

ELECTROLYTIC AND OTHER GRODECS AS ACCURATE NEURON MODELS FOR NEURAL NETWORKS.

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Abstract A grodec is a versatile entity with switched growth-decay pressure behaviour. Grodecs can be used to construct both grodec stack machines and accurate neuron models. A grodec needs an operating fluid, and when this fluid involves ions, pressure changes are voltage changes. A single reservoir-based grodec has the pressure-change properties of a short section of biological membrane, and can give rise to generalized, pressure-change action potentials. A pair of appropriately coupled reservoir-based grodecs can generate a much sharper action potential than can a single grodec.

The action potentials from a single reservoir-based electrolytic grodec based on Na^+ , and from a coupled electrolytic grodec pair with one grodec based on Na^+ and the other based on K^+ , are analysed in detail. So close is the voltage-switching mechanism and behaviour of the coupled electrolytic grodec pair to those of a neuron with dual Na^+ and K^+ ion switching, that it is concluded that a neuron membrane can be regarded conceptually as a Na^+ grodec coupled to a K^+ grodec. Grodecs of any kind can be coupled in excitatory or inhibitory manner to form neural nets.

1.0 Introduction

The concept of a grodec, or growth decay entity, was originally developed as an abstract entity for the construction of parallel grodec stack machines [3] with time fractal behaviour [2, 13, 14, 18].

A grodec is an abstract growth-decay entity that can be physically implemented, and which can undergo a period of pressure growth until a characteristic trigger pressure is reached, at which point a period of pressure decay follows, in such a way that consecutive growth/decay periods can in theory be repeated endlessly and additively. Thus if the growth-period pressure increase exceeds its subsequent decay-period pressure decrease, the pressure in the grodec can grow endlessly, but punctuated by regular periods of pressure decay. Grodecs can be implemented as electric charge (electronic or ionic), gas and liquid pressure devices.

Grodecs can also be combined in interesting ways for generating logical phenomena. For example, certain assemblies of grodecs in parallel can be used for constructing fractal machines where the pressure has time fractal behaviour. In addition, as will be shown in this paper, reservoir-based grodecs also have pressure and fluid-flow properties consistent with the essential voltage (i.e. ionic electrical pressure) and current (ion flow) properties of neuron membranes or neuramines [1, 8, 12, 16]. Thus the neuron action potential can be usefully interpreted as a grodec phenomenon, as will be

demonstrated in this paper. This carries the important implication that grodecos could in principle be used to construct both neural nets, and machines whose function is isomorphic with that of biological brains. The idea of a brain isomorph based on current silicon chip technology has recently become important in investigations into the hard problem of consciousness[4], although the very possibility of such an isomorph has been in doubt [4].

In this paper we begin with an analysis of the operation of a grodec, and with an elementary demonstration of how grodecos can be used to build fractal machines; full details of grodecos and fractal machines are given elsewhere[3]. Then we give a detailed analysis showing how specific kinds of grodec behaviour are closely correlated with neuron membrane behavior. We demonstrate how grodecos of any kind can give rise to a pressure-change 'action potential', that is, a spike of pressure change that can propagate along a chain of grodecos. We also show how electrolytic grodecos, involving Na^+ ions and/or K^+ ions, can give rise to voltage-change action potentials involving a mechanism in principle the same as that in neurons. Finally we show that grodecos can in principle be used to build logical machines.

Grodecos are related to oscillators, but have switching properties; it is in fact legitimate to regard an oscillator as that particular case of a grodec in which growth always equals decay. For every kind of oscillator there is the possibility of constructing a corresponding grodec. Like the oscillator, a grodec has two stores that can each hold a fluid (in the broadest sense of the term), and energy that tends to drive the fluid between the stores. But unlike in an oscillator, in a grodec the link is not two way inertial, but one-way switched, resulting in behaviour that is quite different from that of an oscillator.

1.1 Growth-decay entities or grodecos

A growth decay entity or grodec, as envisaged in this paper, is an abstraction of the simplest examples of the various kinds of physical entity - electrical, hydraulic, gaseous - that can exhibit repeated growth and decay phenomena.

A grodec is formally a 6-tuple $\langle c, L, R, g, k, i \rangle$, capable of reacting to a flow F of some abstract fluid. L and R are variables; the other attributes are constants for a specific grodec. The attributes of the tuple of the grodec are always positive. The flow F can be either positive or negative.

A grodec can be visualized as two balancing stores - a right store and a left store, connected by a flow line in which there is a switch. In diagrams (Figure 1), the left store is depicted as a triangle and the right store as a square. R is the measure of fluid quantity for the right store and L for the left store. The capacitance of the right store can be taken as unity in arbitrary units; the capacitance of the left store is c in the same units, so that essentially c is the relative capacitance of the left store with respect to the right store. Capacitance is taken as the measure of how much fluid is required to generate a unit of pressure on a store, so that quantity of fluid Q relates to pressure P by $Q = cP$. Thus the pressure of the right store must be R and that of the left must be L/c . When the pressures on the two stores are equal, which occurs when $L/c = R$, there is no inclination for fluid to flow between the stores.

Fluid can move from right to left stores only if the pressure difference between right and left stores has climbed to a trigger value g that opens the switch, and which prevents the pressure in the right store rising by more than g with respect to the left store. Once the switch has been triggered a steady flow i from the right store ensues, until both (a) the right store pressure has fallen to the level where $R - L/c = 0$, and (b) the original pressure of the right store has decayed by a fraction $1-k$ of the original pressure difference, that is, by a pressure drop of $g(1-k)$, at which point the switch closes. Thus following triggering of the switch because of a pressure difference g , the two stores will come into pressure equilibrium, that is, $R = L/c$, and the right store pressure will have fallen by $g(1-k)$ and the left store pressure will have risen by gk . (Also, if left store pressure exceeds right store pressure, fluid can leak from left to right until equilibrium is reached.)

We define k as the decay fraction for a grodec, that is, k is that fraction of the trigger pressure g remaining in the right store following a pressure decay when the switch opens.

1.2 Grodec implementations

A passive grodec is an abstraction, and no physical entity is likely to function exactly like a grodec. A grodec merely has the essential properties of similar physical entities. A physical implementation of a grodec, whether electrical, hydraulic or gaseous, may differ in detail, but not in principle, from the abstract grodec. It is the essential features of these physical systems that are abstracted in a grodec, namely that each store can hold a fluid, that the pressure in a store increases with the quantity of fluid, and that a specific threshold pressure difference between the two stores will cause a switch to open and allow fluid to flow from right to left, the flow stopping when (a) the pressure difference falls to a specific level to a specific, which can be zero, and (b) a specific pressure drop of the right store has occurred. In an implementation, the fluid could be an electron gas or hole gas as in a metal or semiconductor, a mobile ion concentration as in an electrolyte, a liquid under gravity, or a gas. In the grodec the pressure in a store is assumed for simplicity to increase linearly with quantity of fluid, but linear increase is not an essential feature; similarly, in a grodec the flow between stores is assumed for simplicity to be constant, but this constant flow also is not an essential feature.

The grodec that occupy the bulk of this paper are electrolytic grodec, which are among the most complex. However, to help grasp the basic grodec concept, both the hydraulic version, and the electronic version are useful. In the hydraulic case the left and right stores are vertical columns containing a liquid under gravitational pressure. The columns are connected by a tube containing a pressure switch. When the pressure difference between the right and left columns reaches g , the switch opens, and fluid flows from right to left, until the height (proportional to pressure) in each is equilibrated. Store capacitance is proportional to column cross-section area.

An electronic version involves a pair of capacitors, one as the left store and one as the right store. One plate of each capacitor is grounded and the other pair of plates are connected via a switch, which allows fluid (electric charge) from right to left only if the voltage difference between the capacitors exceeds the switching voltage (electrical pressure) g .

Later we shall analyse electrolytic grodec, where a grodec can also be viewed as two stores containing an electrolyte (eg water with NaCl) each store separated by a membrane permeable to one ion (eg Cl^-), but permeable to the other ion (Na^+) type (the grodec fluid) only if a trigger pressure difference is exceeded. We shall show that it is reasonable and fruitful to consider a neuron membrane as being constructed as an array of two types of (subtly intertwined) grodec, one based on an Na^+ ions as fluid, and the other based on a K^+ ions as fluid.

2.0 En:1 and En:m functions and the n-level grodec stack

To begin, we examine in detail the behavior of a passive grodec under an inflow F . Then we examine a one-sided 2-level grodec stack, and then extrapolate to a one sided n -level grodec stack, where n is a large integer. As we shall see, under inflow F , depending on the setting for the trigger pressure g of the grodec of the stack, the pressure R of the right store of the grodec stack can trace out the pattern of an En:m function for which $m = 1$. An En:1 function is one made up of growth and decay segments, in which each growth segment subdivides into n segments [2, 3] in an endless fractal manner. A decay segment does not subdivide, as we illustrate presently. The particular value of n in the function depends on the values chosen for g in each grodec of the stack. Finally we look at how grodec machines can exhibit En:m function behaviour, where m is not 1, that is where each decay segment subdivides into m segments.

2.1 A single passive grodec under constant inflow F

A grodec, when fed from the right by a steady flow F , will experience successive periods of growing right store pressure R each terminating when the switch opens, causing a subsequent period of decaying right store pressure R , which in turn terminates when the switch closes, allowing growth to resume. Both the growth and the decay periods are periods of linear pressure change with time (Figure 2). During a decay period fluid flows at a steady rate i from right to left, so that during this period the fluid in the right store decays at a rate $i - F$, if F remains on while the switch is open. However, in practice we shall only consider grodec where the inflow F turns off while any grodec switch is open.

The growth curve for R with regular periods of decay, as in Figure 2, can be computed precisely from the attributes of the grodec. Suppose we initialize $L = R = 0$. At time $t = 0$, the fluid inflow F is turned on positively. The value of R increases at a rate of F units per second for a period of g/F seconds, so that R peaks at a value g at time $t = g/F$. At this point the switch opens, causing the right store pressure R to decay at a rate $i - F$ and the value of L to increase at a rate i . The decay will stop when the right and left store pressures are equal, that is, at time T , measured from the beginning of the decay period, given by:

$$g - T(i - F) = Ti/c$$

that is, at time $T = cg/(ic + i - Fc)$

At this point $R = L/c = Ti/c = gi/(ic + i - Fc)$

Measured from the beginning of the initial growth of the right store the time t at the end of the decay period is

$$t = g/F + cg/(ic + i - Fc)$$

Following the decay, growth in R resumes at rate F until $R = g + gi/(ic + i - Fc)$, at time $t = 2g/F + cg/(ic + i - Fc)$. The ensuing decay period stops at the level given by $R = 2gi/(ic + i - Fc)$, at time $t = 2g/F + 2cg/(ic + i - Fc)$.

In general, following n periods of growth, the peak coordinates for the right store are:

$$R = g + (n-1)gi/(ic + i - Fc) \quad \dots (1)$$

$$t = ng/F + (n-1)cg/(ic + i - Fc) \quad \dots (2)$$

Following n periods of decay, the trough coordinates for the right store are:

$$R = ngi/(ic + i - Fc) \quad \dots (3)$$

$$t = ng/F + ncg/(ic + i - Fc) \quad \dots (4)$$

A simpler and more appealing situation ensues if we assume that F is turned off during a decay period, and this will be assumed in all cases henceforth. If this is the case, following n periods of growth, the peak coordinates for the right store are:

$$R = g + (n-1)gi/(ic + i) = g + (n-1)g/(1+c) \quad \dots (5)$$

$$t = ng/F + (n-1)cg/(ic + i) = ng/F + (n-1)cg/i(c+1) \quad \dots (6)$$

Following n periods of decay, the trough coordinates for the right store are:

$$R = ngi/(ic + i) = ng/(1+c) \quad \dots (7)$$

$$t = ng/F + ncg/(ic + i) = ng/F + ncg/i(1+c) \quad \dots (8)$$

Since a decay period reduces an initial pressure g to $g/(1+c)$, which must also be kg , we must have:

$$k = 1/(1+c)$$

so that we must make the setting on the grodec

$$c = (1-k)/k \quad \dots (9)$$

If k and c in the grodec are set to obey this condition, then the grodec right store pressure will always be reduced by exactly $g(1-k)$ in moving to equilibrium with the left store when the switch is triggered. This condition is not important right now in the more elementary parts of our development, but becomes critical later on. Notice that if $k = 0.5$, then $c = 1$, so that R retraces exactly half of g following a switch trigger (Figure 2a), which is what one would expect. Also if $c > 1$, then the decay fraction k is less than 50% (Figure 2b), and greater than 50% if $c < 1$ (Figure 2c).

