THE UNIVERSITY OF CALGARY

TRANSIENT OPTIMIZATION MODEL FOR SIMPLE PIPELINE SYSTEMS

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Transient Optimization Model for Simple Pipeline Systems," submitted by Duncan M^{c}Innis in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

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Abstract

One of the most complicated unsteady flow problems is the analysis of *fluid transients* (pressure pulses) in pipelines. In the past, optimizing procedures for the design of fluid transmission pipelines have, accordingly, tended to focus on the steady state requirements of the system. Consideration of transients often takes place after the fact, if it is done at all. Yet, the factors—such as pipe diameter, pipe material, wall thickness, valves and their associated controller/actuator systems, fittings and joints, etc.—which ultimately determine the cost of the system are tremendously influenced by the occurrence and behaviour of transients in the pipeline. This means that any "optimal" design which fails to properly account for *water hammer* effects can be, at best, *suboptimal*, and in the worst case, completely inadequate.

The research described herein constitutes an attempt to formulate some rationale which will permit the development of optimizing procedures for simple pipeline systems giving due consideration to the importance of transient effects in determining the ultimate pipeline cost. The proposed model is based on the theory of valve stroking which, for a given physical system with one known or specified boundary condition, can solve for the unknown boundary condition such that the magnitude of the head rise or fall in the pipeline is the lowest attainable for that system. This provides a means of assessing the head-based cost terms that includes the effects of transients. In addition, the response time of the system is included in the specification of the known boundary condition, thus permitting the incorporation of a time-varying, i. e., a control variable.

It appears that the idealized behaviour of the synthesized boundary condition may be difficult to achieve in practice. A number of "sibling" approaches to obviate this drawback have been developed. In addition, several innovative methods and techniques—such as "best-fit" boundary conditions—have been devised for obtaining information concerning the sensitivity of the model and its parameters and variables.

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Contents

List of Tables

List of Figures

List of Symbols

Symbol	Meaning
A	Cross sectional area of pipe
A_v	Initial valve setting area
A'_v	Arbitrary valve setting area
A_w	Pipe wall cross sectional area
a	Sonic velocity of fluid
a	Structural coefficient in linear programming
a_0	Reference wavespeed
a_1	A constant
a_2	A constant
A_{max}	Maximum valve acceleration
A_{min}	Minimum valve acceleration
B	A constant
b	Constant right hand side in linear programming
C_d	Valve coefficient of discharge at initial setting
C'_d	Valve coefficient of discharge at any setting
C_{ijm}	Cost coefficient in linear programming
C_M	Collection of constant terms in compatibility equation
C_P	Collection of constant terms in compatibility equation
C^+	Positive compatibility equation
C'^+	Positive compatibility equation
C^-	Negative compatibility equation
C'^-	Negative compatibility equation
c	Objective function coefficient in linear programming
c_1	Constant accounting for type of pipe anchoring
C	Cost of a linear closure system
CM	Unit cost of reference diameter pipe
CP	Unit cost for installation of reference diameter pipe
C_1	A cost factor
C_2	A cost factor
C_3	A cost factor

Symbol	Meaning
D	Nominal pipe diameter
D_i	Inner pipe diameter
D_o	Outer pipe diameter
D_{ref}	Reference pipe diameter
E	Modulus of elasticity
E	Error sum of squares
e	Base of natural logarithm
e	Pipe wall thickness
F	A force
F	Fibonacci constant
$F1$	Fibonacci constant +1
$F2$	Fibonacci constant
f	D'Arcy-Weisbach friction factor
$\mathcal{F}$	A function
$\mathcal{F}'$	First derivative of a function
$\mathcal{F}\mathcal{T}$	A function
g	Gravitational acceleration
$\mathcal{G}$	A function
H	Piezometric pressure head
H_f	Head lost to friction
H_r	Piezometric pressure head in reservoir
H'	Piezometric head loss across valve
H_0	Piezometric head loss across valve at initial setting
h	Dimensionless head
J	Hydraulic gradient
J	Time step index
K	Bulk modulus of liquid
K_j	Flow consumption at node j

Symbol	Meaning
L	Length of pipeline
L_{ij}	Length of pipe connecting nodes i and j
m	Mass
m	A constant
m	Exponential parameter for equal percentage closure
m'	Exponential parameter for equal percentage closure
N	Number of pipe reaches
NS	Number of pipe sections
n	Number of pipe sections
n	An element in a series
P	Internal fluid pressure
P	Scaling factor for pipe unit costs
PDF	Pipe diameter factor
Q	Volumetric rate of flow
Q'	Discharge at any valve setting
Q_0	Discharge at initial valve setting
R	Friction term coefficient
T	A time
T_c	Duration of valve motion (closure)
$\bar{T}$	A time interval
t	Time or dimensionless time
t_c	Dimensionless duration of valve motion (closure)
V_1	Mean fluid velocity at time 1
V_2	Mean fluid velocity at time 2
v	Mean fluid velocity
V_{max}	Maximum valve velocity
V_{min}	Minimum valve velocity

Symbol	Meaning
VF_1	Cost factor for valve velocity
$\mathsf{VF2}$	Cost factor for valve acceleration
X_{ijm}	Pipe length of diameter m connecting nodes i and j
x	Decision variable
y	A discharge quantity
z	Value of objective function
z'	Constant term in objective function

Greek Symbols

Symbol	Meaning
α	Pipe inclination angle measured from horizontal
α	Constant exponent
β	Constant exponent
β'	Constant exponent
γ	Specific weight of fluid
Δ	A change or incremental change
ϵ	Level of tolerance
η	Turbine efficiency
κ	A constant penalty coefficient
μ	Poisson's ratio
π	Pi
ρ	Dimensionless pipeline constant
ρ_w	Mass density of fluid
σ	Circumferential (hoop) stress
σ_{all}	Allowable circumferential stress
τ	Dimensionless valve parameter
φ	A penalty function constant (either zero or one)
ψ	Parabolic discharge coefficient

Math Symbols

Symbol	Meaning
d	Derivative
∂	Partial differentiation
$\lfloor R \rfloor$	"Floor" (i. e, integer portion) of R
$\int$	Integral

Superscripts/Subscripts

Symbol	Meaning
G	Initial coarse search interval
J	Time step J
M	At point M
P	At point P
P_i	Point P at section i
SS	Steady state
c	Closure
d	At point d
e	At point e
f	At point f
f	Final
i	An index
j	An index
k	An index
m	An index
max	Maximum value
min	Minimum value
n	An index
net	Net or available quantity
o	Outer (diameter)
opt	Optimal value
r	Reservoir
r	An index
res	Residual

Symbol	Meaning
t	Time
tr	Transient duration at reservoir
x	Distance
0	Initial (steady state)

Chapter 1

Introduction

One of the most complicated unsteady flow problems is the analysis of *fluid transients* (pressure pulses) in pipelines. In the past, optimizing procedures for the design of fluid transmission pipelines have, accordingly, tended to focus on the steady state requirements of the system. Consideration of transients often takes place after the fact, if it is done at all. Yet, the factors—such as pipe diameter, pipe material, wall thickness, valves and their associated controller/actuator systems, fittings and joints, etc.—which ultimately determine the cost of the system are tremendously influenced by the occurrence and behaviour of transients in the pipeline. This means that any "optimal" design which fails to properly account for *water hammer* effects can be, at best, *suboptimal*, and in the worst case, completely inadequate.

When referring to transient analysis, the term *optimal* generally denotes certain synthetic valve closure arrangements and their resulting *tau* curves (see Figure 1.3). The tau curves are functions describing the relative size of the valve opening and the variation in the coefficient of discharge as the valve is operated. The procedure by means of which the tau curves are generated is called *valve stroking*. Two distinct, but related, methods of valve stroking currently exist—1) stroking for a specified head [Ruus, 1966] and 2) stroking in a specified time [Propson]. Unless otherwise stated herein valve stroking will always be taken to mean valve stroking in a specified time.

Valves control the rate of flow in a pipe by converting the potential energy (static head) of the fluid to kinetic energy (fluid velocity) for the case of a valve opening, the converse being true for a valve closure. For a valve discharging to the atmosphere under steady flow conditions, the valve equation is simply an orifice discharge relation.

$$Q_0 = C_d A_v \sqrt{2gH_0} \tag{1.1}$$

where Q_0 is the steady state discharge, H_0 is the pressure head at the valve (in other words, the head loss across the valve), A_v is the area of the valve opening and C_d is a discharge coefficient which accounts for real valve losses. For any other valve setting the flowrate is given by

$$Q' = C'_d A'_v \sqrt{2gH'}. \tag{1.2}$$

Expressing this in a nondimensional form gives

$$\frac{Q'}{Q_0} = \tau \sqrt{\frac{H'}{H_0}} \quad \text{where} \quad \tau = \frac{C'_d A'_v}{C_d A_v}. \tag{1.3}$$

In order to alter the rate of flow in a pipe, say from Q_0 to Q', the value of tau must be changed from τ_0 to τ. Since instantaneous valve motions are not physically possible, the changes must take place over some finite period of time and, consequently, the variation in valve area and coefficient of discharge can be described by a τ function. There are an infinite number of possible tau functions which cause the system to arrive at the same endpoint but with potentially very different impacts on the hydraulic performance of the system.

Realizing that $Q = VA$ (where V is the mean fluid velocity in the pipe) then going from Q_1 to Q_2 means that $V_1 \longrightarrow V_2$ in time Δt, and therefore

$$\frac{V_2 - V_1}{\Delta t} = \frac{\Delta V}{\Delta t}.$$

The quantity $(V_2 - V_1)/\Delta t$ is the average acceleration of the fluid over the period Δt. Instantaneous values of acceleration are given by

$$\lim_{\Delta t \to 0} = \frac{dV}{dt} \neq 0$$

and , by Newton's second law, the greater the acceleration, the larger must be the imposed forces[1]. Thus, the more rapid the valve closure (opening) the more extreme is the head rise (fall) in the pipeline. For *nonoptimal* valve motions the pressure oscillates between the high and low values of head (see Figure 1.1) created in response to the valve motion. These are progressively damped out by wall and fluid friction forces, eventually stabilizing at the new steady flow condition.

Valve stroking, on the other hand, produces a highly controlled transient which ends precisely when the valve motion ceases (see Figure 1.2). Both Figure 1.1 and Figure 1.2 are for the same physical system and valve motion duration, but have differently shaped tau curves. If one takes a section in the *head-time* plane (see Figure 1.3) through the surface shown in Figure 1.1 or Figure 1.2, then the area under the resulting curve represents the impulse applied to the fluid as it moves from one flow condition to another.

$$\int_{t_1}^{t_2} F\,dt = \gamma A \int_{t_1}^{t_2} H\,dt \tag{1.4}$$

where F is the force applied to the fluid, γ is the unit weight of the fluid, A is the cross sectional area of the pipe and t is time. The impulse is related to momentum

[1]There exists, however, a limit to the amount of head change that can occur even for an instantaneous valve motion, i.e., for $dt = 0$. This limiting head cannot exceed $\pm a\Delta V/g$.

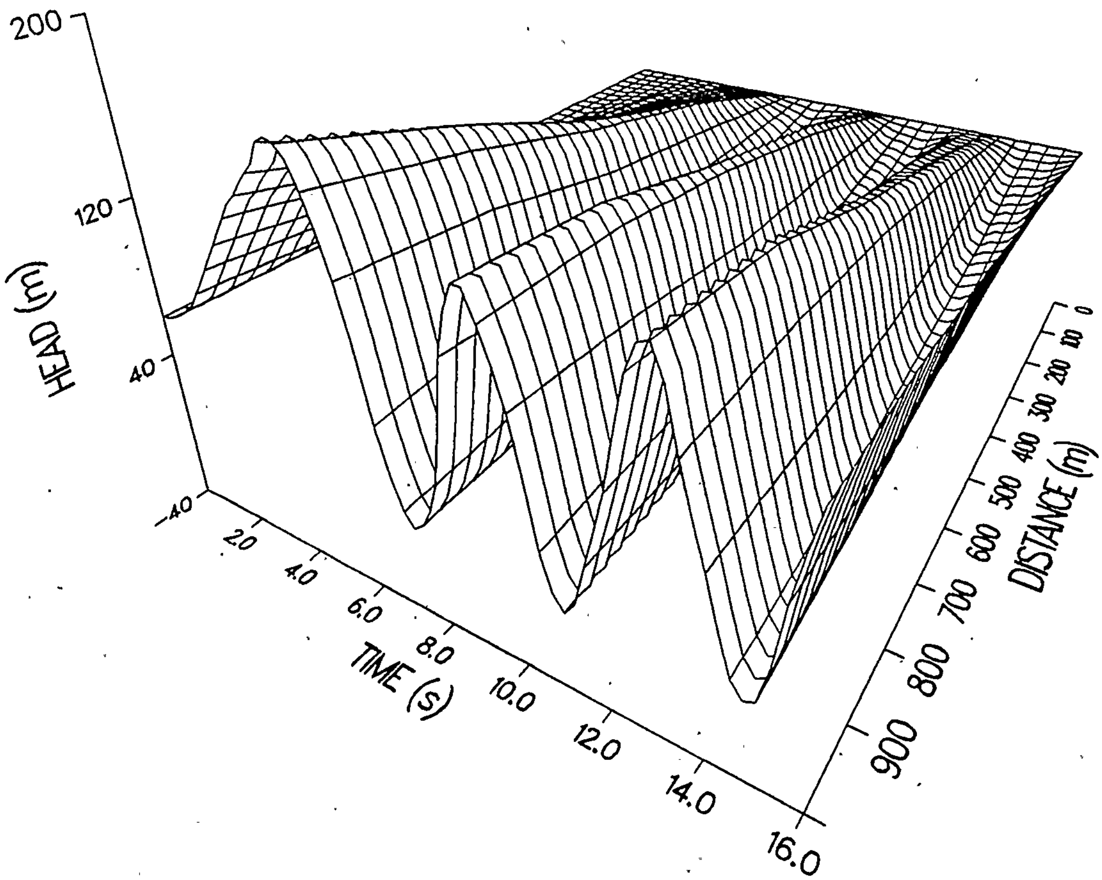


Figure 1.1: Three dimensional representation of a transient produced by an arbitrary valve closure. Duration of valve motion is 6 seconds.

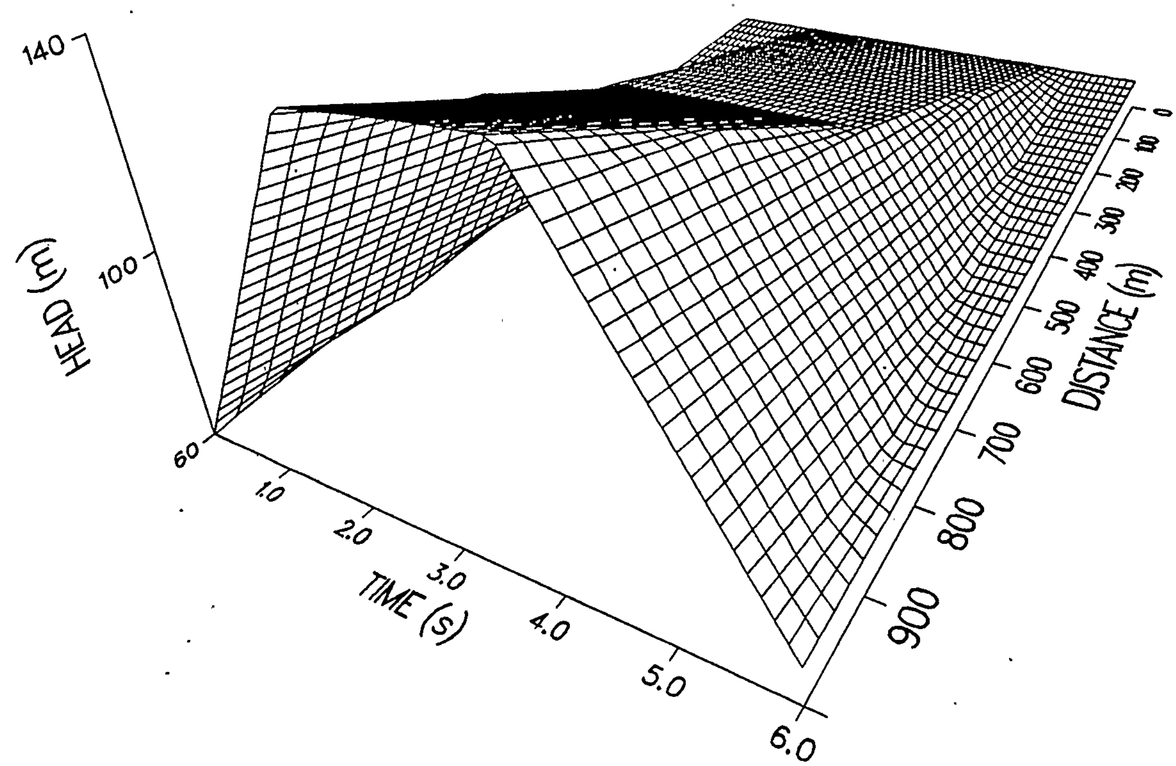


Figure 1.2: Three dimensional representation of a transient produced by valve stroking. Duration of valve motion is 6 seconds.

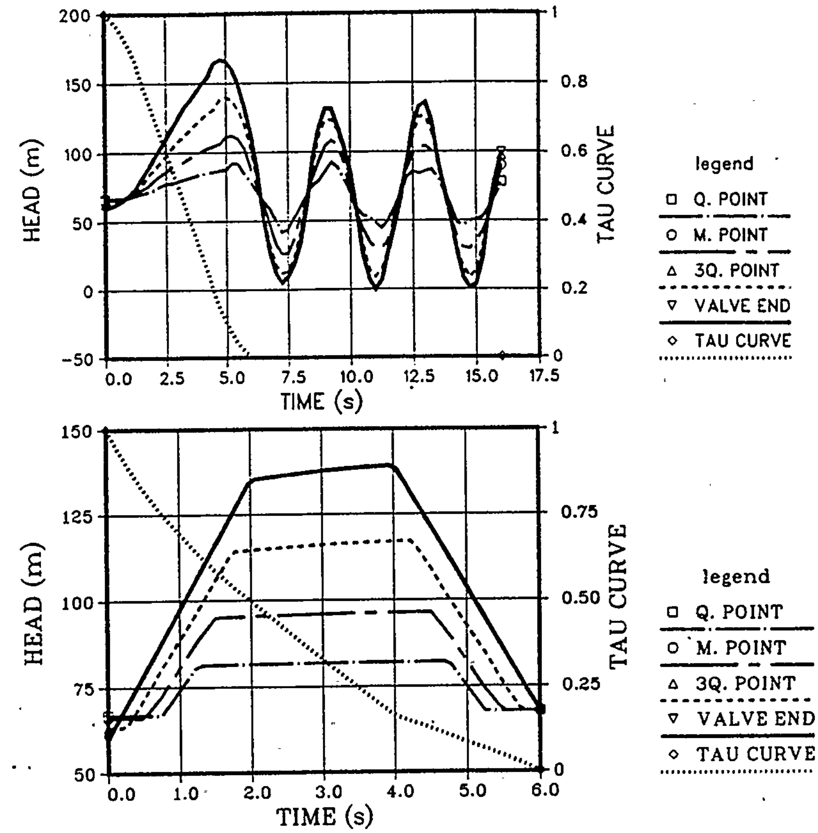


Figure 1.3: *Head-time* profiles for pipe quarterpoint, midpoint, three-quarterpoint and valve end. Tau curve is dotted line. Duration of valve motion is 6 seconds. Top figure shows variation for an arbitrary tau curve; Bottom figure shows variation for an *optimum* tau curve.

by Newton's second law.

$$\int_{t_1}^{t_2} F\, dt = \int_{t_1}^{t_2} m \frac{dv}{dt}\, dt = \int_{t_1}^{t_2} m\, dv. \tag{1.5}$$

For a given system and a given change in the discharge this quantity is a constant. Valve stroking guarantees that, for a fixed duration valve motion, the change in fluid momentum will take place with a minimal (or near minimal) head change. Furthermore, it ensures that no residual transients occur once the valve motion is complete, i.e., that the new steady state is fully obtained. For these reasons the closure arrangement producing this controlled transient has been termed an *optimum* valve closure. Insofar as the head change produced for certain boundary conditions (see Section 3.3 for details) is minimized by valve stroking, use of the word *optimum* is correct. In the context of pipeline design, however, these closure arrangements can in no way be construed as defining a uniquely "best" solution. Rather, they are only one of many factors which may (or may not) contribute to an overall optimal design.

The research described herein constitutes an attempt to formulate some rationale which will permit the development of optimizing procedures for simple pipeline systems which give due consideration to the importance of transient effects in determining the ultimate pipeline cost. The objectives are listed below.

- The identification of relevant cost parameters and cost variables.
- The development of a conceptual framework within which optimizing design procedures can be constructed.
- The formulation of a sensible effectiveness criterion and a meaningful set of constraints incorporating the cost components.
- The definition of the nature of the cost function and the selection of appropriate, efficient methods for its evaluation.
- The consideration of methods which will allow the sensitivity of the solutions to changes in cost parameters to be explored.

Chapter 2 provides some background on optimization theory, existing pipeline optimization methods and a description of nonlinear techniques relevant to the current investigation. Chapter 3 gives the mathematical and hydraulic theory fundamental to the development of the conceptual model proposed by this study. Chapter 4 outlines the nature of the problem, describes the simple system selected for analysis (constant head upstream reservoir with single pipe and valve at the downstream end) and supplies the details of the conceptual model development. The procedures for testing the validity of the model, along with discussion of findings, changes and modifications to the model are presented in Chapter 5. The very important topic of sensitivity analysis is found in Chapter 6 and final conclusions and recommendations issuing out of the investigation are listed in Chapter 7. Two appendices have been given to provide additional detailed information concerning the computer programs used in the model not mentioned in the thesis proper, and for the presentation of some important sensitivity plots.

Chapter 2

Literature Review

In this section a brief introduction to the concept of optimization is presented in conjunction with a, necessarily, simplified overview of some currently applied techniques in the field of pipeline optimization. This will also serve as a literature review of the subject area since, with one exception, no literature really exists on the topic of pipeline optimization with respect to transients. In the final section, a description of nonlinear optimization methods pertinent to this investigation is presented.

The only work known to the author that specifically relates to the idea of optimization procedures and which includes transient phenomena in the process is by [Chiang 1984]. Chiang has developed a procedure which involves the simulation of transients in complex piping systems and uses univariate and gradient search techniques as optimum-seeking strategies. The media can be gas under pressure, liquid or a multiphase continuum. The procedure developed is fairly general and has a wide applicability to mechanical engineering piping systems.

[Wike 1986] refers to control of transients in connection with SCADA system operation but this does not constitute an optimization technique. [Mah 1978] provides a quite general account of the major methods in use in various pipeline industries, along with a discussion of steady and unsteady flow analysis techniques, and alludes to the fact that transients are an important consideration in the design of any pipeline system. However, he does not provide any information suggesting how transients might be incorporated into optimization procedures. This paper has a very extensive bibliography covering the topics of pipeline analysis, design and optimization methods.

Nonlinear optimization methods in pipeline analysis and design are also rather uncommon. One reference to nonlinear methods [De Poli] was unavailable for review. Enough information could be garnered from the abstract, however, to say that it was an application of a penalty function technique for the design and control of water distribution systems. No consideration of transients appears to be involved. [Chara 1984] has developed a very interesting procedure which can be used in linear or nonlinear optimization of operating policies for reservoirs. This method utilizes the concept of *transfer-gain* and is reportedly superior to dynamic programming models in terms of its convergence properties and computational efficiency.

References to other literature in the field of pipeline optimization procedures will be made in Section 2.2 as the various types of existing techniques are surveyed.

2.1 Optimization Concepts and Theory

Perhaps the best way to introduce the concept of optimization is by means of a definition.

Optimization Any process which attempts, by considering a number or group of variables together as interacting components of a system, to obtain a configuration of the components which is "better" with respect to some criterion than any other combination of some, or all, of the elements, may be called an optimization process.

The study of optimization of systems has variously been termed *operations research, systems analysis,* and *management science.* None of these labels is particularly descriptive or informative in relating the nature of this very important and widely used body of knowledge. It is essentially an approach to the design, construction or implementation, maintenance and operation of complex systems which draws on the techniques of pure and applied mathematics in order to provide some rational basis for all types of decision making.

Essential to the integrity of optimization theory is the concept of a *system* and the belief that some means of defining what is "better" or "best" in terms of that system is possible. A system can be thought of as a functional construct divisible into parts or components which can be integrated in various ways to achieve its function. The object of all optimization processes is to find the combination and arrangement of elements which best fulfils the function according to some established criteria.

There are seven steps [Deininger 1975] to followin carrying out any systems analysis study:

1. Formulation of the problem

2. Construction of a mathematical model which describes important system variables.

3. Definition of a *criterion* function or measure of merit.

4. Collection of data to allow the estimation of various parameters in the model.

5. Derivation of optimal solution(s) through formal algorithms.

6. Testing of the model, the solutions and the sensitivity of the parameters.

7. Implementation of the "best" solution.

Steps 4 and 7 only pertain in the case of an actual application of the optimization procedure. They can be omitted for the purposes of this investigation which attempts only to develop and evaluate a conceptual implementation of an optimization model. Carrying out the remaining five steps forms the content of this thesis and, in subsequent chapters, the logical progression through each phase is outlined and discussed.

2.2 Existing Pipeline Optimization Techniques and Models

The scope of the problems to which systems analysis can be applied and the methods used in its application are far too numerous and varied to be discussed in any detail here. Rather, two of the fundamental techniques currently used in the pipeline industry will be outlined in order to provide some acquaintance with the basic ideas of optimal pipeline design.

By far the majority of pipeline optimization techniques fall into either one of two categories—*linear programming* or *dynamic programming* methods. A brief description of each is given along with a number of related references. The remaining optimization methods comprise a variety of techniques such as integer programming, nonlinear techniques and modeling approaches. [Shamir 1979] gives a reasonably complete summary of the major techniques used for steady state optimization of water distribution systems. [Huang 1985] does the same for procedures commonly used in the oil and gas pipeline industry.

2.2.1 Linear programming

Linear programming has long been a popular method for optimizing a system which can be described as a set of linear equations having the form

$$\begin{array}{lrcrcccrl}
\text{maximise } z - z' = & c_1x_1 & + & c_2x_2 & + & \cdots & + & c_nx_n & \\
\text{subject to:} & & & & & & & & \\
& a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & \leq b_1 \\
& a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & \leq b_2 \\
& \vdots & & \vdots & & \ddots & & \vdots & \vdots \\
& a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & \leq b_m
\end{array}$$

where some of the b's may be negative[1] and the problem may also contain equality constraints. The value of the *objective* function, z, is determined by the objective function coefficients (the c_i's) and the *decision* variables (the x_i's). It may also include an initial constant term, z'. The a_{ij}'s are constant coefficients and are often referred to as the *structural* coefficients. The structural coefficients along with the decision variables and right hand side constants (b's) form a set of m linear constraints. In addition, the non-negativity constraints also apply.

$$x_1, x_2, \ldots, x_n \geq 0$$

Linear programming is such a powerful method that strenuous effort is often made to modify nonlinear problems so that they may be handled by this linear solution

[1] In other words they are $\geq$ constraints.

technique.

Linear programming is most often applied to single or branching pipe networks. The objective is to minimize the cost, subject to certain head and delivery requirements, by selecting the diameters and lengths of the various segments of the branches. Let K_j denote the consumption at node j. The heads to be maintained at some or all of the nodes, H_j, must lie within a given range, H_j^{min} to H_j^{max}. The length of each pipe (link) connecting nodes i and j is L_{ij}.

For each link a set of candidate diameters is defined and the decision variables, X_{ijm}, are the lengths of the pipe segment of the m^{th} diameter connecting nodes i and j. Thus, one set of linear constraints can be formulated as

$$\sum_m X_{ijm} = L_{ij} \quad \text{for all } (i,j). \tag{2.1}$$

Assuming that all the consumptions and that the pipe material (i. e., hydraulic resistance characteristics) are known in advance, then the discharges, Q_{ij}, in each link are fixed. Thus, the head loss in the m^{th} segment of the link is

$$\Delta H_{ijm} = J_{ijm} X_{ijm} \quad \text{for all } (i,j,m) \tag{2.2}$$

where J is the hydraulic gradient $\Delta H/L = (fQ^2/2gDA^2)$.

Beginning from a node, s, in the system for which the head is known, for example a reservoir, and proceeding along any path to node, n, for which the head must fall within the specified range, the following set of linear constraints can be generated.

$$H_n^{min} \leq H_s \pm \sum_{(i,j)} \sum_m J_{ijm} X_{ijm} \leq H_n^{max} \tag{2.3}$$

The inner summation is over all segments in a link and the outer summation is over all links along the chosen path.

Many researchers [Shamir 1979] have postulated that the cost of a fixed diameter pipeline is roughly a linear function of its length so that the objective function is formulated as

$$\sum_{(i,j)} \sum_{m} C_{ijm} X_{ijm} \tag{2.4}$$

where the C_{ijm} are the objective function cost coefficients. The optimal solution is obtained by minimizing Equation 2.4 subject to Equations 2.1, 2.3 and non-negativity of the X_{ijm}. This technique can be extended to include pumps and reservoirs in the system, as well as various kinds of operating and maintenance costs.

The major disadvantage of linear programming is that it is restricted to problems for which simple, linear or linearized models can be developed. Some innovations for more complex nonlinear functions have been proposed by Quindry, Shamir, Deb, Watanatada and others. Thus far, linear programming use has been restricted to steady state or quasi-steady state problems.

2.2.2 Dynamic Programming

Dynamic programming is really an algorithmic optimization approach that can be applied to any problem consisting of well defined, sequential stages. State variables are defined at each stage which describe the condition of the system at that stage. Decision variables are input variables which supply information to the system or specify an action to be taken at the stage to which they apply. Stage return functions provide some measure of the effectiveness of a particular decision for any value of the input state variable. [Smith *et al*] give an excellent description of dynamic programming and the reader is referred to this valuable text for more details.

Kally, Liang and others have pioneered the application of dynamic programming in the pipelines area. The method is not so restrictive as linear programming but, unfortunately, becomes computationally unwieldy for problems having two or more state variables. Many attempts have been made to modify the dynamic programming approach in order to improve its efficiency, however, the variations tend to suffer from the same problems or introduce new ones, such as poor convergence properties, which limit the application of this method to relatively simple systems.

2.3 Pertinent Nonlinear Optimization Methods

In spite of the fact that this material anticipates somewhat the course of the investigation and might be more appropriately discussed later on, it is included here since it is consistent with the content of this chapter.

The beauty of nonlinear optimization techniques is that they permit more realistic models of actual phenomena to be created and therefore, one supposes, more realistic solutions to optimization problems. The difficulty in applying them is that these methods are often unreliable and may fail to locate global and local optima or, potentially, even feasible solutions. Nevertheless, they can be successfully implemented for many problems and are often the only methods available for truly nonlinear problems.

Nonlinear methods may be broadly classed as methods for functions of a single variable (univariate) or methods for functions of two or more variables. Within each of these groups two further categories can be defined—*gradient* techniques and *direct* search techniques. All of the foregoing procedures require that an initial trial point be specified. As it will be later shown, depending upon the system being modeled, any or all of these techniques might be employed as optimum-seeking strategies.

2.3.1 Methods for Functions of a Single Variable

Gradient Methods

Numerous texts on numerical or optimization methods can be found which describe this type of optimizing procedure. They are often referred to as methods for obtaining the zeroes of a function or, colloquially, as *root-finders*. They all involve evaluating the derivative(s) of the objective function and therefore can be applied only to functions that are well-behaved and possess a continuous first derivative. Well known examples are the Newton-Raphson and the secant methods.

Direct Search Methods

Although many different search methods exist, attention here is restricted to the Method of Golden Section. This method makes use of an interesting property of the Fibonacci constant, F, which can be obtained from the higher order terms of the series $n_{r+1} = n_r - n_{r-1}$ by

$$F = \frac{n_r}{n_{r+1}} = \frac{n_{r+1}}{n_r} - 1 = 0.618033989. \tag{2.5}$$

The fact that $F^2 = 1 - F$ allows the positioning of trial points in such a way that only a single new point must be added at each iteration, yet the proportions of the subdivided interval of uncertainty remain constant. Let x_1 and x_2 be the endpoints of the interval of uncertainty for some arbitrary nonlinear function, $f(x)$, shown in

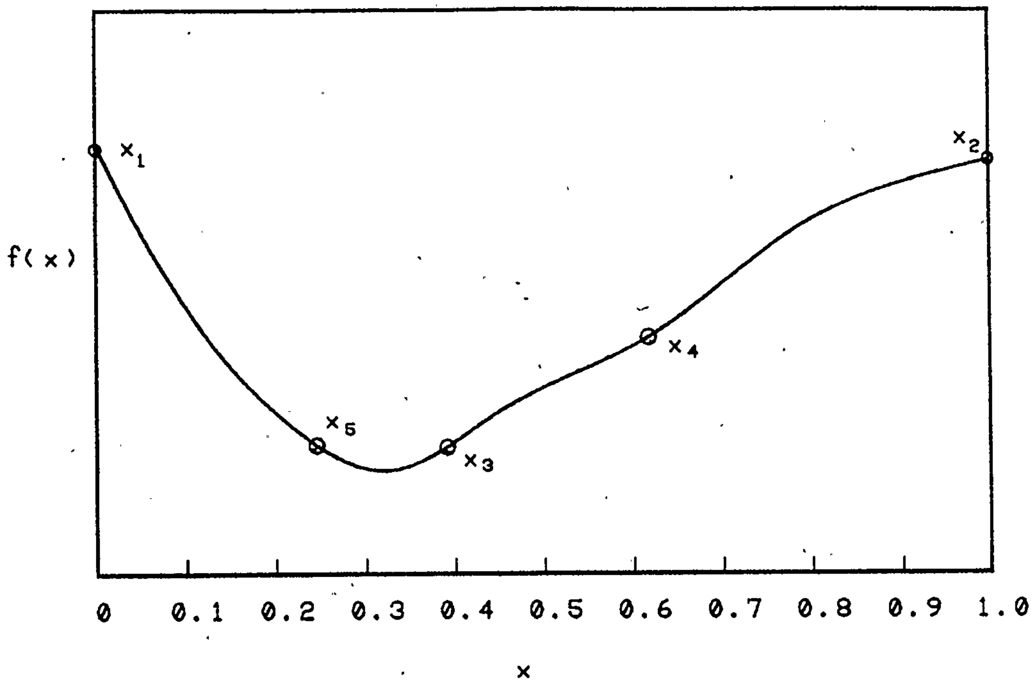


Figure 2.1: Method of Golden Section.

Figure 2.1. Let two interior points, x_3 and x_4, be chosen so that

$$(x_2 - x_3) = (x_4 - x_1) = F(x_2 - x_1). \tag{2.6}$$

The values of the function $f(x)$ are obtained for each of these points. For the sake of illustration, let $f(x_3) < f(x_4)$, then the segment $\overline{x_4\, x_2}$ may be eliminated from further consideration. A fourth point is now added to the remaining segment at x_5 such that

$$(x_4 - x_5) = F(x_4 - x_1). \tag{2.7}$$

The process is repeated at all subsequent iterations until the interval of uncertainty has been reduced to some specified tolerance level. It can be demonstrated that for n iterations $3 + n$ function evaluations must be made and the initial interval of uncertainty is reduced by the factor F^n.

