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STOCHASTIC MODELING AND ANALYSIS OF BUFFERED RANDOM ACCESS LOCAL AREA NETWORKS

by

Abraham Olatunji Fapojuwo

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The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies for acceptance, a thesis entitled *Stochastic Modeling and Analysis of Buffered Random Access Local Area Networks,* submitted by Abraham Olatunji Fapojuwo in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Abstract

The interference problem inherent in random access local area networks (LANs) necessitates a buffer at each network user to temporarily store messages. This dissertation proposes a new methodology for the analysis of a local area network which consists of a finite population of buffered users using the nonpersistent Carrier Sense Multiple Access with Collision Detection (CSMA-CD) protocol for channel access. A new technique is desirable as existing methods become computational intensive and/or inapplicable for large number of network users each equipped with infinite buffer size; and furthermore, there is no technique available for the study of buffering multipacket messages.

The methodology developed in this dissertation is based on: 1) a decomposition approximation, 2) an appropriate state-space modeling of each network user and the shared broadcast channel, 3) a steady-state Markov chain theory, and 4) renewal theory arguments. Within this framework, the methodology offers formal solutions to the following four key problems of practical significance: i) the analysis of multimessage buffer LANs with single-packet messages, ii) the analysis of single-message buffer LANs with multipacket messages, iii) the analysis of multimessage buffer LANs with multipacket messages, and iv) the analysis of interconnected single-message buffer LANs with multipacket messages. To achieve effective use of the channel in transmitting multipacket messages, the gated and limited packet transmission strategies are proposed and analyzed. It is argued that the gated transmission strategy has broad applicability in integrated services local area networks (ISLANs).

Specific new results obtained in the dissertation are: a) steady-state probability generating function for queue length and the mean queue length, b) formulas for mean packet delay and message response time, c) formula for the probability of buffer overflow, d) formula for the throughput, and e) necessary and sufficient condition for system stability. The performance measures are obtained numerically for sample networks of each of the four problems considered and the effects of specific parameters on the system performance are investigated. Discrete-event computer simulation models of the sample networks are constructed and it is found that the numerical results based on the methodology developed are in good agreement with the simulation results.

The significant contribution of the dissertation lies in the potential of the analytic models developed which provide useful insights into the performance of buffered random access LANs.

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To the memory of my father

JACOB FAPOJUWO

who died while I was studying abroad.

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Glossary of Symbols

a	Normalized Propagation Delay
A_k	k stations which were in the empty state at the last transmission completion epoch begin transmission during the collision window of an ongoing transmission period
$A_k^{(i)}$	Input traffic to the i^{th} node during slot $[k,k+1)$
$bin(.,.,.)$	A binomial probability function
b_o	Probability that the bridge buffer is empty
$b(x)$	Probability density function for the duration of a node busy phase
B	Random variable denoting the duration of a node busy phase
BIU	Bus Interface Unit
BR	Generic notation for bridge node
BR_i	Bridge buffer for storing internetwork messages to be transmitted on LAN_i
B_b	Bridge node busy period
$B_c(.,.)$	Average reward earned during an unsuccessful transmission period
B_{ch}	Channel busy period
$B_d(.,.)$	Average reward earned during a dormant slot
B_j	Busy period generated by the j packets in a nodal buffer at the end of a resting period
$B(x)$	Probability distribution function of B
$B_j(x)$	Probability distribution function of B_j
B_o	Voice packet overhead length (in *bits*)
$B_s(.,.)$	Average reward earned during a successful transmission period
B_v	Voice packet data length (in *bits*)
$B_1^{*(j)}(t)$	j-fold convolution of $B_1(t)$ with itself
B^e	Transmission commencement by the bridge node which was in the empty state at the last transmission completion epoch

B^n	Transmission commencement by the bridge node which was in the nonempty state at the last transmission completion epoch
c	Normalized collision resolution period
C	Channel capacity (in *bits/sec*)
C_i	Duration of the i^{th} unsuccessful transmission period
CSMA-CD	Carrier Sense Multiple Access with Collision Detection
CT	Conventional Transmission
d_{ij}	Transition probability that j messages are in the network at the current successful transmission completion epoch given that there were i messages at the last epoch
$d_{ij}^{(k)}$	Transition probability that j messages are in the network at the current successful transmission completion epoch of a message (of length k packets) given that there were i messages at the last epoch
D	Packet delay
$\hat{D}$	Minimum packet delay
D_a	Random access delay for a voice packet
$D_{a_{\max}}$	Maximum value of D_a
D_i	i^{th} channel resensing delay
D_{inter}	Internetwork message response time
D_{intra}	Intranetwork message response time
$D^{(i)}$	D for user i
DAMA	Demand-Assignment Multiple-Access
DFT	Delayed First Transmission
D_M	Message Response time
D_w	Weighted packet delay
$\hat{D}_w$	Minimum weighted packet delay
D_v	Voice Packet Delay
e	Estimator parameter
e_k	Limiting probability of successful transmission of a message of length k packets
e_k'	Normalized value of e_k

$E[x]$	Expectation of x
f_i	Limiting probability that i messages (nonempty nodes)are in the network at an arbitrary transmission completion epoch
f_{ij}	Transition probability that j messages are in the network at the current transmission completion epoch given that there were i messages at the last epoch
f_i^e	Limiting probability that there are i nonempty nodes in the network (given that bridge node is empty) at an arbitrary transmission completion epoch
f_i^n	Limiting probability that there are i nonempty nodes in the network (given that bridge node is nonempty) at an arbitrary transmission completion epoch
$\underline{F}$	Row vector of the f_i's
FDMA	Frequency-Division Multiple-Access
g_j	Probability distribution of message length
g_i'	Probability distribution of unblocked message length
$g_i^{*(k)}$	k-fold convolution of g_i with itself
G	Random variable denoting message length
$G(\underline{z})$	Steady-state joint probability generating function for queue lengths
$G_N(z)$	Probability generating function for queue length at the Markov epochs
GT	Gated Transmission
HOQ	Head of Queue
I_{ch}	Channel idle period (random variable)
I_i	i^{th} node idle period
IEEE	Institute of Electrical and Electronics Engineers
IFT	Immediate First transmission
ISLANs	Integrated Services Local Area Networks
ISO	International Standard Organization
$I(Q_k^{(i)})$	Binary-valued random variable denoting empty/nonempty state of node i at the beginning of slot k
$I(S_k^{(i)})$	Binary-valued random variable indicating channel sensing/nonsensing by node i at the beginning of slot k

I_{BR}	Binary-valued random variable denoting the state of the bridge
I_{B0}	Bridge in empty state ($I_{BR} = 0$)
I_{B1}	Bridge in nonempty state ($I_{BR} = 1$)
I_0	Total delay from a message arrival instant (to an empty node) to its first transmission commencement instant
I_{0_a}	Actual delay from the end of a nodal busy period to the first transmission commencement instant
k_{max}	Maximum number of iterations permissible
K	User buffer size
LAN_i	Local Area Network i
LL	Lower limit
LT	Limited Transmission
L_c	Length of a cycle
L_0	Amount of time spent in state 0 during a cycle
M	Number of stations in the network
M_G	Number of nodes adopting the GT strategy
M_i	Number of stations on LAN_i
M_L	Number of nodes adopting the LT strategy
$M/G/1$	A single server queueing system with exponential interarrival time and general service time
$M^X/G/1$	An $M/G/1$ system with bulk arrivals
n_m	Mean number of times that state m is visited during a cycle
N	Number of messages in a user buffer under steady-state condition
$\tilde{N}$	Number of messages in the network under steady-state condition
N_A	Number of transmission attempts from the end of a nodal busy period up to the beginning of the next successful transmission
N_{BF}	Number of boundary functions
N_i	Number of i-element distinct subsets of a given universal set
$N_{n'}$	Number of packets in the queue of a user at an embedded epoch n'
N_s	Number of channel sensing attempts from the instant a new message arrives to a node in the resting phase up to the first transmission commencement

$\tilde{N}(t)$	Number of messages in the network at time t
OSI	Open System Interconnection
p	Channel sensing probability
$\bar{p}$	Probability that channel is not sensed
$p^{(i)}$	p for user i
p_A	Probability that a nonready user at the current embedded epoch remains nonready at the next epoch
$p_A{}'$	Probability that a ready user at the current embedded epoch remains in the same state at the next epoch
$p_A{}''$	Probability that a ready user (with full buffer) at the current embedded epoch remains ready (with full buffer) at the next epoch
p_B	Probability that a nonready user at the current embedded epoch becomes ready at the next epoch
$p_B{}'$	Probability that a ready user at the current embedded epoch undergoes a transition to the next higher state at the next epoch
p_C	Probability that a ready user successfully transmits the head of queue packet and no packet arrives at the end of the successful transmission period
p_{ij}	Transition probability that j users contend for access to the channel at the current embedded epoch given that i users contended at the last epoch
p_{opt}	Sensing probability at which delay is minimum
$p_n(t, x)\Delta t$	Probability that at time t there are n packets in a nodal queue and the node is undergoing a busy phase and the residual successful transmission time of a packet lies between x and $x+\Delta t$
$\underline{P}$	Row vector of channel steady-state probabilities
P_k	Steady-state probability that k users contend for access to the channel at an arbitrary Markov epoch
P_{of}	Buffer overflow probability
$Pr\{x\}$	Probability of event x
$Pr\{j \mid i\}$	Transition probability that j users will contend for access to the channel at the next Markov epoch given that i users contended at the last epoch
$Pr\{j \mid k,i\}$	Transition probability that j users will contend for access to the channel at the next Markov epoch given that i users contended at the last epoch and k of the i users are recontending.
$\tilde{P}_1$	Probability of transmission commencement by an idle (empty) node with

	success during two consecutive embedded epochs
$\tilde{P}_2$	Probability of transmission commencement by an idle (empty) node without success during two consecutive embedded epochs
$\tilde{P}_3$	Probability of transmission commencement by an active-wait (nonempty) node with success during two consecutive embedded epochs
$\tilde{P}_4$	Probability of transmission commencement by an active-wait (nonempty) node without success during two consecutive embedded epochs
q_j	Probability that a user queue length increases by j packets between two embedded epochs
q_{ij}	Transition probability that j packets are present in a user queue at the current transmission completion epoch given that there were i packets at the last epoch
Q	Queue length of a representative user at an arbitrary slot boundary
QED	which was to be proved
$\underline{Q_k}$	Vector of user queue lengths at the beginning of slot k
$Q_k^{(i)}$	Queue length of user i at the beginning of slot k
Q_n	User queue length immediately after the n^{th} transmission completion epoch
Q_R	Number of packets present in a nodal queue at the end of a nodal resting period
$Q(t)$	Number of packets present in a nodal queue at time t
r_i	Expected reward earned when a user enters state i
$r(x)$	Probability density function for the duration of a nodal resting phase
R	Random variable denoting the duration of a nodal resting phase
R_a	Actual length of a nodal resting phase
$R(x)$	Probability distribution function of R
$\tilde{R}(t)$	Residual resting time of a node in the resting phase
s_{ij}	Transition probability that j messages are in the network at the current successful transmission completion epoch of a minimessage given that there were i messages at the last epoch
s_j	Limiting probability that j messages are in the network at an arbitrary transmission completion epoch of a minimessage
$s_{ij}^{(k)}$	Transition probability that j messages are in the network at the current successful transmission completion epoch of a minimessage (of length k) given that there were i messages at the last epoch

$s(x)$	Probability density function of the packet transmission time
S	Packet transmission time (in seconds)
S_{ch}	Channel throughput
S_{chj}	Normalized throughput generated by stations on LAN_j
$\hat{S}_{chj}$	Estimate of S_{chj}
S'_{chj}	Throughput generated by the stations on LAN_j assuming bridge is deactivated
S_i	Transmission time of the i^{th} packet
$S_k^{(i)}$	Random variable describing channel sensing event by node i at the beginning of slot k
S_m	End of a successful transmission of a minimessage of length k packets
$S(x)$	Probability distribution function of S
$\tilde{S}(t)$	Residual successful transmission time of a packet at time t
S^e	Transmission commencement by a station which was in the empty state at the last transmission completion epoch
S^n	Transmission commencement by a station which was in the nonempty state at the last transmission completion epoch
TDMA	Time-Division Multiple-Access
T_A	Message interarrival time (node idle time)
T_c	Collision resolution period (in slots or seconds)
T_p	Voice packetization time (*msecs*)
T_r	Transmission time of a voice packet (*msecs*)
T_s	Packet transmission time (in slots)
T^c	Transmission is unsuccessful
T^s	Transmission is successful
TQ	Transmit Queue
u	Size of a minimessage
U_n	Number of users contending for access to the channel at the beginning of slot n
$U_{n'}$	Number of users contending for access to the channel at a Markov epoch n'

UL	Upper limit
v_m	Mean successful transmission time of a message given that m nodes are in the active-wait state immediately after a transmission completion epoch
V	System state space vector
V_d	Vocoder rate (in *bits/sec*)
w_m	mean channel idle time from a transmission completion epoch to the next transmission commencement instant
W	Waiting time of an arbitrary packet
W_g	Waiting time due to the position of an arbitrary packet in its message
W_M	Message access delay (Message waiting time)
W_o	Waiting time due to the transmission of other packets ahead of the message containing the tagged packet
W_{ob}	Waiting time of a new message that arrives during a node busy period
W_{or}	Waiting time of a new message that arrives during a node resting period
W_P	Packet access delay
W_1	Access delay of the first packet of a message
$Y^*(\theta)$	Laplace-Stieltjes transform of the distribution function of the random variable Y
$Y^*(z, \theta)$	Generating function for $Y^*(\theta)$
Z_n	Number of messages in the network immediately after the n^{th} transmission completion epoch
$\tilde{Z}_n$	Number of nonempty nodes in the network immediately after the n^{th} transmission completion epoch
$\tilde{Z}(t)$	Number of nonempty nodes in the network at time t
α	Probability of one packet arrival at the end of a successful transmission period
$\overline{\alpha}$	Probability of no packet arrival at the end of a successful transmission period
α_j	Probability of j packet arrivals during a successful transmission period
α_{ji}	Fraction of messages transmitted on LAN_j to be conveyed to LAN_i
α_T	Probability of at least one packet arrival during a successful transmission period
$\alpha(x, y, z)$	Probability that x idle nodes change to the active state during z time units given that y nodes are in the active-wait state at the instant of transmission

	commencement
β	Probability of one packet arrival at the end of an unsuccessful transmission period
$\overline{\beta}$	Probability of no packet arrival at the end of an unsuccessful transmission period
β_j	Probability of j packet arrivals during an unsuccessful transmission period
$\beta(x, k)$	Probability that no idle to active-wait transition occurs during the collision window given that x idle nodes change to active-wait state during k time units
γ	Probability of successful transmission on the channel
$\overline{\gamma}$	Probability of unsuccessful transmission on the channel
$\gamma(y)$	Probability that none of the y active-wait nodes reattempt transmission during a collision window
γ_b	Probability of successful transmission by the bridge
γ_m	Probability of successful transmission of a minimessage
γ_s	Probability of successful transmission by a station
Γ	Probability that an idle node generates a new message of length $\leq K$ packets
δ	Probability that a user buffer contains exactly one packet given the user is ready
δ_{i0}	Kronecker's delta function
δ_k	Probability that there are k packets in a nodal buffer given that the buffer is nonempty
ε	Tolerance factor
$\varepsilon(x, y, k)$	Probability of successful transmission and that x idle nodes change to the active state during k time units
$\varepsilon'(x, y, c)$	Probability of unsuccessful transmission and that x idle nodes change to the active state during c time units
ζ	Mean departure rate of message from the network
η_i	Expected duration that a user spends in state i
θ	Parameter of the Laplace-Stieltjes transform
ι_k	Probability that an arbitrary packet is the k^{th} to be transmitted in a message
κ	Parameter of the exponential distribution
κ_{bi}	Mean retransmission rate by BR_i

κ_i	Mean retransmission rate by a station of LAN_i
λ	Mean message arrival rate (infinite buffer)
λ_{bi}	Mean arrival rate to BR_i
λ_i	Mean message arrival rate to a station of LAN_i
λ'	Effective mean message arrival rate (finite buffer)
λ_{eff}	Effective mean message arrival rate into the network
λ_T	Total input traffic to the network
Λ	Parameter of the message length distribution
μ	Probability that the channel is free
$\bar{\mu}$	Probability that the channel is busy
ν	Probability that the buffer of a user which has just completed a successful transmission contains at least one packet
$\xi(t)$	Phase of a node at time t
Ξ_n	Event that a message is transmitted successfully at the n^{th} transmission completion epoch
Ξ_n'	Event that a minimessage is transmitted successfully at the n^{th} transmission completion epoch
$o(\Delta t)$	Little-oh function, a function that tends to zero faster than Δt
π_j	Limiting probability that j packets are in a user buffer
$\pi_j^{(i)}$	π_j for user i
$\pi_n(t,x)\Delta t$	Probability that at time t there are n packets in the nodal queue and the node is in the resting phase and the residual resting phase lies between x and $x+\Delta t$
ρ	System offered traffic
σ	Probability of one packet arrival in a slot
$\bar{\sigma}$	Probability of no packet arrival in a slot
$\tilde{\sigma}_b$	Probability of a transmission commencement by the bridge before any of the i nonempty stations given that the bridge was empty
σ_G^2	Variance of message length
$\sigma_{G'}^2$	Variance of the lengths of unblocked messages
σ_i	Probability of a transmission commencement by an idle node before any of the i active nodes in the network

$\bar{\sigma}_i$	Probability of a transmission commencement by an idle station before any of the i nonempty stations given that the bridge was empty
τ	Propagation delay in seconds
$\tau(i)$	Sensing probability by a ready user at the next Markov epoch given that i users contended at the last epoch
τ_{av}	Sensing probability by an arbitrary user
$\tau_{av}^{(i)}$	τ_{av} for user i
τ_c	Sensing probability by a user which has just completed an unsuccessful transmission
$\tau_c^{(i)}$	τ_c for user i
τ_s	Sensing probability by a user which has just completed a successful transmission
$\tau_s^{(i)}$	τ_s for user i
υ_b	Probability of a transmission commencement by the bridge before any of the i nonempty stations given that the bridge was nonempty
υ_i	Probability of a transmission commencement by an idle station before any of the i nonempty stations given that the bridge was nonempty
ϕ_i	Limiting probability that i messages are in the network at an arbitrary point in time
Φ_i	Limiting probability that i messages are in the network at an arbitrary transmission completion epoch
ψ_{ij}	Transition probability that i nonempty nodes are in the network at the current transmission completion epoch given that there were i nonempty nodes at the last epoch
ψ_j	Limiting probability that j nonempty nodes are in the network at an arbitrary transmission completion epoch
Ψ	Matrix of transition probabilities, ψ_{ij}
Ω_m	Mean time spent in state m during a cycle
$\lfloor x \rfloor$	Greatest integer smaller than or equal to x (floor of x)
$min(x,y)$	Minimum between x and y
$\in$	is a member of
Π	Product
$\lvert x \rvert$	Absolute value of x

Σ	Summation
$\cap$	Intersection symbol
∞	Infinity

"It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge. ... The most important questions of life are, for the most part, really only problems of probability."

Pierre Simon de Laplace

(Théorie Analytique des Probabilités)

CHAPTER 1

INTRODUCTION

1.1 Theme of the Dissertation

This dissertation solves the interfering queue problem in a random access local area network (LAN) which consists of a finite number of buffered users. Indeed, the tenacious difficulty of the problem makes its exact analysis formidable, hence, the solution offered in this thesis revolves mainly (but not solely) around a decomposition approximation methodology. This analytical framework forms the main contribution of the thesis.

The tasks involved in the solution methodology include stochastic modeling of buffered random access LANs using realistic (practical) assumptions, mathematical analysis of the developed models with techniques from queueing theory and verification of the analytical models by discrete-event simulations.

Finally, the importance of the solution methodology is demonstrated through several numerical examples which provide useful insights into the performance of buffered random access LANs.

1.2 An Overview of Random Access Local Area Networks (LANs)

Basically, a local area network is a communications network which is limited in geographic extent (typically 0.1 - 10 *km* range) and provides a high bandwidth channel (over 1 *Mbit/sec*) for communication among a finite number of geographically distributed users [1-5]. One major characteristic of the communication channel is that only one message can be successfully transmitted at any time instant. However, the geographically distributed users will transmit their messages independent of transmissions by other users. This gives rise to contention (simultaneous demands for the channel) or what is usually referred to as the multiple-access problem. The control algorithm for effective access to the channel by the distributed users is known as the multiple-access protocol. From previous studies in the literature, the control algorithm ranges from no control at all to either fixed control on the one hand or to some form of dynamic control on the other [6-9].

It is noted in [8] that whichever algorithm is employed, there is a price to be paid (you do not gain something for nothing !), either in the form of collisions due to no control, or in idle time due to fixed control or overhead due to dynamic control. This thesis is devoted primarily to random access techniques, which can be defined to be those for which no control algorithm is employed in the sharing of the high speed communication channel, and possesses the advantage of simplicity in its implementation. Note that the focus on only random access techniques eliminates from consideration such schemes as time-division multiple-access (TDMA), frequency-division multiple-access (FDMA), token passing, demand assignment multiple-access (DAMA), and a host of others which are examples of collision-free protocols that have been proposed in the literature [10-14].

The earliest application of random access techniques in a distributed environment is in the area of packet switching over a radio channel [15,16,17]. Recently, the use of random access techniques has spread to bus configured LANs, of which the Carrier Sense Multiple-Access with Collision Detection (CSMA-CD) is the most popular protocol [18]. A well known practical system incorporating the CSMA-CD protocol is Ethernet [19,20] and for which there is currently an IEEE standard [21]. The CSMA-CD protocol operates at the data link layer of the ISO/OSI reference model [22] or the medium access control layer of the IEEE 802 Standard [21,23]. Its operation is described as follows. Each user with a message to transmit first listens to see if the channel is idle ("listen" means to sense the channel for the presence of any ongoing transmission, hence the name "carrier sense"). If the user senses the channel idle, it then transmits its data onto the channel, otherwise if the channel is sensed busy, the user defers its transmission until a later time. However, because of the finite signal propagation delay on the channel and nonzero carrier detection time, a collision may occur when a user (say user i) senses an idle channel and begins to transmit, while another user has already started transmitting a packet that has not yet propagated past user i (thus the "multiple-access"). When a collision occurs (each user is equipped with some means for "collision detection"), all the users involved immediately cease transmission and independently select a random amount of time to wait before initiating a retransmission.

There are three variants of the CSMA-CD protocol: non-persistent, p-persistent, and 1-persistent. In the non-persistent variant, a user upon sensing the channel busy does not persist listening to the channel in order to transmit; rather, it schedules transmission for some future time, according to a retransmission delay distribution. At the scheduled time, if the channel is idle, the user transmits its packet, otherwise it repeats the non-persistent algorithm. For the p-persistent variant, a user sensing the channel idle transmits with probability p and does not with probability $(1-p)$. Upon finding the channel busy, a user performing a p-persistent protocol persists by waiting until the channel becomes idle. The 1-persistent variant is a special case of the p-persistent for which the probability $p = 1$. In this thesis, interest is mainly on the non-persistent variant because it is the most amenable to analytic treatment; however, with modifications, the analysis presented in the thesis can be extended to the other variants.

In much of the work on random access LANs, it is assumed that users can store at most one packet (referred to as unbuffered users) [24, 25]. For the realistic situation when queues of packets are allowed to form at each user (buffered user), then from the viewpoint of queueing analysis, these queues are coupled (or statistically dependent), because, since all the users share one channel, the behavior of one queue will then depend upon the state of the other queues in the network, and this renders the analysis problem quite difficult [26, 27, 28]. In fact, a rather extensive analysis is required in order to study a particular case of only two interfering queues [29] and for more than two queues no exact analysis is yet available. Since exact analysis is so difficult, then it is useful to either seek bounds [8, 30] or apply approximate techniques [31-35]; the latter solution approach is pursued vigorously in this thesis.

A survey of previous work on buffered CSMA-CD LANs is now in order. We note that while a lot of work has been done on CSMA-CD LANs with unbuffered users, only very few studies have appeared on buffered users. For buffered CSMA-CD LANs with infinite buffer size, Takagi and Kleinrock [29] used a joint probability generating function approach to analyze the case of two identical users. Coyle and Liu [31] analyzed the non-slotted non-persistent CSMA-CD system under the assumption that packet buffering is permitted only at one of the system users. Silvester and Lee [32] considered the p-persistent buffered CSMA system where each user is modeled by an $M/G/1$ queue. This approach was recently extended to delayed first transmission (DFT) CSMA-CD by Takine et al [33].

When the user buffer size is finite, previous studies have been reported in [34] and [35]. Tasaka [34] utilized the equilibrium point analysis to obtain approximate results for slotted non-persistent buffered CSMA-CD system while Apostolopoulos and Protonotarios [35] used an approach based on identical statistical behavior of each user to analyze the p-persistent buffered CSMA-CD system.

It is important to note that all the above studies have assumed the buffering of single-packet messages on a particular LAN scenario. The present work provides an analytic treatment which not only is suitable for predicting the performance of buffered (non-persistent) CSMA-CD LANs with single-packet messages but also is applicable in studying the performance of such LANs with users capable of buffering multipacket messages.

1.3 Outline of the Dissertation

1.3.1 Bus LAN Terminologies, Performance Measures and Modeling Methodologies

(i) Bus LAN Terminologies: An illustrative hardware block diagram of a bus configured LAN is shown in Fig. 1.1. It consists of a high speed broadcast channel (bus) and a myriad of network devices which typically ranges from unintelligent terminals to host computers. Intermediate between a network device and the bus is the bus interface unit (BIU) which serves as the connection device. The BIU executes the channel access protocol and provides the necessary data buffers. Fig. 1.2 depicts the associated queueing network block diagram of Fig. 1.1.

Thus far, the word user has been used informally without a precise definition. We note that for data networks in general, the word user may represent a person sitting at the keyboard of a terminal, or it may represent a computer or an application program in a computer or it may stand for a remote controlled printer [36]. As LAN devices also have different forms, then for the sake of brevity and uniformity, the word user in this thesis represents the BIUs of the LAN devices. The words station and node are equivalent terminologies of the word user, these terms are used interchangeably in the ensuing chapters.

(ii) Performance Measures: The three major performance measures of interest in LANs are channel throughput, message (or packet) delay and the conditions required for stability. The throughput is the rate at which messages are success-

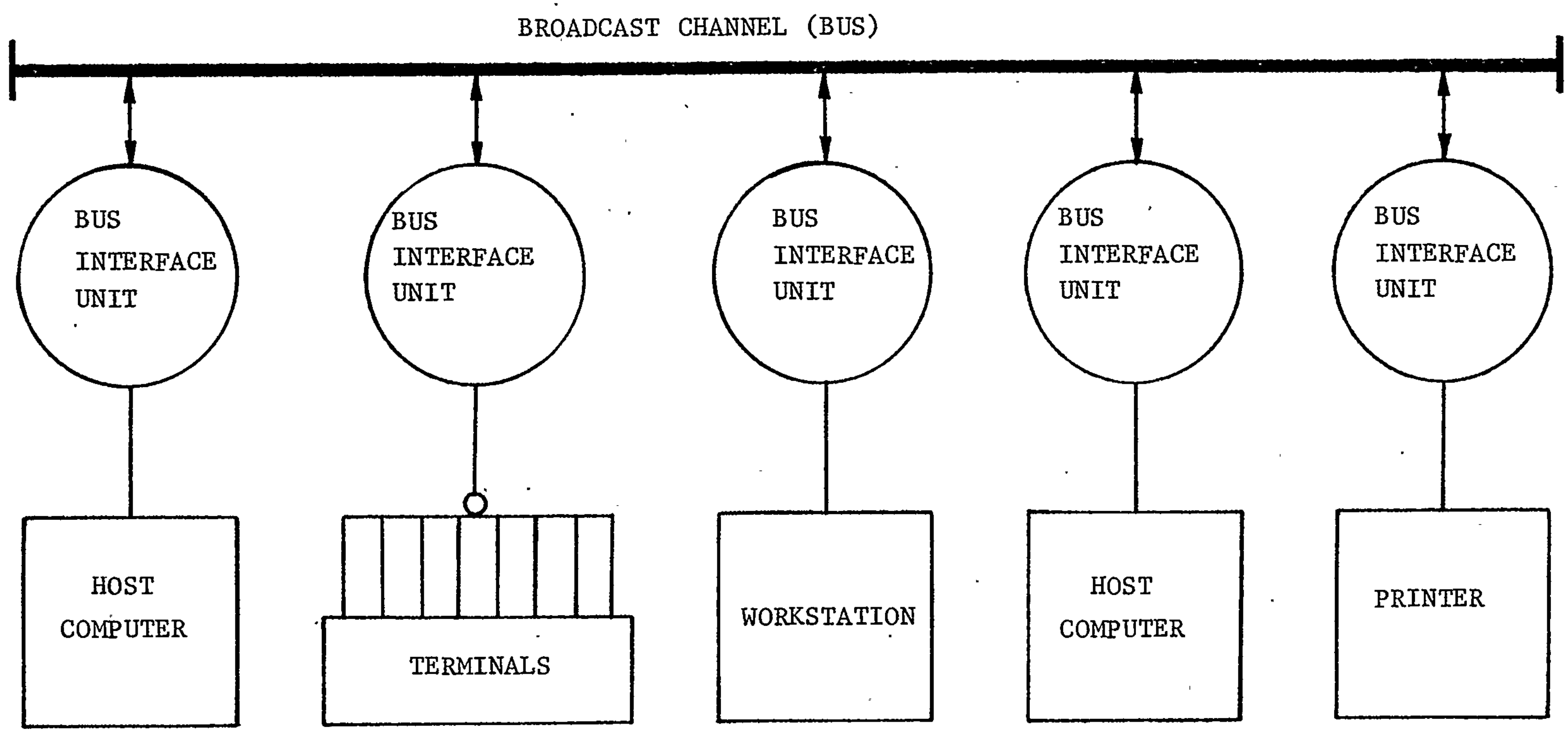


Fig. 1.1. Illustrative hardware block diagram of a bus LAN.

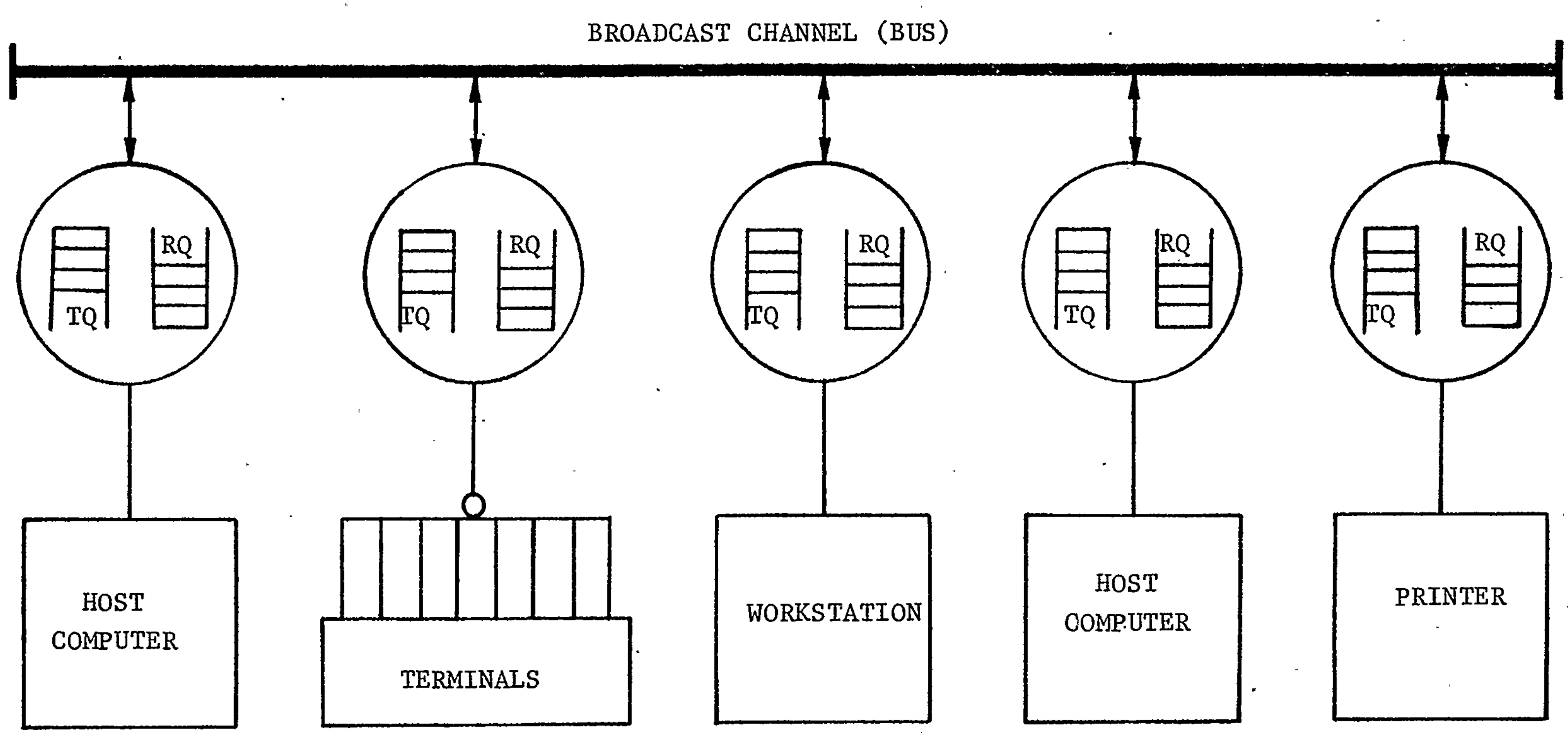


Fig. 1.2. Queueing network block diagram of illustrative bus LAN,
TQ = Transmit Queue, RQ = Receive Queue .

fully transmitted on the channel, and for the most part of the network load, is usually less than the channel capacity because of the loss due to collisions. The message delay is the time interval between the arrival of a message at a source node and its reception at the destination node. Referring to Fig. 1.2, the message delay mainly consists of the waiting time in the transmit queue (TQ) of the source node and the actual transmission time of the message on the channel. Note that the message delay can be expressed as a mean, or as a variance or as the probability that the delay exceeds a fixed value (tail distribution). However, of all the three statistical estimates of delay, only the mean estimate can be obtained in general using the least complicated performance model. Hence, only the mean of the message delay is reported in this thesis. Furthermore, there is a trade-off between the throughput and the message delay: we cannot minimize delay and at the same time maximize throughput. The instability of a random access LAN is manifested by a reduction of the throughput to the lowest levels and a simultaneous increase of the delay to intolerable levels. This obviously is undesirable, hence the problem then is to establish necessary and sufficient conditions for stability.

While the above three measures should not be construed as the sole figures of merit for a LAN, they are the most considered in quantitative performance evaluation and as such are determined in this thesis. Other subtle performance measures include the reliability and robustness of the LAN, LAN maintainability and security, fairness in the sharing of the bus, and the cost of the LAN. The extent to which these factors are pursued varies from organization to organization and is beyond the scope of this thesis.

(iii) Modeling Methodologies: As the LAN studied is a random system, the modeling tools are borrowed from the theory of probability and stochastic processes [37-44], and the main tool for analysis is queueing theory [45-50]. Specifically, the modeling and analysis employ standard concepts in probability theory, probability generating functions, Markov chains, semi-Markov processes, renewal and regenerative arguments, Little's law and some mathematical results from real analysis. In addition, discrete-event computer simulations are conducted to assess the accuracy of the stochastic models [51-54].

1.3.2 The Main Contributions

1. The Decomposition Approximation Methodology

We propose a decomposition approximation methodology as a tractable analytical technique for solving the interfering queue problem in buffered random access LANs. The principles on which the methodology is based are outlined and discussed in Chapter 2.

2. Multimessage buffer LANs with single-packet messages

As stated in Section 1.2, all the previous studies on buffered random access LANs have considered the buffering of only single-packet messages and each of these prior treatments was restricted to a particular network scenario, such as a network consisting of statistically identical users with either finite or infinite buffer size and there is no unified analytical treatment which encompasses all the network scenaria. We propose such a unified analytical framework using the decomposition approximation technique, which not only is applicable to statistically identical (homogeneous) users with finite/infinite buffer size but also is suitable for the analysis of a network consisting of heterogeneous buffered users. We derive expressions for the probability generating function for user queue length, the mean queue length, the channel throughput and the condition for system stability in Chapter 3.

3. Single-message buffer LANs with multipacket messages

Unlike the LANs with single-packet messages considered in Chapter 3, we study in Chapter 4 a more practical LAN whose nodes can store one multipacket message. The gated transmission (GT) and limited transmission (LT) strategies are proposed as more efficient packet transmission strategies over the conventional strategy studied in Chapter 3. For both the GT and LT strategies, the message response time is determined using Markov chain theory and limiting results from regenerative processes, in conjunction with Little's law. The GT strategy also allows us to derive a relationship between the message and packet access delays and furthermore, it is shown that the GT strategy has practical application in integrated services LANs.

4. Multimessage buffer LANs with multipacket messages

As an extension of Chapter 4, we analyze a random access LAN with nodes capable of buffering multipacket messages in Chapter 5 for which to date no reported studies are available. Invoking the decomposition approximation methodology, each network node is independently modeled as an $M^X/G/1$ queueing system with busy and resting periods. The main results obtained from our analysis are the expressions for the probability generating function for queue length, the Laplace-Stieltjes transform of the waiting time and the channel throughput.

5. Interconnected buffered LANs with multipacket messages

Motivated by the limitations of single separate LANs, we study in Chapter 6 the throughput and delay performance of interconnected buffered random access LANs. The previous studies on interconnected LANs have assumed that each node generates (and stores) only single-packet messages. Our study considers the more practical scenario where each node generates (and stores) multipacket messages. The analysis of the interconnected system is performed using some of the results that are already obtained in earlier chapters.

CHAPTER 2

STOCHASTIC MODEL DEVELOPMENT AND PROBLEM FORMULATION

2.1 Introduction

In this chapter, we present a detailed description of the system under investigation. The assumptions required for constructing a tractable analytic model are then stated and for completeness we also discuss the deviation of the assumptions from reality. We choose this as our starting point for two reasons. First, it provides an understanding of the physical system which hopefully will facilitate the abstract task of stochastic modeling. Second, the assumptions will suggest powerful mathematical tools that can be employed in quantitative problem formulation.

The problem of primary interest in the dissertation is then formulated mathematically using the method of probability generating function and solution of stationary equations for Markov chain. Next, we show that these formulations are not amenable to tractable analysis. Finally, the method of decomposition approximation is introduced as a means of obtaining an approximate solution to the problem posed.

2.2 A Detailed System Description

The system under consideration operates in the following way. A finite number of nodes intercommunicate over a single channel, where each node is equipped with a transmit buffer to store arriving messages (Fig. 1.2). For efficient sharing of the channel, all the nodes adopt the non-persistent CSMA-CD protocol. A node can be either in the empty state (no packet is present in the node buffer) or in the nonempty state (at least one packet is present in the node buffer). Upon the arrival of a new message, an empty node changes to the nonempty state and senses the channel either immediately-this implies immediate first transmission (IFT) protocol or after a random delay-that is, delayed first transmission (DFT) protocol. A nonempty node which senses the channel idle immediately begins the transmission of a packet. A transmission is successful if no other node initiates transmission within the collision window from the original transmission commencement instant. After successful transmission, the packet will be deleted from the buffer and queued packets are transmitted on a first-come, first-served basis. On the other hand, when more than one node begins transmission within the collision window, collision occurs. At the end of the collision resolution period, the colliding nodes are rescheduled to sense the channel again after independently selected random timeout intervals.

2.3 Stochastic Model Development

In order to develop a stochastic model for the system, we first specify the channel and the network by the following assumptions.

2.3.1 Modeling Assumptions

For the Channel

A_1 – The channel is error free so that unsuccessful transmissions occur only from collisions.

A_2 – The propagation delay between any two nodes is τ *seconds* which is the maximum one-way propagation delay in the network and is assumed to be very small compared with the packet transmission time.

A_3 – The channel time is either slotted or nonslotted. For the slotted operation, the channel is divided into equal slots of τ *seconds* each.

For the Network

A_4 – The network consists of M nodes intercommunicating via a random access channel.

A_5 – Each node has a buffer capable of storing K, $1 \leq K \leq \infty$, packets.

A_6 – For the slotted operation, a node can sense the channel instantaneously with probability p. All nodes are synchronized so that packet transmission can begin only at the slot boundaries.

A_7 – For slotted channel operation, the message arrival process at each node follows an independent Bernoulli process with mean arrival rate σ messages/slot and the arrival is assumed to occur at the end of a slot. On the other hand, for nonslotted channel operation, messages arrive at each node buffer in accordance with a time-homogeneous Poisson process with rate λ.

A_8 – The network is operating under steady-state condition.

2.3.2 Practical Implications of the Modeling Assumptions

The justification for the assumptions made above are stated as follows:

A_1 – The assumption that the channel is error free implies that there is no channel failure and no transmission error; it is justifiable for high speed communication channels. Hence, unsuccessful transmissions result only from collisions.

A_2 – The assumption of constant (maximum one-way) propagation delay between all pairs of nodes in the network is somewhat pessimistic because in practice, the propagation delay between any two nodes is a random variable which depends on the relative positions of the two nodes in the network. In the case of slotted channel operation where transmissions begin only at the slot boundaries (by assumption A_6), the residual part of a propagation slot is useless anyway, and the above assumption is justifiable. The assumption that the propagation delay is much less than the packet transmission time is a standard requirement for the proper operation of carrier sense based protocols because carrier sensing gives more effective information about the actual state of the channel (bus) when the ratio between the propagation delay and the packet transmission time is a small value.

A_3 – The time-slotted (discrete-time) assumption is made to reduce the length of the contention interval and to simplify our analysis. It is realistic because it matches the time-synchronized configuration of many practical communication systems. However, the nonslotted (continuous-time) channel operation truly represents the actual implementation of the channel.

A_4 – This assumption is valid because realistic communication systems consist of only a finite number of nodes.

A_5 – It is economically infeasible to implement a buffer of infinite capacity. However, the infinite buffer size assumption ensures that no generated message is rejected and it is made for analytic simplicity. Actually, a buffer of finite size is more realistic and also serves as a means of congestion control.

A_6 – The assumption regarding channel sensing being instantaneous implies that the time required to detect whether the channel is busy or idle is negligible. It is convenient to synchronize the nodes connected to a time-slotted channel because the information about the state of the channel is used only at the slot boundaries.

A_7 – In the Bernoulli arrival process assumption, a node generates a message with probability σ and does not with probability $(1 - \sigma)$, the message generations/no generation at the end of successive slots are independent. The Bernoulli message arrival process is valid for discrete-time systems with a finite number of nodes. The Poisson message arrival process assumption is approximately valid for continuous-time systems provided the number of nodes is large ($M \geq 20$) [13]. The Poisson arrival process is a standard assumption supported by a number of observations in queueing and teletraffic theory [49].

A_8 – What we assume here is tantamount to stating that the performance of the system does not depend on the initial state of the system and also after a long time the system has been in operation, the probabilities characterizing the different system variables are invariant with time.

2.4 Problem Formulation

We now formulate quantitatively the interfering queue problem as a *multi-queue* problem. For simplicity reasons, the problem is formulated with respect to a slotted system. Define the system state as the number of packets present in the buffer of node i, $1 \leq i \leq M$, at the beginning of slot n , $n \geq 1$, and denote it by $Q_n^{(i)}$. The system is then described by an $M-tuple$

$$\underline{Q_n} = (Q_n^{(1)}, Q_n^{(2)}, \cdots, Q_n^{(M)}), \ n = 1, 2, \cdots . \tag{2.1}$$

To describe the evolution of the buffer contents at the nodes, it is appropriate to define two indicator random variables. Let

$$I(Q_n^{(i)}) = \begin{cases} 1 \text{ if } Q_n^{(i)} > 0 \\ 0 \text{ if } Q_n^{(i)} = 0 \end{cases} \tag{2.2}$$

and also let

$$I(S_n^{(i)}) = \begin{cases} 1 & \textit{with probability } p \\ 0 & \textit{with probability } (1-p) \end{cases} \tag{2.3}$$

where $S_n^{(i)}$ defines the channel sensing event by node i at the beginning of slot n. Recall that p is the time-invariant probability of sensing the channel at the beginning of a free slot by a nonempty node in a slotted system. Using these definitions and the system description, the stochastic-difference equations governing system evolution for $n = 1, 2, \cdots$, can be written as

$$Q_{n+1}^{(i)} = Q_n^{(i)} - I(S_n^{(i)})\, I(Q_n^{(i)}) \prod_{\substack{j=1 \\ j \neq i}}^{M} (1 - I(S_n^{(j)})\, I(Q_n^{(j)})) + A_n^{(i)} \tag{2.4}$$

where $A_n^{(i)}$ represents the input traffic to the i^{th} node during $[n\ , n+1)$. Notice that $(1 - I(S_n^{(j)})\, I(Q_n^{(j)}))$ is a binary-valued random variable and can be interpreted as the interference indicator at the beginning of slot n, that is, it indicates whether or not node j interferes with node i's transmission. We see from (2.4) that the state of the Q-process at epoch $n+1$ depends only on the state at epoch n, hence the process $\{Q_n^{(i)};\ n \geq 1\}$ is a Markov chain. We have therefore transferred the problem of analyzing the M-dimensional stochastic process (2.1) to the problem of determining the distribution of the M-dimensional Markov chain from which, in principle, the relevant results of the system can be determined. The analysis of the M-dimensional Markov chain can proceed using either the method of generating function or solution of the stationary equations for Markov chain. As will be shown in the following, both approaches lead to analytic complications.

