# THE UNIVERSITY OF CALGARY 

SOFTWARE FOR THE ALIGNMENT OF INDUSTRIAL MACHINERY USING THEODOLITE DIRECTIONAL DATA
by

Brian C. Fuss

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DEPARTMENT OF GEOMATICS ENGINEERING

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Software For the Alignment of Industrial Machinery Using Theodolite Directional Data" submitted by Brian C. Fuss in partial fulfillment of the requirements for the degree of Master of Science.
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[^0]
#### Abstract

Theodolite intersection systems used for industrial situations have proven to be useful tools in the alignment of machinery. In particular, rotating shafts can be operating while an alignment survey is being conducted due to the non-contact nature of using only theodolite directional data.

In this study, an existing theodolite intersection system is investigated and improved upon where limitations have been found. Existing data collection software lacking error detection routines is enhanced while new software is created to generate approximate coordinates and display alignment and correction results in a complete and statistically informative format. Identified in this study is the necessity for some form of deformation analysis if the highest level of result quality is desired in an alignment solution. Laboratory and field testing verified that these improvements have been successful and emphasized how crucial precise survey alignment procedures and data can be.


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To my family-especially my parents, Herb and Helga

## TABLE OF CONTENTS

APPROVAL PAGE ..... ii
ABSTRACT ..... iii
ACKNOWLEDGEMENTS ..... iv
TABLE OF CONTENTS ..... vi
LIST OF TABLES .....  $x$
LIST OF FIGURES ..... xii
NOTATION ..... xv
Chapter
1 Introduction. ..... 1
2 Machinery Alignment ..... 5
2.1 Importance of Proper Machine Alignment ..... 6
2.2 Overview of Some Existing Alignment Systems ..... 9
2.3 Theodolite Intersection Systems ..... 13
3 Summary of Existing Software System ..... 18
3.1 Overview of Software Components ..... 18
3.2 Discussion of Mathematical Models ..... 24
3.3 Tested Applications Using Existing System ..... 28
3.4 Desired Additions or Improvements to Existing System ..... 30
4 Enhancements to COLLECT Software ..... 33
4.1 Summary of Existing System ..... 33
4.2 Theodolite Errors. ..... 36
4.3 New Features Added to COLLECT ..... 42
4.4 Testing and Results: COLLECT ..... 49
4.4.1 Studies of Previously Obtained Data ..... 49
4.4.2 Results of Subsequent Testing ..... 53
5 Approximate Coordinates ..... 56
5.1 Need For Approximate Coordinates ..... 56
5.2 Methods to Gather or Generate Approximate Coordinates ..... 59
5.3 New Approximate Coordinate Software ..... 62
5.3.1 Program Overview ..... 62
5.3.2 Program Module Features ..... 65
5.4 Testing and Results: APPROX ..... 80
5.4.1 Studies of Previously Obtained Data ..... 80
5.4.2 Results of Subsequent Testing ..... 82
6 Shaft Misalignment and Offsets ..... 87
6.1 Alignment Assumptions ..... 87
6.2 Mathematical Modeling ..... 91
6.2.1 Orientation of Shafts Using Direction Numbers ..... 91
6.2.2 Shaft Alignment Mathematical Model ..... 93
6.3 Variance and Covariance Propagation ..... 95
6.3.1 Error Propagation ..... 95
6.3.2 Covariance and Design Matrices ..... 96
6.4 Program Structure ..... 97
6.5 Converting Center Points to Vectors ..... 101
6.6 Testing and Results: SHAFT ..... 104
6.6.1 Studies of Previously Obtained Data ..... 104
6.6.2 Results of Subsequent Testing ..... 108
7 Alignment Testing Using Complete System ..... 113
7.1 The Alignment Test Rig ..... 113
7.2 Alignment of a T2 Theodolite ..... 118
7.3 Other Testing and Results ..... 124
8 Deformation Modeling and Covariance Data ..... 126
8.1 Deformation Analysis ..... 126
8.2 Covariance Data Testing ..... 129
9 Conclusions and Recommendations ..... 133
References ..... 136

## LIST OF TABLES

## Table

4.1 The Effect and Solution of Systematic Errors ..... 38
5.1 DEFAULTS: Function Flow ..... 65
5.2 THEODOLITE SCAN: Function Flow ..... 66
5.3 THEODOLITE FIND: Function Flow ..... 67
5.4 CORRECT OBSERVATION: Function Flow ..... 67
5.5 RELATIVE COORDINATES: Function Flow ..... 68
5.6 ORIENT: Function Flow ..... 70
5.7 SCALE: Function Flow ..... 72
5.8 SIMILARITY: Function Flow ..... 73
5.9 TRANSLATE: Function Flow ..... 74
5.10 MORE POINTS: Function Flow ..... 74
5.11 INTERSECT: Function Flow ..... 76
5.12 RESECT: Function Flow ..... 77
5.13 PRESENT: Function Flow ..... 79
5.14 APPROX Testing: Taped Approximate Coordinates ..... 83
5.15 APPROX Testing: APPROX Generated Approximate Coordinates ..... 84
5.16 APPROX Testing: Adjusted Coordinates ..... 84
6.1 SHAFT Results: Rig Test Number 10 (all shaft radii NOT fixed) ..... 109
6.2 SHAFT Results: Rig Test Number 10 (all shaft radii WERE fixed) ..... 110
6.3 Comparison Between Caliper and SHAFT Output Results ..... 111
7.1 Description of T2 Testing ..... 120
7.2 Comparison of Actual Measured Movements with Theodolite
Intersection Results ..... 121
8.1 Comparison of Full Covariance Deformation Results With Variance-Only Results (Different Datum) ..... 131
8.2 Comparison of Full Covariance Deformation Results With
Variance-Only Results (Same Datum) ..... 131

## LIST OF FIGURES

Figure
1.1 Shaft Alignment Using Theodolite Intersection ..... 2
2.1 Rim and Face Dial Indicator Method ..... 11
2.2 Top View of Width Measurements Using an Optical Jig Transit ..... 12
2.3 Alignment with Laser-Optical Systems ..... 13
2.4 Three-Dimensional Theodolite Intersection ..... 14
3.1 Existing Alignment Software System ..... 20
4.1 Bayly's Theodolite Intersection System ..... 33
4.2 COLLECT's Main Functions ..... 34
4.3 Theodolite Internal Orientation ..... 39
4.4 Single and Station Collimation Errors ..... 40
4.5 Theodolite Pointing Error ..... 41
4.6 New COLLECT Pointing Errors Screen ..... 45
4.7 Warning Screen Reporting Individual Errors ..... 47
4.8 Example of an Instrument Station Error Screen ..... 48
4.9 Kiln Pointing Error Screen. ..... 51
4.10 Steffi Pointing Error Screen ..... 52
4.11 Sample Alignment Rig Pointing Error Screen ..... 54
4.12 Sample Alignment Rig Station Collimation Error Screen ..... 55
5.1 Convergence from Approximate Coordinates ..... 57
5.2 Stadia Method for Scale and Orientation ..... 61
5.3 Coordinate System Used in APPROX ..... 63
5.4 APPROX Program Modules ..... 64
5.5 Theodolite Bearing-Bearing Intersection ..... 69
5.6 Orientation When Shaft Axis is Used ..... 71
5.7 Hausbrandt's Resection Method ..... 78
5.8 Granulator Survey Network ..... 81
5.9 Ensuring Network Targets Significantly Off-line From Other Theodolite ..... 86
6.1 Skewed Plane of Best-Fit Circle ..... 88
6.2 Orientation of Unit Vector Normal to Plane of Circle ..... 89
6.3 The $x, y$, and $z$ Components of Unit Vector a ..... 91
6.4 Direction Cosines ..... 92
6.5 Misalignment Sign Conventions ..... 94
6.6 Graphical Results from SHAFT for LAB5 Data ..... 106
6.7 Graphical Results for Kadon Test Rig Using Vector-II Data ..... 107
6.8 Target Locations on Test Rig ..... 108
7.1 Design Specifications of Co-Developed Test Rig ..... 114
7.2 T2 Test Network ..... 119
7.3 Target Position on Static and T2 Pipes ..... 119
7.4 Orientation of Tested Movements ..... 120
7.5 Human Performance Laboratory Calibration Pyramid ..... 124
7.6 The Quadrant Tangent Method ..... 125
8.1 Measured Compressor Site ..... 128

## NOTATION

$d_{\mathrm{r}} \quad$ relative distance computed via theodolite intersection $D_{i j} \quad$ direction from point $i$ to $j$

| $e_{\mathrm{c} \text { (mean) }}$ | mean of all collimation errors for each theodolite station |
| :---: | :---: |
| $e_{\text {coll }}$ | collimation error at a specific theodolite station |
| $e_{\mathrm{HZ}}$ | error in horizontal readings from mean theodolite station value |
| $e_{\text {STN_COLL }}$ | difference for each station from mean of all station collimation errors |
| $e_{\text {STN_VINDX }}$ | difference for each station from mean of all station vertical index errors |
| $e_{\mathrm{v}}$ | error in vertical reading from mean value at theodolite station |
| $e_{\mathrm{V}(\text { mean })}$ | mean of all vertical index errors for each theodolite station |
| $e_{\text {vindx }}$ | vertical index error at a specific theodolite station |
| $f$ | a function |
| F | factor used in Hausbrandt's Method |
| FACE | array containing either face-I or face-II observations |
| $\mathrm{Hz}_{\mathrm{m}}$ | mean horizontal directions for either face |
| $\mathrm{HZ}_{\text {sum }}$ | sum of all horizontal directions for single theodolite station |
| J | Jacobian matrix |
| $\ell$ | observation vector |
| $\ell_{h}$ | horizontal distance |
| $\mathrm{M}_{\mathrm{xy}}$ | shaft misalignment in xy-plane |
| $\mathrm{M}_{\mathrm{xz}}$ | shaft misalignment in $x z$-plane |
| $n$ | number of pointings |


| $\mathrm{O}_{x y}$ | shaft offset in $x y$-plane |
| :---: | :---: |
| $\mathrm{O}_{x z}$ | shaft offset in $x z$-plane |
| r | radius of a circle |
| rot | angle of network rotation |
| $\hat{\mathbf{r}}$ | vector of residuals |
| s | scale factor from relative and actual network distances |
| STN_COLL | array holding all observations |
| $\mathrm{V}_{\mathrm{m}}$ | mean vertical directions for either face |
| $\mathrm{V}_{\text {sum }}$ | sum of all vertical directions for a single theodolite station |
| $\omega$ | vector misclosures |
| $\omega_{x}$ | vector of differences between initial and current parameter estimates |
| x | right-handed Cartesian coordinate |
| $\mathrm{x}_{0}$ | $x$-coordinate of a line starting point or a circle center |
| $\mathrm{X}_{\text {s }}$ | midpoint of shaft along shaft axis |
| $\bar{\chi}$ | unknowns (e.g. target coordinates) |
| y | right-handed Cartesian coordinate |
| Yo | $y$-coordinate of a line starting point or a circle center |
| z | right-handed Cartesian coordinate |
| zo | $z$-coordinate of a line starting point or a circle center |


| $\alpha$ | direction cosine angle from xz-plane |
| :--- | :--- |
| $\beta$ | direction cosine angle from xy-plane |
| $\gamma$ | direction cosine angle from yz-plane |
| $\hat{\delta}$ | vector of parameter corrections |
| $\sigma_{b}$ | standard deviation of y-direction number |
| $\sigma_{c}$ | standard deviation of z-direction number |
| $\sigma_{d}$ | random centering error |
| $\left(\sigma_{d}\right)_{c}$ | random pointing error |
| $\left(\sigma_{d}\right)_{l}$ | random reading error |
| $\left(\sigma_{d}\right)_{p}$ | total random error |
| $\left(\sigma_{d}\right)_{r}$ | standard deviation of x-coordinate circle center |
| $\left(\sigma_{d}\right)_{\text {total }}$ | standard deviation of y-coordinate circle center |
| $\sigma_{x o}$ | covariance matrix of observations |
| $\sigma_{y o}$ | theodolite bearing |
| $\sigma_{z o}$ |  |
| $\Sigma_{\mathrm{xx}}$ |  |

## Chapter 1

## Introduction

Machinery used in industrial settings such as factories or gas plants must be manufactured and maintained in strict conformity with design specifications. Rotating machinery such as turbine/compressor assemblies, cement kilns, or rollers used in paper mills or printing presses must be aligned to avoid excessive vibration and wear. Lardelli (1987) states that "[t]he main objective of industrial surveying is to construct, assemble and align a workpiece or component as per the relevant drawing to ensure operability."

The alignment of industrial machinery has traditionally relied on the use of various contact methods such as dial indicator gauges, leveling rods, or complicated optical tooling methods (Fuss, 1993). In monitoring machine components that do not rotate, most of these methods provide an inexpensive and adequate solution to the alignment problem. On the other hand, if the actual rotating shafts or other moving parts as well as inaccessible components need to be monitored or aligned, these older methods can be too intrusive or unreliable. With most of these existing techniques, the machinery must be shut-down in order for alignment to take place. This alone can be very expensive in terms of lost revenue due to inoperation. Also, as a consequence of shut-down, the relative alignment between shafts or other machine components will change, at best, by an estimated amount. The ability to align
rotating machinery while operating (hot alignment) has always been an important objective for most designers and operators of rotating machinery.


Figure 1.1 Shaft Alignment Using Theodolite Intersection

One newer method used to align both rotating and non-rotating machinery involves the use of intersecting directional data from theodolites (Figure 1.1). These instruments provide horizontal and vertical directional information that can be manipulated mathematically into a solution for three-
dimensional coordinates of certain targets. These targets can be physical objects such as specially marked stickers placed on static machine components or laser spots projected onto moving components such as a rotating shaft. By using these computed target coordinates, relative positions between important machine structures can be determined and monitored. In the case of rotating shafts, geometric form-fitting is used to find the spatial positions of the axes and thus determine the relationship of one shaft end relative to the other.

In this study, an existing system developed by Donald Bayly (Bayly, 1991) is refined in order to provide more reliable answers to the question of shaft and non-shaft alignment. The existing system consists of hardware enabling one or more theodolites to simultaneously communicate with a portable computer as well as software to interpret and compute the raw directional data into an alignment solution. High accuracies of target positions are necessary for safe, efficient and long term operation of machinery. In the case of rotating shafts, excessive vibration, premature wear of components such as bearings, and reduced power output are symptoms of misalignment. The consequences range from inefficient operation to catastrophic failure (Neale et al., 1991). Target accuracies of 0.025 millimetres in offset and 0.00015 radians ( 30 arc seconds) in rotation are not unreasonable and, depending on the situation, may be necessary (Bayly, 1991).

The primary aim of this investigation is to improve the existing system with error checking and automating the entire process from data collection to a final alignment solution. New software to accomplish these tasks range from adding new functions to Bayly's existing code to writing new programs generating approximate coordinates and displaying a numerical and graphical
alignment solution. The study of advanced surveying techniques necessary for rigorous deformation monitoring is also investigated to determine if the alignment solution can be improved upon.

After discussion of existing alignment systems and software in Chapters Two and Three, chapters on each of the specific programs are presented. In Chapter Four enhancements to Bayly's existing COLLECT software are discussed; in Chapter Five a new program to generate approximate coordinates is presented; and finally new shaft misalignment and offset software is discussed in Chapter Six. Chapter Seven presents testing on the complete system in various settings while Chapter Eight extends the reliability of the whole system by using deformation analysis to determine whether socalled "stable" reference points are in fact stable.

The objective of this study is to create new software to improve the reliability and efficiency of an existing theodolite intersection system. As part of this investigation, the author has learned a great deal about industrial alignment as well as solidifying concepts in geomatics engineering and software development.

## Chapter 2

## Machinery Alignment

The alignment of machine components is vital for efficient and safe operation. A turbine/compressor unit (owned by the Husky Oil Company located at the Stolberg compressor plant west of Red Deer, Alberta) is an example of the consequence of shaft misalignment (Robbins, 1992). In this situation, excessive vibration due to shaft misalignment caused internal warning systems to be activated initiating a full machine shut-down. This situation was observed when the temperature external to the compressor building was very low-below minus $20^{\circ} \mathrm{C}$. The costs associated with this unexpected shut-down were substantial requiring an investigation to be performed. The importance of shaft alignment as well as deformation monitoring was emphasized in this example. (See Chapter 8 for further details.)

In the following sections, details on the importance of alignment are presented along with an overview of existing alignment systems and a specific section on theodolite intersection systems. Investigated literature leaves no question as to the need for less intrusive yet more accurate shaft and nonshaft alignment systems.

### 2.1 Importance of Proper Machine Alignment

Rotating and non-rotating machines need to be aligned periodically. In the case of any moving parts, the tendency for alignment changes to occur is usually relatively high. Due to the continuous and repetitive operation of rotating machinery such as with the turbine/compressor discussed in the introduction, the alignment of rotating machinery is especially important. Mirro (1991) specifies five areas where the alignment of rotating machinery is not only desirable, but also necessary (Mirro, 1991, p. 183):

1. Machinery built before the development of today's powerful design tools in the areas of rotor dynamics and bearing design.
2. Marginal machines that operate satisfactorily until they are rerated, their output resulting in the need for mechanical upgrade.
3. Poorly designed or improperly designed after sales parts that have been retrofitted to existing machines.
4. Units that can be improved via design optimization in order to enhance future operational reliability.
5. Analytical design studies that uncover potential future operational machine problems.

All of these points are important in rotating machinery maintenance. The first three deal with machinery that has been operated for a long period of time since the initial purchase. The last two deal with existing machinery or with newly installed devices. One key point is that alignment begins during formation of design specifications before any machine is built. When
construction is underway, these design tolerances must be followed carefully and should rarely be deviated from without careful study of the ramifications.

The most readily apparent symptom of misalignment in rotating machinery is vibration. Although monitoring this symptom will provide various suggestions as to what alignment problems exist and where they are located, vibration monitoring by its nature is indirect. Most methods more direct than vibration monitoring, however, have had a major disadvantage in that they offer only a cold alignment solution where machinery is shut-down during measurements.

Preventive maintenance approaches are still quite valid as they were decades ago. In some situations, these have given way to predictive maintenance. Cullen (1988) lists three categories of industrial equipment maintenance (Cullen, 1988, p. 71):

## 1. Preventive Maintenance

- carried out at fixed intervals, either based on elapsed calendar time or elapsed fired (operational) hours.


## 2. Predictive Maintenance

- non-periodic maintenance based on equipment condition...determined by trending selected parameters and monitoring vibration and performance


## 3. Failure Maintenance

- un-planned and usually involve little or no data collection.
- equipment is repaired/replaced after failure.
- this type of equipment is usually inexpensive and non critical.

Presently, the desire to switch from preventive to predictive maintenance is a goal of machine operators, especially those operating troublesome machines. Phillips Petroleum Company of Norway is an example of a company that had been actively studying this goal of adding predictive maintenance to their maintenance schedule for machinery used in oil and gas fields in the Norwegian North Sea (Cullen, 1988).

Although predictive maintenance is desirable, the source of the data used in the trending technique could degrade its usefulness. Campbell (1992) and Bloch (1991) emphasize that the manufacturer's predicted machine alignment changes have usually been shown to be unreliable. Cold alignment techniques may miss changes in alignment due to thermal expansion, piping forces, baseplate movement, or changes in radial or axial forces (Neale et al., 1993). Hot and cold alignment as well as the transitions from either state would provide the necessary data to predict alignment changes during various phases of operation assessing performance and machine health.

