Irritating Things

Krista Francis-Poscente
Assistant Professor, Education
St. Mary's University College
Calgary, Alberta
krista.francis@stmu.ab.ca

Sharon Friesen
Associate Professor, Faculty of Education
University of Calgary
Calgary, Alberta
sfriesen@ucalgary.ca

Trevor Pasanen
Graduate Student, Mathematics
University of Alberta

Francis-Poscente, K., Friesen, S, & Pasanen, T. (2011) Irritating things, *Delta-K: Journal of the Mathematics Council of the Alberta Teachers Association 49*(1), 22-7.

Beading with numbers can be an exciting way to engage children's mathematical thinking and develop their proficiency with addition. In this paper, we will explore a hands-on problem, *Irritating Things* (GENA, 2009), which combines color coding with integers from 0-9, to create colourful beaded bracelets. In solving this problem, students will use logical thinking and addition to translate the numbers into beads. The patterns and relationships in this problem can be represented and described with words, tables, numbers and bracelets. The multiple representations will help students organize and communicate their ideas. By working with multiple representations, students will gain flexibility in their thinking and further develop their proficiency with addition. The problem has multiple solutions enabling them to experience numerous ways to solve a problem. Working with problems with multiple solutions and representations give students opportunities to discuss and learn each others' problems solving strategies and solution. This may help them gain deeper understanding of addition, pattern and their own reasoning ability. *Irritating Things* easily lends itself to differentiated instruction, providing all students opportunities for success and challenges.

Irritating Things Problem

There are 10 beads of different colors numbers 0-9.

1. Pick a first and second bead. They can be the same number, or not. For example, pick bead # 6 and bead #7.

2. To get the third bead, add the numbers on the first and second beads. If the sum is more than 9, just use the last (ones) digit of the sum. Adding 6 + 7 equals 13. Using the rules, drop the first digit and use 3.

3. To get the next bead, add the numbers on the last two beads used, and use only the ones digit.

4. Keep going until the first and second beads repeat, in that order.
5. Tie them in a loop to make a bracelet! (Don't use the last two beads since they just repeat the first two beads)

Instructional Strategies

Day 1: Creating a colour coded addition key

Before students begin beading, we suggest that students make their own colour addition chart to help them with the beading addition. In the top row, start with the '+' sign in the uppermost left hand corner. Sequentially, place the numbers 0-9 in each column of the top row. In the left-most column, sequentially place the numbers 0-9. Then fill in the table with the accurate operations and color.

Figure 1: Example of a Colour Coded Addition Key

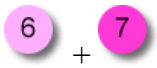
Developing this colour coded addition key should take one 45 minute class. Importantly, all the students should work with the same color codes; ie. a red bead should always be '0.' If everyone has the same colour code, finding addition errors will be much easier. Also, in order to recognize the patterns in the bracelets, all beads have to use the same number representation. See the Appendix for a template that can be used in class.

Day 2: Introducing the problem

Materials Needed:

- Numerous beads with at least 10 different colors. Each group will need approximately 150 different coloured beads. Inexpensive opaque pony beads from craft stores work well.
- Thick cording or shoe laces to thread beads: five strings per pair of students.

Demonstrate to the students how to begin to create their bracelet. Pick two beads 

. Add the numbers on the beads together: 

= 13. There are no beads

that represent '13,' so there is one more step to find the next bead. The rule states to drop off the first digit. When the first digit is removed from '13', the next bead is  . Reinforce how to find the next bead with the Colour Coded Addition Key. A SmartBoard™ is useful for demonstrations with the Colour Coded Addition Key. Circle the row beside the 6 and the column under the 7. The number 3 is the bead that is where the 6 row and 7 column intersect. Students can follow their Colour Coded Addition Key with their fingers.

Figure 2: Example of adding 7 + 6 on the Colour Coded Addition Key

Next, string the third bead on the shoe lace  .

To find the fourth bead, add the last two beads together:  = 10. Again, there is no bead that represents '10.' When the first digit is dropped, a  remains. Demonstrate how to find the fourth bead with the Colour Coded Answer Key. Circle the

row beside the 7 and the column under the 3. The number 0 is the bead where the 7 row and the 3 column intersect.

Figure 3: Example of adding 7 + 3 with the Colour Coded Answer Key

String the fourth bead on the shoe lace 

.

Encourage students to work in pairs, checking their work as they make each new bead on the bracelet. Pairing students together encourages collaboration and makes the problem easier to tackle. Persuade each student to be responsible for the accuracy of the addition. Plan for the bracelet making to take a couple of classes to complete.

Day 4 or 5: Finding the Patterns in Class Discussion

Find bracelets that are the same size. Have students compare their bracelets. Lead them to discover that bracelets of the same size are actually the same bracelet. Starting at with any two beads on the bracelet eventually results in the same bracelet. For

instance, if a student picked  and  , the bracelet would be the same bracelet with the same length if they picked  and  .

