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## Essays on Versioning of Information Goods

## by

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## UNIVERSITY OF CALGARY <br> FACULTY OF GRADUATE STUDIES

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#### Abstract

Large sunk costs of development, negligible costs of reproduction and distribution, and substantial economies of scale make information goods distinct from physical goods. Product differentiation and price discrimination through versioning are common ways firms take advantage of the specific characteristics of information goods. This dissertation consists of three essays focusing on different situations where versioning strategies are implemented by information goods producers. The first essay discusses the interaction between different market segments showing the differences in versioning strategies when moving from horizontal to vertical differentiation as we believe that any product differentiation must be based on existing market segments. The second essay analyzes optimal versioning strategies when treating information goods as "experience goods" whose quality can only be determined through use. The third essay investigates price, quality and versioning strategies that information goods producers use to deter entry and maintain market power.


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## Dedications

To my wife, Feipeng (Fiona) Dou, for her love, a joyful past and a promising future.

## Table of Contents

Approval Page ..... ii
Abstract ..... iii
Acknowledgements ..... iv
Dedications ..... v
Table of Contents ..... vi
1 INTRODUCTION ..... 1
2 ESSAY ONE: MONOPOLY VERSIONING OF INFORMATION GOODS ..... 6
2.1 Introduction ..... 6
2.2 Modeling ..... 9
2.3 Market with Only One Consumer Group ..... 13
2.4 Horizontal Differentiation of Information Goods ..... 15
2.4.1 Multiple Groups Without Threat of Cross-Purchasing ..... 15
2.4.2 Multiple Groups Under Threat of Cross-Purchasing ..... 17
2.5 Vertical Differentiation of Information Goods ..... 21
2.5.1 Multiple Groups and Vertically Differentiated Goods with Un- differentiated Characteristics ..... 21
2.5.2 Vertically Differentiated Goods with Group Related Character- istics ..... 24
2.5.3 Versioning Costs and Segment Aggregation ..... 27
2.6 Conclusions ..... 31
3 ESSAY TWO: EXPERIENCE INFORMATION GOODS: TECH- NOLOGY DIFFUSION AND VERSIONING STRATEGIES ..... 34
3.1 Introduction ..... 34
3.2 Notation and Assumptions ..... 38
3.3 Versioning Strategies with A Lump Sum Payment ..... 40
3.3.1 A Model with Homogeneous Expectations ..... 40
3.3.2 A Model with Heterogeneous Expectations ..... 46
3.3.3 Endogenized Qualities ..... 49
3.4 Versioning Strategies with Periodic License Fees ..... 51
3.4.1 Pessimistic Consumer Expectations ..... 52
3.4.2 Optimistic Consumer Expectations ..... 54
3.5 Conclusions ..... 57
4 ESSAY THREE: VERTICALLY DIFFERENTIATED INFORMA- TION GOODS: MONOPOLY POWER THROUGH VERSIONING ..... 59
4.1 Introduction ..... 59
4.2 Notation and Assumptions ..... 62
4.3 A Monopoly Model ..... 63
4.4 Simultaneous Move Duopoly ..... 66
4.4.1 Simultaneous Move without Versioning ..... 67
4.4.2 Should Firms Version? ..... 68
4.4.3 Comparative Quality Analysis ..... 72
4.4.4 Best Response Functions and Equilibrium Analysis ..... 73
4.5 Sequential Move: Strategic Accommodation and Entry Deterrence ..... 74
4.5.1 Entry Accommodation - A Stackelberg Solution ..... 76
4.5.2 Entry Deterrence ..... 77
4.5.3 Entry Deterrence, Rivalry Clear-out or Coexistence ..... 82
4.5.4 A Numerical Example ..... 84
4.6 Welfare Implications ..... 87
4.7 Conclusions ..... 88
5 CONCLUSIONS ..... 91
6 REFERENCES ..... 95
7 APPENDIX A ..... 99
8 APPENDIX B ..... 103
9 APPENDIX C ..... 106
10 GLOSSARY ..... 109

## List of Tables

Table 4.1: Summary of Key Notation. ..... 64
Table 4.2: A Numerical Example: Simultaneous Game ..... 85
Table 4.3: A Numerical Example: Stackelberg Game ..... 86
Table 4.4: A Numerical Example: Entry Deterrence. ..... 87
Table 4.5: Comparison of Socially Optimal and Monopoly ..... 88

## List of Figures

Figure 2.1: Utility Functions of Consumers in Different Segments. ..... 12
Figure 2.2: Market Interaction under Cross-Purchasing. ..... 20
Figure 2.3: Profits under Vertical Differentiation with Undifferentiated ..... 23
Figure 2.4: Profits under Vertical Differentiation with Group Related. ..... 27
Figure 2.5: Versioning Costs and Aggregation. ..... 29
Figure 3.1: Market Segmentation of Information Goods ..... 41
Figure 3.2: An Upgrade Model. ..... 42
Figure 3.3: A Market When Everyone Upgrades. ..... 44
Figure 3.4: Solutions of the Two-version Situation. ..... 45
Figure 3.5: Market Segmentation of Information Goods for Optimistic Consumers.. ..... 55
Figure 4.1: Two Nash Equilibria in the Simultaneous Game ..... 75
Figure 4.2: A Stackelberg Game Solution. ..... 78

## Chapter 1

## INTRODUCTION

Characterized by large sunk costs of development, and by negligible costs of reproduction and distribution, information goods show substantial economies of scale (Shapiro and Varian, 1999). Jones and Mendelson (2005) categorize information goods as: i) computer software including operation systems, programming tools and applications; ii) online services such as internet search engines and portals; iii) online content such as information provided by Lexis/Nexis, Dow Jones, and Reuters; and iv) other digitalized information goods such as digitalized music, movies and books. An additional unit of an information good can be produced and distributed at negligible cost by allowing it to be downloadable over the Internet (Jones and Mendelson, 2005). Broad adoption of e-commerce, secure and convenient online payments and high-speed internet connections greatly lower the transaction costs.

Another notable feature of information goods is that after the highest quality version has been developed, the creation of its vertically degraded versions is usually less costly. Versioning is to "offer a product line and let users choose the version of the product most appropriate for them (Shapiro and Varian, 1999)", which is often referred to as "the second degree price discrimination". Developments in software engineering have made versioning of most computer software virtually cost-free. Thus, information goods producers can easily provide vertically differentiated products, thereby segmenting the market to maximize profit.

In the first essay, ${ }^{1}$ we follow the self-selection model of consumer choice, emphasiz-

[^0]ing the interaction of market segmentation and product differentiation of information goods as we believe that any product differentiation must be based on existing market segments. We treat vertical differentiation as a special case of horizontal differentiation, and we model the interaction between different segments showing the differences in versioning strategies when moving from horizontal to vertical differentiation. We find that it is optimal to version information goods only if the market is differentiated and cross-purchasing is limited or if characteristics of the information goods are designed for particular segments.

Essential to our model is the definition of an individual consumer taste for quality, and a group taste that is correlated with individual tastes. We find that if there is only one group, then the classic result of no versioning - that is, a single version - found by others, holds. For multiple groups and horizontally differentiated information goods we provide an intuitive condition that results in no cross-purchasing, and consequently a monopolist offers a separate version for each group. Relaxing the no cross-purchasing condition, we find that versioning can still be optimal with the monopolist squeezing lower-taste groups in favor of higher taste and more profitable groups.

Using the same consumer preference structure with correlated individual and group tastes, for multiple groups and vertically differentiated goods we find that a monopolist only offers a single version. However, relative to one consumer group, multiple consumer groups can allow the monopolist to increase profits by trading volume for increased price. We also find that if characteristics of the goods are "group related" - that is, all consumers share the same group tastes for certain characteristics - versioning can be optimal. We then provide an algorithm to determine the optimal number of versions if the costs of producing additional versions are significant.

In the second essay, ${ }^{2}$ we analyze a monopolist's versioning strategies for experience information goods with two different payment arrangements: lump sum payments and periodic license fees. In contrast to "search goods" whose attributes can be determined by inspection without the necessity of use, we treat information goods as "experience goods" whose quality can only be determined through use.

With lump sum payments, consumers pay once to have ownership of the good. Thus the monopolist can offer different versions and provide upgrades to maximize profits. Allowing upgrades makes versioning more complicated. For versioning with upgrades, we examine how producers choose prices for different versions and for upgrades, and how upgrade strategies impact consumers' choices and subsequent versioning strategies. Adopting a two-stage model where consumers can purchase in the first stage and then those that chose a lower quality can upgrade in the second stage, we show that versioning is a useful strategy for the producer to fully reveal the quality of its goods. To maximize profits, the producer generates vertically differentiated versions of its information goods, and designs mechanisms for consumers that purchase lower quality versions to upgrade to the high quality version after having experienced the low quality version and learned its true quality. We find that if consumers have homogeneous initial expectations about quality, then the producers' optimal pricing strategy is to drive all the consumers who buy the low quality version in the first stage to upgrade to the high quality version in the second stage. In this way, consumers that upgrade pay a tax for learning. But if consumers have heterogeneous initial expectations and consumers that are optimistic about quality dominate the market, then only some consumers that purchase the low quality version in the first stage upgrade.

[^1]With periodic license fees, consumers are charged for the usage of the good for a period of time. Thus the monopolist may provide different versions at different periods to attract consumers. We find that for pessimistic consumers, versioning strategies are implemented by the monopolist only if some consumers are already informed of the true quality. For optimistic consumers, the monopolist provides only one version.

With the ease of versioning, product differentiation and pricing strategies of information goods are different from traditional goods, especially in the context of competition. Competition for information goods is more intense than traditional goods and the winners usually dominate the market. In addition, with potential competition, producers of information goods have an incentive to improve quality. They launch their highest quality version, or upgrade the old version, whenever possible, even if they lose money at the margin by cannibalizing the existing market share of the old version (Nault and Vandenbosch, 1996). It is also common for the software producers to release a buggier product early and patch it later to grab the "first mover advantage" in the market (Arora, Caulkins, and Telang, 2006). The subsequent questions are: 1) why do leading producers of information goods dominate their market? and, 2 ) why is a monopoly producer so eager to improve the quality of its information goods under potential competition?

In the third essay, ${ }^{3}$ we analyze price, quality and versioning strategies that information goods producers use to deter entry and maintain market power. We find that under competition, firms provide higher quality information goods with a better "price-quality ratio" than in monopoly. In a Stackelberg game, the leader firm that

[^2]provides the high quality information good decreases its quality level to maintain a first mover advantage. We also show that a monopolist can implement versioning strategies in the low-end market to deter entry, and different versions exist as a signal to prevent potential entry. A vertically differentiated market is often referred to as a "natural oligopoly" for traditional goods, whereas it can be regarded as a "natural monopoly" for information goods.

The rest of the thesis is organized as follows. Chapter 2 analyzes monopoly versioning of information goods. Chapter 3 examines versioning strategies with experience information goods. Competition with information goods is presented in Chapter 4. Chapter 5 concludes the thesis with a brief summary and possible extensions.

## Chapter 2

# ESSAY ONE: MONOPOLY VERSIONING OF INFORMATION GOODS 

### 2.1 Introduction

Information goods such as computer software, online services, online content and digitalized music, movies and books have become an indispensable part of our life. The greatest distinction between information goods and physical goods is reproduction costs where the former incurs large sunk costs of development but negligible costs of reproduction and distribution. Broad adoption of e-commerce, secure and convenient online payments and high-speed Internet connections have greatly lowered transaction costs and made information goods more appealing. In addition to production costs, several other features make information goods different from many other products: due to developments in software engineering, characteristics of information goods can be easily recombined to generate different versions, and information goods are durable goods in that a consumer purchases at most one unit of a specified version of the goods during the overall life cycle.

Price discrimination and product differentiation through versioning are common ways firms take advantage of the specific characteristics of information goods. As consumers can easily compare and select their favorite goods online, third degree price discrimination, which assumes consumers from one segment cannot cross-purchase goods from another segment, is less likely. On the other hand, technologies such as "cookies" adopted in e-commerce make it possible to treat each consumer separately,
thus in principle firms can implement first degree price discrimination. Choudhary, Ghose, Mukopadhyay and Rajan (2005) propose personalized pricing (PP) where firms can perfectly identify valuations of heterogeneous consumers and target them individually with vertically differentiated goods. However, consumers can easily communicate with each other or disguise their patterns to respond strategically to firm's pricing strategy, hence PP has not been widely adopted.

So far the most popular way to sell information goods is where firms offer a menu of goods and prices, and each consumer chooses based on their preference: second degree price discrimination (Shapiro and Varian 1999). Bakos and Brynjolfsson (1999) find that bundling large numbers of unrelated information goods can be profitable, but when different segments of consumers differ systematically in their valuations for goods, simple bundling is not optimal. Sundararajan (2004) shows that for information goods, fixed-fee and usage-based pricing can be used together to maximize a monopolist's profit.

An alternative to bundling and non-linear pricing for information goods is versioning. In previous literature, versioning through vertical differentiation and corresponding pricing strategies are modeled in different contexts such as network externalities (Jing 2002), competition (Jones and Mendelson 2005, Wei and Nault 2006), anti-piracy (Wu, Chen and Anandalingam 2003), and inter-organizational information systems (Nault 1997). They all conclude that versioning is not optimal without certain constraints, consistent with Bhargava and Choudhary (2001). Adopting a quadratic utility function form, Ghose and Sundararajan (2005) propose optimal solutions with multiple versions. Chen and Seshadri (2007) introduce convex reservation utilities when consumers have multiple outside options to explain the existence of multiple versions. Bhargava and Choudhary (2007) examine a more general nonlinear utility function form and propose that versioning is optimal when lower type
consumers have greater ratios of valuations than higher type consumers.
Although the modeling results from previous literature are consistent with many empirical observations, there are other observations that are not effectively explained. The most well-known is Microsoft's Operating Systems. Windows XP has five editions: Home Edition, Professional, Media Center Edition, Tablet PC Edition and Professional x64 Edition. The Home Edition and Professional Edition are vertically differentiated, but the others are horizontally differentiated. The Windows Server 2003 family has six editions and they are not purely vertically differentiated as each edition has its own focus. This also applies to the latest delivery of Windows Vista. Another example is Kurzweil's product line of software-based voice recognition products (Shapiro and Varian 1999). Kurzweil offers seven versions and among them Office Talk is designed for office staff, Law Talk for lawyers and Voice Med for medical staff. Each version is priced differently while all of the versions share a certain amount of common vocabulary (about 20,000 words). The high-end version for surgeons is priced a hundred times higher than the entry-level version. There are numerous other such examples.

Although it is technically possible for firms to generate a "super-version" which contains all the characteristics in their product line and then degrade it to generate vertically differentiated versions, many firms still choose to differentiate their products horizontally whenever possible. Indeed, previous research has shown that a monopolist offering vertically differentiated versions is optimal only under some restrictive conditions. In this work, we treat vertical differentiation as a special case of horizontal differentiation, and model the interaction between different market segments showing how and when monopoly versioning is optimal.

Essential to our model is the definition of an individual consumer taste for quality, and a group taste that is correlated with individual tastes. We find that if there
is only one group, then the classic result of no versioning - that is, a single version - found by others, holds. For multiple groups and horizontally differentiated information goods we provide an intuitive condition that results in no cross-purchasing, and consequently a monopolist offers a separate version for each group. Relaxing the no cross-purchasing condition, we find that versioning can still be optimal with the monopolist squeezing lower-taste groups in favor of higher taste and more profitable groups.

Using the same consumer preference structure with correlated individual and group tastes, for multiple groups and vertically differentiated goods we find that a monopolist only offers a single version. However, relative to one consumer group, multiple consumer groups can allow the monopolist to increase profits by trading volume for increased price. We also find that if characteristics of the goods are "group related" - that is, all consumers share the same group tastes for certain characteristics - versioning can be optimal. We then provide an algorithm to determine the optimal number of versions if the costs of producing additional versions are significant.

The rest of the paper is organized as follows. We set up our notation and assumptions in Section 2.2 and discuss when there is only one group in Section 2.3. We extend the model to investigate situations when there are multiple groups with horizontally differentiated goods in Section 2.4 and with vertically differentiated goods in Section 2.5. Discussion and future research are included in Section 2.6.

### 2.2 Modeling

Following the hedonic hypothesis that "goods are valued for their utility-bearing attributes or characteristics" (Rosen 1974, pp. 34), we define information goods as a set of characteristics. We allow for $M$ characteristics, $\chi=\left\{x_{1}, x_{2}, \ldots, x_{M}\right\}$, and each
good contains a subset of these characteristics, for example $\chi \supseteq X_{1}, X_{2}, \ldots, X_{I}$ where there are $I$ possible information goods. Quality is denoted by $q$ where $q \in[0,+\infty)$. We assume that complexity of information goods does not jeopardize their quality levels and that unused characteristics can be freely disposed of or ignored in use. In other words, the quality of information goods are solely determined by the set of characteristics they include, and more characteristics are better. This is our first assumption.

Assumption 2.1 For two information goods $X_{i}$ and $X_{j}$, if $X_{i} \subseteq X_{j}$, then $q_{i} \leq q_{j}$.

If an information good contains all the characteristics of another good and more, then we call it "vertical differentiation". If two information goods do not include each other, then we call it "horizontal differentiation". Our Assumption 2.1 indicates that quality can be compared between vertically differentiated goods. For horizontally differentiated goods, quality cannot be compared directly.

As in most prior research, we take consumers to be heterogeneous and continuously distributed in their individual taste for quality. We denote the individual consumer taste as $\theta$ which belongs to $\left[\theta_{0}, \theta_{N}\right]$. We assume that $\theta$ has density and cumulative density functions $f(\theta)$ and $F(\theta)$, so that consumers are normalized with a unit population. The density is strictly positive over its support and continuously differentiable. Following Bhargava and Choudhary (2001), Jing (2002) and Sundararajan (2004), we make the following assumption about the distribution of consumer taste:

Assumption 2.2 The reciprocal of the hazard function, $\frac{1-F(\theta)}{f(\theta)}$, is non-increasing in $\theta .{ }^{1}$

[^3]From Assumption 2.2, the function $\frac{\delta-F(\theta)}{f(\theta)}, 0 \leq \delta \leq 1$ is also non-increasing, so we have

$$
\frac{\partial}{\partial \theta}\left[\frac{\delta-F(\theta)}{f(\theta)}\right] \leq 0 \text { for } 0 \leq \delta \leq 1
$$

Essential to our model, we define a second feature that determines an individual's taste for quality: a group taste. Consumers are divided into groups and these groups are correlated with individual tastes which in turn define segments. Consumers with individual taste in segment $\left[\theta_{n-1}, \theta_{n}\right)$ belong to group $n, n \in\{1,2, \cdots, N\}$. Consumers in the same group $n$ share the same group taste $k_{n}$ and higher groups have greater tastes for quality, which means $k_{n+1}>k_{n}$. We represent the taste for quality as a product of the individual and group taste so that it can be represented by $k_{n} \theta$. Without a great loss of generality (as we can rescale $q$ ), we assume a multiplicative relationship in the consumers' willingness to pay $U\left(q, \theta, k_{n}\right)$ between taste $k_{n} \theta$ and quality $q$. This is our third assumption.

Assumption 2.3 The utility function of consumers in group $n$ who purchase information good with quality $q$ is expressed as $U\left(q, \theta, k_{n}\right)=k_{n} \theta q, n \in\{1, \ldots, N\}, \theta \in$ $\left[\theta_{n-1}, \theta_{n}\right)$.

Figure 2.1 illustrates the utility that consumers from different groups receive based on the above utility functions. The combination of individual and group tastes allows us to represent discontinuous consumer heterogeneity.

To provide a separate version for certain consumers may incur additional costs which we refer to as "versioning costs". Versioning costs could include additional development, marketing and managerial costs. Technology development such as software engineering has greatly lowered additional development costs for versioning and broad adoption of e-commerce has minimized additional marketing and managerial costs for providing an extra version. Thus, we make the following limiting assumption


Figure 2.1: Utility Functions of Consumers in Different Segments
about the versioning costs:

Assumption 2.4 Versioning costs are zero after the highest quality information goods have been produced.

We relax this assumption in the last part of Section 2.5 to show how versioning costs may impact versioning strategies.

In segment $n$ where the consumer only chooses between purchasing the good designed for their segment and not purchasing, we define $\tilde{\theta}_{n}$ as the indifferent consumer and the price assignment is

$$
\begin{equation*}
p_{n}=U\left(q_{n}, \tilde{\theta}_{n}, k_{n}\right) \tag{2.1}
\end{equation*}
$$

In segment $n$ where the consumer chooses between purchasing the good designed
for their segment $n$ and a good designed for another segment $i$, we define $\tilde{\theta}_{n}$ as the indifferent consumer and the price assignment is

$$
\begin{equation*}
p_{n}=p_{i}+U\left(q_{n}, \tilde{\theta}_{n}, k_{n}\right)-U\left(q_{i}, \tilde{\theta}_{n}, k_{n}\right) \tag{2.2}
\end{equation*}
$$

In this formulation, the profit maximization problem for a monopolist that serves all $N$ segments is ${ }^{2}$

$$
\max _{\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{N}} \Pi\left(\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{N}\right)=\max _{\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{N}}\left\{\sum_{n=1}^{N} p_{n}\left[F\left(\theta_{n}\right)-F\left(\tilde{\theta}_{n}\right)\right]\right\}, \quad \tilde{\theta}_{n} \in\left[\theta_{n-1}, \theta_{n}\right) .
$$

### 2.3 Market with Only One Consumer Group

If all consumers share the same group taste, then there is only one consumer group in the market. We normalize $k_{1}$ to unity and the utility function is simplified to $U(q, \theta)=\theta q, \quad \theta \in\left[\theta_{0}, \theta_{1}\right]$. With only one group, all the characteristics designed for this group are valued by all consumers.

We assume the monopolist provides $M$ versions of the information good with quality levels $Q=\left(q_{1}, q_{2}, \ldots, q_{M-1}, q_{M}\right)$. Without loss of generality, we assume $q_{1}<q_{2}<\ldots<q_{M-1}<q_{M}$. The highest quality $q_{M}$ is developed first, and the subsequent degraded qualities $q_{M-1}, \ldots, q_{2}, q_{1}$ are produced through versioning. Let $P=\left(p_{1}, p_{2}, \ldots, p_{M-1}, p_{M}\right)$ denote the corresponding prices, and $D\left(P, q_{i}\right)$ denote the demand for the good with quality $q_{i}$ given the price vector $P$.

The provision of $M$ different quality levels divides the market into $M+1$ segments, where the last segment has consumers that do not purchase. In segment 1 where the consumer only chooses between purchasing the good designed for her segment and not purchasing, we define $\tilde{\theta}_{1}$ as the indifferent consumer and the price assignment is

[^4]as in (2.1),
$$
p_{1}=U\left(q_{1}, \tilde{\theta}_{1}\right)=\tilde{\theta}_{1} q_{1}
$$

In segment $i$ (for $2 \leq i \leq M$ ) where the consumer chooses between purchasing the good $q_{i}$ and a good $q_{i-1}$ designed for its closest segment $i-1$, we define $\tilde{\theta}_{i}$ as the indifferent consumer and the price assignment is as in (2.2):

$$
p_{i}=p_{i-1}+U\left(q_{i}, \tilde{\theta}_{i}\right)-U\left(q_{i-1}, \tilde{\theta}_{i}\right)=p_{i-1}+\tilde{\theta}_{i} q_{i}-\tilde{\theta}_{i} q_{i-1}
$$

The monopolist chooses $\tilde{\theta}_{i}$ to maximize its profit. Our profit maximization problem is

$$
\begin{gathered}
\max _{\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{M}}\left\{\left[1-F\left(\tilde{\theta}_{M}\right)\right]\left[\tilde{\theta}_{1} q_{1}+\sum_{j=2}^{M}\left(q_{j}-q_{j-1}\right) \tilde{\theta}_{j}\right]\right. \\
\left.+\sum_{i=2}^{M-1}\left[F\left(\tilde{\theta}_{i+1}\right)-F\left(\tilde{\theta}_{i}\right)\right]\left[\tilde{\theta}_{1} q_{1}+\sum_{j=2}^{i}\left(q_{j}-q_{j-1}\right) \tilde{\theta}_{j}\right]+\left[F\left(\tilde{\theta}_{2}\right)-F\left(\tilde{\theta}_{1}\right)\right] \tilde{\theta}_{1} q_{1}\right\} \\
\ni \quad \theta_{0} \leq \tilde{\theta}_{1} \leq \tilde{\theta}_{2} \leq \ldots \leq \tilde{\theta}_{M} \leq \theta_{1} .
\end{gathered}
$$

The first-order conditions imply that the optimal $\tilde{\theta}_{i}, i \in\{1,2, \ldots, M\}$ satisfies $\tilde{\theta}_{i}=$ $\frac{1-F\left(\tilde{\theta}_{i}\right)}{f\left(\bar{\theta}_{i}\right)}$. Because the inverse hazard function $\frac{1-F(\theta)}{f(\theta)}$ is non-increasing by Assumption 2.2, $\theta=\frac{1-F(\theta)}{f(\theta)}$ has unique solution (considering the constraints, $\tilde{\theta}_{i}$ can be corner solutions, in this case $\tilde{\theta}_{i}=\theta_{0}$ ). It means that all the indifferent consumer types are equal, so that there is a single indifferent consumer. We denote this indifferent consumer as $\theta^{*}$, where

$$
\begin{equation*}
\theta^{*}=\frac{1-F\left(\theta^{*}\right)}{f\left(\theta^{*}\right)} \tag{2.3}
\end{equation*}
$$

The result is that consumers in $\left[\theta^{*}, \theta_{1}\right]$ purchase the highest quality version while consumers in $\left[\theta_{0}, \theta^{*}\right)$ do not purchase. This yields the following theorem:

Proposition 2.1 If there is one consumer group, then it is not optimal to version information goods.

This is the classic one version case, which using our formulation replicates the findings of Jones and Mendelson (2005), Bhargava and Choudhary (2001) and basic argument of Jing (2002), and Wu, Chen and Anandalingam (2003).

### 2.4 Horizontal Differentiation of Information Goods

### 2.4.1 Multiple Groups Without Threat of Cross-Purchasing

For a horizontally differentiated market, consumers from different groups derive value from a shared set of characteristics of the information good, while other characteristics can be tailored to provide value for different groups. We use $X_{a}$ to denote the shared characteristics and $q_{a}$ as the quality index for $X_{a}$. For simplicity, we assume that $X_{a}$ is the only set of characteristics shared by the different goods in the product line, that is

$$
X_{a}=X_{i} \cap X_{j}, \quad \forall i \neq j .
$$

Because $X_{n}$ is the set of characteristics of the information goods for group $n$, we get $X_{n} \cap \bar{X}_{a}$ as the set of special characteristics tailored for group $n$, where $\bar{X}_{a}$ is the complement of $X_{a}$. Only this group values those special characteristics. As a result we have the following utility function

$$
U\left(q_{i}, \theta, k_{n}\right)= \begin{cases}k_{n} \theta q_{i}, & \text { if } n=i \\ k_{n} \theta q_{a}, & \text { if } n \neq i\end{cases}
$$

which means if a consumer from group $n$ purchases the information good tailored for group $n$, then the consumer receives utility $k_{n} \theta q_{n}$. Otherwise, if a consumer purchases the "wrong" information good (one tailored for another group), the consumer only gets utility from the shared characteristics, $k_{n} \theta q_{a}$. Therefore, our definition of horizontal differentiation is this form where the additional characteristics that each group values is mutually exclusive.

To avoid cross-purchasing, which in this case means avoiding consumers preferring the "wrong" information good, we have the following lemma. ${ }^{3}$

Lemma 2.1 A necessary and sufficient condition to prevent cross-purchasing between different groups is

$$
\frac{q_{a}}{q_{n}} \leq \frac{k_{n} \theta_{n}^{*}}{k_{N} \theta_{N}^{*}}, \forall n \in\{1,2, \cdots, N-1\}
$$

where $\theta_{n}^{*}$ is the indifferent consumer when the monopolist maximizes profit in segment $n$ only.

