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Adult Perceptions of the Reform Mathematics Classroom

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ABSTRACT

The purpose of this study was to investigate how three adults, with personal histories reflecting various levels of mathematics anxiety, perceived the present reform of mathematics education. Specifically it focused on what inferences they established regarding the nature of task, discourse and environment as being meaningful. The adults participating each expressed concerns with the subject of mathematics as it was presented to them in the traditional format, and an interest in exposure to the characteristics of a classroom which is more representative of the reform philosophy.

Three adults were interviewed on two different occasions regarding their perceptions of three characteristics of a reform classroom. They were asked specifically to comment on the nature of the tasks, the impact on the discourse of the teacher and the students, and on the environment of a classroom modeling reform features. The data analysis involved an interpretative process which focused on looking for distinctive characteristics of the perceptions.

The results of this study indicate a positive support for the present reform and an awareness of the distinctive changes that this reform has for the teacher of mathematics, and the students studying this subject. The participants perceptions of the reform classroom (a) presented a strong correlation and showed consistency with the N.C.T.M. Standards, (b) indicated an unexpected insightfulness regarding the implications of the changes to the teaching of mathematics, (c) were predictive of the revolutionary changes to the roles of teachers, students, parents or others associated with mathematics education and (d) represented attitudes which valued the changes to the teaching of mathematics because it better supported students needs and educational needs for a changing society.

Acknowledgements

I would like to acknowledge the expert leadership and guidance of Dr. Olive Chapman who, as my supervisor in this thesis, gave more of her time and expertise than I ever expected. Her diligence in keeping me within the time lines, and her friendship and knowledge in keeping me committed to the completion of this thesis, are the reasons I can be proud of the resulting work. I was excited the day that she was assigned to be my supervisor and I have never lost that excitement right up to the day of the oral defense of this topic. I know that I have gained more than just an academic milestone, I have also gained a valued friend.

It would be remiss not to acknowledge the much appreciated support and constant assistance of my wife, Margaret Orsten. Her patience, her understanding, and her sacrifices, during what were sometimes stressful times, were enough to give me the confidence and the desire to complete my thesis. Thank you for being there every day and over a number of challenging years.

Finally, I would like to acknowledge Mrs. B., Mr. M. and Mrs. P. whose real identities will always be secret, except to me, and without whose assistance and positive attitudes this study could never have been completed.

DEDICATION

This thesis is dedicated to my father, Jens Henry Orsten Sr., who died too young and after far too many years of suffering. This occurred before he had the opportunity to display his intelligence and potential to the academic world which I have been given the opportunity to do. Without the advances of modern medicine to assist him, he suffered for over twenty years with a brain tumor which stole his sight, his future, and his opportunity to fulfill his personal goals. Finally it took his pride and self esteem. When he could no longer stand the constant pain, which he had endured for these many years, he died from an operation attempting to make his life bearable. I recognize the goals that I have reached are due to the many traits which I gained from him both genetically, spiritually and in other ways I have come to appreciate.

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CHAPTER ONE

INTRODUCTION

1.1 Prologue

Mathematics education is currently undergoing significant reforms as a result of recommendations from the mathematics research community. (Price, 1995). These reforms are reflected in the Standards, as espoused by the National Council of Teachers of Mathematics (N.C.T.M.,1989, 1991, 1995), and which are being used as guidelines for this reform in North America. This situation will be a challenge for teachers, students, parents, and those indirectly involved with the mathematics classroom. The expectations for the changes to the teaching of mathematics require a shift in philosophy from the traditional one, with its long historical past (Appendix A, p.125), to a one which is more reflective of the reform philosophy. Where previous reforms have had instigators such as the cold war, the space race and international test comparisons, this reform has significant research to support it, and this is reflected in the positive reactions from the mathematical community across North America (Price, p. 489,1995).

One factor influencing the challenges to the reform is how the reform mathematics classroom is perceived in relation to the traditional mathematics classroom. If, for example, the teacher or parent perceives it as meaningful, he or she will be more likely to facilitate the change. Thus, knowing the perceptions of people affected, either directly or indirectly by the reform in the mathematics classroom, will be helpful in its implementation. This thesis is intended to make a contribution to this area of knowledge by focusing on the perception of three adults regarding the characteristics of the reform to the teaching of mathematics. The characteristics being considered are those relating to the nature of task, the nature of discourse involving both teacher and student, and the nature of the classroom environment, in which the task and discourse would occur. These areas are specifically mentioned in the N.C.T.M. Standards as ones needing attention in the changes to the teaching of mathematics within a reform classroom.

1.2 Personal Background Brought to the Study

My interest in conducting this study on the perceptions of the reform classroom evolved from my experiences as: a mathematics teacher, a mathematics department head, a graduate student, and a mathematics facilitator in professional development. The following accounts of specific situations relating to these different responsibilities are presented in order to indicate the personal background and context, which I bring to this study.

My career as a mathematics teacher started about thirty years ago. After my university training, specializing in mathematics education, I was placed in a first year teaching assignment in which, to my surprise, I was offered no mathematics classes. My second year afforded me the opportunity not only to teach mathematics, but also to experience an earlier reform movement (the New Math Era of the sixties). In spite of this reform, and other earlier reform movements, one thing seemed to have been maintained through the expected changes. That was the traditional classroom. As a student I had already lived through this classroom at all levels of my schooling including university teacher-training courses. Thus I had no difficulty in reproducing the expected traditions when assigned by own classes to teach, and I continued to do so for most of my teaching career. This was not a total acceptance of this teaching approach as being "ideal". In fact it was a source of conflict in my second year of teaching Junior High mathematics in grades seven and eight.

I was required to use a textbook called *"Seeing Through Mathematics"* as the teacher resource and the student text. The concern which I experienced with this book, was only exceeded by those of the students. I sensed the frustrations of students sitting everyday in a classroom, listening to me lecturing them, and presenting them with new concepts on the blackboard. I recognized that for many of them yesterday's lecture or lesson was still not clear in their minds and I was attempting to add new knowledge to what was still unclear. After a number of months of trying to teach using the cycle of: 1) review homework, 2) explain a new concept, 3) show applications of the new concept, and 4) assign drill and practice questions for the students to confirm their understanding, I decided to break from this habit, which I had experienced as a student and was now perpetuating as a teacher. I attempted to develop a more individualized program, albeit still traditional one, in which each student would work at their own pace. They would proceed when they felt more confidence and they could see that competence in the topic had been established. In the second year of adopting this variation in the routine of teaching mathematics, the Supervisor of the Department of Mathematics from the Calgary Public School Board came for my assessment for permanent certification. His first question was "Where are the students' textbooks?" I told him that I had taken them away and stacked them on a shelf in the classroom because both the students and I found them frustrating. He informed me that it was my legal responsibility to supply each student with the required text, reprimanded me, and returned to the downtown office. However, for the next three years I continued, with the consent of the administration of my school, to work on developing student confidence and feelings of success with the individualized courses in my mathematics classes.

A few years later I moved to a senior high school and unfortunately, I fell quickly and comfortably back into the initial traditional teaching mode, which I had found to be a concern earlier, and which I had thought to be a questionable method in my junior high teaching. The students in high school were presumably streamed by their various abilities into classes that were expected to better adapt to their diverse backgrounds and different expectations. Coming from grade nine with a wide variety of skills and abilities, they were now directed into three distinct subdivisions with programs that would better meet their individual and their educational needs. This was intended to better address their stages of mathematical development and their academic potential. No longer did it seem necessary for me to address each individual's needs given the streamed populations of homogeneously grouped abilities. The return of the routine of review, teach, model, and assign work characterized the expected roles for high school mathematics instruction. The routine was modeled by my peers' teaching approach and matched by my own. Even though at times I remember having a return of those concerns from junior high, I maintained the expected role, even though I recognized that many of these students had fairly strong, negative, affective reactions to the subject of mathematics. My concerns about the effectiveness of this teaching model, and my empathy for the students for whom mathematics was a compulsory and disenchanting subject, continued to increase over the next few years.

In spite of these concerns, my traditional approach did not change five years later when I moved to a private school with grades from one to twelve. Here I taught the "academic stream" in senior high mathematics courses. Because at the time I was not aware of a teaching alternative, I felt professionally stranded and I maintained the status quo. While with that school I was asked to change my teaching assignment when a number of years later I was requested to teach grade six mathematics. Because our school had grades from one to twelve, it was a fairly unique teaching assignment in which one would be working with the skills of calculus in one period and then move to the upper elementary classroom where development of fractions was a challenge to the students. The challenge to the students became a challenge to me as well. The traditional approach with its habitual patterns of operation did not seem to be suited to this younger, and eager, learner group.

The elementary classroom intensified my questioning of the teaching strategies of the traditional mathematics classroom, and paved the way for me to seek out alternatives to my historical approaches. In particular two elementary teachers had an important impact on my awareness of my teaching pedagogy. Both of these teachers had experiences with, and revealed to me, a very different way of teaching mathematics. It was a very different way from that to which I had grown accustomed. Margaret Orsten, my wife, and Valeen Chow, a close friend, had extensive experiences with division one (grades one to three) and division two (grade four to six), respectively. Valeen Chow had the additional experience of being a mathematics consultant for the elementary grades with the local public school board. Where previously they may have considered themselves as being alienated from giving opinions about my teaching in the secondary grades, I was now in their area of expertise and I was willing to accept their expertise. Fortunately for me they were also very willing to give it. From them I learned about the use of concrete materials or manipulatives and cooperative learning as some of the alternative approaches to the teaching of mathematics.

Because this alternative to teaching created many other unanswered questions for me, I looked for professional activities and opportunities to learn more about the changing philosophy, which I was beginning to develop. Having attended as many workshops as I could find, I slowly began to develop an understanding of this new style of teaching, and the kind of classroom it could create. By no means did I consider myself an expert in this "alternate approach", or even particularly competent, but I was slowly developing a significantly different perspective as to how to teach mathematics. I was excited by these new approaches and immediately implemented them not only in my grade six class, but also in a newly assigned grade four class. At first I was quite concerned about moving even further into the younger grades. But the elementary administrator, who required relief from her teaching load, gave her support that I would do a good job in taking over her class. With these two grades of students I attempted teaching variations with the application of my new knowledge and, after a few years of many successes and also some failures, I was sure that I would never again approach my teaching in the way that I used to. The transition had begun which would eventually alter my teaching approach, not only in my elementary classes, but also in my high school classes.

One particular outcome of being assigned to work with the grade four class had a significant impact on me. As is indicated by the following comments, the effect of my non-traditional approaches to teaching these students was reflected in their attitudes towards the subject of mathematics in a very positive ways. The grade four, home-room teacher informed me that I might not be taking her class again the following year. I wondered at the time if the reason for this was the possibility that I had alienated myself from the students, the parents or both. When I asked her if this was the case she replied: "NO! It's just that the students cheer when I tell them that math class is next and they consider it their favorite subject." Then she added: " They don't cheer for my classes." As part of a student profile for report card purposes, the students had been asked to complete some questions, one of which was to list their favorite subject. To both her surprise and my amazement, they had almost unanimously chosen mathematics as their favorite. The impact of these students' attitudes, towards the subject of mathematics, was to make me realize how the traditional classroom could be a significant factor in the anxiety students experience in mathematics classrooms as they progress from grade to grade.

Since most of my teaching at the high school level continued along the traditional approach, there was now a stronger concern, and a personal contradiction, with my philosophy of how students best learn and the actual classroom practice. I had still not encountered a non-traditional approach for teaching at the high school level, and had not yet made the connection with the approaches that I was using at the elementary level. It seemed at the time that these two distinct levels of students had significant differences in terms of content and therefore required different approaches to teaching. Possibly due to this contention, but definitely due to an increasing academic interest in the investigation of the teaching and learning of mathematics, I requested and was granted a sabbatical.

When I returned to university to undertake graduate studies in curriculum and instruction, my initial research interest was not in teaching strategies or pedagogy. Instead it was on the anxiety that mathematics created for many students. It seemed that an understanding of what influenced mathematics anxiety would provide a basis for knowing how to teach mathematics in a more meaningful way (especially to the specific age of high school students with whom I came in contact). However after reviewing the literature on the topic of anxiety, and also becoming more familiar with the reform movement in mathematics education, my interest shifted to understanding the reform classroom. My excitement with the proposals of the N.C.T.M. Standards (1989,1991,1995) made me curious as to how others, particularly "math anxious" students, would perceive the reform of mathematics education and the resulting impact on students in those classrooms.

My successes with my own elementary classes, and my initial exposure to the mathematics education literature on the reform, gave me the confidence to play an active role in promoting the alternative approach to teaching elementary mathematics. As mathematics department head at my school, I worked with colleagues to make them aware of these approaches, and eventually saw them begin to change their teaching, albeit very slowly. As a workshop presenter for a major publisher of teacher resources of mathematics, I tried to assist elementary teachers who were attempting to change their way of teaching. I spoke at teachers' conventions around my province and shared the knowledge that I had begun to build. This knowledge created the potential for a very different classroom experience for students than that to which many have been exposed.

These experiences made me very aware of a particular dilemma with the elementary teachers. At the beginning of each workshop session and for my own curiosity, I would poll the teachers as to their feelings about the subject of mathematics. Seldom would more than two or three, out of twenty to forty adults, indicate that mathematics was their favorite subject. Many of these same teachers would quickly volunteer to assist with my research when I explained that it related to the topic of math anxiety. In addition to their interest in the topic of anxiety, another concern emerged for me from this simple check. It seemed possible that there was a cycle of fear being consciously, or subconsciously, transferred to their students as a result of their own previous classroom experiences. This was potentially a fear that the teachers were not even aware could be being reproduced in their own students. This fear is likely to exist in many adults who experienced similar classrooms to these teachers, and who left those classrooms with the same feelings of inadequacy from their contact with the traditional approach to the teaching of mathematics.

Adults play an important role in mathematics education not only as teachers but also as parents. From my experience with developing alternative teaching approaches, I realized that how one perceives these reforms has implications and could play an important role in one's acceptance of them. After reviewing the literature, there seemed to be a gap in the available information regarding how adults perceive the present reform mathematics classroom. There has not been significant research within the mathematics community on how this reform will be accepted by the general population. Consequently, I decided to investigate this perception with adults who had a personal history, which reflected a negative attitude towards the learning of mathematics, thereby combining some of the important factors that had strongly influenced my thinking to this point.

1.3 Purpose and Significance of the Study

The purpose of this study is to investigate how three adults, with personal histories reflecting various levels of mathematics anxiety, perceive the present reform of mathematics education. Specifically it will focus on what inferences they establish regarding the nature of task, discourse and environment as being meaningful. Meaningful is being considered as that which the participants would interpret as being valid experiences for students within a reform classroom in comparison to that which they had experienced as students within the traditional classroom. Task, discourse, and environment are three of the four areas addressed in the N.C.T.M. Standards, and are areas that require attention by teachers who are implementing the philosophy of the present reform movement. The adults participating have each expressed concerns with the subject of mathematics as it was presented to them in the traditional format and an interest in exposure to the characteristics of a classroom which is more representative of the reform philosophy.

The usefulness of this study is based on two assumptions. 1) That one's perception of the reform of mathematics education will affect one's support of its implementation, and 2) support by adults (whether teachers or parents, in particular) is necessary to achieve a successful shift to the philosophy of the reform mathematics classroom. This philosophy is one presently grounded in significant research and pedagogy, and is accepted by the mathematical community. If teachers or parents presented with this theory perceive it as just another trend in education, then support will be less likely to occur. If, by contrast, they perceive it as being conceptually sound, success is more likely to occur. Thus the outcome of the study can offer insights into adults' perceptions of the reform of mathematics education. These insights can be useful in providing possible routes that need to be pursued to engage adults in positive ways to make sense of, and potentially support, the continuation of the reform. Additionally, it has important implications regarding communication with parents about the present reform of mathematics, and the impact it will have on their children. It is hoped that the knowledge arising from this research will serve to give a condensed and comprehensive perception of how the reform philosophy is interpreted by adults. This knowledge can then be used to initiate discussion with any adults affected, either directly or indirectly, with the present reform and allow for possible comparison with previous reforms.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

There are three areas of literature which are relevant to this thesis: (1) the area of mathematics anxiety, (2) the present reform movement in mathematics education, and (3) past research which has been carried out in adult perceptions. The literature on mathematics anxiety provides part of the context for the process of identifying the participants who can contribute to this research topic. The present reform of mathematics education provides the context for the instrument to be applied in the interpretation of the data. The literature on adult perceptions provides the context for the research perspective.

2.2 Mathematics Anxiety

The topic of mathematics anxiety has been addressed with more research in the past than recently, and more directly in connection with what would be called by the mathematics community the "traditional philosophy" (Appendix A, p. 125). In terms of the nature of the subject of mathematics, the nature of the learning of mathematics and the nature of the teaching of mathematics, this philosophy had specific impacts. These impacts were on the tasks, or activities occurring in the classroom, on the discourse of both teachers and students, and finally on the environment in which the teaching and learning would take place. All of these are factors in the discussion with respect to mathematics anxiety.

Regarding the traditional classroom, there is available a long history which is consistent with most adults' experiences. It is these experiences which adults use as a context for their discussion about their learning of the subject of mathematics. As a consequence, the traditional classroom created generations of adults with specific memories of their experiences (Chaffe, 1984). More often than not these memories were strong, negative, affective reactions rather than positive memories about specific mathematical concepts or procedures (Tobias, 1978). This resulted in specific beliefs about students' and teachers' roles in the classroom (Schoenfeld, 1985, Garafalo, 1987). Although many survived the traditional classroom, for many it left emotional scars as in documented in the literature about mathematics anxiety.

The research on mathematics anxiety had multiple areas of potential focus including topics like gender or racial issues, pressures and stresses associated with achievement or testing, and classroom interactions between teacher and students on a day to day basis. For example, more recent work by Meece, Eccles and Wigfield (1990) indicated that the students' interpretations of their outcomes and not the actual outcomes themselves have had the strongest impact on the students' affective reactions to the subject of mathematics and their achievement in it. Thus the expectancies to achieve, as the authors defined them, may be more significant than the actual achievement, or the student perceived importance of mathematics.

Other areas of student anxiety related directly to the daily operation of the mathematics classroom. These were given as much attention as were the concerns with assessment, both of which definitely affected students' attitudes.

> The major source of math anxiety is the explain-practice-memorize teaching paradigm. Materials that use a problem solving process in teaching... show a great deal of promise as successful and realistic alternatives. (Greenwood, p.663, 1996)

Authors like Sheila Tobias (1978) and Laurie Buxton (1981) spent many years studying the mathematics anxiety phenomenon in an attempt to solve the problems from within the context of the traditional classroom. From their studies we can know about the classical reactions of students whose existence was often unsuccessful or less meaningful. Many authors found an inverse relationship between higher levels of anxiety and lower levels of achievement (Dew et. al., 1984, Sachs, 1982, Wooley, 1982, Buckley & Ribordy, 1982, Gourgery, 1982, Wright, 1981, Hendel, 1980, Austin-Martin et al., 1980). Without a support system to effectively assist in their learning, many of these students had feelings of helplessness. They found themselves threatened by the very adults to whom they needed to look for help. (Sovchik, et.al., 1981, Chisholm, 1980, Tobias, 1978). Tobias called this helplessness "floundering" while Buxton referred to it as "groping", neither of which fostered academic potential but initiated negative emotional responses.

Both Buxton (1981) and Tobias (1978) noted other compulsive behaviors exhibited by students. They describe how students would act the clown to match the position of being made to look foolish when asked questions in class. Tobias considered the placement of the student in the position of exhibiting "blackboard stupidity" as one leading to demoralization and loss of self-esteem (Tobias, 1978). The impairment of the child's self-image became part of a perpetual cycle in which poor mathematics grades led to poorer self-image which led to again poorer grades and so on (Gourgey, 1982, Sovchik et al., 1981, Tobias, 1978).

> Recently many researchers have documented systematically:
> 1) the failure of mathematics teaching due to its presentations solely as rote manipulation of forms and 2) its role in disenfranchising significant populations from pursuing its study. (Confrey, p.21, 1992)

With control over the classroom and over the content of mathematics, the teacher was identified as being critical to the investigation of the students' anxiety. The authority figure represented by the teacher was the person in whom they needed to confide but became the same person who posed the continued threat to the student's esteem. The authority figure was classified as being threatening to the students in several ways (Buxton, 1981). The first was described as the structural authority as might be seen in the army and would include the parents of the student or the administration of the school. The second was as sapiential authority based on the superior knowledge and expertise which the teacher possessed and exhibited in class. The last was authority was based on the personal charisma of being a mathematics teacher and was assigned by the very nature of the subject of mathematics itself. The development of the idea of MATHEMATICS TEACHER = INTELLIGENCE = POWER attitude was seen by researchers to create uncomfortable reactions in students (Chaffe, 1984, Buxton, 1981). It was noted that the power position, associated with being a mathematics teacher, became for some teachers the basis of their classroom control. They would share their knowledge only with the students with whom they felt they received the appropriate respect. Some even used mathematics as a punishment for inappropriate behavior which added to the development of negative attitudes towards the subject.

Ultimately, for teachers whose maintenance of this power defined their efficacy within the classroom, they would not quickly or easily relinquish the power and control. Alternately, even if they might want to create a trusting environment in which students would want to ask questions, the teachers might find themselves in a compromised situation with the very students they wanted to help. Within this classroom the students were never asked to participate in other than a very structured controlled way (Confrey, p.33, 1992) and anonymity was safety from loss of self-esteem and exposure in front of peers. It was within this context of the traditional classroom that many student attitudes towards the subject and the teacher were built up, and over many years the environment created generations of students who hoped for an anonymous existence (Tobias, 1978).

2.3 The Reform Movement

It may seem to the general population that mathematics education has been under continuous reform during most of this century fueled by factors like modern psychology, the cold war, the space race and international test results, just to mention a few. The present reform of mathematics education is founded in a philosophical perspective that is very different from most previous reforms. This in turn has made its impact on the learning and teaching of mathematics in terms of pedagogy and epistemology (Senger, p.3, 1996). To more clearly understand the present reform movement, it is necessary to expand on the mathematical communities' view of the topics of the nature of mathematics, the learning of mathematics and the teaching of mathematics, as they are perceived within this paradigm shift.

2.3.1 The Reform View of the Nature of Mathematics

Where the traditional perspective was based on the absolutist view of mathematics, the reform perspective is based on the fallibilist view of mathematics (Ernest, p.3, 1991). In looking for the nature of mathematical truths, the absolutist locates them in "a priori' or non-observable knowledge. In contrast the fallibilist view of mathematics finds these truths within the context of "posteriori" or empirical knowledge which is asserted on the basis of human experience.

> These (mathematical ideas) are human formations, essentially related to human actualities and potentialities and thus belong to the concrete unity of the life world. Mathematics is not something we have to look up to. It is right here in front of us at our fingertips. (Husserl, p.170, 1970)

In comparison to the absolutist viewpoint, the fallibilist sees mathematical truth as corrigible and open to revision, correction and addition (Ernest, p.3, 1991). By accepting that mathematics is not a complete or finished product, and by acknowledging the potential error in mathematics, the fallibilist is prepared to consider theory replacement and the natural growth of mathematical knowledge. As a consequence the fallibilist would value concrete over abstract, informal over formal, subjective over objective, discovery over justification, intuition over rationality, emotion over reason, particular over general, practice over theory, and the work of the hand over the work of the brain (Ernest, p.260, 1991). These are all characteristics which are more reflective of the nature of the reform discourse, the reform environment, and are encouraged by the nature of the reform tasks in which students would become engaged.

