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Determination of Efficient Inflationary Levels Under Various Government Policy Perspectives

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Determination of Efficient Inflationary Levels Under Various Government Policy Perspectives" submitted by Richard Gunter Schorn in partial fulfillment of the requirements for the degree of Master of Arts.

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ABSTRACT

This is a study on 'inflationary finance' where optimal inflation levels are derived using various government policy objectives. The first of these policy objectives is real revenue maximization for the government from money creation. The second is the welfare cost minimization of raising revenue from various government financing methods. Lastly, I develop a model and the resulting optimality conditions for a government who wishes to maximize revenues less social costs of its financing methods.

Empirically, I test for how well revenues have been 'smoothed' over time using cointegration and Granger causality testing procedures. By 'smoothing' of revenues I am, in essence, testing for positive correlations between inflation and average tax rates which would portray efficient financing of government expenditures via a policy which seeks to minimize the social costs of raising a given stream of income.

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CHAPTER 1: INTRODUCTION

Governments of today have a wide range of options in terms of raising the revenue needed to finance their extensive expenditures. In addition to being able to levy taxes on almost every kind of economic activity, income and wealth, they may opt for deficit financing and/or may decide to create certain desired levels of money by printing it. The last method is often referred to as 'inflationary finance' because this type of financing will tend to reduce the real value or purchasing power of a given level of money. This follows from economic fundamentals in that if the supply of a particular item increases, for a given normal demand structure, the real worth of that item should fall. Therefore, if money creation is to be an ongoing source of government financing, we can expect persistent inflation. Now another term used synonymously with 'inflationary finance' is 'seigniorage' which specifically means 'the real revenue a government derives from the creation of money'. Originally, the term seigniorage meant "the difference between the face value of coins and the value of the coins as metal"[Shearer, Chant and Bond(1984)] but its meaning has been expanded in economic literature to that of the definition above.

Since monetary growth in the economy not only leads to inflation but also depreciated exchange rates, given everything else, the ability of governments to finance themselves in this way has been expanded in recent years. For much of the early twentieth century, the currencies of major countries were valued on a gold standard system in that these currencies were tied to the price of gold. Therefore, increases in a country's money supply could not occur without a corresponding increase in the world stock of gold, or a decline in overall demand[1]. Between 1959 and the early 1970's, the Bretton Woods 'gold exchange standard' system was established wherein major countries tied their currency value to that of the United States who, in turn, fixed their dollar to the price of gold. Again, monetary control was subject to limitations imposed by these 'official' exchange rates. Since then, however, most countries have adopted floating exchange rates (although member countries of the EEC have recently fixed their exchange rates to other members) and this has led to greater individualism in worldwide monetary growth. Therefore, for countries such as Canada and the United States, there are no limitations on money creation imposed by the international community, except for verbal complaints, so that the feasibility of 'inflationary finance' has been greatly enhanced.

Throughout the discussion presented in this paper, there are basically two assumptions that I will look into or use. The first of these is a real money demand function that is dependant on the level of real income and the nominal interest rate. The essence of this is that as real income rises, the level of real money balances is also assumed to increase as some of this added income is held in liquid form. Also, if real income rises and the level of aggregate purchases rises as a result, then real money demand can be assumed to increase in order to keep the convenience of money balances the same. Nominal interest rates, on the other hand, can be regarded as an opportunity cost of holding money in liquid form and can, therefore, be expected to be negatively related to real money balances in the economy. By liquid money balances, I will basically be referring to currency plus demand or chequable deposits. The second of these assumptions will be that of 'open' inflation. By 'open', I refer to an announced government monetary or inflationary policy that is, indeed, implemented and credible to the private sector. In this way, expectational errors can be omitted from the analysis and we can focus on the actual inflation itself. In addition to this, indexation throughout the economy will generally be assumed so that the redistributive effects of inflation can be ignored.

The contents of this paper are as follows. In the first section, I will look at various methods of analysing government revenue from money creation and show the corresponding inflation or money growth rates which, if implemented, would maximize the government's real proceeds from this type of financing. In expanding on this, I will incorporate the central bank since they have direct control over the money supply and show how, in any specific period, the government's real proceeds from high powered money creation may differ from the total change in the monetary base[2]. The next section analyses welfare cost minimization in the raising of government revenue via alternative financing sources. By keeping the models generalized, the assumption of full indexation can be dropped so that the redistributive costs of inflation can be assumed to be implicitly incorporated into the model. By minimizing the generalized social cost function with respect to an intertemporal budget constraint, it will be seen that, under standard assumptions, inflation rates and tax rates should move together over time since the government should draw on all revenue sources available to them. I then present previous empirical literature on this subject and conclude by performing cointegration and causality tests on CPI inflation and average tax rates for the United States, United Kingdom, France and Canada. From these tests, I will be forced to conclude that there is little support for efficient historical revenue raising policies in France and Canada, some support for cointegration of these variables in the United States and strong support for positive causality from tax rates to inflation rates in the United Kingdom. The analysis is concluded by putting the previous two theoretical sections together to obtain a 'net benefit' type approach where total budgetary revenues less total social costs becomes the objective function for the government. The resulting optimality condition, under such a scenario, will imply that the marginal cost-benefit ratios of different revenue sources be equated which is consistent with what one would expect from the outset.

1. Although a certain country's currency was convertible into gold at an 'official' price, there was a certain flexibility in the private exchange rate due to the costs of handling and shipping gold. In essence, 'gold points' were established in that if a particular country's, say the United States', exchange rate in financial markets appreciated beyond a certain point, foreign payments to the country would be made in gold, rather than dollars, because it would be cheaper to buy the gold at the official price and make payments with it than buying the appreciated currency to make payments. This certain point was referred to as the 'gold import point' because gold would be imported into the United States. The other extreme was known as the 'gold export point'. If gold imports into the country increase, the money supply would also increase because the monetary base would be expanding and this would result in depreciation of the currency in international financial markets. Therefore, although exchange rates were not completely fixed, the range of flexibility was limited by gold handling costs which were not very large. For more on this see Shearer, Chant and Bond(1984).

2. 'High powered' money and the 'monetary base' can be used interchangeably.

CHAPTER 2: GOVERNMENT REVENUE MAXIMIZATION

2.1 INTRODUCTION

Since monetary seigniorage is interpreted as "a government's real revenue from money creation", the most common formulation to derive it, for any time period, is $(dM/dt)/P$. This is because it is assumed that any new money only gets into circulation via government expenditures and this new money is divided by the average aggregate price level in order to put it into real terms. The object of any student of seigniorage is to optimize some function that depends directly or indirectly on the level of money creation in the economy. This could be the real revenue derived by the government in a certain period or a number of periods, the minimization of some welfare cost function, the maximization of some monetary stimulation model for the economy, etc. In this section, I will focus on the former of these by way of a literature review with some analysis of the validity of the arguments raised.

2.2 SETUP

Government revenue from seigniorage can be generated in two ways. The first of these is by taxing the existing real balances in an economy and can be viewed as a type of "inflation tax". The second way seigniorage can be generated is when the monetary authority increases the money supply so as to meet the increased real balances demanded by the population as the economy grows (or other factors that raise the level of real balances such as lower interest rates, etc.). For analytical purposes, (dM/dt)/P can be broken down in the following manner[1]:

$$S_t=\frac{\frac{dM_t}{dt}}{P_t}=\frac{\frac{dM_t}{dt}}{M_t}\frac{M_t}{P_t}=\mu_t f(y_t,R_t)=\mu_t f(y_t,r_t+\pi^e_t)$$

where:
μ_t=currency growth rate in period t.
y_t=real income in period t.
r_t=real interest rate in period t.
π_t=actual inflation in period t.
$\pi^e{}_t$=expected inflation in period t.
R_t=nominal interest rate[2] in period t.
S_t=real government revenue in period t.

In many standard economic models that attempt to analyse seigniorage, the rate of inflation is assumed to approximate the rate of monetary growth in the long run. In such a case, the government's real revenue from money creation in average period t is approximated by $\pi_t(M_t/P_t)$ and is referred to as the "opportunity cost" approach to seigniorage analysis. This is because, if the government announces a permanent expansion of the money growth rate, then it becomes more expensive to hold real money balances in the economy because of the higher inflation (ie: the opportunity cost of holding money increases). This derivation can also be viewed as a form of taxation in that Π can be considered the tax rate which is applied to the tax base, M/P[3]. Under such a scenario, the government optimizes its real revenue by inflating until the elasticity of real money holdings with respect to inflation is unity. This is very easy to show mathematically, if we can assume that the economy is in a long run or steady state position so that $\Pi=\Pi^e=\mu$:

$$S_t=\Pi_t\frac{M_t}{P_t}=\Pi_t f(y_t,R_t)$$

$$\frac{dS_t}{d\Pi_t}=f(y_t,R_t)+\Pi_t f_R\frac{dR_t}{d\Pi_t}=0$$

$$\Pi^*=\frac{-f(y_t,R_t)}{f_\Pi}\Rightarrow\frac{\Pi^*}{m_t}\frac{dm_t}{d\Pi_t}=-1$$

where:
$m_t=M_t/P_t$
$dR/d\Pi=1$ through Fisher effect

Here, a higher inflation rate tends to raise the government's real revenue directly but lower it indirectly as well since real balances fall. The maximum is achieved when the direct gain from an incremental increase in inflation, $f(y_t, R_t)d\pi_t$, is equal to the indirect loss, $\pi_t f_R (dR_t/d\pi_t)d\pi_t$ (ie: this occurs when the percentage increase in the inflation rate is equal to the percentage decline in real money holdings when analysed in absolute values). Therefore, anything that tends to increase the responsiveness of real money holdings to a change in inflation, reduces the optimal rate of inflation[4].

An interesting question now arises. How does the optimizing level of inflation change as the level of real income rises? To answer this, we can now take the derivative $d\pi^*/dy$ (from here on in I will ignore the time subscripts):

$$\frac{d\pi^*}{dy} = \frac{d}{dy}\left[\frac{-f(y,R)}{f_\pi}\right] = -\left[\frac{f_\pi \frac{dm}{dy} - m\frac{df_\pi}{dy}}{f_\pi^2}\right]$$

$$\therefore \frac{d\pi^*}{dy} = \frac{1}{f_\pi^2}\left[-f_\pi \frac{dm}{dy} + m\frac{df_\pi}{dm}\frac{dm}{dy}\right]$$

To determine whether the above function is positive or negative, we need to know the relationship between real money demand and inflation. If the relationship is that of convexity, then $f_\pi = dm/d\pi < 0$ and $df_\pi/dm = d(dm/d\pi)/dm < 0$. This, in turn, would imply two counter effects of dy on $d\pi^*$. The first of these is that if real income increases, then real money demand would rise and the government can gain revenue by accommodating it. This implies a higher optimal inflation rate. The second is that when the income increase causes real money demand to rise, $dm/d\pi$ falls implying more responsiveness of money demand to a change in the rate of inflation. As a result, the optimal inflation rate falls. Therefore the effect is ambiguous and relies on the exact magnitudes of the two effects. However, if a linear relationship was the case, only the first effect would apply and $d\pi^*/dy$ would be strictly positive. If the liquidity curve was concave, then the second effect would reinforce the first and $d\pi^*/dy$ would again be strictly positive and larger than the linear case.

2.3 OTHER CONSIDERATIONS

In reality, however, π may be a bad approximator of μ as other factors such as real economic growth and changes in velocity are also of importance. From the classical identity, MV=PY, we can derive the following relationship between monetary growth and inflation:

$$\mu=\pi+g_y-v$$

or
$\Pi=\mu-g_y+v$

where:
$g_y=(dy/dt)/y$ (growth rate of real income)
$v=(dv/dt)/v$ (growth rate of velocity)

From this identity, we find that growth of the economy or negative growth in velocity implies a lower associated inflation rate for any given rate of monetary expansion. For a given μ, a higher g_y implies that the economy can absorb more of the extra money so that inflation is not as high and a higher v leads to money changing hands more often thereby raising inflation through demand. Therefore, a more accurate representation of seigniorage would be of the following form[5]:

$$S=\mu\frac{M}{P}=(\Pi+g_y-v)\,f(y,R)$$

2.4 LONG RUN STEADY STATE ANALYSIS

Friedman(1971) employs a slightly different approach in his analysis of seigniorage in that a long run steady state is assumed[6] where money demand is equated to money supply. The exact money demand function used is:

$$m^D=\frac{M^D}{NP}=f(y,\Pi)\Rightarrow M^D=NPf(y,\Pi)$$

and this leads to the following functional form for monetary growth[7]:

$$\mu=\frac{d(\ln m^{s})}{dt}=\frac{d(\ln m^{D})}{dt}=g_{N}+\pi+\eta_{my}g_{y}$$

where:
N=population
g_N=growth rate of population
g_y=growth rate of real per capita income
y=per capita real income
η_{my}=elasticity of m with respect to y

Again, we can see how μ may vary from π due to factors such as possible absorption of extra money through positive levels of g_N, g_y and η_{my} which lead to higher money demand. From these monetary conditions, the relevant function to be maximized becomes:

$$S=\frac{M}{P}(g_{N}+\pi+\eta_{my}g_{y})$$

and doing so with respect to inflation results in the optimality condition of:

$$(g_{N}+\pi+\eta_{my}g_{y})\frac{d\log m^{D}}{d\pi}+g_{y}\frac{d\eta_{my}}{d\pi}=-1$$

If $g_y=g_N=0$, then the above equation reduces to $\eta_{m\pi}=-1$ and maximization occurs at this point which is the standard case explored initially. However, when population and real per capita income growth are non-zero and $d\eta_{my}/d\pi=0$, then positive levels of g_N and g_y reduce the optimizing level of inflation. In comparison to the standard case, non-negative values of g_N and g_y imply a higher growth rate of money demand and, therefore, of money supply. The growth rate of money supply is then too high when inflation is still at the standard optimal, $-1/(d\log m^D/d\pi)$, and the inflation rate must be reduced because the optimal monetary growth rate is now set at this standard value of $-1/(d\log m^D/d\pi)$. Therefore, if we can assume that $d\log m^D/d\pi$ has a constant value, then monetary growth becomes fixed at this level so that anything that adds to this monetary growth, such as positive levels of g_N or g_y, will reduce the associated inflation rate accordingly. This differs from the analysis in (2.2) where the change in the optimal inflation rate due to a change in real income levels was analysed. There, it was assumed that the monetary growth rate implied a synonymous level of inflation so that a higher real income level caused real balances to grow and, thus, a higher inflation level under the assumption that the liquidity curve was linear. In Friedman's analysis, growth is incorporated so that a long run level economy is no longer the case as it can now be characterized by a form of steady state growth. Therefore, the actual level of long run real income and, consequently, real balances are not determining forces of optimal monetary growth (except, insofar, as they may affect η_{my}). Rather, it is the growth of long run real income that becomes the relevant factor in optimal monetary growth determination and this μ is no longer synonymous with inflation.

When the assumption that $d\eta_{my}/d\pi=0$ is relaxed, then the results are not so clear cut. If the elasticity is lower at higher rates of inflation, then that would cause the optimal inflation rate to drop. If, on the other hand, $d\eta_{my}/d\pi>0$, then upward pressure would be put on the optimal inflation rate and the overall effect would be ambiguous. Also, the term $d(\log m)/d\pi$ is of extreme importance in the determination of optimal inflation. The more responsive money demand is to a change in inflation, the lower the optimal inflation will be, given everything else. This follows from the fact that real balances will be reduced by more for a given change in inflation and the government has to be wary of this in its policy decisions.

