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Optimal Investment Under Uncertainty: An Application to Abatement Capital

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Optimal Investment Under Uncertainty: An Application to Abatement Capital" submitted by Andrew G. Bradford in partial fulfilment of the requirements for the degree of Master of Arts.

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ABSTRACT

When investments are at least partially irreversible or unexpandable, the firm's ability to respond to changing future conditions is often curtailed. This thesis examines such irreversibilities and unexpandabilities in the context of abatement capital where future taxes on emissions are unknown. A two factor extension of Abel, Dixit, Eberly, and Pindyck's (1995) model of investment under uncertainty is used to evaluate the effects of changes in the various prices of capital and Pigouvian tax rates on the incentives to invest in the two factors. Additionally, the influences of changes in the distribution of future taxes are examined. A particularly striking implication of the results is that, under certain assumptions concerning abatement technology, increases in present or expected future emissions taxes may paradoxically increase the present production of emissions.

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DEDICATION

to my parents

TABLE OF CONTENTS

List of Figures

List of Tables

CHAPTER ONE

1. INTRODUCTION

1.1 Introductory Discussion

Firm level investment decisions are sensitive to a multitude of direct and indirect factors pertaining to the prevailing market conditions, tax regimes, strategic values, and the underlying characteristics of the investment opportunities and their respective magnitudes. Another dimension of complexity is added when one considers the level of certainty, or more appropriately, the absence of certainty regarding many of these factors. In many cases, if conditions turn for the worse, large capital investments cannot simply be undone. For that matter, if conditions turn for the better, it is often costly to expand. Were it the case that investments could be easily undone or expanded, uncertainty would not be problematic since the firm would be able to costlessly react to whatever conditions prevailed. This notion of a curtailed ability to react to future conditions once an investment has been made must be internalized by the firm when making the investment decision in the first place.

Of particular interest are the implications of uncertainty regarding government policy. Regulations change, taxes increase and decrease, and laws are modified, often in ways which are unanticipated by firms. Moreover, future policies can have a bearing on current decisions, particularly if those decisions are partly irreversible or unexpandable. This thesis examines the incentives to invest and the effects of a specific type of policy uncertainty - uncertainty over future taxation of an externality, or bi-product[1] of production. In general, firms have control over two factors which influence the production of an externality - a production factor and abatement capital. The production factor may be any input or combination of inputs used in the production of the firms primary good - for

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[1]These 'bi-products' have several more common names in the literature. Another euphemism is non-market goods. More familiar terms include pollution, emissions, and effluents. I will use all of these terms interchangeably.

example, labour, machines, and other equipment. An increase in the use of this equipment, while increasing production of the primary product, will also increase production of the externality. Abatement capital is equipment like scrubbers, filters, and treatment facilities which mitigate the effects of production capital in the generation of, say, pollution.[2] In many cases, investment in this sort of equipment is at least partially irreversible. Once the abatement capital is installed it can be very costly to remove. Additionally, much of this equipment is highly specific to the plant in which it is employed, implying a reduction in its value to prospective buyers. Also, once the capital is installed it may be difficult or costly to install more (not unlike adding RAM to an existing computer), making the capital investment at least partially unexpandable.

Given these characteristics of abatement capital investment and the uncertainty surrounding future taxation, the firm must consider that the taxes on emissions which make investment in abatement capital optimal *today* , may change in the future. So, in a dynamic framework, how does the firm respond to uncertainty over future taxes when making its investment decisions? Put another way, how does the firm decide upon the appropriate tradeoffs between production capital and abatement capital, and between today's optimal decision for abatement capital and what may be optimal in the future?

The purpose of this thesis is to investigate precisely these issues. Toward this end, a two factor version of Abel *et al*'s (1995) two-period model of investment under uncertainty is employed. The model is then used to evaluate firm level behaviour in response to emissions taxation and uncertainty concerning future taxation.

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[2] The current aggregate level of investment in pollution abatement equipment is not trivial. Rutledge and Leonard (1992) estimate that spending for pollution abatement in the United States was $81 billion in 1990 (1987 dollars).

In order to set up and evaluate such a model, some level of understanding concerning both the nature of Pigouvian taxation *and* the investment behaviour of firms affected by the Pigouvian correction are required. Accordingly, the following two sections respectively explore the current theory and relevant literature pertaining to environmental regulation (specifically the effects of Pigouvian taxation) and investment behaviour.

1.2 Externalities, Pigouvian Corrections, Abatement, and Related Literature

It has long, if not widely, been recognized by economists and policy makers alike that some production bi-products have very real and adverse effects on their recipients. Coupled with this recognition is a set of policy prescriptions designed to mitigate the damaging effects of industrial production. In general, the goal of the policy maker is to strike a balance between the good of the industries and the good of the entities affected by industrial production. This is no easy task since some of the effects of pollution are largely quantitative, such as health related absenteeism and additional costs on water treatment, while others are qualitative, such as a reduction of wildlife population and loss of amenity value. Nevertheless, policy makers have embarked on many and varied remedial schemes with the hopes that the end results will be Pareto improving.

One such policy is a Pigouvian tax.[3] In somewhat broader terms, the firm is imposing an externality on the recipients of pollution. Since the firm does not internalize the full effect of the production of pollution, it will tend to produce a level which is more than what is implied by social optimality. A Pigouvian tax is designed to restore optimality by pricing the effects of pollution external to the firm and forcing their internalization by the firm. Since a cost is imposed for production of the externality, one would expect less to be produced *a priori*.

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[3]Other terms for this type of tax include, corrective tax, emissions tax, pollution tax, or excise tax on emissions.

There are a number of issues with respect to the value of optimal corrective taxes which will not be discussed here other than to say that the optimal tax depends to some extent on the tastes and preferences of the recipients of pollution. As these tastes may change over time, so too may the optimal level of taxation. This is one of the economic factors which contributes to the uncertainty of future taxes. Another may reside with the effects which stock pollutants may have. If the emission is of the type that accumulates over time, (for instance, greenhouse gases or heavy metals in riverbeds) a stock will form. If 'marginal damage' from pollution is increasing in the stock of pollution, taxes may increase over time. If, on the other hand, the marginal damage is decreasing in the stock (e.g. a lake can only become so dead), then taxes may decrease over time. There are of course other factors contributing to the uncertainty of future Pigouvian taxes such as political motivations. Nevertheless, the key notion for the purpose of this analysis is that there are several factors contributing to the uncertainty surrounding future Pigouvian taxes.

A number of studies have been performed which evaluate the effectiveness of corrective taxes in achieving the goal of efficiency. For instance, Hartwick (1990) uses a general equilibrium model to show that the introduction of a Pigouvian tax may result in a social optimum with a higher than initial level of pollution. The model is a two factor, two good, two person model. The intuition behind this result is that a corrective tax drives up the relative price of the good which generates the pollution as well as the factor prices. The agents in the economy each own one of the factors of production and the revenues from the corrective taxes are paid to each of them. Therefore, if one of the agents has a strong taste for the good which causes pollution, and her income increases as a result of a higher price for that good, she will consume more - thus pollution increases. Hartwick points out that the results depend upon the agents each having different preferences over the two goods and having incomes which are supplied by different factors. I would further point out that the results depend upon his method for introducing the tax. In his model the tax was not applied to the pollution directly, rather taxes were applied to the inputs of the production process which causes the pollution.

Kohn (1988) discusses several concepts which are central to this analysis. In particular, he uses a general equilibrium model to examine the effects of abatement on plant scale. His economy has two industries producing two goods in perfectly competitive environments where the output of one good generates emissions which adversely affect the production of the other good. Abatement is modeled as a percentage of total emissions and is a function of abatement inputs as well as the level of production. Kohn identifies two separable effects which cause abatement to influence the average scale of a firm. The first is an *input effect*. Kohn explains that the added costs of inputs used in abatement are comparable to an increase in fixed costs. Given that average cost curves are U-shaped, an increase in these abatement costs effectively increases the minimum efficient scale, thus increasing plant size. The other effect is what Kohn calls an *output effect*. This effect depends on whether the percentage reduction of emission from abatement is increasing or decreasing in the scale of the plant. He shows that if there are positive scale effects for abatement, then the efficient plant scale is further increased. If, however, there are negative scale effects, the input and output effects are in opposition. The model is then considered in the presence of Pigouvian taxes as well as quota-style controls. Again, the input effect increases the minimum efficient scale of the firm. The input effect may be accentuated or diminished by the output effect which functions in response to the average Pigouvian tax per unit of output.

Some studies have empirically searched for an hypothesized reduction in productivity resulting from the imposition of environmental regulation. In particular, Conrad and Morrison (1989) compared the impacts of abatement investment on productivity changes in Canada, The United States, and Germany. Their model used the shadow values from the constraints on emissions production to value the abatement effort. Their overall conclusion was that investment in abatement capital has depressed productivity growth in the three countries, but most notably in North America. Further, these productivity slowdowns were not as severe as previously expected.

These varying methodological frameworks provide an interesting context by which to compare the results from the model in this thesis. The next section continues the background discussion by exploring the investment literature.

1.3 Theories of Investment and the Irreversible Investment Literature

The traditional or *neoclassical* models of investment tend to belie many of the complexities inherent to actual investment decisions. The neoclassical approach to investment valuation dictates a standard protocol for evaluating investment opportunities. This approach prescribes evaluating the present value of the expected flow of revenues (or benefits) net of the present value of the expected flow of expenditures (or costs). The end result is termed the net present value, or more appropriately, the expected net present value. If this value is positive and of greater magnitude than that of competing investment opportunities, the decision should be made to invest. If investment is continuous, investment should be undertaken until the value of the last incremental unit of investment is equal to its unit or incremental cost. That is, invest to the point where marginal benefit equals marginal cost.

Such an approach, however, does not internalize the effects which the arrival of new information may have. The tidy world of neoclassical investment theory does not account for the many vagaries often faced by firms in practice. For instance, consider a firm which has decided to construct a plant (based on its positive expected net present value). Some time after the plant has been producing profitably, the price of one of the major inputs, for which there is no substitute, increases substantially. Management will then have to use this new information to make production decisions concerning a plant which was built on the assumption that expectations correctly and sufficiently characterized the future. In this example the expected flow of expenditures were not realized. This arrival of new information is not explicitly accounted for in the typical net present value calculations. As such, when one considers the vast number of variables which behave stochastically such as input prices, commodity prices, interest rates, inflation, and taxes, it should come as no surprise that the neoclassical paradigm is increasingly considered a poor abstraction of the reality of investment decisions.

Given the above illustration it may be useful to outline the circumstances under which the net present value rule may be considered an appropriate analytical tool for evaluating investment opportunities. Dixit and Pindyck (1994) indicate that one of two implicit assumptions must be satisfied for the net present value (NPV) approach to be valid. First, the investment is reversible. That is, should actual earnings be lower than expected, the initial cost of the investment can be fully recovered. Second, if the investment is irreversible, then the opportunity to invest must be a now or never proposition. In other words, there is no opportunity to wait for new information. Dixit and Pindyck have shown that when the above assumptions are not satisfied (and in many situations they are not), the ability to delay an irreversible investment can profoundly affect the investment decision. The reason is that a firm with an opportunity to invest can be described as holding an option analogous to a financial call option - it has the right but not the obligation to invest in some project at any time, now or in the future. When the firm makes the decision to invest, it is exercising the option and foregoing any possibility of waiting for new information which may have affected the desirability of the investment. Exercising, or killing, an option implies the relinquishment of an asset (the value of the option), hence represents a cost which must be included as a cost of the investment. The NPV rule, therefore, may be modified in such a way as to incorporate the value of the lost option. The new rule would then be to invest when the net present value exceeds zero by an amount no less than the value of keeping the opportunity to invest alive. As such, in the presence of irreversibility and uncertainty, a firm may optimally delay an investment which has a positive NPV.

In the same manner that sunk costs provide the investing firm with a real option analogous to a call option, sunk benefits provide an option analogous to a put option. Notice that the existence of sunk costs allows for (in fact, necessitates) a situation where investment is suboptimal in the presence of a positive NPV. By symmetry, the existence of sunk benefits implies a scenario where it may be advisable to invest even though the net present value of the project is negative. This point is supported by Pindyck (1993) and by Dixit and Pindyck (1994) with an example in environmental policy (interestingly enough). They showed that sunk benefits may exist where environmental damage is irreversible. The argument is quite lucid. The real costs of a species extinction is uncertain but the extinction itself is permanent or irreversible. It may therefore be optimal to incur costs to preserve the species and allow new information to arrive concerning its value even though the present value of the investment in preservation is negative.[4] In other words, allowing extinction today kills the option of allowing extinction in the future.

The notion of sunk costs has material importance to the model outlined in this thesis and merits some discussion. Capital investments may be considered irreversible for several reasons. The first is capital specificity. There are varying degrees to which the level of aggregation of this specificity is applied. For example, marketing and advertising costs are completely sunk but investments in office furniture are not since it is readily resalable. Even if an entire plant were for sale, other firms are likely to be subject to the same market conditions which induced the original firm to want to sell it in the first place. In other words, the most that another firm would be willing to pay for the plant is its NPV less its

[4]"Value" here does not imply strictly commercial value. Rather it implies amenity value, commercial value as well as the marginal value of a diversified ecosystem.

option value, but the operating plant is worth its NPV to the present owner so it is not likely to sell - hence the investment is sunk. The second reason for irreversibility is the 'lemons effect', or more formally, an adverse selection effect. The selling firm has information about the quality of a specific piece of capital but the purchasing firm only has information concerning the average value. As such, the purchasing firm is unlikely to be willing to pay the full value of a relatively high quality unit of capital.

Much of the literature examining irreversible investment under uncertainty employs continuous time models which presume that key parameters follow specified stochastic processes. The tools of stochastic calculus and dynamic programming are then applied to solve a sort of optimal timing problem.

Abel *et al* (1995) provide an alternative framework for analyzing investment behaviour in this environment. They present a simple two-period, one factor model which may be analyzed using more standard and accessible techniques. Moreover, as opposed to the models explored by Dixit and Pindyck (1994), Abel *et al*'s model does not necessarily impose complete irreversibility. On the contrary, the firm can disinvest but the resale price of capital may be less than its acquisition price which makes reversibility a costly decision. Similarly, the firm may expand in later periods but the future cost of capital may be higher than its current acquisition price, making expansion costly. As a result, once uncertainty over future returns is considered, their model accounts for a richer set of options. The notion of expandability in future periods implies a call option - the right but not the obligation to invest in the future. When the firm installs capital which it may later resell, even though it may be costly, it has acquired a put option - the right but not the obligation to disinvest in the future. The internalization of both options in the model shows that options need not always serve to delay investment.

Their model also provides two distinct but equivalent ways of interpreting its features: option pricing theory, where both options are examined separately; and, q-theory, where q summarizes the incentives to invest. They demonstrate that the difference between their model and the standard naive NPV rule is simply that the NPV assumes that expectations will be correct. That is, once the investment is made, it cannot be altered. Conveniently, their model does not depend on the evaluation of option values to illustrate the differences between it and the naive NPV rule, although, approaching the model from this perspective does add some interesting insights.

1.4 Overview of the Thesis

Having provided some background for investment theory and externalities we may now proceed with the thrust of this disquisition. Chapter two presents a two period, two factor model of investment under uncertainty motivated by Abel *et al*'s (1995) model. The key difference from their model, however, is that a second factor, abatement capital, is introduced. This complicates the model but highlights the important interplay between abatement and production capital discussed earlier. A second and substantive difference is in the source of uncertainty. Whereas Abel *et al* model uncertainty in gross returns, my model introduces policy uncertainty - specifically, uncertainty regarding the future level of emissions taxation.

Chapter three explores some of the comparative statics of the model. Here it is shown that while the incentive to invest is decreasing in its own price, there exists ambiguity, or technology dependence, concerning the cross price effects (e.g. the effect that an increase in the cost of production capital has on the incentive to invest in abatement equipment). Another interesting result which arises from this analysis is that an increase in the first period Pigouvian tax has technology dependent influences on the incentives to invest in both types of capital. Moreover, these findings, in particular, are consistent with those of

Kohn (1988). The presence of increasing scale properties in abatement has an increasing effect on the scale of the firm. This has further implications for the effectiveness of corrective policies. However, we show here that in the absence of scale properties in abatement, the introduction of a Pigouvian tax decreases the scale of the firm, which is inconsistent with Kohn's findings. The implications of changes in the distribution of future taxes are also examined and it is shown that increases in expected taxes increase the incentive to invest in abatement capital but the corresponding effect on production capital is ambiguous. The chapter closes with a discussion of policy implications.

