Dakota, United States, and is a well-known network in transportation research which has been studied by many scholars (e.g. Ng, 2012; Xu et al., 2016). Finally, to further demonstrate the applicability of the proposed model, I studied the network of the city of Irvine, California in the United States. In this network, I cover the majority of roads in Irvine, especially the central section and west side of the city. This network has been used as a benchmark for solving many problems including the traffic counting location problem (Chootinan et al., 2005; Xu et al., 2016; Zhou and List, 2010), AVI location problem (Fei et al., 2007; Zhou and List, 2010) and OD estimation problem (Chen et al., 2009; Chootinan and Chen, 2011).

In this work, I assumed that two types of sensors can be installed in a network. Table 2.5 shows the probability of failure and cost of two types of sensor. The two sensor types have a different probability of failure depending on the employed technology and price range. The Traffic Detector Handbook identified different sources of failure for loop detectors which is one commonly used type of counting sensor (Klein et al., 2006). According to this handbook, wire breakage due to pavement failure is one of the primary reasons for loop

detector failure⁸. This handbook, as well as the book entitled "Effects of heavy-vehicle characteristics on pavement response and performance" by Gillespie (1993), emphasized the effect of heavy vehicles on pavement failure. As my focus in this work is on the failure of counting sensors, which also include loop detectors, I took into consideration whether or not a link is often traversed by heavy vehicles. The Federal Highway Administration (FHWA) also defined 15 classes of vehicle among which classes 2 through 13 can be considered as heavy vehicle classes including buses and 2-7 axle trucks (Hallenbeck et al., 2014). The 2015 Urban Mobility Scorecard report (2015) stated that heavy vehicles from classes 3 to 13 constitute up to 7% of traffic load in urban areas in the United States. This percentage shows the maximum load of heavy vehicles which can contribute to pavement failure. In this work, for the sake of simplicity, I assumed that if the percentage of heavy vehicles passes a certain threshold of traffic load (i.e., 3.5%), then the probability of sensor failure on that roadway increases correspondingly. This threshold was selected following the logic that if the HVL exceeds 50% of the maximum loads of heavy vehicles (i.e., 7%), then sensor failure increases accordingly.

⁸ Based on a survey of more than 15,000 loop installations in the state of New York.

	Parameter values				
Sensor type	Probability of failure		Cost per sensor (\times 100\$) ⁹		
	HVL <3.5% TL ¹	HVL >3.5% TL			
1 (Basic)	0.5	0.8	120		
2 (Advanced)	0.3	0.6	180		

Table 2.5 – Data related to the sensors used for the illustrative networks

¹TL stands for total traffic load in the urban area

2.10.1 Full link flow observability: Fishbone network

The minimum number of sensors required to be installed on the links of the Fishbone network is 12 to reach full link flow observability as already discussed in the motivating example. If all 12 sensors are selected from sensor type 1, which has a lower cost compared to sensor type 2, then the total budget required for installing these 12 sensors is 1440\$. This means that the budget cannot be less than 1440\$ in order to reach full link flow observability. However, if the budget cap is more than 1440\$ then some of the sensors to be installed on observed links can be selected from the sensor type 2 category which has a higher cost but offers a lower probability of failure.

Figure 2.4 illustrates the results related to the min-max objective function introduced in Equation (2.16-I) that attempts to minimize the maximum probability of missing the link flow inference of unobserved links. As explained in Section 2.5.1.1, when the sensors are assumed to be identical, this objective function is equivalent to minimizing the maximum

⁹ From Table 2.5 on, cost mentioned in this work should be multiplied by 100.

number of observed links required for the link flow inference of an unobserved link. The vertical axis represents the maximum number of observed links that need to be used for the link flow inference of an unobserved link in the Fishbone network. In this figure, I examine the proposed GA for three different populations having a dissimilar number of chromosomes in their initial pool of solutions. According to Figure 2.4, the first population that has a smaller initial population size, i.e. the lower number of chromosomes in the initial pool, takes a larger number of iterations, 88 iterations, to reach to the optimal solution obtained from the second and third populations with fewer iterations. According to the stopping criteria described in Section 2.8.4, I multiplied the maximum number of iterations among all three populations by four to reach a plateau to determine the number of iterations required to terminate the proposed GA. In this case, as this maximum value belongs to the first population, I multiplied 88 by 4 to obtain the number of sufficient iterations required for minimizing the maximum number of observed links used for the link flow inference of an unobserved link in the Fishbone network. Figure 2.5 illustrates the results related to all three populations when the number of iterations is set to 352. I repeated the equivalent procedure introduced in Figures 2.4 and 2.5 for determining the sufficient number of iterations while using different objective functions in the following tables and figures.


Figure 2.4 – The maximum number of observed links required for link inference of an unobserved link in the Fishbone network using different initial populations



Figure 2.5 – The maximum number of observed links required for link inference of an unobserved link in the Fishbone network using the required number of iterations

Table 2.6 shows the results of applying the min-max and min-sum objective functions for the Fishbone network assuming that all sensors are identical. The objective function defined for the first layout of this table is consistent with Equation (2.16-I) and is defined to minimize the maximum number of observed links required for the link flow inference of an unobserved link. For the second layout, the objective function is defined based on Equation (2.16-II) and aims at minimizing the maximum number of unobserved links whose flow cannot be inferred due to the failure of a sensor. For each objective function, the highlighted value in the corresponding row is the objective function value. For instance, the maximum number of unobserved links for which their flow cannot be inferred due to the failure of a sensor is three according to the highlighted result associated with the second layout. The third layout suggested for the Fishbone network in Table 2.6 is based on the objective function introduced in Equation (2.17), and the results of z_1 and z_2 related to the first and the second layouts, respectively are set as the constraints. The results of the third layout demonstrate improvements in the average number of observed links for the link flow inference of each unobserved link as well as the average number of unobserved links whose link flow inference depends on an observed link when the constraints related to z_1 and z_2 are satisfied. Note that the results presented in Table 2.6 using the GA algorithm are achieved in 5 to 7 seconds.

			NT 6 1		No. of unobserved				
Layout No.	Objective function	Set of unobserved links	No. of obser	ved links ¹	links ²				
			Avg	Max	Avg	Max			
1	Z_1	{2,7,8,11,14,17}	3.83	5	1.92	4			
2	Z ₂	{2,7,10,12,13,16}	4.16	7	2.08	3			
	Z ₃ ,								
3	$\begin{cases} Z_1 \le 5\\ Z_2 \le 3 \end{cases}$	{3,6,7,11,12,17}	3.67	5	1.83	3			

Table 2.6 - Suggested layouts for the Fishbone network using identical sensors

¹ Number of observed links required for the link flow inference of an unobserved link

² Number of unobserved links whose link flow inference depends on an observed link

Figure 2.6 illustrates the number of observed links required for the link flow inference of each unobserved link in different sets of unobserved links for each layout introduced in

Table 2.6. According to this figure, unobserved links 2 and 11 require the maximum number of observed links, i.e., five observed links, for their link flow inference in the first layout of Table 2.6. However, in the second layout of this table, the maximum number of observed links required for the link flow inference of the unobserved link 12 increases to seven. The main source of this difference in the maximum number of observed links for the link flow inference of an unobserved link in the first and in the second layouts relates to the objective functions defined for these layouts. In the first layout, the objective function is consistent with minimizing the maximum number of observed links in each equation used for the link flow inference of unobserved links, while in the second layout, the objective function tends to minimize the maximum number of unobserved links whose link flow inference depends on an observed link. For the third layout, the maximum number of observed links in an equation cannot exceed five observed links as it is defined as a constraint for this layout and according to Figure 2.6, only unobserved link 3 requires five unobserved links for its link flow inference. For a particular layout, if there is no bar for a given link in the horizontal axis of Figure 2.6, then it means that this link is not an unobserved link in that layout. For instance, there is no blue or orange bar for link 6 in Figure 2.6, which means that link 6 is not an unobserved link in layouts 1 and 2.



Figure 2.6 – The number of observed links required for link inference of each unobserved link

Figure 2.7 demonstrates a comparison between the first, second and third layouts in Table 2.6 concerning the appearance of observed links in different equations used for the link flow inference of unobserved links. The reason for this comparison is to study the effect of a sensor's failure on the link flow inference of unobserved links. According to this figure, the observed link 16 appears in four different equations which is the highest number of appearances among other observed links in the first layout. In the case of the second layout, observed links 4, 9, and 17 exist in three different equations used for the link flow inference of unobserved links. For the third layout, the highest number of appearances of an observed link cannot exceed the analogous value for an observed link obtained in the second layout, as it is defined as a constraint for this layout. In this layout, the failure of observed links 15 and 16 can have the highest impact on the link flow inference of unobserved links as they appear in three equations. In Figure 2.7, if there is no bar related to a layout for a given link

in the horizontal axis of this figure, then it conveys that this link is not an observed link in that layout. For instance, there is no blue or orange bar for link 2 in Figure 2.7, which means that link 2 is not an observed link in layouts 1 and 2.



Figure 2.7 – The number of unobserved links requiring each observed link for their link flow inference

Table 2.7 presents the location of sensors in the Fishbone network and the type of sensors to be installed in these locations for three different budgets using the min-sum objective function. In this table, I defined three different budgets of 1500\$, 1700\$, and 2000\$. I assumed that the model shouldn't install more sensors (i.e., more than 12 sensors) as the budget increases, but rather allow the employment of more advanced sensors in the network. This is a valid assumption considering the significant installation cost of sensors in a network. I also assumed the heavy vehicle traffic loads are not high enough to affect the sensors' performance. According to this table, as the budget cap increases, the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors decreases because the model can install more advanced sensors (i.e., sensors with lower

failure probability) in the network. For instance, when the budget increases from 1500\$ to 2000\$, the number of links instrumented with sensor type 2 increases from only one link to eight links. Table 2.7 also compares the relationship between the types of sensor installed on an observed link with the number of appearances of that link in equations used for the link flow inference of unobserved links. According to this comparison, the model tends to install type 2 sensors on the observed links with a higher rate of appearance in different equations in order to minimize the expected number of unobserved links where their flow cannot be inferred due to the failure of a sensor. For instance, when the budget cap is set as 1700\$, links 10, 17, and 18, which appear for the link flow inference of three unobserved links, i.e., are present in three different equations, are instrumented with sensor type 2. All links equipped with sensor type 1 have appeared in the link flow inference equations for the maximum of two unobserved links. Note that the results provided in Table 2.7 employing the proposed GA algorithm are obtained in 5 to 9 seconds.

Budget	Set of observed links equip	oed with sensors	OF ¹				No.	of a	ppe	arai	ice (of ob	serve	d lin	ks in s	separ	ate e	quatio	ons		
(× 100\$)	Type 1	Type 2		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1500	{1,3,4,7,8,9,10,13,14,15,18}	{16}	5.38	2	-	1	2	-	-	1	1	2	2	-	-	2	2	3	3	-	1
1700	{1, 2, 4, 5, 8, 9, 12, 14}	{10,16,17,18}	5.08	2	1	-	2	1	-	-	1	2	3	-	2	-	1	-	2	3	3
2000	{3,13,14,17}	{1,4,5,6,9,10,15,16}	4.70	2	-	1	2	1	1	-	-	2	2	-	-	2	2	3	3	1	-

Table 2.7 – Sensor deployment in the Fishbone network for three different budget caps

¹ OF stands for objective function and it represents the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors

2.10.2 Full link flow observability considering major roads: Sioux Falls network

I investigated the effect of considering major roads on the sensor positioning to reach full link flow observability in the Sioux Falls network. The right side of Figure 2.8 provides a map of the Sioux Falls network. This network is surrounded by highway 90 at the top, and highways 29 and 229 on the left and right sides, respectively. These highways are redcolored, so they can be easily distinguished from the minor roads in the network. I considered these highways as the major roads in this network and assigned a higher weight to them. The left side of Figure 2.8 demonstrates the graphical illustration of this network and employs a color-coding analogous to the right side of the figure to differentiate the major from the minor roads. Moreover, similar to Xu et al. (2016), I did not specify centroid nodes for the Sioux Falls network. This omission of centroid nodes can be justified by long-term counting as indicated by Ng (2013). Ng (2013) argued that centroid nodes can be ignored when traffic count information is collected over an entire day since the origin and destination nodes are interchangeable in the case of daily counts. In other words, a person who leaves home to go to work will return home later. According to this assumption, there are 24 non-centroid nodes in the Sioux Falls network, as well as 76 links which connect these non-centroid nodes. I provide a table that includes the sets of new links associated with the Sioux Falls network in Appendix III. According to Appendix III, out of 24 non-centroid nodes, 23 have non-empty sets of new links, and based on the instruction provided in the concept of new links, from each non-empty set of new links, one link should be selected as an unobserved link. This

means that 23 out of 76 links are selected to not be equipped with sensors in the Sioux Falls network and that the rest of the links should be instrumented with sensors.



Figure 2.8 – Graphical illustration of the Sioux Falls network

To begin with, I construct the set of initial solutions using the original sets of new links using Equation (2.25) as the objective function and Equations (2.18-2.21) as the constraints, while the weight value of major roads is set as 1 and the rest of the roads are given a weight of 0.5. Table 2.8 shows the original sets of new links associated with the non-centroid nodes of the Sioux Falls network. Note that the results using this approach are presented under "*Scenario 1*".

Considering manual assignment of sensors to major roads, I updated the set of new links under two different scenarios named "*Scenario 2*" and "*Scenario 3*". In Scenario 2, I removed the links in the set of major roads from each set of new links and assigned sensors to these links (Please see the column labeled Scenario 2 in Table 2.8). This elimination of major roads results in seven sets out of 23 sets of new links becoming empty sets. Therefore, to construct the set of unobserved links, I need to select more than one link from some of the sets of new links which could result in the matrix of unobserved links becoming a singular matrix. To avoid the creation of a singular matrix, the non-zero determinant condition for the matrix of unobserved links enforced in Equation (2.24) needs to be satisfied. Note that in applying Scenario 2, I ran the proposed GA to generate the initial population, but the GA didn't manage to reach any feasible solution in which the matrix of unobserved links is invertible. This outcome occurred despite using different sizes for the initial population and a considerable number of attempts¹⁰ to generate the initial population, up to 10⁵ attempts.

According to Scenario 3, I attempted to remove the major roads from the sets of new links in a way to avoid these sets becoming empty sets (Please see the column labeled Scenario 3 under 'Updated sets of new links' in Table 2.8). Therefore, under this scenario, I can still use the concept of new links to generate feasible solutions. Using the updated sets of new links in Scenario 3, I could select up to 17 links from the set of major roads to be instrumented with sensors in initial solutions (Please see the last column labeled Scenario 3 in Table 2.8). However, the number of major links to be equipped with sensors can increase

¹⁰ The phrase "number of attempts" refers to the number of times the GA attempted to generate the initial population.

in the subsequent feasible solutions generated by the GA. I then employed the objective function and the constraints introduced in Section 2.6.2 to minimize the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors. Note that for all scenarios, I assumed that the budget is enough to afford 24 sensors of type 2 and 29 sensors of type 1.

		Cata of more limba	Undoted ast		Major roads rei	noved from the
Node	Connected links	Sets of new links	Updated set	s of new links	sets of ne	ew links
		Scenario 1	Scenario 2	Scenario 3	Scenario 2	Scenario 3
1	1,2,3,5	{1, 2, 3, 5}	-	{1}	1,2,3,5	2,3,5
2	1,3,4,14	{4,14}	-	{4}	4,14	14
3	2,5,6,7,8,35	{6,7,8,35}	{6,8}	{6,8}	7,35	7,35
4	6,8,9,10,11,31	{9,10,11,31}	{9,10,11,31}	{9,10,11,31}	-	-
5	9,11,12,13,15,23	{12,13,15,23}	{12,13,15,23}	{12,13,15,23}	-	-
6	4,12,14,15,16,19	{16,19}	-	{16}	16,19	19
7	17,18,20,54	{17,18,20,54}	-	{17}	17,18,20,54	18,20,54
8	16,17,19,20,21,22, 24,47	{21, 22, 24, 47}	{21, 22, 24, 47}	{21, 22, 24, 47}	-	-
9	13,21,23,24,25,26	{25, 26}	{25, 26}	{25, 26}	-	-
10	25,26,27,28,29,30,32,4 3,48,51	{27, 28, 29, 30, 32, 43, 48, 51 }	$ \begin{cases} 27, 28, 29, 30, \\ 32, 43, 48, 51 \end{cases} $	{27, 28, 29, 30, 32, 43, 48, 51 }	-	-

Table 2.8 – Original and updated sets of new links related to the Sioux Falls network

11	10,27,31,32,33,34,36,4 0	{33, 34, 36, 40}	{33, 34, 36, 40}	{33, 34, 36, 40}	-	-
12	7,33,35,36,37,38	{37, 38}	-	{37}	37,38	38
13	37,38,39,74	{39,74}	-	{39}	39,74	74
14	34,40,41,42,44,71	{41, 42, 44, 71}	{41, 42, 44, 71}	{41, 42, 44, 71}	-	-
15	28,41,43,44,45,46,57,6 7	{45, 46, 57, 67}	{45, 46, 57, 67}	{45, 46, 57, 67}	-	-
16	22,29,47,48,49,50, 52,55	{49, 50, 52, 55}	{49, 50, 52, 55}	{49, 50, 52, 55}	-	-
17	30,49,51,52,53,58	{53, 58}	{53,58}	{53,58}	-	-
18	18,50,54,55,56,60	{56,60}	-	{56}	56,60	60
19	45,53,57,58,59,61	{59,61}	{59,61}	{59,61}	-	-
20	56,59,60,61,62,63, 64,68	{62, 63, 64, 68}	<i>{</i> 63, 68 <i>}</i>	{63, 68}	62,64	62,64
21	62,64,65,66,69,75	{65, 66, 69, 75}	{65,69}	<i>{</i> 65, 69 <i>}</i>	66,75	66,75
22	46,63,65,67,68,69, 70,72	{70,72}	{70,72}	{70,72}	-	-
23	42,70,71,72,73,76	{73,76}	{73,76}	{70,72}	-	-
24	39,66,74,75	-	-	-		

Figure 2.9 demonstrates the objective function value, i.e., Equation (2.25), using the proposed GA under Scenarios 1 and 3 in 1,410 iterations. For each scenario, I defined three different initial populations with distinctive population sizes. Figure 2.9 shows that both

scenarios converge to nearly similar results, while the larger population sizes perform slightly better in minimizing the objective function.