Performance and mechanical health are not the same. Performance tests on rotating machinery relate to factors such as pressure, temperature, flow, efficiency and power. By passing all of these tests, mechanical health is still not guaranteed (Godse, 1990). Mechanical health relates to routine tasks such as actual shaft alignment. In other words, an efficient and productive machine may still have alignment problems that, if ignored, may eventually cause failure.

Hashemi (1983) discusses a 500 MW turbine where differential thermal growth of bearing pedestals plus excessive wear and distortion of guide keys cause vertical misalignment. Preventive and predictive maintenance would
have prevented the undesirable results including changes on bearing loads. Vertical and transverse misalignment can cause the loss of bearing integrity, dynamic rotor response, varying coupling stresses, and rotor rubbing (Hashemi, 1983, p. 21).

Most rotating machinery such as turbo-generators should not experience major problems during operation. "There are occasions, however, when only a full scale investigation, using comprehensive instrumentation and calculational procedures can identify the root cause of the problem." (Hashemi, 1983, p. 24) In Chapter 8, this ideal will be integrated in studying the software improvements discussed in preceding chapters by the inclusion of deformation monitoring.

### 2.2 Overview of Some Existing Alignment Systems

In this investigation, enhancements to an existing theodolite intersection system are detailed. This method, in the context of machine alignment at a large scale (i.e. small area), has been refined over the past few years. One approach is to categorize older alignment methods as those that, for the most part, require physical contact with the object being aligned. In contrast, newer methods can be defined as those that have limited or no contact with the machinery. Generally these newer methods (including theodolite intersection) offer substantial improvements over those of the past decade.

There are various methods that have been used for the alignment of rotating machinery for a number of years. Some provide solutions and techniques more applicable to either coaxial or parallel shafts. Most allow only static (cold) alignment yet some indirect methods can be used while the
machinery is operating. Data collection has traditionally required operators and assistants to manually record data on paper while measurements were being made. This lead to reading and recording errors that were difficult to avoid. Presently more and more methods have, to some degree, automatic data collection and analysis capabilities provided by using portable data collectors or computers.

Periodic alignment during shut-down provides a useful but limited ability to eliminate or avoid problems caused by misalignment. As discussed in the previous section, monitoring during operation is more desirable for predictive as well as preventive maintenance. Short term effects such as alignment changes caused by daily external heating and long term effects such as seasonal alignment fluctuations cannot be identified unless full-time monitoring is utilized. Gathering this data for rotating shafts has limited the ability of older methods to simply monitoring changes in static supports or other machine components during operation. If carefully utilized, the data can be used to predict actual shaft alignment in an indirect manner. Users of indirect alignment monitoring devices have been surprised (in many instances) by discrepancies between predictions and the actual alignment readout (Neale et al., 1991). This gap can create problems if the limitations of the method are downplayed or ignored. Existing methods that are used for rotating machinery alignment include:

- dial indicator methods
- Essinger bars
- Dynalign bars
- optical tooling methods
- laser alignment methods

Dial indicators are gauges that are attached to brackets that are placed against particular portions of a shaft or coupling. Figure 2.1 shows one such system known as the rim and face method. The relationship between the shafts on either side of a coupling is measured.


Figure 2.1 Rim and Face Dial Indicator Method

Essinger bars (also known as the Acculign system) are not used directly on the shaft as are dial indicators. Instead, the change in position of a static machine component with respect to the machine foundation is measured. If the foundation itself moves, excessive piping forces could arise causing the machinery to become misaligned (Neale et al., 1991).

Dynalign bars (also known as Dodd Bars) are attached between two static machine components measuring relative changes in position. These systems can be expensive and are time-consuming to use. Movements of the
shaft within the bearings are not detected if this or other non-shaft methods are used.

Optical tooling methods encompass optical levels as well as optical jig transits. Optical levels can be used to measure vertical position, changes in position of static machine components, or the shaft itself if the machinery is not operating. Optical jig transits consist of a telescope that can be rotated to define a vertical plane. The intersection of this plane with a scale rod placed on a non-moving component provide a simple positioning method. A reference target can be used to check whether the jig transit has accidentally moved during measuring (Figure 2.2).


Figure 2.2 Top View of Width Measurements Using an Optical Jig Transit

Laser alignment methods are an advanced equivalent to dial indicators. Some systems provide continuous monitoring of static machine components while others can align the actual shafts if the machinery is shut-down. In Figure 2.3, a set of shafts is aligned across a coupling. On the left, a laser beam emitter is mounted on one shaft. The emitted beam is reflected off a prism (on the right) that is fixed on the other shaft. Changes in the position of the returned laser dot indicate the horizontal and vertical displacement of one shaft relative to the other.


Figure 2.3 Alignment with Laser-Optical Systems

### 2.3 Theodolite Intersection Systems

The use of theodolites to compute coordinates of targets using intersection is not new. Although large scale applications such as industrial alignment are much newer, theodolite intersection techniques have been used for over a century. The principle of theodolite intersection involves the intersection of two known optical rays (i.e. known directions relative to a
common datum). The geometry of the intersection plays the important role in obtaining the desired accuracy of the computed coordinates (see Chapter 3).

In Figure 2.4, a model of the basic geometrical components of theodolite intersection is presented. With a common direction between the two theodolites, the horizontal angles $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$ can be measured. Gravity provides a common vertical direction allowing vertical angles $V_{1}$ and $V_{2}$ to be accurately determined. Using this and other data dealing with the spatial positions of the two instruments, the coordinates of a commonly observed target $(X, Y, Z)$ can be computed.


Figure 2.4 Three-Dimensional Theodolite Intersection

A complete three-dimensional software package should be able to assist the user in collecting the most accurate and reliable data necessary for a useful alignment solution. As mentioned, when aligning rotating machinery the relative positions of the shaft axes must be known very accurately. To compute the positions of these axes, observations on the surface of the shaft can be made. (Observations to the actual axis of rotation are not possible.) By observing a set of points on the arc of the shaft's surface, computation of the relative position of the axis with respect to the surface can be made. Laser spots projected onto the rotating shaft can be used for this purpose.

Theodolite intersection is used to compute the spatial coordinates ( $X, Y, Z$ ) of each target on the shaft. Two well-positioned theodolites are usually adequate to find an accurate solution although more instruments can, to varying degrees, improve the solution. In order for the trigonometric computations to work, the spatial positions of both theodolites have to be known. The best technique to accomplish this in terms of versatility is called free stationing where the instruments can be placed at any convenient location allowing critical machine targets to be easily observed. In order to know where the theodilites are with respect to each other and within the area, resection is used.

Resection is a positioning technique where three or more known targets are observed from an unknown theodolite position. Using geometric relationships, the three-dimensional position of the theodolite can be found. Once two theodolites have been properly set up in front of machinery to be measured, observations are made to these known targets that are usually located on stable wall or floor positions in the vicinity. (Proper set-up
involves various techniques such as leveling, ensuring systematic errors are eliminated, and random errors are minimized; see Chapter 4.) The targets used for resection must be placed on stable and unobstructed structures such as support pillars or other stable locations ideally situated away from the actual moving machinery or related foundations. The targets can be of various types that are adhesive, typically having circular rings with a dot at the center being less than 0.3 millimetres in diameter.

To find these non-shaft target locations, a mathematical adjustment of the network of observations from the two theodolites is conducted. Without knowing the actual theodolite or wall target positions, mathematical optimization is used-typically the least squares method. The technique uses the geometry and relationship between data to find the "best-fit" solution of coordinates for the theodolites and wall targets in question. In order for the least squares method to converge to a solution, approximate target and theodolite coordinates must first be measured. (As part of this investigation, a method for the elimination of manual taping of approximate coordinates is developed and tested in Chapter 5.) Convergence of the method depends on various factors such as the position of each theodolite with respect to each other, as well as each target. It also depends on the type of instruments used and atmospheric conditions. If the approximate coordinates are not close enough to the "true" solution, divergence instead of convergence could result.

Stable wall targets are needed so that a common reference frame exists from which all other measurements can be based. If even one wall target is positioned incorrectly, this error may propagate causing errors in the position of the theodolites. This leads to errors in the intersected shaft target
coordinates eventually leading to erroneous shaft axis positions. Thus misalignments would be reported when in fact none may actually exist. The accuracy of the position of each wall target depends to a large degree on the separation of the theodolite stations. The ideal situation is to have the horizontal angle of intersection of the two theodolite optical rays be at $90^{\circ}$. At this angle any error in the position of either ray is limited in its effect by the other.

Another factor affecting target accuracy is the use of an accurate reference distance. A known distance is required to establish network scale usually by using an invar scale bar. Observations from both theodolites provide coordinates at the ends of the scale bar. By rescaling the network so that the computed distance between the bar's ends match the true value, all coordinates including those on the rotating shaft can be determined accurately. (See Chapter 5.)

One key issue is the importance in not only reporting coordinates for machine and non-machine targets but also providing an assessment of the accuracy of the results. This can be in the form of confidence regions or onedimensional values such as variances or standard deviations of the $\mathrm{X}, \mathrm{Y}$, and Z coordinate components. With this information, a more informed and considered decision as to alignment corrections can be made.

## Chapter 3

## Summary of Existing Software System

The system on which this investigation is based was developed by Donald Bayly (Bayly, 1991). This system was designed to apply theodolite intersection to industrial alignment. Certain software components deal with data collection while others are concerned with adjusting the data or fitting coordinates to geometric forms. In the following sections, details on this existing system are provided. The last section (Section 3.4) explains what additions or enhancements to this existing system are desired.

### 3.1 Overview of Software Components

Bayly's existing system consists of three separate but related software components: COLLECT, ADJUST, and FIT. Details on each of these programs will be covered later in this section. In general, the system uses horizontal and vertical data measured from two or more theodolites directly sent to a laptop computer. The data consists of horizontal directions and vertical angles as discussed in Chapter 2.

The horizontal direction is initially an angular value related to an arbitrary zero direction set within the theodolite. These horizontal values will be unrelated between each theodolite. As an example, zero for the first instrument may be pointing somewhere near south-east while zero for the
other may be near south. Thus the orientation of each theodolite will initially be unknown.

Vertical angles are measured from a horizontal plane upwards while complementary angles (known as zenith angles) are measured in a manner where the reference direction (zero degrees) is straight up-opposite the local gravity vector. This vertical reference direction is considered the same for both theodolites in most industrial alignment situations and thus these angles may be considered vertical directions as well. (An industrial network will usually not exceed twenty metres in size, thus geodetic concerns such as deflections of the vertical will essentially be the same; see Heiskanen and Moritz, 1987.)

This angular data is collected from either theodolite (if using two) and sent to a portable computer via COLLECT (Figure 3.1). Additional information regarding network scale and orientation must be manually added allowing the software to adjust the data and compute three-dimensional coordinates for all target points. Before this process can continue, approximate coordinate data must first have been collected at the site using tape measurements. Depending on the geometry, coordinates accurate to less than one centimetre may be necessary in later adjustments. This would only occur for for points situated at very weak intersections that should be avoided (see Section 5.4.2). Once the adjustment has been completed (using ADJUST) and target coordinates computed, geometric fitting (using FIT) can be used to compute the axes of the shafts. Details on these programs follow.


Figure 3.1 Existing Alignment Software System

## COLLECT

COLLECT is the first of three programs written by Bayly (1991). It is a data acquisition program written specifically for a multi-theodolite interface designed to communicate with two or three theodolites simultaneously. The program consists of three primary functions:

1. Set-up
2. Data collection

## 3. File conversion

The set-up function establishes file names, identifies active channels and communicates with the connected theodolites. The data collection function scans and receives data from the theodolites, performs confirmation of data, and writes the raw data into a file located on the computer's hard-disk. The file conversion function reads this raw data, computes mean horizontal and vertical measurements for each theodolite/target combination, displays a
warning if data was collected on only one face (see below), and saves the data in a format used by the next software module, ADJUST.

To ensure correct measurements are recorded, and to ensure systematic errors inherent in theodolite usage are eliminated, observations should be made with the theodolite telescope positioned directly and then plunged or reversed. This will eliminate a number of systematic errors that could easily occur if the theodolite is not set up and adjusted perfectly. The existing system does not incorporate error checking routines that would check for these errors. (See Chapter 4 for full details.)


#### Abstract

ADIUST ADJUST is a general three-dimensional least squares adjustment program that finds the "best" geometrical solution for a network of observations from theodolites to various targets including those on machinery. This approach to computing coordinates using a network has a number of advantages over another technique called mutual pointing where each theodolite must observe the other very accurately. Mutual pointing can be time-consuming and is open to a number of potential errors. Many of these may be eliminated by using a scale bar centered over one of the theodolites (Grist, 1986; Sivaraman and Delaitre, 1987; Allan, 1987).

Before running ADJUST, the observational data converted by COLLECT must be edited. Using a text editor, a scaled distance and azimuth are added to the data. These act as pseudo-observations since they are, in fact, they are known quantities entered by the user of the program. This first file has a *.obs extension. (It is discussed in more detail in Chapter 4.)


One other file must be created before running ADJUST; one containing the approximate coordinates for every target and theodolite position being used (this file has a *.sta extension). In the existing system, the file is manually created containing the station (target) number and its $\mathrm{X}, \mathrm{Y}$ and Z coordinates. As mentioned previously, this data must be gathered manually.

When ADJUST is executed, the horizontal and vertical observations with the added distance and azimuth data and also the approximate coordinates are all used to find the "best-fit" solution for the network. The resulting output file contains the adjusted coordinates of the targets and the associated standard deviations. Another file can be created reporting details of the adjustment process including redundancy numbers (i.e. geometrical interdependence) as an option.

Advantages of this program include the following (Bayly, 1991, p. 70):

1. Directions, zenith angles, azimuths, spatial distances and height differences are accepted as observations.
2. Initial approximate coordinates can be individually weighted.
3. There is a choice of constraints by weighted coordinates or a free network solution.
4. There is a choice of pre-analysis or adjustment.

Disadvantages are (Bayly, 1991, p. 72):

1. The user must prepare a file of approximate coordinates.
2. No more than 35 three-dimensional stations can be adjusted simultaneously.

Manually adding a file of approximate coordinates can be very time intensive. Measurement of the approximate coordinates can be difficult or impossible depending on the accessibility of target points as discussed in Chapter 5. The actual creation of this file is not difficult but limits the efficiency of the entire intersection system. A more desirable situation is to not interrupt the automatic collection and computation process with manual steps.

## FIT

This is form-fitting software that fits geometric forms to threedimensional points (e.g. lines, planes or circles). The software performs a least squares adjustment of three-dimensional data of points on an object such as the surface of a shaft. When observing actual shafts, points observed on the side can be fitted onto a best-fit circle. FIT allows the user to fix the radius of the shaft if it is known allowing the best-fit circle to be very accurately defined.

The program first reads the coordinate file created by ADJUST using the contained $X, Y$ and $Z$ data as well as the associated standard deviations. The user indicates what form-fitting geometry is desired and can enter various a priori information such as the known radius of the shaft and the associated standard deviation. FIT computes the best-fit circle and plane on which this circle resides allowing the axis of the shaft to be determined.

A limitation of this approach is that the orientation of the plane on which the best-fit circle resides must be orthogonal to the true shaft axis. This in itself is difficult to impose without knowing the position of the shaft axis in
the first place. Another limitation is that FIT reports the orientation of the axes without indicating the relationship between them. In Chapter 6, these limitations are dealt with.

### 3.2 Discussion of Mathematical Models

Except for some hardware routines and other basic calculations, COLLECT does not contain any mathematical models that require further discussion. The programs ADJUST and FIT use various mathematical methods to arrive at the solution of either a best-fit network or best-fit geometric form respectively. In the following discussion, a brief overview of the basic principles used in these programs is presented. This provides an important base from which additions and enhancements have been added by the author. Further information can be found in Bayly (1991) or various other surveying texts.


#### Abstract

ADJUST

Least-squares adjustment involves modeling the effects of the changes in the functions defining the observations with respect to the changes in the related unknowns; $\left(\frac{\partial f}{\partial \bar{\chi}}\right)$. The observations, $\ell$ (horizontal and vertical directions) are a function of the unknowns-target coordinates $(\bar{\chi})$ : $$
\ell=f(\bar{\chi})
$$

Solving for the unknowns (in this case an over determined system where there are more observations than unknowns) a global minimum of the quadratic form of residuals can be found (see below).


The partial derivatives of the functions comprise a matrix called the design matrix (A) which can be used to compute the corrections to the original approximate coordinate estimates. As an example, a direction can be computed as

$$
D_{i j}=\tan ^{-1}\left[\left(x_{j}-x_{i}\right) /\left(y_{j}-y_{i}\right)\right]-d_{o}
$$

where $x$ and $y$ are the coordinates of either point $i$ or point $j$ and $d_{0}$ is an unknown direction corresponding to zero on the theodolite horizontal circle. Thus

$$
\frac{\partial D}{\partial x_{i}}=\frac{-\left(y_{j}-y_{i}\right)}{\ell_{h}^{2}}
$$

where $\ell_{h}^{2}=\sqrt{\Delta x^{2}+\Delta y^{2}}$. Equation 3.3 would form one element of the design matrix. The complete correction to the initial estimates can be presented in a well-known form (e.g. Bayly, 1991, p. 29):

$$
\hat{\delta}=-\left[\mathbf{A}^{\mathrm{T}} \mathbf{C}_{\ell}^{-1} \mathbf{A}+\mathbf{C}_{x}^{-1}\right]\left[\mathbf{A}^{\mathrm{T}} \mathbf{C}_{\ell}^{-1} \omega+\mathbf{C}_{x}^{-1} \omega_{\mathrm{x}}\right]
$$

where
$\hat{\delta}$-vector of corrections to the original estimates of the unknown target coordinates,

A-design matrix obtained by partial differentiation of each observation equation with respect to each unknown coordinate,
$\mathbf{C}_{\ell}$-covariance matrix of the observations,
$C_{x}$-a priori covariance matrix of the coordinates,
$\omega$-vector of misclosures $f(x)-\ell$,
$\omega_{x}$-vector of differences between initial and current coordinate estimates.

In essence, the aim of least squares adjustment is to reduce the remaining small differences between actual observations (i.e. from COLLECT) and the adjusted coordinates found by applying $\hat{\delta}$. Least squares is thus the process where a geometrical solution for coordinates (unknowns) is based on the minimization of the weighted sum of the squares ( $\hat{\mathbf{r}}^{\mathrm{T}} \mathbf{C}_{\mathbf{x}}{ }^{-1} \hat{\mathbf{r}}$ ) of the differences known as residuals, $\hat{r}$

$$
\hat{r}=\mathbf{A} \hat{\delta}+\omega
$$

The advantages of using this method of adjustment are:

- one observation requires one Equation (easily programmable),
- results clearly interpreted through error estimation,
- covariance matrix of coordinates is an important by-product.


## FIT

A similar adjustment approach is used in FIT as was described for ADJUST. The functions being adjusted apply to the particular conditions and constraints appropriate to the particular type of geometric entity being fitted. In particular, for circular form-fitting, one wishes to compute the spatial (three-dimensional) location of the circle (i.e. the shaft) center as well as the orientation of the circle. This is defined by a vector originating at the center of the fitted circle that is also normal to the plane of the circle.

