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Bells, Plates and Multidisciplinary Research: A Multidisciplinary Approach to Controlling the Frequency Response of a Freely Suspended Metal Plate to Approximate the Sound of a Large Church Bell

by

Daryl John Caswell

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Gerontology 0351
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- General 0578
- Ancient 0579
- Medieval 0581
- Modern 0582
- Black 0328
- African 0331
- Asia, Australia and Oceania 0332
- Canadian 0334
- European 0335
- Latin American 0336
- Middle Eastern 0333
- United States 0337

History of Science 0585
Law 0398
Political Science
- General 0615
- International Law and Relations 0616
- Public Administration 0617

Recreation 0814
Social Work 0452
Sociology
- General 0626
- Criminology and Penology 0627
- Demography 0938
- Ethnic and Racial Studies 0631
- Individual and Family Studies 0628
- Industrial and Labor Relations 0629
- Public and Social Welfare 0630
- Social Structure and Development 0700
- Theory and Methods 0344

Transportation 0709
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Women's Studies 0453

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- General 0473
- Agronomy 0285
- Animal Culture and Nutrition 0475
- Animal Pathology 0476
- Food Science and Technology 0359
- Forestry and Wildlife 0478
- Plant Culture 0479
- Plant Pathology 0480
- Plant Physiology 0817
- Range Management 0777
- Wood Technology 0746

Biology
- General 0306
- Anatomy 0287
- Biostatistics 0308
- Botany 0309
- Cell 0379
- Ecology 0329
- Entomology 0353
- Genetics 0369
- Limnology 0793
- Microbiology 0410
- Molecular 0307
- Neuroscience 0317
- Oceanography 0416
- Physiology 0433
- Radiation 0821
- Veterinary Science 0778
- Zoology 0472

Biophysics
- General 0786
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Geochemistry 0996
Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
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Physical Geography 0368
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- Mental Health 0347
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- Obstetrics and Gynecology 0380
- Occupational Health and Therapy 0354
- Ophthalmology 0381
- Pathology 0571
- Pharmacology 0419
- Pharmacy 0572
- Physical Therapy 0382
- Public Health 0573
- Radiology 0574
- Recreation 0575
- Speech Pathology 0460
- Toxicology 0383

Home Economics 0386

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Pure Sciences

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- General 0485
- Agricultural 0749
- Analytical 0486
- Biochemistry 0487
- Inorganic 0488
- Nuclear 0738
- Organic 0490
- Pharmaceutical 0491
- Physical 0494
- Polymer 0495
- Radiation 0754

Mathematics 0405
Physics
- General 0605
- Acoustics 0986
- Astronomy and Astrophysics 0606
- Atmospheric Science 0608
- Atomic 0748
- Electronics and Electricity 0607
- Elementary Particles and High Energy 0798
- Fluid and Plasma 0759
- Molecular 0609
- Nuclear 0610
- Optics 0752
- Radiation 0756
- Solid State 0611

Statistics 0463

Applied Sciences

Applied Mechanics 0346
Computer Science 0984

Engineering
- General 0537
- Aerospace 0538
- Agricultural 0539
- Automotive 0540
- Biomedical 0541
- Chemical 0542
- Civil 0543
- Electronics and Electrical 0544
- Heat and Thermodynamics 0348
- Hydraulic 0545
- Industrial 0546
- Marine 0547
- Materials Science 0794
- Mechanical 0548
- Metallurgy 0743
- Mining 0551
- Nuclear 0552
- Packaging 0549
- Petroleum 0765
- Sanitary and Municipal 0554
- System Science 0790

Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

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General 0621
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  - Généralités 0679
  - Anciennes 0289
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  - Généralités 0401
  - Anciennes 0294
  - Comparée 0295
  - Mediévale 0297
  - Moderne 0298
  - Africaine 0316
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  - Anglaise 0593
  - Asiatique 0305
  - Canadienne (Anglaise) 0352
  - Canadienne (Française) 0355
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- Économie
  - Généralités 0501
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  - Histoire 0509
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- Études canadiennes 0385
- Études féministes 0453
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- Géographie 0366
- Gérontologie 0351
- Gestion des affaires
  - Généralités 0310
  - Administration 0454
  - Banques 0770
  - Comptabilité 0272
  - Marketing 0338
- Histoire
  - Histoire générale 0578
  - Ancienne 0579
  - Médiévale 0581
  - Moderne 0582
  - Histoire des noirs 0328
  - Africaine 0331
  - Canadienne 0334
  - États-Unis 0337
  - Européenne 0335
  - Moyen-orientale 0333
  - Latino-américaine 0336
  - Asie, Australie et Océanie 0332
- Histoire des sciences 0585
- Loisirs 0814
- Planification urbaine et régionale 0999
- Science politique
  - Généralités 0615
  - Administration publique 0617
  - Droit et relations internationales 0616
- Sociologie
  - Généralités 0626
  - Aide et bien-àtre social 0630
  - Criminologie et établissements pénitentiaires 0627
  - Démographie 0938
  - Études de l' individu et de la famille 0628
  - Études des relations interethniques et des relations raciales 0631
  - Structure et développement social 0700
  - Théorie et méthodes. 0344
  - Travail et relations industrielles 0629
- Transports 0709
- Travail social 0452

SCIENCES ET INGÉNIERIE

SCIENCES BIOLOGIQUES
- Agriculture
  - Généralités 0473
  - Agronomie. 0285
  - Alimentation et technologie alimentaire 0359
  - Culture 0479
  - Élevage et alimentation 0475
  - Exploitation des péturages 0777
  - Pathologie animale 0476
  - Pathologie végétale 0480
  - Physiologie végétale 0817
  - Sylviculture et faune 0478
  - Technologie du bois 0746
- Biologie
  - Généralités 0306
  - Anatomie 0287
  - Biologie (Statistiques) 0308
  - Biologie moléculaire 0307
  - Botanique 0309
  - Cellule 0379
  - Écologie 0329
  - Entomologie 0353
  - Génétique 0369
  - Limnologie 0793
  - Microbiologie 0410
  - Neurologie 0317
  - Océanographie 0416
  - Physiologie 0433
  - Radiation 0821
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  - Zoologie 0472
- Biophysique
  - Généralités 0786
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- Biogéochimie 0425
- Géochimie 0996
- Géodésie 0370
- Géographie physique 0368
- Géologie 0372
- Géophysique 0373
- Hydrologie 0388
- Minéralogie 0411
- Océanographie physique 0415
- Paléobotanique 0345
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- Paléontologie 0418
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- Économie domestique 0386
- Sciences de l'environnement 0768
- Sciences de la santé
  - Généralités 0566
  - Administration des hipitaux 0769
  - Alimentation et nutrition 0570
  - Audiologie 0300
  - Chimiothérapie 0992
  - Dentisterie 0567
  - Développement humain 0758
  - Enseignement 0350
  - Immunologie 0982
  - Loisirs 0575
  - Médecine du travail et thérapie 0354
  - Médecine et chirurgie 0564
  - Obstétrique et gynécologie 0380
  - Ophtalmologie 0381
  - Orthophonie 0460
  - Pathologie 0571
  - Pharmacie 0572
  - Pharmacologie 0419
  - Physiothérapie 0382
  - Radiologie 0574
  - Santé mentale 0347
  - Santé publique 0573
  - Soins infirmiers 0569
  - Toxicologie 0383

SCIENCES PHYSIQUES

Sciences Pures
- Chimie
  - Genéralités 0485
  - Biochimie 487
  - Chimie agricole 0749
  - Chimie analytique 0486
  - Chimie minérale 0488
  - Chimie nucléaire 0738
  - Chimie organique 0490
  - Chimie pharmaceutique 0491
  - Physique 0494
  - PolymÇres 0495
  - Radiation 0754
- Mathématiques 0405
- Physique
  - Généralités 0605
  - Acoustique 0986
  - Astronomie et astrophysique 0606
  - Electronique et électricité 0607
  - Fluides et plasma 0759
  - Météorologie 0608
  - Optique 0752
  - Particules (Physique nucléaire) 0798
  - Physique atomique 0748
  - Physique de l'état solide 0611
  - Physique moléculaire 0609
  - Physique nucléaire 0610
  - Radiation 0756
- Statistiques 0463

Sciences Appliqués Et Technologie
- Informatique 0984
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  - Génie civil 0543
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  - Génie industriel 0546
  - Génie mécanique 0548
  - Génie nucléaire 0552
  - Ingénierie des systâmes 0790
  - Mécanique navale 0547
  - Métallurgie 0743
  - Science des matériaux 0794
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- Géotechnologie 0428
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Bells, Plates and Multidisciplinary Research: A Multidisciplinary Approach to Controlling the Frequency Response of a Freely Suspended Metal Plate to Approximate the Sound of a Large Church Bell" submitted by Daryl John Caswell in partial fulfillment of the requirements for the degree of Master of Science.

M. Epstein

Supervisor, Dr. Marcelo Epstein,
Mechanical Engineering

Benno Nigg

Dr. Benno Nigg,
Human Performance
Lab/Mechanical Engineering

G. D. Price

Dr. Glen Price,
Music

O. Vin

Dr. Oleg Vinogradov,
Mechanical Engineering

July 21, 1992

Date

ABSTRACT

Producing the sound of a large bell in an orchestral setting is considered to be one of the most significant unsolved problems of the percussionist. A real bell of the proper frequency range is too large to control in performance and too cumbersome to suspend or transport. The use of bell plates in place of actual bells has had limited success due to the difficulties encountered in producing accurate pitch, sufficient amplitude and a reasonable frequency response. This thesis is based on a novel, multidisciplinary approach which has resulted in the production of bell plates with marked improvement in the most problematic areas. The musical value of the research has been demonstrated by two bell plates designed to produce the sound of distant church bells in the final movement of Hector Berlioz's *Symphonie Fantastique*. The plates were used in performances by both the Calgary Philharmonic and the Vancouver Symphony.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

D = Flexural Rigidity = $\frac{Eh^3}{[12(1-\upsilon^2)]}$

υ = Poisson's ratio

x = horizontal direction

y = vertical direction

W = transverse displacement

ω = frequency in radians per second

ρ = density in kg per unit length

q = forcing function

V = potential energy

U = kinetic energy

m = mass

v = velocity

h = plate thickness

t = time

A, B, C, D = constants

α_r = characteristic functions for the modes of vibration of a plate

l = length

M = bending moment

C = shearing force

E = Young's Modulus of Elasticity (Gpa)

I = Moment of Inertia

C_i = constants

k = stiffness

ξ= normalized displacement in the x direction

η = normalized displacement in the y direction

ϕ = aspect ratio (length over width)

m = mode number of vibration along the long side

n = mode number of vibration along the short side

π = 3.14.....

S = simply supported

INTRODUCTION

Since the age of Galileo, science and art have existed in "two solitudes", with little or no communication between them. Any information transfer which did take place tended to flow from science to the arts. Science was thought to have much to offer the arts but the arts were considered too subjective to be of any use in science. Science has become the language of the 20th century and is still seen to be the most credible source of knowledge. In discounting the arts as lacking objectivity, scientists have denied themselves access to an entire sphere of conceptual and analytical knowledge which has the potential to open doors to the solution of the many difficult problems facing science today.

It is vital that science and art develop a more symbiotic relationship than that which currently exists. The centuries old rift between art and science must be closed and an atmosphere of mutual respect established before real advances in knowledge can be made. Science can only benefit from a full and equal partnership with the arts. Identifying and properly implementing the skills and knowledge that exist in the arts can bring about the solution of scientific problems which have previously defied solution. In addition, the knowledge gained from proper artistic input can often expose weaknesses in scientific theory and point the way to improvements in theory and practice.

The opportunity to test the validity of this approach came in the form of a request from the percussionists of the Calgary Philharmonic

Orchestra to design metal plates which would imitate the sound of large church bells for a performance of Hector Berlioz's "Symphonie Fantastique". Given the vast amount of research involving the vibration of plates, the problem initially appeared to be a straightforward design task employing one of the many variations on the Rayleigh-Ritz Method. The results, however, were totally unsatisfactory. The sound of the plates produced using the theory bore little resemblance to the power and beauty of church bells and the plates were impossible to tune accurately.

Further investigation of the problem revealed apparently insurmountable problems in developing an accurate theoretical model. In fact the possibility of plates imitating bells had long ago been rejected by Helmholtz [1.0]. Still, the percussionists of the Calgary Philharmonic were convinced that this should be possible. The intuitive convictions of the musicians proved, in fact, to be correct, but not by using the usual scientific approach. The path to an effective solution lay in a careful study of the skills, knowledge and intuition of highly trained musicians and the proper implementation of these factors in seeking a better scientific model.

The specific problem to be solved in this thesis is the prediction of the natural frequencies of a freely suspended plate. The original goal of identifying and using the "subjective" knowledge of musicians to compensate for an unworkable theoretical model and arrive at a successful solution has certainly been met, demonstrating the power and potential of a full and equal cooperation between artist and scientist. The final model and the actual prototype were very successful and were used in performances by the Calgary Philharmonic and the Vancouver Symphony. Combining art and science in this way has also raised new questions and pointed out many areas for further research.

CHAPTER 1

HISTORICAL LITERATURE SURVEY

The study of the acoustical/vibrational characteristics of metal plates has a particularly interesting place in the history of acoustics, vibration and music. Pythagoras may have been inspired to begin his numerical investigation of consonance, not by the vibration of a stretched string as is commonly thought, but (according to Boethius) through a chance encounter with blacksmiths pounding on metal plates. The different sizes of hammers used by the blacksmiths happened to produce a pleasing consonance of intervals. These sounds piqued the interest of Pythagoras. He then used a stretched string to develop his theory of tonal relationships which formed the basis of musical development in the western world [1.2].

Controversy developed through the centuries over the rigidity of Pythagoras' numerical relationships because of the musician's intuitive but unarticulated need for greater tonal flexibility. Eventually the conflict resulted in the development of several different methods of tuning such as tempered intonation and just intonation which enabled musicians to respond to the needs of new and more complex musical styles [1.3]. This controversy also contained the seeds of the philosophical point of this thesis: When applying the numerical concepts of science to the intuitive world of the arts, proper communication between the scientist and the artist has the potential to propel analytical methods to new standards.

There is little record of further analytical descriptions of music until the 15th century when Benedetti [1.4] tried to relate musical pitch to the frequency of mechanically observed oscillations. This idea remained undeveloped until the 17th century when Vincenzo Galilei [1.5], Francis Bacon [1.6] and Johannes Kepler [1.7] independently began to study the mathematical nature of sound and vibration. The apparent applicability of mathematics to music has prompted nearly every significant scientist and mathematician from the founders of modern science to the present to try their hand at formulating the behaviour of vibrating bodies and the resulting acoustical phenomena [1.7a].

Especially significant in these early years of modern science are the attitudes of scientists to music and the arts in general. The first identifiable scientists in the modern sense were also musicians or artists. Francis Bacon was a philosopher while Vincenzo Galilei was a lutenist and composer who made substantial contributions to musical theory. Galileo Galilei (son of Vincenzo) was a lutenist as well. Kepler's musical fluency is evident in his *Harmonices Mundi Libri V* [1.7b]. Understanding both the needs of the artist and the mathematical restrictions of Pythagorean harmony, these men consulted musicians and craftsmen. This led them to discover new mathematical relationships which allowed men like Vincenzo Galileo and Gioseffo Zarlino [1.7c] to expand on the arithmetic relationships developed seventeen centuries before.

As science and the arts gradually split into the separate disciplines of today, fewer of the immediate successors of the first men of science saw any need to cultivate skill in the arts. The interest of science in vibration and acoustics became “purely scientific”. Mersenne [1.10] began to relate frequency to the physical dimensions and the state of tension of the vibrating body. Hooke [1.11] affirmed the link between frequency and pitch using a rotating toothed wheel. Huygens began to formulate the concept of overtones and the idea that a body may vibrate with more than one frequency at the same time. Wallis took up the study of vibrational nodes [1.12] while Sauveur examined the phenomenon of beats and the different timbres of sounds having the same frequency. It was, in fact, Sauveur who coined the term ‘acoustique’ in 1700, separating the science of sound from the study of music. This coincided with the denigration of the work of Rameau [1.14] and other musicians by scientists such as Fonatelle [1.13]. Now it was not only unnecessary to cultivate skill and knowledge in the arts, it was undesirable. Sauveur was known to have “neither voice nor ear” but relied on musicians and craftsmen to help him in his acoustic experiments [1.13a].

Into this atmosphere of rift between science and the arts entered a very significant pair of scientists working in the middle of the 1700’s. The work of Daniel Bernoulli and Leonhard Euler clearly showed the error of those scientists who choose to discount the value of the arts. Bernoulli had trained as an organist while Euler, perhaps because of a visual impairment, had little or no training in the arts. Bernoulli is often considered to be the founder of mathematical physics while Euler has the reputation of being the most prolific mathematician of all time. Both men applied their skills to the problem of quantifying the phenomena of acoustics and vibration and shared the credit for developing a theory for the vibration of beams [1.15,1.16].