Figure 2.9 – Results of implementing Scenarios 1 and 3 with different populations for the Sioux Falls network

To investigate the effect of considering major roads on the layout of sensors, I used Equations. (2.17) and (2.25) as the objective functions to observe the possible differences in suggested layouts of sensors in the Sioux Falls network. Figure 2.10 illustrates two optimum layouts of sensors in this network using different objective functions. In layout A, the objective function is as Equation (2.17) to minimize the expected number of unobserved links where their flow cannot be inferred due to the failure of sensors. In layout B, a similar

goal is pursued using Equation (2.25) but also incorporates the relative importance of major roads in finding the location of sensors while using the original sets of new links to generate initial solutions. Figure 2.10 demonstrates that in layout B, the GA assigns more advanced sensors, i.e., sensors with a lower probability of failure, to the major roads. It also minimizes the number of major roads to be included in the set of unobserved links. The reason for this minimization is due to the fact that the probability of missing the link flow inference of an unobserved link is usually higher than the probability of missing the link flow observation of that link as an observed link if more than one link is required for the link flow inference of that link. Note that the results of sensor deployment depicted in Figure 2.10 by employing the proposed GA are achieved in 15 to 25 seconds.



Figure 2.10 - Two optimum layouts of sensors with and without consideration of major roads

Figure 2.11 evaluates the number of major links equipped with sensor type 2, i.e., advanced sensor, for the suggested layouts using Equations (2.17) and (2.25) in each iteration of GA. The initial populations with respect to major roads are generated under Scenarios 1 and 3. In initial populations, 7 links (Scenario 1) and 17 links (Scenario 2) from the set of major roads are instrumented with the sensor type 2. However, the number of links equipped with sensor type 2 converges to the same value under these scenarios as the iterations proceed. According to this figure, through 1,410 iterations, the layouts provided by the consideration of major roads using Equation (2.25) attempt to assign on average more sensors of type 2 to the major roads. For instance, for the last 900 iterations, out of 24 major roads in the Sioux Falls network, on average 20 links are instrumented with sensor type 2 using Equation (2.25) for both Scenarios 1 and 3, while this average drops to 16 links when the concept of major roads is not applied.



Figure 2.11 – Number of major links instrumented with sensor type 2 in Sioux Falls network

Figure 2.12 shows the number of major roads to become unobserved links in layouts generated by the proposed GA under Scenarios 1 and 3 with either the consideration of major roads or not. According to this figure, the number of major roads included in the set of unobserved links when the major links are considered is 25% lower in the last 800 iterations compared to the situation when all links are assumed to be identical.



Figure 2.12 – Number of major links to be included in the set of unobserved links

2.10.3 Full link flow observability considering major roads and heavy vehicle loads: Sioux Falls network

To assess the effect of considering major roads on the layout of sensors, I implemented the proposed model in the Sioux Falls network using Equation (2.25) as the objective functions as well as the updated probability of failure of sensors under high load of heavy vehicles introduced in Table 2.5. I considered the roads going to the depot areas located in the northwest and southwest sections of Sioux Falls as the roads with high HVL (i.e., HVL > 3.5% TL). Figure 2.13 illustrates the optimum layout of sensors, introduced as layout C, in this network using 24 sensors of type 2 and 29 sensors of type 1. This combination of sensors is similar to the one I used in Section 2.10.2. In Figure 2.13, there

are 20 links under high loads of heavy vehicles, indicated by the red double backslashes. In the layout shown, more than 50% of links (i.e., 11 links) with high HVL are equipped with the sensor type 2 and in general, 70% of links (14 links) with high HVL are instrumented with sensors.



Figure 2.13 – Optimum layout of sensors with consideration of major roads and heavy vehicle loads

Table 2.9 evaluates the effect of considering the high HVL on sensor positioning in the Sioux Falls network. In this table, layout B (introduced in Figure 2.10) only considers the major roads, while layout C (depicted in Figure 2.13) considers both the major roads and roads with high HVL (i.e., HVL > 3.5% TL) in the Sioux Falls network. Comparing observed links in layouts B and C in Table 2.9, I see there is a 16.7% increase in deployment of sensor

type 2 on the major links with high HVL in layout C. This means that in considering the effect of HVL on sensor failure, the proposed model attempts to install more advanced sensors on the major links which are also subject to high HVL, in an effort to decrease the adverse effect of high HVL on the link flow observability of observed links and on the link flow inference of unobserved links. Moreover, there is a considerable increase (i.e., 50%) in the number of sensors of type 2 deployed on non-major links with high HVL in layout C compared to layout B. This increase indicates that the model makes a similar effort to instrument links with high HVL with more advanced sensors when these links are not among major links. Eventually, exploring unobserved links in each layout, I found that the number of unobserved links from the set of major links with high HVL decreases in layout C. This is due to the fact that the probability of missing the link flow inference of an unobserved link is usually higher than the probability of missing the flow observation of an observed link; therefore, the proposed model attempts to minimize the number of major links with high HVL in the set of unobserved links. Note that the runtime for achieving the results presented in Table 2.9 is 17 to 19 seconds depending on the size of the initial population for the Sioux Falls network in the proposed algorithm.

Table 2.9 – Sensor deployment considering major links with and without high HVL in Sioux Falls network

		Observed lin	Unobserved links							
Layout	Major links ar	nd HVL>3.5% TL	HVL>	3.5% TL	Major links and HVL>3.5% TL	HVL>3.5% TL				
	Type 1	Type 2	Type 1 Type 2							
В	0	6	8	0	2	4				
С	0	7	3	4	1	5				

2.10.4 Full link flow observability in large networks: Irvine network

In this section, I evaluated the performance of the proposed model for a relatively large network. To do so, I selected the Irvine network shown in top part of Figure 2.14 and also is graphically demonstrated in this figure. The major roads, which mainly comprise the north-south and east-west bound highways, are highlighted in red in the graphical illustration of this figure. The extracted network consists of 162 non-centroid nodes, 496 links, 39 traffic analysis zones (TAZs), and 28 external stations, i.e., there are 67 centroid nodes (28+39)¹¹. In Figure 2.14, 112 out of 496 links are marked as major roads.

¹¹ These data were extracted from the Orange County Transportation Analysis Model



Figure 2.14 – The real map (source google map) and graphical illustration of Irvine network

For the Irvine network, I compared the results of implementing the proposed model to the results obtained by Xu et al. (2016). Xu et al. (2016) reduced the flow variance measurement of unobserved links by minimizing the number of observed links required for flow variance measurement of each unobserved link. In their approach, they attempt to minimize the number of unobserved links connected to each non-centroid node in order to minimize the number of observed links required for flow variance measurement of each unobserved link. As the number of observed links required for variance measurement of unobserved links can be also employed to infer the flow of unobserved links, Xu et al.'s (2016) approach, despite its differences, can be compared to the model proposed in this work. However, to make a comparison, I need to assume that all sensors are identical and that there is no major road in the network. Figure 2.15 shows the average number of observed links required to infer the flow of unobserved links, i.e., Equation (2.10). According to the algorithm procedure explained in Section 2.9, I implemented the proposed model with three different population sizes and 9,136 iterations¹². The horizontal axis in Figure 2.15 depicts the runtime in seconds. According to this figure, the maximum time it takes to reach the optimum value i.e., minimum of the average number of observed links needed to infer the flow of each unobserved link, among all populations, is 8,207 seconds (2.27 hrs). In Figure 2.15, I also marked the time it takes the proposed model to reach the optimum value of Equation (2.10) obtained by Xu et al. (2016). According to Figure 2.15, the maximum time it takes the proposed model to reach this value is 750 seconds. In comparison with Xu et al.'s

¹² The number of iterations was obtained according to the method outlined in Section 9.1.1

(2016) work, the proposed model can successfully decrease the average number of observed links required to infer the flow of unobserved links, although, I was not able to compare the runtime with Xu et al. (2016) as it was not reported in their work.



Figure 2.15 – Results of implementing the proposed model with Equation (2.10) as the objective function in the Irvine network

Table 2.10 shows some of the potential differences in sensor positioning in the Irvine network using the proposed model and the model developed by Xu et al. (2016). This table indicates the number of unobserved links connected to each non-centroid node in the Irvine network for both Xu et al. (2016) and the proposed model. As presented in Table 2.10, Xu et al. (2016) used two objective functions, the min-max and min-sum functions. Their min-sum objective function attempts to minimize the summation of the number of unobserved links

connected to each non-centroid node and their min-max function minimizes the maximum number of unobserved links connected to a non-centroid node. Considering the number of unobserved links connected to each non-centroid node, I can observe that compared to Xu et al. (2016), the proposed model increases the number of non-centroid nodes connected to only one unobserved link. In addition, the number of non-centroid nodes linked to four unobserved links is considerably larger in the results associated with the proposed model. This analysis leads us to the conclusion that the objective functions introduced in Xu et al. (2016) can be modified to focus only on the non-centroid nodes connected to one unobserved link instead of all non-centroid nodes which may attach to more than one unobserved link.

	Objective	# of unobserved links connected to non-									
Model	function		centroid i	iode(s)							
		one	two	three	four						
Xu et al. (2016)	Min-max	64	78	20	0						
	Min-sum	86	56	19	1						
Proposed model	Equation (2.	113	23	22	4						
-	(10)										

 Table 2.10 – Number of unobserved links connected to each non-centroid node

In Table 2.11, I implemented the proposed model in the Irvine network, considering three different sensor assignment scenarios for major roads in the network. In the first scenario, I assigned weights to links and solved the model employing Equation (2.25). On the other hand, in the second and third scenarios, similar to what I implemented in Table 2.8 for the Sioux Falls network, I tried to manually assign advanced sensors in the initial

population of the proposed GA. Similar to the results obtained for the Sioux Falls network, I could not generate a feasible initial population using Scenario 2. However, I can successfully implement Scenarios 1 and 3 and the results are presented in Table 2.11. As shown in Table 2.11, out of 112 major links, 92 (82.1%) are equipped with a more advanced sensor, i.e., sensor type 2, and only 11.6 % of all major links are in the set of unobserved links. Comparing the runtime for Scenarios 1 and 3, I observed that the manual assignment of sensors, while not very practical for large networks, could lead to lower runtime. The reason behind the reduction in runtime under Scenario 3 may relate to the higher number of major links equipped with more advanced sensors in the initial population of Scenario 3 compared to Scenario 1, which facilitates the convergence speed to the optimum solution.

Table 2.11 – Sensor installat	tion in	the Irv	vine netw	ork co	nsidering	major	road	S
-------------------------------	---------	---------	-----------	--------	-----------	-------	------	---

		ks		
Scenario	Observed	l links	Unobserved links	Runtime
	Type 1	Type 2		
Scenario 1	7	92	13	7460
Scenario 3				7133

2.10.5 Full link flow observability with route flow information: Fishbone network

In Section 2.7, I suggested a two-level optimization model to consider the effect of route flow information on the assessment of sensor layouts which lead to full link flow observability. According to Section 2.7, the optimum solutions for full link flow observability should be obtained in the first level optimization. The second level optimization then minimizes the information loss from all routes that provide unique information about

route flows, while guaranteeing full link flow observability and not worsening the objective function value reached in the first level optimization. In this section, I used the Fishbone network to showcase how the two-level optimization model can be implemented, while setting the budget cap as 1700 and assuming the load of heavy vehicles is less than 3.5 % of total traffic load. Following the budget constraint and HVL assumption, Table 2.7 demonstrates only one of the possible optimum layouts of sensors in the Fishbone network. I reapplied the proposed model to find one other possible optimum layout of sensors which results in the same objective function value. Figure 2.16 depicts two optimum sensors layouts in the Fishbone network in which green and blue arrows represent the links equipped with sensor types 1 and 2, respectively. In this figure, layout C is the sensor layout introduced in Table 2.7 and layout D is another possible layout of sensors, with the objective function value for both layouts equal to 5.08. Note that for the Fishbone network, the number of possible optimum layouts which constitute the pool of feasible solutions in the second level optimization was more than two. However, I displayed only two possible layouts to easily describe the procedure for finding the sensor layouts that not only minimize the effect of sensor failure on link flow inference of unobserved links, but also minimize the effect of sensor failure on route flow information gain.



Figure 2.16 – Two optimum layouts of sensors in Fishbone network

Table 2.12 represents four ODs and eight identified routes¹³ between these ODs in the Fishbone network. All of these routes belong to sets R^2 and R^3 introduced in Section 2.7. Routes 1, 5 and 8 belong to R^2 as the set of links {9}, {6,10} and {4,8} are only traversed by these routes, respectively, while the other routes belong to R^3 . All of these routes can contribute to information gain, as the column vectors pertaining to these routes in the linkroute incidence matrix are linearly independent. Table 2.12 also presents the links traversed by each route and instrumented with sensors in layout C or D. The last column of the table shows the probability of missing the route flow observability of each route if sensor positioning follows layout C or D. For instance, route 4 traverses links 1, 5, 11, 15, and 18. Among these links, the set of links {1,5,18} and {1,15} are equipped with sensors in layouts C and D, respectively. The probability of missing the route flow observability of route 4 depends on the *number of sensor-instrumented links traversed by this route* as well as *the*

¹³ For simplicity, in Table 2.12 I introduce only some of all the possible routes between each OD as the set of identified routes

sensor type installed on these sensor-equipped links. In layout C, as both links 1 and 5 are equipped with sensor type 1, and link 18 is instrumented with sensor type 2, then the probability of missing the route flow observability of route 4 equals 0.075^{14} ($0.5^2 \times 0.3 = 0.075$). In layout D, this probability equals 0.15, as links 1 and 15 are instrumented with sensor types 1 and 2, respectively ($0.5 \times 0.3 = 0.15$). Adding the probability of missing the route flow observability of all routes in layouts C and D, I can obtain the objective function value of the second level optimization introduced in Equation (2.31). By doing so, I observe that although having the same objective function value in the first level, the objective function for layouts C and D differs in the second level of the two-level optimization model. In fact, the expected number of routes for which flow will be missed due to the failure of sensors in layout C equals 0.86225, and is higher than the similar expected value for layout D. This means layout D offers more robust sensor positioning compared to layout C if the link-route incidence matrix information is available.

		Sot of a	heared line		Possibility of missing				
OD	Set of involved links in each	Set of 0	observed inn	route flow observability					
	route	C		Γ)	C	D		
		Type 1	Type 2	Type 1	Type 2	C	D		
1-9	Route 1: $\{1, 9, 15, 17\}$	{1,9}	{17}	{1,9}	{15,17}	0.075	0.0225		
	Route 2: {2,12,14,15,17}	{2,12,14}	{17}	{2,14}	{15,17}	0.0375	0.0225		

Table 2.12 –	 Identified 	routes	between	each	OD	and	links	traversed	by	these	routes	in	the	Fishbone
network														

¹⁴ This probability is calculated assuming that heavy vehicle load (HVL) is less than 3.5% of total traffic load

	Route 3: {1, 5, 11, 15, 17}	{1,5}	{17}	{1}	{15,17}	0.075	0.045
1-10	Route 4: {1, 5, 11, 15, 18}	{1,5}	{18}	{1}	{15}	0.075	0.15
	Route 5: {2, 6, 10, 16, 18}	{2}	{10,16,18}	{2,6}	{10,16}	0.0135	0.0225
2-9	Route 6: {3,11,15,17}	-	{17}	-	{15,17}	0.5	0.09
	Route 7: {3,12,14,15,17}	{12,14}	{17}	{14}	{15,17}	0.075	0.045
2-10	Route 8: {4,8,12,16,18}	{4,8,12}	{16,18}	{4}	{16}	0.01125	0.15
					Σ	0.86225	0.5475

2.10.6 Redundant sensors: Fishbone network

The concept of redundant sensors is mainly developed in this work to investigate the location of additional sensors in a network to maintain the full link flow observability when certain sensors already installed on observed links stop functioning. According to the stepwise approach introduced in Section 2.8, I first need to determine the initial location of sensors in a network to be able to consider the combination of failures among them in the second step. In the third step, the location of redundant sensors should be selected for each combination of failure. Finally, in the fourth step, taking into account budget constraints, the sensors are assigned to the locations identified in the third step.