Notice that if the flow F is turned off during decay periods the pressure peak and trough pressures R do not depend on F , but only on the attributes of the grodec. Notice also that if $c = 1$, equations (5) and (7) for peak and trough pressures reduce to:

$$R = g(1 + (n-1)/2) \quad \dots (10)$$

$$R = ng/2 \quad \dots (11)$$

This gives us a series of peak and trough pressures:

$$g, g/2, 3g/2, g, 2g, 3g/2, \dots$$

so that each decay period retraces exactly half of the previous growth period (Figure 2a).

2.2 A one-sided 2 level-grodec stack

In this section we look briefly at a simple system of grodec, namely a one-sided grodec stack, involving just 2 grodec. In such a stack the right stores are common, such that the pressures of the right stores and their fluid quantities are equalized at all times. In such a stack one grodec can be said to be inner with respect to the other or outer grodec. The inner grodec is grodec 1 and the outer grodec 2 (Figure 3). The left stores have the same pressures only when the system is in equilibrium. Remembering that we are dealing with an abstraction with specific rules for the behaviour of that abstraction, the reader may nevertheless be helped by visualizing a grodec in terms of the columns of liquid. We can put two of these liquid column grodec together to form a 2-grodec stack by merging the right columns and having two separate left columns or stores each connected to the common right column or store by a separate tube containing a pressure switch.

If fluid is fed into the right common store, the outer right store and the inner right store of the two grodec automatically have the same increased pressure, but the pressures in the left stores do not change until the appropriate switches open and fluid flows from right to left. Furthermore, the grodec are so selected that threshold pressure differences for opening switches are quite different in the inner and outer grodec. In the inner grodec, the threshold switching pressure difference g_1 is considerably smaller than g_2 for the outer grodec. This means that as the pressure in the right stores increases under a positive inflow F , the threshold pressure difference g_1 for opening the switch to allow the inner left store to equilibrate will be reached long before the threshold pressure difference g_2 is reached for opening the switch for allowing fluid to flow into the outer left store and equilibrate the pressure there.

With this two-grodec stack, the inner grodec 1 can be denoted by $\langle c_1, L_1, R, g_1, k_1, i_1 \rangle$, and the outer grodec 2 is denoted by $\langle c_2, L_2, R, g_2, k_2, i_2 \rangle$. We assume that initially right and left pressures in the grodec are equal, so that $L_1/c_1 = L_2/c_2 = R = H$ where H is the initial pressure (which may also be taken as zero). Suppose now, we begin feeding fluid into a right store at a rate F . A possible trajectory of R is shown in Figure 4a. For this trajectory to actually occur, the constants c_1, g_1, i_1 and c_2, g_2, i_2 must be appropriately set. However, for the moment merely assume that g_2 is much larger than g_1 .

A full analysis of this machine is beyond the scope of this paper [see], but the essentials are as follows. Initially the inner and outer grodec switches are closed and the only fluid flowing into the right stores is due to F . This fluid flowing into the right raises the pressure R until it reaches g_1 , causing the inner switch to open, thus causing the pressure R to fall and L_1 to rise until equilibrium is reached. At this point the inflowing fluid F causes R to resume its increase until the pressure difference between right and left for grodec 1 again reaches g_1 and again we have a decay period, following which R

resumes its climb until the pressure difference between right and left store of the inner grodec is just under g_1 . At this point we have reached the third peak (P) in Figure 4a.

The pressure in the left store of grodec 2 has been so far unaffected and is still at the initial value H. But now, suppose that the pressure R at the third peak is such that the pressure difference between the right and left stores of grodec 2, that is, $R - L_2/c_2$, is equal to g_2 , the threshold for opening the switch in outer grodec 2, to equilibrate the pressure in both stores of grodec 2. If g_2 is thus set, the switch in the outer grodec may be set to open either (a) just before the switch in the inner grodec can open (the "just before opening" or JBO case) or (b) at the same time as the switch in the inner grodec opens (the "same time opening" or STO case" case). We consider only JBO, which is the technically simpler case, in this paper. The opening of the outer switch causes a much longer decay period and a deeper decay of the pressure R, than was the case for the decays associated with the opening of the inner switch as illustrated in Figure 4a, for k equal to 50%. We can also have k less than or equal to 50%, depending on grodec settings.

At the end of this more prolonged decay period (point W1 in Figure 4a), the outer left and right stores of both grodec have equilibrated and the switch in the outer line closes, as does the switch in the inner. At this point (point W1 in the diagram) the pressures in all four store are in equilibrium.

At point W1, conditions are the same as at point W0, except that overall pressure levels are higher. If the right stores continue to be fed by F, the pressure trajectory from W0 to W1 will exactly replicate, from W1 to W2, and continue to replicate, from W2 to W3, and so on. All that has happened is the the inner grodec has modulated the growth and decay periods of the outer grodec, the extent of the modulation depending on the attributes of the grodec

In Figure 4a the grodec attributes are assumed to be such that we get two decay periods in each outer grodec growth period. With different grodec parameters we would get different modulation, such as 3 decay periods within a growth period.

For the growth period modulation in Figure 4a to occur, the following, from equation (5), must hold

$$g_2 = g_1 + (3-1)g_1/(c_1 + 1) \quad \dots (12)$$

This also ensures that the decay factor in the trace (fraction of a previous segment that does not decay) is the same everywhere in the trace in Figure 4a, and that the modulation will replicate exactly when F resumes after equilibrium is reached. The trace in Figure 4a is two subdivision levels of the trace of an E5:1 function. In an E5:1 function growth segments subdivide into 5 smaller segments, 3 growth and 2 decay, and this subdivision continues for ever in fractal manner. We can also have E7:1 functions where each growth segment subdivides into 4 growth segments and 3 decay segments. The general fractal time-function is $E_n:1$ with each growth segment dividing into $(n+1)/2$ growth segments and $(n-1)/2$ decay segments, with no subdivision of decay segments. Here we are dealing with equisegment functions, where a segment subdivides into $(n+1)/2$ equal growth segments and $(n-1)/2$ equal decay segments. Non equisegment functions also exist, but the equisegment functions are the simplest and are the only ones considered in this paper. Other $E_n:m$ functions are covered in [2].

Using equation (11), the condition for any equisegment $E_n:1$ function to be traced out, at two levels of decomposition, must be:

$$g_2 = g_1 + ((n+1)/2 - 1)g_1/(c_1 + 1) \quad \dots(15)$$

Notice that we have substituted $(n+1)/2$ for n in the expression on the right in equation (1), because an $En:1$ function has a sequence $(n+1)/2$ growth periods before the onset of a larger retracement. In terms of a grodec stack tracing an $En:1$ function, there are $(n+1)/2$ growth periods involving pressure on g_1 before a larger decay period where g_2 opens. The above equation assumes that F is turned off during decay periods.

2.3 Grodec stack machines and $En:m$ fractal time functions

For each additional grodec with the proper settings, we can get a further level of decomposition in the equisegment $En:1$ function, so that an n -level grodec stack would be needed to exhibit behaviour consistent with an $En:1$ function to n levels of decomposition. The analysis of such machines is beyond the scope of this paper.

Notice that $En:1$ functions do not have any decomposition in the decay segments. It is possible to construct a grodec machine where the pressures trace is like that in Figure 4a (see Figure 4b) except that it grows downwards. In other words the decay segments decompose but the growth segments do not. The grodec stack machines described above for upward or positive growing $En:1$ functions are called left grodec stack machines. Those for downwards growing or negative $En:1$ functions are called right grodec stack machines and are very similar in operation. We can combine these two types of machines to form a balanced grodec stack machine, in which the pressure trace follows an $En:m$ function, that is, a growth-decay fractal time function in which each growth segment subdivides into n segments and each decay segment subdivides into m segments. The details of balanced grodec stack machines are beyond the scope of this paper. They are complex and exhibit sensitive dependence on initial conditions [6, 7, 19].

Generators and a generation to a few levels of decomposition for $E5:3$ function and the $E7:5$ function are given below. The $E7:5$ function is illustrated graphically in Figure 5.

E5:3 generation

Axiom	Ud
U ->	UdUdU
d ->	DuD
D ->	DuDuD
u ->	UdU

		U				d			1	1
U	d	U	d	U	D	u	D		5	3
UdUdU	DuD	UdUdU	DuD	UdDdU	DuDdU	UdU	DuDdU		13	8
...										

E7:5 generation

Axiom	Ud
U ->	UdUdUdUd
d ->	DuDdU

In the above En:m functions m had the values 3 (in E5:3) and 5 (in E7:5), which means that the decay segments further subdivide. If $m = 1$, as in E5:1 and E7:1 functions, the subdivision is simpler, since the decay segments no longer subdivide:

<i>E5:1 static generation</i>	Axiom	Ud
	U ->	UdUdU
	d ->	d

$$\begin{array}{ccccccc} & & U & & d \\ & U & d & U & d & U & \\ UdUdU & d & UdUdU & d & UdDdU & & d \end{array}$$

<i>E7:1 generation</i>	Axiom	Ud
	U ->	UdUdUdUd
	d ->	d

$$\begin{array}{ccccccc} & & & U & & & d \\ U & d & U & d & U & d & U \\ UdUdUdU & d & UdUdUdU & d & UdUdUdU & d & UdUdUdU \\ \vdots & & & & & & \end{array}$$

The above generations are essentially static or fractal generations [6, 7], since we take an existing pair of growth (U) and decay (d) segments and endlessly subdivide them. But an En:m function can grow endlessly in time. It thus belongs to the general class of self-affine functions [6]. The *dynamic or forward time* generator is somewhat different from the static generator shown above [3].

3.0 The grodec action potentials

As mentioned earlier, both a single grodec and a pair of grodec can exhibit action potential phenomena. The action potential of the single grodec is like that common to many biological membranes, with a sharp initial pressure change and a slower recovery. The action potential with a pair of grodec is more like that of a neuron with both a sharp initial pressure change and a sharp recovery. Many electrically excitable biological membranes have only one switch, corresponding to one grodec, while neurons have two

switches, namely the sodium and potassium switches [1,11,12], corresponding to two grodec.

3.1 Analysis of the action potential in single grodec

Suppose a grodec in which the right store is connected to a large reservoir (large compared to the size of the right store) at the same pressure. Assume also that the line connecting the reservoir to the right store has very low capacity, so that if the right store is emptied, it will be replenished only slowly (at a leak rate) by the reservoir. Suppose that the pressure difference $R-L/c$ is just slightly less than g . Now suppose that a small amount of fluid is pumped into the right store, so that $R-L/c$ reaches g and the grodec triggers. The switch will open and fluid will flow at rate i from right to left, the pressure difference initially between the two stores being g . If t is the time for the two stores to reach pressure equilibrium then:

$$g - ti = ti/c$$

so that

$$t = cg/i(c+1)$$

The drop in pressure for the right store, or the action potential magnitude A , must be

$$A = ti = cg/(c+1)$$

This shows that if c , the capacitance of the left store relative to the right is very large the action potential is almost equal to g ; if c is very small the action potential tends to zero.

Assume that the relative capacitance c of the left reservoir is very large, so that the action potential is approximately g . Once the right and left stores of the grodec have equilibrated, the switch closes, and the right side is replenished slowly from the right reservoir, and the grodec is thus reset, ready to trigger again. Because the replenishment takes some time, there is a refractory period between the times when the grodec can fire. Such a refractory period also occurs in biological membranes. If we assume this replenishment is at a constant rate j , then the time to reset the grodec will be g/j , if j is considerably less than i . This model action potential trace is shown in Figure 6a. In practical implementations of such a model the flows i and j will not be constant but will fall as the pressure difference falls, so that i and j must be taken as average values, where the flows begins large and ends small as equilibrium is approached. Thus the action potential will in practice look more as shown in Figure 6b as opposed to that in Figure 6a. The i and j flows will follow a profile in time as shown in Figure 6c, from which it is apparent that although j begins abruptly large following the switch triggering, j begins to increase as soon as the pressure in R begins to fall. This will have the affect of making the action potential less of a spike and more of a round pulse.

3.2 Analysis of the action potential with two grodec

If we want an action potential with a sharp spike we must use two grodec. This is illustrated by the grodec arrangement in Figure 7a. There are two grodec, a left grodec (grodec G1) and a right grodec (grodec G2). The left store of the grodec G1 and the right store of the grodec G2 are very large compared with the other stores and can be regarded as reservoirs. The right store of grodec G1 and the left store of grodec G2 are common

and have resting pressure R , equal to that of the right store of grodec $G2$. The left store of grodec N is at a pressure slightly less than g units below the pressure R of the right store.

