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# A Sustainable Fiscal Rule to Manage Non-renewable Resource Revenues: Oil Sands as a Second Chance for Alberta

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Sustainable Fiscal Rule to Manage Non-renewable Resource Revenues: Oil Sands as a Second Chance for Alberta" submitted by Lindsay Marie Kendall in partial fulfilment of the requirements of the degree of Master of Arts.

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# Abstract

In this thesis, I examine an optimal fiscal policy for how a government, which has access to non-renewable resource revenues, is able to satisfy issues of intergenerational equity and fiscal sustainability. The optimal fiscal rule restricts the choices available to the government by requiring that it contribute to an oil fund. Within this fiscal rule, the government must follow an appropriate set of behaviours with respect to borrowing, saving and financial liability retirement; behaviours which vary over time as exploitation of the non-renewable resources move through three phases of development. I, then, apply this fiscal policy rule to the Alberta government, taking into account its current fiscal stance and the nature of the oil industry, which is a mix of light crude and the oil sands. I suggest that the Alberta government apply this optimal fiscal policy to manage the non-renewable resource revenues to achieve fiscal sustainability, while at the same time having a richer public sector.

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# Table of Contents

# List of Tables

# List of Figures

# Chapter One

# The Issue

## 1.1 Introduction

Imagine you have just won a "set for life" lottery in which you receive $1,000 every week for the next 25 years of your life. What would you do? Would you save and accumulate some assets and live off that return, providing you with a financial cushion you could live off for the rest of your life? Or would you spend that money as soon as the cheque hits your mailbox? This situation I have just presented to you is like that of any economy that has just discovered a non-renewable resource. The government of that economy now has the ability to save or spend their newly found, but non-renewable, capital. But what is the optimal fiscal policy for a government that discovers it has access to non-renewable resource revenues? My goal in this thesis is to present a model that describes the optimal behaviour for a government bound by a fiscal rule, in which it borrows, saves and eliminates accumulated financial liabilities over time. The goal of this fiscal rule is to provide fiscal sustainability and achieve intergenerational equity.

In the next section, Section 1.2, I provide an overview of the reasons why a resource rich government should save their non-renewable resource revenues. Section 1.3 gives a brief overview of the Government of Alberta's history and their public policy choices, while Section 1.4 reviews the literature. Finally, Section 1.5 presents the issue and the target of the thesis.

### 1.2 Why Should Governments Save?

The first question that must be answered is why should governments go through the process of saving their non-renewable resource revenues? A key feature of non-renewable resources is that they have a finite supply and so will eventually run-out. For this reason, it is important for the rents, or non-renewable resource revenues, which are the royalties received from the sale of these non-renewable resources, to be saved so that future generations will also be able to reap the benefits. In 1976, the Premier of Alberta, Peter Lougheed, introduced a way to save these resource revenues – the Alberta Heritage Saving and Trust Fund (AHSTF). It was established to set aside resource revenues to ensure that future generations of Albertans would have the means to sustain the same standard of living and prosperity as current Albertans. Also in this document, it was set out that the property rights of these non-renewable resources were owned by all Albertans, current and future generations (*A History of the Heritage Fund*, n.d.). Therefore, the Government of Alberta obligated itself to protect the interest not only of those individuals fortunate enough to be living during the period of resource exploitation, but to give those individuals living during the period after the resource was exhausted that same level of prosperity.

The issue of saving non-renewable resources raises questions of moral philosophy. How should all generations of Albertans be treated? Should those individuals alive during resource exploitation be expected to put away savings in order to allow future generations to share in the non-renewable resources benefits? But at what cost to the current generation? Those costs, such as debt incurred, should also be shared across generations. Kneebone (2007) suggests that when the government makes decisions regarding long-term economic development it should consider issues of intergenerational equity. Therefore, to allocate financial resources across generations involves a process of borrowing, thereby bringing resources from the future to the present, and saving, which brings resources from the present to the future. A successful government fiscal policy involves a process of portfolio management.

Another reason why the government should save non-renewable resource revenues is to control fiscal decisions of expenditure and taxation. Throughout much of the history of the Alberta government, increases in non-renewable resource revenues led to increases in government expenditure (this will be looked at in more detail in the next section), and the primary determinant for these increases was public demand. Boothe (1995) discovers that the government used revenues to deliver programs and provide infrastructure which was tempered by the influence of political variables such as the government's share of the popular vote and the size of the legislative majority. The key implication here is that the government had a strong response to the demand for programs. In other words, to curb government expenditure, expenditure control must include influencing the public's opinion on its demand for government programs and this can be accomplished by requiring the current generation to pay for those programs with current taxes.

Another reason for fiscal control is unsustainable spending which is the government's reaction to changes of non-renewable resource revenues. Boothe (1995) discovers that during periods of declining resource revenues, debt accumulates due to public expenditure leaving a tax burden for future generations. In times of increasing resource revenues, program expenditures are increased and are difficult to cut when resource revenues decline. The Alberta government is reacting to temporary fluctuations of the non-renewable resource industry. In order for the government to correct this behaviour, it must go through a period of adjustment and incur the fiscal pains of either increasing taxes or decreasing spending.

An additional reason why a government should save is to prepare for demographically-driven expenditures due to the aging population. This could be increasing government spending on health care. This is primarily saving as a precautionary motive (Posner and Gordon, 2001). An example of this is Norway's Government Pension Fund. The Government of Norway started a fund to pay for the eventual drain on its pension fund due to the aging population.

Posner and Gordon (2001) provide a review of how several governments reliant on non-renewable resource revenues deal with volatility and how these governments have been successful in making the transition from a fiscal policy aimed at program deficit to one aimed at saving. They note that what is important for governments is to legitimize their behaviour to voters. This is done through the use of fiscal anchors, fiscal rewards and the introduction of new budget processes. Using this approach, governments have convinced voters to accept higher tax rates and thereby increase savings (Posner and Gordon, 2001).

## 1.3 A Brief History of the Alberta Government's Savings Plans

The history of the Government of Alberta has been challenged with the volatility of resource revenues, resulting in extreme variations in the effects of its expenditures on governmental services and capital investments. In order to counteract these affects, the government has formulated several plans in the past to manage their non-renewable resource revenues. The following is a brief history of the fiscal rules and savings plans the Alberta Government has used.

In 1972, a new government was elected – the Progressive Conservative party of Alberta led by Peter Lougheed. This party came into power with the first oil price shock in 1973 and is still the governing party in power today. The Government of Alberta has faced fiscal challenges and struggles due to the volatility of oil prices. Therefore, this period marks the inception of the goal for fiscal sustainability and responsibility to ensure intergenerational equity and fairness. But what has led the government to this point; how has this government handled their gift of resource revenues?

Over the period of 1972 to 1990, the Government of Alberta was required to deal with fiscal pressures resulting from growing resource revenues and pressures to expand spending. One of the areas of pressure came from population growth. The population increased by almost 40 per cent over this period, well above the Canadian average of 11 per cent (Boothe, 1995). This most likely was the result of economic expansion of the oil and gas sector, which stimulated job growth and attracted migrants.

Over this entire period, the government of Alberta responded to increases in real per capita resource revenues by increasing real per capita expenditure more readily – as if the increases in resource revenues were permanent. Conversely, when resource revenues decreased, the government responded slowly to decreasing real per capita expenditure – as if falling resource revenues were temporary (Boothe, 1995).

With the large influxes of resource revenues and the pressures of spending, the Government of Alberta did introduce a plan to save for the future. The Premier, Peter Lougheed, believed that there should be an investment plan to set aside sufficient resources so that future generations of Albertans would have a better opportunity to have the same level of standard of living as current generations of Albertans (*A History of the Heritage*

*Fund*, n.d.). Thus, the Alberta Heritage and Savings Trust Fund (AHSTF) were introduced in 1976 as a way to save for the future.

In its inception, $1.5 billion was deposited and a commitment of 30 per cent of all resource revenues was to be put in the fund every fiscal year. However, in response to the crash of oil prices and the introduction of the NEP, in the fiscal year of 1982, the government diverted the interest earned on the AHSTF to the General Revenue Fund (GRF) which would enter directly into the government's budget on the expenditure side. They decreased the payment into the Fund to only 15 per cent of all resource revenues. Furthermore, the government expanded the objectives of the Fund to include short run stabilization of the economy (Boothe, 1995).

The economy was still slow to recover and in 1986, the government abandoned the savings plan of the AHSTF idea altogether. No new resource revenues would enter into the Fund and all resource revenues now entered into the GRF and thus into expenditures (*A History of the Heritage Fund*, n.d.). This gives further evidence that when resource revenues decline, the Alberta government failed to react and decrease program expenditure. This also gives evidence of the need for a savings plan which tempers the government's spending behaviour.

At the end of fiscal year 1992, the Government of Alberta had run deficits for the past nine consecutive years, moving from a next asset position in the fiscal year of 1985 of $12.6 billion to a net debt position of $8.4 billion in the fiscal year of 1994 (Kneebone, 2006). In the election campaign of 1993, the Progressive Conservative party had a new leader, Ralph Klein, and a new campaign – to cut government spending by 20 per cent in nominal terms. The party believed that the government did not have a problem controlling their revenues, but they had a problem controlling their spending. This goal was achieved in the fiscal year of 1996, one year ahead of schedule (Kneebone, 2006). Although it addressed the area of spending, the provincial government still did not believe they had an issue with regards to their resource revenues.

Since the mid-1990s to the current year, the government has experimented with the use of fiscal rules.[1] In the fiscal year of 1993, the Alberta government introduced a new plan for dealing with the volatility of resource revenues. The plan consisted of several fiscal rules, aiming at deficit control and debt reduction. The government's plan was successful – deficits and net debt were eliminated. However, the problem now is that, having achieved these goals, the government is again having trouble regulating their spending. The Alberta government needs a new long-term plan to manage their non-renewable resource revenues.

From the Alberta government's past experience, we have learned that their resource revenues are extremely volatile due to the volatility of non-renewable resource prices – particularly that of oil. Also, when resource revenues are increasing, the government has treated it as a permanent trend and readily increased program spending. However, when resource revenues are declining, the government has treated this as a temporary phenomenon and has been slow to decrease expenditures. Furthermore, the Alberta government has been unsuccessful at maintaining a long-term plan to manage their non-renewable resource revenues. For the Government of Alberta to achieve a goal of fiscal sustainability and intergenerational equity, many analysts have suggested that a savings and expenditure control plan should be made and followed.[2]

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[1] For a survey of these fiscal rules, see Kneebone (2006)
[2] See, for example, Boothe (1995) and Kneebone (2006, 2007)

## 1.4 Literature Review

The above analysis of the Alberta government's past has revealed why a government endowed with a non-renewable resource should save. It suggests that the government has found it difficult to save, especially in times when resource revenues decline. In these times, the Alberta government abandons its savings decision, thereby abandoning its commitment to share non-renewable resource revenues with future generations. The above discussion also proposes that the Alberta government needs to understand the economic rationale behind saving non-renewable resource revenues. The following will be an overview of the literature and the economic rationale for managing non-renewable resource revenues.

One theory on what a government should be doing with its non-renewable resource revenues is due to John Hartwick. Hartwick (1977) looked at the non-renewable resource from a capital stand point – that this resource was a source of capital for the government. The non-renewable resource is finite, once it is discovered it can only be depleted, therefore, if the government consumes the rents from its exploitation than it is actually consuming capital. This process is unsustainable because consuming the capital directly will diminish future consumption. Hartwick (1977) theorizes that the total stock of capital (including non-renewable resources, land, and human capital) should be maintained at a constant level. Thereby, the proceeds from the sale of non-renewable resources need to be saved and invested in other forms of capital, such as a financial asset. In doing so, the government will achieve a constant level of consumption per capita and be able to use the savings from the non-renewable resource rents to fund it once the resource has been exhausted.

The idea of expanding the definition of capital to a more comprehensive one has long been on the minds of economists. Weitzman (1976) expanded his notion of capital when examining the usefulness of national product on a dynamic economy with the welfare concept. The notion of capital was expanded to include such things as resources, knowledge, and research. This was done because all sources of economic growth should be taken into account when evaluating the status of an economy. Implying that what is done to the stock of capital today has some affect on the stock tomorrow because the result of economic activity is the consumption, not the formation, of capital. This idea flowed from Samuelson (1961) wherein he determined that the fundamental notion of social welfare was not in income, but closer to the concept of total wealth, or the present discounted value of future consumption.

Governments have traditionally ignored the influences of other forms of capital on their national accounts, instead focusing on produced forms of capital such as buildings, equipment and infrastructure. Governments reliant on this resource often do not realize the impacts simply because their resource dependence complicates the savings concept due to their definition of capital and, therefore, their national accounts do not include those resources.

Expanding the definition of wealth to include intangible assets, land, non-renewable resources and human capital will, on the one hand, allow for a more thorough analysis of sustainability, but, on the other hand, complicate the government's view of revenues, expenditures and taxation. As was discussed above by other theorists, the concept can be simplified by viewing it as portfolio management. A government transforms non-renewable resource capital into productive assets via investment – like that of Hartwick (1977) in saving resource rents.

Furthermore, the World Bank (2006), in examining the genuine savings concept in their publication *Where is the Wealth of Nations?*, provides a complete explanation for why the definition of wealth should be expanded. A strong link exists between social welfare, wealth and sustainable development in economic theory but for this link to hold, the notion of wealth must be truly all-inclusive of the notion of capital. A development path is sustainable if utility or social welfare does not decline at any point along the path and where social welfare is a measure of inter-temporal well-being. Also, total wealth is a measure of future consumption, which implies that changes in wealth have implications on well-being. Therefore, when examining a government's fiscal stance, the total wealth must not decline at any point along the path for the fiscal policy to be sustainable.

Encompassing a complete definition of wealth is vital to the idea behind a sustainable fiscal policy. If a government chooses to directly consume its non-renewable resource rents, it is on an unsustainable path. To counteract this behaviour, the government should save these rents in other tangible forms of capital and consume a constant level of per capita expenditure and taxation policy. But, how should the government save these rents? Olters (2007) suggests that these types of governments need to develop a forward-looking fiscal strategy, anchored by a fiscal rule, which exchanges one form of capital (the non-renewable resource) into another (the financial asset) – much like managing a financial portfolio. The fiscal rule would be designed such that public expenditure would not exceed permanent income.

Barnett and Ossowski (2002) propose a similar method but, they focus on the non-oil primary deficit or the difference between public expenditures and revenues (i.e. taxation). Accordingly, they devise an appropriate structure for the non-oil primary deficit, modelling after Friedman's permanent income hypothesis for the individual. The basic intuition is that an individual (or in this case the government) desires to smooth consumption over their lifetime and not let it fluctuate with short-run changes in income. This is because the individuals base their consumption on a long-run time horizon and their wealth as lifetime wealth. A consumer faces a finite amount of income over her lifetime and must decide how to allocated their consumption, or spending, over this time period by means of borrowing, saving and debt retirement (Friedman, 1957). In the government's case, their lifetime is longer than one individual's lifetime and thus their decisions affect many generations. Also, the government's consumption is a combination of expenditures and taxation policy or the non-oil primary deficit. The government must choose a non-oil primary deficit based on their wealth, inclusive of non-renewable resource wealth, rather than current income. The government then is able to view the fiscal proceeds of the resource rents not as income but as an investment in their financial asset – the portfolio transaction between non-renewable resource wealth and financial wealth – which, in doing so, preserves government total wealth. The preservation of wealth will also require that the government's consumption, the non-oil primary deficit, in each period be limited to the government's permanent income, which is the implicit return on government wealth. As a result, the non-oil primary deficit is kept constant. The government's choice of the non-oil primary deficit, knowing what its non-renewable resource wealth is, is to set the non-oil deficit equal to the return on its wealth, which achieves fiscal sustainability. To conclude, Barnett and Ossowski (2002) suggest that the government's fiscal policy should be targeted at accumulating substantial assets during the time of oil exploitation to sustain the non-oil primary deficit in the post-oil period.

## 1.5 The Issue

In keeping with the ideas of the theory and the status of the Government of Alberta's fiscal history, I present a fiscal rule for the Government of Alberta to implement and manage their non-renewable resource revenues. This fiscal rule will call for strict government behaviour, outlining the appropriate borrowing, savings and expenditure actions the government needs to take in times of resource revenue abundance and scarcity. In order to find an appropriate fiscal rule, I build a simple dynamic model showing the interrelationships between a government's budget, its net financial liabilities, the stock of financial assets, the flow of non-renewable resources, and the interaction of key macroeconomic variables, in particular the economic growth rate and interest rates, on all of the above. I take a net worth approach to the analysis. This is done to acquire a comprehensive measure of wealth which involves not only the traditional views of capital but also incorporates the role of the government's portfolio management between the non-renewable resource and the financial asset.

