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Three Essays in Industrial Organization and Energy Economics

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by

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Abstract

The first chapter is about market efficiency. Broadly, the primary objective of antitrust policy is to promote efficiency. However, economists do not measure efficiency directly. Rather, they use a less precise concept, market power, because it is thought that market power is more easily measured and correlated with efficiency. The Herfindahl and Lerner indexes are standard measures of market power. This paper explores the circumstances in which these two indexes are and are not reliable indicators of efficiency. In most models, both indexes are often not good proxies of efficiency. In addition, I show how to compute efficiency directly by using firm level output and market price. I conclude that it is not easier to measure market power than efficiency. The second chapter focuses on the dynamic properties of an oligopolistic market structure with stochastic successful probability of creating or maintaining investments for a firm. In each period of a game, a representative firm decides whether to have positioning (creating or maintaining) investments. All established firms compete either in the Bertrand or Cournot framework. By adopting symmetric Market perfect Nash equilibrium, firms choose strategies to maximize the discounted present value of cash flow. We obtain the steady state of the system and the expected duration for each state of the system in different market competitions. We also discover the efficiency level is higher in the Cournot rather than the Bertrand competition with different numbers of active firms in the market. The third chapter develops a theoretical model for the electricity sector in the presence of imperfect competition in a Cournot framework. By allowing market size, industry concentration, and technology to differ, the model incorporates transmission costs and transmission capacity. We identify the condition under which trade benefits a single country or increases global welfare by changing these asymmetric factors for each country. We discover that the expansion of transmission capacity between two countries is not always beneficial for a single country or for global welfare.

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Chapter 1

Market Power VS. Market Efficiency: Towards a Direct Measurement of Market Efficiency

1.1 Introduction

Measurement is a core activity in any social science. Economists deal with complex empirical issues related to various measurements such as price indexes, and productivity. At a microeconomic level, economists and policy makers are interested in knowing the extent to which resources are optimally allocated by markets. Market efficiency is defined as ratio of the realized social surplus to maximum possible social surplus. However, instead of measuring efficiency directly, economists often use a less precise concept - market power - because it is believed that market power is highly correlated with efficiency, so that high market power indicates low market efficiency and vice versa. In the existing literature, the Herfindahl and Lerner indexes are the standard measures of market power. The Herfindahl index is calculated by summing the square of market share of each firm and the Lerner index is the ratio of the markup to market price. One purpose of this paper is to explore the circumstances in which the Herfindahl and Lerner indexes are and are not reliable indicators of efficiency.

Policy makers are concerned with market efficiency. However, in oligopolistic markets such as airlines, railways, electricity, gas, and cement, market efficiency is not always easily attained. Moreover, a slightly loss of efficiency in these industries causes harmful consequences. For example in a 300 (Terawatt hour)/year electricity market, such as Britain, the wholesale market value is 10 billion euros per year. Therefore, each 1% loss of efficiency equals 100 million euros per year.[1] Therefore, I would anticipate that the government would

[1] See: www.ofgem.gov.uk/electricity/wholesale-market/electricity-security-supply

be interested in measures that provide accurate information regarding market efficiency.

Economists often use the Herfindahl index to detect market power instead of measuring efficiency directly. Since 1984, the U.S. Department of Justice (USDOJ) has used the Herfindahl index as a concentration measure for merger reviews. This practice has since been followed by several other regulatory bodies in the U.S., such as the Federal Reserve Board (banking), the Federal Energy Regulatory Commission (electricity), and the Department of Transport (aviation). These governmental bodies use concentration measures as a screening tool. If a market concentration falls into a "safe" level, often no further analysis is pursued. The Herfindahl index number ranges from close to zero to a maximum of 1. A value of 1 indicating a single firm controls 100% of the market. According to the guidelines set forth by the USDOJ an industry with a Herfindahl index lower than 0.1 is considered unconcentrated, and mergers need not be analyzed. An industry with a Herfindahl index between 0.1 and 0.18 is considered moderately concentrated and mergers that create an increase in the Herfindahl index of 0.01 points or more raise competitive concerns and need to be approved. Finally, an industry with a Herfindahl index greater than 0.18 is considered highly concentrated, and mergers causing an increase of greater than 0.005 points raise competitive concerns.[2]

The Herfindahl index is also used as a concentration measure to detect market power in academic research. Researchers misleadingly assume the market power is monotonically related with efficiency. Based on their studies, they conclude that higher concentrated market always experiences lower efficiency.[3] Similarly, the Lerner index has been applied to many industries. For example, the Lerner index has been widely used in the banking and electricity sectors as an indicator of degree of market power. In either sector, these authors argue that market with higher Lerner indexes have lower efficiency.[4] In addition, more studies

[2]For more details refer the U.S. Department of Justice, Horizontal Mergers Guideline, http://www.justice.gov/atr/public/guidelines/hmg-2010.html.

[3]For example, some studies adopt the Herfindahl index to detect market power for the aviation and electricity markets. (e.g.,Hurdle et al. (1989), Borenstein (1990, 1991, 1992), Kim and Singal (1993), Morrison and Winston (1990), Abramowitz and Brown (1993), Joesch and Zick (1994), Schmalensee and Golub (1984), and Cardell et al. (1997).)

[4]Some of the most important studies have been done for Canadian banks, Italian banks, and for samples

corresponding with the Herfindahl and Lerner indexes have been conducted to measure market power.[5] They conclude there is a relationship between market concentration and efficiency. However, I demonstrate by stimulation that there is no strong correlation between them. Moreover, there is no existing literature showing a direct measurement of efficiency highly correlated with concentration.

Even though there are extensive applications and research studies concerning the Herfindahl and Lerner indexes, there have also been many criticisms of these two indexes. The Herfindahl index is mainly criticized for two aspects, first regarding robustness with respect to the definition of the market, and second with respect to the relationship between market power and concentration. The definition of the market has several dimensions, such as geographic dimensions, product characteristic dimensions, and the level of aggregation.[6] Therefore, the concept of "market" can be difficult to define. Even when the market is clearly defined, my paper demonstrates theoretically the Herfindahl index is not a good proxy for efficiency. This is because the economists believe the Herfindahl index is monotonically related with efficiency, but it is not true in reality. The second critique, pointed out by Tirole (1988), is that concentration measures generally ignore many important factors determining market power, such as asymmetries in firms or demand, and costs of entry. Boone and Weigand (2000), Sheffrin (2001), and Blumsack et al. (2002) for example, provided empirical support for Tirole's contribution that the index of market concentration is not a good indicator for efficiency: a lower concentration market may experience lower competition rather than higher competition. However, both theatrical and empirical studies misleadingly considered the profit of firms as a proxy of market inefficiency.

One criticism of the Lerner index is that it requires the use of marginal costs. These,

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of countries in the European Union. (e.g.,Shaffer (1993),Angelini and Cetorelli (2003), Fernandez de Guevara et al. (2005). Maudos and Fernandez de Guevara (2004), and De Guevara and Maudos (2004)) Moreover, von der Fehr and Harbord (1993) derived Lerner indexes from 1990 - 1991 for the two large conventional electricity companies in the England and Wales pool.

[5]Summaries can be found in Weiss (1971, 1974, 1989), Bresnahan (1989), Schmalensee (1989), Scherer and Ross (1990), Martin (1993), Waldman and Jensen (2001), Newmark (2004), White (2006).

[6]The papers are as follows: Hannan (1997), Kwoka Jr (1998), Cesari (2000), Nevo (2000), Lijesen (2004).

however, may be difficult to measure either because data maybe confidential or because a substantial portion of costs may be opportunity costs. Therefore, many studies use price level change, rather than the Lerner index to detect market power. They argue that the market would experience higher market power if the market price increases more and the market would be less efficient. These studies have been conducted in various industries such as, railroads, airlines, electricity, banking, and livestock procurement.[7] Increases in market prices, however, do not necessarily indicate efficiency loss. For example, an increase in market price caused increases in input prices indicates higher costs, but not necessarily a decrease in market efficiency.[8] My paper complements previous literature by showing that it is not necessary to calculate these two indexes if the primary reason to detect market power is to discover market efficiency. In the context of a Cournot framework, I adopt a linear demand function and explore several models of oligopolistic market structures for a homogenous good. A particular market structure is completely described by a number of randomly determined parameters having to do with costs, demand and the number of firms. Then, for each of a large number of market structures within the stochastic model under consideration, I directly calculate market efficiency and the Herfindahl and Lerner indexes. In these calculations, firms have either constant or increasing marginal costs. I then use the data generated by this simulation to assess the reliability of the Herfindahl and Lerner indexes as indicators of efficiency. For most of the models I examine, neither index is a universally reliable indicator of efficiency, and often neither index could be described as a satisfactory indicator of efficiency. The Herfindahl index tends to be better than Lerner index with constant marginal costs, while the Lerner index tends to be better than the Herfindahl index with increasing marginal costs. Most importantly, I compute market efficiency directly by using firm level output and market price, both of which are easy to observe. Therefore, this suggests that researchers ought to concentrate on estimating efficiency directly instead

[7] For more detailed papers refer to Weiss (1989), Audretsch et al. (1992), Baker (1999), Dalkir and Warren-Boulton (2004), Busse and Rysman (2005), Ashenfelter et al. (2006).

[8] Details refer to Stoft (2002)

of rely on the Herfindahl and Lerner indexes which are imperfect indicators of efficiency. Furthermore, by varying the settings as illustrated above, the stochastic models can be examined under different competitions (e.g. Bentrand), differential market goods, non-linear demand function, or more general cost functions. Obviously, in each of these model settings, neither the Herfindahl index nor the Lerner index can be a perfect proxy for efficiency as well.

The paper proceeds as follows: In Section 2, I explore several stochastic models in the context of constant marginal costs and increasing marginal costs. Section 3 shows how market efficiency can be directly calculated in the context of constant marginal costs and increasing marginal costs with n number of firms. To conclude, Section 4 underscores some of the key contributions of this work.

1.2 Herfindahl, Lerner, Efficiency

In this section, I theoretically demonstrate that neither the Herfindahl index of market concentration, H, nor the Lerner index of market power, L, are universally reliable indicators of market efficiency in an oligopolistic market and investigate the degree of reliability of H and L as indicators of efficiency. In addition, I also discover circumstances in which technological progress in terms of reduction of marginal costs worse efficiency.

1.2.1 Methodology

First, let me introduce the methodology that I will apply. I explore a number of stochastic models of oligopolistic market structure for a homogenous good. In all these models, price is a linear function of total quantity as follows:

$$P = A - Q, \qquad A \geq 0$$

where P is price and Q is the total demand per period of time, which equals $\sum_{i=1}^{n} Q_i$. Also, each firm follows Cournot competition. Therefore, I obtain a number of different models. In some, each firm has constant marginal costs so that

$$TC(Q_i) = c_i Q_i$$

where c_i is constant marginal costs. In others

$$TC(Q_i) = \frac{1}{2}\beta_i Q_i^2$$

where β_i is slope of increasing marginal costs. I assume $c_i \geq 0$ and $\beta_i \geq 0$ which indicate non-negative marginal costs. In some models, the market marginal costs are identical, while in others they are asymmetric. In some models, the number of firms is a constant, while in others it is a random variable. Then, I can calculate for any randomly generated market structure, equilibrium quantity, equilibrium price, total surplus in the equilibrium, and the maximum possible surplus for each given market structure. Using this information, for any randomly generated market structure s, I derive H_s (the Herfindahl index), L_s (the Lerner index), and E_s (efficiency, equal to the ratio of equilibrium total surplus to maximum possible total surplus).

When comparing H_s for two markets, say markets 1 and 2, the *standard interpretation* is that when $H_1 < H_2$, market 1 has a higher market efficiency than market 2, or when $H_1 = H_2$, both markets have equal efficiency. Similarly, when comparing L for two markets, the *standard interpretation* is that when $L_1 < L_2$, market 1 has a higher efficiency than market 2, or when when $L_1 = L_2$, the market 1 has the same efficiency as market 2. Notice that the *standard interpretation* of H and L may or may not accurate. In each stochastic model, I will calculate probability that the *standard interpretation* of the Herfindahl index is accurate for two random generated market structures as well as calculating the probability of the *standard interpretation* of the Lerner index is accurate. Naturally, the probability must be greater than 0.5 for consideration as an useful indicator of market efficiency. The closer it is to 1, the more reliable it is; however, as we will see, in some models, probabilities are very close to 0.5 which indicate that the Herfindahl index or the Lerner index has no practical use.

1.2.2 Equilibrium Calculations

In this section, there are two sets of equilibrium calculations. The first set contains all firms with constant marginal costs, and the second set has all firms with increasing marginal costs.

Constant Marginal Costs

Suppose there are n firms in the market and each firm has constant marginal costs, c_i. Each firm competes with quantity and solves the following maximization problem:

$$\underset{Q_i}{Max}\ PQ_i - c_iQ_i$$

I introduce tilde "$\sim$" which indicates the equilibrium value of all firms with constant marginal costs. Then, the equilibrium quantity of firm i, $\tilde{Q}_i$, and equilibrium price $\tilde{P}$ are as follows:

$$\tilde{Q}_i(A, n, c_1, c_2, ..., c_n) = \frac{A - nc_i + \sum_{i=1}^{n} c_i}{n+1}$$

$$\tilde{P}(A, n, c_1, c_2, ..., c_n) = \frac{A + \sum_{i=1}^{n} c_i}{n+1}$$

To obtain non-negative equilibrium quantities, I have to impose the following restriction:

$$\frac{A - nc_i + \sum_{j \neq i}^{n} c_j}{n+1} \geq 0$$

In equilibrium, profits of the two firms and consumer surplus can be written as functions of A, n, and c_i as expressed below:

$$\tilde{\pi}_i(A, n, c_1, c_2, ..., c_n) = \left(\frac{A - (n+1)c_i + \sum_{i=1}^{n} c_i}{n+1} \right)^2$$

$$\widetilde{CS}(A, n, c_1, c_2, ..., c_n) = \frac{1}{2} \left(\frac{nA - \sum_{i=1}^{n} c_i}{n+1} \right)^2$$

By adding $\tilde{\pi}_i$ and $\widetilde{CS}$, I can obtain the aggregate social welfare ($\tilde{W}$) as follows:

$$\tilde{W}(A, n, c_1, c_2, ..., c_n) = \sum_{i=1}^{n} \tilde{\pi}_i + \widetilde{CS}$$

The market efficiency is examined under the index value of equilibrium efficiency $\tilde{E}$ which can be derived as stated below:

$$\tilde{E}(A, n, c_1, c_2, ..., c_n) = \frac{\tilde{W}(A, n, c_1, c_2, ..., c_n)}{\widetilde{TPS}(A, n, c_1, c_2, ..., c_n)} \tag{1.1}$$

Where $\widetilde{TPS}(A, n, c_1, c_2, ..., c_n) = Max\{\frac{(A-c_1)^2}{2}, \frac{(A-c_2)^2}{2}, ..., \frac{(A-c_n)^2}{2}\}$ (*see Appendix* 1.5.1). It is achieved by choosing quantities until market price equals to the lowest marginal costs. Then, the equilibrium efficiency is derived from equilibrium total surplus divided by the total possible social welfare in this market. The Herfindahl index and the Lerner index are two of the most popular measures used to estimate market power. In equilibrium, the Herfindahl index is defined as cited below:

$$\tilde{H}(A, n, c_1, c_2, ..., c_n) = \sum_{i=1}^{n} \tilde{S}_i^2 = \sum_{i=1}^{n} \left(\frac{A - (n+1)c_i + \sum_{i=1}^{n} c_i}{nA - \sum_{i=1}^{n} c_i} \right)^2 \tag{1.2}$$

where $\tilde{S}_i$ is the market share of each firm in equilibrium. In equilibrium the Lerner index is defined as written below:

$$\tilde{L}(A, n, c_1, c_2, ..., c_n) = \frac{\tilde{P} - \sum_{i=1}^{n} \tilde{S}_i MC_i}{\tilde{P}} = \frac{\tilde{H}}{\tilde{\varepsilon}}$$
$$= \left(\sum_{i=1}^{n} \left(\frac{A - (n+1)c_i + \sum_{i=1}^{n} c_i}{nA - \sum_{i=1}^{n} c_i} \right)^2 \right) \left(\frac{nA - \sum_{i=1}^{n} c_i}{A + \sum_{i=1}^{n} c_i} \right) \quad (1.3)$$

Here, $\tilde{\varepsilon}$ represents the elasticity of demand (*see Appendix* 1.5.1).

Increasing Marginal Costs

In analogous fashion, suppose n firms with marginal costs increase linearly with quantity. Each firm follows Cournot competition and solves the following maximization problem:

$$\underset{Q_i}{Max}\ PQ_i - \frac{1}{2}\beta_i Q_i^2$$

The symbol hat "$\wedge$" indicates the equilibrium value of increasing marginal costs firms. Then, the equilibrium quantity of firm i, $\hat{Q}_i$, and equilibrium price $\hat{P}$ are as follows:

$$\hat{Q}_i = \frac{A}{(1+\beta_i)(1+\sum_{i=1}^{n} \frac{1}{1+\beta_i})}$$

$$\hat{P} = \frac{A}{(1+\sum_{i=1}^{n} \frac{1}{1+\beta_i})}$$

Since I have assumed $A \geq 0$ and $\beta_i \geq 0$, the equilibrium quantities and price always have valid value. In the equilibrium market, I derive the total welfare, the Lerner index, and the Herfindahl index when both firms have increasing marginal costs as expressed below:

$$\hat{W}(A, n, \beta_1, \beta_2, ..., \beta_n) = \sum_{i=1}^{n} \hat{\pi}_i + \widehat{CS}$$
$$= \sum_{i=1}^{n} (1 + \frac{1}{2}\beta_i) \left(\frac{A}{(1+\beta_i)(1+\sum_{i=1}^{n} \frac{1}{1+\beta_i})} \right)^2 + \frac{1}{2} \left(\frac{A(\sum_{i=1}^{n} \frac{1}{1+\beta_i})}{(1+\sum_{i=1}^{n} \frac{1}{1+\beta_i})} \right)^2$$

$$\hat{H}(A,n,\beta_1,\beta_2,...,\beta_n)=\sum_{i=1}^{n}\hat{S}_i^2=\sum_{i=1}^{n}\left(\frac{1}{(1+\beta_i)(\sum\limits_{i=1}^{n}\frac{1}{1+\beta_i})}\right)^2 \tag{1.4}$$

$$\hat{L}(A,n,\beta_1,\beta_2,...,\beta_n)=\frac{\hat{H}}{\hat{\varepsilon}}=\left(\sum_{i=1}^{n}\left(\frac{1}{(1+\beta_i)(\sum\limits_{i=1}^{n}\frac{1}{1+\beta_i})}\right)^2\right)\left(\sum_{i=1}^{n}\frac{1}{1+\beta_i}\right) \tag{1.5}$$

The level of market efficiency can be examined as the index as noted below:

$$\hat{E}(A,n,\beta_1,\beta_2,...,\beta_n)=\frac{\hat{W}(A,n,\beta_1,\beta_2,...,\beta_n)}{\widehat{TPS}(A,n,\beta_1,\beta_2,...,\beta_n)} \tag{1.6}$$

where $\widehat{TPS}(A,n,\beta_1,\beta_2,...,\beta_n)=\frac{A^2\left(\sum\limits_{i=1}^{n}\frac{1}{\beta_i}\right)}{2\left(1+\sum\limits_{i=1}^{n}\frac{1}{\beta_i}\right)}$(*see Appendix* 1.5.1). The total possible surplus is derived when each firm chooses to produce level of its output as long as $MC_1(Q_1)=MC_2(Q_2)=...=MC_n(Q_n)=P(\sum\limits_{i=1}^{n}Q_i)$ holds. Obviously, the higher the level of increasing marginal costs in the market, the higher the market price will be and the total possible surplus will decrease.

1.2.3 Market Structures, Constant Marginal Costs

In the context of constant marginal costs, the stochastic models vary in different ways: (1) whether firms have same constant marginal costs, or (2) whether the number of firms is stochastic.

Same Constant Marginal Costs

First, let me consider that all firms have same constant marginal costs, $c=c_1=c_2=...=c_n$, the stochastic model depends on parameters: A, n, and c. A is size of the market with uniformly distributed in range [0, 2]; constant marginal cost, c, is uniformly distributed in range [0, A]; and number of firms, n, is discrete uniformly distributed in range [1, 5]. Then, I derive $\tilde{E}$, $\tilde{H}$, and $\tilde{L}$ in A, n, and c parameter space:

$$\tilde{E}(A,n,c) = \frac{\tilde{W}(A,n,c)}{\widetilde{TPS}(A,n,c)} = \frac{2n+n^2}{(n+1)^2} \tag{1.7}$$

$$\tilde{H}(A,n,c) = \sum_{i=1}^{n} \tilde{S}_i^2 = \frac{1}{n} \tag{1.8}$$

$$\tilde{L}(A,n,c) = \frac{\tilde{H}}{\tilde{\varepsilon}} = \frac{n(A-c)}{n(A+nc)} \tag{1.9}$$

Even a cursory examination of (2.7) and (2.8) reveal that efficiency and the Herfindahl index only depend on the number of firms; one with positive relation and the other with negative. Therefore, I conclude that the *standard interpretation* of $\tilde{H}$ is always accurate.

Proposition 1. *The Herfindahl index is a reliable indicator for market efficiency when we have same constant marginal costs firms in the market.*

Equation (2.9) shows that the Lerner index relies not just on the number of firms, but also the market size and the levels of marginal costs. For this reason, it can not always give qualitatively accurate predictions of efficiency. Moreover, I can also quantify the degree of reliability of the Lerner index as indicator of efficiency. By stimulating 10000 stochastic market structure pairs with these three parameters, I obtain a set of market efficiency indexes and the Herfindahl indexes. When I pick a market efficiency $\tilde{E}_1$ higher than $\tilde{E}_2$ or $\tilde{E}_1$ equal to $\tilde{E}_2$ and the probability of $\tilde{L}_1$ lower than $\tilde{L}_2$ or $\tilde{L}_1$ equal to $\tilde{L}_2$ is 0.55 which is not reliable. If I hold marginal costs level constant, the market efficiency increases and the Lerner index decreases with more competition. Thus, the Lerner index gives qualitatively accurate predictions of market efficiency when c is fixed.

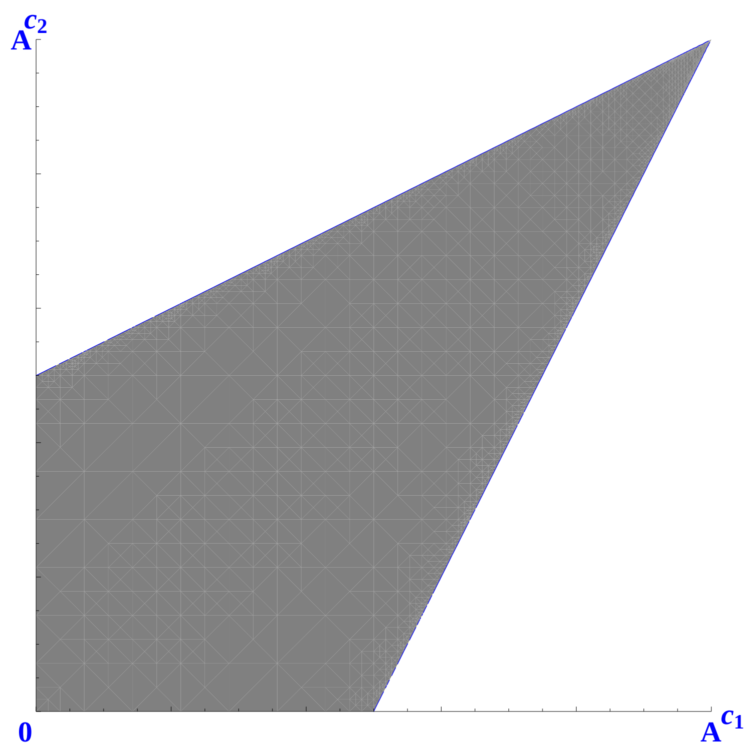
Proposition 2. *With all firms have same constant marginal costs, the Lerner index does not give good prediction for market efficiency and the probability of accuracy is only 0.55. If the levels of constant marginal costs hold, the Lerner index is a good indicator for market efficiency.*

Non-stochastic Number of Firms

In this section, I will explore several models in the constant marginal costs framework as well. In each model, the number of firms are identical in each market and n, is discrete uniformly distributed in range [1, 4]. So I can develop four different models depend on the discrete number of firms. Meanwhile, A is random and uniformly distributed in range [0, 2], and c_i is uniformly distributed in range [0, A]. First, I will construct two firms with asymmetric constant marginal costs model. This gives an explicitly theoretical demonstration in which circumstances the Herfindahl or Lerner are reliable for efficiency and provides insightful intuitions in the two parameters space. I can calculate the equilibrium quantity, price, total surplus as well as maximum possible surplus for this given market structure. In order to obtain non-negative equilibrium quantities, I have to impose the following restriction:

$$\frac{A - 2c_i + c_j}{3} \geq 0,\ i \neq j \text{ and } i, j \in \{1, 2\} \tag{1.10}$$

Figure 1.1: The Permissable Parameter Space with Two Constant Marginal Cost Firms



Therefore, the valid region in the parameters A, c_1 and c_2 space is the shaded areas in

Figure 1.2: The Herfindahl and Efficiency Indexes with Two Constant Marginal Cost Firms

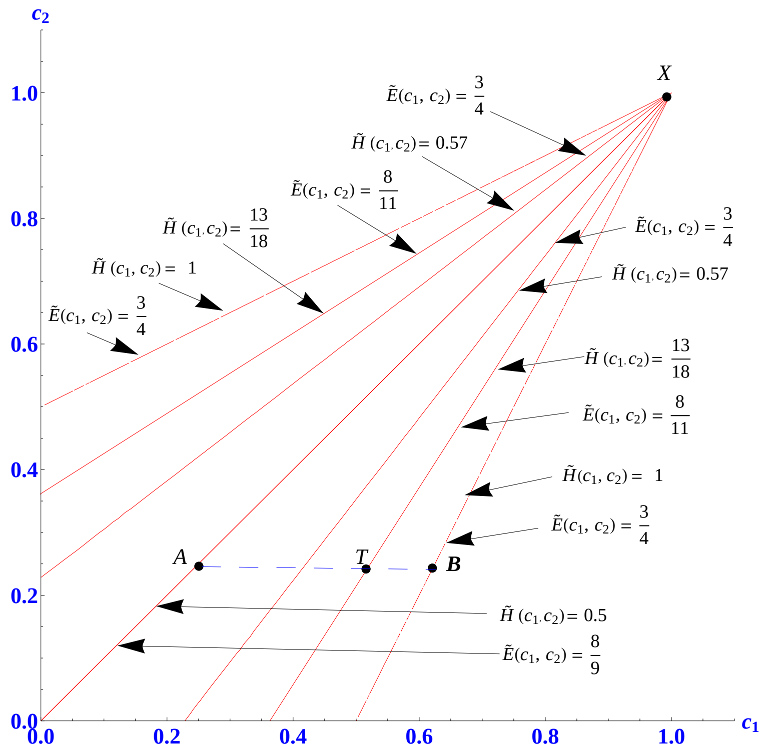


Figure 1.1.

The Herfindahl, Lerner and efficiency can be written as function of A, c_1 and c_2 within the permissable parameter space. With constant market size, I examine how $\tilde{E}$, $\tilde{H}$, and $\tilde{L}$ change when levels of constant marginal costs vary.

In Figure 1.2, I propose an example which plots the diagram in the c_1 and c_2 space. I plot three isoefficiency rays: $\tilde{E}(c_1, c_2) = \frac{8}{11}$, $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and $\tilde{E}(c_1, c_2) = \frac{8}{9}$. The isoefficiency ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ is the minimum value ray; $\tilde{E}(c_1, c_2) = \frac{3}{4}$ is the boundary ray and $\tilde{E}(c_1, c_2) = \frac{8}{9}$ is maximum value ray. All the rays are passing through point X and have constant value of equilibrium efficiency along each ray. $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and $\tilde{E}(c_1, c_2) = \frac{3}{4}$ are symmetric around the 45 degree line, which has $\tilde{E}(c_1, c_2) = \frac{8}{9}$. $\tilde{E}(c_1, c_2) = \frac{3}{4}$ has four segment rays passing through point X. Two rays are boundaries of valid region of parameter space and the other two are located inside the valid region. The minimum value of equilibrium efficiency

$\tilde{E}(c_1, c_2) = \frac{8}{11}$ is associated with two segment rays passing through point X which is located between the boundary ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$, and the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$. Therefore, as the isoefficiency rays deviate farther from 45 degree line, $\tilde{E}$ decreases to its minimum value $\frac{8}{11}$, then increases. In addition, we also plot four different isoHerfindahl rays: $\tilde{H}(c_1, c_2) = \frac{1}{2}$, $\tilde{H}(c_1, c_2) = \frac{13}{18}$, $\tilde{H}(c_1, c_2) = 0.57$ and $\tilde{H}(c_1, c_2) = 1$. Each ray has two segment rays passing through point X, except the curve overlapping the 45 degree line, which has $\tilde{H}(c_1, c_2) = \frac{1}{2}$. $\tilde{H}(c_1, c_2) = 1$ plots the same ray as the boundary ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$, $\tilde{H}(c_1, c_2) = 0.57$ plots the same ray as the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and $\tilde{H}(c_1, c_2) = \frac{13}{18}$ plots the same ray representing the minimum value of market efficiency $\tilde{E}(c_1, c_2) = \frac{8}{11}$. In addition, I can always find the same ray of isoHerfindahl correspondence to the isoefficiency ray. Considering points along each ray, I can conclude that the Herfindahl index gives a qualitatively accurate prediction of market efficiency.

Let me do one further investigation with Figure 1.2. By holding c_2 constant and assuming $c_1 \geq c_2$, I examine how $\tilde{H}$ and $\tilde{E}$ change as c_1 is increased. In this figure, we refer how $\tilde{H}$ and $\tilde{E}$ change alone the horizontal dashed ATB line. This line links points A and B, and is cross to the ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ at point T. In this situation, we have a constant c_2, and the firm 1's technology level declines as we move from point A to B. I find the Herfindahl index increases from A to B. However, the market efficiency decreases from point A to its minimum value T, and then increases afterwards. This is because the decrease in the marginal cost of firm 1 has two effects on market efficiency when firm 2's marginal cost is less or equal than firm 1 and stays constant. First, it has a positive effect on firm 2's profit; however, it has a negative effect on the consumer surplus and its own profit. When c_1 is relatively low, a small increase in firm 1's marginal cost results in a small increase in firm 2's profit; however, it causes a large decrease of consumer surplus and firm 1's profit. The reason is that when firm 1 is sufficiently efficient, firm 2 does not produce large enough. As c_1 continues to increase, the negative effect on consumer surplus and its own profits dominates the effect on firm

2's profits for some time, but is eventually overwhelmed. When c_1 is sufficiently large, the increase in profit of firm 2 resulting from the displacement of production from firm 1 becomes larger than the reduction in consumer surplus and firm 1's profit, because firm 1 does not produce much if their marginal cost is sufficiently high. Therefore, the market becomes more efficient after its minimum point T. In this figure, I find that the market has its maximum efficiency when $c_1 = c_2$ and has its minimum efficiency when $c_1 = \frac{7}{11}c_2 + \frac{4}{11}$. Therefore, I can conclude that transferring more advanced technology for firm 1 which decreases its marginal cost is not always benefit for the market efficiency. If $c_1 < \frac{7}{11}c_2 + \frac{4}{11}$, the decrease of firm 1's marginal cost benefits the market efficiency. Otherwise, it would be worse off. And the market has its optimal efficiency level if all the firms have same constant marginal costs.

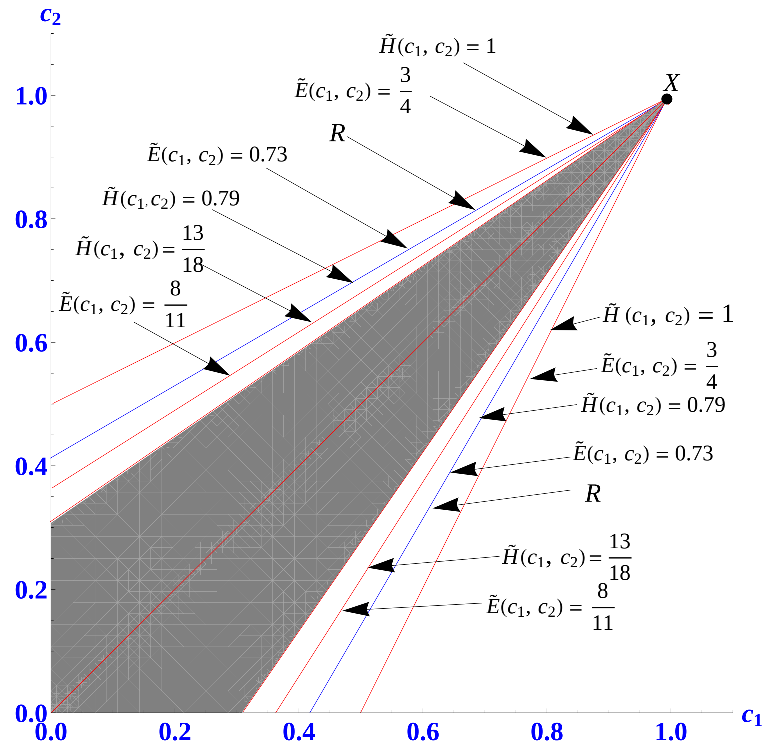
Now, let me pick a reference ray R in four different locations: (1) between the boundary ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the minimum value of equilibrium efficiency $\tilde{E}(c_1, c_2) = \frac{8}{11}$ (2) between the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the maximum value ray $\tilde{E}(c_1, c_2) = \frac{8}{9}$ (3) between the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and (4) at the minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$. Based on the reference point R, I identify all the points in the parameter space that (1) have both a smaller $\tilde{H}$ and a larger $\tilde{E}$ than R, (2) have a larger $\tilde{H}$ and a smaller $\tilde{E}$ than R, or (3) have an identical $\tilde{H}$ and $\tilde{E}$ to R. All of these points are located in the shaded region of the figure. If we compare $\tilde{H}$ for any point in the shaded areas with the $\tilde{H}$ for point R, the *standard interpretation* of $\tilde{H}$ is accurate. Conversely, I can also identify all the points that (1) have both a smaller $\tilde{H}$ and a smaller $\tilde{E}$ than R, or (2) that have a larger $\tilde{H}$ and a larger $\tilde{E}$ than R. These belong to the non-shaded areas. If I compare $\tilde{H}$ for any point in the non-shaded areas with the $\tilde{H}$ for point R, the *standard interpretation* of $\tilde{H}$ is misleading. With variance locations of reference rays R, I plot four figures which give distinct size of shade areas.

Figure 1.3 (a) gives an example of reference ray R in case 1 where the ray is located between the boundary ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the minimum value of equilibrium efficiency

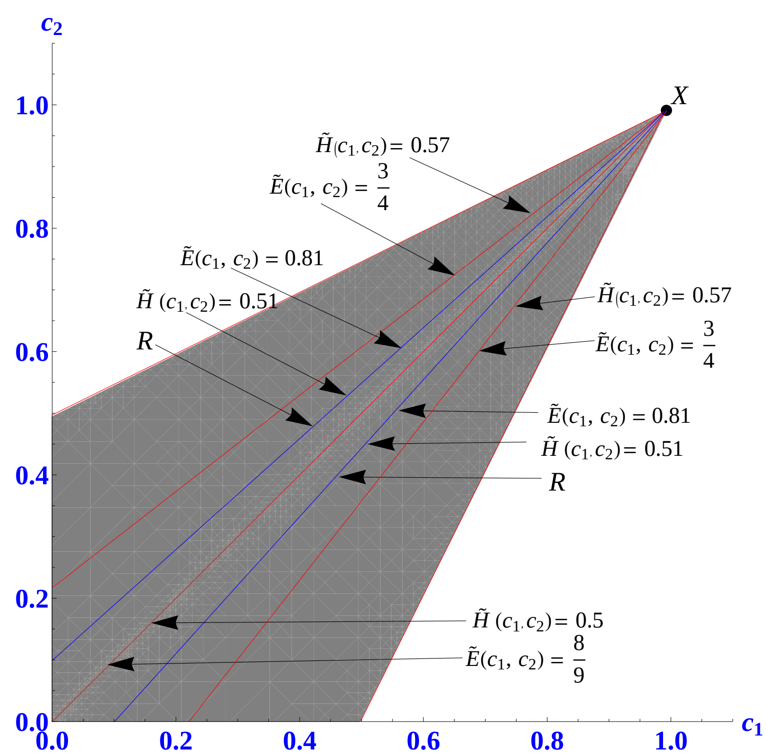
$\tilde{E}(c_1, c_2) = \frac{8}{11}$. The shaded area has its minimum region compare to the other cases which indicates the Herfindahl index gives worst predictions of market efficiency in this case. Figure 1.3 (b) provides an example of reference ray R in case 2 where the ray is located between the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the maximum value ray $\tilde{E}(c_1, c_2) = \frac{8}{9}$. Then, the shaded area would be the entire permissable parameters space. Therefore, I can conclude the Herfindahl index gives best prediction of market efficiency in this case. Moreover, I also provide Figure 1.8 (a) and (b) to illustrate the rest two cases.

Proposition 3. *By holding firm 2's marginal costs constant, the Herfindahl index does not always provide qualitatively accurate prediction of market efficiency with decrease in marginal costs of firm 1.*

Figure 1.3: Two Constant Marginal Cost Firms: Probability of the Herfindahl Index as indicator of Efficiency with Different Reference Points



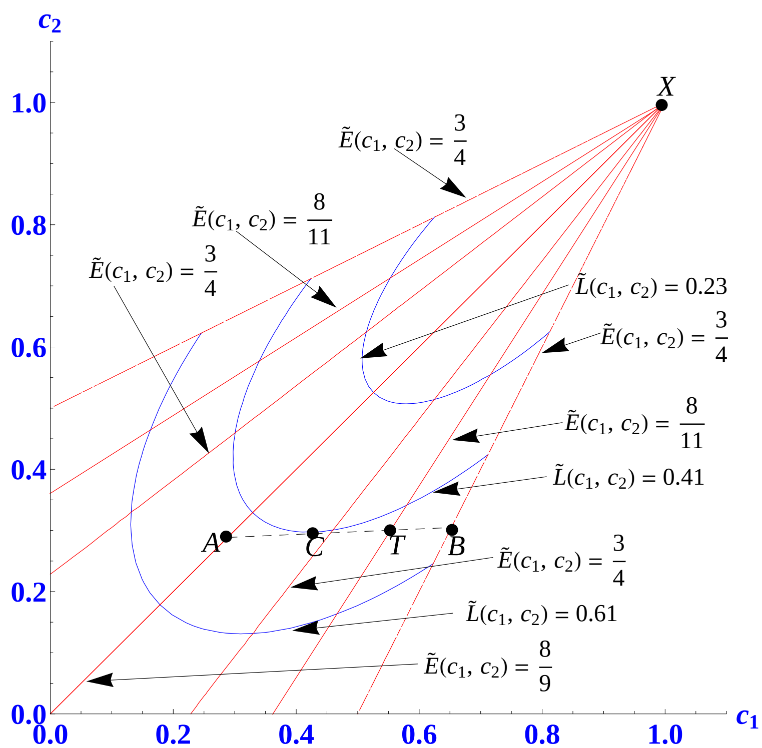
(a)



(b)

Similarly, the Lerner index, $\tilde{L}(c_1, c_2)$ and market efficiency, $\tilde{E}(c_1, c_2)$ can also be expressed as functions of c_1 and c_2. Therefore, I can identify the area where the Lerner index gives qualitatively correct predictions of market efficiency in the c_1 and c_2 space. Figure 1.4 presents and example with three isoLerner curves: $\tilde{L}(c_1, c_2) = 0.2$, $\tilde{L}(c_1, c_2) = 0.4$, and $\tilde{L}(c_1, c_2) = 0.6$. All three curves are convex to origin and symmetric around the 45 degree line. The isoLerner curves for smaller Lerner index values are farther away from the origin. I also plot the three isoefficiency rays from the previous example: $\tilde{E}(c_1, c_2) = \frac{8}{11}$, $\tilde{E}(c_1, c_2) = \frac{3}{4}$, and $\tilde{E}(c_1, c_2) = \frac{8}{9}$. All the rays are passing through point X and have constant value of equilibrium efficiency along each ray.