Lastly, a summation of the methodology, model, and results are provided in chapter four. The results obtained in the previous chapters will be discussed and their relative importance to policy makers explored. The chapter finishes off with a discussion of areas of potential research which could build upon the framework established here.

CHAPTER TWO

2. MODELING INVESTMENT UNDER UNCERTAINTY

This chapter presents a two factor extension of the basic model presented in Abel *et al* (1995) where the two factors are production and abatement capital. Section 2.1 discusses the model in general terms as well as some of the intuitive motivations for its design. The second period optimizing behaviour of the firm is explored in section 2.2. Section 2.3 examines the decisions facing the firm in the first period, some of which have substantive implications for second period returns. The chapter closes with section 2.4 which provides a summary and some concluding remarks.

2.1 General Presentation of the Model

In this chapter a two-period pollution flow model is developed where the firm must make investment decisions for a single production factor, which is simply referred to as either the production factor or production capital, and a pollution abatement factor, which is conveniently referred to as abatement capital[5]. The firm must make investment decisions over both of these factors in both periods. A Pigouvian tax is applied against the firm's production of a non-market good. The expenditure on taxes may be mitigated either by substituting away from the production factor or by employing more abatement equipment. The decisions are further complicated by assuming that the stock of abatement capital is only partially reversible and expandable, both to arbitrary degrees. This is characterized by a resale price on abatement capital which is no greater than the original purchase price and a future purchase price which is no less than the original purchase price. Lastly, there is uncertainty over the second period taxation level. It is implicitly assumed in the extreme cases that the non-market good may be banned (the unit tax is infinitely large) or infinitely subsidized (the unit tax is infinitely negative).

[5]For the convenience of the reader, a complete description of all the variables and functions is included in Appendix A.

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In the first period, the firm installs its production factor, K_1, at unit cost b, and its abatement capital, A_1, at unit cost c. First period returns, $R(K_1)$, are a function of the production factor only. A Pigouvian tax, τ_1, is applied to the production of some non-market good (generally this externality will be some kind of pollutant). Production of the externality is a function of both production factor investment and abatement capital investment, $h(K,A)$, where h is strictly increasing in K and decreasing in A. Hence, in the first period the firm will pay taxes equal to $\tau_1 h(K_1, A_1)$. No *ex ante* restrictions are being placed on the convexities or concavities of the externality production function.

The second period is very similar to the first. Once the stochastic second period tax, τ_2, is realized the firm may adjust the first period production factor level to its optimal second period level, K_2, and earn gross returns $R(K_2)$. The cost of purchasing more of the production factor is assumed to equal the resale price and the first period purchase price, b. That is, the production factor is fully expandable and reversible. Partial reversibility and expandability are modeled via the second period purchase and resale prices of abatement capital. It is assumed that the firm may purchase additional abatement capital at the unit price $c_H \geq c$, making expansion costly. Further, the firm may sell any amount of its abatement capital for the unit price $c_L \leq c$, making reversibility costly. The model is flexible so as to allow any arbitrary degree of irreversibility and expandability. Once the optimal values of the two factors are determined the firm will be assessed the appropriate tax payment, $\tau_2 h(K_2, A_2)$. Notice that the production technologies for the primary and external goods are not time dependent. That is, there is no innovation or technology adoption in the model.

Since the model is solved recursively, the following section will examine the period two optimizing decisions for the firm.

2.2 Second Period Optimizing Behaviour

It should be clear that second period returns must be evaluated in each of three possible regimes, the appropriateness of each depends on the realized level of the second period tax. First, the firm may find that second period taxes are relatively low. Arrival of this 'good news' means that the firm *may* find it optimal to sell some of its abatement capital (and produce more pollution), even at the potentially lower resale price. The flip side is that the firm might find second period taxes to be high relative to expectations. This 'bad news' *may* mean that the firm can reduce its costs by purchasing more abatement capital, even at the possibly higher purchase price. The last case is where second period tax is such that it is optimal to neither purchase nor sell abatement capital. Under such a scenario the stock of abatement capital remains fixed at its first period level, A_1.

This section is devoted to evaluating the second period optimizing behaviour of the firm. First, each of the above three scenarios will be individually assessed. After having assessed the possible scenarios, a criteria will be established which outlines the circumstances under which it is appropriate to buy, sell, or simply maintain the level of abatement capital. This will fully characterize the firm's second period optimal decisions.

To begin, suppose that the level of the second period tax is such that the firm finds it optimal to purchase more abatement capital. In this case, the second period levels of the production factor and abatement equipment are determined by solving the following problem:

$$\underset{K_2, A_2}{MAX}\left\{R(K_2) - b(K_2 - K_1) - c_H(A_2 - A_1) - \tau_2 h(K_2, A_2)\right\}. \tag{1}$$

This expression is the second period net returns to the firm given that it is optimal to purchase additional abatement equipment. They consist of the gross returns from the production factor less (plus) the cost (receipts) from increasing (decreasing) employment in the production factor, less the cost of expanding the stock of abatement equipment above its first period level, and less the taxation on externality production. The first order conditions for maximizing returns are,

$$R'(K_2^*) = b + \tau_2 h_K(K_2^*, A_2^*), \text{ and} \tag{1a}$$

$$-\tau_2 h_A(K_2^*, A_2^*) = c_H. \tag{1b}$$

Necessary condition (1a) tells us that the production factor should be adjusted such that its marginal returns equals its marginal (unit) cost plus the tax cost of its marginal effect on externality production. Condition (1b) has also been written with the marginal benefits on the left hand side (LHS) and the marginal costs on the right hand side (RHS). This condition indicates that abatement capital should be added to the point where the tax savings from the marginal reduction of externality production equals its unit cost.

The three sufficient conditions are,

$$R''(K_2^*) - \tau_2 h_{KK}(K_2^*, A_2^*) < 0, \tag{1c}$$

$$-\tau_2 h_{AA}(K_2^*, A_2^*) < 0, \text{ and} \tag{1d}$$

$$\left[R''(K_2^*) - \tau_2 h_{KK}(K_2^*, A_2^*)\right]\left[-\tau_2 h_{AA}(K_2^*, A_2^*)\right] - \left[-\tau_2 h_{AK}(K_2^*, A_2^*)\right]^2 > 0. \tag{1e}$$

These conditions place certain restrictions on the technologies in the model which are not independent of each other. For instance, from (1c) we notice that we may model increasing returns to scale from the production factor. However, this requires that production of the externality be convex in K. That is, it must be the case that $R''(K_2^*) < \tau_2 h_{KK}(K_2^*, A_2^*)$. So if

$R''(K_2^*) > 0$, then $h_{KK}(K_2^*, A_2^*) > 0$, as long as we confine ourselves to the case where taxes are positive. Further, if taxes are zero, then a solution requires that there are decreasing returns to scale in primary production. It may also be observed in (1d) that a solution requires that production of the externality is strictly convex in A. This is intuitively appealing since it permits the negation of the possibility of purchasing enough abatement equipment to drive externality production below zero.

Conditions (1a) and (1b) comprise two equations in two unknowns and as such, a general solution may be obtained. The general forms of the solution will be $K_2^* = K_2(\tau_2, b, c_H)$ and $A_2^* = A_2(\tau_2, b, c_H)$.

Now suppose that second period taxes were such that the firm found it optimal to disinvest in abatement capital. Maximizing second period net returns requires that the firm solve (2).

$$\underset{K_2, A_2}{MAX}\{R(K_2) - b(K_2 - K_1) + c_L(A_1 - A_2) - \tau_2 h(K_2, A_2)\} \tag{2}$$

Now instead of incurring a unit cost, c_H, for adding abatement capital, the firm receives a unit price, c_L, for disinvesting in it. The necessary conditions for maximizing returns are,

$$R'(K_2^*) = b + \tau_2 h_K(K_2^*, A_2^*), \text{ and} \tag{2a}$$

$$c_L = -\tau_2 h_A(K_2^*, A_2^*). \tag{2b}$$

The optimal level of capital is still determined by the same condition as in (1a). Condition (2b) may be interpreted in two equivalent ways. First, it could read that abatement capital must be chosen such that the marginal tax savings equals its opportunity cost. Second, abatement capital may be sold until the tax value from the ensuing increase in externality production just equals its resale price.

The sufficient conditions are the same as those presented in (1c) through (1e) - a well behaved system requires that they be satisfied whether buying or selling additional abatement capital.

The solution for problem (2) may be generally represented as $K_2^* = K_2(\tau_2, b, c_L)$ and $A_2^* = A_2(\tau_2, b, c_L)$. These are the same functional 'reduced form' representations as those given when the firm further invests in abatement capital except that they are evaluated at the resale price of abatement equipment instead of its purchase price.

The last of the three regimes which must be evaluated is the case where it is neither optimal to buy nor sell abatement equipment. In such cases the production factor is the only choice variable since the stock of abatement capital will be taken as given at A_1. Thus the problem for the firm is to,

$$\underset{K_2}{MAX}\left\{R(K_2) - b(K_2 - K_1) - \tau_2 h(K_2, A_1)\right\}. \tag{3}$$

This is identical to the previous two instances except that $A_2 = A_1$. The first order condition for maximizing net returns is,

$$R'(K_2^*) = b + \tau_2 h_K(K_2^*, A_1). \tag{3a}$$

Again, this is almost identical to (1a) and (2a) with the important difference that the stock of abatement capital is treated exogenously. This means that the only tool available to the firm for reducing its tax expenditures is to alter its employment of the production factor.

The sufficient condition for maximization is given in (3b). As in (3a), the only difference between (3b) and (1c) or (2c) is that it is evaluated at the first period's level of abatement stock.

$$R''(K_2^*) - \tau_2 h_{KK}(K_2^*, A_1) < 0 \tag{3b}$$

The solution to this maximization problem is somewhat different than in the previous two scenarios. The optimal level of the production factor when abatement capital is fixed will be given by $K_2^* = k_2(\tau_2, A_1, b)$.[6] While the level of abatement stock must be considered when choosing production capital, the resale or purchase prices of abatement equipment are no longer relevant.

The next step in the analysis is to consider *when* it is optimal to expand, reverse or simply maintain the first period level of investment in abatement capital. In other words, when and under what circumstances are the previous three scenarios relevant? As suggested earlier, for given resale and purchase prices of abatement capital (c_L and c_H), the decision hinges upon the realized value of the second period Pigouvian tax, τ_2. This portion of the analysis is not unlike the notion of trigger values as in Dixit and Pindyck (1994). However, unlike trigger prices in the irreversible investment literature, there are two trigger prices in this model since it is permitted to reverse investment as well as expand it. Using first order conditions (1b), (2b), and the solution to (3), define two critical values of the second period Pigouvian tax, τ_2^L and τ_2^H as follows,

$$-\tau_2^L h_A\left(k_2\left(\tau_2^L, A_1, b\right), A_1\right) = c_L, \text{ and } -\tau_2^H h_A\left(k_2\left(\tau_2^H, A_1, b\right), A_1\right) = c_H, \tag{4}$$

which may be written in their reduced forms, $\tau_2^L = \tau_2^L(A_1, b, c_L)$ and $\tau_2^H = \tau_2^H(A_1, b, c_H)$.

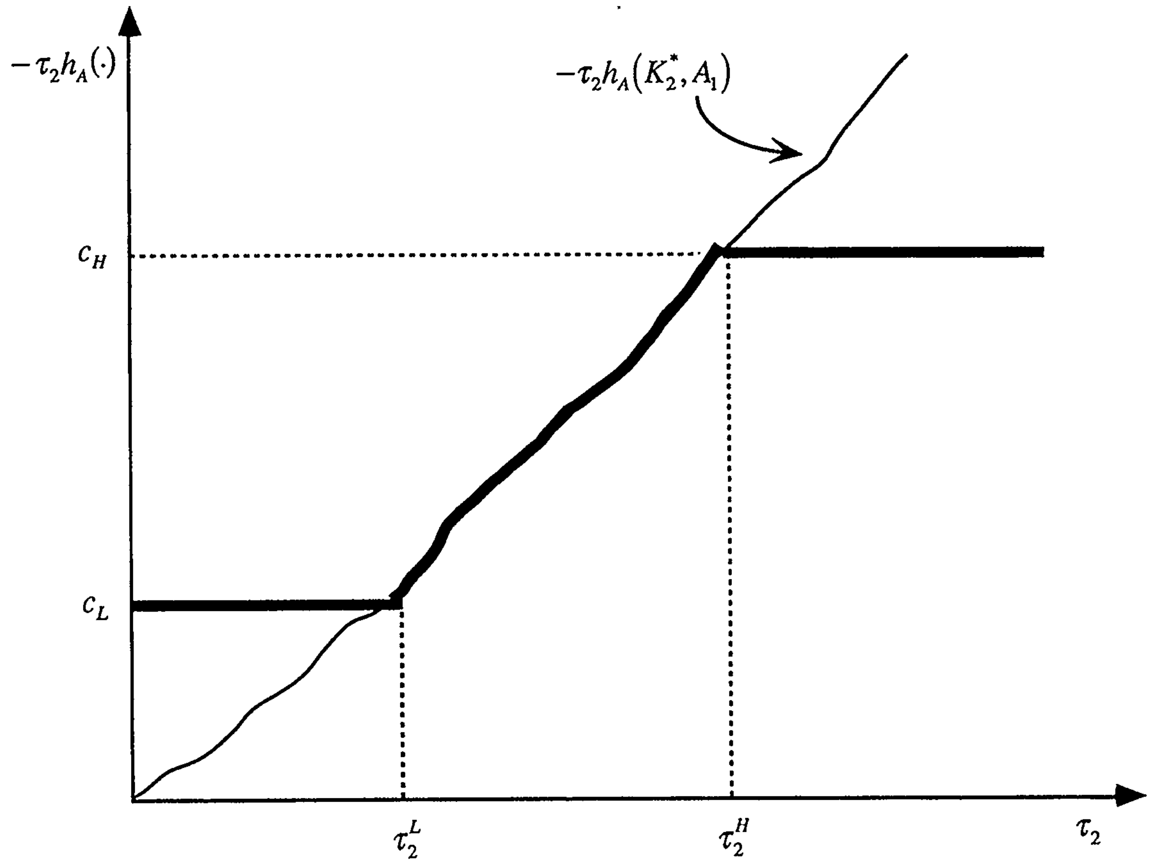
[6]The lower case "k" signifies that this is a different function than those relevant to the cases where abatement capital is endogenous.

These are the values of the tax where it becomes optimal to sell or purchase abatement capital, even at their respective costly prices[7]. Recall that $c_H \geq c \geq c_L$. Using these critical values of τ_2, we may now characterize the firm's behaviour under different tax regimes. If $\tau_2 < \tau_2^L$, it becomes optimal to sell abatement capital until the tax value of the marginal increase in pollution production just equals the resale price, as in (2b). Similarly, when $\tau_2 > \tau_2^H$ the firm would be optimizing by purchasing abatement capital until the value of the tax savings from the marginal reduction in emissions was equal to the purchase price, as in (1b). When $\tau_2^H \geq \tau_2 \geq \tau_2^L$ it is neither optimal to purchase nor sell abatement equipment because its marginal tax savings is greater than its resale price yet lower than the purchase price, so $A_2 = A_1$ within this range.

Figure 1 shows the general relationship between the marginal tax savings from abatement capital and second period Pigouvian taxes. The heavy line tracks this general function, $-\tau_2 h_A\left(K_2^*, A_2^*\right)$. Notice that for values of $\tau_2 < \tau_2^L$ the marginal value of abatement capital will always be set equal to the resale price and for values of $\tau_2 > \tau_2^H$, the level of abatement capital will be chosen such that its marginal tax savings equal the purchase price but for values of τ_2 between these points the marginal valuation will drift upward since adjustment of the abatement stock is not optimal.

