## THE UNIVERSITY OF CALGARY

THREE-DIMENSIONAL ROTATION MEASUREMENTS FOR INDUSTRIAL ALIGNMENT

BY<br>NEDAL NAIM AL-HANBALI

# A THESIS <br> SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN ENGINEERING 

## DEPARTMENT OF GEOMATICS ENGINEERING

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Three-Dimensional Rotation Measurements for Industrial Alignment" submitted by Nedal Naim Al-Hanbali in partial fulfilment of the requirements for the degree of Master of Science in Engineering.


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#### Abstract

The ability to precisely measure the rotations of a point in three dimensions, for hot (operating) alignment situations, is an important and difficult task in industrial alignment. In this thesis, the development and lab testing procedure for a low cost, precise method to measure three-dimensional rotations of a point on a structure, for both cold and hot alignment, is explained and discussed. Three applications, two of which are for hot alignment, are also described.

The method is based on the autoreflection principle and uses the resection method. A one arc second theodolite equipped with an automatic vertical circle compensator as well as two optically-flat, front-silvered mirrors are required to achieve a 6 arc second accuracy. Investigation of this method shows very good reliability and stability under different operational conditions.

The research work also involved a thorough investigation of an electrolytic level tiltmeter. The results, recommendations, and suitable applications for this tiltmeter are discussed.


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## DEDICATION

To my father and mother: Naim and Amal, who are always there when needed: to listen, to lend a hand, to understand, and to care.

To my great teacher Adnan Al-Nahwi, who taught me how to set my goals in this life, to be a good Muslim, and to be a clear thinker.

To my wife, Maha, my partner in life, for her patience and moral support.

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## Chapter 1

## INTRODUCTION

The ability to precisely measure movements of a point in three dimensions is very useful and important in industrial alignment, especially in monitoring hot (operating) alignment changes. Normally, rotations (tilts about horizontal $X$ and $Y$ axes and rotation about a vertical Z-axis) and translations are the key elements used to describe movements of a point in or on a structure. In this research, monitoring rotations of a point on a structure was of interest. This led to the development and lab testing procedure for a low cost, precise method to measure tilts and rotations of a point on a structure.

Originally, the main concern was directed towards choosing a proper instrument to precisely measure tilts of a point. Teskey (1991) used a one arc seconds theodolite with automatic vertical circle compensator as a tiltmeter. The idea worked well except when tilts exceeded the instrument range ( $\pm 2$ arc minutes). This problem led to the use of an electrolytic level transducer tiltmeter which has a higher range, a higher resolution, and it is capable of continuous tilt measurements.

A survey was done to cover all the available instruments and methods used to measure rotations or tilts. These instruments fall into two main categories. The first category is the tiltmeters and the second is the survey instruments: theodolites and levels. Details of these instruments are given in Chapter Two.

The electrolytic level tiltmeter appeared to be a suitable instrument for measuring tilts because of the capabilities mentioned above. A thorough
investigation of an electrolytic level tiltmeter was therefore carried out. Details are given in Chapter Three.

The investigation of the electrolytic level tiltmeter focussed on four areas: linearity, repeatability, drift, and temperature sensitivity. The autocollimation method (see Kissam 1962) was used as a more accurate method for calibration purposes. The investigations show that the tiltmeter cannot be used under variable temperature conditions, which is the case in many industrial applications.

Afterwards it was realized that autocollimation method itself could be used to measure tilts. The method has no range limits and provides high accuracy measurements which are not sensitive to temperature variations. It does, however, have some problems related to ambient lighting and vibrations. This led to the use of the autoreflection method (see Kissam 1962). Here it should also be noted that use of the autocollimation or autoreflection method enables one to also measure rotations about a vertical Z -axis by resecting the theodolite within a reference network.

In Chapter Four, the procedure for the autocollimation and autoreflection methods, the difficulties encountered, and investigations of an error control are discussed and explained. Temperature effects on measurements are also discussed. The method requires a one arc second theodolite (such as Wild T2) equipped with an automatic vertical circle compensator, as well as two optically flat, front silvered mirrors in order to achieve a 6 arc second accuracy for threedimensional rotations.

Chapter Five focusses on developing, verifying, and lab testing the mathematical model for three-dimensional rotation measurements by the autoreflection method. Computer programs implemented to compute threedimensional rotations, as well as refraction and vibration, are also discussed in

Chapter Five. The method is shown to be a low cost, precise, reliable solution to the problem of measuring three-dimensional rotations.

In Chapter Six three applications are described. Two of the applications involve the monitoring of hot (operating) alignment changes in a turbine/compressor combination. The third application is the measurement of tilts and rotations at different points on a buried pipeline. Conclusions and recommendations are given in Chapter Seven.

## Chapter 2

## PRECISE TILT MEASUREMENT INSTRUMENTS

Rotations (tilts around horizontal $X$ and $Y$ axes and rotation around a vertical $Z$ axis) and translations are key elements used to describe the movements of points in or on a structure. The subject of this chapter is an investigation of tilt measurement instruments.

A survey has been done to cover all the available instruments and methods used to measure tilts. These instruments fall into two main categories. The first category is tiltmeters and the second one is survey instruments: theodolites and levels. Both the tiltmeters and the survey instruments can be either mechanical or electrical. More details are given in the following sections.

### 2.1 TILTMETERS

A tiltmeter uses a gravity sensor. The sensor is installed inside a suitable housing. In case of an electronic tiltmeter, part of the sensing device is the transducer which converts any physical change into a corresponding output signal. Automatic data acquisition systems are available for electronic tiltmeters for cases in which continuous tilt measurements are required.

Tiltmeters are used in monitoring tilts in dams, mines, volcanoes, power plants, bridges, and other industrial and scientific facilities. In the Industrial Alignment Project, an electrolytic level tiltmeter has been used to monitor tilts for specific points on buried pipelines.

The precision of a tiltmeter can be expressed either in radians, or arc seconds. Different kinds of tiltmeters are explained briefly in the following sections. Precision and resolution as well as drift and environmental effects (such as temperature variation) are also discussed.

### 2.1.1 Mechanical Tiltmeter

The principle of a mechanical tiltmeter is based on measuring the change of height over a well controlled distance. The tiltmeter consists of a beam with a level bubble and a sensitive dial gage indicator mounted at one end of the beam. Figure (2.1) shows a simple mechanical tiltmeter.


Figure (2.1) : Simple mechanical tiltmeter

The beam is levelled over two reference points using the level bubble. The dial gage indicator monitors any changes in level. The tilt is the measured change in the dial gauge reading divided by the length of the beam.

The zero shift error can be eliminated by taking reverse tiltmeter readings. This will only work if there is a good force centring system installed on the reference points, so that the tiltmeter can be remounted in exactly in the same position. A force centring system can be achieved by using a reference ball affixed to an anchor bar.

Precision of the mechanical tiltmeter depends on the precision of the dial gauge used. Dunnicliff (1988) states that the precision of the dial gauge in a mechanical tiltmeter of 200 mm beam length is approximately $\pm 0.013 \mathrm{~mm}$ which results in a tiltmeter precision of $\pm 13$ arc seconds.

The writer believes that it is very important for the surrounding temperature to be stable within a few degrees for the different measurement epochs; otherwise, the precision of the dial gauge readings will be adversely affected.

### 2.1.2 Tiltmeter with Vibrating Wire Transducer

This tiltmeter is a pendulum, connected to two vibrating wire transducers. When tilt occurs, the pendulum rotates to a new position, causing one of the two vibrating wires to be stretched and the other shortened. This change in length is equal for both wires and opposite in sign. The change of the wire length divided by its length is uniaxial strain. If the strain is measured, the tilt can be calculated. Such a tiltmeter is shown in Figure (2.2).

The frequency of vibration of the wire varies according to the strain, and the relationship is well known from physics. Therefore, measuring the frequency at given epochs will allow one to calculate tilt.


Figure (2.2) : Vibrating wire tiltmeter


Figure (2.3) : Vibrating wire transducer

Figure (2.3) shows the vibrating wire transducer. The steel wire is clamped at one end to a fixed point and at the other end to the pendulum. The wire is tensioned to vibrate at its natural frequency when there is no tilt. An electrical coil is attached near the midpoint of the wire. The electrical coil develops a magnetic field which forces the wire to vibrate. Either the same coil or another one measures the frequency of vibration of the wire. Frequency measurements can be discrete or continuous.

If tilts are required in two perpendicular planes, four vibrating wires will be connected to the pendulum. In each plane, two wires are connected as shown in Figure (2.2). Dunnicliff (1988) states that such tiltmeters have a ranges from $\pm$ 0.1 to 1 degree, with a precision approximately $0.5 \%$ of range, i.e. is $\pm 2$ to $\pm 20$ arc seconds. Temperature effects and drift problems are eliminated due to the fact that corrections will be nominally equal and opposite for two opposing wires.

### 2.1.3 Tiltmeter with Accelerometer Transducer

This tiltmeter is based on a force balance accelerometer: measuring the force required to keep a pendulum from tilting under the force of gravity allows one to compute tilt.

Figure (2.4) shows how a force balance transducer operates. The mass is suspended in a magnetic field of a position detector. When any tilt occurs in the mass, the position detector will sense the motion. Indeed, this motion will change the current in the position detector. This current change is fed back through a servoamplifier to the restoring coil. The restoring coil then generates an electromagnetic force opposite to the gravity force and the mass is kept in the same position. The voltage across a precision resistor measures the induced current change. The voltage is directly proportional to the input force.


Figure (2.4): Force balance transducer

The tiltmeter requires a base plate to establish a reference plane over the point of interest. Anchoring the base plate to the interest point will allow one to take any number of repeated and reverse readings.

The tiltmeter is portable and easy to handle. It does, however, suffer from drift problems. It is also very sensitive to temperature effects, to such an extent that temperature should not change more than a few degrees when tilt measurements are made.

Dunnicliff (1988) gives two examples of such tiltmeters. The first one has a range of about $\pm 30^{\circ}$, but has a precision of about $\pm 50$ arc seconds, with temperature sensitivity of $2-3$ arc seconds per F-degree. The second one has a range of about $\pm 3^{\circ}$, and has a precision of $\pm 1$ arc second, but it is only recommended for use under constant temperature conditions.

This tiltmeter senses tilt with an electrolytic level, similar to a spirit level. When tilt occurs the electrolytic level will produce changes in resistance in response to a rotation of the sensor. The sensor consists of a sealed glass vial filled with a conductive liquid. The vial must be made by using a vacuum forming technique to ensure high precision and low temperature sensitivity. This manufacturing procedure, however, makes the price of the tiltmeter high. There is a cheaper vial from standard glass tube made by softening the glass and bending it to the required radius. This manufacturing procedure creates irregularities in the inside diameter of the tube which reduces the precision and increase the temperature sensitivity dramatically.