2.5 LONG RUN INTERTEMPORAL ANALYSIS

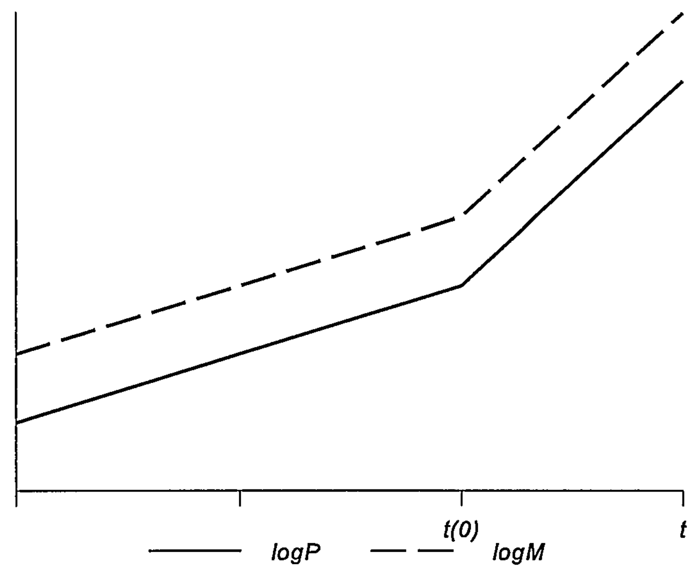
Another common way of analysing seigniorage is by way of an intertemporal analysis that attempts to maximize the present value of the revenue derived from money creation. In this case, such an optimizing configuration would be of the form:

$$\max S=\int \frac{\frac{dM_t}{dt}}{M_t}\frac{M_t}{P_t}e^{-\rho t}dt \text{ s.t. appropriate constraints}$$

ρ=government's social rate of time preference.

If the government's announcement of proposed monetary expansion is credible and, indeed, followed by the monetary authority, then the expected inflation rate should approximately equal the actual inflation rate. In this way, it is not necessary to look at long run average values as expectational lags will not figure into each period's seigniorage revenue from this 'open' inflation.

Auernheimer (1974) explores such an intertemporal optimization function but makes some assumptions about the transition period from an initial inflation level to an optimal rate. In particular he assumes that when a higher rate of inflation is announced, the government immediately buys the amount of real cash balances that are no longer demanded[8]. If we make the simplifying assumption that inflation is equal to the monetary growth rate, it can be graphically shown what Auernheimer's assumption implies:



At t_0, when higher monetary growth is announced, the government buys up the excess supply of money so that $\log M_t$ drops. However, there are no once and for all changes in the price level since the excess money is not being spent on private goods. Therefore, M_t/P_t falls but inflation still equals the money growth rate, ceteris paribus[9]. Some other assumptions that Auernheimer makes are as follows:

1) Constant income elasticity of real money demand.

2) Government is the only issuer of money.

3) $m_t = Ae^{\delta t}e^{[-b(r+\pi)]}$.

4) $d\delta/dt = d\pi/dt = dr/dt = 0$.[10]

5) $\delta<r$.

6) $\rho=r$

where:
$\delta=g_N+g_y g_N$ (growth factor)
g_N=population growth rate
g_y=per capita real income growth rate
r=real interest rate

Here, $S_t=dm_t/dt+\pi_t m_t$[11], so that the optimizing function becomes:

$$\max V_{t=0}=\int m_t(\pi+\delta)e^{-rt}dt+(m_{t=0+}-m_{t=0})$$

where:
$m_t=M_t/P_t$
S_t=real government revenue from seigniorage in period t
$m_{t=0+}$=real cash balances after announcement
$m_{t=0}$=real cash balances before announcement

The second term on the RHS follows from the assumption that government clears any increase or decrease in the economy's real balance position. The idea behind this is that if the government announces a lower rate of inflation, it experiences an initial windfall by supplying the extra cash balances desired and vice versa for a higher announced inflation rate. From this, Auernheimer concludes that the optimal inflation rate is independent of the growth of per capita income. In particular, $\pi^*=1/b-r$ and if we rearrange this optimal level of inflation, and assume that $dR/d\pi=1$, the optimality condition can be rewritten as

$(R/m)(dm/dR)=-1$. Notice that this is the same result as when $R(M/P)$, or the 'opportunity cost' approach to seigniorage with R as the 'tax rate', is maximized under the assumption of a long run steady state position of the economy.

Since the b variable can be explicitly defined as $(-d\log m^{p}/d\pi)$, the above result becomes directly comparable to that of Friedman's analysis. If population and real per capita income growth were non-existent, then Friedman's results were reduced to $\pi^{*}=1/b$. Therefore, Auernheimer's optimal inflation level is smaller than that of standard analyses and Friedman's 'no growth' case. However, the higher the population and per capita real income growth, the less will be the discrepancy between the two and for high enough growth levels, Friedman's optimal inflation level will become smaller than that of Auernheimer's, assuming constant income elasticity of real money demand. This can be more easily seen by viewing the two results simultaneously:

<u>Friedman:</u>

$$\pi^{*}=\frac{1}{b}-g_{N}-\eta_{my}g_{y}$$

Auernheimer:

$$\pi^{*}=\frac{1}{b}-r$$

Now, the inclusion of the term $(m_{t=0+}-m_{t=0})$ so that the average aggregate price level doesn't jump is a unique approach. The government is assumed to buy or sell any excess real cash balances but Auernheimer doesn't fully explain how this is achieved. It could be accomplished through the buying and selling of government inventory (asset) commodities in that if a higher rate of inflation is announced, the government buys up the extra real cash balances no longer desired by selling goods from its inventory. However, these goods must be sold at a "discount", since their value would be bid up by people unloading their monetary balances. Also, all proceeds from the sale must be kept out of circulation. If a lower inflation rate became the government's objective, they could add to their commodity inventory by buying goods at a "premium" financed by printing money. In this way, jumps in the price level could be avoided but both of these methods are somewhat unrealistic as the government doesn't really have a commodity inventory in the sense described.

2.6 CENTRAL BANK INCORPORATION

A more realistic approach to manipulating the money stock is through government open market operations in the bond market. If a higher rate of inflation was announced, people would want to get out of money and the government could accomodate this by selling bonds to the public. Although avoiding general one time price level movements, would this process eradicate any windfall gains or losses from the initial change in m? If inflation increased, the government could sell treasury bonds to the public while keeping the proceeds out of circulation. In this way, the government owes more money but sees nothing for it and would experience a direct loss[12]. However, the key here, once again, is that the money is kept out of circulation[13]. This is all fine and well but what if the decrease in the money supply is achieved by transferring government deposits from the private banks to the central bank? If this occurred then there would not be a windfall loss and the term $(m_{t=0+}-m_{t=0})$ could be omitted from Auernheimer's equation. Such a scenario would lead to a vastly different result. Namely, the optimal inflation rate would become a function of both δ and r:

$$\pi^{*}=\frac{1}{b}-\frac{\delta}{rb}-\delta$$

Again, notice that, in this case, growth reduces the optimal inflation level[14].

One major assumption that has been used throughout this chapter is that new money only gets into circulation via government expenditures. This is not necessarily true as the money supply is directly or indirectly controlled by the central bank in industrialized countries. Even though some central banks are under direct government control[15], this does not necessarily mean that the government would receive all the proceeds from new money creation. For purposes of analysis, a high powered money equation can be derived by setting up an asset-liability structure for the central bank where the monetary base is equated to non-monetary assets minus liabilities. Specifically, the equation is of the following form[16]:

Deposits of Banks + Notes = Securities + Advances + Foreign Assets -Deposits of Government + Float + Other Net Assets

where:
Float=Items in the process of collection - Items in the process of settlement
Deposits of Banks=Reserves

Notes=Currency

From this, we can see how the monetary base can be manipulated without any revenue accruing to the government. The central bank may increase advances to its private counterparts or add to its foreign asset inventory and the only way the government would see any of this new money is through the profit transfer mechanism[17]. Also, the government itself may alter the money supply by transferring funds between its private and public bank accounts and yet not see any change in its revenue position. Therefore, it is evident that all the preceding discussion may have to be altered to take account of the possibility that total seigniorage and government seigniorage may not be synonymous.

Martin Klein and Manfred J.M. Neumann(1989) take the above information and derive what they call an "accounting" approach to the seigniorage issue. They state that during 1987, .77 billion pounds in currency was issued by the bank of England whereas 1.05 billion pounds of "notes and coins" was used in budgetary finance. Also, for Germany, DM11.9 billion in new currency was issued whereas the German government received only DM.3 billion via a transfer from the Bundesbank. This is quite a discrepancy and shows how in any one period, base money creation can be very different from what the government actually uses in its budget[18]. To account for these discrepancies, Klein and Neumann set up their model as follows:

Change in monetary base equation:

$$\frac{dM}{dt}=\frac{dA}{dt}+\frac{dZ}{dt}+\frac{dD}{dt}+e\frac{dF}{dt}+N_m$$

Central bank profits:

$$R=aA+zZ+dD+efF+N_R-V-C$$

Government budget constraint:

$$G-T+bB_T+aA=\frac{dB_T}{dt}+\frac{dA}{dt}+R$$

where:
M=high powered money
A=lending to the government
Z=purchases of government debt in the open market
D=lending to the private sector
e=exchange rate
F=acquiring net international reserves
N_m=change in net balance of all other items
R=profits of central bank
b,a,z,d,f=relevant interest rates
N_R=all other net revenues
V=revaluation losses(gains) on international reserves
C=central bank's operating costs
G=government expenditures
T=total taxes
B_T=total stock of government bonds
$N=N_m-N_R$
B_p=private sector's holdings of government bonds=B_T-Z

From these 3 equations, they derive the following relationship between government and the central bank:

$$G-T+(bB_p-\frac{dB_p}{t})=\frac{dM}{dt}+(dD-\frac{dD}{dt})+e(fF-\frac{dF}{dt})-C-V-N$$

This, in turn, is used to estimate the amount of seigniorage that the government experiences:

$$S_G=\frac{(G-T+bB_p-\frac{dB_p}{dt})}{P}$$

where:
S_G=government seigniorage

Two different concepts of total seigniorage are used for analytical purposes. The first of these is an opportunity cost approach where $S_0=i(M/P)$ ("i" is a certain nominal interest rate; usually the risk free rate in the economy[19]) and S_G is incorporated into this to see if any discrepancies arise. The result is as follows:

$$S_0=S_G+c+(i-d)\frac{D}{P}+(i-f-\frac{\frac{de}{dt}}{e})\frac{eF}{P}+(i-g_{A+Z})\frac{A+Z}{P}$$

where:
$V=-(de/dt)F$
$N_R=N_m=0$
$c=C/P$
$g_{A+Z}=(dA/dt+dZ/dt)/(A+Z)$
b=z

Therefore, S_G may fall short of S_0 in a given period due to a number of factors such as central bank operating costs, lending to the private sector at a rate below that which is competitive or not getting a high enough return on the central bank's foreign assets. In addition to this, a positive value of $(i-g_{A+Z})(A+Z)/P$, in the words of Klein and Neumann, "represents the net loss of seigniorage which results from creating money through credit to the government". To see this more clearly, $(i-g_{A+Z})(A+Z)/P$ can be rewritten in the form $i(A+Z)/P-(dA/dt+dZ/dt)/P$. Here, the interest rate multiplied by the amount of government debt held by the central bank represents a loss of seigniorage for the government but this is offset by the issuance of credit to the government by the central bank. Obviously, once the central banking institution is incorporated into the problem, the opportunity cost approach to seigniorage is not the straightforward proxy that was once envisioned.

The second concept of seigniorage that Klein and Neumann go on to compare in this manner is that of the standard where $S_0=(dM/dt)/P$. They state that this type of "monetary seigniorage measures the actual wealth transfer which the private sector has to make in order to receive base money in the amount of M from the central bank". Now using this type of seigniorage, we get the following:

$$S_0 = S_G + c + v + \frac{\frac{dD}{dt} - dD}{P} + \frac{e(\frac{dF}{dt} - fF)}{P}$$

where:
v=V/P

From this, it can be seen that S_G will fall short of S_0, for a given period, due to a number of factors such as central bank operating costs and revaluation losses on the central bank's foreign reserves. Also, any increase in lending to the private sector or buildup of foreign assets by the bank will contribute positively to (S_0-S_G) as base money will have increased without the government having seen any of it for budgetary finance purposes. However, through the profit transfer mechanism, the government does realize the proceeds from interest earnings on the central bank's private sector lending and foreign asset inventory.

Since differences between private interest rates and those of the central bank can cause government seigniorage to differ from total seigniorage, it might be useful to look at North American history with regards to interest rate policy and its possible implications. Chant, Shearer and Bond(1984) do an in depth analysis of the Canadian financial system and in one part[20] state that "whereas in the U.S., money market rates generally exceeded the discount rate, markedly so in periods of monetary restraint[21], in Canada the bank rate almost always exceeded the comparable money market rates". Therefore, if total seigniorage is taken to be proxied by i(M/P), then the factor (i-d)D/P will tend to be negative for Canada and positive for the U.S. on average. Indeed, this is verified by the table 1 which shows (i-d) figures for Canada and the United States, and the difference between the two, over the period 1959-1994. Given everything else, this implies that S_G will tend to be higher relative to S_0 in Canada than in the United States. If (dM/dt)/P is thought to be the best measure of total seigniorage, then it appears that the same result would still hold. Since money market rates in Canada tend to exceed those of the U.S., the factor (dD/dt-dD)/P should be lower in Canada again implying that S_G will be higher relative to S_0.This conclusion, of course, assumes that everything else is the same between the countries in a relative manner and that D and dD/dt are not heavily influenced by the choice of d, which may be a bit extreme.

By referring to table 2, we see that average (i-d)D/P has negative values in Canada for all the time periods analysed thereby contributing negatively to s_0-s_G. By expressing this term as a percentage of i(M/P), we can proceed with a direct comparison to the same figures for the United States. In doing so, we can see how this factor tends to make s_0-s_G higher in the United States, in a relative manner, when assuming relative equality of all other factors. However, the percentages are extremely low in absolute value implying that the (i-d)D/P portion of this value of total seigniorage is extremely insignificant. When (dD/dt-dD)/P is expressed as a percentage of (dM/dt)/P, the results are not so conclusive as this percentage is higher for the United States, in absolute value, from 1959-1980 but becomes larger for Canada after that. Therefore, for both Canada and the United States, the central bank depository lending factors contribute positively to s_G-s_0, due to their negative values. However, this positive contribution is relatively stronger for the United States up to 1980, but relatively stronger for Canada after that.