The view that mathematics is built around human observations is labeled Empiricism and has two basic premises: first that the concepts of mathematics have empirical origins, and second that the truths of mathematics have empirical justifications. According to Ernest (1991), where the first is considered valid and necessary, the second has the potential to be absurd. The fact, for example, that 999 999 + 1 = 1 000 000 does not have to be observed empirically to be true in the world, but can be understood by theoretical knowledge of number and numeration (Ernest, p.35, 1991). As a consequence to the concern over the second premise, Lakatos created the category of Quasi-empiricism (Lakatos, 1976, 1978) which is based on five theses that include:

1. Mathematics knowledge is fallible
2. Mathematics is hypthetico-deductive
3. History is central
4. Informal mathematics has a paramount importance
5. A theory of knowledge creation is included

(Ernest, p.35-36, 1991)

The fallibility of mathematics is directly associated with the concept that mathematics is a human activity and an acceptance that humans are fallible. The fact that human history is central to mathematics results not only in its growth during certain periods but also in its stagnation at others. It reflects the impact of social life on mathematics and vice versa:

> Mathematics is a dialogue between people tackling mathematical problems. Mathematicians are fallible and their product, including concepts and proofs, can never be considered to be final or perfect but may require negotiations as standards of vigor change, or as new challenges or meanings emerge. As a human activity mathematics can not be viewed in isolation from its history and its application to the sciences and elsewhere. (Ernest, p.35, 1991)

Thus what may prove to be pure mathematics for today may be called the applied mathematics of the real world tomorrow. Because of this connection to human existence and history, fallibilists view mathematics as value laden and carrying with it social, moral and historical values of human development.

> ... constructivism views mathematics as the product of human activity over the course of time. All of the different fields of knowledge are the creation of human beings interconnected with their shared origins and history. Consequently, mathematics, like the rest of knowledge, is culture bound and imbued with the values of its makers and their cultural contexts. A social constructivist history of mathematics has to show what philosophical, social, or political forces drive particular creations or block them. Henry (1971) argues that the creation of calculus was within Descarte's grasp but that he avoided the issues because to approach the infinite would have been considered blasphemous at the time. (Ernest, p.261, 1991)

> Every mathematician must recognize that whatever mathematical activity he may engage in, it is clearly and demonstrably connected with the culture of his time. (Wilder, p.46, 1979)

> One outcome of this view of mathematics is challenge to the cultural domination of abstract, white, male dominance in mathematics. On the contrary, mathematics is a universal human characteristic, which like language, is the cultural birthright of all people. (Ernest, p.263, 1991)

The connections of culture or history are not the focus of this discussion, but their examination does give clearer understanding as to the stimulus for some of the expected changes that will occur in the mathematics classroom as a result of the present reform.

2.3.2 The Reform View of the Learning of Mathematics

Based on the reform perspective of the nature of mathematics, there are implications concerning how the learning of mathematics can be achieved, and how the teaching of mathematics will be affected. These changes are all consistent with a view of mathematics founded in "constructivism". This new paradigm of mathematics instruction describes the creation of conceptual networks that constitute a student's map of reality as "the product of constructive and interpretive activities" on the part of each individual learner (Schifter, p.9, 1993). Whether this product be the mathematical structures of Jean Piaget (Piaget, 1952), the cognitive maps of Richard Skemp (Skemp, 1978) or the radical interpretations of reality associated with Ernst Von Glasersfeld (Von Glasersfeld, 1983, 1990), each of these leaders in mathematical ideology became connected with the principle of constructivism. The opportunity for individual interpretation of one's experiences with the world allows different views as to how this will actually occur. One consistent view of the interpretive nature of constructivism is the need for social interaction. In response to this need one form of constructivist theory emerged called "social constructivism":

> I propose a new philosophy of mathematics call social constructivism... largely an elaboration of pre-existing views of mathematics, notably those of conventionalism and quasi-empiricism. It draws on conventionalism in accepting that human language rules and agreement play a key role in establishing and justifying the truths of mathematics. It takes for the quasi-empiricist its fallibilist epistemology, including the view that mathematical knowledge and concepts develop and change...
> (Ernest, p.42, 1991)

According to Ernest the nature of social contructivism is that it is a descriptive not a prescriptive philosophy. The basis for being labeled as social constructivism comes from three areas. (1) Mathematics is based on linguistic knowledge, conventions and rules requiring language (which is a social construction). (2) Social processes which create interpersonal relationships are essential to turn individual subjective mathematical knowledge into accepted objective knowledge, and (3) objectivity is social as it is the shared product of human minds (Ernest, p.42, 1991). This will have important implications for the task (Appendix B, p.136), the discourse (Appendix B, p.137), and the environment (Appendix B, p.139) for the reform classroom, and as a consequence for the roles of teachers, students and others associated with the classroom operating on the acceptance of the constructivist stance. Each of these characteristics are reflected in the manner in which mathematics is expected to be learned and thus subsequently in the manner in which it will be taught.

The acceptance that mathematics is the product of human experiences and that constructivism is the basis for individual meaning of these experiences has resulted in very different ideas about the learning of mathematics from the perspective of the student.

> We can observe, that when we talk of students' constructive activities, we are emphasizing the cognitive aspect of mathematical learning. It then becomes apparent that we need to complement the discussion by noting that learning is also a process of enculturation....
>
> (Cobb, Yackel & Wood, p.28, 1992)

> <u>Beliefs about students and mathematical learning:</u> Students are active and curious learners who have individual interests and abilities and needs. They come to the classrooms with different knowledge, life experiences, and backgrounds that generated a range of attitudes about mathematics and life. Students learn by attaching meaning to what they do and they must be able to construct their own meaning in mathematics.
>
> (Western Canadian Protocol, p.2, 1995)

The nature of learning in the reform classroom comes as a result of the constructivist philosophy of human experiences and how this has impacted on the theories of learning. The constructivist view of reality believes that knowledge can not be transferred ready made from one person to the next but has to be built up by each individual on the basis of his or her own experiences (Von Glasersfeld, 1990, Confrey, 1990). As one of the earlier proponents of thought based on the constructivist position, Piaget said that unique to the human species, "our intelligence organizes its world by organizing itself" (Piaget, p.35, 1980). As a consequence of this attempt, Von Glasersfeld describes students' learning as the successful organization of his or her own experiences (Von Glasersfeld in Janvier, p.6, 1987). One of the accepted methods in which this organization would occur is based on the premises of Karl Popper.

The three worlds of Popper (Bereiter, p.22, 1994) were described around: (a) world 1 which gave direct information from the five basic senses, (b) world 2 which comprised an individual's subjective interpretations of world 1, and (c) world 3 in which subjective knowledge from world 2 becomes accepted objective knowledge. The last world was considered to have been the product of the human minds, was agreed upon and accepted by the larger segment of the human population. The movement of knowledge, in the subjective to objective conversion of Popper and from a personal perspective to a group acceptance, has been acknowledged by many constructivists. It is eventually in world 3 that the made-by-man subject of mathematics came to exist. Through language and conversation with others the individual learner can take the subjective views of mathematics and adapt, alter or edit these until they become objective. It is around this process of objectifying the subjective knowledge of the individual learner, that the learning of mathematics has become associated with the constructivist theory. As has been expressed by Cobb et al. (1992), learning then becomes the process in which students gradually construct mental representations to best mirror the mathematical features of external representations (Cobb, Yackel & Wood, p.3, 1992). The external world, and the experiencing person's representations of it, became for Von Glasersfeld the extreme or radical form of constructivism (Von Glasersfeld, 1986b). His theory specifies that we can never know the real world, or anyone else's view of that world, but what we have is our own specific and filtered perception of what that world is. In a "fit" or "match" possibility, he states that the closest we can ever achieve is a compatible fit with another person's perspective, and that it will never be a match (Von Glasersfeld, 1984). He accepted and incorporated three ideas from Piaget into his Radical Constructivist viewpoint. First: knowledge is not received passively but is actively built up be a thinking person, second: the cognition of a person is adapting constantly towards a fit or viability of that knowledge, and third: cognition serves specifically the subject's organization of his(her) experiential world. It should not be confused with discovery of an objective ontological reality (Von Glasersfeld, p. 22-3, 1992). He used the term 'viability' as the survival of sustainable knowledge from that which would <u>not</u> endure the survival of the fittest test, and 'perturbations' as the conflict within each individual as they go through the subjective-objective conversion.

> ... it is legitimate to interpret Piaget's work as a social, cultural approach in which he explained the mathematical development of children as self-regulating, autonomous organisms interacting in their environments... Making sense, then, can mean to construct ways and means of operating in a medium to reduce perturbations induced through social interaction. (Von Glasersfeld in Steffe, 1993)

Where Von Glasersfeld had used the term perturbations and Piaget had used the term cognitive dissonance (Piaget, 1978), both were describing the conflict into which the individual's mind was placed when taking subjective knowledge and establishing from it the more permanent objective knowledge.

With learning focusing more on the individual, and his or her interpretation of experiences, then learning will be better accomplished with tasks (Appendix B, p.136) which emphasize the learner's perspective. This will also necessitate a different environment (Appendix B, p.139) that can support learners in meaningful ways in order to 'make sense' of their experiences. Finally, the learner will need access to discourse (Appendix B, p.137) in which the personal subjective interpretations can be presented in a public domain for discussion, criticism, approval, acceptance and finally conversion into the accepted and objective knowledge of the mathematical community.

2.3.3 The Reform View of the Teaching of Mathematics

Where the references on task, discourse and environment cited above give indications as to the changing roles of teachers and students within a mathematics classroom, many of these came as a consequence of the reform view of the teaching of mathematics. As early as Thorndyke it was noted (but ignored) that teaching would not be accomplished by simply telling.

> The commonest error of the gifted scholar, inexperienced in teaching, is to expect pupils to know what they have been told. But teaching is not telling... (Thorndyke in Hergenhahn, p.74, 1984)

This view was repeated by Von Glasersfeld in his comments regarding our use of language as the mode of transmission of knowledge.

> A seasoned users of language, we all tend to develop an unwarranted faith in the efficacy of linguistic communication; we delude ourselves that speech conveys ideas or mental representations.
> (Von Glasersfeld in Confrey, p.485, 1984)

Brown's hermeneutic view of mathematics (Brown, 1994) places the focus on the interpretation of one's experiences, and its significance from the individual's perspective. His view sees a shift in emphasis from mathematics as an externally created body of knowledge to the learner actively engaging in mathematical activity.

> Mathematics is only shareable in discourse and the act of realizing mathematics in discourse brings to it much beyond the bare symbols of a platonic formulation of mathematics (Brown in Ernest, p.147, 1994)

The active participation of the learner in the mathematics learning process has been exemplified in the teaching of mathematics within a constructivist environment. This environment is as a result of intentional and deliberate actions and behaviors on the part of the teacher with the acknowledgment that constructivism pertains specifically to the actions of the learner (Pirie and Kieren, p.506, 1992). According to Pirie and Kieren, the four main attributes of this constructivist environment are as follows:

> 1) Although the teacher may have the intention to move students towards particular mathematics learning goals, she will need to be aware that such progress may not be achieved by some of the students and may be achieved as expected by others.
>
> 2) In creating an environment or providing opportunities for children to modify their mathematical understanding, the teacher will act upon the belief that there are different pathways to similar mathematical understanding.
>
> 3) The teacher will be aware that different people hold different mathematical understandings.
>
> 4) The teacher will know that for any topic there are different levels of understanding, but that these are never achieved 'once and for all'.
> (Pirie and Kieren, p.507-8, 1992)

The change in the environment, in which the learning will take place, shows an increase in the autonomy of the learner and a reduction in the authority of the teacher (Piaget, 1969, Kamii, 1986, De Vries, 1987, Siegel & Bourasi, 1984). In such an environment Von Glasersfeld (1993) describes the responsibility of the teacher as being one in which the student is prodded to make sense of his or her experiences, as he or she develops new concepts in mathematics, and teaching should become much more concerned with understanding (Von Glasersfeld in Ernest, p.224, 1994). The similar expectation for prodding by the teacher, and the social interaction in which the student and teacher would come to operate, was adapted from the theories of Vygotsky (1934) within the framework of social constructivism. He stated that through conversation with others one learns how to have conversations with oneself (Vygotsky, 1934). Social constructivists see this conversation as an opportunity for successful conversion of the subjective knowledge into the objective domain.

The creation of the classroom around the constructivist philosophy, and the changes to teaching as a result of the acceptance of this philosophy, are multiple. One example, the inquiry classroom, outlined by Siegel & Borasi (in Ernest, p.201, 1994) has the following identifiable traits. There is an emphasis on the production of knowledge not the transmission. Learning is a collaborative process within which meanings are negotiated. Students will generate potentially new meanings and develop connections of their ideas to the world. Teachers will support students through the inquiry process and will need to develop stronger skills in listening to students' ideas. With students taking the center stage, the creation of a risk taking classroom will foster student autonomy in which interdependence is essential but independence in the ultimate goal.

Accepting that the classroom is operating with these or similar goals, the teacher's task will be innovative as well. The expectation will not be that the students are simply to discover or rediscover all of the mathematics which has taken generations of human inquiry to accumulate. The teacher will be responsible for choosing tasks, and setting the stages for student inquiry, in which students resolve problematic situations that challenge their current conceptual understanding. The inquiry classroom will focus on exploration, conjecturing, proving the conjectures, and problem solving as active processes (cf. Cobb, Yackel & Wood, 1991, Fennema, Carpenter & Paterson, 1989, Kamii, 1988). These are attributes that are reflective of the nature of task, and discourse represented in the N.C.T.M. Standards (1989, 1991, 1995). As stated by Jack Price, president of N.C.T.M.:

> ... the standards are based on research and on a constructivist theory of learning. This theory proposes that children are not just empty vessels that we fill... Students need time to be actively involved in their learning and time to reflect not only on what they have learned but also on how this learning fits into what they already know... Critics may not agree with the theory, but they can not say that the standards are not research based...
> (Price, p.489, 1995)

As noted by Richards (1991) it will not be serendipitous learning that happens in which students are active learners by accident. It will be by design, and the teacher will have to create tasks and projects which stimulate students to ask questions, pose problems. The teacher will need to provide plans and a structure that supports the development of "informed exploration" and "reflective inquiry" which does not take away the control or the initiative of the student.

In relation to the previous statements teachers need to be aware that mathematics is not embodied by concrete materials, or the tasks or projects which use them. It would instead become embedded in them as the teacher takes the non-contextualized mathematical ideas to be taught and embeds them in the most functional context for student investigation (Brouseau, 1987). In one attempt to define the pedagogy involved, around which teachers would develop an action plan, Simon (1993) described this plan as the "Hypothetical Learning Trajectory".

> This can be seen as the thinking and planning that preceded my instructional interventions in each of the teaching situations... as well as the decisions that I made as a result of the students' thinking.
> (Simon, p.135, 1993)

The three important aspects of this learning trajectory are the goal, the activities and the process. The goal is the specific mathematical skill or concept that the teacher would like the student to attain. The activities discuss actions that require cooperative groups, concrete materials and the conflict in question. They will emphasize the social setting and the negotiation of meaning. The process is considered to be the teacher's prediction of how the students' thinking and understanding will be interpreted and evolve in the context of the activity. All of this is a matter of personal interpretation on the part of the teacher initially as the planner, and on the part of the student as the executor the plan.

With interpretations available to teachers, as part of discussions with colleagues regarding concept development, and available to students as they construct their perceptions of the concepts from the chosen activities, questions will arise in a reform classroom which will need to be addressed. One concern would be the expectation that teachers are attempting to teach mathematics that they have not mastered themselves. On the other hand, if they feel that they have mastery of the concepts from previous experiences, they may not feel comfortable with the requirement or expectations of presenting this material from an inquiry perspective. Teachers may feel alienated from a process in which they have never had first hand experience. These are just a few of the potential concerns which may be raised as the teacher and the classroom adapt to a constructivist position.

> One of the weaknesses of constructivism is the concentration on individual subjectivity, which results in a solipsistic view, blind to the objectively real world that appears right in front of us. A Symbolic Interactionist perspective suggests that although society is built through human activity that expresses subjective meanings, these meanings may become objectified (taken as shared) through social negotiations. This results in the construction of an inter-subjective or taken for granted reality (cf. Berger & Shuckman, 1966, Erickson, 1986). Thus a coordination of a constructivist focus, on the individual construction of meaning, combined with a Symbolic Interactionist focus, on the interpretation of meaning through social interaction, provides a means of studying how the objective world of school mathematics is continually produced and reproduced through subjective human action .
> (Greg, p.46, 1995)

It is the combination of the participants' construction of a reality and the researcher's resulting interpretive process, from social interaction, that will be the basis for this research. It will be the participants' perceptions of the reform classroom that will be the basis for the construction, and their interviews that will allow for an interpretive opportunity. The participants' interpretation is one directed at their perception of the reform classroom. The researcher's interpretation is one directed more at the similarities which the participants' perceptions have with the philosophical paradigm shift reflected in the reform movement. The next section is presented to give some indication as to the research field in terms of adult perceptions and their application in the area of educational research.

2.4 Research on Adult Perceptions

This study is located in the context of research on adult perceptions. Such research covers a wide range of adult experiences. However, a review of a portion of the more recent available literature indicates a significant emphasis in three particular areas. The three areas focused on are 1) the post secondary education of adults, 2) gender issues relating to women in education programs at the adult level, and 3) adults participating in upgrading programs. The data collection tools applied most often are personal interviews, surveys, questionnaires and attitude scales.

In grouping the adult-learner studies, in which the intent was the upgrading of the adult, the strongest emphasis was on student literacy and personal efficacy. The perceptions of the adults regarding their participation in basic adult education programs gave impressions about their experiences as being negative, and regarding school in general as being meaningless (Velasquez, 1990). Another study confirmed that participation in such programs had the greatest impact on the affective domain, not the educational one for which the programs were designed (Walker et al., 1981). This same study made particular reference to the research having an "ability to measure only the perceptions of the impact rather than the actual impact itself". In programs dealing with these particular adults, a similar study by Heaney (1981) came to conclusions regarding the conflict with non-traditional education of adults and the traditional approaches usually applied. This was seen to be as a result of the differences in their perceptions of the reality in which they had to exist and the reality in which they were being educated.

The research on the "at risk" population of rural students in a study by Tompkins and Deloney (1994) used the perceptions of parents, students and educators to identify characteristics unique to the drop-out groups in rural areas of the southern United States. Their results confirmed a significant difference between the perceptions of rural adults from those in metropolitan regions. In a related study on adult literacy, the perceptions of teachers and adult learners showed another disparity when looked at from the functional or fundamental view of these programs (Malicky & Norman, 1995). A number of literacy related studies looked at the concerns resulting from ESL programs, or English as a Second Language (Mikulecky et al., 1996, McKeag, 1993). Perceptions in these studies were directed towards the students' sense of efficacy and tended not to focus on the learning as much as they did on the concept of self-esteem resulting from participation in the programs.

The feminist movement, and the associated concerns around gender issues, continued to be another relevant area for other studies in adult perceptions. The Annual Adult Education Research Conferences held in Saskatoon, Saskatchewan in May, 1992, and in Edmonton, Alberta in May, 1997, contained multiple presentations which drew attention to women's issues in the areas of literacy, higher education, self esteem, and personal improvement programs. Included in these conferences were studies considering such topics as *Women's Perceptions of the Value of Vocational Training Programs* (Davis et al., 1992), *Toward New Perspectives on Women in Adult Education* (Hayes, 1992), *Collegiate Involvement from an Adult Undergraduate Perspective* (Kasworm, 1994) and *Re-mapping Adult Education* (Collard, 1995). There continued to be interest in what women's perspective would be regarding their designated "roles" and the conflict these created with their educational pursuits. As was expressed in the study by Edwards (1993) race, class, and institution influenced women's attempts to balance family and educational objectives.

Women's attempts to become involved in educational programs had little or no effect on their personal responsibilities, especially related to family. Another dominant area of investigation with respect to adult perceptions had some definite overlap with those of women's issues. These studies attempted to address the adult perceptions in an number of areas. Some topics included were those of relationships between professors and students (Lynch & Bishop-Clark, 1994), perceptions by cooperating teachers of non-traditional student teachers (Meloy, 1992), perceptions of the learning environment in colleges of teacher education (Kariuki, 1995), and perceptions of assessment of outstanding teachers by undergraduate and graduate students (Donaldson et al., 1993). One study by Kasworm (1994), on adult perceptions of student involvement, indicated that they were greatly influenced by the value of the program, desire for quality education, the quality of academic learning, the support environment for their learning, and the access to appropriate finances. It was concluded that the students' beliefs and actions about their roles were founded on past educational and personal histories.

Many of the studies cited, and other related studies, made significant contributions to the areas of research in which they were most suited and to which they were being directed. But it was immediately obvious and apparent that there was no significant focus on adult perceptions regarding the subject of mathematics or the present reform to the teaching of mathematics. Once or twice there were indirect references to mathematics within a study, but not as the central focus. Consequently, there seemed to be a gap in the literature on adult perceptions of the mathematics classroom, either past or present. Previously this gap may not have been a concern because of the tendency to accept the traditional mathematics classroom as the status quo. In light of the reform movement in mathematics education, this status quo is being questioned and so are people's perceptions of it. The context of change now gives perceptions greater importance and makes them of more interest to the mathematics educational community. Thus this thesis in unique in that it focuses on a topic that has not been explicitly dealt with in the literature. Finally as was indicated in the study by Walker et al. (1981), it is accepted that the topic of research in this thesis has the ability to measure *only the perceptions of the impact and not the actual impact itself.* It is not physical contact or actual experiences with the classroom (or the students and teacher within that environment), which the participants have had access. Their perceptions are being generated by contact with the research tools chosen by the researcher to reflect the reform philosophy.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Research Perspective

This research has the character of a descriptive qualitative study in which the perceptions of three adults are being investigated. Of particular interest is the potential their perceptions have to contribute to a discussion of the reform of mathematics education. Although each participant made these contributions in a unique and individual manner; the material arising from their conversations will not be treated as individual case studies, but in terms of the information that arose more as a group consensus. The research focused on the nature of their perceptions regarding three characteristics of a reform classroom. Specifically these topics relate to the nature of the task, the discourse and the environment, and were intended to maintain consistency with the research literature and the recommendations from the N.C.T.M. Standards (1989, 1991, 1995). The methodology often associated with this type of research, and applied in this thesis, used personal interviews to gain access to the participants' viewpoints. They were given information that reflected classrooms modeling a reform philosophy, and it was through their reflections on information provided to them that the data was collected. The opportunity was created to gain access to their perceptions of a classroom exhibiting reform characteristics. It was expected that this would give the researcher a unique position from which to discuss the present reform to the teaching of mathematics.

3.2 The Selection of the Participants

The following three characteristics were used to select the participants.

(1) The participants had a personal history with a dislike of mathematics (as a reaction to being a student in a traditional classroom). This was an important feature of any participant in this study because adults who had a dislike of mathematics, or negative experiences within the traditional classroom, would seem to be the greatest proportion of the population. Whether this was representative of fully developed mathematics anxiety, and whether or not it appeared in physical manifestations, it would have left long term psychological manifestations. It is verifiable in the literature that the affective recollections of existence in a traditional mathematics classroom remain in tact many years after the mathematical content of particular classroom experiences have long been forgotten.