Figure (2.5) : Electrolytic level transducer

Figure (2.5) shows the glass vial and how the change in tilt will change the resistances between points $A, B$, and $C$. These resistances act as two arms of a Wheatstone bridge circuit which senses the tilt.

There are several manufactures who produce tiltmeters with electrolytic
level transducers. Dunnicliff (1988) gives specifications of two types, which both use a transducer manufactured by Spectrum Glass and Electronics, Inc. The first one is the Sperry Tilt Sensing System, manufactured with ranges from $\pm 20$ arc seconds to $\pm 45^{\circ}$, with the repeatability of the $\pm 20$ arc seconds version at $\pm 0.3$ arc second. A console and recorder are available for monitoring multiple tiltmeters, and threshold levels can be set individually to provide hazard warning. The second type is manufactured by Applied Geomechanics, Inc. It demonstrated stability of $\pm 0.3$ arc second in a 4 day test. Firm figures for longer term stability are not available. These tiltmeters can be a either single or dual axis. Temperature effects are large; therefore some form of temperature compensation is required.

An electrolytic tiltmeter manufactured by Applied Geomechanics was used to measure tilts in a buried pipeline. Inconsistency in the tiltmeter readings were noticed which led to the investigation of the instrument in terms of repeatability, linearity, drift, and temperature effects. Details of this investigation are given in Chapter Three.

### 2.1.5 Silica Tiltmeter with Photoelectric Cell Transducer

This device is a horizontal pendulum, enclosed in a vacuum chamber. It consists of two silica fibres, stretched vertically, which carry a silver mass. A melted silica frame is used for fixing the silica fibres and transmitting any ground movements to the pendulum; see Figure (2.6).

The transducer is a displacement detector, consisting of a diode and photoelectric cell. The silver mass is located between the diode and the photoelectric cell and has a small hole (window) exactly in line with the diode. The lighted diode will form a spot on the photoelectric cell when diode, window, and cell are in line; See Figure (2.7). A magnetic damper near the mass dampens
the oscillations.


Figure (2.6) : Silica tiltmeter (From Salem 1991)


Figure (2.7) : Photoelectric cell transducer

When the silica frame tilts, the silica fibre transmits the tilt to the pendulum. The result is a rotation of the mass and a displacement of the spot on the photoelectric cell; see Figure (2.8). The displacement is related to the amount of voltage sensed on the cell. The tilt, from the equation of pendulum's motion, is related directly to the displacement on the cell, the natural period of the pendulum, and the distance between the pendulum rotation axis and the cell.


Figure (2.8) Spot displacement and oscillation

According to Saleh (1991), different values for the natural period of the pendulum can be chosen in the calibration process depending on the required resolution and range. The longer the time period, the better resolution and the lower range. For example, a 15 second period will provide resolution of 0.008 arc second and a range of about $\pm 20$ arc seconds, while 3 second period will provide resolution of 0.2 arc second, and a range of about $\pm 7$ minutes.

Saleh (1990) indicates that the instrument can provide 0.1 arc second precision and has a drift of only 0.062 arc second / year. The instrument is suitable
for continuous monitoring. Saleh (1990) used the instrument to detect the effects of the temperature variations on different types of structures. He did not, however, discuss temperature effects on the tiltmeter itself. The writer would expect that the tiltmeter is very sensitive to temperature variations.

### 2.1.6 Mercury Tiltmeter with Air Capacitor Transducer

This device is a liquid balance gauge consisting of two plastic cups, partially filled with mercury, connected by pipes. The distance between the two cups is D. A metal disc is positioned in each cup. Each disc can be screwed down until the spacing between the mercury level and the disc is 0.5 mm . The space is filled with air and serves as a capacitor; see Figure (2.9). The baseline (D), the spacing, and the dimension of the cups are variable depending upon the kind of application, and the degree of precision required.


Figure (2.9) : Mercury tiltmeter

When this arrangement tilts, the mercury level will change in the cups. The tilt will change the capacitances differentially, and the change can be sensed electronically. The tilt is the change in mercury levels in the two cups divided by the baseline distance (D) of the instrument.

Refsum (1988) states that the device must be set up level on a very solid foundation. The tiltmeter can detect tilts in the order of $10^{-9}$ radians ( 0.0002 arc$\mathrm{sec})$. The author did not discuss the precision of the instrument, drift, or the temperature effects. The writer believes that temperature effects on the instrument would be quit large.

### 2.2 SURVEY INSTRUMENTS

### 2.2.1 Precise Level

A level is an instrument which measures the height differences between points. Although both automatic levels and tilting levels can be used for precise levelling, automatic levels are preferred because they can be set up more quickly and are easier to use.

The automatic level has a gravity-referenced prism or mirror compensator to automatically orient the line of sight (line of collimation). Once the bubble of the circular spirit level is centred, the instrument will automatically maintain a horizontal line of sight, even though the instrument is slightly tilted.

Automatic levels can be mechanical or electrical instruments. Cooper (1982) states that both can maintain a horizontal line of sight to at least 1 arc
second. In precise surveying, these levels are equipped with parallel plate micrometers so that the reading accuracy is 0.05 mm . Invar levelling rods must also be used to insure a precise scale which is not affected by temperature variations.

In essence, tilt or rotation between two points is the change in their heights from one epoch to another divided by the distance between the points. Tilt or rotation of a rigid body can therefore be found by measuring the change in height for two points on the rigid body.

In terms of accuracy, if the two points are 1 m apart and the reading accuracy of precise level is about 0.05 mm , the accuracy of the tilt measurement is 10 arc-seconds. There is no limit for the maximum range, except for the fact that one must be able to obtain rod readings.

### 2.2.2 High-Precision Theodolite

A theodolite is a surveying instrument with great versatility; with it one is able to precisely measure horizontal and vertical angles. There are a wide variety of theodolites, differing from each other slightly in design and in reading accuracy. The development of the theodolite occurred over the past century. Before the 1970s, the basic design of the theodolites remained unchanged for over two decades. Recent advances in electronics have resulted in the development of electronic theodolites.

### 2.2.2.1 Automatic Vertical Circle Compensator

The Automatic Vertical Circle Compensator (AVCC) is a device which automatically indexes the vertical circle to zero. It is referred as a 'compensator'
because it compensates for the residual tilts of the vertical axis of the theodolite. The AVCC works only if the plate axis of the theodolite is levelled to within a few minutes of arc.

Two methods can be used to accomplish automatic vertical circle compensation. The first method uses a pendulum to define the direction of gravity (vertical or the plumb line). The second method uses the surface of a liquid to define the local horizontal plane and thus the vertical direction. Further details are given in the following paragraphs.

Pendulum compensator. Vertical is defined by the arm of a free damped pendulum. Figure (2.10) shows the components of such compensator: a pendulum support (1), a pendulum arm (2), a damping system (3), a lens (4) suspended at the end of the pendulum arm, a fixed lens (5), and the reticule (6). Assuming the vertical axis is vertical, an image for the zero mark ( $V$ ) of the vertical circle reading is formed at the reticule (6) by the suspended lens (4) and the fixed lens (5).


Figure (2.10) : Pendulum compensator (From Cooper 1982)

Suppose that the vertical axis is inclined by an angle (e) in the plane of the circle, although the telescope remains horizontal. The compensator will automatically index an image of the zero mark (V) in the reticle instead of the mark ( X ) as shown in Figure (2.10-b). This is because the pendulum keeps the suspended lens horizontal while the fixed lens is inclined with angle (e), and the suspended lens is set at a distance from the pendulum equal to the radius of the circle.

Liquid Compensator. The surface of a small area of a liquid, always assumed horizontal under the gravity effect, is used to define vertical. Figure (2.11) shows how this compensator operates. If the vertical circle is vertical, the image ray of the zero mark (V) will pass through the liquid exactly vertical and will fall directly in the reticle via the prism; see Figure (2.11-a).


Figure (2.11) : Liquid compensator (From Cooper 1982)

Suppose that the circle is inclined by angle (e) to the vertical in the plane of the circle; Figure (2.11-b), while the telescope remains horizontal. The image rays of the zero mark $(\mathrm{V})$ will deviate at the liquid surface so that the ray is perpendicular to the prism, thus falling exactly in the reticle. This works by choosing a liquid with the proper refractive index, and setting a suitable distance between the circle and the upper surface of the liquid.

A variation of the liquid compensator is shown in Figures (2.12) and (2.13). In this variation, light from an LED (light emitting diode) is reflected from a mercury surface (dampened by oil and an optical flat) and sensed by a photodiode array. This tilt sensor can be used as an AVCC in a theodolite or a stand-alone tilt. sensing unit. An example of the latter is the Nivel 20 manufactured by Kern Swiss.

(a)

(b)

Figure (2.12) : Electronic tilt sensor (From Cooper 1982)


Figure (2.13). : Electronic automatic vertical circle compensator (From Cooper 1982)

The resolution of the Nivel 20 instrument is $\pm 0.2$ arc second and the range is $\pm 5.2$ minutes, with linearity error of $\pm$ ( 1.0 arc second $+0.5 \%$ of reading). The instrument was available for only one day. A quick test was performed, which indicated that the tilt sensor is very sensitive to temperature effects.

### 2.2.2.2 Tilt and Rotation Measurements

There are two ways in which a precise theodolite can be used to measure tilts and rotations. The first way is to use the theodolite itself as a tiltmeter; the second way is to use the autocollimation or autoreflection method.

Theodolite as a Tiltmeter. The idea is very simple. Install the theodolite on the point of interest. In each epoch, clamp the telescope at a convenient vertical angle and take the vertical circle reading in the required direction; then rotate the theodolite 180 degrees in the horizontal plane and take another vertical circle reading. Half the difference between the two vertical circle angle readings is the theodolite dislevelment. The tilt between any two epochs is the difference between the dislevelment measurements.

The installation procedure is the most important aspect to insure accurate readings. Teskey (1988) explained one procedure. He wanted to measure the change in tilt along the axis of a concrete column. First, he insured a suitable fixed point on which to install the theodolite. Second, to insure a similar theodolite installation for each epoch, a dedicated tribrach with footscrews permanently locked in the bottom position was used. If the tilt of the column exceeded the working range of the AVCC (about 2 arc-minutes) shims of known thickness were used to bring the theodolite closer to level. More details can be found in Teskey (1988).

This procedure can be used to measure tilts about two orthogonal axes in the horizontal plane. The theodolite can also be used to measure the rotation around the vertical axis using the horizontal circle reading. To do this, the theodolite is used to measure the horizontal angle between a distant point and a close, well-defined point on the same mounting as the theodolite. The difference between the horizontal angles measured in two epochs is the rotation about the
vertical axis.