The action potential in this grodec pair functions as follows. A small initial flow of fluid into the common store, that is, the right store of grodec $G1$, will raise the pressure difference between the stores of grodec $G1$ to g , causing the pressure in the common store to drop fairly abruptly to

$$R - cg/(c + 1)$$

under the influence of the flow i from right to left store. Since c is large for grodec $G1$, this pressure drop is close to:

$$R - g$$

Following this pressure drop the two stores of grodec $G1$ are in equilibrium. The time taken to reach equilibrium is

$$cg/i(c + 1)$$

which, since c is very large, is close to :

$$g/i$$

The pressure trace is illustrated in Figure 7b.

At pressure equilibrium in grodec $G1$ the switch now turns off. But the common store is now at pressure $R-g$ and the right store of grodec $G2$ is at pressure R , so that there is a pressure difference g between them. If g is also the trigger pressure of grodec $G2$, the switch of grodec $G2$ will now abruptly open and the process will be reversed with fluid flowing from the right store of grodec $G2$ to the left store of grodec $G2$. If r is the capacitance of the right store of grodec k relative to the left store of grodec $G2$, the two stores come to equilibrium in a time t given by:

$$tj = g - tj/r$$

$$\text{or} \quad t = gr/j(r+1)$$

$$\text{and at a pressure} \quad R - g + tj$$

$$\text{or} \quad R - g + gr/(r + 1)$$

If the relative capacitance r is very large, that is, if the right store of grodec $G2$ can be regarded as a reservoir, the time tends to g/j and the pressure of the common store tends to R , which brings things back to where they started, thus resetting the system. The abrupt opening of the right switch will bring the pressure of the central store as quickly back to R as it was initially brought to $R-g$ by the left switch, if j and r are comparable - which they can be. Thus we get a spike action potential as illustrated in Figure 7b. In practice the flows would be large initially and small as equilibrium approached, so that i and j are really averages; in such a practical implementation the spike action potential would have slightly more curved shape as shown in Figure 7c.

As we shall see, in neurons two grodec's, coupled essentially like those above, are used. This analysis shows why nature probably evolved the use of two grodec's in neurons. Triggering of the $G1$ -grodec switch alone would be sufficient to produce an action potential. However the recovery period would have to be slow relative to the time taken by the initial voltage pressure collapse. The action potentials of the shape in Figure 6a or 6b would therefore not permit a high frequency of pulses to be generated. To solve this problem nature used the second grodec, the $G2$ grodec, so that when the $G2$ -grodec switch triggers it has the effect of bringing the voltage pressure back to the original resting pressure much more quickly than would otherwise be the case, thus allowing the highest possible pulse frequency in neurons.

However, if grodec is at the basis of the action potential in neurons, then the neuron grodec must be electrolytic grodec. We therefore still have to show how an electrolytic grodec would work, and how they individually and in pairs could generate action potentials involving millivoltage changes, and how this can be engineered within a membrane geometry as in the case of neurons. To fully appreciate the following analysis, electrolytic grodec the reader needs a clear grasp of both fundamental electrical and electrochemical principles.

4.0 Electrolytic grodec and neurons

The analysis in this section is intended demonstrate that membrane and neuron action potentials can be interpreted as examples of naturally occurring grodec phenomena, in which the grodec is electrolytic.

Electrolytic grodec phenomena are possibly the most complex of all grodec phenomena. As in all grodec there are two stores, left and right, connected via a line containing a switch, which is normally closed but which opens when the pressure of fluid in the right store exceed that of the left by a quantity g , the trigger pressure. Unusual complexity is characteristic of an electrolytic grodec because there must always be more than one pressure difference between right and left stores, and thus more than one possible switch.

For example, suppose the stores contain water containing NaCl, and KCl, where it is possible for, water, Na^+ , K^+ and Cl^- , to pass between the stores. That means that we can have a partial pressure difference for the water between right and left (an osmotic pressure), and a distinct partial pressure difference for K^+ , for Na^+ and for Cl^- ; these latter three pressure differences are not the same, since concentrations or partial pressures for K^+ , Na^+ and Cl^- in a store are not the same. In addition, the partial pressure differences for K^+ , Na^+ and Cl^- give rise to electromotive forces (emfs, measured in volts or millivolts) between the stores, one for K^+ , one for Na^+ and one for Cl^- . These electromotive forces may, in conjunction with flow conditions between the stores, cause an overall voltage or electrical pressure difference between the stores, and not necessarily equal to any one of the underlying emfs. This overall voltage between the stores can be measured, and will be affected by the opening of any switch controlling flow of K^+ , Na^+ or Cl^- . The effect of opening the switch will cause the voltage to suddenly change, and then following closing of the switch, return gradually to its original level, in other words an "action potential".

We shall demonstrate the neuron electrical activity or action potential can be interpreted as depending on connected switching phenomena in two grodec, one grodec based on Na^+ ion concentrations or partial pressures, with a switch facilitating sudden Na^+ flow, and another grodec based on K^+ ion concentrations or partial pressures, with a switch facilitating sudden K^+ flow. The essentials of whole phenomenon are in principal (although in much more complex fine detail) those of the pair of grodec described in Section 3. We begin with an analysis of an Na^+ grodec. Then we give an analysis of the behaviour of an Na^+ grodec coupled to a K^+ grodec, and equivalent neuron membrane phenomena.

4.1 The Na⁺ electrolytic grodec

Consider, as in Figure 8a, a membrane separating two stores, the one on the right with a high concentration of NaCl in water, and the one on the left with a low concentration. The membrane is permeable to Na⁺ ions and Cl⁻ ions, but not to water. In addition, the areas of the membrane where the Na⁺ ions can penetrate is different from those where Cl⁻ ions can penetrate. Thus a small area of membrane can let either Na⁺ or Cl⁻ through but not both. [An area of membrane that permits flow of Na⁺ can be accurately visualized as an area containing one or more channels through the membrane, each channel containing immobile negative charges that allow passage (conduction) of only Na⁺ charges; similarly for a Cl⁻ permeable area of membrane.] In addition, the area permeable to a given ion type may be arbitrarily small. Since Na⁺ and Cl⁻ concentrations in each store are the same, the partial pressure driving the Na⁺ ions from the right to left will be the same as the partial pressure driving the Cl⁻ ions from right to left.

Now suppose the permeability of the membrane is different for Cl⁻ and Na⁺, for example greater for Cl⁻. In that case initially a greater number of Cl⁻ ions will get across than Na⁺ ions. This will set up a charge imbalance with the left store negatively charged with respect to the right, and so a voltage difference appears. This voltage will have the effect of slowing down passage of Cl⁻ ions and speeding up passage of Na⁺ ions, so that equal amounts of Na⁺ and Cl⁻ ions cross per time unit, thus preventing any further increase in the initial charge imbalance.

To gain greater insight into this electrical phenomenon, imagine that rearrangement of the membrane channels so that all the Cl⁻ permeable areas are adjacent, and also all the Na⁺ permeable areas (Figure 8b). It should now be apparent that in this arrangement a circular electric current is flowing, being carried by Na⁺ ions in the Na⁺ permeable area, by Cl⁻ ions in the Cl⁻ permeable area and by both Na⁺ and Cl⁻ in the stores. This current is carried entirely ions and no electrons are involved. Such an arrangement can be constructed in the laboratory and is called a Sollner cell. It should be obvious that, depending on the electromotive forces involved and the electrical resistances of the two areas of membrane, a voltage may be set up across the membrane. It should also be obvious that the situation in (a) consists of a large number of such Sollner cells, so that there should be miniature circular currents flowing through adjacent membrane areas of Na⁺ and Cl⁻ permeability.

To analyse the electrical situation in (b), suppose we magnify the membrane in (b) as in Figure 8c. Suppose the left store Na⁺ concentration is n_{Cx} and the right store Na⁺ concentration is n_{Cy} , and that the concentrations from left to right through the membrane are n_{Cx} , n_{C1} , n_{C2} , ..., n_{Ci} , n_{Cy} ; i is an arbitrarily large integer dividing the membrane into i compartments, where the Na⁺ concentration difference between adjacent compartments i and $i-1$ can be slightly positive, slightly negative or just zero. Similarly, the Cl⁻ concentrations from left to right can be taken as c_{Cx} , c_{C1} , c_{C2} , ..., c_{Cj} , c_{Cy} , where the Cl⁻ permeable membrane is divided into j partitions.

Ions in solutions obey the Gas Law $PV = RT$ with respect to partial pressures, volumes and temperatures of one mole of gas. If a gas volume is increased by a small amount dV at constant temperature the work done is PdV , which is $RT(1/V)dV$. The work done for a large volume change is obtained by integrating this, which gives us work

done of $RT \ln(V_2/V_1)$ when a mole of gas expands from volume V_1 to V_2 . This may be expressed as $RT \ln(P_1/P_2)$, if the expansion at constant temperature is from P_1 to P_2 .

Using this expression, if a higher concentration n_8 of Na^+ in one partition expands into a lower concentration n_7 in an adjacent partition to the left, the work done per mole is $RT \ln(n_8/n_7)$. Looking at this electrically, on a per mole basis, when F coulombs of Na^+ ions expands from concentration n_8 into lower concentration n_7 , the work done is $(RT/F) \ln(n_8/n_7) \times F$ joules. (F is the Faraday constant.) Thus there must be an electromotive force $(RT/F) \ln(n_8/n_7)$, for example, driving the Na^+ ions from compartment 8 to compartment 7 (Figure 8c); and there must be a succession of such voltages between each compartment across the membrane, as illustrated in Figure 8d. The total electromotive force for Na^+ across the membrane must be the sum of these small voltages, or:

$$(RT/F) [\ln(n_8/n_7) + \dots + \ln(n_3/n_2) + \ln(n_2/n_1) + \ln(n_1/n_x)]$$

that is:

$$(RT/F) [\ln(n_8 \times n_7 \times \dots \times n_2 \times n_1) / (n_7 \times \dots \times n_2 \times n_1 \times n_x)]$$

that is

$$(RT/F) \ln(n_8/n_x).$$

Consequently, although we cannot know how the minute electromotive forces, and thus the electric fields, are distributed within the Na^+ -permeable part of the membrane, we can say with certainty that the electromotive force nE driving Na^+ across the Na^+ -permeable membrane is

$$nE = (RT/F) \ln(n_8/n_x)$$

This e.m.f thus depends only on the concentration of Na^+ on either side of the membrane. Similarly the electromotive force cE driving Cl^- across the Cl^- -permeable membrane will be

$$cE = -(RT/F) \ln(c_8/c_x).$$

The factor $(RT/F) \ln(C_1/C_2)$ works out as about

$$60 \log(C_1/C_2) \text{ millivolts}$$

in round numbers at a little above room temperature, giving an e.m.f. of about 60 millivolts for each 10-fold concentration difference across the membrane.

Looking at the circular current again in Figure 8c, it is clear that in addition to small concentration difference voltages appearing between the compartments, each compartment must have an electrical resistance to the current flow. Suppose resistances nR_1, nR_2, \dots, nR_i for the compartments of the Na^+ -permeable membrane and cR_1, cR_2, \dots, cR_j for the compartments of the Cl^- -permeable membrane. These small resistances are in series and will simply add, to give us nR for the resistance of the Na^+ -permeable and cR for the Cl^- -permeable membrane.

In this way we get the equivalent circuit in Figure 8e for the entire membrane, with Na^+ ions presumed flowing through resistance nR and Cl^- ions flowing through resistance cR . The Na^+ and Cl^- currents must be equal since there can be only one current in the circuit. From an electrical analysis of this circuit, it is easy to show that the voltage across the membrane must be:

$$V = nE \cdot (cR/(cR + nR)) + cE \cdot (nR/(cR + nR))$$

This is a variant of the Nernst equation [8] for voltages generated under ionic diffusion. It shows that the membrane potential V depends not only on the e.m.f of the ions involved,

but also on the relative resistances or relative conductivities, and thus relative permeabilities, of the ions within the membrane.

[There is a factor of proportionality between ionic electrical conductance and ionic permeability. A somewhat similar and workable (but strictly wrong) equation is also frequently used to analyse electrical potentials across membranes. This is the Goldman equation [1,11,12,16], which makes use of relative permeabilities of the ions, rather than relative conductivities (or resistances). Unfortunately the equation is based on the empirical (but unlikely, and certainly unprovable) assumption that the electric field within the membrane is constant and thus that voltage change within the membrane is linear; for this reason the equation cannot be fully depended upon, and, although widely used, is avoided by some researchers. No such assumption was made in the derivation above of the Nernst equation variant above.]