[7]The assumption that τ_2^H is greater than τ_2^L places an implicit restricting assumption on $h_{AK}(\cdot)$. It must mean that $-\tau_2 h_A\left(K_2^*, A_1\right)$ is increasing in τ_2, which requires that $h_{AK}(\cdot) > \dfrac{h_A(\cdot)}{-\tau_2 k_{2\tau}(\cdot)}$, where $k_{2\tau}(\cdot) < 0$. So $h_{AK}(\cdot)$ is restricted only to the extent that it cannot be 'too negative'. Muloney and Yandle (1984) find $h_{AK}(\cdot)$ to be negative in practice. Indeed, it is difficult to imagine a technology where $h_{AK}(\cdot) > 0$. This would indicate that a given stock of abatement capital will reduce emission by a greater amount on a small plant than on a large plant. This may be the case where small amounts of investment in abatement capital are not mechanically efficient. For instance, smaller effluent filters may not have any effect on larger discharge pipes. However, such systems could always be divided and dealt with in parallel.

FIGURE 1: The Marginal Tax Savings from Abatement Capital



With an understanding of the firm's optimizing decisions for the second period, it is now appropriate to use this information and move backward to the first period. Second period values will simply be evaluated at their optimal levels.

2.3 First Period Optimizing Behaviour

This section explores the firm's first period investment decisions. The full model is presented and the optimal solutions are obtained. The section closes with an intuitive discussion of some of the preliminary results.

The overall objective of the firm is to maximize its total net returns which are the sum of the first and second period net returns. In the first period, decisions concerning the production and abatement factors are made with the understanding that the investment choice for abatement capital may have large implications for second period returns - due to the combined effects of a stochastic second period tax and partial irreversibility and unexpandability. Moreover, the firm "understands" that whatever its first period decisions, it will respond optimally in the second period given the realization of the second period tax parameter, τ_2, which is unknown in the first period. With this understanding, total net returns may be expressed as $V(K_1,A_1)$, where,

$$\begin{aligned}
V(K_1,A_1) &= R(K_1) - bK_1 - cA_1 - \tau_1 h(K_1,A_1) \\
&+ \delta\int_{-\infty}^{\tau_2^L(A_1,b,c_L)}\left[R(K_2(\tau_2,b,c_L)) - b(K_2(\tau_2,b,c_L) - K_1) + c_L(A_1 - A_2(\tau_2,b,c_L))\right]dF(\tau_2) \\
&\qquad - \delta\int_{-\infty}^{\tau_2^L(A_1,b,c_L)}\left[\tau_2 h(K_2(\tau_2,b,c_L), A_2(\tau_2,b,c_L))\right]dF(\tau_2) \\
&+ \delta\int_{\tau_2^L(A_1,b,c_L)}^{\tau_2^H(A_1,b,c_H)}\left[R(k_2(\tau_2,A_1,b)) - b(k_2(\tau_2,A_1,b) - K_1) - \tau_2 h(k_2(\tau_2,A_1,b),A_1)\right]dF(\tau_2) \\
&+ \delta\int_{\tau_2^H(A_1,b,c_H)}^{\infty}\left[R(K_2(\tau_2,b,c_H)) - b(K_2(\tau_2,b,c_H) - K_1) - c_H(A_2(\tau_2,b,c_H) - A_1)\right]dF(\tau_2) \\
&\qquad - \delta\int_{\tau_2^H(A_1,b,c_H)}^{\infty}\left[\tau_2 h(K_2(\tau_2,b,c_H), A_2(\tau_2,b,c_H))\right]dF(\tau_2).
\end{aligned} \tag{5}$$

The objective function given in (5) is simply the sum of first period net returns and second period expected net returns where the second period returns are calculated in each of the three regimes: $\tau_2 < \tau_2^L$; $\tau_2^H \geq \tau_2 \geq \tau_2^L$, and; $\tau_2 > \tau_2^H$. $F(\tau_2)$ is the cumulative probability density function for τ_2 and δ is a discount factor. Notice the production factor is completely expandable and reversible in each of the three regimes. When $\tau_2 < \tau_2^L$, second period net returns are generated such that both the production factor and abatement capital are endogenous and evaluated at the resale price of the abatement capital. The same applies to the case where $\tau_2 > \tau_2^H$ except that the optimal levels of the two factors are evaluated at the purchase price of abatement equipment. When $\tau_2^H \geq \tau_2 \geq \tau_2^L$ it is neither optimal to buy nor sell abatement capital so the stock of abatement equipment is fixed at its first period level. Further, the optimal level of the production factor is a function of first period abatement capital, among other exogenous variables. With investment modeled in this manner, the first period investment decisions have implications for the value of the firm far beyond the first period net returns.

The necessary conditions for optimization are[8],

$$V_K(K_1^*, A_1^*) = R'(K_1^*) - (1-\delta)b - \tau_1 h_K(K_1^*, A_1^*) = 0, \text{ and} \tag{5a}$$

$$\begin{aligned} V_A(K_1^*, A_1^*) = -c - \tau_1 h_A(K_1^*, A_1^*) + \delta c_L F\left(\tau_2^L(A_1^*, b, c_L)\right) + \delta c_H\left[1 - F\left(\tau_2^H(A_1^*, b, c_H)\right)\right] \\ -\delta \int_{\tau_2^L(A_1^*, b, c_L)}^{\tau_2^H(A_1^*, b, c_H)} \tau_2 h_A\left(k_2(\tau_2, A_1^*, b), A_1^*\right) dF(\tau_2) = 0. \end{aligned} \tag{5b}$$

Condition (5a) may be re-written as,

$$R'(K_1^*) = (1-\delta)b + \tau_1 h_K(K_1^*, A_1^*). \tag{5a'}$$

The interpretation of condition (5a') is fairly straight forward. The term on the LHS is the first period marginal gross returns from employing K_1 which must be balanced by the marginal costs represented on the RHS. The first term on the RHS is the purchase price of K_1 net of its discounted value from the second period. The second term on the RHS is the marginal Pigouvian tax cost from employing the factor. Specifically, it is the marginal increase in the production of the externality from employing K_1 times the cost of that extra production, which is simply the marginal tax rate.

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[8]From (5) the reader may note that the limits of integration are themselves functions of the first period abatement factor, A_1. As such there are differential effects on theses limits when evaluating for the optimal level of A_1. While it is true that Leibniz's Rule is the appropriate method of differentiation under such circumstances, appendix B shows that no qualitative or quantitative breaches are committed if these differential effects are ignored. Appendix C shows the derivation of (5b).

Next, condition (5b) may be rearranged to get,

$$-\tau_1 h_A\left(K_1^*, A_1^*\right) - \delta\int_{\tau_2^L(A_1^*,b,c_L)}^{\tau_2^H(A_1^*,b,c_H)} \tau_2 h_A\left(k_2\left(\tau_2, A_1^*, b\right), A_1^*\right) dF(\tau_2) + \delta c_L F\left(\tau_2^L\left(A_1^*, b, c_L\right)\right)$$

$$+\delta c_H\left[1 - F\left(\tau_2^H\left(A_1^*, b, c_H\right)\right)\right] = c. \qquad \text{(5b')}$$

The first term on the LHS is the first period marginal tax savings realized from investing in abatement capital in period 1. Similarly, the second LHS term may be interpreted as the expected present value of the marginal tax savings which are realized when $\tau_2^L \leq \tau_2 \leq \tau_2^H$ times the probability that in fact $\tau_2^L \leq \tau_2 \leq \tau_2^H$, which is the probability that abatement capital is neither purchased nor sold. The third LHS term is the marginal expected present resale value of the equipment times the probability that $\tau_2 < \tau_2^L$, which is the probability that abatement capital is sold. The last LHS term is also a marginal benefit because it is the present marginal value of abatement capital which no longer has to be purchased at the higher price, c_H (because it was already purchased in period 1). This benefit is realized on the probability that $\tau_2 > \tau_2^H$. The RHS term is the immediate and direct marginal cost of abatement capital in period one.

Close inspection of (5b) reveals an interesting feature of partially irreversible and unexpandable investment models in that the shape of the distribution of future taxes is irrelevant above τ_2^H and below τ_2^L. This implies that probability mass could be shifted away from a point just above τ_2^H to a point of much larger taxation (i.e. if bad news arrives it will be very bad) and this will have no impact on the incentive to invest. However, the distribution of taxes between the critical points is of interest to the firm. Abel *et al* (1995) noted that this is a simple extension of Bernanke's (1983) "bad-news principle" to what they call a "Goldilocks principle" - the only region of the probability distribution that affects the incentive to invest is the middle part where news is neither "too hot" nor "too cold".

The second order sufficient conditions are provided in (5c) through (5e).

$$V_{KK}\left(K_1^*,A_1^*\right)=R''\left(K_1^*\right)-\tau_1 h_{KK}\left(K_1^*,A_1^*\right)<0, \tag{5c}$$

$$\begin{aligned}V_{AA}\left(K_1^*,A_1^*\right)&=-\tau_1 h_{AA}\left(K_1^*,A_1^*\right)\\&-\delta\int_{\tau_2^L\left(A_1^*,b,c_L\right)}^{\tau_2^H\left(A_1^*,b,c_H\right)}\left[\tau_2\left[h_{AK}\left(k_2\left(\tau_2,A_1^*,b\right),A_1^*\right)k_{2A}\left(\tau_2,A_1^*,b\right)+h_{AA}\left(k_2\left(\tau_2,A_1^*,b\right),A_1^*\right)\right]\right]dF\left(\tau_2\right)<0,\end{aligned} \tag{5d}$$

and

$$V_{KK}\left(K_1^*,A_1^*\right)V_{AA}\left(K_1^*,A_1^*\right)-\left[V_{AK}\left(K_1^*,A_1^*\right)\right]^2>0, \tag{5e}$$

where $V_{AK}\left(K_1^*,A_1^*\right)=-\tau_1 h_{AK}\left(K_1^*,A_1^*\right)$. The general solution to the model will be of the form, $K_1^*=K_1\left(\tau_1,b,c,c_L,c_H\right)$ and $A_1^*=A_1\left(\tau_1,b,c,c_L,c_H\right)$. As before, there are relatively few restrictions on the technology behind externality production. In fact, the only second order derivative which has been sign restricted is $h_{AA}(\cdot)>0$, (i.e. production of the externality is convex in A). There are still no restrictions on the scale properties of the production technology, however there is an interdependency between these properties and the convexity or concavity of externality production in K, such that $h_{KK}(\cdot)<\frac{R''(\cdot)}{-\tau_1}$. Further, note that $h_{AK}(\cdot)$ is restricted in magnitude only[9]. Verifying this claim requires evaluating $k_{2A}\left(\tau_2,A_1^*,b\right)$. From (3a), implicitly differentiating k_2 with respect to A, we find that

$$k_{2A}\left(\tau_2,A_1^*,b\right)=\frac{\tau_2 h_{AK}\left(k_2\left(\tau_2,A_1^*,b\right),A_1^*\right)}{\left[R''\left(k_2\left(\tau_2,A_1^*,b\right)\right)-\tau_2 h_{KK}\left(k_2\left(\tau_2,A_1^*,b\right),A_1^*\right)\right]}.$$

[9]Now $h_{AK}(\cdot)$ cannot be 'too positive' either.

From (3b), the denominator is unambiguously negative and $V_{AA}(K_1^*, A_1^*)$ (equation (5d) reduces to,

$$-\tau_1 h_{AA}(K_1^*, A_1^*) - \delta \int_{\tau_2^L(A_1^*, b, c_L)}^{\tau_2^H(A_1^*, b, c_H)} \left[\frac{\left[\tau_2 h_{AK}(K_2^*, A_1^*)\right]^2}{\left[R''(K_2^*) - \tau_2 h_{KK}(K_2^*, A_1^*)\right]} + \tau_2 h_{AA}(K_2^*, A_1^*) \right] dF(\tau_2) < 0. \quad \text{(5d')}$$

As is readily observable, the only non-negative term in (5d') is the term involving $h_{AK}(\cdot)$. However, only its magnitude and not its sign is restricted. In other words, the abatement economies to plant scale are irrelevant to the investment decision (in production or abatement capital) and only the magnitude of the value is important. (5d) and (5d') also reiterate the restriction that a solution requires $h_{AA}(\cdot) > 0$.

2.4 Concluding Remarks

This chapter has outlined and presented a two factor version of Abel *et al*'s (1995) two period model of investment under uncertainty. The two factor model differs from their model in several ways. First, here the production factor is presumed to be totally reversible and expandable. Second, the additional second factor is not a production factor, rather it is abatement capital. It is this second factor which is partially irreversible and expandable.

The firm's second period optimizing behaviour was discussed in the context of three different scenarios. The first was where the second period Pigouvian tax was high enough to warrant purchasing more abatement capital, even at the higher price. The second scenario was where taxes were low enough that it was optimal to sell abatement capital, even at the lower price. The last regime was where taxes were such that it was neither optimal to purchase nor sell abatement capital. These results were used to define critical values of the second period tax above and below which changes in the first period stock of abatement capital would occur. Working recursively, the first period investment decisions were explored internalizing the second period optimizing behaviour. Lastly, some of the restrictions on technologies were discussed. It was shown that the model is not very demanding or particular in terms of these technologies.

Chapter 3 will build on the work in this chapter as various comparative static results and ambiguities are evaluated. Many of these ambiguities will arise because of the lack of *ex ante* restrictions placed on the model.

CHAPTER THREE

3. COMPARATIVE STATICS

This chapter explores and assesses the effects and implications of changes in various exogenous factors. Of specific interest are the incentives to invest in production and abatement capital and how they may be altered as a result of movements in first and second period prices, present and expected future taxes, and changes in the level of uncertainty regarding future taxes.

Chapter three is organized as follows: section 3.1 examines the comparative static effects on the incentives to invest from changes in the costs of investment, changes in the cost of reversibility and expansion, and changes in first period taxation level; section 3.2 explores the nature of distributional shifts concerning future taxes and the resulting implications on the incentives to invest; section 3.3 uses the results from the previous two sections to evaluate some possible implications for policy design, and; section 3.4 completes the chapter with some concluding remarks.

3.1 Investment Costs, Resale prices, and Tax Rates

This section seeks to gain understanding into the functional relationships between various prices and taxes and the incentives to invest. This will be accomplished by taking a linear approximation of the model evaluated in the neighbourhood of the optimum. Each comparative static result will be evaluated in turn. The nature and relevance of different technologies will then be discussed and tested on the model.

For convenience, a linear approximation of the model is taken in the neighbourhood of the optimum and represented in matrix notation below[10].

[10]Again, see appendix C for the application of Leibniz's rule.

$$\begin{bmatrix} V_{KK}(\cdot) & V_{AK}(\cdot) \\ V_{AK}(\cdot) & V_{AA}(\cdot) \end{bmatrix} \begin{bmatrix} dK_1^* \\ dA_1^* \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \int_{\tau_2^L}^{\tau_2^H} \tau_2 h_{AK}(\cdot) k_{2b}(\cdot) dF(\tau_2) \end{bmatrix} db + \begin{bmatrix} 0 \\ 1 \end{bmatrix} dc + \begin{bmatrix} 0 \\ -\delta F(\tau_2^L) \end{bmatrix} dc_L$$

$$+ \begin{bmatrix} 0 \\ -\delta[1 - F(\tau_2^H)] \end{bmatrix} dc_H + \begin{bmatrix} h_K(\cdot) \\ h_A(\cdot) \end{bmatrix} d\tau_1 \tag{6}$$

Let $\begin{bmatrix} V_{KK}(\cdot) & V_{AK}(\cdot) \\ V_{AK}(\cdot) & V_{AA}(\cdot) \end{bmatrix} = [H]$, where $V_{AK}(\cdot) = -\tau_1 h_{AK}(K_1^*, A_1^*)$ and condition (5e) ensures that $|H| > 0$. Equations (7) to (16) show the comparative static results along with their respective signs. Each set of comparative statics will be discussed in an attempt to shed some light on the intuition behind the analytic results. We begin first in (7) and (8) with the comparative static effects of the cost of the production factor.