(2) The participants had an ability to be articulate as they spoke about the different aspects under investigation. This criterion was important because it was considered particularly useful to involve participants who would openly, and intelligently, discuss and reflect on the material presented to them. It was also recognized as being productive to the study to have access to the valuable, personal knowledge of these participants through their active conversation in interviews.

(3) The participants' experiences covered different periods from the last thirty years. The choice to use adults as participants for this study was an interest specifically in the adult perspective in this area of research. The representation of three different generations of adults, from the last thirty years, was useful to see how much their perceptions of the reform classroom might be different from each other. Although the purpose of this research was not to attempt to study the

connections of a specific generation to their particular perceptions, it provided the opportunity to look for generalizations about the reform of mathematics from what would qualify as three distinct educational eras.

Based on these criteria, three participants, Mrs. B., Mr. M., and Mrs. P. were selected from a number of adults who volunteered to participate in this research. Mrs. B. graduated from high school thirty years ago, Mrs. P. fifteen years ago and Mr. M. a few months before this research was initiated. The following are brief profiles of each of the three participants.

Mrs. B.

Mrs. B. is a university graduate with a Bachelor of Arts Degree in English and a Bachelor of Education Degree, specializing in High School English. She is the mother of three children all of whom are presently either enrolled in, or graduates of, university programs. Mrs. B. has been a colleague for over ten years at the school where I teach. I had the opportunity to teach all three of her children in different mathematics courses at the secondary level. As the daughter of a famous Canadian Supreme Court judge, she was at one time convinced that she might never get to attend university herself, and mathematics was her major stumbling block. She has been an English, Latin and drama teacher for over seventeen years, and has a great zest for life that is reflected in her approach to teaching. Mrs. B. was the eldest of the participants and had been out of high school for over thirty years. Of the three participants, she was the most classic case of math phobia and anxiety right to the end of her career in high school. Of interest is that she assigns no blame to her teachers, but accepts personal responsibility for her "ineptness" or her inability to be successful. In many of her comments about herself, when volunteering for this research, she echoed the literature regarding classic math anxiety (Tobias, 1978, Buxton, 1981)). She had the greatest fear of being sent to the blackboard and hoped to spend her classroom time in complete anonymity. Her high school graduation year did not give her credits in her final required course in mathematics, even with repeat attempts to pass. Her mathematical education left her with a lifetime desire to "someday know how to solve mathematics (word) problems" and to revisit some of the topics which she never understood when she was a student. As she explained:

> *I seem to remember that back in elementary school that math was actually quite fun and I was very good at learning the times tables and all those kinds of things until I got to grade 6 and I had to deal with problem solving. I think that is probably the root of why I find math very difficult and it was this enormous fear and frustration with trying to figure out.. whatever the problem involved and almost defeating myself because I would get so anxious... part of it was self inflicted because I couldn't get myself to care... mathematics was an endless process of word problems that I couldn't solve...*
> *... to be honest it became something I avoided as much as possible... my reaction to it was to become cavalier about math-- Ah well I didn't care, I wasn't going to use it anyway so, so what!...*
> *I have a list of things I would like to accomplish and someday I would like to feel that I could solve a math* (word) *problem. I still want to do that.*

Mr. M.

As the youngest of the participants, Mr. M. is a very recent graduate from high school who has maintained a part time job, and who had an active athletic involvement throughout his teenage years. At the time of the study, he expressed the desire to someday see himself as a successful university graduate and since the interviews has entered the University of Calgary in a general studies program. He indicated his interest was in becoming a graphic artist with the application of computer knowledge. As a student of mathematics, he presented himself as someone who had great frustrations with the learning of the subject, especially with the routines of review, teach and practice. The focus on independent work directly from textbooks often left him very insecure in his skills, and feeling as though he was floundering with insufficient support. He was not an exceptional achiever in mathematics, but one who still had a view of himself as being a capable learner. Even though he struggled just to pass the last mathematics course, he still felt that the potential was there for him to learn more mathematics. He had a much higher estimation of this mathematical potential than was represented in his formal assessments. His candid comments about his experiences in mathematics relate what he called his "poor preparation for the demands of high school". This created for him significant pressures and stresses, which enhanced his dislike of mathematics. He had seen his elementary experiences as being fun and less structured, but not something that would be applicable at the later grades. His frustrations were most evident when his test evaluations reflected a lack of mastering the "structures of mathematics". His personal solution was to repeat the skills until the structures became clear. The following is an excerpt of Mr. M.'s description of his experiences with mathematics.

> *My experiences have always been lectures and then after lectures, it would have gone right to either homework or maybe a short period of working in class. All of the questions would have been right out of a set textbook and everyone would have been working on the same thing and everything would be set up like that, it would all be done with a textbook. And that's how it's always been as long as I can remember right from grade 1. It's always been out of a textbook and listening to the teacher lecture. I have some trouble working out a textbook but there is like --, the textbook forces a student like myself to repeat a question numerous*

times so maybe it might help understand the question better...
Actually I have quite a bit of interest in math but it's just the way it's taught that really turns -, that turns me off a bit...
I personally think that I have great potential, and I can understand math as it goes along, but it seems anytime it comes to exam time or test time, no matter how much I understand that information, it's just not going to work in a test situation... it just seems overwhelming, all the new information they're trying to teach you and you just don't learn it.

Mrs. P.

Mrs. P. is also a university graduate with a Bachelor of Arts in Social Studies and a Bachelor of Education, specializing as a generalist in elementary division. She holds Educational Diploma Certificates in Mathematics and English. As a member of the Mathematics Implementation Committee of the Calgary Separate School Division, she has been actively involved in the work-shopping of teachers in the reform of mathematics education. Mrs. P. is married with two children. She is presently an elementary teacher with over twelve years of experience mostly in grades one, two and three. She was a member of a graduate class that I took while pursuing my master's degree. She has also been involved in in-service programs for teachers about the reform of mathematics education. As a student, Mrs. P. has been out of the mathematics classroom for about fifteen years. As the representative of the middle generation of the three candidates being interviewed, she did not receive much personal satisfaction with the learning of mathematics and yet it did give her success. Even still she experienced some of the classic fear, and personal frustration, with being in the traditional mathematics classroom. Today she is responsible for teaching mathematics to elementary students at the grade three level. Her personal views about the subject of mathematics are reflected in her teaching and her attitudes towards children's roles in the learning of mathematics. Of the three participants, she has the academic basis and a more complete understanding of the expectations of the reform of mathematics education. The following is an excerpt of Mrs. P.'s description of her experiences with mathematics as a student:

> *... She didn't have much patience and she never taught me how to do it. she just kept showing me one way to do it and I seemed to be getting the right answer but I was always doing it a different way and she didn't like my way. So for a test in math, I would just memorize the questions that had been on the review pages and that would be on the test and that's how I got through math. But I didn't --, I don't remember a thing that I learned... I wasn't good at it. I didn't perceive of myself to be a mathematical person at all and when it got to problem solving, I panicked. I'd say all through school it was the same thing. Teacher modeled and we did the workbook or textbook pages and then we'd correct them or we had homework to do and then we would take them up...*
> *Even when the teacher, you know the men teachers that we had in high school, if he modeled and we were stuck and we needed help, he would come but he was still trying to show me the same way that he had modeled on the board, like he never put it in a new way so that I might be able to figure it out...I didn't learn from the teacher, I actually learned from other kids in my class. You know from what they were doing because someone might have another way of doing something... It was really funny because I was the top kid in one class. I got awards for the top in the class yet, I sometimes look back at that now and I can't --, I don't know how I did it,... so I don't really consider myself to be an A student in math... People thought that I was brilliant but when it came --, it was just memorizing what --, it was just memorizing. That's what got me through my math classes.*

All of the participants were very cooperative during the interviews. They answered all questions without reservation and exhibited enthusiasm towards the role they could have in the study. They quickly established rapport, which developed from the very first interviews, and made the process of focusing on the question of the research easy to address. It was interesting that they were somewhat unsure of the potential for their contributions to have significance to this research, but that they also were quite keen to know the consequences of the ultimate findings. Of the three participants, I was acquainted with the two women prior to the initiation of the study, and they expressed great interest in taking an active role in this research. This relationship assisted the research by establishing a comfortable entry level into the study. The third participant, the young male, was not known to me prior to the study, but seemed to develop a comfortable conversational position immediately from our first contact. None of the participants had contact with each other during this research.

3.3 Data Collection

Data collection consisted solely of separate personal interviews. Each participant was interviewed on two different occasions. Each interview lasted from between forty minutes and one hour. The determination of time and location of the interviews was left to the participants' choice in order to maximize convenience and comfort. Each interview took place in a private and uninterrupted site, and was tape-recorded before being transcribed into hardcopy text, either by myself or a person unassociated with any of the participants. The first interview was structured around three activities representative of reform mathematical tasks. The second interview focused on a 'scenario' to model a reform classroom. The scenario was a story written to give an example of an actual classroom to put the students, the teacher and a mathematical investigation in a real context. As a basis for initiating discussion with the participants, both sets of interviews focused on, and highlighted, the areas of discourse, task, and environment.

3.3.1. Interview #1

3.3.1.(a) Introduction to the Activities

The following is the introductory statement that was used to give the participants an understanding as to their roles and the nature of their participation in this research. Following that are the three activities given to the participants to initiate the conversation. The activities are presented in a similar form to how they were seen by the participants.

As an introduction to your participation in our interviews, I would like to hear about your personal history in mathematics. I would appreciate it if you would relate to me how you experienced mathematics as a student. If you want, you can begin from your earliest experiences and describe them right up to your last mathematics course. It is not the particular knowledge of any mathematical topic that you need to recall but just an overview of your impressions of being a student in a mathematics classroom. Any names you use in relating your story will be edited from the transcript and you will be given a pseudonym to maintain your anonymity in the transcripts that will be made from the interviews. Any questions before I turn on the tape recorder?

For the next part of the interview, I would like you to read and be prepared to give your personal opinions about the nature of three mathematics classroom activities. In your discussion of these activities, it is not your personal knowledge about the concept or topic that is of importance. It is only important that you give your personal opinion and impressions about how you would see changes in a mathematics classroom using one of these activities as the initiator of a new concept. If you can, I would like you to make reference to three specific areas as you discuss these activities.

Those three areas are the TASK, the DISCOURSE as it would been seen from the roles of either the teacher or the student, and how you would see the ENVIRONMENT, in which the teacher and the student would be interacting. If possible, I would like you to predict how you would have operated as an ACTUAL student, had been in this environment, and how you might have expected the teacher to react. I *know that this is purely an imaginative exercise but your opinions and your ideas can be very useful, so again there are no expected or right answers.*

If you can make reference of these ideas to your reading, as you discuss them, and how that would compare or differ from the three activities, that would be especially useful. The comparison between your real experiences and how you would perceive these experiences is the purpose of the exercise. We can read these readings one at a time, and you may speak about each of them separately, or you can combine the information and relate to the ideas as a group. It is your choice. As you speak, I will try not to interrupt but I may ask you to expand on a statement if there is something which catches my attention.

After we complete your discussion of these topics, I have a ***one act play*** *for you to read and have time to think about in preparation for our next interview. The Play is a pretend classroom with fictitious characters who are "acting out" a class including the social interaction or discussion that might have occurred. As you read I would ask that you focus on the following areas for future discussion:*

1) *how you see the activity being used for the development of this concept*
2) *the teacher's role and participation in discourse*
3) *the students' roles and participation in discourse*
4) *how the classroom environment would seem to you*

If you see two different sides to the picture, that is either positive or negative characteristics, could you make mental notes and return with these for our next discussion.

3.3.1(b) Activities

Each of the three activities describes a mathematical investigation in which is embedded an open exploration of a mathematical concept that is specific to a particular grade. During our initial meeting, each participant was asked to read, and speak openly about these three activities one at a time, or as a group. The activities were chosen to model the kinds of tasks used to introduce mathematical concepts in a reform mathematics classroom being based on a problem solving focus (see Appendix B, p.135). A request was again made for the participants to focus on the nature of the tasks, the nature of the discourse emerging, and the nature of the environment in which the task and discourse would occur. The participants were allowed to take the written activities, read them through for a few minutes, and then respond freely. This format generated a necessity for the participants to create a visualization of the classroom in their minds, or a fictitious condition in which they attempted to describe the teacher, the students and the classroom as it might have existed.

The following pages give the three activities in the form in which they were presented to the participants, along with diagrams to represent the actual concrete materials needed for the investigations. The participants were supplied with the actual materials as an opportunity to see the kinds of manipulatives which children would be given as part of the investigation.

ACTIVITY #1:

An Investigation of Quadrilaterals (4 sided shapes) with TANGRAM Tiles

Using the tiles, which come in a TANGRAM set, your objective will be to find and trace as many four sided shapes as you can. Keep a record of each shape and beside each one write information which identifies characteristics which are unique about the shape. You might discuss the lengths of the sides, the types of angles or how the sides relate (For example, are they parallel?). If you know a name or a mathematical term for the shape , label the shape with that term. You do not need to indicate that the shape has 4 sides as ALL OF THE SHAPES will have this characteristic.

Remember that you are allowed to combine one or more of the shapes up to a total of seven. You may work on this problem either with a partner or independently and you might need to ask for BLANK 8.5 x 14 paper from the teacher to record your work on. Be prepared to discuss what you used as characteristics with the class. There are a number of possibilities but each one should use DIFFERENT tiles or DIFFERENT NUMBERS of tiles not just the same tiles rearranged. You cannot duplicate a tile unless the set had two of the same tile in the original 7 pieces. (see the diagram below of the basic TANGRAM set)

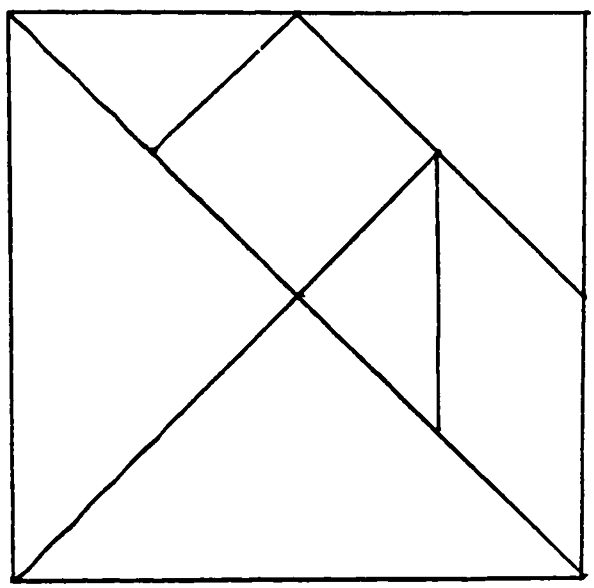

figure 3.3.1 (a)

ACTIVITY # 2:

An Investigation of Flips, Slides and Turns with Pattern Blocks

With a collection of colored pattern blocks as seen in the diagram below, (which represents the 6 possible different geometric figures), your task is find different ways in which these blocks can be fit together on a flat surface. Using two or more pieces, and/or colors, construct a UNIT SHAPE which will fit with itself like pieces of a jigsaw puzzle.

Your objective is to construct flat models like those you might see if you were to look at the geometric patterns in floor tiles or wall paper. The PATTERN should be obvious and one in which a UNIT SHAPE can be determined by another student. The unit shape may have more than one of the same color pattern pieces (as well as using different pieces). If they can find the unit, they will know how to continue the building of the tiling pattern. Test this with other students in the class. There should be no holes or spaces in between the pieces except possibly on the outside edge. You may not have enough pieces to build a large pattern and so, as you work, trace the shapes of the pieces to record your patterns. Color in the patterns with crayons or felt markers.

When you have completed one arrangement with the pieces, you may leave it and try a second or a third unit shape, using different arrangements and different colors of pieces. The eventual project of a MATHEMATICAL TESSELLATION will be assigned after discussion of the student examples. In class the student examples will be used to look for the characteristics of FLIPS, SLIDES and TURNS.

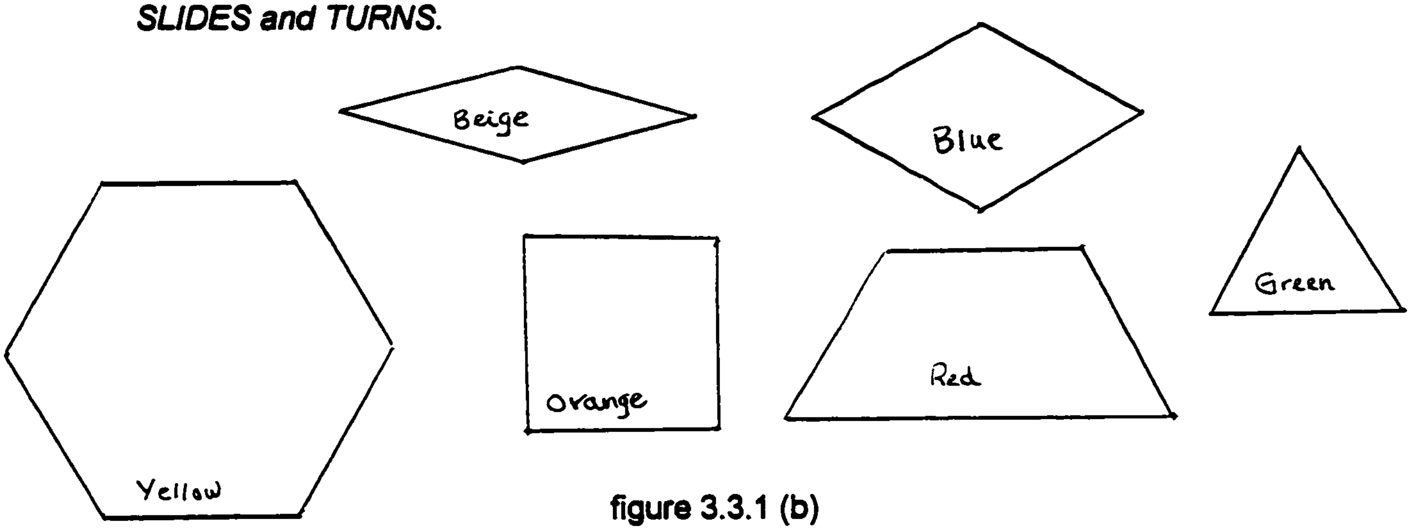

figure 3.3.1 (b)

ACTIVITY #3:

Development of Decimal skills and "*Moving Away from Home!*"

*For the purpose of developing your skills and knowledge in the operations of decimals we will be working on a project called "Moving Away from Home! " This is just a **pretend scenario.** You will be imagining the expectations and requirements of a young student who has just received their first job, and is setting up an apartment with one or more other friends of the same age. The complete details of the project will be introduced as the stages of moving out and setting up your own apartment unfold. Before you are finished you will have gone through the process of finding an apartment through the newspaper and determining the costs involved in maintaining it, including utilities etc. You will be given a fictitious job with a designated hourly wage from which there will be deductions of taxes, insurance, medical etc.*

Throughout this project you will be asked to keep a journal in which you will keep separate pages for a bank account, costs and expenditures for your apartment, details of your wages and pay cheques. In the journal you will keep track of all of your expenditures including both luxuries as well as necessities and make daily reflections of the project as you enter different stages.

*As the initial activity we will be doing a **field trip** to the local food market with a predetermined list of the required foods and other purchases needed to start living on your own. To this list you may add other foods etc. that you think will be necessary to set up the apartment, up to the total of $150. Your task, at the supermarket, is to comparative shop and look at various possible suppliers of the requested goods and also to look at how different SIZES of the product will give you the best use of your money. Remember, you are not really buying anything at the store but only "window shopping". Any mathematical operations or skills you use while at the supermarket should be noted on your page. NO CALCULATORS will be allowed for use during this field trip. As you fill your grocery list with the required purchases, estimations would be valuable, keeping in mind the $150 restriction.*

3.3.1.(c) Reflections on the Activities

After initially being allowed to reflect on their own personal experiences within the mathematics classroom, each of the participants was given the three pages of information modeling the reform classroom mathematics activities. They were to read them, and then to respond. Each participant began the first interview with a very open ended introduction and with an acceptance, or acknowledgement, that the perspective which they brought with them may have been grounded in their past experiences, and not in actual experiences with the reform classroom. It was accepted and understood that each of them had to create a picture of this kind of classroom in their mind, and to envision the environment and the discussion that might develop around each task. The attempt was made by the researcher to allow sufficient latitude to create an unimpeded and relaxed atmosphere. It was their perceptions that were of paramount importance, not their specific knowledge of any particular mathematical concepts. It was important for research purposes that they not feel that, at any time, that it was their mathematical content knowledge that was under investigation, or at any time being evaluated.

As they read the activities and reflected on the nature of a hypothetical classroom, the participants were asked specifically to address the areas of task discourse and environment. They were also encouraged to distinguish between the expected roles of the teacher and the students especially with regard to the anticipated discourse that might have occurred. Finally, they were encouraged to attempt to describe the classroom as if they were seeing a videotape or movie.

3.3.2. Interview #2

3.3.2.(a) Introduction to the Scenario

The second interview was arranged to follow as soon after as was convenient for the participants. This was within a month of the first interview for each of the participants. They were reminded of the three areas, which had been used for the first interview and which focused on discussions of task, discourse and environment. They were informed that this would continue for the second interview. It was important to review with them that what was of interest to this research were their perceptions of the reform to the teaching of mathematics. They were asked again to consider the students' roles, existing within the classroom as they investigated the problem, and how they envisaged the teacher role or the environment, similar to what they had done with the three activities in the first interview.

The participants were reminded that this interview would be structured around reading and relating to a story. They were informed that the story was written to represent a grade four class that was studying the nature of triangles as part of a geometry unit. The scenario, as it was called, was intended to model an investigative activity in mathematics representative of a reform mathematics classroom. They were invited to comment on any aspects of this classroom that caught their attention, whether these were positive or negative characteristics, and were again reassured that their personal mathematical knowledge was not being assessed.

3.3.2.(b) <u>the SCENARIO presented to the Participants</u>

Grade level: Four
Mathematics unit: Geometry
Mathematics concept: What numbers will constitute a triangle?

As the class begins the teacher is holding a plastic box filled with wooden rods of different lengths and different colors, and using the lid of the container he has arranged the rods in increasing lengths from shortest to longest. The class has never seen these materials before, and they give their attention to find out why the teacher has brought them into the class today. The students are used to working with different materials and know that they will have something to do with a mathematical activity.