It is worth mentioning that the fiscal rule I employ is modelled after the one used by the Government of Norway. Norway's economy is like Alberta's in the sense that it is reliant on non-renewable resources. Unlike the Alberta government, Norway has been able to save over USD $401 billion in their Government Pension Fund (GPF) (formally the State Petroleum Fund), a far cry from the AHSTF (Norges Bank Investment Management, 2007). The Fund was established to help alleviate inflationary and exchange rate pressures that the non-renewable resource industry can exert on non-oil exporting industries, in addition to providing a pension fund for the country's aging population. According to Eifert et al. (2002), the Fund acts as a buffer to smooth the fluctuations of resource revenues over the lifetime of the non-renewable resource. When resource revenues accumulate, the government tightens program spending which diminishes domestic demand by reducing domestic inflationary pressures and contains the potential appreciation of the exchange rate, thereby, protecting the non-oil exporting sectors. Alternatively, when resource revenues are declining, the government can draw upon the previously accumulated assets, thereby, alleviating the negative implications on private sector investment and economic growth. This countercyclical fiscal policy results in an indirect relationship between program spending and resource revenues (Eifert et al., 2002).

The remainder of the thesis proceeds as follows: Chapter Two discusses the framework of the basic model and then incorporates the oil fund fiscal rule and its implications. Chapter Three presents a dynamic analysis of the model, while Chapter Four presents a simulation of the model's results and examines the sensitivity of those results. Chapter Five is about second chances for the Alberta government and suggests the appropriate policy the Alberta government should employ in dealing with the oil sands. Chapter Six concludes the thesis.

# Chapter Two

# The Model

## 2.1 Introduction

In the previous chapter, I presented the case for governments with access to non-renewable resources to behave in ways not unlike that of a household. A household has access to income for only a certain fraction of its lifetime. After the household's source of income is exhausted it must survive until death on the revenue generated by the income the household managed to save and by exhausting the accumulated assets (subject to a bequest motive). The goal of the household is to maintain a more or less constant level of consumption during its entire lifetime. During the period when it is earning income, achieving this goal requires that the household initially incur financial liabilities but at some point it must turn to the retirement of financial liabilities, then asset accumulation, and finally asset exhaustion.

In this chapter, I examine a model of government behaviour that mimics the optimal behaviour of a household. Unlike households which have finite lifetimes, governments in some sense live forever. However, a government reliant on non-renewable resource revenue is like a household in the sense it must cope with a temporary source of income. As I will show, the optimal choice for government involves a period where financial liabilities accumulate followed by a period where financial liabilities are retired and then a period of asset accumulation. In the final period, the government maintains spending at a constant level forever and finances it with the investment earned on its accumulated assets.

The analysis of government fiscal sustainability is often based on the level of debt and growth rates. The well-known result that governments borrow at their peril when interest rates exceed the growth rate of the economy is based on a model that assumes a permanent source of revenue. That is, tax revenue is generated by applying a tax rate to private income which is assumed to be permanently available. The analysis of sustainability differs for a government with access to non-renewable revenues which the government can collect only for a period which is short in the long time horizon of a government.

The model presented in this chapter is based on a recent paper by Da Costa and Juan-Ramón (2006) but adapted to Alberta's situation. This model analyzes an economy based on the net worth approach which takes into account the role of both financial and non-financial assets. Total wealth in classical economic theory is estimated as the present value of future consumption. Consumption can come in the form of land, labour, and capital either as produced, natural or intangible capital. Therefore, the net worth approach encompasses a wide-range of wealth allowing me to get a more comprehensive measure of fiscal sustainability.

The purpose of this model is to provide insights into how a non-renewable resource rich economy should behave in order to achieve fiscal sustainability. Fiscal sustainability is defined as a level of net worth where all obligations to current and future generations are met without changing current policy settings. A goal of fiscal sustainability is to use non-renewable resource rents in a way that sustains a constant desirable standard of living. In this model, I incorporate a fiscal rule to bind the government's decision towards fiscal sustainability, intergenerational fairness and equity.

The next section, Section 2.2, provides an overview of the model adapted to Alberta's circumstances. The presentation is based on a simple fiscal rule. Section 2.3 describes steady-states, while Section 2.4 explains the dynamic adjustment process leading to those steady-states. In Section 2.5, I take a closer look at the fiscal rule which constrains the government to an oil fund consistent with the rule employed by Norway. Finally, Section 2.6 concludes this chapter.

## 2.2 The Basic Form of the Model

The basic form of the model is represented by the following six equations. The fiscal analysis of the Alberta government is determined by examining these accounting equations.

$$NW_t = A_t^F + A_t^{NF} - D_t \tag{1}$$

$$NW_t - NW_{t-1} = T_t^* - G_t^* - DEP_t \tag{2}$$

$$T_t^* = T + \rho_t^F A_{t-1}^F + \rho_t^{NF} A_{t-1}^{NF} \tag{3}$$

$$G_t^* = G + iD_{t-1} \tag{4}$$

$$GOPB_t = T_t - G_t \tag{5}$$

$$DEP_t = \delta A_{t-1}^{NF} \tag{6}$$

**Table 1. Definitions**

$NW_t$: Net worth

$A_t^J$: End-of-period stock of assets (J = F, NF; financial and non-financial, respectively)

$D_t$: End-of-period stock of financial liabilities in period t

$T_t^*$: Total revenue, inclusive of investment income, collected in period t

$T_t$: Tax revenue collected in period t

$G_t^*$: Total expenditures, inclusive of debt (financial liabilities) service payments,

| | |
|---|---|
| | in period t |
| $G_t$: | Program spending in period t |
| $DEP_t$: | Depreciation of non-financial assets (i.e. consumption of fixed capital) |
| $GOPB_t$: | Gross operating primary balance of the general government |
| $i$: | Ratio of interest payments to end-of-period nominal financial liabilities |
| $\rho_t^J$: | Ratio of current period return to end-of-period asset |
| $\delta$: | Average depreciation rate of non-financial assets |

With exception of $i$, $\delta$, and $\rho^J$ all variables are measured in nominal dollar values. $i$ represents the effective interest rate paid on outstanding financial liabilities and $\rho^J$ represents the effective rate of return on financial and non-financial assets, *J=(F,NF)*.

Equation (1) defines net wealth at a point in time as the value of the end-of-period stock of financial and non-financial assets less the end-of-period value of financial liabilities. Following Da Costa and Juan-Ramón (2006), I assume that other liabilities are zero[3].

Equation (2) defines the change in net wealth from one period to the next as total revenue (inclusive of investment income), less total expenditures (and interest payments on financial liabilities), less depreciation of non-financial assets.

Equation (3) defines total revenue as composed of total tax revenue plus the rate of return earned on financial and non-financial assets. In practice, total tax revenue would include revenue gained from the application of tax rates on consumption, personal and

[3] Other liabilities include non-contingent liabilities, namely financial derivatives, shares and equity of publicly controlled enterprises, held by the private sector.

corporate income, as well as user fees, transfers received from other levels of government, and non-renewable resource revenue.

Equation (4) defines total expenditures as program spending plus interest payments made on outstanding financial liabilities. In practice, program spending would include expenditures on goods and services, transfers to individuals, and transfers to other levels of government.

Equation (5) defines the gross operating primary balance (deficit) of the general government. This is defined as tax revenue less program spending. In what follows, when expressed as a fraction of GDP, *GOPB* will be treated as a policy choice variable. It measures what the size of the budget surplus would be were it not for investment income generated by previously accumulated assets and the interest costs of carrying previously accumulated liabilities.

Finally, equation (6) defines depreciation of non-financial assets as given by a rate of depreciation applied to the value of non-financial assets.

Substituting equations (1), (3)-(6) into (2), I can write:

$$\left(A_t^F - A_{t-1}^F\right) + \left(A_t^{NF} - A_{t-1}^{NF}\right) - \left(D_t - D_{t-1}\right) = GOPB_t + \rho_t^{NF} A_{t-1}^{NF} + \rho_t^F A_{t-1}^F - iD_{t-1} - \delta A_{t-1}^{NF} \qquad (7)$$

Thus, the change in net wealth (defined on the left hand side) is determined by the size of the primary budget deficit (*GOPB*, a policy choice variable) plus net investment income less depreciation of fixed capital.

The monetary values of end-of-period financial and non-financial assets are equal to the depreciated value of those stocks from the previous period plus net investments in those assets during the current period. Thus:

$$A_t^{NF} = \left(1-\delta\right) A_{t-1}^{NF} + INV_t^{NF} \qquad (8)$$

$$A_t^F = A_{t-1}^F + INV_t^F \tag{9}$$

where $INV_t^{NF}$ and $INV_t^F$ define net purchases of non-financial and financial assets, respectively. Note that there is no depreciation of financial assets. When below I measure $INV_t^{NF}$ and $INV_t^F$ as fractions of GDP, they will be treated as policy choice variables.[4]

It is worth repeating what was noted in the previous chapter that in its annual budget the government does not consider the implications of non-financial assets. Thus, as reported in its annual budget, the government's budget constraint is

$$\left(A_t^F - A_{t-1}^F\right) - \left(D_t - D_{t-1}\right) = GOPB_t + \rho_t^F A_{t-1}^F - iD_{t-1} \tag{7a}$$

which indicates that the change in net financial assets (left hand side) is determined by the size of the primary deficit of the general government plus net investment income earned on net financial assets. My use of (7) rather than (7a) indicates my emphasis on the net worth approach. My approach integrates the flow variables traditionally used in the government's budgeting process, with stock analysis, which introduces financial and non-financial assets into the government's budget. In the next section, I use equations (7) – (9) to analyze the dynamics of the model.

## 2.3 Stability Conditions

To analyze the conditions for stability of the model, I begin by adjusting equations (6) – (8) so that they express values relative to the size of GDP. This does two things. First, it defines monetary values in terms of a useful metric. For example, government financial liabilities of $5 billion may be large or small depending on the size of GDP – the collective

[4] For the model with a simple fiscal rule, investment in financial assets ($INV_t^F$) will be treated as a policy choice variable. However, when I introduce the oil fund fiscal rule, investment in financial assets will be oil revenues – those revenues collected by the government from the sale of oil by private industry.

income of all citizens obligated to meet those financial liabilities. Second, dividing by GDP will introduce the effects of a growing economy into the model of government finances.[5]

The variables that are in terms of GDP are represented by lower case letters, for example, $d_t = D_t / Y_t$ and $a_{t-1}^F = A_{t-1}^F / Y_{t-1}$, where $Y_t$ is GDP at time $t$ and $Y_{t-1}$ is GDP at time $t$-1. In matrix form, the asset accumulation identity, (6), and the two laws of motion equations for financial and non-financial assets, (7) and (8), now become:

$$\begin{pmatrix} d_t \\ a_t^F \\ a_t^{NF} \end{pmatrix} = \begin{pmatrix} \frac{(1+i)}{1+\hat{Y}_t} & -\frac{\rho_t^F}{1+\hat{Y}_t} & -\frac{\rho_t^{NF}}{1+\hat{Y}_t} \\ 0 & \frac{1}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \frac{(1-\delta)}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ a_{t-1}^F \\ a_{t-1}^{NF} \end{pmatrix} + \begin{pmatrix} -gopb_t + inv_t^{NF} + inv_t^F \\ inv_t^F \\ inv_t^{NF} \end{pmatrix} \tag{10}$$

These equations now show the dynamics of financial liabilities, non-financial and financial assets ratios, where the net worth ratio can be inferred from these three equations. See Appendix I for the mathematical derivation of matrix (10). A new variable introduced in (10) is $\hat{Y}_t$ defining the rate of growth in nominal GDP during period $t$.

In writing matrix (10) as I do, I assume that net purchases of financial and non-financial assets and the size of the primary operating balance all expressed relative to GDP ($inv^F$, $inv^{NF}$, and $gopb$, respectively) are exogenous variables. This reflects my treatment

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[5] It is worth commenting that by dividing financial values by GDP, the effects of price inflation have been removed so long as we are willing to assume inflation affects all monetary variables equally. Thus, whether we divide the real or the nominal value of government spending, G, by the real or the nominal value of GDP, makes no difference to the value of the ratio we derive. The Baumol hypothesis suggests that the growth rate of inflation is unequal across sectors and that in particular it may be higher in the public sector. Thus, Baumol (1967) suggests it is inappropriate to divide public sector nominal values by the same price deflator used to deflate private sector nominal values. I ignore that possibility here.

of these variables as policy choice variables and so reflects the characteristics of the simple fiscal rule I am considering here.

The dynamics of the ratio of financial liabilities to GDP (*d*), the ratio of non-financial assets to GDP ($a^{NF}$), and the ratio of financial assets to GDP ($a^{F}$), are determined by the policy choice variables; the government operating primary balance relative to GDP (*gopb*), net investments in non-financial assets relative to GDP ($inv^{NF}$), and net investment in financial assets relative to GDP ($inv^{F}$). In what follows, I assume the government chooses to hold constant the size of the operating primary balance and the size of government net investments in financial and non-financial assets relative to nominal GDP. Our fiscal rule, then, after expressing as a percentage of GDP, requires that the government hold constant the gap between tax revenue and spending on goods and services.[6]

The steady-state value for the ratios of financial liabilities to GDP ($d_{SS}$), net worth to GDP ($nw_{SS}$), and the ratios of non-financial and financial assets to GDP ($a_{SS}^{NF}$, $a_{SS}^{F}$) are determined by holding the three policy-determined variables (*gopb*, $inv^{F}$, and $inv^{NF}$) constant. The steady-state value of net worth as a fraction of GDP ($nw_{SS}$) is determined by adding both non-financial and financial assets ratios minus the financial liabilities ratio (See Appendix II for the mathematical derivation).

$$d_{SS} = \left(\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\right)\left\{\left(\frac{\left(\hat{Y}+\delta-\rho^{NF}\right)}{\hat{Y}+\delta}\right)inv^{NF} + \left(\frac{\left(\hat{Y}-\rho^{F}\right)}{\hat{Y}}\right)inv^{F} - gopb\right\} \tag{11}$$

$$a_{SS}^{F} = \frac{\left(1+\hat{Y}\right)}{\hat{Y}}inv^{F} \tag{12}$$

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[6] Holding the primary balance constant is closely akin to a rule that would require both *G* and *T* to be held constant. That stronger requirement is often advocated as a way of minimizing the excess burden suffered by allowing tax rates and spending propensities to vary.

$$a_{SS}^{NF} = \frac{\left(1+\hat{Y}\right)}{\hat{Y}+\delta} inv^{NF} \tag{13}$$

$$nw_{SS} = \left(\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\right)\left\{\frac{\left(\rho^{NF}-i-\delta\right)}{\hat{Y}+\delta} inv^{NF} + \frac{\left(\rho^{F}-i\right)}{\hat{Y}} inv^{F} + gopb\right\} \tag{14}$$

The steady-state values of financial liabilities, financial assets, and net worth as fractions of GDP are dependant on the rates of return on financial and non-financial assets ($\rho^F$, and $\rho^{NF}$), the interest rate on financial liabilities ($i$), the depreciation rate of non-financial assets ($\delta$), and the growth rate of GDP ($\hat{Y}$). The steady-state value of non-financial assets depends on the depreciation rate and the growth rate of GDP, while the steady-state value of financial assets depends only on the growth rate of GDP.

The stability conditions of the model are $\hat{Y} > i$, $\hat{Y} > 0$, and $\hat{Y} + \delta > 0$ (see Appendix III for the mathematical derivation). We should expect that the second and third conditions will be satisfied in any sensible depiction of the economy as they require only a positive rate of nominal GDP growth and for the rate of depreciation of non-financial capital to be a positive fraction ($0 < \delta < 1$). The first condition for stability, $\hat{Y} > i$, requires that the rate of growth of nominal GDP exceed the interest rate paid on the government's financial liabilities. The intuition here is that should the government run a budget deficit and so issue financial liabilities, the deficit will close and so move the economy into a steady-state only if tax revenues grow faster than the servicing costs of financial liabilities. The former is determined by the rate of growth in the tax base ($\hat{Y}$) while the latter is determined by the interest rate paid on the government's financial liabilities ($i$).

## 2.4 Examples of Dynamic Analysis

The following equations, (15) and (16), will help demonstrate the dynamics of the model with constant government operating balance and investments ratios (our simple fiscal rule), which incorporate the stability conditions. The demonstration is simplified by assuming that the government does not hold nor invest in non-financial assets; thus, $a_t^{NF} = 0$. Therefore, the analysis is solely on the financial liability (*d*) and financial asset ratios ($a^F$). (It will become evident why I am only looking at these variables when I apply the oil fund fiscal rule). The difference equations become:

$$d_t = \frac{1+i}{1+\hat{Y}_t} d_{t-1} - \frac{\rho_t^F}{1+\hat{Y}_t} a_{t-1}^F - gopb_t + inv_t^F \tag{15}$$

$$a_t^F = \frac{1}{1+\hat{Y}_t} a_{t-1}^F + inv_t^F \tag{16}$$

Notice that the financial assets ratio ($a^F$) affects the financial liabilities ratio (*d*) through an inverse relationship; as the financial assets ratio in time *t-1* decreases, the financial liabilities ratio at time *t* increases, holding all else constant. Because the government's return on financial assets has decreased, the government cannot pay more towards the reduction of financial liabilities. The financial liabilities ratio does not affect the financial assets ratio.

The following two scenarios will help to demonstrate the dynamics of this model. The analysis will reveal how the two state variables move after an exogenous shock. In the first scenario, the exogenous shock will be a decrease in the government operating primary balance, while the second scenario is an increase in the investments of the financial asset.

### *2.4.1 A Decrease in the Operating Primary Balance*

In the first scenario, the government decreases its operating primary balance (*gopb*) by, for example, increasing spending relative to GDP. This change occurs while the other policy choice variables, net investments in non-financial assets relative to GDP ($inv^{NF}$) and net investment in financial assets relative to GDP ($inv^{F}$), are held constant.