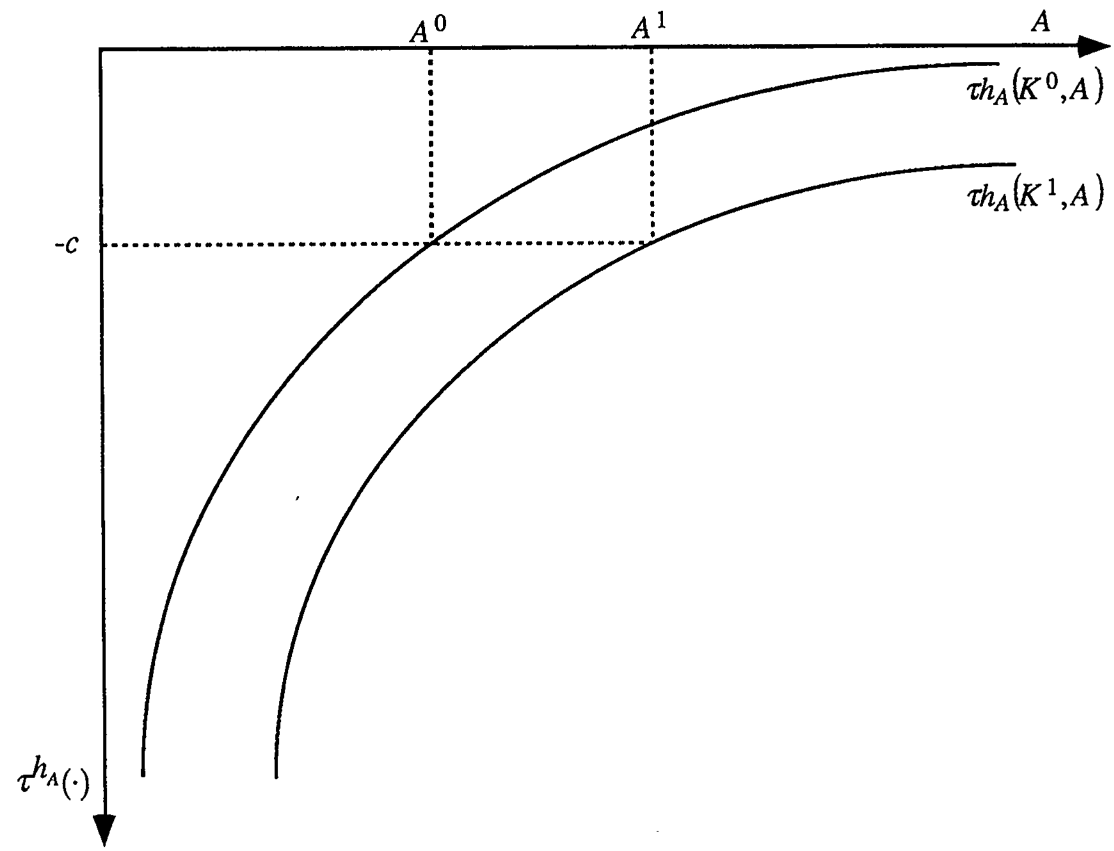
$$\frac{dK_1^*}{db} = \frac{(1-\delta)V_{AA}(\cdot) + \tau_1 h_{AK}(\cdot)\delta \int \tau_2 h_{AK}(\cdot) k_{2b}(\cdot) dF(\tau_2)}{|H|} < 0 \tag{7}$$

$$\frac{dA_1^*}{db} = \frac{(1-\delta)\tau_1 h_{AK}(\cdot) + V_{KK}(\cdot)\delta \int_{\tau_2^L}^{\tau_2^H} \tau_2 h_{AK}(\cdot) k_{2b}(\cdot) dF(\tau_2)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) >=< 0 \tag{8}$$

The evaluation of (7) and (8) depends on our ability to sign $k_{2b}(\cdot)$. Using equation (3a) we find that $k_{2b}(\cdot) = \frac{1}{[R''(\cdot) - \tau_2 h_{kk}(\cdot)]} < 0$. Therefore, an increase in the cost of production capital will decrease the incentive to invest in it. However, an increase in the cost of the production factor has an ambiguous effect on the incentive to invest in abatement capital. Here we find the first evidence of a cross-price ambiguity. The result in (8) shows that this ambiguity arises from the absence of restrictions on the sign of $h_{AK}(\cdot)$. That is, the effect on the incentive to invest in abatement capital is technology dependent in that it is the sign of $h_{AK}(\cdot)$ which determines if decreases in K_1 and increases in A_1 are substitute or complimentary strategies. By inspection, if we assume that the technology indicates that abatement capital mitigates pollution to a lesser degree for higher levels of production factor employment ($h_{AK}(\cdot) > 0$), then increases in the cost of the production factor will increase the incentive to invest in abatement capital. This is because an increase in *b* decreases the incentive to invest in production capital, so, all else being equal, the stock of production capital decreases in both periods. This reduction in the production factor increases the marginal effect of abatement capital on pollution. Hence to optimize via satisfying equation (5b), the firm must increase investment in abatement capital (recall that $h(\cdot)$ is convex in A).

Figure 2 illustrates this phenomenon. As production capital is reduced from K^0 to K^1, the marginal abatement curve shifts downward since $h_{AK}(\cdot) > 0$. In order to maintain optimizing conditions analogous to (1b), (2b), and more to the point, (5b), the firm must invest in more abatement capital as implied by the move from A^0 to A^1.

FIGURE 2: Optimizing Abatement Capital



Had we chosen to assume that $h_{AK}(\cdot) < 0$, the corresponding shifts in the curves from figure 2 would simply have occurred in the opposite direction. In such a case, an increase in the cost of production capital would effectively decrease the first period abatement capital stock.

Equations (9) and (10) show the comparative static effects of the first period cost of abatement capital.

$$\frac{dK_1^*}{dc} = \frac{\tau_1 h_{AK}(\cdot)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) >=< 0 \tag{9}$$

$$\frac{dA_1^*}{dc} = \frac{V_{KK}(\cdot)}{|H|} < 0 \tag{10}$$

Equation (10) demonstrates an intuitively appealing result - increases in the initial cost of abatement capital result in decreases in the initial employment of that capital. However, once again from (9) we find that the corresponding effect on production capital investment is technology dependent. That is, there is ambiguity in the cross-price effect. If we confine ourselves to the case where first period taxes are greater than zero and we further assume that $h_{AK}(\cdot) > 0$, then increases in the initial cost of abatement capital will increase the incentive to invest in production capital. This process may be clarified by reexamining equation (5a'), reproduced here for convenience.

$$R'(K_1^*) = (1-\delta)b + \tau_1 h_K(K_1^*, A_1^*) \tag{5a'}$$

Still assuming that $h_{AK}(\cdot) > 0$, we know that an increase in the initial cost of the abatement factor will decrease its first period employment. This will effectively decrease the tax value of the marginal effect of production capital on the externality production (the second term on the RHS of (5a')). The response is a decrease in the value of the RHS of (5a') which lowers the optimal level of the first period marginal returns. This implies an increase in the incentive to invest in the production factor. Conversely, if the sensitivity of externality production to the production factor is decreasing in A, then, as before, an increase in the first period cost of abatement capital will decrease the incentive to invest in abatement capital, but this decrease in abatement capital investment will increase the value of the RHS of (5a'). The net effect of this is a reduction of the incentive to invest in the production factor.

The following set of comparative statics evaluates the effects of changes in the second period purchase and resale prices of abatement capital.

$$\frac{dA_1^*}{dc_L} = \frac{-V_{KK}(\cdot)\delta F(\tau_2^L)}{|H|} > 0 \tag{11}$$

$$\frac{dA_1^*}{dc_H} = \frac{-V_{KK}(\cdot)\delta[1 - F(\tau_2^H)]}{|H|} > 0 \tag{12}$$

$$\frac{dK_1^*}{dc_L} = \frac{-\tau_1 h_{AK}(\cdot)\delta F(\tau_2^L)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) <=> 0 \tag{13}$$

$$\frac{dK_1^*}{dc_H} = \frac{-\tau_1 h_{AK}(\cdot)\delta[1 - F(\tau_2^H)]}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) <=> 0 \tag{14}$$

Equation (11) indicates that an increase in the second period resale price of abatement capital will amplify the incentive to invest in first period abatement capital. The reasoning behind this result is that an increase in c_L reflects a decrease in the degree of irreversibility of abatement capital. That is, the cost or penalty for investing in 'too much' abatement capital is diminished should future taxes evolve such that $\tau_2 < \tau_2^L$. Notice too how this result characterizes one of the claims from the irreversible investment literature - the presence of sunk costs, or irreversibilities, will delay investment. As c_L decreases, the investment becomes 'more irreversible' the result is that investment will be delayed until it is determined to be necessary in the second period. Similarly, an increase in the future purchase price of abatement equipment results in an increase in the incentive to invest in abatement equipment, as is shown in (12). The logic is similar as well. Increasing the future purchase price effectively decreases the degree of expandability. That is, if the future purchase price of abatement capital increases, the firm will be motivated to purchase more abatement capital to avoid the additional cost or penalty of purchasing more in the second period at the now higher c_H should taxes evolve such that $\tau_2 > \tau_2^H$.

Future purchase and resale price changes have similar effects on production capital as the case where the first period price of abatement capital changes. Again, if $h_{AK}(\cdot) > 0$, then any change in an exogenous variable which increases the incentive to invest in abatement capital will correspondingly decrease the incentive to invest in production capital. On a slightly more intuitive level, if $h_{AK}(\cdot) > 0$, then the sensitivity of pollution production is increasing in A - meaning that a unit increase in production capital contributes a greater amount to the production of the externality at higher levels of abatement capital than at lower levels. As such, if future price changes increase the investment level in first period abatement capital, then production capital will decrease because its marginal contribution to the externality production has increased. Therefore, if $h_{AK}(\cdot) < 0$ then an increase in either the second period resale or purchase price of abatement capital will correspondingly increase the incentive to invest in production capital.

The last set of comparative statics in this section examines the changes in the incentives to invest arising from changes in the first period Pigouvian tax rate. Equations (15) and (16) quantify these resulting effects.

$$\frac{dK_1^*}{d\tau_1} = \frac{h_K(\cdot)V_{AA}(\cdot) + \tau_1 h_{AK}(\cdot)h_A(\cdot)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) <=> \frac{-h_K(\cdot)V_{AA}(\cdot)}{\tau_1 h_A(\cdot)} \tag{15}$$

$$\frac{dA_1^*}{d\tau_1} = \frac{h_A(\cdot)V_{KK}(\cdot) + \tau_1 h_{AK}(\cdot)h_K(\cdot)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) >=< \frac{-h_A(\cdot)V_{KK}(\cdot)}{\tau_1 h_K(\cdot)} \tag{16}$$

The ambiguous results shown here are the result of a synergy between the investment decisions. This synergy is driven by the ubiquitous $h_{AK}(\cdot)$ term. Included in (15) and (16) are the respective critical values for $h_{AK}(\cdot)$. These values suggest that if $h_{AK}(\cdot)$ is positive, or at least not 'too negative', increases in the first period Pigouvian tax rate will decrease the incentive to invest in the production factor and increase the incentive to invest in abatement capital. These ambiguities will be the focus of the discussion in section 3.3. Notice that these findings are at least partially consistent with the findings of Kohn (1988). That is, if there are sufficient increasing scales in abatement ($h_{AK}(\cdot) < 0$), then Pigouvian taxation will increase the scale of the firm. If, however, there are decreasing scales in abatement ($h_{AK}(\cdot) > 0$) then Pigouvian taxation will decrease the scale of the firm. However Kohn noted that in the absence of scale properties, Pigouvian corrections would increase the scale of the firm. If we set $h_{AK}(\cdot) = 0$, however, this model finds quite the opposite result - the introduction of a Pigouvian correction will decrease the scale of the firm.

Table 1 offers a convenient summary of the comparative static results under the two competing assumptions concerning the emissions production technology.

TABLE 1: Summary of Comparative Static Results

	$h_{AK}(\cdot) > 0$		$h_{AK}(\cdot) < 0$	
	dK_1^*	dA_1^*	dK_1^*	dA_1^*
db	neg.	pos.	neg.	neg.
dc	pos.	neg.	neg.	neg.
dc_L	neg.	pos.	pos.	pos.
dc_H	neg.	pos.	pos.	pos.
$d\tau_1$	neg.	pos.	ambiguous	ambiguous

This section has evaluated the comparative static effects of changes in the first period Pigouvian tax as well as various capital prices. In general terms, an increase in the first period price of a given type of capital decreases the incentive to invest in that capital unambiguously. Further, if $h_{AK}(\cdot) > 0$, then an increase in the first period purchase price of a given type of capital *increases* the incentive to invest in the other type of capital. If $h_{AK}(\cdot) < 0$, then the opposite is true - the incentive to invest in the other type of capital will decrease. The more interesting of the results in this section is the ambiguous nature of the effect of an increase in the first period Pigouvian tax. This will be addressed in more detail in section 3.3.

3.2 The Distribution of Future Taxes and the Incentive to Invest

This section analyses the effects that changes in the distribution of future taxes may have on the incentives to invest in current capital. Specifically, we will evaluate the effects of shifts in the cumulative distribution function, $F(\tau_2)$. Of interest here are the effects that both the expected value of the future tax rate and the degree of uncertainty have on the incentives to invest.

In order to proceed, we will assume that future tax rates are distributed normal with mean μ and variance σ^2. Let

$$F(\tau_2,\mu,\sigma) = \int_{-\infty}^{\tau_2} f(\tau_2,\mu,\sigma)d\tau_2$$

where,

$$f(\tau_2,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(\tau_2-\mu)^2/2\sigma^2}. \tag{17}$$

This distribution will be used in the next two subsections where the effects of changes in expected future taxes and the degree of uncertainty will be examined separately.

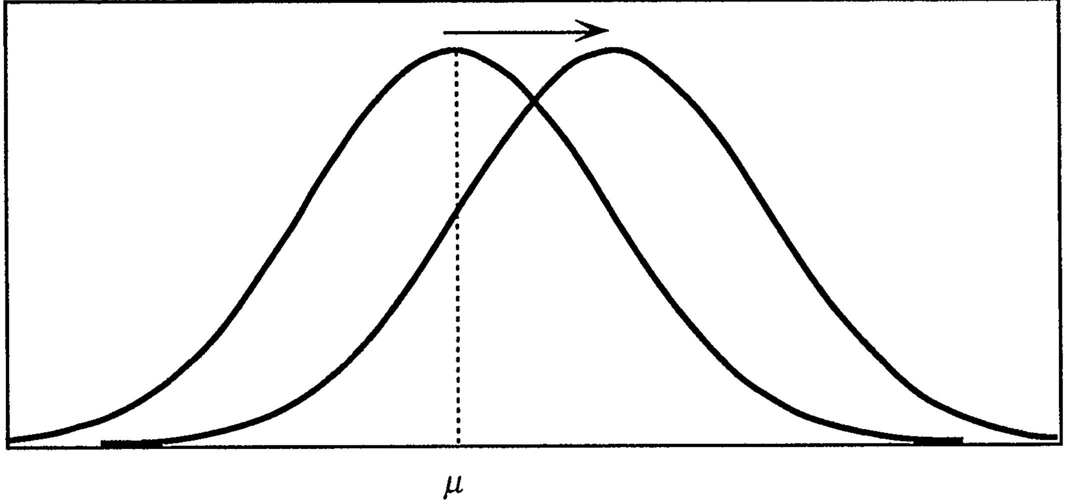
3.2.1 Expected Future Taxation

This subsection explores the effects of a first order shift in the distribution of future taxes on the incentives to invest. Before beginning the evaluation of the comparative statics, it will prove useful to evaluate the partial derivatives of the distribution and cumulative distribution functions with respect to μ. First we begin with $f_\mu(\cdot)$.

$$f_\mu(\cdot) = F_{\tau_2\mu}(\cdot) = \frac{(\tau_2-\mu)}{\sigma^3\sqrt{2\pi}} e^{-(\tau_2-\mu)^2/2\sigma^2}$$

It should be evident that $f_\mu(\cdot) > 0$ for all points where $\tau_2 > \mu$, $f_\mu(\cdot) < 0$ for all points where $\tau_2 < \mu$, and $f_\mu(\cdot) = 0$ if $\tau_2 = \mu$. This is illustrated in figure 4 with a discrete first order shift in a normal distribution.