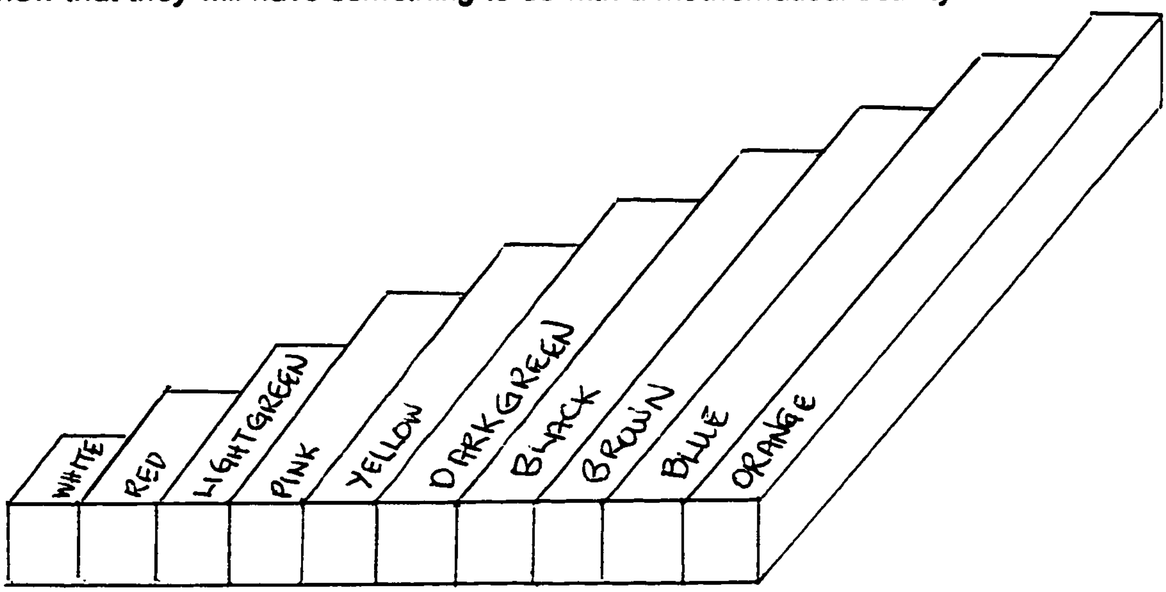

figure 3.3.2 (a)

Teacher: *Good morning, boys and girls. You already know that for the last week or so we have been studying geometric shapes. If I asked you to tell me in your own words what the term **triangle** means, What would you say? Yes, Katherine give us your definition.*

Katherine: *Well a triangle is a shape that has three sides and it can have three sides the same or just two sides the same or maybe none of the sides are the same.*

Teacher. *What do you think Katherine means when she says that two sides are the same or that none of the sides are the same? Yes, Jeff what do you think she is talking about?*

Jeff: *I think she is talking about the length of the sides , if one of the sides is longer or shorter than the others.*

Teacher. *I agree she is talking about how long the sides are and we even used some new words to tell the different types of triangles apart, but that is not as important for today's investigation. Here is the problem that I would like you to solve for me in partners. HOW CAN I TELL IF THREE NUMBERS WILL MAKE A TRIANGLE OR NOT? You will need a set of Cuisenaire rods like this set I am holding up to help you with your investigation. As you look at the arrangement of the colors that I put on the lid you should be able to give some information about the colors. Do you see a pattern or something you would like to tell us about the a arrangement? Yes Josh, what do you see?*

Josh. *Each block is one longer than the one before it.*

Teacher *That's correct Josh, so if asked you about the red block what would you call it?*

Josh. *It's a two!*

Teacher. *And Ellen, if the red one is a two, how long is a pink ?*

Ellen. *It must be a five.*

Teacher. *That's correct and can anyone tell me what the black and the orange would represent?*

Alysha. *I think the black is nine long and the orange is ten long.*

Teacher *Does anybody have another idea? So you all agree with Alysha? That 's good. Now you may want to keep the arrangement of the rods the same as I have it (see figure 3..3.2) just for reference as you work on the problem. Just a few other comments before you get started.*

First : remember it is not the color of the rod that is important ***but the number that it represents.*** *Second: as you explore with the blocks you may need to keep a record of the different combinations which you tried and which ones did make a triangle and which ones did not. If you think you have found a pattern or an answer to the question would you and your partner please write it up in a sentence or two before you come up to see me. Okay off you go.*

Greg. *Can I work alone on the question?*

Teacher. *I'm sorry Greg but I don't have enough sets for individuals so would you please find a partner for today and I would really like to see you discussing your ideas with someone as you investigate the question. Does everybody have a partner and a set of materials? Yes, good.*

Laura. *Can we work outside in the hall?*

Teacher. *Class you may work in middle room, on the floor, on the desks or here on the carpet just as long as you have enough space and room but I would prefer you not wander out into the hall. Make sure you each have a workbook to record your information and a pencil or pen to write with. Okay off you go, you can start.*

The class becomes a hive of activity with students finding their teams, their space and the settling down into the problem. The teacher now looks over the group to insure that all students are organized for working on the task. Walking around and listening to the language of the students, the teacher is looking for confusion about the problem or conflict within groups over use of the materials. As neither seems to be apparent, the teacher continues to circle the groups listening to the student discussions and awaiting the first team to arrive at a conclusion. Jeff and his partner Ian are the first to come running up to the teacher.

Jeff. *We think we know the answer to the question*

Teacher. *Did you write up your solution to the problem in a few sentences?*

Jeff. *NO, can't we just tell you?*

Teacher. *Well, I would prefer you each to write in your own books what you think the answer is and then to trade books and read each others explanation first. Then you may come up and share it with me when you have agreed with the best way to say it.*

Off they go and another team comes running up to declare that if two of the colors are same then it makes a triangle. The teacher then asks them:

Teacher. *So if you have two blacks and an orange what numbers does that give you?*

Students. *Two 9's and a 10?*

Teacher. *And that will make a triangle every time if you have two numbers the same?*

Students (together). *Yes we think so.*

Teacher. *What if I had two reds and an orange? What numbers will that be, have you got those in your chart of the different ways you have tried?*

They answer "No!" and when asked to build a triangle with the blocks from the teacher's set they come across a conflict so they leave to work some more. The class continues with many pairs bringing up their ideas and the teacher asking them if the counter example he chooses will work with their rule. Most of the teams find a flaw with their rule and the teacher example and go back to work to refine their rules. Part way into the investigation the teacher interjects the class to ask a question from the whole group with a response by one of the teams.

Teacher. *AND STOP! Just a quick question group. Nancy and her partner Jill have decided that the rule is ALL THREE SIDES MUST BE THE SAME COLOR will always make a triangle? Does anyone want to respond to their rule?*

Rick. *Well that's true but not all triangles will have all three sides the same length so that rule only works some of the time.*

Teacher. *Excellent, Rick. So we have to look for other information. One of the other teams thought that maybe if two of the sides were equal then you would have a triangle. Does that work all of the time? If it doesn't, when will it work and when won't it work? Can you find a pattern?*

With this new question to add to the original, the students return to their individual discussions and their work. Heidi and Laura come up to check their rule.

Laura. *We think that you can make a triangle with two sides the same, like two oranges and a brown.*

Teacher. *You mean with two 10's and an 8. But what if I gave you two 2's and a 7?*

The two girls take out two red blocks and a black block and find that the triangle can not be made.

Teacher. *You need to check all of the different possibilities and look for the pattern to see if you can find a rule.*

Rosh and his partner come up very excited and say they are sure they know the rule.

Rosh. *We think we have figured out the rule.*

Teacher. *Don't tell me your rule yet. I have three numbers in my head, can you tell me if they will make a triangle? The numbers are 7, 12 and 18.*

Rosh. *Yes they will make a triangle.*

The teacher turns to Rosh's partner and asks if he agrees with Rosh and he says yes he does, so the teacher then asks them to try three new numbers, 12, 15 and 30. Both students say "NO" to that group of numbers and then the teachers asks them to explain why they said yes to the first group and no to the second group.

Rosh. *We decided that if the two smaller numbers don't add up to more than the third number then the three numbers will not make a triangle.*

Teacher. *But what if there are not two smaller numbers? What if only one of the numbers is smaller and the other two numbers are bigger and equal? For example what if I gave you the numbers 10, 15 and 15 would you say yes or no?*

Both students quickly respond "YES" and so the teacher changes the numbers to 10, 100 and 100. The partners look at each other for a few seconds and then are not sure. The teacher asks them to consider the different cases and decide if their rule is still working. If so, they are asked to write up their rule and check it with some other groups to see if it makes sense, but only if the other group has decided they have found the rule in their group.

The pairs of students continue to check with the teacher to see if they have successfully found the rule and in turn they are given the test numbers similar to the first group and a request to formally write their idea in language that their classmates will understand. The students are challenged with numbers which may not be possible with the Cuisenaire Rods they have as materials and they are always asked to justify their "yes" or "no" responses. When the majority of the class has been able to find the rule except for one pair, the teacher calls the class together on the carpet to discuss the conclusions reached by the different teams.

Teacher. *Who would like to share their idea about this problem with the class?*

Katherine. *We know the rule. If you want to make a triangle all you have to do is add the smallest number to the one of the other numbers and if the total is more then it will make a triangle.*

Teacher. *What if one of the numbers is not smaller but all three numbers are equal?*

Alysha. *That will always make a triangle if the three numbers are equal!*

Teacher. *That's true. Can anyone remember the name we gave to that kind of triangle?*

Mike. *That's an equal something triangle.*

Terry *He means equalateral.*

Teacher. *Not bad, actually it is equi lateral. And how will I know if two numbers are equal whether its a triangle and what would we call it then?*

Erin. *It's the same as long as two of the numbers add up to more that the other number. Then it is called an isosceles.*

Jeff. *I think that if you are going to check you should add the numbers two at a time and check the different combinations and then you know for sure. As long as they are always bigger then it is a triangle.*

Teacher. *I think that Jeff has hit the bull's eye with that statement. Then I don't have to worry at all, I have* ***triple checked****. Well group, I think we have solved that problem, so now you have a few minutes left, you can test each other with questions that you make up with three different numbers and take time to discuss the answers or any conflicts with your ideas.*

3.3.2.(c) Participants' Procedure with the Scenario

The participants were requested to discuss, in as much detail as they felt the scenario generated, how the story gave them information about the nature of the students' roles and the teacher's roles in this investigation. The process required that they make direct references to the nature of task, discourse and environment. They had the opportunity of discussing each of the three areas as the reading proceeded, to stop at any point for reflection time, or to have a conversation about what they were reading. In comparison to their reflections on the activities in the first interview, their reflections on the scenario immediately presented an intensity in terms of the responses which they made. This was especially true with respect to the area of discourse of the students. This resulted in clearer and more expansive reflections than those resulting from the activity pages, but there was still consistency in their comments. When they felt they had sufficiently described their perceptions of the activities within a reform classroom, they were allowed to add personal statements about the nature of their involvement in this research. From their reactions to the information that had been used to reflect a reform classroom, they indicated their appreciation for the opportunity to be predictive, and/or to generalize, about the impact of this style of teaching.

3.4 Analysis of the Data

Analysis began by looking for significant patterns, and potentially unique characteristics, which emerged from the participants' discussions. It was their perceptions of the characteristics of classroom, modeling the reform movement in mathematics education, which initiated this research. Therefore, to maintain consistency with the initial intent, the lens for the analysis continued to be placed on the manner in which the participants perceived three particular classroom characteristics (whether being represented in the activities or the scenario). The three areas emphasized the nature of the task (or activity within the classroom), the nature to the discourse (from either the position of the teacher or the student), and finally the nature of the environment in which the activity takes place, and the discourse follows. As expressed earlier, these three facets of the classroom were addressed by the research community as being essential to any potential changes to the teaching of mathematics. They are reflected in the Standards from the N.C.T. M. (Appendix B, p.136).

As both mathematics teacher and researcher, I have to admit that it was difficult to separate my personal experiences with the reform of mathematics teaching, and my personal interpretations of it, from my analysis of the data. Consequently my personal teaching philosophy had an influence on how I read and analyzed the transcripts. The first perusal of the transcripts of the participants' discussions was to distinguish conversations that could be separated into the categories of task, discourse and environment. In an attempt to identify a specific context from which to form a basis for analysis, and regarding the expectations of the reform classroom, it became useful to use what the participants focused on. This tended to be what they found as being "meaningful". The first basis for coding the data from the transcripts was around this quality, and my own knowledge of the characteristics outlined in the Standards. Due to the positive nature of their reactions to the characteristics being displayed in reform classroom, it seemed appropriate to use 'how it was meaningful' as an indicator of their perceptions. In comparison to their traditional classroom experiences, this very positive reaction to the reform classroom emerged as the most applicable basis on which to pursue the analysis.

After designated sections of the transcripts were highlighted as being indicative of being important, these sections were then analyzed to attempt to separate the participants' discussions around the three areas of task, discourse and environment. The objective was an attempt to determine what there was about each sections of transcript that could be identified as being most specific to each of the three categories. In other words, what would be considered meaningful about task that could be different from what would be meaningful about discourse or environment. The selected portions of transcripts were then collected and sorted into subsections with data most relevant to each of the three areas.

Within each subsection, the analysis continued by looking for common themes that could be used to generate categories for classification of unique characteristics. The number of relevant categories was allowed to emerge from the data. There was no preconceived number of categories applied to each of the three topics of task, discourse or environment, but it was considered necessary to separate the discourse of the teacher from that of the students. Each of the resulting categories was then given an appropriate term, or suitable phrase, to capture the essence of the features being portrayed by the chosen portions of transcript. Each category was defined to give the researcher's perspective of how or why the participants found it meaningful.

Having personal knowledge of the characteristics that are assigned by N.C.T.M. to each of the three areas, my objective was to create categories which might be identifiable by other adults as being unique to the teaching of mathematics, especially from within a reform context. These characteristics would most likely not have been observable in classrooms in which they had learned mathematics, and probably not been experienced by the majority of adults. The value that the participants might give to such interpretation is open to debate or discussion as this research project did not return to them for their reactions. This offers a different opportunity which could have been added to the present study and may have increased the depth of analysis possible.

As will be seen in the next chapter, with the presentation of text from the transcribed interviews, each characteristic was not always completely distinct in the participants' responses. They began to overlap and became interdependent. As a result of this, text chosen to suit one category could also be seen to be appropriate in other categories and should be read with this consideration. The personal interpretation applied was one that looked for uniquely distinctive features. These reflect how the participants may have liked "to have seen the activity" in the reform classroom to have been different for them from that which they had experienced in the traditional classroom.

3.5 Limitations of the Study

The opportunity to design a research project in which the emphasis was placed on human participants, and their perceptions regarding the question, has the potential to raise a number of concerns with regard to the methodology. A few of these concerns are presented below to acknowledge the possibility for alternative approaches, or variations in strategy for any further research.

Although the participants showed great enthusiasm and interest in their roles in the research of this topic, it has to be recognized that the nature of the characteristics of a qualitative study may lead to questions. There are the usual concerns in terms of guaranteeing that the participants' perceptions were not influenced by factors not directly related to this study. The research was carried out with an attempt to control outside factors as much as possible but it is recognized that this may not have been entirely achievable.

A second area which emerged, once the analysis began, was with respect to the written statements called the 'activities'. With the scenario, the participants had access to a clearer picture of how the areas of task, discourse and environment would all come together within the context of an actual classroom. This gave them a better opportunity to produce a "video" style representation in their minds than did the activities alone. As this occurred in the second interview with the scenario, it raised a reflective question on the part of the researcher. In the initial interview, should the participants have had an opportunity to personally attempt the activities with the materials and the question under investigation? From a research perspective it raised the concern as to the depth of their understanding of the activities if given only cursory access to the concept. It became immediately obvious that their ability to respond was heightened in the 'scenario' because they had the whole picture of the task, discourse and environment outlined for them. It therefore raised the possibility that deeper meaningful discussion might have followed from actual experience with the reform activities. For future research, what would be of intrigue would be an interest in the adult perception of actual participation in learning mathematics from this alternate teaching strategy.

One other possible concern with this research was the decision not to attempt to introduce gender, cultural, or minority issues into this study. This decision was considered to be important to allow for perceptions that would hopefully not be clouded by other issues not being addressed in this study. Gender issues, or cultural and racial ones, are not being dismissed as being non-existent, or insignificant, but are left to other researchers interested specifically in the impact of the reform of mathematics on these groups.

The last area that potentially raises questions within this research is with respect to the particular participants who volunteered, and were eventually selected for this study. It was their personal history in the mathematics classroom, around which they had varying degrees of success but consistently strong negatives attitudes, which could beg the question. What if strong and successful candidates had been chosen with positive attitudes towards the subject of mathematics? It was researcher's opinion that students, who had not had negative affective reactions to the mathematical experience, would not be representative of the largest portion of the population. From contact with a significant number of possible candidates, who expressed interest in this research, it seemed overwhelmingly the math anxious people who exhibited the greatest interest in the reform to mathematics teaching. It is also proposed as very probable that capable and successful mathematics students would benefit just as much, from the expectations of the reform, as would those who left the mathematics classroom with mostly negative emotive reactions. It was considered more relevant to the research, not how much mathematics students had learned, but how they had learned it. With this interest, and the resulting impact it had on their interpretations of the reform movement, those with a dislike of mathematics were seen to be representative of a much larger segment of the population. In the past, this segment was more indicative of the masses of students who had simply endured the traditional mathematics classroom.

CHAPTER FOUR

RESULTS

Perceptions of the Participants

4.1 Introduction

For two of the three participants, and their statements regarding the characteristics of the reform classroom, it must be accepted that their views reflected their personal opinions not from actual experience, but from their expectations of what might be associated with the changes to the traditional classroom. For the third participant, actual classroom experiences were reflected in many of her comments. It must also be acknowledged that underlying all of the participants' responses were the emotional histories which they associated with the learning of mathematics, and which were depicted in negative attitudes the participants developed towards the subject of mathematics.

The personal histories of each adult, presented previously, are recognized as being strong motivational factors in the interpretations which the participants formulated, or synthesized, from their exposure to the activities chosen to model the reform classroom. The psychological reactions to life in the traditional classroom probably generated interpretations of the reform classroom based more on emotional considerations than ones based on conceptual understanding. Thus, particular skill, knowledge or expertise in mathematics was not of interest in this study. How each participant perceived the task, discourse or environment as being meaningful, within the context of a classroom modeling reform ideas, was the lens for the analysis of the data. How their comments reflected this meaningfulness was the basis of the researcher's interpretation of their perceptions. As a consequence, the results of this study are presented in terms of what the participants considered to be meaningful about the reform classroom in relation to the three focus areas of task, discourse, and environment.

Analysis of the participants' perceptions of the areas of task, discourse and environment provided the opportunity of creating categories which seemed not to be indicative of categories representing traditional mathematical activity or classrooms. What emerged immediately from the participants' perspectives were characteristics which appeared to be more representative of: (1) positive emotive reactions to task, (2) significant changes to the roles in discourse of both teacher or students, and (3) major variations from the classic mathematical classroom environment.

4.2 Participants' Perception of Task

Regarding the nature of task, it seemed most appropriate to define meaningfulness from the participants' opinions in terms of six areas. Initially their opinions would best see the view of reform activities as: (a) being fun or recreational, (b) having a multiple of representative modes, (c) having real life or worldly application, (d) being able to maintain intellectual interest, and finally (e) having the potential for personal fulfillment or satisfaction. An unexpected category relating to task appeared in terms of the creativeness of the teacher. This designation came as a result of the participants' recognition of the expectations from the teacher for finding, or inventing, the types of tasks necessary for an investigative classroom.

4.2(a) Recreational Nature

The participants' reaction to tasks reflected that they valued the recreational nature of tasks, i.e. the removal of qualities usually associated with them being regimented, mundane or habitual, and the assignment of qualities more akin to relaxation, pleasure, amusement, entertainment and even challenge. This made the tasks seem to be interesting, non-threatening, captivating and playful. This was manifested in comments like:

> *... Actually, I really liked them all (the activities) because they're all, they're not doing,-- they're taking away from the usual regime of lectures and working out of textbooks -- is by far an advantage, no matter what it is. If it gets involved in group work, it gets students maybe working together and understanding the function of shapes or the function of numbers, I think it's beneficial where it's just more of an understanding than just trying to absorb lectures or absorb notes. (M.)*

> *... because (the task) it's interesting, its fun, there's physical movement involved, it's visual, its colorful its everything that to me math isn't as a traditional experience. (B.)*

> *... I'd like to work with a partner but to me on this one, it wouldn't make a difference . I could try this one on my own ... it's not threatening but very interesting. It could be challenging because I haven't tried it yet ... but it would be fun because you know that there is a possibility of coming up with something. You should try and play around with these things (P.)*

> *I think (this kind of task based) math is,-- would be fun and I'm finding that I'm enjoying myself , I'm not feeling intimidated. I like what I'm doing and I'm learning at the same time. (P.)*

4.2(b) Presentation Mode

The participants considered the variety of modes of presenting mathematics represented by the tasks to be very useful in enhancing the learning of mathematics. They saw these modes in a variety of ways, i.e., concrete, "scholastic", "idea", abstract and visual. But they emphasized the concrete mode as the following indicates:

> *There are all kinds of activities but they are based on concrete before we can sort of move to the abstract. (P.)*

> *Everyone is experienced with shape ... a very good point in terms of how this mathematical skill is being learned, is that it moves from concrete, into the scholastic, into the idea. (B.)*

> *I'm a very tactile person, I need those pieces ... because I can move them around. I can manipulate them, I can see if it's going to fit the criteria that I've been asked. It's the way I learn, I'm not a visual learner, but I'm visual in the sense that I have to use some things, hands on kinds of things, but to look just at the paper would still be intimidating. I'd still be doing the drawing. (P.)*

> *... and again I think that the whole idea of using colors to represent numbers is very interesting. I envision that when you are going to be tested on this you could "see it" as opposed to just learning rules (B.)*

> *We got to look at it (a study of trigonometric graphs) on a computer which was very interesting because it's very fast and an efficient way of doing it. You could just delete a number, replace a number, anything. You could see immediately what the new shape would be without having to sketch it out and you knew what was sketched out and you could lay patterns on top of each other there on the computer graph paper, you could actually see where it was different shapes that are in relation to the equation. (M.)*

> *Well (the students) they are excited, they -- it's neat. They can see that there are possibilities. They get to work with materials and that is really a lot less threatening than just sitting working on a paper and pencil activity. So you know that they have got something that they get to manipulate and they have someone to work with. That makes it a lot less threatening. (P.)*

> *... but I find this whole idea of looking at mathematics visually, hugely helpful, as opposed to being something that basically involves numbers on the blackboard and letters, which were meaningless to me. This is a yellow hexagon and it's a green triangle and so on, which is infinitely more interesting and more accessible. (B.)*

Thus, they saw comfort in the concrete mode with actual physical materials such as manipulatives, models, calculators or computers to assist in their learning.

4.2(c) Worldliness

The participants greatly appreciated the worldliness or real-life component of tasks. They saw this component in terms of connections they could make to their own real-life experiences and to situations in the world in general. This gave them a perspective of mathematics that was more meaningful in many ways but specifically in comments like:

> *I thought it made perfect sense. The only thing that was kind of – that made me question was like would this be done in a mathematical situation. It seemed like more of a maybe creative like image to me ... It's like new ways to look at math and developing math skills and just – you can apply it to real life which is rarely taught in schools let alone in math. (M.)*

> *Wow! Well it's definitely real world ... One of the things I see in here is, I mean I am actively learning, what it's going to be like out in the real world and I have to figure out how much I need to spend on all of these things and keeping in mind that I'm only making this amount of money. (P.)*

> *Well it takes mathematics right out of the classroom ... Definitely, I think that it is an exciting possibility. (B.)*

4.2(d) Intellectual Nature

From the perspective of the participants, the tasks had an intellectual nature that added to their meaningfulness. They perceived this intellectual nature in terms of the level of mental activity required to complete the tasks. The level of mental activity was stimulating in that it provided challenge, competition and beneficial struggle. As they stated, this should introduce different levels of thinking that stretch beyond simple recall and memorization. They saw this as the opportunity for the students to be involved in discerning, perceptive and insightful activities, from which mathematical knowledge could result.