Lemma 2.1 is the special case of the more general condition $U\left(q_{a}, \theta_{N}^{*}, k_{N}\right) \leq U\left(q_{n}, \theta_{n}^{*}, k_{n}\right)$, $\forall n \in\{1,2, \cdots, N-1\}$. Because consumers with high individual tastes also have high group tastes, consumers in segment $N$ have the highest utility for shared characteristics, and are the most likely to cross-purchase goods designed for other segments. Lemma 1 indicates that the shared characteristics do not provide enough value for consumers in segment $N$ to cross-purchase the goods designed for other segments, and thus guarantees the separation of all the segments. In this situation, we have the following theorem.

Proposition 2.2 With multiple groups and horizontally differentiated goods with shared characteristics, if there is no threat of cross-purchasing, then it is profit maximizing for the monopolist to provide each segment only one version which is tailored for it.

The rationale behind Proposition 2.2 is simple. As compared to the value of the special characteristics, if the value of the shared characteristics is relatively low, then each consumer chooses exactly the right version for their group. In the special case when $q_{a}$ is zero, which means the groups in the market are completely different in

[^5]their taste and quality requirements, we have a "perfect horizontally differentiated market".

### 2.4.2 Multiple Groups Under Threat of Cross-Purchasing

Now we analyze the conditions where Lemma 1 cannot be satisfied. Generally, for consumer group $i$, if we have $\frac{g_{a}}{q_{i}}>\frac{k_{i} \theta_{i}^{*}}{k_{j} \theta_{j}^{*}}, 1 \leq i<j \leq N$, which means $\theta_{j}^{*}$ gets more value from $q_{a}$ than $\theta_{i}^{*}$ gets from $q_{i}$, then there is a threat that part of segment $j$ may prefer to purchase good $X_{i}$ instead of $X_{j}$.

When there is a threat of cross-purchasing, we have the following lemma:

Lemma 2.2 When there is a threat of cross-purchasing: 1. The version with the lowest price is the only version that will be cross-purchased; and 2. For a segment that is under threat of cross-purchasing, all the higher segments are under threat of cross-purchasing and all the higher segments are covered.

We denote the lowest priced segment as segment $i$ and the lowest segment that is under threat of cross-purchasing as segment $j$. As only a higher-end segment can be threatened by an information good designed for a lower-end segment, we have $k_{i}<k_{n}$, for $n \in\{j, j+1, \cdots, N\}$. Under the threat of cross-purchasing, the monopolist changes price constraints in all segments $n$. We denote $\tilde{\theta}_{n}$ as the consumer in segment $n$ that is indifferent between purchasing goods $X_{n}$ and $X_{i}$, and $\tilde{\theta}_{i}$ as the consumer in segment $i$ that is indifferent between purchasing good $X_{i}$ and not purchasing. Then from (2.1) and (2.2), the prices are defined by

$$
\begin{gather*}
p_{i}=k_{i} \tilde{\theta}_{i} q_{i}, \text { for } \tilde{\theta}_{i} \in\left[\theta_{i-1}, \theta_{i}\right) \text {, and }  \tag{2.4}\\
k_{n} \tilde{\theta}_{n} q_{n}-p_{n}=k_{n} \tilde{\theta}_{n} q_{a}-p_{i}, n \in\{j, j+1, \cdots, N\} \text { and } \tilde{\theta}_{n} \in\left[\theta_{n-1}, \theta_{n}\right) \tag{2.5}
\end{gather*}
$$

We know from Lemma 2.2 that all segments higher than segment $j$ are covered. But
segment $j$ need not be covered. We consider these two cases separately.

Case 1: Segment $j$ Is Not Covered In segment $j$, there is another consumer which we denote as $\hat{\theta}_{j}$ that is indifferent between purchasing $X_{i}$ and not purchasing. Thus, we have the additional condition using (2.4)

$$
\begin{equation*}
k_{j} \hat{\theta}_{j} q_{a}=k_{i} \tilde{\theta}_{i} q_{i}, \text { for } \hat{\theta}_{j} \in\left(\theta_{j-1}, \tilde{\theta}_{j}\right) \tag{2.6}
\end{equation*}
$$

Here we have three possible partitions of segment $j$ : consumers in $\left[\tilde{\theta}_{j-1}, \hat{\theta}_{j}\right)$ do not purchase, consumers in $\left[\hat{\theta}_{j}, \tilde{\theta}_{j}\right)$ cross-purchase $X_{i}$, and consumers in $\left[\tilde{\theta}_{j}, \theta_{j}\right]$ purchase $X_{j}$. In segment $i$ consumers in $\left[\tilde{\theta}_{i}, \theta_{i}\right)$ purchase $X_{i}$ and consumers in $\left[\theta_{i-1}, \tilde{\theta}_{i}\right)$ do not purchase. In segment $n$, for $n \in\{j, j+1, \cdots, N\}$, consumers in $\left[\theta_{n-1}, \tilde{\theta}_{n}\right)$ crosspurchase $X_{i}$ and consumers in $\left[\tilde{\theta}_{n}, \theta_{n}\right)$ purchase $X_{n}$.

Substituting (2.4), (2.5) and (2.6) into the profit maximization described in Section 2 (only considering segments that are involved in cross-purchasing), we get

$$
\begin{gathered}
\max _{\tilde{\theta}_{i}, \tilde{\theta}_{j}, \cdots, \hat{\theta}_{N}} \Pi\left(\tilde{\theta}_{i}, \tilde{\theta}_{j}, \cdots, \tilde{\theta}_{N}\right)=\max _{\tilde{\theta}_{i}, \tilde{\theta}_{j}, \cdots, \tilde{\theta}_{N}}\left\{k_{i} \tilde{\theta}_{i} q_{i}\left[F\left(\theta_{i}\right)-F\left(\tilde{\theta}_{i}\right)+F\left(\theta_{j}\right)-F\left(\frac{k_{i} q_{i}}{k_{j} q_{a}} \tilde{\theta}_{i}\right)\right]\right. \\
\left.+\sum_{n=j+1}^{N} k_{i} \tilde{\theta}_{i} q_{i}\left[F\left(\theta_{n}\right)-F\left(\theta_{n-1}\right)\right]+\sum_{n=j}^{N} k_{n} \tilde{\theta}_{n}\left(q_{n}-q_{a}\right)\left[F\left(\theta_{n}\right)-F\left(\tilde{\theta}_{n}\right)\right]\right\} \\
\ni \quad \tilde{\theta}_{i} \in\left[\theta_{i-1}, \theta_{i}\right) \text { and } \tilde{\theta}_{n} \in\left[\theta_{n-1}, \theta_{n}\right), n \in\{j, j+1, \cdots, N\}
\end{gathered}
$$

From the first-order conditions with respect to $\tilde{\theta}_{n}, n \in\{j, j+1, \cdots, N\}$, we have $\tilde{\theta}_{n}^{*}=\frac{F\left(\theta_{n}\right)-F\left(\tilde{\theta}_{n}^{*}\right)}{f\left(\hat{\theta}_{n}^{*}\right)}$. Solving the above equation, we get $\tilde{\theta}_{n}^{*}=\theta_{n}^{*}$, where the latter is the indifferent consumer from Lemma 2.1 when there is no cross-purchasing. This means the market share for good $X_{n}$ is the same as without the threat from good $X_{i}$. However, from (2.5) we know the price $p_{n}$ is lower than when there is no crosspurchasing. Thus, under threat of cross-purchasing from segment $i$, the profit from $\operatorname{good} X_{n}$ decreases.

From the first-order condition with respect to $\tilde{\theta}_{i}$,

$$
\tilde{\theta}_{i}^{*}=\frac{F\left(\theta_{i}\right)-F\left(\tilde{\theta}_{i}^{*}\right)+F\left(\theta_{j}\right)-F\left(\frac{k_{i} q_{i} \tilde{i}_{i}}{k_{j}} \tilde{\theta}_{i}^{*}\right)+\sum_{n=j+1}^{N}\left[F\left(\theta_{n}\right)-F\left(\theta_{n-1}\right)\right]}{f\left(\tilde{\theta}_{i}^{*}\right)+\frac{k_{i} q_{i}}{k_{j} q_{a}} f\left(\frac{k_{i} q_{i}}{k_{j} q_{a}} \tilde{\theta}_{i}^{*}\right)} .
$$

Because $\frac{F\left(\theta_{j}\right)-F(\theta)}{f(\theta)}$ is non-increasing and $\frac{q_{a}}{q_{i}}>\frac{k_{i} \theta_{i}^{*}}{k_{j} \theta_{j}^{*}}$, we have

$$
\frac{F\left(\theta_{j}\right)-F\left(\frac{k_{i} q_{i}}{k_{j} \tilde{q}_{a}} \tilde{\theta}_{i}^{*}\right)}{\frac{k_{i} q_{i}}{k_{j} q_{a}} f\left(\frac{k_{i} q_{i} \tilde{\theta}_{j}^{*} q_{a}}{k_{i}}\right)} \geq \frac{k_{j} q_{a}}{k_{i} q_{i}} \frac{F\left(\theta_{j}\right)-F\left(\theta_{j}^{*}\right)}{f\left(\theta_{j}^{*}\right)}=\frac{k_{j} q_{a}}{k_{i} q_{i}} \theta_{j}^{*}>\frac{k_{i} \theta_{i}^{*} q_{i}}{k_{i} q_{i}}=\theta_{i}^{*} .
$$

And we know $\theta_{i}^{*}=\frac{F\left(\theta_{i}\right)-F\left(\theta_{i}^{*}\right)}{f\left(\theta_{i}^{*}\right)}$. So we have

$$
\tilde{\theta}_{i}^{*}>\frac{F\left(\theta_{i}\right)-F\left(\tilde{\theta}_{i}^{*}\right)+F\left(\theta_{j}\right)-F\left(\frac{k_{i} q_{i}}{k_{j}} \tilde{\theta}_{i}^{*}\right)}{f\left(\tilde{\theta}_{i}^{*}\right)+\frac{k_{i} q_{i}}{k_{j} q_{a}} f\left(\frac{k_{i} q_{i}}{k_{j} q_{a}} \tilde{\theta}_{i}^{*}\right)}>\theta_{i}^{*} .
$$

Case. 2: Segment $j$ Is Covered Here, there are only two partitions of segment $j$ : consumers in $\left[\theta_{j-1}, \tilde{\theta}_{j}\right)$ purchase $X_{i}$ and consumers in $\left[\tilde{\theta}_{j}, \theta_{j}\right]$ purchase $X_{n}$. Other segments are the same as in Case 1. The monopolist's profit maximization is

$$
\begin{gathered}
\max _{\tilde{\theta}_{i}, \tilde{\theta}_{j}, \cdots, \tilde{\theta}_{N}} \Pi\left(\tilde{\theta}_{i}, \tilde{\theta}_{j}, \cdots, \tilde{\theta}_{N}\right)=\max _{\tilde{\theta}_{i}, \tilde{\tilde{j}}_{j}, \cdots, \tilde{\theta}_{N}}\left\{k_{i} \tilde{\theta}_{i} q_{i}\left[F\left(\theta_{i}\right)-F\left(\tilde{\theta}_{i}\right)\right]+\sum_{n=j}^{N} k_{i} \tilde{\theta}_{i} q_{i}\left[F\left(\theta_{n}\right)-F\left(\theta_{n-1}\right)\right]\right. \\
\left.\quad+\sum_{n=j}^{N} k_{n} \tilde{\theta}_{n}\left(q_{n}-q_{a}\right)\left[F\left(\theta_{n}\right)-F\left(\tilde{\theta}_{n}\right)\right]\right\}, \\
\ni \tilde{\theta}_{i} \in\left[\theta_{i-1}, \theta_{i}\right) \text { and } \tilde{\theta}_{n} \in\left[\theta_{n-1}, \theta_{n}\right), n \in\{j, j+1, \cdots, N\} .
\end{gathered}
$$

From the first-order conditions with respect to $\tilde{\theta}_{n}, n \in\{j, j+1, \cdots, N\}$, we still have $\tilde{\theta}_{n}^{*}=\theta_{n}^{*}$. From the first-order condition with respect to $\tilde{\theta}_{i}$,

$$
\tilde{\theta}_{i}^{*}=\frac{F\left(\theta_{i}\right)-F\left(\tilde{\theta}_{i}^{*}\right)+\sum_{n=j}^{N}\left[F\left(\theta_{n}\right)-F\left(\theta_{n-1}\right)\right]}{f\left(\tilde{\theta}_{i}^{*}\right)}
$$

Because $\frac{F\left(\theta_{i}\right)-F(\theta)+\sum_{n=5}^{N}\left[F\left(\theta_{n}\right)-F\left(\theta_{n-1}\right)\right]}{f(\theta)}>\frac{F\left(\theta_{i}\right)-F(\theta)}{f(\theta)}, \forall \theta$, we find that $\tilde{\theta}_{i}^{*}>\theta_{i}^{*}$.
Thus, under both cases we have $\tilde{\theta}_{i}^{*}>\theta_{i}^{*}$ and $\tilde{\theta}_{n}^{*}=\theta_{n}^{*}$. Consequently, the market share for good $X_{i}$ in segment $i$ shrinks in response to the price increase in $X_{i}$ used to mitigate the threat of cross-purchasing from other segments. Moreover, the higher


Figure 2.2: Market Interaction under Cross-Purchasing
$\tilde{\theta}_{i}^{*}$ increases the prices of goods $X_{n}(n \in\{j, j+1, \cdots, N\})$. Figure 2.2 illustrates this situation. Concluding the above analysis, we have the following theorem:

Proposition 2.3 With multiple groups and horizontally differentiated goods with shared characteristics, if there is a threat of cross-purchasing, then the monopolist retains the market share of versions designed for segments that are under the threat of cross-purchasing while shrinking the market share in the segment that serves the version with the lowest price.

What are the consequences if $q_{a} / q_{i}$ increases? Two situations can result:

1. More segments are under threat of cross-purchasing from good $X_{i}$. The sufficient condition for cross-purchasing between segment $i$ and $j$ is $\frac{q_{a}}{q_{i}}>\frac{k_{i} \theta_{i}^{*}}{k_{j} \theta_{j}^{*}}$. And we
know for $k_{i}<k_{j}, \frac{k_{i} \theta_{i}^{*}}{k_{j} \theta_{j}^{*}}<1$. If $q_{a} / q_{i}$ is increasing, then this condition is more likely to be satisfied. Consequently, more high-end segments are involved in cross-purchasing. When $q_{a} / q_{i}$ is close to 1 , all the high-end segments (for those which are higher than segment $i$ ) are under threat of cross-purchasing from $\operatorname{good} X_{i}$.
2. The higher-end segment $j$ is more likely to be covered. The sufficient condition for segment $j$ to be covered is $\frac{k_{i} q_{i} \tilde{\theta}_{i}}{k_{j} q_{a}} \leq \theta_{j-1}$. With the increase of $q_{a} / q_{i}$, this condition is more likely to be satisfied. As a direct consequence, $\tilde{\theta}_{i}^{*}$ is more likely to reach its upper bound $\theta_{i}$, which means segment $i$ is no longer served.

There are two extreme situations for goods with shared characteristics. One is that the shared quality $q_{a}$ is zero, which is the perfect horizontal differentiation situation we discussed in the previous section. The other extreme is that the special quality is zero, which means $q_{h}=q_{a}$, consumers in different groups value all the characteristics of the information good, although at different utility levels, which is a vertical differentiation model. This is the focus of the next section.

### 2.5 Vertical Differentiation of Information Goods

### 2.5.1 Multiple Groups and Vertically Differentiated Goods with Undifferentiated Characteristics

In this section consumers from all groups value all the information goods' characteristics, although they may put different values on those characteristics. These different values depend on the group taste $k_{n}$ and the consumer taste $\theta$.

Using the single version one consumer group result from (2.3) and Proposition 2.1 where $\theta^{*}$ is the indifferent consumer, in our multiple groups context we define the
segment $\theta^{*}$ belongs to as $e$ where $\theta^{*} \in\left[\theta_{e-1}, \theta_{e}\right)$. To simplify our notation we construct the function $H(\theta)=\theta[1-F(\theta)]$ which has a global maximum at $H\left(\theta^{*}\right)$. Consequently, $H(\theta)$ is monotonically increasing for $\theta \in\left[\theta_{0}, \theta^{*}\right]$, and $H(\theta)$ is monotonically decreasing for $\theta \in\left[\theta^{*}, \theta_{N}\right]$.

From the discussion in the previous section, the monopolist serves only the higherend segments and offers only one version. We have the following theorem that determines which customers are served with multiple groups.

Proposition 2.4 With multiple groups and vertically differentiated goods, only the highest quality is provided and versioning is not profit maximizing. Consumers with $\theta \geq \theta_{m}$ are served, where $\theta_{m}$ is defined as

$$
\theta_{m}=\max _{\theta}\left\{k_{e} H\left(\theta^{*}\right), k_{n} H\left(\theta_{n-1}\right) ; n \in\{e+1, e+2, \cdots, N\}\right\}
$$

Proposition 2.4 differs from Proposition 2.1 because of multiple groups defined by their group taste $k_{n}$. The difference comes about because it may be more profitable for the monopolist to only serve segments higher than segment $e$, but at higher prices - prices that can be maintained because $k_{e}<k_{n}$ when $n$ is defined as in Proposition 2.4. Therefore, if $\theta_{m}=\theta^{*}$, then the optimal solution for the monopolist is the same as in Proposition 2.1 where there is only one group. Otherwise, the monopolist shrinks the market to increase profits by trading volume for price, although no versioning is implemented. From this we have the following corollary:

Corollary 2.1 With multiple groups and vertically differentiated goods, monopoly profits are at least as high as when there is only one consumer group.

To illustrate the ideas of a market with vertically differentiated goods, suppose there are only two segments, $k_{1}<k_{2}$, and for consumers in group $1, \theta \in\left[\theta_{0}, \theta_{1}\right)$,
and for consumers in group $2, \theta \in\left[\theta_{1}, \theta_{2}\right]$. Figure 2.3 illustrates the relationship between profits and the division between consumer groups. For Figure 2.3, we define


Figure 2.3: Profits under Vertical Differentiation with Undifferentiated Characteristics
$\theta_{k}=\left\{\theta: k_{2} H(\theta)=k_{1} H\left(\theta^{*}\right), \theta \in\left[\theta^{*}, \theta_{2}\right]\right\}$. Because $H(\theta)$ is monotonically decreasing for $\theta \in\left[\theta^{*}, \theta_{2}\right]$, there is unique $\theta_{k}$. From Proposition 2.4, we know that for $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, $\theta_{m}=\theta^{*}$; for $\theta_{1} \in\left(\theta^{*}, \theta_{k}\right), \theta_{m}=\theta_{1}$; and for $\theta_{1} \in\left[\theta_{k}, \theta_{2}\right], \theta_{m}=\theta^{*}$. Thus we have three regions to consider. When the monopolist does not consider versioning,

1. If $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, then the optimal profit is $\Pi_{1}^{*}=k_{2} q_{h} H\left(\theta^{*}\right)$.
2. If $\theta_{1} \in\left(\theta^{*}, \theta_{k}\right)$, then the optimal profit is $\Pi_{2}^{*}=k_{2} q_{h} H\left(\theta_{1}\right)$.
3. If $\theta_{1} \in\left[\theta_{k}, \theta_{2}\right]$, then the optimal profit is $\Pi_{3}^{*}=k_{1} q_{h} H\left(\theta^{*}\right)$.

Only in the second region does optimal profit depend on $\theta_{1}$. Suppose the monopolist considers versioning, which means a lower quality version $q_{l}$ is developed for group 1, and $q_{h}$ is only for group 2 .

1. If $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, then $\Pi^{*}\left(q_{h}, q_{l}\right)=q_{h} k_{2} H\left(\theta^{*}\right)-q_{l}\left[k_{2} H\left(\theta^{*}\right)-k_{1} H\left(\theta_{1}\right)\right] \leq \Pi_{1}^{*}$. And when $q_{l}=0, \Pi^{*}=\Pi_{1}^{*}$.
2. If $\theta_{1} \in\left(\theta^{*}, \theta_{k}\right)$, then $\Pi^{*}\left(q_{h}, q_{l}\right)=q_{h} k_{2} H\left(\theta_{1}\right)-q_{l}\left[k_{2} H\left(\theta_{1}\right)-k_{1} H\left(\theta^{*}\right)\right] \leq \Pi_{2}^{*}$. And when $q_{l}=0, \Pi^{*}=\Pi_{2}^{*}$.
3. If $\theta_{1} \in\left[\theta_{k}, \theta_{2}\right]$, then $\Pi^{*}\left(q_{h}, q_{l}\right)=q_{h} k_{2} H\left(\theta_{1}\right)+q_{l}\left[k_{1} H\left(\theta^{*}\right)-k_{2} H\left(\theta_{1}\right)\right] \leq \Pi_{3}^{*}$. And when $q_{l}=q_{h}, \Pi^{*}=\Pi_{3}^{*}$.

Given that profits are maximized in the first and second regions when $q_{l}=0$, and in the third region when $q_{l}=q_{h}$, none of the above support versioning. Referring back to the comparison between Proposition 2.1 and 2.4, in the first and third region of Figure 2.3 the monopoly one consumer group solution obtains. In the second region the monopolist serves only higher segments, and profits fall as $\theta^{*}$ rises.

### 2.5.2 Vertically Differentiated Goods with Group Related Characteristics

From our analysis with horizontally differentiated goods, we found that if all characteristics are valued equally by all consumers regardless of group, then versioning is not optimal although there are multiple groups in the market. However, with vertically differentiated goods versioning is optimal with multiple groups and upwardly aggregated preferences. To show this we assume that characteristics are "group related", while still allowing consumers to differ in their individual tastes. This means that if characteristics set $X_{1}$ is related to group 1, then not only consumers in group 1 have group taste $k_{1}$, all the higher taste groups also have group taste $k_{1}$ for the characteristics set $X_{1}$. More generally, if $X_{i}$ is related to group $i$, then consumers that
belong to a higher group have the group taste $k_{i}$ for the characteristics set $X_{i}$, and consumers who belong to a lower group place a zero value on $X_{i}$. The typical example is the Windows Operating Systems. Characteristics designed for the Home Edition are valued by both home users and power users while special features designed for Professional Edition are only valued by power users.

To more concretely define group related, let us suppose that the highest quality information good $X_{h}$ with quality index $q_{h}$ is made up of all group related characteristics $\left\{X_{1}, X_{2}, \cdots, X_{N}\right\}$ with relevant quality indices $\left\{q_{1}, q_{2}, \cdots, q_{N}\right\}$. We assume there is no overlap between any two sets of characteristics, which means $X_{i} \cap X_{j}=\emptyset$ for $i \neq j$. Then the utility function of a consumer in group $i$ for information good $X_{h}$ is

$$
U\left(q_{h}, \theta, k_{i}\right)=\theta \sum_{t=1}^{i} k_{t} q_{t}, \text { for } \theta \in\left[\theta_{i-1}, \theta_{i}\right)
$$

In this case, if versioning is optimal, then versioning strategies must be bottom-up. This means that the higher quality version includes all the characteristics related valued by lower groups. The monopolist's optimal price relationship between any two segments $i$ and $j(j>i)$ with neighboring versions of the information good is

$$
p_{j}=p_{i}+\theta \sum_{t=i+1}^{j} k_{t} q_{t}, \text { for } \theta \in\left[\theta_{j-1}, \theta_{j}\right)
$$

Using this relationship we have the following theorem:

Proposition 2.5 With multiple groups and vertically differentiated goods, if all the characteristics are group related, then

1. Using the definition of $\theta^{*}$ from Proposition 2.1 and segment e which contains $\theta^{*}$ from Proposition 2.4, versioning is optimal only if $\theta^{*}<\theta_{N-1}$.
2. The optimal versioning strategies are

- Any segment $i$ that is lower than $e(i<e)$ is not served,
- Segment e is provided the version with quality $\sum_{t=1}^{e} q_{t}$, the optimal price is $p_{e}=\theta^{*} \sum_{t=1}^{e} k_{t} q_{t}$,
- Any segment $j$ that is higher than $e(j>e)$ is provided a version with quality $\sum_{t=1}^{j} q_{t}$, and the optimal price is $p_{j}=\sum_{t=1}^{e} k_{t} q_{t} \theta^{*}+\sum_{t=e+1}^{j} k_{t} q_{t} \theta_{t-1}$.

Based on the strategies in Proposition 2.5, the total monopoly profit is $\Pi^{*}=$ $\sum_{t=1}^{e} k_{t} q_{t} H\left(\theta^{*}\right)+\sum_{t=e+1}^{N} k_{t} q_{t} H\left(\theta_{t-1}\right)$. More generally, for group related characteristics, when versioning is optimal, all segments that are higher than $e$ are covered.

We extend the two-group example from the previous sub-section to illustrate versioning strategies with group related characteristics. Here we have $q_{1}$ as the quality of group 1 related characteristics $X_{1}$ and $q_{2}$ as the quality of group 2 related characteristics $X_{2}$. Figure 2.4 illustrates the comparison of profits between versioning and not versioning (Because providing $X_{1}$ to both groups is never optimal, we do not show this case).

For Figure 2.4, we know that if $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, then segment 2 is not covered and if $\theta_{1} \in\left(\theta^{*}, \theta_{2}\right)$, then segment 2 is covered. From Proposition 2.5 , we have two regions to consider when the monopolist does not version:

1. If $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, then optimal profit is $\Pi_{1}^{*}=\left(k_{1} q_{1}+k_{2} q_{2}\right) H\left(\theta^{*}\right)$.
2. If $\theta_{1} \in\left(\theta^{*}, \theta_{2}\right)$, then optimal profit is $\Pi_{2}^{*}=\left(k_{1} q_{1}+k_{2} q_{2}\right) H\left(\theta_{1}\right)$.

Only in the second region does optimal profit depend on $\theta_{1}$. Suppose the monopolist considers versioning. Then,

1. If $\theta_{1} \in\left[\theta_{0}, \theta^{*}\right]$, then optimal profit is $\Pi_{1}^{V}=k_{2} q_{2} H\left(\theta^{*}\right)+k_{1} q_{1} H\left(\theta_{1}\right)<\Pi_{1}^{*}$, which means versioning is not optimal;
2. If $\theta_{1} \in\left(\theta^{*}, \theta_{2}\right)$, then optimal profit is $\Pi_{2}^{V}=k_{1} q_{1} H\left(\theta^{*}\right)+k_{2} q_{2} H\left(\theta_{1}\right)>\Pi_{2}^{*}$, which means versioning is optimal.


Figure 2.4: Profits under Vertical Differentiation with Group Related Characteristics

In this second region, where segment 2 is covered by a monopolist with a single version and some consumers in segment 1 also purchase this version, the gains from versioning (in this case, two versions) increase with the proportion of segment 1 that purchase the single version.

### 2.5.3 Versioning Costs and Segment Aggregation

Although variable costs of reproducing information goods are negligible, in certain cases the costs of providing an additional version can be significant. If versioning is optimal and versioning costs are significant, then the monopolist has an incentive to aggregate one or more segments to reduce the number of versions. We use vertical differentiation with group related characteristics discussed in the previous sub-section
to show how versioning costs may impact versioning strategies.
We know from Proposition 2.5 that without considering versioning costs, consumers with taste $\theta \in\left[\theta_{0}, \theta^{*}\right)$ are not served while consumers with taste $\theta \in\left[\theta^{*}, \theta_{N}\right]$ are served with versions according to which segments they belong to. Hence, the served segments are $e, e+1, \cdots, N$. Each segment $i(i \geq e)$ is provided with a different version, the total number of versions provided is $N-e+1$, which we denote as $v_{m}$.