2.4.1 Formulation of Joint Probability Generating Function

If we assume the M-dimensional Markov chain to be ergodic, the steady-state joint generating function for the queue lengths distribution is given by

$$G(\underline{z}) \triangleq G(z_1, z_2, \ldots, z_M) = \lim_{n\to\infty} E\left[\prod_{i=1}^{M} z_i^{Q_n^{(i)}}\right] . \tag{2.5}$$

Assuming knowledge of the explicit expressions for the generating function, we then can appeal to the property of generating functions to derive any moment of the queue lengths and in a straightforward manner determine the average time delays by invoking Little's law [55]. However, in general, $G(\underline{z})$ is expressed in terms of several boundary generating functions (or boundary functions for short) which are defined by

$$G(\underline{0}^J, \underline{z}^{J^c}) = G(z_1, z_2, \ldots, z_M)\,\Big|_{z_i=0,\ i\in J,\ 1\le i\le M} \tag{2.6}$$

where J is a subset of the universal set of all nodes in the system and J^c is the complement of J. In the following, we state and prove a theorem that gives the number of boundary functions required to uniquely determine $G(\underline{z})$. The theorem is a generalization of a proposition in [56].

Theorem 2.1: In a random-access based M-node network where every node can interfere with every other node, the joint probability generating function for queue lengths $G(\underline{z})$, is uniquely determined by $2^M - 1$ boundary functions.

Proof: We introduce the following notations. Denote the set of all nodes in the system by $H = \{ 1, 2, ..., M \}$. Let J_i, $1 \le i \le M$, denote the set of all i-element subsets of H. This means that the elements of a set J_i are themselves sets. Further, denote a generic i-element subset of H by J and let J^c be its complement. For clarification of the above notations, consider the following example when $M = 3$. Then $H = \{1, 2, 3\}$, $J_1 = \{\{1\}, \{2\}, \{3\}\}$, $J_2 = \{\{1,2\}, \{1,3\}, \{2,3\}\}$, and $J_3 = \{\{1,2,3\}\}$. It follows that the set J can represent any of the seven enumerated i-element subsets of H, $1 \le i \le 3$. Suppose $J = \{1,2\}$, then $J^c = \{3\}$.

By the definition of a boundary function (2.6),

$$G(\underline{0}^J, \underline{z}^{J^c}) = G(z_1, z_2, ..., z_M) \mid_{z_i = 0,\ i \in J,\ 1 \le i \le M}$$

and this holds for all the i-element distinct subsets of H. Therefore, the total number of boundary functions, N_{BF} required is just equal to the total number of i-element distinct subsets of H, for all i, $1 \le i \le M$, that is

$$N_{BF} = \sum_{i=1}^{M} N_i$$

where for each i, N_i is the number of i-element distinct subsets of H. But from combinatorics, N_i is the number of distinct ways in which the M elements of H can be arranged into i-element distinct subsets, hence

$$N_{BF} = \sum_{i=1}^{M} \binom{M}{i} = 2^M - 1$$

where the second equality follows easily from a result in combinatorial analysis. QED

It is seen from the above theorem that the number of boundary functions depends on M. We note that even for the case $M = 2$ which requires only three boundary functions, their determination is quite a formidable task and as pointed out in [57] requires a solution of the Riemann-Hilbert boundary value problem. For large M, the number of boundary functions increases and to date there is no known method for determining the boundary functions for $M \geq 3$. Thus, an explicit expression for the generating function cannot be determined for realistic values of M.

2.4.2 Solution of Stationary Equations for Markov Chains

It follows from the ergodicity assumption of the M-dimensional Markov chain that the limiting joint state probabilities for queue lengths can be obtained by solving the stationary equations [47]

$$\lim_{n \to \infty} Pr\{Q_{n+1}^{(1)} = \hat{i}_1, \cdots Q_{n+1}^{(M)} = \hat{i}_M\} = \lim_{n \to \infty} \sum_{i_1} \cdots \sum_{i_M} Pr\{Q_{n+1}^{(1)} = \hat{i}_1, \ldots,$$

$$Q_{n+1}^{(M)} = \hat{i}_M \mid Q_n^{(1)} = i_1, \ldots, Q_n^{(M)} = i_M\} Pr\{Q_n^{(1)} = i_1, \cdots Q_n^{(M)} = i_M\}$$

(2.7)

subject to the constraint

$$\lim_{n \to \infty} \sum_{\hat{i}_1} \cdots \sum_{\hat{i}_M} Pr\{Q_{n+1}^{(1)} = \hat{i}_1, \cdots Q_{n+1}^{(M)} = \hat{i}_M\} = 1 . \tag{2.8}$$

In principle, once the state probabilities are obtained, we can determine the relevant parameters of the system. Unfortunately, there are two basic difficult problems. The first problem is what we call "state-space explosion" which is best illustrated by an example. If we assume that each node has a maximum buffer space of K packets, then from combinatorial analysis, the maximum number of possible states for the process $\{\underline{Q_n}; n \geq 1\}$ (2.1) is $(K+1)^M$. For a typical LAN with $M = 10$, we see that if $K = 1$ (bufferless system) there are 2^{10} possible states and a system with buffering capability with $K>1$ will therefore have a vast number of states. Furthermore, the task of solving (2.7) and (2.8) with very large number of states is tedious even on a CDC CYBER205 Supercomputer [58]. The second problem is a consequence of the first and deals with the enumeration of the transition probabilities (2.7). When the number of system states is very large, it is combinatorially very complex to enumerate all possible transition events. Thus, the solution of stationary equation for Markov chains also seems impractical for buffered LANs.

The explanations presented above demonstrate that the solution of the multiqueue problem using standard techniques is quite complicated and the author believes that there is little (or no !) hope of obtaining exact analytic results in the foreseeable future. To circumvent the above difficulties, resort is made to an approximate analysis.

2.5 Philosophy of the Decomposition Approximation

The basic idea of the decomposition approximation being proposed for solving the multiqueue problem is to break up the network into smaller subsystems. Stated succinctly: consider each node in isolation and then approximate the interaction of a node with the other nodes in the network by the *channel-activity* parameters which have been independently obtained. The decomposition analysis method developed in this thesis is based on the following principles:

(1) Decomposition of the network into subsystems, that is, each node in the network is separately considered as a subsystem.

(2) Approximation of all non-Markovian processes of a subsystem by Markovian processes.

(3) Analysis of the subsystems in isolation.

(4) Analysis of channel-activity parameters.

(5) Approximation of the interaction of a subsystem with the other subsystems via the computation of the channel-activity parameters by an iterative algorithm.

The key points of analysis are Principles 2 and 5. Principle 2 is motivated by the fact that we can construct embedded Markov chains for the stochastic processes $\{Q_n^{(i)} ; n \geq 1 \}, 1 \leq i \leq M$, defined at embedded points which coincide with the beginning of every free slot. Application of renewal-reward arguments then yields the long-run parameters of the processes which are valid at all slot boundaries (or all points in time). Principle 2 becomes invalid when the embedded epochs differ from those defined above and renewal-reward arguments presume that the system is operating under steady-state condition (Assumption A_8). Principle 5 rests on the fact that all the nodes (subsystems) send their information over the same channel . It therefore seems reasonable to approximate the interaction of the nodes by the channel-activity parameters which are the probabilities of successful transmission, unsuccessful transmission and the busy and idle probabilities of the channel. Principle 5 also holds provided the system is operating under steady-state condition.

We conclude this section with the remark that the decomposition approximation technique is not only applicable for solving the interfering queueing problem in random access LANs but can also be used to analyze complex queueing networks consisting of several queues.

2.6 Summary

This chapter describes the operation of a buffered CSMA-CD LAN and the assumptions (including justifications) required for model formulation are also stated. The interfering queue problem is then formulated as a multiqueue problem and it is shown that the problem formulations in terms of probability generating function for queue lengths and solution of stationary equations for Markov chain do not lead to tractable analysis. Thus, in order to overcome the limitation of the standard techniques, a decomposition approximation model is proposed and its basic principles are outlined.

The next chapter illustrates the application of the decomposition approximation principles to the study of multimessage buffer LANs with single-packet messages.

CHAPTER 3

MULTIMESSAGE BUFFER LANs WITH SINGLE-PACKET MESSAGES

3.1 Introduction and Problem Statement

Previous studies on buffered random access LANs have considered the buffering of only single-packet messages [29,31-35] and this trend has continued to date for two reasons. First, it is motivated by practical application such as found in LANs designed primarily for short interactive data or inquiry/response traffic where the information (data burst) sent or received during a communication session is contained in only one packet. Second, even for the case of buffering single-packet messages, the analysis is nontrivial due to the statistical dependence among the user queues. Previous treatments were therefore restricted to a particular system scenario, such as a system consisting of statistically identical users with finite buffer size [34,35] or infinite buffer size [29,31-33] and to the best of the author's knowledge, there is no unified analytical treatment which encompasses all the system scenaria. The purpose of this chapter and one of the main contributions of this thesis is to present such a unified analytical framework which not only is applicable to statistically identical (homogeneous) users with finite/infinite buffer size [59,60] but also is suitable for the analysis of a system consisting of heterogeneous buffered users [61].

For a buffered LAN with nodes capable of storing single-packet messages, the main problem is to determine the system performance measures such as message (packet) delay, channel throughput, probability of buffer overflow (for finite buffer size) and the condition for system stability. Obviously, knowledge of the performance measures are valuable to network designers as such estimates serve as initial indicators of a real system performance. Possibly more significant advantage is the usefulness of the analytic model developed for performance analysis which can be tuned to perform numerous sensitivity studies.

3.2 Model Formulation

Consider a LAN which consists of M homogeneous users sharing a single communication channel. The channel access protocol is the *slotted* non-persistent CSMA-CD protocol and the system operation is as described in Chapter 2. A user can be either ready or nonready. A user is nonready if its input buffer is empty and ready if there is at least one packet in its buffer. A ready user can be in either of two substates: active or dormant. A ready user is active if it is currently transmitting a packet and dormant if it has no transmission in progress. Note that because of the possibility of having more than one user begin transmission at a slot boundary, a user that begins the transmission of a packet is said to contend for access to the channel. Fig. 3.1 illustrates schematically a ready user operation where γ ($\bar{\gamma}$) is the probability of successful (unsuccessful) transmission on the channel, p ($\bar{p}$) is the probability that a ready user senses (does not sense) the channel and μ ($\bar{\mu}$) is the probability that the channel is free (busy). α_j (β_j) is the probability that j packets arrive during a(n) successful (unsuccessful) transmission

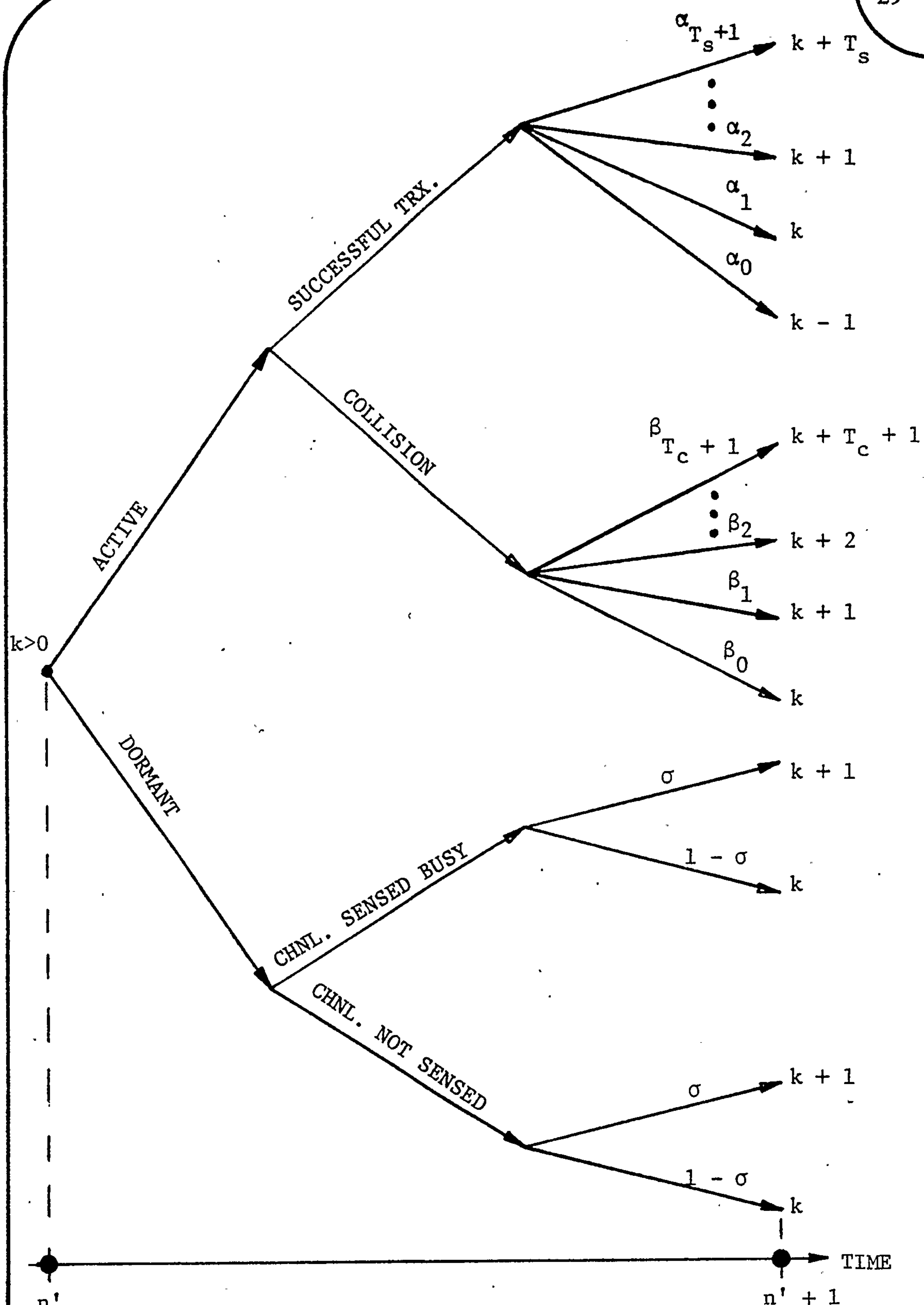


Fig. 3.1. Ready user operation.

period and σ ($\bar{\sigma}$) is the probability of one (no) packet arrival in a slot. In all the probabilities just defined, $\bar{x} = 1 - x$.

We note that the assumptions made in the previous chapter are inadequate to completely formulate a tractable analytic model for the system under consideration. The following additional assumptions are thus introduced:

(a) All users are statistically identical.

(b) Packet lengths are constant and equal to T_s slots, where T_s is an integer and a user can transmit only one packet when it gains sole access to the channel.

(c) Previously collided packets are treated in the same way as the new packets.

(d) From the instant of successful transmission commencement, the channel becomes idle after (T_s + 1) slots. In the case of an unsuccessful transmission, the collision is detected after T_c slots and the channel again becomes idle after (T_c + 1) slots. The collision requirement in CSMA-CD dictates T_c be less than T_s.

Notice that as a slotted channel operation is being considered, hence, from assumption A_7 of Chapter 2 the arrival process is Bernoullian.

The rationale for the additional assumptions are stated below. The identical statistical user behavior assumption is made to reduce the number of system states in the analysis, thereby allowing for the consideration of large size systems. The constant packet length assumption is made to ensure equal transmission time for all packets and also to allow fair sharing of the channel by the users, however, the following analysis can also handle random packet lengths. Assumption (c) implies a delayed first transmission (DFT) protocol which requires that at the beginning of a slot, a ready user senses the channel with probability p and does not with probability $(1 - p)$. Thus, the first channel sensing attempt of any packet (new or old) suffers a geometrically distributed delay with mean $1/p$ slots, and no distinction is made between new and old packets. Conversely, a transmission protocol which distinguishes between new and old packets is called the immediate first transmission (IFT) [62], where a nonready user which has just generated a new packet senses the channel immediately with unity probability. Unfortunately, the analysis of buffered IFT CSMA-CD scheme is formidable [63]; hence, this thesis only considers the DFT protocol. Furthermore, as pointed out in [64] there seems to be no significant difference in performance between the IFT and the DFT protocols in the cases of the (unbuffered) CSMA and CSMA-CD schemes (unlike the ALOHA scheme). Nonetheless, the author believes that the approach developed here can be extended to handle the IFT protocol. By assumption (d), it is evident that the last slot of a transmission period (successful or unsuccessful) allows the end of transmission to be sensed at all stations; and moreover, this also implies that the length of a channel idle period between two successive transmission periods is at least one slot.

Based on the above assumptions, the system state at the beginning of slot $n, n \geq 1$ can now be described by an $(M + 1)$-dimensional vector

$V = \{U_n \,; Q_n^{(1)}, Q_n^{(2)}, \cdots, Q_n^{(M)}\}$ where U_n is the number of users contending for access to the channel and $Q_n^{(i)}$, $i = 1, 2, \cdots, M$, is the number of packets in the buffer of user i. The above definition of system state leads to a reduction in the system state space when compared with previously reported models [34, 65, 66]. However, the modified system state space is still too large to permit exact analysis, therefore, we resort to the decomposition approximation whose principles are outlined in the previous chapter.

By Principle 1, we shall decompose the system into M separate queues with lengths denoted by $Q_n^{(i)}$, $i = 1, 2, \cdots, M$. Furthermore, the process $\{U_n \,, n \geq 1\}$ representing the channel activity is also considered separately. Due to the user homogeneity, it suffices to consider one representative user process, $\{Q_n \,, n \geq 1\}$ and then study the effect of the interaction of the remaining $(M - 1)$ users on the representative user. Each diagram in Fig. 3.2(a) represents the activity of a user in the system and the superposition of all the diagrams in Fig. 3.2(a) yields Fig. 3.2(b) which illustrates the channel activity.

In the following, we study in detail the performance of a network consisting of (i) homogeneous users with infinite buffer size, (ii) homogeneous users with finite buffer size and (iii) heterogeneous users with infinite buffer size. For the three network scenaria, the above model serves as the analytical framework.

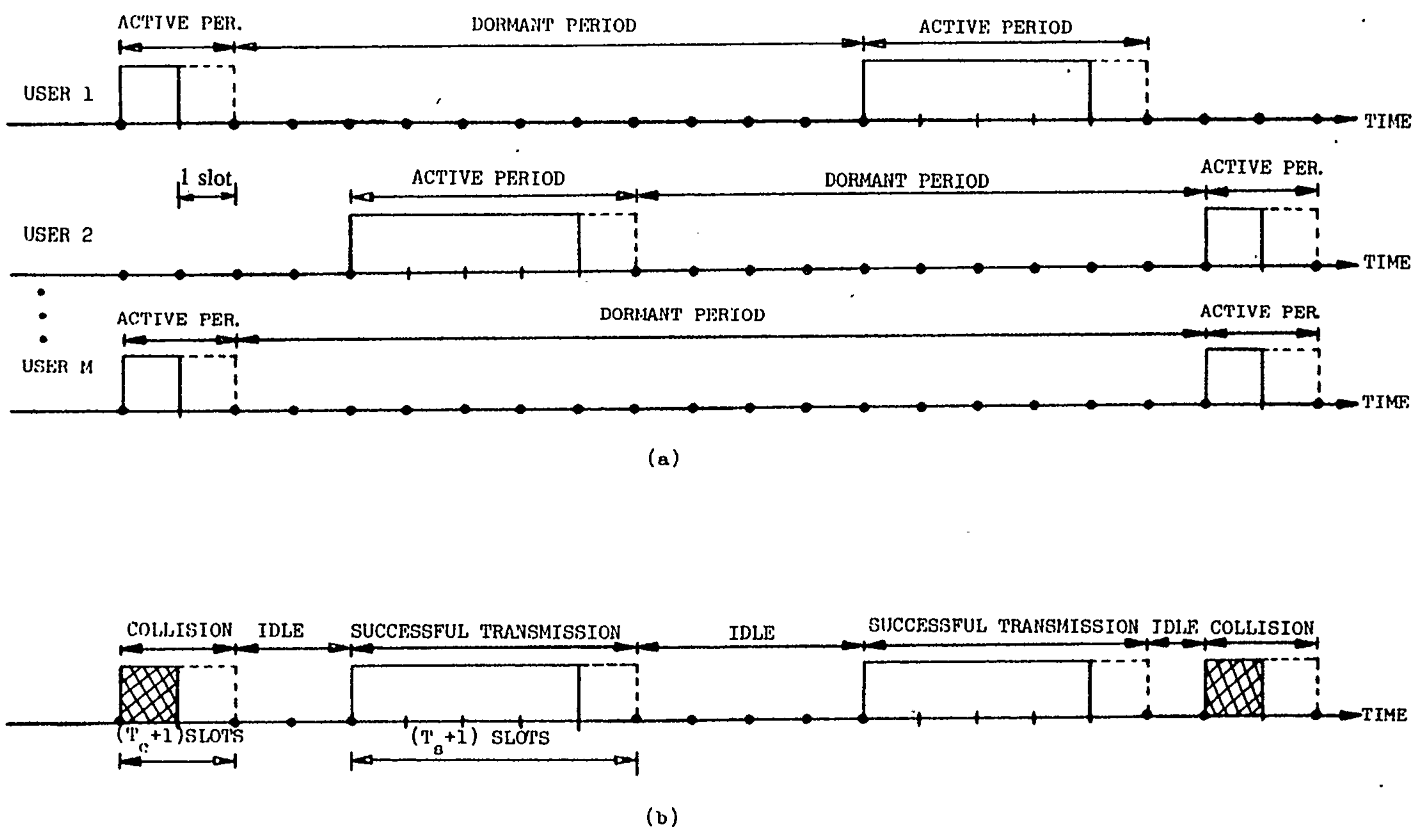


Fig. 3.2(a) Illustration of a ready user state periods
(b) Illustration of the channel state periods
(On each diagram, the embedded Markov epochs are represented by ●).

3.3 Performance Analysis

In this section it is shown how the remaining principles of decomposition approximation are applied to analyze system performance.

3.3.1 Homogeneous Users with Infinite Buffer size

The first scenario assumes that each user has unlimited buffer space. By Principle 2 of the decomposition approximation, we now approximate the user queue length and the channel-activity processes as Markovian processes. We state in passing that the Markov analysis, though nontrivial is analytically tractable and is presented as follows.

3.3.1.1 Approximate User Markov Chain Analysis

The main objective of the user Markov chain analysis is to determine the user queue state distribution from which other relevant user parameters can be found. Consider a representative user activity shown in Fig. 3.2(a) and described by the process $\{Q_n, n \geq 1\}$. The user queue is said to be in state k at the beginning of slot n if $Q_n = k$, $k = 0, 1, 2, \cdots$; where Q_n includes the packet arrival (if any) at the end of slot $n - 1$ but excludes any packet that has successfully completed transmission at the end of slot $n - 1$. (Note that the end of slot $n - 1$ is also the beginning of slot n). Clearly from the discrete-time arrival and departure processes, $\{Q_n, n \geq 1\}$ is a discrete-time stochastic process whose analysis is very difficult to handle because of the task of observing the user queue state at the beginning of every slot. To circumvent this difficulty, we construct an embedded Markov chain (Principle 2), denoted by $N_{n'}$, where $N_{n'}$ represents the number of packets stored in the queue of the representative user at the beginning of embedded epoch n'. The embedded epochs coincide with the beginning of each active period or each dormant slot (represented by shaded dots (•) in Fig. 3.2(a)) and are henceforth referred to as Markov epochs. Let π_j denote the limiting probability that j packets are stored in a user buffer at a Markov epoch. A method of calculating the queue length probability distribution is via the method of generating functions.

A. Generating Function for Queue Length

By definition, the limiting probability generating function for the user queue length distribution at the Markov epochs is

$$G_N(z) = \sum_{j=0}^{\infty} \pi_j \, z^j \tag{3.1}$$

where the π_j's remain to be determined. To do so, we first enumerate all possible transition events and their corresponding transition probabilities. Fig. 3.3 depicts an illustrative user state transition diagram whose transition probabilities are defined (in conjunction with Fig. 3.1) as follows:

$p_A \;=\; Pr\{0|0\}$

$\quad = \; Pr$ {a nonready user at the current Markov epoch remains nonready at the next Markov epoch, that is, no packet arrives at the end of one slot}

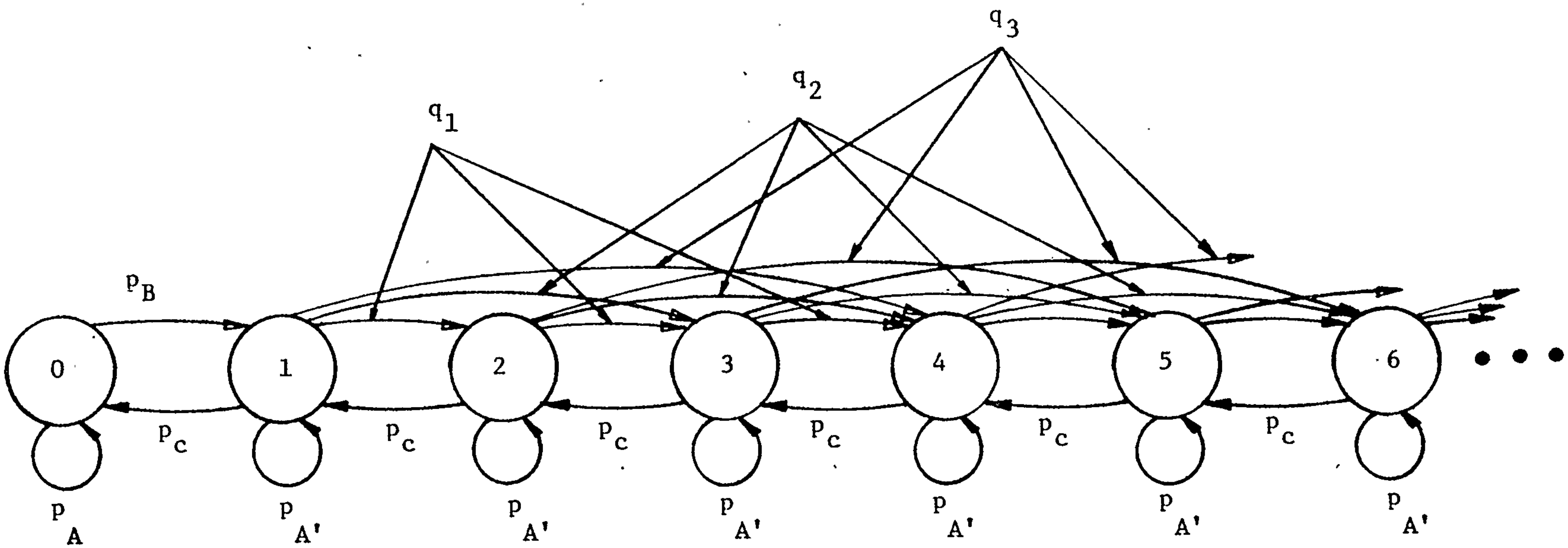
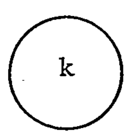


Fig. 3.3. User State Transition Diagram.
(Illustration for the Case T_s = 3 and CSMA-CD Operation)



State k, $k \in \{0, \ldots, \infty\}$

$$= \bar{\sigma} \, . \tag{3.2}$$

$$p_{A'} = Pr\{k \mid k\} \quad , \quad k \geq 1$$

= Pr\{a ready user successfully transmits the HOQ packet and one packet is generated at the end of the successful transmission period or
a ready user unsuccessfully transmits the HOQ packet and no packet is generated at the end of the unsuccessful transmission period or
channel is sensed to be busy and no packet is generated at the end of the slot when the channel is sensed busy or
channel is not sensed at all and no packet is generated at the end of the slot when the channel is not sensed \}

$$= p\mu\gamma\alpha_1 + p\mu\bar{\gamma}\beta_0 + p\bar{\mu}\bar{\sigma} + \bar{p}\bar{\sigma} \, . \tag{3.3}$$

$$p_B = Pr\{1 \mid 0\}$$

= Pr\{ a nonready user at the current Markov epoch becomes ready at the next epoch, that is, a packet is generated at the end of one slot \}

$$= \sigma \, . \tag{3.4}$$

$$p_C = Pr\{(k-1) \mid k\} \quad , \quad k \geq 1$$

= Pr\{ a ready user successfully transmits the HOQ packet and no packet arrives at the end of the successful transmission period \}

$$= p\mu\gamma\alpha_0 \, . \tag{3.5}$$

The q_i's, $i > 0$ in Fig. 3.3 are defined as

$$q_1 = Pr\{(k+1)|k\} \quad , \quad k \geq 1$$

$= Pr\{$ a ready user unsuccessfully transmits the HOQ packet and one packet is generated during the unsuccessful transmission period or a ready user successfully transmits the HOQ packet and two packets are generated during the successful transmission period or channel is sensed to be busy and one packet is generated at the end of the slot when the channel is sensed busy or channel is not sensed at all and one packet is generated at the end of the slot when the channel is not sensed $\}$

$$= p\mu\bar{\gamma}\beta_1 + p\mu\gamma\alpha_2 + p\bar{\mu}\sigma + \bar{p}\sigma \,. \tag{3.6}$$

For the range $1 < j \leq T_c+1$,

$$q_j = Pr\{(k+j)|k\} \quad , \quad k \geq 1$$

$= Pr\{$ a ready user unsuccessfully transmits the HOQ packet and j packets are generated during the unsuccessful transmission period or a ready user successfully transmits the HOQ packet and $(j+1)$ packets are generated during the successful transmission period $\}$

$$= p\mu\bar{\gamma}\beta_j + p\mu\gamma\alpha_{j+1} \tag{3.7}$$

and for the range $T_c + 1 < j \leq T_s$,

$$q_j = Pr\{(k+j)|k\}\ ,\ k \geq 1$$

$$= Pr\{\text{ a ready user successfully transmits the HOQ packet and } (j+1) \text{ packets are generated during the successful transmission period }\}$$

$$= p\mu\gamma\alpha_{j+1}\ . \tag{3.8}$$

Under the assumption that steady-state conditions prevail, the packet flow balance equation is applicable [42], that is,

$$\sum \text{FLOW OUT OF STATE } j \ = \ \sum \text{FLOW INTO STATE j}$$

$$\begin{aligned}
\pi_0 &= p_A\pi_0 + p_C\pi_1 \\
\pi_1 &= p_B\pi_0 + p_{A'}\pi_1 + p_C\pi_2 \\
\pi_k &= q_{k-1}\pi_1 + q_{k-2}\pi_2 + \cdots + q_1\pi_{k-1} + p_{A'}\pi_k + p_C\pi_{k+1}\,,\ 2 \leq k \leq T_s \\
\pi_k &= q_{T_s}\pi_{k-T_s} + q_{T_s-1}\pi_{k-(T_s-1)} + \cdots + q_1\pi_{k-1} \\
&\qquad + p_{A'}\pi_k + p_C\pi_{k+1}\,,\ k \geq T_s + 1\ .
\end{aligned} \tag{3.9}$$

Multiply each equation by z^j, $j \in \{0, ..., \infty\}$, (where j is the subscript of π on the left hand side) and then sum both sides of the resulting equations separately from $j = 0$ to $j = \infty$, we obtain after simplifying

$$G_N(z) = \left[\frac{(p_A + p_B z) - (p_C z^{-1} + p_{A'} + \sum\limits_{i=1}^{T_s} q_i z^i)}{1 - (p_C z^{-1} + p_{A'} + \sum\limits_{i=1}^{T_s} q_i z^i)} \right] \pi_0 \tag{3.10}$$

which on substituting the expressions for the transition probabilities gives

$$G_N(z) = \left[\frac{(\bar{\sigma} + \sigma z) - (p\mu\gamma\alpha_0 z^{-1} + (p\mu\gamma\alpha_1 + p\mu\bar{\gamma}\beta_0 + p\bar{\mu}\bar{\sigma} + \bar{p}\bar{\sigma}) + \sum_{i=1}^{T_s} q_i z^i)}{1 - (p\mu\gamma\alpha_0 z^{-1} + (p\mu\gamma\alpha_1 + p\mu\bar{\gamma}\beta_0 + p\bar{\mu}\bar{\sigma} + \bar{p}\bar{\sigma}) + \sum_{i=1}^{T_s} q_i z^i)} \right] \pi_0 \,. \tag{3.11}$$

Notice that T_c does not appear explicitly in the expression for $G_N(z)$, however, its effect is reflected in the expressions for the q_i's and β_0.

B. Probability that a User is Nonready

As stated before, knowledge of $G_N(z)$ allows us to determine the queue length distribution. To illustrate this point, we shall determine the probability that a user is nonready, π_0 as follows. By using the moment generating function property $G_N(1) = 1$, and applying L'Hospital's rule to (3.10) we get

$$\pi_0 = \frac{p_C - \sum_{i=1}^{T_s} iq_i}{(p_B + p_C) - \sum_{i=1}^{T_s} iq_i} \tag{3.12}$$

and on substituting the values for p_B and p_C from (3.4) and (3.5) respectively, we get

$$\pi_0 = \frac{p\mu\gamma\alpha_0 - \sum_{i=1}^{T_s} iq_i}{(\sigma + p\mu\gamma\alpha_0) - \sum_{i=1}^{T_s} iq_i} \,. \tag{3.13}$$

Having found π_0, it is possible to determine the remaining π_j's by solving (3.9) recursively.

C. Mean Queue Length at the Markov Epochs

The mean queue length at a Markov epoch, $E[N]$, is then calculated by

$$E[N] = \frac{dG_N(z)}{dz}\Big|_{z=1} . \tag{3.14}$$

Equation (3.10) can be expressed in the form

$$G_N(z) = \frac{x(z)}{y(z)} \pi_0 \tag{3.15}$$

where $x(z)$ and $y(z)$ are polynomials in z which are given by

$$x(z) = (p_A + p_B z) - (p_C z^{-1} + p_{A'} + \sum_{i=1}^{T_s} q_i z^i) \tag{3.16a}$$

and

$$y(z) = 1 - p_C z^{-1} - p_{A'} - \sum_{i=1}^{T_s} q_i z^i . \tag{3.16b}$$

respectively. Hence from (3.15)

$$\frac{dG_N(z)}{dz} = \frac{y(z)x'(z)\pi_0 - x(z)\pi_0 y'(z)}{[y(z)]^2} . \tag{3.17}$$

Substituting $z = 1$ into (3.17) gives an indeterminate result, then by applying L'Hospital's rule and after simplifying, we obtain

$$\frac{dG_N(z)}{dz}\Big|_{z=1} = \frac{x''(1)\pi_0 - y''(1)}{2y'(1)} . \tag{3.18}$$

Evaluating the first and second derivatives of (3.16) and then substituting into

(3.18) yields after simplification

$$E[N] = \frac{(2p\,\mu\gamma\alpha_0 + \sum_{i=1}^{T_s} i(i-1)q_i)(1-\pi_0)}{2(p\,\mu\gamma\alpha_0 - \sum_{i=1}^{T_s} iq_i)} . \tag{3.19}$$

It is seen that (3.11), (3.13) and (3.19) are expressed in terms of γ, $\bar{\gamma}$, μ and $\bar{\mu}$ which are henceforth referred to as the channel-activity parameters. It will be shown shortly that these probabilities are functions of the channel steady-state probabilities and this dependence demonstrates the coupling between the user and the channel Markov chains.

3.3.1.2 Approximate Channel Markov Chain Analysis

By Principle 2, we also approximate the process $\{U_n\ , n = 1, 2, 3, \cdots\}$ by its underlying embedded process $U_{n'}$, whose states are determined only at the Markov epochs (represented by • in Fig. 3.2(b)). The Markov epochs correspond to the beginning of each idle slot or each transmission period (successful or unsuccessful) on the channel. Since $U_{n'}$ is a Markov chain with finite state space $\{0, 1, 2, \cdots, M\}$ and is time-homogeneous, irreducible, aperiodic and positive recurrent, a solution exists for the equilibrium probability distribution $\{P_k\ ; k = 0, 1, \cdots, M\}$ of the channel states and can be obtained by solving the following set of linear simultaneous stationary equations [47]

$$\begin{cases} \underline{P} = \underline{P}\,[p_{ij}] \\ \sum_{k=0}^{M} P_k = 1 \end{cases} \tag{3.20}$$

where $\underline{P}$ is a row vector of the channel steady-state probabilities, P_k is the steady-state probability that k users contend for access to the channel at a Markov epoch and $[p_{ij}]$, $i, j \in \{0, 1, 2, \cdots, M\}$ is a square matrix of transition probabilities. p_{ij} is the conditional probability that j users contend for access to the channel at the current Markov epoch given that i users contended at the previous Markov epoch. The main task in (3.20) is the derivation of the expressions for p_{ij}; to do so we proceed as follows:

Consider two successive Markov epochs in Fig. 3.2(b) which for clarity is amplified in Fig. 3.4. Suppose that at Markov epoch n', i users contend for access to the channel while at Markov epoch $n'+1$, j users contend. Recall from the system model formulation that the number of users that sense the channel idle in a slot is equal to the number of users that contend for access to the channel at the next Markov epoch. Then, the transition probability from state i to state j, p_{ij} is defined by

$$p_{ij} \triangleq Pr\{U_{n'+1} = j \,|\, U_{n'} = i\} = Pr\{j\,|\,i\}\,. \tag{3.21}$$

Now, $Pr\{j\,|\,i\}$ can be written as

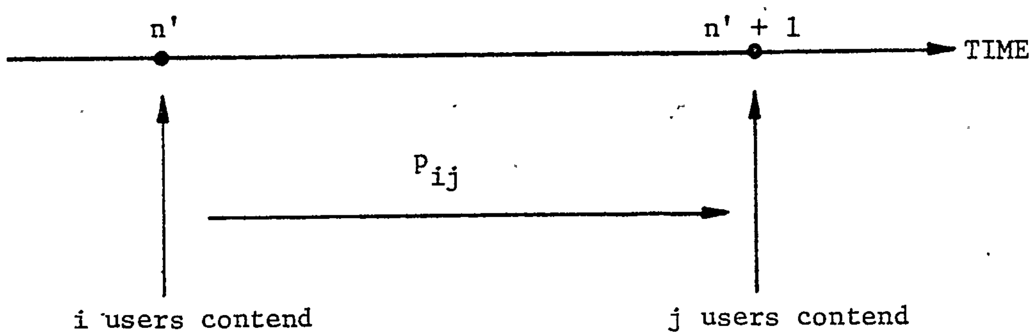


Fig. 3.4. Two Successive Markov Epochs of the Channel State.

$$Pr\{j\,|i\} = \sum_{k=LL}^{UL} Pr\{j,k\,|i\} \tag{3.22}$$

where $Pr\{j,k\,|i\}$ is the conditional probability that j ready users (which include k ready users that contended at epoch n' and are contending again at epoch $n'+1$) contend for access to the channel at epoch $n'+1$ given that i ready users contended at epoch n'. Using conditional probability theory techniques, (3.22) becomes

$$Pr\{j\,|i\} = \sum_{k=LL}^{UL} Pr\{j\,|k,i\}\,Pr\{k\,|i\}\,. \tag{3.23}$$

where UL and LL are respectively the upper and lower limits which remain to be determined. Based on the definition of $Pr\{k\,|i\}$, $Pr\{j\,|k,i\}$ is the conditional probability that given $(M-i)$ nonready/dormant users at epoch n', $(j-k)$ of the $(M-i)$ users will contend at epoch $n'+1$. Hence, (3.23) can be written as

$$p_{ij} = \sum_{k=LL}^{UL} Pr\{(j-k)|(M-i)\}\,Pr\{k\,|i\}\,. \tag{3.24}$$

Noting that all the users behave independently, $Pr\{(j-k)|(M-i)\}$ is binomially distributed with parameters $(M-i)$ and τ_{av} where τ_{av} is the average contention probability by an arbitrary user. Similarly, $Pr\{k\,|i\}$ is binomially distributed with parameters i $(i>0)$ and $\tau(i)$ where $\tau(i)$ is a contention probability which is a function of i. Equation (3.24) then becomes

$$p_{ij} = \sum_{k=LL}^{UL} \begin{bmatrix} M-i \\ j-k \end{bmatrix} \tau_{av}^{j-k}(1-\tau_{av})^{(M-i)-(j-k)} \begin{bmatrix} i \\ k \end{bmatrix} (\tau(i))^{k}(1-\tau(i))^{i-k}\,. \tag{3.25}$$

It now remains to determine τ_{av}, $\tau(i)$ and the summation limits, LL and UL.

Contention Probabilities: Since a nonready user at a Markov epoch will contend for access to the channel with probability zero and a ready user may contend with probability p, then the average contention probability by an arbitrary user (which is either nonready or ready) is given by

$$\tau_{av} = p(1 - \pi_0) . \tag{3.26}$$

To derive the expression for $\tau(i)$, we consider the following cases. Suppose $i = 1$ at epoch n' (Fig. 3.4), this implies a successful transmission and a user which has just completed transmission may contend again at epoch $n' + 1$ provided the user remains in the ready state. This condition occurs if the user buffer contains more than one packet at epoch n' or the user buffer contains only one packet at epoch n' (which has been successfully transmitted) and at least one packet arrives during the successful transmission period. These two mutually exclusive events occur with probability ν defined by

$$\nu = (1 - \delta) + \delta\alpha_T \tag{3.27}$$

where δ is the probability that a user buffer contains exactly one packet conditioned on the user being in ready state, and from the user Markov chain analysis

$$\delta = \frac{\pi_1}{1 - \pi_0} . \tag{3.28}$$

α_T is the probability that at least one packet arrives during the successful transmission period, given by

$$\alpha_T = \sum_{i=1}^{T_s+1} \binom{T_s+1}{i} \sigma^i (1-\sigma)^{T_s+1-i} . \tag{3.29}$$

Since a ready user contends with probability p, $\tau(1)$ then becomes

$$\tau(1) = \tau_s = p\nu . \tag{3.30}$$

For the case $1 < i \le M$ at epoch n', this means that a collision occurred during the ensuing transmission period and no packet is successfully transmitted by any of the contending users. Therefore, a user which contended at epoch n' and suffers a collision remains in the ready state at epoch $n' + 1$ irrespective of whether a new packet arrives or not during the unsuccessful transmission period which began at epoch n'; and any of these i colliding users may contend again at epoch $n' + 1$ with probability p. Hence, $\tau(i)$, $1 < i \le M$, denoted by τ_c is

$$\tau_c = p . \tag{3.31}$$

Summation Limits: To determine the summation limits, it is necessary to first define the parameter, d, by

$$d = (M - i) - (j - k)|_{k=0} \tag{3.32}$$

where d is the number of users in the system which were dormant/nonready at epoch n' and which will also remain dormant/nonready at epoch $n' + 1$, assuming that none of the i users which contended at epoch n' will contend again at epoch $n' + 1$, that is, $k = 0$. There are two cases of interest: (i) $d \ge 0$ and (ii) $d < 0$.

(i) $d \geq 0$: denotes the likelihood that all the j users contending at epoch $n' + 1$ are from those $(M - i)$ users which did not contend at epoch n', hence, k must assume a lower limit (LL) of zero. To determine the upper limit, we note that k must be less than or equal to both i and j, hence the upper limit (UL) for k is equal to $min(i,j)$.

(ii) $d < 0$: implies a negative number of nonready/dormant users at epoch $n' + 1$ which is impossible. Thus, we require that at least $|d|$ of the i users which contended at epoch n' must contend again at epoch $n' + 1$; which is the lower limit for k. As in case (i), the upper limit is $min(i,j)$.