Teskey (1991) shows that the method described above can be used to measure thee-dimensional rotations at a point. The specific application is rotation measurements on a buried pipeline.

Autocollimation/Autoreflection Method. Autocollimation or autoreflection is a procedure that insures the perpendicularity condition between a flat mirror and the theodolite line of sight; Figure (2.14) demonstrates autocollimation and autoreflection. These procedures will be explained in detail in Chapter Four. The autocollimation procedure insures a precision very close to that of the theodolite precision (about 2 arc seconds), whereas autoreflection procedure insures a. precision from 5 to 10 arc seconds.

When tilt/rotation occur in the mirror, the perpendicularity condition is altered, and hence the theodolite must be readjusted to achieve this condition. By taking the horizontal and vertical circle reading and comparing them with the first epoch readings one is able to determine tilts and rotation; see Figure (2.15). The tilt of the mirror about the mirror X-axis is the direct difference between the vertical circle readings of the theodolite for the two epochs. To find the rotation of the mirror about the Z or vertical axis, the horizontal orientation of the theodolite in each epoch must be determined. This orientation can be found by resecting the theodolite from the same reference network in each epoch.

(a) AUTOCOLLIMATION

(b) AUTOREFLECTION

Figure (2.14) : Autocollimation and autoreflection methods


Figure (2.15) : Mirror tilt and rotation

Pointing the theodolite to a mirror will provide rotation and tilt of the mirror along the line of sight of the theodolite. However, the tilt in the direction perpendicular to the line of sight cannot be determined from this mirror. Therefore, two perpendicular mirrors (or at least two mirrors separated by some angle in the horizontal plane) are needed to measure three-dimensional rotation of a point. The development of this method, as well as the lab and field testing, will be explained and discussed in detail in Chapters Five and Six.

## Chapter 3

## INVESTIGATION OF A TILTMETER WITH ELECTROLYTIC LEVEL TRANSDUCER

Tiltmeters are instruments widely used to measure very small tilts. None of these instruments, however, has been thoroughly investigated with regard to error control. In this chapter, the results of the investigations on an electrolytic level transducer tiltmeter (continuous as well as discrete tilt measurement instrument) will be explained and discussed.

According to the manufacturer's specifications, the technical features of the electrolytic level transducer tiltmeter, which have been used for this investigation are:

1- It is a dual-axis, analog output tiltmeter. Figure (3.1) shows the shape and components of the tiltmeter. The tiltmeter model is 701-2, manufactured by APPLIED GEOMECHANICS Inc.

2- The tiltmeter has a temperature sensor in addition to the tilt sensors. A Digital Readout Unit (DRU) (box of dimension $280 \times 230 \times 300 \mathrm{~mm}$ ) is used to make discrete tilt readings.
3- There are two resolution modes for tilt readings: low gain mode and high gain mode. The resolution of both is 0.001 volt which corresponds to 3.84 arc seconds in low gain and 0.36 arc seconds in high gain.
4- The sign conventions for the direction of the tilt (up or down) are illustrated in Figure (3.2).
5- The instrument operates between temperature of $0^{\circ} \mathrm{C}$ to $50^{\circ} \mathrm{C}$.


Figure (3.1) : Electrolytic level transducer tiltmeter (from Applied Geomechanics Inc. 1988)

Further specifications for the tiltmeter are available in the tiltmeter's manual (Applied Geomechanics Inc. 1988).


Figure (3.2) : Sign conventions for the tiltmeter readings

The investigation of the tiltmeter with electrolytic level transducer focuses on four areas:

Repeatability. A voltage output reading will be taken with the tiltmeter placed in a stable force centring device. The tiltmeter will be removed and placed again in the forced centring device. Another voltage output reading will be taken. About 10 repeated readings will be taken at mid range, and the same number of readings will be taken near each end of the range. Results will be summarized by estimated standard deviations of repeated placements and readings.

Linearity. An actual tilt will be introduced through the use of an adjustable machine screw. The tilt will be introduced in different incremental values covering the full range of the tiltmeter. The tilt will also be introduced randomly. To measure the "actual" tilt, the autocollimation method will be used. Linearity of the instrument will be illustrated by graphs, and the standard deviation will
be presented in tabular form.

Drift. Using the same facilities for the linearity test, output voltages will be logged periodically over a long period of time. No actual tilt will be introduced. Results will be summarized in a table.

Temperature Sensitivity. The tiltmeter will be placed in an environmental chamber. Temperature will be varied from $20^{\circ} \mathrm{C}$ to $40^{\circ} \mathrm{C}$ to $0^{\circ} \mathrm{C}$ to $20^{\circ} \mathrm{C}$ by $5^{\circ} \mathrm{C}$ increments, and changes in output voltage (equivalent to apparent changes in tilt) will be recorded. The results will be summarized in graphs. Suggested suitable applications are given in Section (3.4).

### 3.1 REPEATABILITY

In engineering, resolution and precision are completely two different terms. The resolution of an instrument is the smallest division on the instrument readout scale (for digital display: one digit change in the last digit). Precision is the closeness of approach to the arithmetic mean of each reading, for repeated measurements. In practice, an instrument is well designed and manufactured if the precision of the instrument is nearly equal to its resolution.

The precision of the tiltmeter can be found by performing the repeatability test. Ten repeated tiltmeter voltage readings will be taken for the same introduced tilt. The test requires that the tiltmeter be removed, then replaced on it's baseplate, using force centring to reestablish the same position. The force centring is accomplished by setting the hemispherically shaped bases of the three tiltmeter footscrews into V-shaped holes drilled in the baseplate. The plate must be installed on a fixed point, i.e. tilts are not allowed during the test period. The test
will be carried out near the tiltmeter mid range, as well as near each end of the range. During the test, the temperature will be kept constant.

Following is the testing procedure:
1- Install the tiltmeter baseplate on a fixed point.
2- Place and force center the tiltmeter on the baseplate.
3- Adjust the tiltmeter levelling screws so that the readings along the $X$ and $Y$ axes are in the required range.

4- Remove the tiltmeter from the baseplate and then replace it.
5- Wait two to three minutes until the instrument readings are stable.
6- Take the readings along the $X$ and $Y$ axes in both high and low gain modes.
7- Repeat Steps 4 to 6 ten times.
8- Readjust the tiltmeter readings to another range using the levelling screws, and then repeat Steps 4 to 7 .

The graphs of Figure (3.3) show that the deviation of the readings from the mean value for each range are nearly the same. The standard deviation for the X axis readings is $\pm 21.0$ arc seconds ( $\pm 0.0055$ volt), while for the Y-axis readings is $\pm 21.4$ arc seconds ( $\pm 0.0056$ volt). The total standard deviation for both axes is $\pm 21.2$ arc seconds ( $\pm 0.0055$ volt). The repeatability test therefore indicates a precision of $\pm 0.0055$ volts. This is 5.5 times the resolution of the instrument.


Figure (3.3) : Repeatability test in the low gain mode


Figure (3.4) : Repeatability test in the high gain mode

Figure (3.4) shows the graphs for the high gain mode. Although the deviations from the mean value for the different ranges are similar, the precision of the instrument is about 36 times the resolution. The standard deviation for the $X$-axis readings is $\pm 11.6$ arc seconds ( $\pm 0.0321$ volt), and for the $Y$-axis readings is $\pm 14.6$ arc seconds ( $\pm 0.0407$ volt). The total standard deviation for both axes is $\pm 13.1$ arc seconds ( $\pm 0.0363$ volt). This proves that the high gain mode of the tiltmeter does not provide better precision than the low gain mode, it just provides better resolution.The writer believes, however, that if the tiltmeter force centring system were fabricated more precisely, the precision would be better in both low and high gain modes.

### 3.2 LINEARITY

Linearity is the factor relating instrument readings to their corresponding "true" values. A linearity correction, applicable to any instrument reading is often expressed as a percentage value.

The linearity testing procedure will be conducted by introducing an actual tilt through the use of a machine screw threaded through the tiltmeter base. As the screw is raised or lowered different tilts can be introduced. The tiltmeter readings will be compared with the "actual" tilt measured by using the autocollimation method (Kissam 1962). The autocollimation method gives an accuracy of about one arc second (for more details, refer to Chapter Four). Tilt measured by autocollimation will therefore be considered "true" values. The test will be carried out in a small constant-temperature room located in the basement of E block, Engineering Building.

The testing procedure will be divided into two main parts. The first part
is for plotting the linearity diagrams and for determining the linearity correction for the instrument. Tilts here will be introduced in different incremental values covering the full range of the tiltmeter. The second part is for calculating the mean error and the standard deviation of the instrument for different cases. Here different values of tilts will be introduced.

The test requires a fixed point (i.e. tilts are not allowed) on which to install the tiltmeter baseplate, a one arc second theodolite (such as Wild T2), and a precise mirror to which autocollimation measurements are made.

To perform the test, the procedure is as follows:
1- Mount the precise mirror on one axis of the tiltmeter, say the $X$-axis.
2-- Install the tiltmeter baseplate on the fixed point.
3- Place the tiltmeter on the base plate permanently.
4- Adjust the machine screw so that the tiltmeter reading will be at the beginning of the high gain range (about +1.900 volt).

5- Take the tilt readings along the $X$ axis.
6- Follow the autocollimation procedure to measure the actual tilt using the T2 theodolite and the precise mirror.

7- Introduce a new tilt increment along the $X$-axis.
8- Repeat Steps 5 to 6 until the tiltmeter reading is at the end of the range (about -1.900 volt).
9- Repeat Steps 1 to 7 for the low gain mode along the $X$-axis:

LINEARITY


Figure (3.5) : Tiltmeter linearity (high gain)


Figure (3.6) : Linearity, accumulated tilt


Figure (3.7) : Linearity, accumulated tilt errors

LINEARITY


Figure (3.8) : Tiltmeter linearity (low gain)

LINEARITY
AUTOCOLLIMATION AND TILTMETER READINGS


Figure (3.9) : Linearity, accumulated tilt

LINEARITY


Figure (3.10) : Linearity, accumulated tilt errors

The results of the linearity tests for high gain mode are shown in Figures (3.5), (3.6), and (3.7). Figure (3.5) shows the autocollimation readings versus the tiltmeter readings. The plot is nearly a straight line. In Figure (3.6), the accumulated autocollimation and tiltmeter readings are plotted against the voltage. The figure shows very clearly the difference between the two measurements. The difference is shown even more clearly in Figure (3.7). The plot in Figure (3.7) approximates a straight line: the error increases as the tilt angle increases. The linearity error is $3.95 \%$, calculated by dividing the maximum error (58.4 arc seconds) by the measured tilt angle.