With even the most careful observations and with the strongest geometry using well-placed theodolites, small errors will still exist in the size and orientation of the geometric form such as the circle. These errors can be minimized by adjusting the model with over-determinate data (i.e. more
observations than unknowns): Three points on the side of a shaft would provide a unique solution to determine the perimeter of the circle. (A minimum of three points are necessary for any circular solution.) By observing more points, an adjustment can be used to find a better solution for locating the circle. Bayly's method actually involves two steps, first finding the best-fit plane on which the points reside giving a vector defining orientation and then a best-fit sphere with the observed points on the surface. The center of the sphere is then computed (Bayly, 1991, pp. 33-40). (Investigations have revealed that a cylindrical model may provide a more accurate solution when dealing with shafts but will not be particularly necèssary; Harvey, 1991; Obidowski, 1992).

In the case of circular form-fitting, two condition and two constraint equations are used. The condition equations are:

$$
\begin{aligned}
& a x_{i}+b y_{i}+c z_{i}+d=0 \quad \text { (plane) } \\
& \left(x_{i}-x_{o}\right)^{2}+\left(y_{i}-y_{o}\right)^{2}+\left(z_{i}-z_{o}\right)^{2}-r^{2}=0 \text { (sphere) }
\end{aligned}
$$

and the constraints are:

$$
\begin{align*}
& a^{2}+b^{2}+c^{2}-1=0 \text { (vector of unit length) } \\
& a x_{o}+b y_{o}+c z_{o}+d=0 \text { (center on plane) }
\end{align*}
$$

where $\langle a, b, c\rangle$ is a unit vector normal to a plane cutting through this sphere at the center. The intersection of the plane and sphere define a circle; $\left(x_{i}, y_{i}, z_{i}\right)$ is the coordinate of a point on the perimeter of this circle (i.e. the surface of the shaft); $\left(x_{0}, y_{0}, z_{0}\right)$ is a point at the center of the circle; $r$ is the radius of the sphere; and $d$ is a perpendicular distance from the plane of the circle to the coordinate system's origin.

An adjustment model similar to Equation 3.4 is used in FIT to compute the corrections to the approximate coordinates of the above equations. Since there are both condition and constraint equations, two different design matrices are necessary thus making the use of a unified least squares method applicable (Bayly, 1991, pp. 34-35).

### 3.3 Tested Applications Using Existing System

Bayly has successfully used the existing system in various settings. Being non-intrusive, various moving and non-moving objects have been measured and aligned. Although only variance data is propagated through the process from COLLECT to FIT (see Chapter 8), the following tests have proven to be successful (Bayly, 1991):

## ALIGNMENT OF A ROTARY KILN

This problem involved the alignment of a 5 metre diameter kiln (used in the production of cement) under operating conditions of about 2 revolutions per minute in rotation. Using the theodolite intersection system, points on the shaft supports (steel tires supporting the shaft) were measured using a projected laser spot. Using the discussed programs, FIT was used to find the center of each steel tire (located at intervals of about 30 metres). Center coordinate accuracies of 1 to 2 mm were obtained, being about three times the accuracy of most conventional cold alignment techniques. Approximate coordinates were tape measured and final alignment computations were manually calculated.

## ALIGNMENT OF A GRANULATOR

These rotating devices are used in the production of fertilizer. A shaft of about 3 metres in diameter was aligned in addition to trunnions (rollers) supporting the granulator shaft. In this situation, a traditional cold alignment technique would require days to set up although additional time was necessary to compute alignment solutions. Alignment using theodolite intersection with laser spots on the trunnions and shaft found that axes of two of the four trunnions were misaligned with respect to the axis of the granulator drum by about 0.005 radians.

## ALIGNMENT OF A TURBINE-GEARBOX-COMPRESSOR TRAIN

This case involved a train of shafts used in the transmission and processing of natural gas. Using the existing theodolite intersection system, alignment of these shafts was determined. Changes in the three-dimensional coordinates of points between the gearbox and compressor were found to be the same from cold to hot as from hot to cold operating conditions. The changes were as large as 0.90 millimetres in magnitude.

A variety of other applications have been tested using the existing system. The most significant limitation to the existing system is the time necessary to set up, measure, and then compute alignment parameters. In the next section, specific desired improvements to the system are presented.

### 3.4 Desired Additions or Improvements to Existing System

## COLLECT

Rohde mentions several ideal attributes that should be considered in any data acquisition software (Rohde, 1987, pp. 442-443):

## Ease of Use

The software should be self-explanatory without constant reference to a users' guide. An on-line help facility provides assistance in selecting appropriate functions.

## Error Handling

Methods for handling error conditions due to an incorrect keystroke sequence or mechanical problem with the computer or survey instrument mandatory.

## Data Integrity and Security

The original, unprocessed observation data must be preserved, preferably on an external mass memory device. Data files should be protected from accidental erasure or corruption. Only secondary data should be available for editing.

## Consistency and Organization

The procedural and functional segments of the software should be consistent and follow a logical sequence.

## Complexity and Flexibility

The methodology incorporated in the software must be practical and mathematically sound. The ability of the software to adopt to variable observation conditions and constraints is important.

## Adequacy of Printouts

The system should provide concise summaries to detailed listings.

All of these points with exception of error handling are well considered in various areas of Bayly's existing system. Error handling capabilities must be added to COLLECT to ensure instrument or user errors are caught before leaving the job-site. Details of these additions are found in the following chapter.

## ADIUST

The disadvantages of the existing ADJUST software as mentioned in Section 3.1 are (Bayly, 1991, p. 72):

1. The user must prepare a file of approximate coordinates.
2. No more than 35 three dimensional stations can be adjusted simultaneously.

Bayly (1991) suggests four specific areas of improvement desired in the existing system. The first point is listed below. The second reiterates the limitation of only 35 stations, while his third point deals with the data transmission rate through his interface, and the fourth is concerned with
refraction and vibration effects on observations. (These have been investigated by Robbins, 1992 and Al-Hanbali, 1993). Bayly's first point concerned with desired improvements is (Bayly, 1991, p. 129):

> "A program to automatically generate initial approximate coordinates could save time compared to the present procedure: tape measurements, estimation and manual entry of the approximate coordinates file."

In Chapter 5 a new program, APPROX, which automatically generates initial approximate coordinates, is discussed.

## FIT

The form-fitting software FIT will only report the position and orientation of the shaft axes using vectors. A desired alignment solution would require these vectors to be compared as well as the associated errors to be propagated through the model. As an addition to FIT, a new program SHAFT was programmed to accomplish these tasks including adding a graphical display of the results. The development and testing of SHAFT is discussed in Chapter 6.

## Chapter 4

## Enhancements to COLLECT Software

The hardware and s.oftware systems designed by Bayly (1991) were developed to be mutually supportive. In this chapter, a limited portion of the software will be discussed relating to enhancements (see Section 4.3) although most of the program actually involves theodolite communication.

### 4.1 Summary of Existing Features

In Figure 4.1 the general set-up of this system is demonstrated. The basic system consists of two electronic theodolites connected to a multi-theodolite interface subsequently connected to a portable personal computer (PC).


Figure 4.1 Bayly's Theodolite Intersection System (Adapted from Bayly, 1991)

The program COLLECT is used to coordinate and decode signals arriving either from the theodolites or the computer. Thus an operator can control some aspects of data collection from the theodolite (e.g. the user entering target numbers) while other aspects are controlled from the computer such as software mode (see below).

The operator can control essentially all portions of data collection so that any directional data (horizontal and vertical directions) can be automatically transferred from any connected T2000 or T2002 theodolite to the computer for storage and analysis. As mentioned in Chapter 3, the three main tasks of COLLECT are: (1) Set-up of interfacing and communication system, (2) Data collection, and (3) Conversion of the raw data files. These are outlined in Figure 4.2.

```
Set-up:
    - name the data file
    - identify active channels and theodolite station numbers
    - handshaking with theodolites
Data Collection:
    - scan channels, read incoming characters
    - send back data confirmation
    - periodically append raw data to *.tdf file
File Conversion:
    - read raw data file
    - compute mean Hz & V values for each theodolite/target set
    - display warning if target viewed on one face only
    - save averaged observations in *.obs file
```

Figure 4.2 COLLECT's Main Functions (Bayly, 1991, p. 68)

The *.tdf file format (mentioned in Figure 4.2) consists of a number of lines of data where each line corresponds to one observation from either theodolite. These lines include the theodolite and observed target numbers and the horizontal and vertical direction values.

Observations are sent to the computer during the data collection phase by having the theodolite operator set and send the station number of the target being observed from the theodolite control panel. Once all points of interest are observed using direct and reverse theodolite positions, a key can be pressed on the computer to halt data collection.

Next the *.tdf file is used to compute average observations stored in the *.obs file. Here the software determines all common theodolite/station observations and computes a mean horizontal and vertical value for each set. For example, this portion of COLLECT would calculate the mean horizontal and vertical value for all observations from theodolite station $i$ to target $j$. To eliminate a number of potential systematic errors, observations should be made on both faces of the theodolite; in both the direct or reversed positions. (Presently COLLECT will only warn the user if observations are gathered from only one face of the instrument.) Lastly the averaged data is stored in the *.obs file.

Various other systems such as the ROM System (Rohde, 1987) use an integrated collection package that includes orientation of the reference network as part of the data collection system. COLLECT does not do this. The main advantage of systems that integrate orientation is that data is ready for immediate processing. The most significant disadvantage is a loss in the
flexibility of the system in setting various orientation changes in post processing.

### 4.2 Theodolite Errors

The main objective was to enhance COLLECT by adding error checking. In the next section, specific software enhancements are discussed. In this section, relevant background on errors is briefly covered.

Cooper (1987) and Teskey (1991) identify specific random and systematic errors in a theodolite. These can be described as follows:

## Random Errors

Random errors are a measure of uncertainty in theodolite measurements. They have a mean value of zero and will still be present after systematic errors and blunders have been eliminated.

## Systematic Errors

Systematic errors can be eliminated by the direct/reverse procedure as well as dual axis compensation.

## Blunders

Blunders are gross errors usually caused by the operator that can be eliminated by checking collimation and vertical index results. (In the following section, enhancements to COLLECT are presented with the intention of eliminating blunders and monitoring systematic errors.)

The random errors can be described as follows:

Random Reading Error $\left(\sigma_{d}\right)_{r}$ :
Problem: Error caused by reading theodolite directional micrometer readings.

Solution: Using high quality electronic theodolites that automatically read directions (i.e. no manual micrometers).

Random Pointing Error $\left(\sigma_{d}\right)_{p}$ :
Problem: Direction to a target slightly different than previous observation from same theodolite position.

Solution: Use high quality telescope with good magnification abilities plus well defined and visible targets.

Random Centering Error $\left(\sigma_{d}\right)_{c}$ :
Problem: Theodolite and targets not positioned correctly over ground station on subsequent set-ups.

Solution: Instead of using ground targets, use theodolite free stationing and fixed targets.

## Random Leveling Error $\left(\sigma_{d}\right)_{l}$ :

Problem: If theodolite is not level, directional observations incorrect.

Solution: Use automatic vertical circle compensator (AVCC) to reduce errors.

Total Random Error:

$$
\left(\boldsymbol{\sigma}_{d}\right)_{\text {toala }}=\left[\frac{\left(\boldsymbol{\sigma}_{d}\right)_{r}^{2}+\left(\boldsymbol{\sigma}_{d}\right)_{p}^{2}}{n}+\left(\boldsymbol{\sigma}_{d}\right)_{c}^{2}+\left(\boldsymbol{\sigma}_{d}\right)_{l}^{2}\right]^{\frac{1}{2}}
$$

where $n$ is the number of pointings.
Theodolite systematic errors (Table 4.1) can be controlled if the following precautions are taken into account (Cooper, 1987, p. 146):

| Table 4.1 The Effect and Solution of Systematic Errors |  |  |
| :--- | :--- | :--- |
| Type of Error | Effect of Error | Solution |
| Vertical axis not vertical | Error in Hz circle reading | Dual axis <br> compensation |
| Hz collimation error | Error in Hz circle reading | Observe in both <br> direct and reverse <br> positions |
| Cross-hairs inclined | Error in Hz and V readings | Observe in both <br> direct and reverse <br> positions |
| Vertical circle index error | Error in V circle reading | Observe in both <br> direct and reverse <br> positions |
| Existence of parallax | Error in Hz and V readings | Focus x-hairs and <br> adjust equipment |
| Optical plummet error | Variable error in Hz reading | Free stationing |

In Figure 4.3, the relative positions of the theodolite components are illustrated. The line of collimation (telescope axis) is nominally perpendicular to the trunnion axis. Collimation errors will result if the operator observes targets only on one face (i.e. direct or reverse theodolite positions). The orientation of the trunnion and telescopic axes should nominally be perpendicular while the entire instrument should be level.


Figure 4.3 Theodolite Internal Orientation

In the enhancements discussed in the following section, those errors that can be eliminated by observing on both faces (the blunders) are detected and reported within the revised COLLECT program. These, along with reported systematic errors, provide the user with an indication of theodolite adjustment quality as well as the quality of the observations.

In Figure 4.4, individual and average station collimation errors are demonstrated. When the telescopic axis is not perfectly perpendicular to the trunnion axis, an angular error will result. By dividing the difference between the direct and reverse (face-left/face-right) observations in half, a collimation error can be computed. The average of each single collimation error provides an excellent assessment of the relationship between the two indicated axes in the theodolite.


Figure 4.4 Single and Station Collimation Errors

In Figure 4.5, another addition to COLLECT is demonstrated. If the user accidentally points the theodolite to different target than intended, the
deviation of this angular value with respect to other values can be detected and reported. These are known as pointing errors.

## POINTING ERRORS



Figure 4.5 Theodolite Pointing Error

Thus the key is discover and eliminate errors before leaving an alignment job. Other similar software systems offer various error checking abilities. Grist (1986) and Lardelli (1987) describe a method to check for pointing errors using target heights. Rohde (1987) mentions an option in his system to correct for dislevelment and the ability to enter collimation correction values if they can be found beforehand. Bingley (1990) describes systems that contain various levels of data verification and error checking. A number of these systems assume only one face of observations are necessary. As mentioned, this approach is not advisable.

COLLECT has been improved by adding features that compute and present single observation collimation errors, vertical index errors, theodolite station collimation/index errors, and pointing errors. These account for potential errors that can be controlled as outlined in Table 4.1. The use of well-maintained theodolites with dual axis compensation will control the remaining problems not associated with direct and reverse theodolite readings.

### 4.3 New Features Added to COLLECT

As discussed previously, raw observational data is immediately stored in a *.tdf file. This file is never altered or changed in any manner by the added functions (see Rohde (1987), Section 3.4.). An observation set consists of a pair of readings: one horizontal direction and one zenith angle for a sighting from a particular theodolite station to a particular target. For example, one horizontal direction such as $348^{\circ} 24^{\prime} 12.7^{\prime \prime}$ and one vertical value such as $90^{\circ}$ $57^{\prime} 58.0^{\prime \prime}$ are stored for an observation set from theodolite station 201 to target number 10. Originally COLLECT would store these raw observations in one *.tdf file and then, once all data had been gathered, process these into mean observations stored in a *.obs file. If observations were made on only one face (i.e. either direct or reversed positions), the program would simply report this fact without any further actions.

The author, consulting with Donald Bayly, decided to add three important error checks:

- Single point collimation and vertical index errors,
- Station collimation and vertical index errors, and
- Pointing errors.

Each of these were discussed in the previous section. In Table 4.1 in Section 4.2 systematic errors inherent in surveying with theodolites were discussed. Bayly's system uses T2002 theodolites that offer high precision angular readings (0.1 arc second least count) and dual axis compensation. Freestationing is part of the process where the intersection of the vertical, trunnion, and telescope axes define the theodolite's position-not a mark placed on the ground below the instrument (see Figure 4.3). Thus, what remains are those systematic errors that can be controlled by direct and reverse theodolite readings.

The author's additions to COLLECT are almost completely isolated into a C-language function called "create_obs_file". This module computes and saves mean observations into the observation (*.obs) file being compatible with the rest of the intersection system. Originally this function would find all the same observations and compute a mean for each face. The mean of the direct and reverse mean values would be computed and stored in the observation output file. If only one face was used, a warning flag would be set and later a message indicating this would be displayed.

In extending this module, code was added to compute and present horizontal and vertical pointing errors using an interactive display on the PC. The software steps can be summarized as follows:

First the sum of the horizontal and vertical observations are computed for all common observations on the direct face (face-I). For example, the sum of all observations from theodolite station 201 to target 10 observed with the
telescope directly. These sums are stored in variables $\mathrm{Hz}_{\text {sumI }}$ and $\mathrm{V}_{\text {sumI }}$ Similarly for the reversed position (face-II) the variables are $\mathrm{Hz}_{\text {sumII }}$ and $\mathrm{V}_{\text {sumII }}$ - While these values are being computed, each raw observation is stored into an array known as FACE. (A raw or unprocessed observation consists of the theodolite and station numbers plus the horizontal and vertical measurements.)

$$
\mathrm{FACE}=\left[\begin{array}{ccc}
\mathrm{obs} \mathrm{\#} & \mathrm{~Hz} & \mathrm{~V} \\
\vdots & \vdots & \vdots
\end{array}\right]
$$

The observation number (obs\#) is the sequential number of the observation as recorded in the raw data file ( ${ }^{*}$.tdf). Thus the twenty-third observation (23rd record) in the raw data file has observation number 23 . As an observation example, all face-I observations from theodolite station 201 to target 10 are saved in FACE; each on a separate record. Later, when face-II observations are dealt with, FACE will be purged of face-I data and filled with those from face-II. This matrix as well as all others created by the author are dynamically allocated to conserve memory.