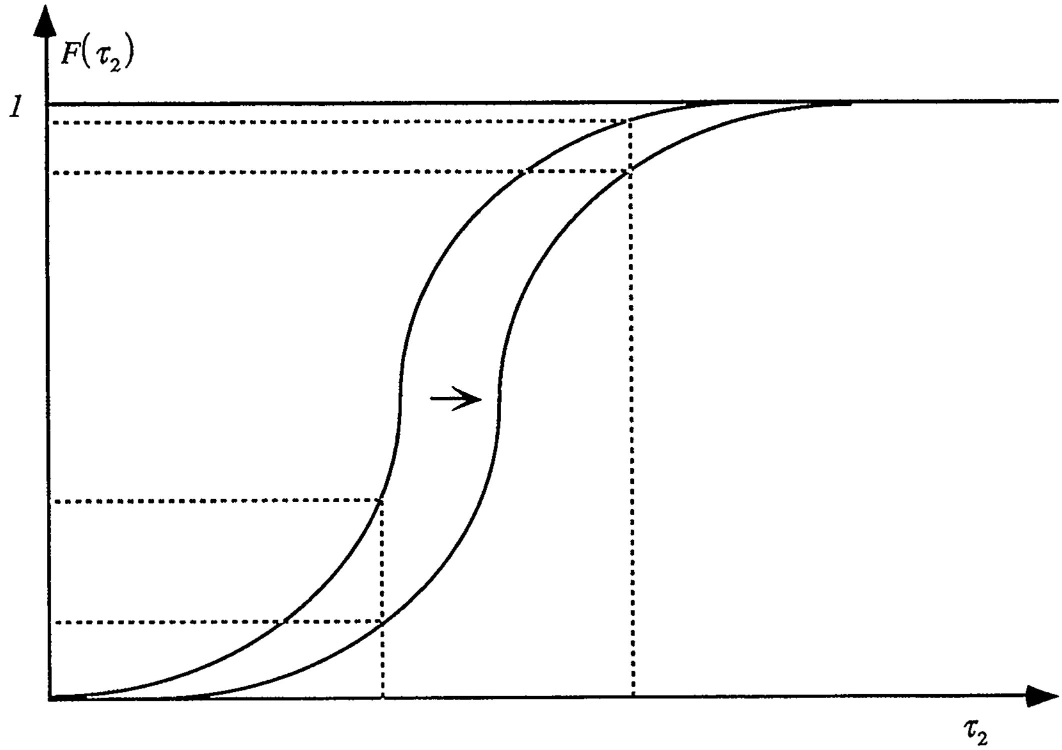
FIGURE 3: A First Order Shift in a Normal Probability Density Function



This foreshadows a potential problem in that the sign of given comparative static may be sensitive to the relative positions of τ_2^L, τ_2^H, with respect to μ.

Now consider $F_\mu(\cdot)$ To evaluate this derivative, we simply note that an increase in the expected future tax simply implies a rightward shift of the cumulative distribution function, as is shown in figure 4.

FIGURE 4: A First Order Shift in a Cumulative Distribution Function



Notice that the rightward shift decreases the probability of observing a lesser value for all points in the distribution. This implies that $F_\mu(\cdot) < 0$ for all values of τ_2.

As in the previous section, we take a linear approximation of the model and conveniently represent it in matrix notation as in (18).

$$\begin{bmatrix} V_{KK}(\cdot) & V_{AK}(\cdot) \\ V_{AK}(\cdot) & V_{AA}(\cdot) \end{bmatrix} \begin{bmatrix} dK_1^* \\ dA_1^* \end{bmatrix} = \begin{bmatrix} 0 \\ \delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2 - c_L F_\mu(\tau_2^L, \mu, \sigma) + c_H F_\mu(\tau_2^H, \mu, \sigma)\right] \end{bmatrix} d\mu \tag{18}$$

The comparative static expressions are as follows,

$$\frac{dA_1^*}{d\mu} = \frac{V_{KK}(\cdot)\delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2 - c_L F_\mu(\tau_2^L, \mu, \sigma) + c_H F_\mu(\tau_2^H, \mu, \sigma)\right]}{|H|} \geq 0, \text{ and} \tag{19}$$

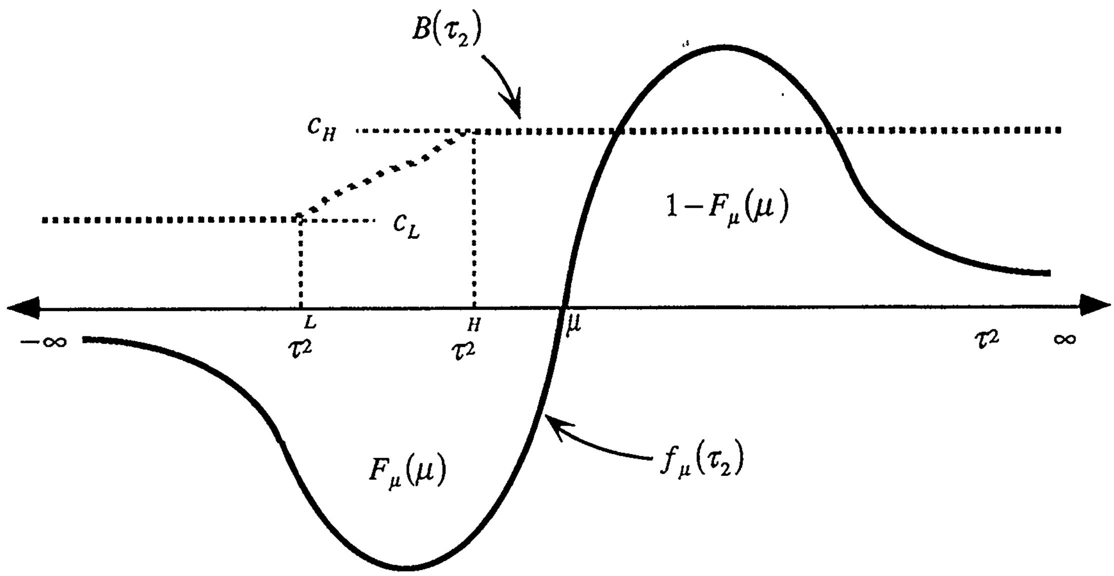
$$\frac{dK_1^*}{d\mu} = \frac{\tau_1 h_{AK}(\cdot)\delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2 - c_L F_\mu\left(\tau_2^L, \mu, \sigma\right) + c_H F_\mu\left(\tau_2^H, \mu, \sigma\right)\right]}{|H|} \geq = \leq 0 \text{ as,}$$

$$h_{AK}(\cdot) <=> 0. \qquad (20)$$

This subsection will continue by first examining the comparative statics to the ends of understanding the full qualitative effects on the incentives to invest. That is, we will determine whether an increase in expected future taxes increases the incentive to invest in the two types of capital. Second, the individual terms in (19) and (20) will be evaluated for their respective individual effects on the incentives to invest.

Casual inspection of the numerators of (19) and (20) reveal few clues concerning their respective signs. We see that each equation contains potentially offsetting terms which make the expression difficult to evaluate. A lesson from Abel *et al* (1995) and Hirshleifer and Riley (1992) may be instructive here. They discussed the problem of a first order shift in terms of stochastic dominance. A similar type of analysis will be performed here. Consider the first order conditions outlined in (1b) and (2b). The first condition indicates that *irrespective of the value of* τ_2 above τ_2^H, $-\tau_2 h_A(\cdot) = c_H$. Similarly, irrespective of the value of τ_2 below τ_2^L, $-\tau_2 h_A(\cdot) = c_L$. Recalling figure 1 may be instructive to this analysis as it maps $-\tau_2 h_A\left(K_2^*, A_1\right)$ against τ_2, clearly showing these important relationships. Figure 1 unambiguously shows that $-\tau_2 h_A\left(K_2^*, A_1\right)$ is non decreasing in τ_2, and, moreover, that it is increasing monotonically over the range τ_2^L to τ_2^H. The graph from figure 1 is slightly modified in figure 5 by the superimposition of the function $f_\mu(\cdot)$.

FIGURE 5: The Effects of a First Order Shift



What may be noticed here is that $-\tau_2 h_A(\cdot)$ is a non decreasing function of τ_2; constant above τ_2^H and below τ_2^L, and increasing in the interval between the two critical points. This means that *irrespective of the mean of* τ_2, the area under this graph to the left of the mean will be smaller than the area under the graph to the right of the mean. Let $B(\tau_2)$ be the function depicted by the dark dashed line in figure 6.[11] Now (19) may be written,

$$\frac{dA_1^*}{d\mu} = \frac{V_{KK}(\cdot)\delta\left[-\int_{-\infty}^{\infty} B(\tau_2) f_\mu(\tau_2) d\tau_2\right]}{|H|} \geq 0. \tag{19'}$$

This may now be signed non-negative because $B(\tau_2)$ is non-decreasing in τ_2, and because $f(\cdot)$ is a symmetric distribution, hence so is $f_\mu(\cdot)$. Indeed, $f_\mu(\cdot)$ is symmetric in the sense that the negative portion of the function below its mean is precisely equal and opposite to the positive portion above its mean. So even though $f_\mu(\cdot)$ flips signs at the mean from

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[11]See appendix D for a proof of the appropriateness of using the composite function $B(\tau_2)$.

negative to positive, the portion of $V_{KK}(\cdot)\delta\left[-\int_{-\infty}^{\infty} B(\tau_2) f_\mu(\tau_2) d\tau_2\right]$ above the mean must be of no lesser magnitude than portion below the mean (because $B(\tau_2)$ is non-decreasing in τ_2). Put differently,

$$\left|V_{KK}(\cdot)\delta\left[-\int_{-\infty}^{\mu} B(\tau_2) f_\mu(\tau_2) d\tau_2\right]\right| \le \left|V_{KK}(\cdot)\delta\left[-\int_{\mu}^{\infty} B(\tau_2) f_\mu(\tau_2) d\tau_2\right]\right|,$$

for all μ. Again, this may be verified visually by inspection of figure 5. The "sum" of $B(\tau_2) f_\mu(\tau_2)$ will be greater than zero regardless of where the mean lies with respect to τ_2^L and τ_2^H. Hence the net effect of an increase in expected future taxes is an increase in the employment of abatement capital today.

This particular analysis clearly demonstrates the nesting of the fully reversible and expandable case. Notice that if $c_L = c_H = c$, then $\tau_2^L = \tau_2^H$ and the effect of a change in expectations on the incentive to invest in abatement capital is zero. This is an intuitively appealing result since, in this case, the firm may costlessly adjust the stock of abatement capital to accommodate future taxes regardless of expectations. However, with the irreversible and unexpandable case, the firm will have limited opportunity to respond to changes in tax regimes as they appear.

Turning to equation (20), it should be noted that the effect of a first order shift of the distribution of future taxes will trivially yield the same qualitative results as the incentive to invest in abatement capital if $h_{AK}(\cdot) < 0$, (that is, investment in production capital will increase), and the opposing result if $h_{AK}(\cdot) > 0$. This effect may have policy implications which will be explored in the following section.

With an understanding of the overall effect of an increase in expected taxes, it may be instructive to explore the individual terms comprising this overall effect. This subsection proceeds by discussing the individual effects of the terms in (19) and (20) and the intuition which may be gleaned from them.

In (19) consider first the following term inside the brackets, $-V_{KK}\delta c_L F_\mu\left(\tau_2^L,\mu,\sigma\right)<0$. This term is unambiguously less than zero. Increasing the expected future tax rate decreases the probability that taxes will evolve in such a way as to be lower than τ_2^L. This is readily verifiable using figure 4. Recall that when realized taxes are less than this critical value, the firm will sell some amount of its abatement equipment at the price c_L. This ability to sell capital represents some degree of reversibility in the event of a 'good state'. When the expected tax rate increases, though, the probability that taxes will evolve such that the firm may sell some of its capital decreases. This represents a probabilistic reduction in the marginal benefits of investing in first period abatement capital since the firm's odds of recouping some of that investment are diminished.

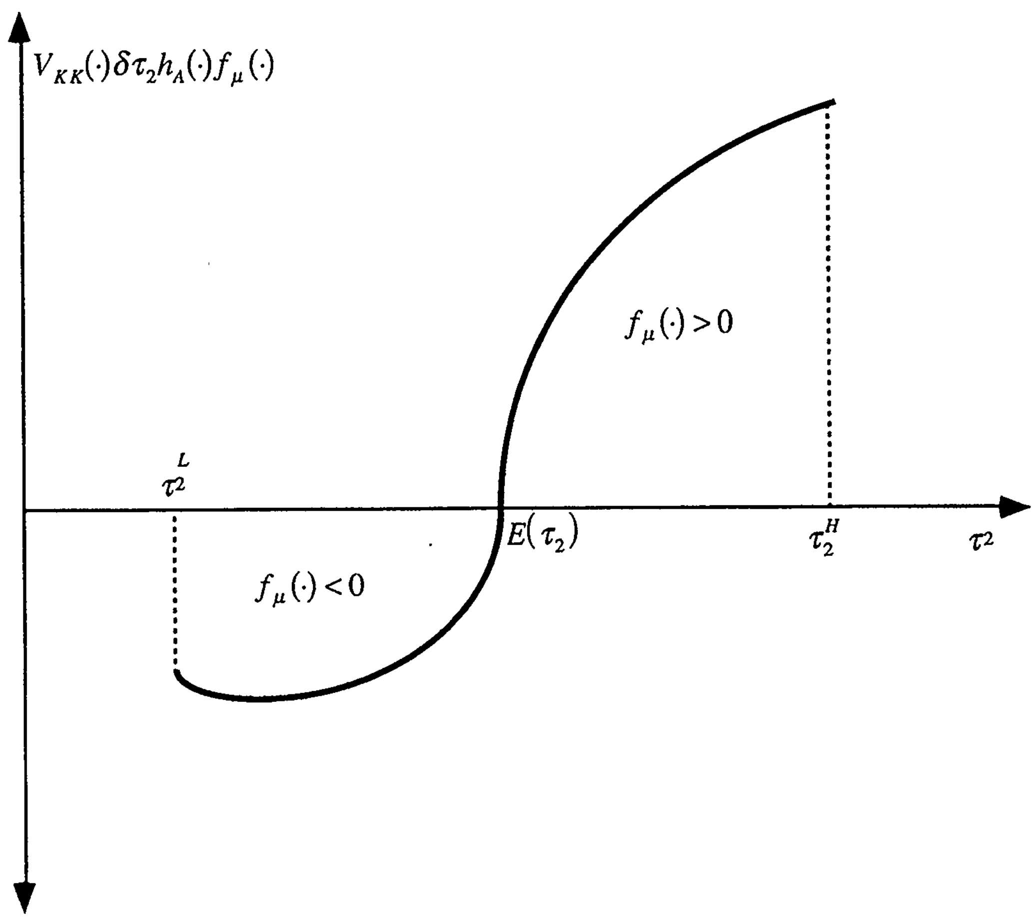
Still in (19) consider the third term inside the brackets, $V_{KK}\delta c_H F_\mu\left(\tau_2^H,\mu,\sigma\right)>0$. This term is unambiguously greater than zero. While it was the case that an increase in the expected value of future taxes decreased the probability of observing a tax rate less than τ_2^L, it increases the probability of observing a tax rate greater than τ_2^H. That is, it has become more likely that future taxes will be high enough to warrant purchasing additional abatement capital. Recall though that it is more costly to purchase abatement capital in the second period than in the first - it is costly to expand. Understanding this, the firm preemptively purchases more abatement capital at the lower first period price, avoiding the higher second period price. In short, since the likelihood of being penalized for carrying insufficient abatement equipment in the second period has increased, the incentive to invest in it today increases.

The sign of the first term in the brackets of (19), $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\mu(\cdot)d\tau_2$ is not as straight forward as the previous two. If the mean or expected value of the future tax lies between the two limits of the integral, then $f_\mu(\cdot)$ changes sign from negative to positive as it passes across the mean. If we impose the restrictive assumption that the mean of future taxes were greater than τ_2^H, then the sign of this term would unambiguously be less than zero, and the effect of this term would be to reduce the incentive to invest in abatement capital. Formally, if $E(\tau_2)\geq\tau_2^H\geq\tau_2^L$ then $f_\mu(\cdot)<0$ for all values of τ_2 between the limits of the integral. $V_{KK}(\cdot)$ and $\tau_2 h_A(\cdot)$ are less than zero for all values of τ_2. Hence $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\mu(\cdot)d\tau_2$ would be less than zero. More intuitively, the effect of this term on the incentive to invest is negative because probability is being reassigned away from it. Put differently, the probability of τ_2 evolving such that $\tau_2^L\leq\tau_2\leq\tau_2^H$ has diminished. As such, the firm is placing less weight on the relevance of the first term to the analysis. This should not be interpreted to mean that if $E(\tau_2)\geq\tau_2^H\geq\tau_2^L$, then an increase in expected taxes will decrease the incentive to invest in abatement capital. On the contrary, more weight in probability has shifted to the likelihood that realized τ_2 will be greater than τ_2^H which means that $V_{KK}\delta c_H F_\mu(\tau_2^H,\mu,\sigma)$ (representing an increase in the incentive to invest) will be large enough to overpower the remaining two negative terms, as was shown above. Again, the disincentive to invest as a result of the $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\mu(\cdot)d\tau_2$ term arises not because of a *direct* disincentive, but because of a decrease in the probability that it is relevant to future profits.