To make it easier to demonstrate how the concept of redundant sensors can be applied to a network, I only consider the Fishbone network to be equipped with redundant sensors. The initial location of sensors is assumed to be according to the third layout determined in Table 2.7. I also assume that all the redundant sensors are identical and can be independently installed in a network. Table 2.13 shows the set of observed links according to the third layout of Table 2.7 as well as the failure probability of the sensors installed on these links. This table also shows the location of redundant sensors while considering the failure of only one sensor among all sensors located on observed links. To obtain the location of redundant sensors, I used Equation (2.17) but dropped the binary variable y_{ij} and the parameter p_f as I assumed all sensors are identical. In total, six links can be nominated for installing the redundant sensors, including links 2, 7, 8, 11, 12 and 18, as these links were not initially instrumented with sensors according to the third layout of Table 2.7.

Links suggested for installing redundant sensors in Table 2.13 are based on the assumption that out of twelve links instrumented with sensors, the sensor installed in only one of these links breaks down. The information provided in Table 2.13 can also be used to determine the expected number of times a link will be selected for installing the redundant sensors. For instance, link 18 is selected to be equipped with redundant sensors if the sensors installed in link 15, 16 or 17 stop working. The probabilities of failure of the sensors installed in these links, i.e., links 15, 16 and 17, are 0.3, 0.3, and 0.5, respectively. Therefore, the expected number of times that link 18 is selected to be instrumented with a redundant sensor is the summation of 0.3,0.3, and 0.5 which is equal to 1.1.

Table 2.13 –	- Possible	location	of redundant	t sensors	with	only o	one se	ensor	failure	among	all	sensors
installed on o	observed li	inks										

Set of observed links	Sensor failure		New observed link
	link location	probability	
{1,3,4,5,6,9,10,13,14,15,16,17}	1	0.3	7
	3	0.5	2

4	0.3	8
5	0.3	7
6	0.3	8
9	0.3	7
10	0.3	8
13	0.5	12
14	0.5	12
15	0.3	18
16	0.3	18
17	0.5	18

In line with the third step introduced in Section 7, Table 2.14 shows the set of links selected to be equipped with redundant sensors for a different combination of failures. This tables also determines, for each combination of sensor failure, the percentage of links missing full link flow observability even with redundant sensors installed in the Fishbone network. For instance, considering a two sensor failure out of twelve installed sensors, I can observe that in one out of the 66 possible combinations of failure, the full link flow observability is impossible even with the installation of redundant sensors. This situation occurs when sensors installed on links 13 and 14 are assumed to break down and these links become unobserved links. As links 13 and 14 are bidirectional links, the column vector associated with these links is linearly dependent. Therefore, the corresponding matrix of unobserved links is not invertible and the system of linear equations for the link flow inference of unobserved links is not determined. Table 2.14 also indicates that as the number of failures increases, the chance of missing the full link flow observability increases correspondingly.

According to the fourth column of Table 2.14, for a certain number of failures, there might be more than one set of links to be equipped with redundant sensors. For instance, when it is assumed that three of twelve sensors installed on observed links break down, then two sets of links, including sets $\{2,7,18\}$ and $\{2,8,18\}$, have an equal chance of being selected as the set of links to be instrumented with redundant sensors. The last column of Table 2.14 demonstrates the expected number of times each set of links will be selected.

Table 2.14 – Results related to redundant sensors in the Fishbone network

Combination of sensor failures	Total possible combinations	% missing full link observability	Set of links to be equipped with redundant sensors	Expected number of selections
One failure	$\binom{12}{1} = 12$	0%	{18}	1.1
Two failures	$\binom{12}{2} = 66$	$\frac{1}{66}$ (1.5 %)	{7,18}, {8,18}	0.99
Three failures	$\binom{12}{3} = 220$	$\frac{14}{220}$ (6.4%)	{2,7,18}, {2,8,18}	0.77
Four failures	$\binom{12}{4} = 495$	$\frac{82}{495}$ (16.6%)	{2,7,8,18}	0.74
Five failures	$\binom{12}{5} = 792$	278 792 (35.1%)	{2,7,8,12,18}	0.67
Six failures	$\binom{12}{6} = 924$	$\frac{601}{924}$ (65%)	{2,7,8,11,12,18}	0.63

In addition to finding the expected number of selections for each set of links to be equipped with redundant sensors in Table 2.14, I show the expected number of selections for individual links considering different combinations of failure in Figure 2.17.

In Figure 2.17, I show the expected number of selections for links, including links 2, 7, 8, 11, 12 and 18, to be instrumented with redundant sensors considering different combinations of failure. The horizontal axis of this figure shows different combinations of failure and the vertical axis represents the expected number of selections associated with each individual link. For instance, according to Table 2.14, in the case of two failures, each of sets $\{7,18\}$ and $\{8,18\}$ have the highest expected number of selections, i.e., 0.99, compared to other sets to be instrumented with redundant sensors. For individual links, the expected number of selections of link 18 is 5.995 and the highest among other links in the case of two failures. The difference in this expected number of selections can be explained by the fact that link 18 might be available in other sets with an expected number of selections that is not as high as it is for sets $\{7,18\}$ and $\{8,18\}$. Therefore, the last column of Table 2.13 does not count the total number of times that link 18 is selected to be equipped with redundant sensors except for the cases when one failure or six failures occur. In these cases, the expected frequency of selection in Figure 2.17 and in the last column of Table 2.14 match as they include only one link (i.e., in case of one failure) or all links (i.e., in the case of six failures) that can be instrumented with redundant sensors.

According to Figure 2.17, link 18 has the highest expected number of selections in each combination of failure. Therefore, according to the fourth step introduced in Section 7, if budget constraints allow, link 18 should be the first link to be equipped with redundant

sensors. Links 8 and 7 are in the second and the third ranks, respectively when comparing the total expected number of selections for all combinations of failure. Note that the expected frequency of selection for a certain link through the different combination of failures represents an irregular pattern, i.e., increasing or decreasing pattern, as it depends on the location of initial sensors, the number of failed sensors, as well as their probability of sensor failure.



Figure 2.17 – Expected number of times a link is selected to be instrumented with sensors

2.11 Discussion on the effectiveness of the proposed model for partial link flow observability

Full link flow observability can be considered as the *ideal* case for link flow observability in a network as the flow of all links of that network can be directly observed or indirectly inferred. However, in the real world, it might not be economically possible to

install enough sensors to reach full link flow observability. Instead, the partial link flow observability problem addresses the situation where there is a smaller number of sensors than the number required to reach full link flow observability. Researchers have introduced various definitions of partial flow observability and more specifically, the partial link flow observability in a network (Viti et al. 2014). Castillo et al. (2011) defined the partial link flow observability problem as the problem of finding a subset of links in a network that should be equipped with sensors in order to make it possible to infer the link flow of a certain number of unobserved links. For instance, in the Fishbone network, the number of unobserved links equals six. A problem which attempts to find the location of sensors in a way to infer the flow of five or a smaller number of unobserved links, is a partial link flow observability problem. Gentili and Mirchandani (2012) defined the number of unobserved links for which flow can be inferred using the information obtained from observed links as *h*, and indicated that partial link flow observability can be studied under different values of h, i.e., different levels of observability. There are other studies that address partial link flow observability when there is a given number of sensors (He, 2013; Ng, 2012). Moreover, to tackle the partial link flow observability problem from a different perspective, Hu et al. (2009) and Ng (2013) address the problem of finding the minimum number of sensors required to reach full link observability when there are already some sensors installed in a network. Concerning the definition proposed by Gentili and Mirchandani (2012), I can also address partial link flow observability using the proposed model. Specifically, as I studied the full link flow observability problem in a network considering the possible failure of sensors, my focus was to minimize the chance of decrease in the level of h if sensor failure
occurs. For instance, Equation (2.17) defined in Section 2.6 minimizes the effect of a sensor failure on link flow inference of unobserved links. In other words, this objective function attempts to assign more advanced sensors on links which appear in more equations required for link flow inference of unobserved links. Accordingly, the chance of reduction in the value of *h* decreases as more advanced sensors, which have lower failure rates, are installed on links with a higher appearance in equations used for link flow inference of unobserved links. For illustration, I selected the sensor location in the Fishbone network associated with layout C introduced in Figure 2.16, to illustrate the possible decrease in the level of *h* if the sensor installed on links 10, 16, 17 and 18 could have the highest impact on the level of observability as their failure can decrease the level of observability from h=6 to h=3. As I can observe in Figure 2.18, the model assigned more advanced sensors (i.e., sensor type 2) on these links to minimize the adverse effect of sensor failure on the level of observability.



Figure 2.18 – The effect of a sensor failure on the level of observability

From a different perspective, the partial link flow observability concerns maximizing the *observability level* for a given number of sensors. Using the proposed model, I can still maximize the observability level, but also minimize the adverse effect of sensor failure on link flow inference of unobserved links. The concept of new links can help me to determine the number of sensors required to reach full link flow observability in a network. Specifically, using the concept of new links, I can determine the number of sensor deficiencies that leads to not enough sensors to reach full link flow observability in a network. As my proposed model is designed to guarantee full link flow observability using an adequate number of sensors, I can adjust the deficiency in the number of sensors required to reach full link observability in a network with some *buffer sensors* that have a failure probability of $1-\varepsilon$ where ε is a very small value. Buffer sensors are not *available sensors* but they make it possible to employ the model to determine the location of available sensors. I employed the Fishbone network to show how the model can be employed for partial link flow observability. Table 2.15 presents the results of sensor location suggested by the model when, for simplicity purposes, all available sensors are assumed to be identical.

I used Equation (2.17) to find the location of sensors, removing the binary variable y_{fi} and the parameter p_f from this equation as I assumed all sensors are identical. In the table, I considered different numbers of available sensors, all of which are less than the number of required sensors, i.e., 12 sensors, to reach full link flow observability. Table 2.15 also evaluates the average number of appearances of observed links instrumented with either available or buffer sensors in equations used for the link flow inference of unobserved links. According to this table, the average number of appearances of available sensors is always higher than the buffer sensors. In the last column of the table, the effect of available sensors on the level of observability, i.e., h, is studied. In this column, I considered the appearance of observed links equipped with buffer sensors in equations required for the link flow inference of unobserved links. If, among the observed links required for the link flow inference of an unobserved link, there is at least one observed link instrumented with buffer sensors, I considered that the flow of that unobserved link cannot be inferred. Comparing the first and the last column of Table 2.15, I see that the model attempts to minimize the effect of a decrease in available sensors on the level of observability. For instance, when there are three deficient sensors in the number of sensors required to reach full link flow observability in the Fishbone network (i.e., there are nine available sensors), the level of observability

decreases to h=4, which means that the flow of only two unobserved links cannot be inferred,

compared to the full link observability condition where h=6.

# of available sensors	Set of links equipped with buffer sensors	Avg. appearance of observed links equipped with buffer sensors	Avg. appearance of observed links equipped with available sensors	Level of observability (h value)
11	{1}	1	1.9	5
9	{2,4,6}	1.3	2	4
7	{2,12,14,16,18}	1.8	2	2

Table 2.15 – Different levels of partial link flow observability considering buffer sensors in the Fishbone network

In Section 2.7 of this study, I discussed the possibility of using the route flow information for evaluating the sensor deployments that lead to full link flow observability. A similar approach can be employed in the model to consider taking advantage of route flow information for the partial observability problem. This means that considering the available sensors and buffer sensors, the model should minimize the effect of available sensor failure on the link flow inference of unobserved links (first level optimization), while attempting to also minimize the effect of sensor failure on route flow information gain of independent routes, i.e., routes in $R^1 \cup R^2 \cup R^3$. Although it is beyond the scope of this work, I believe that combining the problem of partial link flow observability, as well as route flow observability, into a single level optimization model, taking into account the failure of sensors, represents an interesting topic for future research.

2.12 Discussion

. This chapter investigated how the effect of sensor failure on the link flow inference of unobserved links can be considered in identifying the set of observed links in a traffic network. Two types of sensors (i.e., basic and advanced) with different failure probabilities were studied. Methodologically, I considered two contributing factors in determining the location of sensors, namely the probability of missing the link flow inference of unobserved links due to the failure of sensors, as well as the effect of sensor failure on the link flow inference of unobserved links. These two factors are defined as the objective functions in the form of min-max and min-sum functions. I also combined these functions by considering the min-sum function as the objective function while setting a cap for the maximum value of the min-max function. I then proposed GA as a well-known heuristic to solve this problem. To generate the initial solution in the proposed GA, I suggested using the concept of new links to avoid the exhaustive search for constructing the sets of unobserved links. I applied the developed model for three numerical examples including the Fishbone, Sioux Falls, and Irvine networks. The results related to the Fishbone network indicated that the combined objective function formulation leads to better local optimal solutions compared to the situation when the min-sum and min-max functions are employed separately. I further explained how the effect of the above-mentioned factors can be cohesively addressed by minimizing the number of observed links required for the link flow inference of each unobserved link when the sensors are assumed to be identical. However, in the case of nonidentical sensors, the installation of more advanced sensors is required to compensate for the larger number of observed links required for the link flow inference of that unobserved link.

To take into consideration the fact that all links are not equally significant in a network, I also differentiated between major and minor roads to be instrumented with non-identical sensors in a traffic network. I suggested two different approaches to equip major and minor roads with sensors in reaching full link flow observability. The first was to manually equip links in the set of major roads with sensors while also studying the cases where the manually instrumented major links with sensors result in the matrix of unobserved links becoming a singular matrix. The second approach was to keep the constraints already applied to the identical sensors in place while updating the objective function to consider the major roads by incorporating the weights to signify the relative importance of major roads. The results of considering the major roads in identifying the location of sensors in the Sioux Falls network implied that the model attempts to assign more advanced sensors to major roads and to not include these roads in the set of unobserved links. As HVL can influence the failure rate of sensors, I also considered the effect of HVL on major and minor roads in determining the location of sensors in a network. The results of this assessment in the Sioux Falls network suggest that the model attempts both to install more advanced sensors on major roads traversed by a large number of heavy vehicles and to minimize the number of major links with high HVL in the set of unobserved links. The model achieved this goal without affecting the number of major roads with or without HVL in the set of observed links.

I studied the full link flow observability problem in the Irvine network to assess the applicability of the model in a large scale network. For this network, I compared the results obtained with the proposed model to those reported for one of the existing models in the literature, and showed the merit of the current model in handling large size networks as well as decreasing the impact of sensor failure on link flow inference of unobserved links.

The route flow information gain was another topic that I discussed in this work. After introducing different types of routes based on the mutual links traversed by these routes, I proposed a two-level optimization model that can evaluate different sensor layouts which can lead to full link flow observability and minimize the impact of sensor failure on link flow inference of unobserved links. The results related to the Fishbone network demonstrated that sensors failure in two different layouts, having similar sensor failure impact on link flow inference of unobserved links, can have dissimilar effects on route flow observability.

I investigated the location of redundant sensors, i.e. additional sensors which are not initially required for full link flow observability of a traffic network, but which can be installed in a network to maintain the link flow inference of unobserved links in the event of sensor failure. To find the location of redundant sensors, I considered all possible combinations of failure among sensors installed in road links within the network. The results indicated that although considering all combinations of failure among sensors to determine the optimum location of redundant sensors is computationally expensive, there are a substantial number of combinations that prevent full link flow observability and therefore should not be considered in finding the location of redundant sensors.

In the final part of the study, I discussed the possibility of employing the proposed model for the partial link flow observability problem. After providing different definitions of partial link flow observability, I used the Fishbone network to show that the sensor positioning suggested by the proposed model attempts to minimize the chances of decreased levels of link observability in a network. I also proposed the idea of buffer sensors to address the situation where the number of available sensors is not large enough to reach full link flow observability. The results of using buffer sensors for different numbers of sensors available in the Fishbone network demonstrated that the model attempts to install buffer sensors on links with the least number of appearances in equations needed for link flow inference of unobserved links.

CHAPTER 3: MODELING THE EFFECT OF SENSOR FAILURE BEHAVIOR ON THE LOCATION OF COUNTING SENSORS FOR ORIGIN-DESTINATION (OD) MEAN ESTIMATION

3.1 A brief literature review of sensor location for OD estimation purposes

As described in Chapter 2, the NSLP can be divided into two main branches: flow observability and the flow estimation problem (Gentili and Mirchandani, 2012). Models that address the flow observability problem identify the locations of sensors such that the target flow can be uniquely determined. Flow estimation models define certain conditions for the locations of sensors in a traffic network to obtain the best estimation of the target flows. Identifying the location and the number of sensors for accurate determination of OD demand is a seminal step in the OD estimation process (Bianco et al., 2001; Hu et al., 2009).

Based on the four types of sensors defined by Gentili and Mirchandani (2012) (counting sensors, path-ID sensors, image sensors, and vehicle-ID sensors) and the target flows (i.e., link, route, and OD flows), twelve distinctive categories can be considered for the sensor location OD estimation problem (Hadavi and Shafahi, 2016). The list below illustrates examples of the popular categories and the key related literature while some of these categories have received more attention in the literature than others:

- Estimation of OD flows employing traffic count information (Yang et al., 1991; Yang and Zhou, 1998; Doblas and Benitez, 2005; Gan et al., 2005; Eiseman et al., 2006),
- Estimation of link flow using traffic counts (Hu et al., 2009; Castillo et al., 2013),

- Estimation of link flow based on observations provided by image sensors (Bianco et al., 2006),
- Estimation of path flows using path ID sensors information (Gentili and Mirchandani, 2005),
- Estimation of OD flows employing vehicle ID sensors observations (Castillo et al., 2008;
 Hadavi and Shafahi, 2016; Minguez et al., 2010, Zhou and List, 2010), and
- Estimation of path flows based on the traffic information provided by vehicle ID sensors
 (Castillo et al., 2008; Hadavi and Shafahi, 2016; Minguez et al., 2010).