The same test results are shown for the low gain mode in Figures (3.8), (3.9), and (3.10). Here, the behaviour of the linearity error cannot be approximated by a straight line (Figure 3.10). This is the expected behaviour: the error is maximum near the mid range of the instrument, then tends to decrease to zero at the end of the range. The linearity error is $0.5 \%$, calculated by dividing the maximum error (70 arc seconds) by the measured tilt angle.

So far, the results show that the instrument has better linearity in the low gain mode ( $0.5 \%$ ) than in the high gain mode ( $3.95 \%$ ). This is consistent with the results of the repeatability test that the instrument is designed for the low gain mode, and the high gain mode only provides a better resolution.

The second group of tests are conducted to calculate the mean error and the standard deviation for different ranges of measured tilts. First, when there is no limit in the range for measured tilts, the mean error was 2.5 arc seconds, the maximum error was 7 arc seconds, and the standard deviation was $\pm 2.6$ arc seconds in the high gain mode. These results are shown in Figure (3.11). In the low gain mode with no limit in the range for measured tilts, the mean error was 5.5 arc seconds, the maximum error was 25 arc seconds, and the standard deviation was 9.8 arc seconds; see Figure (3.12).

LINEARITY


Figure (3.11) : Random measurement (high gain)

## LINEARITY



Figure (3.12) : Random measurement (low gain)

Figures (3.13) and (3.14) are histograms for high and low gain resolutions to check for randomness of the errors. Figure (3.13) shows that there are systematic errors beside the random errors in the high gain resolution and that the plot cannot be approximated as the normal distribution. However, the behaviour of errors in the low gain resolution is random and very close to the normal
distribution; Figure (3.14).


Figure (3.13) : Histogram for error measurements (high gain)

## HISTOGRAM <br> LOW GAIN RESOLUTION



Figure (3.14) : Histogram for error measurements (low gain)

For completeness, the error behaviour of the measured tilts during the incremental tilt measurements (first group tests) are shown in Figure (3.15) (high gain) and Figure (3.16) (low gain). The two figures show that the behaviour of the tiltmeter is the same for both random and incremental tilt measurements.


Figure (3.15) : Incremental measurement errors (high gain)
LINEARITY INCREMENTAL MEASUREMENT ERRORS


Figure (3.16) : Incremental measurement errors (low gain)

The errors values just quoted for the entire range of the tiltmeter are not actually useful values in most applications. The reason for this is that in most applications tilts do not vary from zero to the maximum values, but tend to be restricted to certain ranges. With this in mind, Table (3.1) was constructed to
show the tiltmeter errors for different ranges in the high gain and low gain modes.

Table (3.1)
Tiltmeter errors

| Tilt | High gain mode | Low gain mode |
| :---: | :---: | :---: |
| 1- Unlimited tilt <br> Max error <br> Mean error <br> Stand. deviation | $\begin{gathered} 7.0 \\ 2.5 \\ \pm 2.6 \end{gathered}$ | $\begin{gathered} 25.0 \\ 5.5 \\ \pm 9.8 \end{gathered}$ |
| 2-Tilt $<50$ <br> Max error <br> Mean error <br> Stand. deviation | $\begin{gathered} 4.6 \\ 0.5 \\ \pm 2.4 \end{gathered}$ |  |
| 3-Tilt > 50 <br> Max error <br> Mean error <br> Stand. deviation | $\begin{gathered} 7.0 \\ 2.4 \\ \pm 3.9 \end{gathered}$ |  |
| 4- Tilt < 200 <br> Max error <br> Mean error <br> Stand. deviation |  | $\begin{gathered} 5.9 \\ 2.2 \\ \pm 2.7 \end{gathered}$ |
| $\begin{aligned} & \text { 5- } 200<\text { Tilt }<400 \\ & \text { Max error } \\ & \text { Mean error } \\ & \text { Stand. deviation } \end{aligned}$ |  | $\begin{gathered} 10.5 \\ 2.9 \\ \pm 4.0 \end{gathered}$ |
| $\begin{aligned} & \text { 6- } 400<\text { Tilt }<1000 \\ & \text { Max error } \\ & \text { Mean error } \\ & \text { Stand. deviation } \end{aligned}$ |  | $\begin{gathered} 13.5 \\ 7.2 \\ \pm 8.5 \\ \hline \end{gathered}$ |
| 7- Tilt $>1000$ <br> Max error <br> Mean error <br> Stand. deviation |  | $\begin{gathered} 38.0 \\ 22.5 \\ \pm 28.0 \\ \hline \end{gathered}$ |

All values in table are in arc seconds.

### 3.3 DRIFT

Drift is measured by the change in the tiltmeter voltage readings over a long period of time when no tilt is introduced. The autocollimation method is used to check if any movements occurred during the test period.

The test requires a stable point, a one arc second theodolite (Wild T 2 ), and two precise mirrors. The procedure for the test is as follows:

1- Mount a precise mirror on the tiltmeter (one along the $X$-axis and one along the Y -axis).

2- Install the tiltmeter baseplate on the stable point.
3- Place the tiltmeter on the baseplate permanently.
4- Take the tilt readings along the $X$ and $Y$ axes for both low and high gain modes.

5- Use the autocollimation procedure to measure the actual tilts along the $X$ and $Y$ axes.

6- Repeat Steps 4 and 5 every two weeks for a period of three months.

The results of the test are shown in Table (3.2). These results show no drift up to week 10 and a small amount of apparent drift thereafter. The writer suspects that the apparent drift after week 10 may have been due to the $A C$ power supply being accidentally disconnected for about one week. During this time the tiltmeter was powered by its DC power supply.

Table (3.2)
Drift test results

| Week <br> No. | Tiltmeter readings |  | Theodolite readings <br> Autocollimation method | Drift |
| :---: | :---: | :---: | :---: | :---: |
|  | High gain | Low gain |  | 0 |
| 0 | No tilt | No tilt | No tilt | 0 |
| 2 | No tilt | No tilt | No tilt | 0 |
| 4 | No tilt | No tilt | No tilt | 0 |
| 6 | No tilt | No tilt | +8 | 0 |
| 8 | +7 | +10 | +9 | 11 |
| 10 | -2 | -3 | +3 | 4 |
| 12 | +2 | +3 |  | 0 |

All values in table are in arc seconds, and are average values for $X$ and $Y$ axes.

### 3.4 TEMPERATURE SENSITIVITY

Temperature effects on an instrument are very important, especially if the instrument is required to operate under variable temperature conditions. Temperature compensation is one solution to this problem.

The testing procedure was designed to focus on the possibility of applying temperature compensation solution. The requirements for the testing are: a fixed point (i.e. tilts are not allowed), and an environmental chamber, in which temperature can be precisely controlled. The procedure for this testing is as follows:

1- Install the tiltmeter baseplate on the fixed point.

2- Place the tiltmeter on the baseplate.
3- Take the tilt readings along the $X$ and $Y$ axes and the temperature reading (about $20^{\circ} \mathrm{C}$, room temperature).
4- Put the tiltmeter in the chamber and decrease the temperature $5^{\circ} \mathrm{C}$.
5- Allow the temperature reading to stabilize for at least 30 minutes.
6- Remove the tiltmeter from the chamber and place it on the baseplate.
7- Take the tiltmeter and temperature readings.
8- Repeat Steps 4 to 7 to $0^{\circ} \mathrm{C}$.
9- Repeat Steps 4 to 7 increasing the temperature from $0^{\circ} \mathrm{C}$ to $40^{\circ} \mathrm{C}$ by $5^{\circ} \mathrm{C}$ increments.

10- Repeat Steps 4 to 7 decreasing the temperature from $40^{\circ} \mathrm{C}$ to room temperature by $5^{\circ} \mathrm{C}$ increments.

The test results (Figures (3.17) and (3.18) show the large effect of varying the temperature on the tiltmeter readings. The test was done only in low gain mode due to problems in installing the base plate. The figures show very clearly that the tiltmeter should not be used under variable temperature conditions. The plots in both figures, and especially the plot for the $Y$ directions, show an erratic behaviour in the readings as the temperature changes. Temperature compensation corrections based on these figures would be very suspect. There seems to be a hysteresis effect in both figures but it is much less apparent in the $Y$ direction readings.


Figure (3.17) : Temperature effects (X-direction)

TEMPERATURE EFFECTS READINGS IN THE Y-DIRECTION


Figure (3.18) : Temperature effects (Y-direction)

### 3.5 SUITABLE APPLICATIONS

The following precautions should be taken when using this tiltmeter:
1- Permanently install the baseplate and take great care in force centring the tiltmeter.

2- If possible, always use an AC power supply. This is especially important for continuous tilt measurement.

3- Do not use the tiltmeter where there will be large variations in temperature.
4- For discrete tilt measurements, ten repeated readings should be taken with the tiltmeter removed and replaced between readings.

Continuous tilt measurements where the tiltmeter is permanently installed is the most suitable application for this instrument. Also, the temperature should be stable within $5^{\circ} \mathrm{C}$, the AC power supply should be connected continuously, and the measured tilt should not be more than 400 arc seconds. Under these conditions an accuracy of about 4 arc seconds can be expected.

The tiltmeter can be used to measure discrete tilts where temperature is stable within $5^{\circ} \mathrm{C}$, a good force centring device is used, and ten repeated readings are taken. Under these conditions an accuracy of 15 arc seconds can be expected.

It should be possible to use the tiltmeter under variable temperature conditions if it is installed in a temperature controlled chamber (Schleppe 1992). The temperature in the chamber would be set at, say, $50^{\circ} \mathrm{C}$ so that heat would always be required to maintain the temperature.

## Chapter 4

# INVESTIGATION OF THE USE OF A HIGH-PRECISION THEODOLITE FOR TILT/ROTATION MEASUREMENT 


#### Abstract

Autocollimation and autoreflection are well known methods, and are described in the optical literature (see, for example, Kissam 1962). These methods, however, have not•been used for tilt/rotation measurement in the industrial environment.


In this chapter, the procedure required for each method, the difficulties encountered, and investigations of the reliability of the measurements will be discussed and explained. Temperature effects on the readings will also be discussed.

### 4.1 AUTOCOLLIMATION METHOD

To collimate is to produce parallel rays. Therefore, to collimate the theodolite line of sight (collimation line) with another axis means to make the line of sight parallel to that axis. In autocollimation, reflected rays will coincide with incident rays. This condition sets the line of sight of the theodolite parallel to the mirror axis; see Figure (4.1).

Autocollimation can be precise to one arc second using a one arc second theodolite (such as Wild T2), an optically flat front silvered mirror, and a collimation device (Wild Heerbrugg Limited 1987). The Wild T2 theodolite used for autocollimation has a precision of one arc second. The mirrors used for
autocollimation were salvaged from an old mirror stereoscope.