Combining the two cases yields the summation limits of (3.25) as

$$LL = \begin{cases} 0 & , \ d \geq 0 \\ |d| & , \ d < 0 \end{cases} \tag{3.33a}$$

and

$$UL = \min(i, j) \ . \tag{3.33b}$$

Based on the above discussion, we can now write (3.25) explicitly for all j, $0 \leq j \leq M$, as follows:

(a) $i = 0$: $LL = 0$ (because $d \geq 0$) and $UL = 0$ (since $i = 0$). Hence (3.25) reduces to

$$p_{ij} = \binom{M}{j} \tau_{av}^{j} (1 - \tau_{av})^{M-j} \quad , \quad i = 0 \ . \tag{3.34}$$

(b) $i = 1$: the limits of summation are given by (3.33), and (3.25) becomes

$$p_{ij} = \sum_{k=LL}^{\min(1,j)} \begin{bmatrix} M-1 \\ j-k \end{bmatrix} \tau_{av}^{j-k} (1-\tau_{av})^{(M-1)-(j-k)} \begin{bmatrix} 1 \\ k \end{bmatrix} \tau_s^k (1-\tau_s)^{1-k} \ , i = 1 \ . \tag{3.35}$$

(c) $1 < i < M$: the limits of summation are given by (3.33), (3.25) then becomes

$$p_{ij} = \sum_{k=LL}^{\min(i,j)} \begin{bmatrix} M-i \\ j-k \end{bmatrix} \tau_{av}^{j-k} (1-\tau_{av})^{(M-i)-(j-k)} \begin{bmatrix} i \\ k \end{bmatrix} \tau_c^k (1-\tau_c)^{i-k} \ , 1 < i < M \ . \tag{3.36}$$

(d) $i = M$: $LL = j$ (because $d = -j$) and $UL = j$ (since j is less than or equal to M) so that (3.25) then reduces to

$$p_{ij} = \begin{bmatrix} M \\ j \end{bmatrix} \tau_c^j (1-\tau_c)^{M-j} \quad , \quad i = M \ . \tag{3.37}$$

In summary, by combining (3.34) to (3.37), we get

$$p_{ij} = \begin{cases} \begin{bmatrix} M \\ j \end{bmatrix} \tau_{av}^j (1-\tau_{av})^{M-j} \ , i = 0 \\ \sum_{k=LL}^{UL} \begin{bmatrix} M-1 \\ j-k \end{bmatrix} \tau_{av}^{j-k} (1-\tau_{av})^{(M-1)-(j-k)} \begin{bmatrix} 1 \\ k \end{bmatrix} \tau_s^k (1-\tau_s)^{1-k} \ , i = 1 \\ \sum_{k=LL}^{UL} \begin{bmatrix} M-i \\ j-k \end{bmatrix} \tau_{av}^{j-k} (1-\tau_{av})^{(M-i)-(j-k)} \begin{bmatrix} i \\ k \end{bmatrix} \tau_c^k (1-\tau_c)^{i-k} \ , 1 < i < M \\ \begin{bmatrix} M \\ j \end{bmatrix} \tau_c^j (1-\tau_c)^{M-j} \ , i = M. \end{cases} \tag{3.38}$$

Having found the p_{ij}'s, we now can compute the vector $\underline{P}$ in (3.20) from which the channel-activity parameters (γ, $\overline{\gamma}$, μ and $\overline{\mu}$) (Principle 4) are evaluated. The probability of successful transmission, γ, is defined as the probability that only one ready user contends for access to the channel given that at least one ready user contends. γ is expressed in terms of the channel steady-state probabilities as

$$\gamma = \frac{P_1}{1 - P_0} \tag{3.39}$$

and the probability of unsuccessful transmission, $\bar{\gamma}$, is

$$\bar{\gamma} = 1 - \gamma = \frac{1 - P_0 - P_1}{1 - P_0}. \tag{3.40}$$

The probability that the channel is idle, μ, is found from the standpoint of renewal theory [43]. Since an idle period consists of a sequence of Bernoulli slots, from Fig. 3.2(b) we see that the number of slots, I_{ch}, in an idle period is geometrically distributed where

$$Pr\{I_{ch} = i\} = P_0^{i-1}(1 - P_0) \quad , \quad i = 1, 2, 3 \cdots \tag{3.41}$$

with mean

$$E[I_{ch}] = \frac{1}{1 - P_0}. \tag{3.42}$$

Note that P_0 is the probability that no user contends for access to the channel at the beginning of a slot. By defining the transmission cycle time as the time interval separating two successive transmission commencement epochs on the channel, the long-run probability that the channel is idle is given by

$$\mu = \frac{E[channel\ idle\ period]}{E[transmission\ cycle]} = \frac{E[I_{ch}]}{\gamma(T_s + 1) + (1 - \gamma)(T_c + 1) + E[I_{ch}]}. \tag{3.43}$$

Using (3.39) and (3.42), μ can be expressed in terms of the channel steady-state probabilities as

$$\mu = \frac{1}{P_1(T_s + 1) + (1 - P_0 - P_1)(T_c + 1) + 1} \tag{3.44}$$

from which $\bar{\mu}(= 1 - \mu)$, the probability that the channel is busy is determined.

3.3.1.3 Performance Measures

In the following, we derive the expressions for the performance measures.

Average Queue Length, $E[Q]$: is the mean number of packets stored in the buffer of a representative user at an arbitrary slot boundary. Recall that the user Markov chain analysis presented above only gives the distribution of the process $\{N_n; n \geq 1\}$ at the embedded epochs which, due to the non-Poissonian input process, is not equal to that of the Q_n-process. We therefore determine $E[Q]$ indirectly using the notion of semi-Markov process with reward [43]. The main idea propounded here is to view the number of packets stored in a user buffer at an arbitrary slot boundary as a reward. The motivation for this idea follows from the application of renewal-reward arguments to an irreducible and ergodic semi-Markov process with reward: a reward is earned whenever a state of the process is entered at a Markov epoch and the reward is earned continuously (in fact at the end of every slot) during the time interval between two successive Markov epochs. We shall then invoke a limiting result from semi-Markov processes with reward, which states that the long-run reward (interpreted here as the mean queue length at an arbitrary slot boundary), $E[Q]$, is given by [43]

$$E[Q] = \frac{\sum_{k=0}^{\infty} \pi_k r_k}{\sum_{k=0}^{\infty} \pi_k \eta_k} \tag{3.45}$$

where the numerator is the expected reward (measured in packets-slots) and the denominator is the expected duration (in slots). When state k is entered at a Markov epoch, r_k is the mean reward accumulated, η_k is the expected duration between two successive Markov epochs and π_k is the steady-state probability that there are exactly k packets in a user buffer at an arbitrary Markov epoch. The π_k's are already determined from the user Markov chain analysis, our next task then is to find η_k and r_k.

(i) Expected Duration: For a nonready user ($k = 0$), the length between two successive Markov epochs is one slot. For a ready user ($k > 0$), the length between two successive Markov epochs is $(T_s + 1)$ slots for a successful transmission (which occurs with probability $p\mu\gamma$), $(T_c + 1)$ slots for an unsuccessful transmission (which occurs with probability $p\mu(1 - \gamma)$) and one slot when a ready user is dormant (with probability $(1 - p\mu)$). Hence for each user state, the expected duration between two successive Markov epochs can be written as

$$\eta_k = \begin{cases} 1 \;, \; k = 0 \\ p\mu\gamma(T_s + 1) + p\mu(1 - \gamma)(T_c + 1) + (1 - p\mu) \;, \; k \geq 1 \;. \end{cases} \tag{3.46}$$

(ii) Expected Reward Accumulated: The expected reward accumulated by a user which has just entered state k is given by

$$r_k = \begin{cases} \sigma \ , \ k = 0 \\ p\mu\gamma B_s(k, T_s) + p\mu(1-\gamma)B_c(k, T_c) + (1-p\mu)B_d(k, 1) \ , \ k \geq 1 \end{cases} \tag{3.47}$$

where, if a user buffer contains k packets at a Markov epoch, $B_s(k, T_s)$ is the average reward accumulated during a successful transmission period, $B_c(k, T_c)$ is the average reward accumulated during an unsuccessful transmission period, and $B_d(k, 1)$ is the average reward accumulated during a dormant slot; which are found to be

$$B_s(k, T_s) = (k-1)(T_s+1) + T_s + \sum_{i=1}^{T_s+1} i\sigma \tag{3.48a}$$

$$B_c(k, T_c) = k(T_c+1) + \sum_{i=1}^{T_c+1} i\sigma \tag{3.48b}$$

and

$$B_d(k, 1) = k + \sigma \tag{3.48c}$$

respectively. We see from (3.48) that the reward accumulated over each time period is due to old packets and new packets that arrive between two successive Markov epochs. Note that σ in (3.47) and (3.48) is the mean number of arrivals per slot and the terms involving the summation in (3.48) represent the expected reward accumulated due to the arrival of a new packet at the end of slot i of a transmission period. Now, substituting (3.46), (3.47) and (3.48) into (3.45) we get after simplification

$$E[Q] = \frac{\pi_0\sigma + K_1 + K_2}{\pi_0 + (1-\pi_0)[p\mu\gamma(T_s+1) + p\mu(1-\gamma)(T_c+1) + (1-p\mu)]} \tag{3.49}$$

where

$$K_1 = [p\mu\gamma(T_s+1) + p\mu(1-\gamma)(T_c+1) + (1-p\mu)]E[N]$$

and

$$\begin{aligned} K_2 &= (1-\pi_0)[p\mu\gamma T_s - p\mu\gamma(T_s+1) + p\mu\gamma\frac{\sigma(T_s+1)(T_s+2)}{2} \\ &+ p\mu(1-\gamma)\frac{\sigma(T_c+1)(T_c+2)}{2} + (1-p\mu)\sigma] . \end{aligned}$$

Mean Packet Delay, $E[D]$: From the mean queue length derived above, application of Little's formula [55] gives the mean packet delay in slots as

$$E[D] = \frac{E[Q]}{\sigma} . \tag{3.50}$$

Throughput, S_{ch}: is defined as the proportion of time the channel is used successfully in a transmission cycle. Using renewal theory arguments,

$$S_{ch} = \frac{E[successful\ transmission\ period]}{E[transmission\ cycle]} = \frac{\gamma T_s}{\gamma(T_s+1) + (1-\gamma)(T_c+1) + E[I_{ch}]} . \tag{3.51}$$

In terms of the channel steady-state probabilities (from (3.39)), S_{ch} is given by

$$S_{ch} = \frac{P_1 T_s}{P_1(T_s+1) + (1-P_0-P_1)(T_c+1) + 1} . \tag{3.52}$$

Condition for System Stability: A rigorous derivation of the necessary and sufficient condition for stability of buffered CSMA-CD systems is extremely difficult. To the best of the author's knowledge, the previous rigorous analyses on stability of buffered random access systems have considered the slotted ALOHA scheme, which is the simplest random access protocol [65, 67, 68]. In this thesis we derive a necessary and sufficient condition for stability based on one possible definition of a stable system: a system is said to be stable for a given arrival rate if the expected delay per packet is finite [69]. By applying this definition, $E[D]$ is finite provided that $E[Q]$ is. But from (3.49) the finiteness of $E[Q]$ requires $E[N]$ to be finite. We then conclude from (3.19) that $E[N]$ is finite provided

$$\sum_{i=1}^{T_s} iq_i < p\mu\gamma\alpha_0 \tag{3.53}$$

which is a necessary and sufficient condition for stability. The left hand side of (3.53) is the mean rate at which packets enter the user queue while the right hand side is the mean rate at which packets depart from the queue. Equation (3.53) states that the user queue is stable if the mean input flow rate is less than the mean output flow rate, and the whole system is stable provided (3.53) holds for all the user queues.

3.3.1.4 Numerical Results and Discussion

(a) Computation Algorithm

The analysis in the previous section reveals that the system performance measures are functions of the user and channel steady-state probabilities and have been shown to be interdependent, hence, they are determined iteratively (Principle 5). The iterative algorithm required is outlined as follows:

Step 1: Select the input parameters and perform initial computation:

(a) select the system parameters - σ, p, M, T_s, and T_c;

(b) compute α_l, $l = 0, 1, \cdots, T_s+1$ and β_m, $m = 0, 1, \cdots, T_c + 1$;

(c) select the initial guess for $\pi_0^{(0)}$ and $\pi_1^{(0)}$.

Step 2: Iteration step - at iteration k, $k = 1, 2, \cdots, k_{\max}$ where $k_{\max}$ is the maximum number of iterations permissible:

(a) compute the channel transition probabilities from (3.38);

(b) solve the channel stationary equation for $\{P_j\}_{j=0}^{M}$ from (3.20);

(c) compute the new values of $\pi_0^{(k)}$ from (3.13) and $\pi_1^{(k)}$ from (3.9).

Step 3: Looping step - if the most recently computed values of π_0 and π_1 are within a specified tolerance of their last computed values, then proceed to Step 4. Otherwise, go back to Step 2.

Step 4: Compute the system performance measures:

(a) compute $E[Q]$ from (3.49), compute $E[D]$ using (3.50) and S_{ch} from (3.52).

Remarks:

(i) The iterative algorithm described above is easily programmed. In Step 2, (3.20) is solved using the Gaussian elimination technique [70]. In Step 3, the L-infinity norm [70] is employed to measure the convergence of π_0 and π_1; and for all the input values chosen in the numerical examples, less than 50 iterations are required to reach convergence of π_0 and π_1 within a tolerance of 10^{-6}. The analytic program is run on a SUN 3/180 system, the typical run time for a point on a performance curve shown below is 5 cpu (central processing unit) seconds.

(ii) In addition to the analytic computation, a discrete-event simulation program has been constructed to determine the user and channel steady-state probabilities along with the system performance measures. Each simulation run is terminated after 30000 packets have been successfully transmitted (rather than terminating the simulation after a fixed period of run time) so as to remove any bias from the simulation results [52]. The simulation program is run on a CDC CYBER 205 Supercomputer where its fast speed has been of great advantage [58] and the typical processing time for one simulation run is 2000 cpu seconds. The estimated values of the performance measures are obtained by averaging the outputs of 10 independent simulation runs.

(b) Discussion on Results

In order to demonstrate the usefulness of the expressions derived for the performance measures, numerical results are obtained for the following set of input parameters (except otherwise stated):

System Population, M: 10.

Packet Transmission time, T_s: 20 *slots*.

Collision resolution period, T_c: 1 *slot*.

Fig. 3.5 shows a plot of the mean queue length versus the system offered traffic, $\rho(=M \sigma T_s)$ where the sensing probability, p is the varying parameter. We see that for each value of p, the mean queue length increases as the offered traffic increases - an intuitive result. Furthermore, we see that in the range of light to moderate offered traffic, the mean queue length decreases as p increases; this is so because for large values of p, the transmission of a packet is attempted as soon as it reaches the head of queue and the transmission is more likely to be successful owing to the low offered traffic; hence, there is a minimal backlog accumulation. Under heavy offered traffic, and at large values of p, the mean queue length grows to a large value due to the backlog of packets at each queue which cannot be cleared as fast as the light load case. Since p is large, most of the users are more likely to contend for access to the channel but this inevitably leads to an increase in the number of collisions on the channel. The large backlog of packets at each user queue is caused by the new packets arriving during the collision periods and

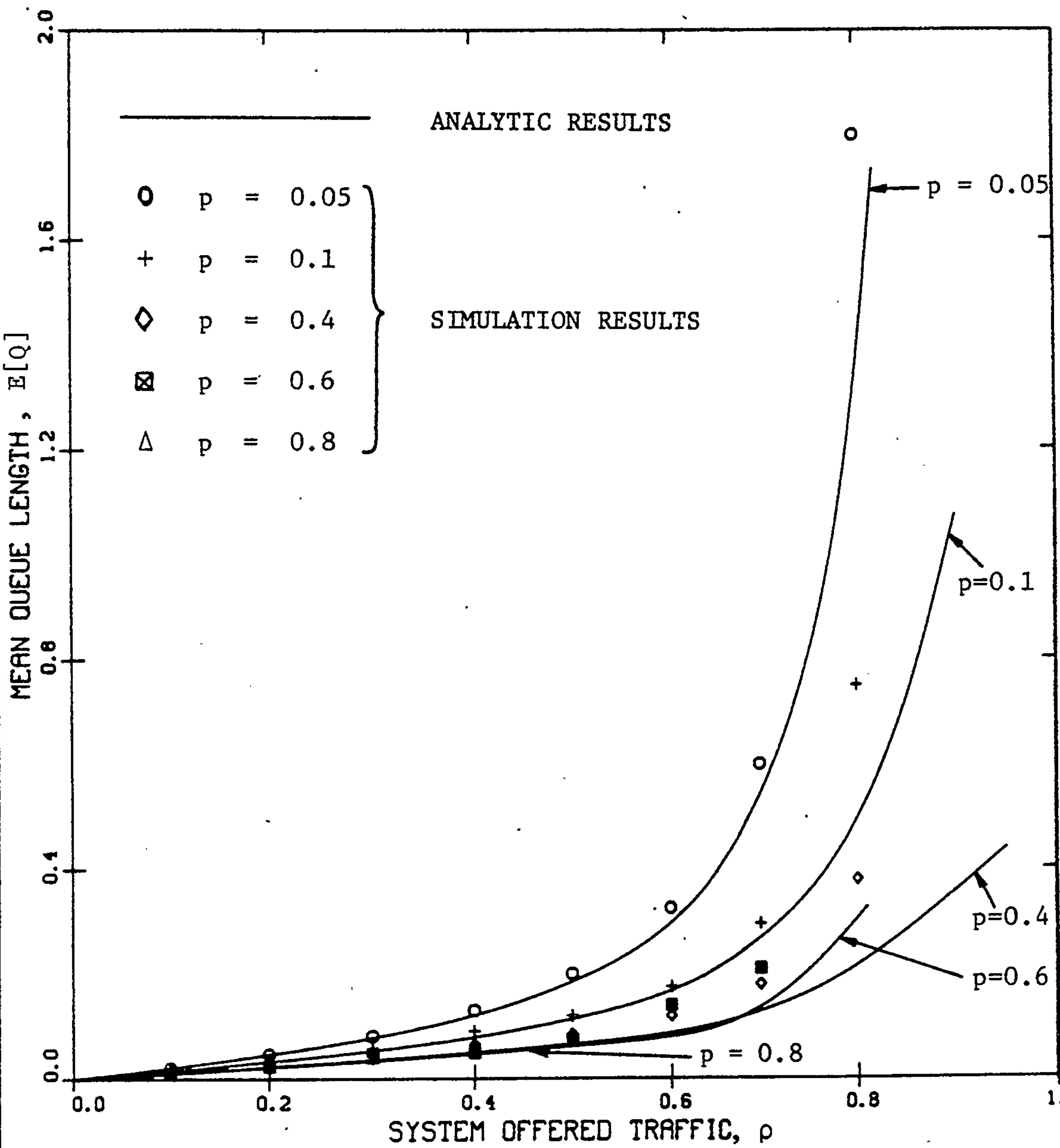


Fig. 3.5. Mean Queue Length versus System Offered Traffic . ($M = 10$, $T_s = 20$ and $T_c = 1$)

timeout intervals. Note the good agreement between the analytic and simulation results especially in the range of light to moderate offered traffic. However, under heavy offered traffic, there is less agreement because the system is no longer stable (that is, the condition for stability (3.53) is violated) and the approximate analysis is not so accurate.

The throughput-delay characteristics shown in Fig. 3.6 illustrate the tradeoff between the mean packet delay and the channel throughput and also facilitate the selection of suitable sensing probability at a given throughput value. We mention in passing that the numerical result obtained for the channel throughput (defined by (3.52)) is found to be equal to the offered traffic for up to 75% offered traffic over the range of p considered. Beyond 75% offered traffic, we find that the throughput is less than the offered traffic where the drop is accounted for by the increase in the number of collisions on the channel, this means that most of the channel time is spent on resolving collisions. As in Fig. 3.5, we see from Fig. 3.6 that for a fixed value of p, an increase in throughput leads to an increase in the mean packet delay (normalized with respect to the packet transmission time, T_s). Under low to moderate throughput values, we observe that an increase in p results in lower mean packet delay. Actually, the mean packet delay consists of the mean packet queueing delay and the packet transmission time where the former depends upon the queue length seen by a newly generated packet, the sensing probability value and the number of retransmission attempts a packet undergoes before its successful transmission begins. In the range of low to moderate throughput (or equivalently

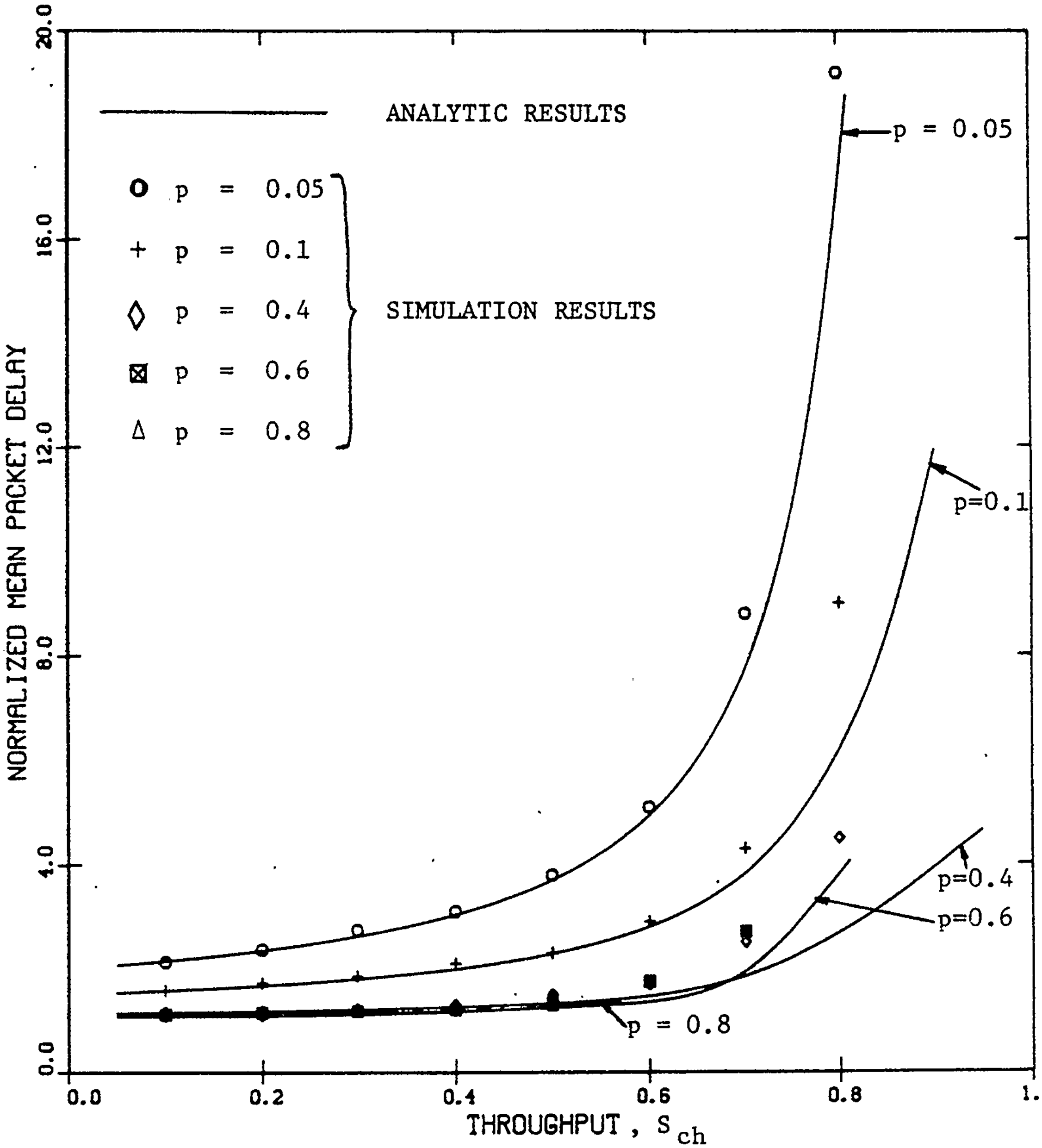


Fig. 3.6. Normalized Mean Packet Delay - Throughput Characteristics ($M = 10$, $T_s = 20$ and $T_c = 1$).

low to moderate offered traffic) and at large values of p, the mean queue length is small and therefore the mean packet delay is dominated by the packet transmission time. However, in the high throughput region when p is large, the delay is no longer small. The large delay is caused by the congestion of the channel which occurs when all the users are constantly in the ready state and they all contend for access to the channel with a high probability (due to large p). The net effect of channel congestion is an increase in the number of collisions (or an increase in the number of retransmissions). Hence, the packet transmission time is dwarfed by the sum of the large mean retransmission delays and the collision resolution periods which then leads to very large mean packet delays. Finally, we observe that the throughput range at which all analytical results agree closely with the simulation results is the range of stable operation of the system. As noted above, the less agreement in the high throughput region signifies system instability and under this condition the approximate analysis becomes inaccurate. It is also important to state that the large delays in the high throughput region is not due to the approximate analysis but is typical for random access protocols.

In Fig. 3.7 we present a plot of the mean packet delay versus the sensing probability with the offered traffic as the varying parameter. For all the curves shown, we see that the packets transmitted when the sensing probability attains low values incur large mean delays owing to buffer congestion which is more pronounced for large values of offered traffic. As the value of p increases, the mean packet delay decreases because the buffer is decongested much more quickly thus preventing

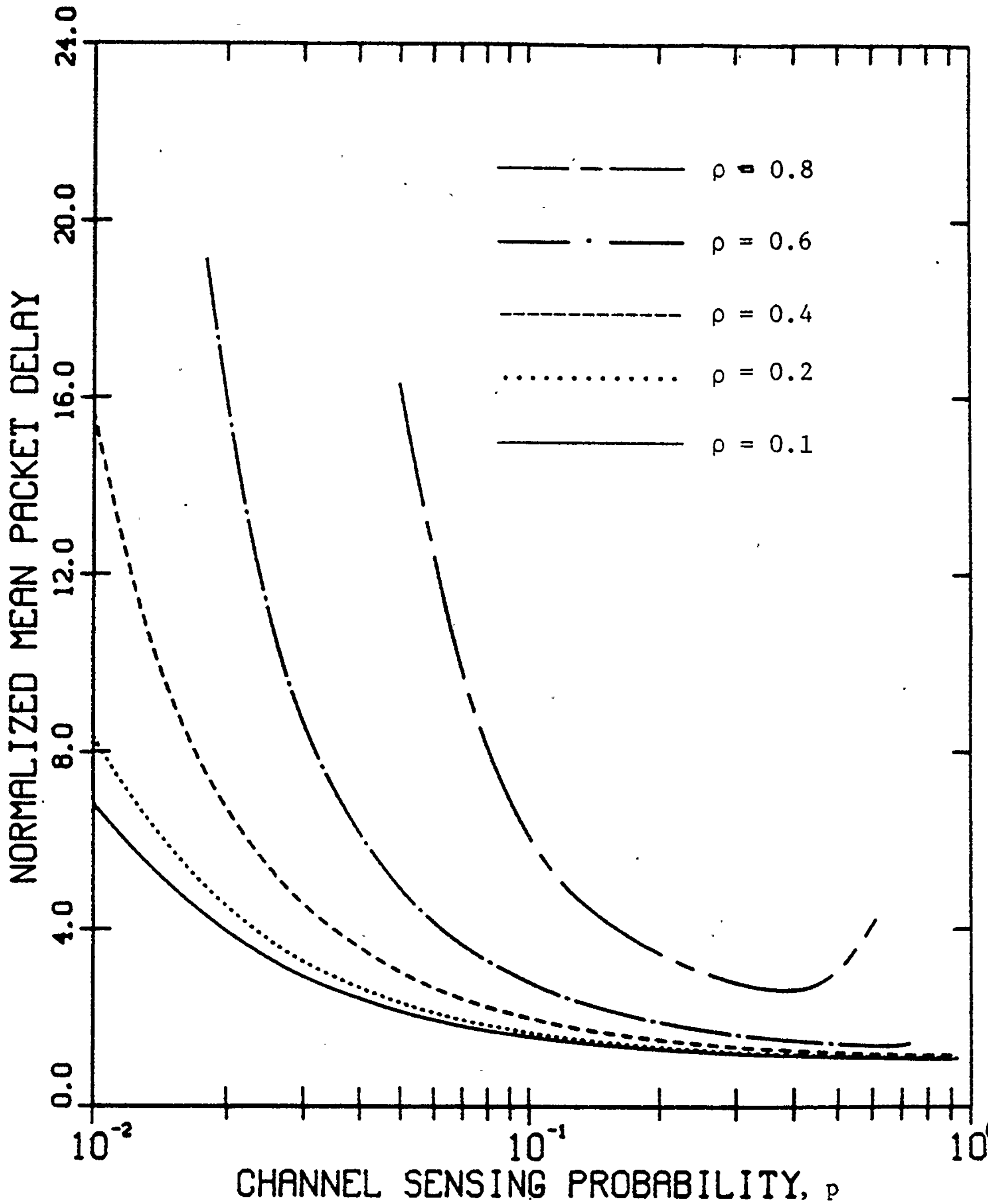


Fig. 3.7. Normalized Mean Packet Delay versus Channel Sensing Probability ($M = 10$, $T_s = 20$ and $T_c = 1$) .

excessive backlog accumulation. For the curves corresponding to moderate (and large) offered traffic, the mean packet delay decreases as p is further increased until a critical value of p (denoted by p_{opt}) at which delay is minimum. Beyond p_{opt}, the delay starts to rise where the increase is caused by channel congestion. We see in Fig. 3.7 that p_{opt} tends to unity for the range $\rho \leq 0.6$.

3.3.2 Homogeneous Users with Finite Buffer Size

In this section we relax the infinite buffer size assumption of Section 3.3.1 so as to arrive at a more realistic system. Unfortunately, the analysis of such a finite buffer size system becomes increasingly difficult. We shall show that with few modifications of the model and analysis presented in Section 3.3.1, the decomposition approximation technique reduces the analytic difficulty.

3.3.2.1 Approximate User Markov Chain Analysis

The user operation (depicted by Fig. 3.1) and its associated state transition diagram (Fig. 3.3) for the infinite buffer size system are not directly applicable to the finite buffer size system because of the need to rewrite the expressions for p_A's for different values of $K < \infty$. The preceding statement requires the repetition of user Markov chain analysis for each K, this not only is cumbersome but also seems impractical. We circumvent this problem by introducing an additional assumption: An active user generates only "one" message with probability α at the end of a successful transmission period and with probability β at the end of an unsuccessful transmission period. By the Bernoullian message arrival process assumption and as shown in Fig. 3.1, we expect that message arrivals during a(n) successful (unsuccessful) transmission period are, in general, more than one. However, the assumption is approximately valid provided σ is very small; for example, in the range $(0 \leq \sigma \leq 0.003)$, the probability that more than one message arrive during a typical successful transmission period becomes small and the assumption is tenable [71]. With σ being much less than unity, α and β are approximately given by $\sigma(T_s + 1)$ and $\sigma(T_c + 1)$ respectively.

A. Generating Function for Queue Length

Following the same analysis procedure as in Section 3.3.1, we derive first the transition probabilities of the modified user state transition diagram (Fig. 3.8). Let $Pr\{N_{n'+1} = k \mid N_{n'} = j\}$, $j, k \in \{0, 1, 2, \cdots, K < \infty\}$, be defined as the transition probability of a user being in state k at the next Markov epoch given that the user is in state j at the current epoch. We obtain

$$p_A = Pr\{0|0\} = \bar{\sigma} \tag{3.54}$$

$$p_{A'} = Pr\{k|k\} = p\mu\gamma\alpha + p\mu\bar{\gamma}\bar{\beta} + p\bar{\mu}\,\bar{\sigma} + \bar{p}\,\bar{\sigma} \quad , \quad 1 \leq k \leq K-1 \tag{3.55}$$

$$p_B = Pr\{1|0\} = \sigma \tag{3.56}$$

$$p_{B'} = Pr\{k+1|k\}$$

$= Pr\{$ either {a ready user unsuccessfully transmits the HOQ packet and

one packet arrives at the end of the unsuccessful transmission

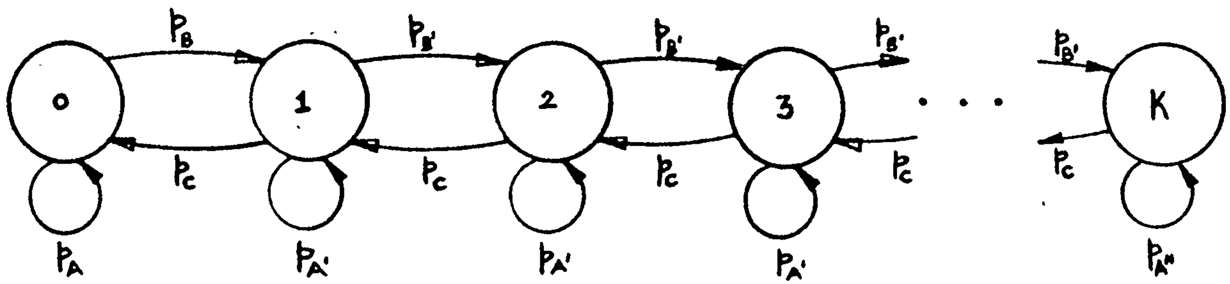


Fig. 3.8. User state transition diagram (finite buffer size).

period}

or {channel is sensed to be busy and one packet arrives at the end of the slot when the channel is sensed busy}

or {channel is not sensed at all and one packet arrives at the end of the slot when the channel is not sensed} }

$$= p\mu\bar{\gamma}\beta + p\bar{\mu}\sigma + \bar{p}\sigma \ , \ 1 \le k \le K - 1 \tag{3.57}$$

$$p_C = Pr\{(k-1)|k\} = p\mu\gamma\bar{\alpha} \ , \ 1 \le k \le K \tag{3.58}$$

$$p_{A''} = Pr\{K|K\}$$

$= Pr\{$ a ready user (with full queue) at the current Markov epoch remains ready (with full queue) at the next epoch}

$$P_{A''} = 1 - p_C = 1 - p\mu\gamma\bar{\alpha} \ . \tag{3.59}$$

Under equilibrium condition, the following balance relations can be written for the user steady-state probabilities:

$$\begin{aligned} \pi_0 &= p_A \, \pi_0 + p_C \, \pi_1 \\ \pi_1 &= p_{A'} \pi_1 + p_B \, \pi_0 + p_C \, \pi_2 \\ \pi_j &= p_{A'} \pi_j + p_{B'} \pi_{j-1} + p_C \, \pi_{j+1} \quad , \quad 2 \le j \le K-1 \\ \pi_K &= p_{A''} \pi_K + p_{B'} \pi_{K-1} \, . \end{aligned} \tag{3.60}$$

Now define the probability generating function (*pgf*) for queue length by

$$G_N(z) = \sum_{j=0}^{K} \pi_j \, z^j \ . \tag{3.61}$$

Using the procedure of Section 3.3.1, we derive the *pgf* for queue length as

$$G_N(z) = \frac{[(p_A - p_{A'}) + (p_B - p_{B'})z - p_C z^{-1}]\pi_0 + [(p_{A''} - p_{A'})z^K - p_{B'} z^{K+1}]\pi_K}{1 - p_{A'} - p_{B'} z - p_C z^{-1}} . \tag{3.62}$$

Equation (3.62) is true for $K = 2, 3, \cdots$ and for $K = 1$, the *pgf* for queue length is obtained if $p_{A'}$ is replaced by $p_{A''}$ and $p_{B'}$ is set to zero. The substitution gives

$$G_N(z) = \frac{[(p_A - p_{A''}) + p_B z - p_C z^{-1}]\,\pi_0}{1 - p_{A''} - p_C z^{-1}} . \tag{3.63}$$

Application of the *pgf* property and L'Hospital's rule to (3.62) and (3.63) gives the expressions for the mean queue length, $E[N]$, at the embedded Markov epochs as

$$E[N] = \begin{cases} \dfrac{p_C(1 - \pi_0)}{p_C - p_{B'}} + \dfrac{[K\,(K-1)(p_{A''} - p_{A'}) - K\,(K+1)\,p_{B'}]\,\pi_K}{2(p_C - p_{B'})} , & K \geq 2 \\ 1 - \pi_0 \; , \quad K = 1 . & \end{cases} \tag{3.64}$$

The knowledge of the *pgf* now permits us to determine the queue length distribution, $\{\pi_j\}_{j=0}^{K}$. Observe that (3.62) contains two unknown user steady-state probabilities π_0 and π_K which are determined as follows. For uniqueness of solution, two equations involving π_0 and π_K are required. One equation is obtained by expanding $G_N(z)$ (3.62) in a Taylor series about $z = 0$ to the term in z^{K+1} and then setting the coefficient of z^{K+1} to zero. Note that it is logical to set the coefficient of z^{K+1} $(=\pi_{K+1})$ to zero because $G_N(z)$ is a polynomial of degree K in z. Another equation is derived using the normalization condition: $G_N(1) = 1$. The two equations obtained are then solved to obtain the expressions for π_0 and π_K. For example, if $K = 2$, the expressions for π_0 and π_2 are derived to be

$$\pi_0 = \frac{(p_{A''} - p_{A'})\, p_C^2\, (p_C - p_{B'})}{D_{nm}} \tag{3.65}$$

and

$$\pi_2 = \frac{\{(1-p_A)(1-p_{A'})^2 - [(1-p_A)p_{B'} + (1-p_{A'})p_B]p_C\}(p_C - p_{B'})}{D_{nm}} \tag{3.66}$$

respectively where the denominator term is

$$\begin{aligned} D_{nm} &= [(1-p_A)p_{B'} + (1-p_{A'})\, p_B]\, p_C\, p_{B'} - p_{B'}(1-p_A)\,(1-p_{A'})^2 \\ &\quad - (p_{A'} - p_{A''})\, p_C^2 (p_C - p_{B'} + p_B)\,. \end{aligned}$$

The other user steady-state probabilities are obtained from the recursive relations (3.60). Using the above procedure, similar expressions can be derived for higher values of K. For the special case where $K = 1$, only the normalization condition need be applied to (3.63) and the expression for π_0 is obtained as

$$\pi_0 = \frac{p_C}{p_B + p_C}\,. \tag{3.67}$$

3.3.2.2 Approximate Channel Markov Chain Analysis

The channel Markov chain analysis for the finite buffer size system is identical to that in Section 3.3.1.2 except that α_T in (3.27) is now replaced by α due to the additional assumption introduced.

3.3.2.3 Performance Measures

The average queue length at an arbitrary slot boundary is also given by (3.49) where now $E[N]$ is defined by (3.64). The average packet delay, $E[D]$ and the channel throughput, S_{ch} are computed from (3.50) and (3.52) respectively. For the finite buffer size system, knowledge of the buffer overflow probability, P_{of} is essential as it represents the fraction of messages rejected due to the buffer being full. For simplicity, we have approximated P_{of} by π_K, (becomes exact for the Poisson input process) the probability that a user buffer is full. Finally, by employing the same argument as in Section 3.3.1.3, we derive the necessary and sufficient condition for system stability as

$$\sigma < \frac{p\mu\gamma}{p\mu\gamma(T_s+1) + p\mu\bar{\gamma}(T_c+1) + (1-p\mu)} . \qquad (3.68)$$

Actually, for the finite buffer size system, the condition given by (3.68) is always satisfied because the queue size is bounded so that there is no possibility of its increasing indefinitely.

3.3.2.4 Numerical Results and Discussion

In the spirit of the decomposition approximation, we first determine the channel-activity parameters by employing the iterative algorithm of Section 3.3.1.4

where the corresponding equations for the finite buffer size system are used. By choosing identical input parameters as in the infinite buffer size system, we obtain system performance characteristics which are discussed in the following. Our objectives here are twofold: (1) discuss the effect of K on throughput-delay characteristics and (2) determine the value of K that will result in an acceptably small number of lost packets.

Fig. 3.9 shows the throughput-delay characteristics for a 10-node system and $p = 0.05$. We see that for $K = 1$, 2 and 3, the mean packet delay is small under low to medium throughput but becomes large in the high throughput range. Another observation is that higher values of K results in larger packet delay (the mean packet delay for the case of infinite queue size ($K = \infty$) is included for comparison). Note the very good agreement between the analytic and simulation results especially under low to medium throughput; but there is less agreement in the high throughput region. The less agreement is accounted for by similar reasons stated in Section 3.3.1.4.

The buffer overflow probability versus the system input traffic ($M \sigma T_S$) is shown in Fig. 3.10 for a 10-node system where the queue size, K, also serves as the varying parameter. Excellent agreement is obtained between the analytic results and the simulation results. We see that low buffer overflow probabilities are obtained for higher values of K. A comparison between Fig. 3.9 and Fig. 3.10 shows the tradeoff between the buffer overflow probability and the mean packet delay for a given K in buffered CSMA-CD systems: an increase of the queue size

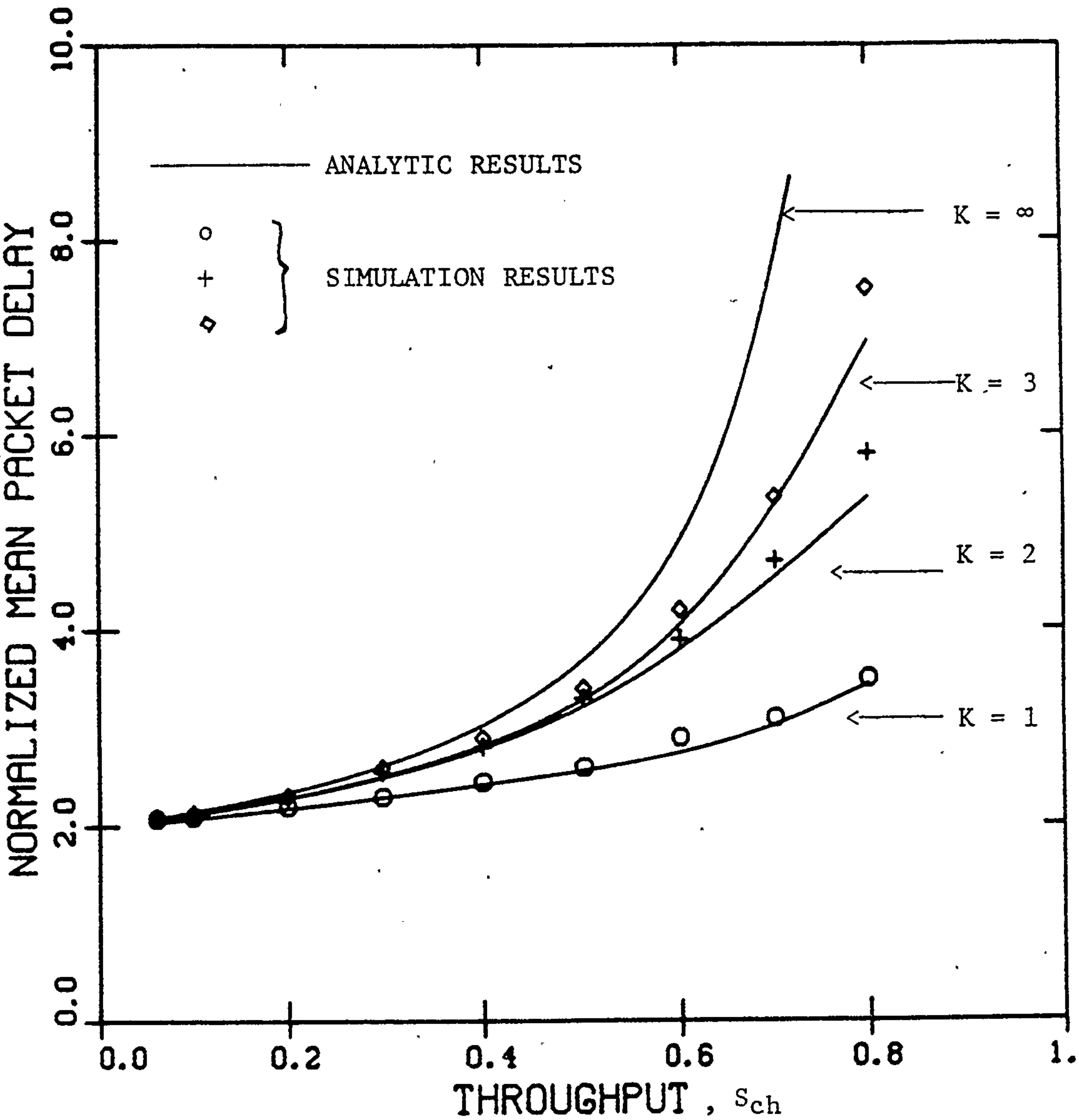


Fig. 3.9. Normalized mean packet delay - throughput characteristics for a 10-node system.

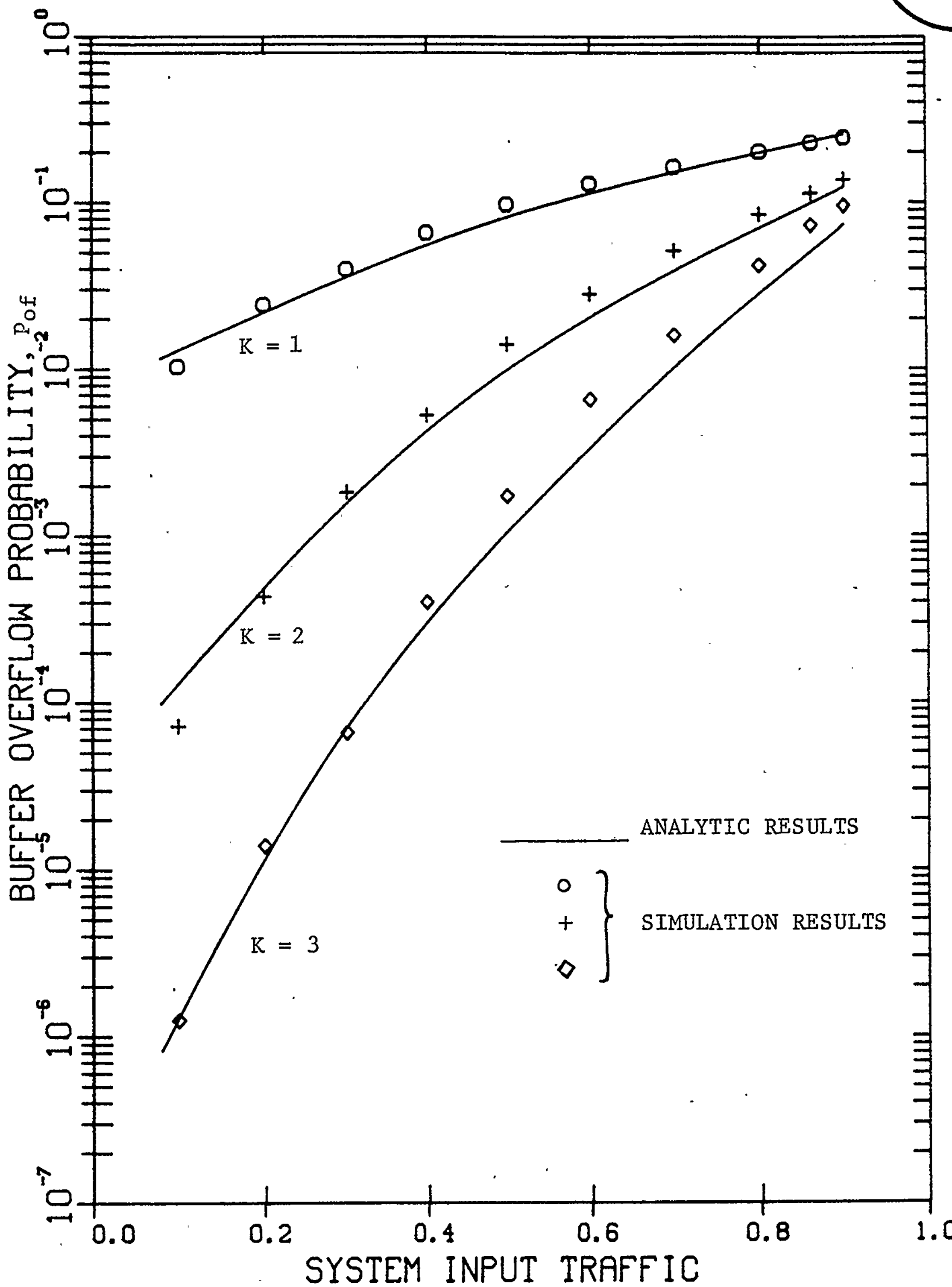


Fig. 3.10. Buffer overflow probability versus System input traffic for a 10-node system .

will reduce the buffer overflow probability but will lead to higher mean packet delay. The author believes that an appropriate choice of queue size is that which satisfies the specified buffer overflow probability as this will result in a small number of lost packets at the expense of an increase in packet delay.