Other studies also discuss the use of a combination of sensors (e.g. both vehicle ID sensors and counting sensors) to identify sensor locations and to estimate target flows (Zhou and List, 2010). Readers can refer to Viti et al. (2014) and Hadavi and Shafahi (2016) for a more extensive literature review on traffic sensor location identification for estimation purposes.

This study employs traffic counts as a source of information for OD estimation, which requires a thorough review of studies that focus on using traffic count information to address the sensor location OD estimation problem.

Chootinan et al. (2005) formulated a bi-objective traffic counting location problem for OD trip estimation. Their proposed model simultaneously employs two contradictory criteria, minimal resource utilization and maximum coverage, to strike a balance between the estimation quality and the coverage cost. Yang et al. (2006) proposed mixed-integer programming to determine the number of OD pairs whose flow can be specified for a given number of sensors. Their proposed formula was able to determine the minimum number of sensors required to specify the OD demands of all OD pairs. In a similar, more recent study, Owais et al. (2019) presented a robust sensor location model that determined the ideal number of sensors and identified their locations while attempting to reduce the maximum possible relative error (MPRE) boundary for an estimated OD matrix.

In general, different measurement indicators are used to evaluate the performance of OD estimation methods. By considering the target OD flows as an OD matrix, Yang et al. (1991) recommended the use of the MPRE index, which evaluates the reliability of an OD estimation. Yang and Zhou (1998) used the MPRE index to formulate four rules for sensor location. Gen et al. (2005) recommended the use of the expected relative error (ERE), which evaluates the expected value of the relative OD estimation error instead of the MPRE, which measures the quality of the OD estimation. More specifically, the MPRE measures the maximum distance between the estimated OD vector and any feasible OD vector in the feasible OD space, while the ERE is the expected distance between the estimated OD vector and a random feasible OD vector. Other studies have proposed other metrics to evaluate the quality of OD estimations. For instance, Bierlaire (2002) recommended the use of total demand scale (TDS) to measure the quality of OD estimation. Ehlert et al. (2006) developed a linear integer model to identify the locations of counting sensors. Their proposed formulation introduces an index of importance for each OD pair and then maximizes the summation of these indices for the covered pairs in the objective function. Ehlert et al. (2006) declared that the LP sub-problem provides an upper bound and can be easily employed for medium- and large-sized networks.

3.2 Example of the motivation behind this research

A small example helps to represent the motivation behind this research. The toy network illustrated in Figure 3.1 has two origins, nodes 1 and 2, and two destinations, nodes 5 and 6. Each origin point can end at one of the possible destinations. Figure 3.1 represents the flow and path between each OD pair.



Figure 3.1 – Toy network

Table 3.1 shows the four origin-destinations, their true OD information¹⁵, and the links traversed by each OD flow in Figure 3.1, respectively. Table 3.1 also shows two possible sensor set locations. Links 1 and 2 should be instrumented with sensors in the first sensor location set, while links 3 and 4 are equipped with sensors in the second location set. For every sensor-equipped link in each location set, the summation of estimated OD demands that traverse that link should be equal to the summation of true OD demands that use the same link assuming no measurement error in the sensors' observations.

For example, link 1 in the first sensor location set is instrumented with a sensor and OD demands 1-5¹⁶ and 1-6, which traverse this link, have true OD demands of 2 and 4, respectively. Therefore, for link 1, the summation of estimated OD demand of ODs 1-5 and

¹⁵ True OD information refers to information that can be used to evaluate the quality of OD estimations.

¹⁶ OD 1-5 represents OD that originate from node 1 and destined to node 5.

1-6 should be equal to the summation of the true OD demand of these ODs, which is, in this case, 4+2=6. Based on this explanation, the last two columns of this table show the estimated OD demand for each OD pair based on whether the sensor location set follows either the first or the second location set.

I employed two known methods, mean square error (MSE) and mean absolute error (MAE), to calculate the OD estimation error. MSE (MAE) calculates the summation of the squared (absolute value) differences between the true and estimated OD demand for each OD pair. With respect to these error measurement methods, the first and the second sets of sensors have equal OD estimation errors of 4 (see the last row in Table 3.1).

 Table 3.1 – True/estimated OD information related to different sensor location sets on the toy network

 presented in Figure 3.1

Origin-destination	True OD demand ¹⁷ Route		Estimated OD demand for sensor location set on the link ¹⁸	
(OD)		Tiouc	1 st set: {1, 2}	2nd set: {3, 4}
1-5	2	1-3-4	3	1
1-6	4	1-3-5	3	5
2-5	4	2-3-4	3	5

¹⁷ The unit for OD demand is (passenger car unit (pcu)/hour (hr)).

¹⁸ These estimated values are presented for the sake of this example. No specific method is employed to use these values.

2-6	6	2-3-5	7	5
OD estimation error (MSE/MAE)			4^{l}	4

¹For the first location set, this value calculated as follows:

$$MSE = (3-2)^{2} + (3-4)^{2} + (3-4)^{2} + (7-6)^{2} = 4$$
$$MAE = |3-2| + |3-4| + |3-4| + |7-6| = 4$$

This work focuses on exploring the effects of sensor failure on sensor deployment scenarios for OD estimation purposes. In other words, I would like to investigate the possible advantages of different sensor locations with consideration of sensor failure, especially while these location sets lead to identical OD estimation errors like the situation described for the first and the second location sets of sensors in Table 3.1.

Table 3.2 shows the estimated OD demand flow information that would be missed if a sensor fails. The table assumes that all sensors are identical and have a similar probability of failure. Note that the **missing OD demand flow information** refers to a situation where it is not possible to estimate the OD demand of a given OD pair. For an arbitrary OD pair, this situation occurs when all the sensors observing the flow of the OD pair break down (refer to the *OD covering* rule by Yang and Zhou (1998)). I can use the first sensor location set as an example, where the sensors are installed on links 1 and 2. The expected flow information loss of OD 1-5 will be 3p where 3 is the estimated demand between origin 1 and destination 5 and p is the probability of not being able to **estimate** the flow of this OD, which occurs when the sensor installed on link 1 fails. Two sensor location sets are presented in Table 3.2. When I compare them, I favor the second location set over the first as it results in less total expected OD demand flow information loss in the event of sensor failure. For any value of p, $p \le 1$, the summation of the expected OD demand loss for the second location set is less than the same value for the first sensor location set: $16p \ge 6p^2 + 10p$.

Origin-destination	Expected demand loss		
(OD)	(Assumption: Identical sensors)		
	1 st set: $\{1, 2\}$	2nd set: {3, 4}	
1-5	3 <i>p</i>	p^2	
1-6	3 <i>p</i>	5 <i>p</i>	
2-5	3 <i>p</i>	$5p^2$	
2-6	7 p	5 <i>p</i>	

Table 3.2 – Expected OD demand loss for each OD pair for each set of sensors

Table 3.2 provides useful information about the missing OD demand probability for individual OD pairs. This information can be used for traffic monitoring applications where some OD demands are of more interest to traffic management authorities than others.

In addition to calculating expected OD demand flow information loss for an **OD pair** due to the failure of sensors that observe that OD pair's flow, I can also determine the aggregate **OD demand flow information loss** due to the failure of a sensor deployed on a sensor-equipped link. Table 3.3 shows the expected OD demand loss for **each sensor-equipped link** in either the first or the second sensor location set. According to Table 3.3,

for instance, OD pairs 1-5 and 1-6 traverse link 1. The expected OD demand that will be missed (cannot be estimated) due to the failure of the sensor installed on this link is 6_p , where $_p$ is the failure probability of the sensor installed on this link and $_6$ is the summation of the estimated OD demand of OD pairs 1-5 and 1-6 according to the OD demand estimation provided by the first sensor location set.

Table 3.3 presents the expected OD demand flow information loss when sensors are considered individually (see the column entitled "Expected OD demand flow information loss on links") and as part of the first and second location sets (see the column entitled "Expected OD demand flow information loss on location set"). The expected OD demand loss on links 3 and 4, for example, is 16p and 6p, respectively. The expected OD demand flow information loss for the second sensor location set, which includes sensors on links 3 and 4, is 10p, which is not equal to the summation of the expected OD demand flow information loss of the links in that location set. The failure of the sensor installed on link 4 in the second sensor location set doesn't affect the OD demand estimation because the total demand can be captured by the sensor deployed on link 3. If the sensor installed on link 3 breaks down, then only the OD demand of OD pairs 1-6 and 2-6 will be missed because OD pairs 1-5 and 2-5 can still be captured by the sensor installed on link 4. According to Table 3.2, the summation of the estimated OD demand for ODs 1-6 and 2-6 is 10. Therefore, the expected OD demand flow information loss for the sensor deployed on link 3 in the second sensor location set is 10p. A failure of the sensor installed on link 4 doesn't affect the OD estimation process, making the OD demand flow information loss for the second sensor location set 10p.

The column in Table 3.3 entitled "Expected OD demand flow information loss on location set" calculates the summation of expected OD demand loss of each sensor in a location set under the assumption that all other sensors in that location set are in working condition. Based on this assumption, OD demand flow information loss due to the failure of a specific sensor can be mitigated as the flow of ODs can still be captured by other sensors in that location set. This calculation is based on the assumptions that all installed sensors have equal failure rates over identical lifetimes and that incidence of failure is an independent event among sensors.

The information provided in Table 3.3 is crucial for identifying links with the highest expected OD demand flow information loss in a traffic network so that those links can be equipped with more advanced sensors that have a lower probability of failure. For instance, although the second sensor location set in Table 3.3 has a lower expected OD demand flow information loss than the first set, link 2 in the first location set has an identical impact on OD demand flow information loss as link 4 has in the second location. Table 3.3 also shows the unique OD demand flow information gain provided by each sensor. In the first location set of sensors, each sensor provides unique OD demand flow information, whereas the sensor installed on link 4 doesn't contribute to unique information in the second location set because all information available by this sensor is also captured elsewhere. When adding additional sensors to a network, this consideration is important for maximizing the OD demand flow information gain.

Observed link	OD demands traversing a link	Expected OD demand flow information loss on links (identical sensors)		Expected OD demand flow information loss on location set (identical sensors)	
	-	1 st set: $\{1, 2\}$	2 nd set: $\{3, 4\}$	1 st set: {1, 2}	2nd set: {3, 4}
1	1-5	6 <i>p</i>			
	2-5	10 n		-	
2	2-6	10 <i>p</i>		_	
	1-5 1-6			16 <i>p</i>	10 <i>p</i>
3	2-5		16 <i>p</i>		
	2-6			_	
4	1-5 2-5		6 <i>p</i>		

Table 3.3 – Total expected flow loss due to the failure of each sensor in the toy network of Figure 3.1

The distinct OD demand flow information loss comparisons presented in Tables 3.2 and 3.3 refer to different viewpoints, including OD-oriented or link-oriented viewpoints that consider sensor failure for OD demand estimation purposes. In the real world, the failure rate of sensors is not necessarily identical and the flow between each OD pair exhibits a more random behavior than a deterministic one.

With respect to the above-mentioned viewpoints, the deployment of non-identical sensors, and the random behavior of OD demands, I propose a new formulation that identifies

more reliable sensor locations in a traffic network to minimize the impact of sensor failure on the OD estimation process.

3.3 Failure rate and mean lifetime

Reliability engineering and survival analysis of systems are primarily concerned with the system's lifetime functionality, which is usually represented as one or more positive random variables. By using a random variable to represent the system's lifetime, I can thoroughly characterize the system by its distribution function. The culmination of a system's lifetime is shown by death, or a terminating event usually known as the failure incident. Knowledge pertaining to the failure behavior of an operating system in a sufficiently small time interval is important to survival analysis of the system. Identifying the failure rate, or the frequency with which a system or a component of a system fails expressed per unit of time is the core component in determining a system's failure behavior. The failure rate of a system or component varies over its life cycle. For example, the failure rate of an automobile's engine in its tenth year of service might be greater than the failure rate in its first year of service.

3.3.1 Failure rate basics

Let $s \ge 0$ be a continuous lifetime random variable with the following cumulative distribution function:

$$F(s) = \begin{cases} \Pr[S \le s] & s \ge 0\\ 0, & s < 0 \end{cases}$$
(3.1)

Unless it is clearly specified, I assume this distribution is proper ($F^{-1}(1) = \infty$ and F(0) = 0) and the support of F(s) is $[0,\infty)$. I can express *s* as the time to failure (destruction) of the system, but other interpretations are also possible. Inter-arrival times in a sequence of ordered incidents or the amount of monotonically-accumulated damage on the failure of a mechanical system are other example instances of the end of a system's lifetime.

This study denotes a system's expected lifetime E[S] as λ and assumes λ to be finite: $\lambda < \infty$. I also assume that F(s) is a continuous function, so the probability density function (PDF) f(s), which is a derivative of s (F'(s) = f(s)), exists everywhere at s. The PDF can be calculated as:

$$f(s) = \lim_{\Delta s \to 0} \frac{\Pr\left[s < S \le s + \Delta s\right]}{\Delta s} = \frac{F\left(s + \Delta s\right) - F(s)}{\Delta s}$$
(3.2)

The PDF can then be used to find the expected lifetime of a system:

$$E[S] = \lambda = \left(\int_{0}^{s} xf(x) dx\right) = \left(sF(s) - \int_{0}^{s} F(x) dx\right)$$
(3.3)

Now I can define the notion of an operating system's failure rate. Considering a time interval $(s, s + \Delta s]$, I am interested in the probability of system failure within this interval knowing that the failure didn't occur prior to time *s*. This probability can be also interpreted as the risk of failure in $(s, s + \Delta s]$ given the stated condition:

$$\lambda(s) = \lim_{\Delta s \to 0} \frac{\Pr\left[s < S \le s + \Delta s \mid S > s\right]}{\Delta s} = \lim_{\Delta s \to 0} \frac{\Pr\left[s < S \le s + \Delta s\right]}{\Pr\left[S > s\right]\Delta s} = \lim_{\Delta s \to 0} \frac{F\left(s + \Delta s\right) - F(s)}{\overline{F}(s)\Delta s} = \frac{f(s)}{\overline{F}(s)}$$
(3.4)

Where $\lambda(s)$ is the failure rate at time *s* when $\Delta s \to 0$. With a sufficiently small Δs , I can also approximate that $\Pr[s < S \le s + \Delta s | S > s] \approx \lambda(s)\Delta s$, which gives a significant interpretation of $\lambda(s)\Delta s$ as an approximate conditional probability of failure at time interval $(s, s + \Delta s]$.

3.3.2 Bath-tub curve :

The bath-tub curve is, in fact, a well-known ubiquitous characteristic of all living things. As a way of illustration, the human life expectancy and a manufactured product's failure times may have many common features in their failure rate profiles as depicted in the bath-tub curve. The bath-tub curve can be segmented in three distinct time phases, including wear-in, useful life and wear-out phases, while each phase corresponds to a distinctive failure mode. The vertical dashed line used in the bath-tub curve shown in Figure 3.2 separates these time phases (Aarset, 1987).



Figure 3.2 – Bath-Tub curve (Aarset, 1987)

The wear-in also is known as the infant mortality is usually short with a decreasing failure rate. The useful life (youth) phase usually inherits a longer period compared to the other two phases among the most apparatus. If it survives the wear-in phase, a device exhibits a constant failure rate in the useful life phase. Eventually, the wear-out (aging) phase is the period that material fatigue, corrosion, embrittlement, etc. occur and result in device failure. At this phase, a device needs an increased regular inspection, special maintenance, and necessary replacement, if required.

3.4 The effect of sensor failure on the OD demand estimation process

In this section, I investigate the significance of sensor failure on the OD estimation process for a traffic network. I began by distinguishing between the effect each sensor's failure would have on the OD demand estimation of ODs that traverse each sensor-equipped link, as well as the effect of sensor failure on OD demand estimation for each OD pair. I discuss these considerations with the assumption that the sensors are not necessarily identical and have dissimilar failure behavior. I assume in this work that sensors are deployed independently, so there is no correlation between the failure of any pair of sensors.

I assume that a traffic network consists of |J| OD pairs and |M| links, where *J* and *M* denote the set of OD pairs and links within the network. Let the binary variable $x_{k,l}$ indicate whether link *l* is instrumented with sensor type $k(k \in K)$, where $\sum_{k \in K} x_{k,l} = 1$ denotes that this link is equipped with a sensor and therefore considered observed, and 0 means the link is not equipped with a sensor. To represent all traffic counting locations in the network, I define set \tilde{M} as a subset of *M*, where $\tilde{M} = \left\{ l \in M \mid \sum_{k \in K} x_{k,l} = 1 \right\}$. Set \tilde{M} is a function with respect to the binary variable $x_{k,l} : \tilde{M} = M(X)$, where $X = \left(\dots, \sum_{k \in K} x_{k,l}, \dots \right)^T$. The random variable v_l represents the stochastic traffic flow on the sensor-equipped link, i.e., observed link, *l* and $\{\tilde{v}_l\}_{(net)}$ is a column vector that denotes the observed traffic flow on the same link that can vary from day to day. The number of rows in the column vector, *n*, represents the number of measurements on link *l* where $\tilde{v}_l^{(n)}$ is the *i*th element of this vector, denoting the *i*th measurement of traffic flow during the peak hour period on link *l*.