To model this problem in a non-trivial way, we suppose that the optimal number of versions provided by the monopolist is at least two, $v_{m} \geq 2$. We define $C(\tau)$ to be the versioning cost function when $\tau$ versions are provided, $\tau \in\left\{1,2, \cdots, v_{m}\right\}$. To make the versioning costs general, we take $C(\tau)$ as non-decreasing, and $C(1)=0$ as when only one version is provided, no versioning costs are incurred. $C(\tau)$ is determined by technical and marketing parameters known to the monopolist.

We define $L(s)$ as the least profit loss function when $s$ segments are aggregated, $s \in\left\{0,1,2, \cdots, v_{m-1}\right\}$. It means when the monopolist aggregates $s$ segments, $L(s)$ is the least possible loss in total profit among all the possible aggregation of $s$ segments. $L(s)$ is strictly increasing with $s$ because the more segments are aggregated, the greater the profit loss. Also we have $L(0)=0$ because when there is no aggregation, there is no profit loss. Figure 2.5 illustrates the profit loss when aggregation occurs.

Now we can start to work through the algorithm for determining the optimal aggregation strategy. First, let us consider the profit loss when the monopolist aggregates one segment.

1. If segments $e$ and $e+1$ are aggregated, then the monopolist has two options:

- Merge segment $e$ into segment $e+1$. The best strategy is to close segment $e$ and serve segment $e+1$ with quality $\sum_{t=1}^{e+1} q_{t}$. Thus, the price for segment


Figure 2.5: Versioning Costs and Aggregation
$e+1$ is increased to $p_{e+1}=\theta_{e} \sum_{t=1}^{e+1} k_{t} q_{t}$, the profit loss is $\Delta \Pi_{e}=\left[H\left(\theta^{*}\right)-\right.$ $\left.H\left(\theta_{e}\right)\right] \sum_{t=1}^{e} k_{t} q_{t}$.

- Merge segment $e+1$ into segment $e$. The best strategy is to serve both segments $e$ and $e+1$ with quality $\sum_{t=1}^{e} q_{t}$. Thus, the price for both segments is $p_{e}=\theta^{*} \sum_{t=1}^{e} k_{t} q_{t}$, the profit loss is $\Delta \Pi_{e+1}=k_{e+1} q_{e+1}\left[H\left(\theta_{e}\right)-H\left(\theta_{e+1}\right)\right]$.

2. If segments $j$ and $j+1$ (where $j \in\{e+1, e+2, \cdots, N-1\}$ ) are aggregated, then the monopolist has two options:

- Merge segment $j$ into segment $j+1$. The best strategy is to serve segment $j+1$ with quality $\sum_{t=1}^{j+1} q_{t}$ with price $p_{j+1}=p_{j-1}+\left[k_{j} q_{j}+k_{j+1} q_{j+1}\right] \theta_{j}$. Thus, consumers in segment $j$ will purchase version for segment $j-1$ at
price $p_{j-1}$. The profit loss is $\Delta \Pi_{j}=k_{j} q_{j}\left[H\left(\theta_{j-1}\right)-H\left(\theta_{j}\right)\right]$.
- Merge segment $j+1$ into segment $j$. The best strategy is to serve both segments $j$ and $j+1$ with quality $\sum_{t=1}^{j} q_{t}$. Thus, the price for both segments is $p_{j}=p_{j-1}+k_{j} q_{j} \theta_{j-1}$, the profit loss is $\Delta \Pi_{j+1}=k_{j+1} q_{j+1}\left[H\left(\theta_{j}\right)-H\left(\theta_{j+1}\right)\right]$.

We define $\Delta \Pi_{m}=\min \left\{\Delta \Pi_{e}, \Delta \Pi_{e+1}, \cdots, \Delta \Pi_{N}\right\}$. Thus we get $L(1)=\Delta \Pi_{m}$, and segment $m$ is merged into its neighboring segment. From the versioning costs function $C(\tau)$, the costs saving due to the aggregation of one segment is $C\left(v_{m}\right)-C\left(v_{m-1}\right)$. If $C\left(v_{m}\right)-C\left(v_{m-1}\right)>L(1)$, which means costs saving due to aggregation is larger than profit loss, then aggregation is optimal, otherwise aggregation is not optimal.

Based on the same logic, our next theorem states our algorithm for the information goods monopolist to determine the optimal aggregation of segments when versioning costs are significant.

Proposition 2.6 The following algorithm determines an optimal solution for the information goods monopolist to aggregate its segments.

1. The following steps are designed for determining $L(s)$.

Denote $j, j \in\{e, e+1, \cdots, N\}$ as the segment to be merged, $\Delta \Pi_{j}$ as the relevant profit loss. Initiate a parameter $s=0$ and a vector $g(s)=0$.

Step 1. If $j=e$, then $\Delta \Pi_{j}=\left[H\left(\theta^{*}\right)-H\left(\theta_{e}\right)\right] \sum_{t=1}^{e} k_{t} q_{t}$.
If $e<j \leq N$, then $\Delta \Pi_{j}=\left[k_{j-1} q_{j-1}-k_{j} q_{j}\right] H\left(\theta_{j}\right)$.
Step 2. If $\Delta \Pi_{e}=\Delta \Pi_{e+1}=\cdots=\Delta \Pi_{N}=0$, then stop.
Otherwise, $s=s+1, g(s)=\left\{j: \min \left(\Delta \Pi_{j}\right.\right.$, for $j \in\{e, e+1, \cdots, N\}$ and $\Delta \Pi_{j}>$ $0)\}, L(s)=\Delta \Pi_{g(s)}$.

Step 3. If $j=e$, then set $k_{j+1} q_{j+1}=k_{j+1} q_{j+1}+k_{j} q_{j}, k_{j} q_{j}=k_{j+1} q_{j+1}, H\left(\theta^{*}\right)=$ $H\left(\theta_{e}\right) ;$

If $e<j<N$, then set $k_{j+1} q_{j+1}=k_{j+1} q_{j+1}+k_{j} q_{j}, k_{j} q_{j}=k_{j+1} q_{j+1}, H\left(\theta_{j-1}\right)=$ $H\left(\theta_{j}\right)$;

If $j=N$, then set $H\left(\theta_{j-1}\right)=H\left(\theta_{j}\right)$;
Return to step 1.

## 2. Determine the optimal aggregation arrangement.

If for all $s \in\{1,2, \cdots, N-1\}, C\left(v_{m}\right)-C\left(v_{m}-s\right) \leq L(s)$, then aggregation is strictly sub-optimal.

Otherwise, we set $l=\max \left\{s: C\left(v_{m}\right)-C\left(v_{m}-s\right)-L(s)\right.$, for $s \in\{1,2, \cdots, N-$
1\}\}. The optimal aggregation arrangement is
a. $l$ segments will be merged.
b. The merged segments are : $g(1), g(2), \cdots, g(l)$.

### 2.6 Conclusions

In this paper we investigated conditions that determine when an information goods monopolist chooses to implement versioning strategies. We showed that versioning strategies are implemented only when different groups of consumers can be clearly defined. In other words, versioning cannot be used to segment a market, rather, versioning strategies must fit the existing market segments. Our optimal versioning strategies are in accordance with Shapiro and Varian's (1999) suggestion that versions should be designed to accentuate the differences between groups in their tastes. We demonstrated that in a horizontally differentiated market, if there is no crosspurchasing, then it is optimal to provide one version for each segment. Otherwise, the monopolist shrinks the market in the lower-end segment to protect profits in the higher-end segments. As a special case of horizontal differentiation, vertical differen-
tiation is not optimal when characteristics of information goods are undifferentiated to different groups. To explain multiple versions in vertical differentiation, we introduced group related characteristics and showed that a lower version is provided only when all higher segments are covered. We further investigated how versioning costs impact versioning strategies and designed an algorithm to determine the optimal aggregation of segments when versioning costs are significant.

Our contribution lies in two aspects. First, although most of the previous research focuses on vertical differentiation, we treat information goods as a combination of characteristics so that we link horizontal differentiation with vertical differentiation. In our model, we made a transition from horizontal differentiation to vertical differentiation and showed how versioning strategies change during the transition. Second, we introduced a group taste to successfully explain the existence of multiple versions. Much of the previous research (Bhargava and Choudary 2001, Jones and Mendelson 2005 , etc.) using linear utility without group tastes found only one version is optimal. Other assumptions such as network externalities (Jing 2002), non-linear utility (Ghose and Sundararajan 2005, Bhargava and Choudhary 2007) and consumers' outside options (Chen and Seshadri 2007) have been needed to show that multiple versions can be optimal. We kept the linear (multiplicative) utility function form because it is easy to understand and convenient for empirical estimation.

There are several limitations in our modeling framework. Our modeling results are based on assumptions such as linear utility and a positive relationship between group taste and individual taste. In addition, the introduction of group related characteristics explains versioning in vertical differentiation, but we recognize that it is a special preference structure. Future research may relax some of these assumptions, and can address additional issues. Previous research demonstrates empirically that in the Internet-based book market used-book sales cannibalize new-book purchases,
increasing welfare (Ghose, Smith and Telang 2006), and there is no similar empirical research on versioning of information goods. In addition, our paper studies monopoly versioning strategies, and we expect that competition (Jones and Mendelson 2005, Wei and Nault 2006, etc.) has the potential to make a significant difference in these strategies.

## Chapter 3

## ESSAY TWO: EXPERIENCE INFORMATION GOODS: TECHNOLOGY DIFFUSION AND VERSIONING STRATEGIES

### 3.1 Introduction

Development of information technology has made information goods popular. Characterized by large sunk costs of development, and negligible costs of reproduction and distribution, information goods are distinct from traditional physical goods (Shapiro and Varian, 1999). Products such as computer software, online contents and digitalized music, movies and books fall into this category (Jones and Mendelson, 2005).

Interestingly, most information goods show features of experience goods (Chellappa and Shivendu, 2005). Different from search goods whose quality can be determined before purchase by actual inspection, quality of experience goods is realized only after consumption (Nelson, 1970, Wilde, 1981). For example, it is difficult for a software producer to describe all the features of the software to communicate its true quality before purchase. Even if the producer includes all the features in the manual, consumers may not appreciate the true value of the software until they actually use it. The more a consumer actually experiences the software, the better she knows its true value.

The concept of experience goods is originally due to Nelson (1970), who contrasts an experience good with a search good. When a new product or service is introduced, potential users typically have imperfect information about the product's attributes,
even though these characteristics may be important to them. A critical source of information about the product is actual experience with it (Nelson, 1970, Wilde, 1981, and Shapiro, 1983). Shapiro (1983) examines pricing of experience goods under both optimistic and pessimistic cases with a multi-period model. He finds that when consumers are optimistic, producers can milk the reputation via a declining price path followed by a jump to a terminal price. And when consumers are pessimistic, producers have to build up the reputation by using a low introductory price followed by a higher regular price. Kim (1992) proposes a two-period model to investigate pricing strategies for experience goods according to the credibility of price precommitment of the producer. His model shows that if the producer can credibly precommit, it is optimal to set a high price in the first period and a low price for the second period. If the price precommitment is not credible, then the results reverse. Other research about experience goods includes Riordan (1986) who constructs a model of monopolistic competition of experience goods, Liebeskind and Rumelt (1989) who analyze market for goods with uncertain product quality, and Villas-Boas who (2006) considers the dynamic competition implications of experience goods. Previous research mostly focuses on non-durable experience goods with repeat purchases. However, information goods are durable goods and consumers buy at most one unit of the good.

In order to communicate the true quality of their information goods, some producers distribute free demonstration versions, and others even send out free trial versions. Recently, Microsoft has adopted a versioning and upgrading strategy for the delivery of Windows Vista. Windows Vista has four versions: Home Basic, Home Premium, Business and Ultimate. Windows Vista anytime upgrade (http://www.microsoft.com) allows consumers to upgrade from a lower version to any of the higher versions by purchasing the corresponding upgrade license.

Without upgrades, providing four versions of Windows Vista is normally referred
to as "versioning". ${ }^{1}$ Versioning is second degree price discrimination: "offer a product line and let users choose the version of the product most appropriate for them" (Shapiro and Varian, 1999). Thus, information goods producers can easily provide vertically differentiated products, thereby segmenting the market to maximize profit (Wei and Nault, 2005). To implement versioning, producers usually produce a flagship version and disable some functionalities to generate subversions. All versions are delivered to separate targeted market segments.

Versioning and pricing strategies of information goods have been studied in various contexts such as network externalities (Jing, 2002), competition (Jones and Mendelson, 2005, Wei and Nault, 2006) and anti-piracy (Wu, Chen and Anandalingam, 2003). They all reach the conclusion that vertical differentiation is not optimal without certain constraints, consistent with Bhargava and Choudhary (2001). Nault (1997) finds that inter-organizational information systems (IOS)-based quality differentiation could effectively separate consumers and reduce competition in a duopoly. Sundararajan (2004) shows that fixed-fee and usage-based pricing can be used together to maximize profits.

Combining experience and information goods together, Chellappa and Shivendu (2005) model pricing and sampling strategies for digital experience goods in vertically segmented markets to manage piracy. They find that piracy losses are more severe for products that do not live up to their hype rather than for those that have been undervalued in the market, thus requiring a greater deterrence investment for the former. To the best of our knowledge, our research is the first to combine experience information goods and versioning strategies.

In this paper, we analyze versioning strategies of experience information goods

[^6]under two different payment arrangements: a lump sum payment arrangement and a periodic license fee arrangement. With lump sum payments, consumers pay once to have ownership of the good. Thus, the monopolist can offer different versions and provide upgrades to maximize profits. Allowing upgrades makes versioning more complicated. For versioning with upgrades, we examine how a monopolist decides prices for different versions and for upgrades, and how upgrade strategies impact consumers' choices and subsequent versioning strategies. Using a two-stage model where consumers can purchase in the first stage and those who chose a lower quality can upgrade in the second stage, we show that versioning is a useful strategy for the monopolist to fully reveal the quality of its goods. To maximize profits, the monopolist generates vertically differentiated versions of its information goods, and designs mechanisms for consumers that purchase lower quality versions to upgrade to the high quality version after having experienced the low quality version and learned its true quality. We find that if consumers have homogeneous initial expectations about quality, then the monopolist's optimal pricing strategy is to drive all the consumers who buy the low quality version in the first stage to upgrade to the high quality version in the second stage. In this way, consumers that upgrade pay a tax for learning. When consumers have heterogeneous expectations, we find that if consumers are pessimistic, then the monopolist's optimal pricing still drives all consumers to upgrade. However, if consumers are optimistic, then under certain conditions, only some consumers that purchased the low quality version in the first stage upgrade. With periodic license fees, consumers are charged for the usage of the good for a period of time. Thus, the monopolist may provide different versions in different periods to attract consumers. We find that for pessimistic consumers, versioning strategies are implemented by the monopolist only if some consumers are already informed of the true quality. While for optimistic consumers, the monopolist provides only one version.

The rest of the paper is organized as follows. We set up our notation and assumptions in Section 3.2. In Section 3.3 we present a two-stage, two-version model of experience information goods with both homogeneous and heterogeneous consumer expectations with lump sum payments. We explore versioning strategies with periodic license fees in Section 3.4. Discussions and future research are included in Section

## 3.5 .

### 3.2 Notation and Assumptions

Our models involve a monopoly producer and heterogeneous consumers. Each consumer purchases either zero or one unit of the good per period. We denote an individual consumer's taste as $\theta$ which is normalized to be in the interval $[0,1]$. We assume that $\theta$ has probability density and cumulative density functions $f(\theta)$ and $F(\theta)$ to set the population to unity. The density is strictly positive over its support and continuously differentiable. Following Bhargava and Choudhary (2001), Jing (2002) and Sundararajan (2004), we make the following assumption about the distribution of consumer tastes:

Assumption 3.1 The reciprocal of the hazard function, $\frac{1-F(\theta)}{f(\theta)}$, is non-increasing in $\theta .{ }^{2}$

We denote the true quality as $q \in[\underline{q}, \bar{q}]$, where $\bar{q}$ is the highest possible quality and $\underline{q}$ is the lowest quality that reasonably can be used. In the lump sum payment arrangement, consumers get utility from ownership of the goods. In the periodic license fee arrangement, consumers receive utility only for the usage of the goods during the specific period. We take a consumer's utility to be multiplicative in taste and quality.

[^7]In that sense, a given consumer has constant marginal value for quality. The information good produced by the monopolist is an experience good in our model, which means before purchasing and using, an uninformed consumer only has incomplete information about quality. The expected quality of the information good before use is denoted by $E(q, \theta)$. After using the good, the informed consumer learns the true quality. This is our second assumption:

Assumption 3.2 For informed consumers, $U(q, \theta)=\theta E(q, \theta)$. For uninformed consumers, $U(q, \theta)=\theta q$.

We denote price of the information good as $p$. Consumers select their favorite good to maximize their surplus $U(q, \theta)-p$.

We assume a monopolist has developed a high quality information good, called the flagship version, which can be easily degraded to generate multiple vertically differentiated sub-versions. In our models, we assume a monopolist provides only two vertically differentiated versions of the information good, high and low, with accordant quality levels $q_{h}$ and $q_{l}$ where $q_{l}<q_{h}$, and prices for these two versions are $p_{h}$ and $p_{l}$, respectively. The high quality version $q_{h}$ is developed first, and the subsequent degraded version $q_{l}$ is generated through versioning. We denote $\theta_{l}$ as the consumer that is indifferent between purchasing $q_{l}$ and not purchasing. Thus, we have

$$
\begin{equation*}
p_{l}=U\left(q_{l}, \theta_{l}\right) \tag{3.1}
\end{equation*}
$$

We denote $\theta_{h}$ as the consumer that is indifferent between purchasing $q_{h}$ and purchasing $q_{l}$. Thus, we have

$$
\begin{equation*}
U\left(q_{h}, \theta_{h}\right)-p_{h}=U\left(q_{l}, \theta_{h}\right)-p_{l} . \tag{3.2}
\end{equation*}
$$

### 3.3 Versioning Strategies with Lump Sum Payments

With lump sum payments, consumers take ownership of the information good after purchasing. This payment arrangement applies to sales of most information goods such as desktop operation systems (Windows series), digital music, movies and pictures, etc. For experience goods that consumers' expectations of quality are different before and after purchase, the monopolist also provides upgrades for consumers who initially purchase the low version to switch to the high version.

For two vertically differentiated versions of an information good with qualities $q_{h}$ and $q_{l}$, we assume that the monopolist provides a mechanism for the consumer to upgrade from the lower version $q_{l}$ to the higher version $q_{h}$. Let $p_{u}$ be the price set by the monopolist for consumers to upgrade from $q_{l}$ to $q_{h}$. We have the following lemma for the prices relationship of different versions and the upgrade.

Lemma 3.1 For a monopolist that offers upgrades, $p_{h}-p_{l} \leq p_{u}$.

Proof. If $p_{h}-p_{l}>p_{u}$, then any consumer who wants to buy $q_{h}$ is better off buying $q_{l}$ instead and upgrading to $q_{u}$.

The monopolist can easily set the upgrade price equal to the price difference of the high and low version without loss of profits. Thus for a profit maximizing monopolist, the price set for an upgrade is not lower than the price difference of the high and low quality versions. In the following, we analyze versioning and upgrading strategies of a monopolist when the consumers' expectations for quality are homogeneous or heterogeneous.

### 3.3.1 A Model with Homogeneous Expectations

Our setting is a two-stage model where consumers choose whether to purchase a good in the first stage, and consumers that chose to purchase a low quality good in the
first stage have the opportunity to upgrade in the second stage. Following Shapiro (1983), we assume for the moment that initial expectations for either version do not depend on consumer taste: $E(q, \theta)=E(q)$. Let the expected qualities for the goods be $E\left(q_{l}\right)=R_{l}$ and $E\left(q_{h}\right)=R_{h}$.

In the first stage, consumers make their purchasing decision based on the expected qualities $R_{h}$ and $R_{l}$. As illustrated in Figure 3.1, the market is divided into three segments. Consumers with tastes lower than $\theta_{l}$ do not purchase, with taste between $\theta_{l}$ and $\theta_{h}$ purchase $q_{l}$ (with expected quality $R_{l}$ in this stage), and with tastes higher than $\theta_{h}$ purchase $q_{h}$ (with expected quality $R_{h}$ in this stage). Consequently, the


Figure 3.1: Market Segmentation of Information Goods
demand for the high quality version $q_{h}$ is $1-F\left(\theta_{h}\right)$ and the demand for the low quality version $q_{l}$ is $F\left(\theta_{h}\right)-F\left(\theta_{l}\right)$.

In the second stage, after purchasing and experiencing the good, consumers know the true quality of the goods. Here we assume that consumers who have purchased and experienced either the high or low quality version know the true quality levels of both versions. Some of the consumers who purchased the low version may upgrade to the high version. We denote $\theta_{u}$ as the consumer that is indifferent between upgrading
or not when the price for the upgrade is $p_{u}$. Thus, we have

$$
\begin{equation*}
U\left(q_{h}, \theta_{u}\right)-p_{u}=U\left(q_{l}, \theta_{u}\right) \tag{3.3}
\end{equation*}
$$

As indicated in Figure 3.2, consumers with tastes between $\theta_{u}$ and $\theta_{h}$ upgrade from the low quality to the high quality version and the demand for the upgrade is $F\left(\theta_{h}\right)-F\left(\theta_{u}\right)$. For upgrading to be a feasible strategy for the monopolist, some


Figure 3.2: An Upgrade Model
consumers must choose to upgrade. The condition that makes upgrading feasible is our second lemma.

Lemma 3.2 For consumers to upgrade, it is necessary that $R_{h}-R_{l}<q_{h}-q_{l}$.

Proof. To allow a positive number of customers to upgrade from the low quality version to the high quality version, we must have $\theta_{u}<\theta_{h}$. From (3.3), we have $\theta_{u}=\frac{p_{u}}{q_{h}-q_{l}}$. And from (3.2), we have $\theta_{h}=\frac{p_{h}-p_{l}}{R_{h}-R_{l}}$. Thus we get $\frac{p_{u}}{q_{h}-q_{l}}<\frac{p_{h}-p_{l}}{R_{h}-R_{l}}$. From Lemma 3.1, we have $p_{u}>p_{h}-p_{l}$, so we have $1<\frac{p_{u}}{p_{h}-p_{l}}<\frac{q_{h}-q_{l}}{R_{h}-R_{l}}$. Thus, we get $R_{h}-R_{l}<q_{h}-q_{l}$.

Lemma 3.2 says that the difference in the true quality of the two versions, learnt after experience, must be greater than the difference in expected quality for any
consumers that chose the low quality good in the first stage to upgrade in the second stage.

The monopolist sets prices $p_{h}, p_{l}$ and $p_{u}$ to maximize profits from both stages. We denote the discount rate for profits in the second stage (as compared to the first stage) as $\delta \in(0,1]$. $\delta$ depends on the time gap between the two stages and the interest rate. The discount rate can also be written as $\delta=e^{-r t}$, where $r$ is the interest rate and $t$ is the time between stages.

From the demand analysis in the previous section, monopoly profits are ${ }^{3}$

$$
\begin{aligned}
\max _{\theta_{h}, \theta_{l}, \theta_{u}} \Pi= & {\left[1-F\left(\theta_{h}\right)\right]\left[\theta_{l} R_{l}+\left[R_{h}-R_{l}\right] \theta_{h}\right]+\left[F\left(\theta_{h}\right)-F\left(\theta_{l}\right)\right] \theta_{l} R_{l} } \\
& +\delta\left[F\left(\theta_{h}\right)-F\left(\theta_{u}\right)\right]\left[q_{h}-q_{l}\right] \theta_{u}, \ni \theta_{l} \leq \theta_{u}
\end{aligned}
$$

where $\theta_{u} \leq \theta_{h}$ is inferred from the proof of Lemma 3.1. The Lagrangian for this problem is $\mathcal{L}=\Pi+\lambda\left(\theta_{u}-\theta_{l}\right)$. Because $\theta_{h}, \theta_{u}$ and $\theta_{l}$ are positive, the first-order (Kuhn-Tucker) conditions are

$$
\begin{gather*}
\mathcal{L}_{\theta_{h}}=\left[1-F\left(\theta_{h}\right)\right]\left[R_{h}-R_{l}\right]-f\left(\theta_{h}\right)\left[R_{h}-R_{l}\right] \theta_{h}+\delta f\left(\theta_{h}\right)\left[q_{h}-q_{l}\right] \theta_{u}=0,  \tag{3.4}\\
\mathcal{L}_{\theta_{u}}=\delta\left[q_{h}-q_{l}\right]\left[F\left(\theta_{h}\right)-F\left(\theta_{u}\right)-f\left(\theta_{u}\right) \theta_{u}\right]+\lambda=0,  \tag{3.5}\\
\mathcal{L}_{\theta_{l}}=\left[1-F\left(\theta_{l}\right)\right] R_{l}-f\left(\theta_{l}\right) \theta_{l} R_{l}-\lambda=0, \quad \text { and }  \tag{3.6}\\
\mathcal{L}_{\lambda}=\theta_{u}-\theta_{l} \geq 0, \text { if }>, \lambda=0 . \tag{3.7}
\end{gather*}
$$

For (3.7), we first assume that the constraint is not binding, which means $\theta_{u}>\theta_{l}$ and $\lambda=0$. Substituting back to (3.5) and (3.6), we have $\theta_{u}=\frac{F\left(\theta_{h}\right)-F\left(\theta_{l}\right)}{f\left(\theta_{u}\right)}$ and $\theta_{l}=\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}$. From Assumption 3.1 that the hazard function is non-decreasing, we get $\theta_{u}<\theta_{l}$, as illustrated in Figure 3.3 below. This violates our constraint (3.7). Therefore, we must

[^8]

Figure 3.3: A Market When Everyone Upgrades
conclude that the constraint is binding, which means $\theta_{u}=\theta_{l}$ and $\lambda>0$. Substituting back to (3.4), (3.5) and (3.6), we have

$$
\begin{equation*}
\theta_{h}-\delta \frac{q_{h}-q_{l}}{R_{h}-R_{l}} \theta_{u}=\frac{1-F\left(\theta_{h}\right)}{f\left(\theta_{h}\right)} \tag{3.8}
\end{equation*}
$$

and

$$
\begin{equation*}
\theta_{u}=\theta_{l}=\frac{R_{l}}{R_{l}+\delta\left[q_{h}-q_{l}\right]} \frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}+\frac{\delta\left[q_{h}-q_{l}\right]}{R_{l}+\delta\left[q_{h}-q_{l}\right]} \frac{F\left(\theta_{h}\right)-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)} . \tag{3.9}
\end{equation*}
$$

From $\theta_{u}=\theta_{l}$, we find that all the consumers who buy the low quality version in the first stage upgrade to the high quality version in the second stage.

We can compare our solution with the case when an upgrade is not offered by the monopolist. In the case of no upgrade, the monopolist's profit function is

$$
\max _{\theta_{h}, \theta_{l}} \Pi=\left[1-F\left(\theta_{h}\right)\right]\left[\theta_{l} R_{l}+\left[R_{h}-R_{l}\right] \theta_{h}\right]+\left[F\left(\theta_{h}\right)-F\left(\theta_{l}\right)\right] \theta_{l} R_{l}
$$

The first-order conditions with respect to $\theta_{h}$ and $\theta_{l}$ generate

$$
\theta_{h}=\frac{1-F\left(\theta_{h}\right)}{f\left(\theta_{h}\right)} \text { and } \theta_{l}=\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}
$$

The non-decreasing (monotone) hazard function means that there is unique solution for $\theta=[1-F(\theta)] / f(\theta)$, which we indicate as $\theta^{*}$. Thus we have $\theta_{h}=\theta_{l}=\theta^{*}$. Consistent with literature in versioning (Bhargava and Choudhary, 2001; Jones and Mendelson, 2005) in absence of offering an upgrade, there is no need for the monopolist to version its information good.