This being said, there are certain results from this

Table 1

	UNITED STATES			CANADA			
DATE	3-MONTH T-BILL RATE	FEDERAL DISCOUNT RATE	i-d(U.S.)	3-MONTH T-BILL RATE	CENTRAL BANK RATE	i-d(CAN.)	[i-d(U.S.)]-[i-d(CAN.)]
1959	3.39	3.36	0.03	4.81	5.13	-0.32	0.35
1960	2.88	3.53	-0.64	3.20	3.54	-0.34	-0.31
1961	2.35	3.00	-0.65	2.81	3.06	-0.25	-0.40
1962	2.77	3.00	-0.23	4.05	4.48	-0.42	0.20
1963	3.16	3.23	-0.07	3.56	3.88	-0.31	0.24
1964	3.55	3.55	0.00	3.75	4.04	-0.29	0.29
1965	3.95	4.04	-0.09	3.98	4.29	-0.31	0.22
1966	4.86	4.50	0.36	5.00	5.17	-0.17	0.53
1967	4.31	4.19	0.12	4.64	4.98	-0.34	0.46
1968	5.34	5.16	0.18	6.27	6.79	-0.52	0.70
1969	6.67	5.87	0.80	7.19	7.46	-0.27	1.06
1970	6.39	5.95	0.44	5.99	7.13	-1.13	1.58
1971	4.33	4.88	-0.55	3.56	5.19	-1.63	1.08
1972	4.07	4.50	-0.43	3.56	4.75	-1.19	0.76
1973	7.03	6.44	0.59	5.47	6.13	-0.66	1.24
1974	7.83	7.83	0.00	7.82	8.50	-0.68	0.68
1975	5.78	6.25	-0.47	7.40	8.50	-1.11	0.63
1976	4.97	5.50	-0.52	8.87	9.29	-0.42	-0.10
1977	5.27	5.46	-0.19	7.33	7.71	-0.38	0.18
1978	7.19	7.46	-0.27	8.68	8.98	-0.30	0.04
1979	10.07	10.28	-0.21	11.69	12.10	-0.42	0.20
1980	11.43	11.77	-0.34	12.79	12.89	-0.10	-0.24
1981	14.03	13.42	0.61	17.72	17.93	-0.21	0.82
1982	10.61	11.02	-0.41	13.66	13.96	-0.30	-0.11
1983	8.61	8.50	0.11	9.31	9.55	-0.24	0.35
1984	9.52	8.80	0.73	11.06	11.31	-0.25	0.98
1985	7.48	7.69	-0.21	9.43	9.65	-0.22	0.00
1986	5.98	6.33	-0.35	8.97	9.21	-0.24	-0.10
1987	5.78	5.66	0.11	8.15	8.40	-0.26	0.37
1988	6.67	6.20	0.47	9.48	9.69	-0.20	0.67
1989	8.11	6.92	1.19	12.05	12.29	-0.24	1.43
1990	7.49	6.98	0.51	12.81	13.05	-0.24	0.75
1991	5.38	5.45	-0.07	8.73	9.03	-0.31	0.23
1992	3.43	3.25	0.18	6.58	6.78	-0.20	0.38
1993	3.00	3.00	0.00	4.84	5.09	-0.24	0.24
1994	4.25	3.60	0.65	5.54	5.77	-0.23	0.88
1995	5.49	5.21	0.28	6.89	7.31	-0.42	0.70
AVERAGE	6.04	5.99	0.04	7.50	7.92	-0.41	0.46

NOTE#1-CANADIAN FIGURES DERIVED FROM STATISTICS CANADA
NOTE#2-UNITED STATES FIGURES DERIVED FROM THE FEDERAL RESERVE
NOTE#3-ALL YEARLY FIGURES ARE 12-MONTH AVERAGES

Table 2

RELATIVE CENTRAL BANK LENDING TO DEPOSITORIES IN HIGH POWERED SEIGNIORAGE EQUATION

COUNTRY	DATE	HIGH POWERED SEIGNIORAGE (dM/dt)/P	[(dD/dt)-dD]/P	[(dD/dt)-dD]/P AS PERCENTAGE OF(dM/dt)/P	OPPORTUNITY COST HIGH POWERED SEIGNIORAGE i(M/P)	(i-d)(D/P)	(i-d)(D/P) AS PERCENTAGE OF i(M/P)
CANADA	1959-1965	6.066181456	-0.017558702	-0.289452304	206.5015575	-0.000465024	-0.000225192
	1966-1970	9.54185033	-0.028248226	-0.296045579	316.0615803	-0.00101683	-0.000321719
	1971-1975	28.33633032	-0.192088762	-0.677888632	402.2924159	-0.021744214	-0.005405077
	1976-1980	23.38606684	-1.21405543	-5.191362183	621.8493906	-0.016766274	-0.002696195
	1981-1985	6.159378081	-2.986870583	-48.49305471	587.4321262	-0.113209072	-0.019271856
	1986-1990	9.850155521	-34.29635099	-348.1808071	543.6900385	-0.332662227	-0.061186007
	1991-1994	8.499863526	-6.216941193	-73.14165897	455.783028	-0.098702074	-0.021655496
UNITED STATES	1959-1965	48.4165668	-19.17179446	-39.59759175	2599.010762	-0.441057251	-0.016970197
	1966-1970	111.5413099	-36.98922409	-33.16190578	3968.619558	1.58928416	0.040046271
	1971-1975	138.1947102	-43.99982584	-31.83900874	4280.308842	0.634880306	0.014832582
	1976-1980	148.312399	-25.04717947	-16.88812239	4674.845292	-0.063093724	-0.001349643
	1981-1985	117.9689006	-44.86964611	-38.03514815	4863.43729	0.90503984	0.018609057
	1986-1990	151.7223801	-23.47412978	-15.47176479	5016.442341	0.783635226	0.015621334
	1991-1994	197.7210079	-3.247674488	-1.642554083	4310.250823	0.074538301	0.001729326

NOTE#1-CANADIAN FIGURES DERIVED FROM STATISTICS CANADA(MILLIONS OF 1986 DOLLARS)
NOTE#2-UNITED STATES FIGURES DERIVED FROM THE FEDERAL RESERVE(MILLIONS OF 1982-1984 DOLLARS)
NOTE#3-THE D VALUES USED HERE ARE TOTAL CENTRAL BANK LENDING TO DEPOSITORY INSTITUTIONS

study that are troubling to me. The first of these is the immense difference between the two seigniorage measures. The reasoning for this could be the arbitrary choice of the nominal interest rate, which I have chosen as the three month treasury bill rate, or it may be the result of using the monetary base as my monetary figure, as Klein and Neumann do, and not M1 or M1A (using these monetary definitions, however, would change the structure of the total seigniorage equations). Another reason could be that since i(M/P) tends to be a 'long run' proxy of total seigniorage, using it for every time period in a historical analysis may be the wrong methodology. Another disturbing item is the huge negative (dD/dt-dD)/P percentages of (dM/dt)/P realized in certain periods (indeed, notice that for Canada between 1986-1990, this percentage skyrockets to -348%). Although these percentages are at least fairly stable in the United States, there is great variance in the corresponding Canadian figures which is largely due to the extremely high levels of Bank of Canada depository lending during the mid to late 1980's. Although I would like to analyse these problems in detail, I am forced by sheer magnitude to include this analysis as only a quick curiosity as full explanations could probably only be attained by analysing the full equations derived by Klein and Neumann (and by possibly using a more comprehensive central bank profit function) and this would be a thesis in itself.

2.7 CONCLUSION

Overall then, Klein and Neumann give a very enlightening analysis of how central bank incorporation into the seigniorage model can have vast consequences on the results of many fine economists due to simplified assumptions. In a one-period model, they show how different government seigniorage can be from either iM/P or (dM/dt)/P with regards to the monetary base. However, over a long run planning horizon, iM/P and (dM/dt)/P may be valid proxies for total seigniorage as any excess(deficiency) of S_0 over S_G may be reversed for some periods so that they average out over time. Also, the definiton of $S_G=(G-T+bB_p-dB_p/dt)/P$ may not be entirely representative of actual government seigniorage and the exclusion of N_R and N_m may not be appropriate as these terms comprise a large portion of a central bank's financial statements.

In the next section, I will switch over to another highly debated topic in monetary economics dealing with the welfare costs associated with inflationary finance. Approaching this topic in a way that culminates these costs with welfare effects of other government revenue raising means such as taxation and debt financing, it will be shown that optimization entails raising revenue through all sources. I will also look at studies that examine how well various national governments have "smoothed" their financing sources in order to cover their expenditures.

1.see, for example, Serletis(1988) pg.361.

2.In actuality, $R=\pi^e+r+r\pi^e$, but since $r\pi^e$ tends to be extremely small, R is approximated by $r+\pi^e$.

3.For more information on this view of an inflation tax, see Bailey(1956).

4.Other models sometimes use R(M/P) as an opportunity cost proxy to seigniorage. In such a case, the optimizing condition becomes:

$$\frac{R}{m}\frac{dm}{dR}=-1$$

5.A long run position is assumed here again so that although monetary growth may vary from $(\pi+g-v)$ in certain periods, the two can be expected to equate over time.

6.The assumption of a steady state implies that actual inflation equals expected inflation.

7.Inferred from the money demand equation is that the real interest rate is a constant. Also, in deriving the growth rate of the money supply, $d\pi/dt$ is assumed to equal zero. This implies that different steady states become incomparable.

8.Vice versa for lower announced inflation rates.

9.Log M_t takes an upward jump when a lower inflation rate is announced.

10.$d\pi/dt=0$ after adjustment occurs.

11.The derivation of this equation stems from the following procedure:

$$m_t=\frac{M_t}{P_t}=Ae^{\delta t}e^{-b(r+\pi)}$$

$$\frac{dm_t}{dt} = \frac{\frac{dM_t}{dt}}{P_t} - \frac{M_t}{P_t}\Pi_t$$

$$\therefore \frac{\frac{dM_t}{dt}}{P_t} = \frac{dm_t}{dt} + \frac{M_t}{P_t}\Pi_t = \frac{dm_t}{dt} + m_t\Pi_t = S_t$$

12.Vice versa for lower announced inflation rates.

13.Here, I ignore the possibility that this borrowing increases the real interest rates along the yield curve. Indeed, the effects may be so small that they would not pass any sensitivity analysis.

14.See Auernheimer(1974) for an analysis of how this result can be considered unstable if the federal government can, indeed, manipulate a commodity inventory readily.

15.Such as Great Britain where Klein and Neumann(1989) point out that the central bank is directly under the control of the government and must contribute to budgetary finance upon demand.

16.Taken from Bond, Chant and Shearer(1984)

17.The profit transfer mechanism is where the central bank gives all net revenues to the government.

18.Although, in the long run, the two may approximate each other.

19.Here, I use 'i' instead of 'R' due to shortness of letters.

20.See pgs.463-465 in Chant, Shearer and Bond(1984).

21.This is because the Federal Reserve discount rate tends to be altered at discrete intervals so that when short term interest rates rise, due supposedly to monetary restraint initiated by the Fed, continuously changing private rates tend to rise above the discount rate at intervals. This factor contributes to the procyclical nature of advances to the chartered banks from the Federal Reserve.

CHAPTER 3: THE MINIMIZATION OF WELFARE COSTS

3.1 INTRODUCTION

In the analysis so far, I have concentrated on inflation strictly from a government's viewpoint since they determine the growth of the money supply in the economy. However, inflation tends to have wide ranging effects on the populace and democratic governments must yield to the wishes of the aggregate or else they will not be able to implement their policies over the long run. Therefore, with regards to seigniorage, the objective may not be to maximize government revenue but rather to minimize the welfare costs associated with raising revenue through various sources such as taxation, debt financing and money creation. The problem with this perspective is in defining the welfare costs associated with the various government financing means.

A cost-benefit analysis of inflation tends to be a complex problem as there are many diverse opinions on the actual effects of inflation. Although it is possible that there are positive economic benefits from increased monetary expansion, on net the costs to society of inflation will be assumed, henceforth, to be positive due to the distributional deadweight losses and loss of consumer surplus because of the higher cost of holding liquid cash balances. I will not go into the social costs associated with taxation and deficit levels as an analysis as such could go on ad finitem so for the purpose of this paper, it will be assumed that taxation levels negatively affect social welfare[1] but the size of the deficit(or surplus) has no direct effect. Therefore, if a social planner attempts to minimize the welfare costs associated with raising a given amount of money, he(they) will obviously go as deep into debt as possible. To correct this situation, an intertemporal budget constraint is imposed on the problem where the present value of future government deficits is zero.

3.2 TAX SMOOTHING

In his paper, "On the Determination of the Public Debt"[2], Robert J. Barro(1979) pursues the above type of problem but does not take the possibility of seigniorage finance into account. Therefore, the objective of the paper is to find optimal tax rates and budget deficits. In particular, Barro sets up his model as follows[3]:

$$Z_t = F(T_t, Y_t) = T_t F\left(\frac{T_t}{Y_t}\right)$$

$$G_t+rb_{t-1}=T_t+(b_t-b_{t-1})$$

$$\sum_{t=0}^{\infty}\frac{G_t}{(1+r)^t}+b_0=\sum_{t=0}^{\infty}\frac{T_t}{(1+r)^t}$$

where:
Z_t=welfare costs(homogeneous)
T_t=total real tax revenue in period t(positive effect on Z_t)
y_t=aggregate real income in t(negative effect on Z_t)
r=real rate of return
b_t=real stock of outstanding public debt in t
G_t=real expenditure of government in t

and the objective becomes:

$$\min Z_t=\sum_{t=0}^{\infty}T_tF(\frac{T_t}{y_t})(1+r)^{-t}$$

subject to:

$$\sum_{t=0}^{\infty}\frac{G_t}{(1+r)^t}+b_0=\sum_{t=0}^{\infty}\frac{T_t}{(1+r)^t}$$

The first order conditions from such a problem determine that the marginal social cost of taxation be equated between time periods and that the tax-income ratio be the same in all periods. Overall then, the level of taxes in each period is determined by the given values of $(y_1,\ldots)$, $(G_1,\ldots)$, r and b_0 with the deficit for each period, b_t-b_{t-1}, found from these taxation levels.

If real income is constant, then taxes over time will be constant and the government budget will always be in balance(ie:$b_t = b_{t-1} = b_0$ so that initial debt is not amortized). On the other hand, if real income is expected to grow at ρ and government expenditures are expected to grow at δ, then taxes must also grow at ρ where $\delta \leq \rho \leq r$ is required to ensure the plausible scenario that $(G/y) < 1$ in every period. Also, the current level of taxation and deficit financing take on the following functional form:

$$t_0 = \left(\frac{r-\rho}{1+\rho}\right)\left(\frac{G_0(1+\delta)}{r-\delta} + b_0\right)$$

$$b_1 - b_0 = \rho b_0 + \frac{(\rho-\delta)}{(r-\delta)} G_1$$

where if ρ does not equal δ:

$$\frac{dt_0}{d\rho} < 0$$

$$\frac{dt_0}{d\delta} > 0$$

$$\frac{d(b_1 - b_0)}{d\rho} > 0$$

$$\frac{d(b_1 - b_0)}{d\delta} < 0$$

Here, the higher the expected growth rate of real income, the less will be the future expected cost of taxation implying that, under optimum, taxation should be deferred to later periods leading to an increase in the current budget deficit. A higher growth rate of government expenditures will lead to higher current taxation as future government financing needs will be collected throughout all periods thereby leading to lower current deficits.

For cases where government spending and/or income deviate from their trend growth rates for a certain number of periods, the same sort of results apply. Taxes must still grow with income and for a positive deviation in economic growth, current taxation levels should be negatively related to the duration of the departure interval(ie: number of years that the "boom" is supposed to last) and positively related to the extent of the boom. Extended periods of abnormally high government expenditures will lead to higher current tax levels as, once again, extra future money needs will be raised throughout all periods.

3.3 REVENUE SMOOTHING

In his paper entitled, "The Optimal Collection of Seigniorage: Theory and Evidence", N. Gregory Mankiw expands on the preceding analysis by including the option of inflationary finance in the government budget but he changes the welfare cost function so that it becomes dependant on the levels of inflation, taxation and income as opposed to only income and taxation. In this case, the government is assumed to minimize:

$$E_t\int_0^{\infty} e^{-\rho s}[f(\tau)+h(\pi)]y\,ds$$

subject to the intertemporal budget constraint:

$$\int_0^{\infty} e^{-\rho s}G_{t+s}ds+B_t=\int_0^{\infty} e^{-\rho s}T_{t+s}ds$$

where now:
$T=\tau y+(\pi+g_y)ky$
$M_t/P_t=ky_t$=real money demand
$(\pi+g_y)ky_t$=seigniorage revenue in period t
$f(\tau)y$=deadweight social losses associated with the tax ($f'>0, f''>0$)
$h(\pi)y$=social cost of inflation ($h'>0, h''>0$)
T_t=total real revenue for the government at time t
G_t=real government expenditure at time t (exogenous)
B_t=real government debt at time t
ρ=real social discount rate (constant)
E_t=expectations operator (with expectation being held at time t)
k=constant

From this problem, the same sort of optimum relationships as Barro's analysis arise. However, since the government has an alternative financing method, namely through the creation of money, not only does the marginal social cost of taxation have to be equated in every period but so does the marginal social cost of inflation. Also, the marginal social costs between the two alternate revenue raising methods must be identically related in every period or else it would be efficient to milk more of the required money out of the less costly source at the margin. Mathematically, these relationships are expressed as follows:

$$E_t f'[\tau_{t+s}] = f'[\tau_t]$$

$$E_t h'[\pi_{t+s}] = h'[\pi_t]$$

$$h'[\pi_t] = k f'[\tau_t]$$

These three equations make up what has come to be known as the "revenue smoothing" hypothesis of public finance in that an efficient government will raise revenue from all sources available to them thereby minimizing the costs on society(ie: taxes and inflation should move together over time).

Thus far, k has been assumed to be a constant but it would be more appropriate if k was made a function of the nominal interest rate or at least the level of inflation in each period. This would be a logical step as we could expect the level of real liquid balances to be related not only to income but also to the "cost" associated with holding this form of asset. Therefore, if we allow the money demand function to take the form $M_t/P_t=k(\pi_t)y_t$, we get the following optimizing conditions[4]:

$$E_t f'[\tau_{t+s}]=f'[\tau_t]$$

$$E_t\psi[\pi_{t+s}]=\psi[\pi_t]$$

$$\psi[\pi_t]=f'[\tau_t]$$

where[5]:

$$\psi(\pi)=\frac{h'(\pi)}{k(\pi)+(\pi+\rho)k'(\pi)}$$

Although more complicated, the same revenue smoothing type relationships still hold in that there are definite intertemporal relationship identities that must be satisfied under optimality.