Figure 1.4: The Lerner and Efficiency Indexes with Two Constant Marginal Cost Firms



In Figure 1.4, I investigate how $\tilde{L}$ and $\tilde{E}$ change alone the horizontal dashed $ACTB$ line. When I hold c_2 constant, the firm 1's technology level declines as the point moves from A to B. I discover that the Lerner index decreases to its minimum point C which is the tangent point to its isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$; however, the market efficiency reaches its minimum point T. Since higher value of the Lerner index indicates lower market efficiency, it is a good proxy of market efficiency only between points C and T. Therefore, I can say with constant marginal costs of firm 2 the Lerner index does not always give a qualitatively accurate prediction of market efficiency as c_1 increases.

Next step of the process involves selecting different reference rays intersecting with isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$. Then, I find reference point R in four different locations: (1) the intersection of the minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$; (2) the intersection of the maximum value ray $\tilde{E}(c_1, c_2) = \frac{8}{9}$ and $\tilde{L}(c_1, c_2) = 0.4$; (3) the intersection of the ray between the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and boundary ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$; (4) the intersection of the ray between the inside region ray $\tilde{E}(c_1, c_2) = \frac{3}{4}$ and maximum value ray $\tilde{E}(c_1, c_2) = \frac{8}{9}$ and the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$. Relative to a reference point R, one can readily identify the portion of the parameter space that (1) have both a smaller $\tilde{L}$ and a larger $\tilde{E}$ than R; (2) have a larger $\tilde{L}$ and a smaller $\tilde{E}$ than R; or (3) have the same $\tilde{L}$ and $\tilde{E}$ as R. I find all these points are in the shaded areas. If I compare $\tilde{L}$ at R with $\tilde{L}$ of any points in the shaded areas, the change in $\tilde{L}$ is qualitatively accurate indicator of the change in $\tilde{E}$. Conversely, $\tilde{L}$ is qualitatively incorrect indicator of market efficiency in the non-shaded areas which composes all the points that (1) have both a smaller $\tilde{L}$ and a smaller $\tilde{E}$ than R; or (2) have a larger $\tilde{L}$ and a larger $\tilde{E}$ than R. Then, the *standard interpretation* of $\tilde{L}$ is misleading.

Figure 1.5 (a) presents the situation of case 1 where the reference point R is located at the intersection of minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$. Then, I could identify the shaded region is above the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$ except the minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and the non-shaded region is below the curve $\tilde{L}(c_1, c_2) = 0.4$. Figure 1.5 (b) shows the situation of case 2 where the reference point R is located at the intersection of maximum value ray $\tilde{E}(c_1, c_2) = \frac{8}{9}$ and the isoLerner curve

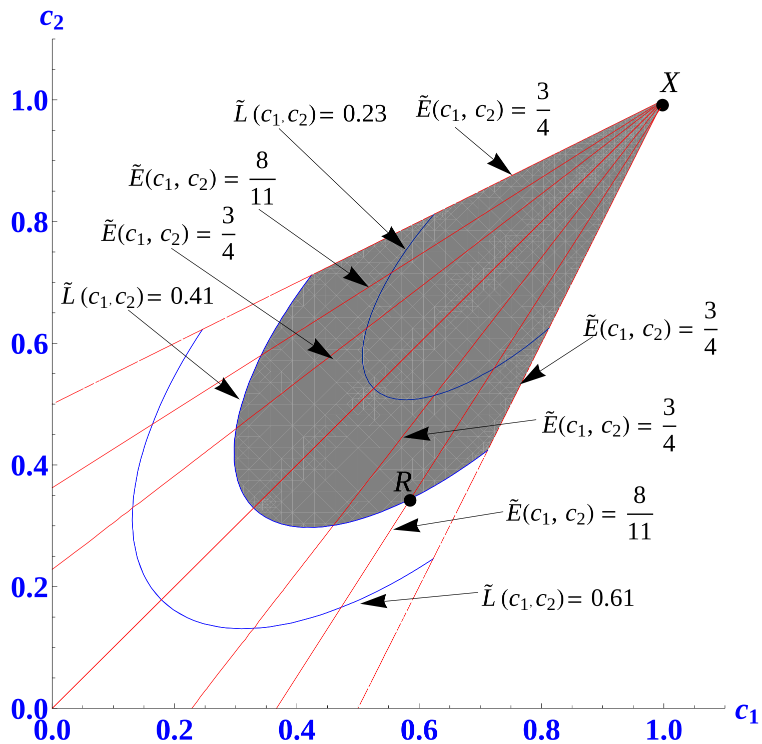
$\tilde{L}(c_1, c_2) = 0.4$. The shaded region and non-shaded region are opposite compare to the case 1. Here, the shaded region is area below the isoLerner curve $\tilde{L}(c_1, c_2) = 0.4$ except the minimum value ray $\tilde{E}(c_1, c_2) = \frac{8}{11}$ and non-shaded region is above this curve. The rest two cases are attached in Figure 1.9.

Proposition 4. *By holding firm 2's marginal costs constant, the Lernner index does not always provide qualitatively accurate prediction of market efficiency with decreases in marginal costs of firm 1 as well.*

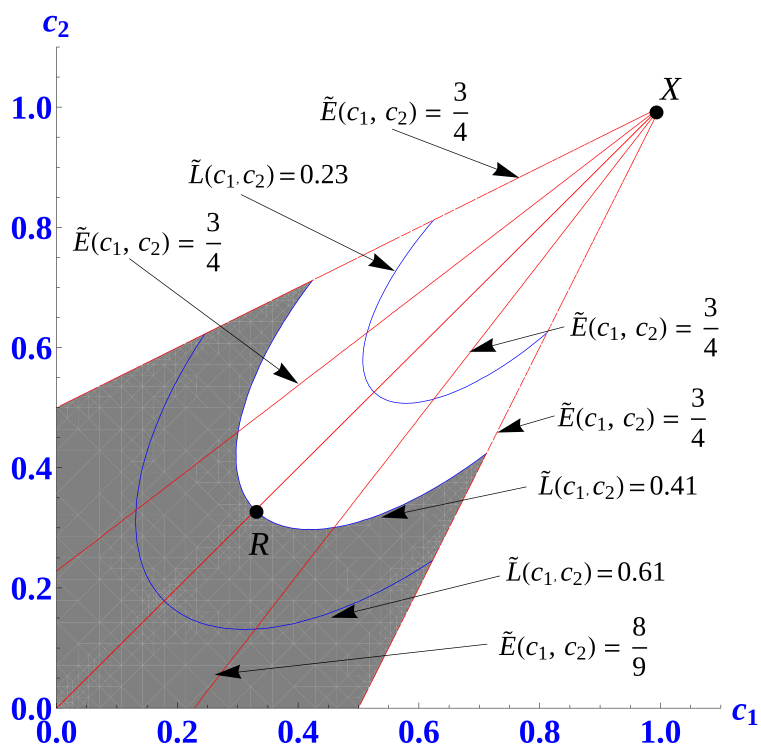
Now, I will use a different angle to illustrate that the Herfindahl index and the Lerner index are often not effective for predicting market efficiency in duopolistic market. Notice that $\tilde{E}$, $\tilde{H}$ and $\tilde{L}$ can be written as function of A, c_1 and c_2. A is uncertain so it is uniformly distributed in range [0, 2]. c_1 and c_2 are uniformly distributed in range [0, A] as well as hold all the quantities positive which is shown in figure 1.1. By simulating 10000 market structures pairs, I would have a set of market efficiency indexes and the Herfindahl indexes. Then, I can deliver the accurate probability of the *standard interpretation* of the Herfindahl index is 0.84. Similarly way, I calculate the probability of the Lerner index as an indicator of market efficiency is only 0.52 which almost equal to randomness.

Like the method described above, I can also calculate the degree of reliability of Herfindahl and Lerner of efficiency for monopolistic, triopolistic, or quadopolistic market. For a monopolistic market, the Herfindahl equals to 1 and efficiency equals to $\frac{3}{4}$; then, the Herfindahl index always gives qualitatively accurate predictions of market efficiency. However, the Lerner index is a stochastic number after stimulation; the probability of the *standard interpretation* of the Lerner index is 0. For the triopolistic and the quadopolistic market cases, the probability of the Herfindahl index qualitatively accurate predictions of market efficiency is 0.79 and 0.74, respectively, and the Lerner index is 0.50 and 0.47 as shown in Table 1.1. I conclude that the Herfindahl index provides better predictions of efficiency than the Lerner index in the context of constant marginal costs and the accuracy of predictions decreases

Figure 1.5: Two Constant Marginal Cost Firms: Probability of the Lerner Index as indicator of Efficiency with Different Reference Points



(a)



(b)

from duopolistic to quadopolistic market for both Herfindahl and Lerner index.

Proposition 5. *The probability of the Herfindahl index as a indicator of efficiency decreases from duopolistic to quadopolistic market. The Lerner index also decreases across these same market structures. The Herfindahl index is a better indicator for efficiency than the Lerner index. The details refer to Table 1.1.*

Table 1.1: The Degree of Reliability of the Herfindahl and Lerner indexes as Indicators of Efficiency in Different Market Structures: Constant Marginal Costs

	(A, c_1)	(A, c_1, c_2)	(A, c_1, c_2, c_3)	(A, c_1, c_2, c_3, c_4)
Herfindahl	1	0.84	0.79	0.74
Lerner	0	0.52	0.50	0.47

Stochastic Number of Firms

With number of firms in each market structure is stochastic, I want to discover the degree of reliability of the Herfindahl index and the Lerner index. Suppose there is equal possibility that the market is monopoly, duopoly, triopoly, or quadopoly. The market size A is uncertain and uniformly distributed in range [0, 2], and c_i is uniformly distributed in range [0, A]. Also, the market output should be valid which is positive. The probability of the Herfindahl index as indicator of efficiency is 0.49 and the probability of the Lerner index is 0.48. Both of these probabilities are close to randomness and not reliable.

Proposition 6. *With random number of firms in each market structure, the probabilities of the Herfindahl index and the Lerner index are 0.49 and 0.48, respectively. Both indexes are misleading indicators for efficiency.*

1.2.4 Market Structure, Increasing Marginal Costs

In the context of increasing marginal costs, the stochastic models vary same manners as in constant marginal costs framework such as: whether marginal costs is asymmetric; whether the number of firms is stochastic.

Same Increasing Marginal costs

First, suppose all firms have same increasing marginal costs, $\beta = \beta_1 = \beta_2 = ... = \beta_n$, then I develop a model which depends on parameter A, β, and n. A is uncertain so it is uniformly distributed in range [0, 2]; n is discrete uniformly distributed in range [1, 5]; and β is uniformly distributed in range [0, 10]. The equilibrium value of efficiency, Herfindahl and Lerner can be written as function of A, β, and n as listed below:

$$\hat{E}(A,n,\beta)=\frac{\hat{W}(A,n,\beta)}{\widehat{TPS}(A,n,\beta)} = \frac{n(\frac{1}{n+1+\beta})^2(1+\frac{1}{2}\beta)+\frac{1}{2}(\frac{n}{n+1+\beta})^2}{\frac{(n\beta^{n-1})}{2(n\beta^{n-1}+\beta^n)}} \tag{1.11}$$

$$\hat{H}(A,n,\beta) = \sum_{i=1}^{n} \hat{S}_i^2 = \frac{1}{n} \tag{1.12}$$

$$\hat{L}(A,n,\beta) = \frac{\hat{H}}{\hat{\varepsilon}} = \frac{1}{1+\beta} \tag{1.13}$$

By comparing (2.11),(2.12) and (2.13), I discover that the Herfindahl index only depends on number of firms, the Lerner index is determined by slope of marginal costs, and market efficiency relies on both. Clearly, $\hat{H}$ and $\hat{L}$ are not always provide qualitatively accurate predictions of market efficiency. To illustrate more explicitly, I plot two figures that show how $\hat{E}$, $\hat{H}$, and $\hat{L}$ change with the levels of n and β. Then, I can find out the areas where the Herfindahl index and Lerner index give effective predictions of market efficiency.

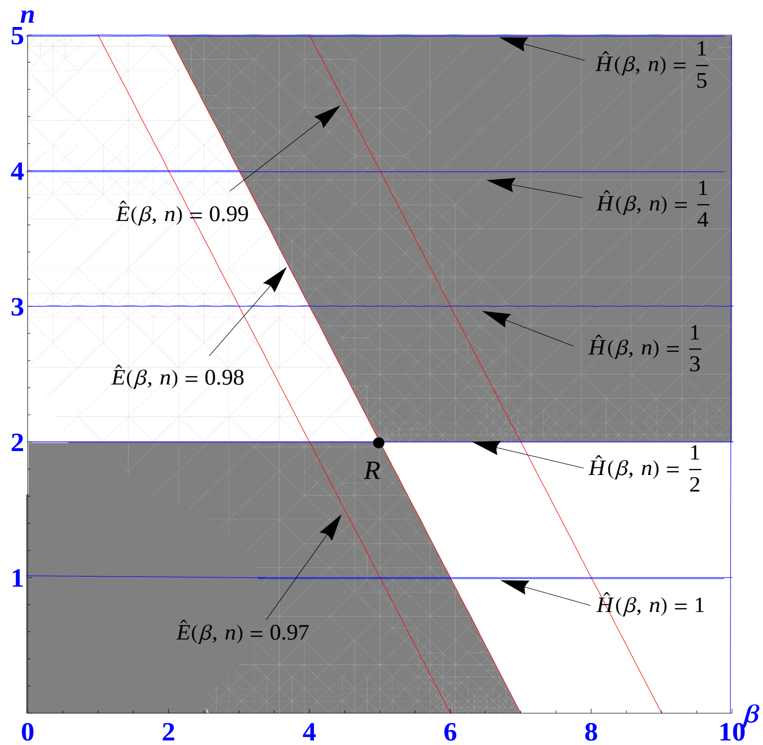
In Figure 1.6 (a), I have three isoefficiency curves: $\hat{E}(n,\beta) = 0.97$, $\hat{E}(n,\beta) = 0.98$, and $\hat{E}(n,\beta) = 0.99$. And these curves increases as more firms enter into the market and higher levels of increasing marginal costs. And I also plot 5 isoHerfindahl rays: $\hat{H}(n,\beta) = 1$, $\hat{H}(n,\beta) = \frac{1}{2}$, $\hat{H}(n,\beta) = \frac{1}{3}$, $\hat{H}(n,\beta) = \frac{1}{4}$, and $\hat{H}(n,\beta) = \frac{1}{5}$, which are constant with different levels of increasing marginal costs and raise with less numbers of firms. If β is held constant, we find the Herfindahl index predicts the market efficiency correctly when the market becomes more competitive; however, the Herfindahl index becomes a misleading indicator of efficiency when marginal cost decreases. Based on the reference point R, I can identify all the points in the parameter space that the *standard interpretation* of $\hat{H}$ is accurate. They are belong to the shaded areas. Conversely, I can also identify all the points that the *standard interpretation* of $\hat{H}$ is misleading which are located in non-shaded areas.

Similarly, Figure 1.6 (b) shows how market efficiency and the Lerner change with the levels of β and n. I plot three isoLerner curves: $\hat{L}(n,\beta) = \frac{1}{3}$, $\hat{L}(n,\beta) = \frac{1}{6}$, and $\hat{L}(n,\beta) = \frac{1}{9}$ instead of 5 isoHerfindahl rays in Figure 1.6 (a). These curves are constant with more firms and raise with lower levels of increasing marginal costs. By holding β constant, the market efficiency increases and Lerner index stays constant with the entry of more firms. This is because the additional firm depresses the price level, as well as the production level for each firm. Therefore, the Lerner index becomes irrelevant with a changing number of firms due to the particular increasing marginal cost function $MC_i = \beta_i * Q_i$. This implies the Lerner index is not an effective index for market efficiency when there is more competition in the market. By holding the number of firms constant, market efficiency increases and the Lerner index decreases with increasing marginal costs. This indicates that the Lerner index is a good proxy for market efficiency when increasing marginal costs decrease. With reference point R, we find the Lerner index yields qualitatively accurate predictions of market efficiency in the shaded areas and the *standard interpretation* of $\hat{L}$ is misleading in non-shaded areas.

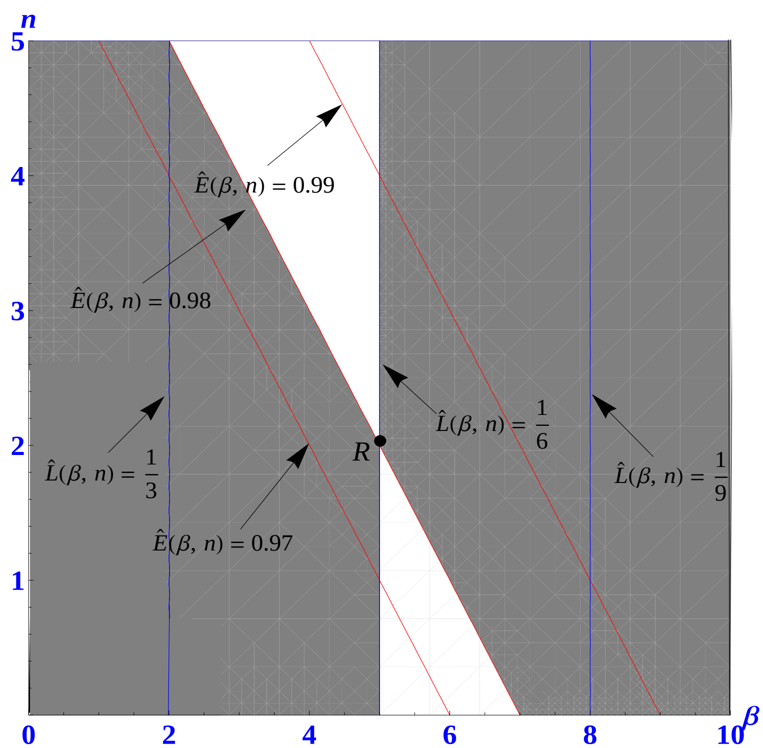
The degree of reliability of the Herfindahl index as indicator of efficiency is 0.80 after simulating 10000 market structure pairs with parameter A, β, and n. The probability of the *standard interpretation* of the Lerner index is 0.92 which is better indicator than Herfindahl.

Proposition 7. *When we hold levels of marginal costs constant, the Herfindahl index predicts market efficiency qualitatively correct with more firms in this market; however, the Lerner index does not. When there is no firm entry or exit in this market, the Lerner index is a good indicator for market efficiency with decreases in technology levels but the Herfindahl index is not. The accurate probabilities of the Herfindahl and Lerner indexes for efficiency are 0.80 and 0.92.*

Figure 1.6: The Probability of the Herfindahl and Lerner as indicators of Efficiency in (n,β) space



(a) Herfindahl and Efficiency in (n,β) space



(b) Lerner and Efficiency in (n,β) space

Non-stochastic Number of Firms

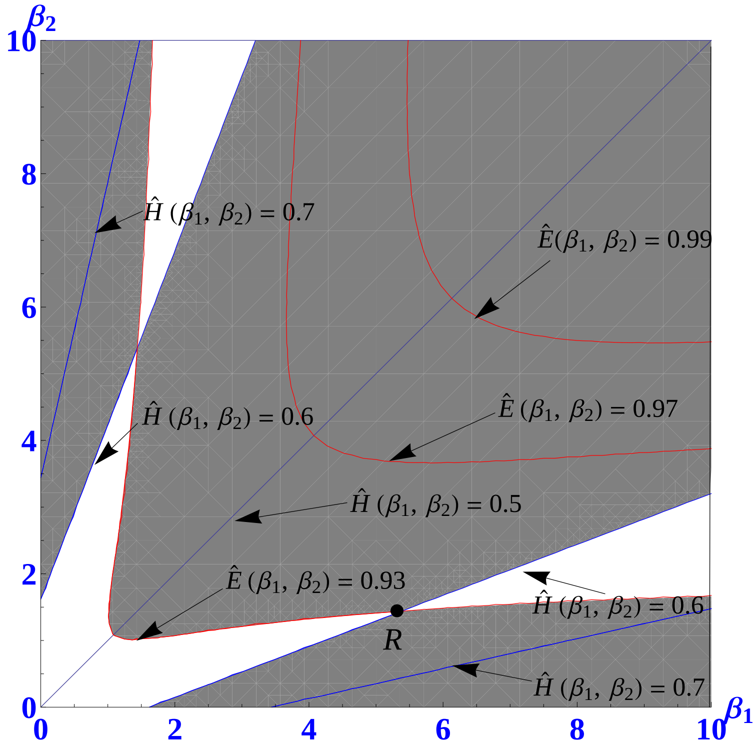
Similar to the context of constant marginal costs, I develop several models with all firms have increasing marginal costs. In each model, the two random generated market structures have identical number of firms as well with n discrete uniformly distributed in range [1, 4], A uniformly distributed in range [0, 2], and β_i is uniformly distributed in range [0, 10]. Then, I can have four stochastic models with different number of firms in each randomly generated market structure. To illustrate explicitly the circumstances in which the *standard interpretation* of $\hat{H}$ and $\hat{L}$ are and are not accurate, I explore the model with two firms. The market efficiency, the Herfindahl index, and the Lerner index can be written as functions of A, β_1, and β_2.

For simplicity, I assume $A = 1$ and both β_1 and β_2 are in the range of [0, 10]. Figure 1.7 (a) shows the case where both firms have their marginal costs increasing with the slopes β_1 and β_2, respectively, instead of having constant marginal costs. I plot the diagram that shows how market efficiency, $\hat{E}(\beta_1, \beta_2)$, and the Herfindahl index, $\hat{H}(\beta_1, \beta_2)$, change with the levels of β_1 and β_2 in order to discover the region where the Herdindahl index gives qualitatively consistent predictions of market efficiency. For example, we plot three isoefficiency curves: $\hat{E}(\beta_1, \beta_2) = 0.93$, $\hat{E}(\beta_1, \beta_2) = 0.97$ and $\hat{E}(\beta_1, \beta_2) = 0.99$. All three curves are convex to origin and symmetric around the 45 degree line. The isoefficiency curves for larger market efficiency are farther away from the origin. Besides, I also plot three different isoHerfindahl curves: $\hat{H}(\beta_1, \beta_2) = 0.5$, $\hat{H}(\beta_1, \beta_2) = 0.6$, and $\hat{H}(\beta_1, \beta_2) = 0.7$. $\hat{H}(\beta_1, \beta_2) = 0.6$ and $\hat{H}(\beta_1, \beta_2) = 0.7$ have two segment rays symmetric around 45 degree line which has $\hat{H}(\beta_1, \beta_2) = 0.5$. As the isoHerfindahl curves deviate farther from the 45 degree line, $\hat{H}$ increases. Based on the reference point R, we identify the Herfindahl index gives qualitatively correct predictions of market efficiency in the region that (1) have both a smaller $\hat{H}$ and a larger $\hat{E}$ than R, or (2) have a larger $\hat{H}$ and a smaller $\hat{E}$ than R. And it does not have correct predictions of market efficiency in the region that (1) have both a larger $\hat{H}$ and a larger $\hat{E}$ than R, or (2) have a smaller $\hat{H}$ and a smaller $\hat{E}$ than R. Therefore, the Herfindahl index is not always an effective index for predicting market efficiency. The shaded region is where predictions are qualitatively correct, and the non-shaded region is misleading. In addition, when the reference point R is at $\beta_1 = \beta_2 = 10$, the Herfindahl index has its best prediction for market efficiency and its worst prediction for market efficiency at $\beta_1 = \beta_2 = 0$. Notice that the efficiency is worse off when technological progress in term of increasing marginal costs decrease. This is because the technological progress causes the reduction of market price benefiting both total welfare and total possible surplus; however, the gain of total possible surplus dominates the gain of total welfare. Therefore, technological progress even loses efficiency. Figure 1.7 (b) uses the same β_1 and β_2 space. It shows the area in which the Lerner index gives qualitatively correct predictions of market efficiency. For instance, relative to the reference point R, the Lerner index gives qualitatively correct predictions of efficiency in the shaded area that (1) have both a smaller $\hat{L}$ and a larger $\hat{E}$ than R, (2) have a larger $\hat{L}$ and a smaller $\hat{E}$ than R, or (3) have an equal $\hat{L}$ and an equal $\hat{E}$ to R. And it gives misleading predictions in the non-shaded area (1) have both a larger $\hat{L}$ and a larger $\hat{E}$ than R, or (2) have a smaller $\hat{L}$ and a smaller $\hat{E}$ than R. Therefore, the Lerner index is not a good proxy for market efficiency as well.

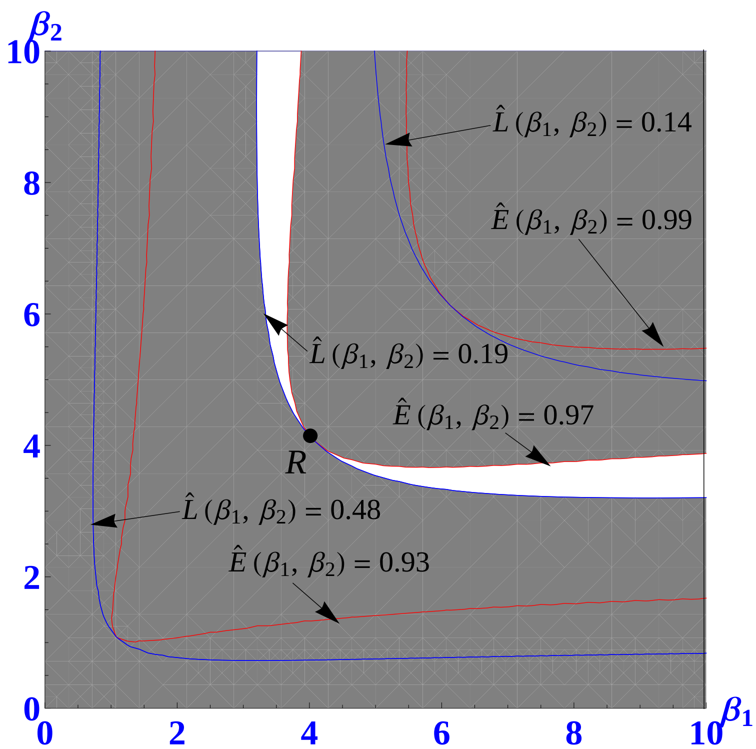
Proposition 8. *The Herfindahl index and the Lerner index do not always have qualitatively correct prediction of market efficiency in β_1 and β_2 space. The higher levels of increasing marginal costs in the market, the higher efficiency will be.*

As reported in Table 1.2, I calculate the probability of the *standard interpretation* of the Herfindahl index and the Lerner index as indicators of efficiency. For a monopolistic market, the Herfindahl index is always 1 but efficiency is stochastic and is positively related with slope of the marginal costs. Also, the Lerner index is stochastic and has negative relation with slope of the marginal costs. Hence, the probability of the accuracy in monopolistic market for the Herfindahl index and the Lerner index is 0 and 1, respectively. Similarly, I

Figure 1.7: The Probability of the Herfindahl and Lerner indexes as indicators of Efficiency with Two Increasing Marginal Cost Firms



(a)



(b)

can calculate a number of sets of efficiency indexes, the Herfindahl indexes, and the Lerner indexes for the duopolistic, triopolistic, and quadopolistic market after stimulation. The probability of the *standard interpretation* of the Herfindahl index is 0.80, 0.76, and 0.60 and the probability of the Lerner index is 0.92, 0.90, and 0.64. Thus, the Lerner index works better for predicting efficiency in the context of increasing marginal costs.

Proposition 9. *The probability of the Herfindahl and Lerner indexes as a indicator of efficiency decreases from duopolistic to quadopolistic market. The Lerner index is a better indicator for efficiency than the Herfindahl index. The details refer to Table 1.2.*

Table 1.2: The Degree of Reliability of the Herfindahl and Lerner indexes as Indicators of Efficiency in Different Market Structures: Increasing Marginal Costs

	(A, β_1)	(A, β_1, β_2)	$(A, \beta_1, \beta_2, \beta_3)$	$(A, \beta_1, \beta_2, \beta_3, \beta_4)$
Herfindahl	0	0.80	0.76	0.60
Lerner	1	0.92	0.90	0.64

Stochastic Number of Firms

I examine the degree of reliability of the Herfindahl and Lerner indexes as indicators of efficiency when number of firms in each market are stochastic. For example , there are four types of market structure: monopoly, duopoly, triopoly, and quadopoly, each of each has equal probability. The market size A is uncertain and uniformly distributed in range [0, 2], and β_i is uniformly distributed in range [0, 10]. After simulating the parameters, I calculate a set of efficiency, Herfindahl and Lerner indexes. The probability of the Herfindahl index as indicator of efficiency is 0.53 and the probability of the Lerner index is 0.51. Neither of them is a good proxy for efficiency.

Proposition 10. *The probabilities of the Herfindahl index and the Lerner index are 0.53 and 0.51, respectively in the context of stochastic number of firms in each market. Both indexes are misleading indicators for efficiency.*

1.3 Calculability

The exogenous variables of market size and magical costs can not be observed directly in the industry level data and I will discover the market efficiency as function of observable variables: the firm level output and market price. By comparing to the Herfindahl index and the Lerner index, the market efficiency index is as easy as calculated. Therefore, if the primary concern of market power is to detect market efficiency, I can measure market efficiency directly without calculating the Herfindahl index or the Lerner index. The calculability formulas of market efficiency for constant and increasing marginal costs are written as follows:

1.3.1 Constant Marginal Costs

Suppose there are n firms in the market all firms have constant marginal costs, the market efficiency is as written below:

$$\tilde{E}(Q_i, Q) = \frac{\tilde{W}(Q_i, Q)}{\widetilde{TPS}(Q_i, Q)} \tag{1.14}$$

$$\tilde{W}(Q_i, Q) = \sum_{i=1}^{n} \tilde{\pi}_i + \widetilde{CS} = \sum_{i=1}^{n} \tilde{Q}_i^2 + \frac{1}{2}\tilde{Q}^2 \tag{1.15}$$

$$\widetilde{TPS}(Q_i, Q) = Max\{\frac{(A - c_i)^2}{2}\} = Max\{\frac{(2\tilde{Q} - \tilde{Q}_i)^2}{2}\},\ i = 1, 2, ..., n \tag{1.16}$$

The profit the market can be expressed as summation of each firm output square its and consumer surplus is derived by half of the market output share. In addition, the total possible market surplus is written as $Max\{\frac{(2\tilde{Q}-\tilde{Q}_i)^2}{2}\}$ which just simply transfer unobservable variables to observable variables: market shares and market output. With this formula, the market efficiency can be calculated directly by using observable variables with constant marginal costs case and I can avoid using other methods to measure market power as proxy of efficiency.

1.3.2 Increasing Marginal Costs

With increasing marginal costs case, the market efficiency can be calculated as follows:

$$\hat{E}(Q_i, Q) = \frac{\hat{W}(Q_i, Q)}{\widehat{TPS}(Q_i, Q)} \tag{1.17}$$

$$\hat{W}(Q_i, Q) = \sum_{i=1}^{n} \hat{\pi}_i + \widehat{CS} = \sum_{i=1}^{n} \hat{Q}_i^2 + \sum_{i=1}^{n} \frac{1}{2}\beta_i \hat{Q}_i^2 + \frac{1}{2}\hat{Q}^2,\ \beta_i = \frac{\hat{P} - \hat{Q}_i}{\hat{Q}_i} \tag{1.18}$$

$$\widehat{TPS}(Q_i, Q) = \frac{A^2}{2\left(\frac{1}{\sum_{i=1}^{n} \frac{1}{\beta_i}} + 1\right)}, \beta_i = \frac{\hat{P} - \hat{Q}_i}{\hat{Q}_i} \tag{1.19}$$

The market profit composes two parts, the first part is the summation of each firm output square and the second part is the summation of each firm output square times its slope of marginal costs. Also, the consumer surplus is half of market output square. Here, the slope of marginal costs for each firm can be expressed by market price and each firm output. Therefore, the market efficiency is easy to calculate when firm level output and market price are available.

1.4 Conclusion

In this paper, I adopt a linear demand function in a Cournot framework and assume the marginal cost for a firm can be either constant or increasing return to scale. Then, I construct a number of stochastic market structure. Each market structure can be written as function of costs, numbers of firms, and demand. Therefore, I would be able to calculate efficiency and the Herfindahl and Lerner index directly by using these parameters.

I argue that neither the Herfindahl index or Lerner index has a monotonic relationship with efficiency. Moreover, the paper theoretically demonstrates the circumstances in which the Herfindahl and Lerner indexes are and are not reliable indicators of efficiency in a number of stochastic oliopolistic market structures. I also quantify the degree of reliability of each of these indexes for efficiency. The results shows that in most models, neither index could be describe as a satisfactory indicator of efficiency. Besides, the Herfindahl index is better than the Lerner index with constant marginal costs, and the Lerner index is better than the Herfindahl index with increasing marginal costs. In addition, efficiency may not benefit with the decrease in levels of constant marginal costs and efficiency becomes even worse with the decrease in levels of increasing marginal costs. Moreover, I derive market efficiency directly by using firm level output and market price, both of which are easy to observe. Therefore, there is no need to rely on the Herfindahl and Lerner indexes which are imperfect indicators of efficiency.

1.5 Appendices

1.5.1 Appendix A: Equilibrium Solution

Derivation of TPS with Constant Marginal Costs

I derive the total possible surplus (TPS) for both firms have constant marginal cost case. Each firm solves the following maximization problem:

$$\underset{Q_i}{Max}\ AQ - \frac{1}{2}Q^2 - \sum_{i=1}^{n} c_i Q_i, \ \ i = 1, 2, ..., n$$

The total possible surplus, $\widetilde{TPS}(A, n, c_1, c_2, ..., c_n) = Max\{\frac{(A-c_1)^2}{2}, \frac{(A-c_2)^2}{2}, ..., \frac{(A-c_n)^2}{2}\}$.

Derivation of Lerner, Herfindahl, and Efficiency with Constant Marginal Constant

First, I will discover the relationship between the Herfindahl index and the Lerner index.