Now let us return to the solutions when the monopolist offers upgrades. From (3.8) we know $\theta_{h}-\delta \theta_{u}\left[q_{h}-q_{l}\right] /\left[R_{h}-R_{l}\right]<\theta_{h}$, thus (3.8) generates the solution $\theta_{h}>\theta^{*}$. From (3.9) we know

$$
\frac{R_{l}}{R_{l}+\delta\left[q_{h}-q_{l}\right]} \frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}+\frac{\delta\left[q_{h}-q_{l}\right]}{R_{l}+\delta\left[q_{h}-q_{l}\right]} \frac{F\left(\theta_{h}\right)-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}<\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}
$$

thus we have $\theta_{l}<\theta^{*}$. These solutions are illustrated in Figure 3.4 below. The result


Figure 3.4: Solutions of the Two-version Situation
is that $\theta_{l}<\theta^{*}<\theta_{h}$, meaning that for experience information goods with upgrades, the monopolist adopts versioning to maximize profits. This implies that an upgrade strategy is profitable.

However, the fact that all the consumers who purchase the low quality version in the first stage upgrade to the high quality version in the second stage indicates that the low quality version serves as a bridge for consumers who purchase the low quality version to learn the true quality of the information good. In the final stage, all the consumers adopt the high quality version. Thus, consumers who purchase the low quality good in the first period pay a $\operatorname{tax} p_{l}+p_{u}-p_{h}$ for learning through experience. To summarize, we have the following proposition.

Proposition 3.1 If all consumers have homogeneous initial expectations about the goods, then the monopolist's optimal strategy involves versioning and upgrades; all consumers who purchased the low quality version in the first stage upgrade to the high quality version in the second stage.

We notice that for homogeneous consumer expectations, demand for $q_{h}$ and $q_{l}$ and upgrade does not depend on whether consumers are optimistic $\left(R_{h}>q_{h}\right.$ and $R_{l}>q_{l}$ ) or pessimistic ( $R_{h}<q_{h}$ and $R_{l}<q_{l}$ ). However, the optimal prices $p_{h}$ and $p_{l}$ are directly related to $R_{h}$ and $R_{l}$. The higher the initial expectations are, the higher are the prices for both versions thus the higher the profits. Hence, positive advertisement that raises expectations can effectively increase profits of the sales of experience information goods. Because price and demand from upgrades is independent of initial quality expectations, so are the profits from upgrades. The only way to increase profits is to shorten the learning process, thus increasing the discount factor $\delta$. Online training may work toward this end.

### 3.3.2 A Model with Heterogeneous Expectations

In this section we relax the assumption that the initial expectations of quality are homogeneous and assume that the initial expected quality of the goods depends on
consumer taste $\theta$, which means before experience, $E\left(q_{h}\right)=R_{h}(\theta)$ and $E\left(q_{l}\right)=R_{l}(\theta)$. We normally would expect that a higher taste consumer has more accurate expectations of the true quality of the goods. Thus we make the following assumption:

Assumption 3.3 As $\theta$ increases, the gap between expected and actual quality decreases: $\forall \theta_{i}>\theta_{j},\left|R\left(\theta_{i}\right)-q_{i}\right|<\left|R\left(\theta_{j}\right)-q_{j}\right|$.

From Lemma 3.2 we know that in order for some consumers to upgrade from the low to high quality version, the real quality gap between the two versions must be nondecreasing with consumer taste. We make the following assumption to extend this result:

Assumption 3.4 The difference between the expected qualities of the high and low versions is nondecreasing in $\theta$, which is

$$
\frac{d\left[R_{h}(\theta)-R_{l}(\theta)\right]}{d \theta} \geq 0
$$

This assumption is reasonable because lower taste consumers are normally not as sensitive to differentiating quality levels.

Substituting the same price relationships (3.1), (3.2) and (3.3) into the profit function, we have

$$
\begin{aligned}
\max _{\theta_{h}, \theta_{l}, \theta_{u}} \Pi= & {\left[1-F\left(\theta_{h}\right)\right]\left[\theta_{l} R_{l}\left(\theta_{l}\right)+\left[R_{h}\left(\theta_{h}\right)-R_{l}\left(\theta_{h}\right)\right] \theta_{h}\right]+\left[F\left(\theta_{h}\right)-F\left(\theta_{l}\right)\right] \theta_{l} R_{l}\left(\theta_{l}\right) } \\
& +\delta\left[F\left(\theta_{h}\right)-F\left(\theta_{u}\right)\right]\left[q_{h}-q_{l}\right] \theta_{u}, \ni \theta_{l} \leq \theta_{u} .
\end{aligned}
$$

The Lagrangian for this problem is $\mathcal{L}=\Pi+\lambda\left[\theta_{u}-\theta_{l}\right]$. Because $\theta_{h}, \theta_{u}$ and $\theta_{l}$ are positive, the first-order (Kuhn-Tucker) conditions are

$$
\mathcal{L}_{\theta_{h}}=-f\left(\theta_{h}\right)\left[R_{h}\left(\theta_{h}\right)-R_{l}\left(\theta_{h}\right)\right] \theta_{h}+\left[1-F\left(\theta_{h}\right)\right]\left[R_{h}\left(\theta_{h}\right)-R_{l}\left(\theta_{h}\right)\right]
$$

$$
\begin{gather*}
+\left[1-F\left(\theta_{h}\right)\right]\left[R_{h}^{\prime}\left(\theta_{h}\right)-R_{l}^{\prime}\left(\theta_{h}\right)\right] \theta_{h}+\delta f\left(\theta_{h}\right)\left[q_{h}-q_{l}\right] \theta_{u}=0,  \tag{3.10}\\
\mathcal{L}_{\theta_{u}}=\delta\left[q_{h}-q_{l}\right]\left[F\left(\theta_{h}\right)-F\left(\theta_{u}\right)-f\left(\theta_{u}\right) \theta_{u}\right]+\lambda=0,  \tag{3.11}\\
\mathcal{L}_{\theta_{l}}=\left[1-F\left(\theta_{l}\right)\right]\left[R_{l}\left(\theta_{l}\right)+\theta_{l} R_{l}^{\prime}\left(\theta_{l}\right)\right]-f\left(\theta_{l}\right) \theta_{l} R_{l}\left(\theta_{l}\right)-\lambda=0, \quad \text { and }  \tag{3.12}\\
\mathcal{L}_{\lambda}=\theta_{u}-\theta_{l} \geq 0, \text { if }>, \lambda=0 . \tag{3.13}
\end{gather*}
$$

From (3.10) we get

$$
\theta_{h}=\frac{1-F\left(\theta_{h}\right)}{f\left(\theta_{h}\right)}\left[1+\theta_{h} \frac{R_{h}^{\prime}\left(\theta_{h}\right)-R_{l}^{\prime}\left(\theta_{h}\right)}{R_{h}\left(\theta_{h}\right)-R_{l}\left(\theta_{h}\right)}\right]+\delta \theta_{u} \frac{q_{h}-q_{l}}{R_{h}\left(\theta_{h}\right)-R_{l}\left(\theta_{h}\right)}>\frac{1-F\left(\theta_{h}\right)}{f\left(\theta_{h}\right)} .
$$

Thus we get $\theta_{h}>\theta^{*}$, where the latter is the indifferent consumer for a monopolist with a single good. From (3.13), we first assume that the constraint is not binding, which means $\theta_{u}>\theta_{l}$ and $\lambda=0$. Substituting back to (3.11) and (3.12), we have $\theta_{u}=\frac{F\left(\theta_{h}\right)-F\left(\theta_{u}\right)}{f\left(\theta_{u}\right)}<\frac{1-F\left(\theta_{u}\right)}{f\left(\theta_{u}\right)}$, from which we get $\theta_{u}<\theta^{*}$, and $\theta_{l}=\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}\left[1+\frac{R_{l}^{\prime}\left(\theta_{l}\right)}{R_{l}\left(\theta_{l}\right)} \theta_{l}\right]$.

To solve $\theta_{l}$, we have to consider two situations. The first is when all the consumers are pessimistic, which means that $R_{l}(\theta)<q_{l}$ and $R_{h}(\theta)<q_{h}$. From Assumption 3.4, we have $R_{l}^{\prime}(\theta)>0$. Hence,

$$
\theta_{l}=\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}\left[1+\frac{R_{l}^{\prime}\left(\theta_{l}\right)}{R_{l}\left(\theta_{l}\right)} \theta_{l}\right]>\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}
$$

which results in $\theta_{l}>\theta^{*}>\theta_{u}$. This violates our constraint (3.13). Therefore, the constraint must be binding, which means $\theta_{l}=\theta_{u}$. This indicates that all consumers who purchased the low version upgrade to the high version, which is identical to the case of homogeneous consumer expectations. Thus, we have the following proposition.

Proposition 3.2 If consumers have heterogeneous initial expectations and are pessimistic, then all the consumers who purchased the low quality version in the first stage upgrade to the high quality version in the second stage.

In the second situation, we assume that all the consumers are optimistic, which means that $R_{l}(\theta)>q_{l}$ and $R_{h}(\theta)>q_{h}$. From Assumption 3.4, we get $R_{l}^{\prime}(\theta)<0$. So
we have

$$
\theta_{l}=\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)}\left[1+\frac{R_{l}^{\prime}\left(\theta_{l}\right)}{R_{l}\left(\theta_{l}\right)} \theta_{l}\right]<\frac{1-F\left(\theta_{l}\right)}{f\left(\theta_{l}\right)} .
$$

Thus we get $\theta_{l}<\theta^{*}$. Since $\theta_{u}$ is also less than $\theta^{*}$, we require an additional condition to determine the relative magnitude of $\theta_{l}$ and $\theta_{u}$. We derive this condition in the following proposition.

Proposition 3.3 If consumers have heterogeneous initial expectations and are optimistic, then a sufficient condition for only some consumers who purchased the low quality version in the first stage to upgrade is that $\forall \theta,\left|\frac{R_{l}^{\prime}(\theta)}{R_{l}(\theta)} \theta\right|>1$.

Proof. Because $\theta_{h}>\theta^{*}$ and $\theta_{l}, \theta_{u}<\theta^{*}$, it is straightforward that $\theta_{l}, \theta_{u}<\theta_{h}$. To get $\theta_{l}<\theta_{u}$, it is equivalent that

$$
\frac{1-F(\theta)}{f(\theta)}\left[1+\frac{R_{l}^{\prime}(\theta)}{R_{l}(\theta)} \theta\right]<\frac{F\left(\theta_{h}\right)-F(\theta)}{f(\theta)}
$$

Simplifying,

$$
-\frac{R_{l}^{\prime}(\theta)}{R_{l}(\theta)} \theta>\frac{1-F\left(\theta_{h}\right)}{1-F(\theta)} .
$$

For $\theta<\theta_{h}$, we have $\frac{1-F\left(\theta_{h}\right)}{1-F(\theta)}<1$. Because $R_{l}^{\prime}(\theta)<0$, we have $-\frac{R_{l}^{\prime}(\theta)}{R_{l}(\theta)} \theta>1$. Thus, $-\frac{R_{l}^{\prime}(\theta)}{R_{l}(\theta)} \theta>\frac{1-F\left(\theta_{h}\right)}{1-F(\theta)}$, which implies $\theta_{l}<\theta_{u}$.

Under the condition in Proposition 3.3, some consumers who purchased the low quality version choose to upgrade to the high quality version, and others continue to use the low version.

### 3.3.3 Endogenized Qualities

In our model, we treat qualities as exogenous variables. Here we discuss the optimal quality levels a monopolist chooses when qualities of the information good are endogenized. To endogenize qualities of both versions, we denote the development cost
function $C(q)$ and versioning cost function $V(q)$. We assume both costs are not decreasing in quality, which means $C^{\prime}(q) \geq 0$ and $V^{\prime}(q) \geq 0$ for $q \in[\underline{q}, \bar{q}]$. Let us denote the optimal indifferent consumers as $\theta_{h}^{*}\left(q_{h}, q_{l}\right), \theta_{l}^{*}\left(q_{h}, q_{l}\right)$ and $\theta_{u}^{*}\left(q_{h}, q_{l}\right)$, respectively.

When qualities are endogenized, the monopoly profits are

$$
N\left(q_{h}, q_{l}\right)=\Pi\left(\theta_{h}^{*}\left(q_{h}, q_{l}\right), \theta_{l}^{*}\left(\dot{q}_{h}, q_{l}\right), \theta_{u}^{*}\left(q_{h}, q_{l}\right)\right)-C\left(q_{h}\right)-V\left(q_{l}\right) .
$$

Dropping the arguments $\left(q_{h}, q_{l}\right)$ of $\Pi$ for convenience, the first order conditions with respect to $q_{h}$ generate

$$
\begin{equation*}
\frac{\partial \Pi}{\partial \theta_{h}^{*}} \frac{\theta_{h}^{*}}{q_{h}}+\frac{\partial \Pi}{\partial \theta_{l}^{*}} \frac{\theta_{l}^{*}}{q_{h}}+\frac{\partial \Pi}{\partial \theta_{u}^{*}} \frac{\theta_{u}^{*}}{q_{h}}+\frac{\partial \Pi}{\partial q_{h}}-C^{\prime}\left(q_{h}\right)=0 . \tag{3.14}
\end{equation*}
$$

Due to the envelope theorem, we have $\partial \Pi / \partial \theta_{l}^{*}=0, \partial \Pi / \partial \theta_{l}^{*}=0$ and $\partial \Pi / \partial \theta_{u}^{*}=0$. For both homogeneous and heterogeneous consumer expectations, we get $\partial \Pi / \partial \theta_{h}=$ $\delta\left[F\left(\theta_{h}^{*}\right)-F\left(\theta_{u}^{*}\right)\right] \theta_{u}^{*}$. Thus,

$$
C^{\prime}\left(q_{h}\right)=\delta\left[F\left(\theta_{h}^{*}\right)-F\left(\theta_{u}^{*}\right)\right] \theta_{u}^{*},
$$

which means the optimal quality of the high version is determined by the development cost function. When we take the first derivative of the net monopoly profits with respect to $q_{l}$, we have

$$
\frac{\partial N\left(q_{h}, q_{l}\right)}{\partial q_{l}}=\frac{\partial \Pi}{\partial \theta_{h}^{*}} \frac{\theta_{h}^{*}}{q_{l}}+\frac{\partial \Pi}{\partial \theta_{l}^{*}} \frac{\theta_{l}^{*}}{q_{l}}+\frac{\partial \Pi}{\partial \theta_{u}^{*}} \frac{\theta_{u}^{*}}{q_{l}}+\frac{\partial \Pi}{\partial q_{l}}-V^{\prime}\left(q_{l}\right)=\frac{\partial \Pi}{\partial q_{l}}-V^{\prime}\left(q_{l}\right) .
$$

For both homogeneous and heterogeneous consumer expectations, we get

$$
\frac{\partial \Pi}{\partial \theta_{l}}=-\delta\left[F\left(\theta_{h}^{*}\right)-F\left(\theta_{u}^{*}\right)\right] \theta_{u}^{*}
$$

so that

$$
\frac{\partial N\left(q_{h}, q_{l}\right)}{\partial q_{l}}=-\delta\left[F\left(\theta_{h}^{*}\right)-F\left(\theta_{u}^{*}\right)\right] \theta_{u}^{*}-V^{\prime}\left(q_{l}\right)<0
$$

It means the lower the quality of the low version, the higher the net monopoly profits and the optimal quality of the low version should be reduced to the lowest quality $\underline{q}$
that reasonably can be used to reveal the quality of the high version. So a feasible solution for the monopolist is to minimize the quality of the low version that contains just sufficient information to reveal the true quality of the information good. It well explains why a monopolist tends to offer trail version or demo.

### 3.4 Versioning Strategies with Periodic License Fees

In the previous model the monopolist charges a lump sum for different versions and consumers take ownership of the good forever. However, information goods are often rented or shared (Varian, 2000). A typical example is computer software such as Norton Antivirus, which can easily be shared with site licenses, license servers and similar technology (http://www.symantec.com). The providers charge license fees by duration of use and consumers receive updates to the information goods during the period the license is in effect. In this section, we analyze the monopolist's versioning strategies when information goods can be charged with periodic license fees.

We assume there is no initial payment added to the periodic license fees. With periodic license fees, consumers make decisions whether to buy or not, and which version to buy for each period. The monopolist applies pricing and versioning strategies to maximize its current value of profits over all periods. For simplicity, we only analyze the case with homogeneous consumer expectations, either optimistic or pessimistic.

We assume all the transactions occur instantly at the beginning of each period. At the beginning of each period, the uninformed consumers have expected quality $R$ and informed consumers know the true quality $q$. We denote $\theta_{\text {old }}$ as the informed consumer with lowest individual taste. Thus, at the beginning of each period, consumers in $\left[0, \theta_{\text {old }}\right)$ expect $R$ and consumers in $\left[\theta_{\text {old }}, 1\right]$ know $q$. In the following periods, consumers who purchased in any previous period know the true quality $q$.

### 3.4.1 Pessimistic Consumer Expectations

For pessimistic consumers, $R<q$, and the monopolist has an incentive to reveal the true quality of the goods (Shapiro, 1983).

Current Period: To start, we consider the situation when the monopolist only maximizes the profits of the current period. Because there are two groups of consumers, a maximum of two versions are provided by the monopolist. For pessimistic expectations, informed consumers always have higher reservation prices than uninformed consumers, thus the market is continuously segmented, as illustrated in Figure 3.1. The monopolist's profit function is

$$
\max _{\theta_{h}, \theta_{l}} \Pi=\left[1-F\left(\theta_{h}\right)\right]\left[\theta_{l} R_{l}+\left[q_{h}-q_{l}\right] \theta_{h}\right]+\left[F\left(\theta_{h}\right)-F\left(\theta_{l}\right)\right] \theta_{l} R_{l} .
$$

For the monopolist to maximize profits of the current period, we have the following corollary (proofs are in Appendix B):

Corollary 3.1 Let $\theta^{*}$ be the unique solution for $\theta=[1-F(\theta)] / f(\theta)$.

1. For $\theta_{\text {old }} \in\left[0, \theta^{*}\right]$, the monopolist provides only $q_{h}$ and $p_{h}=\theta^{*} q_{h}$. The optimal profits are: $\Pi^{*}=\theta^{*} q_{h}\left[1-F\left(\theta^{*}\right)\right]$. At the end of the period, consumers in $\left[\theta_{o l d}, 1\right]$ are informed.
2. For $\theta_{\text {old }} \in\left(\theta^{*}, 1\right]$, the following conditions apply: ${ }^{4}$

- Case 1: If $\frac{\theta_{o l d}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \leq \min \left\{\frac{R_{h}}{q_{h}}, \frac{R_{h}-R_{l}}{q_{h}-q_{l}}\right\}$, then the monopolist provides only $q_{h}$. The optimal profits are: $\Pi_{1}=\theta^{*} R_{h}\left[1-F\left(\theta^{*}\right)\right]$. At the end of the period, consumers in $\left[\theta^{*}, 1\right]$ are informed.

[^9]- Case 2: If $\frac{\theta_{\text {old }}\left[1-F\left(\theta_{\text {old }}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \geq \max \left\{\frac{R_{h}}{q_{h}}, \frac{R_{l}}{q_{l}}\right\}$, then the monopolist provides only $q_{h}$. The optimal profits are: $\Pi_{2}=\theta_{\text {old }} q_{h}\left[1-F\left(\theta_{\text {old }}\right)\right]$. At the end of the period, consumers in $\left[\theta_{\text {old }}, 1\right]$ are informed.
- Case 3: If $\frac{R_{h}-R_{l}}{q_{h}-q_{l}}<\frac{\theta_{o l d}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}<\frac{R_{l}}{q_{l}}$, then the monopolist provides two versions $q_{h}$ and $q_{l}$. The optimal profits are: $\Pi_{3}=\theta^{*} R_{l}\left[1-F\left(\theta^{*}\right)\right]+$ $\theta_{\text {old }}\left[q_{h}-q_{l}\right]\left[1-F\left(\theta_{\text {old }}\right)\right]$. At the end of the period, consumers in $\left[\theta^{*}, 1\right]$ are informed.

Multiple Periods: Now we switch to multiple periods. We first show that there are only two regimes: the initial regime and the stable regime. From the discussions above, we know for $\theta_{\text {old }} \in\left[0, \theta^{*}\right]$, the proportion of informed consumers remain stable because no new consumer purchases during this period. The monopolist earns profits $\Pi^{*}$, which are the highest possible single period profits, for the current period and afterward. For $\theta_{\text {old }} \in\left(\theta^{*}, 1\right]$, in Cases 1 and 3 , at the end of the period, the proportion of informed consumers is extended to $\left[\theta^{*}, 1\right]$, which remains stable thereafter with profits $\Pi^{*}$. Both Cases 1 and 3 are intermediate in the sense that more consumers are informed at the end of the period.

Case 2 is a stable period by itself because the proportion of informed consumers remains the same at the end of this period. But it may not be optimal for the monopolist to maximize profits in the long run because $\Pi_{2}<\Pi^{*}$. The monopolist may sacrifice profits of the current period to maximize the overall profits. The following situations apply:

- If $\Pi_{1} \geq \Pi_{3}$ and $\Pi_{2}-\Pi_{1}<\frac{\delta}{1-\delta}\left[\Pi^{*}-\Pi_{2}\right],{ }^{5}$ then the monopolist provides only $q_{h}$ while extending the market to $\left[\theta^{*}, 1\right]$ which remains stable thereafter.

[^10]- If $\Pi_{1}<\Pi_{3}$ and $\Pi_{2}-\Pi_{3}<\frac{\delta}{1-\delta}\left[\Pi^{*}-\Pi_{2}\right]$, then the monopolist provides two versions $q_{h}$ and $q_{l}$ while extending the market to $\left[\theta^{*}, 1\right]$ which remains stable thereafter.
- Otherwise, the monopolist provides only $q_{h}$ while remaining a market of $\left[\theta_{\text {old }}, 1\right]$ for the first period and periods afterward.

If the initial period starts with $\theta_{\text {old }}=1$, which means there are no informed consumers, then only Case 1 applies. The monopolist earns $\Pi_{1}$ for this period and $\Pi^{*}$ thereafter. Versioning is not implemented. Thus, we have the following proposition for pessimistic consumers:

Proposition 3.4 With pessimistic consumers and periodic license fees, versioning is implemented by the monopolist only if some consumers are already informed of the true quality.

This result applies when a monopolist intends to expand the market. For example, when Adobe Photoshop extends its users from professionals to laymen, professionals know the true quality of the software while the laymen are uninformed and normally pessimistic. In that case versioning attracts more consumers without compromising profits.

### 3.4.2 Optimistic Consumer Expectations

Now we discuss optimistic consumers where $R>q$. With optimistic consumers, the monopolist has an incentive to discourage the dispersion of the quality information (Shapiro, 1983).

Current Period: As before, we assume at the beginning of a period, uninformed consumers in $\left[0, \theta_{\text {old }}\right)$ expect $R$ and informed consumers in $\left[\theta_{\text {old }}, 1\right]$ know $q$. Because
$R>q$, some informed consumers in $\left[\theta_{\text {old }}, 1\right]$ may purchase the low quality version while some uninformed consumers in $\left[0, \theta_{\text {old }}\right)$ may choose the high quality version, in which case the market is discontinuously segmented. As illustrated in Figure 3.5, two versions may divide consumers into six possible segments: consumers in $\left[0, \theta_{l}^{\prime}\right)$ and $\left[\theta_{o l d}, \theta_{l}\right)$ who do not purchase, consumers in $\left[\theta_{l}^{\prime}, \theta_{h}^{\prime}\right)$ and $\left[\theta_{l}, \theta_{h}\right]$ who purchase $q_{l}$ and consumers in $\left[\theta_{h}^{\prime}, \theta_{\text {old }}\right)$ and $\left[\theta_{h}, 1\right]$ who purchase $q_{h}$. Here for informed consumers in $\left[\theta_{o l d}, 1\right]$, we denote $\theta_{l}$ as the consumer that is indifferent between purchasing $q_{l}$ and not purchasing and $\theta_{h}$ as the consumer that is indifferent between purchasing $q_{h}$ and purchasing $q_{l}$. While for uninformed consumers in $\left[0, \theta_{o l d}\right)$, we denote $\theta_{l}^{\prime}$ as the consumer that is indifferent between purchasing $q_{l}$ and not purchasing and $\theta_{h}^{\prime}$ as the consumer that is indifferent between purchasing $q_{h}$ and purchasing $q_{l}$. Corresponding


## Figure 3.5: Market Segmentation of Information Goods for Optimistic Consumers

prices are determined according to (3.1) and (3.2) plus the following two equations:

$$
\begin{gather*}
p_{l}=U\left(R_{l}, \theta_{l}^{\prime}\right), \text { and }  \tag{3.15}\\
U\left(R_{h}, \theta_{h}^{\prime}\right)-p_{h}=U\left(R_{l}, \theta_{h}^{\prime}\right)-p_{l} . \tag{3.16}
\end{gather*}
$$

The monopolist's profit function is

$$
\max _{\theta_{h}, \theta_{l}, \theta_{h}^{\prime}, \theta_{l}^{\prime}} \Pi=p_{h}\left[\left[1-F\left(\theta_{h}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{h}^{\prime}\right)\right]\right]+
$$

$$
\begin{aligned}
& p_{l}\left[\left[F\left(\theta_{h}\right)-F\left(\theta_{l}\right)\right]+\left[F\left(\theta_{h}^{\prime}\right)-F\left(\theta_{l}^{\prime}\right)\right]\right] \\
& \ni 0 \leq \theta_{l}^{\prime} \leq \theta_{h}^{\prime} \leq \theta_{o l d} \leq \theta_{l} \leq \theta_{h} \leq 1
\end{aligned}
$$

For the monopolist to maximize profits in the current period, we have the following corollary (proofs are in Appendix B):

Corollary 3.2 With optimistic consumers, a monopolist that maximizes current period profits provides only one version.

Multiple Periods: Now we discuss when the monopolist chooses to maximize the overall profits. According to Shapiro (1983), when facing optimistic consumers, a monopolist gradually expands the market before it reaches the stable periods after which all consumers who buy are informed consumers. We denote the first stable period as $S_{N}$ and the corresponding profits at this period as $\Pi_{N}$. Because all the consumers who purchase are informed, only one version is provided by the monopolist (Wei and Nault, 2005) and the optimal profits are $\Pi_{N}=\Pi^{*}=\theta^{*} q_{h}\left[1-F\left(\theta^{*}\right)\right]{ }^{6}$ This applies to all the subsequent periods.

Now move a period back to $S_{N-1}$. At period $S_{N-1}$ the monopolist chooses to maximize overall current value of profits $\delta^{N-2} \Pi_{N-1}+\sum_{n=N}^{\infty} \delta^{n-1} \Pi^{*}$, which is equivalent to the monopolist maximizing profits of period $S_{N-1}$ only. According to Corollary 3.2 , only one version is provided. We can trace this process back to any period to get the following proposition:

Proposition 3.5 With optimistic consumers and periodic license fees, a monopolist provides only one version.

With a one-version solution, the pricing strategy for the information goods is

[^11]similar to traditional goods where price declines monotonically to expand the market gradually before it reaches the stable periods when no new consumer buys, as analyzed in Shapiro (1983).