3.3.3 Heterogeneous User with Infinite Buffer

In the analyses of the system scenaria considered thus far, the user homogeneity has been tacitly assumed primarily for analytical convenience. In practice, different users will generate traffic with different arrival rates and/or sense the channel with different probabilities (heterogeneous users). For example, different packet arrival rates among the users are more likely to occur during peak traffic periods and/or different users may sense the channel with dissimilar probabilities depending on the user queue length. Hence, our aim in this section is to investigate the impact of dissimilar packet arrival rates and/or dissimilar channel sensing probabilities on the performance of buffered CSMA-CD LANs using the decomposition approximation.

3.3.3.1 Performance Analysis

The user Markov chain analysis of Section 3.3.1.1 carries over almost exactly to this section except that we now distinguish the parameters p and σ among the users in the system. For example, the parameters p and σ for a user (say user i) are now denoted by $p^{(i)}$ and $\sigma^{(i)}$ respectively.

To analyze the channel Markov chain, extra care is required because, unlike the homogeneous users case, the expressions for p_{ij} for all possible values of i and j cannot be written in a compact form as (3.38). Nevertheless, (3.38) still serves as the basis for writing the p_{ij}'s for the heterogeneous users scenario, but in addition, we must consider the dependence of each user on the other users in the system. As an illustration, the $[p_{ij}]$ for a 2-node (heterogeneous) network is obtained as

$$[p_{ij}] = \begin{bmatrix} \prod_{l=1}^{2}(1-\tau_{av}^{(l)}) & \sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}\tau_{av}^{(l)}(1-\tau_{av}^{(m)}) & \prod_{l=1}^{2}\tau_{av}^{(l)} \\ \frac{1}{2}\sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}(1-\tau_{av}^{(l)})(1-\tau_{s}^{(m)}) & \frac{1}{2}\sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}\tau_{av}^{(l)}(1-\tau_{s}^{(m)})+\frac{1}{2}\sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}(1-\tau_{av}^{(l)})\tau_{s}^{(m)} & \frac{1}{2}\sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}\tau_{av}^{(l)}\tau_{s}^{(m)} \\ \prod_{l=1}^{2}(1-\tau_{c}^{(l)}) & \sum_{l=1}^{2}\prod_{\substack{m=1\\m\neq l}}^{2}\tau_{c}^{(l)}(1-\tau_{c}^{(m)}) & \prod_{l=1}^{2}\tau_{c}^{(l)} \end{bmatrix} \tag{3.69}$$

where the sensing probabilities by a particular user (say user i) are now given by

$$\begin{aligned} \tau_{av}^{(i)} &= p^{(i)}(1-\pi_0^{(i)}) \\ \tau_{c}^{(i)} &= p^{(i)} \\ \tau_{s}^{(i)} &= p^{(i)}\nu^{(i)} \end{aligned} \tag{3.70}$$

the parameters in (3.70) have their usual meaning as stated in Section 3.3.1. It is worth noting that the enumeration of the channel transition probabilities becomes more difficult as M increases.

We can compute identical performance measures which are defined in Section 3.3.1, except that the respective user parameters are now used in the computation. Since, due to dissimilar user parameters, the delay experienced by packets transmitted by different users may now be unequal, it is appropriate to determine the weighted mean packet delay, $E[D_w]$ which is computed from

$$E[D_w] = \frac{\sum_{i=1}^{M} E[D^{(i)}]\, \sigma^{(i)}}{\sum_{i=1}^{M} \sigma^{(i)}} . \tag{3.71}$$

3.3.3.2 Numerical Results and Discussion

The analysis presented above is illustrated by a simple example of a 2-node system where T_s and T_c are as chosen previously. Two investigations were conducted. First, we examine the effects of dissimilar channel sensing probabilities, $p^{(i)}$ (Fig. 3.11 to Fig. 3.13); second, we study the effects of different packet arrival rates, $\sigma^{(i)}$, on the delay-input traffic characteristics (Fig. 3.14). Fig. 3.11 shows the mean packet delay (for $p^{(1)} = 0.1$, $p^{(2)} = 0.05$) versus the packet arrival rate when $\sigma^{(1)} = \sigma^{(2)}$. Note the very good agreement between the analytic and the simulation results, thus validating the approximate analytic technique. We observe that packets transmitted by node 2 incur larger mean delay than those transmitted by node 1, the reason is due to the value of $p^{(1)}$ which is greater than

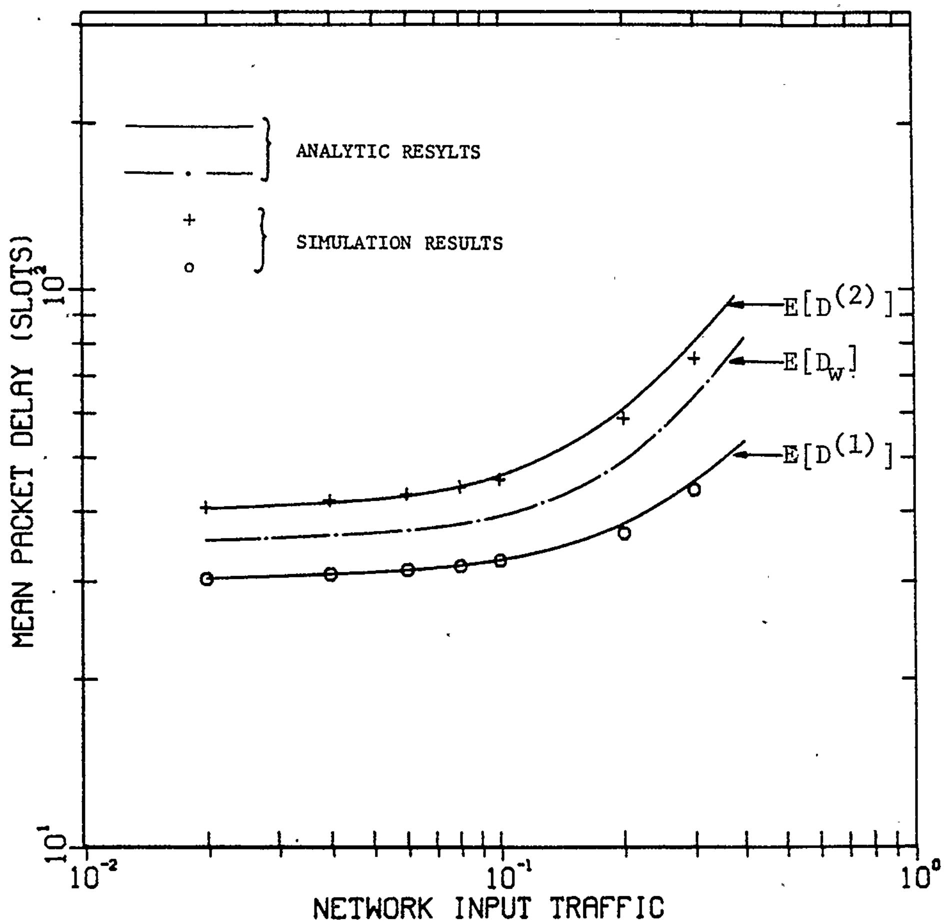


Fig. 3.11. Mean packet delay versus Network input traffic ($p^{(1)} = 0.1$, $p^{(2)} = 0.05$) .

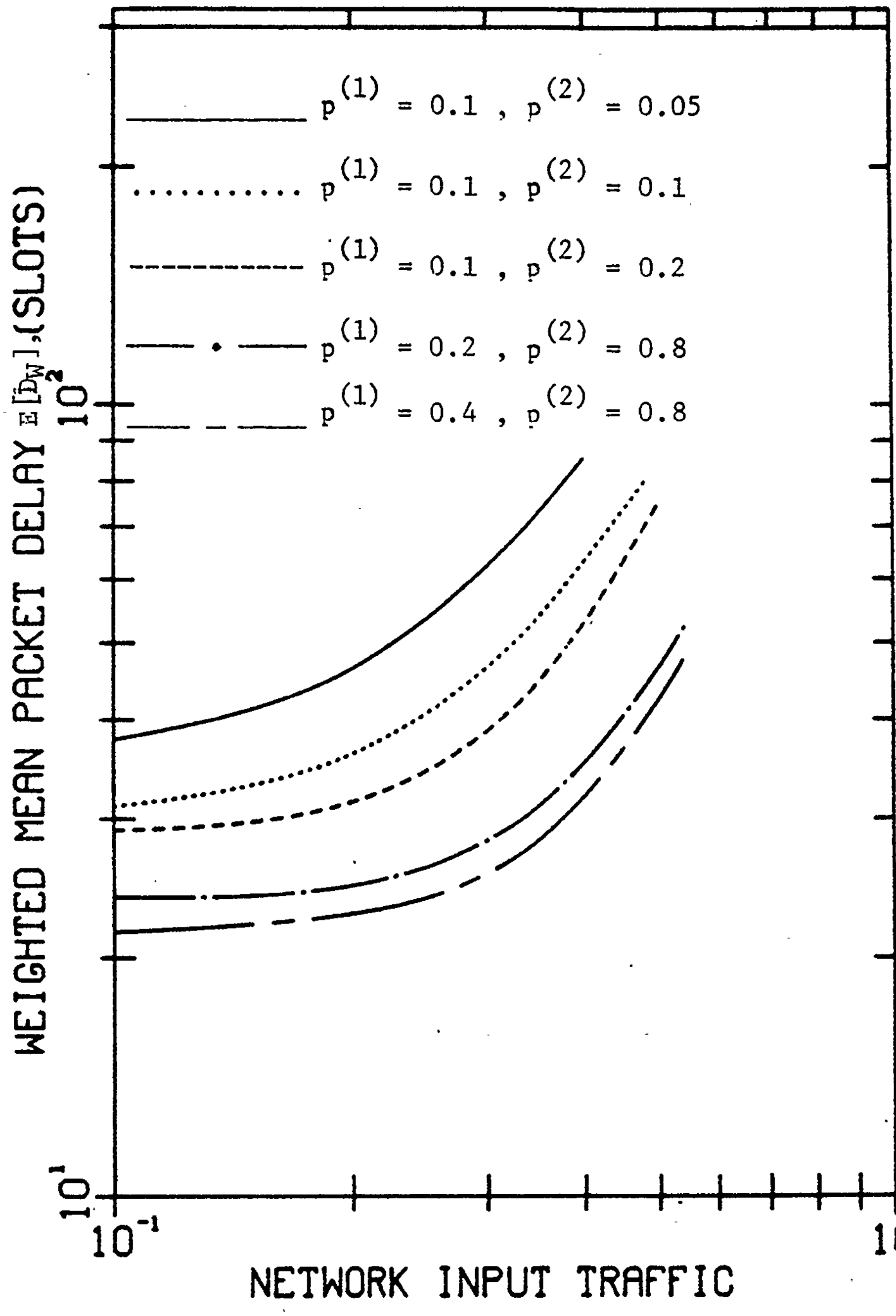


Fig.3.12. Weighted mean packet delay vs. Network input traffic for different combinations of $p^{(1)}$ and $p^{(2)}$.

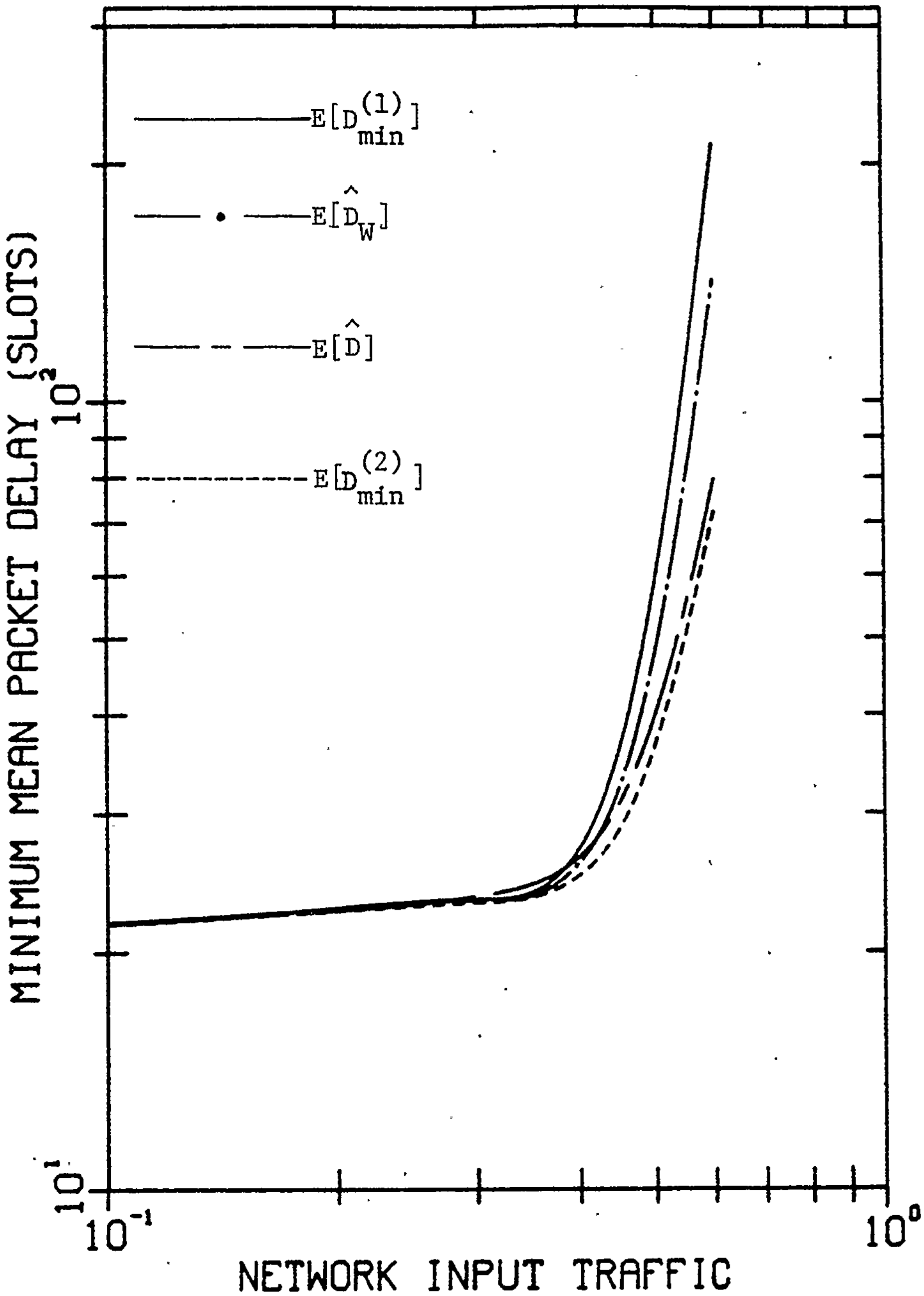


Fig.3.13. Minimum mean packet delay vs. Network input traffic.

$p^{(2)}$. However, it is observed that for all the input traffic considered, the weighted mean packet delay, $E[D_w]$, is lower than the mean delay experienced by the packets transmitted by node 2 (which represents a gain with respect to node 2) but $E[D_w]$ is higher than the mean packet delay for node 1 (seen as a loss with respect to node 1). In Fig. 3.12, we plot $E[D_w]$ for different combinations of the sensing probabilities, the curves show that if $p^{(1)}$ (or ($p^{(2)}$) is kept constant and $p^{(2)}$ (or $p^{(1)}$) is varied, $E[D_w]$ decreases as $p^{(2)}$ (or $p^{(1)}$) increases. We infer from Fig. 3.12 that there is an optimal combination of $p^{(1)}$ and $p^{(2)}$ at which $E[D_w]$ is minimum: set $p^{(2)}$ (or $p^{(1)}$) to unity and then vary $p^{(1)}$ (or $p^{(2)}$). The value of $p^{(1)}$ (or $p^{(2)}$) at which $E[D_w]$ is minimum corresponds to the optimum value of $p^{(1)}$ (or $p^{(2)}$). Fig. 3.13 shows a plot of the minimum weighted mean packet delay ($E[\hat{D}_w]$) versus the network input traffic. The minimum mean packet delay for a homogeneous system ($E[\hat{D}]$) is also included for comparison. We see that the inhomogeneity of users due to dissimilar channel sensing probabilities results in higher delays especially in the medium to heavy network traffic range.

Fig. 3.14 shows the curves of the weighted mean packet delay versus the channel sensing probability ($p^{(1)} = p^{(2)}$) when the network input traffic per slot is 40% ($M\sigma = 0.4$), which is then divided between the two nodes using five sharing ratios. Three observations are made from Fig. 3.14. First, the 10/90% traffic sharing ratio ($\sigma^{(1)} = 0.1M\sigma$, $\sigma^{(2)} = 0.9M\sigma$) gives the highest weighted mean packet

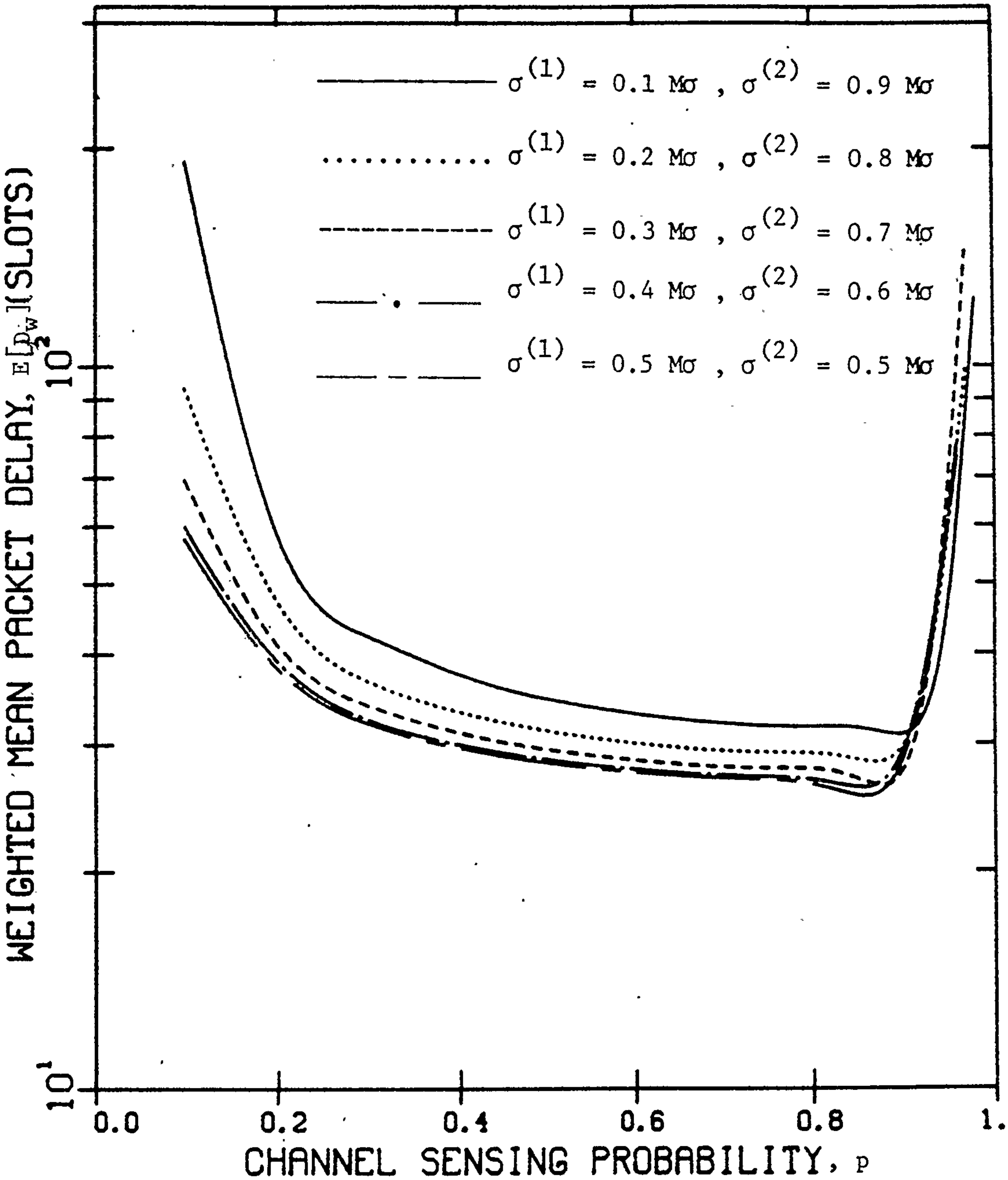


Fig. 3.14. Weighted mean packet delay versus Channel sensing probability (Total system load = 40%).

delay, this is because the weighted mean packet delay tends more towards the mean delay of packets transmitted by node 2 which carries 90% of the network input traffic. Second, as the traffic sharing ratio tends towards the same value, the weighted mean packet delay decreases and the smallest delay is obtained when the network load is shared equally between the two nodes. Third, for each load sharing ratio, the weighted mean packet delay, $E[D_w]$, decreases from a large value to a minimum value and then increases as the channel sensing probability ($p^{(1)} = p^{(2)} = p$) increases. The large value of $E[D_w]$ for small values of p is due to the congestion of the nodal buffer while at high values of p is caused by the congestion of the channel. Fig. 3.14 seems to indicate that at $M\sigma = 0.4$, $E[D_w]$ is minimum somewhere around $0.85 \le p \le 0.9$ for all the sharing ratios considered.

A final comment on the effect of the number of nodes in the network, M on all the performance characteristics shown in this chapter is now in order. A higher (lower) value of M than the one used will cause the characteristics to shift upwards (downwards); this is due to the random control of access to the channel.

3.4 Summary

In this chapter, we have demonstrated the usefulness of the decomposition approximation in analyzing (in a unified manner) the performance of CSMA-CD systems with nodes capable of buffering single-packet messages. The numerical results obtained from the approximate analysis closely match (to a large extent) those from computer simulations, this establishes the accuracy of the approximations introduced in analysis.

The main findings from the numerical examples are tradeoffs which exist between throughput and delay on the one hand and (for finite buffer size) between buffer overflow probability and mean packet delay on the other. It is also found that the performance results based on user homogeneity assumption tend to be overly optimistic especially in moderate to heavy traffic range. For an application of the results obtained to a real network, the author offers the following guidelines: choose the sensing probability to keep the throughput at an acceptable level and simultaneously maintain delay at a tolerable value; select a buffer size to achieve small number of rejected packets.

This chapter (and the previously reported studies in the literature) have addressed the problem of buffering single-packet messages. However, there are LAN applications, for example, host-to-host bulk data transfer and digitized voice transmission that require the buffering of multipacket messages. The remainder of this thesis is concerned with the study of the effects of buffering multipacket messages on CSMA-CD LANs. We begin in the next chapter by examining the performance of single-message buffer LANs with nodes capable of storing multipacket messages.

CHAPTER 4

SINGLE-MESSAGE BUFFER LANs WITH MULTIPACKET MESSAGES

4.1 Introduction and Problem Statement

The study of a LAN whose nodes can store multipacket messages is desirable not only for its practicality but because its analysis is mathematically challenging. Such a LAN is useful for bulk data transfer, for example, in file transfer applications (graphics, high/low resolution images, etc.) where each message is segmented into a random number of packets. In this chapter, we shall concern ourselves with a simplified form of such LANs in which only one message can be stored at each node (the more complex case where more than one message can be stored is studied in the next chapter).

The main problem of interest is the prediction of delays experienced by messages (packets). While message delay is a major performance measure from user's perspective, packet delay is also important for purposes of network dimensioning, for example, the appropriate choice of buffer size at each node depends in part on the tolerable packet delay. In seeking solutions to the above problem, two subproblems are identified. The first is the interfering queue problem. Although we already assume that only one message can be stored at each node, yet, the nodal buffers still interfere with each other (but to a lesser degree) through the contention for access to the channel as each message consists of a random number of packets. Interestingly, exact analysis of such a LAN is possible without recourse to our decomposition approximation technique. The second subproblem deals with the packet transmission strategy to be employed by a node which has gained channel access right. Recall from Chapter 3 that each node gaining channel access right transmits only one packet (even when there are more than one queued packet) and then releases the channel. We shall show that this strategy is not efficient for multipacket message transmission on a CSMA-CD channel. To this end, we propose two packet transmission strategies, namely, gated transmission (GT) and limited transmission (LT) strategies in order to achieve high channel utilization and minimize message (packet) delay [73-74].

To the best of the author's knowledge, there are only two studies on single-(multipacket)message buffer CSMA-CD LANs [25, 72]. Tobagi and Hunt [25] considered a multipacket message as a superpacket whose transmission is successful only if no other node begins transmission during the original transmission's collision window. In an attempt to obtain a more realistic performance result, Heyman [72] modeled the length of a multipacket message by a geometric distribution, but unfortunately, the geometric distribution is restrictive in the sense that it is completely characterized by its mean and thus neglects the effect of the variance of the message length. Hence, in this chapter we model the length of a multipacket message by a general probability distribution which is characterized by its mean and variance so that composite message lengths comprising real-time and non-real-time data can be appropriately modeled. In the following sections, we give a clear exposition of the solutions to the problems outlined above.

4.2 Model Formulation

We adopt the network description and the modeling assumptions of Chapter 2 with the following modifications: the propagation delay, τ seconds is now normalized by the packet transmission time, and is denoted by a time units; the channel time is assumed to be nonslotted and the buffer size K is assumed finite. We also introduce the following additional assumptions.

(i) The length of a message, G, is a random variable which has the probability distribution, $g_j = Pr\{G = j\}$ with mean $E[G]$ and variance σ_G^2. Message lengths are mutually independent and identically distributed.

(ii) Messages arriving at each node constitute a Poisson process with an effective mean arrival rate of $\lambda' = \lambda \sum_{j=1}^{K} g_j$ messages/unit time.

(iii) Upon the arrival of a new message, an empty node becomes nonempty and senses the channel immediately. This implies immediate first transmission (IFT) protocol. Because a node can store at most one message in its buffer, the generation of a new message is inhibited until all the packets of the current message are successfully transmitted.

(iv) If the length of a new message is greater than K, then the message will be rejected (blocked) in its entirety; this acceptance strategy is adopted in order to ensure the integrity of a message once it gets accepted.

(v) The transmission time of a packet is normalized to unity.

(vi) The collision resolution time is c time units where $a < c \ll 1$.

(vii) A collided packet is retransmitted after a random timeout interval which has an exponential distribution with mean $1/\kappa$ time units.

(viii) The message interarrival times, message lengths and retransmission delays are mutually independent random variables.

Based on the above assumptions, we present below the delay analysis of single-message buffer LANs incorporating either the GT or LT strategies.

4.3 Performance Analysis

In order to highlight briefly the need for a more efficient packet transmission strategy for multipacket message transmission, let us suppose that each node adopts the transmission strategy of Chapter 3 (henceforth referred to as the conventional transmission (CT) strategy). The CT strategy implies that there is a random access delay before the successful transmission commencement of every packet of a multipacket message. If we define message response time as the time between message arrival instant and transmission completion instant of the last packet of a message, the message response time will apparently become large due to the sum of the random access delays, along with a concomitant decrease in channel utilization. The author conjectures that these drawbacks can be alleviated by the GT and LT strategies which are described in the following.

4.3.1 The Gated Transmission Strategy

The motivation behind the GT strategy is to minimize the total random access delays incurred by a multipacket message. Under the GT strategy, a node that gains channel access right transmits all the packets present in its buffer at the instant of channel access continuously. Unlike the CT strategy which incurs random access delay for every packet in a message, the GT strategy incurs access delay just once (for the head of queue packet). However, the GT strategy seems to imply unfairness in that nodes with longer messages may hog the channel. We note that in the one-message buffer network under consideration, the unfairness effect is minimized if all the nodes have identical message length distribution and generate traffic with the same intensity.

4.3.1.1 Message Response Time Analysis [74]

In performing the message response time analysis, we shall use the standard embedded Markov chain technique together with the properties resulting from the theory of regenerative processes [42, 43].

By assumption (iii), let ϕ_i denote the steady-state probability that there are i messages, $i \in \{0, 1, 2, \cdots, M\}$, in the network. To obtain the mean message response time, $E[D_M]$, we shall invoke Little's law [55]:

$$E[D_M] = \frac{E[\tilde{N}]}{\lambda_{eff}} = \frac{\sum_{i=0}^{M} i\,\phi_i}{(M - \sum_{i=0}^{M} i\,\phi_i)\lambda}, \tag{4.1}$$

where λ_{eff} is the effective mean arrival rate of messages into the network and $E[\tilde{N}]$ is the mean number of messages in the network. The task of obtaining the mean message response time then reduces to the evaluation of the steady-state probabilities, ϕ_i. To do so, let $\tilde{N}(t)$ be the number of messages in the network at time t so that

$$\phi_i = \lim_{t \to \infty} Pr\{\tilde{N}(t) = i\} \;,\; i \in \{0, 1, 2, \cdots, M\} \tag{4.2}$$

provided the limit exists. However, the modeling and analysis of the continuous time process $\{\tilde{N}(t) \,;\, t \geq 0\}$ is difficult because of the task of observing the process states at all possible times t. To circumvent this difficulty, we shall define a new process which is observed only at specified points (embedded points) in time that coincide with the transmission completion epochs. Denote the new discrete-time process by $\{Z_n \,;\, n \geq 1\}$; where Z_n is the number of messages in the network immediately after the n^{th} transmission completion epoch. As the state of the process at the next epoch depends only on the state at the current epoch, the process $\{Z_n \,;\, n \geq 1\}$ evolves as a Markov chain whose analysis can easily be handled. We define the limiting probability that there are i messages in the network immediately after the n^{th} transmission (successful or unsuccessful) completion epoch by

$$f_i = \lim_{n \to \infty} Pr\{Z_n = i\} \ , \ i \in \{0, 1, 2, \cdots, M\} \tag{4.3}$$

which are determined from the knowledge of the transition probabilities, f_{ij}, between two successive transmission completion epochs where

$$f_{ij} = Pr\{Z_{n+1} = j \mid Z_n = i\} \ , \ i, j \in \{0, 1, 2, \cdots, M\} \, . \tag{4.4}$$

Now, to determine the transition probabilities, it is necessary to examine in detail the possible states of each node in the network. A node can be either idle (buffer empty) or active (buffer nonempty). Furthermore, an active node can be in the active-transmit or active-wait substate. An active node is in the active-transmit substate if it is currently transmitting a packet and in the active-wait substate if it has no transmission in progress. A node in the idle state changes to the active-transmit (active-wait) substate if a new message arrives during the channel idle (busy) period. Conversely, a node in the active-transmit substate changes to the idle (active-wait) state at the end of a successful (unsuccessful) transmission. From (4.4), notice that in order to reach the $(n + 1)^{th}$ transmission completion epoch, at least one of the nodes must initiate transmission after the n^{th} transmission completion epoch. The node(s) initiating transmission(s) may be any of the i nodes in the active-wait state at the n^{th} transmission completion epoch or any of the $(M - i)$ idle nodes which have just changed to active-transmit state. Further, by Markov chain theory, the number of messages in the network immediately after the $(n + 1)^{th}$ transmission completion epoch, j, depends only on the number of messages in the network immediately after the n^{th} transmission completion epoch, i, and any activity during the interval $(n, n+1]$, viz., successful (or unsuccessful) transmission and message arrivals (or no arrivals) to idle nodes. Note that by the gated transmission strategy adopted, the duration of a successful transmission period is k time units if there are k packets, $k \in \{1, 2, \cdots, K\}$, in the buffer at the instant of channel access. From the above explanation, the expressions for f_{ij} can be written concisely as a sum of the probabilities of four mutually exclusive events - $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ and $\tilde{P}_4$ where

$$\tilde{P}_1 = Pr\{\text{transmission commencement by an idle node with success, } Z_{n+1} = j \mid Z_n = i\}$$

$$\tilde{P}_2 = Pr\{\text{transmission commencement by an idle node without success, } Z_{n+1} = j \mid Z_n = i\}$$

$$\tilde{P}_3 = Pr\{\text{transmission commencement by an active-wait node with success, } Z_{n+1} = j \mid Z_n = i\}$$

$$\tilde{P}_4 = Pr\{\text{transmission commencement by an active-wait node without success, } Z_{n+1} = j \mid Z_n = i\}$$

whose explicit expressions are given respectively as

$$\tilde{P}_1 = \sigma_i \sum_{k=1}^{K} \varepsilon(j-i, i, k)\delta_k \ , \ 0 \le i \le M, 0 \le j \le M-1 \tag{4.5a}$$

$$\tilde{P}_2 = \sigma_i \ \varepsilon'(j-i-1, i, c) \ , \ 0 \le i, j \le M \tag{4.5b}$$

$$\tilde{P}_3 = (1-\sigma_i) \sum_{k=1}^{K} \varepsilon(j-i+1, i-1, k)\delta_k \ , \ 1 \le i \le M, 1 \le j \le M-1$$

(4.5c)

$$\tilde{P}_4 = (1-\sigma_i) \varepsilon'(j-i, i-1, c) \ , \ 1 \le i, j \le M \ . \tag{4.5d}$$

From (4.5), given there are i active-wait nodes in the network, σ_i is the probability of a transmission commencement by an idle node before any of the i active-wait nodes. Given that there are y active-wait nodes in the network, $\varepsilon(x, y, k)$ is the probability of successful transmission and that x idle nodes (out of $M-y-1$) change to the active state during a successful transmission period of duration k time units. $\varepsilon'(x, y, c)$ is the probability of unsuccessful transmission and that x idle nodes change to the active state during a collision resolution period of duration c time units. δ_k is the probability that there are k packets in a nodal buffer at a (successful) transmission commencement instant given that the buffer is nonempty. We derive the expressions for σ_i, $\varepsilon(x, y, k)$, $\varepsilon'(x, y, c)$ and δ_k as follows.

(i) $\varepsilon(x, y, k)$: is expressed as a product of three factors,

$$\varepsilon(x, y, k) = \beta(x, k)\, \gamma(y)\, \alpha(x, y+1, k) \ . \tag{4.6}$$

Clearly, a transmission by a node (say node m) is successful if none of the other $(M-1)$ nodes in the network initiate transmission during the collision window (of duration a time units) of node m's transmission period. Let us suppose that the duration of node m's transmission (success) is k time units. Then, $\beta(x, k)$ is the probability that there is no idle to active-transmit transition in the network during the collision window given that x nodes change from idle state to active-wait state during k time units. Since an idle node changes to the active state via the arrival of a message (assumed Poisson), $\beta(x, k)$ is obtained as

$$\beta(x, k) = \left[\frac{e^{-\lambda' a} - e^{-\lambda' k}}{1 - e^{-\lambda' k}} \right]^x . \tag{4.7}$$

Also, given that there are y nodes in the active-wait state when node m begins transmission, $\gamma(y)$ is the probability that none of the y nodes reattempt transmission during the collision window,

$$\gamma(y) = [e^{-\kappa a}]^y , \quad y \in \{0, 1, \cdots, M - 1\} . \tag{4.8}$$

Finally, we derive the expression for $\alpha(x, y + 1, k)$ by the following reasoning. Since there are y nodes in the active-wait state when node m begins transmission, then in all there are $y + 1 (= y')$ active nodes in the network so that $\alpha(x, y + 1, k)$ is just the binomial probability that x idle nodes (out of $M-y-1$) change to the active state during a successful transmission period of duration k time units,

$$\alpha(x, y', k) = \binom{M - y'}{x} \left[1 - e^{-\lambda' k}\right]^x \left[e^{-\lambda' k}\right]^{M-y'-x}, \quad x \in \{0, 1, \cdots, M - y'\}, \tag{4.9}$$

$$y' \in \{0, 1, \cdots, M - 1\} .$$

(ii) $\varepsilon'(x, y, c)$: follows from the definition of $\varepsilon(.,.,.)$

$$\varepsilon'(x, y, c) = [1 - \beta(x, c)\,\gamma(y)]\,\alpha(x, y + 1, c) \tag{4.10}$$

where $\beta(x, c)$, $\gamma(y)$ and $\alpha(x, y + 1, c)$ are given by (4.7), (4.8) and (4.9) respectively with k replaced by c, the collision resolution period. Note that $\varepsilon(x, y, k)$ (also $\varepsilon'(x, y, c)$) is zero if $x < 0$ or $y < 0$ or if x is greater than the number of idle nodes in the network when a node begins transmission.

(iii) σ_i: By the Poisson message arrival process assumption, the message interarrival time at each idle node is exponentially distributed with mean $1/\lambda'$ time units. Also by assumption (*vii*), the retransmission delay for each node in the active-wait state is also exponentially distributed with mean $1/\kappa$ time units. Then, given that i nodes are in the active-wait state immediately after a transmission completion epoch, σ_i is written as

$$\sigma_i = \frac{(M-i)\lambda}{i\kappa + (M-i)\lambda} \tag{4.11}$$

a consequence of the probability that one exponential random variable is smaller than another.

(iv) δ_k: From assumption (*iii*), the presence of k packets in a nodal buffer at the instant of successful transmission commencement simply implies that the message length consists of k packets. Hence,

$$\delta_k = \frac{g_k}{\sum_{l=1}^{K} g_l} . \tag{4.12}$$

We now can write the expressions for the transition probabilities, f_{ij} in a compact form as

$$f_{0j} = \sum_{k=1}^{K} \varepsilon(j, 0, k)\delta_k + \varepsilon'(j-1, 0, c) , \quad 0 \le j \le M \tag{4.13a}$$

$$f_{ij} = \begin{cases} \sigma_i [\sum_{k=1}^{K} \varepsilon(j-i, i, k)\, \delta_k + \varepsilon'(j-i-1, i, c)] \\ + (1-\sigma_i)[\sum_{k=1}^{K} \varepsilon(j-i+1, i-1, k)\, \delta_k + \varepsilon'(j-i, i-1, c)], \\ \qquad\qquad 1 \le i \le M,\, i-1 \le j \le M \\ 0 \ ,\ 1 \le i \le M,\, j < i-1 \ . \end{cases} \tag{4.13b}$$

Note that $f_{ij} = 0$ for $j < i-1$, $i \ge 1$ because at most one message can be successfully transmitted on the channel. Using (4.13), the limiting probabilities $\{f_i\}_{i=0}^{M}$, which are valid at all transmission completion epochs are calculated by the numerical technique of direct forward recursion [75]. In order order to identify those epochs which coincide with the successful transmission completion epochs, we can define a subset of the transition probabilities, f_{ij} as follows

$$Pr\{\Xi_{n+1},\ Z_{n+1} = j \mid Z_n = i\} = \tilde{P}_1 + \tilde{P}_3 \ ,\ 0 \le i \le M, 0 \le j \le M-1 \tag{4.14}$$

where Ξ_n denotes the event that a message is transmitted successfully at the n^{th} transmission completion epoch. Assuming homogeneity, denote the conditional probability $Pr\{\Xi_{n+1},\ Z_{n+1} = j \mid Z_n = i\}$ by d_{ij}, then under steady-state condition, the probability that j messages are in the system immediately after a successful transmission completion epoch, Φ_j is expressed by

$$\Phi_j = \sum_{i=0}^{M} d_{ij}\, f_i \ ,\ 0 \le j \le M-1 \tag{4.15}$$

and since $d_{ij} = 0$ for $j < i - 1$, we obtain

$$\Phi_j = \sum_{i=0}^{j+1} d_{ij} f_i \quad , \; 0 \le j \le M - 1 \,. \tag{4.16}$$

In (4.16), the expressions for the probabilities d_{ij} follow from (4.13) as

$$d_{0j} = \sum_{k=1}^{K} \varepsilon(j, 0, k)\delta_k \quad , \; 0 \le j \le M - 1 \tag{4.17a}$$

$$d_{ij} = \begin{cases} \sigma_i[\sum_{k=1}^{K} \varepsilon(j - i, i, k)\delta_k] + (1 - \sigma_i)[\sum_{k=1}^{K} \varepsilon(j - i + 1, i - 1, k)\delta_k], \\ \qquad\qquad 1 \le i \le M, i - 1 \le j \le M - 1 \\ 0 \;,\; 1 \le i \le M \;,\; j < i - 1 \end{cases} \tag{4.17b}$$

$$d_{iM} = 0 \;,\; 0 \le i \le M \,. \tag{4.17c}$$

where $d_{iM} = 0$ because of Assumption (*iii*).

Having found the Φ_j's, the final task is to determine the ϕ_j's which, as defined previously are valid at all points in time. It is a known fact from queueing theory that the distributions Φ_j's and ϕ_j's are not, in general, equal. Equality holds if and only if the arrival process is Poisson and arrivals and departures occur in unit steps [47]. For the network under study, we know that successfully transmitted messages depart one at a time but message arrivals to the network do not necessarily occur singly because of the possibility of simultaneous arrivals at different nodes. Thus, to relate the Φ_j's to the ϕ_j's, we shall appeal to the conservation theorem which states that under steady-state condition [38]

$$(M - j)\lambda' \phi_j = \zeta\Phi_j \;,\; 0 \le j \le M - 1 \tag{4.18}$$

where ζ is the mean departure rate of message from the network. Equation (4.18) is certainly intuitive, since, in the long-run, the effective mean arrival rate of message into the network must equal the mean departure rate of message from the network. The left hand side follows because under steady-state condition, if there are j messages in the network (with probability ϕ_j), then $(M - j)$ nodes are idle and each has a mean message generation rate of λ'. We see that the linear equations (M in number) defined by (4.18) contain $(M + 1)$ unknowns, namely, ϕ_0, ϕ_1, $\cdots$, ϕ_{M-1} and ζ. Hence, the first step in the solution of (4.18) is to determine ϕ_0 using regenerative arguments, which is applicable because the process $\{N(t), t \geq 0\}$ regenerates whenever $N(t)$ takes the value 0. If we say that a cycle is completed every time the process returns to state 0, then by the theorem of regenerative processes [42],

$$\phi_0 = \lim_{t \to \infty} Pr\{N(t) = 0\} = \frac{E[L_0]}{E[L_c]} \tag{4.19}$$

where L_c and L_0 denote respectively the length of a cycle and the amount of time spent in state 0 during a cycle. L_0 has a mean of

$$E[L_0] = \frac{1}{M\lambda'} \tag{4.20}$$

since when $N(t) = 0$ (or $Z_n = 0$), there are M idle nodes in the network and each generates a new message (of maximum length K packets) in accordance with a Poisson process having a mean arrival rate λ'. To compute the mean length of a cycle, $E[L_c]$ we proceed as follows. If we let Ω_m denote the mean time spent in state m during a cycle and if n_m denotes the mean number of times state m is visited (recall that state m corresponds to the number of messages in the network immediately after a transmission completion epoch) then the mean length of a cycle is expressed as

$$E[L_c] = \sum_{m=0}^{M} n_m \, \Omega_m \, . \tag{4.21}$$

By Markov chain theory and the strong law for renewal processes, n_m is found to be [42]

$$n_m = \frac{f_m}{f_0} \, . \tag{4.22}$$

Ω_m in (4.21) is a sum of two components - w_m and v_m, where w_m is the mean channel idle time between a transmission completion epoch (at which there are m messages in the network) and the next transmission commencement instant, and v_m is the mean successful transmission time of a message. Given that m nodes are in the active-wait state at a transmission completion epoch, the time until one active-wait node begins a retransmission or one idle node (out of $(M - m)$ idle nodes) begins a transmission is exponentially distributed with mean $1/[m\kappa + (M - m)\lambda']$. This follows from the exponential retransmission delay and Poisson assumptions. w_m is then given by

$$w_m = \frac{1}{m\kappa + (M - m)\lambda'} \tag{4.23}$$

and v_m is expressed by

$$
\begin{aligned}
v_m = {} & \sigma_m[(\sum_{k=1}^{K}(k+a)\, e_k')\varepsilon(0, m, a) + (a+c)(1-\varepsilon(0, m, a))] + \\
& (1-\sigma_m)[(\sum_{k=1}^{K}(k+a)e_k')\varepsilon(0, m-1, a) + (a+c)(1-\varepsilon(0, m-1, a))]
\end{aligned}
\tag{4.24}
$$

where the σ_m term is zero when $m = M$ and the $(1-\sigma_m)$ term vanishes when $m = 0$. e_k' is the probability of successful transmission of a message (of length k packets) given that at most K packets of one message can be successfully transmitted consecutively when a node gains channel access right

$$
e_k' = \frac{e_k}{\sum_{l=1}^{K} e_l}, \quad 1 \le k \le K \tag{4.25}
$$

where $e_k = \sum_{j=0}^{M}\left[\sum_{i=0}^{M} d_{ij}^{(k)} f_i\right]$. From (4.17) we can write $d_{ij}^{(k)}$ as

$$
d_{0j}^{(k)} = \varepsilon(j, 0, k)\delta_k \quad , \quad 0 \le j \le M-1 \tag{4.26a}
$$

$$
d_{ij}^{(k)} = \begin{cases} \sigma_i[\varepsilon(j-i, i, k)\,\delta_k] + (1-\sigma_i)[\varepsilon(j-i+1, i-1, k)\,\delta_k], \\ \qquad 1 \le i \le M \; , \; i-1 \le j \le M-1 \\ 0 \; , \; 1 \le i \le M \; , \; j < i-1 \end{cases} \tag{4.26b}
$$

$$
d_{iM}^{(k)} = 0 \; , \; 0 \le i \le M . \tag{4.26c}
$$

Substituting the expressions for $E[L_0]$ and $E[L_c]$ into (4.19) gives

$$
\phi_0 = \frac{f_0}{M\lambda' \sum_{m=0}^{M} f_m \, \Omega_m} . \tag{4.27}
$$

Having determined ϕ_0, we then can solve for the remaining M unknowns using the

M linear equations (4.18) and subsequently, we can compute $E[D_M]$ using (4.1).