The profit function for each firm is as below:

$$\pi_i(Q_i,Q_j) = PQ_i - TC_i$$

$$\frac{\partial \pi_i(Q_i,Q_j)}{\partial Q_i} = \frac{\partial P}{\partial Q}Q_i + P - MC_i$$

At a maximum, the derivative must be zero:

$$\begin{aligned} \frac{\partial P}{\partial Q}Q_i + P - MC_i &= 0 \\ \frac{P - MC_i}{P} &= \frac{S_i}{\varepsilon} \end{aligned}$$

Therefore, I can derive the Lerner index for n firms as below:

$$L = \frac{H}{\varepsilon}$$

where $\varepsilon = -\frac{\partial Q}{\partial P}\frac{P}{Q}$ and $\frac{\partial Q}{\partial P} = -1$. Next, I will derive the Herfindahl, Lerner and efficiency index in the function of parameter space. The maximization of profit function is as expressed below:

$$\underset{Q_i}{Max} PQ_i - c_iQ_i,\ i = 1, 2, ..., n$$

The derivative must be zero in equilibrium:

$$\begin{aligned} P - Q_i - c_i &= 0 \\ A - Q - Q_i - c_i &= 0 \\ nA - nQ - Q - \sum_{i=1}^{n} c_i &= 0 \end{aligned}$$

The equilibrium value of price, firm level output, and market output are as follows:

$$
\begin{aligned}
\tilde{Q}(A,n,c_1,c_2,...,c_n) &= \frac{nA-\sum\limits_{i=1}^{n}c_i}{n+1}\\
\tilde{P}(A,n,c_1,c_2,...,c_n) &= \frac{A+\sum\limits_{i=1}^{n}c_i}{n+1}\\
\tilde{Q}_i(A,n,c_1,c_2,...,c_n) &= \frac{A-(n+1)c_i+\sum\limits_{i=1}^{n}c_i}{n+1}
\end{aligned}
$$

Finally, the equilibrium value of the Herfindahl, Lerner, and Efficiency can be derived as stated below:

$$
\begin{aligned}
\tilde{H}(A,n,c_1,c_2,...,c_n) &= \sum_{i=1}^{n}\left(\frac{A-(n+1)c_i+\sum\limits_{i=1}^{n}c_i}{nA-\sum\limits_{i=1}^{n}c_i}\right)^2\\
\tilde{L}(A,n,c_1,c_2,...,c_n) &= \left(\sum_{i=1}^{n}\left(\frac{A-(n+1)c_i+\sum\limits_{i=1}^{n}c_i}{nA-\sum\limits_{i=1}^{n}c_i}\right)^2\right)\left(\frac{nA-\sum\limits_{i=1}^{n}c_i}{A+\sum\limits_{i=1}^{n}c_i}\right)\\
\tilde{E}(A,n,c_1,c_2,...,c_n) &= \frac{\tilde{W}(A,n,c_1,c_2,...,c_n)}{\widetilde{TPS}(A,n,c_1,c_2,...,c_n)}
\end{aligned}
$$

where $\tilde{W}=\sum\limits_{i=1}^{n}\tilde{\pi}_i+\widetilde{CS}$ and $\widetilde{TPS}=Max\{\frac{(A-c_1)^2}{2},\frac{(A-c_2)^2}{2},...,\frac{(A-c_n)^2}{2}\}$. The firm level profit and consumer surplus are expressed as below:

$$
\begin{aligned}
\tilde{\pi}_i(A,n,c_1,c_2,...,c_n) &= \left(\frac{A-(n+1)c_i+\sum\limits_{i=1}^{n}c_i}{n+1}\right)^2\\
\widehat{CS}(A,n,c_1,c_2,...,c_n) &= \frac{1}{2}\left(\frac{A}{\frac{1}{\sum\limits_{i=1}^{n}\frac{1}{\beta_i}}+\frac{n+1}{n}}\right)^2
\end{aligned}
$$

Derivation of Lerner, Herfindahl, and Efficiency with Increasing Marginal Constant

Each firm maximizes its own profit as below:

$$\underset{Q_i}{Max} PQ_i - \frac{1}{2}\beta_i Q_i^2,\ i = 1, 2, ..., n$$

At a maximum, the derivative must be zero:

$$\begin{aligned} A - Q - Q_i - \beta_i Q_i &= 0 \\ nA - nQ - \sum_{i=1}^{n}(1+\beta_i)Q_i &= 0 \end{aligned}$$

The equilibrium value of price, firm level output, and market output are as noted below:

$$\begin{aligned} \hat{Q}(A, n, \beta_1, \beta_2, ..., \beta_n) &= \frac{A(\sum_{i=1}^{n} \frac{1}{1+\beta_i})}{(1 + \sum_{i=1}^{n} \frac{1}{1+\beta_i})} \\ \hat{P}(A, n, \beta_1, \beta_2, ..., \beta_n) &= \frac{A}{(1 + \sum_{i=1}^{n} \frac{1}{1+\beta_i})} \\ \hat{Q}_i(A, n, \beta_1, \beta_2, ..., \beta_n) &= \frac{A}{(1+\beta_i)(1 + \sum_{i=1}^{n} \frac{1}{1+\beta_i})} \end{aligned}$$

In the context of increasing marginal costs, the equilibrium value of the Herfindahl, Lerner, and Efficiency can be derived as stated below:

$$\begin{aligned} \hat{H}(A, n, \beta_1, \beta_2, ..., \beta_n) &= \sum_{i=1}^{n}\left(\frac{1}{(1+\beta_i)(\sum_{i=1}^{n} \frac{1}{1+\beta_i})}\right)^2 \\ \hat{L}(A, n, \beta_1, \beta_2, ..., \beta_n) &= \left(\sum_{i=1}^{n}\left(\frac{1}{(1+\beta_i)(\sum_{i=1}^{n} \frac{1}{1+\beta_i})}\right)^2\right)\left(\sum_{i=1}^{n}\frac{1}{1+\beta_i}\right) \\ \hat{E}(A, n, \beta_1, \beta_2, ..., \beta_n) &= \frac{\hat{W}(A, n, \beta_1, \beta_2, ..., \beta_n)}{\widehat{TPS}(A, n, \beta_1, \beta_2, ..., \beta_n)} \end{aligned}$$

where $\hat{W} = \sum_{i=1}^{n} \hat{\pi}_i + \widehat{CS}$. The firm level output and consumer surplus are as follows:

$$\hat{\pi}_i(A,n,\beta_1,\beta_2,...,\beta_n) = \sum_{i=1}^{n}(1+\frac{1}{2}\beta_i)\left(\frac{A}{(1+\beta_i)(1+\sum_{i=1}^{n}\frac{1}{1+\beta_i})}\right)^2$$

$$\widetilde{CS}(A,n,\beta_1,\beta_2,...,\beta_n) = \frac{1}{2}\left(\frac{A(\sum_{i=1}^{n}\frac{1}{1+\beta_i})}{(1+\sum_{i=1}^{n}\frac{1}{1+\beta_i})}\right)^2$$

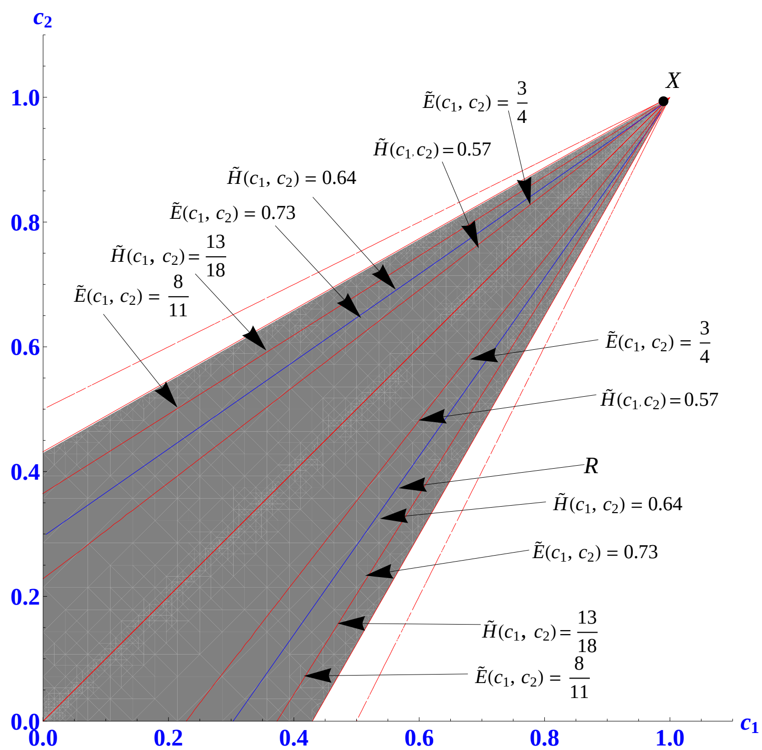
With increasing marginal cost case, each firm solves the following maximization problem:

$$\underset{Q_i}{Max}\ AQ - \frac{1}{2}Q^2 - \frac{1}{2}\sum_{i=1}^{n}\beta_i Q_i^2,\ i = 1,2,...,n$$

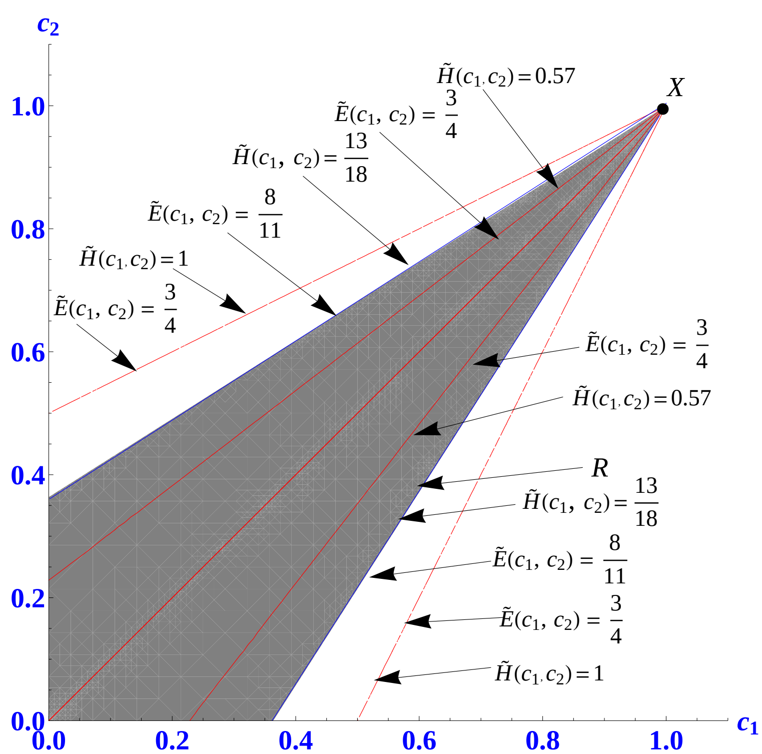
By choosing quantity level for each firm, I derive $\widehat{TPS}$:

$$\widehat{TPS}(A,n,\beta_1,\beta_2,...,\beta_n) = \frac{A^2\left(\sum_{i=1}^{n}\frac{1}{\beta_i}\right)}{2\left(1+\sum_{i=1}^{n}\frac{1}{\beta_i}\right)}$$

Figure 1.8: Two Constant Marginal Cost Firms: Probability of the Herfindahl Index as indicator of Efficiency with Different Reference Points

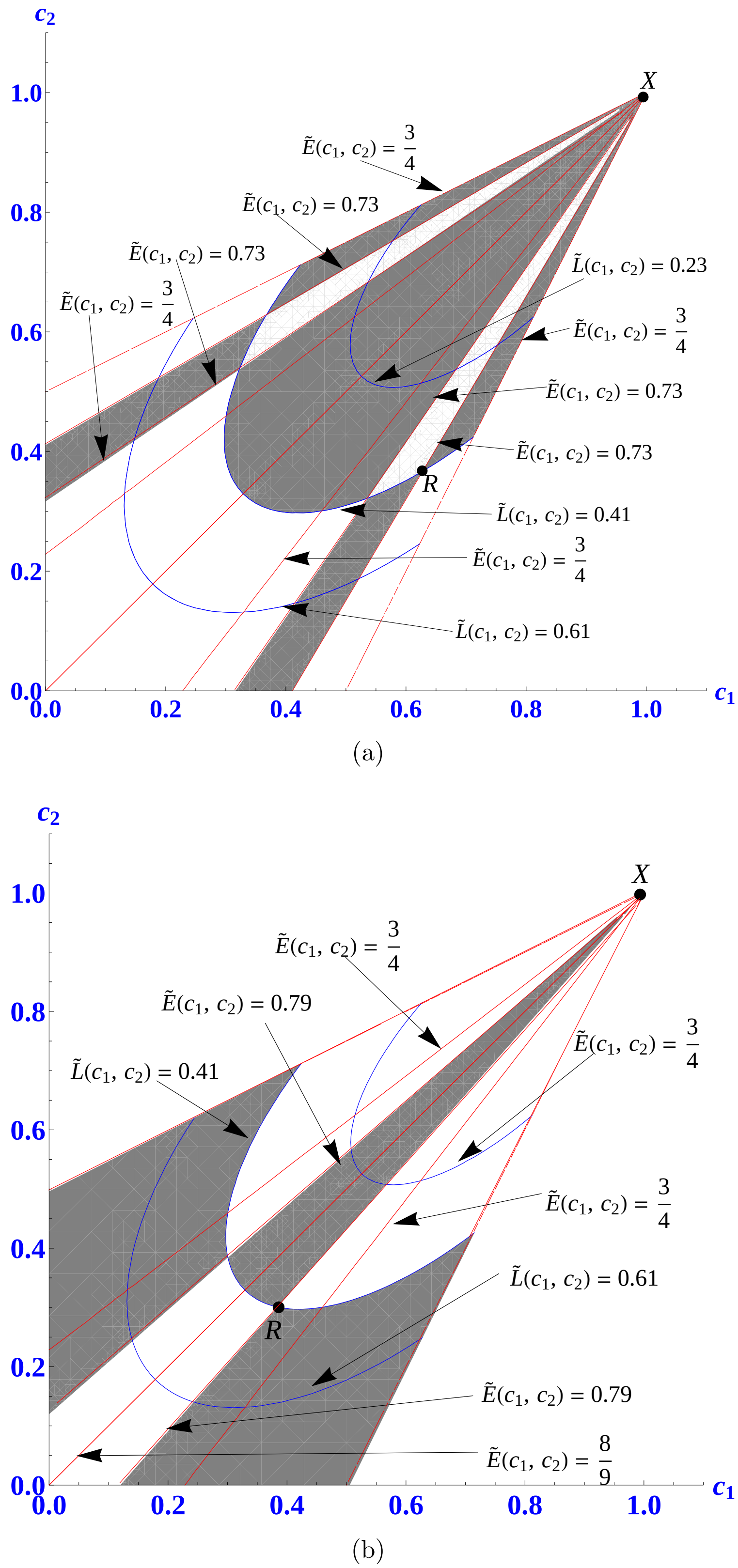


(a)



(b)

Figure 1.9: Two Constant Marginal Cost Firms: Probability of the Lerner Index as indicator of Efficiency with Different Reference Points



Chapter 2

A Dynamic Model of an Oligopolistic Market Structure with Positioning Costs and Different Types of Market Demand

2.1 Introduction

This paper emphasizes the dynamic properties of a modified model, which is based on Bloch et al. (2014). In the paper, we present two major modifications to the model developed by Bloch et al. (2014). The first is that we assume the number of active firms is fixed, or exogenous. The second is that we propose two types of demand in a market, high or low. Bloch et al. (2014) focused on the stochastic of positional investment and market structure. However, they ignored the dynamic properties of their model, which we will emphasize.

In a niche market, the active firms can be established or unestablished. Established firms compete either in the Bertrand or Cournot framework in any period of a game. For welfare comparisons, we introduce as a reference, a third competition - a welfare maximizing alternative. There are various types of positional investments. For example, firms spend money on a new plant, research and development for new products, advertising for products. Unestablished firms decide whether or not to enter into a market with investment costs. These investments are not always successful. For example, in the pharmaceutical, electronic, and automobile industries, firms spend millions of dollars to develop new products, but these may fail in the end. Meanwhile, established firms also need to spend money to maintain their position in the market such as advertising, updating products, and so on. However, this is also not always successful. In addition, the economy is cyclical. So market demand experiences peak in some periods and low in others. However, this is always hard to predict.

With these observations in mind, we built a model which has the probability of success with its positional investments and the two types of market demands, specifically, high and low. In this paper, we focus on the dynamic properties of the model.

Over the last four decades, there have been a number of significant contributors to the industrial organization literature related to our paper including Caves and Porter (1977), Dixit (1979, 1980a), Eaton and Lipsey (1977, 1978a), Schmalensee (1978a), Spence (1977, 1979), Schmalensee (1978b), Eaton and Lipsey (1978b, 1979, 1980), Salop (1979), Dixit (1980b), West (1981), Gilbert and Harris (1984), Sutton (1991), and Bhattacharya and Bloch (2000). In the studies, a range of firm commitment strategies were explored, including percentage of total sunk costs, outlay on the upkeep of sunk assets, research and development investment, advertising, and so on. However, in most of these studies, the firms were considered asymmetric, while in our study, there is no such asymmetry. Moreover, our model adopts the dynamic setting from Tirole and Maskin (1988) instead of a static setting, and employs a symmetric Markov perfect Nash equilibrium (SMPNE) approach. Symmetry requires that we model decisions with respect to positional investment as mixed strategies. We develop an algorithm which allows us to find the market equilibrium.

In this study, we provide some results as follows: First, we present some realizations of market structure when firms use their SMPNE strategies. We discover the patterns of the realization differ significantly in different market competitions. Second, we examine the probabilities of being in a particular state, in times $t > 0$ conditional on being in a particular state at time $t = 0$, when firms use their SMPNE strategies. Third, we show the steady state distribution and calculate the expected duration for each state of the system with different market competitions. We find the value of the expected duration to stay in the state with only one firm and high demand is higher in the Bertrand than Cournot. Fourth, we calculate the steady state of efficiency with various numbers of active firms in the Bertrand and Cournot framework. Moreover, the variation of steady state of the system, efficiency, and the maximum and minimum speed of convergence are also presented when we process 8 different experiments compared to our baseline model. Finally, we show bias estimations of the blind efficiency (this calculation of efficiency ignores stochastic and dynamic features) by comparing with the real efficiency (this calculation of efficiency considers stochastic and dynamic properties). We find the value of real efficiency in the Cournot competition is always higher than in the Bertrands and the blind efficiency has an over estimation in the state of one firm with high demand.

The paper is laid out as follows: In Section 2, we construct a dynamic model of a niche market. To be more specific, for simplicity sake, the consumer demand is linear for differential goods and cost of production is zero. Moreover, the positional investment and type of market demand are also included in the model. Both positional investment and market demand have stochastic property. In the dynamic model, a firm decides whether to have a positioning investment in each period of the game. The Bertrand or Cournot competition is adopted if the firm is established. Besides, the model follows the SMPNE. Based on these assumptions, we obtain the market equilibrium by using an algorithm. In Section 3, we provide some results with analysis and discussion. First, we show the dynamic process with different initial states of the system in different competitions and then calculate efficiency and expected duration. Second, we demonstrate the variations of the steady state of the system, efficiency, and speed of convergence with 8 experiments (the different combination of parameter values) and baseline parameters. Finally, we compare the blind efficiency and real efficiency and identify the bias in each different model. In Section 4, we conclude with the significant contributions of this paper.

2.2 The Model

In a niche market, there are a number of active firms (A) which are able to produce and design goods. Some of the active participants are *established firms*, while others are *unestablished* in a given period. The number of *established firms* is $N \leq A$. A and N are integers; A is exogenous, N is endogenous. The position of firm, g, is either established, E, or unestablished, U, so $g \in \{E, U\}$. Market demand, d, is either high, H, or low, L, in each period. Therefore, $d \in \{H, L\}$. Established firms play either a Cournot, Bertrand, or welfare maximized game within that period, while firms with no established position do not play any oligopoly game. In every period, there is an opportunity for unestablished firms to make an investment in order to establish positions in the market and for established firms to make an investment to maintain their positions in the market.

In each period, a representative firm will be in one of states defined with respect to its own position, the positions of the other active firms and the state of demand. A firm's positional investment, necessary to establish or maintain its position, determines whether it will have an established position or not in the next period, but has no affect on the oligopoly outcome within the current period. Positioning investments of all firms in the current period will determine their positions and the state of oligopoly in the next period. Here, a Markov process is in a space with $4A$ states. We focus on the SMPNE in which firms choose strategies to maximize the discounted present value of cash flow. We look at two versions of this , one where firms choose price in static oligopoly game; and the other where firms choose qualities. In addition, there is a third model in which the objective of firms is to maximize welfare of their market niche.

2.2.1 Positioning Investments

In any period, an unestablished firm can make an investment $I > 0$ to establish a position in the next period with probability P. If it does not choose to invest, it will remain an unestablished firm in the next period. If a firm already has an established position, it can invest $J > 0$ to main that position. Making this investment results in a probability Q of maintaining its established position in the next period; however, failing to spend J leads to a loss of the established position in the next period with probability 1. We assume that

$0 < P < 1$ and $0 < Q < 1$. Notice that the positioning technology involves four parameters: I, J, P, and Q.

There are a number of explanations for the parameters I and J. I could be considered as product development, special purpose capital goods (including product specific human capital) needed to produce the good, and/or an image advertising campaign to launch the good. Similarly, J could be associated with product improvement, maintenance of product specific capital goods, and/or maintenance of the good's image.

In addition, there are two types of market demand in this model. The parameter H associates with probability of high market demand and L associates with probability of low market demand.

There are A active firms in each market niche, each firm has either established or unestablished position and faces either high or low demand, therefore, there are $4A$ states of the dynamic model. We assume $A \in \{1, 2, 3\}$. An active firm in any period can be described as (g, n^E, d), where $g \in \{E, U\}$ means the representative firm's current position, $n^E \in \{0, 1, ..., A\}$ is the the number of the other firms which are currently established, and $d \in \{H, L\}$ is the demand of this market. For instance, state of a representative firm $(U, 2, H)$ means that the state in which the representative firm is unestablished, two of the other active firms are established and the state of the demand is high. In addition, the number of the state of the system is $4A - 2(A - 1)$. The state of system can be presented as (m, d), where m is the number of established firms and d is the demand state. For instance, $(2, L)$ means two established firms with low market demand. For one, two and three active firms' cases, we list the state of a representative firm and the system as follows:

Table 2.1: States of a Representative Firm and the System with One Active Firm

States of a Representative Firm	States of the System
(U, 0, H)	(0, H)
(U, 0, L)	(1, H)
(E, 0, H)	(0, L)
(E, 0, L)	(1, L)

Table 2.2: States of a Representative Firm and the System with Two Active Firms

States of a Representative Firm	States of the System
(U, 0, H)	(0, H)
(U, 1, H)	(1, H)
(U, 0, L)	(2, H)
(U, 1, L)	(0, L)
(E, 0, H)	(1, L)
(E, 1, H)	(2, L)
(E, 0, L)	
(E, 1, L)	

Table 2.3: States of a Representative Firm and the System with Three Active Firms

States of a Representative Firm	States of the System
(U, 0, H)	(0, H)
(U, 1, H)	(1, H)
(U, 2, H)	(2, H)
(U, 0, L)	(3, H)
(U, 1, L)	(0, L)
(U, 2, L)	(1, L)
(E, 0, H)	(2, L)
(E, 1, H)	(3, L)
(E, 2, H)	
(E, 0, L)	
(E, 1, L)	
(E, 2, L)	

Table 2.47 illustrates the convenient notation convention of states. First, we enumerate $g = U$ and $d = H$ of A states, and the number of state (U, n^E, H) is $n^E + 1$. Next, we enumerate $g = U$ and $d = L$ of A states, and the number of state (U, n^E, L) is $A + n^E + 1$, then, we enumerate $g = E$ and $d = H$ of A states, and the number of state (e, n^E, H) is $2A + n^E + 1$. Finally, we enumerate $g = E$ and $d = L$ of A states, and the number of state (e, n^E, L) is $3A + n^e + 1$.

The firm's strategy of the game consists two components. The first component is the positioning action for every decision node in an infinite game and the second component is the price or quantity action for every decision node with established firm. Any decision node of the firm will be in one of $4A$ states, therefore, we can reduce the strategy space by only focusing on the Markovian strategy which has the advantage of the positioning and price (quantity) actions of a firm at any decision node depend only on the firm's state at that decision node. In the dynamic game, the positioning and price (quantity) actions have different role. The positioning actions of firms drive a Markov process that determines their states in the next period, while the price (quantity) actions of established firms determine their profits in that period. Therefore, we use a two step procedure to formulate the value function under the SMPNE: first find equilibrium price and quality of the static oligopoly game and second use the associated equilibrium profit or total surplus to formulate the dynamic game.

2.2.2 The Static Oligopoly Game

Suppose the established firms produce goods that are either undifferentiated or symmetrically differentiated in a symmetric equilibrium. The representative consumer with utility function is as below:

$$U(y, q_1, q_2, ..., q_N) = y + \alpha_d \sum_{i=1,N} q_i - \frac{\beta}{2} \sum_{i=1,N} q_i^2 - \gamma \sum_{i=1,N,j\neq i} q_i q_j \quad d = H \text{ or } L \tag{2.1}$$

Where y is expenditure on a composite good and q_i is the quantity of the goods produced by the i^{th} established firm. We require that $\alpha_d > 0$ and $\beta \geq \gamma > 0$. Here, we assume H for a high demand state and L for a low demand state, $\alpha_H > \alpha_L$. Given this assumption, the inverse demand functions of the representative consumer for the N differentiated goods are

$$p_i = \alpha_d - \beta q_i - \gamma \sum_{j \neq i} q_j, i = 1, N, j = 1, N, d = h \text{ or } l \tag{2.2}$$

We assume there are three models in this paper. First, firms choose quantities to maximize their profit; second, firms choose prices to maximize their profit; third, firms choose prices to maximize total surplus in their market niche as reference points. In this equilibrium, all prices are equal to marginal costs. In addition, we assume that all firms have the same constant marginal cost. For convenience, we set marginal cost equals to zero. The detailed solutions of price and quantity equilibria are provided in the appendix I. If a firm competes under the Cournot framework, the equilibrium quantity, $q_C^*(N, d)$, depends on number of firms, N, and state of demand, d. If a firm competes under the Bertrand framework, the equilibrium quantity, $p_B^*(N, d)$, depends on number of firms, N, state of demand, d. Meanwhile, each established firm in that period gain profit, $R_x(N, d)$, where $x \in \{B, C\}$. In addition, if a firm chooses price in order to maximize total surplus of its market niche, the equilibrium output, $q_W^*(N, d)$, depends on the number of firms and the state of demand as well. The total surplus for market niche $TS(N, d)$ equals to the consumer surplus of this market $CS(N, d)$ since there is no profit for each firm in this circumstance.

Suppose the state of a firm $k \in \{2A+1, ..., 3A\}$, then $g = E$ (it is established), $d = H$ and $n^E = k - 2A$. Consequently, the equilibrium price in any state $k \in \{2A+1, ..., 3A\}$ is $p_B^*(k - 2A, H)$ in the Bertrand framework, equilibrium quantity is $q_C^*(k - 2A, H)$ in the Cournot framework, equilibrium quantity is $q_W^*(k - 2A, H)$ in the welfare maximized framework. Meanwhile, suppose the state of a firm $k \in \{3A + 1, ..., 4A\}$, then $g = E$ (it is established), $d = L$ and $n^E = k - 3A$. The equilibrium price in any state $k \in \{3A + 1, ..., 4A\}$ is $p_B^*(k - 3A, L)$ in the Bertrand framework, equilibrium quantity is $q_C^*(k - 3A, L)$ in the

Cournot framework, and equilibrium quantity is $q_W^*(k-3A,L)$ in the welfare maximized framework. The profit and maximized total surplus in any state $k \in \{2A+1,...,3A\}$ are $R_x(k-2A,H)$ and $TS(k-2A,H)$. Meanwhile, the profit and maximized total surplus in any state $k \in \{3A+1,...,4A\}$ are $R_x(k-3A,L)$ and $TS(k-3A,L)$.

2.2.3 Value Functions and Transition Probabilities

By given a common strategy for all the other active firms, we only focus on the payoff maximizing decisions of a representative firm. The probability of the representative firm s_R^k, $0 \leq s_R^k \leq 1$, make the relevant positioning investment whenever it is in state k; the relevant investment is I when $k \in \{1, 2A\}$ since its position is U, and it is J when $k \in \{2A+1, 4A\}$ since its position is E. The representative firm's positioning strategy is then $S_R = (s_R^1, s_R^2, ..., s_R^{4A})$ and positioning strategy of the other active firm is $S_O = (s_O^1, s_O^2, ..., s_O^{4A})$. The transition matrix $T_{kl}(s_R, s_O)$ denotes the representative firm's probability of transition from state k in any period to state l in the next period and $T(s_R, s_O)$ denotes the entire $4A$ by $4A$ transition matrix. The transition matrix not only depends on the strategy of representative firm and other active firms but also probability of high or low market demand. To illustrate more explicitly, we provide an example that transition matrix is $T_{24}(s_R, s_O)$ and $A = 3$. First notice that when the representative firm is in state 2 $(U, 1, H)$, one of the other two firms is also in state 2 $(U, 1, H)$, and the other one is in state 7 $(E, 0, H)$. The representative firm will be in state 4 $(U, 0, L)$ in the next period if four independent events occur: the representative firm's position remains U, the position of the other firm that is currently in state 2 remains U, the position of the other firm that is currently in state 7 remains U and the market demand shift from high to low. The first of these events will occur with probability $1 - s_R^2 P$, the second with probability $1 - s_O^2 P$, the third with probability $1 - s_O^7 Q$, and the last with probability $1 - H$ so

$$T_{24}(S_R, S_O) = (1 - s_R^2 P)(1 - s_O^2 P)(1 - s_O^7 Q)(1 - H) \tag{2.3}$$

We denote the representative firm's operating profit when it is in state i as $R_x(k-2A,d)$.In states 1 through $2A$, the profit of the firm is zero because the representative firm has a non-established position in these states. $R_x(k-2A,d) > 0$ because the representative firm has an established position and plays the oligopoly game in states $2A+1$ to $4A$. π_x^k is the representative firm's cash flow when it is in state k. We subtract from $R_x(k-2A,d)$ the expected costs associated with its positioning investment.

$$\pi_x^k = 0 - s_R^k I \text{ if } k \in \{1, ..., 2A\},\ x \in \{B, C\} \tag{2.4}$$

$$\pi_x^k = R_x(k-2A,d) - s_R^k J \text{ if } k \in \{2A+1, ..., 4A\}, x \in \{B, C\} \tag{2.5}$$

$V^k((\widehat{S}_R, S_R), S_O)$ is the present value of the representative firm's profit over an infinite time horizon, when it is in state i. Here, $\widehat{S}_R = (\widehat{s}_R^1, \widehat{s}_R^2, ..., \widehat{s}_R^{4A})$ means the representative firm in the current period, S_R means the representative firm in all subsequent periods, and S_O means other firms in current and subsequent periods. The value function can be calculate in the following expression:

$$V^k((\widehat{S}_R, S_R), S_O) = \pi_x^k + D \sum_{j=1,12} T_R^{kj} V^j((\widehat{S}_R, S_R), S_O), k = 1, 4A \tag{2.6}$$

where $D, 0 < D < 1$, is a discount factor. Similarly, we can subtract from $TS(k-2A,d)$ the expected costs associated with its positioning investment.

$$SS^k = 0 - s_R^k I \text{ if } k \in \{1, ..., 2A\} \tag{2.7}$$

$$SS^k = TS(k-2A,d) - s_R^k J \text{ if } k \in \{2A+1, ..., 4A\} \tag{2.8}$$

Then, we can write the present value of maximized total surplus over an infinite time horizon as follows:

$$V^k((\widehat{S}_R, S_R), S_O) = SS^k + D \sum_{j=1,12} T_R^{kj} V^j((\widehat{S}_R, S_R), S_O), k = 1, 4A \tag{2.9}$$

Here, we denote the maximized total surplus for each market niche when it is in state k as $TS(k-2A,d)$. $TS(k-2A,d)>0$ only when $k \in \{2A+1, 4A\}$.

2.2.4 Characterizing the SMPNE

In order to have valid symmetric Markov perfect Nash equilibrium strategy, S^*, There are two equations below to be hold:

$$V^k((S^*,S^*),S^*) \geq V^k((((s^k,S^*_{-k}),S^*),S^*)\forall s^k \in [0,1], \quad \forall k \in \{1,2,...,2A\}. \tag{2.10}$$

Suppose there are some k, $0 < s^{k^*} < 1$, then

$$V^k((S^*,S^*),S^*) = V^k((((s^k,S^*_{-k}),S^*),S^*)\forall s^k \in [0,1] \tag{2.11}$$

s^{k^*} is the firm's current intertemporal maximization problem and (s^k, S^*_{-k}) is the strategy when we replace s^{k^*} by s^k. Suppose $0 < s^{k^*} < 1$, then, the firms's current maximization problem is solved for any $s^k \in [0,1]$.

Then, we will be able to calculate the symmetric Markov perfect Nash equilibrium strategy, $S^* = (s^{1^*}, s^{2^*}, ..., s^{4A^*})$ by using an algorithm in Appendix II.

2.3 Results

In the model, there are 11 exogenous variables: six parameters that control demand of the representative consumer (α_H, α_L, β, γ, h, l), four parameters that govern positioning technology (I, J, P, Q), and a discount factor (D). The model of competition in within period of competition is either Cournot or Bertrand. Besides, we derive maximized total surplus when firms choose prices to maximize consumer surplus in their market niche. Then we will be able to calculate efficiency in Cournot or Bertrand competition with different parameters value as well as different active firms in the market. Our exposition is structured around what we called the baseline parameters for three different competitions. First, we present a variety of results for the baseline parameters in these three different frameworks. Then, we compare results for the different value of parameters.

2.3.1 The Baseline Parameterization

We set the demand parameters to be $\alpha_H = 60$ for the high demand market and $\alpha_L = \frac{60}{\sqrt{2}}$ for the low demand market. Besides, we assume $\beta = 1$, $\gamma = 0.95$, $h = 0.7$, and $l = 0.7$. In addition, the positioning technology parameters are $I = 700$, $J = 200$, $P = 0.8$ and $Q = 0.95$. Table 2.4 shows the high and low demand of equilibrium the profit per each firm, consumer surplus, and total surplus in the Bertrand or Cournot competition for one, two, and three active firms' cases. Moreover, it also presents the equilibrium profit, consumer surplus, and total surplus when firms maximize consumer surplus for their market niche with high or low demand for the one, two, and three active firms' cases. In the welfare maximum framework, the profit always equals to zero with various firm numbers and market demand and the profit decreases much slower with number of firms increase in the Cournot competition than Bertrand. We discover the total surplus obtained from different competitions goes up with the number of firms increase. This implies the gains from variety in this market exceed the losses from extra fixed costs.

Table 2.4: Payoffs in the Bertrand, Cournot, and Welfarel Maximum Frameworks

States of Demand	Number of Active Firms	Payoffs	Bertrand	Cournot	Welfare Maximum
High	One	Profit Per Firm	900	900	0
		Consumer Surplus	450	450	1800
		Total Surplus	1350	1350	1800
	Two	Profit Per Firm	84	414	0
		Consumer Surplus	1674	806	1846
		Total Surplus	1842	1634	1846
	Three	Profit Per Firm	30	237	0
		Consumer Surplus	1770	1030	1862
		Total Surplus	1860	1740	1862
Low	One	Profit Per Firm	450	450	0
		Consumer Surplus	225	225	900
		Total Surplus	675	675	900
	Two	Profit Per Firm	42	207	0
		Consumer Surplus	837	403	923
		Total Surplus	921	817	923
	Three	Profit Per Firm	15	118	0
		Consumer Surplus	885	515	931
		Total Surplus	930	870	931

2.3.2 A Realization of the Process

From table 2.10 to 2.22, we provide an example of the two active firms' case with a realization of the process under the Bertrand, Cournot, or welfare maximum competition in the dynamic world by using baseline parameter values. We impose one of the initial states of the system for each table under each of these 3 competitions. Suppose there are 20 periods in each competition, the patterns of realization differ significantly with different competitions and different initial state of the system. In each period of time, the number of established firms can be the same or different from the previous period. A similar thing happens to the states of the market demand. We discover that two established firms in the market is socially superior in the Cournot competition than Bertrand. This is because the profit per each firm is much lower to have two established firms in the Bertrand competition than Cournot. Therefore, it is more risky for a firm to enter into the market if there is an existing firm in this market for Bertrand than Cournot. Meanwhile, we find one established firm in the market is socially superior in the Bertrand competition than Cournot. In addition, we observe more two established firms in the period of time under the welfare maximum competition than the other two competitions. From the patterns of realization, it is hard to get a firm grip on what is happening. Because of this difficulty, we focus on the dynamic properties of the model.

Table 2.5: Two Active Firms in the Bertrand Framework with the Initial State of the System $(0, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X															X				
1		X	X	X	X	X	X				X	X	X	X	X					X
2								X	X	X							X	X	X	
Market Demand																				
H	X	X	X	X	X									X	X	X	X	X	X	X
L						X	X	X	X	X	X	X	X							
Profit Per Firm	0	900	900	900	900	450	450	42	42	42	450	450	450	900	900	0	84	84	84	900
Consumer Surplus	0	450	450	450	450	225	225	837	837	837	225	225	225	450	450	0	1647	1674	1674	450
Total Surplus	0	1350	1350	1350	1350	675	675	921	921	921	675	675	675	1350	1350	0	1842	1842	1842	1350

Table 2.6: Two Active Firms in the Bertrand Framework with the Initial State of the System $(1, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0									X	X						X				
1	X	X	X	X	X	X	X	X					X	X	X					X
2											X	X					X	X	X	
Market Demand																				
H	X	X	X	X	X	X	X	X								X	X	X	X	X
L									X	X	X	X	X	X	X					
Profit Per Firm	900	900	900	900	900	900	900	900	0	0	42	42	450	450	450	0	84	84	84	900
Consumer Surplus	450	450	450	450	450	450	450	450	0	0	837	837	225	225	225	0	1674	1674	1674	450
Total Surplus	1350	1350	1350	1350	1350	1350	1350	1350	0	0	921	921	675	675	675	0	1842	1842	1842	1350

Table 2.7: Two Active Firms in the Bertrand Framework with the Initial State of the System $(2, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0											X	X								
1				X	X	X	X	X	X	X			X	X	X	X				
2	X	X	X														X	X	X	X
Market Demand																				
H	X	X	X	X	X	X	X										X	X	X	X
L								X	X	X	X	X	X	X	X	X				
Profit Per Firm	84	84	84	900	900	900	900	450	450	450	0	0	450	450	450	450	84	84	84	84
Consumer Surplus	1674	1674	1674	450	450	450	450	225	225	225	0	0	225	225	225	225	1674	1674	1674	1674
Total Surplus	1842	1842	1842	1350	1350	1350	1350	675	675	675	0	0	675	675	675	675	1842	1842	1842	1842

Table 2.8: Two Active Firms in the Bertrand Framework with the Initial State of the System $(0, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X															X				
1		X	X	X	X	X	X				X	X	X	X	X					X
2								X	X	X							X	X	X	
Market Demand																				
H						X	X	X	X	X	X	X	X							
L	X	X	X	X	X									X	X	X	X	X	X	X
Profit Per Firm	0	450	450	450	450	900	900	84	84	84	900	900	900	450	450	0	42	42	42	450
Consumer Surplus	0	225	225	225	225	450	450	1674	1674	1674	450	450	450	225	225	0	837	837	837	225
Total Surplus	0	675	675	675	675	1350	1350	1842	1842	1842	1350	1350	1350	675	675	0	921	921	921	675

Table 2.9: Two Active Firms in the Bertrand Framework with the Initial State of the System $(1, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0									X	X						X				
1	X	X	X	X	X	X	X	X					X	X	X					X
2											X	X					X	X	X	
Market Demand																				
H									X	X	X	X	X	X	X					
L	X	X	X	X	X	X	X	X								X	X	X	X	X
Profit Per Firm	450	450	450	450	450	450	450	450	0	0	84	84	900	900	900	0	42	42	42	450
Consumer Surplus	225	225	225	225	225	225	225	225	0	0	1674	1674	450	450	450	0	837	837	837	225
Total Surplus	675	675	675	675	675	675	1350	675	0	0	1842	1842	1350	1350	1350	0	921	921	921	675

Table 2.10: Two Active Firms in the Bertrand Framework with the Initial State of the System $(2, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0											X	X								
1				X	X	X	X	X	X	X			X	X	X	X				
2	X	X	X														X	X	X	X
Market Demand																				
H								X	X	X	X	X	X	X	X	X				
L	X	X	X	X	X	X	X										X	X	X	X
Profit Per Firm	42	42	42	450	450	450	450	900	900	900	0	0	900	900	900	900	42	42	42	42
Consumer Surplus	837	837	837	225	225	225	225	450	450	450	0	0	450	450	450	450	837	837	837	837
Total Surplus	921	921	921	675	675	675	675	1350	1350	1350	0	0	1350	1350	1350	1350	921	921	921	921

Table 2.11: Two Active Firms in the Cournot Framework with the Initial State of the System $(0, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X															X	X			
1		X							X	X	X							X		
2			X	X	X	X	X	X				X	X	X	X				X	X
Market Demand																				
H	X	X	X	X	X								X	X	X	X	X	X	X	X
L						X	X	X	X	X	X	X								
Profit Per Firm	0	900	414	414	414	207	207	207	450	450	450	207	414	414	414	0	0	900	414	414
Consumer Surplus	0	450	806	806	806	403	403	403	225	225	225	403	806	806	806	0	0	450	806	806
Total Surplus	0	1350	1634	1634	1634	817	817	817	675	675	675	817	1634	1634	1634	0	0	1350	1634	1634

Table 2.12: Two Active Firms in the Cournot Framework with the Initial State of the System $(1, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0								X	X											X
1	X									X	X					X	X	X		
2		X	X	X	X	X	X					X	X	X	X				X	
Market Demand																				
H	X	X	X								X	X	X	X	X	X	X	X	X	X
L				X	X	X	X	X	X	X										
Profit Per Firm	900	414	414	207	207	207	207	0	0	450	900	414	414	414	414	900	900	900	414	0
Consumer Surplus	450	806	806	403	403	403	403	0	0	225	450	806	806	806	806	450	450	450	806	0
Total Surplus	1350	1634	1634	817	817	817	817	0	0	675	1350	1634	1634	1634	1634	1350	1350	1350	1634	0

Table 2.13: Two Active Firms in the Cournot Framework with the Initial State of the System $(2, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0																X	X			
1									X	X	X							X	X	X
2	X	X	X	X	X	X	X	X				X	X	X	X					
Market Demand																				
H	X	X	X	X	X	X	X							X	X	X	X	X	X	X
L								X	X	X	X	X	X							
Profit Per Firm	414	414	414	414	414	414	414	207	450	450	450	207	207	414	414	0	0	900	900	900
Consumer Surplus	806	806	806	806	806	806	806	403	225	225	225	403	403	806	806	0	0	450	450	450
Total Surplus	1634	1634	1634	1634	1634	1634	1634	817	675	675	675	817	817	1634	1634	0	0	1350	1350	1350

Table 2.14: Two Active Firms in the Cournot Framework with the Initial State of the System $(0, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X															X	X			
1		X							X	X	X							X		
2			X	X	X	X	X	X				X	X	X	X				X	X
Market Demand																				
H						X	X	X	X	X	X	X								
L	X	X	X	X	X								X	X	X	X	X	X	X	X
Profit Per Firm	0	450	207	207	207	414	414	414	900	900	900	414	207	207	207	0	0	450	207	207
Consumer Surplus	0	225	403	403	403	806	806	806	450	450	450	806	403	403	403	0	0	225	403	403
Total Surplus	0	675	817	817	817	1634	1634	1634	1350	1350	1350	1634	817	817	817	0	0	675	817	817

Table 2.15: Two Active Firms in the Cournot Framework with the Initial State of the System $(1, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0								X	X											X
1	X									X	X					X	X	X		
2		X	X	X	X	X	X					X	X	X	X				X	
Market Demand																				
H				X	X	X	X	X	X	X										
L	X	X	X								X	X	X	X	X	X	X	X	X	X
Profit Per Firm	450	207	207	414	414	414	414	0	0	900	450	207	207	207	207	450	450	450	207	0
Consumer Surplus	225	403	403	806	806	806	806	0	0	450	225	403	403	403	403	225	225	225	403	0
Total Surplus	675	817	817	1634	1634	1634	1634	0	0	1350	675	817	817	817	817	675	675	675	675	0

Table 2.16: Two Active Firms in the Cournot Framework with the Initial State of the System $(2, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0																X	X			
1									X	X	X							X	X	X
2	X	X	X	X	X	X	X	X				X	X	X	X					
Market Demand																				
H								X	X	X	X	X	X							
L	X	X	X	X	X	X	X							X	X	X	X	X	X	X
Profit Per Firm	207	207	207	207	207	207	207	414	900	900	900	414	414	207	207	0	0	450	450	450
Consumer Surplus	403	403	403	403	403	403	403	806	450	450	450	806	806	403	403	0	0	225	225	225
Total Surplus	817	817	817	817	817	817	817	1634	1350	1350	1350	1634	1634	817	817	0	0	675	675	675

Table 2.17: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(0, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X																			
1							X	X									X			
2		X	X	X	X	X			X	X	X	X	X	X	X	X		X	X	X
Market Demand																				
H	X	X	X												X	X	X	X	X	X
L				X	X	X	X	X	X	X	X	X	X	X						
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	0	1846	1846	923	923	923	900	900	923	923	923	923	923	923	1846	1846	1800	1846	1846	1846
Total Surplus	0	1846	1846	923	923	923	900	900	923	923	923	923	923	923	1846	1846	1800	1846	1846	1846

Table 2.18: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(1, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0							X													
1	X	X														X				
2			X	X	X	X		X	X	X	X	X	X	X	X		X	X	X	X
Market Demand																				
H	X	X	X	X	X	X	X								X	X	X	X	X	X
L								X	X	X	X	X	X	X						
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	1800	1800	1846	1846	1846	1846	0	923	923	923	923	923	923	923	1846	1800	1846	1846	1846	1846
Total Surplus	1800	1800	1846	1846	1846	1846	0	923	923	923	923	923	923	923	1846	1800	1846	1846	1846	1846

Table 2.19: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(2, H)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0										X										
1														X	X	X				
2	X	X	X	X	X	X	X	X	X		X	X	X				X	X	X	X
Market Demand																				
H	X	X	X	X	X											X	X	X	X	X
L						X	X	X	X	X	X	X	X	X	X					
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	1846	1846	1846	1846	1846	923	923	923	923	0	923	923	923	900	900	1800	1846	1846	1846	1846
Total Surplus	1846	1846	1846	1846	1846	923	923	923	923	0	923	923	923	900	900	1800	1846	1846	1846	1846

Table 2.20: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(0, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	X																			
1							X	X									X			
2		X	X	X	X	X			X	X	X	X	X	X	X	X		X	X	X
Market Demand																				
H				X	X	X	X	X	X	X	X	X	X	X						
L	X	X	X												X	X	X	X	X	X
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	0	923	923	1846	1846	1846	1800	1800	1846	1846	1846	1846	1846	1846	923	923	900	923	923	923
Total Surplus	0	923	923	1846	1846	1846	1800	1800	1846	1846	1846	1846	1846	1846	923	923	900	923	923	923

Table 2.21: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(1, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0							X													
1	X	X														X				
2			X	X	X	X		X	X	X	X	X	X	X	X		X	X	X	X
Market Demand																				
H								X	X	X	X	X	X	X						
L	X	X	X	X	X	X	X								X	X	X	X	X	X
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	900	900	923	923	923	923	0	1846	1846	1846	1846	1846	1846	1846	923	900	923	923	923	923
Total Surplus	900	900	923	923	923	923	0	1846	1846	1846	1846	1846	1846	1846	923	900	923	923	923	923

Table 2.22: Two Active Firms in the Welfare Maximum Framework with the Initial State of the System $(2, L)$

Number of Established Firms	Period of Time																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0										X										
1														X	X	X				
2	X	X	X	X	X	X	X	X	X		X	X	X				X	X	X	X
Market Demand																				
H						X	X	X	X	X	X	X	X	X	X					
L	X	X	X	X	X											X	X	X	X	X
Profit Per Firm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Consumer Surplus	923	923	923	923	923	1846	1846	1846	1846	0	1846	1846	1846	1800	1800	900	923	923	923	923
Total Surplus	923	923	923	923	923	1846	1846	1846	1846	0	1846	1846	1846	1800	1800	900	923	923	923	923

2.3.3 Conditional Probabilities

Next, we will give examples for the two active firms' case in the Cournot, Bertrand, or welfare maximum frameworks. The number of states of the system is 6. We present conditional probabilities with different initial states. From the tables in section 2.3.2, it should be clear that the system can go from a particular state to a number of different states, including the state that it is currently in when it is in 1 period, in 2 periods, and so on. This raises an obvious question: what is the probability of going from one state to another in 1 period, in 2 periods, and so forth. We will give an answer to this question below. Here, the probability for a particular state, in times $t > 0$ conditional on being in a particular state at time $t = 0$, is generated when firms use their SMPNE strategies. The firm's expected probability distribution over states in the t periods is calculated as follows:

$$E_t = E[T^*]^t \tag{2.12}$$

E is the initial probability distribution over states which has a 1 for one state and 0s for the others. T^* is the equilibrium transition matrix.