The exogenous shock to *gopb* has no effect on the financial assets ratio ($a^{F}$) because it does not enter into equation (15) and net investment in financial assets relative to GDP ($inv^{F}$) is exogenous. The shock to *gopb* does affect the financial liability ratio ($d$). From equation (14), we see that a decrease in the operating primary balance (as a result, say, of an increase in spending relative to GDP) will increase the financial liabilities ratio. That is, since the government is holding net investments in financial and non-financial assets constant, it cannot finance the increase in spending by retiring assets but only by issuing additional financial liabilities.

The dynamic response of the model to this exogenous decrease in the operating primary balance rides on the relative magnitudes of $\hat{Y}$ and $i$. This is due to reasons discussed earlier in terms of the stability conditions of the model. A decrease in the primary balance requires an increase in financial liabilities. These newly issued financial liabilities carry with it an interest obligation that will increase servicing costs and, all else equal, demand a further increase in financial liabilities in the next period. This dynamic leading to ever larger financial liabilities can be short-circuited only if the government's tax revenue can grow faster than its interest obligations. The growth rate of the financial liabilities service component of the budget and the growth rate of tax revenue (holding tax rates constant) are given by $\hat{Y}$ and $i$. Thus, the dynamic response of the model to this exogenous decrease in the operating primary balance results in convergence only if $\hat{Y} > i$ which is one of the conditions for the model to converge that I discussed earlier.

### *2.4.2 An Investment in Financial Assets*

In the second scenario, it is assumed that the initial state of the financial assets ratio ($a^F$) is at a relatively low level (that is, financial assets are initially small relative to GDP) and the government chooses to increase its investment in the financial asset ($inv^F$) in order to increase its savings fund (the financial assets ratio). This is done while holding constant the operating primary balance (*gopb*) and the non-financial assets ratio ($a^{NF}$).

From equation (15), we see that the financial assets ratio ($a^F$) increases via the direct relationship with investments. The resulting change in the financial liabilities ratio ($d$), on the other hand, is more difficult to determine and is the result of two influences. First, as we see from equation (14), an increase in net investment spending on financial assets ($inv^F$) has a positive effect on financial liabilities ($d$) and this is felt in the same period. With the operating primary balance (*gopb*) and the non-financial assets ratio ($a^{NF}$) held constant, the government must sell financial liabilities to finance the purchase of financial assets. The second influence comes from the fact the purchase of financial assets generates investment income in the following period which, with the operating primary balance (*gopb*) and the non-financial assets ratio ($a^{NF}$) held constant, must cause a reduction in financial liabilities. Thus, as we see from equation (14), the increase in financial assets ($a^F$) will, in the next period, have a negative effect on financial liabilities.

Over time, as the initial increase in the financial assets ratio ($inv^F$) continues to spin off additional investment income, the second influence must eventually dominate and the financial liabilities ratio must fall.

By inspection of (15), the net influence of these two competing effects on financial liabilities ratio ($d_t$) depends on the relative magnitudes of $inv^F$ and $\left(\rho_t^F/\left(1+\hat{Y}_t\right)\right)a_{t-1}^F$. Let us assume that there exists some critical time period, $t_C$, where $inv_t^F = \left(\rho_t^F/\left(1+\hat{Y}_t\right)\right)a_{t-1}^F$. Also assume a constant rate of return on financial assets ($\rho^F$) and a constant growth rate of GDP ($\hat{Y}$) through time. In this critical period an exogenous increase in the financial asset ratio has no effect on the financial liabilities ratio.

Although the second influence of the initial increase in the financial assets ratio ($inv^F$) on the financial liabilities ratio ($d$) must eventually dominate, there must exist a period $t < t_C$ when $inv_t^F > \left(\rho_t^F/\left(1+\hat{Y}_t\right)\right)a_{t-1}^F$. During this period, the financial liabilities ratio increases with the investment of financial assets because the government's investment income has not yet grown sufficiently to generate investment income sufficient to recover the initial outlay to purchase the increase in financial assets. Hence, the government must continue to sell financial liabilities in order to finance the initial purchase of financial assets.

Eventually, at time $t > t_C$, the second influence dominates and we have $inv_t^F < \left(\rho_t^F/\left(1+\hat{Y}_t\right)\right)a_{t-1}^F$. By this time, the initial investment in financial assets has generated sufficient income to halt the need to sell financial liabilities to finance the initial purchase. The financial liabilities ratio ($d$) now falls over time.

In this section, I have illustrated how, assuming the government exogenously chooses values for the operating primary balance (*gopb*), investments in financial assets ($inv^F$), and investments in non-financial assets ($inv^{NF}$), the government's budget responds to certain exogenous events. In particular, I have shown that:

- a decrease in the operating primary balance (as the result of an increase in spending or a cut to taxes) must result in an increase in financial liabilities (*d*).
- an investment in financial assets ($inv^F$) will initially cause the financial liabilities ratio (*d*) to increase but that eventually that ratio must decline.

I will refer to these results in the next section where I examine the implications of the government following a rule for saving the revenue generated from the sale of non-renewable resource revenue.

## 2.5 An Oil Fund Fiscal Rule

As noted in the introduction, many oil rich economies struggle with the idea of fiscal sustainability, which is a main motivation for an oil fund fiscal rule to be introduced by the public sector. One of the key factors of fiscal sustainability is to achieve intergenerational fairness; one generation should not reap the benefits of the non-renewable resource rents over future generations. Thus, an oil fund fiscal rule's main goal is to manage a non-renewable resource-rich economy in such a way as to extend the lifetime of financial assets to sustain the same standard of living that occurred during the period of non-renewable resource exploitation into future generations.

It is important to stress the purpose of the oil fund is to deal with issues of intergenerational equity. This means that the oil fund is not a stabilization fund. A stabilization fund smoothes the impact of fluctuating oil revenues in total revenue and its trend value could be set to zero assuming symmetric shocks to oil revenues.[7] An oil fund is not used for that short-term purpose but rather for meeting the longer-term goal of intergenerational equity.,

In this section, I extend the model to analyze the implications for the government of Alberta of adopting an oil fund fiscal rule similar to that adopted by the government of Norway. The fiscal rule adopted by Norway involves making policy choices with respect to certain fiscal variables (Norges Bank Investment Management, 2007). I will show how this rule affects net worth, financial liabilities, and non-financial and financial assets. I will also show how the fiscal rule entails optimal behaviour in regards to the oil fund and financial liabilities.

A key feature of the fiscal rule I will examine in this section is that non-renewable resource revenue (or "oil revenue") is in the first instance deposited into a savings fund (or an "oil fund"). Consequently, oil revenue is removed from the government's budget except as may be required to meet certain budgetary goals (explained below). In what follows, oil revenue is not counted as part of tax revenue ($T$). Rather, oil revenue enters into the government's budgeting equations via financial assets (the oil fund) in the form of investment in financial assets ($INV^F$).

In analysing my model, I find that this theory is consistent with the permanent income hypothesis for the individual. The permanent income hypothesis states that in the early stage of life, an individual will accumulate debt (financial liabilities) in order to consume products. In the Mature phase of the individual's lifetime, they start accumulating wealth, smoothing consumption across time. As we will see, this is the optimal path for a

[7] The Government of Alberta maintains a Stabilization Fund for that purpose. It also maintains other funds, most notably the Alberta Heritage Savings and Trust Fund, for the purpose of meeting goals of intergenerational equity.

government with resource revenues to obtain fiscal sustainability and intergenerational fairness.

To gain some insight into the workings of the fiscal rule I will be examining, I begin by providing an intuitive explanation of the relationships between certain key elements of the rule.

The basic premise of the oil fund fiscal rule is to restrict the government's behaviour with respect to saving and spending. The fiscal rule consists of the following restrictions:

- In the first instance, 100% of non-renewable resource revenues go directly into a financial asset called the ***oil fund.*** The oil fund also receives an injection in the form of investment income earned on the assets in the fund. The return on investment of the oil fund is what I'll call the ***actual amount.***
- The government must identify the amount of the oil fund that it will allow itself to use to finance current expenditures. This, in part, reflects a guess on the part of the government of the amount of the oil fund it can reliably depend on over the medium-term and so use to finance a constant level of government expenditures. It also reflects some consideration of the appropriate amount of oil revenue that should be spent by the current generation. Let us call this amount the ***cap amount*** – the amount of current expenditures the government allows to be financed by oil revenue and the investment income generated by the oil fund.[8,9]

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[8] The cap amount may or may not be such to eat into the capital of the oil fund.

[9] The Government of Alberta is currently using a fiscal rule defined by the Fiscal Responsibilities Amendment Act of 2005 and the Financial Statutes Amendment Act of 2006. These acts identify the amount of non-renewable resource revenue the government will allow itself to finance current expenditures. This is what we would refer to here as the cap amount. For fiscal year 2007/8 the cap amount identified by the government of Alberta is $5.3 billion. As the return on its savings in 2007/08 is estimated at $2.5 billion, the

- The cap amount is not necessarily sufficient to finance all current expenditures. I define the ***required amount*** as the amount of oil revenue that is required to fill any gap between current tax revenues (net of non-renewable resource revenue) and total current expenditures (inclusive of costs to service financial liabilities) and so prevent the need to borrow and so issue financial liabilities. The required amount, then, measures the size of the non-oil operating deficit.[10]
- The cap amount need not be equal to the required amount. If the cap amount is less than the required amount – so that the government is unable to balance its operating budget – the government must finance this shortfall by selling financial liabilities. An implication of this is that there may be periods when the government is both increasing its financial assets and increasing its financial liabilities. That is, it may be saving and borrowing at the same time. If the cap amount is greater than the required amount – so that the government realizes a surplus in its operating budget – the government must purchase financial assets with the difference.

The following diagrams and hypothetical revenue and expenditure figures will be used to illustrate the workings of the oil fund fiscal rule. I'll consider three scenarios.

**Scenario A:** Let's suppose that the amount of resource revenue earned in the current year is $10 billion. Further suppose that the oil fund held $90 billion at the end of last period. Further assume that:

- the cap amount is $3 billion

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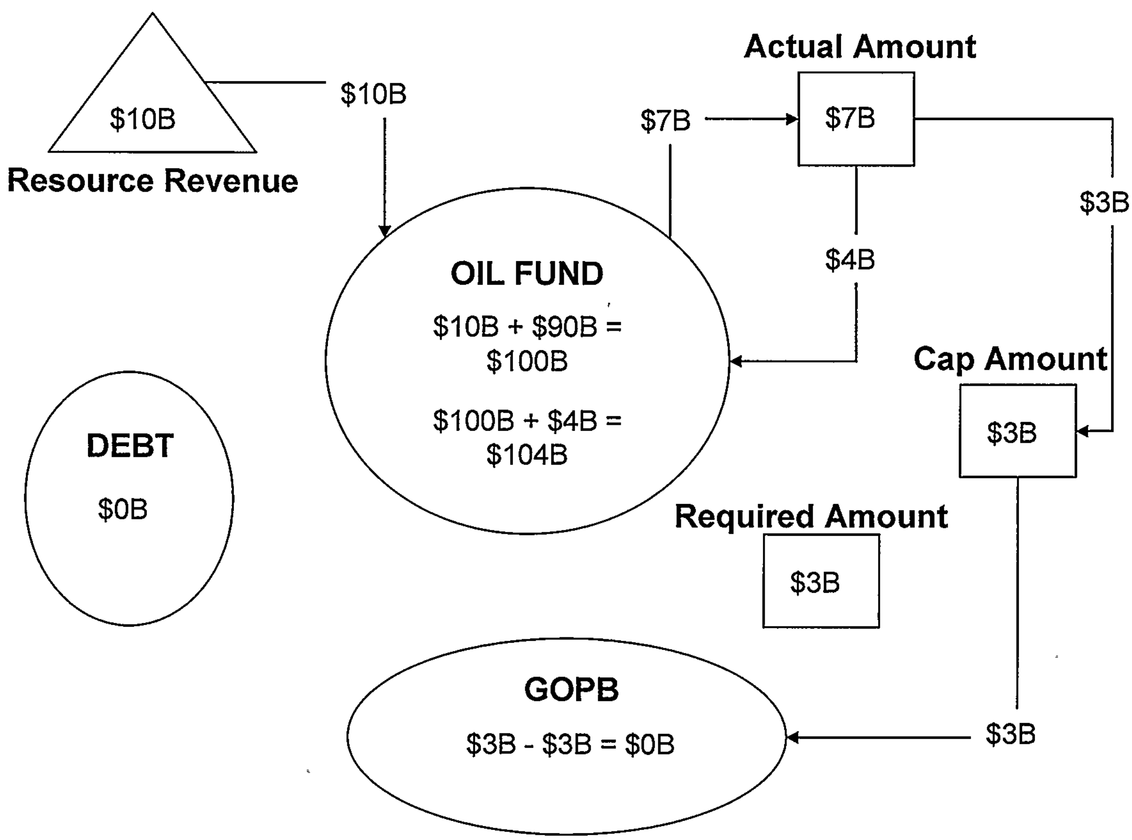
government of Alberta is consuming in the current budget all investment income plus $2.8 billion of the $10 billion in non-renewable resource revenue it expects to collect.

[10] In Section 2.5, I switch the wording of the gross operating primary deficit (or balance) to the non-oil operating deficit. Each variable measures essentially the same thing. However, in the oil fund fiscal rule, the non-oil operating deficit is the measure of the government's current expenditures out of oil wealth.

- the operating balance net of oil revenue (inclusive of the costs to service financial liabilities) is -$3 billion or a non-oil operating deficit, so that the required amount is $3 billion
- the actual amount (the investment return on the oil fund) is $7 billion.

The $90 billion oil fund generated $7 billion of investment income (the actual amount) last period and it receives $10 billion of new oil revenue this period, all of which must be in the first instance deposited in the oil fund. At this point, the oil fund contains $107 billion.

**Figure 1. Scenario A**




The *cap amount* is removed from the oil fund and made available to the government's budget. Because the cap amount is equal to the required amount, the transfer of the cap amount is sufficient to balance the operating budget. No borrowing is required. The oil fund now contains $104 billion.

In this scenario, the actual amount is greater than both the cap and the required amount. As a consequence, $4 billion of the $7 billion of investment income earned by the oil fund remains in the fund.

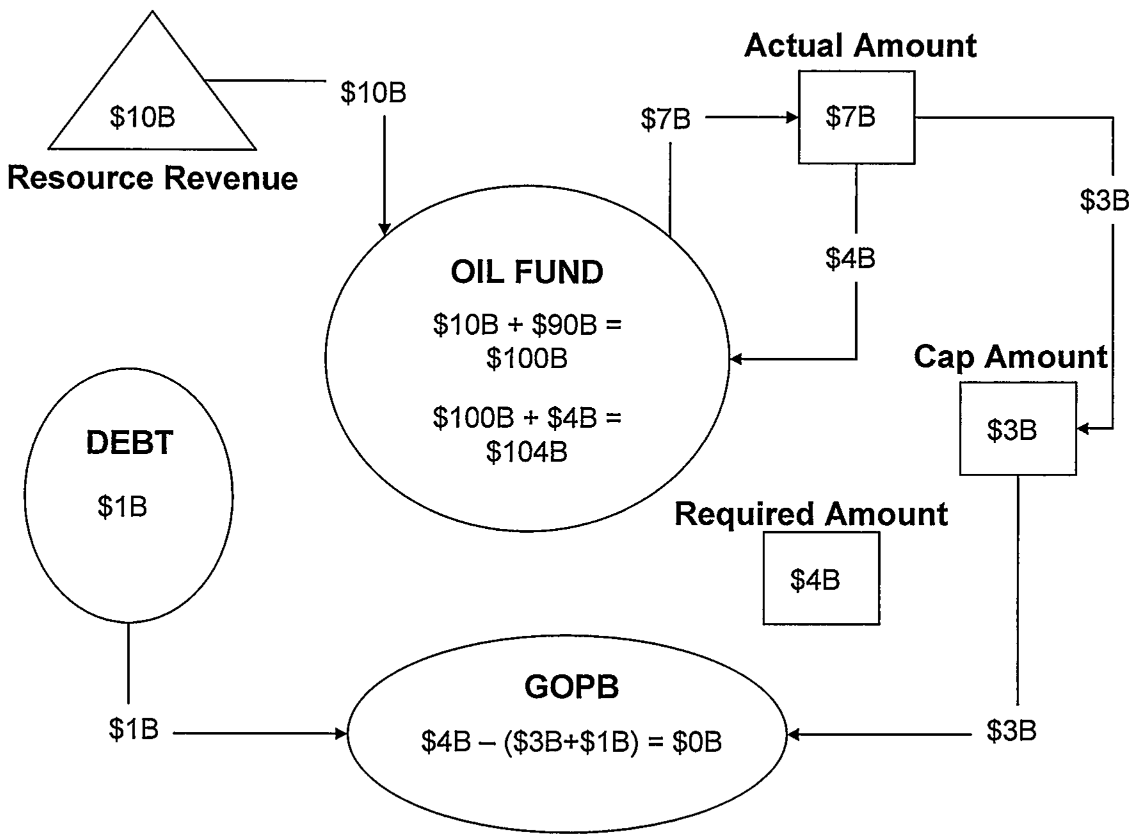
**Scenario B:** Let's again suppose that the amount of resource revenue earned in the current year is $10 billion. Suppose again that the oil fund held $90 billion at the end of last period. Further assume that:

- the cap amount is $3 billion
- the operating balance net of oil revenue (inclusive of the costs to service financial liabilities) is -$4 billion or a non-oil operating deficit, so that the required amount is $4 billion
- the actual amount (the investment return on the oil fund) is $7 billion.