Since results may often depend on the setup of the primary model, I think it would be useful to look at one more welfare cost problem in order to see if the same type of optimizing relations arise. Trehan and Walsh(1990) analyse revenue smoothing but set up their model in a slightly different manner. In particular, they look at a discrete model and analyse the costs of inflation in terms of the benefits of deflation so that the total welfare costs of raising revenue is of the following form:

$$Z=E_t\sum_{i=0}^{\infty} R^{-i}\left[\frac{\tau_{t+i}^{1+\alpha}\phi_{t+i}}{1+\alpha}-\left(\frac{P_{t+i}}{P_{t+1+i}}\right)^{1-\beta}\frac{\epsilon_{t+i}}{1-\beta}\right]$$

where:

$$\frac{\tau_t^{1+\alpha}\phi_t}{1+\alpha}=\textit{excess burden of taxes}$$

$$\left(\frac{P_{t-1}}{P_t}\right)^{1-\beta}\frac{\epsilon_t}{1-\beta}=\textit{benefits of deflation}$$

$$\alpha>0$$

$$\epsilon_t,\phi_t=\textit{stochastic disturbances}$$

and this is subject to the following budget constraint:

$$b_t=Rb_{t-1}+G_t-\tau_t y_t-s_t$$

$$\Rightarrow\sum_{i=0}^{\infty} R^{-i}E_t(\tau_{t+i}y_{t+i}+s_{t+i})=Rb_{t-1}+\sum_{i=0}^{\infty} R^{-i}E_t G_{t+i}$$

where:

G_t=real government expenditures in t (follows exogenous path)

$\tau_t y_t$=real tax revenue in t

$s_t=m_t-m_{t-1}(P_{t-1}/P_t)$=real seigniorage

b_t=outstanding stock of interest bearing government debt at the end of period t. This is assumed to be comprised entirely of one period bonds yielding a real constant return of r.
R_t=1+r=gross interest factor
m=(M/P)

Now, solving the above model for optimal relationships reveals:

$$\frac{\tau_t^{\alpha}\phi_t}{y_t(1+\delta)}=\left(\frac{P_{t-1}}{P_t}\right)^{-\beta}\frac{\epsilon_t}{m_{t-1}(1+\mu)}$$

$$E_t\left[\frac{\tau_{t+1}^{\alpha}\phi_{t+1}}{y_{t+1}}\right]=\frac{\tau_t^{\alpha}\phi_t}{y_t}$$

$$E_t\left[\left(\frac{P_t}{P_{t+1}}\right)^{-\beta}\frac{\epsilon_{t+1}}{m_t}\right]=\left(\frac{P_{t-1}}{P_t}\right)^{-\beta}\frac{\epsilon_t}{m_{t-1}}$$

where:
μ=elasticity of real money demand with respect to $[(P_{t-1}/P_t)-R]$
δ=elasticity of real income with respect to the marginal tax rate

Therefore, for intratemporal optimality, the marginal cost of raising revenue by one dollar from varying τ must equal the marginal benefit from lowering inflation revenues by one dollar. For intertemporal optimality to be realized, the expected marginal distortionary costs have to also be equalized over time. As expected, these relationships are no different than what we have seen from Barro and Mankiw in that the marginal costs of raising revenue between alternate revenue sources should be identically related and equated over time. The only difference is that Trehan and Walsh speak of the costs of inflation in terms of the benefits of deflation which is a unique twist but does not alter the underlying fundamentals of the results.

An extension to the analysis is then done by taking the natural logs of the optimizing conditions and applying a first order Taylor approximation on them resulting in:

$$\ln\tau_t = a_0 + \frac{\beta}{\alpha}\pi_t + \frac{1}{\alpha}[\ln y_t - \ln m_{t-1}] + \frac{1}{\alpha}[\ln\epsilon_t - \ln\phi_t]$$

$$E_t\ln\tau_{t+1} = \ln\tau_t + \frac{1}{\alpha}[E_t\ln y_{t+1} - \ln y_t] - \frac{1}{\alpha}[E_t\ln\phi_{t+1} - \ln\phi_t]$$

$$E_t\pi_{t+1} = \pi_t + \frac{1}{\beta}[E_t\ln m_t - \ln m_{t-1}] - \frac{1}{\beta}[E_t\ln\epsilon_{t+1} - \ln\epsilon_t]$$

where:

$$a_0 = \frac{1}{\alpha}\ln[\frac{(1+\delta)}{(1+\mu)}]$$

$$\pi_t = \ln(\frac{P_t}{P_{t-1}})$$

To me, this is an extremely useful step as it is now very easy to see the interrelationships between taxation and inflation(and between themselves over time) and how other factors readily affect them. For example, the relationship between taxes and inflation is not only dependant on α and β(which affect the social costs of taxation and inflation) but also the difference in their relative tax bases[$\ln y_t$-$\ln m_{t-1}$] and relative costs in terms of current economic distortions[$\ln \epsilon_t$-$\ln \phi_t$]. Also, the optimal tax rate will become a 'random walk' if the expected growth of output is 0 and distortionary tax costs are time invariant in that the best prediction of the future tax rate is its current value. Otherwise, expected growth or lower expected future stochastic distortion costs will tend to raise expected future tax levels if the social planner is working under optimality. With regards to inflation policy, expected future optimal inflation rises above current inflation as real money balances are expanded and/or as distortionary costs of inflationary finance are expected to fall.

3.4 PREVIOUS STATISTICAL LITERATURE

We have now seen how an optimizing government should behave if the social costs of its financing means are to be minimized. But how does this match up in reality? Do governments tend to raise revenues through various means in an efficient way or do they arbitrarily select methods which suit them at the time? For example, a democratic government may be more inclined to impose higher taxes earlier in their reign but resort to inflationary finance as elections near since these costs may not be realized until their possible next term in office. Also, the relevant theory that we have considered deals with an extremely long planning horizon but, in reality, social planners may only be looking at the next few years so that the present value of debt financing does not necessarily have to equal zero. Since debt financing is not included as a cost on society (even though most Canadians feel that it is), this could lead to a great deviation between theory and actuality.

Various tests have been done on the interrelationships between financing methods to see how well the optimization theory stands up in practice. For the United States, Barro(1979) analyses his tax smoothing model by estimating the following equation[6]:

$$\log\left(\frac{B_t}{B_{t-1}}\right)=\alpha_0+\alpha_1\pi_t+\alpha_2\left[\frac{P_t(G_t-\overline{G}_t)}{\overline{B}_t}\right]+\alpha_3\left[\log\left(\frac{Y_t}{\overline{Y}_t}\right)\left(\frac{P_t\overline{G}_t+r\overline{B}_t}{\overline{B}_t}\right)\right]$$

where:

$\overline{G}_t, \overline{Y}_t$*-values along trend growth lines*

$\overline{B}_t$*-average amount of debt for year t (nominal)*

P_tG_t, P_tY_t*-nominal values*

π_t*-average anticipated rate of inflation during t*

The assumption of constant coefficients in the above equation reflects the fact that the trend growth rate of real income(ρ) and the real interest rate(r) are assumed constant as is the duration of temporary expenditure and income deviations(k&η). It is expected that α_1 should be unity as expected inflation should increase the growth rate of nominal debt by an equal amount. Also, Barro expected a positive value for α_2 close to but below unity and the same for α_3 but in a negative manner. After running the regression for the period 1948-1976, the following results were obtained[7]:

α_0=.011(.01)
α_1=1.12(.22)
α_2=.610(.16)
α_3=-1.75(.17)
R^2=.87; SSE=.0124; σ=.022

With $R(1-\theta)$[8] used as a proxy for the anticipated inflation level, the results are extremely similar:

α_0=-.011(.013)
α_1=1.32(.25)
α_2=.77(.15)
α_3=-1.69(.17)
R^2=.88; SSE=.0117; σ=.022

From this, the numbers support the predictions extremely well although the α_3 coefficient is a bit high in absolute value. The high α_3 coefficient of -1.75, when the π variable is used, implies very large countercyclical debt responses in U.S. history "which exceed the amount that would be dictated purely from efficient public finance considerations"[9]. Therefore, it can be concluded that public debt has responded to temporary movements in government spending or aggregate income in a very stable manner. However, the trend growth rate of government spending includes the two world wars in its derivation and this leads to a negative value for the excess of actual government spending over its trend value in all periods between 1948-1976, except for the time of the Korean and Vietnam wars. Although this could be considered appropriate for Barro's analysis between 1916 and 1976, it may not be entirely correct when we are only dealing with the post second world war period as world wars may not be regarded as a regularly occurring phenomenon.

In addition to the above, Barro tests the hypothesis that past debt levels influence current debt levels by adding a variable comprising total real government debt divided by nominal average real income in period (t-1) to the estimating equation but he finds that the coefficient on this variable differs insignificantly from 0. It is, therefore, concluded that the debt income ratio moves randomly over time in response to income and expenditure shocks.

Mankiw(1987) estimates his revenue smoothing model for the United States by using data from 1952 to 1985. Rather than trying to estimate the relevant marginal costs of raising revenue and testing how well actual government policy stands up to the efficiency criterions outlined, he just looks for a positive correlation between inflation (or the 3 month treasury bill rate which he uses as an inflationary proxy) and taxation as, fundamentally, this is what the optimal theory predicts. The variables in question are defined as follows:

INT=3 month treasury bill rate

TAX=avg. tax rate=federal government receipts as a % of G.N.P.
INF=% change in C.P.I.
MAR=marginal average tax rate

Initially, he gets:

$$\underset{(5.9)}{INT=-26.1}+\underset{(.03)}{0.19TIME}+\underset{(.33)}{1.43TAX}$$

with an R^2 of .84. Since the above result does not reveal any meaningful conclusions about serial correlation, Mankiw applies the filter (1-0.5L)[10] to the above equation and obtains:

$$(1-0.5L)INT=\underset{(2.8)}{-11.3}+\underset{(.02)}{0.09TIME}+\underset{(.31)}{1.25(1-0.5L)TAX}$$

Here, the coefficient of determination, R^2, is only .66 but the Durbin Watson statistic does not show statistically significant serial correlation. To test the proposition that both independent and dependent variables may be actually caused by some control variable, the relation is tried in differenced form revealing:

$$\Delta INT=\underset{(.2)}{0.2}+\underset{(.28)}{1.13\Delta TAX}$$

A significant relation between INT and TAX is still realized but the R^2 drops to 0.31 so that only 31% of nominal interest rate movements are explained by average tax rate movements.

In addition to this, Mankiw also attempts to test the following two hypotheses:

1) Higher deficits lead to higher monetary growth and, therefore higher inflation. In this light, higher tax receipts should lower inflation and nominal interest rates if government expenditure is held constant.
2) Interest rates are passively determined by the business cycle.

In doing this, he defines the following:

EXP=federal government expenditure as a fraction of G.N.P.
RU=rate of unemployment

and includes them in the above regressions which result in:

$$\underset{(6.9)}{INT=-28.9}+\underset{(.05)}{0.18TIME}+\underset{(.4)}{1.75TAX}-\underset{(.21)}{0.23EXP}+\underset{(.23)}{0.31RU}$$

$$(1-.5L)INT=\underset{(3.8)}{-9.76}+\underset{(.03)}{.11TIME}+\underset{(.4)}{1.28}(1-.5L)TAX-\underset{(.22)}{.22}(1-.5L)EXP+\underset{(.26)}{.06}(1-.5L)RU$$

$$\Delta INT=\underset{(.21)}{0.23}+\underset{(.38)}{0.86\Delta TAX}-\underset{(.22)}{0.2\Delta EXP}-\underset{(.27)}{0.21\Delta RU}$$

With the inclusion of these variables, the relation between TAX and INT is still significantly positive but neither the expenditure nor the employment variable appear to have a significant relationship with INT.

Lastly, rather than using the nominal interest rate as a proxy for inflation, the percentage change in the C.P.I. is employed as the dependent variable. The results end up being very similar but the coefficients of determination end up being smaller which Mankiw attributes to the "noisiness" of the inflation variable. In particular,

$$\underset{}{INF}=\underset{(11.4)}{-33.1}+\underset{(.06)}{0.14TIME}+\underset{(.64)}{1.80TAX}$$

$$(1-.5L)INF=\underset{(5.0)}{-13.7}+\underset{(.04)}{0.08TIME}+\underset{(.56)}{1.48(1-.5L)TAX}$$

$$\Delta INF=\underset{(.4)}{-0.1}+\underset{(.49)}{1.44\Delta TAX}$$

Poterba and Rotemberg(1990) analyse an efficient revenue raising agenda for a government that wishes to minimize the social costs of their policies and come up with the following optimization condition in which government is assumed to commit to their announced policies:

$$\frac{P_t}{P_{t+1}}=\phi\left[\frac{h'(\theta_{t+1})m_t(1+\eta)}{y_{t+1}(1+\epsilon_\theta)}\right]$$

where:
P_t=price level in time t
h(.)=tax distortion which is increasing and convex in the tax rate
v(.)=benefits from deflation which is increasing and concave
$\phi=(-v')^{-1}$
θ=(tax revenue)/income=tax rate
m_t=real money balances in time t
η=elasticity of money demand with respect to the nominal interest rate

y_t=real income in time t
ϵ_θ=elasticity of income with respect to taxes

This equation is then rewritten in the form:

$$\ln\left(\frac{P_t}{P_{t-1}}\right)=\gamma_0+\gamma_1\ln\theta_t+\gamma_2\ln\left(\frac{m_{t-1}}{y_t}\right)+\mu_t$$

where:

$$\gamma_1=\frac{\alpha}{\beta}$$

$$\gamma_2=\frac{1}{\beta}$$

$$h(\theta_t)=k_1\theta_t^{\alpha+1}$$

$$v\left(\frac{P_{t-1}}{P_t}\right)=k_2\left(\frac{P_{t-1}}{P_t}\right)^{1-\beta}$$

$$k_1,k_2,\alpha,\beta>0 \wedge \mathit{constant}$$

and tested, not to see whether the theory can explain the exact actual policies, since h(.) and v(.) are specifically defined, but rather if it can explain significant movements in the relative variables. Also, it is expected that the level of inflation will positively respond to m_{t-1}/y_t as the theory predicts that the higher this ratio is, the less will be the costs associated with inflation. When the regressions are run for the U.S. from 1946-1986, they get the following results:

IN TREND FORM WITH θ=T/G.N.P.:

$$\ln\left(\frac{P_t}{P_{t-1}}\right)=.572+.320\ln\theta_t+.205\ln\left(\frac{m_{t-1}}{y_t}\right)+.007\mathit{TIME}$$

IN DIFFERENCED FORM WITH θ=T/G.N.P.:

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right)=.009+.334\Delta\ln\theta_t+.294\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

IN TREND FORM WITH θ=WEIGHTED AVG. MARGINAL TAX RATE ON LABOR INCOME (MTR):

$$\ln\left(\frac{P_t}{P_{t-1}}\right)=.382+.177\ln\theta_t+.170\left(\frac{m_{t-1}}{y_t}\right)+.004\mathit{TIME}$$

IN DIFFERENCED FORM WITH θ=MTR:

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right)=.0007+.184\Delta\ln\theta_t+.271\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

Overall then, from the three studies that I have outlined, there seems to be a significantly positive relation between taxation and inflation for the U.S. since the Second World War however these variables are defined. Although the extent of efficient public policy has not been tested, the United States has seemed to follow an efficient path in raising its revenues since the Second World War as they have drawn on all sources available to them in response to expenditure requirements. Indeed, this drawing on various sources was one of the main causes of the abolishment of the Bretton Woods system as U.S. inflation rose above the inflation rates of other countries and the U.S. was unwilling to devalue its currency enough.