First, we want to present the probability distribution of going from any of the 6 states to any of the same 6 states in one period in the Bertrand, Cournot, and welfare maximum frameworks. In table 2.23, we discover the probability to stay in the initial state of the system in time $t = 1$ conditional on being in this particular state at time $t = 0$ is not lower in Bertrand than Cournot only except the initial state of the system is $(2, L)$. Here, the state of the system depends two factors: the number of established firms and the state of the market demand. The steady state can be obtained when the power of the equilibrium transition matrix approaches infinity and the probability distribution becomes constant.

Table 2.23: Conditional Probabilities in the Bertrand, Cournot, and Welfare Maximum Frameworks

	Conditional Probability		
States of the System	Bertrand	Cournot	Welfare Maximum
(0, H)	0.050	0.028	0.028
(1, H)	0.665	0.161	0.161
(2, H)	0.632	0.632	0.632
(0, L)	0.062	0.028	0.028
(1, L)	0.665	0.665	0.161
(2, L)	0.622	0.632	0.632

From table 2.24 to 2.28, we impose different initial states of the system in period 0. By comparing column 9 and column 10 in table 2.26, 2.27, 2.28, and 2.29, we find that the probability distributions are almost identical with different initial states of the system. This looks like some sort of steady state. In other words, the state of the system 10 periods from now has almost nothing to do with the state of the system today.

Table 2.24: Transition Probabilities with Two Active Firms in the Bertrand Framework I

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	1	0.050	0.015	0.014	0.014	0.015	0.016	0.016	0.017	0.017	0.018
(1, H)	0	0.275	0.263	0.262	0.270	0.281	0.293	0.303	0.313	0.321	0.328
(2, H)	0	0.375	0.302	0.256	0.228	0.209	0.194	0.181	0.171	0.162	0.154
(0, L)	0	0.022	0.011	0.012	0.014	0.015	0.016	0.017	0.017	0.018	0.018
(1, L)	0	0.118	0.192	0.233	0.259	0.278	0.292	0.304	0.314	0.323	0.330
(2, L)	0	0.161	0.217	0.223	0.215	0.202	0.190	0.179	0.169	0.160	0.152
(0, H)	0	0.035	0.030	0.027	0.025	0.024	0.024	0.023	0.023	0.023	0.022
(1, H)	1	0.665	0.535	0.478	0.451	0.436	0.426	0.418	0.412	0.407	0.403
(2, H)	0	0.000	0.015	0.027	0.037	0.045	0.053	0.059	0.065	0.070	0.075
(0, L)	0	0.015	0.022	0.024	0.024	0.024	0.024	0.023	0.023	0.023	0.022
(1, L)	0	0.285	0.388	0.421	0.429	0.427	0.423	0.418	0.413	0.408	0.404
(2, L)	0	0.000	0.011	0.023	0.034	0.044	0.051	0.058	0.064	0.069	0.074
(0, H)	0	0.002	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014	0.015
(1, H)	0	0.067	0.104	0.135	0.164	0.190	0.214	0.235	0.254	0.270	0.284
(2, H)	1	0.632	0.472	0.391	0.341	0.305	0.277	0.253	0.233	0.216	0.201
(0, L)	0	0.001	0.003	0.006	0.008	0.010	0.011	0.013	0.014	0.015	0.015
(1, L)	0	0.029	0.077	0.121	0.158	0.189	0.215	0.237	0.256	0.272	0.286
(2, L)	0	0.271	0.340	0.341	0.321	0.296	0.272	0.250	0.230	0.213	0.198

Table 2.25: Transition Probabilities with Two Active Firms in the Bertrand Framework II

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	0	0.026	0.012	0.013	0.014	0.015	0.016	0.017	0.017	0.018	0.018
(1, H)	0	0.125	0.203	0.243	0.268	0.286	0.299	0.310	0.319	0.327	0.333
(2, H)	0	0.148	0.205	0.212	0.204	0.194	0.182	0.172	0.163	0.155	0.148
(0, L)	1	0.062	0.017	0.015	0.015	0.016	0.016	0.017	0.018	0.018	0.018
(1, L)	0	0.292	0.282	0.279	0.285	0.294	0.304	0.313	0.321	0.329	0.335
(2, L)	0	0.346	0.281	0.238	0.213	0.195	0.182	0.171	0.161	0.153	0.147
(0, H)	0	0.015	0.022	0.024	0.024	0.024	0.023	0.023	0.023	0.023	0.022
(1, H)	0	0.285	0.388	0.421	0.429	0.427	0.423	0.417	0.412	0.407	0.403
(2, H)	0	0.000	0.011	0.023	0.034	0.044	0.052	0.059	0.065	0.070	0.074
(0, L)	0	0.035	0.030	0.027	0.025	0.024	0.024	0.023	0.023	0.023	0.023
(1, L)	1	0.665	0.535	0.479	0.452	0.437	0.427	0.419	0.413	0.408	0.404
(2, L)	0	0.000	0.015	0.026	0.035	0.044	0.052	0.058	0.064	0.069	0.073
(0, H)	0	0.001	0.003	0.006	0.008	0.010	0.011	0.013	0.014	0.015	0.015
(1, H)	0	0.032	0.082	0.126	0.162	0.193	0.218	0.239	0.258	0.274	0.287
(2, H)	0	0.267	0.335	0.336	0.317	0.292	0.269	0.247	0.228	0.212	0.197
(0, L)	0	0.002	0.005	0.007	0.009	0.010	0.012	0.013	0.014	0.015	0.016
(1, L)	0	0.076	0.115	0.146	0.174	0.199	0.222	0.243	0.260	0.276	0.289
(2, L)	1	0.622	0.460	0.379	0.330	0.296	0.268	0.245	0.226	0.209	0.195

Table 2.26: Transition Probabilities with Two Active Firms in the Cournot Framework I

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	1	0.028	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
(1, H)	0	0.224	0.106	0.090	0.088	0.088	0.089	0.089	0.090	0.090	0.090
(2, H)	0	0.448	0.469	0.438	0.421	0.413	0.410	0.408	0.407	0.406	0.406
(0, L)	0	0.012	0.005	0.005	0.005	0.006	0.006	0.006	0.006	0.006	0.006
(1, L)	0	0.096	0.110	0.116	0.120	0.124	0.126	0.128	0.129	0.129	0.130
(2, L)	0	0.192	0.304	0.347	0.361	0.366	0.366	0.366	0.365	0.365	0.364
(0, H)	0	0.007	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004	0.004
(1, H)	1	0.161	0.089	0.083	0.083	0.085	0.087	0.088	0.089	0.090	0.090
(2, H)	0	0.532	0.487	0.446	0.426	0.416	0.412	0.409	0.408	0.407	0.406
(0, L)	0	0.003	0.004	0.004	0.005	0.005	0.006	0.006	0.006	0.006	0.006
(1, L)	0	0.069	0.089	0.102	0.112	0.118	0.123	0.125	0.127	0.128	0.129
(2, L)	0	0.228	0.327	0.362	0.371	0.371	0.370	0.368	0.367	0.366	0.365
(0, H)	0	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004
(1, H)	0	0.067	0.069	0.073	0.078	0.082	0.085	0.087	0.088	0.089	0.090
(2, H)	1	0.632	0.509	0.456	0.432	0.420	0.414	0.411	0.409	0.408	0.407
(0, L)	0	0.001	0.002	0.003	0.004	0.005	0.005	0.006	0.006	0.006	0.006
(1, L)	0	0.029	0.060	0.084	0.100	0.111	0.118	0.123	0.125	0.127	0.128
(2, L)	0	0.271	0.358	0.381	0.383	0.379	0.375	0.371	0.369	0.367	0.366

Table 2.27: Transition Probabilities with Two Active Firms in the Cournot Framework II

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	0	0.012	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004
(1, H)	0	0.096	0.110	0.107	0.103	0.099	0.096	0.094	0.093	0.092	0.091
(2, H)	0	0.192	0.304	0.356	0.380	0.392	0.398	0.401	0.403	0.404	0.405
(0, L)	1	0.028	0.010	0.008	0.008	0.007	0.007	0.006	0.006	0.006	0.006
(1, L)	0	0.224	0.198	0.176	0.160	0.149	0.142	0.138	0.135	0.133	0.132
(2, L)	0	0.448	0.372	0.348	0.345	0.349	0.353	0.356	0.359	0.360	0.362
(0, H)	0	0.015	0.013	0.010	0.008	0.006	0.005	0.005	0.004	0.004	0.004
(1, H)	0	0.285	0.242	0.192	0.156	0.133	0.117	0.107	0.101	0.097	0.095
(2, H)	0	0.000	0.165	0.266	0.323	0.356	0.375	0.387	0.394	0.398	0.401
(0, L)	0	0.035	0.025	0.018	0.014	0.011	0.009	0.008	0.007	0.007	0.006
(1, L)	1	0.665	0.471	0.347	0.268	0.217	0.185	0.165	0.152	0.144	0.139
(2, L)	0	0.000	0.084	0.166	0.231	0.277	0.308	0.328	0.341	0.349	0.354
(0, H)	0	0.001	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.004	0.004
(1, H)	0	0.029	0.060	0.075	0.083	0.086	0.088	0.089	0.090	0.090	0.090
(2, H)	0	0.271	0.358	0.390	0.401	0.405	0.406	0.406	0.406	0.406	0.406
(0, L)	0	0.002	0.004	0.005	0.005	0.006	0.006	0.006	0.006	0.006	0.006
(1, L)	0	0.067	0.096	0.111	0.119	0.124	0.126	0.128	0.129	0.129	0.130
(2, L)	1	0.632	0.480	0.416	0.388	0.376	0.370	0.367	0.366	0.365	0.364

Table 2.28: Transition Probabilities with Two Active Firms in the Welfare Maximum Framework I

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	1	0.028	0.004	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	0	0.224	0.085	0.062	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, H)	0	0.448	0.491	0.468	0.454	0.447	0.445	0.444	0.443	0.443	0.443
(0, L)	0	0.012	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	0	0.096	0.062	0.054	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, L)	0	0.192	0.355	0.412	0.431	0.438	0.441	0.442	0.443	0.443	0.443
(0, H)	0	0.007	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	1	0.161	0.074	0.060	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, H)	0	0.532	0.503	0.470	0.454	0.447	0.445	0.444	0.443	0.443	0.443
(0, L)	0	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	0	0.069	0.054	0.053	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, L)	0	0.228	0.364	0.413	0.431	0.438	0.441	0.442	0.443	0.443	0.443
(0, H)	0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	0	0.067	0.063	0.059	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, H)	1	0.632	0.515	0.471	0.454	0.447	0.445	0.444	0.443	0.443	0.443
(0, L)	0	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	0	0.029	0.046	0.052	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, L)	0	0.271	0.373	0.415	0.432	0.438	0.441	0.442	0.443	0.443	0.443

Table 2.29: Transition Probabilities with Two Active Firms in the Welfare Maximum Framework II

States of the System	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
(0, H)	0	0.012	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	0	0.096	0.062	0.054	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, H)	0	0.192	0.355	0.412	0.431	0.438	0.441	0.442	0.443	0.443	0.443
(0, L)	1	0.028	0.004	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	0	0.224	0.085	0.062	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, L)	0	0.448	0.491	0.468	0.454	0.447	0.445	0.444	0.443	0.443	0.443
(0, H)	0	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	0	0.069	0.054	0.053	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, H)	0	0.228	0.364	0.413	0.431	0.438	0.441	0.442	0.443	0.443	0.443
(0, L)	0	0.007	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	1	0.161	0.074	0.060	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, L)	0	0.532	0.503	0.470	0.454	0.447	0.445	0.444	0.443	0.443	0.443
(0, H)	0	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, H)	0	0.029	0.046	0.052	0.054	0.055	0.055	0.055	0.055	0.055	0.055
(2, H)	0	0.271	0.373	0.415	0.432	0.438	0.441	0.442	0.443	0.443	0.443
(0, L)	0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
(1, L)	0	0.067	0.063	0.059	0.057	0.056	0.056	0.055	0.055	0.055	0.055
(2, L)	1	0.632	0.515	0.471	0.454	0.447	0.445	0.444	0.443	0.443	0.443

2.3.4 The Steady State of the System

First, we obtain the steady state of the system by transferring from the steady state of a representative firm. The steady state of a representative firm depends on the state of the representative firm, the number of other established firms, and the state of the market demand. However, the state of the system only focuses on the number of established firms and the state of the demand. Here, each state's steady state presents an estimation of the firm's probability distribution in the far future. This distribution is used to predict the current period if we have no information about the history of the dynamic model. In table 2.30, we discover that the Bertrand has a highest possibility to have zero and one established firm in either high or low demand of the market; welfare maximum model has highest possibility to have two established firms in either high or low demand. Moreover, there is more likely to have two established firms in the Cournot than Bertrand competition. This is because with more firms in the market the payoff for Bertrand is much lower than Cournot.

Table 2.30: Steady State of Two Active Firms

States of the System	Bertrand	Cournot	Welfare Maximum
(0, H)	0.021	0.004	0.002
(1, H)	0.376	0.090	0.055
(2, H)	0.103	0.406	0.443
(0, L)	0.021	0.006	0.001
(1, L)	0.377	0.130	0.055
(2, L)	0.102	0.364	0.443

Similarly, we will also provide the steady state probability for the one and three active firms' cases. The Bertrand and Cournot have the same steady probability in the one active firm' case because in the monopoly market there is no difference to compete in the Bertrand or Cournot. For the three active firms' case, the Bertrand has a highest probability to establish zero and one firm, the Cournot has a highest probability to establish two firms, and the welfare maximum has a highest probability to establish three firms.

Table 2.31: Steady State of One Active Firm

States of the System	Bertrand	Cournot	Welfare Maximum
(0, H)	0.029	0.029	0.029
(1, H)	0.471	0.471	0.471
(0, L)	0.029	0.029	0.029
(1, L)	0.471	0.471	0.471

Table 2.32: Steady State of Three Active Firms

States of the System	Bertrand	Cournot	Welfare Maximum
(0, H)	0.023	0.006	0.000
(1, H)	0.380	0.124	0.009
(2, H)	0.093	0.341	0.116
(3, H)	0.004	0.029	0.375
(0, L)	0.023	0.007	0.000
(1, L)	0.381	0.143	0.007
(2, L)	0.092	0.324	0.096
(3, L)	0.004	0.026	0.397

Then, we also can compute the expected duration of each state of the system in the Bertrand, Cournot, and welfare maximum frameworks. Here, we give an example for the two active firms' case. The formula to calculate expected duration is as follows:

$$ED_i = \frac{1}{1 - CP_i} \tag{2.13}$$

Here, CP_i is the conditional probability for each state of a representative firm and ED_i is the expected duration for each state of a representative firm. By transferring the state of a representative firm to the state of the system, we obtain the expected duration for each state of the system in these different frameworks. In table 2.33, the first column shows the initial states of a representative firm and the initial states of the system. Based on these initial states, we present the conditional probabilities to stay in these initial states with three different competitions. The conditional probabilities in the states of a representative firm are not lower in Bertrand than Cournot except in the state $(E, 1, L)$. Similarly, the conditional probabilities in the states of the system are not lower in Bertrand than Cournot except in the state $(2, L)$. Therefore, we derive the expected duration in the states of a representative firm. Only in the state $(E, 1, L)$ the value of expected duration is lower in Bertrand than Cournot. Meanwhile, the value of expected duration in the states of the system is lower in Bertrand than Cournot only in the state $(2, L)$. In the state of the system, $(1, H)$, the expected duration in the Bertrand framework is maximum (2.985) which is more than twice value in the other two frameworks. In the state of the system, $(1, L)$, the expected durations in the Bertrand and Cournot framework are identical (2.985) and more than twice value in the welfare maximum framework.

Table 2.33: Conditional Probabilities and the Expected Duration With Two Active Firms

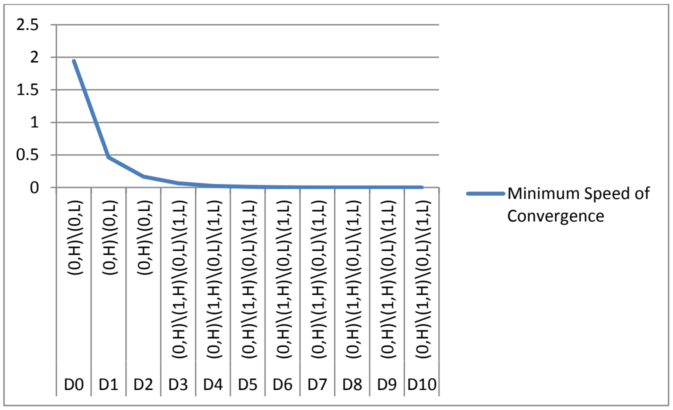
	Conditional Probability			Expected Duration		
States of a Representative Firm	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
(U, 0, H)	0.050	0.028	0.028	1.053	1.029	1.029
(U, 1, H)	0.665	0.133	0.133	2.985	1.153	1.153
(U, 0, L)	0.062	0.028	0.028	1.066	1.029	1.029
(U, 1, L)	0.665	0.665	0.133	2.985	2.985	1.153
(E, 0, H)	0.665	0.133	0.133	2.985	1.153	1.153
(E, 1, H)	0.632	0.632	0.632	2.716	2.716	2.716
(E, 0, L)	0.665	0.665	0.133	2.985	2.985	1.153
(E, 1, L)	0.622	0.632	0.632	2.646	2.716	2.716
	Conditional Probability			Expected Duration		
States of the System	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
(0, H)	0.050	0.028	0.028	1.053	1.029	1.029
(1, H)	0.665	0.161	0.161	2.985	1.192	1.192
(2, H)	0.632	0.632	0.632	2.716	2.716	2.716
(0, L)	0.062	0.028	0.028	1.066	1.029	1.029
(1, L)	0.665	0.665	0.161	2.985	2.985	1.192
(2, L)	0.622	0.632	0.632	2.646	2.716	2.716

2.3.5 The Speed of Convergence to the Steady State

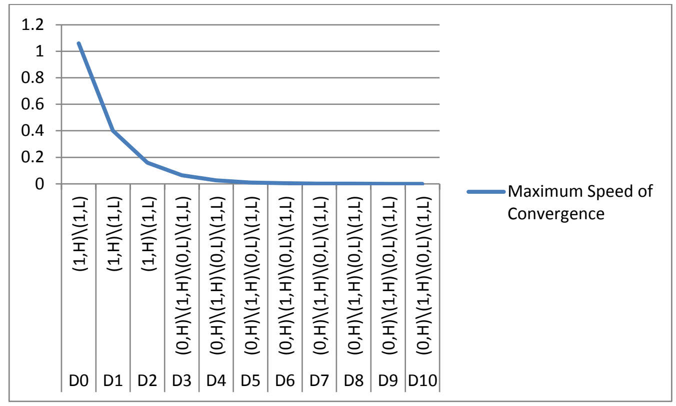
Based on the previous tables in section 2.3.3 and 2.3.4, we calculate the speed of convergence to the steady state of the system for 10 periods. Here, we are measuring and comparing the speed of convergence which is obtained from an initial state by initial state basis. In figure 2.1 to 2.3, the D0 to D10 are calculated from the sum of absolute value of the probability for each state by the power of transition matrix from period 0 to period 10 minus the steady state probability for each state, respectively. Here, the horizontal axis indicates the initial states of the system, which have the minimum or maximum converging speed to the steady states of the system from period 0 to 10. The vertical axis means values for the minimum or maximum of converging speed to the steady states of the system during these 11 periods. Figure 2.1 (a) and (b) show the maximum and minimum speed of convergence to the steady state of the system for the one active firm's case in the Bertrand framework. We discover that the speed of convergence is maximum when the initial states of the system is $(1, H)$ or $(1, L)$ for the first three periods, then has indifference speed of converge with different initial states. In Contrast, the initial states $(0, H)$ or $(0, L)$ has the minimum speed of convergence for the first three periods. We present the minimum and maximum speed of convergence for two active firms' case in figure 2.1 (c) and (d). We discover the initial state of the system $(2, H)$ has the minimum speed of convergence after first period and the initial state $(1, H)$ has the maximum speed of convergence after first two periods. Figure 2.1 (e) and (f) show the maximum and minimum speed of convergence for the three active firms' case. The initial state of the system $(2, H)$ has the minimum speed of convergence after first period and the initial state $(1, H)$ has the maximum speed of convergence after first two periods. If we set the convergence criteria is 0.01, we conclude there is no convergence for the first ten periods under the Bertrand competition with the two or three active firms' cases. Similarly, we present the minimum and maximum speed of convergence for the one, two and three active firms' case under the Cournot competition in figure 2.2 and under the welfare maximum in figure 2.3.

With the convergence criteria 0.01, the one and two active firms' cases are converged with their maximum speed of convergence in the Cournot framework. In the welfare maximum framework, the one, two, and three active firms' cases are all converged with their maximum speed of convergence. In figure 2.4, we show the speed of convergence in the Bertrand, Cournot and welfare maximum framework. The maximum speed of convergence is almost identical for the two and three active firms' case and only converged for one active firm' case in the Bertrand framework. The maximum speed of convergence is quite similar for the one and two active firms' cases in the Cournot framework and converged for both cases. In the welfare maximum framework, the maximum speed of convergence are almost identical for the one, two and three active firms' cases. Therefore, we can list the order of the speed of convergence in term of competition framework as follows: Welfare maximum > Cournot > Bertrand. This indicates firms converge faster to the steady state of the system in the welfare maximum framework than the other two frameworks.

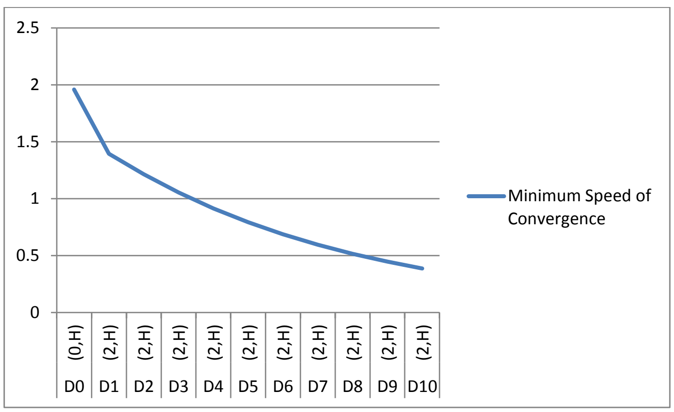
Figure 2.1: The Speed of Convergence in the Bertrand Framework



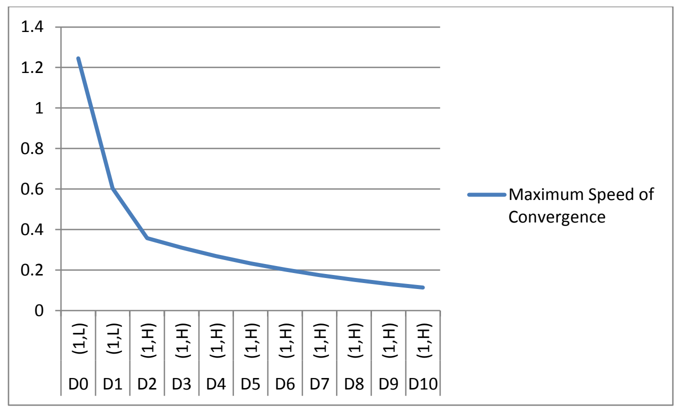
(a) Minimum Speed of Convergence (One Active Firm)



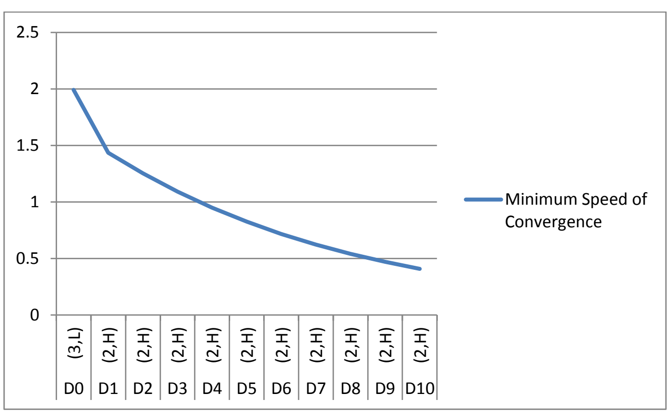
(b) Maximum Speed of Convergence (One Active Firm)



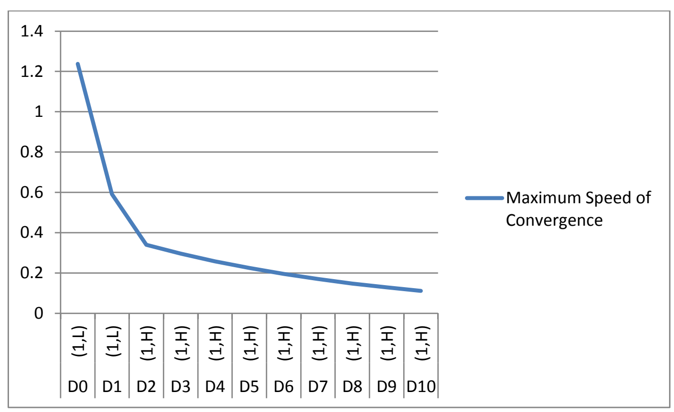
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

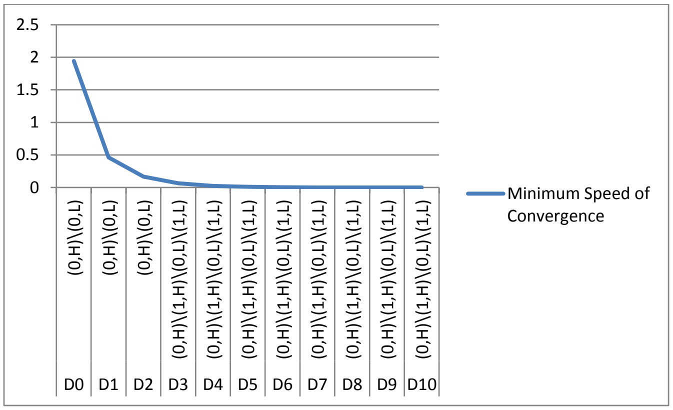


(e) Minimum Speed of Convergence (Three Active Firms)

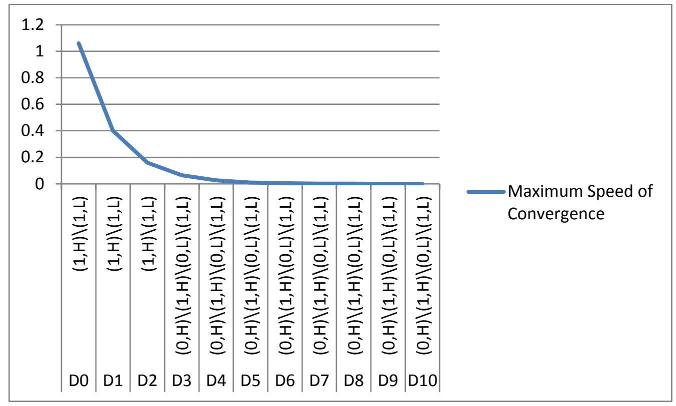


(f) Maximum Speed of Convergence (Three Active Firms)

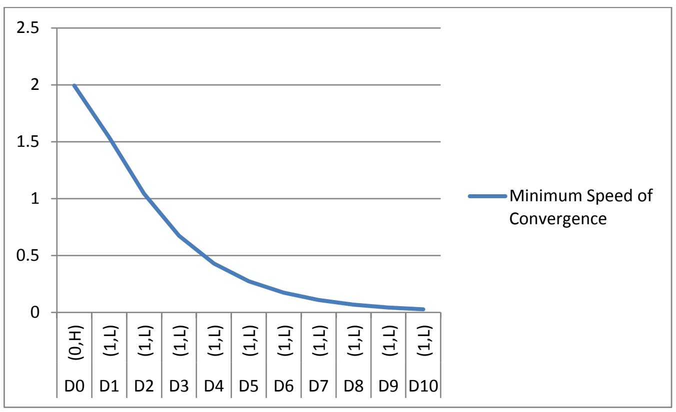
Figure 2.2: The Speed of Convergence in the Cournot Framework



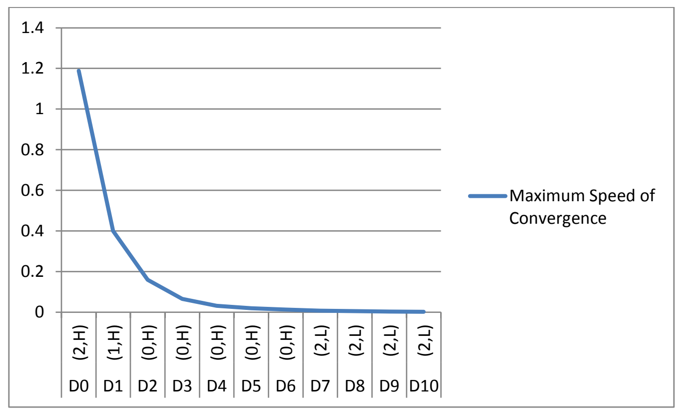
(a) Minimum Speed of Convergence (One Active Firm)



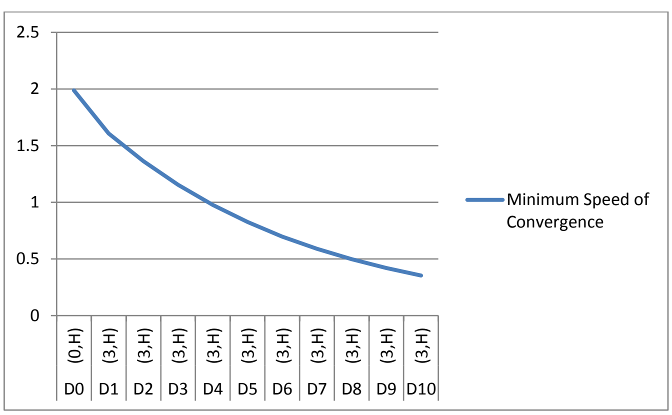
(b) Maximum Speed of Convergence (One Active Firm)



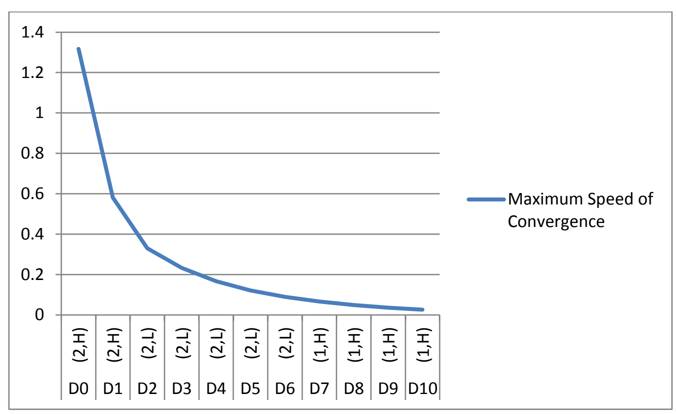
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

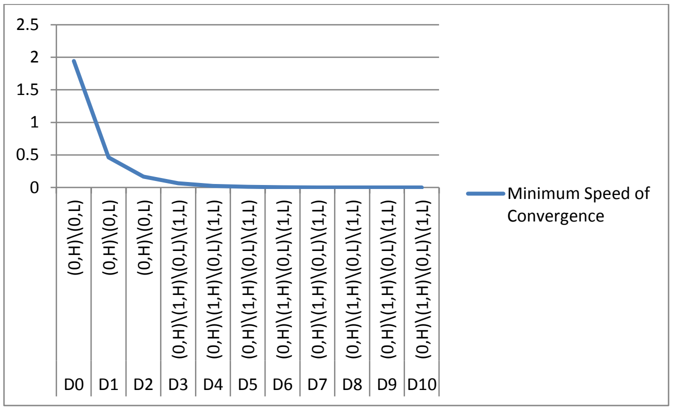


(e) Minimum Speed of Convergence (Three Active Firms)

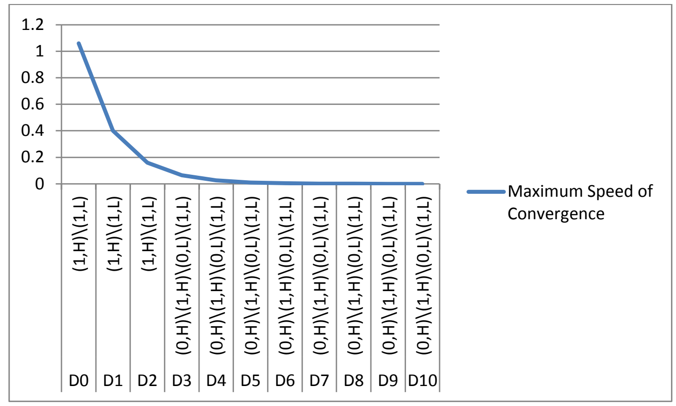


(f) Maximum Speed of Convergence (Three Active Firms)

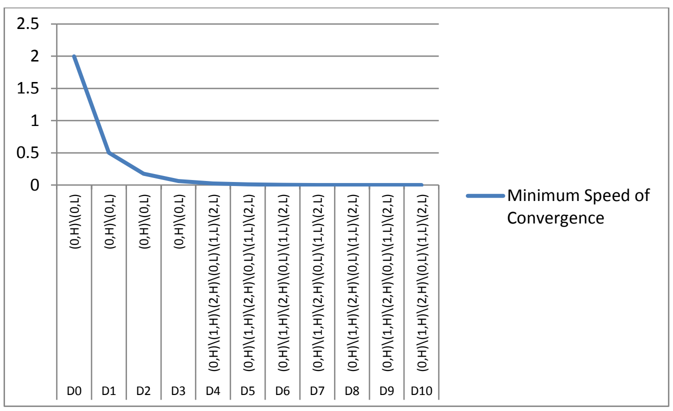
Figure 2.3: The Speed of Convergence in the Welfare Maximum Framework



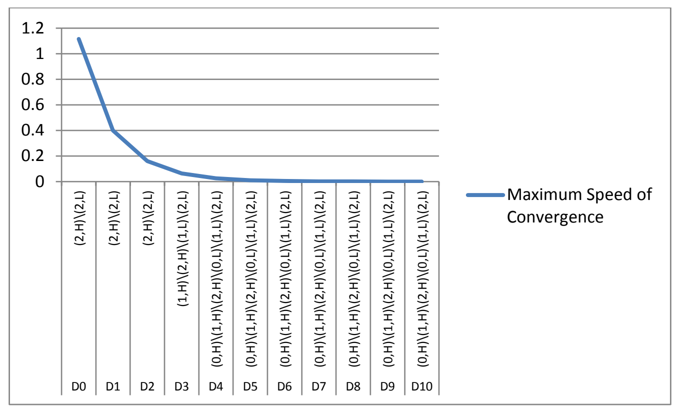
(a) Minimum Speed of Convergence (One Active Firm)



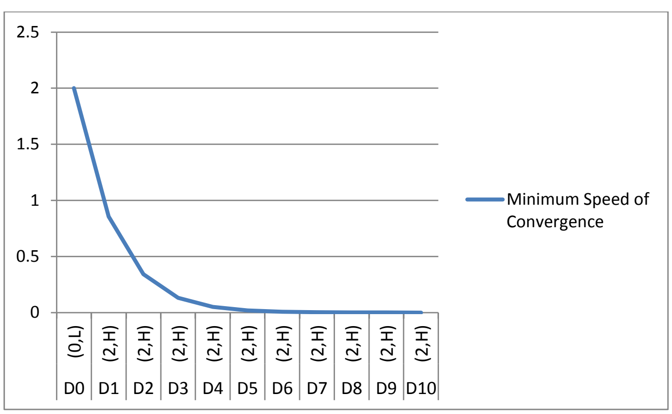
(b) Maximum Speed of Convergence (One Active Firm)



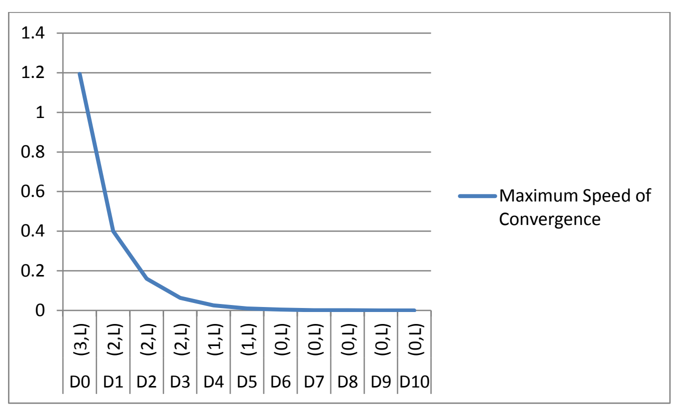
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