As before, the $90 billion oil fund generated $7 billion of investment income (the actual amount) last period and it again receives $10 billion of new oil revenue this period, all of which must be in the first instance deposited in the oil fund. At this point, the oil fund contains $107 billion.

The cap amount is again removed from the oil fund to finance current spending. In this scenario, however, the operating deficit is larger than the cap amount and so the $3 billion transfer is insufficient to balance the operating deficit. To balance the operating deficit the government must issue financial liabilities and so accumulate $1 billion of financial liabilities. It does so even while leaving $4 billion of the $7 billion of investment income earned by the oil fund in the fund. The government, then, simultaneously borrows $1 billion while saving $4 billion.

**Figure 2. Scenario B**



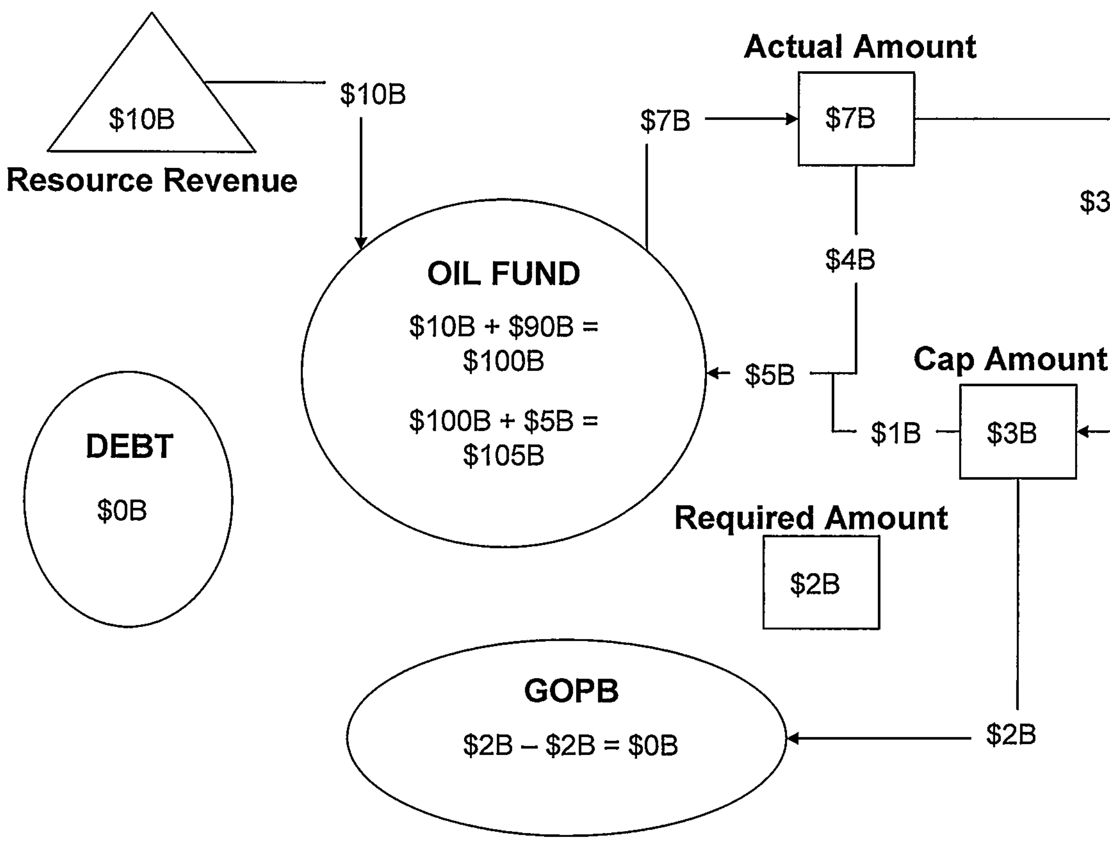

**Scenario C:** Again suppose that the amount of resource revenue earned in the current year is $10 billion and that the oil fund held $90 billion at the end of last period. Further assume that:

- the cap amount is $3 billion
- the operating balance net of oil revenue (inclusive of the costs to service financial liabilities) is -$2 billion or a non-oil operating deficit, so that the required amount is $2 billion
- the actual amount (the investment return on the oil fund) is $7 billion.

As before, the $90 billion oil fund generated $7 billion of investment income (the actual amount) last period and it again receives $10 billion of new oil revenue this period, all of which must be in the first instance deposited in the oil fund. At this point, the oil fund contains $107 billion.

In Scenario C, the cap and actual amounts are, again, what they were in Scenarios A and B, but now, the required amount is only $2 billion. The cap amount, then, is more than sufficient to balance the operating budget so that there is no need to issue financial liabilities. In fact, in this scenario $1 billion of the cap amount is returned to the oil fund. The oil fund in this scenario finishes the period holding $105.

**Figure 3. Scenario C**




The purpose of examining these three scenarios is to gain an appreciation for how the government's choices with respect to the cap amount and the required amount (which it determines by choosing the size of the non-oil operating deficit) govern the accumulation of both financial liabilities and the oil fund. All else equal, increasing the size of the cap amount and increasing the size of the required amount reduces the size of the oil fund. Choosing a cap amount that is less than the required amount causes the government to accumulate financial liabilities even while, in some instances, increasing the size of the oil fund (financial assets). Choosing a cap amount that is greater than the required amount causes the government to accumulate financial assets in the oil fund more quickly.

## 2.6 Conclusion

This chapter presented and discussed a set of accounting relationships describing an economy reliant on non-renewable resource revenue. It also introduced concepts that allow for an examination of the implications a set of fiscal rules has for how the government finances of this economy. Some basic intuition of the implications of these rules has been presented.

In the next chapter, I adjust these accounting relationships to better enable an examination of the implications of the fiscal rules on government finances during the "life" of an oil economy. That is, I adjust the relationships introduced in this chapter to allow me to identify the implications of the fiscal rule for the government's finances throughout the period when oil is extracted. Three periods will be identified: an Initial phase when oil is first discovered, a Mature phase when oil production is at its peak, and a Terminal phase during which the oil resource is exhausted. My examination of the implications for the government's finances during these three phases will rely on numerical simulations.

# Chapter Three

# The Dynamics

## 3.1 Introduction

The previous chapter presented and discussed a set of accounting relationships describing an economy reliant on non-renewable resource revenue. It introduced concepts that allow for an examination of the implications a set of fiscal rules on the government finances of this economy and some basic intuition of the implications of these rules was presented. Three simple scenarios were offered to help understand the implications of the fiscal rule. Those scenarios, however, offered only a "snapshot" of the economy at a point in time.

In this chapter, I adjust these accounting relationships to better enable an examination of the implications of the fiscal rules on government finances during the "life" of an oil economy. This will allow me to identify the implications of the fiscal rule for the government's finances throughout the period when oil is extracted. Three periods will be identified: an Initial phase when oil is first discovered, a Mature phase when oil production is at its peak, and a Terminal phase during which the oil resource is exhausted. My examination of the implications for the government's finances during these three phases will rely on numerical simulations.

In the next section, Section 3.2, I generalize the accounting relationships used in the previous chapter and introduce the rates of return. Section 3.3 will illustrate the dynamics of the oil fund fiscal rule with regards to the three phases of the oil economy, while Section 3.4 will present the resulting equations. Finally, Section 3.5 will conclude the chapter.

### 3.2 Generalizing the Accounting Relationships

The model described in sections 2.2, 2.3, and 2.4 defined rates of return on financial and non-financial assets ($\rho^F$ and $\rho^{NF}$, respectively), the rate of return on financial liabilities ($i$), and the rate of GDP growth ($\hat{Y}$). I abstracted from these considerations when discussing the basic intuition of the effects of the fiscal rule (Section 2.5).

The introduction of rates of return and rates of growth allows us to see the dynamics of the financial liabilities and asset ratios and draw the implications of these dynamic processes for hitting fiscal targets. For instance, as the financial asset (the oil fund) grows, the rate of return does not need to be as high to generate the same dollar amount of investment income as it did when the financial assets was smaller. If the government has a target for the size of the oil fund in dollar terms, then the rate of return demanded of fund managers can fall (perhaps implying less risky asset holdings) or, what I have called the cap amount, can increase and so finance a higher level of spending or lower tax rates.

It is useful to begin by restating the fiscal rule as a way of explaining the roles of these three new variables:

- In the first instance, 100% of non-renewable resource revenues go directly into a financial asset called the ***oil fund***. The oil fund earns investment income, what I have called the ***actual amount***. That investment income expressed as a fraction of the size of the oil fund defines the ***observed rate of return***, $\rho^F$.
- The government must identify the amount of oil revenue and/or investment income it will allow itself to use to finance current expenditures. This is the ***cap amount***. In what follows, I identify the ***cap rate of return*** ($\rho^{CAP}$) as the cap amount expressed as a

fraction of the value of the oil fund. Whether the cap rate of return rises or falls depends on changes in the size of the oil fund and on changes in the cap amount.

- The ***required amount*** is the dollar amount of oil revenue that is required to fill any gap between current tax revenues (net of non-renewable resource revenue) and total current expenditures (inclusive of servicing costs of financial liabilities) and so prevent the need to issue financial liabilities. The required amount, then, measures the size of the non-oil operating deficit. In what follows, I identify the ***required rate of return*** ($\rho^{REQ}$) as the required amount expressed as a fraction of the value of the oil fund. Whether the required rate of return rises or falls depends on changes in the size of the oil fund and on changes in the required amount.
- The cap amount need not be equal to the required amount; it can be less than or greater than the required amount. Consequently, the cap rate of return ($\rho^{CAP}$) can be greater or less than the required rate of return ($\rho^{REQ}$). If the cap rate is less than the required rate – so that the government is unable to balance its operating budget – the government must finance this shortfall by selling financial liabilities. If the cap rate of return is greater than the required rate of return – so that the government realizes a surplus in its operating budget – the government must purchase financial assets with the difference and so add to the oil fund.

The required rate of return on the oil fund ($\rho^{REQ}$) is a hypothetical rate of return on the oil fund that would finance the optimal non-oil deficit ($\overline{gopb}$) and interest payments on outstanding financial liabilities ($id_{t-1}$) and so prevent the need to purchase additional financial liabilities. The required rate of return is determined by equation (17).[11]

$$\rho_t^{REQ} = \frac{-\overline{(gopb)}(1+\hat{Y}_t) + id_{t-1}}{a_{t-1}^F} \tag{17}$$

where the denominator identifies the value of all financial assets (the oil fund) and the numerator identifies the non-oil operating deficit. The required rate of return, then, is determined by the size of the optimal non-oil operating deficit, the stock of financial liabilities, the interest rate paid on financial liabilities, the size of the oil fund at the end of last period, and the rate of growth of GDP.

The cap rate of return is the rate of return required to finance the cap amount – the amount of current expenditure the government has chosen to finance using oil revenue, using the investment income earned on the oil fund, or by borrowing.

$$\rho_t^{CAP} = \frac{(cap\,amount_t)(1+\hat{Y}_t)}{a_{t-1}^F} \tag{18}$$

The cap rate varies with the cap amount, the rate of growth of GDP, and the size of the oil fund.

As well as following the fiscal rule, the government optimizes by smoothing welfare across generations. That is, as discussed in the introduction, it adheres to a version of the permanent income hypothesis. To do so, it chooses an optimal value for the non-oil operating deficit, the difference between tax revenue (exclusive of oil revenue) and

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[11] Equation (17) is determined from the first line in equation (10), taking into account the fiscal rule (all investment goes directly into the oil fund or financial asset) and non-financial assets are removed to simplify. Also, from the definitions, the equation takes into account that the government operating primary balance is this optimal amount ($\overline{gopb}$), the financial assets earns the required amount to finance the government's optimal spending ($\rho^{REQ}$), so that the government does not acquire financial liabilities through this equation ($d_t - d_{t-1} = 0$).

program expenditures. This reflects some consideration of the appropriate share of current spending that ought to be financed by future generations. It also reflects the government's understanding of the implications of this choice for future investment income to be enjoyed by future generations. The shortfall between current taxation and current spending can be covered by oil revenue, by the investment income earned on the oil fund, or by borrowing. These are all equivalent ways of taxing future generations. Expressed as a percentage of GDP this optimal value for the non-oil operating deficit is given by $\overline{gopb}$.

The government's policy choice variables will be the required rate ($\rho^{REQ}$) and the cap rate ($\rho^{CAP}$). The relationship between these variables, relative to the observed rate ($\rho^{F}$) received by the oil fund and given the optimal value of the non-oil operating deficit ($\overline{gopb}$), will determine whether the government builds or shrinks the oil fund ($a^{F}$) and financial liabilities ($d$).

## 3.3 Dynamics of the Oil Fund Fiscal Rule

In this section, I will introduce the three phases of the life of an oil economy. Within these phases, the Alberta government will behave in an optimal way in order to achieve fiscal sustainability. Also, the three phases will govern the interaction between the three rates of returns I introduced in the previous section.

### *3.3.1 Three Phases of the Oil Economy*

The life of an oil economy can be separated into three distinct phases: one in which oil production has begun, the next in which oil production is increasing and then peaks, and finally, one where oil production is declining.

**The Initial Phase:** The first phase of an oil economy's life is the Initial phase. In this phase, oil exploration and discovery has occurred, and extraction of the oil is set to begin. Before oil production starts, the government will lay out details that will regulate output, such as resource royalties. In this way, the government can determine, to some extent, how much they will receive in oil revenues. As the government's goal is fiscal sustainability, it will use this knowledge to determine an optimal level of government spending on goods and services and an optimal level of taxation (as fractions of GDP). These choices are made in consideration of what will be optimal now and into the future, given the extraction of the oil and the fact the oil is a non-renewable resource. In making its spending and taxation choices, the government needs to establish a level of spending and tax rates that will be optimal now and into the future. These choices are reflected in the choice of $\overline{gopb}$ (the optimal non-oil operating balance). Note that as GDP grows over time, $G$ and $T$ must also grow over time to hold the optimal non-oil operating balance constant as a fraction of GDP.

Relating this to the permanent income hypothesis, the government will borrow (i.e., purchase financial liabilities) in order to achieve the optimal level of the non-oil operating balance immediately. Since, the government does not realize the full amount of the oil revenues, that is, the total amount of all oil revenues the government will receive from the total extraction of all the oil discovered, it will need to borrow the funds needed to reach this optimal or "golden" state of the economy ($\overline{gopb}$). The golden state is now the standard of living all current and future generations in the Alberta economy will have, thereby achieving intergenerational equity.

In the Initial phase, the government is borrowing, in other words, the government is accumulating financial liabilities ($d$). Yet, the oil fund fiscal rule requires that the government save its oil revenues in an oil fund ($a^F$). Therefore, the government is establishing a principal in the oil fund ($a^F$) through the addition of oil revenues ($inv^F$) but not through the dynamics of the fiscal rule (which will be explained later on in the section).

**The Mature Phase:** The second phase of an oil economy's life is the Mature phase. The Mature phase is defined by stable and increasing oil production. In this phase, oil production is producing a continuous flow of oil and will reach its peak in production at the end of this period. In the previous phase, the government established the optimal or golden state of the economy ($\overline{gopb}$) as well as a principal in the oil fund ($a^F$).

Relating this to the permanent income hypothesis, the government continues to purchase financial liabilities ($d$). However, the government now is able use the return on the oil fund ($a^F$) to help balance the optimal level of the non-oil operating deficit ($\overline{gopb}$), since the principal has become sufficient. In this phase, the government can put funds towards the retirement of financial liabilities. In the Mature phase, the government is borrowing and saving through the dynamics of the fiscal rule. In other words, financial liabilities ($d$) and the oil fund ($a^F$) are accumulating, but financial liabilities are increasing at a slower rate.

**The Terminal Phase:** The third and final phase of the oil economy's life is the Terminal phase. This phase is defined by the termination of oil production. At the beginning of this phase, oil production is on the decline, and declines throughout the remainder of its lifetime until oil production ceases (it is assumed that no new energy resource technology is introduced to extend the lifetime of the oil reserves infinitely).

In terms of the permanent income hypothesis, the government will only save during this phase. It will no longer borrow but rather use the return on the accumulated assets ($a^F$) to balance the non-oil operating deficit ($\overline{gopb}$). In the Terminal phase, the dynamics of the fiscal rule will only allow for the growth of the oil fund.

### *3.3.2 Relationship between the Rates of Return and the Phases*

Having learned how the government should behave in each of the three phases with respect to the permanent income hypothesis, it is important to relate that behaviour to the three rates of return – the observed rate ($\rho^F$), the cap rate ($\rho^{CAP}$) and the required rate ($\rho^{REQ}$). In this sub-section, I will show how these three rates of return need to behave in order to achieve the optimal behaviour of the government during each phase.