### 3.5 Conclusions

In this paper we treat information goods as experience goods and construct models to explain why a monopolist implements versioning strategies. We show that with lump sum payments, the monopolist offers different versions combined with upgrading. Adopting a two-stage model, we find that if all the consumers have homogeneous initial expectations about the goods' quality, then the monopolist's optimal pricing strategy involves upgrading all the consumers who purchase the low quality version in the first stage to the high quality version in the second stage. In this way, consumers that upgrade pay a tax for learning. When consumers have heterogeneous expectations, we find that if consumers are pessimistic, then the monopolist's optimal pricing still drives all consumers to upgrade. However, if consumers are optimistic, then under certain conditions, the monopolist's optimal pricing strategy induces only some of the consumers that initially purchased low quality versions to upgrade. When quality of the low version can be endogenized, the monopolist minimizes the quality of the low version to maximize profits. With periodic license fees where upgrades cannot be an option, versioning strategies are implemented by the monopolist only if some consumers are already informed of the true quality. But when consumers are optimistic, the monopolist offers only one version.

A limitation of the paper is that we only model a monopoly producer. Whether versioning and upgrading strategies can be applied in a competitive environment is not clear. In previous research we found that versioning strategies can be implemented
by an incumbent firm to deter entry (Wei and Nault, 2006), but this result may not hold when we treat information goods as experience goods and consider upgrading strategies. Another limitation is that in our models we assume that the only way to learn the true quality of the goods is through purchasing, while in the real world, there are many other channels such as "word of mouth" to know the true quality of the goods. On the other side, purchasing of the low quality version may not be sufficient for consumers to learn the quality of the higher versions.

One possible extension of our models is to include network effects in our models. Many information goods such as operating systems and Database management systems display strong positive network effects where the consumers' willingness to pay increases with the total size of the users (Economides, 1996, Sundararajan, 2003, etc.). Network effects may provide the monopolist more incentive to version information goods to expand the user base.

## Chapter 4

# ESSAY THREE: VERTICALLY DIFFERENTIATED INFORMATION GOODS: MONOPOLY POWER THROUGH VERSIONING 

### 4.1 Introduction

Information goods come in many forms. Jones and Mendelson (2005) categorize information goods as computer software including operation systems, programming tools and applications; online services such as internet search engines and portals; online content such as information provided by Lexis/Nexis, Dow Jones, and Reuters; and other digitalized information goods such as digitalized music, movies and books. In each form an additional unit can be produced and distributed at negligible cost either by copying or by allowing it to be downloadable over the Internet. For the latter, broad adoption of e-commerce, secure and convenient online payments and high-speed internet connections greatly lower the transaction costs. Therefore, information goods are characterized by large sunk costs of development and by negligible costs of reproduction and distribution (Shapiro and Varian, 1999).

Another characteristic of information goods is that after the highest quality version has been developed, the costs of creating vertically degraded versions - versions with less functionality - are usually negligible. Versioning is to offer a product in different versions to segment the market and maximize profit, which is often referred to as
second degree price discrimination (Wei and Nault, 2005). Hahn (2001) states that "the functional quality degradation (of software) is an effective consumer screening device, especially when consumers' valuation for each function is negatively correlated (p.1)". Bhargava and Choudhary (2007) reach a similar conclusion under relatively general settings about consumer heterogeneity and utility functions.

With the ease of versioning, product differentiation and pricing strategies for information goods are different from traditional goods, especially in the context of competition. Leaders with information goods usually have substantial market power. As of 2002, Microsoft Windows controlled $97.46 \%$ of the global desktop operation systems market (Windows IT Pro, 2002). Oracle's market share on Linux was $80.6 \%$ in 2005 (www.oracle.com), and according to the Nielsen cabinet the most popular search engine on the web, Google, had a market share of $54 \%$ in 2006, ahead of Yahoo! $(23 \%)$ and MSN (13\%) (www.google.com, 2006). Competition in information goods is more intense than traditional goods in the sense that direct competition can drive prices to zero and both firms lose development costs, but is less intense in the sense that the winners (often the first movers) usually dominate the market. Meanwhile, with potential competition, producers of information goods have strong incentives to improve quality, launching their highest quality version, or upgrading older versions, whenever possible, even if they lose money at the margin by cannibalizing the existing market share of the old version (Nault and Vandenbosch, 1996). It is also common for the software producers to release a buggier product early and patch it later to grab the "first mover advantage" (Arora, Caulkins, and Telang, 2006). In the context of these stylized facts we examine two research questions. The first is to explain in more detail why leaders in information goods dominate their markets. The second is to explain why potential competition motivates a monopolist to improve quality and to version.

Competition with information goods has been investigated in previous research.

Nault (1997) examined quality differentiation using inter-organizational information systems (IOS) and found that IOS could separate consumers and reduce competition in duopoly. Dewan, Jing and Seidmann (2003) developed a duopoly model where firms could produce both standard and customized products, finding that "when firms face a fixed entry cost and adopt customization sequentially, the first follower always achieves an advantage and may be able to deter subsequent entry by choosing its customization scope strategically (p. 1055)". Choudhary, Ghose, Mukopadhyay and Rajan (2005) proposed a personalized pricing (PP) strategy where firms produce vertically differentiated goods and can perfectly identify valuations of heterogeneous consumers. They found that "while PP results in a wider market coverage, it also leads to aggravated price competition between firms (p. 1120)". Empirical research has also examined product and pricing strategies. Nault and Dexter (1995) found that with the adoption of a cardlock IT system, a commercial fueling company successfully differentiated a commodity, maintaining a price premium between $5-12 \%$ of retail. Cottrell and Nault (2004) found that in the microcomputer software industry changes in product variety through new product introductions improve firm performance, but extensions to existing products hinder the performance of the firm and the product. Analyzing Amazon.com, Ghose, Smith and Telang (2006) found that used books are poor substitutes for new books for most customers, but the existence of used book marketplace increases consumer surplus and total welfare. Also using data from Amazon.com, Ghose and Sundararajan (2005) found that an increase in the total number of versions is associated with an increase in the difference in quality between the highest and lowest quality versions.

We use a duopoly model to analyze the price and quality choices for information goods, and examine the effectiveness of versioning strategies as a way for a monopolist to deter entry. We find that under competition firms always provide higher quality
information goods with a better "price-quality ratio" than in monopoly. In addition, as long as the implementation of versions in the market is irrevocable, then in the high-end market (market for the highest quality information goods) a monopolist can strategically set its quality to deter entry, and in the low-end market (market for all lower quality information goods) the monopolist can implement versioning strategies to deter entry. Whereas a vertically differentiated market is often referred to as a "natural oligopoly" for traditional goods (Shaked and Sutton, 1983), because of versioning it can be regarded as a "natural monopoly" for information goods.

Our paper proceeds as follows. We set up our notation and assumptions in Section 4.2, analyze the monopoly producer in Section 4.3, and examine a simultaneous move duopoly in Section 4.4. In Section 4.5, we examine a sequential move duopoly with entry deterrence. Social welfare implications are analyzed in Section 4.6. Discussion and future research are included in Section 4.7.

### 4.2 Notation and Assumptions

In our model, consumers are heterogeneous and uniformly distributed in their individual taste for quality. We denote individual consumer taste as $\theta$ which is normalized to be in the interval $[0,1]$. The consumer taste $\theta$ indicates a consumer's marginal valuation for quality. A consumer has positive utility for one unit only. The total market size is normalized to unity. Consumers select their favorite good with quality $q \in[0, \bar{q}]$ to maximize their consumer surplus $U(q, \theta)-p$, where $p$ is the price of the good and $\bar{q}$ is the boundary quality under technology constraint. We take a consumer's utility to be multiplicative in taste and quality. This is our first assumption:

Assumption 4.1 $U(q, \theta)=\theta q$.

If a firm produces an information good of quality $q$, then it incurs development $\operatorname{cost} C(q)$ and zero marginal cost of reproduction and distribution. The development $\operatorname{cost} C(q)$ is twice differentiable, strictly increasing and strictly convex in $q$ for $q>0$, and zero quality is costless:

Assumption 4.2 For $q=0, C(q)=0$. For $q>0, C^{\prime}(q)>0$ and $C^{\prime \prime}(q)>0$.

Denoting different quality versions with subscripts, after the highest quality $q^{H}$ of the information good is produced, the firm may degrade it to generate a lower quality version $q^{L}$. For any $q>0$, we assume constant versioning costs, $V \in R^{+}$, and there are no versioning costs when $q=0$. Taking the example of software products, we know that after the flagship product is developed, lower quality goods can be generated either by removing, disabling or recombining functions from the flagship product. For the same quality level, we also assume that versioning costs are not higher than development costs.

Assumption $4.30 \leq V \leq C(q), \quad \forall q>0$.

Firms know the distribution of consumers but not their individual type. Thus, only second degree price discrimination is possible. Firms choose price, quality and versioning strategies to maximize profits. This and notation used later are summarized in Table 4.1.

### 4.3 A Monopoly Model

We assume the monopolist provides $N$ versions of the information good with quality levels $Q=\left(q^{1}, q^{2}, \ldots, q^{N-1}, q^{N}\right)$, where $q^{1}>q^{2}>\ldots>q^{N-1}>q^{N}$. The highest quality $q^{1}$ is developed first, and the subsequent degraded qualities $q^{2}, \ldots, q^{N-1}, q^{N}$ are produced through versioning. Let $P=\left(p^{1}, p^{2}, \ldots, p^{N-1}, p^{N}\right)$ denote the corre-

Table 4.1. Summary of Key Notation

| Symbol | Explanation |
| :--- | :--- |
| $U(q, \theta)$ | Utility that consumer $\theta$ gets from information good with quality $q$ |
| $C(q)$ | Cost of developing information good with quality $q$ |
| $V$ | Cost of versioning an information good |
| $\Pi(\cdot)$ | profit function of the firm |
| $p$ | price level of the information good |
| $q$ | quality of the information good |
| $\theta$ | consumer taste for quality |
| $r$ | price-quality ratio |
| $t$ | comparative quality ratio |
| $M$ | monopolist |
| $A, B$ | competing firms who enter the specific market |
| $N$ | Nash Equilibrium point where $q_{A}>q_{B}$ |
| $N^{\prime}$ | Nash Equilibrium point where $q_{A}<q_{B}$ |
| $S$ | Stackelberg point where $q_{A}>q_{B}$ |
| $S^{\prime}$ | Stackelberg point where $q_{A}<q_{B}$ |

*We use superscripts for variables and subscripts for functions to indicate variables and relevant functional forms for firms in different settings.
sponding prices for the above quality levels, and $D\left(P, q^{i}\right)$ denotes the demand for the good with quality $q^{i}$ given the price vector $P$. The monopolist chooses a price-quality schedule to maximize profit.

$$
\max _{P(Q)} \Pi(P(Q))=\max _{P(Q)}\left\{\sum_{i=1}^{N} p^{i} D\left(\dot{P}, q^{i}\right)-C\left(q^{1}\right)-\sum_{i=2}^{N} V\right\} .
$$

The provision of $N$ different quality goods divides the market into $N+1$ segments, where the last segment contains consumers that do not purchase. In segment $N$ where the consumer only chooses between buying the good designed for segment $N$ and not buying, we define $\theta^{N}$ as the indifferent consumer and the price assignment is

$$
p^{N}=U\left(q^{N}, \theta^{N}\right)=\theta^{N} q^{N} .
$$

In segment $i(i<N)$ where the consumer chooses between buying good $q^{i}$ and a good $q^{i+1}$ designed for its closest segment $i+1$, we define $\theta^{i}$ as the indifferent consumer
and the price assignment is

$$
p^{i}=p^{i+1}+U\left(q^{i}, \theta^{i}\right)-U\left(q^{i+1}, \theta^{i}\right)=p^{i+1}+\theta^{i} q^{i}-\theta^{i} q^{i+1} .
$$

In a vertically differentiated market, indifferent consumers only exist between two contiguous segments. Using the price assignments, the indifferent consumer is defined by $\theta^{i}=\left[p^{i}-p^{i+1}\right] /\left[q^{i}-q^{i+1}\right]$, for $i=\{1,2, \ldots, N-1\}$, and $\theta^{N}=p^{N} / q^{N}$. Consumers in segment $\left[\theta^{1}, 1\right]$ buy the good with quality $q^{1}$, consumers in $\left[\theta^{i}, \theta^{i-1}\right), i=\{2, \ldots, N\}$ buy versions with quality $q^{i}$, and consumers in $\left[0, \theta^{N}\right)$ do not buy. The monopolist's profit maximization can be rewritten as

$$
\begin{gathered}
\max _{P(Q)} \Pi(P(Q))= \\
\max _{P(Q)}\left\{p^{1}\left[1-\frac{p^{1}-p^{2}}{q^{1}-q^{2}}\right]+\sum_{i=2}^{N-1} p^{i}\left[\frac{p^{i-1}-p^{i}}{q^{i-1}-q^{i}}-\frac{p^{i}-p^{i+1}}{q^{i}-q^{i+1}}\right]+p^{N}\left[\frac{p^{N-1}-p^{N}}{q^{N-1}-q^{N}}-\frac{p^{N}}{q^{N}}\right]-C\left(q^{1}\right)-\sum_{i=2}^{N} V\right\}, \\
\ni p^{1}>p^{2}>\ldots>p^{N} ; q^{1}>q^{2}>\ldots>q^{N}
\end{gathered}
$$

The first term in the above profit function indicates revenue generated from version $q^{1}$, the second term indicates revenue generated from versions $q^{2}, \cdots, q^{N-1}$, the third term indicates revenue generated from version $q^{N}$. The last two terms indicate development cost for version $q^{1}$ and versioning costs for all the lower versions $q^{2}$ through $q^{N}$. For the optimal price-quality schedule, using the envelope theorem, we have $\partial \Pi(P(Q)) / \partial P=$ 0 (take the partial of $\Pi(\cdot)$ with respect to each $p^{i}$ individually), and the result is

$$
\frac{p^{1}-p^{2}}{q^{1}-q^{2}}=\frac{p^{2}-p^{3}}{q^{2}-q^{3}}=\ldots=\frac{p^{N-1}-p^{N}}{q^{N-1}-q^{N}}=\frac{p^{N}}{q^{N}}=\frac{1}{2},
$$

meaning that all the indifferent consumers are equal. Therefore, except for segment 1, the demand in all the other segments is zero and a profit maximizing monopolist provides only one version. This single-version result replicates the findings of Jones and Mendelson (2005), Bhargava and Choudhary (2001) and basic argument of Jing (2002), and Wu, Chen and Anandalingam (2003) in our formulation.

We denote the optimal price and quality of the only version by $p_{M}$ and $q_{M}$, respectively. The optimal "price-quality ratio" is denoted by $r_{M}=p_{M} / q_{M}$. For the monopolist, we have the following proposition: ${ }^{1}$

Proposition 4.1 (Monopoly) i) A monopolist provides only one version. ii) The necessary condition for a monopolist to profitably launch the good is that the marginal cost of development is greater than the average cost of quality.

With our assumptions the optimal "price-quality ratio" of the good provided by the monopolist is $1 / 2$ and at the optimal price-quality ratio only half of the market is covered. As quality is also chosen, if development costs are sufficiently large, then the monopolist may find it unprofitable to launch the good even at the optimal quality level.

### 4.4 Simultaneous Move Duopoly

In this section we examine the case where two firms $A$ and $B$ are in the market, and each develops their version quality. We take the information goods as vertically differentiated. We formulate our basic model where neither firm considers versioning. Later we show that it is profit maximizing for each firm to provide only one version even when versioning is possible. After the information goods are produced, both firms choose prices. Consumers choose their preferred goods based on the qualities and prices. Thus, our model is a two-stage game where in Stage 1 firms $A$ and $B$ develop information goods with quality levels $q_{A}$ and $q_{B}$, and in Stage 2 the firms compete in prices.

[^12]
### 4.4.1 Simultaneous Move without Versioning

We consider a pure strategy Nash equilibrium of the game. If both firms develop information goods with the same quality level, Bertrand competition drives prices to zero and neither firm gains positive profit. ${ }^{2}$ Without loss of generality, we assume $q_{A}>q_{B}$. The cost for firm $A$ to develop $q_{A}$ is $C_{A}\left(q_{A}\right)$, and for firm $B$ to develop $q_{B}$ is $C_{B}\left(q_{B}\right)$. The cost functions of firms $A$ and $B$ need not be the same. For both firms to have positive share, $p_{A}>p_{B}$.

Let $\theta_{A}$ denote the consumer indifferent between buying goods $q_{A}$ and $q_{B}$, and $\theta_{B}$ denote the consumer indifferent between buying good $q_{B}$ and not buying. Similar to the analysis in the previous section, we have $\theta_{A}=\left[p_{A}-p_{B}\right] /\left[q_{A}-q_{B}\right]$, and $\theta_{B}=p_{B} / q_{B}$. We work backwards to solve the duopoly model.

Stage 2 Firm $A$ and $B$ 's profit functions are

$$
\Pi_{A}\left(p_{A}, p_{B}\right)=p_{A}\left[1-\frac{p_{A}-p_{B}}{q_{A}-q_{B}}\right]-C_{A}\left(q_{A}\right) \text { and } \Pi_{B}\left(p_{A}, p_{B}\right)=p_{B}\left[\frac{p_{A}-p_{B}}{q_{A}-q_{B}}-\frac{p_{B}}{q_{B}}\right]-C_{B}\left(q_{B}\right)
$$

The first-order conditions with respect to own prices yield best response functions ${ }^{3}$

$$
\begin{equation*}
2 p_{A}-p_{B}=q_{A}-q_{B} \text { and } p_{A} /\left[2 p_{B}\right]=q_{A} / q_{B} \tag{4.1}
\end{equation*}
$$

Solving for the equations in (4.1) gives

$$
\begin{equation*}
p_{A}=2 q_{A}\left[\frac{q_{A}-q_{B}}{4 q_{A}-q_{B}}\right] \text { and } p_{B}=q_{B}\left[\frac{q_{A}-q_{B}}{4 q_{A}-q_{B}}\right] \tag{4.2}
\end{equation*}
$$

From (4.2) the market share for $q_{A}$ is $2 q_{A} /\left[4 q_{A}-q_{B}\right]$, which is twice the market share of $q_{B}$.

[^13]Stage 1 Substituting (4.2) back into the profit functions of firms $A$ and $B$, we have

$$
\begin{equation*}
\Pi_{A}\left(q_{A}, q_{B}\right)=4 q_{A}^{2}\left[q_{A}-q_{B}\right] /\left[4 q_{A}-q_{B}\right]^{2}-C_{A}\left(q_{A}\right) \tag{4.3}
\end{equation*}
$$

and

$$
\begin{equation*}
\Pi_{B}\left(q_{A}, q_{B}\right)=q_{A} q_{B}\left[q_{A}-q_{B}\right] /\left[4 q_{A}-q_{B}\right]^{2}-C_{B}\left(q_{B}\right) \tag{4.4}
\end{equation*}
$$

$\Pi_{A}\left(q_{A}, q_{B}\right)$ is concave in $q_{A}$ and $\Pi_{B}\left(q_{A}, q_{B}\right)$ is concave in $q_{B}$ (Proofs are in the Appendix). Firms $A$ and $B$ choose quality levels $q_{A}$ and $q_{B}$ to maximize their profits, thus $\partial \Pi_{A}(\cdot) / \partial q_{A}=0$ and $\partial \Pi_{B}(\cdot) / \partial q_{B}=0$. The equilibrium quality levels $q_{A}$ and $q_{B}$ are implicitly determined by

$$
\begin{equation*}
C_{A}^{\prime}\left(q_{A}\right)=4 q_{A}\left[4 q_{A}^{2}-3 q_{A} q_{B}+2 q_{B}^{2}\right] /\left[4 q_{A}-q_{B}\right]^{3} \tag{4.5}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{B}^{\prime}\left(q_{B}\right)=q_{A}^{2}\left[4 q_{A}-7 q_{B}\right] /\left[4 q_{A}-q_{B}\right]^{3} \tag{4.6}
\end{equation*}
$$

### 4.4.2 Should Firms Version?

In the following, we show that even when versioning is an option for both firms, neither version their information goods. In terms of which firm considers versioning, there are two situations.

Firm $A$ considers versioning. Here we assume firm $A$ develops its high quality version $q_{A}^{H}$ and generates a lower version $q_{A}^{L}$, and firm $B$ develops its quality $q_{B}$. Prices $p_{B}, p_{A}^{H}$ and $p_{A}^{L}$ are set according to Bertrand competition. There are two cases:
Case 1: $q_{A}^{L}<q_{B}<q_{A}^{H}$. Let $\theta_{A}^{H}$ denote the consumer indifferent between buying $q_{A}^{H}$ and $q_{B}, \theta_{B}$ denote the consumer indifferent between buying $q_{B}$ and $q_{A}^{L}$, and $\theta_{A}^{L}$ denote the consumer indifferent between buying $q_{A}^{L}$ and not buying. We have $\theta_{A}^{H}=$ $\left[p_{A}^{H}-p_{B}\right] /\left[q_{A}^{H}-q_{B}\right], \theta_{B}=\left[p_{B}-p_{A}^{L}\right] /\left[q_{B}-q_{A}^{L}\right]$, and $\theta_{A}^{L}=p_{A}^{L} / q_{A}^{L}$. The profit
function of firm $A$ is

$$
\begin{equation*}
\Pi_{A}\left(p_{A}^{H}, p_{A}^{L}, p_{B}\right)=p_{A}^{H}\left[1-\frac{p_{A}^{H}-p_{B}}{q_{A}^{H}-q_{B}}\right]+p_{A}^{L}\left[\frac{p_{B}-p_{A}^{L}}{q_{B}-q_{A}^{L}}-\frac{p_{A}^{L}}{q_{A}^{L}}\right]-C_{A}\left(q_{A}^{H}\right)-V \tag{4.7}
\end{equation*}
$$

The profit function for firm $B$ is

$$
\begin{equation*}
\Pi_{B}\left(p_{A}^{H}, p_{A}^{L}, p_{B}\right)=p_{B}\left[\frac{p_{A}^{H}-p_{B}}{q_{A}^{H}-q_{B}}-\frac{p_{B}-p_{A}^{L}}{q_{B}-q_{A}^{L}}\right]-C_{B}\left(q_{B}\right) \tag{4.8}
\end{equation*}
$$

From the first order conditions of (4.7) with respect to $p_{A}^{H}$ and $p_{A}^{L}$, and of (4.8) with respect to $p_{B}$, we get the best response functions as follows,

$$
\begin{aligned}
& 2 p_{A}^{H}-p_{B}=q_{A}^{H}-q_{B} \\
& -q_{A}^{L} p_{B}+2 q_{B} p_{A}^{L}=0
\end{aligned}
$$

and

$$
\left[q_{B}-q_{A}^{L}\right] p_{A}^{H}-2\left[q_{A}^{H}-q_{A}^{L}\right] p_{B}+\left[q_{A}^{H}-q_{B}\right] p_{A}^{L}=0
$$

Applying the Cramer's Rule, we have

$$
\Lambda_{1}=\left|\begin{array}{lll}
2, & -1, & 0 \\
0, & -q_{A}^{L}, & 2 q_{B} \\
q_{B}-q_{A}^{L}, & -2\left[q_{A}^{H}-q_{A}^{L}\right], & q_{A}^{H}-q_{B}
\end{array}\right|=2\left[4 q_{A}^{H} q_{B}-q_{A}^{H} q_{A}^{L}-q_{B}^{2}-2 q_{B} q_{A}^{L}\right]>0
$$

And we get the equilibrium prices for $p_{A}^{H}, p_{B}$ and $p_{A}^{L}$ as follows,

$$
\begin{gather*}
p_{A}^{H}=\left[q_{A}^{H}-q_{B}\right]\left[4 q_{A}^{H} q_{B}-q_{A}^{H} q_{A}^{L}-3 q_{B} q_{A}^{L}\right] / \Lambda_{1}  \tag{4.9}\\
p_{A}^{L}=q_{A}^{L}\left[q_{A}^{H}-q_{B}\right]\left[q_{B}-q_{A}^{L}\right] / \Lambda_{1} \tag{4.10}
\end{gather*}
$$

and

$$
\begin{equation*}
p_{B}=2 q_{B}\left[q_{A}^{H}-q_{B}\right]\left[q_{B}-q_{A}^{L}\right] / \Lambda_{1} \tag{4.11}
\end{equation*}
$$

Substituting the equilibrium prices (4.9), (4.10) and (4.11) as functions of quality back into the profit function of firm $A$, we have

$$
\Pi_{A}\left(q_{A}^{H}, q_{A}^{L}, q_{B}\right)=\frac{\left[q_{A}^{H}-q_{B}\right]}{\left[\Lambda_{1}\right]^{2}}\left[\left[4 q_{A}^{H} q_{B}-q_{A}^{H} q_{A}^{L}-3 q_{B} q_{A}^{L}\right]^{2}+q_{B} q_{A}^{L}\left[q_{A}^{H}-q_{B}\right]\left[q_{B}-q_{A}^{L}\right]\right]
$$

$$
-C_{A}\left(q_{A}^{H}\right)-V,
$$

where $\Lambda_{1}=2\left[4 q_{A}^{H} q_{B}-q_{A}^{H} q_{A}^{L}-q_{B}^{2}-2 q_{B} q_{A}^{L}\right]$. Taking the partial derivative of $\Pi_{A}(\cdot)$ with respect to $q_{A}^{L}$,

$$
\frac{\partial \Pi_{A}\left(q_{A}^{H}, q_{A}^{L}, q_{B}\right)}{\partial q_{A}^{L}}=\frac{-2 q_{B}^{2}\left[q_{A}^{H}-q_{B}\right]^{2}}{\left[\Lambda_{1}\right]^{3}}\left[20 q_{A}^{H} q_{B}+q_{A}^{H} q_{A}^{L}+q_{B}^{2}-22 q_{B} q_{A}^{L}\right]<0
$$

The negative sign comes from $q_{A}^{L}<q_{B}<q_{A}^{H}$, which means that increasing the quality of its lower version reduces firm $A$ 's profit. Consequently, it is not optimal for firm $A$ to version its information good.