Next the mean horizontal and vertical values are computed separately for each face,

$$
\begin{align*}
& \mathrm{Hz} \mathrm{z}_{\mathrm{m}}=\frac{\mathrm{Hz}_{\mathrm{sum}}}{\# \mathrm{obs}} \\
& \mathrm{~V}_{\mathrm{m}}=\frac{\mathrm{V}_{\mathrm{sum}}}{\# \mathrm{obs}}
\end{align*}
$$

Also computed are the deviations of each observation stored in FACE from the mean values in Equation 4.3 and 4.4. If this difference is larger than a
pre-determined tolerance value, a flag is set. These horizontal and vertical deviations for observation number $n$ in FACE are,

$$
\begin{align*}
& e_{H z}=\mathrm{FACE}_{\mathrm{n}, 2}-\mathrm{Hz}_{\mathrm{m}} \\
& e_{V}=\mathrm{FACE}_{\mathrm{n}, 3}-\mathrm{V}_{\mathrm{m}}
\end{align*}
$$

If any error is found (i.e. $e_{\mathrm{Hz}}$ or $e_{\mathrm{V}}$ exceed tolerances), a pointing errors screen is displayed. Corresponding opposite face observations appear at the same time as all other screen data for user comparison. If screen space is too limited, an option allowing an opposite-face data screen is presented for the user's comparisons.

```
POSSIBLE POINTING ERRORS
FROM 201 TO 10:
FACE 1 (Values with a '*' deviate from mean by at least 10'.)
Mean Hz = 174 2744.9
Mean V = 2592450.3
    obs .tdf line # Hz V
    1 32 * 1713851.6 *259 048.8
    2 51 * 177 16 38.1 * 259 48 51.8
Press '0' to continue,
Or '1' through '2' to omit entire obs.
(Mean values will be recalculated.)
>2
Corresponding FACE 2 Observations: (REFERENCE ONLY)
\begin{tabular}{llll}
.tdf line \# & Hz & V \\
16 & 35139 & 9.4 & 1005923.1
\end{tabular}
```

Figure 4.6 New COLLECT Pointing Errors Screen

This screen appears if either $e_{\mathrm{Hz}}$ or $e_{\mathrm{V}}$ exceeds the tolerance values. In Figure 4.6, an observation from theodolite station 201 to target 10 is displayed due to its value not being near to the mean horizontal and vertical observational values. The values with an asterisk $\left(^{*}\right)$ are flagged as deviating from the mean by 10 arc seconds (" in Figure 4.6); in this case, the set tolerance value. It can be seen that the corresponding face-II value can be compared with the ones simultaneously listed above. In this example, the second observation on faceI is not the compliment of the face-II value. In other words, $177^{\circ} 16^{\prime} 38.1^{\prime \prime}+$ $180^{\circ}=357^{\circ} 16^{\prime} 38.1^{\prime \prime}$ is considerably different than the first observation ( $171^{\circ}$ $38^{\prime} 51.6^{\prime \prime}+180^{\circ}=351^{\circ} 38^{\prime} 51.6^{\prime \prime}$ ). In comparison, the first observation value of $351^{\circ} 38^{\prime} 51.6^{\prime \prime}$ is considerably closer to $351^{\circ} 39^{\prime} 09.4^{\prime \prime}$. Thus observation 2 is removed by entering number " 2 ". The mean values in Equations 4.3 and 4.4 are recomputed with the appropriate FACE record being flagged as empty,

$$
\begin{align*}
& \mathrm{Hz} z_{\text {sum }}=H z_{\text {sum }}-\mathrm{FACE}_{\mathrm{n}, 2} \\
& \mathrm{~V}_{\text {sum }}=\mathrm{V}_{\text {sum }}-\mathrm{FACE}_{\mathrm{n}, 3}
\end{align*}
$$

Next collimation and vertical index errors are computed for the particular observation set being processed (e.g. all observations from 201 to 10). First, if the theodolite user observed on only one face, the $e_{\text {coll }}$ and $e_{\text {vindx }}$ errors are set to zero since differences cannot be computed. If observations on both faces (direct and reversed) are available, these are computed as the mean of the mean horizontal or vertical values from Equations 4.3 and 4.4,

$$
\begin{align*}
& e_{\mathrm{coll}}=\frac{\mathrm{Hz}_{\mathrm{mI}}-\mathrm{Hz}_{\mathrm{mII}}}{2} \\
& e_{\mathrm{vindx}}=\frac{\mathrm{V}_{\mathrm{mI}}-\mathrm{V}_{\mathrm{mII}}}{2}
\end{align*}
$$

A warning is displayed if $e_{\text {coll }}$ or $e_{\text {vindx }}$ exceed a defined tolerance:

Warning: FROM 201 to 10: Horizontal collimation error is 133.2"
Warning: FROM 201 to 10: Vertical index error is $54.3^{\prime \prime}$

Figure 4.7 Warning Screen Reporting Individual Errors

If any error is found (i.e. $e_{\mathrm{Hz}}$ or $e_{\mathrm{V}}$ exceeding tolerances), collimation and vertical index errors are displayed on the screen.

The entire procedure described up to this point is repeated for each observation from the particular theodolite being analyzed. In the above examples, this would be all observations from theodolite station 201. As each new collimation and vertical index error is computed using Equations 4.9 and 4.10, each value is stored in a new array,

$$
\text { STN_COLL }=\left[\begin{array}{cccc}
\text { STN\# } & \text { TARGET\# } & e_{\text {coll }} & e_{\text {vindx }} \\
\vdots & \vdots & \vdots & \vdots
\end{array}\right]
$$

The array holds all observations from all theodolite stations to all targets.
Next, a mean of all the collimation and vertical index errors is found for the theodolite station in question,

$$
\begin{align*}
& e_{c(\text { man })}=\frac{\sum e_{\text {coll }}}{\# \text { targets }} \\
& e_{v(\text { man })}=\frac{\sum e_{\text {indx }}}{\# \text { targets }}
\end{align*}
$$

where $e_{\mathrm{c}}$ and $e_{\mathrm{v}}$ refer to collimation and vertical index errors respectively. The number of targets expressed in the denominator is the number of targets observed from a particular theodolite station such as 201.

The difference between each individual collimation and vertical index error stored in STN_COLL and the mean collimation/vertical index errors is,

$$
\begin{align*}
& e_{\mathrm{stn} \_ \text {coll }}=\mathrm{STN}_{-} \mathrm{COLL}_{\mathrm{n}, 3}-e_{\mathrm{c}(\text { mean })} \\
& e_{\mathrm{stn} \_\mathrm{vindx}}=\mathrm{STN}_{-} \mathrm{COLL}_{\mathrm{n}, 4}-e_{\mathrm{v}(\text { mean })}
\end{align*}
$$

As the entire set of observations for all theodolite stations is processed, the largest $\mathrm{e}_{\text {stn_coll }}$ and largest $\mathrm{e}_{\text {stn_vindx }}$ are stored.

Lastly, the largest station collimation and vertical index errors (Equations 4.14 and 4.15) are displayed for each theodolite observation station. Figure 4.8 gives an example of one of these screens:

Instrument station 201:
Mean Hz collimation error $=10.4^{\prime \prime}$
Mean V index error = 5.1"
Maximum HZ deviation from mean is $15.3^{\prime \prime}$ for observation TO 30 Maximum V deviation from mean is $2.1^{\prime \prime}$ for observation TO 80

Enter 'c' when ready to continue $>c$

Figure 4.8 Example of an Instrument Station Error Screen

### 4.4 Testing and Results: COLLECT

### 4.4.1 Studies of Previously Obtained Data

Additions to COLLECT were developed using various sets of test data-mostly gathered by Bayly and provided to the author. Some of the data was from laboratory testing while other data was collected from actual field sights. The data was used for program development. .This was possible since the original COLLECT functions that were used to record and store raw data were not changed. Changes occurred only within the function to process this data as described in Section 4.3. COLLECT allows one to use existing raw data files (*.tdf) and append new data or process the existing data into the observation file (*.obs) format.

## LAB5

This was the primary set of data used in developing the enhancements to COLLECT (raw data file name lab5.tdf) and was originally collected for an undergraduate laboratory experiment. Two file cabinets were set up with the backs facing one another. Circles, representing opposing shafts, were carefully drawn on two pieces of paper. The pieces of paper were then taped to the backs of the filing cabinets. Targets marked on the perimeter of these two circles were observed using two T2002 theodolites and COLLECT.

No pointing errors were found using the enhanced version of COLLECT (as was expected using this test data). All results computed by COLLECT were manually calculated to verify program output. No individual collimation or vertical index errors were found for a tolerance value set at 20 arc seconds.

The mean collimation and vertical index errors for theodolite station 201 were 2.6 arc seconds and 3.5 arc seconds respectively. For theodolite station 202, the corresponding values were 6.9 and 2.8 arc seconds. These are reasonable values for well maintained instruments and the different values for theodolite stations 201 and 202 reflect that two different theodolites (with internal axes adjusted differently) were used. Since COLLECT uses the averaged values, these small error values were eliminated in the resulting observation file.

## KILN

The next test data (kiln.tdf) used in the development of the enhancements consisted of data gathered from observations on a rotating kiln (Bayly, 1991). For this particular data set, four theodolite stations were occupied using two different T2002 theodolites. Using the raw data file, the new version of COLLECT found a pointing error from station 204 to target 15 in the face-I readings. This screen is illustrated in Figure 4.9.
POSSIBLE POINTING ERRORS:
FROM 204 TO 15:
FACE 1 (Values with a '*' deviate from mean by at least 10 ".)
Mean $\mathrm{Hz}=202411.2$
Mean $V=102535.8$

| obs | .tdf line \# | Hz |  | V |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| 1 | 45 | $* 339$ | 5 | 13.4 | $* 100$ |
| 2 | 58 | $*$ | 43 | 16 | 49.0 |

Press ' 0 ' to continue,
Or '1' through '2' to omit entire obs.
(Mean values will be recalculated.)
$>1$
Corresponding FACE 2 Observations: (REFERENCE ONLY)

| .tdf line \# | Hz | V |
| :--- | :--- | :--- |
| 75 | 2461652.6 | 2555341.1 |

Figure 4.9 Kiln Pointing Error Screen

The erroneous observation (observation 1) was identified by noting the corresponding face-II data displayed at the same time. Only one observation exists in face-II and this corresponds closely to the second observation in faceI. Thus the first observation was eliminated with the desirable result of later screens reporting the mean horizontal collimation error being $1.7^{\prime \prime}$ and the mean vertical circle index error being 3.8" for theodolite station 204. This error could have easily been detected in the field using the enhanced version of COLLECT!

## STEFFI

This data (steffi.tdf) was gathered on a machine used in a shop run by Kadon Electro Mechanical Services Ltd. of Calgary: (The filename originated from tennis star Steffi Graf's name due to the machine being of German origin.)

```
FROM 2 TO 10:
FACE 1 (Values with a '*' deviate from mean by at least 10".)
Mean Hz=212 15 54.9
Mean V = 82 212.9
    obs .tdf line # Hz V
    1 8 * 212 1536.0 82 2 14.2
    2 55 2121638.1 82 2 15.0
    3 135 *21216 7.7 82 2 9.3
    4 141 * 2121610.9 82 2 13.0
Press '0' to continue,
Or '1' through '4' to omit entire obs.
(Mean values will be recalculated.)
>3
Corresponding FACE 2 Observations:(REFERENCE ONLY)
\begin{tabular}{lll}
.tdf line \# & Hz & V \\
18 & 321519.2 & 2775753.7 \\
67 & 321524.0 & 2775751.6
\end{tabular}
```


## Figure 4.10 Steffi Pointing Errors Screen

Upon re-processing the data with the enhanced version of COLLECT, theodolite station 2 (see first line in Figure 4.10) was found to have small inconsistencies: four observations were made on face-I to target 10 (see Figure
4.10) and four on the same face to target 20 (not shown). All other observations contained only two readings per target indicating some type of problem existed in the field when collecting data for these two targets. Referring to Figure 4.10, observation number " 3 " was first eliminated. Upon COLLECT's re-computation of the mean values, observation " 4 " was also found to be erroneous and thus eliminated. This resulted in the re-computed mean values and the remaining individual measurements being within the set tolerance.

Without correcting for these small pointing errors, the maximum horizontal and vertical deviations from the mean values were larger than when corrected. This is the result that one would expect if errors were not removed.

### 4.4.2 Results of Subsequent Testing

The enhanced version of COLLECT has been used successfully by the author several times. In Chapter 7 details on an alignment test rig constructed for testing of simulated shaft misalignments is presented. As part of this testing, COLLECT was used to observe the test rig and the survey network used as a datum. Four theodolite stations were first used to establish this datum but only one T2002 theodolite was available. One observer was used (i.e. the author) so that different observing methodologies between observers would not be a factor in any of the testing. No adjustment was attempted with the theodolite internal axes at any time.

Collimation results were as consistent for every theodolite station as can be expected without any adjustments on the instrument and using only one
observer. Mean horizontal collimation errors of near 5 arc seconds and vertical index errors of about 8.5 arc seconds were found. Target number 80 (see Chapter 7) was difficult to observe from station 204 and the large error of 14.1 arc seconds reflects this. The reason was that theodolite station 204 was observing from very close to the plane of target 80 . The center of the Wild target appeared much more as a disk than as the circle it really was.

A test for rotation about the $y$-axis of the test rig (Chapter 8) revealed a single observational blunder during data collection shown in Figure 4.11:

## POSSIBLE POINTING ERRORS:

## FROM 204 TO 15:

FACE 1 (Values with a ${ }^{\prime *}$ deviate from mean by at least 10 ".)
Mean Hz $=202411.2$
Mean V $=102535.8$

| obs | .tdf line \# | Hz | V |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| 1 | 45 | $* 339$ | 5 | 13.4 | $* 100$ |
| 2 | 58 | $*$ | 43 | 36.8 |  |
| 2 |  | 49.0 | $* 104$ | 6 | 32.9 |

Press ' 0 ' to continue,
Or '1' through ' 2 ' to omit entire obs.
(Mean values will be recalculated.)
$>1$
Corresponding FACE 2 Observations: (REFERENCE ONLY)

| .tdf line \# | Hz | V |
| :--- | :--- | :--- |
| 75 | 2461652.6 | 2555341.1 |

Figure 4.11 Sample Alignment Rig Pointing Error Screen

By using the enhanced version of COLLECT to reveal this error, the use of shaft target 2002 was still possible since an extra (and correct) observation could be made to the point. Without these enhancements, the entire test would have been corrupted without readily knowing how. Specifically, this blunder would have caused the shaft in question to be erroneously mispositioned in three-dimensional space. In Figure 4.12, one of the station error screens is presented after the blunder had been corrected. These values were all very reasonable indicating high quality observational data.

Instrument station 204:
Mean Hz collimation error $=6.7^{\prime \prime}$
Mean V index error $=6.4^{\prime \prime}$
Maximum HZ deviation from mean is 6.7" for observation TO 205 Maximum V deviation from mean is $2.2^{\prime \prime}$ for observation TO 90

Enter ' c ' when ready to continue $>\mathrm{c}$

Figure 4.12 Sample Alignment Rig Station Collimation Error Screen

Other tests using the enhanced version of COLLECT, including applications such as calibrating a structure used in the study of human foot dynamics, are presented in Chapter 7.

## Chapter 5

## Approximate Coordinates

In Chapter 3, some of the mathematical models used in the various software components of the existing system were discussed. The program ADJUST was described as well as the necessary data for a mathematical adjustment to be implemented. In this chapter, reasons for needing approximate coordinates are outlined followed by a description of a new program called APPROX, written by the author. Although this new program is written using file formats applicable to ADJUST, it can also be used to generate approximate coordinates as a stand-alone program.

### 5.1 Need For Approximate Coordinates

With most non-linear models, a solution requires initial approximate values close to the "true" solution in order for mathematical convergence to occur. Being common in many geomatics-type applications, this is a wellknown condition. In Section 3.4, one important addition mentioned as necessary to improve the existing theodolite intersection system (i.e. automatic generation of approximate coordinates) is presented.

The adjustment model discussed in Chapter 3 was an example of a nonlinear programming problem. In general, initial values (such as the coordinates of a target) are found. The adjustment algorithm generates an
improved value which in turn is used as the new approximate value. This iterative process continues until the value approaches the solution. With these types of problems, the process is terminated when the adjusted value becomes close enough to the solution to be considered solved. In Figure 5.1, an idealized demonstration of this adjustment process is presented. If the initial approximate values are too far from the true value, the adjustment solution may incorrectly converge to a local minimum or even continue to diverge.


Figure 5.1 Convergence from Approximate Coordinates

In optimization analysis, an arbitrary point (such as an approximate coordinate) will converge to a solution if the algorithm is closed, has a decent function, and generates a bounded sequence (Luenberger, 1984, p. 193). An important aspect of an iteration algorithm used in this context is that it will in fact generate a sequence that converges to a solution point providing global convergence. Thus the key aspect in dealing with approximate coordinates, as seen in Figure 5.1, is the question of whether the algorithm will eventually converge to a solution, even if initiated reasonably far from it.

Bayly (1991) limits the number of iteration steps to just over 10 before concluding the adjustment process. Depending on the geometrical strength of an industrial survey network, the more accurate the initial approximate coordinates must be. A weak network (i.e. one having very acute or obtuse intersection angles at each target) may require approximate coordinates as accurate as only a few centimetres near the true coordinate values. The main point of this discussion is that accurate approximate coordinates for this or any theodolite intersection system are very important.

Measuring approximate coordinates by taping, as is necessary in Bayly's existing system, can be difficult or impossible to accomplish. Operating machinery that is hot or moving, as well as hard to reach targets may limit the . choice of targets used in an adjustment thus weakening the final shaft alignment solution. If care is lacking, the process of measuring by taping can lead to errors caused by tape sag, skewed angular measurements (e.g. not east-west when assumed to be) and reading errors. Also, manually recording and entering this data into a file creates opportunities for various recording errors.

In Section 5.3, APPROX will be described. Before this, the next section will briefly overview various methods that have traditionally been used to generate approximate coordinate data. Many of these were found to follow similar principles that the author had used yet the degree of sophistication, especially for surveys conducted with more than two theodolites, was found to be limited. Using more than two theodolites enables more machine or shaft points to be observed allowing for a stronger geometrical solution.

### 5.2 Methods to Gather or Generate Approximate Coordinates

One vital difference between a coordinate measuring system that is passive (i.e. theodolite intersection) and one that is active (such as a measuring system relying on measured distances) is that an active system is limited to static, reflective targets (see Gruendig, 1985). In other words, measuring target locations using an EDM (Electronic Distance Measurement) device require special targets to be placed in and around the machinery in question. A passive system such as Bayly's does not require special reflecting targets and thus projected laser spots onto a rotating shaft can be used. As a by-product of using passive systems, approximate coordinates are not readily available. By using a single theodolite system with an EDM, directions and distances can be measured providing new target coordinates while data is collected. The main limitation of this method is that the interdependence between survey data is reduced, thus weakening network redundancy.

To obtain approximate coordinates for theodolite intersection, essentially two methods are available:

1. Taped coordinates.
2. Automatic generation of approximate coordinates.

## 1. TAPED MEASUREMENTS

This is the least sophisticated method to gather approximate coordinates. For a simple network of targets, and with plenty of room to reach them, a tape as well as careful reading and recording of distances can allow relatively accurate three-dimensional approximate coordinates to be recorded. Obstructions between measured points can create plenty of difficulties and hard to reach potential target areas, especially those near dangerous areas, may have to go unused even if they would have provided improved network geometry. Measuring approximate coordinates by taping and manually entering this data into a file can add hours to an otherwise reasonably short survey.

## 2. AUTOMATIC GENERATION OF APPROXIMATE COORDINATES

By using directional data from two theodolites, the intersection of two optical lines of sight can be computed. This is accomplished using traditional bearing-bearing intersection techniques. Initially this seems fairly straightforward to code, but a major problem concerning theodolite orientation comes into play. If one were to use an expensive gyro-theodolite, a known direction (north) would be sensed. Therefore all observations from that theodolite would be true azimuths (Bannister and Raymond, 1977). Thus, by using a gyro-theodolite at every theodolite station, each instrument would be oriented with the other; all would sense the same true north direction. Budgetary concerns presently do not justify the use of these expensive instruments such
as gyro-theodolites where even two high-precision theodolites are a moderately priced alternative. Wild T2002 theodolites cannot sense north and thus they have no common direction. Without common directions, no computations for the bearing-bearing intersection can be made.

Various theodolite intersection systems from the past required the use of two mutually collimated theodolites (Bingley, 1990; Kissam, 1962). By pointing either theodolite to the other, a common direction between the two (at $180^{\circ}$ difference) could be found. This collimation procedure could be difficult and time-consuming. Obstructions would not be allowed to exist between the theodolites. Many of these systems would establish network scale by using an accurate scale bar while other systems would set scale by accurately measuring the distance between the two theodolites with a stadia. In Figure 5.2, the desired distance $D$ is found with basic trigonometry using known distance B (Sivaraman and Delaitre, 1987).


Figure 5.2 Stadia Method for Scale and Orientation

Both the stadia and mutual collimation methods are used to orient the theodolites with respect to each other allowing bearing-bearing intersections to be computed. These methods can be time consuming and are limiting in terms of theodolite placement due to intervisibility being a necessity. Also any error in collimation will effect the directions from either theodolite causing computed angles to be either slightly too large or too small. In fact, for very acute intersection angles (that should be avoided), a small error (of even a few arc seconds) between theodolites will cause several centimetres of error in the position of the target.