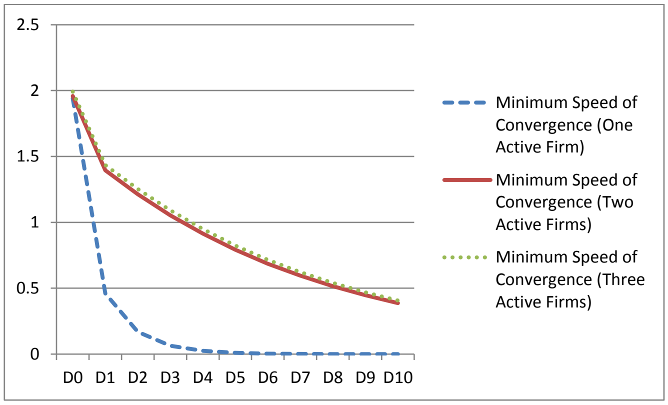


(e) Minimum Speed of Convergence (Three Active Firms)

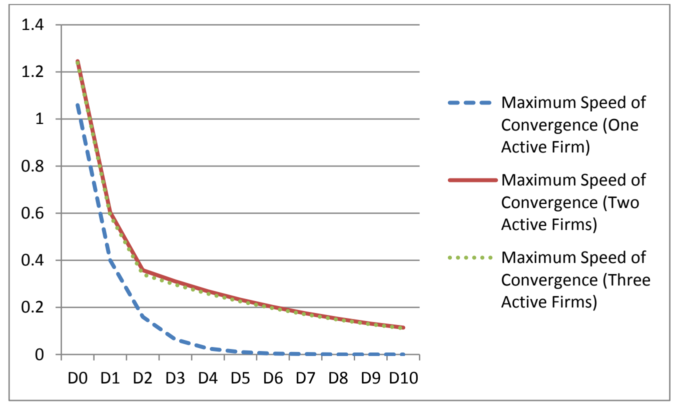


(f) Maximum Speed of Convergence (Three Active Firms)

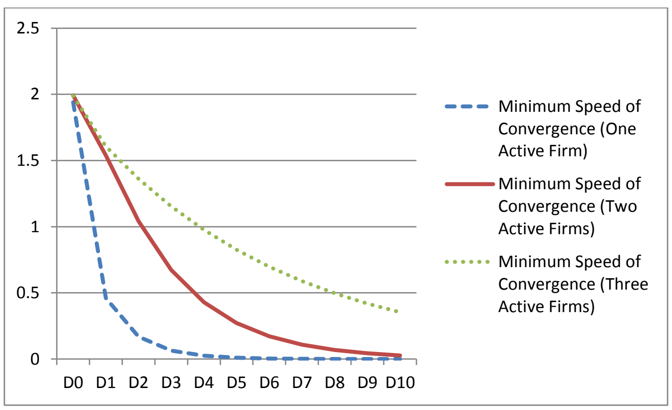
Figure 2.4: The Speed of Convergence in the Bertrand, Cournot and Welfare Maximum Frameworks



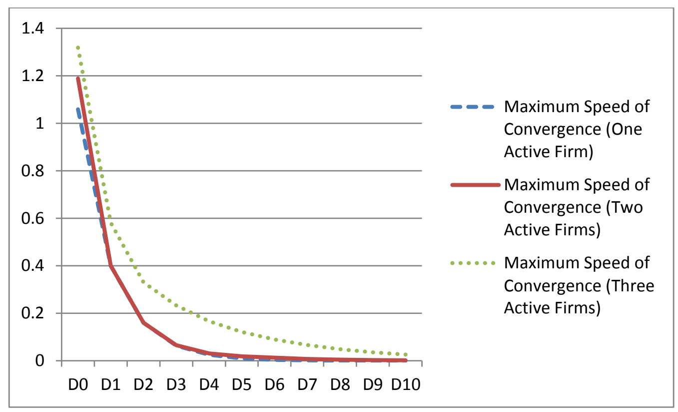
(a) Minimum Speed of Convergence in Bertrand



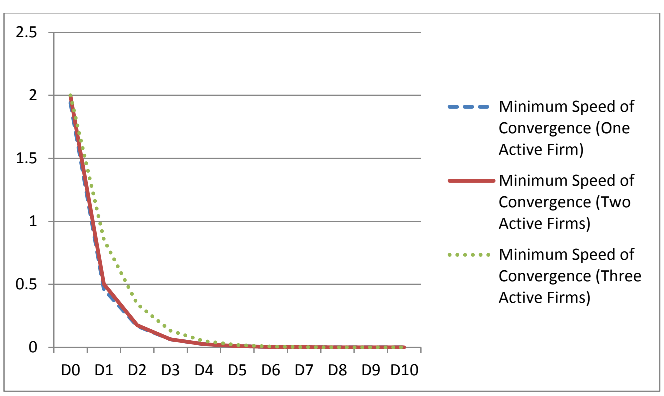
(b) Maximum Speed of Convergence in Bertrand



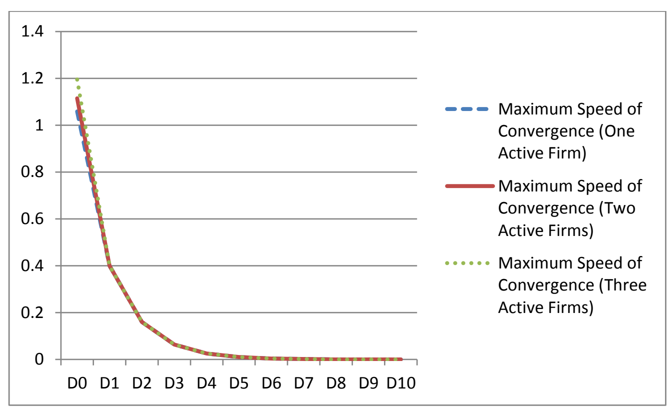
(c) Minimum Speed of Convergence in Cournot



(d) Maximum Speed of Convergence in Cournot



(e) Minimum Speed of Convergence in Welfare Maximum



(f) Maximum Speed of Convergence in Welfare Maximum

2.3.6 Efficiency

With the steady state probability for each state in each competition, we can calculate the expected total surplus and efficiency as below:

$$ESP_J = \sum_{i=1}^{S} Pr_{Ji} * TS_{Ji} \qquad J = \text{Bertrand, Cournot, or welfare maximum} \tag{2.14}$$

Here, ETS_J means the expected total surplus in the Bertrand, Cournot, or welfare maximum framework. Pr_{Ji} and TS_{Ji} are the probability and total surplus for each state in each competition. Therefore, we can calculate the efficiency for Bertrand and Cournot competition as follows:

$$EE_B = \frac{ETS_B}{ETS_{W(\max)}} \tag{2.15}$$

$$EE_C = \frac{ETS_C}{ETS_{W(\max)}} \tag{2.16}$$

We consider $ETS_{W(\max)}$ is the expected welfare maximum surplus for the one, two, and three active firms' cases. ETS_B and ETS_C are considered the expected total surplus in the Bertrand and Cournot framework for one, two, or three active firms' case, respectively. In table 2.34, the one active firm expected total surplus for Bertrand and Cournot are 953. The expected welfare maximum surplus is 1392 in the three active firms' case. Therefore, the one active firm efficiency for Bertrand and Cournot both are 0.68. The two active firm expected total surplus is 1046 for Bertrand and 1170 for Cournot. The two active firms' efficiency is 0.75 for Bertrand and 0.84 for Cournot. 1037 and 1158 are the three active firms expected total surplus for Bertrand and Cournot. The efficiency is 0.74 for Bertrand and 0.83 for Cournot. Here, we introduce Oaxaca decomposition which allows holding either probability distributions in the steady state of the system constant or the total surplus for each state of the system constant. By using this decomposition, we find the probability difference between Bertrand and welfare maximum is 347 and between Cournot and welfare maximum is 123. Meanwhile, the total surplus difference between Bertrand and welfare maximum is 7 and between Cournot and welfare maximum is 110. Therefore, we can conclude that the main loss of efficiency for Bertrand is because of probability difference and main loss of efficiency for Cournot is due the difference for the each state total surplus. This indicates the probability distributions in the steady state of the system are more similar in the Cournot and welfare maximum frameworks than in the Bertrand and welfare maximum frameworks. Meanwhile, we also find that the difference between the total surplus for each state of the system in the Bertrand and welfare maximum frameworks is smaller than the difference between the total surplus for each state of the system in the Cournot and welfare maximum frameworks. In addition, we discover that the two active firms' case has highest efficiency levels for both Bertrand and Cournot with the baseline parameters.

Table 2.34: Expected total surplus and Efficiency with Baseline Parameters

Number of Active Firms	Expected Total Surplus			Efficiency	
	Bertrand	Cournot	Welfare Maximium	Bertrand	Cournot
1	953	953	1271	0.68	0.68
2	1046	1170	1375	0.75	0.84
3	1037	1158	1392	0.74	0.83

2.3.7 Variation of the Parameter Values

In this section, we present 8 different experiments with various parameter values. We want to discover how the steady state of the system, efficiency, and the speed of convergence to the steady state vary with different combination of parameter values. The detail of the 8 experiments are as follows:

First, we want to discover the how is the steady state of the system affected by the variation of parameter values. The following tables represent the change of the steady state of the system by applying these 8 experiments. Table 2.36 and 2.37 reports that with lower

Table 2.35: Values of Parameters for the Basline Model and Eight Experiments

	I	J	P	Q	h	l
Baseline Model	700	200	0.8	0.95	0.7	0.7
Experiment 1	**2000**	200	0.8	0.95	0.7	0.7
Experiment 2	**250**	200	0.8	0.95	0.7	0.7
Experiment 3	700	**500**	0.8	0.95	0.7	0.7
Experiment 4	700	**50**	0.8	0.95	0.7	0.7
Experiment 5	700	200	**0.7**	0.95	0.7	0.7
Experiment 6	700	200	0.8	**0.85**	0.7	0.7
Experiment 7	700	200	0.8	0.95	**0.9**	0.7
Experiment 8	700	200	0.8	0.95	0.7	**0.9**

the investment costs or maintenance costs, there is more likely to establishing additional firms in the two and three active firms' cases. Table 2.38 shows with the lower probability of successful investment or maintenance of firms, there is more likely for firms to exit the market. Therefore, the probability with no firms in the market increases in the one, two, and three active firms' cases. In table 2.39, we present the experiments of increase the probability of high demand and increase the probability of low demand. With the higher probability of high demand, the probability of steady state is higher with high demand state of the system. With the higher probability of low demand, the probability of steady state is lower with high demand state of the system.

Table 2.36: Variation of Steady State of the System with Different I

Number of Active Firms	States of the System	Steady State (Experiment 1)			Steady State (Baseline)			Steady State (Experiment 2)		
		Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
One	(0, H)	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	(1, H)	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471
	(0, L)	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	(1, L)	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471
Two	(0, H)	0.071	0.046	0.002	0.021	0.004	0.002	0.020	0.002	0.002
	(1, H)	0.411	0.420	0.055	0.376	0.090	0.055	0.368	0.055	0.055
	(2, H)	0.018	0.034	0.443	0.103	0.406	0.443	0.113	0.443	0.443
	(0, L)	0.076	0.050	0.002	0.021	0.006	0.002	0.020	0.002	0.002
	(1, L)	0.408	0.417	0.055	0.377	0.130	0.055	0.369	0.055	0.055
	(2, L)	0.017	0.032	0.443	0.102	0.364	0.443	0.112	0.443	0.443
Three	(0, H)	0.087	0.058	0.001	0.023	0.006	0.000	0.018	0.001	0.000
	(1, H)	0.396	0.410	0.037	0.380	0.124	0.008	0.348	0.036	0.009
	(2, H)	0.017	0.031	0.335	0.093	0.341	0.105	0.122	0.335	0.112
	(3, H)	0.000	0.001	0.128	0.004	0.029	0.386	0.011	0.129	0.379
	(0, L)	0.092	0.063	0.001	0.023	0.007	0.000	0.019	0.001	0.000
	(1, L)	0.392	0.407	0.039	0.381	0.143	0.006	0.350	0.037	0.007
	(2, L)	0.016	0.030	0.338	0.092	0.324	0.091	0.121	0.339	0.094
	(3, L)	0.000	0.001	0.122	0.004	0.026	0.403	0.011	0.124	0.399

Table 2.37: Variation of Steady State the System with Different J

Number of Active Firms	States of the System	Steady State (Experiment 3)			Steady State (Baseline)			Steady State (Experiment 4)		
		Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
One	(0, H)	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	(1, H)	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471
	(0, L)	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
	(1, L)	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471	0.471
Two	(0, H)	0.062	0.051	0.002	0.021	0.004	0.002	0.014	0.002	0.002
	(1, H)	0.432	0.439	0.055	0.376	0.090	0.055	0.277	0.055	0.055
	(2, H)	0.006	0.010	0.443	0.103	0.406	0.443	0.209	0.443	0.443
	(0, L)	0.074	0.062	0.002	0.021	0.006	0.002	0.015	0.002	0.002
	(1, L)	0.423	0.432	0.055	0.377	0.130	0.055	0.289	0.055	0.055
	(2, L)	0.003	0.006	0.443	0.102	0.364	0.443	0.196	0.443	0.443
Three	(0, H)	0.076	0.062	0.001	0.023	0.006	0.000	0.013	0.000	0.000
	(1, H)	0.419	0.429	0.046	0.380	0.124	0.008	0.242	0.013	0.009
	(2, H)	0.005	0.009	0.370	0.093	0.341	0.105	0.220	0.143	0.114
	(3, H)	0.000	0.000	0.083	0.004	0.029	0.386	0.025	0.344	0.376
	(0, L)	0.088	0.074	0.002	0.023	0.007	0.000	0.013	0.000	0.000
	(1, L)	0.409	0.420	0.051	0.381	0.143	0.006	0.254	0.017	0.007
	(2, L)	0.003	0.006	0.376	0.092	0.324	0.091	0.208	0.182	0.095
	(3, L)	0.000	0.000	0.072	0.004	0.026	0.403	0.024	0.301	0.398

Table 2.38: Variation of Steady States the System with Different P and Q

Number of Active Firms	States of the System	Steady State (Baseline)			Steady State (Experiment 5)			Steady State (Experiment 6)		
		Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
One	(0, H)	0.029	0.029	0.029	0.033	0.033	0.033	0.079	0.079	0.079
	(1, H)	0.471	0.471	0.471	0.467	0.467	0.467	0.421	0.421	0.421
	(0, L)	0.029	0.029	0.029	0.033	0.033	0.033	0.079	0.079	0.079
	(1, L)	0.471	0.471	0.471	0.467	0.467	0.467	0.421	0.421	0.421
Two	(0, H)	0.021	0.004	0.002	0.022	0.006	0.002	0.064	0.032	0.012
	(1, H)	0.376	0.090	0.055	0.383	0.132	0.062	0.350	0.223	0.133
	(2, H)	0.103	0.406	0.443	0.095	0.361	0.436	0.085	0.244	0.355
	(0, L)	0.021	0.006	0.002	0.022	0.008	0.002	0.066	0.041	0.012
	(1, L)	0.377	0.130	0.055	0.384	0.167	0.062	0.351	0.262	0.133
	(2, L)	0.102	0.364	0.443	0.094	0.325	0.436	0.083	0.197	0.355
Three	(0, H)	0.023	0.006	0.000	0.024	0.009	0.000	0.073	0.042	0.002
	(1, H)	0.380	0.124	0.008	0.386	0.172	0.009	0.347	0.252	0.035
	(2, H)	0.093	0.341	0.105	0.086	0.294	0.107	0.074	0.186	0.175
	(3, H)	0.004	0.029	0.386	0.003	0.024	0.384	0.006	0.020	0.288
	(0, L)	0.023	0.007	0.000	0.024	0.011	0.000	0.075	0.049	0.002
	(1, L)	0.381	0.143	0.006	0.387	0.198	0.007	0.348	0.278	0.033
	(2, L)	0.092	0.324	0.091	0.085	0.269	0.097	0.072	0.156	0.171
	(3, L)	0.004	0.026	0.403	0.003	0.022	0.395	0.005	0.016	0.294

Table 2.39: Variation of Steady States the System with Different h and l

Number of Active Firms	States of the System	Steady State (Baseline)			Steady State (Experiment 7)			Steady State (Experiment 8)		
		Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum	Bertrand	Cournot	Welfare Maximum
One	(0, H)	0.029	0.029	0.029	0.044	0.044	0.044	0.015	0.015	0.015
	(1, H)	0.471	0.471	0.471	0.706	0.706	0.706	0.235	0.235	0.235
	(0, L)	0.029	0.029	0.029	0.015	0.015	0.015	0.044	0.044	0.044
	(1, L)	0.471	0.471	0.471	0.235	0.235	0.235	0.706	0.706	0.706
Two	(0, H)	0.021	0.004	0.002	0.028	0.004	0.003	0.012	0.005	0.001
	(1, H)	0.376	0.090	0.055	0.537	0.101	0.083	0.203	0.092	0.028
	(2, H)	0.103	0.406	0.443	0.184	0.646	0.664	0.035	0.153	0.221
	(0, L)	0.021	0.006	0.002	0.010	0.003	0.001	0.037	0.021	0.003
	(1, L)	0.377	0.130	0.055	0.179	0.064	0.028	0.617	0.395	0.083
	(2, L)	0.102	0.364	0.443	0.061	0.183	0.221	0.096	0.334	0.664
Three	(0, H)	0.023	0.006	0.000	0.030	0.003	0.000	0.013	0.007	0.000
	(1, H)	0.380	0.124	0.008	0.542	0.084	0.007	0.204	0.119	0.002
	(2, H)	0.093	0.341	0.105	0.167	0.554	0.117	0.032	0.116	0.039
	(3, H)	0.004	0.029	0.386	0.011	0.109	0.625	0.002	0.009	0.208
	(0, L)	0.023	0.007	0.000	0.010	0.002	0.000	0.042	0.026	0.000
	(1, L)	0.381	0.143	0.006	0.181	0.039	0.002	0.618	0.439	0.007
	(2, L)	0.092	0.324	0.091	0.055	0.179	0.039	0.087	0.267	0.117
	(3, L)	0.004	0.026	0.403	0.003	0.030	0.208	0.003	0.018	0.625

In table 2.40, we hold all the baseline parameters constant and only change the value of investment I. In the Bertrand and Cournot competitions, both efficiency values increase with lower values of investment costs for two and three active firms' cases. With highest investment cost in experiment 1, one active firm case is optimal in terms of efficiency in both competitions compared to the other two cases. When the investment cost decreases to 700, the two active firms' case has highest efficiency value. In addition, when the investment cost decreases to 250, the three active firms' case becomes to the optimal for both Bertrand and Cournot. Therefore, we conclude that the lower investment costs, the higher number of active firms become optimal in terms of efficiency.

Table 2.40: Increase or Decrease Parameter Value I

Number of Active Firms	Efficiency (Experiment 1)		Efficiency (Baseline)		Efficiency (Experiment 2)	
	Bertrand	Cournot	Bertrand	Cournot	Bertrand	Cournot
1	0.68	0.68	0.68	0.68	0.68	0.68
2	0.64	0.67	0.75	0.84	0.76	0.86
3	0.61	0.65	0.74	0.83	0.77	0.88

Similarly, we change the investment levels of J by holding all the other parameters constant; we obtain the same patterns as we change of the investment level of I. In table 2.41, we find the efficiency is highest for both Bertrand and Cournot in the one active firm' case with $J = 500$ and the efficiency is highest in three active firms' case with $J = 50$.

Table 2.41: Increase or Decrease Parameter Value J

Number of Active Firms	Efficiency (Experiment 3)		Efficiency (Baseline)		Efficiency (Experiment 4)	
	Bertrand	Cournot	Bertrand	Cournot	Bertrand	Cournot
1	0.68	0.68	0.68	0.68	0.68	0.68
2	0.64	0.66	0.75	0.84	0.81	0.86
3	0.62	0.64	0.74	0.83	0.84	0.91

If we decrease the probability of establishing a firm in the market P from 0.8 to 0.7, the Bertrand and Cournot efficiency for the one, two, and three active firms' cases all decrease.

Similar results can be obtained if we decrease probability of maintenance of a firm in the market Q from 0.95 to 0.85.

Table 2.42: Decrease Successful Probability of Creating and Maintaining Investment P and Q

Number of Active Firms	Efficiency (Baseline)		Efficiency (Experiment 5)		Efficiency (Experiment 6)	
	Bertrand	Cournot	Bertrand	Cournot	Bertrand	Cournot
1	0.68	0.68	0.67	0.67	0.62	0.62
2	0.75	0.84	0.74	0.82	0.68	0.75
3	0.74	0.83	0.73	0.81	0.66	0.73

In table 2.43, we show that the increase probability of high demand from 0.7 to 0.9. The efficiency levels for the two, and three active firms' cases increase. In contrast,the efficiency levels for these cases decrease if the probability of low demand increases from 0.7 to 0.9.

Table 2.43: Increase Probability of High h and Low l Market Demand

Number of Active Firms	Efficiency (Baseline)		Efficiency (Experiment 7)		Efficiency (Experiment 8)	
	Bertrand	Cournot	Bertrand	Cournot	Bertrand	Cournot
1	0.68	0.68	0.68	0.68	0.68	0.68
2	0.75	0.84	0.76	0.85	0.73	0.78
3	0.74	0.83	0.75	0.86	0.72	0.77

Finally, we discuss the effects of parameter variations on the minimum and maximum speed of convergence to the steady state of the system with the one, two, and three firms' case in the Bertrand, Cournot, and welfare maximum frameworks. Table 2.5 to 2.8 compare the 8 experiments mentioned above to the baseline parameter value in the Bertrand competition. In table 2.5, we find the higher value of the investment costs, I, the lower minimum speed of convergence for the two and three firms' cases. In contrast, the higher value of I has a higher maximum speed of convergence for the two and three firms' cases. By changing the value of maintenance costs, J, the maximum and minimum speed of convergence both have a faster speed with higher J. In table 2.7, we find that the variation of the successful value of probability investment, P, does not effect the convergence speed; however, the decrease of the successful value of probability maintenance, Q, has a faster minimum and maximum speed of convergence in both two and three firms' cases. Moreover, the change of market demand type has an almost identical impact on minimum speed of convergence for the two and three firms' cases. Table 2.9 and 2.10 show how the minimum and maximum speed of convergence vary with the change of I and J in the Cournot competition. We find that the minimum speed of convergence increases with lower costs and decreases with higher costs in the two and three firms' cases. However, the variation in costs does not have a significant impact on the maximum speed of convergence. In table 2.11, we present that the minimum speed of convergence increases with lower probability Q in the two and three firms' cases. The maximum speed of convergence does not affect by the variation of Q and P. Meanwhile, the variation of h and l does not change the minimum and maximum speed of convergence much for the one, two, and three firms' cases. In table 2.13 and 2.14, we discover that the minimum and maximum speed of convergence decrease with higher costs in the three firms' case and the costs do not affected by the convergence speed for the one and two firms' cases in the welfare maximum framework. With the variation of successful probability of investment and probability of market demand type, the minimum and maximum speed of convergence only sightly change. To see more details, we provide figures in the Appendix.

2.3.8 Real Efficiency VS. Blind Efficiency

In this section, we want to answer the question that what sort of errors are made if regulatory authorities do not see the dynamic properties of the real model. The blind and real efficiency are calculated as below:

$$BE_{ix} = \frac{TS_{ix} - rI - J}{TS_{iW} - rI - J} \qquad x = \text{Bertrand, Cournot} \tag{2.17}$$

$$RE_{ix} = \frac{RTS_{ix}}{RTS_{iW}} = \frac{\sum_{i=1}^{S} Pr_{it} * TS_{ix} * D^t \ - \sum_{i=1}^{N} Pr_{it} * I * D^t \ - \sum_{i=N}^{S} Pr_{Nt} * J * D^t}{\sum_{i=1}^{S} Pr_{it} * TS_{iW} * D^t - \sum_{i=1}^{N} Pr_{it} * I * D^t \ - \sum_{i=N}^{S} Pr_{Nt} * J * D^t} \tag{2.18}$$

TS_{ix} is the expected discounted total surplus in each state of the system for Bertrand or Cournot and TS_{iW} is welfare maximum surplus for each state. BE_x represents the blind efficiency for Bertrand or Cournot in each state. Meanwhile, RE_{ix} is the real efficiency for either Bertrand or Cournot in each state. RTS_{ix} is the real total surplus in the Bertrand or Cournot competition for each state and RTS_{iW} is the welfare maximum surplus for each state. For state i to N, firms are unestablished. For state N to S, all firms are established. In table 2.44, 2.45, and 2.46, we list the real and blind efficiency for the one, two, and three firms' cases. For the one active firm's case, the blind efficiency has a lower estimation of real efficiency in state $(1, L)$ and a higher estimation in state $(1, H)$ in the Bertrand and Cournot framework. For the two active firms' case, the blind efficiency has a lower estimation of real efficiency in state $(1, L)$ for Bertrand and Cournot and a higher estimation of real efficiency in state $(2, H)$ and $(2, L)$ for Bertrand and Cournot. Similarly, the blind efficiency has a lower estimation of real efficiency in state $(1, L)$ and a higher estimation of real efficiency in state $(2, H)$, $(3, H)$, $(2, L)$ and $(3, L)$ for Bertrand and Cournot in the three active firms' case. Therefore, the blind regulatory authority always provides bias estimations of efficiency because the authority ignores the stochastic properties of positional investments and dynamic properties of the model.

Table 2.44: Real Efficiency VS. Blind Efficiency for Each State of the System (One Active Firm)

States of System	Real Efficiency (Bertrand)	Real Efficiency (Cournot)	Blind Efficiency (Bertrand)	Blind Efficiency (Cournot)
(0,H)				
(1,H)	0.697	0.697	0.704	0.704
(0,L)				
(1,L)	0.694	0.694	0.638	0.638

Table 2.45: Real Efficiency VS. Blind Efficiency for Each State of the System (Two Active Firms)

States of System	Real Efficiency (Bertrand)	Real Efficiency (Cournot)	Blind Efficiency (Bertrand)	Blind Efficiency (Cournot)
(0,H)				
(1,H)	0.669	0.827	0.704	0.704
(2,H)	0.829	0.831	0.997	0.865
(0,L)				
(1,L)	0.667	0.779	0.638	0.638
(2,L)	0.819	0.827	0.997	0.836

Table 2.46: Real Efficiency VS. Blind Efficiency for Each State of the System (Three Active Firms)

States of System	Real Efficiency (Bertrand)	Real Efficiency (Cournot)	Blind Efficiency (Bertrand)	Blind Efficiency (Cournot)
(0,H)				
(1,H)	0.651	0.780	0.704	0.704
(2,H)	0.814	0.809	0.997	0.865
(3,H)	0.800	0.855	0.999	0.932
(0,L)				
(1,L)	0.649	0.757	0.638	0.638
(2,L)	0.803	0.804	0.997	0.836
(3,L)	0.789	0.851	0.997	0.905

2.4 Conclusion

This paper extends the model developed in Bloch et al. (2014) with two significant modifications. First, we assume there are two types of demand a niche market. Second, we consider the number of active firms is exogenous; and the numbers are one, two, or three active firms in a market, respectively. In each market niche, firms compete in the Bertrand or Cournot framework, as well as a welfare maximum alternative as a reference within a period. Therefore, the equilibrium price, quantity, profits, and consumer surplus in this period can be obtained. Second, firms decide whether to make positioning investments when they face uncertain market demand. The positioning investments are stochastic therefore; they involve the costs to establish a firm and its probability to success, and the costs to maintenance a firm and its probability to success. Then, we link these two sorts of decisions by the firm through static operating profit of established firms. By employing (SMPNE), we obtain a serial of results with dynamic properties.

First, we present a firm profit, consumer surplus, and total surplus in the Bertrand, Cournot, or welfare maximum framework with different number active firms and types of demand. We find the Bertrand competition has lower profit for a firm in the two or three active firms case and has higher total surplus than Cournot in these two cases.

Second, we show the transition probabilities in the different competitions with different initial states of the system. We discover that the transition probabilities become static in the welfare maximum framework for the first ten periods; however, they keep changing in the Bertrand and Cournot frameworks for the first ten periods.

Third, we find the steady state of the system in the Bertrand, Cournot, and welfare maximum frameworks with different number of active firms. We observe the number of firms is larger in a niche market under the Cournot competition than in Bertrand. This is because the profit for a firm is much higher in Cournot than Bertrand. Therefore, it is more risky for a firm to enter into the market under the Bertrand competition. Moreover, we also calculate the expected duration of the state of the system. In the Bertrand framework, the value of expected duration to stay in state (1, H) is higher than in the Cournot and welfare maximum frameworks for the two active firms case. The value of expected duration is lower in the state (1, L) under the welfare maximum framework than Bertrand and Cournot.

Fourth, we show the speed of convergence in the Bertrand, Cournot, and welfare maximum frameworks. The maximum and minimum speed of convergence is a function of initial states of the system and period of time. We conclude that the maximum speed of convergence meets the convergence criteria (0.01) only in the welfare maximum framework within the first ten periods for one, two, or three active firms case, respectively.

Fifth, we calculate the expected total surplus for Bertrand, Cournot, and welfare maximum. Thus, we can have value of efficiency for Bertrand and Cournot because we set total surplus in the welfare maximum framework as a reference. The efficiency level in Cournot is higher than Bertrand. This is because there is a significant probability difference in the state of the system between Bertrand and welfare maximum based on the analysis of Oaxaca decomposition.

Sixth, we developed 8 different experiments and compare the results of the steady state of the system, efficiency, and the speed of convergence to the steady state to the baseline model. We discover that the higher investment costs reduce numbers of firms into the market. Similarly, the lower probabilities of successful investment also have the same result. Moreover, the efficiency level in the two and three active firms cases increases with lower investment and higher probability of successful investment, as well as higher probability of high demand. In addition, we also present that the variation in I, J, P, and Q have dramatic different effect on the minimum and maximum speed of convergence in the Bertrand, Cournot, and welfare maximum frameworks.

Finally, we illustrate that the important features of the dynamic and stochastic. Suppose the regulate authorities ignore these properties, we have blind efficiency which is biased estimation of efficiency in the one, two, and three active firms cases. The value of real efficiency in the Cournot competition is always higher than in the Bertrands and the blind efficiency has an over estimation in the state of one firm with high demand.

2.5 Appendices

I. Within Period Oligopoly

If firms choose quantities as in Cournot's model, using the inverse demand function from equation 2.2, the profit that a representative firm, firm 1 for simplicity, can be written as

$$\pi_C = q_1(\alpha_d - \beta q_1 - \gamma(q_2 + ... + q_N))$$

where N is the number of established firms. The first order profit maximizing condition is

$$\alpha_d - 2\beta q_1 - \gamma(q_2 + ... + q_N) = 0$$

Of course, in equilibrium, all the qs are the same, so the equilibrium quantity sold by firm 1 to a representative consumer is $\alpha/(2\beta + (N-1)\gamma)$, and the total quantity sold by a representative firm in the Cournot equilibrium, denoted by $q^*(N)$ is

$$q_C^*(N) = \frac{\alpha_d}{2\beta + (N-1)\gamma}$$

For the Cournot case the function $R(N)$ that is as the oligopoly profit of a representative firm in equilibrium, is

$$R_C(N) = \beta(\frac{\alpha_d}{2\beta + (N-1)\gamma})^2$$

Now let us turn to the case where firms choose prices, as in Bertrand's model. Inverting the inverse demand functions in equation (2), we get the following demand functions:

$$q_i = A_N - B_N p_i + C_N \sum_{j \neq i} p_j, i = 1, N$$

where A_N, B_N and C_N are parameters that are defined in the following table:

N	A_N	B_N	C_N
1	$\frac{\alpha_d}{\beta}$	$\frac{1}{\beta}$	
2	$\frac{\alpha_d(\beta-\gamma)}{\beta^2-\gamma^2}$	$\frac{\beta}{\beta^2-\gamma^2}$	$\frac{\gamma}{\beta^2-\gamma^2}$
3	$\frac{\alpha_d(\beta-\gamma)}{\beta^2+\beta\gamma-2\gamma^2}$	$\frac{\beta+\gamma}{\beta^2+\beta\gamma-2\gamma^2}$	$\frac{\gamma}{\beta^2+\beta\gamma-2\gamma^2}$

Profit of a representative firm, again firm 1 for simplicity, is

$$\pi_B = p_1(A_N - B_N p_1 + C_N \sum_{j\neq 1} p_j)$$

The first order profit maximizing condition is

$$A_N - B_N p_1 + C_N \sum_{j\neq 1} p_j = 0$$

It is straight forward to find the equilibrium price, denoted by $p_B^*(N)$, and the oligopoly profit of a representative firm in equilibrium, $R_B(N)$. They are recorded in the following table.

N	$p_B^*(N)$	$R_B(N)$
1	$\frac{\alpha_d}{2}$	$\frac{\alpha_d^2}{4\beta}$
2	$\frac{\alpha_d(\beta-\gamma)}{2\beta-\gamma}$	$\frac{\alpha_d^2\beta(\beta-\gamma)}{(\beta+\gamma)(2\beta-\gamma)^2}$
3	$\frac{\alpha_d(\beta-\gamma)}{2\beta}$	$\frac{\alpha_d^2(\beta+\gamma)(\beta-\gamma)}{(\beta+2\gamma)(2\beta)^2}$

Suppose firm 1 chooses price to maximize consumer surplus in its niche market.

$$CS_1 = (p_1 - mc)q_1 + \frac{1}{2}\left(\frac{A_N + C_N \sum_{j\neq 1} p_j}{B_N} - p_1\right)q_1$$

The first order consumer surplus maximizing condition is

$$-B_N(\frac{1}{2}p_1 - mc + \frac{A_N + C_N\sum_{j\neq 1} p_j}{2B_N}) + \frac{1}{2}q_1 = 0$$

It is easy to obtain the equilibrium quantity, denoted by $q^*_W(N)$, and the total surplus for each market niche, $TS(N)$ equals to consumer surplus because equilibrium price equals to marginal cost in this case. We report equilibrium quantity and total surplus as follows:

N	$q^*_W(N)$	$TS_i(N)$
1	$\frac{\alpha_d}{\beta}$	$\frac{\alpha_d^2}{2\beta}$
2	$\frac{\alpha_d}{\beta+\gamma}$	$\frac{\alpha_d^2}{2(\beta+\gamma)}$
3	$\frac{\alpha_d}{\beta+2\gamma}$	$\frac{\alpha_d^2}{2(\beta+2\gamma)}$

II Algorithm

In order to obtain symmetric Markov perfect Nash equilibrium strategy, we use the following three step producers.

First, we provide the initial value of Markovian strategy $S = (s^1, s^2, ..., s^{4A})$ as well as the parameter value of exogenous variables A, I, J, P, Q, H, L, D, $R(N,d)$, $SP(N,d)$.

Second, suppose we set $\widehat{S}_R = S, S_R = S$ and $S_O = S$ in the system $4A$ value functions 2.6 and 2.9, we will be able to obtain $V^k((S,S),S), k = 1, 4A$. Then, we use these values to calculate s^k and ns^k in the following three criteria:

(1) If $V^k(((1,S_{-k}),S),S) > V^k(((0,S_{-k}),S),S)$,

then $ns^k = \min(1, s^k + \epsilon(V^k(((1,S_{-k}),S),S) - V^k(((0,S_{-k}),S),S)))$.

(2) If $V^k(((1,S_{-k}),S),S) < V^k(((0,S_{-k}),S),S)$,

then $ns^k = \max(0, s^k - \epsilon(V^k(((0,S_{-k}),S),S) - V^k(((1,S_{-k}),S),S)))$.

(3) If $V^k(((1,S_{-k}),S),S) = V^k(((0,S_{-k}),S),S)$,

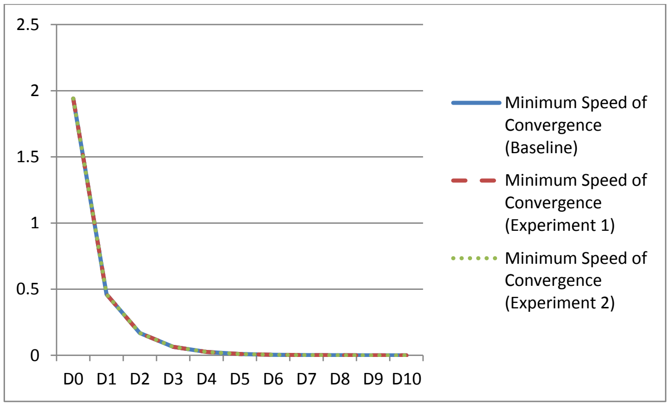
then $ns^k = s^k$. We assume $\epsilon = .000005$ for this exercise.

Third, let us check the condition of convergence as below:

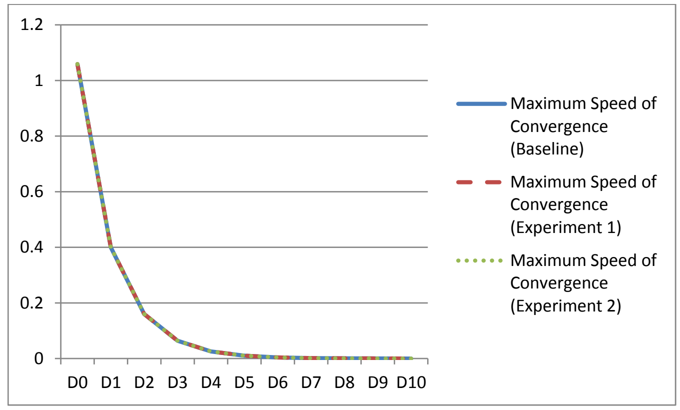
$$\Delta = \sum_{1}^{4A} \left| s^k - ns^k \right|$$

The algorithm is converged to an equilibrium only if $\Delta \leq \delta$. Here, we assume $\delta =$ .0000000001 in practice. Otherwise, we propose S, $(s^1, s^2, ..., s^{2A}) = (ns^1, ns^2, ..., ns^{2A})$, and back to second step until $\Delta \leq \delta$ holds.