Case 2: $q_{B}<q_{A}^{L}<q_{A}^{H}$. Let $\theta_{A}^{H}$ denote the consumer indifferent between buying $q_{A}^{H}$ and $q_{A}^{L}, \theta_{A}^{L}$ denote the consumer indifferent between buying $q_{A}^{L}$ and $q_{B}$, and $\theta_{B}$ denote the consumer indifferent between buying $q_{B}$ and not buying. We have $\theta_{A}^{H}=$ $\left[p_{A}^{H}-p_{A}^{L}\right] /\left[q_{A}^{H}-q_{A}^{L}\right], \theta_{A}^{L}=\left[p_{A}^{L}-p_{B}\right] /\left[q_{A}^{L}-q_{B}\right]$, and $\theta_{B}=p_{B} / q_{B}$. Firm $A$ 's profit function is

$$
\begin{equation*}
\Pi_{A}\left(p_{A}^{H}, p_{A}^{L}, p_{B}\right)=p_{A}^{H}\left[1-\frac{p_{A}^{H}-p_{A}^{L}}{q_{A}^{H}-q_{A}^{L}}\right]+p_{A}^{L}\left[\frac{p_{A}^{H}-p_{A}^{L}}{q_{A}^{H}-q_{A}^{L}}-\frac{p_{A}^{L}-p_{B}}{q_{A}^{L}-q_{B}}\right]-C_{A}\left(q_{A}^{H}\right)-V \tag{4.12}
\end{equation*}
$$

and firm $B$ 's profit function is

$$
\begin{equation*}
\Pi_{B}\left(p_{A}^{H}, p_{A}^{L}, p_{B}\right)=p_{B}\left[\frac{p_{A}^{L}-p_{B}}{q_{A}^{L}-q_{B}}-\frac{p_{B}}{q_{B}}\right]-C_{B}\left(q_{B}\right) \tag{4.13}
\end{equation*}
$$

From the first order conditions of (4.12) with respect to $p_{A}^{H}$ and $p_{A}^{L}$, and of (4.13) with respect to $p_{B}$, we get the best response functions as follows,

$$
\begin{aligned}
& 2 p_{A}^{H}-2 p_{A}^{L}=q_{A}^{H}-q_{A}^{L} \\
& 2 p_{A}^{L}-p_{B}=q_{A}^{L}-q_{B}
\end{aligned}
$$

and

$$
-q_{B} p_{A}^{L}+2 q_{A}^{L} p_{B}=0
$$

Applying the Cramer's Rule, we have

$$
\Lambda_{2}=\left|\begin{array}{lll}
2, & -2, & 0 \\
0, & 2, & -1 \\
0, & -q_{B}, & 2 q_{A}^{L}
\end{array}\right|=2\left[4 q_{A}^{L}-q_{B}\right]>0
$$

and we get the equilibrium prices for $p_{A}^{H}, p_{A}^{L}$ and $p_{B}$ as follows,

$$
\begin{gather*}
p_{A}^{H}=\left[4 q_{A}^{H} q_{A}^{L}-q_{A}^{H} q_{B}-3 q_{A}^{L} q_{B}\right] / \Lambda_{2}  \tag{4.14}\\
p_{A}^{L}=4 q_{A}^{L}\left[q_{A}^{L}-q_{B}\right] / \Lambda_{2} \tag{4.15}
\end{gather*}
$$

and

$$
\begin{equation*}
p_{B}=2 q_{B}\left[q_{A}^{L}-q_{B}\right] / \Lambda_{2} \tag{4.16}
\end{equation*}
$$

Substituting the equilibrium prices (4.14), (4.15) and (4.16) as functions of quality back into the profit function of firm $A$, we have

$$
\Pi_{A}\left(q_{A}^{H}, q_{A}^{L}, q_{B}\right)=\frac{16 q_{A}^{H} q_{A}^{L}\left[q_{A}^{L}-q_{B}\right]+q_{B}\left[q_{A}^{H}-q_{A}^{L}\right]\left[8 q_{A}^{L}+q_{B}\right]}{\left[\Lambda_{2}\right]^{2}}-C_{A}\left(q_{A}^{H}\right)-V
$$

where $\Lambda_{2}=2\left[4 q_{A}^{L}-q_{B}\right]$. Taking the partial derivative of $\Pi_{A}$ with respect to $q_{A}^{L}$, we have,

$$
\frac{\partial \Pi_{A}\left(q_{A}^{H}, q_{A}^{L}, q_{B}\right)}{\partial q_{A}^{L}}=\frac{2\left[q_{B}\right]^{2}\left[20 q_{A}^{L}+q_{B}\right]^{2}}{\left[\Lambda_{2}\right]^{3}}>0
$$

The positive sign comes from $q_{B}<q_{A}^{L}<q_{A}^{H}$, which means that increasing the quality of its lower version monotonically increases firm $A$ 's profit, and firm $A$ sets $q_{A}^{L}=q_{A}^{H}$. So it is still not optimal for firm $A$ to version its information good.

Firm $B$ considers versioning. Here we assume firms $A$ and $B$ develop their highest quality version $q_{A}$ and $q_{B}^{H}$, respectively. Firm $B$ degrades $q_{B}^{H}$ to generate a lower quality version $q_{B}^{L}$. We have $q_{B}^{L}<q_{B}^{H}<q_{A}$. Prices $p_{A}, p_{B}^{H}$ and $p_{B}^{L}$ are set according to Bertrand competition.

Let $\theta_{A}$ denote the consumer indifferent between buying $q_{A}$ and $q_{B}^{H}, \theta_{B}^{H}$ denote the consumer indifferent between buying $q_{B}^{H}$ and $q_{B}^{L}$, and $\theta_{B}^{L}$ denote the consumer indifferent between buying $q_{B}^{L}$ and not buying. We have $\theta_{A}=\left[p_{A}-p_{B}^{H}\right] /\left[q_{A}-q_{B}^{H}\right]$, $\theta_{B}^{H}=\left[p_{B}^{H}-p_{B}^{L}\right] /\left[q_{B}^{H}-q_{B}^{L}\right]$, and $\theta_{B}^{L}=p_{B}^{L} / q_{B}^{L}$. The profit function of firm $A$ is

$$
\Pi_{A}\left(p_{A}, p_{B}^{H}, p_{B}^{L}\right)=p_{A}\left[1-\frac{p_{A}-p_{B}^{H}}{q_{A}-q_{B}^{H}}\right]-C_{A}\left(q_{A}\right)
$$

and the profit function of firm $B$ is

$$
\begin{equation*}
\Pi_{B}\left(p_{A}, p_{B}^{H}, p_{B}^{L}\right)=p_{B}^{H}\left[\frac{p_{A}-p_{B}^{H}}{q_{A}-q_{B}^{H}}-\frac{p_{B}^{H}-p_{B}^{L}}{q_{B}^{H}-q_{B}^{L}}\right]+p_{B}^{L}\left[\frac{p_{B}^{H}-p_{B}^{L}}{q_{B}^{H}-q_{B}^{L}}-\frac{p_{B}^{L}}{q_{B}^{L}}\right]-C_{B}\left(q_{B}^{H}\right)-V \tag{4.17}
\end{equation*}
$$

From the first order conditions of (4.17) with respect to $p_{B}^{L}$, we get

$$
\frac{p_{B}^{H}-p_{B}^{L}}{q_{B}^{H}-q_{B}^{L}}=\frac{p_{B}^{L}}{q_{B}^{L}}
$$

which means that $\theta_{B}^{H}=\theta_{B}^{L}$ and there is no market for $q_{B}^{L}$. So it is not optimal for firm $B$ to version its information good. The above analysis can be extended to the cases when firm $A$ and firm $B$ consider generating multiple versions. Thus, in a simultaneous move duopoly, we have the following proposition:

## Proposition 4.2 (Simultaneous Game) Each firm provides only one version.

### 4.4.3 Comparative Quality Analysis

We denote the equilibrium price-quality ratio of the goods provided by each firm by $r_{j}=p_{j} / q_{j} j \in\{A, B\}$. We also denote the "comparative quality ratio" by $t$ where $t=q_{A} / q_{B}>1$ from $q_{A}>q_{B}$. Thus, the solutions for $p_{A}$ and $p_{B}$ in (4.2) can be rewritten as

$$
r_{A}=2[t-1] /[4 t-1] \text { and } r_{B}=[t-1] /[4 t-1] .
$$

Under our assumptions the optimal price-quality ratio of the good provided by firm $A$ is twice as much as that provided by firm $B$. For $t>1$, we have $r_{A}<1 / 2$ and
$r_{B}<1 / 4$. Using Proposition 1, both firms provide goods with better price-quality ratios than the monopolist.

From (4.2) we get $\theta_{A}=[2 t-1] /[4 t-1]<1 / 2$, thus $1-\theta_{A}=2 t /[4 t-1]>1 / 2$. This indicates that firm $A$ has a market share of more than $1 / 2$, which is larger than that of the monopolist. Also we have $\theta_{B}=[t-1] /[4 t-1]<1 / 4$, thus $\theta_{A}-\theta_{B}=$ $t /[4 t-1]>1 / 4$. This indicates that the total market served is more than $3 / 4$. We know in monopoly only half of the market is served, therefore the total market served expands more than 50 percent in duopoly.

From (4.5) and (4.6), we have $C_{A}^{\prime}\left(q_{A}\right)>1 / 4$ and $C_{B}^{\prime}\left(q_{B}\right)<1 / 16$ for $q_{A}>q_{B}$. If firms have the same development cost, $C_{A}(q)=C_{B}(q)$, then $q_{B}<q_{M}<q_{A}$. This means that the high and low quality firms produce information goods with qualities that bracket the quality chosen by a monopolist. So we have the following proposition:

Proposition 4.3 (Simultaneous vs. Monopoly) i) With equal development cost, the high quality firm produces a higher quality good than a monopolist. ii) Both firms provide goods with better price-quality ratios than in monopoly.

### 4.4.4 Best Response Functions and Equilibrium Analysis

In the following we discuss some characteristics of the best response functions of firm $A$ and $B$. For $q_{A}>q_{B}$, we denote the best response functions of firm $A$ and $B$ by $q_{A}=q_{A}^{*}\left(q_{B}\right)$ and $q_{B}=q_{B}^{*}\left(q_{A}\right)$, respectively. And for $q_{A}<q_{B}$, we denote the best response functions of firm $A$ and $B$ by $q_{A}=q_{A}^{\prime *}\left(q_{B}\right)$ and $q_{B}=q_{B}^{*}\left(q_{A}\right)$, respectively.

The best response function $q_{A}=q_{A}^{*}\left(q_{B}\right)$ is implicitly defined by (4.5). Rewriting (4.5) to emphasize this, we have $C_{A}^{\prime}\left(q_{A}^{*}\right) \equiv 4 q_{A}^{*}\left[4\left[q_{A}^{*}\right]^{2}-3 q_{A}^{*} q_{B}+2 q_{B}^{2}\right] /\left[4 q_{A}^{*}-q_{B}\right]^{3}$.

Taking the first derivative with respect to $q_{B}$, we have

$$
\frac{d q_{A}^{*}(\cdot)}{d q_{B}}=\frac{8 q_{A} q_{B}\left[5 q_{A}+q_{B}\right]}{C_{A}^{\prime \prime}\left(q_{A}\right)\left[4 q_{A}-q_{B}\right]^{4}+8 q_{B}^{2}\left[5 q_{A}+q_{B}\right]}>0
$$

This means that the best response quality $q_{A}$ increases in $q_{B}$. Similarly, the best response function $q_{B}=q_{B}^{*}\left(q_{A}\right)$ is implicitly defined by (4.6), and rewriting gives $C_{B}^{\prime}\left(q_{B}^{*}\right) \equiv q_{A}^{2}\left[4 q_{A}-7 q_{B}^{*}\right] /\left[4 q_{A}-q_{B}^{*}\right]^{3}$. Taking the first derivative with respect to $q_{B}$, we have

$$
\frac{d q_{B}^{*}(\cdot)}{d q_{A}}=\frac{2 q_{A} q_{B}\left[8 q_{A}+7 q_{B}\right]}{C_{B}^{\prime \prime}\left(q_{B}\right)\left[4 q_{A}-q_{B}\right]^{4}+2 q_{A}^{2}\left[8 q_{A}+7 q_{B}\right]}>0
$$

Thus, the best response quality $q_{B}$ increases in $q_{A}$.
The analysis is symmetric for $q_{A}<q_{B}$, where we have the best response functions $q_{A}=q_{A}^{\prime *}\left(q_{B}\right)$ and $q_{B}=q_{B}^{\prime *}\left(q_{A}\right)$. Figure 4.1 depicts the shape of the best response functions. If the competing firms have the same development cost, then there are two equilibria where either firm $A$ or $B$ can provide high quality good while the other firm provides low quality good (Jones and Mendelson, 2005). As is shown in Figure 4.1, if $q_{A}>q_{B}$, then the equilibrium is $N$, and if $q_{A}<q_{B}$, then the equilibrium is $N^{\prime}$. However, if the development cost functions differ for the competing firms, then it is possible that there is only one equilibrium where the firm with the development cost advantage develops the high quality good. For example, firm $B$ with superior technology may find it more profitable to develop the information good with higher quality $q_{B}^{H}$ instead of $q_{B}^{N}$, thus the equilibrium point goes to $N^{\prime}$.

### 4.5 Sequential Move: Strategic Accommodation and Entry Deterrence

In this section we analyze the situation where one firm enters the market earlier than the other. Thus there are three stages: leader chooses quality and whether to version,


Figure 4.1: Two Nash Equilibria in the Simultaneous Game
the follower observes this and chooses quality and whether to enter, and then prices are set. In this sequential duopoly game, the leader can accommodate or deter entry through its choice of quality and whether to version. If the leader accommodates entry, then it is a Stackelberg game where a first mover advantage is obtained by strategically setting quality. Alternatively, the leader may find it profit maximizing to deter entry. In this case, we show that the leader can strategically set quality to deter entry from the high-end market while implementing a versioning strategy to deter entry from the low-end market. Development cost determines whether the leader accommodates or deters entry.

### 4.5.1 Entry Accommodation - A Stackelberg Solution

In the sequential move (Stackelberg) duopoly game, we denote the leader as $A$ and the follower as $B$. Consider first a game of entry accommodation. The leader first develops an information good of quality $q_{A}$ and sets price $p_{A}$. Then the follower determines whether to enter the market. If entry is profitable, the follower determines its best response quality $q_{B}$ and then firms compete in prices. Consumers choose their preferred goods after the qualities and prices are determined.

Working backwards, the leader chooses $q_{A}$ such that $q_{A}>q_{B}$ or $q_{A}<q_{B}$. For vertical differentiation, Jones and Mendelson (2005) show that with a special exponential development cost function form for both firms, the firm with the high quality good gains the greatest profits. We start with the situation where the leader prefers $q_{A}>q_{B}$ first and then discuss the situation where leader chooses $q_{A}<q_{B}$.

We write the follower's best response function as $q_{B}=q_{B}\left(q_{A}\right)$. The quality provided by the leader is determined by

$$
\max _{q_{A}} \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right) .
$$

From the first order condition, we have

$$
\begin{equation*}
\frac{d \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{d q_{A}}=\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{A}}+\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{B}} \frac{d q_{B}}{d q_{A}}=0 \tag{4.18}
\end{equation*}
$$

Because $\Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)$ can be written as $\Pi_{A}\left(q_{A}, q_{B}\right)$ in (4.3), we have

$$
\begin{equation*}
\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\right)}{\partial q_{B}}=-\frac{4 q_{A}^{2}\left[2 q_{A}+q_{B}\right]}{\left[4 q_{A}-q_{B}\right]^{3}}<0 \tag{4.19}
\end{equation*}
$$

The best response function $q_{B}\left(q_{A}\right)$ is determined implicitly by (4.6) and is increasing in $q_{A}$. Thus, we have $d q_{B}\left(q_{A}\right) / d q_{A}>0$. From (4.18), we have $\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right) / \partial q_{A}>0$ at the Stackelberg point.

We know for the simultaneous game, at the Nash equilibrium, $\partial \Pi_{A}\left(q_{A}, q_{B}\right) / \partial q_{A}=$ 0 . Denoting the Stackelberg quality provided by the leader as $q_{A}^{S}$ and the Nash equi-
librium quality from the simultaneous game as $q_{A}^{N}$, from the concavity of $\Pi_{A}\left(q_{A}, q_{B}\right)$ in $q_{A}$, we have $q_{A}^{S}<q_{A}^{N}$. It means with the first mover advantage, the leader lowers quality to increase profit.

This result is interesting. In a traditional Stackelberg game with quantities, the leader increases quantity to gain first mover advantage (Church and Ware, 2000, p.468-470), while in our model of quality and price competition, the leader decreases quality.

If the follower has lower development costs, then the Stackelberg game may have another outcome. As is shown in Figure 4.2, the follower with lower development costs may find it more profitable to develop an information good with higher quality than $q_{A}^{S}$, which means $\Pi_{B}\left(q_{A}^{S}, q_{B}^{S}\right)<\Pi_{B}\left(q_{A}^{S}, q_{B}^{H}\right)$. As shown in Section 4.5.4 numerically, if the follower has "much lower" development costs, then the leader chooses $q_{A}<q_{B}$. For $q_{A}<q_{B}$, we find that as compared to the simultaneous game, with a first mover advantage the leader increases its quality (proofs are in Appendix C).

To summarize the above, we have the following proposition:

Proposition 4.4 (Stackelberg) Compared to a simultaneous game, in a Stackelberg game a leader that provides a high quality good decreases quality, and a leader that provides a low quality good increases quality.

### 4.5.2 Entry Deterrence

In the Stackelberg game above, we assumed that the leader accommodates entry. But the leader can also deter entry with its choice of quality. In the following we show that in a sequential game, the first mover can set information good quality to deter entry from the high-end market while implementing a versioning strategy to deter entry from the low-end market.


Figure 4.2: A Stackelberg Game Solution

## Entry Deterrence from the High-end Market

To begin we analyze potential entry in the high-end market, which means that the follower develops quality $q_{B}>q_{A}$. Once entry occurs, the equilibrium prices and profits of both firms are determined in the same manner as in the simultaneous game. Thus, we have the equilibrium profits as in (4.3) and (4.4) except with $A$ and $B$ reversed.

Taking the total derivative of $\Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right)$ with respect to $q_{A}$, we have

$$
\frac{d \Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{d q_{A}}=\frac{\partial \Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{A}}+\frac{\partial \Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{B}} \frac{d q_{B}}{d q_{A}}
$$

The first term of the right hand side of the equation has the same form as (4.19),
except with $A$ and $B$ reversed, and it is negative. For the second term, from the analysis of the best response functions in Section 4.4, the follower's quality increases when the leader increases quality, so we have $d q_{B} / d q_{A}>0$. Because $\Pi_{B}\left(q_{A}, q_{B}\right)$ is concave in $q_{B}$ and the entry deterrence quality $q_{B}$ is never lower than the Nash equilibrium quality, we get that $\partial \Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right) / \partial q_{B}$ is non-positive. Therefore, $d \Pi_{B}\left(q_{A}, q_{B}\left(q_{A}\right)\right) / d q_{A}$ is negative, which means when $q_{A}$ increases, the follower's profit decreases. Thus, the leader can strategically set quality such that the profit of the follower from the high-end market is zero. This entry deterrence quality is jointly determined by the above profit constraint and the follower's best response function. ${ }^{4}$

When the leader successfully deters entry, it acts as a monopolist. We denote the entry deterrence quality of the leader by $q_{A}^{E}$, and the monopoly quality and profit from Section 4.3 by $q_{M}$ and $\Pi_{M}(q)$, respectively. The entry deterrence quality $q_{A}^{E}$ is not lower than $q_{M}$, otherwise the leader can safely produce at $q_{M}$ and deter entry. Thus we get $q_{M} \leq q_{A}^{E}<q_{B} .{ }^{5}$ From the concavity of $\Pi_{M}(\cdot)$, we have $\Pi_{M}\left(q_{M}\right) \geq$ $\Pi_{M}\left(q_{A}^{E}\right)>\Pi_{M}\left(q_{B}\right)$. With equal development costs, $\Pi_{M}\left(q_{B}\right)>\Pi_{B}\left(q_{A}, q_{B}\right)=0$ because a monopolist always earns more profits. Thus we get the monopoly profits of the leader under entry deterrence are positive, which means the leader can always profitably deter entry.

From discussions in this section, we have the following proposition.

## Proposition 4.5 (Entry Deterrence) i) The leader can strategically set quality

 to deter entry from the high-end market. ii) With equal development costs, the leader can always profitably deter entry.Similar to the "top dog strategy" that overinvestment in capacity makes the leader tougher (Fudenberg and Tirole, 1984) and the threat of a predatory output increase

[^14]after entry made credible by carrying excess capacity prior to entry (Dixit, 1980), with information goods the leader chooses to overinvest in development to deter entry. If the leader has sunk its development costs to produce the entry deterrence quality, then the enhanced quality is always a credible threat to the follower.

## Entry Deterrence from the Low-end Market

When the leader strategically sets quality to deter entry from the high-end market, it opens another door to the follower - entry may occur in the low-end market. In this setting, the leader has already developed its high quality version $q_{A}^{H}$, and generates a low quality version $q_{A}^{L}$ to deter entry from the low-end market. The follower determines whether to enter, and if entry is profitable, then the follower determines its quality $q_{B}$, and firms compete in prices. Consumers select their preferred goods after the qualities and prices of the information goods are determined.

In this model we assume $q_{A}^{L}<q_{B}<q_{A}^{H}{ }^{.}{ }^{6}$ This is the same setting as the Case 1 when firm A considers versioning in Section 4.2, only with different objective that here the leader chooses the quality of its lower quality version, $q_{A}^{L}$, so that the follower gets non-positive profits. Substituting the equilibrium prices into the profit function for firm $B$ in (4.8), we have

$$
\Pi_{B}\left(q_{A}^{H}, q_{B}, q_{A}^{L}\right)=\frac{4 q_{B}^{2}\left[q_{A}^{H}-q_{B}\right]\left[q_{A}^{H}-q_{A}^{L}\right]\left[q_{B}-q_{A}^{L}\right]}{\Lambda^{2}}-C_{B}\left(q_{B}\right)
$$

Taking the partial derivative of $\Pi_{B}(\cdot)$ with respect to $q_{A}^{L}$, we have,

$$
\frac{\partial \Pi_{B}\left(q_{A}^{H}, q_{B}, q_{A}^{L}\right)}{\partial q_{A}^{L}}=\frac{-8 q_{B}^{2}\left[q_{A}^{H}-q_{B}\right]^{2}}{\Lambda_{1}^{3}}\left[2 q_{A}^{H} q_{B}+q_{A}^{H} q_{A}^{L}+q_{B}^{2}-4 q_{B} q_{A}^{L}\right]<0 .
$$

It means the higher the $q_{A}^{L}$, the lower the profit of the follower. Therefore, the leader can use versioning $q_{A}^{L}$ to deter entry.

[^15]In order to effectively deter entry, $q_{A}^{L}$ must be set so that $\Pi_{B}\left(q_{B}\right) \leq 0$. Through the envelope theorem, we have $\partial \Pi_{B}\left(q_{A}^{H}, q_{B}, q_{A}^{L}\right) / \partial q_{B}=0$. Thus, firm A determines $q_{A}^{L}$ by setting the profit of the follower to zero, which is equivalent to

$$
C_{B}^{\prime}\left(q_{B}\right) / q_{B}=\frac{4 q_{B}\left[q_{A}^{H}-q_{B}\right]\left[q_{A}^{H}-q_{A}^{L}\right]\left[q_{B}-q_{A}^{L}\right]}{\Lambda_{1}^{2}}
$$

and the equilibrium quality $q_{B}$ is determined by

$$
\begin{aligned}
C_{B}^{\prime}\left(q_{B}\right)= & \frac{8 q_{B}\left[q_{A}^{H}-q_{A}^{L}\right]}{\Lambda_{1}^{3}}\left\{q_{A}^{H}\left[q_{A}^{H}-q_{B}\right]\left[4 q_{B}^{2}+2\left[q_{A}^{L}\right]^{2}-3 q_{B} q_{A}^{L}\right]\right. \\
& \left.+q_{B} q_{A}^{L}\left[q_{B}-q_{A}^{L}\right]\left[2 q_{B}+q_{A}^{H}\right]-3 q_{B}^{3}\left[q_{A}^{H}-q_{A}^{L}\right]\right\}
\end{aligned}
$$

Clearly, the optimal entry deterrence quality of the sub-version depends on the development costs of the follower. The following proposition concludes this sub-section.

Proposition 4.6 (Entry Deterrence) The leader can generate low quality versions to deter entry from the low-end market.

We denote the comparative quality ratios of $q_{A}^{H}, q_{B}$ with respect to $q_{A}^{L}$ by $t_{H}=$ $q_{A}^{H} / q_{A}^{L}$ and $t_{B}=q_{B} / q_{A}^{L}$. The optimal price-quality ratio of versions $q_{A}^{H}$ and $q_{A}^{L}$ provided by the leader are denoted by $r_{H}=p_{A}^{H} / q_{A}^{H}$ and $r_{L}=p_{A}^{L} / q_{A}^{L}$, respectively. The pricequality ratio of versions $q_{B}$ provided by the follower is denoted by $r_{B}=p_{B} / q_{B}$. From the equilibrium price equations (4.9), (4.10) and (4.11) in the Appendix, we get

$$
r_{H}=\frac{\left[t_{H}-t_{B}\right]\left[4 t_{B}-1-3 t_{B} / t_{H}\right]}{2\left[4 t_{H} t_{B}-t_{H}-2 t_{B}-t_{B}^{2}\right]}, r_{L}=\frac{\left[t_{H}-t_{B}\right]\left[t_{B}-1\right]}{2\left[4 t_{H} t_{B}-t_{H}-2 t_{B}-t_{B}^{2}\right]},
$$

and

$$
r_{B}=\frac{\left[t_{H}-t_{B}\right]\left[t_{B}-1\right]}{\left[4 t_{H} t_{B}-t_{H}-2 t_{B}-t_{B}^{2}\right]}
$$

From the above equations, we have $r_{B}=2 r_{L}$ and $r_{H}>2 r_{B}$. It indicates that the price-quality ratio of the high quality version is more than four times that of the low quality version.

### 4.5.3 Entry Deterrence, Rivalry Clear-out or Coexistence

We know from the previous discussion that the leader can develop higher quality to deter entry from the high-end market and generate versions to deter entry into the low-end market. The key questions are whether it is profit maximizing for the leader to deter entry, and if rivalry already exists in the market, whether it is profit maximizing for one firm to drive its competitor out. If the answer of either of the above questions is "no", then the leader may choose to coexist with its competitor.

## Rivalry Clear-out or Coexistence

We first consider the case where firms $A$ and $B$ are already in the market. with information goods $q_{A}$ and $q_{B}$, and we suppose $q_{A}>q_{B}$. Because the development costs are sunk and there is no marginal cost, a firm will not exit if the price of its good is positive. From Section 4, we know that in equilibrium, profits for firm $A$ and $B$ are (4.3) and (4.4).

Firm $B$ with a lower quality information good cannot drive firm $A$ out of the market. For firm $A$ to drive out firm $B$, it can generate a lower quality version with quality $q_{B}$ and prices of $q_{B}$ go to zero from Bertrand competition. The equilibrium profit for firm $B$ is zero and profit for firm $A$ which we denote by $\Pi_{A}^{C l e a r}\left(q_{A}, q_{B}\right)$, is

$$
\Pi_{A}^{\text {Clear }}\left(q_{A}, q_{B}\right)=\left[q_{A}-q_{B}\right] / 4-C_{A}\left(q_{A}\right)
$$

The first part of the above profit equation is the revenue generated from $q_{A}$ and the second part is the development costs of $q_{A}$. It is straightforward to see that $\Pi_{A}^{\text {Clear }}\left(q_{A}, q_{B}\right)<\Pi_{A}\left(q_{A}, q_{B}\right)$. Therefore, firm $A$ is better off coexisting with firm $B$ because a lower quality version intensifies competition, similar to Judd (1985) and Nault (1997). Therefore, versioning is not optimal in a game when both firms are already in the market.

## Entry Deterrence and Strategic Analysis with Versioning

From the discussion earlier, we know it is always profit maximizing for the leader to generate a lower quality version to deter entry from the low-end market. And, in the high-end market, the sunk costs of development pose a credible threat to deter entry. However, entry deterrence may not be profit maximizing.

Let $q_{A}^{D}$ be the minimum entry deterrence quality in the high-end market, as described in Section 4.5.2. If the leader produces quality less than $q_{A}^{D}$, then entry occurs in the high-end market. As before, let $q_{M}$ be the monopoly quality of the information good from Section 4.3. $q_{M}$ is determined by $C_{A}^{\prime}\left(q_{M}\right)=1 / 4$. If $q_{A}^{D} \leq q_{M}$, then entry in the high-end market is deterred.

If $q_{M}<q_{A}^{D}$, then the leader has to increase quality, which in turn decreases profits, relative to the monopolist. Recall from Section 4.5.1 that the leader may choose a lower quality, allowing the follower to enter with higher quality. Denote this low Stackelberg quality as $q_{A}^{S^{\prime}}$ when accommodating entry in the high end. Let $q_{A}^{D I}$ be the highest quality so that the leader is indifferent between producing high quality to deter entry and low Stackelberg quality $q_{A}^{S^{\prime}} \cdot q_{A}^{D I}$ is determined by $\Pi_{M}\left(q_{A}^{D I}\right)=\Pi_{A}\left(q_{A}^{S^{\prime}}, q_{B}^{S^{\prime}}\right)$. Because $\Pi_{M}\left(q_{A}^{D I}\right)<\Pi_{M}\left(q_{M}\right)$ and concavity of $\Pi_{M}(\cdot)$, we have $q_{A}^{D I}>q_{M}$. If $q_{A}^{D}<q_{A}^{D I}$, which means $\Pi_{M}\left(q_{A}^{D}\right)>\Pi_{M}\left(q_{A}^{D I}\right)=\Pi_{A}\left(q_{A}^{S^{\prime}}, q_{B}^{S^{\prime}}\right)$, then entry deterrence is profit maximizing. Otherwise, it is profit maximizing for the leader to accommodate entry with $q_{A}^{S^{\prime}}$.