Now consider the case where $E(\tau_2)<\tau_2^L\leq\tau_2^H$. In this scenario $f_\mu(\cdot)$ is strictly greater than zero in the range τ_2^L to τ_2^H. As a result, the term $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\mu(\cdot)d\tau_2$ is greater than zero implying that its effect is to increase the incentive to invest in abatement capital given an increase in expected taxes. This term embodies the probabilistic marginal tax savings from investing in abatement capital which are achieved if $\tau_2^L\leq\tau_2\leq\tau_2^H$. Since the probability of $\tau_2^L \le \tau_2 \le \tau_2^H$ now increases with an increase in expected taxes, the relevance of this term to the analysis is augmented.

The last case to consider is where $\tau_2^L \le E(\tau_2) \le \tau_2^H$. Here we have to recognize that when we integrate across τ_2, $f_\mu(\cdot)$ will change its sign from negative to positive. Evaluating $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2$ now requires a clearer understanding of the product of the functions inside the integral. By using figure 5 and isolating the region spanned between τ_2^L and τ_2^H, these individual effects may be illustrated more clearly. We can build on figure 5 by mapping the function $V_{KK}(\cdot)\delta\tau_2 h_A(\cdot) f_\mu(\cdot)$ (which is equal to the function $B(\tau_2)f_\mu(\tau_2)$ evaluated over the same span) directly onto τ_2, where $V_{KK}(\cdot)$ and δ are constant in τ_2. Figure 6 shows this complete relationship.

FIGURE 6: The Product of Functions



For the time being we will confine ourselves to the special case where the limits of integration are equidistant from the mean, as is the case in figure 6. That is $\tau_2^H - E(\tau_2) = E(\tau_2) - \tau_2^L$. Even at a casual glance the reader will notice that the area above the zero line outweighs the area below it. This implies that $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2 > 0$. This is, in fact, the case for all $E(\tau_2)$ such that $\tau_2^L \leq E(\tau_2) \leq \frac{\tau_2^L + \tau_2^H}{2}$. If $E(\tau_2)$ is greater than the midpoint of the two limits then the resulting sign on the term is ambiguous. These claims hinge crucially on two aspects of the model. The first is that $-\tau_2 h_A(K_2^*, A_1)$ is non-decreasing in τ_2. This means that $-\tau_2 h_A(K_2^*, A_1)$ will be greater when evaluated at τ_2^H than it will when evaluated at τ_2^L, further implying that

$$-\int_{E(\tau_2)}^{\tau_2^H} \tau_2 h_A(K_2^*, A_1) d\tau_2 > -\int_{\tau_2^L}^{E(\tau_2)} \tau_2 h_A(K_2^*, A_1) d\tau_2.$$

This simply means that if the mean is the midpoint between τ_2^L and τ_2^H, then, referring to figure 5, the area of the graph between the midpoint and τ_2^H is greater than the area of the graph between τ_2^L and the midpoint. This is reflected in figure 6, where $V_{KK}(\cdot)\delta\tau_2 h_A(\cdot) f_\mu(\cdot)$ evaluated at τ_2^H is greater in absolute value than when evaluated at τ_2^L.

The second aspect of the model which is crucial to the claims is that the distribution is symmetric. This means that

$$\left|\int_{\tau_2^L}^{E(\tau_2)} f_\mu(\cdot) d\tau_2\right| = \left|\int_{E(\tau_2)}^{\tau_2^H} f_\mu(\cdot) d\tau_2\right| \text{ when } \tau_2^H - E(\tau_2) = E(\tau_2) - \tau_2^L.$$

This implies that the probability is not weighing more heavily on one side of the mean than the other over the range τ_2^L to τ_2^H. However, the results of this subsection are general only to the extent that they apply to symmetric distributions. Overall, the economic interpretation is quite simple. As before, if the mean is no greater than the midpoint of τ_2^L and τ_2^H, then an increase in the expected value of the distribution will increase the likelihood that the first term in brackets in (19) is relevant to the firm. That is, the higher will be the probability that τ_2 will be between τ_2^L and τ_2^H, hence more weight will be assigned to that term in the comparative static.

In summation, while separating the three terms in the numerators of comparative static equations (19) and (20), provided no clues concerning the overall signs of the comparative static results, several observations may be made about each term and its contribution to the incentive to invest. One such observation is that the $-V_{KK}\delta c_L F_\mu(\tau_2^L,\mu,\sigma)$ term will always mitigate the incentive to invest when the expected future tax increases. Conversely, the $V_{KK}\delta c_H F_\mu(\tau_2^H,\mu,\sigma)$ term will always increase the incentive to invest when the expected future tax increases. We also noted that, if the expected future tax is no greater than the midpoint between τ_2^L and τ_2^H, then the $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\mu(\cdot) d\tau_2$ term will increase the incentive to invest unambiguously. However, if the expected future tax were between the midpoint of τ_2^L and τ_2^H, and τ_2^H, then nothing could be said in general concerning the term's individual effect on the incentive to invest. Lastly, if the expected future tax were greater than τ_2^H, then the term had a negative impact on the incentive to invest in abatement capital.

Using the notion of stochastic dominance, it was demonstrated that where irreversibilities and unexpandabilities exist, the effect of an increase in future expected taxes increases the incentive to invest in abatement capital. Further, where abatement capital is fully reversible and expandable, a change in future expected taxes has no effect on the incentive to invest in either production or abatement capital.

The next subsection continues this analysis by examining the effects which uncertainty have on the incentive to invest.

3.2.2 The Degree of Uncertainty over Future Taxes - A Mean Preserving Spread

This subsection explores the effects of a second order shift (or mean preserving spread) in the distribution of future taxes. The notion of a mean preserving spread implies that the expected future tax rate remains constant while probability is reassigned to more extreme values. The result is a general decrease in the probability of observing a second period tax near its expected value.

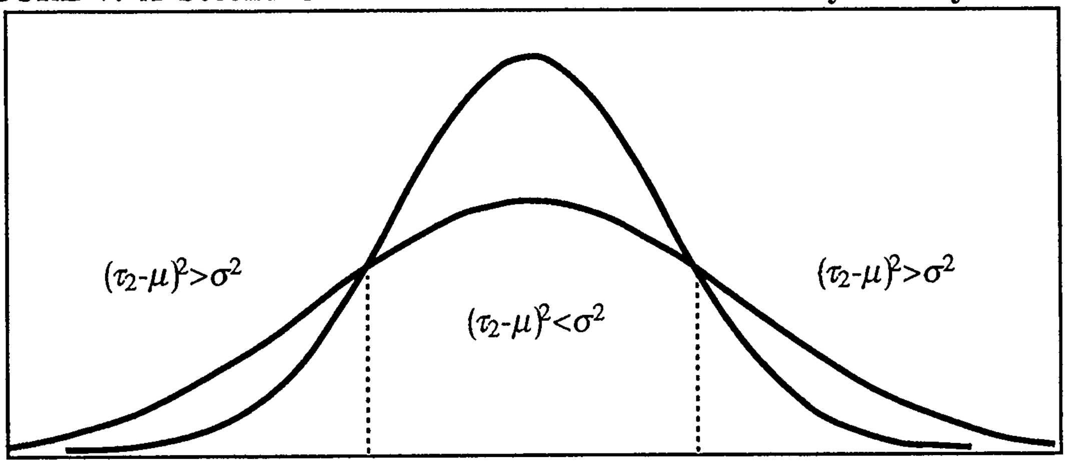
As in the previous subsection, the partial derivatives of the probability density function and cumulative probability functions will be evaluated. The partial derivative of the density function with respect to σ is,[12]

$$f_\sigma(\cdot) = \frac{\left[\frac{(\tau_2 - \mu)^2}{\sigma^2} - 1\right]}{\sigma^2\sqrt{2\pi}} e^{\frac{-(\tau_2-\mu)^2}{2\sigma^2}}.$$

It is now evident that $f_\sigma(\cdot) > 0$ if $(\tau_2 - \mu)^2 > \sigma^2$, and $f_\sigma(\cdot) < 0$ if $(\tau_2 - \mu)^2 < \sigma^2$, where μ is the expected value of the second period tax rate. This effect is illustrated with a discrete second order shift in figure 7.

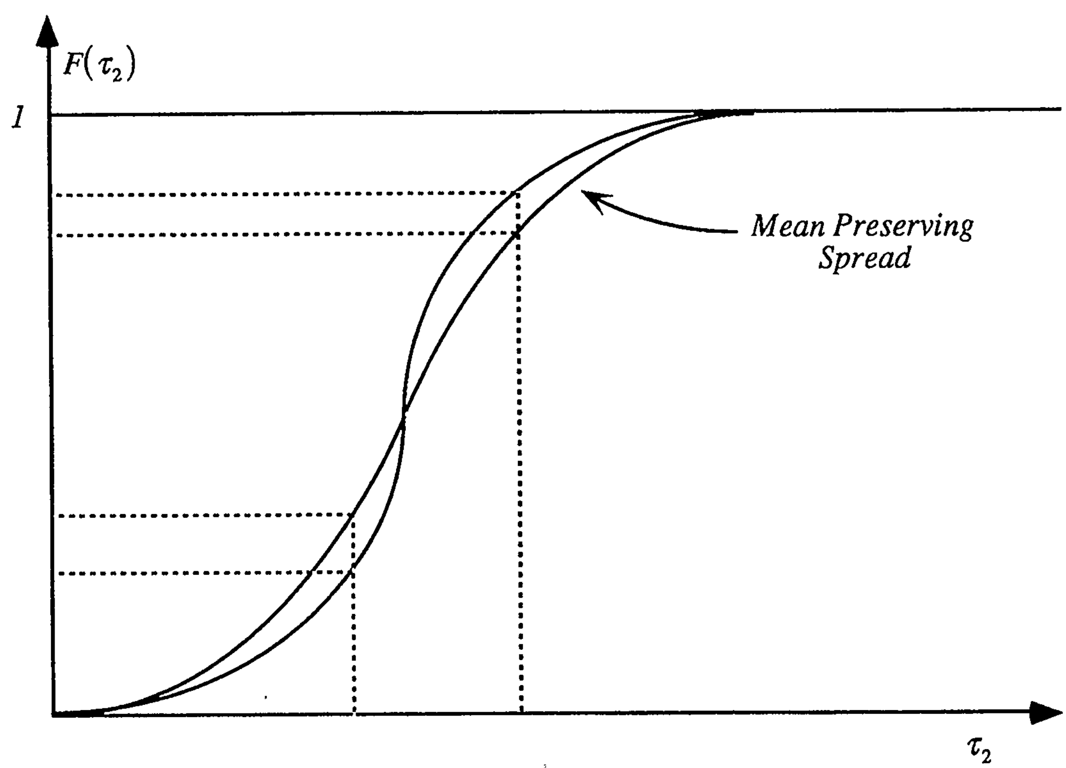
[12]Where σ is a monotonic transformation of the variance, σ^2.

FIGURE 7: A Second Order Shift in a Normal Probability Density Function



Given that we are exploring a symmetric distribution, it must be the case that $F_\sigma(\cdot) > 0$ for all $\tau_2 < \mu$, and $F_\sigma(\cdot) < 0$ for all $\tau_2 > \mu$. This is a characteristic of symmetric distributions which is easily verifiable in figure 8.

FIGURE 8: A Second Order Shift in a Cumulative Distribution Function



We now take a linear approximation of the model and evaluate it in the neighbourhood of the optimum. This is represented in matrix notation in (21).

$$\begin{bmatrix} V_{KK}(\cdot) & V_{AK}(\cdot) \\ V_{AK}(\cdot) & V_{AA}(\cdot) \end{bmatrix} \begin{bmatrix} dK_1^* \\ dA_1^* \end{bmatrix} = \begin{bmatrix} 0 \\ \delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\sigma(\cdot) d\tau_2 - c_L F_\sigma(\tau_2^L, \mu, \sigma) + c_H F_\sigma(\tau_2^H, \mu, \sigma)\right] \end{bmatrix} d\sigma \tag{21}$$

The comparative static expressions are given in (22) and (23) below.

$$\frac{dA_1^*}{d\sigma} = \frac{V_{KK}(\cdot)\delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\sigma(\cdot) d\tau_2 - c_L F_\sigma(\tau_2^L, \mu, \sigma) + c_H F_\sigma(\tau_2^H, \mu, \sigma)\right]}{|H|} >=< 0, \text{ and} \tag{22}$$

$$\frac{dK_1^*}{d\sigma} = \frac{\tau_1 h_{AK}(\cdot)\delta\left[\int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot) f_\sigma(\cdot) d\tau_2 - c_L F_\sigma(\tau_2^L, \mu, \sigma) + c_H F_\sigma(\tau_2^H, \mu, \sigma)\right]}{|H|} >=< 0. \tag{23}$$

Unlike first order shifts, the effects of second order shifts are ambiguous in terms of their effects on either type of capital. As such, this section will proceed by first examining the individual effects of the terms in (22) and (23). Having examined the individual terms, they will be evaluated on aggregate to gain insights as to why the derivatives are not 'signable'.

Beginning with (22), consider the second term inside the brackets, $-V_{KK}(\cdot)\delta c_L F_\sigma(\tau_2^L, \mu, \sigma)$. This term will be greater than zero if $\tau_2^L < \mu$, and less than zero if $\tau_2^L > \mu$. The economic reasoning is as follows: if $\tau_2^L < \mu$ then the probability that τ_2 will evolve such that its realized value is less than τ_2^L will increase as the variance increases. That is, more probability is being allocated to the more extreme values for τ_2.

This means that the firm recognizes that there is a correspondingly greater probability that 'good news' will arrive such that it may sell some amount of its first period abatement capital. This ability to sell abatement capital is a benefit of owning it so the firm's incentive to invest has increased. If, on the other hand, τ_2^L were greater than the expected value of the future tax, then a second order shift in the distribution of future taxes would decrease the probability that τ_2 will be realized with a value less than τ_2^L. This marginal reduction in the probability of 'good news' arriving will signal to the firm to exercise more caution in its first period investment decisions since the probability of a future reversal has diminished. Hence, the incentive to invest in abatement capital will decrease. Curiously, if τ_2^L is the expected future tax, then a second order shift will effect no change (via this term) in the incentive to invest since the probability that τ_2 will be less than τ_2^L will be 0.5 for all variances of the distribution.

Now turn to the third term in brackets in (22), $V_{KK}(\cdot)\delta c_H F_\sigma\left(\tau_2^H, \mu, \sigma\right)$. It should be clear that if $\tau_2^H > \mu$ then this term will be greater than zero, increasing the incentive to purchase abatement capital. Conversely, if $\tau_2^H < \mu$, then a second order shift in the distribution of future taxes will decrease the incentive to invest in abatement capital. If $\tau_2^H > \mu$, then the probability of τ_2 evolving such that it is greater than τ_2^H is increasing in σ. The firm then must take into consideration that the probability of 'bad news' arriving has increased, meaning that it is more likely that the firm will require a higher level of abatement capital in the second period. However, purchasing abatement capital is more expensive in the second period than in the first, so the firm's incentive is to increase its investment in abatement equipment in the first period. If $\tau_2^H < \mu$, though the probability of requiring more abatement capital in the second period is diminished so the firm will have less incentive to invest. Lastly, as with the second term in brackets, if $\tau_2^H = \mu$ there will no change in the probability of requiring additional capital in the second period so the term does not 'influence' the investment decisions.

Analyzing the first term in brackets, $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\sigma(\cdot)d\tau_2$ requires further investigation into the properties of symmetric distributions. Specifically, the following properties will be useful.