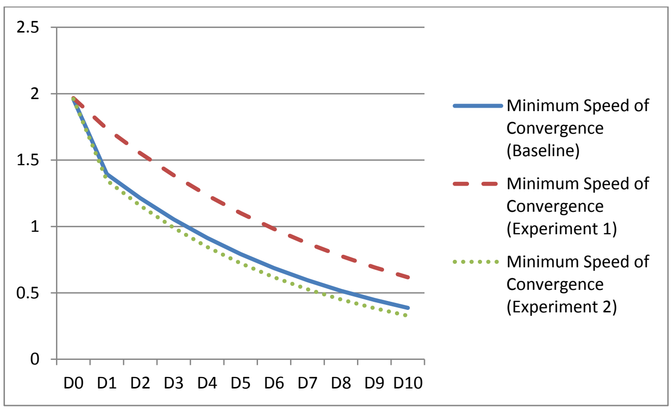
Figure 2.5: The Speed of Convergence with Different I in Bertrand



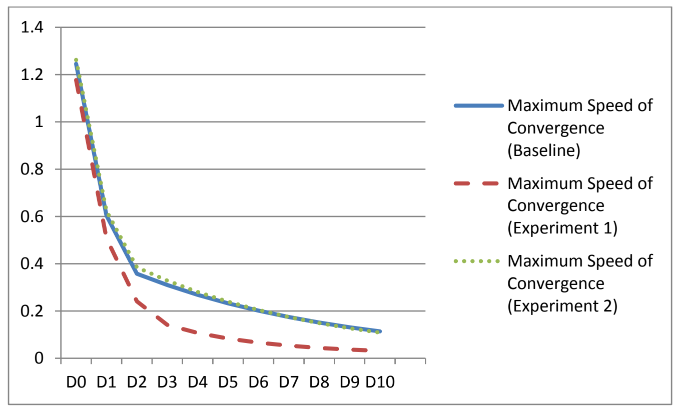
(a) Minimum Speed of Convergence (One Active Firm)



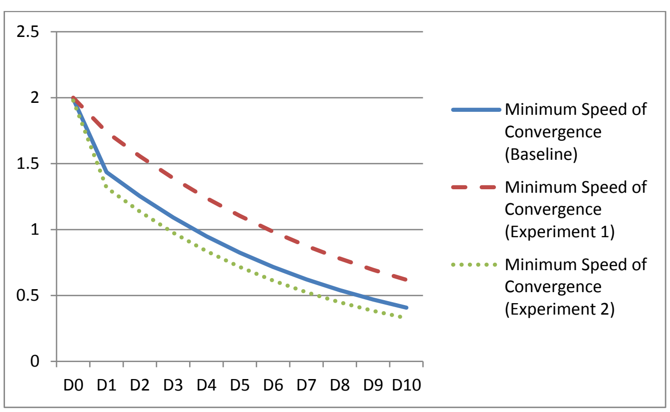
(b) Maximum Speed of Convergence (One Active Firm)



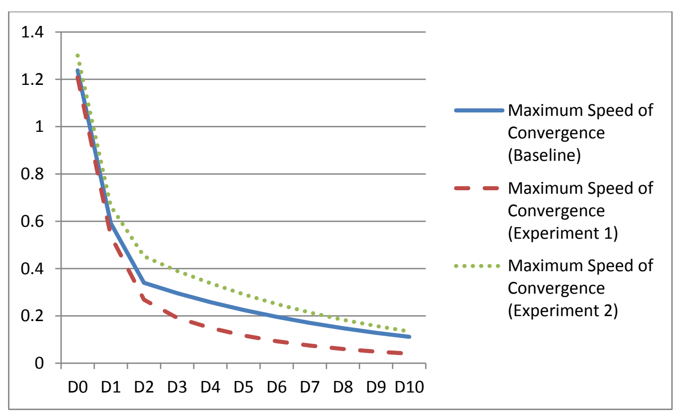
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

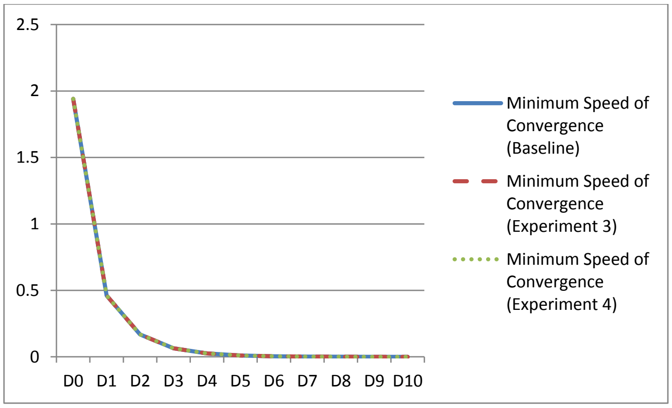


(e) Minimum Speed of Convergence (Three Active Firms)

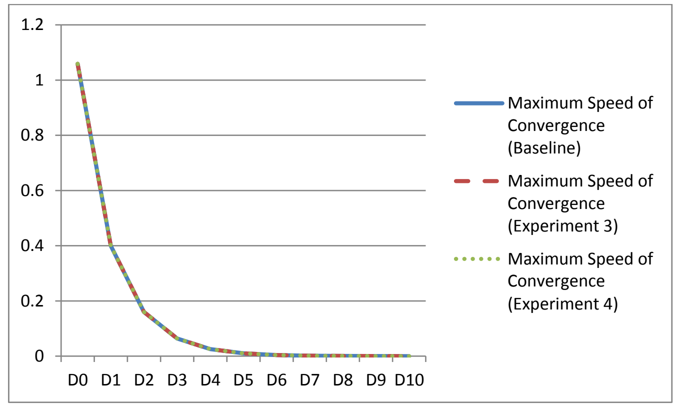


(f) Maximum Speed of Convergence (Three Active Firms)

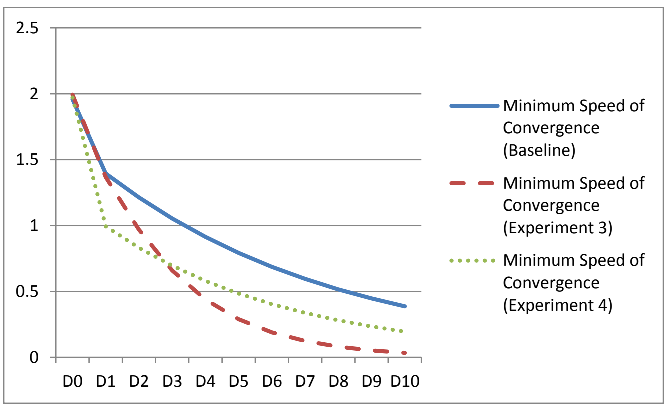
Figure 2.6: The Speed of Convergence with Different J in Bertrand



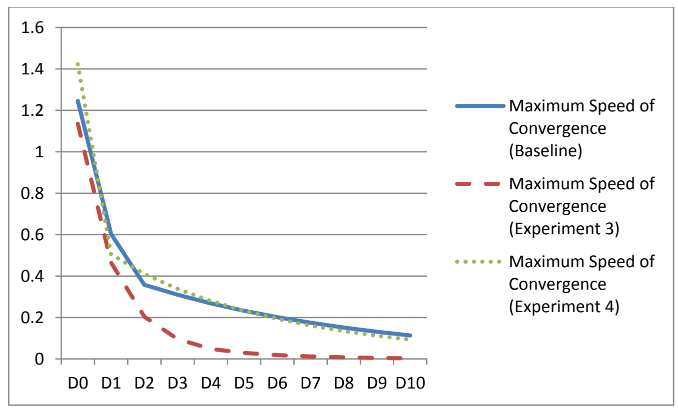
(a) Minimum Speed of Convergence (One Active Firm)



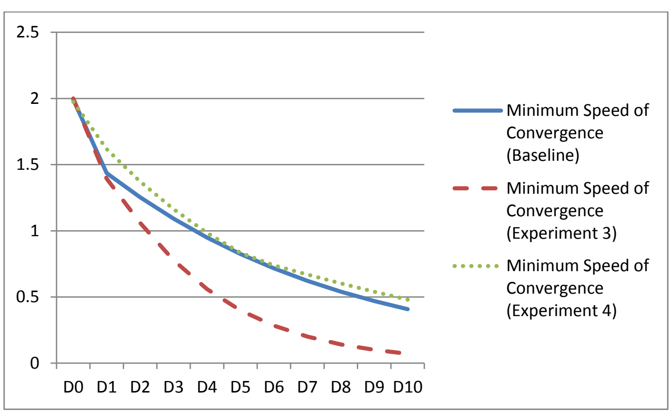
(b) Maximum Speed of Convergence (One Active Firm)



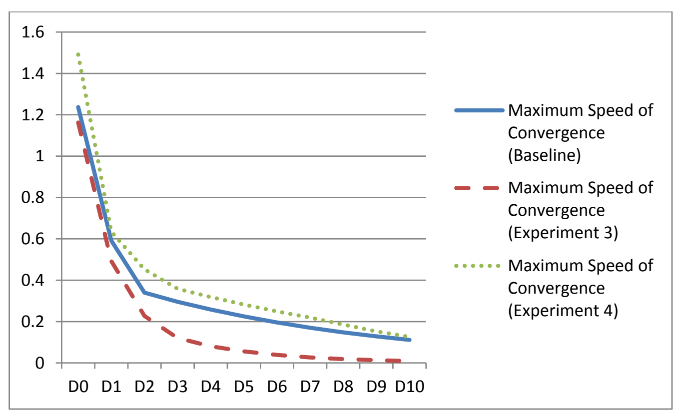
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

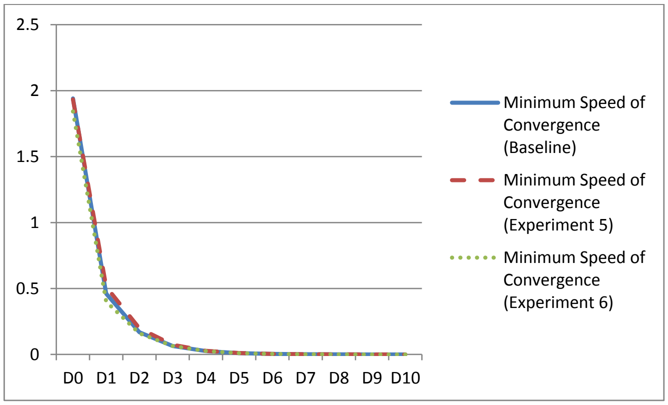


(e) Minimum Speed of Convergence (Three Active Firms)

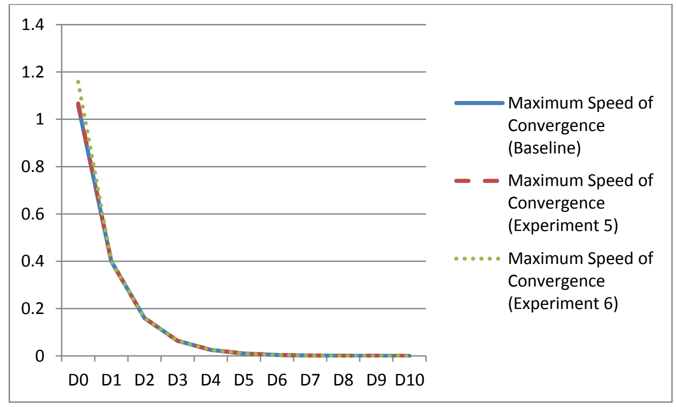


(f) Maximum Speed of Convergence (Three Active Firms)

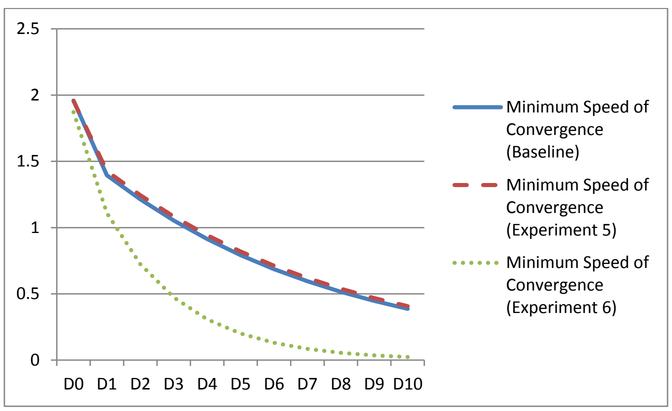
Figure 2.7: The Speed of Convergence with Different P and Q in Bertrand



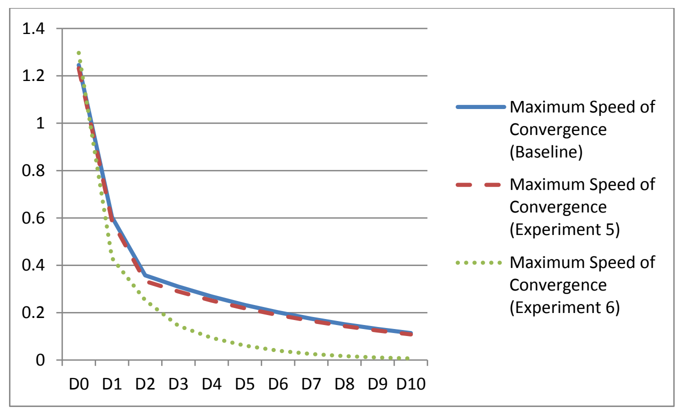
(a) Minimum Speed of Convergence (One Active Firm)



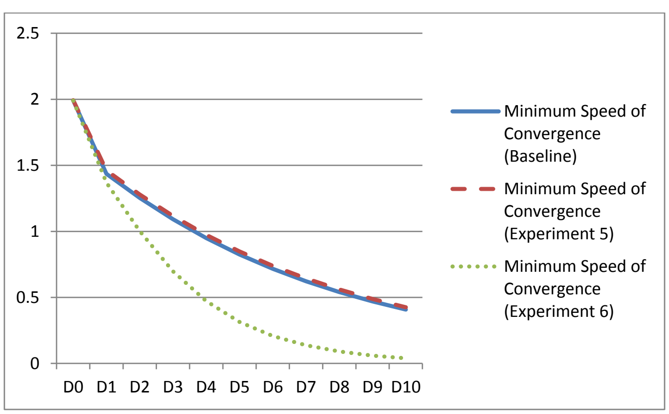
(b) Maximum Speed of Convergence (One Active Firm)



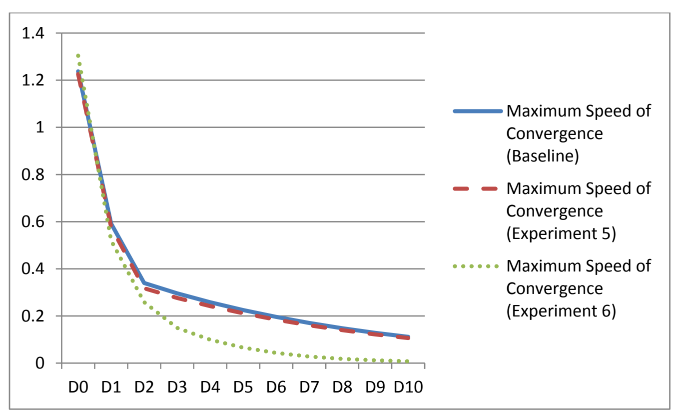
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

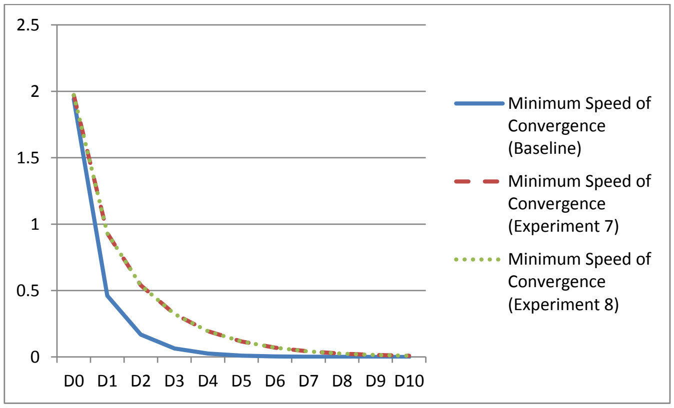


(e) Minimum Speed of Convergence (Three Active Firms)

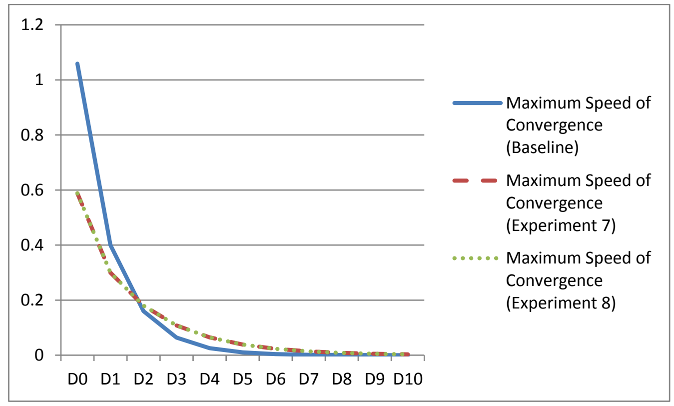


(f) Maximum Speed of Convergence (Three Active Firms)

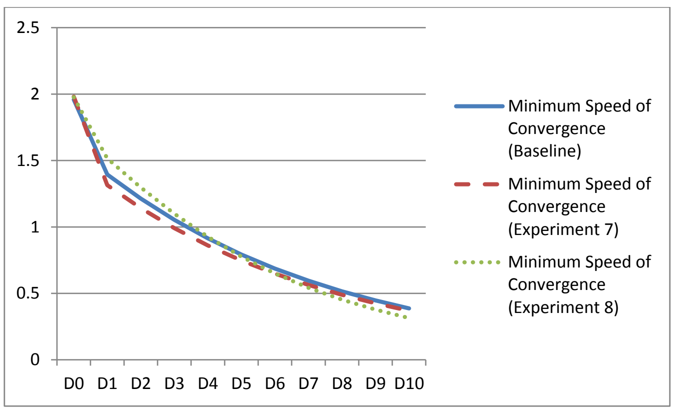
Figure 2.8: The Speed of Convergence with Different h and l in Bertrand



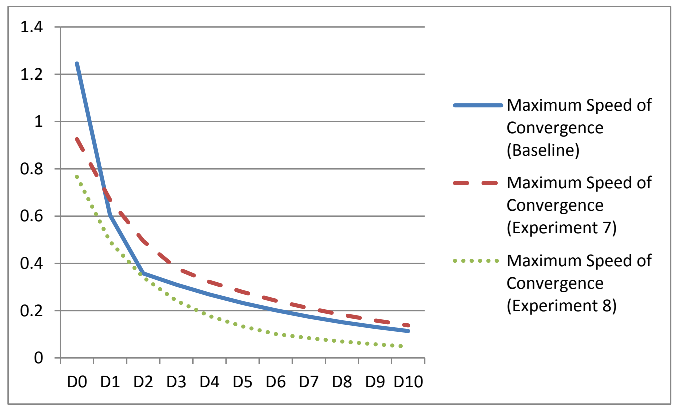
(a) Minimum Speed of Convergence (One Active Firm)



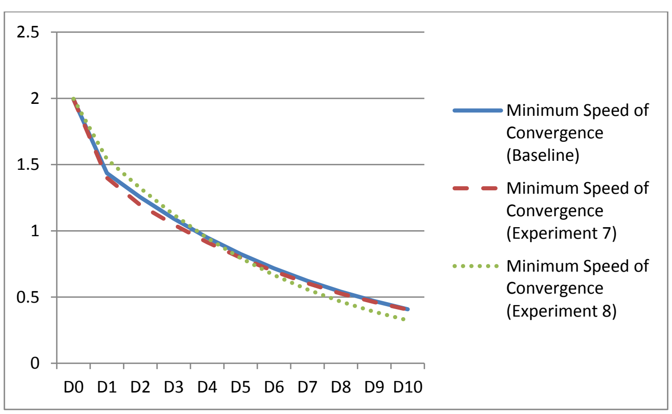
(b) Maximum Speed of Convergence (One Active Firm)



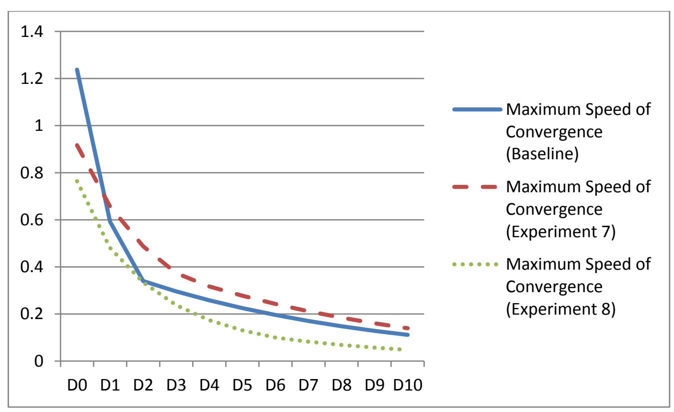
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

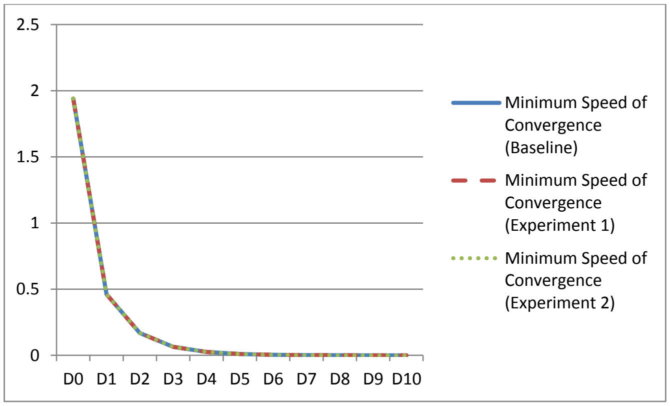


(e) Minimum Speed of Convergence (Three Active Firms)

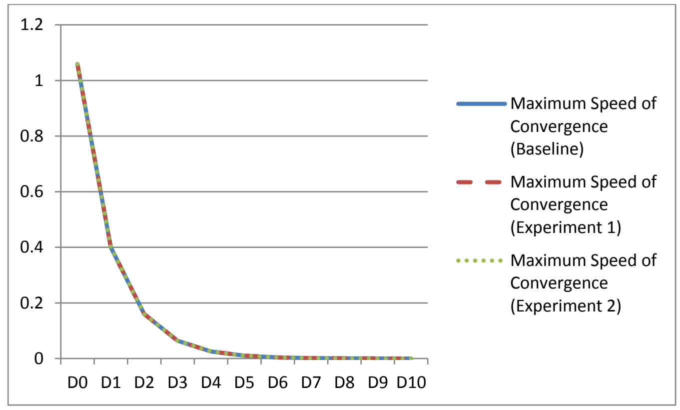


(f) Maximum Speed of Convergence (Three Active Firms)

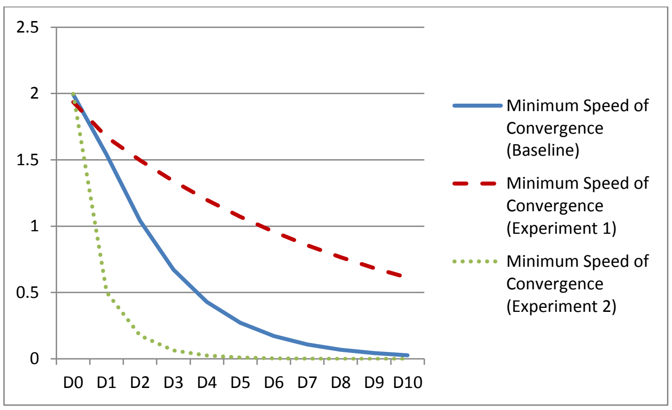
Figure 2.9: The Speed of Convergence with Different I in Cournot



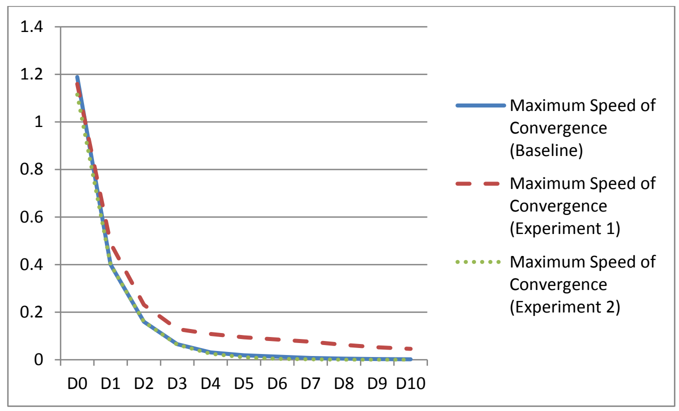
(a) Minimum Speed of Convergence (One Active Firm)



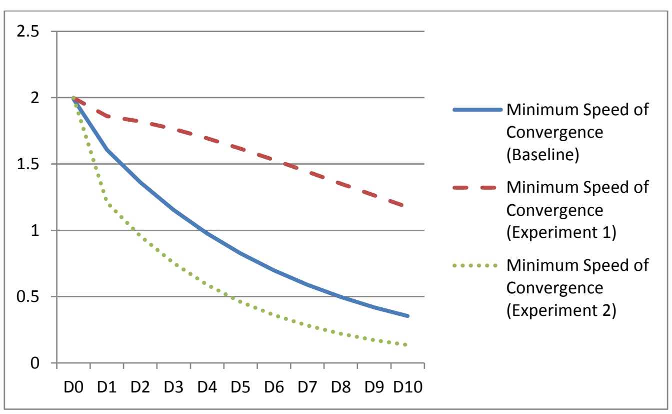
(b) Maximum Speed of Convergence (One Active Firm)



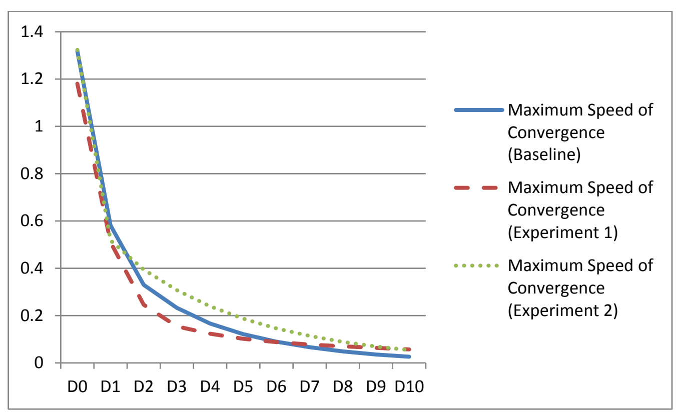
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

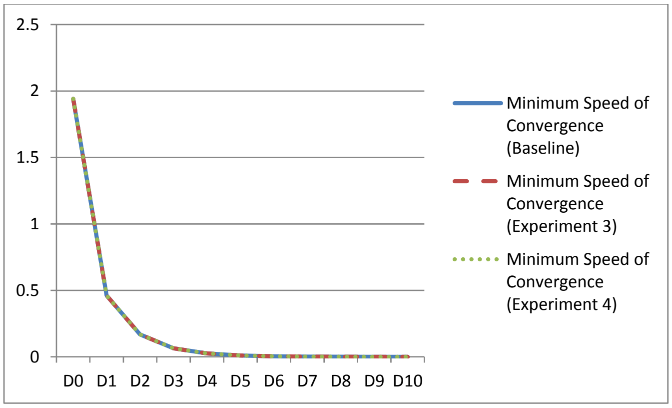


(e) Minimum Speed of Convergence (Three Active Firms)

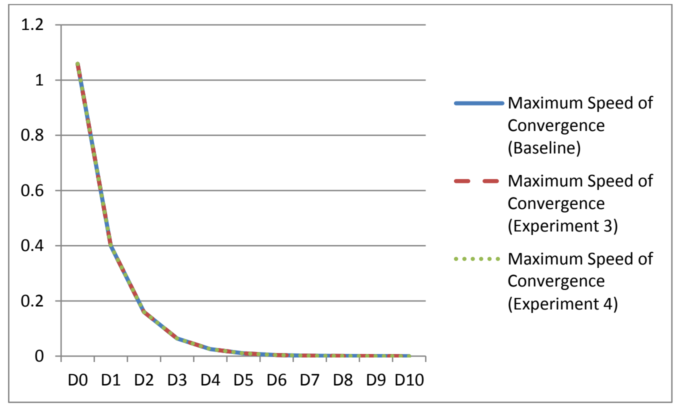


(f) Maximum Speed of Convergence (Three Active Firms)

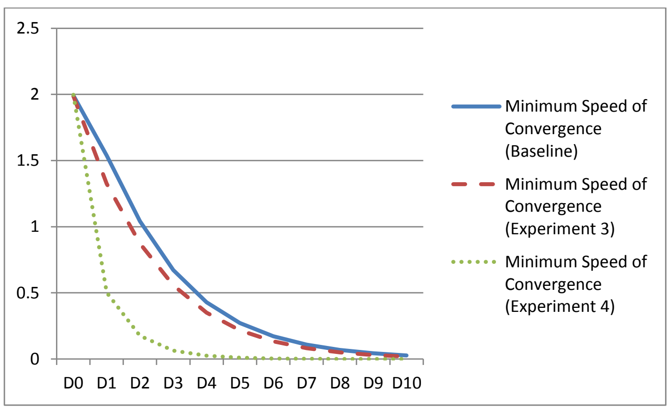
Figure 2.10: The Speed of Convergence with Different J in Cournot



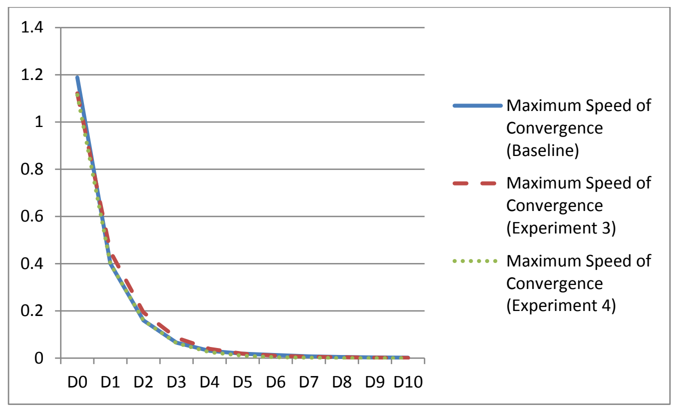
(a) Minimum Speed of Convergence (One Active Firm)



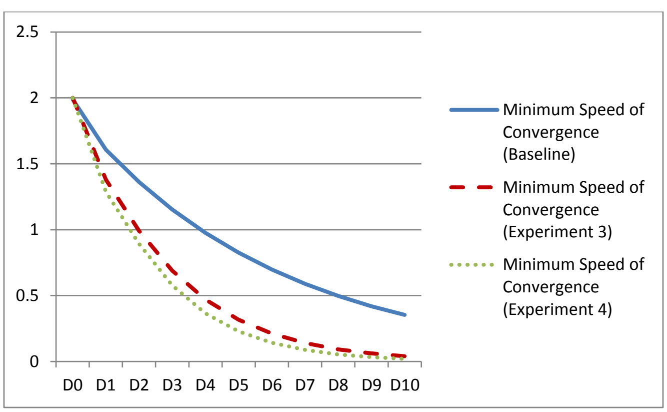
(b) Maximum Speed of Convergence (One Active Firm)



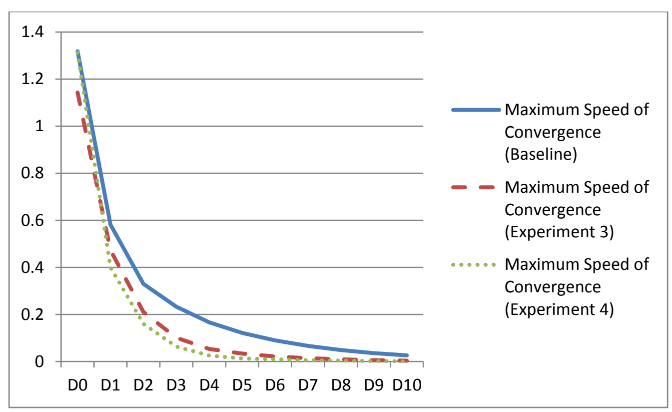
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

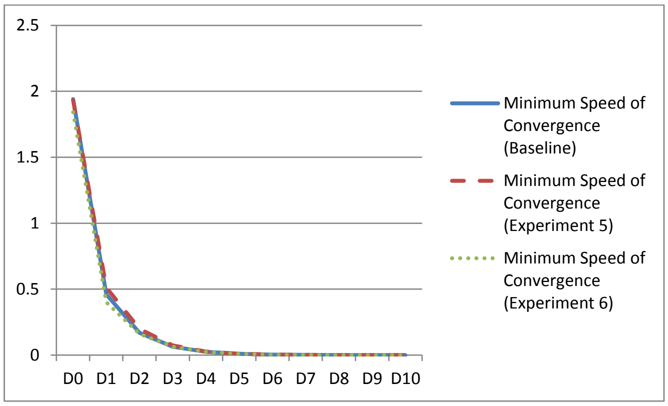


(e) Minimum Speed of Convergence (Three Active Firms)

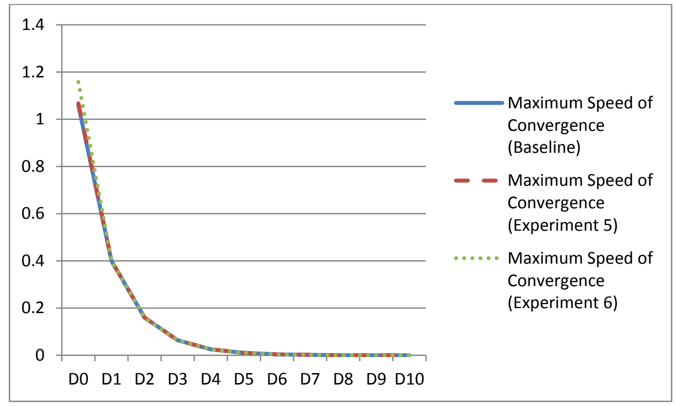


(f) Maximum Speed of Convergence (Three Active Firms)

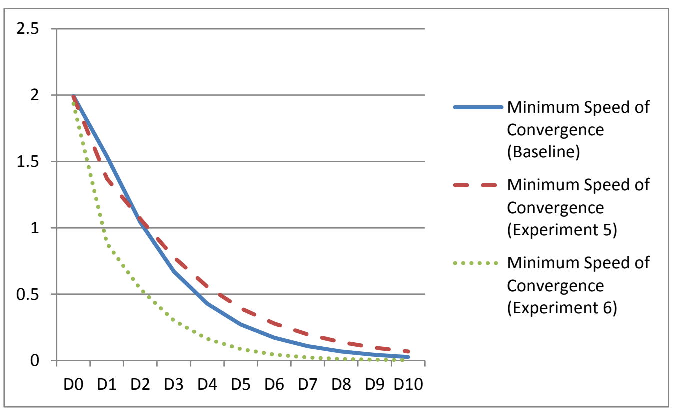
Figure 2.11: The Speed of Convergence with Different P and Q in Cournot



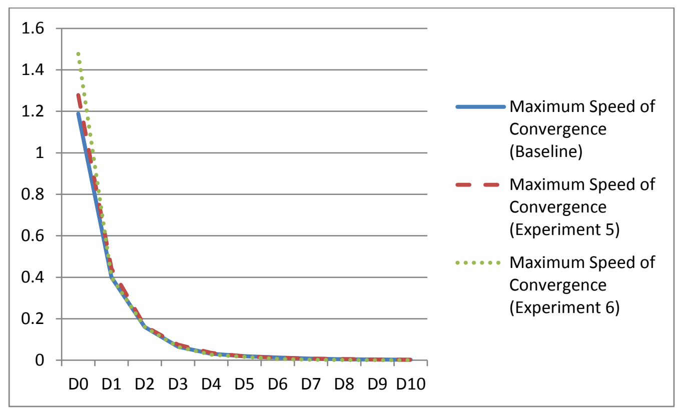
(a) Minimum Speed of Convergence (One Active Firm)



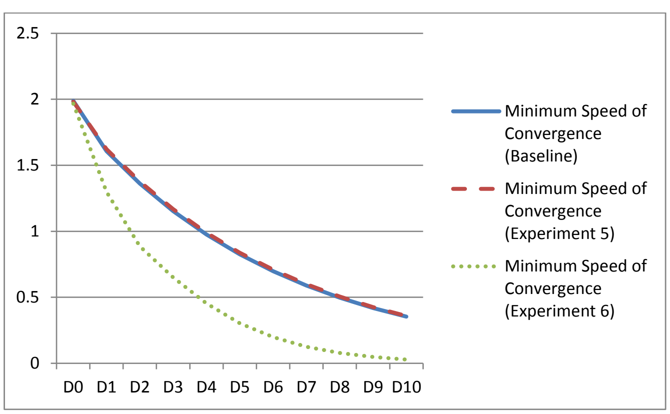
(b) Maximum Speed of Convergence (One Active Firm)



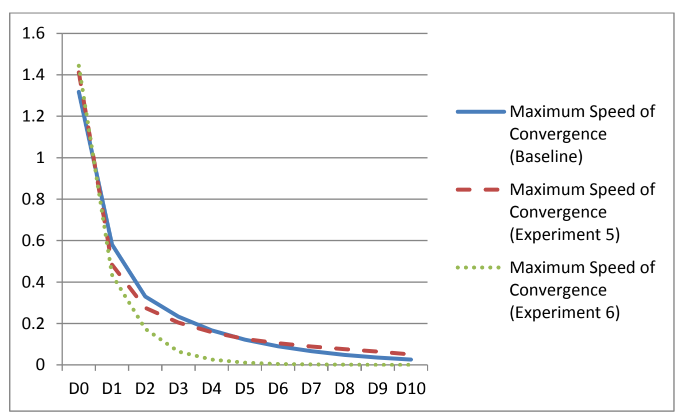
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

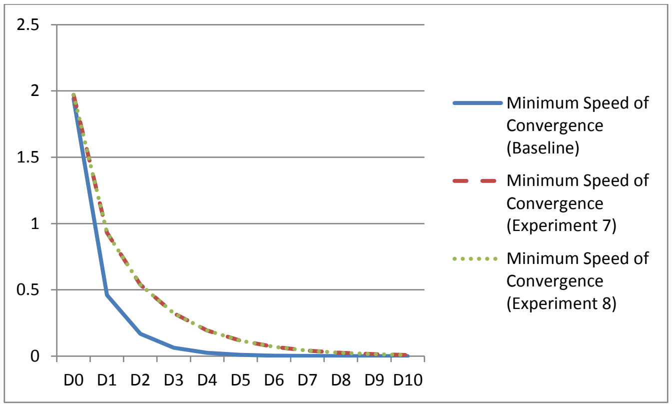


(e) Minimum Speed of Convergence (Three Active Firms)

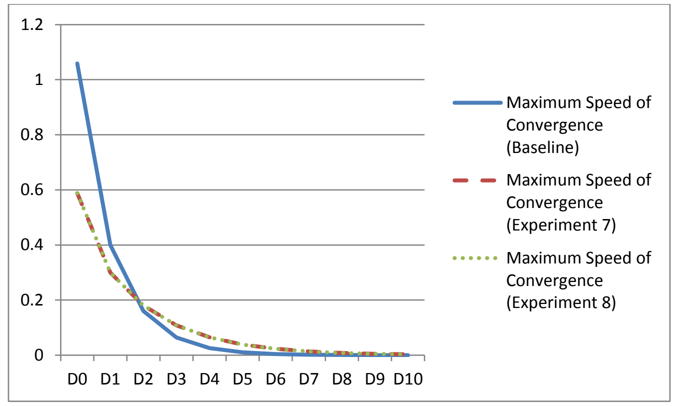


(f) Maximum Speed of Convergence (Three Active Firms)

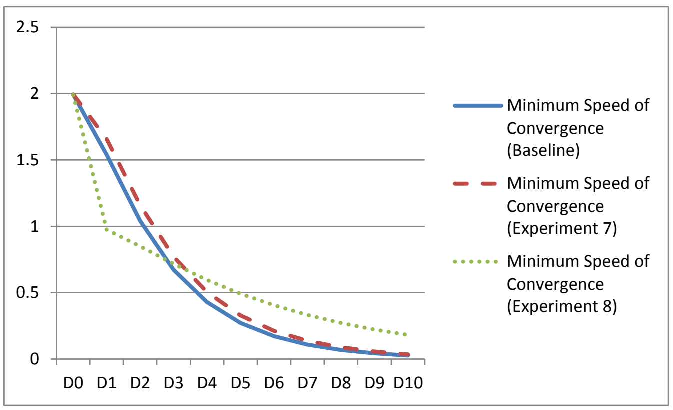
Figure 2.12: The Speed of Convergence with Different h and l in Cournot



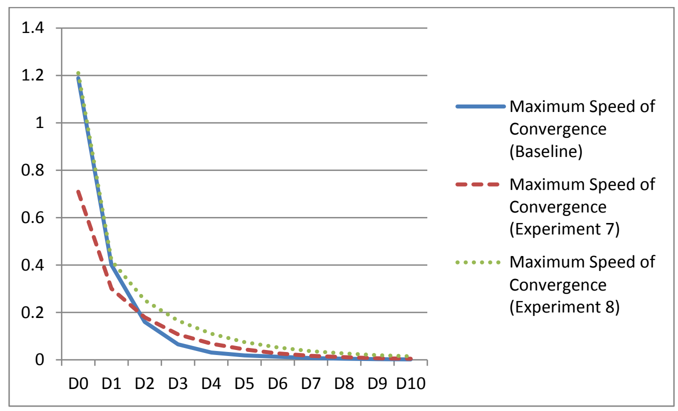
(a) Minimum Speed of Convergence (One Active Firm)



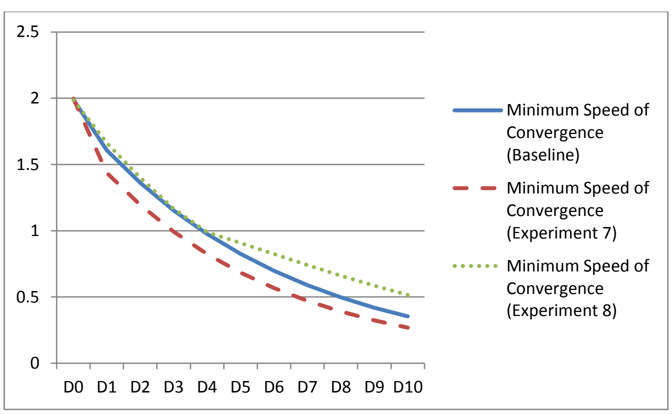
(b) Maximum Speed of Convergence (One Active Firm)