Euler, the mathematician, produced several works describing the mathematical nature of consonance and its effect on the evolution of scales and modes in music [1.16b]. Bernoulli, the musician/mathematical physicist, contributed to the analytical body of knowledge but maintained that there was more to musical sound than these simple relationships. He formulated the concept of sound as a superposition of many resonant modes of the same body and proposed the idea that the varying relative strengths of these modes were responsible for different qualities and timbres of tone.

Although Euler (as well as d'Alembert) developed the differential equations of motion and the general field of mechanics, he was unable to prove or disprove Bernoulli's ideas about the composition of sound. Verification of the concept of superposition would not come for another century although Fourier presented the same basic idea in relation to heat conduction. Euler was very critical of Fourier's work [1.17] and this was likely the reason that Fourier analysis was not developed until many years after Fourier died. The rejection by Euler of Fourier's ideas as well as Bernoulli's claim that Euler's view of musical relationships was too simplistic is a significant example of the rift between science and the arts resulting in an oversimplified analytical model of a physical phenomenon gaining acceptance in spite of the artistically influenced objection that the model does not address the true nature of reality. It took more than one hundred years of research for science to catch up with Bernoulli's (and Fourier's) concept of the complexity of sound which very likely stemmed from Bernoulli's expertise as a musician.

As Bernoulli moved away from pure mathematics to experimental methods supported by mathematical theory, metal plates and beams again became important as experimental models for the study of the physics of vibrating beams and rods. The work of Bernoulli and Euler paved the way for Ernst Chladni (1787) to develop his well known Chladni figures, formed by the migrations of sand on a vibrating brass plate to lines which were believed to represent the nodes of the modes of vibration [1.18]. Chladni first used these plates to verify the Euler-Bernoulli beam theory. The popularity of his figures also opened the door for research into the correlation of nodal patterns with the frequency of vibration. Sophie Germain used the calculus of variations to develop a differential equation for the vibration of plates which was later completed by Lagrange [1.18a].

As more complex relationships which could not be explained by the mathematical theories of the time were discovered, the role of careful observation became increasingly important. The need to accurately measure such parameters as the frequency and amplitude of vibration increased accordingly. The 19th century saw parallel research on the part of Faraday, Helmholtz and Rayleigh to understand the theoretical nature of sound and vibration as well as to develop more precise measurement devices to expose the physical actions of vibration bodies. Once again vibrating metal bars came to the fore as Scheibler developed a series of tuning forks in 1834 which permitted the determination of frequency within 4 hz [1.19]. The frequency standard of A = 440 hz was adopted (ca. 1840) and further advances in measurement using tuning forks and sirens were developed by Helmholtz, Koenig [1.20] and Rayleigh [1.21].

By the middle of the 19th century the interest in the development of acoustical theory and experimentation resulted in the production of the first major treatises on acoustics. Helmholtz's 1863 work *Die Lehre von den Tonemfindungen als Physiologische Grundlage für die Theorie der Musik* was followed by Rayleigh's book *The Theory of Sound* in 1877. From these two works stemmed the development of acoustics into the multi-faceted field that it is today.

The significance of Helmholtz's work was in the coordination of scientific research in acoustics and perception with the musical theory, instruments and styles of the time. Helmholtz was able to look further into the importance of timbre and the detection of harmonics through his development of 'Helmholtz Resonators' and the refinement of Schreibler's Tonometer. While Helmholtz acknowledged the necessity of training the ear as a sound analyzer and was an amateur musician, his work is (by his own admission) restricted by his limited artistic development [1.22]. His references to the 'braying' of brass instruments [1.23] and the 'detestable' sound of hand bell choirs do indeed brand his musical tastes as undeveloped. While his artistically simplistic view of musical sound allowed Helmholtz to see more clearly the fundamentals of the physics of music and the perception of sound, he missed an important opportunity to affirm the importance of the skill and knowledge of the artist in expanding this body of knowledge beyond its fundamental principles. The limitations of the scientific view of music were clearly recognized by the famous german critic Eduard Hanslick (a contemporary of Helmholtz) who stated that "science will never understand the complexity of the musical tone" [1.25].Only recently has the usefulness and importance of the skill and perception of the artist begun to be recognized by scientists and then only by a few [1.24].

Helmholtz made a significant contribution to both science and the arts despite (or perhaps indicated by) the criticism from both sides that he had leaned too far toward the other. He also recognized the potential of trying to understand the perceptions of the artist but admitted that he was "too much of an amateur to be able to approach it" [1.25a].

In the area of plates, Helmholtz successfully identified the similarities and differences between bells and plates: "Both have inharmonic partials but bells have these partials spread out more than plates." [1.26]. Helmholtz was put off any further consideration of the similarities by the seeming inability of a plate to produce a sound which is not a jumble of "secondary tones (which) are so numerous and nearly of the same pitch that most observers would probably fail to separate them satisfactorily" [1.27].

Rayleigh's *Theory of Sound* represents the first comprehensive theoretical development of the physics of sound. It is also the first successful attempt to develop an analytical method for determining the fundamental frequency of a vibrating plate. Rayleigh assumed that the energy of vibration exists only as kinetic and/or potential energy. If damping is ignored there is no change in the total energy of vibration of a body. This concept established a theoretical basis for the analysis of the frequency of vibration which spawned most of the methods used today. Rayleigh used his method to predict the fundamental frequency of a freely suspended circular plate.

Although Rayleigh's work provided the basis, it is actually Ritz's presentation of a paper in 1909 which generally receives the credit as the first real breakthrough in the analysis of a free-free metal plate [1.28]. Almost every method used to date acknowledges a debt to Ritz's addition to Rayleigh's method. Ritz expanded Rayleigh's method to include a better representation of the mode shapes of a vibrating plate. The mode shapes, together with the boundary conditions, permitted a reasonably accurate analytical method which would predict the harmonics as well as the fundamental frequency of a plate under any condition of support. Ritz's method serves to impose additional stiffness factors which result in a higher predicted frequency than actually occurs in practice. Ritz demonstrated the effectiveness of his method by analyzing a free-free square plate.

The completely free plate is difficult to analyze because of the complicated boundary conditions which must be included to solve the equation of motion. Researchers much prefer a clamped or simply supported model which presents less formidable mathematical difficulties. In the twentieth century, freely suspended metal plate analysis seems to undergo a revival every ten or twenty years as a new variation of the Rayleigh-Ritz method surfaces and is then put aside as its limitations are realized. In the 1930's, Mary Waller as well as Andrade and Smith reworked Chladni's research on nodal patterns and discovered that the Chladni figures did not show lines of zero amplitude but instead showed the condition of equilibrium between the acceleration of a point on a plate and the gravitational constant [1.29]. Nothing more happened until 1950 when Young [1.30] and Warburton [1.31] used Ritz's method and more sophisticated mathematics to extend the number of terms and therefore the number of modes and aspect ratios which could be used. In 1969, Leissa surveyed the research on the vibration of plates [1.32] and with the development of computer technology in the 1960's and 1970's presented the possibility of solving even more complex versions of the Rayleigh-Ritz method. Leissa used this technology to more precisely predict the behaviour of a great variety of aspect ratios and support conditions [1.33]. In 1980 Gorman presented an entire book on the free vibration of rectangular plates [1.34]. In this work he develops an extension to the Rayleigh-Ritz method which uses superposition of support methods to achieve still greater accuracy. Gorman also identified several conditions related to high aspect ratio plates which are of fundamental importance in the study of bell plates.

Since the introduction of the Rayleigh-Ritz method, extensive research has been done on the vibration of plates with the notable exception of the free-free plate. Every work which does address this condition does so with an apology for the lack of precision of the results. This indicates that there is much to be done in the analysis of the freely suspended plate.

CHAPTER 2

MULTIDISCIPLINARY CONSIDERATIONS

Research involving different disciplines within the same general area (eg. the sciences) requires the researcher to expand his or her knowledge and vocabulary in order to perform effectively. In bio-mechanics, the engineer must learn physiology as well as the mechanics of materials. Once this expanded knowledge base has been mastered, the resulting research often proves valuable in both disciplines.

Multidisciplinary research between entirely separate areas of knowledge (eg. science and art) involves a much more difficult barrier. Within the sciences or within the arts there exists a common knowledge base and a like-mindedness which facilitates interaction between two different sciences or two different arts. However, this common ground does not exist between science and art. Methods of education and performance evaluation are very different. The popular view has long been that science is objective while the arts are subjective. Science relies on logical process and method while the arts rely on intuition. Much has been said to challenge the posture of objectivity taken by science. However, the focus of this research is to explore the "subjective" or intuitive nature of the arts, to identify the skills and knowledge in a manner which the scientist can understand and to apply this knowledge to the solution of a problem which seems to defy objective analysis.

INTUITION

With this goal in mind, the first step is to examine the phenomenon of intuition. While many great scientists willingly attribute their discoveries to a flash of intuition (regarded as the inexplicable gift of the genius), their ideas will not be accepted until rigorous quantitative analysis provides verification. The arts on the other hand, and particularly the high arts, are entirely driven by intuition. The movement from note to note or brush stroke to brush stroke is intuitively chosen from a myriad of possibilities. As the musical phrase or visual picture takes shape, the process has become so complex and intricate that a quantitative analysis or representation is impossible. The arts must exist solely on the strength of an intuitive result which cannot be quantitatively validated, thus the perception of the arts as "subjective" and therefore of little use in science.

Intuition without quantitative analysis is viewed by science to be subjective, illogical and devoid of credibility in a research environment. This implies that the artist, who relies heavily on highly developed intuition, is also subjective, illogical and devoid of credibility in a research environment. Recently, intuition has been the subject of studies by several leading researchers in the field of psychology [2.1, 2.2, 2.3]. Within even the last two or three years there has been a significant shift in the scientific view of the process and validity of intuition. Rather than being seen as an inexplicable personal gift, intuition is becoming validated as the predictable product of training and practice. Intuition is not a gift but a learned skill requiring hundreds and thousands of hours of learning and experience. The problem with intuition is that it seems to by-pass the logical, methodical scientific process. A person with highly developed intuition seems to jump inexplicably to the answer, leaving those less experienced with the problem to wonder at the seeming lack of logical thought and to question the credibility of the answer.

University of Pittsburgh Professor of Psychology Robert Glaser maintains that "the performance of highly competent individuals indicates the possession of, rapid access to, and effective utilization of an organized body of conceptual and procedural knowledge" [2.4]. The knowledge base of a highly skilled person allows that person to quickly see patterns rather than individual steps or components. It is the rapidity of this pattern recognition which seems, to the non-expert, to defy a logical step-by-step progression.

Pattern recognition is one of the fundamental skills of the highly trained musician as well as the scientist. This is one of the reasons why the training of a musician is such a long and difficult process [2.5]. As well as developing the physical or fine motor skills to perform on a musical instrument, the musician must also, over time, develop a unique set of 'aural templates', specific to his instrument, with which to control tone quality, pitch and rhythm. To the musically unskilled scientist or amateur musician, the results of these unseen skills and processes are assumed to be the 'gift or talent' of the artist.

To relate this idea to the problem at hand, consider the orchestral percussionist, the church bell and the free vibration of a freely suspended metal plate.

THE PERCUSSIONIST

A percussion instrument is, by definition, struck and usually left to vibrate naturally (no forcing function). The percussionist does not control the sound once the vibration is set in motion. This means that the instrument itself possesses a quality of natural resonance or timbre that can be selected but not acted upon by the percussionist. It is the job of the percussionist to constantly seek out new and better sounding instruments. As a result, the skilled percussionist has a very highly developed sense of timbre stemming from the constant refinement of the aural templates against which the tone quality of all his instruments is measured. In addition, the percussionist and particularly the tympanist must be able to hear the pitch of instruments, like the tympani, which often have a very complex and inharmonic frequency response [2.6]. A string player or brass player, for example, has not developed the same aural templates and is usually unable to tune tympani because of the confusing harmonic content.

CHURCH BELLS

The sound of church bells is unmistakable due to their size, shape and material which have evolved throughout the centuries [2.7]. At the same time, each bell sounds different from every other bell. It seems reasonable to say that different listeners will prefer different bells, but there will be no doubt that all are bells. The choice of bell is subjective but the acoustical behaviour of all bells is constant (objective). When comparing the skills and experience (aural templates) of the percussionist to an untrained person, it can be seen that the borderline between the subjective (good or bad) and the objective (bell or not bell) will shift. The years of experience or data gathering of the percussionist will permit a much more refined categorization of bell sounds. For example, is it a Chinese bell or a South American bell? Is it a thin bell or a thick bell? At a much finer level, is it a wedding bell or a funeral bell? The percussionist has a vast amount of experience and expertise which allows him to differentiate between bell sounds in a way that will appear to the untrained person to be intuitive and subjective but which is, in fact, objective and definable.

METAL PLATES

Given that the percussionist possesses the experience and expertise necessary to critique an experimental bell sound, the scientist must now ask the question "Is the musically trained ear superior to a sound analyzer when comparing the sound of a church bell to the sound of a suspended metal plate?" Tests with very good and very poor brass instruments have shown that an FFT sound analyzer, being a mathematically averaging computer, cannot differentiate between subtle tone quality differences easily heard by even an untrained ear. If the desired result is not just a bell-like sound but a distant, eery bell sound calling the witches from their revel, (as in *Symphonie Fantastique*) the trained ear is, at the moment, the only choice. In addition, the ability of the percussionist to pick out the dominant pitch from a harmonically confusing frequency response can be very useful in tuning the bell plates. Once the proper sound is found, the sound analyzer can be invaluable in identifying the most important aspects of the sound which can then be analyzed mathematically.

SCIENTIFIC CONSIDERATIONS IN DEALING WITH MUSICAL PROBLEMS

Having established the potential usefulness of the artist, it is now necessary to clearly define the areas of expertise required and the areas of limitations of both the scientist and the artist. The basic premise is that the goal of the research must be a musically credible result. This goal will be best realized by using the tools of the scientist, which are well documented but only approximate analytical methods, together with the skills of the artist, which are poorly documented or understood but able to cope with an enormous complexity of sensory input. The scientist must be aware that a high degree of complexity must be carried through the analysis in spite of the temptation to simplify the problem. For example, a simpler support method for a metal plate will allow a better analytic result, but the sound will be musically unacceptable. The artist is very skilled at producing sound and critiquing the quality of that sound but the scientist has the analytical tools for identifying and manipulating the useful properties of a musical instrument while minimizing the acoustically undesirable elements.

CHAPTER 3

A STUDY OF THE MUSICALLY TRAINED EAR AND ITS POTENTIAL FOR APPLICATION OUTSIDE OF THE MUSICAL SPHERE

The training of a professional musician spans a period of ten to twenty years, from childhood well into adult life. The focus of this training is on the musical development of the auditory system as well as the coordination of the motor skills necessary to perform on a particular musical instrument. In spite of this long and intensive training period, the musician often functions no better than average in clinical auditory testing [3.1]. In the areas where musicians perform better than average, a minimum of training will bring a non-musician up to the same level. However, to assume that the auditory system of the musician functions no better than normal is to ignore the overwhelming empirical evidence to the contrary. It is more reasonable to say that the auditory skills of the trained ear have not yet been clearly identified. The purpose of this chapter is to shed some light on the problem of identifying the unique qualities of the trained auditory system and to examine the possibility of adapting these skills to use outside of the musical sphere.

> MUSIC: "The science or art of ordering tones or sounds in succession, in combination and in temporal relationships to produce a composition having unity and continuity." [Webster's New Collegiate Dictionary].

To transform Webster's definition of music into a form more suitable for psycho-acoustical analysis, musical sound could be defined as pleasing sounds which possess the elements of pitch, timbre, duration and loudness in a temporal setting. If sound is the language of music, then the musician must be expert at controlling the elements of musical sound. This study is restricted to consideration of the classically trained orchestral musician as this field of music requires the most extensive training.

TRAINING

Every serious student of music receives ear training in order to recognize the fundamental pitch relationships of harmony and melody. Over time these relationships have become increasingly complex. Early music consisted only of melody with no harmonic accompaniment [3.2]. Two or more voices sang in unison. The different frequency ranges of individual voices eventually necessitated the use of the octave. The incorporation of more than one pitch (harmony) began with the introduction of the vocal or instrumental drone. Accidental harmonies resulted and the harmonic interval of the fifth was the first to gain acceptance. From a frequency perspective, harmony progressed from accepting only unison or the octave to accepting fractional relationships such as the fifth (frequency ratio = 3/2) and the fourth (4/3). The melodic and harmonic structures we use today began from these basic concepts.

The student of music must go through the same process of development, learning to recognize, produce and appreciate the musically appropriate pitch relationships [3.3].