Figure (4.1) : Autocollimation

The collimation device is connected to the theodolite telescope in place of the eyepiece, and is equipped with a light source. Its main function is to supply the theodolite with a light source through the telescope to illuminate the crosshairs, so that the reflected image of the crosshairs from the mirror can be easily seen.

To see the reflected image of the crosshairs:
(a) Focus the telescope objective lens to infinity to produce parallel rays.
(b) Turn on the collimation device to illuminate the crosshairs.
(c) Place the theodolite telescope normal to the mirror (It is helpful to set the reflected image of the telescope in the mirror before proceeding to the next step.) (d) Adjust the telescope until the reflected image of the crosshairs can be seen, noting that the reflected image is inverted; see Figure (4.2).

When the reflected image of the crosshairs does not coincide with the real image of the crosshairs as shown in Figure (4.2-a), the theodolite is not yet collimated with the mirror axis. Collimation exists when the condition shown in Figure(4.2-b) is established.


Figure 4.2 Collimation condition

Autocollimation requires that the image of the crosshairs can be seen clearly. This creates two main problems which make this method difficult to apply in industrial alignment monitoring. These are:
1- The image of the crosshairs cannot be seen when the mirror is vibrating (The mirror is mounted on an operating machine.)
2- The image of the crosshairs is not clear when the ambient lighting is intense.

Since the telescope is focused to infinity, this method has the advantage that there is no theoretical limit for the distance between the theodolite and the mirror. This provides good flexibility in high precision surveys. Another advantage is that if any pointing error occurs, the resultant error in the tilt angle measurement will be half this value; see Figure (4.3).


Figure (4.3) : Pointing error

### 4.1.1 Repeatability

The resolution of a Wild T2 theodolite is one arc second. The precision of this theodolite, for a single reading under good conditions, is approximately two arc seconds. This precision value, however, is not guaranteed in measuring tilts and rotations using the autocollimation method. This is due to pointing errors and the degree of flatness of the mirror.

A pointing error occurs when the reflected image of the crosshairs is not very clear. Because of light intensity, vibration, or refraction the image is blurred. This makes it hard to distinguish the exact image of the crosshairs.

Normally, the observer cannot guarantee, for each measurement, that the same point on the mirror is observed. This will produce an error which will depend on the flatness of the front surface of the mirror.

The repeatability test requires the establishment of a reference network, a fixed point (i.e. tilts are not allowed during the test period) on which to install the mirror, and a one arc second theodolite (Wild T2) with the collimation device installed in place of the regular eyepiece. During the test, the temperature must be constant. The reference network as well as the mirror and the theodolite positions are shown in Figure (4.4).


Figure (4.4) : Repeatability test

The test procedure to measure the repeatability of autocollimation measurements is as follows:

1- Install the mirror on the fixed point.
2- Set up the theodolite so that the collimation condition is achieved, and the point of reflection on the mirror is as close as possible to the centre of the mirror.

3- Take three sets of horizontal and vertical circle readings.

4- Resect the position of the theodolite with respect to the reference network, taking measurements to at least four points.
5- Repeat Step 3.
6- Repeat Steps 2 to 5 inclusive for the remaining theodolite positions.

For vertical circle readings, the standard deviation of a single reading was 1.0 arc second and the standard deviation of a 6 reading average was $(1.0 / \sqrt{6}=) 0.4$ arc second. For horizontal circle readings, the standard deviation of a single reading was 3.5 arc seconds, and the standard deviation of a set of 6 reading was $(3.5 / \sqrt{ } 6=) 1.4$ arc seconds. The reason for larger standard deviations for horizontal circle readings is that errors from the resection computations are included, whereas this is not the case for vertical circle readings.

### 4.1.2 Mirror Flatness

Usually the observer does not have time to focus exactly at the centre of the mirror to achieve collimation condition. Sometimes the tilt is so small that the observer does not have to readjust the theodolite position; he or she needs only to readjust the slow motion screws of the theodolite to achieve the new collimation condition. This creates the problem that the area of reflection on the mirror will move. If the surface of the mirror is not extremely flat, this will result in a new error.

The main task of the mirror flatness testing was to determine the reliable pointing area on the mirror. The investigation procedure was similar to that used in the repeatability test. In this test, the pointing was made at different spots on the mirror as well near mirror edges.

The results showed that for the tested mirror ( 30 mm X 30 mm ), if the
pointing region on the mirror was within 10 millimetres of the centre of the mirror, the precision was the same as for the repeatability tests. However, once the pointing area was near any of the edges, the error increased tremendously (to 7 arc second for vertical circle readings and to 15 arc second for horizontal circle readings).

The writer found that the error was high at the edges because the total image of the crosshairs could not be seen. In order to get precise results, the writer would suggest that the entire reflected image be obtained. In the case of the $30 \mathrm{~mm} \times 30 \mathrm{~mm}$ tested mirror, the safe region was within 10 mm of the centre of the mirror.

### 4.2 AUTOREFLECTION METHOD

If a mirror is perpendicular to a telescope line of sight, the telescope image can be seen in the mirror by focusing the telescope to double the distance between the telescope and the mirror; see Figure (4.5).


Figure (4.5) : Autoreflection

Autoreflection refers to the process of achieving the perpendicularity condition with the mirror. To carry out autoreflection, an autoreflection target is fabricated to fit over the barrel of the telescope at the objective lens, so that it acts as a target to coincide with the crosshairs, see Figure (4.6).


Figure (4.6) : Autoreflection target (fabricated ring)

There are different types of targets that can be used; see Allen (1983). The target used by the writer is a ring fabricated with four marks etched at $0^{\circ}, 90^{\circ}$, $180^{\circ}$, and $270^{\circ}$, on the front face. The four marks form imaginary crosshairs. The intersection of the imaginary crosshairs represents the line of sight of the telescope; see Figure (4.7). Etching these four marks in their exact locations is very difficult. However, if the observer uses only two marks separated by $90^{\circ}$, and takes the circle readings for both position of the theodolite, the average value of the two circle readings will eliminate this pointing error.


Figure (4.7) : Perpendicularity condition

The advantage of the autoreflection method over the autocollimation method is that the autoreflection method can be carried out in almost any ambient lighting conditions and is almost unaffected by vibrations of the mirror. Both of these are problems for the autocollimation method.

The main problem associated with using the autoreflection method is the difficulty of achieving the perpendicularity condition. It is much more difficult to match telescope crosshairs with the reflected target marks in autoreflection than it is to match telescope crosshairs with the reflected image of telescope crosshairs in autocollimation.

To measure the precision and accuracy of autoreflection two testing procedures were carried out. The first can be considered an accuracy test with autocollimation providing the "true" value. Direct comparisons were made between corresponding autoreflection and autocollimation vertical and horizontal circle readings, under a variety of environment conditions, both in the lab and the field. The rms (root mean square) value of the discrepancies was 6 arc seconds for
both vertical and horizontal readings.

The second test can be considered a test of precision. It was carried out in exactly the same manner as the repeatability test for autocollimation described in Section (4.1.1). In this test, the standard deviation of a single vertical circle reading was $\pm 1.5$ arc seconds and the standard deviation of a single horizontal circle reading was $\pm 3.5$ arc seconds. Again, resection computations are the reason the standard deviation of the horizontal circle readings was higher than the standard deviation of the vertical circle readings.

One should note that the accuracy test results are appropriate for situations where the environmental conditions(e.g. air temperature) change considerably from one epoch to another, while the precision test results are appropriate for situations in which the environment conditions remain reasonably constant.

### 4.3 TEMPERATURE EFFECTS

There are two major temperature effects that could produce errors in tilt/rotation measurements using autoreflection. The first effect is due to thermal expansion/contraction that occurs between the mirror and the machine body on which the mirror is mounted. The second effect is due to refraction. In the following sections, these effects are discussed. There is also a further discussion on refraction in Chapter Five.

### 4.3.1 Thermal Expansion/Contraction Effects

Thermal expansion occurs when the temperature at the mirror mounting changes from one epoch to another. When two objects made of different materials are fastened together and subjected to temperature changes, differential stresses
are developed, which may cause significant deformations in the composite structure.

In the autoreflection method, the mirror is glued to the body of the machine with a thin layer of quick-setting epoxy; see figure (4.8). The machine body is usually steel. During hot alignment, for example, the temperature of the machine body and mirror increase. This will produce differential stresses which will deform the mirror and possibility result in significant errors in the autoreflection measurements.

(a) Front view

(b) Side view cross-section

Figure (4.8) : A mirror mounted on a machine


Figure (4.9) : The developed moments on the mirror

The deformation pattern for the example just noted is shown in Figure (4.9). Differential stresses will cause bending moments at the edges of the mirror. A simple calculation of these moments was carried out for a temperature change of $40^{\circ} \mathrm{C}$. It was found that for a $30 \mathrm{~mm} \times 30 \mathrm{~mm}$ mirror of 5 mm thickness and a steel body or mounting bracket of 15 mm thickness, the angle of deformation is about 1.0 arc second about the mirror Z -axis and about 0.3 arc second about the mirror Y-axis. The temperature coefficients of the steel and glass used in this calculation are $12 \times 10^{-6}$ and $9 \times 10^{-6}$, and the moduli of elasticity for steel and glass are 206.8 GPa and 69.9 GPa respectively.

This shows that the effect in this case is negligible. However, if the material in the machine body or mounting bracket is aluminum, the effect will be much higher for two reasons. The temperature coefficient for aluminum $\left(23 \times 10^{-6}\right)$ is approximately twice that of steel. Also, the modulus of elasticity of aluminum ( 69 GPa) is approximately one third that for steel. These two effects combined will increase the deformation angles computed above approximately by a factor of 15 .

### 4.3.2 Refraction Effects

Refraction effects in the atmosphere are produced when temperature, pressure, and humidity change from one point to another. The largest refraction effect (about $95 \%$ in most cases) is produced by temperature change. The refraction effects will cause the theodolite line of sight to deviate from the straight line between telescope and target resulting in angular errors.

Fortunately, it has been shown by Bayly (1991) that the largest error will occur when the heat source is close to the theodolite and the smallest error will occur when it is close to the target. This is exactly the case in most industrial problems where the mirror is mounted on the machine body. Also, fortunately, if measurements are being made in steady-state conditions, the refraction error (whatever it is) does not matter as long as it is reasonably constant.

In Chapter Five, a series of tests, with a heat source adjacent to a mirror, are described. The magnitude of errors resulting from refraction effects are given and, for situations where countermeasures might be required, countermeasures are suggested.

### 4.4 DISCUSSION

The results from this chapter show that the autoreflection method is a reliable method for measuring discrete tilts and rotations between epochs because of the following reasons:
1- Essentially unlimited measurement range for tilts and rotations.
2- Accuracy of approximately 6 arc seconds.
3- Insensitive to ambient lighting conditions or target vibration.
4- Insensitive to thermal expansion effects if the machine body or mounting is made of steel.