That's good for the U.S., but what about other major industrialized countries? Poterba and Rotemberg(1990) estimate their same equation (from before for the U.S.) for France, Germany, Japan, and the U.K. and come up with the following regressions:[11]

FRANCE(1948-1985)-

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = -.332 - .681\ln\theta_t + .252\ln\left(\frac{m_{t-1}}{y_t}\right) + .013\,TIME$$

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right) = .011 - .589\Delta\ln\theta_t + .302\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

GERMANY(1954-1984)-

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = .175 - .041\ln\theta_t + .088\ln\left(\frac{m_{t-1}}{y_t}\right) + .0014\,TIME$$

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right) = .0014 - .084\Delta\ln\theta_t + .076\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

JAPAN(1955-1984)-

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = 1.264 + .313\ln\theta_t + .228\ln\left(\frac{m_{t-1}}{y_t}\right) - .008\,TIME$$

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right) = -.011 + .472\Delta\ln\theta_t + .187\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

UNITED KINGDOM(1947-1984)-

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = -.501 - .094\ln\theta_t + .425\ln\left(\frac{m_{t-1}}{y_t}\right) + .015\,TIME$$

$$\Delta\ln\left(\frac{P_t}{P_{t-1}}\right) = .015 - .130\Delta\ln\theta_t + .468\Delta\ln\left(\frac{m_{t-1}}{y_t}\right)$$

3.5 CONCLUSION

The results presented here are not very conducive to our assuming that most governments finance themselves in a way that the social costs resulting are lessened. In particular, France has a strong inverse relationship between tax rate levels and inflation. This could reflect the possibility that the French government finances their extra needs solely by taxes or by inflation depending on the times or the timing in their reign. Japan is the only country out of the four that has a significantly positive relation between taxation and inflation although the standard errors tend to be rather large. However, the money-income ratio coefficients are very similar between all five countries and appear to be quite high for Britain which is interesting as the British central bank is under the complete control of the British government.

1.Administrative costs of taxation can also be assumed to be included.

2.Barro(1979); "On the Determination of the Public Debt"; Journal of Political Economy; University of Chicago; vol.87.

3.Notice that there are no real effects of deficit financing implying that the Ricardian Equivalence Theorem is assumed to hold. Also, $F(T_t/y_t)$ is assumed to be time invariant.

4.This extension also taken from Mankiw(1987).

5.In doing a Hamiltonian optimization procedure, I get g instead of ρ in the denominator of ψ(π).

6.This equation stems from the optimizing condition from earlier in Barro's analysis:

$$\frac{B_1-B_0}{B_0}=\left[\frac{(1+\rho)}{(1+r)}\right]^k\left[\frac{P_1(G_1-\overline{G}_1)}{B_0}\right]-\left[\frac{(1+\rho)}{1+r}\right]^n\left[\frac{P_1\overline{G}_1+rB_0}{B_0}\right]\left[\frac{(Y_1-\overline{Y}_1)}{\overline{Y}_1}\right]+\rho+\pi$$

The reason that taxes are missing from this equation is that it is included implicitly in reduced form.

7.Items in parentheses are the standard errors.

8.R(1-θ)=estimate of the after tax nominal rate of return.

9.Taken from Barro(1979) "On the Determination of the Public Debt"; J.P.E.; University of Chicago; vol. 87; pg.963.

10.This filter was derived by using an autocorrelation coefficient of .5. The L implies the lagged value of the relevant variables.

11.Here, the relevant tax rate is θ=(TAX REVENUE)/G.N.P.

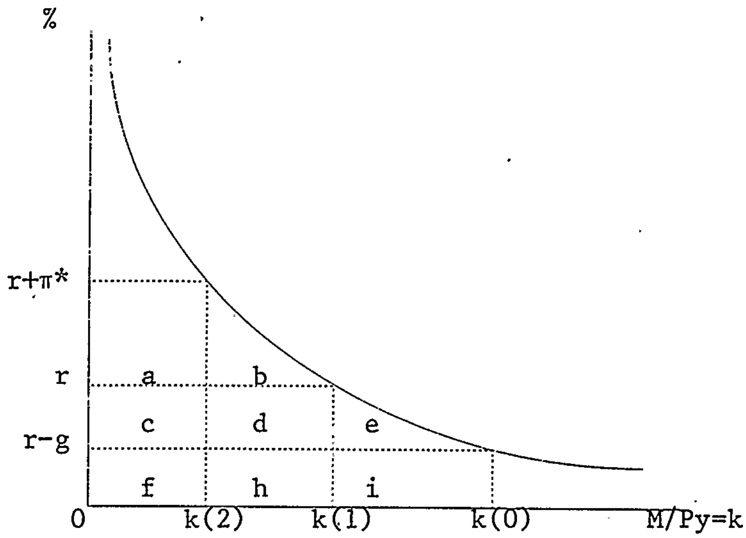
CHAPTER 4: A NET BENEFIT APPROACH TO REVENUE RAISING

4.1 INTRODUCTION

Initially, in this paper, I analysed the effect of money creation and, thereby, inflation on government revenue and how a government could proceed in order to maximize its revenue from monetary policy. From there, I assumed the government was more concerned with minimizing the social costs of their financing methods than maximizing their revenue and it was found that, under optimality, taxation levels and inflation should move together. In this section, I attempt to bring the two concepts together by examining a government that wishes to maximize the difference between revenue gains and welfare losses in choosing their tax and inflation levels. The major problem in looking at things in this perspective is being able to value welfare costs in dollar terms so that they can be compared to government revenue levels. Economically, this means that rather than being an ordinal utility ranking, welfare must be expressed cardinally in dollars so that each possible value carries a specific meaning as opposed to a relative meaning. Since my objective here is just to find common relations between government choice variables and not to derive actual definitive figures, I will assume that welfare costs can, indeed, be valued in this manner.

4.2 RELATIVE COLLECTION COSTS OF 'OPEN' INFLATION

Before I start my analysis, however, I think it would be beneficial to look at how other economists have analysed the costs and benefits of inflationary policy alone. In particular, under the assumption that government embarks on an 'open', credible inflationary policy so that actual inflation equals expected inflation, and that all nominal variables are adjusted accordingly, we can ignore the redistributive and uncertainty effects of this inflation that may be otherwise present. The only cost of inflation that will be considered here is that caused by the change in real money balances in the economy from their presumed non-inflationary levels. If we take a look at the following liquidity preference curve, where the variable on the horizontal axis is real money balances as a fraction of real income:



we can see how M/P responds to changes in the level of nominal interest rates, holding real income, y, constant or inflation, if we hold the real interest rate, r, constant as well (here, I assume a negative convex relation between the nominal interest rate, R, and k).

The liquidity preference curve shows, as Bailey(1956) states, "the subjective marginal rate of substitution of real goods for cash balances for everyone holding the latter". In other words, it shows the tradeoff between real money holdings and real goods foresaken in order to hold these real monetary balances. An inflationary policy amounts to a cost on the holdings of these balances and utility is subsequently reduced as the private sector is forced into holding less balances than they otherwise would (ie: hurting the convenience of having a certain amount of real liquid assets on hand). If we look at the 'open' implementation of the inflation rate, π^*, then the aggregate real loss to society, as a fraction of real income (ignoring the growth rate of real income, g, for now), will be the area under the curve from k_1 to k_2 or, alternatively, area b+d+h. If it can be assumed that seigniorage is proxied by $\pi(M/P)$, in the absence of growth, then seigniorage per unit of real income is equal to area a.

In Bailey's analysis, the welfare loss is expressed per unit of real government revenue so that an inflationary finance 'cost of collection' variable is derived. Graphically, this relative cost would be equal to (b+d+h)/a and can be regarded as an average cost of seigniorage. Using this variable, he then derives 'maximum desirable' rates of inflation for the countries he studied by using a specific Cagan type money demand function and a 7% collection cost figure where the 7% was determined by equating this relative collection cost to those associated with alternative forms of taxation. However, as Tower(1971) points out, it is not the average costs of alternative revenue sources that should be compared, but rather, the marginal costs. This was shown in the section on welfare cost minimization where it was concluded that 'revenue smoothing' would be the optimal method for government financing due to the equating of marginal costs over time. Although my analysis only took total welfare costs into account as opposed to total costs per unit of revenue obtained, marginal values are the standards of economic comparisons and determine the decision criteria under normal market structures. This idea will be looked at in more detail later in this section.

In his 1974 article, Charles D. Cathcart[1] analysed this type of issue but used a different cost determination. In particular, he assumed that money holders not only lose the foregone 'consumer surplus' but also the value of the inflation tax itself. In this way, he sets up a total tax burden per unit of real national income, z, which, from the graph above, would equal area a+b+d+h. From this, government revenue from the inflation per unit of real income is subtracted off to obtain a total 'cost of collection' per unit of real income figure, w, or area b+d+h. Therefore, w, can again be viewed as representative of the loss of utility due to reduced liquid real balances and the corresponding loss of aggregate convenience. However, this variable represents total costs as a fraction of income and not the cost-benefit fraction found in the analyses of Bailey and Tower.

Up to this point, we have not allowed for real income growth in the economy. By reverting back to the graph, it is seen that when this growth, g, is incorporated, government revenue from inflationary finance, as a fraction of real income, will now be equal to area a+c, assuming that relative seigniorage revenue is equal to $(\pi+g)k$. Tower(1971) takes this determination and concludes that the average cost of inflation, as a fraction of the revenue collected, will now be lower as government revenue will now have increased relative to a given total burden on society from this inflation level. Therefore, the 'cost of collection' (relative to actual revenue collected) will now equal $(b+d+h)/(a+c)$. However, although this average cost declines, Tower concludes that the marginal cost of inflationary finance relative to the revenue collected will actually be higher once growth is incorporated. His reasoning for this is "high inflation rates cause the target levels of real cash balances to shrink, and rob the government of part of its normal dividend from the non-inflationary growth of the money supply which economic growth permits". From this then, Tower concludes that growth weakens the case for inflationary finance relative to other government financing methods.

Cathcart(1974) criticizes Tower's analysis for not changing the total burden, per unit of real income, once growth is incorporated. His reasoning is that if the money supply was not being expanded, then inflation would be equal to -g. Therefore, the area under the liquidity preference curve relevant for an analysis on the 'cost of collection' would no longer be that from k_1 to k_2, but rather, from k_0 to k_2. In other words, although government revenue, as a fraction of real income, increases to a+c, the burden on society, as a fraction of real income, will be greater by the area e+i. Overall then, it cannot be concluded offhand whether this average collection cost will be increased or reduced. However, neither of these articles take into consideration the possibility that any real national income growth reduces the corresponding revenue optimizing level of inflation. We saw, in the section on government revenue maximization, that, in the absence of government instantaneously clearing the reduction of real cash balances to avoid initial price level jumps, the optimal inflation rate will be decreasing in the level of real income growth. Therefore, once this growth is incorporated into the model, seigniorage revenue, per unit of income, may be somewhere below a+c and the total burden, per unit of income, would then be less than b+d+h+e+i. Again, however, the effect on the average collection cost from this real economic growth is ambiguous.

What has been presented thus far here has just been a very quick overview of the cost-benefit relationship of 'open' inflationary finance under the assumption of perfect knowledge. Hence forth, I will get back to the analysis originally outlined in the introduction to this section. Although, I will not explicitly use the reduction in real cash balances as a cost on society, I will keep my total cost formulation generalized so one may think of these balance reduction costs as implicitly incorporated. In this way, the cost of the tax rates used may also be considered to implicitly capture the possible subsequent reduction of income or the utility foregone by people switching to 'underground' type activities.

4.3 SETUP

In deriving optimal relationships for a government concerned with maximizing the net gain from its choices of inflation and taxation levels, I assume the following identities:

1) Real Money Demand:

$$\frac{M_t}{P_t}=f(y_t, R_t)$$

2) Seigniorage:

$$\frac{\frac{dM_t}{dt}}{P_t}=(\Pi_t+g_t)\,f(y_t, R_t)$$

3) Welfare Cost Function[2]:

$$Z_t=z\left(\frac{T_t}{y_t}, \Pi_t\right)$$

4) Intertemporal Government Budget Constraint[3]:

$$B+\sum_{t=0}^{T} G_t(1+r)^{-t}=\sum_{t=0}^{T} T_t(1+r)^{-t}+\sum_{t=0}^{T} (\Pi_t+g_t)\,f(y_t, R_t)\,(1+r)^{-t}$$

With the relevant variables defined as:
Π_t=inflation rate in period t.
g_t=growth rate of real income in period t.
y_t=real income in period t.
R_t=nominal rate of interest in period t.
T_t=real tax receipts in period t.
r=real interest rate and assumed constant.
B=initial government real debt at t=0.
G_t=real government expenditure in period t.

From these identities, the intertemporal expected net gain function for a government social planner, in present value terms, becomes[4]:

$$N=E[\sum_{t=0}^{T}[(\Pi_t+g_t)f(y_t,R_t)+T_t-z(\frac{T_t}{y_t},\Pi_t)](1+r)^{-t}]$$

This, then, is what is to be maximized subject to the intertemporal government budget constraint.

4.4 OPTIMALITY

I derive the conditions for intertemporal net gain optimization using a Lagrangian procedure where the relevant condition to be examined is:

$$L=E[\sum_{t=0}^{T}[(\Pi_t+g_t)f(y_t,R_t)+T_t-z(\frac{T_t}{y_t},\Pi_t)](1+r)^{-t}]$$
$$+\lambda[B+\sum_{t=0}^{T}G_t(1+r)^{-t}-\sum_{t=0}^{T}T_t(1+r)^{-t}-\sum_{t=0}^{T}(\Pi_t+g_t)f(y_t,R_t)(1+r)^{-t}]$$

Now differentiating this with respect to the choice variables and equating to zero yields:

$$\frac{dL}{d\Pi_t}=[f(y_t,R_t)+(\Pi_t+g_t)f_R\frac{dR}{d\Pi}-z_\Pi](1+r)^{-t}$$
$$-\lambda[f(y_t,R_t)+(\Pi_t+g_t)f_R\frac{dR}{d\Pi}](1+r)^{-t}=0$$

$$\frac{dL}{dT}=[(\Pi_t+g_t)f_y\frac{dy}{dT}+1-z_{\frac{T}{y}}[\frac{1}{y_t}(1-\epsilon_{y,T})]](1+r)^{-t}$$
$$-\lambda[1+(\Pi_t+g_t)f_y\frac{dy}{dT}](1+r)^{-t}=0$$

where:

$$\epsilon_{y,T}=\frac{dy}{dT}\frac{T}{y}$$

and we can combine these two equations to obtain the following:

$$\lambda=1-\frac{z_{\Pi}}{f(y_t,R_t)+(\Pi_t+g_t)f_R\frac{dR}{d\Pi}}=1-\frac{z_{\frac{T}{y}}[\frac{1}{y_t}(1-\epsilon_{y,T})]}{1+(\Pi_t+g_t)f_y\frac{dy}{dT}}$$

$$\Rightarrow\frac{z_{\Pi}}{f(y_t,R_t)+(\Pi_t+g_t)f_R\frac{dR}{d\Pi}}=\frac{z_{\frac{T}{y}}[\frac{1}{y_t}(1-\epsilon_{y,T})]}{1+(\Pi_t+g_t)f_y\frac{dy}{dT}}$$

This, then, is the condition that the government should estimate and attain in choosing its inflationary policy if the objective is to be realized. However, this expression appears convoluted and it may be hard for the reader to derive a clear picture of what it implies. It is easier to see if we group the variables into respective marginal costs and benefits of taxation and inflation. Explicitly stated, these marginal effects are as follows:

1) Marginal cost of taxation on welfare:

$$MC_t=\frac{dZ_t}{dT_t}=z_{\frac{T}{y}}[\frac{1}{y_t}(1-\epsilon_{y,T})]$$

2) Marginal cost of inflationary finance on welfare:

$$MC_{\Pi}=\frac{dZ_t}{d\Pi_t}=z_{\Pi}$$

3) Marginal benefit of taxation due to effects on seigniorage[5]:

$$MB_T^2=\frac{dS_t}{dT_t}=(\Pi_t+g_t)\,f_y\frac{dy}{dT}<0$$

4) Marginal benefit of inflation through seigniorage:

$$MB_{\Pi}=\frac{dS_t}{d\Pi_t}=f(y_t,R_t)+(\Pi_t+g_t)\,f_R\frac{dR}{d\Pi}$$

5) Marginal benefit of taxation through government revenue:

$$MB_T^1=\frac{dT_t}{dT_t}=1$$

Taking the optimal condition and rewriting it in terms of these marginal effects, we get:

$$\frac{MC_{\Pi}}{MB_{\Pi}}=\frac{MC_T}{MB_T^1+MB_T^2}$$

It is now clearer as to what is being implied here. Optimum values are achieved when the marginal cost benefit ratios of inflation and taxation equate to one another. This makes sense as a one unit increase in inflation should lead to a change in the marginal inflation cost-benefit ratio equal to that of the effect of a one unit increase in taxation on the marginal taxation cost-benefit ratio. If say the LHS was larger, it would be feasible to raise taxation, relative to the level of inflation, and this would raise the RHS and lower the LHS (assuming, of course, that the numerators are increasing and denominators decreasing in their respective revenue sources and ignoring cross effects for now). This follows from economic fundamentals that you milk the less costly or most beneficial source at the margin. When we were only dealing with minimizing the social costs of various means of government financing, the objective was achieved by equating the marginal costs of inflation and taxation. Now that the benefits of inflation and taxation, from a government's standpoint, have been included, it is the ratio of marginal costs to marginal benefits that must be identical for inflation and taxation.