4.3.1.2 Analysis of Message and Packet Access Delays

The message access delay, W_M, is defined as the time between a message arrival instant and the successful transmission commencement instant of the last packet in the message. This definition is reasonable because a message is assumed to be delayed so long as at least one of its packets is still in the buffer. The packet access delay, W_P, is defined as the time between a message arrival instant and successful transmission commencement instant of an arbitrary packet. We shall derive expressions for the mean message and packet access delays as well as their interrelationship.

The problem of comparing batch (message) and customer (packet) delays has been studied before by Halfin [76] and Whitt [77] in the context of queueing systems employing impartial service disciplines. Halfin [76] showed that for a large class of queueing systems where customers arrive in batches, the delay distribution of the last customer in a batch to enter service equals the delay distribution of an arbitrary customer if the batch size has a geometric distribution. Using the notion of stochastic ordering, Whitt [77] provided bounds between the mean delay of an arbitrary customer and that of the last customer in a batch to enter service when the batch size distributions are new better or worse than used in expectation, important concepts in reliability theory [78]. In addition, [77] gave an approximate relationship between the two expected delays, its application requires the knowledge of the mean delay of the first customer in a batch and the first two moments of the specified batch size distribution. Unfortunately, direct application of Whitt's relationship to LANs employing contention-resolving protocols such as the CSMA-CD seems impossible because of the formidable task of determining the mean delay of the first packet of a message. Hence, in the analysis presented below, we use a different approach which reverses the procedure of first determining the mean packet access delay and then the mean message access delay (as described in [76-77]). Specifically, the mean message delay is first obtained from the previously found message response time and then the mean packet access delay is determined. The mean message access delay (or the mean access delay of the last packet of a message), $E[W_M]$, is equal to the mean message response time $E[D_M]$ minus one packet transmission time (a consequence of the GT strategy)

$$E[W_M] = E[D_M] - 1 \tag{4.28}$$

where $E[D_M]$ is given by (4.1). Also, due to the GT strategy, the mean access delay of an arbitrary packet which is the k^{th} to be transmitted in a message is expressed by

$$E[W_P] = E[W_1] + \sum_{k=1}^{K} (k-1)\iota_k \tag{4.29}$$

where $E[W_1]$ is the mean delay of the first packet of a message and ι_k is the probability that an arbitrary packet is the k^{th} to be transmitted in a message where [79]

$$\iota_k = \frac{\sum_{i=k}^{K} g_i'}{E[G']} . \tag{4.30}$$

g_i' $(= g_i / \sum_{j=1}^{K} g_j,\ 1 \le i \le K)$ is the probability distribution of unblocked message lengths with mean $E[G']$. Similarly, we can write the mean access delay of the last packet of a message as

$$E[W_M] = E[W_1] + \sum_{k=1}^{K} (k-1) g_k' . \tag{4.31}$$

Note that in (4.29) and (4.31), $E[W_1]$ is still unknown. By combining (4.28) and (4.31), $E[W_1]$ is obtained as

$$E[W_1] = E[D_M] - 1 - \sum_{k=1}^{K} (k-1) g_k' . \tag{4.32}$$

Substitution of (4.32) into (4.29), and then using the values of g_k' and ι_k gives, after simplification

$$E[W_P] = (E[D_M] - 1) - (E[G'] - 1) + \left[\frac{\sigma^2_{G'} + (E[G'])^2}{2E[G']} - 0.5 \right] \tag{4.33}$$

where $\sigma^2_{G'}$ is the variance of the lengths of unblocked messages. The relationship between the message and access delays then follows from (4.33) as

$$E[W_P] = E[W_M] - (E[G'] - 1) + \left[\frac{\sigma^2_{G'} + (E[G'])^2]}{2E[G']} - 0.5 \right] . \tag{4.34}$$

4.3.2 The Limited Transmission Strategy

Instead of transmitting all the packets of a message when a user gains channel access right, the LT strategy requires the transmission of up to a maximum of u packets where $u > 1$ and is henceforth referred to as a minimessage. In a real network, the value of u is preselected on the basis of the expected network traffic and is assumed to be identical for all the nodes in the network. In this way, the unfairness effect of the GT strategy is minimized, though at the expense of increased access delay. Intuitively, the LT strategy is expected to show an improved performance when compared to the CT.

4.3.2.1 Message Response Time Analysis

The analysis proceeds in the same way as that of Section 4.3.1.1 except for some modifications. First note that the duration of a successful transmission period (neglecting propagation delay) is $min(u,Q)$, where Q is the queue length at a successful transmission commencement instant. Thus, for the LT strategy, transmission completion epoch is defined as

$$\textit{Transmission Completion Epoch} = \begin{cases} \textit{End of an unsuccessful transmission} \\ \qquad\qquad\qquad \textit{or} \\ \textit{End of a successful transmission of } j \\ \qquad \textit{packet}(s), \qquad\qquad 1 \le j \le u\ . \end{cases}$$

The components of the transition probabilities between two successive embedded epochs are given by (4.5) with the following modification. Since the duration of a successful transmission depends on u and the queue length at a successful transmission commencement instant, it is logical to break the expression for $\tilde{P}_1$ (and also $\tilde{P}_3$) into two parts: (i) when the queue length is less than or equal to u, and (ii) when the queue length is greater than u. Expressed mathematically

$$\tilde{P}_1 = \sigma_i[\sum_{k=1}^{u} \varepsilon(j-i, i, k)\delta_k + \varepsilon(j-i-1, i, u)(1-\sum_{k=1}^{u}\delta_k)],$$
$$0 \le i \le M, \; 0 \le j \le M-1 \tag{4.35a}$$

and

$$\tilde{P}_3 = (1-\sigma_i)[\sum_{k=1}^{u} \varepsilon(j-i+1, i-1, k)\delta_k + \varepsilon(j-i, i-1, u)(1-\sum_{k=1}^{u}\delta_k)],$$
$$1 \le i \le M, \; 1 \le j \le M-1 . \tag{4.35b}$$

The f_{ij}'s now become

$$f_{0j} = \sum_{k=1}^{u} \varepsilon(j, 0, k)\delta_k + \varepsilon(j-1, 0, u)(1-\sum_{k=1}^{u}\delta_k) + \varepsilon'(j-1, 0, c),$$
$$0 \le j \le M \tag{4.36a}$$

$$f_{ij} = \begin{cases} \sigma_i[\sum_{k=1}^{u} \varepsilon(j-i, i, k)\,\delta_k + \varepsilon(j-i-1, i, u)(1-\sum_{k=1}^{u}\delta_k) \\ \qquad + \varepsilon'(j-i-1, i, c)] \\ \qquad + \\ (1-\sigma_i)[\sum_{k=1}^{u} \varepsilon(j-i+1, i-1, k)\,\delta_k + \varepsilon(j-i, i-1, u)(1-\sum_{k=1}^{u}\delta_k) \\ \qquad + \varepsilon'(j-i, i, -1, c)] , \quad 1 \le i \le M, i-1 \le j \le M \\ 0 , \quad 1 \le i \le M, j < i-1 . \end{cases} \tag{4.36b}$$

and the f_i's are then determined as before using the direct forward recursion technique. Recall from (4.12) that δ_k was defined in terms of the message length, however, such a definition no longer applies in (4.36) because, at an instant a node gains channel access, its queue length is not necessarily equal to the initial message length, since part of the message might have been successfully transmitted prior to the current channel access. Hence, for the LT strategy, δ_k is redefined in terms of the queue length at a successful transmission commencement instant. To this end, let Q_n denote a representative user queue length immediately after the n^{th} transmission completion epoch. We assume that the process $\{Q_n; n \geq 1\}$ evolves as a discrete-time Markov chain whose limiting buffer state probabilities are defined by

$$\pi_m = \lim_{n \to \infty} Pr\{Q_n = m\} \quad , \quad m = 1, 2, \cdots, K \; . \tag{4.37}$$

Then, δ_k is now given by

$$\delta_k = \frac{\pi_k}{1 - \pi_0} \quad , \quad k = 1, 2, \cdots, u \tag{4.38}$$

where the π_k's remain to be determined. Note however that the π_k's depend implicitly on the value of u, thus as an illustration, if $K = 4$, the state transition diagrams for $u = 1, 2, 3$ and 4 are shown in Fig. 4.1.

The Markov chain $\{Q_n; n \geq 1\}$ has a finite state space and is irreducible, aperiodic and positive recurrent. Hence, the stationary probability distribution for queue length at the embedded epochs exists and is obtained by solving numerically the standard invariant equation for Markov chains [47], where the queue state

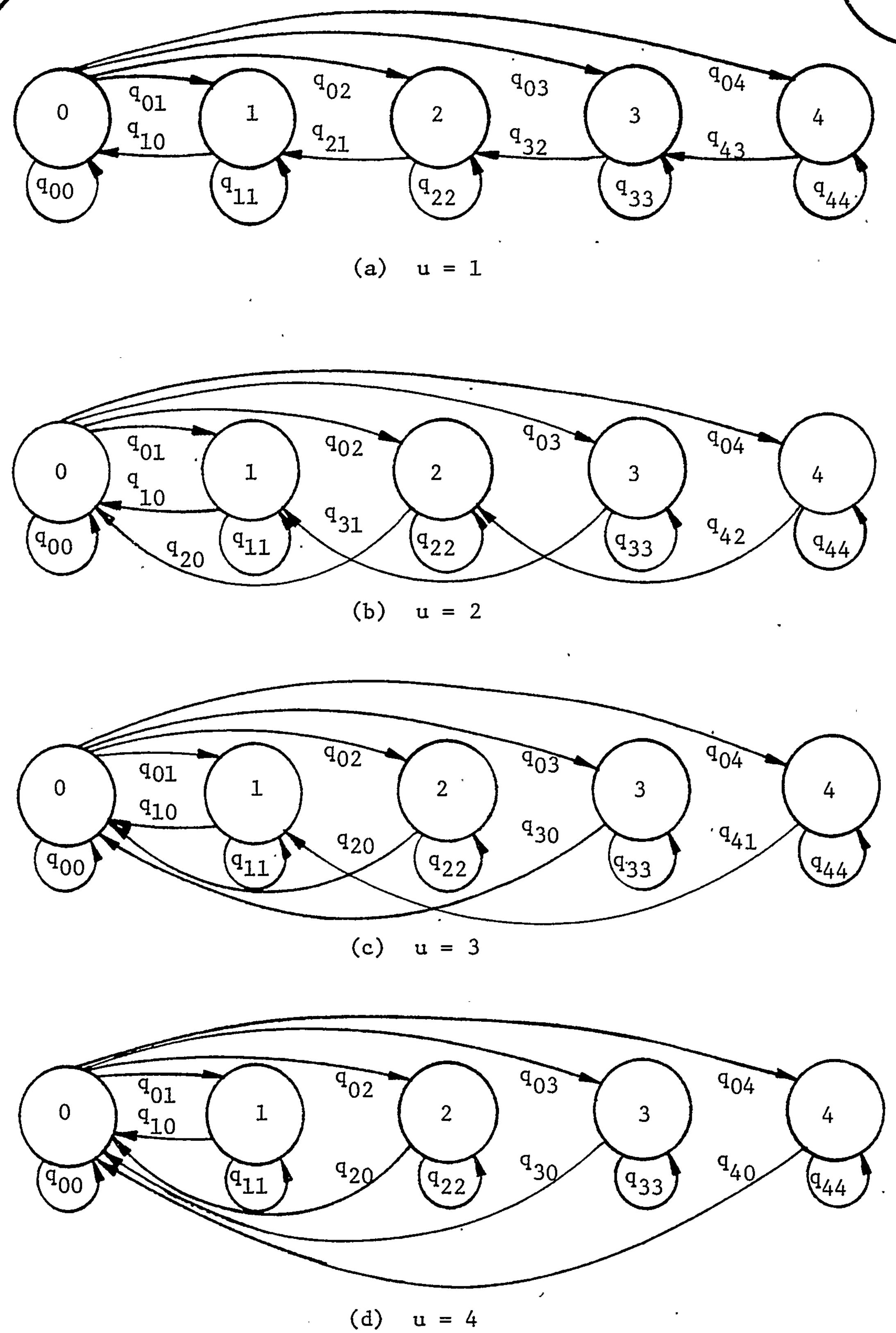


Fig. 4.1. Illustrative state transition diagram when K = 4 and u = 1, 2, 3 and 4.

transition probabilities are given by

$$
\begin{aligned}
q_{0j} &= \begin{cases} \Gamma g_j(1-\gamma_m) + g_{j+u}\gamma_m \ , & 1 \le j \le K-u \\ \\ \Gamma g_j(1-\gamma_m) \ , & K-u < j \le K \end{cases} \\
q_{00} &= \left[1 - \Gamma \sum_{k=1}^{K} g_k\right] + \Gamma\gamma_m \sum_{k=1}^{u} g_k \\
q_{i0} &= \gamma_m \ , \quad 1 \le i \le u \\
q_{i,i-u} &= \gamma_m \ , \quad u < i \le K \\
q_{ii} &= 1-\gamma_m \ , \quad 1 \le i \le K \\
q_{ij} &= 0 \ , \quad otherwise \ .
\end{aligned}
\tag{4.39}
$$

In (4.39), γ_m is the limiting probability of successful transmission of a minimessage and Γ is the probability that an idle node generates a new message (of length $\le K$ packets). To determine γ_m, let Ξ_n' denote the event that a minimessage (of length k, $k \in \{1, 2, \cdots, u\}$ packets) is transmitted successfully at the n^{th} transmission completion epoch. Assuming homogeneity, denote the conditional probability $Pr\{\Xi_{n+1}', Z_{n+1} = j \mid Z_n = i\}$ by s_{ij}, then under steady-state condition, the probability that j messages are in the system immediately after a successful transmission completion epoch, s_j is given by

$$s_j = \sum_{i=0}^{M} s_{ij} f_i \ , \quad 0 \le j \le M \tag{4.40}$$

so that γ_m is

$$\gamma_m = \sum_{j=0}^{M} s_j \, . \tag{4.41}$$

In (4.40), the expression for s_{ij} is written from (4.36) as

$$s_{0j} = \sum_{k=1}^{u} \varepsilon(j, 0, k)\delta_k + \varepsilon(j-1, 0, u)(1 - \sum_{k=1}^{u} \delta_k) \; , \; 0 \le j \le M \tag{4.42a}$$

$$s_{ij} = \begin{cases} \sigma_i [\sum_{k=1}^{u} \varepsilon(j-i, i, k)\delta_k + \varepsilon(j-i-1, i, u)(1 - \sum_{k=1}^{u} \delta_k)] \; + \\ (1-\sigma_i)[\sum_{k=1}^{u} \varepsilon(j-i+1, i-1, k)\delta_k + \varepsilon(j-i, i-1, u)(1 - \sum_{k=1}^{u} \delta_k)] \, , \\ \qquad\qquad 1 \le i \le M \; , i-1 \le j \le M \\ 0 \; , \quad 1 \le i \le M \; , j < i-1 \, . \end{cases} \tag{4.42b}$$

Finally, the expression for Γ in (4.39) is obtained as

$$\Gamma = \sum_{i=0}^{M} \sigma_i f_i \, . \tag{4.43}$$

where the σ_i's are given by (4.11). An examination of (4.36) and (4.39) demonstrates the interdependence between the Q-process and the Z-process: on the one hand, the f_{ij}'s are expressed in terms of the π_i's via δ_k; and on the other hand, the q_{ij}'s are expressed in terms of the f_i's via γ_m. This interdependence is analogous to that seen before between the user and channel Markov chains in Chapter 3. Hence, in the the spirit of principle 5 of the decomposition approximation (Chapter 2), the limiting probabilities $\{f_i\}$ and $\{\pi_i\}$ are determined iteratively.

The remainder of the analysis follows exactly as in Section 4.3.1.1. However, note that in computing the Φ_j's (4.16), the upper limit of summation in (4.17) is replaced by u. Similarly, in (4.24) the upper limit of summation is replaced by u and furthermore, e_k' is now redefined as $e_k' = e_k/\gamma_m$ (note that γ_m is also equal to $\sum_{l=1}^{u} e_l$) where $e_k = \sum_{j=0}^{M}\left[\sum_{i=0}^{M} s_{ij}^{(k)} f_i\right]$. Now, for $k = 1, 2, \cdots, u-1$, $s_{ij}^{(k)}$ is given by

$$s_{0j}^{(k)} = \varepsilon(j, 0, k)\delta_k \quad , \qquad 0 \le j \le M \tag{4.44a}$$

$$s_{ij}^{(k)} = \begin{cases} \sigma_i[\varepsilon(j-i, i, k)\delta_k] + (1-\sigma_i)[\varepsilon(j-i+1, i-1, k)\delta_k] \ , \\ \qquad 1 \le i \le M, \ i-1 \le j \le M \\ 0 \ , \qquad 1 \le i \le M, \ j < i-1 \end{cases} \tag{4.44b}$$

and for $k = u$ we have

$$s_{0j}^{(u)} = \varepsilon(j-1, 0, u)\delta_u + \varepsilon(j, 0, u)(1 - \sum_{k=1}^{u} \delta_k) \quad , \qquad 0 \le j \le M \tag{4.45a}$$

$$s_{ij}^{(u)} = \begin{cases} \sigma_i[\varepsilon(j-i, i, u)\delta_u + \varepsilon(j-i-1, i, u)(1 - \sum_{k=1}^{u} \delta_k)] + \\ (1-\sigma_i)[\varepsilon(j-i+1, i-1, u)\delta_u + \varepsilon(j-i, i, u)(1 - \sum_{k=1}^{u} \delta_k)] \ , \\ \qquad 1 \le i \le M, \ i-1 \le j \le M \\ 0 \ , \qquad 1 \le i \le M, \ j < i-1 \ . \end{cases} \tag{4.45b}$$

Remarks:

1) In the expression for the transition probabilities s_{ij} (4.42), the number of packets that can be transmitted continuously at every channel access is u, however, this is not exactly the case at all times. In fact, the upper bound should be $min(u,Q)$,

where Q is the instantaneous value of the queue length at the instant of channel access by a node. Unfortunately, $min(u,Q)$ is not directly implementable in computation due to lack of prior knowledge of Q. Hence, the approximate upper bound that is used during the computation is $\lfloor E[G'] \rfloor$, the nearest integral value not greater than $E[G']$; note that beyond this value the result predicted by our analysis is inaccurate.

2) Instead of having all the nodes adopt the same strategy, it is possible to divide the nodes into two groups where M_G nodes adopt the GT strategy while M_L nodes employ the LT strategy, we then have what we call the mixed (GT-LT) strategy. Such a scenario has practical application, for example, in integrated services local area networks (ISLANs), which are designed for carrying voice and data traffic over the same channel by packetized transmission [80-83]. In such a network, each voice node employs the GT strategy while each data node uses the LT strategy. It is argued in the following section that the maximum allowable random component of the voice packet delay is minimum if the GT strategy is adopted by the voice nodes.

4.3.3 Analysis of Voice Packet Delay in ISLANs

The voice packet delay, D_v, is a sum of the packetization time, T_p, the channel access delay, D_a, and the packet transmission time, T_r, that is

$$D_v = T_p + D_a + T_r \, . \tag{4.46}$$

$T_p = B_v/V_d$ and $T_r = (B_v+B_o)/C$, where B_v (B_o) is the packet data (overhead) length in *bits*, V_d is the vocoder rate in *bits/sec* and C is the channel capacity in *bits/sec*. D_a is the random access delay which depends on the number of collisions experienced by a packet and the backoff algorithm employed. A requirement for voice traffic is the delivery of a voice packet before the maximum allowable value of D_a (denoted $D_{a_{\max}}$) elapses. For continuity of speech, $D_{a_{\max}}$ must be bounded, a way of achieving this is by choosing a finite buffer size. But, at first thought, a consequence of finite buffer is the rejection of voice packets which implies discontinuity of speech. Hence, for voice applications, we shall impose a condition that no new packet in a talkspurt is lost due to the buffer being full, a necessary condition to ascertain continuity of speech. We implement this condition by discarding the oldest (head of queue) packet when a new packet arrives into a full buffer since at that instant, the head of queue packet has been delayed up to $D_{a_{\max}}$ anyway. Given a packet transmission strategy, the above explanation enables us to express $D_{a_{\max}}$ in terms of T_p, T_r, and K. For the CT strategy,

$$D_{a_{\max}} = KT_p - T_r \tag{4.47a}$$

for the GT strategy,

$$D_{a_{\max}} = K(T_p - T_r) \tag{4.47b}$$

and for the LT strategy,

$$D_{a_{\max}} = KT_p - uT_r \; . \tag{4.47c}$$

It is obvious from (4.47) that $D_{a_{\max}}$ is least for the GT strategy; notice also that

(4.47a) and (4.47b) are special cases of (4.47c).

4.3.4 Performance Comparison of the Transmission Strategies

We present below numerical results obtained from the analyses of the preceding sections. In addition, simulation results are also presented to assess the accuracy of the analytic results. The following input parameters are selected:

Number of nodes in the network, M: 50.

Normalized propagation delay, a: 0.01 *slots*.

Collision resolution period, c: 0.02 *slots*.

Mean packet retransmission rate, κ: 0.5.

Buffer Size, K: 10.

Mean message length, $E[G]$: 5 *packets*.

For Fig. 4.2 only, message lengths are geometrically distributed (but other distributions can as well be used). From the specified values of K and $E[G]$, the mean length of unblocked messages, $E[G']$ is calculated to be 3.8.

Fig. 4.2 is a plot of the mean message response time versus the throughput for the conventional ($u = 1$), limited ($u = 2, 3$) and gated transmission strategies. The throughput is defined (nonrigorously) as the rate of successful transmission of messages, by the steady-state operation assumption, this implies that effective message throughputs are effective message arrival rates, λ_{eff}, so that the normalized effective throughput (or simply throughput) is given by $\lambda_{eff} E[G']$. Note the excellent agreement between the simulation and analytic results. We see that the

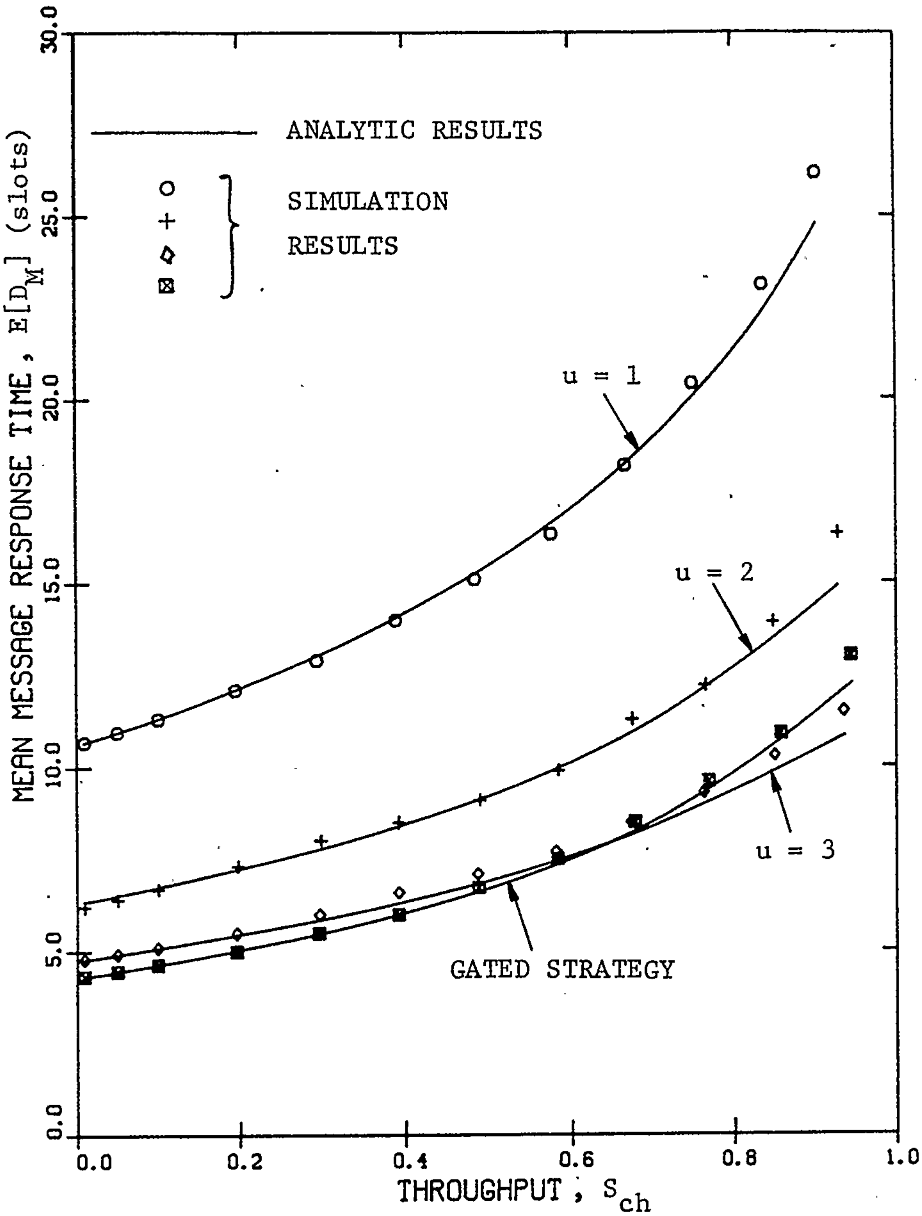


Fig. 4.2. Effect of packet transmission strategies on message response time - throughput characteristics.

CT strategy gives the worst response time, thus confirming our earlier intuition of its unsuitability for multipacket message transmission on a random access channel. For the *LT* strategy, we see that as u increases, there is an improvement in performance and the best response time is obtained using the *GT* strategy. However, our result suggests that the *GT* strategy does not give the least response time over all range of throughput; for this example, beyond approximately 65% throughput, the *LT* strategy ($u = 3$) gives a lower response time than the *GT* strategy. This behavior is explained by first noting that in the high throughput region, the mean retransmission delays incurred by employing either the *LT* strategy ($u = 3$) or *GT* strategy will be approximately equal. Hence, the extra delay of the *GT* strategy is due to the transmission time of the extra packets (and overheads) in a message.

Fig. 4.3 shows a comparison between the mean message and packet access delays (*GT* strategy in vogue) for two message length distributions: geometric and truncated Poisson. We have chosen the truncated Poisson distribution because of its lower variance relative to the 'popular' geometric distribution, which is found to best fit measurement data on real computer communication networks [84]. Notice that the truncated Poisson and geometric distributions are selected here to illustrate the usefulness of the analysis; in a real network, the actual message length distribution will depend largely on the type of application for which the network is designed.

Fig. 4.3 reveals that in the throughput range below 0.8, the mean message access delay is not equal to the mean packet access delay for the two message

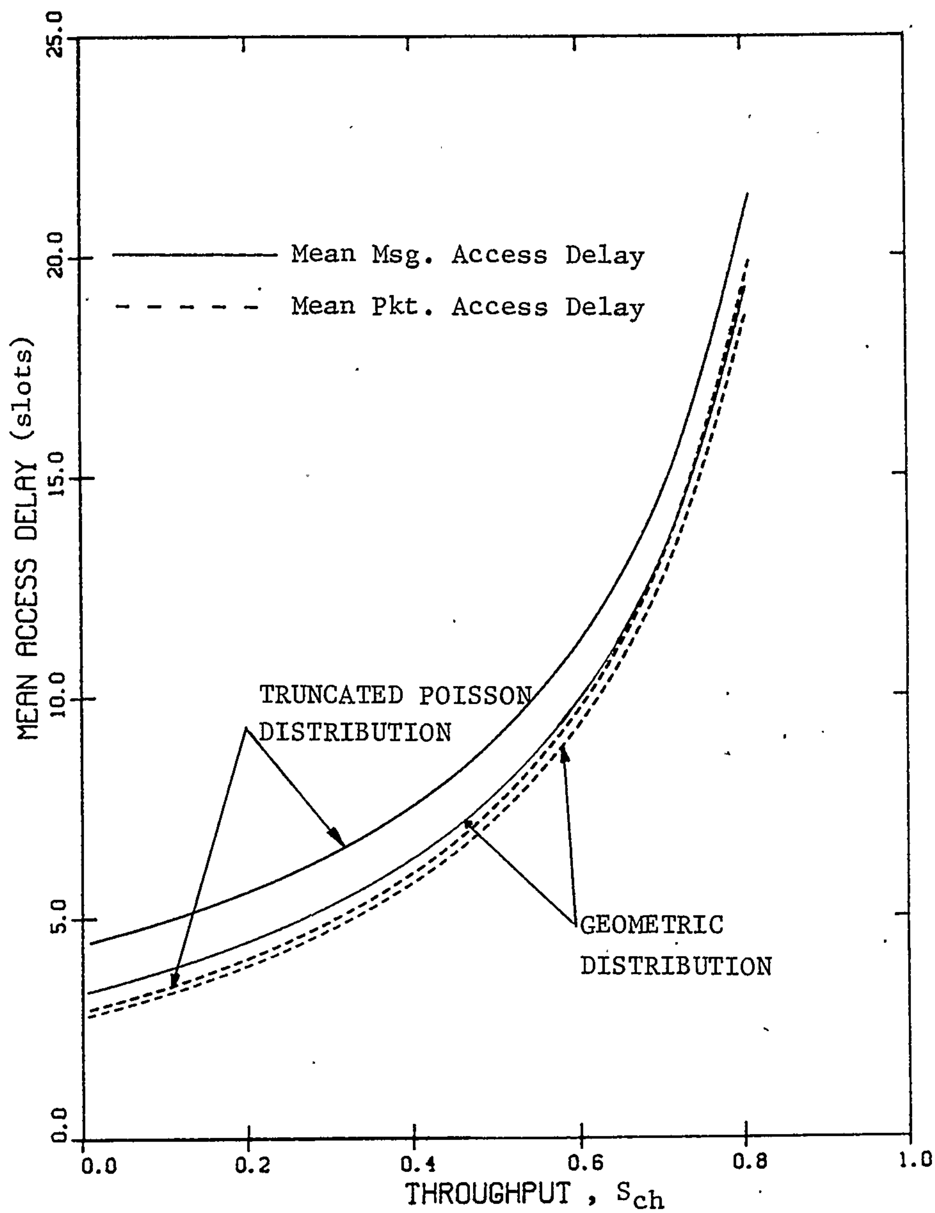


Fig. 4.3. Mean access delay versus throughput (Gated Transmission Strategy)

length distributions, the difference between the access delays being lower for the geometric distribution. A further observation from Fig. 4.3 is that the mean message (packet) access delay for the truncated Poisson distribution is higher than the corresponding delays for the geometric distribution. However, it is found that the variance of geometrically distributed message lengths is higher than that for truncated Poisson distributed message lengths so that the above observation is contrary to the expected result that a higher variance message length naturally leads to higher delay. We explain the above result by the finite buffer size assumption and the whole message acceptance strategy adopted in this chapter. For the geometric distribution, a large proportion of the messages generated are rejected due to their lengths being longer than the buffer space. Conversely, for the lower variance truncated Poisson distribution, only a fewer number of messages will be rejected. Thus, the larger number of messages accepted then leads to an increased message (packet) access delay as compared to that for geometrically distributed messages. Note that the effect of increasing K and/or $E[G]$ leads to an increase in the mean access delays [74].

Fig. 4.4 displays the mean voice packet delay (purely simulation results) versus the number of voice nodes in the network where the nodes employ either the GT or CT strategy. In the simulation model, the channel parameters are based on the IEEE 802.3 standard [21]. Throughout the simulations, each voice node is assumed to be active, that is, it generates an alternate sequence of talkspurts and silent periods. The distributions of talkspurts and silences are assumed exponential

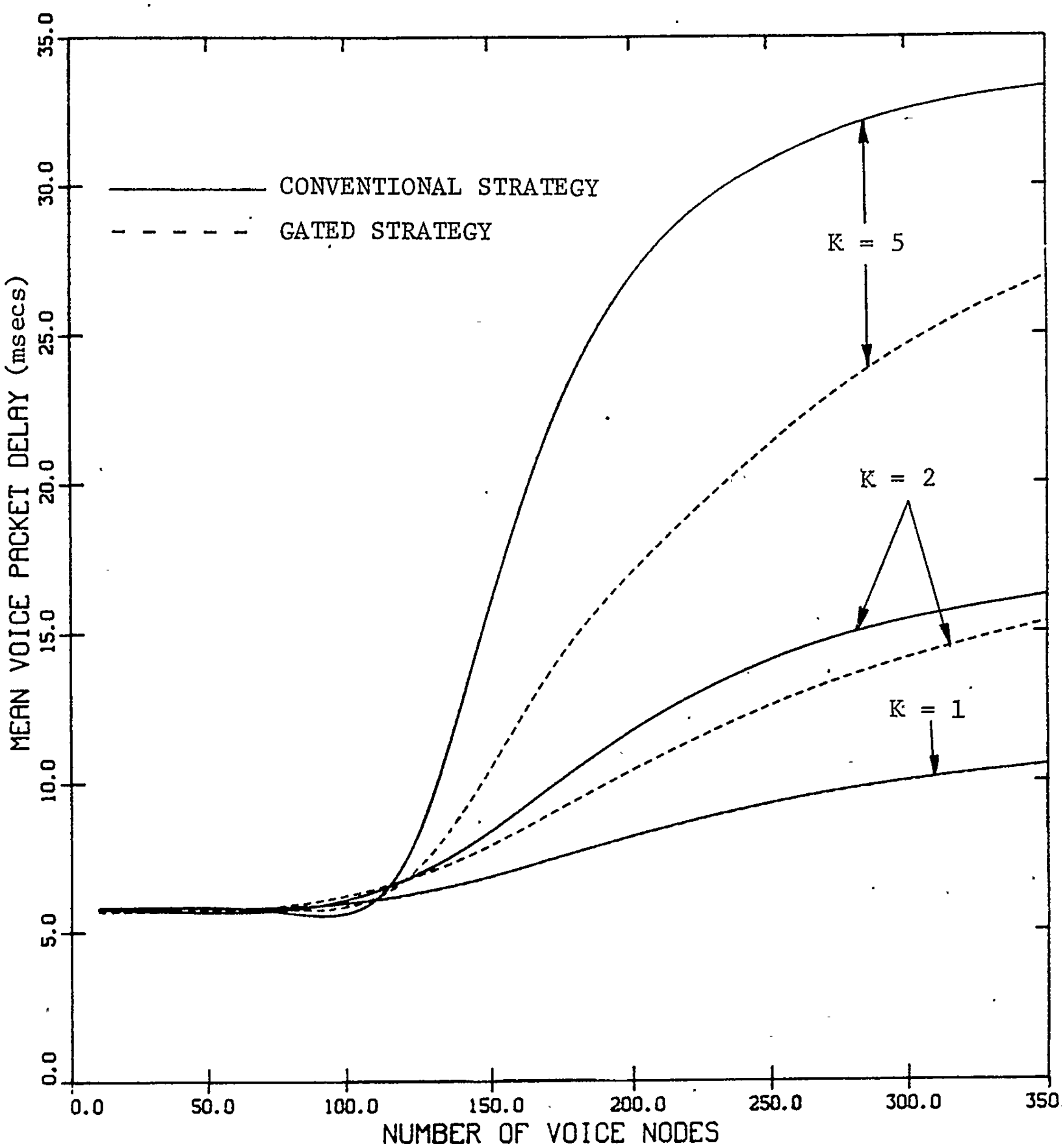


Fig. 4.4. Effect of packet transmission strategy on mean voice packet delay.

with average lengths taken to be 1.34 sec and 1.67 sec respectively [85]. The voice signal generated while in a talkspurt is digitized at a constant rate of 64 *kbits/sec* with a speech activity of approximately 40%. Each voice packet formed consists of 368 *bits* (data) and 208 *bits* (overhead). The above chosen data gives a packetization time of 5.75 *msecs* and the number of packets in a talkspurt (geometrically distributed) has a mean (calculated) of 234 packets.

The voice packet delay is plotted for different values of K, which as seen from (4.47), partly determines the maximum random access delay. We see that for $K = 1$, there is no gain derived by employing either the *GT* or *CT* strategy, which is expected. However, as K increases, the *GT* strategy gives a better delay performance compared to the *CT* strategy; this improvement is more pronounced at higher values of K. Thus, the *GT* strategy appears to be more suitable for real-time multipacket message transmission on a random access channel.

4.4 Summary

The gated and limited transmission (*GT* and *LT*) strategies are proposed in this chapter as possible candidates for transmission of multipacket messages on CSMA-CD LANs. The delay analysis of the two strategies is then performed using Markov chain theory and limiting results from regenerative processes.

The principal findings from this chapter are as follows:

1) the conventional transmission (*CT*) strategy seems unsuitable for multipacket message transmission on CSMA-CD LANs and the application of the *LT* and *GT*

strategies yields an improved delay performance,

2) for low to medium throughput range, the *GT* strategy gives the least mean message response time (compared to *LT* and *CT* strategies), but in the high throughput region, the *LT* strategy (with a high minimessage size) may perform better than the *GT* strategy,

3) the mean access delays are sensitive to the message length distribution and node buffer size, and

4) for voice data transfer application, the *GT* strategy displays a much improved performance over the *CT* strategy for values of buffer size greater than one.

This chapter has examined the performance of CSMA-CD LANs in which only one multipacket message can be stored at each node. The scenario where more than one multipacket message can be stored forms the topic of the next chapter.

CHAPTER 5

MULTIMESSAGE BUFFER LANs WITH MULTIPACKET MESSAGES

5.1 Introduction and Problem Statement

In this chapter, we shall study the performance of a random access LAN having a finite number of nodes each capable of buffering multipacket messages; this as the reader will recall is the higher order scenario of that considered in the preceding chapter. The motivation for studying the current scenario follows naturally from the operation of a real (random access) network to which new messages arrive while a node is waiting to transmit previously queued messages. Hence, the single-message buffer assumption of the last chapter may seem somewhat unrealistic. But, as pointed out in Chapter 3, moving towards reality leads to analytic complications due to the interfering queue problem, this is also the main problem we are faced with in the current chapter. Furthermore, in the multipacket message buffered CSMA-CD LANs considered here, the inherent interfering queue problem is compounded by the modeling of message lengths and how the queued messages are transmitted when a node gains channel access right. Our objective in this chapter then is to provide appropriate solutions to these problems which will, hopefully, enable us to predict the performance of such LANs. We state that there are no known or published prior studies on the above scenario for random access LANs and the current study is the first [86].

5.2 Model Formulation

The model description of Section 4.2 carries over into the present section except for the following changes: the buffer size at each network node is assumed to be infinite, as a consequence, the mean message arrival rate of the Poisson process is now equal to λ. The latter part of assumption (*iii*) in Section 4.2 is relaxed since for the current scenario a node can store more than one multipacket message. Assumption (*iv*) is dropped in its entirety as there is now no restriction on message length. Finally, assumption (*v*) is generalized so that the transmission time of a packet is denoted by a random variable S with distribution function (*DF*) $S(x)$ and probability density function (*pdf*) $s(x)$. We further assume that after successful transmission, a packet will be deleted from the nodal queue and queued messages are transmitted on a first-come, first-served basis. A node that gains channel access right employs the gated transmission strategy and in addition, continues to transmit until its buffer becomes empty and the channel then becomes idle (this transmission strategy is referred to as exhaustive service under the polling access scheme [87]).

Since an exact analysis for the above network model is difficult due to the interaction among the nodal queues, we therefore resort to the decomposition approximation (Chapter 2). Specifically, by principle 1, we shall decompose the network into M separate nodal queues. From the network operation and the modeling assumptions stated above, each node is independently modeled by an $M^X/G/1$ queue undergoing busy and resting phases; and the supplementary variables technique [88-90] is applied for calculation of relevant parameters of the nodal queues. The interaction of the nodes is then modeled by the channel-activity parameters, namely, the probability that the channel is idle, μ and the probability of successful transmission on the channel, γ.

5.3 Performance Analysis

We begin by defining the important terminologies required in the analysis. A node is said to be in the busy phase if it is in the nonempty state (at least one packet is present in the node buffer) and has acquired the channel access right. On the contrary, a node is said to be in the resting phase if (a) it is in the empty state (no message is present in the buffer) or (b) it is in the nonempty state but undergoing backoff or (c) it is in the nonempty state and is currently undergoing unsuccessful transmission (Fig. 5.1). The justification for assuming case (c) as a resting phase is as follows: since the colliding packets will be retransmitted the nodes involved in a collision behave like resting. Transmission initiation by a node occurs if it is in the nonempty state and it senses the channel busy. Transmission commencement by a nonempty node occurs if it senses the channel idle and immediately begins a transmission. The end of a resting phase of a nonempty node occurs if the node commences a transmission and acquires the channel access right.

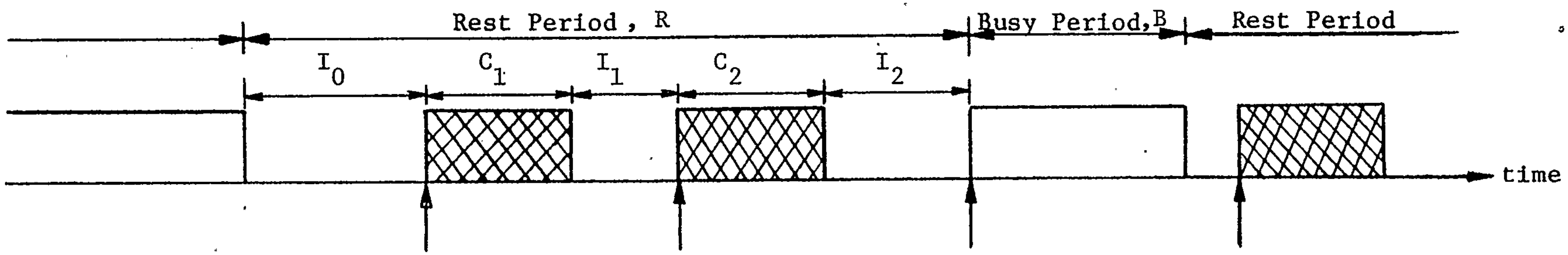
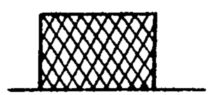
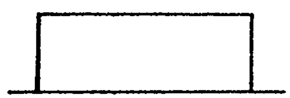


Fig. 5.1. A sequence of busy and rest periods for a node.

 : Transmission commencement instant by a node.

 : Unsuccessful transmission period.

 : Successful transmission (node busy) period.

5.3.1 Generating Function for Nodal Queue Length

In this Section, we derive the generating function for nodal queue length under steady-state condition. We first observe that at an arbitrary time t, a node in the network can be characterized by the following random variables:

$\xi(t)$ = the phase of the node at time t

$$\xi(t) = \begin{cases} 0 \text{ if the node is in the resting phase at time } t \\ 1 \text{ if the node is in the busy phase at time } t \end{cases}$$

$Q(t)$ = number of packets present in the queue at time t

$\tilde{S}(t)$ = residual successful transmission time of a packet at time t

$\tilde{R}(t)$ = residual resting time of a node in the resting phase at time t

In addition, let B be the random variable denoting the duration of a busy phase of a node and let $B(x)(b(x))$ be its DF (pdf). Also, denote the duration of a resting phase of a node by R and let $R(x)(r(x))$ be its DF (pdf).

In general, the single-server queue-length process $\{Q(t), t \geq 0\}$ is non-Markovian because the future behavior of the process cannot be predicted by the current number of packets in the queue. To make the queue length process Markovian, we shall apply the supplementary variable technique whose underlying idea is characterized by the inclusion of one or more additional variable(s) in the non-Markovian process [88]. Application of the supplementary variable technique to the resting and busy phases of a node (Fig. 5.1) results in two bivariate Markovian processes $\{Q(t), \tilde{S}(t)\}$ and $\{Q(t), \tilde{R}(t)\}$ which will then be jointly studied in order to obtain results for the non-Markovian process $\{Q(t)\}$. In the queueing literature, it is usual to define the augmentation variable in terms of the backward recurrence time instead of the forward recurrence time used in the definition of $\tilde{S}(t)$ and $\tilde{R}(t)$ stated above.