From the equations above we see that if the Na^+ resistance nR is very large compared with the Cl^- resistance cR , that is, if the membrane is highly permeable to Cl^- ions and only weakly permeable to Na^+ ions, V tends to cE , the Cl^- e.m.f. Similarly, if the Cl^- resistance is very large compared to the Na^+ resistance, V tends to nE , the Na^+ e.m.f. In other words, the membrane potential tends to the potential of the ion that is very permeable compared to the other ion.

Thus in Figure 8a, if the left store concentration is 1 (millimoles per litre) and the right is 10, the membrane voltage will be +60 millivolts (with the left side positive) if Na^+ is very permeable compared with Cl^- , and -60 millivolts (the left negative) if Cl^- is very permeable compared with Na^+ ; and if Na^+ and Cl^- are equally permeable, that is, if $cR = nR$, then $V = 0$. We will have the same electrical results if the left store has a concentration of 10 and the right 100, or the left 100 and the right 1000. As with all grodec, it is not the quantity of fluid in the stores that determines the pressure between the two stores, but the difference - although with electrolytic grodec we must deal with a logarithmic scale of ion concentration (and thus partial pressure) differences, as opposed to the linear scale in other grodec. [A gas pressure grodec would also require a logarithmic scale of gas quantities.]

In Figure 8, the left store would correspond to the inside of a neuron, with low Na^+ concentration, and the right side to the surrounding interstitial fluid with high Na^+ concentration. In the membrane resting state the Na resistance is very high compared to that of Cl^- , and so the inside of the neuron (left side of the grodec) is negative with respect to the outside by about 60 mV.

Suppose now that we have an Na based grodec with very small stores, and with a high resistance Na^+ membrane separating the stores and containing a switch for Na^+ , and with a 10-fold NaCl concentration difference. In addition suppose that the left store is connected to a large reservoir of the same concentration, as is the right store. Suppose also that the grodec switch (Figure 8f) is such that the Na resistance is high when the switch is open, which is the normal case or 'resting' case, but low when the switch is closed. If the open switch resistance nR is extremely high compared to cR , this will mean that the left side of the grodec is -60mV with respect to the right when the switch is open, but +60mV when the switch is closed; furthermore, when the switch is closed, because the total Na and Cl resistance is low, a high current will flow, that is a high Na flow from right to left and a high Cl^- flow from right to left (Figure 8g). In other words, the closing of the

Na resistance switch simply enables NaCl to flow at a relatively high rate from the high concentration right side to the low concentration left side.

In practice nR will be finite and not infinite. If we assume more realistically that $nR/cR = 19$ when the switch is open, then a small current will be flowing in the loop when the switch is open, and a small amount of NaCl will be leaking through the membrane. The membrane voltage will now be reduced slightly to -55mV (see Figure 8f, or use the Nernst equation variant above). Furthermore if, when the switch closes, nR does not drop to zero, but to about one third of cR , then the membrane voltage will swing by 85mV to only $+30\text{mV}$ instead of to $+60\text{mV}$. At this peak the current in the loop, and thus the flow of NaCl accross the membrane, will have increased about 20 times

In addition, suppose we make the switch dependent on the membrane pressure, that is, the pressure between the two stores, as in a normal grodec. The question is how to arrange that, given the complex array of pressures involved. A little thought will show how it should be done.

When the grodec is in a steady state close to its trigger pressure, with high concentration (partial pressure) difference between right and left, assuming as above that the Na resistance is high (about 19 times cR), but capable of being switched to a low value about $1/3$ of cR , then the left side is negative -55mV with respect to the right as we have seen. This means that under NaCl concentration difference pressure from right to left, Cl initially flows through the Cl- membrane and charges the left side negative in an attempt to pull Na ions through, an attempt that is resisted because of the high Na resistance. In addition, were the high Na resistance to break down, the left side negative potential of -55mV would suddenly collapse to $+30\text{mV}$.

Now, when the grodec is resting, prior to any trigger, if a small electrical current is passed through the membrane (both Na and Cl permeable portions) from left to right, that is, an additional flow of Cl- ions from right to left, this will cause the left side to become more positive, say from -55mV to -50mV . Let us now arrange the resistance so that it is just such a membrane voltage change that activates the grodec trigger. There will then be only two ways to activate the switch. Either (a) increase the Na+ concentration difference between the stores without changing the Cl concentration difference (study Figure 8f), which is consistent with the theory of a grodec that uses Na+ as its active fluid, and which will cause an increased leakage Cl- current through the Cl- resistance from left to right (i.e. Cl- ions from right to left). Or (b), directly pass a (positive) current from left to right, that is a negative (Cl- ion) current from right to left. It is the second method that is of interest in most cases for generation and propagation of action potentials.

Once the Na resistance switch triggers, there will be a sharp shift in membrane V from about -55mV to $+30\text{mV}$, and a large flow of NaCl from right to left, in familiar grodec fashion (Figure 9a). This will rapidly equalize the concentrations in the two stores (if they are small compared to membrane area) so that the Na+ and Cl- electromotive forces both drop to zero, bringing membrane voltage to zero and causing the switch to close. The voltage trace is shown in Figure 9b. A slow diffusion of NaCl from left side to the NaCl reservoir, and from NaCl reservoir to right side, will slowly restore the former concentration difference, bringing the left side back to -55 (Figure 9b) ready to be triggered again. If the diffusion to and from the reservoirs is very slow, it could take a

relatively long time before conditions in the grodec prior to the switched collapse is restored.

4.2 Propagation of an electrolytic grodec action potential

If a large number of these grodec are placed in parallel and connected by resistive connections, we can get the action potential due to triggering of one grodec to propagate along the series of grodec. The situation is equivalent to the equivalent circuit in Figure 9c. Initially all grodec have left side at -55mV with respect to the right, with right side having a 10-fold higher concentration of NaCl than the left. Suppose now we pass a stimulating current from left to right through the top grodec in Figure 9c, sufficient to trigger the switch in that grodec. Some of this stimulating current, but a necessarily lesser current because of the connecting resistors, will flow into the second grodec, and even less into the third, and so on. But these grodec will not receive enough current to trigger them, if the connecting resistances are large enough. Once the first grodec has triggered its left side voltage switches to 30mV , this causes a significant current to flow between the first and second grodec, under a combined voltage of $30 + 55$ or 85mV . The resulting current through the second grodec will be from left to right, or Cl^- from right to left, and will cause the second grodec to trigger. The triggering of the second grodec will likewise trigger the third, and so on. Thus the action potential will propagate along the array of grodec.

[In a neuron with myelination [8,12], we will likewise have the equivalent of an array of grodec in parallel (one for each gap in the myelin), and once a single myelin-gap grodec equivalent triggers, the collapse will propagate along the neuron. The stores of the neuron grodec are the areas very close to the membrane, and the remaining fluid constitutes the reservoirs. Myelination and neurons is discussed later in Section 4.5.]

4.3 Action potential from two electrolytic grodec

As shown above, recovery from an action potential for the single Na^+ grodec depends on the time taken for NaCl to move to and from the store reservoirs. This recovery time can be much reduced by employing two electrolytic grodec in parallel, thus obtaining a sharper action potential.

Consider the two electrolyte compartments separated by a membrane, as in Figure 10a. The membrane is permeable to Cl^- , and only slightly permeable to K^+ and Na^+ ; however, the Na and K resistances can be switched to much lower levels. In addition, as in the earlier grodec, the right compartment has high NaCl concentration and the left low NaCl concentration, with a 10-fold difference. In addition the left has high K_2SO_4 concentration and the right low K_2SO_4 , the difference being 30-fold. [The SO_4^{--} ion is not membrane permeable at all, and is included merely to simplify the following discussion, by providing ions to balance the charges, since an electrolyte is essentially neutral.] The situation in Figure 10a is close to that in a neuron, where the left compartment corresponds to the inside of the neuron, that is, inside with high K^+ and low Na^+ and outside with high Na^+ and low K^+ .

The compartments in Figure 10a, can be rearranged as in Figure 10b, where there are now two membranes, one permeable to NaCl (with low Na permeability and high Cl permeability) and the other membrane permeable to KCl (with low K permeability and high Cl permeability). If we repeat the analysis of the previous section, the situation in Figure 10b has the electrical equivalent circuit in Figure 10c. The top part of the circuit is very close to that given for the Na⁺ grodec in Figure 8f, and the bottom part is the electrical circuit for a K⁺ grodec. The situation in Figure 10b corresponds to two grodec, an Na⁺ grodec with high partial Na⁺ pressure in the right store, and a K⁺ grodec with store order reversed, that is, high K⁺ partial pressure in the left store. This is also depicted in Figure 10d in terms of abstract grodec. In addition the two grodec are coupled, by what must necessarily be negligibly resistive links.

Suppose the Na⁺ grodec works as described in the previous section, and assume, to keep the analysis as simple as possible, that the initial K⁺ resistance kR in the K⁺ grodec is very much greater, say 100-fold, than the Cl resistance cR . Also, although in Figure 10c the Cl⁻ resistance in the Na grodec is $cR/2$, the same as in the K grodec, because the two grodec are non resistively coupled, the effective resistance for Cl is cR as before. A positive current from left to right, or negative Cl⁻ ion current from right to left, will trigger the Na⁺ grodec, as before causing the left side membrane potential to move from about -55mV to a peak +30mV, at which point recovery will set in and the Na⁺ switch closes. However, the new positive voltage accross the membrane will send an (increased) weak positive current from left to right through the K⁺ resistance; the same effect that would occur if the K⁺ concentration in the left (high concentration) side of the K⁺ grodec were increased. Now suppose the K⁺ grodec is initially set such that it triggers if either (a) there occurs a small increase in K⁺ partial pressure difference from left to right, in normal grodec fashion, or (b) a small increased current goes from left to right accross the K⁺ resistance kR (which is the same thing). In that case once the Na⁺ grodec has triggered and brought the left side membrane potential from its initial -55mV to +30 mV and equalizing NaCl concentrations, immediately the K⁺ grodec will trigger, the K⁺ resistance kR dropping suddenly to a value comparable to cR , with the Na⁺ resistance switch closing at the same time, so that the left side membrane potential reverses again, back towards a value equal to the K⁺ e.m.f or -90; but because of the effect of the comaprable Cl resistance cR , the left side mebrane potential actually swings to a value between -55mV and -90mV, more like -70mV. The open K⁺ switch also causes KCl to move from left to right, in a short time transiently equalizing the K⁺ concentrations (the stores are small), which then closes the K⁺ resistance switch. The action potential for this sequence of events is shown in Figure 10e.

Notice that opening the K⁺ resistive switch causes the membrane potential to return quickly to something like -70mV and not its original resting value of -55mV. This overshoot is an inevitable consequence of the coupling of the Na and K grodec, where the the K concentration difference in the K grodec (30-fold) is greater than the Na concentration difference in in the Na grodec (only 10-fold). Exactly the same overshoot phenomenon occurs in neurons where typical concentrations (millimoles per liter) for the membrane permeable ions are approximately [1, 8, 11]:

	inside (left)	outside (right)	difference
Na ⁺	15	150	10-fold

K+	150	5	30-fold
Cl-	10	110	11-fold

which values are much the same as those used in the grodec described above.

Once the membrane potential has returned to and overshoot its initial resting potential of -55mV, all switches are closed, and the reservoirs replenish the stores, and the left side potential drops from its overshoot level of about -70mV to its normal resting level of -55mV. The pair of grodec are now ready to trigger again.

4.4 The neuron action potential interpreted as a grodec phenomenon

We have shown earlier how any grodec can trigger to give rise to a transient pressure pulse, from which it can slowly recover and then trigger again, thus giving rise to a pressure shift "action potential". We have shown that the pressure recovery process can be speeded up by means of a second grodec couple to the first, so that when the first grodec triggers and causes its right store pressure to suddenly drop, this causes the second grodec to trigger to bring the pressure in the right store of the first grodec quickly back to its original level, thus causing a quicker replenishment of the stores of the first grodec.

Then we showed that even an electrolytic ion-partial-pressure difference grodec, such as a Na⁺ grodec, can act in principle like any grodec and give rise to pressure action potentials, where this time the pressure difference change is electrical, i.e. a voltage change. Again to speed up the voltage recovery, and thus to increase the maximum possible triggering frequency a second electrolytic grodec, this time a K⁺ grodec can be used. If we wish to use the same physical store geometry for both grodec, then the second grodec must be based on different ions from the first (K⁺ versus Na⁺). [This special-case geometry with the same stores used for both grodec (i.e. the right store of the first is defined in the same physical space as the left store of the second, and vice versa) corresponds to a neuron.] If we merely share the right store of the first grodec with the left store of the second, then we could produce sharp action potentials using two grodec based on the same ion.