> *There is color and shape involved, which I think is hugely interesting and secondly, I see it as being quite competitive. Be it like doing, what were they called, Rubic's cubes, you know a few years ago, that kind of mental activity I think is hugely appealing to children. (B.)*

> *... lots of preliminary kinds of activities that get them thinking along the lines of problem solving, reasoning and making those connections ... this kind of thinking is critical thinking ... Well because they start thinking along these lines -- they know that it's not just an answer that I am looking for but all those things, all the things that they were doing to get to the answer, right? (P.)*

> *It was kind of stimulating that they could come up with their own ideas ... further more the students were given the opportunity to investigate it among themselves, or in groups, and sort of come up with their own solutions to the problems ... You are kind of, in a way, making it up for yourself and figuring out the problem yourself and doing everything physically ... so its more like self-teaching, kind of idea, so I think that's beneficial for understanding. (M.)*

> *Definitely, sound and significant mathematics for sure because they have done the leg work themselves, and I think that they would be very excited ... um it tests knowledge, interests and experiences because their experiences needn't be vast but everyone has experiences. (B.)*

4.2(e) Fulfillment Potential

The participants perceived a fulfillment potential in the task that made them meaningful in terms of providing encouragement for students to participate in them, to achieve satisfaction from their efforts in completing them, and eventually to see accomplishment and the benefits of their efforts. They saw this quality of task as providing latitude and freedom for students to explore while taking responsibility for their learning. They also saw this in terms of the pleasure and satisfaction that was possible from an "AH! HA!" experience. Where students could be envisioned doing the tasks, the following represents some of the ways in which the fulfillment potential was expressed.

> *... (these activities are the) kinds of tasks that require some playing around with and some time to juggle and so on, but it would be very satisfying if it could be accomplished without teacher assistance. (B.)*
>
> *It's math but it's math in a fun way... instead of sitting ... I'm actually going to participate and doing things is going to help me in my learning ... I can begin to think about things and put things together.... I'm now becoming more active... I'm taking responsibility for coming up with the process here, how I'm going to put this together. (P.)*
>
> *Well I think that it's very interesting to see that sudden leap in logic and coming to the conclusion that you have to have sides that add up to a certain amount in order to construct a triangle. Because I have to admit, initially I thought "Where is all of this leading?" and then suddenly it clicks in and it just makes sense which was really exciting. (B.)*

4.2(f) Creativeness of the Teacher

For the tasks to be meaningful to the student, the participants' perceptions noted a creativeness quality from the teachers. They commented on the implications of the nature of the inventiveness associated with the tasks which the students see in class, and on the necessity of the teachers being resourceful, ingenious, imaginative and enterprising in determining which tasks to use. As they stated, the creative or inventive nature of the kinds of activity, associated with reform tasks, require a divergent thinker with time to develop novel and intriguing investigations which will excite and maintain student interest. Thus from the perceptions of the participants, support for a reform teaching strategy will make these kinds of demands on teachers. Their comments in terms of time and originality to create them, or in terms of finding acceptable resources which will contain them, are reflected in comments like:

> *... well probably some creativity and maybe some motivation to maybe want to help kids learn in a more beneficial way... but if the teachers really enjoy their job and they like what they are doing then I think this sort of creative thinking wouldn't be so hard, if they were really driven to want to teach their students this way. (M.)*

> *That I already visualize in my head as being a bit of a problem. It is going to take enormous inventiveness on the part of the teacher and I would say a huge amount of out of classroom preparation, and of thought into materials, even to the level of being an inventor... I think you are going to have to add new dimensions in teaching teachers how others learn math. (B.)*

> *Well, because our role (the teacher's) has changed and our role is more -- our role used to be easier. We used to be able to assign questions, they'd work on it, if they didn't get it, I would say just do this ... and then we would move on. Now, what happens is that I have to do a lot more planning. I have to be a lot more organized. Before we'd just use the textbook and we would assume the textbook taught everything that we needed to know. (P.)*

> *But I think the biggest thing for teachers -- is when there is already something out there, and it's sound, and it's good , you want to use it. But to find the time to make these up is not always convenient ... and to think about what material is really going to do a good job... the biggest drawback is finding really good quality kinds of activities. (P.)*

4.3 Participants' Perception of Discourse

Where for the most part discourse in the traditional setting was limited to the teacher, great interest on the part of the participants was created when given the opportunity to discuss discourse in a reform setting. They interpreted this as having distinctly different roles for both teacher and students. They saw the historical roles of teacher and student as having been cast aside. In a classroom in which open investigation and intuitive experiences allowed for significant social interaction to develop, active student communication dominated the class time instead of more submissive participation. The ensuing social interaction more obviously reflected a constructivist stance when attempting to determine the novel roles the classroom members would be assigned.

The teacher's role was seen by the participants to be structured around knowing when to intercede, or participate in the students' conversation, and when to give latitude or freedom. For the purpose of this research this will be called 'hands off' and 'hands on'. The other significant role for the teacher came with the expectation from the participants that the teacher provide closure to the activity. They saw this as a necessity for the teacher to orchestrate the summation of the activity under investigation. With the teacher providing an invitation into active discourse in the classroom, the participants saw the students as (1) having significant opportunity for introduction of their ideas, (2) being involved in multiple forms of presentation of their ideas, and (3) being expected or requested to validate these ideas and theories in small, or large, group discussions. For the purposes of the analysis these will be classified as a) Initiation of student ideas, b) Presentation of student ideas and c) Verification of student ideas.

4.3.1 <u>Teacher's Role</u>

Three situations were identified in which the participants considered discourse in terms of the teacher's behaviour to be meaningful: "hands off", "hands on", and closure.

4.3.1(a) "Hands off"

The participants considered a hands-off approach by the teacher to be meaningful in terms of not intervening with the students at specific times when the students were actively working on tasks. They noted the necessity for the teacher to maintain a physical distance, and a state of silence, while students were investigating and exploring mathematical activities. This was to allow autonomy and latitude for the students to develop ideas, and an acknowledgment of the need for student independence and time to investigate the task. Thus they perceived the teacher to be a passive observer 'stepping aside' when necessary and assuming the status of a listener.

> *The teacher had left it to the students on their own to investigate the problem themselves without actually confining them to a set way of thinking, or ways of thinking that they might get caught up in ...*
> *The teacher's role seems to be very passive, and not talking, but just kind of watching and seeing what is going on in the classroom, (M.)*

> *The teacher perhaps touring around and looking, without saying very much, because I think this is one of those kinds of tasks that would require some playing with, and time to juggle and so on ... I would say because this would take a lot of getting used to and in a sense somebody without a large ego, because you are not going to be the teacher at the front of the room lecturing and being the focus anymore. He or she is kind of stepping aside and allowing learning to take place, one on one through the children. (B.)*

I think that it's definitely good to allow struggle, ... it provides strength for the individuals.. If there was no opportunity for struggle with a problem, maybe a mathematical problem such as this one, they would never learn it properly, it would be sort of memorization sort of , like a lot of schools are doing now. (M.)

Well it's allowing them to think, she's -- the questions she is posing are forcing them to think for themselves rather than the, you know, well this is the right answer or this it the way you should be doing it ... Well the number one thing you have to know how to question kids and you really need to know the activity, you need to know what it is you want to get them to learn and you have to decide, I guess, on the materials that you are going to use (P.)

Exactly! As long as the teacher did a really thorough job of explaining this (the task) in the first place. He or she should be able to back off ... ... (there is) not necessarily a teacher space. Perhaps a spot to be while the kids are working. I almost see something like that --, I know this sounds ridiculous perhaps, but almost a bar stool or high chair in the corner where he or she (the teacher) is up, a little elevated, and can see what's going on. An observer! (B.)

4.3.1(b) "Hands on"

In contrast to the hands-off situation described above, the meaningful qualities of discourse, in terms of the teachers' behaviours, also included a hands-on situation in which the teacher should intervene. The participants considered it necessary for the teacher to become an active member of various student groups in the classroom only in a mediation role. In this role he or she models, leads, questions, challenges, shares, and responds in interaction with the students. Thus, they perceived the teacher to be accessible in a more conversational or comfortable way. The role for the teacher becomes the facilitator, coordinator, motivator, supporter, questioner or leader; instead of presented an intensity in terms of the responses which they made. This was someone simply transferring information. These attributes are reflected in the following comments regarding the teacher's active role in discourse:

> *... comfortable and very accessible ... More conversational than we would normally find in a math classroom, less teacher directed and I think that it's accessible, in terms of the way that it's worded, in any rate ... The fact that there is room for discussion I think is very interesting. (B.)*
>
> *... probably the teacher's communication with the students and um ... it seems the teacher was very polite ... it was very supportive speaking that the teacher was doing, It was very clear and didn't provide a lot of complications between the teacher and the students, which was really good. (M.)*
>
> *Again, its very interesting because I see a consistent pattern developing in all of these kinds of tasks ... the teacher is much more of a motivator and a coordinator than merely just an information giver ... asking questions, coaxing them, urging them on to dig deeper, to look more closely ... I think that the teacher would have to be a very strong role player here, going around from group to group, particularly at this age group, to make sure that the math language is being used in one way. (B.)*
>
> *The teacher was kind of supportive of the way, ah ... the way my thinking process worked, that they are trying to stimulate me to go back ... to keep trying until I actually came up with a solution ... (M.)*

4.3.1(c) Providing Closure

The final quality of discourse, which participants considered to be meaningful, was closure, i.e. the role in which the teacher assists in summation of ideas, culmination of the activity, and realization of success for the students. They noted that after sufficient time to carry out the investigation, it was necessary for the students to present their ideas for scrutiny, confirmation and verification by the class groups. They also noted the necessity for the teachers to assign activities or tasks that would allow for the natural arrival at a conclusion at some point in time (where closure seemed an expected occurrence). The following reflects their perceptions on how the teacher brought closure to the investigation.

> *.. they were actually able to come to concrete conclusions, but through their own way of pursuing it rather than being told and just being asked after to repeat it ... because this is going to stick with them better. (B.)*
>
> *Well the discovery thing for me would be, just as they were coming up with things and they were realizing that.... sort of that the light bulb would go on kind of thing. But the structure, the activities, the materials the way that she used them and the way ... what she was getting at, she knew where she wanted them to go. She knew where the lesson was heading. (P.)*
>
> *Um-- the way it think it was discussed in the end and the way that this teacher is able to stimulate the class and the students enough to come with the proper answer was really interesting ... and the discussion, like the wrap up and how it was stated, the teacher never really gave the answer to the problem, as it was stated in the beginning. The students actually came up with the answer... and the discussion it was more of a student discussion with the teacher stimulating the conversation. (M.)*
>
> *... but you know that there is closure at the end because the teacher is pulling it all together ... and you have come up with this ... and this is what we have found, ... so there is closure to it ... (P.)*

4.3.2 Students' Role

Students' roles in discourse were perceived by the participants to be meaningful in three unique situations: 1) initiating their ideas, 2) presenting their ideas and 3) validating their ideas.

4.3.2(a) Initiation of Students' Ideas

The participants' perception of the entry level of the student role in discourse was with respect to the initiation of students' ideas. They saw this in multiple of forms of social interaction including those of student to student or student to teacher. The participants saw the potential benefits of this in terms of student formulation of conjectures and personal theories about the questions being explored. This was reflected in comments like the following.

> *Well the fact that there is discourse for number one ... they feel very free about coming up and saying "OH, we have it!" and they are all really secure about it, feeling very confident, and then when she says "Well what about ...?" and they say, " Oh I never thought about that, well let's go try that". That leads me to believe that there is that sense of ... that they are allowed to take a risk, they are able to try and do it again, and it's not going to mean that they are right or wrong, but that there is something else more to it. So go investigate and see what you can come up with. (P.)*

> *... I think that group work would be a definite benefit, definitely beneficial because it would be coming from two people who are trying to learn and understand math instead of coming from someone who already knows it and they are trying to teach it... (M.)*

> *The fact that they are sharing in groups, partners, right? That they have someone else to bounce ideas off of. There is lots of interaction going on, that's for sure. (P.)*

> *... (the teacher) needs to be setting up situations that allow them to learn at the same time as she walks around or she is monitoring. There needs to be some communication between groups... so that we can see the kinds of learning that took place, this group did this and the kids need to see it. We need to see all the different possibilities because sometimes we don't realize there are other things out there we hadn't thought of ourselves. (P)*

4.3.2(b) Presentation of Students' Ideas

It was considered a meaningful by the participants for the students to have access to a variation in modes of presentation of their ideas. These could be in pictorial, symbolic or physical representations. The solutions conceived by students, or their developing theories, were considered by the participants to be a critical part of the role of students in discourse and these might have occurred in a number of ways as indicated by comments like:

> *... would it not be interesting to take it to a certain point, just say stop and then slide that sheet of paper onto the next person and you get a new set, a new sheet. Yes ... they would swap at some point (B.)*

> *... I'd probably be having a presentation situation where one student could be presenting to the class or maybe a group of students could be presenting it to each other... some sort of presentation involvement ... It would be the students responding or just with demonstrating an answer maybe that they had found and the teacher just putting in on the board or putting it so that all the students would be able to see the answer and maybe work off that answer (M.)*

> *Yes, but they would really be assigned this (the activity) to do and it would be enormously entertaining and they would be chatting about it, well we could do this, we could do that and why don't we try this or whatever. Far more independence established than in a traditional setting. (B.)*

> *We talk about cooperative learning, that's all part of it, right, you know the give and take, the compromise, the assisting one another... its not always going to be mathematics, but what's coming out of it is mathematics, because that's what they have been working on, the task. I can't get over it, this is a great activity! (P.)*

4.3.2(c) Validation

The participants considered it meaningful for the students to have access to verification of their ideas and validation by others as being essential to student discourse, whether this was peers, teacher or both. The perception was that the students in discourse would have to substantiate, support, or verify their ideas as the class attempted to come to conclusions about the topic under investigation. This was expressed in comments like:

> *It would give them all sorts of ability to express themselves that would necessitate expression. It might cause some argument, but on the other hand, its almost competitive which I find really interesting ... it forces them to examine something that's really quite, oh how can I put it ... a very objective kind of assignment becomes more subjective because it's in the eye of the beholder. (B.)*

> *Well what I really like ... the way it was set up with them going and coming back and constantly re-affirming, getting questioned about whether or not it was correct, leading them to decide what the criteria was for making up a triangle. I thought it was excellent ... she always gave them something to make them think ... so it was constantly checking their rule, the one they thought was going to be the answer (P.)*

> *... The thing that caught my attention was the way that the students were approaching the teacher -- the teacher would go back and it seemed that in most cases they (the students) were questioned. The teacher would make a direct question challenging maybe the way they were thinking so that the students would have to go back and apply their theory to the problem the teacher was giving them. It was very interesting, the problem the teacher was giving them usually didn't work or it challenged the way that they were thinking ... (M.)*

> *You know what I see as a problem though, is it's (active student participation in discourse) not always continued. Sometimes I feel like I do it (open investigation) in my elementary grades and then I even look at division II and I don't see it happening ... I see the impact but I don't necessarily think that it's being continued and promoted in other grades (P.)*

4.4. Participants' Perception of the Environment

If the nature of task created the opportunity for discourse, then environment located that task in a physical space and within an emotional context. Analysis of the participants' responses regarding the nature of environment led to other new and novel qualities associated with the classroom. Designated areas and locations of either teacher space, or student spaces, were perceived by the participants to no longer be static or predetermined. Neither would the teacher's access to the available time within a particular class period with more time being made available to the students. In addition, the physical needs of the classroom, in terms of equipment, furniture and accessories, became one of the topics to be addressed in meeting the needs of the reform environment. As a consequence, the atmosphere necessary for the kind of collaborative work, implied with these tasks, also became a quality of environment that emerged from the participants' perceptions.

Thus, there were two ways in which the participants perceived the reform environment to be meaningful: (a) the actual physical sites and (b) freedom or personal control of the student. While the first was seen to address the physical needs of the student, the second was seen to cater to the emotional needs of the student.

4.4(a) Physical Sites

The participants' responses, to the question regarding the environment, drew immediate attention to the various locations in which the mathematics activity could take place. They saw meaningfulness in the bright, colorful and inviting classrooms where there was flexibility in the arrangements of students' desks, and in availability of concrete materials or manipulatives to assist in the development of meaning and understanding. The participants expressed great interest in creating environments that supplied the necessary "props" for the teaching and learning of mathematics. These would have to include more than just the usual desks, books and blackboards and could even require relocation of the sites away from the school itself.

> *I've just envisioned a nice bright classroom with small interesting groupings, lots of room ... I think I'd do desk clusters. I would want an arrangement where the desks can be moved easily into different groupings or basically scratched if I wanted, as for the second task, ... tons and tons of room on the floor ... I think that visuals can make a subject very exciting ... I think that visuals are very important... colors, shapes etc. Yeah, a constant reminder that math is interesting and multi-dimensional... it's an activity more than a math problem, it's an activity and I think that's the way the environment should be created (B.)*

> *... the classroom would be set up, everybody would move their desks together so there would be groups or 2 or 3 desks of students facing each other to work on the task, or side by side ... it seems (the teacher is) more somebody there to answer questions or help but not really giving any input to the students as they were walking by ... the teacher would probably be either up at the front doing something else, marking papers or something, or maybe walking around and looking at the students, observing the students. (M.)*

> *.. there would be kids everywhere. She (the teacher) said some people could be at desks, there would be some people on the floor, they could be out in the hall ... just not wandering out in the hall ... Just working in groups with their partners and the teacher wandering around ... seeing where they are at ... They are all excited , it's neat! (P.)*

> *... I would almost see it as desk free setting, even outside. I think this would be a really fun thing to do on a concrete area, a playground or whatever. (B.)*

> *... the only thing I , am I going to have these actual pieces or do I have to look at the pictures? I am a very tactile person, I need those pieces....*

because I can move them around, I can manipulate them, I can see if it is going to fit the criteria you've asked me ... seeing if it fits the kinds of things that you have asked me to do. (P.)

Bright, colorful, diverse, expensive ... just because I think that visuals can make a subject very exciting ... a constant reminder to them that math is interesting and multi-dimensional ... would it ever happen that for example, from the standpoint that the math supplies that they (the students) need ... that you would ask them to purchase them , that each would have to buy their own? (B.)

One opportunity which surprised the participants, but which became very meaningful to them, was the consideration of taking the class away from the school entirely on a field trip to the supermarket. The relocation of mathematics instruction outside of the classroom gave them the same enthusiasm it might have for other subjects, in which field trips were considered necessary or more usual for those programs.

The field trip, now that's a very interesting experience ... I think that I would' see the same kind of excitement if you are sharing this with someone... at Superstore, we could get this and at Coop we could get that and so on. I think again that it makes it really challenging... ... it takes mathematics right out of the classroom which I think is an exciting possibility. (B.)

... I think it (the field trip) would be better to go as a class as opposed to going on my own and I think what's really neat is that, you know even as an adult, looking at all the different sizes of containers and prices and seeing what is your best buy. You can't do it unless you are at the store ... So it was one of the things that I missed in math , seeing it in real life, like this, bringing it out. (P.)

There are prices on everything everywhere you look and just being able to maybe analyze costs and that would be actually ... I think an excellent way to be able to do estimations. (M.)

The participants also saw a meaningful relocation of mathematics to the home environment with parents being encouraged to take an active role in the student's learning. They expected that this might require an introduction to the parents of the variation in the approaches of the mathematics classroom reflecting a reform philosophy.

> *I think that would be interesting - I think it would be really interesting to take some of these things home and introduce it to a family and say, "Okay, Mom and Dad and cousin so and so, we have to build the shape tonight and we can all be involved." ... I think that they would probably need an introductory session to expose them to what the real advantages of this kind of learning would be. (B.)*

> *You know what I would do, I would really strongly recommend work-shopping of parents, of having them in and having them watching it take place. Actually I would have an evening and have the parents doing some of this (kind of mathematics teaching) and see what would come of it,... because wisdom comes through experience. (B.)*

> *... I started sending problems home, I thought "Oh yes, I can just hear them now. We don't know how to get the answers to these, how do expect the students to get the answers." so I had the parents come and we, -- I worked, I showed them what some of the manipulatives where. I explained what we do with them, why they were important ... But I have parents coming into my classroom a lot and they get to see what's happening on their own and they're very positive about what's happening ... (P.)*

4.4 (b) Freedom and Personal Control

To best support the expected learning outcomes, the participants indicated a need for students to have opportunities for personal control over their time in the classroom, and for the students to have freedom to work in a risk taking environment. To accomplish this the participants saw the teacher's role as pivotal.

You (the teacher) need quite a bit of leadership to organize and build the structure and create the kind of atmosphere where struggle would not be disabling you, but be more beneficial. (M.)

... the teacher still has to create the excitement and the enthusiasm. Part of good teaching is allowing independent learning ... little things that came up that were dealing not only with the mathematical learning but also with the social learning ... that was balanced with the task oriented business at hand (B.)

... I think that learning something as formalized as math could be a lot more interesting and a lot less threatening when you are learning in a group and when you are sharing your language than if you are learning from a teacher ... But it so saying, I don't think there should be options for both at the same time. I think that it is really good that if one has to learn to work in a group or if one has to learn to work independently. There should be a mix of that in math so that you learn to trust your own judgment and not always rely on the group to base your ideas on. (B.)

Well, ... it's very interactive. There are times when they have to do independent work kinds of activities but for the most part, it's in small groups or with a partner. But, I think independent work is important too, because then it, you know, you have to force yourself to think and sometimes when we're with a group or a partner, we have --, we tend to let the others do the thinking for us so I still think it's important once you've done lots of activities, is to have an activity where you are required to think for yourself ... (P.)

Absolutely, yes because fundamentally, that's what you're doing but you're doing it (the mathematical activity) through more physical means, well not more physical, but a mix of physical and mental means as opposed to just mental...
Because they are able to come to concrete conclusion, but through their own way of pursuing it, rather than being told and just being asked to repeat it after... that is going to stick with them better.
... (it requires a teacher) who is prepared to accord children a lot more independence and allow them to sort this through. This would take a lot of patience and in a sense somebody without a large ego because you are not going to be in front of the classroom anymore. He or she is stepping aside and allowing the learning to occur on on one through the children.
I find it very beneficial um ... just the fact that it would be stimulating children in ways that I would want them to look at the world. And maybe

> *see how the world works and create a really good problem solving ability towards dealing with life ... creating wisdom and the problem solving ability instead of just basic knowledge like dividing or subtracting, just out of a textbook. (M.)*

To conclude, the results show that, in general, all of the participants perceived the reform mathematics classroom to be significantly more meaningful than the traditional mathematics classroom, which they had experienced as students. From their comments it would appear to be possible to have their support for this type of investigative mathematics environment, and to see them as facilitators of the reform movement in mathematics education.

CHAPTER FIVE

REFLECTIONS

5.1 Discussion

Reflections of the participants' perceptions regarding the teaching of mathematics within a reform classroom created novel characteristics for task, discourse and environment. These characteristics were neither anticipated nor predictable. The objective of this section is to outline and elaborate on some of these characteristics as they connect to the potential implications for the reform to the teaching of mathematics. This creates interesting and unique alternatives for the students, the parents, the teachers and other adults involved, either directly or indirectly, with mathematics education. It also presents an opportunity to acknowledge and highlight options not previously recognized in the reform movement. This movement is presently in the process of redefining the teaching of mathematics (Western Canadian Protocol, 1995).

In their discussion regarding the nature of task, discourse, and the environment, the participants' perceptions of the reform classroom:

- presented a strong correlation and showed consistency with the N.C.T.M. Standards
- indicated an unexpected insightful awareness regarding the implications of the changes in teaching of mathematics and how it will impact onthe students' learning of mathematics
- were predictive of the revolutionary changes to the roles of students, teachers, and parents

- represented attitudes which valued the changes to the teaching of mathematics because it better supported student needs.