The Method of Golden Section is an extremely efficient search technique and can be used for locating the optimal point of constrained or unconstrained functions. This method has been used very successfully during this investigation and is discussed again in Section 5.3. A more complete description of the procedure can be found in [Smith *et al*].

2.3.2 Methods for Functions of Two or More Variables

The methods available for functions of two or more variables are not fundamentally different from those for a single variable. However, the complexity of the problem and the solution methods is far greater than for the univariate case.

Gradient Methods

A number of gradient techniques have been developed to deal with nonlinear functions. Most of these employ a constant step size and involve the evaluation of derivatives of the function at each trial point, with the objective being to locate the direction of the maximum change in the value of the function. The next trial point is located by moving a distance s, the step size, in this direction. The process is continued until no improvement can be made upon the value of the function at the current value of s. The step size is reduced by some factor and further iterations are performed in the same manner. The process is repeated until the value of the step size is reduced below some tolerance level. The method may be generalized for n variables where the function to be optimized is

$$z = f(\mathbf{x}) \quad \text{where} \quad \mathbf{x} = (x_1, x_2, \ldots, x_n)^T. \tag{2.8}$$

The vector of gradients (for a minimization) is obtained at an initial trial point, $\mathbf{x}_0$, by

$$\mathbf{x}_{r+1} = \mathbf{x}_r - s\mathbf{d}_r \tag{2.9}$$

where

$$d_i = \frac{\partial z/\partial x_i}{\left[\sum_{i=1}^{n} (\partial z/\partial x_i)^2\right]^{\frac{1}{2}}}. \tag{2.10}$$

Direct Search Methods

Search methods can be split into three general groups—enumeration, random search and directed vector searches. The choice of a particular method will depend upon the nature of the function and the computer resources available.

Enumeration This is the most time consuming of the direct search methods but has the distinct advantage of always locating the global optimum, something that none of the other methods guarantees. In the past, its application has been restricted to only computationally "small" problems. However, increasingly available computing power may make this the preferred method for optimizing difficult functions in the near future.

Random Search As the name implies, this method selects trial points for evaluation of the function by a random procedure. It is also a time consuming approach but has been shown to be more efficient than other search methods for many problems. It also can be used as a means for escaping from a local optimum or establishing that a solution is indeed global.

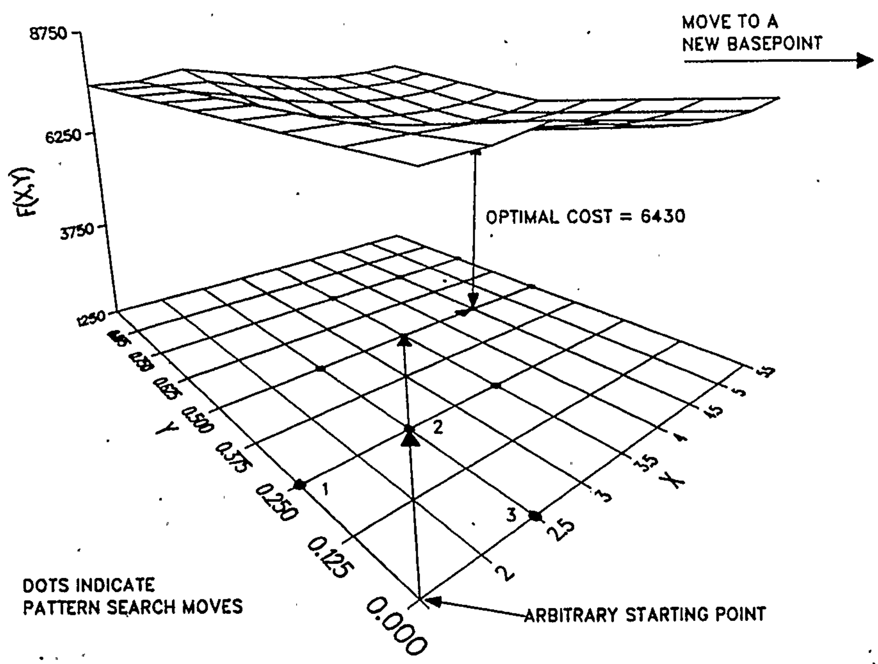


Figure 2.2: Pattern search method.

Directed Vector Search These techniques involve the selection of a set or sequence of directions according to some strategy, with the search for the optimum solution proceeding along the chosen directions. The direction of searching can be changed in accordance with the conditions existing at the current location of the position vector.

A popular and relatively efficient set of algorithms belonging to the category of directed vector searches is called *pattern search* techniques. One such method is shown in Figure 2.2. The initial starting point for the search may be selected by any convenient criterion. The search then proceeds by evaluating the function at three neighbouring points lying parallel to the major axes at some specified grid interval. The point having the lowest value (for a minimization problem) is chosen as the new basepoint and another pattern search is initiated. The process is repeated until no improvement in the value of the objective function can be made over the current basepoint, which may involve several changes in direction and a number of pattern searches. The grid interval is reduced and the cycle continued until the step size has attained some acceptable level of accuracy.

Directed vector search methods are often debilitated by the presence of local optima which "trap" the search within an area and prevent it from locating the global optimum.

2.4 Summary

In this chapter a cursory overview of optimization theory and its application to the field of pipeline analysis and design was presented. A number of nonlinear optimization methods which are relevant to the investigation have also been introduced. The intention has been only to provide some familiarity with the concepts of optimization and the current state of the art so that the developments of this study can be placed in perspective. For a more comprehensive treatment of any of these topics the reader is referred to the sources given in the bibliography.

Chapter 3

Mathematic and Hydraulic Preliminaries

3.1 Governing Equations

The fundamental equations governing the flow of liquids in rigid, closed, prismatic, circular conduits have been developed from consideration of mass conservation and the equation of motion. Excellent accounts of their derivation may be found in [Wylie/Streeter]. The two fundamental equations taken together form a pair of first order, simultaneous, quasi-linear partial differential equations of the hyperbolic type.

$$H_t + VH_x - V\sin\alpha + \frac{a^2}{g}V_x = 0 \tag{3.1}$$

$$V_t + VV_x + gH_x + \frac{fV|V|}{2D} = 0 \tag{3.2}$$

where

H = the piezometric head in the pipe

V = the mean fluid velocity in the pipe

d = the pipe diameter

a = the wavespeed

$$f = \text{the Darcy-Weisbach friction factor}$$
$$g = \text{the acceleration due to gravity}$$
$$x = \text{the distance along the pipe}$$
$$t = \text{time}$$
$$\alpha = \text{the angle of inclination of the pipe}$$

The subscripts denote partial differentiation with respect to the subscript variable. The $V \sin\alpha$ term in Equation 3.1 is generally considered to be a negligible quantity[1]

The pressure, P, at any point in the pipe is simply related to the head by $P = \gamma(H - z)$ where z is the elevation of the centerline of the pipe above some arbitrary datum and γ is the unit weight of the fluid. In these equations several important assumptions are considered to be true:

1. The pipe is flowing full with the minimum pressure always above the vapour pressure of the fluid.

2. The velocity is one dimensional and has a uniform distribution over the pipe cross section.

3. The pressure is considered to have a value equal to that existing at the pipe centerline and is also uniform over the cross section.

4. The frictional resistance of the pipe is the same at any instant as it would be for the corresponding steady flow condition.

[1] In the past, it has been suggested that since the $V \sin\alpha$ term does not appear in the steady state equations, it is reasonable to omit it from the dynamic equations. In fact, [Karney, pers. comm.] it is now believed that this term does not appear in the expression for steady flow due to an improper formulation of the steady state equations thereby negating that argument. In any event, provided that V and/or α are not large, the quantity $V \sin\alpha$ is insignificant and can be discarded.

5. The walls of the pipe are perfectly elastic.

The absolute value signs on the velocity are necessary to ensure that the pipe wall shear stresses always oppose the flow.

3.2 The Method of Characteristics

Equations of the sort indicated in Section 3.1 can be readily transformed into four ordinary differential equations which are amenable to solution by the method of characteristics. In this case, the partial differential equations become

$$0 = \frac{dV}{dt} + \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} \tag{3.3}$$

$$\frac{dx}{dt} = V + a \tag{3.4}$$

$$0 = \frac{dV}{dt} - \frac{g}{a}\frac{dH}{dt} + \frac{fV|V|}{2D} \tag{3.5}$$

$$\frac{dx}{dt} = V - a. \tag{3.6}$$

Equations 3.3 and 3.4 are known as the C^+ equations while Equations 3.5 and 3.6 are called the C^- equations. Equations 3.3 and 3.5 are also referred to as *compatibility* equations and Equations 3.4 and 3.6 are respectively termed the positive and negative *characteristics.* The significance of these equations can be explained as follows. Each compatibility expression is valid along a curve described by its corresponding characteristic. As no mathematical approximations are used in the derivation of the four ordinary differential equations, every solution of the original system (Equations 3.1 and 3.2) is valid for the charateristic system, i. e., Equations 3.3 through 3.6. It can be demonstrated that the converse is also true [Courant]. Before considering the methods for solving this system of equations, it is important to understand some properties regarding uniqueness of solution for certain problem types in the method of characteristics.

3.2.1 Initial Value Problems

The "normal" method of characteristics solution belongs to this category of problems. Figure 3.1 shows two arcs, $\overline{AO}$ and $\overline{BO}$, which lie along the C^- and C^+ characteristics respectively. The positive characteristic passing through point P intersects arc $\overline{AO}$ at A while the negative characteristic going through P intersects arc $\overline{BO}$ at B. The interval AOB is called the domain of dependence and the values of V and H at P depend only on the initial data along that interval. It can be demonstrated that, in the region $AOBPA$, this solution is unique. The arrows (which are drawn along the characteristic directions) on the arcs $\overline{AO}$ and $\overline{BO}$ indicate the number of data which can be prescribed along each arc. The number of arrows entering the region $AOBPA$ indicate the number of data which must be given along each boundary in order for a unique solution to exist in the region of interest. Note that only one datum can be prescribed on each characteristic arc. For a noncharacteristic curve, such as $\overline{AB}$, two data must be known in order for the solution at P to be unique. In the customary method of characteristics analysis two data are specified on the domain of dependence at the points of intersection with the positive and negative characteristics passing through the point P.

Since no reference to direction or orientation with respect to the x–t plane has been made in this analysis, the same observations are equally true for solutions proceeding backward in time, i. e., for the point P' in Figure 3.1. The only considerations for a valid and unique solution are the orientation of the initial value

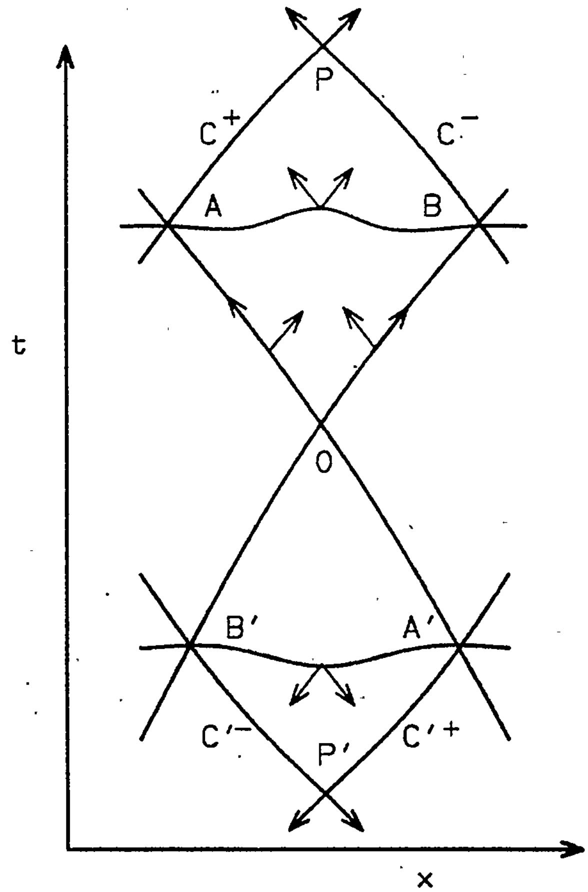


Figure 3.1: Domain of dependence in the x–t plane for an initial value problem.

curves in the solution plane and the nature of the data on those curves.

3.2.2 Boundary Value Problems

The problem described in the previous section is used to advance the method of characteristics solution through time along a *space-like* initial arc (line). Likewise, the solution which forms the basis for the valve stroking procedure is an analogous case which requires that two data be specified at each boundary point on a *time-like* arc (line) in order to promulgate the method of characteristics solution through space. A time-like arc is one which has a characteristic direction on either side of it while a space-like arc has both characteristic directions on the same side[2]. Figure 3.2 shows the situation just described where the interval AOB is the domain of dependence of P and the region $AOBPA$ contains a unique solution for V and H at P. Notice that no data may be prescribed along the arbitrary time-like arc

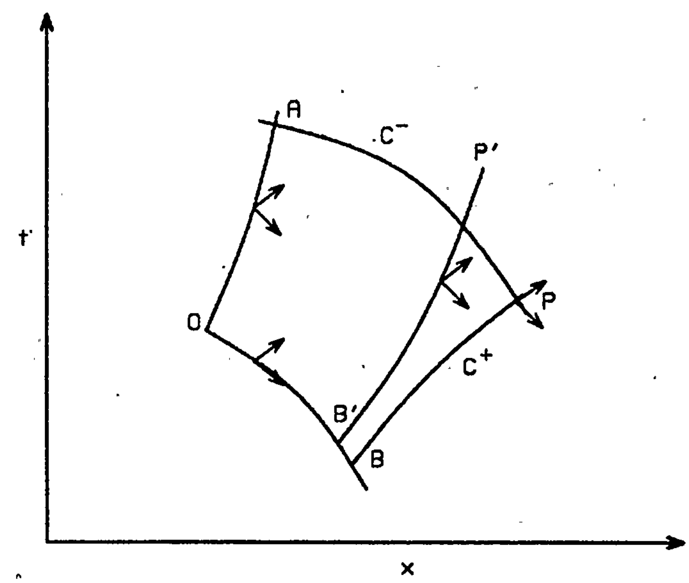


Figure 3.2: Method of characteristics—boundary value problem.

$\overline{B'P'}$ since it lies within the region of unique solution for this problem. Hence, the solution along arc $\overline{B'P'}$ is defined by the same domain of dependence as for P. When $\overline{AO}$ and $\overline{B'P'}$ are parallel to the time axis, and arc $\overline{BO}$ is a noncharacteristic line parallel to the x-axis, the method of characteristics solution can be advanced

[2]A characteristic arc is the limiting case since one characteristic direction is tangent to it.

through space from one of the boundary value curves. This is the approach taken in the valve stroking solution of the method of characteristics.

3.2.3 Finite Difference Equations

Equations 3.4 and 3.6 can be further simplified if one realizes that normal pressure wave celerity in most dense fluids is two or three orders of magnitude greater than the mean fluid velocity in the pipeline. In water, for example, the normal range of background velocity is from about 1–4 metres per second while wavespeed ordinarily falls between 800–1400 metres per second. It is therefore justifiable to say that the characteristic equations may be reduced to

$$\frac{dx}{dt} = \pm a. \tag{3.7}$$

This results in a computationally efficient set of finite difference equations due to the fact that the characteristics are now a pair of complementary straight lines. Figure 3.3 shows the x–t plane for a pipeline divided into N equal reaches, each Δx in length. If a time step is computed as $\Delta t = \Delta x/a$, then the diagonals of the grid will be the positive and negative characteristics emanating from each grid intersection point. Equation 3.3 is valid along the positively sloped characteristic line $\overline{AP}$ with both V and H known at A. Similarly, Equation 3.5 holds along $\overline{BP}$ with both variables known at B. Expressing Equations 3.3 and 3.5 in terms of discharge and multiplying each by $a\,dt/g = dx/g$, produces

$$dH + \frac{a}{gA}dQ + \frac{f}{2gAD^2}Q|Q|dx = 0 \tag{3.8}$$

$$-dH + \frac{a}{gA}dQ + \frac{f}{2gAD^2}Q|Q|dx = 0. \tag{3.9}$$

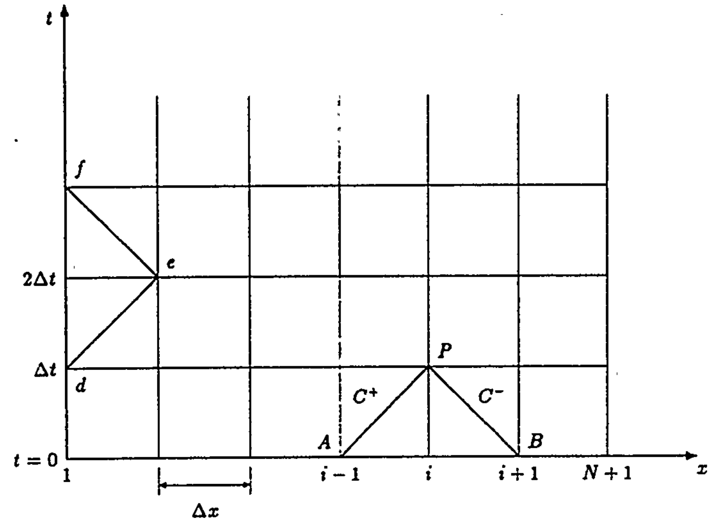


Figure 3.3: x–t grid for single pipeline problems.

Integrating the first equation along its respective characteristic gives

$$\int_{H_A}^{H_P} dH + \frac{a}{gA}\int_{Q_A}^{Q_P} dQ + \frac{f}{2gDA^2}\int_{x_A}^{x_P} Q|Q|dx = 0. \tag{3.10}$$

The manner in which Q varies with x in the last term is not explicitly known so an approximation is employed in order to evaluate the final integral. A first order approximation is adequate for most situations (high friction cases being the major exception), i. e., Q_P is assumed to be equal to Q_A. Noting that, in Figure 3.3, the points A and B are nodes $i-1$ and $i+1$, the following general formulation of Equations 3.8 and 3.9 results.

$$C^+ : \quad H_{P_i} = C_P - BQ_{P_i} \tag{3.11}$$

$$C^- : \quad H_{P_i} = C_M + BQ_{P_i} \tag{3.12}$$

where the constants C_P, C_M, B and R are given by

$$C_P = H_{i-1} + BQ_{i-1} - RQ_{i-1}|Q_{i-1}| \tag{3.13}$$

$$C_M = H_{i+1} - BQ_{i+1} + RQ_{i+1}|Q_{i+1}| \tag{3.14}$$

$$B = \frac{a}{gA} \tag{3.15}$$

$$R = \frac{f\Delta x}{2gDA^2} \tag{3.16}$$

Elimination of Q_{P_i} in Equations 3.11 and 3.12 allows H_{P_i} to be evaluated as

$$H_{P_i} = (C_P + C_M)/2. \tag{3.17}$$

Q_{P_i} can then be determined by back substitution in either Equation 3.11 or 3.12. After the first time step, appropriate boundary conditions must be known in order to complete the solution for a specific time interval. Likewise, if more than one pipe exists in the system, the boundary conditions at the series connections must be available. The subject of boundary conditions is discussed in Section 3.3.3.

3.3 Valve Stroking

Valve stroking differs from the usual time series solution of the method of characteristics in that it is a *synthetic* rather than a *simulation* procedure. In other words, the simulation solution may continue indefinitely through time and ends only because an artificial time limit is placed on it. Valve stroking methods, in contrast, are limited both in space and time and essentially allow the conditions at a particular boundary to be established for a finite time span. This requires that both the initial and the final conditions are known for the system. In addition, one boundary condition is either fully known or specified for the time interval under consideration. Thus, the method of characteristics solution actually proceeds through space from the known to the unknown boundary condition.

The finite difference equations used for the valve stroking procedures are quite similar to those used for the initial value problems. They are derived in the same way but are distinguished by the fact that the equations produce solutions for points backward in time. If one integrates Equation 3.3 along the characteristic line from point d to point e shown in Figure 3.3 and Equation 3.5 from point e to point f (so that the integrations are both occurring in the positive sense of time) the following equations result.

$$C^+ : \quad H_e = H_d - B(Q_e - Q_d) - RQ_d|Q_d| \tag{3.18}$$

$$C^- : \quad H_f = H_e + B(Q_f - Q_e) + RQ_e|Q_e| \tag{3.19}$$

For the majority of practical cases no flow reversal occurs making the absolute value signs unnecessary and the equations may be solved for Q_e.

$$Q_e = \frac{B}{R}\left[1 - \sqrt{1 + \left(\frac{RQ_d}{B}\right)^2 - \frac{R}{B}\left(Q_d + Q_f + \frac{(H_d - H_f)}{B}\right)}\right] \tag{3.20}$$

H_e can now be directly evaluated from Equation 3.18. If the assumption of no reversal of flow does not hold, Equations 3.18 and 3.19 should be solved by a numerical technique such as Newton-Raphson.

The expressions for stroking in the opposite sense are

$$Q_e = \frac{B}{R}\left[1 - \sqrt{1 + \left(\frac{RQ_d}{B}\right)^2 - \frac{R}{B}\left(Q_d + Q_f + \frac{(H_f - H_d)}{B}\right)}\right] \tag{3.21}$$

$$H_e = H_d + B(Q_e - Q_d) + RQ_d^2 \tag{3.22}$$

The manner in which these equations are applied to a system for the two types of valve stroking will be discussed next.

3.3.1 Valve Stroking in a Specified Time

This application of the method of characteristics was first developed by [Propson]. This is actually the more recent method and first appeared some four years after valve stroking for a specified head had been developed. It is, however, a less restrictive and simpler method and will therefore be considered first.

Figure 3.4 depicts the x–t plane for an arbitrary system. The initial conditions

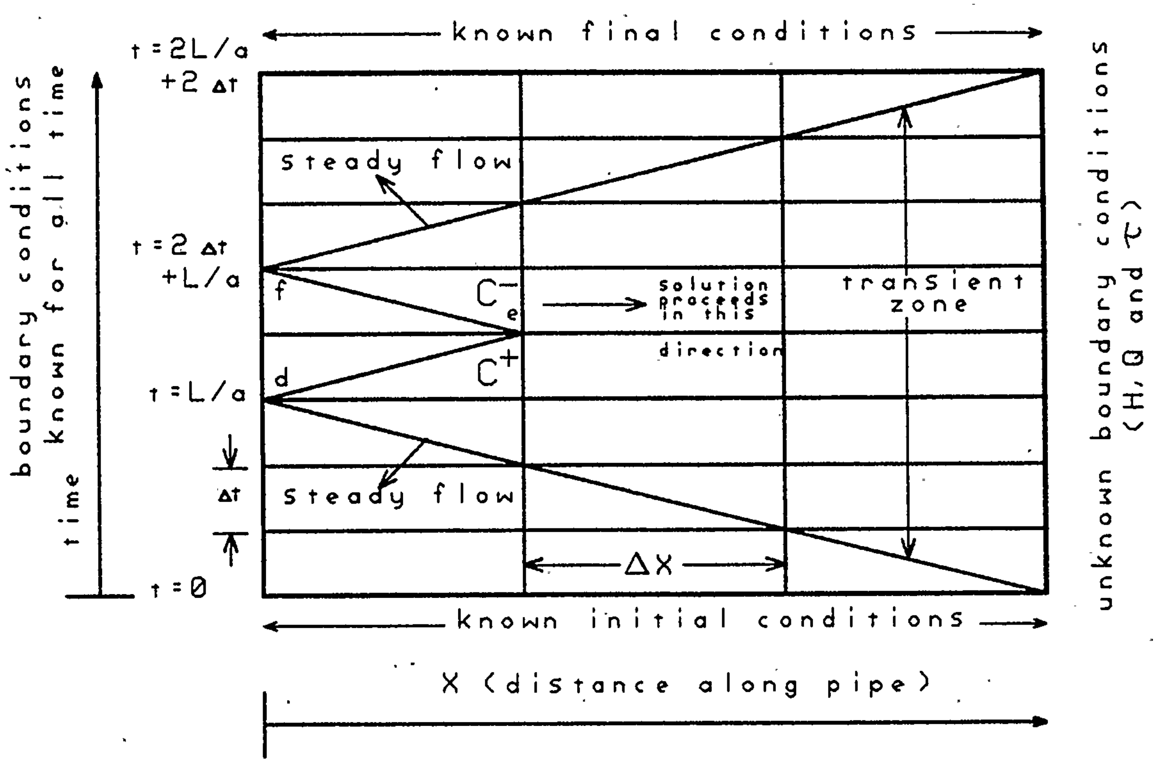


Figure 3.4: Method of characteristics solution for valve stroking in a specified time.

are known along x for $t = 0$ and for $t = 2L/a + 2\Delta t$. There is no significance to the fact that, in the figure, the duration of the transient at the upstream end is $2\Delta t$. It can be any number of Δt's, the only restriction being that the wave cannot return to the downstream boundary in less than $2L/a$ seconds. The boundary conditions are also specified at the upstream section, either by the physical nature of the system or arbitrarily by the analyst. The negatively sloped diagonal line is the negative characteristic emanating from the downstream boundary at time $t = 0$, i. e., when the conditions at the downstream end begin to change. The disturbance reaches the upstream boundary in exactly L/a seconds. The positively sloped diagonal is the characteristic passing through the downstream section at the time when the disturbance has just ended. The region bounded by the two characteristic diagonals is the zone of transient fluid flow. The two triangular areas represent regions of steady fluid flow.

The method of characteristics solution is straightforward. Using Equations 3.20 and 3.18 one solves for Q and H at each spatial node for all time. Once the values of the unknown variables have been determined at one physical section, these values are used to generate solutions at the next section. Note that it is only necessary to solve for Q and H in the transient region of the x–t plane since the upper and lower triangles are already known from the initial and final conditions. The solution proceeds in this manner until the downstream boundary is reached.

3.3.2 Valve Stroking for a Specified Head

This type of valve stroking was first proposed by [Ruus, 1966] as a means of adjusting wicket gates for turbines in hydropower installations. The method was originally developed as a graphical procedure and splits the transient phenomenon into three distinct phases. In the first phase, shown in Figure 3.5, the head at the downstream end is increased to its maximum prescribed value, H_{max}. This phase

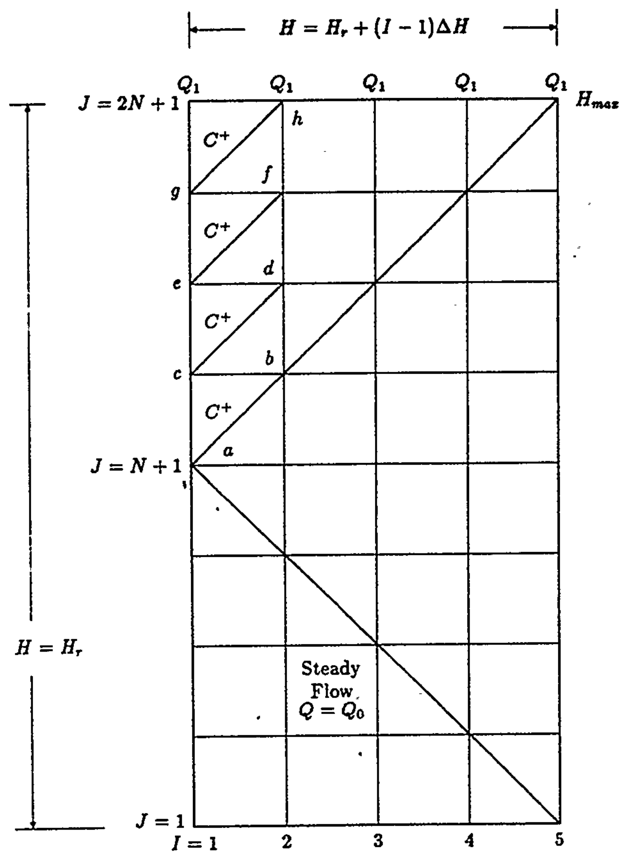


Figure 3.5: Phase I of valve stroking for a specified head.

takes $2L/a$ seconds or, since $dt = \Delta x/a = L/Na$, it requires $2L/a \times Na/L = 2N$ time periods. Thus at timestep $J = 2N + 1$ the hydraulic grade line is straight from H_r at the upstream end to H_{max} at the downstream end. Once again, the upper and lower triangular regions are areas of steady flow (both [Propson] and [Wylie/Streeter] provide proofs for this assertion) and the head at points b, d, f and

h are known and equal to $H_r + \Delta H$. At any given time step, the flow is uniform so it can be seen that $Q_c = Q_b, Q_e = Q_d$ and $Q_g = Q_f$. Rewriting the positive compatibility equation for point a, Q_b may be obtained by

$$Q_b = Q_a - \frac{(\Delta H + RQ_a^2)}{B}. \tag{3.23}$$

Applying the same logic to the time steps from $J = N+1$ to $J = 2N+1$ provides all the upstream boundary condition data needed for Phase I.

During Phase II the head at the downstream end is held constant at H_{max} while the flow is progressively varied in accordance with Equation 3.23. When it has reached its ultimate value the final zone of transient flow is defined by the two long diagonals in Figure 3.6. Observe that the three phases are defined by the changes in head at the downstream end and not by the change in flowrate at the upstream end. Thus, the flow continues to change during Phase II and for a portion of Phase III. During this final period of unsteady flow at the downstream end the head is allowed to adjust to its terminal level. Figure 3.6 shows the relationship between the last two phases. It can be seen that the minimum theoretical time for valve stroking in this manner is $4L/a$ seconds.

This procedure provides all the necessary information for the method of characteristics solution to be carried out. It is performed in an identical fashion to that for valve stroking in a specified time proceeding from the upstream to the downstream boundary. Having examined the two methods of obtaining the solutions for the unknowns Q and H for the grid points in the x–t plane, it is now appropriate to discuss the relevant boundary conditions.

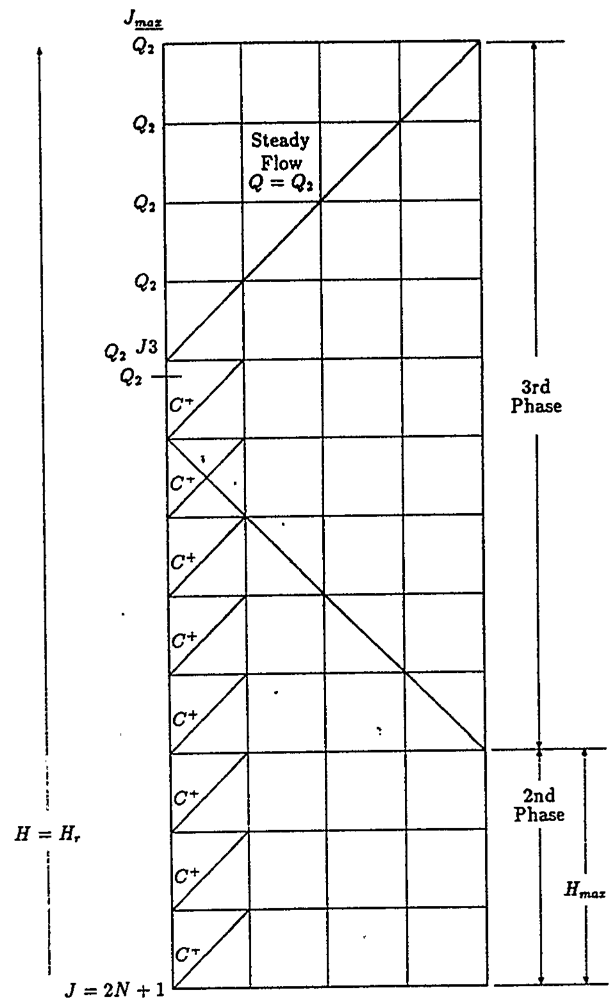


Figure 3.6: Phase II and Phase III of valve stroking for a specified head.

3.3.3 Boundary Conditions

A consideration of all the possible boundary conditions to which the valve stroking procedures can be applied goes far beyond the scope of this investigation. For those interested in such details, [Wylie/Streeter] and [Propson] provide thorough accounts of these. Attention here will be directed to only those conditions pertinent to the constant head upstream reservoir pipeline with a valve at the downstream end. This system contains three possible boundary conditions—the reservoir, the valve and the series pipe connections (if the pipeline consists of more than one pipe).

Upstream Reservoir

This boundary condition has already been implicitly discussed in the description of the valve stroking methods. In both cases, i. e., stroking in a specified time and stroking for a specified head, the pressure at the upstream end is fixed at the reservoir head, H_r. The flowrate in the steady flow zones is equal to either the initial or the final flowrate, depending on which zone is being considered. Only during the transient phase is there any indeterminacy with respect to the discharge. [Propson] suggests that, for stroking times $\leq 4L/a$, the upstream discharge be varied linearly during the central transient condition. For stroking times in excess of $4L/a$ this procedure may not produce the lowest possible head rise for the pipeline, particularly for systems in which viscous effects are important. The recommended procedure for solution in these situations is valve stroking for a specified head wherein the upstream variation in discharge is fixed by the restrictions placed on the head at the downstream end.

Neither of these methods of specifying the upstream discharge is sacred and it must be borne in mind that under certain circumstances, it may be desirable to use some other means of prescribing the flowrate at the reservoir.

Valve at the Downstream End

The relationship between the head, flowrate and the valve motion at the downstream end of the pipeline has been mentioned in Section 1. The value of τ at each time grid line, i.e., for $J = 1, \ldots, (T_{max}/\Delta t) + 1$ is

$$\tau(J) = \frac{Q_{N+1}^{J}}{Q_0\sqrt{H_{N+1}^{J}/H_0}} \tag{3.24}$$

where $N + 1$ is the index of the pipe section at the valve. Note that the datum must be taken through the centerline of the valve for this expression to be valid[3].

Series Pipe Connections

This type of junction is used to incorporate changes in the pipeline diameter or other physical properties such as wavespeed, friction factor, type of anchoring, etc.. Ignoring any minor losses at the connection, the head at the end of one pipe must equal that at the beginning of the next pipe. Continuity also says that the discharges in the two pipes are equal. Thus, the boundary conditions can be very simply stated as $H_{NS}^{J} = H_1^{J}$ and $Q_{NS}^{J} = Q_1^{J}$ where the subscript NS denotes the number of sections in the first pipe and the superscript J refers to the time step index.

[3]It would be more proper to say that the datum must be taken at the level of the downstream reservoir in order for the expression to be correct. Since the valve discharges to the atmosphere the level of the downstream reservoir is effectively at the centerline of the valve.

3.3.4 Valve Stroking and Transient Optimization

The question that should be raised at this point is, "What does valve stroking have to do with optimal design of pipelines?" It has already been demonstrated that the concept of optimal pipeline design is very complex and depends as much on the particular design goals as it does on the physical parameters. Nevertheless, certain fundamental considerations are common to most, if not all, pipeline applications. For the majority of systems, the dominant pipeline cost is the capital expense of the pipe and its installation. The cost of the pipe is an increasing function of the volume of material contained in the pipe cross section. Since the internal pressure is, for larger pipelines, the overriding factor that determines the amount of material needed in the pipe cross section, it is clear that the cost for any given diameter tends to be minimized if the internal pressure can be reduced to the lowest possible value. The valve stroking procedures accomplish exactly that—they guarantee that, for a particular diameter of pipe and valve motion duration, the maximum internal pressures are the minimum attainable for that system. Although this is not the only consideration involved in minimizing the cost of a pressure line, it assures that any solution based on the valve stroking procedure will always be "better" with respect to an infinite number of solutions based on arbitrary valve closures. That is to say, for every unique valve stroking solution an unlimited number of tau curves exist which will produce a larger pressure increase in the pipeline. In spite of the fact that valve stroking may not result in a global minimum, it does provide a rational basis for an optimal design procedure. Even if the suboptimal solution proves too costly in other respects to be implementable, a good starting point has been found from which the designer can proceed using experience and sound engineering judgement to arrive at a reasonable solution.