Up to this point, the training of the musical ear is common to all classically trained musicians. An advanced student must now begin to examine the quality of sounds and phrases specific to his instrument rather than just the correctness of pitch and rhythmic relationships. A sense of timbre is coupled with fine adjustments in pitch, rhythm and loudness to produce the eloquent aural shading typical of the fully developed artist. However, each family of musical instruments requires the development of aural skills which are unique to that family. The pianist, for example, has little control over the piano string once it has been struck. A violinist, on the other hand, can continuously alter the pitch, timbre, duration and amplitude of any given note. Therefore, the pianist must be very adept at balancing the loudness and timing of the beginning of each note or chord but cannot influence the actual tone of the note once it has been sounded. The violinist is, in addition, concerned with developing control of the tone and pitch of individual notes. The variables which require adjustment by the performer depend on the type of instrument. A summary of these variables is presented in Table 3.1. Any attempt to determine the nature of the aural skills of a musician must take this into account.

Table 3.1 exposes the distinct acoustical demands of the various orchestral instruments. Obviously, a double bass player must focus on the opposite end of the frequency scale from the flutist. On a more subtle level, the harpist has more control over the initial response of a note than the pianist because the strings are plucked with the fingers rather than struck by mechanical hammers. Although the instruments are very similar, the harpist must develop an extra dimension in the use of the auditory system in order to exert sufficient control over the instrument. An instrument like the violin permits and requires total control over the pitch. The violinist must therefore develop a very fine pitch sense. Such a high level of pitch recognition is not as critical for the brass player who is assisted in playing the correct pitch by the natural harmonic series of an open pipe [3.4]. The table shows that, while all musicians have the same level of basic ear training, the advanced player must develop special skills related to his particular instrument. This is an important consideration when choosing musicians for the purpose of applying their skills outside of the musical sphere. A tuba player will not be as effective in a high frequency problem as a flute player.

PHYSIOLOGY

Having established the range of acoustical variables which the musician must be able to control, the next step is to examine the auditory system to isolate any factors which may contribute to musical training.

The ear is considered to be a passive sensing device. Unlike a muscle, exercise will not increase stamina, sensitivity or reaction time. However, it cannot be said that all ears are therefore identical. Statistically, there is a range in the effectiveness of different systems from totally deaf to hypersensitive. Auditory systems capable of a high level of training will be those systems without defects. In general it can be said that if an auditory system has a normal physical makeup, it has the potential for musical training and once trained that same system will not differ in any physical sense from the untrained system.

While musicians may possess auditory systems free of developmental defects, damage to those systems is an occupational hazard in the music industry [3.5]. It is interesting to note that, although hearing damage is common amongst musicians, it often does not prevent the musician from performing properly. This presents two important considerations. Musicians are able to compensate for the loss by shifting the focus from the primary sensing mechanism to a secondary mechanism [3.6]. For example, under conditions of extremely loud music in an orchestra, players are able to maintain the proper pitch through the physical sensation of the vibration of the instrument. In addition, successful performance in spite of hearing damage indicates that the skill of the musician is located at a higher processing level of the central nervous system than that of sensation.

The psychology of hearing deals mostly with judging the quality of a sound with respect to pitch, timbre and loudness [3.10]. A paper by McAdams [3.11] refers to the concept of "harmonic template matching" to explain the psychological development of hearing in musicians. The concept of harmonic template matching could be described as the memorization by each musician of a vast series of tonal qualities. For any given musical condition of style, harmony, melody, emotional and structural function, the accomplished artist can draw on this collection of templates, The incoming sound can then be compared to that of the appropriate template in the mind of the performer and feedback adjustments are made to bring the actual sound in line with the conceived sound. The development of these templates is a long process involving extensive exposure and trial and error.

As stated earlier, it is not enough merely to recognize musically appropriate sounds. A musician must develop the psycho-motor control to produce the desired sound. What form this development will take depends on the demands of the particular instrument. For the pianist, a fine sense of touch and timing between the ear and the fingers is essential. For the brass player, the coordination of the ear with the tongue, lips and fingers is required. String players need a fine spatial sense on the fingerboard combined with an even pressure and motion in the bow arm. To this point, isolating the various skills of the musician related to the auditory system has exposed aural skills which (with the exception of quality judgement) can quickly be taught to a subject with no musical ear training. The unique abilities of the musician surface in the coordination of these skills during performance where countless complex acoustical adjustments must be made quickly and continuously. This dynamic sensory-psycho-motor coordination involves every level of the auditory system. Most of the higher levels of functioning are poorly understood and difficult to access.

Until recently, study of the auditory system has been through static, single tone stimuli which do not address the dynamic nature of music and the musician. Static testing removes the musician from his sphere of expertise. Every performer has experienced the drastic reduction in control and quality which occurs when attempting to produce a sound such as the constant repetition of a single tone in a non-musical setting. Given the need for dynamic functioning, it is not surprising that musicians do not perform any better than average in aural testing. Until the auditory system is better understood and dynamic testing methods are developed, the unique qualities of the musically trained ear will continue to defy analysis.

NON-MUSICAL APPLICATION OF THE TRAINED EAR

There is a significant potential in uniting the skills of the scientist and the artist in the field of acoustics and vibration. The overriding problem in such an endeavour lies in the fact that the artist strives to control complexity and randomness while the scientist wants to reduce the problem to its basic principles. In order for the scientist to work successfully with the artist, the scientist must adapt to the need for dynamic licence while the artist must recognize the need for consistency. Given the fundamental conflict and the lack of precise knowledge regarding the functioning of the trained ear, a productive union of the scientist and the artist will be difficult to achieve. However, a few ground rules could pave the way for a successful ensemble.

The restrictions of each type of instrument dictate the skills which will be developed by the musician. The musician should be matched to the problem using information such as that presented in Table 3.1. Again, it would not be productive to involve a double bass player in a high frequency problem.

It is essential to allow the artist to work with the proper tools. Electronically produced, poor quality sound in a static environment will not tap the skills of the artist. Music performed on real instruments will involve the musician at the most productive level.

Real music reproduced by high quality electronics might be effective and would supply a degree of consistency.

Communication between the scientist and the artist is the most important and the most frustrating aspect for both parties. The subjective interpretation of the artist must be reconciled with the scientist's need for concise, factual and consistent data. This can only be achieved in an atmosphere of mutual respect.

The incorporation of these basic guidelines could facilitate the use of the musician in solving problems outside the musical world. The most potentially successful applications of this concept are in the area of room acoustics and musical instrument building. Other applications may surface as the concept becomes better developed.

TABLE 3.1
MUSICAL INSTRUMENT CONTROL CHART

INSTRUMENT	FREQ. RANGE (HZ)	VARIABLES AND CONTROL OF VARIABLES (Total Control= ■, Good= ▮, Poor= ❙, No Control= │)					
		Attack	Decay	Ampl.	Freq.	Timbre	
						Single	Mult.
Piano	27___4186	❙	❙	❙	│	❙	▮
Violin	196__2093	▮	■	■	■	▮	▮
Viola	130__1046	▮	■	■	■	▮	❙
Cello	65____659	▮	■	■	■	▮	❙
Bass	41____246	▮	■	■	■	▮	❙
Oboe	233__1396	▮	■	■	▮	▮	│
Bassoon	58___622	▮	■	■	▮	▮	│
Clarinet	146__1658	▮	■	■	▮	▮	│
Flute	261__2043	❙	■	■	▮	❙	│
Trumpet	174___932	▮	■	■	▮	▮	│
Trombone	82____466	▮	■	■	▮	▮	│
French Hn	61____698	▮	■	■	▮	▮	│
Tuba	41____311	▮	■	■	▮	▮	│
Tympani	87____174	❙	▮	▮	■	❙	│
Marimba	130__2793	❙	▮	▮	│	❙	▮
Harp	32___3176	▮	❙	❙	│	❙	▮
Drums	white or pink	❙	▮	❙	│	❙	│

CHAPTER 4

THE THEORY OF THE VIBRATION OF PLATES

The governing differential equation for the free vibration of a thin plate undergoing small amplitude vibration is:

$$\frac{\partial^4 W(x,y)}{\partial x4} + \frac{2\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} - \frac{\omega^2 \rho}{D} W(x,y) = 0 \qquad (1)$$

where

x = distance along plate width

y = distance along plate length

$W(x,y)$ = transverse displacement of the plate

ρ = mass per unit area of the plate

$D = \dfrac{Eh^3}{12(1-\upsilon^2)}$

E = Young's Modulus of elasticity

h = plate thickness

υ = Poisson's ratio

ω = frequency of plate vibration in radians per second

Equation (1) is based on the simplifying assumptions:

(a) The thickness of the plate is small compared with other dimensions.

(b) No strain is suffered by the middle surface.

(c) A line normal to the middle surface remains straight and normal after deformation.

(d) Load is normal to the surface (if there is a load).

(e) Deflections are small in comparison with the thickness of the plate.

(f) The plate material is linearly elastic.

Exact solutions for equation (1) with appropriate boundary conditions are very difficult to come by, particularly for the freely suspended plate. However, approximate methods do exist which can give good results for the prediction of the natural frequencies of vibration (ω_n, n = 1,2,3,....) for a given plate which obeys the initial assumptions.The prediction of the natural frequencies of vibration of a freely suspended rectangular plate is the focus of this thesis. We can, therefore, restrict the analysis to examining the available methods for determining these natural frequencies.

The additional requirement that the sound of the vibrating plate must be an approximation of the sound of a large church bell will lead to a very complex version of equation (1) which would have to model the structural damping of the modes of vibration as well as the strike of the mallet. Rather than attempt to develop or solve a model of this degree of difficulty, the existing approximate methods will be examined in order to extract the features which, when coordinated with the knowledge of competent musicians, can be used to develop a simpler model.

The methods available for the approximation of ω under various conditions of support stem from the Rayleigh-Ritz method first introduced in 1909 as a modification of an earlier method developed by Lord Rayleigh [4.2].

RAYLEIGH-RITZ METHOD:

Rayleigh's original method was based on the assumption that the energy in a body vibrating as a conservative system would be entirely due to the kinetic energy of motion (U) and, in the case of a plate, the elastic potential energy of bending (V). If a suitable mode shape W(x,y) is assumed and the total amount of energy is considered to be a constant, the frequencies of vibration of the body can be determined. Ritz improved the representation of the mode shape by incorporating a series of functions, all of which conform to the boundary conditions. From Timoshenko [4.3], consider a segment of a plate of uniform thickness h.

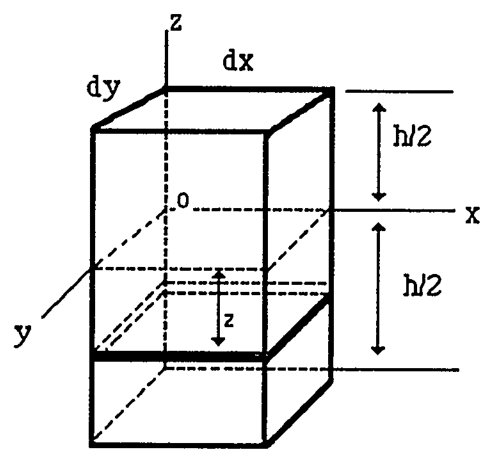


Fig. 4.1

Fig. 4.1 A Plate Element

And, using the following well known relationships:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \tag{3}$$

where

$u_1 = -z\, \partial W/\partial x$

$u_2 = -z\, \partial W/\partial y$

$u_3 = W$

$\lambda = \dfrac{\upsilon E}{(1+\upsilon)(1-2\upsilon)}$

δ_{ij} = Kronecker's Delta

σ_{ij} = stress components (plane stress only)

ε_{ij} = strain components

$G = \dfrac{E}{2(1+\upsilon)}$ (the shear modulus of elasticity)

The potential energy of the shaded strip in Fig. 1 can be represented by:

$$dV = \left(\frac{\varepsilon_{xx}\sigma_x}{2} + \frac{\varepsilon_{yy}\sigma_y}{2} + \varepsilon_{xy}\tau_{xy} \right) dx\, dy\, dz \qquad (4)$$

where

τ_{xy} = shear stress in the x,y plane at level z

Substituting equations (2) and (3) into (4) yields:

$$dV = \frac{Ez^2}{2(1-\upsilon)^2}\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W \partial^2 W}{\partial x^2\, \partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right] dx\, dy\, dz \qquad (5)$$

For a plate of constant thickness h the total potential energy is

$$V = \frac{D}{2}\int\!\!\int\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W \partial^2 W}{\partial x^2\, \partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right] dx\, dy \qquad (6)$$

where

$$D = \frac{Eh^3}{12(1-\upsilon^2)}$$

and the kinetic energy will be

$$U = \frac{\rho h}{2}\int\int\left(\frac{dW}{dt}\right)^2 dxdy \tag{7}$$

In a vibrating body displacement must be a function which changes in time and space [$W = W(x,y,t)$] while obeying the boundary conditions imposed by the support method.

If we assume that

$$W = W_0 \cos \omega t \tag{8}$$

where W_0 is a function of x and y describing the modes of vibration, from equations (6) and (8) the potential energy will be a maximum when $W = W_0$.

$$V_{max} = \frac{D}{2}\int\int\left[\left(\frac{\partial^2 W_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W_0}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W_0 \partial^2 W_0}{\partial x^2 \; \partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W_0}{\partial x \partial y}\right)^2\right]dx\,dy \tag{9}$$

The kinetic energy will be a maximum for

$$\frac{\partial W}{\partial t} = \omega W_0 \tag{10}$$

which gives

$$U_{max} = \frac{1}{2}\rho h \omega^2 \int\int W_0{}^2 dxdy \tag{11}$$

From equations (12) and (14), knowing that $V_{max} = U_{max}$ for synchronous motion of a conservative system:

$$\frac{\frac{D}{2}\int\int\left[\left(\frac{\partial^2 W_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W_0}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W_0 \partial^2 W_0}{\partial x^2 \ \partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W_0}{\partial x \partial y}\right)^2\right] dx\, dy}{\rho \frac{h}{2}\int\int (W_0{}^2 dxdy)} \tag{12}$$

It is difficult to establish exactly where Rayleigh's method ends and Ritz's method begins but the dividing line could be assumed to be that Rayleigh's method relies on a single term to describe the mode shape and satisfy the boundary conditions. This single term gives reasonably accurate results for simple systems such as the simply supported plate. However, for more complex conditions such as the freely suspended plate, the expressions which obey the boundary conditions:

$$\frac{\partial^2 W(x,y)}{\partial X^2} + \upsilon\frac{\partial^2 W(x,y)}{\partial Y^2} = 0 \qquad \frac{\partial^2 W(x,y)}{\partial Y^2} + \upsilon\frac{\partial^2 W(x,y)}{\partial X^2} = 0$$

$$\frac{\partial^3 W(x,y)}{\partial Y^3} + (2-\upsilon)\frac{\partial^3 W(x,y)}{\partial Y \partial X^2} = 0 \qquad \frac{\partial^3 W(x,y)}{\partial X^3} + (2-\upsilon)\frac{\partial^3 W(x,y)}{\partial X \partial Y^2} = 0 \tag{13}$$

are very difficult to formulate and a better approximation of the mode shape is required.

Ritz's method,an extension of Rayleigh's method, was developed to deal with the more complicated support conditions and has dominated this type of analysis since it was first presented in 1909. Using Ritz's extension to Rayleigh's method, W_0 can be presented in the form of a series

$$W_0 = a_1\alpha_1 (x,y) + a_2\alpha_2 (x,y) + a_3\alpha_3 (x,y) + \ldots \tag{14}$$

where α_n (x,y) are suitable functions which satisfy the boundary conditions:

$$\frac{\partial^2\alpha_n}{\partial X^2} + \upsilon\frac{\partial^2\alpha_n}{\partial Y^2} = 0 \qquad \frac{\partial^2\alpha_n}{\partial Y^2} + \upsilon\frac{\partial^2\alpha_n}{\partial X^2} = 0$$

$$\frac{\partial^3\alpha_n}{\partial Y^3} + (2-\upsilon)\frac{\partial^3\alpha_n}{\partial Y\partial X^2} = 0 \qquad \frac{\partial^3\alpha_n}{\partial X^3} + (2-\upsilon)\frac{\partial^3\alpha_n}{\partial X\partial Y^2} = 0 \tag{15}$$

The series approximation for W_0 will take the form

$$W_0 = \sum_{m=1}^{p}\sum_{n=1}^{q} a_{mn}\, X_m(x)Y_n(y) \tag{16}$$

Substituting eq. (14) into (12) will produce an equation which is linear with respect to the coefficients a_{mn}. These coefficients are the only unknowns and must be calculated to give a minimum value for equation (12). This will result in a system of equations of the type:

$$\frac{\partial}{\partial a_{ij}}\int\!\!\int\left[\left(\frac{\partial^2 W_0}{\partial x^2}\right)^2+\left(\frac{\partial^2 W_0}{\partial y^2}\right)^2+2\upsilon\left(\frac{\partial^2 W_0}{\partial x^2}\frac{\partial^2 W_0}{\partial y^2}\right)+2(1-\upsilon)\left(\frac{\partial^2 W_0}{\partial x\partial y}\right)^2-\frac{\omega^2\rho h}{D}W_0{}^2\right] dx\, dy = 0$$

---

(18)

Equating the determinant to zero will give the approximate values for the natural frequencies of vibration.