Finally, if we look at what actually occurs in a neuron during an action potential, we will see that it is exactly the same in principle as what occurs in a pair of Na and K grodec with common stores. But first, a brief summary of neuron characteristics is necessary.

4.5 The neuron action potential

A typical neuron has a main body or the *soma*, with protruding *dendrites* that receive signals, at locations called *synapses*, from other neurons. Signals received at dendrites are integrated in the soma to trigger an action potential that travels along a long *axon* emanating from the soma. At the end of the axon away from the soma and dendrites, the axon typically branches. The axon branches, called *terminations*, end on the dendrites of other neurons (or at muscles), the joint between an axon termination and a dendrite of another neuron being the synapse or coupling location between two neurons. The long axon is a cylinder in which the inside of the cell is separated from the outside by an active

membrane. The axon propagates action potentials, and to speed up propagation, the axon is often enclosed in an insulating *myelin sheath*. In a myelinated neuron, the axon sheath is an electrically insulating myelin except for gaps, like an insulated copper wire where the insulation has been removed (at the gaps) at regular intervals. [These gaps are known as the *nodes of Ranvier*.] At a gap in the sheathing, the outside of the cell is separated from the inside only by the membrane. Electric current and ions can flow through the membrane only at these gaps in the myelin.

Outside the membrane the concentration of Na^+ is high and of K^+ is low and inside Na^+ is low and K^+ is high, so that Na^+ tends to diffuse inward along with other matching negative Cl^- ions, while K^+ tends to migrate out [1,8,12], with ions concentrations similar to those given in the table above, and similar to those used to analyse electrolytic grodec above.

The electrical operation of the neuron is as follows

1. The membrane is resting with inside about -55mV with respect to the outside, with concentration differences for K^+ , Na^+ and Cl^- as given earlier.
2. If a current is passed across the membrane from inside to outside (negative charges from outside to inside) to bring ("reducing") the resting potential of the inside to about -50mV.
3. If the resting potential has reached the threshold (trigger) potential Na^+ channels open allowing much increased flow of Na^+ ions (and Cl^- ions) into the cell from outside.
4. At the same time the inside resting potential shifts rapidly to a peak value typically between +30mV and +40mV, depending on the type of neuron, or about +35mV in general.
5. Then the Na^+ channels close, and the K^+ channels open, causing K^+ to flow and Cl^- to flow out of the cell, and forcing the membrane potential to a value beyond the original resting potential of about 55mV, that is, an "overshoot" to about -65mv. The overshoot then decays back to the original resting.
6. During the action potential, at a myelin gap, the inside positive potential causes a current to flow to the next myelin gap, triggering the membrane at that gap, which in turn causes the next gap to trigger, and so on in an ionic domino effect as the action potential travels from sheathing gap to sheathing gap along the neuron.

From this description, and from the analysis of electrolytic grodec action potentials above, it is clear a myelinated neuron axon can be considered as a sequence of electrolytic grodec pairs, one grodec of the pair having sodium ion concentration as the mobile fluid and the other having potassium ion concentration as the mobile fluid. There is a grodec pair at each sheathing gap or node where the membrane is exposed to both inter and intra cellular fluids, which, because of different ionic conductances through the membrane give rise to electrical potentials across the membrane.

5.0 Grodec logic circuits for use as neural net models

We have shown above that a single grodec has the switching properties of a neuron, except that recovery is slow, and that a pair of grodec can also have the

switching properties of a neuron, but with comparably fast recovery. Either can thus be used as a model of a neuron. It remains to show how these grodec can be used to construct logic circuits. We shall use the single grodec model of a neuron for the sake of simplicity. The logic of the discussion to follow would not be materially different if a pair of grodec were used as a neuron model.

5.1 Stimulation and inhibition of one grodec by another

When used as a neuron model the single grodec, in its "resting" or non active state, is set with its pressure difference just under the switching pressure g . Now when the switch triggers energy is released, which will normally be dissipated as waste heat. However, in a neuron this energy is usually utilized to trigger grodec in a neighbouring sheathing gap, as described above, by passing a stimulating current.

This is one way in which an action potential is triggered in a neighbouring membrane section in nature, namely use the energy released during the action potential to change the pressure in a neighbouring grodec sufficiently to trigger it. Another way is to use the energy released to alter the trigger pressure of a neighbouring grodec. This is what is done at a neuron synapse, where a neurotransmitter chemical is released by one neuron at its terminal (presynaptic), which substance then migrates to the neighbouring neuron dendrite membrane (postsynaptic), alters its trigger pressure downward, and so triggers an action potential localized to where the neurotransmitter is accepted [8, 12, 16].

We can use both methods to form artificial neural nets from grodec, that is, either use of released energy of an action potential to alter the pressure of another grodec and so trigger it, or use of released energy to alter the trigger pressure g of another grodec and so trigger it.

In the discussion of neural net model in the next section, we shall assume that the first method is being used to couple two or more grodec. The details of the coupling do not matter. We might imagine for example that the fluid flow during the pressure collapse generates a small amount of electrical power which is used to pump a small quantity of fluid into the right store of a neighbouring grodec, thus increasing its pressure enough to trigger it. This is positive or stimulative coupling of grodec, where an active grodec causes a quantity of fluid q to be pumped into the right store of a resting grodec.

We can also have negative or inhibitive coupling of two grodec where the energy released by one grodec is used to pump a small quantity of fluid out of the right store resting grodec, reducing its pressure and making it harder to trigger. Note that because of the nature of the resting equilibrium in the grodec, a delivery of a quantity q that does not stimulate the grodec will result in the removal of q to the reservoir shortly afterwards, since delivery of q cause the right store to be out of equilibrium with the reservoir, and cause fluid to move from the right store to the reservoir. Similarly, removal of a quantity q from the right store will result in it being replaced from the reservoir in a short time. This temporary nature of the coupling affect of one grodec on another due to an action potential is necessary if the logic circuits formed from grodec are to work repeatedly.

5.2 Grodec logic gates

Basic And circuit

If grodec A, B and C are each positively coupled to grodec D such each can deliver a fluid quantity q to D, and it take a quantity $3q$ to stimulate the grodec D, then the action potentials A, B and C must all go off at the same time in order to trigger grodec D. In this way an And circuit can be constructed any number of inputs.

Majority circuit

If 5 grodec are positively coupled to a sixth, where the sixth needs $3q$ to trigger, and each of the 5 can deliver q , then the sixth will trigger if a majority, at least 3, of the 5 are triggered. In this way a Majority circuit can be constructed with any number of inputs.

A and not B circuit

If A and B grodec are coupled to grodec C, which needs q to trigger, and if A is positively coupled to deliver q and B negatively coupled to remove q , then if both A and B are triggered, C will not be triggered. C will be triggered only if A alone is triggered, because of the inhibition coming from grodec B.

OR circuit

If A, B, and C grodec are coupled positively to grodec D, each delivering q to D, and if D needs q to trigger, then D will trigger if any one of A, B or C triggers.

A and B but neither C or D

We have this circuit if A and B are each positively coupled to E and each deliver q to E while C and D are each inhibitably coupled and each remove q from E, and E needs $2q$ to trigger. If A and B are triggered, then E will trigger. But if A and B trigger and either C or D also trigger, or both C and D trigger, then E will not trigger.

Not circuit

This one is slightly less obvious, for we need a set of oscillating grodec X and Y, in addition to two grodec A and B for input and output. Grodec A is negatively coupled to B, and can remove q units from B. Grodec X and Y constitute a separate system. X triggers Y and after a delay Y triggers X which, after a delay triggers Y, and so on. But not only does X deliver enough fluid to trigger Y, it is also positively coupled to B and delivers q units to B each time it triggers; B is set so that q units is enough to trigger it.

Thus since the oscillator XY triggers continually, when A is not triggered B will continually trigger, that is, if A false, B is true. However if A triggers, this will inhibit B and so B cannot trigger. But as soon as A stops being triggered B will resume being triggered, that is, if A is true if B is false. This gives us a Not circuit (Figure 8f)

Not And or Nand

Grodec A, B and C are negatively coupled to D. An oscillator grodec X is positively coupled to D, delivering $3q$ to D, where q is just enough to trigger D. Each of A, B, and C remove q units from B. Thus D will trigger continually, so that D is true,

except if all of A, B and C are triggered, that is, except when A, B and C are all true. This is Nand logic, where the output is $(1-ABC)$, when A, B and C can have values 1 or 0.

5.3 Finite-state grodec machines

It has been shown by Minsky [8, 9, 17] and by others in the field of finite-state sequential machines, that relatively few logic elements of the type shown above, which in function are similar to those originally developed by McCulloch and Pitts [15], can be used to build simple networks to carry out conventional computing functions, including short-term memory, with encoding and decoding, as well as arithmetic functions.

A finite state machine [5] can be abstracted as a 6-tuple $M = (Q, S, R, f(Q,S), g(Q,S), q_0)$ where the set Q is the finite set of internal machine states, the set S is the set of possible inputs to the machine, and R is the set of possible outputs. The function $q = f(Q,S)$ is the state-transition function in which q is a member of Q ; it determines the next state of the machine as a function of the current state and the current input, that is, the next state depends on the current input and the current state. The output function $r = g(Q,S)$, where r is a member of R , determines the next output of the machine as a function of the current input and current state. Thus the next output is determined by the current input and the current state. At any time the machine can be reset to an initial state q_0 .

In practical terms a grodec logical machine built from the above logic elements will have an array of input grodec at the input interface, some of which will be triggered by an input that is a member s of S . Also at the output interface of the machine will be an array of grodec, some of which will be triggered to give an output that is a member r of R . To be specific about the operation of a grodec logical machine, we need to consider (a) the state transition time and (b) the nature of the set of allowed inputs S .

Take any pair of coupled grodec of the machine, G_1 and G_2 , such that when G_1 triggers it triggers G_2 . If G_1 being triggered at time zero always results in G_2 being triggered at the same later time p for every grodec pair, such that at time p G_1 has been reset and could be triggered again, and if this is true for every coupled grodec pair in the machine, then p is the propagation time of the machine. When the machine is operating, every p units of time action potentials will be triggered in different grodec of the machine, even with looped grodec or oscillators. For example if G_1 is coupled to G_2 , which is in turn coupled to G_1 , then if this pair is oscillating it will generate action potentials in either grodec G_1 or G_2 at a frequency of $1/2p$ spikes per second, and in both grodec at a frequency of $1/p$ spikes per second. Finally, if the machine has m grodec in sequence between input and output grodec interfaces inclusive, an output action potential will be detected at $k = mp$ units of time after an input stimulus; k is therefore also the time required, on receipt of an input, for the machine to switch to a new state, or the state transition time of the machine, ready to accept a new input.

Consider now the inputs to a grodec logical machine. Suppose the machine has input interface grodec A-K and output interface grodec L-Z. A specific input s_5 , coming with the machine in state q_4 , could involve stimulus of input interface grodec A, C, D and F, for example, at time $4k$. [The initial input s_1 comes at time zero, with the machine in state q_0 .] The stimulus on each input interface grodec due to any input s_n would be a single stimulus (being a transfer of fluid to the grodec sufficient to trigger it)

at a specific time nk , where k is the machine's state transition time and n is an integer. Each of the input interface grodec A, C, D and F would respond to $s5$ at time $4k$ with an action potential at time $4k + p$. The next specific input $s6$ would be at time $5k$ and could be stimulus of input interface grodec B, C, D, G, and K so that each of these grodec would generate an action potential at time $5k + p$, and so on. A specific input $s5$ will propagate through the logic elements of the machine, in time $k = mp$, turning on some oscillating loops and turning off others turned on by earlier inputs, thus placing the machine in a new state $q5$ at time $5k + mp = 6k$ and stimulating an action potential at specific output interface grodec, say N, P and U, that is, the output $r5$. This output $r5$ obviously must depend on the nature of the input $s5$ and the state of the machine at the time of the input, that is, $q4$. Also the new state of the machine $q5$ must obviously depend on the input $s5$ and the original state $q4$. Thus a grodec logic machine of the type described, with a well defined state transition time, satisfies all the conditions for a finite state machine.

Note that the above discussion implies a design feature of action potential based logic elements not found in logic elements based on conventional electronic gates. Suppose a grodec logic machine could be economically designed so that a specific input $s8$ could be processed with g gates in sequence (for a time gp) while all other inputs needed $x + i$ gates in sequence for a time $(x + i)p$. This would mean that one type of input could result in an output much more quickly than would be the case with other inputs. But action potentials do not endure, unlike the voltage states of conventional electronic gates, so that if the machine were to be used as a subcomponent of a larger machine, this design would cause a malfunction, since outputs would be inputs to another machine that would expect them at intervals of k . It would thus be necessary to add i grodec in sequence in an appropriate way to the original machine to ensure that the output from $s8$ appeared exactly k units of time after the input.