I can calculate the sample mean peak hour traffic flow on link *i* using the information obtained from the column vectors of observed traffic flows using the following equation:

$$E(v_l) = \frac{1}{n} \sum_{i=1}^{n} \tilde{v}_l^{(i)} \qquad \qquad \forall l \in \tilde{M}$$
(3.5)

If I represent estimated OD demand for OD pair *j* during the same *peak hour period* (Q_j) as a multivariate random variable in which $E(Q_j) = q_j$ where q_j is the estimated mean OD demand, then Equation (3.6) is an alternative way to calculate v_j :

$$v_l = \sum_{j \in J} t_{l,j} Q_j \qquad \forall l \in \tilde{M}$$
(3.6)

Where $t_{l,j}$ is the proportion of traffic flow from OD *j* that uses link *l*. In this research, I assume that $t_{l,j}$ is *not* a fixed parameter for each link and OD demand¹⁹. The sample mean of peak hour traffic flow associated with link *l* ($E(v_l)$) can be newly represented using Equation (3.7):

$$E(v_l) = E\left(\sum_{j \in J} t_{l,j} Q_j\right) = \sum_{j \in J} t_{l,j} q_j \qquad \forall l \in \tilde{M}$$
(3.7)

Equations (3.6) and (3.7) are both expressed for sensor-equipped links. The sample mean of traffic flows obtained via link traffic counts provides vital information about the traffic flow pattern traversing each link.

3.4.1 Expected OD demand flow information loss of OD pair

The failure of sensors installed on an observed link can adversely affect the OD demand estimation of each OD pair in a traffic network. Studying the effect of sensor failure

¹⁹ In the following sections, I discuss the calculation procedure of the link choice proportion function

on each OD pair is important, as the flows between some OD pairs might be more important for traffic management authorities than the flows between others. The following equation expresses the expected OD demand flow information loss for each OD pair in a traffic network:

$$E^{L}\left(Q_{j}\right) = E^{L}\left(\left(\prod_{k\in K}\left(\frac{f_{k}\left(s\right)}{\overline{F_{k}}\left(s\right)}\right)^{\sum_{l\in M}\varphi_{l,j}x_{k,l}}\right)Q_{j}\right) = \left(\prod_{k\in K}\left(\frac{f_{k}\left(s\right)}{\overline{F_{k}}\left(s\right)}\right)^{\sum_{l\in M}\varphi_{l,j}x_{k,l}}\right)q_{j} \quad \forall j\in J, s\in S$$

$$(3.8)$$

Where the index variable *L* in $E^{L}(Q_{j})$ refers to the OD demand loss of OD pair *j*. The equation $\sum_{l \in M} \varphi_{l,j} x_{k,l}$ is the power of $\left(\frac{f_{k}(s)}{F_{k}(s)}\right)$. It calculates the number of observed links that are equipped with sensor type *k* and traversed by OD pair *j*, where $\varphi_{l,j}$ is the link-OD incidence matrix that signifies whether OD *j* traverses link *l* ($\varphi_{l,j} = 1$) or not ($\varphi_{l,j} = 0$). There is no loss in the estimated demand (q_{j}) in this equation when the multiplication of $\left(\frac{f_{k}(s)}{F_{k}(s)}\right)$ is approximately zero, which means the failure of that sensor has a minimal effect on

the estimated mean OD demand.

3.4.1.1 Relative OD demand flow information gain for each OD pair

In a traffic network, the true OD mean can be used to evaluate the quality of estimated values for each OD demand. The ideal information gain for each OD pair is when the estimated mean OD equals the true OD mean. Any absolute deviation between these two values can be considered information loss for that OD pair. The probability of failure of the sensors installed on links traversed by an OD pair can affect the OD demand information

gain as the higher the failure rate, the lower the information gain for that OD pair. The relative OD demand flow information gain proposed below incorporates both the OD estimation error and the failure of sensors installed on links traversed by each OD pair:

$$\beta_{j,s} = \left(1 - \prod_{k \in K} \left(\frac{f_k(s)}{\lambda^{\max} \overline{F_k}(s)}\right)^{\sum_{l \in M} \phi_{l,j} x_{k,l}} \right) \left(\frac{1}{1 + \left(\frac{\left|q_j - q_j^*\right|}{q_j^*}\right)}\right) q_j^* \qquad \forall j \in J, s \in S$$
(3.9)

Where $\lambda^{\max 20} (\lambda^{\max} > 0)$ is the maximum failure rate that can be achieved in the possible range of *s* while, dividing $\frac{f_k(s)}{\overline{F_k}(s)}$ by λ^{\max} results in a positive value between 0 and 1. Moreover, the

maximum possible information gain equals true mean OD demand (q_j^*) for OD pair j at time s. It is possible to reach this maximum gain where the estimated mean OD demand equals the true mean OD demand $(|q_j - q_j^*| = 0)$ and the probability of missing to estimate

OD demand *j* is approximately zero $\left(\prod_{k \in K} \left(\frac{f_k(s)}{\overline{F_k}(s)}\right)^{\sum_{k \in M} \varphi_{i,j} x_{k,l}} \approx 0\right)$.

In this equation, $\frac{|q_j - q_j^*|}{q_j^*}$ is always greater than or equal to zero, where $q_j^* > 0$.

Consequently, $\frac{1}{1 + \left(\frac{|q_j - q_j^*|}{q_j^*}\right)}$ ranges between 0 and 1 (see Figure 3.3). Moreover,

 $^{^{20}}$ When the failure rate is defined as a value between 0 and 1, then $\,\lambda^{\rm max}\,$ will be 1.

 $\frac{1}{1 + \left(\frac{\left|q_{j} - q_{j}^{*}\right|}{q_{j}^{*}}\right)}$ relates to the relative calculation of OD demand flow information gain with

respect to the true mean OD demand of OD pair *j*, while $\frac{|q_j - q_j^*|}{q_j^*}$ signifies the relative

difference between the estimated and true mean OD demand flow information gain with regards to the quantity of OD demand related to each OD pair.



Figure 3.3 – The graph of the function $f_x = \frac{1}{1+x}$

To elaborate more on relative information gain, let me build upon my example toy network from Section 3.2. Assume that there are two estimated OD demands for OD pairs 1-6 and 2-6 as shown in Table 3.4, which also calculates the relative information gain with respect to each estimated demand using the equations outlined above. Table 3.4 shows that the larger the difference between the true and estimated mean OD demand for each OD pair, the closer the value gets to zero and consequently the less the OD demand flow information is gained. Table 3.4 also shows that the equal absolute difference between the true and estimated OD demand doesn't necessarily lead to equal OD demand flow information gain for different OD pairs, as this value depends on the true OD demand value of each OD pair. For instance, the absolute difference for the *first* OD demand estimations of each OD pair are equal, but these estimations lead to different OD demand flow information gains because

they have different true OD demand values and the $\frac{|q_j - q_j^*|}{q_j^*}$ value is different for each.

Table 3.4 – The OD demand information gain for two OD pairs in Figure 3.1

				Relative OD
				demand flow
Origin-	True OD	Estimated OD	1	information gain
Destination	demand	demand	$1 + \left(\frac{\left q_{j} - q_{j}^{*}\right }{q_{j}^{*}}\right)$	$\left(\frac{1}{1+\left(\frac{\left q_{j}-q_{j}^{*}\right }{q_{j}^{*}}\right)}\right)q_{j}^{*}$
1-6	4	1 st Estimation: 2	0.667	2.667
		2 nd Estimation: 9	0.44	1.76
2-6	6	1 st Estimation: 4	0.75	4.5
		2 nd Estimation: 13	0.462	2.772

In Equation (3.9), $\beta_{j,s}$ becomes a positive value for each OD pair j at time s (

$$0 \le \beta_{j,s} \le q_j^* \quad \forall s, \in S, j \in J \text{) as } \frac{1}{1 + \left(\frac{\left|q_j - q_j^*\right|}{q_j^*}\right)} \text{ and } \left(1 - \prod_{k \in K} \left(\frac{f_k(s)}{\lambda^{\max} \overline{F_k}(s)}\right)^{\sum_{l \in M} \phi_{l,j} x_{k,l}}\right) \text{ range between } 0$$

and 1. I can use $\beta_{j,s}$ to construct a new objective function that maximizes the relative OD demand flow information gain among all OD pairs at time *s*:

$$Z_s^2 = \max\left(\min_{j\in J}\left(\beta_{j,s}\right)\right) \tag{3.10}$$

The objective function introduced in Equation (3.10) considers both the reliability of the OD estimate and the probability that sensors deployed on observed links and traversed by each OD pair at time *s* will fail. To maximize the total minimum possible information loss related to OD demands over the sensors' lifetimes, I can integrate Equation (3.10) with *s* and maximize the resulting equation:

$$Z^{2'} = \max\left(\sum_{j \in J} \left(\int_{0}^{w} (\beta_{j,s}) ds \right) \right)$$
(3.11)

Where w is the time period used as the integration's upper bound. In other words, Equation (3.11) is based on the assumption that the PDFs of all sensors are defined over an equal range of w^{21} . In the following section, I discuss the expected and relative information loss of each observed link due to sensor failure as well as estimation error.

²¹ Depending on the average lifetime of sensors, I can define w in yearly or monthly periods.

3.4.2 Expected OD demand flow information loss on observed link

In addition to considering the impact of sensor failure on the demand estimation loss of each OD pair in a traffic network, I am also interested in studying the effect of sensor failure on the estimation of OD demand while traversing an observed link. In other words, I investigate the effect of failure from each sensor on the total OD demand flow information loss during the OD demand estimation process in a traffic network. I achieve this by calculating the mean OD demand flow information loss for all OD pairs traversing each sensor-equipped link. Equation (3.12) calculates expected OD demand flow information loss related to the sample mean traffic flow on link i at time s:

$$E^{L}(v_{l}) = E^{L}\left(\sum_{j \in J}\sum_{k \in K} \left(\frac{f_{k}(s)}{\overline{F_{k}}(s)}\right) \upsilon_{l,j}\varphi_{l,j}t_{l,j}x_{k,l}Q_{j}\right) = \sum_{j \in J} \left(\sum_{k \in K} \upsilon_{l,j}\varphi_{l,j}t_{l,j}\left(\frac{f_{k}(s)}{\overline{F_{k}}(s)}\right)x_{k,l}\right)q_{j} \qquad \forall l \in \tilde{M}, s \in S$$

(3.12)

Where $f_k(s)$ is the PDF associated with the sensor type k, $\varphi_{l,j}$ is the binary parameter that determines whether OD pair j traverses link l, and $v_{l,j}$ is a binary variable that determines if link l is the only observed link that is traversed by OD pair j^{22}

$$(1-\max_{\substack{l'\in \tilde{M}\\l'\neq l}} \{\varphi_{l',j}\} = \upsilon_{l,j}, \forall j \in J, l \in \tilde{M}$$
). Multiplying $\upsilon_{l,j}$ and $\varphi_{l,j}$ allows me to consider the

uniqueness of the OD demand that traverses each observed link in a network. According to Equation (3.12), the OD demand flow information traversing link i cannot be estimated if

²² Note that $U_{l,j}$ can be 1 for observed link l and OD pair j while this OD also traverses other links which are **unobserved**.

the sensor installed on this link breaks down. Although subset \tilde{M} only considers links equipped with sensors, I must still include a binary variable $x_{k,l}$ to determine the type of sensor that is installed on link l.

3.4.2.1 <u>Relative OD demand flow information gain on an observed link</u>

The relative OD demand gain for an observed link focuses on the opposite of OD demand loss, which occurs due to possible simultaneous sensor failure and estimation error. The true mean OD demand for observed link *i* must satisfy the following equation:

$$E(v_l) = \sum_{j \in J} t_{l,j} q_j^* \qquad \forall l \in \tilde{M}$$
(3.13)

Equation (3.13) represents the relationship between the true mean OD demand and the mean traffic flow on observed link *i*. I need to incorporate $v_{l,j}$ into Equation (3.13) to account for the unique OD demand flow information obtained on observed link *j*:

$$E'(v_l) = \sum_{i \in J} t_{l,j} v_{l,j} q_j^* \qquad \forall l \in \tilde{M}$$
(3.14)

To obtain the relative OD demand gain on the observed link i at time s, I use Equation (3.14) to consider the true OD demand gain, Equation (3.12) to consider the possible estimated OD demand loss on the link, and proposed Equation (3.15) that incorporates both the relative estimation error and the failure of sensors installed on this link:

$$\alpha_{l,s} = \sum_{j \in J} \sum_{k \in K} \left(\left(1 - \frac{f_k(s)}{\lambda^{\max} \overline{F_k}(s)} \right) x_{k,l} \right) \left(\frac{1}{1 + \left(\frac{\left| q_j - q_j^* \right|}{q_j^*} \right)} \right) t_{l,j} \upsilon_{l,j} q_j^* \qquad \forall l \in \tilde{M}, s \in S \qquad (3.15)$$

Where the closer the value to $E'(v_l)$, the more reliable the OD demand information obtained from the sensor installed on link l. The ideal condition is that this equation is equal to $E'(v_l)$, which means that the failure probability of the sensor installed on the link l is zero $\left(\frac{f_k(s)}{\overline{F_k}(s)}=0\right)$ and that the estimated mean OD demand on the link is equal to the true mean OD demand for each OD pair that uses this link $(q_j = q_j^* \rightarrow |q_j - q_j^*| = 0)$.

Equation (3.15) assesses the relative total OD demand flow information gain on link i for all unique OD flows that traverse that link. In my definition, the term **relative** refers to

 $\left(\frac{1}{1+\left(\frac{\left|q_{j}-q_{j}^{*}\right|}{q_{j}^{*}}\right)}\right)q_{j}^{*} \text{ in Equation (3.15), which addresses the$ **relative OD demand information** $}\right)$

gain from the mean OD demand estimation for OD pair *j* with respect to the true mean OD demand of this OD pair. As the true and estimated mean OD demands on link *i* are positive values $(q_j \& q_j^* > 0), 1 + \frac{|q_j - q_j^*|}{q_j^*}$ is therefore always a positive value ranging between 0 and

1. Moreover, $\left(1 - \frac{f_k(s)}{\lambda^{\max}\overline{F_k}(s)}\right)$ is also a positive value in the same range. Therefore, $\alpha_{l,s}$ is

always positive for each observed link at time s, $0 \le \alpha_{l,s} \le E'(v_l) \quad \forall s, \in S, l \in \tilde{M}$. Using $\alpha_{l,s}$, I can define an objective function that maximizes the minimum relative information gain on observed links at time s:

$$Z_{s}^{1} = \max\left(\min_{l \in M}\left(\sum_{j \in J} \alpha_{l,s}\right)\right)$$
(3.16)

The objective function introduced in Equation (3.16) considers both the reliability of the OD estimations that traverse each observed link and the failure probability of the sensors installed on the links at time s. To maximize the total relative information gain on the observed links over the sensors' lifetimes, I can integrate Equation (3.16) with respect to s and then maximize the resulting equation:

$$Z^{1'} = \max\left(\sum_{l \in \tilde{M}} \left(\int_{0}^{w} \left(\sum_{j \in J} \alpha_{l,s} \right) ds \right) \right)$$
(3.17)

The objective functions introduced in Equations (3.10, 3.11, 3.16, and 3.17) can assist me in finding the appropriate formula for the sensor location problem. This will be discussed in the following section.

3.5 Mathematical formulation:

The two objective functions introduced in Sections 3.4.1.1 and 3.4.2.1 focus on maximizing both the relative OD demand flow information gain on each observed link and the OD demand flow information gain for each OD pair. I can use each of these functions

separately or combine them as a bi-objective function using the weighted sums method (WSM) and the ε -constraint method. According to the WSM, objective functions can be combined by using weights to signify the importance of each. The ε -constraint method maintains a singular objective function while treating other objective functions as constraints that define an upper bound/lower bound boundary depending on the natures of the objective functions.

The bi-objective function and constraint(s) using the WSM are shown below:

$$Z_{wsm} = \max\left(w_1 Z^{1'} + w_2 Z^{2'}\right) = \max\left(w_1\left(\frac{\sum\limits_{l\in \tilde{M}} \left(\int\limits_{0}^{w} \left(\sum\limits_{j\in J} \alpha_{l,s}\right) ds\right)\right)}{\sum\limits_{l\in \tilde{M}} E'(v_l)}\right) + w_2\left(\frac{\sum\limits_{j\in J} \left(\int\limits_{0}^{w} \left(\beta_{j,s}\right) ds\right)}{\sum\limits_{j\in J} q_j^*}\right)\right)$$
s.t.,
$$w_1 + w_2 = 1$$

$$(3.18)$$

Where $w_1 \& w_2$ are the weights that imply the importance of Z^{1° and Z^{2° , respectively. The value of these weights can be determined using expert judgment or considering the main focus of sensor installment. The sum of the weights should be equal to 1. In this equation, Z^{1° and Z^{2° are divided by $\sum_{l \in M} E^{l}(v_l)$ and $\sum_{j \in J} q_j^*$ because the objective functions employed by the WSM should be normalized.

There are two possible illustrations of the objective function with respect to the ε constraint because either z^{1} or z^{2} will be employed as an objective function and the other
as a constraint:

Π

Where ∂^1 and ∂^2 are the lower bound boundaries for z^1 and z^2 , respectively.