The valve stroking procedures are also important to the concept of transient optimization in that they provide a convenient means for incorporating constraints relating to control of the system. Finally, it should be noted that the foregoing discussion is only valid if the effectiveness criterion to be optimized is based upon variables which are functions of the internal pressure in the system.

Chapter 4

Conceptual Optimization Model

4.1 Description of System

The phrase *simple pipeline systems* has already been mentioned but no explanation has yet been given as to what this actually means. The system which has been chosen for investigation is the classical constant head upstream reservoir with a single pipe and a valve discharging to the atmosphere at the downstream end. A schematic depiction of this system is shown in Figure 4.1. The following dimensionless quantities are often used in hydraulic transient literature.

$$h_f = \frac{H_f}{H_r} \tag{4.1}$$

$$\rho = \frac{aV_0}{2gH_r} \tag{4.2}$$

There are a number of advantages, both from the theoretical and practical points of view, in studying this particular system.

1. Transient analysis is intrinsically complex and difficult, therefore a system which contains a small number of variables presents fewer impediments to the development of a rigorous, analytical model. Furthermore, significant features of the model are less likely to be obscured or invalidated due to complex interactions between parameters.

2. This system is generally percieved to be the classical starting point for all pipeline transient analyses.

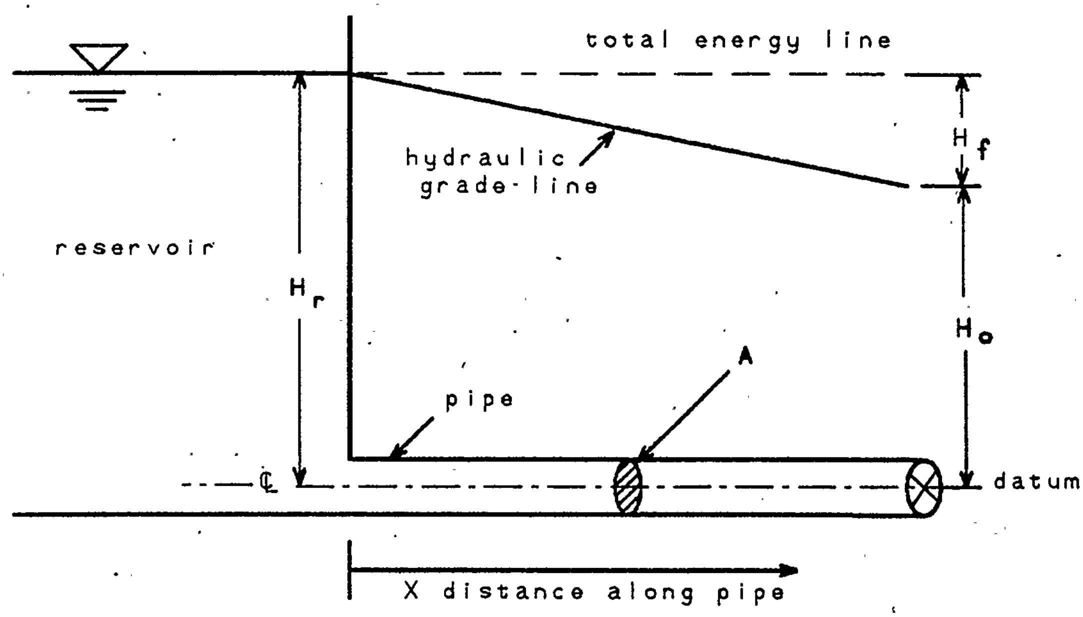


Figure 4.1: Schematic diagram of physical system.

3. Despite its simplicity, the system is very representative of many real physical installations, for example, penstocks in a hydro-power facility, a reservoir-municipal water supply link or a pipeline drawing oil from a large storage tank to a loading site. Even a pump with a relatively flat characteristic curve operating at constant speed could be usefully approximated in this way.

4. The valve stroking procedures which form the hydraulic basis for the model have certain limitations when systems become more complex.

The following assumptions have been made in the transient analysis.

- The system can be divided into homogenous lengths having the following characterisitcs:

  - The pipe diameter is constant.
  - The wall thickness is constant.

- The valve is located at the downstream end of pipe and the datum is set at the level of the downstream reservoir (the centerline of the valve for a valve discharging to the atmosphere).
- The reservoir level remains constant.
- No air pockets exist in the pipe.
- Pipe friction associated with a given unsteady flow follows the quadratic law, i.e., the D'Arcy-Weisbach law.
- The velocity head is negligible.
- Only one valve motion takes place and the motion is a closure.

These assumptions are commonly made in transient analyses and are valid for most pipeline systems. They are not requirements for the development of the optimization procedures proposed in this dissertation but do, however, facilitate the investigation.

4.2 Definition of Cost Factors

It is not essential for the conceptual development of the model to know precise, practical expressions for the cost terms. In fact, since the actual form of cost estimating equations used for a particular installation will be highly site and project specific, it would be a gross error to propose a model having exact functions which should apply to all situations. To demonstrate the utility of a model it suffices only to define the parameters which influence the effectiveness criterion and to determine the manner in which they affect the cost.

The most important factors determining the cost of a pipeline installation are:

- the length of the pipeline
- the diameter (size) of the pipe
- the wall thickness of the pipe
- the pipe material (eg. steel, PVC, concrete, etc.)
- the specification of the system boundary conditions (in this case a discharge-time relation)
- the response time of the system (synonomous with the duration of valve motion for a stroked system).

The significance of each of these factors with respect to the cost function is discussed separately below.

4.2.1 Pipeline Length

It is clear that the cost of material, placement, maintenance, etc., increases in proportion to the length of the pipeline. This has significant ramifications in determining the relative importance of unit costs versus single item expenditures. It also impinges directly on the optimization with respect to transients in that it establishes the return trip wave travel time ($2L/a$) for pressure pulses in the system. This effectively constrains the optimizing procedure in terms of its minimum response time. The reasons for this are explained in the section on valve stroking theory (Section 3.3). There are a number of methods already in existence (and use) to deal with the problem of optimizing costs associated with pipeline length (see Section 2).

4.2.2 Pipe Diameter and Wall Thickness

These two factors have been placed under a single heading because they are not independent of one another. Consider the classical hoop stress equation which describes the circumferential stress in the wall of the pipe.

$$\sigma = \frac{\gamma H D_i}{2e} \tag{4.3}$$

where γ is the unit weight of the fluid, σ is the stress in the pipe wall, H is the static pressure and the other variables are defined in Figure 4.2. In terms of the required wall thickness, Equation 4.3 becomes

$$e = \frac{\gamma H D_i}{2\sigma} \tag{4.4}$$

The greater the head in the pipe, the thicker the wall must be to withstand the pressure forces. Even if the pressure in the pipe is held constant and, let us say, that σ cannot increase since this is to be maintained at some specified design value, σ_{all}, then for a larger diameter pipe the wall thickness must also be increased.

Compounding this phenomenon is the fact that for a unit increase in pipe wall thickness, the cross sectional area of the wall increases nonlinearly with pipe

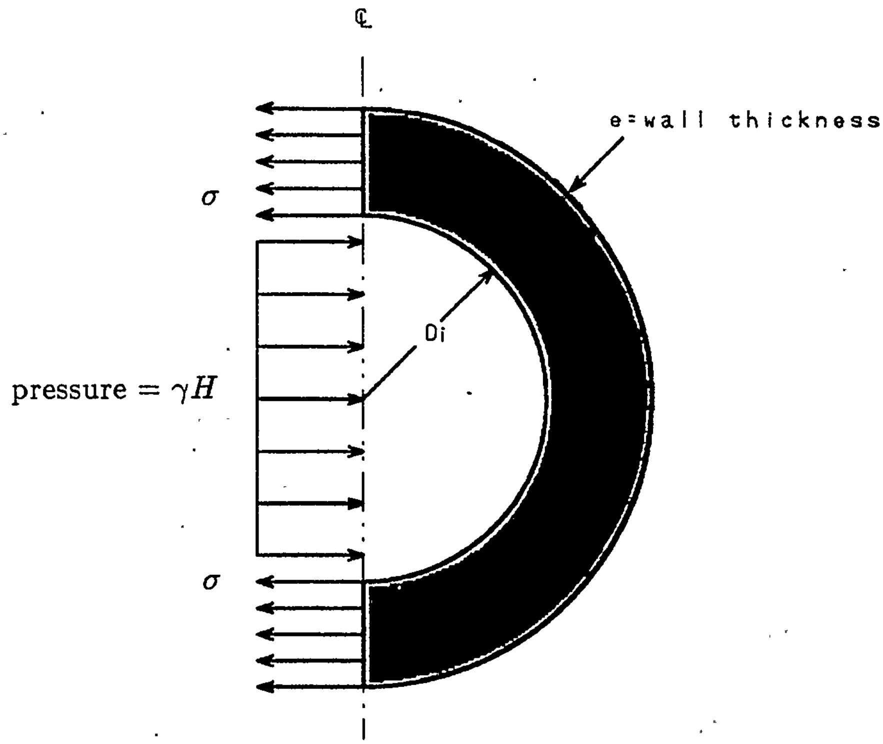


Figure 4.2: Hoop stress in the wall of a pipe.

diameter. The cross sectional area of the pipe can be expressed as

$$A_w = \frac{\pi}{4}(D_o^2 - D_i^2) \qquad \text{where } D_o = D_i + 2e. \tag{4.5}$$

Substituting for D_o gives

$$A_w = \pi(e^2 + eD_i) \tag{4.6}$$

and replacing e by Equation 4.4 produces

$$A_w = \frac{\pi D_i^2}{4}\left(\left(\frac{\gamma H}{\sigma_{all}}\right)^2 + \frac{2\gamma H}{\sigma_{all}}\right). \tag{4.7}$$

The cross sectional area integrated over the length of the pipe is equal to the volume of material required to withstand the pressure exerted by the fluid on the pipe.

It is evident from Equation 4.7 that the wall cross sectional area for a given head varies as the square of the inner diameter of the pipe. This is by far the dominating factor determining the amount of material in the pipe cross section. The factor enclosed in parentheses shows that, for a given size of pipeline, a unit increase in the fluid pressure also means that the wall area must be greater for larger values of D_i. For smaller pipe sizes structural, bedding, backfill, handling and wheel loads outweigh internal pressure requirements in terms of dictating the necessary wall thickness. However, in large diameter pipes, internal pressure is an important design consideration. It is not surprising then that disproportionate amounts of additional material are needed to manufacture large pipes as opposed to smaller ones for the same design head. Taken together, these two factors mean that for a unit increase in head more material is required for a safe design in a larger pipe.

Why then, do engineers not design systems using the smallest possible pipe size needed to carry the required discharge? There can be many reasons for "oversizing" a pipe. Perhaps the most compelling reason for installing "larger than necessary" pipes is the uncertainty of future demands. It may be far cheaper to design a system with capacity in excess of the present need so that costly expansions might be avoided at a later time. Another example is the cost represented by increased head losses due to friction resulting from higher fluid velocities in the system. If, for example, the constant head upstream boundary condition is replaced by a pump (which may also be operating at constant head) then the cost of pumping the fluid is a function of the head losses experienced by the system. Pumps and pumping stations are expensive items and it is desirable to reduce costs by designing the system so that the steady state operating expense is minimized (see Sections 2 and 4.3.4). A further argument in favour of using larger pipe sizes is the adverse conditions that negative pressures in a pipeline can produce, such as cavitation. If the head in a pipe is too low the likelihood of subatmospheric pressures developing in the system increases since even minor transients may produce significant pressure drops. Other reasons might be a minimum required delivery pressure either for a reservoir or to prevent cavitation in a valve under steady state conditions.

The capital investment required for pipelines increases for larger pipe sizes due to the transportation costs and the expenses associated with machinery, forms and even manufacturing methods. In addition, associated pipeline and appurtenant structures, miscellaneous equipment as well as a host of control devices increase nonlinearly in price as the pipe diameter gets bigger. Valves and their associated actuator/controllers become very expensive in larger sizes due to the increased forces, moments and torque acting on them. As a consequence, valves are often placed in a short section of reduced diameter pipe to decrease the capital expense while only marginally increasing the operating costs.

4.2.3 Pipe Material

The unit cost of pipes manufactured from different materials is subject to change depending on market and production conditions. Also, various pipe materials exhibit different types of stress-strain behaviour. Steel, for example, is an elastic material and can withstand a sustained increase in head without suffering a significant reduction in wall thickness (Poisson's effect). In contrast, a visco-elastic material such as polyvinyl chloride (PVC) can resist high pressures for only a short period of time due to creep effects increasing the likelihood of a rupture. Differences in the frictional resistance and the elastic modulus of the material have important consequences in determining the transient response and, hence, the cost of the system.

4.2.4 Specification of the System Boundary Conditions

This topic is discussed in greater detail in Sections 3.3.3 and 4.4.1. Suffice it to say here that the manner in which the system proceeds from one set of conditions to another (normally steady state conditions) has a profound influence on the transients arising in the system. In certain situations, these boundary conditions can be easily specified reducing the complexity of the optimization process while in other circumstances many alternative formulations exist and more elaborate methods in the design procedure are needed to cope with them. In either case, the manner in which boundary conditions are prescribed has a bearing on the ultimate cost of the system.

4.2.5 Response Time of the System

The minimum response time of the system, it has already been stated, is physically fixed by the wavespeed of the fluid and the length of the pipeline. This is simply a restriction imposed by the valve stroking procedure itself. There is, however, no such constraint on the maximum response time of the system. In general, it is advantageous in terms of controlling water hammer to change conditions as slowly as possible subject to whatever restrictions are imposed on the system. Unfortunately, virtually all pipeline installations are required to make adjustments as rapidly as possible either for operational, control or emergency reasons. For example, turbines in a hydro-power facility must be capable of reacting to a total load rejection by the utility power grid. This may be due to broken power lines or any of a number of other causes. When this occurs, the turbines, having lost their brake loads, begin to rotate more and more rapidly. The only way to curb this runaway situation is to reduce the flow of water through the turbines quickly and drastically. The natural consequence of this action is to generate powerful transients in the penstocks. The dilemma is this: if the strength of the penstocks is inadequate then more expensive turbomachinery must be purchased in order to withstand the angular momentum attained before the discharge is reduced. Conversely, if less expensive generating equipment is to be installed then more expensive penstocks must be constructed to ensure the safety of the powerhouse, its equipment and operators.

4.3 Formulation of the Effectiveness Criterion

From the foregoing discussion, it can be observed that the effectiveness criterion must involve a complex interaction of many factors. It can, however, be stated reasonably succinctly in the following form.

$$Cost = Material + Placement + Response\ Time + Operation \tag{4.8}$$

4.3.1 Pipe Material (Mass)

The amount of material at a particular cross section of pipe is directly linked to the maximum pressure occuring at that point in the system. This can be thought of as one contribution from the steady state head and another from the head rise caused by transients. Figure 4.3 shows clearly the steady state and the maximum head envelopes. Generally, however, since the maximum steady state pressure that

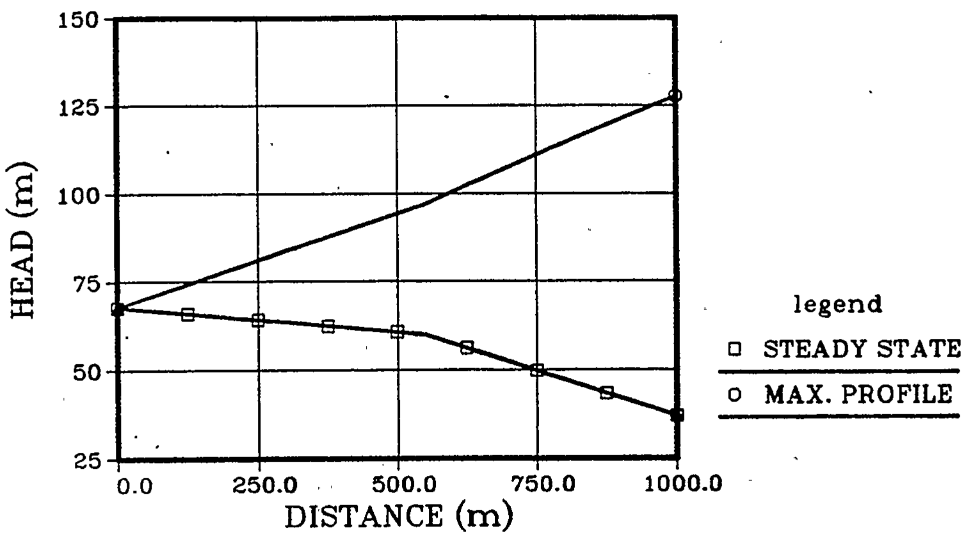


Figure 4.3: Steady state and maximum head lines for simple pipeline system.

can exist in the pipe is the total energy line, this provides a convenient quantity to which all other heads in the system can be referred. The maximum value of head attained at specific sections in the pipe during the transient phase is calculated by the valve stroking procedure. The effect of the pipe diameter can be accounted for by use of a pipe diameter factor or PDF (akin to the *capacity* factor used by cost engineers for preliminary estimates). Note that the form of the PDF factor is very similar to Equation 4.7. A cost term having the following form results.

$$\mathsf{C}_1 \sum_{i=1}^{i=n-1} (\Delta h_{max} + 1)\, \mathsf{PDF}\, \Delta x \tag{4.9}$$

where $\Delta h_{max} = (H_{max} - H_r)/H_r$

H_{max} = maximum head attained at section

H_r = reservoir head (reference head)

n = the number of sections

$\mathsf{PDF} = (D/D_{ref})^{\beta}$

D_{ref} = the reference pipe diameter

D = the pipe diameter

Δx = the reach length

β = an arbitrary exponent

$\mathsf{C}_1 = \mathsf{CM} \times \mathsf{P}$

CM = cost per unit length of D_{ref}

P = scaling factor for unit price flucuations.

Equation 4.9 takes no account of the stress-strain behaviour of the pipe material.

4.3.2 Placement

The placement or installation cost of the pipeline can be simply formulated as

$$\mathsf{C}_2 \sum_{i=1}^{i=n-1} \mathsf{PDF}\,\Delta x \tag{4.10}$$

where $\mathsf{C}_2 = \mathsf{CP} \times \mathsf{P}$

CP=the unit cost of installing D_{ref}

P=scaling factor for unit price variations.

It is clear that the placement cost depends primarily on the physical system itself. Although this commonly leads to the formulation of a cost term which is a linear function of the pipeline length, other factors, such as wall thickness, do introduce nonlinearities into the placement cost. For instance, welding costs increase for a thicker walled pipe.

4.3.3 Response Time

Since the cost associated with this term will most likely be installation dependent, there is little to gain in terms of insight into the problem by using some complicated mathematical expression to evaluate it. Even if no specific aspect of cost can be directly attributed to the response time in a particular system, this generally acts as a constraint on the objective function. For the purposes of this research the following arbitrary, monotonically increasing function was used.

$$(C_3)^{t_c^{\alpha}} \tag{4.11}$$

where

$$\begin{aligned} C_3 &= \text{an arbitrary constant (in cost units)} \\ \alpha &= \text{an arbitrary exponent} \\ t_c &= T_c/(2L/a) \end{aligned}$$

Valve/Controller/Actuator System

If the response time cost term does include specific elements of cost these can be incorporated directly into the cost function in any manner deemed appropriate by the analyst. As an example, consider the cost of the valve/controller/actuator system required at the downstream end of the pipe.

The cost associated with the control valve is a function of the size of the pipe and the precision needed to execute the prescribed valve closure arrangement. The size of the valve can easily be accounted for by a PDF, but the degree of control needed in the system is more difficult to evaluate. Since the tau curve essentially represents the relative motion of the valve, it may be characterized by its velocity and acceleration components. The more rapid and complex a valve motion is, the greater the velocities and accelerations the valve system must be capable of providing. This, in turn, means it will be more expensive. Acceleration is more expensive to achieve in the acuator as it requires sensitive variable speed drivers.

The first and second derivatives of the closure arrangement can be respectively obtained by first and second order Taylor expansions, and in this way, approximations to the velocities and accelerations occurring during the valve motion may be obtained. This leads to the formulation of the following cost term.

$$\left(\mathsf{VF}_1(\mathsf{V}_{max} - \mathsf{V}_{min})^k + \mathsf{VF}_2(\mathsf{A}_{max} - \mathsf{A}_{min})^m + \mathsf{C}\right) \mathsf{PDF}_{valve} \tag{4.12}$$

where $\mathsf{PDF}_{valve} = (D_{valve}/D_{ref})^{\beta'}$

V_{max} = maximum velocity

V_{min} = minimum velocity

A_{max} = maximum acceleration

A_{min} = minimum acceleration

k, m and β' = arbitrary exponents

C = the cost of a linear closure system

VF_1 and VF_2 = factors expressing the relative importance of the velocity and acceleration terms.

The linear closure cost term, C, (generally the cheapest to attain in practice) has been included because the other terms in Equation 4.12 become zero for this case; i.e., $V_{max} - V_{min}$ and $A_{max} - A_{min}$ are equal to zero.

It must be understood that real valves, valve systems and their closure characteristics are enormously complex and it is doubtful that any simple or general formulation could be so easily made. At the same time, it is not necessary to do so in order to incorporate a conceptually valid representation of their effect on the overall cost of a pipeline system.

4.3.4 Operation

A logical approach to evaluating costs of this type would be to preoptimize them using one of the steady state techniques described in Section 2. This is appropriate if either

1. the cost associated with this term is much greater than any other cost component, pumping costs on a short, high capacity line for example.

2. the system is too complex to permit the convenient incorporation of steady state factors in the transient model.

In either case, a "complete" optimization (one that includes water hammer effects) can be achieved by performing separate procedures with the transient model performing a suboptimization role in the overall process.

In some instances the operational costs may include items not normally thought of as maintenance expenditures. For example, a reservoir supplying water to another reservoir by gravity does not seem to have any operational costs other than normal maintenance. However, if the friction losses in the system are thought of as a loss of capacity under normal operating conditions, then an operational consideration analogous to pumping costs can be seen to exist even in this simple situation. This is intrinsically considered in the simple upstream reservoir system in the specification of the required discharge, since parameters are selected such that the discharge constraint is always satisfied. However, this may not be true in every situation and the flowrate itself need not be specified as an equality but rather as a minimum required flowrate or a range of discharges.

As a specific example of an operational cost which can easily be incorporated into the transient model, consider the power output from a hydroelectric installation. The equation for the power produced by a turbine is

$$P = Q\gamma H_{net}\eta \quad \text{where } \eta \text{ is the turbine efficiency.} \tag{4.13}$$

Ignoring, for the moment, that the turbine efficiency is not constant, the amount of power that can be generated is a function of the available head and the discharge. The available head (or net head) is a function of the flowrate and the pipe cross sectional area (diameter).

$$H_{net} = \left(H_0 - \frac{kQ^2}{A^2}\right) \qquad \text{and } k = \text{a constant.} \tag{4.14}$$

The discharge is a function of the pipe diameter and the head losses in the wicket gates. In broad terms, for a larger diameter pipe, more water can pass through the turbine, the lower are the various frictional losses and, hence, the power output of the turbine can be increased. A tradeoff exists, though, between higher pipeline costs for increased capacity and the value of the power generated.

4.4 Cost Variables

The preceeding sections have focused on the parameters which ultimately determine the cost of a pipeline system and it has been demonstrated that they can be used to generate a theoretically meaningful cost function. It is, however, probably not yet clear exactly how these factors are related to the cost variables *per se*. Thus far it has been postulated that

$$Cost = \mathcal{F}\{Material, Placement, Response\ Time, Operation\}. \tag{4.15}$$

If we extract the key mathematical and hydraulic variables from these somewhat nebulous terms a more analytical expression results.

$$Cost = \mathcal{F}\{D_i, \Delta h_{max}, Q_{SS}, t_c, L, a, \text{boundary condition specification}\}. \tag{4.16}$$

These variables have been defined and discussed at length in Section 3.3.

The heads which are produced during the transient are completely determined by the physical system characteristics, the initial and final steady states and the upstream boundary conditions. Likewise, the closure arrangement is fixed by the manner in which the boundary conditions are specified. Therefore, the only variables which are not necessarily set by the specification of the system are the duration of the transient, t_c, the pipe diameter, D_i, the wavespeed, a, and the nature of the boundary conditions. Every other system quantity is either given or can be derived from these variables, and all of these quantities can be manipulated by the valve stroking procedure(s). The pipe diameter and the valve motion duration have been discussed in previous sections.

4.4.1 Boundary Condition Specification

The importance of the nature of the boundary conditions to the optimization model cannot be overemphasized. Each class of problem has its own peculiar dependence on the way in which the starting boundary condition is chosen. In other words, the spatial and temporal characteristics of that point in the system from which the valve stroking procedure begins has a profound influence on the dynamic response of the system. In some instances the choice of starting points is obvious and simple while in others it is completely arbitrary and requires experience, sound judgement and perhaps even research in order to obtain boundary conditions which will provide optimal results.

To illustrate precisely what this entails, consider the following example which involves the same physical system that has been discussed thus far. In this system, one can only stroke from the reservoir. This is because one variable, namely the head, is fixed at the reservoir. Hence, the only variable which can be manipulated is the discharge, Q. One means for manipulating the discharge is shown in Figure 4.4. A parabolic discharge factor, ψ, ranging between zero and unity can be obtained as follows.

$$\psi = \frac{(Q_0 - Q_f)/2 + y}{Q_0 - Q_f} \tag{4.17}$$

where y is measured positive upwards and is the difference between the discharge at $t = t_{tr}/2$ and $\Delta Q/2$. For any given value of ψ, the value of the discharge at the reservoir when the transient phase is half complete ($t = t_{tr}/2$) is

$$Q|_{t=t_{tr}/2} = \psi \frac{Q_0 - Q_f}{2}. \tag{4.18}$$

Knowing these three points on the discharge curve, the flowrate at any time during

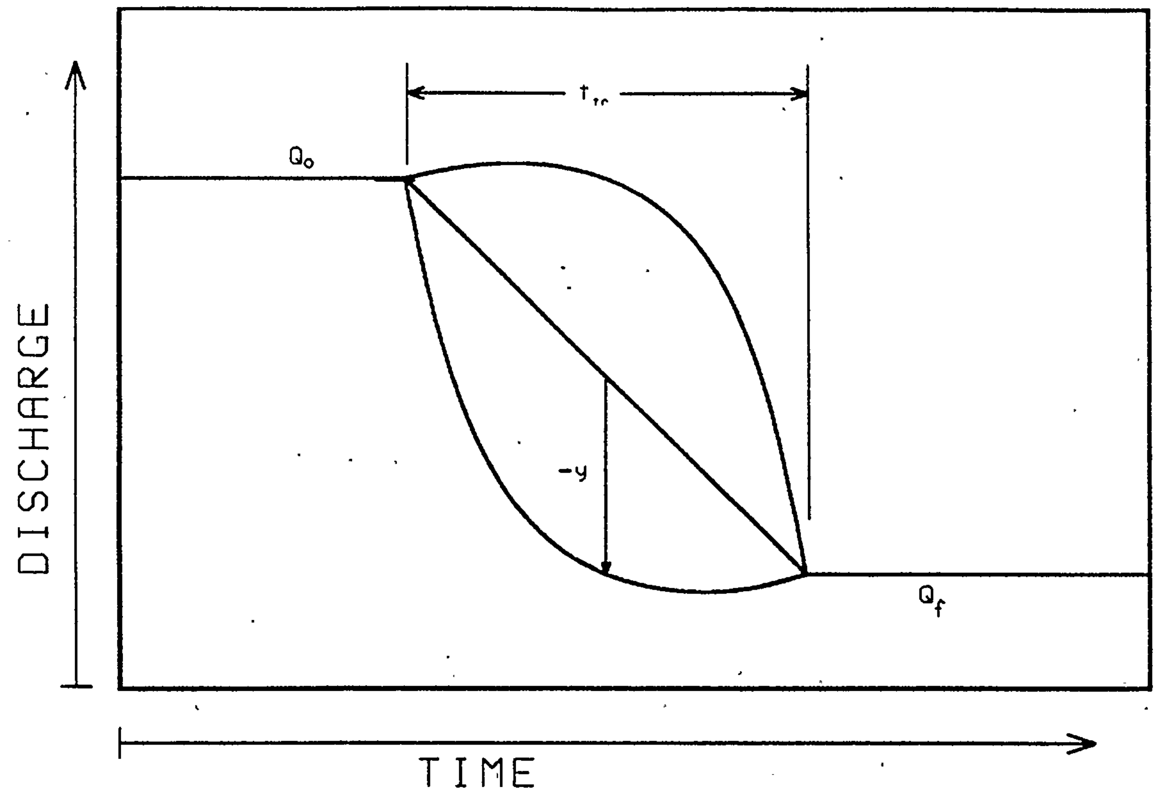


Figure 4.4: Specification of the upstream boundary condition by use of a parabolic discharge function.

the transient phase at the reservoir can be obtained by parabolic interpolation.

Figure 4.5 shows a cost surface for a single pipe diameter resulting from the application of the model to a fictitious system of the sort being considered here. Note the very pronounced effect on cost that has been produced by changing the specification of the upstream boundary condition. It is interesting to see that the minimum cost for a given stroking time is almost always obtained when $\psi =$ 0.5, i.e., when the variation in the discharge is linear. This is not surprising as other researchers[Propson] have demonstrated analytically that for stroking times

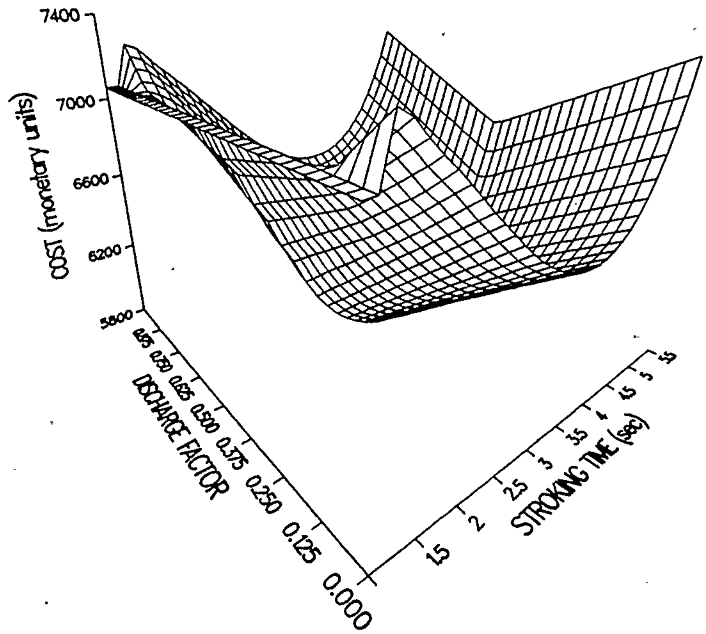


Figure 4.5: Cost surface for parabolically varied upstream discharge. This is for a fixed pipe diameter. Note the distinct "trough" for $\psi = 0.5$.

between $2L/a$ and $4L/a$ seconds, a linear change in the discharge at the reservoir produces the minimum head rise in a frictionless system. It is instructive that this is not so for stroking times in excess of $4L/a$ seconds[Propson][Karney/Ruus 1985] and the manner in which the system proceeds from the initial to the final discharge can be more crucial in these cases.

This example points out very clearly the importance of the boundary condition specification for the system as a factor affecting the cost of the installation. It also provides some indication of how complex matters may become in systems with boundary conditions having two or more degrees of freedom.

In terms of the simple system being investigated here, it is possible to formulate the cost function in terms of the pipe diameter and the response time only.

4.4.2 Wavespeed

In actuality, the wavespeed, a, represents a convenient means for describing a number of physical properties of the fluid, the pipe material and the method of pipe anchoring used for a particular installation. The most general expression for the wavespeed is

$$a = \sqrt{\frac{K/\rho_w}{1 + \frac{KD}{Ee} c_1}} \tag{4.19}$$

where K is the bulk modulus of the fluid, ρ_w is the density of the fluid, E is the elastic modulus of the pipe material, and D and e are the pipe diameter and wall thickness respectively. The constant c_1 takes account of the type of support provided for the pipeline. Typically, three cases are recognized with c_1 defined for each as follows.

Case a The pipeline is anchored at the upstream end only.

$$c_1 = 1 - \frac{\mu}{2} \tag{4.20}$$

Case b The pipeline is anchored against longitudinal movement.

$$c_1 = 1 - \mu^2 \tag{4.21}$$

Case c The pipeline has expansion joints throughout.

$$c_1 = 1 \tag{4.22}$$

The Poisson's ratio for the pipe material is denoted by μ.

Most commonly, $(KD/Ee)c_1 \to 0$ and Equation 4.19 simplifies to $a = \sqrt{K/\rho_w}$ which is the expression for the acoustic wavespeed in a fluid. For this reason a is referred to as the *wavespeed*. As indicated by Equation 4.19, the situation in a pipeline is rather more complicated.

For the majority of hydraulic analyses involving transients, the wavespeed can be considered to be constant. Even in those cases where some uncertainty exists regarding the wavespeed, the solutions of the governing equations, with respect to peak pressures, are relatively insensitive to changes in this parameter. As mentioned in Section 3.2, it is not unusual to deliberately vary the wave celerity by as much as $\pm$ 15% in order to maintain a constant time step for solution by the method of characteristics. Does the same hold true, however, for the valve stroking procedure? Work done by Ruus indicates that maximum and residual pressures obtained by a system are very sensitive to non-ideal valve motion, especially in the final $2L/a$ seconds.

Another concern in this context relates to the ratio of D/e in Equation 4.19. If this ratio is changing during the optimization process, can the variation be properly accounted for by the model? Furthermore, suppose that it is wished to vary the wall thickness along the length of the pipe to take advantage of the reduction in maximum pressure which occurs as one proceeds upstream. In other words, each pipe reach, L_i, could have a different D/e ratio and hence a different wavespeed. How does this affect the optimization procedure?

The answer to the former question is straightforward. The *optimal* nature of the tau curves generated by the valve stroking procedures does not seem to be particularly sensitive to variations in the wavespeed. Figure 4.6 shows the manner in which the heads produced by an optimal valve closure arrangement change when the actual wavespeed of the system differs from that used to obtain the optimum tau curve. In the figure, the maximum head ratio is defined as $H_{max}/H^{a_0}_{max}$ where $H^{a_0}_{max}$ is the maximum head occurring for a_0 and H_{max} is the maximum value of head achieved for an arbitrary wavespeed. The maximum residual head is H^{res}_{max}/H_0 where H^{res}_{max} is the maximum head obtained after the cessation of the valve motion and H_0 is the final steady state head at the valve. The dimensionless wavespeed is simply a/a_0.