4.5 USING TAX RATES INSTEAD OF REAL TAX RECEIPTS

Many people would disagree with using total real tax receipts instead of tax rates as a government choice variable. In this section, therefore, I modify the government intertemporal net gain function so that tax rates become the choice variable instead of the real tax receipts. However, the same type of results can be expected in that the marginal cost-marginal benefit ratios for inflation and taxation should be equated in every period under optimality. To verify this, I modify the welfare cost and government revenue functions in the following way[6]:

$$Z_t = z(\Pi_t, \tau_t, y_t)$$

$$\$_t = (\Pi_t + g_t) f(y_t, R_t) + \tau_t y_t$$

Using these equations, then, the relevant Lagrangian function becomes:

$$L = E\left[\sum_{t=0}^{T} \left[(\Pi_t + g_t) f(y_t, R_t) + \tau_t y_t - z(\Pi_t, \tau_t, y_t)\right](1+r)^{-t}\right] \\ +\lambda\left[\bar{B} + \sum_{t=0}^{T}\left[G_t(1+r)^{-t} - \tau_t y_t (1+r)^{-t}\right] - \sum_{t=0}^{T}(\Pi_t + g_t) f(y_t, R_t)(1+r)^{-t}\right]$$

and solving this for net gain maximization by again differentiating with respect to the government choice variables and equating to zero:

$$\frac{dL}{d\Pi} = \left[f(y_t, R_t) + (\Pi_t + g_t) f_R \frac{dR_t}{d\Pi_t} - z_\Pi\right](1+r)^{-t} \\ -\lambda\left[f(y_t, R_t) + (\Pi_t + g_t) f_R \frac{dR_t}{d\Pi_t}\right](1+r)^{-t} = 0$$

$$\frac{dL}{d\tau} = \left[(\Pi_t + g_t) f_y \frac{dy_t}{d\tau_t} + y_t + \tau_t \frac{dy_t}{d\tau_t} - z_\tau - z_y \frac{dy_t}{d\tau_t}\right](1+r)^{-t} \\ -\lambda\left[y_t + \tau_t \frac{dy_t}{d\tau_t} + (\Pi_t + g_t) f_y \frac{dy_t}{d\tau_t}\right](1+r)^{-t} = 0$$

yields the following optimizing relationships:

$$\lambda=\frac{f(y_t,R_t)+(\Pi_t+g_t)f_R\dfrac{dR_t}{d\Pi_t}-z_\Pi}{f(y_t,R_t)+(\Pi_t+g_t)f_R\dfrac{dR_t}{d\Pi_t}}$$

$$=\frac{(\Pi_t+g_t)f_y\dfrac{dy_t}{d\tau_t}+y_t+\tau_t\dfrac{dy_t}{d\tau_t}-z_\tau-z_y\dfrac{dy_t}{d\tau_t}}{y_t+\tau_t\dfrac{dy_t}{d\tau_t}+(\Pi_t+g_t)f_y\dfrac{dy_t}{d\tau_t}}$$

$$\Rightarrow\frac{z_\Pi}{f(y_t,R_t)+(\Pi_t+g_t)f_R\dfrac{dR_t}{d\Pi_t}}=\frac{z_\tau+z_y\dfrac{dy_t}{d\tau_t}}{y_t+\tau_t\dfrac{dy_t}{d\tau_t}+(\Pi_t+g_t)f_y\dfrac{dy_t}{d\tau_t}}$$

This, then, is the condition which must be met under optimality. Under this scenario, the various marginal benefits and costs are redefined as follows:

$$MB_\Pi=f(y_t,R_t)+(\Pi_t+g_t)f_R\frac{dR_t}{d\Pi_t}$$

$$MC_\Pi=z_\Pi$$

$$MB_\tau^1=y_t+\tau_t\frac{dy_t}{d\tau_t}$$

$$MC_\tau=z_\tau+z_y\frac{dy_t}{d\tau_t}$$

$$MB_{\tau}^{2}=(\pi_t+g_t)\, f_y \frac{dy_t}{d\tau_t}<0$$

Now, once again, rewriting the optimal relation in terms of these marginal costs and benefits we get the exact same relationship as when total real tax receipts were one of the choice variables, namely:

$$\frac{MC_{\pi}}{MB_{\pi}}=\frac{MC_{\tau}}{MB_{\tau}^{1}+MB_{\tau}^{2}}$$

4.6 REVENUE SMOOTHING REVISITED

In chapter 3, it was concluded that taxation and inflation levels should be positively correlated over time if the central government has the intention of minimizing the social costs associated with raising fiscal revenue. Does the same optimization choice procedure still hold when we are dealing with a government whose objective is to maximize the net returns? In other words, should revenue sources still be smoothed over time? If we assume that taxation levels only affect the RHS of the marginal optimization condition and inflation only affects the LHS, then revenue smoothing would be the ideal scenario because both sides tend to be increasing in their relative subscripts so that higher tax levels will tend to lead to higher inflation if equality of the marginal ratios is to be maintained. This follows from standard economic assumptions such as:

1) $dR/d\pi=1$
2) $z_\pi>0$, $z_{\pi\pi}>0$
3) $z_\tau>0$, $z_{\tau\tau}>0$
4) $z_y<0$, $z_{yy}>0$
5) $f_R<0$, $f_{RR}<0$
6) $f_y>0$, $f_{yy}<0$
7) $dy/d\tau<0$, $d^2y/d\tau^2<0$

which imply the following:

$$\frac{dMB_\pi}{d\pi}=2f_R+(\pi+g)\frac{df_R}{dR}<0$$

$$\frac{dMC_\pi}{d\pi}=\frac{dZ_\pi}{d\pi}>0$$

$$\frac{dMC_\tau}{d\tau}=\frac{dZ_\tau}{d\tau}+\frac{dy}{d\tau}\frac{dZ_y}{dy}\frac{dy}{d\tau}+Z_y\frac{d^2y}{d\tau^2}>0$$

$$\frac{dMB_\tau^1}{d\tau}=2\frac{dy}{d\tau}+\tau\frac{d^2y}{d\tau^2}<0$$

$$\frac{dMB_\tau^2}{d\tau}=(\pi+g)\left[\frac{dy}{d\tau}\frac{df_y}{dy}\frac{dy}{d\tau}+f_y\frac{d^2y}{d\tau^2}\right]<0$$

However, due to the setup of the model, there are cross effects[7] that have to be considered. In particular, tax levels not only affect the RHS but also affect the marginal benefit of inflation in that higher taxation tends to have a negative impact on real income thereby leading to a decline in seigniorage revenue for a given inflation level. Mathematically, this is expressed as:

$$\frac{dMB_{\pi}}{d\tau}=f_{y}\frac{dy}{d\tau}<0$$

Inflation also tends to affect both sides because as inflation rises, not only does this put upward pressure on the LHS, but it increases the responsiveness of seigniorage to taxation level changes as well (via real income) and is shown by:

$$\frac{dMB_{\tau}^{2}}{d\Pi}=\frac{d}{d\Pi}[\frac{dS}{dy}\frac{dy}{d\tau}]=f_{y}\frac{dy}{d\tau}<0$$

Therefore, higher taxation tends to raise both the RHS and LHS of the optimizing equality while higher taxation tends to do the same as well. This, however, assumes that $MB_{\pi}>0$ so that seigniorage still rises from increases in inflation, via monetary growth changes (which is a safe assumption as we can expect optimal inflation to be below its revenue maximizing level since added social costs of inflation are now being taken into account within the objective function). As a result of this, there then exists two conditions under which revenue should be smoothed. The first of these is that the effect of tax changes on seigniorage is small relative to their effect on the taxation marginal cost-benefit ratio <u>and</u> the effect of inflationary changes on the responsiveness of seigniorage to tax changes is small relative to their effect on the inflation marginal cost-benefit ratio (accordingly weighted). The second condition is the exact opposite of the first in that the cross effects in both cases are larger than the direct effects. Mathematically, these two conditions can be represented as follows:

$$(1)\quad \frac{d}{d\Pi}\left[\frac{MC_{\Pi}}{MB_{\Pi}}-\frac{MC_{\tau}}{MB_{\tau}^{1}+MB_{\tau}^{2}}\right]>0$$

$$\frac{d}{d\tau}\left[\frac{MC_{\Pi}}{MB_{\Pi}}-\frac{MC_{\tau}}{MB_{\tau}^{1}+MB_{\tau}^{2}}\right]<0$$

$$(2)\quad \frac{d}{d\Pi}\left[\frac{MC_{\Pi}}{MB_{\Pi}}-\frac{MC_{\tau}}{MB_{\tau}^{1}+MB_{\tau}^{2}}\right]<0$$

$$\frac{d}{d\tau}\left[\frac{MC_{\Pi}}{MB_{\Pi}}-\frac{MC_{\tau}}{MB_{\tau}^{1}+MB_{\tau}^{2}}\right]>0$$

If both differentials above have the same sign, then a negative correlation between inflation and tax rates would be the ideal fiscal method that the public authority should strive for. By employing the differentiation implied, the two conditions above can be rewritten as:

$$(1)\quad \frac{[MB_{\tau}^{1}+MB_{\tau}^{2}]^{2}}{[MB_{\pi}]^{2}} > \frac{-MC_{\tau}\dfrac{dMB_{\tau}^{2}}{d\pi}}{MB_{\pi}\dfrac{dMC_{\pi}}{d\pi}-MC_{\pi}\dfrac{dMB_{\pi}}{d\pi}}$$

$$\wedge \frac{[MB_{\tau}^{1}+MB_{\tau}^{2}]^{2}}{[MB_{\pi}]^{2}} < \frac{(MB_{\tau}^{1}+MB_{\tau}^{2})\dfrac{dMC_{\tau}}{d\tau}-MC_{\tau}(\dfrac{dMB_{\tau}^{1}}{d\tau}+\dfrac{dMB_{\tau}^{2}}{d\tau})}{-MC_{\pi}\dfrac{dMB_{\pi}}{d\tau}}$$

$$(2)\quad \frac{[MB_{\tau}^{1}+MB_{\tau}^{2}]^{2}}{[MB_{\pi}]^{2}} < \frac{-MC_{\tau}\dfrac{dMB_{\tau}^{2}}{d\pi}}{MB_{\pi}\dfrac{dMC_{\pi}}{d\pi}-MC_{\pi}\dfrac{dMB_{\pi}}{d\pi}}$$

$$\wedge \frac{[MB_{\tau}^{1}+MB_{\tau}^{2}]^{2}}{[MB_{\pi}]^{2}} > \frac{(MB_{\tau}^{1}+MB_{\tau}^{2})\dfrac{dMC_{\tau}}{d\tau}-MC_{\tau}(\dfrac{dMB_{\tau}^{1}}{d\tau}+\dfrac{dMB_{\tau}^{2}}{d\tau})}{-MC_{\pi}\dfrac{dMB_{\pi}}{d\tau}}$$

From this, the term on the LHS of the above equations can be viewed as a type of determining criterion. Looking at the first condition above, if both inequalities have different signs, then revenue smoothing would be optimal, but if they are determined to have the same sign, revenue smoothing would be the opposite of efficiency. Although, the first assumption that might be made is that the direct effects should outweigh the cross effects, this is an ad hoc assumption and should not necessarily be concluded without an in depth analysis of the welfare cost and money demand functions. Therefore, if maximizing the net gain function outlined initially in this paper is to be the government's objective, there is no clear fiscal policy, which, if implemented, will necessarily achieve the desired goal of marginal cost-benefit ratio equality.

4.6 CONCLUSION

Overall then, optimality for the net benefit model is achieved by equating the marginal cost-benefit ratio of inflation to that of taxation. However, this is no easy task as it becomes necessary to define the exact slopes of the money demand and welfare cost functions with regards to all the variables contained within them. Also, before any marginal cost-benefit procedure can be implemented, the welfare cost structure must be cardinally indexed so that it may be comparable to other cardinal indexes such as total government revenue. Obviously, this is a very difficult assignment for anyone who wishes to implement some statistical evaluation on the net benefit scheme in order to derive optimal fiscal procedures for the government. Therefore, I will exclude any possible empirical review on this subject as such an analysis would have to be concentrated on in detail and, therefore, would be beyond the scope of this paper.

1.Although not considered here, the allowance for lags in expectations such that π^e does not necessarily equal π for a certain period(s) creates interesting results. For a discussion of these types of effects see Cathcart(1974) or game theoretical approaches by Barro(1983) and Barro-Gordon(1983).

2.This welfare cost function is assumed to be positively related to inflation and the tax revenue-income ratio.

3.The inclusion of the variable T_t in this function implies that the government is able to choose the level of real tax receipts in each period as opposed to just the tax rate or rates. Also, the finite time horizon of T is assumed high enough so that this intertemporal budget constraint achieves equality.

4.In looking at the present value of this net gain function, I assume that the social rate of time preference can be approximated by the real interest rate. Basically, this implies that the government's discount factor, in real terms, is equal to that of the private sector's.

5.S_t is defined as the government's real revenue from money creation in time t.

6.Here, I let government revenue in time t, $\$_t$, be equal to the sum of seigniorage revenue and real income tax revenue in that period. In addition to this, I assume that income taxes are the only source of tax revenue for the government and that these tax rates are not progressive but, rather, fixed over income levels. Although simplistic, this assumption allows us to conclude on general results without the model becoming unduly complex.

7.From this point on, I will refer to direct effects of inflation and taxation as those changes which effect their respective marginal cost-benefit ratios directly. Inflationary effects on the taxation marginal cost-benefit ratio and vice versa will be referred to as cross effects.

CHAPTER 5: EVIDENCE ON REVENUE SMOOTHING

5.1 INTRODUCTION

We have now seen many theoretical problems that central governments should take into account when forming their monetary policy. Economists in the past have set certain guidelines that the monetary authority should follow if it wants to maximize seigniorage revenue over time or if the minimization of social welfare costs is to be the objective. I then expanded the previous economic literature and, via a simple model, determined optimal policy under a net benefit scheme which takes both of the above goals into consideration. Now, the only statistical literature to this point has been on welfare cost minimization and, specifically, the degree to which inflation and taxation have moved together over time. These studies found positive correlations for the United States and Japan, a strong negative correlation for France, and weak negative correlations for Germany and Britain. In this section, the intent will be to continue with the analysis on inflation-taxation correlations because it would be beneficial to analyse the time series behaviour of the specific variables and also to include Canada in the empirical investigation.