The choice of the characteristic functions for X(x) and Y(y) is the essential difference between the many variations on the Rayleigh-Ritz method. Two common choices are to model the deflections X(x) and Y(y) with either beam functions or a Fourier series.

CHARACTERISTIC FUNCTION MODELLED AS A SERIES OF BEAM FUNCTIONS.

The derivation of the shape functions of a vibrating beam is presented here in detail because of its usefulness in future developments. From Timoshenko [4.3]:

Modelling a beam of rectangular cross section lying along the x-axis with transverse deflection in the y direction as a result of vibration:

Fig. 4.2

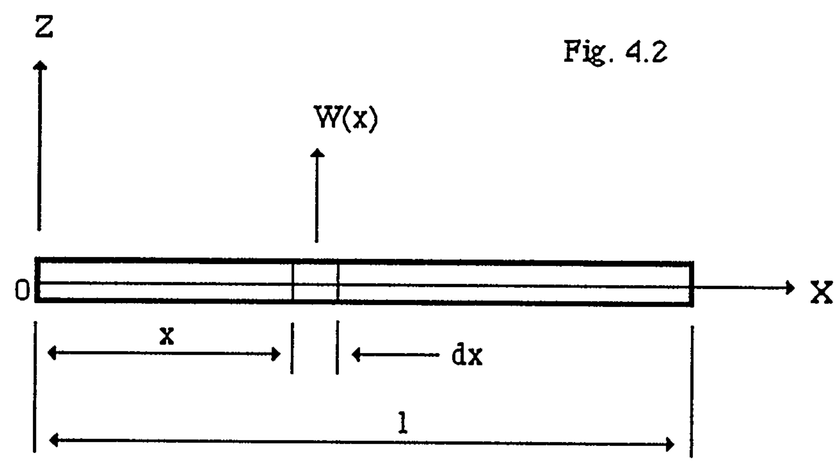


Fig. 4.2 Analytical Model For A Beam

Under the effects of vibration, the element dx will be subjected to internal and external forces as shown in the following free body diagram:

Fig. 4.3

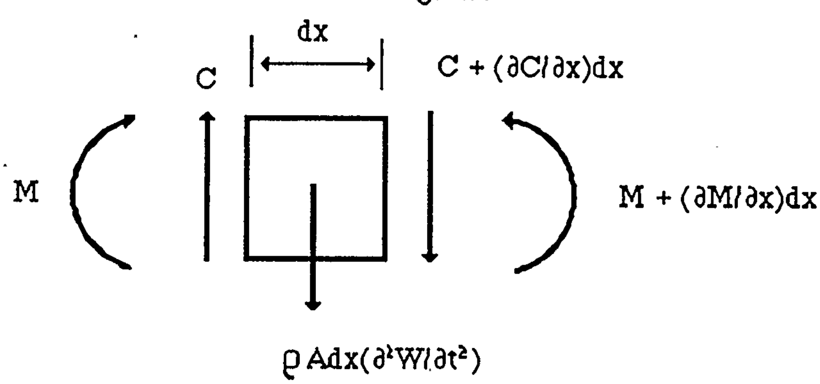


Fig. 4.3 Free Body Diagram Of A Beam Element

Where

- C = shear force
- M = bending moment
- A = cross sectional area of the beam
- $\rho A\,dx$ = mass of element dx

For a beam vibrating transversely (displacement in the y direction) dynamic equilibrium results in:

$$\frac{\partial M}{\partial x}dx - Cdx = 0 \qquad (19)$$

for the balance of moments and

$$\frac{\partial V}{\partial x}dx + \rho A dx\,\frac{\partial^2 W}{\partial t^2} = 0 \qquad (20)$$

for the balance of forces.

From (19) and (20):

$$\frac{\partial^2 M}{\partial x^2}\, dx = -\rho A dx\, \frac{\partial^2 W}{\partial t^2} \tag{21}$$

Now, incorporating the relationship

$$M = EI\frac{\partial^2 W}{\partial x^2} \tag{22}$$

where I = the moment of inertia

and substituting (22) into (21):

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 W}{\partial x^2}\right) dx = -\rho A dx\, \frac{\partial^2 W}{\partial t^2} \tag{23}$$

or

$$EI\, \frac{\partial^4 W}{\partial x^4}\, dx = -\rho A dx\, \frac{\partial^2 W}{\partial t^2} \tag{24}$$

and

$$\frac{\partial^4 W}{\partial x^4} = -\frac{1}{q^2}\frac{\partial^2 W}{\partial t^2} \tag{25}$$

where

$$q = \sqrt{\frac{EI}{\rho A}} \tag{26}$$

Since deflection varies harmonically with time in a normal mode of oscillation,

$$W_i = X\left[A_i\cos(\omega_i t) + B_i\sin(\omega_i t)\right] \tag{27}$$

for the i^{th} mode. For any particular mode, substituting (27) into (25) yields:

$$\frac{d^4 X}{dx^4} - \frac{w^2}{q^2}X = 0 \tag{28}$$

Equation (28) is a fourth order ordinary differential equation. The general solution of (28) can be written in the form:

$$X = C_1\sin(kx) + C_2\cos(kx) + C_3\sinh(kx) + C_4\cosh(kx) \tag{29}$$

where

$$k^2 = \frac{\omega}{q} \tag{30}$$

The constants C_1, C_2, C_3, C_4 are determined by the boundary conditions.

The total deflection response can be expressed as a summation of all the normal modes:

$$W = \sum_{i=1}^{\infty} X_i\left[A_i\cos(\omega_i t) + B_i\sin(\omega_i t)\right] \tag{31}$$

with appropriate coefficients A_i and B_i.

To move now from a beam to a plate, we can superimpose this result in both the x and y directions in eq. (18a) which can then be used in the Rayleigh-Ritz method. Leissa [4.4] used this displacement model with 30 terms (i=30) to predict the frequencies of vibration for plates with aspect ratios of 1 to 2.5 under the 23 possible types of support (two free sides and two simply supported sides, etc.). The case of the completely free plate is also presented although Leissa admits that this is the most poorly behaved analytical model. Some significant results from this model relating to the problem at hand are presented in Table 7.1 and Graph 7.1.

CHARACTERISTIC FUNCTION MODELLED AS A FOURIER SERIES

An extensive work by Gorman [4.5] uses a Lévy type solution which employs a single series of trigonometric functions to describe transverse plate motion. The importance of the aspect ratio (a/b) necessitates the use of a dimensionless form of equation (1):

$$\frac{\partial^4 W(\xi,\eta)}{\partial\eta^4} + 2\phi^2\frac{\partial^4 W(\xi\eta)}{\partial\eta^2\partial\xi^2} + \phi^4\frac{\partial^4 W(\xi,\eta)}{\partial\xi^4} - \phi^4\omega^4 W(\xi,\eta) = 0 \quad (32)$$

where

$$\xi = \frac{x}{a},\ \eta = \frac{y}{b} \text{ and } \phi = \frac{a}{b} \quad \text{(see fig.4.4)}$$

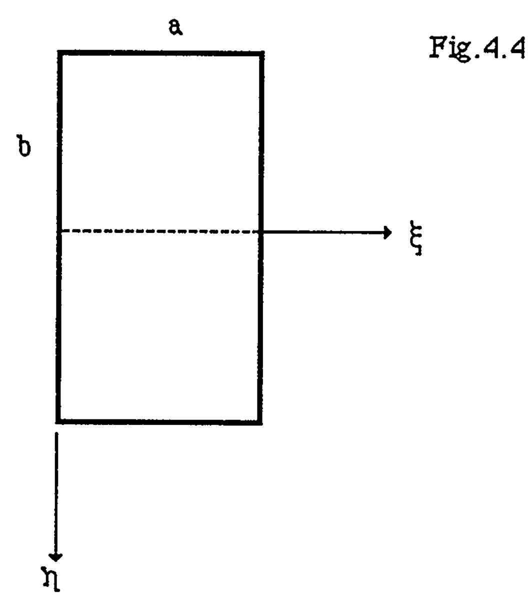


Fig. 4.4 Coordinate System For Analysis Of A Freely Suspended Plate

Transverse motion is represented by

$$W(\xi,\eta) = \sum_{m=1}^{k} Y_m(\eta)\sin(m\pi\xi) \tag{33}$$

where the coefficients Y_m are functions of η.

Substituting equation (33) into (32) will give:

$$\sum_{m=1}^{\infty}\left\{\frac{d^4Y_m(\eta)}{d\eta^4} - 2\phi^2(m\pi)^2\frac{d^2Y_m(\eta)}{d\eta^2} + \phi^4\left[(m\pi)^4 - \omega^4\right]Y_m(\eta)\right\}\sin(m\pi\xi)=0 \tag{34}$$

This is an ordinary fourth order homogeneous differential equation with constant coefficients which will have a solution for each value of m.

$$Y_m(\eta) = A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sin\gamma_m\eta + D_m\cos\gamma_m\eta \tag{35}$$

when $\quad \omega^2 > (m\pi)^2$

and

$$Y_m(\eta) = A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sinh\gamma_m\eta + D_m\cosh\gamma_m \tag{36}$$

when $\quad \omega^2 < (m\pi)^2$

where $\quad \beta_m = \phi\sqrt{\omega^2 + (m\pi)^2} \quad (37)$

and $\quad \gamma_m = \phi\sqrt{\omega^2 - (m\pi)^2} \quad$ or $\quad \phi\sqrt{(m\pi)^2 - \omega^2} \quad (38)$

whichever is real. The constants A_m, B_m, C_m, D_m are determined by the boundary conditions.

Gorman introduces a further refinement in modelling the free vibration of rectangular plates by developing a unique superposition method to simplify the calculations required. The method claims greater accuracy for the completely free plate than any previous method. The results pertaining to the present problem are presented in Tables 6.2 to 6.4.

ANALYSIS OF A SIMPLER CASE:

Knowing that the freely suspended plate is the most difficult to model because of the end conditions, examination of a more approachable support method may offer additional insight into the more complicated case.

A plate which is simply supported on all sides and vibrating in its doubly symmetric modes is the simplest of the free vibration plate problems with simply supported edges.

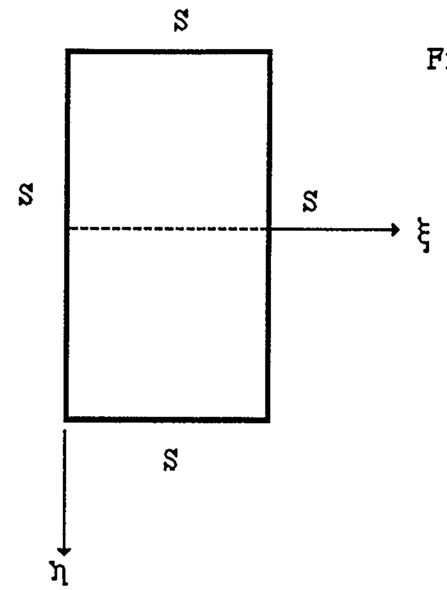


Fig. 4.5 Coordinate System For The Simply Supported Plate

Deleting the antisymmetric terms from equations (35) and (36) yields:

$$Y_m = A_m \cosh\beta_m\eta + D_m \cos\gamma_m\eta \tag{39}$$

for $\omega^2 > (m\pi)^2$ and

$$Y_m(\eta) = A_m \cosh\beta_m\eta + D_m \cosh\gamma_m \tag{40}$$

for $\omega^2 < (m\pi)^2$.

With this model, the boundary conditions at x = 0 and x = 1 are already satisfied and it remains only to enforce the boundary conditions at $\eta = \frac{1}{2}$ or

$$Y_m(\eta) = \frac{d^2Y_m(\eta)}{d\eta^2} = 0 \qquad \Bigg|_{\eta=\frac{1}{2}} \tag{41}$$

The result is two sets of simultaneous homogeneous algebraic equations:

$$A_m \cosh\frac{1}{2}\beta_m + D_m \cos\frac{1}{2}\gamma_m = 0 \tag{42a}$$

$$A_m \beta_m{}^2 \cosh\frac{1}{2}\beta_m + D_m \gamma_m{}^2 \cos\frac{1}{2}\gamma_m = 0 \tag{42b}$$

for $\omega^2 > (m\pi)^2$ and

$$A_m \cosh\frac{1}{2}\beta_m + D_m \cosh\frac{1}{2}\gamma_m = 0 \tag{43a}$$

$$A_m \beta_m{}^2 \cosh\frac{1}{2}\beta_m + D_m \gamma_m{}^2 \cosh\frac{1}{2}\gamma_m = 0 \tag{43b}$$

for $\omega^2 < (m\pi)^2$.

A nontrivial solution for equations (42) requires that the determinant be equal to zero. This in turn will result in an eigenvalue solution of the form:

$$\omega^2 = (m\pi)^2 + \frac{(n\pi)^2}{\phi^2} \tag{44}$$

The implications of equation (44) are fundamental to the solution of the problem at hand.

Two important considerations arise:

1. For a long narrow plate (large ϕ) equation (44) indicates that the modes along the short side (n) will not develop until several modes along the long side (m) have developed. This presents the possibility of suppressing unwanted modes in the frequency response.

2. A large aspect ratio reduces equation (44) to :

$$\omega^2 = \pi^2 \tag{45}$$

for the fundamental frequency of vibration (m=n=1). This implies that a long narrow plate could be modelled as a beam undergoing circular

bending at its fundamental mode. Accuracy could be improved by allowing for the increased stiffness of a narrow plate undergoing circular bending along the long side.
These two implications of equation (44) will have great importance in the eventual solution of the problem of making a plate ring with the approximate frequency response of a large bell.

The approximate methods which have been examined here or which stem from the concepts developed in the Ritz method can give good results when the material obeys the initial assumptions. However, the musical requirement that the plate approximate the sound of a large church bell imposes restrictions which are in opposition to the initial assumptions. The following conflicts arise:

1.The plate is assumed to be a conservative system (negligible damping). While this condition holds as far as predicting the frequencies of vibration of the plate, structural damping is the controlling factor in determining the amplitude balance of the natural frequencies. This overtone balance is critical to determining both the dominant frequency and the tone of the plate. Structural damping is poorly understood and difficult to model. While a mathematical model could be developed the difficulties encountered would be considerable. The aim of this thesis is to find a simple model.

2. The strength of sound required to approximate the sound of a church bell precludes the use of very thin plates. Thicker plates require adjustment to the model to account for a more representative strain relationship. This will violate the assumption that plane sections remain plane.

Although Timoshenko develops a technique for dealing with thick plates (strain varies linearly from the middle fibre) much more work would need to be done in conjunction with the other conflicting constraints.

The problem is further complicated by:

1. The approximate methods are most useful when analyzing a given plate. In this case the shape of the plate is unknown and must be determined given very restrictive conditions. Again, it may be possible to develop a suitable mathematical model and use a computerized iterative procedure to isolate the most probable shapes but this would be a problem of enormous proportion.

2. The means of initiating the vibration is another significant unknown which has a great effect on the sound produced by the plate but which is difficult to model with the necessary precision

CONCLUSIONS:

The available theoretical models of vibrating metal plates have been examined for application to producing plates which approximate the sound of large church bells. While some interesting relationships have been revealed, there remain many complicating restrictions and requirements. Yet these requirements are vital to the success of the project. Viewed from a strictly theoretical position, the problem is one of almost insurmountable complexity. Yet many knowledgeable musicians are of the opinion that a plate can indeed approximate the sound of a large bell.

A method of analysis is required which, like energy methods, will allow the many difficulties to be lumped together and dealt within the context of a unifying concept. This concept might be termed the "musical energy method" and can be developed by combining the ideas presented in Chapters 2, 3 and 4 to fill in the gaps which remain in the mathematical models.

CHAPTER 5

DEVELOPMENT OF AN ARTISTICALLY DRIVEN SOLUTION METHOD

The theoretical analysis of freely suspended vibrating plates leaves the researcher with three major obstacles when using plates to approximate the sound of large church bells.

1. No basis exists for choosing a material or a material thickness. The obvious choice is brass (real bells are made from brass) but large brass plates are known to be very difficult to damp.
2. No basis exists for choosing one aspect ratio over another.
3. No basis exists for the selection of the harmonic structure required. According to the theoretical results all plates suffer from an excess of unharmonic overtones.

The researcher is then left with three choices:

1. Accept Helmholtz's view that plates cannot approximate the sound of large bells.
2. Attempt to develop and solve an extremely complex mathematical model.
3. Find new ways to approach the problem.

The third choice is the obvious one given the premise of this thesis:

Identifying and properly implementing the skills and knowledge that exist in the arts can bring about the solution of scientific problems which have previously defied solution.