Thus, the optimal strategies for the leader are as follows (details are in the appendix):

- If $q_{A}^{D} \leq q_{M}$, then the optimal quality of the information good provided by the leader is $q_{M}$. Versioning is implemented in the low-end market and entry is deterred.
- If $q_{M}<q_{A}^{D}<q_{A}^{D I}$, then the optimal quality of the information good provided by the leader is $q_{A}^{D}$. Versioning is implemented in the low-end market and entry is deterred.
- If $q_{M}<q_{A}^{D I} \leq q_{A}^{D}$, then the quality of the information good provided by the leader is $q_{A}^{S^{\prime}}$. No versioning is implemented and entry is accommodated. The follower quality is $q_{B}^{S^{\prime}}$.


### 4.5.4 A Numerical Example

Here we use a numerical example to illustrate which strategies firms adopt in different situations. Similar to Jones and Mendelson (2005), we assume development costs are quadratic in quality, $C(q)=K q^{2}$. Firms differ in the parameter $K$ : the higher the $K$, the higher are development costs. The indifferent quality $q_{A}^{B I}$ is defined as the quality of the good produced by firm $A$ where firm $B$ is indifferent between producing high and low quality. The indifferent quality $q_{B}^{A I}$ is defined as the quality of the good produced by firm $B$ where firm $A$ is indifferent between producing high and low quality.
Simultaneous Game. In Table 4.2, we show that if $K_{A} / K_{B} \leq 0.63$, then there is only one Nash equilibrium where firm $A$ produces the high quality good while firm $B$ produces the low quality good. For $0.63<K_{A} / K_{B}<1.59$, there are two Nash equilibria where either firm may produce the high quality good while the other firm produces the low quality good. When $K_{A} / K_{B} \geq 1.59$, then there is only one Nash equilibrium where firm $B$ produces the high quality good while firm $A$ produces the low quality good.

Stackelberg Game without Versioning. Even without versioning, in a sequential move duopoly, the leader can take the first mover advantage to reap more profit than in the simultaneous game. We show in Table 3 that if $K_{A} / K_{B}<1.482$, then the leader

Table 4.2: A Numerical Example: Simultaneous Game

|  | $K_{A} / K_{B}$ | $1 / 3$ | 0.63 | 1 | 1.59 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $q_{A}>q_{B}$ | $q_{A}$ | 125.2421 | 125.7666 | 126.6554 | 128.2485 | 129.326 | 131.648 |
|  | $q_{B}$ | 9.5605 | 16.6932 | 24.1193 | 33.2245 | 38.0931 | 46.8165 |
|  | $\Pi_{A}$ | 14.3711 | 13.355 | 12.2193 | 10.7125 | 9.8532 | 8.2175 |
|  | $\Pi_{B}$ | 0.2994 | 0.5257 | 0.7637 | 1.0648 | 1.2316 | 1.5408 |
|  | $q_{A}$ | 15.6055 | 20.9314 | 24.1193 | 26.5422 | 27.4641 | 28.6814 |
|  | $q_{B}$ | 43.8827 | 80.7953 | 126.6554 | 199.9688 | 251.0219 | 375.7263 |
|  | $\Pi_{A}$ | 0.5136 | 0.6701 | 0.7637 | 0.8347 | 0.8616 | 0.8982 |
|  | $\Pi_{B}$ | 2.7392 | 6.7489 | 12.2193 | 21.2345 | 27.571 | 43.1132 |
| Indifferent <br> quality level | $q_{A}^{B I}$ | 26.8955 | 50.8325 | 80.6866 | 128.2917 | 161.3732 | 242.0597 |
|  | $q_{B}^{A I}$ | 80.6866 | 80.6866 | 80.6866 | 80.6866 | 80.6866 | 80.6866 |

* The development cost function is $C(q)=K q^{2}$ and $K_{A}=0.001$.
can choose to produce at the Stackelberg point $S$ as indicated in Figure 2 where the leader produces a higher quality good than the follower. When $K_{A} / K_{B} \geq 1.482$, the leader cannot produce at Stackelberg point $S$ because the follower is better off producing higher quality than the leader at $S$. When $1.482 \leq K_{A} / K_{B}<2.572$, the leader can still get more profit by producing a higher quality good than the follower. To maximize profits, the leader produces at the point where the follower is indifferent between producing a higher quality good or a lower quality good. ${ }^{7}$ When $K_{A} / K_{B}=2.572$, the leader becomes indifferent between producing at the follower's indifferent point or producing at its low Stackelberg point $S^{\prime}$. In that case, the leader should produce at its low Stackelberg point $S^{\prime \prime}$. Thus we get when $K_{A} / K_{B} \geq 2.572$, the leader produces a lower quality good than the follower.

Entry Deterrence with Versioning. As discussed in the previous section, if versioning can deter entry from the low-end market, then the leader can set quality to deter entry from the high-end market and capture monopoly profits. For this specific

[^16]Table 4.3: A Numerical Example: Stackelberg Game

|  | $K_{A} / K_{B}$ | $1 / 2$ | 1 | 1.482 | 2 | 2.572 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $q_{A}>q_{B}$ | $q_{A}$ | 124.4645 | 122.59 | 119.7105 | 115.9129 | 111.5372 | 108.41 |
|  | $q_{B}$ | 13.7145 | 23.913 | 30.973 | 36.1843 | 39.8635 | 41.6273 |
|  | $\Pi_{A}$ | 13.7869 | 12.2352 | 11.0283 | 10.0136 | 9.1666 | 8.677 |
|  | $\Pi_{B}$ | 0.4304 | 0.7577 | 0.993 | 1.1754 | 1.3128 | 1.3835 |
|  | $q_{A}$ | 19.3227 | 24.197 | 26.2612 | 27.4738 | 28.2799 | 28.6908 |
|  | $q_{B}$ | 64.7292 | 126.6667 | 186.5366 | 251.0226 | 322.3302 | 375.7268 |
|  | $\Pi_{A}$ | 0.6159 | 0.7637 | 0.8254 | 0.8616 | 0.8858 | 0.8982 |
|  | $\Pi_{B}$ | 4.8767 | 12.2069 | 19.5666 | 27.5696 | 36.4514 | 43.1119 |
| Indifferent <br> quality level | $q_{A}^{B I}$ | 40.3433 | 80.6866 | 119.5775 | 161.3732 | 207.5259 | 242.0597 |
|  | $q_{B}^{A I}$ | 80.6866 | 80.6866 | 80.6866 | 80.6866 | 80.6866 | 80.6866 |

* The development cost function is $C(q)=K q^{2}$ and $K_{A}=0.001$.
** For the Stackelberg game, we assume firm $A$ moves before firm $B$.
development costs function, we show that if $K_{A} / K_{B} \leq 1.5$, then the leader is "natural monopoly". The leader can safely set the quality of its flagship version the same as when there is no competition and adopts versioning strategies to deter entry from below.

If the follower has much lower development costs, in our case $1.5<K_{A} / K_{B}<$ 2.9563 , then the leader has to increase its quality above the monopoly quality in order to deter entry. If the follower has a substantial development costs advantage, in our case $K_{A} / K_{B} \geq 2.9563$, then entry deterrence is no longer optimal. The leader is better off choosing quality at the lower Stackelberg point $S^{\prime}$ where $q_{A}<q_{B}$, as indicated in Figure 4.2. Through our analysis for the leader with and without versioning, we see that when the leader versions it acts more aggressively, that is, it tends to produce a higher quality good despite a development cost disadvantage.

Table 4.4: A Numerical Example: Entry Deterrence

|  | $K_{A} / K_{B}$ | $1 / 2$ | 1 | $3 / 2$ | 2 | 2.9563 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Stackelberg <br> Game | $q_{A}$ | 19.3227 | 24.197 | 26.3157 | 27.4738 | 28.6542 | 28.6908 |
|  | $q_{B}$ | 64.7292 | 126.6667 | 188.7753 | 251.0226 | 370.2737 | 375.7268 |
|  | $\Pi_{A}$ | 0.6159 | 0.7637 | 0.827 | 0.8616 | 0.8972 | 0.8982 |
|  | $\Pi_{B}$ | 4.8767 | 12.2069 | 19.8436 | 27.5696 | 42.4315 | 43.1119 |
| Entry <br> Deterrence | $q_{A}$ | 41.6667 | 83.3333 | 125 | 166.6667 | 246.3582 | 250 |
|  | $\Pi_{A}$ | 8.6806 | 13.8889 | 15.625 | 13.8889 | 0.8972 | 0 |
|  | $q_{A}$ | 125 | 125 | 125 | 125 | 125 | 125 |
|  | $\Pi_{A}$ | 15.625 | 15.625 | 15.625 | 15.625 | 15.625 | 15.625 |

* The development cost function is $C(q)=K q^{2}$ and $K_{A}=0.001$.


### 4.6 Welfare Implications

Because the marginal cost of producing information goods is zero, to be socially optimal the price of the information good must also be zero. We denote the socially optimal quality by $q^{\circ}$ and the optimal social welfare by $W^{O}$, where $q^{\circ}$ maximizes social welfare $W^{O}$. We know $W^{O}\left(q^{O}\right)=\int_{0}^{1} q^{O} \theta d_{\theta}-C\left(q^{O}\right)$, so the optimal quality is determined by $C^{\prime}\left(q^{O}\right)=1 / 2$. All consumers enjoy $q^{O}$ at price zero with total surplus $q^{O} / 2$, firm incurs negative profit $-C\left(q^{O}\right)$ (the sunk development costs). The optimal social welfare is $W^{O}\left(q^{O}\right)=q^{O} / 2-C\left(q^{O}\right)$.

From Section 4, we know the monopoly quality $q_{M}$ is determined by $C^{\prime}\left(q_{M}\right)=1 / 4$ and price $p_{M}$ is set equal to $q_{M} / 2$. Only half of the consumers in the market enjoy the information good and the total consumer surplus is $q_{M} / 8$. The monopolist profit is $\Pi_{M}=q_{M} / 4-C\left(q_{M}\right)$. The total social welfare is $W_{M}\left(q_{M}\right)=3 q_{M} / 8-C\left(q_{M}\right)$.

The social optimal and monopoly, compared in Table 4.5, represent two extremes where the first focuses on social welfare while the second focuses on the firm profits. At the social optimal, quality, consumer surplus and total social welfare are the highest. The monopolist obtains its profit by serving only half of the market, and the
monopolist earns the highest profit.

Table 4.5. Comparison of Socially Optimal and Monopoly

|  | Socially Optimal | Monopoly |
| :--- | :--- | :--- |
| Quality | $C^{\prime}\left(q^{O}\right)=1 / 2$ | $C^{\prime}\left(q_{M}\right)=1 / 4$ |
| Price | 0 | $q_{M} / 2$ |
| Market Coverage | 1 | $1 / 2$ |
| Consumer Surplus | $q^{O} / 2$ | $q_{M} / 8$ |
| Firm Profit | $-C\left(q^{O}\right)$ | $q_{M} / 4-C\left(q_{M}\right)$ |
| Total Social Welfare | $q^{O} / 2-C\left(q^{O}\right)$ | $3 q_{M} / 8-C\left(q_{M}\right)$ |

In a simultaneous move duopoly, given the same technology, firm $A$ produces $q_{A}$ which is higher than the monopoly $q_{M}$ while firm $B$ produces $q_{B}$ which is lower than the monopoly $q_{M}$. The market coverage of $q_{A}$ is more than $1 / 2$ and the total market coverage is more than $3 / 4$. The total profits of firm $A$ and $B$ are less but the total consumer surplus is higher than that of the monopoly. The social welfare is also higher than in monopoly. In the entry deterrence situation, if the leader accommodates entry, then it is equivalent to the simultaneous move duopoly. If the leader successfully deters entry, it acts like a monopolist. But in this case, the leader usually provides a higher quality than in monopoly without entry, and profits are lower. The consumer surplus is higher in the successful entry deterrence case and the market coverage is the same as in monopoly. In this situation, the social welfare cannot be determined without specifying a development cost function.

### 4.7 Conclusions

This paper focuses on the competition of vertically differentiated information goods. Under assumptions of linear utility function and convex development costs, we explain why leading producers usually dominate the market. First, we reproduce the basic
prior result in our context whereby a monopolist does not version. Next show that under competition, producers always offer information goods with a better pricequality ratio than in monopoly and more of the market is covered. Moreover, in a simultaneous move duopoly neither of the producers version. However, in a sequential game the leader can set quality higher than in monopoly to deter entry from the highend market, and implement versioning to deter entry from the low-end market. Thus, for versioning to occur requires that there be a sequential game, and that in this game it is profit maximizing for the leader to deter rather than accommodate entry.

In examining leader strategies to deter entry, although the sunk costs of development pose a credible threat to deter potential entry from the high-end market, it may not be profit maximizing since quality is set higher than it would be in monopoly. Nonetheless, it is always profit maximizing for the leader to implement versioning strategies to deter entry from the low-end market. However, for versioning to be effective, versioning must be implemented irrevocably because once the follower enters the low-end market, it is no longer optimal for the leader to maintain its lower quality version in the market. Thus, if the lower quality version can be removed, or priced as though it is dominated, then versioning is not a credible threat to deter entry from the low-end market. To make versioning a credible threat - that is, irrevocable, the leader must have some mechanism to tie its lower quality version good with its higher quality version to make the follower believe that the lower quality version good will not be withdrawn from the market post-entry. One suggested mechanism is for the leader to sign long term service contracts with consumers for all the sub-versions.

The key limitations of the paper lie in the functional form of consumers' utility and the distribution of consumers' types, which restricts the generality of our results. Our results rely on the assumptions that a consumer's utility is multiplicative in taste and quality, and that consumers are uniformly distributed in their individual
taste for quality. Further research can generalize the utility function and consumers' distribution. In the meanwhile, there are two possible extensions for this paper. The first one is to consider network externality effect. In that case, the various degraded versions may not just act as a "signal" to deter entry, but effective means to maximize profit (Jing, 2002). The other extension is to consider temporal issues for the development and marketing of information goods: timing may have significant impact on the development costs and the consequent optimal price and quality choices of information goods producers.

## Chapter 5

## CONCLUSIONS

Versioning has been broadly adopted by. information goods producers to segment the market. However, using a linear utility function where consumers' utility is multiplicative in individual taste and quality, previous research shows that versioning is not optimal for profit maximizing producers (Bhargava and Choudary 2001, Jones and Mendelson 2005). To explain the popularity of versioning, some research explores different contexts such as network externalities (Jing 2002), competition (Jones and Mendelson 2005), anti-piracy (Wu, Chen and Anandalingam 2003), and interorganizational information systems (Nault 1997) while others resort to modifying the utility function (Ghose and Sundararajan 2005, Bhargava and Choudary 2007, Chen and Seshadri 2007, etc.). In this thesis, we maintain a linear utility function to analyze versioning strategies of information goods producers in monopoly and under competition.

Our first essay investigates conditions that determine when an information goods monopolist chooses to implement versioning strategies. We show that versioning strategies are implemented only when different groups of consumers can be clearly defined. In other words, versioning cannot be used to segment a market, rather, versioning strategies must fit the existing market segments. Our optimal versioning strategies are in accordance with Shapiro and Varian's (1999) suggestion that versions should be designed to accentuate the differences between groups in their tastes. We demonstrate that in a horizontally differentiated market, if there is no crosspurchasing, then it is optimal to provide one version for each segment. Otherwise, the monopolist shrinks the market in the lower-end segment to protect profits in the
higher-end segments. As a special case of horizontal differentiation, vertical differentiation is not optimal when characteristics of information goods are undifferentiated to different groups. To explain multiple versions in vertical differentiation, we introduce group related characteristics and show that a lower version is provided only when all higher segments are covered. We further investigate how versioning costs impact versioning strategies and design an algorithm to determine the optimal aggregation of segments when versioning costs are significant.

Our second essay treats information goods as experience goods and constructs models to analyze conditions under which a monopolist implements versioning strategies and explain how experience impacts pricing and versioning decisions based on different payment arrangements. With lump sum payments, we show that a monopolist generates vertically differentiated versions of information goods as bridges that lead consumers to experience the goods so that they can appreciate their true quality, and then provide upgrades to consumers who initially purchase a lower quality version. Adopting a two-stage model, we find that if all the consumers have homogeneous initial expectations about the goods' quality, then the monopolist's optimal pricing strategy involves upgrading all the consumers who purchase the low quality version in the first stage to the high quality version in the second stage. In this way, consumers that upgrade pay a tax for learning. When consumers have heterogeneous expectations, we find that if consumers are pessimistic, then the monopolist's optimal pricing still drives all consumers to upgrade. However, if consumers are optimistic, then under certain conditions, the monopolist's optimal pricing strategy induces only some of the consumers that initially purchased the low quality version to upgrade. With periodic license fees, the monopolist may implement versioning strategies to encourage more consumers to realize the true quality of the information goods when facing pessimistic consumers, and only one version should be offered when facing
optimistic consumers.
Our third essay focuses on competition with vertically differentiated information goods. Under assumptions of a linear utility function and convex development costs, we explain why leading producers usually dominate the market. We show that under competition, producers always offer information goods with a better price-quality ratio than in monopoly and more of the market is covered. Moreover, in a simultaneous move duopoly neither of the producers version. However, in a sequential game the leader can set quality higher than in monopoly to deter entry from the high-end market, and implement versioning to deter entry from the low-end market. Thus, for versioning to occur requires that there be a sequential game, and that in this game it is profit maximizing for the leader to deter rather than accommodate entry. In examining strategies to deter entry, although the sunk costs of development pose a credible threat to deter potential entry from the high-end market, it may not be profit maximizing since quality is set higher than it would be in monopoly. Nonetheless, it is always profit maximizing for the leader to implement versioning strategies to deter entry from the low-end market. However, for versioning to be effective, versioning must be implemented irrevocably because once the follower enters the low-end market, it is no longer optimal for the leader to maintain its lower quality version in the market. Thus, if the lower quality version can be removed, or priced as though it is dominated, then versioning is not a credible threat to deter entry from the low-end market. To make versioning a credible threat - that is, irrevocable, the leader must have some mechanism to tie its lower quality version good with its higher quality version to make the follower believe that the lower quality version good will not be withdrawn from the market post-entry. One suggested mechanism is for the leader to sign long term service contracts with consumers for all the sub-versions.

The key limitations of our research lie in the function form of consumers' utility,
which restricts the generality of our results. Our results rely on the assumption that a consumer's utility is multiplicative in taste and quality which may be generalized in future research. Meanwhile, there are two possible extensions. The first one is to include network externalities that are common for a series of information goods. The other extension is to consider temporal issues for the development and marketing of information goods: timing may have significant impact on the development costs and the consequent optimal price and quality choices of information goods producers.

## REFERENCES

Arora, A., J. P. Caulkins, and R. Telang. 2006. Research Note - Sell First, Fix Later: Impact of Patching on Software Quality. Management Science, 52(3), pp. 465-471. Bakos, Y., E. Brynjolfsson. 1999. Bundling Information Goods: Pricing, Profits, and Efficiency. Management Science. 45(12), pp. 1613-1630.

Bhargava, H.K., and V. Choudhary. 2001. Information Goods and Vertical Differentiation. Journal of Management Information Systems, Fall 2001, Vol. 18(2), pp. 89-106.

Bhargava, H. K., V. Choudhary. 2008. When is Versioning Optimal for Information Goods? Management Science. 54(5), pp. 1029-1035.

Chellappa, R.K. and S. Shivendu. 2005. Managing Piracy: Pricing and Sampling Strategies for Digital Experience Goods in Vertically Segmented Markets. Information Systems Research. 16(4), pp. 400-417.

Chen, Y., S. Seshadri. 2007. Product Development and Pricing Strategy for Information Goods Under Heterogeneous Outside Opportunities. Information Systems Research. 18(2), pp. 150-172.

Choudhary, V., Ghose, A., Mukopadhyay, T., and U. Rajan. 2005. Personalized Pricing and Quality Differentiation. Management Science, 51 (7), pp. 1120-1130.

Church, J. R., and R. Ware. 2000. Industrial Organization: A Strategic Approach. McGraw-Hill Companies, Inc.

Cottrell, T., and B. R. Nault. 2004. Product Variety and Firm Survival in the Microcomputer Software Industry. Strategic Management Journal, 25(10), pp. 1005 -1026.

Dewan, R., Jing, B., and A. Seidmann. 2003. Product Customization and Price Competition on the Internet. Management Science, 49 (8), pp. 1055-1070.

Dixit, A. 1980. The Role of Investment in Entry Deterrence. Economic Journal, 90, pp. 95-106.

Economides, N. 1996. The Economics of Networks. International Journal of Industrial Organization, Vol.14, pp. 673-699.

Fudenberg, D., and J. Tirole. 1984. The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look. American Economic Review, Vol. 74, No. 2, pp. 361-366. Ghose, A., M. Smith, and R. Telang. 2006. Internet Exchanges for Used Books: An Empirical Analysis of Product Cannibalization and Welfare Impact. Information Systems Research 17(1), pp. 3-19.

Ghose, A. and A. Sundararajan. 2005. Software Versioning and Quality Degradation? An Exploratory Study of the Evidence. Working Paper. New York University. Google. 2006. Information on Google. (September 6) www.google.com

Hahn, J. H. 2001. Functional Quality Degradation of Software with Network Externalities. Discussion Paper, Keele University.
http://www.microsoft.com
http://www.symantec.com
Jing, B. 2002. Versioning information goods with network externalities. Discussion paper, Stern School of Business, New York University.

Jones, R., and H. Mendelson. 2005. Information Goods: Development, Quality and Competition. Discussion paper, Stanford University.

Judd, K. L. 1985. Credible Spatial Preemption. RAND Journal of Economics, 16(2), pp. 153-166.

Kim, J. 1992. Experience Goods, Expectations and Pricing. Economic Record, 68(200), pp. 7-15.

Liebeskind, J., and R. P. Rumelt. 1989. Markets for Experience Goods with Performance Uncertainty. The RAND Journal of Economics, 20(4), pp. 601-621.

Nault, B. R., and A. S. Dexter. 1995. Added Value and Pricing with Information Technology. MIS Quarterly, 19(4), pp. 449-463.

Nault, B. R. 1997. Quality Differentiation and Adoption Costs: The Case for Interorganizational Information Systems Pricing. Annals of Operations Research, 71, pp. 115-142.

Nault, B.R. and M.B. Vandenbosch. 1996. Eating Your Own Lunch: Protection Through Preemption. Organization Science, 7, 3, pp. 342-358.

Nelson, P. 1970. Information and Consumer Behavior. The Journal of Political Economy, 78 (2), pp. 311-329.

Oracle Corporation. Oracle is Number 1 on Linux. 2006 (September 8). www.oracle.com Riordan, H. R. 1986. Monopolistic Competition with Experience Goods. The Quarterly Journal of Economics, 101(2), pp. 265-180.

Rosen, S. 1974. Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. Journal of Political Economics. 82(1), pp. 34-55.

Shaked, A., and J. Sutton. 1983. Natural Oligopolies. Econometrica. September, 51 (5), pp. 1469-1483.

Shapiro, C. 1983. Optimal Pricing of Experience Goods. The Bell Journal of Economics. Autumn, 14 (2), pp. 497-507.

Shapiro, C., and H. R. Varian.' 1999. Information Rules. Harvard Business School Press.

Sundararajan, A. 2003. Network Effects, Nonlinear Pricing and Entry Deterrence. Working paper, Leonard N. Stern School of Business, New York University. Sundararajan, A. 2004. Nonlinear Pricing of Information Goods. Management Science, 50 (12), pp. 1660-1673.

Varian, H. R. 2000. Buying, Sharing and Renting Information Goods. Journal of Industrial Economics, 48(4), pp. 473-488.

Villas-Boas, J. M. 2006. Dynamic Competition with Experience Goods. Journal of Economics and Management Strategy, Vol. 15, pp. 37-66.

Wei, X., and B. R. Nault. 2005. Product Differentiation and Market Segmentation of Information Goods. Discussion Paper at Workshop on Information Systems and Economics (WISE), Dec. 2005, Irvine, CA.

Wei, X., and B. R. Nault. 2006. Vertically Differentiated Information Goods: Entry Deterrence, Rivalry Clear-out or Coexistence. Proceedings of the 2006 INFORMS Conference on Information Systems and Technology, Pittsburgh, Pennsylvania. Wei, X., and B. R. Nault. 2007. Experience Information Goods: Versioning and Upgrading. Proceedings of the 2007 INFORMS Conference on Information Systems and Technology, Seattle, Washington.

Wilde, L. L. 1981. Information Costs, Duration of Search, and Turnover: Theory and Applications. Journal of of Political Economy, 89 (6), pp. 1122-1141.

Windows IT Pro. 2002. Windows Market Share Rises But PC Sales Flatten. (September 10) www.windowsitpro.com

Wu, S., P. Chen, and G. Anandalingam. 2003. Fighting Information Goods Piracy with Versioning. Proceedings of the International Conference on Information Systems. Seattle, WA.

## APPENDIX A

## Proof of Lemma 2.1

First, as we treat different groups separately, this is the set-up for every group. We can solve for $\theta_{n}^{*}, n \in\{1,2, \cdots, N\}$, which is,

$$
\theta_{n}^{*}=\text { Solve }\left\{\theta=\frac{F\left(\theta_{n}\right)-F(\theta)}{f(\theta)} \text { for } \theta \in\left[\theta_{n-1}, \theta_{n}\right)\right\}
$$

Or $\theta_{n}^{*}=\theta_{n-1}$ if there is no interior solution for the above equation. The optimal price for group $n$ is $p_{n}=U\left(k_{n}, q_{n}, \theta_{n}^{*}\right), n \in\{1,2, \cdots, N\}$. The constraints to prevent cross-purchasing are

$$
U\left(k_{n}, q_{j}, \theta\right)-p_{j} \leq 0, \text { for } n \neq j \text { and } \theta \in\left[\theta_{n-1}, \theta_{n}\right)
$$

We know if cross-purchasing occurs, consumers in the highest group $N$ are the most likely to purchase the good tailored for group $i$ because they get the most utility out of the shared characteristics. To prevent this, we set

$$
U\left(k_{N}, q_{n}, \theta_{N}^{*}\right)-p_{n} \leq 0, n \in\{1,2, \cdots, N-1\}
$$

Solving the above constraints, we get

$$
\frac{q_{a}}{q_{n}} \leq \frac{k_{n} \theta_{n}^{*}}{k_{N} \theta_{N}^{*}}, n \in\{1,2, \cdots, N-1\} .
$$

This is the necessary condition.
When the above conditions are satisfied, consumers in $\left[\theta_{N-1}, \theta_{N}^{*}\right)$ do not purchase because they get negative surplus purchasing either $q_{N}$ or $q_{n}$. Consumers in $\left(\theta_{N}^{*}, \theta_{N}\right]$ prefer $q_{N}$ to $q_{n}$ because they receive greater utility purchasing $q_{N}$ rather than $q_{n}$. This can be shown as follows:

$$
\frac{q_{a}}{q_{n}} \leq \frac{k_{n} \theta_{n}^{*}}{k_{N} \theta_{N}^{*}} \Leftrightarrow-k_{N} \theta_{N}^{*} q_{a} \geq-k_{n} \theta_{n}^{*} q_{n} \Leftrightarrow k_{N} \theta_{N}^{*} q_{N}-k_{N} \theta_{N}^{*} q_{a} \geq k_{N} \theta_{N}^{*} q_{N}-k_{n} \theta_{n}^{*} q_{n}
$$

And we know $k_{N} \theta\left(q_{N}-q_{a}\right)>k_{N} \theta_{n}^{*}\left(q_{N}-q_{a}\right)$. Thus we get, $k_{N} \theta\left(q_{N}-q_{a}\right)>k_{N} \theta_{N}^{*} q_{N}-$ $k_{n} \theta_{n}^{*} q_{n}$. Transforming, we get,

$$
k_{N} \theta q_{N}-k_{N} \theta_{N}^{*} q_{N}>\dot{k}_{N} \theta q_{a}-k_{n} \theta_{n}^{*} q_{n}
$$

Which is exactly

$$
U\left(k_{N}, q_{N}, \theta\right)-p_{N}>U\left(k_{N}, q_{n}, \theta\right)-p_{n}
$$

Thus, cross-purchasing does not occur in group $N$ and any lower group, which proves the sufficient condition.