From the concept of "revenue smoothing", it was determined that the marginal costs of raising revenue associated with different revenue sources should be identically related and equated over time. Therefore, under standard cost assumptions, it would be efficient to have inflation correlate positively with an appropriate tax rate measure. For my analysis, I use C.P.I. inflation and an aggregate average tax rate as opposed to individual tax rates (such as income tax rates, capital gains tax rates, interest income tax rates, sales tax rates, etc.) because I want to analyse and conclude about general relationships between seigniorage and taxation without going too deeply into the components that comprise both these variables. Indeed, a comprehensive study could be achieved by analysing the time series behaviour and marginal costs associated with all of the various taxes and forms of money creation which would be very interesting but beyond the scope of this paper. However, before analysing how inflation and taxation correlate over time, it's wise to look at each independent time series and extract inherent properties. In particular, it can be quite meaningful to determine whether shocks to a time series are temporary, so that unforeseen exogenous effects have no or declining influence on future values of that variable, or permanent, where these effects become locked into future values. Another related concern is whether or not the specific variables in question follow a time trend in that the value of a variable depends on what period it is measured in.

5.2 INTEGRATION

Before investigating the properties of the inflation and taxation time series', I would like to explain a little bit about the concept of stationarity versus non-stationarity with regards to levels, trends and differences. Taking a look at the following relationships between a variable, x_t, and other factors:

$$x_t=c+e_t \qquad (1)$$

$$x_t=c+at+v_t \qquad (2)$$

$$x_t=c+x_{t-1}+\mu_t \qquad (3)$$

where:
c=constant
e_t, v_t, μ_t=white noise disturbances

we can see that, in the first case, x_t can be considered stationary in "levels" because the expected value of x_t will be the constant, c, but will vary around this value due to random disturbances, the expected value of which will be 0. In the second case, x_t can be considered to be "trend" stationary because x_t no longer oscillates around a constant but rather a time trend. If 'a' is considered positive, then x_t will be expected to grow over time at a constant, stationary level but it obviously will not be stationary in levels because it is expected to grow over time. The third case exhibits "difference" stationarity because the expected value of x_t-x_{t-1} will be the constant, c^1. This concept of stationarity leads us into the idea of order integration in which "a series with no deterministic component which has a stationary, invertible, autoregressive moving average process representation after differencing d times, is said to be integrated of order d, denoted $x_t \sim I(d)$" (Engle and Granger(1984)). To see this more clearly, x_t would be I(0) in equation 1 above and I(1) in equation 3. An example of an I(2) series can be represented by:

$$(x_t-x_{t-1})-(x_{t-1}-x_{t-2})=c+e_t$$

or, equivalently,

$$\Delta x_t - \Delta x_{t-1} = c + e_t$$

To help understand the differences between a series that is I(0) and one that is I(1), we can look to the properties associated with them. If a series is I(0), then it will have a finite variance and innovations will only have temporary effects on the variable in question. If, on the other hand, a particular series was integrated of order 1, then its variance will be asymptotically unlimited and innovations will have permanent effects in that x_t will be the sum of all previous changes. Therefore, compared to an I(0) series, an I(1) series will tend to be rather smooth with dominant extended swings[2]. In addition to this, it is interesting to note that the sum of an I(0) and an I(1) series will always be (1):

let:

$$x_t = c + e_t$$

and

$$y_t = a + y_{t-1} + v_t$$

then,

$$z_t = x_t + y_t = a + c + y_{t-1} + e_t + v_t$$

Also, since

$$x_{t-1}=c+e_{t-1}$$

and

$$y_{t-1}=a+y_{t-2}+v_{t-1}$$

it follows that

$$z_{t-1}=x_{t-1}+y_{t-1}=a+c+y_{t-2}+e_{t-1}+v_{t-1}$$

therefore,

$$z_t=a+c+(a+y_{t-2}+v_{t-1})+e_t+v_t$$

$$z_t=a+(a+c+y_{t-2}+v_{t-1})+e_t+v_t$$

$$z_t=a+(z_{t-1}-e_{t-1})+e_t+v_t$$

$$z_t=a+z_{t-1}+\mu_t$$

where:
$\mu_t=e_t-e_{t-1}+v_t$ and is also white noise since it is the linear sum of white noise disturbances

In determining the integration order for the inflation and tax rate time series' in Canada, France, Britain and the United States, the Dickey-Fuller(DF) and Augmented Dickey-Fuller(ADF) unit root tests[3] were used. In particular, these tests are outlined as follows:

$$\Delta x_t = \alpha_0 + \alpha_1 x_{t-1} + \sum_{j=1}^{p} \beta_j \Delta x_{t-j} + \epsilon_t \qquad (4)$$

$$\Delta x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 t + \sum_{j=1}^{p} \beta_j \Delta x_{t-j} + \epsilon_t \qquad (5)$$

Here, there are two types of regressions in that (5) contains a time trend whereas (4) does not. Also, the choice of p is made to ensure that the disturbances are uncorrelated and for p>0, we have what is known as "Augmented Dickey Fuller" tests. For p=0, a "Dickey Fuller" regression is used. Now if we run the above tests and determine that α_1 is not significantly different from 0, then there is evidence for non-stationarity and a unit root[4]. Then, if we first difference Δx_t in equations 4 and 5, it can be determined whether there is evidence of a second unit root and so on. In running these tests for the various countries specified on inflation and the average tax rate, I obtain table 3[5] and will base my conclusions on the ADF tests as these are the ones that attempt to eliminate error correlation. Therefore, when the trend variable is included, I cannot reject non-stationarity for either

TABLE 3

UNIT ROOT TESTS IN LEVELS

			WITHOUT TREND (ACV 10%=-2.57)			WITH TREND (ACV 10%=-3.13)		
COUNTRY	YEARS	VARIABLE	DF	ADF	ADF LAGS	DF	ADF	ADF LAGS
UNITED KINGDOM	1957Q2-1994Q1	CPI INFLATION	-6.3385*	-1.8546	12	-6.3624*	-1.6092	12
		TAXRATE	-11.186*	-2.6834*	12	-14.531*	-2.2645	12
FRANCE	1965Q1-1994Q4	CPI INFLATION	-2.814*	-1.6978	2	-3.0776	-2.1129	2
		TAXRATE	-13.065*	-2.4105	8	-13.163*	-2.5642	8
UNITED STATES	1959Q1-1995Q2	CPI INFLATION	-3.7988*	-2.2101	3	-3.7689*	-2.1305	3
		TAXRATE	-2.2053	-1.8916	1	-3.5421*	-2.8435	1
CANADA	1950Q1-1994Q4	CPI INFLATION	-4.9154*	-1.9341	12	-4.9686*	-1.454	12
		TAXRATE	-3.242*	-2.8193*	1	-3.5517*	-3.0859	1

UNIT ROOT TESTS IN FIRST DIFFERENCES OF LEVELS

			WITHOUT TREND (ACV 10%=-2.57)			WITH TREND (ACV 10%=-3.13)		
COUNTRY	YEARS	VARIABLE	DF	ADF	ADF LAGS	DF	ADF	ADF LAGS
UNITED KINGDOM	1957Q2-1994Q1	CPI INFLATION	-	-3.9133*	12	-	-4.0696*	12
		TAXRATE	-	-3.4522*	12	-	-3.8302*	12
FRANCE	1965Q1-1994Q4	CPI INFLATION	-	-5.1462*	5	-	-5.381*	5
		TAXRATE	-	-5.6838*	8	-	-5.6633*	8
UNITED STATES	1959Q1-1995Q2	CPI INFLATION	-	-4.0487*	12	-	-4.1558*	12
		TAXRATE	-	-4.8202*	6	-	-4.7907*	6
CANADA	1950Q1-1994Q4	CPI INFLATION	-	-4.3408*	12	-	-4.5329*	12
		TAXRATE	-	-5.3524*	6	-	-5.3915*	6

NOTE: * indicates significance of the lagged coefficient
ACV 10% is the asymptotic critical value at the 10% level

variable in any of the countries. When the trend variable is excluded, I can just barely reject non-stationarity in the average tax rates for Canada and Britain. I then go on to test for second unit roots by first differencing the ADF regressions with respect to time and can strongly reject non-stationarity in the first differences of both variables for all countries[6]. Overall, then, I conclude that the tax rate and inflation time series are integrated of order 1 and are therefore characterized by all the properties pertaining to an I(1) series.

5.3 Cointegration

The above discussion of integration orders leads us directly into the concept of cointegration where two or more economic variables, although individually characterized by extensive movements over time, do not wander far apart from each other. If this is the case, then the variables in question are said to form a 'cointegrated system'. An example of such a scenario can be represented by looking at three month and six month futures prices. Although each of these prices can be expected to have high variance, especially as maturity nears, the two cannot drift too far apart from each other due to the presence of arbitrage opportunities. The methodology behind this concept "is to search for a linear combination of individually non-stationary time series' that is itself stationary. Evidence to the contrary provides strong empirical support for the hypothesis that the integrated variables have no inherent tendency to move together over time"[Serletis(1995)].

Algebraically, this can be depicted as follows:

$$z_t = y_t - \beta x_t$$

Here, z_t can be viewed as an equilibrium error and if this is stationary, then we would conclude that y_t and x_t cointegrate. This makes sense as the expected value of z_t can be considered equivalent to zero, or possibly some constant intercept value, and although it may stray from this value for a period or a number of periods, it will be expected to return to it over the long run. However, it is generally required that in such an analysis, y_t and x_t both be integrated of the same order, this being at least the first order. If y_t and x_t "are already stationary so that they are I(0), then the equilibrium error has no distinctive property if it is I(0)"(Engle and Granger (1984)). If z_t was determined to be non-stationary, say I(1), then a positive shock to the y_t series, causing it to rise above x_t for that period, would cause a positive shock on z_t and become engrained in the time series so that the expected value of z, in subsequent periods, would no longer be the same. The result would be the conclusion that y_t and x_t do not form a cointegrated system.

The specific tests that I will use for the cointegration analysis are:

$$TR_t=\alpha_0+\beta_1\pi_t+\mu_t \quad (6)$$

$$TR_t=\alpha_0+\alpha_1 t+\beta_1\pi_t+\mu_t \quad (7)$$

where:
TR_t=average tax rate in period t
π_t=inflation in period t
t=time trend

and from these I will look for stationarity in the residuals as this would imply cointegration between the average tax rate and inflation. In testing for residual stationarity, I once again use the Augmented Dickey Fuller tests[7] and these results are presented in table 4 as well as the R^2 and Durbin Watson statistics from regressions (6) and (7). In

TABLE 4
COINTEGRATION TESTS

REGRESSAND=CPI INFLATION
REGRESSOR=AVERAGE TAXRATE

					RESIDUAL UNIT ROOT t TEST		
COUNTRY	YEARS	SPECIFICATION	COEFFICIENT OF DETERMINATION	DURBIN WATSON	TEST STAT.	ACV 10%	LAGS
UNITED KINGDOM	1957Q2-1994Q1	CONSTANT, NO TREND	0.01226	0.9709	-1.18377	3.04	12
		CONSTANT ,TREND	0.02345	0.9723	-1.7293	3.5	12
FRANCE	1965Q1-1994Q4	CONSTANT, NO TREND	0.01739	0.2491	-1.6603	-3.04	2
		CONSTANT ,TREND	0.07476	0.2603	-2.0401	-3.5	2
UNITED STATES	1959Q1-1995Q2	CONSTANT, NO TREND	0.234	0.4325	-1.634	-3.04	11
		CONSTANT ,TREND	0.3943	0.6282	-2.5286	-3.5	11
CANADA	1950Q1-1994Q4	CONSTANT, NO TREND	0.01718	0.4621	-1.9584	-3.04	12
		CONSTANT ,TREND	0.06699	0.4916	-1.7318	-3.5	12

REGRESSAND=AVERAGE TAXRATE
REGRESSOR=CPI INFLATION

					RESIDUAL UNIT ROOT t TEST		
COUNTRY	YEARS	SPECIFICATION	COEFFICIENT OF DETERMINATION	DURBIN WATSON	TEST STAT.	ACV 10%	LAGS
UNITED KINGDOM	1957Q2-1994Q1	CONSTANT, NO TREND	0.01226	1.907	-2.3771	3.04	12
		CONSTANT ,TREND	0.2363	2.398	-2.3142	-3.5	12
FRANCE	1965Q1-1994Q4	CONSTANT, NO TREND	0.01739	2.355	-2.491	-3.04	8
		CONSTANT ,TREND	.0.02287	2.375	-2.5055	-3.5	8
UNITED STATES	1959Q1-1995Q2	CONSTANT, NO TREND	0.234	0.1811	-0.81061	-3.04	11
		CONSTANT ,TREND	0.8123	0.5657	-3.7453*	-3.5	1
CANADA	1950Q1-1994Q4	CONSTANT, NO TREND	0.01718	0.2002	-2.7861	-3.04	1
		CONSTANT ,TREND	0.2163	0.2512	-3.092	-3.5	1

NOTE #1: CPI LEVELS DERIVED FROM 3 MONTH AVERAGES
NOTE #2: RESIDUAL UNIT ROOT TESTS DO NOT CONTAIN CONSTANT OR TREND TERMS

the first analysis, I use inflation as the dependent variable and in the second, the average tax rate is made dependent, to see if any differences arise. In viewing the residual test statistics, I cannot reject non-stationarity for any country except the U.S. when equation (7) is used and the average tax rate is the dependent variable. From these results, I conclude that inflation and taxation have not formed a historically cointegrated system in the U.K., France or Canada but any strong conclusions for the U.S. are not achieved here.

5.4 Causality Tests

If we can't reject a unit root in the residuals, then the test of inflation on the average tax rate, or vice versa, will suffer from serial correlation. Because I want to also include some Granger type causality tests on inflation and the average tax rate to expand on the cointegration analysis, I have to modify the equations to correct for this serial correlation. This type of algebraic manipulation can be shown as follows. Given:

$$TR_t=\alpha+\beta\pi_t+\mu_t$$

and a first order autocorrelation[8] function of the form:

$$\mu_t=\rho\mu_{t-1}+v_t$$

where v_t=white noise disturbance, we have:

$$TR_t=\alpha+\beta\pi_t+\rho\mu_{t-1}+v_t$$

$$TR_{t-1}=\alpha+\beta\pi_{t-1}+\mu_{t-1}$$

$$\rho TR_{t-1}=\rho\alpha+\rho\beta\pi_{t-1}+\rho\mu_{t-1}$$

$$TR_t-\rho TR_{t-1}=(1-\rho)\alpha+\beta(\pi_t-\rho\pi_{t-1})+v_t \quad (8)$$

Therefore, since I couldn't reject a residual unit root (ie: that ρ=1) for any of the countries[9], (8) above reduces to:

$$\Delta TR_t=\beta\Delta\pi_t+v_t \quad (9)$$

and this is the equation that should be tested in order to obtain BLUE estimators. However, in running the Granger type causality regressions, I look at how past tax rate levels affect current inflationary levels, and vice versa. This would tend to make sense as we would not necessarily expect a change in inflation to be matched by a change in tax rates in a specific quarter but rather to lead up to each other over time. Although inflation can be somewhat changed almost continuously due to monetary policy, tax structures tend to be changed at discrete levels and very intermittently thereby lessening the confusion associated with them.

The specific Granger causality tests that I use here are specified as:

$$\Delta TR_t=\alpha_0+\sum_{i=1}^{r}\alpha_i\Delta TR_{t-i}+\sum_{j=1}^{s}\beta_j\Delta\pi_{t-j}+\mu_t \qquad (10)$$

$$\Delta\pi_t=\alpha_0+\sum_{i=1}^{r}\alpha_i\Delta\pi_{t-i}+\sum_{j=1}^{s}\beta_j\Delta TR_{t-j}+\mu_t \qquad (11)$$

and from these, we can look for directional causality between inflationary changes and tax rate changes. For example, if all the β_j's in (10) are significantly different from zero but are not in equation (11), then it can be concluded that there is causality from inflationary changes to tax rate changes (ie: inflationary changes precede tax rate changes) but not the other way around. An F test can be used to determine the degree of causality where the F statistic observed, if μ_t is white noise, is defined as:

$$F_{actual}=\frac{\dfrac{(SSR_r-SSR_u)}{s}}{\dfrac{SSR_u}{(T-r-s-1)}}$$

where:
SSR_u='unrestricted' residual sum of squares which are obtained from running regression (10) or (11) above.
SSR_r='restricted' residual sum of squares where restricted implies that the overall regression is run under the null hypothesis that the coefficients of all the lagged independent variables are zero.
T=number of observations
s=numerator degrees of freedom
T-r-s-1=denominator degrees of freedom

Taking a look at equation (10), for a specific number of lags and total sum of squares, if the β coefficients are all insignificant, then the SSR_r and SSR_u of its regression can be expected to be fairly close in value. This would lead to a low observed F value compared to if these same coefficients were significantly different from zero. Under the null hypothesis of no causality, an observed F value which is less than its critical value, for these specific degrees of freedom, would cause us to not reject the null and conclude that there is no causality.