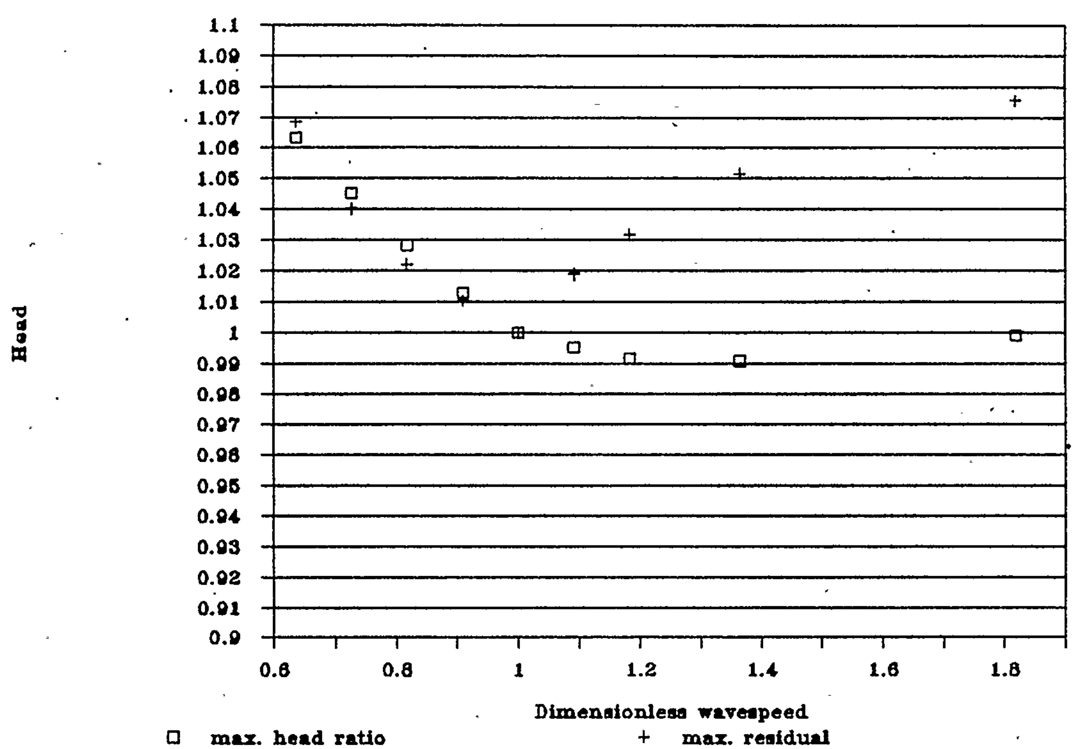


Figure 4.6: Sensitivity of optimum tau curve to variations in wavespeed. $L = 550$ m, $\rho = 0.94$, $T_c = 2.56$, $a_0 = 1100$ m/s, $Q_0 = 2.0$ m^3/s, $Q_f = 1.0$ m^3/s, $H_r = 67.7$ m, $D = 1.5$ m, $f = 0.010$, number of pipe reaches = 8.

The second question is equally easy to answer. If the designer deems it necessary to use variable wavespeeds, they can be trivially handled by the model since only a single variable need be adjusted at each trial. The pipe diameter under consideration is known for each possible solution and the wall thickness can be evaluated by an iterative scheme provided that some means of relating wall thickness to pipe diameter, internal and external pressures and structural considerations is available. The other parameters in Equation 4.19 are all known from the system specifications.

Finally, if it is desired to alter the wall thickness along the length of the conduit, it becomes necessary only to define each sublength as a different pipe in the system. Hence, each reach of pipe can possess a different wavespeed and the pipeline is modeled as a multipipe system. Some adjustment to the wavespeed may be necessary, as previously mentioned, in order to maintain a constant time step for the method of characteristics solution. However, an excellent approximation of the true system will be obtained.

4.5 Constraints

Equality and inequality constraints can easily be incorporated into nonlinear optimization problems by means of either penalty terms or transitional penalty functions. These are extra terms added to the objective function which impose a penalty cost if the constraint is violated. Penalty terms may be expressed in the following general form.

$$z_i = \varphi \kappa_i |\mathcal{G}_i\{x_1, x_2, \ldots, x_n\}| \tag{4.23}$$

where

$$
\begin{aligned}
\mathcal{G}_i\{x_1,\ldots,x_n\} &= \text{the } i^{\text{th}} \text{ constraint function} \\
z_i &= \text{the penalty cost associated with the } i^{\text{th}} \text{ constraint} \\
x_1,\ldots,x_n &= \text{the cost variables of the } i^{\text{th}} \text{ constraint} \\
\varphi &= \begin{cases} 1 & \text{for } \mathcal{G}_i\{x_1,\ldots,x_n\} \neq 0 \\ 0 & \text{for } \mathcal{G}_i\{x_1,\ldots,x_n\} = 0 \end{cases} \\
\kappa_i &= \text{a penalty coefficient, eg. } 10^2
\end{aligned}
$$

In order to avoid the steep-sided valleys that are sometimes produced by penalty terms of this sort, transitional penalty functions which provide a smoother change in the cost function are often employed. This is desirable as it results in a cost surface which is more easily traversed by certain optimum seeking strategies. Transitional penalty functions have the form

$$z_i = \varphi\kappa_i \left(e^{|\mathcal{G}_i\{x_1,\ldots,x_n\}|} - 1\right). \tag{4.24}$$

Figure 4.7 shows the excessive cost increase that a penalty function generates at a constraint boundary. The scale is such that the contours of the cost surface are less prominent making it appear as a low, flat region in the figure.

Apart from any specific constraints imposed on the system by design, there are certain restrictions placed on the cost domain by nature itself. With respect to the response time of the system, it has already been mentioned (Section 4.3.3) that valve stroking theory cannot be applied when the duration of valve motion is less than $2L/a$ seconds. In practice, this is not a severe restriction since the behaviour of the system is not unduly different for valve motions taking place in a shorter period of time. The reason for this is simply that no reflected waves can reach the

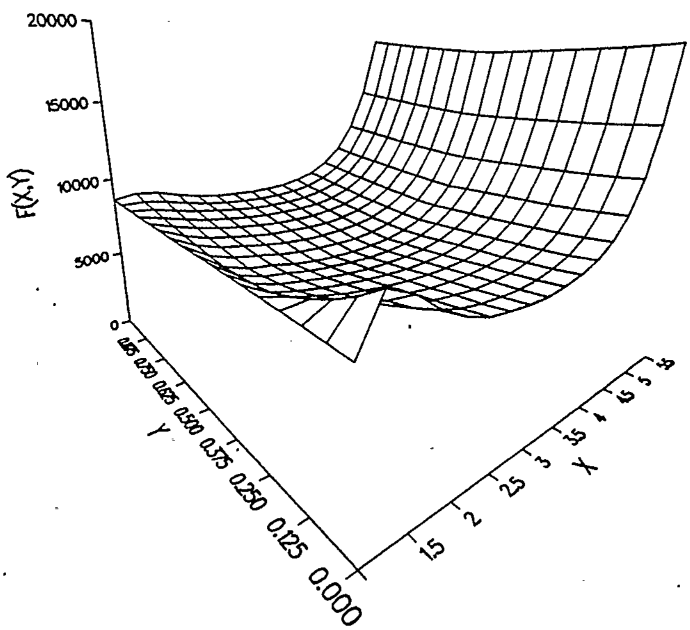


Figure 4.7: Example of a constraint formed by a penalty function.

valve before $2L/a$ seconds have elapsed anyway so no reduction in the maximum head produced at the valve occurs. Some moderation of head can, however, take place in the pipe upstream from the valve since reflected waves will be traversing the length of the pipe starting from the reservoir after L/a seconds have elapsed. In practice valve motions having a duration less than $2L/a$ seconds only rarely take place (undamped check valves being a notable exception).

An upper limit to the valve motion duration can be established by virtue of the fact that, for changes in the system taking longer than about $5(2L/a)$ seconds[Karney/Ruus 1985], rigid water column theory provides acceptable solutions for unsteady flow problems. This does not mean that the effects of fluid elasticity do not exist for longer duration valve motions, but that they are simply not a requirement for an accurate solution. For these reasons, the model described herein restricts its attention to only that part of the unconstrained cost domain which falls between $2L/a \leq t \leq 10L/a$, i. e., $1 \leq t_c \leq 5$.

4.6 Feasibility

In general, *feasibility* in the context of optimization refers to that part of the solution domain which does not violate the constraints imposed upon the problem, either intentionally by the designer or inherently by the physical world. Section 4.5 has already dealt with the latter. However, there are some other restrictions which define the region of feasibilty insofar as the model being described herein is concerned. Certain conditions of flow which may occur in practice, or which result from an inferior formulation of the problem, must also be considered.

- Negative pressures at the valve are definitely possible for certain system and valve configurations. These situations are considered to be unacceptable as potential solutions and are viewed as *infeasible* by this model since they would permit air to be introduced into the pipeline via the valve. Such an occurrence can normally be handled by placing an accumulator at the downstream end of the pipe but as this introduces an unnecessary degree of complexity into the system it will not be considered as a legitimate solution here.

- Equation 3.20 is not valid for situations involving a reversal of flow. If it were necessary to incorporate the possibility of negative flows into the model, Equations 3.18 and 3.19 could be solved numerically using Newton-Raphson or some other method.

- A solution which would be optimal with respect to the transient behaviour of the system can exist, but it may not satisfy the steady state requirements of the system. Hence, any optimal solution must be able to meet the requirements for the desired steady flow characteristics of the system.

4.7 Self-Regulation

It is a relatively simple matter to incorporate the cost of computer analysis directly into the optimization model. Irrespective of the type of optimum seeking strategy employed by the model, the change in the cost function at each move or cycle can be readily evaluated. If the cost of the computer resources needed to locate the next improved value of the objective function exceeds, for an arbitrary number of moves, the improvement in the value of the cost function the program may be terminated. This is probably most important if the cost of a given system is small and the computer analysis constitutes a significant proportion of the overall project cost.

Chapter 5

Model Testing

The model as described in Chapter 4 is an abstract, hypothetical formulation of a generalized mathematical framework which allows for many different implementations of the model depending upon the peculiar needs or desires of the analyst. In this chapter a number of specific versions of the model are described in conjunction with the particular aspects of the investigation for which they were used. Hence, in the discussion that follows, the term *model* should not be understood to indicate some static optimization construction. Rather, it should be realized that the outcome of various testing stages has influenced the form and use of the model in the latter phases. The model is first described as it was initially conceived and subsequent changes are related as they took place during the evolution of the final model concept.

All of the computer versions of the model[1] described were coded in Fortran 77 and have been implemented on the Honeywell DPS8/6 computer at the University of Calgary. A partial, vectorized version of the model was also written in Fortran and run on the University of Calgary's CYBER 205 computer[2].

[1]The various versions of the model are referred to as **TOM1**, **TOM2**, etc., where **TOM** is an acronym for **T**ransient **O**ptimization **M**odel.

[2]Descriptions of the various programs and versions of **TOM** can be found in Appendix A.

5.1 Verification of Valve Stroking Algorithm

The valve stroking algorithm itself will not be described since a complete account of this is given by [Wylie/Streeter]. Interested readers are referred to this excellent text for details concerning computerization of both types of valve stroking.

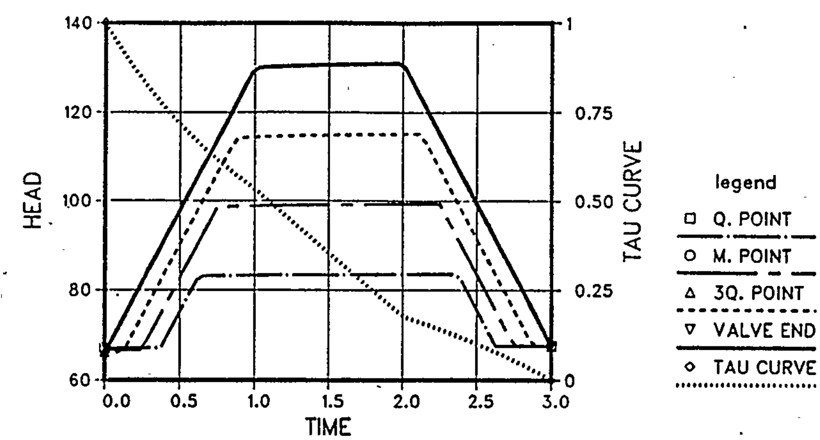
The validity of the optimization model is completely dependent upon the accuracy and correctness of the valve stroking procedures since these are the means by which the physical nature and the hydraulic performance of the system are assessed. Two methods for testing the valve stroking algorithms were used:

1. The optimum closure curve produced by the valve stroking program can be used as the input for another program that performs ordinary method of characteristics time simulation and is known to generate correct results.

2. Experimental data for a valve-stroked closure can be used as a check on the accuracy of the transient response predicted by the valve stroking subroutine.

Figure 5.1 shows the results of one test of the former type. Actual numerical results are given in Table 5.1. Several similar tests were performed on a number of different systems and in each case the method of characteristics simulation gave a system response identical to that predicted by the valve stroking program.

For the latter test, data used by [Propson] in the experimental verfication of his valve stroking procedure were utilized. The specification of the system variables conformed to the laboratory setup and the output of the valve stroking program (Figure 5.2) is compared with the results (both predicted and experimental) obtained by [Propson]. Note that the agreement here, although good, is not perfect.



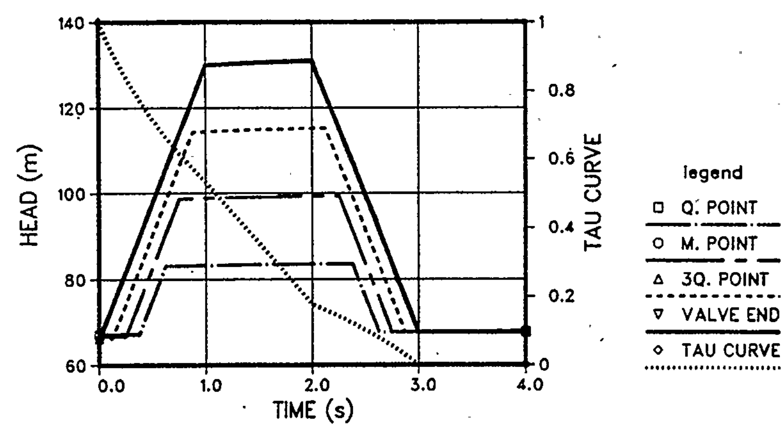


Figure 5.1: Comparison of system response predicted by valve stroking program and ordinary method of characteristics simulation. Top figure—valve stroking response (head in meters); Bottom figure—method of characteristics simulation response. H_r = 67.7 m, Q_0 =1 m^3/s, Q_f = 0 m^3/s, T_c = 3 s, D = 0.75 m, f = 0.010, a = 1100 m/s and L = 550 m, number of pipe reaches = 8. Variation in discharge at the reservoir end is linear with time.

Valve Stroking				MOC[a] Simulation		
Time (s)	τ^b	Head (m)	Q (m^3/s)	Time (s)	Head (m)	Q (m^3/s)
0.00	1.000	65.78	1.00	0.00	65.78	1.00
0.25	0.840	81.78	0.937	0.25	81.78	0.937
0.50	0.717	97.82	0.874	0.50	97.82	0.874
0.75	0.617	113.89	0.811	0.75	113.89	0.811
1.00	0.533	129.99	0.749	1.00	129.99	0.749
1.25	0.443	130.33	0.624	1.25	130.33	0.624
1.50	0.354	130.60	0.499	1.50	130.60	0.499
1.75	0.265	130.82	0.374	1.75	130.82	0.374
2.00	0.177	130.98	0.249	2.00	130.98	0.249
2.25	0.141	115.21	0.187	2.25	115.21	0.187
2.50	0.102	99.40	0.125	2.50	99.40	0.125
2.75	0.055	83.56	0.062	2.75	83.56	0.062
3.00	0.000	67.70	0.000	3.00	67.70	0.000

Table 5.1: Comparison of system response predicted by valve stroking program and ordinary method of characteristics simulation. Heads and discharges are taken at the valve end. $H_r = 67.7$ m, $Q_0 = 1$ m^3/s, $Q_f = 0$ m^3/s, $T_c = 3$ s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 8. Variation in discharge at the reservoir end is linear with time.

[a] Method of Characteristics
[b] For both valve stroking and method of characteristics.

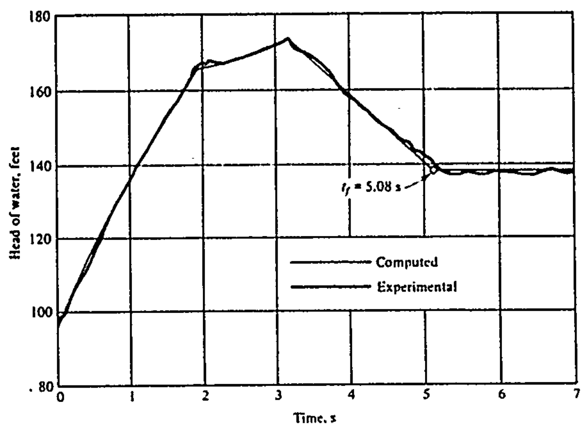
This is likely due to the fact that [Propson] has used a second order approximation to evaluate the friction term in Equation 3.10. In addition, [Propson] does not assume that fully developed turbulent flow exists and accounts for the variation of f with Reynolds number.

The foregoing tests do not absolutely guarantee the correctness of the valve stroking algorithms. They do provide, however, strong support for the assertion that the formulation and implementation of the valve stroking procedures is, in all likelihood, valid. Having established, as well as is possible, the verity of the hydraulic component of the optimization model, further testing of the model is now appropriate.

5.2 Validity of Cost Variables

5.2.1 Pipe Diameter

No testing has specifically been done to demonstrate the importance of pipeline diameter as a valid decision variable. A glance at any of the literature dealing with the topic of pipeline optimization will attest to the importance of this variable as a major factor affecting the cost of a pipeline installation. The problem, in fact, is only to determine what other factors apart from the pipe diameter should be included as key decision variables in the optimization process. The following sections concern themselves with this issue and the validity of pipe diameter as a cost variable is accepted *a priori*.



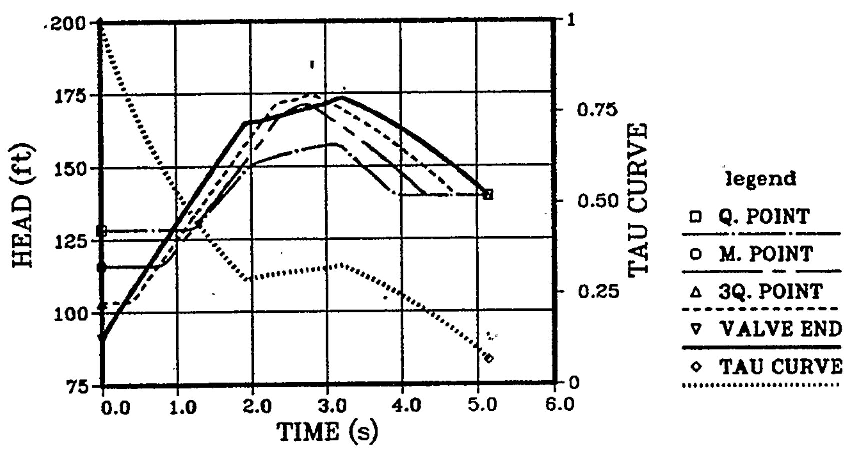


Figure 5.2: Comparison of results for experimental data (top figure—excerpted from [Wylie/Streeter]) and valve stroking procedure (bottom figure). Top figure shows head change at the valve end only. Time of closure (denoted by t_f in the top figure) is 5.08 seconds.

5.2.2 Boundary Condition Specification

The specification of the known boundary condition for the valve stroking procedure has already been mentioned in Section 3.3.3. It was stated that, for the simple system considered in this investigation, the optimal solution almost always occurred when the variation in the discharge at the reservoir during the transient phase was linear with respect to time. For this system, the reservoir end is the only boundary condition which can be fully specified. This must be so because the head at the upstream end is fixed by the level of the water in the reservoir. It would make no sense to prescribe conditions at some other physical location in the system since the valve stroking procedure could produce values of head at the reservoir end of the pipe that would be inconsistent with the assumption of a constant reservoir water level. Therefore, the only consideration in specifying completely the conditions at this boundary is the manner in which the discharge varies with time in going from the initial to the final flowrate.

Referring to Figure 4.4, it can be seen that a linear time-discharge relation produces sharp discontinuities in the slope of the discharge curve at the beginning and the end of the valve motion. It seems intuitive that by eliminating one or both of these instantaneous changes in the fluid velocity, some amelioration of the head rise produced in the system might be achieved. The parabolic discharge function was devised as a means of testing this hypothesis. The objective function used was simply the sum of the individual cost terms given in Chapter 4 i. e.,

$$Cost = \overbrace{\mathsf{C}_1 \sum_{i=1}^{i=n-1} (\Delta h_{max} + 1)\,\mathsf{PDF}\;\Delta x}^{\text{Mass}} + \overbrace{\mathsf{C}_2 \sum_{i=1}^{i=n-1} \mathsf{PDF}\,\Delta x}^{\text{Placement}} + \overbrace{(\mathsf{C}_3)^{t_c^\alpha}}^{\text{Duration}}$$

$$+\overbrace{\left(\mathsf{VF}_1(\mathsf{V}_{max}-\mathsf{V}_{min})^k+\mathsf{VF}_2(\mathsf{A}_{max}-\mathsf{A}_{min})^m+\mathsf{C}\right)\mathsf{PDF}_{valve}}^{\text{Valve System}} \quad (5.1)$$

The values of the various constants used in Equation 5.1 are: $C_1 = 1.0$, $C_2 = 4.0$, $\beta = 1.5$, $D_{ref} = 0.5$ m, $C_3 = 7.0$, $\alpha = 0.8$, $VF_1 = 50.0$, $VF_2 = 80.0$, C$= 0.0$, $\beta' = 1.5$, $k = 1.3$, $m = 2.0$.

Note that no cost term for the operating expenses has been included. This would simply introduce an extra mathematical relation that provides no additional information. In any case, it can be considered to be accounted for in the mass and/or placement cost expressions. For that matter, interactions between some cost factors could permit the combination of other terms, such as the mass and placement expressions. The cost function was evaluated for discrete combinations[3] of stroking time and ψ . Figure 4.5 shows a cost surface generated by the model for a given diameter of pipe, with the parabolic discharge factor, ψ and stroking time as the decision variables. Several tests for different pipe diameters, initial and final flowrates, wavespeeds and friction factors were performed to determine if any reduction in system heads (costs) was possible. In virtually all cases no improvement could be obtained by altering the flowrate in this manner. Table 5.2 shows some typical cost results for one such numerical experiment. Note that for $\psi = 0.5$ the discharge-time relation is linear. Only for stroking times in excess of $4L/a$ seconds was any evidence found to support the contention that a nonlinear variation in discharge could give some improvement in the objective function. For those instances in which the cost function showed some improvement, the value

[3]It should be mentioned that the program **TOM1** which was used to generate the cost surfaces constitutes the simplest optimization model, i.e., enumeration of all feasible solutions.

Discharge Factor ψ	Stroking Time T_c (seconds)							
	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.000	1084	9107	7851	7407	7207	7201	7324	7678
0.125	9245	8136	7420	7035	6850	6836	6991	7383
0.250	8217	7484	7009	6728	6597	6602	6772	7183
0.375	7731	7063	6643	6430	6347	6387	6584	7014
0.500	7611	6904	6400	6201	6141	6202	6417	6863
0.625	7890	7155	6663	6408	6304	6333	6522	6950
0.750	8155	7440	6984	6692	6549	6546	6710	7117
0.875	8450	7784	7316	6997	6831	6800	6937	7320
1.000	8832	8220	7651	7353	7165	7101	7205	7558

Table 5.2: Objective function cost (arbitrary units) for parabolic discharge relation. $H_r = 67.7$ m, $Q_0 = 2$ m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2.

of ψ was very close to 0.5, i.e., the discharge curve was nearly linear. [Propson] has shown mathematically that for a frictionless system, and for stroking times less than $4L/a$ seconds, that the minimum system heads are obtained when the flowrate varies linearly with time. The tests performed in this investigation support this assertion.

These results are perhaps not surprising if one considers the nature of hyperbolic differential equations. Sharp wave fronts which exhibit little or no dispersion are typical for the solutions of these differential equations by the method of characteristics, hence the instantaneous changes in fluid velocity that take place are not at all unrealistic. The significance of these numerical tests is that, for the constant head upstream reservoir system, the specification of the upstream boundary condition can be reduced to a linear variation of the discharge during the transient phase. In other words, the discharge function need not be considered as a decision variable in order to find the optimal value of the objective function. It is important to realize that this is not necessarily so for other systems.

5.2.3 Valve Closure Time

Two cost terms have been associated with the valve motion in the formulation of the objective function given in Equation 5.1. The first term is simply intended to represent the fact that, for critical control operations, the shortest possible duration should provide the greatest cost benefit. As the length of the control operation is increased, it is reasonable to expect the cost benefits to decrease. The second term in the objective function relates specifically (as described in Section 4.3.3) to the cost of the valve and its associated actuator/controller mechanism. Intuitively, one supposes that this term ought to behave in a similar manner, i. e., that the cost of the valve system is a decreasing function of time. In Figure 5.3 the individual cost terms for an arbitrary pipeline system are plotted. The units of cost bear no relation to real costs and are produced by using Equation 5.1. They do, however, give an indication of the nature of the individual cost components and of the objective function itself. The mass[4] cost term is, as one would expect, a nonlinear, decreasing function of time. The placement cost term is constant for a given pipeline diameter and length. The response time cost term is of course, by design, a nonlinear, monotonically increasing function of time. The valve system cost term is also nonlinear, but quite surprisingly, is not a gradually decreasing function of time. Rather, it exhibits extremely high values for valve closure times near the

[4]The expression *head cost term* is synonomous with the mass cost term.

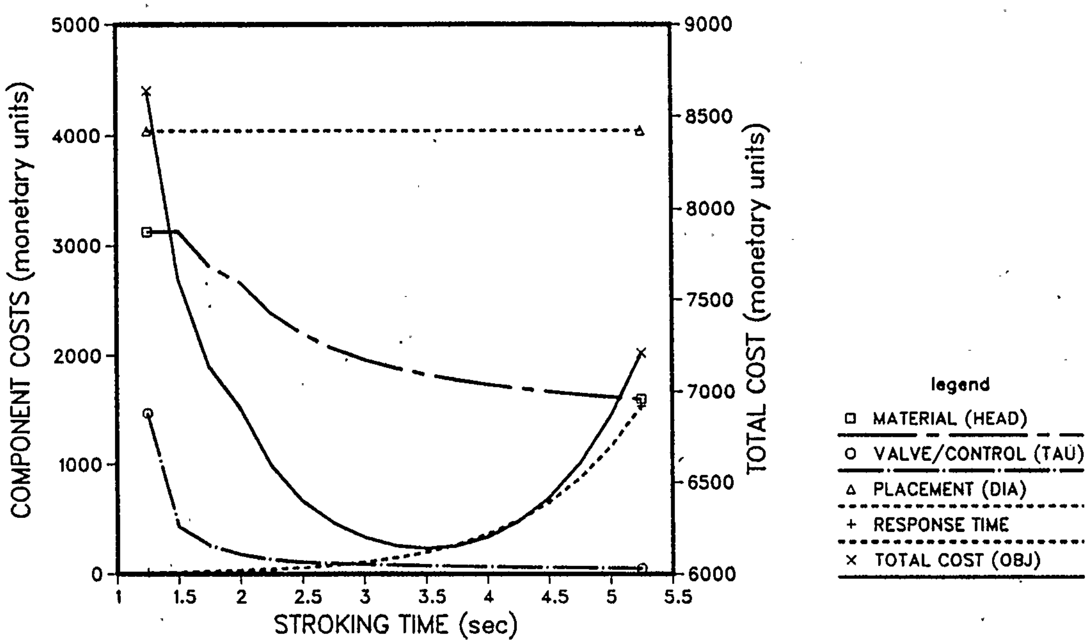


Figure 5.3: Objective function cost terms. $H_r = 67.7$ m, $Q_0 = 2$ m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2.

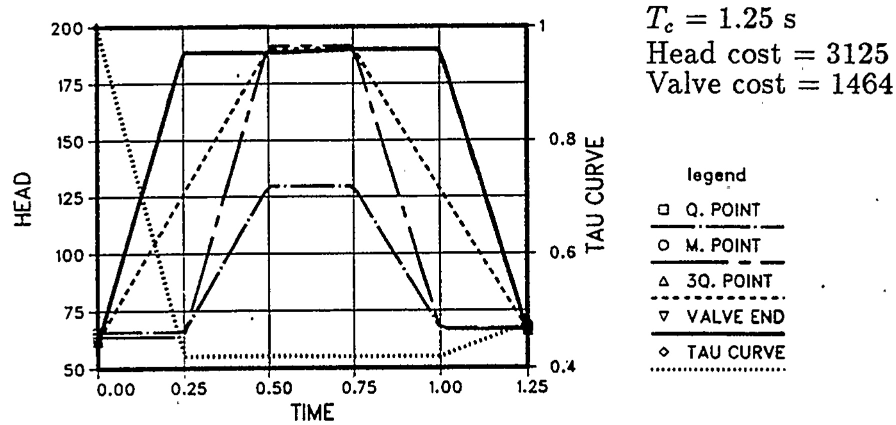


Figure 5.4: Optimal valve stroking—*head-time* plot. $H_r = 67.7$ m, $Q_0 = 2$ m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2. Costs in arbitrary monetary units.

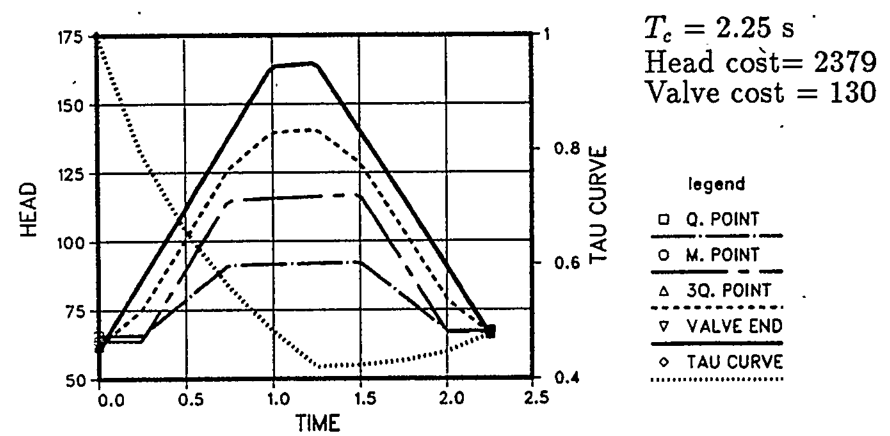


Figure 5.5: Optimal valve stroking—*head-time* plot. $H_r = 67.7$ m, $Q_0 = 2$ m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2. Costs in arbitrary monetary units.

$T_c = 3.25$ s
Head cost = 1885
Valve cost = 81

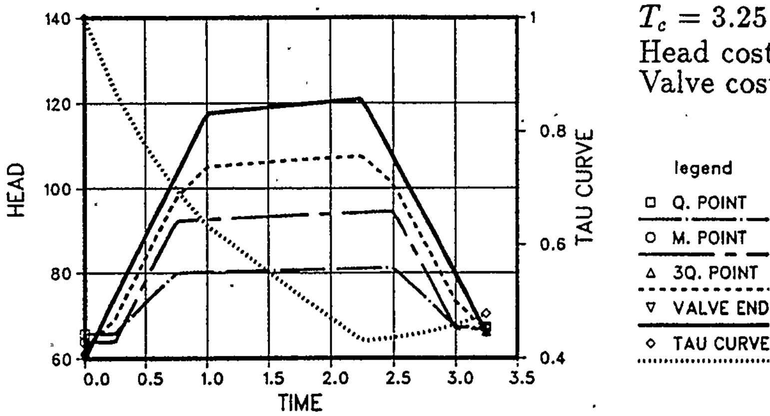


Figure 5.6: Optimal valve stroking—*head-time* plot. $H_r = 67.7$ m, $Q_0 = 2$ m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2. Costs in arbitrary monetary units.

$T_c = 5.25$ s
Head cost = 1594
Valve cost = 49

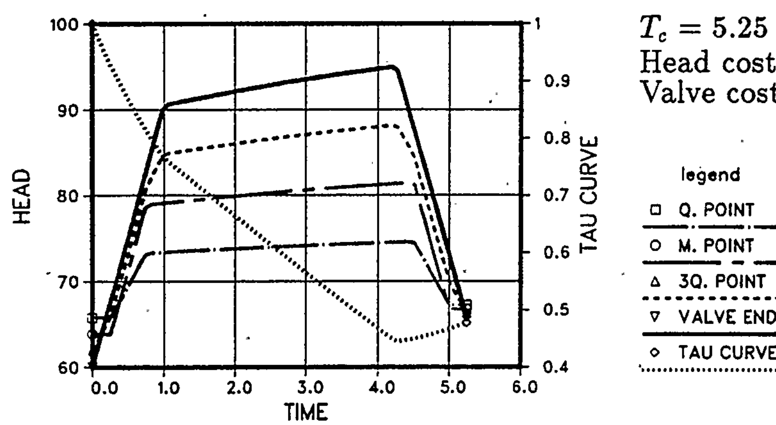


Figure 5.7: Optimal valve stroking—*head-time* plot. $H_r = 67.7$ m, $Q_0 =$X m^3/s, $Q_f = 1$ m^3/s, $D = 0.75$ m, $f = 0.010$, $a = 1100$ m/s and $L = 550$ m, number of pipe reaches = 2. Costs in arbitrary monetary units.

minimum stroking time of $2L/a + \Delta t$ seconds[5], then rapidly drops becoming very flat thereafter. This would seem to indicate that only for quite rapid valve motions does a cost term of this sort exert a significant influence on the overall behaviour of the objective function. It is justifiably arguable whether the proposed expression for the cost of the valve/controller/actuator is a realistic representation of the actual cost behavior of these system components. Regardless of this, the function demonstrates clearly that the valve stroking phenomenon is highly complex and cannot be thought of in oversimplified terms. It behaves in a manner which is not always in accordance with one's intuition and is not easily predictable. Figures 5.4 through 5.7 show the *head-time* curves and *tau* curves for various stroking times. In addition, the values of the head cost term and the valve system cost term are shown.

As an aside, it is worth mentioning that as the number of computational reaches increases, thereby producing a smaller time step, the accuracy of the velocity and acceleration approximations is improved. The valve system cost term can be quite sensitive to changes in the error of the numerical approximations depending on the values of the constant coefficients and exponents used in the cost function. Hence, one should pay strict attention to the size of the error terms generated by numerical approximations and select the number of pipe reaches, i. e., the time step, such that the accuracy of the approximations is consistent with the formulation of the cost term.