We first derive the steady-state differential-difference equations governing each node activity. For the $\{Q(t), \tilde{S}(t)\}$ process, we define $p_n(t, x)\Delta t + o(\Delta t)$, $n > 0$ as the probability that at time t there are n packets in the nodal queue and the node is in the busy phase and the residual successful transmission time of a packet lies between x and $x + \Delta t$, that is

$$p_n(t, x)\Delta t = Pr\{Q(t) = n, x < \tilde{S}(t) \le x + \Delta t, \xi(t) = 1\}, n = 1, 2, \cdots . \quad (5.1a)$$

Similarly, for the $\{Q(t), \tilde{R}(t)\}$ process, we define $\pi_n(t, x)\Delta t + o(\Delta t)$, $n \ge 0$ as the probability that at time t there are n packets in the nodal queue and the node is in the resting phase and the residual resting time lies between x and $x + \Delta t$, that is,

$$\pi_n(t, x)\Delta t = Pr\{Q(t) = n, x < \tilde{R}(t) \le x + \Delta t, \xi(t) = 0\}, n = 0, 1, \cdots . \quad (5.1b)$$

To obtain the differential-difference equations connecting these probabilities, we shall relate the state of the node at time $t + \Delta t$ to the state at time t. The probabilities defined by (5.1a) and (5.1b) give rise to two cases of interest.

Case 1: Node in Busy Phase

For $n > 1$,

$$p_n(t+\Delta t, x) = p_n(t, x+\Delta t)[1 - \lambda\Delta t + o(\Delta t)] + \sum_{k=1}^{n-1} p_{n-k}(t, x+\Delta t)[\lambda\Delta t + o(\Delta t)]g_k + p_{n+1}(t, \Delta t)[s(x)\Delta t] + \pi_n(t, \Delta t)[s(x)\Delta t] \,. \tag{5.2a}$$

The term on the left hand side is the probability that at time $t + \Delta t$, there are n packets in the queue, the node is busy and the residual successful transmission time of a packet is x seconds ($x > 0$). The only four ways to enter this state are

(i) there were n packets in the queue at time t, the node is busy and the packet undergoing successful transmission has a residual transmission of $x + \Delta t$ seconds and there were no message arrivals in $(t, t + \Delta t)$ or

(ii) there were $n - k$ packets in the queue at time t, the node is busy and the packet under transmission has a residual transmission time of $x + \Delta t$ seconds and one message of length k packets with probability g_k, ($k = 1, 2, \cdots, n - 1$), arrived in $(t, t + \Delta t)$ or

(iii) there were $n + 1$ packets in the queue at time t and the node is busy with the residual packet transmission time being equal to Δt and the packet transmission will be completed in $(0, \Delta t) \equiv (x, x + \Delta t)$ or

(iv) there were n packets in the queue at time t and the node is resting with the residual resting time being equal to Δt and the head of queue packet will begin a successful transmission in $(0, \Delta t) \equiv (x, x + \Delta t)$.

Note that in (5.2a), the probability that a packet will complete a transmission in $(0, \Delta t)$ and the probability that a packet will begin a transmission in $(0, \Delta t)$ are equal.

For $n = 1$,

$$p_1(t + \Delta t, x) = p_1(t, x + \Delta t)[1 - \lambda\Delta t + o(\Delta t)] \\ + p_2(t, \Delta t)[s(x)\Delta t] + \pi_1(t, \Delta t)[s(x)\Delta t] \tag{5.2b}$$

where the terms in (5.2b) have the same interpretation as the corresponding terms in (5.2a).

Case 2: Node in Resting Phase

For $n \geq 1$,

$$\pi_n(t + \Delta t, x) = \pi_n(t, x + \Delta t)[1 - \lambda\Delta t + o(\Delta t)] \\ + \sum_{k=1}^{n} \pi_{n-k}(t, x + \Delta t)[\lambda\Delta t + o(\Delta(t)]g_k \ . \tag{5.3a}$$

The term on the left hand side is the probability that at time $t + \Delta t$, there are n packets in the queue and the node is resting with the residual resting time being equal to x seconds. The only two ways to enter this state are

(i) there were n packets in the queue at time t, and the node is resting with the residual resting time being equal to $x + \Delta t$ seconds and there were no arrivals in $(t, t + \Delta t)$ or

(ii) there were $n - k$ packets in the queue at time t, and the node is resting with the residual resting time being equal to $x + \Delta t$ seconds and one message of length k packets arrived in $(t, t + \Delta t)$.

For $n = 0$, we can write

$$\pi_0(t + \Delta t, x) = \pi_0(t, x + \Delta t)[1 - \lambda \Delta t + o(\Delta t)] + \pi_0(t, \Delta t)[r(x)\Delta t] + p_1(t, \Delta t)[r(x)\Delta t]. \tag{5.3b}$$

The left hand term and the first term on the right hand side have the same interpretation as the corresponding terms of (5.3a). The second term on the right hand side can be interpreted as the probability that at time t there were no packets in the queue and the node is resting with the residual resting time being equal to Δt and the resting period will terminate in $(x, x + \Delta t)$ while the third term can be interpreted as the probability that at time t there was one packet in the queue and the node is busy with the residual packet transmission time being equal to Δt and the resting period will begin in $(x, x + \Delta t)$. As in case 1, the probability that a resting period will end in $(x, x + \Delta t)$ and the probability that a resting period will begin in $(x, x + \Delta t)$ are equal. Note that the node activity described above is analogous to the standard $M/G/1$ queueing system with server vacations where the server begins a vacation when the buffer goes empty [91-92]; but in our case the node can be in a resting phase even when the buffer is nonempty. To obtain the steady-state differential-difference equations describing the node activity we adopt the following procedure: Starting from (5.2a), let us expand the left hand side in a

Taylor series about t while ignoring second-order and higher order terms in Δt:

$$p_n(t+\Delta t, x) = p_n(t,x) + \frac{\partial p_n(t,x)}{\partial t}\Delta t + o(\Delta t). \tag{5.4}$$

Substituting this in (5.2a) and rearranging the terms gives

$$\begin{aligned} p_n(t,x) - p_n(t,x+\Delta t) + \frac{\partial p_n(t,x)}{\partial t}\Delta t &= -[\lambda \Delta t] p_n(t, x+\Delta t) \\ &+ \lambda \Delta t \sum_{k=1}^{n-1} p_{n-k}(t, x+\Delta t) g_k + p_{n+1}(t,\Delta t)[s(x)\Delta t] \\ &+ \pi_n(t,\Delta t)[s(x)\Delta t] + o(\Delta t) \end{aligned} \tag{5.5}$$

Dividing both sides by Δt and letting $\Delta t \to 0$ gives

$$\begin{aligned} -\frac{\partial p_n(t,x)}{\partial x} + \frac{\partial p_n(t,x)}{\partial t} &= -\lambda p_n(t,x) + \lambda \sum_{k=1}^{n-1} p_{n-k}(t,x) g_k \\ &+ p_{n+1}(t,0)s(x) + \pi_n(t,0)s(x) . \end{aligned}$$

As $t \to \infty$, using the equilibrium results $\lim\limits_{t\to\infty} \frac{\partial p_n(t,x)}{\partial t} = 0$ and $\lim\limits_{t\to\infty} p_n(t,x) = p_n(x)$ we finally obtain

$$-\frac{dp_n(x)}{dx} = -\lambda p_n(x) + \lambda \sum_{k=1}^{n-1} \pi_{n-k}(x) g_k + p_{n+1}(0)s(x) + \pi_n(0)s(x) \quad , \quad n \geq 2 . \tag{5.6a}$$

Similarly, by applying the above procedure to (5.2b), (5.3a) and (5.3b) we get

$$-\frac{dp_1(x)}{dx} = -\lambda p_1(x) + p_2(0)s(x) + \pi_1(0)s(x) \tag{5.6b}$$

$$-\frac{d\pi_n(x)}{dx} = -\lambda\pi_n(x) + \lambda \sum_{k=1}^{n} g_k \pi_{n-k}(x) \quad , \quad n \geq 1 \tag{5.7a}$$

and

$$-\frac{d\pi_0(x)}{dx} = -\lambda\pi_0(x) + \pi_0(0)r(x) + p_1(0)r(x) \tag{5.7b}$$

respectively. Under steady-state condition, define the Laplace-Stieltjes transforms (LST) of $p_n(x)$, $\pi_n(x)$, $S(x)$ and $R(x)$ respectively by

$$P_n^*(\theta) = \int_0^\infty e^{-\theta x} p_n(x)dx \quad , \quad n = 1, 2, \cdots \tag{5.8a}$$

$$\Pi_n^*(\theta) = \int_0^\infty e^{-\theta x} \pi_n(x)dx \quad , \quad n = 0, 1, \cdots \tag{5.8b}$$

$$S^*(\theta) = \int_0^\infty e^{-\theta x} s(x)dx \tag{5.8c}$$

$$R^*(\theta) = \int_0^\infty e^{-\theta x} r(x)dx \; . \tag{5.8d}$$

Further, let us introduce the generating functions

$$P(z, 0) = \sum_{n=1}^{\infty} p_n(0)z^n \tag{5.9a}$$

$$P^*(z; \theta) = \sum_{n=1}^{\infty} P_n^*(\theta)z^n \tag{5.9b}$$

$$\Pi(z, 0) = \sum_{n=0}^{\infty} \pi_n(0)z^n \tag{5.9c}$$

$$\Pi^*(z,\theta) = \sum_{n=0}^{\infty} \Pi_n^*(\theta) z^n . \tag{5.9d}$$

Then, the steady-state probability generating function for queue length $Q^*(z,\theta)$ is given by

$$Q^*(z,\theta) = P^*(z,\theta) + \Pi^*(z,\theta)$$

which after tedious algebraic manipulations is found to be

$$Q^*(z,\theta) = \left\{ \frac{a_1(z,\theta) + a_2(z,\theta)}{(1 - R^*(\lambda))(z - S^*(\lambda - \lambda G(z)))(\theta - \lambda + \lambda G(z))} \right\} p_1(0) \tag{5.10}$$

where

$$a_1(z,\theta) = z(R^*(\lambda - \lambda G(z)) - 1)(S^*(\lambda - \lambda G(z)) - S^*(\theta))$$

and

$$a_2(z,\theta) = (z - S^*(\lambda - \lambda G(z)))(R^*(\lambda - \lambda G(z)) - R^*(\theta)) .$$

To establish the existence of the limiting probabilities we shall first set θ equal to zero in (5.10)

$$Q^*(z,0) = \left\{ \frac{a_1(z,0) + a_2(z,0)}{(1 - R^*(\lambda))(z - S^*(\lambda - \lambda G(z)))(-\lambda + \lambda G(z))} \right\} p_1(0) . \tag{5.11}$$

Using Takac's Lemma [93, p. 47], we see that if $\lambda E[G]E[S] \le 1$, then a denominator factor of (5.11) $(z - S^*(\lambda - \lambda G(z))) = 0$ has exactly one root z_0 in the region $|z| \le 1$ and this single positive real root lies on the unit circle, that is, $z_0 = 1$. Note that by the analyticity property of probability generating functions the numerator of $Q^*(z,0)$ must be zero at $z_0 = 1$. Since the root z_0 is uniquely determined, we conclude that the steady-state probabilities $\{p_j(0)\}$ and $\{\pi_j(0)\}$ can be determined uniquely. In particular, to obtain $p_1(0)$, we simply set $z = z_0 = 1$ in (5.11) and after applying L'Hospital's rule twice we get

$$p_1(0) = (1 - \lambda E[G]E[S])\frac{(1 - R^*(\lambda))}{E[R]} . \tag{5.12}$$

The mean queue length is obtained using a property of probability generating functions and successive application of L'Hospital's rule (four times to reach convergence) we obtain after tedious manipulations

$$E[Q] = \lambda E[G]E[S] + \frac{\lambda E[G]E[R^2]}{2E[R]} + \frac{\lambda(\lambda(E[G])^2 E[S^2] + (E[G^2] - E[G])E[S])}{2(1 - \lambda E[G]E[S])} . \tag{5.13}$$

Notice that $E[Q]$ is expressed in terms of the first two moments of the node resting period which remain to be determined.

5.3.2 Waiting Time Distribution

The waiting time of an arbitrary packet, W, consists of two components: the waiting time due to the other packets present in the queue when the message of the tagged packet arrives, W_o, and the waiting time due to the position of the arbitrary packet in its message, W_g. The evaluation of W_o depends on whether the message containing the test packet arrives during a nodal busy or resting period. Let $\{W_{ob} | Q = n\}$ denote the waiting time of the new message arriving in a busy period and finding n packets in the queue, $n = 1, 2, 3, \cdots$. We have the conditional event

$\{W_{ob} | Q = n\}$ = Residual transmission time of the packet undergoing transmission

+ $(n - 1)$ packet transmission times of the other packets in the queue.

(5.14a)

Similarly, let $\{W_{or} | Q = n\}$ denote the waiting time of the new message arriving in a resting period and finding n packets in the queue $n = 0, 1, 2, \cdots$. Then

$\{W_{or} | Q = n\}$ = Remaining resting time of the node

+ n packet transmission times of the packets in the queue. (5.14b)

Herein lies the rationale for using the forward recurrence time (as opposed to the usual backward recurrence time) as the supplementary variable: It enables us to determine the LST of the distribution of residual transmission time of the packet undergoing successful transmission when on arrival the new message finds n packets in the queue (given by (5.8a)). Furthermore, we can also determine the LST of the distribution of residual resting period of the node from the arrival instant of the new message which finds n packets in the queue on arrival (given by (5.8b)). Hence, by unconditioning (5.14) with respect to n the LST of the distribution of W_o is given by

$$W_o^*(\theta) = \sum_{n=1}^{\infty} P_n^*(\theta)[S^*(\theta)]^{n-1} + \sum_{n=0}^{\infty} \Pi_n^*(\theta)[S^*(\theta)]^n . \tag{5.15}$$

Using (5.9b) and (5.9d), we can express $W_o^*(\theta)$ as

$$W_o^*(\theta) = \frac{P^*(S^*(\theta),\theta)}{S^*(\theta)} + \Pi^*(S^*(\theta), \theta)$$

and after simplifying, $W_o^*(\theta)$ becomes

$$W_o^*(\theta) = \frac{1 - \lambda E[G]E[S]}{(\theta - \lambda + \lambda G(S^*(\theta)))} \frac{(1 - R^*(\theta))}{E[R]} . \qquad (5.16)$$

To obtain W_g, we shall invoke Burke's result which gives the probability that a packet is in the k^{th} position in its message as [79]

$$\iota_k = \frac{\sum_{i=k}^{\infty} g_i}{E[G]} \qquad (5.17)$$

that is, from the instant when a node begins the successful transmission of the first packet in a message, the waiting time of a test packet which is in the k^{th} position in the message is equal to the sum of the transmission times of the $(k - 1)$ packets ahead of the test packet. Stated mathematically,

$$\{W_g \le t \mid \textit{test packet is in } k^{th} \textit{ position}\} = \{\sum_{i=1}^{k-1} S_i \le t\} , k = 1, 2, \cdots$$

where S_i is the transmission time of the i^{th} packet. Taking the LST of the distribution function of both sides and unconditioning yields

$$W_g^*(\theta) = \sum_{k=1}^{\infty} \iota_k [S^*(\theta)]^{k-1} . \qquad (5.18)$$

where the packet transmission times are assumed to be independent and identically distributed. Substituting (5.17) into (5.18) and after some algebraic manipulations we get

$$W_g^*(\theta) = \frac{1 - G(S^*(\theta))}{E[G](1 - S^*(\theta))} . \qquad (5.19)$$

Thus, the LST of the distribution of W is the product of $W_o^*(\theta)$ and $W_g^*(\theta)$ since

W_o and W_g are independent random variables,

$$W^*(\theta) = \frac{(1-R^*(\theta))(1-\lambda E[G]E[S])}{E[R](\theta-\lambda+\lambda G(S^*(\theta)))} \cdot \frac{(1-G(S^*(\theta)))}{E[G](1-S^*(\theta))} \cdot \tag{5.20}$$

By the LST property, the expression for the mean packet waiting time, $E[W](=E[W_o]+E[W_g])$, is derived from (5.16) and (5.19) as, after simplification

$$E[W] = \frac{\lambda E[G]E[S^2]}{2(1-\lambda E[G]E[S])} + \frac{(E[G^2]-E[G])E[S]}{2E[G](1-\lambda E[G]E[S])} + \frac{E[R^2]}{2E[R]} \cdot \tag{5.21}$$

We remark that the first two terms on the right hand side of (5.21) is the mean waiting time of an arbitrary packet (customer) in an ordinary $M^X/G/1$ single-server queueing system without a rest period [49] while the last term represents the equilibrium mean residual resting time of a node that is currently undergoing a resting phase. Notice here also the dependence of $E[W]$ on the moments of R.

5.3.3 Node Resting Period Distribution

As can be seen from Fig. 5.1, each node in the network alternates between a sequence of 'rest' and 'busy' periods. From the end of a busy period, let N_A be the number of transmission attempts up to the beginning of the next successful transmission (busy period). Clearly, N_A is geometrically distributed with mean $1/\gamma$. We can write the equivalent events (Fig. 5.1)

$$\{R \le t \mid N_A = k\} = \left\{(I_0 + \sum_{i=1}^{k-1}(C_i + I_i)) \le t\right\} \tag{5.22}$$

where C_i , $i > 0$ is the duration of the i^{th} unsuccessful transmission period and

I_i , $i \geq 0$ is the length of the i^{th} node idle period. Taking the LST of the distribution function of both sides

$$E[e^{-\theta R} | N_A = k] = I_0^*(\theta)[I^*(\theta)C^*(\theta)]^{k-1}$$

which follows from the independence of I_0 with the I_i's, $i > 0$ and the fact that the I_i's are independent and identically distributed. By unconditioning and introducing the probability generating function for N_A we have

$$R^*(\theta) = \frac{I_0^*(\theta)}{C^*(\theta)I^*(\theta)} N_A(I^*(\theta)C^*(\theta)) = \frac{\gamma I_0^*(\theta)}{1-(1-\gamma)C^*(\theta)I^*(\theta)}. \tag{5.23}$$

C_i, $i > 0$ is a constant equal to $c + a$ so that $C^*(\theta) = e^{-\theta(c+a)}$ where c and a are respectively the collision resolution period and the normalized propagation delay. From (5.23), we calculate the first two moments of R to be

$$E[R] = -I_0^{*\prime}(0) - \left[\frac{1-\gamma}{\gamma}\right] X^{*\prime}(0) \tag{5.24}$$

and

$$E[R^2] = I_0^{*\prime\prime}(0) + \left[\frac{1-\gamma}{\gamma}\right] X^{*\prime\prime}(0)$$
$$+ 2\left[\frac{1-\gamma}{\gamma}\right] X^{*\prime}(0)\left[I_0^{*\prime}(0) + \left[\frac{1-\gamma}{\gamma}\right] X^{*\prime}(0)\right] \tag{5.25}$$

where $X^*(\theta) = C^*(\theta)I^*(\theta)$, and $I_0^{*\prime}(0)(X^{*\prime}(0))$ and $I_0^{*\prime\prime}(0)(X^{*\prime\prime}(0))$ respectively denote the first and second derivatives of $I_0^*(\theta)(X^*(\theta))$ with respect to θ and evaluated for θ equal to zero. It remains now to determine $I_0^*(\theta)$, $I^*(\theta)$ and γ.

(i) Evaluation of $I_0^*(\theta)$: Since at the end of a busy period the nodal queue is empty, then I_0 (Fig. 5.1) actually consists of the message interarrival time, T_A, plus the total delay before the first transmission commencement instant by a node, for clarity these components are shown in Fig. 5.2(a). However, for the purpose of calculating the queue length and the packet (message) waiting time, I_0 is henceforth taken to be the total delay from the instant the first message arrives at an empty buffer to the first transmission commencement instant, the reason being that the message interarrival time should not be included as part of the waiting time of a message because the message has not arrived yet. From the instant a new message arrives to a node in the resting phase, let N_s denote the number of times the channel is sensed up to the first transmission commencement. For example, if $N_s = k$ then the node must have sensed the channel busy in the first $(k-1)$ transmission initiations while the channel is sensed idle in the last sensing attempt. Thus N_s is geometrically distributed with mean $1/\mu$. Recall from the model formulation that each instant the channel is sensed busy, the next channel sensing occurs after an exponentially distributed delay, D_i. Thus

$$\{I_0 \le t \mid N_s = k\} = \{\sum_{i=1}^{k-1} D_i \le t\} . \tag{5.26}$$

Taking the LST of the distribution function of both sides and unconditioning

$$I_0^*(\theta) = \frac{1}{D^*(\theta)} \sum_{k=1}^{\infty} Pr\{N_s = k\}[D^*(\theta)]^k . \tag{5.27}$$

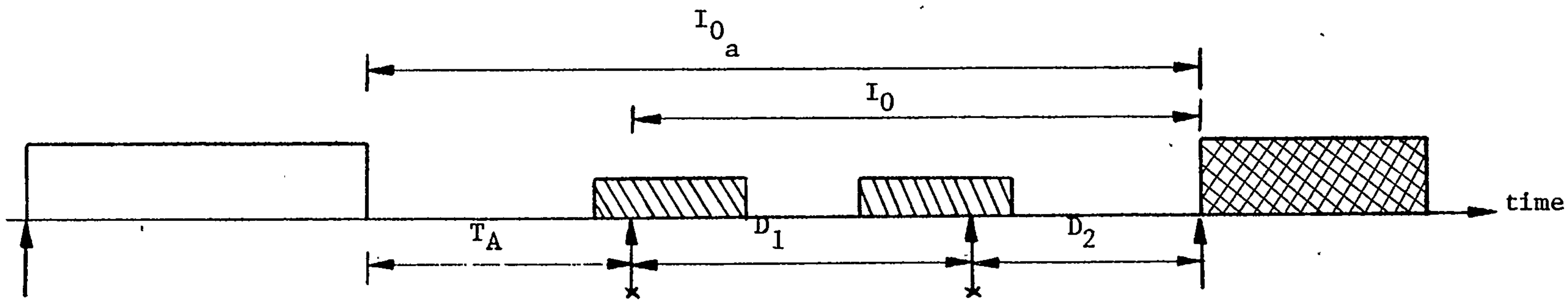


Fig. 5.2(a). Illustration of I_{0_a} and I_0 for a node.

: Transmission initiation by a node.

: Transmission commencement by a node.


: Channel busy period due to transmission(s) by other node(s).

By using the *pgf* of a geometrically distributed random variable we get

$$I_0^*(\theta) = \frac{\mu}{1-(1-\mu)D^*(\theta)} . \tag{5.28}$$

Note that

$$D^*(\theta) = \frac{\kappa}{\theta+\kappa} \tag{5.29}$$

since D_i is exponentially distributed with parameter κ. From (5.28), we find the first two derivatives of $I_0^*(\theta)$, evaluated at $\theta = 0$ as

$$I_0^{*\prime}(0) = \frac{-(1-\mu)}{\kappa\mu} \tag{5.30a}$$

and

$$I_0^{*\prime\prime}(0) = \frac{2(1-\mu)}{\kappa^2\mu^2} . \tag{5.30b}$$

However, μ still remains unknown in (5.30).

(ii) Evaluation of $I^*(\theta)$: Each of the I_i's, $i > 0$ shown in Fig. 5.1 consists of a sum of retransmission delays - the first retransmission delay is the timeout interval caused by the previous collision while the other retransmission delays are due to the channel being sensed busy at subsequent successive transmission initiations before the next transmission commencement instant (Fig. 5.2(b)). Following the same procedure used for deriving the expression for $I_0^*(\theta)$, we find that

$$I^*(\theta) = \frac{\mu D^*(\theta)}{1-(1-\mu)D^*(\theta)} \tag{5.31}$$

where as in (5.28) μ is yet to be determined. With $I^*(\theta)$ determined, the first two

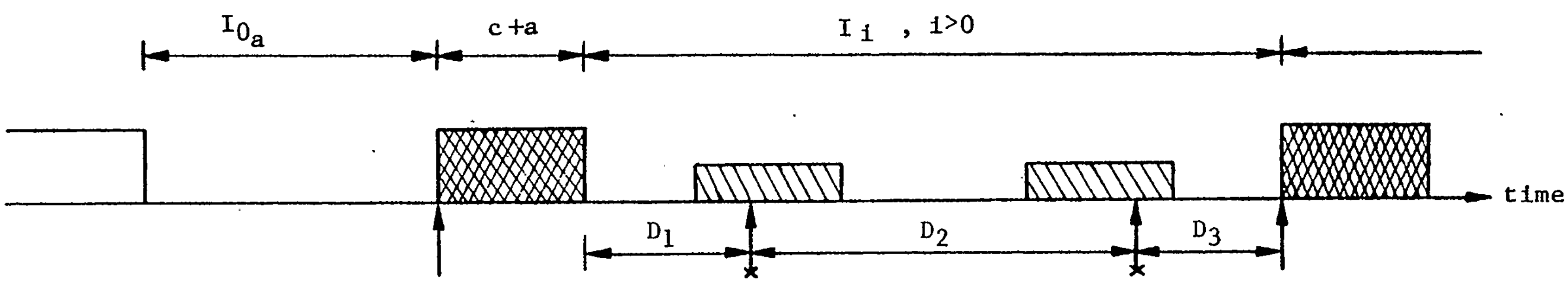


Fig. 5.2(b). Illustration of I_i , $i>0$ for a node.

derivatives of $X^*(\theta)$ at $\theta = 0$ are

$$X^{*\prime}(0) = \frac{-((c+a)\kappa\mu + 1)}{\kappa\mu} \tag{5.32a}$$

and

$$X^{*\prime\prime}(0) = (c+a)^2 + \frac{2(c+a)}{\kappa\mu} + \frac{2}{\kappa^2\mu^2} . \tag{5.32b}$$

(iii) Evaluation of γ: Observe that γ depends on the state of the channel and accounts for the node interactions with the channel. Fig. 5.3 depicts a snapshot of the channel which is obtained from the superposition of all the node activities in the network. Let $\tilde{Z}(t)$ denote the number of nonempty (active) nodes in the network at time t. Also, define an embedded epoch (represented by • in Fig. 5.3) as a time instant of a transmission completion. If we let $\tilde{Z}_n$; $n \geq 1$ denote the number of nonempty nodes in the network immediately after the n^{th} transmission completion epoch, then the process $\{\tilde{Z}_n; n \geq 1\}$ is a Markov chain embedded in the process $\{\tilde{Z}(t); t > 0\}$. Further, let f_i denote the limiting probability that i nonempty nodes are present in the network at an arbitrary embedded epoch, that is,

$$f_i = \lim_{n \to \infty} Pr\{\tilde{Z}_n = i\} , \quad i \in \{0, \cdots, M\} . \tag{5.33}$$

Introduce the vector, $\underline{F} = (f_0, f_1, \cdots, f_M)$. Then the probabilities $\{f_i\}$ are determined by the system of equations [47]

$$\underline{F} = \underline{F}\Psi \tag{5.34}$$

subject to the normalizing condition $\sum_{i=0}^{M} f_i = 1$. Ψ in (5.34) is the matrix of tran-

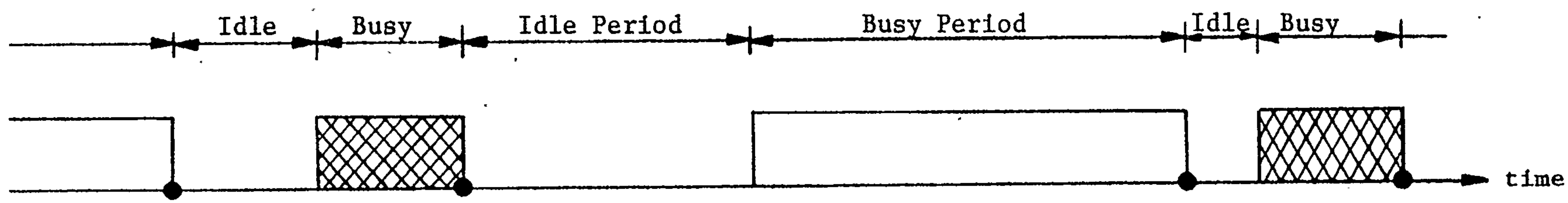


Fig. 5.3. A snapshot of the channel time.

(● denotes an embedded epoch)

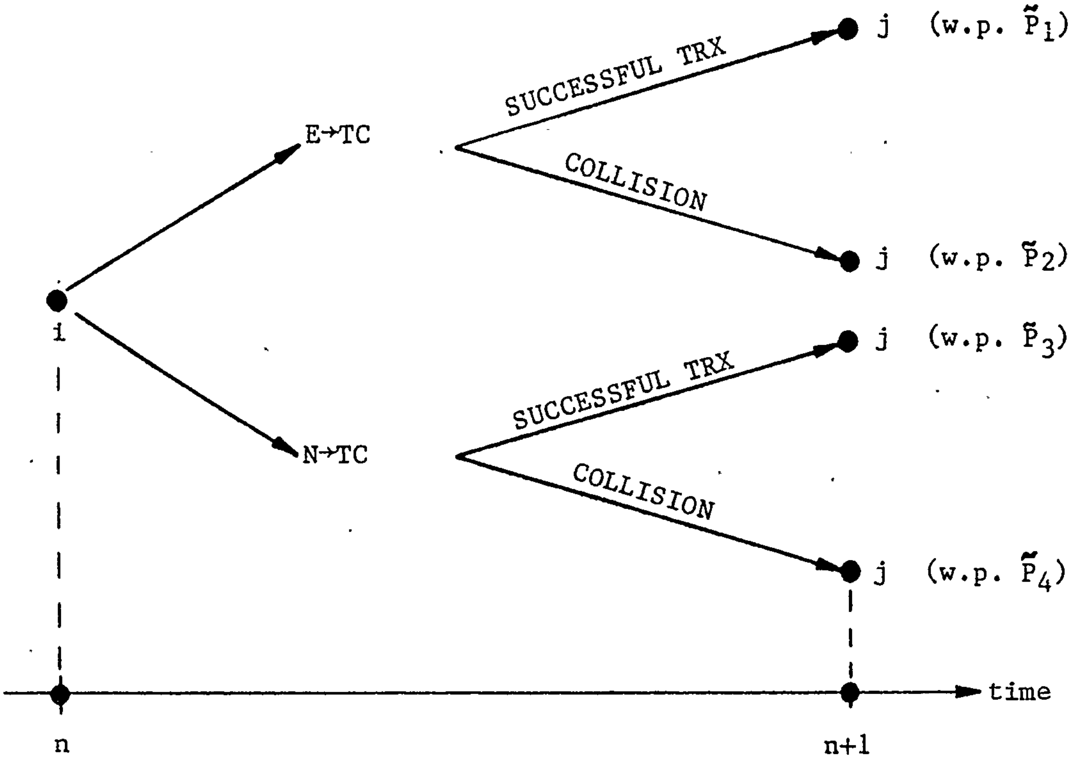


Fig.5.4. Events for a transition.

E→TC : An empty node commences a transmission.

N→TC : A nonempty node commences a transmission.

w.p. : with probability.

sition probabilities ψ_{ij} between two consecutive embedded epochs, where $\psi_{ij} = Pr\{\tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$. From Fig. 5.4 we can write the expression for ψ_{ij} as the sum of the probabilities of four mutually exclusive events - $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ and $\tilde{P}_4$ where

$$\tilde{P}_1 = Pr\{\text{transmission commencement by an empty node with success, } \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\tilde{P}_2 = Pr\{\text{transmission commencement by an empty node without success, } \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\tilde{P}_3 = Pr\{\text{transmission commencement by a nonempty node with success, } \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\tilde{P}_4 = Pr\{\text{transmission commencement by a nonempty node without success, } \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

Note that in the definitions of $\tilde{P}_2$ and $\tilde{P}_4$ the unsuccessful transmission is due to other transmission commencement(s) during the collision window of the original transmission. These other commencement(s) may be from either at least one of the i nonempty nodes or from $k\,(k \geq 1)$ nodes which were empty at epoch n. The explicit expression for $\tilde{P}_1$ is

$$\tilde{P}_1 = \sigma_i \exp[-\{(M - i - 1)\lambda + i\kappa\}a]\, bin(M - i - 1, j - i, (E[B] - a)), \quad 0 \leq i, j \leq M - 1 \tag{5.35a}$$

where the first factor σ_i of the product on the right hand side is the probability of a transmission commencement by an empty node, the second factor is the exponential function which is the probability that none of the i nonempty nodes and the other $(M - i - 1)$ empty nodes commence transmission during the collision window, and the third factor is the probability that $(j - i)$ out of $(M - i - 1)$ empty nodes become nonempty during the mean busy period.

From the explanation on the cause of unsuccessful transmission stated above, the expression for $\tilde{P}_2$ is broken into two parts: a) $k = 0$ and b) $k \geq 1$. For $k = 0$

$$\tilde{P}_{20} = \sigma_i \exp[-(M - i - 1)\lambda a](1 - \exp(-i\kappa a))bin(M - i - 1, j - i - 1, c),$$
$$1 \leq i \leq M - 1, i + 1 \leq j \leq M \qquad (5.35b)$$

where on the right hand side σ_i has the same interpretation as in (5.35a). The second and third factors can be interpreted as the probability that none of the other $(M - i - 1)$ empty nodes commence transmission during the collision window but at least one of the i nonempty nodes does. The fourth term is the probability that $(j - i - 1)$ out of $(M - i - 1)$ empty nodes become nonempty during the collision resolution period. For $k \geq 1$

$$\tilde{P}_{2k} = \sigma_i bin(M - i - 1, k, a)bin(M - i - 1 - k, j - i - 1 - k, c),$$
$$0 \leq i \leq M - 2, i + 2 \leq j \leq M, 1 \leq k \leq j - i - 1 \qquad (5.35c)$$

where σ_i has the same interpretation as stated above. The second factor is the probability that k out of $(M - i - 1)$ empty nodes commence transmission during the collision window and the third factor is the probability that $(j - i - 1 - k)$ out of $(M - i - 1 - k)$ empty nodes become nonempty during the collision resolution period. Combining (5.35b) and (5.35c) yields

$$\tilde{P}_2 = \sum_{k=0}^{j-i-1} \tilde{P}_{2k}$$

Similarly, we can write the expressions for $\tilde{P}_3$ and $\tilde{P}_4$ as

$$\tilde{P}_3 = (1-\sigma_i)\exp[-\{(M-i)\lambda + (i-1)\kappa\}a]bin(M-i, j-i+1, (E[B]-a)),$$
$$1 \le i \le M, 0 \le j \le M-1 \qquad (5.35d)$$

$$\tilde{P}_{40} = (1-\sigma_i)\exp[-(M-i)\lambda a](1-\exp[-(i-1)\kappa a])bin(M-i, j-i, c),$$
$$2 \le i \le M, i \le j \le M \qquad (5.35e)$$

$$P_{4k} = (1-\sigma_i)bin(M-i, k, a)bin(M-i-k, j-i-k, c),$$
$$1 \le i \le M-1, i+1 \le j \le M, 1 \le k \le j-i \qquad (5.35f)$$

and

$$\tilde{P}_4 = \sum_{k=0}^{j-i} \tilde{P}_{4k} .$$

The factors on the right hand side of (5.35d) - (5.35f) can be interpreted in the same way as those of $\tilde{P}_1$ and $\tilde{P}_2$. Note that in (5.35a) - (5.35f), $bin(y, x, \Delta)$ is a binomial distribution with parameters $(y, (1-\exp(-\lambda\Delta))$; furthermore, given that i nonempty nodes are in the network at epoch n, then from the assumptions of exponentially distributed message interarrival time and retransmission delay, $\sigma_i = (M-i)\lambda/[(M-i)\lambda + i\kappa]$. The transition probability between two embedded epochs is then given by

$$\psi_{ij} = \tilde{P}_1 + \tilde{P}_2 + \tilde{P}_3 + \tilde{P}_4 , \quad 0 \le i, j \le M . \qquad (5.36)$$

Observe that the limiting probabilities $\{f_i\}_{i=0}^{M}$ obtained from (5.34) are valid for all transmission completion epochs. In oder to identify those epochs which coincide with the successful transmission completion epochs, we can define a subset of the transition probabilities (5.35) as follows

$$Pr\{\tilde{Z}_{n+1} = j \text{ , } Success \mid \tilde{Z}_n = i\} = \tilde{P}_1 + \tilde{P}_3 \text{ , } 0 \le i \le M, 0 \le j \le M-1 \tag{5.37}$$

so that

$$\psi_j = \sum_{i=0}^{M} \lim_{n \to \infty} Pr\{\tilde{Z}_{n+1} = j \text{ , } Success \mid \tilde{Z}_n = i\} f_i \text{ .} \tag{5.38}$$

The limiting probability of successful transmission on the channel, γ, is then given by

$$\gamma = \sum_{j=0}^{M-1} \psi_j \text{ .} \tag{5.39}$$

Note that for simplification in (5.35a) and (5.35d), instead of the actual length of a busy period the mean busy period $E[B]$ has been used as an approximation. It remains now to determine $E[B]$.

5.3.4 Node Busy Period Distribution

A way of determining $E[B]$ is from the knowledge of the busy period distribution. From Fig. 5.1, the busy period B is the time interval from the end of a resting period until the nodal queue becomes empty. This is equal to the transmission time of all the packets (messages) present in the queue at the end of the resting period (original packets) as well as the transmission time of the messages which arrive during the transmission time of the original packets. Suppose that at the end of a resting period there are j packets in the nodal queue. Let the busy period generated by these j packets be denoted by B_j with distribution $B_j(t)$ and LST $B_j^*(\theta)$. It is shown in [49] that

$$B_j(t) = B_1^{*(j)}(t) \tag{5.40}$$

where $B_1(t)$ is the distribution of the busy period started by a single-packet message and $B_1^{*(j)}(t)$ is the j-fold convolution of $B_1(t)$ with itself. From the definition of busy period given above, it is clear that the busy period generated when $j = 1$ at the end of a resting period is equal to the transmission time of the original single packet plus the sub-busy periods generated by those messages which arrive during the successful transmission time of the original single packet. Expressed mathematically,

$$B_1(t) = \sum_{i=0}^{\infty} \int_{x=0}^{t} \sum_{k=0}^{i} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_i^{*(k)} B_i(t-x)dS(x) \tag{5.41}$$

where $g_i^{*(k)}$ is the k–fold convolution of g_i with itself, with $g_i^{*(0)} = \delta_{i0}$ - the Kronecker's delta function. Using (5.40) in (5.41) and then taking the LST of both sides we find that

$$B_1^*(\theta) = \int_{t=0}^{\infty} \sum_{i=0}^{\infty} \int_{x=0}^{t} \sum_{k=0}^{i} e^{-\theta t} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_i^{*(k)} B_1^{*(i)}(t-x)dS(x)dt \ .$$

By interchanging the order of integration

$$B_1^*(\theta) = \int_{x=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{i} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_i^{*(k)} dS(x) \int_{t=x}^{\infty} e^{-\theta t} B_1^{*(i)}(t-x)dt \ .$$

Now making the transformation $t - x = y$ and then using the convolution property, we get

$$B_1^*(\theta) = \int_{x=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{i} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_i^{*(k)} e^{-\theta x} (B_1^*(\theta))^i dS(x) \ .$$

Interchanging the order of summation and then simplifying gives

$$B_1^*(\theta) = S^*(\theta + \lambda - \lambda G(B_1^*(\theta))) \ . \tag{5.42}$$

Taking the LST on both sides of (5.40) and using the convolution property we have

$$B_j^*(\theta) = [B_1^*(\theta)]^j \ . \tag{5.43}$$

Thus by unconditioning (5.43) we obtain

$$B^*(\theta) = \sum_{j=1}^{\infty} B_j^*(\theta) Pr\{Q_R = j\} \tag{5.44}$$

where Q_R is the number of packets present in the queue at the end of a resting period, R_a, which in this case is defined as the time interval from the instant the node buffer becomes empty to the successful transmission commencement instant. Clearly, by the Poisson arrival process

$$Pr\{Q_R = j\} = \int_{x=0}^{\infty} \sum_{k=0}^{j} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_j^{*(k)} dR_a(x) \ . \tag{5.45}$$

Substituting (5.43) and (5.45) into (5.44) we get

$$B^*(\theta) = \sum_{j=1}^{\infty} [B_1^*(\theta)]^j \int_{x=0}^{\infty} \sum_{k=0}^{j} \frac{(\lambda x)^k e^{-\lambda x}}{k!} g_j^*(k) dR_a(x)$$

which after interchanging the order of summation and simplifying we finally obtain

$$\bar{B}^*(\theta) = R_a^*(\lambda - \lambda G(B_1^*(\theta))) \,. \tag{5.46}$$

$R_a^*(\cdot)$ in (5.46) is also given by (5.23) except that I_0 is now equal to the sum of T_A and the total delay before the first transmission commencement instant by a node (I_{0_a} in Fig. 5.2(a)). The LST of the distribution function of I_{0_a} is then derived to be

$$I_0^*(\theta) = \frac{\mu T_A^*(\theta)}{1 - (1-\mu) D^*(\theta)} \tag{5.47}$$

where $T_A^*(\theta)$ is given by $\lambda/(\theta + \lambda)$. By using (5.46) we obtain the mean node busy period as

$$E[B] = \lambda E[B_1] E[G] E[R_a] \tag{5.48}$$

But from (5.42), $E[B_1] = E[S] \,/\, (1 - \lambda E[G] E[S])$ so that

$$E[B] = \frac{\lambda E[G] E[S] E[R_a]}{1 - \lambda E[G] E[S]} \tag{5.49}$$

which is a function of the first moment of R_a. Equation (5.49) asserts that the node busy period will terminate provided $\lambda E[G] E[S]$ is less than unity which is a necessary and sufficient condition for system stability and this condition was alluded to earlier for the validity of Takac's Lemma (Section 5.3.1). By substituting (5.31) and (5.47) into (5.23), we obtain the first moment of R_a

$$E[R_a] = \frac{\kappa\mu + \lambda(1-\mu)}{\lambda\kappa\mu} + \left[\frac{1-\gamma}{\gamma}\right]\frac{(c+a)\kappa\mu + 1}{\kappa\mu} \tag{5.50}$$

Finally, the probability for the channel being idle is obtained from the standpoint of renewal theory. We see from Fig. 5.3 that the channel alternates between idle and busy periods, so let I_{ch} (B_{ch}) denote the channel idle (busy) period. If we suppose that a cycle is completed at the end of a transmission completion epoch, then the above constitutes an alternating renewal process. By a theorem of alternating renewal processes, the long-run proportion of time for the channel being idle, μ, is given by [42]

$$\mu = \frac{E[I_{ch}]}{E[I_{ch}] + E[B_{ch}]} = \frac{E[I_{ch}]}{E[I_{ch}] + \gamma(E[B] + a) + (1-\gamma)(c+a)}. \tag{5.51}$$

Suppose there are i nonempty nodes in the network immediately after a transmission completion epoch. Since the time interval between a transmission completion epoch and the next transmission commencement instant is exponentially distributed with mean $1/[(M-i)\lambda + i\kappa]$, then the unconditional expected idle period of the channel is given by

$$E[I_{ch}] = \sum_{i=0}^{M} \frac{f_i}{(M-i)\lambda + i\kappa}. \tag{5.52}$$

5.3.5 The Iterative Algorithm

The above analysis shows that there are no explicit expressions for the channel-activity parameters, γ and μ. In fact, from (5.39) and (5.51) both γ and μ depend on $\{f_i\}$; it is also seen from the definition of the transition probabilities that $\{f_i\}$ can be determined from the knowledge of the channel-activity parameters (via $E[B]$) - a node defined parameter). Also from (5.49) and (5.50), $E[B]$ can be obtained through the knowledge of the channel-activity parameters. The preceding statements demonstrate the coupling between the node defined parameter and the channel-activity parameters and we assert that this coupling approximately accounts for the interaction of the nodal queues. By principle 5 of the decomposition approximation, the values of γ and μ are determined iteratively using the following algorithm:

Step 1: Initialization of channel-activity parameters

- select $\gamma^{(0)}$ and $\mu^{(0)}$
- set iteration count $k = 1$

Step 2: Calculation of node parameters

- compute $E[R_a]$ from (5.50) and $E[B]$ from (5.49)

Step 3: Calculation of Transition Probabilities

- if $k = 1$, then compute all transition probabilities using (5.35a) - (5.35f), else compute $\tilde{P}_1$ from (5.35a) and $\tilde{P}_3$ from (5.35d)

Step 4: Calculation of steady-state probabilities

- compute $\underline{F}$ from (5.34)

Step 5: Calculation of $\gamma^{(k)}$ and $\mu^{(k)}$

- compute $\gamma^{(k)}$ from (5.39) and $\mu^{(k)}$ from (5.51)

Step 6: Test for convergence

- if $\max\{|\gamma^{(k)} - \gamma^{(k-1)}|/\gamma^{(k)}, |\mu^{(k)} - \mu^{(k-1)}|/\mu^{(k)}\} < \varepsilon$, then stop

Step 7: Otherwise increase the iteration count and continue

- $k = k + 1$ and go to Step 2.