### 5.3 New Approximate Coordináte Software

While enhancing Bayly's theodolite intersection system, investigation revealed that a new program would be essential to improve the process by integrating output from COLLECT directly to input into ADJUST. APPROX was created with the objective of creating a versatile program that would use existing theodolite directional observations in order to compute approximate coordinates leading to data allowing ADJUST to converge.

### 5.3.1 Program Overview

Program APPROX was written to generate a unique solution of approximate coordinates for a three-dimensional survey network used for industrial alignment, deformation analysis or any other network relying solely on directional observations. The program scans the observation file ( ${ }^{*}$.obs) created by COLLECT (Chapter 4) finding at least two theodolites that observe a known scaled distance as well as each other. Coordinates are calculated for any points observed by these two instruments. Next, rotation and scale
parameters are calculated based on a coordinate system illustrated in Figure

## 5.3:

TOP VIEW
Right-Handed Coordinate System


Figure 5.3 Coordinate System Used in APPROX

Using the computed scale and rotation values, a similarity transformation is implemented. Any points or "extra" theodolites outside this immediate network are positioned (if possible) using a combination of intersection and three-point resection. Finally the approximate coordinate data is saved in a newly created file (*.sta) to be used by ADJUST. Azimuth and slope distance data (orientation and scale) are added to the existing observation file (*.obs) yet at no time is the raw data file (*.tdf) ever altered. In Figure 5.4 the modules forming APPROX are shown. (All software was written in C. In Section 5.3.2, each of these functions will be discussed in more detail.) It is worth emphasizing that accurate mutual pointing of the theodolites is not necessary when using APPROX and that any number of additional theodolites or targets are accounted for.


Figure 5.4 APPROX Program Modules

### 5.3.2 Program Module Features

| Table 5.1 DEFAULTS: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| (user) | - fixed point(s) <br> - scaled distance $d_{m}$ <br> - fixed azimuth | 420 |

## DEFAULTS

This is the first module in APPROX. It is used to obtain necessary data about network orientation from user input. The function first obtains names of input (*.obs) and output (*.sta) files. Then the user identifies one or two fixed points in the network, either being any specified target or one on the end of a shaft (if one is being measured). The user indicates what two points (being the scale bar or other points) define network scale. The program either fixes an azimuth by using the shaft axis or orients the network by setting an azimuth between any two points specified by the user. This function is relatively large due to the variety of scale and azimuth options that could be entered. This enables APPROX to change according to the user's needs and not the user having to change their approach according to the program.

| Table 5.2 THEODOLITE SCAN: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| *.obs file name | obs. to 1st and 2nd <br> end of scaled distance <br> $\#$ theods. that observe <br> each other <br> all theodolite stations | 160 |

## THEODOLITE SCAN

This function is used to scan the observation file on disk and then-save the necessary data in arrays identifying different types of observations. The observation file ( ${ }^{*}$.obs) is opened and scanned as follows:

- all direction observations to the first end of the scaled distance are stored in one array
- all direction observations to the second end of the scaled distance are stored in another
- all reciprocal theodolite observations (i.e. from one theodolite station to another) are then scanned

These observations from the theodolite stored as "theodolite $A$ " to the next one called "theodolite $\mathrm{B}^{\prime \prime}$ (see next function) are stored in a new array only if reciprocal observations exist from theodolite $B$ to $A$. In other words, only theodolites that observe each other are stored at this stage.

| Table 5.3 THEODOLITE FIND: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - obs. to 1st and <br> 2nd end scaled <br> distance <br> - theod. \#'s of those <br> obs. each other | - theodolite A | 80 |

## THEODOLITE FIND

In this module, the objective is to find those theodolites that observe both ends of the scaled distance, specifically eliminating those that may only observe one end. This new list of theodolites is then compared with the list of those that observe each other and matches are found. Once any two theodolites that in fact observe each other and observe both ends of the scaled distance are found, they are assigned as "theodolite A " and "theodolite B ".

| Table 5.4 CORRECT OBSERVATION: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - theods. A and B | - directional obs. <br> from theods. A <br> and B | 130 |
| - target \#'s of ends |  |  |
| of scaled dist. |  |  |$\quad$| - zenith obs. from |
| :--- |
| theod. A |$\quad$

## CORRECT OBSERVATION

At this stage a pair of theodolites that observe each other and the scaled distance should be found; otherwise an error message is displayed. To establish a common orientation, this function will set the azimuth from theodolite A to B as $0^{\circ}$ and from theodolite B to A as $180^{\circ}$. All observations from these theodolites are then "corrected" so that they have a common orientation based on these assigned values.

| Table 5.5 RELATIVE COORDINATES: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - theods. A and B <br> - direction obs. <br> from A and B <br> - zenith obs. from <br> theod. A | - relative point <br> coords. $x_{p}, y_{p}, z_{p}$ <br> stored in array $\mathbf{r}$ | 110 |

## RELATIVE COORDINATES

This function computes the coordinates of points intersected by observations from theodolites A and $\mathrm{B}:\left(x_{p}, y_{p}, z_{p}\right)$. These coordinates are stored in memory for future use. To achieve these bearing-bearing intersection calculations, the coordinates of theodolites $A$ and $B$ must be known although at this stage of the program, they cannot be. Thus theodolite A is assigned the $x, y, z$ Cartesian coordinates $(0,0,0)$ and theodolite $B$ is given
the coordinates $(0,1,0)$. That is, they are set as being one unit apart in the y-direction. (See Figure 5.3 for orientation.)

This process requires a three-way matching algorithm: First all direction observations from theodolite A are matched with their corresponding zenith observations also from $A$. These are then matched with observations from theodolite B. For example, all direction observations from theodolite A to target 10 are compared with zenith observations also from A to 10. Those that match are then compared with all directional observations from B to 10 . (Zenith observations from B are not used since their only contribution in this program would be to verify zenith observations from A.) The actual intersection computation takes only a few lines of code emphasizing how much "book-keeping" is necessary in creating a versatile program. The functions are as follows:


Figure 5.5 Theodolite Bearing-Bearing Intersection

$$
d_{A P}=\frac{\left(x_{B}-x_{A}\right) \cos \phi_{P B}-\left(y_{B}-y_{A}\right) \sin \phi_{P B}}{\sin \left(\phi_{A P}-\phi_{P B}\right)}
$$

with theodolite $A=(0,0,0)$ and theodolite $B=(0,1,0)$,

$$
d_{A P}=\frac{-\sin \phi_{P B}}{\sin \left(\phi_{A P}-\phi_{P B}\right)}
$$

thus, solving for the coordinates of target point $P$,

$$
\begin{align*}
& x_{p}=d_{A P} \sin \phi_{A P} \\
& y_{p}=d_{A P} \cos \phi_{A P} \\
& z_{p}=\frac{d_{A P}}{\tan z_{A P}}
\end{align*}
$$

This intersection algorithm is adapted from Teskey (1988).

| Table 5.6 ORIENT: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - relative coords. <br> $x_{p}, y_{p}, z_{p}$ stored in <br> array r <br> - shaft target <br> coords. (if shaft is <br> present | - rotation angle (rot) | 240 |

## ORIENT

Up to this stage all points in the network that were observed only by theodolites A and B have relative coordinates based on a theodolite separation of one unit. Orientation from theodolite $A$ to $B$ was set at $0^{\circ}$. In this function, the angle necessary to rotate the existing relative network to the position where the user's set azimuth (see DEFAULTS) will be matched within
the relative network is computed. If a shaft is used to define orientation, an approximate midpoint (vertically and transaxially) is computed for either end as demonstrated in Figure 5.6. A vector joins these midpoints and the azimuth of the vector is set at $90^{\circ}$ as was seen in Figure 5.3. (The vector is extended in Figure 5.6 for clarity.)


Figure 5.6 Orientation When Shaft Axis is Used

This is only one specific option where the user has told APPROX to use the shaft axis to define orientation. To identify shaft target points, a constraint indicating that these points must be labeled in the thousands (e.g. 1001 or 2003 as in the figure), is specified.

If the user instead specifies an azimuth between two points, where one or both are not observed by theodolites A and B, a flag is set telling the program that the network cannot be rotated correctly until targets not observed by these two primary theodolites are computed. In other words, if orientation is defined using target points not directly intersected from theodolites $A$ and $B$, a reorientation is necessary once these relative coordinates are found. (See the MORE POINTS module).

| Table 5.7 SCALE: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - relative coords. <br> $x_{p}, y_{p}, z_{p}$ stored in <br> array $\mathbf{r}$ <br> - scaled distance $d_{m}$ <br> entered by user | -scale_factor $s$ | 70 |

## SCALE

To determine a scale factor necessary so that the network will be the correct size, APPROX must compare the relative distance between two points to a known distance that was entered by the user. Initially the scale of the network is unknown (since this known distance was not yet intersected) and thus the distance between theodolites A and B was assumed to be one unit. Now, since relative coordinates have been computed for the ends of the scaled distance, and since the actual scale distance was entered, the scale factor can be solved for:

$$
s=\frac{d_{m}}{d_{r}}
$$

where $s$ is the scale factor, $d_{r}$ is the relative distance and $d_{m}$ is the measured distance entered by the user.

| Table 5.8 SIMILARITY: Function Flow |  |  |
| :---: | :--- | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| rot. angle (rot) | actual coords. stored <br> in $\mathbf{c}$ | 50 |
| scale_factor |  |  |
| rel. coords. $\mathbf{r}$ |  |  |

## SIMILARITY

This function is used to transform the relative network of points intersected by theodolites A and B. The transformation is a two-dimensional similarity (Moffitt and Mikhail, 1980) except translation is not immediately used (see TRANSLATE below). The computed results of this function are coordinates derived using the network scale and rotation variables previously found. The rotation of this network is about theodolite A giving actual untranslated coordinates c in Equation 5.7 ( $z_{\text {new }}$ is scaled and added to c independently):

$$
c=\left[\begin{array}{cc}
a & b \\
-b & a
\end{array}\right] \mathbf{r}
$$

where $\mathbf{c}=\left[\begin{array}{l}x_{\text {new }} \\ y_{\text {new }}\end{array}\right], \mathbf{r}=\left[\begin{array}{l}x_{\text {old }} \\ y_{\text {old }}\end{array}\right] . \mathbf{c}$ is a vector of new coordinates resulting from the transformation; $r$ is a vector of existing relative target coordinates; and,

$$
\begin{array}{ll}
\mathrm{a}=s \cos (r o t) & 5.8 \\
\mathrm{~b}=s \sin (r o t) & 5.9
\end{array}
$$

| Table 5.9 TRANSLATE: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - actual coords. c | - translated actual <br> coords. c | 40 |
| - coords. of fixed |  |  |
| point(s): see |  |  |
| DEFAULTS |  |  |$\quad$|  |
| :--- |

## TRANSLATE

In this module the scaled and rotated network of points is translated so that the user's first fixed point (or the first end of the shaft if specified) will be at the coordinates the user had specified in DEFAULTS. Translation is introduced separately from the similarity transformation due to the need to know the actual distance necessary to translate the fixed point; this can only be found after actual coordinates have been computed.

| Table 5.10 MORE POINTS: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - actual coords. c | - actual coords. c <br> with 'extra' targets <br> and theodolites <br> added | 390 |
| - array of all theods. <br> including those <br> not being A or B | - array containing <br> extra points; i.e. <br> those not in c |  |

## MORE POINTS

This is the most complex module in APPROX. By allowing for additional theodolites and thus additional targets, the versatility of the program is contrasted with many other approximate coordinate programs referred to earlier. MORE POINTS calls two functions regularly:

- RESECT
- INTERSECT

These are discussed below.
The main purpose of MORE POINTS is to calculate the coordinates of any points (either targets or theodolites) that are not part of the network observed from the first two theodolites called $A$ and $B$ in the program. Additional theodolite stations that may have been used in the network to observe otherwise non-visible machine components will add new intersections that may or may not be solvable using theodolites A and B.

Initially two arrays are created: EXTRA and EXPOINT. The first holds all observations either "from" or "to" any targets not observed by theodolites A and B. The second array holds those points that are in fact undefined with the code ensuring that the point is stored only once within this array. The first array is searched and if three "known" points (i.e. exist in c) are found, function RESECT is called to find the coordinates of this additional theodolite. If the point in question (either an unknown theodolite or unknown target) is otherwise observed by two "known" theodolites (A, B or any added by MORE POINTS), the function calls INTERSECT to compute the target position. This
process continues until all undefined points are accounted for or until limitations of the observational data will not allow any further processing.

In order to search the various sets of data that include "known" and "unknown" points, extensive search routines are necessary. Further complications are added since APPROX never assumes that the user has sorted or ordered the data found in the observation file ( ${ }^{*}$.obs). Thus, for example, a zenith observation must be matched with a direction and compared with various arrays as outlined above.

| Table 5.11 INTERSECT: Function Flow <br> (Sub-function for MORE POINTS) |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - all observational <br> data (arrays in <br> memory and *.obs <br> file | - coords. of newly <br> intersected point | 320 |

## INTERSECT

This is a sub-function called by MORE POINTS. The purpose is to find the coordinates of any unknown points using intersection observations from any two known theodolites. The two known theodolites are not necessarily A or B since MORE POINTS adds additional theodolites as they are found. Since one or both of the theodolites may not be $A$ or $B$, orientation of these two instruments must first be established. This is accomplished by computing a bearing between a "known" network target and the additional theodolite. This bearing is used to correct all the observations from this extra theodolite.

INTERSECT is set up to search all possible observational data to intersect a new target. Searches are conducted through the previously discussed arrays as well as searches through the observation file ( ${ }^{*}$.obs) if the arrays in memory do not contain the necessary information. Once the appropriate theodolites and data are found, the actual intersection as demonstrated in Figure 5.5 is finally conducted.

| Table 5.12 RESECT: Function Flow <br> (Sub-function for MORE POINTS) |  |  |
| :---: | :---: | :---: |
| INPUT | . OUTPUT | LINES OF CODE |
| - all observational <br> data (arrays in <br> memory and <br> file. obs | - coords. of newly <br> resected theodolite | 70 |

## RESECT

This function will find the coordinates of a theodolite station knowing the coordinates of three points observed by that instrument. MORE POINTS conducts the actual searches to find these three known targets. The adopted resection algorithm is based on Hausbrandt's method (Teskey, 1988). Figure 5.7, demonstrates the geometry of this method.


Figure 5.7 Hausbrandt's

$$
\begin{align*}
& x_{p}=x_{A}-\frac{\left(f_{1}-f_{2} F\right)}{\left(1+F^{2}\right)} F \\
& y_{p}=y_{A}+\frac{f_{1}-f_{2} F}{1+F^{2}}
\end{align*}
$$

where

$$
\begin{align*}
& f_{1}=\left(y_{B}-y_{A}\right)-\left(x_{B}-x_{A}\right) \cot \alpha_{1} \\
& f_{2}=\left(x_{B}-x_{A}\right)+\left(y_{B}-y_{A}\right) \cot \alpha_{1}, \text { and } \\
& F=\frac{f_{1}-\left(y_{C}-y_{A}\right)+\left(x_{C}-x_{A}\right) \cot \alpha_{2}}{f_{2}-\left(x_{C}-x_{A}\right)-\left(y_{C}-y_{A}\right) \cot \alpha_{2}}
\end{align*}
$$

In Figure 5.7 and Equations 5.10 through 5.14 the two-dimensional coordinates $\left(x_{p}, y_{p}\right)$ of theodolite station P are computed knowing the twodimensional coordinates of points $A, B$, and $C$. (The height or z-position of $P$ is found using zenith data to one of these targets.)

| Table 5.13 PRESENT: Function Flow |  |  |
| :---: | :---: | :---: |
| INPUT | OUTPUT | LINES OF CODE |
| - actual coords. c <br> including points <br> added by MORE <br> POINTS | $-*$.sta: file of <br> approx. coords. <br> - scaled distance <br> data | - <br> created by <br> COLLECT (Ch. 4$)$ |

## PRESENT

This is the last module in APPROX. Before being called, APPROX tests to see whether or not the network needs to be reoriented. (The reader may recall that in ORIENT a flag is set if the initial network-created only by theodolites A and B -did not contain the points that the user specified in establishing orientation.) The purpose of PRESENT is to create the actual file of approximate coordinates (*.sta) using the computed solution contained in the array c. It also updates the observation file (*.obs) for the reasons noted in the following paragraphs.

Before using APPROX, the observer using COLLECT is required to roughly observe between any two theodolites to give a common direction from which APPROX can begin. Since only directional data and not the zenith angles from either theodolite are used to compute the horizontal target coordinates, if an obstruction such as the machine itself exists between the two instruments, an extension pole or other implement could be used
providing the observer from one theodolite a rough direction to the other. Since this data will usually be very inaccurate in terms of the adjustment (i.e. when using ADJUST), it is removed from the observation file so that ADJUST will not diverge. (Note: These same observations are usually accurate enough for APPROX; see the following section.)

Another reason the observation file is updated is to add scale and orientation parameters as mentioned in Chapters 3 and 4. Again, at no point during the execution of APPROX is any data in the raw observation file (*.tdf) ever altered. Thus full data integrity is always ensured.

### 5.4 Testing and Results: APPROX

### 5.4.1 Studies of Previously Obtained Data

Various sets of data were gathered by Bayly as mentioned near the end of the previous chapter. A number of selected cases that had been successfully completed were used in developing APPROX.

Program development revealed that by automatically adding the slope distance and azimuth parameters to the observation file (*.obs), the need to manually edit this file could be eliminated. Testing data such as LAB5 (Section 4.4.1) revealed that the basic theodolite $A /$ theodolite $B$ intersection routines were working correctly. Testing of the additional code necessary for additional theodolites and targets proved to be considerably more involved.

Studying various sets of data such as the granulator survey network (Bayly, 1991) revealed how essential the ability to position additional
theodolite stations or targets would be. In Figure 5.8, this network of seven theodolites and eleven targets proved to be an excellent benchmark test for APPROX.


Figure 5.8 Granulator Survey Network
(Bayly, 1991, p. 111)

When the raw data was originally gathered in the field, no rough betweentheodolite observations were taken. Thus stations 201 and 202 were used by the author in back-calculating the observations from the existing network solution. These observations (that the new version of COLLECT would have automatically added) were placed in the observation file.

The following processes in solving for approximate coordinates were observed in output from APPROX:

- from theodolite station 201 and 202, targets $1,2,3,4,5,6$ and 7 were intersected directly
- theodolite stations 203, 204, 205 and 206 were resected using any three of the above seven known targets
- intersections were computed using any of the six known theodolite stations (i.e. 201, 202, 203, 204, 205 and 206) to targets $8,9,10$ and 11
- finally, using either target 8 or 9 plus the two known targets ( 1 and 3 ), theodolite station 207 was resected.

The computed approximate coordinate results were all within a few millimetres of the actual adjusted coordinates. This is actually too good to be unconditionally accepted. The reason is that the author's back-calculations between stations 201 and 202 were, in essence, perfect. This was because the final adjusted solution was used to compute these and thus the observations were as good as the best mutually pointed network (discussed in Section 5.2) could ever be. These highly accurate results more significantly reveal that APPROX gives correct results since the adjustment only needs to "pull" a few millimetres per target towards the best solution.