5.4 Grodec memory

We have not discussed memory units, although it is clear that short term memory units can be constructed as oscillators, On for True and Off for False, from the logic elements presented above. However, in biological neural networks the evidence is compelling that long term memory is due to changes in synaptic efficiency, that is, coupling efficiency between neurons [12, 16], rather than to conventional logic circuitry.

In terms of grodec, synaptic efficiency is the measure of how many units of fluid are transferred to a coupled grodec by an excited grodec. The more intriguing aspect of synaptic efficiency memory, however, is that in biological systems it happens because of use. If a synapse is used more it becomes more efficient in terms of making a greater contribution to generating an action potential in the axon [8]. We can use grodec to illustrate what this means.

Suppose we have an And grodec circuit with 4 input grodec A, B C and D and one output grodec E, where A, B, C and D each transfer q units to E and E needs q units to trigger. Suppose now, and this is the key point, that the coupling system in the grodec is designed to increase the quantity of fluid transferred to E if the coupling is used much more than usual. Assume an unusually large use of the A and B grodec on

the input of the And circuit so that eventually each of these can transfer $2q$ units of fluid to E instead of the original q units each. This will mean that now any two inputs alone will trigger E, except C and D. In this state we can say a 1 is being stored, and something is remembered. However, suppose now that A and B falling into subsequent abnormal disuse causes the coupling efficiency mechanism to go into reverse. Once more A and B deliver q units each to E, and once more the circuit is a 4 input And. Here there is a process of forgetting, and 0 is stored once more. It is probably because of this kind of adaptability memory, as well as massively parallel operation, that neural machines in nature are so profoundly different in operation from conventional digital computing machines [8,9]. However, it should be at least clear that grodec and grodec machines can serve as the building blocks of conventional neural networks for researching these problems.

6.0 Summary

Grodec is a growth decay entity in which there are two stores that contain a fluid under pressure. The two stores are connected by a flow line in which there is a switch that opens abruptly when the pressure difference reaches a specific trigger pressure. We have analysed the switching behaviour of grodec in detail and we have demonstrated that grodec can be used as the basis of both grodec stack machine, and logical machines that can model neural nets.

It has been shown that a single grodec has the most of the switching properties of a section of biological membrane, and can give rise to action potentials when properly set. A pair of grodec has been shown to be able to generate a much sharper action potential, like those of a neuron with dual Na and K ion switching. In this way we have shown that a neuron membrane can be considered to be a Na grodec coupled to a K grodec.

Grodec can be coupled in positive (excitatory) or negative (inhibitory) manner to form logic circuits and example of such common circuits have been given, so that grodec can be the basis of conventional neural nets.

However, grodec can also be coupled quite differently, to form grodec stack machines. In such machines pressures (possibly electrical) accumulate in periods of growth and decay, so that a central machine pressure parameter exhibits time function behaviour similar to that of a fractal growth-decay time function, such as an $E_n:m$ function.

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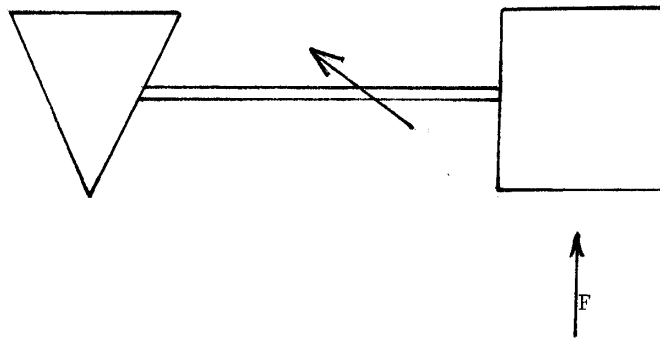


Figure 1

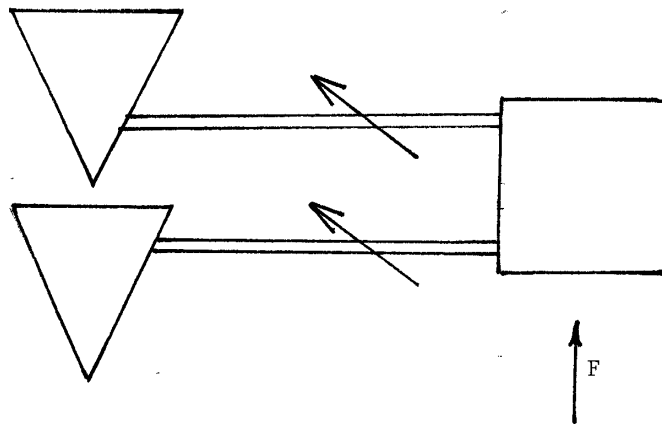
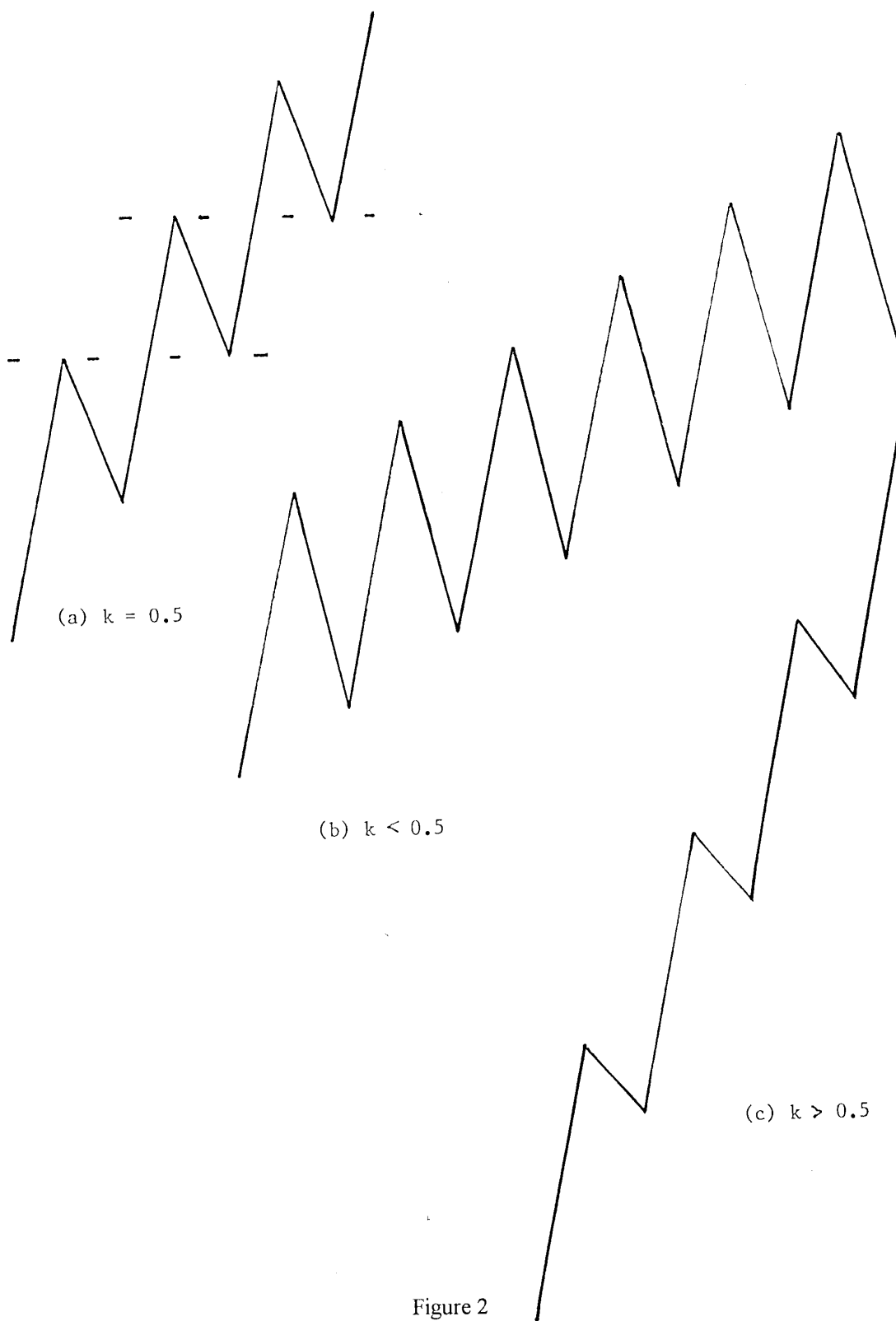


Figure 3



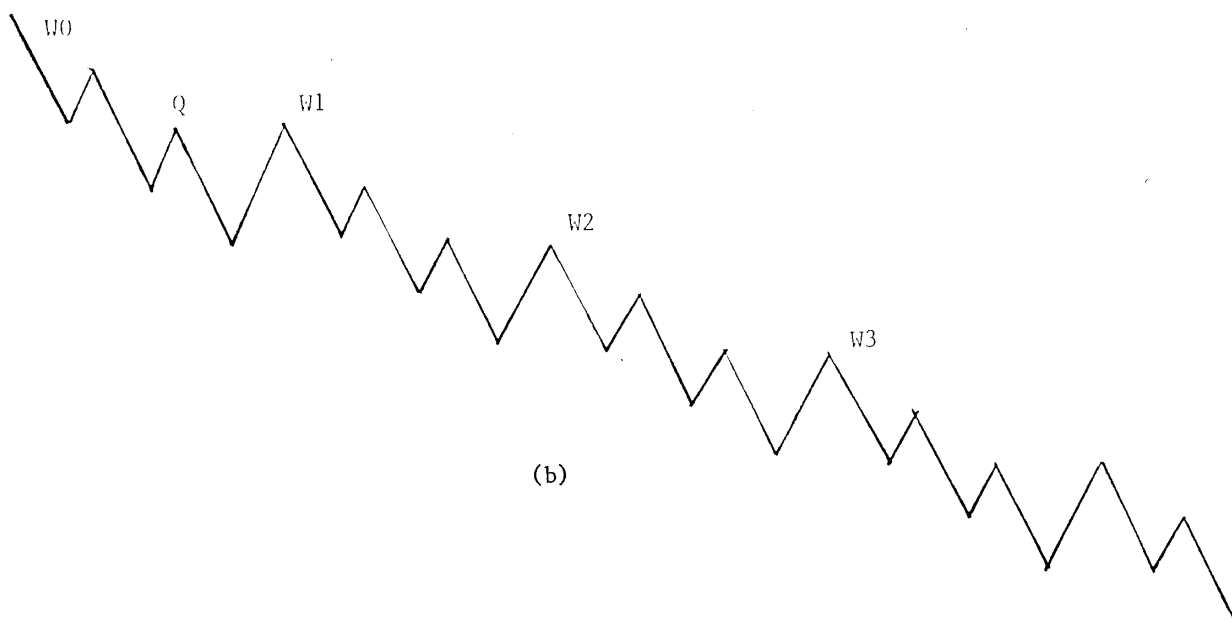
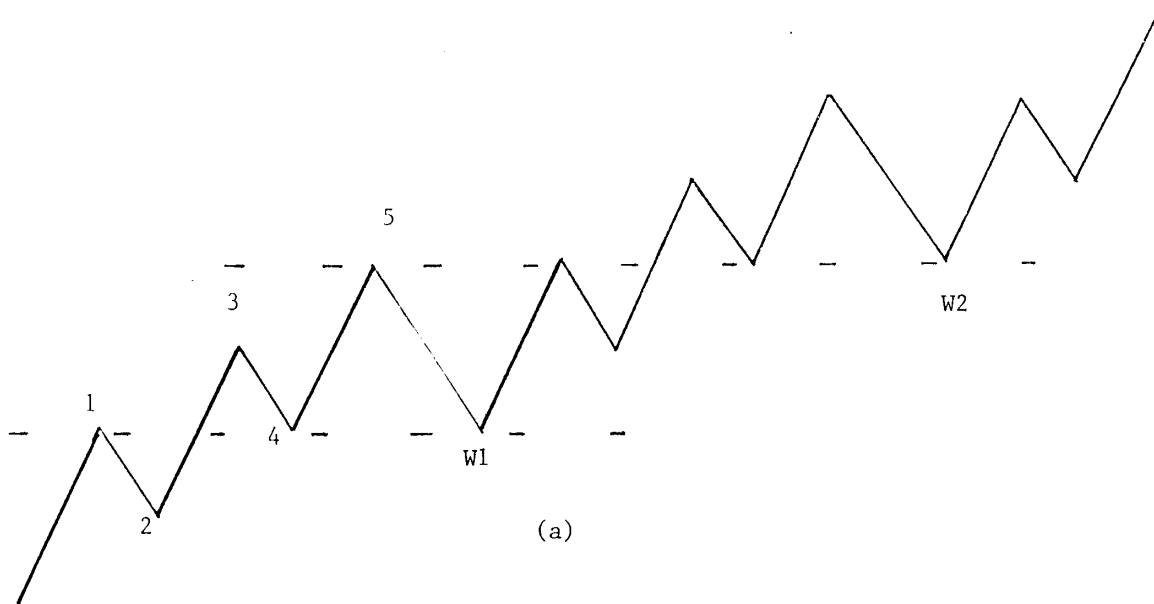