Table 5 shows the results from running (10) and (11) for the four countries analysed previously. In running these regressions, I look at from 1 to 4 lags of the variables on the RHS (ie: r=s=1 for the 1 lag case,..., r=s=4 for the 4 lag case) and obtain the F statistics shown. Since these lag structures are arbitrarily chosen, it is not wise to base

TABLE 5

GRANGER CAUSALITY TESTS ON TAX RATE AND INFLATIONARY CHANGES

				F-RATIO			
COUNTRY	**YEARS**	**REGRESSAND**	**REGRESSOR**	**1-LAG**	**2-LAGS**	**3-LAGS**	**4-LAGS**
U.K.	1957Q4-1994Q1	INFLATION CHANGE	TAX RATE CHANGE	38.625*	21.513*	7.4*	4.684*
		TAX RATE CHANGE	INFLATION CHANGE	0.195	15.527*	1.226	0.712
FRANCE	1965Q3-1994Q4	INFLATION CHANGE	TAX RATE CHANGE	0.078	0.511	0.783	0.621
		TAX RATE CHANGE	INFLATION CHANGE	0.416	0.033	0.719	0.681
U.S.A.	1959Q3-1995Q2	INFLATION CHANGE	TAX RATE CHANGE	3.114	1.841	1.364	1.14
		TAX RATE CHANGE	INFLATION CHANGE	0.0097	0.474	0.808	0.557
CANADA	1950Q3-1994Q4	INFLATION CHANGE	TAX RATE CHANGE	0.2	0.095	0.476	0.516
		TAX RATE CHANGE	INFLATION CHANGE	0.053	1.328	0.903	1.313

NOTE #1: THE YEARS CORRESPOND TO THE FIRST LAG. EVERY SUBSEQUENT LAG HAS ONE LESS OBSERVATION AS THE FIRST IS DROPPED OFF.
NOTE #2: THE * SUPERSCRIPT INDICATES SIGNIFICANCE AT THE 5% LEVEL

any concrete conclusions on the nature of causality. However, it is possible to form generalized conclusions about causality and the appropriate lag structures to be chosen. In particular, the only country which exhibits any strong relationship is the U.K. where tax rate changes appear to precede inflationary changes.

A more scientific approach to determining causality is achieved via the use of a specific criterion that establishes optimal lag lengths to be employed in the analysis. This criterion, which can be referred to as a form of the Akaike Information Criterion (AIC), is defined as:

$$AIC=N[\log(\frac{SSE}{N})]+2K$$

and the basic idea is to choose the number of lags, r and s, which minimize it. In other words, an attempt is made to minimize the appropriately weighted residual sum of squares. In using up to twelve lengths for each of the variables on the RHS of (10) and (11), I decided on the values of r and s reported in table 6. Using these lag structures, I once again compute the F ratios to see if no causality can be rejected. I also report the AIC under the restricted case

TABLE 6

GRANGER CAUSALITY TESTS ON TAX RATE AND INFLATIONARY CHANGES

COUNTRY	YEARS	REGRESSAND	REGRESSOR	UNRESTRICTED AIC	RESTRICTED AIC	F-RATIO
UNITED KINGDOM	1958Q2-1994Q1	INFLATION CHANGE	TAX RATE CHANGE	-1310.624	-1293.515	19.72059*
		r=3	s=1			
	1958Q2-1994Q1	TAX RATE CHANGE	INFLATION CHANGE	-1373.379	-1373.899	1.436377
		r=3	s=1			
FRANCE	1965Q4-1994Q4	INFLATION CHANGE	TAX RATE CHANGE	-1258.153	-1259.58	0.55648
		r=2	s=1			
	1966Q1-1994Q4	TAX RATE CHANGE	INFLATION CHANGE	-1343.049	-1344.272	0.74138
		r=3	s=1			
UNITED STATES	1959Q4-1995Q2	INFLATION CHANGE	TAX RATE CHANGE	-1564.25	-1562.639	3.55475**
		r=2	s=1			
	1959Q3-1995Q2	TAX RATE CHANGE	INFLATION CHANGE	-1551.164	-1553.158	0.00973
		r=1	s=1			
CANADA	1950Q4-1994Q4	INFLATION CHANGE	TAX RATE CHANGE	-1809.242	-1811.119	0.12089
		r=2	s=1			
	1950Q3-1994Q4	TAX RATE CHANGE	INFLATION CHANGE	-1829.18	-1831.127	0.0531457
		r=1	s=1			

NOTE #1: * INDICATES SIGNIFICANCE AT THE 5% LEVEL
** INDICATES SIGNIFICANCE AT THE 10% LEVEL

TABLE 7

REGRESSAND= CHANGE IN INFLATION RATE(Z)
REGRESSOR= CHANGE IN AVG. TAXRATE(X)

COUNTRY	YEARS	VARIABLE	COEFFICIENT	S.E.	t-RATIO
UNITED KINGDOM	1958Q2-1994Q1	ZL	-0.55054	0.07151	-7.699
[.5034]		Z2L	-0.38802	0.07878	-4.925
		Z3L	-0.36317	0.07535	-4.82
		XL	0.17306	0.03897	4.441
		CONSTANT	-9.86E-05	8.65E-04	-0.114
UNITED STATES	1959Q4-1995Q2	ZL	-0.28936	0.07588	-3.813
[.244]		Z2L	-0.44136	0.07491	-5.892
		XL	0.14264	0.07566	1.885
		CONSTANT	-1.37E-05	3.49E-04	-0.03937

NOTE #1: THE L's IMPLY LAGS SUCH THAT L=1 LAG, 2L=2 LAGS, etc.
NOTE #2: TERM IN PARATHESES UNDER COUNTRY IS THE COEFFICIENT OF DETERMINATION

where no values of the regressor are found. Obviously, since the force behind the choice of lags is a minimal AIC, a lower criterion associated with the restricted case would imply no causality between the variables. Thus, in analysing the results, I find no significant causality between average tax rate changes and inflationary changes for France and Canada. However, I do find very significant causality from the tax rate changes to inflationary changes in the United Kingdom's history and weak causality (significant only at the 10% level) in the same manner for the United States. Also reported are the estimated coefficients (and subsequent t-ratios) of these regressions for the U.S and U.K. where it can be seen that any causality can be concluded to be in a positive manner.

5.5 CONCLUSION

Overall, the notion or theory that governments will tend to raise their desired funds in an efficient manner so that average tax rates and inflation, via monetary policy, move together does not stand up to statistical evaluation for Canada and France. However, there is some evidence that these two variables cointegrate in the U.S. when the tax rate is used as the regressand and a trend variable is included but the only thing that can be stated is that the results are not completely conclusive. In performing Granger causality tests, the only strong relationship between tax rate changes and inflationary changes is found in the U.K. where the former of these can be safely concluded to precede the latter. This is an extremely interesting result due to the dependency of the British central bank on the wishes of their government. Whether this causality stems from the government actually drawing on inflationary finance and taxation semi-simultaneously for financing purposes or whether these tax rate changes have direct effects on inflationary changes (ie: raising taxes actually causes higher inflation) is a good question, but again considered beyond the scope of this paper. Nevertheless, I have to say that since this causality strongly occurs in only one country out of the four, a conclusion that tax rate changes directly affect inflationary changes is not soundly based.

1.The variable $x_t=c+\beta x_{t-1}+\mu_t$ where $0<\beta<1$ would also be considered stationary over the long run because, since $\beta<1$, any exogenous shocks to the system would peter out over time and x_t would return to its equilibrium or average level.

2.See Engle and Granger(1984) for an in depth analysis of differing integration orders

3.A series that has a unit root is equivalent to being integrated of order 1, if it has a second unit root, it can be considered integrated of order 2, etc.

4.If non-stationarity cannot be rejected, then "standard asymptotic analysis cannot be used to obtain the distributions of the test statistics". See White(1993) pg.160.

5.All tests herein contained use quarterly data for the relevant time periods specified. For the U.S., France and Britain, this data was obtained via OECD statistics. Canadian data was retrieved from Statistics Canada.

6.10% asymptotic critical values were used in the analysis because I wanted to reduce the probability of TYPE II errors since I expected to find unit roots and, therefore, wanted to reduce the chance of accepting the null hypothesis when it is in fact false.

7.No intercept is used in this test since the expected value of the error term is equal to zero.

8.I use 'autocorrelation' and 'serial correlation' interchangeably.

9.I couldn't explicitly reject a first order autocorrelation coefficient of unity for the U.S., so ,therefore, it probably lies somewhere just below this value. However, for the causality tests that I will subsequently employ, a value of one would not be considered an inappropriate measure.

CHAPTER 6: CONCLUSION

We have now seen various optimal relationships that should be taken into account by central governments if they wish to proceed with inflationary type financing. If the government does not view the costs of 'open' inflation as being very large, they may be more inclined to expand the money supply at a rate which will maximize their seigniorage revenue over time. On the other hand, large perceived inflationary costs should tend to make the degree of monetary growth dependant on the cost structures of alternative revenue raising methods. If it is possible to value the social costs of inflation and various taxation methods in monetary terms, then the best of both worlds can be achieved in that the government, under the conditions outlined in chapter 4, is then able to maximize the net benefits resulting from differing financing methods. In the analysis presented here, it was assumed that government cares as much about the costs on society as they do for their own revenue intake. However, this assumption is not necessary as the net benefit objective function can be easily modified to account for weighting differentials between revenues and welfare costs.

In testing the concept of 'revenue smoothing' for Canada, France, Britain and the United States, I refrained from doing an in depth analysis of the social cost structures of seigniorage and taxation forms and, instead, concentrated on the relationship between C.P.I. inflation and the average aggregate tax rate. Although simplifying matters, the underlying theorem states that since marginal costs from different revenue sources should move together over time, then so to should the relevant variables under standard cost assumptions. In employing co-integration tests on these variables, I could find no strong evidence to reject non-stationarity in the residuals so it cannot be concluded that inflation and taxation rates form a co-integrated system in any of the countries. In performing Granger causality testing procedures on the first differences of these variables, it's found that tax rate changes strongly precede inflation rate changes in only one out of the four countries; namely Britain, whereas reverse 'causality' is not found in any country. The case for revenue smoothing among these four countries can, thus, be concluded to be quite weak. However, by modifying the model or estimating equation accordingly, different results may be achieved. In particular, actual monetary growth may be used as an explanatory variable instead of C.P.I. inflation and/or various tax rates could be used simultaneously in the analysis rather than only an average aggregate tax rate figure.

When analysing government policy, the goal of economists is to derive some efficient path which they then use to either advise the people in power or test to see how well governments have actually implemented the appropriate policies. However, in democratic societies, political considerations are many times dominant over economic considerations resulting in possible ad hoc, short term decision making as opposed to long term strategies. This is exemplified by comparing pre-election to post-election governments with regards to tax hikes and handouts. All the sound economic theory in the world will probably not be enough to deter the government from the views of strong special interest groups and this could lead to great discrepancies between actuality and optimal theory.

REFERENCES

Auernheimer,Leonardo."The Honest Government's Guide to the Revenue From the Creation of Money".Journal of Political Economy,vol.82,1974:pp.598-606.

Bailey,Martin J."The Welfare Cost of Inflationary Finance". Journal of Political Economy,vol.64,April 1956:pp.93-110.

Barro,Robert J."On the Determination of the Public Debt". Journal of Political Economy,vol.87,1979:pp.940-971.

Barro,Robert J."Measuring the Fed's Revenue From Money Creation".Economic Letters,vol.10,1982:pp.327-332.

Barro,Robert J."U.S Deficits Since WWI".Chap.10, Macroeconomic Policy,©1990;Harvard University Press Cambridge,Massuchusetts;London,England.

Barro,Robert J. and Gordon,David B."Rules, Discretion and Reputation in a Model of Monetary Policy".Journal of Monetary Economics,vol.12,1983:pp.101-121.

Barro,Robert J. and Gordon,David B."A Positive Theory of Monetary Policy in a Natural Rate Model".Journal of Political Economy,vol.91,No.4,1983:pp.589-609.

Calvo,Guillermo A."Optimal Seigniorage From Money Creation- An Analysis in Terms of the Optimum Balance of Payments Deficit Problem".Journal of Monetary Economics,vol.4,1978: pp.503-517.

Cathcart,Charles D."Monetary Dynamics, Growth, and the Efficiency of Inflationary Finance".Journal of Money, Credit, and Banking;vol.6,1974:pp.169-190.

Chappell,David."On the Revenue Maximizing Rate of Inflation".Journal of Money, Credit, and Banking;vol.13, No.3,August 1981:pp.391-392.

Dornbusch,Rudiger and Fischer,Stanley."Moderate Inflation". The World Bank Economic Review,vol.7,No.1,1993:pp.1-44.

Engle,Robert F. and Granger,C.W.J."Co-Integration and Error Correction: Representation, Estimation, and Testing". Econometrica,vol.55,1984:pp.251-276.

Fischer,Stanley and Modigliani,Franco."Towards an Understanding of the Real Effects and Costs of Inflation". Weltwirtschaftliches Archives,vol.114,1978:pp.810-833.

Frenkel,Jacob A."Some Dynamic Aspects of the Welfare Cost of Inflationary Finance".Money and Finance in Economic Growth and Development,vol.8,Ronald I. Mckinnon,1976:pp177-195.

Friedman, Milton."Government Revenue From Inflation".Journal of Political Economy,vol.79,1971:pp.846-856.

Ghosh,Atish R."Intertemporal Tax-Smoothing and the Government Budget Surplus: Canada and the United States". Journal of Money, Credit, and Banking;vol.27,No.4,November 1995:pp.1033-1045.

Klein,Martin and Neumann,Manfred J.M."Seigniorage: What Is It and Who Gets It".Weltwirtschaftliches Archives,vol.126, 1990:pp.205-221.

Mankiw,N.Gregory."The Optimal Collection of Seigniorage: Theory and Evidence".Journal of Monetary Economics,vol.20, September 1987:pp.327-342.

Marty,Alvin L. and Chaloupka,Frank J."Optimal Inflation Rates: A Generalization".Journal of Money, Credit, and Banking;vol.20,No.1,February 1988:pp.141-144.

McCallum,Bennett T.Monetary Economics: Theory and Policy. ©1989;Macmillan Publishing Company.

Melnick,Rafi and Sokoler,Meir."The Government's Revenue From Money Creation and the Inflationary Effects of a Decline in the Rate of Growth of G.N.P.".Journal of Monetary Economics,vol.13,1984:pp.225-236.

Mundell,Robert A."Growth, Stability, and Inflationary Finance".Journal of Political Economy,April 1965: pp.97-109.

Poterba, James M. and Rotemberg, Julio J. "Inflation and Taxation With Optimizing Governments". Journal of Money, Credit, and Banking; vol.22, No.1, February 1990.

Serletis, Apostolos. "New Tests of the Theory of Optimal Seigniorage". Economic Letters, vol.27, 1988: pp.361-365.

Serletis, Apostolos. "Export Growth and Canadian Economic Development". Journal of Development Economics, vol.38, 1992: pp.133-145.

Serletis, Apostolos and Zestos, George. "On the Gibson Paradox". (1995) Forthcoming in: Review of International Economics.

Shearer, Ronald A.; Chant, John F. and Bond, David E. The Economics of the Canadian Financial System: Theory, Policy and Institutions. Second Edition, ©1984; Prentice-Hall Canada Inc.

Tower, Edward. "More on the Welfare Cost of Inflationary Finance". Journal of Money, Credit, and Banking; vol.3, November 1971: pp.850-860.

Trehan, Bharat and Walsh, Carl E. "Seigniorage and Tax Smoothing in the United States: 1914-1986". Journal of Monetary Economics, vol.25, 1990: pp.97-112.

White, Kenneth J. Shazam: Econometrics Computer Program. User's Reference Manual Version 7.0, ©1993.