Further developments of the autoreflection method for industrial applications are described in Chapter Five.

## Chapter 5

# DEVELOPMENT AND LAB TESTING OF A TILT/ROTATION MEASUREMENT METHOD FOR INDUSTRIAL ALIGNMENT 

### 5.1 INTRODUCTION

The autoreflection method, using a one arc-sec theodolite and a precise, front silvered mirror, is a very reliable method of measuring tilts and rotation about the mirror axes. However, in most industrial alignment problems, the component that should be monitored is often inaccessible, and the alignment of the mirrors is not necessarily the same as the alignment of the axes of interest.

The first problem is dealt with by assuming that the object of interest is a rigid body. Therefore, rotations at the surface will be the same as rotations at a central axis. The second problem, which requires transformation of axes, is the focus of this chapter. A discussion of refraction and vibration effects concludes the chapter.

### 5.2 MATHEMATICAL MODELS

In three-dimensional space, the rotation matrix R rotates one orthogonal coordinate system to another orthogonal coordinate system using the three rotation angles (alpha $\alpha$, beta $\beta$, gamma $\gamma$ ) between the axes of the two systems; see Vanicek and Krakiwsky (1982). The rotation matrix R is

$$
\begin{equation*}
R(\alpha, \beta, \gamma)=R_{1}(\alpha) R_{2}(\beta) R_{3}(\gamma) \tag{5.1}
\end{equation*}
$$

where:

$$
R_{1}(\alpha)=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{5.2}\\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{array}\right]
$$

$$
R_{2}(\beta)=\left[\begin{array}{ccc}
\cos \beta & 0 & \sin \beta  \tag{5.3}\\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{array}\right]
$$

and

$$
R_{3}(\gamma)=\left[\begin{array}{ccc}
\cos \gamma & \sin \gamma & 0  \tag{5.4}\\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{array}\right]
$$

### 5.2.1 Tilts/Rotations Measured Using Mirrors

Any mirror tilts/rotations alter the perpendicularity (collimation) condition; thus, the horizontal and vertical circle readings of the theodolite change. The following relates the theodolite readings to the tilt and rotation.

The Tilt Angle. The tilt angle (V) is the change in the orientation about the mirror Y-axis; see Figure (5.1). When the mirror tilts from the initial position in epoch 1 to another position in epoch 2, the orientation of the axes change; see Figure (5.2). The theodolite for each epoch measures, using vertical circle reading, the angle between the mirror $X$-axis and the horizontal plane of the theodolite. Therefore, the direct difference between these two angles gives the tilt angle around the
mirror Y -axis.

$$
\begin{equation*}
V=V_{2}-V_{1} \tag{5.5}
\end{equation*}
$$



Figure (5.1) : Theodolite measurements to a mirror


Figure (5.2) : Tilt angle of mirror (3-D view)

The rotation angle. The rotation angle $(\mathrm{H})$ is the change in the orientation of the mirror X-axis about the Z-axis (vertical axis); see Figure (5.1). In each epoch, the horizontal circle of the theodolite is read. This reading can only be used if, for each epoch, the direction (azimuth) of the theodolite's zero circle reading is calculated so that this reading will always relate to a fixed orientation. This is done using the resection method. Details related to azimuth calculations are given in Section (5.4).


Figure (5.3) : Horizontal rotation angle of mirror (top view)

Let the calculated azimuth in the first epoch be $A z_{1}$, and the calculated azimuth in the second epoch be $\mathrm{Az}_{2}$. The required rotation angle is H . From Figure (5.3)

$$
\begin{equation*}
H=180-\theta_{1}-\theta_{2} \tag{5.6}
\end{equation*}
$$

From triangle $A B C$ that contains $\mathrm{Az}_{1}$ :

$$
\begin{equation*}
\theta_{1}=90-A z_{1} \tag{5.7}
\end{equation*}
$$

From triangle $A^{\prime} B^{\prime} C^{\prime}$ that contains $A z_{2}$ :

$$
\begin{equation*}
\theta_{2}=90+A z_{2} \tag{5.8}
\end{equation*}
$$

Substituting equations (5.7) and (5.8) into equation (5.6) :

$$
\begin{equation*}
H=A z_{2}-A z_{1} \tag{5.9}
\end{equation*}
$$

This means that the rotation angle of the mirror around the vertical axis is the difference between the calculated azimuths of the mirror $X$-axis (Obidowski 1991).

One rotation is still missing for three-dimensional rotation measurements at the mirror position; the rotation about the $X$-axis of the mirror. A second mirror can be mounted on the body of the machine so that it's $X$ and $Y$ axes ( $X 2$ and $Y 2$ axes) are perpendicular to the first mirror $X$ and $Y$ axes ( $X 1$ and $Y 1$ axes). The third rotation can then be determined by taking another set of theodolite measurements to the second mirror. The third rotation is the tilt (V2) which is the tilt about the $Y$-axis of the second mirror (Y2-axis) and is equivalent to the tilt about X -axis of the first mirror (X1-axis); see Figure (5.4). Notice that the rotation angle $(\mathrm{H})$ is the same for both mirrors; this allows one to check the computed rotation angles against each other.


Figure (5.4) : Axes of two perpendicular mirrors
5.2.2 Transformation of Tilts and Rotation Using Two Mirrors Perpendicular to Each Other

The only difference between the coordinate system formed by the mirror axes of two perpendicular mirrors and the coordinate system formed by the shaft axes of a machine is the rotation angle gamma ( $\gamma$ ) shown in Figure (5.5). Rotations can be transformed by

$$
\begin{equation*}
R=R_{3}(\gamma) \tag{5.10}
\end{equation*}
$$

Therefore, the tilts and rotation of the shaft axes are

$$
\left\{\begin{array}{l}
S_{x x}  \tag{5.11}\\
S_{y y} \\
S_{z z}
\end{array}\right\}=\left[\begin{array}{ccc}
\cos \gamma & \sin \gamma & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{array}\right]=\left\{\begin{array}{c}
V_{1} \\
V_{2} \\
H
\end{array}\right\}
$$



Figure (5.5) : The coordinate systems of the shaft and the mirrors
5.2.3 Transformation of Tilts and Rotation Using One Mirror Perpendicular to the Shaft Axis and the Other Mirror at an Angle $\gamma$

In many cases, mounting the second mirror axis (X2-axis) perpendicular to the first mirror axis (X1-axis) is not possible. It is always possible, however, to mount the first mirror such that its $X$-axis is perpendicular to the $X$-axis of the shaft. This will directly provide the rotation angle about the Z -axis and the tilt about the $X$-axis of the shaft. The tilt around the $Y$-axis of the shaft cannot be directly measured but it can be calculated.

Let the angle between the second mirror and the shaft $X$-axis be $\gamma$; see Figure (5.6). Let the first mirror be denoted as M1, the second mirror as M2, and assume that there is an imaginary mirror M3 mounted at right angles to mirror M2.


Figure (5.6) : Geometrical determination of the tilt about the shaft Y -axis

From Figure (5.6), the tilt $S_{y y}$ about the shaft $Y$-axis is

$$
\begin{gather*}
S_{y y}=V_{2 y}+V_{3 y}  \tag{5.12}\\
S_{y y}=V_{2} \cos \gamma-V_{3} \sin \gamma \tag{5.13}
\end{gather*}
$$

The tilt $S_{x x}$ about the shaft $X$-axis is

$$
\begin{gather*}
S_{x x}=V_{2 x}+V_{3 x}  \tag{5.14}\\
S_{x x}=V_{2} \sin \gamma+V_{3} \cos \gamma \tag{5.15}
\end{gather*}
$$

But the tilt $S_{x x}$ is equal to the tilt $\left(V_{1}\right)$ measured by the first mirror M1; therefore

$$
\begin{gather*}
V_{1}=S_{x x}=V_{2 x}+V_{3 x}  \tag{5.16}\\
V_{1}=S_{x x}=V_{2} \sin \gamma+V_{3} \cos \gamma  \tag{5.17}\\
V_{3 x}=V_{3} \cos \gamma=V_{1}-V_{2} \sin \gamma \tag{5.18}
\end{gather*}
$$

Therefore $V_{3}$ is given by

$$
\begin{equation*}
V_{3}=\frac{V_{3 x}}{\cos \gamma}=\frac{V_{1}-V_{2} \sin \gamma}{\cos \gamma} \tag{5.19}
\end{equation*}
$$

Substituting equation (5.19) in equation (5.13) results in

$$
\begin{align*}
S_{y y} & =V_{2} \cos \gamma-V_{3} \sin \gamma \\
& =V_{2} \cos \gamma-\left(V_{1}-V_{2} \sin \gamma\right) \tan \gamma  \tag{5.20}\\
& =-V_{1} \tan \gamma+V_{2}(\cos \gamma+\sin \gamma \tan \gamma)
\end{align*}
$$

Therefore, the rotation matrix $R 3$ to transform the tilts and rotation from mirrors measurements to the shaft axis is given by

$$
\left\{\begin{array}{l}
S_{x x}  \tag{5.21}\\
S_{y y} \\
S_{z z}
\end{array}\right\}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
-\tan \gamma & (\cos \gamma+\sin \gamma \tan \gamma) & 0 \\
0 & 0 & 1
\end{array}\right]\left\{\begin{array}{l}
V_{1} \\
V_{2} \\
H
\end{array}\right\}
$$

### 5.3 VERIFICATION OF THE MATHEMATICAL MODELS FOR TILTS/ROTATION

To verify the above mathematical models, tests were carried out in which actual tilts/rotation were introduced and compared with those computed using the mathematical models.

The body of the machine was considered as the telescope of a second T2 theodolite. (The first theodolite is used for the measurements.) The central axis is the telescope line of sight. This means that any movements in the central axis of the body can be determined to a precision of about 1 arc second (theodolite precision).

Transformation Model of Tilts/Rotation.Using Two Mirrors Perpendicular to Each Other

To verify this model, two mirrors were mounted on the body of the telescope at a horizontal angle gamma ( $\gamma$ ) between their axes and the telescope line of sight; see Figure (5.7).