When convergence has been reached, we then compute the first two moments of the resting period from (5.24) and (5.25) and subsequently the mean queue length from (5.13) and the mean packet waiting time from (5.21) as well as the channel throughput, S_{ch}, which is defined as

$$S_{ch} = \frac{\gamma E[B]}{E[I_{ch}] + \gamma(E[B] + a) + (1 - \gamma)(c + a)} . \qquad (5.53)$$

5.3.6 Numerical Results and Discussion

In this section we show the usefulness of the approximate analytic model presented above by an illustrative example. The main objective is to investigate the effects of the mean retransmission rate, the average message size and the message length distribution on the network performance measures such as the mean packet waiting time, the mean message response time and the channel throughput. In addition, a computer simulation model is constructed whose results serve to validate the results obtained from the approximate analysis. The following input parameters were selected:

Number of nodes in the network, M: 20.

Normalized propagation delay, a: 0.05 *slots*.

Collision resolution period, c: 0.1 *slots*.

Packet transmission time, S: 1 *slot*.

Except otherwise stated, the message length is assumed to follow a geometric distribution.

During the computation of the analytic results, the relative error test tolerance for convergence of the iterative algorithm was chosen as 10^{-6}, and it was observed that on average less than 50 iterations were required to reach convergence for all the parameters considered. The simulation model constructed was a discrete-event type and for each simulation experiment, 10 independent runs were performed where each run was terminated after 50000 multipacket messages have been successfully transmitted; however, the simulation outputs were gathered over the last 25000 messages to allow for the transient effect. The estimated values of the performance measures were obtained by averaging the 10 values associated with the 10 independent replications and the approximate 95% confidence intervals were constructed by assuming a *Student−t* distribution with 9 degrees of freedom.

Fig. 5.5 shows the mean packet waiting time - throughput characteristics for various values of retransmission rate, κ and mean message length, $E[G] = 5$ packets. Note the excellent agreement between the approximate analytic and simulation results which are exhibited under low to medium throughput; the lesser agreement in the high throughput range is due to the congestion of the channel. We observe that for a fixed value of $\kappa (\kappa \geq 1)$ and for low to medium throughput, there is on average one message in the buffer and the mean waiting time of an arbitrary

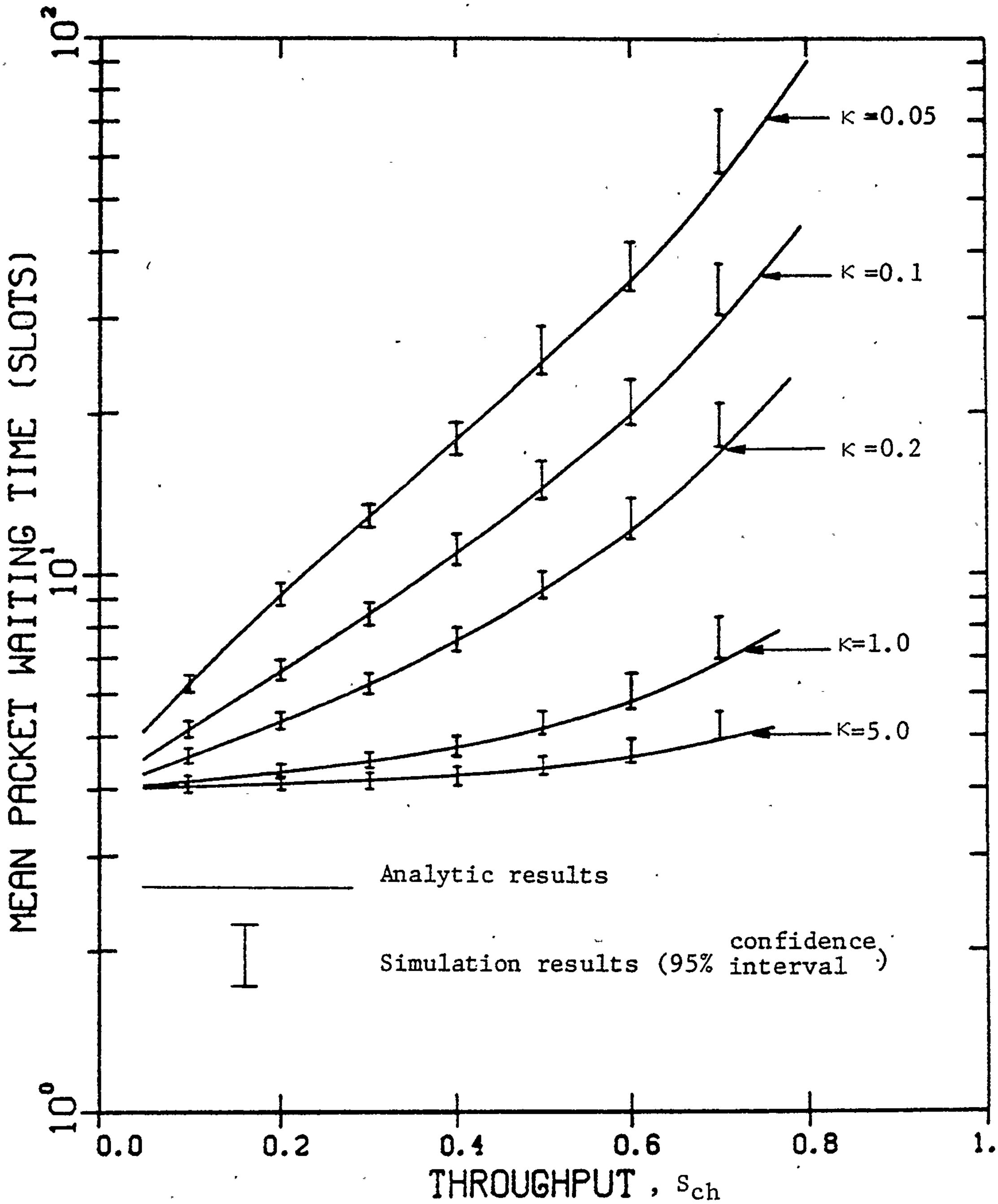


Fig. 5.5. Mean packet waiting time - throughput characteristics

(M = 20,E[G] = 5 packets, Geometric distribution) packet, $E[W]$, is dominated by the waiting time due to the packet's position in its message. In the high throughput range (that is high input traffic), the mean packet waiting time increases and the large delay is accounted for by the retransmission delay (due to numerous collisions) as well as the waiting time of the message containing the test packet where the message waiting time comprises the transmission time of all the queued messages ahead of the test packet's message and the access delay of the head of queued messages. We also see from Fig. 5.5 that the mean packet waiting time decreases as κ increases, which is due to the lower mean retransmission delay.

In Table 5.1 we show the dependence of the mean message response time (time interval from message arrival instant at the source node to the reception instant at the destination node) on the average message size, $E[G]$, where the total input traffic to the network, $\lambda_T(=M\lambda E[G]E[S])$ is the varying parameter and $\kappa = 1$. For a fixed value of λ_T, we see that an increase in $E[G]$ leads to an increase of the mean message response time. Actually for low to medium input traffic when there is relatively no queueing of messages, the mean message response time is approximately equal to the average transmission time of a message ($E[G]$ slots). However, at high input traffic, the number of collisions on the channel increases, thus the additional delay to the message transmission time is largely due to the retransmission delay of the collided messages. We observe in Table 5.2 that for a specified λ_T , the effect of increasing $E[G]$ is to slightly decrease the throughput and this is more pronounced at high values of λ_T; however, the

Table 5.1. Effect of E[G] on Message Response Time.

$E[G]$ \ λ_T	0.1	0.2	0.4	0.6	0.8
	MEAN MESSAGE RESPONSE TIME (slots)				
2	2.1258	2.2833	2.7606	3.7273	6.5886
5	5.1368	5.3034	5.7852	6.6931	8.9779
10	10.1605	10.3505	10.8756	11.8091	13.9945
20	20.2101	20.4498	21.0739	22.0997	24.323

Table 5.2. Effect of E[G] on Channel Throughput.

$E[G]$ \ λ_T	0.1	0.2	0.4	0.6	0.8
	CHANNEL THROUGHPUT				
2	0.10	0.1998	0.3976	0.5895	0.7709
5	0.10	0.1998	0.3976	0.5894	0.7664
10	0.10	0.1997	0.3975	0.5893	0.7652
20	0.10	0.1997	0.3975	0.5893	0.7647

throughput tends to a limiting value at higher values of $E[G]$.

Next, we investigate the effect of the message length distribution on the mean packet waiting time - throughput characteristics. Apart from the popular geometric distribution that is usually employed for modeling message lengths, here we also assume the message lengths to follow a low variance (relative to geometric) truncated Poisson distribution which is defined by $g_i = \Lambda^i/(i!(e^{\Lambda} - 1))$, $i \geq 1$, $\Lambda > 0$. Fig. 5.6 depicts the mean packet waiting time - throughput characteristics obtained when $\kappa = 0.1$ and 1.0, and $E[G] = 5$ packets. We see that for each value of κ and under low to medium throughput, there is a marked difference between the waiting time-throughput curves for the two distributions considered. The difference is explained by the value of the mean waiting time due to the position of an arbitrary packet in a message, $E[W_g]$, which is calculated to be 4 slots and 2.48 slots respectively for the geometric and truncated Poisson distributions (using the first moment of (5.19) and $E[G] = 5$ packets). However, in the high throughput range when the mean packet waiting time is now dominated by the retransmission delay, the mean packet waiting time for the two message length distributions tends to the same value. Note that in the high throughput range, the curves for the case $\kappa = 0.1$ are closer than that for $\kappa = 1$ because of the high mean retransmission delay (10 slots for the former and 1 slot for the latter).

The next set of results deals with the choice of κ which will ensure system stability for a specified network input traffic. Fig. 5.7 shows the mean packet wait-

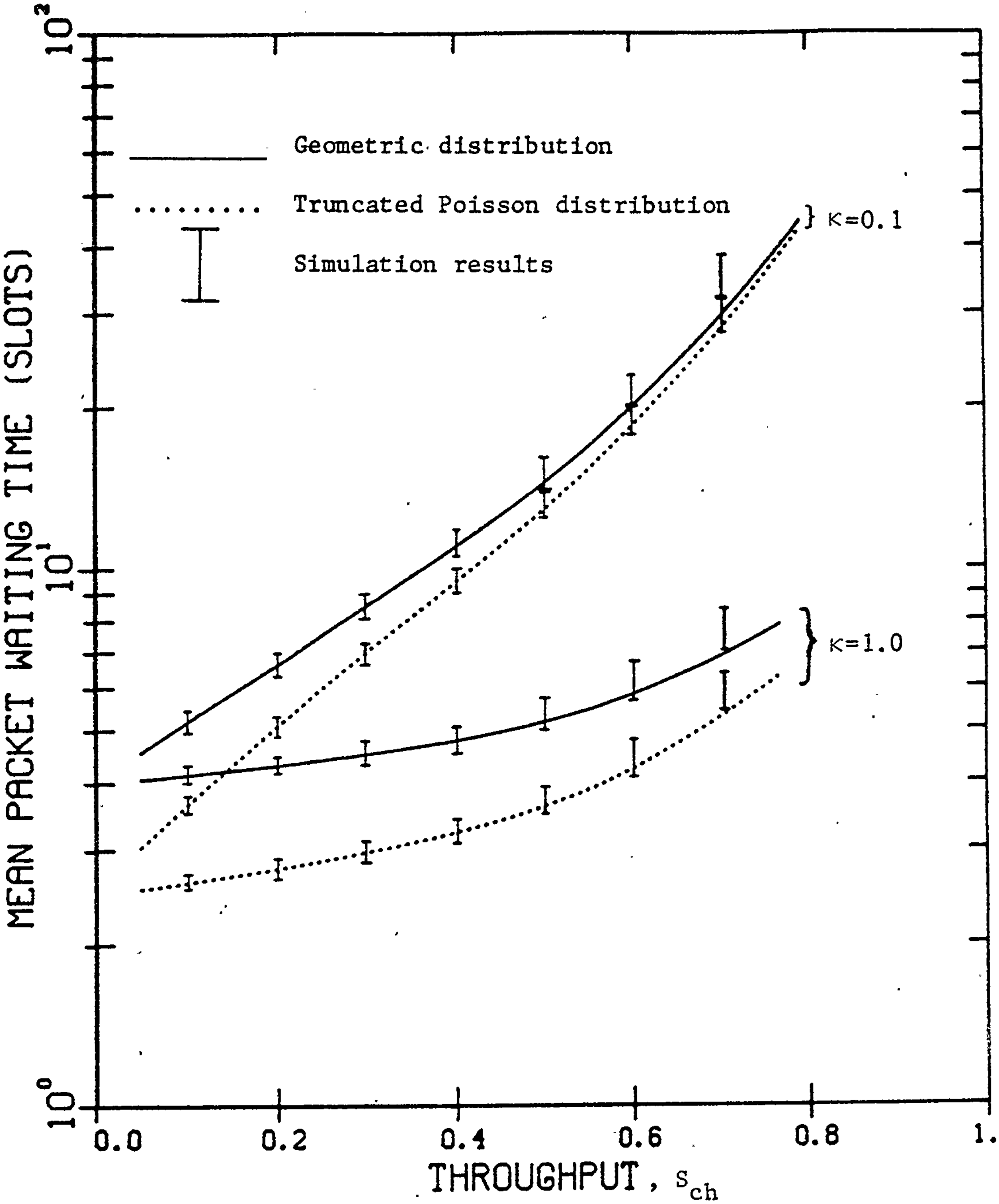


Fig.5.6. Mean Packet waiting time - throughput characteristics
(M = 20, E[G] = 5 packets)

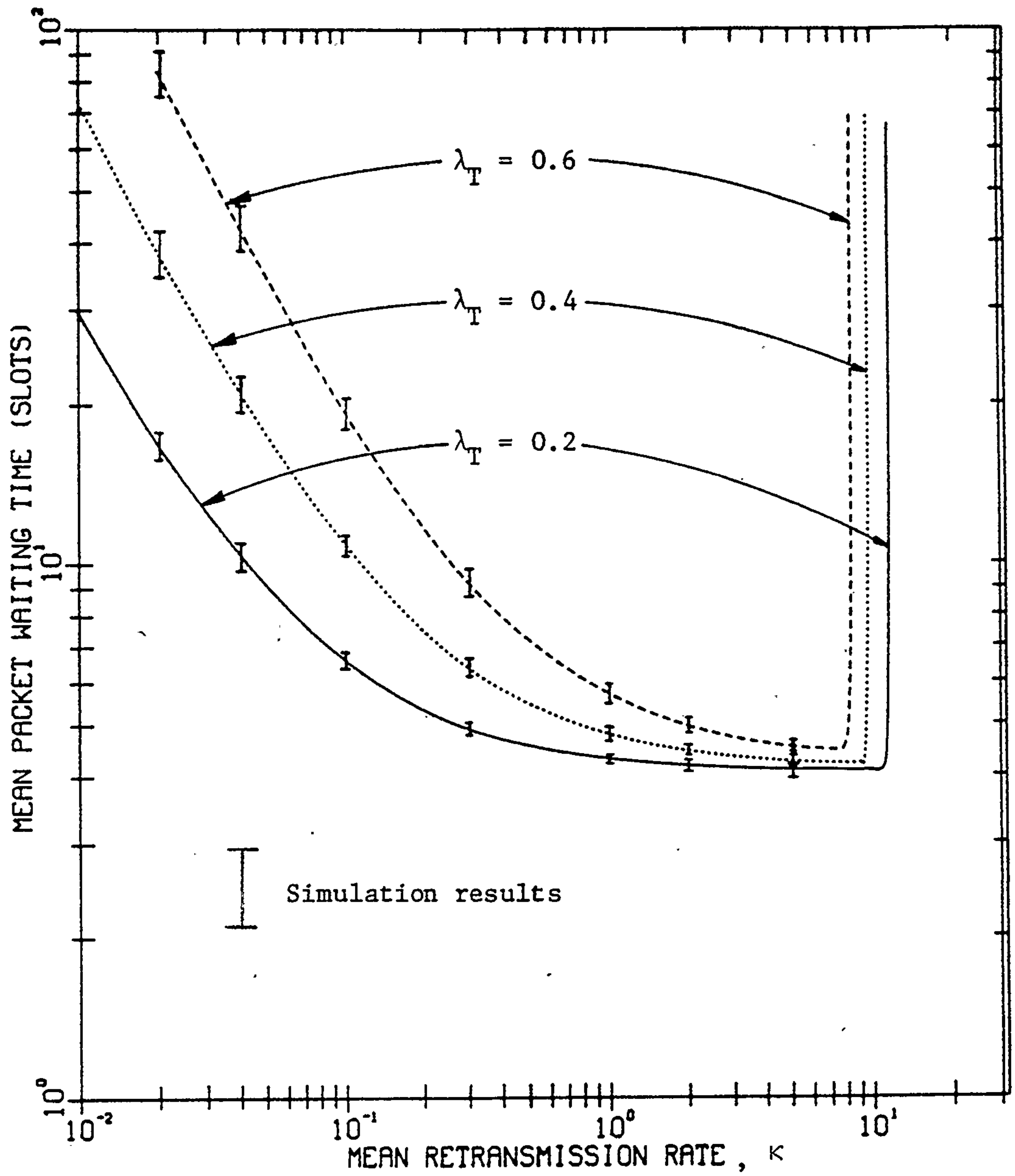


Fig.5.7. Mean packet waiting time Vs. mean retransmission rate (M = 20, E[G] = 5 packets, Geometric distribution)

ing time - mean retransmission rate characteristics when $M = 20$, $E[G] = 5$ packets, and $\lambda_T = 0.2$, 0.4 and 0.6. We see that at a given value of λ_T, very small values of κ leads to large mean packet waiting time. As κ is increased, the mean packet waiting time decreases until a minimum is reached at κ_{opt}, the optimum value of κ. A larger κ than κ_{opt} leads to system instability which is exemplified by very large mean packet waiting time and (although not shown) a decrease of throughput to very small values [94]. From Fig. 5.7 note that as λ_T increases three observations are evident; first, the acceptable region of mean retransmission rate becomes smaller; second, the value of κ_{opt} decreases, $\kappa_{opt} \approx 10$, 9, and 8 for $\lambda_T = 0.2$, 0.4, and 0.6 respectively; and thirdly, the minimum mean packet waiting time increases. From these observations, we conclude that κ_{opt} cannot assume a single value over a specified range of input traffic; it follows then that the retransmission rate must be tracked as the input traffic changes so as to maintain system stability. The minimum mean packet waiting time obtained for this example is shown in Fig. 5.8.

Finally, in Fig. 5.9 we present the mean packet waiting time - retransmission rate characteristics for $\lambda_T = 0.7$, $E[G] = 5$ packets and three values of M: 5, 10 and 20. Note that the three observations made from Fig. 5.7 also apply here. These observations follow from the fundamental characteristic of a random access channel: under heavy traffic condition, an increase in the number of nodes accessing the channel leads to an increase in the number of collisions on the channel and

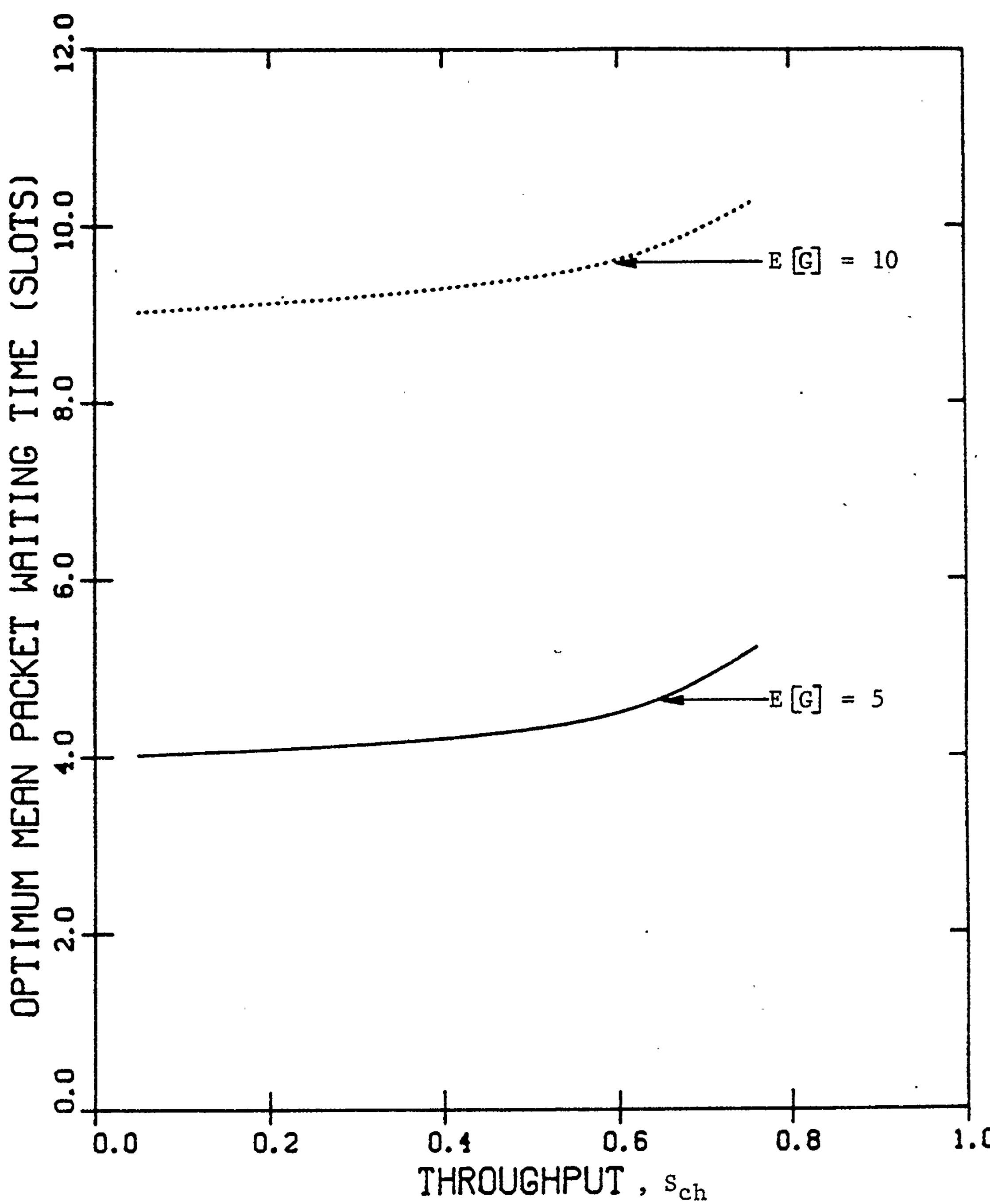


Fig. 5.8. Minimum mean packet waiting time - throughput characteristics (M = 20, Geometric Distribution)

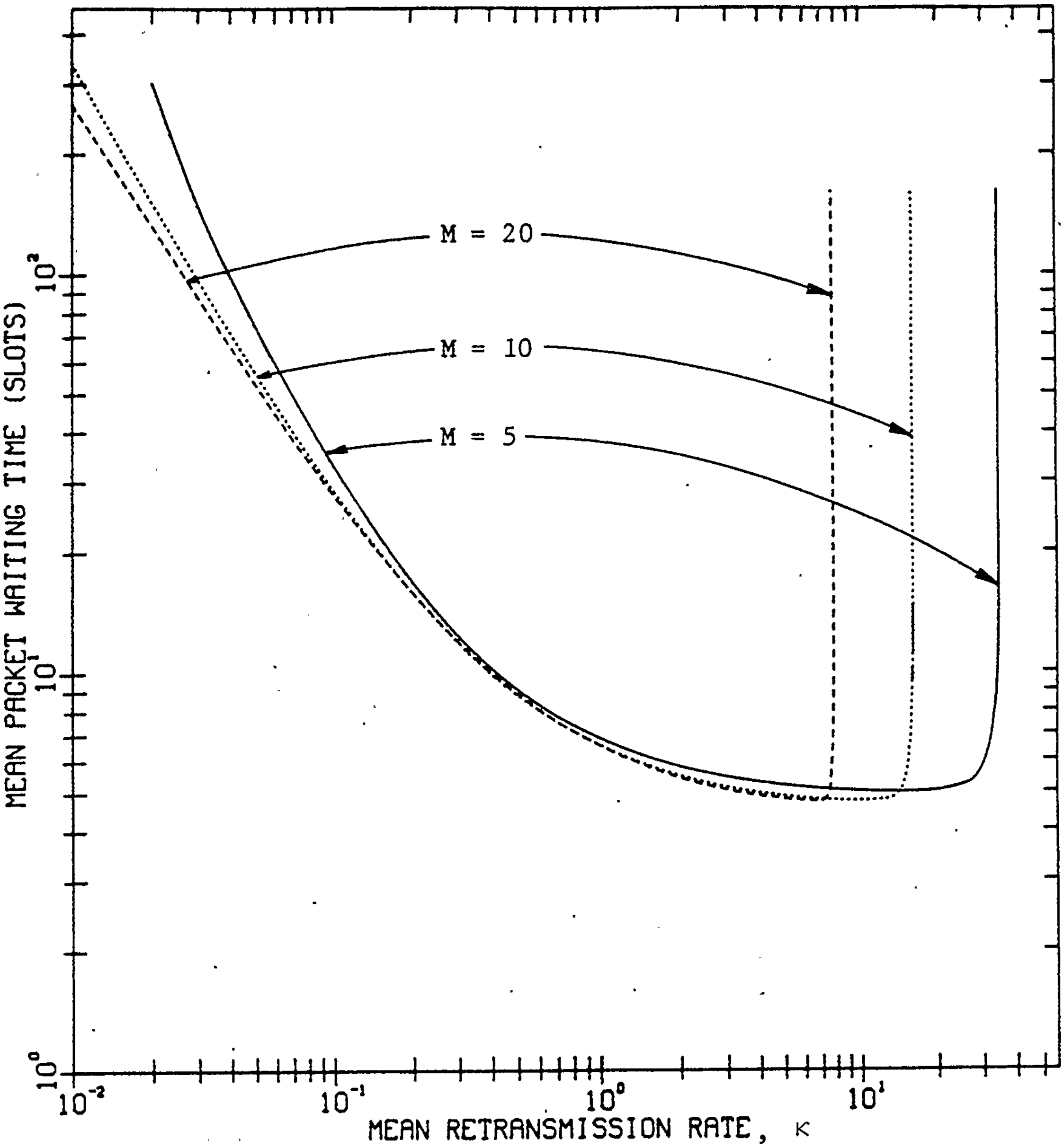


Fig. 5.9. Mean packet waiting time Vs. mean retransmission rate

($E[G] = 5$ packets, Geometric distribution) as we have noted before, the net effect is a high mean packet waiting time. From this explanation, we conclude also that the number of nodes in the network has considerable effect on the value of the retransmission rate which should be reasonably small in order to maintain system stability.

5.4 Summary

In this chapter we have presented the analysis of a CSMA-CD LAN with nodes capable of buffering multipacket messages where each message length may follow any arbitrary probability distribution. The central idea of the analytical technique presented is the decomposition approximation in which each node in the network is independently modeled as an $M^X/G/1$ queueing system with busy and resting periods and the interaction among the nodes is accounted for approximately by the channel-activity parameters which are determined from an iterative algorithm.

We find from the numerical results obtained that the mean packet (message) waiting time is sensitive to the message length statistics as well as to the message length distribution. Furthermore, the study in this chapter reveals that in the presence of variations in the input traffic and/or the number of nodes in the network, the choice of the retransmission rate has a definite effect on the system stability.

CHAPTER 6

INTERCONNECTED BUFFERED LANs WITH MULTIPACKET MESSAGES

6.1 Introduction and Problem Statement

Thus far, we have studied only single buffered random access LANs. The problem with such isolated LANs is that necessary future upgradability may be hampered due to inherent network design limitations. Two examples of such limitations are noteworthy. First, as we have found from the previous Chapters, a single random access LAN to which a very large number of nodes are connected leads to poor performance, and this necessitates limiting the number of network nodes to a reasonable number so as to attain acceptable performance level or reducing the traffic generated per node. Second, a high performance level can be achieved if the geographic coverage of random access LANs is restricted to short distances, for example, the Ethernet standard specifies a segment length of 500 *meters*. A way to overcome the above limitations is by interconnecting LANs together, where the interconnection may be implemented either at the data link layer (with a bridge) or the network layer (with a router) of the ISO/OSI reference model [95]. Note that at the present time, there is no consensus on which of these two interconnecting devices is more efficient, see [95] for details of differing views. Nonetheless, for our purpose here, we shall assume that the interconnection is accomplished with a bridge.

Simplistically, a bridge is a store-and-forward device that interconnects LANs and possesses the transparency (intercommunication among nodes connected to different LANs as if the nodes were on the same LAN) and filtering (isolation of a LAN from traffic which does not need to traverse that LAN) properties. The problem of interest in this chapter then is the performance analysis of bridged CSMA-CD LANs. We note that by appropriate modeling of the individual LANs and the bridge, the analysis of the interconnected system is performed by applying some of the results that are already obtained in the preceding chapters. The very few previous studies on interconnected random access networks are found in [96-99], where for reasons of analytic tractability, all have assumed that nodes on each LAN generate (and store) only single-packet messages. The study presented here differs from the above studies in that the more realistic assumption where each node generates (and stores) multipacket messages is made [100].

6.2 Model Formulation

Consider two CSMA-CD LANs (labeled 1 and 2) as depicted by Fig. 6.1. Each LAN is structured and operates as those studied in Chapter 4, hence the modeling and analysis therein (Section 4.3.1) also apply here. However, for identification purposes, we denote the mean message arrival rate to each node of LAN_i (i = 1, 2) by λ_i and the retransmission rate by κ_i.

To enable stations on one network to communicate with stations on the other, a bridge is introduced. The bridge is capable of receiving messages on one net-

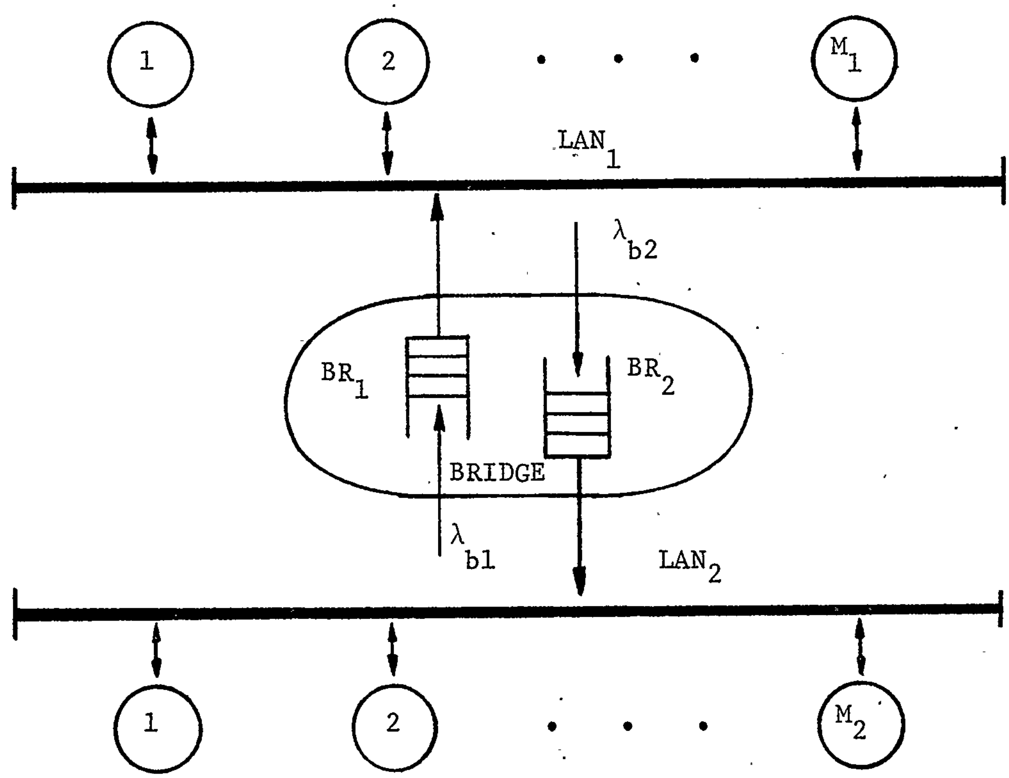
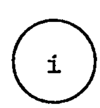


Fig. 6.1. Schematic Representation of interconnected LANs



Station i

work and transmitting them on the other. Such internetwork messages to be thus conveyed are identified by the bridge via an address that is included in each message. It follows then that the bridge consists of two halves: one receiving on network 2 and transmitting on network 1, denoted BR_1; and the other in the reverse direction, denoted BR_2; these two halves operate independently (Fig. 6.1). Since messages may arrive at the bridge faster than they can be transmitted, the bridge must contain buffer space in which messages are temporarily stored. For analytic simplicity, it is assumed that the bridge buffer space is unlimited and internetwork messages arrive at the bridge buffer in accordance with a Poisson process having rate λ_{bi} $(i = 1, 2)$. The arriving internetwork messages are added to the bridge's buffer where the transmission (service) discipline is first-come, first-served. The bridge adheres to the same transmission protocol that the stations on the individual LANs adopt except that the bridge, when it gains channel access right continues to transmit until its buffer goes empty, the reason being to minimize bridge buffer congestion; aside from this, no other preference is given to the bridge. However, if the bridge is involved in a collision, the colliding message will be retransmitted after an exponentially distributed delay having a mean of $1/\kappa_{bi}$.

The above description for the bridge closely fits that of a node in the LANs considered in Chapter 5; thus, the activity of the bridge is modeled as a sequence of cycles each consisting of an alternating busy and resting periods. Then, under the assumption of Poisson (internetwork) message arrival process, the bridge behavior is approximately modeled by an $M^X/G/1$ queueing system with busy and resting periods. In the spirit of the decomposition approximation, the interaction of the bridge with the nodes connected to LAN$_i$ is then approximated by the channel-activity parameters which, as we have seen before are determined iteratively.

6.3 Performance Analysis

The first task in the analysis is the evaluation of the input rates to the two independent halves of the bridge, denoted by λ_{b1} and λ_{b2}. Note that $\lambda_{bi}(i = 1, 2)$ is the rate at which messages to be forwarded to LAN$_i$ (from LAN$_j$, $j = 1, 2$) are arriving at BR_i (Fig. 6.1). Furthermore, observe that the normalized mean arrival rate of messages to be forwarded to LAN$_i$ is given by $E[G']\lambda_{bi}$, $E[G']$ being the average transmission time (in slots) of a message. Now, let S_{chj} be the normalized throughput generated by all stations on LAN$_j$ of which a fraction α_{ji} is to be conveyed to LAN$_i$. Then, under equilibrium condition, the input and output flow rates are equal, that is

$$E[G']\lambda_{bi} = \alpha_{ji} S_{chj} \qquad i, j = 1, 2 ; i \neq j \tag{6.1}$$

Alternatively, the expression for S_{chj} can be derived independently in terms of λ_{bj}. It then follows that in principle we can solve (6.1) simultaneously for λ_{b1} and λ_{b2}; but unfortunately, the determination of the explicit expression for S_{chj} (in terms of λ_{bj}) is formidable so that the algebraic solution of (6.1) seems impossible. This difficulty is overcome by computing λ_{b1} and λ_{b2} numerically using the iterative algorithm given in Section 6.3.2.

6.3.1 Throughput Analysis

In this section, we derive expressions for the throughput of an isolated LAN, S'_{chj} as well as that of interconnected LANs, S_{chj}.

(i) Analysis of S'_{chj}: S'_{chj} is the throughput generated by the stations connected to LAN_j assuming BR_j is deactivated, that is $\lambda_{bj} = 0$. The expression for S'_{chj} is derived from the standpoint of renewal theory [43-44]. Specifically, if we define a transmission cycle as the time interval between two successive transmission completion epochs on the channel, then by the limiting theorem of alternating renewal processes, S'_{chj} is expressed as

$$S'_{chj} = \frac{\gamma E[G']}{E[I_{ch}] + \gamma(E[G'] + a) + (1-\gamma)(c+a)} \tag{6.2}$$

where γ is the probability of successful transmission on the channel and is given by

$$\gamma = \sum_{j=0}^{M-1} \Phi_j \tag{6.3}$$

the Φ_j's being obtained from (4.16). $E[I_{ch}]$ is the mean length of channel idle period which is computed from

$$E[I_{ch}] = \sum_{m=0}^{M} w_m f_m \tag{6.4}$$

where w_m is given by (4.23) and the f_m's are obtained from (4.13). Recall from the previous chapters that a and c in (6.2) are respectively the normalized propagation delay and collision resolution period.

(ii) Analysis of S_{chj}: By adopting similar concept used above in writing (6.2), the throughput generated by stations on LAN$_j$ of an interconnected system, S_{chj}, is given by

$$S_{chj} = \frac{\gamma_s E[G']}{E[I_{ch}] + \gamma_s (E[G'] + a) + \gamma_b (E[B_b] + a) + (1 - \gamma_s - \gamma_b)(c + a)} \tag{6.5}$$

where γ_s (γ_b) is the probability of successful transmission by a station (bridge), and $E[B_b]$ is the mean successful transmission period by the bridge, all of which remain to be determined. Note that the expression for $E[I_{ch}]$ (6.4) is no longer valid in (6.5) because now the bridge is active and its effect is to reduce the throughput generated by the stations on LAN$_j$ (with an attendant increase in delay); this of course is the rationale for the approximation made in Step 3 of the iterative algorithm (Section 6.3.2). The evaluation of the unknown quantities now proceeds as follows.

First, observe that the effective number of nodes contending for access to the channel of LAN$_j$ is now M+1 comprising the M original stations plus the bridge. Since the two halves of the bridge operate independently, we shall focus on LAN$_j$

to which one half of the bridge is connected and henceforth refer to this as the system. Let $\tilde{Z}_n$ denote the number of nonempty nodes in the system immediately after the n^{th} transmission completion epoch. As noted in Chapter 5, the process $\{\tilde{Z}_n, n \geq 1\}$ evolves as a Markov chain so that the next task is to determine its state transition probabilities. In addition to the explanation of Section 5.3.3 for writing the ψ_{ij}'s, it is important to note here that the state transition probability between epoch n and epoch $n+1$ also depends on the state of the bridge (empty or nonempty) at epoch n. The distinction between the bridge and any ordinary station is necessary here because of the difference in the message arrival rates (and also the difference between the retransmission rates). Based on the above, the transition probability between two successive embedded epochs, ψ_{ij} is now defined as

$$\psi_{ij} = E[Pr\{\tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i, I_{BR}\}] \ , \ 0 \leq i, j \leq M+1 \tag{6.6}$$

where the expectation is with respect to I_{BR}, a binary-valued random variable for the state of the bridge,

$$I_{BR} = \begin{cases} 0 \text{ if } BR \text{ is empty} \\ 1 \text{ if } BR \text{ is nonempty} \end{cases} \tag{6.7}$$

and BR is a generic notation for BR_1 or BR_2. For brevity, denote $Pr\{\tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i, I_{BR} = 0\}$ by $[\psi_{ij} \mid I_{B0}]$ and similarly $Pr\{\tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i, I_{BR} = 1\}$ by $[\psi_{ij} \mid I_{B1}]$. In order to further simplify the description of $[\psi_{ij} \mid I_{B0}]$ and $[\psi_{ij} \mid I_{B1}]$, we introduce the following notations:

$S^e(S^n)$: Transmission commencement by a station which was in the empty (nonempty) state at epoch n

$B^e(B^n)$: Transmission commencement by the bridge which was in the empty (nonempty) state at epoch n.

$T^s(T^c)$: Transmission is successful (unsuccessful).

A_k: Within a time units after the beginning of an ongoing transmission period, k stations which were in the empty state at epoch n begin transmission.