Having taken the first step of examining the theory of vibrating plates (Ch.4) and identifying the skills of the "musically trained ear"

(Ch. 3), the next step involves matching the appropriate skill and knowledge of the artist to each of the three problem areas of the analytical model. We first determine which type of musician could best evaluate the particular difficulties encountered.

MATERIAL SELECTION

This is possibly the most difficult connection to make. It requires a musician who has an extensive empirical knowledge of the sound of many different types and sizes of metal. While percussion players perform on many different metal instruments, they require no knowledge about the material itself. For the percussionist, the only question is "Does it produce the proper sound?" Only a percussion instrument manufacturer would have a high degree of knowledge about the possible acoustical behaviour of different materials. But no such manufacturer has been able to produce adequate bell plates. Since an individual of this description was not available, the next choice was to find a person who worked with metal and was a musician as well. Fortunately Paul Lavoie, the Supervisor of the Faculty Machine Shop for the Engineering Department at the University of Calgary, is a welder by trade and an excellent bluegrass guitar player.

When presented with the question "Which material and what approximate thickness of that material would be the best choice for eventually producing a bell-like sound?" he was able to select an array of materials and material thicknesses as possible candidates.

Given that the material choice must be steel, brass or aluminum, the most likely material initially seemed to be brass since large bells are made of brass. However, in the opinion of percussionists, brass plates of a manageable size have a weak sound, probably due to the low modulus of elasticity. The rhythmic damping of such plates is also very difficult to control because of the large transverse displacements, again a result of a low modulus of elasticity.

The opinion of the musician/welder was that steel plates, and more specifically one inch thick steel plate, held the most promise. The higher modulus of elasticity allows for small displacement and powerful, long lasting vibration. Thinner plates dissipate the vibrational energy too quickly and thicker plates "clink" meaning that there is an excess of strong, high harmonics in the frequency response. Aluminum plates are too weak since the modulus of elasticity is even lower than for brass.

CHOOSING A SHAPE FOR THE PLATE

The choice of material and material thickness was made to maximize the strength and duration of vibration while maintaining a playable size of plate. The next step was to select the shape which would give a frequency response closest to that of a bell. Although the musician/welder had opinions about the sound which were formulated from experimentation with a variety of shapes and sizes of plates, the problem now required the input of a musician intimately acquainted with the specific sound required and the various solutions which have been tried (chimes, recordings, synthesizers, local church bells playing at the right moment, etc.). The final solution must be compared to the ideal sound (which may exist only in the mind of the musician) and the best previous attempts. A successful solution will exist between these two limits.

Tim Rawlings, the principal percussionist with the Calgary Philharmonic Orchestra has performed *Symphonie Fantastique* many times and has been actively involved in finding a better approximation for the bell sound. Based on the information in Chapter 3 he is the musician likely to have the most refined 'aural templates' for this particular purpose.

Because the expertise of the musician lies more in evaluation than in prediction, the most effective use of his skills was in evaluating a carefully selected array of shapes which covered the full range of shape possibilities. Although there are an infinite number of shapes, they can be categorized into:

1. Rectangular (from square to beam).
2. Round (from circular to oval).
3. Combinations of straight and curved sides.
4. Plates with holes.

The following selection of plates was presented for evaluation by the percussionist:

TABLE 5.1

Material Selection and Evaluation

MATERIAL	DIMENSIONS (cm)	EVALUATION
Aluminum plate	1.25 X 25.0 X 25.5	dull tone
Circular Steel Plate	1.25 X 12.5(radius)	gong-like
Steel Bar	1.75 X 5.0 X 43.0	chime-like
Steel Plate (slightly rectangular)	0.92 X 23.5 X 15.5	no distinct pitch
Round Steel Bar	0.73(radius) X 32.0	chime-like
Steel Plate (long and narrow)	2.54X25.6X79.3	fairly bell-like
Steel Plate with Round Hole	1.83 X 21.4 X 30.6 Hole: 7.2(radius)	no distinct pitch

The sound of a long narrow plate was promising enough to begin a detailed theoretical study of its vibration characteristics.

DETERMINATION OF THE DOMINANT FREQUENCIES.

As explained in Helmholz [5.1], the main drawback of using plates to approximate the sound of bells is the abundance of overtones which confuse the listener as to the actual pitch of the plate. There are two factors which must be considered here. First, a skilled percussionist is able to exert a considerable amount of control over which overtones dominate. This is achieved by striking the instrument in the most advantageous spot, thereby suppressing unwanted overtones, or by constructing a beater which will dampen or accentuate the appropriate modes of vibration. Secondly, while an untrained listener will hear a jumble of tones coming from an instrument like a bell or a tympani, a skilled musician (in this case Tom Miller, principal tympanist with the CPO) can determine which frequency will dominate in the context of the orchestra.

The tympanist's analysis was compared to the output of an FFT spectrum analyzer. The tympanist was able to correctly identify the frequencies of the overtones of the plate as well as single out the dominant frequencies. The skill of Mr. Miller surpassed the analyzer in that he could also hear fluctuations in pitch due to the different methods of striking the plate. An analyzer fails in these areas because of the mathematical averaging resulting from filtering, sampling and performing the Fast Fourier Transform calculations. Any subtle changes in the signal are lost in this process.

The tympanist was able to verify the choice of the percussionist and the welder. In addition, he was able to isolate the dominant frequencies in the plate and explain that it was the spacing and balance of fundamental and overtones that yielded the bell-like tone. Many times during the research project it was the tympanist who either pointed out problems that the analyzers failed to pick up or identified important frequency relationships which needed to be preserved.

RESULTS

In summary, the contribution of the musicians' skill and knowledge resulted in the selection of a long, narrow, fairly thick steel plate of approximately 0.80 meters in length and one inch in thickness. The success of this exercise rests primarily in ensuring that the missing information for the theoretical model be matched to specific skills which the individual musicians possess. As well as selecting the type of musician most appropriate for the evaluation, the musicians must be allowed to function in a manner which requires them to think and act as musicians rather than as pseudo-scientists or tightly controlled test subjects (see Ch. 3).

THEORETICAL ANALYSIS OF THE MUSICIAN'S INPUT

The use of musicians' specialized knowledge in the designing of metal plates to ring like large church bells has provided a direction for further analysis which would have been impossible using a purely theoretical approach. But the artist alone does not have the technical or theoretical training to complete the solution. While the vast scope of possible materials, sizes and shapes has been narrowed to a manageable range, refining the choices to yield the precise frequency and frequency response required is outside the expertise of the artist.

The problem again becomes theoretical as it is now necessary to find the analytical explanation for the musicians' conclusions in order to predict the dimensions of the required plates.

CHOICE OF MATERIAL AND MATERIAL THICKNESS

The vibrations of the plates must be strong and long lasting. At the same time, excessive transverse displacement which affects the rhythmic control of the plate must be avoided. To return to Rayleigh's view of vibrating bodies in general, the goal is to maximize the energy in the plate while minimizing the transverse displacement (damping effects are ignored). Recalling the equations for maximum potential and kinetic energy in a plate:

$$V_{max} = \frac{D}{2}\int\int\left[\left(\frac{\partial^2 W_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W_0}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W_0}{\partial x^2}\frac{\partial^2 W_0}{\partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W_0}{\partial x \partial y}\right)^2\right]dx\,dy \tag{1}$$

$$U_{max} = \frac{1}{2}\rho h \omega^2 \int\int W_0^{\,2}\,dxdy \tag{2}$$

where

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \tag{3}$$

These equations indicate that the vibrational energy in a plate can be increased without increasing displacement (W) by increasing E, h, or ρ. The advantage of steel over brass or aluminum in terms of E and ρ is obvious from the table below with the exception of brass being more dense than steel.

TABLE 5.2
Comparison of Material Constants

MATERIAL	MODULUS OF ELASTICITY (GPa)	DENSITY (Kg/m^3
Steel	200	7860
Brass (annealed)	96.5	8500
Aluminum	75	2700

However, the increase in the kinetic energy of a brass plate due to its greater density is more than compensated for by the decrease in potential energy resulting from the lower modulus of elasticity. The increase in kinetic energy of a brass plate over a steel plate explains the difficulty encountered in rhythmically damping a brass plate.

The influence of the thickness (h) is significant because it affects the energy in an exponential manner (h^3).

From the equation

$$\omega^2 = \frac{\frac{D}{2}\iint\left[\left(\frac{\partial^2 W_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W_0}{\partial y^2}\right)^2 + 2\upsilon\left(\frac{\partial^2 W_0 \partial^2 W_0}{\partial x^2 \ \partial y^2}\right) + 2(1-\upsilon)\left(\frac{\partial^2 W_0}{\partial x \partial y}\right)^2\right]dx\,dy}{\rho \frac{h}{2}\iint (W_0{}^2 dxdy)} \tag{4}$$

the thickness has a squaring effect on the mode frequencies. This accounts for the "clink" of a plate which is too thick and therefore has a frequency response dominated by high overtones.

The final choice of plate thickness is a critical factor which must be carefully balanced to give a strong vibration without unduly increasing the response frequencies. While the musician's "trained ear" quickly identified the optimum thickness, the effect of plate thickness in relation to the other parameters (E, ρ, length) is an area requiring further research. As explained in Chapter 6 there appears to be a critical relationship between plate thickness and length which has not been previously identified.

CHOICE OF PLATE SHAPE

The solution of the many variations of the Rayleigh-Ritz method (Ch. 4) generally results in an eigenvalue solution of the type:

$$\omega^2 = (m\pi)^2 + \frac{(n\pi)^2}{\phi^2} \tag{5}$$

The long, narrow plate selected by the musicians was chosen because of the absence of unwanted overtones in the frequency response. The desired overtone structure will consist of the interval relationships which characterize the sound of a bell. Acceptable intervals are: the octave (frequency ratio 2:1), the fifth (3:2), the fourth (4:3), the third (5:4) and the minor third (6:5). Any other intervals are perceived to be dissonant or not bell-like by the listener. The goal is to suppress those dissonant modes which Helmholtz identified as the stumbling blocks to achieving a bell-like response from a metal plate.

The circular plate will vibrate according to the relationship

$$\omega = \frac{\theta}{a^2}\sqrt{\frac{D}{\rho h}} \qquad [5.2] \qquad (6)$$

where q, is a constant determined for a given number of nodal diameters and nodal circles.

A bar can be considered as a plate with a very large aspect ratio which results in a vibrational response described by the first term of equation (5).

$$\omega^2 = (m\pi)^2 \qquad (7)$$

In this model, a bar is assumed to vibrate with mode shapes described by circular bending. Variations in the shape of the plate due to combinations of straight and curved sides will affect the frequency response through a slight distortion of the straight sided vibrational pattern. The distortion of the mode shape will affect only the higher frequencies. This can be seen from the basic principle of the Fourier Series. As the vibration pattern departs from a simple sine function, its Fourier series approximation will contain more high frequency components, not more low frequency components.

In general, the addition or removal of small amounts of mass from the basic rectangular shape will result mainly in frequency response changes high up in the sound spectrum **unless** mass is added or removed along nodal lines. The removal or addition of mass along nodal lines has a significant effect on the low frequency components of vibration. This is the result of balancing the change in mass with the change in stiffness along nodal lines. The effect of the change in mass will dominate over the minimal change in stiffness. This balancing is a fine tuning of the basic vibrational relationship:

$$\omega = \sqrt{\frac{k}{m}} \tag{8}$$

This is a common practice in tuning bells and chimes but it also opens up another problem area for theoretical modeling. One of the goals of this project is to eliminate, as much as possible, the need for painstaking and expensive tuning of the plates. Attempting to re-tune a poorly tuned instrument usually compromises the clarity of the sound.

The governing relationships for both the bar and the circular plate (eqs. 6 and 7) offer no theoretical possibility for suppressing unwanted overtones. In addition, the frequency response of the bar or circular plate with respect to the relative amplitude of the overtones cannot be predicted. Only a relationship like equation (5) for a rectangular plate offers the theoretical possibility of permitting the suppression of unwanted overtones. However the prediction of the relative amplitudes of the overtones will still remain a problem. For a large aspect ratio ($\phi >> 1$) several modes will develop along the long side (m) before any modes develop along the short side (n).

SELECTING THE DOMINANT OVERTONES

Once the possibility of suppressing unwanted overtones was established, the remaining problem was adjusting the desired overtones to have a bell-like harmonic interval relationship. The first consideration was to ensure that the plate is not so narrow that the modes on the short side are weak and spaced too far above the fundamental. Secondly, the plate must not be so wide that modes along the short side interfere with the characteristic overtone spacing required.

The literature indicates that:

1. The principle of superposition applies to vibrating plates (linearity).
2. Musically acceptable frequency relationships are:
   - 1:1 (unison)
   - 2:1 (octave)
   - 3:2 (fifth)
   - 4:3 (fourth)
   - 5:4 (major third)
   - 6:5 (minor third)
3. A bell-like frequency response has at least an octave between the fundamental and the first overtone ([5.3] and graph 7.1).

4. A plate with an aspect ratio of 4 or greater will develop almost no modes along the short side [5.4].

From conditions (3) and (2) above, the acceptable frequency relationships must be octave shifted. The interval of a fifth will become an octave plus a fifth, etc. Ignoring the octave and unison as trivial, this results in bell-like frequency relationships of:

3:1 (fifth)

4:1.5 (fourth)

5:2 (major third)

6:2.5 (minor third)

Eliminating 4:1.5 and 5:2 as being too close to the limits, the best choice is essentially a 3:1 frequency ratio. Turning now to condition (1), a plate cut to a 3:1 length/width ratio should produce a reasonable approximation of a bell sound. This is in agreement with the evaluation of the tympanist.

In summary, the qualitative evaluation of the musicians has been reinforced by the governing mathematical relationships for vibrating plates. The solution, at this point, seems to be a one inch thick steel plate with an aspect ratio of 3. This combination of parameters has been shown both musically and theoretically to give strong, long lasting bell-like vibrations with a frequency response which approximates that of a church bell.

Chapter 6

TESTING, PROTOTYPE EVALUATION AND RESULT

The combination of theoretical and artistic input into the problem of using metal plates to approximate the sound of large church bells has resulted thus far in a theoretical solution which neither the artist nor the scientist could have achieved independently. The final test is the production of actual plates and the use of these plates in an orchestral concert. The original goal of this project was to produce two bell plates to imitate the sound of distant church bells in the fifth movement of Hector Berlioz's "Symphonie Fantastique". The musical score calls for a C bell and a G bell (sounding a fourth lower than the C bell). Berlioz had no particular bells in mind and instructs the performers to use several grand pianos if no bells can be found. Even the octave in which the bells should sound is left up to the musicians. Preliminary calculations indicated that plates could be produced in three octaves:

TABLE 6.1

Berlioz Bell Plates in Three Octaves

PLATES	Small		Medium		Large	
	Freq. (hz)	Weight (lbs)	Freq. (hz)	Weight (lbs)	Freq. (hz)	Weight (lbs)
C Plates	260	69	130	135	65	270
G Plates	196	92	98	180	49	360

EVALUATION OF THEORETICAL MODELS

Test plates were cut to check the predicted frequency response from the methods of Leissa, Gorman and an Ansys finite element program (see program 7.1). Aspect ratios were used for which tables already exist (1.0, 2.5, 3.0). The test plates were hot rolled steel plate, flame cut to size and suspended by a polypropylene rope from a hole drilled in the center of one side.The material constants were assumed to be: $E = 200GPa$ and $\upsilon = 0.3$.

The results were:

TABLE 6.2
Prediction of the Fundamental Frequency of a Freely Suspended Vibrating Plate
Steel Plate Dimensions = 0.1222 X 0 1222 X 0.0167m
Aspect Ratio = 1.0
* st = semitones

METHOD	FUNDAMENTAL			1st OVERTONE		
	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*
Actual	3384			6090		
Leissa	3664	8.3	1.5	5378	11.7	2
Gorman	3578	5.7	1.0	5223	14.2	2.5
Ansys	3692	9.1	2	5373	11.8	2

TABLE 6.3
Prediction of the Fundamental Frequency of a Freely Suspended Vibrating Plate

Steel Plate Dimensions = 0.2604 X 0 6509 X 0.0254m
Aspect Ratio = 2.5
* st = semitones

METHOD	FUNDAMENTAL			1st OVERTONE			2nd OVERTONE		
	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*
Actual	326			488			890		
Leissa	315.2	3.3	<1	481.4	1.36	<0.5	875.9	1.6	<0.5
Gorman	309.1	5.2	1.0	469.2	3.8	<1	1010	13.5	2.0
Ansys	314	3.7	0.6	483	1.0	<0.5	899	1.0	<0.5

TABLE 6.4
Prediction of the Fundamental Frequency of a Freely Suspended Vibrating Plate
Steel Plate Dimensions = 0.2604 X 0.7812 X 0.0254m
Aspect Ratio = 3.0
* st = semitones

METHOD	FUNDAMENTAL			1st OVERTONE			2nd OVERTONE		
	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*
Actual	224			429			615		
Gorman	215	4.0	1	389	9.3	1.5	824	33.9	5
Ansys	218	2.6	<0.5	401	6.5	1	625	1.6	<0.5

FUNDAMENTAL FREQUENCY PREDICTION

The results of these three mathematical models fluctuates widely. The results do not obey the assumption that inaccuracies in the model will cause the predicted frequencies to be too high due to the additional stiffness imposed by the model. An examination of the plates used to develop the theory [6.1] indicates that, in order to obey the assumption of negligible shear strain, the plate must be very thin in relation to its length (h/l = 1/100). Since the use of a one inch thick plate has been determined to be critical to the sound quality, it is not feasible to try thinner plates. Yet, the most likely cause of the errors is the thickness of the plates. The use of thick plate theory [6.2] reduces the stiffness by only a small margin and often serves to increase the error rather than decrease it. Altering the assumed values for the material constants does not appreciably affect the results .