It is doubtful whether or not the complex valve motions required by the valve

[5] The Δt seconds of extra time is necessary to ensure that the change in fluid velocity at the reservoir does not occur instantaneously. Although this is theoretically possible, such an event cannot occur in fact.

stroking procedure can be both accurately and economically reproduced by actual valve systems. This also raises some important questions regarding the sensitivity of the valve stroking solutions to departures from the idealized valve closure arrangements. These issues will be addressed in some depth in the section on sensitivity analysis (Section 6). For now, it will be assumed that either the optimal tau curves can be practically achieved or that the ideal solution provides a quasi-optimal point which can then be modified by some means so that a practicable approximation to the solution can be implemented. This being the case, it is now possible to formally define the algorithm which constitutes the optimal solution procedure.

5.3 Prototype Model

The optimization problem can be seen to be reducible to a nonlinear, bivariate cost function with valve closure time, T_c, and pipe diameter, D_j, as its principal decision variables. The model which has been devised and used as a basis for all subsequent versions of **TOM** will be described in an algorithmic format. This form of the model has been coded as the Fortran program **TOM2**.

In the previous section, it has been discovered that the cost function is a nonlinear, three dimensional, continuous cost surface. Although the nonlinear methods outlined in Section 2.3 for functions of two or more variables could be applied to this problem, they are generally not very efficient and do not always guarantee that a global optimum will be found. It is important to realize as well that, although the cost expression is a continuous function of valve closure time, only discrete values of the stroking time may be used. This results from the fact that in order for solution by the method of characteristics to be possible, the closure time must some multiple of the time step. Naturally, by making the time step arbitrarily small, a closer approximation to a continuous function can be achieved. In practice, however, it might be computationally uneconomical to use such small time steps and a reasonable degree of accuracy can be obtained without recourse to minute increments of time. In any case, the size of the time step required to generate an adequate number of points on the tau curve will usually provide a reasonably "continuous" cost function.

Furthermore, as pipes are not normally available in a continuous range of sizes, the pipe diameter may be treated as a discrete variable thus simplifying the problem and permitting a more reliable and efficient method of solution to be used. Steady state requirements usually restrict the number of possible pipe diameters to a relatively narrow range. The customary approach in many pipeline optimization procedures is, therefore, to generate a list of candidate diameters which can each be treated as a separate optimization problem. Thus, a set of local optimal solutions can be generated using univariate techniques and the global optimum can be selected from these by inspection. Examination of the cost function given by Equation 5.1 and shown in Figure 5.3 reveals that it does not have a continuous first derivative. Gradient methods are unsuitable for the solution of such a function and recourse must be taken to univariate search techniques. One of the most efficient and reliable of these is the "Method of Golden Section." This procedure is described in Section 2.3.1 and a complete discussion of the method is provided by [Smith *et al*]. This technique, in a modified form which deals with the discretized nature of the cost function, has been used in the current model.

Figure 5.8 is a schematic depiction of the prototype optimization model. The procedure is summarized in the following algorithm[6].

Algorithm–1

Step 1 Input $Q_0, Q_f, H_r, L_i, z_i, f_i, a_i, n$, number of pipes i, objective function constant coefficients and exponents, and list of candidate diameters, D_j, where $j = 1, \ldots, m$.

Step 2 Adjust wavespeeds to maintain constant time step if more than one pipe exists in system. Set $\text{Cost}_{opt} = \infty$, initialize D_{opt} and T_c^{opt}.

Step 3 For D_j, $j = 1, \ldots, m$, do Steps 4–5.

Step 4 Compute steady state hydraulic grade lines for D_j and check feasibility and/or required head constraints.

- if H_{SS} is infeasible: $j = j + 1$, go to Step 3.
- if H_{SS} is feasible: continue.

Step 5 Use Method of Golden Section to find optimal stroking time, T_c^{opt}.

- if $\text{Cost}_{D_j} < \text{Cost}_{D_{j-1}}$, $\text{Cost}_{opt} = \text{Cost}_{D_j}$, $T_c^{opt} = T_c^j$, $D_{opt} = D_j$.

Step 6 Output Cost_{opt}, D_{opt} and T_c^{opt}.

---

[6] The subscripts and superscripts *opt* refer to the optimal values of their respective variables.

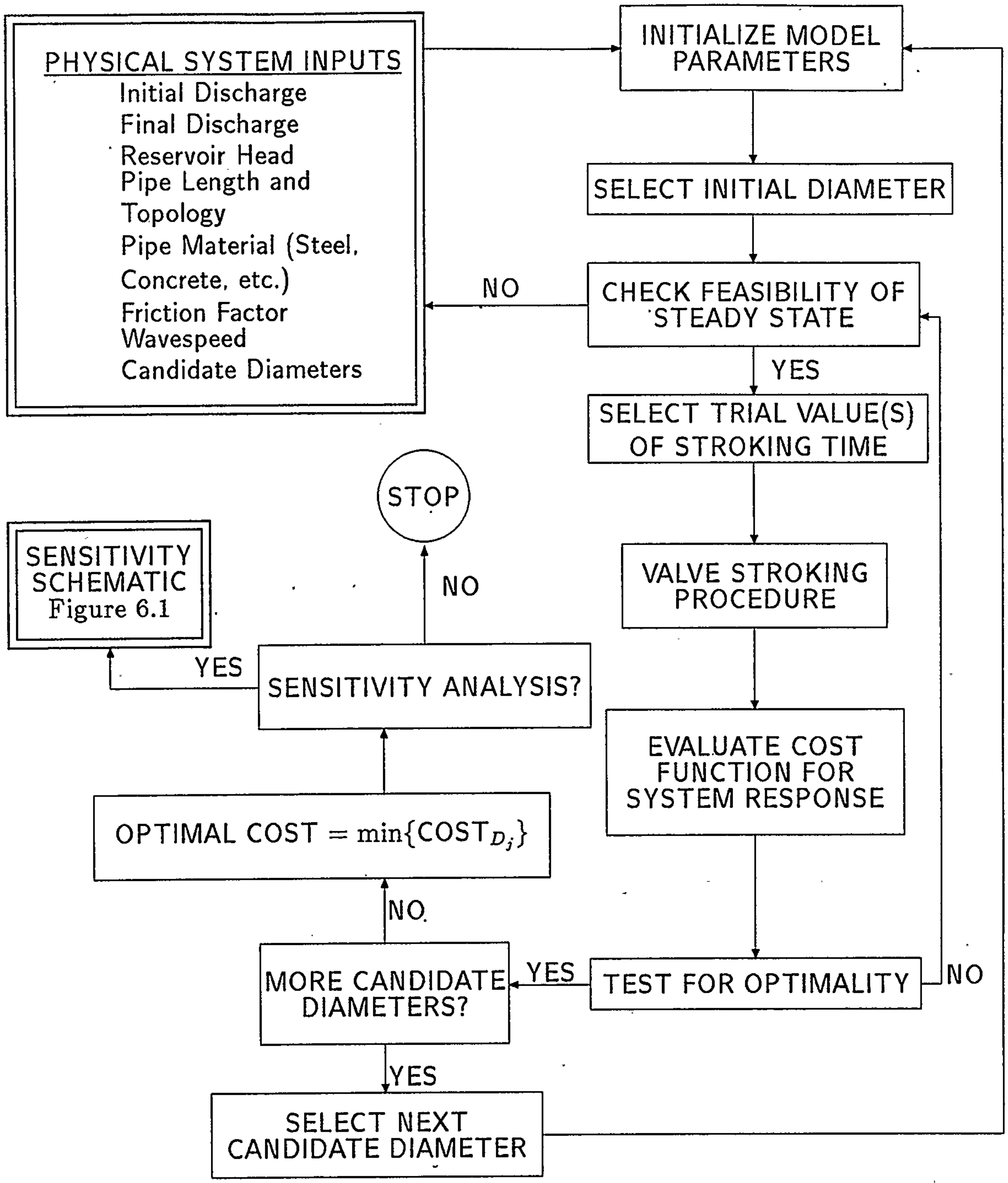


Figure 5.8: Schematic diagram of prototype optimization model

In words, the model functions in the following manner. The description of the physical system forms the input to the model. Constraints are incorporated implicitly by certain system inputs or explicitly as conditional switches in the Fortran program. For example, if a minimum downstream head of 30 m is required, the head at the valve produced by the steady state computations is tested to ensure that this restriction is satisfied for the pipe diameter being considered. If the steady flow conditions are found to be feasible, the program proceeds to locate the minimum cost solution for that candidate diameter using the Method of Golden Section. Each candidate diameter is processed in this fashion with the improved local optimal values of the objective function and the valve closure time being updated after each trial. Once the complete list of feasible diameters has been analysed the optimal values of the cost, pipeline diameter and stroking time are known.

It is appropriate to include the Method of Golden Section algorithm here also since the details of its implementation are different from those ordinarily employed due to discretization of the cost function. This algorithm comprises Step 5 of Algorithm 1.

Algorithm–2

Step 1 Input as in Step 1 of Algorithm 1 and tolerance[7] ϵ.

[7] Ordinarily the value of ϵ will be equal to the time step.

Step 2 Set the following:

Fibonacci factors	$F1 = 1.6180339$	
	$F2 = 0.6180339$	
Function	$\mathcal{FT} = \left\lfloor \frac{F\,T_c}{\Delta t} \right\rfloor \Delta t$	*($\lfloor R \rfloor$ denotes integer part of R)*
Intervals	$\bar{T}_G = 2L/a - \Delta t$	*(initial coarse search interval)*
	$\bar{T}_1 = \bar{T}_G$	
	$\bar{T}_2 = \mathcal{FT}\{F1, \bar{T}_1\}$	
Times	$T_1 = 2L/a + \Delta t$	
	$T_2 = T_1 + \bar{T}_G$	

Step 3 Evaluate objective function (Equation 5.1) to obtain $Cost_{T_1}$, $Cost_{T_2}$.

Step 4 Set $T_4 = T_2 + \bar{T}_2$ and compute $Cost_{T_4}$. *(Start coarse search procedure.)*

- While $Cost_{T_4} < Cost_{T_2}$: *(i. e., while function is decreasing)*

  Set $T_1 = T_2$

  $T_2 = T_4$

  $Cost_{T_1} = Cost_{T_2}$

  $Cost_{T_2} = Cost_{T_4}$

  $\bar{T}_1 = \bar{T}_2$

  $\bar{T}_2 = \mathcal{FT}\{F1, \bar{T}_2\}$

Step 5 Set $\bar{T}_2 = \bar{T}_1$ *(initialize search procedure)*

$\bar{T}_1 = \mathcal{FT}\{F2, \bar{T}_2\}$

$\bar{T}_3 = T_4 - \bar{T}_2$

Compute $Cost_{T_3}$

Step 6 While $\bar{T}_1 > \epsilon$:

- If $Cost_{T_3} < Cost_{T_2}$, Discard segment $\widehat{\bar{T}_1\bar{T}_2}$ by setting

  $\bar{T}_1 = \bar{T}_2$

  $\bar{T}_2 = \bar{T}_3$

  $Cost_{T_1} = Cost_{T_2}$

  $Cost_{T_2} = Cost_{T_3}$

  Go to Step 5.

- Else discard segment $\widehat{\bar{T}_3\bar{T}_4}$ by setting

  $\bar{T}_4 = \bar{T}_3$

  $\bar{T}_3 = \bar{T}_2$

  $Cost_{T_4} = Cost_{T_3}$

  $Cost_{T_3} = Cost_{T_2}$

  $\bar{T}_2 = \bar{T}_1$

  $T_2 = T_1 + \bar{T}_1$

  $\bar{T}_1 = \mathcal{FT}\{F2, \bar{T}_1\}$

  Compute $Cost_{T_2}$

Step 7 Compare the final two function values and select the lower one.

- If $Cost_{T_2} < Cost_{T_3}$, then set $Cost_{opt} = Cost_{T_2}$, $T_c^{opt} = T_2$.

- Else set $Cost_{opt} = Cost_{T_3}$, $T_c^{opt} = T_3$.

Step 8 Output optimum cost, $Cost_{opt}$, and optimal stroking time, T_c^{opt}.

STOP *(procedure completed successfully)*

Diameter D_j (m)	Stroking Time T_c (seconds)							
	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50	5130	4580	4077	3860	3791*	3847	4058	4501
0.75	7182	6727	6293	6113	6066	6137	6359	6811
1.00	9691	9297	8919	8767	8737	8820	9050	9508
1.50	15920	15600	15290	15180	15170	15270	15510	15970

Table 5.3: Objective function costs (arbitrary units) by enumeration. H_r = 67.7 m, Q_0 =2 m^3/s, Q_f = 1 m^3/s, f = 0.010, a = 1100 m/s and L = 550 m, number of pipe reaches = 2.

The accuracy and validity of the solutions produced by this version of the model (program **TOM2**) have been tested by enumerating all feasible solutions for a given system and comparing the results to the solution predicted by the model. In every case the same optimal point results. Table 5.3 shows a summary of the results produced by enumeration of feasible solutions. The output from **TOM2** is listed below.

```
                OPTIMAL VALVE STROKING

NUMBER OF PIPES =  1
NUMBER OF REACHES ON LAST PIPE =  2
INITIAL STEADY STATE DISCHARGE =  2.000
FINAL STEADY STATE DISCHARGE =  1.000
RESERVOIR HEAD  = 67.700
TIME OF TRANSIENT COMPUTATION=  6.00
TIME STEP FOR STORING HEAD CHANGE DURING TRANSIENT = 0.0 SEC.

PIPE NO     LENGTH     WAVE VEL.     FRICTION FACTOR
             ( m)      ( m/s )

    1        550.00     1100.0            0.010


***** PIPE TO BE STROKED FIRST IS PIPE  1 *****

  DIAMETER IS  0.50 m.
```

```
PIPE NO      ADJUSTED WAVE VEL      NO. OF REACHES
                  ( m/s )
   1              1100.0                  2

  COST =  3790.7               TSTROKE = 3.50


  DIAMETER IS  0.75 m.

PIPE NO      ADJUSTED WAVE VEL      NO. OF REACHES
                  ( m/s )
   1              1100.0                  2

  COST =  6065.7               TSTROKE = 3.50


  DIAMETER IS  1.00 m.

PIPE NO      ADJUSTED WAVE VEL      NO. OF REACHES
                  ( m/s )
   1              1100.0                  2

  COST =  8737.0               TSTROKE = 3.50


  DIAMETER IS  1.50 m.

PIPE NO      ADJUSTED WAVE VEL      NO. OF REACHES
                  ( m/s )
   1              1100.0                  2

  COST = 15160.0               TSTROKE = 3.25

  ***************************************************

  OPTIMAL COST =   3790.7 FOR STROKING TIME OF  3.50 s
                          AND A DIAMETER OF  0.5 m.
```

The optimal solution by inspection of Table 5.3 is marked by an asterisk. It can be seen that this value agrees with that produced by the model **TOM2**. It is instructive to note that the optimal solution occurs for the smallest feasible diameter in the list of candidate pipe sizes. This is not surprising if one recalls the relationships between pipe diameter, wall thickness, hoop stress and internal pressure, which were described in Section 4.2.2. The inclusion of a steady state power cost term will force the solutions towards larger diameter pipes if the power costs in the system are significant.

In summary, the model **TOM2** has been been shown to be mathematically well formulated and encoded, and the solutions it produces appear to be valid within the context of the model. However, it can well be argued that the idealized nature of the solution, ideal, that is, with respect to the solved-for boundary condition, may be unattainable in practical terms. In many cases this is likely to be true since the valve stroking procedure knows no restrictions in terms of realistic valve motions and therefore often produces optimal tau curves of a complex and impractical nature. Interestingly enough, in the majority of the physical systems examined, the optimal closure arrangements have been found to reasonably conform to certain readily obtainable, "off-the-shelf" valve closures. This suggests an alternate approach to the optimization process in which the ideal boundary condition is replaced by some approximate form of the closure arrangement which gives the best system performance subject to the constraints of the problem. This topic is dealt with in the following section.

5.4 Best Fit Model

During the course of the investigation it was observed that most of the optimal tau curves generated by the model **TOM2** could be reasonably approximated by either one of two sorts of common valve closure arrangements—*equal percentage* or *bilinear* closures. This prompted the development of another version of the model which has been encoded in the Fortran 77 program **TOM3**.

The reasoning underlying this particular formulation is as follows. Since the cost term (Equation 4.12) poses some difficulty in terms of its ability to accurately represent the true cost of a valve system required to faithfully reproduce the idealized boundary condition, it is logical to remove it from the cost function altogether. The effect on the behaviour of the cost function is not extreme as can be seen by comparing Figure 5.9 with Figure 5.3, which have been produced from identical physical systems. The only difference between the two is that the valve system cost term has been removed from the cost function of Figure 5.9.

In light of the fact that whatever optimal tau curve the model generates is likely to be complex, and therefore uneconomic, a more practical suboptimal solution can perhaps be obtained by simply fitting the optimal closure curve with an easily achievable, less costly tau curve. Except in cases where residual heads are of the utmost importance, i. e., a high degree of transient control is required in reaching and maintaining the new flow conditions, satisfactory system performance can be provided by a "best fit" closure.

Thus, this version of the model optimizes the physical system as before, but without regard to the valve system cost. Having found a solution which is still op-

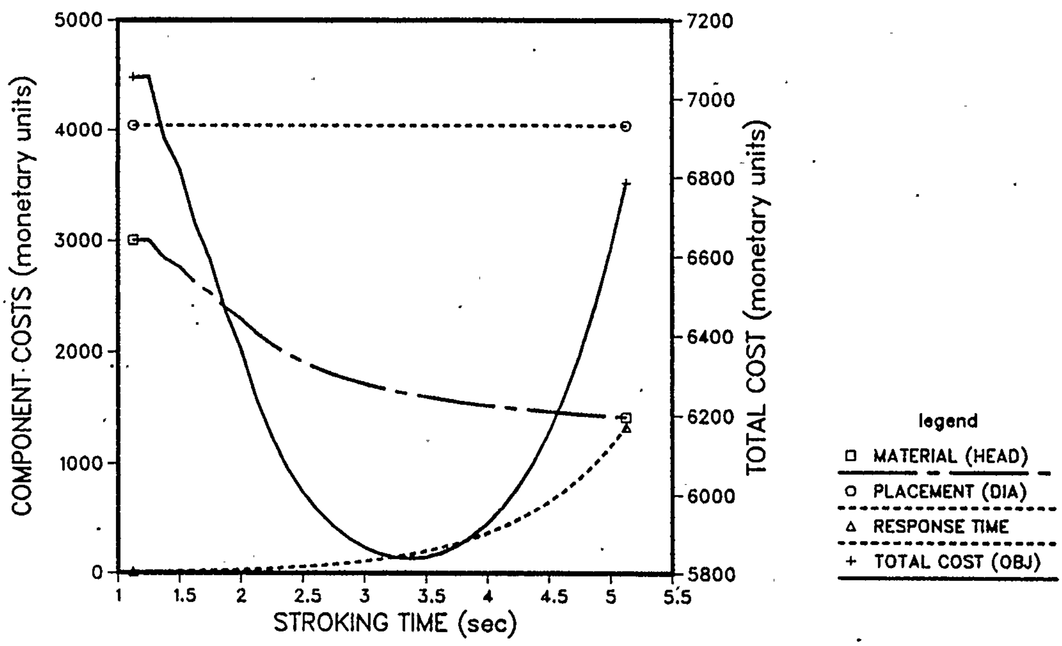


Figure 5.9: Objective function cost terms without valve system. H_r = 67.7 m, Q_0 =2 m^3/s, Q_f = 1 m^3/s, D = 0.75 m, f = 0.010, a = 1100 m/s and L = 550 m, number of pipe reaches = 2.

timal in all other respects, the model simply fits the optimal tau curve with various practical closures and assesses the cost of each alternative. The fitted tau curves are used, along with the relevant system data, as the input for a program (subroutine) which performs ordinary method of characteristics time series simulation and which produces the new system response as its output. Using this information the objective function can be reevaluated. The details of the procedures developed for fitting different types of valve closures to the optimal tau curves are described in subsequent sections.

Some general comments which apply to all of the fitting techniques developed in this investigation can be made. The fitting process is only applied to the portion of the tau curve occuring before the final $2L/a$ seconds. The last $2L/a$ seconds of the closure curve are simply replaced with a straight line from the fitted value of τ at $T_c - 2L/a$ seconds to the final value of τ. The fitting procedures are simplified by the fact that the initial and final values of tau must take their original values in order to produce the initial and final flow conditions.

Figure 5.10 shows the *head-time* curves and the tau curve for a valve stroked system. The various fitted tau curve results are displayed in Figures 5.12 through 5.16 for comparison. For reference purposes, Table 5.4 provides the actual numerical values of the tau curve to be fit. It is prudent at this point to refrain from attempting to make any generalizations regarding the nature and performance of the various fitted boundary conditions since the results for each type of fit can differ considerably depending on the values of the system parameters ρ, H_r, T_c, Q_0 and Q_f, f, a, L and D_i (see Figures B.1 through B.14). As well, it should be stated that, although a correlation coefficient was derived and examined during the

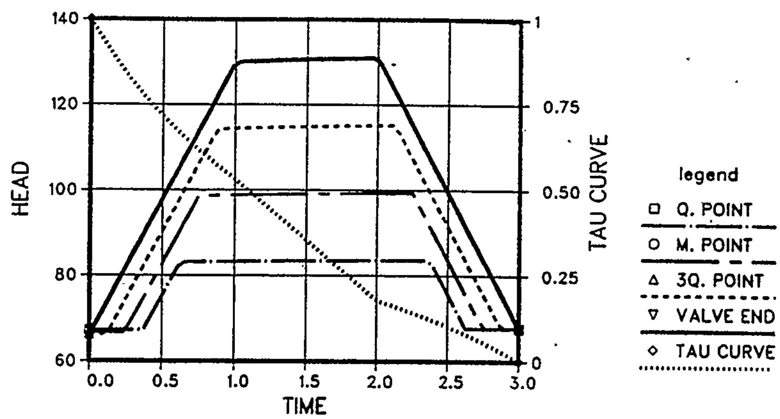


Figure 5.10: Valve stroked system response. $H_r = 67.7$ m, $D_{opt} = 0.75$ m, $T_c^{opt} = 3$ s, $Q_0 = 1$ m^3/s, $Q_f = 0$ m^3/s, $a = 1100$ m/s, $f = 0.010$, $L = 550$ m, number of pipe reaches = 8. All heads in meters, time in seconds.

Time (s)	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Tau	1.000	0.840	0.717	0.617	0.533	0.433	0.354
TIME (s)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Tau	0.265	0.177	0.141	0.102	0.055	0.000	0.000

Table 5.4: Tau values for valve stroked system.

investigation, it did not provide a good indication of the system response, particularly for the minimum head fitting procedures. This is not surprising considering Ruus's statement [B. W. Karney, pers. comm.] that deviations from the optimum tau values during the final $2L/a$ seconds of closure affect the head rise far more than deviations prior to the last $2L/a$ seconds. It was found that the ability of the fitted boundary condition to reproduce the ideal system response could be better measured by dimensionless head rise parameters. These are described in detail in the section on sensitivity analysis and, as little benefit is to be had by repeating that discussion here, no further mention of them shall be made in the current section.

5.4.1 Equal Percentage Fits

The equal percentage closure has been coined the "poor man's optimum closure." It can be approximately produced by a V-notch ball valve using a linear driver. Figure 5.11 shows a number of equal percentage closure curves all having the same time of closure. An expression describing the manner in which tau varies with time during an equal percentage closure is

$$\tau = 10^{m(T/T_c)} \tag{5.2}$$

where m is a negative exponential parameter and the other quantities have been previously defined. The parameter m is a function of h_f, t_c, ρ, and Δh_{max}, as well as whether or not the closure is from a full or partial valve opening. Note that the equal percentage curves can only approach the final value of τ asymtotically. For this reason a linear portion is substituted during the final $2L/a$ seconds of

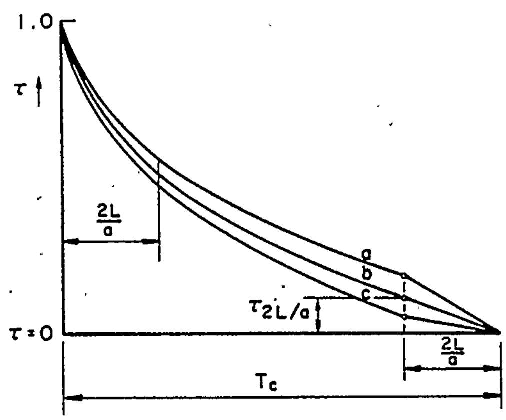


Figure 5.11: Equal percentage closure curves. (Excerpted from [Karney/Ruus 1985]).

the closure time. For a more complete discussion of equal percentage closures the interested reader is referred to [Karney/Ruus 1985].

For the purposes of the model **TOM3**, the object of the equal percentage fitting procedures is to determine the value of the exponential factor m. Two different equal pecentage fitting methods have been used—least squares regression fitting and minimum head (equal peak) fitting.

Equal Percentage Regression Fit

In dealing with regression analysis it simplifies matters a great deal if natural logarithms are used. In the following discussion the exponential factor m is converted to its equivalent natural logarithmic value by

$$m' = \frac{m}{2.3026}. \tag{5.3}$$

Hence, the expression for tau now becomes

$$\tau = e^{m'(T/T_c)}. \tag{5.4}$$

It is very often the case in least squares regression that no exact solution to the normal equation(s) exists when the approximating function is an exponential relation. This is true of Equation 5.4 for which the expression for the error sum of squares is

$$E = \sum_{i=1}^{i=n} \left(\tau_i - e^{m'(T_i/T_c)}\right)^2 \tag{5.5}$$

where n is the number of time grid points over which the tau function is fit, i.e.,

$$n = \frac{T_c - 2L/a}{\Delta t} + 1.$$

It is trivial to show that the normal equation is

$$\frac{dE}{dm'} = 0 = 2\sum_{i=1}^{i=n} \left(\tau_i - e^{m'(T_i/T_c)}\right)\left(-(T_i/T_c)e^{m'(T_i/T_c)}\right) \tag{5.6}$$

for which no exact solution can be found. It is, however, possible, to obtain an approximate solution to the problem by taking the natural logarithm of Equation 5.4,

$$\ln \tau_i = m'\left(\frac{T_i}{T_c}\right) \tag{5.7}$$

for which the solution to the normal equations is

$$m' = \frac{\sum_{i=1}^{i=n}\left((T_i/T_c)\ln \tau_i\right)}{\sum_{i=1}^{i=n}(T_i/T_c)^2}. \tag{5.8}$$

A further refinement has been added in the model **TOM3** by using the Newton-Raphson procedure to improve upon the least squares estimate of the value of m'.

Time (s)	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Tau	1.000	0.841	0.708	0.595	0.501	0.421	0.354
TIME (s)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Tau	0.298	0.251	0.188	0.125	0.063	0.000	0.000

Table 5.5: Tau values for least squares regression fit.

The best value of m' in terms of minimizing the error sum of squares will be a zero of Equation 5.6. If we let Equation 5.6 be denoted by $\mathcal{F}$, then

$$\mathcal{F}' = \sum_{i=1}^{i=n} \left(\frac{T_i}{T_c}\right)^2 T_i e^{m'(T_i/T_c)} + 2\sum_{i=1}^{i=n} \left(\frac{T_i}{T_c}\right)^2 e^{2m'(T_i/T_c)} \tag{5.9}$$

is the first derivative of $\mathcal{F}$. Thus, the procedure is simply to iterate by

$$m'_{k+1} = m'_k - \frac{\mathcal{F}(m'_k)}{\mathcal{F}'(m'_k)} \tag{5.10}$$

until a suitable degree of accuracy has been reached. The initial estimate of m' is provided by the least squares approximation. The final step in the process is to convert m' to m by Equation 5.3.

Figure 5.12 shows the *head-time* plot and tau curve for the least squares regression fit of the optimum system of Figure 5.10. Tau values are given in Table 5.5.

Minimum Head (Equal Peak) Fit

The rationale for this particular fitting procedure is not difficult to comprehend if the discussion in Section 1 regarding the relationship between the impulse applied to the fluid and the area under the *head-time* curve is recalled (see Equation 1.4). At that time it was postulated that the shape of the *head-time* profile which minimized the system head rise would be that which most closely resembled a rectangle.

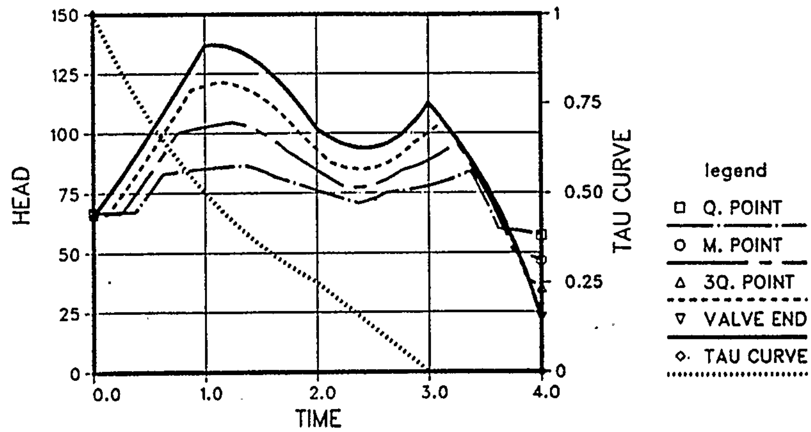


Figure 5.12: System response for equal percentage regression fit. H_r = 67.7 m, D_{opt} = 0.75 m, T_c^{opt} = 3 s, Q_0 = 1 m^3/s, Q_f = 0 m^3/s, a = 1100 m/s, f = 0.010, L = 550 m, number of pipe reaches = 8. Head is in meters, time in seconds.

Examination of Figures 5.13 and 5.16 show that this is indeed the case. Moreover, it can be seen that by maintaining the head at or near its maximum value until the valve motion is complete means that some of the impulse is still being applied beyond the valve closure time. Consequently, an, often, even lower value of the head rise can be achieved by this type of closure than can be obtained by the valve stroking procedure itself. This may seem surprising since valve stroking has always been thought of as guaranteeing the lowest possible head rise for a given system undergoing a change in flow conditions. However, the valve stroking method has placed upon it the additional restriction that *no* residual transients occur once the valve motion has ceased. Since the minimum head fitting procedure is not subject to this constraint, it is sometimes possible to reduce the maximum head

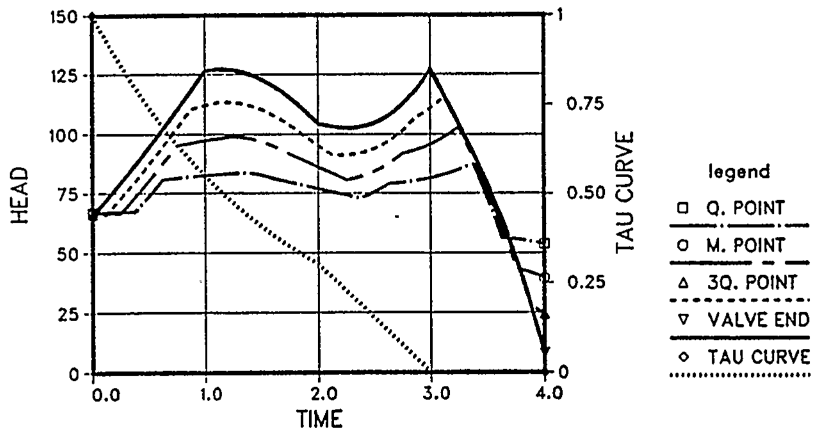


Figure 5.13: System response for equal percentage minimum head fit. $H_r = 67.7$ m, $D_{opt} = 0.75$ m, $T_c^{opt} = 3$ s, $Q_0 = 1$ m^3/s, $Q_f = 0$ m^3/s, $a = 1100$ m/s, $f = 0.010$, $L = 550$ m, number of pipe reaches = 8. Head is in meters, time in seconds

Time (s)	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Tau	1.000	0.861	0.741	0.638	0.549	0.473	0.407
TIME (s)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Tau	0.350	0.301	0.226	0.151	0.075	0.000	0.000

Table 5.6: Tau values for equal percentage minimum head fit.

to a value below the valve stroking head rise. For systems in which the prospect of some residual transient behaviour is not crucial, this type of closure may offer a superior alternative. These comments apply equally to any type of minimum head fit including the bilinear tau curve described in Section 5.4.2.

The minimum head exponential parameter is obtained using the standard Method of Golden Section described in Section 2.3.1. The dependent variable is the maximum system head and the independent quantity is the factor m. The trial value of

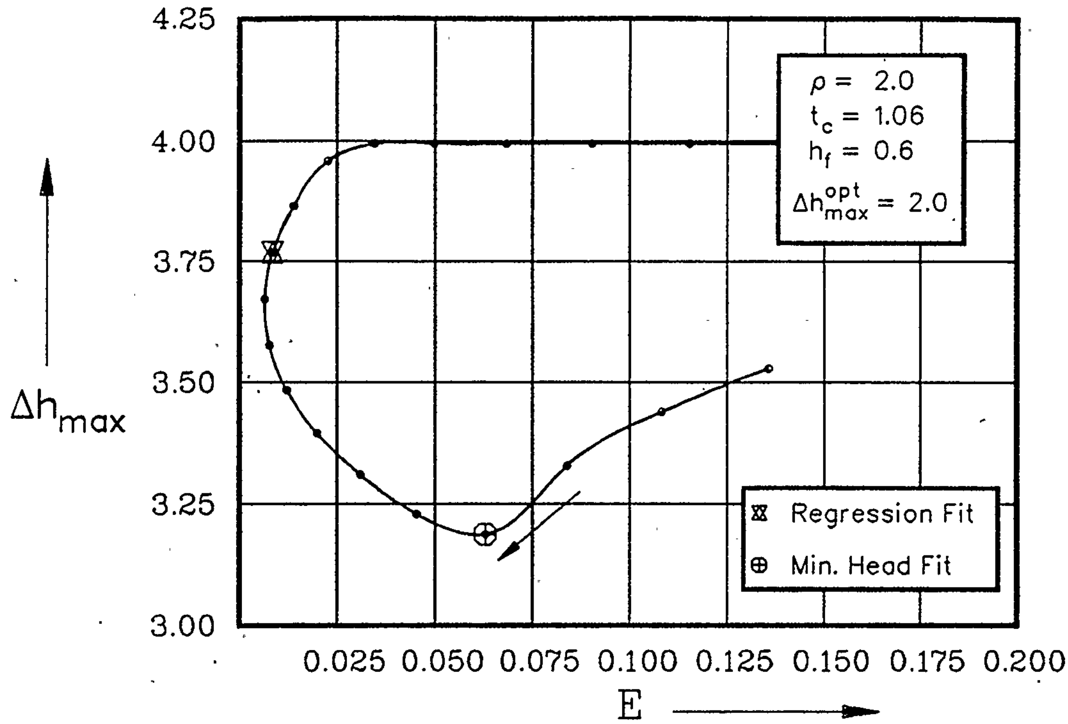


Figure 5.14: Relationship between the exponential parameter m and the system head rise.

m is used to produce the tau curve input for a subroutine which performs ordinary method of characteristics time series simulation and generates the system response. The quantity to be minimized is the maximum head rise. Figure 5.14 shows a plot of the maximum head rise, Δh_{max}, and the error sum of squares, E, for different values of m. The arrow indicates the direction of decreasing value of m. Each point on the curve represents a change of a constant Δm from neighbouring points. It is easy to see from this diagram that a single critical point exists with respect to m corresponding to a minimum value of the head rise. The existence of a single critical value of m for the minimum head and another for the minimum error, E, has been observed in every case that was studied. The points corresponding to the values of m associated with least squares and minimum head fits are marked with symbols. The starting values of m and Δh_{max} are provided by the least squares regression parameters. A suitable value for the increment in m for conducting the coarse search was found to be $m/5$.