Using the above notations, the components of $[\psi_{ij} \mid I_{B0}]$ are defined below:

$$\psi_{ij}^{bse} = Pr\{B^e \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$
$$\psi_{ij}^{ese} = Pr\{S^e \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$
$$\psi_{ij}^{nse} = Pr\{S^n \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$
$$\psi_{ij}^{bce} = Pr\{B^e \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$
$$\psi_{ij}^{ece} = Pr\{S^e \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$
$$\psi_{ij}^{nce} = Pr\{S^n \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

The explicit expressions for the above transition probabilities are (to reduce the complexity of notation, we shall henceforth assume that the two LANs are identical so that $\lambda_1 = \lambda_2 = \lambda$, $\kappa_1 = \kappa_2 = \kappa$, $\lambda_{b1} = \lambda_{b2} = \lambda_b$, and $\kappa_{b1} = \kappa_{b2} = \kappa_b$)

$$\psi_{ij}^{bse} = \tilde{\sigma}_i \tilde{\sigma}_b \exp[-\{(M-i)\lambda + i\kappa\}a]\, bin(M-i, j-i, E[B_b]),$$

$$0 \le i,\, j \le M \qquad (6.8a)$$

$$\psi_{ij}^{ese} = \tilde{\sigma}_i(1-\tilde{\sigma}_b)\exp[-\{(M-i-1)\lambda + \lambda_b + i\kappa\}a]\{bin(M-i-1, j-i, E[G'])$$

$$\cdot \exp(-\lambda_b E[G']) + bin(M-i-1, j-i-1, E[G'])(1-\exp(-\lambda_b E[G']))\},$$

$$0 \le i \le M-1,\, 0 \le j \le M \qquad (6.8b)$$

$$\psi_{ij}^{nse} = (1-\tilde{\sigma}_i)\exp[-\{(M-i)\lambda + \lambda_b + (i-1)\kappa\}a]\{bin(M-i, j-i+1, E[G'])$$

$$\cdot \exp(-\lambda_b E[G']) + bin(M-i, j-i, E[G'])(1-\exp(-\lambda_b E[G']))\},$$

$$1 \le i \le M,\, 0 \le j \le M \qquad (6.8c)$$

$$\psi_{ij,0}^{bce} = \tilde{\sigma}_i \tilde{\sigma}_b \exp[-(M-i)\lambda a](1-\exp(-i\kappa a))\, bin(M-i, j-i-1, c),$$

$$1 \le i \le M,\, i+1 \le j \le M+1 \qquad (6.8d)$$

$$\psi_{ij,k}^{bce} = \tilde{\sigma}_i \tilde{\sigma}_b\, bin(M-i, k, a)\, bin(M-i-k, j-i-1-k, c),$$

$$0 \le i \le M-1,\, 2 \le j \le M+1;\, 1 \le k \le j-i-1 \qquad (6.8e)$$

$$\psi_{ij,0}^{ece} = \tilde{\sigma}_i(1-\tilde{\sigma}_b)\exp[-\{(M-i-1)\lambda + \lambda_b\}a](1-\exp(-i\kappa a))\{bin(M-i-1, j-i-1, c)$$

$$\cdot \exp(-\lambda_b c) + bin(M-i-1, j-i-2, c)(1-\exp(-\lambda_b c))\},$$

$$1 \le i \le M-1,\, i+1 \le j \le M+1 \qquad (6.8f)$$

$$\psi_{ij,k}^{ece} = \tilde{\sigma}_i(1-\tilde{\sigma}_b)[bin(M-i-1,\, k, a)\exp(-\lambda_b a)\{bin(M-i-1-k,\, j-i-1-k,\, c)$$

$$\cdot \exp(-\lambda_b c) + bin(M-i-1-k,\, j-i-2-k,\, c)(1-\exp(-\lambda_b c))\}$$

$$+ bin(M-i-1,\, k-1,\, a)(1-\exp(-\lambda_b a))bin(M-i-k,\, j-i-1-k,\, c)],$$

$$0 \le i \le M-1,\, 2 \le j \le M+1;\, 1 \le k \le j-i-1 \qquad (6.8g)$$

$$\psi_{ij,0}^{nce} = (1-\tilde{\sigma}_i)\exp[-\{(M-i)\lambda + \lambda_b\}a](1-\exp[-(i-1)\kappa a])\{bin(M-i,\, j-i,\, c)$$

$$\cdot \exp(-\lambda_b c) + bin(M-i,\, j-i-1,\, c)(1-\exp(-\lambda_b c))\},$$

$$2 \le i \le M,\, i \le j \le M+1 \qquad (6.8h)$$

$$\psi_{ij,k}^{nce} = (1-\tilde{\sigma}_i)[bin(M-i,\, k,\, a)\exp(-\lambda_b a)\{bin(M-i-k,\, j-i-k,\, c)\exp(-\lambda_b c)$$

$$+ bin(M-i-k,\, j-i-1-k,\, c)(1-\exp(-\lambda_b c))\}$$

$$+ bin(M-i,\, k-1,\, a)(1-\exp(-\lambda_b a))bin(M-i-k+1,\, j-i-k,\, c)],$$

$$1 \le i \le M-1,\, 2 \le j \le M+1;\, 1 \le k \le j-i\ . \qquad (6.8i)$$

Thus,

$$[\psi_{ij}|I_{B\,0}] = \psi_{ij}^{bse} + \psi_{ij}^{ese} + \psi_{ij}^{nse} + \sum_{k=0}^{j-i-1}\psi_{ij,k}^{bce} + \sum_{k=0}^{j-i-1}\psi_{ij,k}^{ece} + \sum_{k=0}^{j-i}\psi_{ij,k}^{nce}\ . \qquad (6.9)$$

Similarly, the definition of the transition probabilities given that the bridge is nonempty, $[\psi_{ij}\,|I_{B\,1}]$ can be stated thusly

$$\psi_{ij}^{bsn} = Pr\{B^n \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\psi_{ij}^{esn} = Pr\{S^e \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\psi_{ij}^{nsn} = Pr\{S^n \cap T^s \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\psi_{ij}^{bcn} = Pr\{B^n \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\psi_{ij}^{ecn} = Pr\{S^e \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

$$\psi_{ij}^{ncn} = Pr\{S^n \cap T^c \cap A_k \cap \tilde{Z}_{n+1} = j \mid \tilde{Z}_n = i\}$$

The expressions for the above transition probabilities can then be written as

$$\psi_{ij}^{bsn} = \upsilon_i \upsilon_b \exp[-\{(M-i+1)\lambda + (i-1)\kappa\}a]\; bin(M-i+1, j-i+1, E[B_b]),$$
$$1 \le i \le M+1,\ 0 \le j \le M \qquad (6.10a)$$

$$\psi_{ij}^{esn} = (1-\upsilon_i)\exp[-\{(M-i)\lambda + (i-1)\kappa + \kappa_b\}a]\; bin(M-i, j-i, E[G']),$$
$$1 \le i, j \le M \qquad (6.10b)$$

$$\psi_{ij}^{nsn} = \upsilon_i(1-\upsilon_b)\exp[-\{(M-i+1)\lambda + (i-2)\kappa + \kappa_b\}a]$$
$$.\ bin(M-i+1, j-i+1, E[G']),\quad 2 \le i \le M+1,\ 1 \le j \le M \qquad (6.10c)$$

$$\psi_{ij,0}^{bcn} = \upsilon_i \upsilon_b \exp[-(M-i+1)\lambda a](1-\exp[-(i-1)\kappa a])bin(M-i+1, j-i, c),$$
$$2 \le i, j \le M+1 \qquad (6.10d)$$

$$\psi_{ij,k}^{bcn} = \upsilon_i \upsilon_b binM-i+1, k, a)\; bin(M-i+1-k, j-i-k, c),$$
$$1 \le i \le M,\ 2 \le j \le M+1;\ 1 \le k \le j-i \qquad (6.10e)$$

$$\psi_{ij,0}^{ecn} = (1-\upsilon_i)(1-\exp[-\{(i-1)\kappa+\kappa_b\}a])\exp[-(M-i)\lambda a]$$

$$\cdot\, bin(M-i,\, j-1,\, c)\,, \quad 1 \le i \le M,\, 2 \le j \le M+1 \qquad (6.10f)$$

$$\psi_{ij,k}^{ecn} = (1-\upsilon_i) bin(M-i,\, k,\, a)\, bin(M-i-k,\, j-i-1-k,\, c),$$

$$1 \le i \le M-1,\, 2 \le j \le M+1;\, 1 \le k \le j-i-1 \qquad (6.10g)$$

$$\psi_{ij,0}^{ncn} = \upsilon_i(1-\upsilon_b)(1-\exp[-\{(i-2)\kappa+\kappa_b\}a])\exp[-(M-i+1)\lambda a]$$

$$\cdot\, bin(M-i+1,\, j-i,\, c),\quad 2 \le i,\, j \le M+1 \qquad (6.10h)$$

$$\psi_{ij,k}^{ncn} = \upsilon_i(1-\upsilon_b)\, bin(M-i+1,\, k,\, a) bin(M-i+1-k,\, j-i-k,\, c),$$

$$2 \le i \le M,\, 2 \le j \le M+1);\, 1 \le k \le j-i \qquad (6.10i)$$

so that

$$[\psi_{ij} | I_{B1}] = \psi_{ij}^{bsn} + \psi_{ij}^{esn} + \psi_{ij}^{nsn} + \sum_{k=0}^{j-i} \psi_{ij,k}^{bcn} + \sum_{k=0}^{j-i-1} \psi_{ij,k}^{ecn} + \sum_{k=0}^{j-i} \psi_{ij,k}^{ncn}\,. \qquad (6.11)$$

The probabilities $\tilde{\sigma}_i$, $\tilde{\sigma}_b$, υ_i and υ_b introduced in (6.8) and (6.10) are defined respectively as follows:

$\tilde{\sigma}_i$ = the probability of a transmission commencement by an idle node before any of the i nonempty stations given that the bridge node is empty

$$= \frac{(M-i)\lambda + \lambda_b}{(M-i)\lambda + \lambda_b + i\kappa} \qquad (6.12a)$$

$\tilde{\sigma}_b$ = probability of a transmission commencement by the bridge node before any of the i nonempty stations given that the bridge node is empty

$$= \frac{\lambda_b}{(M - i)\lambda + \lambda_b} \tag{6.12b}$$

υ_i = probability of a transmission commencement by an idle node before any of the i nonempty nodes given that the bridge node is nonempty

$$= \frac{(i - 1)\kappa + \kappa_b}{(i - 1)\kappa + \kappa_b + (M - i + 1)\lambda} \tag{6.12c}$$

υ_b = probability of a transmission commencement by the bridge node before any of the i nonempty nodes given that the bridge node is nonempty

$$= \frac{\kappa_b}{(i - 1)\kappa + \kappa_b} \,. \tag{6.12d}$$

It is seen from (6.6), (6.8a) and (6.10a) that Ψ (and subsequently the f_i's using (5.34)) are determined completely provided the probability that the bridge node is empty, b_o and the mean bridge busy period, $E[B_b]$ (both of which are henceforth referred to as bridge parameters) are known. From (5.12)

$$b_o \approx p_1(0) = \frac{1 - \lambda_b E[G'] E[S]}{E[R]} R^*(\lambda_b) \tag{6.13}$$

and from (5.49)

$$E[B_b] = \frac{\lambda_b E[G'] E[S] E[R_a]}{1 - \lambda_b E[G'] E[S]} \tag{6.14}$$

where the parameters $E[.]$ and $R^*(.)$ are as defined in Chapter 5 with λ_b, κ_b and γ_b still retaining their earlier definitions but are now defined with respect to the bridge node. Implicit in (6.13) and (6.14) is μ, the probability that the channel is idle, which from the standpoint of renewal theory is given by

$$\mu = \frac{E[I_{ch}]}{E[I_{ch}] + \gamma_s (E[G'] + a) + \gamma_b (E[B_b] + a) + (1 - \gamma_s - \gamma_b)(c + a)} \tag{6.15}$$

and γ_s, γ_b and $E[I_{ch}]$ still remain unknown. Following similar idea used in obtaining (5.38) and (5.39), γ_s and γ_b are found to be

$$\gamma_s = \sum_{j=0}^{M} \left[\sum_{i=0}^{M+1} \psi_{ij}^{ss} f_i \right] \tag{6.16}$$

and

$$\gamma_b = \sum_{j=0}^{M} \left[\sum_{i=0}^{M+1} \psi_{ij}^{sb} f_i \right] \tag{6.17}$$

respectively. In (6.16), ψ_{ij}^{ss} is the transition probability between two consecutive successful transmission completion epochs by a station which formally takes the definition of (6.6) with ψ_{ij} replaced by ψ_{ij}^{ss} and also

$$[\psi_{ij}^{ss} | I_{B\,0}] = \psi_{ij}^{ese} + \psi_{ij}^{nse} \tag{6.18a}$$

and

$$[\psi_{ij}^{ss} | I_{B\,1}] = \psi_{ij}^{esn} + \psi_{ij}^{nsn} . \tag{6.18b}$$

Further, ψ_{ij}^{sb} in (6.17) is the transition probability between two consecutive successful transmission completion epochs by the bridge and similarly takes the form (6.6) with ψ_{ij} replaced by ψ_{ij}^{sb} and

$$[\psi_{ij}^{sb} | I_{B\,0}] = \psi_{ij}^{bse} \tag{6.19a}$$

and

$$[\psi_{ij}^{sb} | I_{B\,1}] = \psi_{ij}^{bsn} . \tag{6.19b}$$

Finally, $E[I_{ch}]$ in (6.5) (and also in (6.15)) is determined using the same argument for deriving (6.4) but now the empty and nonempty states of the bridge node are taken into account. $E[I_{ch}]$ is then expressed as

$$E[I_{ch}] = \sum_{i=0}^{M} \frac{b_o f_i^e}{(M-i)\lambda + \lambda_b + i\kappa} + \sum_{i=1}^{M+1} \frac{(1-b_o) f_i^n}{(M-i+1)\lambda + (i-1)\kappa + \kappa_b} \tag{6.20}$$

where the f_i^e's (f_i^n's) are the limiting probabilities $\lim_{n\to\infty} Pr\{\tilde{Z}_n = i\}$ under the condition that the bridge node is empty (nonempty) and are separately determined by applying (5.34) in conjunction with the corresponding terms of (6.6) for the empty and nonempty states of the bridge.

It is seen from the above analysis that there are no explicit expressions for the channel-activity parameters μ, γ_s, and γ_b. In fact, they are interdependent with b_o and $E[B_b]$. Hence, they are computed by invoking principle 5 of our decomposition approximation (Chapter 2) via an algorithm similar to that outlined in Chapter 5. However, we shall first compute λ_{b1} and λ_{b2} by the following algorithm.

6.3.2 The Iterative Algorithm

Step 1: Compute S'_{chj} from (6.2).

Step 2: Compute $\hat{S}_{chj} = eS'_{chj}$ $(0 < e < 1)$ where $\hat{S}_{chj}$ is the estimate of S_{chj} and e is the estimator that accounts for the effect of BR_j on S'_{chj} assuming BR_j is active. Set iteration count k to 1.

Step 3: Compute $\lambda_{bi}(i \neq j)$ from (6.1) where S_{chj} is approximated by $\hat{S}_{chj}$.

Step 4: Compute S_{chi}, $(i \neq j)$ from (6.5).

Step 5: Compute $\lambda_{bj}(j \neq i)$ using (6.1), with $i \rightarrow j$ and $j \rightarrow i$).

Step 6: Compute $S^{(k)}_{chj}$ $(j \neq i)$ from (6.5).

Step 7: Test for convergence: if $|S^{(k)}_{chj} - \hat{S}_{chj}| < \varepsilon$, stop.

Step 8: Otherwise, $\hat{S}_{chj} = S^{(k)}_{chj}$, increase the iteration count and go to Step 3.

Note that the key steps of the algorithm outlined above are Steps 1, 4 and 6.

6.3.3 Message Response Time Analysis

The message response time in an interconnected system is of two types: a) intranetwork message response time, D_{intra} and b) internetwork message response time, D_{inter}.

a) Intranetwork Message Response Time, D_{intra}*:* The intranetwork message response time is the time interval from the instant a message is generated to the time the message is received at its destination where both the transmitting and receiving stations are on the same LAN. Assuming that the mean number of intranetwork messages in the system is known, then in principle, the mean intranetwork message response time can be determined using Little's law. Unfortunately, the limiting probabilities of state $\{f_i\}$ determined from (6.6) are only useful for calculating the mean number of nonempty nodes in the system which, because the bridge buffer may contain more than one message at a given time instant, is not the same as the mean number of intranetwork messages in the system. Nevertheless, an optimistic (lower bound) intranetwork message response time is obtainable by considering an isolated LAN (with bridge node deactivated), the message response time analysis of such a system has already been presented in Chapter 4 (Section 4.3.1.1).

b) Internetwork Message Response Time, D_{inter}*:* The internetwork message response time consists of the following components: time interval from when a message arrives at a station until the instant the message reaches the bridge buffer (assumed to be equal to the intranetwork message response time, D_{intra}); the message waiting time in the bridge buffer, W_M and the actual message transmission time, $E[G']$ on the channel of the LAN to which the destination node is connected, that is,

$$E[D_{inter}] = E[D_{intra}] + E[W_M] + E[G'] \tag{6.21}$$

Notice that W_M is equal to the total waiting and transmission times of all the packets ahead of the tagged message (equivalent to W_o in Chapter 5). From (5.16), the LST of the distribution function of W_M is given by

$$W_M^*(\theta) = \frac{(1 - \lambda_b E[G']E[S])}{(\theta - \lambda + \lambda G'(S^*(\theta)))} \cdot \frac{(1 - R^*(\theta))}{E[R]} \tag{6.22}$$

with mean

$$E[W_M] = \frac{E[R^2]}{2E[R]} + \frac{\lambda_b E[G']E[S^2]}{2(1 - \lambda_b E[G']E[S])} + \frac{\lambda_b (E[(G')^2] - E[G'])(E[S])^2}{2(1 - \lambda_b E[G']E[S])} \tag{6.23}$$

where $E[R]$ and $E[R^2]$ are given respectively by (5.24) and (5.25) with γ and κ replaced by γ_b and κ_b respectively.

6.3.4 Numerical Results and Discussion

For illustration purposes, the following parameters have been used in the numerical example presented below.

Number of stations on each LAN ($M_1 = M_2$): 10.

Fraction of messages transmitted from LAN_1 to LAN_2, α_{12}: 0.5.

Station retransmission rate, κ: 0.2.

Bridge retransmission rate, κ_b: 0.2.

Mean message size, $E[G]$: 5 *packets*.

Station buffer size, K: 10.

Normalized Propagation delay, a: 0.05 *slots*.

Collision resolution period, c: 0.1 *slots*.

Fig. 6.2 shows the throughput characteristic of the bridged network plotted against the total network input traffic. Note that the throughput plotted here is that generated by all the stations connected to the two LANs. In addition, the throughput characteristic of an equivalent CSMA-CD LAN to the bridged network is also shown for comparison, where the equivalent CSMA-CD LAN is that consisting of $M_1 + M_2$ stations which are connected to the same channel. We see from Fig. 6.2 that at low network load, the throughput performance of the bridged network is identical to that of its equivalent network, but as the network load increases, the throughput of the bridged network becomes better than that of its equivalent LAN. An explanation for this improvement is the possible occurrence of two simultaneous successful (intranetwork) transmissions (one on each LAN) at high network load.

Fig. 6.3 depicts the delay characteristics versus the total network input traffic for the bridged network and its equivalent LAN. The delay shown for the bridged network is that for internetwork messages, which, as seen from Fig. 6.3 is largely dominated by the internetwork message waiting time in the bridge buffer. We also observe that the message delay (message response time) in the equivalent network is much lower than the internetwork message delay which is obviously due to the absence of the waiting time in the bridge buffer. The above observation therefore

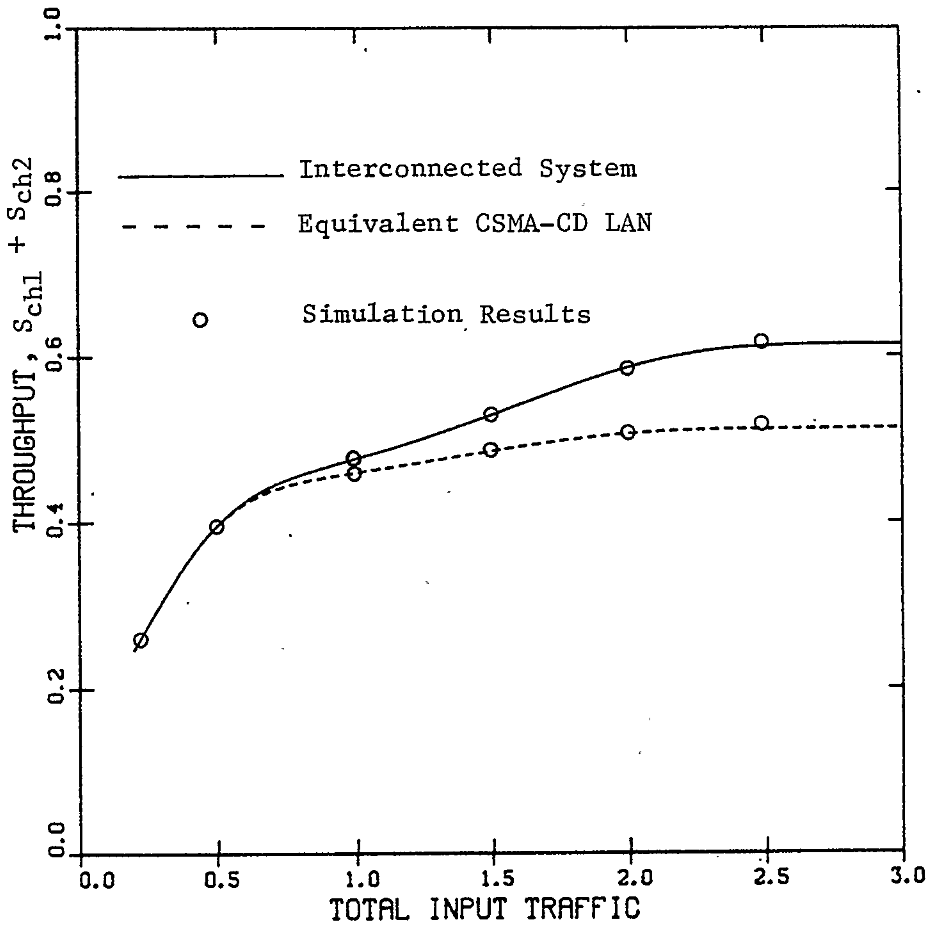


Fig. 6.2. Throughput versus total input traffic

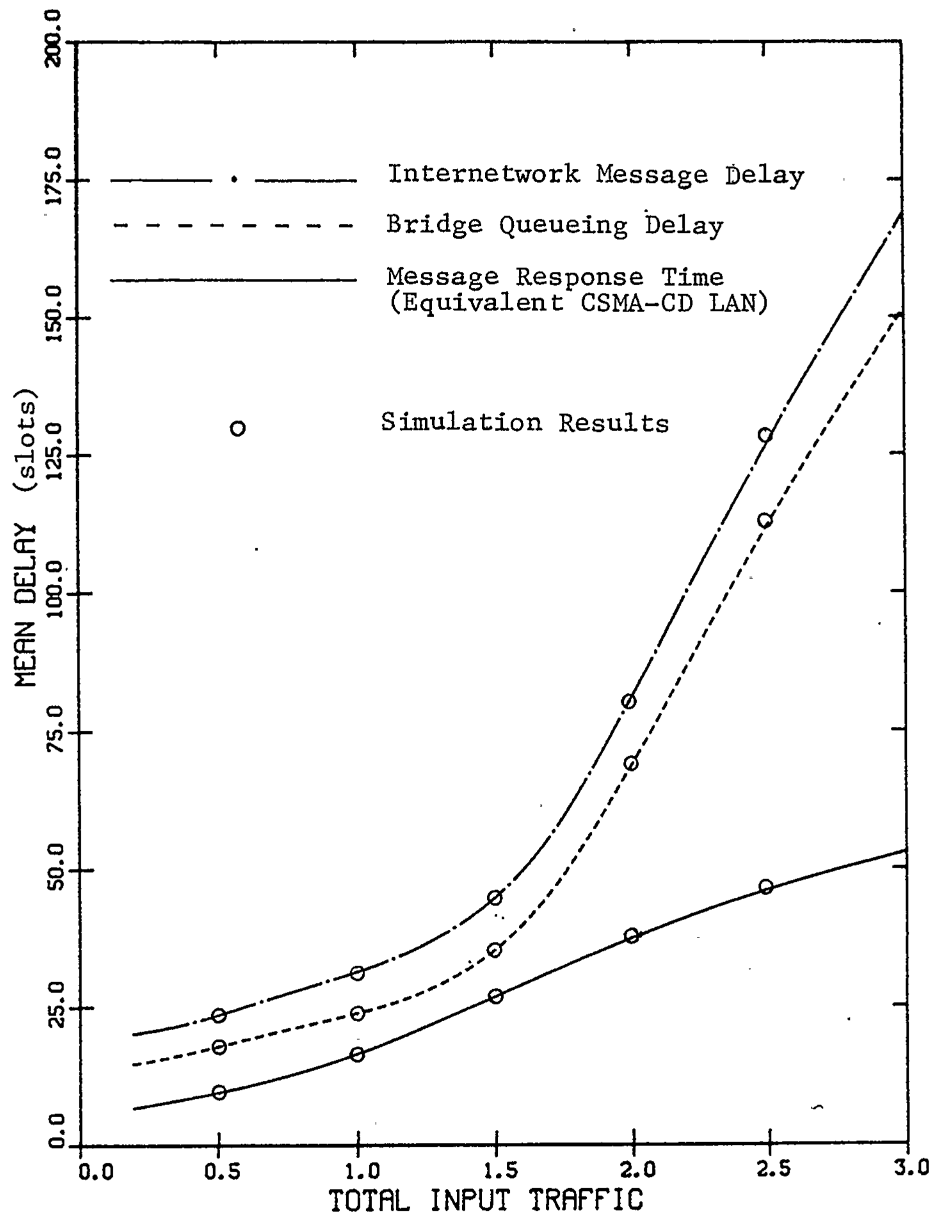


Fig. 6.3. Mean Delay versus total input traffic.

demonstrates a performance penalty of LAN interconnection, however, the author believes that this penalty can be minimized by appropriate choice of the input parameters for the bridge node; for example, selecting high value of bridge retransmission rate and using a finite (instead of an infinite) bridge buffer size, the implementation of the latter suggestion will no doubt lead to increasing analytic complexity.

6.4 Summary

Motivated by the limitations that abound in single separate LANs, this chapter studies the throughput and delay performance of interconnected buffered CSMA-CD LANs. The analysis of the bridged network is performed by applying some of the results of the previous chapters.

The main finding from this study is the improvement in the throughput of the bridged network over that of its equivalent LAN. It is also found that large inter-network message delay is a performance penalty of interconnected LANs, and this delay can be reduced by appropriate choice of bridge node (input) parameters.

CHAPTER 7

CONCLUSIONS AND OPEN PROBLEMS FOR FURTHER RESEARCH

7.1 Conclusions

The central focus of this thesis was on the solution of the interfering queue problem in buffered random access local area networks (LANs). We began by demonstrating that an exact analysis using either the standard method of probability generating function or the solution of the stationary equations for Markov chains was intractable, hence, a decomposition approximation technique was proposed. The versatility of the decomposition approximation methodology in analyzing the performance of buffered CSMA-CD LANs with single-packet as well as multipacket messages distinguishes this dissertation from prior work on buffered CSMA-CD LANs. Specifically, the solution methodology offered formal solutions to the following key problems: 1) analysis of multimessage buffer LANs with single-packet messages, 2) analysis of single-message buffer LANs with multipacket messages, 3) analysis of multimessage buffer LANs with multipacket messages, and 4) analysis of interconnected single-message buffer LANs with multipacket messages. Throughout the thesis, the emphasis was on the quantitative performance evaluation and the main performance measures determined were the throughput, the message delay and the necessary and sufficient condition for stability.

The main contributions and findings of this thesis can be summarized as follows:

- We proposed a decomposition approximation methodology as a tractable analytical technique for solving the interfering queue problem in buffered random access LANs. For each of the key problems considered, we developed a discrete-event simulation model to assess the accuracy of our approximations.

- We showed that for buffered CSMA-CD LANs with single-packet messages, the decomposition approximation technique served as a unified analytical framework for studying the performance of such LANs which consist of homogeneous users, each equipped with finite/infinite buffer size and also of LANs with heterogeneous users. To the best of the author's knowledge, no such unified analytic technique exists to date. We derived expressions for the probability generating function for user queue length, the mean queue length, the channel throughput and the necessary and sufficient condition for system stability.

The main findings from the numerical examples are tradeoffs existing between throughput and delay on the one hand and (for finite buffer size) between buffer overflow probability and mean packet delay on the other. We also found that the performance results based on the user homogeneity assumption tend to be optimistic especially in moderate to heavy traffic range. Our findings can be applied to a real network as follows: choose the sensing probability to keep the throughput at an acceptable level without exceeding the tolerable delay, then select a buffer size to achieve a small number of rejected packets.

- Recently, LANs are being considered for digitized voice transmission in addition to the traditional bulk data transfer. Such data are broken down into messages, each message consisting of more than one packet. We provided the message response time analysis for such "new generation" LANs and for efficient transmission of the multipacket messages, we proposed the gated and limited packet transmission strategies. Using the gated transmission strategy only, we derived a relationship between the message and packet access delays.

Results from this study confirmed our intuition of the unsuitability of the conventional transmission strategy for multipacket message transmission. We found that in the low to medium throughput range, the gated transmission strategy gave the least mean message response time; however, in the high throughput region, the limited transmission strategy (with a high minimessage size) might perform better than the gated strategy. In addition, we saw that the mean access delays were sensitive to the message length distribution and buffer size. For voice data transfer application, we found that the gated transmission strategy displayed a much improved performance over the conventional strategy for value of buffer size greater than one.

- We extended the decomposition approximation principles to analyze multimessage buffer random access LANs with nodes capable of buffering multipacket messages. For this type of LAN, we claim that this study is the first. The main results obtained from our analysis are the expressions for the probability generating function for queue length, the Laplace-Stieltjes transform of the waiting time and the channel throughput.

Our numerical results showed the sensitivity of the mean packet (message) waiting time on the message length statistics as well as on the message length distribution. The results also revealed that in the presence of variations in the input traffic and/or the number of nodes in the network, the choice of the retransmission rate had a definite effect on the system stability.

- We further showed that the main idea of the decomposition approximation had broad applicability in interconnected buffered random access LANs, the study of which constitutes the topic of current research due to the limitations of single separate LANs. We offered the throughput and delay analysis of interconnected buffered CSMA-CD LANs, our work differed from recently reported studies in that the more realistic assumption where each node generates (and stores) multipacket messages was made.

We found an improvement in the throughput of the interconnected LANs over that of its equivalent LAN. However, we saw that the internetwork messages suffered larger delays compared with the message response time in the equivalent LAN, we conjectured that the internetwork delay could be reduced by appropriate selection of bridge node (input) parameters.

Finally, the solution of the interfering queue problem in random access LANs (via the decomposition approximation technique) presented in this thesis represents a fascinating application of probabilistic thinking with elegant mathematical analysis. We believe that this thesis has provided an indepth study of the interfering queue problem in random access LANs. We sincerely hope that this effort will stimulate further research.

7.2 Open Problems For Further Research

The solution to the four key problems addressed in the thesis has unfolded a number of problems that require further investigation. We outline briefly some of these in the following:

1. Dynamic Control Procedure for Buffered CSMA-CD LANs

For simplicity reasons, steady-state condition was tacitly assumed in all our analyses. However, it will be appropriate to analyze the system dynamics under transient condition, this will give a better indication of the actual system behavior but such an analysis is going to be extremely difficult.

2. Stability of Buffered CSMA-CD LANs with Multipacket Messages

While the necessary and sufficient condition for stability was derived for buffered CSMA-CD LANs with single-packet messages, we only discussed numerically the stability issue for buffered CSMA-CD LANs with multipacket messages. We state categorically that the problem of deriving the necessary and sufficient conditions for stability for buffered random access LANs is a formidable one. In fact, it is still a wide open problem in the field of random access communications. It will therefore be desirable to investigate mathematically the condition for stability for multimessage buffer CSMA-CD LANs with multipacket messages as well as other issues. For example, the dynamic control of retransmission rate in the presence of variations in the input traffic and/or number of nodes in order to maintain system stability.

3. Whole versus Partial Message Acceptance Strategy

In Chapters 4 and 6, it was assumed that arriving messages were accepted into the nodal buffer provided the message lengths were smaller than or equal to the available buffer space. An alternative acceptance strategy is to accept parts of a message which will just fill up the empty buffer space (dubbed partial acceptance strategy), which has possible application in real-time data transfer. It will be appropriate to compare the effects of these two acceptance strategies on system performance. However, the author notes that partial acceptance poses a question of what to do with the remaining packets of a partially accepted message while waiting for a free buffer space.

4. Message and Packet Access Delays using Limited Transmission Strategy

In Chapter 4, we were unable to derive a relationship between the message and packet access delays for the limited transmission (LT) strategy because the argument used for the gated transmission (GT) strategy was no longer valid. A problem for further investigation is to seek an approach for deriving the relationship between the two access delays for the LT strategy. Furthermore, the analysis of Chapter 5 also assumed gated-like transmission strategy, but as noted in Chapter 4, this strategy may imply unfairness in the usage of the channel. A topic for further research will be the analysis of buffered LANs with multipacket messages, each node employing the LT strategy. In the author's opinion the LT strategy will, to some extent, ensure a higher degree of fairness than the GT strategy, it will also be desirable to quantify this claim.

5. Parameter Tuning in Interconnected Buffered CSMA-CD LANs.

We presented in Chapter 6 the analysis of the interconnected buffered CSMA-CD LANs using the decomposition approximation. It will be desirable to investigate the effects of varying the bridge node and station parameters on the interconnected LANs performance. In addition, a possible extension of the model used is to incorporate a finite bridge buffer size and also allow the buffering of more than one message at each station. Such a model, though more realistic than the one considered in Chapter 6 will be very difficult to analyze but it is well worth the effort.

REFERENCES

[1] D.D. Clark, K.T. Progran and D.P. Reed, "*An Introduction to local area networks*," Proc. IEEE, vol. 68, no. 11, pp. 1497-1515, 1978.

[2] IEEE Journal on Sel. Areas in Commun., vol. SAC-1, *Special issue on local area networks,* November 1983.

[3] IEEE Communications Magazine, *Special issue on Architectures of local area networks,* vol. 22, no. 8, August 1984.

[4] W. Stallings, *Local Networks: An Introduction.* New York: Macmillan, 1984.

[5] K. Kuemmerle, J.O. Limb and F.A. Tobagi, Editors, *Advances in local area networks,* New York: IEEE Press, 1987.

[6] F.A. Tobagi, "*Multiaccess protocols in packet communication systems,*" IEEE Trans. Commun., vol. COM-28, no. 4, pp. 468-488, 1980.

[7] J.F. Kurose, M. Schwartz and Y. Yemini, "*Multiple-access protocols and time-constrained communication,*" Computing Survey, vol. 16, no. 1, pp. 43-90, March 1984.

[8] L. Kleinrock, "*On queueing problems in random-access communications,*" Information Theory, vol. IT-31, no. 2, pp. 166-175, March 1985.

[9] S. R. Sachs, "*Alternative local area network access protocols,*" IEEE Commun. Magazine, vol. 26, no. 3, pp. 25-45, March 1988.

[10] S.S. Lam, "*Delay Analysis of a time-division multiple access,*" IEEE Trans. Commun., vol. COM-25, no. 12, pp. 1489-1494, Dec. 1977.

[11] I. Rubin, "*Message Delays in FDMA and TDMA Communications channel,*" IEEE Trans. Commun., vol. COM-27, pp. 769-778, 1979.

[12] J.F. Hayes, *Modeling and Analysis of Computer Communication Networks.* New York: Plenum Press, 1984.

[13] J.L. Hammond and P.J.P. O'Reilly, *Performance Analysis of local Computer Networks.* Reading: Addison-Wesley, 1986.

[14] M. Fine and F.A. Tobagi, "*Demand Assignment multiple access schemes in broadcast bus local area networks,*" IEEE Trans. Computer, vol. C-33, pp. 1130-1159, Dec. 1984.

[15] N. Abramson, "*The ALOHA system - Another alternative for computer communications,*" 1970 Fall Joint Computer Conference, AFIPS Conf. Proc., pp. 281-285, vol. 37, Montvale, NJ, AFIPS Press, 1970.

[16] F.A. Tobagi, "*Random access techniques for data transmission over packet switched radio networks,*" Ph.D. dissertation, Tech. Rep. Computer Science Dept., School of Engineering and Applied Science, Univ. of California, Los Angeles, rep. UCLA-ENG 7499, Dec. 1974.

[17] L. Kleinrock and F.A. Tobagi, "*Packet switching in radio channels: part I - Carrier sense multiple access modes and their throughput-delay characteristics,*" IEEE Trans. Commun., vol. COM-23, pp. 1417-1433, Dec. 1975.

[18] R.M. Metcalfe and D.R. Boggs, *Ethernet distributed packet switching for local computer networks,*" Commun. Ass. Computer Machinery, vol. 19, pp. 395-404, 1976.

[19] *The Ethernet, A local area network: Data link layer and physical layer specifications,* Version 1.0, Digital Equipment Corporation, Intel, Xerox, Sept. 30, 1980; Version 2.0, November 1982.

[20] J.F. Shoch, Y.K. Dalal, D.D. Redell and R.C. Crane, "*Ethernet,*" in Advances in local area networks, pp. 28-48, 1987.

[21] "*IEEE Standards for Local Area Networks, Carrier Sense Multiple Access with Collision Detection (CSMA/CD) Access Method an Physical layer Specifications,*" ANSI/IEEE 802.3, 1985.

[22] H. Zimmermann, "*OSI Reference Model - The ISO Model of Architecture for Open Systems Interconnection,*" IEEE Trans. Commun., vol. COM-28, no. 4, pp. 425-432, April 1980.

[23] *IEEE Computer Society Project 802, Local area network Standards,* IEEE, 1981.

[24] S.S. Lam, "*A carrier sense multiple access protocol for local networks,*" Computer Networks, vol. 4, no. 1, pp. 21-32, February 1980.

[25] F.A. Tobagi and V.B. Hunt, "*Performance Analysis of Carrier Sense Multiple Access with Collision Detection,*" Computer Networks, vol. 4, pp. 245-259, 1980.

[26] L. Kleinrock and Y. Yemini, "*Interfering queueing processes in packet-switched broadcast communication,*" in Proceedings of the IFIP Congress, 1980, Tokyo, Japan, pp. 557-562.

[27] K.K. Mittal and A.N. Venetsanopoulos, "*Buffer analysis in random multiple access broadcast communication systems,*" Canadian Electrical Engineering Journal, vol. 10, no. 4, pp. 158-162, 1985.

[28] E. Sykas, D.E. Karvelas and E.N. Protonotarios, "*Queueing analysis of some buffered random multiple access schemes,*" IEEE Trans. Commun., vol. COM-34, no. 8, pp. 790-798, August 1986.

[29] H. Takagi and L. Kleinrock, "*Mean Packet Queueing Delay in a buffered two-user CSMA/CD system,*" IEEE Trans. Commun., vol. COM-33, pp. 1136-1139, 1985.

[30] W. Szpankowski, "*A multiqueue problem: Bounds and approximations,*" in Performance of Computer Communication Systems, pp. 349-364, 1984.

[31] E. Coyle and B. Liu. "*Calculation of the Stability characteristics and buffer requirements of asynchronous CSMA/CD networks,*" in Conf. Record, Intl. Conf. Commun., Philadelphia, PA, 1982, pp. 7F.1.1-7F.1.5.

[32] J. Silvester and I. Lee, "*Performance Modeling of Buffered CSMA - An Iterative Approach,*" in Conf. Record, GLOBECOM '82, Miami, FL, 1982, pp. 1195-1199.

[33] T. Takine, Y. Yakahashi and T. Hasegawa, "*An approximate analysis of a buffered CSMA/CD,*" IEEE Trans. Commun., vol. COM-36, pp. 932-941, 1988.

[34] S. Tasaka, "*Dynamic behavior of a CSMA-CD system with a finite population of buffered users,*" IEEE Trans. Commun., vol. COM-34, pp. 576-586, 1986.

[35] T.K. Apostolopoulos and E.N. Protonotarios, "*Queueing Analysis of buffered CSMA-CD Protocols,*" IEEE Trans. Commun., vol. COM-34, pp. 898-905, 1986.

[36] M. Schwartz, *Telecommunication Networks: Protocols, Modeling and Analysis*. Reading: Addison-Wesley, 1987.

[37] K.L. Chung, *Elementary Probability with Stochastic Processes*. New York: Springer-Verlag, 1979.

[38] S.M. Ross, *Introduction to Probability Models*. London: Academic Press, 1985.

[39] P. Bremaud, *An introduction to Probabilistic Modeling*. New York: Springer-Verlag, 1988.

[40] A. Papoulis, *Probability, Random variables and Stochastic Processes*. New York: McGraw-Hill, 1984.

[41] D.L. Isaacson and R.W. Madsen, *Markov chains: Theory and Applications*. New York: John Wiley, 1976.

[42] S.M. Ross, *Stochastic Processes*. New York: John Wiley, 1983.

[43] D.P. Heyman and M.J. Sobel, *Stochastic Models in Operations Research, vol. 1: Stochastic processes and operating characteristics*. New York: McGraw-Hill, 1982.

[44] D.R. Cox, *Renewal Theory*. London: Methuen & Co., 1962.

[45] D.R. Cox and W.L. Smith, *Queues*. London: Chapman and Hall, 1961.

[46] T.L. Saaty, *Elements of queueing theory with applications*. New York: Dover Publications, 1961.

[47] L. Kleinrock, *Queueing Systems, vol. I: Theory*. New York: John Wiley, 1975.

[48] L. Kleinrock, *Queueing Systems, vol. II: Computer Applications*. New York: John Wiley, 1976.

[49] R.B. Cooper, *Introduction to Queueing Theory*. New York: North Holland, 1981.

[50] D. Gross and C.M. Harris, *Fundamentals of Queueing Theory*. New York: John Wiley, 1985.

[51] W.J. Graybeal and U.W. Pooch, *Simulation: Principles and Methods*. Cambridge: Winthrop Publishers, 1980.

[52] I. Mitrani, *Simulation techniques for discrete event systems*. Cambridge: Cambridge University press, 1982.

[53] G.S. Fishman, *Principles of Discrete-Event Simulation.* New York: John Wiley, 1978.

[54] A. Law and W. Kelton, *Simulation Modeling and Analysis.* New York: McGraw-Hill, 1982.

[55] J. Little, "*A proof of the queueing formula* $L = \lambda W$," Oper. Res., vol. 9, pp. 383-387, March-April 1961.

[56] M. Sidi, "*Discrete-time Priority Queues with partial interference,*" IEEE Journal on Selected Areas in Comm., vol. SAC-5, no 6, pp. 1041-1050, July 1987.

[57] G. Fayolle and R. Iasnogorodski, "*Two Coupled processors: The reduction to Riemann-Hilbert problem,*" in Zeitschrift Fur Wahrscheinlichkeits-Theorie und Venwandte Gebiete. Berlin: Springer-Verlag, pp. 325-351, 1979.

[58] A.O. Fapojuwo and D. Irvine-Halliday, "*Steady-state probabilities in large Markov chains of buffered CSMA-CD systems,*" in Proc. Supercomputing Symposium '87, Calgary, pp. 173-179, June 1987.

[59] A.O. Fapojuwo, W.C. Chan and D. Irvine-Halliday, "*An Iterative Approximation for Modelling Buffered CSMA-CD LANs,*" Journal of Electrical and Electronics Engineering, Australia, 1988, vol. 8, pp. 102-111.

[60] A.O. Fapojuwo, W.C. Chan and D. Irvine-Halliday, "*Approximate Performance Analysis of Symmetric Buffered CSMA and CSMA-CD Systems,*" revised and resubmitted to the IEEE Transactions on Communications, June 1988.

[61] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*Queueing analysis of CSMA-CD LANs with heterogeneous population of buffered users,*" in Proc., IEEE WESCANEX '88 Conference, Saskatoon, May 11-12,1988, pp. 16-26.

[62] F.A. Tobagi, "*Distributions of packet delay and interdeparture time in slotted ALOHA and Carrier Sense Multiple Access,*" Journal of the Assoc. for Computing Machinery, vol. 29, no 4, Oct. 1982, pp. 907-927.

[63] S. Tasaka, "*Performance analysis of multiple access protocols,*" Cambridge: MIT PRESS, 1986.

[64] Y. Onozato and S. Noguchi, "*A unified analysis of steady-state behaviour in random access schemes,*" Computer Networks and ISDN Systems, vol. 10, pp. 111-112, 1985.

[65] T.N. Saadawi and A. Ephremides, "*Analysis, stability, and optimization of slotted ALOHA with a finite number of buffered users,*" IEEE Trans. Automat. Contr., vol. AC-26, pp. 680-689, June 1981.

[66] A. Ephremides and R.-Z. Zhu, "*Delay analysis of interacting queues with an approximate model,*" IEEE Trans. Commun., vol. COM-35, pp. 194-201, February 1987.

[67] G. Fayolle, E. Gelenbe, and J. Labetoulle, "*Stability and optimal control of the packet switching broadcast channel,*" Journal of the Assoc. for Computing Machinery, vol. 24, no 3, pp. 375-377, 1977.

[68] W. Szpankowski, "*Bounds for queue lengths in a contention packet broadcast system,*" IEEE Transactions on Communications, vol. COM-34, pp. 1132-1140, Nov. 1986.

[69] D. Bertsekas and R. Gallager, *Data Networks*. New York: prentice-Hall, 1987.

[70] R.L. Burden and J.D. Faires, *Numerical analysis*. Boston: PWS Publishers, 1985.

[71] T-Y. Yan, Associate Editor on Multiple Access Strategies, IEEE Transactions on Communications, *Personal Communication,* March 1988.

[72] D.P. Heyman, "*The effects of random message sizes on the performance of the CSMA/CD protocol,*" IEEE Trans. Commun., vol. COM-34, pp. 547-553, June 1986.

[73] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*Multipacket message transmission on CSMA-CD LANs using limited and gated strategies,*" accepted for presentation at IEEE Pacific Rim Conference on Communications, Computers and Signal Processing, June 1-2, 1989, Victoria, BC, Canada.

[74] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*Message and packet access delays in CSMA-CD local area networks,*" accepted for presentation at IEEE Conference on Computer Communications (INFOCOM '89), April 24-27, 1989, Ottawa, Ontario, Canada.

[75] M.J. Maron, *Numerical Analysis: A practical approach,* New York: MacMillan Publishing Company, 1987.

[76] S. Halfin, "*Batch delays versus customer delays,*" Bell System Tech. Journal, vol. 62, pp. 2011-2015. Sept. 1983, part 1.

[77] W. Whitt, "*Comparing batch delays and customer delays,*" Bell System Tech. Journal, vol. 62, pp. 2001-2009, Sept. 1983, part 1.

[78] R.E. Barlow and F. Proschan, *Statistical Theory of reliability and life testing,* New York: Holt, 1975.

[79] P.J. Burke, "*Delays in single-server queues with batch input,*" Bell Syst. Tech. J., vol. 54, pp. 830-833, 1975.

[80] G.J. Nutt and D.L. Bayer, "*Performance of CSMA/CD networks under combined voice and data loads,*" IEEE Trans. Commun., vol. COM-30, pp. 6-11, Jan. 1982.

[81] F.A. Tobagi and N. Gonzalez-Cawley, "*On CSMA-CD local networks and voice communication,*" in Proc. IEEE INFOCOM '82, Las Vegas, NV, Mar/April 1982, pp. 122-127.

[82] N.F. Maxemchuk, "*A variation of CSMA/CD that yields movable TDM slots in integrated voice/data local networks,*" Bell Syst. Tech. Journal, vol. 61, no. 7, pp. 1527-1550, Sept. 1982.

[83] J.D. Detreville, "*A simulation-based comparison of voice transmission on CSMA/CD networks and on token buses,*" AT & T Bell Lab. Tech. Journal,

vol. 63, no. 1, pp. 33-55, Jan. 1984.

[84] E. Fuchs and P.E. Jackson, "*Estimates of distributions of random variables for certain computer communications traffic models,*" Communications of the ACM, vol. 13, pp. 752-757, 1970.

[85] P.T. Brady, "*A technique for investigating on-off patterns of speech,*" Bell System Technical Journal, vol. 44, no. 1, pp. 1-22, January 1965.

[86] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*Performance Analysis of a CSMA-CD System with nodes capable of buffering multipacket messages,*" submitted to the IEE Proceedings on Computers and Digital Techniques, November 1988.

[87] H. Takagi, *Analysis of Polling Systems,* Cambridge: The MIT Press, 1986.

[88] D.R. Cox, "*The Analysis of Non-Markovian Stochastic Processes by the Inclusion of Supplementary Variables,*" Proceedings of the Cambridge Philosophical Society, 1955, vol. 51, pp. 433-441.

[89] N.K. Jaiswal, *Priority Queues,* New York: Academic Press, 1968.

[90] M.L. Chaudhry and J.G.C. Templeton, *A first Course in Bulk Queues,* New York: John Wiley, 1983.

[91] S.W. Fuhrmann, "*A Note on the* $M/G/1$ Queue with Server Vacations," Operations Research, 1984, vol. 32, pp. 1368-1373.

[92] Y. Baba, "*On the* $M^X/G/1$ Queue with Vacation Time," Operations Research Letters, 1986, vol. 5, pp. 93-98.

[93] L. Takacs, "*Introduction to the Theory of Queues,*" New York: Oxford University Press, Inc., 1982.

[94] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*Stability Consideration of a buffered CSMA-CD system with multipacket messages,*" IEE Electronics Letters, vol. 25, no. 2, pp. 122-124, January 1989.

[95] IEEE Network Magazine, *Special issue on LAN interconnecting devices,* vol. 2, no. 1, January 1988.

[96] S. Ben-Michael and R. Rom, "*Gatewaying two ALOHA networks,*" in Conf. Rec. INFOCOM'86, pp. 30-33, April 1986.

[97] G.M. Exley and L.F. Merakos, "*Throughput-delay performance of interconnected CSMA local area networks,*" IEEE Journal on Selected areas in Communications, Vol. SAC-5, pp. 1380-1390, December, 1987.

[98] C.C. Ko, W.C. Wong and K.M. Lye, "*Performance of CSMA/CD networks connected by bridges,*" in Conf. Rec. Int. Conf. on Commun., Philadelphia, pp. 0119-0123, June 1988.

[99] J-L.C. Wu, J. Wu and T-C. Lee, "*Performance analysis of interconnected CSMA/CD networks with finite population,*" in Conf. Rec. INFOCOM'88, pp. 1005-1011, April 1988.

[100] A.O. Fapojuwo, D. Irvine-Halliday and W.C. Chan, "*A model for analysis of interconnected buffered CSMA-CD local area networks,*" Summary submitted to the 7^{th} Int. Conf. on Math. and Computer Modeling, November 1988.