From the researcher's interpretation there was an intuitiveness associated with the participants' perceptions of the reform classroom that was consistent with the philosophy of the reform movement. When the N.C.T.M. guidelines for the features of the reform mathematics classroom were compared to the participants' perceptions, it was immediately obvious how strongly they correlated. This was seen in the attributes assigned to each of the areas of the classroom tasks (the student activities), the discourse (whether teacher or student), and the impact of tasks and discourse on the environment in which all of this would occur. Many of these attributes repeat the characteristics outlined in the literature presented in the chapter 2 on the reform classroom, and in the information in Appendix B. This is significant when it is understood that only one of the participants had access to actual documents in which these features are described (i.e.: the N.C.T.M. Standards and the Western Canadian Collaboration in Basic Education). Considering the negative experiences that they had with the learning of mathematics, it was of special interest that their responses were so positive, so consistent with each other and so consistent with the professional mathematics documents. It appears that the learning of mathematics, within the traditional perspective, must have given them sufficient reason to want to critically discuss the nature of the classroom. They were quite motivated by the benefits for the students who they saw resulting from learning mathematics within a reform classroom.

There appeared to be inherent characteristics associated with the kinds of activities or investigations representative of reform task. The participants were able to capture the essence of these. They seemed to be able to be predictive of the kinds of changes that would affect the nature of task, discourse and environment in a reform classroom simply from their exposure to samples of these activities. An awareness of the changes to these three areas did not seem dependent on access to the N.C.T.M. or Western Canadian Protocol documents, but could be accomplished with access to activities chosen as examples of reform tasks. Consequently, support for the reform of the teaching of mathematics will not appear to be difficult to attain if interested adults were given access to similar kinds of information.

There was an insightfulness apparent in the participants' recognition of the implications of the reform. This endorses the inclusion of adult participation in the debate and discussion concerning the expected changes to the teaching of mathematics. If given an opportunity, consistent with those of the participants in this study, members of the general public can take an active role in making informed evaluations about the present reform. It is both useful and essential to encourage adult participation in the transition period from the traditional perspective to one reflecting the reform philosophy.

Based on the experiences of the participants, it appeared to be productive to use *scenarios* as the means of 'educating the public' about the reform in mathematics education. Scenarios written for different grade levels, and representative of different topics within the various mathematical programs, might better permit other adults to make sense of what the expectations for change actually involve. In their discussions of the activities, the participants referred to the necessity of inservicing the parents in relation to the reform perspective. The scenario may be the first step that could effectively begin this inservicing. A possible extension of this implication allows for a whole new area of research which focuses not on the teacher making sense of the reform pedagogy, not on the student affected by the reform pedagogy, but on the parent and how they might become an integral part of the educational community working from a reform perspective.

Accepting that there will be anticipated changes to the roles of teachers, students, parents, and other involved with mathematics education, it was interesting that the participants were as explicit as they were about the nature of the teachers' roles. The separation of these roles into the hands-off and hands-on categories came directly as a consequence of their interpretations of the novel role into which the teacher will be placed. The importance of the teacher taking the class through a summation stage was another responsibility that they assigned to the teacher. These roles place the teacher more in the position of being a coordinator or facilitator than being the information dispenser. The ability of the teacher to orchestrate the actions within the classroom will require the philosophical shift to a reform perspective, and an acceptance of a constructivist framework consistent with the reform philosophy.

> Finally, when we learn to listen, then we become convinced that constructivism is fundamentally right: students learn to think, and they actively build representations in infinite varieties. They find ideas in working purposefully with concrete objects, in talking with each other and in sharing with their teachers.
> (Davis, Noddings & Maher, p.190, 1990)

The extreme difference viewed by the participants, in the part played by the students in the classroom, not only received significant attention, but simultaneously drew them to make revolutionary conclusions about the whole nature of discourse in the classroom. The participants' perceptions specifically challenged the status of the student in the traditional classroom where, more often than not, they were simply passive spectators. The participants implied that to meet the expectations of a reform environment, the students would be responsible for initiating their ideas in an open forum of investigations and explorations where the focus is on the student discussion. The students' developing ideas would require presentations to each other and to the class (in various modes including visual, pictorial, concrete, etc.) for the purpose of demonstrating their perceptions of the concepts being introduced. For students, a final stage required that there be an opportunity to validate and confirm quality conclusions, or to assist others to alter and improve on the nature of their conclusions as their subjective views were transformed into the objective views of the mathematics community. All of this would take place within a community of learners. The impact of these changes, to the roles of teachers and students, was not seen to be confined to the classroom but would create a ripple effect on all those around the school including parents and suppliers of educational resource materials. These are the kinds of expectations consistent with the constructivist positions, which is inherent in the reform philosophy.

> From all of this it should be clear that, whatever else it may be, constructivism is not without consequences. Adopt a constructivist point of view and you will need to change your expectations of schools, of teachers, of "content", of teacher education, and of research methodologies. (Davis, Noddings & Maher, p.190, 1990)

As a consequence of the recognition of the multiple changes in the mathematics classroom, the participants perceived a need for sufficient transition time and a support mechanism for enlightening all of the stakeholders involved. Not only were the inservicing of teachers mentioned by them, but also the inservicing of parents. Finally, from the participants was an implication that the suppliers of educational resources adapt to the needs of the classroom, and to the needs of the teacher adopting a reform philosophy.

Based on the experiences of these participants, their attitudes would give strong support to the present requests for the reform of mathematics education. With the attempt to better meet students' needs, as the focus of attention, their attitudes exhibited very supportive reactions. From their historical reflections on life in the classroom, it was not indicative of the traditional environment that the students' needs were a priority. The routines and procedures of the traditional classroom were more to accommodate the simplicity of the planning for the teacher, or the disciplinary control of the teacher over the students (Appendix A, p.117). The attitudes which this created in the students were not considered to be relevant or of importance. The attitudes which students develop towards the learning of mathematics in the reform classroom were of great importance to the participants. The participants expressed the potential for very positive student attitudes to develop from the learning of mathematics in this very different environment.

As this study began with the participants' emotional reaction to the learning of the mathematics, it was considered important to discuss some of the conclusions arrived a with regard to this area. The participants' reactions to the reform of mathematics gave indications of these emotional reactions in three ways:

1. They reflected strong and very positive, emotional reactions to the potential changes within the reform environment
2. They reflected enthusiasm for the opportunity to participate in this study and to have a voice in an areas where they had experienced less success as students.

3. They confirmed the shift towards the focus on the students, and their learning, and away from the teacher dominance in the classroom.

Where the traditional mathematics classroom had left large segments of the populations with very strong, negative emotional reactions to their mathematical education, the participants' reactions to the examples of the reform mathematical activities had the exact opposite impact. They were extremely positive about the psychological effect the reform classroom experiences would have on students. All three of them made reference to "this kind of experience" as being enjoyable, producing pleasure, and creating enthusiasm towards the learning of new concepts in mathematics. It was as a consequence of these feelings that they indicated a renewed interest which might encourage more students to want to attempt more mathematics, and thus t pursue more classes in mathematics.

The development of positive emotive responses by the students was considered by the participants to be essential. They saw this as a natural consequence of the opportunities presented by the types of activities which can be used in a reform context. If mathematics had a history of being onerous and burdensome, the classroom operating around the reform characteristics had the opportunity of significantly changing that perception. The tasks used to initiate concept development of a reform nature gave the participants very positive experiences and piqued their interest in wanting to learn the associated mathematical skills. Tasks traditionally alienated students from wanting to persist in mathematical pursuits often due to their excessive repetitiveness. This traditional perception of mathematical activity would require the re-education of adults, both parents and teachers, to understand the removal of repetitive and meaningless tasks, and the replacement of these with intriguing and thought provoking ones which would actively engage students.

An immediate personal reaction from the participants was their uncertainty regarding their ability to make significant contributions to research involving the area of mathematics education. This was most probably as a result of their assessments of themselves as unsuccessful learners of mathematics. But, the immediate and obvious enthusiasm, which the participants demonstrated in the interviews, was a major contrast to their experiences in the traditional classroom. That they might have value to this study, and be able to make contributions to the reform of mathematics education, first came as a surprise to them. Later it reflected a new appreciation, as well as an acceptance, of their role in critiquing the expectations of the reform classroom. A conversational situation was created in which they contemplated the modifications to task, discourse, and classroom environment. This offered them the same risk taking option that is expected to be offered to children within a reform classroom. Thus, their perceptions emerged as uninhibited and valuable perceptions of those three areas; task, discourse and the environment within the classroom.

In selecting the participants for this study there were numerous adults who offered to become involved, and who must have had a personal incentive to want to participate in research on mathematics education. The consistent negative experience of the majority of adults with that of the participants chosen, indicates an unvarying and historically rigid experience which many can confirm within a traditional classroom, and one that spans many generations. The implications for a world, which is not stable and unvarying, are approaches to teaching and mathematics education, which attempts to keep pace with the changing times. Preparation of students for the twenty-first century is better accomplished with attitudes and pedagogy that are more reflective of future needs and the necessary adaptations needed for that changing world. If the participants' perceptions are an example of altered attitudes to the subject of mathematics, then the enthusiasm to participate in the research on the teaching of mathematics can potentially be matched with students' enthusiasm to participate in mathematics as a life long pursuit.

The participants' perceptions about discourse and the environment within a reform classroom gave an immediate sense of the priority in the shift of attention from the teacher to the student. This does not mean a failure to recognize the importance of the teachers, but does indicate a focusing of attention on the expectations and participation of the students. The expectations for the different role of the teacher will of necessity have implications for the students. Some of the consequences to these changes in students' roles require further discussion. In comparison to their traditional roles of being empty receptacle needing to be filled, the students are given: a) far more responsibility, b) much greater classroom freedom, c) more opportunity to develop personal meaning, and d) time to make sense of their mathematical experiences.

> From a constructivist perspective, again it is crucial that the children develop their constructive competence through creative experimenting, commenting about their images, ideas, and expectations, and particularly through adapting to adequate issues in interactions with their teachers and their classmates. (Bauserfield, p.472, 1992)

With a new focus on the student as an active learner in the classroom, and an active participant in the discourse in the classroom conversation, an immediate implication relates to the process of introducing students to their new roles. Without effective and well-planned developmental stages for moving students from their previous controlled, site-specific locations and roles, the new freedoms can be counterproductive to the classroom purpose. The participants recognized that the understanding and the acceptance of the responsibility for materials, class time and personal leaning is not going to happen overnight or by chance. They predicted the potential for the change to the student being the center of attention to have counterproductive results if not balanced with the acceptance of the individual's responsibility. They perceived an expectation for students to be accountable for their own independent learning.

> Earlier I stated that personal autonomy is the backbone of the process of construction. Baird and White (1984) argued that a significant improvement in the student learning depends on a "fundamental shift from teacher to student in responsibility for, and control over, learning
> (Confrey, p.115, 1993)

Based on the concerns raised by the participants in their analysis of the changing role of the students, there appears to be an accommodation and an adjustment period necessary for students to productively handle the newly assigned responsibilities.

These discussions initiated thoughts and reflections regarding the reform of mathematics that looked at the opposite perspective, the potential problems. Where the participants' perceptions of the reform classroom gave information which offered positive reactions to the potential changes to task, discourse and the classroom environment, these same perceptions offered the opportunity to present interpretations that reflect the negative as well. There are a few areas that are necessary for inclusion with respect to the manner in which the responses were analyzed to reflect negative impressions. Three of these include:

a) the speculative nature of the participants' responses

b) the lack of understanding of the needed teacher expertise in the creation of successful activities, and

c) a concern over student profiles unsuited to this type of classroom.

Given that the participants' involvement in this research took the form of interviews, and not actual experiences with the reform activities, the depth of their understanding of the impact to this style of teaching is open to debate. It may potentially been more productive to have expected the participants to actively attempt the activities. As was described earlier in the methodology, the objective of the study was not to put the participants under scrutiny regarding their actual mathematical knowledge. Therefore, the speculative nature of their responses was accepted for the value it gave to the study in terms of the interpretations that they gave regarding the reform. It must be acknowledged that having authentic experiences with the activities may have increased the depth and value of their judgments about the various aspects under investigation. What must also be accepted as valid are their personal experiences and stories which they brought to this study and around which they established their interpretations of the reform information presented to them. For future research purposes there may be advantages to adding the actual experiences with the activities into the interviews so that the speculative nature of their discussions could be reduced. It is therefore accepted that their responses have meaning only in terms of the perceptions that the participants were able to create in terms of the nature of an *imagined reform classroom* not an experienced one.

At times the participants showed a lack of understanding of the sophistication and the expertise represented in the nature of the teacher knowledge, either of mathematics or of teaching pedagogy, necessary to develop appropriate activities. The mathematical content knowledge need by the teacher, and also the pedagogical knowledge to put the concept into a useful and productive task, was not as easily recognized by the two participants who were not responsible for actually teaching mathematics. The consequences of successfully embedding the concept so that it seemed simplistic and obvious was noticed only by the participant, who in preparation for teaching her own class, recognized the value and the cleverness of the activity.

> Brousseau (1987) asserts that part of the role of the teacher is to take The non-contextualized mathematical ideas that are to be taught and embed them in a context for student investigation. (Simon, p.119, 1995)
>
> The teacher must design tasks and projects that stimulate the students to ask questions, pose problems, and set goals. Students will not become active learners by accident, but by design, through the use of plans that westructure to guide the exploration and inquiry.
> (Richards in Simon, p.118, 1995)

What was acknowledged by all of the participants was the demand on the teacher time and the degree of creativity necessary for the development of these kinds of activities. As a consequence they hoped for assistance which could remove the onerous task of having each teacher responsible for their own inventive, and unique, tasks and activities. Until there are more teachers comfortable with creating tasks representative of reform tasks, the participants saw a need for support and access to teacher tested ideas. The participants considered this as being essential to the continuation of the reform movement. Without this support, they predicted a greater potential for teacher frustration, and eventual return to a less threatening and more comfortable approach.

> The teacher is ever vulnerable to self- doubt induced by the unpredictability and uncontrollability of human interaction, given this uncertainty, the teacher's sense of efficacy is in continual jeopardy, in danger of attack by resistant and hostile students, angry parents, demanding administrators and dissatisfied colleagues... Teachers who succumb to feelings of inefficiency are likely to suffer debilitating stress and reduction to their effectiveness with students. (Ashton in Smith, p.390, 1996)

> A robust sense of efficacy therefore appears to be important, if not necessary, part of a teacher's belief systems. To teach to the best of their abilities, teachers need to construct and maintain conscious beliefs that link their teaching actions to their students' learning, that project a measure of control over their difficult and complex task, and that allow them to persist in the face of obstacles. (Smith, p.390, 1996)

The depth of understanding, and the meaningful characteristics, which the participants attributed to the nature of task, discourse and environment in a reform context could be questioned as being very cursory and lacking in substance. If one recognized the present limited access to classrooms reflective of the reform philosophy, and given the difficulties with identifying articulate student candidates from actual reform classrooms, it is accepted that the participants' opinions are not indicative of truly reflective information. But in defense of the opinions, and in acceptance of the constructivist stance and philosophy, whatever information they provided was an interpretation of their reality. Their reality was the interviews and the information presented to them in those interviews.

To acknowledge the potential concerns with the research methodology applied, one alternative to the participants' roles in the research could have been to allow them more reflective time between the access to the information and the expectation to respond to that information. This created the possibility for them to have carried on a conversation with others and to have altered or changed their perceptions based on different meanings which may have emerged from that communication. Where intuitive or 'gut feeling' was the object of the interviews, then the procedure applied would best supply that immediate perception. If deeper investigation of their perceptions were expected, then a methodology that allowed for reflective time, and a discussion as to the nature of the interpretive process, would have been more appropriate.

From the youngest and the oldest of the participants there was a concern which seemed to question the potential for the reform classroom to be able to supply the necessary " restrictions and control" over the students' actions in the classroom. They predicted that a particular student profile would take advantage of the less structured environment, possible at the expense of the education of other students, but probably at the expense of the student himself. There was even an assessment about the suitability of this type of classroom for high school classes. It is the researcher's interpretation that this is reflective of the kind of typical "controlled" environment more representative of experiences in the classrooms in which the participants had been taught. It raised some valid questions though. Are investigative classrooms exploring mathematical concepts more viable for certain age groups? Does lack of consistency from one grade to the next, with one teacher operating in a reform mode and the next in a traditional mode, have a negative impact on the learning of the students? Are there certain student types who would not learn as well in a reform environment? These are not questions to which the answers are present within this research but are definite topics for future research.

> Some genuine but very tough questions about teaching follow. Might it not be the case, for example, that some students perform strong acts of construction no mater now the material in presented? And might it also be the case that, while the teacher is encouraging explorations and

> genuine (strong) acts of construction, some students perform weak acts such as quietly waiting for group consensus to occur and then noting the answers? (Noddings, .14, 1996)

Response to the questions raised by the above quote, and from personal experience with the new expectations for the students, is an understanding that students' acceptance of this responsibility is not automatic. The nature of any alterations to the designated teacher roles and student roles introduces a potential for students to "disappear in a group". If anonymity in a traditional classroom was accomplished by playing the silent observer and being indistinguishable from the rest of the class, anonymity can now be masked with the impressions of intense activity but with no productivity resulting form the activity. Implications of this for teachers will be to develop monitoring procedures to be able to track both groups and individuals functioning in a more autonomous environment.

> The teacher will need to create as many and as varied ways of gathering evidence for judging the strength of students' constructions as possible. The result will be that the teacher will create a "case study" of each student. (Confrey, p.112, 1990)

5.2 <u>Conclusions</u>

> The three main attributes of the constructivist learning and teaching models are: a) building a suitable environment for using relevant experiences, b) building up a friendly relationship among students and between students and the teacher, and c) allowing students to work together and to express themselves. (Sabetghadam, p.23, 1996)

First reactions to the results of this study are the acceptance and concurrence of the participants with the expectations of the reform movement in mathematics education. The recognition of the novel roles for teachers, and the participants' recommendations for the students' roles, came as a direct result of the type of activity or task which will be used to introduce mathematical concepts in a reform classroom. This will continue to have implications for the kinds of classrooms in which the activities will take place and the materials, which will be necessary to support the different kind of learning. The opportunity to have access to manipulatives or concrete materials, as well as flexible classroom features, was seen to be essential to support the learning environment. The extension of the mathematical experiences outside of the classroom into the community, and even into the home, was described as being productive and reconnecting mathematics with the students' real world.

An emphasis on the emotive reactions of the participants to the learning of mathematics initiated the interest, and the completion, of this research. It concluded with a significant shift in their attitudes. Each of the adult participants in this study exhibited a renewed interest in having access to mathematics from this new perspective, even to the extent of returning to attempt previously taught concepts again. The desire to fulfill their mathematical potential with meaningful experiences and clearer understanding, was a strong motivation for these adults wanting to revisit topics which were still of intrigue to them. Their enthusiasm for 'seeing' the interactions between teachers and students in the classroom, as presented in the scenario, gave an alternate possibility of exposing adults to the character of a lesson in a reform classroom. Based on the constructivist position that most people learn by doing, the scenario could offer the opportunity to put oneself in the classroom without actually being there. The next step appropriately seems to be creating actual physical experiences with tasks similar to those presented to the students so that adults can understand how to support their children outside of the classroom. Workshop experiences with parents, teachers and other associated parties emerged as a useful technique in assisting adults with the philosophical shift required to make sense of the present reform of mathematics. Support for the continuation of the reform movement appears to be available from many adults if given the opportunity is consistent with the participants of this study.

There are challenges for teachers that are immediately obvious and come as a direct consequence of considering a change to a reform perspective. There will be objections from a variety of sources to "experimenting" with children's' education and expectations for justifying yet another reform. The demands of the shift to this style of teaching will attack the core of many teachers' beliefs about teaching, learning and the nature of mathematics. Most of this will occur in a non- supportive environment where skepticism is high. The conclusions of this researcher validate the acceptance of the challenge by teachers and offer the options for a very different mathematical experience for students entering the twenty-first century.

> ... the explanations and strategies that teachers construct to cope with the tensions and contradictions of the school mathematics traditions are constraining in that they inhibit the envisionment of alternative practices... More generally, it suggests that the school mathematics tradition "works" because the taken-as-shared beliefs and practices of this tradition represent a consistent and coherent worldwide view that enables teachers to fulfill their obligations within this tradition. As such, this suggests that any reform movement that hopes to be successful must challenge the classroom, school, and societal obligations that characterize teachers' roles in the school mathematics tradition. (Greg, p.463, 1995)

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APPENDIX A

THE TRADITIONAL MATHEMATICS CLASSROOM

A.1 Introduction

The intent of this appendix is twofold. First, it provides an historical context for the participants of the study, all of whom experienced only the traditional mathematics classroom as students. Second, it provides a basis for comparison with the reform mathematics classroom to highlight how it is different. In this discussion the traditional classroom will be highlighted in terms of the three factors specified earlier in the objective of the study: task, discourse, and environment. The philosophical perspective that frames these factors, and consequences for students of the traditional environment, are also discussed.

A.2 Traditional Mathematical Tasks

Tasks, in the traditional classroom, have generally been drill and practice exercises associated with the development of a specific topic (for example: an operation in integers). Once the mathematics process has been modeled by the teacher, there is an expectation for students to mimic the process with repetitive variations of questions. From this repetition students are expected to gain mastery of the concept. Thus, these tasks encourage memorization of procedures. The tasks usually have no meaningful connections for students or to the real world. They focus on mathematical concepts in an isolated way that tends to eliminate connections of mathematics within itself or to other disciplines.

This also tends to create disconnected concepts as students proceed from one unit or chapter to another. The goal of these tasks is to practice mathematical concepts, usually in a mindless manner.

There are two groups of these tasks, those in symbolic form and those in verbal or embedded form (i.e., word problems). These forms are related in that the word problems are generally symbolic questions in disguise. They also both serve the same purpose; practice of mathematical concepts in an algorithmic way. However, the word problems have the added and often challenging step of translation from verbal language to mathematical symbols. The translation process is often characterized by types of problems. For example, two variable problems requiring a system of equations would occur in groups. One might cover topics like coin or value questions and then be followed by a section in which students would be looking at distances and velocities of two moving objects. Thus, once a student had learned the type of problem and its related process, higher level thinking was not important. Instead it simply became recall and application of the learned process in "new" situations.

Tasks are usually organized from simple to complex in traditional mathematical textbooks. Since problems are considered to be the most complex, they are generally located at the end of each set of exercises or the chapter as the "applications" of the concepts. Selected answers can often be found in a section at the back of the textbook.