First, let us review the results of the scenarios presented in Section 2.5, but instead introduce the three rates of return. This will illustrate how the relative magnitudes of these rates produce the optimal behaviour of the government.

(1) When $\rho^{REQ} > \rho^F$, the required amount of investment income spun off by the oil fund exceeds the amount realized. As a consequence, the government is unable to fill the gap between current tax revenues (net of non-renewable resource revenue) and total current expenditures (inclusive of costs to service financial liabilities). It must issue financial liabilities.

(2) When $\rho^{REQ} > \rho^{CAP}$, the portion of current expenditures the government has chosen to finance with investment income generated by the oil fund is not sufficient to fill the gap

between current tax revenues (net of non-renewable resource revenue) and total current expenditures (inclusive of costs to service financial liabilities). In this case, the government must issue financial liabilities.

(3) When $\rho^{F} > \rho^{REQ}$, the investment income earned on the oil fund is more than sufficient to fill the gap between current tax revenues (net of non-renewable resource revenue) and total current expenditures (inclusive of costs to service financial liabilities). In this case, the government purchases financial assets and so adds to the oil fund.

(4) When $\rho^{F} > \rho^{CAP}$, the investment income earned on the oil fund is more than sufficient to finance that portion of current expenditures the government has chosen to finance with investment income generated by the oil fund. The excess is retained by the oil fund.

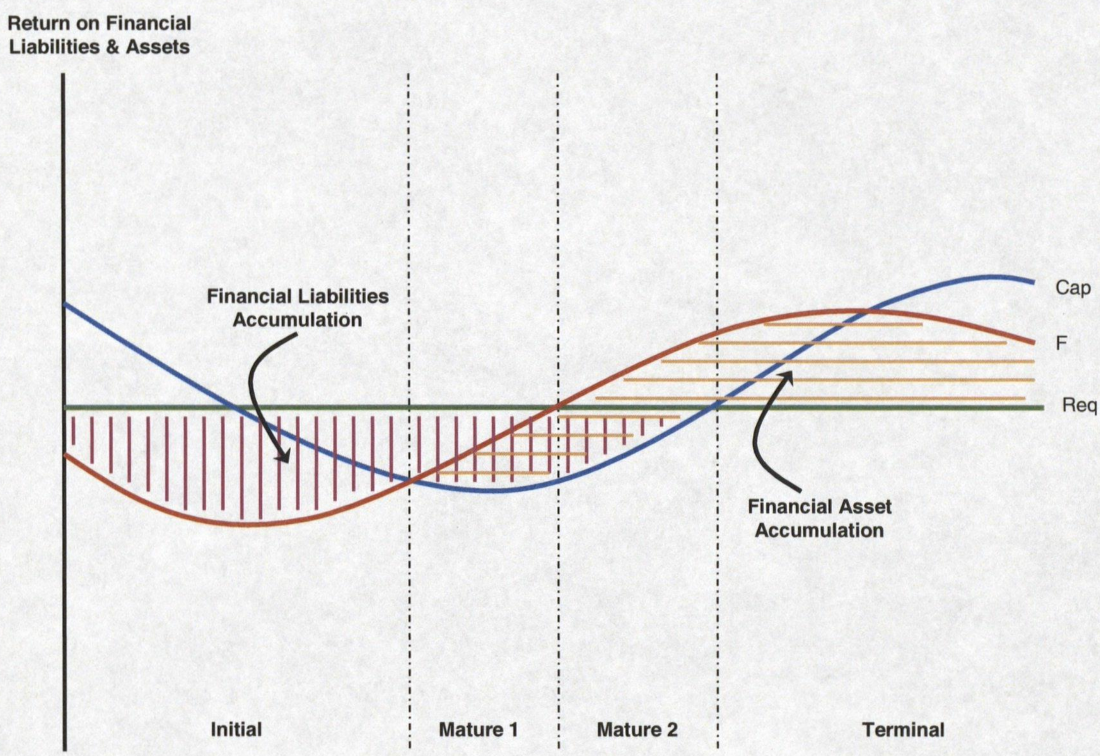
In ***the Initial phase***, I have determined that the government's behaviour should be to borrow in order to balance the non-oil operating deficit and allow the oil fund to establish a principal. To attain this, both the cap rate ($\rho^{CAP}$) and the required rate ($\rho^{REQ}$) should be greater than the observed rate ($\rho^{F}$), as seen in (1) and (2) above.

In ***the Mature phase***, the government's behaviour is to both borrow and save. This phase is unique in that it has two sub-phases. In the first sub-phase, the government will borrow more than it will save, while in the second sub-phase it will save more than it borrows. To achieve the appropriate behaviour in the first sub-phase, the required rate ($\rho^{REQ}$) will be greater then the observed rate ($\rho^{F}$) and greater than the cap rate ($\rho^{CAP}$). The appropriate responses will be (1), (2) and (4). In the second sub-phase, the observed rate ($\rho^{F}$) is greater than the required rate ($\rho^{REQ}$) which is greater than the cap rate ($\rho^{CAP}$), resulting in (2), (3) and (4) above.

Finally, in ***the Terminal phase***, I determined that the government's behaviour should be to only save and use the return from those savings to balance the non-oil operating deficit. To attain this behaviour, the observed rate ( $\rho^{F}$ ) will be greater than both the cap rate ( $\rho^{CAP}$ ) and the required rate ( $\rho^{REQ}$ ), as in (3) and (4) above.

The following figure displays the relationship between the three phases, the three rates of return and how they financial assets and liabilities build up.

**Figure 4. The Relationship between the Rates of Return and the Phases**




Note that in Figure 4 **Cap** refers to the earnings on the cap rate of return; **Req** refers to the earnings on the required rate of return; and **F** refers to the earnings of the observed rate of return.

## 3.4 Equations of the Oil Fund Fiscal Rule

In the previous section, I described the optimal behaviour of the government in each of the three stages in the development of an oil economy. In order to derive the dynamic equations for the model with an oil fund fiscal rule, I introduce two new parameters, $\lambda$ and

$\gamma$, which take values of either 0 or 1. Their purpose is to act as "switches" that will enable me to show how values of financial liabilities, financial assets (the oil fund), and non-financial assets (schools, human capital, infrastructure) vary for the different stages in the oil economy.

The $\lambda$ parameter indicates whether or not the stock of financial liabilities grows over time. Thus,

$\lambda = 1$; financial liabilities accumulate

$\lambda = 0$; financial liabilities do not change

The $\gamma$ parameter indicates whether or not the stock of financial assets (the oil fund) grows over time. Thus,

$\gamma = 1$; financial assets accumulate and the oil fund grows

$\gamma = 0$; financial assets do not change.

The following table will illustrate how the phases of an oil economy, the rates of return, and the parameters interact to produce the government's optimal behaviour – borrowing and/or saving.

**Table 2. Phase and Rate of Return Relationship**

| PHASE | RATES | PARAMETER |
|---|---|---|
| Initial | $\rho^{F} \leq \rho^{CAP} \leq \rho^{REQ}$ | $\lambda = 1;\quad \gamma = 0$ |
| Mature 1 | $\rho^{CAP} \leq \rho^{F} \leq \rho^{REQ}$ | $\lambda = 1;\quad \gamma = 1$ |
| Mature 2 | $\rho^{CAP} \leq \rho^{REQ} \leq \rho^{F}$ | $\lambda = 1;\quad \gamma = 1$ |
| Terminal | $\rho^{REQ} \leq \rho^{CAP} \leq \rho^{F}$ | $\lambda = 0;\quad \gamma = 1$ |

In Appendix IV, I show how incorporating the fiscal rules described in the previous two sections along with new parameters, $\lambda$ and $\gamma$, allows one to write the general laws of motion for the ratios to GDP of financial liabilities, financial assets (the oil fund), and non-financial assets in the following way:

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} - \left(gopb_t - \overline{gopb}\right) - \frac{\rho_t^{NF}}{1+\hat{Y}_t} a_{t-1}^{NF} + \lambda\left(\frac{\rho_t^{REQ} - (1-\gamma)\rho_t^F - \gamma\rho_t^{CAP}}{1+\hat{Y}_t}\right) a_{t-1}^F \tag{19}$$

$$a_t^F = \frac{1}{1+\hat{Y}_t} a_{t-1}^F + \gamma\left(\frac{\rho_t^F - (1-\lambda)\rho_t^{REQ} - \lambda\rho_t^{CAP}}{1+\hat{Y}_t}\right) a_{t-1}^F + inv_t^F \tag{20}$$

$$a_t^{NF} = \frac{(1-\delta)}{1+\hat{Y}_t} a_{t-1}^{NF} + inv_t^{NF} \tag{21}$$

### 3.4 Conclusion

In the above three sections, I have described and illustrated the optimal behaviour for a government, reliant on non-renewable resource revenues, whose goal is to achieve fiscal sustainability. From this behaviour, the government chooses the cap and the required rates of return in order to achieve this optimal outcome. To summarize:

- In the **Initial phase,** the government finds it optimal to borrow in order to finance its current expenditures. In order to do this, the government sets the *required rate* ($\rho^{REQ}$) and the *cap rate* ($\rho^{CAP}$) higher than the *observed rate* ($\rho^F$). This causes financial liabilities ($d$) to accumulate, while the financial asset ($a^F$) only grows through injections of oil revenues ($inv^F$).

- In the **Mature phase,** the government's optimal behaviour is to both borrow and save. Initially, the government will set the *required rate* ($\rho^{REQ}$) greater than the *observed* *rate* ($\rho^F$) which is greater than the *cap rate* ($\rho^{CAP}$). As a result, financial liabilities ($d$) and financial assets ($a^F$) both accumulate, but financial liabilities are increasing at a slower rate. In the second sub-phase, the government will set the *cap rate* ($\rho^{CAP}$) lesser than the *required rate* ($\rho^{REQ}$) which will be less than the *observed rate* ($\rho^F$). At this stage, financial assets ($a^F$) gain over financial liabilities ($d$), which results in positive net worth.

- In the **Terminal phase**, the government will find it optimal to use the return on its savings to finance its current expenditure. In order to do this, the government will set both the *required* ($\rho^{REQ}$) and *cap rates* ($\rho^{CAP}$) lower than the *observed rate* ($\rho^F$). This will cause financial assets ($a^F$) to grow.

In the next chapter I will examine a hypothetical situation for the Alberta government. It describes how the government of Alberta should behave upon the discovery of an oil reserve. The simulation will show what the model's outcome is in terms of the level of financial liabilities, financial assets and net worth. The chapter will also discuss the sensitivity of the results to parameter values such as the level of the non-oil operating deficit.

# Chapter Four

# The Simulations

## 4.1 Introduction

The purpose of the simulation is to illustrate a hypothetical situation in which a new oil reserve is discovered and how the rule, defined in the previous chapters, behaves. I determine what the fiscal situation with regards to financial liabilities, financial assets and net worth, would look like if the government behaved according to the optimal behaviour in the model. I will then discuss the sensitivity of the results to certain assumptions of parameter values, such as the size of the government's non-oil operating deficit.

The chapter is organized as follows. Section 4.2, outlines the assumptions and parameters in the simulation and defines the dynamic equations. Section 4.3 presents the results of the simulation, while section 4.4 displays the sensitivity of the results to the assumption of the size of the government's non-oil operating deficit and discusses the implications. Section 4.5 concludes.

## 4.2 Framework

In order to derive the simulation, I have made several assumptions regarding the parameter values. The interest rate on financial liabilities ($i$), GDP growth rate ($\hat{Y}$), depreciation ($\delta$), the return on non-financial assets ($\rho^{NF}$), and the observed return on financial assets ($\rho^{F}$) represent reasonable estimates of reality, but are used for purely illustrative purposes. The interest rate on financial liabilities ($i$) is equal to 3%, which is the 10-year average of the return on Canadian long-term government bonds. The GDP growth rate ($\hat{Y}$) is assumed to be 5%, which is the average of the growth rate in Alberta's nominal GDP over the past 25 years. The depreciation rate ($\delta$) is assumed to be 10%, in other words, capital is replaced every 10 years in the economy. The return on non-financial assets ($\rho^{NF}$) is 4%, while the observed rate of return on financial assets ($\rho^{F}$) is 6%. All of the above variables are kept constant throughout the lifetime of the oil industry and are in nominal terms.

I am abstracting from all the uncertainty in the economy. In doing so, I am not denying the importance of the stochastic nature of the problem, however, it adds to the complexity. Therefore, I assume the total amount of oil reserves is known. This also allows me to determine the production path of oil which, according to my model will increase until the end of the Mature phase and then decline until oil is exhausted at the end of its lifetime. For the government planner, knowledge about the production path will allow it to apply the optimal behaviour described in the previous chapters. The assumption about the deterministic size of oil reserves might seem unlikely for conventional oil. However, as I will show in Chapter Five, this assumption is likely when dealing with the oil sands.

The initial positions of financial liabilities and financial assets are assumed to be zero. However, I assume that the economy has a positive initial non-financial asset position. In other words, the economy has an initial stock of infrastructure, institutions, and human capital. I also assume that the government maintains a non-oil operating deficit ($gopb$) constant at its optimal level throughout the lifetime of the oil industry ($gopb_t = \overline{gopb}$). As the focus of my simulation is on the dynamics of financial liabilities, financial assets and net worth, I assume investment in non-financial assets ($inv^{NF}$) to be constant over the entire time period. This assumption is designed to give a constant level of non-financial assets.

The government's choice variables are the cap rate ($\rho^{CAP}$) and the required rate ($\rho^{REQ}$). Together with the observed rate ($\rho^{F}$), each of these variables is chosen according to the optimal behaviour which was outlined in the previous chapters. For example, in the Terminal phase, the cap rate is set to be greater than the observed rate, which is in turn greater than the required rate.

In order to determine the investment in financial assets ($inv^{F}$) or oil revenues, I have made assumptions regarding the price of oil, resource royalties and the size of oil reserves. In my simulation, I have chosen the price of oil as well as resource royalties to remain constant across the lifetime of the oil industry. This also decreases the complexity of the problem without affecting the qualitative results. The price of oil is assumed to be \$40 per barrel, while the resource royalty is 5%. As I stated previously, the amount of oil reserves is known and assumed to be 1,000,000 barrels, which will be extracted in the next 200 years. I have determined a production path where oil production increases at a constant rate until peaking at the end of the Mature phase (year 150), and declining at a constant rate until oil ceases to exist at year 200.

In order to perform this simulation, I have to further define the dynamic equations of the model. From the model of the oil fund fiscal rule, we know that both the required rate ($\rho^{REQ}$) and the optimal amount of the non-oil operating deficit ($\overline{gopb}$) jointly determine the optimal amount of how much the Government of Alberta needs to spend.

Since both of these variables are in our dynamic equations (19) and (20), I use equation (17) and solve for the required rate.[12] Recalling equation (17):

$$-\overline{gopb}+\frac{i}{1+\hat{Y}_t}d_{t-1}=\frac{\rho_t^{REQ}}{1+\hat{Y}_t}a_{t-1}^F \tag{17}$$

Solving for the required rate:

$$\rho_t^{REQ}=\frac{d_{t-1}}{a_{t-1}^F}i-\left(1+\hat{Y}_t\right)\frac{\overline{gopb}}{a_{t-1}^F} \tag{17a}$$

Starting with the dynamic equation for financial liabilities, substitute in the above equation into equation (19) to find:

$$d_t=\frac{1+\lambda i}{1+\hat{Y}_t}d_{t-1}-\frac{\rho_t^{NF}}{1+\hat{Y}_t}a_{t-1}^{NF}-\lambda\left(\frac{\left(1-\gamma\right)\rho_t^F+\gamma\rho_t^{CAP}}{1+\hat{Y}_t}\right)a_{t-1}^F-gopb_t+\left(1-\lambda\right)\overline{gopb} \tag{19a}$$

In the above equation of financial liabilities, we can see that in the Initial and Mature phases of an oil industry (when $\lambda=1$), financial liabilities directly finance current expenditures and accumulate. Taking a closer look at the individual phases, in the Initial phase (when $\lambda=1$ and $\gamma=0$), all of the return on the financial assets goes towards retiring financial liabilities. However, the return earned on financial assets is less than the non-oil operating deficit and financial liabilities accumulate.

In the first sub-phase of the Mature phase (when $\lambda=1$ and $\gamma=1$), financial liabilities continue to accumulate only at a slower rate than they did in the previous phase because the return earned on the financial assets is less than the non-oil operating deficit. In the second sub-phase of the Mature phase (when $\lambda=1$ and $\gamma=1$), the return earned on the financial asset is larger than the non-oil operating deficit and financial liabilities

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[12] The required rate of return is used instead of the optimal government non-oil deficit because the former enters into both equations (19) and (20), unlike the latter which only enters into equation (19).

decrease. Financial liabilities decrease in this phase because the size of the financial asset is larger in the second sub-phase than in the first, therefore, the return earned is larger.