Ι

Note that the values of ∂^1 and ∂^2 can possibly contribute to the infeasibility of the optimization problem. We, therefore, must meticulously determine these values to avoid having the optimization problems defined in Equation (3.19) become infeasible. I can do this by solving one of the optimization forms in Equation (3.19) and using the optimal value as the lower bound of the constraint while changing the objective function. In addition to the WSM- and ε -constraint-related constraints, other general constraints should be employed for both methods. These include the budget constraint (Equation (3.20)), the sensor deployment constraint (Equation (3.21)), the OD covering rule (Equation (3.22)), and the constraints associated with the relationship between the true and estimated mean OD demand on observed links (Equation (3.23)).

$$\sum_{l \in M} \sum_{k \in K} x_{k,l} \le \eta$$
(3.20)
$$\sum_{l \in M} x_{k,l} \le 1 \qquad \forall l \in M$$

$$\sum_{k \in K} x_{k,l} \le 1 \qquad \qquad \forall l \in M \tag{3.21}$$

$$\sum_{l\in \tilde{M}}\sum_{k\in K}\varphi_{l,j}x_{k,l} \ge 1 \qquad \forall j \in J$$
(3.22)

$$\sum_{j \in J} t_{l,j} q_j^* - \sum_{j \in J} t_{l,j} q_j = \sum_{j \in J} t_{l,j} \left(q_j^* - q_j \right) = 0 \qquad \forall l \in \tilde{M}$$
(3.23)

Where η is the budget constraint that restricts the number of sensors installed on links in Equation (3.20). The budget in this work also constrains the types of sensors that can be deployed on links, where a larger budget supports the installation of more advanced sensors with lower failure probabilities.

According to Equation (3.21), at most one sensor can be installed on each link in a traffic network. Equation (3.22) addresses the *OD covering* rule, which signifies that the OD demand of each OD pair should be observed by at least one sensor. Equation (3.23) demonstrates that subtracting the sum of the true mean OD demand from the estimated OD demand should result in zero on each observed link l ($\forall l \in \tilde{M}$).

3.5.1 Further considerations for locating sensors in a traffic network:

In this section, I discuss additional considerations that can be studied for identifying ideal sensor locations in a traffic network. I introduce these considerations based on the four rules provided by Yang and Zhou (1998) and Yang et al. (2006). Equation (3.22) from Section 4 satisfies the *OD covering* rule, while I can address sensor failure with a new equation that ensures that the probability of OD demand flow information loss for a certain OD pair due to the failure of sensors is less than a certain rate:
$$\prod_{k \in K} \left(\frac{f_k(s)}{\overline{F_k}(s)} \right)^{\sum_{l \in M}^{Q_{l,j} \times k, l}} \le \gamma \quad \forall j \in J$$
(3.24)

Where the left-hand side of inequality in Equation (3.24) calculates the probability of OD demand loss for OD pair *j* in the event of failure of the sensors installed on links that observe OD demand of this OD pair. The right-hand side of this inequality introduces a value threshold that OD demand flow information loss probability should not surpass. I can further extend Equation (3.24) to apply the constraint only for those OD pairs whose true OD demand is greater than a certain value (q^L):

$$\prod_{k \in K} \left(\frac{f_k(s)}{\overline{F_k}(s)} \right)^{\sum_{i \in M} \varphi_{i,j} x_{k,i}} \le \gamma \quad \forall j \in J \quad if \quad q_j^* \ge q^L$$
(3.25)

This equation can be used to reduce the OD demand information loss for OD pairs with high demand rate. I can address the sensor location priority in a traffic network from three different viewpoints:

Maximum flow fraction: Prioritizing the link locations that can capture the largest OD demand with the fewest OD pairs

Maximum flow intercepting: Prioritizing the link locations whose sum of OD demands from all sensors maximizes the total captured OD demand

Maximal net OD flow captured: Prioritizing the link locations that minimize the OD demand information duplicity

This work's defined objective functions, shown in Equations (3.10 and 3.11), attempt to maximize the OD demand flow information gain for each OD pair, which indirectly addresses the *maximum flow fraction* rule. A similar set of sensors can be used to create link locations that capture the largest demand over the fewest OD pairs, maximizing the OD flow fraction in exchange for a higher relative OD information gain. The proposed objective functions, Equations (3.16 and 3.17), minimize the possibility of missing the unique OD demand flow information gain on each observed link while observing the relative OD demand flow information gain. These functions also address the *maximum flow intercepting* rule as they attempt to shrink the unique OD demand flow information between observed links, thereby maximizing the intersection of captured OD demand flow between observed links.

The *maximal net OD flow captured* rule was not addressed directly in the initial network sensor deployment, but it was considered in my proposed model by putting greater importance on link locations with more unique OD demand flow information; thereby placing more advanced sensors with less failure probability on these links. To employ additional sensors, I can consider the *link independence* rule by putting extra sensors on links that cover more unique OD demand flow, as shown in the following equation:

$$if \qquad \sum_{j \in J} \upsilon_{i,j} \varphi_{i,j} t_{i,j} q_j \ge \sum_{j \in J} \upsilon_{i,j} \varphi_{i,j} t_{i,j} q_j \qquad \Rightarrow \quad \sum_{k \in K} x_{i,k} \ge \sum_{k \in K} x_{i,k} \qquad \forall l^{\dagger} \& l^{\dagger} \in M \setminus \tilde{M}$$
(3.26)

Where the unobserved link l^{\dagger} is favored over link l for sensor installation as it covers less duplicate OD demand information.

3.5.2 Considerations for locating sensors in a traffic network with regard to the availability of route flow information

The following rules primarily depend on the existence of route information, including the link-route incidence matrix and the route-OD incidence matrices. While the main focus of this work is focused on creating a link choice proportion matrix in a network, I can discuss the four latter rules if a traffic network's route information is available. The *route covering* rule developed by Yang et al. (2006) and Cipriani et al. (2006) is an extension of the *OD covering* rule, which imposes the constraint that each route connecting each OD pair should be observed by at least one sensor.

$$\sum_{l \in \tilde{M}} \sum_{k \in K} \phi_{l,r} x_{k,l} \ge 1 \qquad \forall r \in R$$
(3.27)

Where $\phi_{l,r}$ is the binary element of the link-route incidence matrix equal to 1 if route r traverses link l and otherwise equal to 0. Considering the possible failure of sensors, I can update Equation (3.27) to incorporate the expected information gain from route flows. I can set the constraint to create a threshold γ' that either addresses the probability of missing each route's flow (Equation (3.28)), or that only activates if the true route flow exceeds a certain value (Equation (3.29)):

$$\prod_{k \in K} \left(\frac{f_k(s)}{\overline{F_k}(s)} \right)^{\sum_{l \in M} \phi_{l, r^{X_{k, l}}}} \le \gamma' \quad \forall r \in \mathbb{R}$$
(3.28)

$$\prod_{k \in K} \left(\frac{f_k(s)}{\overline{F_k(s)}} \right)_{i \in M}^{\sum \phi_{i, r^{X_{k,l}}}} \le \gamma' \quad \forall r \in R \text{ if } d_r \ge d^L$$
(3.29)

Where d^{L} is the boundary of route flows that activates this equation for each route r if the flow of that route exceeds d^{L} . The *maximal OD demand fraction* rule implies that the ratio between the route flow on the link t of OD pair j and the total flow of that OD demand should be maximized. When route flow information is available, minimal changes can be made to the proposed to cover the *maximal OD demand fraction* rule. These changes include using a link-route incidence matrix and the route-OD incidence matrices in the objective function to maximize the route flow information gain for each OD pair. The updated objective function should also minimize the expected route flow loss for each OD pair concerning sensor failure.

Similar in definition to the *maximal OD flow captured* rule, the *maximal net route flow captured* rule prioritizes sensor locations that capture the largest unique route flows. This rule does not consider double-counting²³ of route flows by sensors installed on observed links.

3.6 Solution algorithm:

In this section, I describe the solution algorithm used in this work. This includes the explanation related to the random link choice proportion and the employed Genetic Algorithm (GA).

²³ Double counting refer to as observing route flows more than once.

3.6.1 Description of OD estimation process with stochastic link choice proportion:

I assumed the link choice proportion inherits a stochastic behavior and needs to be updated through the estimation process. For the estimation process, I used the *maximum entropy* method to estimate the OD demands from the observed link flows that obtain from the sensor assignment²⁴. Note that depending on the type of priori OD available information, other OD estimation methods such as Bayesian estimation, generalized least square or maximum likelihood can be used instead of the maximum entropy (Cascetta, 2009). The choice of the functional form of the OD estimation method might affect the optimal sensor location results; more research needs to be conducted to investigate this impact. Moreover, using the stochastic user equilibrium (SUE), I employ the estimated OD demands to update link flows and the link choice proportion (Please see Figure 3.4). This information obtained from the SUE should be consistent with the observed link flows and link choice proportions. Thus, the link flows and link choice proportions should be updated iteratively until the difference between the estimated link flows obtained from the traffic assignment and observed link flows obtained from traffic counts is less than the tolerance rate which is set as 0.001. With the estimated OD values, I can calculate the OD demand information again from the current sensor assignment and update this assignment to maximize this information gain, if required.

²⁴ For more information on the formulations related maximum entropy, please refer to Van Zuylen and Willumsen (1980)



Figure 3.4 – Flowchart of the sensor assignment with OD demand estimation and stochastic link choice proportions

3.6.2 Genetic Algorithm

Figure 3.5 illustrates the flowchart of the proposed algorithm. Moreover, in what follows, I elaborate on the GA procedures in more detail:



Figure 3.5 – Algorithm Flowchart

3.6.2.1 Chromosome generation and representation

To represent the sensor location in a network, I defined each chromosome length to be equal to the number of links in a network while each cell can demonstrate if a link is instrumented with a sensor or not. For instance, if the j^{th} cell of a chromosome is equal to

1, it means that the sensor type 1 should be installed on the link j while zero means that link j is an unobserved link The population size in this solution algorithm is set to 30 in each iteration.

3.6.2.2 Fitness function

Objective functions, introduced as $Z_{Fitness}$ in Figure 3.5, are the functions in Equations (3.18) and (3.19) are employed as the fitness function of the GA.

3.6.2.3 Mutation and crossover procedures

With respect to the fitness function, the crossover procedure is employed to increase the chance of reproduction of the chromosomes that stand in a higher rank. In addition to crossover, the mutation procedure is employed to guarantee diversity in the generation of the subsequent population.

3.6.2.4 Stopping criteria

Stopping criteria also known as the termination condition determines the stopping point of the generational process in GA. Reaching a fixed number of repetitions, meeting the budget cap, and making no improvement in the fitness function through successive iterations are the common terminating conditions. I used the number of iterations to terminate the proposed GA. This criterion enforces the GA to stop when it reaches a predefined number of iterations. To avoid exhaustive iterations while defining the number of iterations, I implemented the GA with different initial populations and defined the number of iterations as four times the maximum iterations among different populations in which there is no improvement in the fitness function in the successive iterations.

3.7 Numerical examples:

In this section, I implement the proposed model for two illustrative examples including the Fishbone network and Sioux Falls network. Both of these networks have been widely employed in previous studies (Hu et al., 2014; Ng, 2012,2013; Salari et al., 2019; Xu et al., 2016).

Fishbone network: The Fishbone network is a relatively small-sized network with six noncentroid nodes and four centroid nodes (Nodes 1 and 2 are origin nodes and nodes 9 and 10 are destination nodes). This network has eighteen links that connect the origin nodes to destination nodes. The graphical illustration of this network is demonstrated in Figure 3.6.



Figure 3.6 – Fishbone network

I provided the information related to OD nodes, routes between each OD pair and Prior/True OD flows in Table 3.6. According to this table, there are four OD pairs between origin nodes and destination nodes in this network. Table 3.6 also presents the prior OD demand information and the True OD demand flow for each OD pair. Note that the information is randomly generated and is not from any real data source. The rest of the information related to this network including the link flows and link choice proportions are presented in Appendix IV.

OD#	OD nodes	Routes	OD demands	
			Prior	True
1	1-9	1-9-15-17	200	300
		2-11-15-17		
2	1-10	1-9-15-18	350	410
		2-7-9-15-18		
3	2-9	3-12-14-15-17	250	320
		2-6-10-16-17		
4	2-10	4-8-12-16-18	510	470

Table 3.6 Origin-Destination details of Fishbone network

Sioux Falls network: This network belongs to the city of Sioux Falls in the state of South Dakota, Unites States. This relatively mid-sized network consists of 76 links and 24 nodes. Consistent with Hao et al. (2019), I assumed that there exist 30 OD pairs in this network. Figure 3.7 represents a graphical illustration of this network. The information related to Origin-Destination nodes, prior OD demands, and true OD demands is provided in Appendix *V*.

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Figure 3.7 – Sioux Falls network

I assumed that there are two types of sensors introduced as basic and advanced sensors having different life-time distribution and cost. Table 3.7 provides detailed information related to these two types of sensors. According to this table, the basic sensor presents a higher failure rate per given period of time and is less expensive than the advanced sensor. Please note that while the lifetime distributions of sensors follow exponential distributions, their failure rates remain constant. In other words, the failure rates obtaining from Table 3.7 belong to the useful life phase of sensors.

Table 3.7 – Information pertaining to two types of sensors

Sensor type	Parameter values						
	Life-time distribution	Cost per sensor (× 100\$)					
1 (Basic)	Exponential ($\beta = 2$) ²⁵	120					
2 (Advanced)	Exponential ($\beta = 5$)	180					

3.7.1 The implementation of the GA Fishbone network

I generated three different population sizes and implemented the model for 100 iterations to observe the convergence rate behavior for different populations in the Fishbone network. Among the three populations, I explored the maximum number of iterations prior to the steady-state phase when no better solution obtained in subsequent iterations and multiplied this value by four. In Figure 3.8, I illustrate the convergence behavior of the proposed GA with three different population sizes when Equation (3.18) is used as the fitness function ($w_1 = w_2 = 0.5$) and it is assumed there exist four basic sensors to be installed in the network. According to this figure, the maximum number of iterations is set as 272 for the Fishbone network while I can see the GA reaches the maximum value before the 150th iteration for all three populations.

²⁵ This is the time gap between two consecutive events



Figure 3.8 – Convergence of the proposed GA for the Fishbone network using three different populations

3.7.2 Relative OD information gain on links/OD pairs: Fishbone network

In Figure 3.9, I assessed the effect of the number of sensors on the relative OD demand information gain for each OD pair in Fishbone network assuming that all sensors are identical²⁶. In this table, the number of identical sensors ranges from 2 to 8 sensors knowing that two sensors are the minimum number of sensors required to satisfy OD covering rule. I employed Equation (3.11) as the singular objective function to identify the location of sensors, i.e., equivalent to Equation (3.19-II) while there is no cap for the demand information gain on links). In Figure 3.9, each quadrant of the circle belongs to an OD pair which are differentiated with color-coding. For each quadrant, the outer boundary and the

²⁶ It's assumed all sensors are basic sensors.

colored dashed part are the true OD demand and the relative OD demand information gain values of that OD pair, respectively. This means that for each OD pair, the closer the dashed colored area to the boundary of the quadrant, the higher the relative OD demand information gain related to that OD pair. Moreover, according to this figure, as the number of sensors increases, on average, the level of OD demand information gain of the OD pairs increases subsequently (i.e., See the bottom row in Figure 3.9). For instance, as the number of sensors increases from 2 to 6, the relative OD information gain increases from 182.15 to 285.23 which is a closer value to the maximum possible information gain on all OD pairs.



Figure 3.9 – Sensor assignment on Fishbone network

In Table 3.8, I investigate the relative information gain on links. In this table, I defined different combinations of two type of sensors to be located in the Fishbone network. I investigate the effect of the number of sensors on the link information gain while the

objective function is set as Equation (3.19-I). In the first row of this Table, I assumed there is one sensor from each type of sensors. Compared to the first row, the second and third rows of Table 3.8 present a different combination of basic and advanced sensors while the total number of sensors remain unchanged for these two rows. As I can observe in Table 3.8, as the number of the sensors increases from the first row to the second row, the relative information gain on each link increases, correspondingly (Please see the third column of Table 3.8). Moreover, compared with the second row, the number of advanced sensors increases in the last row of this table, and the relative information gain on link 17 increases, subsequently as the model can locate one of the advanced sensors on this link. In this table, I also evaluate the sensor assignment on the observed links. According to the fourth column of this table, in all three rows, the advanced sensor is installed on links that can provide the highest information gain on links, equivalent to Equation (3.19-I), increases as the number of advanced sensors or the number of advanced sensors increases.

Sensor co	ombination	Set of observed links	Relative information	Link(s) instrumented	Total Information gain on
Basic	Advanced		gain on link(s)	with advanced sensor	links
					Equation (3.19-I)
1	1	{17,18}	{0,243.78,632.63}	{18}	876.41
2	1	{5,17,18}	{0,256.79,654.78}	{18}	911.57
1	2	{13,17,18}	{0,390.05,632.64}	{17,18}	1022.68

 Table 3.8 Relative OD information gain on links in Fishbone network

With a closer look at Table 3.8, it can be realized that in the second and last row of this table, links 5 and 13 become observed links, while in fact these links are not traversed by any OD pair. The sensor assignment to these links is due to that the variable $v_{l,j}$ is zero for all links when links 17 and 18 are observed links. Therefore, the model has no preference for selecting links which are traversed by an OD pair over other links. This is an insightful finding that implicitly emphasizes the importance of defining two objective functions in the proposed model. In other words, if I incorporate both objective functions using Equation 3.18 instead of employing a singular objective function, I can have a more relevant sensor assignment on links.