Figure 5.14 and others like it will be discussed in more depth in the section on sensitivity methods. It has been included here only to demonstrate the relationship between the exponential factor m and the system head rise.

5.4.2 Bilinear Fits

The bilinear fit, as the name suggests, simply splits the tau curve into two linear portions. The first linear portion extends from $0 \le t \le T_c - 2L/a$ seconds. The second segment obtains from $T_c - 2L/a < t \le T_c$. The form of the approximating equations are

$$\tau = \begin{cases} a_1 t + \tau_0 & \text{if } 0 \le t \le T_c - 2L/a \\ a_2 \Delta t + \tau_{t'} & \text{if } T_c - 2L/a < t \le T_c \end{cases} \tag{5.11}$$

where

$$\tau_{t'} = a_1 t' + \tau_0, \quad (t' = T_c - 2L/a) \quad \text{and} \quad a_2 = \frac{\tau_f - \tau_{t'}}{2L/a}.$$

The fitting procedures determine the value of a_1 and once this is known the parameters for the last segment may be calculated.

Least Squares Regression Fit

The regression equations for the bilinear fit are very simple.

$$a_1 = \frac{\sum_{i=1}^{i=n}(T_i/T_c)\tau_i - \sum_{i=1}^{i=n}(T_i/T_c)}{\sum_{i=1}^{i=n}(T_i/T_c)^2} \tag{5.12}$$

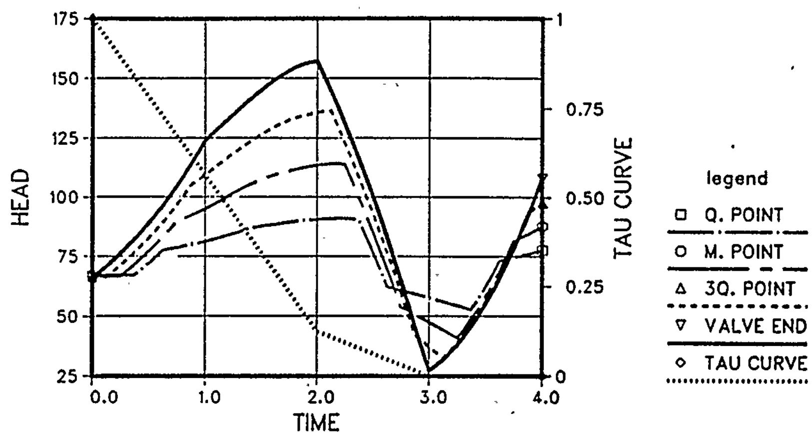


Figure 5.15: System response for least squares bilinear regression fit. $H_r = 67.7$ m, $D_{opt} = 0.75$ m, $T_c^{opt} = 3$ s, $Q_0 = 1$ m^3/s, $Q_f = 0$ m^3/s, $a = 1100$ m/s, $f = 0.010$, $L = 550$ m, number of pipe reaches = 8. Head is in meters, time in seconds.

Time (s)	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Tau	1.000	0.891	0.781	0.672	0.563	0.453	0.344
TIME (s)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Tau	0.235	0.125	0.094	0.063	0.031	0.000	0.000

Table 5.7: Tau values for least squares bilinear regression fit.

Since no transformations are involved in the approximating function, these equations are exact. Figure 5.15 shows the response of the system of Figure 5.10 to the bilinear regression tau curve. Again, for reference, Table 5.7 gives the actual tau values for this fit.

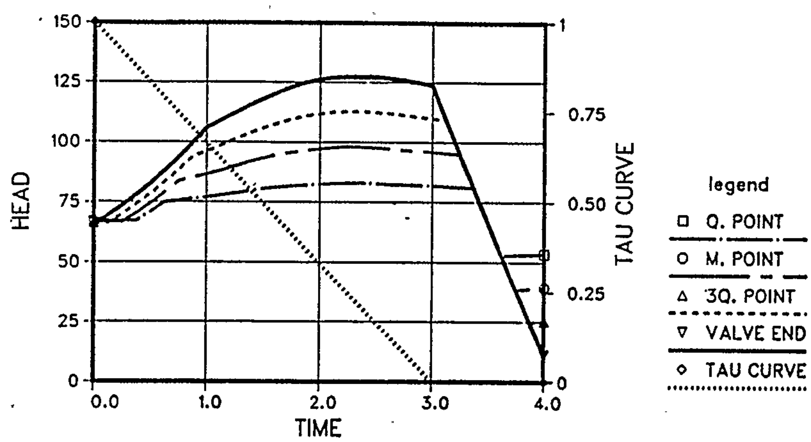


Figure 5.16: System response for minimum head bilinear fit. $H_r = 67.7$ m, $D_{opt} = 0.75$ m, $T_c^{opt} = 3$ s, $Q_0 = 1$ m^3/s, $Q_f = 0$ m^3/s, $a = 1100$ m/s, $f = 0.010$, $L = 550$ m, number of pipe reaches = 8. Head is in meters, time in seconds.

Time (s)	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Tau	1.000	0.916	0.832	0.748	0.664	0.580	0.495
TIME (s)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Tau	0.411	0.327	0.245	0.164	0.082	0.000	0.000

Table 5.8: Tau values for minimum head bilinear fit.

Minimum Head Bilinear Fit

The procedure for the bilinear minimum head fit is analogous to that for the equal percentage one. The only difference lies in the equations that are used for generating the tau values, which are given as Equation 5.11.

Figure 5.16 and Table 5.8 show the details of the system response and tau curve for the bilinear minimum head fit of the system shown in Figure 5.10.

5.5 Extensions for Multi-diameter Pipelines

Although the developments outlined in the previous sections have been restricted to a single diameter pipeline, the procedures can be extended to systems containing pipes of different diameters.

For the general case of n possible pipe diameters and pipe segment lengths, the solution space has $n^2 + 1$ dimensions, the extra decision variable being the response time of the system. The direct search techniques described in Section 2.3.2 provide a suitable solution method for this general problem. By using the linear programming methods outlined in Section 2.2.1, an excellent approximation to the solution can be obtained based on the steady state characteristics of the system. This solution may then be used as a starting point for the nonlinear technique.

A simpler, but not unrealistic, multi-diameter problem involves a pipeline system in which the pipe diameters have a fixed relationship. For example, a pipeline may be composed of n pipes where the diameters are related by an expression of the following kind.

$$D_{i+1} = kD_i \qquad \text{where } k \text{ is a constant factor.} \tag{5.13}$$

The solution to a problem of this sort can readily be obtained by the univariate methods proposed in this dissertation.

5.6 Summary

In taking the conceptual, mathematical model of Section 4 from its neophyte stages, and by virtue of the investigations conducted, through an evolutionary process which allowed the development of a collection of models providing a flexible tool with which the designer/analyst can interact, it is evident that the outcome of the study has been a *heuristic* model(s). The "larger" model embodies a number of similar sub-models, each of which incorporates a specific approach to the problem. It also has within it several layers of analytic suboptimization procedures. The basic analytic model rests on the assumption that provided the solution can be accomplished by the implementation of idealized boundary condition behaviour, a "true" optimal solution exists and can be found by a mathematical process. If the problem is such that this may or may not be true, as is more likely to be the case, additional rules and procedures (for example, the fitted boundary condition solution(s)) may be utilized by means of which other suboptimal solutions can be discovered. Obviously, as knowledge of the processes (and their problems) by which solutions, optimal, suboptimal, or even feasible, are sought increases, more appropriate rules and procedures will be developed. For instance, simply because the valve stroking procedure allows one to arrive at an ideally optimal solution, does not necessarily mean that this is the best possible point for selecting an approximate solution. Perhaps each valve stroking solution could be fit by any number of practicable approximating closure arrangements so that a number of equally good nonideal solutions might be established. It is not difficult to postulate other variations that might prove to be valuable techniques for locating better solutions in optimizing pipeline designs while incorporating transient phenomena.

One final aspect of the optimization process needs to be addressed—sensitivity analysis. Without the means to assess the impact on the model solutions, of changes in the cost function, the constraints or their associated parameters and variables, the utility of such a model is not particularly viable. The world is a highly dynamic place and decisions made on the basis of static information are likely to be unsound. The following section describes some of the procedures and techniques developed to provide answers to the question, "What if...?"

Chapter 6

Sensitivity Analysis

No optimization process is complete without methods for assessing the sensitivity of the model solutions to changes in the parameters, constraints or the objective function. Not only do the actual conditions influencing the problem change but there is always some inaccuracy, uncertainty, or even error, introduced whenever a physical phenomenon is described in purely mathematical terms. In order to evaluate their significance, in terms of providing information for decision making, some means of exploring the ramifications of such changes is mandatory. In this section, a number of techniques and ideas for acquiring sensitivity information will be described. A general account of sensitivity analysis procedures is given, followed by explanations of the more specific methods and the situations to which they may apply.

6.1 General Sensitivity Procedures

In dealing with complex, integrated analytic or heuristic optimization models, it is difficult to find rigorous, mathematical techniques—such as exist for linear programming models—for conducting sensitivity studies. More often, direct, iterative, interactive methods must be used in order to extract information regarding the effects of changing system conditions on the model solutions. Figure 6.1 shows the interrelationships of the various sensitivity elements devised in this investigation.

It can be readily seen that the sensitivity process comprises many individual components interacting with one another in a complex fashion. Various parts and paths may be followed depending upon the nature of the problem and the type of sensitivity information being sought.

The most direct approach to sensitivity analysis constitutes the top loop of the schematic in Figure 6.1, in which one simply makes whatever changes are desired to the model itself and produces a new solution for the modified system. The degree of departure from the original solution indicates how much the original solution has been affected by the alteration of the system[1]. This is appropriate if the changes affect only the values of the decision variables and the objective function. In other words, when the objective function, the constraints or their parameters have been modified.

A similar procedure can be applied if changes are made to the optimal values of the decision variables and the parameters which influence them. For example, if the idealized boundary condition solution is altered in some manner, the modified tau curve can be used as input for the ordinary method of characteristics time series simulation to generate a new system response. The information generated from this procedure can be used to assess the changes in the cost function or the validity of constraints, etc..

The sensitivity procedures outlined in the schematic also show three additional processes—stochastic treatment of closure curves, equivalent system approximations and variation of the "best-fit" tau curve parameters. These are techniques

[1]In Figure 6.1 the arrows entering the OPTIMIZATION MODEL box generally indicate that the objective function is to be reevaluated.

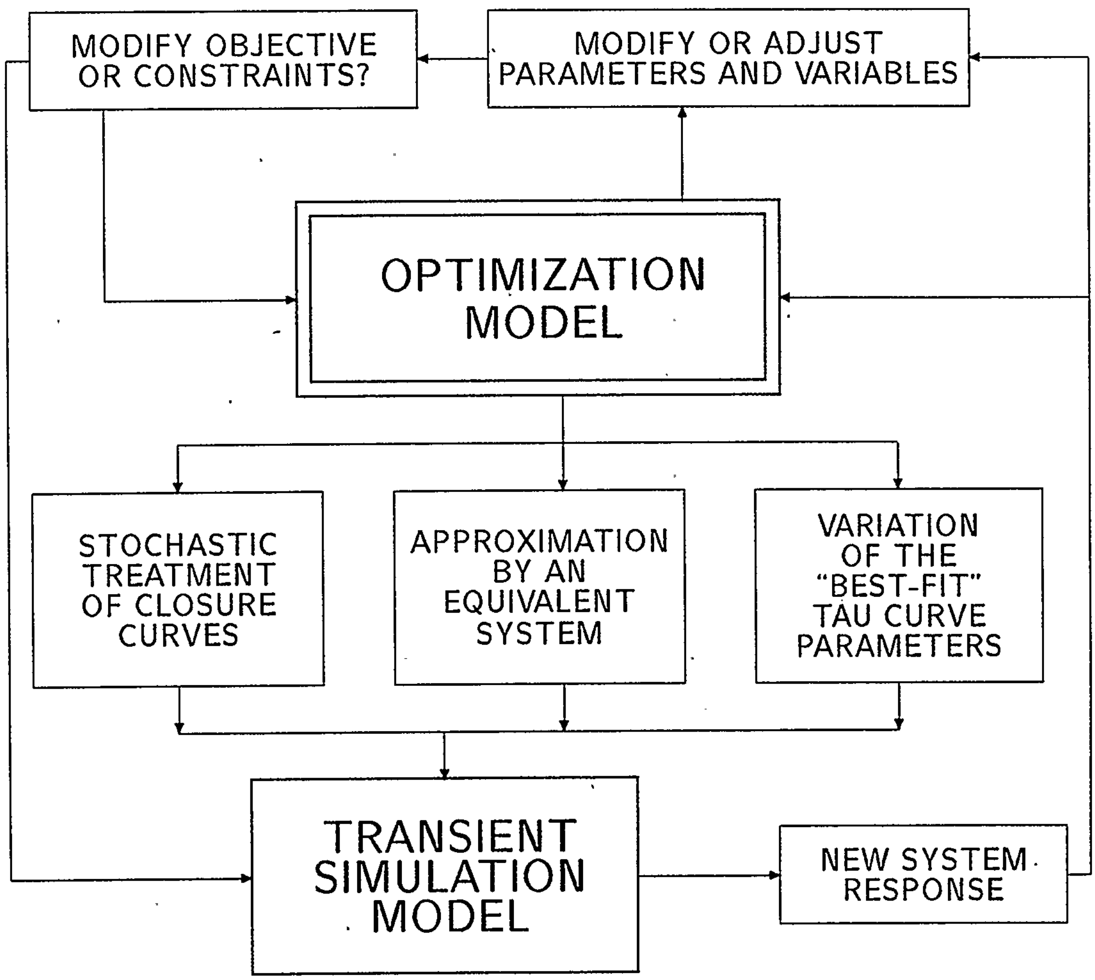


Figure 6.1: Schematic diagram of components of sensitivity analysis.

intended to allow a more specific type of sensitivity analysis pertaining to the cost and hydraulic performance of the system to be analysed. The following sections discuss each in turn.

6.2 Stochastic Treatment of Valve Closure Curves

In practice, no tau curve can be executed with perfect accuracy because of either the physical limitations of the valve system or anomalies introduced by power fluctuations, maintenance problems and other unforeseen or unpredictable events. For many problems of this nature, random departures of the valve system from its ideal or anticipated behaviour can be expected to occur. How do these discrepancies affect the system performance? A simple procedure for evaluating minor, random flucuations in the tau curve would be to simulate the system response using ordinary method of characteristics time series simulation. The tau curve can be subjected to some form of stochastic treatment and the deviant tau curves used as the input for the simulation procedure.

This approach is appropriate for installations in which a high degree of control is required for the safe and proper functioning of the system. Generally, however, it is more important to determine by how much the closure arrangement can be varied before the solution becomes nonoptimal. The general procedures given in the previous section are more suitable for obtaining this kind of information.

6.3 Approximation by an Equivalent System

Very often, in conducting transient analyses, it happens that it is convenient or necessary to use a simplified representation of the pipeline system. Many factors, such as bedding or anchoring conditions, frictional resistance, etc., are not known with certainty. Important parameters, wavespeed for instance, and hence the system behaviour, can be affected by minor variations in the hydraulic properties of the system. In fact, such variations always exist in real installations and any mathematical model used to represent the physical pipeline is, in actuality, only an approximation to the thing itself. Moreover, known physical discontinuities in system properties may force the designer to adopt a simplified representation in order to carry out the analytical procedures. For example, if a fibre-reinforced polyester pipeline has a very short steel section, perhaps at a river crossing, an unreasonably small time step might be needed in order to apply the method of characteristics for transient analysis.

One technique for accomodating the aforementioned difficulties is to create an equivalent system which spans minor variations in system properties while maintaining the numerical requirement of a common time step. The physical length, the momentum distribution and the headloss in the pipeline are retained while wavespeed, frictional resistance and cross sectional area are converted to "equivalent" values. The procedures for carrying out an equivalent system conversion are well known and a full account of these is given by [Wylie/Streeter]. If a variable property system is to be optimized according to the methods put forward in this investigation, in other words, having a single diameter and a constant time step, it must first be transformed into an equivalent system. This equivalent system can then be optimized in the usual manner.

Having obtained a solution for the problem, the question arises, "How sensitive is the solution to the approximations made in order to facilitate the modeling process?" The way in which the sensitivity of the actual system to the "approximate" optimal solution is tested, is simply to simulate the response produced in the actual system by the tau curve generated for the equivalent system representation by the optimization model. Figure 6.2 shows the ideal valve stroking response of an equivalent system. Figure 6.3 shows the response produced by the same "optimal" tau curve in the actual system. The values of the various hydraulic parameters for both systems can be calculated from the data given in the figure captions and the equations in [Wylie/Streeter].

6.4 Sensitivity of "Best-fit" Tau Curves

Sections 6.2 and 6.3 have been included for the sake of providing a complete description of sensitivity procedures. The methods outlined in the current section have been given more attention because they are considered to be of greater practical concern. The concept of fitting the idealized boundary condition solution by various standard closures has been explained in Section 5.4. The objective of the sensitivity studies in this connection has been to find some method for evaluating the effect of variation in the parameters of the approximating equations on the system performance. In addition, a logical and informative means of presenting the sensitivity information was desired. Both goals have been achieved with a high

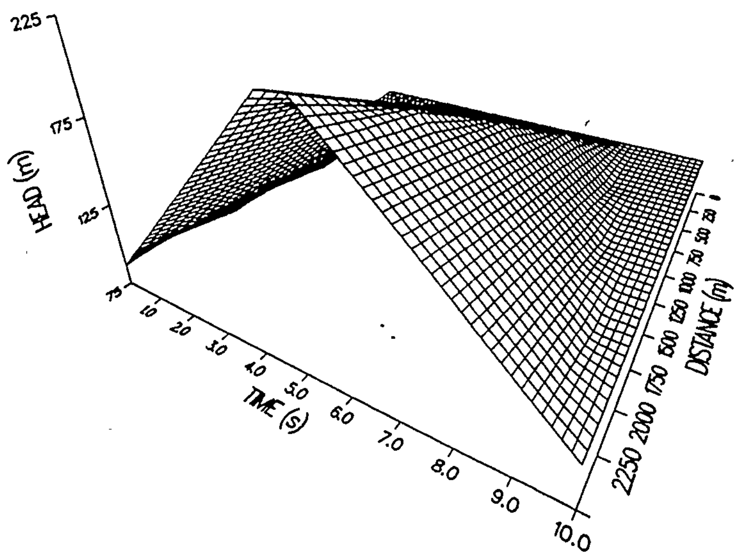


Figure 6.2: Equivalent system response. $H_r = 100$ m, $Q_0 = 2$ m^3/s, $Q_f = 0$ m^3/s, $f = .01735$, $a = 1035$ m/s and $L = 2300$ m, $T_c = 8.889$ s and number of sections $= 4$.

degree of success.

The performance indicators that were selected as being the most meaningful are the dimensionless maximum head rise, Δh_{max}, and the dimensionless maximum residual head rise, Δh^{res}_{max}.

$$\Delta h_{max} = \frac{H_{max} - H_r}{H_r} \tag{6.1}$$

$$\Delta h^{res}_{max} = \frac{H^{res}_{max} - H_r}{H_r} \tag{6.2}$$

The former is the maximum head rise obtained in the system during the valve motion and the latter is the maximum head rise obtained after the valve motion has ended. These quantities have been made dimensionless by referring them to the reservoir head. The reasons for selecting the reservoir head as a reference quantity are

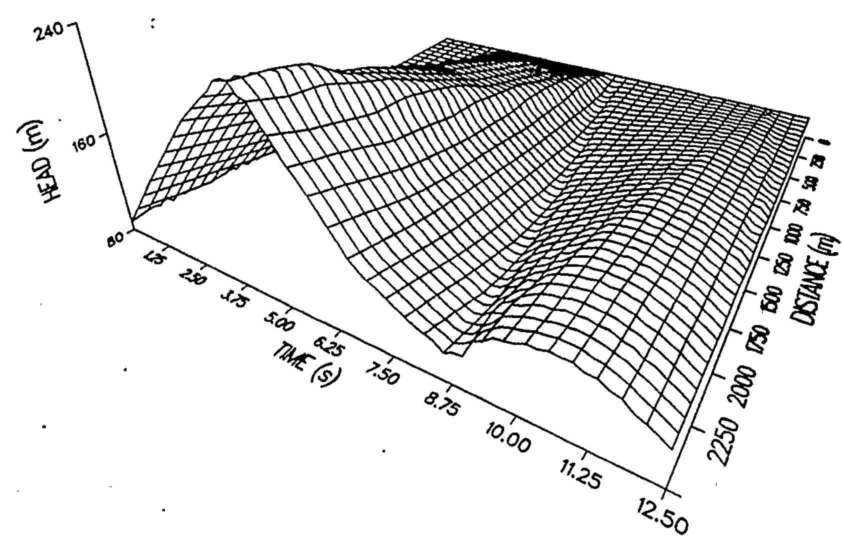


Figure 6.3: Actual system response. H_r = 100 m, Q_0 =2 m^3/s, Q_f = 0 m^3/s, number of pipes = 3, f_1 = .02, a_1 = 1000 m/s, D_1 = 1.2 m and L_1 = 1000 m, f_2 = .01, a_2 = 1200 m/s, D_2 = 1.0 m and L_2 = 800 m, f_3 = .015, a_3 = 900 m/s, D_3 = 0.8 m and L_3 = 500 m. Pipes are numbered consecutively from the reservoir and the equivalent reach length was taken as 2300 m.

1. It provides an absolute frame of reference so that the sensitivity of different systems may be more readily compared.

2. The valve motions which are of critical interest are most often complete closures from full or partial openings. This means that the final steady state is also equal to the level of water in the reservoir.

The dimensionless maximum head rise for the ideal valve stroked system is denoted by Δh_{max}^{opt}. Comparing the values of Δh_{max} and Δh_{max}^{opt} provides some measure of how close to optimal the response is in terms of maximum internal pressure. It does not indicate, however, the shape of the *head-time* profiles, although this can be inferred from the agreement between the two values. The parameter Δh_{max}^{res} indicates how fully the new flow conditions have been obtained.

These parameters are used in conjunction with the dimensionless system parameters ρ, h_f and t_c (defined in Section 4.1) to provide a fairly complete nondimensional representation of the system and its sensitivity to changes in the best-fit parameters m and a_1 defined in Section 5.4. The values of Δh_{max} and Δh_{max}^{res} for a series of m or a_1 values are plotted as dependent variables against the error sum of squares, E, for a particular type of fit. Figures 6.4 and 6.5 are examples of such curves for the equal percentage and bilinear regression fits. Several more sensitivity plots can be found in Appendix B. The values of t_c and ρ are typical for many pipeline installations. The figures show sensitivity curves for a wide range of frictional values.

Each dot on the plot represents one value of the fitting parameters seperated by a constant amount from each neighbouring point. The arrow indicates the di-

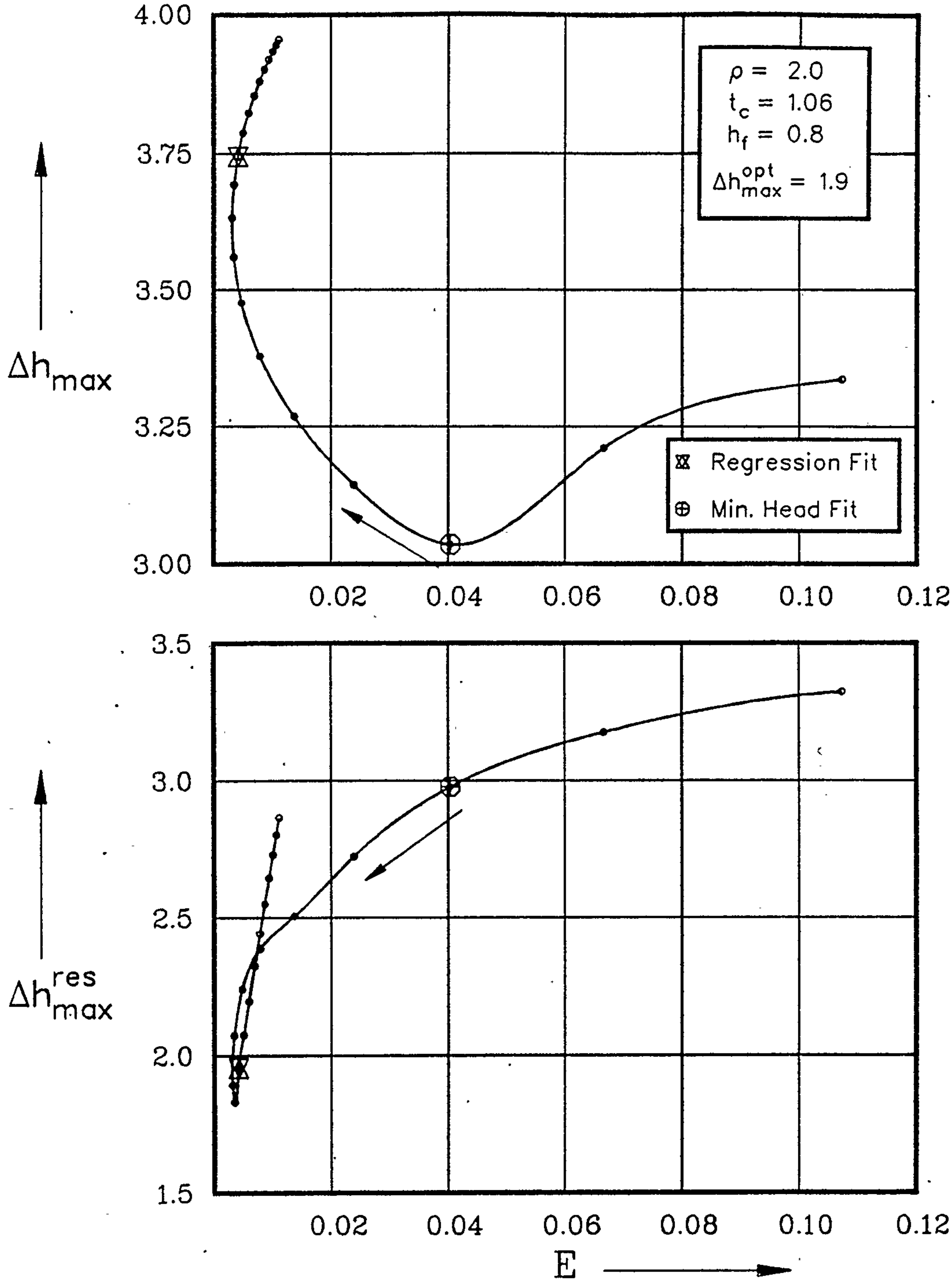


Figure 6.4: "Best-fit" sensitivity plots. Equal percentage regression fit.

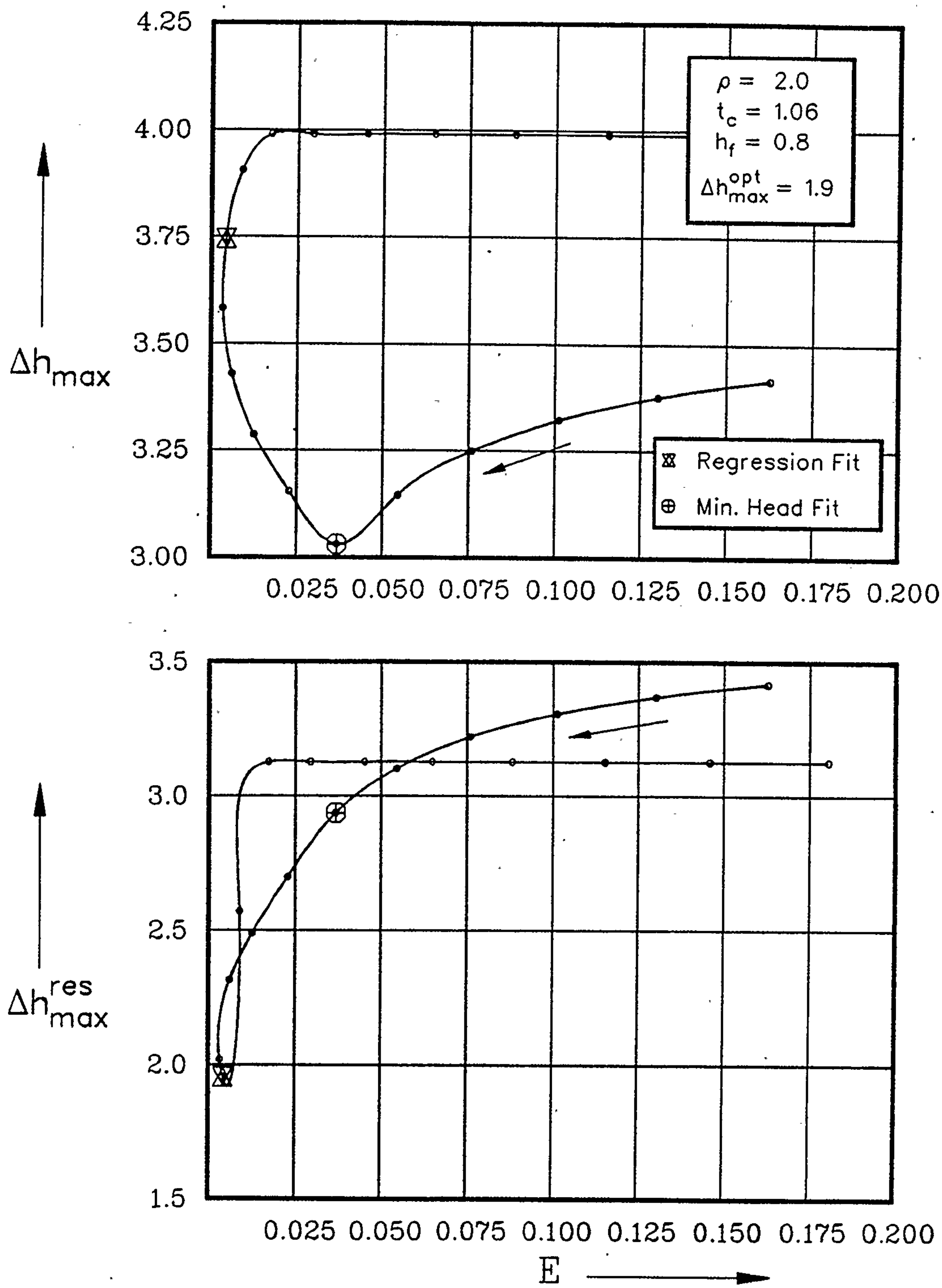


Figure 6.5: 'Best-fit" sensitivity plots. Bilinear regression fit.

rection in which the fit parameter is decreasing, i. e., increasing in absolute value. Depending upon the values of ρ, h_f, t_c, the shape of the optimum tau curve and the type of approximating function, the sensitivity curves can differ remarkably. In virtually every case, however, the curves exhibit two distinct global minima—one with respect to the error sum of squares corresponding to the "true" value of the least squares regression parameter, and another with respect to Δh_{max} corresponding to the value of the minimum head fit parameter.

These plots demonstrate quite clearly that the minimum head closure is seldom the same as the minimum error closure. Those tau curves which more closely approximate the closure arrangement over its latter portion produce lower head rises in the system. They also show that it is not uncommon for the minimum head fit to produce head rises that are lower than can be obtained by valve stroking itself. The plots for the maximum residual head rise are also shown below its corresponding maximum head rise plot. In general, the plots indicate that the least error fits usually are associated with a lower residual head while the minimum head fits tend to have higher values of residual head. This can be misleading since, in some cases, the maximum residual head occurs at the end of the valve motion. Thereafter, the head declines to near the final steady state head with only minor periodic oscillations in the head persisting. This is actually a fairly desirable situation if one need not have fully obtained the new flow condition at the moment the valve motion ends. Perhaps a preferable alternative in evaluating Δh_{max}^{res} would be to look for the maximum residual head starting L/a seconds after the valve motion has ceased. This would restrict the consideration of residual head values to those occuring periodically rather than those occurring during a more or less linear reduction from the maximum head to values nearer the final steady state.

The sensitivity plots presented in this section provide a great deal of information to the analyst in a concise and coherent format. Almost at a glance, one can see what the most effective approximate valve motions are likely to be, both in terms of maximum heads and maximum residual heads. The slope of the curves also conveys information regarding which ranges of fit parameters produce the least change in the head rise for an incremental error. It is not difficult to imagine that, if such curves were available for a wide range of system parameters and different types of valve closures, they could provide valuable guidance in selecting commonly available valve closures which provide more "optimal" system performance than could be had by some arbitrary selection.

6.5 Summary

Sensitivity studies are crucial to the valid and successful implementation of any optimization process. This section has introduced a few ideas regarding some actual and some potential methods for obtaining various types of sensitivity information. Of particular interest are the general methods and the sensitivity curves developed for evaluating the performance of the various "best-fit" tau curves. These provide a fairly complete and workable set of sensitivity tools for the analyst and should make the viability of the model developed in this investigation a more tenable proposition.

Chapter 7

Conclusions and Recommendations

In the past, optimization techniques have focused on the steady state aspects of pipelines almost exclusively. Time-varying characteristics of pipelines are either ignored or treated in a quasi-steady state manner. The internal pressures that the pipe must withstand are of paramount importance in determining the cost of the pipeline. Since the capital cost of the pipeline represents the greatest cost component of most pipeline installations, it seems illogical to neglect fluid transients as the heads produced by these shock phenomena can be many times greater than the anticipated steady state design pressures. The present investigation has attempted to formulate some rationale by means of which optimizing design procedures that properly account for the effect of transients in pipelines can be developed. More precisely, the objectives have been:

- The identification of relevant cost parameters and cost variables.
- The development of a conceptual framework within which optimizing design procedures can be constructed.
- The formulation of a sensible effectiveness criterion and a meaningful set of constraints incorporating the cost components.
- The definition of the nature of the cost function and the selection of appropriate, efficient methods for its evaluation.

- The consideration of methods which will allow the sensitivity of the solutions to changes in cost parameters to be explored.

The outcome of this investigation has been successful in accomplishing these goals and positive in identifying numerous areas of interest for potential future work.

Specific conclusions arising from the study are:

1. The model(s) developed and described in this investigation comprise a heuristic approach to the problem of cost optimization of pipelines with respect to hydraulic transients. They require interaction and input from the designer in order to be fully utilized.

2. If one accepts the premise that no residual transients should occur once the valve motion is complete, then the model **TOM2** based entirely on valve stroking theory is a rigorous mathematical model for optimization with respect to transients.

3. The problem of cost optimization of a pipeline is a highly nonlinear, complex function of many variables.

4. The model(s) employ valid algorithms and produce mathematically correct solutions.

5. The optimal solution in the ideal boundary condition model (**TOM2**) almost always occurs when the variation in the discharge during the transient phase is a linear function of time. This is true for the case of a valve closure in the system composed of a constant head upstream reservoir, single pipe with a valve at the downstream end.