Figure 4

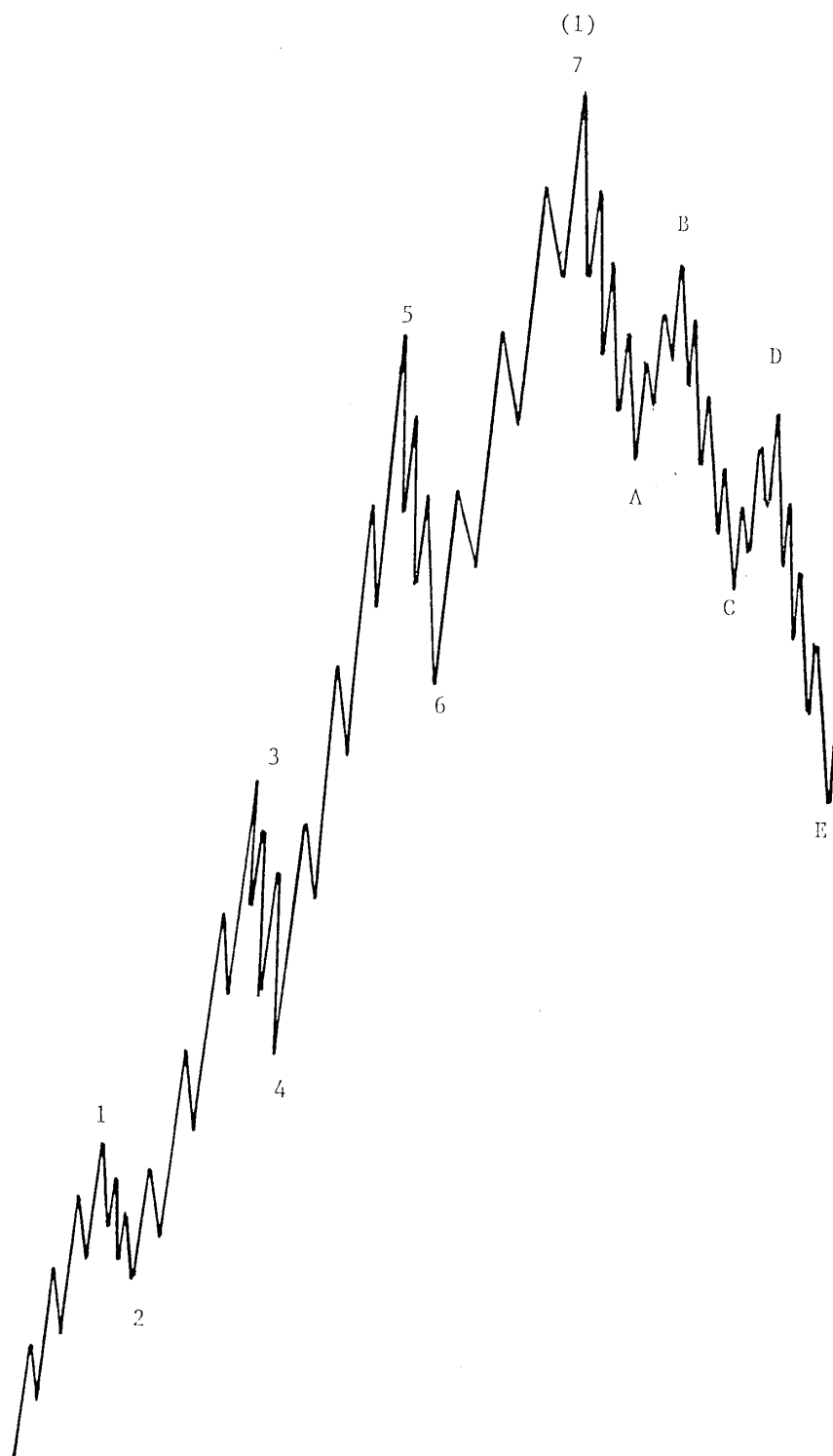


Figure 5 E7:5

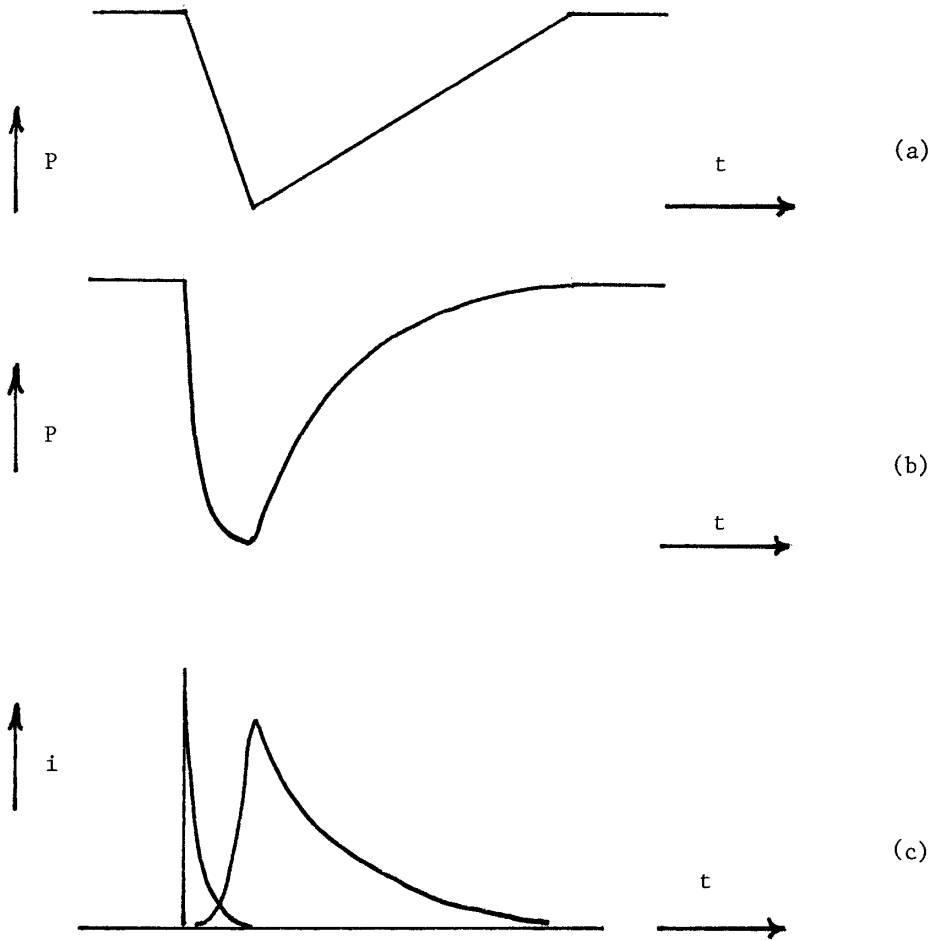


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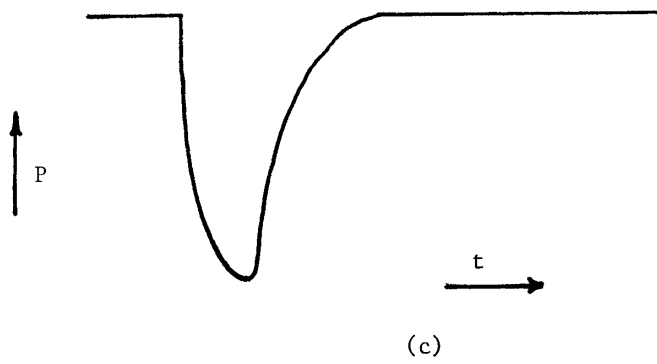
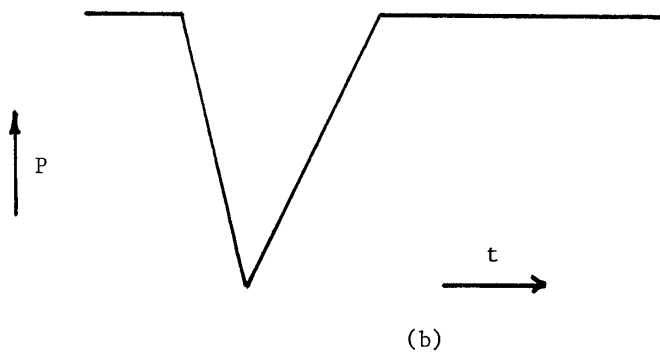
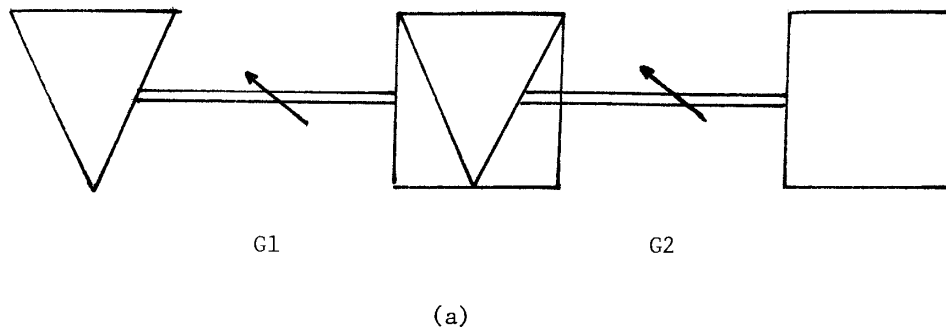