## Proof of Lemma 2.2

We denote the segment with the lowest priced version as segment $i$, the price for the version is $p_{i}$ and a segment that is under threat of cross-purchasing as segment $j$. When cross-purchasing occurs, consumers from segment $j$ that purchase goods designed for any other segments receive utility $k_{j} q_{a} \theta$. Thus, the version designed for the segment with the lowest price is the only version that will be cross-purchased.

If segment $j$ is under threat of cross-purchasing, then we have $k_{j} q_{a} \theta_{j}^{*}-p_{i}>0$. For any segment $n$ that is higher than segment $j$, we have $k_{n}>k_{j}$ and $\theta_{n}^{*}>\theta_{j}^{*}$, and as a consequence $k_{n} q_{a} \theta_{n}^{*}-p_{i}>0$. This means all segments higher than segment $j$ are also under threat of cross-purchasing. Furthermore, for any segment $n$ that is higher than segment $j$, we have $\theta_{n-1}>\theta_{j}^{*}$, so that $k_{n} q_{a} \theta_{n-1}-p_{i}>0$, which means even the lowest taste consumer in segment $n$ has positive surplus purchasing the version designed for segment $i$. Thus, all consumers in segments higher than segment $j$ either purchase the version designed for their segment, or cross-purchase.

## Proof of Proposition 2.4

Suppose the monopolist considers introducing a lower quality $q_{l}$, and offers $q_{h}$ to $\theta \in\left[\theta_{h}, \theta_{N}\right]$, and $q_{l}$ to $\theta \in\left[\theta_{l}, \theta_{h}\right)$. Through Proposition 2.1, we know if versioning
is possible, then $\theta_{h}$ and $\theta_{l}$ must belong to different groups, and we denote them as group $m$ and $v$. Suppose $k_{u}$ is the group taste for $\theta_{h}, k_{v}$ is the group taste for $\theta_{l}$, and $k_{u}>k_{v}$. Then the profit maximization problem is

$$
\max _{\theta_{h}, \theta_{l}} \Pi\left(\theta_{h}, \theta_{l}\right)=\max _{\theta_{h}, \theta_{l}}\left\{k_{u}\left(q_{h}-q_{l}\right) H\left(\theta_{h}\right)+k_{v} q_{l} H\left(\theta_{l}\right)\right\}, \quad \theta_{0} \leq \theta_{l} \leq \theta_{h} \leq \theta_{N}
$$

We know from Proposition 2.1 that there is unique solution $\theta^{*}$ for $\theta=\frac{1-F(\theta)}{f(\theta)}$, for $\theta \in\left[\theta_{0}, \theta_{N}\right]$.

We can transform the above optimization as

$$
\max _{\theta_{h}, \theta_{l}} \Pi\left(\theta_{h}, \theta_{l}\right)=\max _{\theta_{h}, \theta_{l}}\left\{q_{h} k_{u} H\left(\theta_{h}\right)-q_{l}\left[k_{u} H\left(\theta_{h}\right)-k_{v} H\left(\theta_{l}\right)\right]\right\}
$$

We denote

$$
\theta_{m}=\max _{\theta}\left\{k_{e} H\left(\theta^{*}\right), k_{n} H\left(\theta_{n-1}\right)\right\}, \text { for } n>e
$$

where $m$ is the group that $\theta_{m}$ belongs to. Thus, we get

$$
\Pi\left(\theta_{h}, \theta_{l}\right)=q_{h} k_{u} H\left(\theta_{h}\right)-q_{l}\left[k_{u} H\left(\theta_{h}\right)-k_{v} H\left(\theta_{l}\right)\right] \leq k_{m} H\left(\theta_{m}\right) q_{h}
$$

If $k_{u} H\left(\theta_{h}\right)>k_{v} H\left(\theta_{l}\right)$, then we set $m=u$ and $q_{l}=0$. If $k_{u} H\left(\theta_{h}\right) \leq k_{v} H\left(\theta_{l}\right)$, then we set $m=v$ and $q_{l}=q_{h}$. So versioning is not profit maximizing.

## Proof of Proposition 2.5

Consider when each segment is provided with a separate version. For consumers that belong to segment $n$, the quality index is $\sum_{t=1}^{n} q_{t}$. From (2. $\Phi$ ) and (2.2), the price relationship is

$$
\begin{gathered}
p_{1}=U\left(q_{1}, \tilde{\theta}_{1}, k_{1}\right)=k_{1} q_{1} \tilde{\theta}_{1}, \quad \tilde{\theta}_{1} \in\left[\theta_{0}, \theta_{1}\right) \\
p_{t}=p_{t-1}+k_{t} q_{t} \tilde{\theta}_{t}, \quad \tilde{\theta}_{t} \in\left[\theta_{t-1}, \theta_{t}\right) \text { and } t \in\{2, \cdots, N\} .
\end{gathered}
$$

So the profit maximization problem for the monopolist is

$$
\max _{\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{N}} \Pi\left(\tilde{\theta}_{t}\right)=\max _{\tilde{\theta}_{1}, \cdots, \tilde{\theta}_{N}}\left\{\sum_{t=1}^{N} k_{t} q_{t} H\left(\tilde{\theta}_{t}\right)\right\}, \ni \tilde{\theta}_{t} \in\left[\theta_{t-1}, \theta_{t}\right) \text { and } t \in\{1, \cdots, N\}
$$

Because $H(\theta)$ is monotonically increasing for $\theta \in\left[\theta_{0}, \theta^{*}\right)$, for segments that are lower than $e$, the optimal $H\left(\tilde{\theta}_{t}\right)$ is reached when $\lim _{\tilde{\theta}_{t} \rightarrow \theta_{t}} \tilde{\theta}_{t}=\theta_{t}$, which means segments that are lower than $e$ are not served. If $\theta_{N-1} \leq \theta^{*}$, then only segment $N$ is served, which means only one version is provided by the monopolist. Equivalently, we get versioning is optimal only if $\theta^{*}<\theta_{N-1}$.

Now consider when versioning is optimal $\theta^{*}<\theta_{N-1}$. Here only segment $e$ and higher segments are each provided with a separate version and the monopolist's profit maximization becomes
$\max _{\tilde{\theta}_{e}, \cdots, \tilde{\theta}_{N}} \Pi\left(\tilde{\theta}_{t}\right)=\max _{\tilde{\theta}_{e}, \cdots, \bar{\theta}_{N}}\left\{\sum_{t=1}^{e-1} k_{t} q_{t} H\left(\theta^{*}\right)+\sum_{t=e}^{N} k_{t} q_{t} H\left(\tilde{\theta}_{t}\right)\right\}, \ni \tilde{\theta}_{t} \in\left[\theta_{t-1}, \theta_{t}\right)$ and $t \in\{e, \cdots, N\}$.
Because $H(\theta)$ has a global maximum at $\theta^{*}$, for segment $e$, only consumers in $\left[\theta^{*}, \theta_{e}\right.$ ) are served. The optimal quality for segment $e$ is $\sum_{t=1}^{e} q_{t}$ and the optimal price is $p_{e}=\theta^{*} \sum_{t=1}^{e} k_{t} q_{t}$. Moreover, as $H(\theta)$ is monotonically decreasing for $\theta \in\left(\theta^{*}, \theta_{N}\right]$, we get $\tilde{\theta}_{t}^{*}=\theta_{t-1}$ for $t>e$, which means all segments higher than $e$ are covered. The optimal quality for each segment higher than $e$ is $\sum_{t=1}^{j} q_{t}$, and the optimal price is $p_{j}=\sum_{t=1}^{e} k_{t} q_{t} \theta^{*}+\sum_{t=e+1}^{j} k_{t} q_{t} \theta_{t-1}$ for $j \in\{e+1, \cdots, N\}$. Thus we get the optimal profit

$$
\Pi^{*}=\sum_{t=1}^{e} k_{t} q_{t} H\left(\theta^{*}\right)+\sum_{t=e+1}^{N} k_{t} q_{t} H\left(\theta_{t-1}\right)
$$

## APPENDIX B

## Proof of Corollary 1

For $\theta_{\text {old }} \in\left[0, \theta^{*}\right]$, all the consumers in $\left[\theta^{*}, 1\right]$ are informed. The monopolist simply sets $p_{h}=\theta^{*} q_{h}$ to sell the highest quality to informed consumers in $\left[\theta^{*}, 1\right]$ to get optimal profits $\Pi^{*}$. Because all consumers who buy are informed consumers, only one version is provided by the monopolist.

For $\theta_{\text {old }} \in\left(\theta^{*}, 1\right]$, the monopolist has three options: 1) still choose to serve $\left[\theta^{*}, 1\right]$ with $q_{h}, 2$ ) choose to serve only the informed consumers in $\left[\theta_{\text {old }}, 1\right]$ with $q_{h}$, or 3 ) provide two versions $q_{h}$ and $q_{l}$.

For Case 1 , the optimal profits are $\Pi_{1}=\theta^{*} R_{h}\left[1-F\left(\theta^{*}\right)\right]$ when setting price as $\theta^{*} R_{h}$. For Case 2, the optimal profits are $\Pi_{2}=\theta_{\text {old }} q_{h}\left[1-F\left(\theta_{o l d}\right)\right]$ when setting price as $\theta_{\text {old }} q_{h}$. For Case 3, from the monopolist's profit function, we have

$$
\Pi\left(\theta_{h}, \theta_{l}\right)=\theta_{h}\left[q_{h}-q_{l}\right]\left[1-F\left(\theta_{h}\right)\right]+\theta_{l} R_{l}\left[1-F\left(\theta_{l}\right)\right]
$$

Optimal profits are obtained when $\theta_{h}=\theta_{o l d}$ and $\theta_{l}=\theta^{*}$, and $\Pi_{3}=\theta^{*} R_{l}\left[1-F\left(\theta^{*}\right)\right]+$ $\theta_{\text {old }}\left[q_{h}-q_{l}\right]\left[1-F\left(\theta_{\text {old }}\right)\right]$.

The monopolist chooses the optimal profits among the above three cases.

- For Case 1 to be optimal, $\Pi_{1}>\Pi_{2}$ and $\Pi_{1}>\Pi_{3}$. Thus, $\frac{\theta_{o l d}\left[1-F\left(\theta_{o l l}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}<\frac{R_{h}}{q_{h}}$ and $\frac{\theta_{o l l}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}<\frac{R_{h}-R_{l}}{q_{h}-q_{l}}$, which is $\frac{\theta_{o l}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}<\min \left\{\frac{R_{h}}{q_{h}}, \frac{R_{h}-R_{l}}{q_{h}-q_{l}}\right\}$.
- For Case 2 to be optimal, $\Pi_{2}>\Pi_{1}$ and $\Pi_{2}>\Pi_{3}$. Thus, $\frac{\theta_{\text {old }}\left[1-F\left(\theta_{\text {old }}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}>\frac{R_{h}}{q_{h}}$ and $\frac{\theta_{o l d}\left[1-F\left(\theta_{o l a}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}>\frac{R_{l}}{q_{l}}$, which is $\frac{\theta_{o l l}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}>\max \left\{\frac{R_{h}}{q_{h}}, \frac{R_{l}}{q_{l}}\right\}$.
- For Case 3 to be optimal, $\Pi_{3}>\Pi_{1}$ and $\Pi_{3}>\Pi_{2}$. Thus, $\frac{R_{h}-R_{l}}{q_{h}-q_{l}}<\frac{\theta_{o l d}\left(1-F\left(\theta_{\text {old }}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]}<$ $\frac{R_{l}}{q_{l}}$.


## Proof of Corollary 2

When we substitute $p_{h}$ and $p_{l}$ from (3.1) and (3.2), the monopolist's profit function is expressed as

$$
\begin{aligned}
\max _{\theta_{h}, \theta_{l}, \theta_{h}^{\prime}, \theta_{l}^{\prime}} \Pi= & \theta_{h}\left[q_{h}-q_{l}\right]\left[\left[1-F\left(\theta_{h}\right)\right]+\left[F\left(\theta_{\text {old }}\right)-F\left(\theta_{h}^{\prime}\right)\right]\right]+ \\
& \theta_{l} q_{l}\left[\left[1-F\left(\theta_{l}\right)\right]+\left[F\left(\theta_{\text {old }}\right)-F\left(\theta_{l}^{\prime}\right)\right]\right] \\
& \ni 0 \leq \theta_{l}^{\prime} \leq \theta_{h}^{\prime} \leq \theta_{\text {old }} \leq \theta_{l} \leq \theta_{h} \leq 1
\end{aligned}
$$

It is equivalent to maximizing

$$
\begin{equation*}
\max _{\theta_{h}, \theta_{h}^{\prime}} \Pi=\theta_{h}\left[\left[1-F\left(\theta_{h}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{h}^{\prime}\right)\right]\right] \tag{5.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\max _{\theta_{l}, \theta_{l}^{\prime}} \Pi=\theta_{l}\left[\left[1-F\left(\theta_{l}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{l}^{\prime}\right)\right]\right] \tag{5.2}
\end{equation*}
$$

separately.
When we substitute $p_{h}$ and $p_{l}$ from (3.15) and (3.16), the monopolist's profit function can be expressed as

$$
\begin{aligned}
\max _{\theta_{h}, \theta_{l}, \theta_{h}^{\prime}, \theta_{l}^{\prime}} \Pi= & \left.\theta_{h}^{\prime}\left[R_{h}-R_{l}\right]\left[1-F\left(\theta_{h}\right)\right]+\left[F\left(\theta_{\text {old }}\right)-F\left(\theta_{h}^{\prime}\right)\right]\right]+ \\
& \theta_{l}^{\prime} R_{l}\left[\left[1-F\left(\theta_{l}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{l}^{\prime}\right)\right]\right] \\
& \ni 0 \leq \theta_{l}^{\prime} \leq \theta_{h}^{\prime} \leq \theta_{\text {old }} \leq \theta_{l} \leq \theta_{h} \leq 1 .
\end{aligned}
$$

It is equivalent to maximizing

$$
\begin{equation*}
\max _{\theta_{h}, \theta_{h}^{\prime}} \Pi=\theta_{h}^{\prime}\left[\left[1-F\left(\theta_{h}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{h}^{\prime}\right)\right]\right] \tag{5.3}
\end{equation*}
$$

and

$$
\begin{equation*}
\max _{\theta_{l}, \theta_{l}^{\prime}} \Pi=\theta_{l}^{\prime}\left[\left[1-F\left(\theta_{l}\right)\right]+\left[F\left(\theta_{o l d}\right)-F\left(\theta_{l}^{\prime}\right)\right]\right] \tag{5.4}
\end{equation*}
$$

separately.

Comparing (5.1), (5.2), (5.3) and (5.4), we get that (5.1) and (5.2) have exactly the same functional form while (5.3) and (5.4) have exactly the same functional form. Thus we conclude that for the monopolist's profit maximization problem, $\theta_{h}=\theta_{l}$ and $\theta_{h}^{\prime}=\theta_{l}^{\prime}$. It indicates that a monopolist that maximizes the current period profits provides only one version.

## APPENDIX C

## Proof of Proposition 4.1

We already show that in monopoly, only one version is provided. Using the envelope theorem, we get $p_{M} / q_{M}=1 / 2$. Substituting back into the profit function, we have $\Pi_{M}=q_{M} / 4-C\left(q_{M}\right)$. Based on the first order condition, we have $C^{\prime}\left(q_{M}\right)=1 / 4$. For the monopolist to profitably launch the information good, we have $\Pi_{M}=q_{M} / 4-$ $C\left(q_{M}\right)>0$, thus we get $C\left(q_{M}\right) / q_{M}<1 / 4$. So we have $C\left(q_{M}\right) / q_{M}<C^{\prime}\left(q_{M}\right)$.
$\Pi_{A}\left(q_{A}, q_{B}\right)$ is concave in $q_{A}$ and $\Pi_{B}\left(q_{A}, q_{B}\right)$ is concave in $q_{B}$.
Proof. From (4.3), take the second derivative of $\Pi_{A}\left(q_{A}, q_{B}\right)$ with respect to $q_{A}$, we have

$$
\frac{\partial^{2} \Pi_{A}\left(q_{A}, q_{B}\right)}{\partial q_{A}^{2}}=-\frac{8 q_{B}^{2}\left(5 q_{A}+q_{B}\right)}{\left(4 q_{A}-q_{B}\right)^{4}}-C_{A}^{\prime \prime}\left(q_{A}\right)<0
$$

and

$$
\frac{\partial^{2} \Pi_{B}\left(q_{A}, q_{B}\right)}{\partial q_{B}^{2}}=-\frac{2 q_{A}^{2}\left(8 q_{A}+7 q_{B}\right)}{\left(4 q_{A}-q_{B}\right)^{4}}-C_{B}^{\prime \prime}\left(q_{B}\right)<0
$$

Thus we have $\Pi_{A}\left(q_{A}, q_{B}\right)$ is concave in $q_{A}$ and $\Pi_{B}\left(q_{A}, q_{B}\right)$ is concave in $q_{B}$.

## Proof of Proposition 4.3

In the text we show that the leader providing a high quality good decreases its quality in a Stackelberg game. Here we show in detail that the leader providing a low quality good increases its quality in a Stackelberg game. This is the case when $q_{A}<q_{B}$. From (4.4), we can get $\Pi_{A}$ as

$$
\Pi_{A}\left(q_{A}, q_{B}\right)=q_{A} q_{B}\left[q_{B}-q_{A}\right] /\left[4 q_{B}-q_{A}\right]^{2}-C_{A}\left(q_{A}\right)
$$

and we have

$$
\begin{equation*}
\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\right)}{\partial q_{B}}=\frac{q_{A}^{2}\left[q_{A}+2 q_{B}\right]}{\left[4 q_{B}-q_{A}\right]^{3}}>0 \tag{5.5}
\end{equation*}
$$

From (4.5), the best response function $q_{B}\left(q_{A}\right)$ is determined by

$$
C_{B}^{\prime}\left(q_{B}\right)=4 q_{B}\left[4 q_{B}^{2}-3 q_{A} q_{B}+2 q_{A}^{2}\right] /\left[4 q_{B}-q_{A}\right]^{3}
$$

If firm $A$ increases $q_{A}, C_{B}^{\prime}\left(q_{B}\right)$ increases, and so does $q_{B}$. Thus, we derive from the best response function that $d q_{B} / d q_{A}>0$.

At the Stackelberg point

$$
\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{A}}+\frac{\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right)}{\partial q_{B}} \frac{d q_{B}}{d q_{A}}=0
$$

and we have $\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right) / \partial q_{B}>0$ and $d q_{B} / d q_{A}>0$, then $\partial \Pi_{A}\left(q_{A}, q_{B}\left(q_{A}\right)\right) / \partial q_{A}<$ 0.

We know at the Nash equilibrium point, $\partial \Pi_{A}\left(q_{A}, q_{B}\right) / \partial q_{A}=0$. From our analysis of the simultaneous game, we know at the Nash equilibrium, $\partial \Pi_{A}\left(q_{A}, q_{B}\right) / \partial q_{A}=$ 0. Denoting the Stackelberg quality provided by the leader as $q_{A}^{S^{\prime}}$ and the Nash equilibrium quality as $q_{A}^{N^{\prime}}$, from the concavity of $\Pi_{A}\left(q_{A}, q_{B}\right)$ in $q_{A}$ we have $q_{A}^{S^{\prime}}>q_{A}^{N^{\prime}}$. It means with first mover advantage, the leader that provides low quality good increases its quality to increase its profit.

## Entry Deterrence with Versioning.

i) If $q_{A}^{D} \leq q_{M}$, then the leader can safely deter entry at $q_{M}$ and get the optimal profits. Versioning is implemented and the follower is out of the market.
ii) If $q_{M}<q_{A}^{D}<q_{A}^{D I}$, then the leader cannot deter entry at $q_{M}$. It is still optimal for the leader to deter entry and obtain monopoly profits because she still gets more profits than if she chooses to accommodate entry, which is $\Pi_{M}\left(q_{A}^{D}\right)>\Pi_{A}\left(q_{A}^{S^{\prime}}, q_{B}^{S^{\prime}}\right)$. The optimal quality of the information good provided by the leader is thus $q_{A}^{D}$. Versioning is implemented and the follower is out of the market.
iii) If $q_{M}<q_{A}^{D I} \leq q_{A}^{D}$, then the leader is better off accommodating entry because the optimal profits she gets from entry deterrence are less than if she chooses to produce
at $q_{A}^{S^{\prime}}$ to accommodate entry, which is $\Pi_{M}\left(q_{A}^{D}\right)<\Pi_{A}\left(q_{A}^{S^{\prime}}, q_{B}^{S^{\prime}}\right)$. The optimal quality of the information good provided by the leader is thus $q_{A}^{S^{\prime}}$. The corresponding quality of good by the follower is $q_{B}^{S^{\prime}}$. No versioning is implemented.

## GLOSSARY

First degree price discrimination. The case where the firm can extract all the surplus from a heterogeneous set of consumers. It means each consumer pays the maximum he or she is willing to pay. It is also called "perfect price discrimination".

Market segmentation. The aggregating of prospective buyers into groups (segments) that have common needs and will respond similarly to a marketing action.

Product differentiation. A marketing process that showcases the differences between products. Differentiation looks to make a product more attractive by contrasting its unique qualities with other competing products.

Product line. Group of products produced by a firm that are closely related in use and in production and marketing requirements.

Second degree price discrimination. Price discrimination schemes in which the firm knows that consumers differ in ways that are important to the firm but it is unable to identify individual consumers so as to be able to discriminate directly. Thus the firm offers consumers a schedule of prices to choose from.

Software release. The distribution, whether public or private, of an initial or new and upgraded version of a computer software product.

Third degree price discrimination. Price discrimination schemes in which the firm can base prices directly on group identity.

Versioning. To offer a product in different versions for different market segments.


[^0]:    ${ }^{1}$ The original paper was titled "Product Differentiation and Market Segmentation of Information Goods" by Xueqi (David) Wei and Barrie R. Nault, which was presented at the Workshop on Information Systems and Economics (WISE) in Irvine, California, December 2005.

[^1]:    ${ }^{2}$ The original paper was titled "Experience Information Goods: Versioning and Upgrading" by Xueqi (David) Wei, Christian Weiss and Barrie R. Nault, which was presented at the 2007 INFORMS Conference on Information Systems and Technology in Seattle, Washington, November 2007.

[^2]:    ${ }^{3}$ The original paper was titled "Vertically Differentiated Information Goods: Entry Deterrence, Rivalry Clear-out or Coexistence" by Xueqi (David) Wei and Barrie R. Nault, which was presented at the 2006 INFORMS Conference on Information Systems and Technology in Pittsburgh, Pennsylvania, November 2006 (Received "Best Paper Award").

[^3]:    ${ }^{1}$ As discussed in Bhargava and Choudhary (2001), this assumption is satisfied by common distributions such as the uniform, normal, logistic, chi-squared, exponential, and Laplace distributions, and any distribution with increasing density.

[^4]:    ${ }^{2}$ Strictly speaking, a monopolist chooses prices to maximize its profits. But in our formulation a monopolist choosing indifferent consumer types is equivalent. Proofs are available upon request.

[^5]:    ${ }^{3}$ Proofs are provided in Appendix A.

[^6]:    ${ }^{1}$ The demos and trial versions mentioned above can also be treated as a subversion of the final product. In that sense, providing demos and trial versions is versioning as well.

[^7]:    ${ }^{2}$ As discussed in Bhargava and Choudhary (2001), this assumption is satisfied by common distributions such as the uniform, normal, logistic, chi-squared, exponential, and Laplace distributions, and any distribution with increasing density.

[^8]:    ${ }^{3}$ It is equivalent that the monopolist chooses optimal $\theta_{h}, \theta_{l}$ and $\theta_{u}$ instead of $p_{h}, p_{l}$ and $p_{u}$. Proofs are available upon request.

[^9]:    ${ }^{4}$ Conditions in Cases 1, 2 and 3 are exhaustive. For $\frac{R_{h}}{q_{h}} \geq \frac{R_{l}}{q_{l}}$, we have $\frac{R_{h}}{q_{h}} \leq \frac{R_{h}-R_{l}}{q_{h}-q_{l}}$. Case 1 becomes $\frac{\theta_{\text {old }}\left[1-F\left(\theta_{\text {old }}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \leq \frac{R_{h}}{q_{h}}$, Case 2 becomes $\frac{\theta_{\text {olld }}\left[1-F\left(\theta_{\text {old }}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \geq \frac{R_{h}}{q_{h}}$ and Case 3 does not apply. For $\frac{R_{h}}{q_{h}}<\frac{R_{l}}{q_{l}}$, we have $\frac{R_{h}}{q_{h}}>\frac{R_{h}-R_{l}}{q_{h}-q_{l}}$. Case 1 becomes $\frac{\theta_{\text {old }}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \leq \frac{R_{h}-R_{l}}{q_{h}-q_{l}}$ and Case 2 becomes $\frac{\theta_{o l d}\left[1-F\left(\theta_{o l d}\right)\right]}{\theta^{*}\left[1-F\left(\theta^{*}\right)\right]} \geq \frac{R_{l}}{q_{l}}$.

[^10]:    ${ }^{5}$ Note that $\Pi_{2}-\Pi_{1}$ is the profit loss for the current period and $\frac{\delta}{1-\delta}\left[\Pi I^{*}-\Pi_{2}\right]$ is the present value of the profit gains for the following periods. $\delta$ is the discount rate defined in Section 3.3.

[^11]:    ${ }^{6}$ Consumers in $\left[\theta^{*}, 1\right]$ must be informed, otherwise the monopolist can simply set $p=\theta^{*} q_{h}$ to extend the market to $\left[\theta^{*} q_{h} / R_{h}, 1\right]$ with $\Pi_{N}=\theta^{*} q_{h}\left[1-F\left(\theta^{*} q_{h} / R_{h}\right)\right]>\Pi^{*}$.

[^12]:    ${ }^{1}$ All proofs are in Appendix C.

[^13]:    ${ }^{2}$ Both firms providing goods with the same quality is not a Nash equilibrium. If $H$ is the high quality good and $L$ is the low quality good, the possible combinations of goods provided by the two firms are $(H, H),(H, L),(L, H)$ and $(L, L)$. Only $(H, L)$ and $(L, H)$ are Nash equilibria.
    ${ }^{3}$ The sufficient second order conditions are satisfied for this and the remaining optimization problems. Details are available upon request.

[^14]:    ${ }^{4}$ It is the same as equation (4.5), only with $A$ and $B$ reversed.
    ${ }^{5} q_{B}$ is the quality of the entry when $\Pi_{B}\left(q_{A}, q_{B}\right)=0$. Theoretically, entry is deterred in this case.

[^15]:    ${ }^{6}$ One might argue that potential entry may come from an even lower-end market, which means $q_{B}<q_{A}^{L}$. In that case, the leader can generate another lower version to deter entry, with the same mechanism we describe here.

[^16]:    ${ }^{7}$ Strictly speaking, the leader should produce at a quality that is strictly higher than this level to prevent the follower from producing a higher quality good.