### 5.4.2 Results of Subsequent Testing

Since its development, APPROX has been used in a variety of settings. In Chapter 7 details on an alignment test rig are provided. Using this device, six tests were undertaken in July of 1992 in the basement of a residence
building at The University of Calgary. The ambient temperature was static and, being an all concrete basement room, the walls and floor were very stable. Initially a survey was made to establish a network of known points for targets on the concrete walls and pillars within the room. This was an ideal opportunity to test APPROX.

Below are three tables of these results. In Table 5.14, taped approximate coordinates of these targets are presented. These were gathered using a steel tape. In Table 5.15, the approximate coordinates of these same targets generated using APPROX are presented. Lastly, Table 5.16 presents the final adjusted coordinates (expressed to three decimals). Note that theodolite stations 203 and 204 were resected in APPROX while target 40 was intersected.

| Table 5.14 APPROX Testing: Taped Approximate Coordinates |  |  |  |
| :--- | :--- | :--- | :--- |
| POINT | $X(\mathrm{~m})$ | $Y(\mathrm{~m})$ | $\mathrm{Z}(\mathrm{m})$ |
| 10 | 10.000 | 10.000 | 0.140 |
| 20 | 9.660 | 16.420 | 2.590 |
| 30 | 12.400 | 20.370 | 0.190 |
| 40 | 19.980 | 20.500 | 2.670 |
| 50 | 20.180 | 14.750 | 0.310 |
| 60 | 22.770 | 11.600 | 2.170 |
| 70 | 17.190 | 10.000 | 2.720 |
| 80 | 17.190 | 10.000 | 0.250 |
| 90 | 15.800 | 15.700 | 2.700 |

Table 5.15 APPROX Testing: APPROX Generated Approximate Coordinates

| POINT | $X(\mathrm{~m})$ | $Y(\mathrm{~m})$ | $\mathrm{Z}(\mathrm{m})$ |
| :--- | :--- | :--- | :--- |
| 10 | 10.000 | 10.000 | 0.140 |
| 20 | 9.888 | 16.438 | 2.649 |
| 30 | 13.203 | 20.387 | 0.255 |
| 40 | 20.157 | 20.450 | 2.733 |
| 50 | 20.343 | 14.713 | 0.374 |
| 60 | 23.798 | 11.643 | 2.889 |
| 70 | 17.400 | 10.000 | 2.792 |
| 80 | 17.397 | 10.000 | 0.285 |
| 90 | 15.986 | 15.687 | 2.779 |

Table 5.16 APPROX Testing: Adjusted Coordinates (fixed to 3 decimal places)

| POINT | $\mathrm{X}(\mathrm{m})$ | $\mathrm{Y}(\mathrm{m})$ | $\mathrm{Z}(\mathrm{m})$ |
| :--- | :--- | :--- | :--- |
| 10 | 10.000 | 10.000 | 0.140 |
| 20 | 9.660 | 16.387 | 2.589 |
| 30 | 12.963 | 20.358 | 0.186 |
| 40 | 19.946 | 20.493 | 2.671 |
| 50 | 20.171 | 14.729 | 0.304 |
| 60 | 22.744 | 11.663 | 2.710 |
| 70 | 17.191 | 10.000 | 2.712 |
| 80 | 17.185 | 10.000 | 0.234. |
| 90 | 15.771 | 15.640 | 2.707 |

This data shows that a number of coordinates generated by APPROX (primarily the $x$-coordinates) were not as close to the final solution as may have been expected. This is due to the between-theodolite observations
(between 201 and 202) not being very accurate. The reason for this was that only one T2002 theodolite was available during testing, thus temporary points had to be placed on the floor acting as the position of the second theodolite. It was therefore known that either of these two supposedly mutual observations were incorrect by about 5 to 20 centimetres. (The theodolite could not be perfectly positioned over these points due to various factors such as sighting targets on the test rig.) Thus a biasing from 5 to 20 centimetres in the solution was the result.

With the two theodolite positions 201 and 202 being positioned close to parallel to the $x$-axis of the shaft (see Figure 5.3), intersection giving $x$ coordinates were expected to be weakest due to intersection angles being very acute at those targets nearly in line with the theodolites. The above results initiated this hypothesis and subsequent testing (Chapter 7) confirmed it.

The results in Tables 5.14 and 5.15 were both successfully used in ADJUST giving exactly the same results as seen in Table 5.16. The $x$ coordinate of target 60 in Table 5.15 was reported by ADJUST as farthest from the solution. Point 60 is a critical point since it is almost collinear with a line joining theodolite stations 201 and 202 (note Figure 5.9). The intersection angle to target 60 was very acute thus being very weak. With rough betweentheodolite observations, the accuracy of these intersecting lines becomes even more critical since a small orientation error could mathematically move the intersecting rays metres from the true target using APPROX computations.

Further analysis using the six mentioned tests indicate that the weakness of intersection to target 60 was consistent with theodolite placement. Out of these six tests, in which two surveys were conducted within each for a total of
twelve, three did not have coordinates that could be used for subsequent adjustment due to difficulties in generating useful coordinates for target 60.

A similar problem was noticed during analysis of field test data gathered for the compressor site mentioned in Chapter 2. Testing found that two points were almost directly in-line with a line joining the two primary theodolites. (i.e. the first two to be identified in APPROX as A and B.) By using another pair of instruments as primary theodolites, this problem was eliminated.

Overall, this testing has shown that APPROX works correctly and efficiently if the user ensures that there are no target points within about two metres of a line joining the other theodolite as shown in Figure 5.9:


Figure 5.9 Ensuring Network Targets
Significantly Off-line From Other Theodolite

This will depend on overall network geometry and theodolite positions. With a geometrically strong network, a minimum distance less than the indicated 2 meters would be sufficient.

## Chapter 6

## Shaft Misalignment and Offsets

In this chapter, a new program completing the enhancements and additions to Bayly's intersection system is presented. In this program, the desire was to take the adjusted coordinates computed by ADJUST and, after fitting them to a circle (using FIT), have a final program that would compare these results and display the shaft alignment schematically. With this, full error propagation using the standard deviations produced in FIT is presented in the final solution.

### 6.1 Alignment Assumptions

A new program called SHAFT was written to complete the existing theodolite intersection system. Offsets at the shaft midpoint are computed along with misalignment vectors originating from either end of the shaft or shaft train. Alignment computations utilize direction numbers directly from the output files created by FIT. Base corrections (i.e. used to realign the shafts) and error propagation are also derived from the form-fitting results.

These results are based on targets used in the initial survey. In circle fitting, it is assumed that the surface targets on the shaft (laser spots) are each on a perimeter of constant distance from a common center point. In reality, these targets deviate from this constant radius by varying amounts. Form-
fitting using least squares (i.e. FIT) will minimize these effects providing what can be considered a best-fit center point. From this center point a corresponding vector is found that is perpendicular to the plane of the circle.

The orientation of this vector depends on the target points being used. It is assumed that the points are vertically oriented and that the user specifies the correct ones in FIT. If the targets form a best-fit plane that is not vertical (Figure 6.1), the orthogonal vector to this plane will be in error due to not being parallel to the "true" shaft axis.


Figure 6.1 Skewed Plane of BestFit Circle

A cylindrical model (instead of a circle) could offer the advantage of not having to assume that the shaft targets are vertically planar. Such models such as discussed by Harvey (1991) may offer a better solution for shaft axis orientation since all observed targets would be best-fit to the surface of a cylinder. As will be seen in later in this chapter, this cylindrical model is
unnecessary due to a method added to SHAFT that uses only best-fit center points. By connecting these points and forming a unit vector, the positioning of the shaft axes is not dependent on the orientation of the plane defined by the best-fit circle.

Vectors representing either end of the shaft are assumed to be pointing away from each other. This convention is important in dealing with the mathematical model and in the calculations to be discussed in the following sections. Initially FIT orients these vectors according to the numbering scheme of the target points. The center coordinates of the circle are found by intersecting three planes as shown in Figure 6.2. The unit normal vector is oriented according to the cross product of the vectors of the second and the last target points. This may or may not provide vectors that point away from each other. SHAFT checks for this and will compensate by multiplying by negative 1 if necessary.


Figure 6.2 Orientation of Unit Vector Normal to Plane of Circle
(Bayly, 1991)

The orientation of the network containing the shaft is assumed to have the x -axis parallel to the shaft axis as shown in Figure 5.3. This does not have to be exact but the results presented in SHAFT will be easier to interpret if it is close. This means that if the network is oriented as mentioned, there will be no x-translation necessary. Offsets are presented at the midpoint between the two $x$-values defining the origin of the shaft end vectors.

Actual misalignment angles are considered to be fairly small. For angles less than about 0.1 radians $\left(\sim 5^{\circ}\right)$ this assumption is made so direction numbers (from direction cosines; see next section) can be used in the computations. The vertical misalignment values are considered to be small due to machine operation requiring shaft axes to be perpendicular to the direction of gravity. Horizontal values will be larger but normally should not exceed the limit indicated for this assumption.

Lastly, in coaxial alignment, the elimination of soft feet is usually the first step in an alignment process (Neale et al., 1991). This occurs when a corner or other parts of the machine base (or feet) are not all in contact with the foundation or have unequal load sharing. It is assumed that the machine has no soft feet.

### 6.2 Mathematical Modeling

### 6.2.1 Orientation of Shafts Using Direction Numbers

Direction numbers are components of a three-dimensional vector in space. In SHAFT, component vectors are used to simplify the mathematical model avoiding computationally intensive trigonometric functions.

Consider a vector $\mathbf{a}=\left\langle a_{1}, a_{2}, a_{3}\right\rangle$ as shown in Figure 6.3. Assume that this three-dimensional vector has unit length.


Figure 6.3 The $x, y$, and $z$ Components of Unit Vector a

Viewing the components of this vector (as in Figure 6.4) it can be seen how vector a can be represented in the $x, y$, and $z$ directions by using cosine values:


Figure 6.4 Direction Cosines

For small angles (see previous section) the cosine of an angle is essentially equal to the angle itself (in radians). As an example,

$$
\cos \beta=\beta=\frac{a_{2}}{|\mathbf{a}|}=\frac{y_{2}-y_{1}}{|\mathbf{a}|}
$$

FIT generates unit vectors where $\langle a, b, c\rangle$ represent the $x, y$, and $z$ components of the three-dimensional vectors; i.e. the direction numbers. (Note: $\langle a, b, c\rangle$ is the notation used in FIT and SHAFT corresponding to $\left\langle a_{1}, a_{2}, a_{3}\right\rangle$ above.) Misalignments are necessary only for the y and z directions thus only vector components $b$ and $c$ are used in the mathematical functions. The direction of the $x$-component, namely $a$, is only used to establish a common orientation. (Alignment in the $x$-direction is not necessary since it can always be well controlled.) On the fixed end of the shaft, the vector $\langle a, b, c\rangle$ is assumed to be pointing in the negative $x$-direction. If the $x$ component $a$ is not negative, $\langle a, b, c\rangle$ is multiplied by negative 1. This process is similarly used for the other end except the requirement is to have the vector pointing in the positive direction.

### 6.2.2 Shaft Alignment Mathematical Model

The misalignments and offsets of the shaft are calculated for both side and top views. Each of these can be represented by the following four mathematical functions created for SHAFT:

Misalignment in the XY-Plane (Top View):

$$
f_{1}\left(b_{1}, b_{2}\right)=b_{1}+b_{2}
$$

Misalignment in the XZ-Plane (Side View):

$$
f_{2}\left(c_{1}, c_{2}\right)=-c_{1}-c_{2}
$$

Offset in the XY-Plane (Top View):

$$
\begin{align*}
f_{3}\left(y_{1}, y_{2}, b_{1}, b_{2}, x_{s}\right) & =\mathbf{V}_{2}-\mathbf{V}_{1} \\
& =\left(y_{2}-b_{2} x_{s}\right)-\left(y_{1}-b_{1} x_{s}\right) \\
& =y_{2}-y_{1}-x_{s} b_{2}+x_{s} b_{1}
\end{align*}
$$

where $x_{s}=\frac{x_{2}-x_{1}}{2}$ is the midpoint of the shaft.

Offset in the XZ-Plane (Side View):

$$
\begin{align*}
f_{4}\left(z_{1}, z_{2}, c_{1}, c_{2}, x_{s}\right) & =\mathbf{V}_{2}-\mathbf{V}_{1} \\
& =\left(z_{2}-c_{2} x_{s}\right)-\left(z_{1}-c_{1} x_{s}\right) \\
& =z_{2}-z_{1}-x_{s} c_{2}+x_{s} c_{1}
\end{align*}
$$

where $x_{s}=\frac{x_{2}-x_{1}}{2}$ is again the midpoint of the shaft.
$b_{1}$ and $b_{2}$ are the $y$-components of the vectors orthogonal to two circle planes through either shaft end. $c_{1}$ and $c_{2}$ are the $z$-components while $\left(x_{1}, y_{1}, z_{1}\right)$ is the origin of one vector and $\left(x_{2}, y_{2}, z_{2}\right)$ the other. $\mathbf{V}_{1}$ and $\mathbf{V}_{2}$ are the actual vectors.

These functions utilize direction numbers to calculate misalignments and offsets. Equation 6.2 represents the misalignment of the shaft in the $y$ -
direction. If the shaft is not misaligned in that direction, both values of $b$ would either be zero or the same magnitude but of opposite sign. Similarly for Equation 6.3. As an example, if $c_{1}=+5$ (i.e. pointing upwards) and $c_{2}=-5$ (also pointing upwards; see Figure 6.5), the amount of misalignment in the XZ-plane would be $-5-(-5)=0$. This is not an indication that the shaft is perfectly aligned since a possibility of the shaft components being parallel but offset from each other exists. It should be noted that Equations 6.2 and 6.3 use different signs only for the purpose of sign convention. Figure 6.5 demonstrates that in order to present misalignments on one end of the shaft, the vectors $b_{1}$ and $c_{2}$ are mathematically "moved" to the free end and then differenced.


Figure 6.5 Misalignment Sign Conventions

Equations 6.4 and 6.5 are derived by finding the $y$ or $z$ coordinates of each vector at the midpoint of the shaft and then differencing. This provides
the offset at the midpoint with the correct orientation. The slope terms are negative since one is following each vector in the opposite direction from which it is pointing.

### 6.3 Variance and Covariance Propagation

### 6.3.1 Error Propagation

The new program SHAFT utilizes standard deviations presented in the output files created by FIT. These values correspond to each coordinate of the vector origin as well as for each component of the vector itself. Thus the center point $\left(x_{0}, y_{0}, z_{0}\right)$ has three standard deviation values and the vector itself $\langle a, b, c\rangle$ also has three standard deviation values. All of these values are computed by propagating the standard deviations from the original theodolite measurements to the standard deviations of the computed results.

In SHAFT the standard deviations provided by FIT are propagated to the final solution of misalignments and offsets. This is accomplished by calculating the partial derivatives of each function (Section 6.2.2) with respect to each random variable. These equations can be grouped in what is known as the Jacobian matrix having the general form given below:

$$
\mathbf{J}_{x y}=\left|\begin{array}{cccc}
\frac{\partial y_{1}}{\partial x_{1}} & \frac{\partial y_{1}}{\partial x_{2}} & \cdots & \frac{\partial y_{1}}{\partial x_{n}} \\
\frac{\partial y_{2}}{\partial x_{1}} & \frac{\partial y_{2}}{\partial x_{2}} & \cdots & \frac{\partial y_{2}}{\partial x_{n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial y_{m}}{\partial x_{1}} & \frac{\partial y_{m}}{\partial x_{2}} & \cdots & \frac{\partial y_{m}}{\partial x_{n}}
\end{array}\right|
$$

These derivatives are calculated for each of the four functions discussed in the previous section. The propagation is accomplished in the following way (Davis et al., 1981):

$$
\Sigma_{y y}=\mathbf{J}_{x y} \Sigma_{x x} \mathbf{J}_{x y}^{\mathrm{T}}
$$

where $\Sigma_{x x}$ is the covariance matrix of the observations and $\Sigma_{y y}$ of the results. In SHAFT the equation is in the form:

$$
\mathbf{C}_{x}=B C_{\ell} B^{T}
$$

### 6.3.2 Covariance and Design Matrices

The matrix B in Equation 6.8 is referred to as a design matrix. In this case, it has four rows, each row representing one function; and ten columns, each column representing one random variable. (Note again that the $a$ component of each vector is not used because the assumption was made that there is no change in the x -direction.)

$$
\mathbf{B}=\left|\begin{array}{lllll:lllll}
\frac{\partial f_{1}}{\partial x_{1}} & \frac{\partial f_{1}}{\partial y_{1}} & \frac{\partial f_{1}}{\partial z_{1}} & \frac{\partial f_{1}}{\partial b_{1}} & \frac{\partial f_{1}}{\partial c_{1}} & \frac{\partial f_{1}}{\partial x_{2}} & \frac{\partial f_{1}}{\partial y_{2}} & \frac{\partial f_{1}}{\partial z_{2}} & \frac{\partial f_{1}}{\partial b_{2}} & \frac{\partial f_{1}}{\partial c_{2}} \\
\frac{\partial f_{2}}{\partial x_{1}} & \frac{\partial f_{2}}{\partial y_{1}} & \frac{\partial f_{2}}{\partial z_{1}} & \frac{\partial f_{2}}{\partial b_{1}} & \frac{\partial f_{2}}{\partial c_{1}} & \frac{\partial f_{2}}{\partial x_{2}} & \frac{\partial f_{2}}{\partial y_{2}} & \frac{\partial f_{2}}{\partial z_{2}} & \frac{\partial f_{2}}{\partial b_{2}} & \frac{\partial f_{2}}{\partial c_{2}} \\
\frac{\partial f_{3}}{\partial x_{1}} & \frac{\partial f_{3}}{\partial y_{1}} & \frac{\partial f_{3}}{\partial z_{1}} & \frac{\partial f_{3}}{\partial b_{1}} & \frac{\partial f_{3}}{\partial c_{1}} & \frac{\partial f_{3}}{\partial x_{2}} & \frac{\partial f_{3}}{\partial y_{2}} & \frac{\partial f_{3}}{\partial z_{2}} & \frac{\partial f_{3}}{\partial b_{2}} & \frac{\partial f_{3}}{\partial c_{2}} \\
\frac{\partial f_{4}}{\partial x_{1}} & \frac{\partial f_{4}}{\partial y_{1}} & \frac{\partial f_{4}}{\partial z_{1}} & \frac{\partial f_{4}}{\partial b_{1}} & \frac{\partial f_{4}}{\partial c_{1}} & \frac{\partial f_{4}}{\partial x_{2}} & \frac{\partial f_{4}}{\partial y_{2}} & \frac{\partial f_{4}}{\partial z_{2}} & \frac{\partial f_{4}}{\partial b_{2}} & \frac{\partial f_{4}}{\partial c_{2}}
\end{array}\right|
$$

or, by solving for these derivatives using Equations 6.2 through 6.5,

$$
\mathbf{B}=\left|\begin{array}{ccccc:ccccc}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\
\frac{b_{1}-b_{1}}{2} & -1 & 0 & \frac{x_{2}-x_{1}}{2} & 0 & \frac{b_{1}-b_{2}}{2} & 1 & 0 & \frac{x_{1}-x_{2}}{2} & 0 \\
\frac{c_{-}-a_{1}}{2} & 0 & -1 & 0 & \frac{x_{1}-x_{1}}{2} & \frac{q_{1}-c_{2}}{2} & 0 & 1 & 0 & \frac{x_{1}-x_{2}}{2}
\end{array}\right|
$$

The covariance matrix of the observables $\left(\mathbf{C}_{\ell}\right)$ in Equation 6.8 contains only variance values. These variances correspond to the values obtained from FIT. In Chapter 8, testing will show that with these types of industrial surveys, full variance-covariance propagation from COLLECT to FIT (i.e. before using SHAFT) will not add any substantial information to the solution.