$\int_{\tau_2^L}^{\tau_2^H} f_\sigma(\tau_2,\sigma)d\tau_2 \le 0$ if $0 \le \tau_2^H - \mu \le \sigma$ or $-\sigma \le \tau_2^L - \mu \le 0$ or $\tau_2^L \le \mu \le \tau_2^H$, hence

$$V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\sigma(\cdot)d\tau_2 \le 0;$$

$\int_{\tau_2^L}^{\tau_2^H} f_\sigma(\tau_2,\sigma)d\tau_2 \ge 0$ if $\tau_2^H - \mu \le -\sigma$ or $\tau_2^L - \mu \ge \sigma$, hence

$$V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\sigma(\cdot)d\tau_2 \ge 0, \text{ and}$$

$\int_{\tau_2^L}^{\tau_2^H} f_\sigma(\tau_2,\sigma)d\tau_2 >=< 0$ if $\tau_2^L - \mu \le -\sigma$ and $-\sigma < \tau_2^H - \mu < 0$ or

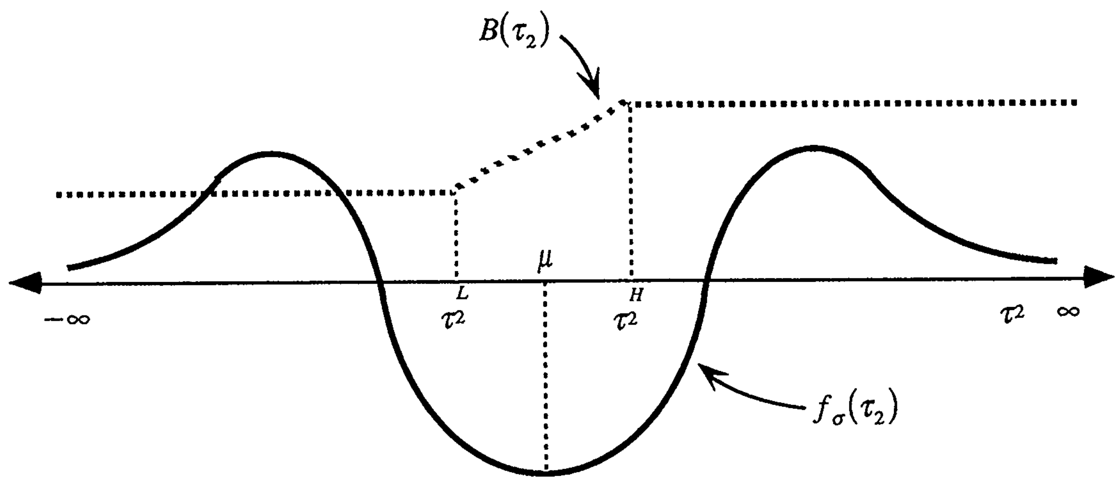
$\tau_2^H - \mu \ge \sigma$ and $\sigma > \tau_2^L - \mu > 0$, hence

$$V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\sigma(\cdot)d\tau_2 >=< 0$$

As has been shown, these properties may be used directly to sign $V_{KK}(\cdot)\delta\int_{\tau_2^L}^{\tau_2^H}\tau_2 h_A(\cdot)f_\sigma(\cdot)d\tau_2$ for all possible combinations of τ_2^L and τ_2^H. Again, the economic rationale is quite simple. The critical points for τ_2^L and τ_2^H given in the above properties are the points where the probability of the future tax evolving such that it is between τ_2^L and τ_2^H is either increasing, decreasing, or ambiguous. So if the probability is decreasing, the relevance of the term to the firm is diminishing and less weight will be placed on it. If the probability is increasing, the relevance of the term is increasing and correspondingly more weight is given to it in the analysis.

Examining each term separately provides some insights to the various forces shaping a firm's decision, but it provides little help in determining the overall effect of a change in the level of uncertainty on investment decisions. Figure 9 presents a familiar graph which clarifies this overall effect in terms of the function $B(\tau_2)$.

FIGURE 9: The Effects of a Second Order Shift



As the functions are represented in figure 9, the positions of τ_2^L and τ_2^H relative to the mean depict the case where the sign of $dA_1^*/d\sigma$ is ambiguous. To see this, recall that the area under $f_\sigma(\tau_2)$ below the zero line is equal to the two areas in the tails above the zero line. As drawn, it is unclear whether the area above the graph outweighs or is outweighed by the area below the graph. However, it is clear that if the mean is greater than τ_2^H, then an increase in uncertainty decreases the incentive to invest in abatement capital. So it would appear as though if a 'bad state' were anticipated, an increase in uncertainty is cause for optimism. Conceptually, the resulting increase in the probability of 'very bad news' (near the right tail) is perfectly offset by the reduction in the probability of 'fairly bad news', while the corresponding increase in the probability of 'good news' reduces the firm's incentive to invest. If, however, the expected tax is less than τ_2^L, then an increase in uncertainty influences the firm to purchase more capital. The reasoning is similar, there is a zero sum increase in the probability of 'good news' but a net increase in the probability of bad news. So the firm will respond by investing in more abatement capital. This is simply a characteristic of the Goldilocks principle - the distribution of good news or bad news is of no concern to the firm.

As before it may be noted that, trivially, the comparative static effect of a second order shift in the distribution of future taxes will have the same qualitative effect on production capital if $h_{AK}(\cdot) < 0$, and the opposing effect if $h_{AK}(\cdot) > 0$.

This subsection explored the various forces which influence the overall investment decisions when the uncertainty over future taxes increases. Each of these forces, represented by the individual terms in equations (22) and (23), have qualitative effects which are sensitive to the proximities and precise locations of the critical points, τ_2^L and τ_2^H, with respect to the expected future tax rate. As such, the terms were exhaustively examined, exploring all of the possible scenarios which could confront the firm. The discussion then turned to the overall effect of changes in uncertainty on the incentive to invest in the various types of capital. It was shown that if the expected tax rate did not reside in the range spanned by τ_2^L and τ_2^H, then the comparative static effects could be signed. Specifically, if expected taxes are greater than the critical point at which the firm will begin to purchase more abatement capital, then the firm will respond to an increase in the level of uncertainty by decreasing its investment in abatement capital. If, on the other hand, expected taxes are lower than the critical point at which the firm will begin selling abatement capital, the firm will respond to an increase in the level of uncertainty by increasing its investment in abatement capital. It was also noted that if $h_{AK}(\cdot) < 0$, then the corresponding effects on the incentive to invest in production capital will be qualitatively the same as the effect on the incentive to invest in abatement capital. If, however,

$h_{AK}(\cdot) > 0$, then the opposite is true. While this conclusion is trivial in terms of the present analysis, it has potentially large implications for policy. The following section examines this as well as other policy implications.

3.3 Implications of Comparative Static Results to Broad Policy Objectives

This section explores the effects of various policy changes in terms of the effectiveness of those changes in meeting its objectives. Specifically, we will examine the effects these policy changes have on the production of the externality. For example, policy makers or regulators presumably wish to curb the production of pollution. It is standard doctrine to believe that increasing the taxes on emissions will reduce the production of those emissions. The present analysis will suggest that the policy makers should be aware of the potential for counter intuitive responses to such policies.

We begin first by exploring the effect which an increase in the first period Pigouvian tax has on the production of pollution. This effect may be characterized by the following comparative static,

$$\frac{dh(K_1^*, A_1^*)}{d\tau_1} = h_K(\cdot)\frac{dK_1^*}{d\tau_1} + h_A(\cdot)\frac{dA_1^*}{d\tau_1}. \tag{24}$$

The sign of this derivative will be unambiguously negative if $\frac{dK_1^*}{d\tau_1} < 0$ and $\frac{dA_1^*}{d\tau_1} > 0$. That is, if the optimal response to an *ex ante* increase in the first period tax is to decrease employment in the production factor and increase investment in abatement capital, then the overall effect will be a reduction in the production of the externality. If either production factor investment increases or abatement capital investment decreases as an optimal response to increased taxes, however, then the net effect on the production of pollution is ambiguous. In the limit, if both investment in production capital increases and investment in abatement capital decreases, then the result of an increase in Pigouvian taxation will unambiguously increase production of the externality!

From (15) and (16) we know that,

$$\frac{dK_1^*}{d\tau_1} = \frac{h_K(\cdot)V_{AA}(\cdot) + \tau_1 h_{AK}(\cdot)h_A(\cdot)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) <=> \frac{-h_K(\cdot)V_{AA}(\cdot)}{\tau_1 h_A(\cdot)}, \text{ and}$$

$$\frac{dA_1^*}{d\tau_1} = \frac{h_A(\cdot)V_{KK}(\cdot) + \tau_1 h_{AK}(\cdot)h_K(\cdot)}{|H|} >=< 0 \text{ as } h_{AK}(\cdot) >=< \frac{-h_A(\cdot)V_{KK}(\cdot)}{\tau_1 h_K(\cdot)}.$$

Not surprisingly, the results of the comparative static depend on the sign and magnitude of $h_{AK}(\cdot)$, or rather, the technology of the externality production. Formally, the effect of an increase in the first period Pigouvian tax on the production of the externality depends upon $h_{AK}(\cdot)$, in the following manner.

$$\frac{dh(\cdot)}{d\tau_1} < 0. \text{ if } h_{AK}(\cdot) > MAX\left\{\frac{-h_K(\cdot)V_{AA}(\cdot)}{\tau_1 h_A(\cdot)}, \frac{-h_A(\cdot)V_{KK}(\cdot)}{\tau_1 h_K(\cdot)}\right\},$$

$$\frac{dh(\cdot)}{d\tau_1} > 0 \text{ if } h_{AK}(\cdot) < MIN\left\{\frac{-h_K(\cdot)V_{AA}(\cdot)}{\tau_1 h_A(\cdot)}, \frac{-h_A(\cdot)V_{KK}(\cdot)}{\tau_1 h_K(\cdot)}\right\}, \text{ and}$$

$$\frac{dh(\cdot)}{d\tau_1} >=< 0, \text{ otherwise.}$$

More to the point, if we substitute the partial effects directly into (24) we may achieve a more precise critical value for $h_{AK}(\cdot)$.

$$\frac{dh(K_1^*, A_1^*)}{d\tau_1} = \frac{V_{AA}(\cdot)h_K(\cdot)^2 + V_{KK}(\cdot)h_A(\cdot)^2 + 2\tau_1 h_{AK}(\cdot)h_K(\cdot)h_A(\cdot)}{|H|} \tag{24'}$$

From this expression, it can be shown that,

$$\frac{dh(\cdot)}{d\tau_1} >=< 0 \text{ as } h_{AK}(\cdot) <=> \frac{-1}{2\tau_1}\left[\frac{h_K(\cdot)V_{AA}(\cdot)}{h_A(\cdot)} + \frac{h_A(\cdot)V_{KK}(\cdot)}{h_K(\cdot)}\right], \tag{25}$$

which is exactly the average of the two critical points given above. This implies that one of production capital or abatement capital may be responding 'normally' to the increase in taxes, while the other is responding in a counterintuitive direction. For example, if abatement capital increases in response to an increase in the first period tax, and production capital also increases, it may be that overall pollution will increase as well. That is, the increase in production capital does more to increase pollution than the increase in abatement capital does to mitigate it. Intuitively, if taxes are increased, then the firm may wish to respond by increasing its investment in abatement capital. Assuming that $h_{AK}(\cdot)$ is negative, however, the sensitivity of pollution production to the level of production capital has declined. That is, one of the marginal costs of employing production capital has decreased, so the firm will respond by increasing investment in the production factor. In some cases this increase in the production factor will more than offset the mitigating effect of the increase in abatement capital. The end result is that the firm, by optimizing, has paradoxically increased the production of some externality in response to an increase in its taxation.

Therefore, and in general, if $h_{AK}(\cdot)$ is positive, or at least not 'too negative', then increases in the first period level of Pigouvian taxation will have the desired and intuitive appealing effect of reducing the amount of emission in the first period. If, however, $h_{AK}(\cdot)$ is 'too negative', then the amount of pollution generated will be increasing in the tax. Further, it may be noticed that the operational definition of 'too negative' depends in part on the initial level of the tax itself. From (25) it is apparent that if the tax is very small, the critical value of $h_{AK}(\cdot)$ is very negative. Conversely, if the tax is very large, the critical value of $h_{AK}(\cdot)$ is not very negative. However, this sensitivity to the level of the tax may be at least partially offset by the firm's optimizing behaviour (i.e. $h_A(\cdot), h_K(\cdot), V_{AA}(\cdot),$, etc. are all functions of τ_1.

A strategy which policy makers may wish to use is simply to threaten the imposition of tougher future regulation in order to encourage behaviour more consistent with certain objectives. In the context of this model, the threat would be in the form of higher future taxation. If the threat is at least partially credible, this may have the effect of increasing the expected level of future taxation. To evaluate the effect of an increase in expected future taxes use equations (19), (20), and (24) along with the function $B(\tau_2)$ to arrive at the following comparative static,

$$\frac{dh(K_1^*, A_1^*)}{d\mu} = \frac{[h_A(\cdot)V_{KK}(\cdot) + \tau_1 h_K(\cdot)h_{AK}(\cdot)]\delta\int_{-\infty}^{\infty} -B(\tau_2)f_\mu(\tau_2)d\tau_2}{|H|} >=< 0, \text{ as,} \tag{26}$$

$$h_{AK}(\cdot) <=> \frac{-h_A(\cdot)V_{KK}(\cdot)}{\tau_1 h_K(\cdot)}.$$

Here we see that the required conditions for an increase in expected future taxes to increase pollution may be slightly less imposing. By examining (16) we find that this critical value for $h_{AK}(\cdot)$ is the same critical value at which increases in first period taxes decreases investment in the abatement factor. Therefore, if $h_{AK}(\cdot)$ is low enough that increases in τ_1 decrease investment in the abatement factor, but not so low that increases in τ_1 increase pollution overall, then increases in expected future taxes will increase pollution unambiguously.

In this section we used the model to explore some of the policy implications of the firm's optimizing behaviour. In the analysis we found that the effect of increased taxes or increases in expected taxes on the production of the externality is technology dependent. That is, it depends on the value of $h_{AK}(\cdot)$. The implications for policy is quite clear -

establishing policy without a clear understanding of the technologies involved may be counter productive to the policy objectives

3.4 Concluding Remarks

The goal set out in this chapter was to evaluate some of the model's more relevant comparative statics. Section 3.1 began by ascertaining the effects of changes in various capital costs, resale costs, and first period taxes on the incentives to invest in both production and abatement capital. Many ambiguities were found, particularly in cross price effects and most notably in the effects of increases in first period taxes. These ambiguities were found to be a result of the function $h_{AK}(\cdot)$. This implies that many of the comparative static results are technology dependent.

Section 3.2 examined the effects of first and second order distributional shifts on the incentives to invest. In particular, it was found that changes in expected taxes increased the incentive to invest in first period abatement capital. The corresponding effect on production capital was found to be dependent on technology. The effects of second order shifts were also explored. These effects are ambiguous in general but, under certain circumstances, may be signed as having either a positive or negative influence on the incentive to invest in abatement capital. Again, the influence on production capital is technology dependent.

The implications for policy and achieving policy objectives were discussed in section 3.3. Here it was found that increases in first period taxes may or may not influence the firm to decrease its production of pollution. In fact, under certain technologies, it may actually increase pollution. It was also shown that increases in expected future taxes may also have the counter intuitive result of increasing the current production of emissions. Moreover, the conditions for this event were weaker than those required for first period taxes to increase pollution.

CHAPTER FOUR

4. SUMMATION AND CONCLUSION

4.1 Overview and Summary of Results

This thesis has used a two period model of investment under uncertainty to investigate the influences of Pigouvian taxation on the incentives to invest. Uncertainty was introduced into the model in the form of unknown future taxes. While the present tax was known with certainty, the firm only had distributional information regarding second period taxes. Investment decision were made over two types of capital, production capital and investment capital. Production capital could be purchased and sold in the second period for the same price as in the first period, making it fully reversible and expandable. Arbitrary levels of irreversibility and unexpandability were imposed for abatement capital - the second period purchase price of abatement capital was no lower than the original purchase price which was no greater than the second period resale price. The irreversibility and unexpandability implied two critical values of the future tax above which it would become optimal to purchase more abatement capital, below which it would become optimal to sell, and for values in between it would be optimal to do neither. This way of modeling investment explicitly internalizes the implications of first period investment decisions on second period returns.

Chapter two established the optimality conditions in the model. It was noted that a solution required several assumptions concerning the pollution generating technology. In particular, it was found that the production of pollution must be convex in abatement equipment - meaning that an additional unit of abatement capital abates more pollution when the stock of abatement capital is small than when it is large. Chapter two also showed that the scale properties of abatement technology, whether increasing or decreasing, were restricted in magnitude. Overall though, the model proved to be quite flexible in terms of both production and abatement technologies.