Another interesting pattern is that when the total lengths of all the bracelets are added: 1 + 3 + 4 + 12 + 20 + 60 = 100. 100 equals the total number of beads in the rows and columns of the colour coded addition key. 100 equals 10 rows time 10 columns for multiplication; the number of different ways beads can pair together.

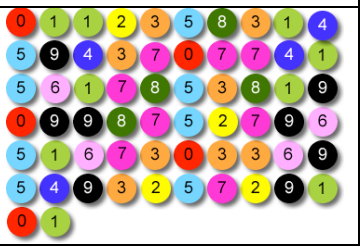
Solutions to Irritating Things

There are exactly six different bracelets of sizes 1, 3, 4, 12, 20 and 60. Expect many errors in addition. If the students have not created a bracelet with a total number of beads not equal to 1,3,4,12,20 or 60, be certain an addition error has been made. Have the students find their error. If available, a few parent volunteers could help students find the error. Once an error has been made, remove all the beads past the error. In our experience, students express dismay when the error is found and they have to pick up from the mistake. However, very quickly they are back beading again.

Now, one idea for differentiating is to encourage a group with slower addition skills to start with one of the smaller bracelets. Pick out the beads for  and  so students will find a bracelet that is twelve beads long or pick out the beads  and  for a pair to find a bracelet that is twenty beads long. They will find success quicker with a smaller bracelet.

When students find their bracelet, there are several more to be found. The students with quicker addition skills can find more bracelets. Ask them to find a different sized bracelet. With differentiation, the entire class can be engaged.

Solution:

# of beads	1	3	4	12	20	60
Sequence	0	0-5-5	4-2-6-8	2-1-3-4-7-1-8-9-7-6-3-9	2-2-4-6-0-6-6-2-8-0-8-8-6-4-0-4-4-8-2-0	

Mapping to the Program of Studies

Beginning in Grade 1 Alberta students start learning addition of numbers with answers up to 20. This problem can easily fit with the addition of integers in grade 1. Concepts of additions continue to be reinforced in further grades with increasing proficiency with higher number answers (Alberta Education, 2007). The colour coded answer key is a guide that can correspond to developing strategies for mental addition. Irritating Things addresses many of the specific outcomes for Grades 1-3 Number Sense.

Similarly, many of the specific outcomes in the General Outcomes Patterns and Relations are addressed when students find the recursive patterns in the bracelets. Students sort beads by number and colour, use beads as manipulatives to create, describe, and extend a circular pattern of number and colour. The colour coded addition key helps

students make predictions and find the next element of the bracelet. In Grade 3, the sequence reinforces Pattern and Variables by solving addition equations.

Importantly, this problem addresses the mathematical processes discussed in the Program of Studies (Alberta Education, 2007). Irritating Things connects addition to pattern, encourages the development of fluency with addition, visualization of addition and pattern, and develops mathematical reasoning through problem solving.

Conclusion

Irritating Things is provides an exciting opportunity to explore addition and pattern with problems solving. Creating a colour coded addition key reinforces the correspondence of number and addition building procedural fluency. Building the bracelets provides a non-pencil-and-paper method for developing proficiency with addition. Finding that each bracelet of the same size is the same bracelet is an exciting discovery of pattern. Importantly, this set of activities is rigorous, demanding and fun.

Irritating things is a rich problem and can be explored in different ways. For instance, what happens when use a different number of beads is used? How about 8 beads? How about 12 beads? Are there more bracelets ? Or are there less? How long is the longest bracelet? How long is the shortest? Using different numbers of beads takes students into operations with a non-ten base. Most of all, have fun exploring this investigation into addition and pattern.

References

Alberta Education. (2007). Mathematics Kindergarten to Grade 9 program of studies. Government of Alberta. Retrieved from http://education.alberta.ca/teachers/program/math/programs.aspx

GENA. (2009). Galileo Educational Network Association. Retrieved July 14, 2009, from http://www.galileo.org/math/puzzles/IrritatingThings.htm

Papert, S. (2002). Hard Fun. Retrieved July 14, 2009, from http://www.papert.org/articles/HardFun.html

Appendix: Colour Coded Addition Key Template

+	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
⓪	○	○	○	○	○	○	○	○	○	○
①	○	○	○	○	○	○	○	○	○	○
②	○	○	○	○	○	○	○	○	○	○
③	○	○	○	○	○	○	○	○	○	○
④	○	○	○	○	○	○	○	○	○	○
⑤	○	○	○	○	○	○	○	○	○	○
⑥	○	○	○	○	○	○	○	○	○	○
⑦	○	○	○	○	○	○	○	○	○	○
⑧	○	○	○	○	○	○	○	○	○	○
⑨	○	○	○	○	○	○	○	○	○	○