The use of a larger one inch plate should give greater accuracy as the thickness to length ratio will be reduced. This condition would eliminate the smallest set of plates. The remaining two larger sets of plates can be modelled with greater accuracy but the rhythmic control of the plates will be more difficult to achieve. The use of superposition to predict a consonant frequency response has met with limited success as shown in Table 6.4. The octave between the fundamental and the first overtone is acceptable but the second overtone is flatter than desired. Since this is not as critical as the frequency of the fundamental, a slight reduction of the short side would bring the frequency response into a more consonant overtone relationship. A thinner model with a slightly enlarged aspect ratio was tested to determine the potential of the two larger sets of plates with respect to prediction accuracy and overtone adjustment. The results are shown in Table 6.5 and Graph 7.4.

TABLE 6.5
Prediction of the Fundamental Frequency of a Freely Suspended Vibrating Plate
Steel Plate Dimensions = 0.2420 X .7800 X .0162m
Aspect Ratio = 3.20
* st = semitones

METHOD	FUNDAMENTAL			1st OVERTONE			2nd OVERTONE		
	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*	Freq. (hz)	Error (%)	Error (st)*
Actual	140			273			392		
Ansys	139.8	0.1	0	275	0.7	0	399	1.7	<0.5

The accuracy of the prediction improved remarkably and the overtone spacing improved as well. However, a new and entirely unexpected result also occurred. The plate would not ring predominantly at the fundamental frequency. Instead, the most audible frequency was the third harmonic. Therefore, rather than ringing at a D, the plate would be perceived to ring at a G above the D (see graph 7.4). Further testing indicated that there may be a critical thickness/length ratio below which the fundamental frequency will not dominate (see graphs 7.2 to 7.11).

Now, rather than eliminating the smaller plates due to large prediction errors, it becomes clear that the smaller plates are the **only** possibility. In that case, a more accurate prediction method must be found.

THICK PLATE FREQUENCY PREDICTION PROBLEM

The necessity of using the smallest set of plates dictated that a solution to the error caused by the thickness of the plate must be dealt with. Returning to the equation which represents the typical vibrating plate solution:

$$\omega^2 = (m\pi)^2 + \frac{(n\pi)^2}{\phi^2} \tag{1}$$

Equation (1) shows that a plate with a large aspect ratio may be treated as a beam undergoing circular bending. This applies only to the fundamental frequency (m=1, f >>1). Since the overall frequency response is now satisfactory, the determination of the frequency of the fundamental mode is all that is required. It should therefore be possible to model the plate as a beam and use Euler's beam theory to more accurately predict the frequency of the fundamental mode. From Thomson [6.3]:

$$\omega_n = (\beta_n l)^2 \sqrt{\frac{EI}{\rho l^4}} \tag{2}$$

Using this equation and the material constants defined previously, the accuracy of the prediction for the fundamental is greatly improved as shown in table 6.6.

TABLE 6.6
Comparison of Beam Theory Predictions Vs
Plate Theory Predictions
Steel Plate Dimensions = (0.7811 X 0.2604 X .0254m)
Aspect Ratio = 3

Method	Frequency (hz)	Error (%)
Actual	224	
Gorman	214.6	4.2
Ansys	218	2.6
Beam Theory	216	3.6

While at this point Ansys has the least error, the method can only analyze a given plate and cannot determine the dimensions required for a given frequency. Beam theory gives the best possible accuracy and applicability combination. A small adjustment to stiffen the mathematical model (E = 210 GPa instead of 200 Gpa) brings the beam theory model into a very high degree of accuracy as shown in Table 6.7.

Table 6.7
Beam Theory Predictions vs
Actual Plates Cut For 'Symphonie Fantastique'
Aspect ratio = 3.2
E = 210 Gpa

Frequency Required (hz)	Predicted Dimensions (m)	Actual Frequency (hz)	Freq. Error (%)
261.63 (C)	0.7186 X 0.2231 X .0254	263	0.5
196.0 (G)	0.8303 X 0.2578 X .0254	197	0.5

The final design required only a single cut to achieve the desired fundamental frequencies. As the plates were cut from a larger plate, it was also convenient to cut plates for the lower octaves to verify the predicted problems and to check for any more unexpected results (see graphs 7.5 to 7.10). From graph 7.5 and 7.10 it is interesting to note that, while thin plate theory predicts the suppression of modes along the short side for a large aspect ratio, only the thick plate actually achieves the suppression.

The plates were used in a performance of Berlioz's "Symphonie Fantastique" by the Calgary Philharmonic Orchestra in March of 1991. The pitch and quality of the sound produced by the plates was appraised very favourably by the conductor (Mario Bernardi), by the percussionists (Tom Miller and Tim Rawlings) and by the audience. Exposing the plates to the very stringent demands of an actual performance identified the need for further research in the areas of controlling the rigid body motion of the plates and avoiding non-linear behaviour under loud performance conditions (heavy hammer blows).

CONCLUSIONS AND FUTURE WORK

The successful design of metal plates which approximate the sound of large church bells would not have been possible without the carefully coordinated input of both musicians and engineers. The project, which initially appeared to be a simple matter of plugging numbers into mathematical models which already exist, quickly became an impossibly complex theoretical problem due to the accuracy of pitch and the quality of sound required. The musicians were equally at a loss because their lack of technical expertise relegated any research to trial and error amidst a vast quantity of variables in material selection and dimension.

The success of this project is primarily due to a very careful study, selection and implementation of the skills and knowledge possessed by the professional musician. With this knowledge in hand, each limitation of the mathematical models could be matched to an appropriate musically derived solution. The theoretical basis of the musically derived solution could then be determined and the impasse of the mathematical limitation overcome. This method has produced the best solution to date of a one hundred and fifty year old problem dismissed as unsolvable by none other than Helmholtz.

The initial problem of designing metal plates to approximate the sound of large church bells was solved although the problems uncovered along the way have identified several areas for further theoretical research. The problems encountered during the course of this project primarily concerned the mathematical tracking of the energy of vibration in a freely suspended plate. The most important of these problems is that of determining the relationship between the vibrational behaviour of a plate and its acoustic emission. Plates which conform to the assumption of thinness (negligible shear stress) will vibrate at the predicted frequencies but the acoustic emission will not dominate at the fundamental frequency. As a result, a thin plate will be perceived to ring at one of the overtones rather than at its fundamental frequency. From the experiments carried out on many different sizes of plates, there appears to be a critical relationship of thickness to length (in this case $h/l \approx 0.03$) below which the fundamental mode will not dominate acoustically.

Another problem has to do with the suppression of modes along the short side of a plate with a high aspect ratio. Although the theory which predicts this is based on the assumption of thinness, it is only effective when the plate violates this assumption. A thick plate ($h/l = 0.035$) will suppress modes on the short side but a thin plate ($h/l = 0.015$) with the same aspect ratio will not suppress these modes.This may also be tied to the relationship of vibration and acoustic emission since effective suppression occurs at the same ratio.

Although the fundamental frequency of plates with this optimum dimensional relationship can be most accurately predicted using beam theory, this is mostly a fortunate coincidence. More work needs to be done to improve the applicability of plate theory. The main difficulty seems to be modelling the shear strain in a plate which is no longer thin. Thick plate theory accounts for shear strain which varies linearly from the centre of the plate. A further refinement would be to allow a cubic relationship which correctly models the lack of shear at the plate surfaces. There may also be nonlinearities resulting from the suspension method and the method of excitation. Nonlinearity due to the method of excitation is evident in the rise in frequency of the G plate when it is played loudly with a soft beater.

Acoustical analysis instrumentation is an area requiring an entirely new direction of research. This is very evident when comparing the output of the best Fast Fourier Transform and digital sound analyzers to the perception of a highly trained musician. All three sensors receive the same signal but the computer driven analyzers lose so much of the signal through filtering, sampling and mathematical averaging that most musically important subtleties are lost. A return to time domain analysis may be the key since the time domain signal contains all of the information required.

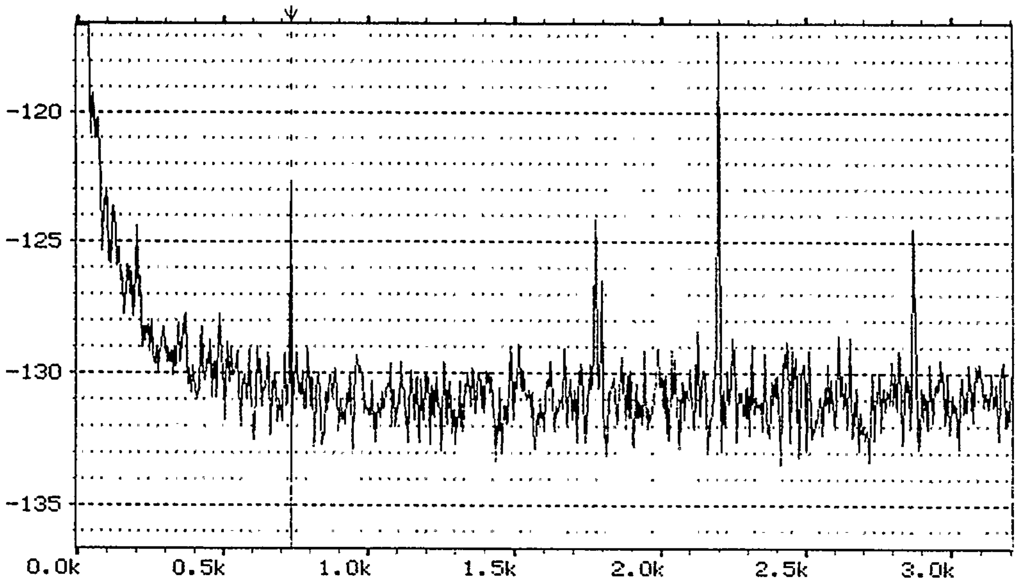
In closing, the author would like to paraphrase a thought taken from Hermann Helmholtz in his work *On The Sensation Of Tone.* Those who prefer mechanical explanations may express their regret at having opened the door for artistic intervention while other critics with more metaphysical interests may reject the methods as too coarsely mechanical. "I hope my critics will excuse me if I conclude from the opposite nature of their objections that I have struck out nearly the right path".

CHAPTER SEVEN
GRAPHS

Graph 7.1
Frequency Response
of a
Small Ship Bell

W9 AUTO SPEC CH.A
Y: -116.6dB /1.00V RMS 20dB
X: 0Hz + 3.2kHz LIN
#A: 50

MAIN Y: -122.7dB
X: 732Hz
TOTAL : -79.8dB/YREF



SETUP W1

MEASUREMENT: CH.A SPECTRUM AVERAGING
TRIGGER: FREE RUN

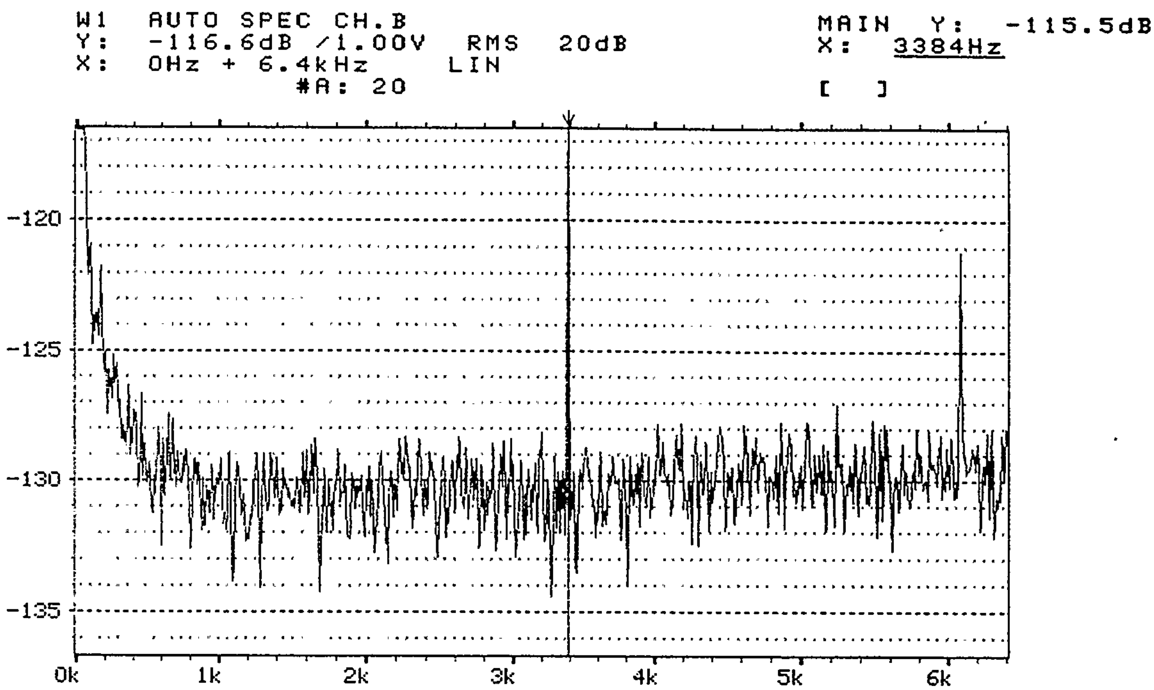
AVERAGING: LIN 50 OVERLAP: MAX

FREQ SPAN: 3.2kHz ΔF:4Hz T:250ms ΔT:122μs
CENTER FREQ: BASEBAND
WEIGHTING: HANNING

CH.A: 30mV + PREAMP FILT:25.6kHz 1V/V
CH.B: 30mV + PREAMP FILT:25.6kHz 1V/V
GENERATOR: DISABLED
SINE GENERATOR FREQ.: 250.000Hz

Graph 7.2
Frequency response
of a
Steel Plate

Dimensions = 0.1222 X 0.1222 X 0.0167m
Aspect Ratio = 1.0
h/l = 0.1370



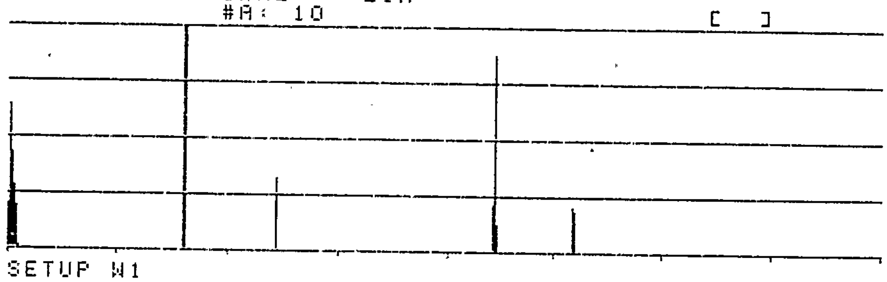
SETUP W1

MEASUREMENT:	CH.B SPECTRUM AVERAGING			
TRIGGER:	FREE RUN			
AVERAGING:	LIN 20	OVERLAP: MAX		
FREQ SPAN:	6.4kHz	ΔF:8Hz	T:125ms	ΔT:61.0μs
CENTER FREQ:	BASEBAND			
WEIGHTING:	HANNING			
CH.A:	30mV +	PREAMP	FILT:25.6kHz	1V/V
CH.B:	30mV +	PREAMP	FILT:25.6kHz	1V/V
GENERATOR:	DISABLED			

Graph 7.3
Frequency Response
of a
Steel Plate

Dimensions = 0.2604 X 0.6509 X 0.0254m
Aspect Ratio = 2.5
h/l = 0.0390

W1 AUTO SPEC CH B   MAIN Y: -44.3dB
Y: -48.5dB /1.00V   RMS 20dB   X: 326Hz
X: 0Hz + 1 6kHz   LIN
#A: 10   [ ]