The result of Equation 6.8 is $\mathbf{C}_{\mathbf{x}}$. This is a square $4 \times 4$ matrix representing the misalignments and offsets in both planes. The covariance terms are small in comparison to the diagonal variances indicating little correlation between individual parameters. The first diagonal term (element $1,1)$ represents the variance for the misalignments in the XY-plane while the second (element 2,2) is for misalignments in the XZ-plane. The third diagonal term (element 3,3) represents the offsets in the XY-plane while the last (element 4,4 ) represents offsets in the XZ-plane. The units are the same as those used to form the covariance matrix of observations. FIT presents the standard deviations of the circle origins and vector components in metres. SHAFT converts these values to millimetres and squares them (variance $=$ (standard deviation ${ }^{2}$ ) before they are used in the diagonal of the $\mathbf{C}_{\ell}$ matrix.

### 6.4 Program Structure

SHAFT is composed of a number of modules that each accomplish a different task. In general, the program reads the data created by FIT, forms the covariance and design matrices, calculates $C_{x}$, and then presents the results. .In the following discussion, details on each of these modules are provided.

## INPUT: (200 LINES)

This is the first function called by the main program. The user is asked to indicate whether two or four input files will be used. If the shaft in question was measured once on either end, form-fitting will produce two separate files. Each file represents the position and orientation of the shaft end with one three-dimensional coordinate ( $x_{0}, y_{0}, z_{0}$ ) and one vector $\langle a, b, c\rangle$. (This is a weaker solution than using four files representing two center points on either end of the shaft.) The user provides the names of these files and they are searched in order to find the relevant values as well as the associated standard deviations. Section 6.5 examines this option more closely.

The user is then asked what units the misalignment and offset values are to be presented with:

1) Angles in RADIANS and distances in MILLIMETRES
2) Angles in DEG/MIN/SEC and distances in MILLIMETRES
3) Angles in MILLIMETRES/METRE and distances in MILLIMETRES
4) Angles in INCHES/FOOT and distances in INCHES

Actual internal calculations within SHAFT are performed using radians and metres.

## CONVERT: ( 125 LINES)

This function is executed only if the user enters four files. The objective is to convert what are to be called VECTOR-II data into VECTOR-I. In other words, to convert from two center points per shaft end to only one center
point and one unit vector. CONVERT propagates the variances for the two center points into the VECTOR-I format as discussed in Section 6.5.

## SOLVE: (40 LINES)

This function performs the actual mathematics that were presented in Section 6.3. The two misalignments (to be called $M_{x y}$ and $M_{x z}$ ) and two offsets (called $O_{x y}$ and $O_{x z}$ ) are computed. The design matrix $B$ is formed according to Equation 6.10. Once the $\mathbf{C}_{\ell}$ matrix is created, Equation 6.8 is solved creating a covariance matrix of the unknowns; specifically of the misalignments and offsets.

## EXTRACT: (60 LINES)

EXTRACT is a function that reads the variance data from the solution matrix and converts these values into the user specified units. It is important to realize that the covariance matrix of the observations $\left(\mathbf{C}_{\ell}\right)$ was populated with variances associated with the unit vectors or converted into millimetres squared. The extracted misalignment values from the solution matrix are initially square-rooted, converting variances into standard deviations. After this, appropriate unit conversion routines are executed.

## PRESENT: (400 LINES)

This last function is fairly large due primarily to various arrays storing the different orientations of the shaft for screen output. (With program updating, less code would have been necessary if a more general graphics calling routine was devised.) PRESENT first reports the covariance matrix in
an unprocessed form. If the user desires, they can inspect this matrix to see how the elements relate to each other. The off-diagonal terms should be very small in comparison to the diagonals.

Following this a detailed screen reporting the misalignments and offsets for each plane is displayed. The orientation is displayed using a right-handed coordinate system with the $z$-axis pointing upwards and the $x$-axis along the shaft as discussed in Section 5.3. Accuracy values in the form of standard deviations are displayed next to the solution values. The specified units are adhered to for all displayed measurements including the standard deviations.

Lastly the results are displayed in a schematic form. This is an idealized representation of the shaft orientation. The fixed end is displayed on the left and the free end on the right side of the screen. (As an example, see Figure 6.6.) The free end is considered as such by convention only. This is valid since it is common to find that one end of the shaft is easier to move than the other (Bayly, 1991). The user indicates what data pertains to the end of the shaft considered fixed (INPUT). Thus the results are in the form of misalignments and offsets with respect to this "fixed" end.

## FOOT: (300 LINES)

This function is an option that can be called if the user wishes to know how much the base or feet (on the free end of the shaft) must be moved in order to be aligned with the fixed end. The results are presented numerically following the sign convention discussed in Section 6.2.2, as well as schematically.

The equations to solve for the corrections necessary to realign the shaft are similar to those discussed in Section 6.2.2 that deal with offsets (Equations 6.4 and 6.5 ). The vector defining the fixed end of the shaft is "followed" from its origin to the foot to be adjusted (position entered by user) in a similar manner as the equations. The other vector does not have a common origin corresponding to the base measurements provided by the user. Thus the slope equation must first subtract the distance between the origins of both vectors from the user provided base distances. For example, if the user indicates that one foot is 2.5 metres from the origin of the fixed shaft end along the $x$-axis, and the distance to the free end vector is 4 metres in the same direction, the $x$-distance from the free vector will be $2.5-4.0=-1.5$ metres, i.e. 1.5 metres along the second vector in the negative $x$-direction.

### 6.5 Converting Center Points to Vectors

Direction numbers provide a convenient and efficient means for calculating shaft misalignments and offsets. Fitted circle output data can be applied directly to the mathematical model used for these computations. Unfortunately the resulting vector may not be perpendicular to the "true" shaft axis as seen in Figure 6.1. Also, if the region where the circle readings were made happens to be slightly deformed (e.g. non-cylindricality of the shaft), results will show misalignments when the shaft may actually be situated in its optimal operating position. For this reason, another means for finding the "true" shaft axis is necessary.

Normally FIT will be used twice after coordinates have been adjusted: once for each form-fitted circle per shaft end. Under these conditions two
vectors as well as their points of origin are reported. SHAFT uses these two files to compute misalignments and offsets as discussed previously. The author refers to these as VECTOR-I type calculations. Another method to find the shaft axis is to measure not two but four regions of the shaft: two on the so-called fixed end and two on the free one. These regions can be strategically positioned according to shaft visibility. This allows for a greater shaft component length to be represented than by using just one crosssection. These so-called VECTOR-II calculations provide a stronger solution as will be shown in Chapter 7.

FIT can be used to produce four files corresponding to each measured region of the shaft. The results will be in the same form as with VECTOR-I; namely ( $x_{0}, y_{0}, z_{0}$ ) and $\langle a, b, c\rangle$ as well as standard deviations and other data such as shaft radii. Using the VECTOR-II option, only the center point data is used in the mathematical model. A pair of center points are joined to form a vector for either end of the shaft. This vector is likely not of unit length and thus must be converted before SHAFT can continue. This is accomplished by dividing the components of the vector with the spatial distance of that component. The x-component is not used for reasons discussed previously except for spatial distance computations:

$$
\begin{align*}
& b=\frac{y_{2}-y_{1}}{d_{12}} \\
& c=\frac{z_{2}-z_{1}}{d_{12}}
\end{align*}
$$

Variances and covariances are propagated in a similar manner as explained in Section 6.3. The $\mathbf{C}_{\ell}$ matrix is composed of three coordinate
variances for each circle on one end of the shaft. Thus for both circles on one shaft end:

$$
\mathbf{C}_{\ell}=\left|\begin{array}{ccc:ccc}
\sigma_{x_{1}}^{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\mu}^{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{z_{1}}^{2} & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{x_{2}}^{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{\mu_{2}}^{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{z_{2}}^{2}
\end{array}\right|
$$

The Jacobian is a $2 \times 6$ matrix obtained from the derivatives of Equations 6.11 and 6.12:

$$
\mathbf{J}=\left|\begin{array}{llll}
\frac{\left(x_{2}-x_{1}\right)\left(y_{2}-y_{1}\right)}{d_{12}^{3}} & \frac{\left(y_{2}-y_{1}\right)^{2}-d_{12}^{2}}{d_{12}^{3}} & \frac{\left(y_{2}-y_{1}\right)\left(z_{2}-z_{1}\right)}{d_{12}^{3}} & \text {-element }{ }_{11} \\
\frac{\left(x_{2}-x_{1}\right)\left(z_{2}-z_{1}\right)}{d_{12}^{3}} & \frac{\left(y_{2}-y_{1}\right)\left(z_{2}-z_{1}\right)}{d_{12}^{3}} & \frac{\left(z_{2}-z_{1}\right)^{2}-d_{12}^{2}}{d_{12}^{3}} & \text {-element } \text { element }_{13} \\
\text {-element } & \text {-element } & \text {-ele }
\end{array}\right|
$$

Thus the variance data can be propagated by:

$$
\mathbf{C}_{\mathbf{x}_{\mathrm{I}}}=\mathbf{J} \mathbf{C}_{\ell} \mathbf{J}^{\mathrm{T}}
$$

The diagonals of the resulting $2 \times 2$ covariance matrix are the variances used to obtain vector components $b$ and $c$ respectively. These values, as well as the solutions for Equations 6.11 and 6.12 are used to form the data set:

$$
\begin{aligned}
& \left(x_{o}, y_{o}, z_{o}\right), \\
& \left(\sigma_{x_{o}}, \sigma_{y_{o}}, \sigma_{z_{o}}\right), \\
& \langle b, c> \\
& \text { and }\left\langle\sigma_{b}, \sigma_{c}\right\rangle .
\end{aligned}
$$

This entire procedure is again repeated for the other end of the shaft. The results can then be used as if we began with two files instead of four as was described in Section 6.4.

### 6.6 Testing and Results: SHAFT

### 6.6.1 Studies of Previously Obtained Data

As with the programs discussed earlier, SHAFT was tested using previously gathered data. During program development, the data from LAB5 was used. One is reminded that instead of a shaft being used, circles were drawn on the backs of two opposing file cabinets. After adjusting the coordinates of points on these perimeters, FIT was used twice to form-fit these points to an adjusted circle leaving two output files. These resulting files contain data describing the center point of each circle as well as a threedimensional vector. SHAFT used these files computing the following covariance matrix of misalignments and offsets:

$$
\begin{gather*}
\mathrm{M}_{x y} \\
\mathbf{C}_{\mathbf{x}}= \\
\begin{array}{cccc|c}
0.40210 & 0.0 e+00 & \mathrm{M}_{x z} & \mathrm{O}_{x y} & \mathrm{O}_{x z} \\
0.0 e+00 & 0.37060 & 0.0 e+01 & 0.0 e+00 & \mathrm{M}_{x y} \\
\hdashline-1.0 e-01 & 0.0 e+00 & 0.45106 & -1.4 e-08 & \mathrm{O}_{x y} \\
0.0 e+00 & -3.6 e-02 & -1.4 e-08 & 0.41506 & \mathrm{O}_{x z}
\end{array}
\end{gather*}
$$

The first two rows and columns represent the misalignments and the remaining elements are offsets. When converted into the relevant output form (indicated by the user), these values indicate that the largest standard deviations are $\pm 0.000634$ radians in misalignment and $\pm 0.672$ millimetres in offset. These are quite reasonable since the survey was in a controlled environment and utilized strong geometry.

To test the VECTOR-II option in this situation, each vector file was duplicated with only the $x$-coordinates changed a small amount towards the
midpoint of the shaft. Thus, when VECTOR-II was implemented, only the four three-dimensional center coordinates and their standard deviations were used. The largest reported standard deviation from SHAFT was a much smaller $\pm 0.00004$ radians in misalignment and $\pm 0.07$ millimetres in offset. This suggests that this technique can provide more accurate results than by using just one fitted circle per shaft end through propagated errors. Figure 6.6 is an example of the results presented by SHAFT. These results were independently confirmed by hand-calculations.

As an independent test, one set of data previously collected by Bayly was found to contain VECTOR-II type data. This data represents measurements gathered using the test rig built by Kadon Electro Mechanical Services Ltd. In these tests a laser spot was used on four separate sections of the rotating shaft providing the measurements necessary for the author to test the VECTOR-II option in SHAFT.

The largest reported standard errors were $\pm 0.00020$ radians in misalignment and $\pm 0.21$ millimetres in offset. Figure 6.7 represents the actual schematic output created by SHAFT.


Figure 6.6 Graphical Results from SHAFT for LAB5 Data


Figure 6.7 Graphical Results for Kadon Test Rig Using Vector-II Data

These preliminary tests were used in the development and initial testing of SHAFT. In the next section the discussion will concentrate on subsequent tests measuring small movements on a test rig co-developed by the author. In Chapter 7 further details on this device are provided.

### 6.6.2 Results of Subsequent Testing

The test rig referred to earlier and described more fully in the next chapter was used to test all the new and enhanced software including SHAFT. (See also Section 5.4.2.) The network was initially adjusted independent of the test rig. Subsequently targets on the rig and network targets were observed and adjusted. The numbering scheme used on the shafts of the test rig are shown in Figure 6.8


Figure 6.8 Target Locations on Test Rig

The left shaft (A) was fixed while the right one (B) was moved by small measured amounts.

The VECTOR-II option was used in most tests. (VECTOR-I proved to be very weak and one significant conclusion from this and other testing indicates that VECTOR-I data should be avoided if possible; see also Chapters 7 and 9.) FIT was used once for each numbered target region on the shafts-namely the 1000's, 2000's, 3000's and 4000's regions. The resulting standard deviations associated with the $x, y$ and $z$ components of the center coordinate of each circle as well as the coordinates themselves were used by SHAFT to compute the misalignments and offsets of pipe $B$ with respect to $A$. Pipe $B$ was moved in various directions or angles by small amounts and changes in the alignment compared with pipe A were computed. Tables 6.1 and 6.2 summarize the results of one test where shaft $B$ was moved from a misaligned to aligned position. The first table represents results obtained when the radii of the shafts were not fixed while the second is for fixed radii that were repeatedly measured using digital calipers.

| Table 6.1 SHAFT Results: Rig Test Number 10 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| (all shaft radii NOT fixed) |  |  |  |  |  |


| Table 6.2 SHAFT Results: Rig Test Number 10 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| (all shaft radii WERE fixed) |  |  |  |  |  |

The terms before and after in the first column refer to the shaft positions before and after attempted alignment. The shaded rows indicate the resulting misalignments after attempting to align the shafts. The last row in either table refers to changes in alignment only. It can be seen that with the same data, Table 6.2 shows much smaller standard deviations thus indicating a more reliable solution.

In testing pure translations using the test rig, discrepancies between the caliper and SHAFT results were all less than 0.26 millimetres. Pure rotational discrepancies were less than 46 arc seconds. On the other hand, in Tests 9 and 10 (see below), the attempt was to have misaligned shafts moved to a realigned position using SHAFT's foot output. The shaft that could be moved was found to clearly be closer to an aligned position but not as good as expected. This was later found to be due to inherent limitations in the rig design to be discussed in Chapter 7. Since the primary effort was to test SHAFT and other software, a new and more reliable test (Section 7.2) was
devised. Table 6.3 is presented summarizing the comparisons between caliper and SHAFT results for the test rig.

| Table 6.3 Comparison Between Caliper and SHAFT |
| :---: | :---: | :---: |
|  |$\quad$| READINGS |  |
| :---: | :---: |
| (mm or deg-min-sec) |  |

The results from Test 10 are better than those from Test 9 due to the initial misalignment of the rig in Test 10 being relatively small. This reduces the effect of erroneous caliper readings. If the shafts are not very close to perfect in initial alignment, the caliper readings will not be purely horizontal or
vertical. Attempting to model for these slope caliper readings concluded with the author discovering that the intrinsic structure of the rig would not allow for proper independent measurements to be taken. Details of this are found in the next chapter.

## Chapter 7

## Alignment Testing Using Complete System

In the previous discussion three new or enhanced programs adding to Bayly's existing theodolite intersection system were discussed. COLLECT included enhancements while APPROX and SHAFT were newly developed programs integrated with Bayly's original three modules (Chapter 3).

In this chapter, a test rig designed in cooperation with fellow graduate student Ray Obidowski is detailed. This rig has already been referred to in each of the proceeding chapters. The first section covers the structure and design of this rig as well as general conclusions reached by the previously discussed testing. In Section 7.2, a supplemental test where the test rig was not used is discussed. This last test was conducted due to limitations found in the test rig design and to determine if the new software was working correctly.

### 7.1 The Alignment Test Rig

This device consists of two aluminum pipes of the same diameter that are positioned spatially using adjustable bolts (see Figure 7.1). One pipe represents the fixed end of an industrial shaft being thought of as the end of a coaxial shaft train that is less adjustable than the other in terms of re-alignment. The fixed end of the test rig is designed to be able to move in


Figure 7.1 Design Specifications of Co-Developed Test Rig
translation along the axial direction although this is not used in the testing. The other end can be positioned with four degrees of freedom: translation in the $z$-direction (i.e. up or down), translation in the y-direction (at right angles to shaft axis), rotation about the $z$-axis and rotation about the $y$-axis. These movements allow the pipe to be positioned in a variety of orientations being either aligned or misaligned with respect to the fixed pipe.

In order for the rig design to conform to the needs of both designers, the decision was made to position four pillars vertically next to either side of the pipe at both ends. These pillars were used as stable reference points from which measurements could be made using calipers in order to have independent measurements of rotations and translations. The calipers were placed between the bolts extending from the pillars and the pipe bolts. Theoretically, by measuring the linear changes in pipe position from four pillars, average values can be computed that represent the rotations and translations of the pipe. Results from testing revealed that even for pure horizontal rotations, the four measurements (i.e. changes in values) had an undesirably large distribution. Attempts to model for this lead to the conclusion that there was no independent verification to determine if the vertical rotation axis of the pipe was centered or even measurable.

In the tests described earlier, this rig was used to determine if the extended theodolite intersection system data could be verified using caliper readings. The old intersection system had been successfully tested (Bayly, 1991) and so using this test rig was more a verification that the new integrated package was in fact working correctly. In this capacity, the tests using the rig
were successful. In terms of using the calipers to validate the data, they were not successful.

Although a number of assumptions about the test rig were shown to be unverifiable (see below), many important discoveries about the new and existing software were made:

1) The author discovered that the new COLLECT software would give erroneous collimation values when the first observation from a new theodolite set-up was set to zero. On investigation, it was found that if the closing direction (i.e. reading of same target again from the same theodolite position) was nearly the same direction but just less than $360^{\circ}$ such as $359^{\circ} 59^{\prime} 47.9^{\prime \prime}$, the collimation was reported as being near $180^{\circ}$ instead of near $0^{\circ}$. This was readily corrected.
2) It was found that by having COLLECT always display collimation errors, one could always see how well the theodolites were internally adjusted.
3) ADJUST was updated (with Bayly's permission for testing purposes) to accept more than just 25 stations. As an example of this limitation, with four regions on the pipes having five targets each, 20 stations out of the 25 were already used up.
4) It was found that using only three or even four points per best-fit circle gave substantially weaker alignment solutions than using five or more.