Figure (5.7) : Two perpendicular mirrors mounted on a theodolite telescope

Following is a description of the test procedure. Figure (5.8) shows a top view of the test.
1- Establish a precise reference network (points 1 to 7 inclusive) such as the one established for the repeatability test in Section (4.1.2).
2- Mount two perpendicular mirrors on the telescope of a T2 theodolite, so that the mirrors and the telescope form a rigid body. Denote this theodolite as (M-T2).
3- Choose two fixed points, and measure the difference between vertical and horizontal circle readings to them using the (M-T2) theodolite. The chosen
points for this test were points $2^{\prime}$ and $2^{\prime \prime}$, close to the reference point 2 ; see Figure (5.8).
4- Resect the position of theodolite (M-T2) using at least four points from the reference network. Calculate the azimuth of the zero horizontal circle reading of the theodolite.
5- Mount the autoreflection ring on the first T2 theodolite. Denote this theodolite as ( $\mathrm{R}-\mathrm{T} 2$ ).
6- Set the line of sight of theodolite (M-T2) to point $2^{\prime}$, and take the horizontal and vertical circle readings.
7- Take autoreflection readings from theodolite ( $\mathrm{R}-\mathrm{T} 2$ ) to mirror M1. Resect theodolite ( $\mathrm{R}-\mathrm{T} 2$ ) position from at least four reference points.

8- Set the line of sight of theodolite ( $\mathrm{M}-\mathrm{T} 2$ ) to point $2^{\prime \prime}$, and take the horizontal and vertical circle readings.
9- Repeat Step 7.
10- Repeat Steps 6 to 9 inclusive for mirror M2.


Figure 5.8 Top view of the verification test

The actual tilts/rotation of the telescope of theodolite (M-T2) were as follows:
tilt $S_{y y}$ about telescope $Y$ axis: 6 min 10 sec
tilt $S_{x x}$ about telescope $X$ axis: 0 min 00 sec
(note that this must be zero)
rotation H about vertical axis: 3 min 52 sec

The tilts/rotation computed from the mathematical model were as follows:
tilt $S_{y y}$ about telescope $Y$ axis: 6 min 12 sec
tilt $S_{x x}$ about telescope $X$ axis: 0 min 02 sec
rotation H about vertical axis: 3 min 53 sec

The maximum discrepancy is 2 arc seconds, with the precision estimated to about 3 arc seconds. The test verifies that the mathematical model is correct.

Transformation Model of Tilts/Rotation Using One Mirror Perpendicular to the Shaft Axis and the Other Mirror at Any Angle $\gamma$

To test this mathematical model, one mirror must be perpendicular to the telescope $X$-axis and the other is at angle gamma $(\gamma)$ to the $X$-axis. The model can be tested by making use of the previous test results, in which the angle $\gamma$ was computed as 9.8 degrees for mirror M1 and 79.2 degrees for mirror M2. Also note again that the actual rotation about the telescope Z-axis is zero. This allows use to make two independent sets of computation about the telescope $Y$-axis: one using mirror M1 and one using mirror M2. The results were as follows:
tilt $S_{y y}$ about telescope $Y$ axis (from mirror M1) : 6 min 8 sec
tilt $S_{y y}$ about telescope $Y$ axis (from mirror M2) : 6 min 24 sec
The discrepancies are 2 arc seconds and 14 arc seconds which can be accepted as verification of the mathematical model.

### 5.4 IMPLEMENTED COMPUTER PROGRAMS

Three computation stages are required to calculate the desired tilts and rotation. These stages are described in the following sections.

### 5.4.1 Reference Network Computations

A commercial software package is used to calculate the coordinates of the reference network. This software package is called MONALYSA (Monitoring Network Analysis and Adjustment) and was developed by Biacs (1988) in an M.Sc. program supervised by W.F. Teskey. The adjustment of the network requires measured directions and vertical angles to the reference network points, and approximate coordinates for these points.

The result of this computation stage are the adjusted coordinates of the reference network points. This information is very important for the next stage where some of these coordinates are used for the determination of the theodolite positions by the resection method.

### 5.4.2 Resection and Adjusted Azimuth Computations

A program was written by the author to calculate the adjusted coordinates of the theodolite using the resection method. For each resection, it is required that directions to at least four points be taken, so that an accurate position of the theodolite can be computed. The program also computes the adjusted azimuth of the zero circle reading of the theodolite and the azimuth of the line normal to the mirror.

The particular resection method implemented is explained in detail in

Kahman (1988). MONALYSA was used to check the output of the program. The program requires the measured directions to reference network points and coordinates of the reference network points. It does not require approximate coordinates for the position of the theodolite; the program will calculate the approximate coordinates from the input information.

### 5.4.3 Tilts/Rotation Computations

Using an ordinary scientific calculator the tilts and rotation of the mirrors and the tilts and rotation about the shaft axes are calculated.

### 5.5 REFRACTION EFFECTS

Tests for refraction effects were carried out because it was realized that these effects would decrease the accuracy of measurements. In an actual application, the mirror usually is glued to the body of the machine (see Figure 5.9) and sometimes the temperature of this body rises because of operating conditions. The body can therefore act as a heat generator.


Figure (5.9) : A mirror glued to the machine body

There are three ways the heat may be transferred from the body: conduction, convection, radiation. Air is a very poor conductive material; therefore, the heat transfer is via only convection and radiation. Each has different effects on the theodolite readings. As the angle between the line of sight and the machine body changes, the refraction effect will also change.

The tests were carried out by placing a ceramic heater (with a fan built in) behind a fixed mirror. In different epochs, theodolite readings were taken, normal to the mirror. It was noticed that when the heater was on, convection effects were dominant, and when the heater was shut off, radiation effects dominant.

In each epoch, two readings were taken: first when the heater was on (convection effects), second, immediately after it was shut off (radiation effects). A static reading was taken before ceramic heater was turned on. Another static reading was taken at the end of the test after everything returned to room temperature.

Two sets of tests were carried out: first when the heater was centered behind the mirror (corresponds to a mirror mounted on a machine body), and the second test when the heater was not centred behind the mirror (corresponds to a mirror mounted at an angle to the machine body); see Figure (5.10).

(a) : Ceramic heater centered

(b) : Ceramic heater off centre

Figure (5.10) : Test for refraction effects

The results are shown in Table (5.1). The values given are the differences with respect to the static values. In all cases, four repeated readings were made with standard deviations computed from these repeated readings in the order of $\pm 3$ arc seconds.

Table (5.1)
Refraction effects on theodolite readings

| Heater <br> Position | Heater On $\quad$ ( Convection |  | Heater Off $=>$ Radiation |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Refraction effect (arc seconds) |  | Refraction effect (arc seconds) |  |
|  | V. Angle | H. Angle | V. Angle | H. Angle |
| Test \# I <br> Centered | +3 | +16 | $+24$ | +8 |
| Test \# II <br> Off centre | +12 | +30 | +38 | $+10$ |

The results of these two tests can be summarized as follows:
1- Radiation effects are larger than convection effects on vertical angles.
2- Convection effects are larger than radiation effects on horizontal angles.
3- Refraction effects (both from convection and radiation) are larger for a mirror mounted at angle to the heat source.

It is important to note that the values given in Table (5.1) are not applicable in situations in which measurements are made on operation machines where the refraction effects (whatever they are) will not vary too much. The values will be applicable where operating conditions change (e.g. startup). In these cases,
refraction corrections could be estimated from measurements made to a point on the machine base, i.e. a point one would expect to remain stable. Another possibility to minimize refraction effects, suggested by Robbins (1992), is to use a fan to mix the air along the line of sight between theodolite and target. This reduces temperature gradients and thus refraction effects.

### 5.6 VIBRATION

Vibration effects are minor and can be ignored according to Bayly (1991). The main reason for this is due to the very small amplitude of vibrations in heavy, high speed, rotating machinery. For example, at a frequency of 60 Hz , peak-to-peak displacements of 0.0013 to 0.0026 mm are considered to be in the "smooth" range while displacements above 0.08 mm are considered to be in the "very rough" range. These small displacements have little effect on the precision of theodolite measurements. In Bayly (1991), two tests were carried out. The first test dealt with vibration effects on targets. It was found that a theodolite angular precision of one arc second was attainable to targets vibrating with typical machinery vibrations. The second test dealt with vibration effects on the theodolite. Here it was found that a two arc second precision was attainable. Obviously, vibration effects on both targets and instrument can be ignored.

### 5.7 DISCUSSION AND SUITABLE APPLICATIONS

The autoreflection method has been shown to be a low cost, precise, reliable, and flexible solution for three-dimensional rotation measurements of a point on a structure. Actual field tests, however, are still required. Two main applications were chosen. The first one was the determination of the change in position of a turbine/compressor coupling in a startup situation. The second one was the determination of the change in alignment of a turbine/compressor combination in an operating situation. A third application involved the monitoring of three-dimensional rotations in buried pipelines. Details of all applications are given in Chapter Six.

## Chapter 6

## FIELD TESTING OF THE METHOD

The final stage in the development of the methodology is field testing at an industrial site. Three field test sites were chosen to apply this method. The following sections will define the problem, describe each site condition, and analyze and discuss the results obtained at each site.

### 6.1 ALIGNMENT MONITORING OF A TURBINE/COMPRESSOR COMBINATION

Turbine/compressor combinations are used by natural gas processing companies. In a gas processing plant, if the turbine/compressor system shuts down during production, there will be a major financial loss. One main reason for shutdown is because of change in coaxial alignment between the turbine and compressor. Therefore, the need to measure these changes is of major concern for these companies. In the following sections two turbine and compressor alignment problems will be discussed.

### 6.1.1 Startup Monitoring (Crossfield)

The first site is the Amoco Canada Gas Plant near Crossfield, Alberta, about 20 km north of Calgary.

### 6.1.1.1 Problem Definition and Conditions

At this plant, a two-stage gas turbine driven compressor was removed, overhauled and replaced. Amoco Canada was interested in the turbinecompressor coaxial alignment changes which occurred during start up (about a six-hour period).

According to manufacturer's specifications, the compressor and the turbine must be misaligned only in the vertical plane by 0.108 inch ( 2.74 mm ) before startup. After startup, due to thermal expansion, the turbine is expected to move until the two machines are coaxially aligned. Normally, the compressor is considered a fixed structure. If the machines are not aligned to within 0.080 inch ( 2.0 mm ) at the coupling, when the machines are running in "hot" condition, large vibrations will result. The large vibrations will then cause the automatic safety system to shut down the turbine.

A high precision electronic theodolite system (Wild T2000) was used to setup the required reference network. Reference network points are required to be stable and are defined by adhesive concentric circle targets. Points 100 to 103 inclusive are located on the concrete base on which the turbine/compressor is mounted, point 104 is located on a steel beam, and point 106 is located on a steel column. To establish the network, directions and zenith angles were measured on all theodolite-to-target lines from three theodolite stations. To provide scale in the network, the distance from 101 to 102 was measured with a steel tape. The same system, with a high-precision electronic distance measurement instrument added, was used to measure to targets on the turbine and compressor and to determine translations of these targets (Teskey et al. 1992).