I study the effect of incorporating two objective functions versus a singular function in Figure 3.10. In this figure, I assigned two advanced and two basic sensors to be installed on links of Fishbone network. The layout on the left side of Figure 3.10 demonstrates the position of the sensors when the objective function is set as Equation (3.19-I). On the right side layout of this figure, however, I employed Equation (3.18) and set the weights w_1 and w_2 as 0.5. Concerning the use of Equation (3.19) as the objective function, the model assigns the advanced sensors to links 17 and 18 and the basic sensors are assigned to links 7 and 13, which are traversed by OD route presented in Table 3.6. In the layout on the right side of Figure 3.10, advanced sensors are assigned to links 18 and 15, while links 17 and 11 are instrumented with basic sensors. In this sensor, as the objective function considers both the information gain on links and for each OD pair, all the sensors are assigned to links 18 and 15 which provide a higher value resulting from the combination of information gain on links and OD pairs.



Figure 3.10 - Sensor assignment on Fishbone network

3.7.2.1 <u>Time-dependent failure rate (wear-out phase): Fishbone network</u>

With respect to the general characteristic of the bath-tub curve, the failure rate of sensors at the wear-out phase exhibits a time-dependent and increasing behavior. To study the behavior of sensors at this phase, I assumed that the basic and advanced sensors introduced in Table 3.5 inherit time-dependent failure rate functions as $\lambda_1(t) = 2t^4$ and $\lambda_{11}(t) = 2t^2$, respectively with a lifetime ranging from 0 to 10^{27} in the wear-out phase. I also assumed that after 10 years of age in the wear-out phase, either basic or advanced sensors will definitely fail. According to my assumption, the failure rate associated with basic sensors, i.e., $\lambda_1(t)$, demonstrates a steeper increasing failure rate compared to the same rate, i.e., $\lambda_{11}(t)$, for more advanced sensors.

²⁷ This range is defined in year

For the sake of simplicity, I assumed there are two basic and two advanced sensors to be deployed in the Fishbone network while these sensors present an increasing failure rate behavior through time. I studied the relative OD information gain on links and for each OD pair with respect to the location of basic and advanced sensors. The model suggests installing the advanced sensors on link 15 and 18 which are traversed by two and three OD pairs, respectively. Moreover, basic sensors should be deployed on links 2 and 12. According to the sensor deployment, contrary to other OD pairs, the last OD pair demand only traverses across one route, yet two of the links, links 12 and 18, in its route are instrumented with sensors. The true OD demand of this OD pair has the largest value and can provide a significant benefit to the total OD demand information gain from sensors. Moreover, the second OD pair has the most benefit from the sensor deployment as there are three sensorequipped links, links 18,15 and 2, from the set of routes in this OD pair. In Figure 3.11, I showed the relative OD demand information gain for each OD pair through the lifetime of sensors in the wear-out phase. As the second and the last OD pairs provide the most information gain, the model attempts to keep the level of gain from these OD pairs higher compared to the other two OD pairs. For instance, comparing the OD demand information gain for OD pairs 1 and 4 in Figure 3.11, I observe that OD demand information gain for OD pair 4 drops to the fifty percent of the maximum OD demand gain at time 8.78 years while for OD pair 1, the fifty percent of maximum OD demand information gain occurs at time 8.3 years.



Figure 3.11 - Relative OD demand information gain on OD pairs of Fishbone network

3.7.3 The convergence of the GA Sioux Falls network

To test the convergence rate per time of the GA, I implemented the proposed model for the Sioux Falls network. According to the procedure explained in Section 3.7.1, the GA is implemented for three different population sizes and the number of iterations is set as 933. I also employed Equation (3.18) as the objective function and set the weights as $w_1 = w_2 = 0.5$. Figure 3.12 represents the convergence pace of the GA for three different populations. According to this figure, the steady-state phase with the highest value of the objective function, i.e., weighted sum value, is reached at the time 671 *s*,1218 *s*, and 2433 *s* for the third, first and second populations, respectively. Moreover, the GA managed to conclude the number of iterations, i.e., 933 iterations, between 7611 *s* and 7763 *s* (around 2.15 hrs) for all three populations. I also observed a gap between the maximum weighted-sum values

obtained from using different populations. This gap can stem from distinctive population sizes. Besides that, the randomness involved in the implementation of a GA which results in reaching the local optimal solution instead of the global solution is another considerable factor that can lead to the possible difference in locally optimal solutions.



Figure 3.12 – Convergence pace of the GA for Sioux Falls network

3.7.4 The effect of prior OD on the relative OD demand information gain: Sioux Falls network

The quality of the prior OD can play a crucial role in the OD demand information gain as it directly affects the OD estimation process. In this section, I studied the effect of the deviation of the prior OD from the true OD on the OD demand information gain. I set three different levels of deviations introduced them as *errors* between the prior and the true OD while the

error is calculated as $\frac{\left|q^* - q^{prior}\right|}{q^*} \times 100$. This means that the higher the error percentage, the

more the difference between the true and the prior OD values. Figure 3.13 showcases the relative OD demand information gain for 90%, 50% and 30% level of error. Moreover, for each OD pair, this figure represents the true OD demand as a yellow-highlighted horizontal bar that determines the *maximum possible OD demand information*. According to Figure 3.13, as the level of error decreases, the level of relative OD demand information gain increases. As a way of illustration, for the fifth OD pair, the true OD demand value is 600 pcu²⁸/hr, the highest and lowest OD demand information gain are 509 pcu/hr and 121 pcu/hr which belong to the lowest and highest level of errors, respectively.



Figure 3.13 – Prior OD effect on the OD demand information gain in Sioux Falls network

²⁸ Passenger car unit

CHAPTER 4: SUMMARY AND CONCLUSIONS

This chapter presents concluding remarks and provides potential directions for new research. Sections 4.1 and 4.2 share an overall research summary and the findings related to Chapters 2 and 3. I suggest areas that may be of interest for future study in Section 4.3.

4.1. Research findings on the flow observability problem

The minimum set of sensor-equipped links in a traffic network for full link flow observability is not necessarily unique since a different set of observed links can lead to a different system of linear equations that can be used for link flow inferences of unobserved links. Different sets of observed links will result in different probabilities of inference loss of the unobserved links in a network in the event of sensor failure. I examined different locations for sets of sensors to assess the resulting flow information loss of unobserved links for each location set when a sensor fails.

In Chapter 2, I introduced two contributing factors for determining sensor locations: the number of observed links required to make link flow inferences for unobserved links and the number of observed links included in the different linear equations for link flow inferences of unobserved links. my min-max and min-sum formulas stem from these two contributing factors. When I assume that the sensors are identical, the min-sum equation can minimize the number of observed links required for link flow inferences of unobserved links and the min-max equation can find the minimum number of observed links necessary to make link inferences of the maximum number of unobserved links.

I also challenged the assumption of similarity between sensors. I assumed a scenario with two types of sensors with different probabilities of failure. Assuming that the sensors are not identical, the two objective functions outlined in Chapter 2 attempt to minimize the value of the expected chance of missing the link flow inferences of unobserved links due to sensor failure through the min-sum of a sensor installed on an observed link. Based on the results, the proposed model attempts to assign more advanced sensors with lower failure rate to links that appear in more link flow inference equations.

Chapter 2 discussed using the weighting method for consideration of major links. I differentiated between more-trafficked arterials and minor roads. The model was designed to minimize the number of major links in the set of unobserved links. More advanced sensors decreased the chance of missing flow observations of those links. I introduced redundant sensors that can be used as backups to reduce the possibility of missing full link flow observability if sensor failure occurs. I also used the proposed model, which was primarily designed for full flow observability, to address the problem of partial observability.

Finally, Chapter 2 discusses the existing definitions of partial observability in the literature. It introduces the concept of buffer sensors, which I use to identify locations where the number of real sensors is insufficient for full link flow observability.

4.1. Research findings on the flow estimation problem

In Chapter 3, I evaluated the effect of sensor failure on the OD estimation process. I introduced the concept of relative information gain during the OD estimation process for each OD pair and for observed, or sensor-instrumented, links. While I was defining the relative information gain, I stressed that in this case, contrary to the flow observability problem, sensor failure doesn't necessarily result in missing the OD flow estimation. I addressed this aspect of the OD estimation problem in prioritizing the relative information gain on links.

Based on the two types of information gain introduced in Chapter 3, I defined two objective functions to maximize the information gain on links and on OD pairs. The normalized versions of these two objective functions are combined to create a bi-objective function using the WSM. As an alternative representation of the mathematical formula, I used the ε -constraint method to employ either information gain on links or OD pairs as the primary objective function while the other function is used as a constraint. I defined two types of sensors to address the possibility of multiple types of existing counting sensors, each with distinctive costs and failure rates, benefitting from different technologies. For the sake of simplicity, I used the exponential distribution to model the age range of these sensors.

I implemented the proposed model on a Fishbone network and reported the results in Chapter 3. Based on the model's output, I learned that when the objective is defined based only on information gain on links or OD pairs, there is a possibility of obtaining misleading results. I demonstrated that the model may assign sensors to links that are not traversed by any OD routes. In such cases, I recommended using the combined version of the two objective functions. The objective function that was used to determine the sensor assignments mattered. Depending on which function was used (either the combination of two functions with WSM or a singular objective function employing the ε -constraint method), the output was one of two different sensor assignments. When the objective function was set to maximize the information gain on links, the model assigned the advanced sensors to links with the highest impacts on relative information gain.

4.1. Future extension

There are numerous possible future directions for this research. I can combine the full link flow observability and the OD estimation problems to identify the location of sensors that satisfies both the full link flow observability and OD estimations with the consideration of sensor failure. I can then separately compare the recommended sensor locations with the locations recommended for OD estimation and the sensor locations for full link flow observability. Alternatively, I can use the concept of sensor failure for other types of sensors, including vehicle-ID and image sensors. More advanced sensors may not suffer from the same failure rates as the comparatively primitive loop detectors, but they can still fail to observe ongoing traffic due to other influencing factors such as weather. I can use that knowledge in future research.

Another interesting extension of this study is considering possible future disruptions in traffic monitoring through connected vehicles. Connected vehicles are becoming information hubs that generate, process, send, and receive vast amounts of data while on the move. With these recent advancements in technology, traffic networks should be capable of supporting all the communications required to enable cooperative data transfer between vehicles and infrastructure. Vehicle to vehicle (V2V) communication involves the data transfer between connected cars in a traffic network, while the vehicle to infrastructure (V2I) communication addresses the data exchange between an information collection center and connected vehicles. Figure 4.1 illustrates a possible V2V and V2I connection in a traffic network with yellow and gray arrows representing V2I and V2V communications, respectively. Simultaneous V2I communication between multiple connected vehicles is possible. This infrastructure can be connected to a traffic management center to transfer the traffic data obtained from V2I connections.



Figure 4.1 – Illustration of data transfer with V2I and V2V communication

Considering that there are still other useful traffic information collection tools such as traffic sensors in current traffic management applications, I can study the effect of introducing new infrastructure, including the possible disruption to current traffic measurement methods like existing traffic sensors. One type of traffic sensor is automated vehicle identification sensors (AVIs), which are also known as vehicle-ID sensors. AVIs encompass license plate recognition (LPR) sensors. Recent technology advancements in the area of vehicular plate detection have allowed LPR sensors to create individualized vehicle path reconstructions for all vehicles they capture.

Although they can provide essential information about the traffic flow patterns in a traffic network, a downside to LPR sensors is that they cannot detect the V2V connectivity and will, therefore, miss this new source of traffic information. By introducing new infrastructure into a traffic network to facilitate V2I connections, I can not only detect V2V connection through V2I, but also reduce the reliance on LPR sensors for traffic information. In fact, the new information sourced through V2I connections can reconstruct vehicle paths, which have traditionally been the purview of LPR sensors. Whether I am adding LPR sensors within the existing infrastructure or employing both LPR sensors and V2I infrastructure in a network, I will need to take a meticulous approach to minimize the cost of installing and maintaining the system while maximizing the traffic flow information gain from the traffic data collection resources.

Appendix I

For identical sensors, if two layouts have the same average number of observed links required for the link flow inference of an unobserved link, i.e., having equal value of r_3 introduced in Equation (2. 10), the expected number of unobserved links whose flow cannot be inferred due to the failure of sensors can be different, i.e., they have different values of r_2 introduced in Equation (2. 9). An example of this scenario would be for instance, if there are two layouts A and B for a network, and the summation of non-zero values of the matrix $-T_u^{-1}T_o$ for each row of layouts A and B are similar except for rows j' and j' in which:

$$\begin{cases} n_{j'}^{A} = n_{j'}^{B} + 1 \\ n_{j''}^{A} = n_{j''}^{B} - 1 \end{cases} \qquad n_{j''}^{A} \le n_{j'}^{A}$$
(I-2.1)

In Equation (I-2.1), $n_{j'}^{A}$, $n_{j'}^{B}$ represent the summation of non-zero values of row j' in matrix $-T_{u}^{-1}T_{o}$ related to layouts A and B, respectively. These two layouts have the same value of Y_{3} and the probability of missing the link flow inference is the same for all unobserved links except for the links j' and j'. If I subtract the Y_{2} related to layout B, shown as $Y_{2}^{(B)}$, from the same value associated with layout A, shown as $Y_{2}^{(A)}$, then I reach the following equation:

$$\begin{aligned} Y_{2}^{(A)} - Y_{2}^{(B)} &= \\ \left[\left(1 - (1 - p)^{n_{j}^{A}} \right) + \left(1 - (1 - p)^{n_{j}^{A}} \right) \right] - \left[\left(1 - (1 - p)^{n_{j}^{B}} \right) + \left(1 - (1 - p)^{n_{j}^{B}} \right) \right] = (1 - p)^{n_{j}^{B}} + (1 - p)^{n_{j}^{B}} - \left[(1 - p)^{n_{j}^{A}} + (1 - p)^{n_{j}^{A}} \right] \\ &\Rightarrow (1 - p)^{n_{j}^{A} + 1} + (1 - p)^{n_{j}^{A} - 1} - \left[(1 - p)^{n_{j}^{A}} + (1 - p)^{n_{j}^{A}} \right] = (1 - p)^{n_{j}^{A}} (-p) + (1 - p)^{n_{j}^{A} - 1} (p) \\ &\Rightarrow (1 - p)^{n_{j}^{A} - 1} \left[(1 - p)^{n_{j}^{A} - n_{j}^{A} - 1} (-p) + p \right] \ge 0 \end{aligned}$$

(I-2.2)

In Equation (I-2.2), $(1-p)^{n_p^A-n_p^A-1}$ is a value between 0 and 1, and therefore the inequality in which $p(1-p)^{n_p^A-n_p^A-1} \le p$ is always valid and I can conclude that $Y_2^{(A)} \ge Y_2^{(B)}$. Therefore, layout B should be preferred over layout A as it has a lower value associated with the expected number of unobserved links for which their flow cannot be inferred due to the failure of sensors, while both layouts have an identical average number of observed links required for the link flow inference of unobserved links.

Appendix II



Fig II-1 Consideration of major roads in the Fishbone network

In the Fishbone network, let's assume that the red highlighted links, including links 1, 2, 3, 4, 9, 10, 11, 12, 15, 16 and 18, are major roads in the network. According to the equation developed by Ng. (2012) and introduced in Section 2.2, 12 links in the Fishbone network should be equipped with sensors to reach full link flow observability. Therefore, the number of major roads is less than the number of links that should be instrumented with sensors, i.e., observed links. If I want to apply the manual approach introduced in Section 2.5.2 to equip the major roads with sensors, all major roads listed above should be considered as observed links, i.e., sensor-equipped links in this network and one of the links not included in the set of major roads should be instrumented with a sensor as well. In this case, among seven links, including links 5, 6, 7, 8, 13, 14, and 17, which are not in a set of major roads, six of them should be selected as the unobserved links. Knowing that links 13 and 14, 5 and 7, and 6 and 8 are three sets of bi-directional links which can create cyclic graphs in a directed network²⁹, I need to select all links from at least two of these three sets to have six links as unobserved

²⁹ A network in which the links have directions

links in the Fishbone network. Therefore, unavoidably, the links selected to be in the set of unobserved links create a cyclic graph³⁰. According to Bapat (2010), the column vectors of a matrix that create a cyclic graph are linearly dependent, and the matrix, while being a square matrix, is a singular one. In what follows, I provide a lemma and a proposition to indicate that the column vectors in the matrix of unobserved links should not induce a cyclic graph:

Lemma 1 Let T be a $n \times m$ node-link incidence matrix that represents a network having m links and n non-centroid nodes. Any selections of column vectors of T are linearly independent if and only if the corresponding links of those selected columns don't induce a cyclic graph.

Proof:

If *k* columns are selected from the matrix *T*, and *p* columns of the *k* columns, $p \le k$, induce a cyclic graph, then Bapat (2010) proved that those *p* column vectors could be used to construct a matrix which is of the form $\begin{bmatrix} B \\ 0 \end{bmatrix}$, where *B* is the $p \times p$ matrix with column sums zero and represents the cyclic graph formed by the columns 1,..., *p*. Therefore, *B* is singular³¹ and the columns 1,..., *p* are linearly dependent. This proof addresses the "only if" part of the *Lemma 1*. For the proof of the other part, please refer to Bapat (2010).

³⁰ A cyclic graph or circular graph is a subset of links that construct a path known as a cycle such that the first node of the path corresponds to the last one (Pemmaraju and Skiena, 2003).

³¹ The determinant of this matrix is zero and therefore is not invertible.