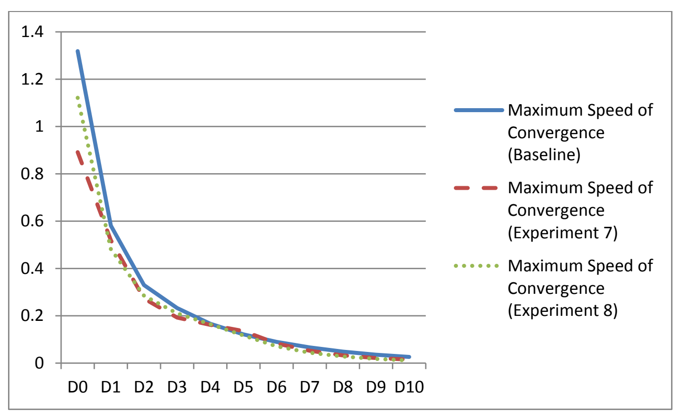
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

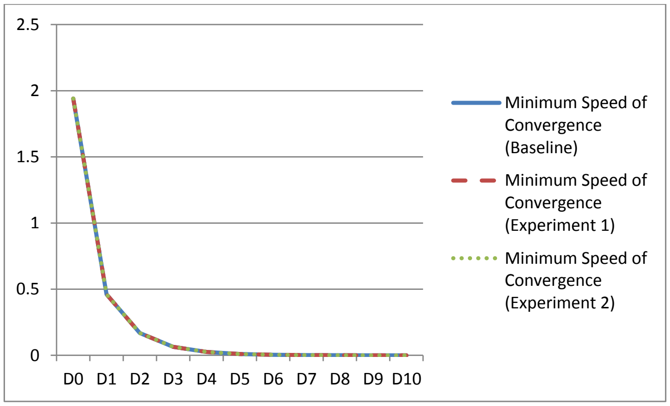


(e) Minimum Speed of Convergence (Three Active Firms)

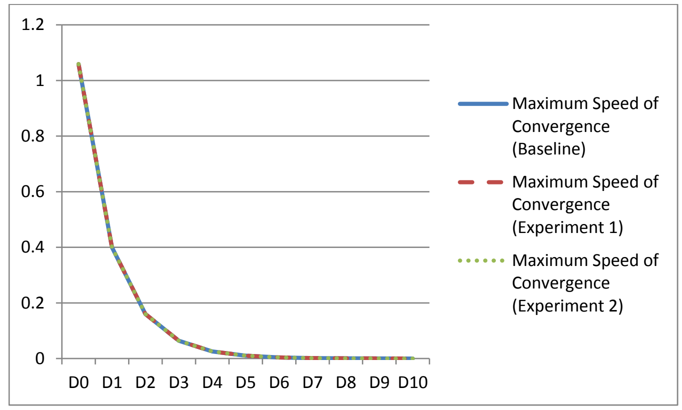


(f) Maximum Speed of Convergence (Three Active Firms)

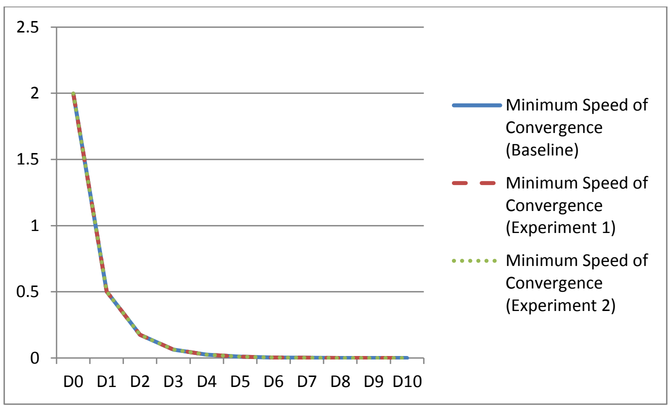
Figure 2.13: The Speed of Convergence with Different I in Welfare Maximum



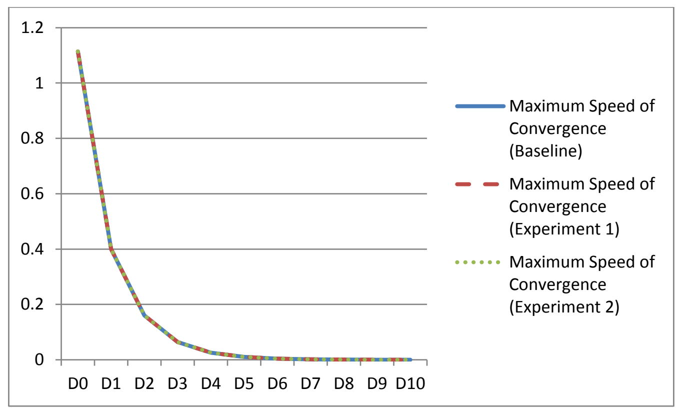
(a) Minimum Speed of Convergence (One Active Firm)



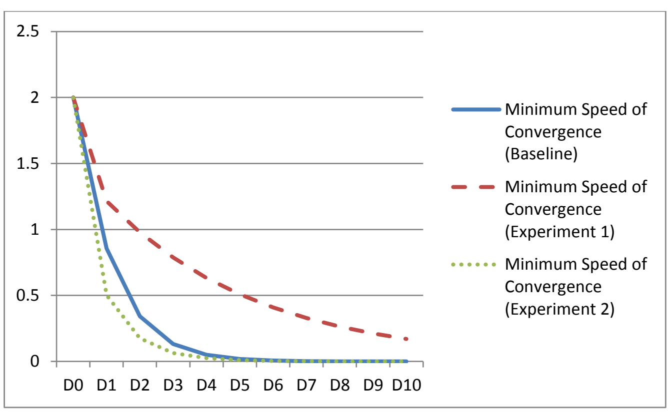
(b) Maximum Speed of Convergence (One Active Firm)



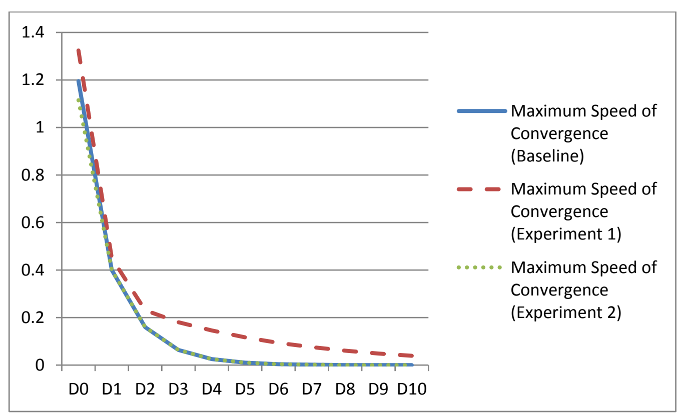
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

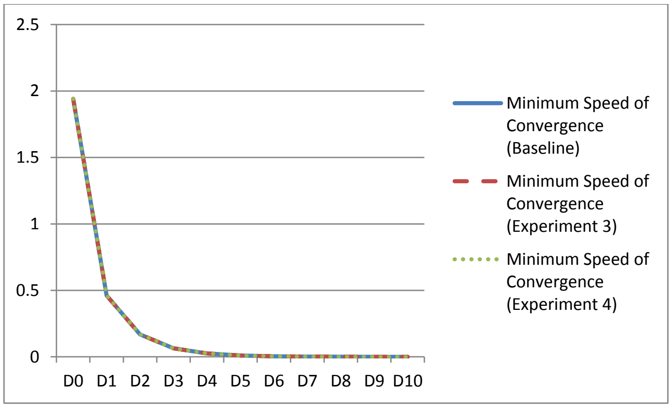


(e) Minimum Speed of Convergence (Three Active Firms)

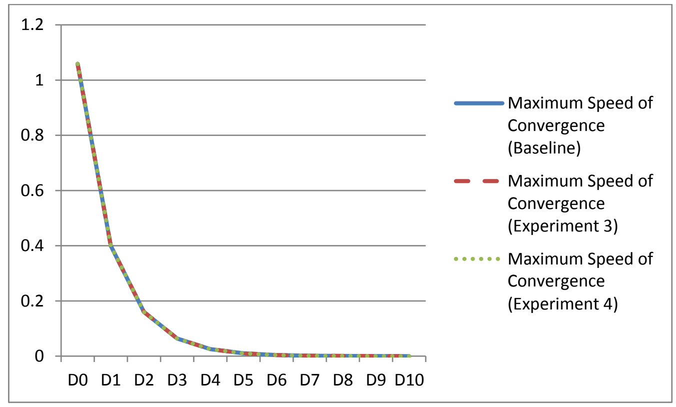


(f) Maximum Speed of Convergence (Three Active Firms)

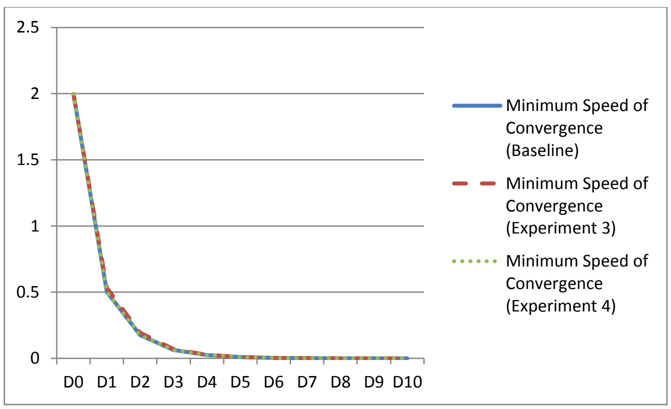
Figure 2.14: The Speed of Convergence with Different J in Welfare Maximum



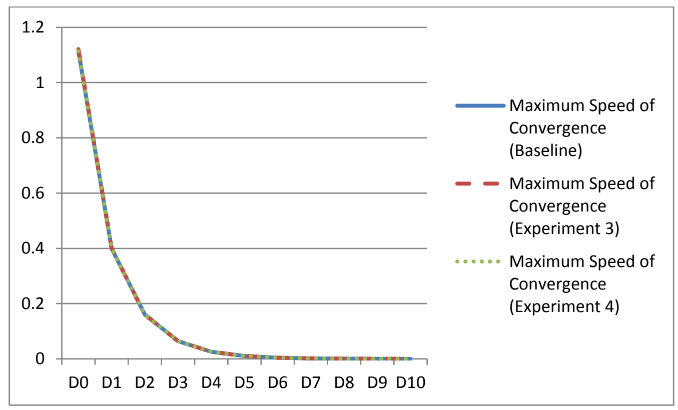
(a) Minimum Speed of Convergence (One Active Firm)



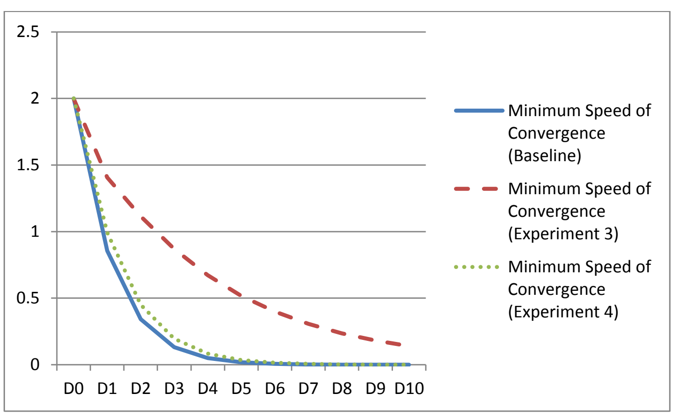
(b) Maximum Speed of Convergence (One Active Firm)



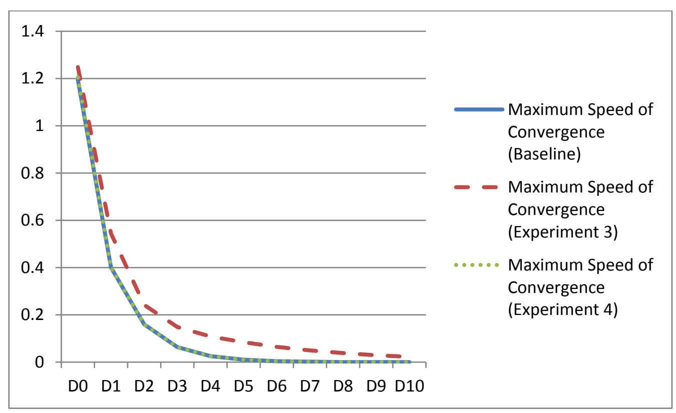
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

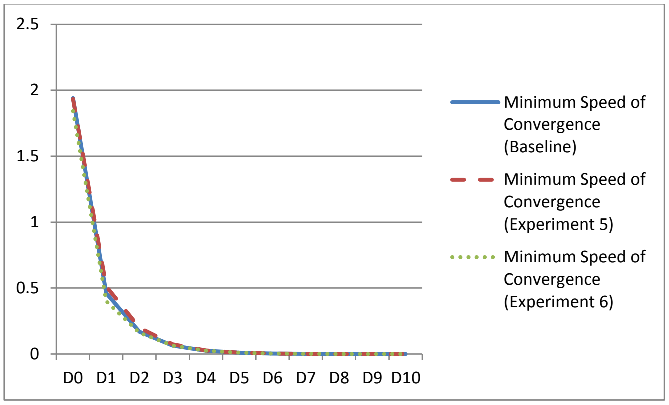


(e) Minimum Speed of Convergence (Three Active Firms)

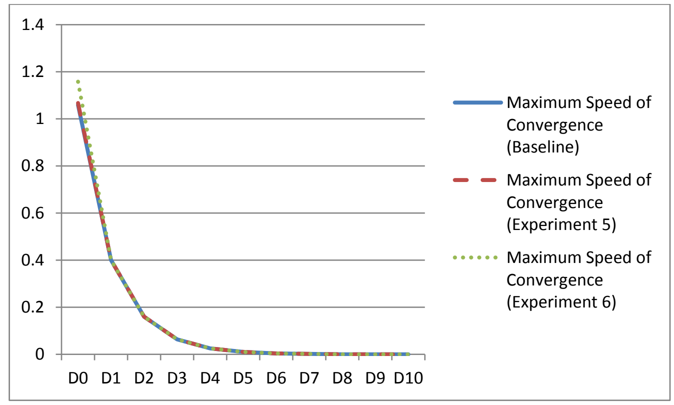


(f) Maximum Speed of Convergence (Three Active Firms)

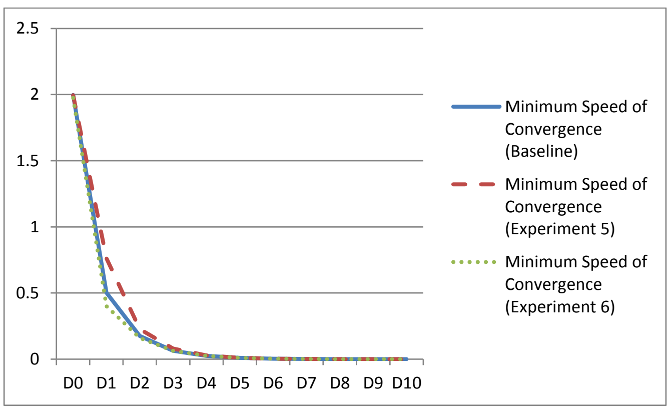
Figure 2.15: The Speed of Convergence with Different P and Q in Welfare Maximum



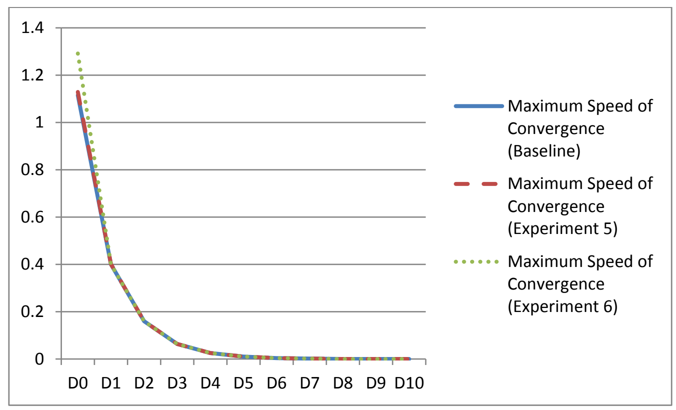
(a) Minimum Speed of Convergence (One Active Firm)



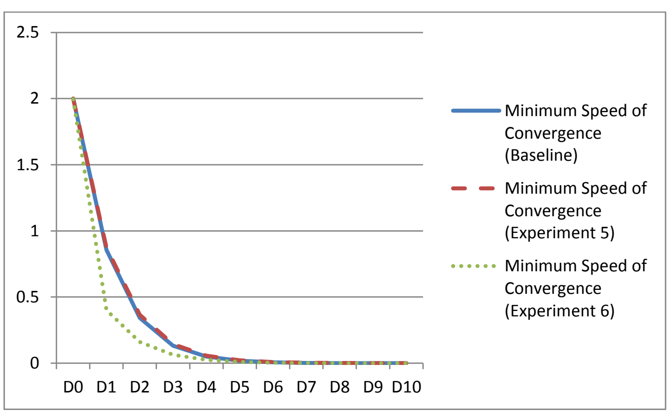
(b) Maximum Speed of Convergence (One Active Firm)



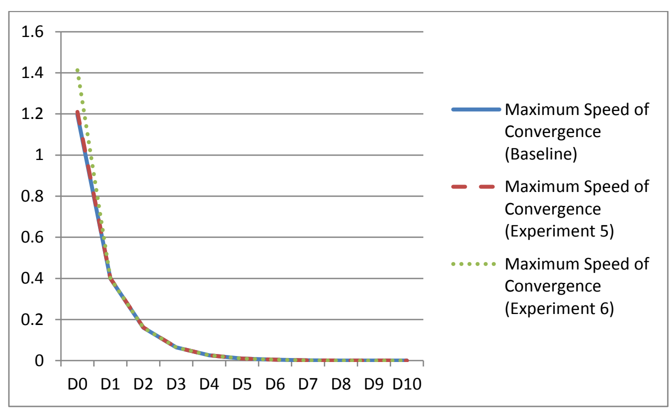
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)

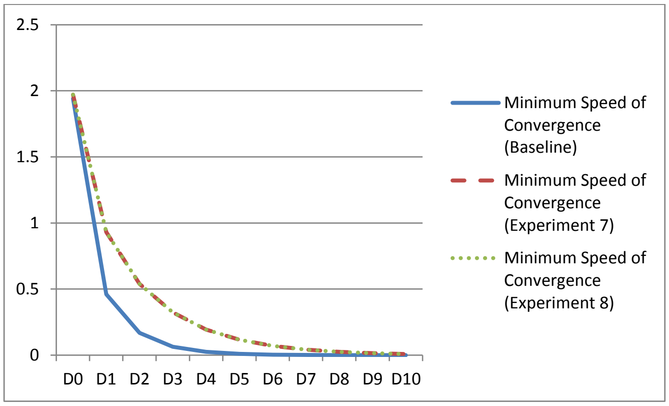


(e) Minimum Speed of Convergence (Three Active Firms)

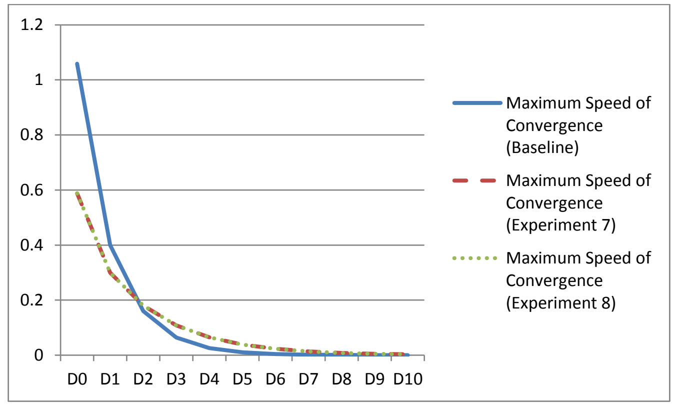


(f) Maximum Speed of Convergence (Three Active Firms)

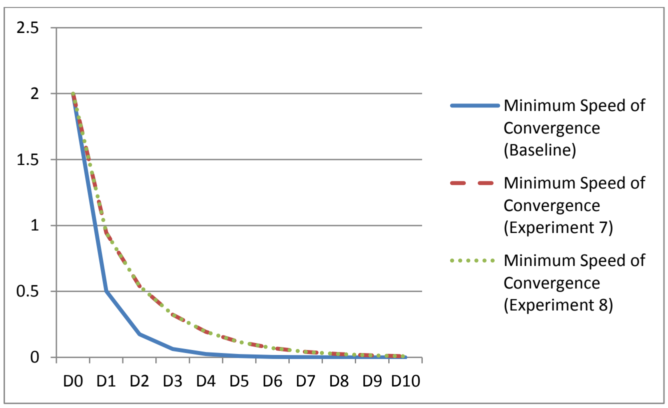
Figure 2.16: The Speed of Convergence with Different h and l in Welfare Maximum



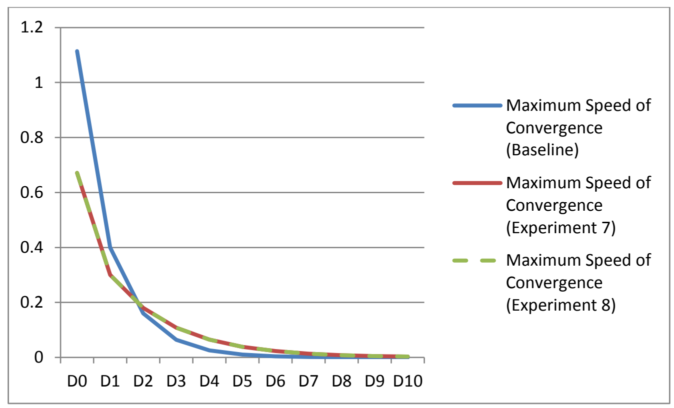
(a) Minimum Speed of Convergence (One Active Firm)



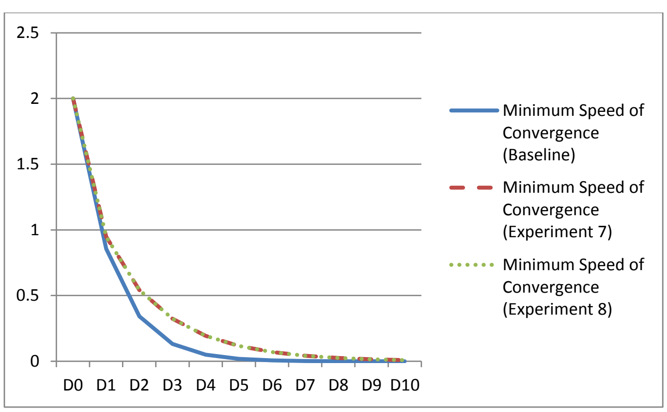
(b) Maximum Speed of Convergence (One Active Firm)



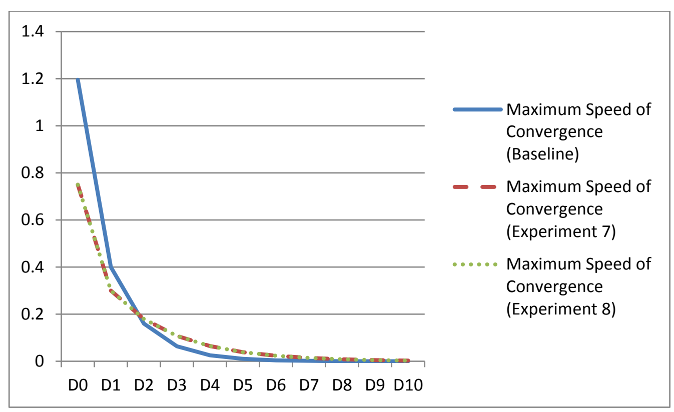
(c) Minimum Speed of Convergence (Two Active Firms)



(d) Maximum Speed of Convergence (Two Active Firms)



(e) Minimum Speed of Convergence (Three Active Firms)



(f) Maximum Speed of Convergence (Three Active Firms)

Table 2.47: Numbering Convention for States

State	$(u,h,0;A)$	$\cdots$	$(u,h,A-1;A)$	$(u,l,0;A)$	$\cdots$	$(u,l,A-1;A)$	$(e,h,0;A)$	$\cdots$	$(e,h,A-1;A)$	$(e,l,0;A)$	$\cdots$	$(e,l,A-1;A)$
Number	1	$\cdots$	A	$A+1$	$\cdots$	$2A$	$2A+1$	$\cdots$	$3A$	$3A+1$	$\cdots$	$4A$

Chapter 3

Transmission Capacity and the Welfare Effects of Electricity Market Interaction

3.1 Introduction

The use of energy is the distinguishing characteristic of modern economies, and in most applications this energy comes in the form of electricity. The trade of electricity between countries or provinces may improve the security of electricity supply, economic efficiency as well as the quality of the environment (Turvey (2006)). Government policy in the electricity sector is a major determinant of both social welfare and the state of the natural environment. With these effects in mind, my research project addresses a series of innovative questions including: Would an increase in competition in the electricity market be beneficial for social welfare? Would the expansion of electricity transmission capacity between countries/provinces benefit welfare?

In this paper, we develop a theoretical trade model which represents the electricity sector. The model discards the assumption of symmetry, and examines the effects of trade when marginal costs, market concentration and country size are all allowed to be heterogeneous. Furthermore, it also incorporates transmission costs and transmission capacity since these are two important factors in the electricity sector. Variation in both may affect the social welfare and environmental quality. Under the theory of comparative advantage, the global welfare gains from trade. This is because each country will export under free trade that commodity which has the lower relative price under autarky. However, firms in the electricity sector have market power, so our analysis depends on imperfect competition. Price will deviate from marginal cost of electricity production under conditions of imperfect competition.

Therefore, in some cases electricity may be imported whose marginal cost of production is lower than the delivered cost (marginal cost plus transmission cost) of other countries. This may lead to a net global decrease in welfare.

By considering the effects of imperfect competition in the electricity sectors, our model relates four stylized facts regarding trade between these two countries/ provinces. First, both are asymmetric in a variety of ways. Secondly, the electricity market is not perfectly competitive. Also, the Heckscher-Ohlin theorem predicts a much lower volume of intra-industry trade than actually takes place. In reality, a large proportion of trade occurs even with similar levels of factor abundance, while the H-O theorem predicts that trade would only result with very different levels of factor abundance. Linder (1961), Helpman and Krugman (1985) used two different models to explain this stylized fact. However, both of these explanations fail to explain why we observe a considerable volume of intra-industry trade even when the product categories are very finely defined. We adopt the model of Brander and Krugman (1983) to explain this fact. In this model, it is assumed that products are homogeneous and there will be cross-hauling even with transportation costs because oligopolistic markets imply that price will be above marginal cost. All these assumptions fit the reality of electricity industry in both provinces. We argue that a province is likely to export a commodity produced by an industry that is relatively less concentrated. This contradicts the assumptions of the H-O theorem, which considers identical concentration across industries; with different degrees of concentration, H-O underestimates the volume of intra-industry trade between provinces. Fourth, Helpman (1987) uses a theory of product diversity to explain that countries benefit more from trade with other countries of a larger size (a central result of the gravity model). Hummels and Levinsohn (1985) note that "relative unanimity of our results suggests that there is something other than monopolistic competition that may be responsible for the empirical success of the gravity model". By modeling asymmetry in market size as well as other parameters, our model produces results that are consistent with the actual empirical relation between trade volume and country size.

Several papers have been written on the economics of electricity transmission. Sertelis and Dormaar (2007) empirically examine whether electricity exports and imports cause price fluctuation in Alberta market. Genc et al. (2011) investigate whether trade has any effect on the price formation process in the Ontario market, and identify interconnected markets that have higher impacts on prices in that market. Focusing on the electricity market, Borenstein et al. (2000) demonstrate that limited transmission capacity can give a firm an incentive to restrict its output in order to congest transmission into its area of dominance; as a result, relatively small investments in transmission may yield surprisingly large payoffs in terms of increased competition. Stoft (1999), Joskow and Tirole (2000), and Gibert et al. (2004) show that, depending upon various circumstances, the *ex ante* allocation of financial transmission rights in a congested network might either enhance or mitigate market power. Consequently, the greater interconnection creates a larger market and should therefore increase competition. However, the achievement of increased market efficiency predicted by economic theory appears to depend upon the features of a particular market and mechanism. With this perspective in mind, our theoretical paper investigates whether free trade with transmission cost of electricity benefit both markets, and whether the expansion of transmission capacity would increase the global social welfare.

This model can be successfully applied to the electricity markets of Alberta (AB) and British Columbia (BC). The main structural features of the markets are as follows: Aggregate production of electricity in these two provinces is similar (10.4% and 10.1% of total Canadian output, respectively), but there is a significant difference in demand that reflects differences in population size [1,2]. The AB electricity market is oligopolistic with five firms supplying 80% of provincial generation capacity, while the BC market is monopolistic since

[1]Statistics Canada 2009, pp. 20-21

[2]"Canada's population estimates: Table 2 Quarterly demographic estimates". Statcan.gc.ca. 2010-06-28. http://www.statcan.gc.ca/daily-quotidien/100628/t100628a2-eng.htm. Retrieved 2010-07-26.

BC Hydro totally dominates the BC industry [3] . Interprovincial trade in electricity is constrained by transmission capacity. Electricity produced in AB involves significant emissions of CO_2 because it is dominated by coal power (72% of total utility-generated power), while production in BC involves far less CO_2 emissions because it uses mainly hydro power (90% of total utility-generated power) [4]. AB firms have lower fixed costs which permit relatively easy entry into the market, but much higher marginal costs of production. At the same time, BC has a carbon tax (since July 1, 2008), while AB has no such tax. In this context, several policy questions deserve attention: Would more competition in BC improve welfare and the natural environment in BC and/or AB? Would it be desirable to expand electricity transmission capacity across the provincial border? Would imposing a pollution tax in AB be beneficial for welfare and the natural environment?

Furthermore, by focusing on the effect of welfare after trade given asymmetries in market size, concentration and marginal costs, this model allows us to identify which parameters yield the highest welfare to a country/province. While other studies have considered trade with transmission costs, transmission constraints and trade in the presence of asymmetries, no paper has combined all three factors. Our model captures three important aspects of real-life electricity markets by incorporating trade with transmission costs, trade with limited capacity, and trade with three different asymmetries between countries/provinces, all at the same time.

Lahiri and Ono (1988) show under oligopoly with uneven technology, elimination of minor firms improves average efficiency of production which benefits welfare, though at the same time it leads to a more oligopolistic market structure in a nation. With this issue in mind, our comprehensive model could make suggestion of the policy question of whether the government should subsidize the cost-efficient industry instead of cost-inefficient one within two interlinked markets. This could happen if relatively similar marginal costs of firms within

[3]Statistics Canada 2009, pp. 20-21

[4]Statistics Canada 2009, pp. 20-21

two countries/ provinces lead subsidized inefficient firms to take the market of efficient firms, and expand to the international market, causing more total pollution and lower total welfare for the two provinces.

3.2 The Model

In this model, we assume there are two countries: large and small. They are denoted by subscripts $i = L, S$. In the small country, electricity demand is $q_S = \gamma(a - bp_S)$, where $0 \leq \gamma \leq 1$. We assume that $a, b > 0$. Note that q_i indicates the total output in each country. We assume there are n symmetric electricity firms in the small country, each of which has a constant marginal cost c_S. The large country has m symmetric electricity firms, each of which has a constant marginal cost c_L. The demand function is $q_L = a - bp_L$, where p_L is the price charged in country L. Firms compete under Cournot conjectures, and the social welfare for each country is a simple sum of the total profits made by symmetric firms, and consumer surplus. We assume the number of firms in each country is exogenously fixed in order to focus on the effects of interest. We also assume that S and L are the only two countries in the world.

3.2.1 Autarky Outcomes

The motivation for finding the autarky outcomes is to compare the results with free trade and limited trade, both of which have transmission costs. Therefore, we will be able to determine whether trade leads to improved welfare, and under which conditions. Let us initially assume that there is no trade between large and small countries. We record equilibrium values for total output of electricity (q_i), price (p_i), and social welfare (W_i) in the following table for each country. The symbol $\vee$ indicates equilibrium values in autarky. $q_L > 0$ requires that $a > bc_L$, and $q_S > 0$ requires that $a > bc_S$. Subject to these constraints, we have price, quantity and welfare for each province.

Large	Small
$\check{q}_L = \frac{m}{m+1}(a - bc_L)$	$\check{q}_S = \frac{\gamma n}{n+1}(a - bc_S)$
$\check{p}_L = \frac{a+mbc_L}{b(m+1)}$	$\check{p}_S = \frac{a+nbc_S}{b(n+1)}$
$\check{W}_L = \frac{m}{b}(\frac{a-bc_L}{m+1})^2(1 + \frac{m}{2})$	$\check{W}_S = \frac{\gamma n}{b}(\frac{a-bc_S}{n+1})^2(1 + \frac{n}{2})$

Given the autarky outcomes above, we are now able to make comparisons with cases of free trade with transmission cost, and trade with both transmission cost and limited transmission capacity to see how market concentration, marginal costs and market size affect welfare in each case, and whether trade leads to superior outcomes in terms of social welfare.

3.2.2 Outcomes for Free Trade with Transmission Costs

In reality, electricity trade is permitted between these two countries. We assume a transmission cost of $\theta \in [0, 1]$ per unit of electricity. The total output produced in each country, $\hat{q}_i$, which includes two components: output consumed within the country $\hat{q}_i^i$ and output exported to the other country $\hat{q}_i^j$. Therefore, the total output of the small country can be written as $\hat{q}_S = \hat{q}_S^S + \hat{q}_S^L$. The price for each country is $\hat{p}_i$. The symbol $\wedge$ indicates equilibrium values in free trade without transmission limits. Each country's welfare $\hat{W}_i$ is now the sum of total profits from electricity consumed within the country $\hat{\pi}_i^i$, total profits from export to the other country $\hat{\pi}_i^j$, and consumer surplus. The firms in each country will only operate if their marginal costs with transmission costs are below the autarky price in the other country; that is, $\hat{q}_S^S > 0$ and $\hat{q}_S^L > 0$ require that $(1+\theta)c_S \leq \frac{a+mbc_L}{b(m+1)}$, and $\hat{q}_L^L > 0$ and $\hat{q}_L^S > 0$ require that $(1+\theta)c_L \leq \frac{a+nbc_S}{b(n+1)}$. Subject to these constraints, output, price and welfare are given as follows:

Large	Small
$\hat{q}_L^L = \frac{m\{a+nb(1+\theta)c_S-(n+1)bc_L\}}{m+n+1}$	$\hat{q}_S^S = \frac{n\{a\gamma+m\gamma b(1+\theta)c_L-(1+m)\gamma bc_S\}}{1+m+n}$
$\hat{q}_L^S = \frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}$	$\hat{q}_S^L = \frac{n\{a+mbc_L-(1+\theta)(m+1)bc_S\}}{m+n+1}$
$\hat{p}_L = \frac{a+n(1+\theta)bc_S+mbc_L}{b(m+n+1)}$	$\hat{p}_S = \frac{a+nbc_S+mb(1+\theta)c_L}{b(1+m+n)}$
$\hat{\pi}_L^S = \frac{m\gamma\{a+nbc_S-(1+n)b(1+\theta)c_L\}^2}{b(1+m+n)^2}$	$\hat{\pi}_S^S = \frac{n\gamma\{a+mb(1+\theta)c_L-(1+m)bc_S\}^2}{b(1+m+n)^2}$
$\hat{\pi}_L^L = \frac{m\{a+nb(1+\theta)c_S-(n+1)bc_L\}^2}{b(m+n+1)^2}$	$\hat{\pi}_S^L = \frac{n\{a+mbc_L-(1+\theta)(m+1)bc_S\}^2}{b(m+n+1)^2}$
$\hat{CS}_L = \frac{\{a(m+n)-n(1+\theta)bc_S-mbc_L\}^2}{2b(m+n+1)^2}$	$\hat{CS}_S = \frac{\gamma\{a(m+n)-nbc_L-(1+\theta)mbc_S\}^2}{2b(1+m+n)^2}$

The net trade balance for the large country is $NTB_L = \hat{q}_L^S - \hat{q}_S^L$. The world welfare is simply the sum of both country;s welfare: $\hat{W} = \hat{W}_L + \hat{W}_S$. We will first consider the welfare of the large country (small country's welfare is symmetric), and then global welfare.

Market Size and Large Country's Welfare

The following derivative shows how the welfare of the large country changes with the market size of the small country:

$$\frac{\partial \hat{W}_L}{\partial \gamma} = \frac{m\{a + nbc_S - (1+n)b(1+\theta)c_L\}^2}{b(1+m+n)^2} > 0 \tag{3.1}$$

Lemma 1. *The welfare of the large country always increases with γ, the market size of the small country.*

The intuition for this lemma is obvious: the larger is the market size of the small country, the higher is the export profit for the large country; therefore, an increase in the market size of the small country benefits large country's welfare.

Market Concentration and Large Country Welfare

The number of firms in the large country has an ambiguous impact on its own welfare. Large country firms' profits earned from inside country and from exports to the small country are maximized in m at the peak $m = n + 1$, however, consumer surplus increases with m.

$$\begin{aligned}
\frac{\partial \hat{W}_L}{\partial m} &= \frac{\partial \hat{\pi}_L^S}{\partial m} + \frac{\partial \hat{\pi}_L^L}{\partial m} + \frac{\partial \hat{C}S_L}{\partial m} \\
&= \frac{\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}^2(n+1-m)}{b(1+m+n)^3} \\
&+ \frac{\{a + nb(1+\theta)c_S - (n+1)bc_L\}^2(n+1-m)}{b(m+n+1)^3} \\
&+ \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{a + nb(1+\theta)c_S - (n+1)bc_L\}}{b(m+n+1)^3}
\end{aligned} \tag{3.2}$$

Lemma 2. *$\hat{W}_L$ has a single peak at $m^* > n + 1$. The optimal number of firms in the large country equals m^* (see appendix 3.4.1)*

Large country's welfare first rises, then decreases with number of firms because there are two different effects in action. First, consumer surplus increases with the number of firms, but at a decreasing rate. Secondly, the increase in the number of firms m initially increases the profits of large country's firms from both in-country consumption and export because of some displacement of production from small to large country's firms. However, as m increases further, the negative effect on profit from additional competition dominates the effect of increased sales. Furthermore, this negative effect also dominates the gain in consumer surplus if m is sufficiently large.

A similar relationship holds between $\hat{W}_L$ and n :

$$\begin{aligned}
\frac{\partial \hat{W}_L}{\partial n} &= \frac{\partial \hat{\pi}_L^S}{\partial n} + \frac{\partial \hat{\pi}_L^L}{\partial n} + \frac{\partial \hat{C}S_L}{\partial n} \\
&= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}\{(1+m)bc_S - m(1+\theta)bc_L - a\}}{b(1+m+n)^3} \\
&+ \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{(m+1)b(1+\theta)c_S - mbc_L - a\}}{b(m+n+1)^3} \\
&+ \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{a - (m+1)(1+\theta)bc_S + mbc_L\}}{b(m+n+1)^3}
\end{aligned} \tag{3.3}$$

Lemma 3. *If small country's marginal cost is very high (i.e. if $(n-m)a + (2n+1)bc_L < n(1+\theta)(1+2m)bc_S$), $\hat{W}_L$ always decreases when n increases. However, if small country's marginal cost is similar to that of the large country, then $\hat{W}_L$ will be U-shaped in n, and the* *lowest welfare will be reached when the number of firms in the small country equals n^* (see appendix 3.4.2). If small country firms' marginal cost is very low, $\hat{W}_L$ always increases as n decreases.*

Why does a higher number of small country firms always increase $\hat{W}_L$ when small country firms' marginal cost is sufficiently low? This is because large country firms produce little when small country firms are sufficiently efficient, so an increase in the number of small country firms only decreases the electricity price in the large country's market, which benefits its consumers. If small country's firms are sufficiently inefficient, the benefit from higher consumer surplus is dominated by the loss of profits from in-country consumption with number of small country firms, therefore, $\hat{W}_L$ always decreases when n increases. However, if firms in both countries have similar marginal costs, $\hat{W}_L$ will be U-shaped in n. There are two effects in this case. First, the increase of n decreases the profits of large country's firms from both in-country consumption and export to the small country because of more competition. However, the increase of n also increases consumer surplus in the large country's market. The first negative effect dominates when n is small, until the level n^* where $\hat{W}_L$ is minimized; then, $\hat{W}_L$ gradually increases with n as the positive effect dominates.