A one arc second theodolite (Wild T2) equipped with an autoreflection target ring was used to measure to autoreflection mirrors on the turbine and
compressor to estimate the three -dimensional rotations. The mirrors, about 30 $\mathrm{mm} \times 30 \mathrm{~mm}$ in size and covered with a protective cap when not in use, were mounted on the compressor and the turbine by using a quick-setting epoxy followed by a silicon adhesive. Figure (6.1) shows the turbine/compressor combination, the positions of the mirrors, and the reference network points.


### 6.1.1.2 Analysis and Results

There were two epochs considered, with the first epoch measurements taken just before start up (static readings). The second measurements were taken 200 minutes after startup (dynamic readings), when the vibrations of the machines were minimal.

A major problem was encountered during the second epoch measurements. The turbine temperature rose until the epoxy between the mirror and the turbine body melted. This meant that it was not possible to determine the threedimension rotations of the turbine.

One of the mirrors on the compressor was mounted at a 45 degree angle with respect to the shaft X -axis. Transforming the measured rotations to the shaft axes (with sign given by the right hand rule) results in:

Rotation around the shaft $Z$ axis(Yaw) $\quad 2^{\prime} 3.5^{\prime \prime}=>0.6 \mathrm{~mm} / \mathrm{m}$ Rotation around the shaft $Y$ axis(Pitch) $-1^{\prime} 32.8^{\prime \prime} \Rightarrow 0.45 \mathrm{~mm} / \mathrm{m}$

The standard deviation of the computed yaw and pitch were approximately 20 and 7 arc seconds, respectively. The total movement at the coupling can be calculated by adding the rotation effect (considering the shaft axis length to be 0.33 m ) to the measured translations. Three-dimensional translations of the compressor were measured at approximately the same location as the rotations. The translation in $Y$ direction was -0.2 mm , and in Z direction was -.18 mm . The results are:
horizontal movements at compressor side of coupling (in $Y$ direction)

$$
=0.2 \mathrm{~mm}+(-0.2 \mathrm{~mm})=0.0 \mathrm{~mm}
$$

vertical movements at compressor side of coupling (in Z direction)
$=0.15 \mathrm{~mm}+(-0.18 \mathrm{~mm})=-0.03 \mathrm{~mm}$

The results show that there was very little movement on the compressor side of the coupling during startup. This means that there must have been the expected thermal growth on the turbine side during startup to offset the initial misalignment in the vertical plane between compressor and turbine. This finding is consistent with vibration measurements taken by the manufacturer's representative during startup which indicated that vibration levels were initially very high and then become very low several hours after startup.

### 6.1.2 Long Term Monitoring (Nordegg)

The second site is the Stolberg Compressor Plant near Nordegg, Alberta, about 150 km west of Red Deer.

### 6.1.2.1 Problem Definition and Conditions

At times during operation of this plant, the automatic safety system shut down the turbine/compressor unit. This happened early in the morning on very cold days $\left(-30^{\circ} \mathrm{C}\right.$ or colder), when the day-to-night temperature difference was $20^{\circ} \mathrm{C}$ or more.

Three sets of measurements were taken, one late afternoon, one early in the morning, and one at noon. A reference network had previously been established using a high precision electronic theodolite (Wild T2002). The network consisted of seven points: points 102 and 103 are located on short concrete columns supporting heavy steel skids on which the turbine and compressor are mounted,
and points $100,101,104,105$ and 106 are located on a monolithic concrete floor (no construction or expansion joints).


Two mirrors were mounted on the compressor, one perpendicular to the compressor axis and the other parallel to the compressor axis. The turbine mirrors were mounted as follows: one parallel to the turbine axis, and one mirror forming an angle $\gamma$ with the turbine axis. All the mirrors were 30 mm square in size and a quick-setting epoxy was used for mounting. A one arc second theodolite (Wild T2) equipped with an autoreflection target was used to make the measurements.

Figure (6.2) shows the turbine/compressor combination, the positions of the mirrors, and the reference network points.

### 6.1.2.2 Analysis and Results

The trip to the Stolberg plant was planned for a time when the day-tonight temperature difference was expected to be $20^{\circ} \mathrm{C}$. Unfortunately, the weather forecast was not accurate and the temperature difference was only $10^{\circ} \mathrm{C}$.

Measurements were taken at three epochs: the first at 6 PM (epoch 1), the second at 4 AM (epoch 2), and the third was at noon (epoch 3). The rotations transformed to the machine axes are shown in Table 6.1.

Table 6.1
Turbine/Compressor rotations

| Machine | Rotation about Z axis (Yaw) <br> (arc seconds) |  | Rotation about Y axis (Pitch) <br> (arc seconds) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Compressor | Turbine | Compressor | Turbine |
|  | +3.1 | +13.0 | +1.0 | +1.0 |
| Epoch 1-3 | +1.4 | +17.2 | +6.4 | +5.8 |

All standard deviations are approximately $\pm 6$ arc seconds.

The results show that the compressor did not move at all since all the monitored rotations are less than the 6 arc sec standard deviation. However, the turbine shows a very small rotation around the Z-axis. This rotation most probably is due to thermal expansion of the machine body and not the actual movements of the turbine shaft. In addition to these rotation measurements, Fuss and Robbins (Fuss 1992) monitored the translations of this turbine/compressor unit over a six month period. They found that while the machines were operating, there were no significant translations. However, when the unit was shut down, they found that there were a significant movements of all points on the turbine and compressor, as well as significant movement of point 102 which one would expect to be stable. It is suspected that the alignment problem may be caused by the fact that the steel skid on which the turbine and compressor are mounted is not stiff enough to resist temperature induced movements in the piping connected to the turbine and compressor.

### 6.2 DEFORMATION MONITORING OF A BURIED PIPELINE

There are three different sites in norther Alberta where three-dimensional deformation monitoring (both translations and rotations) is required on sections of large diameter buried natural gas pipeline. The autoreflection method is being used at two of these sites.

Because of unstable slopes and frost action, very large movements can occur in buried pipelines. These movements are sometimes large enough to cause pipeline failure and shutdown natural gas transmission. To prevent financial loss during an unplanned shutdown, monitoring systems are used. The usual criterion is that if estimated strain exceeds 5000 microstrain, remedial action (e.g. excavation and reburial of the line) will be taken.

Teskey (1991) has used two methods to measure the three-dimensional rotations at different points on a buried pipeline. In the first method, the tilts were too large for a one arc second theodolite (Wild T2) to measure (Section 2.2.2.2). In the second method, the temperature effects on an electrolytic level tiltmeter were much too large (see Chapter 3). The autoreflection method was then used. Two perpendicular mirrors were permanently mounted on each point on the buried pipeline. The required reference network was already in place to monitor the translations and allow for resection in order to compute rotations about vertical axes. Martin Geomatics of Lethbridge did lab testing to find a suitable epoxy and mirrors that could withstand the expected temperature changes (about $-40^{\circ} \mathrm{C}$ to $+30^{\circ} \mathrm{C}$ ).

Good results have been obtained with autoreflection. The accuracy of rotations about orthogonal $X$ and $Y$ axes in the horizontal plane (tilts) is about 15 arc seconds, and the accuracy of rotations about vertical axes is about 30 arc seconds (Martin 1992).

## Chapter 7

## CONCLUSIONS AND RECOMMENDATIONS

The research presented in this thesis resulted in the development of a low cost, precise method to monitor three-dimensional rotations of a point on a structure. The method is based on autoreflection and uses resection within a reference network to determine the rotation about the vertical Z -axis. The method has been shown to be:

1- Flexible: the measurement range is essentially unlimited.
2- Accurate: the accuracy is in the order of $\pm 6$ arc seconds.
3- Reliable: measurements are possible under normal lighting conditions and with vibration levels normally encountered in the industrial environment.
4- Low-cost: only a standard one arc second theodolite (e.g. Wild T2) is required.
5- Insensitive to thermal expansion effects if the machines are made from steel.
6- Insensitive to refraction effects.

Three field tests were performed. Two of the tests involved the monitoring of alignment changes in a turbine/compressor combination. The first involved monitoring the alignment changes at the coupling due to thermal expansion during a startup. The results had a precision of about 10 arc seconds and reflected the expected behaviour at the coupling. The second involved monitoring the effects of varying outside air temperatures on the alignment while a turbine/compressor combination was operating. The results indicated that there were no significant alignment changes. The third application was the measurement of tilts and rotations at different points on a buried pipelines. The
method was successful: 15 arc second precision was obtained for tilts (about horizontal X and Y axes) and 30 arc second precision was obtained for rotations about vertical Z axes.

The research work was also involved a thorough investigation of an electrolytic level tiltmeter. Although the instrument has two resolution modes, high gain ( 0.36 arc second resolution) and the low gain ( 3.84 arc seconds resolution), the tests show that the precision of the instrument is limited by the low gain resolution. The instrument has no drift problems. The main problem with the tiltmeter is that it only provides reliable readings if the temperature remains stable to within $5^{\circ} \mathrm{C}$ during all measurement epochs.

The linearity error is $0.05 \%$ in the low gain mode and $3.95 \%$ in the high gain mode. The accuracy is about $\pm 3$ to 4 arc seconds, if the tilt angle is less than 400 arc seconds whether it is in the low or high gain mode. However, if the tilt angle is more than 400 arc seconds, the high gain mode provides the same accuracy but the low gain mode accuracy decreases. The range of the instrument is $\pm 4$ degrees.

The tiltmeter will perform best when it is used for continuous tilt measurements. Where the tiltmeter is installed permanently, the temperature does not change, the AC power supply is connected continuously, and the measured tilt angle is not more than 400 arc seconds; the accuracy will be $\pm 3$ to 4 arc seconds.

The tiltmeter can be used to measure discrete tilts under the condition of constant temperature, a good force centring device, and 10 repeated readings. These tilts will have an accuracy of approximately $\pm 15$ arc seconds.

## RECOMMENDATIONS

The method developed in this thesis will normally take about 30 . minutes to measure three-dimensional rotations. This time can be reduced to about 10 minutes if the method is modified as follows.

1- Use an interline procedure instead of resection to define the theodolite position.
2- Use a modern electronic theodolite rather than an optically reading theodolite such as a Wild T2.

3- Use a braced tripod with adjustable height for the $Z$ axis, and a translation stage for the $X$ and $Y$ axes.
4- Develop software routines that accept the data directly from the electronic theodolite.

With regard to tilt measurement with tiltmeters rather than theodolites, the results produced by all tiltmeters would improve considerably if the tiltmeter were installed inside a temperature controlled chamber. The temperature in the chamber would be set at some high value (say $50^{\circ} \mathrm{C}$ ) so that some heating would always be required. In this way it would be very easy to control the temperature of the tiltmeter to within $1^{\circ} \mathrm{C}$ and thus almost completely eliminate the errors caused by temperature variation.

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