In An Advanced Course in Algebra (revised edition) the authors state:

> *At the end of each chapter and within some chapters, larger selections of graded exercises are inserted, covering a wider range of idea. It is by working such sets of exercises that a student gains mastery of the subject. The exercises graded "A" are simple and direct in character and should be attempted by all pupils: those*

> *graded "B" make somewhat greater demands but are not beyond the powers of the average pupil; the "C" exercises provide an opportunity for pupils with special ability in mathematics.* (Miller & Rourke, preface, p.vi-vii, 1940)

Although taken from a 1940 textbook, this approach to exercises applies to most texts still in use into the 1990's, particularly at the high school level. Tasks are often grouped according to one concept and they are often presented with a singular way of doing them. If students see these tasks later they encounter difficulties when the tasks have the surface appearance of being different. Thus difficulty in task really meant disguising the concept by introducing different surface features to it. An example question can appear as: "*Given* $x + y = 2$ *and* $x - y = 6$*, find* x *and* y". The concept of a system of simultaneous equations would look different. Compared with: "*Find two numbers such that the sum of the numbers is two and. The difference of the same numbers is 6. Find the two numbers.*" Where the symbolic form was considered to be simple, the latter was considered to be more difficult. Most difficult was a question like: "*The college theater collected $1300 from the sale of 450 tickets. If the tickets were sold at a cost of $2 and $3 per ticket, how many tickets were sold at each price?*" Figure A.1 page 116, shows one sample of traditional kinds of tasks. It was taken from a chapter on Quadratic Functions from a grade 11 textbook (Holt, 1988), and is probably still in use in some high schools today.

In general, tasks in a traditional mathematics classroom have a narrow goal in terms of an educational objective, that being the mastery of mathematical facts and skills (often without meaningful understanding). Thus, for many students, the "learning" is characterized by the condition in which you either know how to do the tasks or you do not know how to do them. Open-ended

Figure A.1

AN EXAMPLE OF TRADITIONAL TASKS

1. Identify the features to be considered in an analysis of the graph of $y= a(x-p)^2 + q$.

2. Using a table of values and the graph of $f(x) = -2(x+3)^2 - 4$, demonstrate and list the effects of the -2, the 3, and the -4 on the basic graph of $f(x) = x^2$.

3. For each parabola, state the direction of opening, the location of the vertex, the equation of the line of symmetry, the x- and y- intercepts (if they exist), and the domain and range.
   a. $f(x) = (x-3)^2 + 2$ b. $f(x) = (x-4)^2 - 8$
   c. $y = -0.7(x+2)^2 - 3$ d. $f(x) = x^2 + 5$
   e. $y = 3(x-4)^2 - 2$ f. $y = -5(x-4)^2 - 4$

4. Sketch each of the parabolas in Exercise 4 using the information found .

5. Sketch the graph of $y = x^2$ after it has been reflected in the x-axis and translated to the right 5 units and upward 2 units. Write a function satisfying these conditions.

6. Describe how each graph would be different from the graph of $y = x^2$.
   a. $y = x^2 + 5$ b. $y = x^2 - 3$
   c. $y = (x+4)^2$ d. $y = (x-8)^2$
   e. $y = -4x^2$ f. $y = 1/2x^2$
   g. $y = 2(x+1)^2$ h. $y = -3(x-2)^2$
   i. $y = (x+2)^2 + 10$ j. $y = (x+3)^2 - 4$
   k. $y = -1/2(x-2)^2$ l. $y = -.5(x+8)^2 - 1$

7. Sketch the graph of $y = 2(x+2)^2 + 2$. Translate the graph to the right 4 units. Write the new equation of the function after the translation.

8. Sketch the graph of $y = -.4(x+3)^2 - 2$. Translate the graph up 4 units. Write the new equation of the function after the translation.

9. Find an equation of each parabola.
   a. has a vertex a (0,4), congruent with the graph of $f(x) = x^2$
   b. has a vertex at (4, -2) and zeros at 2 and 6
   c. has the y-axis as an axis of symmetry, passes through (1, -1) and (-2,5)

10. Write the quadratic function in the form of $y = a(x-p)^2 + q$ which fits the given description for each. Assume that $a = \pm 1$.
    a. opens upward, vertex at (3,0)
    b. opens downward, vertex at (-5,2)
    c. opens upward, vertex at (2,6)
    d. opens downward, vertex at (-1,1)
    e. opens downward, $y= 10$ if $x = 4$ and axis of symmetry is $x = 4$
    f. opens downward, vertex on x-axis axis of symmetry is $x = -3$
    g. opens downward vertex is on the y-axis, has a maximum value for the function of 2.

11. Describe the change in the graphs of $f(x) = 3(x-p)^2 - 2$ as p varies from -3 to 3.

12. Describe the change in the graph of $y = -3(x-4)^2 + q$ as q varies from -4 to +4

13 Sketch and describe each function
    a. $y = 2(x+5)^2$ b. $y = -4x^2$
    c. $y = (x+2)^2 - 2$ d. $y = 3x^2 - 9$
    e. $y = -2(x-3)^2$ f. $y = 5(x+1)^2 - 4$
    g. $y = -1/3(x-1)^2$ h. $y = 1.2(x+2)^2$
    i. $y = -(x+5)^2 - 3$ j. $y = -2x^2 - 3.5$

tasks are often considered to be "enrichment", "extra practice", or "not really math" and are provided only to exceptional students. The majority of students is unlikely to be exposed to such tasks and are denied the opportunity to experience any genuine problem solving situations in mathematics.

Although traditional tasks will still have a role in the future of mathematics education, they can only be viewed as a very narrow part of a broader range of tasks that the reform classroom requires. Thus, their limitations, compared to if used by themselves, will be less problematic. In selected doses, they will still be a useful and valid way for students to confirm their understanding and their mastery of concepts, especially in secondary and post secondary mathematics classes. Ultimately, if students are responsible for their independent presentation of skills and concepts on formal evaluations, such as exams and tests, they will find the practice offered within traditional tasks useful in developing confidence and facility with their acquired knowledge.

A.3 Discourse in a Traditional Mathematics Classroom

Discourse in the traditional mathematics classroom follows a similar perspective to that of tasks. It is generally a highly structured process in which students are not required to think, but to recall facts. Thus, it is easily identifiable and usually quite predictable because it is the teacher who is doing most of the talking. To understand discourse, one needs to understand the routine of the majority of traditional mathematics classroom lessons. The general structure of these mathematics lessons consists of four parts: "demonstrating a procedure, providing practice, correcting students work and re-teaching where necessary"

(Smith, p.393, 1996). An example of a classic lesson would be a process of: (1) checking the answers to the previous day's homework, (2) working some of these examples on the blackboard, (3) introducing new material contained in the new chapter (as part of the ongoing development), (4) modeling characteristic examples of the types of questions in which the particular topic would be found, and (5) assigning practice work for student independent application of the new knowledge and concepts. (Cobb, Yackel, Wood, and McNeal in Greg, p.442, 1995). As Smith (1996) pointed out, this routine simplified the issues of daily lesson planning and classroom management for the teacher. Thus in order to conduct his or her activities with a level of comfort or ease, it seemed to better serve the needs of the teacher and appeared to have little to do with the needs of the student.

Within this classroom routine, discourse, in terms of teacher-student or student-student interaction, was completely controlled by the teacher. With the teacher as the primary focus of attention from the beginning of each class, and with an expectation of silence from the students for most of the class, very little dialogue occurred or was fostered. Teachers can be seen to be filling the blackboard with reams of symbolic notation, while simultaneously carrying on a non-stop monologue. In fact, it could be noted how quickly many traditional mathematics teachers would write when needed. From this teaching position, discourse took as much significance in a symbolic written form as it did in verbalization. The following Vignette of a fictitious grade 11 class, studying the quadratic function, is an illustration of this type of traditional lesson with examples of these restrictive interactions.

A.3.1 Vignette #1

(written by Cynthia Ballheim and Jens H. Orsten)

As the students enter their Mathematics 20 classroom, the teacher is positioned behind her already humming overhead projector. They assume their assigned alphabetical seats in separated rows. Students open their workbooks, and turn to the page in their texts that is written on the board. As the lights are turned off, the students' attention is drawn to the screen and they take a note copying position with pens in hand.

Teacher: Today's lesson will introduce one of the transformations of the quadratic function which you already know characteristically takes the shape of a parabola.

Pointing to an expression of the overhead she begins to describe the algebra which the students are now viewing and copying.

Teacher: The variable 'a' which you see written in the expression a x [f(x)] is a parameter which we want to understand as a transformation of a graphical representation of $f(x) = x^2$

Student (after raising his hand) asks: Why are there two x's?

Teacher: Oh, the first x means multiply so I will use brackets instead.

At this the teacher takes a red marker pen and crosses out the first 'x' and places red brackets around the f(x) expression.

Teacher: OK, any more questions at this point? No, all right, let me continue.

The teacher points to a table of values on the overhead transparency which the students are seeing on the screen.

Teacher: This is the table of values associated with the function $f(x)=x^2$ and beside it you can see the accompanying graph. Notice that for the chosen domain values the range is always positive. You will need this example for comparison so copy the chart and the graph into your notebook.

After a few minutes the first overhead is removed and replaced with a second one to which the teacher points and continues.

Teacher: The new chart represents the introduction of a factor of 2 multiplied by the original x^2. You will notice the effect is in the range in that they are all doubled from the first chart. You can the impact on the graph is that the parabola is skinnier. In mathematical terms we call this transformation an expansion and the expansion factor is 2. Do you understand? (a very short pause). GOOD.

The teacher removes the second transparency page and flashes a third one onto the screen with new numbers, new algebra and a new graph. Some of the students have lost interest in the bright glare of the overhead and are quietly nodding off in the darkness. The teacher takes a quick sip of her coffee and shuffles a little to ease her tense leg muscles. She then continues.

Teacher: You should have noticed immediately that the 'a' value has been replaced with a 1/2 where it used to be 2. This causes the graph to flatten which mathematicians call a compression factor. Add this example to your previous ones.

Those students, still keeping up with the conversation, quickly copy the new example hoping to get all of the information before the teacher removes it from the screen.

Teacher: What do you think would be the effect of a negative value for 'a'? Let me show you with the next overhead page.

The screen changes again and the last overhead for this lesson this day is dropped into position.

Teacher: You can see that the negative factor of 2 reflects the curve over the x-axis but continues the same expansion factor as previously shown. With the knowledge of the three examples that I have given you, and the first comparison mode we started with, you now know the transformation effect of the variable 'a'. From what we have studied before and taking the absolute value of 'a', would your call these transformations reflections, dilatations or translations?

A student raises his hand and responds: Reflection?

Teacher: NO! You are wrong. Cricket what should the correct answer be?

The second student replies "Dilatation" and gets a nod of approval from the teacher.

Teacher: Yes, you will note that I said the absolute value of 'a' so that it would restrict the answer to dilatations . If we let the negative value of 'a' stay in the problems that would introduce the possibility of a reflection. In your text on page 117, you will find examples of this transformation. For homework, complete exercises 1- 25. You may begin now. If you are not working then it must mean that you fully understand this concept and that I can continue on the next topic. If you do not finish during the class time left complete the rest for homework at home.

At this point she turns on the lights, turns off the overhead projector and collects her materials. She moves back to her desk at the front and begins marking paperwork on the desk, occasionally glancing up at the students.

This may seem to be an extremist view of the traditional classroom, but it is necessary to create the contrast with the reform classroom. The vignette draws attention to several aspects of discourse in terms of who talks, and the nature of the talk. The teacher is obviously the dominant talker. The purpose of the talk is to transfer his or her knowledge of mathematics to the student. In this way, classroom discourse is tightly controlled by the teacher who places emphasis on formalized mathematics, that is a collection of facts and procedures. Any discourse that takes place follows teacher initiation, student reply and teacher evaluation patterns (Richards, 1991). Questions posed are to elicit rote or memorized answers from the students in response to direct but factual questions from the teacher. It is usually the case that the teacher already has the expected answer in mind. The teacher's reaction to the students' responses determines their correctness, and consequently also determines whether or not that particular student, or another, will be invited to continue the discourse. (Sirotnik in Greg, p. 443, 1995) Thus, there is no real dialogue or discussion between the teacher and the student, but an "unreflective and routine turn-taking dance" (Confrey, p.33, 1992). The expected and appropriate ways of acting, and reacting, become so habitual in these teacher-student classroom roles that they become almost invisible. (Confrey, 1992)

Students are allowed to talk only in response to the teacher or when they have questions about the procedures being demonstrated. These questions are usually to clarify how the procedures operate, but not why. Students are generally very passive in this environment and are discouraged from interaction with each other. Their only interaction is supposed to be with the teacher to clear up concerns with the procedures, since the teacher is the ultimate authority and judge of what is right or wrong. The teacher and the textbook are the only authorities that the students need, and they should rely totally on them for support of their learning. Demonstrations are the teachers' personal display of mastery over the content for the students, and a reminder of that mastery to themselves. (Schonfeld in Smith, p.392,1972).

The following statement was taken from *Mathematics for Canadians Book 3* by Bowers, Miller and Rourke, a text resource from 1950.

> *A word of Advice to students:*
> *In the pages that follow, you may encounter difficulties. These can be solved either by tackling them aggressively yourself or by going to the teacher for help.*
>
> *Never ask for more help than you need.*
>
> *Always try first to solve the problem by yourself. If you fail, read the text carefully until you grasp clearly the ideas involved. You will find many examples worked in the text. Try them yourself, looking at the given solutions only when absolutely necessary. The next step is to attack the exercises which follow , and show your finished work to your instructor. If you have done your best and failed, then get help.*
> (Bowers, Miller & Rourke, p. xi, 1950)

One can notice immediately the expectation for emphasis on self reliance and classroom operation completely independent of others. With the textbook as one authority containing all the correct examples to be modeled, there was little or no need for discourse, and then only with the teacher. The teacher, who has already modeled some examples for the class, is seen as the last resort and it seems he or she should not be involved until failure is experienced. With no access to discourse with other students, many students were left to struggle within the classroom. The teacher is expected to have covered the concepts by "telling" the students the expected knowledge and by explaining the appropriate procedures. The next stage is expected to be a natural comprehension on the part of the students with the ability to continue on with the exercises. It is intriguing that the authors would even go so far as to suggest that failure should precede help. This set the tone for the environment in which both teacher and the student co-existed.

A.4 The Traditional Mathematics Classroom Environment

Like discourse, environment of the traditional classroom is centered around the teacher. This is reflected as well in Vignette #1. The routines within the environment are so predictable that they could be called rigid and inflexible. Both the teacher's space and the student's space are definable and distinct. The blackboards and the teacher's desk dominate the front of the room with the physical separation of individuals, both teacher and students, as the primary function. The rows of desks which separate students from each other, and the teacher's desk which dominates and regulates the climate, are characteristic of all traditional classrooms. The look and feel of the classroom deters students from taking risks, being involved in investigation, or exhibiting an intrigue or interest in the study of mathematics. Students do not get to formulate their ideas, and test them in the public domain, without serious risk to their feelings of confidence, or to chastisement for breaking the expected silence of the room. Teachers considered it more appropriate for effective thinking, and focused work, that there not be any distracting factors. Classrooms with windows, either to the outside or to the hallway, would be viewed as potentially interrupting the students' concentration on either the presentation by the teacher, or on the required task completion. Large windows to the "world" would only allow for students to be day-dreaming, when they needed to pay the closest of attention to the singular importance of the classroom teacher or task.

As students needed only their workbooks, textbooks, desks and working materials like pencils, erasers, etc. and teachers needed only blackboards, chalk and intermittently an overhead projector with screen; the physical environment of the traditional classroom is generally very Spartan in appearance. Thus, the first impression of the room would be that it is sterile in terms of giving an invitation to the outsider. Any charts or diagrams tend to be formal in content and threatening to the non-mathematical achiever. For a teacher working from a traditionalist perspective, little consideration would be paid in even considering that possibly the environment required any attention. As a consequence the demands of the mathematics department in terms of budgetary expectations for the traditional classroom are quite simplistic

A.5 Philosophical Perspective of the Traditional Mathematics Classroom

The traditional mathematics classroom evolved from what, at the time, seemed to be a very sound perspective of learning and teaching. Within this perspective, the nature of task, discourse, and environment can be justified and will continue to be. Because of this, it is important to provide an overview of this perspective to present the traditional classroom in its appropriate context. This overview will focus on the nature of mathematics in an attempt to account for the essence or qualities of task as they have been presented. It will also focus on the nature of learning and the teaching of mathematics to account for the characteristics of discourse and the environment already outlined.

A.5.1 The Traditional View of the Nature of Mathematics

The nature of mathematics, which is reflected in the approach to teaching in the traditional classroom, is consistent with the "absolutist view of reality". The absolutist view focuses on mathematics as the one and perhaps only realm of certain, unquestioned and objective knowledge (Ernest, p.3, 1991). This is therefore classified as "a priori" knowledge, or knowledge that is built around justifiable beliefs asserted on the basis of reason alone and not observable directly in the real world. With mathematical knowledge acknowledged as being "out there" waiting to be uncovered or discovered, it is classified as (1) specific and immutable, (2) isolated and distinct from other disciplines, (3) rigid and hierarchical in structure, (4) neutral and value free, and (5) based on absolute truths (Ernest, 1991, Confrey, 1993). As a result of this position, mathematics is considered to be a static pre-existing body of accepted truths that is expected to be passed on from generation to generation uninfluenced by mankind. The ongoing process would be one in which the continued discovery of more truths occurs in a purely Platonic realm (Wilder in Lamon, p. 37, 1979).

> The difficulty with mathematics is that it appears not to be conversant with anything outside of itself. It appears to be self-enclosed, complete detached ... (Jardine, p. 187, 1990)

Confrey (1981) classified absolutist positions into two categories where the formal absolutist view of mathematics was:

> the epitome of certainty... concepts in mathematics do not develop, they are discovered.. the previous truths left unchanged by the discovery of the new truth... mathematics proceeding by an accumulation of mathematical truths and as having an inflexible, a priori structure. (Confrey, p. 246-47, 1981)

and contrasted with the progressive absolutist view which saw mathematics as resulting from humans striving for truth and as a :

> ... process of replacement of previous theories by superior theories... Each progressive theory approximating truth more and more precisely. Progress consists of discovering mathematical truths which are not consistent with a theory or not accounted for in a theory, and then extending the theory to account for this larger realm of mathematical phenomenon. (Confrey, p.247-48, 1981)

Both of these resulted from the Platonistic view that the concepts of mathematics have an objective existence which is real but found in an ideal realm independent of humanity and its existence. Doing mathematics became the process of discovering these pre-existing relationships. This would not be accomplished through intuition but through reason and logic, and as a consequence gave rise to three philosophical schools of thought within absolutism: Logicism, Formalism, and Conventionalism.

According to Ernest, the claims of logicism are: (1) all the concepts of mathematics can ultimately be reduced to logical concepts, (2) All mathematical truths can be proved from the axioms and rules of the inferences of logic alone (Ernest, p.8-9, 1991). Formalism, on the other hand, is the view that mathematics is a meaningless formal game played with marks on paper, following specific and restrictive rules. The formalist thesis comprises two claims. (1) Pure mathematics can be expressed as un-interpreted formal systems in which the truths of mathematics are represented by formal theorems, and (2) the safety of these formal systems can be demonstrated in terms of their freedom from inconsistency by means of meta-mathematics (Ernest, p.10, 1991).

Where logicism put value in reason and intellect, and formalism placed its emphasis on the formal written version of mathematical symbols, conventionalism held that mathematical knowledge and truth was based on linguistic conventions. Published posthumously, Wittgenstein made contributions to the philosophy of mathematics in which he claimed it to be a "motley", a collection of language games (Wittgenstein, 1953, 1978). According to him the truth or falsity of propositions depended on our acceptance of the conventional linguistics rules of the games of the times. Unique and culturally distinct languages became the transport vehicle for the transmission of mathematical knowledge. The nature of truth and meaning became tied to language and its development. Where language changed, the sets of conventions and rules also changed especially with respect to informal mathematics (Ernest, p. 32, 1991)

The views of mathematics reflected in the above perspectives validate the nature of tasks used in the traditional classroom. The tasks in which students are expected to participate are representative of facts that make up a formal body of knowledge. As a consequence, it appeared necessary for these tasks to be isolated from other areas of knowledge in terms of their context. Their purpose is to provide students with mathematical truths that have been discovered by mathematicians in a more formal environment.

A.5.2 The Traditional View of the Learning of Mathematics

Much of the theory regarding the learning of mathematics can be traced back to the theories of Psychology. As a result, as psychology went through periods of association with different theories, so also went mathematics education. The nature of learning in the traditional mathematics classroom is consistent with a behaviourist view of learning. This view is linked to the stimulus-response theories in which the learner passively absorbs new knowledge. Two distinct figures whose work significantly influenced the adoption of this view in mathematics education were Edward Lee Thorndyke and William Brownell.

Thorndyke (1922) established three primary laws to learning: the law of Exercise, the law of Readiness, and the law of Effect. The law of Exercise was directed towards repetition. As knowledge, even complex, was built of connections between stimulus and response, learning would consist of establishing, strengthening, and maintaining the required associations. Where the law of Effect stated responses associated with satisfaction are strengthened and those with pain are weakened, the law of Readiness associated a person's satisfaction or annoyance with action or inaction, in the face of the bond's readiness to act or not act. He stated "other things being equal, exercise strengthens the bond between the stimulus and the response" (Thorndyke, p.79, 1962).

> ... in the development of skill... and in the acquisition of facts for relatively permanent retention such as ... the products of numbers and various mathematical formulae or other rules and procedures, the method of practice, especially that known as practice exercises, is the most economical method of learning yet invented.
>
> (Thorndyke, p. 258, 1931)

This practice was the basis for what he thought would lead to the "desirable mechanical features" from the student (Thorndyke, p.78, 1962). It is from this perspective that the students' learning is expected to occur based more on a mechanical feature to the human activity than an intellectual one.

> 1) In the acquisition of a new habit or the learning of an old one we must take care to launch ourselves with as strong and decided initiatives as possible
> 2) Never suffer an exception to occur until the habit is securely rooted in life
> 3) Give the habit exercise (Thorndyke and Gates, p.261, 1931)

The Connectivist school, which came to be associated with the theories of Thorndyke, led to the Structural school of William Brownell. In relation to the learning of arithmetic, Brownell criticized the meaningless form of learning advocated by Thorndyke noting the:

> ... prohibitive task of memorizing multitudes of bonds, the invalidity of product measure as an accurate measure of process, the invalidity of the analysis of adults' performance of arithmetic and the negative effect of the Connectivist theory may have on the later learning of mathematics (Nik Azis Nik Pa, p.12, 1986).

He envisaged arithmetic as becoming meaningful when the child saw the structure of the subject itself. The focus changed from parts being done individually to a view of how the whole of mathematics existed. Therefore the isolated and distinct pieces within a Connectivist school, used for the purpose of building bonds, would only develop more meaning within the context of understanding of the complete structure of the whole for the Structuralist. This was said by Weaver and Suydam (1972) to closely resemble Bruner's (1960) emphasis on teaching the fundamental structure, from which came the "New Math" of the Sixties. This focused on the structure of number sets and set theories.

These theories from Thorndyke and Brownell are consistent with the formalistic view of mathematics and continue to influence how students are expected to learn mathematics. In Thorndyke's case, understanding of the information was not necessary. The goal of learning was to be able to reproduce the information in identical form to that of the teacher and with a minimization of effort. In Brownell's case, understanding the formalistic structure of mathematics became important; but as seen in the failure of the "new math", the traditional classroom has been incapable of reflecting this successfully. Thus, the dominant learning perspective in the traditional classroom is a combination of the stimulus-response theory and the information processing theory. As a consequence of either of these views, Simon notes how the N.C.T.M. Standards(1989,1991,1995) criticize learning in many classrooms as a process in which students passively absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. The improvement of the storage and retrieval system became the basis for the review and practice approach to the teaching of mathematics (Simon, p.139, 1995).