SETUP W1

MEASUREMENT: CH.B SPECTRUM AVERAGING
TRIGGER: FREE RUN

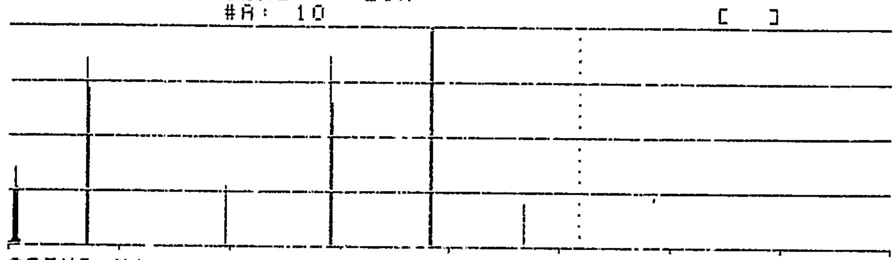
AVERAGING: LIN 10 OVERLAP: MAX

FREQ SPAN: 1.6kHz ΔF:2Hz T:500ms ΔT:244µs
CENTER FREQ: BASEBAND
WEIGHTING: HANNING

CH A: 30mV + 3Hz DIR FILT:6.4kHz 1.24mV/PA
CH.B: 300mV + PREAMP FILT:25.6kHz 1V/V
GENERATOR VARIABLE SINE OFF
SPECIAL PARAMETER : #3: NO OVLD REJECT DEC VALUE: 0
326

Graph 7.4
Frequency Response
of a
Steel Plate

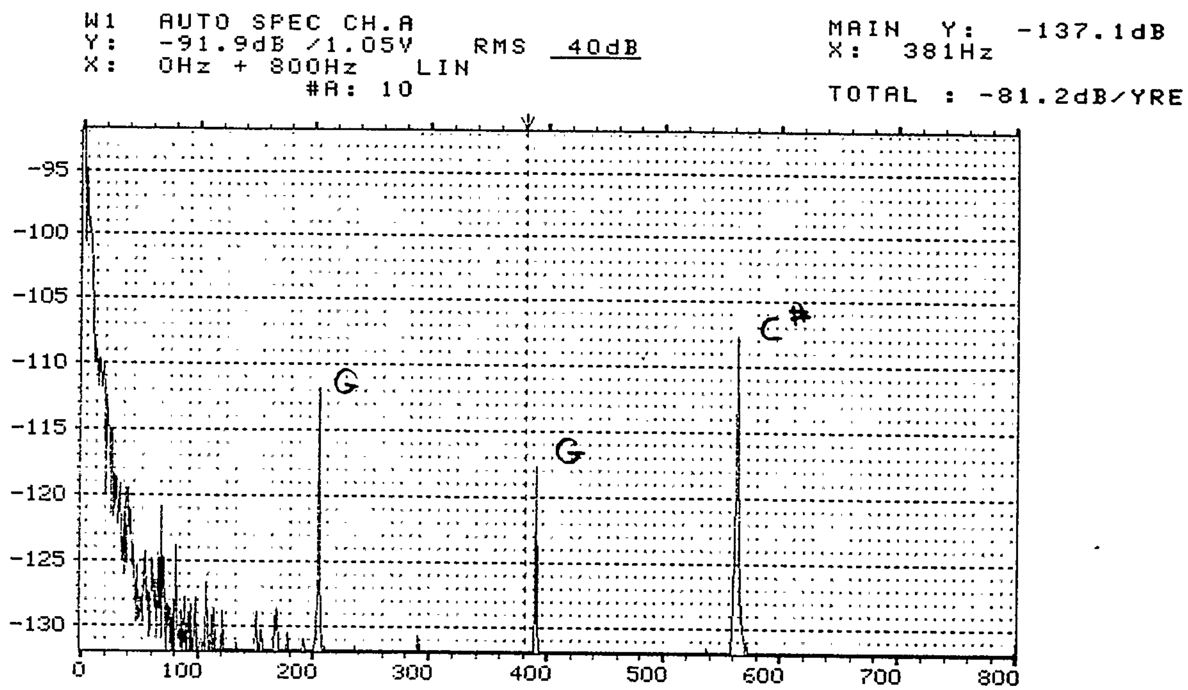
Dimensions = 0.2420 X 0.7800 X .0162m
Aspect Ratio = 3.22
h/l = 0.0277




```
SETUP W1

MEASUREMENT:    CH B SPECTPUM AVERAGING
TRIGGER:        FPEE PUN

AVERAGING:      LIN   10      OVERLAP: MAX

FREQ SPAN:      1.6kHz    ΔF 3Hz        T:500ms     ΔT:244us
CENTER FREQ:    BASEBAND
WEIGHTING:      HANNING

CH.A:           30mV   +   3Hz DIR      FILT:6.4kHz    1.24mV/PA
CH.B:           300mV  +   PREAMP       FILT:25.6kHz   1V/V
GENERATOR:      VARIABLE SINE        OFF
SPECIAL PARAMETER   :   #3: NO OVLD REJECT        DEC VALUE: 0
                                   326
```

Graph 7.5
Frequency Response
of a
Steel Plate

Dimensions = 0.8318 X 0.2583 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 196 hz (G)
h/l = 0.0305

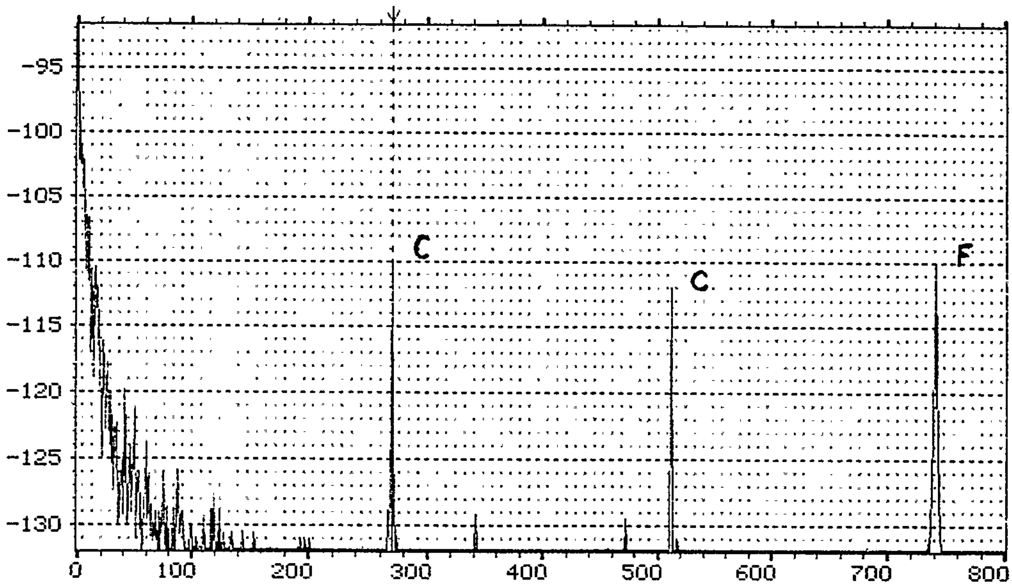


SETUP W1

MEASUREMENT:	CH.A SPECTRUM AVERAGING			
TRIGGER:	FREE RUN			
AVERAGING:	LIN 10	OVERLAP: MAX		
FREQ SPAN:	800Hz	ΔF:1Hz	T:1s	ΔT:488μs
CENTER FREQ:	BASEBAND			
WEIGHTING:	RECTANGULAR			
CH.A:	30mV +	PREAMP	FILT:25.6kHz	1V/V
CH.B:	30mV +	PREAMP	FILT:25.6kHz	1V/V
GENERATOR:	DISABLED			

Graph 7.6
Frequency Response
of a
Steel Plate
Dimensions = 0.7175 X 0.2228 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 261 63 hz (C)
h/l = 0.035

```
W1   AUTO SPEC CH.A                                    MAIN   Y:   -110.0dB
Y:   -91.9dB /1.05V    RMS   40dB                      X:  270Hz
X:   0Hz + 800Hz    LIN
              #A: 10                                   TOTAL  :  -78.2dB/YREF
```

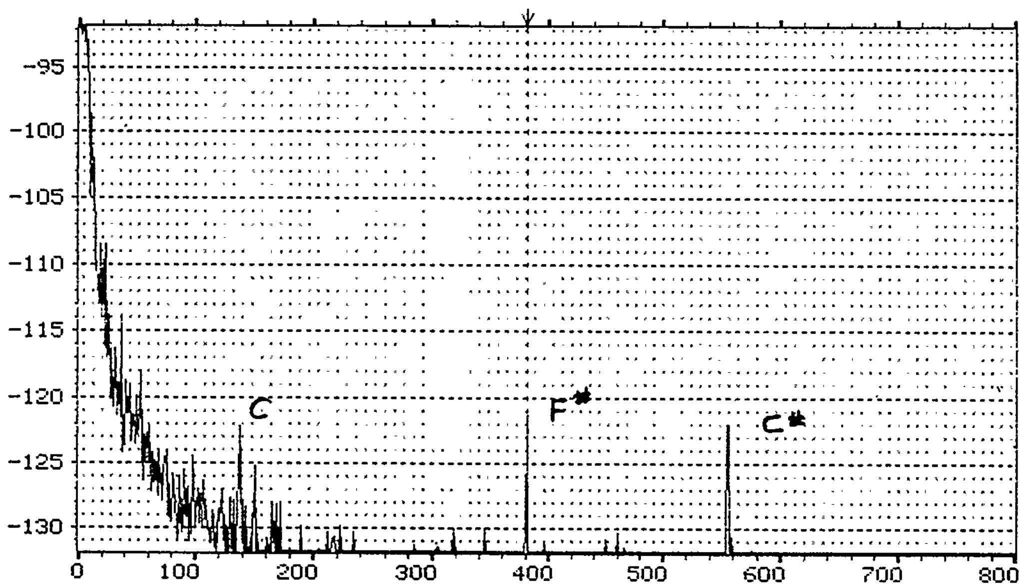



```
SETUP W1

MEASUREMENT:      CH.A SPECTRUM AVERAGING
TRIGGER:          FREE RUN

AVERAGING:        LIN    10      OVERLAP: MAX

FREQ SPAN:        800Hz       ΔF:1Hz        T:1s          ΔT:488µs
CENTER FREQ:      BASEBAND
WEIGHTING:        RECTANGULAR

CH.A:             30mV   +   PREAMP     FILT:25.6kHz   1V/V
CH.B:             30mV   +   PREAMP     FILT:25.6kHz   1V/V
GENERATOR:        DISABLED
```

Graph 7.7
Frequency Response
of a
Steel Plate

Dimensions = 1.004 X 0.3138X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 130 hz (C)
h/l = 0.0253

```
W1   AUTO SPEC CH.A                               MAIN  Y:  -121.2dB
Y:   -91.9dB /1.05V    RMS   40dB                 X:   381Hz
X:   0Hz + 800Hz     LIN
           #A: 10                                 TOTAL : -79.3dB/YREF
```

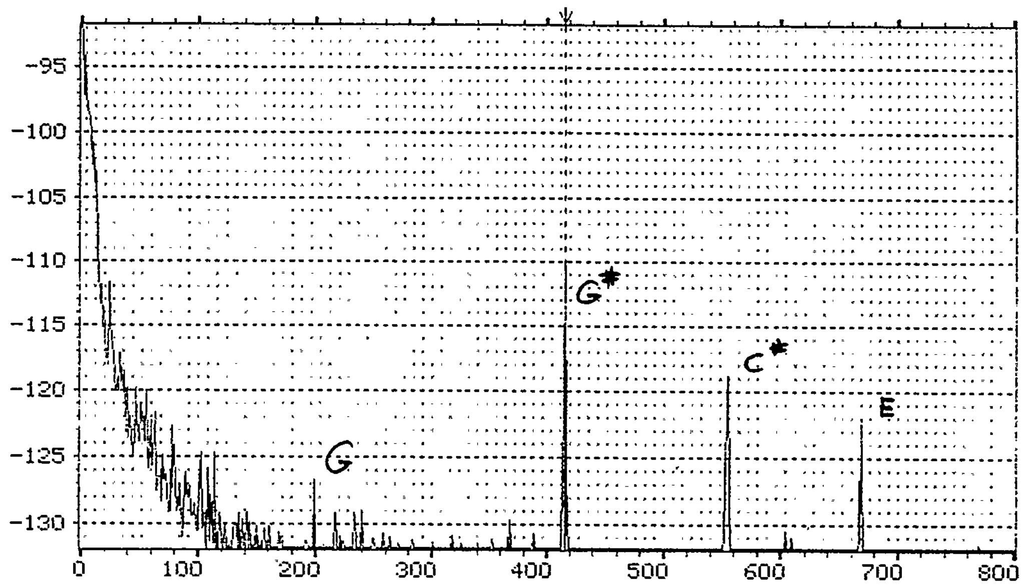



```
SETUP W1

MEASUREMENT:   CH.A SPECTRUM AVERAGING
TRIGGER:       FREE RUN

AVERAGING:     LIN   10     OVERLAP: MAX

FREQ SPAN:     800Hz      ΔF:1Hz        T:1s        ΔT:488μs
CENTER FREQ:   BASEBAND
WEIGHTING:     RECTANGULAR

CH.A:          30mV   +   PREAMP      FILT:25.6kHz   1V/V
CH.B:          30mV   +   PREAMP      FILT:25.6kHz   1V/V
GENERATOR:     DISABLED
```

Graph 7.8
Frequency Response
of a
Steel Plate

Dimensions = 1.160 X 0..3625 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 98 hz (G)
h/l = 0.022

```
W1   AUTO SPEC CH.A                                 MAIN  Y:  -109.9dB
Y:   -91.9dB /1.05V    RMS   40dB                   X:  416Hz
X:   0Hz + 800Hz    LIN
           #A: 10                                   TOTAL  :  -85.2dB/YREF
```

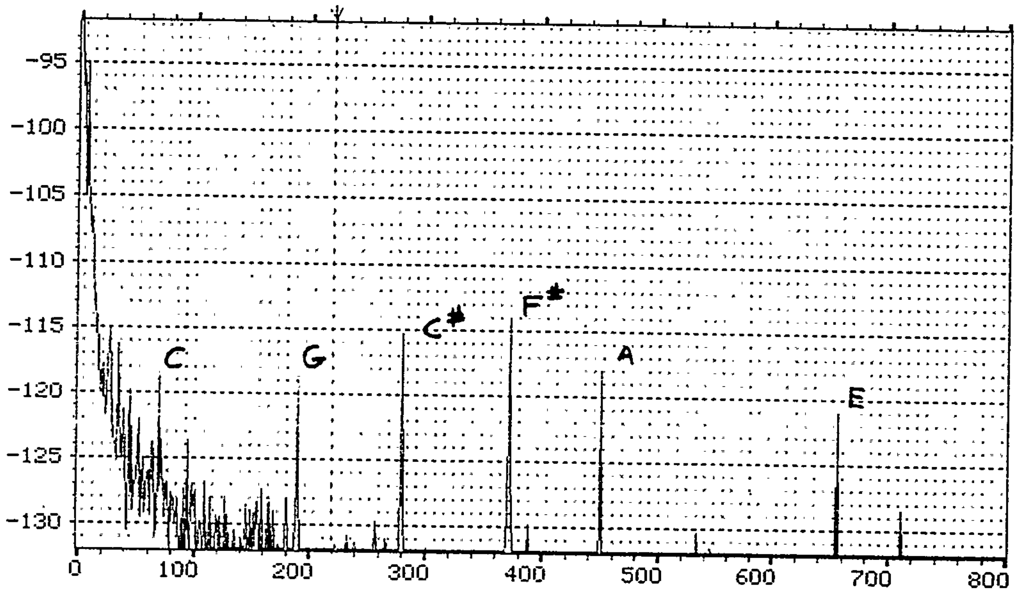



```
SETUP W1

MEASUREMENT:    CH.A SPECTRUM AVERAGING
TRIGGER:        FREE RUN

AVERAGING:      LIN   10      OVERLAP: MAX

FREQ SPAN:      800Hz     ΔF:1Hz       T:1s        ΔT:488μs
CENTER FREQ:    BASEBAND
WEIGHTING:      RECTANGULAR

CH.A:           30mV  +   PREAMP     FILT:25.6kHz   1V/V
CH.B:           30mV  +   PREAMP     FILT:25.6kHz   1V/V
GENERATOR:      DISABLED
```

Graph 7.9
Frequency Response
of a
Steel Plate

Dimensions = 1.4199 X 0..4437 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 65 hz (C)
h/l = 0.0179

```
W1   AUTO SPEC CH.A                                      MAIN   Y:   -137.1dB
Y:   -91.9dB /1.05V     RMS   40dB                       X:  219Hz
X:   0Hz + 800Hz     LIN
              #A: 10                                     TOTAL  :
```