6. Practicable, commonly available valve closure systems can often closely approximate the idealized boundary condition behaviour. In particular, optimal tau curves often resemble equal percentage or bilinear valve closure arrangements.

7. If the restriction imposed by the valve stroking procedure, that the final flow conditions be fully achieved precisely when the valve motion ends, is removed, then it is often possible to obtain a lower value of maximum head rise, Δh_{max}, in the system than is obtainable by valve stroking.

8. The parameters Δh_{max} and Δh^{res}_{max} are good indicators of the characteristics and performance of non-ideal boundary conditions (tau curves).

Many problems have been overcome, much information has been gathered and some new ideas, methods and approaches to the optimization of simple pipelines have been developed. Many more problems have not been solved, our knowledge is still very incomplete and, no doubt, some better or improved techniques for incorporating transient phenomena into optimization procedures will be forthcoming. The following suggestions for future research can be made.

- The model(s) should be tested by attempting to apply it to an actual pipeline installation.

- More information is needed in order to clarify which variables are important in characterizing the performance and cost of valve system options. Is this possible given the current state of valve system technology? Will the developments in electronic controllers, power electronics or computer aided manufacture of valves permit the economic production of accurate, dedicated systems for critical valve motion?

- We can be reasonably certain that the ideal boundary condition solution represents a global optimum since, for each point in the solution domain, only one uniquely optimal tau curve exists. However, if non-ideal boundary conditions are employed, can the existence of a global optimum be assured? Can any general guidelines be established to determine which type of non-optimal valve motion will provide the best results for a given set of dimensionless pipeline parameters?

- Can the ideas and methods presented for the upstream reservoir, single pipe and downstream valve system be readily applied to other types of boundary conditions or to more complex systems?

Clearly, this investigation constitutes only the barest of beginnings for what promises to be a challenging and important area of research and development. This work will be considered to have been a worthwhile effort if it does nothing more than kindle the interest of others, both researchers and practicing professionals, and encourages them to take further steps in the direction initiated by the work of Yao-Chung Chiang and the present study: that is, towards the development of a more comprehensive set of tools with which concerned and dedicated professionals can advance the state of beneficial technology and, hopefully, the condition of all mankind.

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Appendix A

Program Descriptions

The following information has been largely extracted from the documentation of each of the programs listed in this appendix. Copies of any of these programs are available from the authors. The graphics segments contained in the programs utilize the *DISSPLA* (version 9.0 or higher) graphics package from Integrated Software Sytems Corporation of San Diego, California.

A.1 Optimization by Enumeration—Model TOM1

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DATE: 24 MARCH 1986

METHOD: This program calculates transient heads and discharges in a series pipeline due to valve motion at the downstream end. In addition, the dimensionless valve parameter, TAU, required to produce the transient caused by the prescribed boundary conditions is calculated. The Method of Characteristics is used to produce the solution by generating the interior values of head and discharge for all time successively at each physical, computational section. Inputs include the specified duration of the transient (TSTROKE), the initial and final steady state head (H0 and HF) and discharge (Q0 and QF) and the transient discharge at the upstream section for the duration of the transient. The upstream head may be specified as a constant reservoir head (HRES) or alternately be input as discrete values at specific times. The variation in the flowrate during the transient phase is a parabolic function of time and the initial and final discharges. Parameters describing the physical nature of the system include pipe lengths, diameters, D'Arcy - Weisbach friction factors and wavespeeds. Head losses at series pipe connections are considered to be negligible. It is assumed that the valve discharges to the atmosphere and that the hydraulic grade line datum is set at the level of the valve.

An additional feature of the program permits the user to generate output in graphical form as three dimensional time-space-head diagrams, summary profiles of head vs. time and head vs. distance, or any desired combination of these three types of graphs. The character input designations for the various graph types are:

HDT - 3 dimensional head-distance-time

H_T - summary plot of head vs. time at

valve, 3/4 point, midpoint and 1/4 point

H_D - summary plot of head vs. distance for steady flow, maximum head and minimum head

The graphical output may be directed to either a terminal or a plotting device segment by specifying respectively "TERM" or "PLOT" in the plot data input file.

STRUCTURE: This program is structured in six blocks:

BLOCK 1 - Reading and writing the input data

BLOCK 2 - Calculation of pipe constants and steady state conditions.

BLOCK 3 - Transient state calculation

BLOCK 4 - Memory storage update

BLOCK 5 - Determination of maximum and minimum heads

BLOCK 6 - Final print summary

SUBROUTINES:

PARAB - Parabolic interpolation scheme for valve closure

PRELIM - Sets up arrays for storing plot information and computes the interpolation scheme for obtaining plot values

STORES - Stores the values to be plotted at times consistent with the grid mesh size

PLOT3D - Produces the actual plots or plotting files

RTRIM - Returns the length of a character string trimmed of trailing blanks

PRIMARY VARIABLES

REAL:

A	=	wave speed
AR	=	pipe area
B	=	A/(G*AR)
D	=	pipe diameter
DT	=	time step
DTHEAD	=	time increment for storing discrete input heads
F	=	Darcy-Weisbach friction factor
FF	=	F*L/(2*N*G*D*AR**2)
G	=	acceleration due to gravity
H	=	known head at section stroked first
HARRAY	=	array for storing all values of head
HLIMIT	=	allowable max. or min. head for alternate method of valve stroking (Ruus) - not implemented yet
HMAX	=	vector for maximum head at each section
HMIN	=	vector for minimum head at each section
HP	=	unknown head at a section (to be calculated)
HPM1	=	known head at previous timestep
HRES	=	steady state reservoir head

HTMAX = maximum head rise

HTMIN = minimum head rise

HTRANS = discrete input values of head

L = pipe length

Q = known discharge at section stroked first

QARRAY = array for storing all discharges

QF = final steady state discharge

QO = initial steady state discharge

QP = unknown discharge at a section (to be calculated)

QPM1 = known head at previous section

T = time from start of transient at valve

TAUPLT = vector for storing TAU curve

TSTROKE = time interval for computation of transients

INTEGER: IUNITS = 1 for SI and 2 for FPS

IPRINT = print block repeated every IPRINT time steps

M = number of points on tabulated tau curve (Y)

N = number of reaches in a pipe

NP = number of pipes in system

NPMAX = maximum number of pipes in system

NRLP = number of reaches in last pipe (used to find DT)

NSECM = maximum number of sections in one pipe

PIPE = pipe to be stroked first (usually 1, i.e., upstream)

Number of Pipe Reaches	ψ-T_c Grid Size	CPU Time (s)
2	17×17	5.774
4	33×33	58.33
8	65×65	737.0

Table A.1: **TOM1** computer run times on Honeywell DPS8/6. Total number of solution points evaluated is given by the expression in the center column.

```
FORMAT FOR DATA FILE (opt_data):

LINE        DESCRIPTION

1           TITLE  (60 CHARACTERS OR LESS)
2           GRAPHICS SELECTION
3           NP  NRLP  IPRINT IUNITS
4           QO  QF  HRES  PIPE
5           DTHEAD  M  HTRANS(M).....
6           L(1)  D(1)  A(1)  F(1)
+           L(I)  D(I)  A(I)  F(I)
```

Table A.1 shows the performance of the program in terms of central processing unit (CPU) usage for a number of complete runs, but without any graphical output being generated.

A.2 Optimization by Univariate Search—Models TOM2 and TOM3

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METHOD: This program calculates transient heads and discharges in a series pipeline due to valve motion at the downstream end. In addition, the dimensionless valve parameter, TAU, required to produce the transient caused by the prescribed boundary conditions is calculated. The Method of Characteristics is used to produce the solution by generating the interior values of head and discharge for all time successively at each physical, computational section. Inputs include initial and final steady state heads (HO,HF) and discharges (QO and QF) and the transient discharge at the upstream section for the duration of the transient. The upstream head may be specified as a constant reservoir head (HRES) or alternately be input as discrete values at specific times. The variation in the flowrate during the transient phase is a linear function of time and the initial and final discharges. Parameters describing the physical nature of the system include pipe lengths, diameters, D'Arcy - Weisbach friction factors and wavespeeds. Head losses at series pipe connections are considered to be negligible. It is assumed that the valve discharges to the atmosphere and that the hydraulic grade line datum is set at the level of the valve.

The model uses the information generated by the valve stroking procedure to evaluate an effectiveness criterion (cost of system) including terms for steady state power cost, pipe size, length and wall thickness costs, any time dependent costs (i.e. response time) as well as the cost of the associated valve/controller/actuator system. Since "optimal" closures as given by valve stroking are generally not economically feasible in practice, the model generates information regarding the substitution of commonly available valve closure systems (linear, equal percentage and minimum head equal peak) for the theoretical valve closure.

The valve closure information generated by the activities of the model are placed in a file called "tau". This information can be incorporated in a data file of the same type as is used by the subroutine VALVE (or the program VALVE) to produce sensitivity plots summarizing the response of the system to departures from the linear, equal percentage and optimum closure arrangements. The program which produces these plots is called "errplot.fortran".

An additional feature of the program permits the user to generate output in graphical form as three dimensional time-space-head diagrams, summary profiles of head vs. time and head vs. distance, or any desired combination of these three types of graphs. The character input designations for the various graph types are:

HDT - 3 dimensional head-distance-time

H_T - summary plot of head vs time at valve, 3/4 point, midpoint and 1/4 point

H_D - summary plot of head vs distance for steady flow, maximum head and minimum head

The graphical output may be directed to either a terminal or a plotting device segment by specifying respectively "TERM" or "PLOT" in the plot data input file.

SUBROUTINES:

GOLDEN - Optimizes the nonlinear cost function

GOLDEN2 - Determines the exponent associated with the minimum head equal-percentage closure by minimizing the head rise in the system

COST - Calls the individual cost term subroutines (user defined) and evaluates the cost function

VALSTRK - Produces the transient head and valve motion information used to evaluate the associated cost terms

LSQ - Fits linear and equal percentage regression curves to the "optimal" closure.

VALVE - Using the "fitted" closure curves produces new transient head information to reevalvuate the cost function

STOREV - interpolates values in the time dimension for plotting

PRELIM - Sets up arrays for storing plot information and computes the interpolation scheme for obtaining plot values

STORES - Stores the values to be plotted at times consistent with the grid mesh size

PLOT3D - Produces the actual plots or plotting files

RTRIM - Returns the length of a character string trimmed of trailing blanks

PRIMARY VARIABLES

REAL: A = wave speed

AR = pipe area

B = A/G*AR

D = pipe diameter

DT	= time step
DTHEAD	= time increment for storing discrete input heads
F	= Darcy-Weisbach friction factor
FF	= F*L/(2*N*G*D*AR**2)
G	= acceleration due to gravity
H	= known head at section stroked first
HA	= array for storing all values of head
HLIMIT	= allowable max. or min. head for alternate method of valve stroking (Ruus) - not implemented yet
HMAX	= vector for maximum head at each section
HMIN	= vector for minimum head at each section
HP	= unknown head at a section (to be calculated)
HPM1	= known head at previous timestep
HRES	= steady state reservoir head
HTMAX	= maximum head rise
HTMIN	= minimum head rise
HTRANS	= discrete input values of head
L	= pipe length
Q	= known discharge at section stroked first
QA	= array for storing all discharges
QF	= final steady state discharge
Q0	= initial steady state discharge
QP	= unknown discharge at a section (to be calculated)
QPM1	= known head at previous section

T = time from start of transient at valve

TAUPLT = vector for storing TAU curve

TSTROKE = time interval for computation of transients

INTEGER: IUNITS = 1 for SI and 2 for FPS

IPRINT = print block repeated every IPRINT time steps

M = number of points on tabulated tau curve (Y)

N = number of reaches in a pipe

NP = number of pipes in system

NPMAX = maximum number of pipes in system

NRLP = number of reaches in last pipe (used to find DT)

NSECM = maximum number of sections in one pipe

PIPE = pipe to be stroked first (usually 1, i.e., upstream)

FORMAT FOR DATA FILE (opt_data):

LINE	DESCRIPTION
1	TITLE (60 CHARACTERS OR LESS)
2	GRAPHICS SELECTION
3	NP NRLP IPRINT IUNITS
4	QO QF HRES TLAST PIPE

```
5          DTHEAD   M   HTRANS(M).....
6          L(1)   A(1)   F(1)
+          L(I)   A(I)   F(I)
```

Table A.2 provides some indication of the program performance with respect to the problem size.

Program (Model)	Number of Pipe Reaches	CPU Time (s)
TOM2	2	0.709
	4	2.269
	8	10.08
	16	40.71
TOM3	2	2.660
	4	7.820
	8	26.04
	16	94.26

Table A.2: **TOM2** and **TOM3** computer run times on Honeywell DPS8/6.

A.3 Method of Characteristics Simulation—Program VALVE

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DATE:  3 OCTOBER 1985

REVISED BY DUNCAN McINNIS, FEBRUARY/1986

REVISED BY BRYAN KARNEY, MAY/1986
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METHOD: This program calculates the transient head and discharge in a series pipe system caused by opening or closing a valve at the downstream end. The Method of Characteristics is used based on a specified time increment and calculations are continued for a specified time interval (TLAST). Initial conditions are defined by a steady state discharge (QO) and an upstream reservoir head (HRES) which is considered constant for the duration of the transients. Inputs include the known valve motion, the physical description of the pipe system and the value of the basic hydraulic parameters such as the Darcy-Weisbach friction factors for each pipe. Head losses at series pipe connections are considered negligible. It is assumed that the valve discharges to the atmosphere and that the hydraulic grade line datum is set at the level of the valve.

FEBRUARY REVISIONS

An additional feature of the program permits the user to generate output in graphical form as three dimensional time-space-head diagrams, summary profiles of head vs time and head vs distance, or any desired combination of these three types of graphs. The character input designations for the various graph types are:

HDT - 3 dimensional head-distance-time

H_T - summary plot of head vs time at
valve, 3/4 point, midpoint and 1/4 point

H_D - summary plot of head vs distance for
steady flow, maximum head and minimum head

The graphical output may be directed to either a terminal or a plotting device segment by specifying respectively "TERM" or "PLOT" in the plot data input file.

MAY REVISIONS

All control statements formerly used to jump do loops when the do counter values were zero have been eliminated. This means the program must now be compiled with the 'ansi77' option invoked (ansi66 always processes a loop at least once).

STRUCTURE: This program is structured in six blocks:

BLOCK 1 - Reading and writing the input data

BLOCK 2 - Calculation of pipe constants and steady state conditions

BLOCK 3 - A print block

BLOCK 4 - Transient state calculation

BLOCK 5 - Memory storage update

BLOCK 6 - Final print summary

SUBROUTINES:

PARAB - Parabolic interpolation scheme for valve closure

PRELIM - Sets up arrays for storing plot information and computes the interpolation scheme for obtaining plot values

STORES - Stores the values to be plotted at times consistent with the grid mesh size

PLOT3D - Produces the actual plots or plotting files

RTRIM - Returns the length of a character string trimmed of trailing blanks

PRIMARY VARIABLES

REAL: A = wave speed

AR = pipe area

B = A/(G*AR)

D = pipe diameter

DT = time step

DTAU = time step for storing valve's tau curve

F = Darcy-Weisbach friction factor

FF = F*L/(2*N*G*D*AR**2)

G = acceleration due to gravity

H = known head at a section

HP = unknown head at a section (to be calculated)

HRES = steady state reservoir head

HTMAX = maximum head rise

HTMIN = minimum head rise

L = pipe length

Q = known discharge at a section

QP = unknown discharge at a section (to be calculated)

QO = initial steady state discharge

T = time from start of transients

TLAST = time interval for computation of transients

TAUF = final valve position

TAUO = initial valve position

TV = valve closure time

VALVE1= Hss/Qss**2 - valve constant at full gate opening

Y = actual tabulated valve operation curve (tau values)

INTEGER: IUNITS = 1 for SI and 2 for FPS

IPRINT = print block repeated every IPRINT time steps

K = print counter (used with IPRINT)

M = number of points on tabulated tau curve (Y)

N = number of reaches in a pipe

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NP     = number of pipes in system
NPMAX  = maximum number of pipes in system
NRLP   = number of reaches in last pipe (used to find DT)
NSECM  = maximum number of sections in one pipe
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***   NORMAL INPUT FORMAT   ***