Now, insert equation (17a) into equation (20) to find:

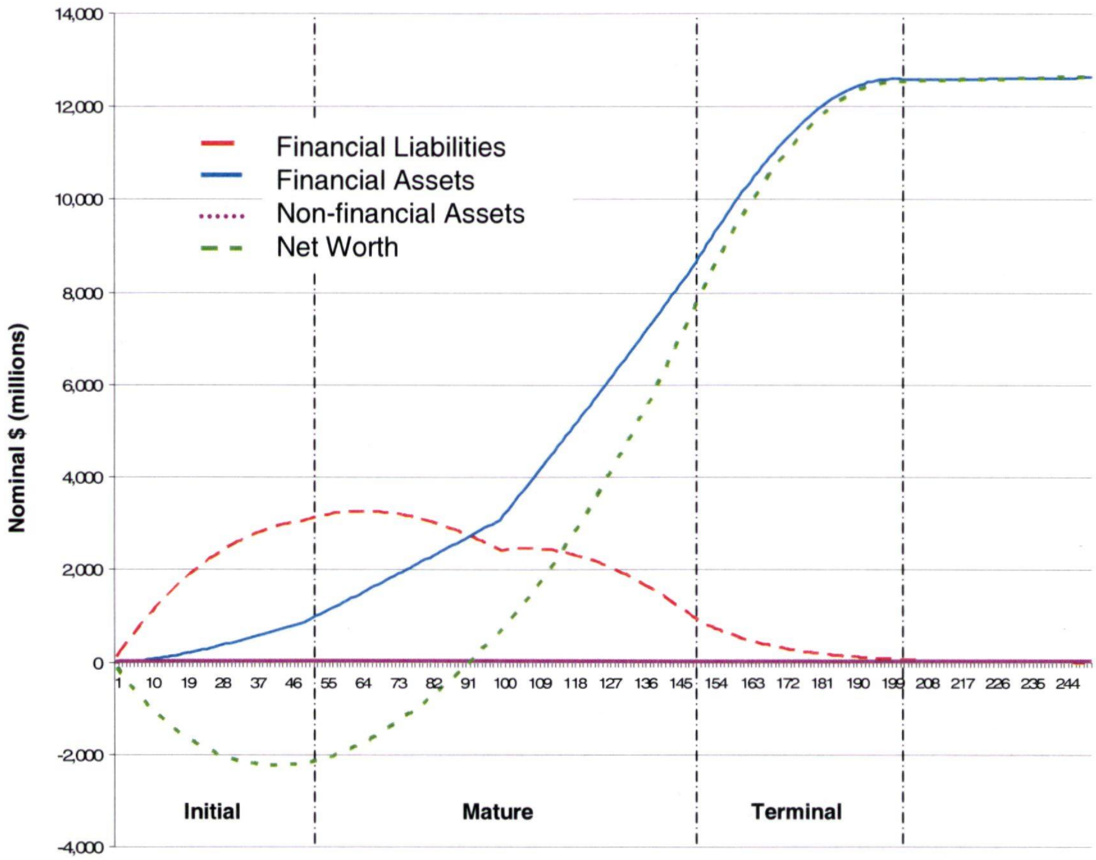
$$a_t^F = \frac{1}{1+\hat{Y}_t} a_{t-1}^F - \frac{\gamma(1-\lambda)i}{1+\hat{Y}_t} d_{t-1} + \gamma \left[ \frac{\rho_t^F - \lambda \rho_t^{CAP}}{1+\hat{Y}_t} \right] a_{t-1}^F + inv_t^F + \gamma(1-\lambda)\overline{gopb} \qquad (20a)$$

Equation (20a) describes the movement of financial assets. In the Initial phase (when $\lambda = 1$ and $\gamma = 0$), only the investment of oil revenues is retained in the fund to accumulate assets. In the Mature phase (when $\lambda = 1$ and $\gamma = 1$), a partial amount of the return earned on the asset is kept in the fund and the financial asset accumulates. Over the course of the Mature phase, the amount of return retained in the fund grows and the financial asset increases further. In the Terminal phase (when $\lambda = 0$ and $\gamma = 1$), all of the return earned is retained in the fund, but the financial asset now finances the non-oil operating deficit. As a result, financial assets continue to accumulate, but to a lesser extent than in the previous phase.

### 4.3 Results

The results of the initial simulation are presented in Figure 5.

**Figure 5. The Base Case Simulation**




In the Initial phase, the government establishes a given level of the non-oil operating deficit and keeps this constant throughout the lifetime of the oil industry ( $\overline{gobp}$ ). This will insure intergenerational equity. In establishing this, the government must be willing to borrow by financing current expenditures with financial liabilities, which allows financial assets (the oil fund) to grow. As a result, in the Initial phase, financial liabilities are increasing at a faster rate than financial assets, and therefore, net worth of the economy is negative throughout this period. One key implication in this phase, for our results to follow the optimal path, is that investment in financial assets needs to be less than the optimal non-oil operating deficit.

As discussed in the previous chapters, the optimal non-oil operating deficit is the government's decision. Understanding its implications on oil for future revenue streams, it must establish a level of spending and tax rates that will be optimal now and into the future. Therefore, investment in financial assets needs to be less than the optimal non-oil operating deficit or else the government could somehow reason that the need for an oil fund is negligible.

In the Mature phase, financial assets rise above financial liabilities as some of the assets are now going towards balancing the non-oil operating deficit. With financial liabilities declining and financial assets increasing, net worth becomes positive in this period.

Looking forward into the Terminal phase, financial liabilities decline towards zero as it no longer finances the optimal non-oil operating deficit. On the other hand, the financial asset now keeps all of its return (the observed rate) and finances both the interest payment on financial liabilities and the optimal non-oil operating deficit. One of the reasons why the financial asset reaches its maximum is due to how its investment is related to oil production. The Mature phase ends with the peak of oil production; therefore, in the Terminal phase oil production is declining. Investment in financial assets (oil revenues) are tied with oil production, and with declining oil production, investment follows suit and the financial assets reach an inflection point. Throughout the Terminal phase, there is positive net worth.

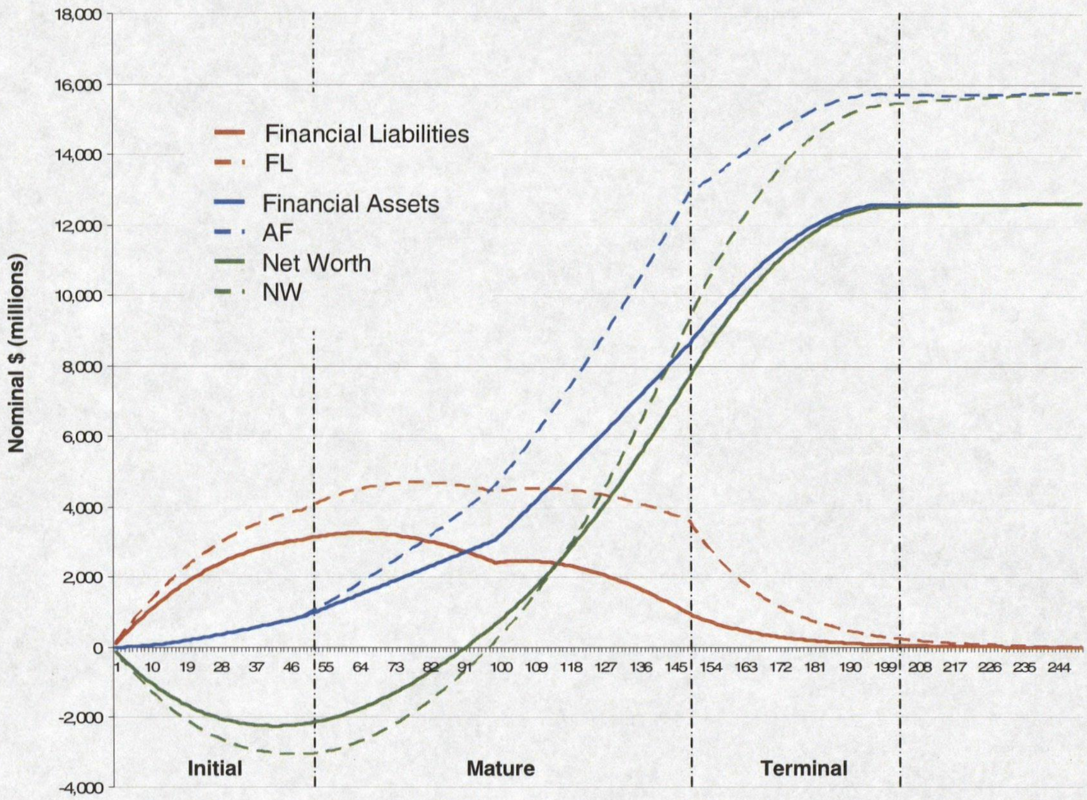
At the end of the Terminal phase, oil has been exhausted. As is shown in Figure 5, net worth and financial assets continue to grow, although at a lesser rate and financial liabilities approach zero. Both net worth and financial assets are sustainable. That is to say, the return on financial assets will be able to finance the non-oil operating deficit into the future. It is important to view the magnitude of financial assets relative to the size of the non-oil operating deficit. In Figure 4, the size of financial assets is 20 times the size of the non-oil operating deficit and so, assuming a 5% rate of return on financial assets, is able to finance the non-oil operating deficit forever.

### 4.4 Sensitivity of Results

The results of the simulation above illustrate the dynamics of financial liabilities, financial and non-financial assets and net worth. However, what would have been the implications had the Government of Alberta chosen a "richer" public sector? The following simulation will demonstrate the implications of the government increasing their non-oil operating deficit ($\overline{gobp}$) and the resulting dynamics.

The government can achieve this richer public sector either by increased spending or decreased taxation to achieve a higher level of the non-oil operating deficit. In this simulation, Case A, I have chosen a non-oil operating deficit which is 22.5% higher than the Base Case, while keeping all other variables constant. This means that I have allowed program spending to be higher and/or current taxes to be lower so that the *gap* between government expenditure and taxation will be 22.5% greater. The results are presented in Figure 6, where the solid lines are the Base Case presented in the previous section and the dotted lines belong to Case A with a larger non-oil operating deficit.

**Figure 6. Comparison of the Base Case and Case A Simulations**




It is important to note that I kept the government's behaviour consistent with optimizing behaviour. The government continued to operate by choosing over the optimal behaviour of the cap rate ( $\rho^{CAP}$ ) and the non-oil operating deficit ( $gopb$ ). However, in adjusting the non-oil operating deficit, I have adjusted the government's behaviour towards it, so that the value of the cap rate of return has been adjusted. Since the government has chosen a richer public sector, their behaviour must be adjusted with regards to the cap rate, in order to gather enough savings during the Mature phase so that the richer public sector is fiscally sustainable in the long run.

In the Initial phase, Case A requires that the government be willing to borrow more than in the Base Case in order to finance the higher level of the non-oil operating deficit. Here, financial liabilities increase more, which forces net worth to decline further.

Financial assets, however, remain consistent between the two cases because, in the Initial phase, financial assets are not affected by the choice variables.

The Mature phase is the key phase and will determine whether the government will be able to sustain its fiscal choices. The government has chosen a higher level of the non-oil operating deficit – that is, it has chosen what I have called a richer public sector – and, as such, increased the level of financial liabilities. Therefore, the government needs to start saving more quickly in order to achieve a greater amount of financial assets and a sustainable, higher level of net worth. In this phase, financial assets increase at a faster rate than they did in the previous case. Financial liabilities remain larger than they did in the Base Case, but follow the same behaviour and decline through the Mature phase. This must be true because the government has been forced to save more and in doing so, has had to slow down its retirement of financial liabilities.

In the Mature phase, the key difference between the two cases is that financial liabilities remain at high levels longer in Case A because the government needs to concentrate on gaining a sufficient amount of financial assets in order to sustain expenditures after oil has been exhausted.

In the Terminal phase, both cases follow the same behaviour, financial assets and net worth peak and financial liabilities decline to zero. However, in Case A, financial assets settle at a higher level than in the Base Case because of the decision to maintain a higher level of the non-oil operating deficit. This also resulted in both a greater amount of financial liabilities and net worth. In both cases, financial assets and net worth plateau when oil has been exhausted. As a result, the government's choice and optimal behaviour has resulted in a sustainable fiscal policy, regardless of the amount of the non-oil operating deficit.

A key implication of the above simulations is that in Case A the government needed to drastically increase their savings compared with the Base Case. This meant that in the Mature phase, the government needed to decrease their cap rate further to allow more of the return on financial assets to be kept in the oil fund. The implication of decreasing the cap rate of the return is that the government decreases the amount of oil revenue they are allowing to enter into general revenue and, thus, used towards current expenditures. Therefore, to cover the difference between the amount needed to finance the non-oil operating deficit (the required rate) and the amount available to be used from the oil fund (the cap rate) is larger in Case A than in the Base Case. This results in financial liabilities increasing further in order to finance the difference. Employing larger reserves of financial liabilities also resulted in a longer time period to eliminate financial liabilities. Furthermore, from the above simulations, I conclude that both are sustainable. In the long run, both financial assets and net worth remain positive and at a constant level after oil reserves are exhausted.

## 4.5 Conclusion

This chapter has presented two simulations which illustrate a hypothetical situation in which a new oil reserve is discovered and demonstrates what the fiscal environment is with regards to financial liabilities, financial assets and net worth. In the first simulation, the Base Case, I showed how a government would behave with a given level of the non-oil operating deficit. In the second simulation, Case A, I increased the level of the non-oil operating deficit to see how financial liabilities, financial assets and net worth would react. I determined that with a richer public sector, the government would need to be more willing to borrow (i.e. the level of financial liabilities will be greater) and in the Mature phase, be more willing to save.

One question still remains unanswered: does either one of these cases represent Alberta's fiscal environment? If not, then what does the Government of Alberta need to do? In the next chapter I will examine the status of Alberta's oil industry and government policy with regards to financial liabilities, financial assets and net worth, evaluate the government's fiscal stance and suggest how the Alberta government should manage its non-renewable resource revenues.

# Chapter Five

# Dealing with the Oil Sands

## 5.1 Introduction

In the preceding chapters, I determined the appropriate behaviour of a government, endowed with a non-renewable resource, to borrow and save financial assets and liabilities and achieve fiscal sustainability. Alberta's current fiscal environment is similar to the model, in that the debt has been eliminated but financial liabilities are positive and there exists savings in the form of financial assets. Therefore, Alberta is in a positive net worth position. However, it differs in the magnitude of financial assets attained and net worth. Another difference exists between the model and Alberta's oil industry. Alberta has both light, sweet crude and the oil sands. Each of these oil reserves should be treated differently in evaluating Alberta's current fiscal environment with regards to the model. In this chapter I discuss these differences. In the focal point of the chapter, I explain how the Alberta government should deal with the oil sands.

The chapter will begin with Section 5.2 where I discuss Alberta's oil industries (light crude and oil sands), the Alberta government's policy, and evaluate the government's current fiscal stance. In Section 5.3, I propose an appropriate behaviour the Government of Alberta should take in dealing with the oil sands, while Section 5.4 concludes the chapter.

## 5.2 Discussion of Alberta's Oil Industry and Government Policy

Alberta's oil industry began with the discovery of light, sweet crude oil at Leduc in 1947. Since then, the reserves for light crude oil have expanded and contracted, which is one of the main challenges the government had when planning on how to save oil revenues. In 1994, light crude oil production peaked and has steadily been declining ever since.[13] Therefore, light crude oil is in the Terminal phase.

According to the model, the Alberta government should have financial liabilities declining towards zero and should by now have saved sufficient financial assets to sustain the non-oil operating deficit into the future. In this sense, the government's current fiscal situation is at least broadly consistent with what the model suggests is appropriate for a government in the Terminal phase of oil production. That is, the government has significantly reduced, though not eliminated, its financial liabilities. Although financial liabilities remain, the government has eliminated net debt. Thus, although there remains a positive level of financial liabilities, financial assets are larger than this. But, is the stock of financial assets large enough to generate sufficient income to support the current level of non-oil operating deficit into the future?

At the end of the fiscal year in 2007, the government had $55 billion stored away in financial assets. The government requires $5.3 billion in non-renewable resource revenues to balance the budget, thus, the non-oil deficit is $5.3 billion (Government of Alberta 2007). Assuming a 6% rate of return on financial assets (which is the similar to rate in the simulations), the government needs net financial assets equal to $88 billion to sustain the current non-oil operating deficit after oil exhaustion. From this simple calculation, the Alberta government's current fiscal stance is unsustainable and incompatible with what the model suggests is appropriate for a government in the Terminal phase of oil production.

[13] Light and medium crude oil production as reported by the Canadian Association of Petroleum Producers (CAPP) in 1994 was 907 thousand barrels per day. In 2006, production was 521 thousand barrels per day. Therefore, production of medium and light crude has been declining since 1994. CAPP does not separate light and medium crude oil in their historical production statistics.

Happily, the Alberta government has a second chance. The light crude oil industry is in the Terminal phase, but Alberta still has an ace up its sleeve: the oil sands. This oil industry is in its Initial phase and more importantly, the oil sands reserves are relatively well known. The oil sands industry is more like a mining and manufacturing industry than traditional oil recovery. The size of the oil sands reserve is known, all that is required is to mine and refine it into a useful product. This factor lends itself quite nicely to planning for the Alberta government.

Back in 1947, the amount of light crude oil reserves was unknown and therefore, managing the oil revenues was difficult. The oil sands reserves are relatively well-known, as is the knowledge regarding production and the time periods for the three phases of the oil industry. For that reason, the government should be able to plan and manage the oil sands revenues better. In the following section, I propose an optimal policy for the Alberta government to follow with respect to the oil sands.