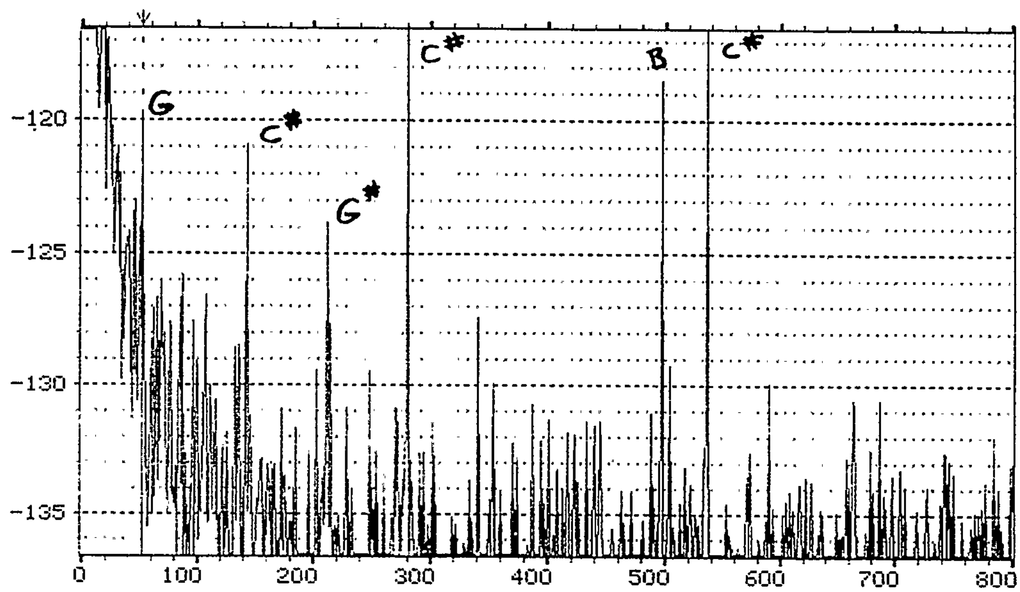



```
SETUP W1

MEASUREMENT:    CH.A SPECTRUM AVERAGING
TRIGGER:        FREE RUN

AVERAGING:      LIN    10      OVERLAP: MAX

FREQ SPAN:      800Hz      ΔF:1Hz        T:1s         ΔT:488µs
CENTER FREQ:    BASEBAND
WEIGHTING:      RECTANGULAR

CH.A:           30mV    +    PREAMP      FILT:25.6kHz   1V/V
CH.B:           30mV    +    PREAMP      FILT:25.6kHz   1V/V
GENERATOR:      DISABLED
```

Graph 7.10
Frequency Response
of a
Steel Plate

Dimensions = 1.6405 X 0.5126 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 49 hz (G)
h/l = 0.0155

```
W1   AUTO SPEC CH.A                    STORED      MAIN  Y:   -119.6dB
Y:   -116.6dB /1.05V  RMS  20dB                    X:  52Hz
X:   0Hz + 800Hz   LIN
SETUP S1      #A: 10
```

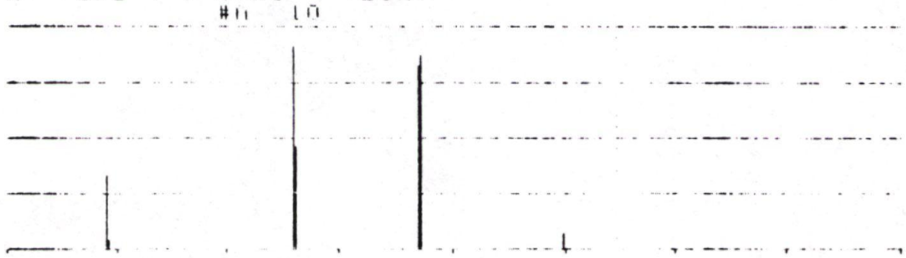



```
SETUP W1

MEASUREMENT:    CH.A SPECTRUM AVERAGING
TRIGGER:        FREE RUN

AVERAGING:      LIN    10      OVERLAP: MAX

FREQ SPAN:      800Hz      ΔF:1Hz        T:1s          ΔT:488μs
CENTER FREQ:    BASEBAND
WEIGHTING:      RECTANGULAR

CH.A:           30mV   +   PREAMP      FILT:25.6kHz   1V/V
CH.B:           30mV   +   PREAMP      FILT:25.6kHz   1V/V
GENERATOR:      DISABLED
```

Graph 7.11
Frequency Response
of a
Steel Plate

Dimensions = 0.852 X 0.2730 X 0.0254m
Aspect Ratio = 3.22
Predicted Fundamental Frequency = 182 hz (G)
h/l = 0.0298

```
W1   AUTO SPEC CH B                              MAIN  Y:  -109.3dB
Y:   -41.9dB  1.00V    RMS  20dB                 X:  1094Hz
X:   0Hz + 1.6kHz         LIN
               #A: 10
```



```
SETUP H1

MEASUREMENT       CH B SPECTRUM AVERAGING
TRIGGER           FREE RUN

AVERAGING         LIN    10        OVERLAP  MAX

FREQ SPAN:        1.6kHz     ΔF:2Hz        T:500ms      ΔT:244us
CENTER FREQ       BASEBAND
WEIGHTING         HANNING

CH A:             30mV   +    3Hz DIF     FILT:6.4kHz    1.24mV/PA
CH B:             300mV  +    PREAMP      FILT:25.6kHz   1V/V
GENERATOR:        VARIABLE SINE         OFF
SPECIAL PARAMETER          #3: NO OVLD REJECT         DEC VALUE: 0
                                        326
```

PROGRAM 7.1
Ansys Finite Element Computer Program
for
Determination of the Frequencies of Vibration
of
A Freely Suspended Rectangular Plate

```
$ansys$44
/prep7
/title, square
et,1,63
kan,2
kay,2,7
kay,3,0
dens,1,7860
r,1,0.0167
ex,1,200e9
n,1
n,9,0.122
fill
ngen,9,10,1,9,1,0,0.01525
e,1,2,12,11
egen,8,1,1,1
egen,8,10,1,8,1
nsel,node,63,67,2
nasel,node,23,27,2
nasel,node,43,47,4
nasel,node,1,9,2
nasel,node,81,89,2
```

```
nasel,node,1,81,20
nasel,node,9,89,20
m,all,ux
m,all,uy
m,all,uz
nall
iter,1,1,1
afwrite
fini
/input,27
fini
```

THESIS REFERENCES

1.0 **von Helmholtz, Hermann H.:** 1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*)
Reprint ed. New York: Dover, 1954 pp 55-6,72

1.2 **van der Waerden, B.L.**: 'Die Harmonielehre der Pythagoräer', *Hermes*, ***lxxviii*** (1943),163

1.3 **Cohen, H.L.:** '*Quantifying Music*', (D. Reidel, Dordrecht, 1984)

1.4 **Dostrovsky, S.:** 'Physics of Music', *The New Grove Dictionary of Music and Musicians*, ed. Stanley Sadie (Macmillan, London, 1980)

1.5 **Palisca, Claude V.**: 'Vincenzo Galelei', *The New Grove Dictionary of Music and Musicians*, ed. Stanley Sadie (Macmillan, London, 1980)

1.6 **Bacon, F.**: '*Sylva sylvarum*' (London, 1627)

1.7 **Kepler, J.**: '*Harmonice mundi libri **V***' (Linz, 1619)

1.7a **Truesdell, C., Bell, J.F.,Dostrovsky, S.**: 'The Physics of Music', *The New Grove Dictionary of Music and Musicians*, ed. Stanley Sadie (Macmillan, London, 1980)

1.7b **Cohen, H.L.:** '*Quantifying Music*', (D. Reidel, Dordrecht, 1984), p.13

1.7c **Cohen, H.L.:** '*Quantifying Music*', (D. Reidel, Dordrecht, 1984), p.4

1.8 **Sauveur, J.**: '*Systême general des intervalles des sons*', Histoire de l'Académie royale des sciences [1701](Paris, 1704), 123

1.10 **Mersenne, M.**: *Harmonie universelle* (Paris, 1636-7/R1963; Eng. trans., 1957)

1.11 **Hooke, R.**: *Micrographia* (London, 1665)

1.12 **Wallis, J.**: 'On the Trembling of Consonant Strings: a New Musical Discovery'. *Philosophical Transactions*, **xii**(1677),839

1.13 **[B. Le B. de Fontenelle]**: 'Sur un nouveau systême de musique', *Histoire de l'Académie royale des sciences* [1701] (Paris, 1704) 123

1.13a **Sauveur, Joseph:** *'Collected Writings On Musical Acoustics'* , ed. Rudolph Rasch (Diapason Press, Utrecht, 1984) p. 14

1.14 **Rameau, J.P.:** *Nouveau systême de musique théorique* (Paris, 1726)

1.15 **Bernoulli, D.**: 'Theoremata de oscillationibus corporum filo flexibili connexorum et catenae verticaliter suspensae', *Commentationes Academiae scientiarum Petropoli* ***vi*** (1732-3), 108

1.16 **L. Euler, L.**: 'De minimis oscillationibus corporum tam rigidorum quam flexibilium methodus nova et facilis', *Commentationes Academiae scientiarum Petropoli,* ***vii***(1734-5),99; repr. in Opera Omnia, **ii**/10,17
______: "Dissertatio physica de sono" (Basle, 1727); repr. in *Opera Omnia,* **iii**/1.182

1.17 **Bell, Eric** : *Men of Mathematics,* (Simon & Schuster: New York,1937) pp. 197

1.18 **Chladni, E.F.F.**: *Entdeckungen über die Theorie des Klanges* (Leipzig, 1787) pp. 8

1.18a **Szilard, Rudolph:** *Theory and Analysis of Plates,* (Prentice-Hall, Inc. 1974)

1.19 **Scheibler, H.**: *Der physikalische und musikalische Tonmesser* (Essen, 1834)

1.20 **Koenig, R.**: 'Bemerkungen über die Klangfarbe', *Annalen der Physik und Chemie,* new ser., **xiv** (1881),369

1.21 **Strutt, J.W. [Lord Rayleigh]:** *The Theory of Sound,* 2nd ed., vol. 1,2 (Macmillan: London) 1894, reprinted (Dover: New York), 1945

1.22 **Helmholtz, H. von:**1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The SensationsOf Tone*)
Reprint ed. New York: Dover, 1954 pp 210

1.23 **Helmholtz, H. von:**1877: *Die Lehre von den Tonempfindungenals physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*) Reprint ed. New York: Dover, 1954 pp 73

1.24 **Clynes, Manfred ;Evans, J.R.**: *Rhythm in Psychological, Linguistic, and Musical Processes*, (Thomas: Springfield, 1986. **Manfred Clynes**: *Music, Mind and Brain*, (Plenum: New York, 1983)

1.25 **Hanslick, Eduard**: *On The Musically Beautiful*, trans. G. Payzant (Hackett: Indianapolis, 1986)

1.25a **von Helmholtz, H.** :1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*) Reprint ed. New York: Dover, 1954 pp 371

1.26 **von Helmholtz, H.**:1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*) Reprint ed. New York: Dover, 1954 pp 72

1.27 **von Helmholtz, H.:** 1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*) Reprint ed. New York: Dover, 1954 pp. 55

1.28 **Ritz, W.**: 'Theorie der Transversalschwingungen einer quadratischen Platte mit freien Rändern, *Annalen der Physik*, fourth series,1909; vol. 28, p. 737

1.29 **Waller, Mary**: *Chladni Figures, A Study in Symmetry*, (G. Bell and Sons: London, 1961)

__________: 'Vibrations of Free Square Plates', *Proc. Phys. Soc.*,1939 vol 51, p. 831, .

__________: 'Vibrations of Free Rectangular Plates', *Proc. Phys. Soc.*,1949 vol 62, p. 277

1.30 **Young, D.**: 'Vibration of Rectangular Plates by the Ritz Method', *Journal of Applied Mechanics, Trans. A.S.M.E.*, 1950; vol. 72, p.448

1.31 **Warburton, G.B.**: 'The Vibration of Rectangular Plates', *Proceedings of the Institute of Mechanical Engineers*, 1950; ser. A, 168, pp. 371-384

1.32 **Leissa, A.W.**: 'Vibration of Plates', *NASA SP-160*, 1969

1.33 **Leissa, A.W.**: 'The Free Vibration of Rectangular Plates', *Journal of Sound and Vibration*, 1973; 31(3), pp.257-293

1.34 **Gorman, D.J.**: *Free Vibration of Rectangular Plates*, (Elsevier: New York,1982)

2.1 **Kaplan, C.A., and Simon, H.A.**: 'In Search of Insight', *Cognitive Psychology*, (July)1990, Vol 22(3), pp. 374-419.

2.2 **Simon, H.A.**: 'Invariants in Human Behaviour', *Annual Review of Psychology*, 1990, Vol. 41, pp. 1-19.

2.3 **Kleinmuntz, Benjamin**: 'Why We Still Use Our Heads Instead of Formulas: Toward an Integrated Approach', *Psychological Bulletin*, (May) 1990, vol. 107(3) pp. 296-310.

2.4 **Benderly, B.L.**: 'Intuition', *Psychology Today*, (Sept) 1989, pp.36-40

2.5 **McAdams, S.**: 'Harmonic Template Matching', *Music, Mind and Brain: Spectral Fusion And The Creation Of Auditory Images*, M. Clynes ed. (New York: Plenum)1982

2.6 **Benade, A.**: *Fundamentals of Musical Acoustics*, (Oxford: New York) 1976, pp. 143-145.

2.7 **Rossing, T.D.** (ed.): *Acoustics Of Bells*, (Van Nostrand Reinhold: New York), 1984

3.1 **Plomp, R.**: *Aspects of Tone Sensation*, (Academic: New York) 1976

3.2 **Grout, D.J.**: *A History of Western Music*, (Norton:New York), 1960

3.3 **Hindemith, P.**: *Elementary Training For Musicians*, (Schott: Mainz), 1974

3.4 **Benade, A.**: *Fundamentals of Musical Acoustics*, (Oxford: New York) 1976, pp. 391-430

3.5 **Woodford, D.**: 'Hearing Problems in Orchestras', *Acoustics Australia*, 15, pp.78-79

3.6 **Woodford, D.**: 'Hearing Problems in Orchestras', *Acoustics Australia*, 15, pp.78-79

3.7 **Yost, W.A.**: *Fundamentals of Hearing*, (CBS College: New York), 1985

3.8 **Sachs, M.B.**: 'Effects of Efferent Stimulation', In B. Moore, R. Patterson (Eds.), *Auditory Frequency Selectivity* (Plenum: New York), 1975, p. 125

3.9 **Thompson, R.F.**: *Introduction to Physiological Psychology*, (Harper &Row: New York), 1975, p. 103

3.10 **Plomp, R.**: *Aspects of Tone Sensation*, (Academic: New York) 1976

3.11 **McAdams, S.**: 'Harmonic Template Matching', *Music, Mind and Brain: Spectral Fusion And The Creation Of Auditory Images*, M. Clynes ed.(New York: Plenum)1982

4.1 **Timoshenko, S. and Woinowski-Krieger, W.**: *Theory of Plates and Shells*, 2nd ed., (McGraw-Hill: New York), 1959

4.2 **Strutt, J.W. [Lord Rayleigh]:** *The Theory of Sound* , 2nd ed., vol. 1,2 (Macmillan: London) 1894, reprinted (Dover: New York), 1945

4.3 **Timoshenko, S., Young, D.H. and Weaver, W.Jr.**: *Vibration Problems in Engineering*,4th ed. (Wiley & Sons: New York), 1974, p.415

4.4 **Leissa, A.W.:** 'The Free Vibration of Rectangular Plates', *Journal of Sound and Vibration*, 1973; 31(3), pp.257-293

4.5 **Gorman, D.J.**: *Free Vibration of Rectangular Plates*, (Elsevier: New York,1982)

5.1 **Helmholtz, Hermann H. von:** 1877: *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Brunswick, 1863, many subsequent edns.; Eng. trans. by A. J. Ellis, 1875, many subsequent edns., as *On The Sensations Of Tone*)
Reprint ed. New York: Dover, 1954 pp 55-6,72.

5.2 **Timoshenko, S., Young, D.H. and Weaver, W.Jr.**: *Vibration Problems in Engineering*, 4th ed. (Wiley & Sons: New York), 1974, p.500

5.3 **Rossing, T.D.** (ed.): *Acoustics Of Bells*, (Van Nostrand Reinhold: New York), 1984

5.4 **Waller, Mary**: *Chladni Figures, A Study in Symmetry*, (G. Bell and Sons: London, 1961)

6.1 **Ritz, W.**: 'Theorie der Transversalschwingungen einer quadratischen Platte mit freien Rändern, *Annalen der Physik*, fourth series, 1909; vol. 28, p. 737

6.2 **Timoshenko, S., Young, D.H. and Weaver, W.Jr.**: *Vibration Problems in Engineering*, 4th ed. (Wiley & Sons: New York), 1974, p.432

6.3 **Thomson, W.T.**: *Theory Of Vibrations With Applications*, 3rd ed. (Prentice Hall: New Jersey), 1988, p.223