 1:     TITLE
 2:     GRAPHICS   HDT   H_T   H_D
 3:     NP   NRLP   IPRINT   IUNITS
 4:     QO   HRES   TLAST
 5:     VALVE1   TAUO   TAUF   TV   DTAU   M
 6:     M * [ Y(I) ]
 7:     NP * [ L   D   A   F ]
```

Appendix B

Sensitivity Plots

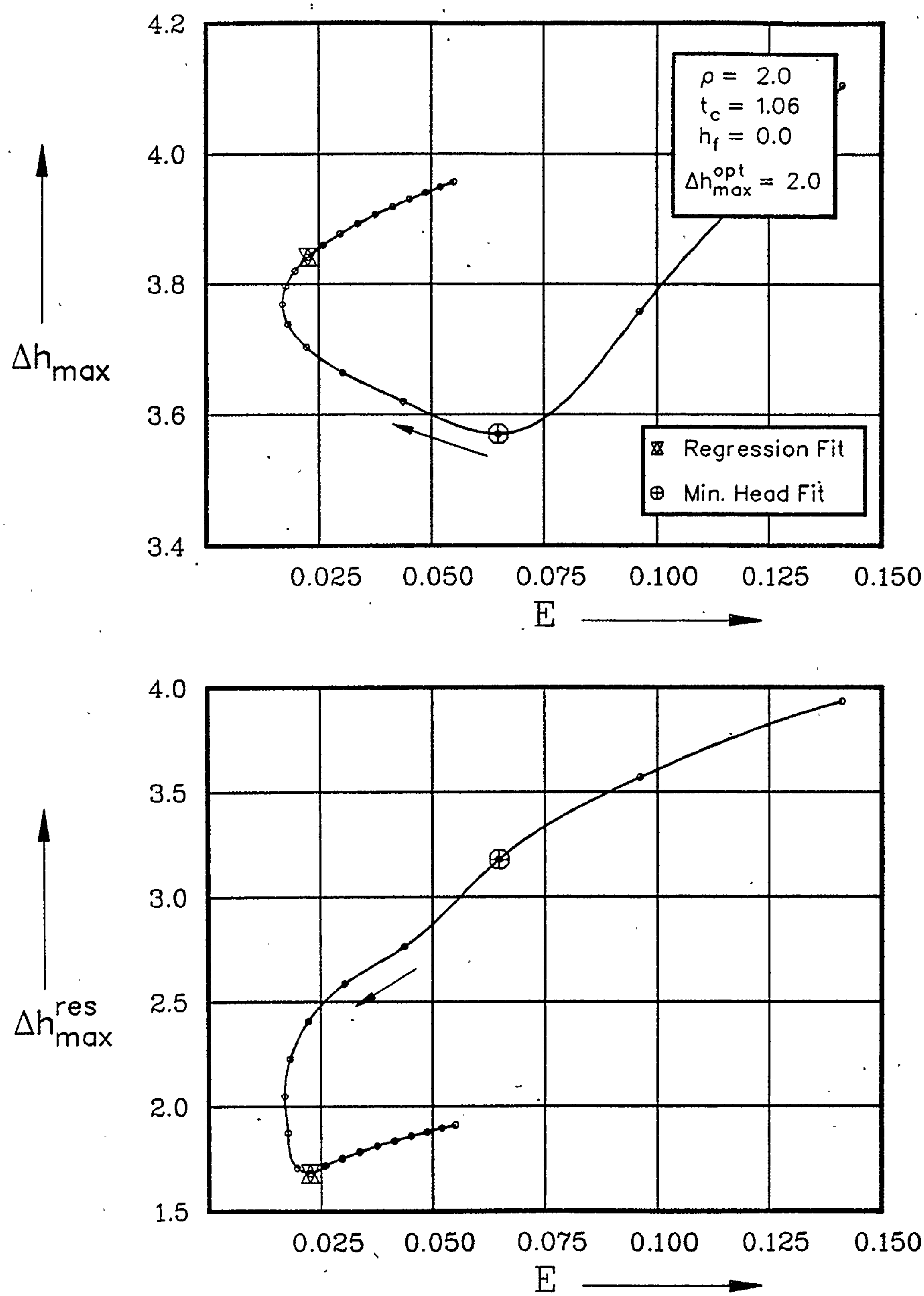

Figure B.1: "Best-fit" sensitivity plots. Frictionless system. Rapid equal percentage closure.

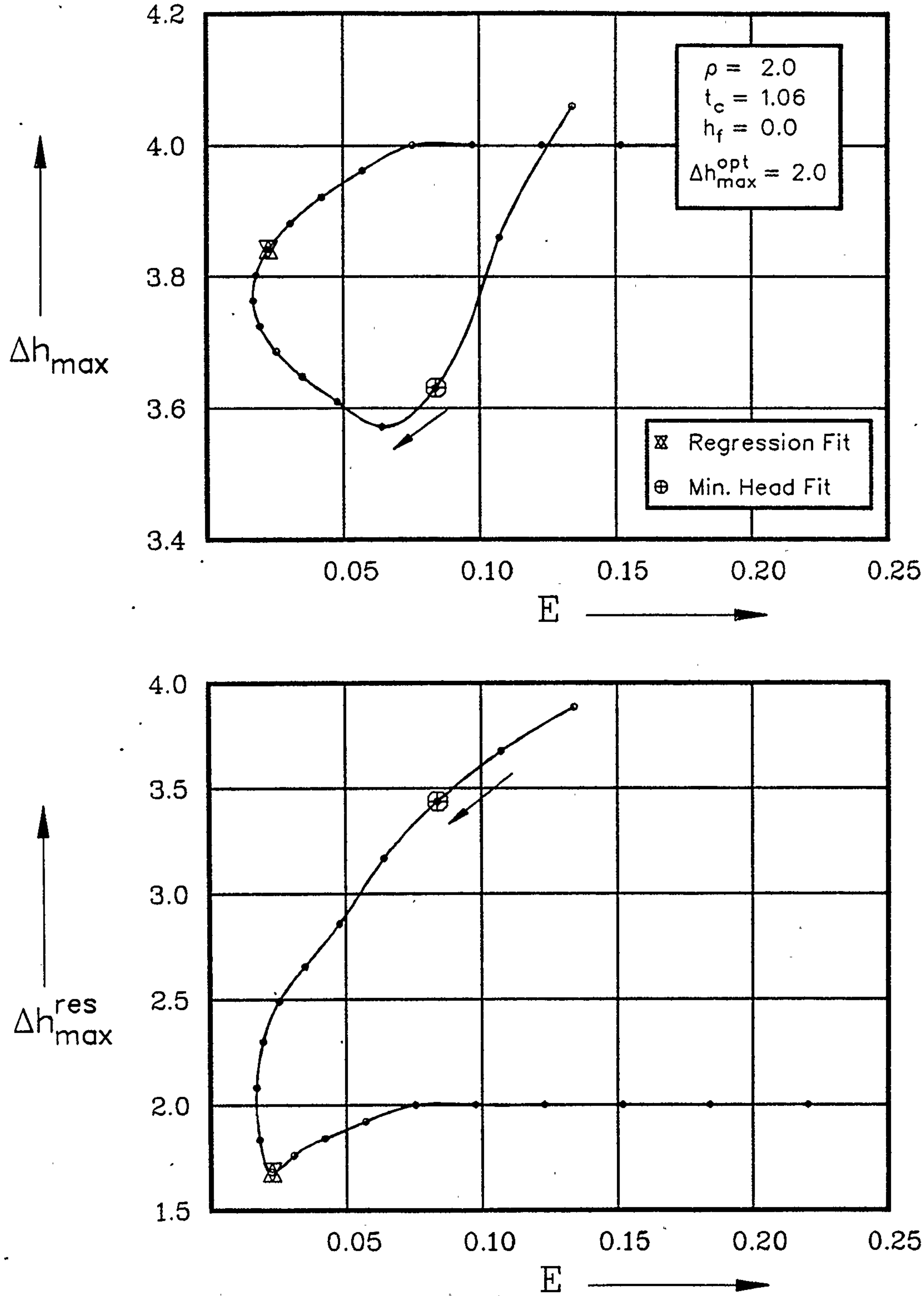

Figure B.2: "Best-fit" sensitivity plots. Frictionless system. Rapid bilinear closure.

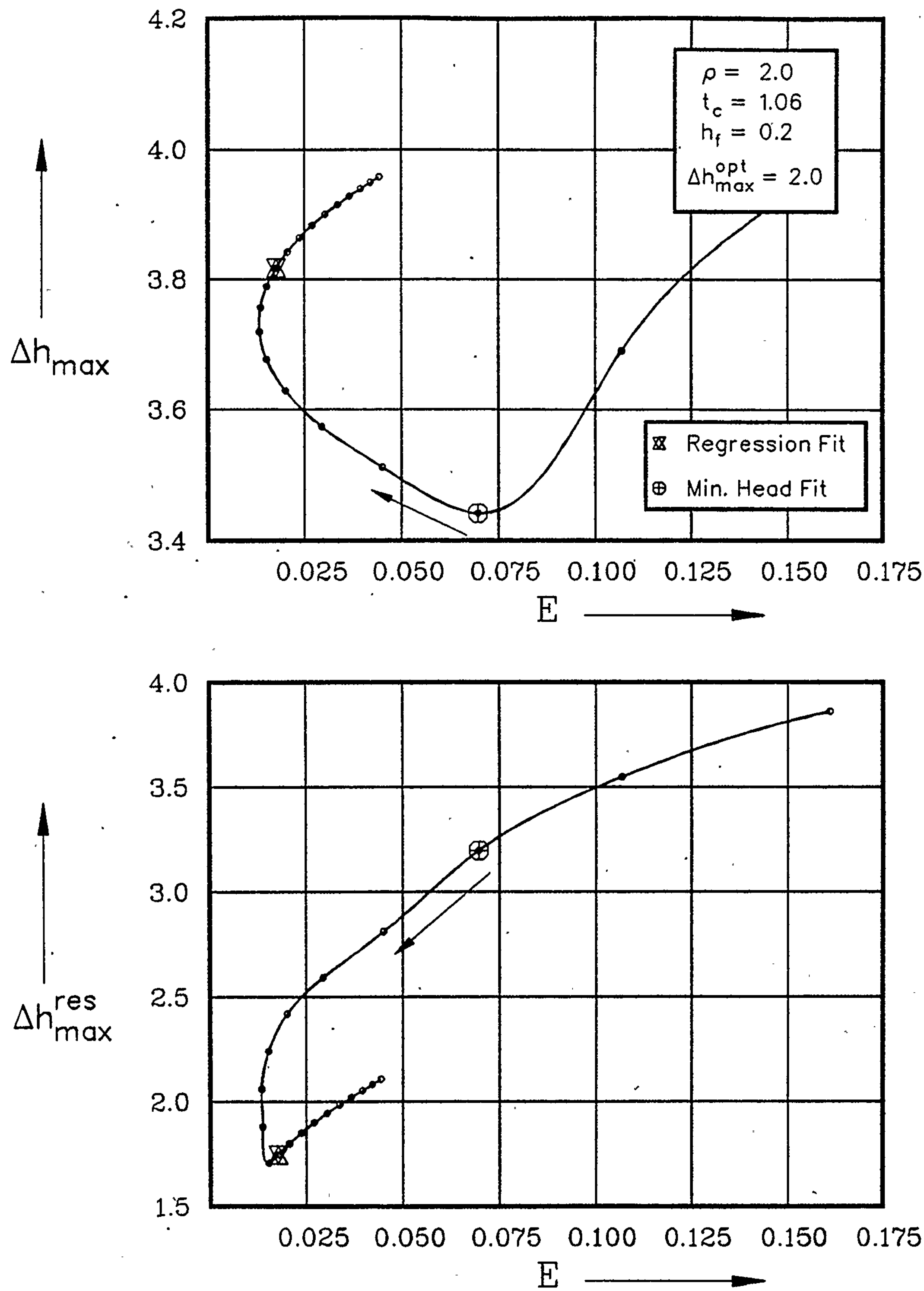

Figure B.3: "Best-fit" sensitivity plots. Low friction system. Rapid equal percentage closure.

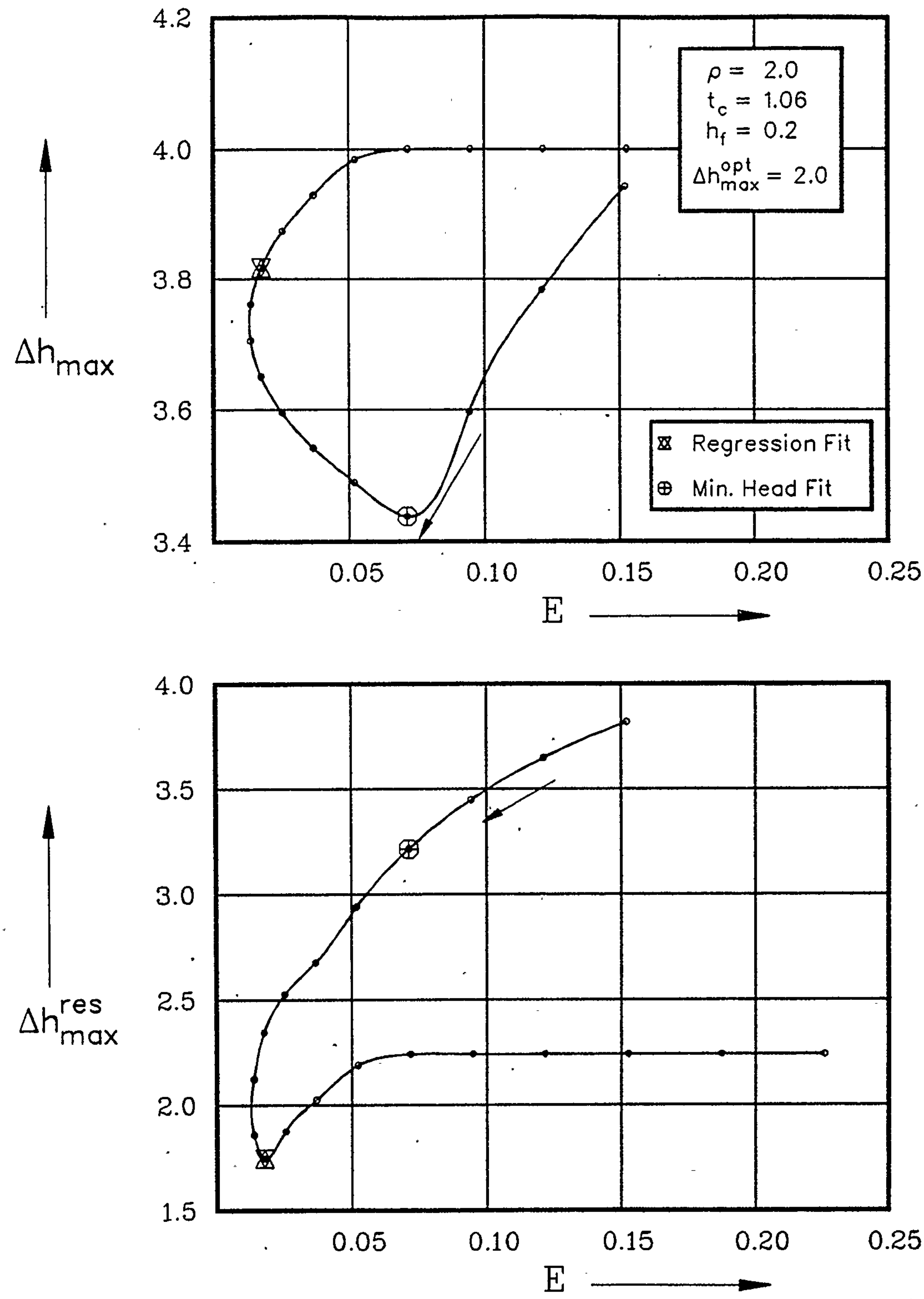

Figure B.4: "Best-fit" sensitivity plots. Low friction system. Rapid bilinear closure.

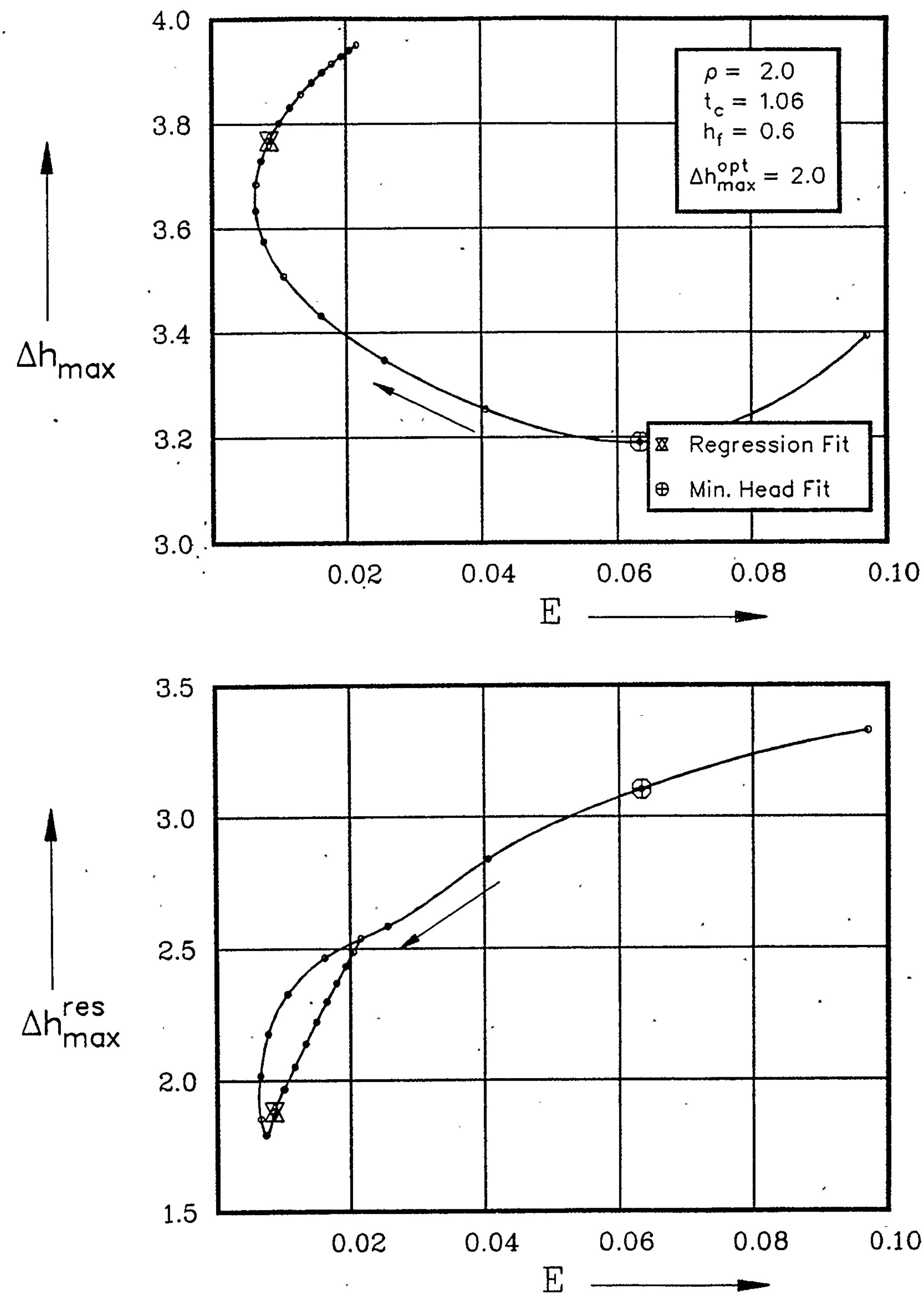

Figure B.5: "Best-fit" sensitivity plots. High friction system. Rapid equal percentage closure.

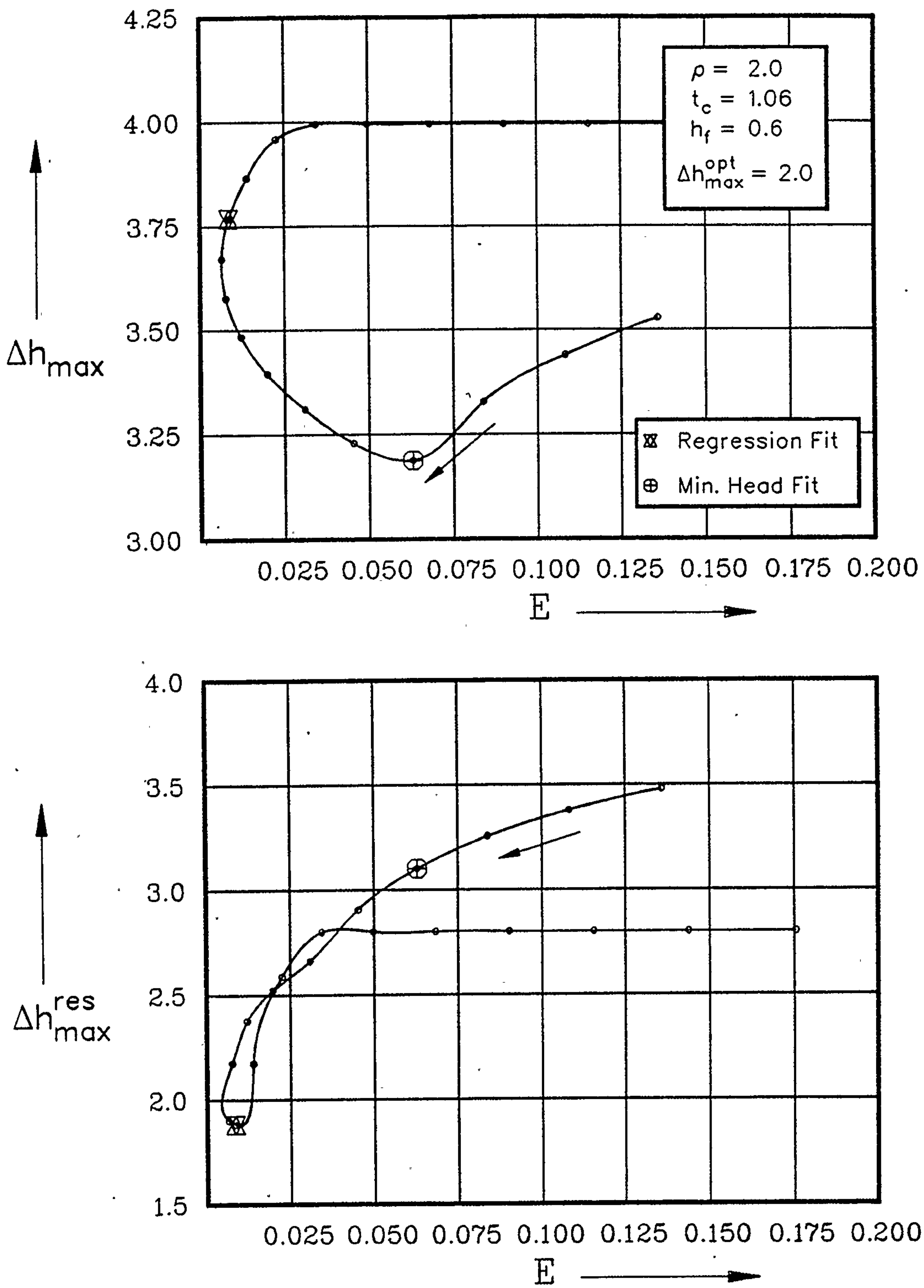

Figure B.6: "Best-fit" sensitivity plots. High friction system. Rapid bilinear closure.

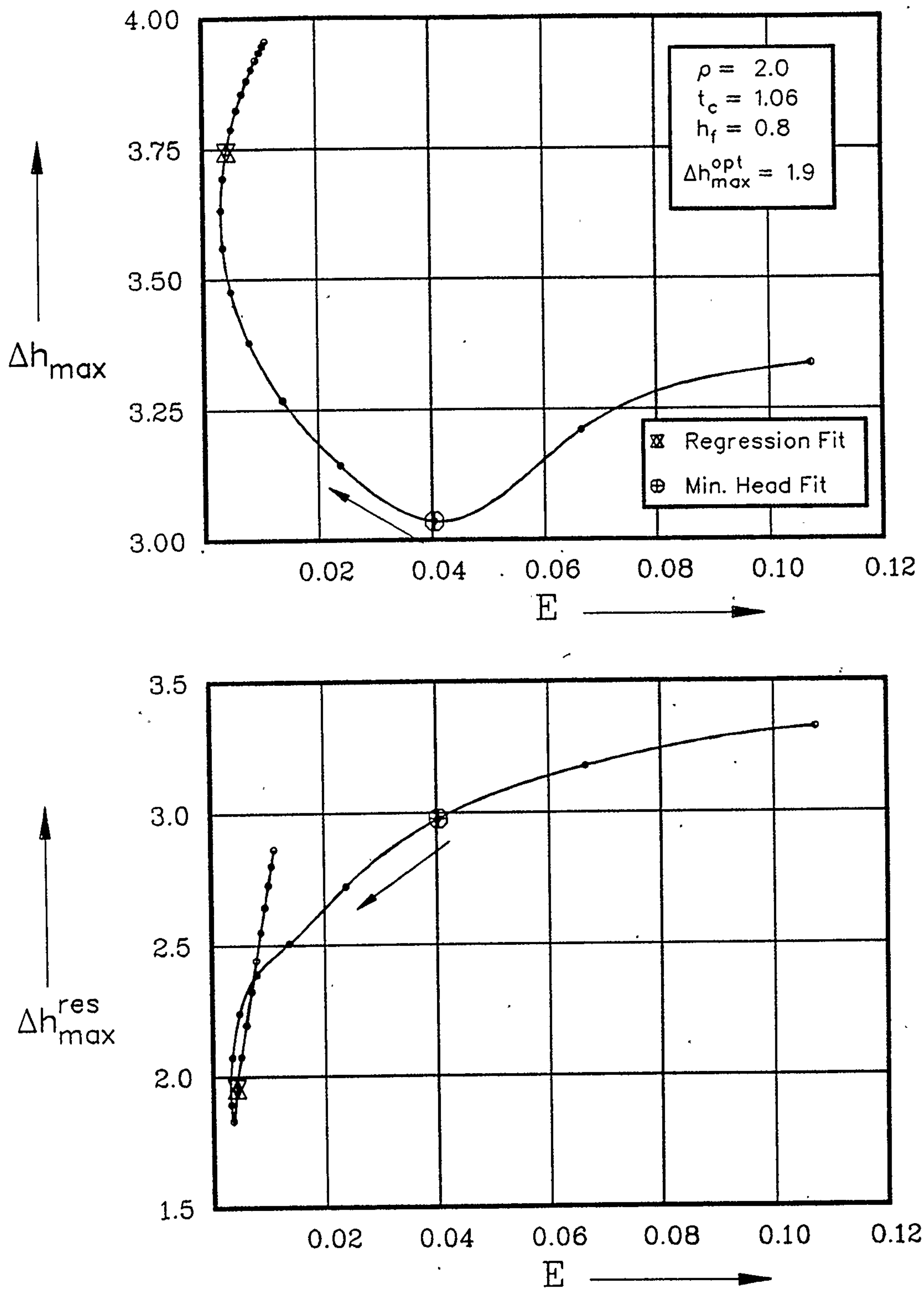

Figure B.7: "Best-fit" sensitivity plots. Very high friction system. Rapid equal percentage closure.

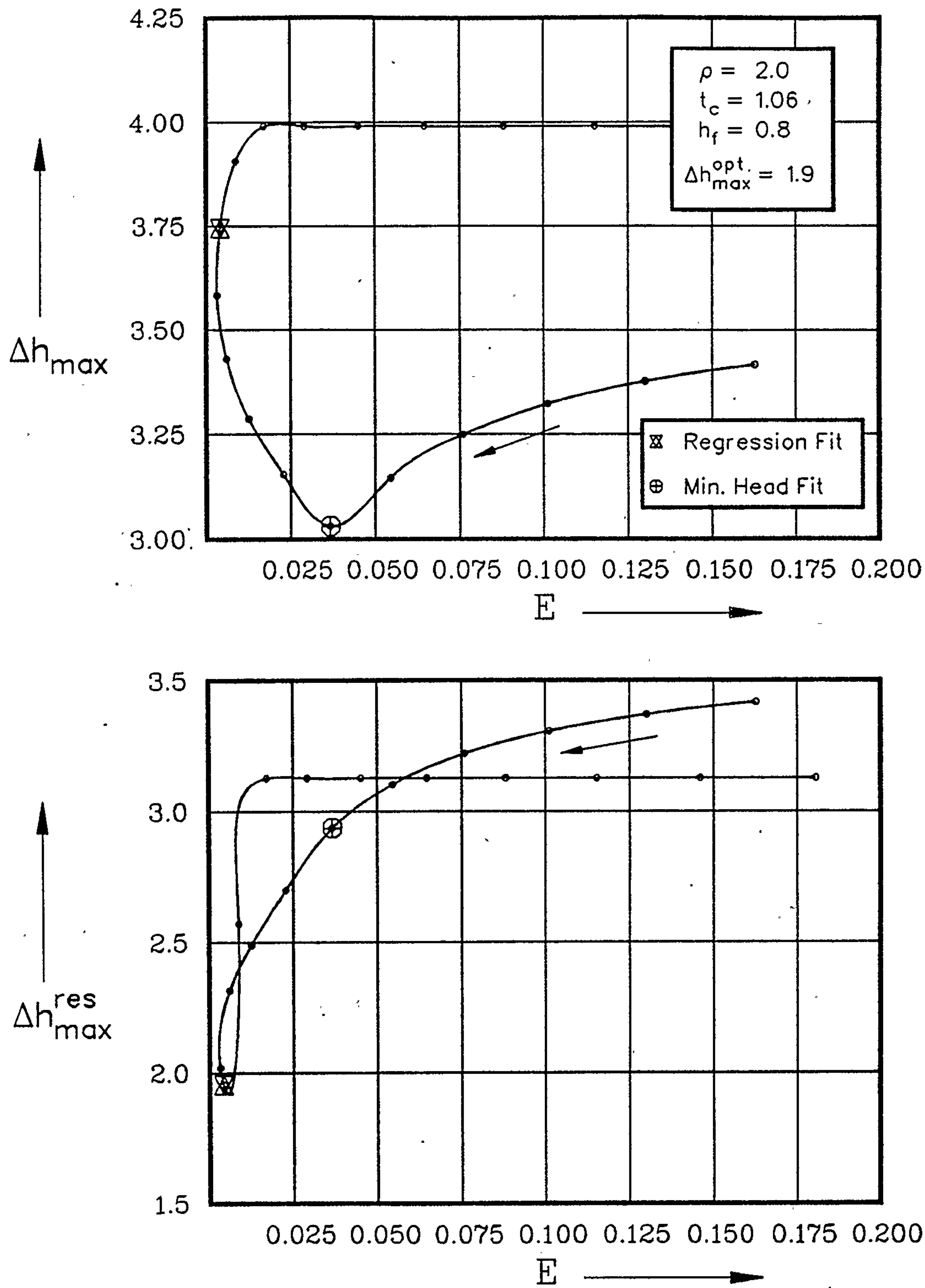

Figure B.8: "Best-fit" sensitivity plots. Very high friction system. Rapid bilinear closure.

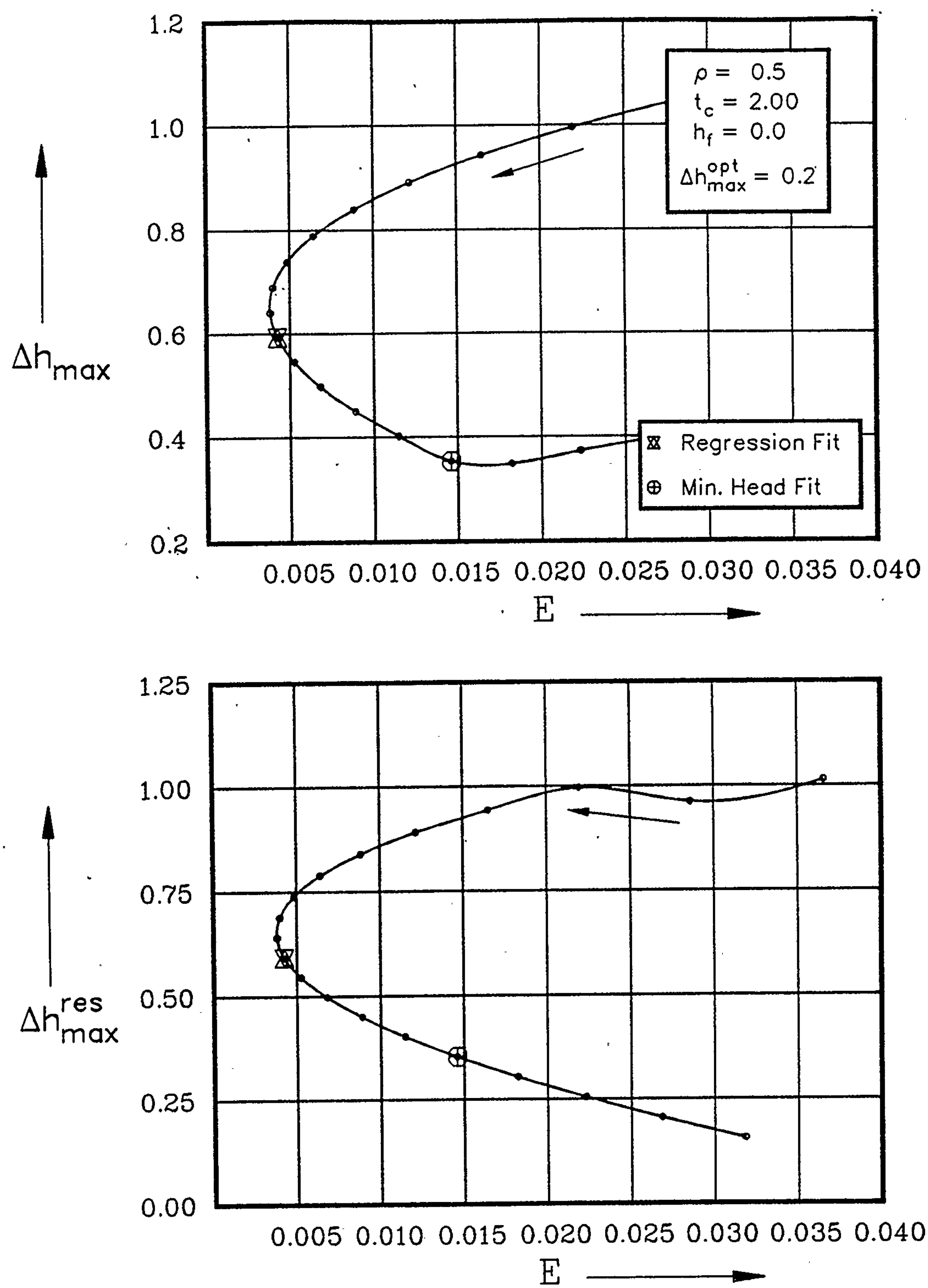

Figure B.9: "Best-fit" sensitivity plots. Frictionless system. Moderate equal percentage closure.

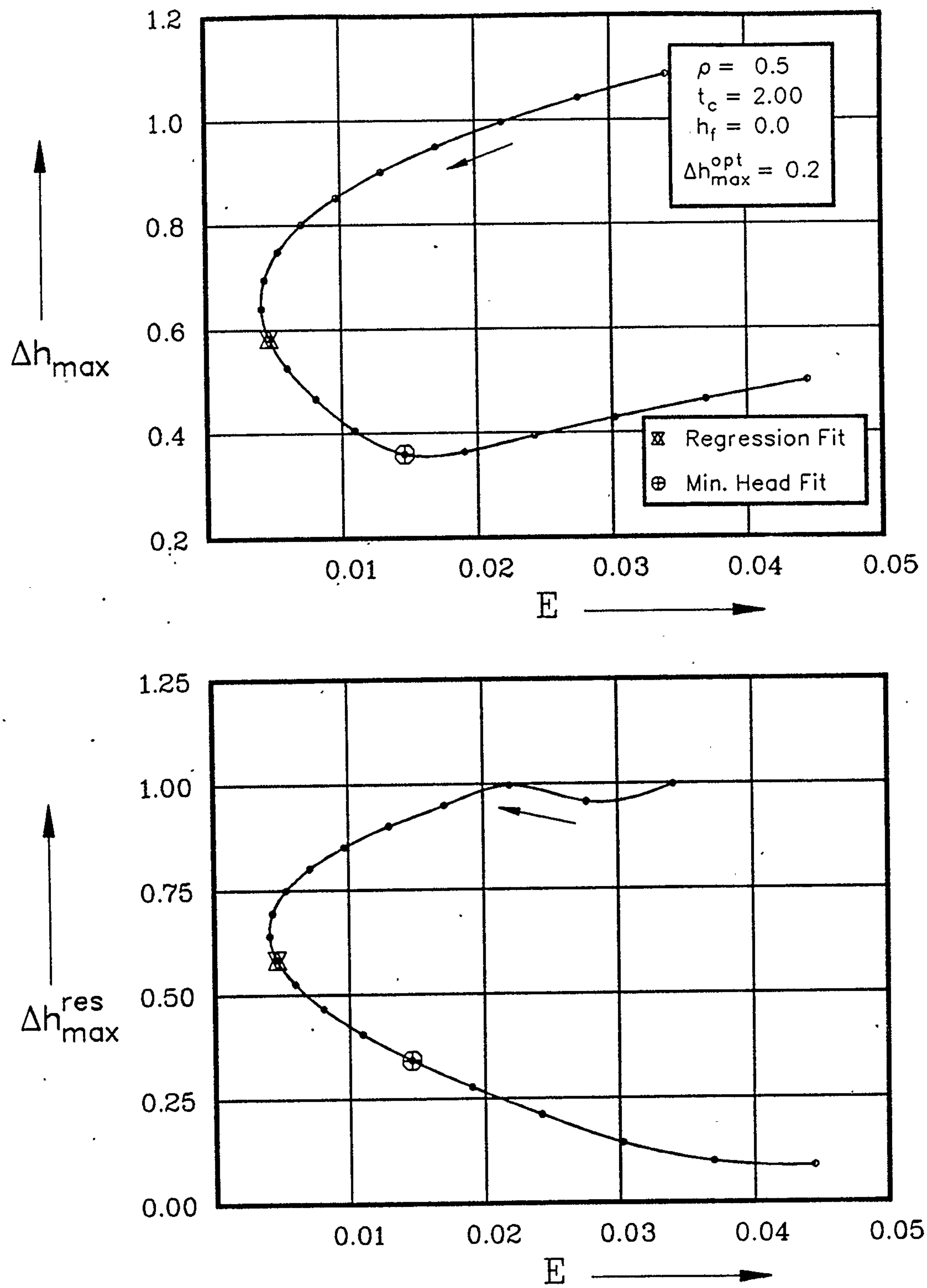

Figure B.10: "Best-fit" sensitivity plots. Frictionless system. Moderate bilinear closure.

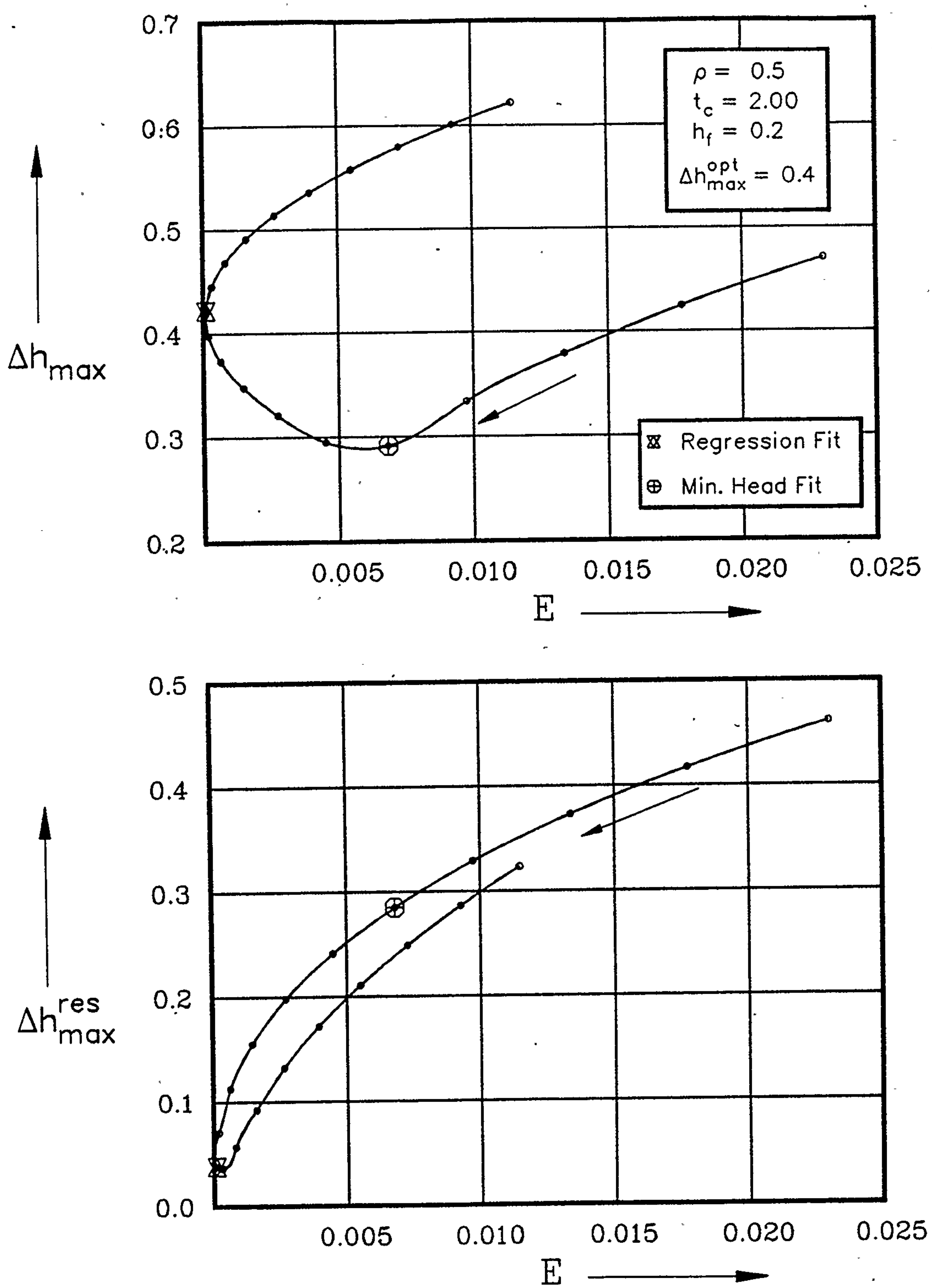

Figure B.11: "Best-fit" sensitivity plots. Low friction system. Moderate equal percentage closure.

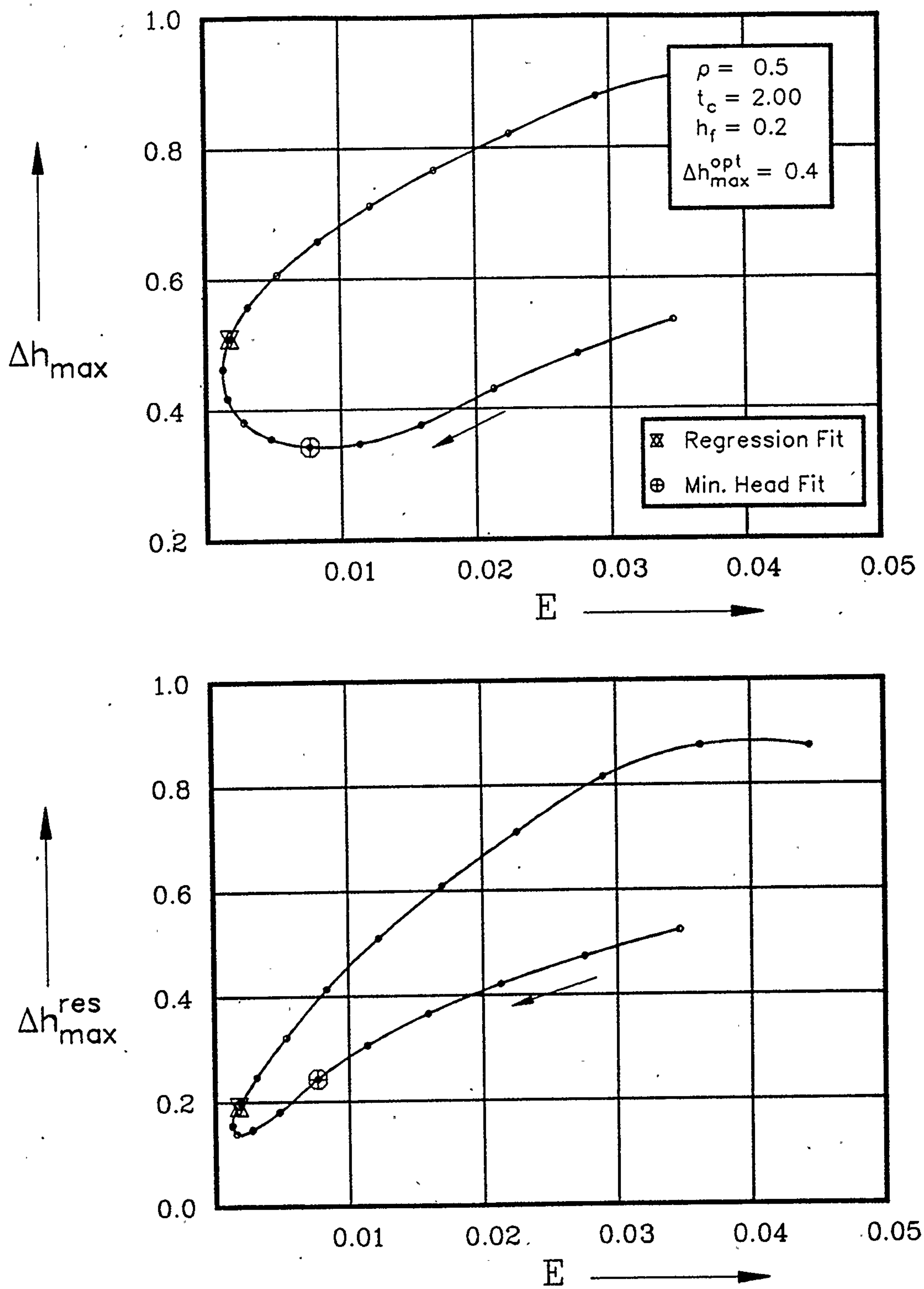

Figure B.12: "Best-fit" sensitivity plots. Low friction system. Moderate bilinear closure.

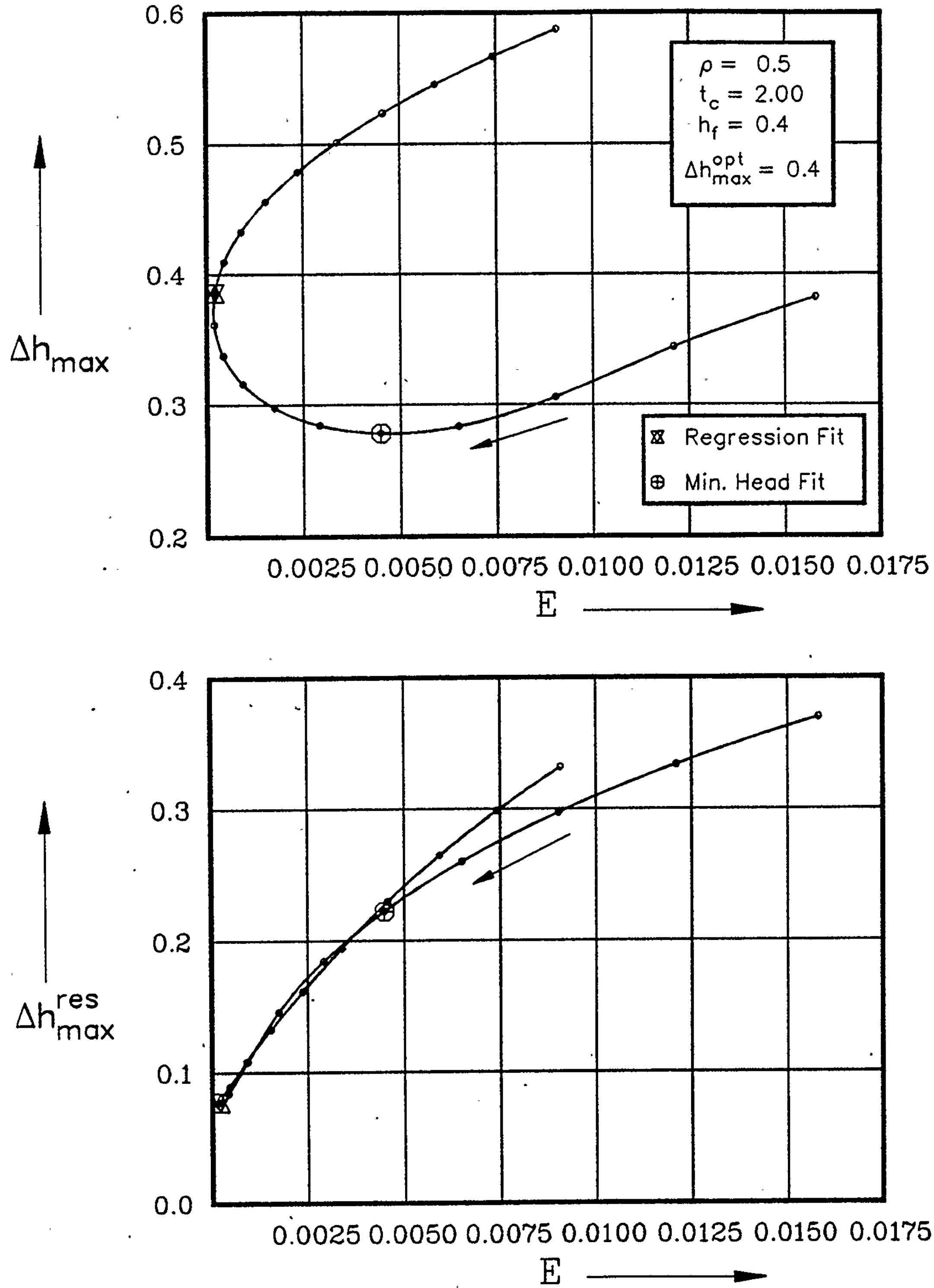

Figure B.13: "Best-fit" sensitivity plots. Medium friction system. Moderate equal percentage closure.

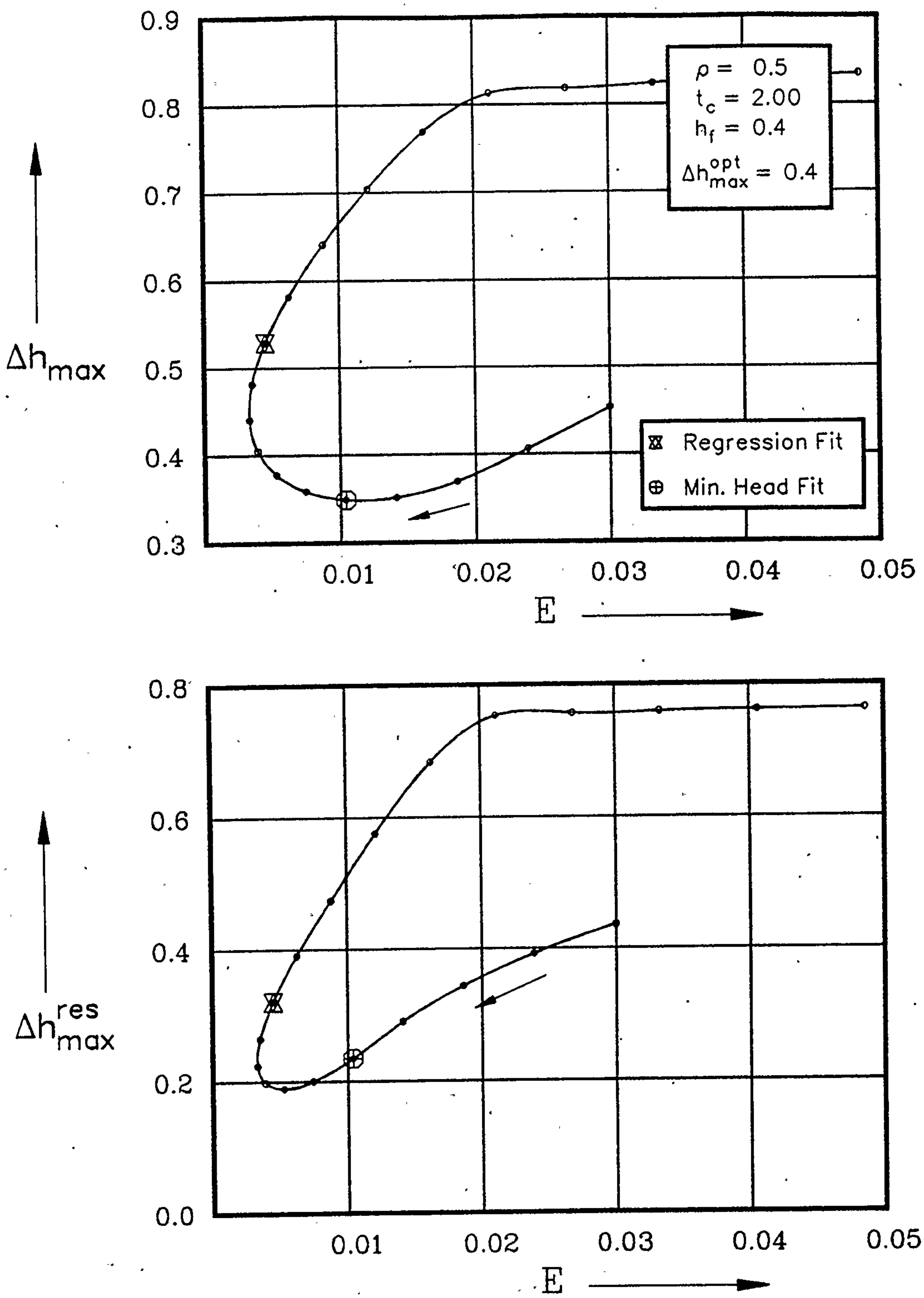

Figure B.14: "Best-fit" sensitivity plots. Medium friction system. Moderate bilinear closure.