### 5.3 Dealing with the Oil Sands

The policy I suggest is the model of the oil fund fiscal rule presented in the preceding chapters. As discussed, this policy requires that the government restricts their behaviour by employing a fiscal rule. In doing so, this allows the government to achieve the optimal behaviour of saving and borrowing, much like the permanent income hypothesis of the individual. In adapting this model to Alberta's situation, I have taken into account the two distinct oil industries – light crude and the oil sands. As the light crude industry is now in the Terminal phase, there is nothing more I can propose, what is done is done. Nevertheless, Alberta does have an established standard of living which can be attributed to this industry, which I will call the Silver level. This represents the Silver level of government spending and taxation policy and therefore, the Silver level of the non-oil operating deficit ($gopb$). I have assumed this level has been achieved solely by the light crude industry and not the oil sands.

Now that I have established the past, I will form the government's policy for the future. As the development of the oil sands is in the Initial phase, the first thing the government may do is to build up the economy to a new, richer standard of living, what I will call the Golden level. This level is achieved either by decreasing taxes or increasing expenditures. This level, like the previous one, will represent the Golden level of the non-oil operating deficit ($\overline{gopb}$). However, according to the fiscal rule, the government must be willing to finance this Golden level by borrowing. But, there is already an established level of the government's fiscal policy, the Silver level, which was a product of the light crude industry.

As a result, the government will finance the *difference* between the Golden level and the Silver level, or the incremental amount, by borrowing. In order to borrow, the government will increase the level of financial liabilities ($d$). This is achieved by setting the required rate ($\rho^{REQ}$) and cap rate ($\rho^{CAP}$) above the observed rate ($\rho^{F}$). In setting the cap rate, the government identifies the amount of the oil fund that it will allow itself to use to finance current expenditures. As discussed in Chapter Two, the required rate is that amount of the oil fund needed to balance the non-oil operating deficit. It is important to note that the government, in choosing the Golden level, has indirectly chosen the required rate which is now determined in each time period.

Meanwhile, still in the Initial phase, the government creates an oil fund ($a^{F}$), which will accumulate through injections of oil revenues ($inv^{F}$). Recall from Chapter Two, that

100% of oil revenues must first go directly into the fund and only investment income earned on the oil fund can be used to finance current expenditures. In this Initial phase, the oil fund or financial assets ($a^F$) only grows via these oil revenue injections.

In the ***Initial phase***, the appropriate behaviour of the government will be to set the cap rate ($\rho^{CAP}$) such that it is below the required rate ($\rho^{REQ}$) and above the observed rate ($\rho^F$). This causes financial liabilities ($d$) to accumulate.

In the ***Mature phase***, which is defined by constant production, the government's optimal behaviour is to both borrow and save. Initially, the government will set the cap rate ($\rho^{CAP}$) below the required rate ($\rho^{REQ}$), which is below the observed rate ($\rho^F$). As a result, financial liabilities ($d$) accumulate but at a slower rate. Net worth at this stage is approaching zero. In the second sub-phase, the government will set the cap rate ($\rho^{CAP}$) less than the required rate ($\rho^{REQ}$) which will be less than the observed rate ($\rho^F$). At this stage, net worth becomes positive as financial assets ($a^F$) gain over financial liabilities ($d$).

The ***Terminal phase*** is defined by declining production. In this phase, the optimal behaviour of the government is to set the cap rate ($\rho^{CAP}$) above the observed rate ($\rho^F$), which will be greater than the required rate ($\rho^{REQ}$). In doing so, financial assets ($a^F$) continue to accumulate until they peak, while financial liabilities ($d$) approach zero, resulting in sustained positive net worth.

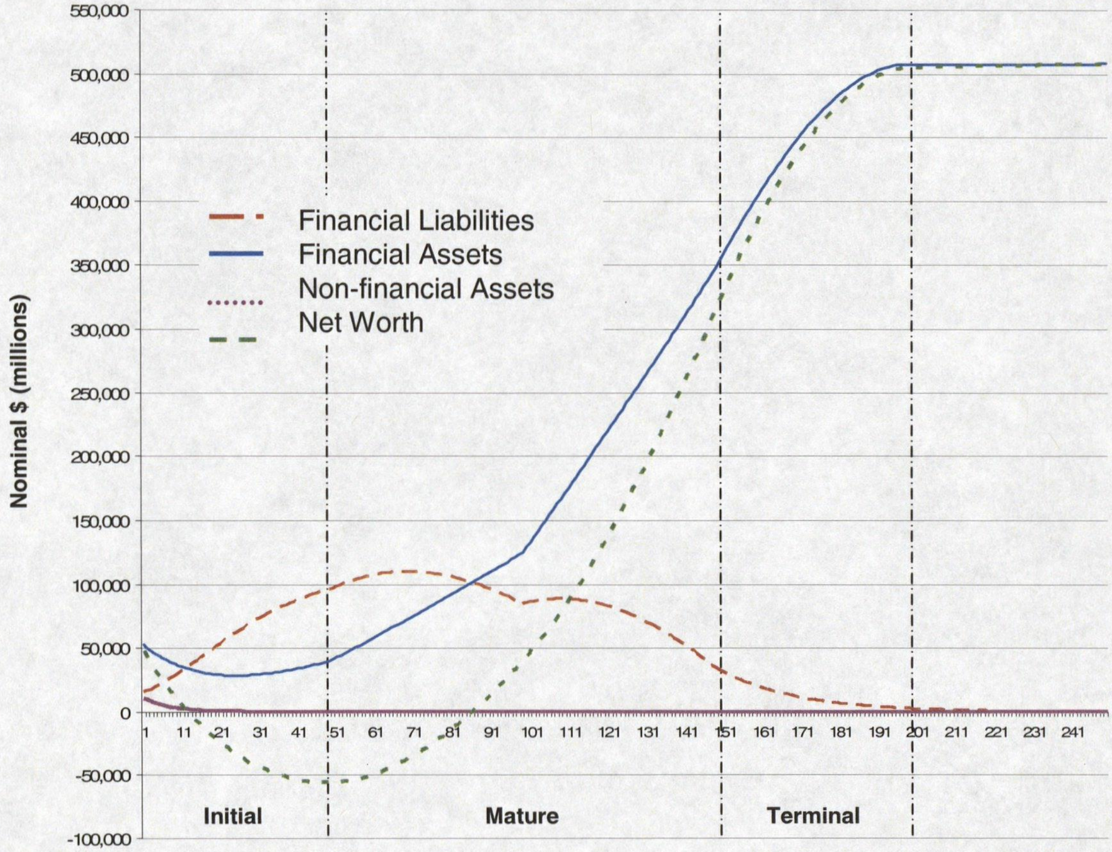
In order to derive the government's behaviour with regards to the oil sands, I have used the Alberta government's current fiscal stance obtained from the 2007 Consolidated Financial Statements. For the initial amounts of financial liabilities, financial assets and non-financial assets, I have assumed the amounts reported by the government as \$15 billion, $55 billion, and $14 billion, respectively. Also, the difference between expenditures and revenues in the fiscal year 2007 was approximately $3.5 billion (excluding non-renewable resource revenues), which, I am assuming, is the Silver level of the non-oil operating deficit. This is the fiscal stance with light crude.

With the oil sands, the government is able to obtain a higher level of the non-oil operating deficit than the one achieved with light crude. As described above, the government initiates a new fiscal stance with the oil sands and moves to a Golden level of the non-oil operating deficit, which I have determined is $4.8 billion, a 37% gain over the Silver level. The government is able to do so because of the large amount of oil sands reserves. I have assumed the oil sands reserves to be 315 billion barrels of oil.[14] Furthermore, I have used the same extraction process as the two previous simulations, where oil production increases at a constant rate for the next 150 years and declines at a constant rate until year 200, when the oil sands cease to exist.

The following figure illustrates how the government's optimal behaviour produces sustainable results.

[14] This value obtained from Alberta Energy (n.d.) is the ultimate potential of the oil sands in 2006.

**Figure 7. The Oil Sands Simulation**




## 5.4 Conclusion

In the above sections, I have devised a plan for the Government of Alberta to manage their non-renewable resource revenues. This plan illustrates the optimal behaviour for a government reliant on non-renewable resource revenues, whose goal is to achieve fiscal sustainability. This plan takes into account Alberta's economic environment which consists of both an oil industry in its Terminal phase (light crude) and an oil industry in its Initial phase (the oil sands). As the light crude industry is at its end, the plan formulates around Alberta's second chance at fiscal sustainability: the oil sands. The first step the government should take is to build up the economy to its Golden state, the Golden level of the optimal non-oil operating deficit. The government must also be willing to borrow in order to finance the incremental expenditure. From there, the Alberta government must tie its hands by establishing a fiscal rule and follow the appropriate behaviour in setting its cap rate. As I have demonstrated, this results in a sustainable amount of financial assets and positive, constant net worth.

# Chapter Six

# The Conclusion

One would think that a government which has access to an immense amount of revenue, independent of a taxation policy, would be more well off than other governments without this access. On the contrary, this type of government has a much more difficult experience in managing their fiscal policy. A government endowed with non-renewable resource rents has an obligation to maintain an appropriate amount of expenditure and taxation policy to both current and future generations as they are the owners of the resource. In this nature, the government needs to form a plan to manage the non-renewable resource, which provides a sustainable fiscal policy so that all generations can benefit from those revenues earned on the resource equally.

What I have examined in this thesis is a framework for an optimal fiscal policy for how a government which has access to non-renewable resource revenues can simultaneously realize goals of intergenerational equity and fiscal sustainability. The government needs to bind its behaviour by a fiscal rule in which it establishes an oil fund. This fiscal rule forces the government to place all resource revenues into the oil fund, and only use the return earned on the fund towards balancing their non-oil operating deficit, the difference between expenditures and taxation. As the literature suggests, the government should use this fund as a portfolio transaction, where it takes non-renewable resource capital and converts it into financial capital. Since the government only uses the return earned on the fund, it achieves sustainability.

Within this fiscal rule to manage non-renewable resource revenues, the government must follow an appropriate set of behaviours which are regulated by the phases of the oil industry. This behaviour mimics that of the individual in Friedman's permanent income hypothesis (Friedman, 1957). In the Initial phase, the government has to be willing to borrow in order to finance its non-oil operating deficit. In the Mature phase, the government continues to borrow, but begins to save more than it is borrowing. Finally, in the Terminal phase of the oil industry, the government has established a sufficient amount of financial assets and retired its financial liabilities. Also, the return earned on the financial assets is enough so that it can finance the non-oil operating deficit after the non-renewable resource is exhausted.

In applying this optimal plan to the Government of Alberta, I have examined its current fiscal stance as well as the nature of the oil industry within the province. The oil industry within Alberta is unique, in that there are two distinct types of oil – light crude and the oil sands. Light crude is in the Terminal phase of its lifetime because oil production is declining. Therefore, the government should have acquired sufficient financial assets to sustain the non-oil operating deficit according to the optimal plan described above. However, the government has failed in this respect. This is likely due to the fact the government was unable to plan properly because light crude oil reserves were relatively unknown upon their discovery in 1947.

Now, looking at the oil sands, these reserves are relatively known. This fact lends itself quite nicely to deriving a plan to manage these non-renewable resource revenues. In knowing the total amount of reserves, the government can determine a production cycle which will help it plan, not only how much the non-oil operating deficit will be, but the time frame for its borrowing and savings periods. With the knowledge of the amount of oil sands reserves, the government can apply this approach more easily than they could have back in 1947 with the discovery of light crude in Leduc. Overall, this fiscal rule and appropriate behaviours gives the Alberta Government a concept in which they can utilize when analysing how to manage the oil sands.

Further areas of research would expand this framework in the following way: introducing price volatility, allowing for royalty reviews, and defining an exact value of the non-oil operating deficit. In particular;

- In this thesis, I have used a very long time horizon and therefore ignored price volatility. If I introduced short-run price volatility, I would anticipate the government would save all windfalls (when prices rise) and borrow to finance drawbacks (when prices drop). Using Figure 7, this would cause the lines of both financial liabilities and financial assets to experience some minor fluctuations. However, the overall trend-lines of these variables should remain consistent with the results.
- This framework does not take into account changes in the royalty regime. If the government increased royalties, I would expect that investment in the oil fund would increase and the government would save more. This change would introduce changes similar to those that result from introducing price volatility.
- The purpose of this thesis was to provide the government with an appropriate framework for how it can move forward and form a plan to manage their non-renewable resource revenues. I did not explicitly determine the optimal level of the non-oil operating balance. Therefore, in applying this to the Alberta government, more research and detailed calibration of the model will need to be done to determine this optimal level.

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# Appendix I

## Basic Form of the Model

In this appendix, I present the basic form of the model and the derivation of matrix (10) that is used in Chapter Two.

Set of identities essential to the net worth approach to fiscal analysis are:

$$\left(A_t^F - A_{t-1}^F\right) + \left(A_t^{NF} - A_{t-1}^{NF}\right) - \left(D_t - D_{t-1}\right) = GOPB_t + \rho_t^{NF} A_{t-1}^{NF} + \rho_t^F A_{t-1}^F - iD_{t-1} - \delta A_{t-1}^{NF} \qquad (6)$$

$$A_t^{NF} = \left(1-\delta\right) A_{t-1}^{NF} + INV_t^{NF} \qquad (7)$$

$$A_t^F = A_{t-1}^F + INV_t^F \qquad (8)$$

Substitute equations (7) and (8) into equation (6):

$$\begin{aligned} &A_{t-1}^F + INV_t^F - A_{t-1}^F + \left(1-\delta\right) A_{t-1}^{NF} + INV_t^{NF} - A_{t-1}^{NF} - D_t + D_{t-1} \\ &\quad = GOPB_t + \rho_t^F A_{t-1}^F + \rho_t^{NF} A_{t-1}^{NF} - iD_{t-1} - \delta A_{t-1}^{NF} \end{aligned}$$

Simply the above equation for $D_t$:

$$-D_t = GOPB_t + \rho_t^F A_{t-1}^F + \rho_t^{NF} A_{t-1}^{NF} - \left(1+i\right) D_{t-1} - INV_t^F - INV_t^{NF}$$

Multiply through by (-1) and rearrange:

$$D_t = -GOPB_t - \rho_t^F A_{t-1}^F - \rho_t^{NF} A_{t-1}^{NF} + \left(1+i\right) D_{t-1} + INV_t^F + INV_t^{NF}$$

$$D_t = \left(1+i\right) D_{t-1} - \rho_t^F A_{t-1}^F - \rho_t^{NF} A_{t-1}^{NF} - GOPB_t + INV_t^F + INV_t^{NF}$$

To make the above equation in terms of GDP, multiply by ${Y_{t-1}}/{Y_t} = {1}/{1+\hat{Y}_t}$ where $\hat{Y}_t$ is growth rate of GDP.

$$\frac{Y_{t-1}}{Y_t} D_t = \frac{Y_{t-1}}{Y_t}\left(1+i\right) D_{t-1} - \frac{Y_{t-1}}{Y_t} \rho_t^F A_{t-1}^F - \frac{Y_{t-1}}{Y_t} \rho_t^{NF} A_{t-1}^{NF} - \frac{Y_{t-1}}{Y_t} GOPB_t + \frac{Y_{t-1}}{Y_t} INV_t^F + \frac{Y_{t-1}}{Y_t} INV_t^{NF}$$

I will denote the lower case letter of the variable to be in terms of GDP.