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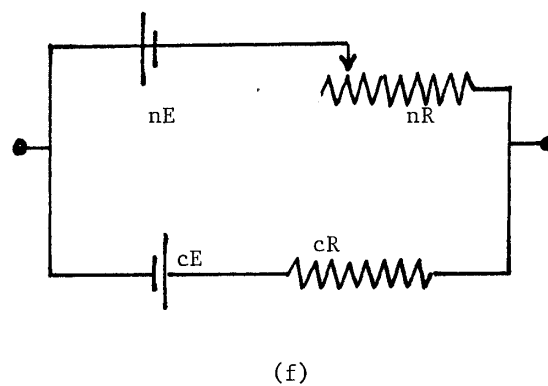
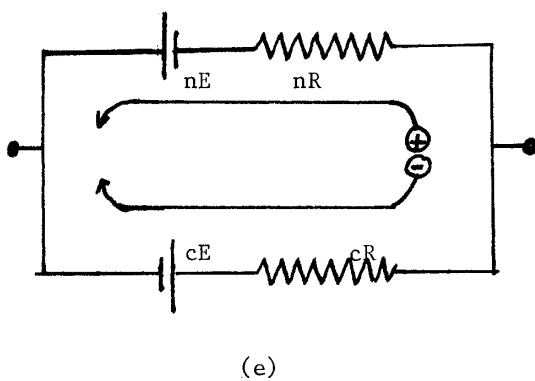
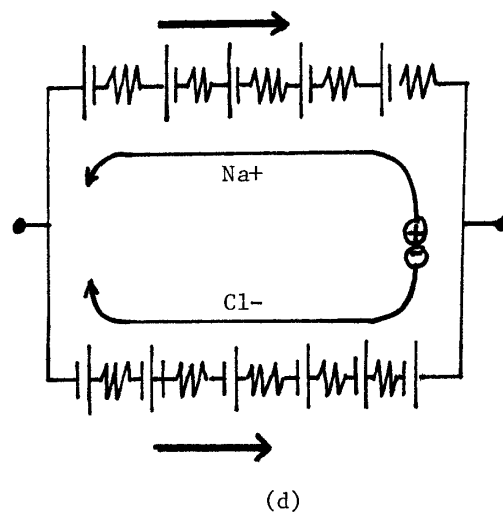
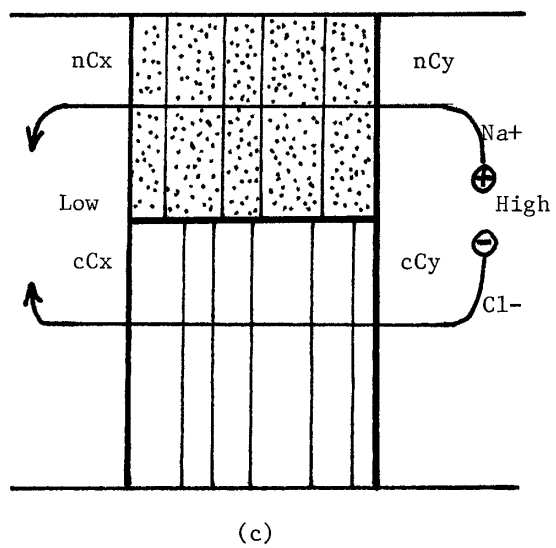
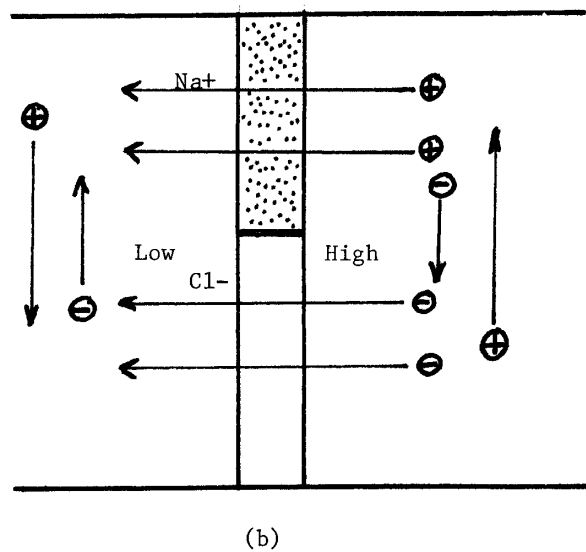
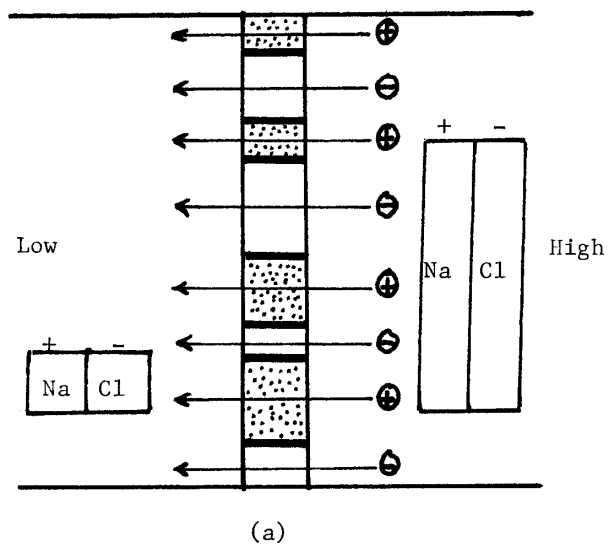


Figure 8

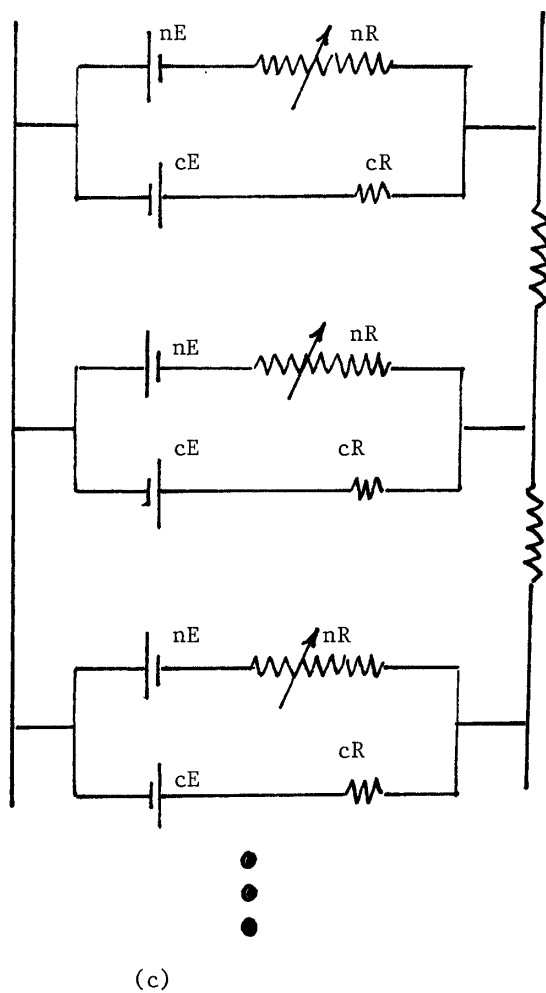
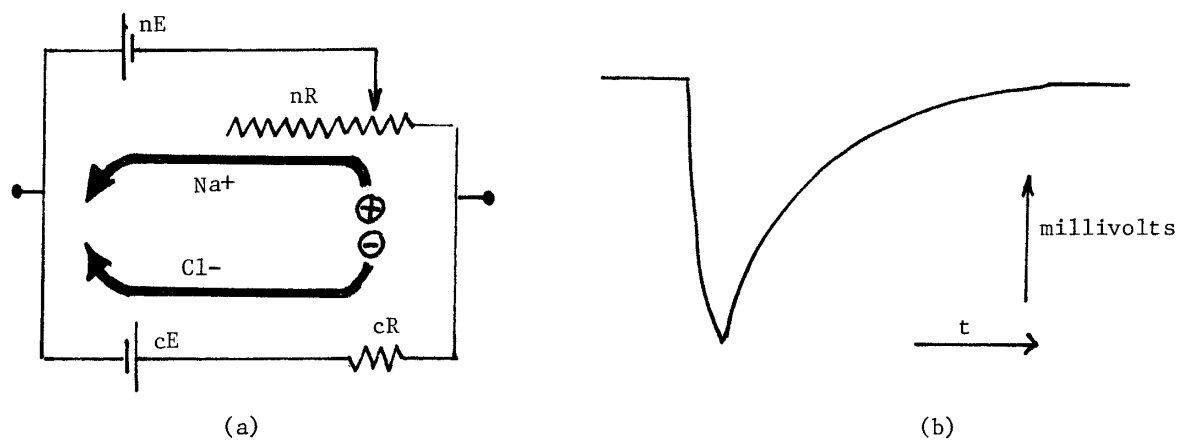
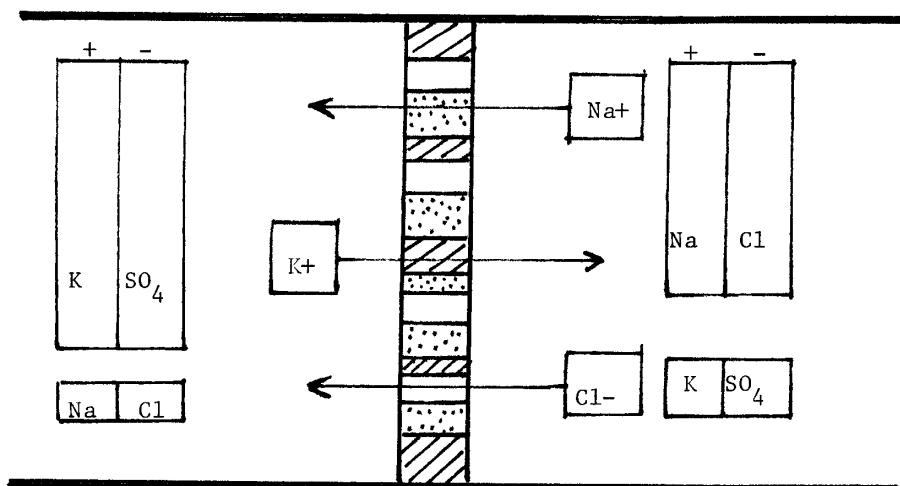
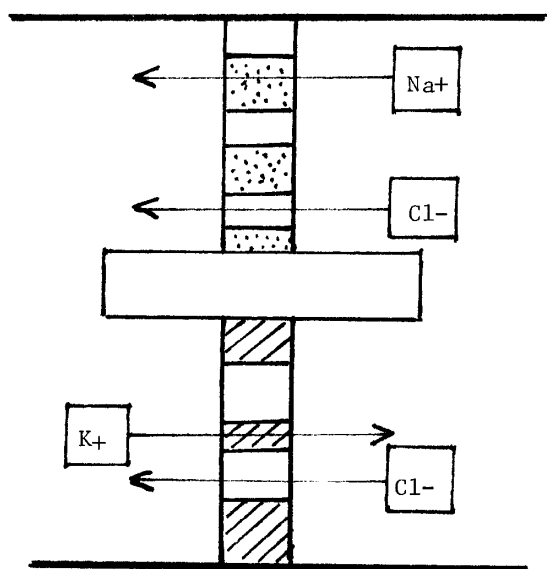


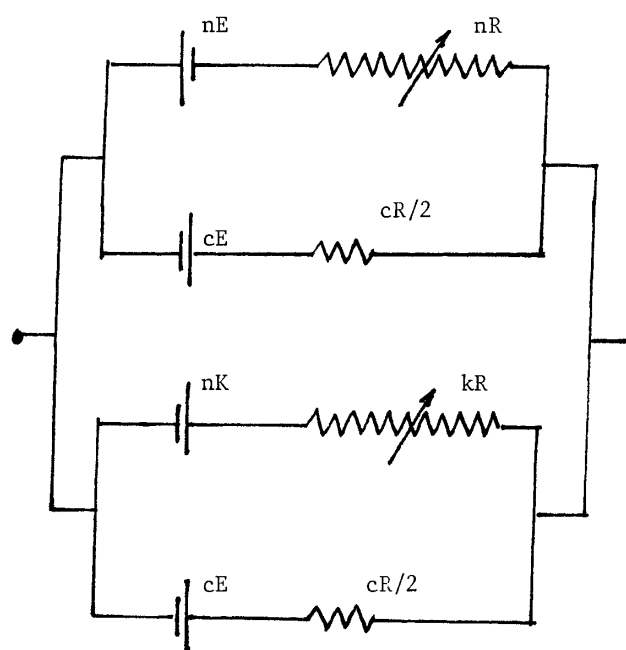
Figure 9



(a)

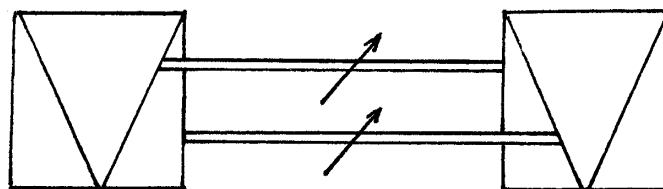


(b)

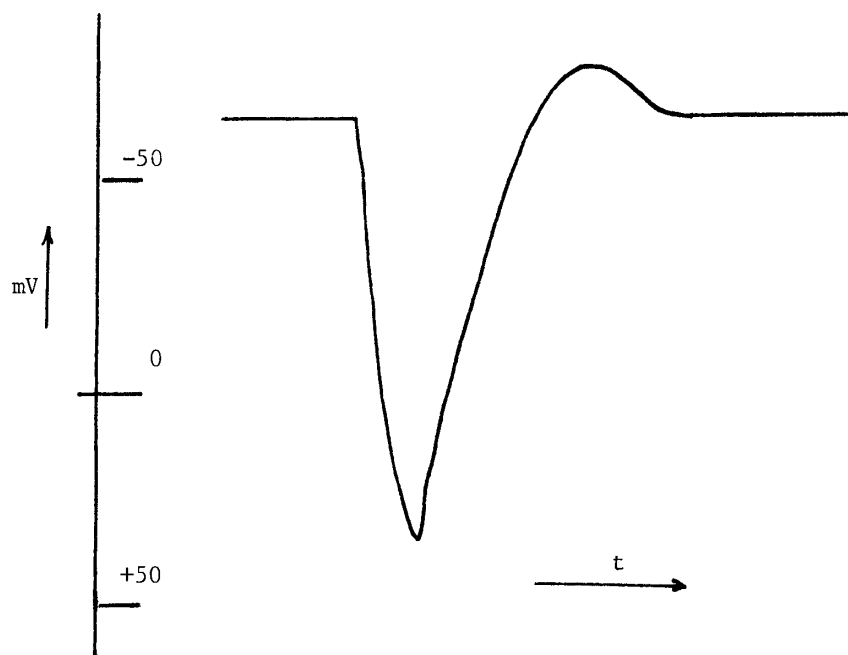


(c)

Figure 10



(d)



(e)

Figure 10