Based on the *Lemma 1*, further criteria should be met to assure the matrix of unobserved links, T_{u} , obtained from the set of new links is an invertible matrix:

Proposition 1:

The links selected from each set of new links to form the matrix of unobserved links T_u , should not induce a cyclic graph in order to be invertible.

Proof:

A square matrix that has linearly independent column vectors is invertible. Matrix T_u is a square matrix as already discussed by Xu et al. (2016). Moreover, based on the *Lemma 1*, the column vectors of T_u are linearly independent if they do not form a cyclic graph. Therefore, the square matrix T_u which has linearly independent column vectors can be inverted and used to infer the flow of unobserved links.

Appendix III

The following table presents the connected links and the sets of new links related to each non-centroid node in the Sioux Falls network. The table is borrowed from Table 2.5 in Xu et al. (2016).

Non-centroid node	Connected Links	New links
1	1,2,3,5	1,2,3,5
2	1,3,4,14	4,14
3	2,5,6,7,8,35	6,7,8,35
4	6,8,9,10,11,31	9,10,11,31
5	9,11,12,13,15,23	12,13,15,23
6	4,12,14,15,16,19	16,19
7	17,18,20,54	17,18,20,54
8	16,17,19,20,21,22,24,47	21,22,24,47
9	13,21,23,24,25,26	25,26
10	25,26,27,28,29,30,32,43,48,51	27,28,29,30,32,43,48,51
11	10,27,31,32,33,34,36,40	33,34,36,40
12	7,33,35,36,37,38	37,38
13	37,38,39,74	39,74
14	34,40,41,42,44,71	41,42,44,71
15	28,41,43,44,45,46,57,67	45,46,57,67
16	22,29,47,48,49,50,52,55	49,50,52,55
17	30,49,51,52,53,58	53,58
18	18,50,54,55,56,60	56,60

Table III-1-The sets of new links associated with the non-centroid nodes in the Sioux Falls network

19	45,53,57,58,59,61	59,61	
20	56,59,60,61,62,63,64,68	62,63,64,68	
21	62,64,65,66,69,75	65,66,69,75	
22	46,63,65,67,68,69,70,72	70,72	
23	42,70,71,72,73,76	73,76	
24	39,66,74,75	-	

Appendix IV

Table I-1 demonstrates the link flow on each link of the Fishbone network. The flow on a link depends on the amount of OD flows that traverse that link.

Table IV-1 – Link flows in the Fishbone network

Link ID	1	2	3	4	5	6	7	8	9
Link	585	125	325	465	320	105	195	225	460
flow									
Link	10	11	12	13	14	15	16	17	18
ID									
Link	405	420	215	270	240	850	650	620	880
flow									

To use stochastic link choice proportion, I need to define the initial link choice proportion between OD demands. In Table I-2, the number of rows and columns represent the number of links and OD pairs in the Fishbone network, respectively. Each cell which is the intersection of a row (representative of a link) and a column (representative of an OD pair) demonstrates the proportion of the flow of an OD pair that uses that specific link.

Table IV-2 – Initial link choice proportion

OD	1	2	3	4
Link				
1	0.75	0.56	0	0
2	0.25	0.44	0	0
3	0	0	0.68	0.51
4	0	0	0.32	0.49
5	0.33	0.27	0	0
6	0	0.14	0.21	0
7	0.11	0	0.1	0.24
8	0	0	0.09	0.2
9	0.53	0.29	0.1	0.24
10	0	0.14	0.44	0.28
11	0.47	0.31	0.12	0.27
12	0	0.26	0.34	0.2
13	0.57	0.31	0	0
14	0	0	0.1	0
15	0.43	0.29	0.32	0.51
16	0.24	0.71	0.68	0.49
17	1	0	1	0
18	0	1	0	1
Appendix V

The origin and destination nodes and the true OD demands for Sioux Falls network are presented in Table V-1. According to Table V-1, there are 30 OD pairs in the Sioux Falls network while nodes 1,6,11,13,18,22 acts as both origin and destination nodes for different OD pairs. For the listed OD pairs, the true OD demand ranges between 240 pcu/hr (OD3, OD19&OD24) and 840 pcu/hr (OD9, OD10, OD15, OD18, OD20&OD28).Note that the origin and destination nodes and the true OD demand values are adopted from Fu et al. (2019).

Table V-1 – OD demand information in Sioux Falls network

OD#	OD nodes	True OD demands (pcu/hr)
1	1-6	480
2	1-11	600
3	1-13	240
4	1-18	600
5	1-22	600
6	6-1	360
7	6-11	600
8	6-13	600
9	6-18	840
10	6-22	840
11	11-1	360
12	11-6	480
13	11-13	480
14	11-18	720
15	11-22	840
16	13-1	360
17	13-6	600
18	13-11	840
19	13-18	240
20	13-22	840
21	18-1	480
22	18-6	480
23	18-11	600
24	18-13	240
25	18-22	840
26	22-1	480
27	22-6	360
28	22-11	840
29	22-13	720
30	22-18	480

Appendix VI

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REFERENCES

Aarset, M. V., 1987. How to identify a bathtub hazard rate. IEEE Transactions on Reliability, 36(1), 106-108.

Bapat, R. B., 2010. Graphs and matrices. New York: Springer.

Bianco, L., Confessore, G., Reverberi, P., 2001. A network based model for traffic sensor location with implications on O/D matrix estimates. Transportation Science 35(1), 50-60.

Bianco, L., Confessore, G., Gentili, M., 2006. Combinatorial aspects of the sensor location problem. Annals of Operations Research 144(1), 201-234.

Bianco, L., Cerrone, C., Cerulli, R., Gentili, M., 2014. Locating sensors to observe network arc flows: exact and heuristic approaches. Computers & Operations Research 46, 12-22.

Bierlaire, M. 2002. The total demand scale: a new measure of quality for static and dynamic origin–destination trip tables. Transportation Research Part B: Methodological, 36(9), 837-850.

Cascetta, E., & Postorino, M. N. 2001. Fixed point approaches to the estimation of O/D matrices using traffic counts on congested networks. Transportation science, 35(2), 134-147.

Cascetta, E. 2009. Transportation systems analysis: models and applications (Vol. 29). Springer Science & Business Media.

Castillo, E., Conejo, A. J., Menéndez, J. M., Jiménez, P., 2008. The observability problem in traffic network models. Computer-Aided Civil and Infrastructure Engineering 23(3), 208-222.

Castillo, E., Jimenez, P., Menéndez, J. M., Conejo, A. J., 2008. The observability problem in traffic models: algebraic and topological methods. IEEE Transactions on Intelligent Transportation Systems 9(2), 275-287.

Castillo, E., Gallego, I., Sanchez-Cambronero, S., Rivas, A., 2010. Matrix tools for general observability analysis in traffic networks. IEEE Transactions on Intelligent Transportation Systems 11(4), 799-813.

Castillo, E., Gallego, I., Menéndez, J. M., Jiménez, P., 2011. Link flow estimation in traffic networks on the basis of link flow observations. Journal of Intelligent Transportation Systems 15(4), 205-222.

Castillo, E., Nogal, M., Rivas, A., Sánchez-Cambronero, S., 2013. Observability of traffic networks. Optimal location of counting and scanning devices. Transportmetrica B: Transport Dynamics 1(1), 68-102.

Castillo, E., Calviño, A., Lo, H. K., Menéndez, J. M., Grande, Z., 2014. Non-planar holegenerated networks and link flow observability based on link counters. Transportation Research Part B: Methodological 68, 239-261.

Chen, A., Pravinvongvuth, S., Chootinan, P., Lee, M., Recker, W., 2007. Strategies for selecting additional traffic counts for improving OD trip table estimation. Transportmetrica 3(3), 191-211.

Chen, A., Chootinan, P., Recker, W., 2009. Norm approximation method for handling traffic count inconsistencies in path flow estimator. Transportation Research Part B: Methodological 43(8-9), 852-872.

Chootinan, P., Chen, A., Yang, H., 2005. A bi-objective traffic counting location problem for origin-destination trip table estimation. Transportmetrica 1(1), 65-80.

Chootinan, P., Chen, A., 2011. Confidence interval estimation for path flow estimator. Transportation Research Part B: Methodological 45(10), 1680-1698.

Cipriani, E., Fusco, G., Gori, S., & Petrelli, M. 2006. Heuristic methods for the optimal location of road traffic monitoring. In 2006 IEEE Intelligent Transportation Systems Conference, 1072-1077

Danczyk, A., Di, X., Liu, H. X., 2016. A probabilistic optimization model for allocating freeway sensors. Transportation Research Part C: Emerging Technologies 67, 378-398.

Doblas, J., & Benitez, F. G. 2005. An approach to estimating and updating origin–destination matrices based upon traffic counts preserving the prior structure of a survey matrix. Transportation Research Part B: Methodological, 39(7), 565-591.

Ehlert, A., Bell, M. G., Grosso, S., 2006. The optimisation of traffic count locations in road networks. Transportation Research Part B: Methodological 40(6), 460-479.

Eisenman, S. M., Fei, X., Zhou, X., & Mahmassani, H. S. 2006. Number and location of sensors for real-time network traffic estimation and prediction: Sensitivity analysis. Transportation Research Record, 1964(1), 253-259.

Federal Highway Administration, 2006. Traffic Detector Handbook: Third Edition— Volume II. Publication FHWA-HRT-06-139, Turner-Fairbank Highway Research Center, McLean, VA. Fei, X., Mahmassani, H., Eisenman, S., 2007. Sensor coverage and location for real-time traffic prediction in large-scale networks. Transportation Research Record: Journal of the Transportation Research Board (2039), 1-15.

Fei, X., Mahmassani, H. S., Murray-Tuite, P., 2013. Vehicular network sensor placement optimization under uncertainty. Transportation Research Part C: Emerging Technologies 29, 14-31.

Fei, X., Mahmassani, H. S., 2011. Structural analysis of near-optimal sensor locations for a stochastic large-scale network. Transportation Research Part C: Emerging Technologies 19(3), 440-453.

Fu, C., Zhu, N., Ling, S., Ma, S., Huang, Y., 2016. Heterogeneous sensor location model for path reconstruction. Transportation Research Part B: Methodological 91, 77-97.

Fu, C., Zhu, N., Ma, S., 2017. A stochastic program approach for path reconstruction oriented sensor location model. Transportation Research Part B: Methodological 102, 210-237.

Hao, Fu., Lam, W.H.K., Hu, S., Xu, X.P., Lo, H.P., Biyu, C., Sze, N.N., 2019. Optimization of Traffic Count Locations for Estimation of Travel Demands with Covariance between Origin-Destination Flows. Under revision in Journal of Transportation Research C. Gan, L., Yang, H., & Wong, S. C. 2005. Traffic counting location and error bound in origindestination matrix estimation problems. Journal of Transportation Engineering, 131(7), 524-534.

Gentili, M., Mirchandani, P. B., 2012. Locating sensors on traffic networks: Models, challenges and research opportunities. Transportation Research Part C: Emerging Technologies 24, 227-255.

Gentili, M., Mirchandani, P. B., 2018. Review of optimal sensor location models for travel time estimation. Transportation Research Part C: Emerging Technologies, 90, 74-96.

Gillespie, T. D.,1993. Effects of heavy-vehicle characteristics on pavement response and performance. Transportation Research Board. National Academy Press, Washington, D.C. United States.

Gómez, P., Menéndez, M., & Mérida-Casermeiro, E. 2015. Evaluation of trade-offs between two data sources for the accurate estimation of origin–destination matrices. Transportmetrica B: Transport Dynamics, 3(3), 222-245.

Guan, J., Aral, M. M., 1999. Progressive genetic algorithm for solution of optimization problems with nonlinear equality and inequality constraints. Applied Mathematical Modelling 23(4), 329-343.

Hadavi, M., Shafahi, Y., 2016. Vehicle identification sensor models for origin–destination estimation. Transportation Research Part B: Methodological 89, 82-106.

Hallenbeck, M. E., Selezneva, O. I., Quinley, R., 2014. Verification, refinement, and applicability of long-term pavement performance vehicle classification rules (No. FHWA-HRT-13-091). United States. Federal Highway Administration. Office of Infrastructure Research and Development.

He, S. X., 2013. A graphical approach to identify sensor locations for link flow inference. Transportation Research Part B: Methodological 51, 65-76.

Hu, S. R., Peeta, S., Chu, C. H., 2009. Identification of vehicle sensor locations for linkbased network traffic applications. Transportation Research Part B: Methodological 43(8), 873-894.

Klein, L. A., Mills, M. K., Gibson, D., Klein, L. A., 2006. Traffic detector handbook: Volume II (No. FHWA-HRT-06-139). United States. Federal Highway Administration.

Li, X., Ouyang, Y., 2011. Reliable sensor deployment for network traffic surveillance. Transportation research part B: methodological 45(1), 218-231.

Lam, W. H. K., & Lo, H. P. 1990. Accuracy of OD estimates from traffic counts. Traffic engineering & control, 31(6).

Larsson, T., Lundgren, J. T., & Peterson, A. 2010. Allocation of Link Flow Detectors for Origin-Destination Matrix Estimation—A Comparative Study. Computer-Aided Civil and Infrastructure Engineering, 25(2), 116-131.

Mínguez, R., Sánchez-Cambronero, S., Castillo, E., Jiménez, P., 2010. Optimal traffic plate scanning location for OD trip matrix and route estimation in road networks. Transportation Research Part B: Methodological 44(2), 282-298.

Morrison, D. R., Martonosi, S. E., 2015. Characteristics of optimal solutions to the sensor location problem. Annals of Operations Research 226(1), 463-478.

Ng, M., 2012. Synergistic sensor location for link flow inference without path enumeration: A node-based approach. Transportation Research Part B: Methodological 46(6), 781-788.

Ng, M., 2013. Partial link flow observability in the presence of initial sensors: Solution without path enumeration. Transportation research part E: logistics and transportation review, 51, 62-66.

Owais, M., Moussa, G. S., & Hussain, K. F. 2019. Sensor location model for O/D estimation: Multi-criteria meta-heuristics approach. Operations Research Perspectives, 6, 100100.

Pemmaraju, S., Skiena, S. 2003. Computational Discrete Mathematics: Combinatorics and Graph Theory with Mathematica[®]. Cambridge university press.

Rinaldi, M., Viti, F., 2017. Exact and approximate route set generation for resilient partial observability in sensor location problems. Transportation Research Part B: Methodological 105, 86-119.

Salari, M., Kattan, L., Lam, W. H. K, Lo, H. P., & Esfeh, M. A. 2019. Optimization of traffic sensor location for complete link flow observability in traffic network considering sensor failure. Transportation Research Part B: Methodological, 121, 216-251.

Schrank, D., Eisele, B., Lomax, T., Bak, J., 2015. 2015 Urban mobility scorecard. Texas A&M Transportation Institute, Texas, United States.

Van Zuylen, H. J., Willumsen, L. G., 1980. The most likely trip matrix estimated from traffic counts. Transportation Research Part B, 14 (3), 281-293.

Viti, F., Rinaldi, M., Corman, F., Tampère, C. M. 2014. Assessing partial observability in network sensor location problems. Transportation research part B: methodological 70, 65-89.

Wang, N., Gentili, M., Mirchandani, P., 2012. Model to locate sensors for estimation of static origin-destination volumes given prior flow information. Transportation Research Record: Journal of the Transportation Research Board (2283), 67-73.

Wang, N., Mirchandani, P., 2013. Sensor location model to optimize origin-destination estimation with a bayesian statistical procedure. Transportation Research Record: Journal of the Transportation Research Board (2334), 29-39.

Xing, T., Zhou, X., & Taylor, J. 2013. Designing heterogeneous sensor networks for estimating and predicting path travel time dynamics: An information-theoretic modeling approach. Transportation Research Part B: Methodological, 57, 66-90.

Xu, X., Lo, H. K., Chen, A., Castillo, E., 2016. Robust network sensor location for complete link flow observability under uncertainty. Transportation Research Part B: Methodological 88, 1-20. Yang, H., Iida, Y., & Sasaki, T. 1991. An analysis of the reliability of an origin-destination trip matrix estimated from traffic counts. Transportation Research Part B: Methodological, 25(5), 351-363.

Yang, H., & Zhou, J. 1998. Optimal traffic counting locations for origin–destination matrix estimation. Transportation Research Part B: Methodological, 32(2), 109-126.

Yang, H., H. Bell, M. G., 1998. Models and algorithms for road network design: a review and some new developments. Transport Reviews 18(3), 257-278.

Yang, H., Yang, C., & Gan, L. 2006. Models and algorithms for the screen line-based trafficcounting location problems. *Computers & Operations Research*, *33*(3), 836-858.

Yang, Y., & Fan, Y. 2015. Data dependent input control for origin–destination demand estimation using observability analysis. Transportation Research Part B: Methodological, 78, 385-403.

Yang, Y., Fan, Y., & Wets, R. J. 2018. Stochastic travel demand estimation: Improving network identifiability using multi-day observation sets. Transportation Research Part B: Methodological, 107, 192-211.

Zhan, F., Wan, X., Cheng, Y., Ran, B., 2018. Methods for Multi-Type Sensor Allocations Along a Freeway Corridor. IEEE Intelligent Transportation Systems Magazine 10(2), 134-149.

Zhou, X., List, G. F., 2010. An information-theoretic sensor location model for traffic origindestination demand estimation applications. Transportation Science 44(2), 254-273.

Zhu, N., Ma, S., Zheng, L. (2017). Travel time estimation oriented freeway sensor placement problem considering sensor failure. Journal of Intelligent Transportation Systems 21(1), 26-40.