For example, $d_t = {}^{D_t}\!/_{Y_t}$:

$$Y_{t-1}d_t = \frac{Y_{t-1}}{Y_t}(1+i)D_{t-1} - \frac{Y_{t-1}}{Y_t}\rho_t^F A_{t-1}^F - \frac{Y_{t-1}}{Y_t}\rho_t^{NF} A_{t-1}^{NF} - Y_{t-1}gopb_t + Y_{t-1}inv_t^F + Y_{t-1}inv_t^{NF}$$

Remember that ${}^{Y_{t-1}}\!/_{Y_t} = {}^{1}\!/_{1+\hat{Y}_t}$:

$$Y_{t-1}d_t = \frac{1+i}{1+\hat{Y}_t}D_{t-1} - \frac{\rho_t^F}{1+\hat{Y}_t}A_{t-1}^F - \frac{\rho_t^{NF}}{1+\hat{Y}_t}A_{t-1}^{NF} - Y_{t-1}gopb_t + Y_{t-1}inv_t^F + Y_{t-1}inv_t^{NF}$$

Divide through by $Y_{t-1}$:

$$d_t = \frac{1+i}{1+\hat{Y}_t}d_{t-1} - \frac{\rho_t^F}{1+\hat{Y}_t}a_{t-1}^F - \frac{\rho_t^{NF}}{1+\hat{Y}_t}a_{t-1}^{NF} - gopb_t + inv_t^F + inv_t^{NF} \tag{A}$$

Where the lower case values at time $t$-1 are also in terms of GDP, for example:

$$d_{t-1} = {}^{D_{t-1}}\!/_{Y_{t-1}}$$

Equations (7) and (8) should also be stated in terms of GDP. Starting with equation (7), first, multiply through by ${}^{Y_{t-1}}\!/_{Y_t} = {}^{1}\!/_{1+\hat{Y}_t}$ where $\hat{Y}_t$ is growth rate of GDP.

$$\frac{Y_{t-1}}{Y_t}A_t^{NF} = \frac{Y_{t-1}}{Y_t}(1-\delta)A_{t-1}^{NF} + \frac{Y_{t-1}}{Y_t}INV_t^{NF}$$

Again, the lower case letter of the variable will denote the variable in terms of GDP.

$$Y_{t-1}a_t^{NF} = \frac{Y_{t-1}}{Y_t}(1-\delta)A_{t-1}^{NF} + Y_{t-1}inv_t^{NF}$$

Remember that ${}^{Y_{t-1}}\!/_{Y_t} = {}^{1}\!/_{1+\hat{Y}_t}$:

$$Y_{t-1}a_t^{NF} = \frac{1-\delta}{1+\hat{Y}_t}A_{t-1}^{NF} + Y_{t-1}inv_t^{NF}$$

Divide through by $Y_{t-1}$:

$$a_t^{NF} = \frac{1-\delta}{1+\hat{Y}_t} a_{t-1}^{NF} + inv_t^{NF} \tag{B}$$

For equation (8), follow the same instructions to find equation (C).

$$\frac{Y_{t-1}}{Y_t} A_t^F = \frac{Y_{t-1}}{Y_t} A_{t-1}^F + \frac{Y_{t-1}}{Y_t} INV_t^F$$

$$Y_{t-1} a_t^F = \frac{Y_{t-1}}{Y_t} A_{t-1}^F + Y_{t-1} inv_t^F$$

$$Y_{t-1} a_t^F = \frac{1}{1+\hat{Y}_t} A_{t-1}^F + Y_{t-1} inv_t^F$$

$$a_t^F = \frac{1}{1+\hat{Y}_t} a_{t-1}^F + inv_t^F \tag{C}$$

Combining equations (A), (B), and (C), I derive equation (10):

$$\begin{pmatrix} d_t \\ a_t^F \\ a_t^{NF} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{(1+i)}{1+\hat{Y}_t} & -\frac{\rho_t^F}{1+\hat{Y}_t} & -\frac{\rho_t^{NF}}{1+\hat{Y}_t} \\ 0 & \frac{1}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \frac{(1-\delta)}{1+\hat{Y}_t} \end{pmatrix}}_{A} \begin{pmatrix} d_{t-1} \\ a_{t-1}^F \\ a_{t-1}^{NF} \end{pmatrix} + \begin{pmatrix} -gopb_t + inv_t^{NF} + inv_t^F \\ inv_t^F \\ inv_t^{NF} \end{pmatrix} \tag{10}$$

# Appendix II

## Steady State Equilibrium of the Basic Form

This appendix determines the steady-state equilibrium values of financial liabilities, non-financial and financial assets and net worth – equations (11) to (14) in Chapter Two. To determine these steady-state equilibrium values, I assume that the three exogenous variables (the operating balance and investment in both non-financial and financial assets) are constant.

*Non-financial assets ratio*

In the steady-state, non-financial assets ratio is constant. Thus, we have:

$$a_t^{NF} = a_{t-1}^{NF} = a_{SS}^{NF}$$

Simplify for $a_{SS}^{NF}$:

$$a_{SS}^{NF} - \frac{(1-\delta)}{1+\hat{Y}} a_{SS}^{NF} = inv^{NF}$$

$$\frac{\left(1+\hat{Y}-1+\delta\right)}{1+\hat{Y}} a_{SS}^{NF} = inv^{NF}$$

$$\frac{\left(\hat{Y}+\delta\right)}{1+\hat{Y}} a_{SS}^{NF} = inv^{NF}$$

Solve for the steady-state non-financial assets ratio:

$$a_{SS}^{NF} = \frac{\left(1+\hat{Y}\right)}{\hat{Y}+\delta} inv^{NF}$$

The steady-state of non-financial assets ratio is dependent on its own investment in the asset, the growth rate of GDP and the rate of depreciation.

*Financial assets ratio*

Like non-financial assets, in the steady-state, financial assets ratio is equal across all time periods:

$$a_t^F = a_{t-1}^F = a_{SS}^F$$

Simply for $a_{SS}^F$:

$$a_{SS}^F - \frac{1}{1+\hat{Y}} a_{SS}^F = inv^F$$

$$\frac{\left(1+\hat{Y}-1\right)}{1+\hat{Y}} a_{SS}^F = inv^F$$

$$\frac{\left(\hat{Y}\right)}{1+\hat{Y}} a_{SS}^F = inv^F$$

Solve for the steady-state:

$$a_{SS}^F = \frac{\left(1+\hat{Y}\right)}{\hat{Y}} inv^F$$

As in the non-financial case, the steady-state financial assets ratio is dependent on its own investments and the growth rate of GDP.

*Financial liabilities ratio*

Again, in the steady-state, the financial liabilities ratio at time $t$ is equal to the financial liabilities ratio at time *t-1*.

$$d_t = d_{t-1} = d_{SS}$$

Simplify for $d_{SS}$:

$$d_{SS} = inv^{NF} + inv^{F} + \frac{(1+i)}{1+\hat{Y}} d_{SS} - gopb - \frac{\rho^{NF}}{1+\hat{Y}} a_{SS}^{NF} - \frac{\rho^{F}}{1+\hat{Y}} a_{SS}^{F}$$

$$d_{SS}\left[1 - \frac{(1+i)}{1+\hat{Y}}\right] = inv^{NF} + inv^{F} - gopb - \frac{\rho^{NF}}{1+\hat{Y}}\left(\frac{(1+\hat{Y})}{\hat{Y}+\delta}\right) inv^{NF} - \frac{\rho^{F}}{1+\hat{Y}}\left(\frac{(1+\hat{Y})}{\hat{Y}}\right) inv^{F}$$

$$d_{SS}\left[\frac{(\hat{Y}-i)}{1+\hat{Y}}\right] = \left(\frac{(\hat{Y}+\delta-\rho^{NF})}{\hat{Y}+\delta}\right) inv^{NF} + \left(\frac{(\hat{Y}-\rho^{F})}{\hat{Y}}\right) inv^{F} - gopb$$

Solve for the steady-state financial liabilities ratio:

$$d_{SS} = \frac{(1+\hat{Y})}{\hat{Y}-i}\left[\left(\frac{(\hat{Y}+\delta-\rho^{NF})}{\hat{Y}+\delta}\right) inv^{NF} + \left(\frac{(\hat{Y}-\rho^{F})}{\hat{Y}}\right) inv^{F} - gopb\right]$$

Here, we see that the steady-state financial liabilities ratio is directly dependent on the investment in financial and non-financial assets and is inversely dependent on the gross operating primary balance. This reflects that investments in the assets and the gross operating primary balance are financed with financial liabilities. The steady-state financial liabilities ratio is also dependent on the interest rate on financial liabilities, the growth rate of GDP, the depreciation rate and the return on non-financial assets and the return on financial assets.

*Net worth ratio*

The steady-state for the net worth ratio is equal to the sum of the steady-state financial and non-financial assets ratios minus the financial liabilities ratio.

$$nw_{SS} = a_{SS}^{NF} + a_{SS}^{F} - d_{SS}$$

Simply for $inv^{NF}$, $inv^{F}$, $gopb$:

$$nw_{SS}=\frac{\left(1+\hat{Y}\right)}{\hat{Y}+\delta}inv^{NF}+\frac{\left(1+\hat{Y}\right)}{\hat{Y}}inv^{F}-\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\left[\left(\frac{\left(\hat{Y}+\delta-\rho^{NF}\right)}{\hat{Y}+\delta}\right)inv^{NF}+\left(\frac{\left(\hat{Y}-\rho^{F}\right)}{\hat{Y}}\right)inv^{F}-gopb\right]$$

$$nw_{SS}=\left[\frac{\left(1+\hat{Y}\right)}{\hat{Y}+\delta}-\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\left(\frac{\left(\hat{Y}+\delta-\rho^{NF}\right)}{\hat{Y}+\delta}\right)\right]inv^{NF}$$

$$+\left[\frac{\left(1+\hat{Y}\right)}{\hat{Y}}-\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\left(\frac{\left(\hat{Y}-\rho^{F}\right)}{\hat{Y}}\right)\right]inv^{F}+\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}gopb$$

Solve for the steady-state net worth ratio:

$$nw_{SS}=\left(\frac{\left(1+\hat{Y}\right)}{\hat{Y}-i}\right)\left\{\frac{\left(\rho^{NF}-i-\delta\right)}{\hat{Y}+\delta}inv^{NF}+\frac{\left(\rho^{F}-i\right)}{\hat{Y}}inv^{F}+gopb\right\}$$

Like that of the steady-state financial liabilities ratio, the steady-state net worth ratio is dependent on investments in financial and non-financial assets, the gross operating primary balance, the interest rate on financial liabilities, the depreciation rate and the rate of returns on both financial and non-financial assets ratio.

# Appendix III

## Stability Conditions of the Basic Form

In this appendix, I determine the stability conditions for the basic form of the model. The stability conditions are determined by finding the eigenvalues in the large matrix $A$ in equation (9). The eigenvalues are determined by finding the determinants of matrix $A$ minus the identity matrix made up of the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ along the diagonal.

$$|A - \lambda I| = 0$$

Substituting in matrix $A$ and solving to find:

$$\begin{vmatrix} \frac{(1+i)}{1+\hat{Y}_t} - \lambda_1 & -\frac{\rho_t^F}{1+\hat{Y}_t} & -\frac{\rho_t^{NF}}{1+\hat{Y}_t} \\ 0 & \frac{1}{1+\hat{Y}_t} - \lambda_2 & 0 \\ 0 & 0 & \frac{(1-\delta)}{1+\hat{Y}_t} - \lambda_3 \end{vmatrix} = 0$$

Solve for the eigenvalues:

$$\frac{(1+i)}{1+\hat{Y}_t} - \lambda_1 = 0 \qquad \frac{1}{1+\hat{Y}_t} - \lambda_2 = 0 \qquad \frac{(1-\delta)}{1+\hat{Y}_t} - \lambda_3 = 0$$

Stability is determined when the eigenvalues are less than one, thus the following are the conditions for stability.

$$\lambda_1 < 1 \quad \Rightarrow \quad i < \hat{Y}_t \qquad \lambda_2 < 1 \quad \Rightarrow \quad 0 < \hat{Y}_t \qquad \lambda_3 < 1 \quad \Rightarrow \quad 0 < \hat{Y}_t + \delta$$

# Appendix IV

# Oil Fund Fiscal Rule

In this appendix, I derive the equations of financial liabilities (equation 19) and financial assets (equation 20) of the model for the oil fund fiscal rule in Chapter Three.

Equation (20) is the oil fund fiscal rule. All oil revenues (or investment in financial assets) are invested and enter directly into the oil fund (or the financial asset). Also, the relationships between the three rates of return, along with the parameters, are introduced into the equation.

Equation (19) is a little more complicated. First, we realized that the difference between the simple model and the oil fund model is that the latter does not finance investment in all types of assets with financial liabilities. Therefore, these variables are removed from the equation and follow the oil fund fiscal rule.

In order to determined equation (19), take equation (17) and insert it into equation (18). Equation (17) is obtained when no new financial liabilities are acquired, but the government acquired these liabilities to pay for current expenditures. However, taking away equation (18) from (17) is the investment income the government has allowed itself to use towards financing current expenditures, or the cap amount. Inserting equation (18) into (17) takes the form:

$$\frac{i}{1+\hat{Y}_t} d_{t-1} = \frac{\rho_t^{REQ} - \rho_t^{CAP}}{1+\hat{Y}_t} a_{t-1}^F + \overline{gopb}$$

Substitute the above equation into the first equation in (10), obeying the fiscal rule and simplify:

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} + \frac{i}{1+\hat{Y}_t} d_{t-1} - \frac{\rho_t^F}{1+\hat{Y}_t} a_{t-1}^F - \frac{\rho_t^{NF}}{1+\hat{Y}_t} a_{t-1}^{NF} - gopb_t$$

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} + \frac{\rho_t^{REQ} - \rho_t^{CAP}}{1+\hat{Y}_t} a_{t-1}^F + \overline{gopb} - \frac{\rho_t^F}{1+\hat{Y}_t} a_{t-1}^F - \frac{\rho_t^{NF}}{1+\hat{Y}_t} a_{t-1}^{NF} - gopb_t$$

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} - \left(gopb_t - \overline{gopb}\right) + \frac{\rho_t^{REQ} - \rho_t^F - \rho_t^{CAP}}{1+\hat{Y}_t} a_{t-1}^F - \frac{\rho_t^{NF}}{1+\hat{Y}_t} a_{t-1}^{NF}$$

Introduce the parameters which take into account the three phases of an oil industry:

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} - \left(gopb_t - \overline{gopb}\right) - \frac{\rho_t^{NF}}{1+\hat{Y}_t} a_{t-1}^{NF} + \lambda \left( \frac{\rho_t^{REQ} - (1-\gamma)\rho_t^F - \gamma\rho_t^{CAP}}{1+\hat{Y}_t} \right) a_{t-1}^F \tag{19}$$

Equations (19), (20), and (21) make-up the following matrix:

$$\begin{pmatrix} d_t \\ a_t^F \\ a_t^{NF} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{1+\hat{Y}_t} & \lambda\left(\frac{\rho_t^{REQ} - (1-\gamma)\rho_t^F + \gamma\rho_t^{CAP}}{1+\hat{Y}_t}\right) & -\frac{\rho_t^{NF}}{1+\hat{Y}_t} \\ 0 & \frac{1}{1+\hat{Y}_t} + \gamma\left(\frac{\rho_t^F - (1-\lambda)\rho_t^{REQ} - \lambda\rho_t^{CAP}}{1+\hat{Y}_t}\right) & 0 \\ 0 & 0 & \frac{(1-\delta)}{1+\hat{Y}_t} \end{pmatrix}}_{B} \begin{pmatrix} d_{t-1} \\ a_{t-1}^F \\ a_{t-1}^{NF} \end{pmatrix} + \begin{pmatrix} \overline{gopb} - gopb_t \\ inv_t^F \\ inv_t^{NF} \end{pmatrix}$$

# Appendix V

## Stability Conditions of the Oil Fund Fiscal Rule

In this appendix, I determine the stability conditions of the model for the oil fund fiscal rule in each of the three phases: the Initial, Mature and Terminal phases.

The stability conditions are determined by finding the eigenvalues in the large matrix $B$ in equation (F). The eigenvalues are determined by finding the determinants of matrix $B$ minus the identity matrix made up of the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ along the diagonal.

$$|B-\lambda I| = 0$$

Substituting in matrix $B$ and solving to find:

$$\begin{vmatrix} \frac{1}{1+\hat{Y}_t}-\lambda_1 & \lambda\left(\frac{\rho_t^{REQ}-(1-\gamma)\rho_t^F+\gamma\rho_t^{CAP}}{1+\hat{Y}_t}\right) & -\frac{\rho_t^{NF}}{1+\hat{Y}_t} \\ 0 & \frac{1}{1+\hat{Y}_t}+\gamma\left(\frac{\rho_t^F-(1-\lambda)\rho_t^{REQ}-\lambda\rho_t^{CAP}}{1+\hat{Y}_t}\right)-\lambda_2 & 0 \\ 0 & 0 & \frac{(1-\delta)}{1+\hat{Y}_t}-\lambda_3 \end{vmatrix} = 0$$

In all three phases of the oil industry, $\lambda_1$ and $\lambda_3$ are the same. The eigenvalues are:

$$\frac{1}{1+\hat{Y}_t}-\lambda_1 = 0 \qquad \frac{(1-\delta)}{1+\hat{Y}_t}-\lambda_3 = 0$$

Stability is determined when the eigenvalues are less than one, thus the following are the conditions for stability in all phases of the oil industry:

$$\lambda_1 < 1 \Rightarrow 0 < \hat{Y} \qquad \lambda_3 < 1 \Rightarrow 0 < \hat{Y}_t + \delta$$

As $\lambda$ and $\gamma$ determine the different phases, they also influence the second stability condition. In the Initial phase, $\lambda = 1$ and $\gamma = 0$, and the second eigenvalue is:

$$\frac{1}{1+\hat{Y}_t} - \lambda_2^I = 0$$

As stability is determined when the eigenvalue is less than one, the second condition for stability in the Initial phase of the oil industry is:

$$\lambda_2^I < 1 \Rightarrow 0 < \hat{Y}$$

In the Mature phase, $\lambda = 1$ and $\gamma = 1$, and the second eigenvalue is:

$$\frac{1+\rho_t^F - \rho_t^{CAP}}{1+\hat{Y}_t} - \lambda_2^M = 0$$

The second stability condition is found when the eigenvalue is less than one, thus we have the following stability condition in the Mature phase of the oil industry:

$$\lambda_2^M < 1 \Rightarrow \rho_t^F - \rho_t^{CAP} < \hat{Y}_t$$

In the Terminal phase we know that $\lambda = 0$ and $\gamma = 1$, and the second eigenvalue is:

$$\frac{1+\rho_t^F - \rho_t^{REQ}}{1+\hat{Y}_t} - \lambda_2^T = 0$$

Stability is, again, determined when the eigenvalue is less than zero, thus the following is the second condition for stability in the Terminal phase of the oil industry:

$$\lambda_2^T < 1 \Rightarrow \rho_t^F - \rho_t^{REQ} < \hat{Y}_t$$