Marginal Costs and Large Country Welfare

We can demonstrate that large country's welfare always increases when large country firms' marginal costs decrease:

$$\begin{aligned}
\frac{\partial \hat{W}_L}{\partial c_L} &= \frac{\partial \hat{\pi}_L^S}{\partial c_B} + \frac{\partial \hat{\pi}_L^L}{\partial c_B} + \frac{\partial \hat{C}S_L}{\partial c_B} \\
&= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}\{-(1+n)b(1+\theta))\}}{b(1+m+n)^2} \\
&\quad + \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{-(n+1)b\}}{b(m+n+1)^2} \\
&\quad + \frac{2\{a(m+n) - n(1+\theta)bc_S - mbc_L\}(-mb)}{b(m+n+1)^2} \\
&< 0
\end{aligned} \tag{3.4}$$

where $-(1+n)b(1+\theta) < 0, -(n+1)b < 0$,and $-mb < 0$. Therefore, $\frac{\partial W_L^T}{\partial c_L} < 0$

Lemma 4. *The total welfare of the large country always increases when large country firms' marginal cost c_L decreases.*

The relationship between large country welfare and small country firms' marginal cost is ambiguous, as shown here:

$$\begin{aligned} \frac{\partial \hat{W}_L}{\partial c_S} &= \frac{\partial \hat{\pi}_L^S}{\partial c_S} + \frac{\partial \hat{\pi}_L^L}{\partial c_S} + \frac{\partial \hat{C}S_L}{\partial c_S} \\ &= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}(nb)}{b(1+m+n)^2} \\ &\quad + \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{nb(1+\theta)\}}{b(m+n+1)^2} \\ &\quad + \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{-n(1+\theta)bc\}}{b(m+n+1)^2} \end{aligned} \tag{3.5}$$

Lemma 5. *$\hat{W}_L$ is a U-shaped function in c_S; it reaches its minimum when small country firms' marginal costs equals c_S^*. If $c_S > c_S^*$, $\hat{W}_L$ increases when c_S increases.*

Here, $c_S^* = \frac{bmc_L(1+2\gamma+2n(1+\gamma))(1+\theta)+a(n+n\theta-m(1+2\gamma+\theta))}{bn((1+\theta)^2+2m(\gamma+(1+\theta)^2))}$

The marginal cost of small country firms has two effect on $\hat{W}_L$. First, it has a positive effect on large country firms' profits from in-country consumption and export to the small country; however, it has a negative effect on consumer surplus in large country since it increases the electricity price in large country's market. When c_S is very low, a small increase in small country firms' marginal cost results in a small increase in large country firms' profits however, it has large decrease of consumer surplus in the large country. The reason is that when small country firms are sufficiently efficient, large country firms produce little. A small increase of c_S has a small effect on profits in large country but has large effect on consumer surplus. As c_S continue to increase, the effect on consumer surplus dominates the effect on profits for some time, but is eventually overwhelmed. When c_S is sufficiently large, the increase in profits from the displacement of production from small country firms is larger than the reduction in consumer surplus in the large country market, because small country firms do not produce much if their marginal costs are sufficiently high. Therefore, $\hat{W}_L$ increases with c_S if $c_S > c_S^*$.

Transmission Cost and Large Country Welfare

$$\begin{aligned}
\frac{\partial \hat{W}_L}{\partial \theta} &= \frac{\partial \hat{\pi}_L^S}{\partial \theta} + \frac{\partial \hat{\pi}_L^L}{\partial \theta} + \frac{\partial \hat{C}S_L}{\partial \theta} \\
&= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}\{-(1+n)bc_L\}}{b(1+m+n)^2} \\
&\quad + \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{nbc_S\}}{b(m+n+1)^2} \\
&\quad + \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{-nbc_S\}}{b(m+n+1)^2}
\end{aligned} \tag{3.6}$$

Lemma 6. *When c_L is sufficiently high ($n\{a+mbc_L-(1+\theta)(m+1)bc_S\} > m\{a+nb(1+\theta)c_S-(n+1)bc_L\}$, and a higher θ decreases $\hat{W}_L$. When c_L is sufficiently low, $\hat{W}_L$ always increases with θ. $\hat{W}_L$ is a U-shaped function in θ when the marginal costs of firms in both countries are similar. The welfare of the large country reaches its minimum when transmission costs equal θ^* (see appendix 3.4.3).*

To understand this relationship, θ has different effects on large country firms' profits from in-country consumption and exports, and also an impact on the consumer surplus. θ has a positive effect on profits earned inside the country by making it costly for small country firms to compete in the large country's market. However, θ has a negative effect on export profits by the same reasoning. In addition, θ has a negative effect on consumer surplus since it increases the price in the large country's market. When c_L is sufficiently high, a small increase in θ produces a large loss in consumer surplus, which dominates the positive effect on profits earned inside the large country. When c_L is sufficiently low, the increase in in-country profits starts to dominate the negative effects on consumer surplus and export profits from the small country as θ increases, so that $\hat{W}_L$ increases as θ increases.

The analysis above indicates that free trade with transmission costs benefits the large country most if its marginal cost is either much higher or much lower than that of the small country. Minimum welfare will be reached if the marginal costs of both countries are relatively similar. In addition, the welfare of the large country is positively correlated with the market size of the small country. The degree to which they benefit from trade will be a function of market concentration in both large and small countries electricity industries, as well as transmission costs.

Corollary 1. *The net trade balance for the large country is positively related to small country's market size and marginal costs, and also to number of large country's firms, and negatively related to the marginal cost of large country's firms and the number of small country's firms.*

The corollary above indicates that if the government of the large country preferred a larger net trade balance with the small country, it could subsidize electricity firms in order to lower their marginal cost, or increase the number of firms within the country.

Market Size and Global Welfare

There are two effects here. The first stems from the size of the world. Total welfare always increases when the world market grows. The other is a composition effect, which we can observe by controlling the size of the world, $\gamma + 1$. Here $\frac{\hat{W}}{\gamma+1}$ indicates total welfare per capita each country's industry is able to generate.

$$\frac{\partial \frac{\hat{W}}{\gamma+1}}{\partial \gamma} = \frac{1}{(1+\gamma)^2}\{\frac{\hat{\pi}_L^S}{\gamma} + \frac{\hat{\pi}_S^S}{\gamma} + \frac{\hat{C}S_S}{\gamma} - \hat{\pi}_L^L - \hat{\pi}_S^L - \hat{C}S_L\} \tag{3.7}$$

Lemma 7. *If the adjusted benefit from the small country's market* $\frac{1}{\gamma}\left(\hat{\pi}_L^S + \hat{\pi}_S^S + \hat{C}S_S\right)$ *is greater than the benefit from the large country's market* $(\hat{\pi}_L^L + \hat{\pi}_S^L + \hat{C}S_L)$*, an increase in the market size of the small country will increase global welfare.*

The implication of this lemma is that if the sum of adjusted profit earned within the small country market and the consumer surplus of small country is greater than that of the large country, increasing small country's market size will benefit the global welfare.

Market Concentration and Global Welfare

This section explains how an increase in the number of relatively efficient firms affects global welfare.

Lemma 8. *If the sum of marginal cost and transmission cost is lower for the large country than for the small country, we can achieve a higher global welfare by increasing the number of firms in the large country.*

Proof. Suppose large country's firms have a lower delivered cost (marginal cost plus transmission cost) compared to the small country's firms, i.e. $(1+\theta)c_L \leq c_S$

Then we have the following results:

$$\begin{aligned} \frac{\partial \hat{W}}{\partial m} &= \frac{\gamma\vartheta^2 + \vartheta(n^2+mn+n)\{\gamma bc_S - \gamma(1+\theta)bc_L\}}{b(1+m+n)^3} \\ &\quad + \frac{\mu^2 + \mu(n^2+mn+n)\{(1+\theta)bc_S - bc_L\}}{b(1+m+n)^3} \\ &> 0 \end{aligned} \tag{3.8}$$

□

Here $\vartheta = a + nbc_S - (1+\theta)(1+n)bc_L > 0, \mu = a + nb(1+\theta)c_S - (n+1)bc_L > 0$

Lemma 9. *If large country firms have a higher marginal cost than small country's firms, the global welfare may increase or decrease with the number of large country's firms. The welfare reaches a minimum level m^* (see appendix 3.4.4).*

If the large country is relatively inefficient compared to the small country, the global welfare will be U-shaped in m. This is caused by the action of two opposite effects. The positive effect on welfare stems from an increase in total output resulting from an increase in the number of large country firms. On the negative side, inefficient firms will displace some efficient firms from the market, leading to an increase in the average marginal cost. The negative effect on welfare dominates when m is relatively small; welfare starts to increase again once number of large country firms exceeds m^*.

Marginal Costs and Global Welfare

The effect is similar to that of a change in concentration, because an increase in firm efficiency benefits social welfare.

Lemma 10. *A decrease in the marginal cost of the large country firms will increase welfare if the delivered costs of the large country are lower than the marginal costs of the small country's firms.*

Proof. (see appendix 3.4.5) □

Lemma 11. *Global welfare will be U-shaped if the delivered costs of the large country are greater than the small country's marginal cost.*

Proof. If $(1+\theta)c_L > c_S$, the global welfare will be a U-shaped function of c_L, which reaches its minimum when $c_L^* = \frac{bc_S n(3+2m+2n)(1+\gamma)(1+\theta)+a(2+m+n)(1+\gamma+\gamma\theta)}{b(m+2mn+2(1+n)^2)(1+\gamma(1+\theta)^2)}$. The case is symmetric when $(1+\theta)c_S > c_L$. □

One implication of this lemma is that a small increase of c_L when $(1+\theta)c_L > c_S$ decreases global welfare as we expected; however, with continued growth in c_L, welfare will start increasing because production of inefficient firms will be replaced by that of efficient firms in small country. Therefore, a decrease in c_L is not always beneficial for global welfare. If $c_L > c_L^*$, a decrease in the marginal costs in the large country decreases the global welfare.

Transmission Costs and Global Welfare

$$\frac{\partial \hat{W}}{\partial \theta} = \frac{\partial \hat{W}_L}{\partial \theta} + \frac{\partial \hat{W}_S}{\partial \theta} \tag{3.9}$$

The transmission costs have a U-shaped relationship with global welfare: the reason is similar to lemma 6. Global welfare reaches its minimum at θ^*(see appendix 3.4.6).

3.2.3 When is free trade with transmission costs better than autarky?

We have examined the relationship between welfare and market size, marginal costs, and market concentration. In the next section, we will conduct a comparative analysis of autarky and free trade in order to uncover the effect of free trade on welfare. Here we assume the number of firms is constant from autarky to free trade. Venables (1985) considered there is a reduction of number of firms and expansion of their output after trade. However, in a short time period, we need not to expect electricity firms to exit from production since they have invested large sunk costs.

Large Country Welfare Differential

The change in large country's welfare caused by the introduction of free trade is defined as $\Delta W_L = \hat{W}_L - \check{W}_L$.

Large Country Welfare Differential and Market Size

Consider how the change in the market size affects large country welfare's differential:

$$\begin{aligned} \frac{\partial \Delta W_L}{\partial \gamma} &= \frac{\partial \hat{W}_L}{\partial \gamma} \\ &= \frac{m\{a + nbc_S - (1+n)b(1+\theta)c_L\}^2}{b(1+m+n)^2} > 0 \end{aligned} \tag{3.10}$$

Lemma 12. *The welfare differential for the large country always increases with the market size of small country.*

This lemma is generalization of a result obtained by Markusen (1981) who found that assuming $m = n = 1$ and $c_L = c_S$, the trade will always increase total world real income, but the large country may experience a welfare loss. In the presence of differences in market size, industry concentration and marginal costs, we can draw important conclusion: a country always prefers to have free trade with another country of a larger market size. The larger the market of the trade partner, the more welfare can be gained from trade.

Large Country Welfare Differential and Market Concentration

First, we will show how the number of firms in the large country's market affects its own welfare differential:

$$\begin{aligned}
\frac{\partial \Delta W_L}{\partial m} &= \frac{\partial \hat{W}_L}{\partial m} - \frac{\partial \check{W}_L}{\partial m} \qquad (3.11)\\
&= \frac{\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}^2(n+1-m)}{b(1+m+n)^3} \\
&\quad + \frac{\{a + nb(1+\theta)c_S - (n+1)bc_L\}^2(n+1-m)}{b(m+n+1)^3} \\
&\quad + \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{a + nb(1+\theta)c_S - (n+1)bc_L\}}{b(m+n+1)^3} \\
&\quad - \frac{(a - bc_L)^2}{b(m+1)^3}
\end{aligned}$$

Lemma 13. *In free trade with transmission costs, large country's welfare increases while m increases in autarky, and reaches its maximum at m^*. Therefore, large country's welfare differential peaks at $\hat{m} < m^*$.*

The implication of this lemma is that the welfare differential peaks when the number of firms in the large country is $\hat{m}$. The reason is that a sufficient number of firms decreases profits in the case of free trade with transmission costs. The optimal number of firms in the welfare differential case is less than the optimal number of firms in the case of free trade with transmission costs, since welfare increases with the number of firms in autarky.

The effect of the number of firms in small country's market on large country's welfare differential is the same as the effect of the number of small country firms on large country welfare under free trade with transmission costs, because the number of firms in the small country does not affect large country's welfare under autarky.

$$\begin{aligned}
\frac{\partial \Delta W_L}{\partial n} &= \frac{\partial \hat{W}_L}{\partial n} \qquad (3.12)\\
&= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}\{(1+m)bc_S - m(1+\theta)bc_L - a\}}{b(1+m+n)^3} \\
&\quad + \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{(m+1)b(1+\theta)c_S - mbc_L - a\}}{b(m+n+1)^3} \\
&\quad + \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{a - (m+1)(1+\theta)bc_S + mbc_L\}}{b(m+n+1)^3}
\end{aligned}$$

The welfare differential of the large country will be a U-shaped function of the number of small country's firms. The reason is the same as in lemma 3.

Large Country Welfare Differential and Marginal Costs

We now show how large country's marginal cost affects its own welfare differential:

$$\begin{aligned}
\frac{\partial \Delta W_L}{\partial c_L} &= \frac{\partial \hat{W}_L}{\partial c_L} - \frac{\partial \check{W}_L}{\partial c_L} \\
&= \frac{2m\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}\{-(1+n)b(1+\theta))\}}{b(1+m+n)^2} \\
&\quad + \frac{2m\{a + nb(1+\theta)c_S - (n+1)bc_L\}\{-(n+1)b\}}{b(m+n+1)^2} \\
&\quad + \frac{2\{a(m+n) - n(1+\theta)bc_S - mbc_L\}(-mb)}{b(m+n+1)^2} \\
&\quad + \frac{(a - bc_S)(m^2+m)}{(m+1)^2}
\end{aligned} \tag{3.13}$$

The welfare differential of the large country is U-shaped in marginal costs of its own firms. It reaches a minimum at $\hat{c}_L$ (see appendix 3.4.7)

Therefore, if $c_L > \hat{c}_L$, a higher in c_L increases large country's welfare differential. If $\hat{c}_L < 0$, the increase of c_L always has a beneficial effect on large country's welfare differential. The marginal cost of the small country has the same effect on large country's welfare differential as on the welfare of the large country under free trade with transmission costs.

$$\frac{\partial \Delta W_L}{\partial c_S} = \frac{\partial \hat{W}_L}{\partial c_S} \tag{3.14}$$

Therefore, large country's welfare differential is U-shaped in c_S, for the same reason as in lemma 4.

Welfare Differential and Transmission Costs for the Large Country

Electricity transmission cost of has the same effect on large country's welfare differential and on its own welfare under free trade with transmission costs, because transmission costs do not play any role in the autarky scenario. Therefore, $\frac{\partial \Delta W_L}{\partial \theta} = \frac{\partial \hat{W}_L}{\partial \theta}$ and the implication is the same as in Lemma 6.

3.2.4 Global Welfare Differential

We now investigate the change in global welfare as the world moves from autarky to free trade with transmission costs. The global welfare differential is defined as $\Delta W = \Delta W_L + \Delta W_{S.}$

Global Welfare Differential and Market Size

$$\frac{\partial \frac{\Delta W}{\gamma+1}}{\partial \gamma}=\frac{1}{(1+\gamma)^2}\{\frac{\hat{\pi}_L^S}{\gamma}+\frac{\hat{\pi}_S^S}{\gamma}+\frac{\hat{C}S_S}{\gamma}+\check{W}_L-\hat{\pi}_L^L-\hat{\pi}_S^L-\hat{C}S_L-\frac{\check{W}_S}{\gamma}\} \tag{3.15}$$

If the sum of the adjusted benefit obtained from small country's market under free trade with transmission costs and the welfare of the large country under autarky is greater than the sum of the benefit obtained from the large country's market under free trade with transmission costs and the adjusted welfare of the small country under autarky, the increase in γ increases global per capita welfare differential. Therefore, if profit earned within the small country's market plus consumer surplus of the small country is greater than that of the large country's under free trade, and large country's welfare is higher than adjusted small country's welfare under autarky, it is better for the world to have trade with an increase in the small country's market size.

Global Welfare Differential and Market Concentration

First, we will show how the number of firms in large country affects the global welfare differential:

$$\begin{aligned}
\frac{\partial \Delta W}{\partial m} &= \frac{\partial \Delta W_L}{\partial m} + \frac{\partial \Delta W_S}{\partial m} \qquad (3.16)\\
&= \frac{\gamma\{a + nbc_S - (1+n)b(1+\theta)c_L\}^2(n+1-m)}{b(1+m+n)^3}\\
&+ \frac{\{a + nb(1+\theta)c_S - (n+1)bc_L\}^2(n+1-m)}{b(m+n+1)^3}\\
&+ \frac{\{a(m+n) - n(1+\theta)bc_S - mbc_L\}\{a + nb(1+\theta)c_S - (n+1)bc_L\}}{b(m+n+1)^3}\\
&- \frac{(a-bc_L)^2}{b(m+1)^3}\\
&+ \frac{2n\gamma\{a + mb(1+\theta)c_L - (1+m)bc_S\}\{(1+n)b(1+\theta)c_L - nbc_S - a\}}{b(1+m+n)^3}\\
&+ \frac{2n\{a + mbc_L - (1+m)(1+\theta)bc_S\}\{(1+n)bc_L - nb(1+\theta)c_S - a\}}{b(1+m+n)^3}\\
&+ \frac{\gamma\{a(m+n) - nbc_S - (1+\theta)mbc_L\}\{a + nbc_S - (n+1)(1+\theta)bc_L\}}{b(m+n+1)^3}
\end{aligned}$$

The global welfare differential is U-shaped in the number of firms from the large country. It reaches its minimum at some intermediate number of large country firms, then grows gradually. The minimum of ΔW can be positive or negative, depending on the values of variables involved.

Global Welfare Differential and Marginal Costs

The effect of marginal costs of the large country on the global welfare differential is similar:

$$\frac{\partial \Delta W}{\partial c_L} = \frac{\partial \Delta W_L}{\partial c_L} + \frac{\partial \Delta W_S}{\partial c_L} \qquad (3.17)$$

ΔW is a U-shaped function of c_L. It reaches its minimum at $\tilde{c}_L$ (see appendix 8)

For small intermediate value of c_L, ΔW can be very small or even negative.

Global Welfare Differential and Transmission Costs

The effect of transmission costs on the global welfare differential:

$$\frac{\partial \Delta W}{\partial \theta} = \frac{\partial \Delta W_L}{\partial \theta} + \frac{\partial \Delta W_S}{\partial \theta} \qquad (3.18)$$

ΔW is a U-shaped function of θ. It reached its minimum at $\tilde{\theta}$ (see appendix 9) If $\theta > \tilde{\theta}$, ΔW always increases when transmission costs rise. ΔW could be positive or negative when θ is at some intermediate level.

3.2.5 Free Trade with Transmission Costs and Capacity

If the net trade balance of the small country, $NTB_S = q_S^L - q_L^S > 0$,we derive $\frac{n\{a+mbc_L-(1+\theta)(m+1)bc_S\}}{m+n+1} - \frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} > 0$. Under this condition, we can say that the small country is a net exporter of electricity. In addition, if we assume there is a fixed transmission capacity $\bar{Q}$ between two countries and $\hat{q}_S^L - \hat{q}_L^S > \bar{Q}$, then the net trade balance of the small country becomes $NTB_S = \hat{q}_S^L - \hat{q}_L^S = \bar{Q}$. Under the case of free trade with transmission costs and limited capacity, the total output for each country $\bar{q}_i$ is simply sum of total output consumed within the country $\bar{q}_i^i$, and output exported to the other country $\bar{q}_i^j$. The total output for the small country can be written as $\bar{q}_S = \bar{q}_S^S + \bar{q}_S^L$. The price here can be written as $\bar{p}_i$. Here, the symbol $-$ indicates equilibrium values in free trade with transmission costs and limit transmission capacity. The welfare in each country $\bar{W}_i$ is the sum of total profits from electricity consumed within the country $\bar{\pi}_i^i$, total profits from export to the other country $\bar{\pi}_i^j$, and consumer surplus $\bar{C}S_i$.

Large
$\bar{q}_L^L = \frac{m\{a-\bar{Q}-bc_L\}}{(m+1)} - \frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}$
$\bar{q}_L^S = \frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}$
$\bar{p}_L = \frac{a-\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)}-\frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}+\bar{Q}\}}{b}$
$\bar{\pi}_L^S = \frac{m\gamma\{a+nbc_S-(1+n)b(1+\theta)c_L\}^2}{b(1+m+n)^2}$
$\bar{\pi}_L^L = \{\frac{a-\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)}-\frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}+\bar{Q}\}}{b} - c_L\}*$
$\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)} - \frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}\}$
$\bar{C}S_L = \frac{1}{2}\{\frac{a}{b} - \frac{a-\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)}-\frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}+\bar{Q}\}}{b}\}*$
$\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)} - \frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)} + \frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} + \bar{Q}\}$

Small
$\bar{q}_S^S = \frac{n\{a\gamma+m\gamma b(1+\theta)c_L-(1+m)\gamma bc_S\}}{1+m+n}$
$\bar{q}_S^L = \frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} + \bar{Q}$
$\bar{p}_S = \frac{a+nbc_S+mb(1+\theta)c_L}{b(1+m+n)}$
$\bar{\pi}_S^S = \frac{n\gamma\{a+mb(1+\theta)c_L-(1+m)bc_S\}^2}{b(1+m+n)^2}$
$\bar{\pi}_S^L = \{\frac{a-\{\frac{m\{a-\bar{Q}-bc_L\}}{(m+1)}-\frac{m^2\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{(m+n+1)(m+1)}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n}+\bar{Q}\}}{b} - (1+\theta)c_S\}*$
$\{\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} + \bar{Q}\}$
$\bar{C}S_S = \frac{\gamma\{a(m+n)-nbc_S-(1+\theta)mbc_L\}^2}{2b(1+m+n)^2}$

Small Country's Welfare and Transmission Capacity

$$\begin{aligned}\frac{\partial \bar{W}_S}{\partial Q} &= \frac{\partial \bar{\pi}_S^L}{\partial Q} \\ &= \frac{\{a(m+n+1) - ma\gamma - mn\gamma bc_S - \ell\}}{b(m+n+1)(m+1)} \\ &\quad + \frac{-m\gamma\{a + nbc_S - (n+1)(1+\theta)bc_L\}}{b(m+n+1)(m+1)}\end{aligned} \tag{3.19}$$

where, $\ell = \{(1+\theta)b(m+n+1)(m+1)c_S + m(m+n+1)bc_L + m(1+\theta)(n+1)\gamma bc_L -$

$\bar{Q}(m+n+1)\}$ The optimal capacity is $\bar{Q}^*$ (See appendix 10).

Lemma 14. *If $\bar{Q}^*+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} < \hat{q}_S^L$, then $a(m+1)-ma\gamma+m(m+1)bc_L+m(1+\theta)(n+1)\gamma bc_L < mn\gamma bc_S+(1+\theta)b(m+1)(m+1)c_S$. The welfare of the small country increases as capacity increases until $\bar{Q}^*$, then decreases until the point $\bar{Q}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} = \hat{q}_S^L$, after which the welfare stays constant even as transmission capacity increases.*

Small country's welfare increases, as the capacity of transmission increases when capacity is small. This is because the positive effect on output dominates the negative effect on price when capacity is small. Therefore, small country's export profit increases. When we further increase capacity after $\bar{Q}^*$, welfare decreases with higher capacity. This is because the increase in capacity does not produce large increases in output for export from the small country, since its marginal cost is relatively higher than in the large country. Therefore, additional output takes over only a small part of the large country's market as transmission capacity increases. On the other hand, the increase in capacity decreases the small country's electricity price. Therefore, the export profit of the small country decreases with a sufficiently large increase in capacity. Further increase in capacity when $\bar{Q}+\frac{m\{a\gamma+n\gamma bc_S-(1+\theta)(1+n)\gamma bc_L\}}{1+m+n} > \hat{q}_S^L$ does not affect small country welfare since no matter what level of capacity, small country's firms always choose their optimal level of export output at $\hat{q}_S^L$.

Large Country's Welfare and Transmission Capacity

$$\begin{aligned}\frac{\partial \bar{W}_L}{\partial Q} &= \frac{\partial \bar{\pi}_L^L}{\partial Q}+\frac{\partial \bar{C}S_L}{\partial Q} \\ &= \frac{-m\bar{Q}\{a(m+n+1)-ma\gamma-mn\gamma bc_S+\Upsilon\}}{b(m+n+1)(m+1)^2} \\ &\quad+\frac{-\bar{Q}\{am(m+n+1)-m^2a\gamma-m^2n\gamma bc_S+\Lambda\}}{b(m+n+1)(m+1)^2} \\ &\quad+\frac{\bar{Q}\{a(m+n+1)(m+1)-a(m+n+1)+\Psi\}}{b(m+n+1)(m+1)^2} \\ &\quad+\frac{\bar{Q}\{am(m+n+1)+ma\gamma+\Omega\}}{b(m+n+1)(m+1)^2}\end{aligned} \tag{3.20}$$

where,$\Upsilon = m(1+\theta)(n+1)\gamma bc_L - (m+n+1)bc_L - (m+n+1)\bar{Q}$, $\Lambda = m^2(1+\theta)(n+1)\gamma bc_L - m(m+n+1)bc_L - m(m+n+1)\bar{Q}$, $\Psi = ma\gamma + mn\gamma bc_S - m(1+\theta)(n+1)\gamma bc_L - (m+n+1)bc_L + (m+n+1)\bar{Q}$, and $\Omega = mn\gamma bc_S - m(1+\theta)(n+1)\gamma bc_L - (m+n+1)bc_L + (m+n+1)\bar{Q}$ The large country welfare reached its minimum when capacity is $\bar{Q}^* = \frac{ma\gamma - m(1+\theta)(n+1)\gamma bc_L + nm\gamma bc_S}{(m+n+1)}$

Lemma 15. *If $\bar{q}_L^{L*} < \hat{q}_L^L$, then $m^2nbc_L + 2m^2(1+\theta)(n+1)\gamma bc_L + mna - 2m^2a\gamma < (m+n+1)mnb(1+\theta)c_S + 2m^2n\gamma bc_S$. The welfare of the large country is constant when capacity increases until a point where $\bar{q}_L^{L1} = \hat{q}_L^L$, when welfare starts decreasing with higher capacity until it reaches $\bar{Q}^*$, after this point, the welfare of the large country increases with capacity.*

The reason is that when capacity is very small, large country's firms always produce $\hat{q}_L^L$ as it under unlimited capacity case, because small transmission capacity does not affect in-country consumption of electricity produced in large country. When we further increase capacity, the welfare of the large country decreases since the loss of $\bar{\pi}_L^L$ is higher than $\bar{C}S_L$ when marginal cost of large country's firms is relatively lower than the cost of small country's firms. If we increase capacity beyond $\bar{Q}^*$, the benefit to consumer surplus in the large country dominates the loss of profit from in-country consumption. This is because sufficiently high capacity brings higher marginal benefits to consumer surplus than marginal losses of profits.

3.3 Conclusion

This paper combines asymmetries in market size, degree of competition and technology of two regions with transmission costs and capacity constraints in the electricity market. In this framework, we are able to identify the conditions under which trade benefits a single country or increases global welfare when one of these factors changes for each country.

The paper finds that the welfare of a country can be improved if it trades electricity with a larger country. Empirical evidence supports the finding that a large country has little incentive to open trade with smaller country; at the same time, however, it may benefit from a multilateral trade agreement with several small countries.

The paper also argues that a government subsidy to inefficient firms in its own country might decrease the global welfare. Countries sometimes set a tariff to protect their inefficient firms to allow them to grow and become competitive in world market. However, as we showed, this might harm the global welfare since inefficient firms take over the market of efficient firms, leading to an increase in the global average market cost. In contrast, subsidizing relatively efficient firms can help them take over the production of inefficient firms, increasing the global welfare.

In addition, we also discover that the expansion of transmission capacity between two countries is not always beneficial to a single country or to global welfare. Sometimes, the expansion of capacity may make a country worse off since the benefit from its consumer surplus is dominated by the loss of profit. At some range of transmission capacity, its expansion does not affect a country's welfare because the firms are not constrained by the capacity in their choice of optimal output level, producing the same amount as they would with free trade. The resulting welfare effects have important policy implications for two countries in negotiating their expense contributions for expansion of transmission capacity between them.

3.4 Appendices

3.4.1

$$m^* = \frac{\nu + b^2\left(-c_S c_L n(1+n)(1+2n+2(1+n)\gamma)(1+\theta) + c_L^2(1+n)^3\left(1+r(1+\theta)^2\right) + c_S^2 n^2\left(\gamma + n\left(\gamma+(1+\theta)^2\right)\right)\right)}{a^2\gamma + ab(c_S n(1+2\gamma+\theta) - c_L(n+2\gamma+2n\gamma+2(1+n)r\theta)) + \eta}$$

where $\nu = a^2(1+\gamma+n(2+r)) + ab(-c_L(1+n)(2+3n+2\gamma+2n\gamma+2(1+n)\gamma\theta) + c_S n(1+2\gamma+\theta+n(3+2\gamma+3\theta)))$,

$\eta = b^2\left(-c_S c_L n(1+2\gamma+2n(1+\gamma))(1+\theta) + c_A^2 n^2\left(\gamma+(1+\theta)^2\right) + \varrho\right)$, and $\varrho = c_L^2(1+n)\left(\gamma(1+\theta)^2 + n\left(1+\gamma(1+\theta)^2\right)\right)$

3.4.2

$$n^* = \frac{m\left(a^2(1+2\gamma)+ab(-c_S(1+m)(1+2\gamma+\theta)+c_L(-1+m)(1+2\gamma(1+\theta)))+b^2c_L\left(c_S(1+m)(1+2\gamma)(1+\theta)-c_Lm\left(1+2\gamma(1+\theta)^2\right)\right)\right)}{a^2-ab(c_S(2+3m+2m\gamma+2\theta+3m\theta)-c_Lm(3+2r(1+\theta)))+\lambda}$$

Where $\lambda = b^2\left(-c_Sc_Lm(3+2\gamma+4m(1+\gamma))(1+\theta)+\varepsilon\right)$ and $\varepsilon = 2c_L^2m^2\left(1+\gamma(1+\theta)^2\right)+c_S^2(1+m)\left((1+\theta)^2+2m\left(\gamma+(1+\theta)^2\right)\right)$

3.4.3

$$\theta^* = \frac{nc_S(a(-m+n)+b(-c_s(1+2m)n+c_L(m+2mn)))+2c_Lm(1+n)(a-b(c_L-c_Sn+c_Ln))\gamma}{bc_S^2(1+2m)n^2+2bc_L^2m(1+n)^2\gamma}$$

3.4.4

$$\text{m}^* = \frac{-a^2(1+r)+ab\{-c_Sn(3+n)(1+r+\theta)+c_L(1+n)(2+n)(1+r+r\theta)\}}{bn\{a(c_s(1+r+\theta)-c_L(1+r+r\theta))+b-c_Sc_L(1+2n)(1+r)(1+\theta)+c_S^2n(r+(1+\theta)^2)+c_L^2(1+n)(1+r(1+\theta)^2)\}} + \frac{-b^2\{-c_Sc_Ln(1+n)(3+2n)(1+r)(1+\theta)+c_S^2n^2(2+n)\left(r+(1+\theta)^2\right)+c_L^2(1+n)^3 1+r(1+\theta)^2\}}{bn\{a(c_S(1+r+\theta)-c_L(1+r+r\theta))+b-c_Sc_L(1+2n)(1+r)(1+\theta)+c_S^2n(r+(1+\theta)^2)+c_L^2(1+n)(1+r(1+\theta)^2)\}}$$

3.4.5

If we assume $(1+\theta)c_L < c_{S,}$ then we could find the following:

$$\begin{aligned}
\frac{\partial \hat{W}}{\partial c_L} &= -\frac{2m(1+\theta)\gamma(1+n+m)a+2mn(1+\theta)\gamma(1+n+m)bc_S-2m(1+n)^2(1+\theta)^2\gamma bc_L-2nm^2(1+\theta)^2\gamma bc_L-2m^2(1+\theta)^2\gamma bc_L}{(1+m+n)^2} \\
&\quad -\frac{2m(1+n+m)a+2mn(1+n+m)(1+\theta)bc_S-2m(1+n)^2bc_L-2nm^2bc_L-2m^2bc_L}{(1+m+n)^2} \\
&< -\frac{2m(1+\theta)\gamma(1+n+m)a+2mn(1+\theta)\gamma(1+n+m)bc_S-2m(1+n)^2(1+\theta)\gamma bc_S-2nm^2(1+\theta)\gamma bc_S-2m^2(1+\theta)\gamma bc_S}{(1+m+n)^2} \\
&\quad -\frac{2m(1+n+m)a+2mn(1+n+m)bc_S-2m(1+n)^2bc_S-2nm^2bc_S-2m^2bc_S}{(1+m+n)^2} \\
&= \quad -\frac{2m(1+\theta)\gamma(a-bc_S)}{(1+m+n)} - \frac{2m(a-b(1+\theta)c_S)}{(1+m+n)} < 0
\end{aligned}$$

3.4.6

$$\theta^* = \frac{-2mrabc_L-2nabc_S-a(m+n)nbc_S-a(m+n)mbc_L-\jmath}{-2n^2(1+m)b^2c_S^2-m^2(1-2n\gamma)b^2c_L^2-2m\gamma(1+n^2)b^2c_L^2+2n(1+m^2)b^2c_S^2} - 1$$

where, $\jmath = \{2mn(1+n)(1+\gamma)b^2c_Sc_L - 2mn(1+m)(1+\gamma)b^2c_Sc_L - 2mn(1+\gamma)b^2c_Sc_L\}$

3.4.7

$$\hat{c}_L = \frac{-\frac{2a\left(1+\frac{m}{2}\right)m}{(1+m)^2}+\frac{2am(1+n)}{(1+m+n)^2}+\frac{am(m+n)}{(1+m+n)^2}-\frac{bc_S mn(1+\theta)}{(1+m+n)^2}+\frac{2bc_S mn(1+n)(1+\theta)}{(1+m+n)^2}+\frac{2am(1+n)\gamma(1+\theta)}{(1+m+n)^2}+\frac{2bc_S mn(1+n)\gamma(1+\theta)}{(1+m+n)^2}}{-\frac{2b\left(1+\frac{m}{2}\right)m}{(1+m)^2}+\frac{bm^2}{(1+m+n)^2}+\frac{2bm(1+n)^2}{(1+m+n)^2}+\frac{2bm(1+n)^2\gamma(1+\theta)^2}{(1+m+n)^2}}$$

3.4.8

$$\tilde{c}_L = \frac{\xi+\frac{2bc_S mn(1+n)(1+\theta)}{(1+m+n)^2}-\frac{2amn\gamma(1+\theta)}{(1+m+n)^2}-\frac{bc_S mn\gamma(1+\theta)}{(1+m+n)^2}+\frac{2bc_S m(1+m)n\gamma(1+\theta)}{(1+m+n)^2}+\frac{2am(1+n)r(1+\theta)}{(1+m+n)^2}+\frac{2bc_S mn(1+n)\gamma(1+\theta)}{(1+m+n)^2}}{-\frac{2b\left(1+\frac{m}{2}\right)m}{(1+m)^2}+\frac{bm^2}{(1+m+n)^2}+\frac{2bm^2n}{(1+m+n)^2}+\frac{2bm(1+n)^2}{(1+m+n)^2}-\frac{2b\left(1+\frac{n}{2}\right)n\gamma}{(1+n)^2}+\frac{bm^2r(1+\theta)^2}{(1+m+n)^2}+\frac{2bm^2nr(1+\theta)^2}{(1+m+n)^2}+\frac{2bm(1+n)^2\gamma(1+\theta)^2}{(1+m+n)^2}}$$

where, $\xi = -\frac{2a\left(1+\frac{m}{2}\right)m}{(1+m)^2} - \frac{2amn}{(1+m+n)^2} + \frac{2am(1+n)}{(1+m+n)^2} + \frac{am(m+n)}{(1+m+n)^2} - \frac{2a\left(1+\frac{n}{2}\right)n\gamma}{(1+n)^2} - \frac{bc_S mn(1+\theta)}{(1+m+n)^2} + \frac{2bc_S m(1+m)n(1+\theta)}{(1+m+n)^2}$

3.4.9

$$\tilde{\theta} = \frac{\varkappa-\frac{bc_S c_L mn\gamma}{(1+m+n)^2}-\frac{2bc_L^2 m^2 n\gamma}{(1+m+n)^2}+\frac{2bc_S c_L m(1+m)n\gamma}{(1+m+n)^2}+\frac{2ac_L m(1+n)\gamma}{(1+m+n)^2}+\frac{2bc_S c_L mn(1+n)\gamma}{(1+m+n)^2}-\frac{2bc_L^2 m(1+n)^2\gamma}{(1+m+n)^2}+\frac{ac_L m(m+n)\gamma}{(1+m+n)^2}}{\frac{2bc_S^2(1+m)^2 n}{(1+m+n)^2}+\frac{bc_S^2 n^2}{(1+m+n)^2}+\frac{2bc_S^2 mn^2}{(1+m+n)^2}+\frac{bc_L^2 m^2\gamma}{(1+m+n)^2}+\frac{2bc_L^2 m^2 n\gamma}{(1+m+n)^2}+\frac{2bc_L^2 m(1+n)^2\gamma}{(1+m+n)^2}}$$ where

$\varkappa = -\frac{2ac_S mn}{(1+m+n)^2} - \frac{bc_S c_L mn}{(1+m+n)^2} + \frac{2ac_S(1+m)n}{(1+m+n)^2} + \frac{2bc_S c_L m(1+m)n}{(1+m+n)^2} - \frac{2bc_S^2(1+m)^2 n}{(1+m+n)^2} - \frac{bc_S^2 n^2}{(1+m+n)^2} - \frac{2bc_S^2 mn^2}{(1+m+n)^2} + \frac{2bc_S c_L mn(1+n)}{(1+m+n)^2} + \frac{ac_S n(m+n)}{(1+m+n)^2} - \frac{bc_L^2 m^2\gamma}{(1+m+n)^2} - \frac{2ac_L mn\gamma}{(1+m+n)^2}$.

3.4.10

$$\bar{Q}^* = \frac{a(m+n+1)-2ma\gamma-2mn\gamma bc_S-(1+\theta)b(m+n+1)(m+1)c_S+m(m+n+1)bc_L+2m(1+\theta)(n+1)\gamma bc_L}{(m+n+1)}$$

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