This perspective validates the focus on the mastery of skills, procedures and facts in the learning of mathematics. It is a continuation of the absolutist perspective in which students' personal experiences are ignored. As a result, it is a perspective which makes sense in relation to the absolutist view of mathematics; and together this view, with the associated theories on learning, provide a solid framework to support the behaviours previously described in the traditional classroom. This position is also reflected in the manner is which discourse was controlled by the teacher in the traditional classroom. The student access to discourse is restricted to clarification of procedures and the environment is expected to be one in which the student operates very independently from other students. All of these characteristics are obvious from the previous sections on task, discourse and environment. The most immediate implications of this acceptance of this model of learning are with respect to the nature of the teaching of mathematics.

A.5.3 The Traditional View of the Teaching of Mathematics

The nature of teaching, that was reflected in the traditional mathematics classroom, was determined by the perspective of the nature of mathematics and the learning of mathematics as previously described. These perspectives set up boundaries regarding what appropriate teaching strategies would effectively satisfy the absolutist orientation. Thus, they provide well defined prescriptions of how the teaching of mathematics should be carried out, and what specifically the teacher should do. Brownell summarized this relationship between the nature of mathematics (arithmetics in particular), learning and teaching as follows:

> ... arithmetic consists of a vast host of unrelated facts and relatively independent skills. The pupil acquires the facts by repeating them over and over... until he can perform the required operation automatically The teacher need give little time to instructing the pupil in the meaning of what he is learning. (Brownell, p.2, 1935)

However, although Brownell's approach to teaching structural meaning included the use of concrete materials and practical applications ((McKillop and Davis, p.16, 1980), to be consistent with the general absolutist/behaviourist framework, this was interpreted as the teacher doing the modeling while the students continued to be passive observers. The emphasis was maintained in language based teacher transmission of information, with the students' activity mimicking or copying what had been modeled by the teacher.

The teacher-centered perspective of teaching restricts the teacher's behaviour to telling, i.e. the teacher is given authority over the mathematical content. Thus, instruction constitutes a linguistic domain of discourse with its own culture, and insures the teacher-centered conversation (Richards, 1991). With the teacher and textbook as authorities, interactions are reduced to emphasis on the transfer of knowledge. Thus teacher talk exceeds student talk; teacher has whole class focus rather than focus on individuals; teacher is in control of class time; and teacher has importance as actor with students as audience, seats arranged in rows like a theater (Cuban in Gregg, p.48, 1992). This allows for transfer of mathematical knowledge as absolute truths and has the expectation of seeing student development as mastery of these truths.

From within the framework of the traditional mathematics classroom, there appears to be sound rationale for the mode of operation. But, these occurrences have had adverse effects on many students that eventually led to a challenge of the status quo, and also to recommendations for reform of that classroom.

APPENDIX B

TASK, DISCOURSE AND THE ENVIRONMENT IN THE REFORM MATHEMATICS CLASSROOM

B.1 Introduction

The reform mathematics classroom is significantly different than the traditional classroom. While the latter has a long history from which to discuss its characteristics, the former is only now beginning to slowly become a reality. This is especially true of the reform environment as it might be found in the high school grades. As a consequence, at the present there is more information about the reform classroom from a theoretical than a practical level. This is because school curricula have only recently started to change to reflect the reform movement, and jurisdictions are in the process of making the changes mandatory. In Alberta, for example, the mathematics curriculum, as prescribed by the Western Canadian Protocol, is strongly based on the N.C.T.M. Standards (1989, 1991, 1995) and the resulting changes will be implemented in different stages. Elementary grades began implementation in 1997, Junior High began in 1996 with continuation in 1997 and Senior High has planned implementation from 1998 to the year 2000.

Because of this time frame, one of the ways in which the reform classroom can be understood is in terms of what is currently being prescribed by the N.C.T.M. Professional Standards (1989,1991,1995). This appendix is a summary of the recommendations from those Standards regarding task, discourse, and environment (so as to be consistent with the objective of this study). It reflects the philosophical perspective of the reform classroom as it is seen from within a review of available literature and relevant research.

The N.C.T.M. Professional Teaching Standards (1991) define task, discourse, and environment in relation to the central focus of the reform of mathematics education, that being the *development of mathematical power for all students.* Within this document it is noted:

> Mathematical power includes the ability to explore, conjecture, and reason logically; to solve non-routine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use qualitative and spatial information in problem solving and in making decisions. Students' flexibility, perseverance, interest, curiosity and inventiveness also affect the realization of mathematical power.
> (NCTM, p.1,1991)

This requires a radically different nature for task, discourse and environment from those found in the traditional classroom. According to NCTM (1991), it requires a shift:

- toward classrooms as mathematical communities- away from classrooms simply as a collection of individuals
- toward logic and mathematical evidence as verification- away from the teacher as the sole authority for right answers
- toward mathematical reasoning- away from merely memorizing procedures
- toward conjecturing, inventing, and problem solving- away from an emphasis on mechanistic answer-finding
- toward connecting mathematics, its ideas and its applications- away from treating mathematics as a body of isolated concepts and procedures (p.3)

B.2 Tasks in a Reform Classroom

The N.C.T.M. Standards describe tasks as projects, problems, exercises, constructions, applications and so on (1991, p. 24). These tasks should be based on:

- sound and significant mathematics,
- knowledge of students' understandings, interests and experiences,
- knowledge of the ways that diverse students learn mathematics.

They should also:

- engage students' intellect,
- develop students' mathematical understandings and skills,
- stimulate students to make connections and develop a coherent framework for mathematical ideas,
- call for problem formulation, problem solving, and mathematical reasoning,
- promote communication about mathematics,
- represent mathematics as an ongoing human activity,
- display sensitivity to, and draw on, students' diverse background experiences and dispositions,
- promote the development of all students' dispositions to do mathematics. (Standard 1, p.25)

In general, tasks reflect four characteristics from the Curriculum and Evaluation Standards, i.e. mathematics as problem solving, connections, communication and reasoning. Thus, significant emphasis is placed on tasks involving: a) talking and writing about mathematics, b) inductive and deductive reasoning, c) application within and outside of mathematics, and d) open ended problem solving investigations. Examples of these kinds of tasks for various grades are included on pages 141-44.

B.3 Discourse in the Reform Mathematics Classroom

As presented in the N.C.T.M. Professional Standards (1991), discourse will be found divided into two distinct areas as the teacher discourse, and the students' discourse. Regarding the first of these, the Standards say that the teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their answers orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to when to model, when to lead, and when to let the student struggle with a difficulty. (Standard 2, p. 35)

Regarding students' role in discourse, the teacher of mathematics should promote classroom discourse in which students-

- listen to one another, and question the teacher and each other,
- use a variety of tools to reason, make connections, solve problems, and communicate,
- initiate problems and questions,
- make conjectures and present solutions,
- explore examples and counter examples to investigate a conjecture,
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures and answers,
- rely on mathematical evidence and argument to determine validity. (Standard 3, p. 45)

The tools of the mathematics classroom are changing and so that the teacher of mathematics can enhance discourse, he or she should encourage and accept the use of:

- computers, calculators, and other technology,
- concrete materials used as models,
- pictures, diagrams, tables and graphs,
- invented and conventional terms and symbols,
- metaphors, analogies, and stories,
- oral presentations and dramatizations,
- written hypotheses. explanations and arguments. (Standard 4, p.52)

The Vignette #2 (page 145) is given as an example of the difference in the discourse as it might have been expected to occur with the teacher and students in a reform classroom. The same mathematics topic applied in Vignette #1 (p.119) is also used there to facilitate comparison of the traditional and reform contexts. It should be noticeable that the student is encouraged to take much more prevalent role in the conversation, and the language used by the teacher attempts to enhance and continue the discussion.

B.4 Reform Mathematics Classroom Environment

Finally regarding the environment in which teacher and student are involved in the study of mathematics, the expectations based on the NCTM Standards (1991) are that the teacher of mathematics will create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematical concepts and grapple with significant ideas and problems,
- using the physical space and materials in ways that facilitate student learning of mathematics,
- providing a context that encourages the development of mathematical skill and proficiency,
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions.

And by consistently expecting and encouraging students to:

- take intellectual risks by raising questions and formulating conjectures;
- work independently and/or collaboratively to make sense of mathematics;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument (Standard 5, p. 57)

The attempt to operate in a reform classroom, with the classic traditional settings of rows of desks separated from each other and distinguishable from that location which defined the 'teacher space', would seem counter productive. Student mobility, at one point in time, and proximity, at other times, needs to be seen as part of a different classroom arrangement for student working spaces, in addition to other teaching tools required in the classroom. The teacher can also be seen as being very mobile and not restricted to, or in control of, that teaching space (which would have previously been easily recognizable to the outsider). This environment will need to determine the nature of physical objects (like computers, concrete materials, posted student work, etc.) which contribute to the student comfort with the learning environment.

As well as considering the physical changes, which may need to be addressed in the reform environment, there is also a need to consider the nature of the social and emotional environment of the classroom. Risk taking by students brings with it the expectation for the creation of a trusting and respectful environment, not only from the teacher's stance, but also from the students' reactions to each other as well. The individual will need to learn to take responsibility for himself, or herself, compared to the traditional classroom in which the teacher took responsibility for all, by setting all of the procedures and the expectations for the mode of operation.

<u>SAMPLE TASKS</u>

<u>Example 1</u> [Grade 6]

<u>Project: Moving Away From Home.</u>

This is a fictitious situation in which the student will be pretending to be old enough to move out and live on their own or with some other students. The objective of the project is to put mathematics into a real world connotation with real world applications. Aspects of the project have been chosen to model as closely as possible those real world experiences, which could occur if this scenario were to actually take place.

As part of the unit on the study of decimals, and operations involving this group of numbers, you will be working on a project simulating a student moving into an apartment with some other friends. You will be taking on a part time job as a grocery packer with a Calgary COOP store and be receiving a weekly paycheque. From this paycheque will be deducted the appropriate amounts relating to: income taxes, unemployment insurance premiums, health care, etc. Each week you will calculate the different deductions mentioned and make out you own paycheques, to be signed by the teacher. As well, you will be responsible for maintaining you own bankbook with deposits and withdrawals. You will find and pay for: rental of an apartment, utilities (telephone, television, heat etc.), food and associated costs of keeping an apartment (as you determine the costs of items such as those classified as luxury items), and finally set aside the savings portion of each cheque (as you build an account to attend university).In preparation for moving out, you will search the newspapers to find potential accommodation. Your group can equally share the costs of damage deposit, hook up fees, monthly food and apartment costs. All information relating to these costs will be kept in a journal in which you are also asked to comment about the experiences on a regular basis. The journal should be organized in identifiable sections covering each of the areas mentioned and each person is *responsible for their own journal. Every Friday, with the role of the dice, events will occur in which you may be given over time opportunities, lose days to illness, lose money from your wallet being stolen, get a donation of money from your grandpa, have to buy gifts for special events like birthdays etc. This will create the scenario for interruptions to the expected, as happens in real life. As part of knowing the real world we will be doing a field trip to a major grocery chain to do comparative shopping in setting up house in the new apartment. Your group will get a total of $150.00 to do the shopping.*

<u>Example 2</u> [Grade 7]

<u>Application</u> : <u>Understanding Addition of Integers</u>.

In an introduction to the use of integers in the world students are encouraged to share their personal experiences and understanding of integers. These may include topics such as temperature or bank accounts. The class can then continue with the following investigation.

You will be given two sets of ten Unifix cubes in two different colors. The colors are not important. Let one color represent positive numbers and the other one represent negative numbers. Your job is to use the cubes to explain the ***addition*** *of positive and negative numbers. Write your explanation and include diagrams showing different cases.*

<u>Extension</u>: If you can see a use for the blocks to understand the operation of addition, how might they be used to understand subtraction?

Example 3 [Grade 9]

Application : Factoring of trinomials in the form "x^2+bx+c"

Students' previous experiences with algebra tiles showing the multiplication of two binomials can be linked to the multiplication of two numbers in our base ten system. It may be desirable to model an example with the base ten materials to assist in the connection. A discussion with students should emphasize the term factors, and how they are identifiable in the model, in a diagram, or in the symbolic notation. How the factors relate to the shape of the rectangle needs to be clear in the students' minds before moving into the abstract concepts created by the algebra.

With the set of algebra tiles provided, your task is to investigate different rectangles, which represent the multiplication of two binomials. You will be working only with the sides that are all the same color. The other, colored, side allows for negative representations of "x^2", "x" or a constant like "5", but we will leave those for another day. Look for patterns which would assist in finding the FACTORS of the rectangles (which are representative of factors of the trinomials). Keep a record of those shapes that generated the rectangular array and what factors would be associated with the shapes. Can you make a generalization of the patterns which would allow you to work the problem from a symbolic notation?

Example 4: [Grade 11]

Investigation: Transformations of the basic functions.

The students may have to be reminded how geometric transformations were investigated as flips, slides and turns in previous grades. They may also want to discuss how they see these transformations within a context outside of the classroom (for example, as applications of the concepts in tessellation art). It might be helpful to introduce the terminology of translations, dilatations, and reflections before starting this investigation. This task requires *access to a computer classroom environment and the public domain software of Transformations of Functions written by Francis Sommerville of the Calgary Public School Board.*

Once operational within the program, you will notice that you have a choice of functions available for study. Start with the basic quadratic, $y = a(x - k)^2 + p$. By moving the scroll bars representing the constants 'a', 'k' and 'p', you can select values from a range of numbers and; graphs will be drawn for you for each set of selected values. By working with each of the constants separately, your task is to determine the effects of the changes to the graph with each constant independently. You will want to keep a record of your findings, which you should be prepared to discuss with the class.

Extension: *if you have time or are ready to proceed, you may change more than one value in the function simultaneously but again look for patterns in the effects of the changes. Keep a record of anything that allows you to predict the nature of the graph. Next class we will look at other functions available in this program including the reciprocal, the cubic, the exponential and the square root.*

Each of these tasks, projects and applications has been chosen to serve as an example of how a reform classroom would interpret the engagement of students into a mathematical activity; and around which significant mathematics would be expected to be learned.

<u>Vignette #2</u> (written by J.H.Orsten)

REFORM CLASSROOM DISCUSSION

As the students enter the classroom, the teacher hands them a prepared page with some partially completed tables and empty graphing sections. They talk quietly as they assume their seats, prepare their required materials, and wait for the teacher to start.

Teacher: Today's lesson will develop the idea of one of the transformations of a function. The model we will use today is based on the quadratic function. Can anybody tell me from previous experience what graphical representation is characteristic of the quadratic?

As a number of hands are raised to respond, the teacher asks Maya to give her answer and also to come to the overhead projector and show the graph she would expect for the equation $y = x^2$. She completes a table, in which the values of "x'" are already written on the left of a table, and as she plots points from the completed chart she responds that she recognizes the shape as a parabola.

Teacher: Does anyone have a different idea other than parabola? No, so you all agree with Maya and her picture on the screen.

Aaron: But what would happen if the equation didn't have a squared value? What if it had another exponent like 3 or 4?

Teacher: Does anyone want to respond to Aaron? Yes, Julia.

Julia: When I was playing with my graphing mode calculator, I tried to see what the graphs of different algebra looked like but it gets very confusing to determine the other shapes.

Teacher: That's true Julia, the procedure for identifying the character of other graphs is a study for grade 12 and is more difficult than it first seems. We could take a look at some of the other *conditions after we have established some understanding of the "x" squared condition first. We will leave that for next year or at least until we have completed our introductory work with thex^2 condition.*

Commenting on the expressions shown on the overhead screen, she continues the discussion by referencing three different equations $f(x)= x^2$, $f(x) = 2x^2$ and $f(x) = .5x^2$ as they appear at the top of three charts.

Teacher: Thank you Julia, would you like to continue as our overhead writer?

Julia seems comfortable in her present role and so the teacher continues.

Teacher: We are interested in the impact of the different coefficients on the graph of $f(x) = x^2$. Are you clear as to what is meant by coefficient?

A student responds "Is that the value that is added after the x^2?"

Teacher: Can anyone answer John's question?

Saleem: Well, yes it could be that a constant value is being added after the squared term, but this time it means the constant that is being multiplied in front of the x^2.

Teacher: What do you think about Saleem's explanation? Any discussion or other comments? No, that must be because we all agree with Saleem's explanation. Then let me add the following information. We will use the term coefficient when it precedes a variable , and is being multiplied , and the term constant when it follows after the variable, either being added or subtracted. Any other concerns before we continue?

There is a short pause while the teacher waits for any students to raise other questions.

Teacher: All right. Please read the instructions at the top of the work page and complete the remaining charts and corresponding graphs. You may work on this in pairs or individually. Julia would you please complete the chart on the overhead for us, Thank you.

The students begin to work with calculators to complete the three tables of values that have been prepared for them on the work page, and they can be seen to be making sketches of the corresponding graphs on the graph spaces beside each table. The teacher gives the students a ten minute time frame to complete the calculations.

Teacher: You should have had enough time to complete the charts and make the appropriate sketches by now. Are you ready to discuss your findings?

Most of the students affirm they are done at which point the teacher turns their attention to the charts on the overhead projector. The teacher asks Julia to point to the second chart and then questions:

Teacher: Does anyone have any values different from the ones appearing on the first table which was described before by Julia? Good, then what about the second chart?

Matthew: Our chart has a value of 16 beside $x=2$ *in the* $2x^2$ *table.*

Teacher: Does anyone have a possible reason why Matthew's value is different?

Sheri: I think he forgot about the BEDMAS rules for order of operations. He should have squared first and then multiplied by 2.

Teacher: Does Sheri's explanation make sense to you Matthew?

Matthew: Oh yeah, if I square it and then multiply by 2, I get 8 like your chart.

He and his partner correct their values to match the ones on the screen.

Teacher: If you compare the second graph to the very first one of x^2, what conclusions would you make about the effect of the coefficient 2 on the graph.

Sally: The graph seems to be skinnier when you multiply by two.

Teacher: Correct, that's one way of putting it. How would you then describe the effect of the 1/2 on the graph of x^2 in comparison?

Greg: The graph gets fatter or wider than the first one.

Teacher: That terminology describes the actual picture but we have more precise language to discuss the impact of these coefficients. We call these values expansion factors and compression factors. Which of these so you think best describes the second and third examples, the ones which are affected by the values of 2 and 1/2?

Marita: I think that the 2 compresses the graph and the 1/2 expands the graph because compressing would be thinner and expanding would make it look bigger.

Teacher: That's very possible Marita, is there any problem with this explanation or another possibility?

Ken: If the number is being multiplied times the f(x) then multiplying by 2 would seem to me to be expanding and taking 1/2 would seem to be compressing the function, and not the graph.

Teacher: Now that we have two different opinions on the floor, there will be a need to decide which would be the description which we can agree upon. Anyone want to make a suggestion about the choice?

Marita: I want to change my idea and use Ken's instead because he gave a good explanation which I thinks fits the conditions.

The teacher waits for any further conversation to emerge but mostly sees nods of approval with Marita's acceptance of an alternate viewpoint. Then she continues with an introduction to the language necessary for this investigation.

Teacher: What we have been looking at here are examples of two possible types of transformations contained within one of the three possible categories of reflections, dilatations, or translations. Expansions and compressions are examples of dilatations. So we call the dilatation factor the value of 'a' , but I have another question. Do you think there is a generalization which can be made about the value of a > 1 compared to values of a between 0 and 1?

Brian: I guess that if the number is greater than 1 then it is an expansion factor and if the value is between 0 and 1 then it is a compression factor.

Teacher: What determined what expansion meant compared to compression in your estimation of these words, Brian?

Brian: I called it expansion if it made the graph goes higher faster and compression if the graph was squashed closer to the x-axis."

Teacher: That would be a good explanation of the changing values for 'a' and is going to work for positive values of x. Next we will add one more complication to the condition. Marita, you can go back to your work group if you would like. I'll take over as writer on the overhead . Then the next question is, what do you think the effect of a negative number will be as a coefficient? Anyone want to make a prediction?

Ken: I think that the negative won't have any effect because squaring a negative number is the same as squaring a positive number.

Teacher: Well, let's leave the answer to that until after the investigation.

The teacher hands out a second prepared work page on which the students find the same format with the values pre-chosen on the 'x' side and blanks for the students to fill in on the f(x) sides. Beside each table is a grid on which the students can sketch the graphs after they have completed their calculations. The new charts have the expressions $-1/2\,x^2$, $-2\,x^2$ *and* $-1.5\,x^2$.

Teacher: Again with your calculators would you work out the values for f(x) and sketch each of the graphs on the grids provided. You should be able to discuss your findings in about 10 minutes.

The students return to their individual work and, after the time has elapsed, the teacher directs their attention once more to her completed charts on the overhead screen.

Teacher: How would you describe the impact of the negative values on the function of f(x) = x2?

Mary: It seems like the graph is upside down now.

Teacher: That would be a possible way of explaining it. Did the values of 1/2 or 1.5 or 2 have the same impact as before?

Ken: Yes they got skinnier and wider just like before.

Teacher: You mean that the -2 was an expansion factor and the -1/2 was a compression factor similar to the first cases. How do you think we can mathematically define the impact of the negative on the a-value?

Mary: I think we could call the impact of the negative a reflection because it is like a mirror image reflected over the x-axis.

Teacher: I like the word reflection that you used because that is the term which we want to apply to describe that change in the graph. Now we have two different impacts of a changing value of 'a' in the original of y = a [f(x)]. Hopefully at this point you are be able to predict, without making a table of values, how a sketch of a parabola will look for an equation like f(x) = -3x2. I would expect that you would be able to give a verbal description of how a graph of f(x) = -3/4 x2 would look in comparison. Next class we will be investigating the third kind of transformation which is called the translation. For homework would you please do exercises page 102: numbers 1,2,3,4,6, and 8 parts a, b, and c from each question.

The students open their texts to the page, there is only a little discussion within this classroom but most students begin working independently on their work. The teacher returns to her desk to start marking and then, while the students are attempting to apply their new knowledge she adds:

Teacher: If you have any questions, you may bring them up to me for clarification and assistance, or you can check with someone sitting near by. Calculators are allowed for this assignment and will be allowed on tests regarding this topic.

The remainder of the class is fairly quiet as the students work, but does have ongoing conversation in small groups. When the bell rings the students collect their materials and break into much louder conversations as they exit the room.

As can be seen from the conversations above, the reform emphasis is not on the teacher talk, but equally, if not more so on the student talk. This may be oral, or written, but it is an opportunity in which student ideas are given credence and respect as the students attempt to validate their opinions through the development of new mathematical skills. Correctness is not the prime requisite of answers, or responses, as much as is variation of ideas and acceptance of those variations. Within the constraints of the topic under investigation, some ideas pass the test of verification, some may require alterations to the ideas, and still others may have to be discarded; but it is the student who decides how and what to discard, not the teacher. There is an opportunity for, and an expectation that, students will use mathematical language where appropriate as they hear that language modeled by the teacher. There should also be an acceptance of non-formal or invented language as students move into unfamiliar territory, and in preparation for formalization. Although not present in the vignette, there are times to the outsider that discourse in the classroom may seem out of control and in total chaos. But again if risk taking is to be encouraged amongst the students, then during these times the enthusiasm must be allowed to overflow the normal guidelines of conversation. The ultimate result of any and all discourse must eventually focus on development of meaning, and of making sense of the task and its related mathematical concepts. To achieve this purpose the environment in which task and discourse occur becomes a very salient feature of the reform classroom.

IMAGE EVALUATION
TEST TARGET (QA-3)

150mm

6"

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