Chapter two also discussed Abel *et al*'s (1995) extension of Bernanke's (1983) "bad-news principle". The distribution of future taxation above and below the critical points is irrelevant to the firm's investment decisions. In other words, even if the firm knows that when high taxes are realized, they will be very high, investment decisions will not be influenced. This provides the context for Abel *et al*'s (1995) "Goldilocks principle" - the only news of interest is that which is neither too hot nor too cold.

Comparative statics were investigated in chapter three. The first section examined the effects on the incentives to invest in the two types of capital from changes in the first period tax rate and the various investment costs. It was found in general that the incentive to invest in capital was decreasing in its own first period price but there were ambiguous, or technology dependent, cross price effects. In particular, these cross price effects change qualitatively depending on whether there exist increasing or decreasing scale properties in abatement technology.

Changes in the degrees of irreversibility and unexpandability were characterized by examining the effects of changes in the second period purchase and resale prices of abatement capital. Increases in the resale price of abatement equipment reflects a decrease in the degree of irreversibility of investment. Since the cost of 'over investing' is diminished, the effect is to increase the incentive purchase abatement capital in the first period. Similarly, increases in the future purchase price of abatement equipment *increases* the degree of unexpandability. This provides the firm with an additional incentive to invest in abatement capital in the first period to avoid an increased penalty should taxes be high enough to warrant purchasing additional abatement equipment. As with first period prices, the cross price effects are characterized by a dependence on the scale properties of abatement capital.

The influences which first period tax levels have on the incentives to invest were also examined. It was found that these influences were ambiguous for both types of capital, again sensitive to the scale properties of abatement. The implication of this finding is that if there exist substantial increasing returns to scale in abatement equipment, increases in the first period Pigouvian tax will paradoxically increase the firm's production of pollution. Thus it would bode well for policy makers to have an intimate understanding of current abatement technologies.

The distributional effects of second period taxes were also explored in a comparative static analysis. Here it was shown that an increase in expected taxes increased the incentive to invest in first period abatement capital irrespective of the expected future tax. This increase in the incentive to invest is comprised of several competing incentives, each of which was explored. Again, the influence on the incentive to invest in production capital was found to be technology dependent. An implication of these findings is that if increasing present taxes decreases investment in abatement capital, then, irrespective of whether it increases the production of first period pollution, an increase in expected future taxes will unambiguously increase first period pollution.

The effects of an increase in the variance of future taxes were also examined. Here we found that the influence on the incentives to invest is ambiguous. In particular, if expected future taxes are no less than the critical value of the tax at which the firm purchases more abatement capital, then the effect is to reduce the incentive to invest in abatement capital. Conversely, if 'good news' is expected, then an increase in the variance of expected taxes increases the firm's incentive to invest in abatement capital. This is an application of the

Goldilocks principle of irreversible and unexpandable investment. Again, the influence on the incentive to invest in production capital is dependent on the abatement technology.

4.2 Suggestions for Future Research

This section explores some of the avenues for research which may build upon the insights and understandings gained in this thesis. Some of these imply greater departures from this model than others.

One such avenue would be to remove some of the investment freedom the firm enjoys in production capital. That is, remove the assumption that it is completely expandable and reversible and treat it in a similar manner to abatement capital. However, this may require an additional layer of uncertainty, rendering the model intractable.

It may prove interesting to infuse some short-run supply characteristics into the costs of abatement capital. For instance, if taxes evolve such that they are higher than expected, the cost of purchasing additional abatement capital may increase with the ensuing increase in demand. The net effect of this is to endogenize the future purchase and resale prices of abatement capital as functions of the future Pigouvian tax.

Another avenue for research explores the regulatory alternatives available to the policy makers. Indeed, a Pigouvian tax is not the only tool which may be used to correct for the inefficiencies of externality production. For example, regulation may come in the form of 'command and control' - quotas on the generation of pollution. Indeed, this may provide some valuable insights into the policies currently in practice. In many cases, future quotas are uncertain, while expectations and the degree of uncertainty may be well defined. If we model non-compliance and penalties, how then will a firm respond to such uncertainty?

We may find that firms will produce at levels above or below their present quotas due to the nature of investment decisions under uncertainty and irreversibility/unexpandability.

An alternative to quotas is to examine the effects on the incentives to invest when the firm must enter into an emissions market to purchase emissions permits. Given that many asset prices have been shown to contain random elements in their price evolution, it would seem that the framework in this model would lend itself to such a policy analysis. Allowing the firm to bank permits may have additional implications for investment strategy.

Abel *et al* (1995) showed how their model may be interpreted to show the respective values of the put and call options associated with investment. Such an analysis, while equivalent to the approach used here, may provide some additional insights into the investment strategies discussed in this thesis.

Yet another area which could benefit from research is that of the policy maker's derivation of the optimal tax. Most research in optimal tax policy does not recognize the effects of uncertainty and irreversibility on firm level investment decisions. Internalizing these factors may yield some important results, both in application and academically. Indeed this framework could be generalized to accommodate research into general optimal tax theory. What may be found is that another dimension of optimal tax policy is required given the role which uncertainty has been shown to have in investment decisions.

APPENDIX A

TABLES OF VARIABLES AND FUNCTIONS

TABLE 2: Description of Variables

Variable	Description and Alternative Names
A_t	The stock of abatement capital in period t - also called abatement equipment and abatement factor.
b	The purchase and resale price of the production factor in both periods.
c	The unit cost of abatement capital in the first period.
c_H	The purchase price of abatement capital in the second period.
c_L	The resale price of abatement equipment in the second period.
K_t	The stock of the production factor in period t - also called production capital.
τ_1	The first period Pigouvian tax rate to be applied to the production of the non market good (or externality).
τ_2	The second period Pigouvian tax rate. The tax is stochastic and is not known until the second period.
τ_2^H	This is the determined critical tax rate at which it becomes optimal to begin purchasing additional abatement capital.
τ_2^L	This is the determined critical tax rate at which it becomes optimal to begin selling extraneous abatement capital.
σ	The variance of the distribution of future Pigouvian tax rates.
μ	The expected value, or mean, of the future Pigouvian tax rate - also written $E(\tau_2)$.
δ	A discount factor.

TABLE 3: Description of Functions

Function	Description
$R(\cdot)$	The time invariant gross returns from employing the production factor.
$h(\cdot)$	The time invariant externality production function, increasing in production capital and decreasing in abatement capital.
$A_1(\cdot)$	A 'reduced form' function characterizing the optimal first period stock of abatement capital as a function of the exogenous variables.
$A_2(\cdot)$	A 'reduced form' function characterizing the optimal second period stock of abatement capital as a function of the exogenous variables.
$\tau_2^L(\cdot)$	The critical tax where it becomes optimal to sell abatement capital as a function of several exogenous variables and first period abatement capital.
$\tau_2^H(\cdot)$	The critical tax where it becomes optimal to purchase abatement capital as a function of several exogenous variables and first period abatement capital.
$F(\cdot)$	The cumulative distribution function for second period taxes.
$f(\cdot)$	The probability density function for second period taxes
$K_1(\cdot)$	A 'reduced form' equation characterizing the optimal stock of the first period production factor as a function of exogenous variables.
$k_2(\cdot)$	The function characterizing the optimal second period stock of the production factor if abatement capital is held constant at first period levels.
$K_2(\cdot)$	The function describing the optimal second period stock of production capital where second period abatement capital levels are endogenous.

APPENDIX B

THE APPLICATION OF LEIBNIZ'S RULE

We may formally write Leibniz's as follows:

Given, $$g(a,b,s)=\int_{a(s)}^{b(s)} h(s,x)dx \text{, then} \tag{B1}$$

$$\frac{\partial g(a,b,s)}{\partial s}=-h(s,a)a'(s)+h(s,b)b'(s)+\int_{a(s)}^{b(s)}\frac{\partial h(s,x)}{\partial s}dx. \tag{B2}$$

Consider the integration from negative infinity to infinity, using this rule.

$$G(a,b,s)=\int_{-\infty}^{\infty} h(s,x)dx=\int_{-\infty}^{a(s)} h(s,x)dx+\int_{a(s)}^{b(s)} h(s,x)dx+\int_{b(s)}^{\infty} h(s,x)dx. \tag{B3}$$

Employing Leibniz's rule yields,

$$\begin{aligned}\frac{\partial g(a,b,s)}{\partial s}&=h(s,a)a'(s)+\int_{-\infty}^{a(s)}\frac{\partial h(s,x)}{\partial s}dx-h(s,a)a'(s)+h(s,b)b'(s)\\&\quad+\int_{a(s)}^{b(s)}\frac{\partial h(s,x)}{\partial s}dx-h(s,b)b'(s)+\int_{b(s)}^{\infty}\frac{\partial h(s,x)}{\partial s}dx\end{aligned} \tag{B4}$$

Which may be rewritten,

$$\begin{aligned}\frac{\partial g(a,b,s)}{\partial s}&=\int_{-\infty}^{a(s)}\frac{\partial h(s,x)}{\partial s}dx+\int_{a(s)}^{b(s)}\frac{\partial h(s,x)}{\partial s}dx+\int_{b(s)}^{\infty}\frac{\partial h(s,x)}{\partial s}dx\\&=\int_{-\infty}^{\infty}\frac{\partial h(s,x)}{\partial s}dx\end{aligned} \tag{B5}$$

This is precisely the problem in the present analysis. The applicability of the rule depends upon the following features of the model. First,

$$A_2\left(\tau_2^L(A_1,b,c_L),b,c_L\right)=A_1=A_2\left(\tau_2^H(A_1,b,c_H),b,c_H\right),$$

$$k_2\left(\tau_2^L(A_1,b,c_L),A_1,b\right)=K_2\left(\tau_2^L(A_1,b,c_L),b,c_L\right), \text{ and}$$

$$k_2\left(\tau_2^H(A_1,b,c_H),A_1,b\right)=K_2\left(\tau_2^H(A_1,b,c_H),b,c_H\right).$$

This implies that, like the example of Leibniz's rule given above in (B4) to (B5), the values of the functions on either side of the limits τ_2^L and τ_2^H are equal. Thus the differential effects on the limits of integration cancel out on either side. So while the differential effects are present, their effects may be ignored.

APPENDIX C

THE ABATEMENT CAPITAL FIRST ORDER CONDITION

The objective function is shown for convenience.

$$
\begin{aligned}
V(K_1,A_1) &= R(K_1) - bK_1 - cA_1 - \tau_1 h(K_1,A_1) \\
&+ \delta\int_{-\infty}^{\tau_2^L(A_1,b,c_L)}\left[R(K_2(\tau_2,b,c_L)) - b(K_2(\tau_2,b,c_L)-K_1) + c_L(A_1 - A_2(\tau_2,b,c_L))\right]dF(\tau_2) \\
&\qquad - \delta\int_{-\infty}^{\tau_2^L(A_1,b,c_L)}\left[\tau_2 h(K_2(\tau_2,b,c_L),A_2(\tau_2,b,c_L))\right]dF(\tau_2) \\
&+ \delta\int_{\tau_2^L(A_1,b,c_L)}^{\tau_2^H(A_1,b,c_H)}\left[R(k_2(\tau_2,A_1,b)) - b(k_2(\tau_2,A_1,b)-K_1) - \tau_2 h(k_2(\tau_2,A_1,b),A_1)\right]dF(\tau_2) \\
&+ \delta\int_{\tau_2^H(A_1,b,c_H)}^{\infty}\left[R(K_2(\tau_2,b,c_H)) - b(K_2(\tau_2,b,c_H)-K_1) - c_H(A_2(\tau_2,b,c_H)-A_1)\right]dF(\tau_2) \\
&\qquad - \delta\int_{\tau_2^H(A_1,b,c_H)}^{\infty}\left[\tau_2 h(K_2(\tau_2,b,c_H),A_2(\tau_2,b,c_H))\right]dF(\tau_2).
\end{aligned}
\tag{5}
$$

$V_A(K_1^*,A_1^*)$ may be expressed (ignoring Leibniz's rule),

$$
\begin{aligned}
V_A(K_1^*,A_1^*) &= cA_1 - \tau_1 h_A(K_1,A_1) + \delta c_L F(\tau_2^L) + \delta c_H\left[1 - F(\tau_2^H)\right] \\
&+ \delta\int_{\tau_2^L(A_1,b,c_L)}^{\tau_2^H(A_1,b,c_H)}\left[\left[R'(K_2^*) - b - \tau_2 h_K(K_2^*,A_1)\right]k_{2A}(\tau_2,A_1,b) - \tau_2 h_A(K_2^*,A_1)\right]dF(\tau_2) = 0
\end{aligned}
\tag{C1}
$$

Equation (3a) asserts that as a condition of maximization, $R'(K_2^*) = b + \tau_2 h_K(K_2^*,A_1)$. So (C1) reduces to (5b),

$$
\begin{aligned}
V_A(K_1^*,A_1^*) &= -c - \tau_1 h_A(K_1^*,A_1^*) + \delta c_L F(\tau_2^L(A_1^*,b,c_L)) + \delta c_H\left[1 - F(\tau_2^H(A_1^*,b,c_H))\right] \\
&\qquad - \delta\int_{\tau_2^L(A_1^*,b,c_L)}^{\tau_2^H(A_1^*,b,c_H)} \tau_2 h_A(k_2(\tau_2,A_1^*,b),A_1^*)\,dF(\tau_2) = 0
\end{aligned}
$$

In effect, we can ignore the changes brought about in $k_2(\cdot)$ by changes in A_1, because $k_2(\cdot)$ is already being evaluated optimally. This is an application of the Envelope Theorem.

APPENDIX D

THE VALIDITY OF THE COMPOSITE FUNCTION

Take the negative of the heavy line (which we call $B(\tau_2)$)from figure 6 and integrate across its probability density function:

$$-\int_{-\infty}^{\infty} B(\tau_2)f(\tau_2)d\tau_2 = -\int_{-\infty}^{\tau_2^L} c_L f(\tau_2)d\tau_2 + \int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot)f(\tau_2)d\tau_2 - \int_{\tau_2^H}^{\infty} c_H f(\tau_2)d\tau_2 \tag{D1}$$

Simplify,

$$-\int_{-\infty}^{\infty} B(\tau_2)f(\tau_2)d\tau_2 = \int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot)f(\tau_2)d\tau_2 - c_L F(\tau_2^L) - c_H\left[1 - F(\tau_2^H)\right] \tag{D2}$$

Last, take the derivative with respect to the mean of the distribution.

$$-\int_{-\infty}^{\infty} B(\tau_2)f_\mu(\tau_2)d\tau_2 = \int_{\tau_2^L}^{\tau_2^H} \tau_2 h_A(\cdot)f_\mu(\tau_2)d\tau_2 - c_L F_\mu(\tau_2^L) + c_H F_\mu(\tau_2^H) \tag{D3}$$

Once again, this is precisely the three terms found in the brackets of (19).

BIBLIOGRAPHY

Abel, A.B., A.K. Dixit, J.C. Eberly, and R.S. Pindyck (1995). "Options, The Value of Capital, And Investment". *NBER Working Paper* No. 5227.

Bernanke, B.S. (1983). "Irreversibility, Uncertainty, and Cyclical Investment". *The Quarterly Journal of Economics* 98(1), 85-106.

Conrad, K., and C.J. Morrison (1989). "The Impact of Pollution Abatement Investment on Productivity Change: An Empirical Comparison of the U.S., Germany, and Canada". *Southern Economic Journal* 55(3), 684-698.

Dixit, A.K. and R.S. Pindyck (1994). *Investment Under Uncertainty*. Princeton: Princeton University Press.

Hartwick, J.M. (1990). "Increasing Pollution with Optimal Corrective Taxes". *Queen's University Institute for Economic Research Discussion Paper* #795.

Hirshleifer, J. and J.G. Riley (1992). *The Analytics of Uncertainty and Information*, New York: Cambridge University Press.

Kohn, R.E. (1988). "Efficient Scale of the Pollution Abating Firm". *Land Economics* 64(1), 53-61.

Muloney, M.T. and B. Yandle (1984). "Estimation of the Cost of Air Pollution Control Regulation". *Journal of Environmental Economics and Management* 11 (September), 244-263.

Pindyck, R.S. (1993). "Sunk Costs and Sunk Benefits in Environmental Policy." Unpublished Working Paper, Massachusetts Institute of Technology, May.

Ramachandran, B., and K.S. Lau (1991). *Functional Equations in Probability Theory*, San Diego: Academic Press.

Rutledge G.L. and M.L. Leonard (1992). "Pollution Abatement and Control Expenditures, 1972-90". *Survey of Current Business* June, 25-41.