Additional discoveries were related to the structure of the test rig as discussed above. Table 6.3 presented a strong indication of these problems
where the differences between the intersection system and the hand-measured readings were generally too large. Study of the structure of the test rig lead to the following assumptions about the rig being highly in question:

1) The assumption that caliper readings would either be purely horizontal or vertical. In fact, with the movable pipe being even one or two millimetres out of alignment, measurements from the pillars to the pipe would actually be slope values.
2) The assumption that the test rig pipes were centered with respect to the four horizontal bolts. In fact, measurements of position changes on these bolts revealed that the pipe was not perfectly centered between the four pillars.
3) The assumption that all the pillars were the same height and exactly vertical. In fact, this was questionable since the base on which they were attached was only stable after being placed on a stable surface. The base was flexible enough for the positions of the pillars to always be in question.

Due to the difficulty in obtaining reliable caliper measurements, these tests using the rig are considered only as a good trial method in discovering and eliminating bugs in the new and enhanced software: A better series of tests was examined as discussed in the next section.

### 7.2 Alignment of a T2 Theodolite

This series of tests uses the same software that was used on the alignment test rig with minor revisions discussed in the previous section being applied. The experience gained from the first series of tests allowed the author to apply some of the ideas in this series, such as avoiding targets being placed too close to the line between the two instruments as seen in Figure 5.9.

These tests consist of two plastic pipes acting as a coaxial shaft that requires alignment. One pipe is mounted on a tripod and secured using tape while the other is mounted on the top of a T2 Wild Theodolite telescope. Any movements of the telescope would be the same movement experienced by the attached pipe. This enables relative translations and rotations of the pipe to be monitored independent of the theodolite intersection system.

The network used for this series of T 2 tests consisted of two theodolite observation stations 201 and 202 as well as a 2 metre scale bar (ends 101 and 102) and six wall targets at various heights (see Figure 7.2). A set of five targets was placed on either end of each of the two pipes for a total of 20 "shaft" targets (see Figure 7.3). Although three targets would provide a unique center point, five were used for added redundancy giving better positioning from the geometric adjustments in FIT.


Figure 7.2 T2 Test Network


Figure 7.3 Target Position on Static and T2 Pipes
Unlike the tests in Section 7.1, these tests were set up such that all twenty shaft targets were visible from both theodolite stations. This made it possible to use more than the minimum number of observations to improve
the solution. The diameters of all four pipe ends were measured with the calipers rated as 0.01 mm in resolution. Over twenty readings were made on each pipe end to obtain standard deviations that could be used to fix the pipe radii in FIT. Four specific tests were conducted as summarized in Table 7.1. Figure 7.4 shows the orientation of these movements as related to Figure 7.2.

| Table 7.1 Description of T2 Testing |  |  |
| :---: | :---: | :---: |
| TEST | TESTED MOVEMENT | DESCRIPTION |
| A | Y-axis translation | Perpendicular to shaft <br> axis |
| B | Y-axis rotation | In vertical plane <br> containing shaft axis |
| C | Z-axis rotation | In horizontal plane <br> containing shaft axis |
| D | all | Misaligned to aligned |



Figure 7.4 Orientation of Tested Movements

A translation stage was used for $y$-translation and the T 2 telescope could be used to rotate the pipe about the $Y$ and $Z$ axes. The readings on the translation stage provided 0.01 millimetre resolution measurements in translation while the T 2 theodolite micrometer gave angular values interpreted up to one tenth of an arc second. Vertical translation was not possible with the available translation stage. As described earlier, x-translation is not necessary for testing since this is not a problem when dealing with coaxial shaft alignment. Table 7.2 is a summary of the results comparing the "actual" measured shaft movements with the results obtained using the enhanced theodolite intersection system.

| Table 7.2 Comparison of Actual Measured Movements with Theodolite Intersection Results |  |  |  |
| :---: | :---: | :---: | :---: |
| TEST | "ACTUAL" <br> MOVEMENT <br> ( $\mathrm{mm} / \mathrm{dd}-\mathrm{mm}-\mathrm{ss}$ ) | THEODOLITE INTERSECTION RESULTS ( $\mathrm{mm} / \mathrm{dd}-\mathrm{mm}-\mathrm{ss}$ ) | DIFFERENCE ( $\mathrm{mm} / \mathrm{mm}$-ss) |
| A | $\begin{aligned} & 1.00 \\ & ( \pm 0.014) \end{aligned}$ | $\begin{aligned} & 1.45 \\ & ( \pm 5.00) \end{aligned}$ | 0.45 |
| B | $\begin{aligned} & 00-01-00.0 \\ & ( \pm 00-02.8) \\ & \hline \end{aligned}$ | $\begin{gathered} 00-00-53.1 \\ ( \pm 11-25.8) \\ \hline \end{gathered}$ | 00-06.9 |
| C | $\begin{aligned} & 00-00-21.5 \\ & ( \pm 00-02.8) \\ & \hline \end{aligned}$ | $\begin{aligned} & 00-01-33.0 \\ & ( \pm 03-34.4) \\ & \hline \end{aligned}$ | 01-11.5 |
| D <br> Y-trans <br> Y-axis rot. <br> Z-axis rot. | $\begin{aligned} & 3.44( \pm 0.014) \\ & 00-22-06.6( \pm 00-02.8) \\ & 00-06-39.0( \pm 00-02.8) \end{aligned}$ | $\begin{aligned} & 3.46( \pm 1.00) \\ & 00-22-33.0( \pm 01-27.1) \\ & 00-07-33.5( \pm 03-34.7) \end{aligned}$ | $\begin{aligned} & 0.02 \\ & 00-26.4 \\ & 00-54.5 \end{aligned}$ |

All the results in Table 7.2 were obtained using the VECTOR-II option. The VECTOR-I option (i.e. using only the 1000's and 4000's circle data) was tested and the results were very weak in comparison to Table. 7.2. Thus the author again suggests avoiding the use of less than two best-fit circles per shaft end for critical alignment situations.

With the results in Table 7.2 , the radii of the two pipes were measured and fixed as described earlier. Depending on the strength of the survey network as well as other factors (such as target visibility), fixing the radii was shown to improve results by as much as $80 \%$ when comparing related statistical confidences. The following observations can be made regarding these results:

1) Tests A and B had used a softer plastic pipe to simulate a shaft. Thus the measured radii were not as reliable (higher variances) and the shape of the pipe was found to not be cylindrical.
2) The only test area available was in a room on the fourth floor of The University of Calgary Engineering building. During observations of the test network, small vibrations were visible from the building causing the focused targets to appear slightly blurry.
3) Sunlight through windows (that could not be perfectly shaded) caused uneven heating in various parts of the room.
4) The simulated shafts were made of PVC and ABS plastic. The PVC pipe was white and the ABS black. Uneven heating may have caused unanticipated movements of the pipes.
5) The scale bar itself was unavoidably placed within direct sunlight at various times. Since the bar material was invar, this effect would have been small.
6) T2 theodolite readings were difficult to observe with the pipe placed on the telescope.
7) The instrument used for target observation was the less accurate Wild T2000; a T2002 theodolite (i.e. with dual axis compensation) was not available at the time thus making this point very significant.
8) The accuracy of the T 2 was about 2 arc seconds horizontally and 1 to 2 arc seconds vertically.

These eight points, especially the seventh, can justify a large portion of the discrepancies in Table 7.2. Test D was favorable due to extra care being taken in avoiding weak geometry.

Fixing the azimuth from points 101 to 102 made apparent alignment accuracies smaller. This also occurs when the scale bar is situated closer to the shaft. The first point is explained by the fact that targets 101 and 102 are special targets that are on the ends of an accurately known distance-the scale bar. Thus target 102 is extremely well positioned with respect to target 101 in distance and azimuth. This creates a strong reference datum from which computations are based. As to the second point, the size of each target confidence region will increase the further one gets from the fixed points when using conventional minimal constraints (i.e. in Bayly's ADJUST). Thus, when the scale bar is near the shaft, the apparent accuracies of targets on the shaft are better giving a seemingly stronger alignment solution.

### 7.3 Other Testing and Results

A good production type test was conducted for The University of Calgary's Human Performance Laboratory in the Department of Physical Education. They designed this structure to correlate measurements taken on human feet during walking. This was to be accomplished through the use of charged coupled device cameras (CCDs). The cameras were to be positioned below a clear floor to study the bottom of test subject's feet. A calibration structure was built to establish scale but plastic target spheres within the structure had to be accurately positioned before actual tests could begin. Figure 7.5 is a simplified representation of this calibration pyramid.


Figure 7.5 Human Performance Laboratory Calibration Pyramid

The plastic spheres within the steel structure were about 2 centimetres in diameter. The author was asked to position each sphere accurate to a least 0.1 millimetres using the enhanced theodolite intersection system as follows: COLLECT $\rightarrow$ APPROX $\rightarrow$ ADJUST. Scale was set using the calipers mentioned earlier; they were set at a preset distance repeatable to 0.01 millimetres. The spheres were observed in a similar manner surveyors have traditionally used
to position the sun: The quadrant tangent method. In Figure 7.6 the face-left and face-right positions (direct and reverse) of the theodolite cross-hairs on the sphere are shown. This method ensures that the center of each target is always observed correctly, no matter from which observation station.

Face Left Observation


Face Right Observation


Figure 7.6 The Quadrant Tangent Method

The results gave three-dimensional target positions accurate to an average of close to 0.02 millimetres. This was an excellent test for both the enhanced version of COLLECT and the new program APPROX. COLLECT reported large collimation errors due to the nature of the quadrant method without affecting the resulting data. One pointing error was found immediately and easily corrected saving what otherwise would have been hours of effort if a new survey would have been necessary. APPROX was very helpful especially because of the difficulty in measuring within this calibration structure due to very limited space for scales or other manual taping devices.

## Chapter 8

Deformation Modeling and Covariance Data

In this chapter, a summary of an investigation dealing with the use of network and shaft point data to conduct deformation analysis is presented. As discussed in previous chapters, an independent network is established to provide a stable datum or reference set from which shaft positioning can be based. Thus any actual shaft positions can be compared with previous epoch data providing a valuable monitoring method allowing one to model shaft alignment changes. If the so-called stable network targets suffer from small movements that go undetected, the movements will propagate through the software unnoticed giving erroneous reports of shaft movements or misalignments. It is therefore vital to ensure that if repeated surveys are conducted on any machinery, the reference network be stable or have those points that are unstable isolated from the analysis. The following two sections outline this investigation and include testing on how significant covariance data is to the statistical results when dealing with these types of survey networks.

### 8.1 Deformation Analysis

Biacs (1989) discusses software and testing for deformation analysis in his M.Sc. thesis. This software was available to the author for confirming results
obtained at the previously mentioned compressor site west of Red Deer, Alberta. One program in particular (called GEODEAN), can analyze multiepoch data and determine whether target points should be deemed as having moved significantly using congruency test procedures (Biacs, 1989). The author created some basic software to convert data file formats from Biacs' program to Bayly's program and vice versa. This was conducted in order to use adjusted coordinates from ADJUST in Biacs' deformation analysis software GEODEAN. The software was successfully used in this conversion process.

The compressor layout is presented in Figure 8.1. The datum points assumed to be stable were numbered in the hundreds while machine points were numbered under 10. Actual shaft measurements were not possible in this case due to bearing housings having to be in place for operating machine safety. Two surveys were made to test for moved points (Robbins, 1992). The first epoch data was gathered as the machinery was running while the second was gathered during shut-down.

An initial deformation analysis was performed by Robbins (1992) and confirmed by the author using GEODEAN. In this analysis, movements of various points from epoch 1 to epoch 2 were detected. The results indicate that both epochs provide comparable solutions in terms of precision as would be expected when using essentially the same datum. After deformation analysis (discussed below), the surprising result was that only a few network points thought to be stable were actually so. Without detecting and eliminating those datum points found to be unstable, unreliable reports of machine movements would result.


Figure 8.1 Measured Compressor Site (Robbins, 1992)

Various datum configurations were tried with one result of some interest: Point 106 was removed from the datum and replaced with 102. Using this datum, global congruency failed yet single point tests for targets 100, 101, and 102 passed. This is reasonable especially if one notes that three of these
points are situated on one side of the network. This analysis indicates that point 102 was in fact unstable causing the global test for congruency to fail. This movement "pollutes" point 106 (which is stable) causing the local test to fail at that point. In other words, targets 100, 101, and 106 were found to actually be the best possible points that could be used as a stable datum between epochs. From this, a general conclusion can be made: Extremely careful analysis of simple coordinate differences can provide excellent results if and only if datum congruency is considered before obtaining the data. This may be in the form of ensuring truly stable network points are available, or by applying rigorous deformation analysis. By simply using results from SHAFT at various epochs, changes in alignment could be reported when none actually exist. A stable datum ensures these alignment results are correct.

### 8.2 Covariance Data Testing

In some typical deformation networks, such as those used to monitor dam movements, "object" points can be occupied thus acting not only as theodolite stations but as target stations. This creates a dependency among observations to these occupied stations and results in high covariances associated with the computed coordinates. In industrial alignment situations discussed in this thesis, the object points (e.g. points on a shaft) almost always cannot be occupied. This results in low covariances associated with the computed coordinates.

The author investigated if the lack of covariance propagation in Bayly's existing software should be rectified. In essence: Was the propagation of only
variance data adequate? Would statistical results including confidence in reported shaft alignment be in question?

The compressor site data discussed above was used to test the effects of removing only covariances from the data files while leaving only variances. (Initial data, including data used in this present test, were hand recorded. This data (both with and without covariances) was adjusted in Biacs' MONALYSA software (Biacs, 1989) since full variance/covariance propagation was possible.) Without covariance data, the stable datum discussed above (100, 101, and 106) increased by one target station: 103. Station 103 being added to the datum was reasonable since the original reported movements (i.e. with covariances) were only slightly significant statistically in the vertical direction and statistically stable in the other directions. All other object point values were within 0.06 millimetres of the original datum. In general, the boundary values (confidence regions in one dimension) in the northing ( Y ) and easting $(X)$ directions were smaller without covariance data while the boundary values for heights were larger. Of all the object boundary values, the largest difference from this datum (i.e. using only variances) to the full variance/covariance matrix solution was 0.08 millimetres for a change in northing of point 1. This is certainly an acceptable difference.

Datum target point 102 was of particular importance due to being located on one of the support pillars of the compressor (Figure 8.1). The following data (Table 8.1) presents the full matrix results (datum $=100,101$, and 106) compared with the variance-only results (datum $=100,101,103$, and 106). The results demonstrate that since the datum is not the same, not only the boundary values but also the movements themselves will be slightly different.

| Table 8.1 Comparison of Full Covariance Deformation Results With Variance-Only Results (Different Datum) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MATRIX | $\begin{gathered} \Delta \mathrm{N} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bN} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \Delta \mathrm{E} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bE} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \Delta \mathrm{H} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bH} \\ (\mathrm{~mm}) \end{gathered}$ |
| FULL | 0.18 | 0.19 | 0.42 | 0.25 | -0.14 | 0.12 |
| VARIANCES ONLY | 0.20 | 0.21 | 0.43 | 0.20 | -0.09 | 0.14 |

The author decided to establish the exact datum as the original by eliminating point 103. This time the results for point movements match the full matrix results exactly as would be expected using the same datum. The northing and easting boundary values were very close to the full matrix results while the height boundary values were generally larger by about 0.09 millimetres. Table 8.2 reports movements of point 102 but this time using the same datum in both the full matrix and variance-only results.

| Table 8.2 Comparison of Full Covariance Deformation Results With Variance-Only Results (Same Datum) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MATRIX | $\begin{gathered} \Delta \mathrm{N} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bN} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \Delta \mathrm{E} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bE} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \Delta \mathrm{H} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{bH} \\ (\mathrm{~mm}) \end{gathered}$ |
| FULL | 0.18 | 0.19 | 0.42 | 0.25 | -0.14 | 0.12 |
| VARIANCES ONLY | 0.18 | 0.24 | 0.42 | 0.24 | -0.14 | 0.16 |

With these types of industrial networks, covariance data contributes very little to the deformation solution and can therefore be excluded from the software chain. Adding the ability to propagate covariances as well as variances to Bayly's existing system was not deemed necessary after this investigation.

## Chapter 9

## Conclusions and Recommendations

Theodolite intersection offers an accurate and reliable non-contact method to align industrial machinery. In this investigation, an existing system was studied and subsequently enhanced to create a cohesive and efficient series of software modules.

Vital error detection routines were successfully added to the existing program COLLECT. It was noted that random errors can be reduced by using high quality theodolites with dual axis compensation and free stationing techniques. Systematic errors are similarly reduced or eliminated by also stipulating that the theodolites must be used on both faces. This provided the necessary data to warn the user whether collimation or vertical index errors existed as well as reporting pointing errors. Data integrity was never compromised. Testing had shown that these additions to COLLECT were invaluable in avoiding costly errors especially before leaving any job-site.

To ensure adjustment convergence, software was created to automatically generate approximate coordinates. This eliminated the need to manually measure target coordinates and create the appropriate file for the adjustment program. A large amount of code in APPROX was necessary to accommodate more than two theodolites in a network (often necessary to observe various parts of shafts or other machinery). Routines were created for coordinate
matching of various types of observational data thus detecting if all appropriate data existed or not. Resection and bearing-bearing intersection were used to position targets or theodolites in a repetitive process. This process would search every possible observation or coordinate to locate these additional points.

Testing of APPROX revealed that although proper mutual pointing between any two theodolites in the network is not necessary, one should attempt to be as accurate as possible or avoid geometrically weak targets inline with the two instruments. The time saved in gathering approximate coordinates was substantial and the automatic scale and orientation of the network eliminated the need to manually edit files created by COLLECT.

SHAFT was written to provide a final informative solution on shaft alignment. Data in the form of direction numbers was used and full variance/covariance propagation was incorporated to provide an assessment of the quality of the alignment solution. Graphical output was added to provide an unambiguous representation of the solution and inform the user what corrections would be necessary for re-alignment. An option to allow the user to use four instead of only two best-fit shaft circles proved to be a more reliable method for solving misalignment. This was evident in the included statistical data.

An alignment test rig was constructed with the intention of allowing for comparison of caliper-measured translations and rotations with results obtained from the enhanced theodolite intersection system. Comparisons were excellent for translation while rotations proved to be troublesome. Investigation revealed that the rig structure was not conducive to measuring
accurate rotations using calipers against stationary pillars. Thus a subsequent test of the new software was conducted on a set of pipes, one of which was placed on the top of a T2 theodolite telescope. Results were substantially improved and the new software worked successfully. The alignment of a calibration pyramid provided an unexpected test of the abilities of COLLECT and APPROX.

Investigations into deformation analysis revealed that simple coordinate differencing between epochs at an industrial site leave room for dangerous unexpected movements of so-called stable targets to corrupt the final alignment solution. Testing at a compressor site indicated that these unexpected movements will cause errors in the final shaft alignment solution. Other testing revealed that full variance/covariance propagation beginning with observational data was not necessary in these types of industrial settings where network targets are not occupied by theodolites.

Future efforts in industrial alignment should concentrate an expanding these types of non-contact methods by using improved instrumentation or by way of other methods such as photogrammetry or laser scanning. The author feels that investigation into these areas will lead to even more efficient and accurate industrial alignment systems. By incorporating directional data with full network distance information, an extremely "solid" adjustment solution will result. This would also eliminate the need for complex software to determine approximate coordinate data.

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